BITS F218: GENERAL MATHEMATICS III

BITS Pilani Hyderabad Campus
First Semester, 2022- 2023
Mid-semester Examination (Closed Book)

Date: November 1, 2022 | **Duration:** **11.00 am - 12.30 pm**
Total Marks: **30**

This question paper contains 6 questions. Answer all questions.
All notations have their usual meaning as per the Text book.
Any standard results can be used without proof. Only provide the precise statement of the result.

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1. State **TRUE** or **FALSE** with proper justification for each of the following statements: **[3× 2]**

   (a) The sum of the diagonal entries of a skew-symmetric matrix is zero.

   (b) There exists a linear system with exactly 199 distinct solutions.

   (c) Let A be an 3×3 matrix. If B is the matrix obtained from A by replacing the 1^{st} column a_1 of A by $3a_1$ then the systems $Ax = 0$ and $Bx = 0$ are equivalent.

2. Use the Gauss Jordan method to find the solution of the following linear system:

$$2x + y = 2, \quad x + y + z = 4, \quad x + z = 8.$$

   Hence show that Span $\{(2, 1, 0), (1, 1, 1), (1, 0, 1)\} = \mathbf{R}^3$. **[3 + 2]**

3. Consider the matrix $A = \begin{bmatrix} 1 & -2 & 2 \\ -5 & 10 & -9 \\ -3 & 6 & c \end{bmatrix}$. Can you choose the parameter c so that the Rank$(A) = 3$? **[3]**

4. Let P be any $n \times n$ invertible matrix. If u, v, w are linearly independent vectors in $\mathbf{R}^n$, then show that $Pu, P(u - v), P(v + w)$ are linearly independent. **[3]**

5. Consider

$$W = \left\{(x_1, x_2, x_3, x_4) \in \mathbf{R}^4 \ : \ x_4 = 2x_2 - x_1 + x_3\right\}.$$

   Show that W is a subspace of $\mathbf{R}^4$. Find a basis for W. Also, determine the dimension of W. **[3+ 3 + 1]**

6. Find the linear transformation $T : \mathbf{R}^3 \to \mathbf{R}^2$ such that $T(1, 1, 0) = (2, 5)$, $T(1, -1, 0) = (1, -2)$ and $T(1, 0, 1) = (3, 7)$. Also, find the matrix of the linear transformation T with respect to the ordered basis $\mathcal{B} = \{(1, 1, 0), (1, -1, 0), (1, 0, 1)\}$ and $\mathcal{C} = \{(1, 0), (0, 1)\}$, respectively. **[3 + 3]**

END