The Performance of
CIVIL AIRCRAFT

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BY

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PREFACE

THE performance of present-day medium altitude aircraft is sufficient to offer the public an adequately fast and reliable form of transport. Failures in reliability are largely due to the absence of proper ground facilities, signalling and meteorological aids. This state of affairs is perhaps natural, since from the beginning of aviation's history, performance has received the most consideration from all concerned. Out of this period of thought and study, of experiment and research, have arisen certain methods of attacking the problems and applying the lessons of performance. It is with these lessons that this book is mostly concerned: it is *not* concerned with the allied problems of meteorology, navigation, radar, etc.; nor with the commercial exploitation of the aircraft's potentialities.

It is an axiom of aircraft publications that anything which is written to-day will be out of date by the time it is printed to-morrow. At the present time, however, there exists a short lull before the full advent of very fast, high altitude machines (discussed in Chapter XVII); and it is perhaps worth while, therefore, to summarize the more elementary facts which we already know about aircraft already in existence. This is done with the knowledge that a scientific approach to the subject will not be universally accepted as necessary. Such a remark applies particularly to what is usually described as Cruise Control. Take-off Procedure and Cruise Control are at present, like organized religion, more honoured in the breach than the observance. Any treatise which deals with such subjects is likely to be received with varying amounts of disfavour; and to invite comments such as the following—

From the *Pilot*: "That's not the way we *fly* the aircraft, anyway."

From the *Designer*: "Too elementary, and most misleading."

From the *Operator*: "All very interesting, but—— really!"

Such a reception is discouraging; yet there is need for there to be some common ground between the three gentlemen mentioned. For all of them, the great objectives must be—

SAFETY ECONOMY PUNCTUALITY

and any publication whose aim is to describe (however imperfectly) how these aims can be achieved is worthy of a little notice. Pilots may not relish mathematics; technicians may dislike associating with "popular" explanations; operators may be content to buy the best aeroplanes and continue charging expensive fares; but in the future all three of them will find themselves giving very careful consideration to the practical performance characteristics of the aircraft used in Commercial Aviation. To ensure *Safety*, I.C.A.O. (the International Civil Aviation Organization) is already on the march. The remorseless pressure of current events, and the insistent demand for cheaper flying, will force the issue of *Economy*; international competition, and the improvement in radio and navigational aids, will take care of *Punctuality*.

In any case, the Pilot, the Designer, and Operator have their own sources of technical information. The general public, or rather that portion of the public who are interested in aircraft, are not so fortunately placed; and it is to them that this book is mainly addressed.

Part I, Preliminary, is a summary of some of the elementary principles involved. Without this knowledge, the statements made in Part II will prove unnecessarily difficult to follow.

Part II deals with Take-off, Climbing, Air Traffic, Range Calculations, Allowances and Reserves, Flight Planning, and a number of other related matters. It is assumed throughout Part II that we are dealing with existing commercial aircraft, powered with piston engines.

Part III is concerned with aircraft having gas-turbine engines, and is incomplete. In the present state of practical knowledge this is bound to be the case. Nevertheless an attempt has been made to summarize the main cruising characteristics of such aircraft, and to predict some of the developments in high altitude flying.

The Appendices provide information which is not readily available elsewhere.

The book contains over 100 diagrams, many in the form of graphs or charts. Performance calculations rely very largely on the accurate reading of charts, and the intelligent use of elementary mathematics. However regrettable this may be, it is a fact which cannot be ignored. It is satisfactory to know, however, that the procedures adopted rest upon some logical basis. Readers who aim at more than a superficial knowledge of the subject will find valuable practice in doing for themselves the sort of simple calculations with which the text is liberally sprinkled, and are invited to communicate any interesting examples of the use of the methods described.

CONTENTS

APPENDIX II

APPENDIX III

APPENDIX IV

APPENDIX V

ILLUSTRATIONS

LIST OF ABBREVIATIONS AND SYMBOLS

Abbreviations

A. & A.E.E.	Aircraft and Armament Experimental Establishment
a.m.p.g.	air miles per gallon
a.m.p.lb.	air miles per pound
A.N.D.	Air Navigation Directions
a.n.m.p.g.	air-sea miles per gallon
A.R.B.	Air Registration Board
A.S.I.	air-speed indicator
A.S.I.R.	air-speed indicator reading
A.T.C.	Air Traffic Control
B.C.A.R.	British Civil Airworthiness Requirements
B.E.A.	British European Airways
b.h.p.	brake horse-power
b.m.e.p.	brake mean effective pressure
B.O.A.C.	British Overseas Airways Corporation
B.Th.U.	British thermal unit
C	centigrade
C. of A.	Certificate of Airworthiness
C.A.S.	calibrated air-speed
c.g.	centre of gravity
C.H.U.	centigrade heat unit
C.P.	constant power
C.S.	constant speed
E.A.S.	equivalent air-speed
E.G.M.	equivalent geographical mileage
eng.	engine
E.T.A.	estimated time of arrival
E.T.D.	estimated time of departure
ft	foot
ft^2	square foot
ft^3	cubic foot
ft/sec	feet per second
ft/sec^2	feet per second per second
g	gramme
gal	gallon
g.m.p.g.	ground miles per gallon
G.S.	ground speed
h.p.	horse-power
hr	hour
I.C.A.N.	International Convention of Air Navigation
I.C.A.O.	International Civil Aviation Organization
i.e.r.p.m.	indicated engine revolutions per minute
I.F.R.	instrument flight rules
i.h.p.	indicated horse-power
in.	inch
$in.^2$	square inch
kg	kilogramme
km	kilometre
kt	knot
lb	pound
m	metre
m.a.p.	mean atmospheric pressure
max.	maximum
M.C.A.	Ministry of Civil Aviation
m.e.p.	mean effective pressure
mm	millimetre
m.p.h.	miles per hour
N.A.C.A.	National Advisory Council for Aeronautics
N.G.T.E.	National Gas Turbine Establishment
n.m.	nautical mile
n.m.p.h.	nautical miles per hour
O.A.T.	Outside Air Temperature
oz	ounce
P.E.	position error
R.A.E.	Royal Aircraft Establishment
R.A.S.	rectified air-speed
r.p.m.	revolutions per minute
sec	second
S.F.C.	specific fuel consumption
S.L.	sea-level
sp. gr.	specific gravity
T.A.S.	true air-speed
t.h.p.	thrust horse-power
V.F.R.	visual flight rules

Symbols

a_0 speed of sound
A aspect ratio
b_0 span
c_f specific consumption
C calorific value
C_D drag coefficient
C_{D_Z} theoretical drag coefficient at zero lift
C_L lift coefficient
D drag
$D_{S.L.}$ drag at sea level
D_{100} drag at 100 ft/sec
e N.A.C.A. efficiency factor
E fuel flow
f acceleration
f fuel flow in lb/sec
F_t total fuel required for take-off
g acceleration of gravity
G gradient of climb
G_1 a.m.p.g. at W_1
h height
H_d density height
H_p pressure height
J Joule's equivalent
K constant
K_p specific heat at constant pressure
K_v specific heat at constant volume
L unit of length
L lift
m mass
M unit of mass
M_a air mass flow
M_g gas mass flow
n number of engines
N revolutions per minute
O equivalent geographical mileage
p pressure
P force
P_f power absorbed by friction
P_s power absorbed by the supercharger
r distance to alternate
R range
R rate of climb
s distance
S area of plate
S_W wing area
t time
T absolute temperature
T thrust
T unit of time
T_A actual absolute air temperature
T_S standard absolute air temperature
T_0 standard temperature at sea-level
u initial velocity
v final velocity
v headwind componemt
V specific volume
V true air-speed
V_c vertical component of climbing speed
V_c critical speed
V_j jet velocity relative to aircraft
V_g ground velocity
V_i equivalent air-speed
$V_{i_{md}}$ indicated speed for minimum drag
$V_{i_{mp}}$ speed for minimum power
V_j jet velocity when aircraft is at rest
$V_{i_{mcc}}$ minimum speed for comfortable continuous cruising
V_{mc} minimum control speed on climb
V_{mcg} minimum control speed on ground
V_r indicated air-speed
V_{ss} safety speed
V_{s_0} safety speed with engine off
V_{s_1} stalling speed, engine off
W weight
W_D disposable load
W_f weight of fuel
W_L landing weight
W_m mean weight
W_p payload
W_{rf} weight of fuel reserve
W_s standard mean weight
W_{TO} take-off weight
W_1 starting weight
z vertical distance
ε downwash angle
π relative pressure
ρ density
ρ_0 standard density
σ relative density
τ relative temperature
η propeller efficiency
η_p propulsive efficiency

PART I—PRELIMINARY

CHAPTER I

Elementary Mechanics

1.1. Speed

SPEED is a measure of the rate at which a body moves. To define the rate completely, we need to know the direction and line of action: a speed so defined is known as a *velocity*.

(Movement in a straight line is sometimes referred to as having a "linear" velocity, to distinguish it from rotational or "angular" velocity.)

An aeroplane which is flying north, in still air, may encounter an east wind: the resultant velocity is west of north, and can be determined by the parallelogram law. This is not surprising, since velocity has position, direction, and size, and the path of the aeroplane can be represented by a straight line drawn on the chart, in a certain direction and having a certain length. Any other quantities which can similarly be represented by straight lines, are known as *vectors*—common examples are force, velocity, acceleration, momentum—and all of them can be compounded by the parallelogram law.

Quantities such as mass, energy, work, power are not vectors.

1.2. Newton's First Law

In the absence of any resultant force, a body will either remain at rest or else move with constant linear velocity.

If you throw a stone along the surface of a smooth sheet of ice, it moves with nearly constant velocity. If friction could be eliminated altogether, the stone would continue with unabated speed until it reached the end of the ice. A rocket hurtling through the cosmic void, unaffected by the presence of any gases or any gravitational force, would continue for ever with undiminished velocity.

An aeroplane can fly at constant speed, although it is acted on by a number of forces: this result is achieved by a complete balance of the forces—thrust cancelling drag, and lift cancelling weight—so that there is still no *resultant* force.

It is important to notice that Newton's first law implies a converse. If a body does *not* move in a straight line with constant velocity then there must be a resultant force causing the deviation.

1.3. Acceleration

Acceleration is a measure of the rate of increase of velocity; deceleration, or retardation, is a measure of the rate of decrease of velocity. (By convention, the first is + and the second is —.) Acceleration is, like velocity, a vector; but it is not necessary for the velocity and acceleration of a body to be both in the same direction. For example, an aeroplane doing a banked turn has a velocity along the tangent to its circular path, and an acceleration towards the centre of the circle.

Accelerations, like velocities, are rarely constant. Even if a man bales out from an aeroplane, his acceleration before the parachute opens is not really constant, since the increase in his speed increases the air resistance; but it is customary, as a first approximation, to regard the acceleration of gravity as constant. The acceleration of the piston of a reciprocating engine is very far from constant: it is, in fact, greatest when the velocity of the piston is zero, and is zero when the velocity is greatest.

In the case of bodies whose accelerations are really constant, the following simple equations can be proved to hold good:

If u = velocity at the beginning of the motion, or *initial velocity*,

v = velocity at the end of the motion, or *final velocity*,

t = time from the beginning to the end,

s = distance covered,

f = acceleration (given a — sign if it is a retardation),

then

$$\bar{v} = \bar{u} + \bar{f}t$$

$$\bar{s} = \bar{u}t + \tfrac{1}{2}\bar{f}t^2 = \tfrac{1}{2}(\bar{u} + \bar{v})t$$

$$v^2 = u^2 + 2fs.$$

The bar - over the symbols implies that the quantities are vectors; e.g. in the first equation, v, u and f must all be measured in the same line or direction.

1.4. Units

The units commonly used for measuring velocity are: feet per second, metres per second, feet per minute (for climbing), miles per hour, knots or nautical miles per hour, kilometres per hour.

$$1 \text{ knot} = 6\,080 \text{ ft per hr}$$

$$1 \text{ m.p.h.} = 5\,280 \text{ ft per hr}$$

To convert knots to m.p.h., multiply by 1·15.

To convert m.p.h. to knots, divide by 1·15 or multiply by 0·87.

$$60 \text{ m.p.h.} = 88 \text{ ft per sec}$$

To convert ft per sec to m.p.h., multiply by $\frac{15}{22}$.

$$1 \text{ metre} = 39{\cdot}37 \text{ inches}$$

$$66 \text{ kilometres} = 41 \text{ statute miles}$$

To convert kilometres per hour to m.p.h., multiply by $\frac{5}{8}$.

1.5. Acceleration of Gravity

We shall see later that acceleration is caused by "force." In particular, gravitational force causes the so-called "acceleration of gravity." Now this force varies inversely as the square of the distance from the centre of the earth (for bodies outside the surface); and hence this force is greater at the poles, where the earth's radius is less, than it is at the Equator.

The acceleration of gravity is denoted by g. Its numerical value in foot-second units in the latitude of London is about 32·19. At the Equator it is about 32·091.

Since bodies at the Equator are travelling in a circle, due to the earth's rotation, this also has a bearing on the value of g. The radius of the circle is the earth's radius, about 3 960 miles; and the complete circle is described once every (sidereal) day; so that the angular velocity of a body at the Equator is about $2\pi/86\,400$ radians per second. This circular motion demands the presence of a force towards the centre of the circle, i.e. bodies at the Equator will lose in weight. This means that the acceleration of gravity is reduced by about $\frac{1}{289}$ of itself at the Equator; and there are similar though smaller decreases at other latitudes until the pole is reached.

It appears, therefore, that g is less at the Equator for two reasons: primarily because of the increased radius, and secondarily because of the rotation of the earth.

1.6. Mass

The mass of a body is usually defined as the quantity of matter in the body. This of course assumes that matter is indestructible, that none of it can be dissipated in radiation, and that its value is unaffected by speed.

Direct measurement of *mass* is not easy. In most cases its existence becomes obvious only through the evidence of *weight*. Yet mass remains constant whereas weight varies. For example, a pound of tea, containing a certain number of tea leaves, will still be a pound of tea on the surface of the moon; but if we weighed it with a spring balance, it would be nothing like a pound weight on the moon. This does not mean that the bag containing the tea has been leaking: it means that the acceleration of lunar gravity is a lot smaller than on earth.

The units in which mass is measured are quite arbitrary. Scientists use the *gramme*, and 454 grammes, approximately, make up the *pound*. A standard pound mass, kept free from corrosion and interference, is available in London; and the magnitude of other masses could be determined by balancing one against the other, and employing the principle of moments.

In aircraft work the unit of mass is the *slug*, which is equal to 32·174 pounds. For example, an aircraft of mass 32 174 lb will also have a mass of 1 000 slugs. If the same aeroplane were to be put on the surface of the moon, we could still find its mass in slugs, by either of these two methods—

(i) Balance it against the standard pound, or a number of such pounds, using suitable levers, scale-pans, etc.; and divide the answer in pounds by 32·174.

(ii) Weigh it with a spring balance, or its equivalent, i.e. find the force with which it is attracted to the centre of the moon; and then divide by *the local value of g* at the particular spot on the moon's surface.

Mass is not a vector.

1.7. Momentum

The linear momentum of a body is the product of its mass and its velocity. The product is measured in "units of momentum," which differ according to the units used for mass and velocity.

There is also another type of momentum, known as *angular momentum* or *moment of momentum*, in which we take moments of linear momentum.

Momentum is a vector, owing to velocity being a vector.

1.8. Newton's Third Law of Motion

To every action there is always an equal and contrary reaction; or, the mutual actions of any two bodies are always equal and oppositely directed.

This law tells us that any action between the component parts of a system of bodies, such as a number of particles, whether due to attraction at a distance or to actual contact, cannot affect the momentum of the system as a whole. For if two of the bodies A and B act on each other, the reaction of A on B is equal and opposite to the action of B on A; the action and reaction therefore generate equal and opposite amounts of momentum in the two bodies. Hence the total momentum reckoned in any fixed direction is unaltered. This constitutes a very important principle known as the *Conservation of Linear Momentum*—

In any system of attracting or colliding or exploding bodies, the linear momentum in any fixed direction remains unaltered unless there is an external force acting in that direction.

Example 1. A ten-ton truck on a railway line, travelling at 5 m.p.h., strikes an eight-ton stationary truck; and the two trucks move on together. What is the speed of the two trucks?

The original momentum is $10 \times 5 = 50$ units of momentum.

The momentum after the collision is 18V, where V is the unknown speed.

$$\therefore \qquad 18V = 50$$

$$V = \underline{\underline{2\tfrac{7}{9} \text{ m.p.h.}}}$$

Example 2. A bullet weighing 1 oz is fired with a velocity of 576 ft/sec into a block of wood of mass 4 lb, lying on a smooth table. Find the velocity with which the block and bullet move after the bullet has become embedded in the block.

The mass of the bullet is $\frac{1}{16} \times \frac{1}{32}$ slugs, so that its momentum is $\frac{1}{16 \times 32} \times 576$ units, or $\frac{9}{8}$. Hence, the velocity of the block is V where

$$\frac{4\frac{1}{16}V}{32} = \frac{9}{8}$$

$$V = \underline{\underline{8{\cdot}86 \text{ ft/sec}}} \text{ (approx.)}$$

Example 3. A man of mass 200 lb dives in the upstream direction off the stern of a boat of mass 2 000 lb, travelling at 5 ft/sec downstream. If the initial velocity of the man, relative to the water, is 10 ft/sec, find the new velocity of the boat after the man has dived.

The original momentum of the boat and man is 2 200 × 5 units.

The new momentum of the boat is 2 000V.

The new momentum of the man is 200 × 10, in the opposite direction.

Hence
$$2\,000V - 200 \times 10 = 2\,200 \times 5$$

$$V - 1 = \frac{2\,200 \times 5}{2\,000}$$

$$\therefore \quad V = \underline{\underline{6\tfrac{1}{2} \text{ ft/sec}}}$$

1.9. Dimensions

The units of mass, length, and time are called *fundamental units*, since the units of other quantities, such as speed, force, etc., can be expressed in terms of them. If we denote the units of length, mass and time by L, M, T, the dimensions of other physical quantities are, for instance,

Velocity $\frac{L}{T}$

Acceleration $\frac{L}{T^2}$

Momentum $\frac{ML}{T}$

Density $\frac{M}{L^3}$

Angular velocity $\frac{1}{T}$

We shall add to this list later (see Appendix I). It is worth noticing now, however, that we are here provided with a useful check as to whether a formula is possible or reasonable. Take, for instance,

$$v^2 = u^2 + 2fs, \text{ mentioned above.}$$

The dimensions of v^2 and u^2 are $\frac{L^2}{T^2}$; and the dimensions of $2fs$ are $\frac{L}{T^2} \times L$ or $\frac{L^2}{T^2}$, so that all the terms in the equation have the same dimensions.

1.10. Force

Force is any cause which produces or tends to produce a change in the existing state of a body. We are guided to an appreciation of the meaning of force by our own personal experience of its effects.

Newton's Second Law states that change of momentum per unit time is proportional to the impressed force, and takes place in the direction of the straight line in which the force acts. That is,

Force $\propto$, rate of change of mv,

$\propto$, $m \times$ rate of change of v, if m is constant

$\propto$, $m \times f$, where f is the acceleration.

The unit of force is that force which, acting on unit mass, generates in it unit acceleration.

When the unit of mass is the slug (= 32·174 lb mass), and the units of space and time are the foot and second, the unit force is called the pound, i.e. 1 lb gives a mass of 1 slug an acceleration of 1 ft per sec per sec.

Thus, if the force be P lb, the mass m slugs, and the acceleration f ft/sec^2,

$$P = mf$$

Example 1. Find the force, assumed constant, necessary to give an aircraft of mass 32 000 lb, on the runway, a velocity of 100 ft/sec in a distance of 1 000 ft, in addition to overcoming the drag. Assume a constant acceleration.

Since $v^2 = u^2 + 2fs$, put $u = 0$, $v = 100$, $s = 1\,000$,

hence $f = 5$ ft/sec^2. If the force be P lb,

$$P = \frac{32\,000}{32 \cdot 174} \times 5 = \underline{\underline{4\,973 \text{ lb}}} \text{ (approx.)}$$

Since no work is done against gravity, it is not surprising that this answer is independent of the local value of g. Consider, however, the answers obtainable in the two systems of units which, to many people, are more familiar.

(*a*) *The Scientific System.* Here

$$P = mf$$

where m is the mass in pounds and P is the force in poundals.

In our case $P = 5 \times 32\,000 = 160\,000$ poundals

$$= \frac{160\,000}{g} \text{ pounds weight,}$$

which means that, if the force causing the aircraft to move was contained in a long rope which passed over a smooth pulley at the end of the runway, then the weight hanging freely from this end of the rope would be $\frac{160\,000}{g}$ pounds weight.

(*b*) *The Engineers' System.* Here

$$\frac{P}{W} = \frac{f}{g}$$

where P and W are both measured in the same units. Using pounds weight we get the same answer as in (*a*).

Using pounds,

$$W = \frac{32\,000}{32{\cdot}174} \times g \text{ (see § **1.11**).}$$

hence

$$P = \frac{32\,000}{32{\cdot}174} \times 5 = \underline{4\,973 \text{ lb.}}$$

It should be noted that this force of 4 973 lb is in fact the surplus thrust, over and above that necessary to cancel the drag. This example can also be done by a consideration of kinetic energy: see later.

The equation $P = mf$ can also be written

Force = rate of destruction (or creation) of momentum

Example 2. Water (of which 1 ft^3 weighs $62\frac{1}{2}$ lb) issues from a circular pipe of 3 in. diameter with a velocity of 15 ft/sec. Find the mass of water discharged per minute. If the water impinges directly upon a plane, and its momentum is thereby wholly destroyed, what is the pressure of the jet upon the plane?

The area of the cross-section of the pipe is $\frac{\pi}{4} \times \frac{1}{16}$ ft^2, and a column 15 ft long is discharged every second.

The volume per minute $= \frac{\pi}{64} \times 15 \times 60$ ft^3

and its mass

$$= \frac{\pi \times 15 \times 60 \times 125}{64 \times 2} \text{ lb}$$

$$= 2\,762 \text{ lb (nearly)}$$

$$= \frac{2\,762}{32 \cdot 174} \text{ slugs}$$

$$= \underline{\underline{85 \cdot 8 \text{ slugs}}}$$

The momentum destroyed per second is therefore

$$\frac{85 \cdot 8 \times 15}{60} \text{ slug-ft units,}$$

i.e. the pressure $= \underline{\underline{21 \cdot 5 \text{ lb}}}$

Example 3. A jet of 800 lb of air is driven backwards by an aircraft propeller, and the *change* of velocity imparted is 160 ft/sec. Find the thrust developed.

Change of momentum of air backwards, per second,

$$= \frac{800 \times 160}{32 \cdot 174} \text{ lb} = \underline{\underline{4\,000 \text{ lb}}} \text{ thrust (approx.)}$$

Note that the same result would be obtained by 80 lb of air with a relative velocity of 1 600 ft/sec, which corresponds to the case of the gas turbine engine's jet.

Suppose a flat plate of area S ft^2 is placed in an airstream of velocity V ft/sec at right angles to the plate. The volume of air striking the plate is

$$VS \text{ ft}^3/\text{sec}$$

If ρ be the mass in slugs of a cubic foot of air, the mass of air is

$$\rho VS \text{ slugs}$$

so that the momentum of the air striking the plate is

$$\rho SV^2 \text{ slugs-ft/sec}$$

Some fraction of this momentum, say $K\rho SV^2$, is destroyed, and this is equal to the force on the air; which by Newton's third law is equal to the force on the plate.

This argument is at the basis of the Lift and Drag formulae in Aerodynamics.

Moreover, if an airstream applies a lift L to an aerofoil, the aerofoil must (by Newton's third law) apply a force L to the airstream in a downward direction, and therefore (by Newton's second law) impart a downward momentum to the airstream. In other words, the airstream is deflected downwards so that it has a vertical component to its velocity. The angle through which the airstream is deflected relative to its direction before it meets the aerofoil is called the *downwash* angle, and is usually denoted by the Greek letter ε.

The dimensions of force are $M \times \frac{L}{T^2}$; which are also the dimensions of rate of change of momentum.

1.11. Weight

Weight is the force with which the earth attracts unit mass of matter. Since the mass is constant, the weight will vary for different values of g. If the mass be m lb, then the weight is

$$\frac{m}{32{\cdot}174} \times g \text{ pounds}$$

since it is found that the acceleration of gravity (in a vacuum) is independent of mass.

Both force and weight are vectors.

1.12. Work

When a force moves its point of application it is said to do work, and the measure of the work is the product of the force and the distance through which the point of application moves in the direction of the force.

Work is measured in foot-pounds. It is not a vector. Its dimensions are $\frac{ML^2}{T^2}$.

1.13. Power

Power is the rate of doing work, i.e. the work done in unit time. The British unit of power is the *horse-power*, which is 550 ft-lb/sec, or 33 000 ft-lb/min.

(The C.G.S. unit of power is the *watt*; 1 h.p. = 746 watts, approx.) The dimensions of power are $\frac{ML^2}{T^3}$.

If a force of P lb keeps its point of application moving in the direction of the force with uniform speed v ft/sec, the work done per second, or power, is Pv ft-lb, and the h.p. is $Pv/550$. For example, an aeroplane travelling with true speed V ft/sec as the result of a thrust T lb, equal and opposite to a drag D lb, will require a thrust horse-power of TV/550. If V is measured in m.p.h., the necessary thrust h.p. is TV/375, since 88 ft/sec = 60 m.p.h.

If an aircraft of weight W lb is given a vertical climbing speed of V_c ft/min, the power required will be

$$\frac{WV_c}{550 \times 60} = \frac{WV_c}{33\,000}.$$

1.14. Energy

The energy of a body is its capacity for doing work. Since the energy of a body is measured by the work it can do, the units of energy will be the same as those of work.

1.15. Kinetic Energy

The *kinetic energy* of a body is the energy it possesses in virtue of its motion, and is measured by the amount of work which it does in coming to rest. Consider a body of mass m slugs moving with velocity v ft/sec, and suppose it is brought to rest by a constant force P lb which produces in it a retardation f ft/sec²; then $P = mf$. Let x ft be the distance covered by the body before it comes to rest, then

$$0 = v^2 - 2fx$$

$$\therefore \quad fx = \tfrac{1}{2}v^2$$

Now the work done by the body in foot-pounds is

$$Px = mfx = \tfrac{1}{2}mv^2$$

$$\therefore \quad \text{Kinetic energy of the body} = \tfrac{1}{2}mv^2$$

If m is measured in slugs, and v in ft/sec, kinetic energy will be in ft-lb.

Example 1. Find the constant force necessary to give an aircraft of mass 32 000 lb, on the runway, a velocity of 100 ft/sec in a distance of 1 000 ft.

Suppose the force be P lb.

The required kinetic energy $= \frac{1}{2} \times \frac{32\,000}{32{\cdot}174} \times 100^2$ ft-lb.

The work done to create this energy $= P \times 1\,000$ ft-lb.

$$\therefore \quad P = \tfrac{1}{2} \times \frac{32\,000}{32{\cdot}174} \times \frac{100^2}{1\,000} = \underline{\underline{4\,973 \text{ lb.}}} \text{ (approx.)}$$

More generally, if T is the thrust on an aircraft from rest to "unstick," and s is the distance covered on the runway,

$$Ts = \tfrac{1}{2}mv^2$$

where m is the aircraft's mass in slugs, and v is the speed at unstick. In this formula T is in practice not constant, so to find s we find the area under the $\frac{1}{T}$, v^2 curve.

1.16. Potential Energy

The *potential energy* of a body is the work it can do in moving from its actual position to some standard position.

One example of this is the cold-water cistern in the roof of a private house. When a tap below is turned on, the water flows along the pipe and out of the tap, thereby losing potential energy and gaining kinetic energy. The work that has been done in pumping that amount of work up to the cistern, from the level of the tap, will be equal to the kinetic energy of the water that comes out of the tap. This of course assumes that no energy is lost in heat or friction with the pipe: with these reservations, it is true that *the sum of the kinetic and potential energies of a system of bodies is constant.* This principle, known as *the conservation of energy,* has a wide application in mechanics. It does not apply to motions in which collisions or explosions occur, since work is then lost in deformation of the material, heat, etc.

If z is the vertical distance through which the body is lifted and $\frac{mg}{32{\cdot}174}$ is the weight of the body of mass m lb, then the work done in lifting will be $\frac{m.g.z.}{32{\cdot}174}$ ft-lb. For a body of mass m lb, or $\frac{m}{32{\cdot}174}$ slugs, the kinetic energy is $\frac{mv^2}{2 \times 32{\cdot}174}$. We can now cancel the number 32·174 and write $\frac{1}{2}\frac{mv^2}{g} + mz$ $=$ const. if m is in pounds, or $\frac{1}{2}mv^2 + mgz =$ const. if m is in slugs, both answers being in foot-pounds; and g having its local value.

Example 1. A mass of 5 lb at the end of a string 1 ft long, which is fixed to a nail at the other end, is held so that the string is horizontal. The mass is then released. Find its speed when the string is vertical.

In the horizontal position, the kinetic energy of the mass is zero and the potential energy, measured relative to the vertical position, is 5×1 ft-lb. In the vertical position, the kinetic energy is $\frac{1}{2} \frac{5}{g} v^2$ ft-lb, and the potential energy is zero.

Hence
$$0 + 5 \times 1 = \tfrac{1}{2} \frac{5}{g} v^2 + 0$$
$$\text{or } v = \sqrt{2g} = \underline{\underline{8 \text{ ft/sec}}} \text{ (approx.)}$$

1.17. Other Forms of Energy

In the example above referring to the house water system, we were concerned with only the mechanical energy of the liquid. This assumes that there is no change in pressure, temperature, etc.

Bernoulli's theorem for incompressible fluids can be written as

$$\tfrac{1}{2} \frac{mv^2}{g} + \frac{mp}{\rho} + mz = \text{constant}$$

in which m is again in pounds, and p and ρ are the pressure and density of the fluid, the latter in pounds per cubic foot. If there is no appreciable loss or gain of potential energy, we obtain

$$\tfrac{1}{2}\rho v^2 + p = \text{constant}$$

where ρ is now in slugs per cubic foot.

This total energy per unit mass, $p + \frac{1}{2}\rho v^2$, is known as the *total head* of the fluid; p is the *pressure head* and $\frac{1}{2}\rho v^2$ the *dynamic head.*

In the case of gases, it was established by Joule that mechanical energy can be converted into heat and vice versa; and the internal energy of a given quantity of gas is dependent only upon its temperature. To convert from one form of energy to the other, we remember that a centigrade heat unit (C.H.U.) equals 1 400 ft-lb (and a B.Th.U. equals 778 ft-lb). This number 1 400 is usually called *Joule's equivalent* J. Thus a kinetic energy of $\frac{1}{2} \frac{v^2}{g}$ ft-lb/lb can be written as $\frac{1}{2} \frac{v^2}{gJ}$ C.H.U./lb. In a common thermodynamic process, known as the *adiabatic,* in which no heat is received or given up to the outside, the total energy will then be

$$\frac{v^2}{2gJ} + k_p\mathrm{T} = \text{constant}$$

where k_p is the specific heat of the gas at constant pressure, T is the absolute temperature, and $k_p\mathrm{T}$ is (by definition of k_p) a measure of the heat energy per unit mass.

1.18. The Gas Law

The link between Bernoulli's theorem and this last energy equation can be found by remembering the law for a perfect gas. This can be written as

$$\frac{p\mathrm{V}}{\mathrm{T}} = \text{constant} = \mathrm{R}, \text{ say}$$

where p is the pressure in pounds per square foot, V is the specific volume in cubic feet per pound. It is shown in treatises on thermodynamics that $\mathrm{R} = \mathrm{J}(k_p - k_v)$ where k_p and k_v are the specific heats of the gas, at constant pressure and constant volume respectively. For the majority of air/fuel ratios with which we are concerned, or for air, $k_p - k_v = 0{\cdot}0684$, so that

$$\frac{p\mathrm{V}}{\mathrm{T}} = 95{\cdot}6$$

If p is measured in pounds per square inch, and $\rho = 1/\mathrm{V}$ in pounds per cubic foot,

$$\frac{p\mathrm{V}}{\mathrm{T}} = \frac{95{\cdot}6}{144} = 0{\cdot}665$$

so that $$\rho = \frac{144}{95{\cdot}6}\frac{p}{\mathrm{T}} = 1{\cdot}5\frac{p}{\mathrm{T}}$$

Using the relative densities, pressure, and temperature defined in the next chapter, this boils down to

$$\frac{p_1}{p_2} = \frac{\rho_1}{\rho_2} \cdot \frac{\mathrm{T}_1}{\mathrm{T}_2}$$

or $$\pi = \sigma\tau$$

which we refer to later as the *equation of state.*

1.19. Temperature-Velocity Relationship

For an adiabatic flow process

$$\frac{v_1^2}{2gJ} + k_pT_1 = \frac{v_2^2}{2gJ} + k_pT_2$$

so that for complete diffusion, with $v_2 = 0$,

$$\frac{v_1^2}{2gJ} = k_p(T_2 - T_1)$$

Writing $\theta_v = T_2 - T_p$, the *temperature equivalent of a velocity,*

$$v_1^2 = 2gJk_p\theta_v$$

Taking $k_p = 0{\cdot}240,\ \theta_v = \left(\frac{v_1}{147}\right)^2$ for air;

$k_p = 0{\cdot}276,\ \theta_v = \left(\frac{v_1}{158}\right)^2$ for exhaust gases.

CHAPTER II

Elementary Aerodynamics

2.1. Pressure Height and Density Height

2.1.1. TEMPERATURE. The *temperature* of the air is measured in degrees centigrade, 0°C corresponding to the freezing point of water. We may also measure temperature in *degrees absolute*, obtained by adding 273°C to the ordinary centigrade reading. Since — 273°C is the zero on the absolute scale, so + 15°C = 288° absolute. Zero degrees on the absolute scale is that temperature at which, in theory, solid bodies would shrink into nothingness.

The *relative temperature* τ is the ratio of the actual temperature on the absolute scale, to the *standard temperature* on the absolute scale, taken to be 15°C = 288° absolute.

Thus, for instance, for — 5°C, $\tau = \frac{268}{288} = 0{\cdot}93$, and for + 25°C, $\tau = \frac{298}{288} = 1{\cdot}035$, so that τ can be either less or greater than unity.

2.1.2. PRESSURE. There are a number of ways of measuring *atmospheric pressure*. Four of the commonest are: (1) in pounds per square inch, (2) in inches of mercury, (3) in millimetres of mercury, (4) in millibars.

The first two are commonly used to measure manifold pressure, or m.a.p., and boost. The *standard pressure* at sea-level, at standard temperature, is taken to be one of the following seven equivalent figures—

760 mm mercury
29·9212 in. of mercury
33·8985 ft of water
10 332·276 kg/m^2
14·69601 lb/$in.^2$
2 116·225 lb/ft^2
1 013·2 millibars

To convert, for instance, 35 in. Hg. m.a.p. to a boost measurement, we see that it is evidently

$$\frac{35}{29{\cdot}9212} \times 14{\cdot}69601 \text{ lb/in.}^2 = 17{\cdot}2 \text{ (approx.)} = 2{\cdot}5 \text{ lb boost}$$

(since it is 2·5 lb/in.² above the standard 14·7).

The *relative pressure* π is defined to be the ratio of the actual pressure to the standard pressure, both measured in the same system of units. Thus for a pressure of 1 000 millibars at standard temperature,

$$\pi = \frac{1\,000}{1\,013{\cdot}3} = 0{\cdot}9874$$

2.1.3. THE DENSITY. It will be noticed that we have decided that one way to measure pressure is in pounds per square inch; by inference we are measuring force in pounds. This means that mass will have to be measured in slugs (see Chapter I). Since density measures mass per unit volume, the corresponding unit of density will be slugs per cubic foot or slugs per cubic inch. Thus if we divide the mass in pounds of a cubic foot of dry air (at standard temperature and pressure) by 32·174, we obtain the standard density

$$\rho_0 = 0{\cdot}002378 \text{ slugs/ft}^3$$
$$= \tfrac{1}{421} \text{ slugs/ft}^3 \text{ (approx.)}$$

The *relative density* σ is defined to be the ratio of the actual *density* to the standard density, both measured in the same system of units. For example, if $\rho = 0{\cdot}001756$,

$$\sigma = \frac{\rho}{\rho_0} = \frac{0{\cdot}001756}{0{\cdot}002378} = 0{\cdot}7384$$

This ratio will usually be less than unity.

2.1.4. THE GAS LAW. The quantities τ, π, σ, are connected by certain natural laws.

For an isothermal expansion

$$\pi = \sigma$$

For an adiabatic expansion

$$\pi = \sigma^{1{\cdot}4}$$

All three are related by the *equation of state*

$$\pi = \sigma\tau$$

The equation of state enables us to find σ if π and τ are known, on any required system of units, e.g.

$$\sigma = \frac{9{\cdot}63p_1}{273 + t} \text{ where } p_1 \text{ is pressure in inches of mercury}$$

$$= \frac{0{\cdot}1368p_2}{273 + t} \text{ where } p_2 \text{ is pressure in pounds per square inch}$$

$$= \frac{288p_3}{1\,013(273 + t)} \text{ where } p_3 \text{ is pressure in millibars}$$

t is temperature in degrees C.

2.1.5. I.C.A.N. Atmosphere. The above natural laws must be clearly distinguished from the man-made laws which apply to what is known as the *standard atmosphere*. This conception, sponsored by I.C.A.N. (now replaced by I.C.A.O.), postulates an atmosphere which behaves itself in an orderly fashion. Starting with a sea-level temperature of 15°C, the temperature is taken to fall at the steady rate of 1·98°C per 1 000 ft, up to a height of 36 089 ft (11 000 m), after which the temperature is taken to remain constant at — 56·46°C. This ideal picture is not very far from the average state of affairs in temperate latitudes, though parts of the stratosphere filled with ozone are likely to be a good deal hotter than — 56·46°C. (For conditions in Arctic and tropical climates, see Appendix III.)

The I.C.A.N. *lapse rate* of temperature just described can be written in the form of an equation.

$$T = T_0 - \frac{1{\cdot}98h}{1\,000},$$ where h is measured in feet and T_0 is the standard temperature at sea-level,

or T absolute $= 288 - 1{\cdot}98H$, if $H = \dfrac{h}{1\,000}$,

so that the relative temperature $\tau = 1 - \dfrac{1{\cdot}98H}{288} = 1 - aH$, say,

where $a = \dfrac{1{\cdot}98}{288} = 0{\cdot}006875.$

The necessity for an artificial picture of this kind can be appreciated when we realize that flights are carried out under all sorts of conditions of temperature, pressure, and density, and the results cannot be usefully compared unless the results obtained for aircraft speed, engine power, carburettor temperature, etc., are reduced to some standard set of atmospheric conditions.

2.1.6. Relations between π, σ, and Height. In order to relate the relative pressure π and density σ to height, we introduce the normal hydrostatic relation

$$dp = -g\rho dh$$

which may be written in the form

$d\pi = -b\sigma d\mathrm{H}$, where b is a constant equal to 0·036135. One way to check this number is to plot the curve of π against H (using the figures in Appendix II), and so estimate $-b = \frac{1}{\sigma}\cdot\frac{d\pi}{d\mathrm{H}}$. Alternatively

$$\frac{dp}{dh} = -g\rho \text{ is equivalent to } \frac{dp}{d\mathrm{H}} = -1\,000 g\rho;$$

and $\frac{1}{\sigma}\frac{d\pi}{d\mathrm{H}} = -b$ is equivalent to $\frac{dp}{d\mathrm{H}} = -bp_0\frac{\rho}{\rho_0}$.

Whence $b = \frac{1\,000g \times 0{\cdot}002378}{2116{\cdot}225}$, taking the values at S.L. already given.

But $$\pi = \sigma\tau = \sigma(1 - a\mathrm{H})$$

$$\therefore \qquad \frac{d\pi}{\pi} = \frac{-bd\mathrm{H}}{1 - a\mathrm{H}}$$

or $$\log\pi = \frac{b}{a}\log(1 - a\mathrm{H})$$

(both terms vanishing when H = 0).

$$\therefore \qquad \pi = (1 - a\mathrm{H})^{b/a}$$

which is usually written as

$$\pi = \tau^n = (1 - a\mathrm{H})^n$$

where $n = \frac{0{\cdot}036135}{0{\cdot}006875} = 5{\cdot}256$.

$$\sigma = \tau^{n-1} = (1 - a\mathrm{H})^{n-1}$$

where $n - 1 = 4{\cdot}256$.

This could be written the other way round as

$$\tau = \sigma^{0{\cdot}2350} = \pi^{0{\cdot}1903}$$

These equations satisfy the *change of state* relation

$$\pi = \sigma\tau$$

Since, moreover, the two values of H are equal, under standard conditions,

$$\pi^{\frac{1}{n}} = \tau^{\frac{1}{n-1}}$$

or

$$\pi = \sigma^{\frac{n}{n-1}} = \sigma^{1\cdot235}$$

which is a man-made convention comparable with the law of adiabatic expansion

$$\pi = \sigma^{1\cdot4}$$

2.1.7. Approximations for σ. The relation $\sigma = (1 - a\text{H})^{n-1}$ is a little complicated to work out in a hurry, and the following approximation

$$\sigma = \frac{66 - \text{H}}{66 + \text{H}}$$

is usually sufficiently accurate. A corresponding approximate formula for $\sqrt{\sigma}$ is

$$\sqrt{\sigma} = \frac{132 - \text{H}}{132 + \text{H}}$$

For example, at 10 000 ft, the answers $\sigma = 0{\cdot}738$, $\sqrt{\sigma} = 0{\cdot}859$ given by these formulae agree, to three places of decimals, with that obtained from

$$\sigma = \left(1 - \frac{19{\cdot}8}{288}\right)^{4\cdot256}$$

by the use of four-figure logarithm tables; and the labour involved in the latter calculation is considerably greater.

Numerous other approximations for σ, $\sqrt{\sigma}$, etc., have been devised (see Appendix II).

2.1.8. Pressure Height. The I.C.A.N. formula for π can be reversed so as to read

$$\text{H} = \frac{1}{a}(1 - \pi^{\frac{1}{n}})$$

which gives the actual height if π is "standard." If π is not standard, the value

$$\text{H}_p = \frac{1}{a}(1 - \pi^{\frac{1}{n}})$$

gives the *pressure height*, i.e. the actual height at which the relative pressure has its actual value.

This can be illustrated by an example. At an actual height of 10 000 ft, the standard value of π is 0·6877. If, however, the actual value of π is 0·6790, which corresponds to an actual height of 10 500 ft, then the latter is the *pressure height* corresponding to an actual height of 10 000 ft and an actual pressure of 20·18 in. of mercury.

The altimeter is an instrument which, provided the zero is properly adjusted, measures pressure height, since its functioning depends upon the actual barometric pressure. As long as we realize that it is not necessarily the same as actual height, pressure height is a useful fact to know, since it affects the behaviour of the engine.

2.1.9. Density Height. The I.C.A.N. formula for σ can be reversed to read

$$H = \frac{1}{a}(1 - \sigma^{\frac{1}{n-1}})$$

which gives the actual height if σ is standard. If σ is not standard, the value

$$H_d = \frac{1}{a}(1 - \sigma^{\frac{1}{n-1}})$$

gives the *density height*, i.e. the actual height at which the density has its actual value. If for instance at some particular actual height the temperature is more than standard, we should expect the density at this point to be less than standard, and to this lesser density there corresponds a greater height on the I.C.A.N. scale: this greater height is the density height at the point in question.

The density height is important because the drag and lift of the aircraft are directly proportional to density. Instead of finding density directly, it is usual to find the pressure height and the outside air temperature; if the latter is not standard we make a correction and so find the density height and then use the standard density for this height.

2.1.10. Correction Charts. The correction to be made can easily be read off the charts supplied with the majority of control manuals. Their method of construction can be understood from the following argument—

Suppose that the relative pressure π is fixed, and that there is a variation in σ and τ, still governed by $\pi = \sigma\tau$. This is quite possible if the I.C.A.N. lapse rate no longer holds.

The pressure height is still given by

$$H = \frac{1}{a}(1 - \pi^{\frac{1}{n}}),$$

but there is an alteration ΔH in density height given by

$$\Delta H = \frac{1}{a}\left(- \frac{1}{n-1}\,\sigma^{\frac{1}{n-1}-1}\,\Delta\sigma\right)$$

Also $$0 = \sigma\Delta\tau + \tau\Delta\sigma$$

Hence $$\Delta H = \frac{1}{a}\,\frac{1}{n-1}\,\sigma^{\frac{1}{n-1}}\,\frac{\Delta\tau}{\tau}$$

$$= \frac{1}{a}\,\frac{1}{n-1}\,(1 - aH)\,\frac{\Delta\tau}{\tau}$$

If, for instance, $H = 10$ ($h = 10\,000$ ft), where the standard temperature is $-4{\cdot}8°$, $\tau = \dfrac{268{\cdot}2}{288}$.

$$\therefore \qquad \Delta H = \frac{288}{1{\cdot}98} \times \frac{1}{4{\cdot}256}\left(1 - \frac{19{\cdot}8}{288}\right)\frac{288\Delta\tau}{268{\cdot}2}$$

or $$\Delta H = 0{\cdot}1187 \times 288 \times \Delta\tau$$

Hence for $\Delta H = 1$, $288\Delta\tau = 8{\cdot}428$, and so $\Delta T = 8{\cdot}428$°C, i.e. the density height is 1 000 ft more for every 8·428°C above standard.

The figure 8·428°C varies slightly with height, but we shall not be far wrong if we take 8°C per 1 000 ft. With this rough rule, two examples are as follows—

(1) At 20 000 ft, standard temperature = − 24·6°C. Suppose actual temperature = − 10·0°C. Then density height

$$= \frac{14{\cdot}6}{8} \times 1\,000 + 20\,000 = 21\,830 \text{ ft}$$

(2) At sea level, standard temperature = 15°C. Suppose actual temperature = 0°C. Then density height

$$= 0 - \tfrac{15}{8} \times 1\,000 = -1\,870 \text{ ft}$$

These answers will be found to agree very well with those read off Fig. 1 (*a*), which is used in this way.

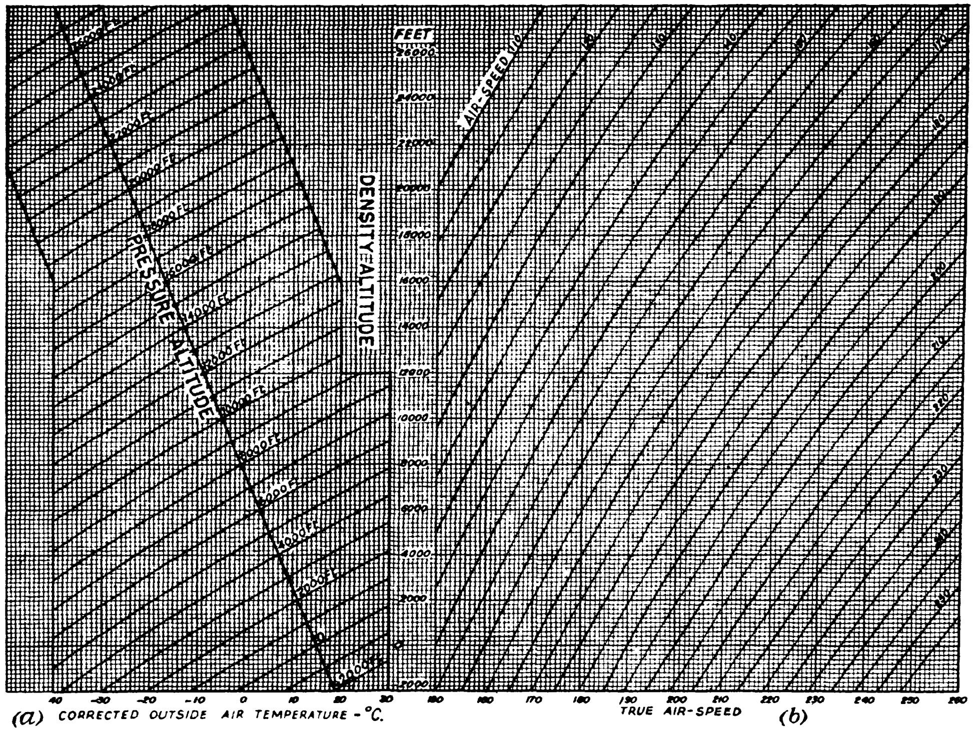


FIG. 1. CORRECTION CHARTS

(*a*) Pressure height and density height. (*b*) True air-speed and equivalent air-speed.

Proceed along the lapse line (Fig. 1 (*a*)) until we reach the oblique line corresponding to the given pressure height. Read vertically downwards to find the standard temperature at this height. If the actual temperature is not standard, proceed along the oblique pressure height line until reaching the point on the line vertically above the actual temperature. Then read off horizontally the density height.

2.1.11. QUANTITIES VARYING WITH $\sqrt{\sigma}$. At a fixed pressure height, $\sigma \propto \frac{1}{\tau}$, i.e. $\sqrt{\sigma} \propto \frac{1}{\sqrt{\tau}}$. This fact is at the bottom of another example of correcting for temperature changes.

Differentiating logarithmically,

$$\frac{\Delta\sqrt{\sigma}}{\sqrt{\sigma}} = -\tfrac{1}{2}\frac{\Delta\tau}{\tau}$$

i.e. percentage change in

$$\sqrt{\sigma} = -50\frac{\Delta\tau}{\tau}$$

so that a 1 per cent decrease in $\sqrt{\sigma}$ corresponds to an increase of temperature of ΔT where $50\Delta T = 288$ at sea-level, and $50\Delta T = 268$ at 10 000 ft, i.e. $\Delta T = 5{\cdot}76$°C at sea-level, and $\Delta T = 5{\cdot}36$°C at 10 000 ft. Any quantity, such as indicated speed, which depends upon $\sqrt{\sigma}$ can be corrected for temperature at a given pressure height by—

increasing the quantity by 1 per cent for from 5°C to 6°C drop in temperature,

decreasing the quantity by 1 per cent for from 5°C to 6°C rise in temperature.

It may appear tedious to labour this point, but aircraft are frequently flown in tropical or Arctic conditions, and so it is important to understand the general effect of non-standard temperatures. The serious effect on take-off is discussed further in Chapter V.

The numerical statements so far made can be shown to be consistent by the following example.

Suppose we are at a pressure height of 10 000 ft, and the temperature is 5·36°C above standard. Then the density height is more than 10 000 ft by the amount $\frac{5{\cdot}36}{8{\cdot}43} \times 1\,000 = 635$ ft.

Now $$\sqrt{\sigma} = \frac{132 - H}{132 + H} = 1 - \frac{H}{66} \text{ (approx.)}$$

i.e. $$\Delta\sqrt{\sigma} = \frac{-\Delta H}{66} = \frac{-0{\cdot}635}{66},$$

so that quantities depending upon $\sqrt{\sigma}$ will decrease by $\frac{63{\cdot}5}{66}$ or approximately 1 per cent for a 5·36°C rise in temperature.

2.2. True Speed and Indicated Speed, and Corrections

2.2.1. TRUE AND EQUIVALENT AIR-SPEEDS. The aircraft A.S.I. measures the difference between total head and atmospheric pressure, i.e. it measures the dynamic pressure $\frac{1}{2}\rho V^2$ lb/ft² due to the local relative velocity V, or *true air-speed* (T.A.S.) in feet per second where ρ is the local density in slugs per cubic foot.

It is not difficult to construct an instrument which converts the variations in V^2 into linear divisions on a dial, but evidently this will be useless if ρ also varies. We therefore write

$$\tfrac{1}{2}\rho V^2 = \tfrac{1}{2}\rho_0 V_i^2$$

where ρ_0 is the standard sea-level density; then the instrument can be made to read the variations in V_i, known as the *equivalent air-speed*, or E.A.S.

The correction between true air-speed V and equivalent air-speed V_i is

$$V^2 = \frac{\rho_0}{\rho} V_i^2$$

or $$V = \frac{1}{\sqrt{\sigma}} V_i$$

so that the true speed can be found only when we know both the instrument speed and the relative density. Fig. 1 (*b*) is a graph of $V_i = \sqrt{\sigma} V$.

2.2.2. CORRECTIONS. It is important that V should be known accurately, and we must therefore be certain of the various corrections to the instrument speed and $\sqrt{\sigma}$. The A.S.I. reading (A.S.I.R.) should be corrected for—

(i) Instrument error, if any.

(ii) Position error. This varies with speed and with the type of aircraft. Care must be taken to give the correction the right sign, e.g. from the accompanying graph, Fig. 2, showing the P.E. for a four-engined transport, the correction at $V_i = 140$ knots will be 2·4 knots, and at $V_i = 240$ knots will be $-$ 2 knots, for a weight of 90 000 lb.

(iii) Compressibility error. This arises at all speeds, but at heights below 10 000 ft it is usually negligible for speeds less than 300 m.p.h., when it is no longer true that the difference

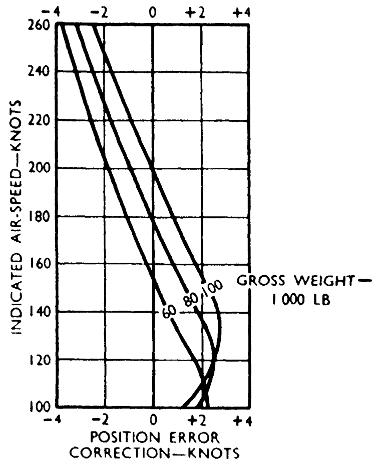


FIG. 2. POSITION ERROR CORRECTION

between total head and static pressure is equal to $\frac{1}{2}\rho_0 V_i^2$. There is an increase of total head with Mach number, based on Bernoulli's equation for compressible flow.

If V_r = indicated air-speed, corrected for instrument error only, and V_i = equivalent air-speed = T.A.S. $\times \sqrt{\sigma}$, it is found that, approximately,

$$V_r^2\left(1 + \frac{1}{4}\frac{V_r^2}{a_0^2}\right) = V_i^2\left(1 + \frac{1}{4}\frac{V_i^2}{\pi a_0^2} + \frac{1}{40}\frac{V_i^4}{\pi^2 a_0^4}\right)$$

(π = relative pressure)

where a_0 is the speed of sound in I.C.A.N. conditions at sea-level.

By repeated approximation this reduces to

$$V_i = V_r\left[1 - \frac{V_r^2}{8a_0^2}\left(\frac{1}{\pi} - 1\right) - \frac{V_r^4}{128a_0^4}\left(1 + \frac{6}{\pi} - \frac{27}{5\pi^2}\right)\right],$$

i.e. the E.A.S. is less than that given on the instrument.

This correction can be calculated, or read off a graph such as Fig. 3 and is independent of the aircraft in use. At a height of, say, 40 000 ft the error is considerable.

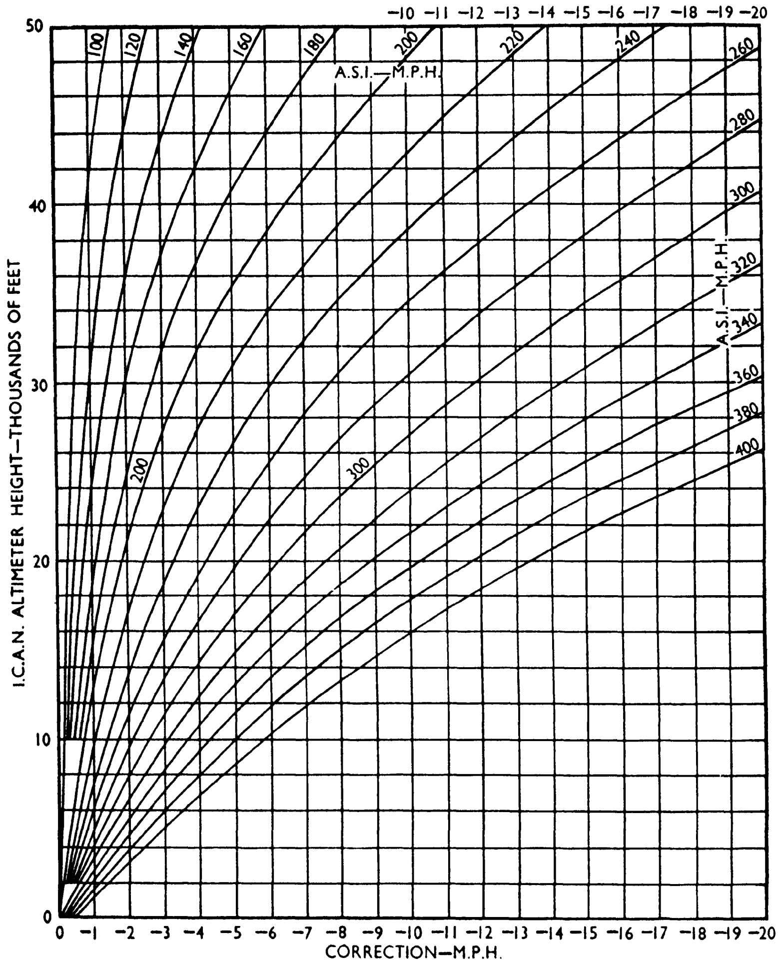


Fig. 3. Compressibility Error to Speed

Compressibility also affects the thermometer readings. A thermometer carried externally on an aeroplane measures the temperature of the air in contact with the bulb, which has been compressed adiabatically by being brought to rest relative to the bulb. The temperature of this air on the bulb is thus greater than that of the surrounding atmosphere; the amount of the error depends upon the instrument, and is found experimentally by plotting temperature against V_i^2. The error can then be plotted in a graph such as Fig. 4.

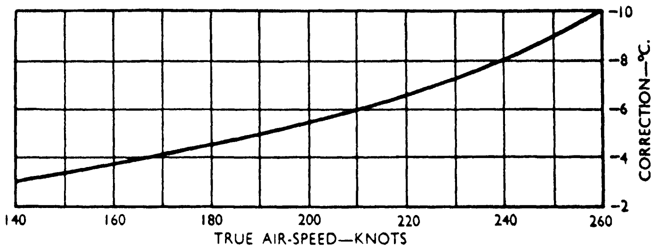


FIG. 4. COMPRESSIBILITY CORRECTION FOR OUTSIDE AIR TEMPERATURE

2.2.3. SPEEDS. After corrections (i) and (ii) have been made, the resulting speed is sometimes known as the *rectified air-speed* (R.A.S.); after correction (iii) has been made, we get the *calibrated air-speed*, or C.A.S. In what follows we shall assume that the calibrated air-speed (C.A.S.) and equivalent air-speed (E.A.S.) are identical.

This speed is usually measured in knots, and it will be convenient to remember that

$$88 \text{ ft/sec} = 60 \text{ m.p.h.} = 52{\cdot}2 \text{ kt}$$

and that knots can be converted into miles per hour by multiplying by 1·15. More precisely,

$$1 \text{ knot} = 1{\cdot}151553 \text{ statute miles an hour}$$
$$= 1{\cdot}688944 \text{ ft/sec}$$

2.2.4. SUMMARY. We have already explained, in the first section of this chapter, how to correct $\sqrt{\sigma}$ for non-standard temperature. For those who prefer charts to calculations, the first thing to do is to find the density height from Fig. 1 (*a*), and then the value of V in terms of V_i can be read from Fig. 1 (*b*), which is simply a graphical form of $V_i = \sqrt{\sigma}V$. Or the latter work can be done on a navigator's computer. If we prefer simple arithmetic, find $\sqrt{\sigma}$ from a table or from $\sqrt{\sigma} = \dfrac{132 - H}{132 + H}$

and decrease its value by 1 per cent for every 5·5°C rise in temperature.

2.3. The Drag Curve: Wind Tunnel and Flight Tests

2.3.1. THE DRAG FORMULA. The contribution of the air-frame to the performance of any aircraft is mainly dependent upon the drag D, given by

$$D = \tfrac{1}{2}\rho V^2 . S_W . C_D = \tfrac{1}{2}\rho_0 V_i^2 . S_W . C_D \text{ pounds}$$

in which S_W is the wing area in square feet, V and V_i are measured in feet per second, and C_D is the non-dimensional *drag coefficient* which is a function of the angle of attack, but is independent of wing area, speed, and density. Other small contributions to the drag (e.g. cooling) will at present be neglected, and no account will be taken of any incidental thrust in the opposite direction, such as that due to the exhaust.

2.3.2. THE LIFT FORMULA. During level flight the lift obtainable from the wings, etc., is expressible in a similar form. Taking the weight W equal to the lift,

$$W = \tfrac{1}{2}\rho V^2 S_W . C_L = \tfrac{1}{2}\rho_0 V_i^2 S_W . C_L$$

in which C_L is the *lift coefficient.* Over a large part of its range, C_L is linear with angle of attack, varying from zero at the small negative *no-lift angle* to about unity for an un-flapped wing. Lowering the flaps has the effect of increasing the maximum value of C_L, i.e. it decreases the stalling speed, and there is usually a slight increase in the slope of the lift curve, and a considerable increase in the negative value of the no-lift angle.

2.3.3. THE LINEAR RELATION BETWEEN C_D AND C_L^2. The drag coefficient C_D can be found in two ways; firstly by tests of a model in a wind tunnel. It is usual to plot C_D against C_L, giving a *polar*, or more often against C_L^2. In the latter case the result is the straight line graph

$$C_D = C_{D_z} + \frac{1}{\pi A e} C_L^2 \ (\pi = 3{\cdot}1416 \text{ approx.})$$

where A is the aspect ratio, *e* the N.A.C.A. *efficiency factor* and C_{D_z} is the value of C_D where the straight line joining the experimental points is produced back to cut the axis at $C_L = 0$. It is found in practice that, for the majority of conventional aerofoils such as the N.A.C.A. 230 series used in aircraft like the Tudor and the Hermes, the straight line law is deviated from for the larger values of C_L. The "cleanest" aircraft have the lowest values of C_{D_z}: a reasonable value is 0·025. War-time bombers, with their numerous excrescences, had higher values of C_{D_z}. In the case of low-drag aerofoils, whose greatest thickness is farther back from the nose, the straight line graph is more closely followed for the larger values of C_L.

2.3.4. The Calculation of Drag. By incorporating the equation for W, the drag equation is

$$D = \tfrac{1}{2}\rho_0 S_W C_{D_z} V_i^2 + \frac{W^2}{\tfrac{1}{2}\rho_0 S_W \pi A e} \times \frac{1}{V_i^2}$$

or

$$D = aV_i^2 + \frac{bW^2}{V_i^2}$$

where

$$a = \tfrac{1}{2}\rho_0 S_W C_{D_z},\ b = \frac{1}{\tfrac{1}{2}\rho_0 S_W \pi A e}$$

$$(\text{writing } f = S_W \times C_{D_z},\ a = \tfrac{1}{2}\rho_0 f)$$

The first of these drag terms is the *parasitic* drag, and the second is the *induced* drag, which decreases as V_i increases and the aspect ratio is made larger, and increases with the square of the weight.

When performing this experiment in a wind tunnel, the value of ρ_0, S_W, A are known beforehand, and the values of C_{D_z} and e are found from the graph by measuring respectively, the intercept of the straight line, and its gradient.

There are, of course, errors due to scale effect, which seriously affects $C_{L\max}$, but has little effect on the slope of the lift curve. Scale effect on C_{D_z} is uncertain.

2.3.5. Flight Testing. The relation between C_D and C_L^2 can also be found by *flight testing* the actual aircraft. There are considerable difficulties in this method, and great care must be taken to reduce the experimental results to standard conditions. But in general outline the process is as follows: By using a torquemeter it is possible to find the b.h.p. delivered by the engines, using a relation of the type

$$\text{b.h.p.} = \frac{\text{r.p.m.} \times \text{torquemeter reading}}{\text{constant}}\ (\text{lb/in.}^2)$$

or a suitable conversion dial. It is also possible to estimate the propeller efficiency for the conditions (speed and b.h.p.) of the flight; then

$$\text{b.h.p.} = \frac{\text{t.h.p.}}{\eta} = \frac{DV}{375\eta}$$

where D is the drag in pounds, V is the T.A.S. in m.p.h. If a reliable thrustmeter is fitted, we can dispense with the necessity

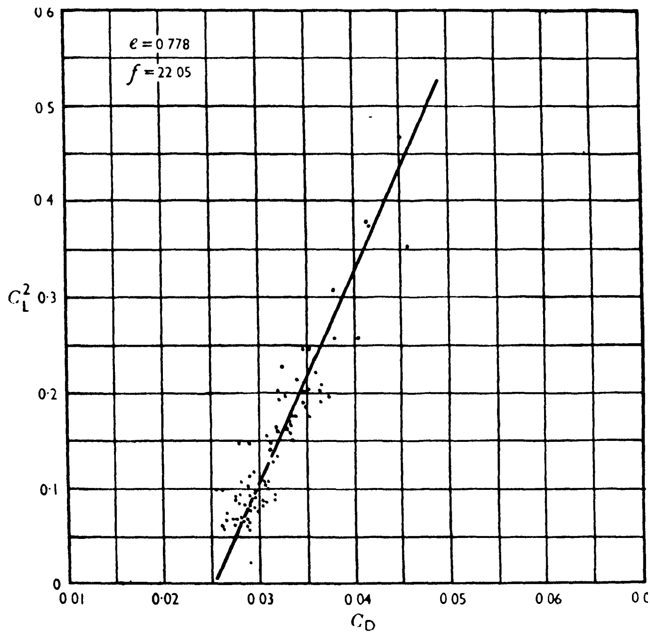


FIG. 5. POLAR CURVE $C_L{}^2$ VERSUS C_D

for estimating the propeller efficiency. In certain cases we can deduce the b.h.p. and dispense with a torquemeter, by using the engine power charts. Since the weight and height are known (see below) we can deduce the corresponding values of C_L and V_i, and plot C_D against $C_L{}^2$ as before. Fig. 5 shows the $C_D/C_L{}^2$ straight line obtained by flight testing a modern prototype two-engined transport. The apparent scatter of the points should not cause alarm. There are various reasons why the "constants" e and C_{D_z} may not remain invariable throughout a series of test flights, but the variations in these quantities will not seriously affect subsequent results.

We stated above that the weight was known. If there is an accurate record of the fuel used, this is the case; and it is usual to correct all results to a standard "mean weight" for the type being tested.

When no propeller efficiency curves are available, the following method may be used. Consider the quantities

$$P_{i_w} = \frac{\text{b.h.p.}\sqrt{\sigma}}{\left(\frac{W}{W_s}\right)^{\frac{3}{2}}}, \qquad N_{i_w} = \frac{N\sqrt{\sigma}}{\left(\frac{W}{W_s}\right)^{\frac{1}{2}}}, \qquad V_{i_w} = \frac{V\sqrt{\sigma}}{\left(\frac{W}{W_s}\right)^{\frac{1}{2}}} = \frac{V_i}{\left(\frac{W}{W_s}\right)^{\frac{1}{2}}}$$

where P = power, N = r.p.m., V = T.A.S., W_s = standard mean weight.

Now in level flight

$$\eta \times \text{b.h.p.} \propto C_D \cdot \frac{V_i^3}{\sqrt{\sigma}} \text{ (cf. § 2.4.4.)}$$

$$\therefore \qquad \eta \times \text{b.h.p.} \times \sqrt{\sigma} \propto C_D V_i^3$$

Dividing by $\left(\frac{W}{W_s}\right)^{\frac{3}{2}}$ we get $\eta P_{i_w} \propto C_D V^3_{i_w}$

Now if $\qquad C_D = C_{D_Z} + \frac{C^2_L}{\pi A e},$

and $\qquad C_L = \frac{W}{\frac{1}{2}\rho_0 S_w V^2_i}, \; C_L \propto \frac{1}{\left(V_{i_w}\right)^2}.$

We know that $\qquad \eta = F(C_p, J)$ and $C_p \propto \frac{\text{b.h.p.}}{N^2\sigma}$

Dividing top and bottom by $\left(\frac{W}{W_s}\right)^{\frac{3}{2}}$, $C_p \propto \frac{P_{i_w}}{\left(N_{i_w}\right)^2}.$

Also

$$J \propto \frac{V\sqrt{\sigma}}{N\sqrt{\sigma}}, \text{ hence } J \propto \frac{V_{i_w}}{N_{i_w}}, \text{ hence } \eta = F\left(P_{i_w}, V_{i_w}, N_{i_w}\right)$$

$$\therefore \; F\left(N_{i_w}, P_{i_w}, V_{i_w}\right) = 0.$$

The three quantities can be plotted on a "carpet" and standard conditions picked off as required.

2.3.6. THE DRAG CURVE. Whichever method is used, we can now plot a family of curves of drag against indicated speed, weight being the parameter which distinguishes the members of the family. As an example, consider the twin-engined aircraft for which $C_{D_Z} = 0{\cdot}025$, $S_W = 900$, $e = 0{\cdot}8$, $A = 8$, $f = 22{\cdot}5$. Then

$$a = \tfrac{1}{2} \times \frac{1}{421} \times 22{\cdot}5 = 0{\cdot}0267,\ b = \frac{2 \times 421}{900\pi \times 6{\cdot}4} = 0{\cdot}0465$$

so that the drag $D = 0{\cdot}0267\, V_i^2 + 0{\cdot}0465 \dfrac{W^2}{V_i^2}$.

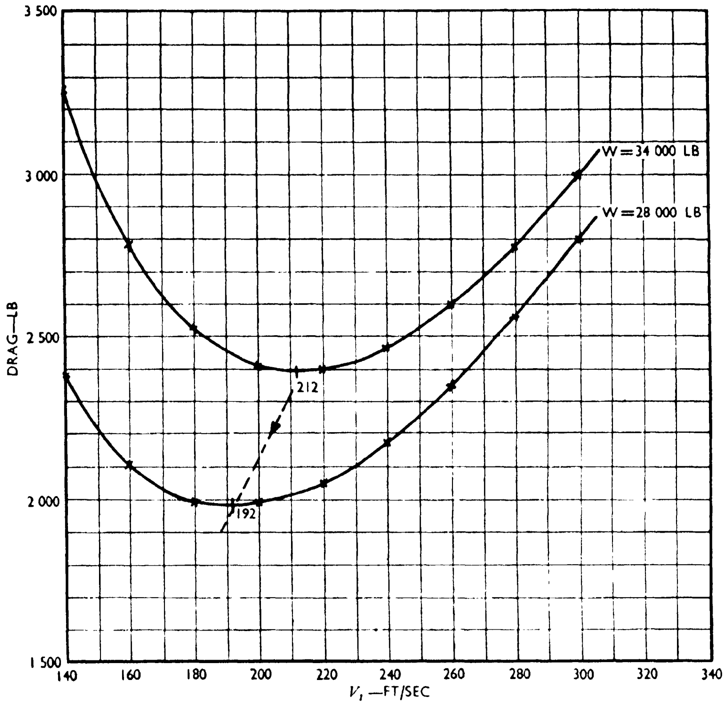


FIG. 6. DRAG CURVES

Now calculate the drag for two values of W, 28 000 lb and 34 000 lb, for values of V_i between 140 and 300 ft/sec. The curves obtained are shown in Fig. 6.

2.3.7. THE SPEED FOR MINIMUM DRAG. It is apparent from the curves that, for each weight, there is an indicated speed for minimum drag. This is given by

$$V_{i_{md}} = \left(\frac{1}{\pi A e C_{D_Z}}\right)^{\frac{1}{4}} \left(\frac{W}{\frac{1}{2}\rho_0 S_W}\right)^{\frac{1}{2}} = \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{W} \text{ ft/sec},$$

giving 192 ft/sec for 28 000 lb, and 212 ft/sec for 34 000 lb.

	V_i	140	160	180	200	220	240	260	280	300	400	440
	V_i^2	19 600	25 600	32 400	40 000	48 400	57 600	67 600	78 400	90 000	160 000	193 600
	$0{\cdot}0267\ V_i^2$	524	684	865	1 069	1 290	1 539	1 805	2 090	2 400	4 270	5 170
W = 28 000	$\frac{W}{V_i}$	200	175	155·6	140	127·3	116·7	107·8	100	93·4	—	—
	$\frac{W^2}{V_i^2}$	40 000	30 630	24 230	19 600	16 200	13 620	11 620	10 000	8 724	—	—
	$0{\cdot}0465\ \frac{W^2}{V_i^2}$	1 860	1 425	1 127	910	753	633	540	465	405	—	—
	D	2 384	2 109	1 992	1 979	2 043	2 172	2 345	2 555	2 805	—	—
W = 34 000	$\frac{W}{V_i}$	243	212·5	189	170	154·6	141·6	131	121·6	113·4	85	77·5
	$\frac{W^2}{V_i^2}$	59 050	45 150	35 720	28 910	23 910	20 040	17 160	14 790	12 860	7 240	6 006
	$0{\cdot}0465\ \frac{W^2}{V_i^2}$	2 750	2 110	1 660	1 342	1 110	931	789	687	598	336	279
	D	3 274	2 794	2 525	2 411	2 400	2 470	2 603	2 777	2 998	4 606	5 249

One noticeable feature of the curves is the large increase in drag at low speeds.

As will be seen shortly, the speed for maximum range (for a piston-engined aircraft only) is just a bit more than the speed for minimum drag—or the speed for maximum L/D, which is the same thing.

We note that the minimum drag speed varies with the square root of the weight, but that during this variation the angle of attack remains constant. We have

$$\frac{C_D}{C_L} = \frac{1}{C_L} \cdot C_{D_Z} + \frac{1}{\pi Ae} \cdot C_L \text{ a minimum when}$$

$$C_L{}^2 = \pi AeC_{D_Z}, \; C_L = \sqrt{\pi AeC_{D_Z}}$$

$$\therefore \qquad C_D = 2C_{D_Z}$$

Since C_L is independent of W, α is also constant.

Substituting this value of C_L,

$$W = \tfrac{1}{2}\rho_0 V_i{}^2 S_W \sqrt{\pi AeC_{D_Z}}$$

$$\therefore \qquad V_L = \frac{W^{\frac{1}{2}}}{(\frac{1}{2}\rho_0 S_W)^{\frac{1}{2}}(\pi AeC_{D_Z})^{\frac{1}{4}}}$$

and the maximum value of L/D is equal to $\frac{1}{2}\sqrt{\dfrac{\pi Ae}{C_{D_Z}}}$.

For example, if $C_{D_Z} = 0{\cdot}025$, $\pi Ae = 20$, $C_L = \sqrt{0{\cdot}5} = 0{\cdot}707$, and the maximum value of L/D is 14·14.

2.4. The Power Curve

2.4.1. Calculation of B.H.P. The brake horse-power required from all engines is given by

$$\text{b.h.p.} = \frac{DV}{550\eta}, \text{ where } \eta = \text{propeller efficiency}$$

$$= \frac{1}{550\sqrt{\sigma}\eta}\left(aV_i{}^3 + \frac{bW^2}{V_i}\right)$$

If we neglect the variation of η with speed and take it as 0·8 throughout, and plot the total b.h.p. for two weights at 10 000 ft, where $\sqrt{\sigma} = 0{\cdot}859$, we get a pair of curves similar to those in Fig. 6.

The result is shown in Fig. 7.

		100	120	140	160	180	200	220	240	260
	V_i	100	120	140	160	180	200	220	240	260
	V_i^2	10 000	14 400	19 600	25 600	32 400	40 000	48 400	57 600	67 600
	$0{\cdot}0267 \times V_i^2$	267	384	524	684	865	1 069	1 290	1 539	1 805
W = 28 000	$\frac{W}{V_i}$	280	233·5	200	175	155·6	140	127·3	116·7	107·8
	$\frac{W^2}{V_i^2}$	78 400	54 520	40 000	30 630	24 230	19 600	16 200	13 620	11 620
	$0{\cdot}0465 \times \frac{W^2}{V_i^2}$	3 640	2 540	1 860	1 425	1 127	910	753	633	540
	D	3 907	2 924	2 384	2 109	1 992	1 979	2 043	2 172	2 345
	V	1 165	139·5	163	186·5	210	233	256	280	302
	$\frac{V}{550}$	0·264	0·317	0·370	0·424	0·478	0·530	0·582	0·635	0·686
	b.h.p.	1 032	930	883	895	954	1 050	1 190	1 380	1 610
W = 34 000	$\frac{W}{V_i}$	340	283	243	212·5	189	170	154·6	141·6	131
	$\frac{W^2}{V_i^2}$	115 600	80 090	59 050	45 150	35 720	28 900	23 910	20 040	17 160
	$0{\cdot}0465 \times \frac{W^2}{V_i^2}$	5 380	3 720	2 750	2 110	1 660	1 342	1 110	931	798
	D	5 647	4 104	3 274	2 794	2 525	2 411	2 400	2 470	2 603
	b.h.p.	1 452	1 300	1 210	1 185	1 220	1 276	1 392	1 570	1 785

2.4.2. THE SPEED FOR MINIMUM POWER. At each weight there is a speed for minimum power, given by

$$V_{i_{mp}} = \sqrt[4]{\frac{1}{3\pi A e C_{D_Z}}}\sqrt{\frac{W}{\frac{1}{2}\rho_0 S_W}} \text{ ft/sec}$$

$$= 11{\cdot}28\left(\frac{W^2}{efb_0^{\,2}}\right)^{\frac{1}{4}} \text{ m.p.h. (where } b_0 \text{ is the span)},$$

giving 146 ft/sec for 28 000 lb and 161 ft/sec for 34 000 lb.

So that $V_{i_{md}} = 3^{\frac{1}{4}} \times V_{i_{mp}} = 1{\cdot}316 V_{i_{mp}}$,

i.e. the speed for minimum drag is 31·6 per cent higher than the speed for minimum power.

Let us look for a moment at the point P on the 28 000 lb curve in Fig. 7, where the tangent from the origin touches the curve. This is where b.h.p./V is least, i.e. D is least, so that both $V_{i_{mp}}$ and $V_{i_{md}}$ can be found from the power curve.

The indicated speed for minimum power should be regarded as a basic constant for the particular aeroplane.

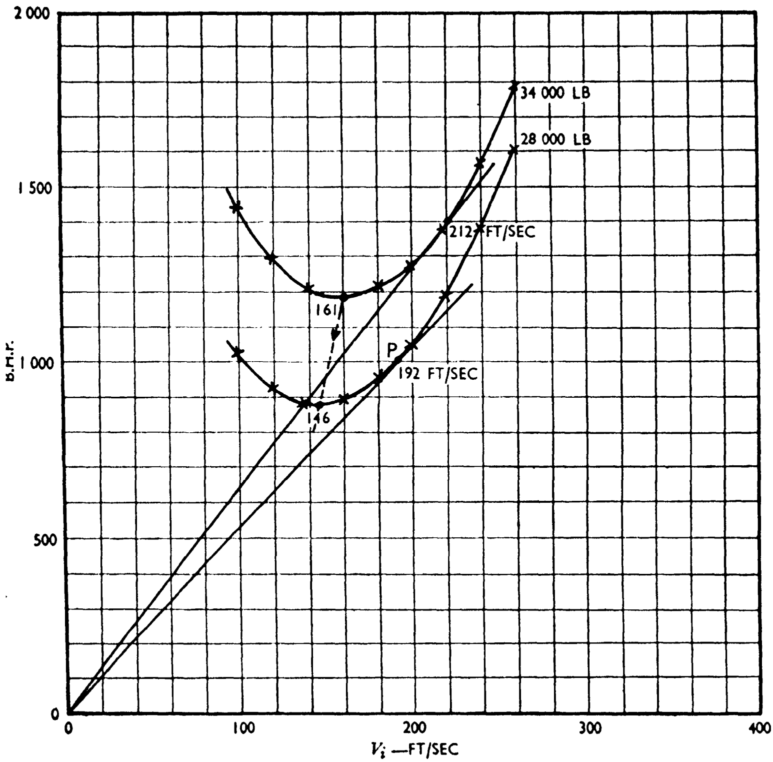


FIG. 7. POWER REQUIRED CURVES

It is assumed that $\eta = 0{\cdot}8$ and $\sqrt{\sigma} = 0{\cdot}859$ (10 000 ft).

It is evidently

(i) the speed for maximum endurance, because the fuel flow in gallons per hour is proportional to the b.h.p.,

(ii) the best climbing speed near the ceiling. At one particular setting of r.p.m. and boost, the power available from all engines will be roughly constant over the speed range, and the curve for power available will cross the power required curve drawn as in Fig. 8.

For a given weight and engine setting, the reserve power available for climbing is given by the difference in the ordinates of the two curves, and this is clearly a maximum round about $V_{i_{mp}}$.

2.4.3. Gas-turbine-engined Aircraft. Note that the above remark applies only to piston-engined aircraft. For a gas-turbine engine, the b.h.p. available is roughly proportional to

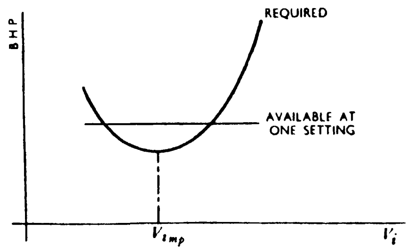


Fig. 8. Available and Required Power: Piston Engines

the speed, since the thrust is roughly constant; hence the greatest surplus b.h.p. will occur at a speed in the neighbourhood of P (Fig. 9). This means that the best climbing speed is

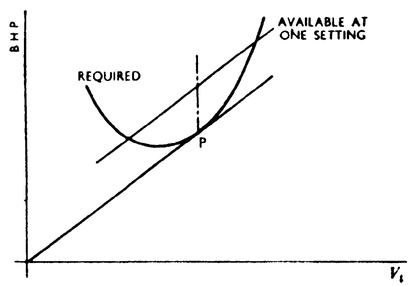


Fig. 9. Available and Required Power: Gas-turbine Engines

round about the speed for minimum drag, i.e. 32 per cent higher than the best climbing speed for piston-engined aircraft.

2.4.4. Performance Estimation. It should be pointed out that the above method of drawing the power-speed curve is not that usually followed in the manufacturer's design office.

One common method of procedure is based upon the following reasoning—

Since $W = \frac{1}{2}\rho V^2 S_W C_L$, then $W^{\frac{3}{2}} = (\frac{1}{2}\rho S_W)^{\frac{3}{2}} C_L^{\frac{3}{2}} V^3$.

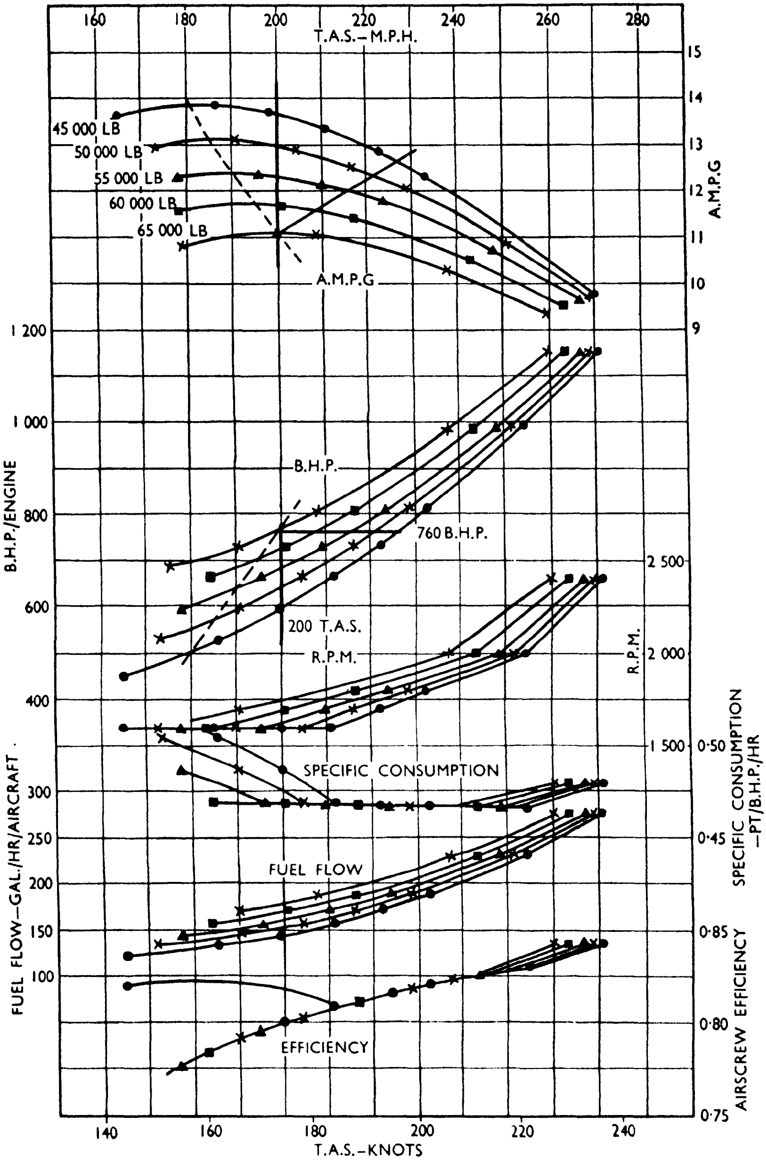


FIG. 10. CRUISING DATA AT 10 000 FT
Three-blade airscrews.

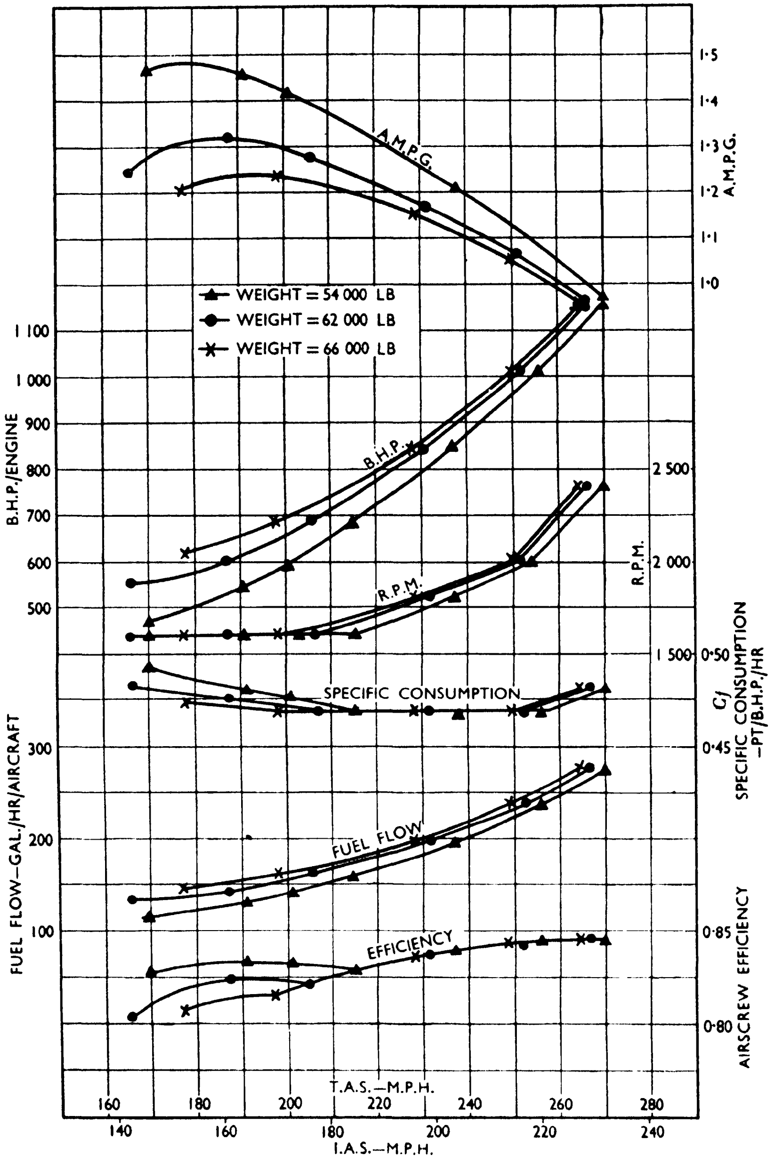


FIG. 11. CRUISING DATA AT 10 000 FT
Four-blade airscrews.

Power required

$$= \frac{DV}{550} = C_D \frac{\frac{1}{2}\rho V^3 S_W}{550} = C_D \frac{\frac{1}{2}\rho S_W}{550} \times \frac{W^{\frac{3}{2}}}{(\frac{1}{2}\rho S_W)^{\frac{3}{2}} C_L^{\frac{3}{2}}}$$

$$= \frac{C_D}{C_L^{\frac{3}{2}}} \times \frac{1}{(\frac{1}{2}\rho S_W)^{\frac{1}{2}}} \times \frac{W^{\frac{3}{2}}}{550}$$

Power required per engine $\times \sqrt{\sigma}$

$$= \frac{C_D}{C_L^{\frac{3}{2}}} \cdot \frac{1}{(\frac{1}{2}\rho S_W)^{\frac{1}{2}}} \frac{W^{\frac{3}{2}}}{550} \times \frac{1}{\text{number of engines}}$$

We can thus draw a family of "$\sqrt{\sigma} \times$ drag power" curves against indicated speed $\times \sqrt{\sigma}$ for different weights, based on our wind tunnel information about the variation of C_L and C_D with angle of attack and hence with speed.

Across this family of curves is plotted another family. The b.h.p. available per engine, for various settings of boost and r.p.m., is multiplied by $\eta\sqrt{\sigma}$, and again plotted against $V_i\sqrt{\sigma}$. The latter family will cut the drag power curve for one particular weight in a series of points at different speeds; we take the right-hand intersections, where, for constant level speed, power required = power available, and so decide what engine settings and b.h.p. are required for each speed. An advantage of this method is that, for all heights, only one set of drag power curves is necessary.

The results can be re-plotted in a composite graph like Figs. 10 and 11. Here the lower curve shows propeller efficiency, calculated at different weights and engine speeds. The middle graph shows r.p.m. This remains at the lowest allowable, 1 600 r.p.m. for the lower speeds, and then rises to 2 400 at maximum weak mixture power. The b.h.p. required from each engine shows a steady rise from (for the heaviest weight) 620 to 1 150. The remaining curves will be commented upon later.

CHAPTER III

Piston Engines

3.1. Analysis of Power

3.1.1. INDICATED HORSE-POWER. From the fundamental formula

$$\text{i.h.p.} = \frac{\text{PALEC}}{33\,000}$$

where P = m.e.p. in pounds per square inch,
A = piston area, square inches,
L = length of stroke, in feet,
E = number of expansion strokes per minute,
C = number of cylinders,

it is easy enough to show that, for a particular engine,

$$\text{i.h.p.} \propto \text{K} \times \text{m.e.p.} \times \text{r.p.m.}$$

The m.e.p. is affected by the mixture strength (or air/fuel ratio), and also by the volumetric efficiency, which is the ratio of the weight of mixture to the cylinders during the induction stroke to the weight of charge required to fill the swept volume at standard temperature and pressure. This is where the supercharger comes into the picture.

Put in rather simpler words, this amounts to saying that—
Power is governed by the variation of three main quantities,

(*a*) air/fuel ratio,
(*b*) r.p.m., or engine speed,
(*c*) boost, i.e. the pressure in the induction system measured above or below atmospheric pressure.

3.1.2. AIR/FUEL RATIO. The first, air/fuel ratio, is quickly disposed of. If the boost and r.p.m. are kept constant, the specific consumption varies with b.h.p. in the manner shown in Fig. 12. The minimum specific consumption is obtained at an air/fuel ratio of about 16, but there is not much variation over the "weak mixture" range; and, in fact, the diffuser in the carburettor is designed to keep a constant mixture strength for all combinations of r.p.m. and boost within the cruising range.

As cruising will usually be done using weak mixture, we are not going to get much help from this source.

3.1.3. BOOST AND R.P.M. The object of a supercharger is to raise the pressure in the induction pipe so that it is equal to, or greater than, atmospheric pressure. (We can measure this manifold pressure either in inches of mercury or pounds of boost above or below standard, e.g. we saw in § **2.1.2.** that 35 in. Hg m.a.p. = + 2·5 lb boost.) At sea-level, raising the

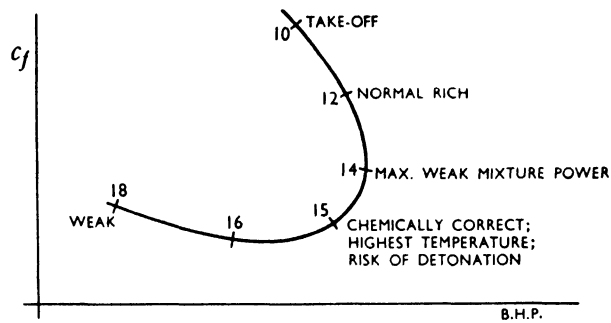


FIG. 12. SPECIFIC CONSUMPTION VERSUS B.H.P. AT DIFFERENT AIR/FUEL RATIOS

pressure above atmospheric gives additional power, but too much boost is liable to wreck the engine, and consequently the throttle is not fully open.

As climbing proceeds, if the engine speed (i.e. r.p.m.) is kept constant, then the amount of supercharge is also kept constant by means of an automatic boost control, until a height is reached where the throttle is fully open. During this process the power output is actually increased because the lower air density at altitude reduces the exhaust back pressure which in turn gives better scavenging, thus increasing the weight of charge per stroke; and also because the lower air temperature at altitude gives better cooling, and a higher density to the induced charge.

We have said above that the throttle is not fully open at sea-level. The following simplified numerical example will explain this fact—

Imagine that at constant climbing r.p.m., the boost pressure is + 7 lb/in.^{2}, and full-throttle height is 7 000 ft, at which height the atmospheric pressure is 11·3 lb/in.^{2} The actual pressure in the induction system is $14{\cdot}7 + 7 = 21{\cdot}7$ lb/in.^{2} Thus the supercharger increases the pressure of the charge from $11{\cdot}3$ lb/in.^{2} to $21{\cdot}7$ lb/in.^{2}, and the pressure ratio is $\frac{21{\cdot}7}{11{\cdot}3}$ or $1{\cdot}9$. This ratio is a function of r.p.m.; hence at the same r.p.m., *at sea-level*, the outlet pressure will be $14{\cdot}7 \times 1{\cdot}9 = 27{\cdot}9$ lb/in.^{2}, i.e. the resulting boost pressure is $27{\cdot}9 - 14{\cdot}7 = +\ 13{\cdot}2$ lb, which would soon cause serious trouble.

On reaching the *full-throttle height* for this particular boost, and r.p.m., the power will then start to fall as height is further increased, since the manifold pressure starts to drop.

A simple test to check whether the engine is actually at full throttle or not is as follows:

When the aircraft is in steady flight with given throttle and r.p.m. settings, pull the r.p.m. lever slowly back and watch the boost gauge. If the boost falls, the engine is at full throttle; if it remains constant, the engine is throttled. In the latter event, the drop in r.p.m. before the boost begins to fall is a measure of the degree of throttling at the original setting.

3.2. Engine Power Charts

3.2.1. Rudimentary Charts. The foregoing remarks can be illustrated by considering Fig. 13, which shows a rudimentary engine power chart.

Considering first of all the sea-level diagram on the left, the vertical scale on the right-hand side shows the b.h.p. obtainable by various combinations of r.p.m. and boost. It appears, for instance, that 1 600 r.p.m. and 34 in. m.a.p. is about equivalent to 2 200 r.p.m. and 26 in. m.a.p., and it is obviously necessary to decide which pair of settings is preferable.

This point is settled in a later paragraph of this chapter, where it is shown that low r.p.m. and high boost is the best combination, provided that the r.p.m. is above a certain minimum below which unpleasant vibrations may occur.

Turning now to the right-hand part of the engine power chart in Fig. 13, it appears, for instance, that the full-throttle height for the 1 600 r.p.m., 32 in. m.a.p. combination is 5 000 ft, and that at greater height the power will fall unless the r.p.m. is increased.

The slope of all the lines giving the increase of power, for a particular r.p.m. and m.a.p., from sea-level to the full-throttle height, is roughly constant, as is clearly shown by the slopes of the dotted lines. For example, the dotted line joining 2 000 r.p.m. and 32 in. m.a.p. on the sea-level scale to the intersection of the same settings on the altitude scale shows a gradual increase of power from 620 b.h.p. at sea-level to 720 b.h.p. at 12 000 ft.

Note that the height to be used is the pressure height.

3.2.2. ENGINE POWER CHARTS. Figs. 14, 15, and 16 are comprehensive power charts for the "Hercules 100, 101, 110," series for "M" and "S" supercharger ratios.

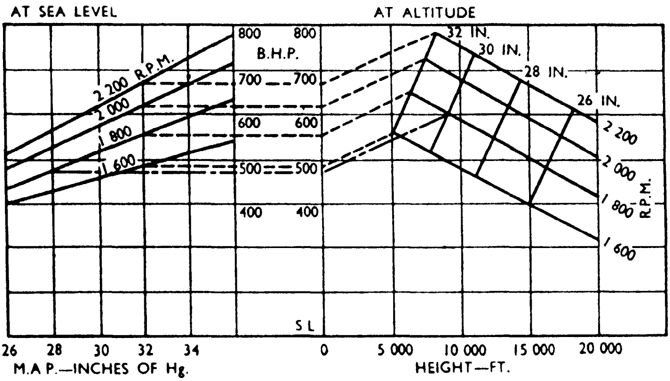


FIG. 13. RUDIMENTARY POWER CHART

These are typical of power charts issued by engine manufacturers, and enable the relation between boost, revs., and engine power for a given altitude to be obtained over the full power range of the engine. In order to understand how it is built up, and how it is used, we shall consider one line on Fig. 14, the one labelled 2 400 r.p.m. + 6 lb/in.² This shows that if the engine is run at 2 400 r.p.m. at sea-level and throttled to give + 6 lb/in.² boost, it will give 1 420 b.h.p., and as altitude is increased at 2 400 r.p.m., the throttle being progressively opened to maintain + 6 lb/in.² boost, the b.h.p. will increase to 1 510 at 7 800 ft where the throttle will be fully open. At altitudes beyond this the line shows how the b.h.p. will fall off, and the lines crossing it indicate the values of the decreasing boost.

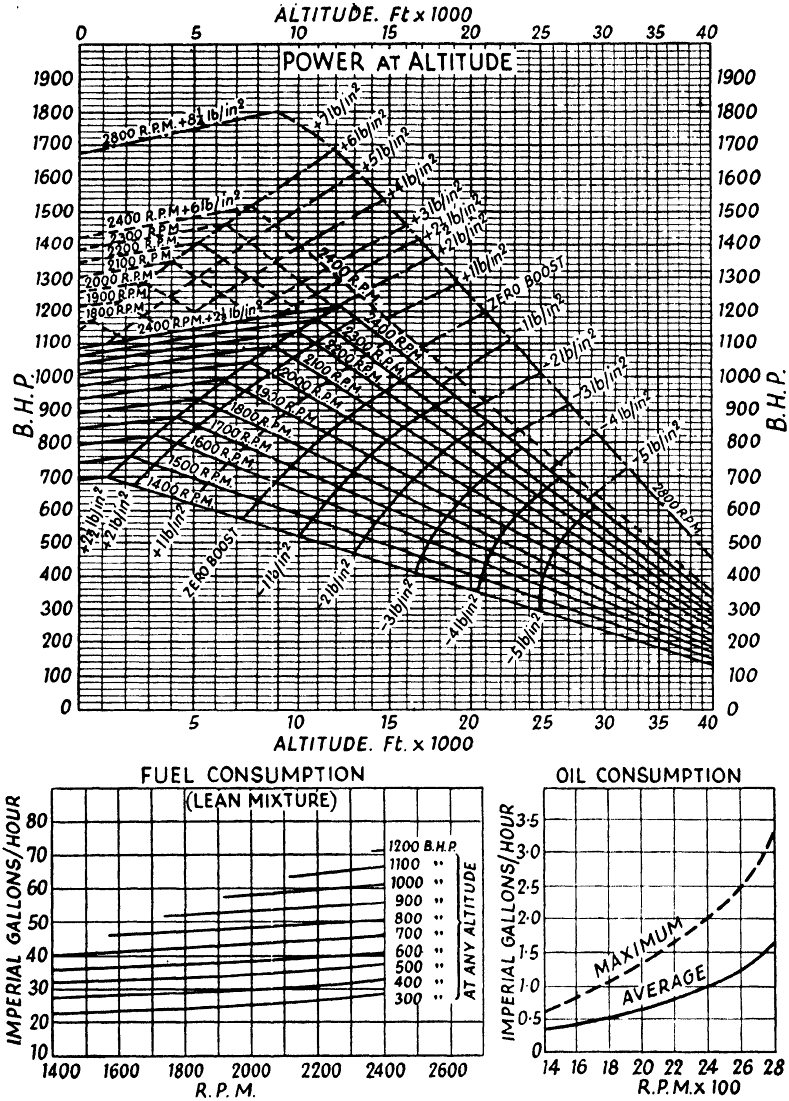


Fig. 14. Power Chart

Performance curves for "Hercules 100" in "M" gear.
D.E.D. 2 000 corrections; open exhaust and without ram; standard temperature and pressure conditions; fuel—100 octane, 130 grade.
Maximum power for—

- ——— - ——— Take-off—5-minute limit.
- - — - — ——— Combat flight—5-minute limit.
- — — — — — Normal climb. Emergency cruising—1-hour limit. Continuous cruising—rich mixture.
- —————— Continuous cruising—lean mixture.

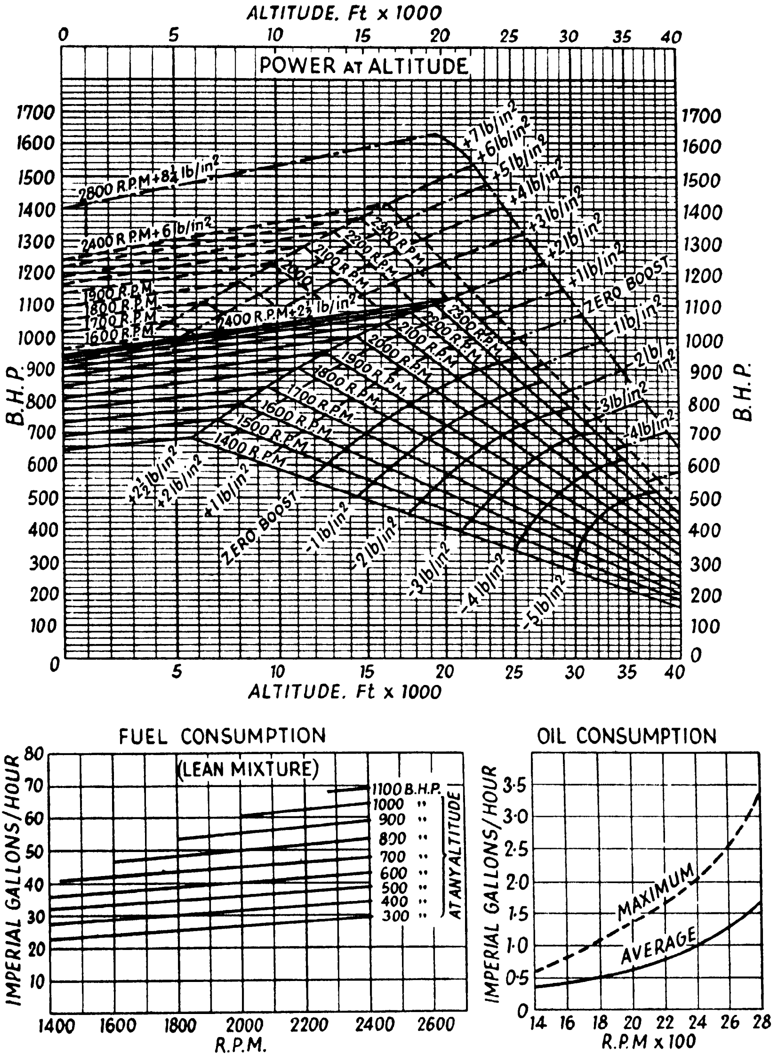


FIG. 15. POWER CHART

Performance curves for "Hercules 100" in "S" ratio.
D.E.D. 2 000 corrections; open exhaust and without ram; standard temperature and pressure conditions; fuel—100 octane, 130 grade.
Maximum power for—

- Combat flight—5-minute limit.
- Normal climb.
- Emergency cruising—1-hour limit.
- Continuous cruising—rich mixture.
- Continuous cruising—lean mixture.

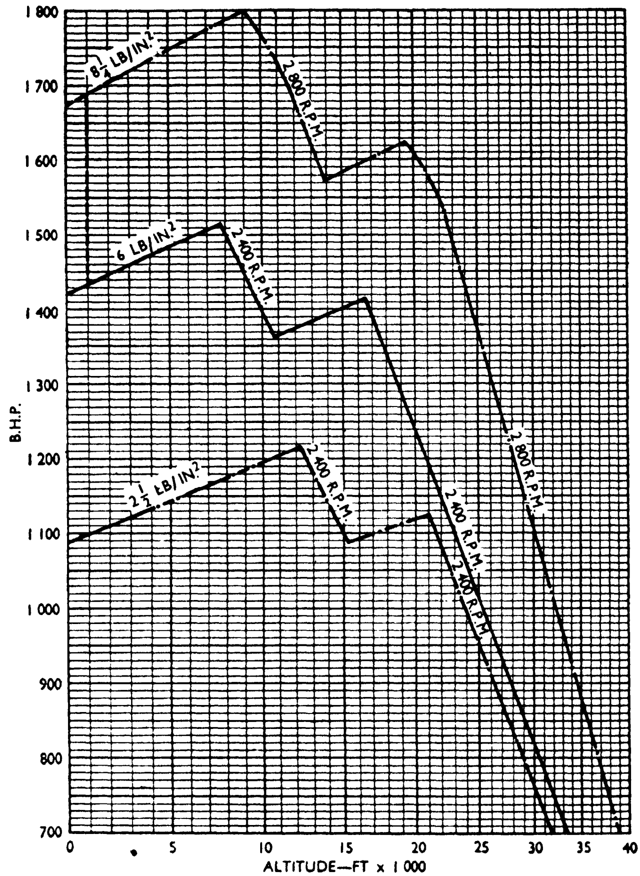


FIG. 16. POWER CHART

Power at various altitudes for "Hercules 100, 101."
D.E.D. 2 000 corrections; open exhaust and without ram; standard temperature and pressure conditions; fuel—100 octane, 130 grade.
Maximum power for—

——— - ——— Take-off—5-minute limit.
— — - —— — Combat flight—5-minute limit.
— -- — — — { Normal climb. Emergency cruising—1-hour limit. Continuous cruising—rich mixture.
————— Continuous cruising—lean mixture.

We can find, for instance, the power at 5 000 ft pressure altitude given by 2 800 r.p.m. and + 6 lb/in.^{2} boost. The sea-level chart tells us that 1 540 b.p.h. are obtained at sea-level, and the top line of the altitude chart shows that at 2 800 r.p.m. full throttle, 6 lb/in.^{2} boost is obtained at 12 000 ft, giving 1 670 b.h.p. Joining these two points will give the 2 800 r.p.m. + 6 lb/in.^{2} line, and where this cuts 5 000 ft we read off that 1 600 b.h.p. is obtained. However, this line is not shown on the chart as for this engine 2 400 r.p.m. and 6 lb/in.^{2} boost are the maxima permitted for continuous use in rich mixture. The line we considered and the series below it refer to continuous cruising at r.p.m. of 2 400 and below, at the maximum permitted boost, rich mixture.

It is not possible to read off the power obtained at say 2 600 r.p.m. + 4 lb/in.^{2} boost as this setting would not, or at least should not, be used, for a reduction in power would be made by decreasing r.p.m., not boost, in order to maintain the maximum b.m.e.p., and hence the best fuel consumption.

The top line on the altitude chart gives the maximum power obtainable from the engine, permitted for short periods only, at the maxima of 2 800 r.p.m. and + 8 lb/in.^{2} boost. This is normally used for take-off only.

The bottom series of curves relates to continuous cruising in weak mixture, starting with the permitted maxima of 2 400 r.p.m. + $2\frac{1}{2}$ lb/in.^{2} boost. This is the series of curves in which we are mainly interested for cruising purposes, and it will be noted that the sea-level chart gives the power obtainable at various r.p.m. over the whole range of boost pressures. This enables us to read off the power given by boost and rev. combinations not given directly on the altitude chart. This is done by drawing in the constant boost line, as in the previous example, by joining the point given by the sea-level power to the full-throttle point for the particular revs. and boost and noting where it cuts the required altitude.

For cruising purposes, then, we could extract this part of both charts, but it can be further simplified by omitting the sea-level chart; for we should not need to use settings other than those given directly by the altitude chart, as they would be less economical of fuel than the latter. For engines fitted with automatic boost control, the power given by the chart can be and r.p.m.; and is evidently somewhat similar in the previous diagram (Fig. 20) but upside down, since a.m.p.g. $\propto \frac{1}{c_f}$, and other quantities.

Suppose the aircraft is at its cruising height, and that 1 800 is the minimum r.p.m. permitted. Starting from the point *D* where the speed is least, an increase in speed is best obtained by keeping the minimum r.p.m. and increasing the boost, as shown by the line *DC*. At *C* we reach a point where, for the particular height in question, full-throttle conditions come into

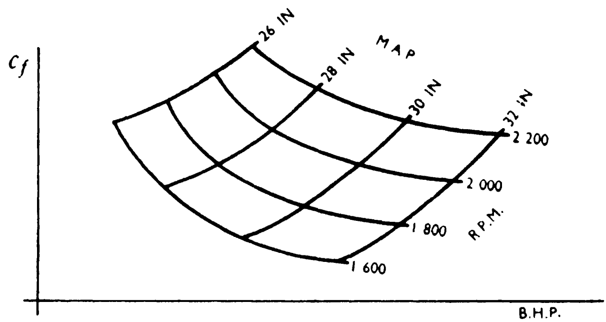


FIG. 20. SPECIFIC CONSUMPTION VERSUS B.H.P. AT DIFFERING R.P.M. AND BOOST

operation, and for a further increase in power (i.e. speed) it will be necessary to increase both r.p.m. and boost. (This necessity is easily seen from studying the altitude part of the engine power chart in Fig. 13.) At *B* we reach the maximum boost obtainable, and a further increase in speed necessitates increases in r.p.m. along the line *BA*.

We thus see the reason for the angles at *C* and *B*. Note that at a lower height we might be able to obtain + 2 boost at 1 800 r.p.m.; whereas at a considerably greater height, the whole of the a.m.p.g. line might be at full throttle.

If we now turn to the Cruising Data in Figs. 10 and 11, we see how—as already described—the specific consumption gets less as boost is increased at constant r.p.m., and how it remains constant at full-throttle conditions until maximum boost is reached, whereupon specific consumption again increases. We thus see the reason for c_f being least at a considerably higher speed than that for minimum drag, leading to the

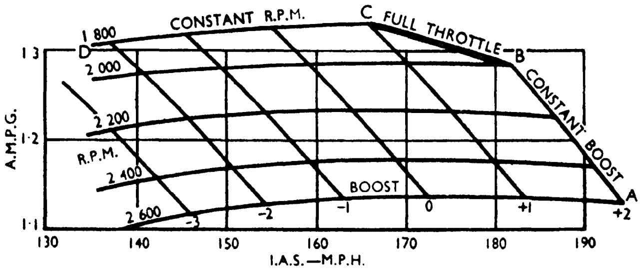


FIG. 21. A.M.P.G. GRID

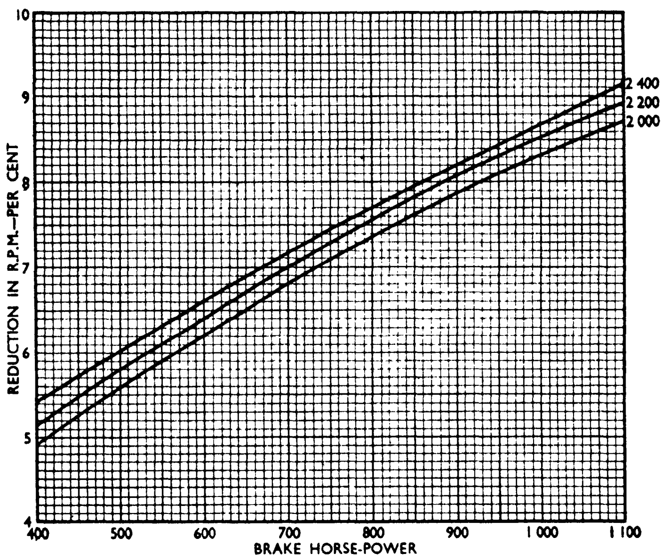


FIG. 22. CHOICE OF SUPERCHARGER GEAR FOR "HERCULES 100"

Minimum percentage reduction in engine r.p.m. for an improvement in fuel economy to occur on changing from "M" to "S" gear at constant power.

maximum range speed being above the minimum drag speed as described in the last chapter.

Fig. 21 also illustrates another important point. If it was possible to produce the line *BC* backwards to the left, considerably higher a.m.p.g. would be attained. But r.p.m. limitations prevent this, and as a result it is not possible to run the engine at full-throttle conditions at speeds in the neighbourhood of the minimum drag speed. This effect is often noticeable at the lower heights, and explains why the optimum

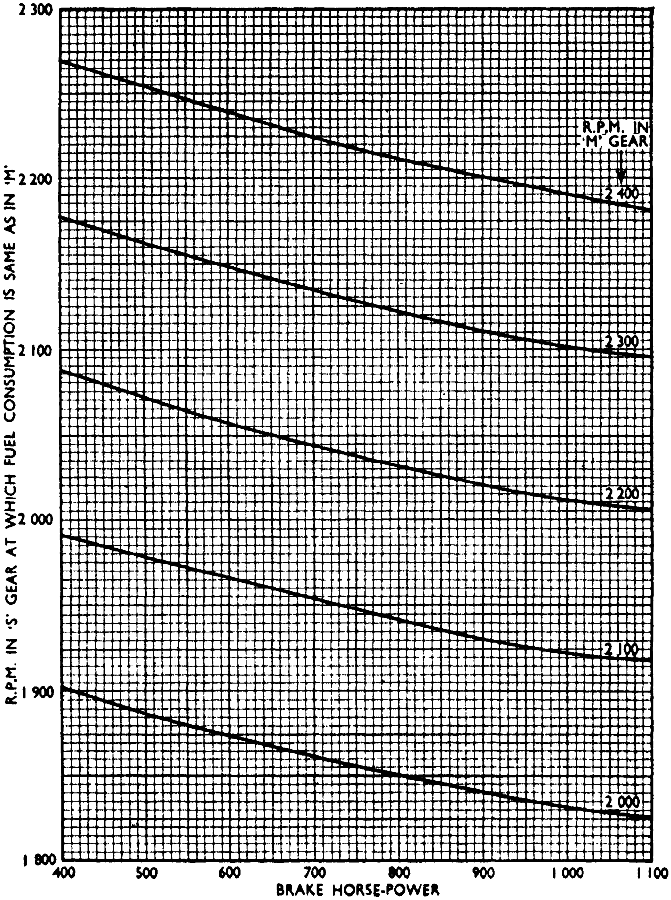


FIG. 23. CHOICE OF SUPERCHARGER GEAR FOR "HERCULES 100"
"S" gear r.p.m. below which fuel consumption is reduced.

height is usually the lowest height at which the minimum drag speed corresponds to full-throttle conditions for the engine; or rather, corresponds to full-throttle conditions at minimum r.p.m.

3.4. Two-speed Superchargers

In order to attain the increased boost desirable at considerable height, many engines are fitted with two-speed superchargers. The use of a higher gear allows a given speed to be maintained at higher boosts and lower r.p.m., which tends to improve the specific fuel consumption. But the use of high gear, at given boost and r.p.m., tends by itself to make specific fuel consumption worse, mainly on account of the increased power required to drive the supercharger. The two effects therefore oppose each other, and only tests can decide which will have the greater effect. In this connexion it is interesting to study Figs. 22 and 23 which show under what conditions, for the "Hercules 100" engine, the use of the higher "S" gear is more economical than that of the lower "M" gear.

3.5. B.M.E.P.

On a previous page we gave the fundamental formula

$$\text{i.h.p.} = \frac{PALEC}{33\,000}$$

This can be written as

$$\text{b.h.p.} = \text{b.m.e.p.} \times \text{r.p.m.} \times \text{a constant}$$

where b.m.e.p. is a measure known as the *brake mean effective pressure*, of the pressure inside the cylinders corresponding to a certain b.h.p. If we write the above result as

$$\text{b.m.e.p.} = k \times \frac{\text{b.h.p.}}{\text{r.p.m.}} \quad \text{(where } k \text{ depends on the particular engine)}$$

certain consequences follow.

It is undesirable for the b.m.e.p., using weak mixture, to be above a certain value, except for short periods of time, and so there is an upper limit to allowable b.h.p. at maximum r.p.m. and a *lower limit* to r.p.m. at a fixed b.h.p.

If we decide to operate at a fixed maximum b.m.e.p., we are led to some modification of the "low r.p.m.—high boost" rule. This is evident from the specific fuel consumption grid of Fig. 24, on which constant b.m.e.p. lines have been drawn; or from a study of the tabulation shown on page 63.

For a fixed b.m.e.p. there is thus a particular r.p.m., not always the lowest, which gives minimum specific consumption; and we shall lose in engine efficiency, and may lose in b.h.p., if we do not operate at higher r.p.m. The torquemeter reading, on those aircraft fitted with torquemeters, gives a measure of the b.m.e.p.; and if we operate at the maximum b.m.e.p., and

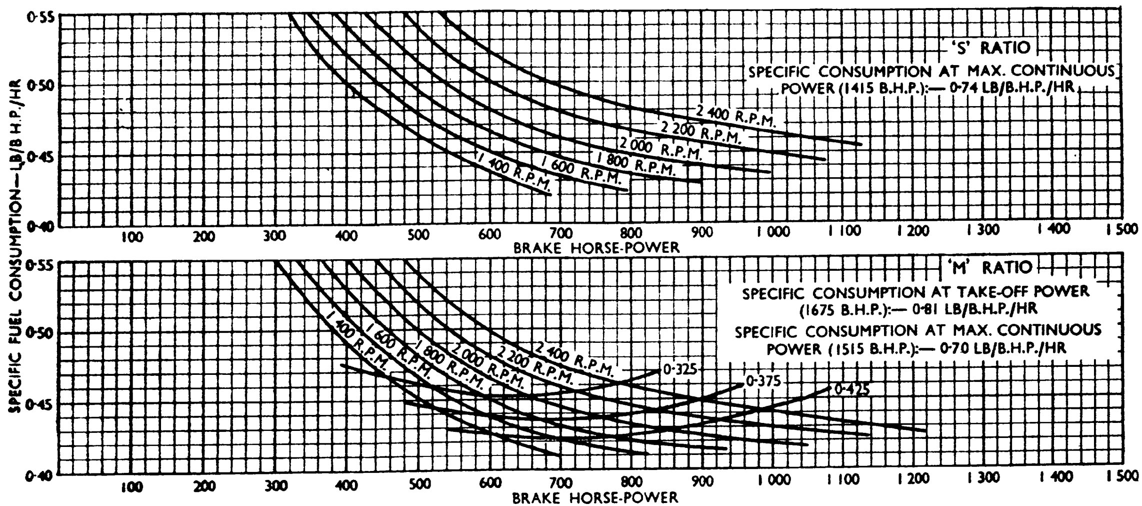


Fig. 24. Specific Fuel Consumption Grid for "Hercules 100"

Standard temperature and pressure conditions; fuel—100 octane, 130 grade. Specific fuel consumptions apply for any altitude and boost with the lean mixture range.

choose the lowest r.p.m. to give the desired b.h.p., then we can be fairly certain of attaining minimum c_f.

(c_f is measured in lb/b.h.p./hr.)

$\frac{\text{b.m.e.p.}}{k}$	r.p.m.	1 400	1 600	1 800	2 000	2 200	2 400
0·375	b.h.p.	525	600	675	750	825	900
	c_f	0·445	0·438	(0·436)	0·437	0·443	0·45
0·425	b.h.p.	595	680	765	850	935	1 020
	c_f	0·426	0·424	(0·424)	0·429	0·434	0·44
0·435	b.h.p.	455	520	585	650	715	780
	c_f	0·467	0·460	0·454	(0·453)	0·456	0·461

3.6. Propeller Efficiency

The third factor in the a.m.p.g. formula is η, the propeller efficiency. As the constant speed propeller is designed to allow optimum r.p.m., the variations in η should not be large; say, from 0·75 on the climb to 0·86 cruising at full throttle and the higher speeds. Propellers are unsuitable for very high speeds, owing to the loss of efficiency due to compressibility effects at the tips, and it is questionable whether the goal of 0·9 is now worth working for.

Propeller manufacturers supply elaborate methods for estimating the efficiency under any combination of speed, r.p.m. and b.h.p., and these are useful in forming preliminary estimates of performance. These should, however, be carefully checked in flight, by the use of a thrustmeter. A typical variation of η with speed, at constant height, is shown in Figs. 10 and 11.

CHAPTER IV

Gas-turbine Engines

4.1. General

A TURBINE, driven by the exhaust gases of a piston engine and connected to a blower, in order to maintain the sea-level power of the engine at altitude, was originally conceived by Rateau. This idea was carried a step further, to the practical flight stage, by the R.A.E., Farnborough, and then abandoned. But Wright Field, with General Electric, in the U.S.A., brought it to a high state of development and it is now fitted to a large number of American aircraft.

The advent of the "Whittle" turbine/jet unit has carried the turbine forward to a position where, after relatively few years of intensive development, it has virtually taken the place of the piston engine in pursuit aircraft and now threatens the piston engine in all other fields of aviation, with the possible, but only temporary, exception of the requirement for fuel economy.

The "Whittle" is an internal combustion engine and it drives its own blower which discharges to combustion chambers located, integrally, between blower and turbine.

The gas turbine, like the piston engine, is basically an air pump, but, whereas the piston engine has a timed or controlled respiratory cycle, with its inherent mechanical limitations, the gas turbine does not suffer these restrictions. It works on a continuous (untimed) combustion cycle and since its mechanical system is rotational it can deal with vastly more air than the piston engine.

There are certain disadvantages which prevent it from taking the place of the piston engine completely. The operating temperatures of the latter are much higher than those at present possible with the gas turbine. The maximum working temperature within the piston engine, according to compression ratio and mixture strength, etc., varies from about 2 500°C to 3 000°C absolute. Further, the working fluid of air and hydro-carbon fuel is not diluted with the excess air, which is necessary in the case of the gas turbine where, at present, a temperature of 1 100°C absolute, at the turbine entry, cannot be exceeded, due to material limitations.

It will be appreciated, therefore, that these temperature limitations prevent the gas turbine from attaining the thermal efficiency of the piston engine. But this will not necessarily be so in future. In any case, the gas turbine has other advantages.

The turbine/jet engine is considerably lighter than the piston engine of equivalent power, both in regard to its "dry" and "installed" weights. It does not need any coolant because it is internally cooled by its own excess air. Present gas turbines run on ball and roller bearings so that a large quantity of oil is unnecessary and oil cooling is not required.

The specific weight of a simple turbine/jet engine is about 0·3 lb per lb of static thrust. Propeller/turbines have somewhat higher weight, between 0·75 lb and 0·90 lb per equivalent shaft horse-power.

Future turbines, both of the "jet" and also the propeller or power types, will tend to become more complex when designed for high fuel economy, and a specific weight of 1 lb to 1·5 lb per equivalent shaft horse-power is to be expected in the latter (propeller) case. These figures are related to maximum power at sea-level.

4.2. Types of Gas Turbine

Gas turbines may be divided into two general categories.

(1) Turbine/jets, or "pure jets."

(2) Propeller or power turbines, or "prop. jets."

In the case of (1) there is, naturally, the "Whittle" type which consists of a single-stage, double-sided, centrifugal compressor and single-stage turbine. One variant of this is the De Havilland "Goblin," which has a single-stage and single-sided compressor.

The performance curves in Figs. 25 and 26 show the results obtained at one stage in their development; today the figures are higher.

The Armstrong-Siddeley "Python" has an axial flow compressor directly coupled to a two-stage turbine. The propeller is driven through a double-reduction gear, at the front end.

The pressure ratio of the compressor is 5/1, and the propeller shaft, plus 1 150 lb thrust (static) at the jet pipe. The operating speed is 8 000 r.p.m. The weight is 3 140 lb, without propeller.

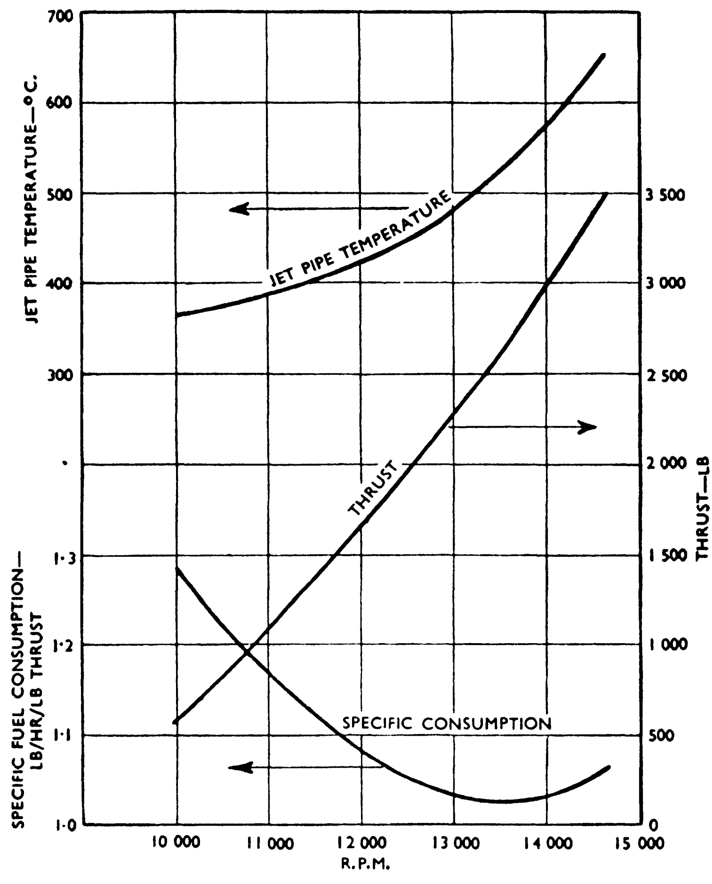


FIG. 25. PERFORMANCE CURVE FOR "DERWENT V" ENGINE

The Bristol "Theseus" has an axial flow and a centrifugal compressor, in series. The centrifugal compressor is located between the axial compressor and the combustion chambers. The theoretical advantages claimed for this arrangement are that the centrifugal compressor assists the axial compressor over its relatively narrow operating range, particularly during the starting of the engine, thus avoiding instability and promoting flexibility of operation. There is one important advantage in the location of the centrifugal compressor. The air from the discharge of the axial compressor must be turned sharply

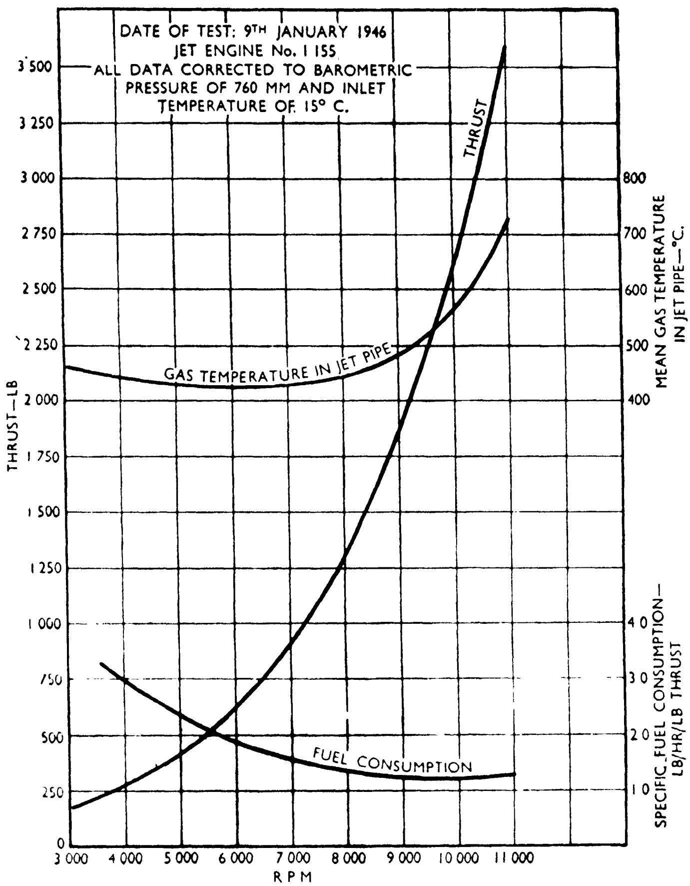


FIG. 26. PERFORMANCE CURVE FOR "GOBLIN" ENGINE

when separate combustion chambers are used; if a centrifugal compressor is considered necessary for other reasons it is a very convenient substitute for sharp pipe bends and also provides an increase in total pressure ratio.

Both the axial and centrifugal compressors of the "Theseus" are driven by a two-stage turbine. There is another turbine wheel fitted to a shaft which passes through the hollow shaft of the two-stage turbine and compressors. This turbine wheel independently drives the propeller through double reduction gearing.

In the particular case of the "Theseus," by the mechanical separation of the propeller turbine from the compressor, some flexibility is obtained, such as, for example, overspeeding the propeller for take-off. This flexibility also enables the pilot to use a lower propeller speed when landing and taxying.

The separation of the propeller from the compressors is also a help in reducing the power required for starting, because no power is absorbed in turning the propeller.

The "Theseus" engine has been designed to incorporate a heat exchanger for improved fuel economy. A reduction to the order of 10 per cent in specific fuel consumption will be achieved at small expense in power, but appreciable increase in weight. The compressors (axial and centrifugal) have an overall pressure ratio of 4·35/1. The shaft horse-power, at the propeller, was 1 950 h.p. with 500 lb thrust (static) in the jet pipe according to early tests. The engine speed is 9 000 r.p.m. and its weight is 2 500 lb, without propeller.

In the Rolls-Royce "Clyde" the first turbine wheel drives the centrifugal compressor and the second turbine drives the axial compressor and, also, the propeller, through double-reduction gearing. The arrangement is for contra-rotating propellers.

The power is 3 000 h.p. at the propeller shaft, with 1 200 lb thrust (static) in the jet pipe. The engine speed is 6 000 r.p.m. The "dry" weight, without propeller, is about 2 500 lb.

4.3. The Constant Pressure Cycle

The gas turbine works on the "constant pressure" thermodynamic cycle. This can be demonstrated more clearly by a temperature/entropy diagram, which is shown in Fig. 27.

Referring to the diagram; the full line (0, 1′, 2′, 3′, 4′, 5′) represents the theoretical cycle, assuming no losses. Atmospheric air, in a condition represented by point 0, is raised in temperature and pressure to conditions corresponding to point 1′ by the effect of "ram." Point 2′ is reached after the air has passed through the compressor. From 2′ to 3′ heat is added at constant pressure, by an amount proportional to the temperature difference, C. The gas is then expanded from 3′ to 5′ where the pressure is, again, atmospheric.

The energy available in the expansion is proportional to the temperature drop from 3′ to 5′. A proportion of this energy 3′ to 4′, equal to 1′ to 2′, is required to drive the compressor. And the remaining energy 4′ to 5′ proportional to A, may be used, in jet form, to propel the aircraft, or some may be converted into shaft power by means of a turbine.

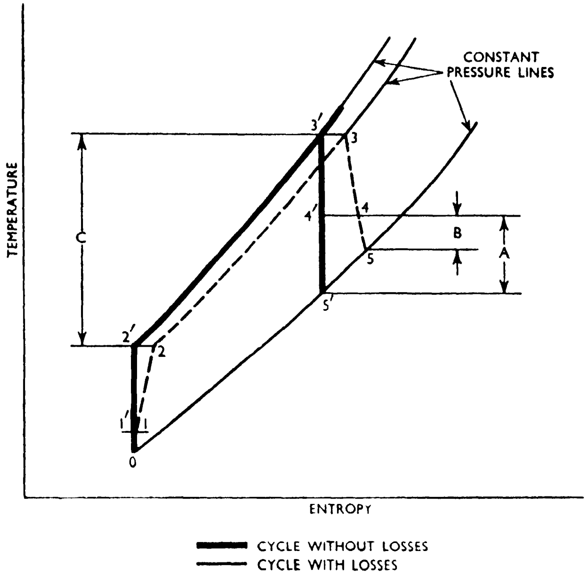


FIG. 27. ENTROPY DIAGRAM FOR GAS TURBINE

The actual cycle, with losses taken into account, is shown by the dotted line (0, 1, 2, 3, 4, 5). With the same input of work into the compressor, the resultant pressure at 2 is lower than 2′. Because of this lower pressure ratio and, also due to the losses during expansion, the available energy in expansion, 3 to 5, is much smaller.

The work required to drive the compressor is, however, unaffected and, consequently, the useful net energy available, 4 to 5, proportional to B, is much reduced. The thermal input for both the cycles shown is proportional to C. Therefore, the thermodynamic efficiency of the cycle, with no losses, is A/C and, with losses, is B/C.

The diagram (Fig. 27) shows the great importance of achieving low losses in the compressor, turbine and combustion chamber to obtain good efficiency in the gas turbine as a whole.

4.4. The Effect of Pressure Ratio on Thrust and S.F.C.

The temperature/entropy diagram may be used to illustrate the effect of increasing compression ratio and gas temperature

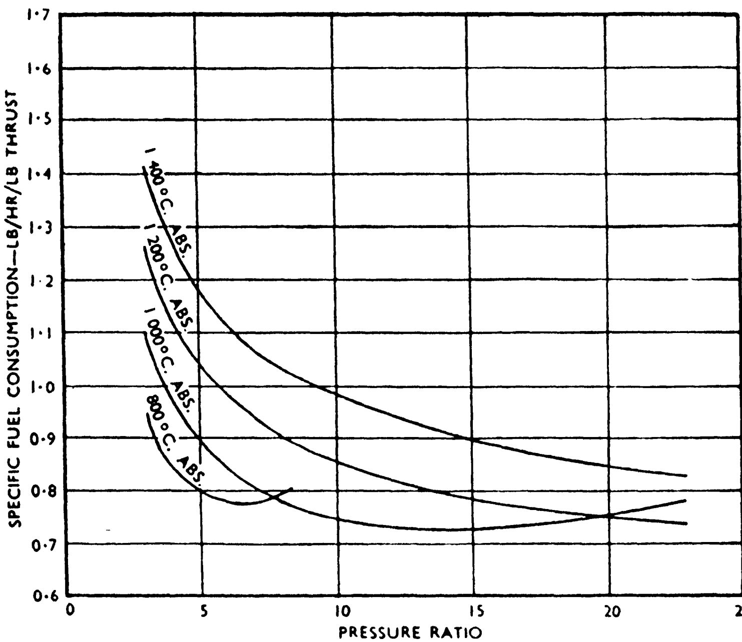


FIG. 28. SPECIFIC FUEL CONSUMPTION VERSUS PRESSURE RATIO: JET ENGINE

at the inlet to the turbine. Figs. 28, 29, 30 and 31 show some results of calculations of these effects.

Output per unit rate of air flow and specific fuel consumption are shown on these figures, for simple jet and also propeller turbines. They are drawn for sea-level static conditions. In the case of the propeller turbine, it will be seen that, as the turbine inlet temperature is increased, the specific consumption decreases, and the specific output rises. As the pressure ratio is increased, at a given turbine inlet temperature, the specific consumption passes through a minimum and the specific output through a maximum. But it will be noted that these two optimum points do not occur at the same pressure ratio. This is particularly marked in the curves for 800°C absolute and 1 000°C absolute, and is due to the increasing effect of losses in compression and expansion which offset any thermodynamic gain.

The effect of pressure ratio and temperature on the performance of simple jet engines, shown in Figs. 28 and 29, is com-

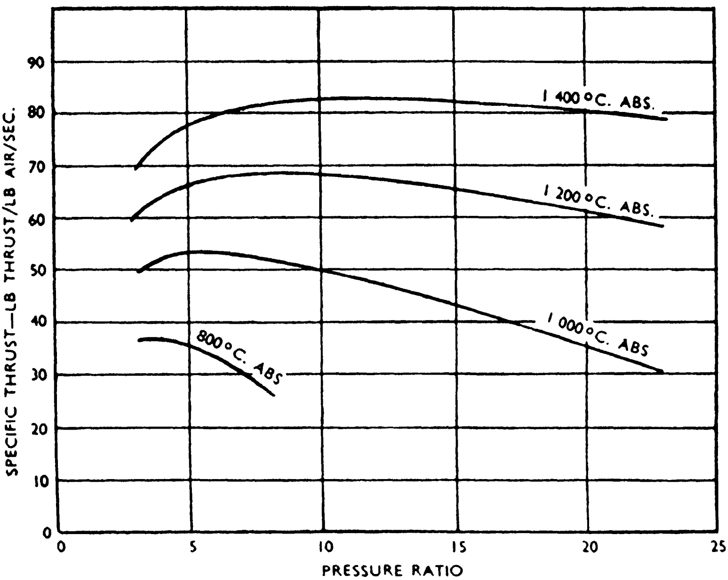


FIG. 29. SPECIFIC THRUST VERSUS PRESSURE RATIO: JET ENGINE

plicated by the additional factor of propulsive efficiency. The specific thrust varies in approximately the same way as the specific power output of propeller turbines. The specific consumption, however, generally rises as the temperature increases. This is the result of decreases in propulsive efficiency, which occur when the jet velocity is increased.

At any given temperature there is an optimum pressure ratio, which is shown most definitely on the curves for 800°C absolute. But, for temperatures now used, this occurs at a large value of the pressure ratio, which is above that giving the maximum thrust.

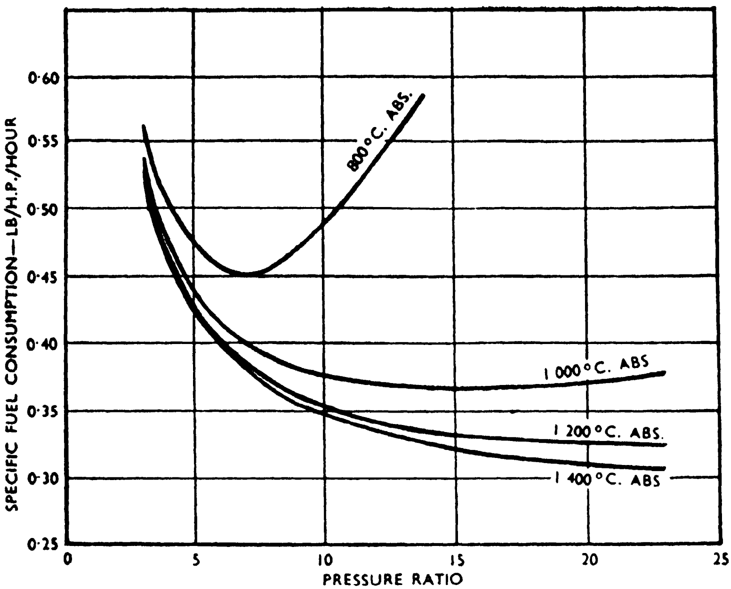


FIG. 30. SPECIFIC FUEL CONSUMPTION VERSUS PRESSURE RATIO: PROP. JET

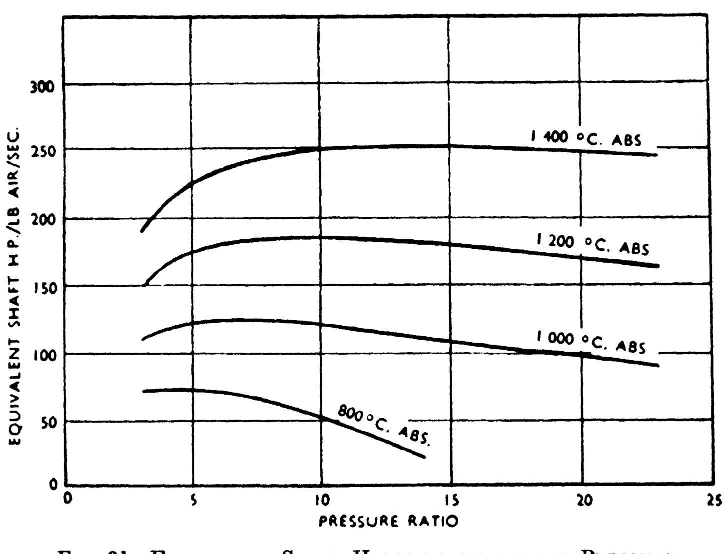


FIG. 31. EQUIVALENT SHAFT HORSE-POWER VERSUS PRESSURE RATIO: PROP. JET

Up to the present time, we have been using pressure ratios between 4/1 and 6/1 for simple jet engines and, although higher pressure ratios are desirable, for thermodynamic reasons, the increased weight and greater complication involved may largely counterbalance any improvement in thermodynamic efficiency.

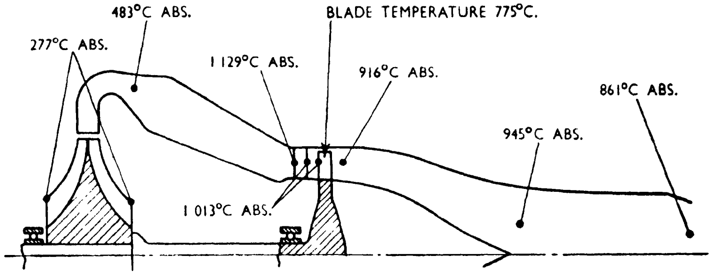


FIG. 32. TEMPERATURES IN A GAS-TURBINE ENGINE

4.5. Working Conditions

It will, perhaps, be of interest to give some idea of the working conditions inside a simple jet engine of the "Whittle" type, when running at normal maximum speed. In order to

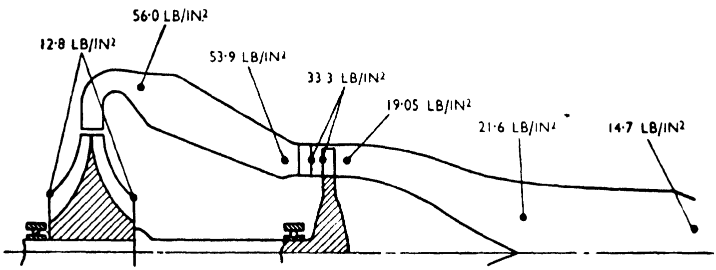


FIG. 33. PRESSURES IN A GAS-TURBINE ENGINE

make this as clear as possible, Figs. 32, 33 and 34 show a section, through one combustion chamber, of this type of engine, giving the particular conditions existing throughout its system when it is running. Fig. 32 shows the temperature conditions, from the compressor inlet to the jet pipe. Fig. 33 gives the pressure distribution and Fig. 34 the Mach number at the various points.

The air consumption of the gas-turbine/jet engine is enormous and the accepted figure for normal assessment is 1 lb of air per second for every 50 lb of thrust. This means that the "Derwent V" engine, giving 4 000 lb thrust, consumes about 2 tons of air per minute, or over 120 tons per hour. With compressors of higher efficiency and when turbines can work at higher gas temperatures, we should be able to obtain 60 lb of thrust per lb of air per second.

As mentioned earlier the gas turbine cannot operate at the relatively high temperatures of the normal piston engine. This is due to the present limitations of material and cooling. Except in the primary zone, the overall air/fuel ratio is very high and

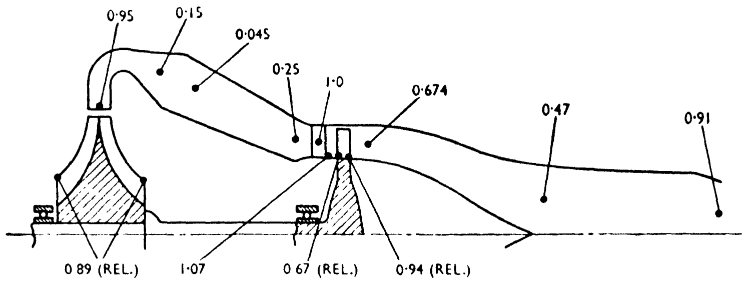


FIG. 34. MACH NUMBERS IN A GAS-TURBINE ENGINE

in the region of 60–70/1, compared with that for the piston engine, which approaches the value for chemically correct combustion, of about 15/1.

It will be understood that in the case of the simple jet engine, the turbine wheel is employed only to drive the compressor for pumping the large mass of air, which, after the addition of heat in the form of fuel, is ejected through the propelling nozzle at high speed—approximately 1 600 ft/sec. The reaction of the hot gases provides the propulsive effort.

4.6. The Propeller Turbine

4.6.1. GENERAL PROPERTIES. The propeller or power turbine is designed so that more of the power, or gas energy, is absorbed by the turbine wheel, or wheels, than is the case with the jet turbine. This provides power, both for the compressor and the propeller. But less jet energy is then available and the gases leave the propelling nozzle at reduced velocity.

4.6.2. DIVISION OF POWER. Propeller turbines can be designed to meet a fairly wide range of aircraft operating conditions, and the power at the propeller shaft and in the form of jet energy can be proportioned accordingly; for instance, 90 per cent shaft horse-power and 10 per cent jet thrust, or 75 per cent shaft horse-power and 25 per cent jet thrust, etc.

4.6.3. Saving of Heating Losses. Anyone who has seen a jet engine operate knows how much heat and velocity energy is imparted to the jet. All of this heat energy is wasted, and even at present-day World Record speeds nearly half of the velocity energy is wasted also. At present-day transport speeds the fraction is more than two-thirds of the velocity energy. To increase the speeds of civil transports may be an obvious answer, but this may not be so easy as might be imagined. The inefficiency of a jet engine under such conditions, is, therefore, obvious, and hence it does not comply with the second main requirement of civil aviation, namely, economy of operation.

Three main techniques are known at present whereby the exhaust-heat loss can be minimized: (*a*) the use of the heat exchanger, either recuperator or regenerator; (*b*) the use of a high turbine expansion ratio entailing a correspondingly high compression ratio; (*c*) the closed cycle systems, which permit alteration of the working medium. To these should possibly be added that most interesting device, the Brown-Boveri "Comprex" system. There is considerable doubt as to the practicability of the closed cycle in aviation, necessitating, as it does, not one but several heat exchangers. The choice, therefore, rests between the use of a heat exchanger and high compression.

4.6.4. Heat Exchangers. Two fundamental features, weight and volume, impose restrictions on the design and utilization of the heat exchanger, which makes its practical realization more difficult and a little less attractive. The additional weight of such an assembly must be more than offset by a reduction of fuel consumed to show any economy, and that a time factor is thereby introduced. This time of flight, which is a function, therefore, of the heat-exchanger weight and efficiency, will determine the usefulness and application of the turbine-heat-exchanger combination.

The weight permissible for the heat exchanger must, therefore, be the absolute minimum, since the efficiency is limited by other considerations. Likewise, the other restricting feature of volume is also difficult to define simply, but, nevertheless, is very real to the designer, both in the geometric layout of the design and its subsequent influence on the power plant performance.

Practical heat exchangers will cause a certain restriction to both the "charge air" and the "exhaust gas" flow, the results of which will, to a certain extent, offset their ideal advantages. So far only the case of a fixed adiabatic efficiency of 85 per cent for the turbine and 88 per cent stage efficiency for the compressor has been considered, these figures being a fairly reasonable top limit at the present state of the art when operating at the design conditions. The efficiency of a gas turbine plant is dependent on the ratio of peak permissible gas temperature to atmospheric temperature. The case so far considered is where the air temperature is 15°C and the peak temperature 800°C. It will be noted that the efficiency is materially improved either with increased maximum temperature or reduced air temperature.

4.6.5. Pressure Ratio. When designing for the altitude condition it is necessary to compromise to a certain extent between optimum performance and optimum fuel consumption. The present-day permissible gas temperatures are of the order of 800°C, and for this value the optimum compression ratios are in the region of 6·5 : 1 and 9·0 : 1, the compromise value being in the neighbourhood of 8·5 : 1 having regard to altitude conditions.

4.6.6. Discharge Velocity. The fact that most of the work in the gases is taken out in the turbine in order to provide power for the compressor and propeller means that the temperature drop across the jet nozzle is of a very low order, resulting in a relatively low discharge velocity. This is by no means a disadvantage, since, for the aircraft speeds at which the propeller turbine is designed to operate, a low discharge velocity is required in order to obtain the maximum Froude efficiency at this speed.

As the hot gases pass through the turbine wheels their temperature falls in proportion to the work done. Since all the blades could presumably withstand the same temperatures, it has been suggested that if some form of reheating between stages could be resorted to, either an improved efficiency or power output would be possible. When a separate turbine wheel to drive the propeller is used, the obvious place for re-heating would be between the compressor and propeller-turbine wheels.

4.6.7. DESIGN CONSIDERATIONS. The need for avoiding bends in the air flow after compression is not so acute, as the high pressure air has to be slowed down considerably before entering combustion chambers of present-day types. In the case of engines fitted with heat exchangers, a further deceleration of the flow is inevitable, and it is fortunate that this reduces to a very small order the losses consequent upon "looping" this flow, which is unavoidable in an engine fitted with a heat exchanger. This "looping" may also be adopted where it is desired to shorten the engine, as a straight-through turbine, whilst of very small diameter, tends to considerable length.

Another very important factor controlling the design layout is the power at take-off; the propeller may obtain its power either directly from the compressor/turbine system or from a free power turbine, which derives its energy from the exhaust gases of a separate compressor/turbine gas generator, of which Bristol turbines were at one time the only aeronautical exponents.

A further question to be settled in relation to the power transmission is that of propeller location. Although no pusher turbine installations are known as yet, the gas turbine lends itself very conveniently to pusher propeller installation, with, however, a most important reservation on the score of vibration.

4.6.8. NEED FOR AXIAL COMPRESSORS. It will be noted that, with the exception of the Rolls-Royce "Trent" engine, all current propeller turbines use an axial compressor, although in three instances this type has been used in conjunction with a centrifugal impeller. For sizes smaller than 1 000 h.p., however, the air consumption would not appear to warrant the complication of an axial compressor, and in the case of the Bristol "Janus" engine, centrifugal impellers are the only means of compression.

When we turn to small propeller turbines, i.e. those of under 1 000 h.p., some modification of the relative merits of the axial and centrifugal compressors is, however, to be anticipated. The low air flow required coupled with the fact that the same compression ratio (and, therefore, the same number of stages) has to be preserved, means that the aspect ratio of the blades is very low, and the blade-tip clearances may consequently have to be greater in proportion to the blade height. Both these facts tend to reduce the efficiency of the small axial. To this must be added the fact that the small compressor contains just as many parts as the large one and, therefore, would cost nearly as much. The compressor system for these small turbines is, therefore, likely to consist solely of centrifugal compressors, and it is probable that, in the interest of efficiency, there should be at least two stages.

A very marked increase in axial-compressor performance and efficiency is unlikely in the immediate future until such time as the Mach number effect has been mastered. There is a very direct analogy in this respect with aircraft performance in which supersonic speeds have yet to be obtained without the inherent detrimental consequences of increased drag.

4.6.9. Reduction Gearing. The reduction gear ratio has changed from about 2 : 1 to 8 : 1 or 10 : 1, thus necessitating quite radical changes in the general layout of the gearing. Moreover, input-shaft speeds have risen to nearly four times those used in current reciprocating-engine gearing, and since the minimum gear blank size cannot be reduced proportionately, pitch-line velocities are extremely high. In consequence, many turbine gears show the effects of very high-speed vibration, though it is difficult to indicate definitely the sources responsible.

4.7. The Ducted Fan

There is also the turbine/jet engine with an augmenter or ducted fan. This takes the form of an axial type fan in an annular duct surrounding the main turbine. The fan can be connected to a ring of turbine blades, which are located in the gas stream behind the turbine proper. Another method is to drive the fan mechanically direct or through gearing and locate it at or near the air entry of the compressor. The fan duct surrounds the engine and is carried rearwards, where it can discharge as an annular jet or into the jet pipe proper, to mix with gases there. One of the advantages of locating the fan at the compressor entry is that some degree of "ram" or supercharge is given to the air entering the compressor.

The advantages claimed for the ducted fan, or augmenter, are that it provides greater thrust than the simple jet during take-off and also the climb of the aircraft. But the thrust advantage over the ordinary jet decreases with forward speed, dropping to zero at about 550 m.p.h. The ducted fan in its simplest form has a lower weight than the propeller and less mechanical complication. But it is probable that, in order to obtain the best from the ducted fan, it will tend to become more complicated and may have to have blades with automatic pitch variation. In addition, variable area propelling nozzles will have to be provided. Further, the ducted fan will not have the complete flexibility of the propeller, but this disadvantage could, to some extent, be overcome by burning fuel in the duct, behind the fan, to increase the thrust, similar to burning fuel in the jet pipe, which has already been tried with encouraging results. However, it is very uneconomical to burn fuel in this manner. The control problems of the ducted fan engine may be formidable and the aircraft designer must first be reasonably satisfied that it gives him the means of producing a cleaner aircraft. It should be less noisy than a propeller.

4.8. Fuel Consumption

It is still necessary to improve the fuel consumption of the simple jet engine. This can be done by (*a*) higher efficiency compressors, (*b*) combustion chambers having a still lower pressure drop, and (*c*) more efficient turbines. The latter, (*c*), so far as the simple jet is concerned, does not have an important influence upon fuel consumption, within 1 or 2 per cent of the optimum efficiency. But, when related to propeller turbines, the efficiency is of very great importance; a difference of 2 or 3 per cent in turbine efficiency, in this case, will affect the fuel consumption by as much as 8 or 9 per cent.

In the case of (*a*), the axial compressor may supersede the centrifugal type, because the axial offers the advantages of higher pressure ratios and higher adiabatic efficiency. But it is not yet absolutely clear that this will be universally the case. The centrifugal compressor of the two-stage type should be capable of development, to give a pressure ratio of 6 : 1 (sea-level) with an adiabatic efficiency of 80 per cent. An axial compressor for the same pressure ratio would, to-day, give an adiabatic efficiency of, say, 85 per cent, but an efficiency of 88 per cent is considered attainable, eventually.

4.9. Axial or Centrifugal ?

Axial compressor turbines offer opportunities for small diameter, for the same power, relative to the centrifugal types, particularly when the annular combustion chamber is used. It is now considered that the former will not weigh any more than the latter, which was not the case a year or so ago. Actually, there are, in existence, axial type engines weighing no more than their centrifugal counterparts.

Some disadvantages of the axial type are that it is more costly to produce and it is probably more critical to accidental damage because of its many light blades. Also, it is critical to the condition of its blades which, if dirt builds up in their surfaces, generally causes a serious loss of efficiency and drop in performance.

It is doubtful, however, whether there is any advantage in exceeding a pressure ratio of about 6 : 1 for the centrifugal compressor of the simple jet engine, because the efficiency of compression tends to fall beyond this value. However, in the case of the propeller turbine, as we have already seen, high pressure ratios are very necessary to achieve high operating economy and the axial compressor is the only way to obtain high pressure ratios, together with relatively high adiabatic efficiency. Compounding, i.e. using compressors in series, will probably be necessary at the high pressure ratios in order to obtain the maximum degree of flexibility and to avoid stalling at low rotational speeds. It is very necessary to obtain as high an adiabatic efficiency as possible, otherwise the power required to drive the compressor will increase disproportionately.

In small short-range aircraft of the high speed pursuit type, the length of the axial compressor may be more embarrassing than the larger diameter of the centrifugal compressor. Because, if the engine is placed in the fuselage, its diameter may matter little but its length may seriously prejudice the arrangement of guns and fuel tanks. The air intakes of the engine, which are of paramount importance, must not suffer loss in efficiency, even for the gun installation. It is not now practicable to put the fuel and guns in the thin wings of the high-speed jet fighter.

Thus, it may be necessary, in some cases, to mount engines on the wings in order to provide adequate power and to leave space in the fuselage for fuel and military equipment. In this position the air entry conditions for the engines will also be very good.

4.10. Combustion Chambers

Practically all British engines use the separate combustion system, as opposed to the annular type. The advantages are that it is a relatively simple matter to test one unit (chamber) and know that it is representative of those eventually to be fitted to the engine. Also, it takes only a fraction of the total equivalent engine air to test one of a number of chambers. The annular combustion chamber, however, requires the total air necessary for the engine and, therefore, is a much more expensive test consideration. It is unsatisfactory to test sections of an annular chamber since it is not possible to reproduce the correct flow conditions. Because of these difficulties and a previous lack of full-scale air supply, the annular combustion chamber has received less development than the multiple or separate type. The advantages of the annular system are its small relative diameter and the fact that it should have a lower pressure drop than the multiple system.

If and when a successful and durable annular combustion chamber is developed, full advantage can then be taken of the small diameter of the axial type compressor, and it should be possible to build a jet engine giving about 3 300 lb thrust for a diameter of 24 in.

4.11. Design Problems

However, the problems of size and weight bring in other factors. We have discussed the efficiency of the gas turbine and the influence of compression ratio and temperature on its efficiency and power output. But, in order to design compact and light turbine engines, it is necessary to obtain the largest output of work per stage of both the compressor and the turbine. It is also necessary to have the highest flow rates possible, in order to obtain an engine of minimum total cross-sectional area. To achieve this, high flow velocities through the components are required, extending well into the compressibility region. In addition, high blade speeds are desirable, but this obviously leads to high stresses. Consequently, the design, as in all mechanical engineering, must be a compromise, which, in this case, is one between the stressing of the turbine and its blading and the maximum flow velocities or Mach numbers which can be obtained without any appreciable loss in efficiency.

The gas turbine is a "full-throttle" engine and normally runs at or near maximum conditions. It has, therefore, the characteristics of such engines and the power falls off from sea-level to altitude. The turbine is also critical to air inlet temperature and it is very important to ensure that, when meeting a new aircraft requirement, allowance is made for the take-off and climb conditions in hot climates and also, under similar conditions, from high altitude aerodromes.

For instance, an aircraft flying from Europe to Australia could lose anything up to 20 per cent of its thrust at take-off from the Far Eastern aerodromes en route.

The size or capacity of the engine must, therefore, be accurately assessed for all the known and possible aircraft operating conditions. Further discussion on these points will be found in Chapters XVI and XVII, but some considerations which apply to the propeller turbine may be of interest here.

4.12. Operating Conditions

4.12.1. Optimum Conditions. To obtain optimum operating conditions for a propeller gas turbine, there are three main factors which can be varied, either independently or in some predetermined relationship to each other. These are: (*a*) the fuel flow; (*b*) the propeller pitch; (*c*) the exhaust-nozzle area. Other quantities could be made variable for control purposes, such as compressor blade angles, turbine-nozzle areas, reduction-gear ratios, etc., but in general it is found that the added complexity of these devices can so far be avoided by suitable design of the engine and suitable correlation of the three main variables listed above.

4.12.2. Fuel Control. The control of the fuel supplied to the engine can be arranged to cater for the first requirement of the control system; i.e. the prevention of operating conditions in which the limitations of the engine design are exceeded. In addition, the fuel supplied for a given "throttle" setting can be automatically adjusted to allow for the effects of forward speed and altitude.

4.12.3. Propeller Control. The propeller pitch may now be varied independently to secure optimum propeller conditions or optimum propeller turbine conditions. A conventional constant-speed unit may be employed in which the propeller speed setting is independent of the throttle setting or interlinked with it to give single-lever control. Alternatively, the pitch may be selected to give a prescribed turbine r.p.m. as in the Bristol "Theseus" control system, where the pitch-change mechanism maintains a constant ratio between compressor and propeller turbine speeds.

4.12.4. Position of Jet Pipe. In the larger aircraft more generally suited to the propeller turbine, the disposal of a large undercarriage is usually arranged within the power-plant nacelles. This precludes a straight exit and may divert the jet-pipe run to avoid the tyre; such division can, fortunately, be done without serious consequences on engine performance, as the jet-pipe gas velocity can be comparatively low. The consequential loss in a bend at gas speeds of the order of 600 ft/sec (as compared with 1 600 ft/sec or so in the jet propulsion engine) is small, particularly as this mainly affects only about 20 per cent of the effective thrust. An alternative is to bifurcate the jet pipe and discharge at an angle either side of the nacelle.

4.12.5. Starting. The turbine-engine starting system is still a long way from a satisfactory solution inasmuch as the requirements include that of a prolonged steady load, which is a considerable drain on any electric system. In the case of a propeller coupled to the compressor, the power required for starting is much greater, and this is another argument in favour of the separation of the two drives. The main disadvantages of electric starting are to be found in the military field, but, even so, efforts are, and should be, made towards the development of self-contained systems, such as slow-burning cartridge turbine starters.

4.12.6. Scaling-up. Although it has been postulated that the gas turbine lends itself readily to "scaling-up" for changes in power of a given basic design and hence rapid production of a range of sizes, this is, perhaps, only strictly true for a narrow range and is mainly applicable to the passage areas of the working fluid. Mechanical design problems associated with the stiffness of larger diaphragms and casings, increased bearing sizes and capacities, and sometimes geometrical disposition of units, combine to make such scaling-up less amenable to design adjustments than at first seemed possible.

Additionally, and very much more important, is the time factor involved in the manufacture of new, and perhaps larger, components and their subsequent development in combination with other components of an engine. This development period, by no means unknown in the more general reciprocating-engine field, still remains with us in turbine manufacture, perhaps more so due to the extreme accuracy and advanced technique required.

4.13. Future Development

It is paradoxically true that the propeller turbine stands less in need of development of a radical nature than of consolidation of its present possibilities, and there can be no doubt that absolute priority in this case should be given to improvement in thermal efficiency with consequent reduction in fuel consumption. The propeller turbine has already quite sufficient advantage in respect of weight, bulk, lack of vibration, lack of noise, cheapness and safety of fuel, etc., to establish an overwhelming case for its adoption were it not for its present disadvantages in the matter of fuel consumption, and this must, therefore, be rectified before attention is given to further improvement of the former attributes.

As regards the propeller itself, this would appear to be quite capable of coping with the demands made by the turbine, at least in sizes up to 10 000 h.p. Above this power it may prove difficult to maintain the present specific weight, which, according to dimensional theory, should increase, but the advent of the hollow-steel blade (perhaps with an internal structure) will do much to avoid this limitation. Where speed is concerned, there is now a fair amount of evidence that propeller efficiency can be maintained at a high figure up to speeds in the 500–550 m.p.h. region, above which jet propulsion may be expected to prevail except in very special cases.

PART II—THE PERFORMANCE OF PISTON-ENGINED AIRCRAFT

CHAPTER V

Take-off and Landing: Temperature Accountability

5.1. The Hazards

THE hazards to which an aeroplane may be subject at take-off are of three main kinds—

(*a*) Engine failure.

(*b*) Adverse climatic conditions, including high temperatures, crosswinds.

(*c*) Adverse aerodrome conditions, such as inadequate length, long wet grass, high altitude, etc.

5.2. Engine Failure

5.2.1. THE DESIGN. Certain steps can be taken, in the design stage, to lessen the possibility or the consequences of engine failure. These include—

(1) Fitting engines having a high degree of reliability.

(2) Reducing the "arm" of the dead engine drag: a step that can be taken with pure jet engines more readily than with propellers.

(3) Never installing less than four engines.

(4) Using a tricycle undercarriage.

(5) Fitting an adequate rudder, and an adequate fin.

5.2.2. NEED FOR A DRILL. The above are some of the things which the designer can do. The aeroplane is built and goes to be tested. This process takes many months. Not only must the aircraft satisfy certain standards of controlability laid down by I.C.A.O. and A.R.B., but in addition it is necessary to find out the "critical speeds," etc., peculiar to the aircraft. With a knowledge of these speeds, a Drill can be formulated for the pilot to follow. We can then affirm that, if the A.R.B. requirements are satisfied, and if the pilot follows the Drill, the aircraft is as safe as it can be under normal climatic and airfield conditions.

5.2.3. THE IMPORTANT SPEEDS. After an engine failure during some stage of the take-off and climbaway, there are three important questions to be answered—

(1) Is the aeroplane controllable

(*a*) on the ground?

(*b*) in the air?

(2) If the answer to (1) is yes, will the aircraft use more or less aerodrome distance by re-landing than by climbing (on the remaining engines) to a height of at least 50 ft?

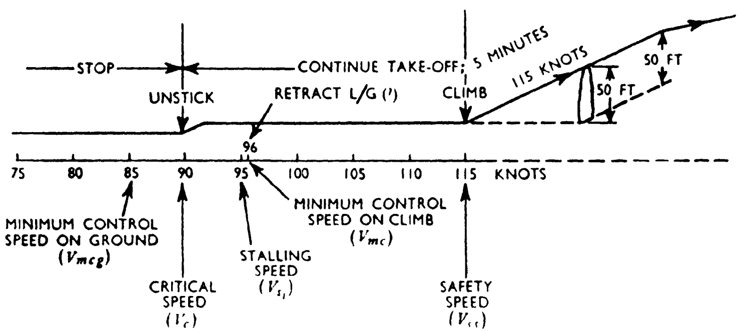


FIG. 35. SPEEDS AT TAKE-OFF

(3) If the climb is attempted, at what minimum speed should it be started?

The sort of answers we should expect to these questions can be seen most clearly by reference to a typical case, shown in Fig. 35. The figures marked on the horizontal line are speeds, in knots; and what we want to know is how to fix the *critical speed*, after which the take-off should be continued; and the *safety speed*, after which the climb should be started, and continued up to the limit of five minutes.

5.2.4. STATE OF THE AIRCRAFT. We will assume that the above speeds are to be found—

(*a*) at take-off weight,

(*b*) at the most adverse c.g. position,

(*c*) at sea-level I.C.A.N. conditions,

(*d*) with the undercarriage in a number of positions,

(*e*) with the dead engine windmilling (the pilot is hoping that the dead engine may pick up again), (*f*) (when the aircraft is on the ground), with no differential braking (as this will reduce take-off performance), and also that

(*g*) the angle of bank, if any is used, shall be small,

(*h*) the flying control loads shall at all times be reasonable, and easy to apply quickly.

5.2.5. CONTROL ON THE GROUND. Now let us focus our attention on the aircraft which is accelerating along the runway; and which is liable to have a cut engine at any time.

While the aircraft is still on the ground, the effect of a cut of a starboard engine will be to cause a yaw to starboard, which can only be overcome (in the absence of differential braking) by the use of the rudder. A tricycle undercarriage will of course help; and will also put the aircraft in a more favourable attitude for the effective operation of the rudder. If there is no blanketing, the restoring moment of the fin and rudder is proportional to some power of the speed, i.e. the aircraft must reach a certain speed before rudder control can hope to keep it straight, and, furthermore, we must guard against overswing following rudder correction.

If the aircraft is just clear of the ground, the rudder can be augmented by the use of the ailerons. If the dead engine is banked up, by as little as two or three degrees, the bank introduces sideslip, and the sideslip acts on the fin to keep the aeroplane straight. (This assumes that the fin chosen has sufficient area not to be stalled by the operation of the rudder.) These considerations lead us to the necessity for determining the

Minimum Control Speed on the ground or V_{mcg}

which is the lowest speed at which (allowing a certain margin for differences between aircraft of the same type and for airspeed indicator errors) the aircraft can be controlled on or near the ground.

The answer to this, as to all other speeds yet to be mentioned, will be affected by some, if not all, of the following—

(*a*) aeroplane weight,

(*b*) aerodrome altitude and air temperature,

(*c*) take-off surface, i.e. coefficient of friction,

(*d*) take-off surface gradient,

(*e*) wind velocity parallel to the direction of take-off,

(*f*) wind velocity at right angles to the direction of take-off.

As regards (*e*), answers given for conditions with or against the wind must be "reduced" to the no-wind condition.

As regards (*f*), the minimum lateral wind component is taken to be a 10 m.p.h. or 8 knots, i.e. the aircraft must be controllable in a side wind of at least 8 knots.

Tests on our particular aircraft show that, with specified values of (*a*) and (*b*) above (i.e. weight and c.g. position), and with any cross wind up to 8 knots, there is enough directional control and enough acceleration for the take-off, even if the engine cuts at 75 knots. Allowing a certain margin, for reasons already stated, we will fix the minimum control speed on the ground as 85 knots, i.e.

$$V_{mcg} = 85 \text{ knots}$$

and it is evident that the pilot should not attempt to continue the take-off if engine failure occurs before this speed has been reached.

5.2.6. THE CRITICAL POINT AND CRITICAL SPEED. It does not necessarily follow, however, that at a speed greater than 85 knots the pilot should automatically continue to take-off. We can appreciate this best by considering the four possibilities shown in Fig. 36.

In each case the point of engine failure has been marked with a small circle; and the diagrammatic behaviour of the aeroplane after failure is shown by a double line.

In case (1) the engine fails at 75 knots, and the pilot immediately takes the necessary action to stop the aeroplane.

In case (2), when engine failure occurs at 85 knots, we allow him the alternative of either stopping or continuing; and it is found that he will use less length of runway if he stops than if he accelerates to the "safety speed" of 115 knots (see later), and then climbs to 50 ft.

In case (3), engine failure does not occur until he reaches 110 knots, and in that case more aerodrome length is required to bring him to a stop than if he continued accelerating to 115 knots and then climbed to 50 ft.

In case (4), where engine failure occurs after he has reached the safety speed, the same remarks apply as in case (3).

Now it is evident that between cases (1) and (2) on the one hand and cases (3) and (4) on the other hand there exists a speed for which the length of aerodrome required will be the same whichever alternative the pilot adopts. If engine failure occurs at this *critical speed* V_c, the corresponding point on the aerodrome is known as the *critical point*; *the distance from the start to the critical point, and from there on to the stop, is the maximum length of the aerodrome he is likely to want, whenever*

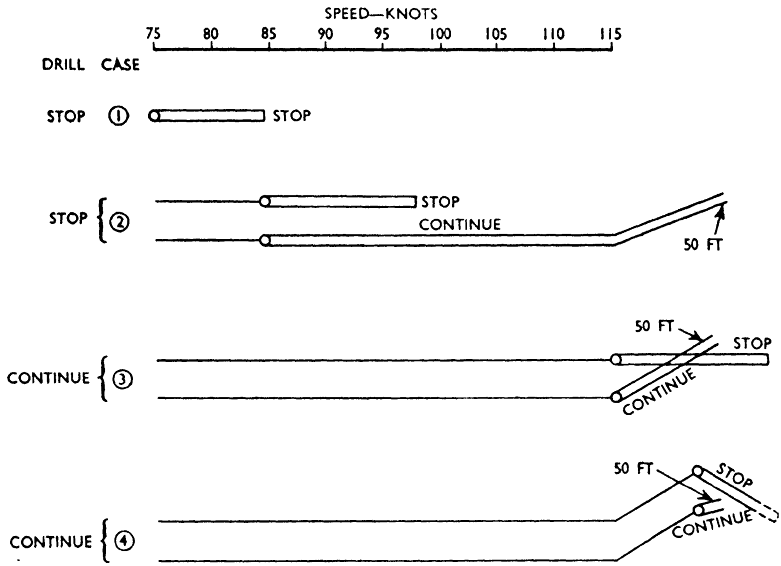


FIG. 36. ENGINE FAILURE AT VARIOUS SPEEDS: THE DRILL

engine failure occurs; assuming that he follows the STOP or CONTINUE drill above described.

The critical speed can be more or less than the safety speed, depending upon the qualities of the controls and the take-off power. A low critical speed suggests that the brakes are poor, or that V_{mcg} is low; a high critical speed suggests that the climbing capacity of the aeroplane is not good. This point is made clearer by a study of Fig. 37 which shows how the critical point can be determined by finding the intersection of the graphs showing, after an engine failure,

(*a*) distance to accelerate and stop, for various speeds,

(*b*) total distance required to clear 50 ft, if the climb is started (with one engine dead) after reaching the safety speed.

If the brakes are poor, curve (*a*) will have a steeper gradient, and will intersect curve (*b*) at a lower speed. If the take-off characteristics are poor, curve (*b*) will be lifted bodily upwards, and will intersect (*a*) at a higher speed.

The Airworthiness Standards recommended by I.C.A.O.* and the A.R.B. British Civil Airworthiness Requirements† both state that the critical point and speed are to be selected by the manufacturer, and as a result of all the tests which the prototype undergoes. I.C.A.O. further state: "The pilot shall

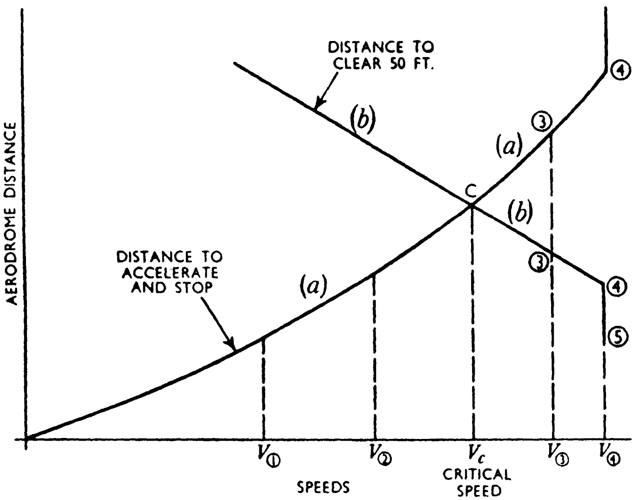


FIG. 37. CURVES TO FIND CRITICAL SPEED

be provided with a ready and reliable means (e.g. air-speed indicator reading) of determining when the critical point has been reached"; and the A.R.B. Civil Airworthiness Requirements add that, after failure of the engine at the critical point, "the following action, if practicable, may be taken: coarse re-trimming of the aeroplane, initiation of landing gear retraction, adjusting the power of the remaining engines to emergency maximum power, adjusting the inoperative airscrew to a coarse-pitch (but not feathered) position." This, however, is not in entire agreement with I.C.A.O. who require the landing gear

* "Proposed 1947 Edition of Airworthiness (AIR) Standards and Recommended Practices," March, 1947 (Ref. Doc. 3031, AIR/181), *P.I.C.A.O. Airworthiness Division, Second Session Final Report*, Volume III.

† "Performance Requirements: Take-off, Climb, and Landing (Landplane)," *Air Registration Board—British Civil Airworthiness Requirements DI*, Issue 4.

to remain extended throughout the period between critical speed and safety speed. These differences will be considered later.

5.2.7. Accelerate-and-Stop Point. This is the point on the runway at which the aeroplane can safely be brought to a stop, when the engine has failed at the critical point and the attempt to take off is abandoned. The corresponding distance, from start to finish, is the *accelerate-and-stop distance.*

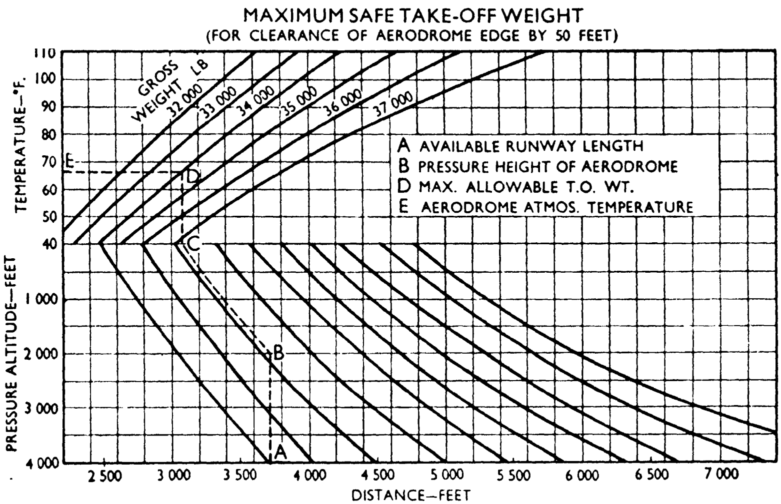


Fig. 38. Chart for Finding Take-off Distances

5.2.8. Size of the Aerodrome. It has already been stated that, if the pilot follows the drill—if he comes to a stop after the engine fails at less than critical speed, and continues the take-off when the engine fails at more than critical speed—then the maximum length of runway required is a known distance; called the accelerate-and-stop distance. This distance has an obvious bearing on the suitability of an aeroplane for a particular aerodrome, or of an aerodrome for a particular aircraft.

Referring again to Fig. 37, we can see that the total aerodrome distance required will be made less (1) by decreasing the gradient of curve (*a*), i.e. by improving the brakes; or (2) by bringing the curve (*b*) lower down, i.e. by improving the take-off performance. Any circumstance which causes a deterioration in take-off performance will increase the size of aerodrome required for safety.

It is rapidly becoming normal procedure for operators (or manufacturers) to supply pilots with a simple nomographic chart giving, for a variety of take-off weights, temperatures, and runway surfaces, the take-off distance with all engines functioning. Such a chart is shown in Fig. 38; and the answer is exemplified in Fig. 37 by the ordinate of the point (5). It should be equally possible to supply pilots and flight supervisors with a chart giving the critical distance for the engine-

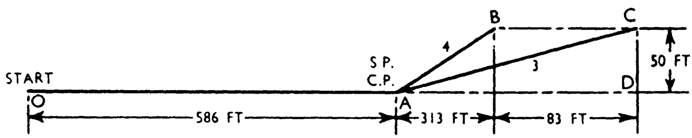


FIG. 39. ENGINE FAILURE AT THE TAKE-OFF SAFETY SPEED

cut case—the ordinate of the point C in Fig. 37, under the same variety of conditions.

5.2.9. CHOICE OF CRITICAL POINT. Before a Certificate of Airworthiness is granted, the relevant facts about the aircraft's take-off behaviour must be known, and entered into the Flight Manual (to be) carried in each aircraft. Now the critical point can be selected by the applicant for a C. of A., and he may decide that it may be worth while to alter slightly the accelerate-and-stop distance—by taking the critical point not exactly as defined in § **5.2.4,** in order to get greater simplicity.

To give an example, for a certain modern aeroplane of A.U.W. 16 500 lb, the selected critical speed is the same as the known safety speed and the aircraft's take-off can be illustrated diagrammatically as in Fig. 39.

A is the vital point, 586 ft from the start *O*, and the speed here is 89 knots. If an engine fails between *O* and *A*, the take-off is abandoned, and the aircraft brought to rest at some point along *OAD*—the maximum distance, for failure at *A* being at *D*, a total accelerate-stop-distance *OD* of 982 ft. If an engine does not fail between *O* and *A*, there is no attempt to climb until *A* is reached; and thereafter the climb starts. If an engine fails on the early part of this climb, the aircraft will reach 50 ft at some point between the extremes *B* and *C* (a distance of 83 ft). *Note*, however, that the point *C* will not be vertically above *D* unless *A* is the real critical point as previously defined.

5.2.10. MINIMUM CONTROL SPEED ON THE CLIMB, V_{mc}. So far we have tacitly assumed that, on reaching the safety speed, (*a*) the aircraft is controllable on the climb, and (*b*) the remaining engines give a satisfactory climb performance. Neither of these is necessarily true.

The minimum control speed on the climb, V_{mc}, is such that, if one engine fails suddenly during a steady climb at take-off power, control of the aircraft can still be maintained. By "control" we mean straight flight, either with zero yaw or with any bank up to 5°; and the aeroplane is assumed to have its landing gear retracted, to be trimmed for take-off and have its flaps in the take-off position.

I.C.A.O. add further to this definition: "From the time at which the engine (becomes) inoperative to the time at which recovery is complete, it (should) not require exceptional skill, alertness, or strength on the part of the pilot to prevent any loss of altitude or any change of heading in excess of 20°, nor shall the aeroplane assume any dangerous attitude."

As speed increases the effectiveness of the rudder can be assumed to increase, and so I.C.A.O. stipulate that V_{mc} shall not be more than 30 per cent above the engine-off stalling speed (V_{s_1}); and they further require that the force on the rudder pedals shall not exceed 180 lb.

It will now be clear why the safety speed is taken higher than the unstick speed; a steady climb should not be attempted until it is quite certain that the aircraft is controllable should an engine fail on the climb.

In the two examples we have chosen, the values of V_{mc} and other speeds all in knots are as follows—

Fig.	Take-off Weight	V_{mc}	V_{s_1}	$1{\cdot}3V_{s_1}$	Take-off Safety Speed	Critical Speed V_c	V_{mcg}
35	60 000	96	95	123	115	90	85
39	16 500	80·5	74	96·2	89	89	—

5.2.11. Variation of Critical Point. We have remarked in § **5.2.9** that the applicant for a C. of A. is at liberty to define the critical point so that the accelerate-stop distance is not exactly the same as the distance to climb 50 ft. It may happen, however, that these distances cannot be made equal, i.e. that the curves in Fig. 37 do not meet. If V_{mcg} and V_{mc} are close together, and controllability is the important criterion, V_c will tend to be high; and in consequence the accelerate-stop distance will almost certainly be longer than the distance to climb to 50 ft. In such cases the critical point is not the point for equal distances for "stop" and "go," but rather for a minimum value of the accelerate-stop distance—"minimum" because we cannot allow the critical speed to be below that for adequate controllability.

5.2.12. Rate and Gradient of Climb. Before we can finally decide on the take-off safety speed, we must be certain that the acceleration and rate of climb are adequate. In order to allow for variations of temperature and aerodrome altitude, the stipulated rate of climb must be fairly severe. The conditions laid down by I.C.A.O. refer to an aeroplane with landing gear retracted, and with flaps in differing positions, and are as follows—

(*a*) *All Engines Operating.* The steady rate of climb at 5 000 ft shall be not less, in feet per minute, than $5{\cdot}5V_{s_1}$, where V_{s_1} is the engine-off stalling speed with flaps up in miles per hour.

(*b*) *One Engine Inoperative.* At each altitude within the range for which a take-off weight is to be established, the steady rate of climb, in feet per minute, shall not be less than $0{\cdot}035\ V_{s1}^2$ or $0{\cdot}38\ V_{s_1}^{1{\cdot}5}$, where V_{s1} is the engine-off stalling speed, flaps in the take-off position expressed in miles per hour. (Here it is assumed that the propeller of the dead engine is windmilling in the fully fine position.) If the landing gear is extended, the rate of climb is not to be less than 50 ft per minute.

The conditions laid down by A.R.B. are less mathematical and can be summarized as follows—

	Rate of Climb	*Gradient of Climb*
(*a*) All engines operating	300 ft/min	5%
(*b*) One engine inoperative	200 ft/min	3%

These assume that the landing gear is retracted, and the propeller of the dead engine is feathered.

To see how these conditions work out in practice, we will consider again the aeroplane of Fig. 39 for which the following flight test results are available.

	V_{s_1}	I.C.A.O. Requirement	A.R.B. Require-ment		Flight Test Result	
(*a*) All engines operating	88·4	$5{\cdot}5 \times 88{\cdot}4 = 486$ ft/min	300 ft/min	5%	620 ft/min	7%
(*b*) One engine inoperative	74·0	$0{\cdot}035 \times 74^2 = 191{\cdot}5$ ft/min $0{\cdot}38 \times 74^{1{\cdot}5} = 241$ ft/min	200 ft/min	3%	340 ft/min	4%

5.2.13. Acceleration at Take-off. If the engine fails between the critical speed and the take-off safety speed (assuming the critical speed occurs first), the pilot is told to accelerate until the safety speed is reached. Can this be done? There is obviously a need for a minimum standard of acceleration. Direct measurements of time and distance or speed, leading to acceleration, are not very reliable, and it is suggested that we use the relation

$$f = \frac{Rg}{60V}\ \text{ft/sec}^2 = \frac{gG}{100}\ \text{ft/sec}^2$$

where $g = 32{\cdot}2$ ft/sec²,
f = acceleration in feet per second per second,
R = rate of climb in feet per minute,
G = gradient of climb, expressed as a percentage,
V = true speed in feet per second.

For example, 360 ft/min at 115 knots equals 1 ft/sec² or 0·7 m.p.h./sec.

5.2.14. Effect of Take-off Weight. Variations of temperature or aerodrome altitude may reduce the rate of climb and acceleration, and the only cure that can be applied immediately is to reduce the take-off weight. Pilots must therefore be provided with an easy means of determining the result of these adverse conditions, so that they can take appropriate action if the I.C.A.O. or A.R.B. requirements are not likely to be met. A reduction of about 4 per cent in take-off weight, for instance, may improve the rate of climb by as much as 50 ft/min. Probably the best way to attack the problem is to provide the pilot with simple nomographic charts (such as Fig. 38) which indicate, both for all engines operating and for one engine dead, the anticipated rate of climb under all conditions of loading and temperature; and the safety speed may have to be modified to suit exceptional conditions.

5.2.15. TAKE-OFF SAFETY SPEED. The drill provided for the pilot instructs him not to start a steady climb until the safety speed has been reached. We have already discussed some of the matters which affect the choice of the best safety speed, and we are now in a position to formulate exact conditions. They can be grouped under three headings.

Heading	I.C.A.O. Requirement	A.R.B. Requirement
(a) *Margin above stalling speed*	$1{\cdot}20V_{s_1}$ for aircraft with two engines; $1{\cdot}15V_{s_1}$ for aircraft with more than two engines	$1{\cdot}20V_{s_1}$ for two- or three-engined aircraft; $1{\cdot}15V_{s_1}$ for aircraft with more than three engines
Examples:		
Aircraft of Fig. 35	$1{\cdot}15 \times 95 = 109$ knots	$1{\cdot}15 \times 95 = 109$ knots
Aircraft of Fig. 39	$1{\cdot}15 \times 70{\cdot}4 = 81$ m.p.h.	$1{\cdot}15 \times 61{\cdot}5 = 70{\cdot}5$ knots
(b) *Margin above V_{mc}, the minimum control speed on the climb*	$1{\cdot}10V_{mc}$	$1{\cdot}10V_{mc}$
Examples:		
Aircraft of Fig. 35	$1{\cdot}10 \times 96 = 106$ knots	$1{\cdot}10 \times 96 = 106$ knots
Aircraft of Fig. 39	$1{\cdot}10 \times 80{\cdot}5 = 88{\cdot}5$ m.p.h.	$1{\cdot}10 \times 70{\cdot}7 = 77{\cdot}7$ knots
(c) *Sufficient rate and gradient of climb*	See § 5.2.12 above	See § 5.2.12 above
Examples:		
Aircraft of Fig. 35	115 knots	
Aircraft of Fig. 39	78 knots = 89 m.p.h.	
Selected Safety Speed is the greatest of (a), (b), (c)	Aircraft of Fig. 35 = 115 knots; Aircraft of Fig. 39 = 89 m.p.h.	

5.2.16. UNDERCARRIAGE RETRACTION. There is some difference of opinion as to when the retraction of the undercarriage should begin. A.R.B. suggests the critical point, whereas I.C.A.O. regulations are that the undercarriage should remain extended throughout the time required to reach the critical point or take-off safety speed, whichever time is the greater. This appears to be the logical decision; if the critical point occurs after the safety speed point, the aircraft has to re-land and needs its landing gear, and if the critical point occurs before the safety speed point, a tricycle undercarriage may still be of value to give directional control until the climb is started.

5.2.17. BAULKED LANDING. We may perhaps quote here the relevant I.C.A.O. recommendation.

At each altitude within the range for which a maximum landing weight is to be established, the steady rate of climb shall be not less, in feet per minute (metres per second), than 200 or $0{\cdot}07\ V_{s_0}^2$, whichever is greater, V_{s_0} being expressed in miles per hour, with—

(*a*) engines operating within the take-off power limitations;

(*b*) landing gear extended;

(*c*) landing flaps in the appropriate landing position except insofar as they are retracted by completely automatic means as power is applied, without

(i) involving dangerous change of trim, sudden change of angle of attack, or adverse change in the flight path;

(ii) operation exceeding either three seconds, or the time required to increase the power of the engines safely from windmilling conditions to maximum take-off power, whichever is the greater;

if automatic wing flap retraction is provided, compliance with the conditions prescribed in (i) and (ii) shall be demonstrated for all speeds between $1{\cdot}2V_{s_0}$ and the maximum permissible speed with the wing flaps in the landing position;

(*d*) cowl flaps and radiator shutters in the position recommended by the applicant for normal use in a final approach to a landing;

(*e*) aeroplane weight equal to the maximum landing weight appropriate to the altitude.

5.2.18. LANDING WITH ONE ENGINE DEAD. We again quote the relevant I.C.A.O. recommendation.

At each altitude within the range for which a maximum landing weight is to be established, the steady rate of climb shall be not less in feet per minute than $0{\cdot}035\ V_{s_1}^2$, V_{s_1} being expressed in miles per hour, with—

(*a*) critical engine inoperative, its propeller stopped;

(*b*) remaining engines operating within the take-off power limitations;

(*c*) landing gear retracted;

(*d*) wing flap in the position recommended by the applicant for a preliminary approach with the critical engine inoperative;

(*e*) cowl flaps and radiator shutters in the position recommended by the applicant for normal use during a preliminary approach to a landing;

(*f*) aeroplane weight equal to the maximum landing weight appropriate to the altitude.

Note. The main purpose of the above is to ensure to the greatest practicable extent that, in the event of engine failure, the aeroplane will be able to maintain the altitude required to clear all obstacles and to reach a suitable landing area, thus minimizing the possibility of, and hazards associated with, an emergency landing.

5.3. Adverse Climatic and Altitude Conditions

5.3.1. Effect of Temperature and Height. Recent tests have shown that the I.C.A.O. regulations in respect of take-off do not appear to have sufficient margins to cover take-off with one engine failed in sub-tropical conditions—especially for aerodromes above sea-level.

The comments of P. H. Hufton deserve quoting in this matter*—

"There is a rather subtle point here, besides the obvious fact that sub-tropical temperatures are above temperate conditions. In the I.C.A.O. regulations, the take-off climb requirement of $0{\cdot}035\ V_{s_1}^2$ is asked for in a standard atmosphere: e.g. if you are at 5 000 ft aerodrome, you calculate the performance for 5 000 ft and 5°C. Now there seems little reason for associating with the air immediately above an aerodrome at an altitude above sea-level of x ft, the temperature corresponding to a standard atmosphere of x ft. Fairly obviously there will be a large effect of the presence of the ground on the temperatures and the temperature might be considerably more than those in the standard atmosphere.

"Thus at Salisbury, Southern Rhodesia, we have the upper

* P. A. Hufton, M.Sc., "Testing Civil Aircraft," *Journal of the Royal Aeronautical Society*, No. 439 (July, 1947).

curve of Fig. 40 for maximum daily temperatures, while for Croydon we have the lower curve. You will note that at Salisbury, the 'official' standard temperature is exceeded every day in the year, while at Croydon the official standard temperature for sea-level of 15°C is only exceeded 139 days out of 365. Finally, the temperature which is exceeded only one day out of the 365 is 30° higher than standard temperature at Salisbury, while it is only 14° higher than standard at Croydon. Thus if

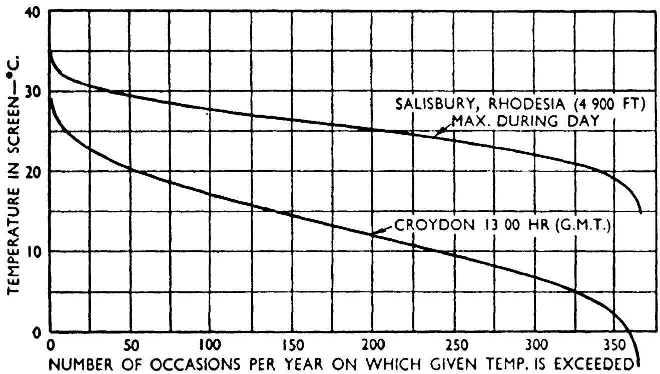


Fig. 40. Variations of Temperature: Frequency with which Ground Temperatures are Exceeded

Daily maximum occurs about 13·00. Daily minimum occurs just before dawn.

we assume we can tolerate, and only just tolerate, the drop of performance which may occur at Croydon one day in 365, then we shall be well below a tolerable level at Salisbury this one day in 365.

"To show how important this is, let us think of the DC-3. The present requirements allow it a rate of climb at take-off at sea-level in standard atmosphere of 270 ft/min, i.e. a gradient of 2·8 per cent. This falls to 130 ft/min on one day in 365 at Croydon, i.e. a gradient of 1·2 per cent. This is bad enough—Croydon clearances are barely adequate for this, and this would certainly count as one of the hazards referred to above. At Salisbury, however, the rate of climb would be negative, i.e. the aircraft would not climb at all.

"We have methods for correcting the performance of piston-engined aircraft for temperature, and at first sight it looks as though all we need is to put this into reverse. However, what is good enough for correction may not be good enough for prediction, especially when payload depends so vitally on the aircraft performance. It may be well, therefore, that actual tests of the aircraft under tropical conditions will be essential if we are to get the accurate information which we all need. This is becoming increasingly true with the more refined reciprocating engines with their many different types of temperature and pressure controls. How these will affect the performance of the engine under tropical conditions is difficult to foresee, hence actual tests must be made.

"This state of affairs is even more true for turbine engines. While we have a beautifully simple method of non-dimensional plotting for the pure-jet engine, we have only just touched the fringe of the complications that may exist for the turbine-propeller engine, and it will be some time before we can venture to forecast what the effect of temperature will be on performance of these engines. Even for the simple jet we often find that, unless tests are actually made over a wide range of temperatures and pressures, it is impossible to use the non-dimensional methods for predicting performance. Thus, to obtain jet take-off performance at altitudes above sea-level, it seems absolutely essential at the moment actually to measure the performance at high altitudes—which obviously means using a high altitude aerodrome.

"Finally, as indicated above, the establishment of the cooling characteristics of aircraft engines may well demand tests under tropical conditions with the more complicated systems now being considered.

"Thus it appears at the moment as though the trials of civil aircraft may well need to include some tests in the tropics, possibly at both a high and low altitude aerodrome."

5.3.2. TEMPERATURE ACCOUNTABILITY. This has been defined by one authority* as "the taking into account, more accurately than is now done, of variations in an aircraft's performance arising from the fact that the atmosphere in which it flies will almost always be different from the theoretical standard atmosphere on which the performance estimates of aircraft are at present based."

* K. T. Spencer and P. A. Hufton, "Modern Operational Factors Affecting Airworthiness," *Anglo-American Conference*, 1947, *Proceedings*.

A Committee of I.C.A.O. has been at work on the question; but in general it can be said that the usual "density height" calculations will be sufficient to provide reliable information about cruising, particularly at the considerable heights contemplated in the future. Where, however, the safety of the aircraft is seriously prejudiced by deviations from I.C.A.N. conditions, is at take-off and landing.

The problem is most acute when assessing maximum safe take-off weight. Current practice is to schedule take-off performance for a range of airfield altitudes, assuming that the variation of atmospheric density, pressure and temperature with height conforms to the standard (I.C.A.N.) atmosphere. In an extreme case the real atmosphere may have very different characteristics.

Current international rules permit either of two methods to be used: the "geographical altitude method" or the "equivalent density altitude method." To show how the former could err, the following comparison may be given—

	Standard Atmosphere	Tropical Airfield Atmosphere
Density (slugs/ft³)	0·002378	0·002154
Pressure (millibars)	1 013·2	1 013·2
Temperature (°C)	15	45

The second method takes the standard atmosphere scheduled performance at 3 350 ft and assumes that this will obtain at the tropical sea-level airfield. But at that airfield the temperature is 45°C, whereas at 3 350 ft in the standard atmosphere it is only 8·4°C. Tabulating—

	Density (slugs/ft³)	Pressure (millibars)	Temperature (°C)
Real values at sea-level tropical airfield	0·002154	1 013·2	45
Standard atmosphere values at sea-level	0·002378	1 013·2	15
Standard atmosphere values at 3 350 ft	0·002154	896·6	8·4

The variation of engine power with atmospheric density, pressure and temperature is complicated. No one law covers even all conventional types of piston engine, and the turbo-jet introduces a still wider range of variations. To illustrate this fact, two types of aircraft may be examined: type *A*, a twin-engined machine with piston engines operating below super-charge height, and *B*, a four-engined aircraft with turbo-jets. The calculated take-off distances to clear 50 ft with all engines running, are for aircraft *A*: geographical altitude method, 865 yd; real performance, 985 yd. For aircraft *B* under the same conditions, the corresponding figures are 1 410 yd and 1 985 yd. The weight reduction needed, for aircraft *A*, to keep the one-engine-out angle of climb constant at 0·0260 in real atmosphere is 10·5 per cent.

The first difficulty is the ascertainment of the aircraft's performance, not only under standard atmospheric conditions but also under sufficient range of non-standard.

The second difficulty is to present the scheduled information in a way in which it can be easily grasped. For any standard or non-standard atmospheric conditions the pilot must be able to ascertain at once, without calculations, the pertinent performance characteristics.

The third difficulty is to decide what values of density, pressure, and temperature are to be taken.

The first difficulty is not serious, and the second difficulty—presenting the scheduled information so that it can be used easily by the operator—could be overcome. It means scheduling the variation of aircraft performance with temperature as well as with altitude, wind, weight and so on. One way of doing this for the two parameters, temperature and altitude, is shown in Fig. 38. The graph (Fig. 38) can be used to give the maximum safe take-off weight as limited by any one of the several I.C.A.O. operating rules. For any given airfield and time the pilot will know the temperature, pressure, and available runway length. Assuming the latter is represented by point *A* in the graph, a vertical line is drawn from *A* to *B*, *B* representing the pressure altitude of the airfield. *BC* is drawn parallel to the nearest of the family of curves and a vertical drawn from *C* to the point *D* level with the atmospheric temperature *E*. Then the curve on which *D* falls—in this case 34 000—gives the allowable take-off weight.

This leaves, then, only the third difficulty: what values of density, pressure and temperature should be used at any particular time and place?

The many possible answers to this all fall within two extremes. At one extreme is the scheduling for individual airfields of long-term average values of density, pressure and temperature.

At the other extreme is the determination, immediately prior to the take-off, of the real values of the density, pressure, and temperature. This, from the technical point of view, is the right thing, but it may be considered to place an unnecessary burden on the pilot, particularly at well-established aerodromes.

5.4. Summary

The problems considered in the preceding paragraphs are not growing less as the years go by. Some typical figures given by M. B. Morgan may be of interest—

Typical	Year 1930	Year 1947
(1) Wing loading	14 lb/ft²	70 lb/ft²
(2) Power loading	16 lb/h.p.	10 lb/h.p.
(3) Take-off distance to 50 ft	500 yd	1 500 yd
(4) Take-off safety speed	100 m.p.h. (1939)	135 m.p.h.

The increase in (3) is partly accounted for by the movement to improve the aircraft's safety after a possible engine failure.

The increase in (4) would have been much larger were it not for the concurrent increase in the area of fins and rudders, together with the use of assistors, etc., to cope with the increased pedal loads.

Sooner or later a halt will have to be called in this process of enlargement. That is a problem for the designer, and one which is being tackled energetically from a great variety of angles. The operator and pilot can register their protests, but must for the present await future developments with as much hope as they can muster. In the meantime it is vitally necessary that all pilots and flight supervisors should be thoroughly conversant with the regulations laid down for avoiding accidents at take-off and landing, and should appreciate the reasons that lie behind the various drills. Without Vision the people perisheth; without Safety there is no real future for Commercial Aviation.

5.5. High Lift Devices

In order to comply with the airworthiness requirements, the aircraft must be provided with adequate flaps, so that the necessary value of C_{Lmax} can be attained.

The distance covered from rest to climbing speed varies inversely as C_{Lmax}, while the distance on the climb to 50 ft varies as the induced drag and thus as C_{Lmax}^2; hence there is an optimum value of C_{Lmax}. For a typical aircraft, of wing loading 50 lb/ft², the optimum C_{Lmax} rises from 2·6 to over 3·5 as power loading (in lb/b.h.p.) is reduced from 16 to 6. For the same aircraft with one engine cut, the optimum C_{Lmax} ranges from 2·25 to over 3·5 as power loading decreases from 14 to 6: for a power loading of 16, the climb is impossible. I.C.A.O. one-engine-out rate of climb requirements are satisfied if the power loading (on 4-engined aircraft) is less than 12·5.

The landing distance, from 50 ft to stop, is also important, as wing loading increases. Assuming that sufficient engine is used on the approach to reduce the gliding angle to 3°, and the maximum ground deceleration is taken as $0{\cdot}25g$, it is found that the landing distance (for C_{Lmax} of 2·5) varies between 1 800 ft and 4 800 ft; as the wing loading is increased from 20 to 70 lb/ft², and these figures are typical. Larger lift coefficients are limited by the I.C.A.O. requirement, which specifies a minimum rate of climb with the flaps in the landing position.

CHAPTER VI

Climbing and Descent

6.1. Best Climbing Speed

THE highest climbing speed, at full climbing power, is something which has to be found by flight testing each aircraft, though it is theoretically apparent from the following equation, or from our previous argument,

$$550\sqrt{\sigma}\eta \times \text{b.h.p.} = aV_i^3 + \frac{bW^2}{V_i} + WV_c\sqrt{\sigma}$$

(where V_c is the vertical component of climbing speed) that V_c will be greatest when V_i is in the neighbourhood of $V_{i_{mp}}$. For an aircraft with an engine having only one supercharger speed, it is found that the best climbing speed varies little from ground level to full-throttle height, and then falls away from this height to the ceiling, more or less linearly. With two-speed superchargers, the best climbing speed varies little between ground level and the full-throttle height in the higher supercharger gears. In both cases the final speed at the ceiling is round about $V_{i_{mp}}$.

The highest climbing speed for a gas-turbined aircraft is the speed for maximum excess thrust power, and will be approximately the minimum drag speed at the ceiling, often about 150 m.p.h. I.A.S. for present-day aircraft, and increasing as the available excess thrust increases. The best climbing speed will almost invariably be higher than for piston-engined aircraft; and in consequence the maximum angle of steady climb may be rather flat.

As an example of the speeds laid down for a particular aircraft, we find that, for the Constellation, the recommended I.A.S. is 160 knots for heights below 5 000 ft; and that above this height the recommended climbing speed is gradually reduced until it reaches 130 knots at 20 000 ft.

The reason why $V_{i_{mp}}$ is impracticable at the lower heights is that the engine is running at a high power, and at low altitudes adequate cooling is not forthcoming.

Moreover, from the point of view of fuel economy, the highest climbing speed is wasteful. If we climb using maximum weak mixture, and at a speed near that for minimum drag, we shall be saving fuel and may obviate having to open the cooling gills.

We of course need to know also the vertical rate of climb associated with these forward speeds; this is usually not more than 400 ft/min except where tropical conditions require a quicker ascent. When we have decided on the operating height, the time taken to reach this height can easily be found.

6.2. Fuel Consumption

6.2.1. FORMULAE. As regards the fuel consumed on the climb, we must first of all fix the fuel allowance for run-up, taxying, and take-off. Added to this there will be fuel consumed in proportion to the operating height. The total fuel required for take-off and climb is thus of the form

$$F_c = A + Bh, \text{ where } h \text{ is the height in feet}$$

Examples of this, worked out from actual flight tests, are

(i) Constellation, $F_c = 205 + 0{\cdot}095\,h$ lb.
(ii) Viking, $F_c = 108 + 0{\cdot}029\,h$ lb.

Graphs of these results are shown in Figs. 41 and 42.

6.2.2. TABULATION. In practice a more convenient method for the actual climb is to tabulate the forward distance, time, and fuel required, and similarly for the descent. A typical case is given below.

Climb

Climb at 400 ft/min, and 160 knots C.A.S. Mean T.A.S. = 172 knots.

Height (of Climb (ft)	Time Taken (min)	Fuel Used (Imp. gal)	Forward Distance Covered (nautical miles)						
			Headwind Component (knots)			Still Air	Tailwind Component (knots)		
			60	40	20		20	40	60
2 000	5	10	9	11	12	14	16	17	19
4 000	10	20	18	21	24	28	32	35	38
6 000	15	30	28	33	38	43	48	53	58
8 000	20	40	38	45	51	58	65	71	78
10 000	25	50	46	55	63	71	79	87	96

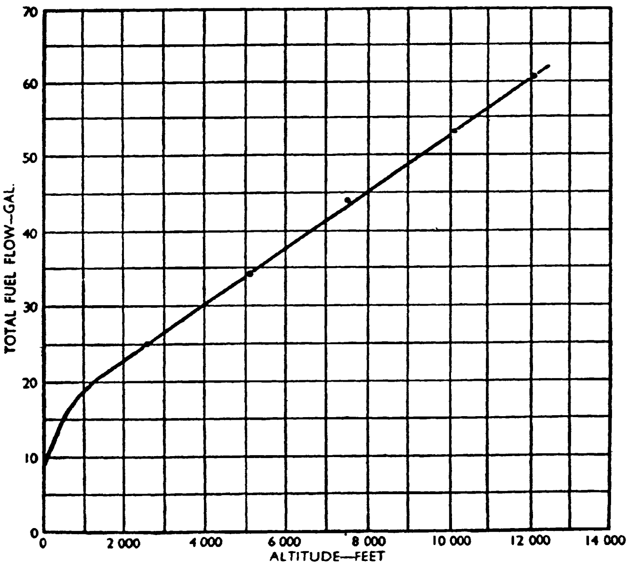


FIG. 41. CLIMBING DATA: VIKING

Total fuel consumed during climb versus height of climb. (Taxi and take-off allowance included.)

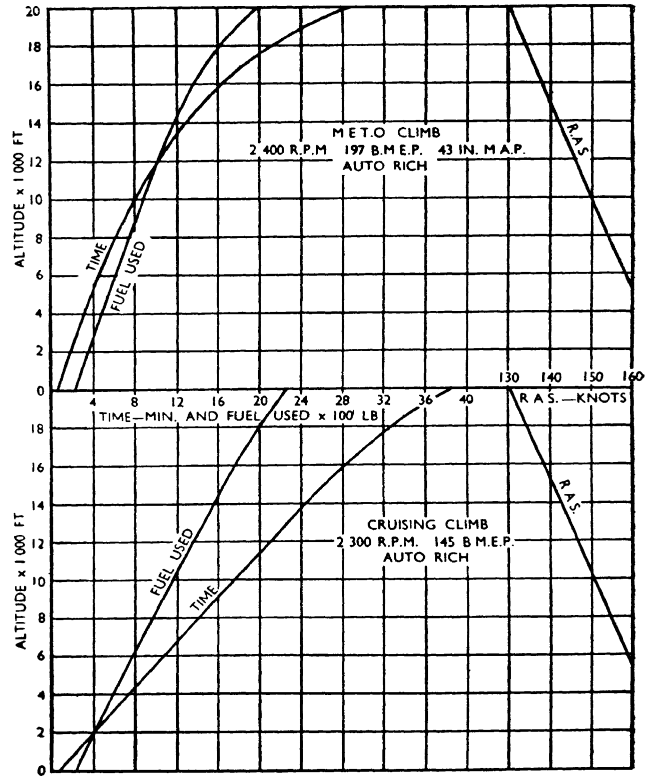


FIG. 42. CLIMBING DATA: LOCKHEED 49–51 CONSTELLATION

Descent

Descent at 300 ft/min, and 160 knots C.A.S. Mean T.A.S. = 172 knots.

Height from which Descent is Made (ft)	Time Taken (min)	Fuel Used (Imp. gal)	Forward Distance Covered (nautical miles)						
			Headwind Component (knots)			Still Air	Tailwind Component (knots)		
			60	40	20		20	40	60
10 000	33	43	62	73	84	95	106	117	128
8 000	27	35	51	60	69	78	87	96	105
6 000	20	26	37	44	50	57	63	70	77
4 000	13	17	25	29	33	38	43	47	51
2 000	6	8	13	15	17	19	21	23	25

6.3. Allowances

Let us now examine each part of the climb and descent in more detail, giving results applicable to most piston-engined aircraft.

	Phase	Time Allowance	Fuel Allowance
1	Start . .	Less than a minute	½ lb
2	Warm-up . .	*Nil* (done before passengers enter)	24 lb per engine
3	Taxi. . .	3 min, average	1½ lb per engine per minute
4	Take-off . .	3 min, average	Assume 12 lb per engine per minute (but available from makers' consumption charts)
5	Climb . .	$\frac{h}{300}$ to $\frac{h}{500}$ min according to rate climb (h the height in feet)	Take $7\left(\frac{h}{300}\right)$ lb to $7\left(\frac{h}{500}\right)$ per engine according to rate of climb
6	Glide . .	$\frac{h}{200}$ to $\frac{h}{300}$ min according to rate of descent. Forward speed same as cruising speed	Take as identical with cruise at op. height
7	Circuit . .	15 to 45 min	Fuel used at cruising power
8 9 10	Land Taxi Stop } . .	3 min	18 lb
11	Total for Manœuvring .	10 min	

6.4. Equivalent Geographical Mileage

The problem of climb and descent is sometimes tackled in a slightly different way.

Suppose that *ABDC* in Fig. 43 represents the path of the actual aircraft travelling from *A* to *C*. Imagine a fictitious aircraft which starts at *A′* at cruising height, reaches *B′* (coincident with *B*) at the same time that the real aircraft reaches *B*, and reaches *C′* at the same time that the real aircraft reaches the destination *C*. The fictitious aircraft flies at the same mean speed as the actual aircraft.

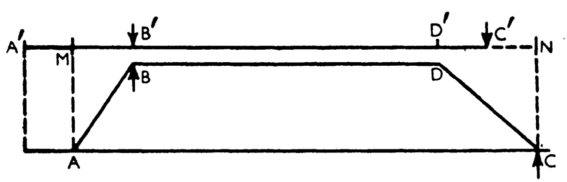


FIG. 43. EQUIVALENT GEOGRAPHICAL MILEAGE

The distance $A'C' = AC + A'M - C'N$ is called the *equivalent geographical mileage* or E.G.M., where $A'M$ is the *E.G.M. on climb* and $C'N$ is the *E.G.M. on descent.*

The fuel used by the real aircraft between *A* and *B* will be greater than that used by the fictitious aircraft between *A′* and *B′*; this is referred to as the "fuel lost" on the climb. The fuel used between *D* and *C* will be greater or less than that used between *D′* and *C′*; this is the "fuel lost" on the descent.

For example, for a Constellation, climbing to 15 000 ft and thereafter using constant power.

Route	*Forward Speed*	*Time*	*Fuel used*
AB	172 knots	26 min	1 635 lb
A′B′	216 knots	26 min	1 000 lb

Distance $MB' = 75$ naut. miles, distance $A'B' = 94$ naut. miles; so that the equivalent geographical mileage on the climb, $A'M$, is 19 miles; and the lost fuel on the climb is 635 lb.

Both equivalent geographical mileage and lost fuel are independent of wind speed, and are therefore best obtained from a consideration of still air conditions. So long as a constant rate of climb is maintained, lost fuel per engine is independent of gross weight and the drag characteristics of the aircraft. It may be calculated from the engine power and consumption charts. Equivalent geographical mileage is dependent upon gross weight, the drag characteristics, and the variation of climb power with height.

For any particular aircraft, charts can be prepared giving E.G.M. and lost fuel, on climb and glide for a variety of take-off weights. When these are done, it will be easy to apply these quickly for any particular height and weight.

CHAPTER VII

Air Traffic

7.1. Airport Traffic Control

7.1.1. GENERAL. The code of laws governing the handling of aircraft began with A.N.D., or Air Navigation Directions, issued in 1919. Since then many new regulations have been added, and others have been amended or cancelled. These regulations cover such diverse matters as the size of the letters giving the registration marks of the aircraft, the form of the C. of A., the licensing of pilots, etc., and the responsibilities of the ground engineer. Those who are interested should consult the relevant M.C.A. papers. In this chapter we attempt to give some picture of the rules regarding air traffic; the general body of laws we refer to as "Civil Air Regulations." All aeronautical activities at any airport, or landing area, and all flying or aircraft departing from or arriving at an airport in the air space which constitutes an airport approach zone or air traffic zone, are conducted in conformity with the current pertinent provisions of the M.C.A. Regulations or appropriate authority.

Pilots should bear in mind that they have the privilege of asking for a change in instructions if they believe another course of action will be safer than the one requested by an airport traffic tower.

When flying in visual flight rule weather conditions, it is the direct responsibility of the pilot to avoid collision with other aircraft. Under such conditions, the information and instructions issued by the control tower are intended to aid pilots to the fullest extent in avoiding collisions. In this connexion a clearance issued by a tower (such as "*Cleared to Land*") by either radio or visual signal is permissive in nature and does not relieve the pilot from exercising a reasonable degree of caution in executing the provisions of the clearance. However, such clearances will not be issued unless, in the opinion of the tower, the anticipated action can be safely completed from a collision hazard standpoint if reasonable caution is exercised by the pilot.

7.1.2. Taxying. No person should taxi an aircraft until he has ascertained through information furnished by airport attendants, or otherwise, that there will be no danger of collision with any person or object in the immediate area.

At airports where a control tower is in operation, it is the duty of a pilot to obtain clearance before taxying on to or across any runway.

A uniform system of hand signals covering operation and movement of aircraft on the ground is available.

7.1.3. Landing and Take-off. Pilots of aircraft must not land or take-off at a landing area, where an airport traffic control tower is in operation, contrary to instructions received by radio or visual signals from the tower. If a pilot does not receive either radio or visual signal from the tower when approaching for a landing, he must make at least one complete circle of the airport, conforming to the traffic pattern, and then start approach for a landing. The pilot should be on the alert for a signal from the tower and maintain a watch for other aircraft.

Aircraft approaching for a landing should, unless impracticable, maintain a straight approach course for the last 1 000 ft before crossing the airport boundary.

Landings and take-offs should be made on the runway most nearly aligned with the wind or, when winds are light, in the direction indicated by a controlled T or similar device unless otherwise authorized by an airport traffic control tower.

No turn should be made after take-off until the airport boundary has been reached and the pilot has attained an altitude of at least 500 ft and has ascertained that there will be no danger in turning into the path of a following aircraft, unless exceptions are authorized by an airport traffic control tower.

Aircraft departing from or arriving at a controlled airport will take precedence over other air traffic within the airport traffic zone of such airport.

7.1.4. Traffic and Taxi Patterns. Airport traffic control tower personnel are guided by certain standards in the control of air traffic to insure the orderly flow of traffic on the landing area and in the air space surrounding the landing area. These standards take the form of definite patterns for the landing area concerned. Air traffic controllers issue such specific instructions to individual aircraft as are necessary to insure that the aircraft generally follow the desired flight path (traffic patterns) when flying in the airport traffic zone and the proper taxi route (taxi pattern) when on the ground.

Traffic and taxi patterns are prepared by the appropriate authority in collaboration with local representatives of the airport management. The controller bases his instructions on these patterns to obtain the desired flow of traffic in the vicinity of the airport.

The following terminology for the portions of a visual approach to a landing has been adopted as standard for use by control towers and pilots.

(1) *Downwind leg*—that portion of the approach parallel to, but in the opposite direction to, the landing.

(2) *Base leg*—that portion of the approach at right angles to the landing direction on the downwind side of the airport.

(3) *Final approach*—that portion of the approach from the last turn into the landing direction until contact is made with the airport.

7.2. Intercommunication

7.2.1. LIGHT SIGNALS PROCEDURES FOR AIRPORT TRAFFIC CONTROL. The following procedures are used by air traffic control towers in the control of aircraft not equipped with radio. These same procedures are used to control aircraft equipped with radio if radio contacts cannot be established.

Airport traffic control personnel use a directive traffic control signal which emits an intense narrow beam of a selected colour (either red or green) when controlling traffic by light signals. The normal range of the signal in good weather is 10 miles in daytime and 15 miles at night and is readily discernible to the pilot of any aircraft that is visible to the controller.

Although the traffic signal light offers the advantage that some control may be exercised over non-radio equipped aircraft, all pilots should be cognizant of the disadvantages which are—

(1) The pilot may not be looking at the control tower at the time a signal is directed towards him.

(2) The instructions transmitted by a light signal are very limited since only approval or disapproval of a pilot's anticipated actions may be transmitted. No supplementary or explanatory information may be transmitted except by the use of the "General Warning Signal" which advises the pilot to be on the alert.

7.2.2. Aircraft In-bound. When an aircraft is in flight—

(i) A green light from a directed traffic control light means, "Cleared to Land."

(ii) A red light from a directed traffic control light means, "Give Way to other Aircraft and Continue Circling."

(iii) A series of red flashes from a directed traffic control light means, "Aerodrome Unsafe, Do not Land."

(iv) A series of green flashes, "Return for Landing."

During the hours of darkness, a pilot wishing to land should turn on a landing light as he approaches the airport unless he has already been given a green light.

A series of flashes of a landing light by a pilot intending to land means—

(*a*) If the floodlight is on, the pilot wants it turned off.

(*b*) If the floodlight is off, the pilot wants it turned on.

Pilots should acknowledge light signals by rocking their wings during the hours of daylight or by blinking their landing lights during the hours of darkness.

7.2.3. Aircraft on the Airport. During the hours of darkness, a pilot wishing to attract the attention of the air traffic control tower operator should turn on a landing light and taxi the aircraft in position so that light is visible to the tower operator. The landing light should remain on until appropriate signals are received from the tower.

When a pilot is taxying—

(i) A red light from a directed traffic control light means "Stop."

(ii) A series of red flashes from a directed traffic control light means, "Taxi Back to Hangar Line."

(iii) A series of green flashes from a directed traffic control light means, "Cleared to Continue Taxying."

When a pilot is in position for take-off—

(iv) A red light from a directed traffic control light means, "Clear the Runway Immediately and Wait."

(v) A green light from a directed traffic control light means, "Cleared for Take-off."

Pilots should acknowledge light signals by moving the ailerons of rudder during the hours of daylight or by blinking the landing lights during the hours of darkness.

7.2.4. General Warning Signal. A series of alternating red and green flashes from a directed traffic control light is used as a general warning signal to advise a pilot to be on the alert for hazardous or unusual conditions.

7.2.5. Suspension of Visual Flight Rule Operations. During the hours of daylight, the lighting of the rotating beacon or flashing amber light, and during the hours of darkness, flashing lights outlining the traffic direction indicator (tetrahedron, wind tee or other device), mean that flying in accordance with visual flight rules has been suspended.

7.3. Radio-telephone Communication

The following phraseologies and procedures are used in all radio-telephone communications with aeronautical ground stations—

7.3.1. Identification of Ground Stations. Control towers are identified during radio-telephone communications by the name of the airport followed by the word "Tower" as, for example, "Brimpton Tower," "London Tower," etc. Towers operating approach control service are identified as: "London Approach Control," "Hurn Approach Control," etc., when issuing clearances over the voice feature of the radio range or other approach control channel.

7.3.2. Call-up and Replies. The call-up procedure used in radio-telephone communications consists of the following—

Item	*Example*
(*a*) Designation of the station called	George Baker Fox
(*b*) This is	This is
(*c*) Designation of the calling station	Brimpton Tower
(*d*) Invitation to reply	Over

The reply to an initial call-up consists of—

Item	*Example*
(*a*) Designation of the station called	Brimpton Tower
(*b*) This is	This is
(*c*) Designation of the answering station	George Baker Fox
(*d*) Invitation to reply	Over

Communication is initiated by call-up and reply when—

(*a*) Communication has not been established.

(*b*) Previous contact has been terminated.

After contact has been established in accordance with the above, a second call-up followed immediately by the message should be made.

If it is reasonably certain that the aircraft will receive the initial call-up, an airport traffic control tower may follow the first call-up with the message without waiting for a reply from the aircraft. However, pilots of aircraft should remember that an airport traffic control tower may be receiving messages from several aircraft simultaneously, and therefore the pilot should always receive an "invitation to reply" (Over) from the tower before proceeding with a message.

After communication has been definitely established, continuous intercommunication may be conducted without further call-up or identification other than preceding message with the aircraft identification of the aircraft concerned until termination of the contact.

7.3.3. TERMINATION OF COMMUNICATION. An aircraft should acknowledge receipt of a radio-telephone message by transmitting the "*Aircraft Identification*" followed by the word "*Roger.*" The word "*Out*" shall also be used when conversation is ended and no response is expected.

Examples: "*George Baker Fox, Roger, Out.*"

7.3.4. AIRCRAFT DEPARTING. The pilot will call the control tower when ready to taxi out. The body of the message should include (location) permission to taxi.

Example—

Brimpton Tower, this is George Baker Fox, on ramp or at dispersal. Permission to taxi. Over.

George Baker Fox, this is Brimpton Tower. Clear to taxi. Runway twenty-four, wind, etc., altimeter, etc. (and any other relevant information). Over.

George Baker Fox. Roger. Out.

Pilots desiring to file Flight Plans should do so through the Flight Control Officer. To avoid radio congestion, pilots should not file Flight Plans by radio when other means are available.

If an airway or area traffic control clearance is necessary, the A.T.C. Officer will relay the clearance to the pilot as follows—

Tower: George Baker Fox. A.T.C. clears you to Uxbridge control. Cruise at three thousand. Leave zone, etc. Over.

Aircraft: George Baker Fox. Cleared to Uxbridge Control. Cruise at three thousand feet, etc. Over.

Tower: George Baker Fox. Roger.

After the A.T.C. clearance has been issued and acknowledged, the pilot has completed cockpit drill and is ready for immediate take-off.

Example—

Brimpton Tower, this is George Baker Fox. Permission to take-off. Over.

George Baker Fox, this is Brimpton Tower. Cleared for take-off (and any relevant flying directions) or Hold your Position. Over.

George Baker Fox. Roger. Out.

After the take-off the pilot normally has no occasion to use his transmitter again except to acknowledge receipt of further information or instructions from the control tower.

The pilot should continue to guard the control tower frequency until he leaves the airport traffic zone, or airport approach zone. When the pilot must use the navigational feature of the radio range or Beacon immediately after take-off, the control tower will clear him to leave tower frequency as soon as possible.

7.3.5. Aircraft Arriving. A pilot flying under visual flight rule conditions should call the control tower for local traffic information and landing instructions when approximately 15 miles from the airport of destination. A pilot flying under instrument flight rule conditions should not call the control tower until an appropriate clearance has been received from Air Traffic Control.

The body of the message should include—

(*a*) Geographical position.
(*b*) Time (optional).
(*c*) Flight altitude of the aircraft.
(*d*) Request for information or instructions—if pertinent.

Example—

Brimpton Tower, this is George Baker Fox. Approaching you from 15 *miles N.E. at three thousand V.F.R. Landing Instructions. Over.*

The airport traffic control tower will then acknowledge this message and issue an appropriate clearance such as "cleared to enter traffic pattern." Clearance to enter traffic pattern informs the pilot that traffic exists in the traffic pattern and authorizes entry into the traffic pattern but does not constitute landing authority. Wind information and number of runway in use is included in this clearance to assist the pilot in making his approach for entry into the traffic pattern but clearance to land is ordinarily withheld until the aircraft is in sight of the control tower and no conflicting traffic will interfere with the landing.

Example—

Tower: George Baker Fox 15 *miles N.E. at three thousand. Cleared to enter traffic pattern (or join circuit at height). Wind, speed and direction. Runway in use, etc. Q.F.E. in M.B.S. Over.*

Aircraft: G.B.F. Roger.

A clearance to land is given, after a pilot reports in the airport traffic zone or when he is sighted from the control tower. The pilot should report to the control tower immediately on entry into the traffic pattern if the control tower has not previously sighted the aircraft and issued landing instructions. The pilot reports and the tower replies by issuing landing clearance if practicable, or suitable instructions.

Example—

Tower: George Baker Fox. Three miles N.W. at 1 000 *ft. Cleared to land.*

Wind information and runway number will again be supplied if a revision to the information previously given is necessary. The pilot should acknowledge and indicate compliance with instructions. If one or more preceding aircraft are approaching for a landing, or are in the traffic pattern for landing instructions, the Air Traffic Controller will issue a landing sequence number.

Example—

George Baker Fox. Three miles N.W. at 1 000 ft. Number TWO to land. Follow B.O.A.C. DAKOTA, etc.

After the preceding aircraft completes landing, the air traffic controller will then issue clearance to land. After a pilot has landed the air traffic controller will furnish any necessary instructions relative to taxying. This control will be continued until the pilot has parked his aircraft. The control tower operator will initiate calls to in-bound aircraft which have not called the tower as soon as such aircraft are observed.

At the conclusion of each flight the Flight Control Officer at the aerodrome of landing will send an arrival message through the area control to other area controls concerned.

7.4. Visual Flight Rules

7.4.1. Whenever an aircraft is being operated in weather conditions equal to or better than the basic minimum specified in Civil Air Regulations such conditions of flight are known as *Visual Flight Rules*, abbreviated V.F.R. The entire responsibility of avoiding other aircraft under these conditions rests with the pilot who is required to maintain at least those basic minimums specified in the Civil Air Regulations (not including the exceptions which may be authorized by Air Traffic Control).

Flight in accordance with Visual Flight Rules in weather conditions less than the basic minimums (but not less than the reductions to those minimums which may be authorized by Air Traffic Control) is known as *Controlled Visual Flight Rule Flight*. These flights will be individually authorized in the same manner as Instrument Flight Rule Flights as explained in a later section.

7.4.2. FLIGHT PLANS. A Flight Plan is not normally required for V.F.R. flight, but if desired or if required by current regulations, may be submitted to the nearest airway traffic control centre, airport traffic control tower or airway communications station either in person or by Flight Control Officer. Flight Plans may be filed by radio if no other means are available, but this practice should be avoided whenever possible to reduce congestion of radio channels. The flight plan shall contain the following items—

(*a*) Identification of aircraft and pilot.

Example: *Dakota GABCD Smith.*

(*b*) Number of aircraft if a formation flight.

Example: *Two Dakotas.*

(*c*) Point of departure, proposed cruising altitude, route of flight and point of first intended landing.

Example: *Portsmouth Visual Flight Rules (V.F.R.) to Heath Row.*

(*d*) Proposed indicated airspeed.

(*e*) Usable radio equipment carried in aircraft.

(*f*) Proposed time of departure.

Example: *Departing Portsmouth* 1405.

(*g*) Estimated elapsed time in hours in minutes.

In connexion with item (*c*) of the flight plan, a visual flight rule flight plan may specify "V.F.R." only at a cruising altitude. The use of this term in lieu of an actual altitude indicates that no traffic control clearance is desired and that the pilot intends to fly in weather conditions equal to the basic minimums in Civil Air Regulations. The only report required of a pilot submitting this type of flight plan is the hourly position report and an arrival report.

7.4.3. Basic Visual Flight Rule Minimum. Flight may be conducted without a traffic control clearance (i.e. on a V.F.R. flight plan or with no flight plan) under flight conditions better than the following minimums—

Within Tower Control Zone. Aircraft may be operated in an aerodrome control zone at all altitudes if visibility is at least 3 miles and clouds are avoided by at least 500 ft vertically and 2 000 ft horizontally.

Within Airport Control Zone. Aircraft may be operated in an airport control zone but outside of a Tower Control zone at altitudes above 700 ft if visibility is at least 3 miles and clouds are avoided by at least 500 ft vertically and 2 000 ft horizontally. Flight at altitudes below 700 ft may be conducted with visibility down to 1 mile.

Within Air Traffic Control Area. Aircraft may be operated in a control area but outside of an airport control zone at altitudes above 700 ft if the visibility is at least 3 miles and clouds are avoided by at least 500 ft vertically and 2 000 ft horizontally. Flight at altitudes below 700 ft may be conducted with visibility down to 1 mile and need only remain clear of clouds (no specified distance).

Aircraft may be operated outside of Tower Control Zones, Airway Traffic Control Areas, Airport Control Zones and outside of those portions of Airport Control Zones lying within control areas at any altitude if the visibility is at least 1 mile. Aircraft operating at more than 700 ft above the surface are required to maintain at least 500 ft vertical and 2 000 ft horizontal separation from all clouds; however, aircraft below 700 ft need only remain clear of clouds.

7.4.4. FLIGHT ALTITUDES. It is recommended that V.F.R. flights operating at altitudes more than 700 ft above the surface be conducted at flight levels specified in the following sections.

7.5. Controlled V.F.R. Flight

Traffic conditions permitting, Air Traffic Control can authorize visual flight rule flight in weather conditions less than the basic V.F.R. minimums. The exceptions permitted include controlled V.F.R. flight within Aerodrome Control Zones at all altitudes with less than the specified visibility and cloud clearance minimums; within control areas at altitudes above 700 ft above the surface with less than 3 miles visibility, but not less than 1 mile; within control zones below 700 ft at less than the cloud clearance minimums. All visual flight rule flights must remain "clear of clouds" regardless of authorization received from Air Traffic Control.

Controlled V.F.R. flights are authorized by Air Traffic Control at the specific request of the pilot concerned, provided actual and anticipated traffic conditions permit. Clearance of each such flight depends entirely upon local traffic conditions, whether the aircraft is equipped with functioning two-way radio, and/or the extent of the flight proposed. For example, an aircraft equipped with two-way radio may be authorized to enter the aerodrome control zone when the visibility is less than 1 mile; or, when the restricting weather element is known to be a local condition, an aircraft without radio may be completed within 15 minutes (or the time allowed for a standard instrument approach) after passing over the radio range station on the initial approach, or within 15 miles after being issued approach clearance under conditions of approach sequence assignment, a pilot shall report this information and obtain further instructions from Air Traffic Control. Air Traffic Control then will determine whether the pilot will be allowed another immediate attempt or instruct him to stand by on a designated leg of the range at an assigned altitude until other aircraft in line have landed or taken off. This decision will be based upon existing conditions such as remaining fuel, weather trend, etc. A decision to route an aircraft to an alternate airport will be made by the pilot after conferring with the Air Traffic Control personnel concerned.

7.6.14. STANDARD INSTRUMENT APPROACH. An aircraft instructed to make a Standard Instrument Approach on a specified radio range shall cross the range station at the approved Initial Approach Altitude, proceed out the Approach course of the radio range for a period of 4 minutes, make procedure turn and descent to cross the range station at the approved Final Approach Altitude. Descent shall then be made to the Minimum Altitude and, if the ground is sufficiently visible, landing may be made. If the ground is not visible, the aircraft shall proceed in accordance with the established Missed Approach Procedure or as directed by Air Traffic Control. The approved Initial and Final Approach Altitudes, unless otherwise specified by Air Traffic Control, shall be those specified by appropriate authority.

If visual reference to the ground is established before completion of the approach procedure, the entire procedure must nevertheless be executed unless the pilot requests and is granted clearance to proceed to the airport.

7.7. Air Traffic Control in Great Britain

A Notice to Airmen, No. 250 of 1947, is of great importance to all those concerned with air line, charter, service and private flying. The basis of the plan introduced by the Notice is the formation of two Control Areas, covering most of Southern England, in which all aircraft between 3 000 ft and 10 000 ft (5 000 ft and 10 000 ft in the Western Area), when flying under instrument flying rules, must conform with certain laid-down regulations.

The object of the introduction of this plan is to promote greater safety and regularity of air traffic. Its effective application depends largely on the co-operation of the air-line operators, the military services, and other users, and on the communication navigational and instrument landing aids which they decide to carry in their aircraft. In this respect the plan is a development of existing procedures, and does not appear to call for the use of additional equipment. In fact, it acknowledges the limitations of existing aids by not calling for their use.

The controller is still in advisory capacity and his instructions are not mandatory. However, under the 1923 statute, and later under new legislation which has been introduced to simplify procedure, action may be taken against a pilot who by disregarding the controller's instructions without good reason prejudices the safety of aircraft.

The Notice to Airmen specifies a number of Flight Information Regions, Control Areas and Control Zones, within which aircraft have to adhere to certain rules, according to the weather in which they are flying. If the pilot of an aircraft keeps quite clear of cloud and can see well ahead (the limits are defined) he need only worry about the Visual Flight Rules (V.F.R.). These are nothing more than the ordinary rules of the air which have been a statutory requirement since 1923. If, however, for any reason such as general weather conditions, maintaining schedule, etc., he has to fly in or near cloud, or in poor visibility, then the pilot must obey the Instrument Flying Rules (I.F.R.), which are additional to V.F.R.

When flying in one of the five U.K. Flight Information Regions (irrespective of the height), aircraft may be supplied with information on weather, serviceability of aerodromes and equipment, and details of other known aircraft's movements. This service is rendered by the Control Unit—known as Area Control—on whom also rests the onus for alerting the search and rescue organization when necessary.

Within Flight Information Regions there may be areas of great traffic density, where the need for more stringent control is necessary in the interests of safety. Such areas have been least. This is at a speed about 32 per cent higher than the speed for minimum drag.

In symbols,

$$\text{a.m.p. lb.} = \frac{V_i}{\sqrt{\sigma}\, c_f'\, D} = \frac{V_i}{\left(aV_i^2 + \dfrac{bW^2}{V_i^2}\right)\sqrt{\sigma}\, c_f'}$$

which is greatest when $V_i = (3)^{\frac{1}{4}} \left(\frac{b}{a}\right)^{\frac{1}{4}} \sqrt{W}$, i.e.

V_i (jet max. a.m.p. lb.) $= 1{\cdot}316 \times V_i$ (piston max. a.m.p.g.)

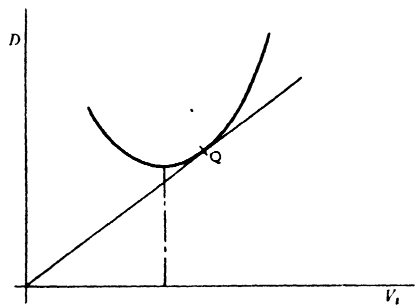


FIG. 45. POINT OF GREATEST $\frac{V_i}{D}$

The above formula illustrates another point in connexion with jet engines; namely that, so long as c'_f remains roughly constant, it pays to fly high.

8.4. Minimum Drag

We have already found that

$$\text{a.m.p.g.} = \frac{3\,000}{D}\frac{\eta}{c_f}$$

and we know that, for a given weight, the drag is constant for a particular *indicated* speed. As height changes, we should normally fly at a constant indicated speed, in order to keep the same angle of attack. To maintain the same *true* speed will be less satisfactory; the drag may decrease with height, and so may the b.h.p., but such a decrease is by no means obvious. The manner of the variation of D and b.h.p. with height, at a given true speed and weight, can be seen from the equations

$$D = a\sigma V^2 + \frac{bW^2}{\sigma V^2}$$

$$\text{b.h.p.} = \frac{1}{550\eta}\left(a\sigma V^3 + \frac{bW^2}{\sigma V}\right)$$

Provided that the true speed V is *above* the speed for minimum drag, the term $a\sigma V^2$ will be larger than $bW^2/\sigma V^2$ and there will be an optimum σ for minimum D at fixed V and W. Taking the example of the last chapter, $a = 0{\cdot}0267$, $b = 0{\cdot}0465$ and the minimum drag true speed at sea-level was 212 ft/sec. Increase this by 10 per cent, to 233 ft/sec. Then the drag at sea-level will be

$$D_{SL} = 0{\cdot}0267 \times 233^2 + \frac{0{\cdot}0465 \times 34\,000^2}{233^2}$$

$$= 1\,450 + 980 = 2\,430 \text{ lb}$$

At 21 500 ft where $\sigma = \frac{1}{2}$ (approx.),

$$D_{21\,500} = 725 + 1\,960 = 2\,685 \text{ lb}$$

so that the drag is *greater* at this height, and so also the b.h.p. required. The best height to fly will be when

$$1\,450 - \frac{980}{\sigma^2} = 0 \text{ or } \underline{\sigma = 0{\cdot}822}, \text{ height} = \underline{\underline{6\,500 \text{ ft}}}$$

where $\quad D_{6\,500} = 1\,190 + 1\,190 = 2\,380$ lb

so that the saving in drag is only 50 lb, and in b.h.p. only about 26. These savings will, of course, be more noticeable at higher speeds; but enough has been said to show that, to obtain marked improvement in a.m.p.g. with height, we must examine the possibility of decreasing c_f, the specific consumption.

8.5. Variation with Height

Speaking generally, it would be expected (as explained in § **3.3.2**) that the optimum height for a.m.p.g. is usually the lowest height at which the power required for the minimum drag speed corresponds to full-throttle conditions at minimum r.p.m. Below the full-throttle height, the engine itself is not running at its best; above full-throttle height we have to cope with the density drop; which cannot be met entirely by further supercharging owing to the fuel used for giving extra r.p.m. to the compressor.

The existence of an optimum height can be seen from

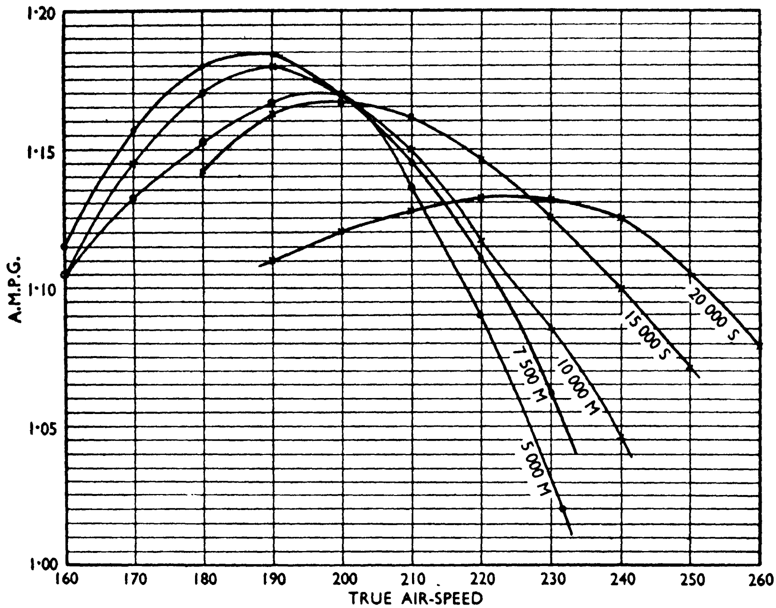


FIG. 46. VARIATION OF A.M.P.G. WITH HEIGHT

Four-engined aircraft.

Figs. 46, 47, and 48, which are all experimental results. Fig. 46 shows the a.m.p.g. curves for a four-engined transport at a certain weight at a number of different heights, plotted against T.A.S. Fig. 47 shows the maximum a.m.p.g. for a two-engined transport plotted against height, for a number of different weights. Fig. 48 shows the a.m.p.g. for the same aircraft as Fig. 47 at a certain fixed power plotted against height, for a number of different weights; and it will be noted that the total variation is here much less. An improvement of a.m.p.g. from 2·0 to over 2·5 is obtained by climbing from 2 000 ft to 10 000 ft, if flying at maximum range speed (at 26 000 lb); whereas the improvement with constant

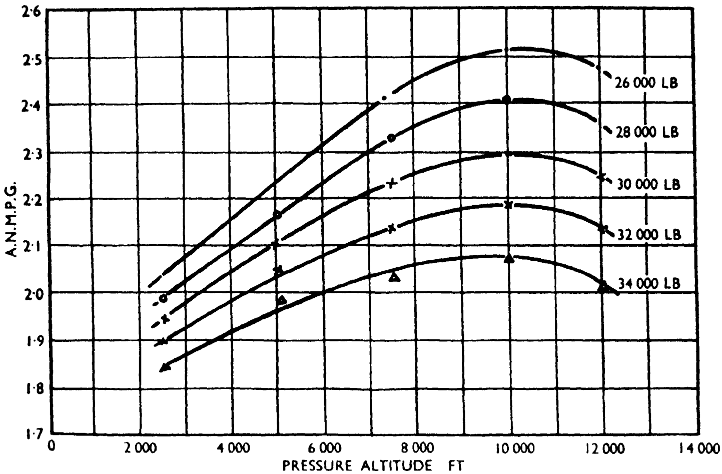


FIG. 47. VARIATION OF A.N.M.P.G. WITH HEIGHT: MAXIMUM RANGE CONDITIONS

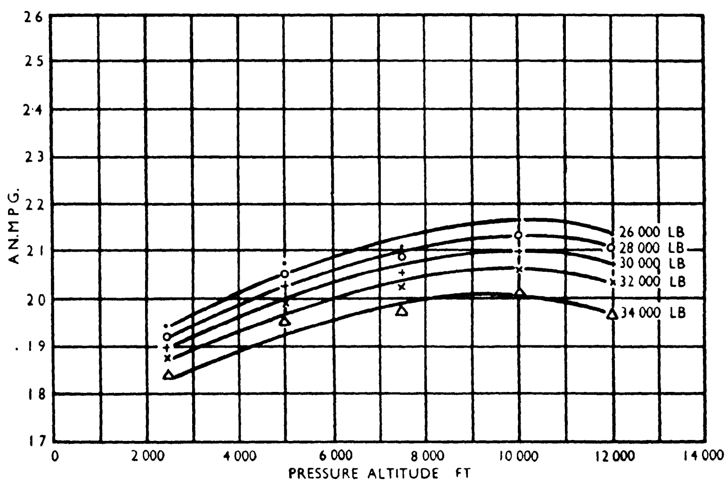


FIG. 48. VARIATION OF A.N.M.P.G. WITH HEIGHT

49 per cent M.E.T.O. power (760 b.h.p.)

power cruising is only from 1·94 to 2·17—a difference of 0·2 instead of 0·5. In other words, the economy to be effected by maximum range flying is likely to be largely nullified if the wrong height has to be chosen.

8.6. General Description of Grid

We now choose a particular height and draw a "grid" of a.n.m.p.g. curves for different weights. In the majority of cases they will appear something like Fig. 49.

They are flat-topped curves, with these two main characteristics—

(i) as W decreases, max. a.m.p.g. increases,
(ii) as W decreases, the speed for max. a.m.p.g. decreases.

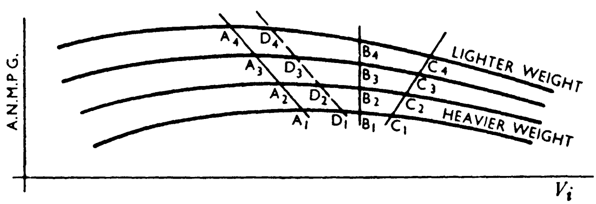


FIG. 49. TYPICAL A.M.P.G. GRID

The a.m.p.g. grid is the foundation of all cruise control regulations, charts, and tabulations. The airframe manufacturers must construct such graphs in the first instance at four or five chosen heights, in order to get some idea of the estimated performance; and later these are checked and modified by tests of the prototype and production aircraft. As soon as the grid is authenticated, it can be used as it stands for all types of flight planning, or it can be used to construct other charts which may be more readily followed in practice. Before explaining how this is done, it is as well to consider in what way the grid will be modified by headwinds and tailwinds.

8.7. Headwinds

8.7.1. EFFECT ON A.M.P.G. The method can be best illustrated by an example. The following are a set of figures applicable to an aircraft of 65 000 lb at 10 000 ft.

V, T.A.S. (m.p.h.)	190	200	210	220	230	240	250	260
E, Fuel Flow (gal/hr)	172	180	190	202	218	235	254	277
$\frac{V}{E}$ (a.m.p.g.)	1·105	1·115	1·105	1·09	1·055	1·022	0·985	0·94

These points give a maximum a.m.p.g. at a T.A.S. of about 200 m.p.h.

Against a headwind v m.p.h., when $v = 40$, the ground m.p.g. will be $\frac{V - v}{E}$, so that we continue the tabulation as follows—

$V - v$, Ground Speed	150	160	170	180	190	200	210	220
$\frac{V - v}{E}$ (g.m.p.g.)	0·873	0·889	0·895	0·892	0·874	0·852	0·826	0·795

The maximum g.m.p.g. now occurs at about 210 m.p.h.

This is equal to $200\left(1 + \frac{10}{200}\right) = V\left(1 + \frac{v}{4V}\right)$

The maximum g.m.p.g. is reduced from 1·115 to 0·895, which is approximately the original figure multiplied by $1 - \frac{40}{200}$, i.e. $1 - \frac{v}{V}$.

These two results can be stated as follows—

(i) For maximum range, increase the T.A.S. from V to $V\left(1 + \frac{v}{4V}\right)$, more accurately $V\left(1 + \frac{v}{4V} + \frac{7v^2}{32V^2}\right)$, so that the new actual ground speed is now $V\left(1 - \frac{3v}{4V}\right)$.

(ii) Modified g.m.p.g. = g.m.p.g. with no headwind $\times\left(1 - \frac{v}{V}\right)$; more accurately the factor is $\left(1 - \frac{v}{V} + \frac{v^2}{8V^2}\right)$.

If the above process is repeated for the other weights, we have a fresh grid which can be used as before, or for fresh lines on our charts.

8.7.2. TABLE OF WIND COMPONENTS. In the previous paragraph we have spoken of headwinds and tailwinds. Beam winds can be dealt with by using a table of wind components such as is given on p. 149.

8.8. Methods of Cruising

Three common methods of operating commercial aircraft are as follows—

(*a*) Maximum Range Flying,
(*b*) Constant Air-speed Flying,
(*c*) Constant Power Flying,

and we should know a little about each in turn.

8.9. Maximum Range Flying

As fuel is consumed, and the weight of the aircraft decreases, we fly at the speeds typified by the points A_1, A_2, A_3, A_4 of the a.m.p.g. grid in Fig. 49.

This requires us to lower speed as the trip proceeds. This is of course a nuisance, and on short journeys the saving of fuel is negligible. On long journeys there are three further disadvantages—

(i) Towards the end of the journey the economical speed may be less than $V_{i_{mcc}}$, the *minimum speed for comfortable continuous cruising*, which is the speed fixed by the flight test pilots as a result of experience, and below which it may become difficult to control the aircraft and regain height lost through gusty or turbulent conditions.

In the opinion of the A. & A.E.E., Boscombe Down, the average value of $V_{i_{mcc}} = 1{\cdot}35V_{i_{mp}}$. (It is also taken, in some cases, as $1{\cdot}5V_{s_1}$), whereas we have seen that

$$V_{i_{\text{max. range}}} = 1{\cdot}395V_{i_{mp}} (= 1{\cdot}06 \times 1{\cdot}316 \times V_{i_{mp}}),$$

so that normally $V_{i_{\text{max. range}}} > V_{i_{mcc}}$, but this is by no means always so, particularly at the lighter weights. It may happen, for instance, that one engine of a twin-engined aircraft fails. If we then operate the other engine at maximum weak mixture power, thereby not increasing the fuel consumption, we may find that at some stage of the journey the speed falls below $V_{i_{mcc}}$.

(ii) If the aircraft encounters a considerable headwind, such

Angle Between Wind and Track Directions	Wind Speed in Knots											
	10	20	30	40	50		60		70		80	
	Aircraft True Air-speed in Knots											
	All Speeds	All Speeds	All Speeds	All Speeds	120–170	170–220	120–170	170–220	120–170	170–220	120–170	170–220
0	- 10	- 20	- 30	- 40	- 50	- 50	- 60	- 60	- 70	- 70	- 80	- 80
5 or 355	- 10	- 20	- 30	- 40	- 50	- 50	- 60	- 60	- 70	- 70	- 80	- 80
10 or 350	- 10	- 20	- 30	- 40	- 50	- 50	- 60	- 60	- 70	- 70	- 80	- 80
15 or 345	- 10	- 20	- 29	- 39	- 49	- 49	- 59	- 59	- 69	- 69	- 79	- 79
20 or 340	- 10	- 19	- 29	- 39	- 49	- 49	- 59	- 59	- 69	- 69	- 79	- 79
25 or 335	- 9	- 19	- 28	- 38	- 47	- 47	- 57	- 57	- 67	- 67	- 77	- 77
30 or 330	- 9	- 18	- 27	- 36	- 47	- 46	- 56	- 55	- 66	- 65	- 76	- 75
35 or 325	- 8	- 17	- 25	- 35	- 45	- 44	- 54	- 52	- 64	- 62	- 74	- 72
40 or 320	- 8	- 16	- 24	- 33	- 42	- 41	- 51	- 49	- 62	- 59	- 72	- 68
45 or 315	- 7	- 15	- 22	- 31	- 40	- 39	- 49	- 47	- 59	- 56	- 69	- 65
50 or 310	- 7	- 14	- 21	- 29	- 38	- 36	- 46	- 43	- 56	- 53	- 66	- 61
55 or 305	- 6	- 12	- 19	- 27	- 36	- 33	- 43	- 40	- 53	- 49	- 63	- 57
60 or 300	- 5	11	- 18	- 24	- 33	- 31	- 40	- 37	- 50	- 46	- 60	- 53
65 or 295	5	- 10	- 15	- 22	- 30	- 27	- 36	- 33	- 46	- 41	- 55	- 47
70 or 290	- 4	- 8	- 13	- 19	26	- 24	- 32	- 29	- 41	- 36	- 49	- 42
75 or 285	3	- 6	- 10	- 16	- 23	- 20	- 28	- 24	- 36	- 31	- 44	- 37
80 or 280	2	- 5	- 8	- 13	- 18	- 15	- 24	- 20	- 31	- 26	- 39	- 31
85 or 275	1	- 3	6	- 10	- 14	- 11	- 19	- 15	- 25	- 20	- 32	- 24
90 or 270	0	- 1	- 3	- 5	- 10	- 6	- 13	- 9	- 19	- 13	- 26	- 17
95 or 265	+ 1	0	- 1	- 3	- 6	- 2	- 8	- 4	- 13	- 7	- 18	- 10
100 or 260	+ 2	+ 2	+ 2	+ 1	- 1	+ 2	- 3	+ 1	- 7	- 1	- 11	- 3
105 or 255	+ 3	+ 4	+ 4	+ 4	+ 4	+ 7	+ 2	+ 6	0	+ 6	3	+ 4
110 or 250	+ 3	+ 6	+ 8	+ 8	+ 9	+ 11	+ 8	+ 12	+ 7	+ 12	+ 5	+ 12
115 or 245	+ 4	+ 7	+ 10	+ 12	+ 14	+ 15	+ 14	+ 18	+ 14	+ 19	+ 14	+ 20
120 or 240	+ 5	+ 8	+ 12	+ 15	+ 18	+ 19	+ 20	+ 23	+ 21	+ 26	+ 22	+ 27
125 or 235	+ 6	+ 11	+ 15	+ 19	+ 23	+ 24	+ 25	+ 29	+ 27	+ 32	+ 29	+ 34
130 or 230	+ 6	+ 12	+ 17	+ 23	+ 27	+ 28	+ 31	+ 34	+ 34	+ 38	+ 37	+ 42
135 or 225	+ 7	+ 13	+ 20	+ 25	+ 31	+ 32	+ 35	+ 38	+ 40	+ 43	+ 44	+ 48
140 or 220	+ 7	+ 14	+ 22	+ 28	+ 34	+ 35	+ 40	+ 42	+ 45	+ 48	+ 51	+ 54
145 or 215	+ 8	+ 16	+ 24	+ 30	+ 37	+ 38	+ 44	+ 46	+ 50	+ 52	+ 57	+ 60
150 or 210	+ 8	+ 16	+ 25	+ 33	+ 41	+ 41	+ 48	+ 49	+ 56	+ 57	+ 63	+ 65
155 or 205	+ 9	+ 17	+ 26	+ 34	+ 43	+ 43	+ 51	+ 52	+ 60	+ 61	+ 68	+ 69
160 or 200	+ 9	+ 18	+ 27	+ 36	+ 45	+ 45	+ 54	+ 55	+ 63	+ 64	+ 71	+ 72
165 or 195	+ 10	+ 19	+ 29	+ 38	+ 47	+ 47	+ 56	+ 57	+ 66	+ 67	+ 75	+ 75
170 or 190	+ 10	+ 20	+ 30	+ 40	+ 49	+ 49	+ 58	+ 59	+ 68	+ 69	+ 78	+ 78
175 or 185	+ 10	+ 20	+ 30	+ 40	+ 50	+ 50	+ 60	+ 60	+ 70	+ 70	+ 80	+ 80
180	+ 10	+ 20	+ 30	+ 40	+ 50	+ 50	+ 60	+ 60	+ 70	+ 70	+ 80	+ 80

as may occur on the west-bound Atlantic crossing, the economical true air-speed rises $\left(\text{we found } V = \left(1 + \frac{v}{4V}\right)V\right)$. If this alteration of speed is not made, the aircraft will not be flying at maximum g.m.p.g. but at a lower speed, and so the trouble taken to reduce speed as fuel is consumed will be wasted energy.

(iii) The average ground speed over the distance is less than by any other method.

For these three reasons it is sometimes necessary to instruct pilots to fly at a speed some 10 to 20 per cent higher than that for maximum a.m.p.g., as shown by the points D_1, D_2, D_3, D_4, in the grid of Fig. 49. There will then be no danger of sinking below the minimum speed for comfortable cruising, and no alteration need be made should unexpectedly high headwinds be encountered. The disadvantage of altering speed as we enter each fresh zone will of course remain; and since we have agreed to jettison maximum economy of fuel, navigational simplicity suggests that it might be better to adopt the second alternative below.

In studying the performance estimates prepared by manufacturers, it is advisable to find out if the figures for range and payload are based on flight at maximum range. If they are, modifications may be necessary before deciding how the aircraft will fit in with a commercial route.

8.10. Constant Air-speed Flying

This is typified by the points B_1, B_2, B_3, B_4 in Fig. 49. When we come to consider current cruise control manuals in more detail, we shall find that the loss of payload, as compared with maximum range flying, is not very considerable, and may be negligible against a strong headwind. In any case, the consumption of fuel is considerably less than that resulting from the third method.

8.11. Constant B.H.P. Flying

This is typified by the points C_1, C_2, C_3, C_4 in Fig. 49. On a long trip, particularly with a tailwind, the waste of fuel may be large. As against this, the method has two advantages—

(i) It is easier for the pilot and flight engineer.

(ii) There may be considerable saving in *time* for the journey.

8.11.1. Choice of Power. It should be realized that the *actual* power to be used is a matter of choice. Most charts at present in use are constructed for a fixed power, assuming

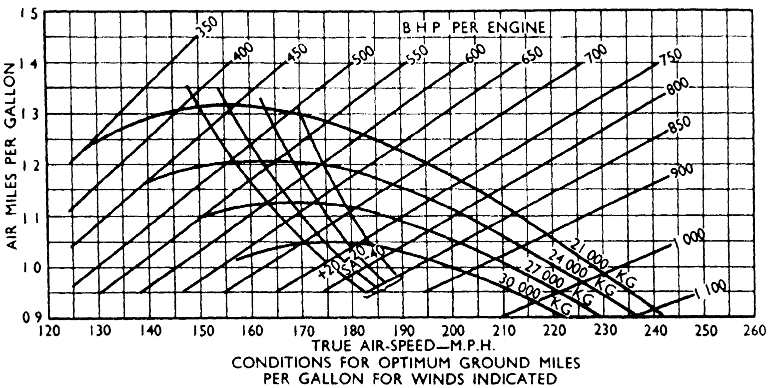


Fig. 50. A.M.P.G. Grid: 2 000 ft

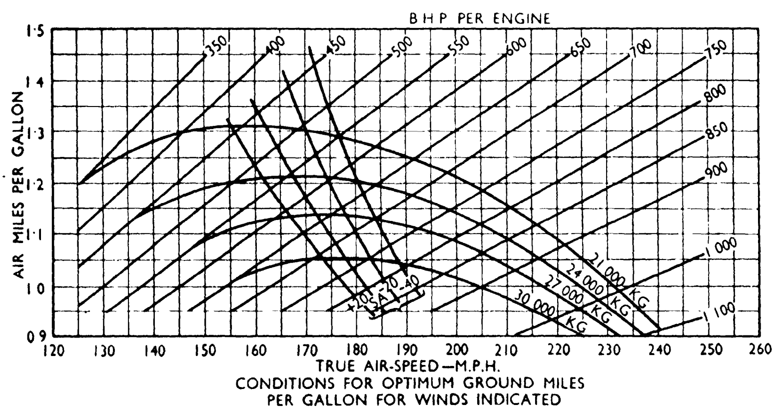


Fig. 51. A.M.P.G. Grid: 4 000 ft

maximum take-off weight, and a series of journeys of about the same distance. But a free-lance operator might be called upon to undertake journeys of greatly varying distance, and with all sorts of loading conditions. He would then strive for "maximum payload miles per hour," a method which attempts to strike a balance between the claims of speed and payload. This point is considered in more detail when dealing with payload calculations in Chapter XII, where it is shown that the optimum constant b.h.p. is considerably reduced as the range to be covered increases. This aspect of the matter

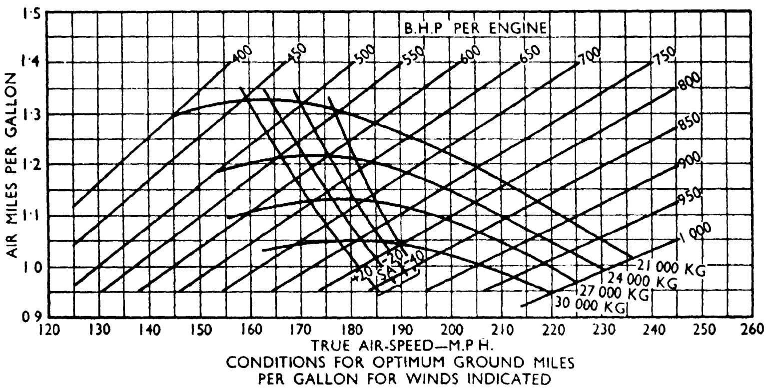


Fig. 52. A.M.P.G. Grid: 6 000 ft

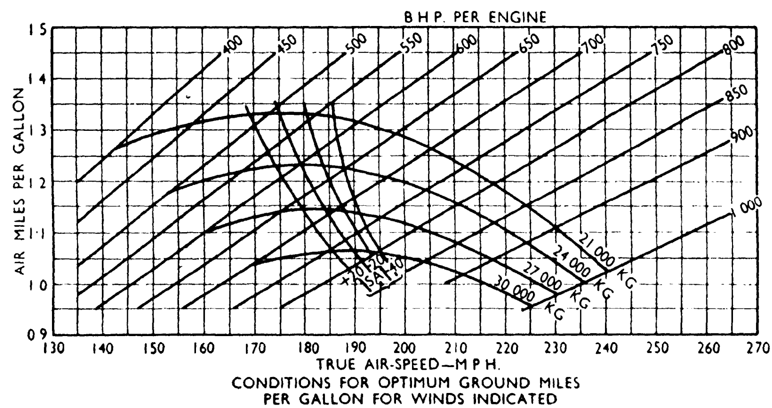


Fig. 53. A.M.P.G. Grid: 8 000 ft

has received less attention than it should have done in this country.

8.12. Examples of Complete Grids

Figs. 50–56 show grids drawn for an aircraft at a converted four-engined bomber aircraft at a number of heights. As we shall see in the next chapter, these grids are in themselves

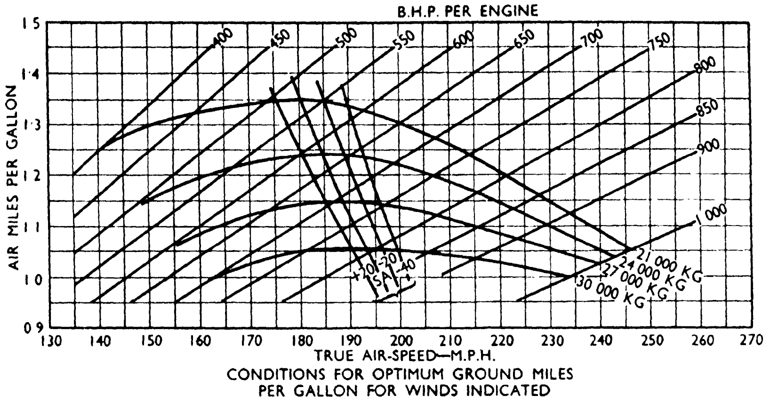


Fig. 54. A.M.P.G. Grid: 10 000 ft

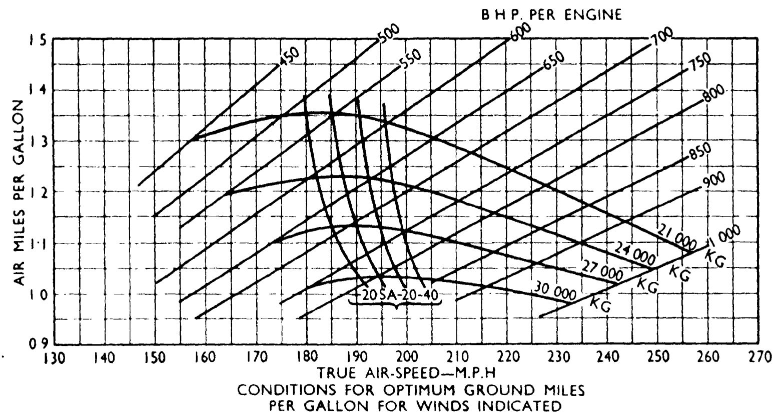


Fig. 55. A.M.P.G. Grid: 12 000 ft

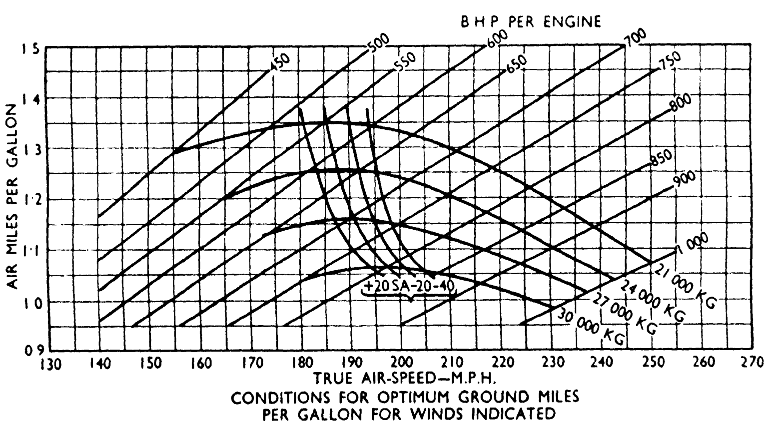


Fig 56. A.M.P.G. Grid: 14 000 ft

sufficient to provide all the necessary information for range calculations, which can, however, be summarized into one diagram, Fig. 60.

8.13. Range Formulae

The range for a given quantity of fuel, or the fuel required for a given range, are usually found by the graphical methods described in Chapters IX and X. In certain circumstances, however, it may be an advantage to have a formula. One that is often quoted is known as *Breguet's Formula.*

Suppose W = take-off weight,

W_t = weight t sec after take-off,

W_f = weight of fuel consumed for whole distance,

W_{ft} = weight of fuel consumed after t sec,

c_f = pounds of fuel consumed per h.p. per hour,

$(\text{b.h.p.})_t$ = brake horse-power available at time t.

Assume that c_f and η are constant. At any time t

$$\eta(\text{b.h.p.})_t = \frac{DV}{550} = \frac{W_t V}{\frac{L}{D} \cdot 550}$$

But in a short interval of time δt, weight of fuel consumed is

$$\delta W_{ft} = \frac{c_f(\text{b.h.p.})_t \cdot \delta t}{3\,600}$$

$$= \frac{c_f \cdot W_t V}{3\,600 \times 550\eta(L/D)}\,\delta t$$

$$\therefore \qquad V\delta t = 1\,980\,000\,\frac{\eta}{c_f} \cdot \frac{L}{D} \cdot \frac{\delta Q_t}{W_t}$$

But if R is the range, $\delta R = V\delta t$.

$$\therefore \qquad \delta R = 1\,980\,000\,\frac{\eta}{c_f} \cdot \frac{L}{D} \cdot \frac{\delta Q_t}{W_t}$$

$$= 1\,980\,000\,\frac{\eta}{c_f} \cdot \frac{L}{D} \cdot \frac{\delta Q_t}{W - W_{ft}}$$

$$\therefore \qquad R = 1\,980\,000\,\frac{\eta}{c_f} \cdot \frac{L}{D} \log_e \frac{W}{W - W_f}\ \text{ft}$$

$$= 375 \frac{\eta}{c_f} \cdot \frac{L}{D} \cdot \log_e \cdot \frac{W}{W - W_f} \text{ miles}$$

$$= 863{\cdot}5 \frac{\eta}{c_f} \times \frac{L}{D} \log_{10} \frac{W}{W - W_f} \text{ miles}$$

If we fly at maximum L/D, this formula gives the range for minimum drag, assuming that η/c_f is constant.

For short ranges W_f/W is small, and we can write

$$R = 375 \frac{\eta}{c_f} \cdot \frac{L}{D} \cdot \left(\frac{W_f}{W} + \frac{1}{2} \frac{W_f^2}{W^2} + \ldots\right).$$

Breguet's formula can be turned round so as to read

$$W_f = W[1 - e^{\frac{R}{375} \cdot \frac{c_f}{\eta} \cdot \frac{D}{L}}],$$

or $$W_f = W\left[\frac{R}{375} \cdot \frac{c_f}{\eta} \cdot \frac{D}{L} - \frac{1}{2}\left(\frac{R}{375} \cdot \frac{c_f}{\eta} \cdot \frac{D}{L}\right)^2 \ldots\right] \text{ approx.}$$

Further range formulae, of a rather more practical value, are developed in §§ **9.4, 10.3.2,** and **14.3.**

CHAPTER IX

Cruising (II)

9.1. Range Calculations by the Mean Weight Method

THERE are two main methods of doing range calculations; this chapter describes the first and Chapter X the second. Both are based on an application of the information contained in the a.m.p.g. grids.

Let us assume that the air is still, and that the operating height has been chosen, as a compromise between international regulations, meteorological conditions, and the most economical height for the particular aircraft. If the temperature is non-standard, we need to convert this to a density height. We may then arrive at some answer like 8 800 ft, in which case it will be necessary to interpolate between results obtained from a.m.p.g. grids supplied for, say, 5 000 ft and 10 000 ft.

9.1.1. MEAN WEIGHT. Suppose that W_1 = weight after the aircraft has climbed to the operating height; and that W_f = weight of fuel consumed for the whole journey, except that for climb and descent. Then we define the *mean weight* as $W_m = W_1 - \frac{1}{2}W_f$, and we make the assumption that all the necessary answers can be found by taking the weight as remaining constant and equal to W_m throughout the journey at operating height.

9.2. Constant Power

This method is particularly simple to use when the aircraft is operating at constant power, on a time schedule; because constant power necessitates a constant fuel flow, provided the r.p.m. variations are small, and so the fuel used for the journey is obtainable at once. An example will make this clear.

Given:	Method	Constant power
	Range	500 miles
	Density height	10 000 ft
	Fuel flow at constant power .	209 gal/hr
	Weight on first reaching 10 000 ft	29 200 kg
	Schedule time	2·25 hr
	Tailwind	20 m.p.h.

Required:: Constant b.h.p. at which to fly.

Then:	Fuel used	2·25 × 209 = 470 gal = 1 540 kg
	Mean weight	29 200 − 770 = 28 430 kg
	Mean ground speed . . .	$\frac{500}{2\cdot25}$ = 222 m.p.h.
	Mean true air-speed . . .	202 m.p.h.

Now turn to the a.m.p.g. grid for this particular aircraft (Fig. 54), and fix the point corresponding to T.A.S. 202 m.p.h.

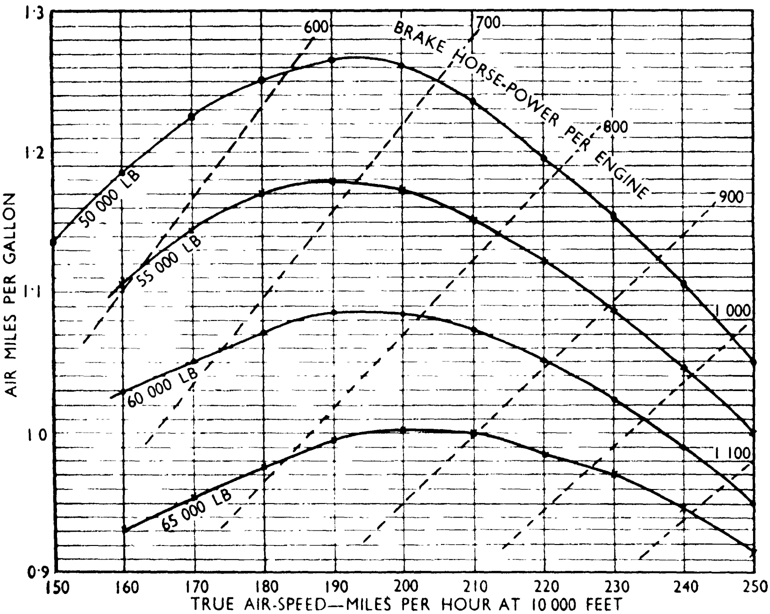


FIG. 57. A.M.P.G. GRID: FOUR-ENGINED AIRCRAFT AT 10 000 FT —"M" GEAR

and a weight of 28 430 kg. We can now read off the b.h.p. required and the actual a.m.p.g., which should check with the fuel flow consumption (cf. Fig. 60).

Knowing the b.h.p., the pilot can then find from his engine chart the correct boost and r.p.m. In practice this is often the only chart he uses.

Given the b.h.p., we can reverse the process to find the schedule time. This may be necessary if one engine fails. In that case we fly at maximum weak mixture power; e.g. at a certain height for a twin-engined aircraft this may be 1 230 b.h.p. We then imagine that each engine is working at 615 b.h.p.

9.3. Constant Speed

When the fuel flow is not constant, the method is less direct. If, for instance, we are operating at constant speed, the argument might be as follows—

Given:	Method	Constant speed of 210 m.p.h.
	Range	500 miles
	Density height	10 000 ft
	Weight on first reaching op. height	65 000 lb
	Headwind . . . . *nil*	
Required:	Fuel for journey	
Then:	Time at op. height	$\frac{500}{210} = 2{\cdot}38$ hr

Turning now to the a.m.p.g. grid (Fig. 57), it appears that, along the 210 m.p.h. constant speed line, a.m.p.g. at 65 000 lb is 1·0, a.m.p.g. at 50 000 lb is 1·235;

hence fuel used at 65 000 lb is 500 gal, fuel used at 50 000 lb is

$\frac{500}{1{\cdot}235} = 405$ gal;

hence fuel used at 65 000 lb is 3 600 lb, fuel used at 50 000 lb is 2 940 lb.

The fuel actually used will obviously lie between 3 600 lb and 2 940 lb.

We will now find the actual distances that can be covered with each of these fuel weights, starting at 65 000 lb.

Suppose we use 500 gal = 3 600 lb.

Mean weight = 65 000 − 1 800 = 63 200 lb.

A.m.p.g. at 63 200 lb (from grid, along 210 m.p.h. line) = 1·0275.

Distance covered = 1·0275 × 500 = 514 miles.

Again, suppose we use 405 gal = 2 940 lb.

Mean weight = 65 000 − 1 470 = 63 530 lb.

A.m.p.g. at 63 530 lb (from grid, along 210 m.p.h. line) = 1·02.

Distance covered = 1·02 × 405 = 413 miles.

Assume that the variation of gallons with distance is linear between these two points; we can then interpolate to find OX, the gallons for 500 miles.

Clearly $\frac{RS}{PS} = \frac{QT}{PT}$, in Fig. 58, or $\frac{RS}{87} = \frac{95}{101}$ or $RS = \frac{87 \times 95}{101} = 82,$

so that RN = 82 and 405 = 487 gal = 3 500 lb.

(*Check:* Mean weight = 65 000 − 1 750 = 63 250 lb.
A.m.p.g. at 63 250 = 1·025.
Distance covered = 487 × 1·025 = 500 miles.)

A more satisfactory method of doing such calculations is described in § **10.3.1.**

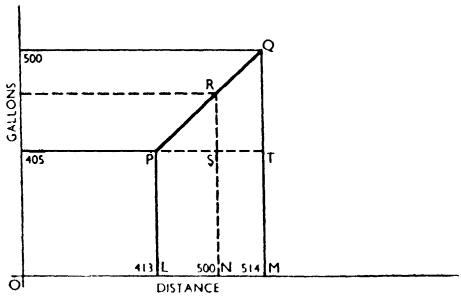


FIG. 58. INTERPOLATION DIAGRAM

9.4. Maximum Range

When flying for maximum range at the peaks of the grid curves the process can be simplified by assuming that the maximum a.m.p.g. is inversely proportional to the weight,

or $$\text{a.m.p.g.}_{\text{max}} \propto \frac{1}{W}$$

If G be taken to stand for a.m.p.g.$_{\text{max}}$, the above means

$$G \times W = K, \text{ a constant}$$

(This is not true in practice; a better empirical rule is found to be $G \times W^{0.88} = K$, based on the assumption $\eta/c_f \propto V^n$.)

Using the first assumption, here is an appropriate example—

Given:	Method	Maximum range
	Range	500 miles = R, say
	Density height	10 000 ft
	Weight on first reaching op. height .	65 000 lb, or W_1 lb, say
	Headwind.	*nil*

Required: Fuel used and time taken.

Suppose that W_f is the weight of fuel used for the journey. Then the mean weight $W_m = W_1 - \frac{1}{2}W_f$ and the fuel in gallons $= \frac{W_f}{7 \cdot 2}$. Let G_m = a.m.p.g. at the mean weight W_m.

From an inspection of the grid (Fig. 57) we find that $G \times W = 65\,200$, say, then $G_m(W_1 - \frac{1}{2}W_f) = 65\,200 = K$, say.

But
$$\frac{W_f}{7 \cdot 2} = \frac{R}{G_m}$$

Hence
$$\frac{7 \cdot 2R}{W_f}(W_1 - \tfrac{1}{2}W_f) = K$$

$$W_f = \frac{7 \cdot 2RW_1}{3 \cdot 6R + K} \text{ lb}$$

With our numbers this gives

$$W_f = \frac{7 \cdot 2 \times 500 \times 65\,000}{1\,800 + 65\,200} \text{ lb}$$
$$= \underline{\underline{485 \text{ gal}}} \text{ or } \underline{\underline{3\,490 \text{ lb}}}$$

The mean weight is now $W_1 - \frac{1}{2}W_f = 63\,255$ lb. Looking at the grid (Fig. 57) again, and reading along the line joining the peaks of the curves, we find the T.A.S. at this mean weight is about 200 m.p.h. so that the time for the journey is $2\frac{1}{2}$ hours.

The formula we have just used for W_f can be re-written, approximately, as

$$W_f = \frac{R}{G_1}\left(1 - \frac{3 \cdot 6R}{G_1 W_1}\right) 7 \cdot 2 \text{ lb}$$

in which we have replaced K by W_1G_1, where G_1 is the a.m.p.g. at the starting weight W_1.

A more accurate formula for max. range flying, based on the relation $\frac{\eta}{c_f} \propto V^n$ already referred to, is

$$W_f = \frac{R}{G_1}\left(1 - \frac{3 \cdot 2R}{G_1 W_1}\right) 7 \cdot 2 \text{ lb}$$

$$[= 7 \cdot 2\ (497 \cdot 6 - 12 \cdot 2) = 7 \cdot 2 \times 485 \cdot 4 \text{ in our case}]$$

9.5. Headwinds

In the case of flying against a headwind at maximum range, if the true speeds are raised as described in Chapter VIII, the above formulae for W_f can still be used if we replace R by $R/\left(1 - \frac{v}{V} - \frac{1}{8}\frac{v^2}{V^2}\right)$ where v is the headwind and V is the "maximum range" true speed at the mean weight when there is no headwind.

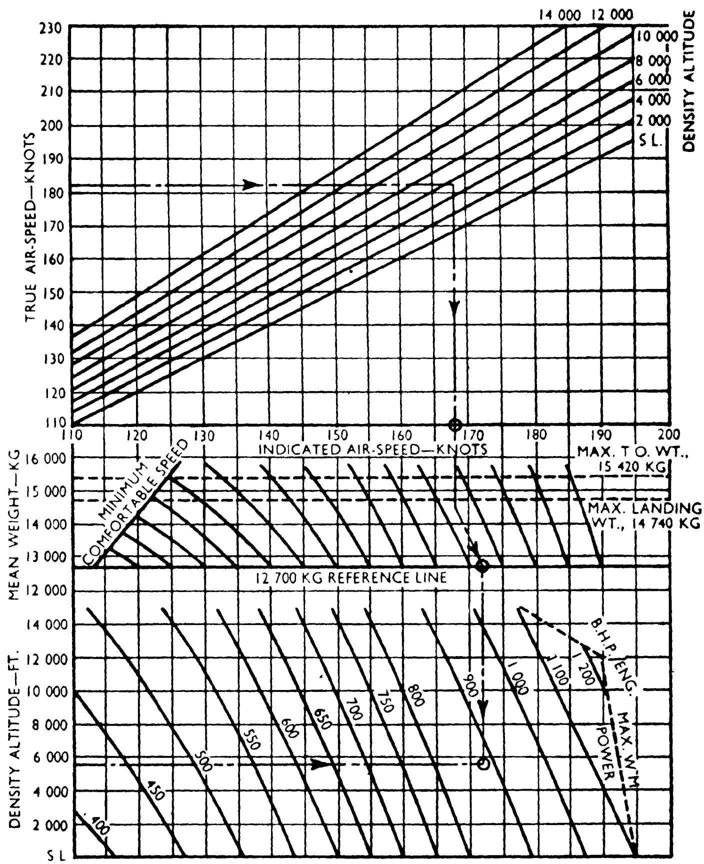
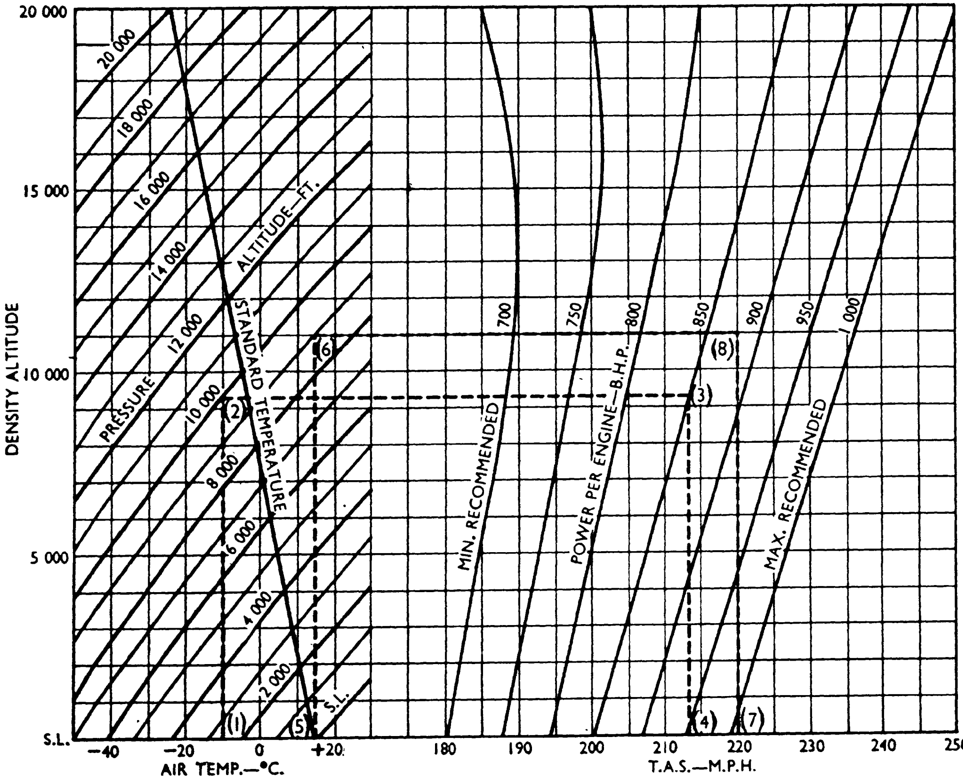


FIG. 59. COMPOSITE CRUISING CHART: TWO-ENGINED AIRCRAFT

9.6. Conclusion

The mean weight method will enable us to construct any desired form of cruising chart. By assuming a series of values for fuel used, we calculate the range covered, and the resulting curve between range and fuel can be used as the basis for a



CRUISING POWER REQUIRED
VS TRUE AIR-SPEED AND ALTITUDE
AT AVERAGE CRUISING WEIGHT

Average weight = 27 000 kg
(59 525 lb)

Problem 1

Find T.A.S. for 850 b.h.p. 10 000 ft P.A. + − 10° O.A.T.

Answer

Follow dotted lines through points 1, 2, 3, 4.
Average T.A.S. = 213·5 m.p.h.

Problem 2

Find av. power for T.A.S. = 220 m.p.h. and 9 000 ft P.A. with + 15° C.O.A.T.

Answer

Follow dotted line 5, 6, 8, then 7 and 8. At point 8, h.p. = 875.

FIG. 60. CRUISING POWER REQUIRED CHART: FOUR-ENGINED AIRCRAFT

Fuel Consumed chart. In a similar manner we can find the speeds at each point, and so construct a curve giving the time taken. It is doubtful, however, whether this method is as satisfactory as that described in the next chapter.

It is pertinent to inquire whether the mean weight method is accurate. Although there is no doubt that it slightly underestimates the amount of fuel required, the difference is usually negligible. For reasonably quick calculations it is far the best method, since it avoids the necessity for constructing elaborate charts and the tendency (often overlooked) to misread the charts after they have been constructed. The charts used on many airline routes are only a.m.p.g. grids in a disguised form. For instance, Fig. 59 dispenses with an a.m.p.g. grid, but the relation between speed, mean weight, and b.h.p. found from the chart is the same thing in a different setting. The same remark applies to Fig. 60.

CHAPTER X

Cruising (III)

10.1. Range Calculations by the Integral Method

THE information supplied by a.m.p.g. grids can be transformed so as to supply us with graphs of related quantities, such as distance gone against time out, fuel used against distance, fuel

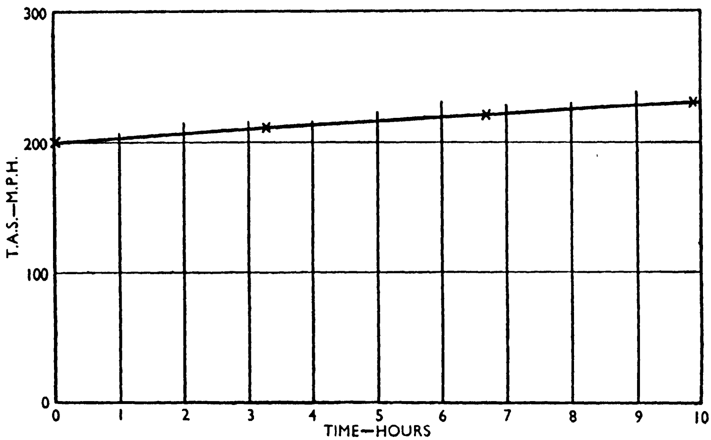


FIG. 61. CONSTANT POWER: VARIATION OF SPEED WITH TIME

used against time, etc., and the final results shown in standard types of charts. We propose to show how these charts can be constructed for each of the three methods of cruise control, by using the a.m.p.g. grid shown in Fig. 57.

10.2. Constant Power, 850 b.h.p. per Engine

10.2.1. CALCULATIONS. Reading along the 850 b.h.p. line (Fig. 57), the basic information is

Weight W	65 000	60 000	55 000	50 000
a.m.p.g.	1·00375	1·07	1·1175	1·155
T.A.S. (m.p.h.) . . .	198·8	211·9	221·2	230·0

We now make the assumption that operating at 850 b.h.p./ engine involves a fuel flow of 210 gal/hour on all engines; this is 1 512 lb/hour. Now proceed with the tabulation—

Change in Weight	0	5 000	10 000	15 000
Time $= \frac{\text{Change of Wt}}{1\ 512}$ (hr)	0	3·30	6·72	9·93

The resulting speed-time curve can be plotted (see Fig. 61). To find the distance covered during each hour, we need to calculate the area under the graph for each hourly zone, and can tabulate the results as follows—

Time	0	1	2	3	4	5	6	7	8	9	10
Distance covered per Hour	0	200	204	208	212	215	217·5	220	223	226	228
Total Distance	0	200	404	612	824	1 039	1 256·5	1 476·5	1 699·5	1 925·5	2 153·5
T.A.S.	199	202	206	210	213	216	219	222	224	227	230

There is no need to plot a.m.p.g. against time, as we have already assumed a fuel flow of 210 gal/hour, but we may as well insert the weight of fuel used after each hour, in order to complete the table.

Time	0	1	2	3	4	5	6	7	8	9	10
Wt. of Fuel Used (lb)	0	1 512	3 024	4 536	6 048	7 560	9 072	10 584	12 096	13 608	15 120

10.2.2. Distance-Speed Chart. We now have all the information required for constructing the charts. First of all, we need to show the change of speed with distance gone, neglecting the climb, drawn accurately in Fig. 62. On the same chart we should also show the corresponding speeds at all other heights, at intervals of 5 000 ft.

The main object of this first chart is to provide a preliminary estimate of the time taken for each part of the journey. For example, suppose one zone lies between 800 and 1 000 miles, then the average true air-speed at mid-zone (read from the chart) is 214·2 m.p.h., so that the time taken (in still air) is $\frac{200}{214\cdot2} = 0\cdot935$ hr $= 56$ min.

10.2.3. TIME-SPEED CHART. When making a complete flight plan, it may be more convenient to use a time-speed chart, which gives us the speeds to be used at various times from the start. We already have all the necessary information, which is

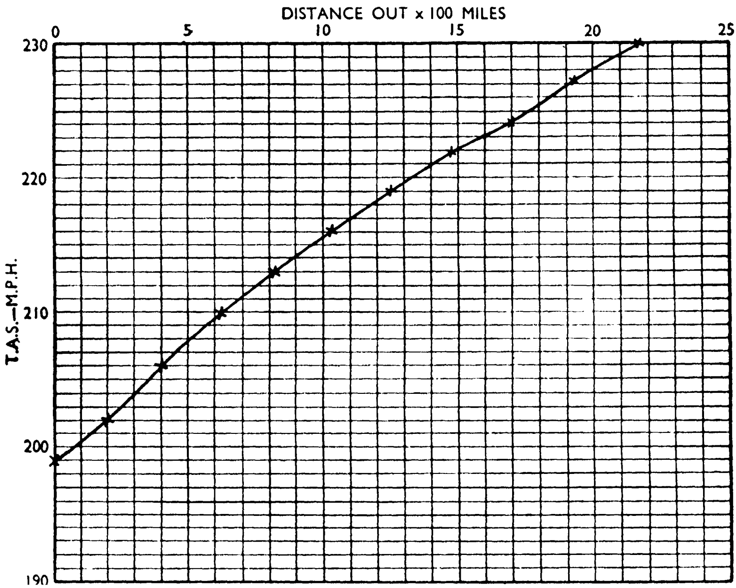


FIG. 62. CONSTANT POWER: VARIATION OF SPEED WITH DISTANCE

shown in Fig. 61. The use of Figs. 61 and 62 will be discussed at greater length in Chapter XII.

10.2.4. FUEL CHART. The second chart (Fig. 63) is primarily used for finding the fuel used for the whole trip, and is constructed as follows: The "Fuel Used" line is drawn first, by relating time out with fuel used in accordance with the tabulation above. On the top of the graph, now insert a distance out scale, starting at any convenient origin. Then the points fixing the "distance" line can be plotted in accordance with the tabulated relations between distance and time. Neither the "fuel used" nor the "distance" lines are necessarily straight;

in the present case, of constant power, the former will be, but not the latter.

To use the chart, start at a point on the top distance out

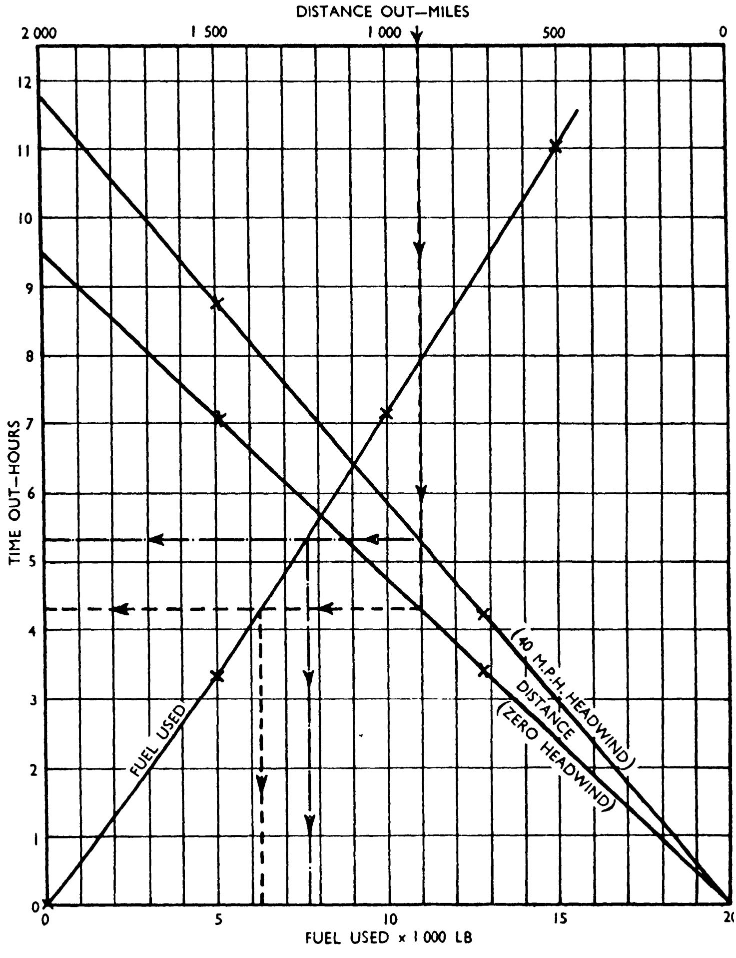


FIG. 63. CONSTANT POWER: FUEL CHART

scale, e.g. 900 miles. Drop vertically down to the "distance line," and then horizontally (to the right or left), to cut the "fuel used" line. From here we move horizontally to find the time, and vertically downwards to find the fuel used, getting in our example 4·35 hr and 6 600 lb of fuel. The time so obtained should check with that found from the first chart. From the first chart, the speeds at 0 and 900 miles are 199 m.p.h. and 214 m.p.h. respectively, giving a mean speed of 206·5 m.p.h., so that the time is $\frac{900}{206 \cdot 5} = 4 \cdot 35$ hr, as before.

We may also start from the time scale on the left, and read the fuel directly.

In order to allow for headwinds and tailwinds, further "distance" lines have to be added. We shall continue to fly at the same constant b.h.p. so that the only quantities to be altered will be the ground speeds, thereby altering the relation between distance and time, but not that between fuel used and time. As an example, take the case of a 40 m.p.h. headwind. The ground-speed time tabulation is now—

Ground Speed	159	172	181	190
Time	0	3·30	6·72	9·93
Hence the Distances are Given by—				
Distance	0	546·2	603·6	595·4
Total Distance	0	546·2	1 149·8	1 745·2

These four points are sufficient to establish the new 40 m.p.h. headwind "distance" line; under these headwind conditions, the chart is read exactly as before except for the use of the new "distance" line.

The charts constructed have one disadvantage, they are only true for 850 b.h.p. Under certain circumstances we might operate the engines at a lower power without losing much time, and thereby increase the possible payload.

A further alternative is provided by the case of engine failure when we might decide to operate at the same constant power on the remaining engines. In the case of a four-engined aircraft, this is equivalent to $\frac{3}{4} \times 850$ per engine on four engines (neglecting any extra drag), i.e. 637·5 b.h.p. This might lead to speeds below $V_{i_{mcc}}$, so that a higher b.h.p. on the remaining engines may be preferred. In either case, the chart constructed for 850 b.h.p. four-engined operation will be of no use to us.

10.3. Constant Speed: 210 m.p.h. T.A.S.

10.3.1. CALCULATIONS. Reading along the 210 m.p.h. T.A.S. line in Fig. 57, the basic information is as follows—

Weight W	65 000	60 000	55 000	50 000
a.m.p.g.	1·000	1·0725	1·150	1·235

The first chart drawn in the case of constant power is no longer necessary; the fuel chart can be constructed from the following tabulation—

Fuel Used (lb)	0	5 000	10 000	15 000
a.m.p.lb. $\left(\frac{\text{a.m.p.g.}}{7\cdot 2}\right)$	0·139	0·149	0·160	0·172

To get the distance gone we plot a.m.p.lb. against fuel used (lb), finding the area under each part of the curve, as shown in Fig. 64.

Distance Gone	0	720	772·5	830
Total Distance	0	720	1 492·5	2 322·5
Time at 210 m.p.h.	0	3·43	7·12	11·06
Time at 170 m.p.h.	0	4·23	8·79	13·66

From these results we can again construct the fuel chart, as before. This chart appears in Fig. 65. This time the "distance" line is straight, but the "fuel used" line is a curve. It will be noted that, for the 900 mile journey, there is a distinct saving in fuel but no loss of time, as compared with the case of constant power. The headwind and tailwind "distance" lines can be inserted as before.

10.3.2. MATHEMATICAL ALTERNATIVE. When we are operating at constant speed, the power required is a function of the weight, of the form

$$\text{b.h.p.} = A + BW^2$$

Moreover, if we assume that $\frac{\eta}{c_f}$ is a function of the speed, then

$$\text{a.m.p.g.} = \frac{1}{\lambda + \mu W^2}$$

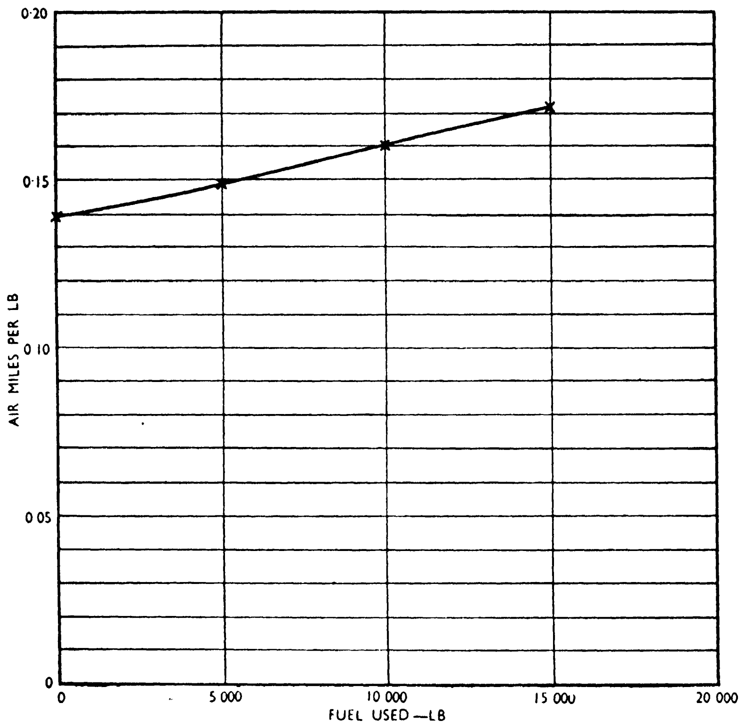


FIG. 64. CONSTANT SPEED: A.M.P.LB. VERSUS FUEL USED

and it will be found that the figures given at the head of this paragraph for a.m.p.g. against W satisfy this relation by taking $\lambda = 0{\cdot}5323$, $\mu = 1{\cdot}11 \times 10^{-10}$.

For example, if

$$W = 60\,000,\ \lambda + \mu W^2 = 0{\cdot}5323 + 1{\cdot}11 \times 10^{-10} \times 10^8 \times 36$$

$$= 0{\cdot}5323 + 0{\cdot}3996 = 0{\cdot}9319$$

$$\text{a.m.p.g.} = \frac{1}{0{\cdot}9319} = \underline{1{\cdot}073}$$

It would thus be possible to obtain the distance by direct integration, as

$$\int_{65\,000}^{50\,000} \frac{1}{7{\cdot}2}\left(\frac{1}{\lambda + \mu W^2}\right) dW$$

$$= \frac{10^5}{7{\cdot}2\sqrt{1{\cdot}11}\sqrt{0{\cdot}5323}}\left[\tan^{-1}\sqrt{\frac{1{\cdot}11}{0{\cdot}5323}} \times \frac{W}{10^5}\right]_{50\,000}^{65\,000}$$

for our numbers = 2 322 miles, for 15 000 lb of fuel.

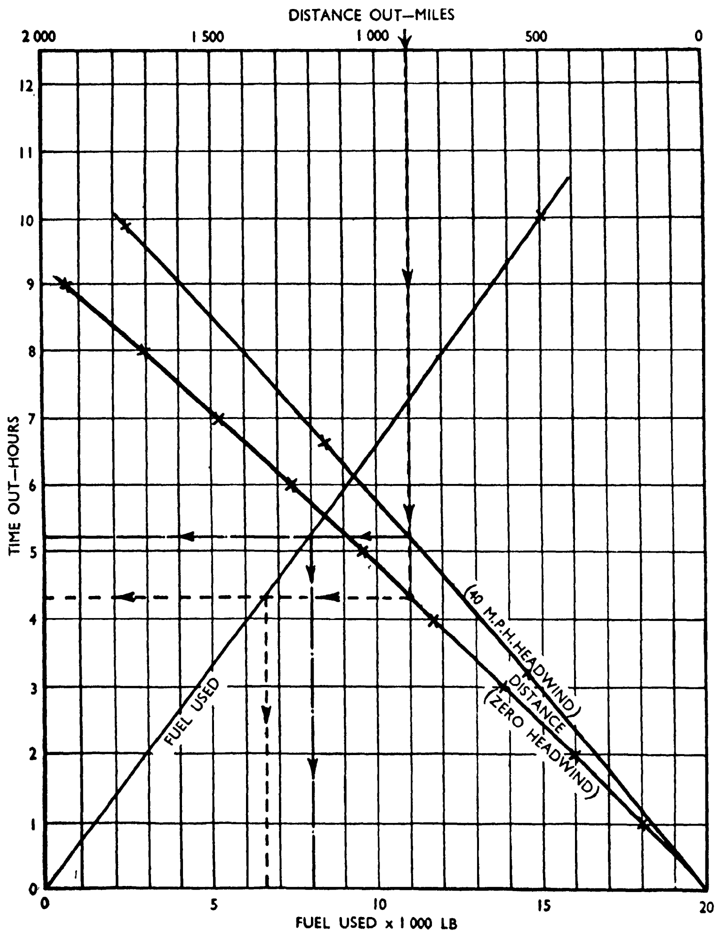


Fig. 65. Constant Speed: Fuel Chart

Four engines; 210 m.p.h., T.A.S.

10.4. Maximum Range

This time the basic information from the grid in Fig. 57, reading along the line of peaks, is

Weight W . .	65 000	60 000	55 000	50 000
a.m.p.g. max. .	1·005	1·0875	1·180	1·2675
T.A.S. (m.p.h.) .	201	195·2	190	189
$\frac{1}{\text{T.A.S.}}$ . . .	0·004975	0·005123	0·005263	0·005291

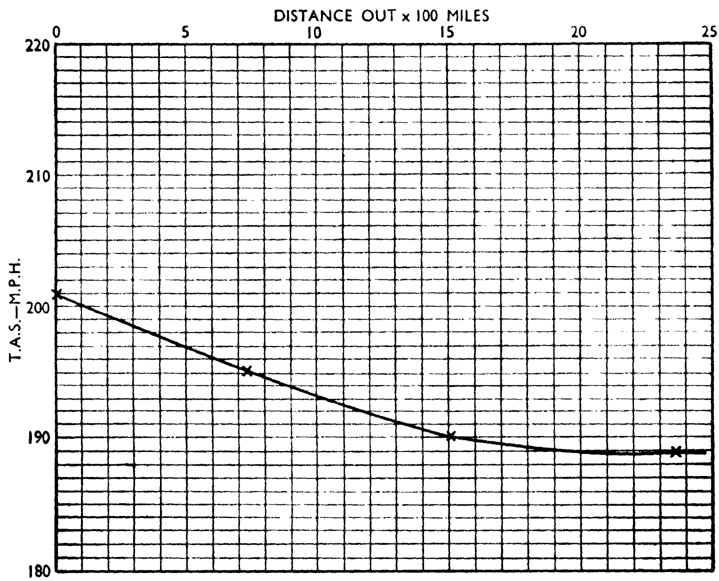


FIG. 66. MAXIMUM RANGE: VARIATION OF OPTIMUM SPEED WITH DISTANCE

The distance gone can be found as in § **10.3.1.**

a.m.p.lb.	0·1395	0·151	0·164	0·176
Distance Gone . . .	0	726·25	787·5	850·0
Total Distance . . .	0	726·25	1 513·75	2 363·75

To find the time for each stage, we need to find the area under the $\frac{1}{\text{speed}}$ against distance curve $\left(\text{since time} = \int \frac{dt}{dx}\,dx\right)$.

Time for Each Stage . .	0	3·668	4·089	4·485
Total Time . . .	0	3·668	7·757	12·242

Mean Speed = 194

The two types of chart can now be plotted as before, assuming no headwind. The first gives the T.A.S. at any distance out, so

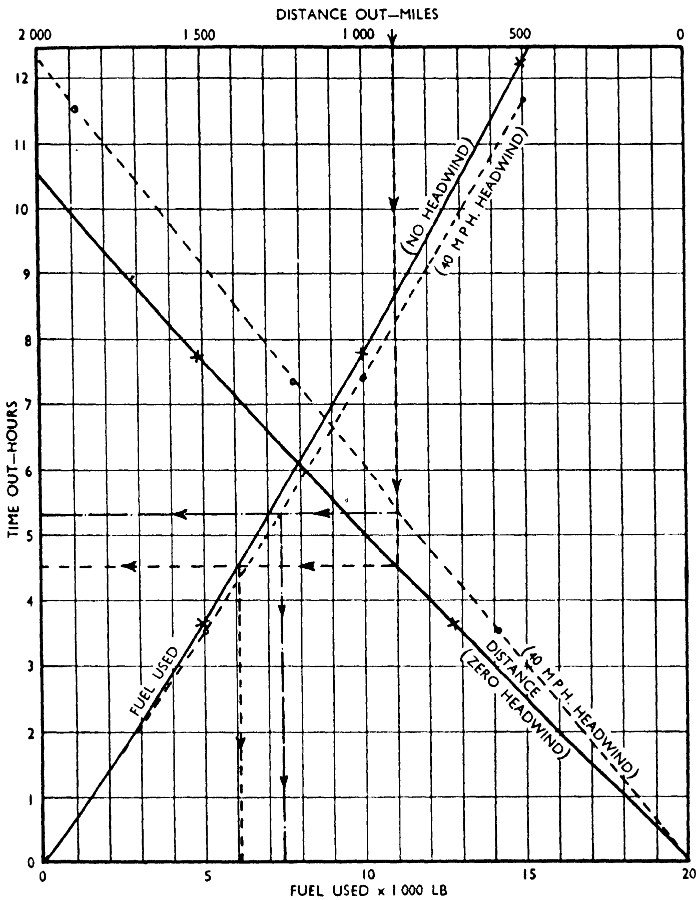


FIG. 67. MAXIMUM RANGE: FUEL CHART

that the time for each zone can be calculated, see Fig. 66. The second is the usual fuel chart, see Fig. 67. Compared with the constant speed case, the journey of 900 miles has taken 0·25 hr = 15 min longer, and the fuel saved is 200 lb or about 28 gal.

10.5. Maximum Range with a Headwind

Since the economical speed against a headwind is different from that in still air, both the "distance" and the "fuel used" lines will be altered. Our best method is probably to construct a fresh grid, as follows—

Weight (lb)	65 000				60 000			
T.A.S. (m.p.h.)	190	200	210	220	190	200	210	220
a.m.p.g. No headwind	0·995	1·005	1·0	0·985	1·085	1·085	1·0725	1·0525
Fuel Flow $E = \frac{T.A.S.}{a.m.p.g.}$	191	199	210	225	175	189	196	209

Weight (lb)	55 000				50 000			
T.A.S. (m.p.h.)	190	200	210	220	190	200	210	220
a.m.p.g. No headwind	1·1775	1·1725	1·15	1·1225	1·265	1·26	1·235	1·195
Fuel Flow $E = \frac{T.A.S.}{a.m.p.g.}$	161	170·5	182	196	150	158·5	170	184

Suppose there is a headwind of 40 m.p.h.

Weight (lb)	65 000				60 000			
Ground Speed	150	160	170	180	150	160	170	180
g.m.p.g. $= \frac{G.S.}{E}$	0·785	0·805	0·81	0·809	0·857	0·87	0·87	0·861

Weight (lb)	55 000				50 000			
Ground Speed	150	160	170	180	150	160	170	180
g.m.p.g. $= \frac{G.S.}{E}$	0·932	0·94	0·935	0·92	1·0	1·01	1·0	0·98

The basic information from the new grid (Fig. 68) is now—

Weight, W	65 000	60 000	55 000	50 000
g.m.p.g. (max.) . . .	0·813	0·874	0·94	1·01
Ground Speed, G.S. . .	166	164	162	160
$\frac{1}{G.S.}$	0·006024	0·006098	0·006173	0·00625
g.m.p.lb	0·113	0·121	0·130	0·140
Distance Gone . . .	0	585	627·5	675
Total Distance . . .	0	585	1 212·5	1 887·5
Time for Each Stage . .	0	3·546	3·850	4·192
Total Time . . .	0	3·546	7·396	11·588

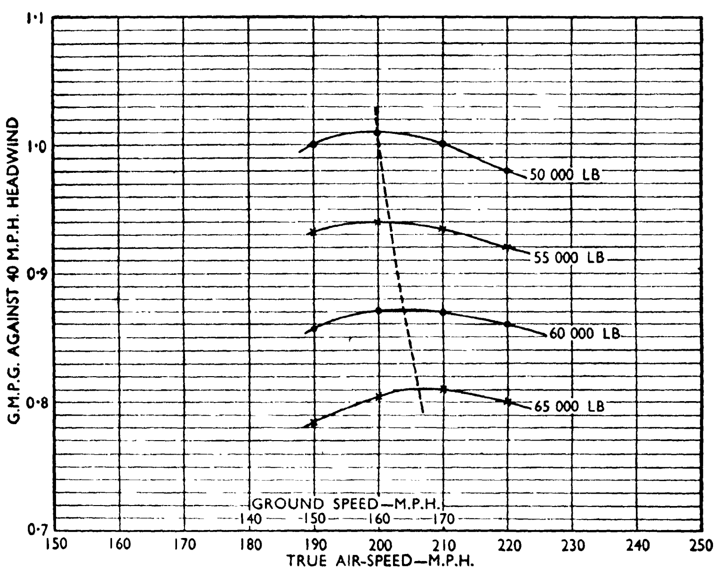


Fig. 68. Maximum Range: A.M.P.G. against a Headwind of 40 m.p.h.

Four-engined aircraft at 10 000 ft in "M" gear.

The new "distance" and "fuel used" lines are plotted on the original chart (Fig. 67). It will be noticed that for the specimen distance of 900 miles, compared with still air, the time has increased by 48 min, and the fuel consumed by 1 300 lb. The total fuel consumed is now 7 400 lb. Flying at constant speed against this headwind, the fuel consumed would have been 7 700 lb, so that the trouble involved can scarcely be said to have been repaid.

10.6. Comparison of Methods

A complete comparison of the answers obtained for 900 miles is as follows—

Method	Constant Power		Constant Speed		Max. Range	
Wind	No headwind	40 m.p.h. headwind	No headwind	40 m.p.h. headwind	No headwind	40 m.p.h. headwind
Time	hr 4 min 21	hr 5 min 15	hr 4 min 18	hr 5 min 18	hr 4 min 33	hr 5 min 21
Fuel Used (lb)	6 600	8 000	6 300	7 700	6 100	7 400
Fuel Used in Excess of Max. Range Fuel	500	600	200	300	0	0
Time Saved compared with Max. Range	min 12	min 6	min 15	min 3	min 0	min 0

This comparison is worth study; not because the results are typical, but because only by careful examination of such figures can the operator decide what procedure is most suitable for the sort of trips he has to cover.

10.7. Maximum Range Speed, Increased by a Fixed Percentage

Another interesting comparison to make is that between—

(1) Constant speed, in still air.

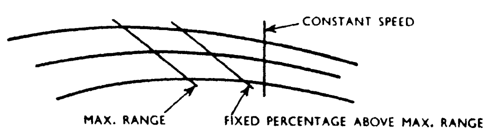


FIG. 69. FIXED PERCENTAGE ABOVE MAXIMUM RANGE SPEED

(2) Maximum range against that fictitious headwind which gives the same schedule speed for the trip, i.e. the max. range speed for each weight, but with the speed increased by a constant percentage. In order to start such an investigation, we must first of all decide what happens to the a.m.p.g. when the speed is increased by a fixed percentage above that for maximum range.

In the case of the grid used in this chapter, it can be proved mathematically, or by inspection of the grid, that the alteration in a.m.p.g. is related to the increase in V as follows—

Percentage Increase in V .	5	10	15	20	25
Percentage Decrease in a.m.p.g. . . .	0.50	1·93	4·06	6·77	9·94

(These satisfy an empirical relation of the form percentage decrease in a.m.p.g. = 0·026 × (percentage increase in V)$^{(1\cdot 87)}$.)

If G is max. range a.m.p.g. and G_1 is a.m.p.g. at v m.p.h. higher, and V_E is max. range speed, then

$$\frac{G_1}{G} = 1 - (2 + n)^2 \frac{v^2}{V_E{}^2}$$

where n is about ¼, as previously defined, is a formula sometimes used.

Take as an example the case of a 15 per cent increase in V above max. range speed, with a corresponding 4 per cent decrease in G (a.m.p.g.). Reading from the grid in Fig. 57—

W	65 000	60 000	55 000	50 000
G_{max}	1·005	1·0875	1·180	1·2675
$V_{G_{max}}$	201	195·2	190	189
$1\cdot 15V_{G_{max}}$	231	225	219	228
$\frac{1}{1\cdot 15V_{G_{max}}}$	0·00432	0·00444	0·00456	0·00458
$0\cdot 96G_{max}$	0·965	1·043	1·131	1·215
a.m.p.lb	0·134	0·145	0·157	0·169
Distance Gone . . .	0	697·5	755	815
Total Distance . . .	0	697·5	1 452·5	2 267·5
Time for Each Stage . .	0	3·055	3·397	3·725
Total Time . . .	0	3·055	6·452	10·177

I.e. the mean speed is 222·8 m.p.h.

We now find the effect of flying at a constant speed of 222·8 m.p.h. From the grid in Fig. 57—

W	65 000	60 000	55 000	50 000
a.m.p.g.	0·981	1·046	1·112	1·183
a.m.p.lb	0·136	0·145	0·155	0·164
Distance	0	702·5	750	797·5
Total Distance	0	702·5	1 452·5	2 250·0
Time	0	3·153	6·519	10·098

Now the fuel used for 2 250 miles in the former case will be approximately

$$1\,500 \times \frac{2\,250}{2\,267{\cdot}5}\ \text{lb} = 14\,885\ \text{lb} = (15\,000 - 115)\ \text{lb}$$

i.e. for the trip of 2 250 miles in 10·098 hr, there is a saving of 115 lb (or 16 gal) by flying at 15 per cent above max. range speed instead of at constant speed. This method is thus scarcely a practical proposition.

10.8. Allowances for Climb

The charts we have just constructed were made on the assumption that the starting weight at cruising height was 65 000 lb. In reality, of course, the starting weight will depend upon the height, if we assume a fixed take-off weight. This means that the charts for each height should actually be drawn with the correct starting weight for that height.

This refinement can be seen by looking at the charts in Figs. 70–78 for a well-known transatlantic aircraft. The first chart, Fig. 70, showing true air-speed at various distances out does not assume the same form as our Fig. 61 until a certain distance out, equal to the forward distance achieved by climbing to that particular height; and the first cruising speed shown corresponds, not to the take-off weight, but to the weight on first reaching this height.

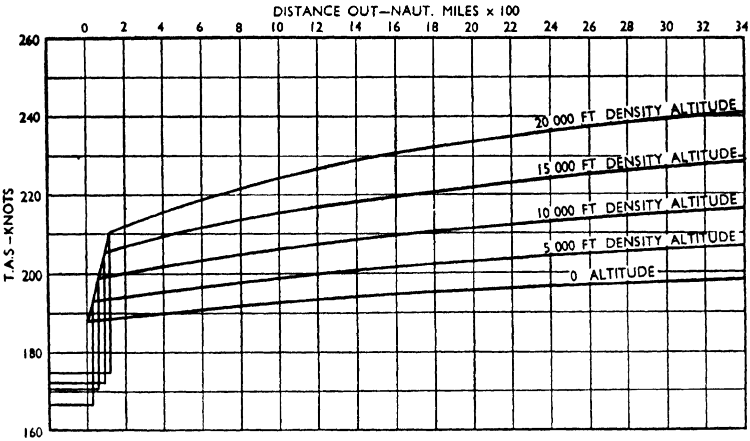


FIG. 70. CONSTANT POWER CRUISING: T.A.S. VERSUS DISTANCE OUT: LONG RANGE; 1 000 B.H.P.

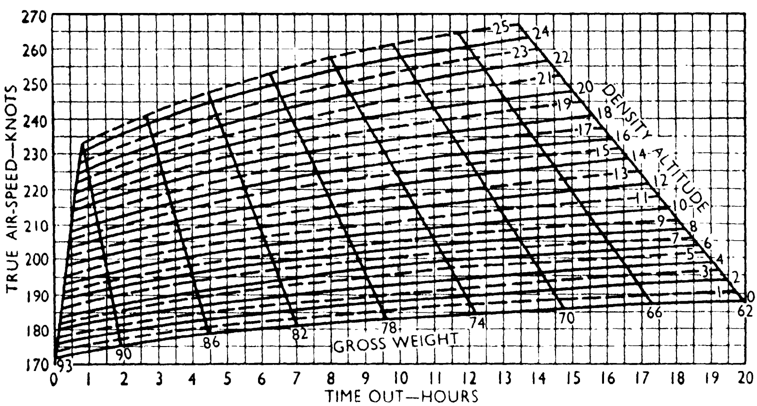


FIG. 71. CONSTANT POWER: T.A.S. VERSUS TIME OUT

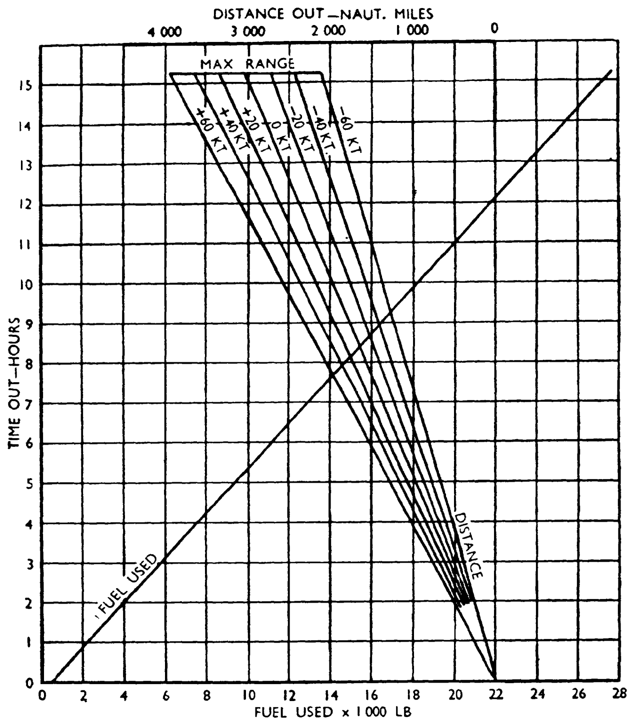


FIG. 72. FUEL CHART: TRANSATLANTIC AIRCRAFT: CONSTANT POWER (1 000 B.H.P.): LONG RANGE: 5 000 FT, DENSITY ALTITUDE

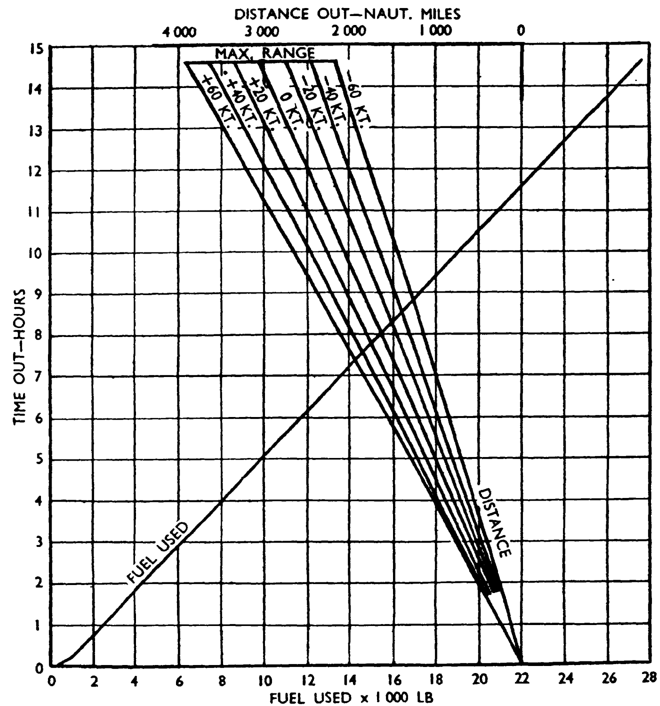


FIG. 73. FUEL CHART: TRANSATLANTIC AIRCRAFT: CONSTANT POWER (1 000 B.H.P.): LONG RANGE: 10 000 FT, DENSITY ALTITUDE

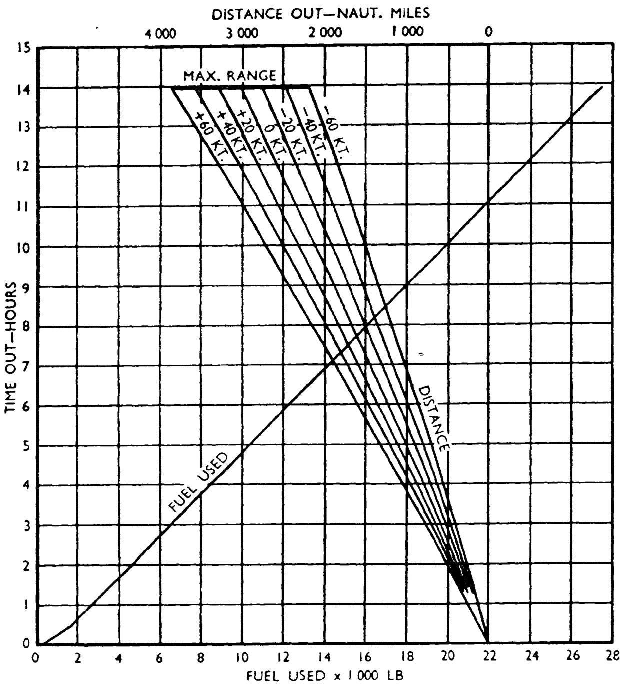


FIG. 74. FUEL CHART: TRANSATLANTIC AIRCRAFT: CONSTANT POWER (1 000 B.H.P.): LONG RANGE: 15 000 FT, DENSITY ALTITUDE

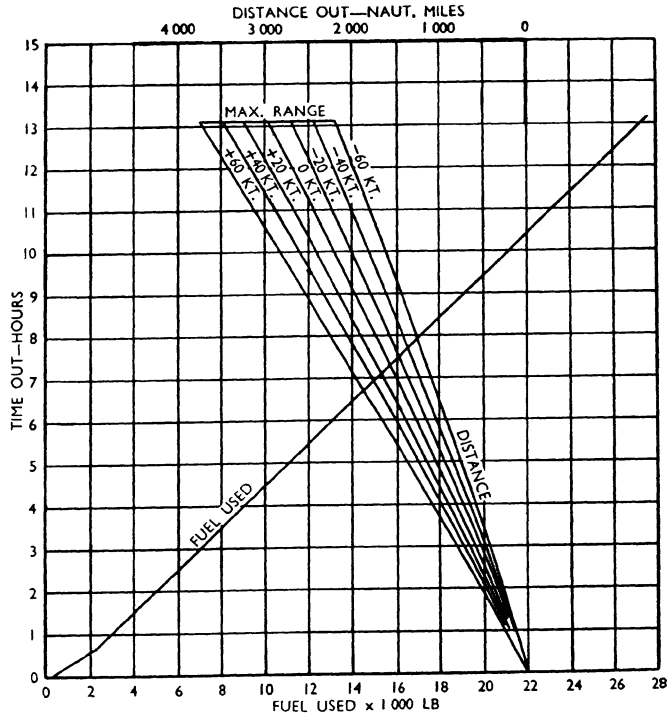


FIG. 75. FUEL CHART: TRANSATLANTIC AIRCRAFT: CONSTANT POWER (1 000 B.H.P.): LONG RANGE: 20 000 FT, DENSITY ALTITUDE

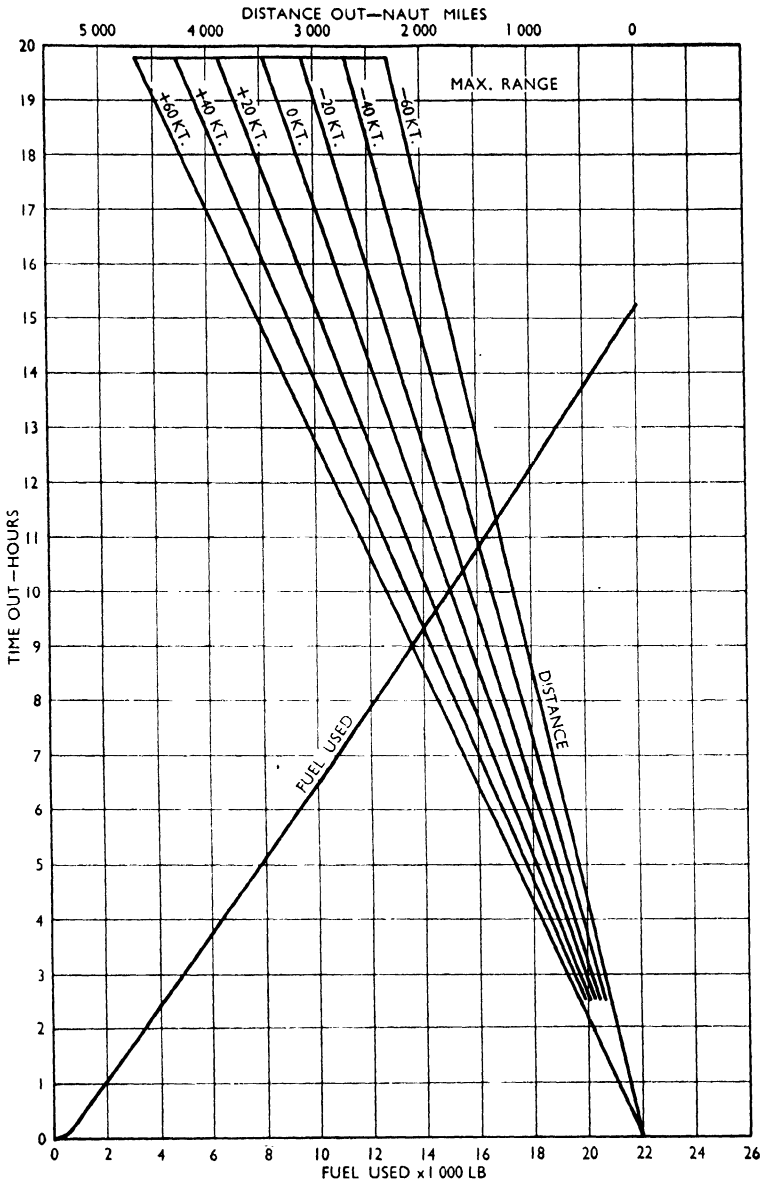


FIG. 76. FUEL CHART: TRANSATLANTIC AIRCRAFT: CONSTANT AIR-SPEED, 165 KT: 5 000 FT, DENSITY ALTITUDE

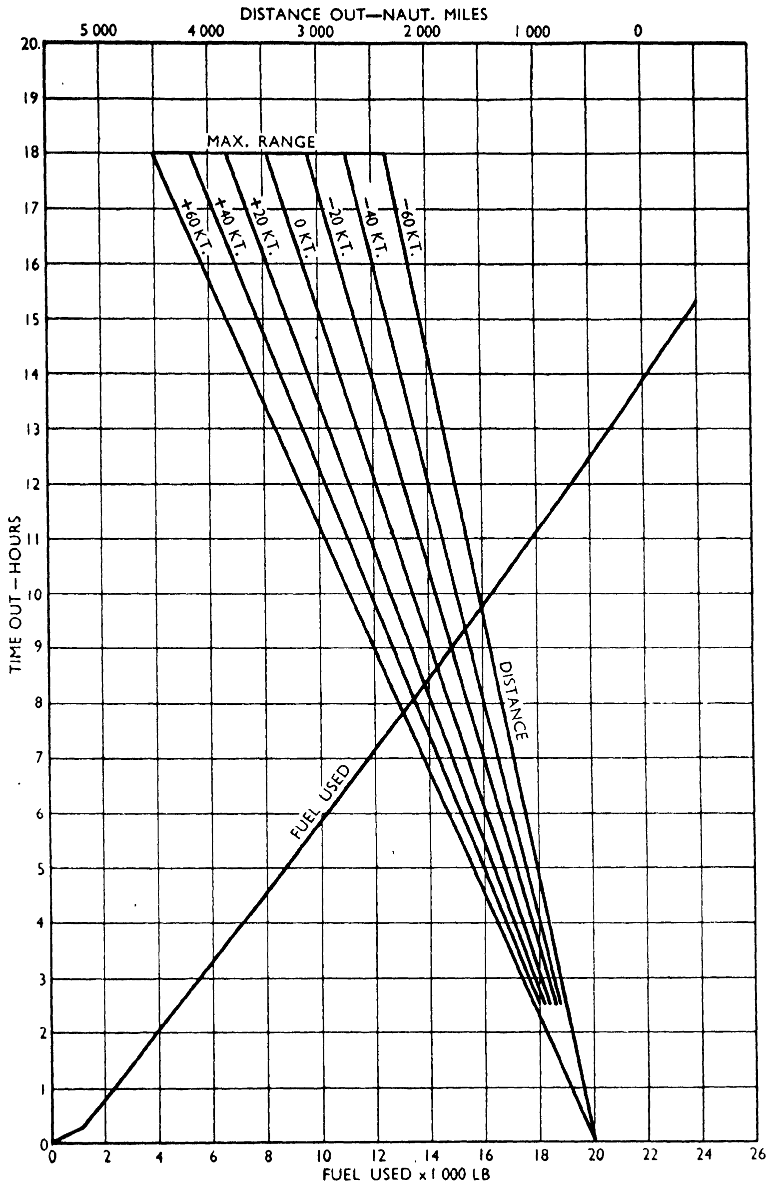


FIG. 77. FUEL CHART: TRANSATLANTIC AIRCRAFT: CONSTANT AIR-SPEED, 165 KT: 10 000 FT, DENSITY ALTITUDE

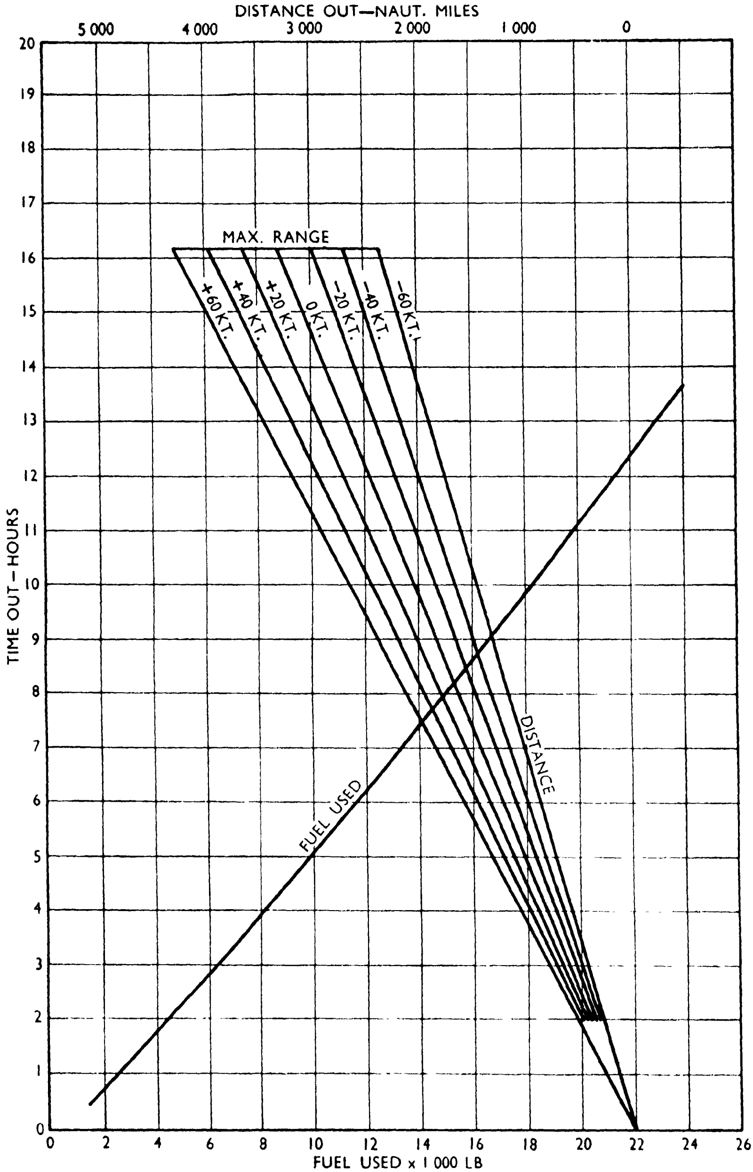


FIG. 78. FUEL CHART: TRANSATLANTIC AIRCRAFT: CONSTANT AIR-SPEED, 165 KT: 15 000 FT, DENSITY ALTITUDE

For a similar reason the "fuel used" line in the second type of chart (Figs. 72–78) has a small kink at the beginning, corresponding to the fuel allowance for take-off and climb.

10.9. Alteration in Take-off Weight

One of the great advantages of the integral type of chart is the fact that alterations in take-off weight can easily be allowed for. Suppose, for instance, the take-off weight is 85 000 lb instead of 90 000 lb in the case of the transatlantic aircraft. The difference is equivalent to using 5 000 lb of fuel before starting.

The time on the climb will be the same as before; the fuel used on the climb will be $\frac{85}{90}$ of that found from the climb data. The fuel used at cruising height is found by *starting* the trip at the point on the distance and time scales corresponding to 5 000 lb of fuel.

The same method can be used if it is necessary to alter the method of cruise control or the height during the course of the journey. The fuel actually used in the first part of the journey gives us a fictitious time and fictitious distance from which to start the second part of the journey under the new conditions. A detailed example of this procedure is worked out in Chapter XII, on Flight Planning.

10.10. Engine Failure

For a four-engined aircraft, any alterations in the settings of the other three engines, made necessary by the failure of one, will depend upon (*a*) the cruise control method in force before failure, (*b*) the objective after failure. For constant power operation, schedule can only be kept by multiplying the power output of the other three by $\frac{4}{3}$. If this can be done without using rich mixture, the total fuel flow will not necessarily increase, but it is more usual to leave the three good engines at the same power, and re-calculate the E.T.A., or else to change over to constant speed, or a fixed percentage above maximum range.

If operating beforehand at constant speed, we can keep schedule, i.e. maintain the same speed, by increasing the power from the remaining engines. This will cause, at the heavier weights, some decrease in a.m.p.g., and a new chart for these

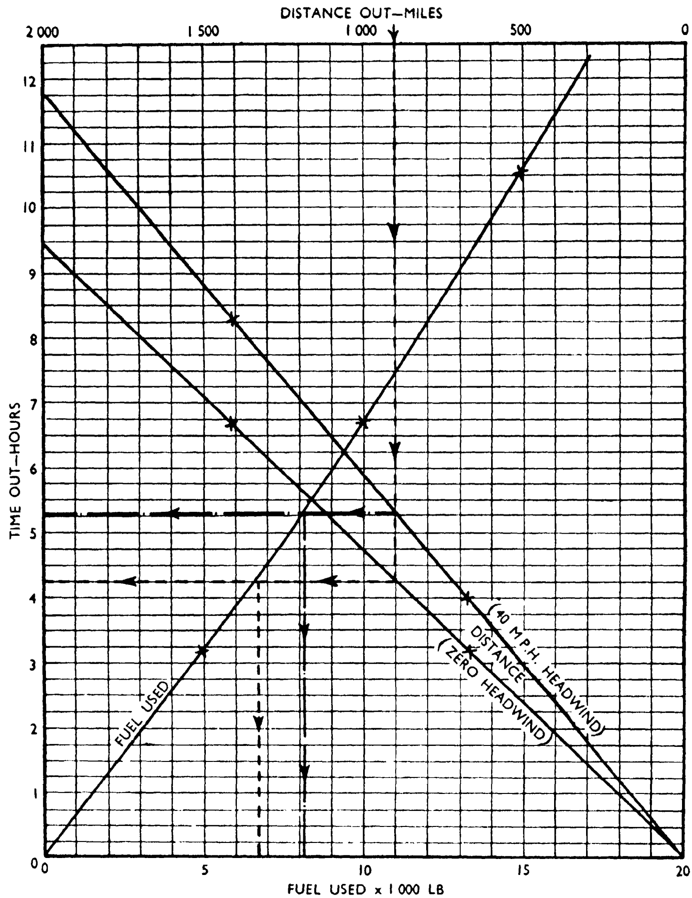


FIG. 79. FUEL CHART: CONSTANT SPEED (210 M.P.H.): THREE ENGINES

conditions will have to be constructed. From Fig. 57, at 210 m.p.h.—

Weight W	65 000	60 000	55 000	50 000
a.m.p.g.	1·0000	1·0725	1·150	1·235
b.h.p./Eng.	900	839	780	726
Fuel Flow/Eng. . . .	53	50	47	44
Fuel Flow, 4 Engs. . .	212	200	188	176
New b.h.p./Eng. . . .	1 200	1 120	1 040	968
Fuel Flow/Eng. . . .	72	67	62	57
Fuel Flow, 3 Engs. . .	216	201	186	171
Fuel Flow, with Drag Factor.	228	212	196	180
New a.m.p.g. . . .	0·930	1·012	1·104	1·208
Fuel Used	0	5 000	10 000	15 000
a.m.p.lb	0·129	0·141	0·1535	0·168
Distance Gone . . .	0	675	736	804
Total Distance . . .	0	675	1 411	2 215
Time at 210 m.p.h. . .	0	3·21	3·50	3·83
Total Time . . .	0	3·21	6·71	10·54
Time at 170 m.p.h. . .	0	3·97	8·32	13·00

It appears from the new chart (see Fig. 79) that three-engine operation uses slightly more fuel than constant power (cf. Fig. 63).

CHAPTER XI

Fuel Allowances and Reserves

11.1. General

THE previous chapter dealt with range calculations for the journey on a particular day. Over any prescribed route, there will be daily variations in headwind and conditions at the destination airport; but apart from these variations the fuel allowances will be the same throughout the year. This chapter will deal with these allowances, as well as the effect of headwind on schedule time and payload.

The three variables—

(1) Time of flight,
(2) Fuel required,
(3) Payload capacity,

are closely interlinked, and that they must be found in that order. When all three are known for a particular set of conditions, we can extend the results to cover more general or average conditions, thereby providing the operator with a *route analysis* on which his future planning can be based.

We have already seen that the time of flight, depends upon the method of cruise control, the distance, and the wind.

11.2. Headwinds

For any particular journey, knowledge of the headwind will lead to a perfectly definite answer for the time. When, however, we wish to know the *average* time for this particular journey, the answer must depend upon the *average* headwind to be found during the course of a year. In other words, the *schedule time* for the trip can only be found by examining a large number of different cases, and then deciding what is the least schedule time it should be possible to maintain with a high degree of regularity.

The maximum headwind component occurring on 85 per cent of the days is known as the *average maximum adverse wind*, and is used (for propeller driven aircraft) for the purpose of fixing the probable payload. Owing to the fact that the aircraft will not always carry maximum payload, the difference being made up by extra fuel, the regularity of the resulting services should be higher than 85 per cent—something in the region of 93–95 per cent is to be expected.

This average maximum adverse wind will of course be more than the actual wind on many occasions; so that for purposes other than fixing the payload and schedule time—such as estimating the year's expenses—we can use the average headwind, i.e. approximately the maximum wind reached on only 50 per cent of the days.

For long routes, covering zones of widely varying conditions, the accuracy of the schedule time will depend largely upon the winds encountered over the last few zones, so that it will not be necessary to take the average maximum adverse wind over the whole journey.

It is evident that there is considerable need for reliable information about the headwinds on any particular route. Unless this information is forthcoming, flight planning and route analysis may be very misleading.

11.3. Navigational Deviation

In order to cover the possibility of faulty navigation, resulting in the aircraft flying farther than planned, 2 per cent or less is added to the actual distance; this addition, however, is not made to the equivalent geographical mileage on the climb and ascent, nor to the further distance to the alternate.

11.4. Airframe Depreciation

In making schedules, it is usual to reduce the supposed mean air-speed by 2 per cent or less, to allow for variations between individual aircraft and increase of the drag due to various causes.

11.5. Carburettor Maladjustment

Another common source of error, particularly applicable to engines, with automatic boost control, is the anticipated fuel flow; and a 3 per cent allowance is made for this purpose.

11.6. Fuel Remaining in Tanks

On large aircraft, a reserve of 10 minutes' fuel at cruising consumption is at present allowed in *each* tank from which petrol is directly drawn from the engines at any stage in the flight: this precaution is made in order to avoid the danger of momentary engine failure during gentle manœuvres.

11.7. Route Analysis

We are now in a position to draw up a *Route Analysis* for any particular journey, and will do so for the imaginary trip for which a preliminary flight plan is given in Chapter XII.

ROUTE ANALYSIS SHEET

1. Service	
2. Aircraft	
3. Sector	
4. Alternate destination	
5. Prescribed stage length	2 000 miles or R miles
6. Plus 2% for navigational deviation	2 040 miles or 1·02R miles
7. Further distance to alternate	100 miles or r miles
8. Total geographical distance	2 140 miles or (1·02R + r) miles
9. Cruising height	10 000 ft
10. Method of cruise control	Constant Power

TIME AND FUEL ESTIMATE

No.	*In Symbols*	*Example*	*Unit*
11. Equivalent geographical mileage for climb and glide	0	14	miles
12. Total E.G.M. to destination (6 + 11)	1·02R	2 054	miles
13. Average cruising T.A.S.	V	215	m.p.h
14. Airframe depreciation (2%)	− 0·02V	− 5	m.p.h.
15. Headwind component	$-v$	− 10	m.p.h
16. Average ground speed (13–14–15)	$0{\cdot}98V - v$	200	m.p.h.
17. Equivalent cruising time to destination (12 ÷ 16)	$\dfrac{1{\cdot}02R}{0{\cdot}98V - v}$	10·27	hr
18. Allowance for ground manœuvring (10 min)	$\frac{1}{6}$	0·17	hr
19. Allowance for final circuit (15 min)	$\frac{1}{4}$	0·25	hr
20. Schedule time (17 + 18 + 19)	$\dfrac{1{\cdot}02R}{0{\cdot}98V - v} + \dfrac{5}{12}$	10·69	hr
21. Schedule block to block speed (5 ÷ 20)	$\dfrac{R}{\dfrac{1{\cdot}02R}{0{\cdot}98V - v} + \dfrac{5}{12}}$	187	m.p.h.

TIME AND FUEL ESTIMATE—(*contd.*)

No.	*In Symbols*	*Example*	*Unit*
22. Average cruising fuel flow	E	1 512	lb/hr
23. Carburettor maladjustment (3%)	0·03E	45	lb/hr
24. Average fuel flow (22 + 23)	1·03E	1 557	lb/hr
25. Fuel for equivalent time (17 × 24)	$\frac{1{\cdot}05RE}{0{\cdot}98V - v}$	15 990	lb
26. "Lost fuel" allowance	*nil*	800	lb
27. Final circuit allowance (22 ÷ 4)	$\frac{1}{4}E$	378	lb
28. Fuel to destination (25 + 26 + 27)	$\frac{1{\cdot}05RE}{0{\cdot}98V - v} + \frac{1}{4}E$	17 168	lb
29. Additional time to alternate (7 ÷ 16)	$\frac{r}{0{\cdot}98V - v}$	0·5	hr
30. Additional fuel to alternate (29 × 24)	$\frac{1{\cdot}03Er}{0{\cdot}98V - v}$	779	lb
31. Allowance for stand-off at alternate (22 ÷ 4)	$\frac{1}{4}E$	378	lb
32. Fuel to remain in tanks	$\frac{1}{6}E$	252	lb
33. Total fuel required (28 + 30 + 31 + 32) (which can be turned in gallons and compared with the normal tankage)	$\frac{1{\cdot}05RE + 1{\cdot}03rE}{0{\cdot}98V - v} + \frac{2}{3}E$	18 577	lb
34. Percentage fuel reserves (100 × $\frac{33}{28}$ − 100)	$100\left(\frac{12{\cdot}36r + 4{\cdot}9V - 5v}{12{\cdot}6R + 2{\cdot}94V - 3v}\right)$	8	%
35. Equivalent still air mileage (13 × 33 ÷ 22)	$\left(\frac{1{\cdot}05R + 1{\cdot}03r}{0{\cdot}98V - v}\right)V + \frac{2}{3}V$	2 495	miles

11.8. Effect of Allowances

It appears from this analysis that the various allowances have increased the effective distance from 2 000 miles to 2 495 miles, i.e. by as much as 25 per cent. We can examine the reason for this, slightly simplifying the analysis.

Again let V = average T.A.S.,
v = average headwind,
E = pound per hour = fuel flow, all engines,
R = prescribed stage length,
r = further distance to alternate.

As above, suppose that the E.G.M. on climb and glide, and the lost fuel, are both zero; this assumption is reasonable in practice. Then we can tabulate the figures found above as follows—

6 and 12. $1{\cdot}02\text{R}$

16. $0{\cdot}98\text{V} - v$

17. $\dfrac{1{\cdot}02\text{R}}{0{\cdot}98\text{V} - v} = 1{\cdot}04\,\dfrac{\text{R}}{\text{V}}\left(1 + \dfrac{v}{\text{V}}\right)$ (approx.)

20. $1{\cdot}04\,\dfrac{\text{R}}{\text{V}}\left(1 + \dfrac{v}{\text{V}}\right) + 0{\cdot}42$, the schedule time.

21. $\dfrac{\text{R}}{1{\cdot}04\,\dfrac{\text{R}}{\text{V}}\left(1 + \dfrac{v}{\text{V}}\right) + 0{\cdot}42}$

24. $1{\cdot}03\text{E}$

25. $1{\cdot}03\,\text{E} \times 1{\cdot}04\,\dfrac{\text{R}}{\text{V}}\left(1 + \dfrac{v}{\text{V}}\right) = 1{\cdot}07\text{E}\,\dfrac{\text{R}}{\text{V}}\left(1 + \dfrac{v}{\text{V}}\right)$ (approx.)

26. Nil

27. $\dfrac{\text{E}}{4}$

28. $1{\cdot}07\text{E}\,\dfrac{\text{R}}{\text{V}}\left(1 + \dfrac{v}{\text{V}}\right) + \dfrac{\text{E}}{4}$

29. $\dfrac{r}{0{\cdot}98\text{V} - v} = 1{\cdot}02\,\dfrac{r}{\text{V}}\left(1 + \dfrac{v}{\text{V}}\right)$ (approx.)

30. $1{\cdot}05\text{E}\,\dfrac{r}{\text{V}}\left(1 + \dfrac{v}{\text{V}}\right)$ (approx.)

31. $\dfrac{\text{E}}{4}$

32. $\dfrac{\text{E}}{6}$

33. $1{\cdot}07\,\dfrac{\text{ER}}{\text{V}}\left(1 + \dfrac{v}{\text{V}}\right) + \dfrac{\text{E}}{4} + 1{\cdot}05\,\dfrac{\text{E}r}{\text{V}}\left(1 + \dfrac{v}{\text{V}}\right) + \tfrac{5}{12}\text{E}$

$= \dfrac{\text{E}}{\text{V}}\left(1 + \dfrac{v}{\text{V}}\right)(1{\cdot}07\text{R} + 1{\cdot}05r) + \tfrac{2}{3}\text{E}$, the total petrol load.

35. $\left(1 + \dfrac{v}{\text{V}}\right)(1{\cdot}07\text{R} + 1{\cdot}05r) + \tfrac{2}{3}\text{V}$, the equivalent still air mileage.

Hence $35 - R = 0{\cdot}07R + 1{\cdot}05r + \frac{2}{3}V + \frac{v}{V}(1{\cdot}07R + 1{\cdot}05r)$, giving in our numerical example

$$
\begin{aligned}
0{\cdot}07R &= 140 \text{ (3 per cent, 2 per cent, 2 per cent)} \\
1{\cdot}05r &= 105 \text{ (3 per cent, 2 per cent, for alternate)} \\
\tfrac{2}{3}V &= 143 \text{ (stand off, tank reserve)} \\
\frac{v}{V}(1{\cdot}07R + 1{\cdot}05r) &= 105 \text{ (headwind)} \\
&\overline{\ 493}
\end{aligned}
$$

so that the increases from the four causes are roughly of equal importance.

11.9. Equivalent Still Air Mileage, etc.

We can thus analyse the components of equivalent still air mileage by saying that is found from R by adding—

(1) 7 per cent of R, for { navigational errors, airframe depreciation, carburettor maladjustment.

(2) 105 per cent of r, for the alternate, the increase of 5 per cent being for { airframe depreciation, carburettor maladjustment.

(3) $\frac{2}{3}$ of V, for stand-off, and tank reserves.

(4) $\frac{v}{V}(1{\cdot}07R + 1{\cdot}05r)$ for headwinds.

Some of these increases can, and should be, reduced, but (3) is liable to increase as traffic density increases, and (4) may be large over routes like the west-bound Atlantic.

It may be noted that our answer is considerably in excess of that given by some authorities. For instance, Mentzer and Nourse ("Some Economic Aspects of Transport Aeroplane Performance," *R.Ae.Soc.Journal*, June, 1940) merely allow 2 per cent for navigational errors, and 0·20 hr for taxying and manœuvring, so that they give

$$1{\cdot}02\frac{R}{V} + 0{\cdot}26$$

in place of our value for *schedule time*

$$1{\cdot}04\,\frac{R}{V}\left(1 + \frac{v}{V}\right) + 0{\cdot}42.$$

Again, R. K. Pierson ("Design Factors of Civil Aircraft Affecting the Operating Cost," *R.Ae.Soc.Journal*, Jan., 1945), gives as *equivalent still air distance*

$$1{\cdot}02R + 60$$

in place of our result

$$1{\cdot}07R + 1{\cdot}05r + \tfrac{2}{3}V + \frac{v}{V}\,(1{\cdot}07R + 1{\cdot}05r).$$

11.10. Total Petrol Load

The main importance in these differences lies in the fact that *the total petrol load* required in our tabulation is

$$E\,.\,\frac{1}{V}\left(1 + \frac{v}{V}\right)(1{\cdot}07R + 1{\cdot}05r) + \tfrac{2}{3}E,$$

whereas that required by the authors mentioned is considerably less. The consequence is that the *payload* is seriously affected, as will be seen in Chapter XIII.

Note. In the case of constant speed or maximum range operation when E is not constant, we can still use this result if E is found by dividing the weight of fuel at cruising height found from the chart (disregarding all allowances and the alternate), by the time for the distance at cruising height found from the chart (again disregarding allowances).

CHAPTER XII

Flight Planning

12.1. The Pre-flight Plan

12.1.1. METHOD. Time spent in pre-flight planning, before the aircraft gets near the runway, is well repaid. To introduce the subject we will perform a calculation by a method which is not actually used in practice.

First of all we must know which type of cruise control is to be adopted—constant speed (and if so, what speed), maximum range, or constant power (and if so, what power is most suitable to (*a*) the length of the journey, (*b*) the engines).

We are then ready to receive the weather forecast. Armed with this we are in a position to

(*a*) select the altitude, or altitudes,
(*b*) decide on the total time,
(*c*) decide on the total fuel.

12.1.2. ZONES. We divide the flight into zones, the length of which (preferably 200–400 miles) should be determined by the following factors: change of course, radio facilities, landmarks. The first zone is, of course, subdivided into climb and cruise. At one of the four or so altitudes selected as possible, we tabulate as shown on p. 196.

12.1.3. CLIMB. Referring to climb tabulation or chart for aircraft in question, we find average T.A.S. = 160 m.p.h., time = 17 min, fuel used = 1 155 lb. By usual methods, or computor, ground speed = 151 m.p.h.; hence forward distance on climb = 43 miles. Remainder of zone to cover = 257 miles.

12.1.4. REMAINDER OF FIRST ZONE. Turning to the first chart for constant power (Fig. 62), we find that the speeds at beginning and end are 199·6 and 204. Mean T.A.S., 201·8. The ground speed is now 195·7, from computor. Hence the time for this part of the first zone is $\frac{257}{195{\cdot}7}$ or 1 hr 19 min (also to be found from Fig. 61).

Referring now to the second chart (Fig. 63), the fuel used

Method : Constant Power

Height : 10 000 ft

Zone No.	Zone Distance (miles)	Total Distance (miles)	True Course	Wind	Temp.	Height Density (ft)	T.A.S. (m.p.h.)	G.S. (m.p.h.)	Zone Time (hr min)	Total Time (hr min)	Zone Fuel (lb)	Total Fuel (lb)
1 (Climb)	43	43	45	330/20	− 2	Taken as 10 000	160	151	0 17	0 17	1 155	1 155
1 (Cruise)	257	300	45	330/20	− 1	,,	201·8	195·7	1 19	1 36	1 900	3 055
2 ,,	250	550	45	315/15	− 1	,,	206·4	205	1 13	2 49	1 800	4 855
3 ,,	325	875	45	315/15	− 6	,,	209·8	209	1 33	4 22	2 500	7 355
4 ,,	210	1 085	50	300/10	− 6	,,	214·4	211	1 0	5 22	1 400	8 755
5 ,,	415	1 500	50	290/15	− 7	,,	219·6	218	1 54	7 16	2 800	11 555
6 ,,	500	2 000	50	280/30	− 8	,,	225·1	226	2 13	9 29	3 200	14 755

between 43 and 300 miles out is 2 300 — 400 = 1 900 lb and the time taken is 1·5 — 0·2 = 1·3 hr = 1 hr 18 min, thus checking the result obtained from the first chart.

(*Note.* In the case of Figs. 70–78, climb is allowed for, and the above subtraction is not necessary.)

Should the density height be very different from 10 000 ft, it will be necessary to use two pairs of charts at heights above and below the actual density height, and interpolate the required answers for time and fuel used. In our case, this is not necessary.

12.1.5. SECOND ZONE. From the first chart (Fig. 62), the mean T.A.S. is $\frac{204 + 208\cdot8}{2} = 206\cdot4$, so that the ground speed $= 205$ and the zone time $\frac{250}{205} = 1$ hr 13 min. From the second chart (Fig. 63), the fuel used is 1 800 lb and the time taken 1 hr 12 min.

In the case of constant power, the fuel used can be found without the second chart, by multiplying the zone time by the fuel flow at the chosen power.

We can now complete the picture for the whole journey at the chosen height.

We should then repeat the process at a number of other heights, with their appropriate winds and temperatures, and select the height giving the most favourable results. We may, of course, decide to fly at different heights for different zones. An example of this, in which fictitious time and fictitious fuel and figures are required, is shown in § **12.4**.

12.2. Diversions and Allowances

When the height, or heights, have been selected, the plan should be extended to cover diversion to the one or more possible alternates, should the original destination airport prove unserviceable. To the total fuel then required should be added the fuel required for stand-off, navigational errors, etc.; a fuller account of which has been given in the previous chapter.

12.3. Engine Failure

The cruise control method to be adopted in the case of engine failure should be known beforehand. If the aircraft is originally operating at constant power, we can (*a*) stay at the same power on the remaining engines, thereby reducing the fuel flow but increasing the time for the journey, or (*b*) change over to operation at constant speed, or (*c*) fly at the maximum range speed appropriate to the expected headwind. If the aircraft is originally operating at constant speed, it is usual to keep this speed by increasing the power on the remaining engines, a case already worked out. Over short distances, particularly with twin-engined aircraft, we may run the other engine at maximum weak mixture power and let the speed look after itself.

12.3.1. Flight Graphs. In all cases it is a wise precaution to prepare beforehand a flight graph, from which it is easy to predict, as soon as an engine fails, what will be the consequences of carrying on to the destination, or of returning to the start.

In order to explain how a *flight graph* can be built up from the three-engine chart, we propose to give a somewhat simplified example of the journey already described. Using the three-engined chart in Fig. 79, the results can be tabulated as follows—

	(Climb)	(Cruise)					
Zone	1	1	2	3	4	5	6
Zone Distance (miles)	43	257	250	325	210	415	500
Total Distance (miles)	43	300	550	875	1 085	1 500	2 000
Headwind or Tailwind	10	− 20	− 30	− 40	− 20	+ 10	+ 20
T.A.S. (m.p.h.)	160	210	210	210	210	210	210
Ground Speed Forward (m.p.h.)	150	190	180	170	190	220	230
Ground Speed Backward (m.p.h.)	170	230	240	250	230	200	190
Zone Time Forward (hr min)	0 29	1 35	1 39	1 91	1 11	1 89	2 18
Total Time Forward (hr min.)	0 29	1 64	3 03	4 94	6 05	7 94	10 12
Zone Time Backward (hr min.)	0 25	1 12	1 04	1 30	0 92	2 07	2 63
Total Time Backward (hr min.)	0 25	1 37	2 41	3 71	4 63	6 70	9 33
Zone Fuel Used Fwd. (lb)	1 160	2 100	2 200	2 900	1 600	2 500	2 800
Total Fuel Used Fwd. (lb)	1 160	3 260	5 460	8 360	9 960	12 460	15 260
Total Fuel Left Forward (carrying 17 000 lb)	15 840	13 740	11 540	8 640	7 040	4 540	1 740
Total Fuel Required for Backward (lb)	200	1 600	3 600	5 700	7 100	9 900	13 200

If we plot the last two rows, on a distance basis, we have a picture of the fuel position at any distance from the start (see Fig. 80). Should engine failure not occur until a certain distance out, the curve giving "fuel required for return" will enable us to find out quickly what the position is, if we also plot the "fuel still left" curve for the four-engine case on the same graph.

We can also plot on a time basis, i.e. draw the graphs relating distance and fuel with Greenwich Mean Time. In practice we can plot the "fuel still left" curve as the flight proceeds, relying on the previously plotted "fuel required for return" curve. It will be noticed that the figures given for fuel are "round numbers," because it is difficult to read the chart with greater

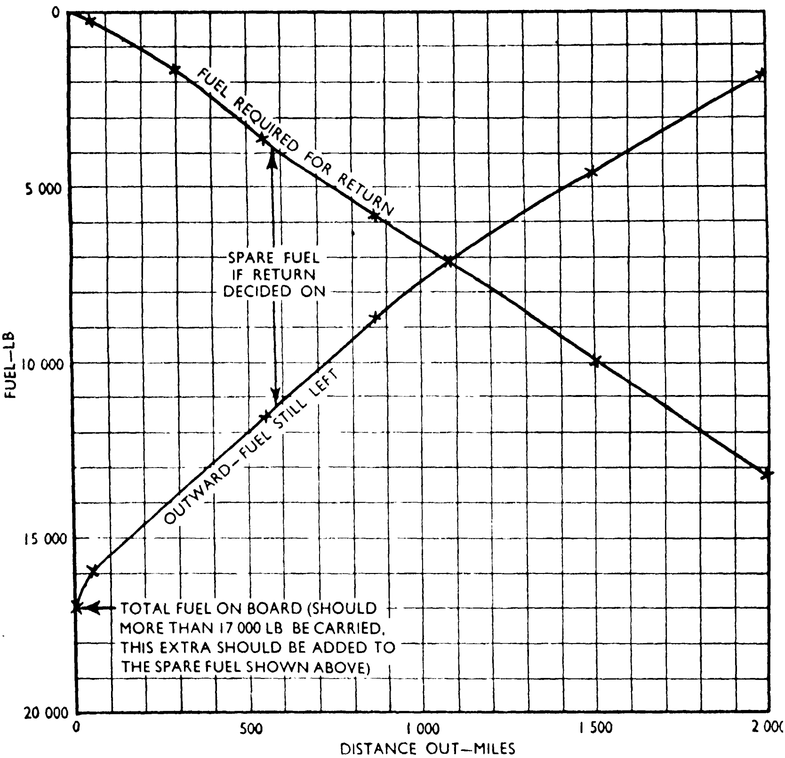


FIG. 80. THREE-ENGINE FLIGHT GRAPH

accuracy. This difficulty could be overcome by comprehensive tabulation of the fuel data. (Many pilots and operators prefer tabulation to charts.)

12.4. Change of Height and Cruise Control Method

A good deal of practice in the use of the charts is advisable, and we now give a further example using Figs. 42 and 70–78.

(*Note.* Figures in 4th column, or corrected ones, are found from the charts.)

Narrative	Figure Given	Relevant Chart	Figure Deduced
12.4.1. At Start			
Take-off weight	90 000 lb	—	—
Operational height	15 000 ft	—	—
Direction	Eastbound	—	—
Cruising procedure	Constant power	—	—
Tailwind forecast	20 knots	—	—
O.A.T. forecast	− 15°C	—	—
Track distance	2 500 naut. miles	—	—
Density height	—	Correction chart, Fig. 1	15 000 ft
12.4.2. Cruising Climb	2 300 r.p.m. auto-rich		
Fuel used	—	Climb tabulation	1 635 lb
Time	—	,,	26 min = 0·43 hr
I.A.S.	—	,,	140 knots
Mean T.A.S.	—	,,	172 knots
Ground speed	—	—	192 knots
Distance covered	—	—	83 naut. miles
12.4.3. Flight Plan for Whole Distance of 2 500 Miles			
Mean T.A.S.	—	Fig. 70, C.P. 15 000 ft	216 knots
Ground speed	—	—	236 knots
Time at oper. height	—	—	10·21 hr
Total time	—	—	10·66 hr
Check on total time	—	Fig. 74, C.P.	10·75 hr
Total fuel used	—	,,	21 400 lb
"Navigational deviation"	Add 2%	Usage	21 800 lb
Fuel for 1 hr stand-off	1 320 lb	,,	23 120 lb
Carburettor maladjustment	Add 3%	,,	23 800 lb
Fuel actually carried	24 000 lb	—	—
12.4.4. After going 1 000 Miles, Aircraft Recalled to Base and Descends to 14 000 ft for Return			
Revised Flight Plan for First 1 000 Miles			
Mean T.A.S.	—	Fig. 70, C.P.	211 knots
Ground speed	—	—	231 knots
Time at oper. height	—	—	3·97 hr
Total time	—	—	4·4 hr
Check on total time	—	Fig. 74, C.P.	4·5 hr
Total fuel used	—	,,	9 400 lb
12.4.5. Return Flight. First 500 Miles at Constant Speed, Against Revised Headwind			
Constant air speed	165 knots	Usage	—
New headwind	60 knots	—	—
(Enter chart at "fuel used 9 400 lb" point, using 15 000 ft chart.)			
Fictitious time	—	C.S. 15 000 ft chart, Fig. 78	4·75 hr
Fictitious distance	—	,, ,,	700 naut. miles
(Now travel a further 500 miles)			
Total fuel now	—	,, ,,	15 400 lb
Fuel for these 500 miles	—	—	6 000 lb
Total time now	—	—	8·30 hr
Time for these 500 miles	—	—	355 hr
12.4.6. Descent to 10 000 ft for Remaining 500 Miles, Against Re-revised Headwind of 50 Knots			
Time of descent	—	—	—
Fuel used in descent	—	—	•
Outside air temperature	− 5°	—	—
Density height	—	Correction chart, Fig. 1	10 000 ft

Narrative	Figure Given	Relevant Chart	Figure Deduced
12.4.7. Diversion to Alternate 700 Miles Instead of 500 Miles			
Case (i): Constant Air-speed, Four Engines			
Fictitious time (for 15 400 lb fuel)	—	C.S. 10 000 ft, Fig. 77	9·4 hr
Fictitious distance	—	,,	1 350 naut.miles
Total distance	—	—	2 070 naut.miles
Total time	—	C.S. 10 000 ft, Fig. 77	14·5 hr
Time for last 700 miles	—	—	5·1 hr
Total fuel	—	C.S. 10 000 ft, Fig. 77	22 800 lb
Fuel for last 700 miles	—	—	7 400 lb
Fuel available for stand-off	—	—	1 200 lb, less*
Case (ii): Maximum Range, Four Engines			
Fictitious time (for 15 400 lb fuel)	—	Max. R. 10 000 ft	9·55 hr
Fictitious distance	—	,, ,,	1 300 naut.miles
Total distance	—	—	2 000 naut.miles
Total time	—	Max. R. 10 000 ft	14·85 hr
Time for last 700 miles	—	—	5·3 hr
Total fuel	—	Max. R. 10 000 ft	22 200 lb
Fuel for last 700 miles	—	—	6 800 lb
Fuel available for stand-off	—	—	1 800 lb, less*
Mean T.A.S. for 700 miles	—	—	—
Ground speed for 700 miles	—	—	—
Check on time for 700 miles	—	—	—
Case (iii): Constant Speed on Three Engines			
Fictitious time (for 15 400 lb fuel)	—	3-engine chart	8·85 hr
Fictitious distance	—	—	1·250 naut.miles
Total distance	—	—	1 950 naut.miles
Total time	—	—	140 hr
Time for last 700 miles	—	—	5·15 hr
Total fuel	—	—	23 000 lb
Fuel for last 700 miles	—	—	7 600 lb
Fuel available for stand-off	—	—	1 000 lb, less*
12.4.8. Summary of E.T.A.S.			
Case (i)	—	—	—
Case (ii)	—	—	—
Case (iii)	—	—	—

12.5. A Complete Flight Plan

The transatlantic aircraft charts give sufficient information for the purpose of making a complete flight plan, in our case from Gander to Prestwick.

The meteorological service provides us with a map of isobars and fronts, from which the flight forecast is derived, together with the terminal and alternate forecasts.

We can now go ahead with the flight plan, using constant power, which is shown in detail on pages 202–3. Attention is called to the allowances calculated in § **12.5.3** (see Chapter XI).

The student is advised to work through the plan in detail, and check all the numbers given against those which he finds

FLIGHT PLAN

AIRCRAFT: *Trans.*	DATE: *25th Sept. /46*	SERV. No. —	FROM: *Shannon*	TO: *Gander*
Route: *G/C*	Proc. *Const. Speed*	Gr. Wt. *90 000 lb*	E.T.D. *0245* Z	E.T.A. *1352* Z
Alternates-Term	(1) *Dartmouth*	(2)	Ret. (1) *Shannon*	(2) *PWK*
Sunrise	(1) Z	(2) Z	(1) Z	(2) Z
Sunset	(1) Z	(2) Z	(1) Z	(2) Z

Terminal Destination Dist. *1 720* nm Time *11* h *07* m	(*a*) Fuel to Destination	*17 040* lb
	(*b*) Add 160 lb for ea. flight plan hour (max. 1 600 lb) (*1 780 lb*)	*1 600* lb
Furthest Terminal Alt. Dist. *452* nm Time *2* h *40* m	(*c*) Fuel to Alternate ( )	*3 700* lb
	(*d*) Add 25% of fuel to alternate ( )	*925* lb
	(*e*) Add stand-off ( )	*1 320* lb
Fuel on board per loadsheet gal	(*f*) Add residual fuel (*600*)	*600* lb
" " lb		
Flight endurance 4 engines h m	Fuel per loadsheet (less 600)	lb
Furthest Ret. Alt. Dist. nm Time h m	(Fuel required *25 185*) (Stand-off *1 320*)	lb
Calculate Point No Return using *13* h *18* m	Endurance given by fuel of	*20 450* lb
Point of No Return is *7* h *46* m	or *1 188* nms	

Zone No.	To	Press. Alt.	Air Temp.	Den. Alt.	T.A.S. Kt	Wind Kt	Track °T	True Co	W Var.	Mag Co	G.S. Kt	Zone Dist	Zone Time	Total Time	E.T.A.	Fuel Used
2/3	*DR Ht.*				*168*	*235/32*	*279*	*271*	*16*	*287*	*142*	*223*: *28½*	*12*	*12*		
3	*15W*	*6 710*	*+ 8*	*7 510*	*185*	*240/35*	*279*	*272*	*16*	*288*	*157*	*194½*	*74½*	*86½*		
4	*20W*	*6 800*	*+ 8*	*7 600*	*185*	*270/35*	*272*	*272*	*20*	*292*	*150*	*180*	*72*	*158½*		
5	*25W*	*6 710*	*+ 4*	*7 110*	*183*	*280/45*	*272*	*274*	*22*	*296*	*140*	*179*	*76½*	*235*		
6	*30W*	*6 650*	*+ 4*	*7 050*	*183*	*290/38*	*264*	*269*	*25*	*294*	*150*	*181*	*72½*	*307½*		
7	*35*	*6 560*	*+ 4*	*6 960*	*183*	*300/35*	*264*	*271*	*27*	*298*	*154*	*181*	*70½*	*378*		
8	*40*	*6 440*	*+ 4*	*6 840*	*183*	*300/34*	*256*	*264*	*29*	*293*	*156*	*189*	*72½*	*450½*		
9	*45*	*6 290*	*+ 5*	*6 490*	*181*	*300/34*	*256*	*264*	*30*	*294*	*154*	*191*	*74½*	*525*		
10	*50*	*6 200*	*+ 5*	*6 400*	*181*	*280/18*	*249*	*252*	*31*	*283*	*168*	*205*	*73½*	*598½*		
11	*Gander*	*6 230*	*+ 7*	*6 630*	*182*	*250/15*	*249*	*249*	*30*	*279*	*167*	*191*	*68½*	*667*		
Av.	*Density alt.*	*7 000 ft*										*1 720*	*667*			*17 040*
	Dart-mouth	*6 440*	*+ 7*	*6 840*	*183*	*240/15*	*235*	*236*			*170*	*452*	*160*			*3 700*
													827			

Signed . Navigator Signed . Captain

from the charts. Complete agreement cannot be expected, because of the thick lines used on the charts.

The following route information is given for the various zones—

From	To	Track (T)	Naut. Miles	Variation
ROUTE—				
Shannon (10°W)	15 00 W	279	223	16 W
15 00 W	20 00 W	272	180	20 W
20 00 W	25 00 W	272	179	22 W
25 00 W	30 00 W	264	181	25 W
30 00 W	35 00 W	264	181	27 W
35 00 W	40 00 W	256	189	29 W
40 00 W	45 00 W	253	191	30 W
45 00 W	50 00 W	249	205	31 W
50 00 W	Gander (54°W)	249	191	30 W
			1 720	
ALTERNATES—				
Gander	Sydney (60°W)	233	286	
Gander	Halifax (Dartmouth) (63°W)	235	453	

12.5.1. CHOICE OF HEIGHT. As a result of the meteorological conditions and the characteristics of the aircraft, a flight in the neighbourhood of 7 000 ft is decided upon.

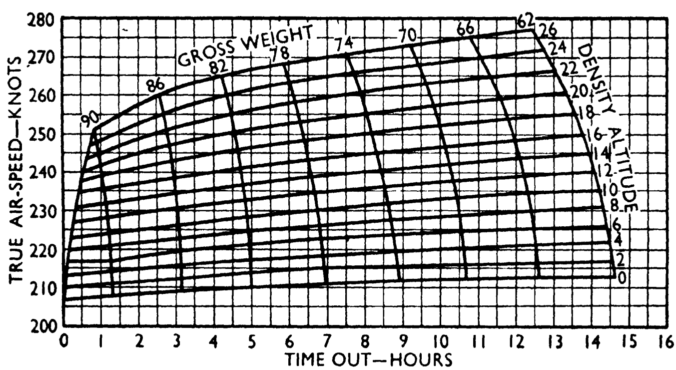


FIG. 81. FLIGHT PLAN FOR TRANSATLANTIC AIRCRAFT AT 93 000 LB A.U.W.

True air-speed versus time out; constant power: short range.

12.5.2. Charts Used. The charts used are those for the transatlantic aircraft at constant speed (Figs. 76, 77, 78).

12.5.3. Allowances

Fuel allowance for air distance with – 50 kt wind component and 1½% navigational deviation	20 450 lb
Fuel allowance to furthest alternate (Dartmouth) . .	4 350 lb
Stand-off allowance for 1 hr	1 320 lb
Residual fuel, i.e. reserve	600 lb
Total fuel allowance	26 720 lb
Less run-up, take-off, and taxying allowance included on range charts	300 lb
	26 420 lb

12.5.4. Winds and Temperatures at Different Heights.

Zone	03	°C	04	°C	05	°C	06	°C	07	°C
2 000′ M.S.L.	230/30		290/20		290/38		290/38		300/35	
6 000′ M.S.L.	240/35	8	270/35	8	280/45	4	290/38	4	300/35	4
8 000′ M.S.L.	240/38	4	260/40	4	270/48	0	300/40	0	310/38	0
10 000′ M.S.L.	250/40	0	250/45	0	270/50	4	300/40	– 4	310/38	– 3
M.S.L. Press.	1 006		1 003		1 006		1 008		1 011	

Zone	08	°3	09	°C	10	°C	11	°C	12	°C
2 000′ M.S.L.	300/30	4	300/30		270/15		230/10		210/10	
6 000′ M.S.L.	300/34	5	300/34	5	280/18	5	250/15	7	240/15	7
8 000′ M.S.L.	310/36	1	300/36	2	290/20	3	270/18	5	260/18	5
10 000′ M.S.L.	310/38	- 2	300/38	– 2	290/20	0	280/20	4	270/20	4
M.S.L. Press.	1 015		1 020		1 023		1 022		1 020	

CHAPTER XIII

Payload

13.1. Disposable Load

IF we add to the unladen weight of the aircraft (= empty weight + furnishings + equipment and essential stores), the weight of the necessary oil, together with the weight of the crew and their baggage, and then subtract this total from the maximum take-off weight, we arrive at the *disposable load* = *fuel load* + *payload.*

The proportion of fuel to payload will evidently depend upon the distance to be covered, in a manner shown roughly by the diagram in Fig. 82.

13.2. Maximum Payload for Short Distances

In reality, however, this is not the case, because of the restriction on landing weight, which is usually less than the take-off weight; so that our diagram now takes the shape shown in Fig. 83.

Maximum payload = *disposable load less* (*take-off weight* − *landing weight*), and this maximum payload is attainable for any range for which the fuel required does not exceed (take-off weight − landing weight). For example, take-off weight 65 000 lb, landing weight 58 000 lb, disposable load 20 000 lb, hence maximum payload = 20 000 − 7 000 = 13 000 lb, obtainable for any range that does not require more than 7 000 lb of fuel.

13.3. Maximum Payload in General

The general case can be put into symbolic form, as follows—

Let W_{TO} = take-off weight,

W_L = landing weight,

W_E = weight of empty aircraft, including fixtures and crew,

W_D = disposable load, made up as follows—

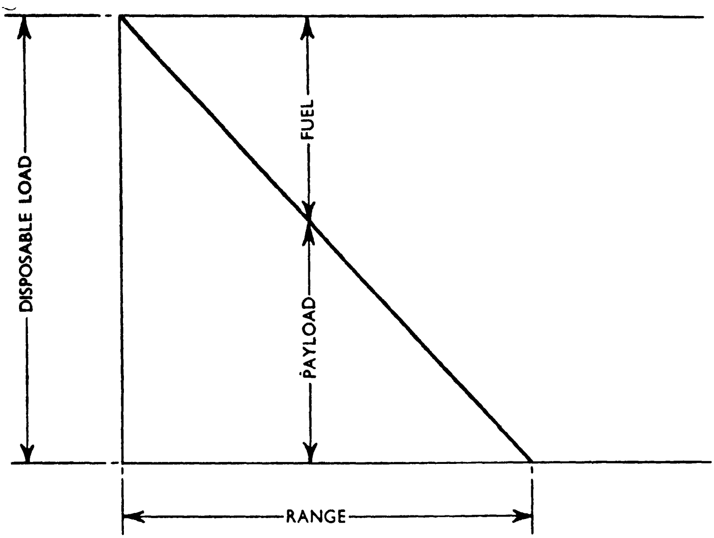


FIG. 82. DISPOSABLE LOAD VERSUS RANGE

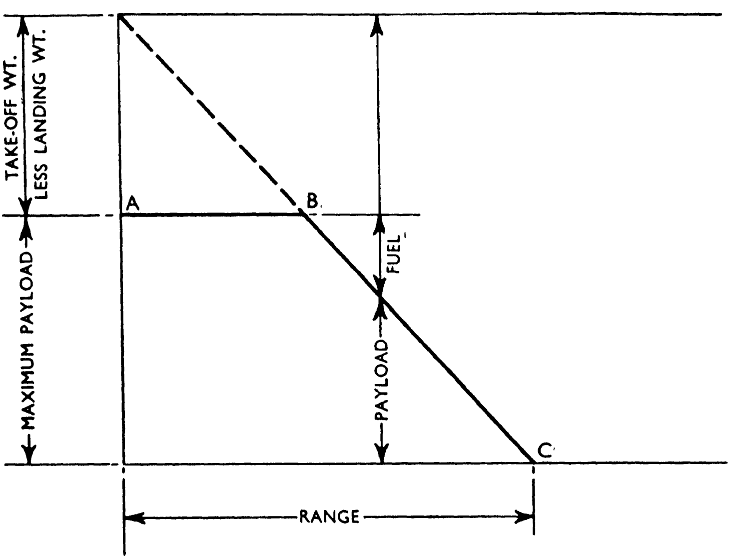


FIG. 83. DISPOSABLE LOAD VERSUS RANGE

W_D the disposable load:
- W_f = weight of fuel (at 7·2 lb or 3·27 kg/gal),
- W_{rf} = weight of reserve fuel (at 7·2 lb or 3·27 kg/gal),
- W_p = payload.

Then the maximum payload is the *lesser* of the two quantities $W_{p_1} : W_{p_2}$ given below (assuming that the volumetric restriction of 12 lb/ft³ is borne in mind)—

(1) $W_{p_1} \leq W_{TO} - W_E - W_{rf} - W_f$

(2) $W_{p_2} \leq W_L - W_E - W_{rf}$

Equation (2) operates over the portion *AB* of the diagram (Fig. 83) and equation (1) over the portion *BC*. For example,

if W_{TO} = 65 000 lb,
W_L = 58 000 lb,
W_E = 45 000 lb,
W_f = 17 000 lb, for 2 000 miles,
W_{rf} = 1 500 lb,

then the maximum payload, for small ranges, is

$$W_L - W_E - W_{rf} = 11\,500 \text{ lb}$$

and the payload for the range of 2 000 miles is

$$W_{TO} - W_E - W_{rf} - W_f = 1\,500 \text{ lb}$$

(assuming that the fuel tank capacity is sufficient).

13.4. Nomograms

The payload, disposable load, and all-up weight are related by a simple nomogram. A typical example is shown in Fig. 84.

13.5. Minimum Weights of Passengers and Crew

In order to assess the number of passengers that can be carried, a Notice to Airmen dated 21st April, 1947, gives the following weights for the different classes—

	kg	lb
For adult males (including crew)	75	165
For adult females (including crew)	65	143
For children between 2 and 12.	39	85
For infants under 2	8	17

To this we may have to add hand baggage at 10 lb per passenger seat, and heavy baggage at 66 lb per seat.

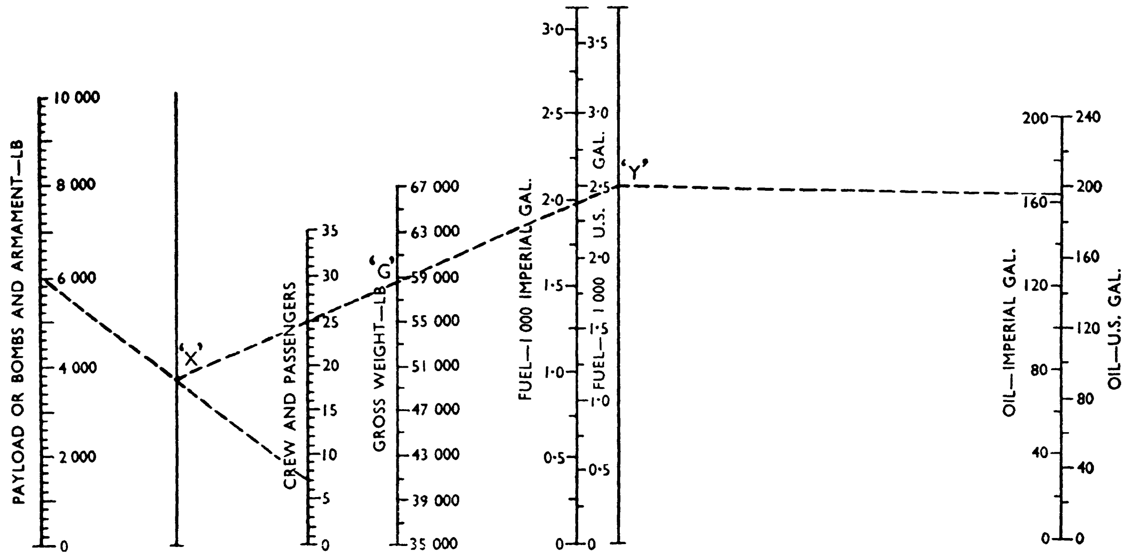


FIG. 84. GROSS WEIGHT NOMOGRAM

Instructions: To determine ground weight, connect armament load and number of crew to get "X." Connect fuel and oil to get "Y." Connect "X" and "Y." Read gross weight at "G."

13.6. Effect of Fuel on Payload

We have seen that the payload for long ranges is

$$W_{TO} - W_E - W_f - W_{rf} \ldots \text{(from § **13.3**)}$$

and that, in general,

$$W_f + W_{rf} = \frac{E}{V}\left(1 + \frac{v}{V}\right)(1{\cdot}07R + 1{\cdot}05r) + \tfrac{2}{3}E \ldots$$

(from § **11.8**)

The payload can thus be increased in two ways—

(1) By increasing $W_{TO} - W_E$.

(2) By decreasing E, the average fuel flow in lb per hour.

Suggestions for achieving (1) are discussed at the end of Chapter XIV.

A worth-while reduction in E can only be achieved by strict adherence to some form of cruise control.

CHAPTER XIV

Choice of Range: Re-fuelling in Flight

14.1. Choice of Range

It will be observed that the three variables which fix the payload for a particular aircraft are (*a*) take-off weight, (*b*) speed, (*c*) range. The earlier chapters have explained how the method of cruise control, i.e. the choice of speed, affects the payload. We must now consider the effect of range, which has a far more devastating effect than speed on the earning capacity of an airline.

14.2. Fuel Costs per Ton-mile

As a numerical example, consider the case of a well-known four-engined flying boat (take-off weight 56 000 lb) flying at a constant speed of 130 knots. Assume a disposable load of 18 000 lb, and no headwind, and disregard allowances. Then at 10 000 lb we have the following figures (see Fig. 85)—

R Distance (in naut. miles)	F_1 Fuel (gal)	F_2 Fuel (lb)	P Payload	Cost per nautical ton-mile $= \frac{4\,480\,F_1}{RP} \times 12$ pence (at 2s. per gal)
500	590	4 250	13 750	4·6 pence
1 000	1 125	8 100	9 900	6·1 ,,
1 500	1 585	11 400	6 600	8·6 ,,
2 000	2 010	14 500	3 500	15·5 ,,

There is thus an enormous increase in cost per ton-mile at the longer range, and it would obviously be ridiculous to operate the aircraft over these distances. In a typical case the curves are as shown in Fig. 86, from which it is clear that, for a particular aircraft, there is a certain range above which it is highly uneconomic to operate.

14.3. Optimum Range

The range should not only be below a certain maximum, but it should also be above a certain minimum. We can best see this by calculating the fuel costs and the receipts in terms of

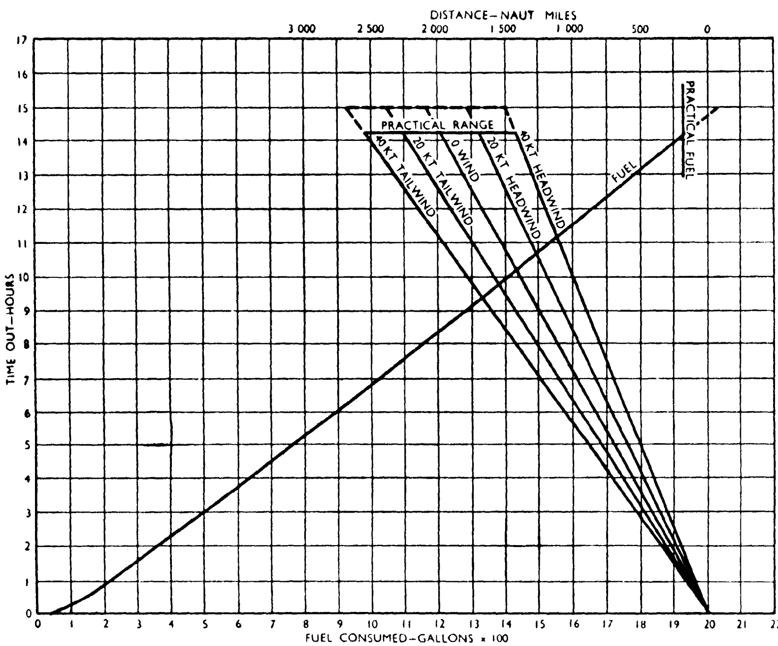


Fig. 85. Fuel Chart: Four-engined Flying Boat
Constant air-speed cruising at 120 kt R.A.S.: 10 000 ft density. Take-off gross weight = 56 000 lb.

the range. It is then easy to see at what point the costs per ton-mile become uneconomic, and whether there is any advantage in breaking down a long range into shorter stages; also, whether the application of flight re-fuelling has much advantage.

Let W = take-off weight,

W_D = disposable load,

W_p = payload,

W_f = weight of fuel,

$W_D = W_p + W_f$,

W_m = mean weight $= W - \frac{1}{2}W_f$,

V = true air-speed, m.p.h. or n.m.p.h.,

a, b = constants,
R = range, miles or nautical miles,
p = payment per pound per mile,

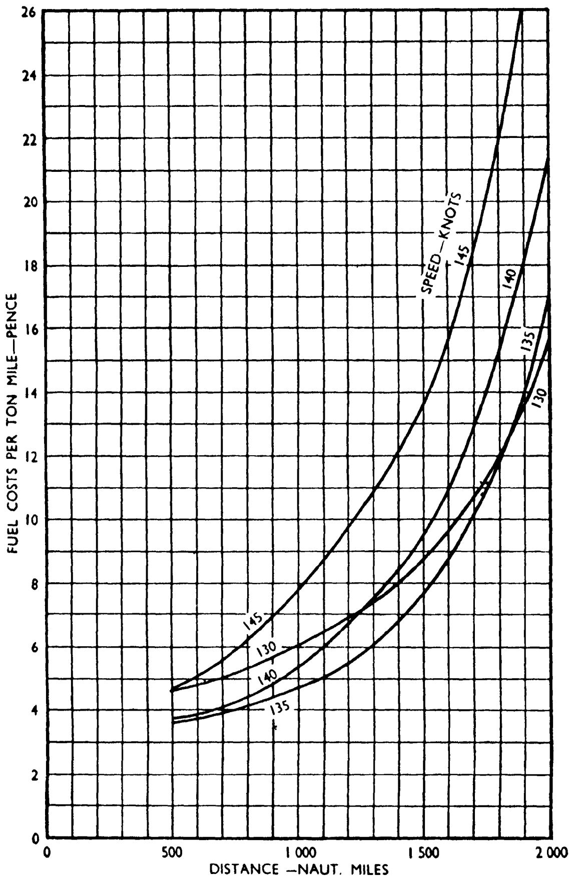


FIG. 86. COSTS PER TON-MILE VERSUS RANGE

r = price of pound of fuel,

$$c = aV^2 + \frac{bW^2}{V^2}.$$

Now the a.m.p.g. formula is equivalent, at constant V (and therefore constant η/c_f), to

$$\frac{W_f}{R} = aV^2 + \frac{bW_m^2}{V^2} = aV^2 + \frac{b}{V^2}\left(W^2 - WW_f + \frac{W_f^2}{4}\right).$$

By solving the quadratic in W_f, after reduction and approximation,

$$W_f = cR\left(1 - \frac{RbW}{V^2}\right).$$

$$\therefore \qquad W_p = W_D \doteq cR\left(1 - \frac{RbW}{V^2}\right).$$

$$\therefore \qquad \text{Receipts} = Rp\left[W_D - cR\left(1 - \frac{RbW}{V^2}\right)\right]$$

and fuel costs $= Rrc\left(1 - \dfrac{RbW}{V^2}\right).$

$$\text{Costs per ton-mile} = \frac{rc\left(1 - \dfrac{RbW}{V^2}\right) 2\,240}{W_D - cR\left(1 - \dfrac{RbW}{V^2}\right)}$$

Receipts per ton-mile $= 2\,240p$.

If we take the example already mentioned in this chapter in § **14.2**—

R	500	1 000	1 500	2 000
W_f	4 250	8 100	11 400	14 500

These figures give $a = 0{\cdot}00022$, $b = 0{\cdot}000028$, $c = 8{\cdot}92$ and $\dfrac{bW}{V^2} = 0{\cdot}000093$, for with these numbers we get

R	500	1 000	1 500	2 000
0·000093R	0·0465	0·093	0·1395	0·186
1–0·000093R	0·9535	0·907	0·8605	0·814
R(1–0·000093R)	476·75	907	1 290·75	1 628
W_f = 8·92R (1–0·000093R)	4 250	8 000	11 400	14 500
Payload W_p	13 750	9 900	6 600	3 500
Fuel costs	4 250r	8 100r	11 400r	14 500r
Receipts	6 875 000p	9 900 000p	9 900 000p	7 000 000p
Costs per ton-mile	1·385r	1·83r	2·58r	4·64r

If one passenger plus baggage = 400 lb, and we charge 6d. per passenger per mile; and if the petrol costs 2s. per gal, then

$$p = £\tfrac{1}{16000}; \quad r = £\tfrac{1}{72}.$$

We then get the following results—

R	500	1 000	1 500	2 000	
Fuel costs (£)	59	112	158	201	
Receipts (£)	430	618	618	437	See Fig. 87
Profits (£)	371	506	460	236	
Costs per ton-mile (pence)	4·6	6·1	8·6	15·5	See Fig. 88
Receipts per ton-mile (pence)	33·6	33·6	33·6	33·6	

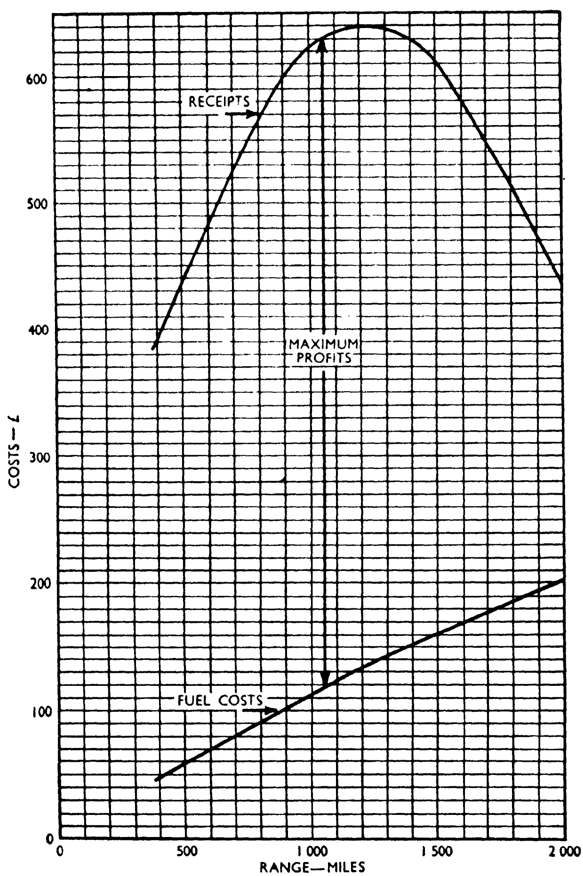


FIG. 87. COSTS VERSUS RANGE

This calculation reveals the existence of a range for which the profits = receipts minus fuel costs is greatest. For

$$\text{profit} = \mathrm{R}p\left[\mathrm{W}_{\mathrm{D}} - c\mathrm{R}\left(1 - \frac{\mathrm{R}b\mathrm{W}}{\mathrm{V}^2}\right)\right] - \mathrm{R}rc\left(1 - \frac{\mathrm{R}b\mathrm{W}}{\mathrm{V}^2}\right).$$

At a first approximation, write

$$\text{profits} = \mathrm{R}p(\mathrm{W}_{\mathrm{D}} - c\mathrm{R}) - \mathrm{R}rc$$

which is greatest when

$$\mathrm{R} = \frac{p\mathrm{W}_{\mathrm{D}} - rc}{2pc} = \frac{\mathrm{W}_{\mathrm{D}}}{2c} - \frac{r}{2p},$$

i.e. the optimum range is approximately

$$\frac{\mathrm{W}_{\mathrm{D}}}{2c} = \frac{\mathrm{W}_{\mathrm{D}}}{2\left(a\mathrm{V}^2 + \dfrac{b\mathrm{W}^2}{\mathrm{V}^2}\right)}$$

i.e. about half the range for zero payload.

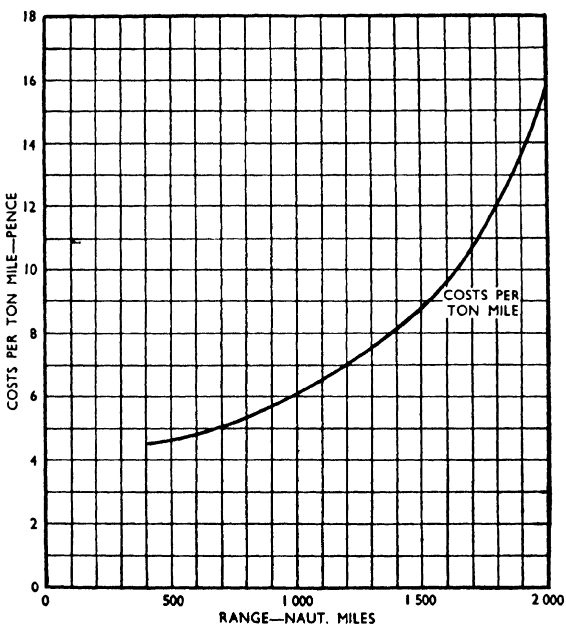


FIG. 88. COSTS PER TON-MILE VERSUS RANGE

14.4. Number of Stops En Route

These results have a considerable bearing on both trans-oceanic and trans-continental air travel.

As an example we will consider two alternative ways of covering, with the aircraft just dealt with, a sea journey of (*a*) 2 000 miles, (*b*) 1 000 miles. In all cases we assume that the payload is the maximum permissible, i.e. that we do not carry more petrol than is necessary, and that the take-off weight is always the maximum of 56 000 lb. In cases (2) and (4) it is assumed that flight re-fuelling takes place at halfway, i.e. that an airborne tanker does the necessary re-fuelling in flight.

Case	Distance	Fuel Costs	Receipts	Profits
(1)	2 000	£201	£437	£ 236
(2)	1 000 + 1 000	2 × 112 = £224	2 × 618 = £1 236	£1 012
(3)	1 000	£112	£618	£ 506
(4)	500 + 500	£118	£860	£ 742

To the costs in cases (2) and (4) must be added the re-fuelling costs, which are independent of the range. If we assume these to be £236, then—

(*a*) difference between profits in cases (1) and (2) = £1 012 − £236 − £236 = £540;

(*b*) difference between profits in cases (3) and (4) = £742 − £506 − £236 = £0.

The conclusion we now reach is this: if the range is considerably more than the optimum for the particular aircraft, as in the case of trans-oceanic flight, it will pay to practise flight re-fuelling.

If, on the other hand, the total range is in the neighbourhood of the optimum, it may not pay to break the trip up into shorter stages. And, if in this case shorter stages are necessary for traffic and commercial reasons, it will be an advantage to choose a different aircraft, one for which the optimum range is equal to the average of the stage distances.

It is desirable to make it clear at this point that the optimum range described—

(1) gives the maximum profit *per trip*, and *not* per mile

(cost per mile inevitably increases with range; but profits per trip have a maximum at the optimum),

(2) bears no relation to the range *OM* on the payload diagram (Fig. 83) at which payload begins to decrease with increase of range, i.e. the minimum range for immediate landing.

For the flying boat, for instance, the permissible landing weight is as much as 54 000 lb, so that the maximum payload is (18 000 − 2 000) lb = 16 000 lb, and the range corresponding to 2 000 lb of fuel is only about 250 miles; whereas the optimum range, as we have seen, is about 1 000 miles.

14.5. Re-fuelling in Flight

The application of radar Rebecca and Eureka units has made possible the establishment of initial contact in any weather conditions—current research may even make re-fuelling contact ossible where weather conditions are too bad to permit visual act. The tanker may accompany the airliner to the opera-al point; but if contact is to be made by interception, the anker takes up station at the approximate rendezvous point to await the arrival of the airliner. The aircraft are in constant R/T communication, initially through a ground station and then in direct communication when airborne. When they are between 50 and 80 miles apart, the tanker can "home" by Rebecca on the airliner's Eureka beacon. The method of interception from this point is best illustrated by the accompanying diagrams, Fig. 89, A. to I. In addition, all tankers are equipped with radio and compass, and manned by expert navigators.

Once visual contact is established, re-fuelling contact can be made in a matter of minutes, and the method has been proved reliable. The whole operation of making contact, passing 1 000 gal of fuel at a mean rate of 120 gal/min, and breaking away, can be accomplished in 15 to 20 min.

With the aircraft at present in use as tankers, fuel has been successfully transferred at flight speeds of 200 m.p.h. (I.A.S.) and there is little reason to doubt that if it should prove desirable to re-fuel at even greater flight speeds, the equipment could be readily modified to make this possible.

It is claimed that the receiver equipment, as fitted in airliners, is quite simple, takes up very little space, and weighs little more than 300 lb. It merely comprises the reception coupling in the stern; a pipeline from the coupling to the fuel tanks; a hauling line with which to haul in the hose; and a windlass for hauling the line.

The advantages claimed for flight re-fuelling are many; it is capable of greatly increasing the payload on established long non-stop stages; permitting long non-stop stages where they are impracticable without re-fuelling in flight; improving the safety of take-off and climb when carrying full payload; and providing reserve fuel in the air when required.

14.6. Choice of Speed

We have seen in Chapter VIII that, for maximum range with a given amount of fuel, we should decrease the speed so that it is always at the peak of the a.m.p.g. curve for the instantaneous weight. We also found that, in the majority of cases, it was more practical to fly at constant speed. This speed usually fixed irrespective of the range, although ideally it s be near the maximum range speed for the mean wei responding to the particular trip.

Now in Chapter II we saw that the maximum range sp

$$\left(\frac{b}{a}\right)^{\frac{1}{4}}\sqrt{W_m}, \text{ or } \left(\frac{b}{a}\right)^{\frac{1}{4}}\left(\frac{2+n}{2-n}\right)^{\frac{1}{4}}\sqrt{W_m},$$

if account be taken of the variation of η/c_f. (*Note.* These values of a and b are different from those used in the present chapter, but the ratio b/a is the same in both cases.) If W be the take-off weight, the best speed for a range R is

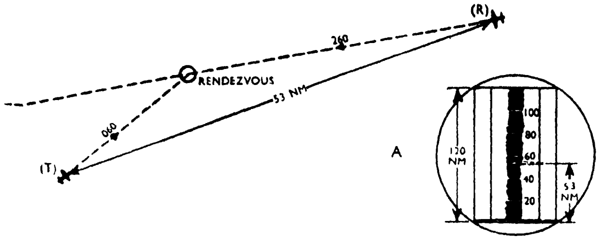
$$\left(\frac{b}{a}\right)^{\frac{1}{4}}\sqrt{W}\left(1-\frac{\sqrt{ab}}{2}R\right)$$

as will now be proved. In practice we can draw a series of costs per ton-mile curves with speed as the abscissa, and range as the parameter, and pick off the minimum on each curve (see Fig. 90).

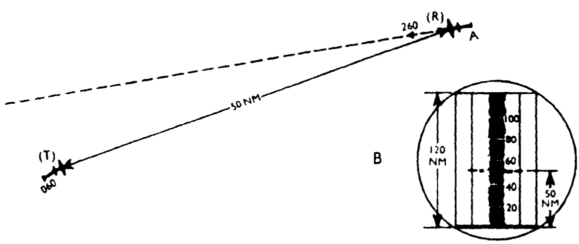
We have seen that, approximately,

$$\frac{W_f}{R}=\left(aV^2+\frac{bW^2}{V^2}\right)\left(1-\frac{RbW}{V^2}\right), \text{ for constant } \eta/c_f$$

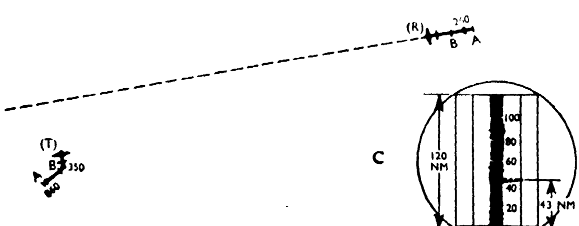
$$\therefore\ \frac{W_f}{R}=\left(ax+\frac{bW^2}{x}\right)\left(1-\frac{RbW}{x}\right), \text{ if } x=V^2.$$



INITIAL CONTACT, Time: + 00 min. RECEIVER (R): Proceeding to rendezvous at 9 000 ft on 260 deg at 150 knots. TANKER (T): Proceeding to rendezvous on 050 deg when navigator observes receiver's Eureka beacon on Rebecca indicator at 53 miles' range. Radar contact is now established. Rebecca signal indicates receiver to starboard.

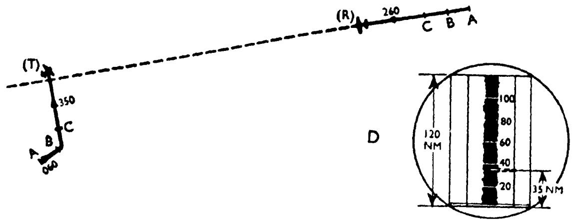


POSITION FIXING, Time: + 01 min. RECEIVER (R): Course unchanged. TANKER (T): Turns until receiver appears dead ahead on Rebecca indicator; navigator logs range, course, time, Tanker turns to fly at right angles to receiver's course.

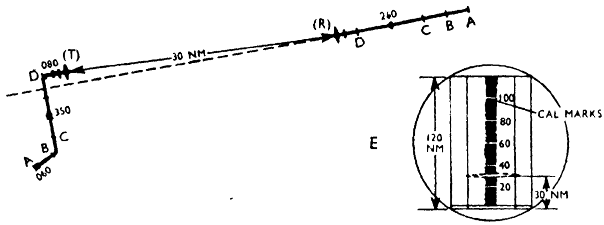


FLYING TOWARDS RECEIVER'S TRACK, Time: + 02 min. RECEIVER (R): Course unchanged, pilot notifies Tanker of true course, I.A.S. and altitude. TANKER (T): Acknowledges information and adjusts altitude to fly 500 ft above receiver (9 500 ft). Rebecca signal indicates receiver indicator to starboard.

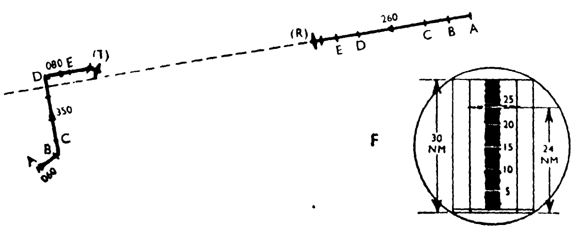
FIG. 89. RE-FUELLING IN FLIGHT



TURNING ON TO RECEIVER'S RECIPROCAL, Time: + 05 min. RECEIVER (R): Course unchanged. TANKER (T): Has flown computed time and turns on to receiver's reciprocal; Rebecca signal shows receiver to starboard.

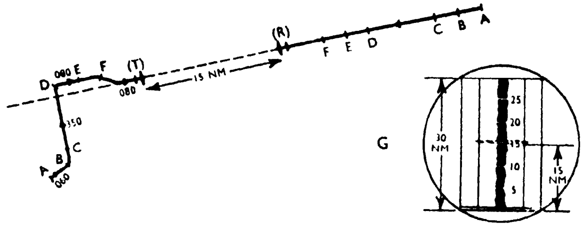


CHECKING POSITION, Time: + 06 min. RECEIVER (R): Course unchanged. TANKER (T): Has settled on receiver's reciprocal. Rebecca signal indicates receiver slightly to starboard; receiver's track is overshot.

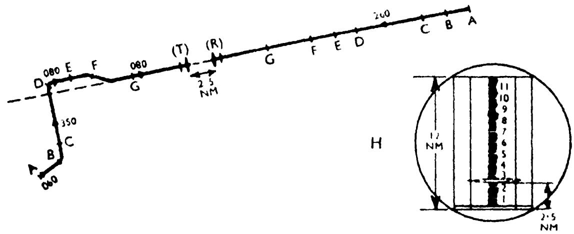


DOG-LEG ON TO RECEIVER'S TRACK, Time: + 07 min. RECEIVER (R): Course unchanged. TANKER (T): Dog-leg to starboard to fly on receiver's track. Navigator now on 30-mile range for greater accuracy; signal changes first to a port indication.

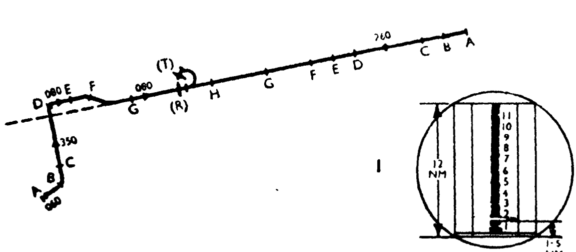
FIG. 89 (*contd.*). RE-FUELLING IN FLIGHT



Closing in, Time: + 09 min. **Receiver** (R): Course unchanged. **Tanker** (T): Flying on receiver's reciprocal at 9 500 ft; receiver dead ahead on indicator, range 15 miles. Navigator continually informs pilot of range, etc.



Turning to Formate, Time: + 11·5 min. **Receiver** (R): Course unchanged. **Tanker** (T): Navigator now on 12-mile range; at 2 to 4 miles range pilot turns to formate on receiver.



Formating on Receiver, Time: + 13 min. **Receiver** (R): Course unchanged. **Tanker** (T): Turn almost completed; moving into contact position; Rebecca indicates receiver to port.

Fig. 89 (*contd.*). **Re-fuelling in Flight**

This has a minimum for

$$ax^3 - bW^2x + 2Rb^2W^3 = 0.$$

The approximate solution to this is

$$x = V^2 = \sqrt{\frac{b}{a}}\,W\left(1 - \sqrt{ab}\,R\right).$$

For example, for the aircraft previously considered in this chapter: $a = 0{\cdot}00022$, $b = 0{\cdot}000028$; $W = 56\,000$.

Let $$R = 1\,000$$

$\therefore$ $$V = 141(1 - 0{\cdot}039)$$
$$= 136 \text{ knots},$$

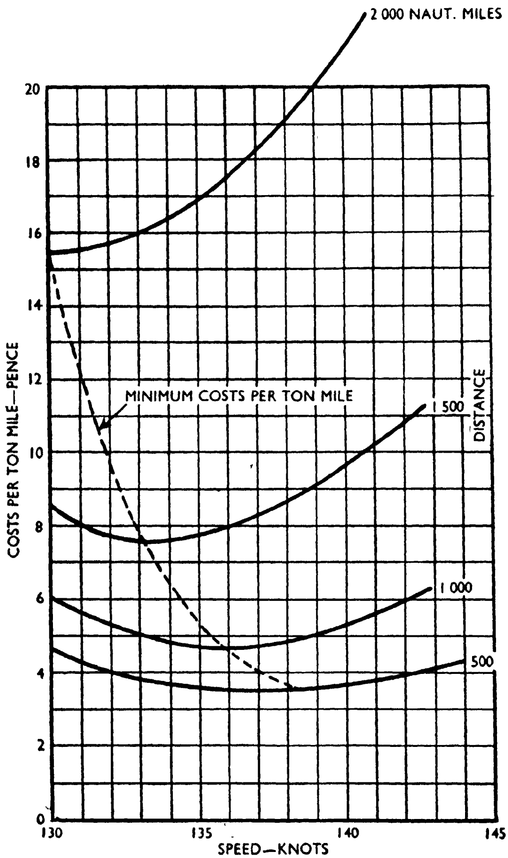


FIG. 90. COSTS PER TON-MILE VERSUS SPEED

at the height for which a and b were determined. Alterations in height can be allowed for by altering a and b.

Similar examples can be worked out for other aircraft. A far more complicated method of assessing the economic speed is given in Appendix IV.

14.7. Maximum Payload Miles per Hour

The lesson of the previous paragraph is that, if sufficient charts are available, the costs can be reduced over long ranges by reducing the mean speed. This rather obvious result has already been touched upon when mentioning the idea of striving for maximum payload miles per hour, i.e. striking a balance between maximum payload and minimum time for the journey.

Fig. 91 is a composite graph which could be used to decide (for a given height) at what b.h.p. to operate for trips of different lengths. For distances up to about 1 500 miles, the higher b.h.p.s give the greatest product of payload and speed; but for longer distances there is a distinct drop in this product.

14.8. Choice of All-up Weight

The weight W_E of the empty aircraft, plus fixtures, oil and crew, is inherent in the design. The take-off weight W_{TO} is found by adding W_D, the weight of payload and fuel. If it was possible to increase W_D without increasing the structure weight or engine weight, we should have a larger W_{TO} and either a longer range for the same payload or a bigger payload for the same range.

In the example already chosen, $W_{TO} = 65\,000$ lb, $W_E = 45\,000$ lb, so that $\frac{W_{TO}}{W_E} = 1{\cdot}44$. Would it be possible to increase this ratio? Champions of all-wing aircraft believe that it is. Removal of the fuselage and tail reduces the structure weight, and gas turbines increase the power-weight ratio of the engines; while the value of C_{D_Z} is reduced very considerably.

Imagine a flight in which all the fuel W_f is used up, and disregard W_{rf}, so that

$$W_L = W_{TO} - W_f = W_E,$$

or $\quad W_2 = W_1 - W_f = W_E$, writing $W_1 = W_{TO}$; $W_2 = W_L$,

i.e. $$W_f = W_1 - W_2.$$

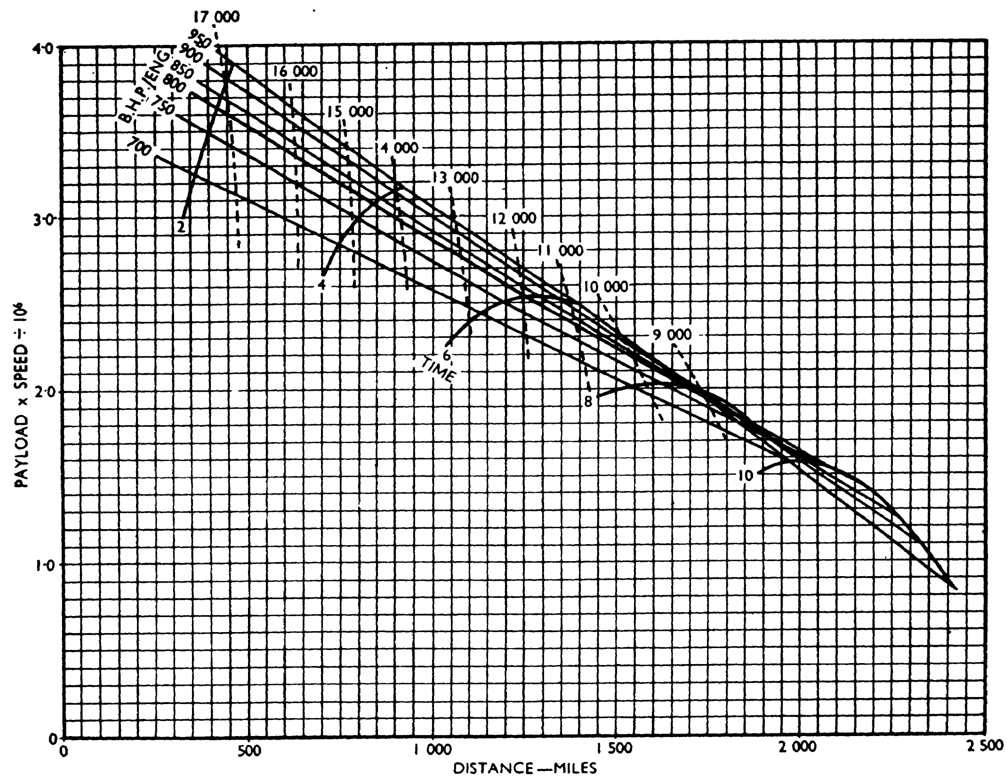


Fig. 91. Payload Miles per Hour versus Range

Now the formula in § **14.3** is

$$\frac{W_f}{R} = aV^2 + \frac{b}{V^2}\left[W^2 - WW_f + \frac{W_f^2}{4}\right],$$

or

$$\frac{W_1 - W_2}{R} = aV^2 + \frac{b}{V^2}[W_1^2 - W_1(W_1 - W_2) + \tfrac{1}{4}(W_1 - W_2)^2]$$

or $$R = \frac{W_1 - W_2}{aV^2 + \dfrac{b}{4V^2}(W_1 + W_2)^2}$$

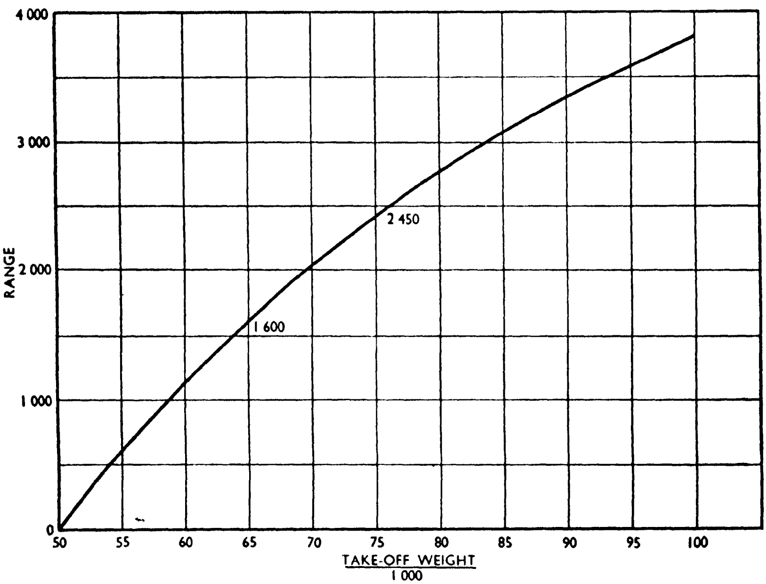


FIG. 92. RANGE VERSUS TAKE-OFF WEIGHT FOR A GIVEN TARE WEIGHT

Let us consider an aircraft for which $W_2 = 50\ 000$ lb, $a = 0{\cdot}00022$, $b = 0{\cdot}000028$, $V = 130$; and *assume that an increase in* W_1 *can be obtained without any increase in* W_2, i.e. that structure weight and engine weight do not have to be increased in order to increase the payload.

Write $W = 1\ 000x$, then

$$R = \frac{(x - 50)1\ 000}{3{\cdot}72 + 0{\cdot}042\left(\dfrac{x}{10} + 5\right)^2}$$

We can then construct the following table—

x	55	60	65	70	75	80	85	90	100
R	600	1 135	1 620	2 040	2 430	2 770	3 080	3 340	3 800

which is plotted in Fig. 92. The results from this curve are rather striking. By increasing the take-off weight from 65 000 lb to 75 000 lb, or 15·4 per cent, we increase the range from 1 600 miles to 2 450 miles, or 53 per cent. If the take-off weight could be pushed up to 85 000 lb (30·8 per cent), the range goes up to 3 100 miles (94 per cent).

	Objective
Designer	To increase the ratio $\frac{W_{TO}}{W_E}$, and thus increase maximum range.
Operator	To choose the aircraft whose designed *maximum* range is about double the longest hop on the journey.

Such figures cannot be attained in practice because of the increase in structure and engine weight, but it is clear from this and the earlier sections of this chapter that two most important objectives of designer and operator should be as above.

CHAPTER XV

Economic Aspects of Airline Operation

15.1. Preliminary

It is not one of the objects of this book to enter into the economics of airline operation: a complete discussion would inevitably introduce topics having no relation whatever to aircraft performance. Yet there remain certain aspects of the matter which are of interest to the engineer, and some of these are summarized below.

Attempts have recently been made, notably by P. G. Masefield,* to take the financial statements of the world's airlines and lick them into some sort of shape; so that it is possible to see what are the main items of expenditure. These praiseworthy attempts are not in any way assisted by confused efforts to show that the results follow mysterious theoretical laws; and for this reason: the conditions under which these expenses were incurred—in America, a closely-knit continental route-system; in Europe and Asia, a gallant but clearly hopeless attempt to open up the world's air routes with aircraft totally unsuited to the purpose—are not likely to recur; so that many of Masefield's figures refer to artificial and exceptional conditions.

Moreover, as we shall see, there are still far too many unknowns boiling away in the pot; so that it is idle to make any definite assertions about the future until experience has shown us the taste of these ingredients.

15.2. Cost of the Aircraft

15.2.1. Overall Cost. In a recent lecture (G. R. Edwards; "Problems in the Development of a New Aeroplane, *Journal, R.Ae.S.*, March, 1949), the chief designer of Vickers has given figures to show that the industry's selling price of a new aircraft is about £3 per lb of gross weight.

* "Some Economic Factors in Civil Aviation," *Journal R.Ae.Soc.*, Oct., 1948.

15.2.2. DEVELOPMENT COST. Note that this selling price is attained after, not before, the manufacturer has spent £35 per lb of gross weight on development and £10 per lb of gross weight on production tooling; so that an aircraft of 100 000 lb costs £4 000 000 before any of the production models are sold. The total cost of development is actually still higher, because it is difficult to assess in terms of money the value of the research work being carried on at the official establishments, or of the students—heavily subsidized by the Government—who later become members of the design teams involved.

Nothing can, of course, replace this research and development; but the magnitude of its financial burden may stagger the layman, particularly if he belongs to an older generation, and remembers how the aircraft firms were despised and neglected in the years between the first and second world wars. At that time there was not the same realization of the vital part which aircraft must play in national defence.

15.2.3. REPLACEMENT. Since a new aircraft is so expensive, it is not surprising that the British Airline Corporations are asking for aircraft which can be kept in service for ten years. Whether this desire can be fulfilled in practice remains to be seen. No doubt the aircraft will still fly in ten years' time, but will the customer, the flying public, demand something more up-to-date? Are future technical developments likely to be so rapid that a ten-year-old aeroplane will prove to be as inadequate as a hundred-year-old locomotive?

15.3. The Initial Design

15.3.1. AERODYNAMICS AND PERFORMANCES. One of the greatest contributions towards lowering the initial cost of an aeroplane is to make no mistakes in the main features of the original design. For example, the first prototype of a much publicized recent aircraft had too small a fin, an elementary mistake which might perhaps have been avoided by more careful thinking.* A larger fin was built, and this necessitated adding a piece to the tailplane, which now had a sharp leading edge. Even so, the handling qualities of the aircraft on the ground were poor; in the tail-down position the rudder was

See *Aircraft Engineering*, March, 1949, for an article "Choice of Fin Area and Dihedral," by F. B. Baker.

out of the slipstream and was useless for purposes of control. When the aircraft was taken to Africa for tropical tests, it was found that the a.m.p.g. figures were far below their estimated values: whether this was due to some defect in the engines or to the unexpectedly high drag of the airframe is a question not worth pursuing now. Perhaps the biggest mistake which was made, however, was to regard this particular aircraft as suitable for the flight across the Atlantic. We have already seen that, in fuel costs alone, it is foolish to exceed the optimum range (unless, of course, a satisfactory system of flight refuelling is in operation). And this aircraft's performance figures, even in the early optimistic days, never gave promise of providing a payload sufficiently large to make the Atlantic trip an economic success.

15.3.2. THE VICKERS "VISCOUNT." Other recent British aircraft have not been so unlucky. After one small initial blunder, the Hermes series of aircraft have been highly successful; the drag has proved less, and the performance of the Bristol engines better, than was anticipated. Among gas-turbine aircraft, the "Comet," "Hermes V," and "Viscount" all appear to have been "right first time." In the case of the last-named aircraft, the designer has published a summary of the considerations which dictated the ultimate form of the aircraft; and extracts from these are given below.

The aircraft was required to carry 32 passengers and their luggage at a cruising speed of 240 knots over a still-air range of 900 nautical miles. Full I.C.A.O. standards were required, in conjunction with an aerodrome length of 1 600 yards. Propeller-turbines were to be the power units.

> "The wing area, aspect ratio, and plan form, together with the type of flaps used, were dictated by a compromise between aerodrome size and the optimum performance in cruising flight. Large flaps and a relatively small wing were proved to be more efficient than normal flaps and an increased wing area. The span was largely determined by the fuselage diameter plus four propeller discs, bearing in mind that no large overlap between inners and outers is permissible, and that an adequate propeller clearance at the fuselage must be achieved in order to preserve low noise level.
>
> "The choice of propeller diameter and reduction gear ratio was influenced by the maintenance of a low cruising tip speed (600 ft/sec) from noise considerations, although it

was achieved at the expense of a relatively low thrust per horse-power at take-off. (This top-speed problem is inherent in single-shaft propeller turbines with their cruising r.p.m. at 90 to 95 per cent of the take-off r.p.m.) The position of the outboard engine and its subsequent effect upon plan form was influenced by considerations of the off-set torque in the engine-cut case, and the necessity to keep the ailerons clear of the slipstream.

"The size and shape of the body were dictated by the accommodation for thirty-two passengers and their luggage, the provision of a good pilot's view, and adequate ground clearance for a tail-down approach. For this size of aeroplane, low wing junction was chosen to ensure a smooth cabin floor, and to provide the shortest undercarriage possible with the chosen propeller diameter. This also enables the aircraft services to be housed outside the passenger cabin and provides increased safety under ditching conditions. The usual difficulties which are normally expected to occur at the junction of a circular fuselage and a low wing were subsequently solved in the wind tunnel by the use of fairly large rear fillets and a small nose fillet. The high cruising speed also enabled the wing to be set at a relatively small angle to the fuselage.

"The relation between the wing, body, and tail was decided by a consideration of the centre of gravity position and the required longitudinal stability. Because of the long overhung nacelles and the large fuselage, the aerodynamic centre had succeeded in moving forward, almost to the leading edge of the wing. This necessitated a large tail and encouraged every effort to keep the centre of gravity of the aircraft forward and to restrict to a minimum C.G. movements caused by load variations. The interior layout with its central pantry and two lavatories was a factor in achieving this.

"Universal difficulties in obtaining satisfactory stability and elevator control with tailplanes in the slipstream made this particular design team very cautious in the vertical disposition of the tail. The outcome was to raise the tailplane by sweeping up the fuselage and employing dihedral until the tailplane was clear of the slipstream and high in the downwash. A predicted gain of 0·1 S.M.C. in the allowable aft C.G. position was attained by the addition of the dihedral alone. This also kept the tail clear of the area of bad flow at the extremities of the flaps when in the fully down position.

"The long overhung nacelles are a feature of small diameter, close-cowled propeller-turbine power plants. The accessories have to be mounted behind the engine instead of around it, and the clearance of the jet pipe from the wing structure is also contributory to the nacelle length. A short curved jet pipe was chosen, in preference to a straight pipe running through the wing, on consideration of the following—

"(1) Fire risk. (2) Loss of power due to the large pipe. (3) Undercarriage problems. (4) Noise level. (5) Structural difficulty in the wing.

"The undercarriage problem in (3) was at first resolved by an undercarriage retracting sideways into the fuselage, but, because of the large hole cut in the pressure cabin wing intersection by the twin wheels, this was abandoned in favour of a conventional fore-and-aft chassis. The tricycle undercarriage was fitted with nose-wheel steering, a feature considered essential in an aircraft of this type powered by propeller-turbines.

"Being a civil aircraft, the safety aspect was allowed to influence the whole design. In addition to a low wing loading and four engines, high-lift flaps, to give low landing speeds, were installed. The tricycle undercarriage was fitted with twin wheels and duplicated brakes, the paraffin was housed in crashproof tanks, and all the windows were made emergency exits. The elliptical shape of doors and windows required the minimum reinforcing in a pressure cabin.

"Choice of controls is mainly a matter of personal opinion and recent experience. This aircraft was small enough to control manually, and sealed balance ailerons and inset hinge type elevators and rudder were chosen.

"The size and shape of the body was dictated by the accommodation for thirty-two passengers and their luggage, the provision of a good view for the pilot and ground clearance for a tail-down approach. Two large entrance doors were provided in the fuselage, one forward and one aft, allowing flexibility in internal layout without subsequent structural change. The propeller-turbines necessitated a pressure cabin; investigation into both circular and double-bubble cross sections ended in favour of the former.

"In order to allow unrestricted rate of climb and descent without discomfort, the aircraft was pressurized to $6\frac{1}{2}$ lb/sq. in., the equivalent of 8 000 ft cabin conditions at an actual altitude of 30 000 ft. This high pressure difference provides cabin heating adequate to meet arctic conditions without the use of combustion heaters. Refrigeration for sub-tropical conditions was provided by a turbo-expander. Temperature was controlled thermostatically. A generous supply of air, well distributed, obviates resorting to air re-circulation.

"The framing of the climb requirement in terms of the stalling speed undoubtedly lends weight to the need for high-lift flaps. It is fully appreciated that some types of high-lift flaps can completely offset this advantage by shallow gliding angles during the approach and long floating periods. Double-slotted flaps were chosen in this case, as wind tunnel tests indicated that they gave rise to greater drag than other high-lift devices, and the available information showed that the change of trim caused by lowering them was in fact extremely satisfactory. The need for high drag during the approach is undoubtedly justified in view of the high minimum idling thrust obtainable on propeller-turbine engines.

"Maintenance was built into the aeroplane from the beginning. Since much electric power and equipment was essential for the radio and radar, it was thought that it would be unfortunate to add other services employing different techniques and fluids, and so it was made an all-electric aeroplane. The available heat in the exhaust gases was used to provide the source of thermal de-icing."

15.3.3. OTHER CONSIDERATIONS. During the controversy about the "Tudor," there was some comment on the time and money wasted by having to incorporate many trivial new modifications into the design: the colour of the lavatory seat, one might say, caused as much trouble as the cruising speed. While such ridiculous disputes can cause exasperated amusement, they suggest that liaison between operator and manufacturer may have been directed into the wrong channels. The right channel, without any doubt, was to make sure that the aircraft provided had the lowest possible maintenance costs, because, in the words of N. E. Rowe, "The aeroplane is the revenue earner. If, because of its inherent bad qualities, such as low relative payload, high unserviceability, poor maintenance features, and so on, its direct cost of operation is high, then no amount of administrative excellence in reducing overheads will make the operations economic."

15.4. Maintenance

15.4.1. EASY SERVICING. There has always been much criticism of aircraft design from maintenance engineers whose work is made unnecessarily difficult because servicing problems have not been given sufficient attention during the design stage. There is no excuse for inaccessibility, or for such comments as "hydraulic, pneumatic, nitrogen, and other charging points are scattered all over the aircraft." Ideally, of course, the matters which require most attention should be the easiest for the service engineer to reach. In the case of mechanisms which require testing, it is not always easy to say which ones are likely to be the least reliable.

15.4.2. RELIABILITY. The reliability of components can only be judged by experience in actual flight; but it is becoming increasingly possible for a manufacturer to state: "This component will last so many hours without attention," without risk of being proved a liar. Sealed components, such as, for instance, carburettors, must not be touched by the maintenance engineer: they can only be replaced. As reliability improves, it should be possible to aim at a state of affairs in which the engineer's only job is servicing—cleaning filters, topping up oil levels, and so on. When this happy day arrives, it should be possible to save money on inspection.

15.4.3. INSPECTIONS. The schedule of daily inspections, 50-hour, 100-hour, etc., inspections, is one which has a two-fold object: (1) to detect any fault in the aircraft, and (2) to make it unnecessary for an aircraft to be withdrawn for months at a time in order to undergo an overhaul. Since the C. of A. has to be renewed each year, such overhauls were at one time necessary. Regular inspections have become traditional, but if we examine the accident statistics, there is little reason for supposing that the safety of flight has been materially improved by so much inspection. Since safety in flight is, after all, the main consideration, it might be as well to pay most attention to those defects which have a direct bearing on safety, and those where trouble can be confidently expected.

15.4.4. SUMMARY. We may summarize these statements by quoting some recent remarks by H. S. Crabtree.

"A reduction in maintenance time, and therefore in costs, can be achieved by a combination of several methods—

"(*a*) Increasing the efficiency of the methods and means of maintenance and overhaul work on aircraft.

"(*b*) Reducing the amount of work to be done.

"(*c*) Making what work has to be done easier to accomplish.

"One suggested method revolves round the changing of units at specific periods of time, based on knowledge obtained in various ways of the safe life of such units. Although this has been the aim of most of the leading airline engineers for a number of years, coupled with the development of a 'snag shooting' technique which is based on trial and error methods applied in logical sequence of possibilities for each trouble that arises, progress has been slow, possibly because many airline engineers have been forced to work in conditions of factual ignorance.

VALUE OF RECORDS. "The basis of all such aims, however, depends on factual evidence. This can be collected by the accurate recording and careful practical analysis by the aircraft operator of his day-to-day engineering experience. Such a collection of facts can be the only solid basis of all thinking by airlines, not only on their own engineering organization, practices and procedures; but also on the engineering requirements of their new replacement aircraft.

"With accurate and careful analysis of this previous experience, the operator should be able to reduce on existing aircraft his maintenance work to a level where he can be assured that no unnecessary work is called for on his maintenance schedules, or done on his aircraft.

"By constant review of the maintenance schedules and the engineering records, tendencies and trends can be noted and steps taken by changing the schedules on existing engineering practices and procedures and taking possible modification action to meet foreseen, and thus often calculable, developments. Aircraft servicing will be working on facts, and will be immediately aware of changes due to increasing life and experience. It will no longer be dependent upon the opinions of a number of junior inspection staff working with little factual guidance.

"But real progress towards reducing aircraft servicing time will never be achieved until both the aircraft operators and the manufacturers appreciate that such reduction will only be achieved by designing aircraft, especially the detailed engineering aspects of aircraft, around this accumulated past experience of the operators.

"It is the collection and presentation of this experience in which the manufacturer of a new type of aircraft is interested. Yet if the operator is to gain the utmost benefit possible from the study of his own engineering results, it is the constant reviewing of this experience as thrown up by his technical records and cost statistics, that must be for ever prompting him and guiding him to make changes in his engineering procedures.

"The rule of an operator's engineering department should be to plan his future engineering requirements by working in the present time on the accumulated experience of the past.

15.5. Technical and Cost Records

"The true basis of any efficient business is the collection and presentation, in as simple a form as possible, of reliable technical cost records which can be used by all executive and managerial staff as a guide and measure for all decisions, organizations and procedures.

"In air transport such records can be greatly simplified by using man-hours and material (spares) costs as cost indices. Such costs, to be of real use, must be closely tied into the current technical operational problems of the fleet being used; and their form must therefore be laid down by the engineering department itself.

"For example, if on an existing fleet of aircraft persistent trouble arises on, say, pressurized cabin windows, the airline engineer wants to know the statistical position, i.e. the number of average window changes for 1 000 hours of flying, the average service life of the windows replaced, and the tendencies for these two figures to deteriorate with increasing age of the aircraft, or to increase as a result of technical development work and subsequent modification action he may have initiated.

"Besides these technical statistics, he must also have the cost index, in man-hours expended on windows per 1 000 hours of aircraft life and for the average life of the windows, as well as the material costs. As in the statistical records, he must have these cost indices cast up in such a manner that he can see whether his costs caused by the trouble are increasing or decreasing. His incidents of window troubles, in fact, may be increasing, yet his costs through careful planning action may be decreasing—or, equally probable, although through his own development and resulting modification action his incidents may be decreasing, his costs may be on the increase through bad planning and bad technical control of his hangars and workshops.

"The intelligent and constant watch of these four essential sets of figures is the real basis for all solid engineering progress. With these tools, the airline engineer can assess the sources of the greatest unnecessary expense, and of the greatest delay on the ground and can establish at any one moment the units, systems, and parts of his particular aircraft which are the chief cause of engineering labour and effort and hence, a limit to increasing aircraft utilization. He can direct the energies of his assistants to the improvement of the worst of these difficulties and he can note the necessity of the avoidance of similar mistakes on future aircraft.

"Above all, the airline engineer can measure the efforts of his staff in solving current problems by watching his cost indices and technical statistics. He is in the correct position of judging the results of his own decisions and directions and measuring the failure or success of his efforts or those of his staff, from the actual cost and technical results thrown up.

"Much engineering development work depends for the final results on trial and error and is limited by financial considerations. It is essential in air transport that those who initiate suggestions for technical improvement should be responsible for carrying out their suggestions and should be able to see and judge at first hand the results obtained. Second-hand development work initiated by a staff in no way responsible for carrying out the work suggested, nor held responsible for obtaining the results they may have claimed, cannot lead to real efficiency and true progress.

15.6. New Ideas and Equipment

"During the past fifteen years, and particularly the past ten years, the airline engineer has seen the influx of a great number of new ideas and, as a result the embodiment in his new aircraft of equipment in ever-increasing quantity and complexity. In general the modern aircraft have become more difficult to 'keep flying' because of the heavy increase of work necessary on the increasing technical gear to be serviced. No one objects to new equipment that produces a dividend, either in increased safety, payload, range or speed. But it is evident that much new equipment does not necessarily produce such a dividend. There are even instances of new equipment that could only have found its way on to civil transport aircraft because of some theoretical idea, the application of which has never been checked against its true effect upon the safety and payload of the aircraft, still less against its likely effect upon the servicing requirements of the aircraft.

"Although costs and statistical records of previous experience cannot be used as a satisfactory measure to decide whether a new piece of equipment can justify itself on an aircraft, they can at least be checked for any possible pointer of likely effects on the servicing of the aircraft. Such action will often give a valuable indication of its best application to attain the cheapest possible servicing.

"The simplification of civil transport aircraft is therefore of importance to the future of air transport. Simplification is likely to prove a far more difficult, and indeed more worth while, aim to achieve than the facile agreement to add this or that further complication in order to improve a certain point, or often overcome a difficulty already caused by the application of a previous complication.

"A well-known British designer was overheard to say that he had come to the conclusion that, apart from the fact that he and his staff were simple people, they would have to design simple aeroplanes, because his production department only employed simple people. He might well have added that the operators have long since realized that they too are only simple people employing simple people, and that the advent of simple aeroplanes might well change the whole financial structure of the world's airlines and their operations."

15.7. Utilization

We have seen that one of the big unknowns in fixing the price of airline operation is the cost of maintenance. When the British corporations are criticized for their deficits, it is often forgotten that in some cases they had to provide the staffs of engineers, collect the equipment, and sometimes make the aerodromes, before scheduled services could begin. This was pioneer work; other airlines have shared in the benefits without sharing the initial costs.

Now the cost of maintenance is interlinked with the utilization rate. Most of the aircraft of the future are going to travel faster; and on the face of it this means that one particular aircraft should be able to increase its flying time in the course of a year. The time put forward as a goal to aim at is 3 000 hours a year, which is certainly not achieved by airline aircraft in Europe or Asia at present; it means that the aircraft is like the sentry, 24 hours on and 48 hours off, since there are nearly 9 000 hours in a year. The two factors which are likely to defeat this goal are bad weather and poor reliability, the latter necessitating excessive overhaul time. The Berlin Air Lift showed—admittedly with piston-engined aircraft—that bad weather can be defeated; and it seems that it is now almost time for the aircraft constructors to demonstrate, by actual flights, that their products are so reliable as not to require the full quota of tests at present demanded by A.R.B.

Utilization also depends upon passenger response.

15.8. Passenger Response

Examination of airline accounts, both here and in America, makes it clear that so far there have not been sufficient paying passengers to make air traffic a profitable concern. In order to extract more money from the public, three courses are open to the operator—

(1) To charge more per mile. On those routes where the passengers are wealthy, or are being paid for out of an expenses account, such a policy might be successful, but this could only be done where this particular airline had a monopoly of the route.

(2) To charge less per mile. Anyone privileged to sit in the easy chairs provided for the original "Tudors" must have come to the conclusion that the corporation believed that excessive luxury was indispensable in air travel. As speeds increase, and block times decrease, the need for such pampering of the passengers surely disappears. It should in fact be possible to reduce the fares on all the popular routes, such as those across the Atlantic. A route such as London–Baghdad is not one which would be wildly popular whatever the fare; but it seems fairly certain that, on suitable routes, a "tourist" rate could scarcely fail to be a success if the policy was enthusiastically pursued.

(3) To increase the number of journeys per day, or per week. Up to the present this alternative would have been impossible owing to a lack of suitable aircraft, but it is a natural corollary of higher speed and higher utilization rate.

Speed also enters into the argument for another reason. Whatever the advocates of flying boats may say, there seems little doubt that the public will usually choose the faster of two alternative services; and may even pay extra for the privilege.

As we cannot reasonably divide an aircraft cabin into first and third classes, like a train, the reasonable thing to do seems to be to divide the services into two sorts: fast, luxury travel, for those who can afford it, or who have a special reason for requiring speed, and less rapid, less luxurious travel for those with smaller purses or those who are not in such a tearing hurry. In a very short time we shall have the aircraft for both sorts of journey; though whether the piston-engine or the prop. jet should be the power unit for the slower journey is a matter which the experts will have to fight out among themselves.

15.9. Flying Boats

Flying boats are costly to run, partly because they are slow, and partly because flying boat bases are used by so few aircraft, while an adequate base staff must be maintained throughout the year. Yet boats are popular, though it is difficult to appreciate the fascination of flying across countless square miles of empty ocean. (This is perhap a poor argument, however, since land flight at an average height of 30 000 ft will not allow the passenger to see anything but square miles of empty cloud.) On certain specialized routes boats will probably continue to be used; but it is difficult to believe that they have a great future before them.

15.10. Size of Aircraft

As already pointed out in Chapter XIV, there is an ideal aircraft for every route; and the all-up weight must be chosen with this in mind. On this assumption it is interesting to inquire whether there is any substance in the criticisms of the Bristol "Brabazon." We are told that there are only two runways in the world where such an aircraft can land; that the development costs have been out of all proportion to the results; and that, by analogy with the "Queen Mary" and "Queen Elizabeth," the provision of such a monster can never be an economic success—and is, in fact, a gesture of prestige value only, intended to catch those millionaires who are willing to pay for the privilege of travelling in the "largest aeroplane in history" (which it certainly is not).

Such an inquiry must remain unanswered, until flying tests have shown what a.m.p.g. and what disposable load are likely to be realized in practice; but such figures as are available suggest that the "Brabazon" is just about the size best suited to a transatlantic aircraft for economic running. It is quite true that the question of aerodrome length makes it difficult to provide suitable alternates; and it is also quite true that the development costs have been terrific; this was bound to happen in the case of an aircraft incorporating so many new features.

No verdict on the "Brabazon" can be given at present, but we can say that it was a bold venture, and that it would be refreshingly new to hear of an aircraft whose economic range was more than its range in service, and not vastly less, as is much more frequent.

There remains the question whether two aircraft carrying, say, 50 passengers each, would not in practice be more economic than one aircraft carrying 100; not for reason of fuel costs, but because of maintenance, servicing, and other allied reasons. This is a matter which deserves further close study by those who are in command of sufficient reliable up-to-date information on which to found a judgment.

15.11. Summary

We have seen that in assessing airline costs in the future there are two major unknowns—

(1) cost of maintenance,
(2) passenger response,

and that a third minor unknown—

(3) utilization rate, for economic running, is intimately bound up with the first two, and with a fourth variable

speed, whose size we are in a position to prophesy with some accuracy.

Other variables, such as fare charged per passenger per mile, degree of luxury, ratio of disposable load to tare weight, and choice of all-up weight to suit the range, are matters which can be fixed beforehand; and their determination should not cause great difficulty.

What is now needed, therefore, is bold experimentation in methods of reducing maintenance costs, and a forward policy in attracting so far untapped sources of passenger revenue. Slogans and exhortations, demanding that the people should become "air-minded," are not of much value, because the actual experience of flying in pressurized saloons at colossal heights is never likely to be one which, in itself, will be thought exhilarating by anyone except a small percentage of enthusiasts. If, however, such travel can be done safely, at high speed and in comfort, with reasonable punctuality, and at a cost which does not compare too unfavourably with other forms of transport, there is no doubt that the airline booking offices can look forward to having long queues outside their doors.

PART III—THE PERFORMANCE OF GAS-TURBINE-ENGINED AIRCRAFT

CHAPTER XVI

Cruising Performance Characteristics

16.1. Introductory

As stated in Chapter IV, gas-turbine engines fitted to aircraft are of two kinds—

(*a*) the turbo-jet, or *pure* jet, e.g. de Havilland "Comet";
(*b*) the turbo-propeller, or *prop.* jet, e.g. Vickers "Viscount" and Handley-Page "Hermes V."

The two engines have the same essential components; but in case (*b*) the thrust is provided by the slipstream of one or more propellers, which take most of the energy out of the turbine; whereas in case (*a*) the turbine itself ejects a stream of hot gas at high velocity.

Engine (*a*) is the only one for which a considerable amount of flying experience is available; so that what we have to say about engine (*b*) is to some extent conjectural. As far as cruise control is concerned, however, the main characteristics of (*b*) are reasonably clear, and can in most cases be deduced by an extension of the known characteristics of (*a*). It is, of course, true that the development of heat exchangers, thrust augmenters, reduction gear, de-icing equipment, pressurizing, and so on, is still in its early stages; but it is certain that these practical problems and difficulties will eventually be solved satisfactorily, and the prop. jet of the future will behave very much as described below. At one time it was thought that engine (*a*)'s comparatively high fuel consumption, poor propulsive efficiency at take-off and low speeds, and its need for flight at uneconomically great heights would debar it from commercial exploitation; while engine (*b*) had the further advantage that the aircraft's landing run could be reduced by the use of reversible pitch propellers; but the battle between the two types is by no means settled yet.

16.2. The Pure Jet: Net Thrust. (a) Effect of Forward Speed

The forward thrust of the jet is due to the rate of change of momentum of the hot gases in the opposite direction.

Let V_j = jet velocity when aircraft is at rest, ft/sec,

V_f = jet velocity relative to aircraft when in motion, ft/sec

(for reasons for difference, see below),

V = true speed of aircraft, ft/sec,

M_g = gas mass flow, lb/sec,

M_a = air mass flow, lb/sec.

For a stationary engine, the thrust P is given by

$$P = \frac{M_g}{g} V_f \text{ lb}$$

When the aircraft is in motion, the rate of change of momentum is

$$\frac{M_g}{g} V_f - \frac{M_a}{g} V$$

$$= \frac{M}{g}(V_f - V), \text{ approx., writing } M = M_a = M_g$$

For a given M and V_f the thrust thus appears to decrease as forward speed V increases.

This, however, is not the end of the story. Due to ram compression the air temperature at the inlet to the engine is atmospheric temperature $+ \frac{(V)^2}{(147)^2}$ (see § **1.19**). Hence the pressure $=$ atmospheric pressure $\left[\text{atmospheric temperature} + \frac{(V)^2}{(147)^2}\right]^{\frac{\gamma}{\gamma-1}}$.

For the same engine r.p.m. as in the static case, the compressor pressure ratio will be the same, hence the impact compression causes a rise in the outlet pressure (about 30 per cent at 500 m.p.h.).

For a given r.p.m. the work required per pound of air is constant, and hence the pressure drop through the turbine will be the same. Thus the pressure in the jet pipe is higher than in the static case, giving rise to a higher final jet velocity. The proportional increase is very roughly as

$$V_f = V_j\left[1 + \left(\frac{V}{V_j}\right)^2\right].$$

Also there will be an increase in mass flow due to the increase in air density to ρ' say, at the compressor inlet.

The net thrust is then

$$\frac{M}{\gamma}\frac{\rho'}{\rho}(V_f - V)\left[1 + \left(\frac{V}{V_j}\right)^2 - \left(\frac{V}{V_j}\right)\right], \text{ where } \frac{\rho'}{\rho} = 1 + \frac{1}{\tau}\left(\frac{V}{147}\right)^2.$$

The subtracted term $\frac{M}{g}\frac{\rho'}{\rho}(V_f - V)\frac{V}{V_j}$ may be described as the *intake momentum drag.*

For an engine having a jet velocity in the region of 1 800 ft/sec, the term $\left[1 + \left(\frac{V}{V_j}\right)^2 - \frac{V}{V_j}\right]$ decreases up to 600 m.p.h. and then increases again. At the same time there is a continual increase in air density ratio $\frac{\rho'}{\rho}$ as V increases.

We thus arrive at the first important property of a pure jet. *For a given r.p.m. and height, thrust is approximately constant, but has a minimum somewhere between* 0 *and* 600 *m.p.h.*

This result is illustrated by the curves in Fig. 93 ; from which it will be seen that the fall off in net thrust with speed is less severe as height increases. These curves were plotted from the following figures for an early type of gas-turbine engine.

Height (ft)	0			20 000			40 000		
True Speed (m.p.h.)	0	300	600	0	300	600	0	300	600
Net Thrust (lb)	1 960	1 620	1 670	1 177	1 040	1 140	585	526	633
Specific Fuel Consumption	1·095	1·38	1·545	1·04	1·25	1·37	1·00	1·195	1·285
Fuel Flow (lb/hr)	2 150	2 240	2 580	1 223	1 302	1 565	587	630	813
Air Miles per Lb	0	0·134	0·233	0	0·230	0·383	0	0·476	0·738

16.3. The Pure Jet : Net Thrust. (b) Effect of Height

Decrease in temperature and pressure affect the thrust in two inter-related ways.

(1) There is a change in mass flow due to the density change;

i.e. M decreases roughly as the relative density σ: this effect causes a decrease in thrust with height.

(2) The change in temperature has a marked effect on

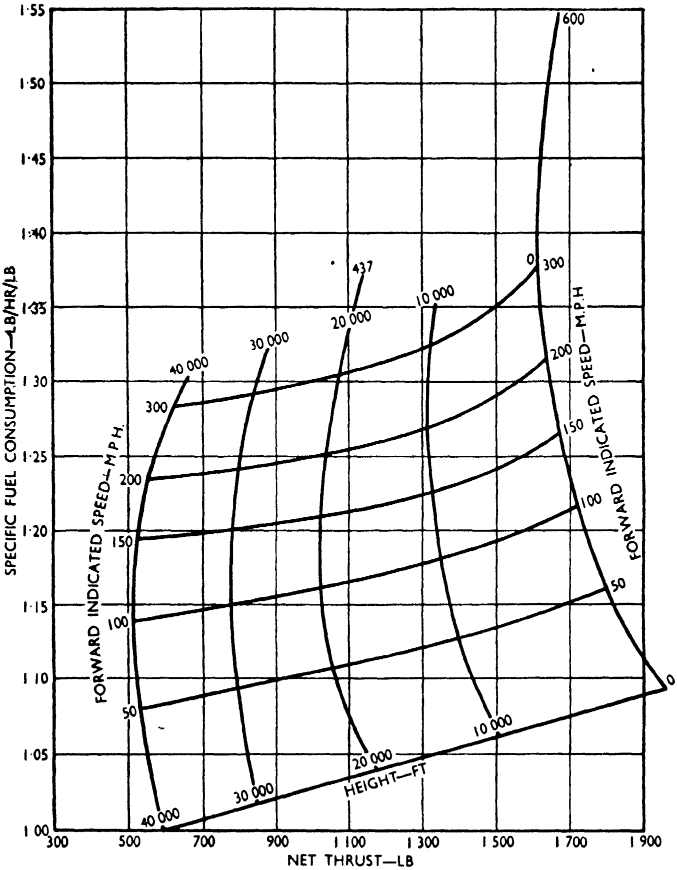


FIG. 93. CHARACTERISTIC CURVES FOR A GAS-TURBINE PURE JET

Specific consumption versus thrust at various heights and indicated speeds and constant r.p.m.

pressure ratio in the compressor, which increases (at a given r.p.m.) as the temperature falls. This pressure increase causes the jet velocity to rise, and so compensates to some extent for the loss of thrust in (1).

The decline in net thrust with height is less marked at high speeds, as is illustrated by the figures in the above table.

The temperature effect in (2) suggests a considerable fall in specific fuel consumption with height (considered in more detail later); but the decrease is lessened by a fall-off in the efficiencies of the compressor and combustion chamber.

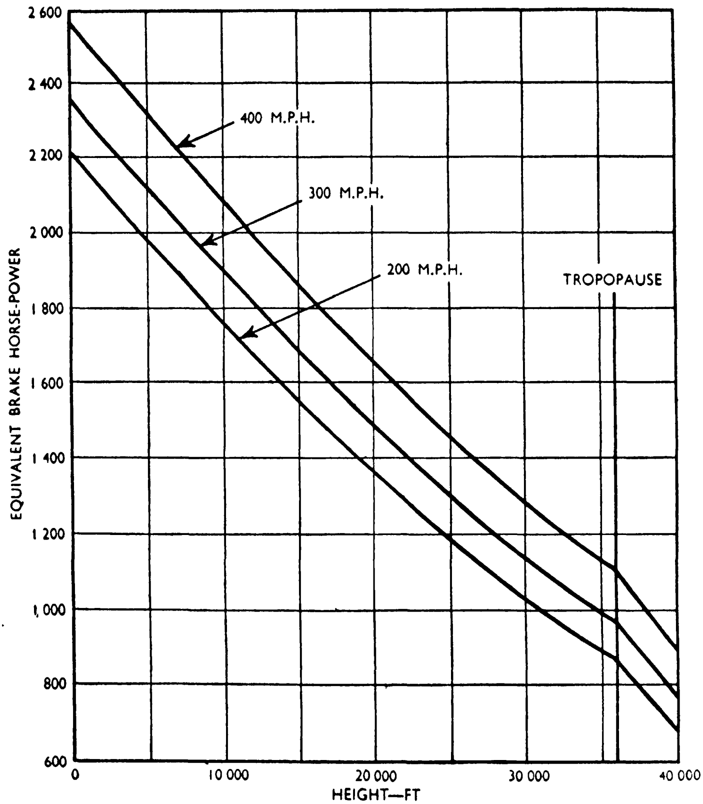


FIG. 94. GAS-TURBINE PROP. JET: MAXIMUM POWER AGAINST HEIGHT AT VARIOUS SPEEDS

We can summarize these remarks by saying—

The net thrust of a pure jet, for a given r.p.m. and speed, decreases with height. Typical figures can be read off Fig. 93.

16.4. The Prop. Jet: Equivalent B.H.P. (a) Effect of Forward Speed

If the thrust of a pure jet unit for a given r.p.m. and height is roughly constant, then the power to be derived from it—

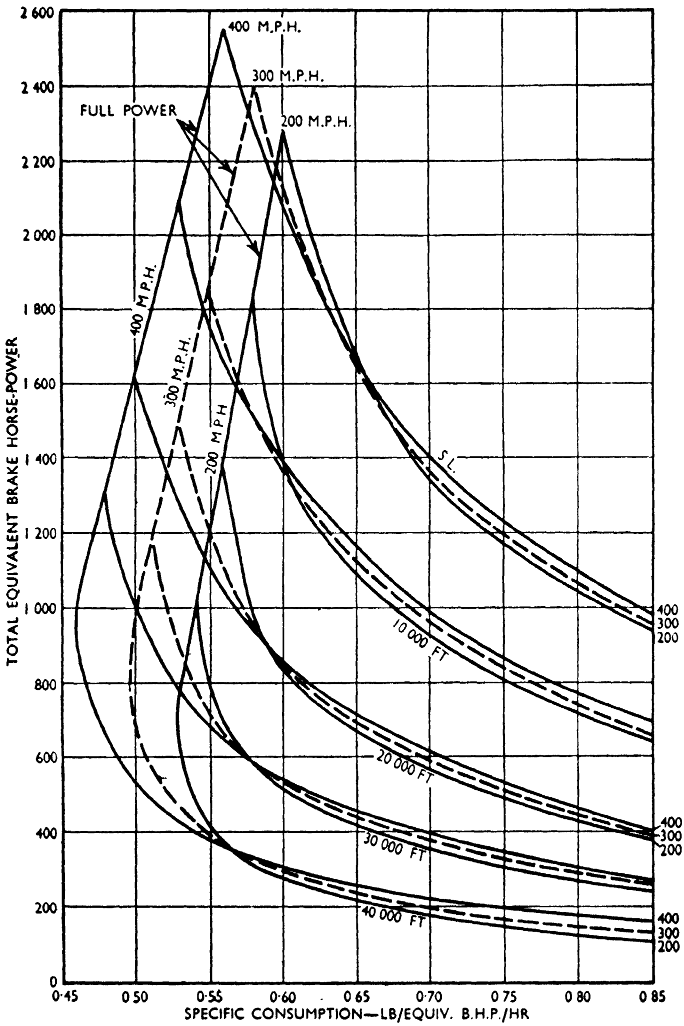


FIG. 95. GAS-TURBINE PROP. JET: SPECIFIC CONSUMPTION VERSUS POWER AT VARIOUS HEIGHTS AND SPEEDS

depending upon the product of thrust and forward speed—may be expected to increase with speed.

Some of the energy of the turbine is used to drive the compressor. If the greater part of the energy remaining in the turbine—or in another turbine geared to the compressor turbine—is used to drive a propeller, then the resulting prop. jet shows an increase of power with speed; but, as will be seen from Figs. 94 and 95, the increase is not large.

16.5. The Prop. Jet: Equivalent B.H.P. (b) Effect of Height

We have seen that the thrust of a pure jet, for a given r.p.m. and speed, decreases with height. We should thus expect the equivalent b.h.p. of a prop. jet to behave similarly. This can be seen in Figs. 94 and 95. In Fig. 95, for instance, the b.h.p. at 400 m.p.h. is reduced from 2 560 at sea-level to 1 100 at 36 000 ft.

16.6. The Pure Jet: Power Equations and Propulsive Efficiency

Once again using the symbols of § **16.2**, the *gas horse power* or *jet power* of a pure jet, when the engine is stationary,

$$= \frac{1}{2}\frac{M}{g}\frac{V_f^2}{550}\text{ h.p.} = \text{gross thrust} \times \frac{V_f}{1\,100},$$

which for a particular engine works out to be 3 100 h.p. approx.

When the aircraft is moving, the useful propelling power (or *thrust power*) = thrust × forward velocity

$$= \frac{FV}{550} = (\text{gross thrust} - \text{intake drag})\frac{V}{550}$$

$$= \frac{M}{g}(V_f - V)\frac{V}{550}\text{ h.p. approx.}$$

For an aircraft at 500 m.p.h., this is roughly 1 500 h.p., so that only half the jet power is useful at this speed.

The jet power when the aircraft is moving can be regarded as the sum of

(*a*) *the power taken from the atmosphere*

$$= \frac{1}{2}\frac{M}{g}\frac{V^2}{550} = \text{intake drag} \times \frac{V}{1\,100}$$

$$= 600\text{ h.p. at } 500\text{ m.p.h.}$$

(*b*) *the power developed by the engine*

$$= \frac{1}{2}\frac{M}{g}\frac{(V_f^2 - V^2)}{550}$$

$$= \frac{(\text{gross thrust} \times V_f - \text{intake drag} \times V)}{1\,100}$$

$$= F\frac{(V_f + V)}{1\,100}$$

$$= 2\,500 \text{ h.p. at } 500 \text{ m.p.h.}$$

This engine power can itself be subdivided into thrust power, already considered, and—

$$\textit{wake power} = \frac{1}{2}\frac{M}{g}\frac{(V_f - V)^2}{550}$$

$$= \frac{F(V_f - V)}{1\,100},$$

which accounts for the missing 1 000 h.p.

These results are found from the following figures—

G.T. = 2 000, I.D. = 900, F = 1 100, V = 750 ft/sec, V_f = 1 700 ft/sec;

and they can be tabulated like this—

Thrust horse power	1 500	
Wake power	1 000	
Power from engine		2 500
Power from atmosphere		600
		3 100

Two further "powers," which should be understood are—

(i) *compressor power* $= \frac{M_a J K_p \Delta T}{550}$,

(ii) *fuel power* $= \frac{fCJ}{550}$.

With the following notation—

J is Joule's Equivalent (1 400 ft-lb per C.H.U.),

K_p = specific heat at constant pressure,
ΔT = temperature rise in the compressor (°C),
f = fuel flow, lb/sec,
M_a = air mass flow, lb/sec,
C = calorific value, C.H.U./lb.

The latter power is required in order to measure

(a) the *engine thermal efficiency* $= \dfrac{\text{engine power}}{\text{fuel power}}$,

(b) the *thrust thermal efficiency* $= \dfrac{\text{thrust power}}{\text{fuel power}}$.

Note that the various proposed methods of thrust augmentation, e.g. cooling the intake air by using liquids such as tetrachloride, after burning, or re-heating, and injection of water into the combustion chamber, are all designed to increase these efficiencies.

16.7. Propulsive Efficiency

The apparent paradox disclosed by the above figures for thrust power, etc., is the explanation of the low *propulsive efficiency* of the pure jet.

The propulsive efficiency

$$\eta_p = \frac{\text{useful power}}{\text{engine power}}$$

$$= \frac{M}{g}\,\frac{V(V_f - V)}{\frac{1}{2}\frac{M}{g}(V_f^2 - V^2)} = \frac{2}{1 + \dfrac{V_f}{V}}.$$

It is readily seen that η_p will be small if V_f is much greater than V. Note that the formula for η_p is equally true for a propeller, where V_f is the velocity of the slipstream; but the slipstream consists of a large mass of air thrown backwards at a velocity not much greater than that of the aircraft; while the jet from a gas turbine consists of a small mass thrown back with high velocity. Since the mass flow $M = \rho AV$, where A is the cross-sectional area, one way of increasing the jet's propulsive efficiency is to increase its area—and this is what propellers (and ducted fans) set out to do. Another method would be to decrease the jet pipe velocity; but in attempting to do this we should encounter an entirely different set of difficulties, and to some extent be fighting against a leading characteristic of the gas turbine engine.

Fig. 96 shows a comparison of propulsive efficiencies for a typical propeller, pure jet, and rocket, taken at 20 000 ft. The

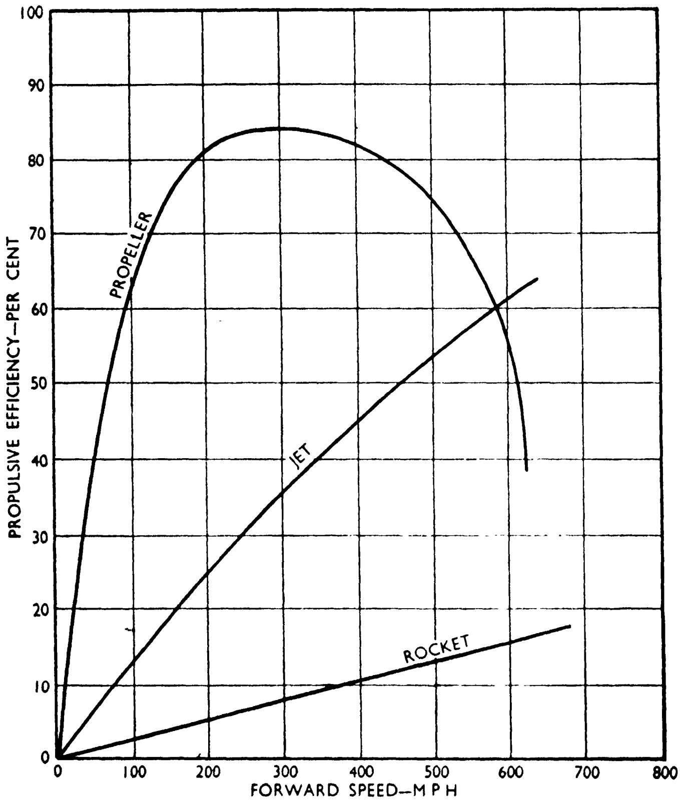


FIG. 96. COMPARISON OF PROPULSIVE EFFICIENCIES AT 20 000 FT

propeller and jet curves cross at 585 m.p.h., where the efficiency of the propeller is dropping rapidly owing to compressibility, and is given by the curve as 60 per cent at this speed. More efficient propellers may be designed in the future, but this will not alter the nature of the difference between the two methods of propulsion.

We can now see why the pure jet aircraft was at one time thought to need an impossibly large take-off distance: Fig. 96 shows a propulsive efficiency of only 13 per cent at 100 m.p.h. and it is even worse at commencement of take-off. If the useful thrust available at low speeds is increased by fitting more powerful engines, the aircraft may be grossly over-powdered for cruising.

16.8. The Pure Jet: Specific Consumption

There are three main factors which influence the specific consumption of a pure jet engine,

(*a*) forward speed of the aircraft,
(*b*) height,
(*c*) r.p.m. or engine speed.

16.8.1. VARIATION OF FORWARD SPEED AT A GIVEN HEIGHT AND R.P.M. The temperature rise due to ram causes a slight increase in the compressor outlet temperature, and thus affects the fuel flow. Less fuel is required per pound of air as speed increases; but since the mass flow is greater, there is actually an increase in specific fuel consumption (expressed in pounds of fuel per pound thrust per hour). Put in another way—

$$\text{S.F.C. } c_f' = \frac{\text{fuel used}}{\text{thrust}} = \frac{Eg}{M(V_f - V)}, \text{ approx.}$$

where E is the fuel flow in pounds per hour. The ram effect causes a reduction in E/M, but this is offset by the increase in V.

Some idea of the size of the increase can be got by studying Fig. 93 which shows these results for maximum r.p.m.

	Sea-level			20 000 ft			40 000 ft		
True Air-speed	200	300	450	200	300	450	200	300	450
Specific Fuel Consumption	1·315	1·38	1·46	1·187	1·275	1·312	1·137	1·187	1·244

For a given thrust and height, the same gradual increase of c_f' with speed can also be observed in Fig. 97. It is seen that the percentage increase with *indicated* or *equivalent air-speed* is much the same at all heights.

Note also that if the speed is reduced without lowering the r.p.m.—as in climbing—there will be a reduction of specific consumption.

16.8.2. Variation of Height, at a Given Forward Speed and R.P.M. The low temperature at height causes a decrease in

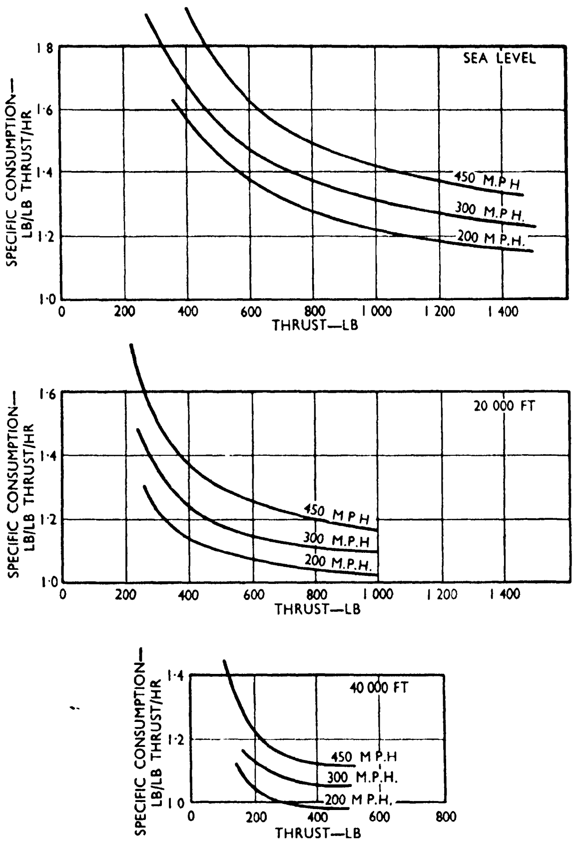


Fig. 97. Pure Jet: Specific Consumption versus Thrust at Various Speeds and Altitudes

the efficiency of the compressor. Also the lower pressures at the burners cause a deterioration in their atomizing characteristics. These two effects work in opposition to the *barostat*, which keeps the air/fuel ratio constant as the density decreases; and also to the increase in pressure ratio with lowering temperature; and as a result the fall in specific consumption with height is not so considerable as might be expected.

The degree of decrease in c_f' to be expected may be seen from Fig. 93 which is the maximum r.p.m. case. A more instructive comparison is provided by Fig. 98. Curve (1) shows the decrease in c_f' with height at maximum r.p.m. corresponding to Fig. 93. Curve (2) is also at maximum r.p.m. but at a much lower speed. This is the case of "high thrust and low speed," as on the climb; and we notice that the decrease in c_f' with height is now negligible. Curve (3) is the one which shows the most marked decrease of c_f'; described as the "typical economical cruising case," it shows the effect of height at r.p.m. considerably less than the maximum. Since curves (1) and (3) cross at 16 000 ft, it appears that the lower r.p.m. may be preferred above this height. This point is considered in the next paragraph.

16.8.3. Variation of R.P.M. at a Given Forward Speed and Height. One of the leading characteristics of the gas turbine engine is its dislike of low engine speeds. We can illustrate this point by a set of figures taken from some test-bed results for the Power-Jets W2/700 engine—

R.P.M.	Fuel Power	Engine Power	Compressor Power	Thermal Efficiency	Fuel per H.P. (lb/h.p./hr)	Specific Fuel Consumption (lb/hr/lb)	Gross Thrust (lb)
10 000	—	—	—	—	—	1·82	400
14 000	9 200	925	2 250	10·05	1·36	1·51	960
15 250	11 700	1 572	3 100	13·45	1·02	1·20	1 330
16 750	15 850	2 400	4 330	15·15	0·40	1·19	1 800

The same sort of variation in c_f' can be seen in the upper sea-level curve of Fig. 99; but it is noticeable that the corresponding curve at 20 000 ft is almost flat.

It is thus seen that, at sea-level, r.p.m. has a serious effect on thrust and specific fuel consumption. If the aircraft is to fly low, its engine speed for cruising should never be below 85 per cent of the maximum r.p.m. If aerodynamic reasons require a smaller thrust, then the aircraft is fitted with the wrong engines. This point is further discussed under the heading of a.m.p. lb in § **16.10.**

16.9. The Prop. Jet: Variation of Specific Consumption

We should expect the prop. jet to behave in a manner similar to the pure jet. The specific consumption c_f is now given in lb/equiv. b.h.p./hr. Turning to Fig. 95, for some typical figures, we find—

Speed (m.p.h.)	Specific Fuel Consumption at Maximum r.p.m.		
	At Sea-level	20 000 ft	40 000 ft
200 . . .	0·60	0·56	0·54
300 . . .	0·58	0·53	0·51
400 . . .	0·56	0·50	0·48

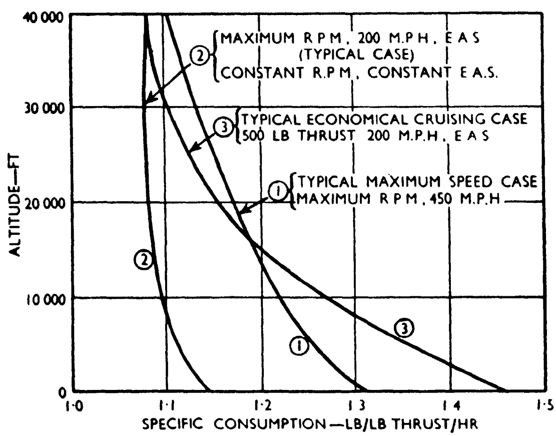


FIG. 98. PURE JET: SPECIFIC CONSUMPTION VERSUS HEIGHT UNDER VARIOUS CONDITIONS

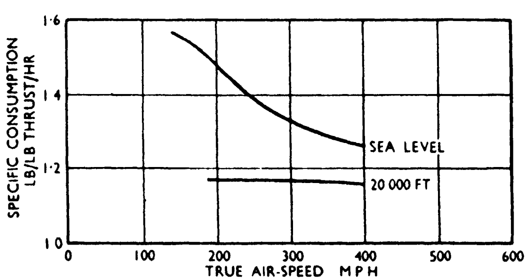


FIG. 99. PURE JET: EFFECT OF THROTTLING ON SPECIFIC CONSUMPTION

At maximum r.p.m. there is thus—

(i) a *decrease* of specific fuel consumption with forward speed,

(ii) a decrease of specific fuel consumption with height.

At less than maximum r.p.m., however, forward speed tends to increase c_f, as in the case of the pure jet.

16.10. The Pure Jet: Air Miles per Pound

If E is the fuel flow in lb per hour, n the number of engines, D the drag in lb, and c_f' the specific fuel consumption per engine in lb/lb thrust/hr, then the air miles per lb

$$= \frac{V}{E} = \frac{V}{nc_f' \frac{D}{n}} = \frac{V_i}{\sqrt{\sigma} c_f' D}$$

since the thrust required from each engine will be equal to $\frac{1}{n} \times D$.

For a fixed height and a fixed value of c_f', V_i/D is a maximum at the point on the drag-indicated speed curve when the tangent passes through the origin (see Fig. 100). If we write $D = aV_i^2 + \frac{bW^2}{V_i^2}$, then the corresponding value of V_i is found to be

$$V_{i0} = (3)^{\frac{1}{4}} \frac{(b)^{\frac{1}{4}}}{(a)^{\frac{1}{4}}} \sqrt{W}$$

which is 31·6 per cent above the indicated speed for minimum drag.

Now we have found in Chapter II that a typical aircraft, of weight 34 000 lb, has a minimum drag at an indicated speed of 212 ft/sec = 144 m.p.h.; so that the speed for maximum V_i/D is 190 m.p.h. Also, the drag at this speed is about 2 800 lb.

It appears at first sight that the engine characteristics, demanding high r.p.m., would over-rule the aerodynamic argument for maximum V_i/D speed—in other words, we might expect the actual variation in c_f' to call for a considerably higher speed. In order to test this we will find the air miles per pound (*a*) at $V_i = 190$, (*b*) at $V_i = 273$ (= 400 ft/sec).

In order to do this, we will assume that four pure jet engines are installed, and that the specific consumption behaves as shown in Fig. 97.

	Height (ft)	True Speed V (m.p.h.)	Total Drag D (lb)	Thrust per Engine	Specific Consumption c_f'	$\frac{V}{D}$	Air Miles per Lb $\frac{V}{c_f'D}$
V_i = 190 m.p.h.	Sea-level .	190	2 800	700	1·30	0·068	0·0520
	20 000	260	2 800	700	1·10	0·093	0·0844
	40 000	385	2 800	700	1·05	0·137	0·132
V_i = 273 m.p.h.	Sea-level .	273	4 600	1 150	1·25	0·0595	0·0472
	20 000	375	4 600	1 150	1·14	0·082	0·0716
	40 000	517	4 600	1 150	—	0·113	—

We have now got, at three different heights, two points on the a.m.p.lb. curves for the aircraft under consideration. The

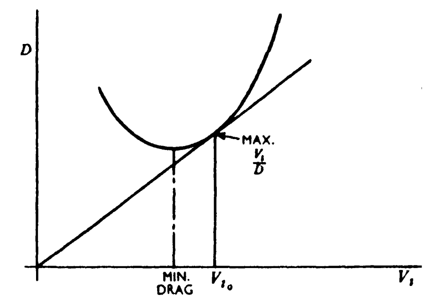


FIG. 100. POINT OF GREATEST $\frac{V_i}{D}$

process can, of course, be repeated for other weights, and other speeds. The points of maximum a.m.p.lb. on these curves will not always come exactly at the maximum V_i/D speed; but they will certainly be in the neighbourhood. In fact, although c_f' increases with lower engine speed and thrust, the effect of this on a.m.p.lb. is annulled by the necessity for maximum V_i/D.

It might of course happen that, on considering the power available from the engines for take-off and climb, it was found to be inadequate. One's natural instinct would be to install more powerful engines. Having done this, the thrust per engine in the cruising case (i.e. 700 lb) might demand such a severe cut in r.p.m. that c_f' was seriously affected. This is quite a likely dilemma, because of the low propulsive efficiency of a pure jet at low speeds. Safety is more important than fuel economy, so nothing must be done to lengthen the take-off run; or to make matters difficult if one engine should cut.

16.11. Suggestions for Improvement

Some suggestions for overcoming the difficulty are as follows—

16.11.1. CRUISING AT GREAT HEIGHT. We saw in the example just given that the thrust required to maintain maximum V_i/D speed was obtainable with a much higher r.p.m. at 20 000 ft than at sea-level. This is due to the drop in maximum thrust, and to the increase in true speed, as height increases. For a given combination of aircraft and engine, it should be quite easy to select the optimum height where the maximum V_i/D speed can be maintained for, say, 90 per cent of maximum r.p.m.

This point is so important that it is worth looking at Fig. 93 in order to see how such an optimum height could be found. The grid of Fig. 93 refers to a different engine from Fig. 97; and is actually drawn for maximum r.p.m. conditions. We will assume, however, in order to save the necessity for a further graph, that Fig. 93 is true for 90 per cent maximum r.p.m.; and our object is to read off the graph the height at which a thrust of 700 lb corresponds to an indicated speed of 190 m.p.h. The answer is about 34 000 ft, where the value of c_f' is 1·23; giving an a.m.p.lb of

$$\frac{190}{0{\cdot}5763} \times \frac{1}{1{\cdot}23} \times \frac{1}{2\,800} = \underline{\underline{0{\cdot}096}}$$

Had we not bothered with the aerodynamic requirement for maximum V_i/D and flown at $V_i = 273$ m.p.h., for a thrust of 1 150 lb, the optimum height would have been reduced to 17 000 ft, and $c_f' = 1{\cdot}295$, giving an a.m.p.lb. of

$$\frac{273}{0{\cdot}7675} \times \frac{1}{1{\cdot}295} \times \frac{1}{46\,000} = \underline{\underline{0{\cdot}060}}$$

It is evident that the choice of optimum height is very important. The extra fuel and time required for climbing to the greater height can be made up on all but the shortest ranges. In particular, the higher *true* speed at the greater height insures a saving of time for the whole trip.

16.11.2. Assisted Take-off. An example of these are rockets. For a short period such as five minutes, the extra weight of rockets is negligible. After take-off the rocket is jettisoned, and this extra power is no longer available. This state of affairs is not so serious as it sounds, because at the considerable heights at which the aircraft is likely to do most of its cruising, there will be plenty of surplus power.

Since a gas turbine loses a large proportion of its maximum power in a tropical climate, and there is always the possibility of an engine cut, it is advisable to have at least four engines. Even though the power of the four engines may be adequate for take-off, it might be a good thing to carry rockets in order to cope with a combination of emergencies.

We should also add that many of the difficulties of long take-off runs by pure jets can be solved by water injection.

16.11.3. Resting of Two Engines. We might choose the four engines so that four would be adequate for take-off, and two—working at 90 per cent maximum r.p.m.—sufficient for cruising at maximum V_i/D at the lower heights, where the variations of c_f' are greater. On the climb we meet the conditions of low speed and high thrust which the engine likes; and at considerable heights any reduction in r.p.m. has less effect on c_f'.

16.11.4. Raising the Minimum Drag Speed. We found in §§ **2.4** and **16.10** that the maximum V_i/D speed is approximately

$$\frac{19\cdot6\sqrt{W}}{(efb_0^2)^{\frac{1}{4}}} \text{ m.p.h.}$$

where e = N.A.C.A. efficiency factor defined by

$$C_D = C_{D_Z} + \frac{1}{\pi Ae} \cdot C_L^2,$$

and $f = C_{D_Z} \cdot S_W$,

b_0 = span,

W = weight.

It is clear that modification of the aircraft might increase this speed, and therefore allow us to mate optimum r.p.m. with optimum speed more easily. One way of doing this would be to decrease C_{D_Z}. For example, if C_{D_Z} could be reduced from, say, 0·025 to 0·015, leaving the other numbers unchanged, this would result in an 18 per cent increase in minimum drag and maximum V_i/D speeds. Such a large decrease in C_{D_Z} is not impossible: values as low as 0·011 can be achieved with all-wing aircraft.

(Note that a similar result would be achieved by decreasing the aspect ratio; but this could only be done at the expense of total drag, thus defeating our object.)

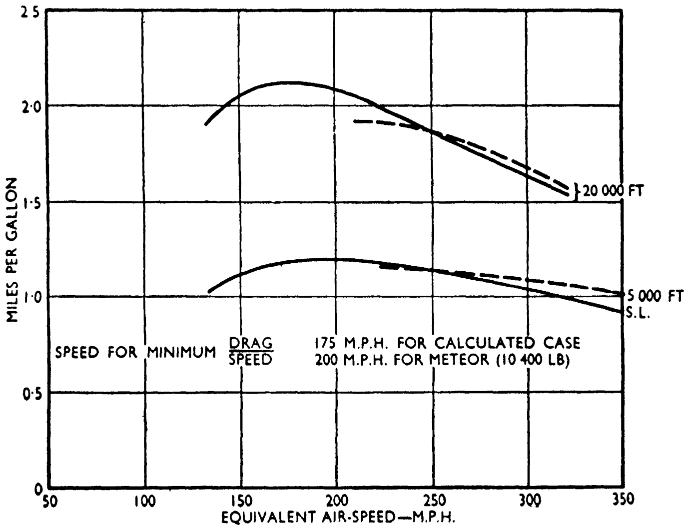


FIG. 101. PURE JET: MILES PER GALLON VERSUS SPEED AT VARIOUS HEIGHTS

——————— Calculated for a typical jet aircraft.
— — — — — Experimental (meteor).

As a further illustration of the economy effected by flying at considerable height, we may study Fig. 101.

16.12. The Prop. Jet: Air Miles per Pound

For an aircraft driven by propellers,

$$\text{a.m.p.lb} = \frac{\text{speed}}{\text{fuel flow}} = \frac{V}{E} \text{ (E in lb per hour)},$$

and the b.h.p. engine required $= \dfrac{\dfrac{D}{n} V}{375\eta}$ (n number of engines),

where $E = nc_f \times \text{b.h.p.},$

so that $$\text{a.m.p.lb.} = \frac{375}{D}\frac{\eta}{c_f}.$$

In the case of piston-engined aircraft we found that D had its minimum at a fairly low speed, and that c_f/η had its minimum at a fairly high speed; and that, on the average, a speed about 6 per cent above minimum drag speed would give maximum a.m.p.lb.

We should expect a somewhat similar state of affairs in the present case. This is not the place to do a complete performance estimation; the curves in Figs. 94 and 95 will be sufficient for our present purpose.

For the sake of comfort and to save time, we select an indicated speed a short way above minimum drag speed; and we shall then exemplify the following properties—

(1) *For the chosen speed at cruising r.p.m. there is an optimum height for maximum a.m.p.lb.; and this height is much greater than the average optimum height for piston-engined aircraft.*

(2) *Increasing the indicated speed, at the same r.p.m., will lower the optimum height and decrease the a.m.p.lb. found in* (1).

(3) *Decreasing the r.p.m. at any speed will decrease the a.m.p.lb. found in* (1).

The minimum drag indicated speed for the aircraft already mentioned is about 150 m.p.h. and the drag is then about 2 400 lb. For reasons already stated we propose to select 200 m.p.h., when the drag is 3 000 lb.

Suppose that the aircraft is fitted with *two* engines of powers given by Figs. 94 and 95, or that, when cruising, only two engines out of a possible four are in use.

We now have—

$$V_{i_{md}} = 200 \text{ m.p.h.},$$

$$V = \frac{200}{\sqrt{\sigma}} \text{ m.p.h.},$$

$$D = 3\,000 \text{ lb, approx.}$$

$$\text{b.h.p./engine} = \frac{DV}{375} \times \tfrac{1}{2}$$

$$= \frac{1\,500}{375} \times \frac{200}{\sqrt{\sigma}} \times \frac{1}{0{\cdot}8}, \text{ taking } \eta = 0{\cdot}8$$

$$= \frac{1\,000}{\sqrt{\sigma}}.$$

Now tabulate at various heights.

Height (ft)	S.L.	10 000	20 000	22 000	30 000	40 000
$\sqrt{\sigma}$	1	0·859	0·729	0·707	0·612	0·495
b.h.p./engine	1 000	1 165	1 375	1 410	1 635	2 010
V	200	254	275	283	328	405

The curves in Fig. 94 are actually drawn for maximum power. To obviate the necessity for further curves, we will assume that the curves there drawn are correct for 90 per cent maximum r.p.m.

After a little juggling it is seen that, at maximum power, the point corresponding to 22 000 ft 1 410 b.h.p. is (approx.) on the 283 m.p.h. line.

Now turn to Fig. 95 to obtain the specific fuel consumption and it will be found that $c_f = 0{\cdot}53$.

Hence $$c_f D = 1\,590$$

$$\text{a.m.p.lb.} = \frac{375 \times 0{\cdot}8}{1\,590} = \underline{\underline{0{\cdot}189}}.$$

(This equals 1·51 a.m.p.g. if kerosene has a sp. gr. of 0·8.)

To show the effect of increasing the velocity, still using maximum r.p.m., suppose

$$V_i = 273 \text{ m.p.h.},$$

$$V = \frac{273}{\sqrt{\sigma}},$$

$$D = 4\,600 \text{ lb},$$

$$\text{b.h.p./engine} = \frac{2\,090}{\sqrt{\sigma}}.$$

Again tabulate at various heights.

Height (ft)	S.L.	10 000	4 000	3 000
$\sqrt{\sigma}$	1	0·859	0·942	0·957
b.h.p./engine	2 090	2 440	2 220	2 180
V	273	316	290	286

It appears from Fig. 94 that the figures 3 000 ft, 2 180 b.h.p., 286 m.p.h. fit the curve.

Turning to Fig. 95 we find $c_f = 0{\cdot}57$

$$\text{Hence } c_f\text{D} = 2\ 620$$
$$\text{a.m.p.lb.} = 0{\cdot}114$$
$$= (0{\cdot}912 \text{ a.m.p.g.})$$

It is interesting to note that the increase in indicated speed has not resulted in the saving of cruising time, since the true speeds are practically the same. The fuel (and time) absorbed in climbing to 22 000 ft, as against 3 000 ft, has of course to be added; but, even for a trip of 500 miles, there is a difference in fuel consumption for cruising of approximately 1 700 lb, and there will be greater savings for longer ranges.

We note also that the optimum a.m.p.lb. is not so terrible when compared with corresponding figures for the same aircraft equipped with piston engines. The actual figures are—

2·0 a.m.p.g. at 10 000 ft for the piston engine,

1·51 a.m.p.g. at 22 000 ft for the prop. jet.

The second alternative is to fly at the first chosen speed, 200 m.p.h., but at considerably less than maximum r.p.m. We can obtain an approximate tabulation as follows—

Height (ft)	S.L.	10 000	15 000
b.h.p./engine	1 000	1 165	1 260
V	200	234	252
c_f	0·825	0·635	0·59

After that we are entering the region of cruising r.p.m. This alternative would only be selected if the range was small enough to make a climb to 22 000 ft uneconomic; or if the headwind at the greater height was more severe.

16.13. The Pure Jet: Effect of Aircraft Weight

Since the maximum V_i/D speed varies with

$$\sqrt{\text{weight of aircraft}}$$

there is actually a fall in this speed as the flight proceeds. So far we have ignored this effect, and have tacitly assumed that it will be sufficient to choose the maximum V_i/D speed for the *mean weight.* Greater fuel economy could be effected if the speed were gradually reduced during flight. This can be done in two ways—

(i) by increasing height as the flight proceeds,
(ii) by reducing r.p.m. as the flight proceeds.

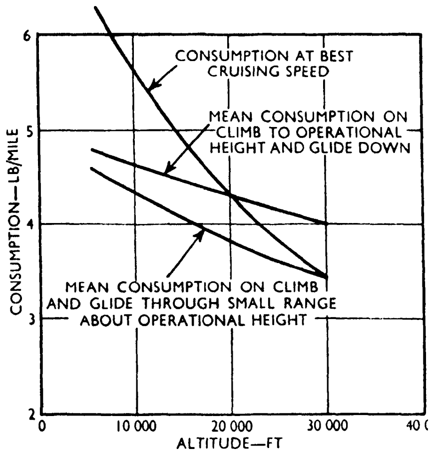


FIG. 102. PURE JET: CONSUMPTION VERSUS HEIGHT

The mean consumption on climb and glide is compared with the cruising consumption. $W = 40$ lb/ft^2: $A = 8$.

The effect of (ii) may well be to cancel out the beneficial effect of flying at the correct V_i/D speed; and (i) is a method which would probably not appeal to pilots or operators. The saving effected by (i) will in most cases be small; but the possibility should be examined in relation to any particular aircraft and engine combination.

16.14. The Pure Jet: Climbing

Since the thrust of a pure jet is roughly constant at all speeds, the curve of thrust horse-power against speed is a line which is approximately parallel to the tangent from the origin to the drag-power curve. This tangent touches the power curve at the speed for minimum drag; hence, near the ceiling, the best climbing speed of the jet aircraft, being the speed for maximum excess thrust horse-power, is approximately the minimum drag speed. This is higher than for conventional aircraft which, near the ceiling, have a best climbing speed in the neighbourhood of the speed for minimum power.

A greater forward speed on the climb has a tendency to

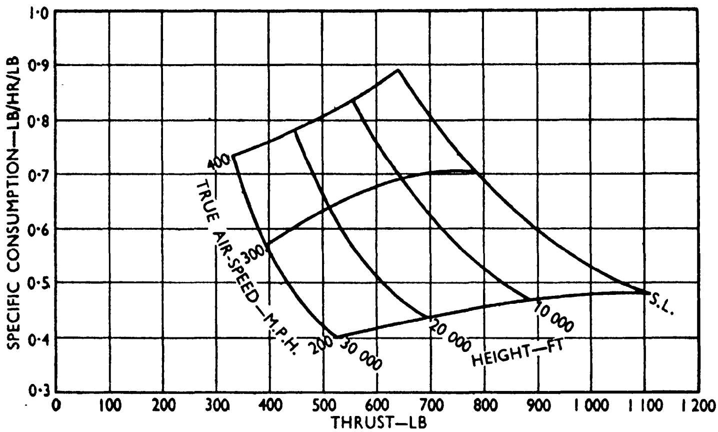


FIG. 103. PURE JET: CHARACTERISTIC CURVES

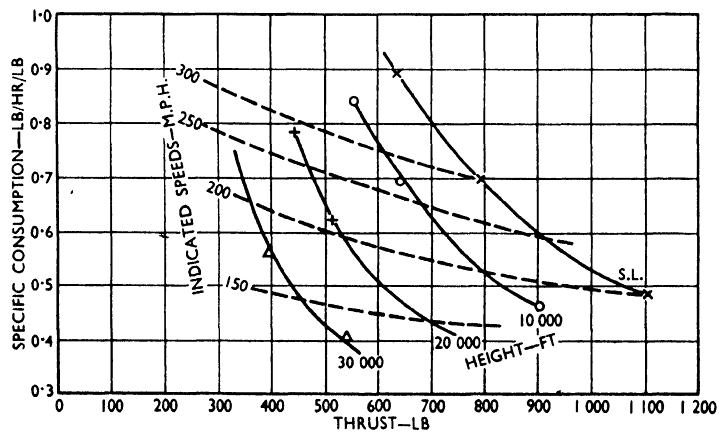


FIG. 104. PROP. JET: CHARACTERISTIC CURVES

flatten the angle of climb, which is an advantage for civil aircraft; although the present restriction of 400 ft/min for the vertical rate of climb may be relaxed when all aircraft are pressurized, as soon as they leave the control zone at 5 000 ft or less.

Now with a reasonably high forward speed on the climb and glide, we are covering a considerable forward distance; hence the penalty for having to climb to great heights for cruising is not so serious as might be supposed. It is found that the mean miles per gallon for climb, cruise, and glide may even be more than for cruising the whole distance at the chosen height. This point is illustrated in Fig. 102. The reason for this is that on the climb, the engine is operating at high thrust and low forward speed, under which conditions the specific consumption is the lowest possible.

CHAPTER XVII

High Altitude Flight

17.1. Future Trends

WE have seen in Chapter XVI that gas-turbine-engined aircraft will attain reasonable fuel economy only if flown *high* and *fast*. Aircraft already designed and flown, intended for airline operation in the next few years include—

1. *The Bristol Brabazon I, Mark II.* A 300 000 lb landplane with eight propeller turbines, cruising at 30 000–40 000 ft.

2. *The Saunders Roe S.R.* 45. A 300 000 lb flying-boat with ten propeller turbines, cruising at 30 000 to 40 000 ft.

3. *The De Havilland DH* 106. A 100 000 lb landplane with four jet turbines, cruising at about 500 m.p.h.

4. *The Handley-Page Hermes V.* A 90 000 lb landplane with four propeller turbines.

5. *The Vickers Viscount.* A 40 000 lb landplane with four propeller turbines.

If these are to be the long-distance aircraft of the future, it is worth while considering what advantages, if any, the increase in speed and height are likely to cause.

17.2. Increase in Speed

The attainment of higher speeds may be expected to bring about some of the following—

(i) Increased aircraft utilization, i.e. number of commercial operations both per year and per major overhaul and inspection. This should lead to reduced maintenance charges per flight, and the use of a smaller number of aircraft on a given route.

(ii) The partial elimination of night stops and the need for night stop facilities and staff.

(iii) The elimination of relief aircrew on long operations, thus leading to a reduction in the total aircrew required.

(iv) Increased regularity against adverse winds, and hence reduced schedule tolerances, leading to smaller fuel reserves and increased payload.

17.3. High Altitude Flying

The better weather conditions at high altitude should give—

(i) Increased regularity, due to the avoidance of bad weather at lower heights.

(ii) Increased passenger comfort, due to the absence of vibration and the better weather.

(iii) Reduced crew fatigue.

17.4. Disadvantages

17.4.1. TECHNICAL. We have already indicated, in Chapter XVI, some of the technical problems which gas-turbine engines have to face. At high altitudes we shall obviously need—

(*a*) Efficient pressurization and cabin atmosphere control;
(*b*) Reliable de-icing devices;
(*c*) Reliable combustion;
(*d*) Gust warnings;

and practical experience with these matters has so far been inadequate. Added to these, as speeds increase, there will be problems of stability and control, and in the strength of materials.

17.4.2. COST. Moreover we must not forget that the cost of development for gas turbine engines is at present very high, and will remain so until mass-production methods can be introduced more generally, when designs become stabilized.

17.4.3. METEOROLOGICAL. There are also certain meteorological questions to which no certain answers can be given at present—such as gust formation, the wind strengths and frequencies, and the humidity and temperature, at high altitudes.

17.5. Physiological Problems

17.5.1. PASSENGER COMFORT. We have stated in Chapter XVI that the present limitations in rate of climb and rate of descent are likely to be removed—in fact, *must* be removed, if the aircraft is to reach 30 000 ft or more in a reasonable time, thereby allowing it to take full advantage of the fuel economy and greater true air-speed at high altitude.

We need to know a good deal more about passenger reaction to decompression, to high frequency noises from the jet, to rapid changes in climatic conditions, etc. If rocket assistance is to be used at take-off, in order to allow the use of aerodromes which are small or in tropical climates; but knowledge of its effect on passenger comfort is at present very limited.

17.5.2. AIRCREW. We have also to remember that, unless alternative oxygen apparatus is available, the pilot and engineer will be flying with the knowledge that any failure in the pressurization system will demand very rapid remedial measures. As speeds increase towards the local speed of sound, the pilot's response to the approach of the "vice-speed" may not, without special instruments or training, be as quick as it should be.

17.6. Compressibility Effects

The behaviour of aircraft at Mach numbers at which compressibility effects become apparent is already beginning to be known. At the critical Mach number a noticeable change of longitudinal trim occurs, usually to nose down, often accompanied by buffeting, and the nose or either wing may drop as shock waves build up and a stall develops. In addition to pure compressibility effects, an initial change of trim may occur before the critical Mach number is reached. This initial trim change, which may have either a nose down or a nose up tendency, is primarily due to distortions of the airframe, particularly of the tail surfaces, and is more marked at low altitudes.

The speed of sound decreases as the air temperature falls, and the Mach number at which compressibility effects are encountered consequently occurs at decreasing true air-speeds (and even more rapidly decreasing indicated air-speeds) as height is gained—at least up to the tropopause at about 36 000 ft. This involves imposing a sliding scale of maximum permissible indicated air-speed, decreasing with altitude. To meet this difficulty, an instrument known as a Mach meter has been evolved. All the pilot needs to know is the limiting Mach number. This limiting Mach number will be slightly less than the critical Mach number at which local shock stalling begins to occur. The pilot thus has one number to remember, and one instrument to watch; and he is not expected to read any charts or do any calculations.

Fig. 105 shows for a typical transport of wing loading 50 lb/ft², and thickness chord ratio of 17·5 per cent,

(*a*) how the operating C_L, T.A.S., and Mach number change as heights are increased;

(*b*) the boundary at which we may expect compressibility effects first to show themselves;

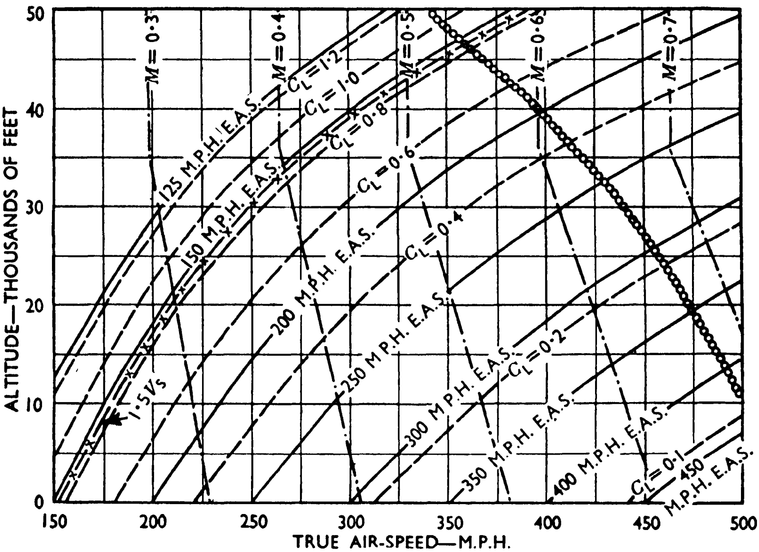


FIG. 105. EFFECT OF COMPRESSIBILITY OF HIGH ALTITUDE OPERATION OF TYPICAL TRANSPORT AIRCRAFT

———————— Constant E.A.S.
—— —— —— Constant C_L.
——— — ——— — Constant Mach number.
—— x —— x —— Minimum speed for comfortable cruising, $1{\cdot}5V_{S_0}$.
oooooooooooooooo Critical Mach number for 17½ per cent thickness chord ratio, high speed aerofoil.

(*c*) the lowest comfortable cruising speed, taken as 50 per cent above the stalling speed.

It will be seen from the diagram that the range of cruising speeds between (*b*) and (*c*) is cut down enormously at high altitude; and if such an aircraft is to cruise at 350 m.p.h. T.A.S. at 40 000 ft, the chance of avoiding compressibility effects is small.

APPENDIX I

Dimensions

We have already seen, in § **1.9**, that the "dimensions" of a quantity are the three fundamentals, viz. mass, length, and time, of which it is composed. Every quantity used in elementary aeronautics can be reduced to dimensions of these three fundamentals. Some of these are given in the table below—

Symbol	Name	Dimensions: Mass M	Dimensions: Length L	Dimensions: Time T	Units
L	Length		1		ft
M	Mass	1			slug
T	Time			1	sec
	Area		2		ft^2
	Volume		3		ft^3
v	Specific Volume	−1	3		ft^3/slug
V	Velocity		1	−1	ft/sec
	Acceleration		1	−2	ft/sec^2
N, Ω	Angular Velocity			−1	radn/sec, r.p.m.
	Angular Acceleration			−2	radn/sec^2
M^1	Mass Flow (air)	1		−1	slugs/sec, lb/sec, lb/hr
Q	Fuel Flow	1	2	−3	C.H.U./sec, ft-lb/sec
ρ	Density	1	−3		slugs/ft^3
	Momentum	1	1	−1	slug-ft/sec
F	Force	1	1	−2	lb
	Angular Momentum	1	2	−1	slug-ft^2/sec
	Torque, Couple, Moment	1	2	−2	lb ft
	Energy, Work	1	2	−2	ft-lb
W	Power	1	2	−3	ft-lb/sec, horse-power
θ	Temperature		2	−2	(ft/sec)^2
P	Pressure	1	−1	−2	lb/ft^2
G	Air Miles per Pound	−1	1		miles/lb
c_f	Specific Fuel Consumption		−2	2	lb/b.h.p./hr
μ	Coefficient of Viscosity	1	−1	−1	slugs/ft sec
$\nu = \frac{\mu}{\rho}$	Kinematic Coefficient of Viscosity	—	2	−1	ft^2/sec

The dimensions taken for pressure and temperature are important, because they lead us to formulate certain combinations of quantities which are *non-dimensional,* i.e. have no dimensions in length, mass or time. Consider for instance the ratio—

$$\frac{P}{\rho\theta}$$

where
$P =$ pressure,
$\rho =$ density,
$\theta =$ absolute temperature.

If we write down the dimensions of the separate items—

$$P = ML^{-1}T^{-2}; \ \rho = ML^{-3}; \ \theta = L^2T^{-2}.$$

So that
$$\frac{P}{\rho\theta} = M^\circ L^\circ T^\circ.$$

It follows that $\frac{P}{\rho\theta}$ has "no dimensions" and is a constant; this is the first Gas Law.

In the more general case, we can regard the quantities—

$D =$ a length,
$P =$ a pressure,
$\theta =$ a temperature,

as independent variables; and all other quantities such as r.p.m., speed, mass flow, force, power, etc., as dependent variables; and discover the exact nature of the dependence by making the dependent variable non-dimensional. This, of course, assumes that no other independent variables, such as viscosity, need be considered.

For instance, if we wish to know how power varies with length, pressure, and temperature, we take the following four quantities—

Power, $W = ML^2T^{-3}$
Length, $D = L$
Pressure, $P = ML^{-1}T^{-2}$
Temperature, $\theta = L^2T^{-2}$

and multiply them together, with arbitrary indices a, b, c for the last three. The result is—

$$(ML^2T^{-3})(L)^a(ML^{-1}T^{-2})^b(L^2T^{-2})^c$$

If this product is to be non-dimensional, the indices of M, L, and T must separately be zero, i.e.—

For M, $1 + b = 0$ $\quad\therefore b = -1.$

For L, $2 + a - b + 2c = 0 \quad \therefore a = -2.$
For T, $-3 - 2b - 2c = 0 \quad \therefore c = -\tfrac{1}{2}.$

Hence $WD^{-2}\rho^{-1}\theta^{-\frac{1}{2}}$ is non-dimensional;

or $\dfrac{W}{D^2\rho\sqrt{\theta}}$ is a constant, with the assumptions made above.

Here is a list of some of the results obtained in this way—

$$\frac{ND}{\sqrt{\theta}} = \text{constant, or } \frac{N_2D_2}{\sqrt{\theta_2}} = \frac{N_1D_1}{\sqrt{\theta_1}}.$$

$$\frac{V}{\sqrt{\theta}} = \text{constant, or } \frac{V_2}{\sqrt{\theta_2}} = \frac{V_1}{\sqrt{\theta_1}}.$$

$$\frac{M'\sqrt{\theta}}{D^2\rho} = \text{constant, or } \frac{M_2'\sqrt{\theta_2}}{D_2^{\,2}\rho_2} = \frac{M'_1\sqrt{\theta_1}}{D_1^{\,2}\rho_1}.$$

$$\frac{F}{D^2\rho} = \text{constant, or } \frac{F_2}{D_2^{\,2}\rho_2} = \frac{F_1}{D_1^{\,2}\rho_1}.$$

$$\frac{W \text{ or } Q}{D^2\rho\sqrt{\theta}} = \text{constant, or } \frac{W_2 \text{ or } Q_2}{D_2^{\,2}\rho_2\sqrt{\theta_2}} = \frac{W_1 \text{ or } Q_1}{D_1^{\,2}\rho_1\sqrt{\theta_1}}.$$

Note. Fuel flow Q has the same dimensions as power W because we regard fuel flow as work per unit time; the work arising from the calorific value of the fuel, i.e. we consider the fuel after combustion, not before.

The first consequence of this analysis is to provide us with relations between some of the standard quantities. For instance, the relation $\dfrac{M'\sqrt{\theta}}{D^2\rho}$ = constant is attained under the choked final nozzle conditions of a gas turbine engine where M′ is the mass flow, D^2 the area and θ and P are absolute temperatures and pressures.

More generally the method leads us to look for relations between the non-dimensional quantities and hence, when searching for information, to draw graphs, between these quantities. If F stands for jet thrust and α for aircraft attitude, we can for instance show that these must be functional relations of the type—

$$\frac{F}{D^2P} = f_1\left(\frac{V}{\sqrt{\theta}}, \frac{ND}{\sqrt{\theta}}, \alpha\right).$$

$$\frac{Q}{D^2P\sqrt{\theta}} = f_2\left(\frac{V}{\sqrt{\theta}}, \frac{ND}{\sqrt{\theta}}, \alpha\right).$$

$$\frac{\text{Drag}}{D^2P} = f_3\left(\frac{V}{\sqrt{\theta}}, \alpha\right).$$

When a flight test is carried out, or an engine is run on a test bed, the variations in local conditions of pressure and temperature will be ironed out if the results are plotted non-dimensionally. The best way to do this is usually obvious. Take, for example, lift, as found in a wind tunnel. From the relation F/D^2P = constant, this might at first sight suggest that we plot $\frac{\text{lift}}{\text{wing area} \times \text{pressure}}$ against incidence. But $\frac{\text{lift}}{\text{pressure}}$ has the same dimensions as $\frac{\text{lift}}{\rho\theta} = \frac{\text{lift}}{\rho V^2}$, which is easier to measure, and we therefore plot $C_L = \frac{\text{lift}}{\frac{1}{2}\rho V^2 S}$ against α.

Then the resulting curve of C_L against α will be independent of temperature and pressure variations. How far it will depend on changes in viscosity is discussed later.

In many of the applications of the method, when we are dealing with one particular aircraft or engine, the length dimension D can be omitted.

There is, of course, an alternative method of plotting test results which does not require us to use the non-dimensional form. This involves, broadly speaking, calculating each reading as a ratio to the value of the quantity at standard conditions. This has the effect of correcting the quantity for the existence of non-standard conditions. In the table on page 279 are shown some examples of—

(*a*) the non-dimensional form (second column),
(*b*) the corrected form (third column).

A more general application of the method is provided by Rayleigh's formula for aerodynamic force. If it is agreed that this force must depend upon pressure variation (i.e. density and velocity), the skin friction (i.e. viscosity and velocity),

Quantity	Non-dimensional form	Corrected form
Atmospheric temperature, θ_0 .	$\frac{\theta_0}{288}$	$288\theta_0$
Barometric pressure, p_0 . .	$\frac{p_0}{30}$	30·0
Any temperature, θ . .	θ/θ_0	$\theta \times \frac{288}{\theta_0}$
Any pressure, p . . .	$\frac{p}{p_0}$	$p \times \frac{30}{p_0}$
Speed, V	$V/\sqrt{\theta_0}$	$V \times \sqrt{\frac{288}{\theta_0}}$
Angular velocity, N . .	$N/\sqrt{\theta_0}$	$N \times \sqrt{\frac{288}{\theta_0}}$
Air mass flow, M . . .	$\frac{M\sqrt{\theta_0}}{p_0}$	$M \times \frac{300}{p_0}\sqrt{\frac{\theta_0}{288}}$
Force of thrust, F. . .	$\frac{F}{p_0}$	$F \times \frac{30}{p_0}$
Fuel flow or power, W . .	$\frac{W}{p_0\sqrt{\theta_0}}$	$W \times \frac{30}{p_0}\sqrt{\frac{288}{\theta_0}}$

together with some representative length, then the independent variables are (for a given angle of attack) now

Density ρ	Dimensions	ML^{-3}
Velocity V	,,	LT^{-1}
Kinematical viscosity $\nu = \frac{\mu}{\rho}$	,,	L^2T^{-1}
Length D	,,	L

and consequently the product of force (dimensions MLT^{-2}) and suitable powers of the four independent variables must be non-dimensional. Hence $MLT^{-2}(ML^{-3})^a(LT^{-1})^bL^c(L^2T^{-1})^d$ is non-dimensional.

For M : $1 + a = 0.$

L : $1 - 3a + b + c + 2d = 0.$

T : $-2 - b - d = 0.$

$$\therefore \qquad a = -1;\ b = -2 - d;\ c = -2 - d$$

$$\therefore \qquad \frac{\text{Aerodynamic force}}{\rho V^{2+d} D^{2+d} \nu^{-d}} = \text{a constant, say K;}$$

$$\text{or aerodynamic force} = K\rho V^2D^2\left(\frac{VD}{\nu}\right)^d$$

$$= K\rho V^2D^2 f\left(\frac{VD}{\nu}\right)$$

where $f(VD)/\nu$ is some function of the ratio VD/ν. This ratio, usually written as Vl/ν, is the "Reynolds' Number." If the lift force on a wing is measured under two different conditions, keeping ρV^2D^2 the same, the answers obtained will not be identical unless the Reynolds' Number also remains the same. This explains why wind tunnel measurement of force and moment are not always repeated in full-scale tests.

The most important force measurement which may be so affected is drag. We have seen that

$$\text{drag} = \tfrac{1}{2}\rho V^2 S_W \times C_D$$

and it is possible to plot the drag coefficient C_D against incidence. But C_D, although non-dimensional, will vary with Reynolds' Number, and the amount of the variation will depend on the size of the object whose drag is being measured. If, for instance, external fuel tanks or a pannier are added to an aircraft whose drag is already known, the Reynolds' Number of the new parts will be different from the Reynolds' Number of the aircraft without the parts; so that it is hazardous to rely on wind tunnel results to estimate the full-scale value of drag.

Sometimes the drag is stated, not as a coefficient but as D_{100}, which means the drag in pounds at 100 ft/sec. This is a way of comparing drags which eliminates the velocity effect only. The value of D_{100} may have more meaning to an engineer than a coefficient C_D; but its use does not remove the difficulties associated with differing Reynolds' Numbers.

Returning to Rayleigh's formula, the most general case would be that in which we consider *six* independent variables,

$$V,\ D,\ g,\ \nu,\ a,\ \rho$$

where g is the local acceleration of gravity, and a is the local velocity of sound.

By a method similar to that given above we can show that aerodynamic force is expressible as

$$K\rho V^2D^2 f\left(\frac{V^2}{gD}, \frac{VD}{\nu}, \frac{V}{a}\right).$$

The parameter $\frac{V}{a}$ is known as the *Mach Number*; its effect on life, drag, and moment is a matter of very great importance.

We have dealt with engine testing and aerodynamics. There remains the general performance of the aircraft, which can be regarded as a function of $w = \frac{\text{weight}}{\text{area}}$, the wing loading; P the power of the engine; D′ the propeller diameter; n the r.p.m., then it can be shown in a similar way that the level speed is a function of the type

$$\sqrt{\frac{w}{\sigma}}\,f\left(\frac{P}{W}\sqrt{\frac{\sigma}{w}},\ nD'\sqrt{\frac{\sigma}{w}}\right) \qquad \begin{pmatrix} W = \text{weight} \\ \sigma\ = \text{relative density} \end{pmatrix}$$

where the parameters $\frac{P}{W}\sqrt{\frac{\sigma}{w}}$, $nD'\sqrt{\frac{\sigma}{w}}$ are themselves non-dimensional. There is thus a theoretical basis for the type of plotting carried out in performance analysis and reduction. If there is no variation in P, n or D′, this reduces to the result: level speed $= K\sqrt{\frac{w}{\sigma}}$ which is sufficient to deal with simple charges in loading and air density.

APPENDIX II

The I.C.A.N. Standard Atmosphere (English Units)

h	ρ	σ	$\sqrt{\sigma}$	p(lb/ft^2)	p(in. Hg)	$\pi = p/p_0$	t°C	$\nu \times 10^4$	a(ft/sec)
0	0·002378	1·0000	1·0000	2 116	29·92	1·0000	15·00	1·56	1 117
500	0·002343	0·9855	0·9927	2 079	29·38	0·9821	14·01	1·58	1 115
1 000	0·002309	0·9710	0·9854	2 041	28·86	0·9644	13·02	1·60	1 113
1 500	0·002275	0·9568	0·9782	2 004	28·33	0·9469	12·03	1·62	1 111
2 000	0·002242	0·9428	0·9710	1 968	27·82	0·9298	11·04	1·64	1 109
2 500	0·002209	0·9288	0·9638	1 932	27·31	0·9129	10·05	1·66	1 107
3 000	0·002176	0·9151	0·9566	1 897	26·81	0·8967	9·06	1·68	1 105
3 500	0·002144	0·9015	0·9495	1 862	26·32	0·8798	8·07	1·70	1 103
4 000	0·002112	0·8881	0·9424	1 828	25·84	0·8636	7·08	1·72	1 102
4 500	0·002080	0·8748	0·9353	1 794	25·36	0·8477	6·09	1·74	1 100
5 000	0·002049	0·8616	0·9282	1 761	24·89	0·8320	5·09	1·77	1 098
5 500	0·002018	0·8487	0·9213	1 728	24·43	0·8165	4·10	1·79	1 096
6 000	0·001988	0·8358	0·9142	1 696	23·98	0·8013	3·12	1·81	1 094
6 500	0·001957	0·8232	0·9073	1 664	23·53	0·7863	2·13	1·83	1 092
7 000	0·001928	0·8106	0·9003	1 633	23·09	0·7716	1·14	1·86	1 090
7 500	0·001898	0·7982	0·8934	1 602	22·65	0·7571	0·14	1·88	1 088
8 000	0·001869	0·7859	0·8865	1 572	22·22	0·7427	− 0·84	1·90	1 086
8 500	0·001840	0·7738	0·8797	1 542	21·80	0·7286	− 1·83	1·93	1 084
9 000	0·001812	0·7619	0·8728	1 513	21·39	0·7147	− 2·82	1·95	1 082
9 500	0·001784	0·7501	0·8661	1 484	20·98	0·7011	− 3·81	1·98	1 080
10 000	0·001756	0·7384	0·8593	1 455	20·58	0·6876	− 4·80	2·00	1 078
10 500	0·001728	0·7269	0·8525	1 437	20·18	0·6743	− 5·79	2·03	1 076
11 000	0·001702	0·7154	0·8458	1 400	19·79	0·6614	− 6·78	2·05	1 074
11 500	0·001675	0·7042	0·8392	1 373	19·41	0·6486	− 7·77	2·08	1 072
12 000	0·001648	0·6931	0·8325	1 346	19·03	0·6359	− 8·76	2·11	1 070
12 500	0·001622	0·6821	0·8259	1 319	18·65	0·6234	− 9·75	2·13	1 068
13 000	0·001596	0·6712	0·8193	1 294	18·28	0·6112	− 10·74	2·16	1 066
13 500	0·001570	0·6685	0·8127	1 268	17·93	0·5992	− 11·73	2·19	1 064
14 000	0·001545	0·6499	0·8062	1 243	17·57	0·5873	− 12·72	2·22	1 062
14 500	0·001520	0·6394	0·7996	1 218	17·22	0·5757	− 13·73	2·25	1 060
15 000	0·001496	0·6291	0·7932	1 194	16·88	0·5642	− 14·70	2·28	1 058
15 500	0·001472	0·6189	0·7867	1 170	16·54	0·5530	− 15·71	2·31	1 056
16 000	0·001448	0·6088	0·7803	1 147	16·21	0·5418	− 16·68	2·34	1 054
16 500	0·001424	0·5988	0·7738	1 124	15·89	0·5309	− 17·69	2·37	1 052
17 000	0·001401	0·5891	0·7675	1 101	15·57	0·5205	− 18·66	2·40	1 050
17 500	0·001378	0·5793	0·7611	1 079	15·25	0·5097	− 19·67	2·44	1 048
18 000	0·001355	0·5698	0·7549	1 057	14·94	0·4992	− 20·64	2·47	1 046
18 500	0·001333	0·5603	0·7485	1 035	14·63	0·4891	− 21·65	2·50	1 044
19 000	0·001311	0·5509	0·7422	1 014	14·33	0·4790	− 22·62	2·54	1 041
19 500	0·001289	0·5418	0·7361	993	14·03	0·4691	− 23·63	2·57	1 039
20 000	0·001267	0·5327	0·7291	973	13·74	0·4594	− 24·60	2·61	1 037
20 500	0·001246	0·5237	0·7237	953	13·48	0·4500	− 25·59	2·64	1 035
21 000	0·001225	0·5148	0·7175	933	13·19	0·4406	− 26·58	2·68	1 033
21 500	0·001204	0·5061	0·7114	913	12·91	0·4315	− 27·57	2·72	1 031
22 000	0·001183	0·4974	0·7053	894	12·64	0·4223	− 28·56	2·76	1 029
22 500	0·001163	0·4889	0·6992	875	12·37	0·4135	− 29·55	2·80	1 027
23 000	0·001143	0·4805	0·6932	856	12·10	0·4047	− 30·54	2·83	1 025
23 500	0·001123	0·4721	0·6871	838	11·85	0·3961	− 31·53	2·87	1 023
24 000	0·001103	0·4640	0·6812	820	11·59	0·3876	− 32·52	2·92	1 021
24 500	0·001085	0·4559	0·6752	803	11·35	0·3794	− 33·51	2·96	1 019
25 000	0·001065	0·4480	0·6693	785	11·10	0·3711	− 34·50	3·00	1 017

h	ρ	σ	$\sqrt{\sigma}$	p(lb/ft²)	p(in. Hg)	$\pi = p/p_0$	t°C	$\nu \times 10^4$	a(ft/sec)
25 500	0·001046	0·4401	0·6634	769	10·87	0·3631	– 35·49	3·04	1 015
26 000	0·001028	0·4323	0·6575	752	10·63	0·3552	– 36·48	3·09	1 012
26 500	0·001010	0·4247	0·6517	735	10·39	0·3475	– 37·47	3·13	1 010
27 000	0·000992	0·4171	0·6458	719	10·17	0·3399	– 38·46	3·18	1 008
27 500	0·000974	0·4097	0·6401	704	10·00	0·3325	– 39·45	3·22	1 006
28 000	0·000957	0·4023	0·6343	688	9·73	0·3251	– 40·44	3·27	1 004
28 500	0·000940	0·3951	0·6286	673	9·51	0·3179	– 41·43	3·32	1 001
29 000	0·000922	0·3879	0·6228	658	9·30	0·3108	– 42·42	3·37	999
29 500	0·000906	0·3809	0·6172	643	9·09	0·3039	– 43·41	3·42	997
30 000	0·000889	0·3740	0·6116	629	8·89	0·2970	– 44·40	3·47	995
30 500	0·000873	0·3671	0·6059	614	8·68	0·2900	– 45·39	3·52	993
31 000	0·000857	0·3603	0·6003	600	8·48	0·2837	– 46·38	3·57	991
31 500	0·000842	0·3537	0·5947	587	8·29	0·2773	– 47·37	3·62	989
32 000	0·000826	0·3472	0·5892	573	8·10	0·2709	– 48·36	3·68	987
32 500	0·000811	0·3408	0·5838	560	7·92	0·2647	– 49·35	3·73	985
33 000	0·000795	0·3343	0·5782	547	7·74	0·2586	– 50·34	3·78	982
33 500	0·000780	0·3280	0·5727	535	7·56	0·2526	– 51·33	3·84	980
34 000	0·000765	0·3218	0·5673	522	7·38	0·2467	– 52·32	3·91	978
34 500	0·000750	0·3108	0·5575	510	7·21	0·2410	– 53·31	3·97	975
35 000	0·000736	0·3098	0·5566	498	7·04	0·2353	– 54·30	4·04	973
35 332	0·000727	0·3057	0·5529	490	6·92	0·2314	– 55·00	4·06	971
35 500	0·000722	0·3036	0·5510	482	6·82	0·2278	– 55·00	4·09	971
36 000	0·000704	0·2967	0·5442	474	6·71	0·2242	↑	4·20	↑
36 500	0·000687	0·2893	0·5379	463	6·55	0·2189		4·30	
37 000	0·000671	0·2824	0·5314	452	6·39	0·2137		4·40	
37 500	0·000655	0·2758	0·5252	442	6·24	0·2087		4·51	
38 000	0·000640	0·2692	0·5188	431	6·09	0·2037		4·62	
38 500	0·000625	0·2629	0·5127	421	5·95	0·1990		5·73	
39 000	0·000610	0·2566	0·5066	411	5·81	0·1943		4·84	
39 500	0·000596	0·2506	0·5006	401	5·68	0·1899	↓	4·96	↓
40 000	0·000582	0·2447	0·4947	392	5·54	0·1852	– 55·00	5·08	971

APPROXIMATE EQUATIONS FOR THE STANDARD ATMOSPHERE

	Altitude and Percentage Error					
	5 000 ft	10 000 ft	15 000 ft	20 000 ft	25 000 ft	30 000 ft
σ, Standard Value	0·8616	0·7384	0·6291	0·5327	0·4480	0·3740
$1 - \frac{h}{40\ 000}$	0·8750	0·7500	0·6250	0·5000	—	—
Per cent error	+ 1·55	+ 1·55	− 0·66	− 6·14	—	—
$1 - \frac{h}{34\ 160} + \left(\frac{h}{59\ 000}\right)^2$	0·8608	0·7360	0·6255	0·5292	0·4475	0·3800
Per cent error	− 0·09	− 0·33	− 0·57	− 0·66	− 0·11	+ 1·60
$e^{-\left(\frac{h}{33\ 000}\right)}$	0·8600	0·7365	0·6408	0·5450	0·4683	0·4025
Per cent error	− 0·19	− 0·26	+ 2·02	+ 2·31	+ 4·53	+ 7·62
$e^{-\left(\frac{h}{34\ 160 - 0{\cdot}12h}\right)}$	0·8615	0·7385	0·6290	0·5325	0·4483	0·3745
Per cent error	− 0·01	+ 0·01	− 0·01	− 0·04	+ 0·07	+ 0·13
$\frac{33\ 600 - 0{\cdot}53h}{33\ 600 + 0{\cdot}47h}$	0·8612	0·7388	0·6308	0·5348	0·4486	0·3712
Per cent error	− 0·05	+ 0·05	+ 0·27	+ 0·40	+ 0·13	− 0·75
$\log_e \left(\frac{91\ 300}{33\ 600 + h}\right)$	0·8616	0·7390	0·6304	0·5330	0·4440	0·3612
Per cent error	0	+ 0·08	+ 0·21	+ 0·06	− 0·89	− 3·43

APPROXIMATE EQUATIONS FOR THE STANDARD ATMOSPHERE—(*contd.*)

	Altitude and Percentage Error					
	5 000 ft	10 000 ft	15 000 ft	20 000 ft	25 000 ft	30 000 ft
$\frac{1}{\sqrt{\sigma}}$ Standard Value	1·0773	1·1637	1·2608	1·3701	1·4940	1·6352
$1 + \frac{h}{60\,000}$	1·0833	1·167	1·250	1·333	1·417	—
Per cent error	+ 0·56	+ 0·26	− 0·86	− 2·69	− 5·17	—
$1 + \frac{h}{50\,000}$	1·100	1·200	1·300	1·400	1·500	1·600
Per cent error	+ 2·11	+ 2·86	+ 3·11	+ 2·18	+ 0·40	− 2·15
$1 + \frac{h}{68\,320} + \left(\frac{h}{68\,320}\right)^2$	1·078	1·168	1·268	1·378	1·500	1·622
Per cent error	+ 0·06	+ 0·37	+ 0·56	+ 0·60	+ 0·40	− 0·81
$\frac{68\,320 + 0{\cdot}293h}{68\,320 - 0{\cdot}707h}$	1·0772	1·1634	1·2599	1·3691	1·4936	1·6368
Per cent error	− 0·009	0·03	− 0·07	− 0·07	− 0·03	+ 0·10
$e^{\left(\frac{h}{63{,}000}\right)}$	10·079	1·164	1·255	1·354	1·461	—
Per cent error	+ 0·13	0·00	− 0·43	− 1·17	− 2·21	-
$e^{\left(\frac{h}{66\,000}\right)}$	1·083	1·172	1·269	1·374	1·487	1·610
Per cent error	+ 0·49	+ 0·72	+ 0·64	+ 0·27	− 0·47	− 1·54
$e^{\left(\frac{h}{68\,320 - 0{\cdot}24h}\right)}$	1·0773	1·1639	1·2608	1·3704	1·494	1·635
Per cent error	0·00	+ 0·02	0·00	+ 0·02	0·00	0·00
$\text{Log}_e\left(\frac{68\,000 - 0{\cdot}14h}{25\,000 - 0{\cdot}42h}\right)$	1·078	1·164	1·260	1·368	1·491	1·637
Per cent error	+ 0·06	0·00	− 0·07	− 0·15	− 0·20	+ 0·13

APPENDIX III

Standard Climates

A.R.B. Design Conditions

FOR those interested in an attempt at a precise definition of standard climates, the following extract from the B.C.A.R. may be of value.

The Temperate, Tropical, and Arctic climates are defined by—

(1) The temperature envelopes enclosed by the appropriate maximum and minimum temperature lines of Fig. 106, from zero feet to the selected height (e.g. the temperatures appropriate to 0–30 000 ft in the Standard Temperate Climate are those within the envelope *A*, *B*, *C*, *D*, in Fig. 106).

(2) Every point included in these envelopes is associated with a relative humidity range of 20 to 100 per cent; except that in the conditions represented by the area *E*, *F*, *G* in Fig. 106 the relative humidities shall be assumed to vary from 100 per cent maximum and 20 per cent minimum respectively at the line *EF* to the value appropriate to the height at the line *GF*. The value of relative humidity on the line *GF* shall be taken to vary linearly from 100 per cent maximum and 20 per cent minimum at *F* to some lower values at *G* (given here as 10 per cent maximum and 2 per cent minimum).

(3) Every point included in these envelopes is associated with the International standard pressure (I.C.A.N.) appropriate to the height, as shown in Table I.

(4) Every point included in these envelopes is associated with the density corresponding to the temperature, pressure and humidity; extreme values are given in Table I.

These conditions do not cover variation of pressure from the International standard. This shall be allowed for by assuming a variation of pressure 5 per cent above and below the International standard pressure (I.C.A.N.) associated with the International standard temperature (I.C.A.N.).

The standard climatic conditions are intended primarily for

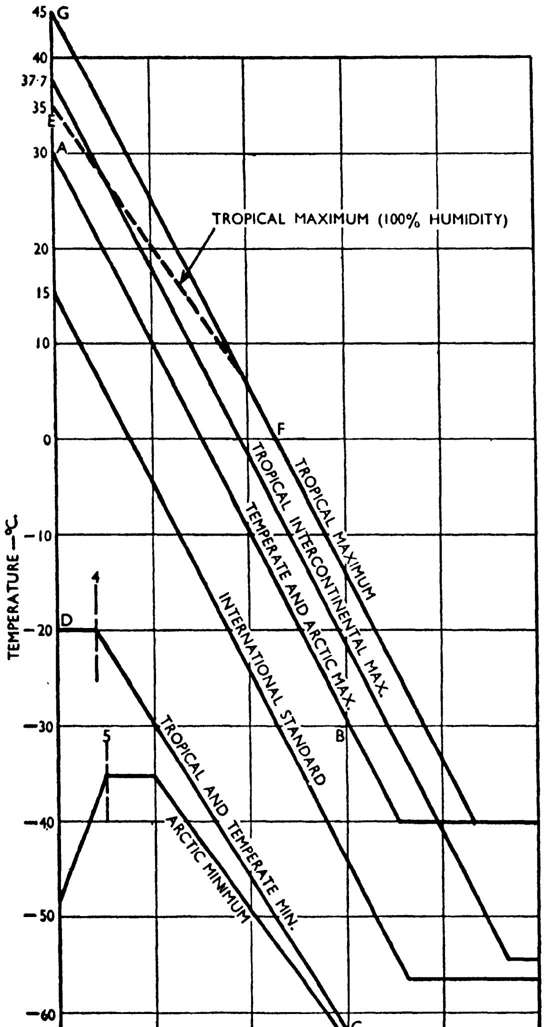


FIG. 106. TEMPERATURE LAPSE RATE IN VARIOUS STANDARD CLIMATES

TABLE I

Relative Pressures and Densities

Air density at zero height (barometer, 29·92 in.; temp., 15°C) is 0·07652 lb/ft.^{3}

Height, Feet, Pressure Basis	Relative Pressures (I.C.A.N.)	Relative Densities Associated with Conditions Stated				
		International Standard (I.C.A.N.)	Tropical Maximum	Temperate and Arctic Maximum	Tropical and Temperate Minimum	Arctic Minimum
0	1·000	1·000	0·905	0·950	1·14	1·29
1 000	0·964	0·971	0·880	0·923	1·10	1·23
2 000	0·930	0·943	0·855	0·896	1·06	1·17
3 000	0·896	0·915	0·828	0·868	1·02	1·11
4 000	0·864	0·888	0·803	0·843	0·982	1.06
5 000	0·832	0·862	0·778	0·818	0·955	1·01
6 000	0·801	0·836	0·754	0·794	0·923	0·970
7 000	0·771	0·810	0·730	0·768	0·893	0·932
8 000	0·743	0·786	0·709	0·746	0·867	0·900
10 000	0·688	0·730	0·664	0·700	0·813	0·832
12 000	0·636	0·693	0·622	0·655	0·762	0·778
14 000	0·587	0·650	0·583	0·615	0·714	0·727
16 000	0·542	0·609	0·545	0·575	0·657	0·680
18 000	0·499	0·570	0·509	0·537	0·622	0·632
20 000	0·459	0·532	0·475	0·502	0·582	0·589
22 000	0·422	0·498	0·444	0·469	0·543	0·551
24 000	0·387	0·464	0·413	0·437	0·503	0·509
26 000	0·355	0·432	0·383	0·406	0·468	0·473
28 000	0·325	0·402	0·355	0·377	0·435	0·437
30 000	0·297	0·374	0·331	0·351	0·403	0·406
32 000	0·271	0·347	0·306	0·325	0·374	0·376
33 000	0·258	0·334	0·294	0·312	0·358	0·360
34 000	0·247	0·322	0·284	0·301	0·348	0·347
35 000	0·235	0·310	0·272	0·290	0·333	
36 000	0·224	0·298	0·263	0·278	0·319	
37 000	0·214	0·284	0·252	0·264	0·303	
38 000	0·204	0·271	0·243	0·252	0·289	
39 000	0·194	0·258	0·233	0·240	0·275	
40 000	0·185	0·246	0·224	0·229	0·262	
41 000	0·176	0·234	0·215	0·217	0·249	
42 000	0·168	0·223	0·207	0·208	0·238	
44 000	0·153	0·203	0·188		0·217	
46 000	0·139	0·184	0·172		0·197	
48 000	0·126	0·167	0·155		0·178	
50 000	0·114	0·152	0·140		0·162	

use in designing aeroplane structure and equipment which should remain airworthy when subjected to the appropriate conditions.

Aeroplane performance will vary considerably within the defined climates. It is not intended that any one stated performance should be achievable throughout the whole envelope of conditions, but rather that sufficient performance data should be scheduled for an operator to determine the performance which will be achieved in particular conditions.

The climatic conditions given are conditions of the free atmosphere. The temperatures achieved in an aeroplane in these atmospheric conditions may be considerably higher. In the absence of precise information as to surface finish, ventilation, and type of engine, etc., the following maximum ambient temperatures should be assumed—

	Temperate and Arctic	Tropical
In the interior of an aeroplane	45°C	60°C
For portions of the outer covering liable to be in the sun and parts attached directly to such covering.	55°C	80°C
In an engine compartment for parts not attached directly to the engine. Parts connected to the engine may attain higher temperatures . .	100°C	100°C

APPENDIX IV

Cruising Speed

SEVERAL alternative methods have been suggested for finding the T.A.S. for cruising.

Barthe's Method

Consider the equation of § **2.4.1**, slightly modified,

$$550\eta P = \left(a\sigma V^3 + \frac{bW^2}{\sigma V}\right)$$

when P is the b.h.p. supplied to the propellers, V is the T.A.S. in feet per second, and W is the instantaneous weight, and

$$a = \tfrac{1}{2}\rho_0 S_W C_{D_Z},\ b = \frac{1}{\tfrac{1}{2}\rho_0 S_W \pi A e},\ \text{as before.}$$

The power equation is equivalent to the quartic

$$a\sigma V^4 + \frac{bW^2}{\sigma} - 550\eta PV = 0$$

or

$$V^4 - \alpha V + \beta^2 = 0$$

$$\text{where } \alpha = \frac{550\eta P}{a\sigma} = \frac{550\eta P}{\tfrac{1}{2}\rho_0 S_W C_{D_Z}\sigma} = \frac{0{\cdot}4625 \times 10^6 \eta P}{C_{D_Z} \, . \, \sigma \, . \, S_W}$$

$$\text{and } \quad \beta = \sqrt{\frac{bW^2}{a\sigma^2}} = \sqrt{\frac{2W^2}{\tfrac{1}{4}\pi\rho_0{}^2 S_W{}^2 A e C_{D_Z}\sigma^2}} = \frac{0{\cdot}04744 \times 10^4 W}{\sigma S_W \sqrt{AeC_{D_Z}}}.$$

The solution can now be shown in the form of a chart (Fig. 107) giving the value of V for different values of α and β. The method is amenable to changes in the parameter. If we are using Constant Power, at a given height, we can estimate the mean weight for the trip from the fuel flow; hence α and β are known, and the answer found for V will be the mean T.A.S.

Methods Introducing *n* and *p*

We have seen in § **2.3.7** how to find the indicated speed for minimum drag

$$V_{i_{md}} = \left(\frac{1}{\pi A e C_{D_Z}}\right)^{\frac{1}{4}} \left(\frac{W}{\frac{1}{2}\rho_0 S_W}\right)^{\frac{1}{2}} = \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{W} \text{ ft/sec}$$

so that the T.A.S. for minimum drag, at a height given by σ, is

$$u = \frac{1}{\sqrt{\sigma}}\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{W} \text{ ft/sec}$$

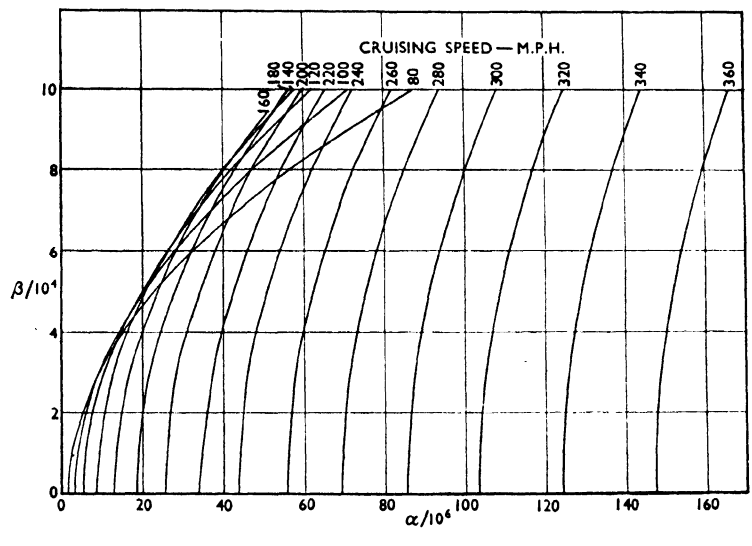


FIG. 107. CHART SHOWING VALUES OF V FOR DIFFERENT VALUES OF α AND β

We can use § **2.4.1**. to find the thrust power P_u required for this speed. Suppose that V is the actual T.A.S. ft/sec and B_V the actual b.h.p.; and define the ratios p and n by

$$p = \frac{\eta B_V}{P_u}$$

$$n = \frac{V}{u}$$

related by the equation $p = \frac{1}{2}\left(n^3 + \frac{1}{n}\right)$.

If u and v are taken in miles per hour, the procedure is as follows—

Fig. 108 shows a chart for finding u,

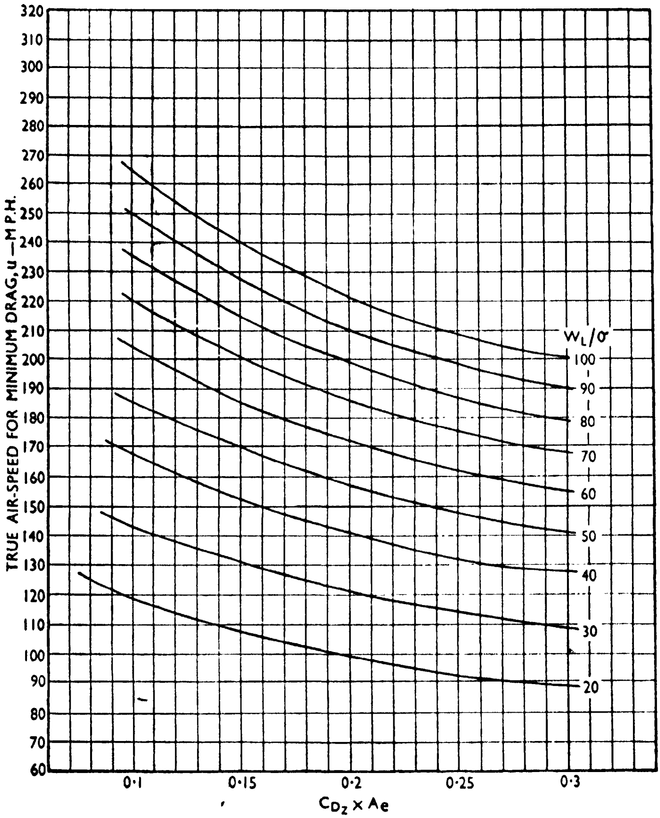


FIG. 108. VARIATION OF MINIMUM DRAG SPEED WITH $C_{D_Z} \times Ae$ AND W_L/σ

where $$W_L = \text{wing loading} = \frac{W}{S_W}.$$

Now calculate $$P_u = \frac{Wu}{332\left(\dfrac{Ae}{C_{D_Z}}\right)^{\frac{1}{2}}}$$

and hence find $$p = \frac{\eta B_V}{P_u}.$$

Now use Fig. 109 to find n. The T.A.S., in m.p.h. can then be found from

$$V = nu.$$

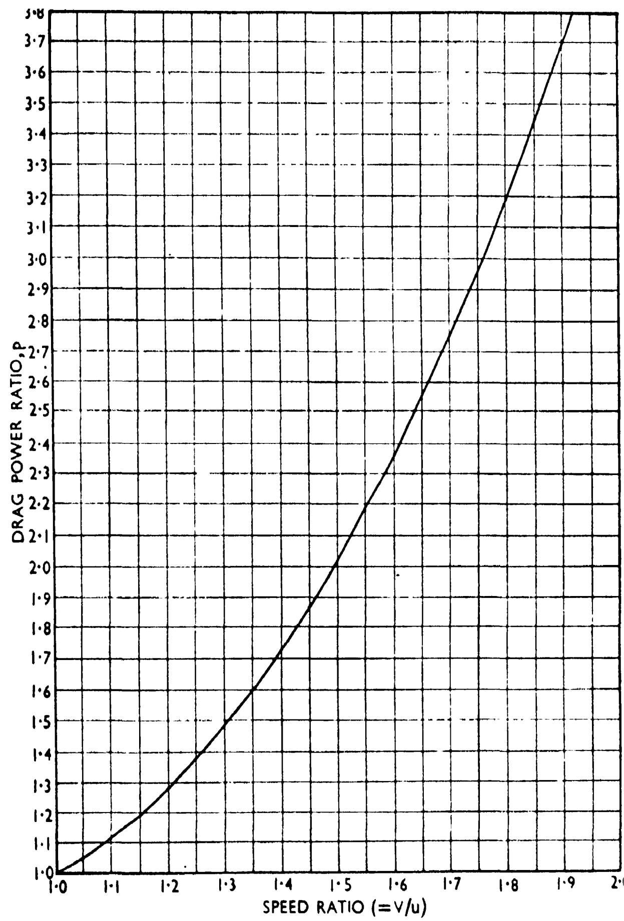


FIG. 109. VARIATION OF DRAG POWER RATIO, p, WITH SPEED RATIO, n

This method is extremely tedious and would only be tolerable in cases when u, n, and p were required for other purposes. To obtain an answer that means anything at all in practice, we have again got to start with B_V and estimate W_m.

APPENDIX V

Elementary Questions for the Student

SOME of the answers to the elementary questions which follow can be found in the preceding text; the remainder are put in to remind the reader of certain facts in basic aerodynamics and piston-engine theory, which should be general knowledge.

1. Theory of Flight

(*a*) *Lift*

(1) Explain why all sections of a finite aerofoil operate at less than the geometrical angle of attack.

(2) Explain the term "downwash," and the effect of downwash on the setting of the tail-plane of a conventional aircraft.

(3) Explain the object and function of—
  (*a*) slotted flaps, at take-off;
  (*b*) split flaps, on the approach;
  (*c*) wing-tip slots;
  (*d*) spoilers, on the upper surface of the mainplane.

(4) What is likely to be the effect of the propeller slip-stream on—
  (*a*) rudder effectiveness;
  (*b*) engine-off stalling speed?

(5) Explain what is meant by the "aerodynamic centre" of an aerofoil section.

(*b*) *Drag*

(1) Explain what is meant by "skin friction drag," and how it may be lessened.

(2) Explain the meaning of: (*a*) "form drag";
  (b) "interference drag."

(3) Draw a rough graph to show how total drag varies with indicated speed. What is the effect of (*a*) height, (*b*) weight, on the true speed for minimum drag?

(4) Give examples of how the drag of an aircraft may be intentionally increased, and describe their effects on the "gliding angle."

(5) Explain the terms: (*a*) "cooling drag";
  (*b*) "intake momentum drag";
 and describe under what conditions each is liable to increase.

(*c*) *Performance*

(1) Explain what is meant by the "critical speed" of an aircraft taking off with one engine dead.

(2) What considerations are required to fix the "safety speed" of an aircraft?

(3) What extra precautions are necessary at take-off—
  (*a*) on a wet, grassy runway;
  (*b*) on an aerodrome in a tropical climate;
  (*c*) on an aerodrome several thousand feet above sea-level?

(4) The best forward speed of an aircraft on the climb varies between the minimum-drag speed at sea-level and the minimum-power speed near the ceiling: explain the reasons for this.
(5) What is meant by the "minimum speed for comfortable cruising"? Under what conditions may the aircraft be flown at a lower speed?

2. The Atmosphere

(*a*) *The Gas Laws*

(1) Distinguish between isothermal. isentropic, and polytropic expansions.
(2) State the relation between pressure, density, and temperature on any consistent system of units (which must be clearly defined).
(3) Explain the statement: "the standard density of dry air is 0·002378 slugs per cubic foot."
(4) Explain how to convert a manifold pressure in inches of mercury into a plus or minus boost pressure in lb per square inch, and give an example.
(5) Explain why the temperature of the air on the bulb of a thermometer in a moving aircraft is liable to be greater than the temperature of the surrounding air.

(*b*) *The I.C.A.N. Atmosphere*

(1) Give the I.C.A.N. lapse rate of temperature (*a*) in the troposphere; (*b*) in the stratosphere; and sketch curves to show how I.C.A.N. pressure and density vary with height.
(2) Give reasons for the necessity for having a standard atmosphere.
(3) Explain the term "relative density," and why it is important in aircraft performance investigations.
(4) Explain the difference between "pressure height" and "density height," and for what purposes each is used.
(5) Explain the term "Mach Number," and why the local velocity of sound depends upon temperature.

(*c*) *Speeds*

(1) Explain the difference between "true air-speed" and "equivalent air-speed."
(2) Explain the cause of "position error," and what must be done to allow for it.
(3) State in general terms the effect of compressibility on the A.S.I. reading and briefly explain the reasons.
(4) Give suitable conversion factors for turning feet per second, miles per hour, kilometres per hour, and knots into each other.

3. Power Plants

(*a*) *Piston Engines*

(1) Explain what is meant by the statement: In the ideal four-stroke petrol engine cycle, heat is supplied and rejected "at constant volume."
(2) Explain "thermal efficiency," and how it depends upon compression ratio. What considerations set an upper limit to compression ratio?
(3) Explain what is meant by the "octane rating" of a fuel. How can the octane number be raised?
(4) Give a brief explanation of: (*a*) detonation;
(*b*) pre-ignition.
(5) Give a brief explanation of the object and operation of single-speed supercharging.
(6) State the requirements of an aircraft carburettor. Indicate those where the choke-type carburettor is unsatisfactory.

(7) Give exact definitions of: (*a*) rated boost;
(*b*) international r.p.m.;
(*c*) rated altitude;
(*d*) rated, or international, power.

(8) Give reasons why specific fuel consumption is lowered by using low r.p.m. and high boost.

(9) Explain the main objects of a two-speed supercharger, and the probable effects of using the higher gear on fuel consumption.

(10) Explain why air miles per gallon depend upon (*a*) aircraft drag; (*b*) propeller efficiency; (*c*) engine specific fuel consumption.

(11) Explain the advantages and disadvantages of cruising—

(*a*) at constant power;
(*b*) at constant indicated air-speed;
(*c*) at maximum air miles per gallon.

(12) Explain what is meant by the "mean weight" of an aircraft, and how its use facilitates range calculations.

(*b*) *Gas-turbine Engines*

(1) Explain what is meant by saying that the ideal gas-turbine engine operates on the "constant pressure" cycle.

(2) Explain the object of the compressor; and indicate briefly what limits the practicable pressure ratio.

(3) Give a list of the more important advantages of the gas-turbine engine, when compared with the piston engine.

(4) Explain the term "propulsive efficiency," and on what main factors it depends; and sketch a rough graph comparing the propulsive efficiency of jet and propeller as speed varies.

(5) Sketch a typical combustion chamber, and indicate how it deals with the large air/fuel ratio involved.

(6) Explain the function of the turbine assembly; and indicate briefly the main manufacturing difficulties in its production.

(7) State and explain the effect of aircraft height on specific fuel consumption.

(8) Explain the meaning of "ram," and how it is utilized in the gas-turbine engine.

(9) State the effects of forward speed on jet thrust; and of climbing on specific fuel consumption.

(10) State the operating conditions best suited to the gas-turbine engine, to ensure maximum fuel economy.

(11) Give a list of the main engine controls and engine instruments required in the cockpit of a gas-turbine-engined aircraft.

4. Propellers

(1) Fill in the blanks in the following statements—

"Constant speed airscrews incorporate a mechanism which insures that the.. of the airscrew is maintained at a predetermined value, whatever the forward speed,.............................. ...., and.................... may be; and this is achieved by adjusting the.................. ....of the blades."

(2) Describe the main features of a constant speed unit.

(3) Give reasons why there exists an optimum diameter of blade for a particular aircraft.

(4) Explain why a reduction gear is necessary between engine and propeller.

(5) State some of the advantages of contra-rotating airscrews.

5. Flying Controls and Stability

(1) Cross out the incorrect statements in the third and fourth columns of the following table—

Control Surface	Aircraft Movement	Movement of Control Surface	Movement of Control Column or Pedal
Aileron	Turn to port	1. Port aileron up, stbd. aileron down 2. Port aileron down, stbd. aileron up	1. Wheel clockwise 2. Wheel anti-clockwise
Elevator	Climb	1. Elevator down 2. Elevator up	1. Column forward 2. Column backwards
Rudder	Turn to port	1. Rudder(s) to port 2. Rudder(s) To stbd.	1. Left pedal down 2. Right pedal down

(2) Explain the object of a large fin area, and what bearing this has on the area of the rudder. (Consider in particular an aircraft flying with one engine dead.)

(3) Explain what is meant by the "hinge moment" on a control surface, and describe how the hinge moment can be reduced to zero by means of a "trim tab."

(4) Explain the working of *three* of the following methods of lightening the control loads by means of aerodynamic balance: nose balance, horn balance, geared tabs, spring tabs.

(5) Explain the object of: (*a*) "frise" ailerons,
(*b*) ailerons with differential gearing.

(6) Explain the reason for a control surface being mass-balanced dynamically, and how this can be achieved in practice.

(7) Define the term "static longitudinal stability," and explain what is meant by the "neutral point."

(8) Distinguish between "stick-fixed" and "stick-free" conditions of an aircraft; and explain why an aeroplane flying at high speed is approximately in the stick-free condition.

(9) Explain the meaning and cause of "auto-rotation."

(10) Explain the effect of dihedral during a sideslip.

(11) Explain the meaning and origin of "tail-buffeting."

(12) What corrective measures would you apply if an aircraft tended to fly with one wing low?

6. Loading

(1) Explain the necessity for a forward limit to c.g. position, and the probable effects of exceeding the limit.

(2) Explain the necessity for a rear limit to c.g. position, and the probable effects of exceeding the limit.

(3) At what periods during a flight, between take-off and touch-down, will incorrect loading be most noticeable to the pilot?

(4) What steps must be taken to make sure that the aircraft will be correctly loaded at the end of a flight, as well as at the beginning?

(5) What do you understand by static longitudinal stability?

(6) In the absence of a loading diagram or slide rule, how would you verify that the distribution of the loads will retain the c.g. within the limits laid down in the C. of A., for a complete flight?

(7) If the c.g. of an aircraft is correct, what effects would you expect if you
(a) exceeded the maximum permissible take-off weight?
(b) exceeded the maximum permissible landing weight?

(8) A weighbridge is large enough to take (a) the main wheels, (b) the nose wheel or tailwheel, but not all three together. Explain how (for *either* a tricycle *or* conventional landing gear, but not for both) you would find the c.g. position, tail down.

(9) Explain how to convert 1 gal of—
(a) 100 octane petrol,
(b) kerosene,
(c) engine oil,
into (i) pounds, (ii) kilograms; at standard temperature and pressure.

(10) Explain the terms: (a) "disposable load,"
(b) "wing loading,"
(c) "power loading,"
(d) "volumetric restriction on loading."

(11) What considerations have to be borne in mind when fixing—
(a) the maximum take-off weight,
(b) the maximum landing weight.

(12) Give a rough sketch showing how payload may be expected to vary with range, and explain why the maximum payload is constant over the shorter ranges.

INDEX