MANUAL OF
BUILDING SCIENCE

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PREFACE

THE foundations of the Building Industry were laid many hundreds and even thousands of years ago in the earliest civilizations known to man. In Egypt, two thousand years before Christ, there were craftsmen with a tradition of fine building producing not only fine architecture but works of art, and the achievements of the Greek and Roman civilizations are still a source of inspiration to us, but, just as in other old-established industries, the traditional methods, although ensuring a certain standard of craftsmanship, unfortunately also had the effect of limiting the development of new methods. There has been much opposition to the use of scientific methods in Building and it is only of recent years that a scientific approach to the problems of the industry has been accepted and that an attempt has been made to develop the fundamental theory underlying Building technique. Many of the traditional methods have been found to be scientifically correct, but others have been shown to be obstructive and wasteful; but perhaps the most important result of the new scientific approach has been the production of new non-traditional methods of building which allow full scope to the special properties of the new materials which Science has made available. The work of the Building Research Station is particularly noteworthy and the reader is recommended to study the excellent pamphlets and bulletins issued by this body which, it should be added, are both interesting and clearly written.

To obtain the best results from the new methods of building a higher degree of knowledge of applied science is desirable among those now entering the industry. The Technical Schools and Colleges of this country are attempting to ensure this by the provision of full-time Building courses and by part-time classes for the National Certificates and by Craft apprentices courses. It is the purpose of this book to provide the elementary science necessary as a basis for the study of the various technical subjects forming part of these courses. Inevitably we have had to present our material, drawn from many branches of science, in an abridged form and have included only that which seemed to form the minimum requirement for the basis of a technical education for the building student. To those who, having read this book, wish to learn more of any particular subject, we recommend the comprehensive standard works on the various branches of science.

JOHN F. DOUGLAS
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SYMBOLS

Acceleration due to gravity	g	Stress, direct	f
		shear	q
Linear acceleration	f	tensile	t
Mass	m	Temperature	t
Modulus of elasticity, Bulk	K	Thermal conductivity	k
Modulus of elasticity, Rigidity	N	Time	t
Modulus of elasticity, Young's	E	Velocity	v
Modulus of section	Z	Weight	w
Moment of inertia	I		
Moment of resistance	M		
Poisson's ratio	$\frac{1}{m}$		
Radius of gyration	k		
Specific heat	c		
Strain	e		
Stress, compressive	c		

Greek Symbols

Coefficient of linear expansion	α
Density	ρ
Refractive index	μ
Surface tension	γ

ABBREVIATIONS

British Thermal Unit	B.Th.U.	Litre	gal
Calorie	g. cal	Metre	m
Candle-power	c.p.	Minute	min
Centimetre	cm	Pound	lb
Cubic centimetre	cm^3	Second	sec
Cubic foot	ft^3	Specific heat	sp. ht.
Cubic inch	$in.^3$	Square centimetre	cm^2
Cubic yard	yd^3	Square foot	ft^2
Cycles per second	c/s	Square inch	$in.^2$
Foot	ft	Square metre	m^2
Gallon	gal	Square yard	yd^2
Gramme	g	Temperature	temp
Horse-power	h.p.	Ton	t
Hour	hr	Velocity	vel
Hundredweight	cwt	Watt	W
Inch	in.	Weight	wt

THE GREEK ALPHABET

Α	α	alpha (a)
Β	β	beta (b)
Γ	γ	gamma (g)
Δ	δ	delta (d)
Ε	ε	epsilon (ĕ)
Ζ	ζ	zeta (z)
Η	η	eta (ē)
Θ	θ	theta (th)
Ι	ι	iota (i)
Κ	κ	kappa (k)
Λ	λ	lambda (l)
Μ	μ	mu (m)
Ν	ν	nu (n)
Ξ	ξ	xi (x)
Ο	ο	omicron (ŏ)
Π	π	pi (p)
Ρ	ρ	rho (r)
Σ	σ, ς	sigma (s)
Τ	τ	tau (t)
Υ	υ	upsilon (u)
Φ	φ	phi (ph)
Χ	χ	chi (ch, as in loch)
Ψ	ψ	psi (ps)
Ω	ω	omega (ō)

CHAPTER I

SCIENCE AND THE BUILDER

RIGHT from the beginning of his history Man has had to build to protect himself from the elements and from his enemies. His early efforts were crude and often unsuccessful but he learnt from his failures and, in time, the craft of the builder gradually evolved. This craft was based on the methods which had been found to be successful in practice. Man had no understanding of the potentialities of his materials and was still unable to predict what would

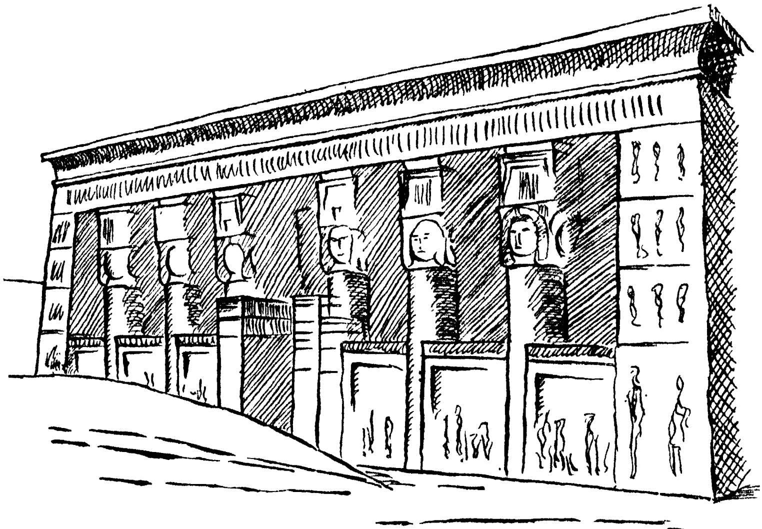


FIG. 1. TEMPLE OF HATHOR, DENDERAH

happen if he departed from the accepted rules of construction. But the demand grew for larger and more ambitious buildings. With increased size came new structural problems, needing new solutions. Design by trial and error alone became impossible because the waste of labour and material and the possible loss of life entailed in the collapse of these structures could not be tolerated. To meet the ever-increasing demands on his skill the builder had to turn scientist, study his materials and improve them, use new materials such as cast iron, steel, and concrete, and devise methods of design which would enable him to solve, safely and economically, those problems which the old craftsmen could only tackle by the long and expensive method of trial and error.

The gradual evolution of building science is recorded in the history of architecture. The earliest dwellings were probably the rock caves of the hillsides and the natural arbours formed from the overhanging branches of trees. These probably inspired the huts of timber and stone of prehistoric times. Few records exist of the development of building before 3900 B.C., but by that time the rule of the Pharaohs had begun in Egypt and the art of architecture was well developed. Construction was carried out with massive stone columns and lintels, some impression of which can be obtained

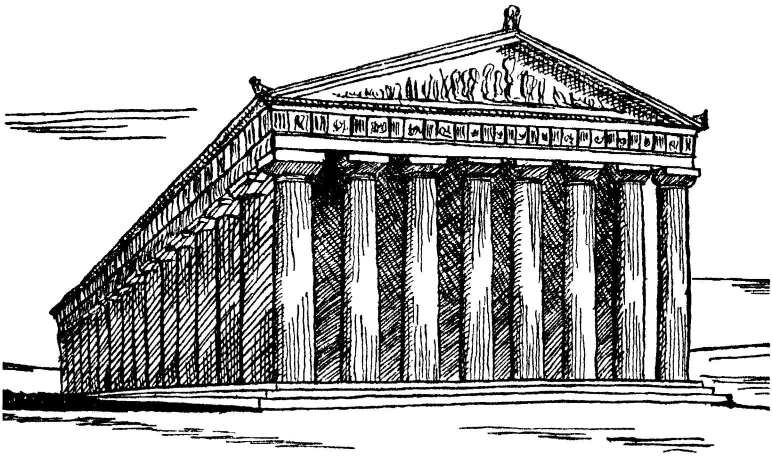


FIG. 2. THE PARTHENON, ATHENS

from the temple of Hathor, Denderah, shown in Fig. 1. This *trabeated* construction was brought to perfection by the Greeks, culminating in masterpieces such as the Parthenon, Fig. 2. A notable step forward in structural science was marked by the use of the arch by the Romans and the development of the vaulted roof and of such fine domes as that of the Pantheon, Fig. 3, which exceeds 140 ft in diameter. The Romans also developed the science of bridge building to such a pitch that one bridge, over the Danube, had individual spans of 170 ft and was 150 ft high and 60 ft wide.

In the years following the fall of the Roman empire experiments were made on the balancing of arch thrusts and the concentration of loads on isolated piers rather than on the wall structure. Eventually a scientific framework of piers, arches, and *flying buttresses* was evolved giving to Gothic architecture a lightness which can hardly be surpassed, and a good example of this is shown in Fig. 4. With the Victorian era came new materials and methods. Cast iron made possible that amazing building, the original Crystal Palace, Fig. 5, with its standardized prefabricated units, and with the twentieth century has come the use of steel and concrete with possibilities which are not yet exhausted.

The great complexity of our modern civilization has brought new

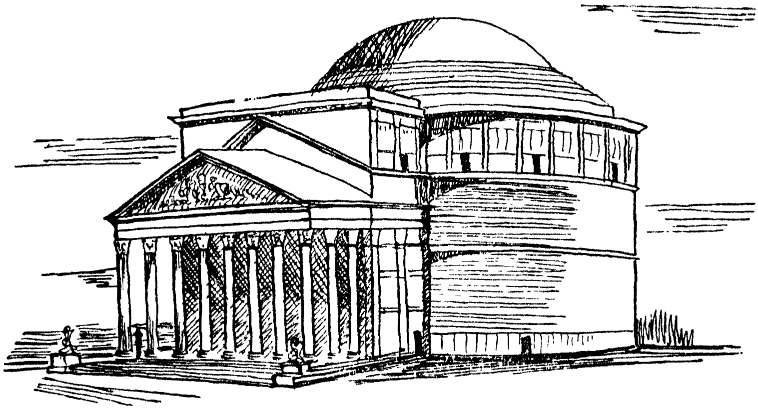


FIG. 3. THE PANTHEON, ROME

problems to the builder. It is no longer sufficient that a building should have four walls and a roof; it must be heated, properly lighted and provided with services such as water, electricity, and

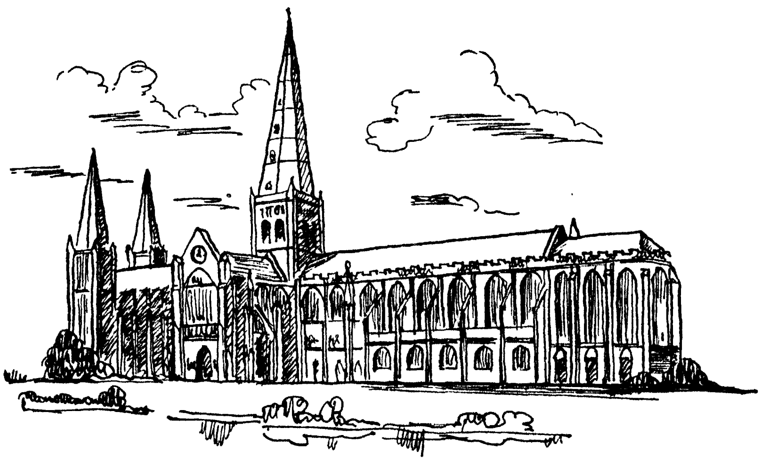


FIG. 4. LICHFIELD CATHEDRAL

all manner of mechanical plant. All these matters lie in the field of applied science, and so it is the purpose of this book to give the reader an understanding of fundamental scientific principles and to explain their application to building.

Requirements for a Building

Before proceeding it will be of value to make a review of the function of a modern building and the requirements which it has to satisfy.

Protection. The primary function of a building is to provide a place where the routine of daily life can be carried on secure from wind and weather and, where necessary, with privacy. There must be walls and a roof to keep out rain and snow in winter and sun and

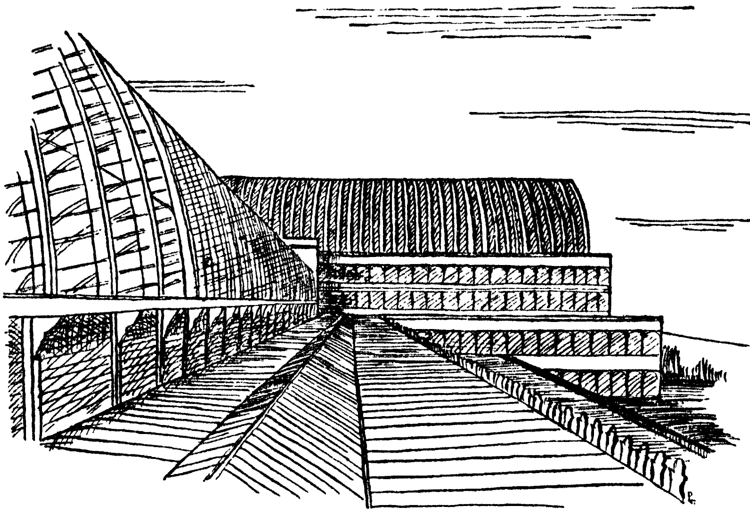


Fig. 5. Crystal Palace

dust in summer. The complete exclusion of moisture still remains a problem, although much ingenuity has been shown in the development of cavity walls and damp-proof courses. In this respect the scientific study of permeability and capillarity has been of great assistance.

Heating and Heat Insulation. For comfort the interior must be maintained at a temperature which is neither too high nor too low. In cold weather some method of heating must be provided, and it must be confessed that the satisfactory solution of this problem has not yet been reached, although much progress has been made since the days when the sole source of heat was an open fire burning in the middle of the room, with a hole in the roof for the smoke to escape.

To the problem of heating must be added that of heat insulation, a subject which has been much neglected until recently. It is of little use attempting to warm a room if all the time the heat supplied is escaping through the walls and ceiling. This is clearly wasteful, and scientific knowledge of heat insulation has been applied to prevent it. Insulation is also required to keep the interior cool in hot weather by preventing the inflow of heat from outside. In the past thick walls provided this insulation to a certain extent, but new methods and new materials now allow a more economical solution.

Structural Strength. It is most important that a building should be soundly constructed and have adequate strength so that there may be no danger of collapse. In small buildings this can be ensured by using excessive amounts of material although such a solution is wasteful. In larger structures the extra material is not only uneconomical, but is also an added liability because the structure has to support its own weight in addition to the loads it is designed to carry. Thus in the case of the Golden Gate Bridge or the Sydney Harbour Bridge the traffic passing over the bridge forms but a small proportion of the total load to be carried. Much of the strength of the structure is needed to carry its own dead weight.

The object of the builder must be to provide the lightest possible structure that will carry the load with safety. The study of such subjects as the Theory of Structures and Strength of Materials has helped to achieve this aim. The former enables the effects of the applied loads to be found at all parts of the structure, and the latter deals with the ability of the material to resist these effects. Together they allow an estimate to be made of the size and shape of the component members of a structure necessary to ensure that it has adequate strength.

Water Supply and Drainage. Our ancestors were content to take their water supply from the nearest stream or from the village pump. The presence of an adequate natural water supply largely controlled the siting of their towns and villages. Nowadays there is a universal demand for a piped supply of pure water. Indeed it is essential in towns, where the concentration of population makes the local water supply inadequate and liable to contamination from the refuse of town life. The need is equally great in the country, both for the convenience of the housewife and the use of the farmer. An adequate supply of water for live stock and for farm purposes is a prime need of efficient agriculture.

The actual provision of the main water supply is the business of the water engineer, but the builder is concerned with the proper design and construction of the pipework within the building, and for this work an elementary knowledge of the science of Hydraulics is necessary. An adequate system of drainage must also be provided to remove waste water and sewage. Where a main sewage system is available the work is restricted to the choice of suitable pipe sizes and gradients and the proper arrangement of drains. In isolated areas the builder may be called on to construct small sewage disposal plants, and here again acquaintance with elementary scientific ideas will be helpful.

Mechanical and Electrical Services. The growing complexity of modern civilization has brought with it the use of many mechanical and electrical appliances. Electrical wiring systems for light, heat, power, and telephones are needed in modern buildings. Provision must also be made for gas supply, central heating, and ventilation. Most large blocks of buildings have lifts and some large department stores employ escalators. Although these services are usually designed and supplied by specialist firms the building technician, both in the drawing office and on the site, must make provision for their installation and should have a general knowledge of the way in which they function. Considerable delay and expense can be caused by failure to provide adequate ducts for cables, while mechanical plant can be rendered inefficient by incorrect installation.

Sound Insulation. The development of motor and air traffic and the use of loud-speakers has created a noise problem which has been aggravated by the trend in structural design towards the use of lighter construction, having poor sound insulation properties. The position is so serious that new methods of construction are being sought, and sound insulation promises to be a major factor in the planning and design of urban buildings. Under these conditions a study of sound and its behaviour is of value.

Lighting. Too little consideration has been given in the past to the proper siting of buildings and the provision of adequate window space so that sufficient daylight and sunlight should enter each room. Design has been based largely on empirical rules and on consideration of architectural effect, frequently resulting in parts of the building being cold and dark. It is now possible to treat daylighting and sunlighting scientifically and so obtain more satisfactory solutions.

Limitations of Science Applied to Building

An attempt has been made in this chapter to show that a knowledge of scientific principles can assist the builder, but it is necessary to point out that the results obtained by applying these principles must sometimes be treated with caution. The conditions of actual building operations are so variable that they can only be treated scientifically by making assumptions about materials and circumstances which may not be true. As a result, calculations based on these assumptions cannot yield an answer that is correct, although it may be very nearly so. Thus in using such an answer the assumptions upon which it is based should always be borne in mind, and, from a numerical point of view, it will usually be sufficient to work to three significant figures.

SECTION I—MATTER

CHAPTER II

PHYSICAL FORMS OF MATTER

ALL the substances used in building, such as bricks, mortar, wood, and glass, are forms of matter. The characteristics of matter are that it has mass and that it occupies space, and is thus distinguished from such things as heat or light, which are forms of energy.

Solids, Liquids and Gases. As there are many kinds of matter, it is convenient to distinguish between them according to their physical and chemical properties. Most materials may be classified as belonging, under normal conditions, to one of three main physical groups, the *solids*, *liquids*, or *gases*, although some substances have properties which are intermediate between solid and liquid; for example, pitch, and others, which are finely divided solids, may behave on occasion like liquids. This latter fact supports the theory (page 12) that all matter is composed of small particles, and that the difference between solid, liquid and gas lies in the strength or weakness of the forces holding the particles together.

In a *solid* these particles are bound together closely in a compact mass, giving the material a rigid form which retains its shape and, in some cases, e.g. steel, will only deform slightly under substantial forces. If the deforming force is not too great, many solids will resume their original shape when the force is removed and are said to be *elastic*; but, if the force is sufficiently large, the material may be permanently deformed. Some materials, such as lead, show little elasticity; they are *plastic*, and readily take a permanent set or deformation. Other solids, such as cast iron, do not become plastic even under heavy loads, but are *brittle* and crack suddenly.

The particles in a *liquid* are less closely knit, but the forces between them are still sufficient to bind them together into one continuous mass. Liquids lack rigidity and flow under their own weight, but, unlike gases, a given amount of liquid occupies a fixed volume, which alters very little even when great pressure is applied; indeed, for most purposes liquids may be regarded as incompressible.

A *gas* is composed of particles which are so loosely associated that they flow easily in all directions, expanding to fill any vessel, even though it contains another gas, in which case the two gases mingle or *diffuse* to produce a uniform mixture. Even when one gas is much heavier than the other, they will not separate into distinct layers, but will diffuse in this way.

Changes of State. Under given physical conditions a substance will fall into one of the above categories, but it is possible to make it assume a different state by varying its temperature or pressure, and so altering the forces linking the particles of the material together. To take a well-known example, *water* is a liquid at ordinary temperatures under atmospheric pressure, but by lowering the temperature it can be converted into *ice*, which is a solid. If, however, the temperature is raised or the pressure reduced sufficiently, the water will be turned into *steam*, a gaseous vapour. In either case the composition of the material remains the same, only its physical state being changed. This change is purely temporary, since, on restoring the original conditions, the water resumes its liquid form. Heat plays a great part in these changes of state, a large amount of heat being absorbed in converting water to steam, or given out in turning water to ice. (See page 145.)

Crystalline and Non-crystalline Solids. Many solids exist in crystalline form, the particles composing them being arranged in an orderly manner, thus giving to pieces of the material definite geometrical forms varying according to the nature of the substance. Granite, a well-known building stone, is composed of an interlocked mass of such crystals, those of each of its component materials—quartz, felspar and mica—having their own regular shape, as shown in Fig. 6. Metals also have crystalline structure, and break with a crystalline fracture in which the individual crystals can be seen with a lens.

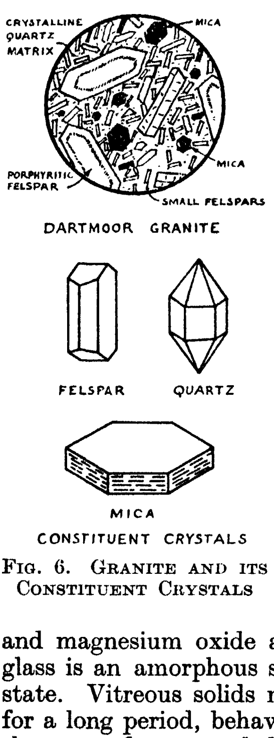


FIG. 6. GRANITE AND ITS CONSTITUENT CRYSTALS

Substances which do not crystallize are called *amorphous* and may be either powders or solids. Powdered charcoal and magnesium oxide are examples of amorphous powders while glass is an amorphous solid and is in what is known as a *vitreous* state. Vitreous solids may sometimes flow if subjected to a load for a long period, behaving like liquids of very high viscosity, and they are often regarded as *super-cooled liquids*. Thus, glass in its clear state is a super-cooled liquid, but it will sometimes crystallize to form an opaque solid. Recently, crystalline glass has been used commercially as a building material under the name of vitreolite. The fracture of vitreous solids is *conchoidal*, revealing a smooth and glassy curved surface.

Pure Substances, Mixtures and Compounds. Materials, such as granite, are composed of a number of substances and will show different properties in different parts. A *pure substance* exhibits the same properties throughout and is said to be *homogeneous*.

All parts of a sheet of glass have the same properties and so do all the particles in a bowl of water.

Granite is a *mixture*, since its properties vary from point to point, and it is said to be *heterogeneous*. In a mixture the constituent substances can be present in any desired proportions, and it is usually possible to see the separate particles of the individual substances either by eye or with a lens. Each component retains its own properties and can be separated from the mixture by mechanical means. If iron filings and sulphur are mixed, they can be separated either by collecting the filings with a magnet or by shaking with water and allowing the heavier iron to sink to the bottom.

Mixtures must be distinguished from *compounds* that are formed by the chemical union of their components. If iron filings and sulphur are heated together in a test tube they glow brightly and react to form a dark grey compound. No trace of the individual particles of the original substances can be seen, the material is non-magnetic, and the components cannot be separated out. A new homogeneous substance, iron sulphide, has been formed. The difference between the mixture and the compound of iron and sulphur may be emphasized by adding dilute hydrochloric acid first to the mixture and then to the compound. The iron in the mixture will react with the acid to produce hydrogen, which is odourless, but with the iron sulphide sulphuretted hydrogen will be produced, which has a very characteristic "bad eggs" smell.

Solutions. A special kind of mixture is a *solution* of one substance in another. The dissolved substance, or *solute*, is intimately mixed with the *solvent* in such a finely divided form that there is no tendency for it to settle out. The result is homogeneous, the individual particles not being distinguishable even under a powerful lens. Yet the solution is not a compound since it retains some of the individual properties of its components and can be split up by physical means without resort to chemical action. If a solution of salt in water is heated the water can be evaporated away and condensed as pure water, leaving a residue of salt. Also, up to a certain limit, the substances can be dissolved in any proportions, again indicating that the solution is a mixture. When, however, the solution has reached a certain strength, no more of the solute will dissolve and the solution is said to be *saturated*. The amount of solute dissolved is dependent on the temperature and the strength of a solution at saturation point can often be increased by heating.

The best known solutions are those of solids in liquids. Water is an extremely good solvent for many substances, and for this reason it is difficult to obtain really pure water. Its solvent action on lead is of importance in plumbing, as certain types of water may become poisonous as a result of standing in lead pipes. For those solids which are insoluble in water many other liquid solvents are in use, such as oil and alcohol, which are used in the manufacture of paints and varnishes.

Solutions of gases in liquids are also frequent. Rainwater in industrial areas dissolves carbon dioxide and sulphur dioxide present in the smoky air, thus becoming acid and causing serious decay to building stones, especially limestone.

Less common are solutions of solids in solids, which obviously do not occur under normal conditions. Mutual solution can occur if the solids are mixed together in molten form, as, for example, in the case of brass, which is a solidified solution of copper and zinc. Bronze is similarly formed from copper and tin, while solder is a solid solution of lead and tin. Solid solutions of metals are called *alloys* and often exhibit properties quite unlike the properties of the component metals.

EXERCISES

1. Explain the difference between a solid, a liquid and a gas, giving an example of each.
2. A solid may be elastic, plastic, or brittle. What is meant by these terms? Give an example of each type of solid.
3. What is the difference between a crystalline and an amorphous solid? Illustrate your answer by references to common building materials.
4. If a sample of sand is suspected of containing clay, how could you separate the sand and the clay from the mixture?
5. How does a mixture differ from a compound?
6. Explain the meaning of the term solution. Give examples of solvents commonly used in the building industry.
7. What is meant by the strength of a solution? Is there any limit to the strength of the solution that can be made from a given substance in a given solvent?
8. How can solutions of solids in solids be made? Give examples of such solutions.

CHAPTER III

STRUCTURE OF MATTER

A LARGE number of materials are used in the construction of even a small building and, in the larger field of Nature as a whole, the variety of substances is overwhelming.

Elements. To bring order to this chaotic abundance attempts have been made from the earliest times to find a small number of simple substances which can be combined together to form this great variety. The first expression of this idea appears in Greek philosophy with the theory that all things were compounded from four elements, fire, air, water and earth, but little progress was made until the development of chemical analysis enabled compound substances to be broken up and the simple substances from which they are made were identified. By chemical and other methods about ninety of these *elements* have been discovered and eighty-six of them are listed in Table I. These substances cannot be decomposed by chemical action and from their combination all forms of matter can be built up. Many elements are extremely rare; only twenty (marked with an asterisk in Table I) are of common occurrence in building materials. Estimates show that about 98 per cent of the crust of the earth, to a depth of 10 miles, consists of compounds of only eight elements, nearly 50 per cent of the material in these compounds being oxygen.

Molecules and Atoms. The theory that substances are made up of a large number of small particles has already been mentioned, and it may be understood more clearly by taking an example. A brick might be broken and crushed in a mortar, then each grain crushed into still finer particles, and the process continued until, in imagination, the particles were so small that they could not be further divided without losing their identity. These final particles, invisible to the eye even under the most powerful microscope, are called *molecules*, and are the smallest mass of a substance capable of independent physical existence.

Now the material of the brick taken as an example is a compound and is composed of several different elements, and each of these elements must be present in the molecule in its proper proportion if the molecule is to be recognized as a particle of brick. Thus molecules must be built up of other smaller particles or *atoms* of the elements. These atoms are the smallest particles of matter and they remain undivided in all chemical actions, but the molecule remains the smallest particle normally capable of independent existence. The molecule of an element is not necessarily a single atom and is, in fact, usually a group of atoms, all of the same kind. For example, argon, a rare gas, is *monatomic* having only one atom in each molecule, but oxygen is *diatomic* with two atoms to the molecule.

TABLE I

THE ELEMENTS: THEIR CHEMICAL SYMBOLS AND ATOMIC WEIGHTS

Element	Symbol	Atomic Weight	Element	Symbol	Atomic Weight
*Aluminium	Al	26·97	Neodymium	Nd	144·27
Antimony	Sb	121·76	Neon	Ne	20·18
Argon	A	39·94	*Nickel	Ni	58·69
Arsenic	As	74·91	Niobium	Nb	92·91
Barium	Ba	137·36	*Nitrogen	N	14·01
Beryllium	Be	9·02	Osmium'	Os	190·2
Bismuth	Bi	209·00	*Oxygen	O	16·00
Boron	B	10·82	Palladium	Pd	106·7
Bromine	Br	79·92	*Phosphorus	P	30·98
Cadmium	Cd	112·41	Platinum	Pt	195·23
Caesium	Cs	132·91	*Potassium	K	39·09
*Calcium	Ca	40·08	Praseodymium	Pr	140·92
*Carbon	C	12·01	Protoactinium	Pa	231
Cerium	Ge	140·13	Radium	Ra	226·05
*Chlorine	Cl	35·46	Radon	Rn	222
*Chromium	Cr	52·01	Rhenium	Re	186·31
Cobalt	Co	58·94	Rhodium	Rh	102·91
*Copper	Cu	63·57	Rubidium	Rb	85·48
Dysprosium	Dy	162·46	Ruthenium	Ru	101·7
Erbium	Er	167·2	Samarium	Sm	150·43
Europium	Eu	152·0	Scandium	Sc	45·10
Fluorine	F	19·00	Selenium	Se	78·96
Gadolinium	Gd	156·9	*Silicon	Si	28·06
Gallium	Ga	69·72	Silver	Ag	107·88
Germanium	Ge	72·60	*Sodium	Na	22·99
Gold	Au	197·2	Strontium	Sr	87·63
Hafnium	Hf	178·6	*Sulphur	S	32·06
Helium	He	4·00	Tantalum	Ta	180·88
Holmium	Ho	164·94	Tellurium	Te	127·61
*Hydrogen	H	1·008	Terbium	Tb	159·2
Indium	In	114·76	Thallium	Tl	204·39
Iodine	I	126·92	Thorium	Th	232·12
Iridium	Ir	193·1	Thulium	Tm	169·4
*Iron	Fe	55·85	*Tin	Sn	118·70
Krypton	Kr	83·7	Titanium	Ti	47·90
Lanthanum	La	138·92	Tungsten	W	183·92
*Lead	Pb	207·21	Uranium	U	238·07
Lithium	Li	6·94	Vanadium	V	50·95
Lutecium	Lu	174·99	Xenon	Xe	131·3
*Magnesium	Mg	24·32	Ytterbium	Yb	173·04
Manganese	Mn	54·93	Yttrium	Y	88·92
Mercury	Hg	200·61	*Zinc	Zn	65·38
Molybdenum	Mo	95·95	Zirconium	Zr	91·22

* Of common occurrence in building materials

The atomic theory of the structure of matter was given definite form by John Dalton about 1802, who asserted that—

1. Chemical elements are composed of very minute particles, or atoms, which remain undivided in all chemical changes.

2. Each kind of atom has a definite weight. Different elements have atoms of different weights.

3. Chemical combination occurs by union of the atoms of the elements in simple numerical proportions.

These simple assumptions explain the laws of chemical combination satisfactorily. Since atoms are indestructible in chemical changes the total number of atoms present remains constant, and the total quantity of matter must also remain constant. This is in accordance with the chemical *Law of Indestructibility of Matter*, which states that matter cannot be destroyed.

Again, the molecules of a compound are made up of numbers of atoms of two or more elements, but the molecules are all alike and therefore each must contain the same proportion of atoms of each element. Thus in forming a given compound the proportions in which the elements combine will not vary from molecule to molecule, but will always be constant. This is the *Law of Constant Proportions.*

Atomic Weights. Different elements have atoms of different weights but the weight of a single atom is extremely small, that of a hydrogen atom being $1{\cdot}66 \times 10^{-24}$ g (1 lb $= 453{\cdot}6$ g). Dalton, realizing this difficulty, investigated the relative weights of the atoms, taking the weight of hydrogen, the lightest atom, as unity. In the modern system of *atomic weights* the weight of an oxygen atom is taken as 16, and the atomic weight of an element is n if the ratio of the weight of an atom of the element to an atom of oxygen is $n : 16$. On this scale the atomic weight of hydrogen is 1·008. Other atomic weights are listed in Table I.

Molecular Weights. Using the above scale of relative weights in which the weight of an oxygen atom is 16, the relative weight of a molecule of a substance will be the sum of the atomic weights of its component atoms. There are two atoms in a molecule of oxygen, therefore the *molecular weight* of oxygen is $2 \times 16 = 32$. Again, two atoms of hydrogen (atomic weight $= 1$, approximately) combine with one atom of oxygen (atomic weight $= 16$) to form one molecule of water, which will therefore have a molecular weight of $(2 \times 1) + 16 = 18$. Note that the scale of molecular weights is based upon a weight of 16 for the atom, not the molecule, of oxygen.

Chemical Symbols. For chemical formulae and calculations the names of the elements are abbreviated to the symbols listed in Table I. In this system, due to Berzelius, each element is represented by the initial letter, sometimes with another letter, of its Latin name. Thus copper is represented by the symbol Cu, derived from the first two letters of its Latin name, cuprum. The symbol of an element has, also, a quantitative meaning, representing one atom, or one atomic weight, of the element. Thus O represents one atom, or 16 parts by weight, of oxygen. To represent more than one atom of an element combined in a molecule a small subscript figure is added; O_2 means two atoms of oxygen combined to form one molecule, O_3 represents three atoms of oxygen forming one molecule of ozone.

Chemical Formulae. The molecule of a compound is symbolized by a formula in which the symbols of its constituent elements are written down side by side with a subscript number after each symbol, indicating the number of atoms of that element present in the molecule. Thus a molecule of chalk contains one atom of calcium (Ca), one atom of carbon (C), and three atoms of oxygen (O), and will be represented by the formula $CaCO_3$. To represent more than one molecule a whole number is placed in front of the formula; two molecules of chalk being indicated by $2CaCO_3$. Sometimes certain atoms are associated in groups within the molecule and, where a number of such groups are present in the molecule of a compound, the atoms forming the group are enclosed in a bracket and the number of groups present is indicated by a small subscript number placed after the closing bracket. One molecule of slaked lime (calcium hydroxide) is composed of one atom of calcium, two of oxygen, and two of hydrogen, but the oxygen and hydrogen are combined in *hydroxyl* (OH) groups making the formula of calcium hydroxide $Ca(OH)_2$.

Certain substances, in forming crystals, require to be intimately associated with water. Each molecule of sodium carbonate, in its crystalline form, has ten molecules of such *water of crystallization* associated with it and this is indicated by the formula $Na_2CO_3 . 10H_2O$, the full stop in the formula indicating that the water is not chemically combined but only loosely associated with the sodium carbonate. This can be shown by heating the crystals and driving off the water of crystallization leaving dry, or *anhydrous*, sodium carbonate in the form of a white powder.

The molecular formula of a vapour or a gas may be taken as representing one part by volume, since by Avogadro's hypothesis, equal volumes of all gases and vapours under the same conditions of temperature and pressure contain the same number of molecules.

Chemical Equations. Chemical changes or reactions may be represented by equations in which each of the substances taking part is represented by its formula on the left-hand side and the products of the reaction are shown on the right-hand side. Such an equation not only represents the reaction, but is quantitative, showing the number of molecules of each substance involved and therefore the relative weights present. To comply with the law of indestructibility of matter the equation must balance, every atom shown on the left-hand side reappearing on the right. For example, hydrogen and oxygen combine to form water, the equation representing this being—

$$\underset{\text{2 molecules of hydrogen}}{2H_2} + \underset{\text{1 molecule of oxygen}}{O_2} = \underset{\text{2 molecules of water}}{2H_2O}$$

Although there are different numbers of molecules on each side, this equation is balanced, since there are four hydrogen and two oxygen atoms on each side.

Even if it balances an equation is not necessarily correct unless the reaction shown is possible. The equation—

$$\underset{\text{Lead}}{Pb} + \underset{\text{Tin}}{Sn} = Pb.Sn$$

is not correct because, although lead and tin may be melted together to form solder, no chemical action takes place, solder being a mixture and not a compound. Again the equation—

$$H_2 + O = H_2O$$

balances and represents an actual chemical reaction, but it is still incorrect because, under normal circumstances, oxygen atoms do not exist singly, but are combined in pairs to form an oxygen molecule O_2.

Chemical Calculations. Since chemical equations are quantitative, the relative weight of substances involved in a reaction can be found. Taking the equation $2H_2 + O_2 = 2H_2O$, the relative weight of a hydrogen molecule $H_2 = 2 \times 1 = 2$, of an oxygen molecule $O_2 = 2 \times 16 = 32$, and of a water molecule $H_2O = (2 \times 1) + 16 = 18$. Rewriting the equation to give the relative weights—

$$\begin{array}{ccccc} 2H_2 & + & O_2 & = & 2H_2O \\ 2 \times 2 & & 32 & & 2 \times 18 \end{array}$$

or, in terms of parts by weight—

4 parts of hydrogen + 32 parts of oxygen = 36 parts of water. Thus, to make 36 lb of water, 4 lb of hydrogen must combine with 32 lb of oxygen.

Example. The burning of limestone to form quicklime is represented by the equation—

$$\underset{\substack{\text{Limestone} \\ \text{(calcium carbonate)}}}{CaCO_3} = \underset{\substack{\text{Quicklime} \\ \text{(calcium oxide)}}}{CaO} + \underset{\substack{\text{Carbon} \\ \text{dioxide}}}{CO_2}$$

Find the weights of quicklime and carbon dioxide produced from 50 t of limestone.

Writing the molecular weights down for each substance—

$$\begin{array}{ccccc} CaCO_3 & = & CaO & + & CO_2 \\ (40 + 12 + 3 \times 16) & & (40 + 16) & & (12 + 2 \times 16) \\ = 100 & & = 56 & & = 44 \end{array}$$

$\therefore$ 100 parts by weight of $CaCO_3$ produce 56 parts of CaO and 44 parts of CO_2

and 50 t of limestone will produce 28 t of quicklime and 22 t of carbon dioxide.

EXERCISES

1. What is an element? Name eight important elements found in the earth's crust.

2. What is the smallest mass of a substance normally capable of independent existence and what is the smallest particle taking part in a chemical reaction?

3. Outline Dalton's atomic theory, explaining the implications of his assumptions.

4. Explain the meaning of *(a)* atomic weight, *(b)* molecular weight. Calculate the molecular weight of copper sulphate, $CuSO_4$.

5. What requirements must be satisfied before a chemical equation can be said to be correct?

6. When hydrochloric acid acts on marble, carbon dioxide is formed according to the equation—

$$CaCO_3 + 2HCl = CaCl_2 + H_2O + CO_2$$

What weight of carbon dioxide is produced if 2 lb of marble are converted to $CaCl_2$?

7. Calcium sulphate, which causes permanent hardness in water, can be removed by adding sodium carbonate (washing soda)—

$CaSO_4$	+	Na_2CO_3	=	$CaCO_3$	+	Na_2SO_4
Calcium sulphate		Sodium carbonate (washing soda)		Chalk (precipitated)		Sodium sulphate

How much soda is required to remove 10 lb of calcium sulphate?

8. Calculate the molecular weight and percentage composition of crystalline sodium carbonate if its formula is $Na_2CO_3.10H_2O$.

9. A paraffin candle weighs 100 g and is composed of 82 per cent carbon and 18 per cent hydrogen. Find the weight of the products formed when the candle is completely burnt.

CHAPTER IV

CHEMICAL REACTIONS

Combustion in Air

Air is one of the commonest and most important substances on the earth, entering into many of the natural chemical reactions which make life possible. One of these important chemical reactions is burning, or combustion.

Combustion. It is common knowledge that if the air supply to a domestic fire is restricted by closing the air inlet and damping the fire down with coal dust, combustion will be slowed down and the fire may even go out. If additional air is pumped into the fire with a bellows, as is done in a blacksmith's forge, burning will be fiercer and more rapid. These facts show that air is essential to combustion. Now air is not an element, but is a mixture of a number of gases. To find which of these gases assists combustion we may perform the following experiment.

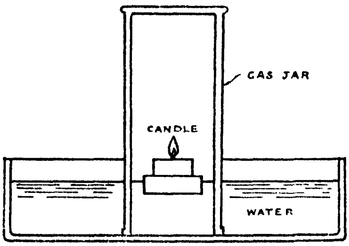


Fig. 7. Burning a Candle in Air

Fix a short candle or a nightlight on a piece of wood and float it in a trough containing water, Fig. 7. Light it and cover it with an inverted gas jar, immediately marking the water level on the side of the jar. The light will burn for a short time and then go out, and the water level will be found to have risen about one-fifth of the way up the jar. Clearly one-fifth of the air has taken part in the reaction, but the other four-fifths are incapable of supporting combustion. The active one-fifth of the air is *oxygen*, and the remaining four-fifths are composed of *nitrogen*. Thus air at the earth's surface is a mixture of oxygen and nitrogen in the proportions of one to four by volume and in addition there will usually be traces of other gases present as impurities.

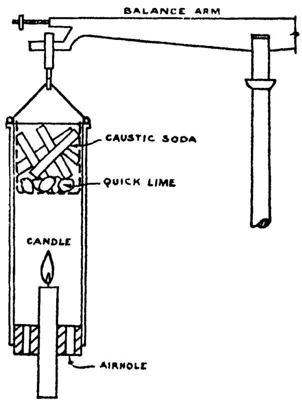


Fig. 8. Retaining the Products of Combustion of a Candle

Law of Indestructibility of Matter. During combustion, as for instance in the case of the burning of the candle in the last experiment, it may seem as if destruction of matter has occurred. The following experiment will show that this is not the case.

Fix a small candle in a cork, with holes on either side to admit air, and insert it into a glass tube, Fig. 8, on the top of which a piece of wire gauze is supported containing sticks of caustic soda and a few pieces of quicklime. Counterpoise the whole apparatus on the arm of a balance and light the candle. In a short time the balance arm carrying the candle will sink, showing that the weight of the products of combustion, absorbed by the soda and lime, is greater than the loss of weight of the candle. This increase in weight is due to oxygen being taken from the air during the process of combustion.

To test the actual changes of weight caused by a chemical change the reaction must be carried out in a closed space, so that none of the materials taking part, nor any of the products of the reaction, can escape.

Put a small piece of white phosphorus in a round-bottomed flask, tightly closed with a rubber stopper, and weigh the whole. Warm the flask gently until the phosphorus ignites and, when combustion has ceased, allow the flask to cool and then reweigh it. The weight will be unchanged and the experiment shows that, although a very evident chemical reaction has occurred, the total weight of matter in the flask has remained constant.

From experiments such as these a general conclusion has been drawn which is known as the *law of indestructibility of matter*. It states that "*in a chemical reaction, matter cannot be produced or destroyed, but only changed from one form to another.*"

Fire-resisting Construction. Since air is essential to combustion, outbreaks of fire can, to a certain extent, be controlled by preventing the free access of air to the section of the building which is burning. This may be done by dividing the premises up into compartments separated from each other by self-closing, fire-resisting doors, and, where possible, using fire-resisting glazing such as wired glass in metal frames, which will not shatter under heat and so will not admit air from outside. Staircases and lift-shafts should be separated from the rest of the building by doors because they provide an easy path for fire to spread from floor to floor and, acting as chimneys, set up a draught which intensifies the fire. Suitable building materials should be used in construction, and in this respect fire-resisting materials are not necessarily those which are incombustible. Timber, although itself inflammable, may have a high degree of fire resistance if it is of sufficient thickness and has no sharp corners exposed to the flames. Large-section wooden beams will char on the surface, but this charring prevents the free access of air to the wood beneath and either reduces or prevents further combustion. On the other hand steelwork, which is incombustible, requires careful protection from fire by encasement with concrete or other insulating materials because the large expansions which can occur when steel is heated may cause the steelwork to buckle or the joints to fail, resulting in a collapse of the structure.

Oxidation and Oxides

Oxidation. In combustion, substances combine with oxygen rapidly with the production of considerable heat and light, but for many materials the reaction occurs more slowly, without burning and the production of flame. The rusting of iron is an example of this slower process of *oxidation* and the following experiment will serve to show its similarity (on a less violent scale) to the process of combustion.

Wet the inside of a large test-tube and pour some iron filings into it, so that they adhere to its sides. Support the tube upside down in a beaker of water and mark the level of the water inside the tube. After leaving the apparatus for a week, the iron will be found to have rusted and the water level will have risen in the tube. Mark the new level on the tube. It will be found that about one-fifth of the air has been used up. Close the mouth of the tube with the thumb and remove it from the water. Test the residual air by introducing a lighted taper into the tube; the taper will be extinguished, showing that rusting of the iron has removed the oxygen from the air.

Oxides. Oxygen combines readily with many substances to form *oxides*, which are compounds of single elements with oxygen. Several oxides may be formed by the same element according to the conditions under which the reaction occurs. To distinguish between them the oxide containing less oxygen is given the termination *-ous* and that containing more oxygen *-ic*. Thus the oxides of iron are *ferrous oxide* (FeO), *ferric oxide* (Fe_2O_3), which is used as a pigment (*Venetian red*) and for polishing (*jewellers' rouge*), and *tri-ferric tetroxide* (Fe_3O_4) often called magnetic oxide. Another method used for distinguishing different oxides of the same element is to add a prefix to the word oxide showing the number of oxygen atoms present. Thus sulphur forms sulphur dioxide (SO_2) and sulphur trioxide (SO_3).

Alkalis, Acids and Salts

Basic Oxides, Bases, and Alkalis. The oxides of metals are known as *basic oxides* and when combined with water they form *bases* or *hydroxides*.—

$$\underset{\text{Sodium oxide}}{Na_2O} + \underset{\text{Water}}{H_2O} = \underset{\substack{\text{Sodium hydroxide}\\\text{(caustic soda)}}}{2Na(OH)}$$

Hydroxides of sodium, potassium, and other metals are sometimes called *alkalis*. One of the alkalis must be mentioned specially because it differs from the others in not being produced from a metal. *Ammonia* (NH_3) is formed from nitrogen and hydrogen, but it behaves as a base and reacts with acids to form *salts*, which all contain the ammonium group or radical (NH_4).

Properties of Alkalis. The presence of an alkali is easily detected because it will turn red litmus blue. Alkaline solutions feel soapy when rubbed between the fingers, the effect being more pronounced in the case of the stronger alkalis. The strength of an alkali refers to its properties and the readiness with which it takes part in a reaction rather than to its dilution with water. Ammonia is a comparatively weak alkali, but caustic soda is very strong and can cause severe burns to the skin by its corrosive action on the tissues. An important characteristic property is that all alkalis react with acids to form chemical salts, water also being formed during the process.

Acidic Oxides and Acids. When non-metals are oxidized they form *acidic oxides* or acid anhydrides, which combine with water to form acids. Here again the suffixes *-ous* and *-ic* are used to differentiate between acids formed from the same non-metal, but which contain less or more oxygen. Thus sulphur forms two oxides—

$$\underset{\text{Sulphur}}{S} + \underset{\text{Oxygen}}{O_2} = \underset{\text{Sulphurous anhydride}}{SO_2}$$

$$\underset{\text{Sulphur}}{2S} + \underset{\text{Oxygen}}{3\,O_2} = \underset{\text{Sulphuric anhydride}}{2SO_3}$$

and these will yield two acids—

$$\underset{\text{Sulphurous anhydride}}{SO_2} + \underset{\text{Water}}{H_2O} = \underset{\text{Sulphurous acid}}{H_2SO_3}$$

$$\underset{\text{Sulphuric anhydride}}{SO_3} + \underset{\text{Water}}{H_2O} = \underset{\text{Sulphuric acid}}{H_2SO_4}$$

All acids contain hydrogen and nearly all contain oxygen, but one important exception is hydrochloric acid (HCl) which is composed of hydrogen and chlorine only.

Properties of Acids. Acids have a sharp, sour taste, and should *not* be tasted indiscriminately since the stronger acids, such as sulphuric acid, nitric acid, and hydrochloric acid, are corrosive and will "burn" both skin and tissues. The weaker acids are responsible for the sharpness of fruit, citric acid being present in lemon juice and another weak acid, acetic acid, giving the sour taste to vinegar. The terms "weak" and "strong" applied to acids refer to their properties, a strong acid being more active than a weak one, and this classification is independent of the amount of water with which the acid may be diluted. Acid solutions containing little water are called *concentrated* while those containing a large proportion of water are said to be *dilute*.

All acids, whether weak or strong, concentrated or dilute, will turn blue litmus red. With alkalis they react to give salts, water being formed in the process.

Salts. When acids and bases react together a salt is formed, water also being produced, e.g.—

$2NaOH$	+	H_2SO_4	=	Na_2SO_4	+	$2H_2O$
Sodium hydroxide		Sulphurid acid		Sodium sulphate		Water

It is impossible to lay down any general properties for such salts. Some are soluble, others are insoluble, their colours vary and their tastes differ, but one classification may be made according to the acids from which they are formed. Just as the acids containing less or more oxygen were distinguished by the terminations *-ous* and *-ic*, so the corresponding salts of these acids are given the suffixes *-ite* and *-ate*. Sulphur forms both sulphurous and sulphuric acids and the corresponding salts which would be formed with sodium hydroxide are sodium sulphite and sodium sulphate.

The process of formation of a salt consists of the replacement of the hydrogen in the acid by a metal or radical. This replacement may be observed experimentally, as certain salts can be formed directly by the action of the acid on a metal, free hydrogen being given off in the process, e.g.—

Zn	+	H_2SO_4	=	$ZnSO_4$	+	H_2
Zinc		Sulphuric acid		Zinc sulphate		Hydrogen

When an acid contains only one atom of hydrogen in its molecule it can form only one class of salt and the acid is said to be *mono-basic*. Hydrochloric acid is an example, forming only one set of salts, the chlorides, in which all the hydrogen is replaced by a metal, e.g.—

$NaOH$	+	HCl	=	$NaCl$	+	H_2O
Sodium hydroxide		Hydrochloric acid		Sodium chloride		Water

If an acid has in its molecule two atoms of hydrogen which can be replaced, it is *di-basic* and can form two classes of salts. Either all the hydrogen may be replaced to form a *normal salt* or only one atom may be replaced giving an *acid salt*, e.g.—

$2NaOH$	+	H_2CO_3	=	Na_2CO_3	+	$2H_2O$
Sodium hydroxide		Carbonic acid		Sodium carbonate (normal salt)'		Water
$NaOH$	+	H_2CO_3	=	$NaHCO_3$	+	H_2O
Sodium hydroxide		Carbonic acid		Sodium bicarbonate (acid salt)		Water

Sometimes the acid salt is known as sodium *hydrogen* carbonate.

EXERCISES

1. Are the products of combustion, resulting from the burning of a candle in air, lighter or heavier than the original candle? How can your answer be reconciled with the law of indestructibility of matter?

2. How can a knowledge of the process of combustion be applied to the construction of fire-resisting buildings?

3. Compare the processes of combustion and slow oxidation, giving illustrations.

4. Define the terms: basic oxide, base, alkali. Describe the properties of alkalis.

5. Explain the use of the terms: weak, strong, dilute and concentrated, in connection with acids and their solutions.

6. How are acids formed and what are their general properties?

7. What are salts? Give three examples of these substances.

8. How may salts be classified? How does an acid salt differ from a normal salt?

CHAPTER V

PHYSICAL PROPERTIES OF MATTER

THAT which occupies space is *matter*, and the quantity of space occupied is the *volume* of the matter present. The unit of measurement in the British system of units is the cubic foot or, for fluids, the gallon, which is defined as the volume of 10 lb of water weighed in air at 60° F. There are approximately 6¼ gal to the cubic foot. In the Metric system the unit of volume is the cubic metre or, for fluids, the litre.

The *weight* of any volume of matter is the force, due to gravitation, with which the earth attracts that body towards its centre. The *mass* of any volume of a substance is the amount of matter in that volume and is, therefore, proportional to the number of molecules in the body. At any given distance from the centre of the earth, the weight of a body is also proportional to the number of molecules, so that, at any one place, the weight of a body is proportional to its mass. On the other hand, the gravitational force on, and therefore the weight of, a body is inversely proportional to the square of its distance from the centre of the earth, so that as a body is lifted from the surface of the earth its *weight decreases*, but its *mass remains constant*. This could be verified by weighing a bag of nails on a spring balance in an aeroplane just before taking off and again at a height of, say, 10,000 ft. The second weight would be found to be less than the first, although the bag would still contain the same number of nails and therefore its mass would be unaltered. In performing this experiment a pair of scales could not be used since the gravitational pull would diminish equally on the bag and the weights and the bag of nails would therefore appear to weigh the same as before.

Weight and mass are thus essentially different. Mass is the amount of matter in a body; weight is the force exerted by the gravitational pull of the earth. The units employed to measure mass are the *pound* in the British system and the *gramme* in the metric system. In scientific work the corresponding units of force are the *poundal* and the *dyne*. The poundal is a very small unit and in commercial and building practice the pound is also employed as the unit of force.

Density. To compare the relative lightness or heaviness of different materials, or of the same material under different conditions, it is necessary to determine the weights of equal volumes of the specimens. This is most conveniently done by measuring the weight per unit volume, or *density* of each specimen. Thus—

$$\text{Density} = \frac{\text{weight}}{\text{volume}}$$

The units adopted in building practice will be lb/ft^3, the corresponding metric units being g/cm^3.

Specific Gravity. Since the units in which density is stated vary, it is sometimes more convenient to know the ratio of the density of a material to that of a standard substance. The standard substance used throughout the world is pure water and the relative density or *specific gravity* of any substance is the ratio of the density of that substance to the density of pure water at 4° C. This value is just a ratio and is independent of the system of units used. The specific gravity of pure water is unity. If the specific gravity of a substance is known, then the density of that substance in any required system of units is equal to the product of its specific gravity and the density of water in the required units.

EXAMPLE. The specific gravity of limestone is 2·75, find its density in pounds per ft^3 and in grammes per cm^3, if the density of water is 62·4 lb/ft^3 in British units or 1 g/cm^3 in metric units.

$$\begin{aligned} \text{Density} &= \text{specific gravity} \times \text{density of water} \\ &= 2{\cdot}75 \times 62{\cdot}4 = 172 \text{ lb/ft}^3 \\ \text{or} \quad &= 2{\cdot}75 \times 1 \quad = 2{\cdot}75 \text{ g/cm}^3 \end{aligned}$$

Relative densities can be used conveniently to compare the relative weights of fitments of the same type made from different materials. For example the specific gravities of aluminium alloy and steel are 3·4 and 7·8 respectively, so that the relative density of alloy to steel is $\frac{3{\cdot}4}{7{\cdot}8} = 0{\cdot}44$. Hence the same fitting made in aluminium alloy would have only 0·44 of the weight it would have if made of steel. The specific gravities of common materials are given in Table II.

Relative Densities of Gases. Relative densities with respect to air or hydrogen are used in work with gases. In studying certain questions of ventilation it is useful to know that the relative density of carbon dioxide to air is 1·53 and that of water vapour is 0·62.

Applications to Building. The following examples show the importance of the determination of densities and specific gravities in building work—

1. The determination of the weights of structures such as walls or floors.
2. The determination of the strength of solutions, such as the magnesium chloride solution used in magnesium oxychloride cement (magnesite). The success of this material depends on careful control of the specific gravity of the solution.
3. The determination of the porosity of building materials.
4. The testing of materials for purity. If the density found experimentally does not agree with the known value for the pure substance then adulteration is suspected. If the impurity is known and its density determined, then the amount present in the material can be calculated.

TABLE II

SPECIFIC GRAVITIES AND DENSITIES OF COMMON MATERIALS

Material	Specific Gravity and Density g/cm³	Density lb/ft³
Metals—		
Aluminium	2·7	169
Copper	8·9	556
Lead	11·3	707
Steel	7·8	487
Tin	7·3	456
Zinc	7·2	449
Stones—		
Granite	2·5	156
Limestone	2·0–2·7	125–170
Marble	2·7	170
Sandstone	2·2–2·4	140–150
Woods—		
Beech	0·74	46
Fir	0·48–0·61	30–38
Oak	0·74–0·96	46–60
Pine	0·43–0·54	27–34
Miscellaneous—		
Asphalt	1·4	88
Brick, common	2·0	125
pressed	2·4	150
soft	1·6	100
Brickwork	1·8	115
Cement	1·4	90
Chalk	1·8–2·6	112–162
Clay	2·2	135
Concrete, coke breeze	1·4	90
plain	2·2	140
reinforced	2·4	150
Cork	0·26	16
Glass	2·6	160
Gravel	1·7	110
Ice	0·92	57
Linseed oil	0·94	58·6
Sand, dry	1·6	100
Turpentine	0·87	54·3
Water, fresh	1·00	62·4
sea	1·025	64·0
White lead	3·1	197

DETERMINATION OF DENSITIES AND SPECIFIC GRAVITIES

(1) **By Direct Measurement.** The weight of a specimen can always be determined by direct measurement to a reasonable degree of accuracy, and if the volume of the specimen can be found then the density is given by—

$$\text{Density} = \frac{\text{weight}}{\text{volume}}$$

For regular shaped solids, volume can be determined by measurement and calculation. If the solid is irregular, its volume may be found by measuring the volume of water displaced when it is totally immersed. This may be done in a displacement can, Fig. 9, which is filled with water up to the level of the overflow. A measuring cylinder is placed under the outlet and the specimen is then carefully lowered into the can, the displaced water being collected in the measuring cylinder and its volume read off directly or determined by weighing and dividing the weight of water collected by the density of water.

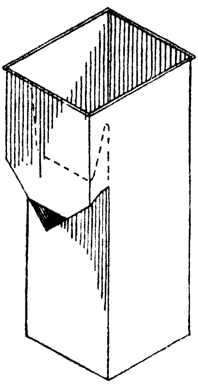


FIG. 9. DISPLACEMENT CAN FOR MEASURING THE VOLUME OF SOLIDS

(2) **By Applying Archimedes's Principle.** When a body is suspended in a liquid an upward thrust is exerted upon it by the surrounding liquid. According to Archimedes, this *upthrust* is equal to the weight of liquid displaced by the body, that is to say, the weight of an equal volume of liquid if the body is completely immersed. A body weighed when immersed in a liquid will thus suffer an apparent loss of weight.

If w_1 = weight of body weighed in air

w_2 = weight of body weighed when totally immersed in water

Then, $w_1 - w_2$ = weight of water displaced

= weight of an equal volume of water

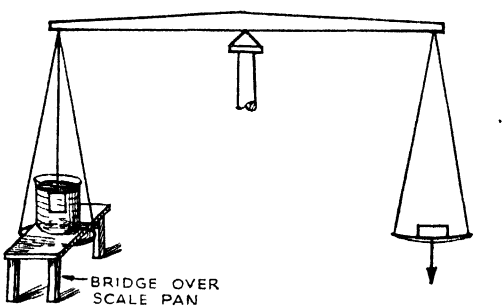


FIG. 10. MEASUREMENT OF SPECIFIC GRAVITY BY THE PRINCIPLE OF ARCHIMEDES

$$\text{But, Specific gravity} = \frac{\text{weight of body}}{\text{weight of an equal volume of water}}$$

$$= \frac{w_1}{w_1 - w_2}$$

In order to use an ordinary balance to measure w_2 a wooden bridge is fitted over the left-hand scale pan, as shown in Fig. 10. A beaker full of water is placed on the bridge and the body is suspended from the hook of the balance.

For materials lighter than water a sinker is used to immerse the specimen completely, and the results must be corrected for the weight of water displaced by the sinker. For materials soluble in water, other liquids of known specific gravity may be used, thus

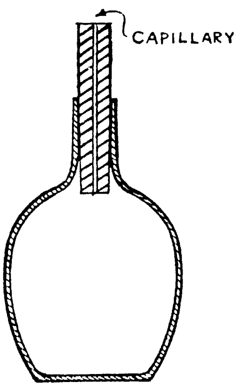


FIG. 11. SPECIFIC GRAVITY BOTTLE

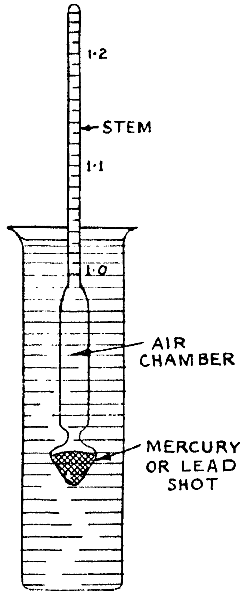


FIG. 12. FLOAT HYDROMETER

enabling the weight of liquid displaced to be converted to the weight of an equal volume of water.

(3) **By Specific Gravity Bottle.** One of the most widely used methods of determining the density of a liquid is the *specific gravity bottle* illustrated in Fig. 11. It is fitted with a carefully ground glass stopper through which passes a fine bore. To find the specific gravity of a liquid, the clean dry bottle, with stopper, is weighed. It is then filled with the given liquid and the stopper inserted, allowing any surplus liquid to be expelled through the capillary. The outside of the bottle is carefully dried and the whole is reweighed. After all traces of the liquid have been thoroughly washed out, the last step is repeated using water instead of the liquid. By subtracting the weight of the empty bottle and stopper,

the weights of equal volumes of liquid and water can be found and the specific gravity of the liquid is then obtained by dividing the weight of the liquid by the weight of the equal volume of water. The accuracy of this method, even for a fairly viscous liquid, depends upon that of the balance, and should therefore be of a high order.

(4) **By Hydrometer.** In industry, the usual method of determining the specific gravity of a liquid is to employ a *hydrometer*, which is simply a graduated float. A common hydrometer is illustrated in Fig. 12, but in many manufacturing processes special modifications, especially methods of graduation, have been introduced. The hydrometer used by milk inspectors is called a *lactometer* and has the density of milk containing the minimum legal amount of cream very prominently marked on its scale. Similarly, the brewer still uses the *Twaddell scale*, as originally used by the inventor of the hydrometer, to check the alcoholic content of the liquor.

The instrument consists of three parts: a stem (usually graduated) a floating chamber, and a loaded keel. The purpose of the latter is not only to keep the instrument vertical, but to control its weight, which can be done by varying the amount of mercury or small lead shot used. Applying the principle of Archimedes, there will be an upthrust U on the instrument which will be equal to the weight of liquid which it displaces, but since the instrument floats in the liquid—

$$\text{Weight of instrument } w = \text{upthrust } U$$

In all forms of common hydrometer, w is constant and hence the value of U is also constant.

$$\text{Now, Density} = \frac{\text{weight}}{\text{volume}}$$

$$\text{Or,} \quad \text{Volume of liquid displaced} = \frac{\text{weight of liquid}}{\text{density of liquid}}$$

As the weight of liquid displaced is equal to U and therefore constant, the volume of the displaced liquid must vary inversely as its density, so that the instrument will displace a smaller volume of a heavier liquid than of a less dense one. The distance to which the instrument sinks into the liquid is therefore an indication of the density of the liquid and the stem may be calibrated so that the density may be read off a scale against the level of the surface of the liquid. Common hydrometers used for general work in laboratories are arranged in nests of a number of instruments covering the full range of specific gravities over which this type of instrument will give accurate results. After taking a reading it is advisable to tap the hydrometer further into the liquid and let it come to rest again so that the first reading may be checked. This is particularly important with more viscous liquids, since the surface tension of the liquid may be great enough to "hold" the stem and cause the instrument to come to rest over a range of positions. For such liquids the specific gravity bottle is a more satisfactory instrument.

Surface Tension

Molecular Forces at the Surface of a Liquid. The force of attraction between two molecules varies inversely as the square of their distance apart and therefore decreases rapidly as the distance between them increases, becoming negligible when this distance exceeds a certain value. This limit is reached in the case of gases. The molecules of a liquid are close together and attract each other with equal forces in all directions so that the resultant force on a molecule within the liquid is zero. In the case of a molecule on the surface of a liquid, there will be a force of attraction pulling it into the liquid, but no balancing force pulling it out of the surface. The unbalanced resultant of such forces produces in the surface a tendency to become as small as possible. It behaves as though there were an elastic skin separating the liquid from the air, the force exerted in this surface skin being known as the *surface tension.*

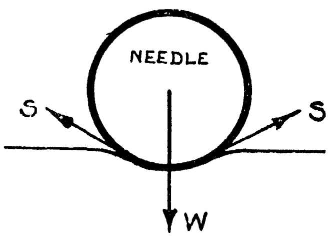


Fig. 13. Needle Floating, Supported by Surface Tension

The realism of this analogy of an elastic skin can be illustrated by an experiment in which a steel needle is "floated" on the surface of water. Put the needle on a piece of blotting paper and place the paper on the surface of some water contained in a beaker. The paper quickly absorbs water and sinks, leaving the steel supported by the "skin" or surface tension of the water. As the specific gravity of the steel is 7·8 its weight is very much greater than the upward thrust of the water on it, so that the needle is kept afloat by the pull exerted by the surface tension, as shown in Fig. 13. If the surface of the water is even momentarily broken by a slight disturbance the surface tension will no longer balance the other forces and the needle will sink immediately.

A further example of the action of surface tension is that a cup of tea may be overfilled to the extent of about one-eighth of an inch without its spilling over.

Meniscus. Difficulty is found in reading liquid gauges of all kinds because of the curved surface of the liquid in the gauge glass. With most liquids this forms a concave *meniscus*, but with a few heavy liquids, notably mercury, the meniscus is convex. The correct gauge reading in these cases is taken as shown in Figs. 14 and 15.

The concave meniscus is formed when the glass of the containing vessel attracts the molecules at the surface of the liquid with a greater force than the latter are attracted by the liquid molecules beneath them. The convex meniscus is formed when the molecular attraction of the liquid on the molecules at its surface is greater than the attraction of the walls of the vessel. This explains why mercury cannot "wet" the walls of a glass vessel while water will do so unless the surface has been greased to decrease its attraction.

Capillary Action. Fig. 14 also illustrates that when a fine bore tube (called a *capillary tube*) is dipped into water, the surface tension pulls the water up the tube beyond the level of that in the containing vessel. The water wets the inner surface of the tube and there will be an upward pull due to surface tension which draws the water up the tube until the weight of the column of liquid above the

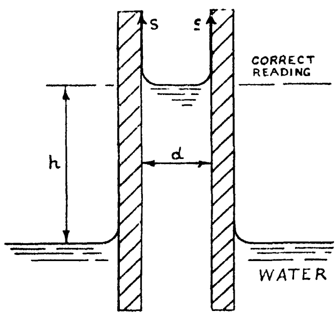


FIG. 14. CAPILLARY ACTION

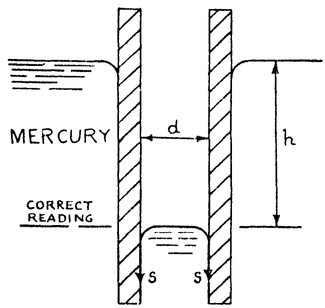


FIG. 15. DEPRESSION OF MERCURY IN A CAPILLARY TUBE

general surface level outside counterbalances the surface tension. If d is the diameter of the bore and h the height of the column supported, ρ the density of the liquid and γ the surface tension, then—

Weight of column of liquid = upward pull due to surface tension

$$\text{Volume of column} \times \text{density of liquid} = \gamma \times \text{circumference of tube}$$

Therefore $$\frac{\pi d^2}{4} \times h \times \rho = \gamma \times \pi d.$$

Therefore $$h = \frac{4\pi d\gamma}{\pi d^2 \rho} = \frac{4\gamma}{\rho d}$$

Hence the smaller the diameter of the capillary the higher the liquid will rise. This upward movement against gravity is called *capillarity*, and is of very great importance to the builder in his use of porous materials and when fine channels are formed in overlapping or butting materials.

Fig. 15 shows the opposite effect of mercury in a capillary tube where the attraction of glass for mercury is less than that of mercury for itself, so that the surface tension pushes down the level of the mercury in the tube below that in the vessel outside.

Capillary action in building materials can be demonstrated by means of the apparatus shown in Fig. 16. The specimen is supported on a glass cup *A* which can be filled with water from a reservoir *R* through a tee-piece *T*. The other arm of the tee-piece is connected to a horizontal glass capillary tube *C* level with the top of the cup. The tap *S* is opened to allow water to fill cup *A* and capillary tube *C* completely and is then closed. Water passes by capillary action

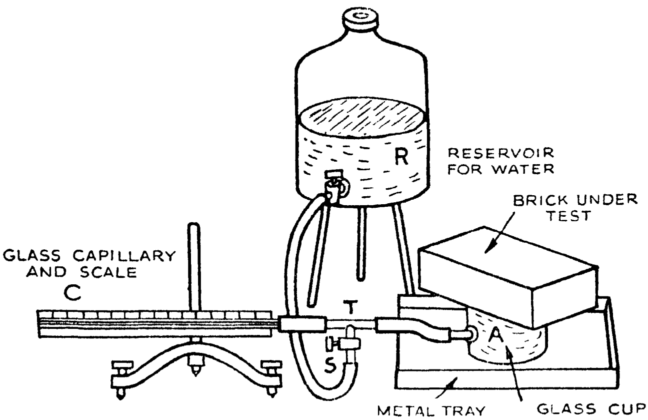


FIG. 16. CAPILLARITY IN BUILDING MATERIALS

into the specimen from the cup *A* and is replaced by water drawn from the capillary tube, so that the thread of water in the capillary tube is seen to move slowly along the tube.

LABORATORY WORK

EXPERIMENT 1. *Determination of density by direct measurement*

Determine the density of a specimen of stone or concrete by direct measurement, using a displacement can to determine the volume as explained on page 26.

EXPERIMENT 2. *Determination of specific gravity by applying the principle of Archimedes*

Using the same specimen as in Expt. 1 determino its specific gravity by the method explained on page 26. Taking the density of water as 62·4 lb/ft^3 compare your result with that obtained in the previous experiment.

EXPERIMENT 3. *Determination of specific gravity by specific gravity bottle*

Use a specific gravity bottle to determine the specific gravity of paraffin, carrying out the experiment as described on page 27.

EXPERIMENT 4. *Use of the float hydrometer*

Check the value obtained for the specific gravity of paraffin in Expt. 3 by using a suitable form of float hydrometer.

EXERCISES

1. Estimate the weight of a 12 in. × 12 in. pine timber 16 ft long, if its specific gravity is 0·56.

2. Calculate the specific gravity of powdered brick from the following results—

Weight of specific gravity bottle	= 30·60 g
Weight of bottle + powder	= 54·77 g
Weight of bottle + powder + water	= 102·17 g
Weight of bottle filled with water only	= 88·49 g

3. Applying Archimedes' Principle, find the volume and density of a piece of the same brick as was used in Question 2, given after saturating it with water the following results—

Weight of saturated piece of brick in air	= 34·35 g
Weight of saturated piece of brick in water	= 19·95 g

4. Explain how to make up a solution of magnesium chloride from crystals of the salt so that its strength is such that its specific gravity is 1·54.

5. What is surface tension? Explain how it affects the readings of liquid gauges.

6. Explain the action of a hydrometer. On what principle is it based?

7. What is capillarity and what is capillary action? Explain how it arises.

8. Give examples of precautions taken in building construction to prevent the passage of water by capillary action.

CHAPTER VI

POROUS MATERIALS

MANY building materials absorb water or water vapour, while others are impervious, and it is necessary for the builder to have a knowledge of the mechanism and extent of this absorption. If dry specimens of the first group, such as timber, brick, concrete, mortar, and some natural stones, are examined under a microscope, the solid material will be seen to be permeated by small pockets of air. In timber these pores are geometrical in shape and are more

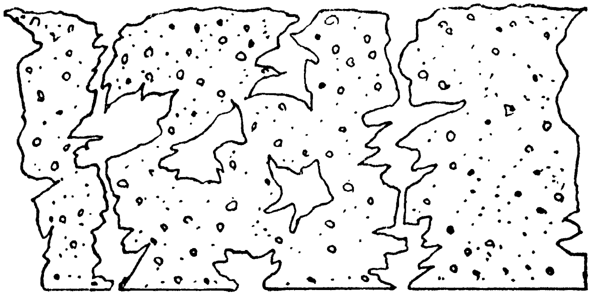


FIG. 17. POROUS STRUCTURE OF BRICK

or less evenly arranged, but in brick and concrete they are very irregular in size and shape. Some are linked together by narrow channels, or *capillaries*, which continue right through the material, while others form isolated pockets surrounded by the solid. If the solid is itself impervious, water cannot enter these latter cavities, but will only penetrate through the capillaries to fill some of the pockets. Fig, 17 represents the cross-section of a specimen of brick in which both types of porosity—linked cavities and isolated pockets—exist.

Flow of water through the material may be caused by—

1. Force of gravity
2. Capillary action
3. Pressure—

(*a*) Due to driving rain

(*b*) Due to a head of water

(*c*) Due to thermal expansion of water already absorbed at a lower temperature.

It is very important that in building science a clear distinction is drawn between porosity or the total pore space in the material, water absorption, permeability, and capillarity.

Porosity. The porosity of a material is the ratio between the total volume of pore space in a given specimen of the material and the actual volume of the specimen. It is usually expressed as a percentage.

Method of Measuring Porosity. To determine the porosity of a London Stock brick, for example, select a typical specimen and break off two lumps each about the size of a brazil nut. Powder one very finely with a pestle and mortar and dry the other in an oven. To ensure that all moisture is expelled, the latter specimen should first be weighed and then reweighed after drying and the heating and reweighing continued until the weight remains constant. This final weight is recorded. The dried specimen is now placed in water and thoroughly saturated, a process which is assisted by boiling it in water for about half an hour and then cooling it quickly. It is then reweighed totally immersed in water and weighed again in a saturated condition in air. From these readings the apparent loss of weight in water can be determined and the volume of water displaced calculated, using the principle of Archimedes. Let—

Weight of dried lump of brick		= 13·69 g
Weight of saturated lump in air	$= a$	= 14·50 g
Weight of saturated lump in water	$= b$	= 7·23 g
Weight of water displaced	$= a - b$	= 7·27 g
Volume of water displaced		= 7·27 cm^3
Volume of lump of brick		= 7·27 cm^3

The specific gravity or density of the powdered brick is determined by using the specific gravity bottle which is first weighed clean and dry and then about one-third filled with the powder and weighed again. The bottle is then carefully filled with water so that no powder is washed out and shaken to expel the air from between the particles. The air bubbles collect as a foam below the stopper and can be removed by filling up with water and pushing out the surplus by replacing the stopper. The bottle, together with the powder and water, is now weighed and, finally, the powder is thoroughly washed out and the bottle refilled with water and weighed again.

Weight of specific gravity bottle		$= a$	= 30·57 g
Weight of bottle plus powder		$= b$	= 38·74 g
Weight of powder	$= b - a$	$= c$	= 8·17 g
Weight of bottle plus powder and water		$= d$	= 93·56 g
Weight of water to fill bottle containing powder	$= d - b$	$= e$	= 54·82 g
Weight of bottle plus water		$= f$	= 88·52 g
Weight of water to fill empty bottle	$= f - a$	$= g$	= 57·95 g
Weight of water equal in volume to powder	$= g - e$	$= h$	= 3·13 g
Density of powder		$= \frac{c}{h}$	**= 2·61 g/cm³**

The porosity can now be determined from the measurements made on the first lump of brick—

$$\text{Volume of solid matter in first lump, excluding pore space} = \frac{\text{weight of dried lump}}{\text{density of powder}}$$

$$= \frac{13{\cdot}69}{2{\cdot}61}\ \text{cm}^3 = 5{\cdot}25\ \text{cm}^3$$

$$\text{Volume of pore space in first lump} = \begin{Bmatrix}\text{volume of}\\ \text{whole}\\ \text{lump}\end{Bmatrix} - \begin{Bmatrix}\text{volume of}\\ \text{solid}\\ \text{matter}\\ \text{only}\end{Bmatrix}$$

$$= 7{\cdot}27 - 5{\cdot}25 = 2{\cdot}02\ \text{cm}^3$$

$$\text{Percentage porosity of sample} = \frac{\text{volume of pore space}}{\text{volume of solid matter}} \times 100$$

$$= \frac{2{\cdot}02}{7{\cdot}27} \times 100 = 27{\cdot}8\%$$

Porosity in Building Materials. The thermal conductivity of any material is largely dependent on its percentage porosity so that if it is sufficiently strong a very porous material is preferred because it is a better heat insulator. Where there is a possibility of condensation of water vapour, a porous material will absorb the moisture rapidly so that water will not form drops and run on the surface. There are, however, other considerations which have to be taken into account in the choice of a material as, apart from the question of cost, the circumstances under which the material is to be used decide whether certain properties are advantageous or otherwise.

Measurement of Water Absorption. There is no simple relationship between porosity and water absorption, for we have already seen that there may be some pores which cannot absorb water. To measure the water absorption of a material, a sample should be taken and dried until its weight remains constant. Now boil the sample in water for about an hour and then place it immediately in cold water. Wipe the specimen and reweigh it. The increase in weight must be the weight of water absorbed and can be expressed as a percentage of the weight when dry, this percentage indicating the water absorption of the material.

Water Absorption and Weathering. Research carried out at the Building Research Station shows that when water first comes into contact with a dry porous material, it is very rapidly absorbed by capillary action. The rate of absorption decreases considerably, until after two or three hours the material is saturated. There is then a fairly constant flow of water through the material. The rapid absorption by the dry material is referred to as *suction*.

Rain falling upon the dry surface of clay roof tiles or brick walling is absorbed by suction into the pores of the material, but is quickly removed again by evaporation as soon as the humidity (see page 147)

of the air decreases. The material will act efficiently as a barrier against moisture as long as it does not become saturated. Thus, in spite of their porous nature, materials with a high suction will weather well against frost and rain, since the water will not be retained for long in the pores. The actual amount of water absorbed is apparently of minor importance. Where suction is slow, evaporation is slow and, if frost should follow rain, there is a greater danger that disintegration, flaking, or splitting will take place when the water expands on freezing.

Permeability. The permeability of a substance is a measure of the rate of flow of water through the saturated material and depends more upon the pore structure than upon the pore space, see Fig. 17. Thus sandy, wire-cut bricks and Fleitons have fairly high porosities and permeabilities but London Stocks, which have about the same porosity as wire cuts, are more than twice as permeable. Engineering bricks have both low porosity and low permeability, but hard yellow bricks also have a low permeability although they have a porosity of about 40 per cent.

Measurement of Permeability. A standard test for the measurement of the permeability of clay or marl plain roofing tiles (see B.S.S. No. 402—1930) and of concrete plain roofing tiles (see B.S.S. No. 473—1932) is recommended by the British Standards Institution, and it can be adapted for use with other materials. The standard test is as follows—

"From every batch of 10,000 tiles (or part thereof) three tiles shall be tested in the following manner: Each tile to be tested shall be dried at a temperature of approximately 194° F. (90° C.) to 212° F. (100° C.) to constant weight, then waxed on to a special metal cover with Faraday wax, as shown in Fig. 18. The upper surface and the sides of the tiles shall be coated with wax as far as the four sides of the metal cover, to which an air cock is fitted. Tubing connecting the interior of the metal cover to a head of water, *R*, and to the calibrated capillary tube, *C*, is also attached. The rate of flow into the specimen at any time after the commencement of the test is deduced from the rate at which the water travels along the calibrated capillary tube, *C*. Whilst this rate of flow is being observed with a stop-watch the tap, *T*, is closed. In the interval between readings this tap is opened and water supplied to the specimen from the reservoir, *R*.

"The rate of flow through the specimen at the end of 24 hr shall not exceed that indicated by a rate of flow of 4 in. per minute along a capillary tube of 1 mm bore under a head of 20 cm (approx. 8 in.). The average of three tiles tested shall be taken. Should this test fail a further three tiles shall be selected and the test repeated. Should the second test fail, then the batch of tiles may be rejected."

If it is desired only to obtain relative values of the permeabilities of different materials, the above specification need not be rigidly adhered to. The authors have found that, for tests on cement mortar specimens of very low permeability, a pressure greater than that due to 20 cm of water was desirable. The standard value of 20 cm was chosen in the standard test because it is approximately equal to the pressure set up during a heavy storm.

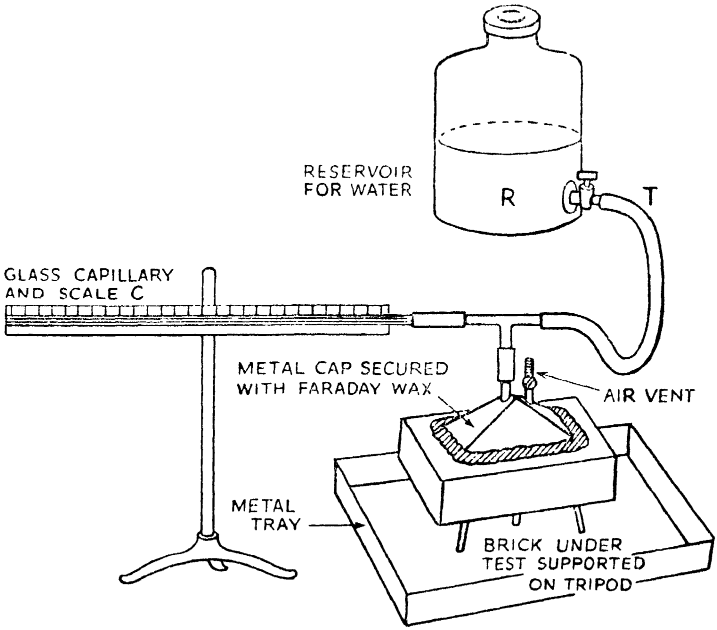


FIG. 18. APPARATUS FOR PERMEABILITY TEST

Permeability of Walls. The permeability of a wall is not necessarily the same as that of the bricks and mortar from which it is built. Even a rich cement mortar may allow considerable passage of water through capillaries due to lack of adhesion between the mortar and the brick or stone. The Building Research Station recommends that the properties of the mortar should correspond to those of the units forming the wall, and that, for all except the most dense and impervious materials, a cement–lime mortar not richer than 1 : 1 : 6 (cement : lime : sand) is to be preferred.

LABORATORY WORK

EXPERIMENT 1. *Measurement of porosity of a Fletton brick*

Select a specimen of Fletton brick and determine its porosity, using the method described on page 34.

EXPERIMENT 2. *Determination of water absorption*

Take a specimen of the same brick and find its percentage water absorption as described on page 35.

EXPERIMENT 3. *Measurement of permeability*

Carry out the standard test described on page 36 to determine the permeability of a plain clay roofing tile.

EXERCISES

1. Distinguish clearly between porosity, water absorption, permeability and capillarity.
2. What is the importance of porosity in building materials?
3. What is meant by "suction"? How does it affect weathering and waterproofness of materials?
4. What is permeability? How is it related to porosity?
5. Why is the permeability of a brick wall not necessarily the same as that of the bricks and mortar of which it is composed?
6. Give an example from normal building construction of the precautions which must be taken against capillarity.

SECTION II—STRESS ANALYSIS AND DESIGN OF STRUCTURES

CHAPTER VII

CONCURRENT AND NON-CONCURRENT FORCES

One of the most important preliminaries in building is the estimation of the strength required of the various parts of a structure and the *direction* in which that strength is required.

Forces

Definition, Units of Force. A force is defined as "that which changes, or tends to change, a body's state of rest or of uniform motion in a straight line." Every body is subjected to a force, known as its weight, caused by the vertical gravitational pull of the earth upon it. In Britain the unit employed for measuring weight is the *pound*, abbreviated to lb, which is defined as the weight of a standard cylinder of platinum kept by the Standards Dept. of the Board of Trade. The *gramme* (g) is the metric unit and is the weight of one-thousandth of a litre of pure water at a temperature of 4° C. (Table III shows the relation between English and metric units.) Not all forces are vertical, but these same units are used whatever the direction of the force may be.

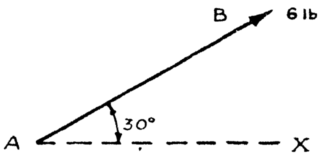


Fig. 19. Description of a Force

TABLE III

Relation Between English and Metric Units

1 inch	= 2·540 centimetres	1 litre	= 61·03 cubic inches
1 foot	= 0·3048 metres	1 ounce	= 28·35 grammes
1 yard	= 0·9144 metres	1 pound	= 453·6 grammes
1 mile	= 1·609 kilometres	1 ton	= 1016 kilogrammes

Description of a Force. To describe a force completely four things must be known—

(*a*) Its size or magnitude
(*b*) The point at which it acts
(*c*) The line along which it acts
(*d*) Its sense, whether it exerts a push or a pull at its point of application.

Thus the force in Fig. 19 would be fully described as "a force of 6 lb acting at A, along a line AB inclined at 30° to AX, and exerting a pull at A."

Since, mathematically, a point is infinitely small it would be impossible to apply a force at a point. In practice the point of application is taken as the place at which the force may be considered to be concentrated without altering its effect on the body to which it is applied, as a whole.

Graphical Representation. A mathematical quantity can be represented graphically in magnitude by the length of a line and in direction by the inclination of that line to a base line. Certain quantities, such as volume or time, do not involve the idea of direction and are known as *scalar quantities*, while forces, velocities, and similar quantities that do involve direction are called *vector quantities*. Referring again to Fig. 19, if AB is drawn of length, say,

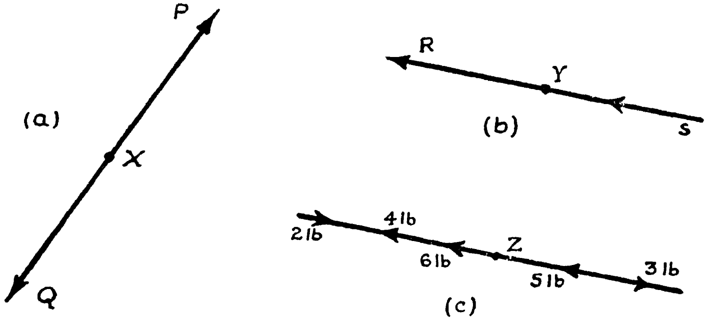


FIG. 20. RESULTANT OF FORCES IN THE SAME STRAIGHT LINE

3 in. at a given angle to the base line AX, then AB represents the pull of 6 lb in direction and magnitude to a scale of $\frac{1}{2}$ in. to 1 lb, the sense being indicated by the arrowhead. AB is referred to as a *vector*.

Equilibrium of Forces. If a number of forces act on a body in such a way that they balance and do not affect the state of motion of the body these forces are said to be in *equilibrium*. If they do not balance, they will tend to move the body in some direction. The single force which could be substituted for the original system of forces without altering the effect is known as the *resultant* of that system. The force required to keep such a system in equilibrium is called the *equilibrant* and will be equal and opposite to the resultant.

CONCURRENT FORCES

Resultant of Forces Acting in the Same Straight Line. If a number of forces act at a point, and in the same straight line, their resultant will be the *algebraic sum* of the forces, that is the sum taking into account the direction in which the forces act. Thus, in Fig. 20 (a), if the force P acts in the positive direction Q will be acting in the negative.

$\therefore$ Resultant of P and Q = algebraic sum of P and Q

$$= P - Q$$

But, in Fig. 20 (*b*), R and S are acting in the same direction at Y

$$\therefore \text{Resultant of } R \text{ and } S = R + S$$

And, in Fig. 20 (*c*), if forces of 2 lb, 4 lb, 6 lb, 5 lb, and 3 lb, act as shown at Z.

$$\text{Then resultant force} = -2 + 4 + 6 + 5 - 3 \text{ lb} = 10 \text{ lb}$$

In each case the equilibrant will be equal and opposite to the resultant.

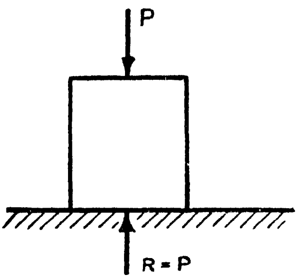


FIG. 21. REACTION BETWEEN A SURFACE AND A BODY

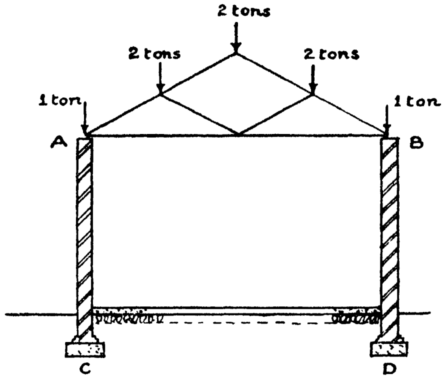


FIG. 22. REACTIONS IN A BUILDING

One special form of equilibrant must be mentioned. In Fig. 21 a force P acts on a body supported by a surface. Since the body remains at rest the surface must exert some force R on the body such that, whatever the value of P, the resultant of R and P is zero for if this were not so there would be an unbalanced force which would set the body in motion. The name *reaction* is given to R and, since the resultant of P and R is zero—

$$R = P$$

Since all structures remain at rest, both as a whole and with respect to their individual parts, the loads which they carry must be balanced by reactions at their points of support. Thus in Fig. 22 there will be reactions at A and B exerted by the walls on the roof truss and also at C and D between the ground and the footings. These reactions must never be forgotten when deciding the forces acting on a given structure.

Resultant of Two Intersecting Forces. When two intersecting forces act at a point O, Fig. 23, they tend to move the point partly in the direction of P and partly in that of Q. The same effect could be produced by a single force R acting in a direction between that of P and Q. Such a force would be the resultant of P and Q.

The magnitude and direction of R can be found graphically by a construction known as the *parallelogram of forces*. Choose a suitable scale and mark off OA along OP to represent P and OB along OQ to represent Q. Draw AC parallel to OB and BC parallel to OA to form the parallelogram $OACB$. Then the diagonal OC represents

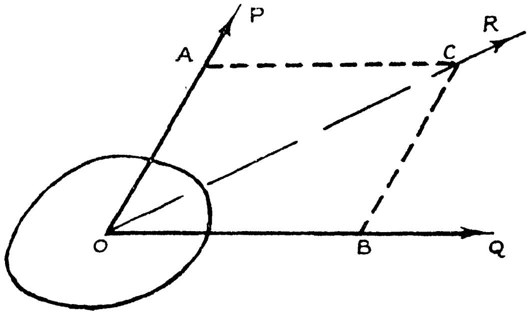


FIG. 23. RESULTANT OF TWO INTERSECTING FORCES

the resultant R in magnitude and direction to the same scale as was used for P and Q. Care must be taken in selecting the correct diagonal. The parallelogram must be constructed so that P, Q, and R all pass through O and act either all towards O or all away from O. If the equilibrium is required it will be equal and opposite to the resultant R.

Resolution of a Force into Components. It is sometimes convenient to split a single force into *components*. Thus any force P,

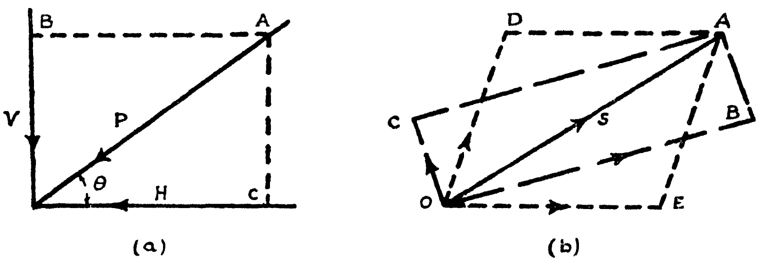


FIG. 24. RESOLUTION OF FORCES INTO TWO COMPONENTS

Fig. 24 (*a*), can be split into horizontal and vertical components H and V by drawing OA to represent the force P to scale and completing the parallelogram $OBAC$. Then OC represents the horizontal component H and OB the vertical component V. Alternatively the values of H and V can be calculated, for if angle $AOC = \theta$

Then $$V = P \sin \theta \text{ and } H = P \cos \theta.$$

A single force can be split into two components in any required directions; thus in Fig. 24 (*b*) the force *S* could be split into components *OC* and *OB* or *OD* and *OE* or components in any other two directions desired.

Triangle of Forces. It is not necessary to draw the whole parallelogram to find the resultant of two forces. Thus, in Fig, 23, it is sufficient to draw the triangle *OBC*, making the sides *OB* and *BC* represent the forces in magnitude and direction. The third side *OC* will represent the resultant in magnitude and direction. Again, referring to Fig 25. (*a*), to find the equilibrant of forces *P* and *Q*,

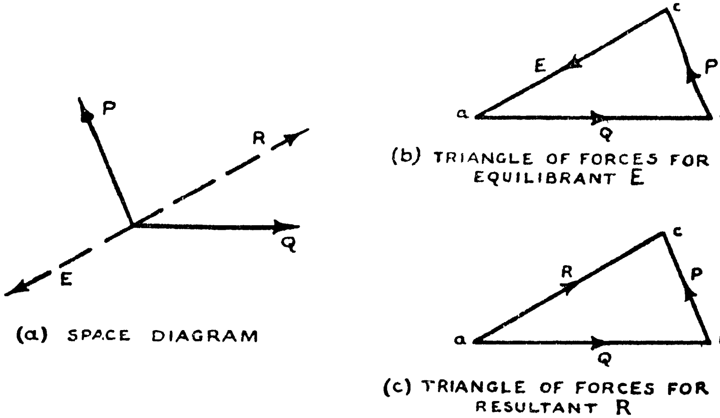


FIG. 25. RESULTANT AND EQUILIBRANT BY TRIANGLE OF FORCES

draw the triangle of forces *abc* making *ab* parallel to *Q* and representing it to scale and *bc* parallel and proportional to *P*. Then *ca* represents the required equilibrant *E*, or—

$$Q : P : E = ab : bc : ca$$

Note that the sides of the triangle have been written down to show the sense in which the forces act, *ca* meaning that *E* acts from *c* to *a*. It should also be noticed that, when finding the equilibrant, the arrows representing the directions of action of the forces follow consecutively round the triangle, Fig. 25 (*b*), whilst if the resultant of *P* and *Q* were required, Fig. 25 (*c*), its sense would be opposite to that of *E* and would act the opposite way round the triangle to *P* and *Q*.

Stating the principle of the triangle of forces in words: "If two concurrent forces are kept in equilibrium by a third force, the three forces can be represented, completely, by the sides of a triangle taken in order and drawn parallel to their directions."

The converse of this is also true, namely that: "If three forces can be represented in magnitude, direction and sense by the sides of a triangle taken in order, the forces are in equilibrium."

This latter statement is useful in determining whether three given forces meeting at a point are in equilibrium. Thus in Fig. 26 draw *ab* representing the 8 lb force, *bc* representing the 6 lb force, and *cd* the 7 lb force. It will be seen that the triangle of forces does not close and the forces are not in equilibrium. To balance the

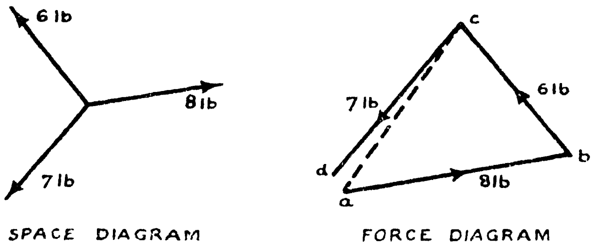


FIG. 26. TESTING THE EQUILIBRIUM OF THREE CONCURRENT FORCES

6 lb and 8 lb forces the 7 lb force must be replaced by a force represented by *ca*, in this case 7·4 lb, acting at *O* parallel to *ca*, its sense being from *c* to *a*.

Bow's Notation. A convenient method of notation in this type of problem is Bow's notation. The spaces between the forces are lettered *A*, *B*, and *C*, Fig. 27, and the force *P* is described as *ab*

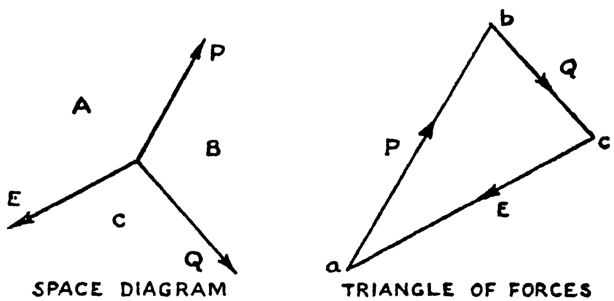


FIG. 27. USE OF BOW'S NOTATION

since it lies between space *A* and space *B*, and it will be represented by side *ab* in the triangle of forces. Similarly *Q* is called *bc* and *E* is called *ca*. The lettering of the spaces may be carried out either clockwise or anti-clockwise.

Resultant of more than Two Concurrent Forces. A solution of this problem could be obtained by application of the parallelogram of forces. Select two of the forces *L* and *M*, Fig. 28, and draw a parallelogram to find their resultant R_1. The system of three forces is thus reduced to two, R_1 and *N*, and their resultant R_2 can be found by drawing a second parallelogram as shown. R_2 will then be the resultant of the original forces, L, M, and N. The method can be applied to any number of forces, but is rather clumsy.

Polygon of Forces. A neater solution is given by a method developed from the repeated application of the triangle of forces.

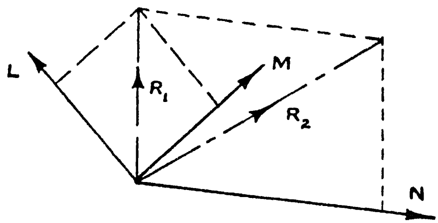


FIG. 28. RESULTANT OF MORE THAN TWO CONCURRENT FORCES

The space diagram, Fig. 29, shows four concurrent forces, S, T, U, and V acting at O. Select two forces, S and T, and draw *ab* in the force diagram parallel to S and *bc* parallel to T, representing them in magnitude, direction, and sense. Then *ac*, the closing side of the triangle *abc*, represents the resultant R_1 of S and T. Now find the resultant of R_1 and U by drawing *cd*, to the same scale, representing U and parallel to it. Then *ad*, the closing side of triangle *acd* repre-

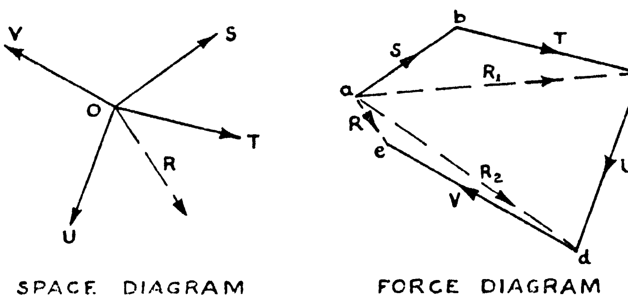


FIG. 29. POLYGON OF FORCES

sents the resultant R_2 of S, T, and U. Finally draw *de* parallel and proportional to V and complete the triangle *ade*. Then *ae* is proportional and parallel to the resultant R of R_2 and V, that is to say that R is the resultant of S, T, U, and V. The resultant R can now be shown on the space diagram acting through O and parallel to *ae*.

The polygon *abcde* is known as the polygon of forces and can be constructed directly by taking the forces S, T, U, and V in order and drawing *ab*, *bc*, *cd*, *de* parallel and proportional to them with the arrows denoting the sense of the forces following round in the same direction. The closing side *ae* is then drawn to give the magnitude and direction of the resultant and the arrow denoting its sense will act the opposite way round the polygon to S, T, U, and V.

Note that, if the original forces had been in equilibrium, the resultant would be zero and the vectors representing the forces would form a closed polygon, as shown in Fig. 30, which also demonstrates the use of Bow's notation. It can therefore be stated that

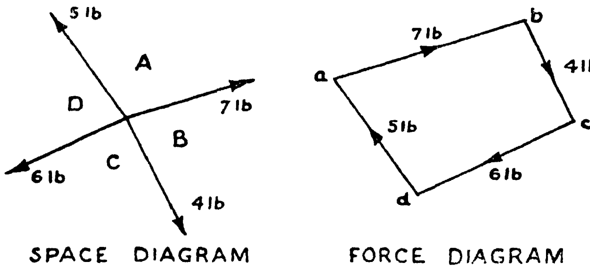


FIG. 30. POLYGON FOR FORCES IN EQUILIBRIUM

"If a number of concurrent forces in the same plane can be represented by a polygon whose sides are parallel and proportional to the given forces, then the system of forces is in equilibrium."

NON-CONCURRENT FORCES

If the lines of action of a number of forces do not meet at a common point they are said to form a *non-concurrent system of forces*. Such a system is shown in Fig. 31, and the equilibrant can

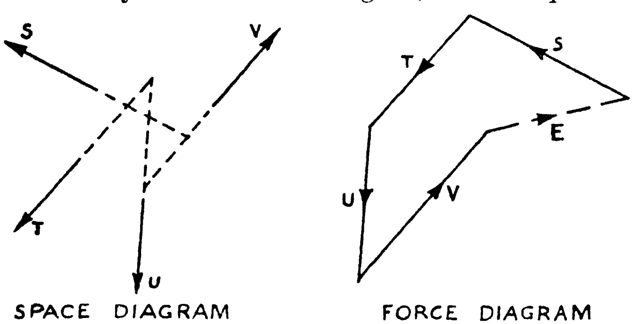


FIG. 31. NON-CONCURRENT FORCES

be found in magnitude, direction and sense by drawing the force polygon, as shown, in the same way as for concurrent forces, but, as there is no common meeting point for the forces, the point of action of E is not known.

Link Polygon. To find the point of action of the equilibrant of several non-concurrent forces a construction, known as the *link* *polygon* or *funicular polygon*, is used. Referring to Fig. 32, select any point O inside or outside the force polygon and join Oa, Ob, Oc, Od, and Oe. Then Oba forms a triangle of forces and Ob and Oa represent the forces p_1 and p_2 required to balance the force S. Such forces can be provided by links or ties AE and AB, shown on the space diagram drawn through A parallel to Oa and Ob respectively. Similarly force T can be balanced by the force p_2 exerted by the link BA and a force p_3 proportional and parallel to Oc in the force polygon, supplied by the link BC. Force U is balanced by p_3 in the link CB and p_4 in the link CD drawn parallel to Od and V is balanced by p_4 in link DC and p_5 in link DE drawn parallel to

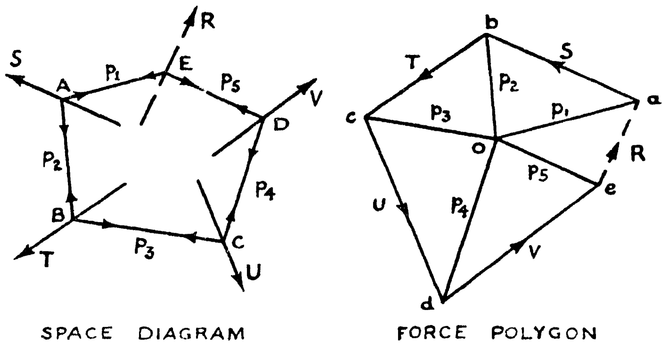


FIG. 32. LINK POLYGON

Oe. The only unbalanced forces in the system are now p_1 and p_5 the original forces being held in equilibrium by the links. Referring to the force polygon, Oae is a triangle of forces the sides being proportional to p_1, p_5, and R, which are therefore three concurrent forces. The force R is the equilibrant of p_1 and p_5, and therefore of the whole system, and since it is concurrent with p_1 and p_5 it must pass through the intersection of their lines of action, which will be at E, where links DE and AE meet. The figure $ABCDE$ is called the link polygon.

Thus to find the equilibrant of a system of non-concurrent forces it is necessary—

(*a*) To draw the force polygon to find the magnitude, direction, and sense of the equilibrant.

(*b*) To draw the link polygon to find a point on the line of action of the equilibrant.

The resultant of such a system will be equal and opposite to the equilibrant.

It can also be stated that: For a non-concurrent system of forces to be in equilibrium, both the force polygon and the link polygon must close.

LABORATORY WORK

EXPERIMENT 1. *Verification of the parallelogram of forces*

Use the apparatus shown in Fig. 33 consisting of a drawing board fixed to the wall and three pulleys, A, B and C, which can be fastened in any required position on the edge of the board. A piece of paper is fixed on the

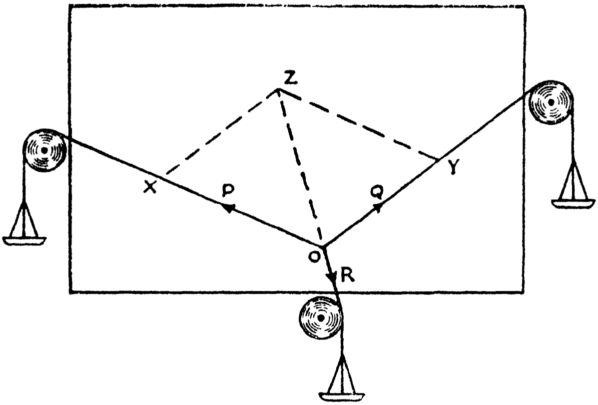


FIG. 33. VERIFICATION OF PARALLELOGRAM OF FORCES

board and three strings are tied together and their other ends are passed over the pulleys to carry known weights P, Q and R. The strings will take up a position such that the forces P and Q are held in equilibrium by the force R. Mark the position of the strings on the paper and remove it from the

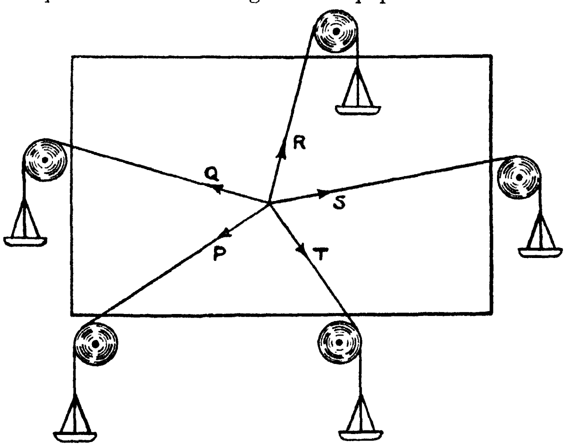


FIG. 34. VERIFICATION OF POLYGON OF FORCES

board. Choose a suitable scale and mark off OX and OY to represent P and Q. Complete the parallelogram $OXZY$ and measure the diagonal OZ. This will be found to represent the force R to the chosen scale and to be in line with the line of action of R.

EXPERIMENT 2. *The polygon of forces*

Use the same apparatus as in Expt. 1, but with additional pulleys as shown. Five strings are joined together and their other ends are passed over pulleys to carry the known weights P, Q. R, S, T, Fig. 34. The strings take up positions such that the system is in equilibrium. Mark these positions on the paper, remove the paper, and, choosing a suitable scale, draw the polygon of forces. As the forces are in equilibrium the polygon will be found to close.

EXPERIMENT 3. *Wall crane*

A model wall crane, similar to those used in warehouses, is shown in Fig. 35 (*a*). For laboratory purposes spring balances are fitted in the jib BC and

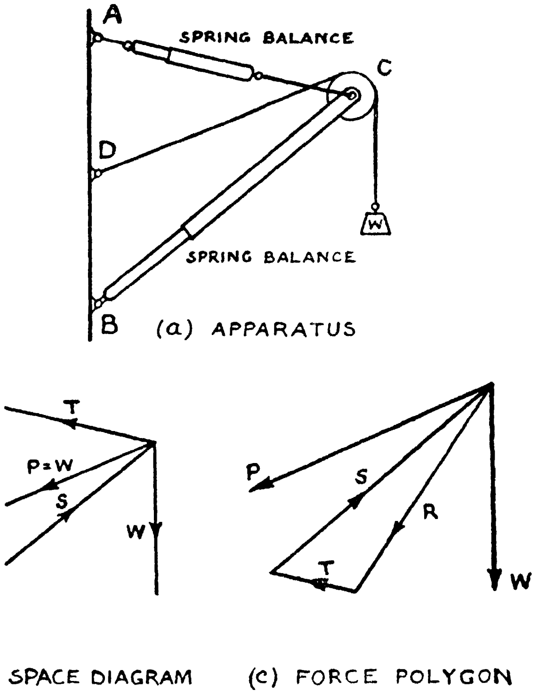


FIG. 35. LABORATORY MODEL WALL CRANE

the tie AC and the cord supporting the load W is secured to the wall at D instead of being connected to a winch. When the load W is applied the forces S in AC and T in BC can be read on the balances.

To check these values measure up the model and draw a line diagram showing the direction of the forces acting at C, Fig. 35 (*b*). The measurements must be made while the model is loaded as its shape will alter due to the compression or extension of the spring balances. Now draw the force polygon, Fig. 35 (*c*), first compounding W and P to give R and from it check the values of S and T.

EXPERIMENT 4. *Sheer legs.*

A laboratory model of a pair of sheer legs is shown in Fig. 36. A spring balance forms part of each leg and another is connected in the backstay. Measure the forces in the legs and backstay when the load *W* is applied, measure up the model and check the results by drawing.

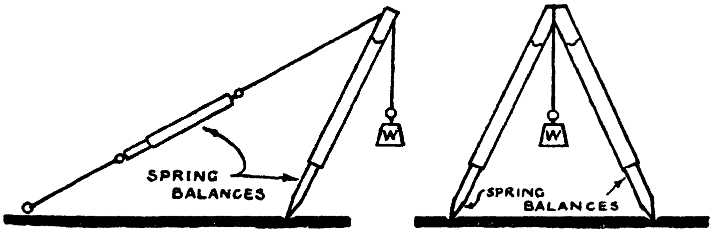


FIG. 36. LABORATORY MODEL SHEER LEGS

EXERCISES

1. Two pulls are applied at a point, one of 6 lb and the other of 10 lb. Find, graphically, the magnitude and direction of the resultant when the forces are inclined at (*a*) 90°, (*b*) 45°, (*c*) 30° to each other.

2. Two horizontal wires are attached to the top of a pole, the angle between the wires being 120°. Find the resultant force on the pole, in magnitude and direction, if the pulls in the wires are 100 lb and 120 lb respectively.

3. In Fig. 29, $S = 12$ lb, $T = 20$ lb, $U = 15$ lb, $V = 17$ lb, angle $SOT = 60°$, angle $TOU = 100°$, and angle $UOV = 80°$. Find the magnitude and direction of the resultant force.

4. A uniform plank, *AB*, is 6 ft long, weighs 80 lb and is inclined at 40° to the vertical. Its lower end *A* is hinged to a support, while a light chain is fastened to a ring 4 ft vertically above *A* and to a point on the plank 1 ft from *B*. Find the tension in the chain and the magnitude and direction of the reaction of the hinge at *A*.

5. A truck is pulled along a track by a rope inclined at 30° to its line of motion. If the pull in the rope is 240 lb what will be the force exerted (*a*) along the track, (*b*) at right angles to the track?

6. A square plate, of 2 ft side, *ABCD*, has forces of 4 lb, 5 lb, 3 lb, and 6 lb acting on the edges *AB*, *BC*, *CD*, and *DA* respectively. If all the forces act in the same direction round the plate, find the magnitude, direction and point of action of the resultant.

7. A vertical post, 12 ft high, is subjected to pulls from four wires all lying in the same vertical plane and on the same side of the post. These pulls are 50 lb acting horizontally at the top, 80 lb acting at 60° to the vertical and downwards at a point 10 ft from the ground, 75 lb acting at 45° to the vertical and downwards at a point 8 ft from the ground, and 40 lb acting downwards at 30° to the vertical at a point 6 ft from the ground. Find the magnitude, inclination to the vertical, and point of action of the resultant.

8. The jib, *BC*, Fig. 35, of a wall crane is 21 ft long, the length of the tie *AC* is 12 ft, the distance *AB* is 15 ft and *BD* is 7 ft. Find the forces in the tie and the jib if a load of 6 cwt is suspended from the crane.

CHAPTER VIII

PARALLEL FORCES, MOMENTS AND COUPLES

Parallel Forces

A SPECIAL case of a non-concurrent system of forces is that in which all the forces are parallel. With a little ingenuity the resultant of such a system can be found by repeated application of the parallelogram of forces.

Resultant by Parallelogram of Forces. The method is demonstrated for the case of two parallel forces P and Q in Fig. 37. If

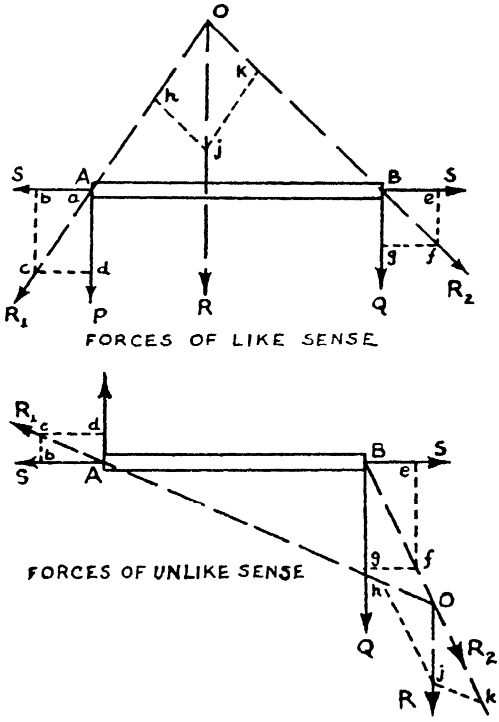


Fig. 37. Resultant of Parallel Forces by Parallelogram of Forces

two equal and opposite forces S acting in the same straight line AB are applied at A and B, they will balance out and not affect the final result. Now at A there are forces P and S acting, the resultant R_1

of which can be found by constructing the parallelogram of forces *Abcd.* Similarly, R_2, the resultant of Q and S acting at B, can be found by constructing the parallelogram *Befg.* Thus two concurrent forces R_1 and R_2 can be substituted for P and Q. The resultant R of R_1 and R_2 can be found by drawing the parallelogram of forces

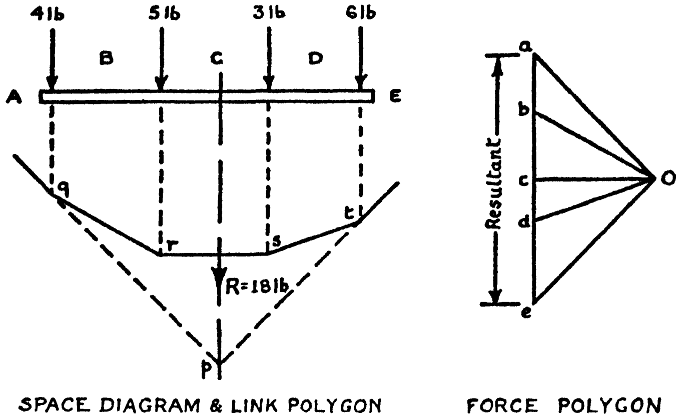


FIG. 38. RESULTANT OF LIKE PARALLEL FORCES BY THE POLYGON OF FORCES

Ohjk and R will also be the resultant of P and Q. It will be found that R is equal to the sum of P and Q, if these forces are of like sense, or the difference if P and Q are of unlike sense, and it also acts parallel to them.

Polygon of Forces. A simple solution for a number of parallel forces may be obtained by drawing the polygon of forces, as shown in Fig. 38.

Bow's notation has been adopted and the forces shown are

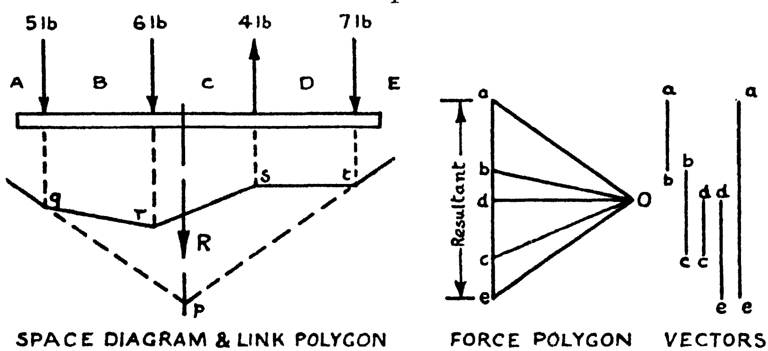


FIG. 39. RESULTANT OF UNLIKE PARALLEL FORCES BY THE POLYGON OF FORCES

$AB = 4$ lb, $BC = 5$ lb, $CD = 3$ lb, and $DE = 6$ lb. On drawing the force polygon it will be found to be a straight line and the magnitude and direction of the resultant will be given by $ea = 18$ lb. Now choose any pole O, join Oa, Ob, Oc, Od, Oe, and construct the link polygon by drawing pq, qr, rs, st, tp parallel to Oa, Ob, Oc, Od, Oe respectively. Then the resultant will act through p. A straight line vector diagram is sometimes called a load line. Although only a single line is drawn the vector ae should be considered as lying side by side with the other vectors. Fig. 39 shows the case when all the forces do not act with the same sense. Note that the load line doubles back on itself where the force CD occurs. The actual vectors in their relative positions are shown to the right of the force polygon.

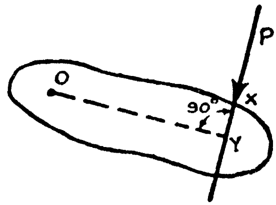


FIG. 40. MOMENT OF A FORCE

MOMENTS

Moment of a Force. Consideration has been given, so far, to the tendency of a force to move a body as a whole along its line of action. Now if a force P, Fig. 40, acts at a point X on a body pivoted at O, the body will tend to rotate about O. P is then said to have a *moment* about O. This moment depends on the magnitude of P and the distance of O from its line of action, and is measured by multiplying the size of the force by the perpendicular distance of O from its line of action—

$$\text{Moment of } P \text{ about } O = P \times OY$$

The units of measurement will be the product of the units of P and of O. Thus if $P = 5$ lb and $OY = 10$ in.

$$\text{Moment of } P \text{ about } O = 5 \times 10 = 50 \text{ lb-in.}$$

In referring to the moment of a force it is necessary to state the point about which the moment is taken, in this case O, since the force will have a moment about any point which does not lie in its line of action. If the point does lie in its line of action, the perpendicular distance OY is zero and there is no moment about that point.

The sense of a moment can be given by the direction in which it tends to cause rotation. If this is, to the observer, in the same direction as the hands of a clock, the moment is said to be *clockwise* and is usually called *positive*; if in the reverse direction, the moment is *anti-clockwise* or *negative*.

Principle of Moments. If several forces all tend to rotate a body in the same direction the resultant moment about a given point will be the sum of the moments of each individual force about that point. If some forces exert clockwise moments and others anti-clockwise moments, the resultant moment will be the difference of the sum of the clockwise moments and that of the anti-clockwise moments, that is to say the resultant moment will be the *algebraic sum* of the moments of each separate force.

If the resultant is zero, the body will be in equilibrium from the point of view of rotation. This fact forms the *Principle of Moments* which states that: "If a body is in equilibrium with regard to rotation, the algebraic sum of the moments of the forces acting on that body about any point must be zero."

Resultant of Two Parallel Forces by Principle of Moments. In Fig. 41 (a) two parallel forces P and Q, of like sense, act on a body and the perpendicular distance between their lines of action is L.

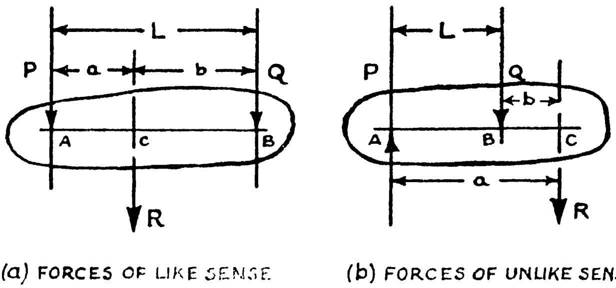


Fig. 41. Resultant of Two Unequal Forces by the Principle of Moments

As there are no horizontal forces the resultant R must act vertically. Suppose it to act through some point C at a distance a from P and b from Q. Then if the action of R is to represent that of P and Q correctly—

Moment of R about any point = sum of the moments of P and Q about the same point.

Take moments about C, the point about which R has no moment—

Then $$0 = Qb - Pa$$

or $$\frac{a}{b} = \frac{Q}{P} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Hence the resultant lies nearer to the bigger force, and from this equation the position of C can be found.

Also, taking moments about B

$$Rb = PL = P(a + b)$$

$$R = P\frac{a}{b} + P$$

But, from equation (1),

$$Q = P\frac{a}{b}$$

$$\therefore R = Q + P$$

or Resultant = sum of the forces acting.

Now consider the case where P and Q are of opposite sense, Fig. 41 (b). As before, assume R to act through some point C at a distance a from P and b from Q. Again the sum of the moments of P and Q about C must be zero. This can only occur if C does not lie between P and Q, for if P and Q are on opposite sides of C their moments will be of the same sense and therefore the sum of these moments cannot be zero.

Taking moments about C—

$$Pa - Qb = 0$$

$$\therefore \frac{a}{b} = \frac{Q}{P}$$

Thus in this case also the resultant acts nearer to the larger force.

Also taking moments about B—

$$Rb = P(a - b)$$

$$R = P\frac{a}{b} - P$$

$$= Q - P$$

i.e. Resultant = difference of the forces acting.

We may summarize these results as follows—

1. The resultant is equal to the sum of the given forces, if they are of like sense; or their difference, if they are of unlike sense.

2. The resultant is parallel to the given forces and lies between them, if they are of like sense, and outside them if they are of unlike sense.

3. The resultant acts through a point nearer to the larger force and so placed that the perpendicular distances between the lines of the resultant and the given forces are inversely proportional to the magnitude of the given forces.

Couples

Couples. The resultant of two equal and opposite parallel forces, Fig. 42, cannot be found by the method given above. In this case $Q = P$ and, referring to Fig. 41 (b)—

$$R = Q - P = P - P = 0$$

also

$$\frac{a}{b} = \frac{Q}{P} = \frac{P}{P}$$

$$\therefore a = b$$

Clearly these results are not applicable and therefore it may be concluded that there is no single force which can be the resultant of two equal and opposite parallel forces. Such a system will produce rotation only and the two forces are said to form a *couple*.

Moment of a Couple. The moment of a couple about any point will be the algebraic sum of the moments of the individual forces. In Fig. 42, P and P are the parallel forces and d is the perpendicular distance between them, referred to as the *arm* of the couple.

Taking moments about A and calling clockwise moments positive

$$\text{Moment of couple about } A = 0 + Pd = Pd$$

Again taking moments about B—

$$\text{Moment of couple about } B = Pd + 0 = Pd$$

Now take any point C lying between the forces—

$$\text{Moment of couple about } C = Pa + P(d - a) = Pd$$

And take any point D lying outside the forces—

$$\text{Moment of couple about } D = P\,(d + x) - Px = Pd$$

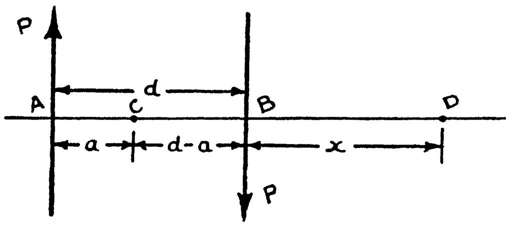


FIG. 42. THE MOMENT OF A COUPLE

In each case the moment of the couple is Pd, hence: "The moment of a couple is the same about any point in its plane and is the product of one of the forces and the arm of the couple."

Equilibrant of a Couple. Since no single force can be the resultant of a couple, no single force can be its equilibrant. A couple can only be balanced by an equal and opposite couple acting in the same plane as, or in a plane parallel to, that of the original couple. This is evident from the Principle of Moments which requires that, for equilibrium the sum of the moments in all directions shall be zero.

Magnitude of Forces Forming a Couple. Since the moment of a couple is the product of one force and the arm of the couple, it is clear that the forces representing a couple of a given moment can be chosen at will, provided that a suitable value is given to the arm of the couple.

EXAMPLE. A couple has a moment of 80 lb-in. If the forces constituting the couple are each 10 lb, what is the lever arm of the couple? If the lever arm were 6 in., what is the value of each force?

Let d be the arm of the couple: then

$$Pd = 80 \text{ lb-in.}$$

Putting $$P = 10 \text{ lb}$$

$$10d = 80 \text{ lb-in.}$$

$$d = \underline{\underline{8 \text{ in.}}}$$

Again, $Pd = 80$ lb-in.
Putting $d = 6$ in.

$$6P = 80 \text{ lb-in.}$$
$$P = \underline{\underline{13\tfrac{1}{3} \text{ lb}}}$$

Given Force Replaced by a Force and a Couple. When a force acts on a body it is sometimes convenient to consider it to be applied at another point and to add a couple to keep the system in equilibrium. Thus, in Fig. 43, the force P acts at a point A but it is desired to treat it as acting at B. At B apply equal and opposite forces P and P'. Since these forces balance the resultant of the whole system is still P acting at A. Now P acting at A and P'

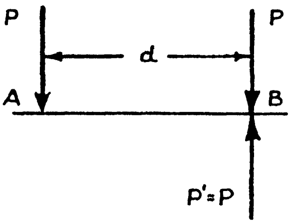


FIG. 43. REPLACEMENT OF A FORCE BY A FORCE AND A COUPLE

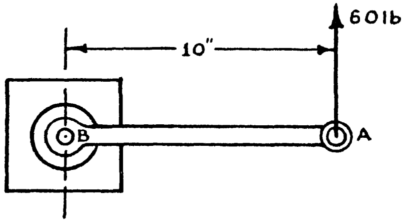


FIG. 44. TURNING MOMENT ON A CRANK HANDLE

acting at B form a couple, since $P = P'$ and, if d is the perpendicular distance between them, the moment of the couple is Pd. Thus the system can be represented by a couple of moment Pd and a force P equal to the original force acting at the new position.

EXAMPLE. A force of 60 lb is applied to the crank-handle of a car, as shown in Fig. 44. Find the turning moment on the crankshaft and the upward force exerted on the bearing.

Transferring the 60 lb force from A to B

Force exerted on the bearing $= \underline{\underline{60 \text{ lb}}}$

Turning moment $= 60 \times 10 = \underline{\underline{600 \text{ lb-in.}}}$

Resultant of Parallel Forces by Moments. The resultant of a number of parallel forces can be found conveniently by taking moments. In Fig. 45, forces w_1, w_2, w_3, w_4, and w_5 act perpendicular to the line AB at distances x_1, x_2, x_3, x_4, and x_5 from A. The resultant R of these forces must act so that its moment about any point will be equal to that due to the system of forces. Also the resultant force due to R must equal the algebraic sum of the forces w_1, w_2, w_3, w_4, w_5

$$\therefore \; R = w_1 + w_2 - w_3 + w_4 - w_5$$

Taking moments about A, let R act at a distance d from A.

Then $$Rd = w_1x_1 + w_2x_2 - w_3x_3 + w_4x_4 - w_5x_5$$

From these two equations the value and position of R can be found.

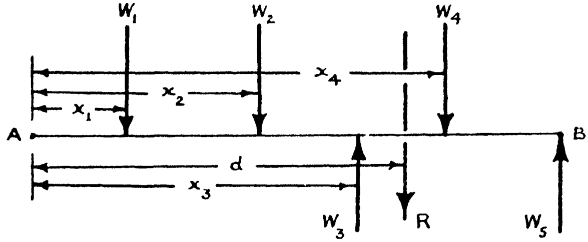


FIG. 45. RESULTANT OF PARALLEL FORCES BY THE PRINCIPLE OF MOMENTS

EXAMPLE. Forces of 10 lb, 8 lb, 15 lb and 12 lb act perpendicular to the line AB in the positions shown in Fig. 46. Find the position and magnitude of the resultant of the system.

Let R be the resultant force.

Then $R = 10 + 8 - 15 + 12 = 15$ lb.

If R acts at a distance x from A, then, taking moments about A—

$$Rx = 8 \times 2 - 15 \times 5 + 12 \times 9 \text{ lb-ft}$$
$$= 16 - 75 + 108 \text{ lb-ft}$$
$$= 49 \text{ lb-ft}$$

Putting $R = 15$ lb

$$15x = 49$$
$$x = 3\tfrac{4}{15} \text{ ft}$$

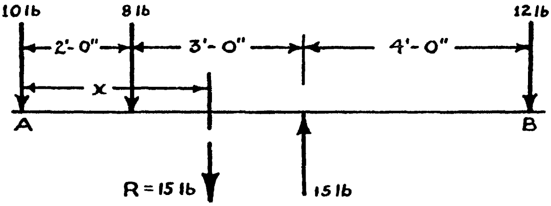


FIG. 46. RESULTANT OF FOUR PARALLEL FORCES

GENERAL CONDITIONS OF EQUILIBRIUM OF A SYSTEM OF FORCES

The findings of Chapters VII and VIII can now be combined together to lay down general conditions which must be fulfilled if a system of forces acting on a body is to remain in equilibrium. Stated very briefly these are—

1. There must be no resultant force acting in any direction.
2. The algebraic sum of the moments of the forces about any point must be zero, i.e. the clockwise and anti-clockwise moments must balance.

These conditions are self-evident, for if condition (1) is not fulfilled the body will move in the direction of the resultant force, and if condition (2) is not obeyed the body will start to rotate under the influence of the unbalanced moment.

Equilibrium for a Uniplanar System of Forces. Four uniplanar forces which are to be tested for equilibrium are shown in Fig. 47.

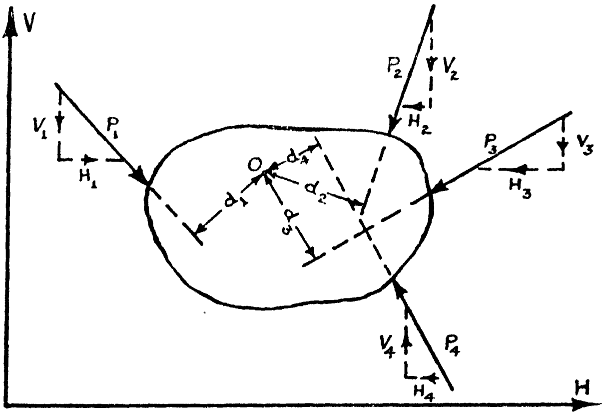


FIG. 47. EQUILIBRIUM OF UNIPLANAR FORCES

Applying condition (1) there must be no resultant force in any direction. To find the resultant select two directions, V and H, at right angles and split the forces P_1, P_2, P_3, and P_4 into components V_1, H_1; V_2, H_2; V_3, H_3; V_4, H_4 in these directions. Then H_1, H_2, H_3, H_4 are a series of parallel forces and their resultant will be the component of the resultant of the whole system in the H direction.

Resultant of the H components $= H_1 - H_2 - H_3 + H_4$. This may be written ΣH, read as *sigma* H, meaning the algebraic sum of all the forces acting in the H direction.

For equilibrium the resultant of the whole system is zero and therefore there can be no component in the H direction.

$$\therefore \ \Sigma H = 0$$

Similarly there must be no component of the resultant in the V direction

$$\therefore \ \Sigma V = 0$$

Again, take any point O and let the perpendicular distances from the lines of action of P_1, P_2, P_3, P_4 to O be d_1, d_2, d_3, and d_4. Then, applying condition (2)

$$P_1d_1 - P_2d_2 - P_3d_3 + P_4d_4 = 0$$

This may be written

$$\Sigma M = 0$$

where ΣM means the algebraic sum of the moments of the forces about any point.

Any number of uniplanar forces can be dealt with in the same way and will be in equilibrium if the three conditions,

$$\Sigma H = 0, \ \Sigma V = 0, \ \Sigma M = 0,$$

are fulfilled.

LABORATORY WORK

EXPERIMENT 1. *Principle of moments*

A wooden disc is pivoted at its centre O, Fig. 48, so that it will rotate freely in a vertical plane. It has a number of holes on its face into which pins can be fixed. Attach cords to the pins and lead the cords over pulleys to carry weights W_1, W_2, W_3 and W_4. Allow the disc to come to its position of equilibrium under these forces. Measure the distances d_1, d_2, d_3 and d_4 from O to the lines of action of the forces. Calculate the moment of each force and verify that the sum of the clockwise moments is equal to the sum of the anticlockwise moments.

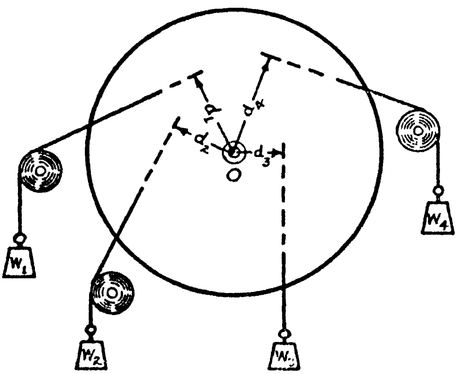


FIG. 48. VERIFICATION OF THE PRINCIPLE OF MOMENTS

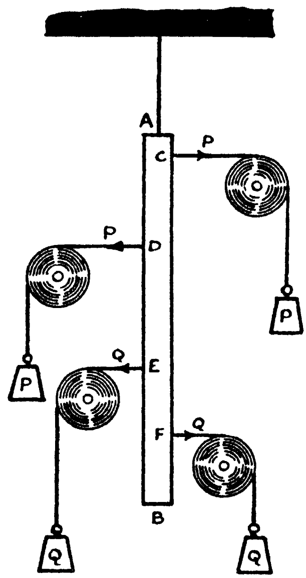


FIG. 49. EQUILIBRIUM OF EQUAL AND OPPOSITE COUPLES

EXPERIMENT 2. *Equilibrium of equal and opposite couples*

Suspend a light rod, AB, Fig. 49, by a string. Apply, by cords, pulleys and weights, two equal and opposite forces, P and P, at points C and D and a second pair, Q and Q, at points E and F. Adjust these forces so that—

$$P \times CD = Q \times EF$$

The rod is now under two equal opposing couples. Check that it remains at rest in its original vertical position.

EXPERIMENT 3. *Equilibrant of a number of parallel forces*

Support a metre rule by means of a spring balance, Fig. 50, and adjust a number of loads W_1, W_2, W_3, W_4, until the rule is in equilibrium. The pull

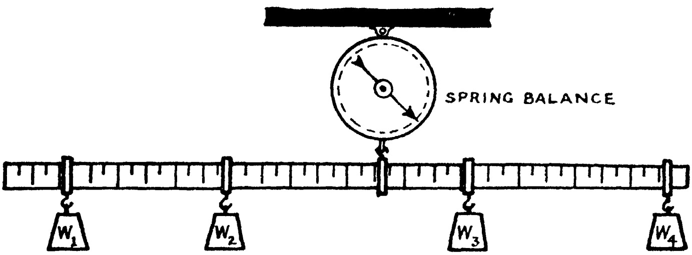


FIG. 50. EQUILIBRANT OF A NUMBER OF PARALLEL FORCES BY EXPERIMENT

of the spring balance is now acting as the equilibrant of the loads. Note the values and positions of the loads and check the results by calculation using the principle of moments.

EXERCISES

1. A beam of 32 ft span carries vertical loads of $1\frac{1}{2}$, 2, 3, and $4\frac{1}{2}$ cwt at distances of 4, 8, 16, and 24 ft from its end A. Find, by using the link polygon, the magnitude and point of action of the resultant of these forces.

2. In Fig. 45, $W_1 = 500$ lb, $W_2 = 350$ lb, $W_3 = 600$ lb, $W_4 = 550$ lb, and $W_5 = 700$ lb. If $x_1 = 2$ ft, $x_2 = 5$ ft, $x_3 = 9$ ft, $x_4 = 11$ ft, and $x_5 = 14$ ft, find the resultant of the system of forces in magnitude and also its point of action.

3. Find the magnitude and position of the resultant of two unlike parallel forces of 5 t and 3 t if the perpendicular distance between them is 8 ft.

4. What is meant by the following terms: couple, moment, arm of a couple? If the moment of a couple is 650 lb-ft and its arm is 15 ft, find the magnitude of the forces causing it.

5. A gate is supported by hinges which are 2 ft apart vertically and which share the vertical reaction supporting the gate equally between them. If the weight of the gate is 150 lb which may be taken as acting vertically at a distance of 21 in. from the axis of the hinges, calculate the magnitude and direction of the reaction at each hinge.

6. In Fig. 49, the value of the forces P is 15 lb and the distance CD is 7 in. Calculate the value of the forces Q if EF is 5 in.

7. A light rod, 2 ft long, is hinged at one end and supported horizontally by a string attached to the other end and inclined at 45° to the horizontal. If loads of 3 lb, 4 lb and 2 lb are placed on the rod at distances of 6 in., 11 in., and 18 in., from the hinge respectfully, calculate the magnitude and direction of the reaction at the hinge and the pull in the string.

8. A force of 45 lb is applied to a spanner and produces a turning moment of 405 lb-in. on a nut. What is the length of the spanner and what force will be exerted on the nut?

CHAPTER IX

CENTRE OF GRAVITY AND CENTRE OF AREA

Centre of Parallel Forces. If forces P and Q, Fig. 51, act on a rod AB, the point of action C of their resultant R can be found as explained in Chapter VIII and

$$P : Q = BC : AC \quad . \quad . \quad . \quad . \quad (1)$$

Now if the lines of action of P and Q are inclined, as at P' and Q', the resultant R' will act at some point X on DE such that—

$$P : Q = EX : DX$$

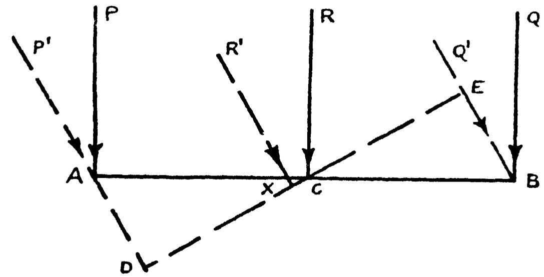


FIG. 51. CENTRE OF PARALLEL FORCES

therefore, from equation (1)—

$$BC : AC = EX : DX \quad . \quad . \quad . \quad . \quad (2)$$

Now triangles ACD and ECB are similar

$$\therefore\ BC : AC = EC : CD$$

hence, from equation (2)—

$$EC : CD = EX : DX$$

Thus points C and X must coincide and R' passes through C showing that the point of action of the resultant has not been altered by the inclination of the forces. The point C is called the *centre* of the parallel forces P and Q.

Centre of Gravity. A body can be considered to consist of a large number of particles, each of which is subjected to a vertical gravitational pull, and the total weight of the body will be the resultant of all these parallel pulls. Now if the body is rotated the direction of the gravitational forces, relative to a fixed line on the body, will be altered, but, as shown above, there will be one point on the line of action of the resultant which will be common to both positions. This point is known as the *centre of gravity* (abbreviated to c.g.) of the body and, whatever the position of the body, it will be through this point that the resultant weight will act. Also, since this point is on the line of action of their resultant, the algebraic sum of the moments of the weights of the particles constituting the body, taken about the centre of gravity, will be zero.

Centre of Gravity of a Lamina, by Experiment. If a thin sheet, or lamina, Fig. 52, is suspended by a cord at A, the only forces acting on it will be the pull of the string acting up and the weight acting down through the centre of gravity. Since the lamina hangs in equilibrium these forces must act in the same straight line, the centre of gravity lying vertically below the point of suspension. Using a plumb-bob the line AB can be drawn on the surface of the lamina and the centre of gravity must lie on this line. Now suspend the lamina from a point C and repeat the process, drawing the line CD. The point G at the intersection of AB and CD is the centre of gravity of the lamina. This can be checked by hanging the lamina from another point E and verifying that EF also passes through G.

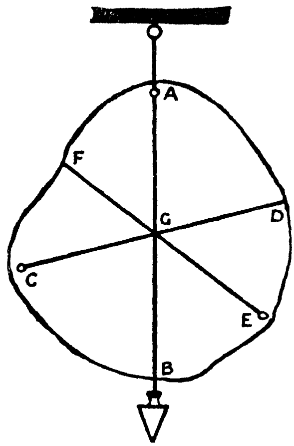


FIG. 52. CENTRE OF GRAVITY OF A LAMINA

Centre of Gravity of a Number of Particles. The principle of moments may be used to find the centre of gravity of a number of

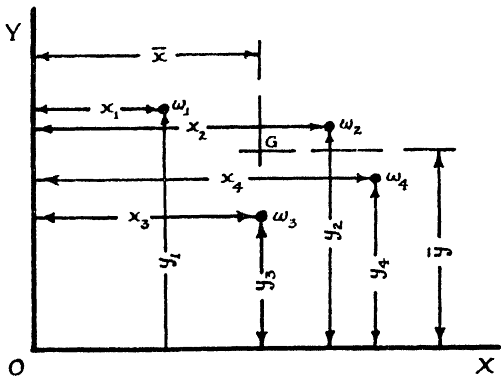


FIG. 53. CENTRE OF GRAVITY OF A NUMBER OF PARTICLES

particles, of weight w_1, w_2, w_3, w_4, etc. Take co-ordinate axes OX and OY, Fig. 53, and suppose the co-ordinates of w_1, w_2, etc. to be (x_1, y_1), (x_2, y_2), etc., as shown. The centre of gravity of the system will be at some point G, having co-ordinates $(\bar{x}, \bar{y})$ such that the moment of the total weight, acting at G, about any axis will be equal to the sum of the moments, about the same axis, of each particle taken separately.

$$\text{Total weight} = w_1 + w_2 + w_3 + w_4 + \dots = \Sigma w$$

therefore, taking moments about OY—

$$\bar{x} \times \Sigma w = w_1 x_1 + w_2 x_2 + w_3 x_3 + w_4 x_4 + \dots = \Sigma wx$$

$$\bar{x} = \frac{\Sigma wx}{\Sigma w}$$

Similarly, taking moments about OX—

$$\bar{y} \times \Sigma w = w_1 y_1 + w_2 y_2 + w_3 y_3 + w_4 y_4 + \dots = \Sigma wy$$

$$\bar{y} = \frac{\Sigma wy}{\Sigma w}$$

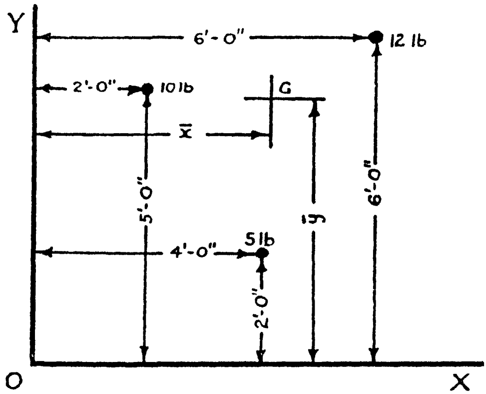


FIG. 54. CENTRE OF GRAVITY OF THREE LOADS

EXAMPLE. Find the centre of gravity of the three loads, shown in Fig. 54 relative to the axes OX and OY.

$$\text{Total load } \Sigma w = 10 + 12 + 5 \text{ lb}$$
$$= 27 \text{ lb}$$

Let G be $\bar{x}$ feet from OY and $\bar{y}$ feet from OX.

Taking moments about OY—

$$27\,\bar{x} = 10 \times 2 + 12 \times 6 + 5 \times 4 \text{ lb-ft}$$
$$= 20 + 72 + 20 = 112 \text{ lb-ft}$$
$$\bar{x} = \tfrac{112}{27} = 4\tfrac{4}{27} \text{ ft}$$

Taking moments about OX—

$$27\,\bar{y} = 10 \times 5 + 12 \times 6 + 5 \times 2 \text{ lb-ft}$$
$$= 50 + 72 + 10 = 132 \text{ lb-ft}$$
$$\bar{y} = \tfrac{132}{27} = 4\tfrac{8}{9} \text{ ft}$$

Position of centre of gravity is:

$4\frac{4}{27}$ ft from OY

$4\frac{8}{9}$ ft from OX

Centre of Gravity of a Lamina. The above method can be applied to find the centre of gravity of a uniform lamina. Suppose that the lamina in Fig. 55 weighs w pounds per square foot. Take any small element of area a, the co-ordinates of the centre of which are (x, y) relative to the OY and OX axes.

$$\begin{aligned} \text{Weight of element} &= \text{weight per unit area} \times \text{area} \\ &= wa \end{aligned}$$

Taking moments about OY—

$$\text{Moment of element} = wax$$

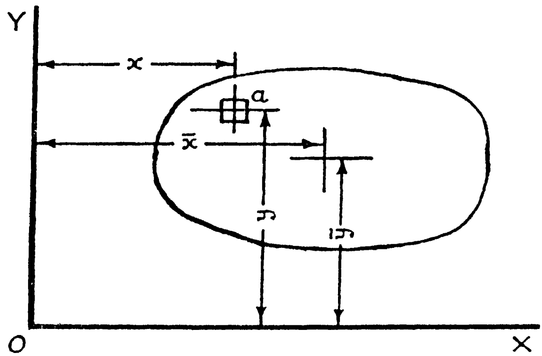


FIG. 55. CENTRE OF GRAVITY OF A LAMINA

Now the sum of the moments of all such small elements making up the lamina must equal the moment of the total weight acting at the centre of gravity G whose co-ordinates are $(\bar{x}, \bar{y})$. Thus if A = total area of lamina

$$\text{weight of lamina} = wA$$

and therefore,

$$\Sigma wax = wA\bar{x}$$

$$\bar{x} = \frac{\Sigma wax}{wA}$$

or, since w is a constant—

$$\bar{x} = \frac{\Sigma ax}{A}$$

Similarly—

$$\bar{y} = \frac{\Sigma ay}{A}$$

Note that the weight per unit area of the lamina does not affect the position of the centre of gravity relative to the OX and OY axes.

Centroid, Centre of Area. If the lamina is made very thin it becomes, in fact, a geometrical figure. The results obtained above are still applicable, since they involve neither the thickness nor the weight per unit area of the lamina. Thus the co-ordinates of the point G for such a figure are still given by

$$\bar{x} = \frac{\Sigma ax}{A}, \qquad \bar{y} = \frac{\Sigma ay}{A}$$

As a plane figure has no mass, and therefore cannot be acted upon by gravity, the term *centroid* or *centre of area* is used for this point instead of centre of gravity. Thus just as, for a body, the c.g. is

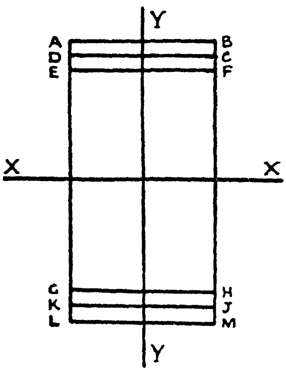


FIG. 56. CENTRE OF AREA OF A RECTANGLE

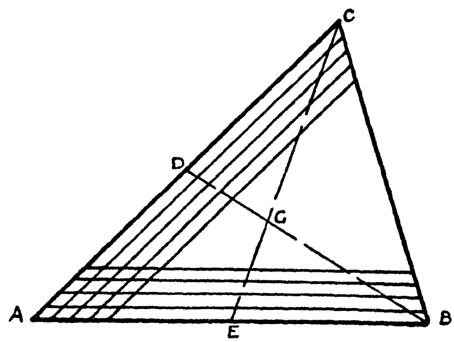


FIG. 57. CENTROID OF A TRIANGLE

the point at which the resultant weight acts and about which the algebraic sum of the moments of the weights of the particles composing the body is zero, so, for a geometrical area, the centroid is that point at which the area may be considered to be concentrated.

The terms ax and ay are the product of distance and area and are known as *moments of area*. The name *first moment of area* is given to the terms Σax and Σay, and, since as before, taking moments about G,

$$\Sigma ax = 0 \text{ and } \Sigma ay = 0$$

the first moment of area of a figure about an axis through its centroid is zero.

Centroid of a Rectangle. The centroid of a rectangle lies at the intersection of the lines joining the mid-points of its sides. Thus, in Fig. 56, the centroid lies at the intersection of XX and YY.

To verify this divide the rectangle into an even number of strips of uniform depth parallel to XX. Calling distances above XX positive and below XX negative, the moments of those strips above will be positive and of those below negative. Strips *ABCD* and *JKLM* are of equal area and equidistant from *XX*, therefore they have equal and opposite moments of area and so Σay for these two strips is zero. This will be the case with all other pairs of strips; therefore the first moment of area about *XX*, the line joining the mid-points of the sides *AL* and *BM*, is *zero* and the centroid lies on *XX*. Similarly, by taking strips parallel to *YY*, it can be shown that the centroid lies on *YY*. Hence the centroid lies at the intersection of *XX* and *YY*. In practice this point is found more easily by finding the point of intersection of the diagonals.

Centroid of a Circular Area. It is evident that the centroid both of a circular area and of the circumference of a circle will lie at their

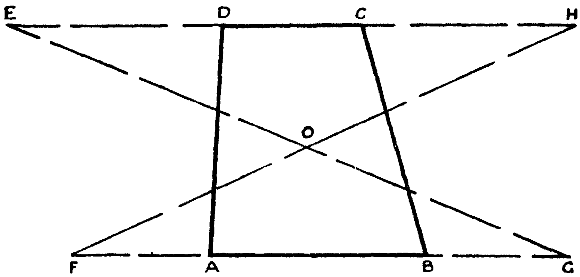


FIG. 58. CENTROID OF A TRAPEZIUM

geometrical centres. A proof for this, on the same lines as that for the rectangle given above, may be derived as a useful exercise for the reader.

Centroid of a Triangle. Imagine the triangle to be divided into strips parallel to *AC*, Fig. 57. The centroid of each strip will lie at the centre of its length and all these centres will lie on the line *BD* which is the median bisecting *AC*. Therefore the centroid lies somewhere in *BD*. Similarly, taking strips parallel to *AB*, the centroid must lie somewhere in the median *CE* bisecting *AB*. Thus the centroid of a triangle lies at the point of intersection of the medians. By geometry it can be shown that *EG* is one-third of *EC*, and that the perpendicular height from the base to the centroid is one-third of the perpendicular height of the triangle.

Centroid of a Trapezium. To find the centroid of a trapezium *ABCD*, Fig. 58, of which *AB* and *CD* are the parallel sides, produce *CD* and *AB* as shown, making *DE* and *CH* equal to *AB* and *AF* and *BG* equal to *CD*. Join *EG* and *FH* so that they intersect at *G* which will be the centroid of the original trapezium.

Centroid of a Built-up Figure, by Moments. The centroid of an area built up from simple geometrical figures can be determined by taking moments of area of the component figures about two axes at right angles. The method employed can be best understood by studying the following examples.

EXAMPLE 1. Find the distance of the centroid of the area $ABCDEIJK$, Fig. 59, from the sides AB and AK.

Let the centroid G be a distance $\bar{x}$ from AB and $\bar{y}$ from AK.
Divide the figure into three rectangles $AHJK$, $EFHI$, and $BCDF$.
Area of whole figure = sum of areas of three rectangles.

$$= (3 \times 1) + (4 \times 1\tfrac{1}{2}) + (4 \times 2) \text{ in.}^2$$
$$= 3 + 6 + 8 = 17 \text{ in.}^2$$

Also,

Area of whole figure × distance of centroid from a given axis = sum of moments of area of the three rectangles about that axis.

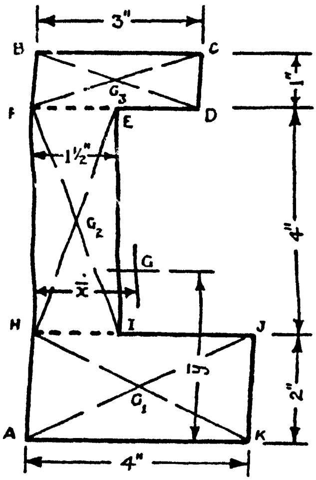


FIG. 59. CENTROID OF A BUILT-UP FIGURE

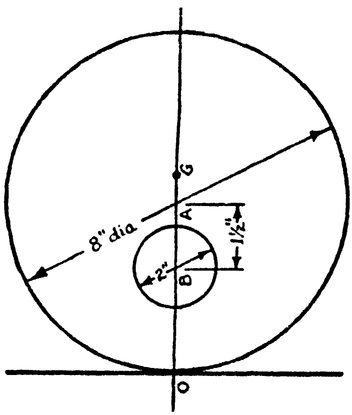


FIG. 60. CENTROID OF A FIGURE WITH A PART REMOVED

Now, for the purpose of taking moments, the area of each rectangle can be considered to act at its centroid which will be, respectively, G_1, G_2, or G_3.

Thus, taking moments about AB—

$$17\bar{x} = (3 \times 1) \times 1\tfrac{1}{2} + (4 \times 1\tfrac{1}{2}) \times \tfrac{3}{4} + (4 \times 2) \times 2$$
$$= 4\tfrac{1}{2} + 4\tfrac{1}{2} + 16 = 25$$
$$\bar{x} = \tfrac{25}{17} = \underline{\underline{1\tfrac{8}{17} \text{ in.}}}$$

Similarly, taking moments about AK—

$$17\bar{y} = (3 \times 1) \times 6\tfrac{1}{2} + (4 \times 1\tfrac{1}{2}) \times 4 + (4 \times 2) \times 1$$
$$= 19\tfrac{1}{2} + 24 + 8$$
$$= 51\tfrac{1}{2}$$
$$\bar{y} = \frac{51\tfrac{1}{2}}{17} = \underline{\underline{3\tfrac{1}{34} \text{ in.}}}$$

EXAMPLE 2. A circular area, centre A, 8 in. diameter, Fig. 60, has a small circle centre B, 2 in. diameter, removed. Find the centre of area of the remainder if $AB = 1\frac{1}{2}$ in.

It is evident from the symmetry of the figure that the centroid G of the remainder will lie on OX, the diameter passing through A and B. It now remains to locate G on this diameter.

Let OY be the tangent to the circle at O; then, taking moments about OY.

Area of remainder $\times$ GO = moment of area of original circle about OY — moment of area of small circle.

$$\left(\frac{\pi}{4}\times 8^2 - \frac{\pi}{4}\times 2^2\right)\times GO = \left(\frac{\pi}{4}\times 8^2\right)\times 4 - \left(\frac{\pi}{4}\times 2^2\right)\times 2\tfrac{1}{2}$$

$$(8^2 - 2^2)\times GO = 8^2\times 4 - 2^2 \times 2\tfrac{1}{2}$$

$$60\,GO = 256 - 10 = 246$$

$$GO = \tfrac{246}{60} = 4{\cdot}1 \text{ in.}$$

or $$AG = GO - 4 = \underline{\underline{0{\cdot}1 \text{ in.}}}$$

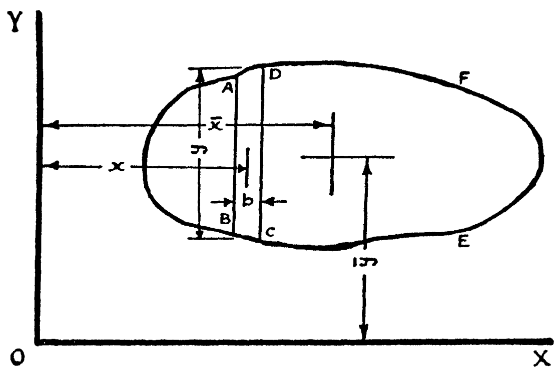


FIG. 61. CENTROID OF AN IRREGULAR PLANE FIGURE

Note that *if an area is symmetrical about an axis the centroid will lie on that axis.* This fact can often be used, as above, to save unnecessary labour.

Centroid of an Irregular Plane Figure. To find the distance of the centroid of an irregular plane figure, $ABCEFD$, Fig. 61, from co-ordinate axes OX and OY, consider a thin strip $ABCD$, parallel to OY, of area a, whose centroid is at a distance x from OY.

First moment of area of strip about $OY = ax$

Now if y is the average width, parallel to OY, of the strip and b is its breadth;

$$a = by$$

$\therefore$ First moment of area of strip about $OY = byx$.

And, if the whole area is divided into a number of such strips;

First moment of whole area about OY = sum of first moments of all the strips.

$= \Sigma byx$

If the strips are small, y can be taken as the mean width of the strip and x can be taken as the distance from OY to the centre of the strip. Thus b, y, and x can all be measured and byx can be found for each strip and summed to give the first moment of area of the whole figure. The work is best tabulated as follows—

Strip Number	Breadth of strip $= b$	Mean width $= y$	Area $a = by$	x	ax
			Σa		Σax

The position of the centroid from OY is then given by—

$$\bar{x} = \frac{\Sigma ax}{\Sigma a}, \text{ since } a = \text{area of whole figure.}$$

The process is then repeated taking strips parallel to OX to find the value of $\bar{y}$ the distance of the centroid from OX.

LABORATORY WORK

EXPERIMENT. *To find the centroid of a figure experimentally.*

Draw the figure on stiff cardboard or sheet metal and cut it out. Hang the sheet up, as shown in Fig. 52, and find its centroid by the method described above (page 63) for determining the centre of gravity of a lamina.

Check your results graphically by the method given on page 69.

EXERCISES

1. Weights of 10 lb, 12 lb, 8 lb, and 9 lb are situated respectively at the corners A, B, C and D of a square. Find the position of the centre of gravity of these loads with respect to the sides AB and AD.

2. An L-shaped lamina is 4 in. high, 3 in. wide, and each leg is ½ in. thick. Find its centre of gravity.

3. A circular lamina, 9 in. in diameter, has a square hole of 2 in. side cut out of it. If the resulting area is symmetrical about a diameter and one side of the square passes through the centre of the circle, find the position of its centre of gravity.

4. The arm of a T-section is 6 in. wide × ½ in. thick and the overall depth of the T is 8 in. If the thickness of the stem is ¾ in., find the position of the centre of area of the section.

5. The base *AB* of a trapezium, Fig. 58, is 8 in. long, the perpendicular distance between *AB* and *CD* is 5 in. and angles *DAB* and *ABC* are 80° and 70° respectively. Find the position of the centre of area of the figure by the construction (page 67) and check your result by dividing the figure into strips.

6. A cast-iron beam has an overall depth of 12 in. The top flange is 6 in wide × 1 in. thick, the bottom flange is 10 in. wide × 1½ in. thick, and the thickness of the web is 1 in. Find the position of the centre of area of a cross-section of this beam.

7. Three weights of 4 lb, 8 lb and 12 lb respectively, act at the corners *A*, *B* and *C* of an equilateral triangle. Find the position of their centre of gravity.

8. A rod carries loads of 10 lb, 12 lb, 9 lb and 5 lb at distances of 3 in., 12 in., 20 in., and 30 in. from one end. Find the point at which the rod will balance. Neglect the weight of the rod.

CHAPTER X

LOAD BEARING STRUCTURES AND THEIR EQUILIBRIUM

THE previous chapters have been devoted to the study of forces without special thought as to the things to which these forces may be applied.

Load Bearing Structures. The particular aspect which must now be considered is the effect of such forces upon load bearing structures, by which is meant any assemblage of bars, blocks or other materials capable of withstanding a system of loads. Such structures fall, roughly, into two main groups—

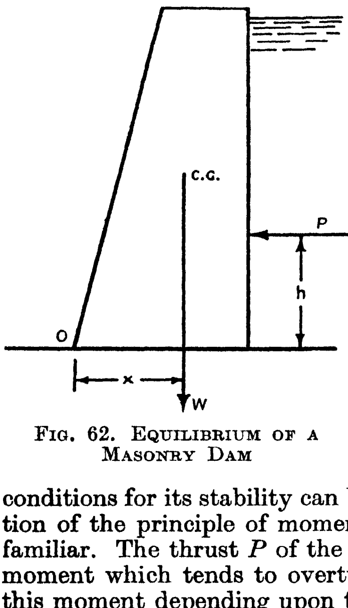


FIG. 62. EQUILIBRIUM OF A MASONRY DAM

1. Mass structures which depend upon their weight for their stability.
2. Stress structures, depending partly upon the strength and arrangement of the materials from which they are made, and partly upon the distribution of the internal forces which are set up in these materials to resist the applied loads.

Mass Structures. Since the stability of a mass structure depends upon its weight it will usually be a solid mass of heavy material. The masonry dam, shown in Fig. 62, is a typical example and the conditions for its stability can be determined readily by the application of the principle of moments with which the reader is already familiar. The thrust P of the water retained by the dam creates a moment which tends to overturn it about its toe O, the value of this moment depending upon the height h above the base at which the thrust acts. The weight w of the structure, acting downwards through its centre of gravity, will cause an opposing moment wx, where x is the perpendicular distance from the line of action of w to O. For the limiting condition when the dam is about to overturn,

$$Ph = wx$$

For design purposes the value of Ph must be made less than wx in order to provide a margin of safety.

Stress Structures. Frames, trusses and lattice girders, beams and plate girders, portals and arches are all relatively light-weight structures which depend for their strength upon the way in which the internal forces, or stresses, in the structure are distributed and upon the ability of the materials from which they are made to withstand these stresses. There is no single way of dealing with these structures, but the reader will find some of the above types dealt with in detail in later chapters.

Reactions. The purpose of any load bearing member of a structure is to transmit the loads which act upon it to certain fixed points. Thus the loads acting upon the roof truss, Fig. 63 (*a*), are

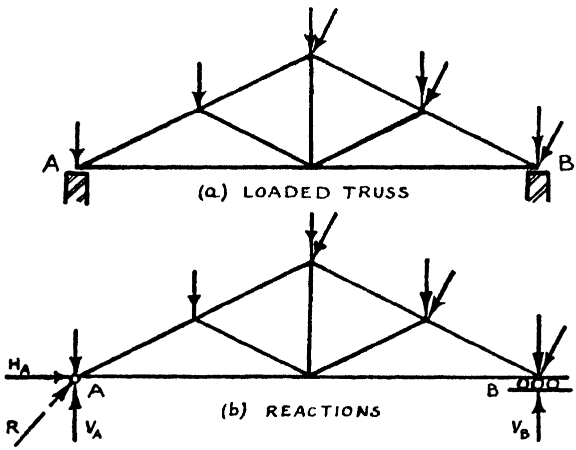


FIG. 63. REACTIONS OF A ROOF TRUSS

transmitted to the supporting piers at *A* and *B*. Since the truss remains at rest, the horizontal and vertical components of the loads must be balanced by forces, known as *reactions*, exerted between the piers and the truss at *A* and *B*. The general conditions of equilibrium, found in Chapter VIII, are—

Algebraic sum of horizontal forces = 0

Algebraic sum of vertical forces = 0

Algebraic sum of the moments of all forces about any point = 0

and these may be applied to the equilibrium of the truss, the reactions being the unknown quantities and the applied loads the known. Since three algebraic equations can only yield solutions for three unknowns, only three reactions are required to maintain the truss in equilibrium. These reactions are shown in Fig. 63 (*b*), and comprise horizontal and vertical reactions at *A* and a single vertical reaction at *B*, which might be provided by a pin point at A and frictionless rollers at B. Note that the horizontal and vertical reactions at A may be considered as components of an inclined reaction R shown dotted.

Redundant Reactions. The bridge girder, shown in Fig. 64, is pinned at the left-hand abutment A and runs on rollers at B, but it is also supported on a pier in mid-stream at C. Four reactive forces are acting, namely vertical and horizontal components, V_A and H_A, at A, a vertical reaction V_B at B and a vertical reaction V_C at C. Thus there are four unknown forces, but only three equations which can be written down from consideration of the statical equilibrium of the girder. As four equations are required to give solutions for four unknowns, the reactions cannot be determined

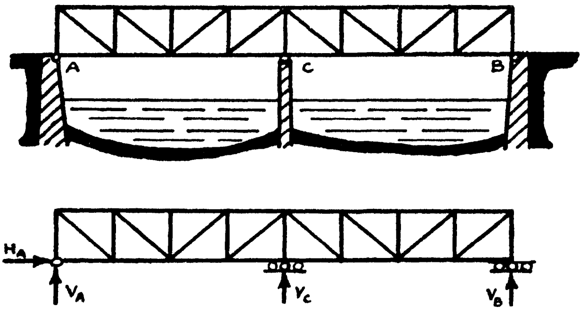


FIG. 64. BRIDGE GIRDER WITH REDUNDANT REACTION

by statics alone, and the problem is said to be *statically indeterminate*. If, however, one of the vertical reactions were to be omitted, for instance V_C if the pier were removed, the problem could be solved statically; therefore V_C is called a *redundant reaction*. Even if H_A is zero the girder will still have a redundant reaction, for one of the equations, $\Sigma H = 0$, deals only with horizontal forces, and there will still be three vertical reactions and only two equations to determine them, namely $\Sigma V = 0$ and $\Sigma M = 0$. In the work which follows only statically determinate problems will be dealt with, but it is desirable to know that there are cases which cannot be solved by statics alone.

Determination of Reactions by Calculation. A beam of length L, Fig. 65, is supported on knife edges at its ends A and B, in which state it is said to be *simply supported*. A vertical load w is applied at a point C, a distance a from A and b from B, and for simplicity the weight of the beam will be considered to be negligible compared to w. Vertical reactions R_A and R_B will be produced at A and B and, since w has no horizontal component, there will be no horizontal reaction.

For equilibrium—

$$\text{Sum of moments about any point} = 0$$

Taking moments about B,

$$R_A \times L = w \times b$$

$$\therefore R_A = w \frac{b}{L}$$

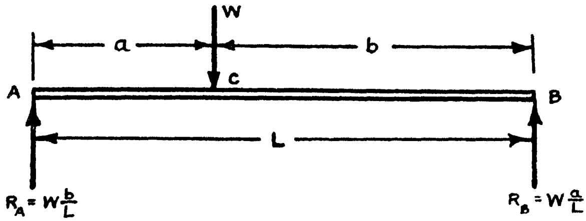


FIG. 65. REACTIONS DUE TO A SINGLE POINT LOAD

To find R_B, either take moments about A giving,

$$R_B \times L = w \times a$$

$$R_B = w \frac{a}{L}$$

or apply the condition,

$$\text{Sum of vertical forces} = 0$$

$$\therefore R_A + R_B = w$$

substituting $R_A = \frac{wb}{L}$,

$$R_B = w - \frac{wb}{L} = w\left(\frac{L - b}{L}\right)$$

but, $$L - b = a$$

$$\therefore R_B = w \frac{a}{L}$$

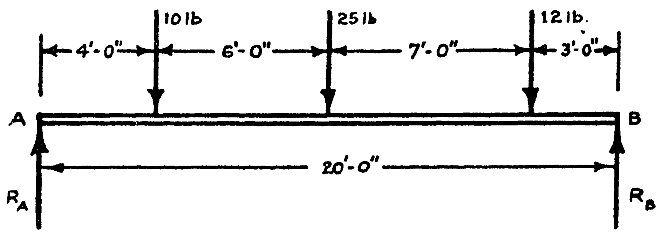


FIG. 66. REACTIONS DUE TO THREE POINT LOADS

This method can be extended to deal with any number of loads as can be seen in the following example.

EXAMPLE. A beam 20 ft long, Fig. 66, carries loads of 10 lb, 25 lb and 12 lb as shown, and is simply supported at its ends A and B. Neglecting the weight of the beam, calculate the reactions at A and B.

Taking moments about B—

Sum of clockwise moments — sum of anti-clockwise moments = 0

$$(R_A \times 20) - (10 \times 16 + 25 \times 10 + 12 \times 3) = 0$$

$$20\,R_A = 160 + 250 + 36 = 446 \text{ lb-ft}$$

$$R_A = \tfrac{446}{20} = \underline{\underline{22 \cdot 3 \text{ lb}}}$$

also, Algebraic sum of vertical forces = 0

$$R_A + R_B - 10 - 25 - 12 = 0$$

$$R_B = 47 - R_A$$

$$= 47 - 22 \cdot 3 = \underline{\underline{24 \cdot 7 \text{ lb}}}$$

Reactions for Uniformly Distributed Loads. So far the loads have been considered to be concentrated at individual points but in many cases the load is *uniformly distributed* along the beam. Thus, in

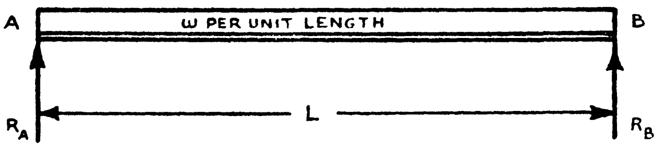


FIG. 67. REACTIONS DUE TO A UNIFORMLY DISTRIBUTED LOAD

Fig. 67 a uniformly distributed load of w per unit length is carried by a beam of length L.

Total load on beam = wL

This may be considered to be acting at the centre of gravity of the load, i.e. at mid-span. The determination of R_A and R_B now proceeds as before.

Taking moments about B—

$$R_A \times L = wL \times \frac{L}{2}$$

$$\therefore\ R_A = \frac{wL}{2}$$

Also

$$R_A + R_B = wL$$

$$\therefore\ R_B = \frac{wL}{2}$$

The following numerical examples show the method of dealing with distributed loads which do not cover the whole span and with combinations of distributed and point loads. Uniformly distributed loads are of common occurrence in practice since all structures have to carry their own weight, which is usually a load of this type, in addition to the other superimposed loads.

EXAMPLE 1. A beam 18 ft long, Fig. 68, weighing 40 lb per foot run, is simply supported at its ends A and B and carries a further distributed load of 60 lb per foot run over a length of 6 ft from A. Find the reactions R_A and R_B.

Centre of gravity of 40 lb/ft load is at 9 ft from A.

Centre of gravity of 60 lb/ft load is at 3 ft from A.

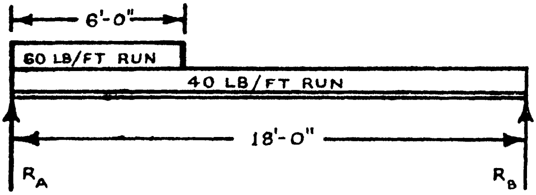


FIG. 68. REACTIONS FOR A COMBINATION OF UNIFORMLY DISTRIBUTED LOADS

Taking moments about B—

$$R_A \times 18 = (40 \times 18) \times 9 + (60 \times 6) \times 15 \text{ lb-ft}$$
$$= 6480 + 5400 = 11{,}880 \text{ lb-ft}$$
$$R_A = \underline{\underline{660 \text{ lb}}}$$

Also

$$R_A + R_B = \text{sum of loads}$$
$$= 40 \times 18 + 60 \times 6 \text{ lb}$$
$$= 720 + 360 = 1080 \text{ lb}$$
$$R_B = 1080 - 660 = \underline{\underline{420 \text{ lb}}}$$

EXAMPLE 2. A beam 20 ft long, Fig. 69, is supported at A and B, 17 ft apart, and overhangs 3 ft at the left-hand end. It carries a uniformly distributed load of 40 lb per foot run over a distance of 8 ft from the left-hand end and loads of 100 lb and 200 lb at 9 ft and 3 ft from the right-hand end. Find the reactions at A and B.

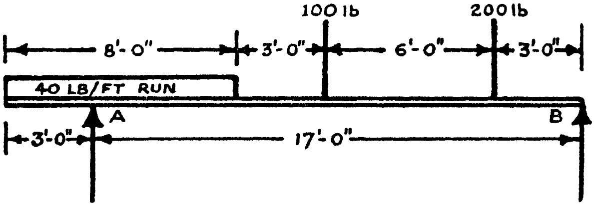


FIG. 69. REACTIONS DUE TO A COMBINATION OF POINT LOADS AND UNIFORMLY DISTRIBUTED LOADS

Centre of gravity of the distributed load is 4 ft from the left-hand end or 1 ft to the right of A.

Taking moments about B—

$$R_A \times 17 = (40 \times 8) \times 16 + 100 \times 9 + 200 \times 3 \text{ lb-ft}$$
$$= 5120 + 900 + 600 = 6620 \text{ lb-ft}$$
$$R_A = \underline{\underline{389 \text{ lb}}}$$

Also

$$R_A + R_B = 40 \times 8 + 100 + 200 \text{ lb}$$
$$R_B = 620 - R_A = 620 - 389 \text{ lb}$$
$$= \underline{\underline{231 \text{ lb}}}$$

Reactions Determined by Link Polygon. When a truss or girder carries a number of loads, as in Fig. 70 (*a*), the reactions can be determined by using the link polygon described in Chapter VII,

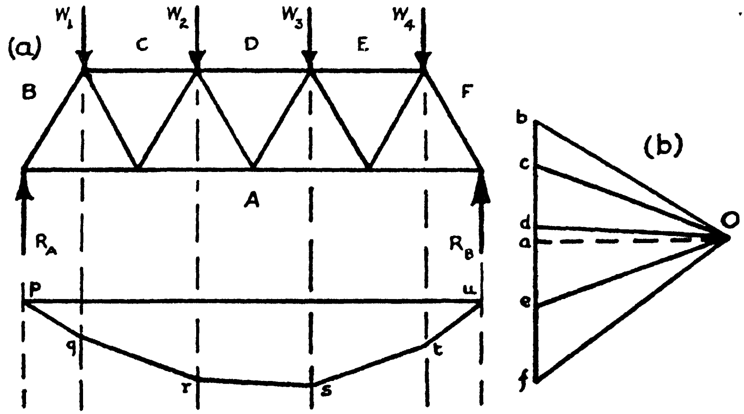


FIG. 70. REACTIONS DETERMINED BY LINK POLYGON

since the loads and reactions form a system of non-concurrent forces which are in equilibrium. Adopting Bow's notation, draw the force diagram, Fig. 70 (*b*), which will be the straight line *bcdef*. Choose any pole O and draw *Ob*, *Oc*, *Od*, *Oe*, *Of*. Going back to Fig. 70 (*a*) project the lines of action of the forces and reactions downwards and, starting from p on the line of R_A, construct the link polygon *pqrstu*, drawing *pq* parallel to *Ob*, *qr* parallel to *Oc*, and so on. Complete the link polygon by joining *pu*, then, by drawing *Oa* parallel to *pu*, the position of a can be located in the force diagram and *ab* and *af* represent R_A and R_B to the scale chosen.

LABORATORY WORK

EXPERIMENT 1. *Experimental measurement of beam reactions.*

A light wooden beam, Fig. 71, is supported on two spring balances set a known distance apart. Loads w_1, w_2, etc., are hung at various positions on

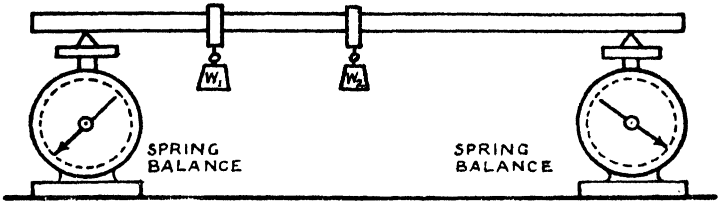


FIG. 71. EXPERIMENTAL DETERMINATION OF REACTIONS

the beam. Record the reactions as measured on the spring balances and check the results by calculation.

EXERCISES

1. A uniform beam, 20 ft long, weighs $1\frac{1}{2}$ t and is supported at its ends A and B. A uniformly distributed load of $\frac{1}{2}$ t/ft run extends over a length of 9 ft from A and a concentrated load of 3 t is applied at a point 5 ft from B. Calculate the reactions at A and B.

2. A beam, 12 ft long, carries loads of 2 t, 8 t and 4 t at distances of 3 ft, 6 ft and 9 ft, respectively, from its left-hand end. Calculate the reactions if the beam is supported at its ends.

3. A beam AB, 30 ft long, is supported at two points, C and D, 20 ft apart, and has an overhang AC of 6 ft. The beam carries a load of 9 cwt at A, a uniformly distributed load of 2 cwt per foot run between C and D, and a point load of 12 cwt at B. Determine the value of the reactions at C and D.

4. A horizontal platform is supported on three piers, A, B and C, forming a triangle in plan. $AB = 6$ ft, $AC = 8$ ft, $BC = 8$ ft. The centre of gravity of the platform and the load carried by it is 5 ft from A and 4 ft from B. Find the proportion of the load carried by each of the three piers. Show that if there were four piers instead of three, the reactions could not be determined without further information.

5. In Fig. 70 (*a*), the truss carries the following loads: $BC = 3000$ lb; $CD = 6000$ lb; $DE = 7500$ lb; $EF = 5000$ lb. Determine the reactions R_A and R_B if the truss has a span of 32 ft and the inclined members are at an angle of 60° to the horizontal.

6. A beam, 18 ft span, carries loads of 3 t, 5 t and 7 t at distances of 3 ft, 7 ft and 12 ft, respectively, from one support. Find the reactions at the supports by a graphical method, neglecting the weight of the beam.

7. A roof truss ABC is supported by a pin-joint at A and rests on a flat plate at B. The rafters AC and BC are inclined at 30° to the horizontal and the span AB of the truss is 30 ft. A load of 4000 lb is uniformly distributed along AC and acts at right angles to the rafter. Determine the reactions at A and B.

8. A flat equilateral triangular plate, of 4 ft side, is supported horizontally by three legs, one at each corner. A vertical force of 150 lb is applied to the plate at a point which is 3 ft from one leg and 2 ft from another. Find the force in each leg.

CHAPTER XI

STRESS AND STRAIN

So far only external forces have been dealt with, that is the loads and reactions which act upon a structure; but it is clearly not enough that the structure should be in equilibrium under these forces, it must also be strong enough to resist them.

Stress. The tie bar, shown in Fig. 72, has a force P applied at one end and is fixed securely to the wall at the other end. As the bar cannot move the reaction R at the wall must be equal and opposite to P. Now suppose that the bar is cut into two portions A and B at section XX. Portion A would immediately move away

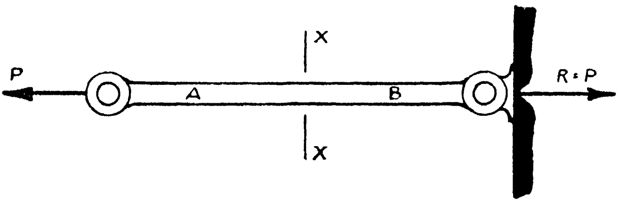


FIG. 72. STRESS IN A TIE BAR

unless a foice P were applied at its right-hand end. Thus, in the uncut bar, portion B must exert a pull on portion A equal and opposite to the external force P. This internal force exerted by one part of the bar on the other is called a *stress*. The pull P is said to cause a stress in the bar which is withstood by the cohesion of the material.

If the area of cross-section at XX is A the stress per unit area will be $\frac{P}{A}$. This is known as the *intensity of stress* or *unit stress* at XX and is usually denoted by f.

$$\text{Intensity of stress} = \frac{\text{load}}{\text{cross-sectional area}}$$

or

$$f = \frac{P}{A}$$

The units used for measuring intensity of stress will be those of force per unit area such as lb/in.^{2}, tons/ft^2, kg/cm^2, according to the system of units employed.

Although the term stress means, properly, the total internal force transmitted across a section of a body, it is frequently used as an abbreviation for intensity of stress. This practice should only be adopted where there is no likelihood of confusion and the appropriate units, e.g. lb for stress and lb/in.^{2} for intensity of stress, should be given to ensure clarity.

Direct Stresses. If a member of a structure is subjected to forces acting axially along it, as in Fig. 73, it is said to be under *direct*

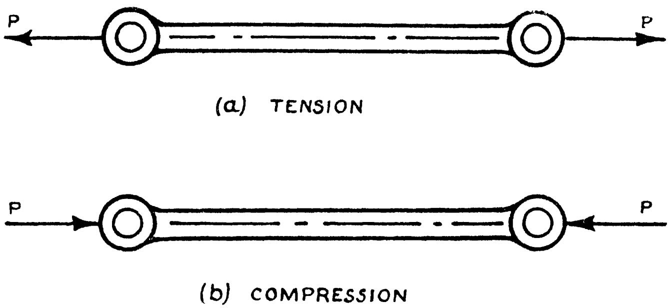


FIG. 73. DIRECT STRESSES

stress. If the applied force P tends to stretch the member, as in Fig. 73 (a), it causes a *tensile stress* which has an intensity $t = \frac{P}{A}$ where A is the cross-sectional area of the member taken at right angles to its axis.

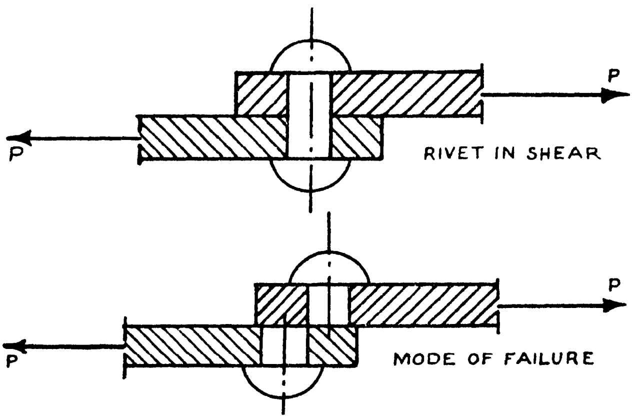


FIG. 74. SHEAR STRESS

If the applied force tends to crush the member, as in Fig. 73 (b), it causes a *compressive stress* which will also have an intensity $c = \frac{P}{A}$.

Tangential or Shear Stress. Shearing stress occurs when a material is subjected to opposing forces which are not co-axial. Fig. 74

shows two plates riveted together and subjected to equal and opposite forces P which do not act in the same straight line. The right-hand plate exerts a pull equal to P on the left-hand plate, which is transmitted across section AB by the material of the rivet. Failure would occur by sliding of the two parts of the rivet along the section AB as indicated in the lower sketch. The material of the rivet is said to be in *shear* along AB and the intensity of shear stress $q = \frac{P}{A}$ where A is the *cross-sectional area* of the rivet at section AB.

Ultimate Stress, Working Stress and Factor of Safety. There is a limit to the intensity of stress which a given material can withstand. When this *ultimate stress* is exceeded the material fails or breaks; thus the greatest load which a member of a structure can withstand will be equal to the product of the ultimate stress and the cross-sectional area. In practice it is not desirable to stress any material to the limit as provision should be made for unforeseen load. For this reason a lower intensity of stress, known as the *working stress*, is adopted as a maximum for design purposes. The ratio $\frac{\text{ultimate stress}}{\text{working stress}}$ is known as the *factor of safety*, and may have a value of from 2 to 4. Thus if a material fails at a stress intensity of 27 t/in.^{2} and a factor of safety of 3 is adopted, the working stress must not exceed $\frac{27}{3} = 9$ t/in.^{2}

EXAMPLE 1. A tie-bar, of rectangular section, 2 in. wide, carries a pull of 12 t. If the permissible working stress is not to exceed 8 t/in.^{2}, what must be the thickness of the bar?

$$\text{Sectional area required} = \frac{\text{load}}{\text{permissible stress}} = \frac{12}{8} = 1\tfrac{1}{2}\text{ in.}^2$$

$$\text{Thickness} = \frac{\text{area}}{\text{width}} = \frac{1\frac{1}{2}}{2} = \underline{\underline{\tfrac{3}{4}\text{ in.}}}$$

EXAMPLE 2. A hollow cast-iron column is of 3 in. internal and 4 in. external diameter and carries a load of 60 t. Calculate the intensity of stress in the material and the factor of safety if cast iron fails in compression at 45 t/in.^{2}

$$\text{Cross-sectional area} = \frac{\pi}{4}(4^2 - 3^2)\text{ in.}^2 = 5\tfrac{1}{2}\text{ in.}^2$$

$$\therefore \text{Intensity of stress} = \frac{\text{load}}{\text{area}} = \frac{60}{5\frac{1}{2}} = \underline{\underline{10{\cdot}45\text{ t/in.}^2}}$$

$$\text{Factor of safety} = \frac{\text{ultimate stress}}{\text{working stress}}$$

$$= \frac{45}{10 \cdot 45} = \underline{\underline{4 \cdot 3}}$$

EXAMPLE 3. Two ½ in. thick mild-steel plates are riveted together, as shown in Fig. 74. The load transmitted across the joint is 10 t. Assuming that the joint fails by shearing of the rivets, how many ¾ in. diameter rivets are required if the working stress in the rivets is not to exceed 5 t/in.^{2}?

$$\text{Cross-sectional area of } \tfrac{3}{4} \text{ in. rivet} = 0 \cdot 44 \text{ in.}^2$$

$$\begin{aligned} \text{Load transmitted by one rivet} &= \text{stress} \times \text{area} \\ &= 5 \times 0 \cdot 44 \text{ t} \\ &= 2 \cdot 2 \text{ t} \end{aligned}$$

$$\therefore \text{ No. of rivets required} = \frac{10}{2 \cdot 2} = \underline{\underline{5 \text{ rivets.}}}$$

Strain. Whenever a body is loaded or subjected to stress it will become deformed, although in some cases this may not be detected by the eye. This alteration of form and dimensions is called strain and each of the stresses described above produces its own special type of strain.

Longitudinal Strain. When a member is subjected to a direct tension or compression it undergoes a tensile or compressive longitudinal strain which is measured as the change in length of the member per unit length; or

$$\text{Strain} = \frac{\text{final length} - \text{original length}}{\text{original length}}$$

Since it is measured as a ratio, strain has no units or dimensions but is a pure number.

EXAMPLE. Under load a member 3 ft long shortens by ½ in., what compressive strain has occurred?

$$\text{Strain} = \frac{\text{change in length}}{\text{original length}}$$

$$= \frac{\tfrac{1}{2}}{3 \times 12} = \frac{1}{72}$$

Note that both change in length and original length must be expressed in the same units, in this case inches.

Transverse Strain. When a material is in tension or compression not only is its length altered, but also its transverse dimensions, i.e. width and thickness. A bar in tension becomes thinner and one in compression grows thicker. These changes are called *transverse strains* and are measured in a similar way to longitudinal strains.

$$\text{Transverse strain} = \frac{\text{change in width}}{\text{original width}}$$

It has been found that, for a given material, the ratio of transverse strain to longitudinal strain is a fixed quantity, known as *Poisson's ratio*, denoted by $\frac{1}{m}$.

Thus,
$$\frac{1}{m} = \frac{\text{transverse strain}}{\text{longitudinal strain}}$$

For common metals m has a value of from 3 to 4.

Volumetric Strain. If a body is subjected to a uniform pressure over its whole surface, as in the case of a submarine lying at the bottom of the ocean, its volume will change slightly and it will suffer volumetric strain.

If, V_1 = original volume of the body.

V_2 = new volume of the body.

$$\text{Volumetric strain} = \frac{V_2 - V_1}{V_1}$$

Shear Strain. The action of a shear stress differs from that of a direct stress in that it causes a change in the shape of the body.

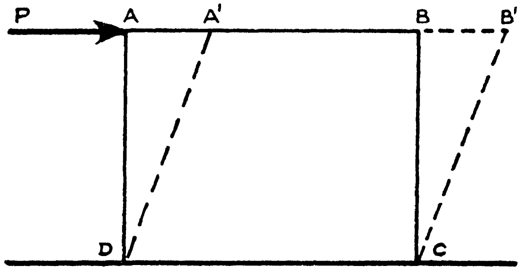


FIG. 75. SHEAR STRAIN

Thus if a shearing force P is applied to the face AB of the rectangular block $ABCD$, Fig. 75, it will deform the rectangle to a rhombus $A'B'CD$. The shear strain is measured as the angle θ in radians through which AD rotates in deforming to $A'D$. Note that a radian is the angle subtended at the centre of a circle by an arc of length equal to the radius: 2π radians are equal to 360°. For metals θ is very small and it is sufficiently accurate to take $\theta = \tan\theta$ giving

$$\text{Shear strain} = \frac{AA'}{AD}$$

Elasticity. When a body is loaded and becomes strained, mechanical work is expended since the point of application of the load will have moved. When the load is removed the body tries to return to its original form and the work done in deforming it may be recovered. A material is said to have perfect *elasticity* if, when a straining force is removed, it returns completely to its original shape and dimensions and if the energy given out during recovery is equal to that used in straining it.

Many materials are nearly perfectly elastic provided that they are not stressed beyond a certain limit depending on the nature of the material and the type of stress applied. This limiting stress is known as the *elastic limit*, and if it is exceeded the material will not recover its original form and dimensions completely but is said to have suffered a *permanent set.*

Hooke's Law, Modulus of Elasticity. Experiments have led to the formulation of a law which states: "Strains are proportional to the stresses producing them, provided that the elastic limit is not exceeded." Known as *Hooke's law*, after its discoverer, it is very closely obeyed by most metals. It may be stated as—

$$\text{Strain} \propto \text{stress}$$

or

$$\frac{\text{Stress}}{\text{Strain}} = \text{constant}$$

This constant is called the *modulus of elasticity* and is known by various names according to the type of stress involved. The units in which it is measured will be the same as those used for stress since strain is a pure number.

For direct stresses, tensile or compressive, this constant is *Young's modulus*, denoted by E, and is given by—

$$E = \frac{\text{direct stress}}{\text{longitudinal strain}}$$

The name *bulk modulus* is given to the constant if the material suffers volumetric strain.

$$\text{Bulk modulus } K = \frac{\text{pressure stress}}{\text{volumetric strain}}$$

For shear stress the constant is known as the *modulus of rigidity* and,

$$\text{Modulus of rigidity } N = \frac{\text{shear stress}}{\text{shear strain}}$$

These elastic moduli are of great importance for, if the value of the appropriate modulus is known, the strain produced by a given stress, or the stress required to cause a given strain, can be calculated.

EXAMPLE 1. A mild-steel rod, $1\frac{1}{2}$ in. in diameter, is subjected to a tension of 15 t. What strain will occur and what will be its extension if its length is 10 ft? Young's modulus E for mild steel = 13,000 t/in.²

$$\text{Cross-sectional area of rod} = \frac{\pi}{4}(1\cdot5)^2 = 1\cdot77 \text{ in.}^2$$

$$\text{Tension in rod} = 15 \text{ t}$$

$$\text{Tensile stress intensity} = \frac{15}{1\cdot77} = 8\cdot5 \text{ t/in.}^2$$

$$\text{Strain} = \frac{\text{stress}}{\text{E}} = \frac{8\cdot5}{13{,}000} = \underline{\underline{0\cdot00065}}$$

$$\text{Extension of 10 ft length} = \text{length} \times \text{strain}$$
$$= 10 \times 12 \times 0\cdot00065 = \underline{\underline{0\cdot078 \text{ in.}}}$$

EXAMPLE 2. A tie bar, 2 in. wide × $\frac{3}{8}$ in. thick × 12 ft long, is $\frac{1}{16}$ in. too short. What stress will be induced in the bar if it is forced into position and what force will be required? Take E for the material of the bar as 27×10^6 lb/in.²

To force the tie into position it must be stretched $\frac{1}{16}$ in.

$$\therefore \text{Strain} = \frac{\frac{1}{16}}{12 \times 12} = \frac{1}{16 \times 144}$$

$$\text{Stress} = \text{E} \times \text{strain}$$

$$\therefore \text{Stress in tie} = \frac{27 \times 10^6}{16 \times 144} = \underline{\underline{11{,}700 \text{ lb/in.}^2}}$$

$$\text{Force required to strain tie} = \text{stress} \times \text{area}$$
$$= 11{,}700 \times 2 \times \tfrac{3}{8} \text{ lb}$$
$$= \underline{\underline{8800 \text{ lb or } 3\cdot9 \text{ t.}}}$$

This example serves to show that all parts of a structure must be made accurately to size and that attempts to force ill-fitting members into position may set up high stresses in them which will reduce the load which they can carry safely. Similar large stresses may be set up if structures are not able to expand or contract freely under changes of temperature, as may be seen from the following simple example.

EXAMPLE. A brass bar is heated to 160° F. and has its ends clamped rigidly. It is now cooled to 50° F. What stress will be set up in the bar if no change of length occurs? For brass E = 5700 t/in.² and coefficient of expansion = 0·00001 per ° F.

Fall in temperature = 160 − 50 = 110° F. If bar were free to contract—

$$\text{Change in length per unit length} = 110 \times 0\cdot00001$$
$$= 0\cdot0011$$

Since bar cannot contract—

Tensile strain due to stress in bar = 0·0011

$\therefore$ Stress = E $\times$ strain = 5700 $\times$ 0·0011 t/in.^{2}

Tensile stress in bar = $\underline{\underline{6{\cdot}27 \text{ t/in.}^2}}$

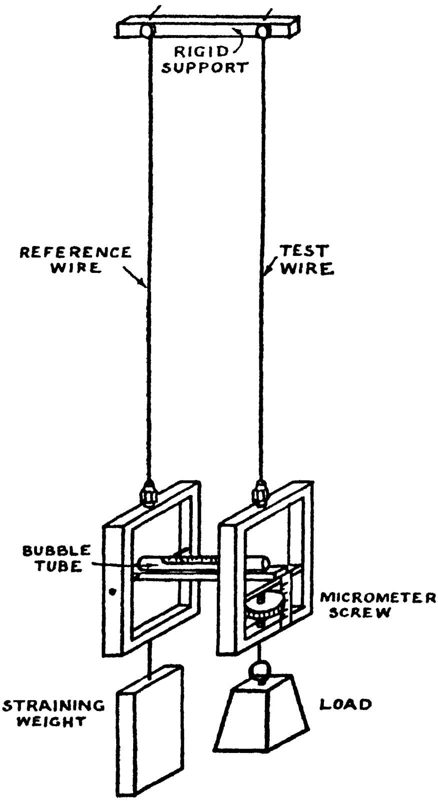


FIG. 76. SEARLE'S APPARATUS FOR DETERMINING YOUNG'S MODULUS FOR A WIRE

LABORATORY WORK

EXPERIMENT 1. *Determination of Young's modulus for a wire using Searle's apparatus*

The apparatus used is shown in Fig. 76. Two wires about 6 ft long support a bubble tube. The left-hand wire is kept taut by a straining weight and forms a standard against which the extension of the wire under test can be measured. One end of a bubble tube is pivoted at the centre of a ring suspended from the standard wire and the other end is supported on the point of a micrometer screw carried by the test wire.

Adjust the micrometer until the bubble is at the centre of its run and read the micrometer. On applying a load, the test wire will extend and the bubble will cease to be central. Bring it back to its original position by turning the micrometer screw. The distance moved by the point of the screw will be the extension of the wire under the given load and is given by the difference in the two readings of the micrometer.

Take a series of readings of load and extension in this way for increasing and decreasing loads. Measure the original length and diameter of the test wire, and calculate the tensile stress and strain in the wire for each reading. Plot a graph of stress against strain, drawing a mean straight line through the points. The slope of this line will give the mean value of $\frac{\text{stress}}{\text{strain}}$, i.e. of Young's modulus.

EXPERIMENT 2. *Tensile test to destruction*

The test piece used may be either rectangular or circular in cross-section and is about 14 in. long, having a parallel-sided centre section 9 in. long and enlarged ends which enable it to be held in the grips of the testing machine, Fig. 77. By turning the loading handle the lower grip can be moved down-

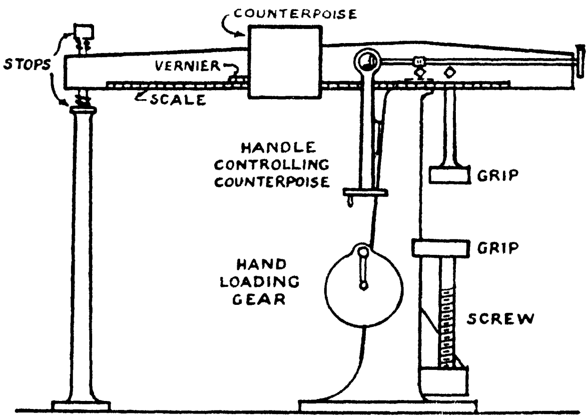


FIG. 77. TENSILE TESTING MACHINE

wards. The pull so exerted is measured by moving the counterpoise along the beam of the machine until the beam just floats free of the stops. The pull on the specimen can then be read off on the scale.

Determine the dimensions of the test piece at four or five points and from these calculate the average area of cross-section. Centre punch two marks 8 in. apart on the specimen and then fix the specimen in the testing machine. Apply the load gradually and for each load measure the extension on the 8 in. gauge length using callipers and a steel rule. This extension will be small until the *yield point* is reached. Continue the process until the maximum load is developed after which the specimen will be found to continue to extend rapidly, the beam of the testing machine falling, indicating a reduction of load. By running the counterpoise back try to keep the beam level until fracture occurs. This will give a rough value of the breaking load.

From the values so obtained plot a curve of tensile stress against strain and determine the yield stress, ultimate stress and breaking stress. A typical curve for mild steel is shown in Fig. 78.

EXERCISES

1. A round tie bar carries a pull of 14 t. Calculate the diameter of the bar if the safe stress is 8 t/in.^{2}

If the bar is 12 ft long, how much will it extend under this load? E = 13,000 t/in.^{2}

2. A steel bar, 4 in. wide, $\frac{1}{2}$ in. thick, and 20 ft long, carries a pull of 12 t. Find its extension in length and contraction in width and thickness under this load. E = 13,500 t/in.^{2}; $m = 3{\cdot}5$.

3. A $1\frac{1}{2}$ in. diameter tie bar, 20 ft long, lengthens $\frac{1}{16}$ in. under a pull of 7 t.

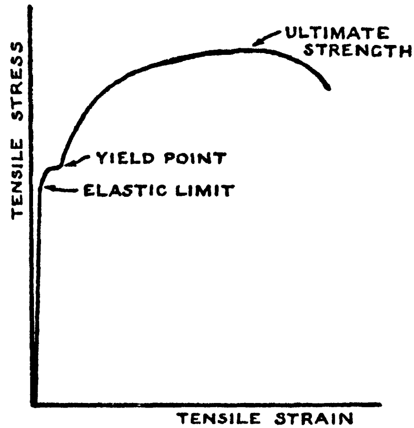


FIG. 78. STRESS-STRAIN CURVE FOR A TENSILE TEST TO DESTRUCTION

Find the intensity of tensile stress in the bar and the value of Young's modulus for the material.

4. A brass rod, 3 ft long and $\frac{1}{2}$ in. diameter, is cooled from a temperature of 180° F to 70° F. What force is required to prevent any change of length? E = 5700 t/in.^{2}; coefficient of expansion = 0·00001 per °F.

5. A short concrete column, 12 in.^{2}, is reinforced with four 1 in. diameter steel bars. Find the load which the column can carry if the stress in the concrete is not to exceed 600 lb/in.^{2} What will be the stress in the steel? Ratio of Young's modulus for steel to that for concrete is 18.

6. A load of 2500 lb is supported by three long parallel wires, equal in length and in the same vertical plane. The middle wire is of steel and the outer wires are of brass, each wire having a cross-sectional area of $\frac{1}{4}$ in.^{2} The wires are adjusted so that each carries one-third of the load and then a further load of 7500 lb is added. Calculate the stress in each wire.

E for steel = 30×10^6 lb/in.^{2};
E for brass = 12×10^6 lb/in.^{2}.

7. A column is carried on a square base slab which transmits a load of 66 t to the ground beneath it. If the safe bearing pressure on the ground is 3 t/ft^2, find the minimum size of the slab on plan.

8. Determine Young's modulus from the following results of a tensile test—

Diameter of specimen = $\frac{3}{4}$ in.
Gauge length over which extension is measured = 4 in.
Extension under load = 0·002 in.
Load causing extension = 2·9 t.

CHAPTER XII

PLANE FRAMES, TRUSSES AND LATTICE GIRDERS

An arrangement of bars connected together constitutes a *framework* and is able to resist loads partly by virtue of the strength of the individual members and partly as a result of their arrangement.

Frames. Examples found in building work are the trusses used for carrying roofs having large clear spans and also the lattice girders often employed for bridges. In design it is necessary to

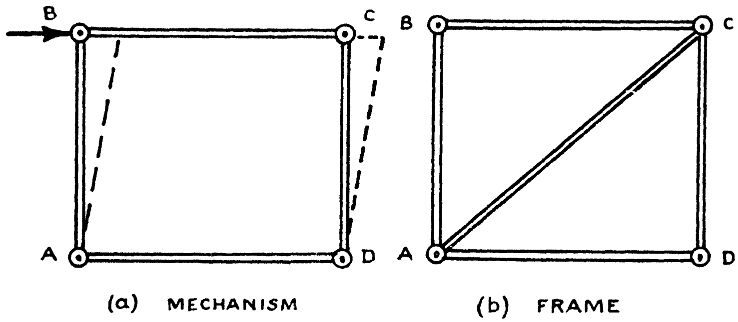


FIG. 79. A MECHANISM AND A FRAME

calculate the loads carried by the frame and the internal forces or stresses in each member. Members of suitable cross-section can then be chosen so that the intensity of stress does not exceed that which the material can withstand safely, and these members must be connected together by joints of sufficient strength to transmit the stresses from one member to another.

Plane Frames, Space Frames. If all the bars of a frame lie in the same plane, as for example in a roof truss, the resulting structure is known as a *plane frame,* and can resist only those forces acting in its own plane. If the bars lie in more than one plane, they form a *space frame* and are able to resist loads acting in all directions—a typical example of such a space frame is the lattice work pylon used to carry electrical transmission lines.

Number of Bars Required in a Plane Frame. The arrangement of four bars, shown in Fig. 79 (a), pinned together at A, B, C, and D, does not form a framework. If a load is applied at B the bars will move freely and the whole assembly is, in fact, a mechanism, and requires a fifth bar AC, Fig. 79 (b), to convert it into a frame capable of withstanding loads.

To ensure that, in designing a frame, the correct number of bars is provided to fix each joint in its proper position, it is useful to have a rule connecting the number of joints and the number of bars. The simplest frame is a triangle which has three joints and three bars, Fig. 80. To brace another joint D to the frame *ABC*

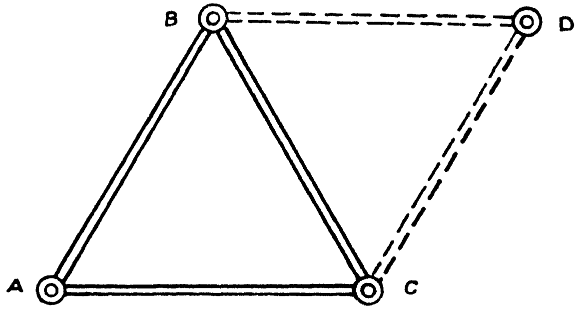


FIG. 80. NUMBER OF BARS REQUIRED TO FORM A FRAME

two more bars will be required and for each additional joint an extra two bars will be needed. Thus, in a plane frame having n joints—

Number of bars required for first three joints $= 3$
Number of bars required for remaining $(n - 3)$ joints at 2 per joint $= 2(n - 3)$
Number of bars required for a frame having n joints $= 2n - 3$

Not only must there be the correct number of bars in the frame, but they must be properly arranged. Thus if a frame has 8 nodes

FIG. 81. CORRECT AND INCORRECT ARRANGEMENT OF BARS

or joints, 13 bars are required to brace them together. In Fig. 81 (*a*) these bars are satisfactorily arranged, but in Fig. 81 (*b*), although the correct number of bars is provided, they are incorrectly disposed and the two right-hand joints are not properly braced. From inspection of these two cases it can be seen that the frame is properly braced when the bars form a continuous series of triangles. The frame is then said to be *triangulated*.

Frames Connected to Fixed Points. When a frame is connected to fixed points, such as *A* and *B*, Fig. 82, the process is equivalent to bracing extra nodes to an existing frame. Thus if a plane frame has n joints excluding the points of attachment, only $2n$ bars are required. In Fig. 82 there are five joints excluding A and B and ten bars are required.

Stress Determination in Frame Members. As will be shown later, the stresses in the members of a frame which is just stiff, i.e. has the correct number of bars correctly disposed, can be determined by applying the ordinary laws of statics. Such a frame is therefore said to be *statically determinate.* A frame such as that shown in Fig. 83 has a greater number of members than is required to make it just stiff since the structure would still be adequately braced if one of the diagonals was omitted. In this case the stresses in the members cannot be determined by statics but will depend upon their elastic properties. The frame is therefore called *statically indeterminate* and is said to have a *redundant* member. In this book only statically determinate frames will be dealt with.

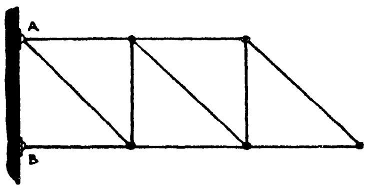


FIG. 82. FRAME BRACED TO FIXED POINTS

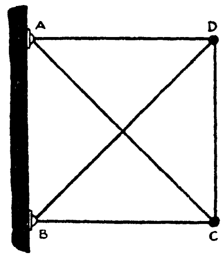


FIG. 83. STATICALLY INDETERMINATE FRAME

STRESS ANALYSIS

Basic Assumptions. To simplify the calculation of the stresses set up in the members of a frame certain assumptions are made, although they are not necessarily carried out in practice. These assumptions are—

1. That the frame is statically determinate, having no redundant members or reactions.
2. That the frame has the correct number of members, calculated as described above, and that it is properly triangulated.
3. That the members are connected by pin-joints and that each member extends only from one joint to the next.
4. That the centre-lines through the centroids of the members pass through the centre of the pin-joints.
5. That the external loads act only at the joints and that their lines of action pass through the centres of the joints.

Calculation of Loads. In practice the loads carried by a truss or girder rarely act only at the joints or panel points but, in accordance with assumption (5), for design purposes the total load is calculated and then split up and allocated to the various joints. The method can be explained by considering a roof carried by a number of trusses, Fig. 84. The truss *ABC* may be assumed to carry the load from half of the bay in front of it and half of the bay beyond it, as shown shaded. If the roof load is equivalent to 50 lb/ft^2 for the horizontal area it covers—

$$\text{Total load carried by the truss} = 20 \times (5 + 6) \times 59 \text{ lb}$$
$$= 11{,}000 \text{ lb}$$

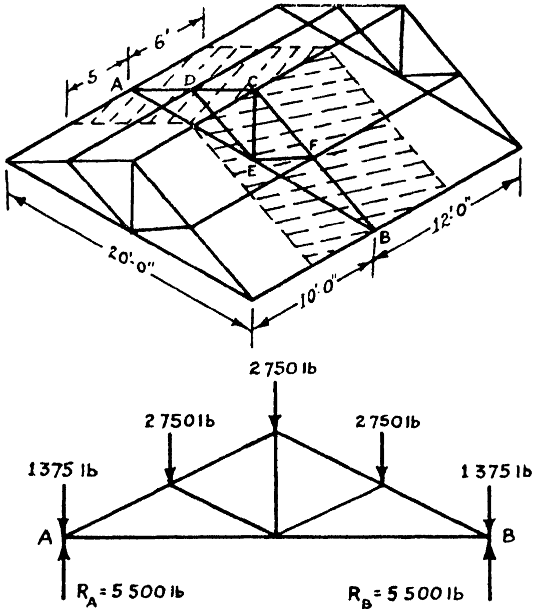


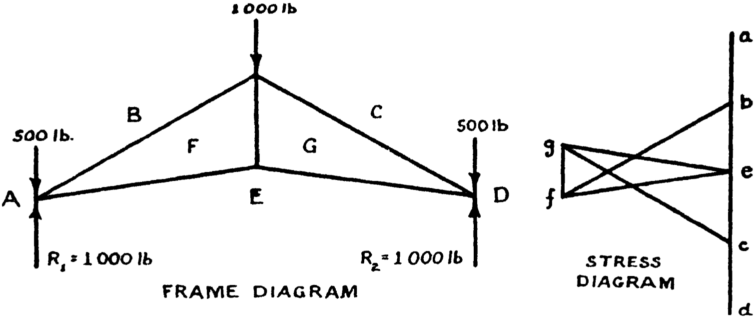
FIG. 84. LOAD CARRIED BY ROOF TRUSS

This vertical load will be applied to the joints *A*, *D*, *C*, *F*, and *B*, each joint taking a proportion of the load according to the length of the member on either side of it. Thus the loads at *A* and *B* will be half those at *D*, *C*, and *F* as shown in the lower diagram.

Graphical Stress Analysis. It has already been shown that for a structure to be stable the external loads and reactions must be in equilibrium, and that this fact may be employed to calculate the values of the reactions. Similarly any part of the structure must also be in equilibrium under the action of the external forces and the internal stresses acting in that part. Thus the joint at *A* in Fig. 84 must be in equilibrium under the external loads and reactions at *A* and the stresses in *AD* and *AE*. Thus if a polygon of forces were drawn for the joint *A* the value of the stresses in *AD* and *AE* could be found and, by applying the process to each joint in turn, the stresses in every member could be found. In practice the work can be carried out in a single *stress diagram* in the manner explained in the examples below.

EXAMPLE 1. Find the stresses in each member of the roof truss shown in Fig. 85.

Bow's notation may be used for both external and internal forces, the spaces being lettered as shown.



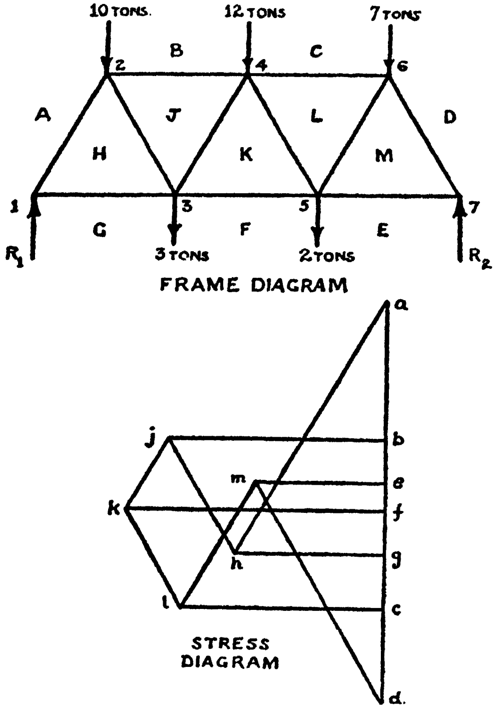
Member	BF	CG	EG	EF	FG
Tension (lb)	—	—	1200	1200	390
Comp (lb)	1370	1370	—	—	—

FIG. 85. STRESS DIAGRAM FOR A ROOF TRUSS

Since all loads are vertical and are symmetrically disposed the polygon for the external forces is a straight line *a b c d e* and the reactions *ae* and *de* are equal, thus fixing the position of *e*.

Taking the joint at the left-hand support, of the four forces acting the two external forces are completely known and the direction of the internal forces in *BF* and *FE* is also known. The polygon of forces *a b f e* can now be completed by drawing *bf* parallel to *BF* and *ef* parallel to *EF* thus enabling the magnitude of the forces in *BF* and *EF* to be found by measuring *bf* and *ef*. To determine whether these forces are pushes or pulls note the way in which the forces follow round the polygon; thus, since *ab* acts downwards, *bf* acts downwards and to the left and *fe* acts upwards and to the right, hence member *BF* exerts a push on the joint and is in compression and *FE* exerts a pull on the joint and is in tension.

Now take the top joint of the truss and complete the polygon *bcgf* to find the values of the forces in *FG* and *GC*. Finally complete the polygon for the right-hand joint to find the force in *GE*. The points *g* and *e* in this polgyon have already been fixed in the diagram and therefore form a check on the accuracy of the work for, if the line joining *e* to *g* is not parallel to the member *GE*, some error of drawing has occurred. This fact that the diagrams are self-checking is the principal advantage of the graphical method of stress analysis. If the loading is symmetrical labour can be saved by constructing only half the diagram, for the load in corresponding members in



Member	AH	BJ	CL	DM	EM	FK	GH	HJ	JK	KL	LM
Tension (tons)	—	—	—	—	9·2	18·2	10·6	9·4	—	—	10·2
Comp. (tons)	21·2	15·3	14·2	18·4	—	—	—	—	5·8	8·0	—

FIG. 86. STRESS DIAGRAM FOR A WARREN GIRDER

each half of the truss will be the same. Thus in Fig. 85 the loads in *EF* and *EG* are identical and so are those in *BF* and *GC*.

EXAMPLE 2. Find the stresses in each member of the Warren girder shown in Fig. 86.

As the loads are not symmetrical the reactions R_1 and R_2 must be calculated, or the position of the point *f* in the polygon for the external loads must be found by drawing the link polygon.

Taking moments about the right-hand support—

$$36\ R_1 = 10 \times 30 + 3 \times 24 + 12 \times 18 + 2 \times 12 + 7 \times 6$$
$$= 300 + 72 + 216 + 24 + 42 \text{ t-ft}$$
$$= 654 \text{ t-ft}$$
$$R_1 = 18{\cdot}2 \text{ t}$$
$$\text{And } R_2 = (10 + 3 + 12 + 2 + 7) - 18{\cdot}2$$
$$= 15{\cdot}8 \text{ t.}$$

Bow's notation may be used, lettering the spaces as shown. Take the loads and reactions in order, clockwise, round the truss and draw to scale the polygon, or load line, for the external forces *a b c d e f g*. Start at joint (1) and draw *ah* and *gh* parallel to members *AH* and *GH*, thus fixing *h* in the stress diagram.

Now take joint (2) and locate *j* in the same way and continue the process taking joints (4), (5), and (6) in turn.

Check that the final line *dm* is parallel to *DM* in the frame diagram.

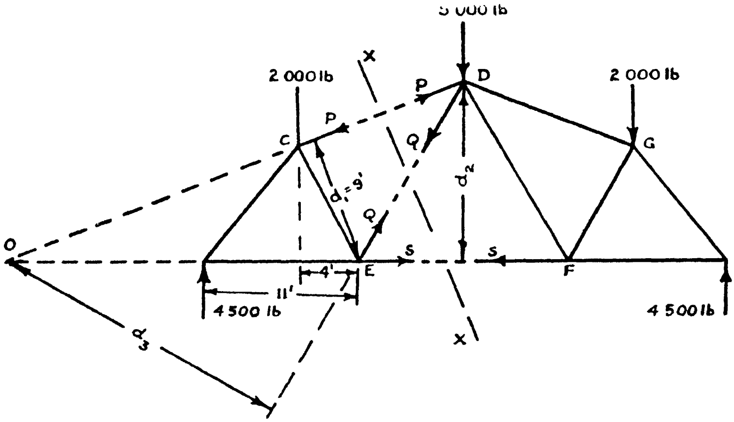


FIG. 87. STRESSING BY THE METHOD OF SECTIONS

By scaling off the stress diagram determine the stress in each member and, by inspection of the polygon for each joint, determine which members are in tension and which in compression. Note that normally the members forming the top chord of the girder, *AH*, *BJ*, *CL*, and *DM*, are in compression, while those forming the bottom chord, *GH*, *FK*, and *EM*, are in tension.

Stressing by the Method of Sections. Stresses in the members of a frame can be determined by calculation, using the method of sections. To find the stresses in the members *CD*, *DE*, and *EF* in Fig. 87, first calculate the reactions at *A* and *B* in the normal way. Now assume that the truss is cut along a line *XX* passing through the members in which the stress is to be found. The two parts of the truss can be maintained in equilibrium by applying external forces P, Q, and S to replace the internal forces previously exerted on the joints by the cut members. The value of these forces can be found by taking moments about suitable points.

To find the value of P take moments about E, the point at which Q and S meet and about which they have no moment. Then, considering the equilibrium of the left-hand side of the truss—

$$Pd_1 = 4500 \times 11 - 2000 \times 4 \text{ lb-ft}$$
$$= 41{,}500 \text{ lb-ft}$$

where d_1 = perpendicular distance from line of action of P to E.
$$= 9 \text{ ft}$$
$$\therefore P = \frac{41{,}500}{9} = 4611 \text{ lb}$$

The value of S can be found by considering the right-hand part and taking moments about D, the point at which P and Q meet. Similarly for Q take moments about G the point at which P and S meet. The lengths of the perpendiculars d_1, d_2, and d_3 are easily measured from the drawing of the frame.

This method is very convenient if it is desired to find the stress in a few members only since it avoids the necessity of drawing a complete stress diagram. It can also be applied to the stressing of the whole frame by taking a number of different sections, but in that case a stress diagram would probably be simpler.

Design of Frames

Choice of Members. The intensity of stress in the members of a frame must not exceed the safe working value for the material used, and so, once the stresses in the members have been found, the required cross-sectional area of each member can be calculated. Thus if the stress in a given member is 20 t and the allowable working stress intensity is 8 t/in.^{2} then the cross-sectional area of that member must not be less than $2\frac{1}{2}$ in.^{2}, and may be provided in a number of different ways depending upon whether the member is a strut or a tie and upon practical considerations of manufacture. In steelwork it is desirable to use channels or angles for compression members because their stiffness helps to prevent buckling; flats, rods or light angles may be used for tension members.

Riveted Joints. In steelwork, riveted joints may be used for making permanent connections between plates or rolled sections. In the *lap joint*, Fig. 88, the two plates are overlapped and are connected by rows of rivets, more than one row being used if necessary. In the *butt joint*, Fig. 89, the plates are butted together, edge to edge, and a cover plate is riveted to both plates on one or both sides. In making riveted joints the holes are either punched or drilled in the plates to a slightly larger diameter than that of the rivet. The red-hot rivet is put in position and the head formed by hand or pneumatic hammer or by a hydraulic riveting machine, during which process the rivet is expanded to fill the hole.

Failure of a riveted joint may occur in one of four ways—

(*a*) By the rivet being placed too near the edge of the plate so that it tears out when the load is applied. To prevent this the

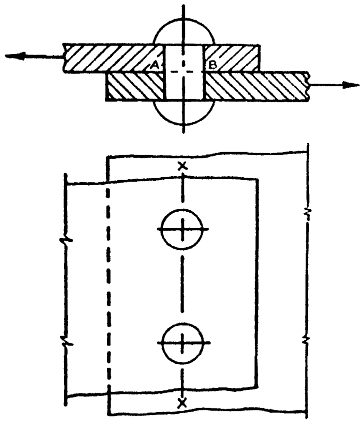


FIG. 88. RIVETED LAP JOINT

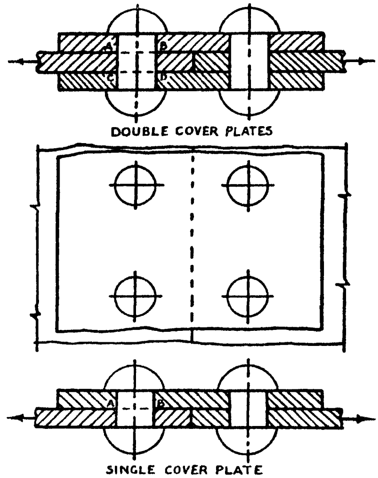


FIG. 89. SINGLE- AND DOUBLE-COVER BUTT JOINTS

centre of the rivet hole should be not less than one and a half times the rivet diameter from the edge of the plate.

(*b*) By the material of the plate failing in compression as it bears against the rivet.

Strength of rivet and plate in bearing

= allowable intensity of bearing stress × projected area of hole

= allowable intensity of bearing stress × thickness of plate × diameter of rivet.

(*c*) By shearing of the rivet. In the lap joint, Fig. 88, shearing will occur along the section *AB*. The rivet is said to be in *single shear* and—

Strength of rivet in single shear

= allowable intensity of shear stress × cross-sectional area of rivet.

In the single-covered butt joint, in tension, the rivets are also in single shear; in the double-covered butt joint, however, the rivets are in *double shear*, since the stress is transmitted across both sections AB and CD. Thus—

Strength of rivet in double shear

= 2 × strength in single shear

= 2 × cross-sectional area of rivet × allowable intensity of shear stress.

For a butt joint in compression the stress will be transmitted across the edges of the plate in contact, provided they fit, and the rivets will not be under shear stress at all.

(*d*) By tearing of one of the plates along the line XX, Fig. 88, where the plate has been weakened by the rivet holes. For this reason the area of the rivet hole must be deducted from the gross cross-sectional area of the plate when calculating its strength in tension. In compression no deduction need be made since the stress is assumed to be transmitted by the edge of the holes bearing upon the rivets.

In design the working strength of a riveted joint will be the lowest value given under the above four conditions. The ratio of the strength of the joint to the strength of the full section of the member is known as the *efficiency* of the joint and is usually expressed as a percentage.

EXAMPLE. Design a double-covered butt joint to transmit an axial tension of 39 t. The thickness of the plates to be joined is $\frac{5}{8}$ in., rivet diameter $\frac{7}{8}$ in., and allowable working stresses are: in shear, $f_s = 6$ t/in.^{2}; in bearing, $f_b = 12$ t/in.^{2}; in tension, $f_t = 8$ t/in.^{2}. Find also the efficiency of the joint.

To find the number of rivets required—

$$\text{Double shear value of one rivet} = 2 \cdot \frac{\pi d^2}{4} \cdot f_s$$

$$= \frac{2 \times \pi + (\frac{7}{8})^2}{4} = 7{\cdot}2 \text{ t}$$

$$\text{Bearing value of one rivet} = f_b \cdot \text{t} \cdot d$$

$$= 12 \times \tfrac{5}{8} \times \tfrac{7}{8} = 6{\cdot}56 \text{ t}$$

∴ Strength of rivet = 6·56 t

Number of rivets required on each side of joint

$$= \frac{\text{Total pull}}{\text{Rivet strength}} = \frac{39}{6{\cdot}56} = 6$$

These rivets will be arranged in the leading rivet formation shown in Fig. 90.

To find the width of the plates to be joined—

The plates must be wide enough to prevent failure in tension at either of sections (1), (2), or (3).

At section (1)

$$\text{Pull transmitted} = 39 \text{ t}$$
$$\text{Strength of plate} = t(B - d) \,.\, f_t$$
$$\tfrac{5}{8}\,(B - \tfrac{15}{16}) \times 8 = 39$$
$$(B - \tfrac{15}{16}) = 7{\cdot}8$$
$$B = 8\tfrac{3}{4} \text{ in.}$$

At section (2)

$$\text{Pull transmitted} = 39 - 6{\cdot}56 = 32{\cdot}44 \text{ t}$$
$$\text{Strength of plate} = t(B - 2d) \,.\, f_t$$
$$\tfrac{5}{8}\,(B - 1\tfrac{7}{8}) \times 8 = 32{\cdot}44$$
$$(B - 1\tfrac{7}{8}) = 6{\cdot}49$$
$$B = 8\tfrac{3}{8} \text{ in.}$$

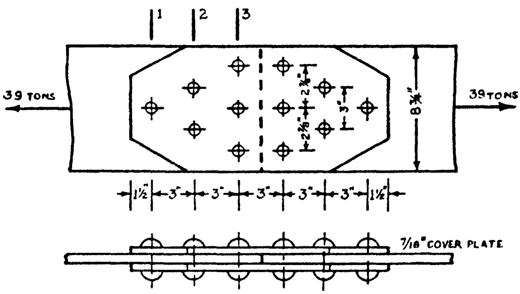


FIG. 90. DESIGN FOR A DOUBLE-COVER BUTT JOINT

At section (3)

$$\text{Pull transmitted} = 39 - 3 \times 6{\cdot}56 = 19{\cdot}32 \text{ t}$$
$$\text{Strength of plate} = t\,(B - 3d) \,.\, f_t$$
$$\tfrac{5}{8}\,(B - 2\tfrac{13}{16}) \times 8 = 19{\cdot}32$$
$$(B - 2\tfrac{13}{16}) = 3{\cdot}86$$
$$B = 6\tfrac{3}{4} \text{ in.}$$

Necessary width of plate $= 8\tfrac{3}{4}$ in.

To find thickness of cover plates—

Weakest section of cover plates is section (3).

If T = thickness of single cover plate

$$(B - 3d) \,.\, 2T \,.\, f_t = 39$$
$$(8\tfrac{3}{4} - 2\tfrac{13}{16}) \,.\, 2T \,.\, 8 = 39$$
$$T = \tfrac{13}{32}, \text{ say } \tfrac{7}{16} \text{ in.}$$

The full details of the joint will now be as shown in Fig. 90.

To find the efficiency of the joint—

Rivet strength of joint $= 6 \times 6{\cdot}56 = 39{\cdot}36$ t

Tearing strength, section (1) $= \frac{5}{8}\,(8\frac{3}{4} - \frac{15}{16}) \times 8 = 39{\cdot}06$ t

Cover plate strength, section (3)

$$= (8\tfrac{3}{4} - 2\tfrac{13}{16}) \times 2 \times \tfrac{7}{16} \times 8 = 41{\cdot}56 \text{ t}$$

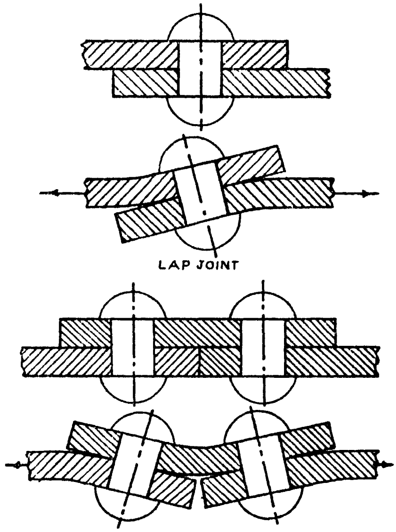


FIG. 91. BENDING ACTION AT A LAP JOINT AND A SINGLE-COVER BUTT JOINT

Therefore

Strength of joint $= 39{\cdot}06$ t (The least of the above values.)

Strength of solid plate $= 8\frac{3}{4} \times \frac{5}{8} \times 8$ t

$= 43\frac{3}{4}$ t

Efficiency of joint $= \dfrac{\text{strength of joint}}{\text{strength of solid plate}} \times 100$ per cent

$= \dfrac{39{\cdot}06}{43{\cdot}75} = \underline{\underline{89{\cdot}5 \text{ per cent.}}}$

Riveted joints should never be so designed that the rivets are in tension, for their heads are not necessarily reliable. Loading should always be a push or a pull along the axes of the plates and should put the rivets in shear. Lap joints and single-cover butt joints suffer from a bending action, Fig. 91, since the forces acting are parallel and not in the same straight line. For this reason double-cover butt joints are preferable for both tension and compression members since they do not suffer from this defect.

Riveted Connection for Roof Truss. A typical connection for a roof truss member is shown in Fig. 92. A steel gusset plate has been

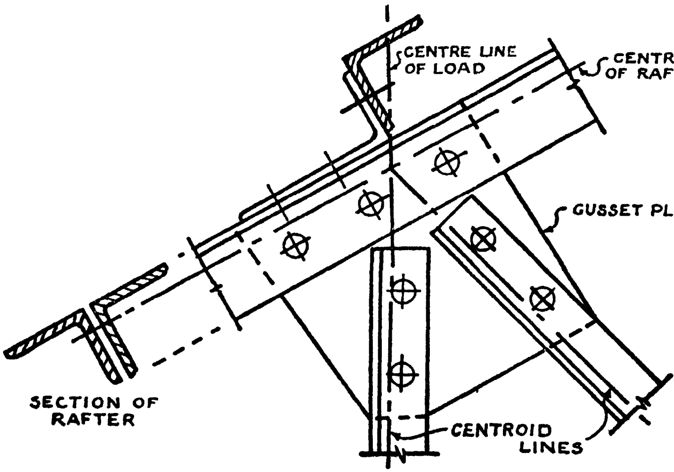


FIG. 92. CONNECTION FOR ROOF TRUSS MEMBERS

riveted to the rafter and the other members are secured to the plate. Note that the lines through the centroids of each member and the line of action of the load intersect at a common point (see page 92).

EXERCISES

1. In the truss shown in Fig. 93, $AB = BC = CD = 5$ ft; $DE = 1$ ft 6 in.; $CF = 2$ ft 6 in.; $BG = 4$ ft; $AH = 6$ ft. If a load of 1 t is applied at D calculate the stresses in all parts of the truss.

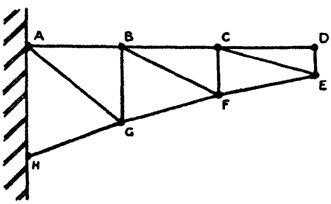


FIG. 93. CANTILEVER TRUSS

2. Calculate the forces in all the members of the truss shown in Fig. 94 under the given loading.

3. Calculate the forces in all the members of the truss shown in Fig. 95.

4. Find the stresses in all the members of the roof truss shown in Fig. 96.

5. Find by the method of sections the stresses in the members BC, BF, and FG in the cantilever, Fig. 93, if a load of 2 t is applied at D together with a load of 1 t at C.

6. Calculate the strength of a $\frac{7}{8}$ in. rivet in double shear, if the allowable shear stress is 5 t/in.^{2} and compare this value with the strength of the same rivet in bearing in a $\frac{5}{8}$ in. plate, if the allowable bearing stress is 12 t/in.^{2}

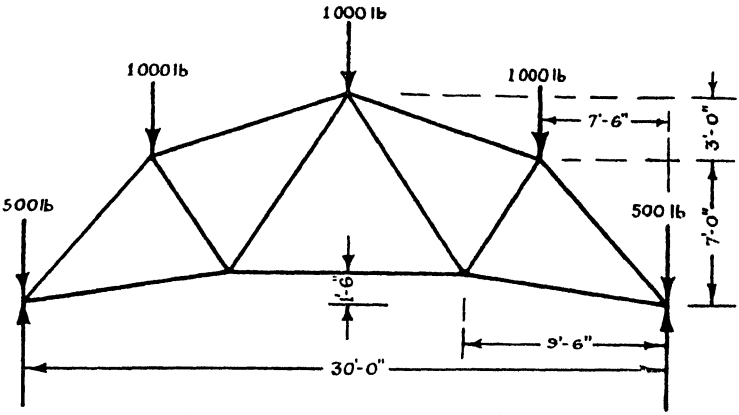


FIG. 94. NON-PARALLEL TYPE GIRDER

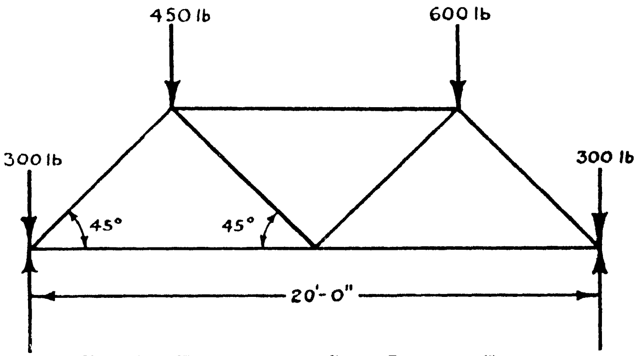


FIG. 95. TWENTY-FOOT SPAN LATTICE GIRDER

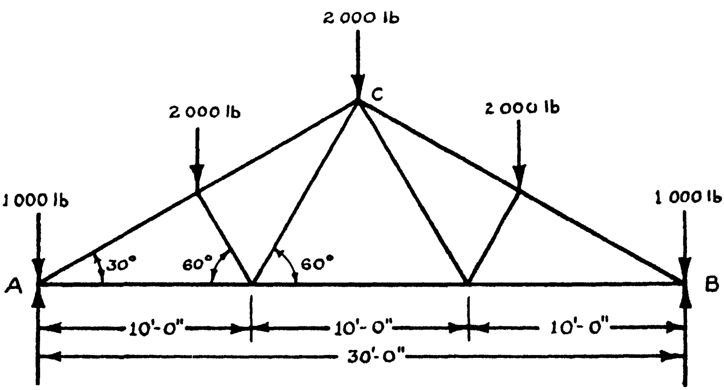


FIG. 96. THIRTY-FOOT SPAN ROOF TRUSS

7. Two $\frac{1}{2}$ in. plates are to be connected by a double-covered butt joint, which is to transmit a load of 20 t. The rivets used are to be $\frac{5}{8}$ in. diameter and will require $\frac{11}{16}$ in. holes. If the allowable stresses are: shear stress $f_s = 6$ t/in.^{2}; bearing stress $f_b = 12$ t/in.^{2}; tensile stress $f_t = 8$ t/in.^{2}, calculate the number of rivets required, the necessary width of the plates and the thickness of the cover plates. Determine the efficiency of the joint.

8. For the roof truss in Fig. 96, choose suitable sizes for all the members if the intensity of stress in compression members is not to exceed 5 t/in.^{2} and that in tension members 8 t/in.^{2}

Design also a suitable riveted joint for the four members meeting at C.

CHAPTER XIII

BENDING OF BEAMS

A MEMBER, usually greater in length than in other dimensions, which is acted upon by loads applied transversely to its length, is called a *beam*. When loaded a beam suffers strain, known as *flexure* or bending; if initially straight it becomes curved, or, if curved originally, its curvature will be altered. The term *joist* is often used

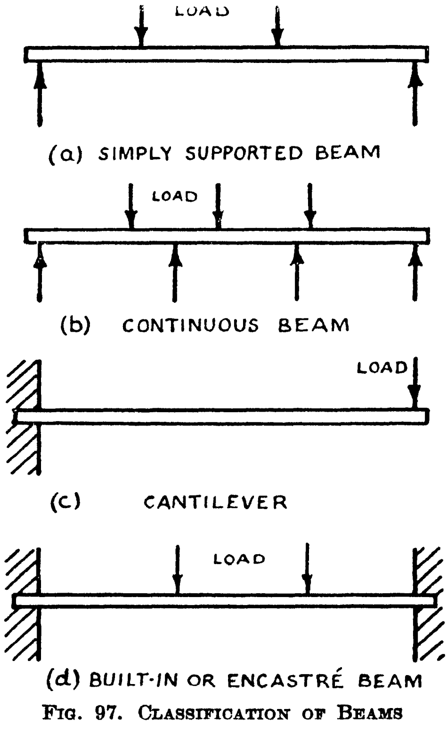


FIG. 97. CLASSIFICATION OF BEAMS

to describe a beam of moderate size made from one piece of material while beams of larger size, built up of several parts, are known as *girders*.

Beams are also classified according to their mode of support. If a beam rests on two supports, Fig. 97 (*a*), but is not fixed to them, it is said to be *simply supported*. If an intermediate support is provided, Fig. 97 (*b*), it is called a *continuous beam*. A beam supported at one end only, Fig. 97 (*c*), is known as a *cantilever*, but if built-in at both ends, Fig. 97 (*d*), it is said to be *encastré* or *built-in*. In this chapter only simply supported beams and cantilevers will be dealt with and the work will also be confined to cases of pure bending where there are no components of the external forces which act along the beam and all forces are applied in the plane in which the beam bends.

Shear Force and Bending Moment

Nature of the Stresses in a Beam. Before inquiring into the stresses set up inside a beam it is necessary to complete the knowledge of the external forces acting by determining the values of the reactions. This is done by the method described in Chapter X, and the following example will serve to remind the reader.

Example. A beam AB of 15 ft span, Fig. 98, is simply supported at its ends and carries loads of 2 t and 3 t at distances of 5 ft from A and 4 ft from B respectively. Calculate the reactions R_A and R_B.

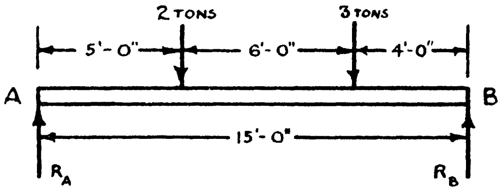


Fig. 98. Calculation of Beam Reactions

For the beam to be in equilibrium—

Algebraic sum of the moments about any point = 0. Taking moments about B

$$15 \times R_A - 2 \times 10 - 3 \times 4 = 0$$
$$15\,R_A = 32$$
$$R_A = 2{\cdot}13 \text{ t}$$

Also, algebraic sum of vertical forces = 0

$$R_A + R_B - 2 - 3 = 0$$
$$R_B = 5 - R_A = 5 - 2{\cdot}13$$
$$R_B = 2{\cdot}87 \text{ t}$$

In Fig. 99 (*a*), a beam of span L carries a central point load w. The reactions at the supports will be $\frac{w}{2}$. To find what internal forces are acting in the beam at a section XX a distance x from the left-hand end, imagine the beam to be cut through at that section. To maintain the left-hand portion in equilibrium a force F, Fig. 99 (*b*), would have to be applied to prevent vertical motion and, clearly, $F = \frac{w}{2}$. Thus, in the uncut beam, the right-hand portion must exert a force F on the left-hand portion. Similarly the left-hand portion exerts a force F' on the right-hand and since there can be no unbalanced internal force $F' = F$. This can also be seen by considering the equilibrium of the right-hand portion—

$$F' + \frac{w}{2} - w = 0$$

$$F' = \frac{w}{2} = F$$

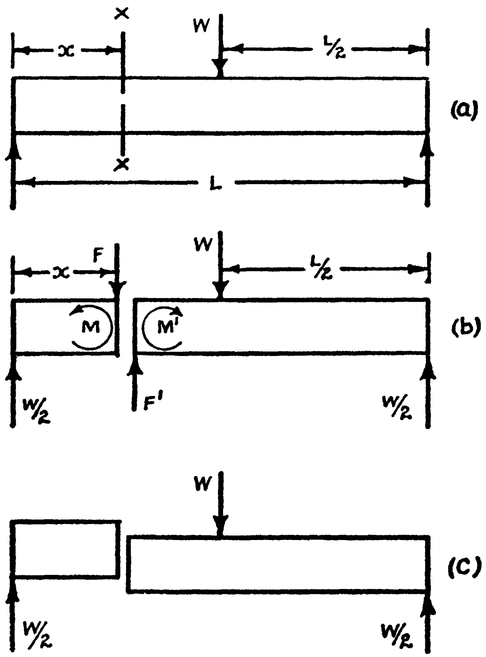


FIG. 99. INTERNAL EQUILIBRIUM OF A BEAM

These internal stresses are set up by a tendency for the beam to shear as indicated in Fig. 99 (*c*). There is, therefore, said to be a *shearing force F* at this section. The magnitude of this shearing force is given by the rule—

"The shearing force at any section is equal to the algebraic sum of the forces acting to one side of that section."

Referring again to Fig. 99 (*b*), it will be seen that although the left-hand portion is in equilibrium with regard to vertical forces, it is not so from the point of view of moments. The reaction and the force F form a clockwise couple so that, to maintain equilibrium, a moment M is required. Thus in the uncut beam the right-hand portion must exert a moment M on the left-hand portion such that $M = w\frac{x}{2}$. Similarly the left-hand portion exerts an equal and opposite moment M' on the right-hand. This internal resisting moment is known as the *moment of resistance* of the beam and its numerical value can be found by considering either one or other part of the beam. Thus—

$$\text{For the left-hand } M = \frac{w}{2} \cdot x$$

$$\text{For the right hand } M = M' = \frac{w}{2}(L - x) - w\left(\frac{L}{2} - x\right)$$

$$= \frac{w}{2} \cdot x$$

Note that the right-hand side of the equation is, in each case, the algebraic sum of the moments, about section XX, of the forces acting to one side of the section. This sum is called the applied *bending moment* at XX. From the above comes the following rule—

"The bending moment at any section across a beam is the algebraic sum of the moments, about that section, of all the forces acting to one side of that section."

Sign Convention for Shear Force and Bending Moment. It has already been seen that numerically it does not matter whether the left-hand or the right-hand side of the section is considered when calculating either shear force or bending moment; but, although F and F' are numerically equal, they act in opposite directions. Similarly the moments M and M' are also equal and opposite. But both F and F' produce the same effect since they both tend to resist the external loads which are pushing the left-hand side of the beam up and the right-hand side down. Again, M and M' are both tending to resist the applied bending moment which is causing the beam to sag. New sign conventions are therefore necessary. That for shearing force is based upon the direction in which the external loads tend to move the two parts of the beam and is shown in Fig. 100. For bending the convention is shown in Fig. 101. If the applied bending moment makes the beam sag it is positive; if it makes the beam *hog*, or become convex upwards, the moment is negative.

Shear Force and Bending Moment Diagrams. A correctly designed beam must be strong enough to withstand the shear force and bending moment occurring at every section. As an aid to this work it is often convenient to write down equations or draw graphs giving the values of these quantities for any distance from one end of the beam. Such graphs are called *shear force diagrams* (S.F. diagrams) and *bending moment diagrams* (B.M. diagrams) and can be drawn by calculating values at a number of sections of the beam and plotting these as ordinates on a base line representing the length of the beam. The points so obtained are joined up with curves or straight lines as applicable. In many cases it is possible to express the shear force or bending moment in terms of equations applicable over part or all of the span thus simplifying the work

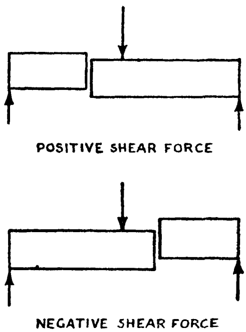


FIG. 100. SIGN CONVENTION FOR SHEAR FORCE

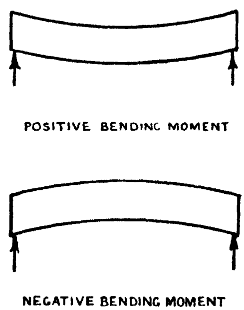


FIG. 101. SIGN CONVENTION FOR BENDING MOMENT

of plotting. The S.F. and B.M. diagrams worked out in the following paragraphs for several important cases will serve to demonstrate the method.

Cantilever with Point Load at the End. A cantilever of length L carries a point load w at its free end, Fig. 102. Take any section at a distance x from the free end, then the shear force at this section F_x will be the sum of the loads to one side of the section. Choosing the right-hand side this gives: $F_x = + w$. This will apply wherever the section is taken and the S.F. diagram will be as shown in Fig. 102.

The bending moment M_x at this section will be the moment of the forces acting to one side and, again choosing the right-hand side: $M_x = - w\, x$, the negative sign indicating that the cantilever is bent so that it is convex on its upper side. This equation will apply all along the cantilever and the B.M. diagram will be a straight line, Fig. 102, such that $M_x = 0$ at the free end where $x = 0$ and $M_x = - w\, L$ at the support where $x = L$.

Cantilever with Uniformly Distributed Load. Fig. 103 shows a cantilever of length L carrying a uniformly distributed load of w per unit length (for example 2 t per ft run). Again take any section at a distance x from the free end. Shear force $F_x =$ sum of loads to one side of the section.

$$\begin{aligned}\text{Load on right-hand side} &= \text{Load per unit length} \times \text{length from free end to section}\\ &= wx\\ \therefore\ F_x &= wx\end{aligned}$$

Thus the shear force diagram will be a straight line, Fig. 103, running from $F_x = 0$ at the free end where $x = 0$ to $F_x = w\,L$ at the support where $x = L$.

Similarly the bending moment M_x is equal to the moment of all the load to one side of the section.

Moment about section of all load to the right

= Total load on length x × distance of c.g. of this load from the section.

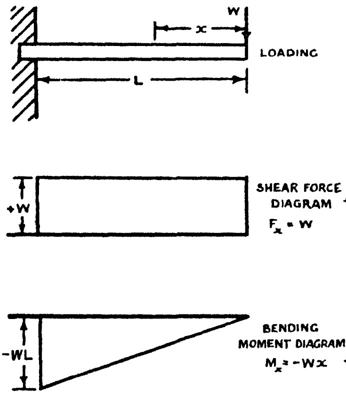


FIG. 102. CANTILEVER CARRYING A POINT LOAD

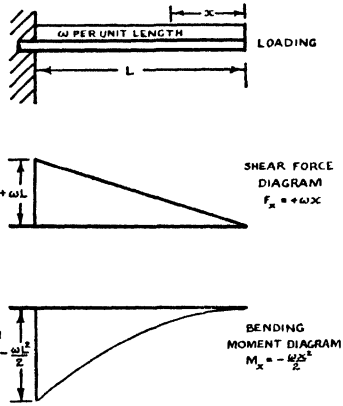


FIG. 103. CANTILEVER CARRYING A UNIFORMLY DISTRIBUTED LOAD

Since the load is uniform the centre of gravity of the portion of the load to the right of the section will be at a distance $\frac{x}{2}$ from it.

Moment about section of all load to the right

$$= w\,x\,.\,\frac{x}{2}$$

$$\therefore\; M_x = -\,\frac{wx^2}{2}$$

This bending moment is negative since the cantilever tends to become convex on its upper side. This equation gives a parabolic curve for the B.M. diagram, Fig. 103, running from $M_x = 0$ at the free end where $x = 0$ to $M_x = -\,\frac{wL^2}{2}$ at the support.

Simply Supported Beam, with Central Point Load. Fig. 104 (a) shows a beam of length L simply supported at A and B and carrying a load w at its mid-point C. The reactions at A and B will each be $\frac{w}{2}$.

Take any section X at a distance x from A, then if X lies between A and C the only load acting to the left of X is the reaction at A.

Thus, between A and C—

$$F_x = + \frac{w}{2}, \text{ and is constant from } A \text{ to } C, \text{ as shown in Fig. 104}(b)$$

and,

$$M_x = + \frac{w}{2} x, \text{ giving a straight line rising from } M_x = 0 \text{ at } A,$$

$$\text{to } M_x = \frac{wL}{4} \text{ when } x = \frac{L}{2} \text{ at } C, \text{ as shown in Fig. 104 } (c).$$

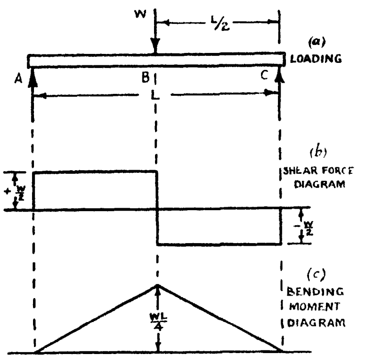


FIG. 104. BEAM, SIMPLY SUPPORTED, WITH CENTRAL POINT LOAD

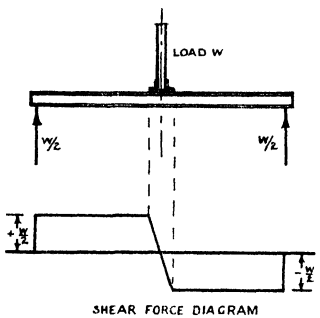


FIG. 105. APPLICATION OF A POINT LOAD IN PRACTICE

Between C and B the point load w has to be taken into account and a discontinuity is thus formed in the diagram. For this portion—

$F_x = -\frac{w}{2}$ and $M_x = \frac{w}{2}(L - x)$ completing the diagrams as shown in Figs. 104 (b, c). Note that in moving from one side of the load to the other the shearing force has changed from $+\frac{w}{2}$ to $-\frac{w}{2}$, or by an amount equal to the point load. The occurrence of both positive and negative values of shear force at C may trouble the reader, but it should be remembered that a point load cannot be applied in practice, the actual situation being nearer to that shown in Fig. 105.

Simply Supported Beam, Load Uniformly Distributed. A beam of length L, Fig. 106, is simply supported at A and B, and carries a uniformly distributed load of w per unit length.

Total load carried $= wL$

$$R_A = \frac{wL}{2} \text{ and } R_B = \frac{wL}{2}$$

Take any section X at a distance x from A.

Then, Load on length $AX = wx$

and the c.g. of this load will be at the mid-point of AX or at $\frac{x}{2}$ from X.

Considering shearing force—

$$F_x = R_A - w\,x = \frac{wL}{2} - wx$$

$$= w\left(\frac{L}{2} - x\right)$$

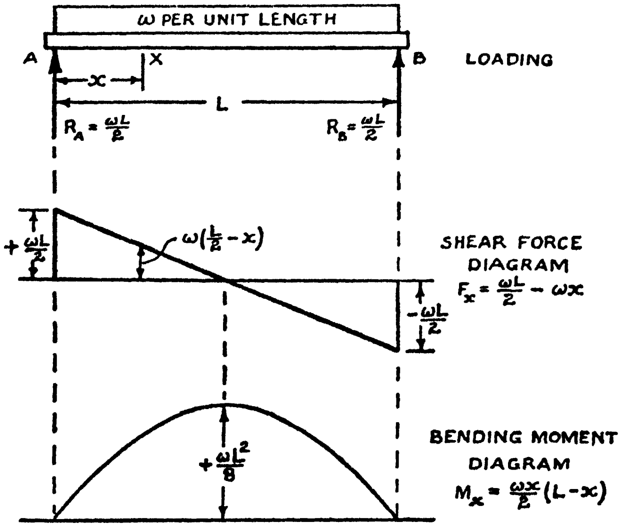


FIG. 106. BEAM, SIMPLY SUPPORTED, WITH UNIFORMLY DISTRIBUTED LOAD

Thus the S.F. diagram, Fig. 106, runs from $+\frac{wL}{2}$ at A when $x = 0$ to $-\frac{wL}{2}$ at B when $x = L$ and $F_x = 0$ at mid-span.

For bending moment—

$$M_x = R_A\,x - w\,x\,.\,\frac{x}{2} = \frac{wx}{2}\,(L - x)$$

Thus the B.M. diagram is a parabola, Fig. 106, and $M_x = 0$ at A, when $x = 0$, and at B, when $x = L$, with a maximum value of $M_x = \frac{wL^2}{8}$ at mid-span when $x = \frac{L}{2}$.

Beam Carrying Several Loads. The shear force diagram for such a case may be obtained either by plotting the S.F. diagram for each load taken separately and then adding the ordinates to make the complete diagram, or the combined diagram may be found directly by applying the rule that the shear force at a given section is equal

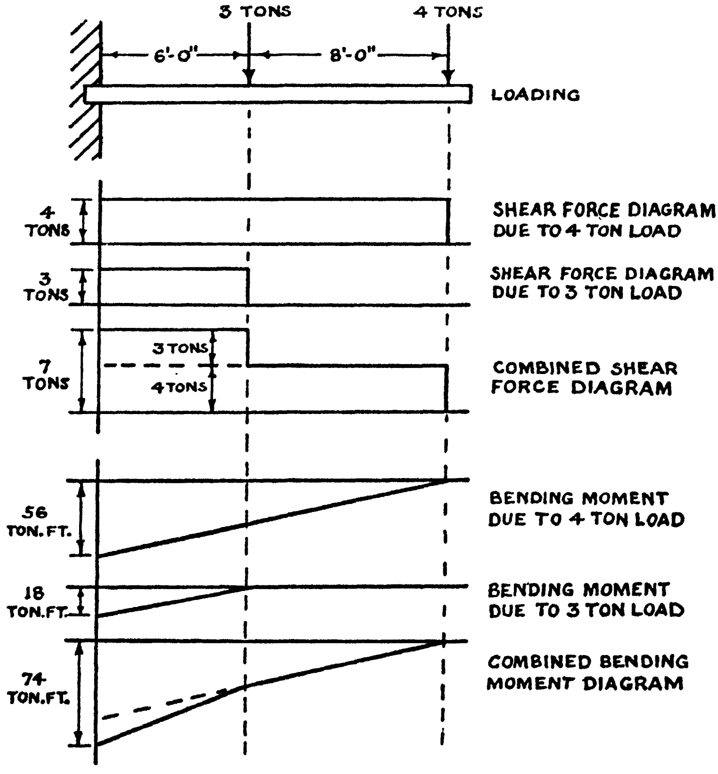


FIG. 107. BEAM CARRYING A NUMBER OF POINT LOADS

to the algebraic sum of the loads to one side of the section. Similarly the bending moment diagram can be built up by adding the ordinates of the B.M. diagrams for each load taken separately or may be found directly by applying the rule that the bending moment at a given section is equal to the algebraic sum of the moments of the loads to one side of the section.

EXAMPLE. A cantilever 14 ft long carries a load of 4 t at its free end and a load of 3 t, 6 ft from its support. Draw the S.F. and B.M. diagrams.

Fig. 107 shows the method of building up the diagrams. Usually it is not necessary to draw the individual diagrams separately, the combined diagram being built up by plotting one part on top of another instead. Note that in moving from one side of the load to the other the S.F. diagram changes by an amount equal to the load and the B.M. diagram also has a discontinuity at this point.

Position of Maximum Bending Moment. Referring to Figs. 104 and 106, it can be seen that the point at which the bending moment is a maximum is also the point at which the shear force is zero. Therefore when looking for the position of maximum bending moment for design purposes, find the point at which the shear force is zero, or, in the case of point loads, where it changes from positive to negative.

LABORATORY WORK

EXPERIMENT 1. *Experimental determination of bending moments*

The apparatus, Fig. 108, consists of a 6 ft wooden beam, 1 in. wide by 3 in. deep, with a hinge at a distance of 2 ft from one end. To prevent the beam

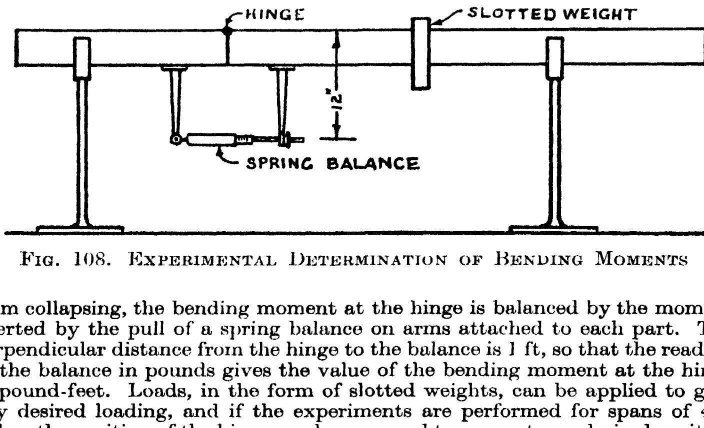


FIG. 108. EXPERIMENTAL DETERMINATION OF BENDING MOMENTS

from collapsing, the bending moment at the hinge is balanced by the moment exerted by the pull of a spring balance on arms attached to each part. The perpendicular distance from the hinge to the balance is 1 ft, so that the reading of the balance in pounds gives the value of the bending moment at the hinge in pound-feet. Loads, in the form of slotted weights, can be applied to give any desired loading, and if the experiments are performed for spans of 4 ft or less the position of the hinge can be arranged to come at any desired position between the support and mid-span.

With the aid of this apparatus determine the bending moments at several points along the span for a given system of loads and check the results by calculation.

EXPERIMENT 2. *Experimental determination of shear force*

The apparatus for this experiment, Fig. 109, consists of a wooden beam similar to that used in Expt. (1) but with the arm supporting the spring balance so arranged that the balance measures the shear force at the point at which the beam is cut.

Using this apparatus determine the shear force at several points along the span for a given system of loads and check the results by calculation.

EXERCISES

1. A beam, 20 ft in span, simply supported at its ends, carries a load of 5 t at its centre, a load of 4 t at 5 ft from one end and a load of 8 t at 6 ft from the other end. Draw the shear force and bending moment diagrams.

2. A beam, 16 ft long, is supported at two points 8 ft apart and overhangs 4 ft at each end. If the beam carries a uniformly distributed load of 2 cwt per ft run, draw the shear force and bending moment diagrams and determine the greatest value of the bending moment.

3. A beam *AB*, 28 ft long, is supported at points *C* and *D*, 6 ft from the left-hand and right-hand ends. It carries a central point load of 18 t together with a uniform load of 1 t per ft run over the lengths *AC* and *DB*. Draw the shear force and bending moment diagrams.

4. A beam, 20 ft long, is supported at its ends and carries a uniformly distributed load of 3000 lb per ft run over its whole length together with a

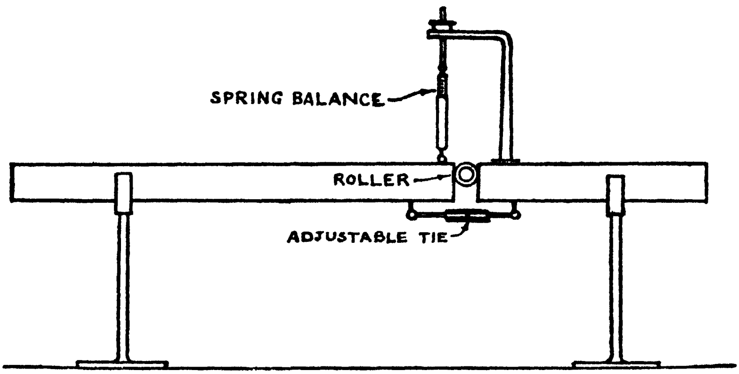


FIG. 109. EXPERIMENTAL DETERMINATION OF SHEARING FORCE

uniform load of 5000 lb per ft run over a distance of 6 ft starting at the left-hand end. Draw shear force and bending moment diagrams and determine the value of the bending moment (*a*) at the centre, (*b*) at its maximum value.

5. A wall 8 ft high has to resist a wind pressure of 30 lb/ft² on the upper half of its height. Taking a length of wall 1 ft long, draw the bending moment and shear force diagrams.

6. A beam, 30 ft long, overhangs its supports by 5 ft at each end and carries a load of 3 t at each end. Draw the shear force and bending moment diagrams and comment on the results.

7. A beam, 35 ft long, overhangs its supports by 5 ft at the left-hand end and 10 ft at the right-hand end. If it carries a uniformly distributed load of 500 lb per ft run, determine the value and position of the maximum bending moment.

8. A beam of 10 ft span carries a length of wall, triangular in elevation, increasing in height from zero at the left-hand end to 5 ft at the right-hand end. If the wall weighs 144 lb/ft³ and is 9 in. thick, determine the position and value of the maximum bending moment.

CHAPTER XIV

STRESS DUE TO BENDING

A BEAM or cantilever must, at each section along its length, be strong enough to resist the shear force and bending moment at that section, and so it is necessary to calculate the stresses set up, and to see that the allowable intensity of stress for the material is not exceeded at any point of the cross-section.

Nature of Stresses Caused by Bending. The stresses are of two kinds: shear stresses due to the shear force and longitudinal stresses due to the bending moment. The problem is complicated by the

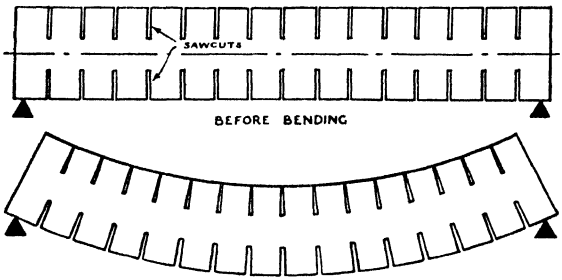


FIG. 110. LONGITUDINAL COMPRESSION AND TENSION CAUSED BY BENDING

fact that the intensity of stress is not the same for all points of a given cross-section. This may be shown experimentally by making a number of saw-cuts in the top and bottom of a timber beam, Fig. 110. When the beam is loaded the saw-cuts on top will be found to close, indicating that the material is in compression, while those on the bottom will open showing that there is tension.

Thus there is a variation from compressive strain to tensile strain, and therefore from compression to tension in the intensity of longitudinal stress. The intensity of shear stress is also found to vary, though in a different manner.

Longitudinal Stresses in a Beam. It has already been seen that stresses are set up along the length of a beam and that they are of opposite kinds at the top and bottom surfaces. Since the external loads are applied at right angles to the length of the beam they can have no components along the beam, it is therefore the bending moment which is resisted by the longitudinal stresses at any section. Also, since the stress changes from tension to compression in the depth of the beam, there must be some horizontal plane in the beam where the changeover occurs and the stress is zero. This plane is called the *neutral plane*, Fig. 111, and it intersects each cross-section in a line known as the *neutral axis* (N.A.). Before investigating the relation between the longitudinal stresses and the bending moment at a given section the following assumptions must be made—

1. That the beam is not stressed beyond the elastic limit and that Hooke's law is obeyed, i.e. stress is proportional to strain.
2. Young's modulus of elasticity is the same in tension and compression.
3. A plane cross-section before bending remains plane after bending and does not warp.
4. That there is no resultant axial force acting at any cross-section.

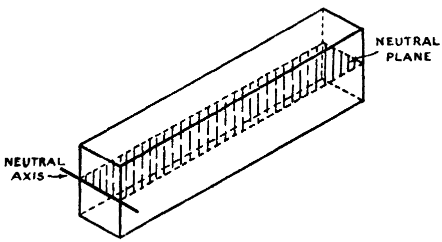


FIG. 111. NEUTRAL PLANE AND NEUTRAL AXIS OF A BEAM

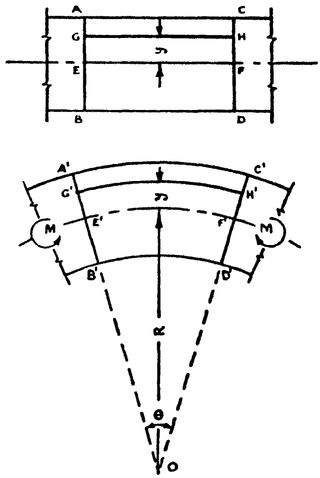


FIG. 112. CALCULATION OF STRESS DUE TO BENDING

5. The cross-section of the beam is symmetrical about an axis through its centroid in the plane of bending.

In Fig. 112 AB and CD are two adjacent cross-sections in a beam at the ends of which two equal and opposite couples M are applied. This bending moment will stretch the fibres above the neutral axis and compress those below it causing the sections AB and CD to take up the positions $A'B'$ and $C'D'$. They will no longer be parallel but will be inclined to each other at a small angle θ and will meet at O. The neutral plane EF will remain unchanged in length but will form an arc of a circle of radius R as at $E'F'$.

Take any layer of fibres GH at a distance y from EF. After bending it will form an arc $G'H'$ of radius $R + y$.

$$\text{Strain in layer } G'H' = e = \frac{G'H' - GH}{GH}$$

$$\text{Length of arc } G'H' = \text{radius} \times \text{angle } G'OH' \text{ in radians.}$$

$$= (R + y)\theta$$

and

$$E'F' = R\theta$$

but $$GH = EF = E'F'$$

$$\therefore e = \frac{(R + y)\theta - R\theta}{R\theta} = \frac{y}{R}$$

But longitudinal stress intensity f in $G'H'$

$$= \text{E} \times \text{strain in } G'H'$$

$$f = \text{E} \cdot e = \text{E} \cdot \frac{y}{R} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

or $$\frac{f}{y} = \frac{\text{E}}{R} = \text{constant for a given beam and B.M.} \quad . \quad . \quad (2)$$

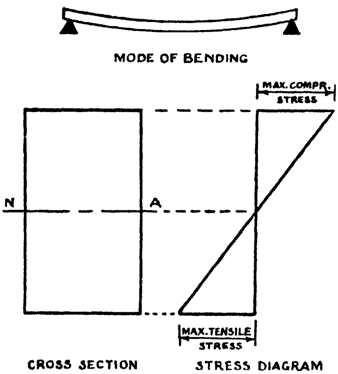


FIG. 113. VARIATION OF STRESS INTENSITY THROUGH A BEAM

Hence the longitudinal stress intensity f in a fibre is directly proportional to the distance y of the fibre from the neutral axis and will be tensile on one side of the neutral axis and compressive on the other. This variation is shown in Fig. 113, in the form of a stress diagram.

Position of the Neutral Axis. Before proceeding to find the relation between the intensity of stress and the applied bending moment the position of the neutral axis must be found. Since the longitudinal stresses to one side of the section form a couple, the neutral axis must be so placed that the total tensile stress is equal to the total compressive stress. Fig. 114 shows the cross-section of a beam, which may be of any shape symmetrical about an axis YY through the centroid in the plane of bending. Let CD be any thin strip of area a parallel to the neutral axis N.A. and at a distance y from it.

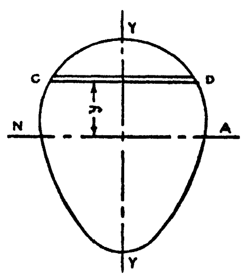


FIG. 114. LOCATION OF NEUTRAL AXIS

If the strip is very thin the intensity of stress on CD may be taken as constant and equal to f.

Total force on CD = stress intensity $\times$ area $= f\,a$ but, from equation (1)—

$$f = \text{E} \cdot \frac{y}{R}$$

$$\therefore \text{ Force on } CD = \frac{\text{E}}{R} \cdot ay$$

If the whole area is divided into similar strips, those on one side of the neutral axis will be in compression and those on the other side in tension and, since there can be no resultant thrust—

Total tension = total compression

or Algebraic sum of the forces in all strips = 0

This may be written—

$$\Sigma \frac{E}{R} ay = 0$$

or since E and R are constants for any given case,

$$\frac{E}{R} \Sigma ay = 0$$

Now E and R will not be zero;

therefore, $\Sigma ay = 0$

It has already been shown, in Chapter IX, that $\Sigma ay = 0$ when y is measured from an axis through the centroid of the section.

∴ The neutral axis passes through the centroid of the section.

Moment of Resistance. It remains to find the relation between the applied bending moment and the stress set up in the beam. For equilibrium—

Applied bending moment M = moment of resistance. Considering the strip CD again—

$$\text{Moment about N.A. of force on } CD = \left(\frac{E}{R} \cdot ay\right) \cdot y$$

$$= \frac{E}{R} \cdot ay^2$$

For any other such strip a similar expression holds and all such moments will be positive whether the strip is above or below the neutral axis since y^2 is positive irrespective of the sign of y. The sum of all these moments will give the moment of resistance of the section.

$$\text{Moment of resistance} = \frac{E}{R} \Sigma ay^2$$

The term Σay^2 is the *second moment of area* of the section but is often called the *moment of inertia* and is usually denoted by I

$$\therefore \text{ Moment of resistance} = \frac{E}{R} I = \text{applied bending moment M}$$

$$\therefore \frac{M}{I} = \frac{E}{R}$$

but, from equation (2),

$$\frac{f}{y} = \frac{E}{R}$$

$$\therefore \frac{M}{I} = \frac{f}{y} = \frac{E}{R}$$

EXAMPLE. A beam, symmetrical about its neutral axis, has a second moment of area, or moment of inertia, of 200 inch units and is 10 in. deep. If the maximum stress intensity is not to exceed 8 t/in.², find the greatest bending moment that the beam can resist.

Maximum stress occurs at the outside fibres, 5 in. from the neutral axis. $\therefore\ y = 5$ in.

Maximum allowable stress intensity $= f = 8$ t/in.²

Second moment of area $= \mathrm{I} = 200$ inch units.

Applying the formula

$$\frac{\mathrm{M}}{\mathrm{I}} = \frac{f}{y}$$

$$\mathrm{M} = \frac{f\mathrm{I}}{y} = \frac{8 \times 200}{5} = \underline{\underline{320 \text{ t-in.}}}$$

PROPERTIES OF BEAM SECTIONS

Modulus of Section. The greatest stress intensity will occur in the fibres furthest away from the neutral axis and this will control the design of the beam. If the stress intensity is not to exceed a value p, the moment of resistance will be given by—

$$\mathrm{M} = p\frac{\mathrm{I}}{y_{max}}$$

where y_{max} is the distance to the furthest fibre. Both I and y_{max} are constants of the beam section and the term $\frac{\mathrm{I}}{y_{max}}$ is known as the *section modulus*, denoted by Z.

Thus, $\mathrm{M} = p\,\mathrm{Z}$

In the example just given, I $= 200$ inch units.

$y_{max} = 5$ in.

therefore, $\mathrm{Z} = \frac{200}{5} = 40$ inch units.

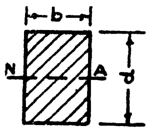
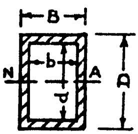
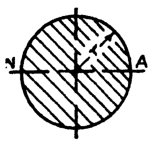
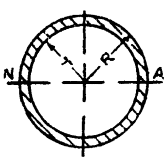
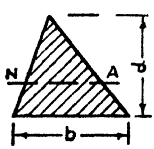
Radius of Gyration. Just as, in dealing with first moments of area, an area may be considered to be concentrated at its centroid, so, for second moments of area, an area A may be supposed to act at some radius k from its centroid such that $A\,k^2 =$ Second moment of the area A about its centroid. This distance k is known as the *radius of gyration* of the area.

Properties of Common Sections. Table IV gives the values of the second moment of area, radius of gyration and section modulus for geometrical sections which occur frequently.

Properties of Rolled Steel Sections. Rolled-steel sections are used extensively in building construction and are manufactured in a range of standard sizes to the forms shown in Fig. 115. The properties of these sections are tabulated for convenience, as shown in Tables V*a*, *b*, and *c*.

TABLE IV

PROPERTIES OF COMMON SECTIONS

Section		Area	Depth to N.A. from top	I_{NA}	k^2_{NA}	Z
Rectangle		bd	$\frac{d}{2}$	$\frac{bd^3}{12}$	$\frac{d^2}{12}$	$\frac{bd^2}{6}$
Hollow rectangle		$BD - bd$	$\frac{D}{2}$	$\left(\frac{BD^3}{12} - \frac{bd^3}{12}\right)$	$\frac{BD^3 - bd^3}{12(BD - bd)}$	$\frac{BD^3 - bd^3}{6D}$
Circle		πr^2	r	$\frac{\pi r^4}{4}$	$\frac{r^2}{4}$	$\frac{\pi r^3}{4}$
Hollow circle		$\pi(R^2 - r^2)$	R	$\frac{\pi}{4}(R^4 - r^4)$	$\frac{R^2 + r^2}{4}$	$\frac{\pi(R^4 - r^4)}{4R}$
Triangle		$\frac{1}{2} bd$	$\frac{2}{3} d$	$\frac{bd^3}{36}$	$\frac{d^2}{18}$	$\frac{bd^2}{24}$

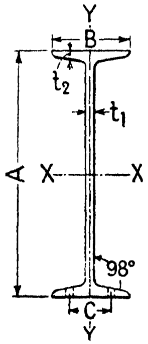


TABLE V (*a*)

PROPERTIES OF BRITISH STANDARD BEAMS

All dimensions in inch units

Flange drilling		
Flange width B	Centres of holes C	Rivet diam.
8	4¾	⅞
7½	4½	⅞
7	4	⅞
6½	3¾	⅞
6	3½	¾
5	2¾	¾
4½	2½	¾
4	2¼	⅝
3	1½	½
1¾	⅞	¼
1½	⅝	¼

Size A × B	Weight lb/ft run	Area in.^{2}	Standard thickness		Moment of Inertia		Section modulus	
			Web t_1	Flange t_2	About X–X	About Y–Y	About X–X	About Y–Y
24 × 7½	95	27·94	0·57	1·011	2533·04	62·54	211·09	16·68
22 × 7	75	22·06	0·50	0·834	1676·80	41·07	152·44	11·73
20 × 7½	89	26·19	0·60	1·010	1672·85	62·54	167·29	16·68
20 × 6½	65	19·12	0·45	0·820	1226·17	32·56	122·62	10·02
18 × 8	80	23·53	0·50	0·950	1292·07	69·43	143·56	17·36
18 × 7	75	22·09	0·55	0·928	1151·18	46·56	127·91	13·30
18 × 6	55	16·18	0·42	0·757	841·76	23·64	93·53	7·88
16 × 8	75	22·06	0·48	0·938	973·91	68·30	121·74	17·08
16 × 6	62	18·21	0·55	0·847	725·05	27·14	90·63	9·05
16 × 6	50	14·71	0·40	0·726	618·09	22·47	77·26	7·49
15 × 6	45	13·24	0·38	0·655	491·91	19·87	65·59	6·62
15 × 5	42	12·36	0·42	0·647	428·49	11·81	57·13	4·72
14 × 8	70	20·59	0·46	0·920	705·58	66·67	100·80	16·67
14 × 6	57	16·78	0·50	0·873	533·34	27·94	76·19	9·31
14 × 6	46	13·59	0·40	0·698	442·57	21·45	63·22	7·15
13 × 5	35	10·30	0·35	0·604	283·51	10·82	43·62	4·33
12 × 8	65	19·12	0·43	0·904	487·77	65·18	81·30	16·30
12 × 6	54	15·89	0·50	0·883	375·77	28·28	62·63	9·43
12 × 6	44	13·00	0·40	0·717	316·76	22·12	52·79	7·37
12 × 5	32	9·45	0·35	0·550	221·07	9·69	36·84	3·88
10 × 8	55	16·18	0·40	0·783	288·69	54·74	57·74	13·69
10 × 6	40	11·77	0·36	0·709	204·80	21·76	40·96	7·25
10 × 5	30	8·85	0·36	0·552	146·23	9·73	29·25	3·89
10 × 4½	25	7·35	0·30	0·505	122·34	6·49	24·47	2·88
9 × 7	50	14·71	0·40	0·825	208·13	40·17	46·25	11·48
9 × 4	21	6·18	0·30	0·457	81·13	4·15	18·03	2·07
8 × 6	35	10·30	0·35	0·648	115·06	19·54	28·76	6·51
8 × 5	28	8·28	0·35	0·575	89·69	10·19	22·42	4·08
8 × 4	18	5·30	0·28	0·398	55·63	3·51	13·91	1·75
7 × 4	16	4·75	0·25	0·387	39·51	3·37	11·29	1·69
6 × 5	25	7·37	0·41	0·520	43·69	9·10	14·56	3·64
6 × 4½	20	5·89	0·37	0·431	34·71	5·40	11·57	2·40
6 × 3	12	3·53	0·23	0·377	20·99	1·46	7·00	0·97
5 × 4½	20	5·88	0·29	0·513	25·03	6·59	10·01	2·93
5 × 3	11	3·26	0·22	0·376	13·68	1·45	5·47	0·97
4¾ × 1¾	6·5	1·91	0·18	0·325	6·73	0·26	2·83	0·30
4 × 3	10	2·94	0·24	0·347	7·79	1·33	3·89	0·88
4 × 1¾	5	1·47	0·17	0·239	3·66	0·19	1·83	0·21
3 × 3	8·5	2·52	0·20	0·332	3·81	1·25	2·54	0·83
3 × 1½	4	1·18	0·16	0·249	1·66	0·13	1·11	0·17

(By courtesy of Messrs. Dorman Long & Co., Ltd.)

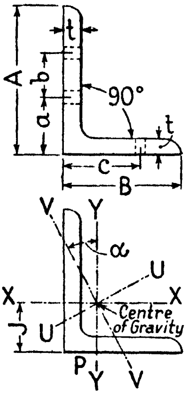


TABLE V (*b*)

PROPERTIES OF BRITISH STANDARD UNEQUAL ANGLES

All dimensions in inch units

Spacing of holes

Leg	*a*	*b*	*c*
7	2½	3	4
6½	2½	2½	3¾
6	2¼	2¼	3½
5½	2¼	2	3¼
5	2	1¾	3
4½	—	—	2½
4	—	—	2¼
3½	—	—	2
3	—	—	1¾
2¾	—	—	1⅝
2½	—	—	1⅜
2¼	—	—	1¼
2	—	—	1⅛
1¾	—	—	1
1½	—	—	⅞
1¼	—	—	¾

Size and Thickness $A \times B \times t$	Area in.²	Weight lb/ft run	Dimensions		Moment of Inertia				Tan α
			J	*P*	About *X–X*	About *Y–Y*	About *U–U*	About *V–V*	
7 × 3½ × ⅝	6·17	20·99	2·55	0·81	30·53	5·14	32·31	3·35	0·26
× ½	5·00	17·00	2·50	0·76	25·07	4·27	26·59	2·75	0·26
× ⅜	3·80	12·91	2·44	0·71	19·28	3·31	20·47	2·12	0·26
6 × 4 × ¾	6·94	23·59	2·06	1·07	24·26	8·53	27·75	5·04	0·43
× ⅝	5·86	19·93	2·02	1·02	20·82	7·37	23·90	4·29	0·43
× ½	4·75	16·16	1·97	0·97	17·14	6·11	19·73	3·52	0·44
6 × 3½ × ⅝	5·55	18·86	2·11	0·87	19·85	4·96	21·73	3·08	0·33
× ½	4·50	15·30	2·06	0·82	16·36	4·18	17·95	2·53	0·34
× ⅜	3·42	11·63	2·01	0·77	12·62	3·21	13·87	1·96	0·34
6 × 3 × ⅝	5·24	17·80	2·22	0·73	18·80	3·14	19·86	2·07	0·25
× ½	4·25	14·45	2·17	0·68	15·51	2·62	16·43	1·70	0·26
× ⅜	3·24	11·00	2·12	0·63	11·99	2·05	12·72	1·32	0·26
5 × 4 × ⅝	5·24	17·80	1·61	1·11	12·44	7·02	15·79	3·67	0·62
× ½	4·25	14·45	1·56	1·06	10·29	5·83	13·11	3·01	0·62
× ⅜	3·24	11·00	1·51	1·01	7·97	4·53	10·18	2·32	0·63
5 × 3½ × ⅝	4·92	16·74	1·69	0·94	11·89	4·74	13·91	2·72	0·47
× ½	4·00	13·61	1·64	0·90	9·84	3·96	11·56	2·24	0·48
× ⅜	3·05	10·37	1·59	0·85	7·63	3·09	8·99	1·73	0·48
5 × 3 × ½	3·75	12·75	1·73	0·74	9·33	2·51	10·31	1·54	0·35
× ⅜	2·86	9·73	1·68	0·69	7·25	1·97	8·03	1·19	0·36
4½ × 3 × ½	3·50	11·91	1·52	0·78	6·94	2·45	7·96	1·43	0·43
× ⅜	2·67	9·09	1·47	0·73	5·41	1·92	6·22	1·11	0·44
4 × 3½ × ⅝	4·30	14·61	1·29	1·04	6·28	4·45	8·55	2·19	0·74
× ½	3·50	11·91	1·24	0·99	5·24	3·72	7·16	1·79	0·75
× ⅜	2·67	9·09	1·19	0·94	4·09	2·91	5·61	1·39	0·75
4 × 3 × ½	3·25	11·05	1·32	0·82	4·97	2·37	6·04	1·30	0·54
× ⅜	2·49	8·45	1·27	0·77	3·89	1·87	4·75	1·01	0·55
4 × 2½ × ⅜	2·30	7·81	1·36	0·61	3·66	1·10	4·10	0·66	0·38
× ¼	1·56	5·32	1·30	0·56	2·54	0·77	2·85	0·46	0·39
3½ × 3 × ½	3·00	10·20	1·12	0·87	3·40	2·28	4·55	1·13	0·71
× ⅜	2·30	7·81	1·07	0·82	2·67	1·80	3·59	0·88	0·72
× ¼	1·56	5·32	1·01	0·77	1·86	1·26	2·50	0·61	0·72
3½ × 2½ × ⅜	2·11	7·17	1·15	0·65	2·51	1·06	2·98	0·59	0·49
× ¼	1·44	4·89	1·09	0·60	1·75	0·74	2·08	0·41	0·50
3 × 2½ × ⅜	1·92	6·54	0·94	0·70	1·62	1·02	2·13	0·51	0·67
× ¼	1·31	4·47	0·89	0·65	1·14	0·72	1·50	0·36	0·68
3 × 2 × ⅜	1·73	5·90	1·03	0·53	1·50	0·53	1·72	0·31	0·42
× ¼	1·19	4·04	0·98	0·48	1·06	0·38	1·22	0·22	0·43
2½ × 2 × ⅜	1·55	5·26	0·82	0·57	0·89	0·50	1·13	0·27	0·61
× ¼	1·06	3·61	0·77	0·53	0·63	0·36	0·81	0·19	0·62

(*By courtesy of Messrs. Dorman Long & Co., Ltd.*)

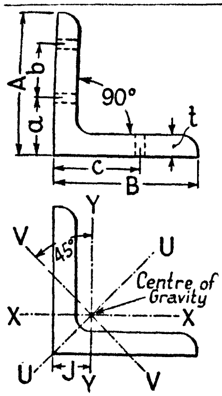


TABLE V (*c*)

PROPERTIES OF BRITISH STANDARD EQUAL ANGLES

All dimensions in inch units

Spacing of holes			
Leg	*a*	*b*	*c*
7	2½	3	4
6½	2½	2½	3¾
6	2¼	2¼	3½
5½	2¼	2	3¼
5	2	1¾	3
4½	—	—	2½
4	—	—	2¼
3½	—	—	2
3	—	—	1¾
2¾	—	—	1⅝
2½	—	—	1⅜
2¼	—	—	1¼
2	—	—	1⅛
1¾	—	—	1
1½	—	—	⅞
1¼	—	—	¾

Size and Thickness *A* × *B* × *t*	Area in.²	Weight lb/ft run	Dimension *J*	Moment of Inertia About *X–X*	About *U–U*	About *V–V*
8 × 8 × 1	15·00	51·01	2·35	87·85	139·40	36·31
× ⅞	13·24	45·00	2·30	78·44	124·65	32·23
× ¾	11·44	38·89	2·25	68·58	109·11	28·06
× ⅝	9·61	32·68	2·20	58·26	92·75	23·78
7 × 7 × 1	13·00	44·20	2·10	57·46	90·97	23·96
× ⅞	11·48	39·05	2·06	51·45	81·64	21·27
× ¾	9·94	33·79	2·01	45·12	71·72	18·53
× ⅝	8·36	28·42	1·96	38·45	61·19	15·72
× ½	6·75	22·95	1·91	31·42	50·02	12·82
6 × 6 × ⅞	9·73	31·10	1·81	31·51	49·86	13·16
× ¾	8·44	28·69	1·76	27·74	44·01	11·47
× ⅝	7·11	24·17	1·71	23·73	37·73	9·74
× ½	5·75	19·55	1·66	19·48	30·99	7·96
× ⅜	4·36	14·82	1·61	14·95	23·79	6·11
5 × 5 × ¾	6·94	23·59	1·51	15·54	24·57	6·50
× ⅝	5·86	19·93	1·47	13·37	21·21	5·53
× ½	4·75	16·16	1·42	11·04	17·55	4·53
× ⅜	3·61	12·28	1·37	8·53	13·57	3·49
4½ × 4½ × ¾	6·19	21·04	1·39	11·08	17·47	4·68
× ⅝	5·24	17·80	1·34	9·56	15·15	3·98
× ½	4·25	14·45	1·29	7·92	12·59	3·26
× ⅜	3·24	11·00	1·24	6·15	9·78	2·52
4 × 4 × ¾	5·44	18·49	1·26	7·57	11·89	3·25
× ⅝	4·61	15·68	1·22	6·56	10·37	2·76
× ½	3·75	12·75	1·17	5·46	8·66	2·26
× ⅜	2·86	9·73	1·12	4·26	6·77	1·75
3½ × 3½ × ⅝	3·99	13·55	1·09	4·27	6·72	1·82
× ½	3·25	11·05	1·05	3·57	5·65	1·49
× ⅜	2·49	8·45	1·00	2·80	4·45	1·15
× ¼	1·69	5·74	0·95	1·94	3·09	0·80
3 × 3 × ½	2·75	9·35	0·92	2·18	3·44	0·92
× ⅜	2·11	7·17	0·88	1·72	2·73	0·71
× ¼	1·44	4·89	0·83	1·20	1·91	0·49
2½ × 2½ × ½	2·25	7·65	0·80	1·21	1·89	0·52
× ⅜	1·73	5·90	0·75	0·96	1·52	0·40
× ¼	1·19	4·04	0·70	0·68	1·08	0·28
2 × 2 × ⅜	1·36	4·62	0·63	0·47	0·74	0·20
× ¼	0·94	3·19	0·58	0·34	0·53	0·14
1½ × 1½ × ¼	0·69	2·34	0·46	0·13	0·21	0·06
1¼ × 1¼ × ⅛	0·30	1·01	0·34	0·04	0·06	0·02

(*By courtesy of Messrs. Dorman Long & Co., Ltd.*)

Parallel Axes Rule for Second Moments of Area. The formulae in Table IV give the second moments of area of the sections about an axis through their centroids, but it is often necessary to find their value about some other axis. Thus in Fig. 116 let I_{XX} be the second moment of area of the figure about an axis XX, and suppose that it is desired to find the second moment of area I_{ZZ} about an axis ZZ parallel to XX at a distance d from it.

Consider a small area a at a perpendicular distance y from XX—

Second moment of this area about $XX = ay^2$

$\therefore$ Second moment of whole area about $XX = \Sigma ay^2$

Also taking moments about ZZ—

Second moment of small area about $ZZ = a(y + d)^2$

$= ay^2 + 2ayd + ad^2$

Summing for the whole area—

Second moment of whole area

$$\text{about } ZZ = I_{ZZ} = \Sigma ay^2 + 2d\,\Sigma ay + d^2\Sigma a$$

d and d^2 being taken outside the summation sign since d is a constant.

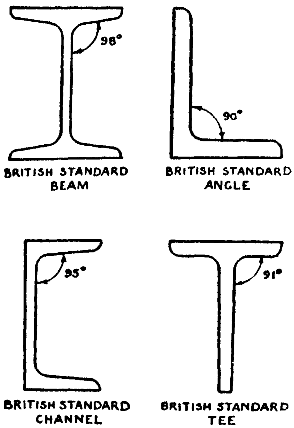


FIG. 115. ROLLED STEEL SECTIONS

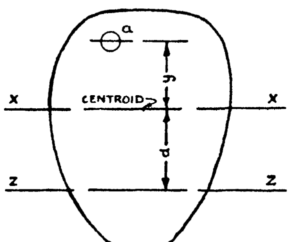


FIG. 116. PARALLEL AXES RULE FOR SECOND MOMENTS OF AREA

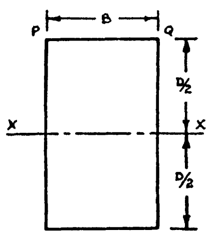


FIG. 117. SECOND MOMENT OF AREA OF A RECTANGLE—ABOUT ITS EDGE

But, $\Sigma ay^2 = I_{XX}$

and Σay = first moment of figure about an axis through its centroid $= 0$

Also Σa = area of whole figure $= A$

$$I_{ZZ} = I_{XX} + Ad^2$$

EXAMPLE. If the second moment of area of a rectangle, Fig. 117, is $\frac{BD^3}{12}$ about an axis XX through its centroid, find its value about an edge PQ.

$$I_{PQ} = I_{XX} + Ad^2$$

But,
$$I_{XX} = \frac{BD^3}{12},\ A = BD \text{ and } d = \frac{D}{2}$$

$$\therefore\ I_{PQ} = \frac{BD^3}{12} + BD\left(\frac{D}{2}\right)^2 = \frac{BD^3}{3}$$

Built-up Sections. Where steel beams of great size are required, plates and rolled steel sections are often riveted or welded together. The second moment of area of the compound section can be found by using the parallel axes rule.

EXAMPLE 1. A compound beam consists of a 12 in. × 5 in. × 32 lb rolled-steel joist (I_{XX} = 221·07 in. units), with an 8 in × 1 in. plate riveted to each flange. What is the second moment of area of the built-up section? Neglect rivet holes.

Since the section is symmetrical the centroid of the built-up section will lie on the XX axis of the joist.

I_{XX} of whole section = I_{XX} of joist + 2 (I_{XX} of one plate).

Area of one plate = 8 × 1 = 8 in.^{2}

Distance of centroid of plate from XX axis = 6·5 in.

Second moment of area of plate about an axis through its own centroid parallel to XX $= \frac{bd^3}{12} = \frac{8 \times 1^3}{12}$

= 0·67 inch units

$\therefore\ I_{XX}$ of one plate = 0·67 + 8 × (6·5)^2 inch units

= 0·67 + 338 inch units

= 338·67 inch units

I_{XX} of whole section = 221·07 + 2 × 338·67

= 888·4 inch units.

EXAMPLE 2. Find the second moment of area of the section in Fig. 118 about an axis XX through its centroid.

Since the section is symmetrical the XX axis will be 7 in. from the top.

The second moment of area can be found in either of two ways.

(a) I_{XX} of whole section

= I_{XX} rectangle $ABGH$ − I_{XX} rectangle $MLKJ$

− I_{XX} rectangle $DEFG$.

or, since rectangle $MLKJ$ = rectangle $DEFG$

$$\begin{aligned} I_{XX} \text{ of whole section} &= I_{XX} \text{ rectangle } ABGH \\ &\quad - 2\,(I_{XX} \text{ rectangle } DEFG) \\ &= \frac{6 \times 14^3}{12} - \frac{2 \times 2\frac{5}{8} \times 13^3}{12} \text{ inch units} \\ &= 1372 - 960{\cdot}4 \text{ inch units} \\ &= 411{\cdot}6 \text{ inch units.} \end{aligned}$$

(*b*) I_{XX} of whole section

$$\begin{aligned} &= I_{XX} \text{ rectangle } LDEK + 2(I_{XX} \text{ of one flange}) \\ &= \frac{\frac{3}{4} \times 13^3}{12} + 2\left\{\frac{6 \times (\frac{1}{2})^3}{12} + 3 \times (6\tfrac{3}{4})^2\right\} \\ &= 137{\cdot}6 + 2\,(0{\cdot}06 + 136{\cdot}95) \\ &= 137{\cdot}6 + 274 = \underline{\underline{411{\cdot}6 \text{ inch units.}}} \end{aligned}$$

Design of Beams

Method of Design. The process of designing a beam can now be summarized—

(*a*) Calculate the maximum values of bending moment and shearing force.

(*b*) Knowing the maximum bending moment M, calculate the section modulus Z required to keep the bending stress within the allowable intensity f, using the formula $M = fZ$.

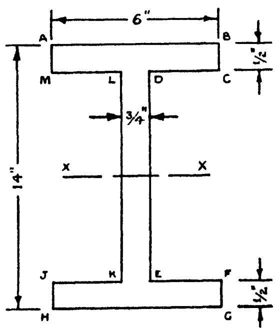


Fig. 118. Second Moment of Area of an I-Section

(*c*) Select a suitable section having a value of Z equal to, or greater than, that required. For rectangular sections $Z = \frac{bd^2}{6}$ and suitable values of b and d must be chosen. For rolled-steel joists a section having a suitable value of Z can be looked up in the tables.

(*d*) Check the shear stress intensity. For rolled steel sections the shear stress distribution is such that the whole shear force may be assumed to be resisted by the web area only.

Example 1. A beam, 12 ft long, carries a uniformly distributed load of 1 t/ft run. Find a suitable rolled-steel joist if the allowable bending stress is not to exceed 8 t/in.² and the shear stress 5 t/in.²

$$\text{Max. bending moment} = \frac{wL^2}{8} = \frac{1 \times 144}{8} = 18 \text{ t-ft}$$

$$= 216 \text{ t-in.}$$

$$\therefore \text{ Z required} = \frac{M}{f} = \frac{216}{8} = 27 \text{ inch units.}$$

Referring to tables, possible suitable sections are—

8 in. × 6 in. × 35 lb joist, Z = 28·76 inch units.
10 in. × 5 in. × 30 lb joist, Z = 29·25 inch units.

The 10 in. × 5 in. joist is deeper and will therefore deflect less than the 8 in. × 6 in. joist, also, since the cost depends upon the weight per foot the 10 in. × 5 in. joist will be cheaper. Therefore, if there is sufficient room, the 10 in. × 5 in. joist should be chosen.

Checking for shear—

$$\text{Max. shear force} = \frac{wL}{2} = 6 \text{ t}$$

$$\text{Web area of 10 in.} \times \text{5 in. joist} = 0{\cdot}36 \times 8{\cdot}9 = 3{\cdot}2 \text{ in.}^2$$

$$\therefore \text{ Intensity of shear stress} = \frac{6}{3{\cdot}2} = 1{\cdot}9 \text{ t/in.}^2$$

This is satisfactory.

Selected section: 10 in. × 5 in. × 30 lb joist.

EXAMPLE 2. If a timber joist was used instead of the steel joist in the previous example, what would be a suitable section if the bending stress must not exceed $\frac{1}{2}$ t/in.^{2}?

As before: Max. bending moment = 216 t-in.

$$\therefore \text{ Z required} = \frac{M}{f} = \frac{216}{\frac{1}{2}} = 432 \text{ inch units.}$$

For a rectangular section $Z = \frac{bd^2}{6}$

where b = breadth of beam
d = depth of beam.

$$\therefore \frac{bd^2}{6} = 432$$

$$bd^2 = 2592$$

A value of either b or d must be chosen which will suit the particular circumstances.

Take, say, $b = 12$ in.

Then $12d^2 = 2592$

$$d^2 = 216$$

$$d = 14{\cdot}6 \text{ in.}$$

Timber section required = 15 in. × 12 in.

EXERCISES

1. Calculate the safe central point load that a timber beam 6 in. deep by 2 in. wide could carry over a span of 8 ft if the allowable stress due to bending must not exceed 900 lb/in.^{2}

2. A cantilever 4 ft long is made from timber 12 in. deep and 6 in. wide with a density of 40 lb/ft^3. It carries a concentrated load of 1500 lb at its mid-point and a uniform load of 40 lb per ft run. If the working stress in the timber is not to exceed 900 lb/in.^{2}, what additional load could be carried?

3. A timber joist, 9 in. deep by 3 in. wide, carries a uniformly distributed load across a span of 10 ft. If the stress intensity in the timber is not to exceed 700 lb/in.^{2}, calculate the load in pounds per foot run which the beam can carry.

4. A built up beam consists of a 10 in. $\times$ 6 in. $\times$ 40 lb rolled-steel joist ($I_{XX} = 204{\cdot}8$ in. units) with an 8 in. $\times$ $\frac{3}{4}$ in. plate riveted to each flange. Calculate the second moment of area, or moment of inertia, of this section, neglecting rivet holes.

5. A cast-iron beam has an I-section with a top flange 3 in. broad, bottom flange 9 in. broad and an overall depth of 12 in. If the thickness of metal at all points is $\frac{3}{4}$ in., calculate the position of the neutral axis and the second moment of area of the section.

6. Select a suitable rolled-steel joist for a simply supported beam carrying a uniformly distributed load of 3 cwt per foot run and a central point load of 4 t over a span of 12 ft if the maximum stress due to bending is not to exceed 8 t/in.^{2}

7. A timber beam of rectangular section is required to carry a uniformly distributed load of 112 lb per foot run over a span of 10 ft. Select suitable dimensions for this beam if the stress due to bending is not to exceed 900 lb/in.^{2}

8. Calculate the safe central point load which a 24 in. $\times$ $7\frac{1}{2}$ in. $\times$ 95 lb rolled-steel joist can carry over a span of 20 ft, (*a*) if the weight of the joist is neglected, (*b*) taking the weight of the joist into account.

SECTION III—ELEMENTARY DYNAMICS AND SIMPLE MACHINES

CHAPTER XV

LINEAR MOTION, WORK AND ENERGY

While primarily concerned with the stability of stationary structures, the building technician requires an elementary knowledge of dynamics (the study of moving bodies) to enable him to understand the mechanical plant which he employs and to employ it to the best advantage.

Linear Motion

Distance-Time Diagram. A body travelling in a straight line is said to have a *linear motion* which can be represented by drawing

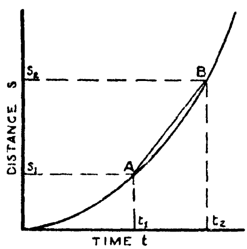


Fig. 119. Distance-time Diagram for a Moving Body

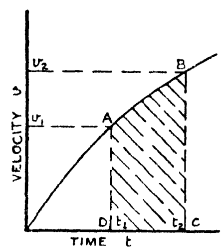


Fig. 120. Velocity-time Curve for a Moving Body

a graph showing the distance, s, from a fixed point at any given time t, since it left that point. This graph is called a *distance-time* diagram, Fig. 119.

Velocity. Velocity is the rate at which a body is changing its position. Thus, if its distance from a fixed point at time t_1 is s_1 and at time t_2 is s_2—

$$\text{Velocity of body} = \frac{s_2 - s_1}{t_2 - t_1}$$

It is possible that the velocity may vary over the interval $t_2 - t_1$ so that $\frac{s_2 - s_1}{t_2 - t_1}$ is only the average velocity over this interval. From Fig. 119 it can be seen that this term represents the slope of the line AB. If B approaches A and finally coincides with it, the slope of AB will become the slope of the tangent to the distance-time curve at A and will represent the instantaneous velocity at A.

If the velocity is variable, a *velocity-time* curve, such as that shown in Fig. 120, can be drawn to represent this variation. Velocity is measured in feet per second and similar units.

Acceleration. Acceleration is the rate of change of velocity with respect to time. The average acceleration in the interval between t_1 and t_2, Fig. 120, will be $\frac{v_2 - v_1}{t_2 - t_1}$ and the instantaneous acceleration at A will be equal to the slope of the tangent to the velocity-time curve at A. Acceleration is measured in feet per second per second or ft/sec.^{2} The acceleration due to gravity is denoted by g and is equal to 32·2 ft/sec.^{2}

Area under Velocity-Time Curve. The distance moved by a body is the product of its velocity and the time during which it moves. If the velocity is variable, as in Fig. 120, the distance moved in the time interval from t_1 to t_2 can be found by measuring the area $ABCD$ under the velocity-time curve between the ordinates t_1 and t_2.

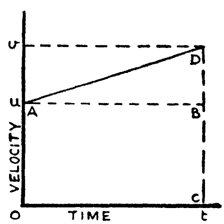


FIG. 121. VELOCITY-TIME CURVE — BODY MOVING WITH UNIFORM ACCELERATION

Uniform Velocity. If a body having linear motion travels for a time, t, with uniform velocity, v, the distance, s, will be given by—

$$s = v\,t$$

Uniform Acceleration. The velocity-time diagram for a body moving with uniform acceleration is shown in Fig. 121. If u is the initial velocity, then after time, t, the velocity will have increased by an amount $f \times t$ where f is the acceleration.

$$\text{Final velocity: } v = u + f \times t \quad . \quad . \quad . \quad (1)$$

Again, to determine the distance, s, travelled in time t—

$$\begin{aligned} s &= \text{Area under velocity-time diagram} \\ &= \text{Area } OABC + \text{Area } ABD \\ &= ut + \tfrac{1}{2}(v - u)t \end{aligned}$$

Or, substituting for v from equation (1)

$$s = ut + \tfrac{1}{2}ft^2 \quad .. \quad . \quad . \quad . \quad (2)$$

Again, in equation (2), substituting for t from equation (1)

$$\begin{aligned} s &= u\,\frac{(v-u)}{f} + \tfrac{1}{2}f\,\frac{(v-u)^2}{f^2} \\ &= \frac{2uv - 2u^2 + v^2 - 2uv + u^2}{2f} \end{aligned}$$

Or $$v^2 - u^2 = 2fs \quad . \quad . \quad . \quad . \quad (3)$$

Composition and Resolution of Velocities. Velocity has both magnitude and direction, i.e. it is a vector quantity. As such, velocities can be compounded and resolved by means of triangles and polygons of velocities, just as triangles and polygons of forces are used for the composition and resolution of forces (page 43).

EXAMPLE. A ball is thrown upwards from the foot of a building 40 ft high and reaches a height of 12 ft above the level of its flat roof. If the ball finally falls on to the roof, find its total time of flight and its initial velocity. Acceleration due to gravity = 32·2 ft/sec².

If, t_1 = Time of upward flight in seconds
t_2 = Time of downward flight in seconds
u = Initial velocity in feet per second.

For upward flight, applying the equation $v^2 - u^2 = 2fs$ and putting $f = -32{\cdot}2$ ft/sec.², since gravity exerts a decelerating force, $s = 52$ ft the total upward flight and $v = 0$, since the ball has no velocity at the top of its flight—

$$u^2 = 2 \times 32{\cdot}2 \times 52 = 3350$$

$$\therefore \text{ Initial velocity } u = \underline{\underline{57{\cdot}8 \text{ ft/sec.}}}$$

To find the time of upward flight, apply the equation $v = u + ft$ putting $v = 0$, $u = 57{\cdot}8$ ft/sec, and $f = -32{\cdot}2$ ft/sec²—

$$t_1 = \frac{0 - 57{\cdot}8}{-32{\cdot}2} = 1{\cdot}8 \text{ sec}$$

To find the time of downward flight use the equation $s = ut + \frac{1}{2}ft^2$ putting $s = 12$ ft, $u = 0$, and $f = +32{\cdot}2$ ft/sec², since gravity has an accelerating effect. Then,

$$12 = 0 + \frac{32{\cdot}2}{2} \times t_2^2$$

$$t_2^2 = \frac{2 \times 12}{32{\cdot}2} = 0{\cdot}74$$

$$t_2 = 0{\cdot}86 \text{ sec}$$

$$\text{Total time of flight} = t_1 + t_2 = \underline{\underline{2{\cdot}66 \text{ sec}}}$$

Newton's Laws of Motion. Three fundamental laws, due to Sir Isaac Newton, whose name they bear, form the basis of the study of dynamics—

1. *Every body continues in its state of rest or of uniform motion in a straight line except in so far as it is compelled by forces to change that state.*

2. *The rate of change of momentum of a body produced by a force is proportional to the applied force and takes place in the direction in which the force acts.*

The *momentum* of a body is the product of its mass and velocity so that the rate of change of momentum will be the product of its mass and the rate of change of velocity (i.e. acceleration). Thus this second law gives the relation between the impressed force and the acceleration which it produces—

$$\text{Force} = \text{mass} \times \text{acceleration}$$

3. *To every action there is always an equal and opposite reaction.* This law shows that it is impossible for a single force to act by itself. For example, to exert a pull on a string its other end must be fixed.

Work, Power and Energy

Work. Work is done when the application of a force causes its point of action to move in the direction in which the force is acting. If a body remains at rest under a system of forces, no work is done. Work is measured as the product of the force and the distance through which it moves in the direction of its line of action; the units employed are of the form *force* × *distance*, the British unit being the *foot-pound.*

Power. Power is the work done in unit time or the rate of doing work. The British unit is the *horse-power*, which is a rate of working of 33,000 ft-lb/min or 550 ft-lb/sec. Thus the horse-power in any given case can be found by dividing the work done per minute by 33,000.

The metric unit is the *Watt*, known to most people as the unit of electrical power, being the rate of working when one ampere flows under a potential difference of one volt. The product of current in amperes and the voltage will give the power in watts. One horse-power is equivalent to 746 W.

Energy. The energy of a body is its *capacity for doing work* and is measured in terms of the work which could be performed thus having the same units. There are many different forms of energy such as heat energy, electrical energy, sound energy, and chemical energy. Dynamics is concerned with mechanical energy which may be of two forms: *potential* energy and *kinetic* energy.

Potential Energy. In so far as a body can do work by falling, it can have potential energy by virtue of its weight and its position above any standard reference level. If a body weighing w lb is situated h feet above datum, then

$$\text{Potential energy} = wh \text{ ft-lb}$$

Kinetic Energy. A moving body has kinetic energy by virtue of its motion, which is measured in terms of the amount of work which must be done to bring it to rest. Supposing that a body of weight w $\left(\text{i.e. mass } \frac{w}{g}, \text{ where } g = \text{acceleration due to gravity}\right)$ is brought to rest by a force P, then the retardation f produced can be calculated from Newton's second law—

$$\text{Force} = \text{mass} \times \text{retardation}$$

$$\therefore\ P = \frac{w}{g}f$$

If the force brings the body to rest in a distance x, then, since the initial velocity is v and the final velocity zero

$$v^2 - 0 = 2fx$$

or

$$fx = \tfrac{1}{2}v^2$$

But work done by force $= Px = \dfrac{w}{g}fx$

Kinetic energy of body $= \dfrac{w}{g}fx = \tfrac{1}{2}\dfrac{w}{g}v^2$

This value will be in practical units.

Principle of Conservation of Energy. The following general law is found to be true for all forms of energy—

"*Energy cannot be created or destroyed, but can only be converted from one form to another.*"

There is, however, no guarantee that all of a given quantity of one kind of energy can be converted into the equivalent amount of another particular type of energy. For example, in mechanical plant it is desired to convert the chemical energy of the fuel into mechanical energy. The process is only partially successful, but the remainder of the chemical energy is not destroyed since it appears in other forms, principally as heat.

EXERCISES

1. A train travelling from one station to another 1·6 miles away starts by accelerating at the rate of 25 m.p.h. in 0·5 min until it reaches a speed of 30 m.p.h. This speed is maintained until on approaching the second station the brakes are applied to produce a retardation of 3 ft/sec^2 to bring the train to rest in the station. What is the time taken for the journey?
2. A body, falling freely, passes two points 30 ft apart in one-fifth of a second. From what height above the upper point did it start and how long had it been travelling?
3. A boat starts to cross a river 90 yd wide in a direction at 90° to its banks, but is swept down stream by the current to a point 40 yd further down than the point it should have reached. If the boat takes 3 min to cross, what is the speed of the current?
4. A pump raises 45 t of water per hour to a height of 50 ft. How much work does the pump do in one minute? What is its horse-power?
5. A barge was towed 100 ft by a rope inclined at an angle of 30° to its direction of motion. If the pull in the rope was 215 lb, how much work was done?
6. A truck of weight 20 t moving at 6 ft/sec was brought to rest by buffers in a distance of 8 in. What was the original kinetic energy of the truck and what average force was exerted by the buffers?
7. A car, in which the resistance to motion on the level may be considered to be the same at all speeds, runs steadily on the level at 20 m.p.h. It now commences to climb a rise of 1 in 12. If the energy supplied by the engine is still the same, how far will the car travel along the rising road?
8. A vessel weighing 10,000 t having a speed of 30 ft/sec is slowed down to a speed of 10 ft/sec in a distance of 4000 ft. What is the average resistance to motion?

CHAPTER XVI

SIMPLE MACHINES

A MACHINE is a device which takes in energy in one form and delivers it in another form more suited for the purpose in view. Many lifting machines, such as cranes, pulley blocks and hoists, are used in building practice to enable large loads to be handled by means of comparatively weak forces.

Mechanical Advantage. This advantage (the handling of large loads by small forces) is termed the mechanical advantage of the machine, If a given load can be lifted by applying a certain force or effort—

$$\text{Mechanical advantage} = \frac{\text{load overcome}}{\text{effort applied}}$$

Velocity Ratio. In using a set of pulley blocks it will be seen that the rope pulled by the operator moves very much faster than the load which is being raised. The *velocity ratio* of the machine is equal to $\frac{\text{velocity of effort}}{\text{velocity of load}}$. These velocities will be proportional to the distances moved in a given time by the load and effort respectively, so that—

$$\text{Velocity ratio} = \frac{\text{distance moved by effort}}{\text{distance moved by the load}}$$

Efficiency. It has already been seen that, although a small effort can be made to overcome a large load, the distance moved by the effort will be much greater than that moved by the load. According to the Law of Conservation of Energy, the energy supplied by the effort must be at least equal to the work done on the load. In practice a certain amount of the energy supplied will be dissipated through friction in the machine in the form of heat so that the total energy put into the machine will be greater than the work done by the machine.

$$\text{Work given to machine} = \text{effort} \times \text{distance moved by effort}$$
$$\text{Work done by machine} = \text{load} \times \text{distance moved by load}$$

$$\text{Efficiency} = \frac{\text{work done by machine}}{\text{work given to machine}}$$
$$= \frac{\text{load} \times \text{distance moved by load}}{\text{effort} \times \text{distance moved by effort}}$$
$$= \frac{\text{mechanical advantage}}{\text{velocity ratio}}$$

The efficiency cannot be greater than unity. It is sometimes expressed as a percentage. The efficiency of a machine is not necessarily constant but may vary with the load.

Ideal Effort. In an ideal machine, completely free from friction, all the work supplied would be usefully employed. The effort which would be required under these conditions of 100 per cent efficiency is called the ideal effort—

$$\text{Work supplied} = \text{work done by machine}$$

$$\therefore \text{ Ideal effort} \times \text{distance moved by effort}$$
$$= \text{load} \times \text{distance moved by load}$$

$$\text{Ideal effort} = \text{load} \times \frac{\text{distance moved by load}}{\text{distance moved by effort}}$$
$$= \frac{\text{Load}}{\text{velocity ratio}}$$

A practical machine is not 100 per cent efficient. The difference between the ideal effort and the actual effort required is used in overcoming friction and is called the friction effort.

EXAMPLE. A load of 200 lb is raised 2 in. vertically with the help of lifting tackle by exerting a force of 15 lb through a distance of 6 ft. What is the mechanical advantage, velocity ratio, and efficiency of the tackle, and what is the force used in overcoming friction?

$$\text{Mechanical advantage} = \frac{\text{load raised}}{\text{Effort}} = \frac{200}{15} = \underline{\underline{13{\cdot}3}}$$

$$\text{Velocity ratio} = \frac{\text{distance moved by effort}}{\text{distance moved by load}} = \frac{6 \times 12}{2} = \underline{\underline{36}}$$

$$\text{Efficiency} = \frac{\text{mechanical advantage}}{\text{velocity ratio}} = \frac{13{\cdot}3}{36}$$
$$= \underline{\underline{0{\cdot}37}} \text{ or } \underline{\underline{37 \text{ per cent}}}$$

$$\text{Ideal effort} = \frac{\text{load}}{\text{velocity ratio}} = \frac{200}{36} = \underline{\underline{5{\cdot}55 \text{ lb}}}$$

$$\text{Friction effort} = \text{actual effort} - \text{ideal effort}$$
$$= 15 - 5{\cdot}55 = \underline{\underline{9{\cdot}45 \text{ lb}}}$$

Characteristic Curves for a Machine. Although the velocity ratio of a machine is always constant, the mechanical advantage, efficiency, ideal effort and friction effort will vary according to the load. If the values of effort for a range of loads are determined experimentally, curves can be plotted for mechanical advantage, efficiency, ideal effort, and friction effort on a base representing load from which the values of these quantities can be found for any given load. A typical set of results is shown in Fig. 122.

Load lb .	30	50	75	100	125	150	175	200	225	250
Effort lb .	5·0	6·2	7·7	9·1	10·5	12·1	13·6	15·0	16·5	18·0
Mech. Adv.	6·0	8·1	9·7	11·0	11·9	12·4	12·9	13·3	13·6	13·9
Ideal effort	0·8	1·2	1·9	2·5	3·1	3·8	4·4	5·0	5·6	6·2
Friction effort	4·2	5·0	5·8	6·6	7·4	8·3	9·2	10·0	10·9	11·8
Efficiency .	15	20	24	28	30	31	32	33	34	35

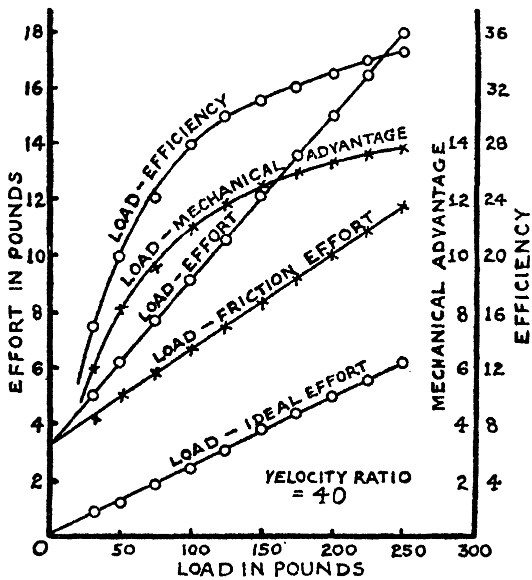


FIG. 122. CHARACTERISTIC CURVES FOR A SIMPLE MACHINE

Studying these curves it will be seen that the load-effort, load-ideal effort and load-friction effort graphs are straight lines showing that effort, ideal effort, and friction effort are directly proportional to the load. The load-mechanical advantage and load-efficiency curves are similar in form, the curves rising steeply at first, but becoming flatter as the load increases. Maximum mechanical advantage and maximum efficiency are obtained at maximum load, because at low loads a large proportion of the effort is spent in overcoming internal friction in the machine, while at higher loads, although the friction effort has increased, it forms a smaller proportion of the total effort.

Pulley Blocks. Fig. 123 shows a common form of lifting tackle consisting of a two-sheave and a three-sheave pulley block. The effort is exerted on the "fall" at P and the load is suspended at w.

If the effort moves through a distance d, each rope in the system shortens by the same amount, in this case $\frac{d}{5}$, and the load will also rise by a distance $\frac{d}{5}$.

$$\text{Velocity ratio} = \frac{\text{distance moved by effort}}{\text{distance moved by load}}$$

$$= \frac{d}{\frac{d}{5}} = 5 = \text{number of supporting ropes}$$

The mechanical advantage will depend upon the internal friction in the system and will be equal to $\frac{w}{P}$.

Weston Differential Pulley Block. This type of block is shown in Fig. 124 and consists of a single lower pulley carrying the load w

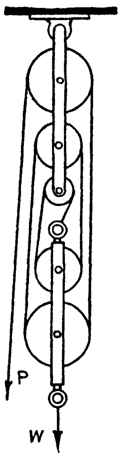


FIG. 123. PULLEY BLOCKS

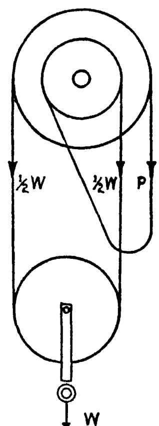


FIG. 124. WESTON DIFFERENTIAL PULLEY BLOCK

which is supported by an endless chain from an upper pulley which has two sections of slightly different diameter. The links of the chain passing round these pulleys engage in recesses which prevent slipping. The effort P is applied to one side of a loop in the chain as shown. The weight w is supported equally by the two vertical sections of chain which will therefore carry loads of $\frac{1}{2}w$ each.

For one revolution of the upper pulley,

Distance moved by effort $P = 2\pi R$

Loss of chain between top and bottom blocks $= 2\pi\,(R - r)$

Distance moved by load $= \pi (R - r)$

$$\text{Velocity ratio} = \frac{2\pi R}{\pi(R - r)} = \frac{2R}{(R - r)}$$

As before, the mechanical advantage will depend upon the friction.

LABORATORY WORK

EXPERIMENT. *Lifting Machines*

Students should carry out experiments on simple machines available in the laboratory and draw characteristic curves. This work should include test on—

1. Pulley blocks.
2. Screw jack.
3. Simple crane or winch.

It is often difficult to decide what the exact effort required for a given load should be. The correct value will be that which, when set in motion, will travel with uniform velocity either up or down.

EXERCISES

1. Explain the terms: Mechanical advantage, velocity ratio, efficiency, and ideal effort.

A wheel and axle have radii 12 in. and 3½ in. respectively. If a force of 12 lb applied to a rope wound round the wheel is sufficient to raise a weight of 28 lb suspended from a rope wound round the axle, calculate the value of the above quantities.

2. Given a pair of four- and five-sheave blocks, it is found that a pull of 120 lb will raise a load of 800 lb. Calculate the value of the velocity ratio, mechanical advantage, and efficiency at this load.

3. A screw jack is used to raise a load of one ton. The screw rises ½ in. per revolution and a force of 20 lb, exerted at the end of a tommy bar 30 in. long, is required to turn the screw. Find the efficiency of the jack at this load.

4. In a Weston differential pulley block the two diameters of the top sheave are 9 in. and 9½ in. respectively. What is the velocity ratio?

If a pull of 71 lb is required to raise a load of 15 cwt find the mechanical advantage and efficiency at this load.

5. A load of 5 t is placed on a smooth plane inclined at an angle of 3° to the horizontal. Calculate the force which must be applied parallel to the plane to keep the load in position and find the mechanical advantage of this arrangement.

6. A lifting machine has a velocity ratio of 15. A test gave the following results—

Load (lb)	30	40	50	60	70	80	90	100
Effort (lb)	7·0	8·6	10·1	11·6	13·2	14·7	16·1	17·6

Draw curves, to a base of load, for: Mechanical advantage, ideal effort, friction effort, and efficiency.

7. An effort of 10 lb is sufficient to raise a load of 15 lb using a wheel and axle. If the radius of the axle is 6 in. and the efficiency of the machine is 90 per cent, what is the radius of the wheel?

8. A load of 288 lb can be raised by a pull of 60 lb using a pair of three- and four-sheave pulley blocks. Calculate the velocity ratio, mechanical advantage, and efficiency at this load.

SECTION IV—HEAT AND HEAT TRANSMISSION

CHAPTER XVII

HEAT

A KNOWLEDGE of the fundamental principles and theories of heat is essential for the proper study of the heating and, to a certain extent, the ventilation of buildings.

Nature of Heat. Heat is a form of energy and can be produced by a change from other forms, as in the following cases—

1. From chemical energy—when a fuel burns.
2. From mechanical energy—the spark produced by a lighter flint when the wheel is turned.
3. From electrical energy—when the heater of an electric radiator glows.

As a form of energy heat has the capacity to do work by causing the circulation of water in water heating systems and the rising of warm air currents within a room or twisting and buckling large steel joists with ease when a building takes fire.

Temperature. It is necessary to distinguish between the words *heat* and *temperature.* If two identical heaters were placed in the centres of two rooms, one small, the other large, but in all other respects identical, then after a given time both rooms would have received equal amounts of heat energy, but the smaller room would be found to be at a higher temperature than the larger. The word *temperature* is used to indicate the concentration of heat.

Again, although a cup full of boiling water will have a temperature far greater than that of a bath full of cold water, the latter may contain considerably more heat. The water in the bath would melt a greater weight of ice than the boiling water in the cup. The water in the cup is at a higher temperature than that in the bath because the smaller amount of heat in the former is more concentrated than that in the latter.

Absolute Zero of Temperature. When heat energy is given to a body it is converted into kinetic energy, increasing the speed of vibration of the molecules and usually causing a rise of temperature. Conversely, as heat leaves a body it loses internal energy, the molecules vibrate more slowly and the temperature drops. These changes are accompanied by changes in size, normally the body tends to expand as its temperature rises and to contract as it falls. If a body could lose all its heat its molecules would come to rest, there would be no concentration of heat energy and the temperature would have reached an *absolute zero.* It will be seen later (page 157) that this absolute zero of temperature, though it has never been reached, lies 273 degrees below the zero on the centigrade scale (i.e. at $-273°$ C). In building practice, it is the amount of heat gained or lost by a body, rather than the total amount of heat contained, which is important.

Measurement of Temperature

Temperature is commonly measured by means of *thermometers*, which depend for their action upon the expansion or contraction of their constituent materials in taking up the temperature of the substance under investigation. For determining the temperature of ovens or small kilns, rods of brass or other material with a high coefficient of expansion may be used, being arranged so that the changes in length of the rod are magnified mechanically and control the movement of a pointer over a graduated scale. Liquid thermometers are used for lower temperatures, the changes in volume of the liquid in a thin-walled bulb at the end of a thick-walled capillary tube causing a thread of liquid to move along a scale engraved on the capillary tube. Mercury and alcohol are suitable liquids since they expand regularly, but as the freezing and boiling points of mercury are respectively $-39°$ C and 356° C and those of alcohol are $-130°$ C and 78° C the range of each type of thermometer is limited.

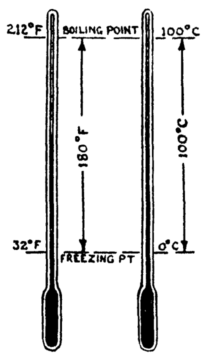


Fig. 125.
Fahrenheit and Centigrade Temperature Scales

Temperature Scales. Two scales of temperature are in use in the building industry, the *Fahrenheit* and the *centigrade*. Both scales are based on the same two fixed points—the imaginary or real position of the expanding material at the freezing and boiling points of water. On the Fahrenheit scale these correspond to 32° and 212° respectively and on the centigrade scale to 0° and 100°. The diagram, Fig. 125, shows that the degree Fahrenheit is smaller than the degree centigrade—

$$180°\ F = 100°\ C \text{ or } 1°\ F = \tfrac{5}{9}°\ C$$

To convert a reading on one scale to the corresponding reading on the other the difference in zero readings must be taken into account—

(*a*) To convert a reading of $t°$ F to the centigrade scale. No. of Fahrenheit degrees above freezing point $= t - 32$.

$\therefore$ No. of centigrade degrees above freezing point $= \frac{5}{9}(t - 32)$

Reading on centigrade scale $= \frac{5}{9}(t - 32)$

(*b*) To convert a reading of $T°$ C to the Fahrenheit scale. No. of centigrade degrees above freezing point $= T$.

$\therefore$ No. of Fahrenheit degrees above freezing point $= \frac{9}{5} T$

Reading on Fahrenheit scale $= \frac{9}{5} T + 32$.

Pyrometers. These are instruments used for the measurement of very high temperatures. Electrical pyrometers depend either on the fact that the resistance of a metallic wire (platinum or a platinum alloy in many cases) changes with temperature or in the case of thermo-electric pyrometers upon the electromotive force set up when the junction of two dissimilar metals is heated. The magnitude of the effect depends upon the temperature. Pairs of metals used in this way constitute *thermo-couples.* Suitable pairs of metals are—

Platinum in contact with alloys of platinum and rhodium or iridium.

Nickel in contact with an alloy of nickel and chromium.

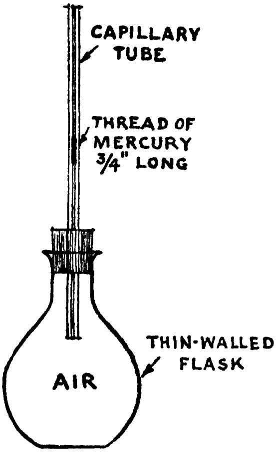


FIG. 126. AIR THERMOMETER

It is a great advantage if the pyrometer is not directly subjected to the temperature to be measured. In one, devised by Féry, heat rays are concentrated on the thermo-couple by means of a concave mirror. Where the pyrometer is placed directly in the furnace it is protected by a thin fireclay tube or thermo-couple cover. The change of temperature is read from the movement of the needle of a galvanometer (page 163) which may be placed in a convenient position remote from the actual furnace.

Air Thermometer. Small differences of temperature can be measured with an air thermometer, Fig. 126. Air has a high coefficient of expansion and as its volume changes with temperature the thread of mercury in the capillary tube moves up or down. As with liquid thermometers, the sensitivity of the instrument will depend partly upon the fineness of the bore of the capillary.

Maximum and Minimum Temperatures. In testing the effectiveness of a heating installation records of maximum and minimum temperatures are required. These are automatically registered on a special *maximum and minimum thermometer*, one type of which is illustrated in Fig. 127.

MEASUREMENT OF HEAT

The amount of heat energy gained or lost by a body is proportional both to its mass and to its gain or loss of temperature—

$$H \propto m \times t$$

Thus if a gas geyser requires a certain quantity of gas to raise the temperature of a given quantity of water by a certain amount, it will require twice that amount of gas to raise the temperature of twice that mass of water by the same amount, or, alternatively, to raise the temperature of the same mass of water by twice that amount.

Specific Heat. Another factor to be taken into account is the nature of the material itself. Equal masses of different materials require different amounts of heat to raise their temperature by the same amount. They are said to have different *thermal capacities*. The thermal capacity of a substance at any temperature is the quantity of heat required to raise the temperature of the substance by one degree. It is more convenient to compare the amount of heat required to cause a given temperature rise in a material with the amount of heat required to produce the same temperature rise in the same mass of a standard substance. The ratio of these two figures is called the *specific heat* of the material. Pure water is the standard chosen and water thus has a specific heat of unity. It is a convenient standard because it can be obtained in a reasonably pure state by distillation and also has a higher specific heat than most substances. Table VI gives values for the specific heats of common substances. It is the high specific heat of water which makes it so suitable for conveying heat within buildings, since for a given fall of temperature it will give out a very large quantity of heat.

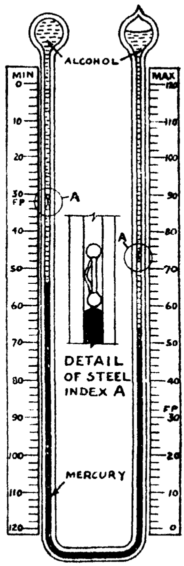


FIG. 127. MAXIMUM AND MINIMUM THERMOMETER

TABLE VI

SPECIFIC HEATS

Aluminium	0·212	Asbestos	0·20
Brass	0·09	Ebonite	0·40
Copper	0·094	Glass	0·19
Lead	0·031	Ice	0·50
Steel	0·115	Marble	0·21
Zinc	0·094	Quartz	0·17
Firebrick	0·20	Wood	0·42

Units of Heat. A convenient unit of heat is the amount required to raise unit mass of water through unit rise of temperature. In the metric system, used in laboratory work, the unit is the *calorie* which is the amount of heat required to raise one gramme of water through one degree centigrade. In the British system the practical unit is the *British thermal unit* (B.Th.U.) which is defined as the amount of heat required to raise one pound of pure water through one degree Fahrenheit. In gas production the unit employed is the *Therm*, whuich is a measure of the heating value of the gas and is equal to 100,000 B.Th.U.

To calculate the amount of heat gained or lost by a body find the product of its weight, specific heat, and its gain or loss of temperature—

EXAMPLE. Calculate the heat required to raise the temperature of a length of steel pipe, weighing 1 t, by 160° F if the specific heat of the material is 0·12.

$$\text{Amount of heat required} = 2240 \times 0{\cdot}12 \times 160 \text{ B.Th.U.}$$
$$= \underline{\underline{47{,}000}} \text{ B.Th.U.}$$

Determination of Specific Heat. The simplest method of determining the specific heat of a substance is that known as the *method of mixtures*. A specimen of the material is broken into pieces small enough to slide into a boiling tube. The tube is heated in a water bath until the temperature of the material throughout its whole volume is that of the boiling water. A plug of cotton wool should be placed in the end of the tube to prevent steam wetting the specimen.

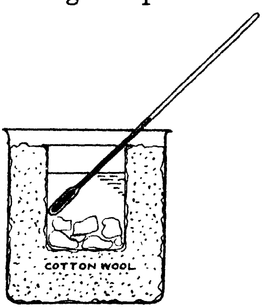


FIG. 128. INSULATED CALORIMETER

Meanwhile a clean dry calorimeter (a simple cylindrical copper vessel) is weighed. It is partially filled with water and reweighed to determine the weight of water added. The calorimeter and its contents are surrounded with an insulating shield, Fig. 128. When the material in the boiling tube has been thoroughly heated its temperature and the temperature of the water in the calorimeter are noted and the specimen is transferred quickly from the boiling tube to the calorimeter. The mixture is then stirred carefully with the thermometer until it registers a constant temperature, which should be noted. When the calorimeter is sufficiently cool it is reweighed, with its contents. The following readings and results are for a test on asbestos sheeting—

Weight of calorimeter $= m_c = 52{\cdot}40$ g
Weight of calorimeter + cold water $= 92{\cdot}97$ g
Weight of cold water $= m_{cw} = 40{\cdot}57$ g
Temperature of cold water and calorimeter $= 16{\cdot}8°$ C
Temperature of heated specimen $= 99°$ C
Temperature of mixture $= 23{\cdot}4°$ C
Rise of temperature of water and calorimeter $= t_1 = 6{\cdot}6°$ C
Fall of temperature of specimen $= t_2 = 75{\cdot}6°$ C
Weight of calorimeter + mixture $= 108{\cdot}58$ g
Weight of specimen $= m_s = 15{\cdot}61$ g

Assuming that no heat is lost during the experiment—

$$\underbrace{\text{Heat gained}}_{\substack{\text{By cold water} \quad \text{By calorimeter} \\ m_{cw} \times c_{cw} \times t_1 + m_c \times c_c \times t_1}} = \underbrace{\text{heat lost}}_{\substack{\text{By specimen} \\ m_s \times c_s \times t_2}}$$

where, c_{cw}, c_c, and c_s are the specific heats of cold water, calorimeter, and specimen respectively. Taking $c_c = 0{\cdot}1$

$$t_1 (m_{cw} + 0{\cdot}1\ m_c) = m_s \times c_s \times t_2$$

$$c_s = \frac{(m_{cw} + 0{\cdot}1\ m_c)t_1}{m_s t_2}$$

$$= \frac{(40{\cdot}57 + 0{\cdot}1 \times 52{\cdot}40) \times 6{\cdot}6}{15{\cdot}61 \times 75{\cdot}6}$$

∴ Specific heat of specimen $= \underline{0{\cdot}26}$

The above heat equation is a corollary of the Law of Conservation of Energy. Any radiation or other heat loss from the calorimeter will, of course, constitute an error, but, with properly designed apparatus this should be small.

Most calorimeters are made of copper because this material is a good conductor of heat and the whole calorimeter will rapidly assume the temperature of its contents, so that t_1 will be the rise of temperature of both. The product of the weight of the calorimeter and the specific heat of the material of which it is made will be a constant for each particular calorimeter and is referred to as its *water equivalent.*

Latent Heat. Normally, when addition of heat energy increases the molecular energy of a material, its temperature rises, but if the material is at its melting point or boiling point its temperature will remain constant and the additional energy will cause it to change its state from solid to liquid or from liquid to gas or vapour. No matter at what rate the heat is supplied the temperature will not rise until the whole of the material has changed from one state to the other. Thus under normal conditions of pressure the temperature of a mixture of ice and water will always be 32° F and the temperature of steam in the presence of boiling water will be 212° F irrespective of the amount of heat being supplied. The heat used to cause the change of state is said to be *latent* in the liquid or gas formed and is given out again when the liquid freezes or the steam is condensed. The latent heat of fusion (or solidification) and the latent heat of vaporization (or condensation) is measured in calories per gramme or B.Th.U. per pound. Under normal conditions the latent heat of ice is 80 g.cal/g and that of steam is 536 g.cal/g (966 B.Th.U./lb). Hence at 100° C, 536 cal are required to convert 1 g of water into steam at the same temperature.

The *total heat* of a substance is made up therefore of the *sensible heat*, which causes rise of temperature, and the *latent heat*, which has been expended in causing change of state.

LABORATORY WORK

EXPERIMENT 1. *Liquid thermometers*

Make a mercury thermometer by blowing a bulb at the end of a fine capillary tube. Fill with mercury and seal the end. Calibrate the thermometer by means of ice and steam.

EXPERIMENT 2. *Air thermometer*

Make an air thermometer similar to that shown in Fig. 126. Calibrate it against a mercury thermometer.

EXPERIMENT 3. *Specific heat*

Carry out an experiment to determine the specific heat of a solid using the method of mixtures described on page 144.

EXPERIMENT 4. *Latent heat*

Determine the latent heat of steam experimentally, using the method of mixtures.

EXERCISES

1. Convert the following temperatures on the centigrade scale to the corresponding readings on the Fahrenheit scale: 1450° C, 71° C, − 273° C.
2. Convert the following temperatures on the Fahrenheit scale to the corresponding readings on the centigrade scale: 65° F, 98·4° F, 0° F.
3. Draw a graph to show the relationship between Fahrenheit scale readings and centigrade scale readings, and from it determine the temperature at which both scales agree.
4. Find the relationship between the units of heat by calculating the number of calories that are equivalent to one British thermal unit. 1 lb = 453·6 g.
5. How much heat is required per minute from an immersion heater to raise the temperature of 50 gal of water at 40° F to 80° F in 10 min. How much would be lost in heating its containing tank made of galvanized steel (sp. heat 0·11) weighing 120 lb. Assume that the tank is sufficiently insulated to make radiation losses negligible.
6. Calculate the specific heat of a specimen for which the following readings were obtained experimentally—

Weight of copper calorimeter	= 48·86 g
Weight of calorimeter + cold water	= 69·33 g
Temperature of cold water	= 16·8° C
Temperature of mixture	= 47·5° C
Temperature of specimen	= 100° C
Weight of calorimeter + water + specimen	= 126·35 g

7. Explain fully how you would find the specific heat of a liquid.

150 g of oil heated to 95° C are dropped into 100 g of water, contained in a copper calorimeter weighing 150 g, at a temperature of 17° C. If the resulting temperature is 39·6° C and the specific heat of copper is 0·1, find the specific heat of the oil.

8. A copper calorimeter weighing 130 g contains 240 g of water at 25° C. After adding a piece of dry ice the temperature becomes steady at 8° C. If the latent heat of fusion of ice is 80 g.cal/g and the specific heat of copper is 0·1, what was the weight of ice added?

CHAPTER XVIII

EVAPORATION, HUMIDITY AND MELTING POINT

EVEN at temperatures just above freezing point the molecules of a liquid are in a state of agitation and may, on reaching the surface, pass out into the air above.

Evaporation and Boiling. This explains the phenomenon of evaporation by which a liquid may be converted to vapour at a temperature below boiling point. In the process of boiling, the molecules vaporize inside the liquid forming bubbles, the vapour being produced more rapidly. In either case the latent heat is the same, but, in evaporation there need be no external source of heat energy, the heat being drawn from the remainder of the liquid which will, in turn, draw heat from its surroundings.

The foregoing explains why damp walls make a room cold, for the heat necessary to evaporate the moisture will be drawn from the room itself or alternatively a large amount of fuel must be wasted in supplying it. Dampness also increases the humidity of the air (see below) making the room uncomfortable. On the other hand, a loggia can be cooled down in hot weather by liberally watering the floor.

Water Vapour in the Atmosphere. Although the amount of water vapour in the atmosphere is only about 1 per cent, variations above or below the normal amount at any given temperature are of great importance, controlling to a large extent the comfort of a room and the condensation of moisture on walls or other surfaces. Timber, unless painted or otherwise waterproofed, is also affected both during seasoning and in actual use since it readily absorbs or loses moisture according to atmospheric conditions.

The mass of water vapour in unit volume of air is referred to as the *humidity* of the atmosphere from which the sample is drawn, and is usually measured in grammes per cubic metre. At any given temperature and pressure there is a maximum amount of water vapour that can diffuse or mix with the air. When this is present the air is said to be *saturated*. If it is then cooled, it tends to become supersaturated and tiny particles of liquid water may be deposited in the form of a mist or dew.

Relative Humidity. The relative humidity of air is the ratio of the amount of water vapour present to the amount required to saturate the same volume of air at the same temperature. In England the relative humidity out-of-doors is never less than 30 per cent and this value may reach 100 per cent in damp, "muggy" weather even though no rain falls. Rooms should be kept at a humidity of 50 to 60 per cent by adequate ventilation to ensure comfortable conditions.

Dew Point. When air is gradually cooled, the temperature at which particles of liquid water first form is called the dew point, its value depending upon the relative humidity of the air at its original temperature. In nature this gradual cooling often occurs after sunset as the ground and the air immediately above it is cooled by radiation. The first appearance of dew takes place when the temperature has fallen to the dew point.

Hygrometers. Instruments used for the measurement of the relative humidity of air are called *hygrometers* and operate on one of the following principles—

1. The measurement of the dew point.
2. The rate of evaporation from the bulb of a thermometer surrounded by a wet muslin bag.
3. The change in length of a human hair which alters with relative humidity.

Daniell's gold-leaf hygrometer is described in most Physics textbooks but is of little use to the builder as it is difficult to read and liable to error. Regnault's hygrometer is more satisfactory but is still essentially a laboratory instrument and not suitable for industrial use. The dew-point hygrometer designed by Griffiths for use in cold-storage plants consists of a nickel-plated block of copper cooled by a flow of brine from the refrigerating system of the plant. A thermometer is placed with its bulb close to the surface of the block and temperatures are taken at the instant of the first formation of dew on the nickel surface and its disappearance when the flow of brine is stopped. The mean of these two values is taken as the dew point.

Wet-and-Dry Bulb Hygrometers. For most applied work instruments of the form shown in Fig. 129 are used. Thermometer A records the temperature of the air in the usual way. The bulb of thermometer B is enclosed in a muslin bag kept wet by the water reservoir C, and will show the temperature to which its bulb is reduced by the evaporation of water from the muslin bag. The rate of evaporation and therefore the resultant temperature of B depends upon the relative humidity of the air so that the difference in readings of the dry and wet bulb thermometers is a measure of the relative humidity. Table VII gives the values of relative humidity for degree differences between the wet and dry bulb temperatures over a practical range of air temperatures. These values are obtained by using a Regnault hygrometer as a standard. If both thermometers register the same temperature the air must be saturated and the relative humidity will be 100 per cent.

EXAMPLE. Find the relative humidity of the air in a room if the wet bulb of a hygrometer reads 50° F when the dry bulb records 56·5° F.

Difference between wet and dry bulb temperatures $= 6{\cdot}5^\circ$ F

$= \frac{5}{9} \times 6{\cdot}5 = 3{\cdot}6^\circ$ C

Wet bulb temperature $= 50^\circ \text{ F} = 10^\circ$ C

Dry bulb temperature $= 56{\cdot}5^\circ \text{ F} = 13{\cdot}6^\circ$ C

From table—

Relative humidity at 13·6° C for a difference of 3° C = 82%
Relative humidity at 13·6° C for a difference of 4° C = 77%

Relative humidity for a difference of 3·6° C

$$= 82 - \tfrac{6}{10}(82 - 77)$$
$$= 82 - 3$$
$$= \underline{\underline{79\ \%}}$$

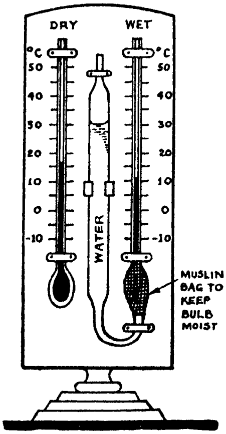


FIG. 129. WET-AND-DRY BULB HYGROMETER

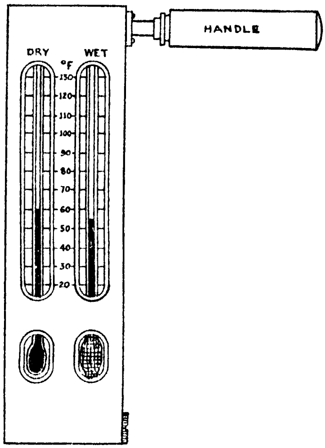


FIG. 130. SLING PSYCHROMETER

Psychrometers. For the readings of wet-and-dry bulb hygrometers to be reliable the air should circulate freely round the bulbs with a velocity of not less than 10 ft/sec. Such ventilated hygrometers are called *psychrometers*, a practical form being the sling psychrometer (Fig. 130) in which the two thermometers are mounted on a frame so that they can be rotated by whirling in the hand. In use the instrument is whirled for 30 sec and the temperature of the wet bulb is read immediately after stopping. The average of several such readings should be taken. To check the air speed the number of turns per minute can be counted and the radius of whirl measured, thus enabling the speed of the bulbs to be calculated.

Hair Hygrometer. A clean human hair, free from grease, is mounted so that its change of length under varying conditions of humidity may be magnified and used to move a pointer over a scale. The graduations are calibrated against a standard hygrometer to give direct readings of relative humidity but require periodical checking, since if the tensile stress in the hair becomes too great it may be permanently deformed thus causing error in

TABLE VII

PERCENTAGE RELATIVE HUMIDITIES FROM READINGS OF WET-AND-DRY BULB HYGROMETERS

Dry Bulb Temperature C°	Difference between wet-and-dry bulb temperatures (° C)									
	1	2	3	4	5	6	7	8	9	10
2	92	87								
4	93	87	80	75						
6	93	87	81	76	70	66				
8	93	87	81	76	71	66	61	58		
10	93	87	82	76	71	66	62	58	53	50
12	93	88	82	76	71	66	62	58	54	50
14	93	88	82	77	72	67	63	58	54	51
16	93	88	82	77	72	68	63	59	55	51
18	94	88	83	77	72	68	63	59	55	52
20	94	89	83	78	73	69	64	60	56	53
25	94	89	84	79	74	70	66	63	60	56
30	95	90	85	80	76	72	68	64	61	58
35	95	90	85	81	77	73	70	66	63	60

the readings. This instrument has the advantage of giving direct readings and, unlike the wet and dry bulb hygrometer, can be used at temperatures below freezing point.

Condensation in Buildings. The amount of water vapour required to saturate air increases considerably as the temperature rises. If the air in a room is saturated, or nearly so, and is at a higher temperature than the surfaces of walls and windows, condensation will take place. In a dwelling-house these conditions most frequently occur in kitchens, sculleries, and bathrooms, but the effect of condensation can be minimized by preventing the air becoming too hot by correct ventilation and by using materials of low thermal conductivity wherever possible.

If the material of a surface is absorbent, condensation may not become visible until it is saturated. Steel window frames are usually the first surfaces to show droplets of water and glass soon becomes misted, the steel being at a lower temperature than the air of the room because of its good conductivity and the low temperature outside. Steel is also non-absorbent, and all condensed water remains on the inner surface and eventually runs down to the sill. Painted or varnished surfaces are also non-absorbent, but plasters and certain types of distempers will absorb moisture and are therefore slow to show condensation.

FREEZING POINTS AND MELTING POINTS

The Freezing of Water. Water belongs to a comparatively small group of substances that expand on solidifying, ice having a volume 9 per cent greater than that of the water from which it is formed.

If this increase in volume is resisted, large forces are set up, tending to burst the containing vessel or pipe, equal in magnitude to the force which would have to be exerted in a testing machine to cause the same strain. With lead, copper, cast iron, and steel the failing stress may be reached if the whole of the water in a pipe freezes. On the other hand, it should be remembered that the specific heat of water is very high, and a large amount of heat must be abstracted from the water in a pipe before ice is formed. Thus if builders would protect all water pipes from cold winds by lagging them with a heat insulating material, the rate of loss of heat could be made so slow that burst pipes would be of rare occurrence.

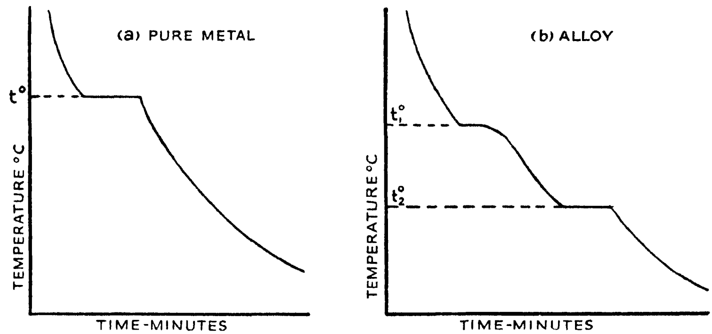


FIG. 131. TEMPERATURE-TIME CURVES FOR DETERMINING MELTING-POINTS

Melting Points. The melting point of a chemically simple substance, such as ice, lead or tin, is a very definite temperature which can be determined with accuracy, but other substances such as alloys, bitumen, glass, or clay, pass through a viscous or plastic stage.

To Find the Melting Point of a Pure Metal or Alloy. The pure metal or alloy is melted in a fireclay crucible (a cast-iron crucible may be used for many metals, e.g. lead or tin). When properly melted, the material is allowed to cool and temperature readings are taken every minute. A mercury thermometer reading to 350° C is suitable for experiments with lead or solder but should be protected by a fireclay sheath or thermo-couple cover. For higher temperatures a pyrometer is required.

Graphs of temperature against time are drawn for each sample, the results being similar to those shown in Fig. 131 (*a* and *b*). In Fig. 131 (*a*) the melting point or solidification temperature of lead is seen to be $t°$ (327° C), for, over a period of minutes, the lead loses heat to the surrounding air, but its temperature remains constant. Fig. 131 (*b*) shows that the alloy (plumber's solder) begins to solidify at $t_1°$ (about 250° C) since at this point the graph shows the temperature to be constant for a short period during which some latent heat is evidently given up. The alloy now becomes plastic, consisting of an increasing number of solid particles mixed with the liquid, and finally solidifies completely at $t_2°$ (183° C). Since the melting point of tin is 232° C, this particular lead–tin alloy could not be used as tinman's solder as it begins to solidify at a higher temperature than the melting point of tin. During the plastic stage it is in the right physical condition to enable a plumber to "wipe his joint."

Slight differences in the percentage composition of an alloy may cause considerable changes in its physical properties. In the case of lead and tin, an alloy containing 60 per cent of tin begins to solidify at about 188° C and finishes at 183° C, while a change in the tin content to 40 per cent gives an alloy which starts to solidify at 245° C. The former would be suitable for tinman's solder which must set quickly and be at a temperature below the melting point of tin. All tin–lead alloys finish solidifying at 183° C but vary considerably in the temperatures at which they commence to solidify (from 327° C to 183° C). A solder containing 63 per cent of tin behaves exactly like a pure metal in that it has a *definite* melting point of 183° C.

LABORATORY WORK

Experiment 1. *Wet-and-dry bulb hygrometer*

The student should use a wet-and-dry bulb hygrometer to determine the humidity of the atmosphere.

Experiment 2. *Determination of melting points*

Determine the melting point of a pure metal using the method described on page 151. Repeat the experiment using an alloy instead of a pure metal and note the difference in the results.

EXERCISES

1. What is meant by the relative humidity of the air? Describe how it may be measured.
2. What is meant by the term dew point? Describe a method of determining the dew point.
3. Why is the temperature indicated by a thermometer usually lowered when a piece of damp muslin is wrapped round the bulb? On what does the extent by which the temperature is lowered depend? Is it possible for such a wet bulb not to show a fall in temperature?
4. Explain what happens when water freezes. What is the importance of this action in building?
5. Describe how you would find the melting point of a solid. How would your results vary if the solid was an alloy and not a pure metal?
6. What is the difference between boiling and evaporation? How is the boiling point affected by changing the pressure on the surface of a liquid?

CHAPTER XIX

THERMAL EXPANSION OF SOLIDS, LIQUIDS AND GASES

As heat energy is given to a material, the kinetic energy of its molecules increases and the body as a whole tends to expand. Conversely, on cooling the material it will tend to contract. Water is an important exception because, although it obeys the general rule above 4° C, it tends to *contract* when heated from 0° C to 4° C. Water has its maximum density at 4° C so that colder water will be found nearer the surface if the temperature falls below this point. This phenomenon helps to prevent volumes of water, large or small, from freezing solid, since a crust of ice will first form on the surface and, being a bad conductor of heat, will help to prevent further solidification, the crust increasing in thickness only slowly.

Expansion of Solids. The expansion of solids can be demonstrated in the laboratory with the aid of a brass sphere of such a diameter that it will just pass through a brass ring when both are at the same temperature. If the ball is heated sufficiently it will expand so that it will rest supported by the ring but, as the ball cools, its own contraction and the expansion of the ring due to the heat it receives will soon allow the ball to pass through again.

In engineering structures special expansion joints must be designed to allow for such changes in dimensions. The steel cantilever sections of the Forth Bridge, for example, have rocking posts placed between them which can move slightly out of the vertical to allow for temperature changes and roller bearings are provided at each end of the bridge to allow freedom for expansion in the end sections.

Similarly, in reinforced concrete construction special expansion joints are required and, in fact, the method of reinforcing concrete is only possible because the two materials, concrete and steel, have approximately equal coefficients of expansion (see Table VIII).

Linear Coefficients of Expansion. The amount of expansion or contraction of a material due to a change of temperature is proportional to its dimensions and to the change of temperature. The coefficient of expansion of the material is defined as the change of unit dimension due to unit difference of temperature. Where expansion must be considered in building work most of the structural units concerned have one dimension much greater than the other two (e.g. a beam) so that only change of length need be calculated, change of sectional area being negligible. For such a case the coefficient of expansion required will be the *coefficient of linear expansion*, α, of the material which is defined as the increase in unit length of that material for a temperature rise of one degree. Providing that both the change in length and the original length are measured in the same units, the value of the coefficient will not be affected by the units used. The value per degree Fahrenheit will be five-ninths of that per degree centigrade. Typical values are given in Table VIII.

TABLE VIII

COEFFICIENTS OF THERMAL EXPANSION PER DEGREE CENTIGRADE

Material	Coefficient
Metals (linear)—	
Aluminium	$2{\cdot}4 \times 10^{-5}$
Brass	$1{\cdot}9 \times 10^{-5}$
Copper	$1{\cdot}7 \times 10^{-5}$
Invar	$0{\cdot}09 \times 10^{-5}$
Lead	$2{\cdot}9 \times 10^{-5}$
Solder	$2{\cdot}5 \times 10^{-5}$
Steel	$1{\cdot}2 \times 10^{-5}$
Steel (stainless)	$1{\cdot}0 \times 10^{-5}$
Zinc	$2{\cdot}6 \times 10^{-5}$
Silica (quartz)	$0{\cdot}04 \times 10^{-5}$
Liquids (volume)—	
Alcohol	122×10^{-5}
Ether	163×10^{-5}
Ice	$15{\cdot}3 \times 10^{-5}$
Mercury	18×10^{-5}
Water	see Table IX
Gases (volume)—	
Air	366×10^{-5}
Stones (linear)—	
Granite	$0{\cdot}8 \times 10^{-5}$
Limestones	$0{\cdot}25 \times 10^{-5}$ to $0{\cdot}85 \times 10^{-5}$

Material	Coefficient
Stones (linear)—	
Marbles	$0{\cdot}15 \times 10^{-5}$ to $1{\cdot}00 \times 10^{-5}$
Sandstones	$0{\cdot}7 \times 10^{-5}$ to $1{\cdot}2 \times 10^{-5}$
Woods (linear)—	
Beech (along the grain)	$0{\cdot}54 \times 10^{-5}$
Beech (across the grain)	$6{\cdot}0 \times 10^{-5}$
Oak (along the grain)	$0{\cdot}25 \times 10^{-5}$
Oak (across the grain)	$5{\cdot}4 \times 10^{-5}$
Pine (along the grain)	$0{\cdot}54 \times 10^{5-}$
Miscellaneous (linear)—	
Bakelite	$2{\cdot}8 \times 10^{-5}$
Brick	$0{\cdot}95 \times 10^{-5}$
Concrete	$1{\cdot}4 \times 10^{-5}$
Ebonite	$7{\cdot}0 \times 10^{-5}$
Glass	$0{\cdot}81 \times 10^{-5}$
Glass (Pyrex)	$0{\cdot}3 \times 10^{-5}$
Rubber	$20{\cdot}0 \times 10^{-5}$

Superficial and Cubical Coefficients. If it is necessary to calculate the change in area of a body with relatively small thickness, such as a roof surface, the superficial coefficient of expansion, which is the change in unit area per degree, will be approximately twice the linear coefficient of expansion. For changes in volume, the change of volume per degree is the cubical coefficient of expansion and is equal to three times the linear value (approximately).

Materials with Low Coefficients. From Table VIII, it can be seen that *Invar* (a special nickel steel) and *quartz* (Silica, SiO_2) have coefficients of expansion which are, for practical purposes, zero. Invar is used for the manufacture of standard measuring rods and tapes while quartz is employed for vessels that are subjected to sudden extremes of heat or cold. If a quartz crucible is heated and plunged into cold water it will not break, as would one made of glass, because there is practically no contraction. *Borosilicate glass* marketed under the name of "Pyrex" also has a very low coefficient of expansion and is used for heat-resisting glassware in the laboratory and kitchen.

Creep in Lead Roofs. The creep of lead on pitched roofs is due to alternating expansion and contraction between day and night temperatures. Owing to the weight of the lead, expansion will take place down the slope and when the sheet contracts on cooling its weight will again act to prevent the sheet from returning to its original position. To minimize the tendency to creep the roof area must be broken up by expansion joints of the form shown in Fig. 132.

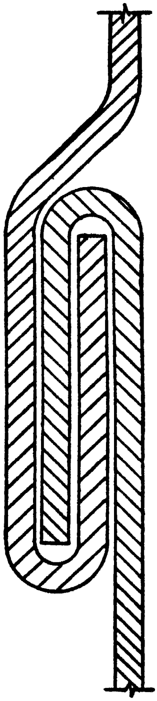


FIG. 132. EXPANSION JOINT IN LEAD SHEETING

TABLE IX

COEFFICIENTS OF CUBIC EXPANSION OF WATER PER DEGREE CENTIGRADE

Temperature Range °C	Coefficient of Cubic expansion
5° to 10°	$5 \cdot 3 \times 10^{-5}$
10° to 20°	15×10^{-5}
20° to 40°	30×10^{-5}
40° to 60°	46×10^{-5}
60° to 80°	59×10^{-5}
80° to 100°	70×10^{-5}

Expansion of Liquids. Expansions of liquids are always expressed in cubical or volume coefficients, their values being much higher than those of solids and, with the exception of mercury, varying considerably with changes of temperature. Water is a special example of such irregularity as Table IX shows, especially since below 4° C it *expands* with *decrease* of temperature.

Circulation Due to Change of Density. Owing to increase of volume due to thermal expansion, the density of a liquid when hot will be less than that when cold. This is made use of for the purpose of obtaining circulation of water in domestic heating systems and can be demonstrated by the following simple experiment.

Water is heated in the boiler *A* (the glass flask), Fig. 133, and a supply tube runs from the top of the water to the top of the hot water cistern *B* (glass bottle). The return pipe *C* leaves near the bottom of *B* and returns by a circuitous route, as in a domestic system, to the bottom of the boiler. Some potassium permanganate crystals are introduced into *B* to show the flow of the current set up. The bottom layers of water in the boiler expand on receiving heat energy, become lighter and rise to the top of the boiler, whence they rise by the supply pipe *D* to the top of the cistern *B*. Colder water from the bottom of cistern *B* sinks down the pipe *C* and enters the bottom of the boiler. This movement can be seen by watching the stream of water dyed by the permanganate crystals. The cold water entering the boiler becomes heated in its turn and a steady circulation is set up, called a *convection* current, gradually heating the water in *B*.

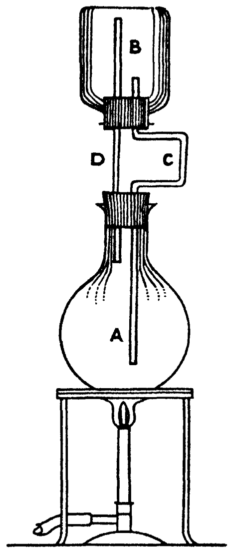


FIG. 133. CIRCULATION OF WATER DUE TO CONVECTION

Force Causing Circulation. It is interesting to take an example and calculate the force causing circulation in such a system—

Vertical height from bottom of boiler to top of cistern	= 20 ft = 610 cm
Sectional area of each pipe	= 1·55 in.^{2} = 10 cm^2
Temperature of water in down pipe	= 10° C
Temperature of water in up pipe	= 60° C
Density of water at 10° C	= 0·9997 g/cm^3
Density of water at 60° C	= 0·9832 g/cm^3
∴ Weight of water in cold pipe	= 610 × 10 × 0·9997 g
∴ Weight of water in hot pipe	= 610 × 10 × 0·9832 g
Difference of weight of water in the two pipes causing flow	= 6100 × 0·0165 g
	= 100 g = 0·22 lb

Thus a force of nearly $\frac{1}{4}$ lb is formed by the expansion of the water and this is sufficient to cause a good circulation, as water flows readily under the action of small pressure differences. Since most available tables of densities give values in metric units it is often convenient to carry out calculations in such metric units and convert to the British system as required.

Expansion of Gases. The simple air thermometer, illustrated in Fig. 126, page 142, shows that the coefficient of cubical expansion for air is very much higher than that of any liquid. For all permanent gases, or mixtures of such gases (e.g. the atmosphere) this coefficient has the same value, namely 366×10^{-5} or $\frac{1}{273}$ per ° C of the volume at 0° C. Thus—

$$\text{Volume of air at } 0° \text{ C} = V \text{ cm}^3$$
$$\text{Volume of air at } -1° \text{ C} = V - \tfrac{1}{273} V \text{ cm}^3$$

It is interesting to carry this to the limiting case by reducing the temperature to − 273° C. Then, theoretically,

$$\text{Volume of air at } -273° \text{ C} = V - \tfrac{273}{273} V = 0 \text{ cm}^3$$

In practice the gas will liquefy before reaching this temperature.

Similarly if the above volume of air were to be heated to + 273° C its volume would be doubled. Thus the density of air or any gas decreases very rapidly as its temperature rises so that convection currents and draughts of relatively high speeds are set up wherever temperature differences occur. Quite a small change of temperature may cause a high wind. Convection currents are of importance to the architect in the design of all systems of ventilation, both natural and artificial.

EXERCISES

1. A steel roof truss with a span of 40 ft is placed in position on a day when the temperature is 50° F. What would be the expansion of its main tie (in inches) on a day when its temperature is 100° F? Does this amount of expansion make it necessary to leave one end of the truss free?

2. A steel railway track was laid when the temperature was at freezing point. What should the sum of the gaps between the rail lengths be in 100 miles of track if the gaps would just close when the temperature rose to 122° F? (Assume that the ends of the *track* are fixed.)

3. A steel member of a structure with its ends fixed has its temperature increased by 45° F. Calculate the value of the stress set up in this member, which is 20 ft long, if the coefficient of linear expansion of steel is 0·000007 per ° F and the value of Young's modulus is 13,000 t/in.^{2}

4. Give four illustrations in building work where attention has to be given to thermal expansion or contraction, and two examples where this property may be usefully applied.

5. Prove that the superficial coefficient of expansion is *approximately* twice, and the cubical coefficient three times, the linear coefficient of expansion for any material.

6. Road engineers limit the length of each concrete bay to 50 ft. What is the total range of movement in this length for a range of temperature between 0° F and 120° F? What would be the superficial expansion over the same range of temperature for a road 30 ft wide?

CHAPTER XX

TRANSMISSION OF HEAT

UNTIL quite recently very little attention has been given to the proper control of temperature within a building. English architects and builders have been particularly remiss in this respect, partly because of our equable climate, partly because of our traditional building materials and methods, and partly from a prejudice against the application of pure science to practical work. Experience in other countries has convinced many people that it is unnecessary to suffer rooms which are hot and oppressive, or cold and draughty, and the introduction of new materials—concrete, steel, fibre board, and slag wool—has made it necessary for the builder to review the whole of his knowledge of heat transmission. Thus in a steel frame structure the walls carry no load and can therefore be designed with special attention to the provision of heat insulation rather than structural strength. To deal with the problems of heating and heat insulation it is essential to understand the methods of transmission of heat and, at least, to be able to compare the rates at which different building materials can conduct heat. It is also necessary to determine the speeds at which convection currents will circulate and to calculate the ability of any surface to radiate or absorb heat waves.

Nature of Heat. It was long assumed that heat was a material body which passed from places at higher temperature to those at a lower temperature, either by contact or through some intervening medium. This theory was disproved by Count Rumford at the close of the eighteenth century in his explanation of the generation of heat by friction, which he suggested could only be accounted for by regarding heat as a *mode of motion*. This theory endorses the molecular theory of the construction of matter and leads to satisfactory explanations of the transmission of heat by *conduction* and *convection*. *Radiation*, the third method of conveying heat, is explained by the wave theory of transmission of energy through the ether.

CONDUCTION

Heat is transmitted through a body by conduction, each molecule passing some of its energy to a neighbouring molecule possessing less, thus raising the latter's temperature, the heat passing through the body from parts at a higher temperature to those at a lower until some condition of equilibrium is set up. The ability to pass on heat energy in this way varies considerably with different materials—silver and copper conducting heat very rapidly and therefore being known as good *conductors*, while air is a poor conductor (or good *insulator*).

Conductors and Insulators. There is a marked similarity between the ability of materials to conduct (or insulate) both heat and electricity, although unfortunately there are no non-conductors of heat. There are some exceptions to this rule; water, for instance, is a fair conductor of electricity but a very poor conductor of heat. This can be shown by holding in the hand the bottom of a test tube holding about two inches of water and heating the top half inch with the flame of a bunsen burner. The water will soon boil at the top but may not even reach blood heat at the bottom.

All metals are good conductors of heat and all materials containing a large proportion, by volume, of air are good insulators. Examples of the latter of interest to the builder are—

(*a*) Fibrous substances—glass wool, asbestos.

(*b*) Powders—sawdust, cork granules.

(*c*) Materials of high porosity—clay bricks, foamed slag.

The heat insulation value of a wall will therefore be increased by designing it with a cavity or by using hollow blocks. Although water is a bad conductor in comparison with many structural materials it is thirty times as good as air (see Table X), so that a damp wall will conduct heat away far more rapidly than the same wall when dry.

TABLE X

THERMAL CONDUCTIVITIES (k) OF COMMON MATERIALS IN CALORIES/CM^2/SEC/CM THICKNESS/° C

Material	k	Material	k
Aluminium	0·50	Air	0·00005
Copper	0·92	Ebonite	0·0004
Zinc	0·26	Firebrick	0·0012
Lead	0·08	Glass	0·002
Steel	0·16	Ice	0·005
Marble	0·007	Soil	0·0004
Quartz	0·024	Slag wool	0·0001
Sandstone	0·025	Cork	0·0001
Water	0·0015	Cork powder	0·000086

Thermal Conductivity. The factors controlling the amount of heat conducted through a material are—

(*a*) The cross-sectional area through which the heat can pass, A.

(*b*) The thickness of the material, d.

(*c*) The temperature difference between the inner and outer surfaces ($\theta_1 - \theta_2$).

(*d*) The conductivity of the given material.

(*e*) The time for which the heat passes, t.

The total amount of heat passing, Q, is directly proportional to the area A, the temperature difference $(\theta_1 - \theta_2)$ and the time t and inversely proportional to the distance d; thus—

$$Q \propto \frac{A \, . \, (\theta_1 - \theta_2) \, . \, t}{d}$$

or

$$Q = \frac{A \, . \, (\theta_1 - \theta_2) \, . \, t}{d} \times k$$

where k is a constant called the coefficient of thermal conductivity. The metric system of units is usually employed and the coefficient of thermal conductivity is then given as the quantity of heat, in calories, that would have to pass per second through a thickness of one centimetre of a given substance over an area of one square centimetre to cause a temperature difference of one degree centigrade between the two outer surfaces. *Coefficients of Thermal Conductivity* for various materials are given in Table X (metric units), and Table XI (British units), the latter giving values of k for various building materials.

TABLE XI

THERMAL CONDUCTIVITIES OF BUILDING MATERIALS IN B.TH.U./FT^2/HR/INCH THICKNESS/° F

Material	k	Material	k
Concrete—		Plaster—	
Gravel 1 : 2 : 4 . .	7·0	Light . . .	2·0
Clinker 1 : 2 : 4 . .	2·8	Medium . . .	5·0
Foamed slag 1 : 2 : 4 .	2·2	Very dense . .	8·0
Pumice 1 : 2 : 4 . .	1·4	Fibre boards . .	0·4
Solid brick walls—		Plaster boards . .	1·1
Flettons . . .	6·3	Asbestos cement . .	2·0
London Stocks . .	6·0	Cork	0·3
Sand lime . . .	9·0	Soft wood . . .	1·0

EXAMPLE 1. Determine the quantity of heat conducted per hour through the windows of a one-room building if the total area of glass is 3·8 m.^{2} The thickness of the glass is 3 mm, the temperature of the room is 18° C, and the outside temperature is 5° C.

$$Q = \frac{A \, . \, (\theta_1 - \theta_2) \, . \, t}{d} \times k$$

$$= \frac{3{\cdot}8 \times 10{,}000 \times 13 \times 3600 \times 0{\cdot}002}{0{\cdot}3} \text{ g. cal.}$$

$$= 12{,}000{,}000 \text{ g. cal. (approx.)}$$

or converting from the metric system to the British system

$$Q = \frac{12{,}000{,}000}{252} = \underline{\underline{47{,}000 \text{ B.Th.U.}}}$$

In building work British units generally replace the scientific metric system and the thermal conductivity of a material is the quantity of heat in B.Th.U. passing through a slab of material one inch thick, per hour, over an area of one square foot due to a temperature difference of one degree Fahrenheit between the two outer surfaces. By converting each unit involved, it can be shown that the value of k in British units is, approximately, 2900 times the value of k in metric units.

Thus, from Table X, the metric value of k for glass is 0·002 and therefore the corresponding value in British units is $0{\cdot}002 \times 2900 = 5{\cdot}8$ B.Th.U./ft²/hr/in./° F.

EXAMPLE 2. Assuming that the total area of wall in the room mentioned in Example 1 is 126 ft² and that the walls are of London Stock bricks, 9 in., thick; determine the quantity of heat conducted through them per hour. The value of k in British units for London Stock bricks is 6·1 if the bricks are dry.

$$Q = k \times \frac{A(\theta_1 - \theta_2)t}{d}$$

$$= \frac{6{\cdot}1 \times 126}{9} \times \{(18 - 5) \times \tfrac{9}{5}\} \times 1 \text{ B.Th.U.}$$

$$= 2000 \text{ B.Th.U.}$$

The temperature difference has been converted from degrees centigrade to degrees Fahrenheit and has been assumed to be the same for all four walls.

Air-to-Air Coefficients of Heat Transmission. Comparing the results of these two examples, it is interesting to note that, considering conduction losses only, 96 per cent of the total heat loss will occur through the glass of the windows, according to calculation. In practice it has been found that the calculated loss through the glass is far too great, the reason being that the flow of heat from the warmed air inside the building to the cold air outside is governed not only by the conductivity of the material, but also by the rate at which the warm air transfers heat to the inner surface and the cold air takes up heat from the outside. This latter factor involves the absorption and emission of heat at the surfaces by radiation or by convection currents. To avoid introducing these complexities into the calculation of heat lost by conductivity, the heating engineer measures the *air-to-air coefficients of heat transmission*, U, the values of which are fixed by experiment and experience.

Total loss of heat through a wall or window

$$= U\,A\,(\theta_1 - \theta_2) \text{ B.Th.U./hr}$$

where U = air-to-air coefficient of heat transfer through a given thickness of the material.

A = area of wall or window in square feet.

$(\theta_1 - \theta_2)$ = temperature difference in degrees Fahrenheit.

A method of fixing the values of U for various building materials and structures is explained, with the necessary charts and tables, in *Principles of Modern Building*, Vol. I, published by H.M. Stationery Office.

In the case of glass and of thin roof sheetings, the air-to-air coefficient U depends less on the value of k than on the ability of the material to absorb or radiate heat at its surface.

Convection

Heat energy can be transmitted through a fluid (a liquid or a gas) by the flow of warmer streams of molecules through the remaining volume of the fluid, thus setting up convection currents. Professor Clerk-Maxwell defined convection as "the motion of the hot body carrying its heat with it." Heat cannot be convected through a solid because the particles of the solid cannot circulate. An explanation of the circulation of convection currents has already been given on page 155 in the section dealing with thermal expansion of liquids.

Radiation

Radiant Heat. Heat may travel from a hot body to a colder one by radiation, in the form of a transverse wave of very short wavelength of the order of $\frac{1}{30,000}$ in., of the same nature as a light wave, which can pass through an intervening medium without heating it. Temperature is only increased when radiation is absorbed by a material; if it is reflected at the surface of a medium or is transmitted through it there is no increase of temperature. Whereas conduction and convection pass on heat by changes of molecular motion, radiation travels as an ether wave and is only apparent as heat when it has been absorbed. Radiant heat, or *infra-red rays*, behaves in the same way as light waves (except that it is not visible to the eye) travelling with the same speed and obeying the same laws of reflection and refraction.

To demonstrate that radiant heat does not require air for its transmission, place a screen in front of an electric fire and suspend a thermometer behind it. When the screen is removed the thermometer reading will rise very rapidly as it absorbs the radiant heat falling upon it. Now place the thermometer in the receiver of an air pump, exhaust the air and repeat the experiment. Again it will be found that when the screen is removed the thermometer reading will rise just as rapidly as before. This shows that the radiant heat passes as easily through a vacuum as it does through air.

Radiation from Various Surfaces. As radiation takes place from the surface of one body and is incident on the surface of another, it is important to study the effects of different kinds of surfaces on their ability to emit and absorb radiant heat. The emission of heat can be studied by means of a *thermopile* (which is a battery of thermocouples), the heat rays incident on an absorbent surface in the instrument being converted into electrical energy which is measured by means of a galvanometer.

The emissive efficiency of different surfaces can be compared by using *Leslie cubes*. These are hollow tinplate cubes in which water

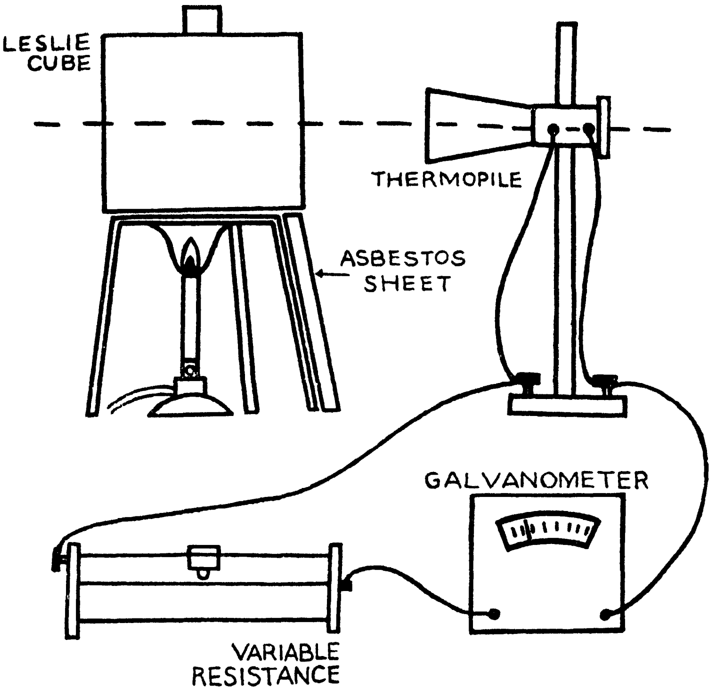


FIG. 134. DETERMINATION OF EMISSIVE EFFICIENCY BY LESLIE CUBE AND THERMOPILE

is kept boiling throughout the experiment so that all surfaces of the cube are at the same temperature. The different surfaces can be treated with different coloured paints, lamp-black, or left as polished tinplate. The heat radiated is compared by means of a thermopile placed at a standard distance from each face in turn. It will be found that for equal degrees of polish a black surface gives the greatest heat emission. A rough matt surface, given by lamp-black, is the most efficient radiating surface of all. Fig. 134 shows the lay-out of the apparatus used. Note that an asbestos sheet is so placed as to prevent radiant heat from the burner affecting the thermopile.

Coefficients of Emission. If the emissive value of lamp-black is fixed at unity the relative emissive values of other types of surface will be found by dividing the galvanometer reading for lamp-black into each of the other readings. Some values of *coefficients of emission* are given below—

Lamp-black	1·00
White lead	0·15
Metals	0·13
Polished silver	0·02

The values given in this table could equally well have been headed *coefficients of absorption* as the absorptive power of any body is equal to its emissive power. In other words a good radiator is an equally good absorber and conversely a bad radiator, which is usually a good reflector, will absorb very little heat.

Materials which Transmit Radiant Heat. Just as a lamp-black surface will absorb most of the heat received and a polished silver surface will reflect practically all incident heat rays, so a few materials, notably rock salt and quartz, will transmit most of the radiation received. Glass transmits a high percentage of the heat rays received from a source at a high temperature, such as the sun or a red fire, but will not transmit radiations from a source of low temperature. This is probably due to differences in wavelength of infra-red heat waves. This "discriminating" effect explains why glass houses trap the heat of the sun's rays. The radiant heat from the sun passes through the glass and is absorbed by the various bodies inside the glass house, but the radiation from these low temperature surfaces is either reflected back or absorbed at the inner surface of the glass. Thus during the day the heat from the sun is stored up, raising the internal temperature above that of the outside air. At night this heat gradually leaks away by conduction through the glass and then by radiation from its outer surface. This effect should be considered when a fireplace is positioned opposite a window.

Insulation Against Heat from the Sun. The values for coefficients of emission given on this page were obtained from a Leslie cube and the radiations were therefore from a low temperature source. It is important that the builder should not assume the same relative values for heat emitted from the sun. Experiments have shown that a *white* painted surface absorbs the least heat, when exposed to the sun, being even better in this respect than polished metal. Thus for a thin roof, the best insulation against the sun's radiant heat would be obtained by painting the outer surface white and having an under surface of polished metal or a metallic paint.

Central Heating "Radiators." Hot water or steam radiators would be more appropriately named "convection heaters," since of the total heat which they pass into the room not more than 30 per cent is transmitted by radiation. Most of the heat is used to warm the air in contact with the warm surfaces by conduction. Convection currents are thus set up which circulate the heat throughout the room. Thus, although black surfaces have the highest coefficient of emission, the type or colour of the surface of a hot water or steam radiator will make but little difference to its efficiency as a heater.

EXERCISES

1. Name and explain the three ways in which heat energy can be transmitted. Give examples of each method that occurs in a domestic hot-water radiator system.
2. Discuss the advantages and disadvantages of lagging the hot-water storage tank in a domestic hot water system.
3. Show that the value of k in British units for any given material is 2900 times the value of k in metric units.
4. Calculate the quantity of heat conducted through a concrete (1 : 2 : 4) wall 12 ft × 8 ft × 5 in. thick over a period of 12 hr if the thermal conductivity of the concrete is 7 B.Th.U./ft²/hr/° F, and the difference in temperature between the two surfaces of the wall is 30° F.
5. Compare, over equal times and for the same differences of temperature, the quantity of heat conducted through the 9 in. solid brick walls of a single room building 20 ft × 15 ft × 10 ft high with that conducted through the window, which is 8 ft × 6 ft if the thickness of the glass is ¼ in.
6. What do you understand by convection? What is the importance of convection currents in nature and what use is made of them in heating systems?

SECTION V—LIGHT AND OPTICS

CHAPTER XXI

LIGHT AND THE LAWS OF REFLECTION

LIGHT is a form of energy consisting of disturbances in the ether which travel with a velocity of 186,000 miles/sec, and since these disturbances travel in straight lines through any given medium, light is said to be propagated rectilinearly. Light waves can pass freely through such substances as air, glass or water which are termed *transparent*, but cannot pass through metals, wood or other *opaque* substances (unless these are reduced to extremely thin slices, as, for example, in the preparation of slides for microscopic examination). Certain other substances, while not passing light freely, transmit a certain proportion and are said to be *translucent*, but the light is diffused and objects cannot be seen clearly; ground glass, some papers, and milky liquids belong to this class.

Light waves which pass through transparent substances are said to be *transmitted*, those striking opaque objects are *reflected*, while those falling on translucent materials are partly transmitted, partly reflected, and partly absorbed. A beam of light falling, or *incident* on an object with an irregular surface, will be scattered in all directions. Such irregular or *diffuse* reflection will occur from distempered walls and ceiling surfaces. A surface which is smooth and polished will reflect the beam regularly without diffusion so that an object viewed in a flat or *plane* mirror can be seen clearly without distortion. Even a good mirror will, however, produce a small amount of scattering through irregular reflection.

Reflection Factors. When light is reflected at any surface, a certain percentage will be absorbed. This will apply even to the best mirror, which cannot be made a perfect reflector although the amount of light absorbed will be small compared with the amount reflected. With other substances the proportion reflected will be smaller. The percentage of incident light reflected from a surface is called the *reflection factor*. Values from some common surfaces are given in Table XII.

Shadows. Because light is propagated in straight lines, any opaque body placed between a light source and a screen will form a shadow, the nature of which will depend upon the size of the source of light. If the source is so small that the rays of light appear to radiate from a point, the shadow will be uniformly dark, of the same shape as the object and will possess sharp edges, but its size will depend upon the relative positions of source, object, and screen, Fig. 135 (*a*).

If the source is smaller than the object, yet of comparable size, Fig. 135 (*b*), the shadow produced on a screen will be composed of

TABLE XII

REFLECTION FACTORS INDICATING PERCENTAGE OF LIGHT REFLECTED FROM THE SURFACE

Material	%	Material	%
White drawing paper	82		
Good white paint	75		
Yellow wall-paper	45	Good white reflector for a lamp (scatters 65%, reflects regularly 15%)	80
Dark brown wall-paper	14	Silver behind glass	85
Black cloth	1	Mercury behind glass	73
Silver in front of glass	92	Stainless steel	60

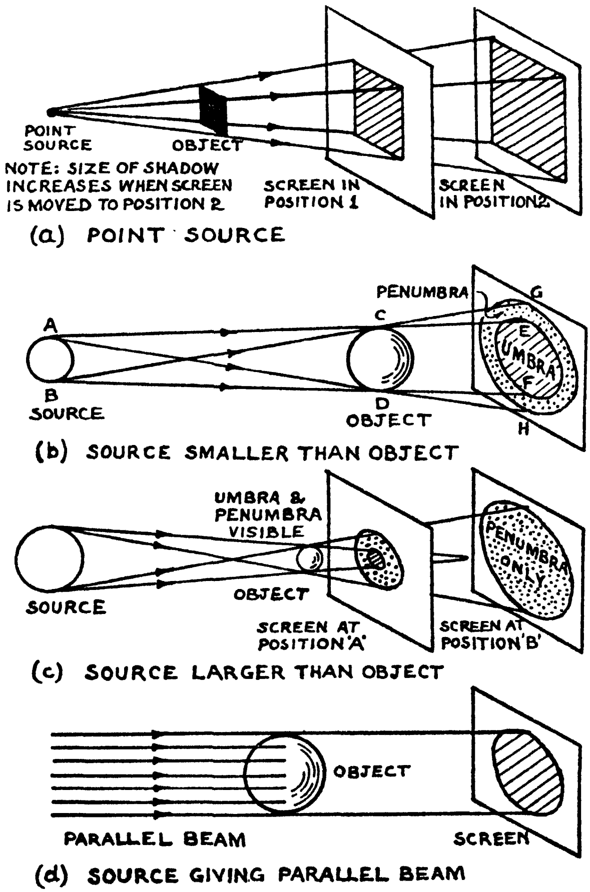


FIG. 135. PRODUCTION OF SHADOWS

two parts. No light from the source can reach the portion lying between *E* and *F* and a dense black shadow, called the *umbra*, will be produced, having the same shape as the object and increasing in size as the screen is moved away from the object. Joining *BC* and *AD* and producing these rays to meet the screen at *G* and *H*, all points lying outside the circle *GH* will receive light from all parts of the source, but points lying inside the circle *GH* will only receive light from a portion of the light source and will be in a zone of partial shadow, called the *penumbra*. The density of this shadow will graduate from full darkness at *E* to full light at *G*.

A similar state of affairs exists when the light source is larger than the object, Fig. 135 (*c*), but in this case the umbral cone converges and the umbra becomes smaller as the screen is moved away from the object. Thus when the screen is at *A* both umbra and penumbra are visible, but when it is placed at *B* the umbra vanishes.

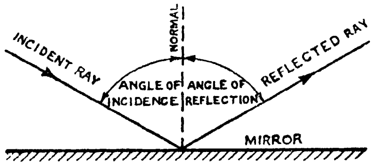


FIG. 136. REFLECTION AT A PLANE MIRROR

Finally, if the source of light is made to produce a beam of parallel rays, Fig. 135 (*d*), an object intercepting the beam will form a shadow with clearly defined edges, of the same size as the object, irrespective of the position of the screen.

REFLECTION

Laws of Reflection. When a ray of light strikes a mirror it is reflected. If the incident ray strikes the mirror normally, i.e. at right angles to the surface, the reflected ray will return along the same path. If the incident ray strikes the mirror at an angle, Fig. 136, it will be so reflected that the angle of incidence between the incident ray and the normal will be equal to the angle of reflection between the reflected ray and the normal. The reflected ray will lie on the opposite side of the normal to the incident ray.

The laws of regular reflection are—

1. The incident ray and the reflected ray make equal angles with the normal at the point of contact.

2. The reflected ray lies in the same plane as the incident ray and the normal, but is on the opposite side of the normal to the incident ray.

Reflection at a Plane Mirror. When an object is held in front of a plane mirror an *image* appears to be formed somewhere behind the surface of the mirror. To find out how this image is formed and where it is situated, consider an object *O*, Fig. 137, in front of a plane mirror *XY*. Rays of light *OA* and *OB* will be reflected along the paths *AC* and *BD* respectively and since for each ray—

$$\text{Angle of incidence} = \text{Angle of reflection}$$
$$\angle OAF = \angle FAC$$
and $$\angle OBG = \angle GBD$$

where AF and BG are the normals to the surface of the mirror at A and B respectively.

If the lines CA and DB are produced they will converge to the point I behind the mirror. Thus to the eye, light originating at O

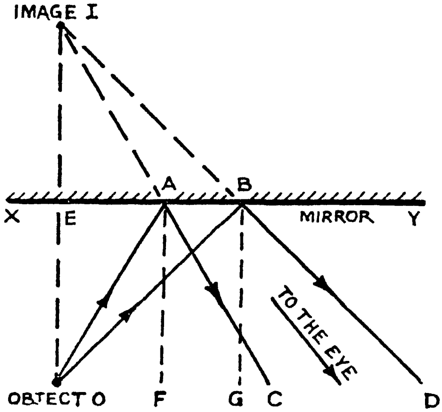


FIG. 137. FORMATION OF AN IMAGE BY A PLANE MIRROR

appears to come from a point I behind the mirror and I will be the image of O.

To determine the position of I join OI and let it cut the mirror plane XY at E.

Now $$\angle OAF = \angle FAC$$
and $$\angle EAF = \angle FAB = 90°$$
$$\therefore \angle EAO = \angle CAB = \text{vert. opp. } \angle EAI$$

similarly, since the angle of incidence $\angle OBG$ is equal to the angle of reflection $\angle GBD$: $\angle ABO = \angle ABI$.

Thus in the triangles OAB and IAB

$$AB \text{ is common}$$
$$\angle ABO = \angle ABI$$
$$\angle OAB = 180° - \angle EAO = 180° - \angle EAI = \angle IAB$$

Therefore triangles OAB and IAB are congruent.

$$\therefore OA = IA$$

Also in triangles OAE and IAE

$$\angle EAO = \angle EAI$$
$$OA = IA$$
$$AE \text{ is common}$$

$\therefore$ triangles OAE and IAE are congruent.

$$\therefore\ OE = IE$$

and $\angle OEA = \angle IEA = 90°$ since OI is a straight line

Thus the image I is situated as far behind the mirror, perpendicularly, as the object is in front.

Nature of the Image in a Plane Mirror. Since the image lies behind the mirror, no light can actually come from it, it only appears to do so. The image is said to be a *virtual* image to distinguish it from a *real* image from which light can actually come. A real image is one which can be formed upon a screen, a virtual image cannot be so formed.

The image observed in a plane mirror will be found to be *erect vertically*, that is to say points at the top of the object will be at the top of the image, but it will be *laterally inverted* so that a point observed to be on the right-hand side when facing the object will appear to be on the left-hand side of the image. This is best understood by looking at the reflection of a printed page in a mirror. The letters will be found to be the right way up, but will run from right to left in the image instead of from left to right.

Since lines joining each point on the image to the corresponding point on the object are perpendicular to the surface of the mirror the dimensions of image and object will be the same. Summarizing the above—

1. A plane mirror forms a virtual image.
2. The image is situated at the same distance perpendicularly, behind the mirror, as the object is in front.
3. The image is erect but laterally inverted.
4. The image is the same size as the object.

Deviation of Light by Reflection. A ray of light AB, Fig. 138, striking an inclined mirror PQ will be deviated and leave in the direction BC, the angle of deviation being $\angle DBC$.

If, Angle of incidence $= \theta$

then, since angle of incidence = angle of reflection

$$\angle ABC = 2\theta.$$

$$\begin{aligned}\therefore\ \text{angle of deviation} &= 180° - 2\theta \\ &= 2\,(90° - \theta) \\ &= 2\angle ABP\end{aligned}$$

i.e. a ray of light reflected from the surface of a plane mirror will be deviated through an angle equal to *twice* the angle between the incident ray and the mirror.

Deviation by Reflection at Two Mirrors. If the ray of light strikes a second mirror RS, Fig. 139, it will be deviated again and will leave along the path CD. Applying the above rule at each mirror—

Deviation at mirror $PQ = 2\angle ABP$

or, since $\angle ABP = \angle CBO$

Deviation at mirror $PQ = 2\angle CBO$
Deviation at mirror $RS = 2\angle BCO$

$$\text{Total deviation} = 2(\angle CBO + \angle BCO)$$
$$= 2(180° - \angle BOC)$$
$$= 360° - 2\angle BOC$$

i.e. Deviation due to successive reflection from two inclined mirrors will be equal to 360° minus twice the angles between the mirrors.

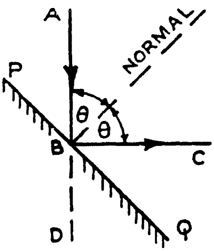


Fig. 138. Deviation of Light by Reflection

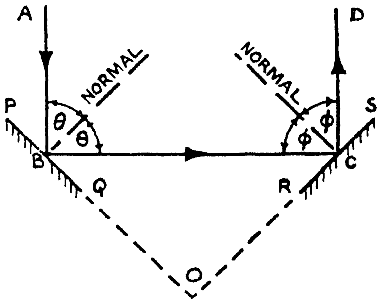


Fig. 139. Deviation of Light by Reflection at Two Plane Mirrors

If the two mirrors are at right angles, then—

$$\text{Angle of deviation} = 360° - 2 \times 90° = 180°$$

and the ray is reflected parallel to its original path but in the opposite direction.

If, on the other hand, the two mirrors are parallel, the angle between them is 180° and—

$$\text{Angle of deviation} = 360° - 2 \times 180° = 0°$$

so that the ray is then reflected parallel to, and in the same direction as, the original path. This is made use of in the *periscope*, used for seeing over obstacles, the action of which can be seen from Fig. 140.

Deviation of the Reflected Ray Due to Rotation of a Plane Mirror. A ray of light AB, Fig. 141, striking a mirror PQ, will be reflected along BC so that, if BN is normal to PQ—

$$\text{Angle of incidence } \angle ABN = \text{angle of reflection } \angle NBC.$$

If the mirror is rotated through an angle α to the position $P'Q'$ the new normal will be BN' and $\angle N'BN = \angle PBP' = \alpha$.

The ray AB will now be reflected along the path BD such that—

$$\text{Angle of incidence } \angle ABN' = \text{angle of reflection } \angle N'BD$$

$$\text{Deviation of reflected ray due to rotation of mirror} = \angle CBD$$
$$= \angle ABC - \angle ABD$$

But, $\angle ABC = \angle ABN + \angle NBC = 2\angle ABN$

$$\angle ABD = \angle ABN' + \angle N'BD = 2\angle ABN'$$

$$\therefore \angle ABC - \angle ABD = 2(\angle ABN - \angle ABN')$$

$$= 2\angle N'BN = 2\alpha$$

$\therefore$ Deviation of reflected ray due to angular rotation $\alpha = 2\alpha$
or, Deviation of reflected ray = twice angle of rotation of mirror.

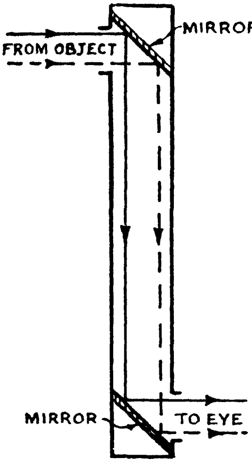


FIG. 140. SIMPLE PERISCOPE

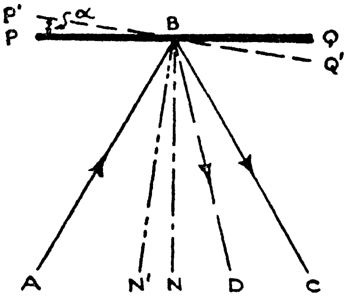


FIG. 141. DEVIATION OF RAY DUE TO ROTATION OF A PLANE MIRROR

This result is made use of for the measurement of angular rotation in many scientific instruments. A ray of light is focused on to a small mirror, fixed to the axis of the rotating portion, and reflected on to a distant scale. From the motion on the scale and the known distance from the scale to the mirror, the angular rotation of the moving part can be calculated.

LABORATORY WORK

EXPERIMENT 1. *Location of images*

(*a*) *Sighting method.* The position I, Fig. 142, of the image of an object O can be determined by inserting two pins P_1 and P_2 and lining in two other pins P_3 and P_4 to form sight lines on the image. The lines P_3P_1 and P_4P_2 are produced until they intersect at a point which must then be the position of the image.

(*b*) *Parallax method.* Two things which coincide will always appear to be together irrespective of the angle or distance from which they are viewed. If two objects in the same straight line from the observer do not coincide, they will appear to move relative to one another if the observer moves out of the straight line, this being called parallax motion. In the first case, where the objects coincide, there is *no parallax.* The position of a point on an image can be located with the aid of a pin, taller than the mirror, which is moved about behind the mirror until there is no parallax between the part of the pin seen over the top of the mirror and the point on the image viewed in the mirror. The position of the image will then coincide with that of the pin.

Using the above methods locate the position of the image in a plane mirror and verify the rules given above (pages 168, 170).

EXPERIMENT 2. *Deviation of light by reflection*

For experiments on deviation a smoke box or an optical disc is desirable. Using such apparatus, verify the rules for deviation of light rays by a single mirror, by two mirrors, and by a rotating mirror.

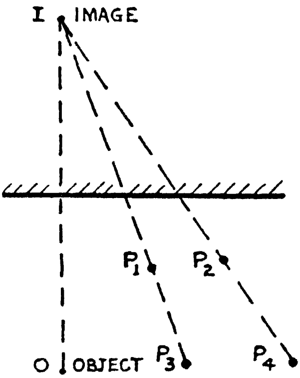


FIG. 142. LOCATION OF AN IMAGE BY SIGHTING

EXERCISES

1. State the laws of reflection of light and explain how images are formed in a plane mirror. How can it be shown (*a*) theoretically, (*b*) experimentally, that the distances of object and image from a plane mirror are equal?

2. Distinguish between a real and virtual image. How could you arrange two plane mirrors to enable you to see the back of your head? Show by a diagram the path of the light rays.

3. A man, whose eye is 5 ft 10 in. above ground level, walks towards a lake with the moon, which is 30° above the horizon, directly in front of him. If the ground level is 12 in. above the level of the water, how far from the lake will the man be when he first sees the reflection of the moon in the water?

4. A boy 4 ft 3 in. in height is standing at a distance of 3 ft from a vertical mirror 2 ft in length, the upper edge of which is 5 ft from the floor. Taking the boy's eye level to be 4 ft above the floor, show by a diagram how much of his height he can see in the mirror and how much of the mirror is useful to him.

5. Two mirrors are placed 12 in. apart with their reflecting surfaces facing each other and a point source of light is placed 3 in. from the first mirror and 9 in. from the second. Show on a diagram drawn to scale the positions of some of the images formed in the mirrors and draw a cone of rays from the object to an eye, placed between the mirrors, which is viewing the third image behind the first mirror.

6. A horizontal beam of light AB falls normally on the centre B of a plane mirror placed in a vertical position. It is required to produce a reflected beam BC such that $BA = BC = 1$ m and $AC = 2{\cdot}5$ cm. Find approximately the angle through which the mirror must be rotated.

7. A horizontal beam of light falls on a mirror and is reflected at right angles horizontally, forming a spot of light on a screen 1 m distant from the mirror. Sketch the arrangement and calculate through what angle the mirror must be rotated to make the spot move a distance of 10 cm horizontally on the screen.

8. An object is held 4 in. from one of a pair of parallel mirrors. If the distance between the second nearest image in one mirror and the second nearest image seen in the other mirror is 60 in., what is the distance between the two mirrors?

CHAPTER XXII

REFLECTION OF LIGHT AT CURVED SURFACES

MIRRORS with curved surfaces are in frequent use; for example, as reflectors to headlamps or searchlights. In their simplest form the reflecting surface forms part of a sphere and such *spherical mirrors* may be either *concave* or *convex*. In a concave mirror the interior of the curve acts as a reflector while in the convex mirror reflection occurs from the outside surface of the sphere. The centre of the sphere is called the *centre of curvature* of the mirror and a section through the mirror passing through this point is a *principal section* of the mirror.

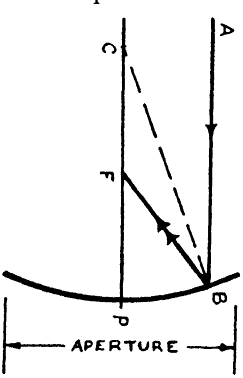


FIG. 143. PRINCIPAL SECTION THROUGH A CONCAVE SPHERICAL MIRROR

Fig. 143 is a principal section of a concave spherical mirror, the centre of curvature being at C. The midpoint P of the reflecting surface is called the *pole* of the mirror and the line CP the *principal axis*. The extreme width of the mirror is called the *aperture*.

Reflection of a Ray at a Curved Surface. A ray of light AB, Fig. 143, striking the curved surface at B will be reflected along some path BF. Considering a very small element of the mirror surface at B it may be regarded as a very small plane mirror inclined to the incident ray AB at the same angle as the tangent to the curved surface at B. Thus the incident and reflected rays AB and BF will make equal angles with the normal to the tangent at B. For a spherical mirror the normal to the tangent is the line joining B to the centre of curvature and referring to Fig. 143 $\angle ABC = \angle CBF$.

CONCAVE MIRRORS

Focus of a Concave Mirror. If the ray AB, Fig. 143, is parallel to the principal axis PC of a concave spherical mirror—

$$\angle ABC = \angle BCF$$

but $\quad \angle ABC = \angle CBF$

hence $\quad \angle CBF = \angle BCF$ and $FC = FB$

If the aperture of the mirror is small, B will be so near to P that FB can be considered equal to FP;

therefore, $\quad FC = FP$

or F is the midpoint of CP.

Similarly any other ray parallel to AB will be reflected through the midpoint F of CP, which is called the *principal focus* of the mirror.

Thus: *All rays parallel to the principal axis of a concave spherical mirror of small aperture will be reflected through the principal focus.*

Conversely: *Rays of light emanating from a point source at the principal focus of such a mirror will be reflected as a parallel beam travelling parallel to the principal axis.*

The distance FP from the principal focus to the pole is the *focal length* of the mirror, denoted by f, and if r is the radius of curvature of the mirror, then, since $CP = r$, $f = \frac{1}{2} r$ or, focal length $= \frac{1}{2}$ radius of curvature.

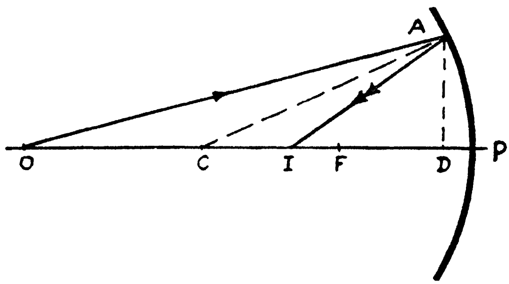


FIG. 144. CONJUGATE FOCI OF A CONCAVE SPHERICAL MIRROR

Conjugate Foci. If O, Fig. 144, is a point source of light on the principal axis CP of a concave mirror, on the opposite side of C to P, any ray OA incident on the mirror at A will be reflected along AI so that—

$$\angle OAC = \angle CAI$$

Now, in triangle AIC—

$$\text{exterior angle } AIP = \angle ICA + \angle CAI \quad . \quad . \quad (1)$$

and, in triangle COA—

$$\text{exterior angle } ICA = \angle COA + \angle OAC$$
$$= \angle COA + \angle CAI \quad . \quad . \quad (2)$$

subtracting (1) from (2)

$$\angle AIP - \angle ICA = \angle ICA - \angle COA$$

or

$$\angle AIP + \angle COA = 2\angle ICA \quad . \quad . \quad . \quad . \quad (3)$$

If the aperture of the mirror is small, these angles will be small and equation (3) can be written—

$$\tan \angle AIP + \tan \angle COA = 2 \tan \angle ICA$$

or

$$\frac{AD}{ID} + \frac{AD}{OD} = \frac{2AD}{CD}$$

Again, if the aperture is small, D and P coincide (approximately, therefore

$$\frac{AD}{IP} + \frac{AD}{OP} = \frac{2AD}{CP}$$

$$\frac{1}{IP} + \frac{1}{OP} = \frac{2}{CP}$$

Substituting the conventional symbols: $v = IP$, $u = OP$ and $r = CP$ = radius of curvature—

$$\frac{1}{v} + \frac{1}{u} = \frac{2}{r}$$

or, since $r = 2f$ where f is the focal length—

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

Now f is a constant, so, for any fixed position of O, the value of v will be constant and all rays leaving O will pass through I. A *real image* of O will be formed at I and could be observed on a screen held there. Conversely a source of light at I would produce a real image at O.

The points O and I are called *conjugate foci.*

Image Formed by a Spherical Mirror. The position and size of an image can be determined diagrammatically by drawing two rays from each point on the object and finding the point where the reflected paths of each pair intersect. The work is simplified by remembering the following rules—

1. Rays parallel to the principal axis will be reflected through the principal focus.
2. Rays through the principal focus will be reflected parallel to the principal axis.
3. Rays through the centre of curvature will strike the mirror normally and be reflected back along their own path.

Images Formed by a Concave Mirror. The image formed by a concave mirror will change according to the position of the object. The diagrams, Fig. 145, drawn by the method explained above, show the various types of image which may be produced.

(*a*) Object beyond the centre of curvature on the side remote from the mirror—a real, inverted, diminished image is formed between the centre of curvature and the principal focus.

(*b*) Object at the centre of curvature—a real, inverted image, of the same size as the object is formed at the centre of curvature.

(*c*) Object between centre of curvature and principal focus—a real, inverted, magnified image is formed beyond the centre of curvature.

(*d*) Object between principal focus and mirror—a virtual, erect, magnified image is formed behind the mirror.

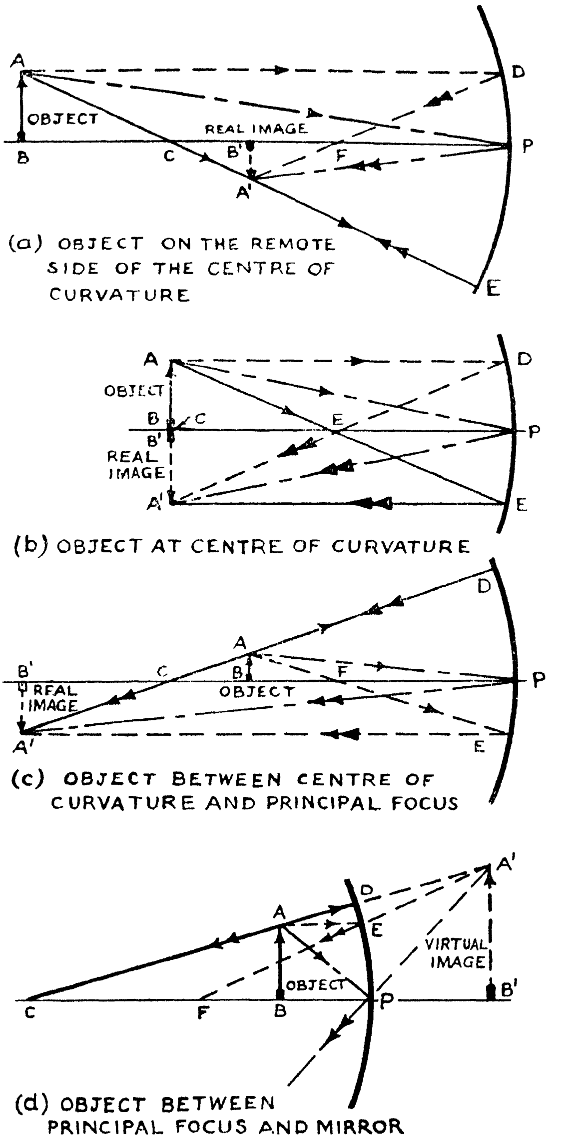


FIG. 145. IMAGES FORMED BY A CONCAVE MIRROR

Magnification of a Concave Mirror. The ratio $\frac{\text{height of image}}{\text{height of object}}$ is called the *magnification* of the image. If the ray APA', Fig. 145, is drawn for any case, triangles ABP and $A'B'P$ will be similar, since $\angle APB = \angle A'PB'$ and $\angle PBA = \angle PB'A'$; therefore,

$$\frac{\text{Height of image}}{\text{Height of object}} = \frac{A'B'}{AB} = \frac{PB'}{PB} = \frac{\text{image distance}}{\text{object distance}} = \frac{v}{u}$$

$$\therefore \text{ Magnification} = \frac{v}{u}$$

Convex Mirrors

Focus of a Convex Mirror. Reflection at the surface of a convex spherical mirror occurs in the same way as at a concave mirror,

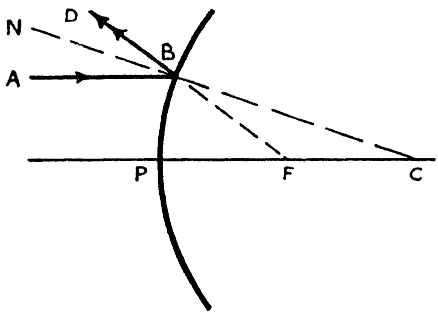


Fig. 146. Principal Section Through a Convex Spherical Mirror

the incident and reflected rays, AB and BD, Fig. 146, making equal angles with the line CB produced, C being the centre of curvature, but parallel rays are not converged to a focus, being reflected as a diverging beam instead.

If AB is a ray parallel to the principal axis, then—

$$\angle ABN = \angle DBN = \text{vert. opp. } \angle FBC$$

Also, since AB and PC are parallel—

$$\angle ABN = \angle FCB$$

$$\therefore \angle FCB = \angle FBC \text{ and } FB = FC$$

or, if the mirror is of small aperture—

$$FP = FC \text{ so that } F \text{ lies midway between } P \text{ and } C,$$

and since FBD is a straight line, the reflected ray BD appears to come from F. Similarly all other rays parallel to the principal axis will appear to come from F. Thus the principal focus F of a convex mirror *lies behind the mirror midway between the pole and the centre of curvature.*

Conjugate Foci. A ray from an object at O, Fig. 147, striking the mirror at A will be reflected along the path AE as if it came from some point I; that is to say that a virtual image of O will be formed at I. If C is the centre of curvature, join CA and produce it to N.

Then— $\angle OAN = \angle NAE = \text{vert. opp. } \angle CAI$

In triangle AIC—

$$\text{ext. angle } \angle AIP = \angle ICA + \angle CAI \quad . \quad . \quad (1)$$

and in triangle OAC

$$\angle ICA = \angle OAN - \angle AOC$$
$$= \angle CAI - \angle AOC \quad . \quad . \quad . \quad (2)$$

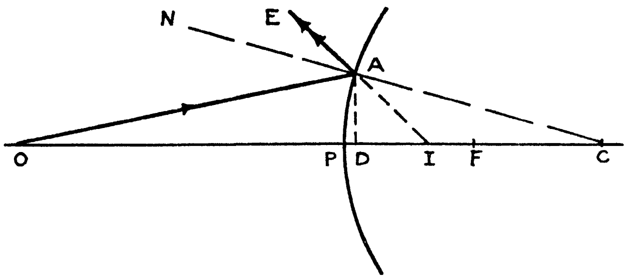


FIG. 147. CONJUGATE FOCI OF A CONVEX MIRROR

Subtracting equation (2) from equation (1)—

$$\angle AIP - \angle ICA = \angle ICA + \angle AOC$$
$$\angle AIP - \angle AOC = 2 \angle ICA$$

If the aperture of the mirror is small, these angles will be small, and the equation may be written—

$$\tan \angle AIP - \tan \angle AOC = 2 \tan \angle ICA$$

or,
$$\frac{AD}{ID} - \frac{AD}{OD} = \frac{2AD}{CD}$$

or, since P and D coincide approximately—

$$\frac{1}{IP} - \frac{1}{OP} = \frac{2}{CP} \quad . \quad . \quad . \quad . \quad (3)$$

If the distance of object and image from the mirror are u and v respectively and distances behind the mirror are considered to be negative, then—

$$IP = -v, \; OP = u,$$
$$CP = \text{radius of curvature} = -r$$
$$= -2f \text{ where } f = \text{focal length } FP$$

Equation (3) now becomes—

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

This equation is identical with that obtained for a concave mirror. As before I and O are conjugate foci.

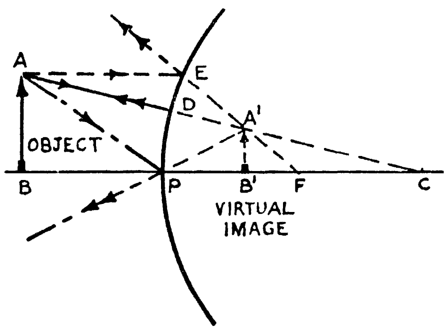


Fig. 148. Image Formed by a Convex Mirror

Image Formed by a Convex Mirror. As shown in Fig. 148, the image produced by a convex mirror will always be virtual, erect, and diminished and, as before—

$$\text{Magnification} = \frac{A'B'}{AB} = \frac{PB'}{PB} = \frac{v}{u}$$

LABORATORY WORK

Using an optical disc or a smoke box, the student should study the reflection of light at curved surfaces. This work can also be carried out on a drawing board using part-cylindrical mirrors (which can be stood on the board) and using pins as objects.

EXERCISES

1. Define the centre of curvature and the principal focus of a concave mirror, and show that the radius of curvature is equal to twice the focal length.

2. What is meant by (*a*) a real image, (*b*) a virtual image? An object is placed 1·5 cm in front of a concave mirror of focal length 4·0 cm, find the position and magnification of the image formed.

3. An object is (*a*) 40 in., (*b*) 20 in. (*c*) 5 in. in front of a convex mirror of 20 in. radius of curvature. What is the position and magnification of the image in each case?

4. State the laws of reflection of light and show by diagrams how these laws account for the formation of images by plane and spherical mirrors. An object placed 10 in. from a spherical mirror produces a virtual image 15 in. from the mirror. Find the radius of curvature and determine whether the mirror is concave or convex.

5. A concave mirror of 36 cm radius gives a real image of an object, enlarged three times. Through what distance and in what direction must the object be moved in order to form a virtual image enlarged three times?

6. An object is set up 20 in. from a concave mirror and a real image is also produced at 20 in. from the mirror. When the object is moved 5 in. farther away from the mirror what happens to the image? Find its position, nature, and size if the object is 2 in. tall. Where must the object be placed to produce a virtual image 4 in. tall? Illustrate your answers with diagrams.

CHAPTER XXIII

REFRACTION OF LIGHT

WHEN light travels from one transparent medium into another, say from air into glass, it will be reflected to a small extent at the surface of separation, but most of it will pass on from the air into the glass. If the light strikes the air-glass surface normally, it will continue to travel through the glass undeviated, but if it strikes obliquely, Fig. 149, the rays will be bent, or *refracted*, in this case

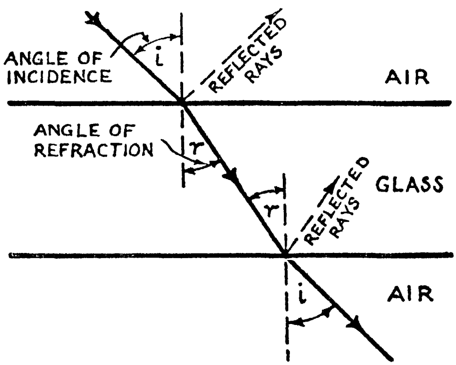


FIG. 149. REFRACTION OF A LIGHT RAY

toward the normal so that the angle of incidence i is greater than the angle of refraction r. When the ray travels out of the glass into the air, there will again be a small amount of reflection, but most of the light will emerge and be bent this time in the reverse direction. If the faces of the glass block are parallel, the emergent ray will issue parallel to the original incident ray but will be laterally displaced.

Whether the light is bent towards or away from the normal at each surface will depend upon the optical properties of the two media. When light entering a second medium is bent towards the normal, the second medium is said to be *optically denser* than the first; but if the light is bent away from the normal, the second medium is said to be optically rarer. Optical density is usually, but not always, related to the physical density of a substance.

Laws of Refraction. The process of refraction obeys certain laws, the first of which states that the refracted ray lies in the same plane as the incident ray and the normal to the surface but is on the opposite side of the normal to the incident ray.

The second law, discovered by Snell in 1621, states that, for any value of the angle of incidence, the ratio

$$\frac{\text{sine of angle of incidence}}{\text{sine of angle of refraction}} \text{ is a constant, or } \frac{\sin i}{\sin r} = \mu$$

where μ is a constant, called the *refractive index*, dependent on the nature of the two media; typical values for some common media are given below—

Air to water	1·33	Air to canada balsam	1·53
,, window glass	1·53	,, methylated spirit	1·36
,, crown glass	1·53 to 1·61	,, paraffin	1·44
,, flint glass	1·62 to 1·79		

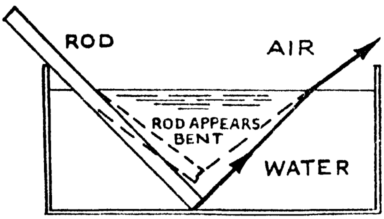


FIG. 150. APPARENT BENDING OF A ROD IMMERSED IN WATER

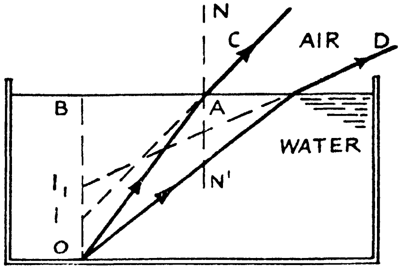


FIG. 151. APPARENT DEPTH DUE TO REFRACTION

Apparent Depth due to Refraction. As a result of refraction the bottom of a pool of water will appear to be nearer to the surface than it really is, a fact which can easily be demonstrated by immersing a straight rod in the water, Fig. 150; the rod will appear to be bent upwards below the water as shown.

Referring to Fig. 151, a ray OA, from a point O at a depth OB below the surface of the water, will be bent away from the normal NAN' at A as it passes from the denser medium, water, to the optically rarer medium, air, and will follow the path AC. To an eye at C the ray from O appears to come from a point I on CA produced and the observer sees an image of O at an apparent depth BI.

$$\text{By the laws of refraction } \frac{\sin \angle NAC}{\sin \angle OAN'} = {}_{\text{air}}\mu_{\text{water}}$$

But, since NA is parallel to OB,

$$\angle NAC = \angle AIB \text{ and } \angle OAN' = \angle BOA$$

$$\therefore {}_{\text{air}}\mu_{\text{water}} = \frac{\sin \angle AIB}{\sin \angle BOA} = \frac{\frac{AB}{IA}}{\frac{AB}{OA}} = \frac{OA}{IA}$$

If A is close to B, then, approximately, $OA = OB$ and $IA = IB$

$$\therefore {}_{\text{air}}\mu_{\text{water}} = \frac{OB}{IB} = \frac{\text{real depth}}{\text{apparent depth}}$$

Since $${}_{\text{air}}\mu_{\text{water}} = \tfrac{4}{3}$$

Apparent depth in water $= \frac{3}{4}$ real depth.

As the obliquity of the line of sight increases, IA and OA cease to be nearly equal to IB and OB, with the result that the object appears to be still nearer the surface than it really is, and an eye placed at D will see the object as if it were at I_1. This effect can be seen when looking towards the deep end of a swimming bath. The bottom will appear to curve upwards although it is actually sloping downwards.

Critical Angle and Total Reflection. If a ray of light AO, Fig. 152, passes from an optically denser medium (e.g. glass) to an opti-

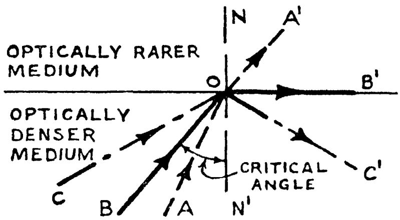


FIG. 152. CRITICAL ANGLE AND TOTAL INTERNAL REFLECTION

cally rarer medium (e.g. air) it will be bent away from the normal along the path OA', the angle $\angle NOA'$ being greater than the angle $\angle AON'$. If the angle of incidence is gradually increased a position will be reached when the incident ray BO is refracted along a path OB' coincident with the surface of separation. For this condition the angle of incidence $\angle BON'$ is known as the *critical angle.* If the angle of incidence is increased beyond the critical angle, the rays of light, such as CO, cannot be refracted into the rarer medium and so must be reflected at the surface along the path OC'. Such rays are said to undergo *total internal refection.* Refraction from one medium into another, which is optically rarer, can only occur if the angle of incidence is less than the critical angle.

Calculation of the Critical Angle. To calculate the critical angle for two given media, say glass and air—

$${}_{\text{glass}}\mu_{\text{air}} = \frac{\text{sine of angle of incidence } \angle BON'}{\text{sine of angle of refraction } \angle NOB'}$$

For the critical angle; $\angle NOB' = 90°$

and; angle of incidence $\angle BON'$ = critical angle

$$\therefore \frac{\text{sine of critical angle}}{\sin 90°} = {}_{\text{glass}}\mu_{\text{air}}$$

and since sin 90° = 1

$$\text{sine of critical angle} = {}_{\text{glass}}\mu_{\text{air}} = \frac{1}{{}_{\text{air}}\mu_{\text{glass}}}$$

Taking

$${}_{\text{air}}\mu_{\text{glass}} = 1{\cdot}5 \text{ approx.}$$

$$\text{sine of critical angle} = \frac{1}{1{\cdot}5} = 0{\cdot}667$$

$$\text{critical angle} = 42^\circ \text{ approx.}$$

Prisms

A prism is a portion of a transparent medium, usually glass, which is bounded by two faces which are not parallel. Fig. 153 shows an ordinary triangular refracting prism. The angle between the faces is known as the *refracting angle.*

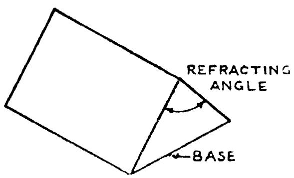


Fig. 153. Triangular Refracting Prism

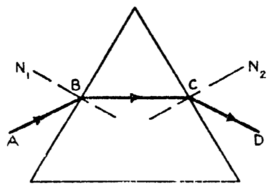


Fig. 154. Refraction of Light by a Prism

A ray of light striking the face of a prism obliquely, Fig. 154, will be refracted towards the normal as it enters the glass. It will travel on through the glass and on leaving will be refracted away from the normal, and, as a result of its passage through the prism, the ray will become deviated from its original path. This deviation varies with the position of the incident ray relative to the prism and will be found to be a minimum when the refracted ray BC, within the prism, is parallel to the base. For this *position of minimum deviation* the angles $\angle N_1BA$ and $\angle N_2CD$ are equal. For any other position of the prism the deviation will be increased.

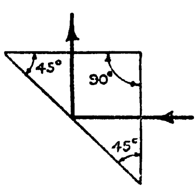


Fig. 155. Right-angled Prism used for Reflection

Total Reflection Prisms. Since the critical angle for glass is about 42°, rays travelling in glass which strike a glass-air surface at an angle of incidence of more than 42° undergo total internal reflection. A prism with a refracting angle of 90° and other angles of 45° can be used for reflection, as shown in Fig. 155, a ray entering one of the faces normally is undeviated and will strike the inclined face at an angle of incidence of 45°, which is greater than the critical angle. It will be reflected and turned through 90° so that it meets the other face normally and passes out without further deviation.

Fig. 156 shows the use of a right-angled prism for inverting a beam of light by total internal reflection.

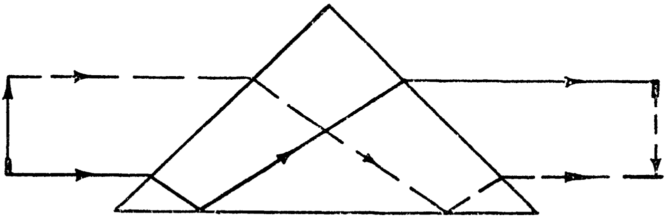


FIG. 156. RIGHT-ANGLED PRISM USED FOR INVERTING A BEAM

LABORATORY WORK

EXPERIMENT 1. *To determine the refractive index from air to glass*

For this experiment a slab of glass, about 4 in. × 3 in. × 1 in. high, is placed on a piece of drawing paper. Draw the outline of the block, *ABCD*, Fig. 157, on the paper and insert two pins P_1 and P_2 on the far side. Looking through the glass slab from the side *CD*, insert two pins P_3 and P_4 so that they appear to be in line with P_1 and P_2 viewed through the glass. Draw the line P_1P_2 to cut *AB* in *E* and P_4P_3 to cut *CD* in *F*. Join *EF* and draw *NEN'* normal to *AB*. Then $\angle P_1EN$ is the angle of incidence and $\angle N'EF$ is the angle of refraction at *E*.

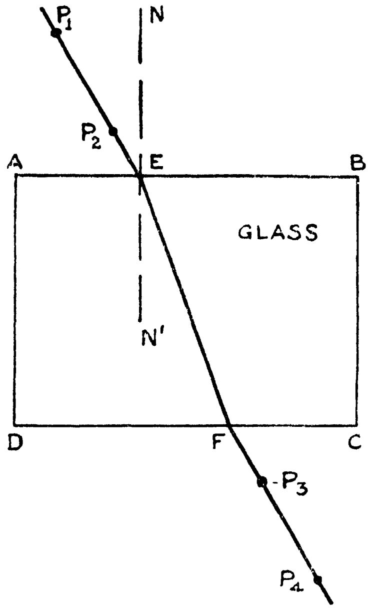


FIG. 157. DETERMINATION OF THE REFRACTIVE INDEX OF GLASS

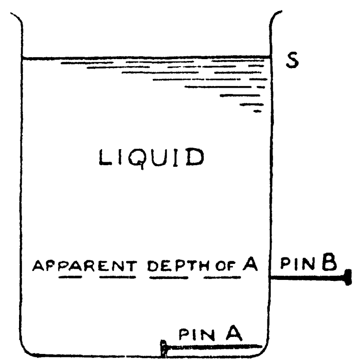


FIG. 158. DETERMINATION OF THE REFRACTIVE INDEX OF A LIQUID

$$_{\text{air}}\mu_{\text{glass}} = \frac{\sin \angle P_1EN}{\sin \angle N'EF}$$

Verify also that the line P_1P_2 is parallel to P_3P_4 and that

$$_{\text{air}}\mu_{\text{glass}} = \frac{1}{_{\text{glass}}\mu_{\text{air}}}.$$

EXPERIMENT 2. *To determine the refractive index of a transparent liquid*

Place a pin A, Fig. 158, at the bottom of a beaker filled with the liquid. Looking down on this pin, move a second pin B up or down the outside of the beaker until it appears to touch the image of A seen through the liquid. Then—

$$_{\text{air}}\mu_{\text{liquid}} = \frac{\text{real depth}}{\text{apparent depth}} = \frac{SA}{SB}$$

EXPERIMENT 3. *Refraction through a prism*

By means of a smoke box or optical disc, study the refraction of a beam of light through a prism and verify that the minimum deviation of the beam occurs when the refracted ray inside the prism makes equal angles with its sides.

Study also the use of a prism for total reflection.

EXERCISES

1. A lamp is placed on the bottom of a rectangular swimming bath, 4 ft deep, which is filled to the brim with water. The lamp is 3 ft from one end of the bath and the shadow of that end is 6 ft high on a vertical wall which is 8 ft from the same edge. Calculate the refractive index of the water.

2. Describe how you would measure the refractive index of water contained in a beaker.

3. A ray of light, travelling in glass, strikes its surface, making an angle with the normal of (*a*) 30°, (*b*) 60°. Show by diagrams the subsequent path in each case. Refractive index of glass is 1·5.

4. Define *refractive index* and *critical angle*.

Construct the path of a ray of light through a rectangular block of glass 3 in. thick, when the angle of incidence is 50° and the refractive index from glass to air is 2/3.

5. A cylindrical measuring jar, 38 cm high and 5 cm in diameter, is filled with water to a depth of 36 cm. Explain why, to an observer looking over the side, the depth of water appears to be less than 36 cm and calculate the apparent depth if the refractive index is 4/3.

Account for the fact that the lower part of the glass under the water appears to be silvered.

6. A ray of light in air is incident obliquely on the surface of a plate of glass. Indicate on a diagram the paths of the reflected and refracted rays and mark the angles of incidence, reflection, and refraction.

Explain the meaning of total internal reflection and show the use of a prism to invert the image of an object.

7. ABC is a principal section of an isosceles right-angled prism, the right angle being at B. A ray of light EF, parallel to AC, falls on the side AB at a point midway between A and B. Trace, graphically, the path of the ray through the prism. Refractive index from air to glass is 3/2.

8. A man is standing 8 ft away from the edge of a bath which is full to the brim with water ($\mu = 4/3$) and which is 6 ft deep. The eye of the man is 6 ft above water level. As he looks into the water at the rows of tiles running transversely across the bottom of the bath, several rows near the edge are invisible to him. If the tiles are 6 in. wide, how many rows of tiles are out of view?

CHAPTER XXIV

LENSES

A LENS is made of a transparent refracting medium, such as glass or certain plastics, and is bounded by two surfaces, both of which may be curved or one may be curved and the other plane.

All lenses fall into one of two classes: *convex* or converging lenses which are thicker in the centre, and *concave* or diverging lenses which are thinner in the centre than at their edges. Various

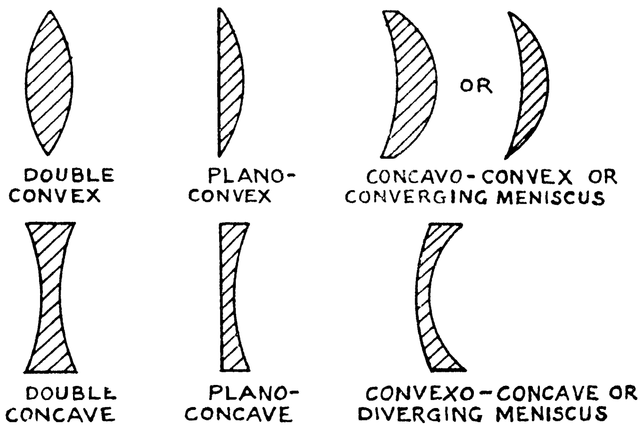


FIG. 159. CLASSIFICATION OF LENSES

types of lenses are shown in Fig. 159. The lens surfaces are usually parts of spheres although they may sometimes be cylindrical or aspherical. For spherical lenses the line joining the centres of curvature of the two surfaces is called the *principal axis* of the lens. When drawing lens diagrams it is usual to take a *principal section*, which is a plane passing through the principal axis.

Principal Focus. If a beam of parallel light falls on a convex lens in a direction parallel to its principal axis it will, in passing through the lens, be converged to a point F, Fig. 160 (*a*), on the axis known as the *principal focus* of the lens. In the case of a concave lens, the rays will not converge but will diverge as if they originated at a point F which again is the principal focus of the lens, Fig. 160 (*b*). Since for a concave lens F lies in front of the lens, it is called a *virtual* focus. The focal length of a lens is the distance from F to the lens position, or, since the lens is assumed to be thin, to the lens surface.

The converse to the above is true. If a source of light is placed at the principal focus of a convex lens, the rays, after refraction, will appear as a parallel beam. Similarly, rays converging to the

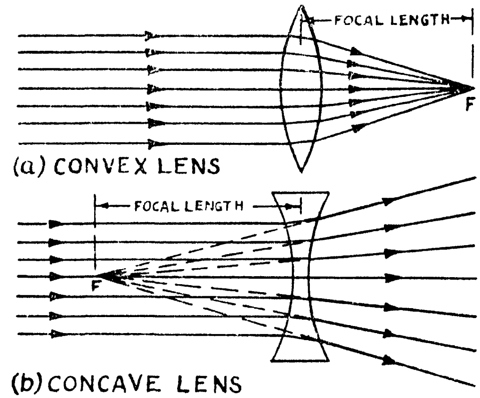


FIG. 160. PRINCIPAL FOCI OF CONVEX AND CONCAVE LENSES

principal focus of a concave lens will also be refracted as a parallel beam.

Optical Centre of a Lens. A ray *AB*, Fig. 161, incident on a lens will be refracted at *B* on entering and again at *C* on leaving. If the incident ray *AB* and emergent ray *CD* are parallel it can be shown geometrically that the ray must pass through a fixed point *O*, the optical centre of the lens, which is at the geometrical centre of the lens if the surfaces are of the same curvature. The ray *ABCD* will receive a lateral displacement as shown, but for thin lenses this will be slight, and it may be assumed that rays passing through the optical centre of the lens continue in the same straight line.

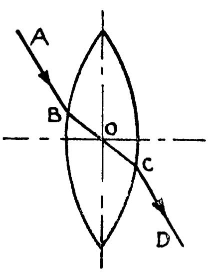


FIG. 161. OPTICAL CENTRE OF A LENS

Images Formed by a Lens. Diagrams showing the position and character of the image produced by a lens can be drawn in a manner similar to that used for mirrors. The work is simplified if it is remembered that—

1. Rays incident in a direction parallel to the principal axis will, after refraction, pass through or appear to have come from, the principal focus according to whether the lens is convex or concave.
2. Rays proceeding through the optical centre are undisturbed.
3. Rays for which the original path passes through, or would pass through, the principal focus will, after refraction, emerge parallel to the principal axis.

Thus by plotting the path of any two rays from a point on the object the position of the image of this point can be found, since it will lie at the point of intersection of the two rays after refraction. If the rays actually intersect, a real image is formed which could be shown on a screen. If the refracted paths diverge, they must

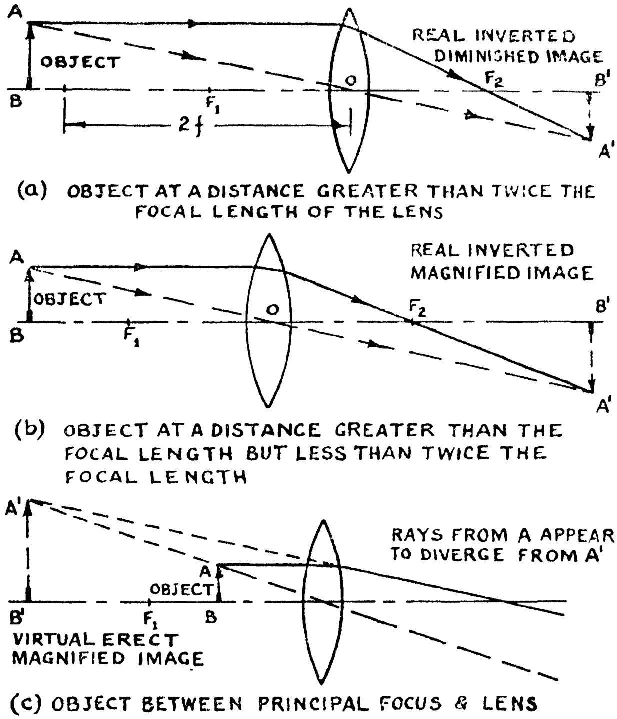


Fig. 162. Images Produced by a Convex Lens

be produced backwards to find the point of intersection and the image produced will be virtual.

Images Produced by a Convex Lens. The type of image formed will depend upon the relative positions of object and lens. Diagrams for the various cases are shown in Fig. 162.

(*a*) Object at a distance greater than twice the focal length—a real, inverted, diminished image is formed.

(*b*) Object at a distance greater than the focal length but less than twice the focal length—a real, inverted, magnified image is formed.

(*c*) Object between principal focus and lens—a virtual, erect, magnified image is formed.

The use of a convex lens as a simple microscope or magnifying glass is an example of case (*c*).

Image Produced by a Concave Lens. For all positions of the object, a concave lens produces a virtual, erect and diminished image, Fig. 163.

Optical Power. Lenses vary in their ability to converge or diverge light. A lens with a short focal length clearly has a greater power of deviating light rays than a lens with a long focal length. The power of a lens is therefore inversely proportional to its focal length.

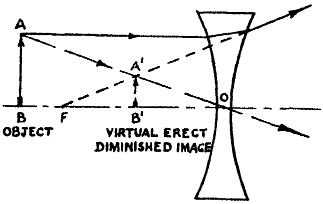


FIG. 163. IMAGE PRODUCED BY A CONCAVE LENS

Optical power, denoted by F, is measured as the reciprocal of the focal length f and if f is measured in metres the unit of power is the *dioptre*, abbreviated to D.

A lens of 1 m focal length has a power of 1 D.

A lens of 20 cm focal length ($\frac{1}{5}$ metre) has a power—

$$F = \frac{1}{f} = 1 \div \tfrac{1}{5} = 5\text{ D}$$

Lens Formula. There is a simple relation, similar to that for concave and convex mirrors, between the distance from the lens of the object and its image and the focal length of the lens. By adopting a suitable sign convention, this relation can be expressed in the form of a single formula covering both convex and concave lenses, and both real and virtual images. The required conventions are—

1. Distances of real objects and images from the lens are positive.
2. Distance of a virtual image is negative.
3. A converging lens has a positive focal length and power.
4. A diverging lens has a negative focal length and power.

Then, putting: u = object distance from lens
v = image distance from lens
f = focal length
F = power of lens.

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} = F$$

Magnification. The magnification of an image is the ratio of the height of the image to the height of the object. Thus in Figs. 162 (*a*, *b* and *c*)—

$$\text{Magnification} = \frac{A'B'}{AB}\text{, or since triangles } ABO \text{ and } A'B'O \text{ are similar,}$$

$$= \frac{B'O}{BO} = \frac{\text{image distance}}{\text{object distance}} = \frac{v}{u}$$

EXAMPLE. A lens is to be employed to project an image on a screen 3 ft away from the object. If the image is to be three times as large as the object, (*a*) what kind of lens is required, (*b*) where must it be placed, (*c*) what must be its focal length?

(*a*) To produce a real image, a convex lens is required.
(*b*) Magnification = 3

$$\therefore \frac{\text{image distance } v}{\text{object distance } u} = 3 \text{ or } v = 3u \qquad (1)$$

But, $u + v = 3$ ft

Substituting for v from equation (1),

$$4u = 3 \text{ ft}$$
$$u = \tfrac{3}{4} \text{ ft}$$
$$v = 3 - \tfrac{3}{4} = 2\tfrac{1}{4} \text{ ft}$$

Lens must be placed 9 in. from the object and 2 ft 3 in. from the screen.

(*c*) If f = focal length of the lens.

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} = \frac{4}{3} + \frac{4}{9} = \frac{16}{9}$$
$$= +\frac{9}{16} \text{ ft} = +6\tfrac{3}{4} \text{ in.}$$

A convex lens of $6\frac{3}{4}$ in. focal length is required.

LABORATORY WORK

EXPERIMENT 1. *Determination of the focal length of a converging lens by direct measurement*

A pin is held above the centre of the lens which is placed flat on a plane mirror. The pin is moved nearer or farther from the lens until there is no parallax motion between the pin and its inverted image. The pin is then at the principal focus of the lens. Rays from the pin, after passing through the lens, are parallel and, striking the mirror normally, are reflected back, still parallel, so that passing through the lens again they converge to the principal focus. Thus the distance of the pin above the lens surface is equal to the focal length of the lens.

EXPERIMENT 2. *Verification of the lens formula*

Taking a number of positions of object relative to the lens, verify the lens formula—

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

EXPERIMENT 3. *Determination of the focal length of a lens by the lens formula*

For any given position of the object, locate the position of the image, either on a screen if real or by sighting or parallax if virtual, and calculate the focal length from the lens formula.

EXERCISES

1. A convex lens forms a real image of an object on a screen placed 1 m from the object. If the magnification is 3, calculate the focal length and power of the lens, and its distance from the object.

2. An illuminated object of height 2 in., stands on the principal axis of a double convex lens and at a distance of 14 in. from it. If the lens has a focal length of 10 in., obtain, graphically, the distance at which a screen must be placed to receive an image of the object. Determine its magnification.

3. Find the position, size and character of the image produced by a convex lens, of 7 in. focal length, of an object, 2 in. tall, placed (*a*) 4 in., (*b*) 10 in. from the lens. Illustrate your answer with diagrams.

4. An object is placed perpendicular to the axis of a convex lens, 10 in. focal length, and 6 in. from the lens. Show by a diagram the path of the rays by which the object is seen by an eye on the axis of the lens and 20 in. from it. What is the least diameter of lens that will allow the eye to see the whole of a 2 in. object at the same time?

5. What is meant by the focal length of a lens?

A bright object, 12 in. from a convex lens, gives a real image on a screen 8 in. beyond the lens. The object is moved 6 in. towards the lens. Where must the screen now be placed to receive the image?

6. What will be the effect of a converging lens, of power 10 D, upon a beam of light (*a*) diverging from a point 15 cm from the lens, (*b*) converging to a point 20 cm beyond the lens, (*c*) diverging from a point 5 cm from the lens?

7. Light, diverging from a point on the principal axis 30 cm in front of a lens, converges after passing through the lens to a point 10 cm behind the lens. Calculate the focal length of the lens and draw a diagram illustrating the paths of two rays.

8. It is desired to form on a screen an image 20 cm tall of an object which is 4 cm tall. The distance between object and screen is 30 in. What focal length should a convex lens have to bring this about and where should the lens be placed?

CHAPTER XXV

COLOUR

LIGHT has been defined as a form of energy transmitted by disturbances set up in the ether. These disturbances may vary in wavelength, producing different colour sensations to the eye. Red light has a long wavelength, while that of violet light is shorter, and other colours have wavelengths lying in between. Ordinary white light is a compound of all the different colours in proper proportions.

If white light is used in experiments with simple lenses it will be noticed that the images produced are edged with colour. This is because the various colours composing the white light are refracted

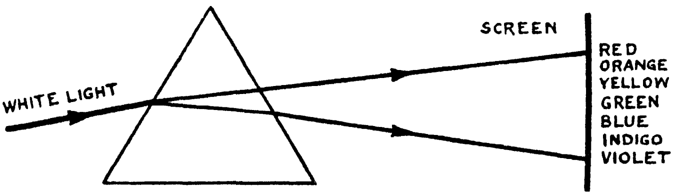


FIG. 164. CHROMATIC DISPERSION OF WHITE LIGHT

by the lens to a different extent; components like violet, with short wavelengths, are bent more sharply than those colours, such as red, which are of longer wavelength. This colour fringing is called *chromatic aberration*, and is a serious defect in a single lens, but can be overcome by using a compound lens made by combining a convex and concave lens manufactured from different types of glass.

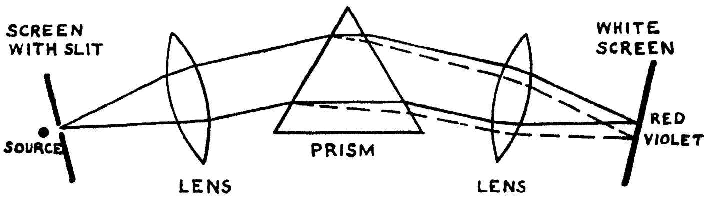


FIG. 165. PRODUCTION OF A PURE SPECTRUM

The Spectrum. Newton first discovered that white light was a compound of various colours and, by passing a beam of white light through a prism, split it into its separate components which appeared as a band of colours imperceptibly shading from one to another, Fig. 164. This band of colours is called a *spectrum*, and the white light is said to have undergone *chromatic dispersion*.

In Newton's original experiment the colours tended to overlap. A pure spectrum, in which the overlapping of the colours does not occur, can be produced with the help of lenses, Fig. 165.

Newton's Colour Disc. To verify that white was a mixture of colours, Newton took a circular card and divided it into a number of sectors, painting each sector with a different colour and making the area of each coloured sector proportional to the amount of that colour in white light. When spun rapidly the colours merged so that the eye experienced the sensation of white light.

Colour of Opaque Bodies. Since any normal object is invisible in the dark, it must ordinarily be visible by the reflection from its surface of light falling upon it from an external source. Thus its colour will depend upon the colour of the light which it reflects. A red material, viewed in white light, appears red because it reflects principally the red component of the white light and absorbs most of the other component colours. This can be verified by viewing the material under green light. It will appear black, because there is no red light present to be reflected and all other light is absorbed. Thus the colour of opaque bodies is due to the colour of the component of white light which they reflect. A black body is one which reflects none of the colours of the spectrum and so will appear black under light of any colour.

Colour of Transparent Substances. A transparent substance viewed by transmitted light owes its colour to the absorption of some components of white light and the transmission of the remainder. Thus a red liquid possesses this colour because only red light is transmitted through it.

Complementary Colours. Two colours which together make white light are called complementary colours. Examples are blue and yellowish-red, red and bluish-green, orange and greenish-blue. When white light falls on a coloured object one colour is reflected and its complementary colour is absorbed or transmitted. Thus a piece of gold-leaf appears a reddish-yellow by reflected light, but if the leaf is thin enough, it will be seen to transmit the complementary colour blue.

Primary Colours. White light can be split up into three primary colours, red, yellow, and blue, which can be mixed together, in the proper proportions, to produce light of any desired hue. Instruments working on this principle are used for colour matching and a system, based upon the three primary colours, is also employed for giving an infinite variation of colour effect in theatrical stage lighting.

LABORATORY WORK

EXPERIMENT 1. *Formation of a spectrum*

Using a prism, repeat Newton's experiment to produce a spectrum.

Repeat the experiment using lenses in addition to the prism to produce a pure spectrum.

EXPERIMENT 2. *Newton's colour disc*

Divide a circular card into sectors and paint each sector with one of the spectral colours, the area of each sector being made proportional to the amount of that colour in white light. Rotate the card rapidly and note the merging of the colours into a greyish white.

EXERCISES

1. How can it be shown that white light can be split up into coloured components? How do these components differ?
2. Explain how to produce a pure spectrum.
3. Describe an experiment to show how white light may be formed by mixing coloured light.
4. Upon what does the colour of an opaque body depend? Explain why a red object appears red under white light but black under green light.
5. What are complementary colours? Give examples.

CHAPTER XXVI

ILLUMINATION AND PHOTOMETRY

In modern building practice it is now recognized as being most desirable to have a set of rules giving the illumination requisite for various conditions. Such a set of rules—scientifically based—is now being established, and this chapter deals with some of the fundamentals and methods of measurement underlying the development of the system.

Illuminating Power. Because sources of light vary in their power of illumination, a standard of comparison is essential. Historically, this was provided by the British Standard Candle made to a definite specification from spermacetti wax, burning at the rate of 120 g/hr; after which the unit of illuminating power, the *candle power*, is named. A more satisfactory primary standard is the Hefner lamp burning amyl acetate, which is of 0·9 c.p. in a horizontal direction, or the Vernon Harcourt pentane lamp, used in America, which is of 10 c.p. measured in a horizontal direction.

In industrial work it is more convenient to use an electric lamp as a secondary standard, which in this country can be calibrated against a primary standard at the National Physical Laboratory. Providing the electrical supply is properly regulated and the filament temperature is kept well below the normal operating temperature of an ordinary lamp, the characteristics of such a secondary standard will remain constant for many hours of operation.

Luminous Flux. From a light source there is a continuous light energy flow, or *luminous flux*, which is measured in *lumens*: one lumen is the amount of light flux received from a standard candle on an area of one square foot every part of which is one foot distant from the candle.

Intensity of Illumination. The intensity of illumination of a surface is the amount of light energy received per unit area per unit time, and is measured by the luminous flux received per unit area. The unit of measurement is the *foot-candle*, and is the intensity produced when one lumen is spread over one square foot, i.e. the intensity produced on a surface all parts of which are one foot away from a light source of one candle power.

Surface Brightness. While intensity of illumination measures the amount of light received, the surface brightness of an object is the measure of the light which it reflects back and will depend not only on the intensity of illumination falling upon it but also upon its reflection factor (page 167). The unit of measurement is the *equivalent foot-candle* and—

$$\text{Surface brightness in equivalent foot-candles} = \text{reflection factor} \times \text{intensity of illumination in foot-candles}$$

A different unit is used for the measurement of the brightness of the surface of a light source, which is measured by dividing the total candle power of the source by the area of its radiating surface, the result being expressed in candles per square inch.

Inverse Square Law. A point source of light radiates light energy equally in all directions so that there would be a uniform intensity of illumination on all parts of a spherical surface with the source at its centre. If the radius of such a spherical surface is increased, the total luminous flux remains constant but the area over which it is spread is greater, causing a progressive reduction in intensity of illumination in direct proportion to the increase of area. Since the area of a spherical surface is proportional to the square of its radius, the intensity of illumination will vary inversely as the square of the distance of the surface from the source.

It should be noted that this inverse square law is only applicable to light emanating from a point source.

Cosine Law. If light falls obliquely on a surface, the inverse square law must be modified because the area on which the light falls is increased. Lambert showed that in this case—

$$\text{Intensity of illumination} = \frac{\text{illuminating power of source}}{(\text{distance of source from surface})^2}$$

$\times \cos \alpha$ where α = angle of incidence of light.

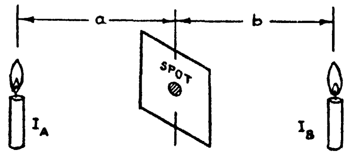


FIG. 166. BUNSEN'S GREASE-SPOT PHOTOMETER

PHOTOMETRY

An instrument used for the comparison of the illuminating power of two sources is called a *photometer*. If one of the sources is a standard of known candle power it can be used to determine the power of the second source. The method employed can best be understood from a description of a very simple form of photometer.

Bunsen's Grease-spot Photometer. A spot of oil or grease is dropped on to a piece of white paper which is then placed between the two sources to be compared (Fig. 166). If the paper is viewed from one side, the grease spot will appear brighter or darker than the surrounding paper according to whether the intensity of illumination from the source in front of the paper is smaller or greater than that from the source on the other side. If the intensities on both sides of the paper are the same the grease spot will disappear.

If, when the spot disappears, the distance from the paper to the source of power I_A is a and from the source of power I_B is b, then— Intensity due to I_A = intensity due to I_B

or, applying the inverse square law—

$$\frac{I_A}{a^2} = \frac{I_B}{b^2} \quad \therefore \quad \frac{I_A}{I_B} = \frac{a^2}{b^2}$$

Thus, if the intensity I_A is known, the intensity of the second source I_B can be determined.

The accuracy of this type of photometer is improved by arranging two mirrors so that both sides of the paper can be viewed simultaneously. The photometer is then adjusted until the grease spot has the same appearance on both sides of the paper.

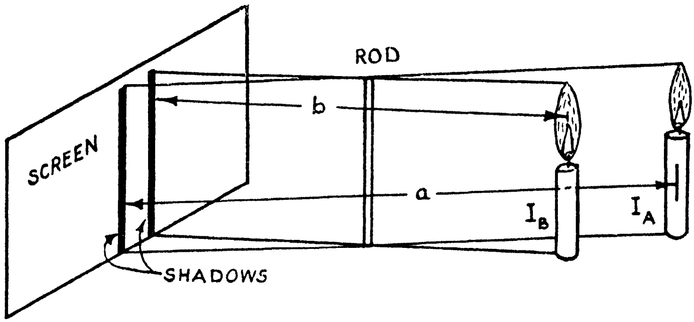


FIG. 167. RUMFORD'S SHADOW PHOTOMETER

Rumford's Shadow Photometer. The two sources to be compared are arranged so that each throws a shadow of a thin stick on a screen, Fig. 167, but each illuminates a shadow thrown by the other. The sources are moved until the two adjacent shadows appear of equal intensity, the illuminating power of the sources then being directly proportional to the square of their distances from the screen (*not* the stick), i.e.

$$\frac{I_A}{I_B} = \frac{a^2}{b^2}$$

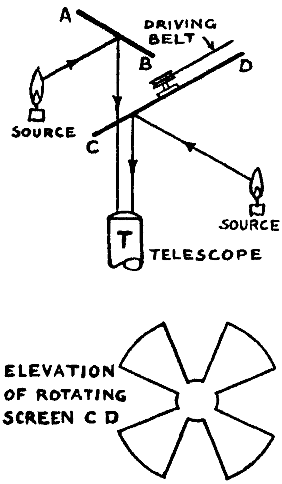


FIG. 168. ABNEY'S FLICKER PHOTOMETER

Abney's Flicker Photometer. In this instrument one source illuminates a white screen AB (Fig. 168), while the other illuminates a rotating screen CD cut in the form of a Maltese Cross. If the two surfaces are unequally illuminated the observer at the telescope T will observe a flicker as he sees alternately the arms of the wheel CD and the surface of the screen AB. This flicker will fall to a minimum when the two surfaces are equally illuminated.

This photometer can be used successfully in comparing light sources of different colours for, although at low speeds of rotation there will be flicker both of colour and of changing intensity, at higher speeds the colour flicker disappears and the observer sees a single tint of flickering intensity. The wheel should rotate at a speed just sufficient to prevent colour flicker; it must not rotate too quickly or else the intensity flicker will not be appreciated. The equalizing of the intensities of illumination is made either by adjusting the distances of the sources or by interposing filters.

Philips's Foot-candle Meter. This meter is based upon the grease-spot photometer, but gives direct readings of illumination intensity in foot-candles. A series of circular "grease spots" are formed in an opaque white surface (Fig. 169) which is illuminated

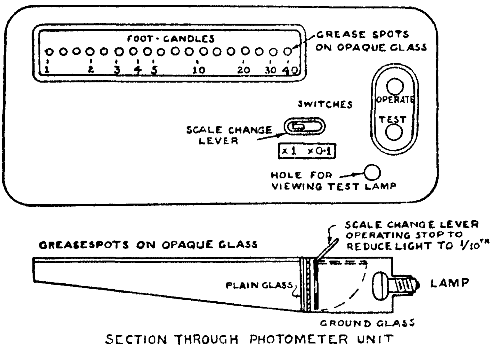


FIG. 169. PHILIPS'S FOOT-CANDLE METER

from above by the outside light to be measured and from below by a lamp, run from dry cells, which acts as a standard. The brightness of illumination of successive grease spots decreases as they get farther away from the lamp. Viewed from above, one grease spot can be found which most nearly matches the brightness of the white surface, the remainder appearing either brighter or darker. The illumination in foot candles corresponding to the selected grease spot is read directly off the scale. By interposing a stop in front of the lamp the range of the instrument can be reduced and its sensitivity increased.

Integrating Photometers. The candle power of a source is not necessarily the same in all directions and is therefore not an index of its total illuminating power. The mean value of the candle power of a source is called its *mean spherical candle power* and its total illuminating power can be measured in lumens. Since a source of one c.p. radiates a luminous flux of one lumen through one square foot of surface one foot away from the source, the total illuminating power of a source of one mean spherical candle power will be 4π lumens. The total illuminating power of a source can be measured experimentally by means of an integrating photometer, a common form consisting of a hollow sphere, about 5 ft in diameter and whitened inside, at the centre of which the lamp under test is fixed. There is a small opening in the wall of the sphere through which a white screen, interposed between the source and the opening, can be viewed through a telescope. The side of the screen, seen in the telescope, will be illuminated by light diffused from the walls of the sphere with an intensity which will therefore depend upon the mean spherical candle power of the source. For the purpose of measurement the telescope may be arranged so that in half the field of view a screen illuminated by a standard lamp is seen. The intensities of illumination of the two halves of the field of view are matched by varying the voltage across the standard lamp or by introducing filters. The photometer must be calibrated by means of a lamp of known mean spherical candle power. It is not necessary to know the reflecting power of the walls of the sphere, but this should be high to ensure proper diffusion and the illumination of the screen by diffused light from all parts of the source.

LABORATORY WORK

The student should construct and set up for himself a simple photometer of the Bunsen grease spot or Rumford shadow photometer type and should use it for the comparison of the illuminating power of two sources.

Where possible, other types of photometer should also be examined and used.

EXERCISES

1. Describe how you would compare the illuminating powers of two pocket torches by means of a shadow photometer. Would it be necessary to use a dark room for the experiment? Give reasons.

2. Explain the terms *illuminating power* and *intensity of illumination* and define the units in which each is measured.

At what distance from a screen must a lamp of 50 c.p. be placed to give the same illumination as a 30 c.p. lamp at 3 ft distance?

3. Two lamps of 16 and 24 c.p. are 5 ft apart. A screen is placed between them 2 ft from the weaker lamp. Compare the intensity of illumination on the two sides of the screen. Where must the screen be placed to give equal illumination on both sides?

4. A lamp is 3 ft directly above one end of a table 4 ft long. Compare the intensities of illumination at the two ends of the table.

5. Two lamps A and B produce equal illuminations on a screen when placed at distances of 3 and 2 ft respectively from it. Compare the candle powers of the lamps.

If the distance from A to the screen is increased to 4 ft, where must B be placed to give equal illumination?

6. Three sources of light are of equal candle power. One is placed 2 ft from a grease-spot photometer, calculate how far away the other two together must be placed to balance the first photometrically.

7. Describe a form of flicker photometer. What are its advantages?

8. What is meant by mean spherical candle power? Describe an instrument by which it may be measured.

SECTION VI—SOUND

CHAPTER XXVII

THE NATURE OF SOUND

SOUND requires air or some other material medium for its transmission. This fact can be verified by placing an electric bell in a glass vessel and, after setting the bell ringing, gradually removing the air from the vessel with a vacuum pump. The sound will gradually die away as the air is removed. Normally sound is carried by air, but it can also travel in other materials such as water, wood, steel, or concrete.

Sound Waves. The energy of the *sound wave* sets the particles of the medium in motion, causing them to oscillate backwards and forwards alternately compressing and stretching the material, thus setting up a *longitudinal pressure wave* which travels through the material. In the case of a solid medium such as wood or steel, it is clear that the sound wave is transmitted without any movement of the medium as a whole, each particle transferring its energy to the next but not travelling along with the sound wave. This is also the case with media such as air or water; there is no movement of the medium as a whole when sounds are transmitted.

The sound waves generated by a continuously vibrating body, such as a tuning fork, consist of alternate *compressions* and *rarefactions*, that is to say regions of high and low pressure, which travel in all directions away from the source with a velocity depending upon the density and elasticity of the medium. In air this velocity is about 1100 ft/sec. The distance from one compression to the next is called the *wavelength* of the sound.

Frequency and Pitch. During the transmission of a continuous sound wave, each particle of the medium goes through a series of movements which it repeats at regular intervals. A complete series is known as a *cycle*, and the number of repetitions of this cycle made by a particle in the medium in one second is called the *frequency* of the sound and is measured in cycles per second.

The velocity of sound in a given medium is independent of its frequency, therefore if the frequency is increased, the regions of compression and rarefaction will be more numerous and closer together so that the wavelength will decrease. The relation between the frequency, wavelength and velocity of sound is given by—

$$\text{Frequency} \times \text{wavelength} = \text{velocity}$$

The term pitch is used in connection with musical notes; a high shrill note is said to have a high pitch. High pitch is associated with sound of high frequency and low pitch with that of low frequency. Doubling the frequency of a musical note is equivalent to raising its pitch by one octave.

Intensity. The intensity of a sound is a measure of its energy and is independent of the ear receiving it. Intensity is defined as the rate of flow of sound energy across unit area normal to the direction in which the sound wave travels and the absolute unit of measurement is the erg per second per square centimetre or the micro-watt per square centimetre. In practice it is more often necessary to compare the intensities of sounds, and for this purpose the *Bel* or its sub-multiple the *decibel* is used. This gives a relative measure of intensity in energy units on a logarithmic scale. Two sounds of intensities I_1 and I_2 in energy units have a difference of intensity of $\log_{10}\left(\frac{I_1}{I_2}\right)$ bels or $10 \log_{10}\left(\frac{I_1}{I_2}\right)$ decibels. Thus if $I_1 = 2 I_2$, then,

$$\text{Difference of intensity} = 10 \log_{10} 2 = 3{\cdot}01 \text{ db}$$

This scale is adopted because, in perceiving changes of intensity with the human ear, it is not the actual increase in sound energy which is important but the fractional increase. The decibel is practically the smallest change in intensity that the ear can distinguish.

Loudness. It is necessary to distinguish between the intensity of a sound, as measured in terms of energy, and the loudness of a sound as judged by the ear. The ear is sensitive to sounds ranging in frequency from about 20 to 20,000 c/s but, unfortunately, the degree of sensitivity varies throughout this range, being poor at low frequencies and improving with increasing frequency to a

TABLE XIII

EQUIVALENT LOUDNESS OF COMMON SOUNDS

Type of noise	Equivalent loudness in phons
Threshold of hearing	0
Whisper, 5 ft away	10
Quiet garden	30
Residential road	40
Ordinary speaking voice	65
City street	70
Railway carriage noise	80
Loud music, motor horns	90
Pneumatic drill, 10 ft distant	100
Riveting machines	110
Aeroplane engine, 20 ft distant	120
Threshold of pain	130

maximum at about 1000 c/s, after which it falls off again. The same mechanical energy put into sounds of different frequencies will, therefore, not produce the same sensation of loudness to the ear. Where it is desired to measure loudness as opposed to intensity, a scale of *equivalent loudness* is used in which the loudness of the sound is compared aurally by the observer with that of a standard pure tone of a frequency of 1000 c/s. The observer adjusts the intensity of the standard tone until it appears to be of equal loudness to the sound to be measured. The intensity of the standard tone is then a measure of the loudness of the sound. The unit employed for the measurement of equivalent loudness is the *phon*, and a sound is said to have an equivalent loudness of n phons if it is judged by a normal observer to be as loud as a 1000 c/s pure tone of which the intensity is n decibels above a fixed zero of 10^{-16} w/cm^2, a level which is almost the threshold of hearing at 1000 c/s. Table XIII indicates the loudness of some ordinary noises in phons.

Inverse Square Law. If a source of sound radiates energy in all directions the sound waves travel outwards as a sphere of ever increasing radius. Thus the sound energy becomes spread over a greater area and there must be a progressive reduction in intensity in proportion to this increase in area. Since the area of a sphere is proportional to the square of its radius, the intensity of sound will be inversely proportional to the square of the distance of the point of observation from the source.

EXAMPLE. What will be the difference between the intensity of a sound at two points A and B, if A is 10 ft from its source and B is 20 ft? Express this difference in decibels.

Let I_A be the intensity at 10 ft from the source

I_B be the intensity at 20 ft from the source

Then,
$$\frac{I_A}{I_B} = \frac{20^2}{10^2} = 4$$

Intensity at A is four times as great as at B.

$$\text{Difference in intensity in decibels} = 10 \log_{10} \frac{I_A}{I_B}$$
$$= 10 \log_{10} 4 = 10 \times 0{\cdot}6 = 6 \text{ db}$$

EXERCISES

1. What is a sound wave? Explain how sound travels through a material.
2. Define the meaning of the terms *wavelength* and *frequency*. What is the relation between the wavelength, frequency and velocity of a sound?
3. Distinguish between the intensity of a sound and its loudness. What is (*a*) the absolute unit, (*b*) the comparative unit, for the measurement of intensity?
4. What is the advantage of using the decibel scale when measuring intensities?

If sound A is four times as loud as sound B, what is the difference of their intensities in decibels?

5. Define *equivalent loudness* and explain why it is necessary to differentiate between the loudness and intensity of a sound.

6. What is a *phon*? Explain how the equivalent loudness of a sound can be determined.

7. What will be the difference of intensity in decibels of a given sound at two points 50 ft and 120 ft respectively from the source?

8. There is a difference of 6 db in the intensity of a sound at two points A and B, 50 ft apart. If the source and points A and B lie in the same straight line and A is nearer to the source than B, calculate the distance from A to the source.

CHAPTER XXVIII

THE BEHAVIOUR OF SOUND WAVES

In many ways sound waves behave like light waves since they can be reflected, refracted and even made to cast acoustic shadows, but the wavelength of sound is so much greater than that of light that these effects can only be observed on a large scale. Table XIV shows the wavelengths of some typical sounds. Comparing these with the wavelength of light which is measured in millionths of an inch, and remembering that the size of the reflectors and other apparatus used must be large compared with the wavelength, it is soon clear why, for instance, light may be reflected from a very small mirror but a reflector for sound must be very large.

TABLE XIV

Frequencies and Wavelengths of Typical Sounds

Sound	Frequency	Wavelength (approx.)
Highest audible note	30,000	0·4 in.
Highest note on piano	3500	3·2 in.
Woman's voice	280	4 ft
Man's voice	140	8 ft
Lowest note on the piano	27	40·8 ft
Lowest audible note	20	55 ft

Acoustic Shadows. Although sound finds its way easily around a small obstacle if the obstruction is large enough, a well-defined acoustic shadow can be formed. Thus a large building acts as a screen behind which the intensity of sound will be greatly reduced, and even a row of trees or a thick hedge may cause a reduction in intensity of 5 db and may be used to shelter buildings which would otherwise be exposed to too great a noise. As might be expected, acoustic shadows are found to be more definite for high frequencies, since the wavelength is smaller compared with the size of the obstacle.

Reflection. Sound waves striking a large surface, such as a wall, are reflected in the same manner and according to the same laws as light waves. The effect is clearer if the surface is smooth and polished, for rough surfaces tend to absorb or scatter the waves. Reflected sound waves are called echoes and can easily be observed by standing well back from a cliff or large building and shouting. After a short interval the reflected sound will return in the form of an echo. The time between the original shout and the echo will be that required for the sound to travel, at 1100 ft/sec, the double journey to the reflecting surface and back.

Reflection of Sound from a Flat Surface. If a ray of sound SC (Fig. 170) travels from a source S and is reflected from a surface AB—

$$\text{Angle of incidence } \angle SCN = \text{angle of reflection } \angle DCN$$

and the reflected ray CD will appear to come from an image of the source at S', which, as shown in the case of light (page 168), will

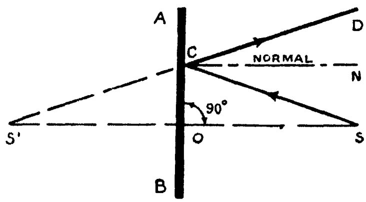


FIG. 170. REFLECTION OF A RAY AT A FLAT SURFACE

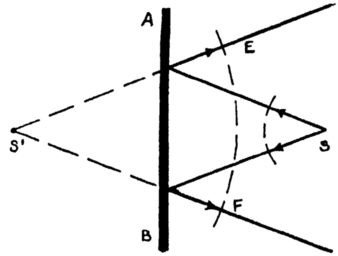


FIG. 171. REFLECTION OF A WAVE FRONT AT A FLAT SURFACE

lie on the line SOS' normal to the surface AB and at a distance OS' behind AB equal to the distance OS of the original source in front of AB.

In practice sound travels outwards from a point source in spherical waves and on striking a reflector the whole wave front will be reflected. This is shown in Fig. 171 for the portion CD of a wave front emanating from a point source S. All the rays from S will be reflected from AB as if they originated at S' and the new wave front EF can be set out as an arc of a circle with centre S'.

Reflection of Sound at a Curved Surface. If a ray of sound SC (Fig. 172) from a source S strikes a curved reflector of any shape

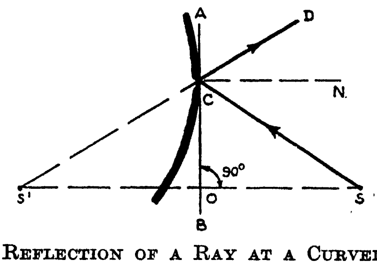


FIG. 172. REFLECTION OF A RAY AT A CURVED SURFACE

at C it will be reflected so that the incident and reflected rays make equal angles with the normal CN to the tangent AB to the curved surface at C. In fact it will be reflected as if AB was a flat reflecting surface and, as shown above, the reflected ray will appear to come from an image S' of the source lying on the normal SOS' to AB, the tangent at C, and so placed that $OS = OS'$.

The reflected wave front for sound from a source S striking any curved surface can be plotted by drawing the paths of a number of

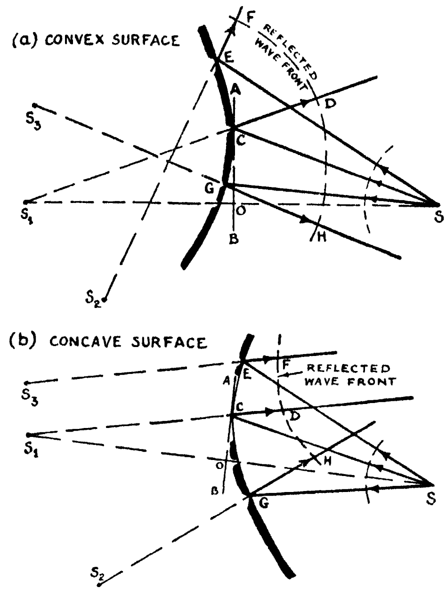


FIG. 173. REFLECTION OF A WAVE FRONT AT A CURVED SURFACE

rays from the source, as shown in the diagrams for a convex surface and also a concave surface in Fig. 173 (*a* and *b*). The ray SC, in either case, will be reflected along the path CD as if it came from an image source S_1, the position of which is found by taking the tangent AB at C, drawing SOS_1 at right angles to AB and making $OS_1 = OS$. Similarly rays SE and SG will be reflected along paths EF and GH as if they came from image sources S_2 and S_3 respectively. Since all points on the new wave front must be at the same distance from the source it only remains to fix the points D, F, and H so that the lengths SCD, SEF, SGH are equal, or since $SC = S_1C$, $SE = S_2E$ and $SG = S_3G$, this may be done by making $S_1D = S_2F = S_3H$. The reflected wave front now passes through D, F and H.

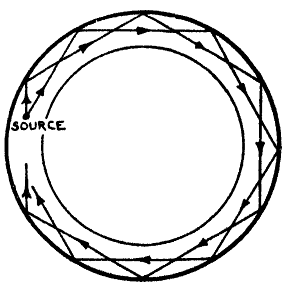


FIG. 174. REFLECTIONS IN A CIRCULAR GALLERY

An interesting example of sound reflection in curved buildings may be found in the Whispering Gallery of St. Paul's Cathedral. A whisper near the wall can be heard everywhere around the circumference because many of the reflected rays keep close to the wall, Fig. 174.

Refraction. Sound, like light, is deviated or refracted on passing from one medium to another of different density. Tyndal showed, experimentally, that the sound produced by a whistle could be focused to a point by means of a soap bubble filled with carbon dioxide. If, however, the velocities of sound in the two media are markedly different, the sound tends to be reflected back at the interface instead of passing into the second medium. When a sound wave in air strikes the surface of water, most of the sound energy is reflected and very little passes into the water. For this reason sound carries well across open water.

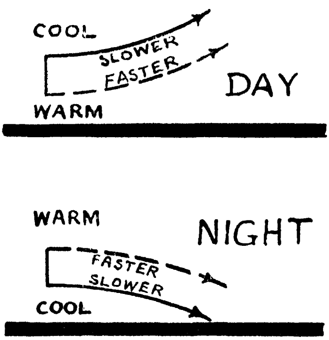


FIG. 175. REFRACTION OF SOUND IN THE ATMOSPHERE

Refraction in the Atmosphere. Sound travels with a greater velocity in warm air than in cold and will tend to be refracted when the atmosphere is not at a uniform temperature. During the day the air at ground level becomes heated by radiation from the ground and is relatively warmer than the air above. Sound waves near the ground will travel more quickly and the wave front will be deviated upwards, Fig. 175, so that the sound may not be heard even at a relatively short distance away at ground level. At night the process is reversed, the air above being warmer than that at ground level. Thus rays which might otherwise tend to travel upwards are refracted back to ground level and the sound will be audible at a considerable distance away.

A similar effect is produced by differences of wind velocity, which is usually greater above the ground than at ground level. The velocity of the wind is added to that of the sound waves, so that sound waves travelling downwind go faster above the ground than at ground level. Their wave front tends to bend down towards the ground where it is reflected and the same process starts again. Thus sounds carried downwind can be heard at ground level for a considerable distance. Sounds travelling upwind are slowed down more above the ground than at ground level so that the wave front tends to be bent upwards and, within a short distance, the sound becomes inaudible at ground level.

Interference. In the diagram, Fig. 176, S_1 and S_2 are two sources generating sound waves. At a certain instant the regions of high pressure will correspond with the full lines and the regions of low pressure with the dotted lines in the diagram. As the waves overlap, there will be points, marked *A*, where the compression due to one wave will coincide with the rarefaction due to the other, each thus tending to annul the other. Similarly there will be other points *B* where the two waves reinforce each other. This effect is known as

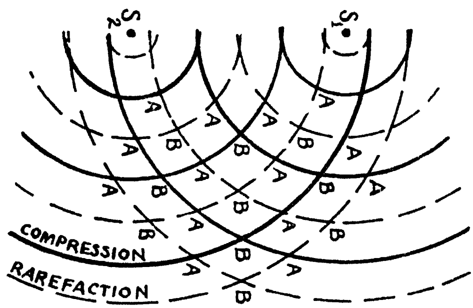


FIG. 176. INTERFERENCE BETWEEN TWO SOUND SOURCES

interference, and alternate bands of quietness and loudness, known as *interference bands*, are formed.

The effect may be observed with a tuning fork. When the prongs of the fork move outwards a compression starts in the direction *OA* and *OB*, Fig. 177, but simultaneously there will be a rarefaction

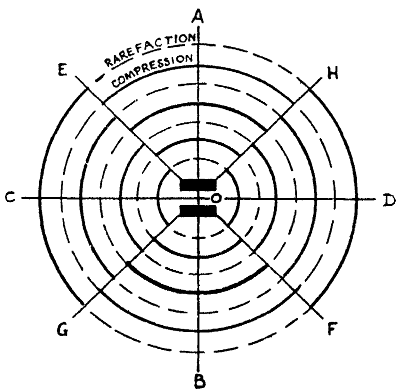


FIG. 177. INTERFERENCE EFFECT WITH A TUNING FORK

along the directions *OC* and *OD*. Similarly as the prongs move in the effect is reversed. Thus two sets of waves are set up which are half a wavelength out of phase with each other and, where they meet along the lines *OE*, *OH*, *OF*, *OG*, a rarefaction due to one will coincide with a compression due to the other and partially annul it, so causing a reduction in the intensity of sound. If a vibrating fork is rotated slowly, the intensity of sound will be heard to vary from a maximum along the axes AB and CD to a minimum along the axes EF and GH.

Beats. Interference will also occur wherever two sets of sound waves are superimposed and the resulting effect can be determined

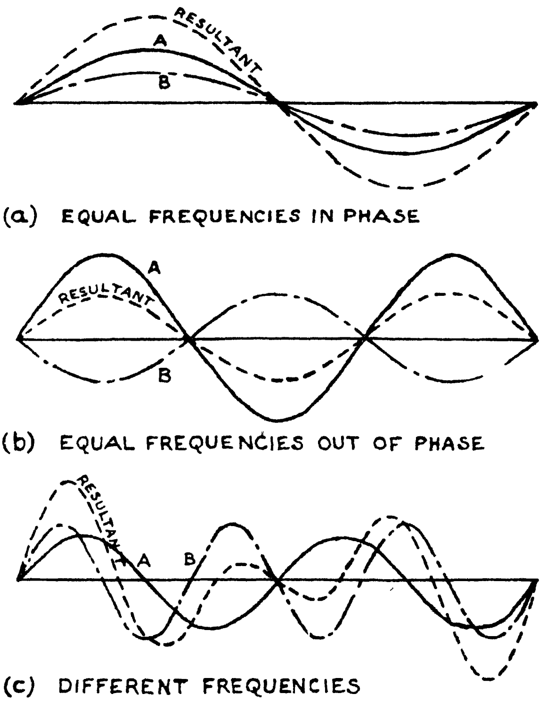


FIG. 178. SUPERIMPOSED SOUND WAVES

by representing the waves graphically, plotting amplitude against time, and finding the curve representing the new wave by adding the amplitudes (ordinates) algebraically. For sounds of equal frequency in the same phase, Fig. 178 (*a*), the resultant wave will be of the same frequency but of greater amplitude. For sounds of equal frequency in opposite phase, the amplitude will be reduced, Fig. 178 (*b*). An interesting case occurs when the sounds have different frequencies. In Fig. 178 (*c*) the waves have frequencies of 3 and 4 c/s and it will be seen that the amplitude of the resultant wave varies from a maximum to a minimum and back to a maximum in unit time causing the resultant sound to be pulsating. These alternations in intensity are called *beats*, and in the example shown the beats will occur with a frequency of one per second. For any two notes of different frequency the number of beats per second, or the *beat frequency*, is equal to the difference between the frequencies of the two notes.

Counting beats is a useful method of comparing frequencies of notes or of tuning one note to another. In the latter case the two notes will be in tune when there are no beats.

Resonance. A tuning fork held in the hand gives out a comparatively feeble sound but, if the stem is placed on the top of a table or other surface which can act as a sounding board, the sound is much louder. The vibrations of the fork are passed on to the sounding board, which is able to set a much greater amount of air in motion than the prongs of the fork alone. The vibrations in the sounding board are called *forced vibrations*, and will have the same frequency as the forcing vibrations of the fork.

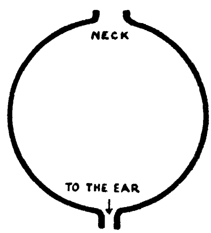


FIG. 179. HELMHOLTZ RESONATOR

All bodies have a natural frequency of vibration and if the forcing frequency is equal to the natural frequency of the body, vibrations of great amplitude will build up and *resonance* occurs. It is for this reason that marching columns must break step while crossing a suspension bridge. The beat of their marching might coincide with the natural frequency of vibration of the bridge and so build up dangerously large oscillations in the structure. An extremely interesting case, in which resonance played a part, was the failure of the Tacoma Narrows suspension bridge. Vibrations, started in the structure by high winds, built up rapidly until the whole bridge was oscillating in a most alarming manner and finally collapsed.

Enclosed volumes of air also have their own particular periods of vibration and will resonate to sound waves having the same frequency. Such resonance is utilized in organ pipes and wind instruments; the shorter the column of air set in motion the higher the frequency to which it resonates. Resonance also accounts for the variation in sound as a jug is filled with water. As the amount of air in the jug is reduced, its periodic time of vibration decreases and it resonates to vibrations of higher frequency.

Resonance was employed by Helmholtz for the analysis of sound by the use of a series of glass globes of the form shown in Fig. 179. These *Helmholtz resonators* respond very sensitively to their own particular frequency and any complex musical sound can be split into its components by finding which resonators resound to it.

Absorption. Sound falling on a surface is partly reflected, as explained above, partly absorbed and partly transmitted. Absorption is due to the transformation of sound energy into other forms of energy, such as heat, in the material upon which it falls. When considering the acoustics of halls and rooms, absorption and transmission are sometimes grouped together under the heading of total absorption, since sound energy is lost by both means. A typical example is an open window, which clearly gives 100 per cent transmission, but is often taken to have 100 per cent absorption.

The ratio of the sound energy absorbed by a material to the total energy falling upon it is called its *absorption coefficient* and, for a given material, is dependent upon the frequency and the angle of incidence of the sound. In an auditorium sound is incident in all directions at random and the coefficient under these conditions is called the *reverberation coefficient of absorption*. Draped and porous materials absorb principally at high frequencies, resilient materials and carpeting at medium frequencies, and stretched membranes and resonating panels at low frequencies. Even at a given frequency, the absorption coefficient is not a fixed constant for a material, but is dependent on the thickness and method of mounting.

The product of the area of a surface and its absorption coefficient is called the absorption of the surface and, in this country and the U.S.A., is measured in *square foot units* or *sabins*. In the case of individual items, such as pieces of furniture or persons, it is only possible to deal with the absorption of each item as a whole. For an auditorium the total absorption will be the sum of the absorptions of all surfaces and of the furniture and audience. Each person in the audience acts as a sound absorbent and, in order that the acoustic conditions should not be affected by the size of the audience, it is desirable that the seats should be upholstered in a soft material, so that, whether empty or filled, they may have similar sound absorbing properties.

Transmission. The fraction of the incident sound energy transmitted through a partition is called the *transmission coefficient* and is also affected by the frequency of the sound. The *reverberation transmission coefficient*, for sound incident in all directions at random, varies from 10^{-1} to 10^{-8} for common types of building partitions. Since the sound insulating properties of partitions are of particular interest, it is preferable to deal with *sound reduction factors*.

$$\text{Sound reduction factor} = 10 \log_{10} \frac{1}{\text{transmission coefficient}}$$

The unit of measurement is the decibel.

Reverberation. In an ordinary room or hall, sound waves will undergo repeated reflections from floor, walls and ceiling, and, although at each surface some energy will be absorbed, the sound may undergo two or three hundred reflections before becoming inaudible. This will result in a considerable prolongation of the sound, known as *reverberation*, which, if not excessive, is desirable as it assists the spread of sound, and, if the reflected rays reinforce the direct rays, will increase its loudness. It is reverberation which makes the addressing of an audience in a hall far less exhausting than speaking in the open air. With music reasonable reverberation is also desirable as it gives added richness. Excessive reverberation causes the sound to become confused, especially with speech. The sound of one syllable may be so loudly prolonged that it mixes with that of the following syllables making the whole unintelligible.

For the purposes of measurement, the reverberation time of an enclosure is that time which must elapse after the stopping of a steady note before the intensity of sound in the enclosure falls to one millionth of its initial value, i.e. by 60 db, and the unit of measurement is the second. This time depends upon the volume of the enclosure and the total absorption, including contents and audience. In the design of lecture rooms and auditoria, reverberation is of great importance. Optimum reverberation times for auditoria for different purposes are shown in Table XV. The

TABLE XV

OPTIMUM REVERBERATION TIMES FOR AUDITORIA

Volume of auditorium ft^3	Optimum reverberation time in secs at 512 c/s			
	Music and Speech	Choral Music	Music	Speech
50,000	1·20	1·68	1·38	1·00
100,000	1·45	2·00	1·67	1·23
200,000	1·80	2·50	2·10	1·55
300,000	2·00	2·80	2·30	1·70
400,000	2·10	2·94	2·40	1·80
500,000	2·20	3·10	2·50	1·87
600,000	2·30	3·20	2·65	1·95
800,000	2·40	3·36	2·76	2·05
1,000,000	2·50	3·50	2·88	2·13

reverberation period of a hall cannot be increased without structural alteration, but if it is too long it can be reduced by the use of special sound absorbing materials which can often be applied in the form of decorative panels.

EXERCISES

1. A man, standing between two parallel cliffs, fires a rifle and hears one echo after $1\frac{1}{2}$ sec, another after $2\frac{1}{2}$ sec, and another after 4 sec. Explain how these echoes reach him and calculate the distance of the two cliffs from the man taking the velocity of sound as 1120 ft/sec.
2. The timekeeper for a 100 yd race stands by the finishing tape and presses his stop watch on hearing the report of the starting pistol. What would be the difference in timing if he had started his watch on seeing the flash of the pistol? Velocity of sound = 1100 ft/sec.
3. Explain why sounds may often be heard more clearly (*a*) at night, (*b*) over open water.
4. Explain what is meant by interference. How does it occur?
5. What are beats? Two notes are heard together and beats are heard at a frequency of 6 c/s. If the frequency of one note is 512 c/s, what is the frequency of the other?

6. What is absorption? Define the reverberation coefficient of absorption and state what materials you would employ for the absorption of (*a*) high frequencies, (*b*) medium frequencies, (*c*) low frequencies.

7. Define the terms *reverberation transmission coefficient* and *sound reduction factor.*

Explain the importance of the sound reduction factor in building work.

8. What is reverberation? Is it desirable or undesirable and how is it measured?

ANSWERS TO EXERCISES

Chapter III (p. 15)

6. 0·88 lb. **7.** 7·79 lb. **8.** Mol. wt. 286; 16·1 per cent Na, 4·2 per cent C, 72·7 per cent O_2, 7·0 per cent H_2. **9.** 301 g CO_2; 162 g H_2O.

Chapter V (p. 32)

1. 559 lb. **2.** 2·30. **3.** 14·40 cm^3; 2·39 g/cm^3.

Chapter VII (p. 50)

1. (*a*) 11·7 lb at 31° to the 10 lb force; (*b*) 14·7 lb at 16·5° to the 10 lb force; (*c*) 15·5 lb at 11·2° to the 10 lb force. **2.** 111·5 lb at 68·8° to 100 lb force. **3.** 13·75 lb at 38·5° to *T*. **4.** 46·7 lb; 84 lb at 33·5° to vertical. **5** (*a*) 207·8 lb; (*b*) 120 lb. **6.** 1·41 lb at 45° to all forces, acting towards *B* through a point $\frac{10}{11}$ ft from 6 lb force and $\frac{8}{7}$ ft from 4 lb force. **7.** 230 lb at 33·5° to vertical, 9·52 ft from base. **8.** 2·8 cwt; 12·6 cwt.

Chapter VIII (p. 61)

1. 11 cwt at $16\frac{2}{11}$ ft from *A*. **2.** 100 lb in direction of W_1 acting 6·4 ft to left of *A*. **3.** 2 t at 12 ft from 5 t force. **4.** $43\frac{1}{3}$ lb. **5.** 151·5 lb thrust at 29° 42′ to horizontal. **6.** 21 lb. **7.** 6·38 lb at 50° 12′ to horizontal. **8.** 9 in; 45 lb.

Chapter IX (p. 70)

1. 5·22 in. from *AB*, 6·16 in. from *AD*. **2.** 0·83 in. from 4 in. side, 1·33 in. from 3 in. side. **3.** 0·067 in. from centre of circle. **4.** 2·86 in. from top of Tee. **6.** 4·56 in. from bottom. **7.** 0·73 *L* from *A*, 0·60 *L* from *B*, 0·44 *L* from *C*. **8.** 14 in. from the end.

Chapter X (p. 79)

1. R_A = 19·25 t; R_B = 18·25 t. **2.** R_A = 6·5 t; R_B = 7·5 t. **3.** R_C = 29·3 cwt; R_D = 31·7 cwt. **4.** *A* = 0·17 W; *B* = 0·40 W; *C* = 0·43 W. **5.** R_A = 9800 lb; R_B = 11,700 lb. **6.** 7·89 t; 7·11 t. **7.** R_A = 3050 lb at 41° to vertical; R_B = 1159 lb. **8.** 21·6 lb; 64·2 lb; 64·2 lb.

Chapter XI (p. 89)

1. 1·49 in.; 0·0886 in. **2.** 0·107 in.; 0·00051 in.; 0·000064 in. **3.** 3·96 $t/in.^2$; 15,200 $t/in.^2$ **4.** 1·23 t. **5.** 52·9 t; 10,800 $lb/in.^2$ **6.** Bending 10,000 $lb/in.^2$; Shear 20,000 $lb/in.^2$ **7.** 4·7 ft^2. **8.** 13,140 $t/in.^2$

Chapter XII (p. 102)

1. *AB* = 2·5 t, *BC* = 2·0 t, *CD* = 0, *DE* = − 1·0 t, *EF* = − 2·05 t, *FG* = − 2·61 t, *GH* = − 2·7 t, *CE* = + 2·1 t, *CF* = − 0·6 t, *BF* = + 0·55 t, *BG* = − 0·25 t, *AG* = 0. **2.** Topchord; − 2650 lb, − 2100 lb, − 2100 lb, − 2650 lb: bottom chord; + 1970 lb, + 1780 lb, + 1970 lb: diagonal; + 50 lb, + 350 lb, + 350 lb, + 50 lb. **3.** Top chord; − 690 lb, − 526 lb, − 794 lb: diagonal; + 54 lb, − 54 lb: bottom chord; 487·5 lb, 564 lb. **4.** Top chord; − 6000 lb, − 5000 lb, − 5000 lb, − 6000 lb: bottom chord; + 5200 lb, + 3470 lb, + 5200 lb: diagonal; − 1730 lb, + 1730 lb, + 1730 lb, − 1730 lb. **5.** *BC* = + 4 t, *BF* = + 2·46 t; *FG* = − 6·45 t. **6.** 6·01 t; 6·57 t. **7.** 6 rivets; $6\frac{3}{8}$ in.; $\frac{1}{4}$ in.; 78·5 per cent.

Chapter XIII (p. 114)

2. 16 cwt-ft. **4.** 198,500 lb-ft at 8·5 ft from L.H. support. **7.** Greatest B.M., 25,000 lb-ft at R.H. support. Maximum B.M., 10,400 lb-ft at 8·16 ft from L.H. support. **8.** 3460 lb-ft at 5·78 ft from L.H. end.

Chapter XIV (p. 128)

1. 450 lb. **2.** 1830 lb. **3.** 189 lb per foot run. **4.** 549·6 in. units. **5.** 4·28 in from bottom; 320 in. units. **6.** Z required = 21·9 in. units. **7.** Z required = 18·7 in. units. **8.** (*a*) 28·2 t; (*b*) 27·8 t.

Chapter XV (p. 134)

1. 3·63 min. **2.** 334 ft; 4·55 sec. **3.** 0·665 ft/sec. **4.** 84,000 ft-lb/min; 2·55 h.p. **5.** 18,600 ft-lb. **6.** 25,000 ft-lb; 37,500 lb. **7.** 160 ft. **8.** 31 t.

Chapter XVI (p. 139)

1. M.A. = 2·33; V.R. = 3·43; Eff. = 68 per cent; I.E. = 8·17 lb. **2.** M.A. = 7·67; V.R. = 9; Eff. = 74·1 per cent. **3.** 29·7 per cent. **4.** V.R. = 74; M.A. = 23·6; Eff. = 31·9 per cent. **5.** 0·26 t; M.A. = 19·2. **7.** 10 in. **8.** V.R. = 7; M.A. = 4·8; Eff. = 68·6 per cent.

Chapter XVII (p. 146)

1. 2642° F; 159·8° F; −459·4° F. **2.** 18·3° C; 36·9° C; −17·8° C. **4.** 252 g. cal. **5.** 2000 B.Th.U.; 52·8 B.Th.U./min. **6.** 0·26. **7.** 0·314. **8.** 48·8 g.

Chapter XIX (p. 157)

1. 0·29 in.; Yes. **2.** 106 yd. **3.** 4·1 t/in.^{2} **6.** 0·56 in.; 2·8 ft^2.

Chapter XX (p. 165)

4. 48,400 B.Th.U. **5.** 0·42 through walls as compared with windows.

Chapter XXI (p. 173)

3. 14·2 ft. **6.** 0·715°. **7.** 2·86°. **8.** $8\frac{2}{3}$ in.

Chapter XXII (p. 180)

2. 2·38 cm; 1·59. **3.** (*a*) $13\frac{1}{3}$ in.; $\frac{1}{3}$; (*b*) 20 in., 1; (*c*) −10 in., 2. **4.** 60 in. concave. **5.** 12 cm towards mirror. **6.** (*a*) $16\frac{2}{3}$ cm from mirror, real, inverted, $1\frac{1}{3}$ in.; (*b*) 5 in.

Chapter XXIII (p. 186)

1. $\frac{4}{3}$. **5.** 27 cm. **8.** 9.

Chapter XXIV (p. 192)

1. $18\frac{3}{4}$ cm; $5\frac{1}{3}$ D; 25 cm. **2.** 35 in.; 2·5. **3.** (*a*) $9\frac{1}{3}$ in. on same side of lens. $4\frac{2}{3}$ in. high; (*b*) $23\frac{1}{3}$ in. on opposite side, $4\frac{2}{3}$ in. high. **4.** $2\frac{6}{7}$ in. **5.** 16 in, **6.** (*a*) real image, 20 cm from lens; (*b*) real image, $6\frac{2}{3}$ cm from lens; (*c*) virtual image, 10 cm from lens. **7.** 15 cm. **8.** $4\frac{1}{6}$ cm; 5 cm from object.

Chapter XXVI (p. 200)

2. 3·87 ft. **3.** 3 to 2; 2·24 ft from weaker. **4.** 4·63 to 1. **5.** 4 to 9. **6.** 1 ft.

Chapter XXVII (p. 203)

4. 6 db. **7.** 7·6 db. **8.** 50 ft.

Chapter XXVIII (p. 213)

1. 840 ft; 1400 ft. **2.** 0·273 sec. **5.** 506 c/s or 518 c/s.

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