BITS F218: GENERAL MATHEMATICS III

BITS Pilani Hyderabad Campus

First Semester, 2022 – 2023

Comprehensive Examination (Closed Book)

Date: December 20, 2022 **Duration: 2 pm - 5 pm**

Total Marks: 40

This question paper contains 9 questions. Answer all questions.
All notations have their usual meaning as per the Text book.
Any standard results can be used without proof. Only provide the precise statement of the result.

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1. Find the value of k such that the following system is solvable by the Cramer's Rule:

$$x + y = 1, \quad x + 3y - z = 5, \quad 5x + 3y + kz = -3.$$

Also, find the unique solution if it exists. **[3]**

2. Let $V = \mathbf{R}^2$. Define

$$\begin{aligned} (x_1, y_1) + (x_2, y_2) &= (x_1 - x_2 + 2,\ y_1 + y_2) \\ a \cdot (x_1, y_1) &= (ax_1,\ ay_1) \quad \text{for all} \quad (x_1, y_1), (x_2, y_2) \in \mathbf{R}^2 \quad \text{and} \quad a \in \mathbf{R}. \end{aligned}$$

Examine whether V is a vector space over $\mathbf{R}$ with respect to the given vector addition and scalar multiplication. **[4]**

3. Let $T : \mathbf{R}^3 \to M_{22}$ defined by $T(x, y, z) = \begin{bmatrix} x + y & 0 \\ y & 2y + z \end{bmatrix}$ for all $(x, y, z) \in \mathbf{R}^3$. Examine whether T is a linear transformation on $\mathbf{R}^3$? **[3]**

4. Solve the following LPP by the simplex method

$$\text{Maximize } z = 2x_1 - 3x_2$$

$$\begin{aligned} \text{subject to} \quad -x_1 + x_2 &\leq 2 \\ 2x_1 - x_2 &\leq 2 \\ x_1 + x_2 &\geq -2 \\ x_1 &\geq 0, \ x_2 \text{ unrestricted in sign.} \end{aligned}$$

In the optimal simplex table, the z- row coefficient corresponding to one nonbasic variable is zero. Does it imply that the problem has an alternate optimal solution? If so, find the solution, or else explain why not.

[3+1]

5. Solve the following LPP by the Dual-Simplex Method

$$\text{Minimize } z = 3x_1 + 2x_2 + 4x_3$$

$$\begin{aligned} \text{subject to} \quad 2x_1 + x_2 + 3x_3 &= 60 \\ 3x_1 + 3x_2 + 5x_3 &\geq 120 \\ x_1, x_2, x_3 &\geq 0. \end{aligned}$$

[4]

6. Consider the following LPP

$$\text{Maximize } z = 7x_1 + 11x_2 - 9x_3$$

$$\begin{aligned} \text{subject to} \quad 3x_1 + 8x_2 - 5x_3 &\geq -14 \\ 5x_1 - 2x_2 + 6x_3 &= 7 \\ 2x_1 + 4x_2 &\leq 19 \\ x_1, x_2 &\geq 0, \quad x_3 \text{ unrestricted in sign.} \end{aligned}$$

(a) Construct the dual problem.

(b) If any primal problem has infeasible solution, what can you say about the solution of the corresponding dual problem. Demonstrate your conclusion with examples by constructing pair of primal and dual problems, each with two decision variables and two functional constraints. **[2 + 4]**

7. Consider the following LPP

$$\text{Maximize } z = 3x_1 + 2x_2 + 5x_3$$

$$\begin{aligned} \text{subject to} \quad x_1 + 2x_2 + x_3 &\leq 43 \\ 3x_1 + 2x_3 &\leq 46 \\ x_1 + 4x_2 &\leq 42 \\ x_1, x_2, x_3 &\geq 0. \end{aligned}$$

and its associated optimal tableau is (with s_1, s_2, s_3 are the slack variables corresponding to the constraints 1, 2 and 3, respectively):

Basic	x_1	x_2	x_3	s_1	s_2	s_3	Solution
z	4	0	0	1	2	0	135
x_2	$-\frac{1}{4}$	1	0	$\frac{1}{2}$	$-\frac{1}{4}$	0	10
x_3	$\frac{3}{2}$	0	1	0	$\frac{1}{2}$	0	23
s_3	2	0	0	-2	1	1	2

Using the post-optimal analysis discuss the effect on the feasibility/optimality of the solution of the above LPP for each of the following changes. Further determine the optimal solution (if it exists) for each of the cases when the following modifications are proposed in the above LPP:

(a) Change the R.H.S vector $b = (43 \;\; 46 \;\; 42)^T$ to $b' = (40 \;\; 44 \;\; 52)^T$.

(b) Consider the new objective function as Maximize $z = 9x_1 + 3x_2 + 4x_3$ **[2 +6]**

8. Five persons A, B, C, D, E are assigned to work on four different machines I, II, III, IV, V. The following table shows how long it takes for a specific person to finish a job at a specific machine. Here, the rows in the following table signifies the persons A, B, C, D and E and column signifies the machines I, II, III, IV and V

5	5	7	4	8
6	5	8	3	7
6	8	9	5	10
7	6	6	3	6
6	7	10	6	11

Find the optimal allocation so that the job is completed in minimum time using the Hungarian method.

[4]

9. Use the Vogel's approximation method to find an initial basic feasible solution for the following transportation problem. **[4]**

					Supply
	5	2	4	3	10
	6	4	9	5	40
	2	3	8	1	35
Demand	15	20	30	50	

END