ELECTRIC POWER TRANSMISSION

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PREFACE TO THIRD EDITION

The first edition of this book was published in 1913 under the title "Overhead Electric Power Transmission." Its suitability as a college text had not been seriously considered; but, owing to its adoption in a large number of technical schools and colleges, certain changes and additions were made in the second edition with a view to enhancing its value as a college text without detracting from its usefulness in the field of practical engineering. The additions included an entire chapter treating of underground power conductors which led to a change in the title of the book.

In this third edition, the contents have not been appreciably altered, but the old matter has been rearranged and revised and, when additions have appeared desirable, it has generally been found possible to omit something of less importance in order that the size of the book might remain practically unchanged. The most important omissions are the sample specifications for pole and tower lines which appeared as appendices in the second edition. These have been eliminated in order to keep the size of the book within reasonable limits, and also to be consistent in treating only of the fundamental principles and scientific laws which determine the correct design of power lines, leaving the consideration of practical details of construction, maintenance, operation, and similar subjects to the many excellent books now available which deal with these aspects of electrical power transmission.

Systems of distribution, whether in town or country, are not touched upon—the subjects dealt with cover only straight long-distance overhead transmission. It is true that, when treating of lightning protection, it is the machinery in the station buildings rather than the line itself that the various devices referred to are intended to protect; and, when considering the most economical system of transmission under given circumstances, a thorough knowledge of the requirements and possibilities in the arrangement of generating and transforming stations is assumed; but these engineering aspects of a complete scheme of power development are not included in the scope of this book.

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LIST OF SYMBOLS

A = area of cross-section (usually square inches).
A = cross-section of dielectric circuit (square centimeters).
a = percentage to cover interest, depreciation, etc.
a = temperature-elongation coefficient.
a = temperature coefficient of resistance at 0°C.
a = reactive component of voltage drop.
$a = \frac{d}{2r}$ in capacity formulas.
$a' = \frac{12d}{2r}$ on chart giving reactance of parallel wires (p. 243).
(a) = permittance (capacity) of three-core cable as measured between one core and the remaining two cores connected to lead sheath.

B = capacity susceptance = ωC (mho).
B = capacity susceptance (line to neutral) per mile of conductor.
B = barometric pressure; inches of mercury.
B. & S. = Brown and Sharpe wire gage.
b = "in-phase" component of voltage drop.
b = barometric pressure (centimeters or inches of mercury).
(b) = permittance (capacity) of three-core cable as measured between any two cores.

C = electrostatic capacity, or permittance (farad).
C_{mf} = electrostatic capacity, or permittance (microfarad).
C_c = cost per pound of conductor material.
C_e = cost per kilowatt-year of energy lost in transmission.
C_n, C_e = capacities of the imaginary condensers indicated in Fig. 135.
C_{ne} = effective equivalent capacity, core to neutral, of insulated cable.

D = depreciation, per cent.
D = butt diameter of wood pole.
D = dielectric flux density (coulombs per square centimeter).
d = distance between centers of parallel conductors.
d = diameter of wood pole.
d = diameter of overhead conductor (inch), used in wind-pressure formulas.
d_g = diameter of wood pole at ground level.
d_w = diameter of wood pole at weakest point.

E = modulus of elasticity.
E = electromotive force (e.m.f.); difference of potential (volts); voltage between wires at receiving end of line.

E_a = voltage, line to ground, on a.c. system (in comparison with d.c. system).

$\left.\begin{matrix}E_a\\E_b\end{matrix}\right\}$ = the "imaginary" and "real" components, respectively, of the open-circuit sending-end voltage (line to neutral).

E_n = voltage between line conductor and neutral at receiving end of line or section of line.

E_{na} = $\frac{1}{2}(E_n + V_n)$ approximately.

E_o = disruptive critical voltage (corona).

E_s = spark-over pressure, wire to neutral (kilovolts).

E_v = visual critical voltage (corona).

e = e.m.f., or potential difference (usually volts).

e.m.f. = electromotive force.

F = wind pressure in pounds per square foot.

F = a factor in "skin effect" formula.

f = frequency (number of cycles per second).

G = potential gradient (volts per centimeter).

G = shunted leakage conductance, line to neutral (mho).

H = height of pole or tower.

h = depth of tower footing below ground level (feet).

h = difference of elevation between two points of support of overhead wire (feet).

I = current (ampere).

I = moment of inertia of section.

I_a = current in three-phase line (in comparison with d.c. transmission).

I_a = "wattless" or "reactive" or "imaginary" component of line current.

I_b = "in-phase" or "active" or "real" component of line current.

I_c = charging current of condenser (ampere).

I_r = current at receiving end of line (ampere).

I_s = current at sending end of line or section of line (ampere).

I_s = current in arc to ground (see Fig. 118).

I_x = current through Petersen coil (Fig. 118).

j = $\sqrt{-1}$, a symbol indicating that the vector quantity which it precedes is measured in a direction parallel to the "imaginary" or Y axis.

K = specific electrostatic capacity of air = 8.84×10^{-14} (farads per centimeter cube).

K = a constant (temperature rise of cables).

K_A, K_B, K_C, K_e, = constants defined in text.

k = resistance of conductor material per circular mil-foot (ohm).

k = 1.5 times the weight in pounds of a cubic inch of overhead conductor.

k = a numerical constant (steel-tower design).

k = dielectric constant or relative permittivity.

k = "skin effect" multiplier.
k = heat conductivity of insulating material (watts per centimeter cube per degree Centigrade).
kv. = kilovolts.
kv.-a. = kilovolt-amperes.
kw. = kilowatts.

L = distance of transmission (mile).
$\mathbf{L}$ = inductance, or flux linkage per unit current (henry).
l = a length.
l = length of wire or cable (centimeter).
l = length of span—horizontal distance between points of support (foot).
l = length of unsupported portion of column in compression (inch).
l = length of path of dielectric flux (centimeter).
l' = straight-line distance between points of support of overhead wire (foot).
l_A and l_B (refer to Fig. 24).

m = a multiplier.
$m = \frac{P_h}{w}$ (used in catenary formulas).
(m) = area expressed in circular mils.
m and n (coordinates of center of circle in Fig. 111).
m_o and m_v (correction factors in corona formulas of Art. 103).

n = a number.
n = the number of units in a string of suspension-type insulators.
n = number of phases or conductors of a polyphase system.
n = estimated "life" in years of plant or equipment (depreciation formula).
n = "loading factor" = ratio

$$\frac{\text{resultant loading per foot of wire}}{\text{loading per foot due to weight of wire alone}}.$$

P = pull, tension, force—tension in suspended wire (pound).
P = "in-phase" power, or "active" power (formula (116)).
P_B = tension in overhead wire at point B; usually the maximum tension in the span (pound).
P_h = horizontal component of tension in overhead conductor (pound)
p = wind pressure per foot length of wire (pound).

Q = "reactive" power, in formula (116).

R = resistance (ohm); resistance of one conductor of a transmission line; resistance per mile of single conductor.
R = a radius.
R = radius at ground level of conical section of earth (foot).

R_a = resistance per mile of single conductor of three-phase system (in comparison with d.c. transmission).
R_i = insulation resistance of 1 mile of cable (megohm).
R_o = resistance at temperature of 0°.
R_p = joint resistance of all conductors of a transmission line connected in parallel.
R_t = resistance at temperature of t°.
R_1 = the "real" component of the fictitious impedance Z_1.
r = a radius.
r = radius of cross-section of cylindrical conductor.
r = radius of circle in diagram of Fig. 111.
r = least radius of gyration (inch).
r = equivalent radius of area of tower footing (foot).
r = rate of interest, per cent.

S = stress (pounds per square inch).
s = vertical sag of overhead conductor (foot).
s' = maximum deflection of overhead conductor from straight line joining points of support which are at different elevations (foot).

t = temperature (usually degrees Fahrenheit, but consult text).
t = temperature rise, or increase of temperature.
t = radial thickness of ice coating on wires (inch).
t = constant defining natural taper of wood pole (Art. 54).

V = wind velocity (miles per hour).
V = volume of frustum of cone (cubic feet).
V = voltage between conductors at sending end of line.
V_n = voltage between conductor and neutral at sending end of line.

W = power (watt).
W = load, in watts, at receiving end of line (in voltage drop formulas).
W = weight; force of gravity (pound).
w = line losses (watt).
w = weight per foot length of overhead wire (pound).
w_m = weight of conductor per circular mil per mile.

X = reactance (ohm); reactance of one conductor of a transmission line; reactance per mile of conductor.
X_1 = the "imaginary" component of the fictitious impedance Z_1.

Y = shunted admittance, line to neutral, per mile of conductor (mho).

Z = section modulus.
Z = impedance (ohm); impedance per conductor; impedance per mile of conductor.
$Z_1 = K_b = R_1 + jX_1$ = the "fictitious" impedance, per conductor, of a long transmission line.

δ = deflection of top of pole or tower (inch).
δ = air-density factor in corona formulas.
ϵ = base of natural system of logarithms = 2.7183.
η = efficiency.
θ = an angle; cos θ = power factor of load at receiving end of line.
θ = angle between straight line joining points of support of overhead wire and horizontal line in same vertical plane.
θ = angle of natural slope of earth.
θ = angle of deviation from straight line at corner pole (Fig. 44).
θ_s = power-factor angle at sending end of line or section of line.
θ_s = sum of several power-factor angles (formula (115)).
λ = length of wire in span.
μ = magnetic permeability.
π = 3.1416 (approximately).
ρ = resistivity, or specific resistance (megohms per centimeter cube).
Φ = magnetic flux (maxwell).
φ = an angle; cos φ = power factor of insulated cables on open circuit; also, in Chap. II, cos φ = power factor of load at sending end of line.
Ψ = dielectric flux (coulomb).
ω = $2\pi f$.

ELECTRIC POWER TRANSMISSION

CHAPTER I

TRANSMISSION LINE PROBLEMS

1. Advantages of Electrical Transmission of Energy.—When a metallic conductor of resistance R ohms carries an electric current of I amp., the rate at which energy is dissipated in the form of heat is I^2R watts. For a given distance of transmission, this loss may be reduced by making R small, *i.e.*, by using a conductor of large cross-section; but this will obviously increase the weight and cost of the conductor, and a limit is soon reached where the increased cost of the line is of greater importance from the economic view point than the cost of the energy losses. If, on the other hand, the current I is reduced, the total amount of energy transmitted is also reduced *unless the transmission voltage is raised* to offset the reduction of current.

Consider an electric generator developing W watts at a pressure of E volts and transmitting energy a distance of L miles through a transmission line of resistance R ohms per mile. Let I be the amperes of current in the line; then the line losses are $I^2(RL)$ watts, and the power delivered at the distant or receiving end is $W - I^2(RL)$. Suppose, now, that the generator is replaced by another machine giving the same output of W watts, but at a pressure ten times as great; the current will then be $\frac{1}{10}I$ and the line losses $\frac{1}{100}I^2(RL)$. It, therefore, follows that the increased voltage of transmission makes it possible to transmit the same amount of energy with the same line losses one hundred times as far if the same size of conductor is used, or ten times as far if a conductor of one-tenth the original cross-section is used, the economical relation between current and size of conductor being a question to be discussed in a later chapter. The facts that it is possible to insulate transmission lines for very high voltages, and also that the a.c. transformer is an ideal device for raising and lowering the voltage at the sending and receiving ends of a transmission line, account for the use of electricity in preference to all other agencies for the long-distance transmission of energy.

2. Reasons for Long-distance Transmission of Energy.—The sight of overhead electric power lines has become so common of late years that we are inclined to overlook the reasons for their presence. In the early days of electricity, small power stations were placed as close as possible to the centers of population and used mainly in connection with electric lighting. The a.c. transformer made it possible to utilize water power sites and transmit the energy of waterfalls to the centers of population; but the bigger developments in electric transmission originated with the introduction of the polyphase induction motor which greatly increased the demand for electric energy, not only for lighting purposes, but also for transformation into mechanical energy for manufacturing and other purposes.

With the increase of voltages and the correspondingly reduced losses in the transmission line, the distance of transmission increased; new water power sites became available and, moreover, it soon became evident that the small coal-burning power station could not compete economically with the larger and more efficient power station erected where coal and water for condensing purposes were most easily obtained, and this notwithstanding the fact that the larger generating station has to reckon with the additional power losses in transmission and the cost of the necessary transmission lines. The large power station is available to supply the rural load due to the growing demand by outlying farms for light and mechanical power, and also to transmit large amounts of power over considerable distances for the electrification of railroads.

Considerable advantages are to be gained by the interconnection, through high-tension lines, of hydroelectric and steam-operated plants, and even of distant generating stations whether hydroelectric or coal burning. These advantages are due not only to the possibility of dividing the total load for the most economical operation of the machinery in the several stations, but also to the increased facilities for maintaining continuity of service even in the event of a portion of the transmission system being temporarily interrupted.

From the successful operation since 1923 of the first 220,000-volt transmission line on the system of the Southern California

Edison Company, it is but a step to the so-called superpower network which, through interconnections between large power stations, may be expected, at no very distant date, to reach from the Pacific to the Atlantic coast.

3. Outline of Subject.—Energy can be transmitted electrically by conductors placed either above or below ground. The cost of a system of underground insulated cables is always higher than that of an equivalent overhead transmission; but there are conditions, especially in Europe, under which overhead wires are not desirable or permissible, and the whole or a portion of the transmission line must then be placed underground.

An overhead electric power transmission line, consisting as it does of wires stretched between insulators on poles or structures the main purpose of which is to maintain the conductors at a proper distance above the ground level, may appear at first sight to be a very simple piece of engineering work. It is indeed true that the erection of an overhead line of moderate length, capable of giving good service on a comparatively low-pressure system, does not present any insurmountable difficulties to a man of ordinary engineering ability; but whether or not such a line will be the best possible line for the particular duty required of it depends very much upon the knowledge, skill, and experience of the designer. By the best line should be understood a line which is not only substantially and lastingly constructed, but in connection with which economic considerations have not been overlooked.

It is an easy matter to design a bridge of ample strength for the load it has to carry, or a transmission line with conductors of so large a size, insulators with so large a factor of safety, and supports so closely spaced and so strong, that the electrical losses will be small and the risk of mechanical failure almost nil; but neither the bridge nor the transmission line will reflect credit on the designing engineer unless he has had before him constantly the commercial aspect of the work entrusted to him, and has so chosen or designed the various parts and combined these in the completed whole, that all economic requirements are as nearly as possible fulfilled.

Efficiency of service, which includes reasonably good voltage regulation and freedom from interruptions, must necessarily be merged into the all-important question of cost. By duplicating an overhead transmission line and providing two separate pole lines, preferably on different and widely separated rights of way, insurance is provided against interruption of service over an extended period of time; but whether or not such duplicate lines shall be erected must be decided on purely economic grounds.

A knowledge of the country through which an overhead transmission line is to be carried is essential to the proper design of the line and its supporting structures. Without a knowledge of the natural obstacles to be reckoned with, including the direction and probable force of wind storms, and whether or not these may occur at times when the wires are coated with ice, the nature of the supports and the economical length of span cannot properly be determined. On the Pacific coast, where there is rarely, if ever, an appreciable deposit of sleet on overhead conductors, it is possible that the spacing of supports may generally be greater than in countries where the climatic conditions are less favorable. At the same time, it has been observed, in districts where the winters are severe and sleet formation on conductors is of frequent occurrence, that the effects of storms in winter on wires heavily weighted with ice, and offering a largely increased surface to the wind, are less severe than in summer when much higher wind velocities are sometimes attained. These examples are here mentioned to emphasize the necessity for a thorough investigation of local conditions before starting upon the detailed design of a proposed transmission line.

Assuming that it is proposed to transmit energy electrically from a point where the power can be cheaply generated to an industrial or populous center where there is a demand for it, a straight line drawn on the map between these two points will indicate the route which, with possibly slight deviations to avoid great differences in ground level, would require the smallest amount of conductor material and the fewest poles or supporting structures. There may be natural obstacles to the construction of so straight a line, as for instance lakes that cannot be spanned, or mountains that cannot be climbed; but even the shortest route which natural conditions would render possible is by no means necessarily the best one to adopt. The right of way for the whole or part of the proposed line may have to be purchased, and the cost will often depend upon the route selected. By making a detour which will add to the length of the line, it may be possible to avoid crossing privately owned lands where a high annual payment may be demanded for the right to erect and maintain poles or towers. Again, by paralleling railroads or highways, the advantage of ease of access for construction and maintenance may outweigh the disadvantage of increased length. A slightly circuitous route may take the transmission line near to towns or districts where a demand for power may be expected in the near future; and such possibilities should be taken into account. The engineer in charge of the preliminary survey work (a section of transmission line engineering which is not dealt with in this book) should bear all such points in mind and compare the possibilities of alternative routes. On a long and necessarily costly transmission line, it is rarely possible to spend too much time and thought on the preliminary work. Money so spent is usually well spent, and will result in ultimate economies.

Apart from capital investment and power efficiency, a factor of the greatest importance, almost without exception, is efficiency and continuity of service. Electrical troubles may be due to faulty insulation, or they may have their origin in lightning or in switching operations causing high-frequency oscillations and abnormally high voltages, leading to fracture of insulators or breakdown of machinery. Troubles are more likely to be due to mechanical defects or mechanical injuries sometimes difficult to foresee and guard against. Trees may fall across the line, landslides may occur and overturn supports, or severe floods may wash away pole foundations; and against such possibilities the engineer must, by the exercise of judgment and foresight, endeavor to protect his work. Other causes of mechanical failure are storms of exceptional violence, either with or without a heavy coating of ice on the conductors. When strong winds blow across ice-coated wires, the danger is not only that the wires themselves may break, but also that the resulting horizontal loading of the poles or towers may be great enough to break or overturn them.

The preceding remarks all tend to the conclusion that the electrical calculations of a transmission line must be supplemented by calculations to determine the strength of the line considered as a mechanical structure and, further, that the economic aspect of the problem must on no account be overlooked, but on the contrary be kept continually in mind, which is merely stating, in connection with a particular kind of work, what is true of almost every engineering undertaking.

These three divisions of the subject, economic, mechanical, and electrical, are somewhat arbitrary, since each must actually take account of the other two; but, so far as possible, they will be kept separate in the succeeding chapters. What the designing engineer should bear in mind is that the electric transmission line is actually a mechanical structure (whether above or below the ground) designed to transmit energy electrically from one place to another, that it must be designed for a certain maximum power with (usually) a specified limit to the permissible voltage variation at the receiving end, and that it must perform these functions economically with the least possible risk of interruption to continuous service.

4. Systems of Transmission. Line Efficiency.—Although the three-phase a.c. system is almost universally used for long-distance transmission, there are some instances of transmission by single-phase currents and also (in Europe) by continuous currents. For transmitting power over very long distances the voltage will be high and the conductors must be carried overhead on tall towers. The cost of an underground system would greatly exceed that of the overhead transmission, even if the cables could be insulated for such high voltages as would be necessary to keep the current and line losses within reasonable limits. There is, however, a use for underground and submarine cables in connection with power transmission for comparatively short distances if the voltage is not too high; this subject will be taken up in detail in Chap. XIII.

The system adopted will affect the design of the generating plant and of the motors or other devices through which the electric energy at the receiving end of the line is converted for industrial purposes or public utility; but, in this chapter, references to alternative systems will be made only for the purpose of comparing them in the matter of line efficiency.

The principal cause of loss of power in a transmission line is the resistance of the conductors. For a given section of conductor, the power dissipated in the form of heat in overcoming the ohmic resistance is proportional to the square of the current. A definite amount of power can, therefore, be transmitted with less loss when the voltage is high than when it is low; but, on each particular transmission, there is a limit to the pressure beyond which there is nothing to be gained in the matter of economy. This limit is determined by the cost of generating and transforming apparatus (which will be greater for the higher voltages), by the greater cost of insulators and of the line supports—owing to the larger spacing required between wires—and also, when extra-high pressures are reached, by the fact that the power dissipated is no longer confined to the I^2R losses in the conductors but occurs also in the form of leakage current over insulators and in the air surrounding the conductors. For the present it is assumed that a total amount of power amounting to W watts has to be transmitted over conductors of known resistance; and losses through leakage or corona will be considered negligible.

The efficiency of transmission is defined as

$$\frac{\text{Output at receiving end}}{\text{Input at sending end}}$$

and if w = total losses in the line, expressed in watts, the line efficiency is

$$\eta = \frac{W}{W + w}$$

which (for accurate slide-rule calculations) may, with advantage, be written

$$\eta = 1 - \frac{w}{W + w} \tag{1}$$

where W stands for the power actually delivered at the receiving end, expressed in watts.

5. Transmission by Continuous Currents.—Let E = voltage between outgoing and return wires at the receiving end of the line; then the power delivered is $W = EI$, and if R = resistance in ohms of one of the two conductors, the voltage at the sending end of the line must be $E + I(2R)$, and the loss of power is $w = I^2(2R)$ watts. The line efficiency, by formula (1), is

$$\eta = 1 - \frac{I^2(2R)}{EI + I^2(2R)}$$

or

$$\eta = 1 - \frac{2IR}{E + 2IR}$$

Example 1. *D.C. Transmission.*—The demand at the end of a d.c. transmission line 6,500 ft. long is 54 kw. at 600 volts. The line losses must be within 10 per cent of the delivered power. Calculate the size of wire required, the voltage at generating end, and the efficiency of transmission at full load.

The current in the line is $\frac{5,4000}{600} = 90$ amp. The permissible line losses are $54 \times 0.1 = 5.4$ kw. The line resistance must not exceed $\frac{5{,}400}{(90)^2} = 0.667$ ohm, or 0.3335 ohm per single conductor of length 6,500 ft. The permissible resistance per mile is, therefore, $0.3335 \times \frac{5{,}280}{6{,}500} = 0.271$ ohm. From the wire table on page 81, the nearest standard size of conductor is found to be No. 4/0 copper, with a resistance of 0.272 ohm per mile. The line drop is $90\left(0.272 \times 2 \times \frac{6{,}500}{5{,}280}\right) = 60.2$ volts, whence the pressure at sending end = 660.2 volts. The efficiency of transmission is

$$\eta = 1 - \frac{60.2}{660.2} = 0.909$$

The fact that continuous currents are not extensively used for the transmission of power to a distance is due mainly to the difficulty of providing sufficiently high pressures to render such transmission economical, and also to the necessity for using rotary machines with commutators to convert the transmitted energy into convenient form at the distant end of the line. The modern aspect of long-distance transmission by means of continuous currents will, however, be dealt with at some length in Chap. XII.

6. Transmission by Single-phase Alternating Currents.—The advantage that alternating currents have over direct currents is in the ease with which pressure transformations can be effected by means of static converters. On a constant-potential system, the distribution of power in scattered districts, at any voltage desired by the consumer, is a very simple matter.

In a single-phase two-wire transmission, the conditions would be similar to those of a d.c. transmission if not only the load, but the line also, could be considered as being without inductance or electrostatic capacity. The current and the line losses would then be the same as if the transmission were by continuous instead of alternating currents.

In practice the inductance must always be reckoned with where alternating currents are used; this inductance is not only that introduced by the load (usually consisting in large part of induction motors), but is partly in the line itself, owing to the loop formed by the outgoing and return conductors. The charging current due to the capacity of the line is of less account on low-voltage transmissions, but becomes of considerable importance on long lines working at high pressures. The effects of inductance and capacity will be explained later.

Another difference between a.c. and d.c. transmission is that an alternating current has the effect of apparently increasing the resistance of the conductor; this is due to the uneven distribution of the current over the cross-section of the conductor. This phenomenon is known as the *skin effect:* it is negligible in conductors of small diameters and at low frequencies. In the case of large conductors, especially at the higher frequencies, it should be taken into account. The manner in which this is done is explained in Chap. IX.

7. Transmission by Two-phase Currents.—If four separate wires are run from the sending station to the receiving station, the calculations are made as for two independent single-phase circuits. The fact that the currents in these two circuits have a phase difference between them of a quarter period, or 90 deg., suggests that two of the conductors might be combined into a single return conductor of a cross-section intermediate between that of a single conductor and that of the two conductors in parallel. Assuming a balanced load, the current relations in such a three-wire two-phase system would be as shown in Fig. 1, where the current I_3 in the common conductor is the vectorial sum of the currents I_1 and I_2. It has been drawn equal but opposite to the dotted resultant OA because it is generally convenient to assume the direction of all currents to be positive when flowing away from the source of supply, in which case the (vectorial) sum of the currents in the three lines must be zero, or

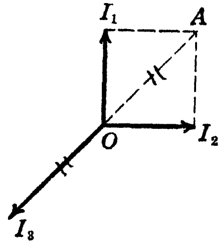


FIG. 1.

$$\dot{I}_1 + \dot{I}_2 + \dot{I}_3 = 0$$

The current I_3 in the common return conductor is seen to have a magnitude equal to $\sqrt{2}$ times either of the two (equal) currents in the other wires, and this suggests a possible saving of copper, but as a matter of fact the resistance of the common conductor has a disturbing effect upon the phase relations of the two-phase system, and the few long-distance transmissions in operation on the two-phase system have four wires and do not use a common return conductor.

8. Transmission by Three-phase Currents.—If six separate conductors are run from generating to receiving station, as indicated in Fig. 2, the transmission is equivalent to three independent single-phase two-wire circuits; and if E_n is the potential difference at the terminals of each circuit, and I the current in each wire, the total power transmitted will be

$$W = 3(E_n \times I)$$

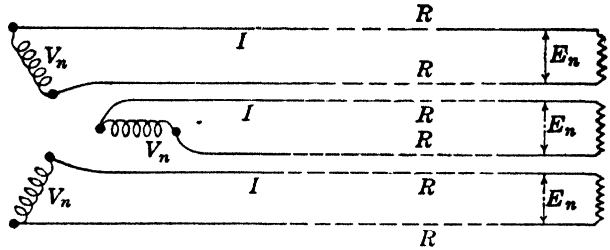


FIG. 2.—Three-phase transmission with six wires.

the assumption being that the power factor of the load is unity and that the inductance and capacity of the line are of negligible amount.

The pressure lost in transmission will be $2R \times I$, and the total power lost in the three lines will be $3 \times I^2 \times 2R$.

Consider now the arrangement as in Fig. 3, where the three circuits have a common terminal at each end of the transmission

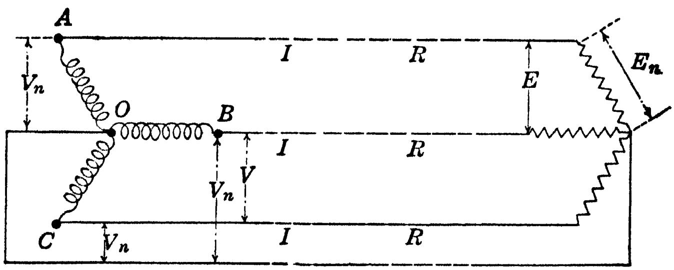


FIG. 3.—Three-phase transmission with three wires.

and three of the wires of the six-wire transmission are replaced by a common return conductor. The pressure at the receiving end, between each of the three terminals and the common return, or neutral point, is still E_n volts; and the total power transmitted is still $W = 3(E_n \times I)$; but, owing to the fact that the sum of the three outgoing currents is zero (since they differ in phase by 120 time-degrees, as shown in Fig. 4, and any one current, such as OB, is exactly equal and opposite to the resultant of the other two currents), there will be no current flowing in the common return conductor, which can, therefore, be omitted; and it follows that both pressure drop and I^2R losses in the lines are reduced to *one-half* of what they were with the arrangement of three separate circuits; the power loss in the lines being now $3I^2R$. This clearly shows how the transmission by three-phase currents is more economical as regards line losses than single-phase transmission. But it must not be overlooked that, in order to obtain a reduction by half of the weight of copper in the lines, the pressure between the wires is greater on a three-phase system than on a single-phase system transmitting the same amount of power.

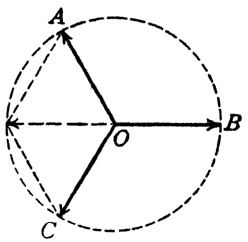


FIG. 4.—Vector diagram of currents in three-phase transmission.

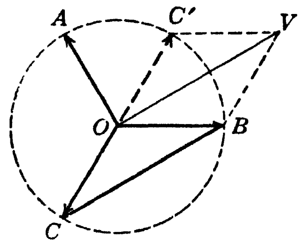


FIG. 5.—Vector diagram of e.m.fs. in three-phase star-connected system.

Thus, the pressure V (Fig. 3) between any two of the three transmission wires is the *difference* between two of the star voltages, as indicated in Fig. 5. Here the e.m.fs. in the three sections of the alternator windings are represented by the vectors OA, OB, and OC; and since the e.m.f. V between any two terminals, such as B and C (Fig. 3), is the resultant of the e.m.fs. acting in the two windings OB and OC connected in series, one of these (as OC) must be subtracted from the other (OB). Thus the resultant is the vector OV (Fig. 5), obtained by *adding* to the vector OB an imaginary vector OC' exactly equal but opposite to OC. This resultant is evidently equal and parallel to the line CB joining the ends of the two vectors OB and OC, and since the angle $C'OV$ is 30 deg., the length OV is equal to $2OC$ cos 30 deg., or $\sqrt{3}$ times the length of any one of the vectors representing e.m.fs. between conductor and neutral point. Thus,

$$V = 1.732V_n$$

and since the resistance drop is the same in all three conductors, a similar condition exists at the receiving end of the line, and we may also write

$$E = 1.732E_n \qquad (2)$$

The *power* of a three-phase circuit, which is three times $E_n \times I$, can evidently also be written

$$W = 3\left(\frac{E}{\sqrt{3}} \times I\right)$$

or

$$W = \sqrt{3}EI \qquad (3)$$

where E is the pressure between any two of the three wires.[1]

9. Line Losses on Any Polyphase System.—Apart from all questions of voltage, or necessary insulation and spacing between adjacent conductors and between the conductors and the supporting structures, the total I^2R losses will be the sum of the losses occurring in each conductor of the transmission system. Each wire may be considered as the outgoing conductor of a two-wire single-phase system in which the return wire has no resistance. Thus, in a balanced three-phase system as illustrated in Fig. 3, wherein the common return wire is not required (since it carries no current), the total losses in the transmission wires are

$$3(I^2 \times R)$$

But this may be written

$$(3I)^2 \times \frac{R}{3}$$

which shows that the total losses can be calculated by adding the currents in the respective conductors regardless of phase relations, and considering this total current as being transmitted over a single wire of the same weight or cross-section as would be obtained by connecting the individual conductors in parallel. This applies to any polyphase system with wires of equal resistance carrying equal amounts of current.

If I is the current per wire, n the number of wires (or phases), and E_n the "star" voltage or pressure measured between any one wire and neutral, the total power transmitted is

$$W = E_nI \times n$$

provided the power factor is unity. On the assumption of a balanced load, with the current lagging behind the voltage by

[1] The *power factor* (cos θ) does not appear in this formula because, owing to the assumed absence of inductance or capacity from both line and load, it is equal to unity.

the same number of time-degrees on each phase of all the systems, no complication will arise if the power factor of the load is taken into account. The total power transmitted in every case may, therefore, be written

$$W = E_nI \cos \theta \times n \tag{4}$$

If R is the resistance of each line conductor, the total line loss for any system will be

$$w = I^2R \times n$$

Still neglecting the inductance and capacity of the line itself, the *percentage power lost in transmission is*

$$\frac{w}{W} \times 100$$

If the loss w be expressed in terms of the total power W, it will be found that this ratio can be put in an interesting form. The symbol R_p will be used to denote the joint resistance of all the conductors in parallel; that is to say,

$$R_p = \frac{R}{n}$$

The power lost is

$$\begin{aligned} w &= nI^2R \\ &= n^2I^2R_p \end{aligned} \tag{5}$$

but for n^2 may be substituted its equivalent value

$$n^2 = \frac{W^2}{E_n{}^2I^2(\cos \theta)^2}$$

obtained from formula (4), whence formula (5) becomes

$$w = \frac{W^2R_p}{E_n{}^2(\cos \theta)^2}$$

which shows how, for any given amount of power transmitted at a given pressure, the I^2R loss is directly proportional to the joint resistance of all the conductors and inversely proportional to the square of the power factor of the load.

By substituting this value of w in the ratio for percentage I^2R loss, the latter quantity becomes

$$\left.\begin{matrix}\text{Percentage power lost in } \textit{any} \\ \text{balanced polyphase system}\end{matrix}\right\} = \frac{WR_p}{E_n{}^2 \cos^2 \theta} \times 100 \tag{6}$$

These formulas show very clearly the advantages of high power factor where economy of transmission is important.

10. Present-day Practices and Tendencies.—As already mentioned, the a.c. three-phase system with three conductors is well established and likely to be used for longer distances of transmission and even higher voltages than at present. The wires are carried on wood, steel, or concrete poles when the distances are short, the conductors being tied to pin-type insulators mounted a few feet apart on cross-arms bolted to the supporting poles. The average distance between poles for such medium-voltage lines will range from 150 to 400 ft. With the longer lines and transmission pressures above 50,000 volts, the suspension type of insulator is used, and when the pressure reaches such values as 110,000 and 220,000 volts, very tall steel towers are necessary to support the wires a considerable distance above the ground. The spacing between the towers is also very much greater than for the lower voltages, and the steel cross-arms from which the insulators are suspended must be carefully designed to withstand the stresses to which they may be subjected. The span, or distance between towers, is much greater than on the pole lines for lower voltages, the average number of towers to the mile being sometimes not more than five. Very much longer spans are used in special cases, as for river crossings. These involve problems for the mechanical engineer which may be more difficult to solve than those relating to the insulation of the high-voltage conductors.

The rare instances in which insulated cables have to be used on long-distance transmission schemes include sections where the electric energy has to be taken across water which cannot be spanned by overhead conductors. There are examples of submarine power transmission in San Francisco where 12,000-volt cables have been laid across the bay and 11,000-volt cables across the Golden Gate.

11. Frequencies and Transmission Pressures.—A frequency of 125 cycles per second was common in connection with the earlier a.c. systems in America: but this was later reduced to 60 cycles. Unfortunately a frequency of 25 cycles was also used, mainly because the earlier designs of synchronous converter operated better on the lower frequencies. We, therefore, have two frequencies which have become more or less standard, namely 60 and 25 cycles per second; but the latter is gradually giving way to 60 cycles, which is becoming the standard in America. In Europe where the same confusion existed in the past owing to the multiplicity of frequencies, the recent tendency has been toward a single frequency of 50 cycles per second.

In the matter of voltages there has also been a lack of standardization: but the more usual transmission voltages are given below. It is true that the simplicity of the a.c. transformer and the ease with which it can be adapted to transform from any one voltage to any other render the lack of standardization of voltages less objectionable than the adoption of many different frequencies.

USUAL VOLTAGES BETWEEN WIRES OF THREE-PHASE TRANSMISSION SYSTEMS

2,200 (or 2,300)
4,400 (or 4,600)
6,600 (or 6,900)
11,000
13,200 (being twice 6,600)
22,000
33,000
44,000
66,000
88,000
110,000
132,000 (being twice 66,000)
154,000
220,000

The star connection is usual on the high-tension side of the transformers connected to the line, and the neutral point is usually grounded at the sending end of the line. The chief arguments in favor of grounding the neutral are (1) that the difference of potential between any conductor and the supporting structure or earth remains unaltered, and cannot become excessive in the event of the grounding of a high-tension conductor, and (2) that it is possible to detect instantly, and disconnect by automatic devices or otherwise, any portion of the system that may become accidentally grounded. The chief objection is that under such conditions the grounding of any one conductor causes a short-circuit, and even if disconnected by the opening of a switch, leads to an interruption of supply. By inserting a resistance between the neutral and the ground connection, the current through the fault can be limited to just so large an amount as may be necessary to operate an automatic device, or give an indication that there is a fault on the line. In some recent installations a low resistance is inserted in the connection between the neutral point and ground, but there is no uniformity of practice in this respect.

12. Substations and Tie Lines.—Great progress has been made in recent years in the construction of outdoor substations on high-voltage a.c. systems. Large self-cooling transformer units are now being made for use out of doors, and switchgear, both of the oil-break and air-break type, is being designed to give satisfaction under the most severe weather conditions. It is usual to house the low-tension switchgear, but the advantages of having high-voltage busbars, disconnecting switches, lightning arresters, and choke coils in the open, where ample clearances may be provided at comparatively small cost, are fully recognized.

Many outdoor substations have been erected and are giving entire satisfaction. These stations naturally cost less than the indoor substation requiring a large building to house the transformers and high-tension switchgear. A reduction of at least 20 per cent in cost may generally be realized. Outdoor substations appear to be essentially an American development: very few are to be found in Europe; but this may be accounted for partly by the greater distances of transmission and higher voltages in America.

The design and equipment of substations on high-voltage transmission systems are beyond the scope of this book; but a few remarks concerning the high-tension lines connecting such stations together and with the main source of supply may not be out of place.

The transmission of energy between substations in a thickly populated area, or between small towns and villages served by a central power station, involves many questions which do not arise in connection with a straight long-distance transmission, the object of which is to convey energy from a place, such as a waterfall, where power is cheap, to the centers of population, where there is a demand for it. In the former case, the problems to be solved are generally similar to those of distributing systems in large cities, except that the pressures are much higher on account of the greater distances to be covered. In the latter case—that of straight long-distance transmission, with few, if any, branch connections between the generating station and the substation from which the energy is distributed at a lower pressure—the chief considerations are those involved in the choice of transmission voltage and a line construction which shall combine economy with such safety factors and protective devices as may be necessary to insure, so far as may be humanly possible, uninterrupted service.

When the system embodies a number of substations and two or more generating stations, the proper arrangement of the transmission lines becomes a matter involving very careful study on the part of the engineer. Energy may be obtained at intermediate stations by tapping a radial line, *i.e.*, a direct line connecting the point of supply with the point of delivery; but tie lines between two substations are also of great value in permitting equalization of load on two or more radial lines. The accompanying diagrams show three possible arrangements of lines supplying energy from a single generating station. Figure 6

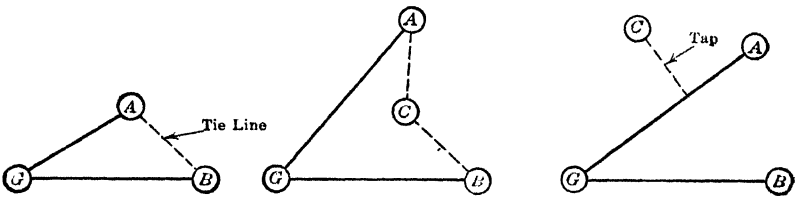


FIG. 6. FIG. 7. FIG. 8.

FIGS. 6, 7, and 8.—Typical arrangements of substations and tie lines.

shows a tie line between two substations, the purpose of which is to control the loading of the two radial lines and also to maintain supply in the event of a breakdown on one of the direct lines. Figures 7 and 8 show how a third substation *C* of relatively small capacity may be supplied with energy.

13. Synopsis of Succeeding Chapters.—When planning either a new transmission line or extensions to an existing system, the engineer has many problems to solve before he can be sure that the proposed line will fulfil, in the best and most economical manner, the purpose for which it is to be built.

The question of economy is indeed a very important one; it will determine the choice of voltage, the size of conductor to carry a particular current, and such questions as the length of span which, if short, will require many insulators and supports, and, if long, fewer insulators but taller and more costly supporting towers.

Having arrived at some general conclusions regarding the type of line, transmission voltage, and route to be followed, the mechanical strength and stability of the line should be carefully considered. These mechanical problems include the strength and durability of the conductors themselves and the supporting poles or towers; they must take into consideration the possible effects of strong winds, sleet and ice deposits, and the effects of temperature changes on the sag of the wires and the tension at the points of support.

The electrical problems include the very important question of insulation for high voltages, the calculation of transmission losses, voltage regulation, and methods of voltage control. The effects of the reactance and capacity of the lines themselves, when the transmission is by alternating currents, involve a considerable amount of calculation when great accuracy is required. In connection with the higher pressures, it is also necessary to consider possible corona formation and calculate the additional line losses due to corona. Finally, protection against interruption to service due to lightning and similar causes must also be classed among the problems with which the electrical engineer is particularly concerned.

These various matters will be taken up in succeeding chapters generally in the same order as here referred to. They will be followed by a short chapter dealing with the possibilities of long-distance transmission by continuous currents, and another devoted exclusively to the consideration of underground and submarine cables for high-voltage transmission. It would seem, however, that before taking up the subject of transmission line economics, it would be advisable to devote a few pages to the consideration and calculation of short lines operated at comparatively low voltages, in order that the reader who may not be familiar with the simpler problems involved in connection with such lines may be better prepared to solve the more difficult problems of electric transmission. Chapter II will, therefore, deal briefly with the simple calculations of short, low-voltage overhead lines.

CHAPTER II

CALCULATIONS FOR SHORT TRANSMISSION LINES

14. Effect of Line Reactance on the Transmission of Alternating Currents.—When the transmission of energy is by alternating currents in parallel wires of considerable length the difference between the conditions at the sending and receiving ends of the line will depend not only upon the resistance of the transmission circuit, but also on its reactance and electrostatic capacity. The effects of capacity, being negligible on short lines, will not be considered here; but it is evident that two or more metallic conductors, separated by a dielectric (air), may form condensers of considerable capacity when the wires run parallel for many miles, and the charging or condenser current may be of considerable magnitude when the alternating pressure between conductors is high. The effect of line reactance cannot be overlooked even on short lines, and it is proposed to explain briefly how this reactance affects the regulation or pressure drop of a.c. transmission lines.

On account of the necessary space between the wires, the loops formed between outgoing and return conductors are of considerable area on a long-distance transmission; and the changing flux of induction in these loops will generate counter e.m.fs. in the conductors, which may be of considerable importance, especially in regard to their effect on the voltage regulation. Whether dealing with single-phase or polyphase transmissions, it will be found convenient to make calculations on single conductors only. Thus, instead of considering the resistance of the *complete circuit* (which is not convenient in the case of polyphase transmissions), the resistance of one conductor only or the resistance *per mile* of single conductor is considered, and the ohmic voltage drop calculated for that portion of the complete circuit only. Similarly, in the matter of the counter e.m.f. due to the self-induction of the line, calculations are based, not on the total flux of induction in the loop or loops formed by outward-going and return wires, but on that portion of the total flux which is included between the center line of any one conductor and the *neutral* plane or line. Thus the induced volts per single conductor, or per mile of single conductor, may be calculated, and the resulting total voltage drop may be computed for each conductor independently of the others. In the case of a single-phase two-wire transmission, the total loss of pressure is evidently just twice the amount so arrived at for a single conductor. In a polyphase transmission, due attention has to be paid to the phase relations between the currents in the various conductors; but the same principle holds good, and calculations of any polyphase transmission can be made by considering each conductor separately, as will be explained later.

The induced volts will be directly proportional to the current and will depend on the diameter of the wire and its distance from the return conductors. This will be again referred to in Chap. IX, but for the present the induced pressure may be calculated by means of the formula:

$$\left.\begin{array}{l}\text{Volts induced per mile}\\ \text{of single conductor}\end{array}\right\} = 0.00466 \times f \times I \times \log_{10}\left(1.285\frac{d}{r}\right) \quad (7)$$

where d and r stand, respectively, for the distance between outward and return (parallel) conductors and the radius or half diameter of the wire, these being expressed in the same units. The frequency f is expressed in cycles per second, and the current I, in amperes. In nearly all handbooks for the use of electrical engineers, tables are published giving inductive pressure drop for different diameters and spacings of wires; the assumption being always, as in the case of formula (7), that the current variation is in accordance with the simple harmonic law (sine-wave). The special case of magnetic conductors such as iron or steel will be referred to in Chap. IX.

15. Fundamental Vector Diagram for A.C. Transmission Lines. Capacity Neglected.—In the diagram Fig. 9, the various quantities are represented as follows:

- OA, or I, is the current vector.
- OB, or E_n, is the vector corresponding to the pressure (wire to neutral) at the receiving end.
- θ is the time angle by which the current lags behind the pressure at the receiving end; cos θ being the power factor of the load.

BC, or *IR*, which is drawn parallel to *OA*, is the quantity $I \times R$; being the voltage component required at the generating end to compensate for ohmic drop of pressure in one conductor.

CD, or *IX*, which is drawn at right angles to *OA*, is the quantity calculated by formula (7), being the voltage component required at the generating end to compensate for loss of pressure due to the inductive reactance of one conductor.

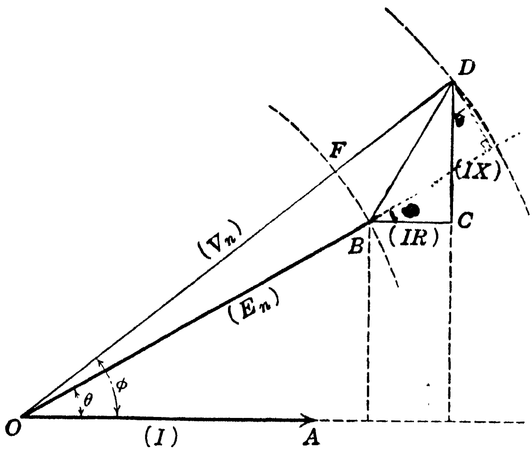


FIG. 9.—Vector diagram for line calculations—capacity neglected.

BD is the sum of the vectors *BC* and *CD*, being the total additional voltage required at the generating end to compensate for the impedance of one conductor.

OD, or V_n, is the vector corresponding to the pressure (wire to neutral) at the generator end of the line, required to maintain the pressure (E_n) at the receiving end when the current in the conductor is *I* amperes.

φ is the time angle by which the current lags behind the pressure at the generating end, cos φ being the power factor of the total load as measured at the generating end.

FD is the (arithmetical) difference between pressures at generating and receiving ends of the conductor.

The *percentage* loss of pressure is, therefore,

$$100 \times \frac{\text{length } FD}{\text{length } OF}$$

$$= 100 \times \frac{\text{length } FD}{\text{length } OB}$$

(the dotted circles being described from the center O).

All graphical solutions of pressure regulation on short transmission lines are based on this fundamental diagram. Some of them give results that are theoretically correct, while in others certain assumptions are made to simplify the construction without introducing any appreciable error in the solution of practical problems.

Graphical and semi-graphical methods of predetermining the voltage regulation of transmission lines are often convenient, but, unless constantly used for the solution of actual problems, they are not likely to prove time savers, and for the purpose of this text, mathematical solutions by means of easily derived formulas would seem to be preferable.

The required trigonometrical functions and vector relations are readily obtained from Fig. 9. In the first place, the functions of the angle φ are:

$$\sin \varphi = \frac{IX + E_n \sin \theta}{V_n} \tag{8}$$

$$\cos \varphi = \frac{IR + E_n \cos \theta}{V_n} \tag{9}$$

$$\tan \varphi = \frac{IX + E_n \sin \theta}{IR + E_n \cos \theta} \tag{10}$$

From formula (9) it is seen that the required voltage at the generating end is

$$V_n = \frac{IR + E_n \cos \theta}{\cos \varphi} \tag{11}$$

and the volts required to overcome ohmic resistance are

$$IR = V_n \cos \varphi - E_n \cos \theta \tag{12}$$

The line efficiency is

$$\eta = 1 - \frac{IR}{E_n \cos \theta + IR} \tag{13}$$

As an example of the use of these formulas, assume, in the first place, that the *material*, *size*, and *spacing* of conductors is known, also, in all cases, the power factor of the load ($\cos \theta$), and, therefore, the other trigonometrical functions of the angle θ, such as $\sin \theta$. Under these conditions, the quantities IR and IX

can readily be calculated, and formula (10) can be used to obtain $\tan \varphi$; thence the angle φ and $\cos \varphi$ (the power factor at the generating end). Then, by formula (11), the required voltage (V_n) at the generating end is easily obtained.

Assume, in the second place, that the size of the conductors has to be determined. The spacing of conductors and the frequency being known, the induced volts IX can be calculated approximately by estimating the value of r for use in formula (7). The more correct estimate of the size of conductor will be based on the *required regulation,* or total voltage drop. The voltage at the generating end is, therefore,

$$V_n = E_n + \text{allowable voltage drop per conductor}.$$

Now, since IR is not definitely known, formula (8) will have to be used. This gives the value of $\sin \varphi$ with a sufficient degree of accuracy even if quite an appreciable error has been made in estimating the size of conductor for the purpose of calculating the inductive drop IX. Having determined the angle φ, the function $\cos \varphi$ can be obtained from trigonometrical tables; and then, by using formula (12), the ohmic drop can be calculated. Thus the proper size of wire for use under given conditions may be determined.

If the *power loss* in the line is the determining quantity, regardless of the voltage regulation, then, since this loss depends only on the voltage IR (the current I being assumed constant), the resistance and size of conductor are readily ascertained, and the unknown quantities may be calculated as in the case first considered.

16. Examples of Short-line Calculations. *Example* 2.—A single-phase line 6.5 miles long consists of two No. 6 B. & S. gage copper wires strung on insulators spaced 30 in. apart. The line operates at 4,600 volts, 60 cycles, and delivers 30 kv.-a. at 80 per cent power factor, the load being concentrated at the receiving end of the line. Required the voltage and power factor at the sending end of the line, and the line efficiency.

The current in the wires is $I = \dfrac{30{,}000}{4{,}600} = 6.52$ amp.

The voltage between wire and neutral is $\dfrac{4{,}600}{2} = 2{,}300$

The diameter of a No. 6 wire (refer to table on p. 81) is 0.162 in., whence the radius of the cross-section is $r = 0.081$ in. and, by formula (7),[1] the inductive reactance per single conductor of length 6.5 miles is $X = 6.5 \times 0.00466 \times 60 \times \log\left(1.285\frac{30}{0.081}\right) = 4.86$ ohms; whence the reactive voltage drop per conductor is

$$IX = 6.52 \times 4.86 = 31.7 \text{ volts.}$$

The resistance per mile of No. 6 copper wire (see table on p. 81) is 2.14 ohms, whence the resistance voltage drop per conductor is

$$IR = 6.52 \times 2.14 \times 6.5 = 90.6 \text{ volts.}$$

From trigonometric tables[2] we find that when $\cos\theta = 0.8$, $\sin\theta = 0.6$, and we may use formula (10) to calculate the power-factor angle at the sending end. Thus

$$\tan\varphi = \frac{31.7 + (2{,}300 \times 0.6)}{90.6 + (2{,}300 \times 0.8)} = 0.731$$

whence $\cos\varphi = 0.807$, which is the power factor at the sending end of the line.

By formula (11), the voltage to neutral at the generating end is

$$V_n = \frac{90.6 + (2{,}300 \times 0.8)}{0.807} = 2{,}390 \text{ volts}$$

giving 4,780 volts between wires, which is 3.91 per cent higher than the receiving-end voltage.

By formula (13), the line efficiency is

$$\eta = 1 - \frac{90.6}{(2{,}300 \times 0.8) + 90.6} = 0.953$$

The loss of power due to wire resistance, expressed as a percentage of the *delivered* power, is $100 \times \frac{90.6}{(2{,}300 \times 0.8)} = 4.93$.

Example 3.—A single-phase load of 15 kw. at 220 volts is required at the receiving end of a transmission line 700 ft. long. The frequency is 60, the power factor of the load is 0.8, and the spacing between the transmission wires is to be 2 ft. What size of copper conductor should be used in order that the voltage regulation shall be within 7 per cent?

[1] A chart giving the reactance in ohms per mile of single conductor will be found on page 243. The handbooks for electrical engineers and other books publish tables giving reactance values for frequencies of 60 and 25 cycles. Very good tables of transmission line reactances are included in Dwight's "Transmission Line Formulas."

[2] A short table giving the connection between the reactive factor ($\sin\theta$) and the power factor ($\cos\theta$) will be found on p. 28.

The maximum permissible difference of voltage between sending and receiving ends of line is $\frac{220 \times 7}{100} = 15.4$, or 7.7 volts per conductor. Therefore, the maximum permissible value of the e.m.f. between wire and neutral at the sending end is

$$V_n = 110 + 7.7 = 117.7 \text{ volts.}$$

The current in the line is

$$I = \frac{W}{E \cos \theta} = \frac{15{,}000}{220 \times 0.8} = 85.2 \text{ amp.}$$

and if the loss of pressure were due to the resistance only, the ohms of a single conductor would be $R = \frac{7.7}{85.2} = 0.0904$, and the resistance per mile would be $\frac{0.0904 \times 5{,}280}{700} = 0.682$ ohm. Referring to the table on page 81, it is seen that No. 1 wire would be large enough if there were no additional drop due to the reactance of the transmission line. It will be noticed that the current density in so small a wire as No. 1 would be very high (1,300 amp. per square inch), and it is probable that a wire at least as large as No. 00 will be required. This size may be assumed for the purpose of estimating the IX drop, which is only slightly affected by small changes in the diameter of the conductors. With a 2-ft. spacing, No. 00 stranded cable has a reactance of 0.615 ohm per mile at 60 cycles per second, and since the length of the line is $\frac{700}{5{,}280} = 0.1325$ mile, the reactive voltage drop per conductor is

$$IX = 85.2 \times 0.615 \times 0.1325 = 6.94 \text{ volts.}$$

By formula (8),

$$\sin \varphi = \frac{6.94 + (110 \times 0.6)}{117.7} = 0.62$$

whence $\cos \varphi = 0.785$, and by formula (12), $IR = 85.2\,R = (117.7 \times 0.785) - (110 \times 0.8)$, whence $R = 0.0505$ ohm. The conductor should, therefore, have a resistance not exceeding $\frac{0.0505 \times 5{,}280}{700} = 0.381$ ohm per mile. Referring to the wire table, it will be seen that a No. 000 conductor will be necessary, since the resistance of No. 00 would produce a voltage regulation exceeding the specified limit. The determination of the exact voltage drop using this size of wire can be made in the same manner as in working out Example 2.

Problems of the same type as Example 3 are by no means uncommon in engineering work. They are not solved by merely inserting numerical values in formulas and performing certain mathematical operations upon them. Thought and judgment are required, and assumptions have frequently to be made, the correctness of which must afterwards be checked. In this particular problem (Example 3), it would have been easy to specify conditions which could not have been satisfied. If the transmission distance had been slightly longer, or the load somewhat greater, the reactance voltage drop alone might have exceeded the specified total drop. The only possible solution then would have been to split up the single circuit into two or more parallel circuits (to reduce the reactive drop); but the proper course would involve a change in the specification, either permitting a higher transmission voltage to be used (thus reducing the current), or modifying the requirements in respect to line regulation.

17. Short-line Calculations without Using Trigonometric Tables.—Referring again to Fig. 9, the power factor of the load being $\cos\theta$ and the axis of reference OA (*i.e.*, the current vector), the voltage E_n may be expressed in terms of its two components: the in-phase or energy component $E_n \cos\theta$, and the reactive component $E_n \sin\theta$. Knowing $\cos\theta$ (the power factor of the load), the quantity $\sin\theta$ is easily calculated without reference to trigonometric tables because

$$\sin\theta = \sqrt{1 - (\cos\theta)^2} \tag{14}$$

Also, from inspection of Fig. 9, it is obvious that, in place of formula (11), which includes the quantity $\cos\varphi$, it is possible to write

$$V_n = \sqrt{(E_n \cos\theta + IR)^2 + (E_n \sin\theta + IX)^2} \tag{15}$$

from which the voltage V_n can be obtained without first having to determine the angle φ by means of formula (10). Having calculated V_n, the power factor at the sending end of line is then directly obtained by using formula (9).

Example 4. *Short Single-phase Transmission Line.*—Assuming the same conditions as in Example 2, the solutions of the problem may be obtained as follows without referring to trigonometric tables.

As previously calculated:

$$I = 6.52;\ E_n = 2{,}300;\ IX = 31.7;\ IR = 88.5$$

The power factor being $\cos\theta = 0.8$, the reactive factor, by formula (14), is $\sin\theta = \sqrt{1 - (0.8)^2} = 0.6$, and by formula (15), $V_n = \sqrt{[(2{,}300 \times 0.8) + 88.5]^2 + [(2{,}300 \times 0.6) + 31.7]^2} = 2{,}390$ (nearly).[1]

By formula (9), the power factor at the sending end of line is

$$\cos\varphi = \frac{88.5 + (2{,}300 \times 0.8)}{2{,}390} = 0.807.$$

These results are the same as obtained by the method of Example 2.

Graphical Solution.—The solution of problems based on the vector diagram of Fig. 9 may be obtained by drawing the vectors to scale and taking measurements from the finished drawing. This method is often very convenient and yields good results if the drawing is accurately made to a reasonably large scale. In the diagram used for the solution of a practical problem, the impedance triangle *BCD* would be very much smaller relatively to the length of the vectors E_n and V_n than in Fig. 9; but that is no reason why results of sufficient accuracy for practical purposes should not be obtained, provided the drawing is carefully made.

When working out Example 4, the formula (15) based on Fig. 9 was used in order to illustrate a method of calculation which avoids the use of trigonometrical tables. This formula is not, however, very convenient for slide rule calculations because it is necessary to extract the square root of the sum of two large quantities. The vector V_n is the sum of the three vectors *OB*, *BC*, and *CD* (of Fig. 9), and since the last two are nearly always small in relation to *OB*, it follows that the angle *DOB* is also small. Imagine a perpendicular to be dropped from the point *D* on to the line *OB* extended beyond the point *B*. It is then easy to see that

$$V_n = \frac{E_n + IR\cos\theta + IX\sin\theta}{\cos(\phi - \theta)},$$

but the cosine of a small angle is approximately unity and, for

[1] Since the slide rule is used in the solution of nearly all the numerical examples in this book, the number of significant figures in the answers will be such as can be read off a 10-in. slide rule. It is very rarely, if ever, that the electrical engineer is justified in carrying out his calculations so as to include more figures than can be read off a 10-in. slide rule. If he uses methods of calculation which will give him more figures in his answers, such additional figures are usually non-significant, or meaningless.

practical calculations on short lines, it is possible to use the approximate formula

$$V_n = E_n + IR\cos\theta + IX\sin\theta \qquad (15a)$$

which is very convenient for slide rule calculations.

Even if the angle $(\phi - \theta)$ is as large as 7 deg., the error in using formula (15*a*) is within 1 per cent.

The accompanying table gives the relation between the power factor ($\cos\theta$) and the reactive factor ($\sin\theta$) for a limited number of values. The column headed $\tan\theta$ gives the ratio $\frac{\sin\theta}{\cos\theta}$.

TABLE GIVING RELATION BETWEEN POWER FACTOR (COS θ) AND REACTIVE FACTOR (SIN θ)

Cos θ	Sin θ	Tan θ	Cos θ	Sin θ	Tan θ
1.00	0.000	0.000	0.76	0.650	0.855
0.98	0.199	0.203	0.74	0.673	0.909
0.96	0.280	0.292	0.72	0.694	0.964
0.94	0.341	0.363	0.70	0.714	1.020
0.92	0.392	0.426	0.68	0.733	1.078
0.90	0.436	0.484	0.66	0.751	1.138
0.88	0.475	0.540	0.64	0.768	1.200
0.86	0.510	0.593	0.62	0.785	1.266
0.84	0.543	0.646	0.60	0.800	1.333
0.82	0.572	0.698	0.58	0.815	1.404
0.80	0.600	0.750	0.56	0.828	1.479
0.78	0.626	0.802	0.54	0.842	1.558

18. Use of Fundamental Diagram for Three-phase Calculations.—The vector diagram (Fig. 10) shows the relative phases of current and e.m.f. for a three-phase system with balanced load when the power factor is unity. Here the three current vectors are OA, OB, and OC. The "star" voltages are

$$Oa = Ob = Oc = E_n$$

each being in phase with the corresponding line current; and the voltages measured between the three conductors of the transmission line are

$$ab = bc = ca = E = \sqrt{3}E_n$$

The total power transmitted is

$$\begin{aligned} W &= \sqrt{3}E \times I \\ &= 3E_n \times I \\ &= 3\,(OA \times Oa) \end{aligned}$$

In Fig. 11 the diagram has been drawn for an inductive load. Here there is a certain displacement of the current phases with reference to the e.m.f. phases. It will be noticed that the vertices of the e.m.f. triangle no longer lie on the current lines as in the preceding diagram. The three current vectors still subtend the same angle of 120 deg. with each other; but they have been moved bodily round (in the direction of retardation) through an angle θ. The total power is evidently no longer equal to three times $OA \times Oa$, but to $3 \times OA' \times Oa$, where OA' is the projection of OA and Oa; and $\cos \theta$ is the power factor of the three-phase load.

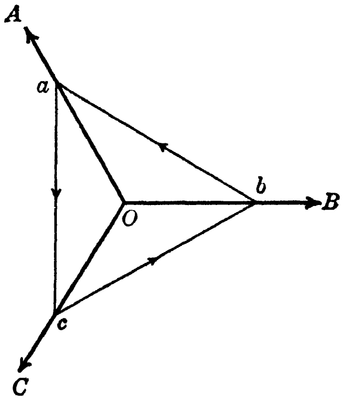


FIG. 10.—Vector diagram for three-phase system on non-inductive load.

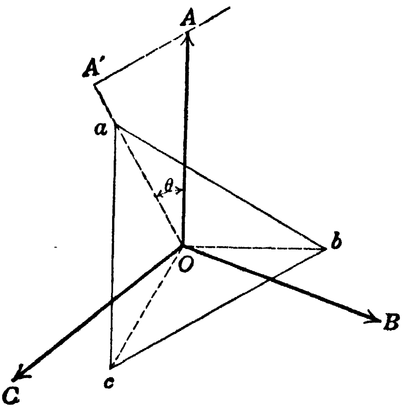


FIG. 11.—Vector diagram for three-phase system on partly inductive load.

It is a simple matter to complete this diagram by taking into account the effects of resistance and reactance of the line, because when the calculations for resistance drop and induced volts are made *per conductor* as previously explained, the construction can be carried out for each phase exactly as explained when describing the fundamental diagram (Art. 15). It is only necessary to bear in mind that OA and Oa, in Fig. 11, correspond to OA and OB in Fig. 9. When this construction has been carried out for each of the three phases, there will be a new set of star vectors which, when their ends are joined, will form a new e.m.f. triangle representing the necessary pressures at the generating end. This is shown in Fig. 12, where *am* and *md* are the vectors representing the required e.m.f. components to counteract the ohmic drop and reactive voltage, respectively, due to the current OA. The same construction is supposed to be followed for the other two phases, and the resulting triangle def indicates not only the magnitude of the potential differences between wires at the generating end, but also their phase relations with the other quantities. Thus the power factor at the generating end is not cos θ, but cos φ, all as explained in connection with Fig. 9.

It is true that a symmetrical arrangement of conductors has been assumed; that is to say, the three conductors are supposed to occupy the vertices of an equilateral triangle, in which case the magnetic flux due to the current in one of the wires will

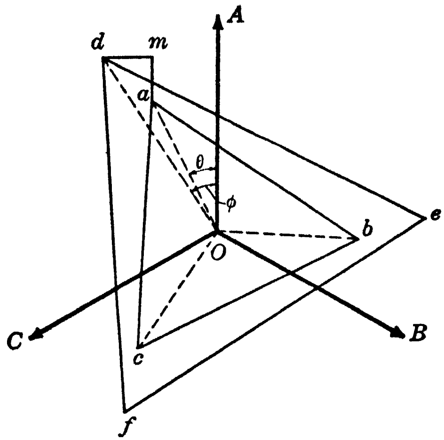


FIG. 12.—Vector diagram for three-phase transmission (capacity current neglected).

neither increase nor decrease the amount of induction through the loop formed by the other two wires; or, in other words, the whole of the current in any one conductor may be considered as returning at a distance from this conductor equal to the side of the equilateral triangle. As a matter of fact, if the wires are arranged in any other practical manner, the effect of the induction due to any one wire on the loop formed by the other two wires is usually small; but it is always possible to substitute for the actual arrangement of the conductors an *equivalent* arrangement with the wires at the vertices of an equilateral triangle, so that the calculations can still be made by considering each of the three conductors separately in relation to an imaginary neutral return conductor of zero impedance. The required correction for unsymmetrical arrangements of conductors will be taken up in Chap. IX.

19. Three-phase Lines to Serve Rural Districts.—Of late years there has been a very rapid development in electric lines designed to provide light and power for outlying farms and other consumers at a considerable distance from the centers of population. These rural lines are usually three-phase and differ from the distribution systems in urban districts mainly in the greater distances to be covered and the higher voltages necessary to cover these greater distances economically.

The most common pressures for these rural or farm lines are 6,600 volts, 11,000 volts, and as high as 13,200 volts when the distances are great. Even if a pressure of 2,200 or 2,300 volts would seem to be sufficient for immediate purposes, as when a small amount of power has to be transmitted a distance of only a few miles, it is best to insulate the lines for a higher voltage which is almost certain to be required as the demand for power and the distance from the point of supply become greater. The tendency today is toward higher pressures on rural lines, with a view to future developments.

The conductors are usually spaced 30 in. apart and are carried on pin-type insulators mounted on 30-ft. wood poles. Steel poles sometimes take the place of wood poles, especially when it is desired to use long spans with a view to reducing the number of poles and insulators. The chief causes of interruption on rural lines are insulator failures and the blowing of transformer fuses. By lengthening the average distance between poles, fewer insulators are required and the chances of failure through faulty insulation are reduced. The earlier lines constructed for this class of service usually consisted of No. 6 soft-drawn copper wire on 25-ft. poles spaced 125 ft. apart. This close spacing of the poles offers no apparent advantage over longer spans with 30-ft. and even 35-ft. poles. The average span on straight runs may then be as long as 250 ft., the wire being hard-drawn copper (not smaller than No. 6 B. & S. gage) or copper-clad steel, or occasionally steel-cored aluminum cable. Shorter spans and guyed poles with double cross-arms are used at the corners where there is a sharp change in the direction of the line.

A comparatively low transmission pressure, such as 2,300 volts, has the advantage that standard transformers are available for outputs as low as 1.5 kv.-a., whereas 11,000-volt and 13,200-volt transformers are not usually built in smaller sizes than 2.5 kv.-a.

A four-wire three-phase system suitable for short-distance transmission is illustrated by Fig. 13. This system gives a 4,000-volt three-phase supply between the three terminals of the star connection, and also 2,300 volts for single-phase service between any one of the three phases and the fourth wire which returns to the neutral point of the system. This point is usually grounded at the supply end of the line.

Example 5. Three-phase 6,600-volt Transmission.—(*a*) What distance can 300 kw. be transmitted by 60-cycle three-phase currents at 6,600 volts using No. 4 copper wires, with a loss of power in the lines not exceeding 30 kw. when the power factor of the load is 92 per cent? (*b*) What pressure between wires is

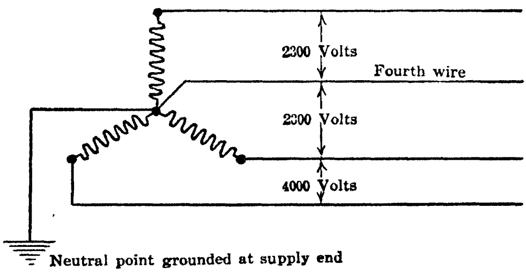


FIG. 13.—Four-wire three-phase system.

necessary at the sending end of the line to maintain 6,600 volts at the receiving end under full-load conditions, the spacing between wires being 30 in.?

Solution (*a*).—The total output is $\sqrt{3}EI \cos\theta = 300{,}000$ watts, wherein $E = 6{,}600$ and $\cos\theta = 0.92$. Solving for the current, $I = 30.9$ amp. If R = the resistance in ohms of one of the three conductors, the total line losses are $3\ (I^2R) = 30{,}000$ watts, whence $R = 10.46$ ohms. Referring to the wire table on page 81, the ohms per mile of No. 4 copper wire = 1.35, whence the maximum permissible length of the transmission line is $\frac{10.46}{1.35} =$ 8 miles (nearly).

Solution (*b*).—Instead of using the formulas of Arts. 15 and 17, an approximate answer to this problem can be obtained by drawing to scale the vector diagram of Fig. 9 (p. 21). In order to obtain V_n, numerical values must be found for the angle θ, the voltage E_n, the resistance drop IR, and the reactance drop IX.

The angle θ can be drawn without reference to trigonometric tables, as will be explained later.

The length OB is $E_n = \frac{6,600}{\sqrt{3}} = 3,810$

The length BC is $IR = 30.9 \times 10.46 = 324$

The length CD (or the numerical value of IX) may be obtained as follows:

From the wire table on page 81, the diameter of No. 4 wire $= 0.204$ in. The spacing between wires being 30 in., the ratio $\frac{\text{spacing (in feet)}}{\text{diameter (in inches)}}$ is $\frac{2.5}{0.204} = 12.25$, which permits of the reactance being obtained from the chart on page 243. Thus, at 100 cycles, $X = 1.2$ ohms per mile, and since the frequency in the

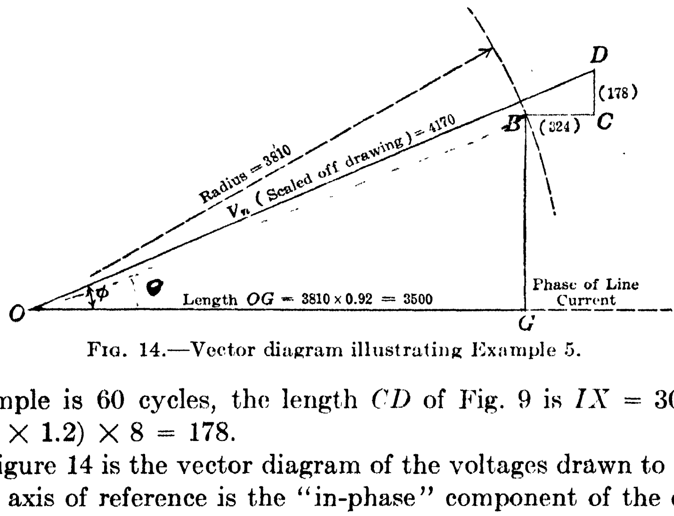


FIG. 14.—Vector diagram illustrating Example 5.

example is 60 cycles, the length CD of Fig. 9 is $IX = 30.9 \times (0.6 \times 1.2) \times 8 = 178$.

Figure 14 is the vector diagram of the voltages drawn to scale. The axis of reference is the "in-phase" component of the e.m.f. at the receiving end, $OG = E_n \cos \theta = 3,500$. At the point G erect the perpendicular GB on OG, and with a radius of length $E_n = 3,810$ describe the dotted arc with center at O. This locates the point B. Draw $BC = 324$ parallel to OG (this being the IR drop), and $CD = 178$ perpendicular to OG (this being the IX drop). The pressure between wire and neutral at the sending end of line must, therefore, be $V_n = OD$, which scales 4,170 volts, and the required pressure between wires is $\sqrt{3} \times 4,170 = 7,230$ volts.

From an inspection of Fig. 14 it is seen that the point D falls very nearly on the prolongation of the vector OB (not drawn), or, in other words, that the power-factor angle φ at the sending end of the line differs very little from the power-factor angle θ at the load end of the line. The very simple approximate formula (15a) on page 28 will, therefore, yield an answer which will be fully as accurate as the result obtained by scaling a length off the drawing. Thus: $\sin\theta = \sqrt{1 - (0.92)^2} = 0.392$, and $V_n = 3{,}810 + (324 \times 0.92) + (178 \times 0.392) = 4{,}177.8$, or, say, 4,178 volts instead of 4,170 volts obtained by scaling off the diagram.

20. The Problems of Long-distance High-voltage Transmission.—Before considering the economic aspects of electric power transmission, it may be well to discuss briefly the present practices and tendencies in the design of long-distance lines. When large amounts of energy have to be transmitted over great distances, the cost of conductors and the losses in the line would be excessive unless very high transmission voltages were used. It is possible to build today a single three-phase line capable of transmitting 150,000 kw. a distance of 250 miles with a pressure of 220,000 volts between conductors. An undertaking of this magnitude involves the study and solution of many problems, including those enumerated below.

1. Mechanical problems, dealing mainly with the strength of the conductors and insulators and supporting structures. With the large size of conductors and long spans used on modern long-distance lines, the towers become very high and the tension at dead-ending points very great. Careful attention must be given to the design of towers and their foundations, but these problems are more easily solved than the design of insulators suitable for so high a working pressure as 220,000 volts and yet of sufficient strength to withstand the maximum possible tension in the conductors. The mechanical strength of suspension-type porcelain insulators has gradually increased, and units are now available suitable for a working load of 6,000 lb.

2. Electrical problems, dealing with the adequate insulation of high-tension lines, possibilities and effects of corona formation, reactance and electrostatic capacity of long lines, regulation of voltage and power factor, and protective devices to minimize the possibility of service interruption.

Strings of suspension-type insulators, consisting of 14 separate units in series, are used successfully on 220,000-volt lines, but a wide spacing between wires is necessary, and this involves great stresses on the supporting cross-arms, requiring extra-strong and somewhat costly steel towers.

3. Economic problems, dealing with the most suitable material and size of conductor (*i.e.*, the selection of a conductor in which the losses have been determined on an economic basis), the proper voltage for an assumed amount of power and a given distance of transmission, the most economical distance between towers, and the question of duplication of lines or of carrying more than one circuit on the same tower line. These problems are closely connected with the mechanical and electrical features of transmission line design, and also with such considerations as durability, continuity of service, protection from lightning and other possible causes of electrical disturbances.

In view of the development of the so-called superpower systems and networks, no important new line should be planned without taking into account neighboring systems of transmission and the possibilities of ultimate interconnection. Main lines of the present day may become tie lines in the future, which may involve the maintenance of constant pressure between wires, notwithstanding changes of load and even reversal of the flow of energy. The use of synchronous machinery to control power factor and of special variable-ratio transformers to control the voltage must be considered in connection with modern transmission systems. These various aspects of transmission line engineering will be taken up in the following chapters.

CHAPTER III

ECONOMIC PRINCIPLES

21. Introductory.—In the construction of electrical plant and machinery, such as generators, transformers, and switching apparatus, the economic conditions are, as it were, automatically fulfilled, owing to the competition between manufacturers, each one of which is a specialist in his own particular line of business. This competition, it should be observed, is not merely in the matter of factory cost or selling price, but in factory cost plus efficiency and durability. It is not necessarily the cheapest or the most costly manufactured article that wins in the long run, but the one which is commercially best suited to the needs of the user.

The cost per mile of a finished transmission line, whether by overhead or underground conductors, is not all-important. It may frequently be said to be of importance only in so far as it influences the annual cost of the line, which annual cost is understood to include interest on the capital sum expended on the line. If a heavy section of copper is used for the conductors, the loss of energy in overcoming resistance will be less than with a lighter section, but the initial cost will be greater. There is only one particular size of conductor which is economically the right size for any given line operating under definite conditions, and this is by no means easy to determine, notwithstanding the apparent simplicity of what is usually referred to as Kelvin's law.

When considering any scheme of power transmission from a generating plant of limited output, it is important to bear in mind that it does not pay to cover a distance greater than that within which there is a reasonable prospect of supplying all the power available at the generating station. The importance of this principle should be fairly obvious; yet there are instances which prove that it has been disregarded.

On the other hand, where the energy available appears to be in excess of the probable demand within a reasonable radius from the generating station, the possibilities of subsequent requirements greatly exceeding the immediate demand must not be overlooked, and a transmission system on a large scale, designed to satisfy future conditions, may be desirable. Each case must be studied separately, on account of the variable nature of the conditions in different localities. The easiest part of the problem is the designing of an economical transmission line, that is to say, a line that will give the best return for capital invested, on the assumptions of a given amount of energy to be delivered at a given point where a given price will be paid for it. The real difficulty lies in estimating not only the immediate demand, but also the probable future demand and its rate of growth, in order that proper provision may be made to avoid unnecessary waste in remodeling or reconstructing the original transmission line.

As a general rule, it is certain that money spent on preliminary surveys and calculations before the work is actually started is well spent. It is a mistake to suppose that a transmission system of any magnitude can be erected economically and give good service unless as much engineering knowledge and judgment enters into its construction as into the manufacture of the generators and transformers.

It is well to bear in mind that the introduction of automatic switches and similar devices designed to save labor and insure the rapid changing over of the load from a faulty section to a sound section on a duplicated transmission line is likely to lead to unlooked-for troubles; and even the generous provision of lightning arresters, especially on the extra-high-tension lines, is not always good policy. Simplicity and the avoidance of unnecessary joints, rubbing contacts (as in switches or cutouts), fuses in the stations, and spark gaps or arresters along the line should generally be aimed at; but there will always be exceptions to such rules.

22. Economy in Design.—Since the transmission system which is the cheapest in first cost is not necessarily the most economical, it may be well to consider briefly what is meant by the most economical transmission line. The cost of energy lost in transmission may be calculated approximately and expressed as a definite annual expenditure of money if the amount of the energy losses and the cost of this energy are known. The total amount of money spent on the line itself, including (if required) the cost of the step-up and step-down transformers at the sending and the receiving ends, is either known or may be estimated without much difficulty, and, in order to compare this with the cost of the lost energy, it may also be put on an annual basis by making a proper allowance for some or all of the following items:

a. Interest on capital invested,

b. Depreciation, which is dependent upon the "life" of the plant and may include what is usually understood by "obsolescence,"

c. Taxes,

d. Insurance,

e. Maintenance.

Thus two distinct items of cost have to be accounted for: (1) the yearly cost of the energy lost in transmission and (2) the yearly cost of the physical plant or property consisting of the line itself and (usually) the step-up and step-down transformers, and also such portions of the generating plant itself as may be considered necessary to supply the power losses in the line. Assuming that the line is designed to perform its function efficiently, then it is obvious that the engineer's task is accomplished when he has constructed a transmission line for which the sum of the cost items (1) and (2) is a minimum.

The problem, of which the above is a general statement, cannot be solved by the simple substitution of numerical values in mathematical formulas, however imposing and worthy of respect these may appear. That is because there are too many variables and possible alternatives and indefinite quantities to be taken account of. The problem is one requiring engineering experience and judgment, and it is usually solved by making estimates for several alternative designs and selecting the one which conforms most nearly to the requirement of minimum total annual cost.

Among the many considerations affecting the design of an efficient and economical transmission line may be mentioned: the location of the line; the choice of system of transmission, frequency, voltage, and type of construction; the most economical spacing of poles or towers, and the most economical material and cross-section of the conductors. The question of duplication of circuits, either on the same pole line, or on two-pole lines running parallel on the same right of way, or, again, on two lines following different routes, is largely an economical one. Another matter of considerable importance is the control of power factor by varying the excitation of synchronous machinery connected to the system.

This is not to be decided entirely on the basis of annual cost, because of its relation to voltage regulation (to be explained in a later chapter), but, nevertheless, it should be noted that although improved power factor will reduce the I^2R losses (by reducing the current for a given power output), the cost of the synchronous condensers required to bring about this result may counterbalance the saving in the cost of the energy losses.

23. Length of Span.—The choice of system and frequency need not be discussed here because the a.c. three-phase system has proved its superiority for transmission purposes, and 60 cycles is becoming the standard frequency in this country. It is the frequency which is almost certain to be adopted by all the so-called superpower networks of the near future, and all new lines should, if possible, be planned for a frequency of 60. The possibilities of transmitting by continuous currents will be considered in a later chapter.

All long-distance transmission must necessarily be by overhead conductors because of the higher cost of underground cables and the difficulties of insulation for very high voltages. The structures for supporting overhead conductors may be of wood, steel, or reinforced concrete. The wood supports may consist of single poles spaced 120 to 300 ft. apart, or they may be A or H frames built up of two poles suitably braced, and capable of supporting longer spans. The steel poles may be of simple tubular type, or may be built up of several tubes or angles with the necessary bracing. The more common construction for high-pressure transmission lines consists of light-braced towers with wide rectangular bases, except where the "flexible" type of structure is adopted. These flexible towers are modeled generally on the A and H types of double wood-pole supports. It is by no means an easy matter to decide upon the most suitable type of supporting structure to be used on any particular transmission scheme. In some cases a composite line including two or more types of support may be found advantageous. Among the factors influencing the choice of the supporting structures may be mentioned the character of the country, the means and facilities of transport, climatic conditions, the nature of the soil, and the scarcity or otherwise of suitable timber in the district through which the line will pass. The type of supporting structure for overhead conductors, together with the height, strength, and cost of the individual pole or tower, will be dependent upon the span or distance between the supports. The determination of the average length of span is indeed a very important economic question. The material of the conductor will, to some extent, influence the choice of span length, because aluminum conductors will usually have a greater summer sag than copper conductors, and this will necessitate higher supports to give the same clearance above ground at the lowest point of the span. In considering span length, the first cost of the individual support is not the only question which has to be taken into account; the cost of maintenance is almost equally important. The longer the span, the fewer will be the points of support; and if the line is well designed and constructed, there should be less trouble through faults at insulators. On the other hand, the cost of repairs when the spans are long is generally considerably higher than on short-span lines. Again, where rent has to be paid for poles placed on private property, it is generally the rent per pole apart from the size of pole which has to be considered, and this is another factor in the determination of the best length of span. In level country, the economic span for high-voltage lines is usually between 600 and 800 ft. The greater the tensile strength of the conductor, the longer will be the economic span. Thus, where 600 ft. might be the best span for aluminum conductors, 800 ft. would probably be best for copper, and even longer spans for steel-cored stranded aluminum conductors. Average spans of 750 to 850 ft. are not unusual on 110,000-volt transmission lines, and the tendency on high-voltage systems is to use conductors of high tensile strength on long spans, in order to reduce as far as possible the number of insulators. Obviously, in hilly or undulating country, very much longer spans are permissible and, by careful planning of the route and tower locations, considerable saving in cost may be effected by the use of long spans without increasing the height of the towers. On short low-voltage lines, large spans may not prove to be economical. It is sometimes advantageous to increase the number of supports in order that these may be so light in weight as to be easily handled and quickly erected. The fact that repairs can be carried out more easily and at less cost on short-span lines should not be overlooked when deciding upon the average span length.

The height of towers in level country depends upon (1) the minimum clearance between the lowest conductor and ground when the sag is greatest; (2) the voltage, since this has an effect on the spacing of the conductors and also to some extent on the clearance above ground level; and (3) the maximum sag. This last is determined by the length of span, the material and size of the conductors, the range of temperatures, and climatic conditions generally.

24. Cost of Transmission Line Supports.—Formulas have been proposed for estimating the height and cost of steel poles and towers, but all such formulas must necessarily be confined to one particular type or design, and they are likely to be used without due regard to their limitations. The weight and cost of the individual tower will depend upon the force tending to overturn the tower, and the height above ground, or leverage, at which this force acts. A strong wind blowing across the line will exert pressure on the tower structure itself, and in addition to this there will be the pressure on the wires which will be transmitted to the tower at the points of attachment of the insulators. Thus, the number and size of the conductors and the length of span are very important factors in determining the tower design. The longer spans involve a greater sag of the wires and, therefore, taller towers to give the same clearance above ground. High voltage, with greater spacing between wires and (usually) greater clearance above ground, also requires taller towers and heavier cross-arm structures. Given that, in all cases, the most economical span is adopted, it might be possible to evolve a formula giving approximate tower weights or costs in terms of the transmission voltage, together with the number, material, and size of the conductors; but a formula of this type is usually of service only to the experienced engineer who can interpret it with judgment based on past experience. As a rough guide to the necessary tower height, the following formula may be used:

$$H = 30 + 0.3 \text{ kv.} \tag{16}$$

where H is the distance in feet from the ground level to the top of the tower, and kv. stands for the pressure between wires in kilovolts. The weight of towers for two-circuit three-phase lines operating at about 88,000 volts may be anything from 3,000 to 6,000 lb., depending upon the diameter of the conductors and the length of span. For rough preliminary estimates, the cost of steel towers erected in position may be taken at about $8 per 100 lb. (which includes 1.5 cts. per pound for erection), while the latticed steel poles which can be handled and erected much in the same manner as wood poles would cost from 6 to 7 cts. per pound erected.

The cost of foundations for towers varies greatly. For fairly high steel towers with wide square bases in soil not requiring the use of concrete, the cost of excavating, setting legs, and back-filling, not including the erection of the towers on the prepared foundations, will generally be between $20 and $40; but, taking an average of all towers, including dead-ending and corner structures, the allowance for foundations should be about one-sixth of the cost of the towers.

Cost of Wood Poles.—The price of wood poles at the present time is greatly in excess of what it was ten years ago. It depends upon the kind and quality of the wood. For the purpose of preliminary estimates, the approximate prices of chestnut or cedar poles are given below:

30 ft	$12
35 ft	15
40 ft	18
45 ft	22
50 ft	26
55 ft	30
60 ft	34

An amount varying from $6 to $15 should be allowed to cover unloading, hauling to site, "framing," digging holes, and erecting.

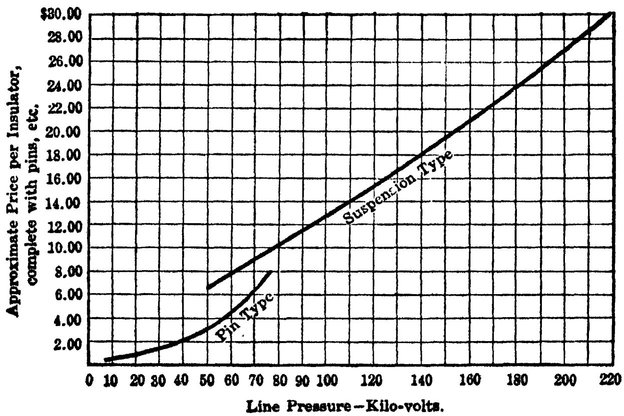


FIG. 15.—Approximate cost of line insulators.

Cost of Insulators.—Makers' catalogues and current price lists should be consulted when getting out preliminary estimates, but the curve (Fig. 15) gives approximate average prices of insulators complete with pins or suspension links. The prices are per insulator, or per string of suspension-type units. The suspension type of insulator is almost universally used for pressures above 50,000 volts, and even in many instances for lower voltages.

25. Total Cost of Transmission Lines.—The approximate costs given in this chapter are not intended to take the place of manufacturers' catalogues and current quotations. The cost of labor is a variable quantity, but recent estimates and actual costs of finished work may frequently be found in the technical journals. It is proposed to consider in this article the manner in which the most economical design of transmission line may be determined; the fact that conditions of transportation of materials and the quality of labor differ widely makes it impossible to give exact cost figures even for a limited period. It would seem, however, that the following examples of preliminary estimate for a complete line may be useful as a basis on which similar estimates may be made.

Preliminary Estimate 1

Wood-pole transmission line, 20 miles long, carrying one three-phase line. Line pressure 22,000 volts. Average span 176 ft. There is no grounded overhead guard wire; but two telephone wires are carried on the same set of poles. An allowance of 20 per cent is made for extra insulators and fixtures to permit of doubling these on corner poles and in other selected positions.

Materials (Excluding Conductors), per Mile

30 creosoted cedar poles, 35 ft. long, 8 in. in diameter at top	$ 430
36 cross-arms 3½ by 4½ in. by 4 ft. long	25
72 galvanized-iron braces, 1¼ by ¼ by 28 in. long	7
The necessary galvanized screws, bolts, and washers	10
1,200 ft. galvanized seven-strand 5/16-in. guy wire	17
10 anchor rods with nuts and washers and the necessary timber for anchor logs	25
The necessary galvanized guy clamps with bolts and standard thimbles for guy wire	4
16 galvanized-iron lightning conductors with bolts	5
16 ground plates or galvanized-iron pipes	15
Sundry materials, including allowance for breakages and contingencies	25
60 telephone-wire insulators (glass) with brackets (wood) and 5-in. nails	10
108 high-tension porcelain insulators	65
72 galvanized-iron insulator pins	17
36 special pole-top insulator pins with bolts	21
Total material cost (excluding conductors), per mile	$ 676

LABOR COST, PER MILE

Distributing materials along the line, trimming poles, cutting gains, drilling holes, setting cross-arms, digging holes and erecting poles, including the necessary guying...	$ 200
Fixing insulators and stringing wires, including telephone line	140
Supervision and sundry small labor items	40
Loss and depreciation of tools	20
Management and preliminary engineering work	40
Cost per mile for charges other than materials, excluding cost of right of way, clearing right of way, and interest on capital invested during construction period	$ 440
Total cost per mile (material and labor), excluding cost of conductor material	$1,116

CONDUCTORS

16,000 ft. No. 1 hard-drawn copper; 700 ft. No. 4 soft copper (for ties); 10,800 ft. No. 10 copper (for telephone circuit); 4,550 lb. at $20 per 100 lb	910
Total cost per mile of finished line, excluding cost of right of way	$2,026

PRELIMINARY ESTIMATE 2

Flexible-type steel-tower line, 60 miles long, with two sets of three-phase conductors. Line pressure 80,000 volts. Average span 480 ft. Spacing between wires 8½ ft. A Siemens-Martin steel cable, acting as grounded guard wire, joins the tops of all towers. Insulators of the suspension type. No telephone wires. Cost of right of way not included in estimate.

MATERIALS (EXCLUDING CONDUCTORS)

10 flexible-type, galvanized-steel, A-frame towers at $120	$1,200	
1 galvanized-steel strain tower	220	
Concrete foundations where necessary	100	
5,600 ft. $\frac{7}{16}$-in. galvanized Siemens-Martin steel strand cable for guard wire and head guys on half-mile flexible towers	220	
4 anchor rods, complete with clamps and thimbles for guy wire	10	
90 sets of suspension-type insulators, including strain insulators and small allowance for breakages, complete with clamps	900	
Sundry small items or special material	50	
Total material cost per mile of line		**$2,700**

the labor on erection and stringing of wires, is independent of the actual size of conductor, then the only variable item in the capital expenditure is directly proportional to the cross-section (or weight) of the conductor, and since the I^2R losses (for a given current) are inversely proportional to the conductor cross-section, the law of maximum economy is greatly simplified and in fact becomes Kelvin's law, which may be expressed as follows:

The most economical section of a conductor is that which makes the annual cost of the I^2R losses equal to the annual interest on the capital cost of the conductor material, plus the necessary annual allowance for depreciation. The cross-section should, therefore, be determined solely by the current which the conductor has to carry, and not by the length of the line or an arbitrary limit of the percentage full-load pressure drop. If there are reasons which make a large pressure drop undesirable, then, if necessary, economy must be sacrificed, and the line calculated on the basis of regulation only.

If (m) stands for the size or cross-section of the conductor (circular mils), the annual cost of the metal in the conductors per mile of line may be written $K_c(m)$, where K_c is a constant determined by the cost per pound of the conductors, the number of conductors in the line, the interest on the purchase price of the conductors, the estimated "life" of the line (depreciation allowance), and the probable scrap value of the conductors at the time when they will have to be replaced.

The annual cost of the energy wasted in I^2R loss per mile may be written $\frac{K_e}{(m)}$, where K_e is a constant determined by the known or estimated value of I^2, and the unit cost of the wasted energy.

The total annual cost is

$$y = K_c(m) + \frac{K_e}{(m)}$$

from which it is seen that the annual cost will be high not only when (m) is very large (high cost of conductors), but also when (m) is very small (high cost of wasted energy). In order to obtain the minimum value of y,

$$\frac{dy}{d(m)} = K_c - \frac{K_e}{(m)^2} = 0$$

Multiplying both sides of the equation by (m), and transposing, the condition of minimum annual cost is obtained:

$$\frac{K_e}{(m)} = K_c(m) \tag{17}$$

which is Kelvin's law as stated above.

28. Economic *IR* Drop.—It is not generally realized that when the size of a conductor is determined by the application of Kelvin's law, the ohmic drop of pressure per unit length of conductor is independent of the actual voltage or the current to be carried and, therefore, bears no definite relation to the total amount of power to be transmitted. The economic data and assumptions alone determine the ohmic drop in volts per unit length of conductor, and this will be a constant quantity whatever the number of conductors or system of electric transmission adopted, the total amount of power to be transmitted, or the voltage ultimately decided upon. This fact very considerably simplifies the problem in its earlier stages.

The equation (17) may be written in the form

$$(m) = \sqrt{\frac{K_e}{K_c}}$$

by which the economical cross-section of conductor could be calculated if the quantities K_e and K_c could be correctly evaluated.

Annual Charges Depending upon Cost of Conductors.

Let C_c = cost per pound of conductor;

a = total percentage to be allowed for interest, depreciation, etc., per annum;

w_m = weight, in pounds, per mile of conductor per circular mil of cross-section;

(m) = cross-section of conductor in circular mils.

Then

$$\left.\begin{array}{l}\text{Cost per mile of conductor}\\ \text{expressed as an annual charge}\end{array}\right\} = \frac{a}{100} \times C_c \times w_m \times (m) \tag{18}$$

Annual Cost of Lost Energy.

Let C_e = cost per kilowatt-year of the I^2R energy losses;

I = amperes of current in the conductor, being the known or estimated value of current which, when squared and multiplied by the conductor resistance, will represent the average rate at which energy is being wasted in the form of I^2R loss;

R = the resistance, in ohms, per mile of conductor, $= k \times \frac{5,280}{(m)}$ where k is the resistance per circular mil-foot.

Then

$$\left.\begin{array}{l}\text{Annual cost of energy loss}\\ \text{per mile of conductor}\end{array}\right\} = C_e \times \frac{I^2k \times 5,280}{1,000 \times (m)} \quad (19)$$

For the condition of minimum cost, it is merely necessary to equate (18) and (19) as indicated by equation (17). This gives the economical cross-section of the conductor in circular mils per ampere as

$$\frac{(m)}{I} = \sqrt{\frac{528k}{w_m}} \times \sqrt{\frac{C_e}{aC_c}} \quad (20)$$

For stranded copper conductors with $w_m = 0.016$ and $k = 10.8$ at 20°C., this becomes

$$\text{Circular mils per ampere (copper)} = 597\sqrt{\frac{C_e}{aC_c}} \quad (20a)$$

For stranded aluminum conductors, with $w_m = 0.0048$ and $k = 17.5$ at 20°C., this becomes

$$\text{Circular mils per ampere (aluminum)} = 1,385\sqrt{\frac{C_e}{aC_c}} \quad (20b)$$

Since the resistance per mile of conductor is $R = k\frac{5,280}{(m)}$, substitute for (m) in equation (20) its value in terms of k and R, which leads to the expression,

$$\left.\begin{array}{l}\text{Economic } IR \text{ drop per mile}\\ \text{of conductor}\end{array}\right\} = \sqrt{52,800\ kw_m} \times \sqrt{\frac{aC_c}{C_e}} \quad (21)$$

For stranded copper conductors at about 20°C. the economic voltage drop per mile is

$$\text{Economic } IR \text{ drop (copper)} = 95.5\sqrt{\frac{aC_c}{C_e}} \quad (21a)$$

and for stranded aluminum cable at the same temperature,

$$\text{Economic } IR \text{ drop (aluminum)} = 66.5\sqrt{\frac{aC_c}{C_e}} \quad (21b)$$

29. Economic Voltage and Calculation of Conductor Sizes.—Having ascertained what will be the most economical resistance drop of pressure per mile of conductor without reference to the total amount of power to be transmitted, the size of the conductor cannot be determined unless the value of the current is known, and this will depend upon the pressure at which the energy will be transmitted.

If the cost of the conductors forming the transmission line, and of the I^2R losses therein, were the only considerations, a high voltage would in all cases be desirable on account of the corresponding reduction of current for a given amount of energy to be transmitted. But, apart from the extra cost of the line due to the better insulation and wider spacing of wires required by the higher pressures, the cost of generation and transformation of high-pressure energy must be taken into account, and as the extra cost per kilowatt of equipment for generating at high pressures will depend largely upon the total output required, it follows that the most economical pressure will bear some relation to the total power to be transmitted. This is apart from the distance of transmission, which is the most important factor governing the choice of voltage. If the distance is great, it is obvious that the reduction of material cost and power losses in the line due to the employment of higher pressures will be relatively of far greater importance than the increased cost of plant in generating and transforming stations. On the other hand, the employment of very high pressures even on a comparatively long line might not be justified if the total amount of power to be transmitted were very small.

As a first approximation, the writer has found the following formula useful in getting out preliminary estimates; the line voltage given by the formula agrees generally with modern practice.

$$\text{Line pressure (kilovolts)} = 5.5\sqrt{L + \frac{\text{kw.}}{100}} \qquad (22)$$

This formula may be used for estimating the probable economical transmission voltage on lines over 20 miles in length. The symbol L stands for the distance of transmission in miles, while kw. stands for the estimated maximum number of kilowatts that will have to be transmitted over one pole or tower line.

Given the amount of power to be transmitted and the length of line, it is possible with the aid of formula (22) to select the nearest standard voltage and proceed with the calculations for current and size of conductor; but it is necessary always to bear in mind that a transmission line cannot be considered by itself; it must be treated as part of a complete scheme of transmission and distribution, and the best voltage to use on any given system can generally be arrived at only by a method of trial and error, taking into account the cost of the various parts of the complete system as influenced by alterations in the transmission voltage. No accurate formula can be evolved which would be applicable to all the varied conditions encountered in actual work; but a practical method of attaining the required end will be explained later.

Example 6. *Illustrating Quick Method of Determining Economic Size of Conductors.*—For the purpose of working out a practical example, the following assumptions have been made:

Total kilowatts to be transmitted = 12,000.

System, three-phase.

Power factor = 0.8.

Distance of transmission = 120 miles.

Copper conductors to be used, the cost C_c being 20 cts. per pound.

Percentage to be taken to cover depreciation and annual interest on cost of copper, a = 12.5.

Estimated cost of wasted energy per kilowatt-year, C_e = $22.

The economic voltage drop per mile of single conductor will be, by formula (21*a*),

$$IR = 95.5\sqrt{\frac{12.5 \times 0.2}{22}}$$
$$= 32.2 \text{ volts}$$

The transmission voltage as given by formula (22) is

$$\text{Kilovolts} = 5.5\sqrt{120 + \frac{12,000}{100}}$$
$$= 85$$

or, say, 88,000 volts at the receiving end.

The current per conductor will be

$$I = \frac{\text{watts}}{\sqrt{3} \times E \times \cos\theta}$$
$$= \frac{12,000,000}{\sqrt{3} \times 88,000 \times 0.8}$$
$$= 98.5 \text{ amp.}$$

Resistance of conductor per mile $= \frac{IR}{I} = \frac{32.2}{98.5} = 0.327$ ohm, and since No. 3-0 B. & S. wire has approximately this resistance per mile, that is the standard size which should be adopted unless a more careful study of the complete scheme should lead to a different decision in regard to the pressure of transmission.

Since, for a given amount of power to be transmitted, the current will vary inversely as the pressure, it follows that the resistance per mile of conductor to give the economic voltage drop per mile (32.2 volts in this particular example) will be directly proportional to the pressure at which the power is transmitted. Thus, if 110,000 volts were found to be a more economical pressure than 88,000, the ohms per mile of conductor would be $\frac{0.32 \times 110}{88} = 0.4$, the nearest standard size being No. 2-0 (ohms per mile = about 0.41 at 60°F.).

Power Lost in Line.—The total I^2R loss in the three conductors, based on the calculated value of the resistance, is

$$3 \times \text{length of line} \times I \times \text{economical } IR \text{ drop per mile}$$
$$= 3 \times 120 \times 98.5 \times 32.2$$
$$= 1{,}140 \text{ kw.}$$

on the assumption that a transmission pressure of 88,000 volts is adopted; and since the total kilowatts transmitted are 12,000, the *percentage* power loss is

$$\frac{1{,}140 \times 100}{12{,}000} = 9.5 \text{ per cent}$$

Voltage Regulation.—The power loss expressed as a percentage of the volt-amperes transmitted is $9.5 \times 0.8 = 7.6$ per cent, which is also the IR drop in the line conductors expressed as a percentage of the line voltage. This figure alone does not, however, give much indication as to what will be the actual regulation of the line, as the effects of inductance and electrostatic capacity must be taken into account and the resultant difference of pressure between the transmitting and receiving ends of the line calculated as explained in Art. 18 of Chap. II for short lines, and more fully in Chap. IX where the effects of capacity are considered. The resultant pressure drop may be found to be excessive; it may be such as cannot readily be dealt with in a practical scheme, and in such a case the economy of the line may have to be sacrificed by putting in larger conductors.

It is obvious that other conditions may render it inexpedient or impossible to adopt the most economical size of conductor as calculated by the application of Kelvin's law, but in such cases experience and common sense will usually indicate the proper course to follow. If the economic size of wire is small, it is possible, but not probable, that there may be trouble due to excessive heating. A want of mechanical strength, or loss of power due to corona formation, is more likely to lead to the selection of a conductor diameter larger than the "economic size." If, on the other hand, the conductor diameter is very large, there may be difficulties in handling and in taking the strain on the individual insulators. The remedy in this case is obviously to subdivide the single circuit into two or more parallel circuits, and, in fact, there are many advantages in doing so rather than running very heavy single conductors.

Again, even from the economic point of view, the case might arise of a temporary installation intended to give a quick return on capital invested, and an exceptionally small size of wire giving a large I^2R loss might produce the best results. This, however, leads to the consideration of the most important factor in the whole problem, namely, the correctness of the estimates of costs, depreciation allowances, and power transmitted, upon which the value of the calculated results will mainly depend. It is here that the experience, foresight, and sound judgment of the engineer must necessarily play an important part, and it is not possible in this chapter to do more than draw attention to important considerations which must not be overlooked.

30. Estimation of Amount and Cost of Energy Wasted in Conductors.—The correct value of the power transmitted, from which to determine the value of the current I in formula (19) of Art. 28, is frequently very difficult to estimate. This is a point which is best considered when determining the cost of the wasted energy. It is, however, clear that the annual amount of energy wasted will depend not only on the average value of I^2, but also on the *time* during which the average amount of power may be considered as being transmitted by the wires. If, therefore, it is desired to estimate accurately the amount of energy wasted annually in the lines, a probable load curve for the year should be drawn and the average I^2 calculated therefrom. This will give a value for I which, if considered as flowing in the wires continuously throughout the year, will lead to a certain watt-hour or yearly energy loss, the cost of which it is desired to know. In practice the value of I, being the square root of the average I^2 taken throughout the year, is about five-eighths of the peak-load current. This is considerably less than the value obtained in the numerical Example 8 to follow, wherein the assumed load conditions are exceptional.

Now, the annual cost of production of an additional electrical horsepower, considered apart from the total cost of production, is always difficult, if not impossible, to estimate accurately, but where coal is the source of energy, there is at least the extra cost of coal consumed to be taken into account when estimating the production cost of the lost energy. The case is different in a water-power generating station, where the cost of running the station at full output is very little in excess of the cost of running at one-quarter or one-tenth of maximum output, and it is even more difficult to decide upon a figure which shall represent the cost of wasted energy (C_e in the calculations) with sufficient accuracy to make the calculations of the economic conductor of real practical value.

There are two points in connection with water-power propositions which must never be lost sight of:

1. If the amount of water power available is limited, while the demand for power is unlimited, the cost C_e of the wasted energy may be taken at the price which the user would actually be prepared to pay for it were it available for useful purposes.

2. If the water power is unlimited as compared with the demand for power, the cost of wasted energy is practically *nil*, except for the fact that a generating plant has to be installed of a somewhat larger capacity than would otherwise be necessary; and the works cost of the wasted energy must, of course, include a reasonable percentage to cover interest and depreciation on this extra plant.

31. Estimation of Percentage to Cover Annual Interest and Depreciation on Conductors.—So far as interest is concerned, if cash is to be paid for the conductors, the figure to be taken for interest on capital should be on a par with the expected percentage profit on the complete undertaking; but if the conductors are mortgaged, it is the annual amount of the mortgage which should be taken.

In regard to depreciation, the probable life of the conductor must be estimated, and this, to a certain extent, may depend upon the life of the transmission line considered as a whole.

The manner in which the probable scrap value of the conductors at the expiration of the "life" period affects the percentage to be allowed for depreciation will be explained in Art. 33.

32. Sundry Cost Items Affecting Economic Transmission Voltage.—It should be clearly understood that the foregoing articles deal only with the determination of the correct size of conductors based on certain assumptions as regards voltage and power to be transmitted. The cost of generating and transforming plant and buildings, *as influenced by the voltage*, must be carefully considered, together with the type and cost of pole line, so far as these are influenced by the size of the conductors. The character of the country, too, will have some bearing on the design of the transmission line, and the final choice of voltage may depend to some extent upon whether a wood-pole line with comparatively short spans and (preferably) small spacing between wires is likely to be more economical than a line with steel towers which will permit of longer spans with wider spacing between wires. In other words, the total cost of the whole undertaking and the total annual losses of energy from all sources, as influenced by any change of voltage, must be considered before the line pressure as given by formula (22) can be definitively adopted as being the most economical for the undertaking considered as a whole.

When figuring on the best voltage for any particular scheme, the capital cost of all works, buildings, or apparatus, which is likely to be influenced by the transmission line pressure, together with all operating and maintenance charges which may be similarly influenced, must be taken into account. It will usually be found convenient to reduce all such costs or differences of cost to the basis of annual charges.

The cost of a generating station of medium size, complete with all plant and machinery, but not including transmission line, may be anything from $50 to $250 per kilowatt installed. It will depend on total output, that is, on the size of the station, on location, and transport and labor facilities. The cost of a hydroelectric station will depend on the head of water, the amount of rock excavation, the size of dam, length of tunnels and penstocks, etc.

The figures given in the accompanying table are approximate costs per kilowatt (excluding transmission line and all hydraulic works outside the power station buildings) for a medium-head hydroelectric development suitable for a total output in the neighborhood of 10,000 kw. to be transmitted over two outgoing three-phase feeders. The usefulness of these figures lies mainly in the indication they give of the probable differences in cost with the variation of transmission line pressure.

APPROXIMATE COST OF HYDROELECTRIC PLANT

	Transmission line voltage		
	33,000	66,000	110,000
Power station building, including excavations	$ 9.90	$10.00	$10.10
Receiving station buildings including outdoor substations (if any)	2.95	3.00	3.15
Switchgear (both ends of line)	1.75	2.00	3.00
Lightning arresters	0.55	1.00	2.00
Transformers (both ends of line)	8.20	9.50	12.00
Generators and exciters	15.00	15.00	15.00
Cables in buildings, entering bushings, etc	2.00	2.00	2.50
Crane, sundries and accessories, including cost of preliminary work	4.80	5.00	5.25
Turbines and hydraulic equipment	16.00	16.00	16.00
Total cost per kilowatt	$61.15	$63.50	$69.00

Annual Charges Depending on Voltage.—The annual charges which should be considered when determining the most economical transmission voltage may be summarized as follows:

1. A percentage (interest and depreciation) on all capital expenditure, whether for generating station, transmission lines, or receiving stations, which is not constant for all voltages.

2. The yearly cost of the energy loss in the transmission line.

3. The yearly cost of energy loss in generators and transformers (the efficiency of the electrical plant will not necessarily be the same for all voltages).

4. The yearly cost of maintenance and operation. This may depend upon length of spans in transmission line, and on the necessary plant, switchgear, etc. to be attended to and kept in working order.

33. Depreciation.—Depreciation is the loss of value or commercial utility due to deterioration with age. The term may be used to cover loss of value resulting from very different causes.

A distinction should be made between *natural* and *functional* depreciation.

Natural or physical depreciation is the loss of value due to physical or chemical changes which, in time, will render the machine or plant practically useless. Atmospheric changes, alternations of heat and cold, wear and tear, erosion, rust, decay, and electrolysis are causes of natural depreciation.

Functional depreciation is the loss of value due to the fact that, with the lapse of time, the machine, plant, or structure under consideration does not function as efficiently as when it was first put into use, or as efficiently as it should function to compete with improved methods or apparatus. It may become inadequate owing to rapid growth in the demand for the service which it is intended to render; or it may become obsolete. Thus, functional depreciation may be due either to *inadequacy* or to *obsolescence*. A machine or structure becomes obsolete owing to scientific or artistic developments, *i.e.*, inventions. It is practically impossible to predict a future state of development in any branch of engineering, and the proper amount to allow for depreciation is largely a matter of guesswork based upon previous experience. A sinking fund for the creation of a depreciation reserve should be formed by placing annually at compound interest a certain sum of money which, at the end of the estimated life of the structure or plant, will reproduce the sum originally invested. The formula for calculating depreciation is

$$D = \frac{100r}{(1 + r)^n - 1} \qquad (23)$$

where r is the interest rate, n is the estimated "life" of the plant or machine, being the number of years during which interest is to accumulate, and D is the amount in dollars to be set aside at the end of each year in order to produce $100 at the end of n years.

The accompanying table gives values of D for interest rates of 4, 5, and 6 per cent. It is assumed that, at the expiration of the term of years, the value of the works or materials under consideration will be nil.

Although it is rarely necessary to consider scrap values, an exception should be made in the case of copper conductors for transmission lines. If P is the price originally paid for the conductors including the cost of erection, and S is the estimated net value of the scrap copper at the end of n years, it follows that the sum to be provided by the annual allowance for depreciation

Depreciation Table

(On basis of 4, 5, or 6 per cent compound interest earned by money put aside annually)

Life, years	Depreciation, per cent		
	4 per cent interest	5 per cent interest	6 per cent interest
5	18.45	18.05	17.75
6	15.10	14.70	14.30
7	12.65	12.25	11.90
8	10.85	10.45	10.10
10	8.32	7.95	7.58
12	6.65	6.28	5.92
14	5.47	5.10	4.76
16	4.58	4.23	3.90
18	3.90	3.55	3.24
20	3.35	3.02	2.72
22	2.92	2.60	2.30
24	2.56	2.25	1.97
26	2.25	1.96	1.69
28	2.00	1.71	1.46
30	1.78	1.505	1.265
32	1.595	1.325	1.100
34	1.430	1.175	0.960
36	1.285	1.045	0.840
40	1.050	0.828	0.646
45	0.827	0.627	0.470
50	0.655	0.477	0.344

is no longer P, but $P - S$. Therefore the percentage depreciation should be

$$D' = D\left(\frac{P - S}{P}\right) \tag{24}$$

where D is the depreciation as given by the table or calculated by formula (23), and D' is the proper depreciation to allow when the scrap value is taken into account.

Example 7. *Illustrating Application of Formula* (24).—The cost of No. 00 copper conductor, delivered on site, is 20 cts. per pound, and the cost of stringing the three conductors of a three-phase transmission is \$100 per mile. The "life" of the line is estimated at 16 years at the termination of which the scrap value of the salvaged copper is assumed to be 13 cts. per pound. The estimated cost of labor salvaging the copper is \$50 per mile of line. From the wire tables the weight per mile of No. 00 copper conductor is found to be 2,165 lb. The cost per pound for erection is, therefore, $\frac{100}{2{,}165 \times 3} = 1.53$ cts., and the labor cost of salvaging the copper is 0.765 ct. per lb.

Assuming 5 per cent interest on the money set aside annually, the percentage depreciation as given in the table is 4.23 which, when corrected by formula (24), gives

$$D' = 4.23\left(\frac{21.53 - 12.235}{21.53}\right) = 2.025$$

This would, therefore, be the proper percentage to allow for depreciation on the basis of the estimated value of the scrap copper.

The "life" in years of any part of a machine or structure is very difficult to estimate. It is here that the distinction between natural and functional depreciation becomes important, because whichever one appears to indicate the shortest life should be considered to the exclusion of the other. Thus, the life of wood poles—liable to decay and attacks by insects—will lead to the allowance for natural depreciation being larger than for functional depreciation, and the latter can, therefore, be ignored. But there are many kinds of plants, such as generators of inefficient design or insufficient capacity, of which the life determined on a basis of functional depreciation is shorter than the probable period during which the cost of maintenance and repairs would not be excessive, and it is then the natural depreciation which should be ignored.

34. Method of Determining Most Economical Transmission Voltage. *Example* 8.—Consider the case of a typical medium-head hydroelectric power station:

Distance of transmission = 50 miles.

Three-phase line with copper conductors.

Cost of copper conductors = $20 per 100 lb.

Power demanded = 15,000 hp. or 11,200 kw. (It is assumed that this power will be required continuously day and night for industrial purposes, and that it is the probable limit of the water power available.)

Power factor = 0.8.

Selling price of power = $21 per horsepower-year.

Interest on capital invested; allow 7 per cent.

The economic IR drop per mile of single conductor as given by formula (21*a*) is

$$IR = 95.5\sqrt{\frac{aC_c}{C_e}}$$

Where C_c is the price in dollars of 1-lb. weight of conductor (in this example $C_c = 0.2$), a is the percentage to cover annual depreciation and interest on cost of conductors, and C_e is the cost per kilowatt-year of the wasted energy. The proper value for a may be arrived at by estimating the term of years corresponding to the life of the conductors, at the end of which they are supposed to be of no value. Taking 16 years as the life of the conductors, the depreciation to be allowed according to the table is 4.23, which would be the proper value to take on the basis of 5 per cent compound interest if the copper wire has no scrap value at the end of this time. It is very difficult to estimate the scrap value of conductors 16 years ahead. Apart from the market quotations which may then determine the price per pound of the metal, the fact that the transmission line is very likely a long way from the place where there is a demand for the copper must not be overlooked. Not only must the labor cost of removing the wires from the poles, together with the transportation charges, be deducted from the price obtainable for the copper, but a further deduction should usually be made to cover, in whole or in part, the cost of stringing the new conductors. Assuming the net amount likely to be obtained from the sale of the scrap copper to be $6 per 100 lb., the proper allowance for depreciation will be

$$\frac{4.23 \times 14}{20} = 2.96$$

which makes

$$a = 7 + 2.96 = \text{say, } 10 \text{ per cent.}$$

With regard to C_e, if the demand for power were equal to the available supply from the time the power plant is put into operation, the works cost of waste power would be the same as the selling price; but, if it is assumed that the supply exceeds the demand during the first 4 years of operation, and that the cost of waste power during this period is only $7 per horsepower-year,[1] the average cost of wasted power during the 16 year life of the conductors should be arrived at by estimating the current and power loss for each year that the plant is in operation.

[1] The actual works cost of the wasted power is always difficult to determine exactly. It must, however, be remembered that even with unlimited power, and no appreciable increase in maintenance and operating charges with increase of losses, the greater capital cost of the plant installed to provide this waste power has to be taken into account and expressed in the form of an annual charge per kilowatt wasted, whether this waste occurs in the generating and transforming plant or in the line itself.

A first approximation to the required line voltage may be obtained by formula (22):

$$\text{Kilovolts} = 5.5\sqrt{\text{distance in miles} + \frac{\text{kilowatts}}{100}}$$

$$= 5.5\sqrt{50 + \frac{11{,}200}{100}}$$

$$= 70$$

The current will be

$$I = \frac{\text{kilowatts transmitted}}{\sqrt{3} \times 70 \times 0.8}$$

$$= \frac{\text{kilowatts}}{97}.$$

The line losses will be proportional to I^2, and in order to arrive at a suitable value for C_e for use in formula (21*a*), the demand for power during the first 4 years of operation (before the hydraulic plant is utilized to its possible limit) should be estimated, and a table constructed as below. An average figure may be assumed for power supplied during any given period of 12 months.

1	2	3	4
Period	Estimated kilowatts demanded	Current I	$I^2 \times$ years
First year	4,000	41.3	1,710
Second year	5,000	51.6	2,660
Third year	6,000	61.8	3,820
Fourth year	8,000	82.5	6,800
Fifth to sixteenth year	11,200	115.5	160,000

Total = 174,990 or, say, 175,000

The total of the figures in the last column covering the 4 years during which the cost of waste power is estimated at $7 per horsepower-year, is 14,990, or, say, 15,000, as compared with 160,000 for the period of 12 years during which the cost of the wasted power will be $21 per horsepower-year. A reasonable value to take for C_e is, therefore,

$$C_e = \frac{(15 \times 7) + (160 \times 21)}{175 \times 0.746} = \$26.5$$

where the figure 0.746 is merely for the purpose of converting cost per horsepower into cost per kilowatt.

The economic resistance pressure drop, by formula (21*a*) is, therefore,

$$= 95.5\sqrt{\frac{10 \times 0.2}{26.5}}$$
$$= 26.2 \text{ volts per mile}$$

It is well to note that the economic voltage drop does not correspond, in this particular example, to the full-load ohmic drop of pressure. The current which causes the ohmic drop of 26.2 volts per mile may be calculated as follows. The average value of I^2 is the total of column 4 in the above table, divided by the number of years, namely, $\frac{175,000}{16} = 10,930$, the square root of which is 104.5, and this is the figure for current to be used in the preliminary power-loss calculations, instead of 115.5 which is the full-load current. The line may, therefore, be considered as transmitting continuously $\sqrt{3} \times 70 \times 0.8 \times 104.5 = 10,130$ or, say, 10,000 kilowatts.

On the basis of a current $I = 104.5$ amp. and an IR drop of 26.2 volts per mile, it follows that the conductor should have a resistance of $\frac{26.2}{104.5} = 0.251$ ohm per mile. The nearest standard size of copper conductor is No. 0000 with a resistance of 0.26 ohm at 60°F. The weight per mile is 3,448 lb.; therefore, at 20 cts. per pound, the cost per mile is $3,448 \times 0.2 = \$689.60$ and the annual cost of the conductor material for the complete 50-mile three-phase line will be $0.09 \times 689.6 \times 3 \times 50 = \$9,300$. The total annual charges, including losses, assuming Kelvin's law of economy to be satisfied, will be twice this amount, or $18,600.

For a particular value of the economic IR drop—in this example 26.2 volts per mile of conductor—the proper size of wire for any other transmission voltage is easily found. The economic resistance per mile for a pressure of 70,000 volts (current = 104.5 amp.) was found to be 0.251 ohm. Therefore, for any other line voltage E, the current would be $104.5 \times \frac{70,000}{E}$. If the line voltage is 88,000 instead of 70,000, the economic ohms per mile would be 0.315, the nearest standard size of conductor being No. 000, with a weight of 2,730 lb. per mile. The total annual charges on the line would be $18,600 \times \frac{2,730}{3,448} = \$14,700$.

By raising the voltage to 110,000, the economic resistance will be $0.251 \times \frac{110}{70} = 0.394$ ohm per mile, the nearest standard size of copper conductor being No. 00 with a weight of 2,165 lb. per mile and a total annual cost of $18{,}600 \times \frac{2{,}165}{3{,}448} = \$11{,}650$.

Thus, by increasing the transmission line pressure from 70 to 110 kv., an annual saving of $18{,}600 - 11{,}650 = \$6{,}950$ is effected; but it remains to be seen whether or not the increased cost of other portions of the complete plant, *due to the raising of the line pressure*, will be less than the saving due to reduced weight of copper and smaller I^2R losses in the line.

In order to take into account the first cost, life, annual maintenance, and operating charges of every portion of the complete undertaking which may be affected by a change in the transmission voltage, the costs—worked out on an annual basis—may be arranged in tabular form as here shown. This tabulation shows clearly that the total charges for the 70,000-volt equipment are lower than for the two higher voltage equipments. The total annual costs and the trend of the various cost items indicate that a voltage appreciably lower than 70,000 would almost certainly lead to a higher annual cost. Thus, a pressure in the neighborhood of 70 kw., for which standard apparatus could be purchased, is the most economical for the particular transmission line considered in this example.

It will be understood that the accompanying estimate of total annual charges of the three selected voltages does not include any items other than those that are liable to vary with changes in the line voltage. An estimate covering the complete undertaking would, in addition to the items named, have to take account of riparian rights for dam, reservoir, etc.; preliminary legal and other expenses; cost of providing proper access for materials to site of works; dam and hydraulic works outside station building; turbines; electric generators and exciters; auxiliary plant; sundries and contingencies.

In the case of a short-distance transmission with a line pressure not exceeding 13,200 volts, and the possibility of winding the generators for the full pressure, the relative costs and efficiencies of generators wound for different voltages should be taken into account.

EXAMPLE 8.— SUMMARY OF COSTS AT DIFFERENT VOLTAGES

Portion of complete undertaking affected by change of voltage	Estimated life, years	Depreciation (from tables)	Depreciation plus 6 per cent interest	Total costs			Annual charges		
				70 kv.	88 kv.	110 kv.	70 kv.	88 kv.	110 kv.
Line conductors (copper) of economic section	16	.....	.....	....	....	....	$18,600	$14,700	$11,650
Steel-tower transmission line (without conductors)	20	3.02	9.02	$137,500	$171,000	$205,000	12,400	15,400	18,500
Generating station buildings	40	0.828	6.828	100,000	100,300	100,800	6,830	6,850	6,880
Substation buildings	26	1.96	7.96	30,000	30,500	31,200	2,390	2,430	2,480
Transformers (both ends of line)	20	3.02	9.02	100,000	106,000	120,000	9,020	9,560	10,800
Switchgear, lightning arresters, cables in buildings, and entering bushings	14	5.10	11.10	50,000	58,000	73,000	5,550	6,440	8,100
Assume unaltered: Yearly cost of energy lost in generators and transformers; Yearly cost of operation and maintenance; Right of way and clearing.						Total annual charges	$54,790	$55,380	$58,410

35. Economics of Power-factor Correction.—If energy is transmitted through a circuit of resistance R ohms, the loss of power is $w = I^2R$, while the useful power is $W = EI \cos \theta$ where E = the voltage, I = the current in the circuit, and $\cos \theta$ = the power factor. Thus $I = \frac{W}{E \cos \theta}$, and the losses in the circuit may be written $w = \frac{W^2R}{E^2 \cos^2 \theta}$ which is similar to formula (6) developed in Art. 9 of Chap. I. Given a definite amount of power transmitted at constant voltage, the I^2R losses are seen to be inversely proportional to the square of the power factor. Thus, if the losses are 100 kw. when the power factor is 0.65, they may be reduced to $100 \times \frac{(65)^2}{(95)^2} = 46.7$ kw. if the power factor can be raised to 0.95.

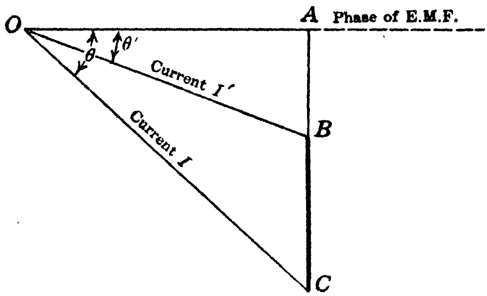


FIG. 17.—Vector diagram for power factor correction.

If, on the other hand, the losses in the circuit remain the same, an improvement in power factor means an increase in the kilowatt capacity of the generating plant and transmission line.

A low power factor, which is due to inductive reactance in the circuit, may be corrected by installing condensers to take a leading current component which will partly or wholly balance the lagging current component due to inductive reactance. Static condensers may be used, or synchronous condensers, or synchronous motors with variable field excitation so that leading current components may be drawn from the line.

Condensers are rarely used for the sole purpose of reducing I^2R losses; the question of power factor control on large high-voltage systems is usually considered in connection with pressure regulation. From the economic point of view, the fundamental principle involved is simply this: Will the entire annual cost due to power-factor correction by the installation of condensers be less than the annual saving due to reduction of energy losses? As an alternative, the possibility of larger conductors should be considered, and the cost of this method of reducing I^2R loss compared with that of the condenser equipment.

Calculation of Required Condenser Capacity.—In the diagram Fig. 17, lines drawn horizontally represent in-phase components of current (or power), while lines drawn vertically represent reactive components of current (or power). If $\cos\theta$ is the power factor before correction and $\cos\theta'$ is the power factor after correction, it follows that the reactive condenser current (or volt-ampere capacity of the condenser) is indicated by the length BC in the diagram.[1]

The delivered power is $W = EI\cos\theta = EI'\cos\theta'$, and the ratio of the total (line) currents is

$$\frac{I'}{I} = \frac{\cos\theta}{\cos\theta'}$$

The ratio of I^2R losses before and after power-factor correction is

$$\frac{I'^2}{I^2} = \left(\frac{\cos\theta}{\cos\theta'}\right)^2 \qquad (25)$$

for the same kilowatts of delivered power.

The reactive current taken by the condensers is

$$BC = OA(\tan\theta - \tan\theta')$$

or

Condenser capacity, in kilovolt-amperes =

power delivered, in kilowatts $(\tan\theta - \tan\theta')$ (26)

Calculations for power-factor correction may conveniently be made without reference to trigonometrical tables by sketching a diagram such as Fig. 17 and then calculating the in-phase and reactive components separately.

Example 9. *Reduction of Losses by Power-factor Correction.*—Suppose a 10,000-kw. load of 65 per cent (lagging) power factor is delivered through a transmission line in which the losses are 10 per cent, or 1,000 kw. What is the necessary kilovolt-ampere capacity of static or synchronous condensers to be connected at the receiving end in parallel with the load, in order that the line losses shall be reduced to 700 kw.?

Referring to Fig. 17, and considering the vectors to represent kilovolt-amperes instead of amperes, note that if the kilowatts

[1] A more detailed treatment of the effects due to capacity in a circuit containing resistance and inductive reactance will be found in Chap. IX.

are $OA = 10{,}000$, the vector OC is $\frac{10{,}000}{0.65} = 15{,}400$, and the reactive kilovolt-amperes with the original power factor are

$$AC = \sqrt{(15{,}400)^2 - (10{,}000)^2} = 11{,}700$$

In order to obtain the desired reduction of losses, write

$$(OB)^2 = \frac{700}{1{,}000}(OC)^2$$

or

$$(OA)^2 + (AB)^2 = 0.7(OC)^2$$

Thus $AB = \sqrt{0.7(15{,}400)^2 - (10{,}000)^2} = 8{,}120$ kv.-a.

The reactive (leading) kilovolt-amperes required, being the condenser capacity to be provided, is therefore $(AC - AB) = 11{,}700 - 8{,}120 = 3{,}580$ kv.-a.

Example 10. *Comparative Costs of Condenser Equipment and Increased Size of Conductor.*—Consider a 66,000-volt three-phase transmission line 50 miles long supplying a 10,000-kw. load of 70 per cent power factor. The line consists of three No. 0000 copper conductors, the resistance per mile of single conductor (at 60°F.) being 0.26 ohm.

The current per conductor is $I = \frac{10{,}000}{\sqrt{3} \times 66 \times 0.7} = 125$ amp., and the total I^2R losses are $\frac{3 \times (125)^2 \times (0.26 \times 50)}{1{,}000} = 610$ kw.

Assuming the cost per kilowatt-year of the energy losses to be $25, the total *annual cost* of the energy lost in transmission is $610 \times 25 = \$15{,}250$.

Now suppose power-factor-correcting apparatus to be installed at the receiving end of the line to raise the power factor from 0.7 to 0.95, and see (1) what saving of annual cost (if any) can be effected by this means, and (2) how the installation of condensers compares with additional copper in the line if it is desired to provide for a heavier load at the receiving end.

1. Referring to Fig. 17 and formula (26), the factor by which the kilowatt load at the receiving end must be multiplied to obtain the necessary kilovolt-ampere capacity of the corrective apparatus (synchronous condenser) is the quantity ($\tan\theta - \tan\theta'$). If it is desired to avoid reference to trigonometrical tables, write

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{1 - \cos^2\theta}}{\cos\theta}$$

$$= \frac{\sqrt{1 - 0.49}}{0.7} = 1.02$$

Similarly,

$$\tan \theta' = \frac{\sqrt{1 - (0.95)^2}}{0.95} = 0.329$$

whence $\tan \theta - \tan \theta' = 0.691$

and the required capacity of the condenser equipment = 0.691 × 10,000 = 6,910 kv.-a.

On the basis of 12 per cent for interest and depreciation, the annual cost of this equipment would be approximately as follows:

7,000 kv.-a. synchronous condenser, in position, with necessary switchgear, etc., at $5 per kilovolt-ampere...	$35,000	
Interest and depreciation, 12 per cent..........................		$4,200
Cost of energy losses in condenser, say, 180 × 25.................		4,500
Attendance and upkeep of additional plant......................		300
Total annual cost of power-factor-correcting equipment.........		$9,000

Now see if this expenditure will be less than the saving effected by reduction of transmission line losses (the saving in transformer losses will be neglected as these are a small percentage of the line losses). By formula (25), the I^2R losses, after power-factor correction, will be $610 \times \frac{(70)^2}{(95)^2} = 330$ kw. (approximately), the saving being $610 - 330 = 280$ kw. The annual saving due to reduction of transmission line losses is, therefore, $280 \times 25 =$ $7,000, which, on the basis of the above figures, indicates that it would not be economical to install condenser equipment merely for the purpose of reducing the line losses.

2. If it is assumed that the line losses remain as before, namely 610 kw., the installation of condenser equipment at a cost of $9,000 per annum makes it possible to transmit a greater amount of energy, namely, $10{,}000 \times \frac{95}{70} = 13{,}550$ kw., or an increase of 3,550 kw. Consider, now, whether or not it would be more economical to accomplish this by increasing the size of the conductors.

With the same transmission voltage and the original power factor at the receiving end of the line, the current corresponding to the greater load is $125 \times \frac{13{,}550}{10{,}000} = 169.5$ amp., and for the same I^2R losses, the resistance per mile must be $0.26 \times \frac{(125)^2}{(169.5)^2} =$ 0.1415 ohm. Since it would be difficult to replace the existing conductors without interruption of service, it will probably be advisable to string up another set of three conductors and connect these in parallel with the existing line. Assume that the necessary cross-arms are provided for a duplex line and that it is merely necessary to provide the insulators and additional wires. The calculated increase of cross-sectional area being very nearly equal to that of the existing line, it will be desirable to duplicate the line with the same size of conductor, namely No. 0000 stranded copper. The total weight of additional copper required will be 3,448 × 50 × 3 = 517,000 lb. Allowing 8 per cent for interest and depreciation, which takes account of the scrap value of the copper at the termination of the estimated "life" period (see Art. 33, Example 7), the annual cost of the conductors at 20 cts. per pound (which includes erection) is 0.08 × 0.2 × 517,000 = $8,270. The annual cost of the additional insulators will be about $1,400, making a total of $9,670 as against the $9,000 for the power-factor-correcting apparatus. Although these figures are slightly in favor of the condenser equipment, it is necessary to bear in mind that increase of conductor cross-section is a better guarantee of satisfactory and continuous service than the installation of machinery which may accidentally fall out of synchronism or give trouble through hot bearings, or other abnormal causes. It is true that static condensers are not subject to such accidents, but they would be too costly to use on large power systems, and, moreover, they have not the advantage of the synchronous condenser in which different values of leading power factor may be obtained by merely varying the field excitation.

In concluding this article on power-factor correction, it may be well to mention that this is a very important feature of modern transmission systems, and that the examples of calculations here given do not cover the ground thoroughly. They serve to illustrate the manner in which such problems should be solved, namely by comparing the annual costs of alternative schemes. Mathematical formulas are of little use in this connection because of the large number of items involved and the many forms which the problem assumes in practice. The economic reasons for installing condensers may be summed up as follows:

1. Reduction of transmission line and transformer losses by raising the power factor.

2. Reduction of the volt-ampere load on the generating plant, thus permitting a larger amount of useful power to be transmitted.

3. Regulation of voltage on long-distance transmission lines (this particular function of the condenser load will be taken up in Chap. X).

Apart from voltage regulation, the main points to be considered in determining the proper type of corrective apparatus and the amount of power-factor correction which will be economical are:

1. The annual cost of the corrective equipment.
2. The annual cost of the energy losses in same.
3. The annual saving through the reduction of line losses.
4. The annual saving in transformer losses.
5. The annual increase (if any) of operating expense due to the installation of power-factor-correcting devices.
6. The annual cost of other means (not involving condenser equipment) which will achieve the same end, *i.e.*, reduction of line losses with or without an increase in the amount of energy transmitted. The possibilities of raising the transmission voltage (to reduce the current) and of increasing the cross-section of the conductors (to reduce the resistance) should, therefore, be considered.

CHAPTER IV

MECHANICAL PRINCIPLES—OVERHEAD CONDUCTORS

36. Materials.—Under ordinary circumstances, the choice of material for the conductors of an overhead high-tension transmission line lies between copper and aluminum. Under certain conditions, as for the transmission of continuous currents or when the price of more suitable materials is abnormally high, galvanized iron or steel may prove satisfactory and economical; and compound wires or cables, such as copper-clad steel, and aluminum cables with galvanized steel core, are used where great mechanical strength is of more importance than high conductivity. Tables of resistances, sizes, and weights, and other physical properties of the materials will be found in the various engineering handbooks and manufacturers' catalogues; and only such particulars will be given here as may be useful for preliminary calculations.

Copper.—It is probably safe to assert that, *apart from the question of cost*, the high conductivity combined with the great strength and elasticity of hard-drawn copper gives this material the advantage over all others for use on the average high-tension electric transmission line.

The ultimate tensile strength of hard-drawn copper is greater per square inch of section in the smaller wires, being approximately as follows:

Gage number, B. & S.	Diameter, inch	Breaking stress, pounds per square inch
2	0.258	58,000
4	0.204	60,000
6	0.162	62,000
8	0.128	64,000
10	0.102	65,000
12	0.081	66,000
14	0.064	67,000
16	0.051	67,500

A stranded cable, in which the pitch is usually between 12 and 16 diameters of the cable, will generally break under a load slightly smaller than the combined breaking loads of the individual wires. The tensile strength of a stranded cable should, however, not be less than 90 per cent of the combined strengths of the single wires.

The elastic limit of hard-drawn copper wires is about 60 per cent of the breaking stress; but it may be as high as 70 per cent and even 75 per cent of the ultimate stress.

Aluminum.—The conductivity of hard-drawn aluminum wire is between 60 and 61½ per cent by Matthiessen's standard; pure copper being 100 per cent. The weight of an aluminum conductor is almost exactly half that of the copper conductor of equal resistance, and it is about 77 per cent as strong as the equivalent copper cable (safe working stress). Comparing aluminum of 61 per cent conductivity with copper of 97 per cent conductivity, the diameter of the equivalent aluminum cable would be 1.26 times the diameter of the copper cable.

The ultimate tensile stress of hard-drawn aluminum wire depends upon the size of wire and hardness; if carried beyond a certain point, high tensile strength is a disadvantage, because the conductivity is lowered and the wire becomes "short."

Approximate values for the ultimate stress of aluminum wires are given below:

Gage number, B. & S.	Diameter, inch	Breaking stress, pounds per square inch
4	0.204	23,000
6	0.162	24,000
8	0.128	25,500
10	0.102	27,000
12	0.081	29,000
14	0.064	32,000

It is usual to apply the same factor as for copper in estimating the ultimate strength of a stranded aluminum cable, this being assumed 90 per cent of the combined strength of the individual wires forming the strand. The elastic limit of hard-drawn aluminum wire is from 50 to 70 per cent of the breaking stress.

Iron and Steel.—The ordinary commercial galvanized-steel strand cable, as used for guy wires, has a breaking strength averaging 70,000 lb. per square inch, and a conductivity of about 11½ per cent by Matthiessen's scale. When used to convey alternating currents, the high permeability of iron increases the so-called skin effect, with the result that the resistance to the flow of current may be greatly increased, depending upon the size of the cable and the frequency. Apart from the greater loss of voltage due to apparent increase of *resistance* when iron wires are used with alternating currents, the loss of pressure due to increased *reactance* must also be taken into account. The *external* reactance is the same for a given diameter and spacing of wires whatever may be the material; but the *internal* reactance will obviously be much greater for a "magnetic" than for a "non-magnetic" conductor.

Experience with iron wires as conductors in Germany during the war has shown this material to be unsatisfactory, not only because of its insuitable electrical properties, but also because it is troublesome and costly to erect, the galvanizing is easily damaged, and joints are difficult to make.

Copper-clad Steel.—By welding a coating of copper on a steel wire, a compound wire known as hard-drawn copper-clad steel wire is produced. This has been well tested, and experience has shown it to be an excellent material for many purposes. The wire can be made up in the form of cables if desired, which, when used as conductors for overhead transmissions, will have greater strength than cables made entirely of copper, and lower resistance than cables made entirely of steel. The two metals are intimately and permanently welded together by means of a special copper-iron alloy, and the relative quantities so adjusted that the finished wire has a conductivity of 30 to 40 per cent of a copper wire of the same diameter. The ultimate tensile strength of commercial copper-clad wire of various sizes is approximately as below:

Gage number, B. & S.	Diameter, inch	Breaking load, pounds	Number of times stronger than copper of same diameter
000	0.410	8,200	1.2
0	0.325	5,600	1.24
2	0.258	4,000	1.33
4	0.204	2,700	1.35
6	0.162	1,800	1.40
8	0.128	1,200	1.45
10	0.102	800	1.50

By using a core of high-carbon steel, an exceptionally strong copper-clad steel wire has been produced which has a strength more than twice as great as hard-drawn copper of the same diameter. For extra-long spans, copper-clad steel compares favorably with bronze (silicon-bronze or phosphor-bronze) which is sometimes used—particularly in Europe—where high tensile strength is required.

Stranded Cables with Steel Wire Core.—The central wire of a stranded conductor may be galvanized steel, or a small-diameter steel cable may be used for the core. This increases the strength, especially in the case of aluminum cables, and a compound conductor of this sort is useful for long spans on an aluminum wire transmission line.

It is usual to neglect the current-carrying capacity of the steel core, and calculate the conductivity on the assumption that all the current is carried by the strands of the higher conductivity metal. Composite cables can be made of steel and copper wires, but the strength of hard-drawn copper is so great that the gain due to the addition of the steel core is comparatively small.

Seven-strand aluminum-steel cables are made with one central steel-wire and six aluminum wires, while 37-strand and 61-strand cables are made with central cores consisting of a seven-strand steel cable.

37. Physical Constants and Sizes of Commercial Conductors.—The accompanying tables gives the most important physical constants for various conductor materials. It will be noticed that aluminum has a larger temperature coefficient than copper. This has an important bearing on the economic length of span; the difference in sag between summer and winter temperatures is often considerable with aluminum conductors, but this difference is, of course, more noticeable on the shorter spans such as occur with a wood-pole construction: on long spans, the difference in sag due to temperature changes is very small, whatever metal is used.

An argument often advanced in favor of aluminum conductors is that the weight of these, for any given transmission scheme, is only about half that of copper. This is certainly an advantage in the handling of the wire, but otherwise it is at least counterbalanced by the fact that the wind effect is greater on the increased diameter and that the towers must often be higher than if copper is used, partly on account of the higher coefficient of expansion of aluminum, but mainly because of the lower permissible stress. The advantage of lighter weight is largely discounted by the fact that the equivalent aluminum conductor can only be drawn up to a tension equal to about three-quarters of the permissible

Physical Constants of Conductor Materials

Properties of conductor materials	Copper, hard-drawn stranded	Aluminum, hard-drawn stranded	Copper-clad steel, 40 per cent	Copper-clad steel, 30 per cent	Siemens-Martin steel, galvanized	Iron, E. B. B. grade, galvanized
Breaking stress, pounds per square inch of cross-section	50,000 to 65,000	22,000 to 30,000	60,000 to 100,000	60,000 to 100,000	60,000 to 80,000	
Breaking stress (average)	57,000	26,000	80,000	80,000	70,000	55,000
Maximum working stress (average)	28,000	13,000	40,000	40,000	33,000	26,000
Elastic limit (pounds per square inch)	35,000 to 40,000	14,000 to 17,000	50,000	50,000	38,000	30,000
[1] Modulus or coefficient of elasticity (Young's modulus) = E	15×10^6	9×10^6	19×10^6	20×10^6	29×10^6	25×10^6
Coefficient of linear expansion of wire per degree Fahrenheit = a	9.3×10^{-6}	12.8×10^{-5}	7×10^{-6}	6.8×10^{-6}	6.6×10^{-6}	6.6×10^{-6}
[2] Weight per foot per circular mil, pounds	3.06×10^{-6}	0.94×10^{-6}	2.85×10^{-6}	2.85×10^{-6}	2.66×10^{-6}	2.66×10^{-6}
[3] Coefficient k	0.486	0.148	0.447	0.447	0.427	0.423
[4] Resistance; ohms per mile per circular mil at 68°F. (20°C.)	57,500	93,200	143,000	191,000	660,000	344,000
Relative resistance	1.00	1.62	2.5	3.33	11.5	6.00
Relative conductivity	1.00	0.617	0.40	0.30	0.087	0.167

[1] Being ratio $\frac{\text{stress in pounds per square inch}}{\text{extension per unit length}}$.

[2] To obtain weight per foot of any size of wire, multiply these figures by the cross-sectional area expressed in circular mils.

[3] Used in sag calculations. k = 1.5 times the weight per cubic inch of conductor.

[4] To obtain resistance per mile of any size of wire, divide these figures by the number of circular mils in the cross-section.

maximum tension of the copper cable. On the other hand, the larger diameter of the aluminum cable may be an advantage on very high-pressure transmissions, because it raises the critical voltage at which corona losses become appreciable.

The physical constants given in the table include Young's modulus of elasticity (E) which is not quite the same for stranded cables as for solid wires. This quantity (E) is the constant in the expression $\delta l = \frac{Sl}{E}$ where δl is the increase in length of a wire of length l when submitted to a tensile stress of S lb. per square inch. This coefficient is approximately constant for all values of stress not exceeding the elastic limit, but in the case of stranded conductors it will depend somewhat upon the number of strands in the cable and the "lay" or pitch of the strands. Some engineers use a figure for Young's modulus as low as 12×10^6 for stranded copper conductors, but experience has shown that, after stretching or being submitted to a cycle of tensile loads due to strong winds or ice deposits, a stranded cable has very much the same modulus of elasticity as a solid wire. The danger of using a figure much lower than the suggested 15×10^6 as given in the table is that the actual tensions in the conductor may be greater than the calculated values. This uncertainty regarding the exact value of the ratio between load and elongation serves to emphasize the fact that mathematical niceties in engineering calculations are not always justified and may involve a waste of time and labor. The engineer should always bear in mind the nature of his data and the probable accuracy of his assumptions before using any but the simplest methods for solving his problems. This is especially true of the mechanical design of transmission lines where reasonable factors of safety are used, and it is impossible to predict the exact maximum load to which the structures may be submitted under severe climatic conditions.

The table of physical constants does not include steel-cored aluminum cables because the constants for these composite conductors depend upon the number of strands and the relative cross-section of iron and aluminum. In a seven-strand cable in which all the wires are of the same material, the center wire is subjected to a greater stress than the outer wires, and it is likely to break first. In steel-cored aluminum cables, the center wire or core is made of high-tensile steel capable of taking, if necessary, the whole of the tension in the conductor. The ultimate stress in the steel core is about 160,000 lb. per square inch, and the modulus of elasticity (up to an elastic limit of about 120,000 lb. per square inch) is $E = 29 \times 10^6$. The modulus for aluminum is about 9×10^6.

Modulus of Elasticity of Composite Cable.

Let S_s = stress in the steel core (pounds per square inch) and S_a = stress in the aluminum wires. Then, since the elastic elongation under load of a given length of the composite cable will be the same for the steel and for the aluminum,

$$\frac{S_s}{29 \times 10^6} = \frac{S_a}{9 \times 10^6}$$

whence $S_a = 0.31\ S_s$, the meaning of which is that for stresses within the elastic limit of either material, the stress in the aluminum wire will be 0.31 times the stress in the steel core. Thus, if the load on the cable is such as to stress the aluminum up to its elastic limit of, say, 15,000 lb. per square inch, the stress in the steel core will be about 48,000 lb. per square inch, which is also within the elastic limit of the high-grade steel used in these cables. The total tension in the cable is divided between the steel and aluminum strands in the ratio $\frac{1 \times 48{,}000}{6 \times 15{,}000} = 0.533$, from which it follows that the steel core takes about 35 per cent of the total tension while the aluminum wires take the balance of 65 per cent.

In order to calculate the modulus of elasticity of a seven-strand cable in which the cross-section of the steel is A and the cross-section of the aluminum is $6A$, write for the extension of the steel core under a stress S_s

$$\delta l = \frac{S_s l}{29 \times 10^6} \qquad (a)$$

Similarly, for the same extension of the composite cable,

$$\delta l = \left(\frac{P}{7A}\right)\frac{l}{E} \qquad (b)$$

where E is the required modulus and P is the total pull (tension in pounds) in the cable. This pull may be expressed in terms of the unit stresses by writing:

$$\begin{aligned} P &= AS_s + 6AS_a \\ &= AS_s + 6A \times 0.31S_s \\ &= 2.86\ AS_s \end{aligned}$$

Substituting in (*b*) and equating (*a*) and (*b*),

$$E = 11.8 \times 10^6 \text{ or, say, } 12 \times 10^6.$$

The modulus for composite cables with more than seven strands may be calculated in a similar manner. The following figures relating to steel-core aluminum cables are supplementary to the constants in the table on page 77.

PHYSICAL CONSTANTS OF STEEL-CORE ALUMINUM CABLE

	7-strand	37-strand	61-strand
Breaking stress, pounds per square inch (average)	47,000	53,000	43,000
Elastic limit, pounds per square inch	30,000	35,000	27,000
Modulus of elasticity (E)	12×10^6	13×10^6	11.5×10^6
Coefficient of linear expansion per degree Fahrenheit (a)	10.5×10^{-6}	10×10^{-6}	10.9×10^{-6}
Weight[1] per mile per circular mil	0.00735	0.00826	0.0068
Weight[1] per foot per circular mil	1.39×10^{-6}	1.56×10^{-6}	1.29×10^{-6}

[1] Multiply these figures by the circular mils of cross-section of the aluminum only.

It is customary to express the sizes of composite aluminum-steel conductors in terms of the aluminum cross-section only (in circular mils); the conductivity of the steel core is neglected and the resistance per circular mil is therefore assumed the same as for aluminum conductors without steel cores.

Seven-strand steel-core aluminum cables would be used for sizes up to No. 0000 (211,600 circular mils); 37-strand up to 350,000 or even as large as 500,000 circular mils, while 61-strand cables may have as much as 900,000 circular mils of aluminum cross-section.

Resistance of Standard Sizes of Conductors.—The accompanying wire table gives the approximate resistances and weights of the usual sizes of copper and aluminum cables and wires as used for overhead conductors. The resistances are calculated for a temperature of 68°F. (20°C.).

Resistance and Weight of Conductors at 68°F. (20°C.)
(Stranded conductors)

Size, circular mils and B. & S. gage numbers	Diameter, inch	Area of cross-section		Copper		Aluminum	
		Circular mils	Square inch	Ohms per mile	Weight per 1,000 ft., pounds	Ohms per mile	Weight per 1,000 ft., pounds
1,000,000	1.150	1,000,000	0.7854	0.0575	3,060	0.093	940
950,000	1.125	950,000	0.7461	0.0605	2,910	0.098	893
900,000	1.100	900,000	0.7068	0.0638	2,750	0.103	846
850,000	1.060	850,000	0.6676	0.0676	2,600	0.109	800
800,000	1.035	800,000	0.6283	0.0719	2,450	0.116	752
750,000	1.000	750,000	0.5890	0.0766	2,300	0.124	705
700,000	0.965	700,000	0.5498	0.0822	2,140	0.133	658
650,000	0.930	650,000	0.5105	0.0884	1,990	0.143	611
600,000	0.895	600,000	0.4712	0.0957	1,840	0.155	564
550,000	0.855	550,000	0.4320	0.1045	1,680	0.169	517
500,000	0.813	500,000	0.3927	0.115	1,530	0.186	470
450,000	0.771	450,000	0.3534	0.128	1,380	0.207	423
400,000	0.725	400,000	0.3141	0.144	1,225	0.233	376
350,000	0.680	350,000	0.2750	0.164	1,070	0.266	329
300,000	0.630	300,000	0.2356	0.192	920	0.310	282
250,000	0.575	250,000	0.1963	0.230	765	0.373	235
0000	0.522	211,600	0.1662	0.272	647	0.430	199
000	0.465	167,800	0.1318	0.343	514	0.556	158
00	0.414	133,100	0.1045	0.432	408	0.700	125
0	0.369	105,560	0.0830	0.545	323	0.882	99
1	0.328	83,690	0.0657	0.687	256	1.11	78.7
2	0.290	66,370	0.0521	0.868	203	1.40	62.4
3	0.260	52,630	0.0413	1.09	161	1.77	49.5
4	0.232	41,740	0.0328	1.38	128	2.23	39.2
5	0.207	33,090	0.0260	1.74	101	2.82	31.1
6	0.185	26,250	0.0206	2.19	81	3.54	24.7

(Single-wire conductors[1])

Size, circular mils and B. & S. gage numbers	Diameter, inch	Circular mils	Square inch	Copper ohms per mile	Copper weight per 1,000 ft., pounds	Aluminum ohms per mile	Aluminum weight per 1,000 ft., pounds
1	0.289	83,690	0.0657	0.673	251	1.09	77.0
2	0.258	66,370	0.0521	0.850	199	1.38	61.1
3	0.229	52,630	0.0413	1.07	158	1.74	48.6
4	0.204	41,740	0.0328	1.35	125	2.19	38.5
5	0.182	33,090	0.0260	1.70	99	2.77	30.5
6	0.162	26,250	0.0206	2.14	79	3.48	24.2

[1] Single wires of aluminum are very rarely used for overhead conductors.

38. Factor of Safety. Joints and Ties.—The use of too high a factor of safety is not good engineering; it leads to uneconomical and unsatisfactory designs and may even stand in the way of progressive development, as indeed was the case in England until quite recently. A factor of safety has little meaning unless it is considered in connection with what is to be understood by the maximum working load. The practice on the continent of America is to estimate the maximum probable load under the worst probable wind and ice conditions and provide for a tension not exceeding the elastic limit of the material under these extreme conditions. This practically amounts to a factor of safety of 2, which is sufficient for overhead conductors, bearing in mind that these would not necessarily be injured even if stretched slightly beyond their elastic limit. The calculation of stresses due to ice and wind loads will be taken up later.

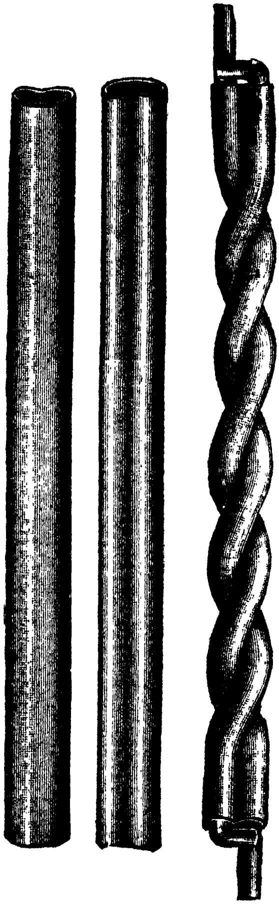


FIG. 18.—Type of joint for overhead conductors.

Joints in the conductor are usually made with the McIntyre sleeve as illustrated in Fig. 18 which shows a joint in a solid copper wire. The two ends of the cable are laid side by side in the sleeve which is from 20 to 30 in. long and of the same material as the conductor. Three complete twists are put into the joint by means of special clamps bolted to each end of the sleeve. These torsion splices have proved themselves entirely satisfactory, but they cannot be made easily on very large cables without injury to the strands. Suitable joints for large cables have been developed, one of these being the compression joint in which the cable ends are butted together in a cast sleeve which is then compressed at intervals along its length between specially shaped jaws of a portable hydraulic press. The metal is caused to flow into a solid block which results in a very durable and satisfactory joint.

Single-wire conductors and the smaller sizes of stranded cables supported by pin-type insulators are usually tied to the insulators by annealed wire of the same material as the conductor, but of smaller diameter. For a No. 4 conductor, a No. 6 tie wire would be used. For larger conductors, such as No. 0 to No. 0000, a No. 2 tie wire would be suitable. A description of the various ties as used in practice is beyond the scope of this book.

The attachment of the conductor to the lowest unit of a string of suspension insulators is usually made by means of special clamps. Much attention has been given to the design of these clamps in order to avoid damage to the conductor through bending at or near the point of attachment to the insulator. Manufacturers' catalogues should be consulted for particulars of the latest designs.

Trouble has been experienced owing to mechanical vibrations in overhead conductors. This is not due to exceptionally strong winds, but occurs where the conditions of weight, stiffness, tension, etc. are such a to establish a kind of mechanical resonance. Vibrations may be set up sufficient to cause injury, if not actual breakage, near the point of attachment to the insulator clamps. The manner in which such injury may be prevented by attaching weights through flexible fastenings to the conductor near the points of suspension is explained by G. H. Stockbridge in the *Electrical World* of Dec. 26, 1925, and June 19, 1926.

39. Stresses in Overhead Conductors.—If a wire is stretched between two fixed points A and B lying in the same horizontal

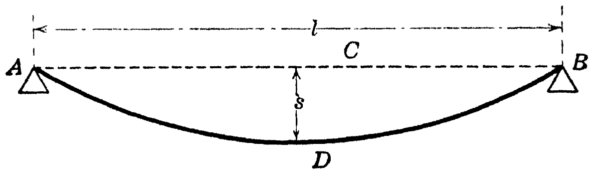


FIG. 19.—Wire hanging between two supports at the same elevation.

plane, and separated by a distance of l ft., there will be a certain *sag* of s ft. in the wire. This sag or deflection from the horizontal line AB will be greatest at the center of the span, and its value for a given length of span (l ft.) will depend upon the weight of the wire and the tension with which it has been drawn up. If the wire were perfectly uniform in cross-section, inextensible, and perfectly flexible, the curve ADB (Fig. 19) would be a catenary. It should be observed, however, that in this and subsequent diagrams, the sag s is shown much larger relatively to the span l than it would be on most practical transmission lines; and as the span l is generally very little shorter than the length of the wire between the suspension points A and B, no appreciable error is introduced by assuming the weight of the wire to be distributed uniformly along the horizontal line *ACB* instead of along the curve *ADB*. On this assumption the curve *ADB* becomes a parabola, and as the calculations are more easily made on the assumption of a parabolic curve than with the possibly more correct catenary, it is customary to use the formulas relating to the parabola for the solution of sag-tension problems. On very long spans, even with fairly large conductors, the actual curves assumed by the wires hanging in still air under the influence of their own weight only will usually approximate more nearly to the catenary than to the parabola, and in calculations on very long spans, such as river crossings with single spans exceeding 1,500 ft. in length, the catenary is the more correct curve to work with; but owing to the many more or less arbitrary assumptions that must necessarily be made regarding load conditions and physical constants, refinements of calculation in these and similar engineering problems are rarely justified unless they involve no additional time or labor.

In the articles immediately following, no definite shape of curve is assumed for the form taken by a wire suspended freely between two fixed supports, but the formulas for sag and tension calculations are developed by considering the weight of a perfectly flexible wire to be distributed uniformly over the straight-line distance between the two supports.

40. Graphical Statics Applied to Transmission Line Calculations.—Consider a mass of any irregular shape, suspended from the fixed points *A* and *B* by two perfectly flexible ties *AC* and *BD*, as shown in Fig. 20. The total weight may be represented by the force P_G acting vertically downward through the center of gravity O' and balanced by the forces P_A and P_B in the suspension cords. Since the mass is at rest, the total downward force P_G may be considered as equal and opposite to the total upward force which is the resultant of the forces P_A and P_B in the suspension cords, and, except for the special case of parallel forces, the lines *AC*, *BD*, and $O'P_G$ will necessarily meet at a common point *O*. The conditions to be satisfied by a system of forces in equilibrium are:

1. The vectorial sum of all the forces and reactions must be zero.

2. The sum of all moments taken about any point must be zero.

Condition 1 is illustrated by the closed triangle OPN of Fig. 20 in which the vectors OP, PN, and NO, respectively, represent in magnitude and direction the three forces P_G, P_B, and P_A. The length MN is a measure of the horizontal reactions at the points of support, necessarily equal, but opposite in direction regardless of whether or not the points A and B are at the same elevation. The vertical component of the reaction at the point A is MO, and at the point B, it is PM.

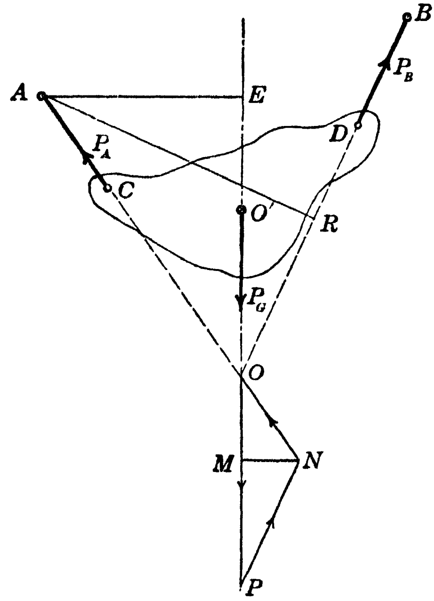


FIG. 20.—Mass suspended from two points—diagram of forces.

Condition 2 may be illustrated by taking moments about the point A where the moment due to the force P_A is zero. The condition for equilibrium at this point is, therefore, that the (clockwise) moment of the force P_G acting at the distance AE must be equal to the (counterclockwise) moment of the force P_B acting at a distance AR. Thus

$$P_G \times AE = P_B \times AR$$

whence the tension in the cord DB, expressed in terms of the weight P_G, is seen to be

$$P_B = P_G \times \frac{AE}{AR}$$

Consider now the portion AB (Fig. 21) of a perfectly flexible wire stretched between two fixed supports which are not necessarily the points A and B. The forces P_A and P_B will act in the direction of the wire at the points A and B or, in other words, the vectors AP_A and BP_B are tangential to the curve formed by the suspended wire.

The force of gravity $O'W$ acts through the center of gravity of the element of wire considered, and all three forces meet at a common point O. The vector diagram of the component forces is shown in the right-hand portion of Fig. 21.

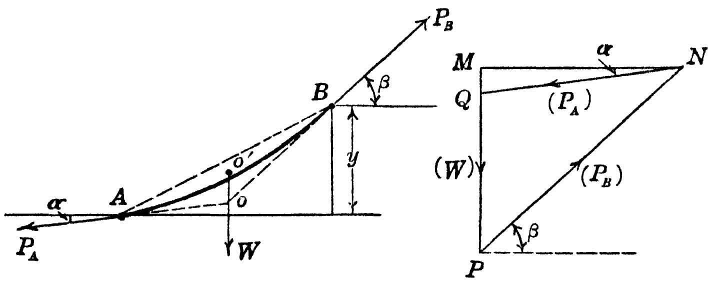


FIG. 21.—Diagram of forces in suspended wire.

Suppose the weight W, the magnitude of the force P_A, and the angle α to be known, and it is desired to find the magnitude and direction of the force P_B. From the vector diagram

$$MN = P_A \cos\alpha = P_B \cos\beta$$

whence

$$P_B = P_A \frac{\cos\alpha}{\cos\beta} \tag{27}$$

To solve for the angle β:

$$\tan\beta = \frac{MP}{MN} = \frac{P_A \sin\alpha + W}{P_A \cos\alpha} \tag{28}$$

or

$$\tan\beta = \tan\alpha + \frac{W}{P_A \cos\alpha} \tag{28a}$$

which, for the special case of $\alpha = 0$, becomes

$$(\text{when } \alpha = 0)\ \tan\beta = \frac{W}{P_A} \tag{29}$$

If it is desired to express β in terms of P_B instead of P_A, write

$$(\text{when } \alpha = 0)\ \sin\beta = \frac{W}{P_B} \tag{29a}$$

The above formulas are general and are independent of the exact position of the center of gravity O' of the section of conductor considered. It will be observed, however, that in order to express the difference in level (y) between the two points A and B in terms of the known quantities (the three forces W, P_A, and P_B), it is necessary to determine the position of the force of gravity W. This will depend upon the shape of the curve of the suspended wire, and the problem will be taken up later when developing the formulas for the catenary.

41. Relation between Sag and Tension in Overhead Conductors.—Consider a wire suspended between two fixed points A and B (Fig. 22) spaced l ft. apart horizontally with a difference of elevation of h ft. The angle θ between the straight line AB and the horizontal is, therefore, $\theta = \tan^{-1}\frac{h}{l}$. The maximum sag or deflection of the wire from the straight line AB being always small relatively to the length l, the total weight of the wire is assumed to be wl' where w = weight per foot of length and l' = straight-line distance between the points of suspension. There will be some point C on the straight line AB where the deflection ($s' = CD$) of the wire will be a maximum. This will be where the direction of the wire is parallel to AB. Let P = tension in wire at position of maximum deflection. Then, taking moments about A,

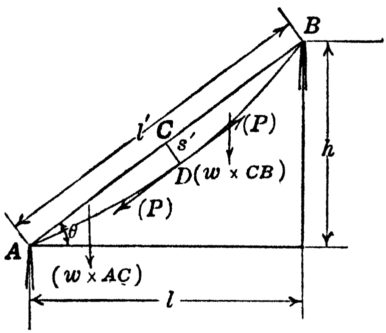


FIG. 22.—Suspended wire with supports at different elevations.

$$w(AC) \times \frac{(AC)\cos\theta}{2} = P \times s'$$

Taking moments about B,

$$w(CB) \times \frac{(CB)\cos\theta}{2} = P \times s'$$

Equating these two expressions, and noting that $AC = CB = \frac{1}{2}l' = \frac{l}{2\cos\theta}$,

it follows that

$$\text{Maximum deflection (at center of span) } s' = \frac{wl^2}{8P\cos\theta} \quad (30)$$

For the usual condition of supports on the same level (or even approximately on the same level), merely put $\theta = 0$, and the maximum sag (at center of span) is then

$$s = \frac{wl^2}{8P_h} \tag{30a}$$

where P_h is the horizontal component of the tension.

If S = tensile stress in the material, in pounds per square inch of cross-section, and A = area of cross-section of conductor in square inches, the quantity SA may be substituted for P in the above formulas. This leads to the expression

$$s = \left(\frac{w}{8A}\right)\frac{l^2}{S}$$

or

$$s = k\frac{l^2}{S} \tag{30b}$$

where k is constant for a given material. The numerical value of k is seen to be one-eighth of the weight in pounds of 12 cu. in. of conductor material, or 1.5 times the weight of a cubic inch. Values of k for different materials are given in the table on page 77.

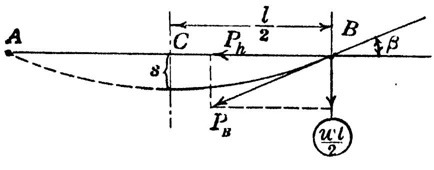


FIG. 23.—Suspended wire with supports at same elevation.

It is very rarely necessary to calculate the tension at any other point than the center of the span because, unless the sag is very great, the tension as approximately the same at all points in the span. The maximum tension occurs at the highest point of support where the angle β made by the wire with the horizontal is greatest. The relation between the maximum tension (at point of support) and the horizontal component of the tension when supports are at the same elevation is illustrated by Fig. 23. Considering one-half of the total span, it is seen that the whole of the weight of the wire in the half span CB acts vertically downward at the point of support B, whence

$$\tan \beta = \frac{wl}{2P_h}$$

which is the same as formula (29) seeing that the weight of wire in the section considered is $W = \frac{wl}{2}$, where w is the weight per foot assumed to be uniformly distributed over the straight-line distance AB. The maximum pull at B is

$$P_B = \frac{P_h}{\cos \beta}$$

a relation which is given by formula (27) and illustrated by Fig. 23.

If it is desired to avoid the use of trigonometrical formulas, the tension at the point B may be written:

$$P_B = \sqrt{P_h^2 + \left(\frac{wl}{2}\right)^2}$$

which simplifies into

$$P_B = P_h + ws \qquad (31)^1$$

The ratio of maximum tension (at point of support) to the tension at center of span, when the two supports are at the same elevation, is

$$\frac{P_B}{P_h} = \frac{P_h + ws}{P_h}$$

$$= 1 + \frac{8s^2}{l^2} \text{ (approximately)} \qquad (32)$$

Example 11. *Sag-tension Calculations.*—Calculate the sag (at center of span) of a copper conductor strung between supports, at the same elevation, spaced 400 ft. apart, given that the tensile stress in the material is 28,000 lb. per square inch.

Let A be the cross-section of the conductor in square inches; then the weight per foot of length is $w = A \times 12 \times 0.32$, where 0.32 is the weight per cubic inch of copper. The tension in the conductor is $P = 28{,}000 \times A$. Assume this to be the pull at center of span, and by formula (30a)

$$s = \frac{A \times 12 \times 0.32 \times (400)^2}{8 \times 28{,}000 \times A} = 2.75 \text{ ft.}$$

[1] When the quantity δ is small as compared with unity, it is permissible to write:

$$(1 + \delta)^m = 1 + m\delta$$

where m is any number. Thus:

$$(1 + \delta)^{1/2} = 1 + \frac{1}{2}\delta$$

and if b is small relatively to a in the expression $\sqrt{a^2 + b^2}$, write:

$$\sqrt{a^2 + b^2} = a\sqrt{1 + \left(\frac{b}{a}\right)^2} = a\left(1 + \frac{b^2}{2a^2}\right) = a + \frac{b^2}{2a}$$

which explains how the formula (31) has been obtained.

It is assumed that the stress of 28,000 lb. per square inch is at the center of the span because, as previously mentioned, the stress is usually approximately the same at all points of an overhead conductor. The maximum stress (at points of suspension) is given by formula (32) thus:

$$\frac{P_B}{A} = 28{,}000 + 28{,}000 \times 8\left(\frac{2.75}{400}\right)^2$$
$$= 28{,}010.6 \text{ lb. per square inch}$$

which is sufficiently near to the specified 28,000 lb. per square inch to justify the use of this figure for the stress at center of span.

42. Supports at Different Elevations.—In Fig. 24, the difference in elevation of the supports is h ft. The span measured horizontally is l ft., and the straight-line distance between the supports A and B is l' ft. If θ is the angle which the line AB makes with the horizontal,

$$\tan \theta = \frac{h}{l}$$

and

$$\cos \theta = \frac{l}{l'}$$

The weight of wire per foot is w lb., and it is assumed that the total weight is wl' lb., distributed uniformly over the straight-line distance $l' = AB$. This force acts through the point C midway between A and B. The other known quantity is the magnitude (but not the direction) of the maximum tension in the wire: This is the force P_B acting through the highest point of support (B).

Draw the vertical line OP_G to represent the total force of gravity (wl'). Through its center C draw the line AB (or a line parallel to AB) making an angle θ with the horizontal. Then from O as a center describe an arc of radius OP_B equal to the known force P_B. Its intersection with AC locates the head of the vector OP_B. Complete the parallelogram of forces and drop the perpendiculars P_AM and P_BN on to the vertical OP_G. These are a measure of the (equal) horizontal components of the reactions at the two points of support, and also of the total tension in the wire at the lowest point D. The length MO is the vertical component of the reaction at the lower support A; while NO is the vertical reaction at B. Their sum is, of course, equal to OP_G. The angle β which the vector P_B makes with the horizontal is the slope of the wire where it leaves the point B.

The construction as above described satisfies the fundamental conditions of equilibrium, namely, that all vertical forces shall balance; that all horizontal forces shall balance, and that the sum of all moments taken about any point shall be equal to zero.

In practice it is not convenient to solve such problems by actual measurement of quantities plotted to scale on drawing paper, because the horizontal components of the forces are usually very much greater than the vertical components. A trigonometrical solution is, therefore, desirable.

In Fig. 24, the known quantities are the two sides OP_B and OC of the triangle OCP_B of which the angle OCP_B is also known, being equal to $\theta + 90$ deg.

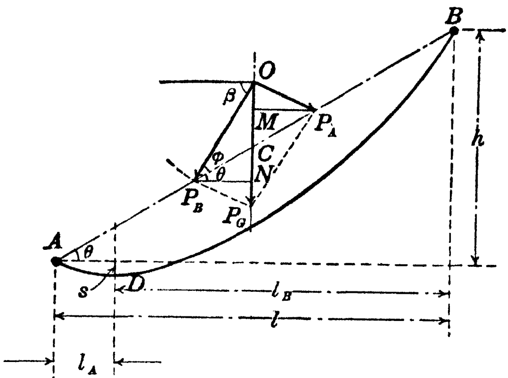


FIG. 24.—Diagram of forces—supports at different elevations.

The angle OP_BC or φ may be calculated from the relation

$$\sin \varphi = \frac{OC}{P_B} \sin (90 + \theta) = \frac{wl'}{2P_B} \cos \theta$$
$$= \frac{wl}{2P_B}$$

and the angle β which the wire at the upper support makes with the horizontal is, therefore,

$$\beta = \theta + \sin^{-1} \left(\frac{wl}{2P_B}\right) \qquad (33)$$

With the aid of trigonometrical tables or a slide rule, the values of $\sin \beta$ and $\cos \beta$ can be obtained, and the other components of the force diagram readily calculated. Thus, the horizontal component at either point of support, which is also the total tension in the wire at the point (if any) where the slope is zero, is

$$P_h = P_B \cos \beta \qquad (34)$$

which is a particular form of the general formula (27). The vertical component of the reaction at the highest point B, where this reaction is greatest, is

$$P_{BV} = NO = P_B \sin \beta \tag{35}$$

The weight supported by the lower pole is

$$P_{AV} = MO = wl' - P_{BV} \tag{36}$$

This may, obviously, be a negative quantity, *i.e.*, the wire may exert an upward pull on the lower support. In that case the sag s below the point A will be zero, and this would correspond to the usual condition on a steep incline, or on a moderate incline if the spans are short.

Position of Lowest Point of Span. Supports at Different Elevations.—The position of the lowest point D in the span will be determined by the vertical weight carried by each of the two supports. Thus, referring to Fig. 24, the vertical component of the forces acting at B is simply the weight of that portion of the wire comprised between the support B and the point D where the tension in the wire has no vertical component. The horizontal distance of the point D from B (in feet) is

$$l_B = l \times \frac{ON}{OP_G}$$

$$= \frac{lP_{BV}}{wl'}$$

$$= \frac{P_{BV} \cos \theta}{w} \tag{37}$$

This may give a distance for l_B which is greater than l. In that case the support A would be the lowest point in the span. The horizontal distance from the lower point of support (A) to the point D is $l_A = l - l_B$; which shows that l_A will be a negative quantity when l_B is greater than l.

If it is desired to express these formulas in terms of the horizontal component (P_h) of the tension, a reference to the force diagram in Fig. 24 will show that

$$P_G = 2\ (P_{BV} - P_h \tan \theta)$$

and since $P_G = wl'$, the vertical component of the tension at the upper support is

$$P_{BV} = \frac{wl'}{2} + P_h \tan \theta$$

$$= \frac{wl'}{2} + \frac{hP_h}{l} \tag{38}$$

Substituting in formula (37),

$$l_B = \frac{l}{2} + \frac{hP_h}{wl'} \tag{39}$$

Similarly

$$l_A = \frac{l}{2} - \frac{hP_h}{wl'} \tag{40}$$

These formulas can be solved without reference to trigonometrical tables, because, although $l' = \frac{l}{\cos \theta}$, it can also be expressed as $l' = \sqrt{l^2 + h^2} = l + \frac{h^2}{2l}$ when h is small in relation to l.

Formula (40) shows that if $\frac{hP_h}{wl'}$ is equal to $\frac{l}{2}$, the lowest point of the wire coincides with the lower support A, while, if the second term in the equation is greater than the first, l_A is negative, and there is a possibility of the resultant *upward* pull on the lower insulator at A being greater than the downward pull due to the weight of the wires in the adjoining span. It is well to bear this point in mind when considering an abrupt change in the grade of a transmission line.

Calculation of Vertical Sag with Supports at Different Elevations. The maximum sag or deflection at the center of a span, whether or not the points of support are at different elevations, is given by formula (30) which was developed in Art. 41. When it is desired to calculate the vertical sag of the wire below the points of support, the formula (30*a*) as calculated for spans with supports on the same level may be used for spans on an incline, provided the distance l_B of Fig. 24 is considered as half of a level span of which the sag is $(s + h)$. Thus,

$$(s + h) = \frac{wl_B^2}{2P_h \cos \theta} \tag{41}$$

Similarly

$$s = \frac{wl_A^2}{2P_h \cos \theta} \tag{42}$$

The appearance of cos θ in the denominator of formulas (41) and (42) is due to the fact that the weight of wire per foot of span *measured horizontally* has been assumed to be $\frac{w}{\cos \theta}$.

Example 12. *Illustrating Use of Formulas.*—Consider a span of 1,740 ft. measured horizontally, with a difference of level of

190 ft. between points of support. The conductor is copper cable of a cross-section of 350,000 circular mils, weighing 1,070 lb. per 1,000 ft. Calculate vertical and horizontal components of forces, also sag and position of lowest point of span, if the maximum tension in the cable due to its own weight only is 3,580 lb.

The known quantities are:

$$l = 1{,}740$$
$$h = 190$$
$$w = 1.07$$
$$P_B = 3{,}580$$

The unknown quantities are calculated as follows:

$$\tan \theta = \frac{190}{1{,}740} = 0.1092$$
$$\theta = 6° \; 14'$$
$$\cos \theta = 0.9941$$
$$\sin \varphi = \frac{wl}{2 P_B} = \frac{1.07 \times 1{,}740}{2 \times 3{,}580} = 0.26$$
$$\varphi = 15° \; 4'$$

By formula (33) $\beta = \theta + \varphi = 21° \; 18'$

$$\cos \beta = 0.9317$$
$$\sin \beta = 0.3632$$

By (34) $P_h = 3{,}580 \times 0.9317 = 3335 \text{ lb.}$

By (35) $P_{BV} = 3{,}580 \times 0.3632 = 1300 \text{ lb.}$

By (37) $l_B = \dfrac{1{,}300 \times 0.9941}{1.07} = 1207 \text{ ft.}$

By (41) $(s + h) = \dfrac{1.07 \times (1{,}207)^2}{2 \times 3{,}335 \times 0.9941} = 235 \text{ ft.}$

Thus the sag below the bottom support will be 235 − 190 = 45 ft. when the maximum tension is 3,580 lb.

If it is desired to avoid the use of trigonometrical tables, the procedure would be as follows; but the assumption now made is that the known tension is the horizontal component, or $P_h =$ 3,335 lb.

The straight-line distance between the two points of support is $l' = \sqrt{l^2 + h^2} = l + \dfrac{h^2}{2l} = 1{,}740 + \dfrac{(190)^2}{2 \times 1{,}740} = 1{,}750.4$ ft.

By formula (39)

$$l_B = \frac{1{,}740}{2} + \frac{190 \times 3{,}335}{1.07 \times 1{,}750.4} = 1{,}208 \text{ ft.}$$

and $\qquad l_A = 1{,}740 - 1{,}208 = 532$ ft.

The sag s measured from the lower support is given by formula (42) wherein $\cos\theta$ can be replaced by $\frac{l}{l'}$. Thus,

$$s = \frac{1.07(532)^2 \times 1{,}750.4}{2 \times 3{,}335 \times 1{,}740} = 45.7 \text{ ft.}$$

No high degree of accuracy is claimed for the results of the numerical examples, the calculations being made on the slide rule.

43. Extra-long Spans. The Catenary Formulas.—All the formulas developed in Arts. 41 and 42 are based upon the assumption that the weight of the suspended wire is distributed uniformly over the straight-line distance between the two points of support. This assumption is justified because, in practice, the maximum deflection of the suspended wire from the straight line is only a small percentage of the span length. It is only when the distance between supports is very great, resulting in a maximum deflection which is a large percentage of the total span, that appreciable errors may be introduced unless more accurate formulas are used.

The weight of a suspended wire is actually distributed uniformly along the wire itself and not along the straight-line distance between the points of support, and if in both cases the assumption of perfect flexibility is made, the formulas derived for the straight-line distribution of load are those of the parabola, while the more correct formulas are those of the catenary.

Although it is only on very rare occasions that it is necessary or even desirable to use the catenary formulas in the mechanical design of electric transmission lines, it seems nevertheless advisable to show how these formulas may be derived. Attempts to use formulas without knowing something about their origin and derivation, and without being familiar with the mathematical operations involved, will frequently lead to inaccuracy in results. It is, therefore, proposed to develop the catenary formulas in the same manner as the formulas based on the parabola were developed in Art. 41.

The curve in Fig. 25 represents the shape assumed by a perfectly flexible wire which is suspended between two fixed supports and of which the total weight is w lb. per foot of length. This weight may be that of the wire itself without additional load, or it may include an additional uniformly distributed load due to wind pressure or ice loading.

Let O be the lowest point of the suspended wire, and B any other point on the curve. Since the wire is assumed perfectly flexible, the forces P_h and P_B act in the direction of the tangents to the curve at the points O and B, respectively. The force P_h will, therefore, act in a horizontal direction which is that of the axis OX in Fig. 25, while there will be an angle β between the direction of the force P_B and the horizontal.

The weight of the section of wire OB is $W = w\lambda$ where λ is the length of the curve OB. This length of wire is in equilibrium, and the three forces P_h, P_B, and W will, therefore, meet at a common point M. The vector diagram of the three forces, moreover, forms a closed triangle as indicated in the right-hand portion of Fig. 25.

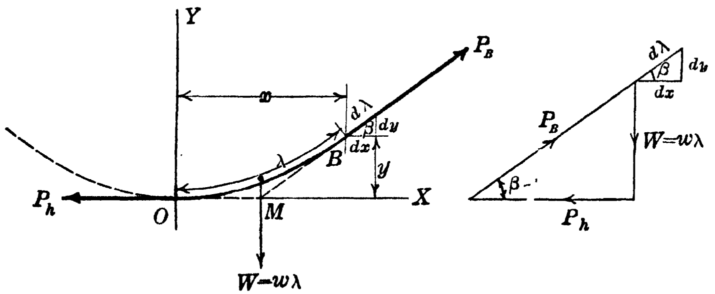


FIG. 25.—Illustrating development of caternary formulas.

A small increment of length $d\lambda$ in the direction of the force P_B leads to an increase of the ordinate y of amount

$$\begin{aligned} dy &= d\lambda \times \frac{w\lambda}{P_B} \\ &= d\lambda \times \frac{w\lambda}{\sqrt{(w\lambda)^2 + P_h^2}} \\ &= d\lambda\left(\frac{\lambda}{\sqrt{\lambda^2 + \left(\frac{P_h}{w}\right)^2}}\right) \end{aligned}$$

whence

$$y = \sqrt{\lambda^2 + \left(\frac{P_h}{w}\right)^2} + C$$

When $\lambda = 0$, $y = 0$, therefore $C = -\left(\frac{P_h}{w}\right)$, and if m is put in place of the quantity $\frac{P_h}{w}$, $y + m = \sqrt{\lambda^2 + m^2}$ (43)

This relation between the ordinate of the point considered and the length of the curve comprised between this point and the point where the tangent to the curve is horizontal is illustrated graphically in Fig. 26.

Draw BT parallel to the vertical (Y) axis, of length $BT = y + m = y_m$. On this line as a diameter, describe the semicircle TSB. With T as a center, describe an arc of radius $TS = m$, thus locating the point S where the two arcs cross. Join BS and note that, since BST is a right-angled triangle, with the side $ST = m$ and the hypotenuse $BT = m + y$, it follows that the

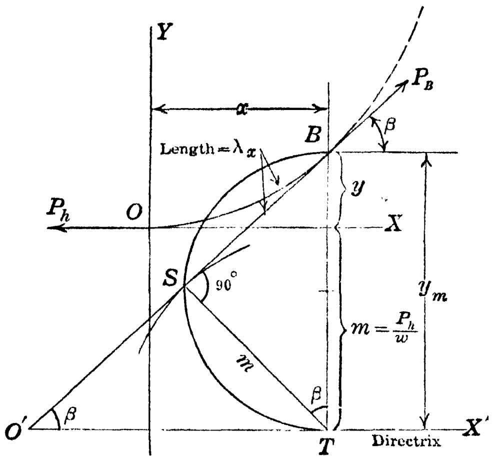


FIG. 26.—Diagram illustrating formula (43).

side $SB = \lambda$ = length of curve OB, because this length, as expressed by formula (43), is $\lambda = \sqrt{(y + m)^2 - m^2}$. Note also that angle BTS = angle $BO'T = \beta$, and $\tan \beta = \frac{\lambda}{m}$, which proves the line SB to be the tangent to the curve at the point B, because, from Fig. 25,

$$\tan \beta = \frac{w\lambda}{P_h} = \frac{w\lambda}{wm} = \frac{\lambda}{m}.$$

The formula (43) is of little practical value unless the length (λ) of the wire is expressed in terms of the distance x. Returning to Fig. 25, and proceeding as in solving for y,

$$dx = d\lambda \frac{P_h}{P_B} = d\lambda \frac{P_h}{\sqrt{(w\lambda)^2 + P_h^2}}$$

Dividing numerator and denominator by w, and substituting m for $\frac{P_h}{w}$,

$$dx = d\lambda \frac{m}{\sqrt{\lambda^2 + m^2}}$$

Thus,

$$x = m \int \frac{d\lambda}{\sqrt{\lambda^2 + m^2}} = m[\log_\epsilon(\lambda + \sqrt{\lambda^2 + m^2}) + C]$$

when $\lambda = 0$, $x = 0$, therefore $C = -\log_\epsilon m$.

and
$$x = m \log_\epsilon \left(\frac{\lambda + \sqrt{\lambda^2 + m^2}}{m} \right) \tag{44}$$

In order to solve for λ, write:

$$\epsilon^{\frac{x}{m}} = \frac{\lambda + \sqrt{\lambda^2 + m^2}}{m}$$

or
$$m\epsilon^{\frac{x}{m}} - \lambda = \sqrt{\lambda^2 + m^2}$$

Squaring both sides of this equation and solving for λ,

$$\lambda = \frac{m}{2}\left[\epsilon^{\frac{x}{m}} - \epsilon^{-\frac{x}{m}} \right] \tag{45}$$

which may also be written,

$$\lambda = m \sinh \frac{x}{m} \tag{45a}$$

It is now possible to express the vertical sag (y) in terms of x and m by substituting for λ in formula (43) its value as given by formulas (45) or (45a). Thus

$$y + m = \sqrt{\lambda^2 + m^2}$$

wherein the value of λ^2, as given by formula (45a), is $m^2 \sinh^2 \frac{x}{m}$, whence

$$y + m = m\sqrt{\sinh^2 \frac{x}{m} + 1}$$

$$= m\sqrt{\cosh^2 \frac{x}{m}}$$

$$= m \cosh \frac{x}{m} \tag{46}$$

or
$$y = m\left(\cosh \frac{x}{m} - 1 \right) \tag{46a}$$

In exponential form,

$$y + m = \frac{m}{2}\left(\epsilon^{\frac{x}{m}} + \epsilon^{-\frac{x}{m}} \right) \tag{46b}$$

wherein $\epsilon = 2.7183$ (approximately) is the base of the Napierian or natural system of logarithms.

To express the tension P_B in terms of x, m, and w, we have, from Fig. 25,

$$P_B = \frac{P_h}{\cos \beta}$$

but $P_h = wm$ and, from Fig. 26, $\cos \beta = \frac{m}{m + y}$

Therefore, $$P_B = w(m + y) = wm \cosh \frac{x}{m} \tag{47}$$

These formulas are easily solved with the aid of tables of hyperbolic functions or of the exponential functions ϵ^x and ϵ^{-x}. It will be observed, however, that the factor m which appears in these formulas has the value $\frac{P_h}{w}$ and it will, therefore, change with every change of tension in the cable when the loading of w lb. per foot is constant. Notwithstanding the apparent simplicity of the catenary formulas, they become cumbersome and unhandy as soon as the problem to be solved involves changes of tension due to temperature variations. For spans of ordinary length, the approximate formulas based on the parabola are to be preferred, and, with short spans, the increased accuracy of the catenary formulas is purely hypothetical because of the practical difficulty of obtaining exact values by interpolation between numbers read from tables.

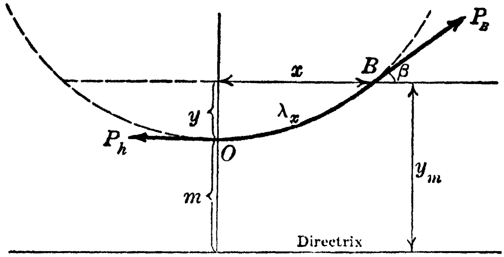
For convenience of reference, the formulas of both parabola and catenary have been assembled in the accompanying table.

Example 13. Illustrating Use of Sag-tension Formulas.—Consider a copper cable of 350,000 circular mils cross-section suspended between two supports on the same level, spaced 2,000 ft. apart. The cable hangs under the influence of its own weight only (no additional load due to wind pressure or ice loading). Under these conditions, it is desired to calculate the maximum sag (at center of span) when the maximum stress in the material is 14,000 lb. per square inch.

The cross-section of the cable is 0.275 sq. in. Therefore, the maximum permissible tension (at the points of support) is

$$P_B = 14,000 \times 0.275 = \text{(say) } 3,800 \text{ lb.}$$

The weight of the cable per foot of length is $w = 1.07$ lb.



Length of curve from O to B = λ_x
Weight per foot of cable (including extra load, if any) = w

ASSEMBLED FORMULAS FOR SAG-TENSION CALCULATIONS

Formula number	Parabola	Catenary	Formula number
	$y = \frac{x^2}{4m}$	$y_m = \frac{m}{2}\left(\epsilon^{\frac{x}{m}} + \epsilon^{-\frac{x}{m}}\right)$	(46b)
		$= m \cosh \frac{x}{m}$	(46)
(30a)	$= \frac{wx^2}{2P_h}$	$y = m\left(\cosh \frac{x}{m} - 1\right)$	(46a)
	Length of cable (O to B)		
	$\lambda_x = x + \frac{2y^2}{3x}$ (approximately)	$\lambda_x = \frac{m}{2}\left(\epsilon^{\frac{x}{m}} - \epsilon^{-\frac{x}{m}}\right)$	(45)
		$= m \sinh \frac{x}{m}$	(45a)
	Horizontal component of tension		
(30a)	$P_h = \frac{wx^2}{2y}$	$P_h = wm$, therefore $m = \left(\frac{P_h}{w}\right)$	
(34)	$= P_B \cos \beta$	$= P_B \cos \beta$	(34)
(29)	$\beta = \tan^{-1} \frac{wx}{P_h}$	$\beta = \tan^{-1} \frac{w\lambda_x}{P_h}$	(29)
(29a)	$= \sin^{-1} \frac{wx}{P_B}$	$= \sin^{-1} \frac{w\lambda_x}{P_B}$	(29a)
	Vertical component of tension at point B		
	$P_{BV} = wx$ (approx.)	$P_{BV} = w\lambda_x$	
(35)	$= P_B \sin \beta$	$= P_B \sin \beta$	(35)
(29)	$= P_h \tan \beta$	$= P_h \tan \beta$	(29)
	Tension along axis of cable at point B		
(31)	$P_B = P_h + wy$	$P_B = wy_m = P_h + wy$	(47)
	$= w\left(\frac{x^2}{2y} + y\right)$	$= wm \cosh \frac{x}{m}$	(47)
(34)	$= \frac{P_h}{\cos \beta}$	$= \frac{P_h}{\cos \beta}$	(34)

Solving for the sag (s) with the approximate formulas, by formula (29a)

$$\sin \beta = \frac{wl}{2P_B} = \frac{1.07 \times 2{,}000}{2 \times 3{,}800} = 0.282$$

whence $\cos \beta = 0.959$

The horizontal component of the tension, by formula (34), is

$$P_h = P_B \cos \beta = 3{,}800 \times 0.959 = 3{,}640 \text{ lb.}$$

Then, by formula (30a),

$$s = \frac{wl^2}{8P_h} = \frac{1.07(2{,}000)^2}{8 \times 3{,}640} = 147 \text{ ft.}$$

Now check this with the formulas based on the catenary. Use the figure of 3,640 lb., obtained from the formulas based on the parabola, as a trial value for the horizontal component of the tension in the cable, and solve for sag and maximum tension as follows:

$$x = \frac{\text{span}}{2} = 1{,}000 \text{ ft.}$$

$$w = 1.07 \text{ lb. per foot}$$

$$P_h = 3{,}640 \text{ lb.}$$

$$m = \frac{P_h}{w} = \frac{3{,}640}{1.07} = 3{,}400$$

$$\frac{x}{m} = \frac{1{,}000}{3{,}400} = 0.294$$

From tables of hyperbolic functions,

$$\cosh \frac{x}{m} = 1.04354$$

By formula (46a)

$$\text{sag} = y = m\left(\cosh \frac{x}{m} - 1\right)$$

$$= 3{,}400 \times 0.04354 = 148 \text{ ft.}$$

By formula (47), the maximum pull (at point of support) is

$$P_B = P_h + wy$$

$$= 3{,}640 + (1.07 \times 148) = 3{,}798 \text{ lb.}$$

This is almost exactly the specified maximum tension of 3,800 lb., showing the assumed horizontal component of tension (P_h = 3,640) to be correct. The maximum sag is 148 ft., as compared with 147 ft. calculated by the approximate formulas, which indicates that, even with this 2,000-ft. span, the catenary curve approximates so nearly to a parabola that no appreciable error is introduced by using the formulas based on the parabola.

The maximum sag or deflection from the straight line between the two supports is approximately $\frac{148 \times 100}{2{,}000} = 7.4$ per cent of the span. If the maximum sag, as calculated by the approximate formulas exceeds 10 per cent of the span length, the calculations should be checked with the more accurate formulas of the catenary. When the sag is a large percentage of the distance between points of support, the formulas based on the parabola no longer apply, and the more accurate formulas should be used.

44. Examples of Extra-long Spans. Maximum Possible Single Span.—Where power lines cross rivers or other waterways, exceptionally long single spans are sometimes necessary. The Pacific Light and Power Corporation has a 2,871-ft. span at Sunland, Calif., the conductors consisting of aluminum wires of 0.475 sq. in. total cross-section reinforced by a central steel core of 0.062 sq. in. cross-section. Another long single span is the Missouri River crossing of the Mississippi River Power Company, which is 3,182 ft. long; the conductors consist of copper wires of 207,600 circular mils total cross-section laid on a central steel core of 0.275 sq. in. cross-section.

The crossing of the Mississippi River by a single span 4,279 ft. long with a difference of elevation of 185.5 ft. between the two points of support is described in the *Transactions* A.I.E.E. (Vol. 44, p. 378), 1925. This particular long span is also referred to by L. E. Imlay in his article, "Mechanical Characteristics of Transmission Lines," *Electric Journal*, December, 1925.

The Cheat River crossing of the West Virginia-Maryland Power Company, at Beaver Hole, consists of a single span 4,317 ft. long with a difference of elevation of 225 ft. between points of support. The conductor consists of a 19-strand cable with seven strands of copper-clad steel for the center core, and twelve strands of hard-drawn copper outside. A description of this long span by C. F. Sheakley, will be found in the *Electrical World* of June 21, 1924.

The Pacific Gas and Electric Company's crossing of the Carquinez Strait by a single span 4,427 ft. long is of particular interest because post-type insulators are used, with the porcelain in compression. For description, refer to the *Transactions* A.I.-E.E. (Vol. 42, p. 977), 1923.

The 110,000-volt line of the Shawinigan Water and Power Company crosses the St. Lawrence River by single span 4,800 ft. long, at a point about 20 miles from Three Rivers, Quebec. The supporting towers are 350 ft. high and 60 ft. square at the base. For details of construction, refer to paper by S. Svenningson, *Transactions* A.I.E.E. (Vol. 37, part 2, p. 1653).

The 150,000-volt line of the Knoxville Power Company crosses the Little Tennessee River near the mouth of the Cheoah River (North Carolina) by a single span 5,010 ft. long. There are four conductors of 500,000 circular mils aluminum cable with steel core 7/8 in. in diameter. The outside diameter of the cable is 1.175 in. For description, refer to paper by T. Varney, *Transactions* A.I.E.E., June, 1920; also L. E. Imlay, "Mechanical Characteristics of Transmission Lines," *Electric Journal*, December, 1925.

The longest power transmission span in the world is 6,240 ft. long. It crosses the Narrows at Puget Sound, Washington, and consists of six plow-steel cables each of which weighs 11 tons. These cables form part of two 110,000-volt circuits of the Cushman development (Tacoma). This extra-long span is described in the *Journal* A.I.E.E., p. 1296, December, 1925.

The fact that spans exceeding 1 mile in length have actually been erected raises the question of ultimate limits to the length of single spans. An inspection of the formulas giving relation between sag and tension will show that, by allowing a large sag, the horizontal component of the tension (or the tension in wire at lowest point of span) may be reduced. But when the span is very large, the weight of the cable comprised between the point of support and the lowest point of span may become excessive. This vertical component of the tension has to be added (vectorially) to the horizontal component to determine the maximum pull (at point of support), and there is a particular relation between sag and span which will cause this resultant pull to be a minimum. In order to determine this relation, it is necessary to use formulas based on the catenary, and it has been shown[1] that the minimum tension at point of support occurs when the sag is 0.337 times the span and the angle which the tangent to the cable at the point of support makes with the horizontal is $\beta = 56.5$ deg. Under these conditions formula (34) gives

$$P_h = P_B \cos \beta = 0.552\, P_B$$

[1] AUSTIN, F. E., in *Elec. Rev.*, p. 4, July 3, 1920. Also SCHENKEL, H., *Elektrotechn. Zeit.*, p. 720, July 7, 1921.

Also, by formula (47)

$$P_B = P_h + wy$$
$$= 0.552\,P_B + w \times 0.337l$$

whence $l = \frac{1.33}{w} P_B$. Thus, if the permissible maximum tension (P_B) and the weight per foot of the cable (w) are known, the maximum possible length of span (l) with supports at the same elevation is easily calculated.

Consider, for instance, a copper cable in which the working stress is not to exceed 30,000 lb. per square inch and assume that the extra load due to wind or ice deposits has the effect of increasing the weight of the cable 50 per cent. If A is the cross-sectional area in square inches; the total permissible tension is $P_B = 30{,}000\,A$. The weight per foot per inch of cross-section of a stranded copper conductor is about 3.88 lb. Then the weight per foot of the loaded conductor is $w = 1.5 \times 3.88 \times A$. The maximum permissible length of span is, therefore,

$$l = \frac{1.33 \times 30{,}000A}{1.5 \times 3.88A} = 6{,}850 \text{ ft.}$$

By using stronger materials, such as bronze or steel, much longer spans are possible; but unless the natural conditions of the ground are such as to allow for the large sag (in this instance $0.337 \times 6{,}850 = 2{,}310$ ft.), very tall towers would be necessary.

The possibility of carrying an overhead 130,000-volt line across the Straits of Messina, between the island of Sicily and the mainland, has been under consideration. At the narrowest point, the distance is over 2 miles. Assume a separation between towers of 11,500 ft., and see if a single span of this length would be possible. There is no doubt that a material of very great tensile strength would be needed, because the sag would have to be as small as possible; but steel with a breaking stress of 160,000 lb. per square inch is easily obtainable. The working stress may be half this amount, or even appreciably greater before reaching the elastic limit. Assume the stress in the wire at the lowest point of the span to be 80,000 lb. per square inch. The weight per cubic inch of steel is about 0.285 lb., and if the cable is of large cross-section, the increased loading due to wind pressure would not exceed about 10 per cent. Then, if A is the cross-section of the wire in square inches, the horizontal component of the tension is

$$P_h = A \times 80{,}000$$

and the weight per foot (loaded) is

$$w = 1.1 \times A \times 12 \times 0.285 = 3.76A$$

Then, by formula (30*a*), the sag would be

$$s = \frac{3.76\ A\ (11{,}500)^2}{8 \times 80{,}000\ A} = 777 \text{ ft.}$$

Since this is considerably less than 10 per cent of the span length, there is no need to check these calculations with the formulas based on the catenary.

By formula (31), the maximum pull (at points of suspension) is

$$P_B = S_B \times A = (80{,}000 \times A) + (3.76\ A \times 777)$$

when the maximum stress is $S_B = 82{,}920$ lb. per square inch, which is not excessive. It follows that although very tall towers may be necessary in order to allow clearance for shipping in addition to the sag of (about) 777 ft., the crossing of these straits by overhead conductors is not impossible, even if the high cost should prove it to be inadvisable from the economic point of view.

CHAPTER V

OVERHEAD CONDUCTORS—SAG AND STRESS CALCULATIONS (*Continued*)

45. Abnormal Stresses in Wires Due to Wind and Ice.—When a wire hangs between horizontal supports in still air under the influence of its own weight only, the tension at center of span for a definite distance between supports and a definite sag at center will be proportional to the weight per foot length of the wire. When loaded with sleet or ice, or subject to wind pressures, or a combination of both these additional loads, the calculations of the proper sag to allow are based on the assumption that the weight per foot is no longer that of the wire only, but n times this amount; where n is a factor depending upon the character and amount of the abnormal loading that the wire is likely to be subjected to.

Ice and Sleet.—The effect of snow and sleet adhering to the wires and forming an ice coating of variable thickness is to add to the dead weight of the wires and offer an increased surface to wind blowing across the line. Sleet will generally collect with slightly greater thickness near the lowest point of the span, but it is usual to assume that the extra vertical loading is uniformly distributed over the whole length of the span. Max N. Collbohm[1] says that in the winter of 1908–1909, in Wisconsin and neighboring states, the snowstorm of January 28 covered nearly all overhead wires with sleet and snow 2½ to 4 in. in diameter. The temperature went down to 4°F. below zero, while a wind velocity of 40 miles per hour was recorded. These conditions were, of course, exceptional; at the same time an average coating of sleet and snow weighing ½ lb. per foot of wire is not unusual. A coating of ice ½ in. thick on wires running through districts where sleet and low temperatures are common is generally allowed for in calculations. Sometimes this is increased to a radial thickness of ¾ in. Sleet deposits 6 in. in diameter have actually been observed on some of the steel conductors of the Central

[1] *Elec. World*, p. 734, Mar. 25, 1909.

Colorado Power Company. The weather conditions in some of the passes through which this line is carried are, however, particularly severe. In New England, a storm of exceptional severity in November, 1921, resulted in sleet formations on the No. 2 B. & S. solid copper wires of the Connecticut Power Company's lines of which the measured cross-section was from 5 to 6 sq. in.

Taking the weight of sleet at 57.3 lb. per cubic foot, the total weight per foot of the loaded conductor is $w + 1.25t(d + t)$ where w is the weight of the wire, t the radial thickness of sleet (assumed to be of circular section), and d the diameter of the uncoated wire; these dimensions being expressed in inches. Thus, in the case of a No. 0 B. & S. gage wire, d equals 0.325 in., and assuming that t equals 0.5 in., the weight of the ice coating is $1.25 \times 0.5 \times 0.825 = 0.516$ lb., or about ½ lb. per foot run, which is a common allowance to make for ice loads on wires of average size.

When wires are hung vertically one above the other, in the manner frequently adopted in the earlier constructions for double-circuit lines, there is the possibility of sleet falling off one of the lower wires, while the upper wire remains heavily loaded and with considerable sag. This might cause the lower wire to rise into contact with the upper wire. With this possibility in mind, the disposition or spacing can be made so that short-circuits due to this cause are not likely to occur.

Sleet storms are not infrequently followed by low temperatures and high winds. The loading—especially when the conductors are of small diameter—is then likely to be so great that it may be false economy to guard against it by increasing the factor of safety. Attempts have been made to prevent the formation of sleet deposits on overhead power conductors by passing sufficient current through them to raise the temperature above that at which the deposit will form. On many systems it is possible to increase the current in the lines by controlling the power factor of the load. Users of power might assist in maintaining service during severe sleet storms by underexciting synchronous motors connected to the transmission system. If sufficient current cannot be obtained by such means, the procedure on systems where there is a duplicate line would be to throw all the load on one line and pass a suitable current through the spare line which would be disconnected from the load and short-circuited at the receiving end.

Wind Pressures.—The pressures due to winds of high velocity acting on poles and wires in a direction approximately at right angles to the transmission line are of great importance, both where sleet formation is possible, and in districts where sleet cannot form. In the latter case, the velocity of the wind is frequently greater than in the colder districts; but, on the other hand, a moderate wind acting on the larger diameter of ice-coated wires will generally lead to the greatest stressing of the conductor material. The maximum wind velocities rarely occur at the lowest temperatures, but falling temperatures and rising wind are not unusual after sleet storms. In mountainous districts a transmission line may be subjected at certain points to gusts of wind blowing almost vertically downward; the pressure in such a case, being directly added to the weight of wire and the ice load, may lead to more serious results than an even stronger wind blowing horizontally across the line.

Numerous observations have been made on wind pressures, and it is found that the pressure exerted on small surfaces is proportional to the square of the wind velocity. It will also depend to some small extent upon the density of the air, or the barometric pressure; but the correction for barometric pressure is usually not worth making.

Wind Velocity.—It is well to distinguish between indicated and true wind velocities. The U. S. Weather Bureau observations are made with the cup anemometer, and wind velocities over short periods of time are calculated on the assumption that the velocity of the cups is one-third of the true velocity of the wind, for great and small velocities alike. This assumption is not justifiable, and a correction must, therefore, be made in order to convert the Weather Bureau recorded velocities into true velocities. Unless otherwise stated, when wind velocity is referred to, this must be understood to be the true velocity.

Formulas for Wind Pressures.—The formula proposed by the U. S. Weather Bureau (Prof. C. F. Marvin), giving pressure in pounds per square foot on small flat surfaces normal to the direction of the wind, is

$$F = 0.004\frac{B}{30}V^2 \tag{48}$$

where B is the barometric reading in inches, and V is the wind velocity in miles per hour.

A similar formula, known as Langley's formula, is

$$F = 0.0036V^2$$

In the case of cylindrical wires, the pressure per square foot of projected area is less than on flat surfaces. The formula proposed by H. W. Buck[1] and generally conceded to be correct is

$$F = 0.0025V^2 \tag{49}$$

where F is the pressure per square foot of projected surface of a cylindrical wire. A more convenient form of expression for this relation is

$$p = \frac{dV^2}{5{,}000} \tag{50}$$

where p is the pressure per foot length of the wire and d is the diameter of the wire in inches; and the denominator, although

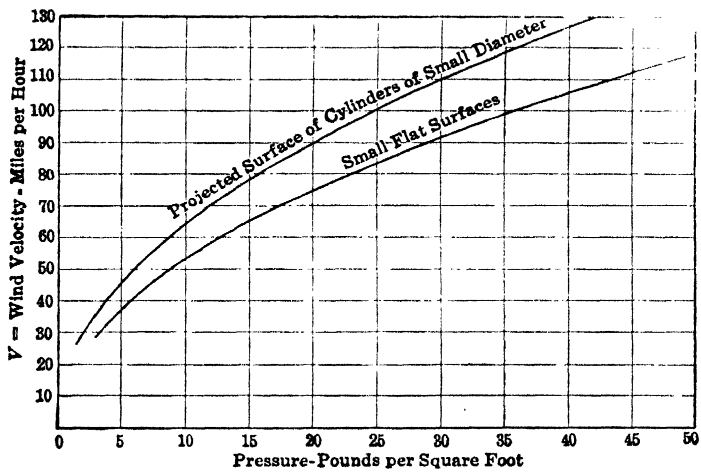


FIG. 27.—Curves giving relation between wind velocity and resulting pressuer.

not exactly as obtained from Mr. Buck's equation, is an easily remembered round number, which is very close to the average of many experimental results. The upper curve in Fig. 27 has been plotted from Mr. Buck's formula for cylindrical wires; while the lower curve for pressures on flat surfaces gives the relation between pressure and wind velocity according to Professor Langley's formula.

Relation between Wind Velocity and Height above Ground.—Owing to the resistance offered by the ground surface, the force

[1] In paper read at the World's Fair, St. Louis, 1904.

of the wind is not so great near the ground as at high altitudes, and greater maximum wind pressures on wires must be allowed for when the line is carried on high steel structures than in the case of the average wood-pole line, as used for the lower voltage transmissions.

F. F. Fowle[1] has given valuable particulars based on maximum wind velocities at different elevations, observed in Chicago. It would appear, from his measurements, that if the maximum wind velocity is 35 miles per hour at an elevation of 30 ft. above ground, the maximum wind velocity in miles per hour would be about 55 at 100 ft., 80 at 200 ft., and 100 at 280 ft. above ground level.

Mr. Fowle suggests that overhead line calculations might be based on a maximum wind velocity of 47 miles per hour for ordinary steel tower construction, and 40 miles for wood-pole lines. These are probably safe limits, especially in climates where this maximum wind pressure is considered as acting on ice-coated wires. In exposed positions, and where the line runs through wide stretches of open country, it is well to allow a maximum of 60 for steel tower lines, and 50 for wood-pole lines. A very complete collection of wind-pressure formulas and recorded wind velocities at different elevations will be found in the paper by C. F. Elwell in the *Journal* of the (British) Institution of Electrical Engineers, of March, 1923 (Vol. 61, p. 407). This paper has particular reference to the proper allowance for wind pressure in the design of tall steel towers.

For the purpose of estimating the probable maximum stresses in wires and supporting structures, three classes of loading are considered in America, the selection of any one as the basis for calculations depending upon the locality.[2]

Class A: Temperature 0°F.; no ice deposit; wind pressure 15 lb. per square foot of projected area.

Class B: Temperature 0°F.; ice coating ½ in. radial thickness; wind pressure 8 lb. per square foot of projected area of ice-coated wire.

Class C: Temperature 0°F.; ice coating ¾ in. radial thickness; wind pressure 11 lb. per square foot of projected area of ice-coated wire.

[1] "A Study of Sleet Loads and Wind Velocities," *Elec. World*, Oct. 27, 1910.

[2] Refer to the 4th edition of the National Electric Safety Code for a modified form of these loading assumptions.

A safety factor of 2 is usually allowed for wires, and 3 to 4 for wood poles.[1]

In Great Britain it is usual to base calculations on a temperature of 22°F., an ice coating of ½ in. radial thickness for high-voltage lines, ¼ in. radial for low-voltage lines, and a wind pressure of 8 lb. per square foot on the projected area of the ice-coated wires. The factor of safety for conductors is 2; for wood poles 3.5; and for steel structures 2.5, based upon the elastic limit of members in tension or the crippling load of members in compression.

In nearly all countries special rules and regulations apply to crossings of main roads and railway lines where extra safety is required. There are also districts, especially at high altitudes in mountainous country, where exceptionally severe weather conditions render higher factors of safety desirable.

Swaying of Wires in Strong Winds.—If a transmission line is well designed and constructed, all the wires of one span will generally be found to swing synchronously in any wind. Under exceptional conditions, however, trouble is liable to occur through wires swinging together, even when all details of design and construction have received careful attention. Troubles of this description are more likely to be met with when the spans are long and the sag in the wires necessarily large, and for this reason the spacing between wires must increase with increase of span length, irrespective of voltage considerations. Copper conductors are decidedly less likely to swing out of synchronism than aluminum conductors; not only because the latter have usually to be strung with a greater sag, but also because of the lightness of the material. Aluminum conductors of small section are easily shaken by sudden gusts of wind, and a little difference in sag will in all probability lead to non-synchronous swinging. It must not be overlooked that wires, after erection, do not always remain equally taut. This may be due to many causes, such as a slight slipping in the ties, straining of insulator pins on cross-arms, unequal ice loading, or local faults in the wires themselves. Again, it has been observed that during snowstorms all the wires do not always become coated to an equal extent, and such a want of uniformity the ice coating may well lead to wires being blown together in a strong wind.

[1] Refer to Art. 65 in Chap. VI for discussion of the expression "factor of safety" as applied to steel towers.

On the high-tension transmission system of the Central Colorado Power Company, with spans averaging 730 ft., the lines cross some very exposed positions at the openings to canyons, and the excessively strong winds that occur at such points have been known to mix up the conductors. It was found necessary to dead-end the line at each tower, guy the towers, and increase the tension in the wires to a point near the elastic limit of the material, steel being used where necessary in lieu of copper.

46. Calculation of Total Stress in Overhead Wires.—The formula (30*a*) of Art. 41 gives the relation between tension, sag, and weight for a wire strung between supports a known distance apart. Thus, if the tension P is known or assumed, the sag can readily be calculated. Now consider how this tension can be computed, not only when the wire carries an increased vertical load in the form of ice deposit, but also when the effect of wind blowing across the line increases the total stress. If w is the weight per foot of the conductor without the extra load due to wind and ice, write nw for total loading in pounds per foot run of the wire, where n is the loading factor which takes account of the extra load on the wire under the most severe weather conditions likely to be encountered in the district where the transmission line is erected.

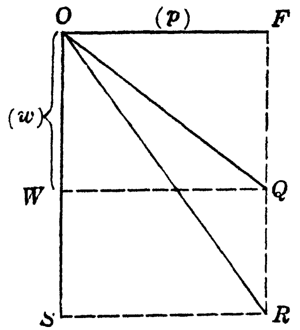


FIG. 28.—Diagram of forces acting on overhead wire.

It is usual to assume that the wind pressure acts in a horizontal direction and that the total load on a conductor is the resultant of two forces, one acting vertically downward due to weight of wire, together with added weight of sleet or ice, if any, and one acting horizontally due to the wind pressure. These forces are indicated in Fig. 28 where $OF = p$ represents the wind pressure, $OW = w$ the weight of the conductor, and WS the added weight of ice. The resultant pressure OR is equal to $\sqrt{(OF)^2 + (OS)^2}$. If the line runs through a country where sleet does not form on the wires, the maximum resultant pressure is OQ instead of OR if the assumed maximum force due to wind is the same in both cases.

The diagram Fig. 29 gives values of the multiplier n (*i.e.*, the ratio $\frac{OQ}{OW}$ of Fig. 28) corresponding to various wind velocities for standard sizes of solid copper conductors on the assumption that there can be no ice formation on the wires, while Fig. 30

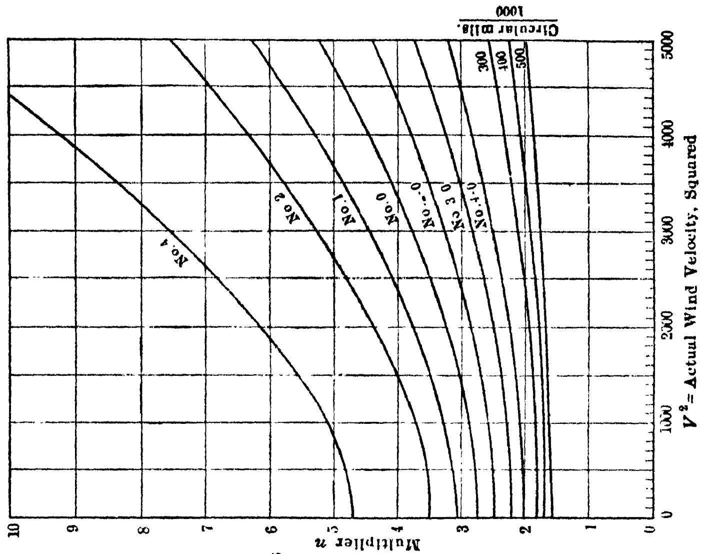


FIG. 30.—Chart giving factor n for ice-coated copper conductors.

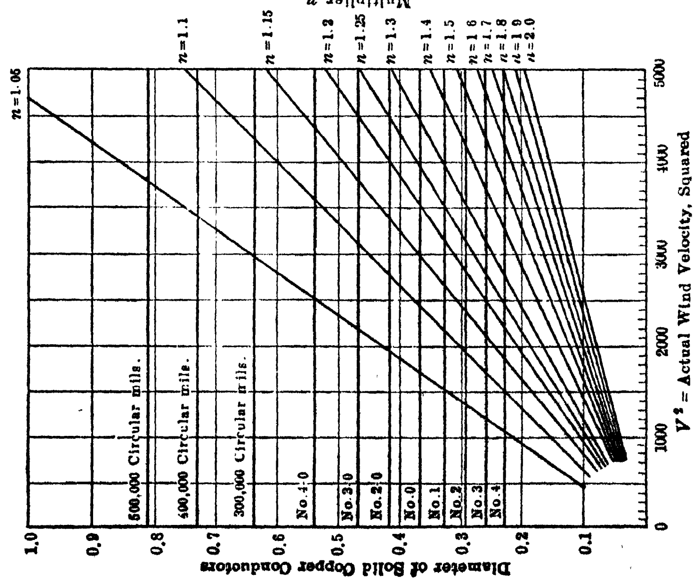


FIG. 29.—Chart giving factor n for copper conductors without ice loading.

gives values of n $\left(i.e., \text{ the ratio } \dfrac{OR}{OW}\right)$ for copper conductors when the weight is increased by a coating of ice 0.5 in. thick with a

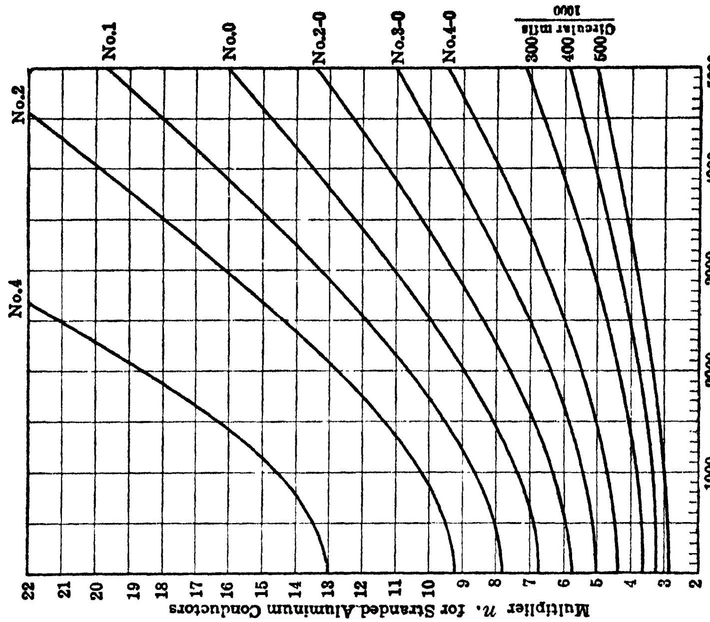


FIG. 32.—Chart giving factor n for ice-coated aluminum conductors

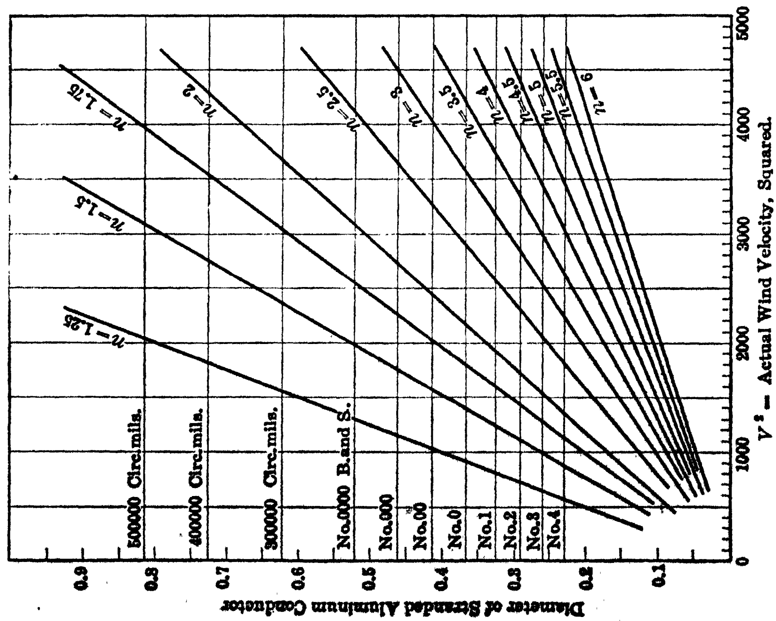


FIG. 31.—Chart giving factor n for aluminum conductors without ice loading.

correspondingly greater wind effect due to the increased diameter. The curves of Figs. 31 and 32 give similar relations but for conductors of aluminum instead of copper.

The formula used for the calculation of wind pressure in connection with these diagrams is

$$p = (d + 2t)V^2 \div 4{,}820$$

where d is the diameter in inches of the conductor, t is the radial thickness of ice deposit (if any), V is the actual wind velocity in miles per hour, and p is the wind pressure in pounds per foot length of conductor.

This is the more correct form of the formula (50) already given. When using the diagrams, it should be noted that the distances plotted horizontally represent the squares of the wind velocities, and the sizes of the conductors are expressed in equivalent B. & S. gage numbers or in circular mils for the larger sizes.

The curves of Fig. 29 are correct for solid copper wires; the values of n for stranded conductors would be somewhat greater because of the larger surface presented to the wind for the same vertical loading. The error introduced by using Fig. 29 for stranded cables is not of great practical importance.

The curves of Fig. 31 are approximately correct for stranded aluminum conductors; but since the actual diameter will vary with the method of stranding, these charts are intended only for the use of practical engineers who are not interested in mathematical niceties. The calculations on the basis of the assumptions previously made are, however, very simple. Thus, if α stands for the angle SOQ or SOR as the case may be (see Fig. 28), write:

$$\tan \alpha = \frac{\text{horizontal loading}}{\text{vertical loading}}$$

whence α and, therefore, $\sec \alpha$ can be obtained from trigonometrical tables. This last quantity being the ratio $\frac{\text{resultant loading}}{\text{vertical loading}}$ is the required factor n when ice loading is not considered. The correction to be made when the vertical load includes ice deposit is simple and obvious.

The special case of solid wires without ice coating can be treated as follows:

Wind pressure per foot length..........$p = \frac{dV^2}{4{,}820}$

Vertical load per foot length (copper)...$w = 3.02d^2$

Vertical load per foot length (aluminum) $w = 0.92d^2$

$$n = \frac{\sqrt{w^2 + p^2}}{w} = \sqrt{1 + \frac{p^2}{w^2}} \qquad (51)$$

whence, for *solid* wires *without ice deposit*,

$$n \text{ (for copper)} = \sqrt{1 + \frac{V^4}{d^2 \times 21{,}200 \times 10^4}} \qquad (52)$$

$$n \text{ (for aluminum)} = \sqrt{1 + \frac{V^4}{d^2 \times 1{,}965 \times 10^4}} \qquad (53)$$

Example: What is the ratio of the resultant load with actual wind velocity of 70 miles per hour, to normal load with wire hanging in still air, in the case of a No. 6 B. & S. copper conductor?

The diameter is $d = 0.162$ in., and by formula (52)

$$n = \sqrt{1 + \frac{(7)^4 \times 10^4}{(0.162)^2 \times 21{,}200 \times 10^4}} = 2.3$$

When an overhead wire is loaded with ice or subjected to strong winds blowing across the line, the effect on the tension and maximum sag is the same as if it were replaced by another wire suspended in still air (without ice coating) but weighing nw lb. per foot instead of w lb. per foot. The sag s expressed in terms of the span l and the horizontal component of the tension P_h is,

$$s = \frac{wnl^2}{8P_h}$$

which becomes the formula (30*a*) of Art. 41, Chap. IV, when $n = 1$. Similarly the general form of the formula (30*b*) is

$$s = \frac{knl^2}{S}$$

where S is the stress in pounds per square inch in the wire.

Although these relations between sag and tension or sag and stress are extremely simple, the calculation of the sag resulting from wind or ice loading in a particular span of wire suspended from two fixed points is not easy. An increase of load per foot length of wire from w to nw lb. produces a certain elastic lengthening of the wire which determines the resulting sag and, therefore, the resulting tension. If there were no elastic elongation, the sag would remain unaltered, and the stress or tension would be

n times as great in the loaded as in the unloaded wire. The fact that account must be taken of the lengthening of the wire with increase of stress tends to complicate the problem. Since changes of temperature affect the length of the wire and, therefore, the sag and tension much in the same manner as changes of loading, the solution of the problem will be considered in connection with the effects of temperature variations.

47. Effect of Temperature Variations on Overhead Wires.—If a is the temperature coefficient of the material of the conductor (see the table of constants in Chap. IV, p. 77), then the length of the wire when the temperature is raised t degrees is

$$\lambda_t = \lambda_o(1 + at) \tag{54}$$

in which λ_o is the original length of the wire. Formulas for calculating the length of the wire in an overhead line, whether considered as taking the shape of the parabola or catenary, were given in Chap. IV (refer to collected formulas on p. 100). The approximate formula for the length of a parabolic arc is sufficiently accurate for practical purposes, and the length of cable in a span between fixed supports l ft. apart may be calculated by the formula

$$\lambda = l + \frac{8s^2}{3l} \tag{55}$$

where s is the sag at center of span, in feet.

If it is desired to eliminate s (the sag) from the formula, this quantity may be expressed in terms of the tension P_h or the stress S. By making the necessary substitutions,

$$\lambda = l + \frac{w^2l^3}{24P_h^2} \tag{55a}$$

and

$$\lambda = l + \frac{8k^2l^3}{3S^2} \tag{55b}$$

The formula (54) giving the increase of length with increase of temperature assumes that the wire is unstressed, or that the stress remains unaltered notwithstanding the increase in temperature. This, however, is not the case with an overhead conductor. As indicated by formula (30a), the tension in a wire suspended horizontally between two fixed supports is almost exactly proportional to the square of the span and inversely proportional to the sag at center of span. The effect of temperature variation is to alter the length of wire and, therefore, the amount of sag and tension. With a reduction of temperature, the length of wire will decrease; this will cause an increase in the tension, but owing to the fact that the wire will stretch under the influence of the increased tension, the sag at the lower temperatures will be somewhat greater than it would be if there were no elongation of the wires with increase of stress. All sag-temperature calculations, whatever the method adopted, must, therefore, take into account not only the effect of elongation with increase of temperature, but also the effect of the elastic contraction of the wire with increase of sag.

If P is the tension in a wire of cross-section A, and if E is the elastic modulus (as given for various materials in the table on p. 77), then the elongation of the wire due to the tension P is

$$\Delta\lambda = \frac{P \times \lambda}{A \times E}$$

the original length of the unstressed wire being λ.

If instead of $\frac{P}{A}$ the letter S be used to denote the stress in pounds per square inch of cross-section, the formula becomes

$$\Delta\lambda = \lambda \times \frac{S}{E} \tag{56}$$

It is customary to assume that the material of the conductors is perfectly elastic up to a certain critical stress known as the elastic limit; that is to say, if the application of a certain stress produces a strain represented by $\Delta\lambda$, it is assumed that, on the removal of the stress, the conductor will contract to its original length λ, and that this process of elongation and contraction follows a straight-line law. This is not scientifically correct, because, on removal of load, the amount of contraction is not directly proportional to the decrease of stress; but the departure from the straight-line law is not considerable, and no serious error is introduced by disregarding refinements of this nature.

A matter of greater importance is the fact that, when stranded conductors are used, the ratio between stress and strain is not correctly given by the modulus E as calculated from tests made on solid wires. The modulus for stranded cables will depend upon the number of strands, the "lay" of the strands, whether the central core is of metal or hemp, etc., and it should be determined by actual tests on samples of the completed cable; but since it will depend largely on the stress to which the cable has been subjected, and will ultimately differ little from the coefficient for solid wires, it is usual in sag calculations to use the same value of E for both solid and stranded conductors.

48. Calculation of Sags and Tensions under Any Conditions of Load and Temperature.—It is assumed that the conductor is strung between two fixed supports on the same level, and that the material of the conductor is not strained beyond the elastic limit. The meaning of the symbols used is as follows:

l = length of span, or straight-line distance between points of support, in feet.

s = sag or deflection at center of span, in feet.

S = stress in conductor at lowest point of span, in pounds per square inch of cross-section.

λ = length of conductor between the two points of support.

t = temperature, in degrees Fahrenheit.

a = coefficient of linear expansion of the conductor per degree Fahrenheit.

E = modulus, or coefficient, of elasticity, being the ratio of stress in pounds per square inch to corresponding extension per unit length.

w = weight of conductor in pounds per foot of length.

n = loading factor, being a multiplier depending upon weather conditions (wind and ice) and the material and size of the conductor (defined in Art. 46).

wn = resultant or total load in pounds per foot length, including wind pressure and ice (if any).

k = a constant depending upon the material of the conductor. It is equal to $\frac{w}{8A}$ or to 1.5 times the weight in pounds of a cubic inch of the conductor material.

S_c, t_c, n_c = known or assumed values of stress, temperature, and loading factor, upon which calculations for different conditions of temperature and loading are based. The other known quantities are, by formula (30*b*),

$$s_c = \frac{kn_c l^2}{S_c}, \text{ and by formula (55),}$$

$$\lambda_c = l + \frac{8s_c^2}{3l}.$$

Numerical values for the quantities a, E, and k will be found in the tables of physical constants (pp. 77 and 80).

With an increase of temperature from t_c to t, a wire of length λ_c ft. will increase in length by an amount

$$\Delta\lambda_c = a(t - t_c)\lambda_c \text{ ft.}$$

With an increase of stress from S_c to S lb. per square inch, a wire of length λ_c ft. will increase in length by an amount

$$\Delta\lambda_c = \lambda_c \frac{S - S_c}{E}$$

The resulting total extension when a wire is subjected to change of temperature and change of tension is, therefore,

$$\lambda - \lambda_c = \lambda_c\left[a(t - t_c) + \frac{S - S_c}{E}\right]$$

In order to obtain an equation expressing the sag in terms of temperature and loading conditions, make the following substitutions:

By formula (55) $\qquad \lambda = l + \dfrac{8s^2}{3l}$

By formula (30*b*) $\qquad S_c = \dfrac{kn_c l^2}{s_c}$

and $\qquad S = \dfrac{knl^2}{s}$

The equation may then be written

$$l + \frac{8s^2}{3l} - \lambda_c = \lambda_c a(t - t_c) + \frac{\lambda_c kl^2}{E}\left(\frac{n}{s} - \frac{n_c}{s_c}\right)$$

In order to solve for the new sag (s), multiply both sides of the equation by $\dfrac{3ls}{8}$ and collect together all the terms containing s. The equation then takes the form,

$$s^3 - s\left(\frac{3l}{8}\right)\left[\lambda_c - l + \lambda_c a(t - t_c) - \frac{l^2\lambda_c kn_c}{Es_c}\right] = \frac{3l^2\lambda_c kn}{8E} \quad (57)$$

The only unknown quantity in this equation is s, and a solution may be obtained by trial for any desired values of the new temperature (t) and the new loading factor (n).

A more convenient form of the equation (57) may be obtained as follows: Note that the quantity $\lambda_c - l$ may be replaced by its equivalent $\dfrac{8s_c^2}{3l}$; also that no serious error is introduced by substituting l for λ_c in the three remaining places where λ_c appears in the equation. This is because of the small difference between the lengths λ_c and l when the sag s_c is small relatively to the span l.

After making these substitutions, it will be found that the equation connecting sag, temperature, and loading may be written as below, a form which is better adapted than equation (57) to the solution of practical problems.

$$(t - t_c) = \frac{8}{3al^2}(s^2 - s_c^2) + \frac{l^2k}{Ea}\left(\frac{n_c}{s_c} - \frac{n}{s}\right) \qquad (58)$$

This equation is of the form,

$$(t - t_c) = A(s^2 - s_c^2) + B\left(\frac{n_c}{s_c} - \frac{n}{s}\right) \qquad (58a)$$

where the known quantities are t_c, n, n_c, s_c and

$$A = \frac{8}{3al^2}$$

and

$$B = \frac{kl^2}{Ea}$$

For every assumed value of t the corresponding value of s may be calculated, but by far the quickest way of obtaining a solution is to assume values of s which do not differ very much from the known sag s_c, and then solve for the change of temperature $(t - t_c)$. It may be well to point out that since equations (57) and (58) are both based on the assumption that the suspended cable takes the form of a parabola, they are not applicable to the special case of a cable having a very large sag relatively to the span. Equation (58) is sufficiently accurate for the solution of nearly all practical problems. If the sag is large in relation to the span, equation (57) will give more accurate results. If the sag or deflection at center of span exceeds 10 or 12 per cent of the distance between supports, an accurate solution can be obtained only by using the formulas based on the catenary. Even in that case a saving of time will usually be effected by using equation (58) in the first instance and then checking results with the catenary formulas.

Example 14. *Calculations for Sag-temperature Chart for Use of Construction Engineers in the Field.*—The given particulars are as follows:

Horizontal span, l = 480 ft., with rigid supports on the same level.

Conductors, stranded aluminum No. 00 B. & S.

Maximum stress, S_c = 14,000 lb. per square inch, with a combined load of 0.5-in. ice coating and 47-mile wind at a temperature t_c = −20°F.

The other numerical quantities needed for solving equation (58*a*) are obtained as follows:

Loading Factors.—The square of the wind velocity is $V^2 = (47)^2 = 2{,}210$, and, from Fig. 32, $n_c = 8$. The value of the loading factor under normal conditions when the wires are being erected (no ice or wind) is $n = 1$.

Sag under Maximum Load Conditions.—By formula (30^b),*

$$s_c = \frac{kn_c l^2}{S_c} = \frac{0.148 \times 8 \times (480)^2}{14{,}000} = 19.5 \text{ ft.}$$

where the figure 0.148 is the value of the coefficient k obtained from the table of physical constants on page 77. Other needed constants, taken from the same table, are

$$a = 12.8 \times 10^{-6}$$

and

$$E = 9 \times 10^6$$

whence, for use in formula (58^a),

$$A = \frac{8 \times 10^6}{3 \times 12.8 \times (480)^2} = 0.905$$

and

$$B = \frac{0.148 \times (480)^2 \times 10^6}{9 \times 10^6 \times 12.8} = 296$$

Assume any value for s which does not differ very much from s_c. Try $s = 18.5$; then, by equation (58*a*),

$$(t - t_c) = 0.905(\overline{18.5}^2 - \overline{19.5}^2) + 296\left(\frac{8}{19.5} - \frac{1}{18.5}\right)$$

whence

$$t = 71 - 20 = 51°\text{F.}$$

Similar calculations made for other assumed values of s give the following results:

When $s = 16.5$ ft., $t = -14$°F.
When $s = 18.5$ ft., $t = +51$°F.
When $s = 20.5$ ft., $t = +123$°F.

These results have been used in plotting the curve of Fig. 33, which gives the men in the field all needed information for the correct stringing of the conductors whatever may be the temperature at the time when the work is carried out. The figures on the curve, indicating tension in the wire, have been calculated by formula (30*a*). The weight per foot of the conductor (w in the formula) is obtained from the table on page 81, where $w = 0.125$ for a stranded No. 00 aluminum conductor.

* As modified on p. 116.

When the spans are short, the sag, under all temperature conditions, is very small, and it is more convenient to disregard the sag, and merely pull the wires until the required tension is indicated on a dynamometer. A suitable chart for such work is shown in Fig. 34, from which the proper stringing tensions at different temperatures may be read.

Example 15. *Illustrating Sag-temperature Calculations.*—Consider a seven-strand No. 000 B. & S. gage steel-cored aluminum conductor suspended between two fixed points (at the same eleva-

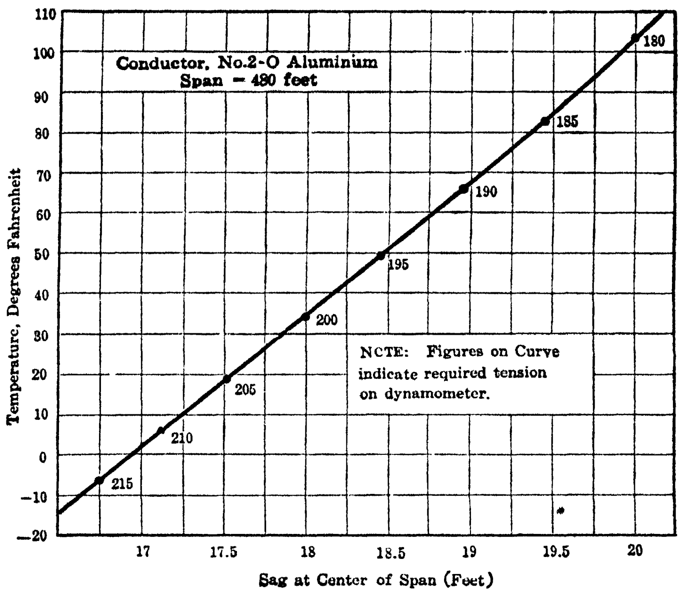


FIG. 33.—Sag-temperature curve illustrating Example 14.

tion) with a distance between them, $l = 400$ ft. It is specified that the tension in the cable must not exceed 3,000 lb. at a temperature of 0°F. with an ice covering of $\frac{1}{4}$ in. radial thickness and a wind pressure of 8 lb. per square foot of projected area. The problem is to calculate (*a*) the wind velocity at a temperature of −20°F. which will produce the same maximum tension when there is no ice deposit on the conductors, and (*b*) the temperature at which the unloaded wire, hanging in still air, will have the same sag as under the specified conditions for maximum loading at 0°F.

Solution (*a*).—If the loading factor *n* which will make the tension the same as in the ice-coated wire can be calculated, it will be easy to determine the wind velocity which produces this loading. First calculate the value of n_c, *i.e.*, the loading factor under the specified conditions, because the curves in the text do not refer to aluminum conductors with steel core.

Particulars relating to steel-cored aluminum cables can always be obtained from manufacturers' catalogues, but the diameter of a seven-strand composite cable may be calculated as follows. The center strand is of steel and there are six aluminum strands of which the total cross-section is 167,800 circular mil

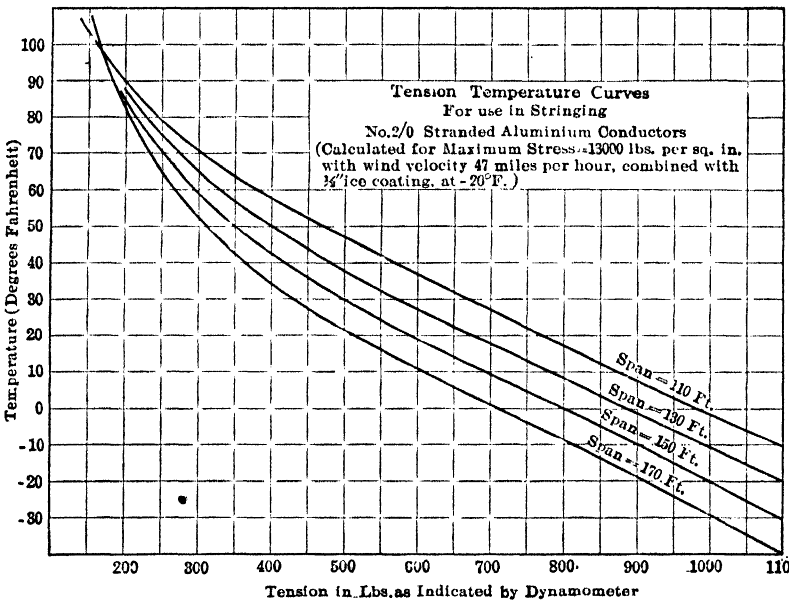


FIG. 34.—Chart giving tension at which wires should be strung.

(refer to wire table on p. 81). The diameter of one-strand is, therefore, $\sqrt{\frac{167,800}{6}} = 167$ mils, and the diameter of the seven-strand cable is $3 \times 0.167 = 0.501$ in. The projected area per foot length of the ice-coated wire is, therefore, 0.0834 sq. ft., and the horizontal pressure due to wind is $p = 0.0834 \times 8 = 0.667$ lb. per foot length of wire. For the vertical loading, the weight of wire without ice is $w = 1.39 \times 10^{-6} \times 167,800 = 0.233$ lb. per foot, where the figure 1.39 is taken from the table of physical constants of steel-cored aluminum conductors on page 80. The weight of ice deposit (see Art. 45, p. 107) is $1.25t\,(d + t) =$

1.25 × 0.25 (0.501 + 0.25) = 0.235 lb. per foot. The total vertical load under the specified conditions of maximum loading is, therefore, 0.233 + 0.235 = 0.468 lb. per foot, and the loading factor (see Art. 46, bottom of p. 113) is

$$n_c = \frac{\sqrt{(0.468)^2 + (0.667)^2}}{0.233} = 3.5$$

The sag s_c corresponding to the specified maximum load conditions is now easily calculated by formula (30a). The horizontal component of the tension is usually so nearly equal to the maximum tension (at point of support) that it is permissible to put $P_h = 3{,}000$ in the formula. Thus

$$s_c = \frac{(wn_c)l^2}{8P_h} = \frac{0.233 \times 3.5(400)^2}{8 \times 3{,}000} = 5.44 \text{ ft.}$$

If it is desired to correct for the difference between the maximum pull (P_B) and the horizontal pull (P_h), this difference, by formula (31), is equal to the product of the sag and the weight per unit length of the conductor. In this instance, the difference is approximately 5.44 (0.233 × 3.5) = 4½ lb., so that the horizontal component of tension P_h is more nearly equal to 2,995 than 3,000 lb.; but the correction is not worth making.

Since one condition of this problem is that the maximum tension shall be the same under the two conditions considered (with and without ice loading), write

$$P_h = \frac{wn_cl^2}{8s_c} = \frac{wnl^2}{8s}$$

whence

$$\frac{n}{s} = \frac{n_c}{s_c} = \frac{3.5}{5.44}$$

the meaning of which is that if the stress in the wire remains constant, the ratio of loading factor to sag will also remain unaltered.

In order to solve for the new loading factor n, first calculate the new sag s. Referring to equation (58a) expressing the relation between temperature, sag, and loading, it is evident that the last term becomes zero, and

$$(t - t_c) = \frac{8}{3al^2}(s^2 - s_c^2)$$

The numerical value for a is 10.5×10^{-6} as given in the table of physical constants for steel-cored aluminum conductors (p. 80). Thus,

$$-20 - 0 = \frac{8 \times 10^6}{3 \times 10.5(400)^2}[s^2 - (5.44)^2]$$

whence $s = 4.12$, which makes it possible to calculate the loading factor because $\frac{n}{4.12} = \frac{3.5}{5.44}$, whence $n = 2.65$.

The wind velocity which will have the effect of making the total load on the wire 2.65 times what it is when there is no wind may be calculated by formula (51) if the horizontal component of pressure $\frac{dV^2}{4820}$ is put in place of p. Thus

$$2.65 = \sqrt{1 + \left(\frac{0.501V^2}{0.233 \times 4{,}820}\right)^2}$$

whence $V = 74$ miles per hour (approximately).

The answer to part (a) of the problem is that a 74-mile wind blowing across the line when the temperature is -20°F. will stress the material to the same extent as the combined effect of ¼ in. ice deposit and a wind pressure of 8 lb. per square foot (about 60 miles per hour[1]) acting on the ice-coated wires at a temperature of 0°F.

Solution (b).—Referring again to formula (58a), since there is no change in the sag, the first term on the right-hand side of the equation becomes zero, and since $n = 1$,

$$t - t_c = \frac{kl^2}{s_c Ea}(n_c - 1)$$

which can be solved for t.

Substituting for $\frac{kl^2}{s_c}$ its equivalent $\frac{S_c}{n_c}$,

$$t = \frac{S_c}{a \times E}\left(1 - \frac{1}{n_c}\right) + t_c$$

where S_c is the stress in pounds per square inch under loaded conditions. The total cross-section of the cable is $\frac{7}{6} \times 167{,}800 = 196{,}000$ circular mils, or 0.154 sq. in., whence $S_c = \frac{3{,}000}{0.154} = 19{,}500$ lb. per square inch. The modulus of elasticity (E) may

[1] Read off Fig. 27, p. 109.

be obtained from the table of physical constants on page 80, where its numerical value is given as 12×10^6. Thus

$$t = \frac{19{,}500 \times 10^6}{10.5 \times 12 \times 10^6}\left(1 - \frac{1}{3.5}\right) + 0$$
$$= 110.5°\text{F}.$$

This is a temperature which the wire will rarely, if ever, attain; but there are instances—especially in connection with aluminum conductors without steel cores,—where the summer sag (without wind) is greater than the maximum deflection of the loaded conductor under storm conditions at low temperatures.

49. Sag-temperature Calculations with Supports at Different Elevations.—The manner in which tensions and sags can be calculated for wires suspended between supports at different elevations was explained in Art. 42, Chap. IV, but the main concern now is the effect of temperature variations on the sag (and tension) of a particular span.

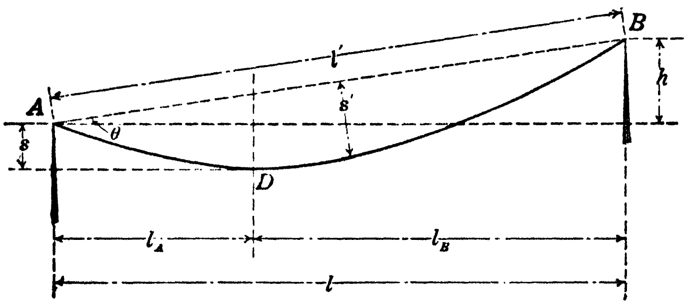


FIG. 35.—Wire hung between supports at different elevations.

Consider, in the first place, the arrangement illustrated by Fig. 35, where the relation of length of span to difference of elevation is such that there is a sag s of the wire below the level of the lower support A. It is desired to determine the manner in which the vertical sag s varies with change of temperature. Usually the required tension in a wire at the time of stringing is determined by the maximum possible tension under conditions of greatest loading, *i.e.*, at low temperature with strong wind blowing, and including ice deposit in districts where ice may be expected. The charts prepared for the engineers erecting the conductors assume no ice loading and usually no wind, so that it is convenient to calculate in the first place the temperature at which the sag will have some particular value after the abnormal loading due to wind and ice (if any) has been removed. The calculations are simplified by assuming the same sag (or deflection at center of span) as when the wire is subjected to the maximum loading. This is what was done for a span with supports at the same elevation in *Solution* (*b*) of the preceding Example 15. The writer has found this method very useful in simplifying calculations, and instead of using formula (57) or (58) as a starting point, the same result will be obtained by a step by step process of reasoning which is easy to follow.

The relation between sag and stress, as expressed by formula (30*b*), is

$$S = k\frac{l^2}{s}$$

when a wire hangs between supports l ft. apart in still air. The general form of this equation is

$$S = \frac{nkl^2}{s}$$

where n is what has been defined as the loading factor. It follows that, *if there is no change in the sag,* the tensile stress S in the material will be n times as great when the wire is loaded than when this load is removed. Obviously, this simple ratio between tensions does not hold *unless the temperature of the wire is increased so that its length remains unaltered.*

Let t_o be the temperature at which the loaded conductor is stressed to the maximum working limit of S_m lb. per square inch of cross-section. When the temperature is raised from t_o to t_c°F., the *increase* of length due to change of temperature is

$$\Delta\lambda = \lambda a(t_c - t_o)$$

and the *decrease* of length due to elastic contraction with reduction of stress is

$$\Delta\lambda = \frac{\lambda}{E}S_m\left(1 - \frac{1}{n}\right)$$

For the condition of no change in length, *i.e.*, constant sag, it is necessary merely to equate these quantities and solve for t_c. Thus, the temperature at which the unloaded cable will have the same maximum deflection as the loaded cable at a lower temperature t_o is

$$t_c = \frac{S_m}{E \times a}\left(1 - \frac{1}{n}\right) + t_o \tag{59}$$

The curves of Fig. 36 give approximate values for $t_c - t_o$ calculated by formula (59) for different materials and loading factors.

Example 16. *Sag-temperature Calculations with Supports at Different Elevations.*—Suppose the given data to be as follows:

Conductor, No. 4/0 stranded copper.

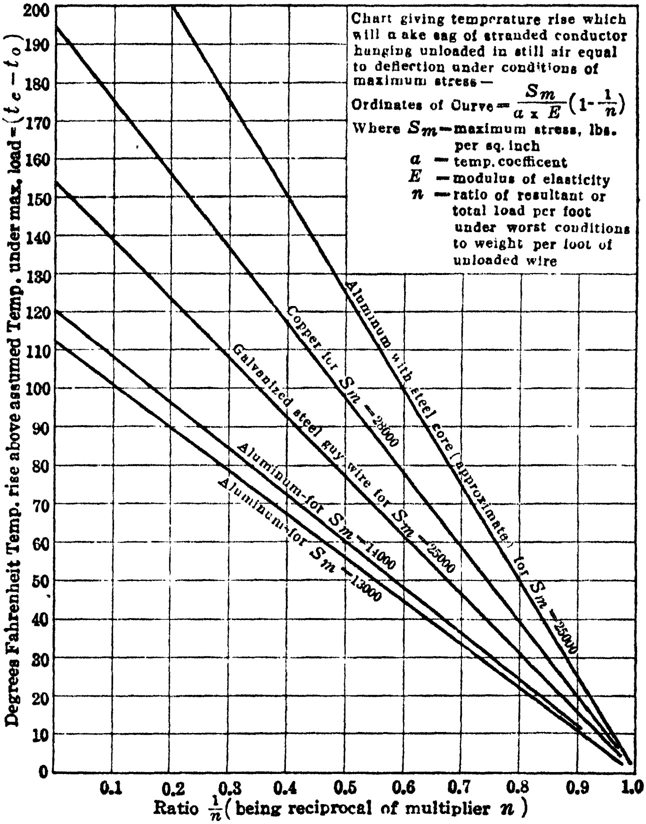


FIG. 36.—Chart for determining temperature (t_c) at which unloaded wire has same sag as loaded wire at temperature t_o.

Cross-sectional area, A = 0.1662 sq. in. } From table, page 81
Weight per foot run, w = 0.647 }

Modulus of elasticity, $E = 15 \times 10^6$ } From table, page 77
Coefficient of linear expansion, $a = 9.3 \times 10^{-6}$ }

Maximum permissible stress, S_m = 28,000 lb. per square inch with ½ in. ice deposit and 55-mile wind at 22°F.

Horizontal distance between points of support, l = 1,000 ft.

Difference of elevation (h in Fig. 35) = 30 ft.

On account of the small difference of elevation compared with the span length, it is permissible to consider the straight-line distance l' between points of support as being the same as the horizontal distance. Actually,[1]

$$l' = \frac{l}{\cos\theta} = l + \frac{h^2}{2l}$$

but the correction is not worth making in this instance. Note

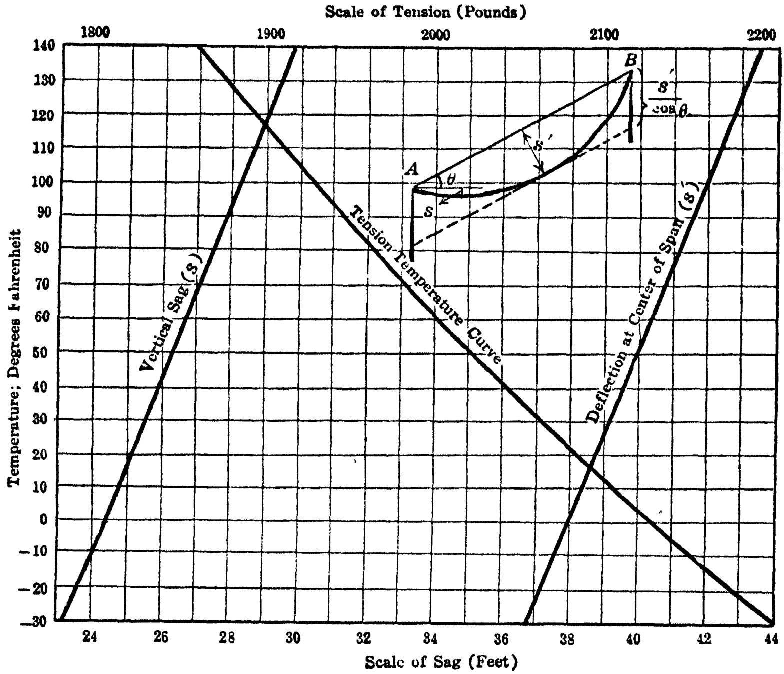


FIG. 37.—Sag-temperature curves illustrating Example 16.

also that, for the same reason, namely, because $\cos\theta$ is very nearly unity, the horizontal component of the tension is very nearly equal to the tension in the direction AB (Fig. 35). No appreciable error will be introduced by assuming the horizontal component of tension under conditions of maximum load to be $P_h = 28{,}000 \times 0.1662 = 4{,}650$ lb. The loading factor, as read off Fig. 30 (p. 113), is approximately $n = 2.5$; therefore, the tension in the wire at the temperature t_c when the wire hangs

[1] See footnote on p. 89.

freely without additional loading, but *with the same sag as under maximum load conditions*, is $P_c = \frac{4,650}{2.5} = 1,860$ lb.

The increase of temperature which will bring about this condition, as read off Fig. 36, is approximately $(t_c - t_o) = 117°F.$, so that the actual temperature would have to be $117 + 22 = 139°F$. Since this is a very high temperature, the tension in the unloaded wire will usually be greater than 1,860 lb. and three arbitrary values will be assumed for P_h, namely 2,000, 2,100, and 2,200 respectively, and then the corresponding sags will be calculated by the step-by-step method followed in the accompanying table.

The curves of Fig. 37 give the results of the calculations in graphical form. In practice, the deflection s' at center of span is easily measured by sighting between two points on the supporting towers $\frac{s'}{\cos\theta}$ ft. below the points of attachment of the wire.

NUMERICAL EXAMPLE 16. CALCULATIONS FOR SAG-TEMPERATURE CURVES

a. Assumed tension P in direction AB...	$P_c = 1,860$	2,000	2,100	2,200
b. Horizontal component of tension, $P_h = P \cos\theta$ (approximately equal to P in this example)	1,860	2,000	2,100	2,200
c. Maximum deflection from straight line AB. Formula (30), $s' = \frac{wl^2}{8P_h} = \frac{81,000}{P_h}$ ft..........	43.5	40.5	38.6	36.8
d. By formula (40), $l_A = 500 - \frac{30P_h}{0.647 \times 1,000} = 500 - \frac{P_h}{21.5}$ ft........	413.5	407	402.3	397 7
e. Vertical sag; by formula (42); $s = 0.323\frac{l_A^2}{P_h}$ ft..........	29.7	26.8	24.9	23.2
f. Length of wire; by formula (55), $\lambda = l' + \frac{8(s')^2}{3l'}$ (approx.) $= 1,000 + \frac{(\text{Item } c)^2}{375}$ ft..........	$\lambda_c = 1,005.05$	1,004.38	1,003.97	1,003.61
g. Total elongation, $\lambda - \lambda_c$ ft..........	0	−0.67	−1.08	−1.44
h. Elongation due to stress variation; by formula (56), substituting l' for λ; $\Delta\lambda_c = \frac{l'}{E}\left(\frac{P - P_c}{A}\right) = \frac{P - P_c}{2,490}$ ft...	0	0.056	0.096	0.136
i. Elongation due to change of temperature = $(g) - (h)$ ft..........	0	−0.726	−1.176	−1.576
j. Increase of temperature $(t - t_c) = \frac{\text{Item } (i)}{a \times \lambda}$ = Item $(i) \times 107.5$ (approximately)..........	0°F.	−78°F.	−126°F.	−169°F.
k. Temperature $t = t_c + (j) = 139 + (j)$	$t_c = 139°F.$	61°F.	13°F.	− 30°F.

Example 17. Short-span Line on Steep Incline.—When a transmission line is run up the side of a hill, and the difference in elevation of the successive supports is large in comparison with

the span, the sag-temperature curves based on the assumption of supports on the same level are no longer applicable. The manner in which correct sag-temperature curves may be calculated for a line on a steep incline is illustrated by the following numerical example.

Given: Conductor, No. 0 B. & S. solid copper
Diameter, $d = 0.325$ in.
Cross-section, $A = 0.083$ sq. in.
Weight of wire per foot run, $w = 0.32$ lb.
Maximum tension in wire = 915 lb.[1]
Wind velocity, 78 miles per hour, giving a pressure of 15 lb. per square foot of projected surface of wire at a temperature of 22°F. No ice coating.
Horizontal distance between supports, $l = 190$ ft.
Difference of elevation between supports, $h = 140$ ft.

Referring to Fig. 38, it is seen that the straight-line distance AB is equal to $\frac{l}{\cos\theta}$ or $\sqrt{(195)^2 + (140)^2} = 240$ ft.

In order to calculate the maximum deflection s' at center of span when there is no extra loading due to wind and ice, use the fundamental relation

$$\text{Sag} = \frac{(\text{span})^2 \times (\text{weight per unit length})}{8 \times (\text{tension in wire})}$$

When the line is on the slope, as indicated in Fig. 38, the load per foot run is not w, but $w \cos\theta$; then, if P = tension in wire at center of span,

$$s' = \frac{(AB)^2 w \cos\theta}{8P} \tag{60}$$

which is simply another way of writing the fundamental formula (30) which was developed in Art. 41, Chap. IV.

When the effect of extra loading due to wind and ice deposits is to be considered, the loading factor as calculated in Art. 46 for level spans is not applicable. When a horizontal wind blows at right angles to the line AB of Fig. 38, the total loading due to gravity, resolved in a direction perpendicular to the line AB, is

[1] This includes a large factor of safety. A low value for the tension has been assumed in order to increase the amount of sag and bring out more clearly the difference between spans on a slope and on level ground.

(w + ice) $\cos \theta$ per foot run; and if p is the pressure of the wind (per foot of wire) blowing across the line, the resultant loading is

$$\sqrt{p^2 + (w + \text{ice})^2 \cos^2 \theta} \text{ lb. per ft.}$$

whence, by definition, the loading factor is

$$n = \frac{\sqrt{p^2 + (w + \text{ice})^2 \cos^2 \theta}}{w \cos \theta} \tag{61}$$

This is the general form of the formula (51) which is true only for the special case of $\theta = 0$ and no ice deposit. It is the factor by which $w \cos \theta$ must be multiplied to obtain the total loading

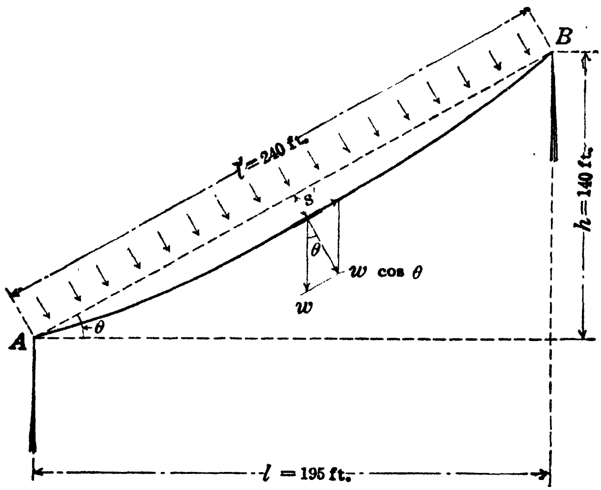


FIG. 38.—Transmission line on steep incline.

per foot of wire. The general expression for the maximum deflection of a wire suspended between two fixed points A and B is, therefore,

$$s' = \frac{(AB)^2\sqrt{p^2 + (w + \text{ice})^2 \cos^2 \theta}}{8P} \tag{62}$$

The numerical values of these quantities are

$$AB = 240$$

$$p = 15 \times \frac{0.325}{12} = 0.406$$

$$w = 0.32$$

Ice loading $= 0$

$$\cos \theta = \frac{195}{240} = 0.8125$$

$$P = 915$$

Substituting in formula (62), the deflection is found to be $s' =$ 3.8 ft.

This is the maximum sag under loaded conditions. When the extra load is removed, and the temperature raised *until the sag is again* 3.8 *ft.*, the tension in the wire will be $\frac{P}{n}$ wherein n has the value as given by formula (61), which in this example is 1.85. Therefore $P_c = \frac{915}{1.85} = 494$ lb., and the temperature at which this

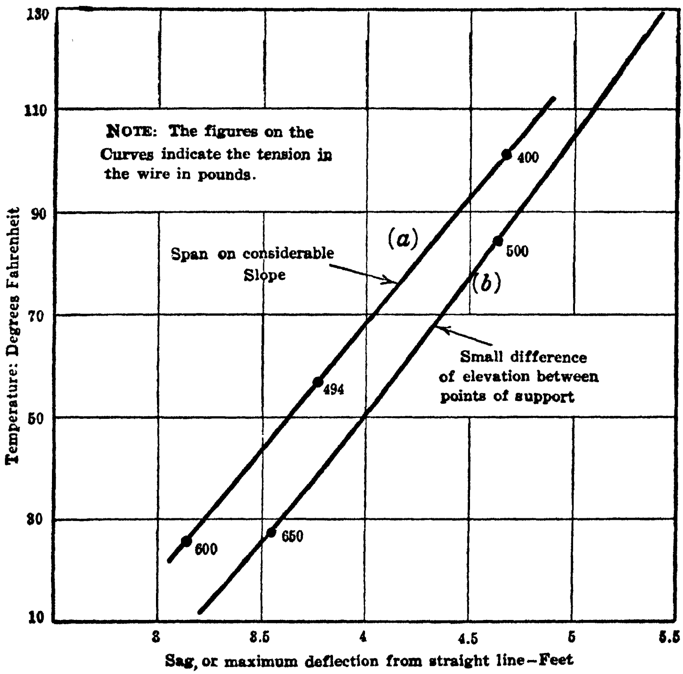


FIG. 39.—Sag-temperature curves illustrating Example 17.

particular condition of sag and tension (in the unloaded wire) will occur is, by formula (59),

$$t_c = \frac{915 \times 10^6}{0.083 \times 15 \times 10^6 \times 9.3}\left(1 - \frac{1}{1.85}\right) + 22$$
$$= 58°\text{F. (approximately)}$$

The temperature corresponding to other (assumed) values of sag may be calculated by means of formula (58*a*), bearing in mind that it is necessary to put $l' = AB$ in place of l, and $k \cos \theta$

$\left(= \frac{w \cos \theta}{8A}\right)$ in place of k. The method of procedure has already been illustrated by Example 14.

The curve marked (*a*) in Fig. 39 gives the results of the calculations. The second curve, marked (*b*), gives the relation between sag and temperature for an exactly similar span except that the points of support are approximately at the same elevation. The separation between points of support, the conditions of maximum loading, and the permissible maximum tension are the same in both cases, but, owing to the difference in the loading factors, the relation between temperature and sag, or temperature and tension, when the load is removed, is not the same for the line on an incline as for the level span. The loading factor calculated for the level span is $n = 1.58$.

50. Wire Tensions When Successive Spans are not of the Same Length.—When stringing overhead wires, it is customary to pull up a number of spans at one time, the proper tension as recorded on the dynamometer being that calculated for one particular length of span and the particular temperature at which the work is carried out. This method is entirely satisfactory when all the spans in the section are equal, but the calculation of the proper tension in the wire at the time of stringing is less simple when the spans are not all of the same length.

Consider, first, a line with unequal spans and wire tied securely to pin-type insulators. If the wires could be strung under weather conditions which will stress the material to the safe working limit, it would be correct to pull the wire to this particular tension and then tie securely at each insulator. It should be noted, however, that, with removal of the additional load, with or without change of temperature, there will be unbalanced pulls tending to bend the insulator pins *unless the spans are all of the same length*. There is one exception to this statement, namely that at a particular temperature which the writer has elsewhere[1] referred to as the "critical temperature," the tension in the unloaded wire will be the same in all spans irrespective of their length. This particular temperature is that at which the unloaded wire, hanging in still air, will have a sag equal to the maximum deflection of the loaded wire under the specified conditions which produce the maximum safe stress in the material.

[1] "Sags and Tensions of Overhead Conductors," *Elec. World*, vol. 59, p. 1021, May 11, 1912. See also *Solution* (*b*) of *Example* 15 on p. 126.

This "critical temperature" may be calculated by the formula

$$t_c = \frac{S_m}{a \times E}\left(1 - \frac{w}{(nw)}\right) + t_o \qquad (59)$$

where S_m = maximum stress in the material (pounds per square inch)

E = modulus of elasticity

a = temperature-elongation coefficient

w = weight per foot of unloaded wire

(nw) = total load per foot of wire, including wind pressure and ice (if any)

t_o = temperature at which maximum load occurs.

Figure 34 illustrates a particular case in which the "critical temperature" is about 98°F., and if the wire were strung at this temperature, it would be a matter of indifference whether or not the spans were all of the same length. When stringing at any other temperature, the question arises as to which of the several curves will indicate the correct tension at which to pull up the wires when the spans in a particular section are not all of the same length.

If it is admitted that a small lateral movement in the direction of line is possible at points of attachment—such as actually occurs when the suspension type of insulator is used—the tensions in the consecutive spans of unequal length will be approximately the same at all temperatures. The relation among tension P, span l, and sag s is

$$P = \frac{wl^2}{8s} \qquad (30a)$$

where P is in pounds, l and s in feet, and w is the weight, in pounds, per foot length of wire.

Any tendency to unbalance the pull on the two sides of a point of attachment will simply cause this point to move until the ratio $l^2 \div s$ is the same in all the spans irrespective of length. It is thus seen that, in a series of spans of different lengths, but in which the points of suspension intermediate between the two strain tower attachments are capable of small horizontal motion, thus equalizing the tension in adjoining spans, the ratio $\frac{l^2}{s}$ will be approximately the same at all temperatures.

Imagine, now, that the actual series of spans is replaced by a number of spans *all of the same length*, such that the total length of wire in the section and the tension at any particular temperature are the same as in the actual arrangement. The important difference between the imaginary series of "equivalent" spans and the actual series is that, since the equivalent spans are all of the same length, there will be *no movement of the point of attachment* with change of temperature (and stress). The intermediate points of support may, therefore, be considered fixed in position, and calculations based on a single "equivalent" span may be used for determining stringing tensions which will apply not only to the series of imaginary "equivalent" spans but to the actual arrangement also.

Calculation of Equivalent Span.—Given a wire of weight w lb. per foot, suspended horizontally between two points l ft. apart, the length of wire in the span is

$$\lambda = l + \frac{w^2 l^3}{24P^2} \qquad (55a)$$

where P is the tension in the wire (pounds).

The total length of wire in a section consisting of several spans of lengths l_1, l_2 . . . etc. will be

$$\lambda_1 + \lambda_2 + \ . \ . \ . \text{ etc.} = \Sigma l + \frac{w^2}{24P^2} \Sigma l^3$$

It is assumed that slipping at points of attachment is possible, or that suspension-type insulators are used, permitting the small lateral movement of the points of support which will occur with changes of temperature if the spans are of unequal length. Consider the actual arrangement of spans to be replaced by a series of n_e "equivalent" spans all of the same length l_e. At any given temperature, the total length of wire in the section and the tension in the wire must be the same as in the actual series of spans, a condition which is expressed by the equation

$$n_e\lambda_e = n_e\left[l_e + \frac{w^2 l_e^{\,3}}{24P^2}\right] = \Sigma l + \frac{w^2}{24P^2} \Sigma l^3$$

Note that $n_e l_e = \Sigma l$, which reduces the equation to a simple form from which the length of the equivalent span is seen to be

$$l_e = \sqrt{\frac{\Sigma l^3}{\Sigma l}} \qquad (62a)$$

This is the length of span which should be used for calculating the stringing tension at various temperatures. If the wires in the section consisting of several unequal spans are pulled to the tension indicated by these calculations, they will not be overstressed when the weather conditions are such as to produce the specified or assumed maximum load.

The use of the *average* span in these calculations is incorrect, although with small differences of length among individual spans the error is not of any practical importance. Moreover, with very great differences of length in adjoining spans, it is customary to dead-end the wires at the junction of the two spans, so that an exact formula for the "equivalent" span is not often needed. The following numerical example shows that there may be an appreciable difference between the *average* and *equivalent* span lengths.

Consider four spans in the dead-ended section, of lengths 300, 300, 800, and 400 ft., respectively. The average length of span is 450 ft., but the equivalent length calculated by formula (62*a*) is 592 ft.

CHAPTER VI

TRANSMISSION LINE SUPPORTS

51. General Considerations. Types of Transmission Line Supports.—The supporting poles or structures for overhead electric power transmission lines are of various kinds. Where the ordinary wood telegraph pole or a larger single pole of similar type is not suitable, double poles of the "A" or "H" type, or even braced wooden towers of considerable height and strength, may sometimes be used with advantage. Under certain conditions it may be economical to use steel poles or light masts of latticed steel, even for comparatively short spans; and poles of reinforced concrete have much to recommend them. But for long spans, and the wide spacing of wires necessary with the higher pressures, steel towers, either of the rigid or flexible type, will generally be required.

The decision as to the best type of line to adopt is not easily or quickly arrived at. The problem is mainly an economic one, and the decision will depend, not only on the first cost of the various types of line construction, but also on the probable life of the line and the cost of maintenance.

It is necessary to make up many preliminary estimates of the completed line, and these must obviously include, not only the cost of the various types of supporting structure delivered at points along the line, but also the cost of foundations and erection. Again, even if a suitable kind of wood is readily available in the district to be traversed by the transmission line, it is possible that the cost of seasoning the poles, and treating them with preservative compounds to ensure a reasonably long life, may render the use of steel structures more economical even for comparatively low pressures. The use of latticed steel poles, from 30 to 40 ft. high, capable of being shipped and handled in one piece, appears to be gaining favor in districts where ultimate economy over wood poles can be shown to result from the adoption of these light steel structures. The life of a steel tower line depends somewhat on climatic conditions. In Great Britain the dampness of the climate, together with the impurities in the atmosphere in the neighborhood of manufacturing and populous districts, renders light steel structures less durable than in America (except, perhaps, on the Pacific coast, where special precautions are required to guard against rapid corrosion due to the prevalence of fogs and moisture). Not only has the iron-work protected by paint to be repainted on the average every 3 years, but the spans must usually be short, as the private ownership of valuable property renders the construction of a straight transmission line with long equal spans almost impossible in the United Kingdom. These conditions are all in favor of the employment of selected and well-creosoted wood poles, the life of which may be 30 years or more.

When making comparisons between wood and steel for transmission line supports, it is not only the matter of first cost that has to be considered. Steel structures have the advantage of being invulnerable to prairie and forest fires; moreover, owing to the longer spans rendered possible by the stronger and taller supports, there is less chance of stoppages owing to broken insulators, and less leakage loss over the surface of insulators. A fact that is often overlooked is that the size of conductor limits the practical length of span; for instance, with a small conductor such as a No. 4 B. & S., it would not be wise to have spans much above 250 or 300 ft. This suggests what is frequently found to be the case, namely, that the total cost of a line may be reduced by using a conductor of rather larger section than the electrical calculations would indicate as being necessary, because the stronger cable permits of a wider spacing of the supporting towers.

52. Wood-pole Lines.—Among the varieties of straight-growing timber used for pole lines on the American continent may be mentioned cedar, chestnut, oak, cypress, juniper, pine, tamarack, fir, redwood, spruce, and locust. In England the wood poles are usually of Baltic pine or red fir from Sweden, Norway and Russia. The woods used for the cross-arms carrying the insulators include Norway pine, yellow pine, cypress or Douglas fir, oak, chestnut, and locust.

Probably the best wood for poles is cedar; but chestnut also makes excellent durable poles. Much depends, however, on the nature of the soil, and, generally speaking, poles cut from native timber will be more durable than poles of otherwise equally good quality grown under different conditions of soil and climate.

With the more extended adoption of preservative treatments (to be referred to later), the inferior kinds of timber which under ordinary conditions would decay rapidly, will become of relatively greater value, and with the growing scarcity of the better kinds of timber, it is probable that poles of yellow pine, tamarack, and Douglas fir will be used more extensively in the future.

The trees should be felled during the winter months, and after being peeled and trimmed should be allowed to season for a period of at least 12 months.

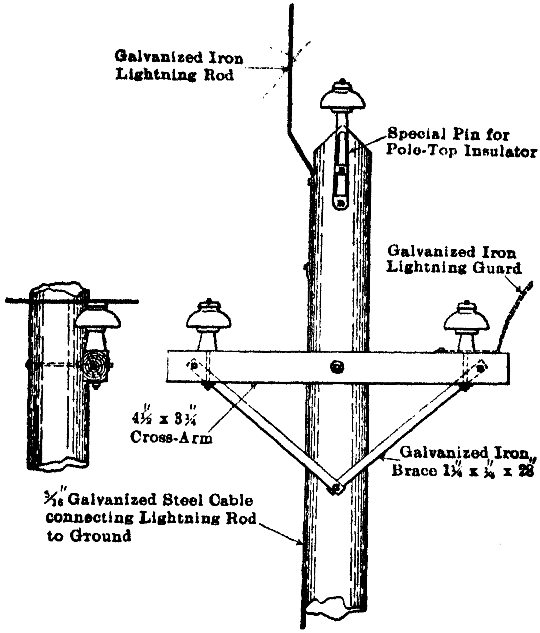


FIG. 40.—Pole-top details.

Single three-phase lines transmitting energy at about 22,000 or 33,000 volts are frequently carried on single wood poles having a top measurement of 7 or 8 in. in diameter. The separation between wires would be about 40 in, for 250 ft. spans. and 36 in. for an average span of 200 ft. Figure 40 shows a pole top arranged to carry a 22,000-volt three-phase line without overhead guard wire, but with lightning rod attached. It is not customary to provide every pole with a lightning rod, but unless these rods are spaced at frequent intervals (about 400 ft. apart), they will not afford much protection to the line. In exposed positions and at angles, pieces of bent flat iron may be fitted with advantage on the cross-arm near the insulators, as shown by the dotted lines in Fig. 40. These pieces serve the double purpose of hook guard in case of the wire slipping off the insulator, and of additional protection against lightning. A discharge from the line tends to leap across to this grounded metal horn over the surface of the insulator, thus frequently preventing the piercing or shattering of insulators.

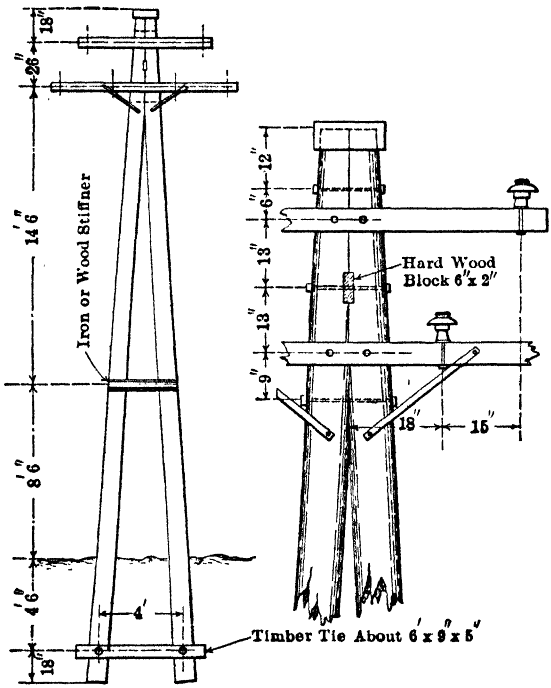


FIG. 41.—Typical "A" frame construction for duplicate three-phase line.

A simple A-frame construction for a duplicate three-phase line operating at 11,000 volts, is shown in Fig. 41. This is a much stronger construction than the single pole line and permits of longer spans, even when carrying fairly heavy conductors. When only three wires have to be carried, a more economical construction consists in using two poles with a single cross-arm joining the tops of the poles. No bracing of the cross-arms is necessary, and no extra bracing need be provided between the poles. There are many instances of 110,000-volt three-phase lines being carried on wood-pole structures of this type, the wires being carried by suspension-type insulators attached to the center and two ends of the single cross-arm. A modern example of this construction is the 110,000-volt line of the California Oregon Power Company, between its Copco plant and Delta, Calif., a distance of 77½ miles. Three stranded copper cables of 250,000 circular mils cross-section are carried on suspension insulators attached to a single horizontal cross-arm of untreated Douglas fir, measuring 6 in. by 8 in. by 22 ft. long. Butt-treated Idaho cedar poles, averaging 60 ft. in length are used, and the spans range from an average of 600 ft. to a maximum of 1,500 ft.

A similar construction has been adopted by the Washington Water Power Company for the recent extensive development of their 110,000-volt transmission lines. The suspension insulators are spaced 10 ft. apart on 22-ft. cross-arms bolted near the top of two 50-ft. butt-treated cedar poles set 8 ft. in the ground. On most of these lines two 5/16-in. stranded galvanized-steel ground wires are attached to the tops of the poles, partly to provide protection from lightning, but also to increase the mechanical strength of the line. One of the earliest instances of wood poles used for high-pressure power transmission is the 100,000-volt three-phase transmission line of the Montana Power Company, where each supporting structure consists of two 45-ft. cedar poles connected at the top by a single cross-arm, but provided with no additional bracing or stiffening members between the poles. This line runs from Great Falls to Deer Lodge, Mont., a distance of 140 miles; it consists of three No. 4/0 copper conductors, and supplies power to the Chicago, Milwaukee & St. Paul Railway.

In districts where wood poles are available, economy may be effected by erecting wooden structures in place of the steel strain towers which are more generally used. A simple wooden structure consisting of only three main poles is used by the San Diego Consolidated Gas and Electric Company for carrying spans up to 4,437 ft. long.[1]

A still more remarkable instance of engineers having had the courage to depart from what is generally considered common

[1] See detailed description by K. B. Ayers, *Elec. World*, vol. 84, p. 114, July 19, 1924.

practice is the 1,611-ft. river crossing on the line between Astoria, and Flavel, Ore., where wood-pole towers 135 ft. high have been used.

53. Life of Wood Poles. Preservative Treatment.—It is not easy to estimate the probable life of poles, because this will depend not only on the kind of timber, but also on the nature of the soil, climatic conditions, the time of seasoning, whether or not the poles have received treatment with preservative compounds, and the nature of such treatment.

In England the life of well-seasoned, creosoted poles may be about 35 years in good soil, and from 18 to 20 years in poor soil. On the American continent, where untreated poles have been used in large numbers, the average life is probably about 12 years. The better woods, such as cedar and chestnut, might last on the average 12 to 14 years, while juniper and pine might have to be replaced in 6 to 9 years. On certain lines where untreated poles of unsuitable timber have been erected in poor soil, or where destructive insects are particularly active, the poles have had to be replaced in less than 4 years.

The effect of preservative treatment upon the life of wood poles is very noticeable, especially in the case of the poorer qualities of timber. The average life of butt-treated cedar and chestnut poles is 20 years, and even pine, when properly treated with preservative compound, may have a life of 20 years in a dry climate.

There is a case on record of two 70-ft. Western red cedar poles originally installed in 1905 which were found to be in such good condition that they were removed (after 20 years' use) and reerected upon another site. The life of a pole depends largely upon the facility with which water can drain away from the portion buried in the ground. Marshy soil is generally bad for wood poles, and also ground that is alternately wet and dry.

Preservative Treatment of Poles.—Many chemical solutions and methods of forcing them into the wood have been tried and used with varying success; but it is generally conceded that treatment with coal-tar creosote oil gives the best protection against decay; and its cost is probably lower than that of any other satisfactory treatment. Corrosive sublimate (bichloride of mercury) is used in the "kyanizing process" of wood preservation, and zinc chloride is satisfactory under certain conditions. It is cheaper than creosote, but it is soluble in water and is, therefore, of little use for the treatment of pole butts except when the poles are to be erected in dry soils.

There are three recognized methods of applying creosote oil in the preservative treatment of wood poles:

a. The high-pressure treatment (Bethel system).
b. The open-tank treatment.
c. Brush treatments.

High-pressure Treatment.—This is undoubtedly the best, but it is also the most costly. The poles, after being trimmed and framed, are placed in large treating cylinders capable of being hermetically closed. If the poles are green or wet, they are first subjected, in these cylinders, to a steaming process from 3 to 8 hours, the steam being admitted under a pressure of 12 to 20 lb. The steam is then blown off, and the treating cylinder is exhausted, the vacuum being maintained for a period of 1 to 2 hours. Immediately afterward the creosote is forced in under pressure at a temperature of 140 to 200°F. Seasoned timber is not subjected to the steaming process, but the temperature inside the treating cylinder is raised by means of heating coils to about 150°F. prior to the filling process.

The poles will absorb from a minimum of 10 lb. to a maximum of 15 lb. of oil per cubic foot. The softer and more porous woods will absorb the most oil; but on the other hand, the benefit such woods derive from the treatment in the matter of increased life is more marked than in the case of the closer-grained timber.

Open-tank Treatment.—The butts of the poles are placed in the creosote oil, which is preferably heated to a temperature of 200 to 220°F. They are maintained in this bath for a period of 1 to 3 hours, after which they are placed in cold oil for another period of 1 to 3 hours. This process will permit of a complete penetration of the sapwood to a height of about 2 ft. above ground level. When properly carried out it is capable of giving very satisfactory results. The open-tank process is especially applicable to the treatment of the more durable kinds of timber, such as cedar and chestnut.

Brush Treatment.—The oil is applied hot with hard brushes, a second coat being applied after the first has soaked in. The temperature of the oil should be about 200°F. This method of application of the oil is the cheapest and the least effective, but it affords some protection when the wood is well seasoned and dry. A penetration of about 1/8 in. may be secured. There is little advantage to be gained by the external application of preservative compounds to green timber; indeed, the sealing up of the surface of such timber, by enclosing the fermentative juices, may even lead to more rapid decay. The brush treatment cannot be applied to poles which are set in the winter months in cold climates, as the frost would so harden the surface of the poles that there would be no absorption of the preservative liquid.

The reader who desires to go further into the important question of wood-pole preservation, is referred to the excellent book by Howard F. Weiss, "The Preservation of Structural Timber."[1] It seems hardly necessary to point out that the saving effected by prolonging the life of the poles will usually justify the cost of the special treatment. Given reliable data regarding costs and probable life of treated and untreated poles, the necessary calculations are easily made.

54. Weight of Wood Poles.—For the purpose of estimating the costs of transport and handling of poles, the weight may be calculated on the assumption that the pole is of circular section and of uniform taper, such that the diameter D at the bottom is equal to the diameter d at the top *plus* a quantity tH, of which H is the distance between the two sections considered, and t is a constant depending on the taper and, therefore, on the kind of wood. Some approximate (average) values of t together with average weight per cubic foot of various kinds of *dry* timber will be found in the table on the following page, from which the value of t for cedar is seen to be 0.016 (the height H being understood to be expressed in *inches*), and the weight per cubic foot, 35 lb.

The *volume* of a frustum of right circular cone is

$$Volume = \frac{H}{3} \times \frac{\pi}{4}(D^2 + Dd + d^2)$$

but $$D = d + tH$$
and the formula becomes

$$Volume = \frac{\pi H}{12}(3d^2 + t^2H^2 + 3dtH) \text{ cubic inches}$$

[1] McGraw-Hill Book Company, Inc. See also Rhodes, F. L. and Hosford, R. F., "Preservative Treatment of Telephone Poles," *Trans.* A.I.-E.E., vol. 34, part 2, p. 2549, October, 1915. Valuable information on preservative treatments is given in the U. S. Forest Service Circ. 84, 98, 147, etc., Dept. Agr.

Constants for Wood Poles

Kind of wood	Weight per cubic foot, pounds approximately	Natural taper t, average	Modulus of rupture S[1]	Modulus of elasticity E[2]
Southern white cedar (juniper)	25	0.014	3,700	
Northern white cedar	23	0.014	4,000	
American Eastern white cedar	35	0.016	4,000	700,000
Spruce	27	.....	4,500	1,300,000
White pine	26	.....	4,500	1,000,000
Red pine	34	....	5,000	
Douglas fir[3]	34	.....	6,500	1,400,000
Norway pine	..	.....	7,000	1,400,000
Idaho cedar	23	0.01	6,000	
Chestnut	42	0.013	6,000	900,000

[1] Being stress in pounds per square inch at moment of rupture under bending conditions.

[2] Inch units. Average figures, which must be considered approximate only.

[3] This name is intended to cover yellow fir, red fir, Western fir, Washington fir, Oregon fir, northwest and west-coast fir.

By using this formula and putting for H the value $65 \times 12 = 780$ in., and for d the value 7 in., the weight of a pole of American Eastern white cedar measuring 7 in. in diameter at top and 65 ft. overall length works out at 2,330 lb.

55. Strength and Elasticity of Wood Poles.—Apart from the dead weight to be supported by the poles of a transmission line—which will include not only the fixtures and the conductors themselves, but also the added weight of sleet or ice in climates where ice formation is possible—the stresses to be withstood include the resultant pull of the wires in adjoining spans, and the wind pressure on poles and wires. It is customary to disregard the dead weight or column loading, except when the spans are large and the conductors numerous and heavy. A formula for approximate calculation of loads carried by poles when acting as struts or columns will be given later. The pull due to the conductors on corner poles is usually met by guying these poles, by which means the pull tending to bend the pole is largely converted into an increased vertical downward pressure; but even on straight runs there may be stresses due to unequal lengths of span which will cause a difference in the tensions on each side of the pole. The most important stresses to which the poles are subjected, apart from such accidents as are due to falling trees or the severing of all wires in one span, are those caused by strong winds blowing across the line. The resulting pressure at pole top due to strong winds acting on long spans of ice-coated wires may be very great, and the poles must be strong enough to resist this.[1]

For the purpose of making strength and deflection calculations, the pole may be considered as a truncated cone of circular section, firmly fixed in the ground at the thick end, with a load near the small end in the form of a single concentrated resultant horizontal pull. The calculation is, therefore, exactly the same as for a beam fixed at one end and loaded at the other. Such a beam, if it exceeds a certain length depending upon the amount of taper, will not break at the point where the bending moment is greatest (*i.e.*, at the ground level), because, in a beam of circular section and uniform taper, the stresses in the material are not necessarily greatest at this point, as will be shown later. The ordinary telegraph or electric lighting pole usually breaks at a point about 5 ft. above ground unless the butt has been weakened by decay. Some engineers insist that the weak section of the pole should be taken at the ground line. There is no objection to this somewhat unscientific assumption, provided a reasonable allowance is made for the reduction of diameter at the ground level owing to decay.

Calculations on strengths and deflections of wood poles cannot be made with the same accuracy as in the case of steel structures; and the constants in the table of Art. 54 are averages only for approximate calculations. The factor of safety generally used on the American continent is 4, both for poles and cross-arms. The maximum wind pressure is taken at 30 lb. per square foot of flat surface, and 18 lb. per square foot of projected surface of smooth cylinders of not very large diameter.

Calculation of Pole Strengths.—The relation between the externally applied load and the stresses in the fibers of the wood is:

Bending moment = stress in fibers most remote from neutral axis × section factor, or $M_B = S \times Z$, where the section modulus Z is the ratio

$$\frac{\text{moment of inertia of section}}{\text{distance of center of gravity from edge of section}}.$$

[1] See Art. 45 in Chap. V.

If P is the force in pounds applied at a point distant x in. from the cross-section A (see Fig. 42), then

$$M_B = Px \text{ lb.-in.}$$

And if the stress S is expressed in pounds per square inch, and the section is assumed circular,

$$Px = S \times \frac{\pi d^3}{32}$$

But it is assumed that the diameter at any point x in. below the section of diameter d is $d + tx$, therefore,

$$S = \frac{32P}{\pi} \times \frac{x}{(d + tx)^3} \qquad (63)$$

In order to find the position of the cross-section at which the pole is most likely to break—that is to say, where the fiber stress is a maximum—it is necessary to differentiate the last equation with respect to x, and find the value of x which makes this differential equal to zero. This gives

$$x = \frac{d}{2t}$$

for the point where the stress S is a maximum. The position of this cross-section is evidently not always at ground level. If this value of x is greater than H, then the maximum fiber stress will be at ground level, and it is calculated by substituting H for x in formula (63).

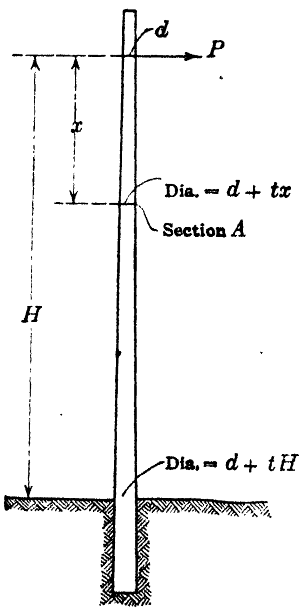


FIG. 42.—Wood pole with horizontal load near top.

The diameter of the pole at the weakest point is

$$\begin{aligned} d_w &= d + tx \\ &= d + t\left(\frac{d}{2t}\right) \\ &= 1.5d \end{aligned}$$

and it is only when the diameter at ground level is greater than one and a half times the diameter where the pull is applied that the pole may be expected to break above ground level.

If the stress S, the taper t, and the pole-top diameter d are known, the load P is readily calculated as follows:

$$\text{Bending moment} = P \times x$$

$$\text{Resisting moment} = S \times \frac{\pi d_w^3}{32}$$

But $x = \frac{d}{2t}$ and $d_w = 1.5d$

therefore

$$\frac{Pd}{2t} = S \times \frac{\pi \times (1.5d)^3}{32}$$

whence

$$P = \frac{2tS\pi \times 3.375 \times d^3}{32 \times d}$$

$$= 0.662 \times S \times t \times d^2 \qquad (64)$$

Similarly, if the pull P is known, the pole-top diameter should be

$$d = \sqrt{\frac{P}{0.662 \times S \times t}} \qquad (65)$$

Example 18. *Strength of Wood Poles.*—Consider a pole of Eastern white cedar designed to sustain a pull of 500 lb. applied 27 ft. above ground level. The average breaking stress (from table of constants) is 4,000 lb. per square inch, and assuming a factor of safety of 6, the safe working stress is S = 660 lb. per square inch.

The other numerical values are

$$P = 500 \text{ lb.}$$
$$H = 27 \text{ ft.}$$
$$t(\text{from table}) = 0.016$$

By formula (65)

$$d = \sqrt{\frac{500}{0.662 \times 660 \times 0.016}}$$

$$= 8.46 \text{ in.}$$

and $$d_w = 1.5 \times 8.46 = 12.7 \text{ in.}$$

The distance below point of application of load of the section where fiber stress is a maximum is

$$x = \frac{d}{2t} = 264 \text{ in.} = 22 \text{ ft.}$$

Therefore, this pole, if subject to a load about six times greater than the maximum working load, may be expected to break 27 − 22 = 5 ft. above ground level, unless the cross-section at ground level has been weakened by decay.

Double-pole supports of the type illustrated in Fig. 41 will be twice as strong as each of the component poles in resisting stresses applied in the direction of the line; but they will be able to withstand about five times as great a load as the single pole when the stresses are in a direction at right angles to the direction of the line. When loaded in this manner up to the breaking point, these double poles of the A type usually fail through the buckling of the member in compression due to initial want of straightness. The strength of both the A and the H type of pole structure can to some extent be increased by the addition of suitable bracing.

56. Deflection of Wood Poles.—It is now generally recognized that there are advantages in having transmission line supports with flexible or elastic properties. The ordinary single wood pole is very elastic, and will return very nearly to its original form after having been deflected considerably by abnormal stresses. The figures given for the elastic modulus in the table previously referred to are subject to correction for different qualities and samples of the same timber. It is well to make a few experiments on the actual poles to be used if accuracy in calculated result is desired. The double-pole structures of the A or H type will have about half the deflection of the single poles in the direction of the line, and, of course, very much less in a direction at right angles to the line. An A pole of usual construction with the two poles subtending an angle of 6½ deg. will deflect only about one-fiftieth of the amount of the single-pole deflection under the same transverse loading. The movement is usually dependent upon the amount of slip between the two poles at top, which again depends upon the angle subtended by the poles.

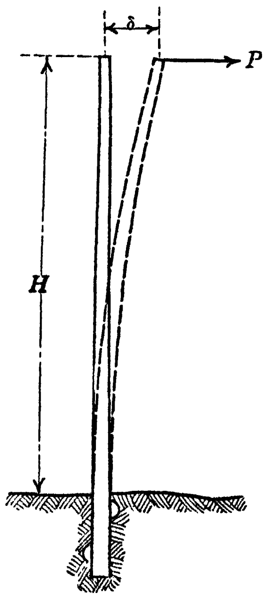


FIG. 43.—Deflection of wood pole.

Calculation of Pole Deflections.—Assume that the pole is fixed firmly in the ground, and that there is no yielding of foundations. The load P being applied in a horizontal direction at the top end, as indicated in Fig. 43, the pole may be considered as a simple cantilever, the deflection of which, *if the section were uniform throughout the entire length*, would be

$$\delta = \frac{PH^3}{3EI}$$

where δ and H are in inches; I is the moment of inertia of the section, and E is Young's modulus (pounds per square inch). For a circular section $I = \frac{\pi d^4}{64}$ where d is the diameter of the (cylindrical) pole in inches. The formula then becomes

$$\delta = 6.78\frac{PH^3}{Ed^4} \qquad (66)$$

If P is evenly distributed, as would be the case with a uniform wind pressure on the pole surface, regardless of other loads, the deflection would be

$$\delta = \frac{PH^3}{8EI}$$

but it is best to consider the wind pressure on pole surface as a single equivalent load concentrated at pole top and added to the load due to wind pressure on the wires. When estimating the probable value of this equivalent load, it should be remembered that the wind pressure is not evenly distributed along the length of the pole, since the wind velocity at ground level is comparatively small and increases with the height above ground surface.

The formula (66) assumes a constant diameter throughout length of pole, and the question, therefore, arises as to where the measurement of diameter should be made on an actual pole. It can be shown that, on the assumption of a uniform taper, the quantity d^4 in formula (66) should be replaced by $(d_g^3 \times d_1)$ where d_g is the diameter at ground level and d_1 is the diameter where the force P is applied.

Example. Pole-top Deflection under Load.—Using the same figures as in the example illustrating strength calculations,

$P = 500$ lb. $\qquad H = 27 \times 12 = 324$ in.
$d_1 = 8.46$ in. $\qquad t = 0.016$
$d_g = 8.46 + (0.016 \times 324) = 13.64$ in.
$E = 700{,}000$

Then

$$\delta = \frac{6.78 \times P \times H^3}{E(d_0^3 \times d_1)}$$

$$= \frac{6.78 \times 500 \times (324)^3}{700{,}000 \times (13.64)^3 \times 8.46}$$

$$= 7.7 \text{ in (approx.)}$$

When possible it is well to make tests on a few actual poles; then for similar poles of the same material subject to the same loading

$$\delta \propto \frac{H^3}{d_0^3 \times d_1}$$

57. Pole Foundations.—A permanent deflection of the pole, when the stresses are abnormal, may occur owing to the yielding of the earth foundation; but this is unusual if the poles are properly set in good ground. If the holes are dug to the depths as given by the following formulas, this will be in accordance with usual practice:

DEPTH OF HOLES FOR WOOD POLES (FEET)

In good ground on straight runs...	3	+ 1/15 (total length of pole, in feet)
At corners in good ground, or on straight runs in soft ground....	4 1/4	+ 1/18 (total length of pole, in feet)
At corners in soft ground........	5	+ 1/18 (total length of pole, in feet)

These depths are such as would be adopted on a well-designed pole line and need not be exceeded except in special cases. In marshy or otherwise unsatisfactory ground, special means must be adopted to provide a reasonably good setting for the pole butts.

Loam and gravel, and even sand, or a mixture of these, provide a firm foundation for poles. A pole that is properly set should break before the foundations will yield to any appreciable extent. Even if there should be a movement of the pole butt in the ground with excessive horizontal load at pole top, this will result in a firmer packing of the earth, which will then be better fitted to resist any further movement.

Firm sand, gravel, or loam will withstand a pressure of about 4 tons per square foot; but only half this resistance should be reckoned on in the case of damp sand, moist loam, or loose gravel.

Proper supervision is necessary to insure that the earth shall be packed firmly around the pole when refilling the holes.

58. Spacing of Poles at Corners. Guy Wires.—In order to reduce the stresses, not only on the pole itself, but also on the insulator pins and cross-arms, it is usual to shorten up the spans on each side of the corner pole. The reduction in length of span will depend upon the amount by which the direction of the wires departs from the straight run. A rough-and-ready rule is to reduce the span length 1½ per cent for each degree of deviation from the straight line. For angles less than 5 deg., it is not necessary to alter the span.

It is not advisable to turn more than 25 deg. on one pole, and whenever the side strain is likely to be excessive, double cross-arms and insulators should be used. By giving proper attention to the matter of guying and to the mechanical construction generally, it is not difficult to meet all requirements at points where a change of direction occurs.

A safe plan is to assume that a corner pole must carry the full load without breaking if the guy wire or wires should fail to take their proper share of the load; but all corner poles should be propped or guyed for extra safety, and to avoid the unsightly appearance of poles bent under heavy side stresses or set at an angle with the vertical.

Sometimes when sharp corners have to be turned, the spans on each side are dead ended on poles with double fixtures. Such poles are head guyed, and the span adjoining the guyed pole is usually shortened, being not more than three-fifths of the average spacing.

The non-synchronous swaying of wires in a high wind, although uncommon, sometimes occurs on wood-pole lines, being aggravated by the difference in the natural period of oscillation of poles and wires. This trouble can generally be cured by guying one or more of the poles at the place where the wires have been found to swing non-synchronously.

Guy wires should be of galvanized stranded steel cable, the breaking strength of which should preferably not exceed about 34 tons per square inch. The reason for this limitation of strength is that the high-strength steel is usually too hard to allow of proper handling and finishing off.

Load to Be Carried by Corner Poles.—Let P = total tension in pounds, including ice loading (if any), of all the wires on each side of the corner pole, and let θ = angle of deviation as indicated in Fig. 44; then, if there is no wind blowing across the line, the resultant pull at the pole top in the direction OP_R is

$$P_R = 2P \sin\left(\frac{\theta}{2}\right) \tag{67}$$

If there is wind blowing across the line in a direction such as to increase the load at the pole top, the wires will no longer hang vertically but will be deflected by the force of the wind as indicated by the dotted curves in Fig. 44. The total pull exerted by the wires at the pole top will now be the resultant of

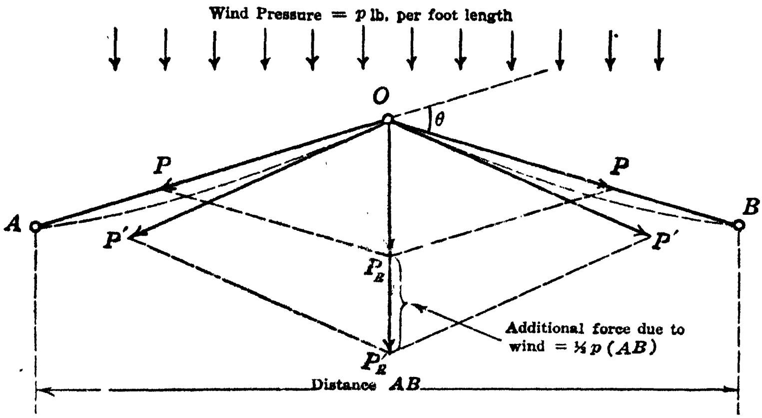


FIG. 44.—Diagram of forces at corner pole.

the two forces OP', and if p = the wind pressure per foot of wire (with or without ice coating as the case may be),

$$P_R' = 2P \sin\left(\frac{\theta}{2}\right) + \frac{1}{2}p(AB) \tag{67a}$$

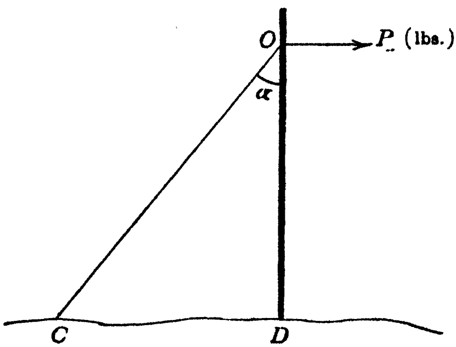


FIG. 45.—Diagram illustrating tension in guy wire.

The stress in the guy wire is easily calculated when the angle α (Fig. 45) which the wire makes with the vertical is known. If P_R is the side pull as calculated by formula (67) or (67a), then

$$\text{Tension in guy wire} = \frac{P_R}{\sin \alpha}$$

$$= P_R \times \frac{OC}{CD} \tag{68}$$

59. Props or Struts. Wood Poles in Compression.—Sometimes it is difficult or impossible to provide guy wires in certain locations; or impurities in the atmosphere may render the use of props or push braces preferable to guy wires. In such cases it is necessary to know approximately what load a wooden pole will support in compression, that is to say, when used or considered as a column. Instead of using the values of unit stress S as given in the table on page 147, the ultimate stress which a wood column will stand in compression should be calculated by the empirical formula:

$$\text{Stress in compression (pounds per square inch)} = S\left(1 - \frac{l}{60d}\right) \quad (69)$$

where l is the length in inches, and d is the diameter or least thickness at the center of the strut.

Example.—Calculate the safe load for a prop of Douglas fir 8 in. in diameter and 10 ft. long, assuming a safety factor of 6.

By formula (69) the breaking stress will be

$$6{,}500\left(1 - \frac{120}{60 \times 8}\right) = 4{,}870 \text{ lb. per square inch}$$

and the maximum safe load will be

$$\frac{4{,}870}{6} \times \frac{\pi}{4} \times 64 = 41{,}000 \text{ lb.}$$

60. Reinforced-concrete Poles.—As substitutes for wood poles supporting overhead wires, concrete poles and latticed steel masts are used. The full advantage of the galvanized or painted steel structure is best realized in the high towers with long spans, such as are used for the transmission of electric energy at high pressures. The use of portland cement for molded poles of moderate height is by no means new; the experimental stage has long ago been passed, and with the deplorable but no less rapid depletion of our forests and the incomparably longer life of the concrete poles, these will probably be used in increasing numbers in the future.

There is much to be said in favor of the wood pole when the right kind of timber, properly seasoned and treated, is used; but, apart from the general unsightliness of wood poles in urban districts, their life is uncertain and always comparatively short.

Concrete poles must be reinforced with iron or steel which, being entirely embedded in the concrete, is protected from the destructive action of air and water.

Longitudinal rods or bars of iron can be placed exactly where required to strengthen those parts of the pole section that will be in tension, and the concrete, filling up the spaces between the reinforcing rods, takes the place of all bracing and stiffening members of the ordinary steel structure in an almost perfect manner. It is probably at this time generally admitted that iron embedded in cement will last almost indefinitely without suffering any deterioration. When excavating for the foundations of the General Post Office in London, England, some old Roman brick-work was discovered in which the hoop-iron bonds were still bright and in perfect condition. The life of a concrete pole is, in fact, almost unlimited, a consideration which should not be overlooked when estimating the relative costs of different kinds of supporting structures. It requires no painting and practically no attention, once it is erected. If any small cracks should at any time develop, they can readily be filled with cement.

Concrete poles are usually molded in a horizontal position, the forms being removed after 3 or 4 days. After a period of seasoning lasting from 2 to 3 weeks, they are erected in the same manner as wood poles. The great weight of concrete poles is probably the most serious objection to their more general adoption in the place of wood poles, where the latter are not readily obtainable or where their appearance is unsightly.

Attempts to mold poles on site in a vertical position and so avoid part of the great cost of transportation have not always met with success. When molding on site, the forms are set up immediately over the hole previously prepared for the pole base. They are set truly vertical and temporarily guyed, the reinforcing inside the form being held together and in position by whatever means of tying or bracing may be adopted. Sometimes iron wire is used, but more uniform results are obtained by using specially designed iron distance pieces with the required spacing between them. The concrete is raised to the top of the mold by any suitable and economic means (preferably direct from the concrete mixer by an arrangement equivalent to the ordinary grain elevator) and is dropped in. By this means the hole in the ground is entirely filled with concrete. No tamping is required, a firm hold being obtained, since the ground immediately surrounding the concrete base has not been disturbed.

The best quality of crushed stone and sand should be used, the usual proportions being: cement, 1 part; sand, 2 parts; crushed stone, 3 or 4 parts, not too large to pass through a 3/4-in. screen. The mixture used for the poles on the Pennsylvania Railroad is 1.5:2:4. When gravel is used, the mixture may be 1 part of portland cement to 5 parts of gravel, provided that the latter is graded, including sand, and that the largest pieces are of a size to pass through a 3/4-in. screen.

The chief trouble that occurs in connection with vertical molding of poles in position is due to the fact that the mixture must be very wet to insure that it will fill the mold completely, and the pressure near the base of a tall pole is, therefore, very great, resulting frequently in leakage of water which carries with it some of the cement. The top of the pole is thus weakened owing to lack of cement.

Another objection to molding on site, whether in a vertical or horizontal position, is the fact that more material is generally used than would be necessary for a pole of the required strength if this were manufactured under expert supervision where permanent modern machinery has been installed for obtaining the best results with the smallest permissible cross-section.

Since a reinforced-concrete pole of solid section is uneconomical in material, a removable iron core is usually provided in connection with the manufacture of the larger poles. This core is freed by pulling it a small distance a few hours after the concrete has been poured; it is entirely removed after 8 or 9 hours. The most satisfactory method of manufacturing concrete poles with hollow cores consists in forcing the concrete against the sides of the mold by rotating the entire form at a high speed while the concrete is setting. The reinforcing steel is rigidly secured in position inside the form which is placed with the axis horizontal. The proper amount of concrete is poured in, and the complete form is rotated until the concrete is set. A very dense shell of concrete is thus obtained, leaving a hollow space through the center.

61. Strength and Stiffness of Concrete Poles.—When designing a concrete pole to withstand a definite maximum horizontal load applied near the top, the pole is treated as a beam fixed at one end and loaded at the other. The calculations are very simple if certain assumptions are made, these being as follows:

1. Every plane section remains a plane section after bending.
2. The tension is taken by the reinforcing rods.
3. The concrete adheres perfectly to the steel rods.

4. The modulus of elasticity of concrete is constant within the usual limits of stress.

The ultimate crushing stress of the concrete may be taken at from 2,000 to 2,600 lb. per square inch. The reinforcing bars should be covered with concrete to a depth of not less than 1 in. The effect of keeping the reinforcing bars under tension while the concrete is poured in the mold and until it has hardened sufficiently to support the strain itself has been tried and found to improve the performance of the poles, but it is doubtful whether the extra apparatus and labor required are justifiable on economic grounds. When subjected to excessive load, a concrete pole will generally yield by the crushing of the material in the base near ground level; but, unless it is pulled out of its foundations, it will not fall to the ground.

The comparative rigidity of concrete poles cannot be said to be a point in their favor, as the flexibility and elasticity of wood poles and some forms of steel structures are features of undoubted advantage under certain conditions. On the other hand, the degree of deflection of concrete poles before breaking is remarkable. The elastic limit is variable, and no exact figure can be given for the elastic modulus of cement concrete; it may be as low as 1,000,000, but for a 1:2:4 mixture 2,000,000 may be taken as a good average figure for approximate calculations.

Some tests made on 30-ft. concrete poles gave deflections of from 3 to 4 in. at a point near the top of pole, when subjected to a test load equal to about double the maximum working load.[1] Another series of tests made in England on some 44-ft. poles of hollow section, 17 in. square at the base and 8 in. square at the top (inside dimensions 13 in. and 4 in., respectively), with loads applied 38½ ft. above ground level, gave a deflection of 66 in. under a horizontal load of 10,500 lb., and the permanent set on removal of load was 21 in. The pole did not fail completely until the deflection was 78 in.

Some recent tests made in Germany on hollow poles 61 ft. long, measuring 9 in. in diameter at top and over 19 in. in diameter at the base, withstood a deflection of 4½ ft. at the top without permanent set or the appearance of cracks in the concrete.

[1] These poles were probably of large cross-section. Some tests made on poles measuring 10 in. square at the base and 32 ft. high gave a deflection of just over 2 ft. with a horizontal load of 2,000 lb. applied near the top.

The illustration Fig. 46 shows a concrete pole of hollow section suitable for carrying six transmission wires on two wooden cross-arms. The pole is 35 ft. long overall, about 6 ft. being buried in the ground. With a top measurement of 7 in. square and a taper to give an increase of 1 in. width for every 5 ft. of length, the size at the bottom will be 14 in. square. The drawing shows a section through the hollow pole taken at a point about 4 ft. above the ground level. Iron spacing pieces, as here shown, or their equivalent, must be placed at intervals to hold the longitu-

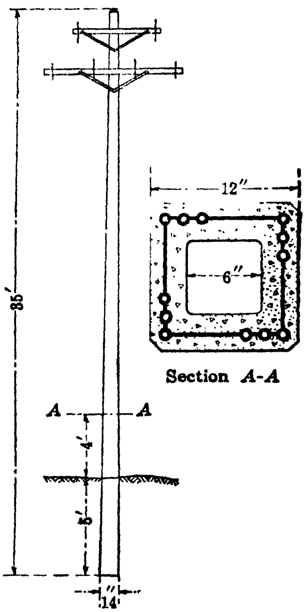


FIG. 46.—Concrete pole of hollow section.

dinal steel reinforcing bars in the proper position. The number of rods will vary with the distance below the point of application of the load. The bending moment to be resisted at every point being known and the taper of the pole decided upon, the amount of reinforcing required at any given section is easily calculated. The weight of a pole as illustrated would be about 2,700 lb. without fixtures. The reinforcing rods and spacing rings would account for approximately one-seventh of the total weight. A factor of safety of four is generally employed in strength calculations of reinforced concrete poles. In some cases the calculations have been based on a safety factor of 5; but there appears to be no justification for using so large a factor.

Weight of Concrete Poles.—The weight of reinforced-concrete poles is necessarily considerable, and this adds appreciably to their final cost erected on site, especially when the distance of transportation from the place of manufacture is considerable. It will generally be found that, unless the poles are erected at no great distance from the factory, the cost will be prohibitive. On the other hand, the fact that the "life" of concrete poles is practically unlimited must not be overlooked when comparing their undoubtedly high first cost with that of wood poles. A recent instance of concrete poles being apparently found more economical than the usual steel towers is the 80-mile 132,000-volt trunk line between Trollhättan and Västeras, Sweden, where each supporting structure consists of two 61-ft. hollow reinforced-concrete poles, each pole weighing 4 tons. All these concrete poles (about 800) have been shipped from the factories in Germany to their respective sites in Sweden.

The weight of shorter poles, such as are used to replace wood poles on comparatively low-voltage lines, are approximately as stated below:

Overall length	Top diameter, inches	Base diameter, inches	Approximate weight, pounds
25	6	12	1,350
30	6	13¼	1,750
35	6	14¼	2,300
40	6	15½	3,000

These weights are based on the hollow type of construction, and a wall thickness with suitable reinforcing rods so that each pole will be able to carry safely a horizontal pull of 1,200 to 1,300 lb. applied about 2 ft. below the top.

62. Steel Poles and Small-base Steel Masts.—As a substitute for wood poles, light steel structures that can be shipped and erected in one piece appear to be gaining favor. Small amounts of energy at comparatively low voltages can be transmitted over distances of 20 to 30 miles by overhead wires supported on steel poles at a cost which need be no higher, and is sometimes even lower, than if the less durable and less sturdy wood-pole construction is adopted. One type of steel pole for small lines is the Bates One-piece Expanded Steel Pole of which Fig. 47 is an example. These

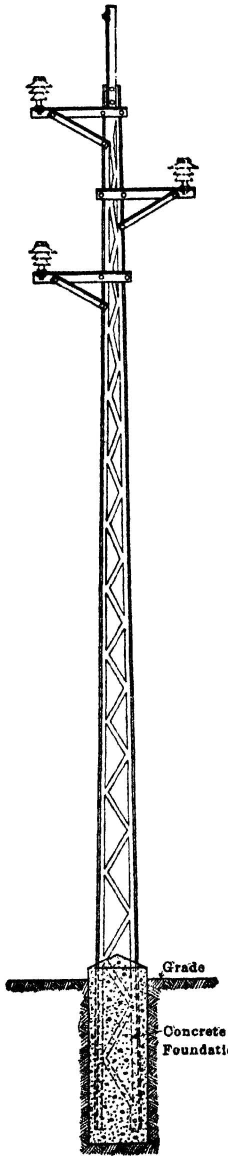


FIG. 47.—Bates one-piece expanded steel transmission pole.

poles are manufactured by the Bates Expanded Steel Truss Company, of Chicago, the pole being made in one piece without bolted or riveted lattice work. Steel poles of the type under discussion, which are intended to be handled much in the same manner as wood poles, usually range from 25 to 35 feet in length and are suitable for spans between 200 and 300 feet.

Figure 48 shows a typical form of small-base latticed steel mast as used on the transmission lines of the Iowa Railway and Light Company, Cedar Rapids, Iowa.[1] Fabricated steel masts

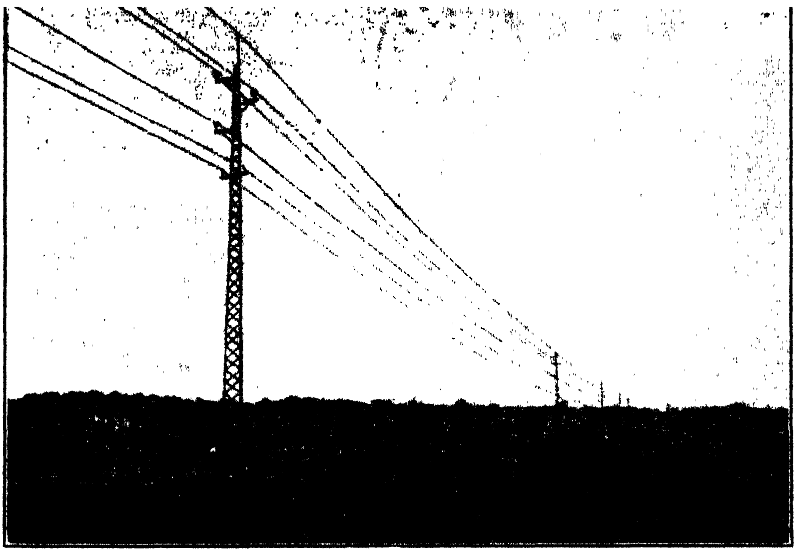


FIG. 48.—Typical transmission line carried on latticed steel masts.

with square bases very much smaller than those of the more usual steel tower construction are used more generally in Europe than in America, even for main lines at fairly high voltages. This design is very satisfactory with spans of about 400 ft. or even longer, if correctly designed and erected on suitable foundations. Small-base fabricated steel masts 70 ft. high carrying two three-phase circuits with spans from 600 to 700 ft. long are not uncommon on the continent of Europe.

[1] Photo supplied by the Ohio Brass Company of Mansfield, Ohio.

CHAPTER VII

TRANSMISSION LINE SUPPORTS (*Continued*)

63. Flexible-type A-frame Supports.—Although calculations of stresses in transmission lines are usually based on the assumption that the ends of each span are firmly secured to rigid supports this condition is rarely fulfilled in practice. There is some "give" about the poles or towers, especially when the line is not absolutely straight, and the insulator pins will bend slightly and relieve the stress when this tends to reach the point at which the elastic elongation of the wires will be exceeded. Then, again, the wires will usually slip in the ties at the insulators, even if these ties are not specially designed to yield or break before damage is done to the insulators or supporting structures. The use of the suspension type of insulator, which is now customary for the higher voltages, adds considerably to the flexibility of the line.

In regard to the towers themselves, all steel structures for dead-ending lines or sections of lines are necessarily rigid, and the usual light windmill type of tower with wide base is also without any appreciable flexibility. The latticed steel masts are slightly more flexible, and the elastic properties of the ordinary wood pole are well known. The deflection of a wood pole may be considerable, and yet the pole will resume its normal shape when the extra stress is removed. There is much to be said in favor of so-called flexible steel structures; that is to say, of steel supports designed to have flexibility in the direction of the line, without great strength to resist stresses in this direction; but with the requisite strength in a direction normal to the line, to resist the side stresses due to wind pressures on the wires and the supports themselves.

Such a design of support has the important advantage of being cheaper than the rigid tower construction, in addition to which it gives flexibility where this is advantageous, with the necessary strength and stiffness where required. The economy is not only in the cost of the tower itself but in the greater ease of transport over rough country, the preparation of the ground, and erection.

Probably the most severe stresses which a transmission line should be capable of withstanding are those due to the breakages of wires. Such breakages may be caused by abnormal wind pressures, by trees falling across the line, or by a burnout due to any cause. Suddenly applied stresses such as are caused by the breaking of some or all of the wires in one span are best met by being absorbed gradually into a flexible system. The supports on each side of the wrecked span will bend toward the adjoining spans because the combined pull of all the wires in the adjoining spans is greater than the pull of the remaining wires, if any, in the wrecked span. This movement of the pole top results in a reduction of tension in the wires of the adjoining span owing to the increased sag of these wires; there will be an appreciable deflection of the second and third poles beyond the break, but the amount of these successive deflections will decrease at a very rapid rate and will rarely be noticeable beyond the fourth or fifth pole. It is obvious that, as the remaining wires in the faulty span tighten up, the stress increases; but the combined pull of these wires on the pole top is smaller than it was before the accident, since it is assisted by the pull of the deflected poles, and these joint forces are balanced by the combined pull of all the wires in the adjoining sound span, which pull, as previously mentioned, is smaller than it was under normal conditions.

The greater the flexibility of the supports in the direction of the line, the smaller will be the extra load which any one support will be called upon to withstand; on the other hand, it is usual to provide anchoring towers of rigid design about every mile on straight runs, and also at angles, in addition to which every fifth or sixth flexible tower may be head guyed in both directions.

It is not unusual to carry a galvanized Siemens-Martin steel strand cable above the high-tension conductors on the tops of the steel structures. This has the double advantage of securely, but not rigidly, tying together the supports, and of providing considerable protection against the effects of lightning. The disadvantages are increased cost and possible—but not probable—danger of the grounded wire falling on to the conductors and causing interruption of supply.

The dead-end towers should be capable of withstanding the combined pull of all the wires on one side only, when these are loaded to the expected maximum limit, without the foundations yielding or the structure being stressed beyond the elastic limit.

The flexible supports must withstand, with a reasonable factor of safety, the dead weight of conductors, etc., and the expected maximum side pressures; but in the direction of the line their strength must necessarily be small; otherwise the condition of flexibility cannot be satisfied.

It is easy to design braced A-frame or H-frame steel structures of sufficient strength to withstand the dead load and lateral pressure and yet have great flexibility, with correspondingly reduced strength, in the direction of the line. Great care must be used in designing a line of this type so that strength and durability shall not be sacrificed to lightness and flexibility without very carefully considering the problem in all its aspects. The possible *twisting* of these flexible structures while subjected to a strong wind blowing across the line must not be overlooked. Suppose, for instance, there is inequality of tension in the wires on the two sides of a structure, or, for any reason, unbalanced pulls on the cross-arms; then the section passing through the two upright members of the A frame is no longer a plane at right angles to the direction of the line, and the strength of the structure to resist transverse loading is appreciably reduced. Moreover, the load such a distorted structure is able to carry safely cannot easily be calculated. This is one objection to the so-called flexible type of steel supporting structure. As an approximate indication of present-day practice in arriving at the load in the direction of the line for which flexible supports should be designed, it may be stated that a load of from one-twentieth to one-tenth of the total load for which the rigid-strain towers are designed should not stress the intermediate flexible structures beyond the elastic limit. It is well to bear in mind that at the moment of rupture of one or more wires on a "flexible" transmission line the resulting stresses in the structures and remaining wires will be in the nature of waves or surges until the new condition of equilibrium is attained, and the maximum stresses immediately following a rupture will generally exceed the final value.

The mathematics required for the exact determination of stresses and deflections in a transmission line consisting of a series of flexible poles is of a very high order, even when many assumptions are made which practical conditions may not justify; but the limiting steady values of these stresses and deflections can be calculated in the manner which was first described by the writer in the *Electrical World* of July 13, 1912,[1] and reprinted in the first and second editions of this book. With the growing use of the suspension type of insulator, even when the transmission voltage is not exceptionally high, the relatively small elastic yielding of the steel supports under unbalanced longitudinal

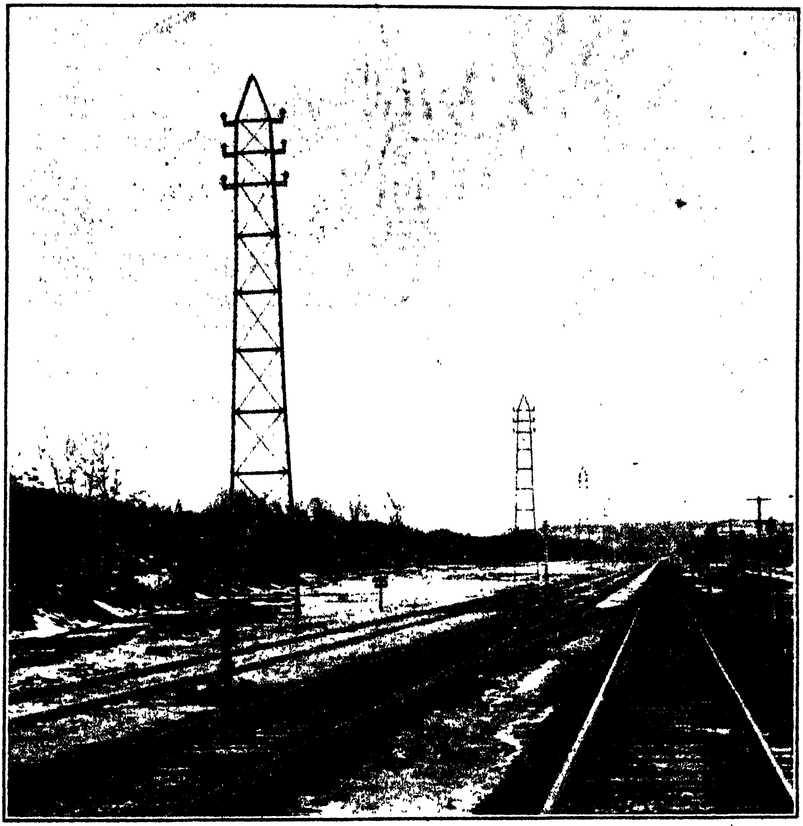


FIG. 49.—Flexible steel tower transmission line.

stress is of small importance, and it is, therefore, not proposed to study in detail the deflections and stresses in the "flexible" steel tower line due to the severing of one or more wires in a particular span. The illustration (Fig. 49) kindly supplied by Archbold Brady and Company shows a common form of "flexible" high-voltage transmission line following a railway.

[1] Vol. 60, p. 97.

64. Rigid Wide-base Steel Towers.—With the coming of high voltages and increased distances between wires and between the conductors and the ground, taller supports become necessary. With a view to saving cost on supports and insulators, spans have been increased, thus necessitating still taller towers. The result of these tendencies is the modern wide-base steel tower of substantial design and often of considerable height.

The parts of large steel towers for important transmission lines are usually galvanized, while for temporary constructions the parts are made of lighter section material and almost invariably painted. It does not follow that paint instead of zinc is not desirable as a protective coating on the iron work of important lines. The problem is mainly an economic one, but a steel structure may unquestionably be made to last almost any length of time if it is thoroughly painted every 3 or 4 years, especially if particular attention is paid to the footings where they emerge from the concrete foundation. These, if not galvanized, should be painted every year. On important high-voltage lines where it would be dangerous for men to work near the live wires, the steel parts of the structure near the wires should be galvanized even if the remainder of the tower is painted.

Height of Towers. Length of Span.—In Art. 24, Chap. III, the effects of span length and transmission voltage on the height and weight of the supporting structures was briefly discussed, and what follows has special reference to the square-base steel towers as used for transmitting large amounts of energy at high voltages. The best or most economical spacing between supporting structures is less easily determined than might at first be supposed. A long span means few towers and fewer insulators per mile of line, but it also means a greater sag in the wires and higher and stronger towers. There will be a particular length of span under given conditions, involving the number, size, and material of the conductors, beyond which any further increase would lead to the cost of the finished line being greater than with the shorter span. This is what, in the past, has determined the proper spacing of supports in those rather exceptional cases where the engineer has not been content merely to follow precedent, but has had the courage to make his own calculations, and the faith to believe in their correctness.

The economical length of span on steel tower lines for pressures up to 110,000 volts usually lies between 600 and 900 ft. For higher pressures, up to 220,000 volts, longer spans are economical, the usual average length being from 900 to 1,300 ft. The average distance between towers on the 110,000-volt lines of both the Pacific Gas and Electric Company and the Mississippi River Power Company is 800 ft.; but the present-day tendency is all in the direction of larger spans, even on the lower-voltage lines. Calculations made by the engineers of the Southern California Edison Company indicate that about 1,250 ft. is an economical span for its 220,000-volt lines. Spans of 1,050 ft. are used on the 132,000-volt main lines supplying energy from the coal fields to Melbourne, Australia.

Ultimate economy and continuity of service are the important factors in determining changes and improvements in transmission line construction. Even if the first cost of a line with very large spacing of towers may be somewhat higher than if shorter spans were adopted, there are still reasons in favor of the longer spacing. The cost of maintenance has been found by experience to be appreciably less with the long-span construction, and interruptions to service are fewer. This may be attributed partly to the fact that the number of points of attachment to insulators is smaller with the long spans, and that variations of temperature do not produce such great changes of stress in the wires as when shorter spans are used.

The height and cost of the supporting structures increase with the increase of voltage because of the greater spacing required between wires of different phases and between all the conductors and the (grounded) towers. It is also customary to allow a greater minimum clearance between the lowest wire and ground on high-voltage than on medium-voltage lines, and this again tends greatly to increase the height and cost of individual towers on the extra-high-tension lines. There are no controlling reasons for any particular clearance to ground in the open country, but every state or district has special regulations regarding minimum clearances at road, railway, and river crossings. Apart from such particular points where shorter spans and special construction may be necessary, the usual clearances allowed between ground and the lowest point of the wire are 20 ft. for overhead lines up to 66,000 volts, increasing to 26 ft. for 132,000 volts, and 30 ft. for 220,000 volt lines. The question of clearances between conductors will be taken up in Chap. VIII in connection with the problems of insulation.

Two types of transmission towers as used on the 220,000-volt Pitt River line of the Mount Shasta Power Corporation (a subsidiary of the Pacific Gas and Electric Company) are shown in Fig. 50.[1] A more usual construction for double-circuit three-

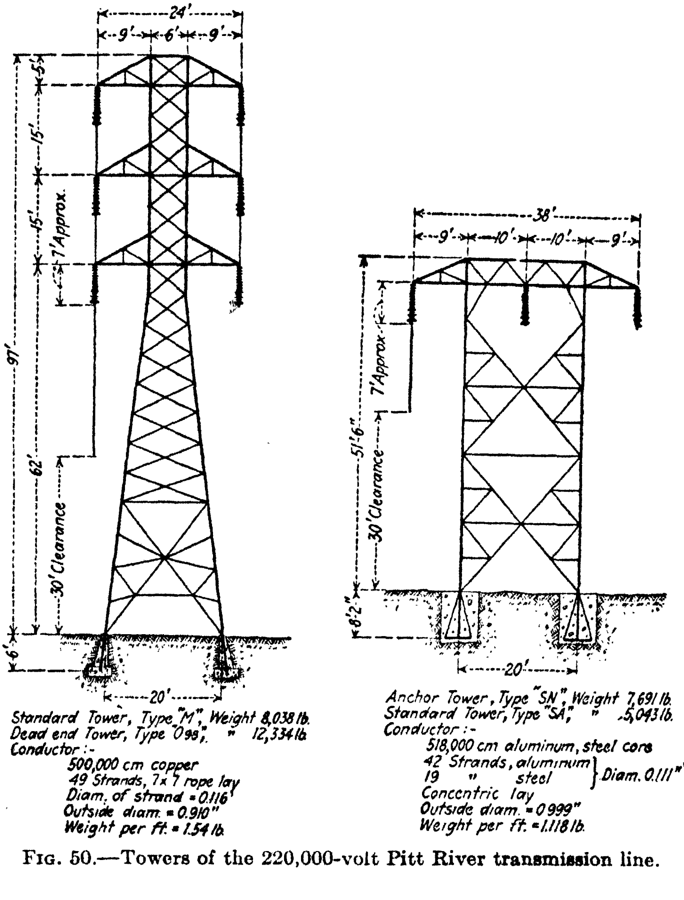


Fig. 50.—Towers of the 220,000-volt Pitt River transmission line.

phase lines is shown in Fig. 51 where the middle cross-arm is seen to be longer than the other two. This illustration is reproduced from photograph kindly supplied by the Ohio Brass Company of Mansfield, Ohio. The towers were designed and constructed by the American Bridge Company of Pittsburgh for the American Gas and Electric Company's 130,000-volt transmission line

[1] Reproduced by permission, from the article by F. G. Baum and S. Barfoed, *Elec. World*, Jan. 27, 1923.

between Wheeling, W. Va., and Canton, Ohio. The six conductors are each of 200,000 circular mils cross-section, and the two grounded guard wires are of the same size. The line is 55 miles long with an average length of span of 580 ft. The reason for the greater separation between the two middle wires is that

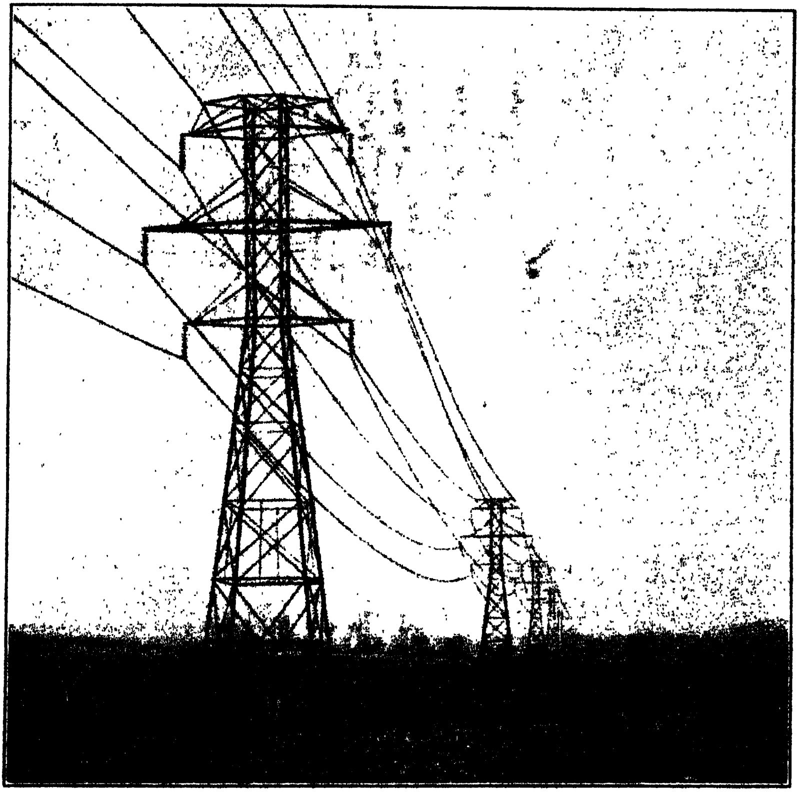


FIG. 51.—Typical steel tower transmission line.

there is less risk of contact between wires owing to unequal vertical loading by ice deposits. A good example of large steel towers is to be found in the 100,000-volt transmission line of the Great Western Power Company of California. Two three-phase circuits are carried on these towers, the vertical spacing between the cross-arms being 10 ft. There are three cross-arms, each carrying two conductors—one at each end. The horizontal spacing between wires is 17 ft. on the two upper cross-arms and 18 ft. on the lower cross-arm, which is 51 ft. above ground level. No conductor is closer than 6 ft. 5 in. to the steel structures, this being the minimum clearance in the horizontal direction. The average distance between towers is 750 ft., and they are joined at the top by a grounded guard wire 5 ft. above the bottom of the highest cross-arm. The base of the tower measures 17 ft. square, the parts underground being separate pieces of steel, buried to a depth of 6 ft., to which the tower proper is bolted after being assembled and erected on site.

The size of base in relation to the height of the tower may usually be determined by the designer, who will decide upon a dimension which leads to economy in material; but there are occasions when the configuration of the ground, or the cost of right of way may lead to the adoption of base dimensions smaller than might be desirable if the cost of the tower and its foundations were the only considerations.

65. Loads to Be Resisted by Towers.—The maximum load which a tower must be designed to withstand will depend upon the number and size of wires to be carried and the estimated ice coating and wind velocity. Apart from the wind pressure on the structure itself, the loading in a direction transverse to the line will be equal to the resultant wind pressure on all the wires (which may or may not be ice coated, depending on the climate); the effective length of each wire being the distance between supports.

In the direction of the line, the forces are normally very nearly balanced, but in the event of one or more wires breaking, the unbalanced load may be considerable, and it is well to design the towers, if possible, to withstand the stresses imposed upon them if two-thirds of all the conductors in one span are severed. It must not be overlooked that if the wires break in one span only, the cross-arm, if pin-type insulators are used, will be subjected to a twisting moment; and if the break in the wires is at one end only of the cross-arm, the whole tower is subjected to torsional strain. With conductors of large cross-section as used on some of the modern main lines, very heavy and costly towers would be necessary if failure of the tower is not to result from failure of the conductors in one span. For this reason some engineers prefer to use a type of conductor clamp with a gripping power adjusted to permit slipping of the wire when the pull exceeds a certain predetermined limit.

The vertical or dead loads consist of the weight of the tower itself and the wires of one span, with possible increase in weight due to sleet or ice. The cross-arms must be of ample strength to take all vertical loads including weight of insulators, with a margin to cover the extra weight of men working on the tower. The approximate weight of insulators is given in the following table:

Working pressure (line voltage)	Approximate weight of insulator, pounds
Pin type:	
22,000	6
44,000	16
66,000	32
Suspension type:	
66,000	44
88,000	60
110,000	80
150,000	110
220,000	140

Particulars regarding wind pressures were discussed in Art. 45 of Chap. V. The wind velocity rarely exceeds 80 miles per hour either on the American continent or in Europe. Tornadoes and cyclones are not considered, because attempts to construct overhead lines strong enough to withstand them would rarely be justified. In regions where sleet deposits are to be expected, it is generally sufficient to allow for the Class B loading on the wires (see p. 110). For the wind pressure on the towers it is customary to neglect any possible ice deposit and allow for a wind pressure of 13 lb. per square inch on 1.5 times the exposed area of one face of the tower.

In regions where strong winds may be expected, but where sleet deposits do not occur, a maximum wind velocity of 76 miles per hour seems a reasonable assumption. This corresponds to a pressure of 21 lb. per square foot of flat surfaces on towers, and 14 lb. per square foot of projected surface of wires and cylindrical poles. The total transverse load is dependent upon the length of span, which must be determined with due regard to economic considerations.

Factor of Safety.—Since there is a possibility of misunderstanding regarding the exact meaning of this term, it should be observed that what the purchaser of a transmission line tower wants to know is the increase over the assumed maximum loading (or specified loading) at which the tower may be expected to fail. It is the business of the designer to choose unit stresses in the various members of the structure which will comply with the requirements in the matter of safety factors. From the purchaser's point of view, it is incorrect to define safety factor as the ratio of ultimate unit tensile stress in the material to unit stress used in the design calculations, because failure of the whole structure will usually occur when the elastic limit in any one compression member is exceeded. This will usually result in bending or buckling and the complete collapse of the whole structure. Thus, when the unit stress is about 60 per cent of the ultimate stress, the structure is likely to fail, so that the actual safety factor of a tower designed with ultimate unit stress 2.5 times the working unit stress would not be 2.5 but $2.5 \times 0.6 = 1.5$ approximately.[1]

Although the location, height, and probable loading of each individual tower in a transmission line might seem to call for many special designs leading to economy of material, it is usually found that three types of structure are sufficient to meet the requirements of the average transmission line, except for river crossings or exceptionally long spans which occasionally require extra-strong or extra-tall supporting structures. The so-called rigid type of steel tower is, therefore, generally supplied to withstand the maximum loading under the following conditions:

1. Standard towers on straight runs, with average spacing between towers.
2. Semi-strain towers for use with normal spans on angles up to 15 deg. or on straight runs with spans longer than the average.
3. Strain or dead-ending towers for angles between 15 and 45 deg. and for the terminals of line sections. These towers are capable of withstanding the pull of all the wires on one side only.

By providing each type of tower with extension bases to increase the height when necessary, it is generally possible to plan the transmission line so as to require very few, if any, additional structures of special design.

[1] On account of the ambiguity of the term "factor of safety" when applied to structural steel designs, the American Bridge Company use the expression "possible overload" to indicate the amount of load, in excess of the specified design load, which a transmission tower may be expected to withstand before failure.

66. Design of Steel Towers.—No attempt will be made in these pages to go into the details of steel tower design. This is a matter which should be left to the manufacturer; but there are numerous books available for the reader who wishes to go further into this subject.[1] At the same time, it may be well to point out that thin metal, even for bracing members of a steel tower, should be avoided. Structures made of few pieces of comparatively heavy section steel will generally prove more durable than those built of a larger number of lighter parts. The tendency is toward the avoidance of light-weight members, although, on the other hand, cross-bracing at fairly frequent intervals cannot be omitted without having the unsupported sections of main members in compression longer than would be desirable or safe. The present practice is to avoid using material less than $\frac{3}{16}$ in. thick, even for web members. For permanent structures that are to be painted, *i.e.*, not galvanized, some engineers specify a minimum thickness of $\frac{1}{4}$ in. Main members should never be made of material less than $\frac{1}{4}$ in. thick, and where large-angle sections, such as 4 by 4 in., are used for the corner legs, it would be advisable to have a minimum thickness of $\frac{5}{16}$ in. When rods are used for tension members in place of flat straps or angles, a minimum diameter of $\frac{5}{8}$ in. is customary.

The most economical design of tower to withstand the probable loads that it will be subject to, and to satisfy local conditions, including such considerations as transport and erection facilities, is a problem deserving close attention on the part of the engineer responsible for the design of the transmission line. A study of the probable loads to be resisted under the worst weather conditions will enable the designing engineer to specify certain test loads which will insure that the finished structure will be strong enough to fulfil the practical requirements. The proper value of these test loads and their distribution or point of application should be determined only after mature consideration. The cost of a tower—apart from the height, which is a function of

[1] Painton, E. T., "Mechanical Design of Overhead Electrical Transmission Lines," Chapman & Hall, Ltd.

Kapper, F., "Overhead Transmission Lines and Distributing Circuits," D. Van Nostrand Company.

Wolfe, W. S., "Graphical Analysis," McGraw-Hill Book Company, Inc.

Hool, G. A. and Kinne, W. S., "Stresses in Framed Structures," McGraw-Hill Book Company, Inc.

the length of span—is determined largely by the specifications of test loads. A specification calling for tests that are unnecessarily severe is just as true an indication of incompetence on the part of the designing engineer as a specification giving test conditions that will result in a tower too weak for the actual requirements.

The calculation of stresses in the various members of so simple a structure as a transmission line tower is not a difficult matter, especially if graphical or semi-graphical methods are adopted. If the designing engineer will make sketches of two or three alternative designs likely to fulfil the required conditions, he should be able quickly to calculate the approximate value of the stresses in the principal members, and so obtain a rough idea of the relative weights and costs of alternative designs. The danger of leaving the problem entirely in the hands of the manufacturer is that the latter is always tempted to put forward a design of which he has perhaps made a specialty, and which may have given entire satisfaction in practice without necessarily being the best type of structure for the purpose, or being entirely suitable for use under different conditions.

Stresses in Compression Members of Tower Structures.—The failure of steel towers under excessive loads is almost invariably due to the buckling of the main leg angles in compression. The designer should, therefore, pay special attention to the proportioning of compression members in the structure. Without going into a discussion of the many empirical formulas used for determining the loads that struts or columns can withstand, it may be said that, for tower designs, the "straight-line" formula, as suggested by Burr, is quite satisfactory provided the ratio $l \div r$ lies between 40 and 200; this last figure corresponds to a length of compression member not exceeding about twenty times the width of flange. This formula is

$$S_{\text{comp.}} = K - k\left(\frac{l}{r}\right)$$

where l is the length, in inches, of unsupported portion of compression member,

r is the least radius of gyration, in inches,

$$= \sqrt{\frac{\text{moment of inertia}}{\text{area of section}}}$$

$S_{\text{comp.}}$ is the unit stress (pounds per square inch) in the column,

k is a constant determined experimentally,

K is the unit crushing stress which would be allowed if the column were very short. It has already been pointed out that a fabricated steel structure such as a transmission line tower may be expected to fail when the stress in one of the members exceeds the elastic limit of the material. Failure of members in compression must, therefore, be expected to occur before the unit stress K attains the ultimate crushing stress which a short column would be able to withstand.

Many formulas are in use, the following being a selection from among the best known:

"Straight-line" formulas giving unit stress to produce permanent deformation—and probable failure—of steel columns:

$$S_{\text{comp.}} = 27{,}000 - 90\frac{l}{r} \text{ for values of } \frac{l}{r} \text{ above } 150 \qquad (70)$$

$$S_{\text{comp.}} = 35{,}000 - 120\frac{l}{r} \text{ for values of } \frac{l}{r} \text{ below } 150 \qquad (71)$$

$$S_{\text{comp.}} = 44{,}000 - 163\frac{l}{r} \qquad (72)$$

this last being the Tetmajor formula in connection with which it is stated that $\frac{l}{r}$ shall not be greater than 105.

Formulas for *safe working stress* in structural steel members in compression:

$$S_{\text{comp.}} = 18{,}000 - 60\frac{l}{r} \qquad (73)$$

which is used in England in connection with bridge construction:

$$S_{\text{comp.}} = 24{,}000 - 60\frac{l}{r} \qquad (74)$$

which is the formula suggested by the National Electric Light Association of America.

In the year 1917 the American Bridge Company of Pittsburgh made a series of tests on angle sections in compression and they have developed "straight-line" working formulas based on the results of these tests. Assuming that the tower may be expected to fail when carrying an overload of 90 to 96 per cent, the unit stress in compression members (pounds per square inch) is,

$$20{,}000 - 85\frac{l}{r} \text{ for values of } \frac{l}{r} \text{ up to } 150$$

and

$$15{,}500 - 55\frac{l}{r} \text{ for values of } \frac{l}{r} \text{ between } 150 \text{ and } 250.$$

Assuming the steel to withstand 60,000 lb. per square inch in compression when the column is very short ($\frac{l}{r}$ very nearly equal to zero), it follows that what is sometimes referred to as the "factor of safety," being the ratio of ultimate to working unit stress, is 3.33 for formula (73) and 2.5 for formula (74); but these figures are not a correct indication of what overload the tower will stand before collapse, because a tower designed on the basis of formula (73) may be expected to fail when the load is actually only 50 per cent in excess of the specified maximum loading upon which the calculations are based (compare formulas (73) and (70)).

In using any of the above formulas, it is important to note that they do not apply to very long columns in which the *slenderness ratio* $\frac{l}{r}$ is appreciably greater than 200. It is customary to choose for the various members of the tower structure a standard section of structural steel of such size as to limit this ratio $\frac{l}{r}$ to 120 for main members and 200 for lateral and secondary members such as cross-braces.

The fact that, for a given cross-sectional area, the *shape* of the section is an important factor in determining the stiffness and ultimate strength of the members in compression, suggests that, where lightness and economy of material are of great importance, a section of structural steel having a large moment of inertia per square inch of cross-section should be chosen. The standard sections of rolled angles or tees are sometimes replaced by steel tubes.

As an example of the relative economy of the tubular form and other forms of section, when used as comparatively long struts, a steel tube 7 in. in internal diameter, 1/8 in. thick, weighing 10 lb. per foot, will be as efficient in resisting compression as a steel angle 7 1/2 by 7 1/2 by 1/2 in. thick, weighing 25 lb. per foot, or as an I beam 8 by 6 by 1/2 in. thick, weighing 35 lb. per foot. So large a tube would not be required except in very high towers; a tube from 4 to 5 in. in diameter would generally be large enough for the main members of a transmission line tower up to 100 ft. high.

Although supporting structures built of tubes would seem to lead to economy of material, there are certain practical objections to the use of tubes in the place of the more usual angle sections.

It is almost impossible to prevent water getting into the tubes and causing rusting of the interior surfaces. Trouble has been experienced through the freezing of water in the main members causing cracks in the tubes, and the difficulty of making satisfactory joints between the various members of the tower are much greater than in designs using the standard sections of structural steel. This difficulty has to a large extent been overcome by the more extended application of electric welding processes, and welded wireless towers 150 ft. high have been successfully constructed of steel pipe.[1]

67. Outline of Usual Procedure for Calculating Stresses in Tower Members.—The illustration, Fig. 52, which is reproduced by kind permission of the Shawinigan Water and Power Company, and the Canadian Bridge Company, Limited, shows a typical square-base galvanized-steel tower as used on the Three Rivers line of the Shawinigan Water and Power Company of Montreal. These towers are designed to carry six aluminum conductors of nineteen-strand 200,000 circular mils cable, each being supported by seven suspension disks of the Ohio Brass Company's standard type. In addition to the conductors, there are two ground wires of 3/8-in. stranded Siemens-Martin steel cable attached to the points (1) at each end of the upper cross-arm. The line is built for 100,000 volts.

The method of procedure in calculating stresses is to make a sketch showing the points of application, and the vertical and horizontal components, of the outer forces. Then indicate by arrows the assumed horizontal and vertical components of the reactions, using the suffixes R and L to indicate the direction or assumed direction of the horizontal components. Since the whole structure is in equilibrium under the influence of the various loads and reactions, it is merely necessary to see that the three following conditions are satisfied at any point considered:

1. The sum of all vertical force components = zero.
2. The sum of all horizontal force components = zero.
3. The sum of all moments about any point = zero.

When taking moments in any particular plane, all those in a clockwise direction would be considered positive and those in a counterclockwise direction negative. All joints are considered as frictionless pivots, which assumption is, of course, not strictly correct, especially in the case of riveted joints. It is usually

[1] *Eng. News-Record*, p. 650, Apr. 20, 1922.

an easy matter to choose a section through the structure in such a position that the stresses in a given bar can readily be calculated by applying one or more of the three equations of equilibrium.

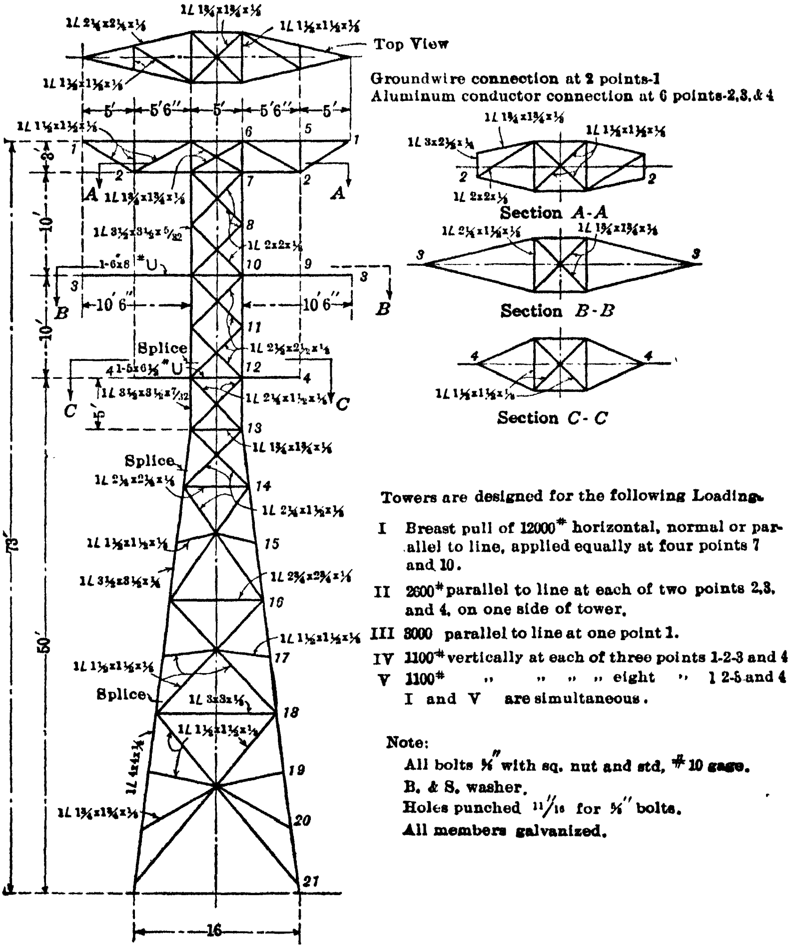


FIG. 52.—Steel tower with members of angle section.

The sketch, Fig. 53, will serve to illustrate the method usually followed in calculating the stresses in the main members of a tower structure such as the one shown in Fig. 52. The loading considered is that corresponding to the condition of test loads *I* and *V* applied simultaneously.

The point at which the horizontal breast pull of 12,000 lb. is applied corresponds approximately to the point 65 ft. above ground level where the corner legs would meet if produced beyond the points (13). The weight of the tower (which it is supposed has not yet been designed in detail) is taken at 4,000 lb., and this, together with the test load V, gives a resultant vertical loading of 12,800 lb. applied somewhere on the center line of the tower.

Consider a section such as XY which cuts only three members, namely, the leg A at ground level, the leg B just above the joint O', and the diagonal brace C.

Select a point O where the members A and C meet, and consider the moments, in the plane of the paper, which are produced about this point by the external forces and the reactions in the members severed by the imaginary section XY. It is obvious that the stresses in A and in C have no effect on the tendency of the part of the structure above the section line to rotate on the point O, and the whole of the externally applied turning moment must be resisted by the stress in the member B. Therefore,

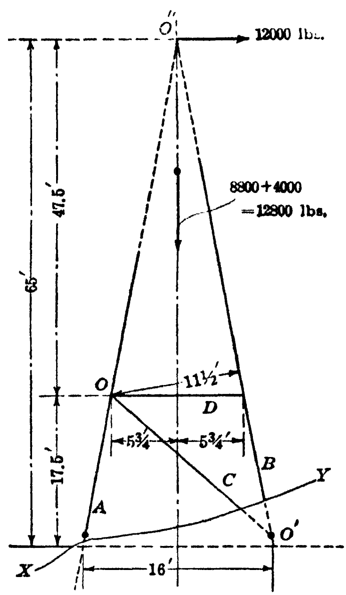


FIG. 53.—Sketch for calculation of stresses in tower members.

$$(12{,}800 \times 5.75) + (12{,}000 \times 47.5) - (x \times 11.5) = 0$$

from which it is found that $x = 56{,}000$ lb.

Since there are two members B taking the whole crushing stress, the total load tending to crush the one member B is 28,000 lb. The length of the unsupported portion of this member is 5.5 ft. or 66 in. The cross-section of 4- by 4- by ¼-in. angle is 1.93 sq. in. and the least radius of gyration, $r = 0.79$. The test load should not strain the tower beyond the elastic limit. Using the formula (71), the ultimate stress is

$$S_{\text{comp.}} = 35{,}000 - 120\left(\frac{66}{0.79}\right)$$
$$= 25{,}000$$

This corner member is, therefore, capable of supporting, just before collapse, a compressive load of $1.93 \times 25{,}000 = 48{,}300$ lb. It should be of ample strength to resist the test load of 28,000 lb. without permanent deformation.

Turning now to the uplifting force acting in the member A and tending to pull up the foundation, the center from which the moments are calculated is shifted to the point O' where the members C and B meet. The equation of moments is now

$$(12{,}000 \times 65) - (12{,}800 \times 8) - (x \times 16) = 0$$

whence $x = 42{,}300$

and the tension in one corner angle A is 21,150 lb.

The above example briefly describes what is known as the method of moments. It has been assumed that the tower side under consideration lies in the same plane as the external forces; but the error introduced is practically negligible. It is an easy matter, if desired, to make the necessary correction.

When calculating the stresses in a diagonal member such as C of Fig. 53, the moments would be taken about the point O'', which is the junction of the members A and B; but in that case the actual loads on cross-arms and the wind pressure on the side of the tower would have to be taken into account and substituted for the concentrated test load of 12,000 lb. at the point O'', which does not produce any stress in the brace C so long as the corner angle A remains truly straight and exerts no lateral pressure at the point O. The method of moments can usually be applied for all sections of a tower structure if the imaginary dividing planes are properly placed. The counter members or ties that are not in tension under the conditions of loading considered are usually assumed to be non-existent, *i.e.*, to serve no useful purpose as compression members.

When computing the stresses in the flexible A-frame steel structures, it is assumed that the structure remains always normal to the line in a vertical plane; but unbalanced forces in the conductors will actually deflect the frame from this position and so reduce its possible resistance to transverse loads. It is practically impossible to calculate the strength of the distorted frame, and although flexibility in the direction of the line is usually a desirable feature of this type of structure, it is very important to design the so-called flexible steel towers so that they will not be deflected unduly by such torsional loads as they may be subjected to at times when strong winds are blowing across the line. This matter of the possible weakening of this type of structure in a direction transverse to the line has already been referred to in Art. 63 (see p. 166).

Light-weight Steel Towers.—The steel used in the construction of transmission line towers is usually what is known as Class A basic open-hearth steel, as used for steel buildings: its ultimate strength is between 60,000 and 70,000 lb. per square inch, but stronger material is obtainable and may be used when light weight is of great importance. Since it is not the cost of the tower at the factory, but its ultimate cost erected on site which is of interest to the purchaser, it is obvious that light weight, even at a considerably higher price per pound of material, may sometimes be desirable.

Stiffness of Steel Towers. Deflection under Load.—The deflection of the top of a transmission line tower of the ordinary light "windmill" type with wide square base, when bolted to rigid foundations and subjected to a horizontal load such as to stress the material to nearly the elastic limit, might be from 2 to 5 in. With regard to the two-legged or "flexible" type of tower, if this is of uniform cross-section, it may be treated as a beam fixed at one end and free at the other end. If the resultant pull can be considered as a single concentrated load of P lb. applied in a horizontal direction, at a point H in. above ground level, the deflection, in inches, will be

$$\delta = \frac{PH^3}{3EI}$$

where E is the elastic modulus for steel (about 29,000,000, being the ratio of the stress in pounds per square inch to the extension per unit length), and I is the moment of inertia of the horizontal section of the structure.

68. Tower Foundations.—The upward pull of the tower legs, which was found in the above example to amount to 21,150 lb., has to be resisted by the foundation. The weight of concrete may be taken at 140 lb. per cubic foot, and of good earth at 100 lb., the volume of the earth to be lifted being calculated at the angle of repose, which may be about 30 or 33 deg. with the vertical, as indicated in Fig. 54. If the footing of a tower is in gravel, or a mixture of sand and loam tightly packed, there is actually a far greater resistance to the pulling up of the footings than that which is offered by the mere weight of the footings with prism of earth as calculated in the usual way. Even if the calculated frustum of earth be considered free to move in an upward direction, the angle of repose for firm earth may be taken at 55 deg. instead of the usual 30 deg. which refers to coarse sand, dry clay, or damp earth. For wet clay this angle is about 20 deg., and for very fine sand it may be as small as 16 deg.

When concrete has to be used, it is sometimes cheaper to reinforce it with steel of an inverted T form, as this makes a lighter construction than a solid block of concrete, and an equally good hold is obtained owing to the increased weight of the packed

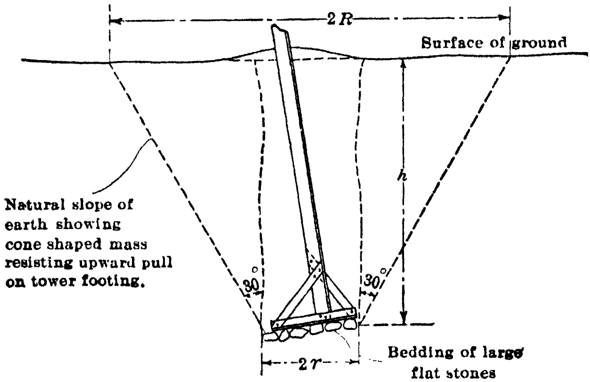


FIG. 54.—Foundation for steel tower anchor stub.

earth which has to be lifted. At the same time it must not be forgotten that the digging of a large hole 5 to 8 ft. deep is considerably more costly than the digging of a hole about 2 ft. square, and this extra cost in erection must be taken account of in designing the footings. In marshy or loose soil, or where the right of way is likely to be flooded, special attention should be paid to the design of durable foundations. Concrete footings with or without piles, or rock-filled crib work may be necessary; it is a matter requiring sound judgment and, preferably, previous experience on the part of the engineer in charge of construction. Crumbling hillsides are best avoided; it is extremely difficult to guard against damage by land slides or even snow slides when towers are erected on the steep slopes of hills.

The use of concrete adds considerably to the cost of foundations and it should be avoided if possible; on the other hand, it is not easy to design foundations to resist a given uplift without an exact knowledge of the soil conditions at the site of the tower. For the greatest economy of foundation, it is necessary that the designer obtain reliable information on this point.

Assuming an average angle of slope of 30 deg., as indicated in Fig. 54, and a weight of soil of 100 lb. per cubic foot, the depth of foundation may be calculated as follows:

Let h = depth of footing below ground level, in feet.
r = equivalent radius of footing area, in feet.
R = radius (in feet) at ground level of conical section of earth to be lifted.
θ = angle of natural slope of earth.

The volume of frustum of cone to be lifted is

$$V = \frac{\pi}{3}h(r^2 + R^2 + rR) \qquad (75)$$

or if $r + h \tan \theta$ be put in the place of R,

$$V = \frac{\pi}{3}h(3r^2 + h^2 \tan^2 \theta + 3rh \tan \theta) \qquad (76)$$

If $\theta = 30$ deg., $\tan \theta = 0.5774$ and (approximately),

$$V = \pi h(r^2 + 0.11h^2 + 0.58rh) \qquad (77)$$

If $r = 1$ ft., and $h = 7$ ft., the volume of earth to be lifted, by formula (77) is then $V = 230$ whence the weight of earth to be lifted is approximately 23,000 lb. As previously mentioned, if the soil is firm, this method of calculation usually gives results well below actual values of pull required to uplift the footing. Under the conditions upon which this example has been based, it is probable that the footing would not move until the pull was about 30,000 lb.; there would then be a packing of the soil immediately above the footing, and a final pull of about 40,000 lb. might be necessary to uproot the stub and footing.

On account of the uncertainty of the data used in calculations on tower anchorages, it is advisable to allow a factor of safety about 25 per cent greater than that used in the design of the towers themselves. Thus, if the tower itself may be expected to fail when the loading is 50 per cent in excess of the maximum load which it is ever likely to be called upon to withstand, a factor of safety of $1.5 \times 1.25 = 1.875$ or, say, 2 should be used in the calculations for the footings. If the factor of safety used in the tower design[1] is 2, a factor of 2.5 should be used in calculating the holding power of the anchorage.

Some types of concrete footings for the usual wide-base steel tower are shown in Fig. 55. It is important to shape the top surface of the concrete around the steel stubs so as to avoid the formation of water pockets which would ultimately cause corrosion of the metal at this point.

When concrete is not used, the design of the anchors is a matter that should receive very careful consideration. If towers are to be subjected to load tests, these tests should, if possible, be conducted on a tower set on its anchors, as used in the field, because

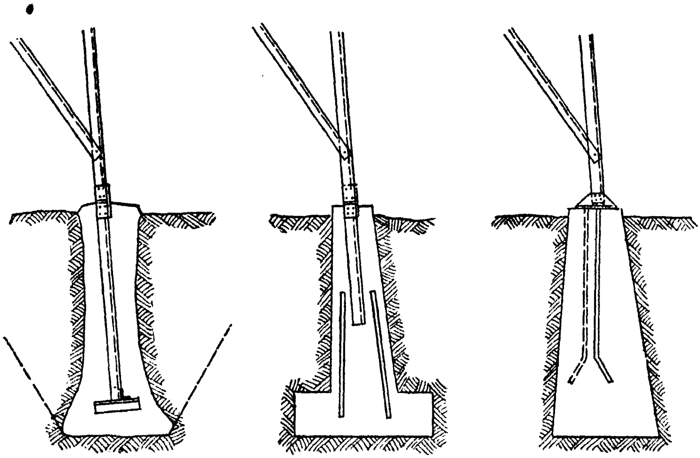


FIG. 55.—Types of tower footings in concrete.

the strength is to an appreciable extent dependent upon the method of attachment of the tower's legs to the anchor stubs.

The design shown in Fig. 54 has the disadvantage that there is a possibility of *bending* forces causing distortion or breaking of the straight unsupported length of angle iron between the footing and the upper end of the stub which is bolted to the corner member of the tower. This possibility is illustrated by Fig. 56. Assuming the usual square-base tower with four anchor stubs, the resultant force tending to uproot the anchor stub *OF* is the vector *OR* of which the horizontal component is $OH = \frac{1}{4}P$, and the

[1] Reference is here made to the true factor of safety and not to the so-called "safety factor" based upon ratio of ultimate unit stress to unit stress used in design calculations (see Art. 65).

vertical component is $OV = \frac{1}{2}P\left(\frac{h}{b}\right) - \frac{1}{4}W$. Note that this resultant force is not necessarily in the direction FO, and that it may be resolved into the two components OL, acting along the axis of the stub, and OB in a direction perpendicular to OF. It is this component which may cause bending in the event of the earth yielding near the ground level. Even a slight displacement

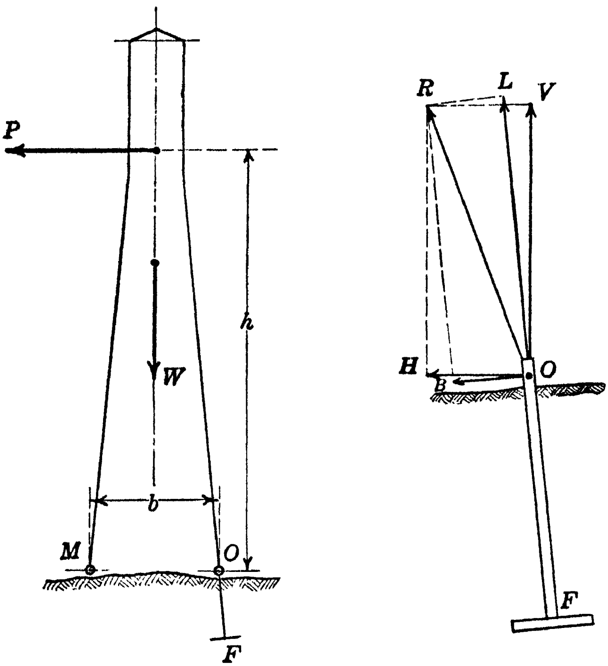


FIG. 56.—Illustrating possibility of bending forces in tower footings.

of the point O would result in abnormal stresses in the tower members, and this might lead to failure of the whole structure. The tendency to avoid concrete footings in order to reduce erection costs is not unreasonable provided proper attention is paid to the design of footings. If the single angle section with grillage and stiffening braces at bottom, as shown in Fig. 54, is replaced by three angle members forming a tripod connecting a triangular plate of adequate area to a common point on the main anchor stub only a short distance below the ground level, a satisfactory tower footing is obtained.

Another type of footing for use without concrete is the design of the American Bridge Company, which is illustrated in Fig. 57. This virtually has the effect of continuing the lower braces of the tower, together with the main corner member, to a point about 3 ft. below the ground surface, where any stresses tending to move the footing in a horizontal direction are taken by short lengths of channel section bolted to the two sides of the main stub.

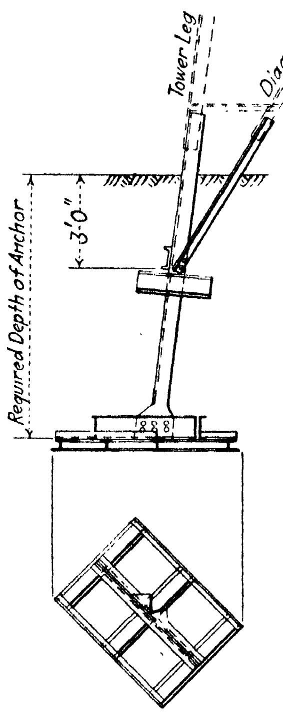


FIG. 57.—Special design of tower footing.

When structural steel, whether galvanized or not, is embedded in concrete, its life is practically unlimited; but there is always the possibility of corrosion when the galvanized-steel footings fabricated from standard structural sections are directly in contact with the soil. In many instances the corrosion has been found to be no greater below than above ground; but when the soil is of an alkaline nature, corrosion may be very rapid.

A point of importance in the design of tower foundations is the area of the footing in contact with the earth. In the case of the upward pull on the stubs connected to the tower members in tension, the weight of the cone of earth to be lifted may be ample to provide the desired factor of safety; but movement of the tower foundations may occur through the packing of the earth due to excessive unit pressure over the upper surface of the footing, and this movement may be appreciable notwithstanding that there may be no disturbance of the ground surface. A surface of not less than 1 sq. ft. for every 10,000 lb. of the vertical force which will pull out the anchors should be provided, unless the nature of the soil is such as to justify a reduction of this allowance.

The Malone Anchor.—The trouble referred to above, namely the compression of the earth above or below the footing without disturbance of the ground surface is not likely to occur when the

"Malone anchor" is used.[1] This type of tower footing consists of a drilled hole with an enlargement at the bottom sprung by dynamite and filled with concrete into which the anchor stub is pushed and held in position until the concrete has set. A hole 5 or 6 in. in diameter can be bored to a considerable depth at a much lower cost than that of the excavations required in connection with the usual methods of setting anchor stubs. Dynamite is exploded near the bottom of the bored hole, and this produces a cavity, usually nearly spherical in shape and approximately 2 ft. in diameter, into which the concrete is poured after the anchor leg—usually a straight length of angle steel—has been correctly set in position. The bored hole is also filled with concrete, but the amount required for a given holding-down pull is very much less than for concrete footings of the ordinary type.

69. Guys for Steel Towers.—There is an unexplained prejudice against the guying of steel towers where extra strength to resist lateral loads is required. By giving proper attention to the method of guying, and inspecting the line at regular intervals, there is no apparent reason why this fairly obvious device to save the extra cost of special structures should not prove entirely satisfactory. The so-called rigid tower with wide base and costly foundations is a somewhat clumsy device to take care of the enormous stresses at points where high-strength conductors are dead ended or where the line turns at a sharp angle. Undoubtedly there are situations where natural obstacles or difficulties of right of way render guying impossible or uneconomical and in such situations the self-supporting, very strong, and very costly steel tower will have to be used, but there are many instances where economy might have been effected if the fairly obvious device of guying corner or terminal structures had been resorted to. It has already been stated that the maximum deflection under load of the square-base rigid type of steel tower does not exceed a few inches, and it would, therefore, be folly to design such a structure of insufficient strength to resist lateral loads and then "strengthen" the tower by long oblique guys without being sure that the yield of the guy anchorages and the elastic extension of the guys themselves would be appreciably less than the deflection of the tower which would stress the members of the structure beyond the elastic limit of the material. In other

[1] *Elec. World*, p. 524, Mar. 3, 1923.

words, a guyed tower should not have a wide base bolted to immovable foundations, but should be designed as a column, to withstand vertical loads, with a narrow base or even (if necessary) a pivoted base so that the guy wires would take their full share of the load. There are many transmission lines using the flexible type of steel support with head guys and corner guys provided where needed that have cost less and given quite as good service as the lines with wide-base rigid towers and longer spans. It is probable that the latter construction will hold its own in connection with important extra-high-voltage developments, but when economy is considered—as it always should be, by the engineer—it is possible that, where special structures are required to resist extra-heavy lateral loads, guyed towers may ultimately take the place of the monstrous self-supporting structures which otherwise would be necessary.

70. Layout and Erection of Overhead Transmission Lines.—This book is not intended to give practical advice to construction engineers or the men actually engaged in the work of erecting poles or towers and stringing wires. A competent construction engineer, with experience in handling men and materials in the field, should be given a free hand in planning and executing the work of erecting a power transmission line; and such a man will not derive much assistance from books. On the other hand, there are some excellent books available dealing with the more practical side of transmission line engineering. These include the various electrical engineering handbooks. The reader desiring information on the methods ordinarily adopted in carrying out the details of construction is referred to these other sources of information; also to the papers and articles which appear from time to time in the journals of the engineering societies and in the technical press.[1]

The principal reason for referring to these matters in this place is to emphasize the importance of devoting much time and

[1] LUNDQUIST, R. A., "Transmission Line Construction," McGraw-Hill Book Company, Inc.

COOMBS, R. D., "Pole and Tower Lines," McGraw-Hill Book Company, Inc.

PAINTON, E. T., "Mechanical Design of Electric Transmission Lines," Chapman & Hall, Ltd.

Refer also to manufacturers' publications for useful details and recommendations in connection with the construction and erection of overhead transmission lines.

thought to the various details of overhead line construction *before the work is actually started.* The proper setting out of the line is among the most important matters connected with overhead construction. If a line is not carefully surveyed and planned in every detail, it will often be impossible to get good and reliable service from it. This does not mean that the commercial aspect of the undertaking is not of prime importance; on the contrary, it is the only aspect from which an engineering undertaking of the kind under consideration should be viewed. But this is not equivalent to saying that a small first cost is always desirable, or that even a short low-voltage transmission line can be constructed and operated economically by persons without engineering skill and experience. It is an easy matter to find examples of lines that have cost too much; but it is not impossible to find the transmission line that has cost too little—in the first instance.

At the present time new power developments at the higher voltages, and extensions on a large scale of existing power lines, are generally in the hands of competent engineers and are very thoroughly studied; but this is not always the case with the smaller and lower voltage undertakings, some of which are not good examples of engineering skill and foresight, and, although perhaps cheaply constructed in the first instance, do not fulfil adequately or economically the purpose for which they were intended.

Determining Position of Supports on Uneven Ground.—The lowest point of the span is not necessarily the point at which the wires come closest to the ground. When there is doubt as to the proper location of the supports in rough country, the method illustrated in Fig. 58 will be found very convenient. The curve *a* is the parabola corresponding to the required tension in the particular wire to be used. The ratio of the scale of feet for vertical measurements to the scale for horizontal measurements should be about 10 to 1. The dotted curves *b* and *c* are exactly similar to *a*, but the vertical distance *ab* represents the minimum allowable clearance between conductor and ground, while the vertical distance *ac* is the height above ground level of the point of attachment of the lowest wires to the standard transmission pole or tower. These curves should be drawn on transparent paper; they can then be moved about over a profile of the ground to be spanned, drawn to the same scale as the curves, until the best location for the supports is found. The point P where the curve b touches the ground line is seen to be far removed from the lowest point of the parabola, in the example illustrated in Fig. 58. A little practice will make the finding of the points A and B an easy matter, even if the length of span, or distance between A and B, must be kept between close limits.

This method is particularly applicable to long-span lines carried over rough country.

In connection with the approximate location of the line and supporting structures, in districts for which detailed maps are

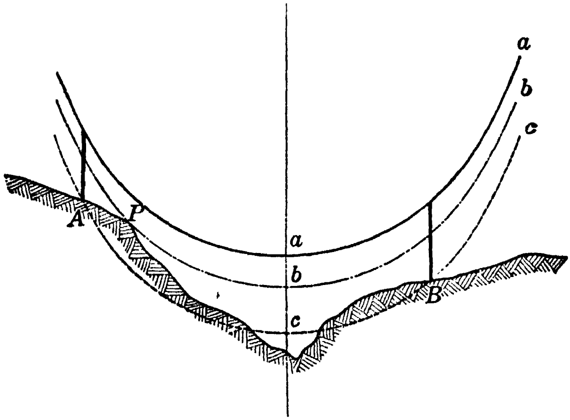


FIG. 58.—Method of locating position of towers in rough country.

not available, a series of photographs of the proposed route taken from an aeroplane by an expert in this new application of photography to land surveying will sometimes shorten the time and reduce the cost of preliminary surveys.

A chapter treating of the mechanical features of overhead transmission lines might be expected to include the design and construction of outdoor substations and even such special structures as transposition towers. The latter usually consist of simple modifications to standard towers, the exact nature of which will depend upon the ingenuity of the transmission line engineer who will usually decide upon details of construction in consultation with the manufacturer. The design and equipment of outdoor substations are subjects of growing importance which require far more space than could be devoted to them here. The technical press, during the last few years, has described a number of modern outdoor substations, the important features of which are the proper arrangement of the various pieces of apparatus in relation to each other and to the bus bars, and the provision of adequate insulation between high-voltage metal work and the grounded steel structure. The design of the supporting structure is not a very difficult matter once the manner of attaching the insulators to the steel work and the necessary clearances for insulation purposes have been decided upon.

CHAPTER VIII

INSULATION OF OVERHEAD TRANSMISSION LINES

71. Insulation Problems. Materials.—The economic transmission of large amounts of electric energy over considerable distances involves the use of very high voltages. The reduction of the current to be transmitted by increasing the pressure of transmission effects not only a saving in the weight and cost of conductors, but also keeps down the I^2R losses and improves the voltage regulation of the line by preventing excessive IR and IX, voltage drop. The material most commonly used for

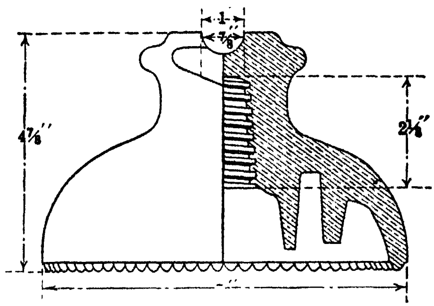


FIG. 59.—Single piece glass insulator. Average weight per piece, 4¾ lb. Made for standard 1 in. and special 1⅜ in. pins. Voltages—Test—Dry 86,200, Wet 50,100, Line 17,000.

insulators on high-tension overhead lines is porcelain; but glass, which is cheaper than porcelain, may sometimes be used to advantage on the lower-voltage lines. Figure 59 shows a glass line insulator manufactured by the Hemingray Glass Company of Muncie, Ind. Glass is a material of high resistance and dielectric strength, but it is mechanically weaker than porcelain and generally more liable to injury from sudden and extreme variations of temperature. The Pyrex glass insulators, recently put on the market by the Corning Glass Works of Corning, N. Y., are able to withstand power arcs without cracking or suffering permanent injury. The Missouri River Electric and Power Company of Helena, Mont., has used glass insulators for working pressures up to 70,000 volts. These insulators are mounted on wood supporting pins treated with paraffin in a vacuum. It is true that the climate is dry and the air free from dust or salt which might cause trouble with so high a voltage if the insulators were installed under less favorable conditions. Porcelain is more commonly used for the higher pressures than glass, but for voltages, up to 25,000, glass insulators have given satisfaction, not only in this country, but also in France where they are used in considerable numbers for the lower-voltage power transmission lines.

For the suspension type of insulator, as used for the higher pressures, porcelain appears to be the best material available at present, although it is far from being ideally suited to the requirements.

Insulators used on high-voltage transmission lines are the result of successive developments and improvements in the design and manufacture of insulators originally used for supporting telegraph wires and the low-voltage conductors of the earlier overhead lighting systems. Of recent years the study of line insulation problems has been conducted on scientific lines, and the later developments are mainly the outcome of a more thorough understanding of the fundamental principles involved in their design.

The design of insulators for the lower voltages is a comparatively simple matter, the difficulties becoming greater with the increase of pressure, although the introduction of the suspension type, which permits of many units being connected in series, has considerably simplified the problem.

It is important to bear in mind that every insulator is necessarily a more or less complicated condenser, and it can generally be thought of as consisting of a number of separate condensers in series, the dielectric being alternately air and porcelain. The current passing from line to ground is partly a leakage current over the surfaces (the leakage through the porcelain being generally negligible) and partly a capacity current. This capacity current spreads itself over the high-resistance surfaces of the insulator material in a way which depends upon the surface conductivity and on the spacing and disposition of the various parts. It is well to keep the electrostatic capacity as low as possible, but it is of equal if not greater importance to distribute it by a scientific arrangement of the component parts so that abnormal stresses will not occur locally. These may puncture or damage the insulator at one particular point, while a more carefully designed insulator of lighter weight may withstand a greater total breakdown pressure, because proper attention has been given to this important matter of capacity distribution.

The effect of rain on the exposed surfaces of an insulator is to increase the capacity, and this will generally lower the flash-over point, but the increased surface conductivity has the effect of equalizing the potential distribution. In the case of a large number of condensers in series, such as occurs especially with the suspension type of insulator, it has actually been observed that this equalizing of the potential distribution may cause the flash-over pressure of the wet insulator to be no lower than the flash-over pressure of the same insulator when dry.

If the distribution of dielectric flux could be easily determined in the case of the rather complicated shapes and varying thicknesses of dielectric which occur in high-tension line insulators, it would be an easy matter to predict the performance of new types and sizes under specified conditions. Although the dielectric circuit can conveniently be treated in a manner analogous to the engineer's treatment of the magnetic circuit, there is always difficulty (except in the simplest cases) in predetermining the amount and direction of the lines of flux or stress.

It is not proposed to take up much space in discussing details of design or in explaining exactly how an insulator should be proportioned to fulfil given requirements, but recent progress and the understanding of present-day problems depend so much on an understanding of the flux distribution and stresses in the dielectric circuit that a brief review of the fundamental relations governing flux distribution in and around insulators will not be out of place.

72. The Dielectric Circuit.—The fundamental law of the dielectric circuit is

$$\Psi = EC \tag{78}$$

where Ψ is the total dielectric flux (in coulombs) in a space of which the *permittance* or *capacity* is C farads, when the potential difference producing this flux is E volts.

If a small element of flux is considered, *i.e.*, a "tube" of induction, of length l cm. and cross-section A sq. cm., over which the flux Ψ is evenly distributed, the flux density, in coulombs per square centimeter, is

$$D = \frac{\Psi}{A} \tag{79}$$

If the difference of potential between the two ends of the path l cm. long is E volts, the capacity (or permittance) is proportional to $\frac{A}{l}$, exactly as in the analogous case of the magnetic circuit of which the *permeance* is directly proportional to A and inversely proportional to l.

With the proper constants inserted,

$$\text{Electrostatic capacity} = C = \left(\frac{10^9}{4\pi(3 \times 10^{10})^2}\right)\frac{kA}{l} \text{ farads} \tag{80}$$

wherein the numerical multiplier results from the choice of units. The factor k is the relative specific inductive capacity, or dielectric constant, of the material ($k = 1$ in air), while the unit for l and A is the centimeter. This expression for capacity may conveniently be rewritten

$$C_{\text{mf.}} = \frac{8.84}{10^8}\frac{kA}{l} \text{ microfarads} \tag{81}$$

Approximate values for k are given in the accompanying table:

DIELECTRIC CONSTANTS AND DISRUPTIVE VOLTAGES

Material	Dielectric constant k	Dielectric strength, kilovolts per centimeter
Air	1	22
Porcelain	4.5	110
Glass	5 to 10	90
Transformer oil	2.4	80
Paraffin	1.9 to 2.3	100

The figures in the last column of the table are approximate only; they indicate the virtual or r.m.s. value of the (sinusoidal) alternating voltage which may be expected to break down a slab of the material 1 cm. thick placed between two large flat metal electrodes. What is usually understood by the expression "disruptive gradient" is obtained by multiplying the values in the table by $\sqrt{2}$.

Returning to formula (79), let *electric flux*, or *quantity* of electricity Ψ be expressed in terms of capacity and e.m.f., with a view to determining the relation between flux density and electric stress. Formula (80) may be written

$$C = (Kk)\frac{A}{l} \text{ farads} \tag{80a}$$

where K stands for the numerical constant. Substituting in equation (78),

$$\Psi = E(Kk)\frac{A}{l}$$

whence

$$D = Kk \times \left(\frac{E}{l}\right)$$

Since $\frac{E}{l}$ is the potential gradient, or volts drop per centimeter, denoted by the symbol G,

$$D = Kk \times G \tag{82}$$

The analogous expression for the magnetic circuit is $B = \mu H$.

In the dielectric circuit, *electric flux density = e.m.f. per centimeter × "conductance" of the material to dielectric flux*, while, in the magnetic circuit, *magnetic flux density = m.m.f. per centimeter × "conductance" of the material to magnetic flux.*

Since the electric stress or voltage gradient G is directly proportional (in a given material) to the flux density D, it follows that, when the concentration of the flux tubes is such as to produce a certain maximum density at any point, breakdown of the insulation will occur at this point. Whether or not the rupture will extend entirely through the insulation will depend upon the value of the flux density (consequently the potential gradient) immediately beyond the limits of the local breakdown.

Before illustrating the application of the above principles to the design of insulators, it will be advisable to assemble and define the quantities which are of interest to the engineer in making practical calculations.

E, e = e.m.f. or potential difference (volts)

l = length, measured along line of force (centimeters)

A = area of equipotential surface perpendicular to lines of force (square centimeters)

$G = \frac{de}{dl}$ = potential gradient (volts per centimeter)

C = capacity or permittance (farads or microfarads)

(Farads $= \frac{\text{coulombs}}{\text{volts}}$ = flux per unit e.m.f.)

K = constant $= 8.84 \times 10^{-14}$ (farads per centimeter cube, being the specific capacity of air)

k = dielectric constant, or relative specific capacity, or relative permittivity ($k = 1$ for air)

Ψ = dielectric flux, or electrostatic induction ($\Psi = CE = AD$ coulombs)

D = flux density $= \frac{\Psi}{A} = (Kk)G$ (coulombs per square centimeter)

Calculation of Condenser Capacity.—Imagine two parallel metal plates, as in Fig. 60, connected to the opposite terminals of a continuous current source of supply. The area of each plate is A sq. cm. and the separation between plates is l cm., the dielectric or material between the two surfaces being air. The edges of the plates should be rounded off to avoid concentration of flux lines. If the area A is large in comparison with the distance l, a uniform distribution of the flux Ψ may be assumed in the air gap, the density being $D = \frac{\Psi}{A}$.

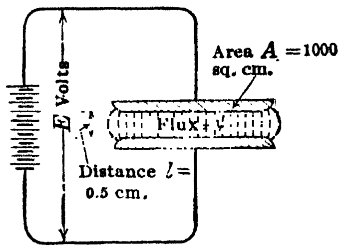


FIG. 60.—Simple plate condenser.

By formula (81), the capacity is $C_{\text{mf.}} = \frac{8.84 \times A}{10^8 \times l}$ mf., since the dielectric constant (k) of air is 1. Assuming numerical values, let A = 1,000 sq. cm., and l = 0.5 cm.

Then $$C = \frac{8.84 \times 1{,}000}{10^{14} \times 0.5} = 1.77 \times 10^{-10} \text{ farads}$$

If E = 10,000 volts, the potential gradient will be $G = \frac{10{,}000}{0.5}$ = 20,000 volts per centimeter. There will be no disruptive discharge, however, because a gradient of 31,000 volts per centimeter is necessary to cause breakdown in air.

By formula (78), the total dielectric flux is $\Psi = 10{,}000 \times 1.77 \times 10^{-10} = 1.77 \times 10^{-6}$ coulombs.

Dielectric Paths in Series.—Assume that a 0.3-cm. plate of glass is inserted between the electrodes of the condenser shown in

Fig. 60. The modified arrangement is illustrated by Fig. 61. On first thought it might appear that this arrangement would improve the insulation, but care must always be taken when putting layers of insulating materials of different specific inductive capacity in series, as this example will illustrate. In addition to the elastance[1] of a 0.3-cm. layer of glass, there is the elastance of two layers of air, of which the total thickness is 0.2 cm. Assuming that the value of the dielectric constant k for the particular quality of glass used is 7, and that G_g and G_a are the potential gradients in the glass and air, respectively, then, by formula (82), $KG_a = 7KG_g$, whence $G_a = 7G_g$.

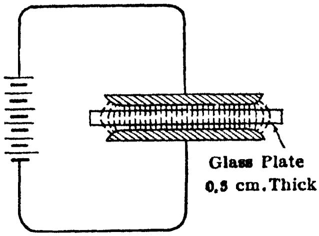


FIG. 61.—Plate condenser with electrodes separated by air and glass.

Taking the total potential difference between electrodes as 10,000 volts, the same as used in considering Fig. 60, $E = 10{,}000 = 0.2G_a + 0.3G_g$, whence $G_g = 5{,}880$ volts per centimeter, and $G_a = 41{,}100$ volts per centimeter. Such a high gradient as 41,100 would break down the layers of air and would manifest itself by a bluish electrical discharge between the metal plates and the glass. On the other hand, the gradient of 5,880 volts per centimeter would be far below the stress necessary to rupture the glass. It should be observed that the introduction of the glass plate has appreciably increased the capacity of the condenser. For example, with the same voltage ($E = 10{,}000$) as before, the total flux is now $\Psi = AD = 1{,}000\ (8.84 \times 10^{-14} \times 41{,}000) = 3.63 \times 10^{-6}$ coulombs. This value is about double the value calculated with only air between the condenser plates.

Conditions Determining Flash-over.—Whether or not spark-over will occur depends not only upon the condition of the surface (clean or dirty, dry or damp), but also upon the shape and position of the terminals or conductors. It is, therefore, almost impossible to determine, other than by actual test, what will happen in the case of any departure from standard practice.

For a given material and surface condition (clean, dry, dirty, or damp), whether or not a flash-over will occur depends upon the voltage gradient or difference of potential per unit length of the

[1] In the dielectric circuit, *elastance* is the reciprocal of *permittance* (or *capacitance*). Elastance $= \frac{1}{C}$.

surface. Thus, if two electrodes are separated by an insulator providing a leakage path over a surface of length l, it is impossible to predetermine the voltage which will cause a flash-over without a knowledge of the *shape* of the insulator surface as well as its nature and condition. In Fig. 62 a thin disc of insulating material is used to separate two metal electrodes, the length of the leakage path over the surface being l. In Fig. 63 the same electrodes are separated by a thicker block of solid insulation, arranged to provide a leakage path of exactly the same length l, as in Fig. 62. Flash-over will occur at a lower voltage around the thin disc of Fig. 62 than over the thicker insulation of Fig. 63. This is because the flux concentration due to the proximity of the terminals in Fig. 62 begins by breaking down the layers of air

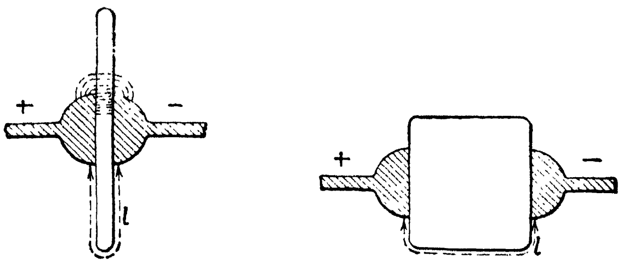


FIG. 62. FIG. 63.

FIGS. 62 and 63.—Leakage path over surface of insulator.

around the edges of the electrodes at a considerably lower potential difference than will be required for the thicker insulating block of Fig. 63. The effect of this incipient breakdown is, virtually, to make a conductor of the air around the edges of the metal electrodes and so shorten the effective creepage distance around the edges of the plate. That the phenomenon of so-called surface leakage may be considered as being mainly one of flux concentration or potential gradient is further illustrated by Figs. 64, 65, and 66. These sketches all show the same separation between the two large flat-plate electrodes. The density of the flux in the air gap of Fig. 64 (as represented graphically by the spacing of the vertical flux lines) will be proportional to the potential difference between the electrodes. In Fig. 65 the electrodes are shown mounted on the ends of a cylinder of some insulating material of which the dielectric constant (or relative permittivity) is assumed to be 4. The flux density in the solid insulation will, therefore, be four times as great as in the air outside the cylinder. The dielectric field has not otherwise been disturbed, and the potential gradient at all parts on the surface of the solid insulation is, therefore, $\frac{E}{l}$, where E is the potential difference between the electrodes. It has been found experimentally that, under these conditions, a good insulator with a clean, dry surface is able to stand just before breakdown a gradient as high as 24,000 volts (r.m.s. value) per inch. If we allow a safety factor of 3, it follows that, with a separation of only 5 in., the arrangement of Fig. 65 would be suitable for a working pressure of 40,000 volts.

Figure 66 shows the solid cylinder of Fig. 65 replaced by a cylindrical insulator provided with projecting flanges, and an attempt has been made to indicate the flux distribution by the

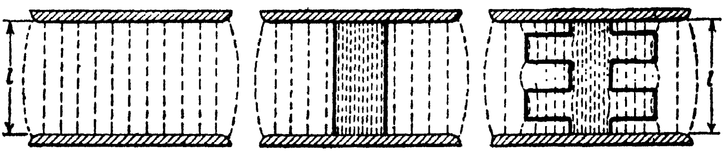


FIG. 64. FIG. 65. FIG. 66.

FIGS. 64, 65 and 66.—Illustrating effect of shape of insulator on flux concentration.

spacing of the vertical flux lines. It is evident that, although the distance over the surface of the insulator has been increased, the flux density—and, therefore, the voltage gradient—in the air spaces has also considerably increased. The air will, therefore, be stressed beyond the breakdown limit with a smaller potential difference between the electrodes than in the case of Fig. 64 or Fig. 65. The layers of solid dielectric of high permittivity placed across the path of the flux lines have reduced the effective distance between the electrodes and so lowered the elastance (or increased the capacity) of the air paths. Although a brush discharge between the flanges may not always be objectionable, and may not lead to a complete flash-over, it is generally preferable to avoid the occurrence of such discharges at voltages near the normal working pressure of the insulator. Thus, the conclusion to be drawn from this discussion is that it is usually desirable to shape the surface of an insulator to conform with the shape of the flux paths, because this will cause the potential gradient at the surface to be a minimum.

When it is necessary to guard against wet, as in overhead line insulators, sheds or petticoats, either of metal or insulating material, are necessary, but their effect is to *shorten* rather than lengthen the effective surface distance; that is to say, they may increase the actual length of the surface creepage distance, but they also increase the tendency for the layers of air to break down over the portions of the surface between the sheds, as illustrated by Fig. 66.

73. Design of Insulators—Wall and Roof Outlets.—The design of an insulator, to comply with any given specification, is a matter which concerns the manufacturer, who has been compelled of late years, owing to the rapid increase of working pressures, to devote his attention to the principles underlying the correct and economical design of insulators for high-pressure lines, and who is, therefore, something of a specialist on this particular subject. The transmission line engineer should understand the principles underlying the correct design of overhead insulators; but it is suggested that however great his knowledge of the subject, he will be well advised to leave details of design to the manufacturer, and to use standard types when possible.

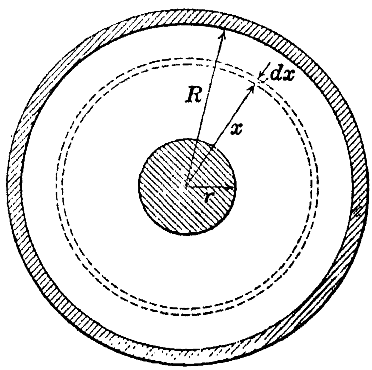


FIG. 67.—Section through insulating bushing.

The manner in which the fundamental principles, briefly discussed in the preceding article, may be applied to the design of high-voltage insulators[1] is most easily illustrated in connection with concentric cylinders of insulating materials such as enter into the construction of roof and wall bushings. It is for this reason that such bushings will be considered before describing the more important pin-type and suspension-type line insulators.

Figure 67 is a section through a conductor of radius r separated by insulating material of specific inductive capacity k from a concentric metal cylinder of radius R.

[1] Questions relating to insulation with special reference to design problems are treated in more detail in the writer's "Elements of Electrical Design," McGraw-Hill Book Company, Inc., but some of the material in this and succeeding articles (including a few illustrations) has been taken from "Elements of Electrical Design."

The equipotential surfaces will be cylinders, and the flux density over the surface of any cylinder of radius x and of unit length, say, 1 cm., will be $D = \frac{\Psi}{2\pi x}$.

By formula (82), the potential gradient is

$$G = \frac{D}{Kk} = \frac{\Psi}{2\pi x Kk} \tag{83}$$

In order to express this relation in terms of the total voltage E, it is necessary to substitute for the symbol Ψ its equivalent $E \times C$, and calculate the capacity C of the condenser formed by the rod and the concentric tube. Considering a number of concentric shells in series, the *elastance* may be written as follows:

$$\frac{1}{C} = \int_r^R \frac{dx}{2\pi x Kk} = \frac{1}{2\pi Kk} \log_\epsilon \frac{R}{r} \tag{84}$$

Substituting in (83),

$$G = \frac{E}{x \log_\epsilon \frac{R}{r}} \text{ volts per centimeter} \tag{85}$$

the maximum value of which is at the surface of the inner conductor, where

$$G_{\text{max.}} = \frac{E}{r \log_\epsilon \frac{R}{r}} \tag{86}$$

This formula is of some value in determining the thickness of insulation necessary to avoid overstressing the dielectric; but it is not strictly applicable to wall bushings in which the outer metal surface is short as compared with the diameter of the opening. The advantage of having a fairly large value for r is indicated by formula (86), and a good arrangement is to use a hollow tube for the high-tension terminal.

Solid porcelain bushings with either smooth or corrugated surfaces may be used for any pressure up to about 40,000 volts. In designing plain porcelain bushings it is important to see that the potential gradient in the air space between the metal rod and the insulator is not likely to cause brush discharge, as this would lead to chemical action, and a green deposit of copper nitrate upon the rod.

Example 19. *Design of Porcelain Wall Bushing.*—It is desired to design a porcelain insulating bushing to take a ¾-in. copper rod through an inside partition wall. The working pressure is 33,000 volts alternating.

In order to avoid the formation of corona or luminous discharge in the air space between the copper rod and inside surface of insulating tube, it is necessary to calculate the voltage gradient at the surface of the rod where the stress will be a maximum.

The voltage gradient which will cause corona or brush discharge at the surface of a rod of radius r cm., according to Mr. Peek,[1] is $31\left(1 + \frac{0.3}{\sqrt{r}}\right)$ kv. This, in the case of a rod of radius ⅜ in. or 0.95 cm., amounts to a breakdown pressure (r.m.s. value) of 28.6 kv. per centimeter; but, since it is proposed to use the formulas developed for long concentric cylinders, which are not quite correct for concentric bushings with short outer metallic cylinders, it will be safer to reduce this figure by about 10 per cent, and use 25.7 kv. as the maximum gradient (r.m.s. value) allowable at the surface of the conductor.

This is a case of two capacities in series, the first between the high-tension rod and the inner surface of the porcelain, with air as the dielectric, and the second between the inner and outer cylindrical surfaces of the porcelain. The dielectric flux being the same in both, and since $\Psi = EC$, it follows that the potential difference across each condenser will be inversely proportional to the capacity, or directly proportional to the elastance as given by formula (84).

Let E_a and E_p stand for the pressure drop (kilovolts) across the air gap and porcelain sleeve, respectively, the total e.m.f. being

$$E = E_a + E_p = 33 \text{ kv.} \qquad (a)$$

Let C_a and C_p stand for the electrostatic capacities of the air space and porcelain sleeve, respectively.

Then
$$E_a \times C_a = E_p \times C_p$$

and
$$E_p = E_a\left(\frac{C_a}{C_p}\right) \qquad (b)$$

Let r = radius of conductor = 0.95 cm.

R_1 = inside radius of porcelain bushing.

R_2 = outside radius of porcelain bushing (in contact with grounded metal cylinder).

[1] PEEK, F. W., JR., "Dielectric Phenomena," McGraw-Hill Book Company, Inc.

k = the dielectric constant for the porcelain = 4.5 (for the air gap, k = 1).

G_a = maximum voltage gradient (r.m.s. value) at surface of rod = 25.7 kv.

Then, by formula (86),

$$E_a = G_a r \log_\epsilon \left(\frac{R_1}{r}\right) \qquad (c)$$

Inserting in (b) the capacities in terms of dimensions and permittivities, as given by formula (84), and making the necessary substitutions and simplifications, the following expression is obtained,

$$\frac{E}{G_a r} = \log_\epsilon \left(\frac{R_1}{r}\right) + \frac{1}{k} \log_\epsilon \left(\frac{R_2}{R_1}\right) \qquad (d)$$

in which the only unknown quantities are R_1 and R_2.

Assume one or more reasonable values for the radius of the hole (R_1) in the insulating tube and then solve for R_2. The case

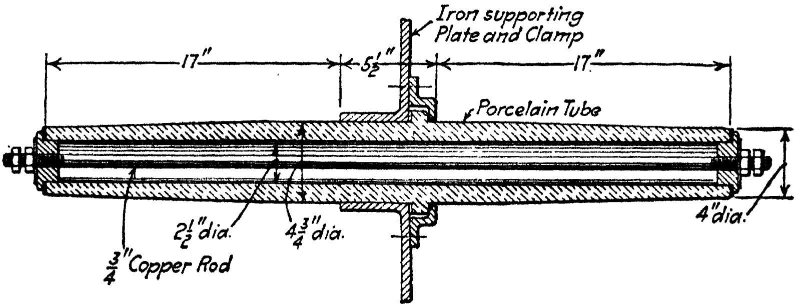


FIG. 68.—High-voltage porcelain bushing—Example 19.

of condensers in series was discussed in connection with Fig. 61, and it is certain that a very small air clearance around the rod will lead to a high voltage gradient and probably brush discharge. On the other hand, a very large air gap will cause the overall diameter of the bushing to be greater than is necessary. Assume a hole 2½ in. in diameter through the porcelain tube (R_1 = 3.18 cm.) and solve for R_2 in the equation (d). The answer is R_2 = 6 cm., approximately, which makes the outside diameter of the porcelain tube about 4¾ in. The pressure drop across the air gap, as given by (c), is E_a = 28 kv., leaving only 33 − 28 = 5 kv. pressure drop in the porcelain bushing.

It will be found that a very small reduction in the size of hole through the porcelain sleeve will lead to an *increase* in the overall diameter, while a larger hole, 7 cm. in diameter, would result in a *smaller* overall diameter, but the calculated thickness of the porcelain would then be too small for a practical design.

The length of the porcelain insulator is determined by allowing a leakage distance over the surface of the porcelain of approximately ½(kv. + 1) in. Corrugations are not necessary, and a smooth surface will be satisfactory; the projection at each end beyond the grounded metal supporting plate and clamp should, therefore, be about 0.5(33 + 1) = 17 in. A section through the completed bushing is shown in Fig. 68. A roof bushing supplied by the Locke Insulator Corporation for the same working pressure of 33,000 volts is shown in Fig. 69. The dry flash-over voltage of this bushing is 101,000 and the wet flash-over is 80,000.

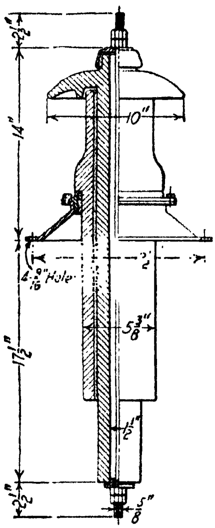


FIG. 69.—Roof bushing for use on 33,000-volt circuit.

Oil-filled Bushings.—The advantage of replacing the air space between the copper rod and the porcelain tube by some insulating compound having a dielectric constant (k) greater than unity may be illustrated by supposing the space surrounding the metal rod to be filled with an insulating oil or compound of the same permittivity as porcelain. The normal working pressure is to be 33,000 volts as before, and it can be assumed that the insulator must withstand a test of 80,000 volts alternating without breakdown, allowing a safety factor of 1.25. The breakdown gradient for the oil or compound will be about 100 kv. per centimeter (r.m.s. value).

Since k is assumed to have the same numerical value (4.5) for both insulating materials (compound and porcelain), formula (86) may be used; but this refers to long concentric cylinders, and since the potential gradient in bushings with short outer metallic sleeve may be 10 per cent greater than as calculated by this formula, it is necessary to introduce the factor 0.9 and write

$$0.9 \times 100 = \frac{80 \times 1.25}{0.95 \times \log_e \dfrac{R_2}{0.95}}$$

whence $R_2 = 3.06$ cm., and the external diameter of the porcelain bushing need not exceed 2½ in., which is appreciably smaller than the design with air clearance between the copper rod and the porcelain. It will be remembered that the reason for the larger diameter of bushing in the first design was to avoid brush discharge or corona formation at the surface of the conductor.

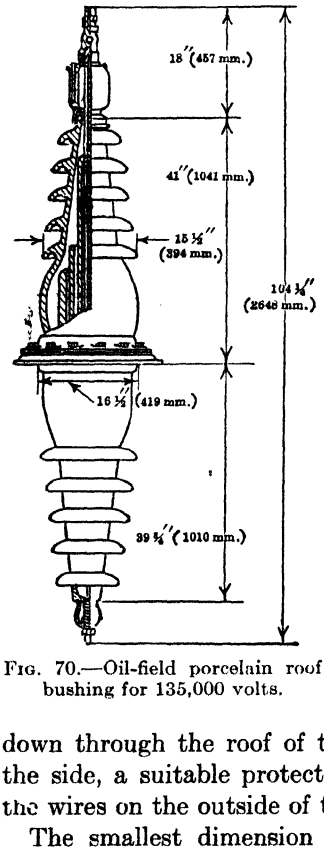


FIG. 70.—Oil-field porcelain roof bushing for 135,000 volts.

Figure 70 shows an oil-filled porcelain roof bushing made by the Ohio Brass Company for a working pressure of 135,000 volts. An oil reservoir is provided at the top to allow for expansion. The weight of this insulator without oil or compound filling is 630 lb. A bushing of this type for use on a 220,000-volt circuit would have an overall length of 11 ft. or more.

One method of bringing the high-tension conductor through the wall of a building is illustrated by Fig. 71, which shows the overhead wire anchored to the wall by a string of suspension insulators, and passing through a standard porcelain wall bushing as manufactured by the Ohio Brass Company.

When the climate and weather conditions are favorable, it is well to avoid bushings entirely. In such cases, the wires cannot be brought down through the roof of the building, but they must enter at the side, a suitable protecting hood or roof being placed above the wires on the outside of the building.

The smallest dimension of the opening in brick, stone, or concrete wall should preferably not be less than as given below:

Line pressure, volts	Width of wall opening, feet
22,000	1½
33,000	2
44,000	2½
66,000	3½
88,000	4¾
110,000	6
132,000	7
220,000	11

On each side of the wall opening, the conductor is carried by line insulators, of the pin or suspension type, as the voltage may require, these being so arranged as to maintain the conductor in the center of the opening, with a slight downward incline toward the outside of the building to prevent raindrops being carried to the inside.

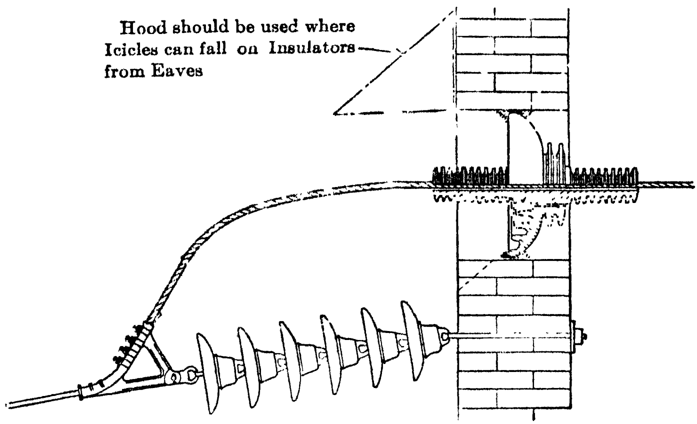


FIG. 71.—Wall bushing for transmission line wires.

74. Pin-type Line Insulators.—The laws of the dielectric circuit were explained in Art. 72, and under the headings *Dielectric Paths in Series* and *Conditions Determining Flash-over*, some fundamental principles of particular importance in connection with insulator design were discussed.

As previously mentioned, the sheds or petticoats, with intervening air spaces which separate the wire at line potential from the pole or cross-arm which is usually at ground potential, may be thought of as a number of condensers in series. If I_c is the charging current, and E is the potential difference causing this flow of current through a condenser of capacity C,

$$I_c = 2\pi fEC^1$$

and when the frequency is constant, $E \propto \frac{I_c}{C}$. Thus, when a number of condensers are connected in series, the current I_c is the same through all the condensers, and the potential difference across any one condenser is *inversely proportional to its capacity*, whence the importance of care in design to avoid too great a stress where the thickness of air or porcelain, measured

[1] See footnote on p. 219.

normally to the direction of the flux lines, is not sufficient to prevent high flux densities and disruptive gradients with a comparatively small potential difference across the insulator as a whole.

Example of Insulator Design.—In order to show how the fundamental principles of the dielectric circuit may be applied to the design of line insulators, consider the line conductor and the tie wire to be replaced by a large, flat metallic plate, which has to be insulated from a metal sphere representing the grounded support of the insulator. This is shown in Fig. 72. The flux lines in the plane of the drawing are arcs of circles which enter the sphere normally to the surface and of which the centers all lie on the

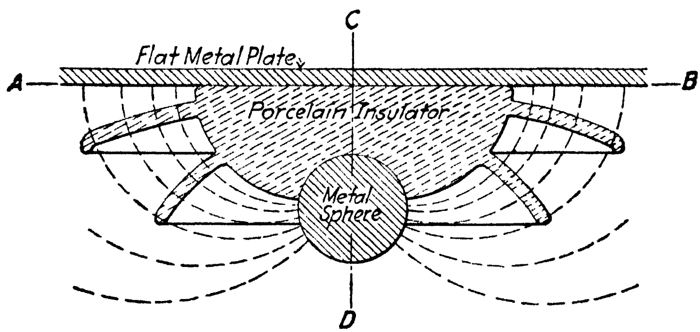


FIG. 72.—Hypothetical pin-type insulator.

line AB in the plane of the flat plate electrode. The equipotential surfaces are normal to the flux lines, being spherical surfaces with their centers on the vertical axis CD. The rain sheds shown in the figure have purposely been shaped to follow equipotential surfaces. The body of the porcelain has been shaped so that the surface follows the path of the flux lines, for reasons explained in Art. 72 when dealing with surface leakage. By making the rain sheds coincide with equipotential surfaces, the flux distribution is not altered. By shaping these differently and placing them close or far apart, it is possible to vary the capacity of the condensers formed by the layers of air between the sheds and between the inner shed and the metal sphere. This enables the designer to modify the potential gradient in one or more of the air spaces between the sheds; it also enables him to control to some extent the ratio between dry and wet flash-over. The greatest voltage gradient in the porcelain between the electrodes occurs at the surface of the metal sphere on the line CD, where the distance from plate to sphere is shortest. The thickness at this point must be sufficient to insure that flash-over will occur before the porcelain is stressed to the breakdown limit.

This hypothetical design of a pin-type line insulator illustrates the manner in which modern insulators may be studied intelligently in relation to the known laws of the dielectric circuit. The exact voltage that such a design would withstand without flash-over under wet and dry conditions, respectively, must be determined experimentally. The design of practical line insulators is based largely on knowledge gained through failures of more or less imperfect earlier designs.

With the pin type of insulator, a number of sheds or petticoats hanging close to the pin, with small air spaces between them, will not be effectual, because, although the leakage path may be long, the capacity is high. The remedy consists in spreading the petticoats away from the pin, the outer shed in some designs being almost horizontal. This outer shed has, in some cases, been replaced by a metal shield. Apart from the advantage of lightness, which permits of a thin metal shed being made of larger diameter than would be permissible if the material were porcelain, the charging current will spread itself more uniformly over the surface of the outer shed, and so prevent the concentration of potential at the point where the conductor is tied to the insulator. An insulator of this type may flash over at a somewhat lower pressure than if the upper hood were of porcelain, but under wet conditions the flash-over pressure may be higher.

The insulator shown in Fig. 73 is not a modern design. The inside shell is too close to the pin and will not take its proper share of the stress when flash-over occurs. Moreover, the air gaps between the sheds just below the cemented joints are too short and the voltage gradient will reach the breakdown limit of the air at these points when the total voltage on the insulator is comparatively low. In modern designs of pin-type insulator, as, for instance, the Faradoid type as manufactured by the Westinghouse Electric and Manufacturing Company (see Fig. 74), the tendency is to increase the thickness of porcelain around the end of the grounded supporting pin, to shape the outside of the porcelain (neglecting the sheds) so as to follow as nearly as possible the direction of the flux lines, to make the sheds as thin as possible and shape them so as to follow approximately the equipotential surfaces, and to aim at about the same capacity between the several sheds. These developments are in accordance with theoretical conclusions based on a knowledge of the fundamental laws of the dielectric circuit as given in the earlier portions of this chapter.

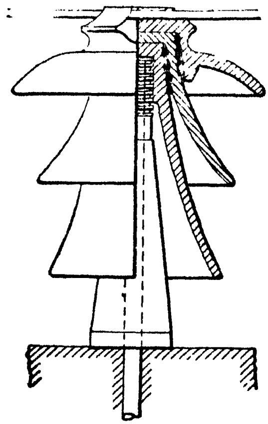


FIG. 73.—Pin-type insulator.

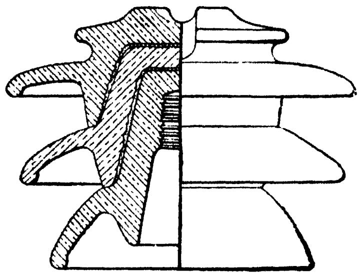


FIG. 74.—Three-part line insulator of the "Faradoid" type.

Pin-type insulators are available for pressures up to 70,000 volts,[1] but the suspension type, made up of two of more units,

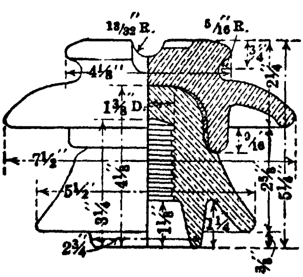


FIG. 75.

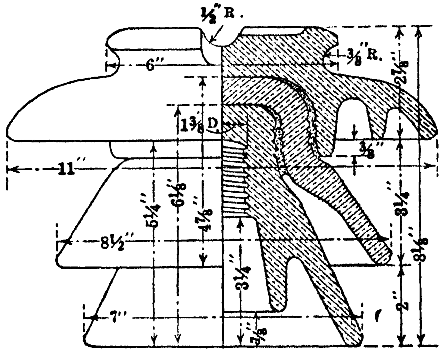


FIG. 76.

Pin-type insulators.

will generally be more satisfactory and economical for pressures above 50,000 volts. The pin type of insulator becomes too heavy

[1] Pin-type insulators of modern design have been in successful operation in Canada on the 90,000-volt lines of the Toronto Power Company since October, 1918. For an interesting account by Paul Ackerman, of experience with these insulators, see *Elec. World*, vol. 80, p. 1439, Dec. 30, 1922.

and costly when designed for the higher voltages, and, owing to the great length of the supporting pin, the bending moment near the point of attachment to the cross-arm tends to become excessive. With conductors of large cross-section, the suspension type of insulator is preferable, even for comparatively low transmission pressures.

Wooden supporting pins are not recommended for high-tension work; they are rarely used on lines working at pressures above 33,000 volts. Metal pins are generally preferable. Trouble due to puncture of the porcelain between the (grounded) pin and the conductor or tie wire has practically been eliminated in the modern designs, which have a greater thickness of porcelain at this point.

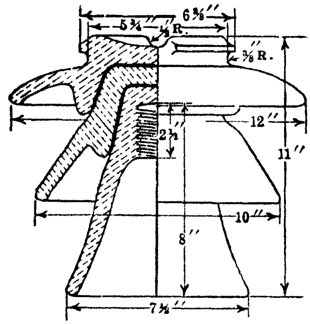


FIG. 77.—Pin-type insulator for 60,000 volts.

Figures 75 and 76 illustrate typical pin-type insulators made by R. Thomas and Sons of East Liverpool, Ohio. Figure 77 is one of the high-voltage designs of the Locke Insulator Corporation of Baltimore, Md. The leading particulars of these insulators, as furnished by the makers, are as follows:

	Fig. 75	Fig. 76	Fig. 77
Normal rating (line pressure), volts	27,000	50,000	60,000
Dry arc-over, volts	89,000	155,000	175,000
Wet arc-over, volts	54,000	102,000	120,000
Leakage distance, inches	12.0	28.0	30.0
Net weight, each, pounds	6.0	22.3	25.0

The sphere gap is used in the measurement of the arc-over voltages.

75. Suspension-type Insulators.—With the *suspension type* of insulator, the conductor is hung below the point of support (which is usually grounded) at the end of a string of insulator units connected to one another by metal links. The potential difference which will cause a flash-over or breakdown on such a series of insulators will not be in direct proportion to the number of insulators in the string. This is due to the unequal distribution of the potential differences, which is again a question of relative capacities. The design of the individual units may appear to be good, and yet a string of such insulators, if these are not specially designed to fulfil certain requirements, may give surprisingly unsatisfactory results. A factor of importance is the ratio $\frac{\text{mutual capacity}}{\text{capacity to ground}}$ which determines the potential distribution; and this ratio will depend not only on the shape and size of the porcelain units, but also on the metal caps or means of attachment, and the spacing between units.

A modern design of suspension-type insulator is shown in Fig. 78. This is one of the types made by the Locke Insulator

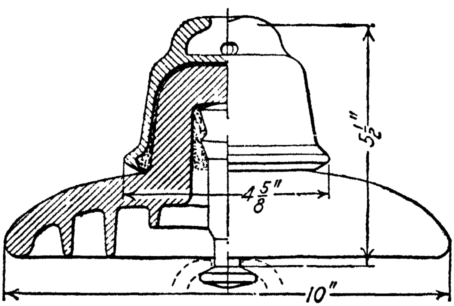


Fig. 78.—Unit of suspension-type insulator.

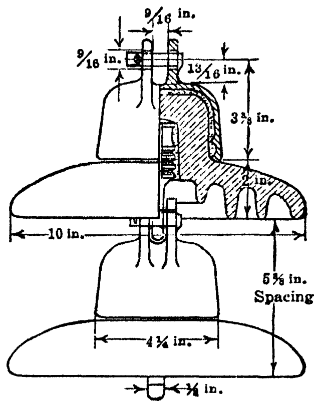


Fig. 79.—Suspension type insulator.

Corporation. It weighs $10\frac{1}{2}$ lb., is fitted with the ball and socket form of attachment, and has a guaranteed mechanical strength of 9,000 lb. The electrical performance of these insulators is not guaranteed by the manufacturers because the test pressures (wet and dry) will depend upon the conditions under which the insulator is used, including the number of units in a string, whether or not guard rings are used or metal shields to equalize the potential distribution, and similar considerations.

A type of suspension insulator made by R. Thomas and Sons is illustrated in Fig. 79. The weight of the single unit is 12 lb.; the dry flash-over is 84,000 volts, and the wet flash-over, 48,000 volts. The flash-over voltages for strings of these insulators, as given by the manufacturers, are as follows:

	Dry, kilovolts	Wet, kilovolts
String of 3 units	194	118
String of 5 units	290	188
String of 7 units	380	260

A string of Hewlett insulator units, as manufactured by the Locke Insulator Corporation is shown in Fig. 80. The assembled insulator is suitable for use on a 132,000-volt line and includes arcing horns at top and grading shield at line potential. The manner in which this grading shield tends to equalize the drop of potential across the individual units in the string will be explained in the following article. The link type of unit was originated by E. W. Hewlett mainly with the object of having the porcelain in compression instead of in tension as in the cap-and-link type of suspension unit. Another advantage claimed for it is that it will stand without damage

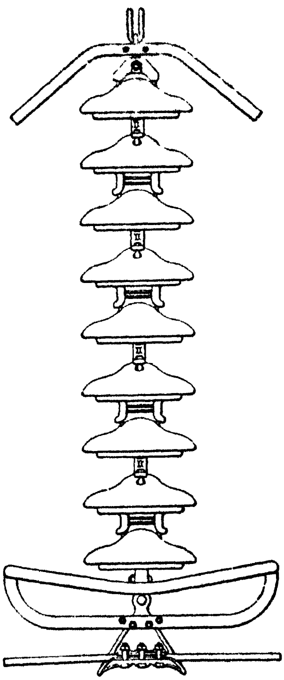


FIG. 80.—Hewlett type suspension insulator with arcing horns and grading shield.

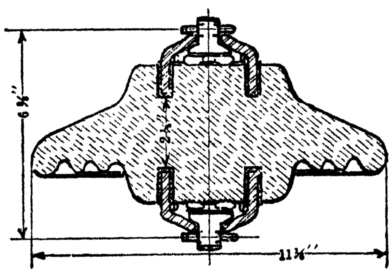


FIG. 81.—Jeffery-Dewitt suspension insulator unit.

sudden changes of temperature. It may be immersed in boiling water and transferred to ice water and back again a number of times without injury.

Figure 81 shows a section through a single unit of the suspension insulator manufactured by the Jeffery-Dewitt Insulator

Company of Kenova, W. Va. The special features of this design are (1) thick porcelain, to resist puncture and the shattering effects of power arcs, and also breakage during transit and malicious breakage by shots or stones; (2) a pressed-steel spider anchored in small corrugated holes by a resilient alloy which is unaffected by changes of moisture. It is claimed for this insulator that it will stand great changes of temperature much better than the cap-and-pin type of unit in which failure has freqently occurred through cracks in the porcelain caused by unequal expansion of the porcelain, cement, and metal clamps. The test by repeated transfer from boiling to ice-cold water, referred to in connection with the Hewlett insulator, can be made on the Jeffery-Dewitt unit without injury to the porcelain.

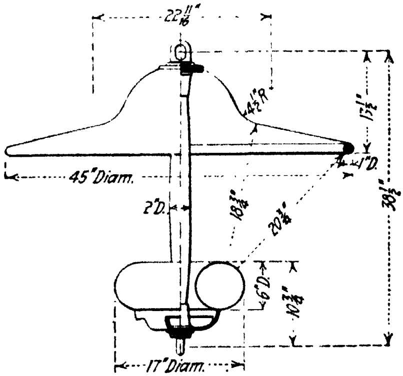


FIG. 82.—New design of suspension insulator unit.

A new design of suspension insulator has recently been developed by Prof. Harold B. Smith.[1] The use of porcelain is avoided, and two metallic terminal members, shaped to provide the required flux distribution, are joined by a single insulating mechanical strain member of treated wood or other suitable material. The dimensions of a single unit suitable for a working pressure of 110,000 volts are given in Fig. 82. This design is of particular interest because it is based upon a thorough understanding of the fundamental principles which determine flux distribution in the dielectric circuit, whereas the porcelain insulator was, until recent years, experimentally developed without much attention being paid to the theory underlying the distribution of the dielectric field within and around the insulator. At the

[1] *Transactions* A.I.E.E., 1924, Vol. 43, p. 1263.

present time this theory is well understood; but, nevertheless, porcelain is not an ideal material for the insulation of overhead lines, and every serious attempt to avoid its use should be carefully investigated. By the proper shaping of the metallic terminal members, Professor Smith has been able to place the single insulating member in what he calls a hollow electric field, that is to say, a field surrounded symmetrically along its axis by a stronger field of higher average and maximum potential gradient. Any arc due to overvoltage will, therefore, strike between the lower metal ring and the edge of the upper metal hood, thus avoiding the insulating strain member at the center of the "hollow" field.

76. Potential Distribution over String of Suspension-type Insulators.—A string of suspension-type insulator units may be thought of as a succession of metal parts separated by insulating material. The distribution of the potential drop between successive metal connecting pieces will depend upon the electrostatic capacities of the condensers formed by these metal parts and (1) the neighboring metal parts in the chain of insulators, (2) the earth or grounded supporting structure, (3) the high-tension conductor. Assuming for the moment that the capacities (2) and (3) are negligible, it is necessary merely to consider the simple case of a number of condensers in series between the high-voltage line and the (grounded) point of support, in which case the potential difference across each unit in the chain would be inversely proportional to the capacity of the individual unit. If, therefore, all the units in the chain were of the same size and type, there would be a uniform drop of potential along the string, provided the capacities to ground and to the high-tension conductor were negligible. It is only in rare instances that it is permissible to neglect the effect of these capacity currents which are in parallel with the main condenser current passing through the series of insulator units; but in order to simplify the problem, consider, in the first instance, only two capacities; namely, (1) that of the insulator unit itself (dielectric usually porcelain) and (2) that formed by the connecting metal work and ground (dielectric air).

Example 20. *Potential Distribution over Four-unit String of Suspension Insulators.*—The metal work (cap and link or bolt) on each side of the porcelain disc constitutes one terminal of a condenser which has a capacity C farads relatively to ground or grounded tower, and a capacity mC relatively to the metal work (cap and link or bolt) on either side of the porcelain disc. This is shown diagrammatically in Fig. 83 where a, b, and c are imaginary condensers, each of capacity C farads, between the ground and the insulating metal caps, etc., while the condensers of capacity mC farads, formed by each individual unit, are numbered (1), (2), (3), and (4). The effects of *surface leakage* and *corona* will

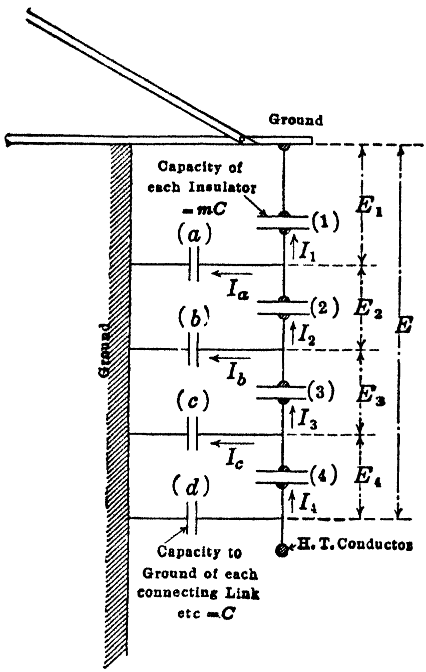


FIG. 83.—Diagrammatic representation of 4-unit suspension type insulator.

be neglected. The former would tend to equalize the potential drop across the insulator units, while the latter—in addition to causing actual leakage losses through the air—might alter the capacity of the units subjected to the higher pressures. No error of appreciable magnitude is likely to be made by neglecting these items.

Assuming the voltage across the unit nearest to the grounded cross-arm to be $E_1 = 10{,}000$ volts, and the ratio $\frac{\text{mutual capacity}}{\text{capacity to ground}}$ to be $m = 10$, write

$$E_1 = 10{,}000 \text{ volts}$$

The capacity current[1] for condenser (1) is

$$I_1 = 2\pi f E_1(mC) = \omega \times 10{,}000 \times 10 \times C$$

Similarly $I_a = 2\pi f E_1 C = \omega \times 10{,}000 \times C$

whence

$$I_2 = (I_1 + I_a) = \omega E_1(m + 1)C = \omega \times 10{,}000 \times 11 \times C$$

and

$$E_2 = E_1\left(\frac{I_2}{I_1}\right) = E_1 \times \frac{11}{10} = 11{,}000 \text{ volts}$$

$$I_b = \omega(E_1 + E_2)C$$

whence

$$I_3 = (I_2 + I_b) = \omega[E_1(m + 2) + E_2]C = \omega \times 131{,}000 \times C$$

and

$$E_3 = E_1\left(\frac{I_3}{I_1}\right) = 10{,}000 \times \frac{131{,}000}{100{,}000} = 13{,}100 \text{ volts}$$

$$I_c = \omega(E_1 + E_2 + E_3)C$$

whence

$$I_4 = (I_3 + I_c) = \omega[E_1(m + 3) + 2E_2 + E_3]C = \omega \times 165{,}000 \times C$$

and

$$E_4 = E_1\left(\frac{I_4}{I_1}\right) = 10{,}000 \times \frac{165{,}100}{100{,}000} = 16{,}510 \text{ volts}$$

[1] Assuming a sine-wave of impressed e.m.f., it is easy to calculate the charging current of a condenser of known capacity. The fundamental law of the dielectric circuit is

$$\Psi = E_{max.} \times C \tag{78}$$

where Ψ is the maximum value of the dielectric flux, expressed in coulombs; $E_{max.}$ is the maximum value of the alternating voltage; and C is the capacity (or *permittance*) of the condenser, expressed in farads.

The charge, or quantity, of electricity, *i.e.*, the dielectric flux, will reach its maximum value Ψ at the instant when the charging current is changing its direction, that is to say, when the current is zero, and, since *quantity of electricity = current × time*, write Ψ = average value of charging current (in amperes) during one-quarter period × time (in seconds) of one-quarter period

$$= \left(\frac{2\sqrt{2}}{\pi}\right) I_c \times \frac{1}{4f}$$

where I_c stands for the virtual or r.m.s. value of the charging current, on the sine-wave assumption. Let E stand for the virtual value of the voltage across the condenser of capacity C farads, then $E_{max.} = \sqrt{2}E$, and formula (78) becomes

$$\frac{2\sqrt{2}I_c}{4\pi f} = \sqrt{2}E \times C$$

whence

$$I_c = 2\pi f E C$$

which is the well-known formula for calculating capacity current when sinusoidal wave shapes are assumed.

The total potential difference across the string of four insulators is, therefore, not $4E_1 = 40{,}000$, but $E = E_1 + E_2 + E_3 + E_4 = 50{,}610$ volts.

The stress across the insulator (No. 4) nearest to the high-tension conductor is $E_4 = 16{,}510$. Assume that it will flash over with twice this pressure, or $2E_4 = 33{,}020$ volts; then the pressure which will start a flash-over across the string of four insulators is $2E$, or 101,220 volts, which is less than four times $2E_4$, or 132,080 volts.

The fact that the ratio (m) of the mutual capacity to the capacity between the suspension link and ground is an important factor in determining the distribution of potential over the insulator string suggests the importance, not only of the shape and surface area of the metal fixtures on the individual insulators, but also the length of the connecting link, or spacing between insulators.

When the mutual capacity is small compared with the ground capacity, there soon comes a point beyond which it is useless to put more insulators in the string. Short strings, of well-designed and properly spaced units, will often be more effective and less costly than a longer series of insulators of which the single units may have excellent insulating properties, but may not have been specially designed for the particular requirements. It is sometimes possible to increase the arc-over voltage of a string of insulators by *reducing* the distance between consecutive units; a fact that it is difficult to understand unless the importance of the proper capacity distribution has been realized.

Example 21. *Effect of Varying Ratio between Series and Parallel Capacities.*—In the preceding numerical example, the ratio $m = \frac{\textit{mutual capacity}}{\textit{capacity to ground}}$ was taken as 10. It will be of interest to plot a set of curves showing how the potential distribution is affected by changes in this ratio. Assume for this ratio the values $m = 1$, $m = 10$, and $m = 100$, and plot curves for a string of five units, showing the percentage potential drop measured from the (grounded) point of suspension.

Following the method used in calculating the numerical values of the potential drop across successive units in Example 20, it is possible to express these potential differences in terms of the voltage E_1 across insulator No. 1 (at the grounded end of the string) and ratio m between the series and shunt capacities as shown diagrammatically in Fig. 83. The calculated values are as follows:

$$E_2 = E_1\left(1 + \frac{1}{m}\right)$$

$$E_3 = E_1\left(1 + \frac{3}{m} + \frac{1}{m^2}\right)$$

$$E_4 = E_1\left(1 + \frac{6}{m} + \frac{5}{m^2} + \frac{1}{m^3}\right)$$

$$E_5 = E_1\left(1 + \frac{10}{m} + \frac{15}{m^2} + \frac{7}{m^3} + \frac{1}{m^4}\right)$$

Substituting for m the assumed values $m = 1$, $m = 10$, $m = 100$, and also m = infinity, values as tabulated below are obtained for a total potential difference of $E = 100$ across the string of 5 units.

	E_1	E_2	E_3	E_4	E_5	E
$m = 1$........	1.82	3.63	9.10	23.65	61.8	100
$m = 10$.......	13.85	15.25	18.1	22.9	29.9	100
$m = 100$......	19.2	19.4	19.8	20.4	21.2	100
$m = \infty$.......	20	20	20	20	20	100

These calculated values have been used for plotting the curves of Fig. 84 which show very clearly how the capacity between the metal connecting links and ground is the cause of unequal distribution of the potential drop along the string of insulator units. The assumption of $m = 10$ yields results approximating to those obtained in practice with units of the type shown in Figs. 78 and 79, but the curves of Fig. 84 do not quite correctly represent practical conditions since the calculations take no account of capacity between the metal work of the insulators and the high-tension conductor. Owing to the smaller capacity of the Hewlett type of insulator and the relatively greater capacity between the connecting metal work and ground, the inequality of the pressures across individual units of a string as illustrated in Fig. 80, but *without the static shield below the bottom unit,* would be more marked than in the case of the cap-and-pin type of unit.

Since a uniform pressure drop is desirable, and this condition could be obtained with a series of similar units, provided the shunted capacities (*a*), (*b*), (*c*) of Fig. 83 could be eliminated, it is obvious that if the metal connecting-pieces were to be replaced by connectors of some insulating material having sufficient mechanical strength, a uniform distribution of voltage along the string would result. This construction is advocated by L. Perrin and E. Piernet[1] who have produced suspension insulators having a strength of about 6,000 lb. with non-metallic connecting pieces between the porcelain elements.

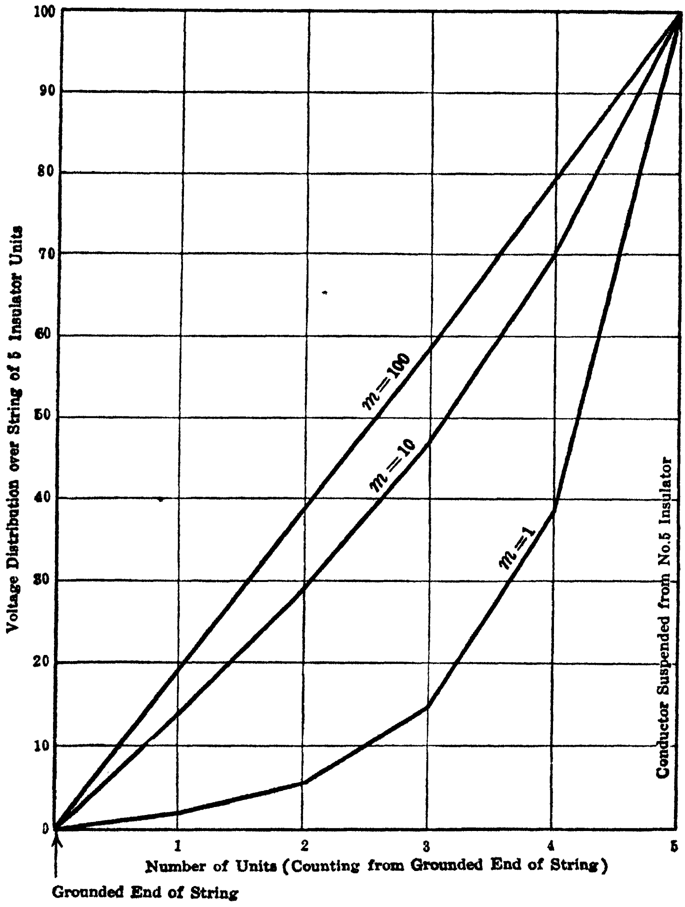


FIG. 84.—Curves illustrating Example 21.

Grading by Determining the Capacity of Each Unit According to Its Position in the String.—The diagram Fig. 85 represents a string of suspension insulator units in which the capacities (*a*), (*b*), (*c*), etc. between the connecting metal work and ground are

[1] *Rev. Gén. d' Élec.*, pp. 716–720, May 13, 1922.

all supposed to have the same value C. The capacity of unit No. 1 (at the grounded end of the string) is mC, and it is proposed to determine the capacities of the other units in the string which will cause the potential across each unit to be the same. The effect of the capacities between the connecting links and the high-tension line will be neglected.

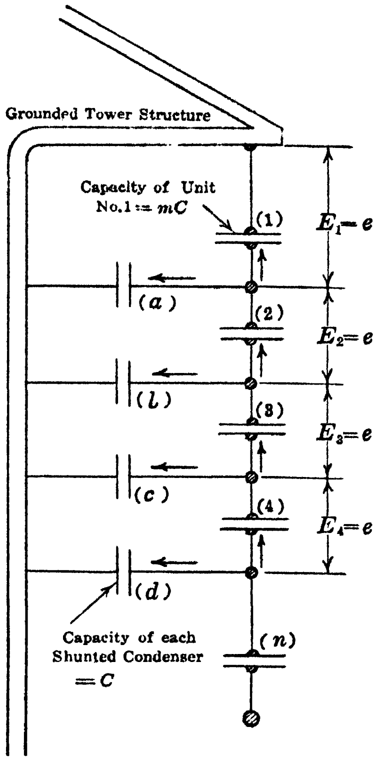


FIG. 85.—Diagram of suspension type insulator with equal potential drop across units.

Let e stand for the voltage across each unit; then the condition to be satisfied is that $E_1 = E_2 = \ . \ . \ . = E_n = e$.

The capacity current through the condenser formed by unit No. 1 is

$$I_1 = \omega emC$$

The currents through the condensers (a), (b), (c), etc. are

$$I_a = \omega eC$$
$$I_b = \omega(2e)C$$
$$I_c = \omega(3e)C$$

and so on, the increase of current being simply proportional to the distance of the unit from the grounded end of the string.

The currents through the successive units must be

$$I_2 = I_1 + I_a$$
$$I_3 = I_1 + I_a + I_b = I_1 + (1 + 2)I_a$$
$$I_4 = I_1 + I_a + I_b + I_c = I_1 + (1 + 2 + 3)I_a$$

and so on, which leads to the general expression

$$I_n = I_1 + [1 + 2 + 3 + \ \ldots \ + (n - 1)]I_a$$
$$= \omega emC + \omega e[1 + 2 + 3 + \ \ldots \ + (n - 1)]C$$

The required capacity of the n^{th} unit is obtained by dividing the current by $2\pi f$ times the voltage across the condenser, or by ωe, whence

Capacity of n^{th} unit $= mC + [1 + 2 + 3 + \ \ldots \ + (n - 1)]C$

As an example, if $m = 10$, the capacities of the several unit insulators to produce uniform potential drop over the whole length of the string would have to be as follows:

First unit	10C
Second unit	11C
Third unit	13C
Fourth unit	16C
Fifth unit	20C
Tenth unit	55C

Thus, for this particular value of the ratio m, and *assuming the capacity currents between high-tension line and connecting links to be negligible*, it is seen that the capacity of unit 5 should be twice that of unit 1, and if the string consists of 10 units, the capacity of the unit nearest to the high-tension conductor should be five and a half times that of unit 1.

The objection to this method of grading suspension insulators is that a number of different kinds or sizes of unit must be carried in stock, and mistakes are likely to be made when units have to be replaced. Without altering the thickness of the porcelain dielectric between the metal attachments, the capacity of the unit is easily changed by varying the area of the metal surfaces in contact with one or both sides of the porcelain disc: but fairly satisfactory results may be obtained by using two or three different types of suspension unit in the same string. F. Buske[1] has obtained good results by building up strings of 10 units with three cap-and-pin-type (large capacity) units near the line end of

[1] *Electrotech. Zeit.*, May 12, 1921.

the string, the remainder being Hewlett-type (small capacity) units.

Grading by Means of Static Shields.—In the preceding discussions, the effect of the high-voltage line conductor upon the potential distribution in a line of suspension units has been

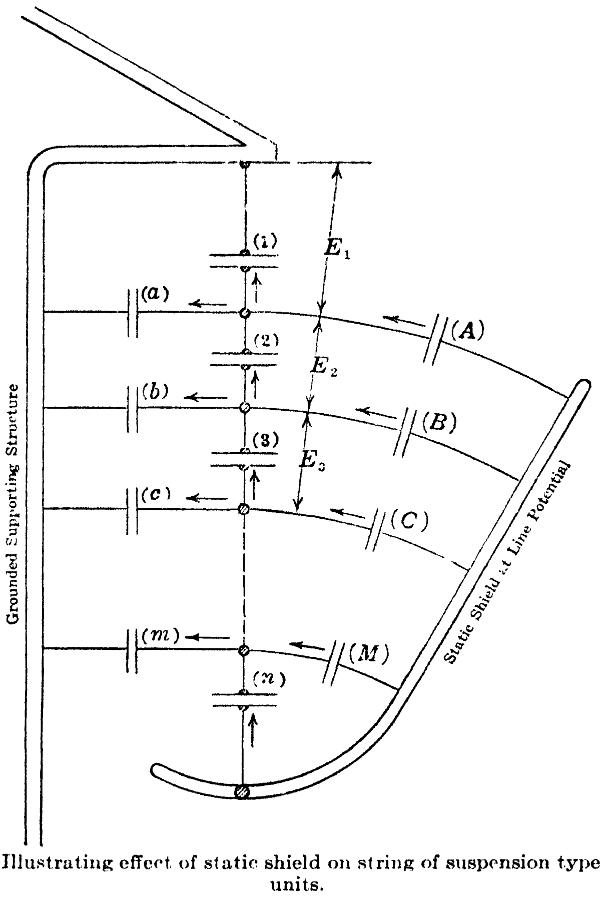


FIG. 86.—Illustrating effect of static shield on string of suspension type insulator units.

neglected. Ordinarily this effect is much smaller than that caused by the proximity of the grounded tower and the very large surface of the ground itself; but if attachments known as static rings or shields are provided, the effect of the high-tension conductor upon units other than the one to which it is connected may be increased to any desired extent. The diagram Fig. 86 shows that the static shield will have an appreciable effect upon the distribution of the capacity currents and, therefore, upon the potential gradient. Assuming that all the units in the string are of the same type and size and that not only the capacities (1), (2), (3), etc. are equal, but also that the capacities (*a*), (*b*), (*c*), etc. to ground are equal, determine the necessary capacities (*A*), (*B*), (*C*), etc. of Fig. 86 which will produce uniform potential drop over a string of *n* units.

The currents through the units are

$$I_2 = I_1 + I_a - I_A$$
$$I_3 = I_2 + I_b - I_B$$

and so on; but in order to satisfy the condition $E_1 = E_2 = \dots = E_n = e$, there must be the *same current* through all the units (which are assumed to have the same capacity), whence

$$I_A = I_a$$
$$I_B = I_b, \text{ etc.}$$

or if C stands for any one of the (equal) capacities (*a*), (*b*), (*c*), etc.,

$$\omega(n - 1)eC_A = \omega eC$$
$$\omega(n - 2)eC_B = \omega(2e)C$$

and so on, which leads to the general expression

$$C_M = (n - 1)C$$

where C_M stands for the capacity between the static shield and the metal work connecting the n^{th} insulator with the unit immediately above it. The proper shape of shield has to be developed experimentally; but by this method, or a combination of the several methods of grading which have been referred to, it is possible to control the potential distribution over a string of insulators of any practical length. A shield of elliptical shape, of which a side view is shown in Fig. 80, has been found to give very satisfactory results in practice. There should be no difficulty in transmitting at pressures appreciably in excess of 220,000 volts with properly graded strings of suspension-type insulators, even if there should be no radical change in the design of the insulator units.

Figure 87[1] shows the combination of different types and sizes of insulator unit, together with grading shield over the lower unit, as used on the 220,000-volt Pitt River line of the Pacific Gas and Electric Company.

[1] Reproduced by permission from the *Elec. World*, Jan. 27, 1923.

The proper sizes, types, and arrangement of insulator units and static shields in a suspension insulator for high-voltage transmission, as illustrated in Fig. 87, must necessarily involve a good deal of experimentation because the various capacities to be considered in the calculations are not known exactly, and they depend, moreover, upon the position and method of attachment of the insulator relative to the supporting structure. This does not mean that the proper understanding of the principles determining

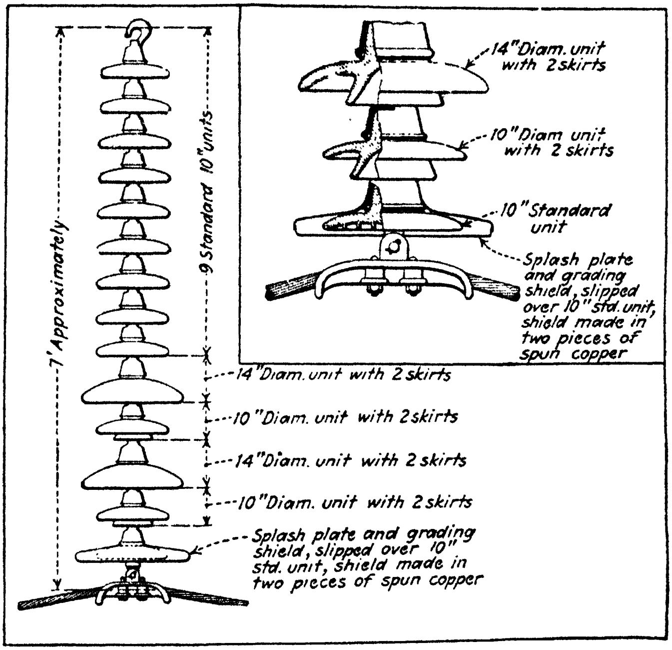


FIG. 87.—Suspension type insulator with units of different sizes.

the voltage distribution over a string of insulator units is unnecessary, because, although a suitable insulator might, with good luck, result from extended and costly trials with various combinations of parts, the problems of high-voltage line insulation cannot be solved efficiently except through the thorough understanding and intelligent application of the principles which have been referred to and briefly discussed in this article.

77. Manufacture of Porcelain Insulators. Mechanical Strength. Porosity.—Since an appreciable amount of space has been devoted in preceding articles to the discussion of the proper shapes of porcelain insulators and the relative positions of the several units entering into the design of multiple-part insulators, it might be supposed that the manufacturer is concerned mainly with the form into which the porcelain should be molded. Actually it is with improvements in the material itself that the manufacturer has been chiefly concerned in his effort to produce a porcelain of high insulating qualities which will be sufficiently strong mechanically to withstand the heavy loads to which it is submitted in practice, and capable of withstanding without deterioration the severe weather conditions, involving frequent changes of temperature and humidity, to which it is subject.

The solution of these problems involves a profound study of ceramics and considerable experience in the manufacture of porcelain. All that need be said here is that great improvements have been made in late years in the production of non-porous porcelain of increasing mechanical strength. Porcelain as used for line insulators has a strength of about 40,000 lb. per square inch in compression and 1,500 lb. per square inch in tension; but special mixtures have been produced with a strength as high as 65,000 lb. per square inch in compression and 12,500 lb. per square inch in tension.[1]

Since porcelain is a material which has no appreciable deformation before rupture under mechanical stress, the finished insulator may be tested without fear of injury with loads not very much smaller than the ultimate breaking load. The fact that porcelain insulators are now made which can stand sudden changes of temperature of 100°C. without developing cracks has already been mentioned, and future improvements in the development of porcelain bodies will be such as to eliminate if possible the causes of the deterioration observed in many insulators after erection. There is also a need for some reliable means of detecting faulty insulators before they leave the factory. Present methods of testing insulators before erection will not be discussed here.

The ultimate strength of modern suspension-type insulators is usually between 8,000 and 9,000 lb., a suitable working load for such units being from 2,000 to 3,000 lb. Extra-strong units for dead-ending purposes or for taking the pull at corners are made with an ultimate strength of 14,000 to 16,000 lb. suitable for working loads of 4,000 to 4,500 lb. There is even a design of high-strength 10-in. unit which is suitable for a working load up to 6,000 lb.

[1] Peaslee, W. D. A., in *Trans.* A.I.E.E., vol. 39, part 2, p. 1179.

78. Factors of Safety. Elimination of Faulty Insulators.—When selecting insulators and deciding upon the spacing and arrangement of conductors suitable for a given voltage, the factor of safety to cover abnormal pressure rises is a matter of great importance, since it is obviously bad engineering to provide insulation in excess of what experience has shown to be a reasonable safeguard against interruption of service. Generally speaking, the insulators should, when dry withstand a pressure test of two and a half to three times the working pressure to ground, applied for 5 minutes, and a wet test of not less than twice the working pressure. This would sometimes be considered too small a margin of safety; but the ratio between the test pressure and working pressure will depend upon whether the line voltage is high or low. Overvoltages due to lightning are about the same whatever may be the operating voltage of the line, and the factors of safety used in the selection of insulators for low-voltage lines should, therefore, be much higher than for high-voltage lines. Obviously, there are economic limits to the amount of insulation which would be justified on transmission lines for comparatively low voltages, and the result is that interruptions and troubles of all kinds caused by lightning are much more frequent on lines operating at pressures between 22,000 and 66,000 volts than on the recently erected 220,000-volt lines, and even on 110,000-volt lines. The following safety factors, representing ratio between wet-test pressure and working pressure, are generally in accordance with usual practice; but the engineer should use his judgment in a matter of this sort. It is clear that, on the coast, where gales and salt sea mists are prevalent, the factor of safety should be rather higher than in a district where the climatic conditions are more favorable. The effects of high altitude—to be referred to later—must also be taken into account.

Working pressure, voltage between line wires	Factor of safety, wet test
22,000	2¾
44,000	2½
66,000	2¼
88,000	2
110,000 and above	1¾ to 2

As the wet or "rain" test will give different results, depending on the method of conducting the test, there should be a clear understanding between the purchaser and manufacturer on this point. A very common specification is that the spray shall be directed at an angle of 45 deg. under a pressure of 40 lb. per square inch at the nozzles, the flow being regulated to give a precipitation of 1 in. in 5 minutes. The method of attaching test wire and ground connection to the insulator should also be clearly defined. The test pressure is usually measured by means of the sphere gap.

Effect of Altitude.—The insulators on a transmission line erected at high altitudes will flash over with a lower voltage than if the line were erected at sea level. The reason for this is the reduced air pressure at the higher elevation. The flash-over voltage will not be exactly proportional to the barometric pressure because the electrostatic field is not uniform, but depends upon the type and design of the insulator. Mr. Peek gives some results of experimental work on various standard designs of insulator in his book "Dielectric Phenomena" previously referred to; but since the departures from the theoretical relation for uniform dielectric fields is very small, the correction for altitude can safely be made by assuming the flash-over voltage to be directly proportional to the barometric pressure.

As an example: Suppose the flash-over voltage of a pin-type insulator is found to be 100 kv. on test at sea level; then, if used on a transmission line erected at an elevation of 8,000 ft., it would be likely to flash over with a pressure of only $100 \times \frac{22}{29.9} = 73.5$ kv. (The proportional figures are the approximate barometric pressures in inches of mercury at an altitude of 8,000 ft. and at sea level.)

Detecting Faulty Insulators while Line is in Operation.—A matter which is now receiving attention, but is not settled or standardized, is the best means of detecting faults in insulators while in use. It is only of recent years that accurate data on the "life" of high-tension insulators are becoming available; and the continued action of alternate heat and cold, dryness and dampness on the porcelain—or rather on the complete assembly of porcelain, metal, and cement—is found to necessitate a large percentage of replacements after a line has been in operation many years.

The causes of rapid deterioration—especially after several years of service—are being investigated and eliminated as far as possible in the later designs; but in the meanwhile, lines that have been in use for a considerable time are giving trouble in the matter of insulation, involving increasing vigilance and activity on the part of the operating staff. There is room for improvement in the methods now available for detecting incipient faults in line insulators without interrupting the supply or disconnecting insulators from the live wires. The so-called "buzz-stick" method as used for locating defective insulators is described by T. F. Johnson, Jr. in the *Electrical World*, Sept. 13, 1919.[1] This method and also the "spark-stick" method of detecting defective insulators while the transmission line is in use, are referred to by Prof. H. J. Ryan in the *Electrical World*, Jan. 31, 1920.[2] Results of megger tests made on suspension insulators on the Great Western Power Company's lines since 1908 are given by C. F. Benham in a paper published in the *Transactions* A.I.E.E.[3] The investigation seems to indicate that expansion stresses due to temperature changes have been the cause of more failures than porosity of the porcelain. Standard insulator designs of the present day appear to withstand the severe conditions of service very much better than the insulators of 10 or 15 years ago.

79. Spacing of Overhead Conductors.—It is difficult to lay down rules for the proper spacing of overhead conductors. The question has been settled in the past by the individual engineer who has usually striven to be "on the safe side" in the matter of possible discharges between wires under abnormal conditions such as strong and variable winds. The result is that great differences are to be found in the wire spacings in different countries and on different transmission systems in the same country.

The spacing of the conductors should be determined by considerations partly electrical and partly mechanical. With the longer spans, the spacing should be greater than with short spans, apart from voltage considerations. The material and diameter of the conductors should also be taken into account when deciding upon the spacing, because a small wire—especially if of aluminum—having a small weight in relation to the area presented to a cross-wind, will swing out of the vertical plane farther than a conductor of large cross-section. Usually wires will swing

[1] Vol. 74, p. 568.
[2] Vol. 75, p. 255.
[3] Vol. 42, p. 981.

synchronously in a wind; but with long spans and small wires, there is always the possibility of the wires swinging non-synchronously, and the size of wire, and also the maximum sag at center of span, are factors which should be taken into account in determining the distance apart at which they shall be strung. A horizontal separation equal to something between one and one and a half times the sag at the temperature corresponding to the season of highest wind velocities should be sufficient to prevent wires swinging within sparking distance of each other, the closer spacing being used with copper conductors of large diameter.

Formulas have been published purporting to give spacings based on data obtained from existing transmission lines, but these formulas usually fail to take account of span length. It is obvious that the spacing between conductors at a given line pressure should be greater with long span lengths than when the distance between supports is short. This does not mean that the spacing between conductors should be varied with every change in span length, but a formula representing the usual spacings under given conditions should take account of the average length of span.

The following formula gives the approximate separation between overhead conductors in inches: the symbol l stands for the average span length in feet, and kv. is the line pressure between conductors in kilovolts.

$$\text{Distance between conductors (inches)} = 20 + \frac{\text{kv.}}{1.4} + \left(\frac{l}{120}\right)^2 \quad (87)$$

This formula may be used as a guide in arriving at a suitable value for the *horizontal* spacing for any line voltage and for spans between 200 and 1,100 ft. The *vertical* spacing may be from three-quarters to two-thirds of the horizontal spacing, but it is usually undesirable to suspend wires in the same vertical plane, especially in locations where sleet and ice deposits are likely to occur.

Clearance between Conductor and Pole or Tower.—If the distance in inches, between the high-voltage conductor and the (grounded) metal work of the supporting structure is not less than 5 + 0.35 kv. this will be generally in accordance with what experience has shown to be a suitable clearance. The symbol kv. stands for the transmission pressure, in kilovolts between lines, and not between line and ground.

In the case of suspension-type insulators, it is well to arrange for the clearance, even under conditions of greatest deflection caused by high winds, to be not less than the sparking distance over the string of insulator units.

Clearance above Ground.—It is customary to allow a distance between ground and the nearest point on the high-tension overhead conductors of not less than 20 ft. A minimum clearance of 22 to 24 ft. is often specified. When the voltage is very high, as on the recent 220,000-volt lines, a minimum clearance of 30 ft. between wire and ground is usual. Even on the lower voltages, when the height above ground is about 22 ft. where the wires are carried over fields or open country, a minimum clearance of about 30 ft. is often specified for important railway or road crossings or in towns and villages.

CHAPTER IX

ELECTRICAL PRINCIPLES AND CALCULATIONS

80. Resistance of Transmission Lines. Variation with Temperature.—The effect of line resistance on transmission losses was referred to in Arts. 1 and 4 of Chap. I, and a numerical example was worked out, showing how the voltage drop and line efficiency of a d.c. transmission depend upon the conductor resistance. In Art. 8, the three-phase transmission line was considered, and the manner in which conductor resistance determines the percentage power loss on any polyphase system was discussed in Art. 9. The physical characteristics of conductor materials were dealt with in Arts. 37 and 38 of Chap. IV, and the wire table on page 81 may be used for calculating the resistance of transmission lines. This table gives resistances which are approximately correct at a temperature of 68°F. (20°C)., and the proper correction must be applied to determine the resistance at any other temperature. The change of resistance of metallic conductors with temperature is approximately according to a straight-line law within certain limits of temperature variation. Thus, if $100a$ stands for the percentage increase of resistance per degree Centigrade rise above a given temperature at which the resistance is known,

$$R_t = R_o(1 + at) \tag{88}$$

where R_o = resistance at the initial temperature.
t = temperature rise above initial temperature.
R_t = resistance at the higher temperature.

When t is measured in degrees Centigrade, and R_o is the resistance at 0°C., the value of a for all pure metals is about 0.00425. This coefficient is known as the "temperature coefficient of resistance at 0°C." The relation expressed by formula (88) is represented graphically in Figs. 88 and 89 where the straight lines giving the relation between resistance and temperature for copper and aluminum are seen to meet the zero resistance ordinate at the point A. The distance OA in Fig. 88 is obtained by putting $R_t = 0$ in formula (88), whence

$$0 = R_o(1 + 0.00425t)$$

and
$$t(=OA) = -\frac{1}{0.00425} = -235°C.$$

This temperature is generally known as the "inferred absolute zero." Suppose the resistance of an aluminum conductor is known to be R_1 ohms at a temperature of t_1°C.; it will be seen from an inspection of the similar triangles in Fig. 88 that the resistance R_2 at some other temperature t_2 will be

$$R_2 = R_1\left(\frac{235 + t_2}{235 + t_1}\right) \tag{89}$$

where t_1 and t_2 are the temperatures in degrees Centigrade.

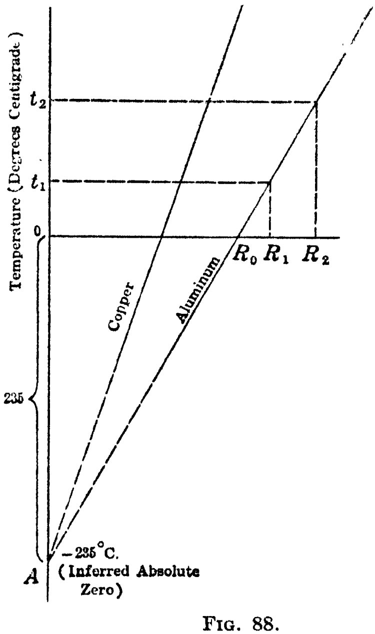


FIG. 88.

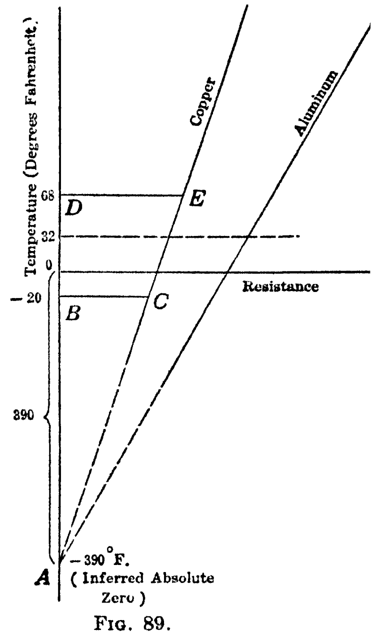


FIG. 89.

FIGS. 88 and 89.—Diagrams illustrating relation between resistance of pure metals and temperature.

When temperatures are read on the Fahrenheit scale, the multiplying ratio must be obtained from Fig. 89 in which the "inferred" absolute zero is

$$-\left[\left(235 \times \frac{9}{5}\right) - 32\right] = -390°F. \text{ (approximately)}$$

and the formula for temperature correction of resistance is now

$$R_2 = R_1\left(\frac{390 + t_2}{390 + t_1}\right) \tag{90}$$

where t_1 and t_2 are temperatures in degrees Fahrenheit.

Example 22. *Change of Resistance with Temperature.*—A stranded copper conductor of 300,000 circular mils cross-section carries a current of 140 amp. Required the I^2R losses in a 10-mile length of this conductor at a temperature of −20°F.

The length in feet is 10 × 5,280 = 52,800.

The resistance in ohms per 1,000 ft. at 68°F. (taken from the table on p. 81) is 0.192, whence the resistance of the conductor at 68°F. is 0.192 × 52.8 = 10.15 ohms. By inspection of Fig. 89, the ratio of the resistances is seen to be

$$\frac{R_{-20}}{R_{68}} = \frac{R_{-20}}{10.15} = \frac{BC}{DE} = \frac{AB}{AD} = \frac{(390 - 20)}{(390 + 68)}$$

whence

$$R_{-20} = 10.15\left(\frac{390 - 20}{390 + 68}\right) = 8.2 \text{ ohms}$$

which is, of course, the result obtained by using formula (90).

The total I^2R losses at a temperature of −20°F. are, therefore, $(140)^2 \times 8.2 = 160{,}700$ watts.

The inferred absolute zero, as given on the diagrams Figs. 88 and 89, is approximately correct for copper-clad steel conductors. It may also be used in calculating the resistances of commercial iron and mild steel wires or cables for ranges of temperature between −20 and +40°C., but the mean temperature coefficient for iron when taken over a greater range of temperature is not the same as for copper and aluminum: moreover, it will depend largely upon the quality of the iron wire. For very pure iron, the temperature coefficient of resistance referred to 0°C. (being a in formula (88)) is about 0.0063 over a range from 0 to 100°C., while hard steel has a temperature coefficient at 0°C. of about a = 0.0016 for a range between 0 and 35°C.

81. Skin Effect.—Imagine a straight length of cable of fairly large cross-section, through which a steady continuous current is flowing, the return circuit being a considerable distance away. The magnetic induction due to this current will not be only in the non-conducting medium surrounding the wire, but a certain amount—due to the current in the central portion of the cable—will be in the substance of the conductor itself. In other words, the magnetic flux surrounding one of the central strands of the cable will be greater than that which surrounds a strand of equal length situated near the surface. It follows that if the circuit be now broken, the current will die away more quickly near the surface of the conductor than at the center; and, for the same reason, on closing the circuit again, the current will spread from the surface inward.

If, now, the conductor be supposed to convey an alternating current, it is evident that, with a sufficiently high frequency (or even with a low frequency if the conductor be of large cross-section), the current will not have time to penetrate to the interior, but will reside chiefly near the surface. This crowding of the current toward the outside portions of the conductor has the effect of apparently increasing the resistance; and it follows that if I is the total current in a cable of ohmic resistance R, the power lost in watts would no longer be I^2R, as in the case of a steady current, but I^2R', where R'—which stands for the apparent resistance of the conductor—is k times greater than R, its true resistance. The multiplier k may be read off the diagram Fig. 90, or if preferred, it can be calculated by means of the formula

$$k = \frac{1 + \sqrt{1 + F^2}}{2} \tag{91}$$

where F is a factor proportional to the vertical distances on the diagram, that is to say, to the quantity *area of cross-section* $\times$ *frequency.* The value of F for copper is

$$F = 0.0105d^2f$$

and for aluminum,

$$F = 0.0063d^2f$$

where d is the diameter of the conductor in inches, and f is the frequency in periods per second. This formula and the curves of Fig. 90 are based on the assumption that the return current is at an infinite distance; but this assumption introduces no appreciable error when dealing with overhead transmission lines.

It will be observed that so long as the product d^2f remains unaltered, the multiplier k is constant provided the material remains the same. Thus if, when doubling the frequency, the sectional area of the (circular) conductor is halved, the ratio $\frac{\text{resistance to alternating currents}}{\text{resistance to continuous currents}}$ remains unaltered.

In regard to the *material* of the conductor, the value of F in the formula is directly proportional to the specific conductivity of the metal so long as the frequency remains constant.

Thus if F (or the value of the ordinate in the diagram, Fig. 90) is known for a conductor of given diameter, made of copper, its value for any other "non-magnetic" material is given by the ratio

$$\frac{\text{conductivity of metal of conductor}}{\text{conductivity of copper}}$$

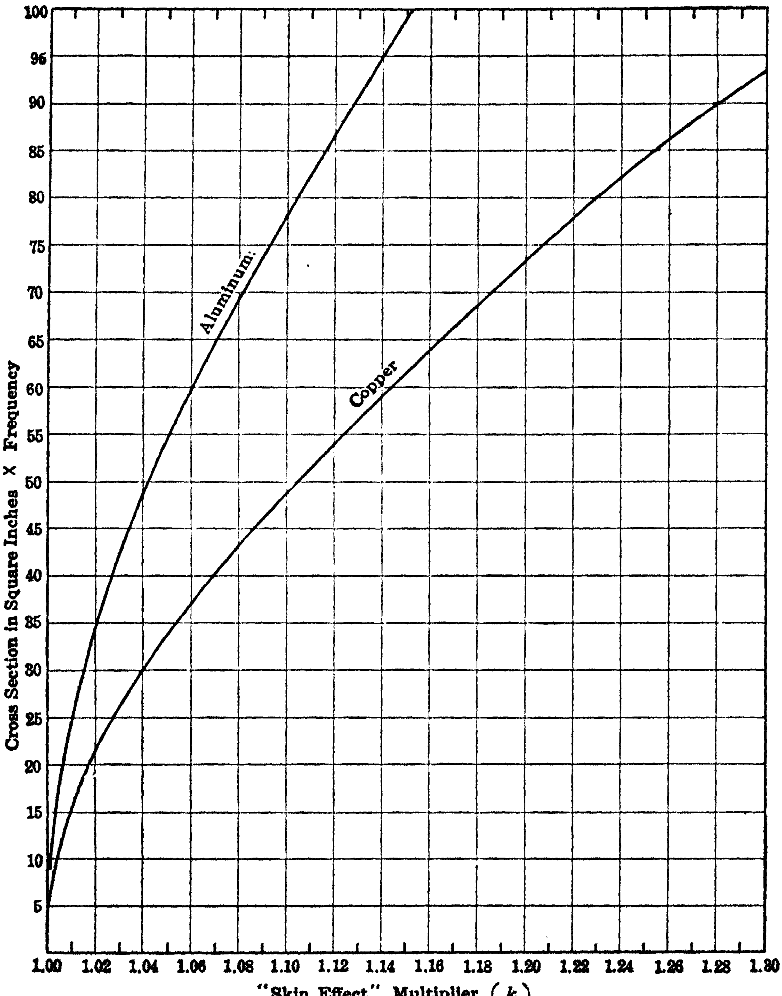


Fig. 90.—Diagram giving "skin effect" coefficient for circular wires.

If the conductor is of iron (or other "magnetic" material), the value of k may be much greater than this ratio would indicate.

It is a not uncommon belief that when aluminum conductors are used in place of copper, the larger diameter necessary to give the same conductivity will lead to a greater loss through "skin effect"; but the above multiplying ratio makes it clear that the percentage increase of losses with alternating currents of the same frequency will be independent of the material of the conductor (iron excepted), because the greater sectional area necessary to maintain the same ohmic resistance of the lines when a wire of lower conductivity is used is evidently exactly balanced by the higher specific resistance of the metal.

The development of exact formulas for the "skin-effect" multiplier will be found in H. B. Dwight's "Transmission Line Formulas."

82. Inductance of Transmission Lines.—For the purpose of calculating the flux of induction outside a straight cylindrical

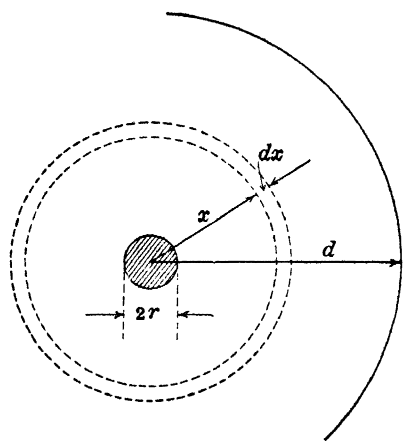


FIG. 91.—Flux surrounding single conductor of circular cross-section.

conductor, it is permissible to assume that the current is concentrated on the center line of the wire. The lines of magnetic induction surrounding a long straight wire carrying an electric current of which the return path is at a considerable distance will be in the form of circles concentric with the conductor. The number of lines, or flux in maxwells, contained between any two imaginary concentric cylinders, of average radius x cm., and axial length l cm. (see Fig. 91) will be the product of the magnetomotive force by the permeance, or

$$d\Phi = \frac{4\pi}{10}I \times \frac{l \times dx \times \mu}{2\pi x}$$
$$= \frac{2Il\mu}{10} \times \frac{dx}{x}$$

where I is the current in the wire, μ is the permeability, and dx is the separation between the cylinders, in centimeters.

Assuming dx to become smaller and smaller without limit, and putting $\mu = 1$ (for the condition of flux lines in air), the expression for the total flux *outside* the conductor, up to a limiting distance of d cm., is

$$\Phi = \frac{2Il}{10}\int_r^d \frac{dx}{x}$$
$$= \frac{2Il}{10}\log_\epsilon\left(\frac{d}{r}\right) \qquad (92)$$

where r is the radius of the conductor, in centimeters.

Effect of Taking into Account the Return Conductor.—The effective flux surrounding any single conductor of a transmission

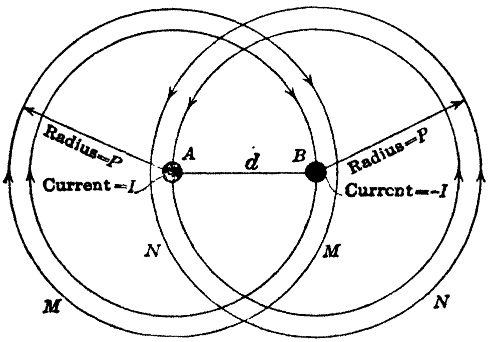


FIG. 92.—Magnetic lines of force around two parallel conductors.

system will depend upon the distance of the parallel return conductor or conductors.

Consider, first, the loop formed by two parallel conductors of circular cross-section, one carrying the outgoing current I and the other carrying the return current $-I$ (see Fig. 92). The flux due to the current I in the conductor A may be considered as extending indefinitely throughout space, with ever-weakening intensity as the distance from the conductor increases, and the same argument applies to the flux surrounding the return conductor B, the only difference being that if the direction of the flux round A be considered positive, that which surrounds B will be in a negative direction.[1] It follows that the whole of the magnetic flux due to the current I in A, which is situated at a distance greater than the distance d between centers of the outgoing and return conductors, is exactly neutralized by the flux due to the current $-I$ in B. Thus, in Fig. 92, it will be seen that the flux of induction surrounding A up to a distance d is not neutralized by the current $-I$ in the conductor B; but any magnetic line, such as M, situated at a greater distance P from the center of the conductor A, is exactly neutralized by the magnetic line N, due to the return current in conductor B, since it also surrounds the conductor A, but in a direction opposite to that of the line M. It follows that the total *effective flux* surrounding A—that is, the resultant flux which will give rise to an induced e.m.f. in the conductor when carrying an alternating current—is merely that portion of the total self-produced flux included between the surface of the conductor and the surface of an imaginary cylinder concentric with the conductor, and of radius d equal to the distance between the centers of the outgoing and return conductors. The formula (92) may, therefore, be used for calculating the flux which is effective in producing an e.m.f. of self-induction in a single straight conductor when the whole of the return current is situated at a distance d from the center of the conductor. The flux as calculated by formula (92) is that which surrounds the wire, and when the current to which it owes its existence alternates in direction, an e.m.f. of self-induction will be induced in the conductor.

Since 100,000,000 lines cut per second generate 1 volt and the total effective flux surrounding the conductor is twice created and twice destroyed in the time of one complete period, the *mean* value of the e.m.f. of self-induction will be $\frac{4\Phi f}{10^8}$ volts. The

[1] The conception of two distinct systems of flux lines occupying the same space, is not always justified since it is the *resultant* flux distribution due to the several components of the magnetizing force that should properly be considered; but where the permeability (μ) is constant, as in air, B is proportional to H, and there can be no valid reason why the resultant flux of induction may not be pictured as the sum of two or more component systems of flux lines.

virtual or r.m.s. value, on the sine-wave assumption, is $\frac{\pi}{2\sqrt{2}}$ or 1.11 times this quantity, whence

$$\text{Induced volts} = \frac{4.44\Phi f}{10^8} \tag{93}$$

If I stands for the virtual value of the alternating current, the maximum value of Φ, by formula (92), will be

$$\Phi = \frac{2(\sqrt{2}I)l}{10} \log_e \left(\frac{d}{r}\right)$$

Substituting in formula (93) after replacing l by the number of centimeters in a mile, and converting the Napierian logarithms into common logarithms,

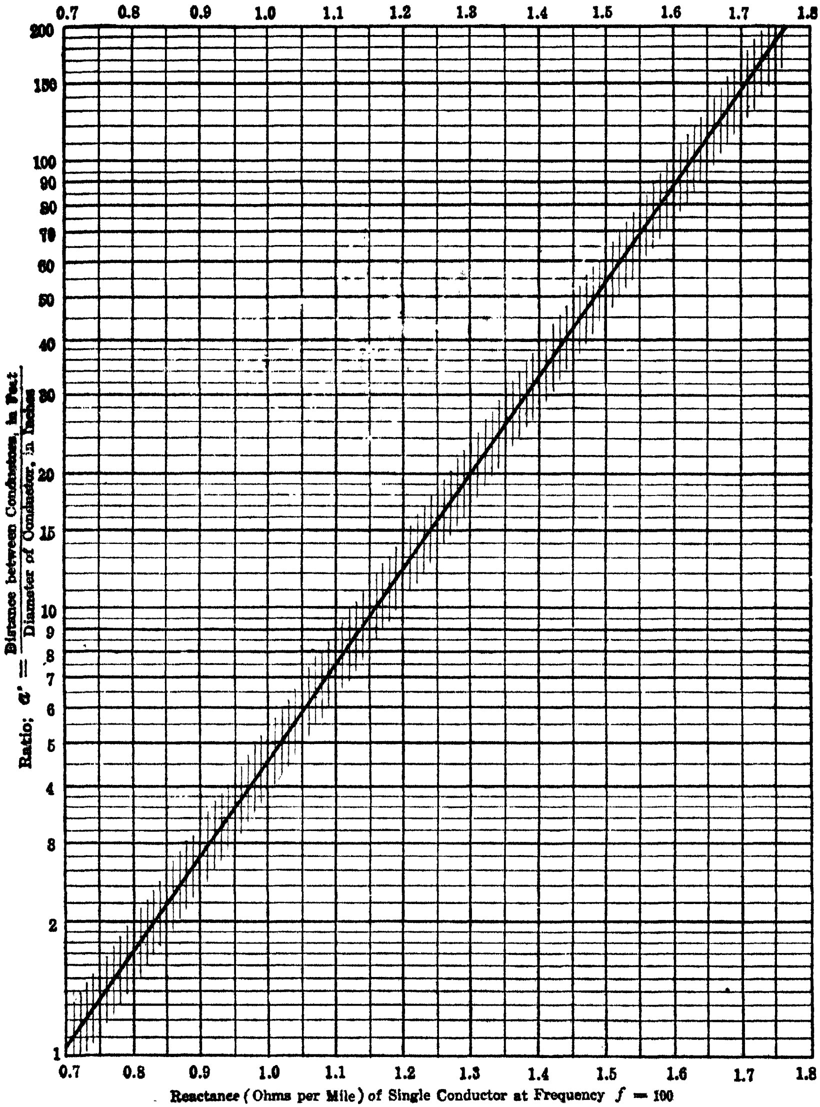
$$\text{Volts induced per mile of conductor} = 0.00466 fI \log \left(\frac{d}{r}\right) \tag{94}$$

Effect of Flux Lines in the Material of the Conductor.—The formula (94) is approximately correct for conductors of overhead transmission lines when these are of "non-magnetic" material; but it should be slightly modified to take into account the effect of all the flux lines *within* the material of the conductor. This additional drop of pressure is not easily calculated because different amounts of flux link with different portions of the conductor. It is obvious that a portion of the conductor near the surface is surrounded by fewer flux lines than a portion near the center of the cross-section. The result is that the e.m.f. induced per unit length of conductor is not the same throughout the cross-section. This suggests the possibility of eddy currents in the wire; but what actually takes place is a distribution of the current density over the cross-section such that the total impedance drop—or the IR drop added (vectorially) to the IX drop—will have the same value at all parts of the conductor cross-section. The correct calculation of the internal reactance drop for cylindrical conductors is given in Prof. H. B. Dwight's book "Transmission Line Formulas."

The result is that the inductive pressure drop is actually somewhat greater than as indicated by formula (94), which neglects the internal flux. The corrected formula is

$$\left.\begin{array}{l}\text{Volts induced per mile}\\ \text{of single conductor}\end{array}\right\} = fI\left[0.00466 \log \frac{d}{r} + 0.000506\right] \tag{95}$$

$$= 0.00466 fI \left[\log \frac{d}{r} + \frac{0.000506}{0.00466}\right]$$



Example Illustrating Use of Chart.—Required, the reactance per conductor of a transmission line 26 miles long consisting of No. 1 wires spaced 5 ft. apart, the frequency being 50. The diameter of No. 1 wire (from Table on page 81) is 0.289 in., whence $\frac{5}{0.289} = 17.3$ and we read 1.27 off chart. The required answer is therefore $1.27 \times 0.5 \times 26 = 16.5$ ohms.

The equivalent spacing of three wires which do not occupy the vertices of an equilateral triangle, but which are regularly transposed throughout the length of the transmission, is $\sqrt[3]{abc}$, where a, b and c are the respective distances between centers of wires, in feet.

The antilogarithm of the constant in the brackets is 1.285, and it is more convenient to write the formula

$$\left.\begin{array}{l}\text{Reactive voltage drop (IX)}\\ \text{per mile of single conductor}\end{array}\right\} = 0.00466fI \log\left(1.285\frac{d}{r}\right) \quad (96)$$

which is the same as formula (7) already given in Chap. II. The reactance of stranded cables is slightly less than that of solid conductors of the same cross-section, owing to the fact that the overall diameter of the cable is greater than that of the solid wire.

Tables giving inductive reactance in ohms per mile for different spacings and sizes of wires are given in the "Standard Handbook for Electrical Engineers;" these figures, when multiplied by the value of the current flowing in the conductor, give the induced volts as calculated by formula (96).

For quick reference and rapid calculations, the chart on page 243 will be found convenient. It gives the reactance of a single conductor in ohms per mile for a frequency of 100. In order to obtain the reactive drop in volts in a single conductor of length L miles, carrying a current of I amp. at a frequency of f cycles per second, it is necessary merely to mutliply the value read off the chart by $\frac{fLI}{100}$.

83. Iron as a Material for Transmission Line Conductors.—The European war, by limiting the supply of copper and aluminum available in Germany and by causing an abnormal increase in the price of these metals all over the world, led electrical engineers to consider the possibility of using other metals as conductors of electricity. Zinc has been used in Germany for insulated wires and cables; but it is mechanically weak and generally unsuitable for overhead transmission lines.

When considering the economic advantages of using iron or steel conductors, it is necessary to take into account: (*a*) the cost of the material at the place where it is to be used; (*b*) the "life" of galvanized-iron wires or cables as compared with that of copper and aluminum; (*c*) the energy losses in transmission; (*d*) the voltage regulation, and the increased cost, if any, of maintaining the pressure within specified limits at the receiving end of the line.

Under item (*b*) the tendency to rust, especially when the conductor is in the form of stranded cable, and the possibility of local hardening resulting in crystallization at or near the points of attachment to insulators must not be overlooked.

Under item (*c*) the greatly increased "skin effect" with alternating currents must be taken into account as well as the higher specific resistance which requires a larger cross-section of iron than of copper wire even when the transmission is by continuous currents.

Under item (*d*) the *internal inductance* of the wire, which is almost negligible with copper or aluminum, becomes a matter of considerable importance owing to the greatly increased magnetic flux in the material of the conductor when iron or steel is used.

Although cables of extra-high-strength steel wire are occasionally used for long spans—such as river crossings—on important overhead lines transmitting large amounts of energy, this material would not be satisfactory as a substitute for copper or aluminum except on comparatively short sections of the entire line.

Very rarely may the use of iron wire for power transmission purposes be justified on economic grounds. Galvanized-iron or steel wire has a limited usefulness in connection with short temporary lines when the current to be carried is small (not exceeding 10 or 12 amp.). The fact that iron or steel wires may be stretched tighter than copper or aluminum permits of longer spans being used, and this tends still further to reduce the initial cost of the line.

On account of the wide variations in the electric and magnetic qualities of the different grades of iron and steel wire, it is practically impossible to predetermine losses and pressure regulation with a high degree of accuracy.

Data and curves referring to iron wire were published in the second edition of this book, but owing to the very small demand for such data, they have been omitted from this edition, and the reader who may be interested in iron wire for overhead lines is referred to other sources of information.[1]

[1] One of the most convenient and comprehensive publications dealing with iron as a conductor material for overhead lines is the booklet issued by the Indiana Steel and Wire Company of Muncie, Ind. Other references are:

The Handbook on Overhead Line Construction, published by the National Electric Light Association.

The Handbooks for Electrical Engineers.

Paper 252 by J. M. MILLER, issued by the Bureau of Standards at Washington, D. C.

Articles by Prof. W. J. RYAN, *Elec. Rev.* (Chicago), vol. 71, p. 496, Sept. 22, 1917, and also vol. 72, p. 713.

Articles in the *Elec. World* by L. W. W. MORROW, July 14, 1917; by C. E. OAKES and P. A. SAHM, July 27, 1918; by R. W. GODDARD, Apr. 27, 1920; by H. S. RUSH, Sept. 10, 1921.

Article by E. KURTZ, *Elec. Rev.* (Chicago), Aug. 6, 1921.

84. Inherent Regulation of Short Transmission Lines.—The fundamental vector diagram Fig. 9 which was described in Art. 15 of Chap. II is reproduced here for convenience. It gives the correct relations between the components of voltage when the effects due to electrostatic capacity are so small as to be negligible.

The formula (11) which was developed in Art. 15 gives the relation between the pressure V_n at the generating end of the line and the pressure E_n at the receiving end in terms of the IR drop and the angles θ and ϕ. Numerical examples were worked out in Art. 16 based on the vector relations as illustrated by Fig. 9.

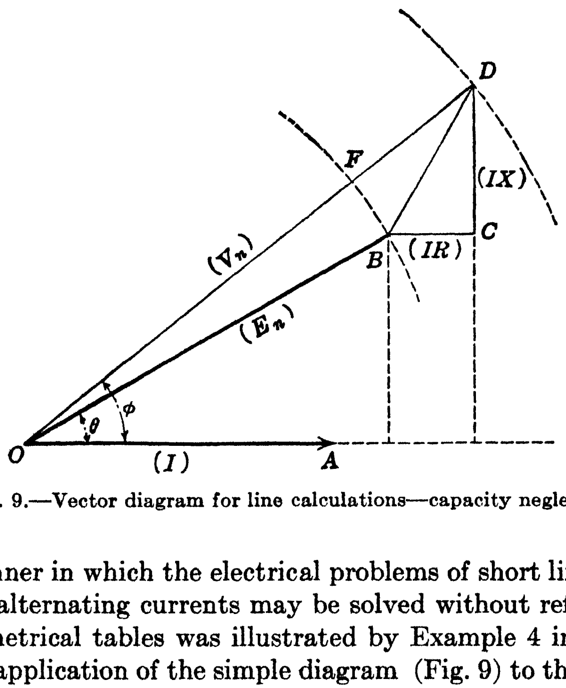


FIG. 9.—Vector diagram for line calculations—capacity neglected.

The manner in which the electrical problems of short lines transmitting alternating currents may be solved without reference to trigonometrical tables was illustrated by Example 4 in Art. 17, and the application of the simple diagram (Fig. 9) to three-phase calculations was explained in Art. 18 and illustrated by Example 5 in which the solution of the problem was obtained by drawing the vector diagram to scale.

The percentage drop of pressure on a transmission line may be defined as the difference between the sending-end and the receiving-end pressures expressed as a percentage of the receiving-end pressure. Thus, in terms of the vectors of Fig. 9,

$$\text{Per cent pressure drop} = \frac{V_n - E_n}{E_n} \times 100$$

and this is the same as the inherent *regulation* when the capacity current between the parallel conductors of the transmission line is so small as to be negligible. The reason why this is not true when capacity is an important factor, as in long high-voltage lines, will be explained later.

Assuming a three-phase transmission with the three conductors arranged to occupy the vertices of an equilateral triangle, any one conductor may be thought of as carrying the outgoing current which returns by the two other conductors at a distance d which is the separation between any two of the wires. Thus, not only the IR drop but the IX drop also will be the same in all three conductors, and the voltage between wires at the sending end will be $\sqrt{3}V_n$ while the voltage between wires at the receiving end will be $\sqrt{3}E_n$ as illustrated by Fig. 12 in Art. 18. This is true whether the system is star connected with a neutral point which may or may not be grounded at one end of the line, or delta connected, in which case an imaginary neutral point may be used whenever this leads to a simplification of the calculations.

85. Inductance of Overhead Transmission Lines with Unequal Spacing of Wires.—The current in any one conductor of an electric transmission line, whether the system be single phase or polyphase, may be considered as the sum of a number of component currents which are exactly equal, but opposite in direction, to the currents in the remaining conductors. The induced e.m.f. in any one conductor of an a.c. transmission may, therefore, be considered as due to a resultant magnetic flux which is the summation of the component fluxes in the two or more single-phase circuits formed by one particular conductor and each of the other (parallel) conductors.[1]

The magnetic flux surrounding a straight wire of circular cross-section, up to a distance d cm. from the center of the wire, as given by formula (92), is

$$\Phi = 0.2lI \log_\epsilon \frac{d}{r} \text{ maxwells}$$

where l = length of conductor in centimeters.
I = current, in amperes.
r = radius of cross-section of wire, in centimeters.

[1] This method of calculating the inductance of transmission lines with any arrangement of parallel conductors was the basis of articles contributed by the writer to the *Elec. World* and published in the issues of May 23, 1908, and Sept. 15, 1910.

This formula assumes the permeability of the medium surrounding the wire to be unity ($\mu = 1$), and it applies, therefore, to a single wire of any system of overhead electric power transmission. The formula does not take account of the flux of induction inside the conductor. If it is desired to calculate the self-induced e.m.f. when the current is alternating with a frequency of f cycles per second use formula (95) or (96) which takes account of the flux lines inside the material of the conductor when this is "non-magnetic" such as copper or aluminum.

Since, as previously stated, any one wire of a power transmission line may be thought of as carrying the outgoing current, the return current—equal in amount, but opposite in direction—being carried by the one or more remaining conductors, it follows that if all of the current I returns at a distance d from the center of the outgoing conductor, the formula (95) may be used for calculating the total IX voltage drop in the single wire. This formula is, therefore, applicable to a two-wire single-phase transmission and also to a three-phase transmission with the three conductors occupying the vertices of an equilateral triangle; but it is not applicable to a single-phase system in which the return current is divided between two or more conductors situated at different distances from the outgoing wire, or to a three-wire three-phase transmission in which the wires do not occupy the vertices of an equilateral triangle.

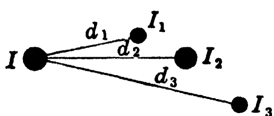


FIG. 93.—Section through four parallel conductors.

In Fig. 93, the total outgoing current I is supposed to be the current in one of the several parallel conductors, while the total return current is divided between all the remaining conductors of the transmission line, the condition being that

$$\dot{I} = -(\dot{I}_1 + \dot{I}_2 + \dot{I}_3 \quad . \quad . \quad . \quad + \dot{I}_n)$$

Let d_1, d_2, d_3, etc. be the distances between the centers of the several conductors carrying the return current and the conductor carrying the outgoing current, and note that the total flux surrounding the latter conductor may be considered as the sum of the component fluxes, namely, the flux due to a current $-I_1$ returning at a distance d_1, the flux due to a current $-I_2$ returning at a distance d_2, and so on, for any number of components of the total current I. Each of these separate components of the total flux can readily be calculated by means of formula (92), and the expression for the total flux surrounding a conductor in which the current I returns along a number of parallel conductors, as indicated in Fig. 93, becomes

$$\Phi = 0.2l\left[-\dot{I}_1 \log_\epsilon \frac{d_1}{r} - \dot{I}_2 \log_\epsilon \frac{d_2}{r} \quad . \quad . \quad . \quad - \dot{I}_n \log_\epsilon \frac{d_n}{r}\right] \quad . \quad . \quad . \quad (97)$$

where r stands for the radius of cross-section of the particular conductor which carries what will be thought of as the outgoing current I.

When energy is transmitted by polyphase currents, with any number of conductors, the algebraic sum of the currents in the conductors must, at any given instant, be equal to zero. Any one conductor may be looked upon as carrying the outgoing current, while the remaining conductors together carry the return current. Formula (97) can, therefore, be used for calculating the effective flux of induction surrounding any one conductor in a polyphase transmission, whatever may be the arrangement of the conductors. Obviously, the phase relations between the several components of flux must be taken into account, since it is the vectorial sum of the quantities in the formula that is required.

86. Effect of Transposing Conductors of Three-phase Transmission.—When the conductors of a three-wire three-phase

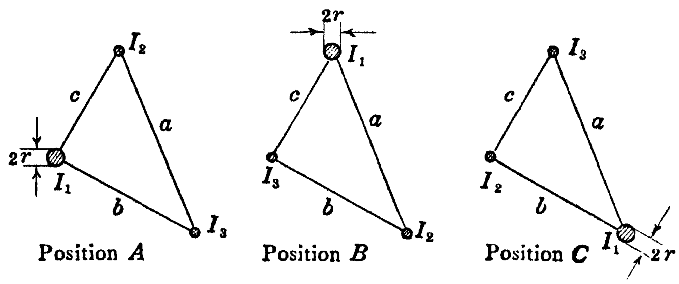


FIG. 94.—Illustrating effect of transposing three parallel conductors.

transmission are not equally spaced, that is to say, when they do not occupy the corners of an equilateral triangle, the IX drop (or induced e.m.f.) will not be the same in all three conductors. If each conductor of such a transmission line is made to occupy, in turn, the positions originally occupied by the remaining two conductors for a distance equal to one-third of the total distance of transmission, it is obvious that the out-of-balance effect will be corrected.

Figure 94 shows the arrangement of three conductors with unequal spacings, a, b, and c. The three conductors carrying currents I_1, I_2, and I_3, respectively, occupy each of the positions (A), (B), and (C) over one-third of the total distance of transmission. Since the induced e.m.f. will necessarily be the same in the three conductors, it may be calculated by considering any one wire and adding the flux components as calculated by formula (97) for each of the three positions indicated in Fig. 94. Consider the total flux which surrounds 1-cm. length of the conductor carrying the current I_1 amp.:

$$\text{In position } (A)\ \Phi_1 = 0.2\left[-\dot{I}_3 \log_\epsilon \frac{b}{r} - \dot{I}_2 \log_\epsilon \frac{c}{r}\right]$$

$$\text{In position } (B)\ \Phi_1 = 0.2\left[-\dot{I}_3 \log_\epsilon \frac{c}{r} - \dot{I}_2 \log_\epsilon \frac{a}{r}\right]$$

$$\text{In position } (C)\ \Phi_1 = 0.2\left[-\dot{I}_3 \log_\epsilon \frac{a}{r} - \dot{I}_2 \log_\epsilon \frac{b}{r}\right]$$

One-third of the sum of these three flux values will be the average flux surrounding the conductor per centimeter of the entire length of the line, whence

$$\text{Average flux per centimeter, } \Phi = \frac{0.2}{3}\left[-(\dot{I}_3 + \dot{I}_2) \log_\epsilon \frac{abc}{r^3}\right]$$

But in place of $-(\dot{I}_3 + \dot{I}_2)$, put I_1; and in place of $\frac{1}{3} \log_\epsilon \frac{abc}{r^3}$, put $\log_\epsilon \frac{\sqrt[3]{abc}}{r}$, whence

$$\text{Average flux per centimeter, } \Phi = 0.2 I_1 \log_\epsilon \frac{\sqrt[3]{abc}}{r} \text{ maxwells.}$$

This does not include the flux inside the material of the conductor, but if the "equivalent spacing" $\sqrt[3]{abc}$ is substituted for d in formula (95), the reactive voltage per mile in any one of the three wires of diameter $2r$ may be written

$$(IX) = 0.00466 fI \log_\epsilon \frac{\sqrt[3]{abc}}{r} + 0.000506 fI \qquad (98)$$

Thus, whatever may be the distances a, b, and c, between the three points of support of a three-wire three-phase transmission with wires regularly transposed, it is always possible to calculate the impedance of the transmission line by considering each of the three wires as one conductor of a single-phase circuit with the current returning by one or more imaginary parallel conductors at a distance equal to $\sqrt[3]{abc}$.

Example 23. *Inherent Regulation of Transmission Line with Unequal Spacing of Conductors.*—Consider a three-phase overhead line 24 miles long, supplying energy at 22,000 volts with a frequency of 60 cycles. The conductors consist of No. 0000 B. & S. stranded copper, spaced in a horizontal plane with a separation of 4 ft. between the center wire and each of the outer wires. It is desired to calculate the maximum load in kilowatts which can be transmitted by this line when the power factor of the load is 0.78, given that the inherent regulation (or percentage pressure drop) must not exceed 10 per cent when the temperature of the wires is 90°F. The amount of the capacity current will be very small and may be neglected.

The required data are collected below:

Pressure between lines at receiving end, $E = 22{,}000$ volts
Frequency, $f = 60$
Power factor of load, $\cos\theta = 0.78$
Length of line, $L = 24$ miles

From table on page 81:
- Diameter of No. 0000 stranded conductor, $2r = 0.522$ in.
- Area of cross-section of wire = 0.1662 sq. in.
- Resistance, ohms per mile, at 68°F. = 0.272

By formula (90), page 235, the resistance per mile at 90°F. will be

$$R = 0.272\left(\frac{390 + 90}{390 + 68}\right) = 0.285 \text{ ohm}$$

The increase of resistance due to skin effect need not be taken into account because the product *area* × *frequency* is 0.1662 × 60 = 6 (approximately), and on referring to the curve for copper in Fig. 90 (p. 238), it is seen that the skin-effect multiplier is so small as to be negligible.

The spacings between wires, in inches, are $a = 48$, $b = 48$, $c = 96$, and the equivalent spacing is

$$d = \sqrt[3]{48 \times 48 \times 96} = 60.5 \text{ in.}$$

Using this value in formula (98), or for d in formula (96) on page 244, $\frac{d}{r} = \frac{60.5}{0.261} = 232$ which may be used in the formula: or with d in feet and $2r$ in the denominator, the ratio becomes $a' = \frac{60.5}{12 \times 0.522} = 9.66$ which may be used for reading reactance per mile from the chart on page 243. The result is

Reactance per mile of single conductor, $X = 0.695$ ohm. The required regulation or percentage voltage drop is

$$100 \times \frac{V - E}{E}$$

where V stands for the voltage between wires at the sending end of the line. The quantity $(V - E)$ is equal to $\sqrt{3}(V_n - E_n)$ where V_n and E_n are star voltages (wire to neutral) at the sending and receiving ends, respectively. If the simplified formula (15*a*) on page 28 is used, giving the voltage drop *per wire*, the loss of pressure as measured *between wires* is

$$(V - E) = \sqrt{3}IL(R \cos \theta + X \sin \theta)$$

But since it is not important to consider the current I except as it affects the total amount of power that can be transmitted, it is possible substitute for it the equivalent quantity $\frac{W}{\sqrt{3}E \cos \theta}$ wherein W is the load (in watts) at the receiving end. Thus

$$(V - E) = \frac{WL(R \cos \theta + X \sin \theta)}{E \cos \theta} \text{ volts (approximately)} \quad (99)[1]$$

or, if preferred, this formula for pressure drop on a three-phase line (wire to wire, *not* wire to neutral) may be written

$$(V - E) = \frac{WL(R + X \tan \theta)}{E} \text{ volts (approximately)} \quad (100)$$

The expression for the percentage voltage drop (or regulation, if the line is short and capacity current may be neglected) is

$$\frac{100(V - E)}{E} = \frac{100WL(R + X \tan \theta)}{E^2} \quad (101)$$

Values for $\tan \theta$ corresponding to any given power factor ($\cos \theta$) may be obtained from trigonometric tables, or calculated from the relation

$$\tan \theta = \frac{\sqrt{1 - \cos^2 \theta}}{\cos \theta}$$

A short table of trigonometric functions will be found on page 28.

[1] This is identical with the formula published by C. E. Warsaw in the *Elec. World*, Jan. 29, 1921.

Applying formula (101) to the solution of this particular problem,

$$W = \frac{\text{per cent pressure drop} \times E^2}{100L(R + X \tan \theta)}$$

$$= \frac{10 \times (22)^2 \times 10^6}{100 \times 24 \times (0.285 + 0.695 \times 0.802)} = 2{,}400{,}000$$

The required answer is that the permissible maximum load at the receiving end is 2,400 kw. A load in excess of this would cause the inherent regulation to be greater than 10 per cent.

87. Inductance of Parallel Circuits as Affected by Position of Conductors.[1]—By means of formula (97) it is possible to calculate the flux surrounding any one conductor of a power transmission line even when more than one circuit is carried on the same pole line. Figure 95 shows the wires of two three-phase circuits occupying the corners of a regular hexagon with equal sides of length *d*. These circuits are supposed to be in parallel and to be carrying a balanced load, so that the amperes of current will be the same in all six conductors. Owing to the symmetrical arrangement of the wires, with phase rotation in the same direction (1, 2, 3) in the two parallel circuits, the total effective flux around any one conductor will be the same (except in regard to phase) as that which surrounds any one of the other five conductors. The calculations for induced e.m.f. (or for resultant flux) may, therefore, be made for any conductor such as the one which carries the current A_1. In applying formula (95), it is necessary merely to remember that the current A_1 may be considered as the sum of five component currents which are exactly equal, but opposite in phase, to the currents in the five remaining wires.

Referring to Fig. (95), let the symbols A_1, A_2, A_3 stand for the currents in one of the parallel three-phase circuits, while B_1, B_2, B_3 stand for the (equal) currents in the other circuit. Then $\dot{A}_1$ is equal to $\dot{B}_1$, not only in magnitude, but also in phase. Similarly,

$$\dot{A}_2 = \dot{B}_2$$

and

$$\dot{A}_3 = \dot{B}_3$$

[1] Reprinted with a few alterations from an article contributed by the writer to the *Jour. Franklin Inst.*, September, 1926.

The several components of the current A_1 are

$$-(\dot{B}_3 + \dot{A}_2) = -(\dot{A}_3 + \dot{A}_2) = +\dot{A}_1 \text{ returning at a distance } d$$
$$-(\dot{B}_2 + \dot{A}_3) = -(\dot{A}_2 + \dot{A}_3) = +\dot{A}_1 \text{ returning at a distance } \sqrt{3}d$$
$$-\dot{B}_1 = -\dot{A}_1 \text{ returning at a distance } 2d$$

whence, by formula (97),

$$\Phi = 0.2l\left[A_1 \log_\epsilon \frac{d}{r} + A_1 \log_\epsilon \frac{\sqrt{3}d}{r} - A_1 \log_\epsilon \frac{2d}{r}\right]$$
$$= 0.2lA_1 \log_\epsilon \frac{\sqrt{3}d}{2r} \qquad (102)$$

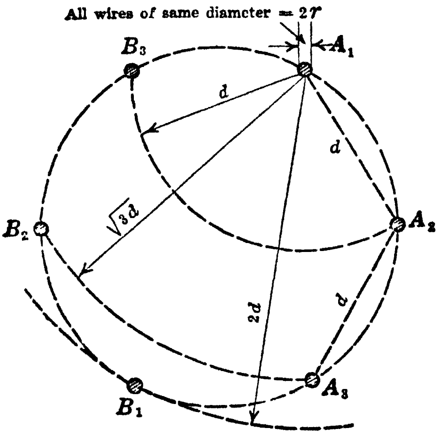


FIG. 95.

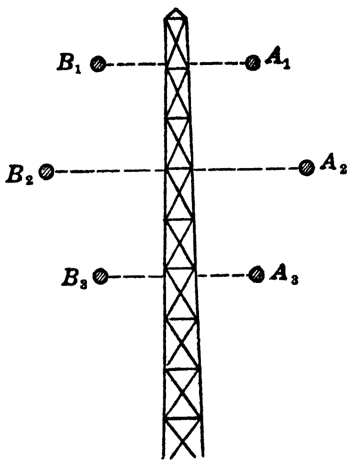


FIG. 96.

FIGS. 95 and 96.—Alternative arrangement of wires when two three-phase circuits are carried on same pole line.

If two separate three-phase lines with a considerable distance between them were substituted for the arrangement of Fig. 95, the flux surrounding any one of the six conductors would be

$$\Phi = 0.2lA_1 \log_\epsilon \frac{d}{r} \qquad (103)$$

assuming the triangular (equidistant) arrangement with a separation d between wires. This is *greater* than the flux as given by formula (102). If a distance d = 120 in. and a radius of conductor cross-section $r = 0.4$ in. be assumed, the ratio of the induced e.m.fs. for the alternative arrangements will be

$$\frac{\log \frac{120}{0.4}}{\log \frac{\sqrt{3} \times 120}{2 \times 0.4}} = 1.025$$

Although, in this example, the reactive drop with the two separate pole lines is only 2½ per cent greater than with all six wires on the same pole line, the fact remains that with the proper arrangement of wires on a single-pole line, two parallel three-phase circuits may be carried with a *smaller* reactive drop than if the two circuits were carried on separate pole lines with the same minimum spacing between wires.[1] By applying formula (97), it is easy to calculate the flux surrounding any one of the conductors, whatever may be the spacing between them, and when the circuits are arranged as in Fig. 96, it will be found that there is considerable unbalance of induced e.m.fs. and that the average reactance drop is appreciably greater than with the phases arranged as in Fig. 95, or with the two circuits carried on separate pole lines.

88. Pressure Available at Intermediate Points on a Transmission Line.—Referring again to Fig. 9 (reproduced on page 246), the volts per phase at the generating end are V_n and at the receiving end, E_n, the total drop being $(V_n - E_n)$ volts. It does not follow, however, that the pressure available at a point halfway along the line will be $V_n - \left(\frac{V_n - E_n}{2}\right)$ because the power factor is rarely the same at all points.

The method of calculating the pressure available at some intermediate point L' miles from the supply station on a line of total length L miles, when the effects of capacity are negligible, is illustrated in Fig. 97 where $C'C = BC\left(\frac{L'}{L}\right)$ and $OD - OD'$ is a measure of the voltage drop between the supply end and the point considered. The power-factor angle at this point will be ψ, which can be calculated by making the required changes

[1] This was pointed out by the writer in an article which was published in the *Elec. World*, Sept. 15, 1910, and, more recently, H. B. Dwight has very clearly shown (*Elec. World*, Jan. 12, 1924) that if two parallel three-phase circuit are carried on the same pole line, the arrangement shown in Fig. 95 is decidedly preferable to the more usual arrangement of Fig. 96.

in the formulas of Article 15. Thus, formula (11) would be written

$$OD' = \frac{BC' + E_n \cos \theta}{\cos \psi}$$

and the procedure throughout is exactly the same as if calculating the required generating-end voltage on a line $(L - L')$ miles in length, to give E_n volts at the receiving end when delivering I amp. at a power factor $\cos \theta$.

Such calculations are usually unnecessary refinements, and, except on very long lines, the error introduced by assuming a

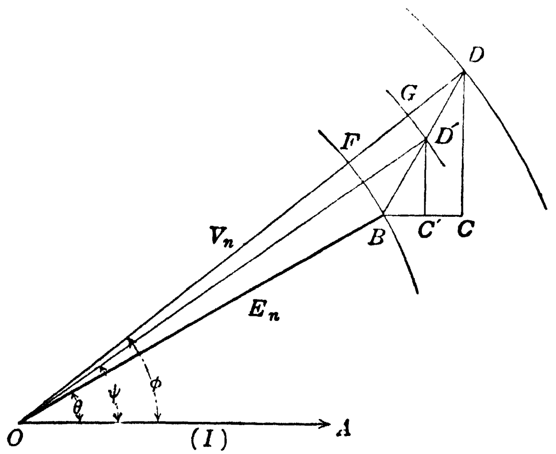


FIG. 97.—Vector diagram showing pressure at intermediate point on transmission line.

uniform drop of pressure along the line is rarely of any practical importance.

The manner in which the values of pressure drop, as calculated for a single conductor, are used in determining the inherent regulation of three-phase lines was explained in Art. 18, Chap. II.

89. Electrostatic Capacity of Overhead Lines.—Any arrangement of two conductors of electricity separated by an insulator forms a condenser of which the capacity will depend upon the spacing of the conductors and the nature of the dielectric between them. In the case of overhead conductors running parallel to each other and to the surface of the ground over a considerable distance, the electrostatic capacity between the individual conductors, and between these conductors and earth, becomes a matter of importance.

The relation between the electrostatic capacity[1] of a condenser and the dielectric flux produced by a given potential difference was briefly discussed in Art. 72, Chap. VIII. With a sine-wave alternating voltage E, the charging current, in amperes, is

$$I_c = 2\pi f E C \tag{104}$$

where C is the capacity of the condenser, in farads, and f is the frequency, in cycles per second. The development of this formula is given in Art. 76 (see footnote on p. 219). This charging current, as is well known, *leads* the applied or impressed e.m.f. by one-quarter period, and if the electrostatic capacity per unit length of a transmission line is known, the capacity current which passes from wire to wire through the dielectric (air) is easily calculated. The total current in the line is obtained by adding (vectorially) the capacity current and the load current.

The exact formula[2] which gives the capacity in microfarads per mile between two cylindrical parallel wires in air is

$$C_{\text{mf.}} = \frac{0.0194}{\log\left[a + \sqrt{a^2 - 1}\right]} \tag{105}$$

where $a = \dfrac{d}{2r}$, d being the distance between centers of wires and r the radius of cross-section of the (circular) conductor, expressed in the same units as d. It is more generally useful to consider the capacity as being measured between one wire and the neutral potential surface. This will be *twice* the value of the capacity as measured between the two wires; but when calculating the charging current, it is the voltage *between wire and neutral surface* that must be taken, if this latter value of the capacity is used.

The formula (105) may be put into another form which is very convenient if tables of hyperbolic functions are available.

[1] The words *permittance* and—more recently—*capacitance*, when applied to a condenser or the dielectric field between conductors, have the same meaning as *electrostatic capacity* or, briefly, *capacity*. The word *permittance* was suggested by Heaviside, who chose it for the very good reason that it connotes in the dielectric circuit the same quality as the words *admittance* and *permeance* in the electric and magnetic circuits. All three words mean generally the same thing, namely, the quality or attribute which admits—allows to penetrate or pass through—while *capacitance* suggests this no more forcibly than *capacity*. The word *condensance*, although the latest aspirant to philological fame, appears to have nothing to recommend it apart from its debatable euphony.

[2] The derivation of these formulas is given in the majority of textbooks treating of alternating currents.

In the formula (105) common logarithms are referred to in the denominator; but by making the proper correction to the numerator and substituting Napierian logs, the denominator becomes $\log_e (a + \sqrt{a^2 - 1})$, which is the quantity of which the hyperbolic cosine is a. Thus, the inverse hyperbolic cosine of a, or $\cosh^{-1} a$, is the equivalent of $\log_e (a + \sqrt{a^2 - 1})$; and with the corrected numerator, the formula (105) becomes

$$C_{\text{mf.}} = \frac{0.0447}{\cosh^{-1} a} \tag{106}$$

If the capacity per mile of single conductor, measured between wire and neutral, is required, the numerators of these formulas must be doubled, and the correct formula may be written either

$$C_{\text{mf.}} = \frac{0.0388}{\log (a + \sqrt{a^2 - 1})} \tag{107}$$

or

$$C_{\text{mf.}} = \frac{0.0895}{\cosh^{-1} a} \tag{108}$$

When the distance d between wires is large as compared with the diameter $(2r)$ of the conductor—a condition which obtains on all high-voltage overhead power lines—the quantity $\sqrt{a^2 - 1}$ in formula (107) is very nearly equal to a, and the simplified formula becomes

$$C_{\text{mf.}} = \frac{0.0388}{\log 2a}$$

or

$$C_{\text{mf.}} = \frac{0.0388}{\log \frac{d}{r}} \tag{109}$$

This formula gives the capacity in microfarads per mile of conductor, as measured between the wire and the imaginary neutral plane, with sufficient accuracy for calculating the charging current in any practical high-voltage overhead power transmission line. Formula (109) gives results very slightly smaller than formula (107) or (108); but when a is large, that is to say, when the distance between wires is many times the diameter, the error is negligible. The error becomes appreciable only if a is less than 10, and even if a is as small as 4 (a quite impossible state of things on an overhead transmission with bare wires), the error would be only 0.8 per cent.

Relation between Permittance and Inductance of Overhead Lines.—It is possible to express the charging current on overhead lines in terms of the reactance. This is due to the fact that there is an approximately constant relation between the inductance and the capacity, which is independent of the size and spacing of the conductors. The formulas (94) and (95) of Art. 82 give values for the quantity IX, that is to say, for the product of *current* and *reactance*. Formula (94) takes no account of the magnetic flux within the material of the conductor, and, if the inductance based upon the *external reactance* only is considered, a simple relation between permittance and inductance is obtained.

If **L** stands for the inductance (coefficient of self-induction) in henrys per mile of single conductor, and X is the reactance in ohms per mile, then, on the sine-wave assumption

$$X = 2\pi f\mathbf{L}$$

whence, if the value given by formula (94) namely, $X = 0.00466f \log \frac{d}{r}$, is substituted for X, the approximate value of the inductance per mile of single conductor is

$$\mathbf{L} = 0.000741 \log \frac{d}{r} \text{ henrys} \tag{110}$$

Comparing this expression with formula (109) giving the capacity in microfarads per mile of wire, it is seen that the product of these two quantities is a constant, namely,

$$C_{mf.}\mathbf{L} = \frac{1}{34{,}700} \tag{111}$$

By using the exact formulas for $C_{mf.}$ and **L**, and taking an average of this product between the usual limits of the ratio $\frac{d}{r}$,

$$C_{mf.}\mathbf{L} = \frac{1}{33{,}600} \tag{112}$$

This formula is generally more exact than formula (111), which neglects the magnetic flux inside the conductor. By expressing **L** in terms of X and the frequency (f)

$$C_{mf.} = \frac{2\pi f}{33{,}600X} \text{ microfarads} \tag{113}$$

If it is desired to express the charging current per mile in terms of the reactance per mile it is necessary merely to substitute the above expression for $C_{mf.}$ in formula (104), which gives

$$I_c = \frac{f^2E}{8.5X \times 10^8} \text{ amp. (approximate) per mile of conductor} \tag{114}$$

This formula for calculating the magnitude of the charging current, when multiplied by the length of the line in miles, will give the charging current in the line at the generating end. The result is usually smaller than the value obtained by measurement on actual lines. The reason is that the assumption of sinusoidal impressed e.m.f. is rarely justified, and the irregularities and peaks in the actual pressure wave may cause an increase of charging current amounting to 20 or even 40 per cent of the calculated value. These considerations emphasize the absurdity of devoting a considerable amount of time to mathematical refinements, or of using complicated formulas of which the increased accuracy is of no practical value seeing that they are based on assumptions that are never realized in an actual transmission system.

90. Capacity of Three-phase Lines.—The formulas in the last article give the capacity between two parallel wires as measured from wire to neutral, and in the case of a single-phase transmission, the capacity between the two wires would, as it were, consist of two such capacities in series and would, therefore, measure *half* the value given by these formulas, all as previously mentioned. It should, however, be noted that it makes no difference which value of the capacity is taken for the purpose of calculating the charging current, provided proper attention is paid to the potential difference available for charging the condenser. In the case of the single-phase transmission, the pressure available for charging the two imaginary condensers in series is exactly twice the pressure between one wire and neutral (see Fig. 98).

Note that if a third parallel wire (3) is placed anywhere in the neutral plane equidistant from the wires (1) and (2), any electric charge on this wire will affect wires (1) and (2) equally, and it will, therefore, *have no effect upon the permittance C as measured between the wires* (1) *and* (2).

Consider, now, a three-phase transmission with the three conductors occupying the vertices of an equilateral triangle[1] as

[1] With an irregular arrangement of the three conductors having spacings a, b, and c, the *equivalent* equilateral triangle will have sides equal in length to the geometric mean of the three actual distances between wires, or $d_{equiv.} = \sqrt[3]{abc}$, as proved in Art. 86 when considering the inductance of irregularly spaced conductors. The fact that this condition holds for capacity calculations as well as for reactance calculations may be inferred from the expression for capacity in terms of reactance as indicated by formula (113) in the preceding article. A complete proof is given in H. B. Dwight's "Transmission Line Formulas."

shown diagrammatically in Fig. 99. Since the capacity between any two conductors is unaffected by the presence of the third conductor, the capacity C as measured between any one wire and neutral is the same as for the single-phase arrangement provided the separation d between conductors and the radius r of conductor cross-section remain unaltered. The usual representation of this condition, with the imaginary capacity C between each wire and neutral (which provides for the two capacities in series between any two wires), is shown in Fig. 100.

Although the capacities C as measured between one wire and neutral are the same in Figs. 98 and 100, it does not follow that the charging current per wire is the same for the three-phase as

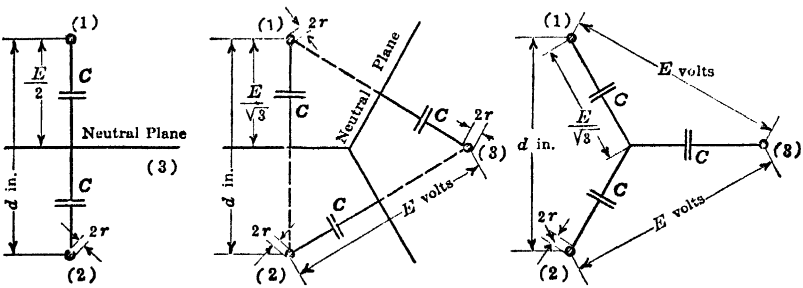


FIG. 98. FIG. 99. FIG. 100.

FIGS. 98, 99 and 100.—Diagrams illustrating electrostatic capacity of overhead transmission lines.

for the single-phase arrangement with the same voltage E between wires. The charging current will depend upon the voltage between wire and neutral, which is $\frac{E}{2}$ for the single-phase transmission and $\frac{E}{\sqrt{3}}$ for the three-phase system. The representation of the capacities between the wires of a three-phase system, as shown in Fig. 100, has the great advantage that a three-phase, or indeed any polyphase, system may be treated as a combination of several single-phase systems, and the calculation of the capacity current per wire becomes a simple matter.

The capacity between each wire and ground is relatively small and may always be neglected in calculations on overhead transmission lines. It is, therefore, only when great refinements and scientific accuracy are aimed at that the simple diagram of Fig. 100 is no longer applicable.

91. Electrical Calculations for Lines with Appreciable Capacity. By means of formula (107) or (109), the capacity from wire to neutral may be calculated for any transmission line or section of line of length L miles. The charging current I_c, which leads the voltage by one-quarter period, may be calculated by formula (104) or (114), and, by adding this current vectorially to the load current I, the total current in the line at the supply end of the section is obtained. The current at the receiving end is merely the load current I, since the whole of the charging current I_c has passed out of the wire over the length L miles between the sending and the receiving ends of the line.

A long transmission line may be divided into a number of sections of length $AB = L$ miles, as indicated in Fig. 101. If R and X stand, respectively, for the resistance and reactance per

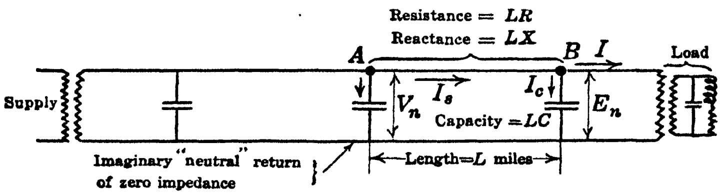


FIG. 101.—Long transmission line divided into sections.

mile of wire, the resistance per single conductor L miles long is LR ohms, the reactance is LX ohms, and the capacity susceptance[1] (on the sine-wave assumption) is $2\pi fLC$ mhos, where C stands for the capacity between wire and neutral per mile (expressed in farads).

When a long transmission line is considered as a succession of unit sections, such as the length AB of Fig. 101, it is necessary merely to imagine a very large number of short sections to understand how a diagram such as Fig. 101 may be used to study the voltage and current relations in an actual transmission line where the inductance and capacity are not concentrated at any particular points but are distributed over the entire length of the line. By imagining the unit sections to be made smaller and smaller without limit, correct formulas may be developed,[2] but since the simpler formulas, based on the assumption of concentrated capacities, are sufficiently accurate for nearly all calculations on power transmission lines, these will be developed in the

[1] The ratio of charging current to voltage.

[2] The development of these formulas is given in Chap. X.

first instance and used to illustrate the effects of capacity when the line is of sufficient length to render these effects appreciable.

The vector diagram Fig. 102 applies to any line or section of line, such as AB of Fig. 101, with the capacity LC (between wire and neutral) supposed to be concentrated at the receiving or "load" end of the section considered. The total line current I_s is, therefore, equal to the (vectorial) sum of the charging current I_c and the current I taken out of the line at the receiving end of the section.

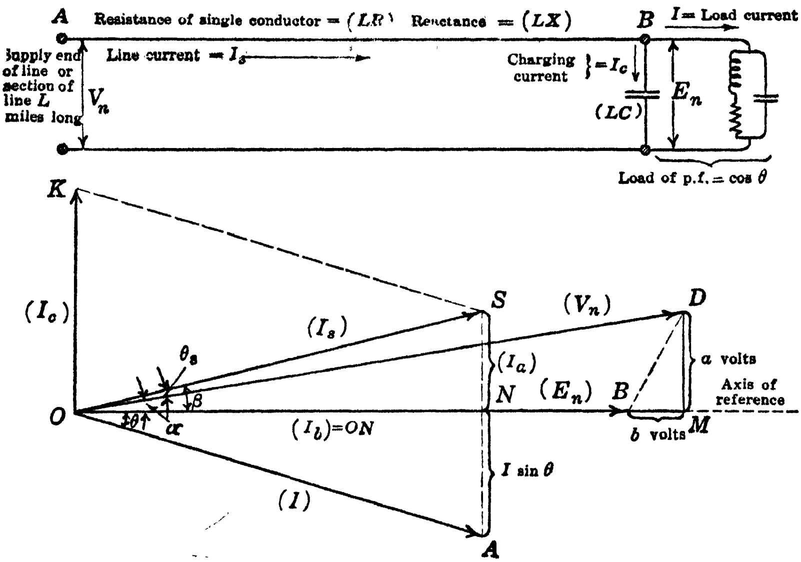


FIG. 102.—Vector diagram of long transmission line.

The vector OK, representing the charging current, is drawn 90 deg. in advance of the receiving-end voltage (E_n), while the vector OA representing the "load" current is drawn θ deg. behind the receiving-end voltage, the power factor of the load being cos θ (lagging). Obviously, when considering a section of the entire line as in Fig. 101, with A as the sending end and B as the receiving end of the section, the expressions "load" and "power factor of load" must be understood to include with the load proper the portion of the transmission line between the point B and the end of the line. The vector OS (Fig. 102) is the total current in the line, being the sum of I_c and I. The manner in which the voltage (V_n) and the power factor (cos θ_s) at the sending end of the section are calculated will be understood from the step-by step development of the formulas, which follows.

Calculation of Sending-end Voltage Based on Voltage and Load at the Receiving End.

(1) $I_a = NS = I_c + I \sin \theta$

(Note that $\sin \theta$ is *negative* when current lags.)

(2) $I_b = ON = I \cos \theta$

(3) $a = MD = I_b(LX) + I_a(LR)$

(When a is *positive*, plot *above* reference axis.)

(4) $b = BM = I_b(LR) - I_a(LX)$

(When b is *positive*, plot to *right* of point B.)

(5) $OM = E_n + b$

(6) $\tan \alpha = \dfrac{a}{(E_n + b)}$

(When $\tan \alpha$ is negative, the angle α is also negative.)

(7) $\alpha =$
(8) $\cos \alpha =$ } (Consult trigonometric tables.)

(9) $V_n = OD = \dfrac{(E_n + b)}{\cos \alpha}$ (This is the voltage—wire to neutral—at sending end.)

If preferred, the expression for this voltage may be written $V_n = \sqrt{(E_n + b)^2 + a^2}$, which simplifies into the convenient formula

$$(9^a)\quad V_n = (E_n + b) + \frac{a^2}{2(E_n + b)} \qquad (115)$$

This approximation is justified because a is small relatively to $(E_n + b)$. Refer to footnote on page 89 of Chap. IV.

Calculation of Current and Power Factor at Sending End.

(10) $\tan \beta = \dfrac{I_a}{I_b}$

(11) $\beta =$
(12) $\cos \beta =$ } (Consult trigonometric tables.)

(Note that when $I_c = 0$, $\tan \beta = \tan \theta$, and $\beta = \theta$.)

(13) $I_s = OS = \dfrac{I_b}{\cos \beta}$

(14) $\theta_s = \beta - \alpha$

(If θ_s is *negative*, current lags behind sending-end voltage.)

(15) $\cos \theta_s =$ (Consult trigonometric tables.) This is the power factor at the sending end of section. It will be a *leading* or *lagging* power factor depending upon whether θ_s is *positive* or *negative*.

Calculation of Line Losses.

(16) Watts lost in a single conductor = power input per phase *minus* power output per phase

$$= V_n I_s \cos \theta_s - E_n I \cos \theta$$
$$= I_s^2 (LR)$$

Example 24.—Electrical Calculations Taking Account of Capacity Current.—The data used in these calculations are as follows:

System, three-phase

Line pressure at the receiving end, $E = 88{,}000$ volts

"Star" voltage, $E_n = \frac{88{,}000}{\sqrt{3}} = 50{,}810$

Frequency, $f = 60$

Load $= 22{,}000$ kv.-a.

Power factor of load, $\cos \theta = 0.8$ (lagging)

Length of transmission line, $L = 100$ miles

Conductors, No. 0000 B. & S., stranded copper

Equivalent spacing $= 9$ ft. ($d = 108$ in.)

From the wire table on page 81, the resistance at 68°F. is found to be $R = 0.272$ ohm per mile. By formula (96),[1] the reactance per mile is

$$X = 0.00466 \times 60 \times \log \left(1.285 \times \frac{108}{0.261}\right) = 0.762 \text{ ohm.}$$

By formula (109), the capacity (wire to neutral) is

$C_{\text{mf.}} = \frac{0.0388}{\log \frac{108}{0.261}} = 0.01485$ mf. per mile, and the capacity susceptance per mile is

$$B = 2\pi \times 60 \times 0.01485 \times 10^{-6} = 5.6 \times 10^{-6} \text{ mhos}$$

The load current is

$$I = \frac{22{,}000}{\sqrt{3} \times 88{,}000} = 144.5 \text{ amp.}$$

By way of illustrating the method of procedure based upon the vector diagram Fig. 102, assume the whole of the electrostatic capacity to be concentrated at the center of the line, as indicated in Fig. 103, and calculate the conditions at the sending end by considering the line as consisting of two sections each ½ L miles long. First calculate the conditions at the halfway point, noting that the 50-mile section connected to the load is supposed

[1] If preferred, the chart on p. 243 may be used.

to be without capacity, so that it would be possible to use the formulas for short lines as developed in Chap. II; but the same results will be obtained by following item by item the procedure based upon the vector diagram Fig. 102, merely observing that $I_c = 0$.

(1) $I_a = 0 - 144.5 \times 0.6 = -86.7$ amp.

(2) $I_b = 144.5 \times 0.8 = 115.6$ amp.

(3) $a = (115.6 \times 50 \times 0.762) - (86.7 \times 50 \times 0.272) = 3{,}220$ volts

(4) $b = (115.6 \times 50 \times 0.272) + (86.7 \times 50 \times 0.762) = 4{,}870$ volts

(5) $(E_n + b) = 50{,}810 + 4{,}870 = 55{,}680$ volts

(6) $\tan \alpha = \dfrac{3{,}220}{55{,}680} = 0.0578$

(7) $\alpha = 3° \ 19'$

(9^a) $E_n' = 55{,}680 + \dfrac{(3{,}220)^2}{2 \times 55{,}680} = 55{,}770$

which is the voltage across the condenser supposed to be connected between line and neutral plane at the midway point.

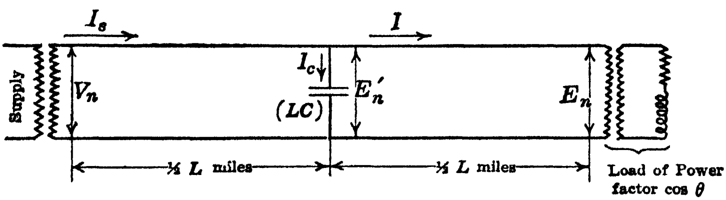


FIG. 103.—Diagram illustrating Example 24.

(13) The current in this section of the line is the load current $I = 144.5$ amp.

(14) $\theta_s = \beta - \alpha = \theta - \alpha = -36° \ 52' - 3° \ 19' = -40° \ 11'$

(15) $\cos \theta_s = 0.764$ (lagging) $[\sin \theta = -0.645]$

This is the power factor at the midway point on the line.

(16) The line losses per conductor in this section of the line are

$$I^2R = (144.5)^2 \times 50 \times 0.272 = 284{,}000 \text{ watts}$$

Repeating the calculations for the 50-mile section at the sending end of the line, it is necessary merely to follow the same procedure, noting that the load current has the same value as before, namely $I = 144.5$ amp., but the voltage (line to neutral) at the receiving end of the section is now $E_n = E_n' = 55{,}770$, and the new "load" power factor is $\cos \theta = \cos \theta_s = 0.764$. The calculations follow:

With a voltage $E_n = 55{,}770$ across a condenser of susceptance $100 \times 5.6 \times 10^{-6}$ mhos, the charging current is

$$I_c = 100 \times 5.6 \times 10^{-6} \times 55{,}770 = 31.2 \text{ amp.}$$

(1) $I_a = 31.2 - 144.5 \times 0.645 = -62$ amp.

(2) $I_b = 144.5 \times 0.764 = 110.5$ amp.

(3) $a = (110.5 \times 50 \times 0.762) - (61.3 \times 50 \times 0.272) = 3{,}380$ volts

(4) $b = (110.5 \times 50 \times 0.272) + (61.3 \times 50 \times 0.762) = 3{,}840$ volts

(5) $(E_n' + b) = 59{,}610$

(6) $\tan \alpha = \dfrac{3{,}380}{59{,}610} = 0.0566$

(7) $\alpha = 3° \, 14'$

(9^a) $V_n = 59{,}610 + \dfrac{(3{,}380)^2}{2 \times 59{,}610} = 59{,}710$ volts

which is the required voltage (wire to neutral) at the sending end of line. The pressure as measured between wires is $\sqrt{3}V_n = 103{,}400$ volts.

(10) $\tan \beta = -\dfrac{62}{110.5} = -0.561$

(11) $\beta = -29° \, 17'$

(12) $\cos \beta = 0.872$

(13) $I_s = \dfrac{110.5}{0.872} = 126.5$ amp.

(14) $\theta_s = -29° \, 17' - 3° \, 14' = -32° \, 31'$

(15) $\cos \theta_s = 0.843$

This is the power factor (lagging) at the sending end of the line.

(16) The I^2R loss per conductor is

$$(126.5)^2 \times 50 \times 0.272 = 217{,}000 \text{ watts}$$

The total losses in the three conductors (both sections) are $3(217 + 284) = 1{,}503$ kw., which is 8.5 per cent of the power delivered at the receiving end of the line.

The line drop per wire is $59{,}710 - 50{,}810 = 8{,}900$ volts, which is 17.5 per cent of the pressure at the receiving end.

The difference between sending-end and receiving-end voltages as calculated in this numerical example for a line 100 miles long, with the whole of the capacity supposed to be concentrated at the center of the line, differs by less than one-half of 1 per cent from the line drop as calculated on the basis of uniformly distributed inductance and capacity. So small an error is entirely negligible in calculations of practical transmission lines, especially as many assumptions have to be made, including the probable amount and power factor of the load, the temperature, and, therefore, the resistance of the conductors, and the wave shape of the impressed e.m.f. The very fact that sinusoidal wave shapes are assumed in nearly all cases, even when the so-called exact formulas are used, and that the amount of the charging current depends largely upon the wave shape, would seem to indicate that refinements of calculation are unnecessary and may even be undesirable. Calculations on lines 150 miles long may be made without appreciable error on the assumption of a single condenser load at the center of the line, as indicated in Fig. 103, and by imagining

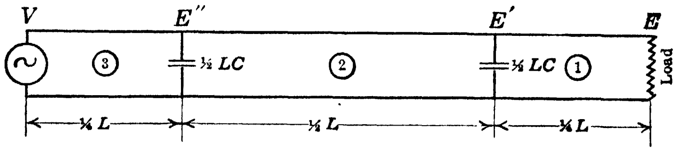


FIG. 104.—Diagram of long line divided into three sections.

a longer line (up to about 300 miles) to consist of two sections similar to the line of Fig. 103, the diagram Fig. 104 is obtained, with the total permittance divided equally between two condensers. By calculating E' at the sending end of section (1), E'' at the sending end of section (2), and finally V at the sending end of section (3), the conditions at the sending end of the line may be determined in three successive steps with a probable error of less than one-half of 1 per cent. As a matter of fact, a straight transmission without branches or intermediate tappings will rarely be found, in practice, of so great a length as 300 miles, and, therefore, the transmission line engineer has very little use for the correct, but less familiar, formulas based on the condition of uniformly distributed inductance and capacity; but since these formulas are of very great value in other fields, and may be preferred by those familiar with the mathematics involved and the use of tables of hyperbolic functions, their development and application will be taken up in the following chapter.

92. Power Factor of Load Consisting of Several Circuits in Parallel.—When the load connected to a particular supply circuit is made up of a number of separate smaller loads—such as motors of different sizes and power factors, lighting circuits, etc.—it is necessary to add these loads in order to determine not only the total actual power, in watts, but also the volt-amperes or the power factor of the total load, so that the current in the supply circuit may be calculated. The summation of loads of different power factors must also be made when a transmission line feeds into a point where one or more branch lines take loads of different amounts and power factors. Assuming the apparent power (volt-amperes) and the true power (watts) to be known in connection with each component load, the summation of these separate loads, all connected in parallel across a single supply circuit, is easily carried out in the following manner:

Let P_1, P_2, P_3, etc. stand for the true power of the respective component loads, and let Q_1, Q_2, Q_3, etc. stand for the corresponding "reactive power"; then the volt-amperes or apparent power of any individual load, as for instance load 2, is $(EI)_2 = \sqrt{P_2^2 + Q_2^2}$ and the power factor of this load is $\cos\theta_2 = \dfrac{P_2}{\sqrt{P_2^2 + Q_2^2}}$.

Similarly, if $\cos\theta_s$ = power factor of the total load, write

$$\cos\theta_s = \frac{\Sigma P}{\sqrt{(\Sigma P)^2 + (\Sigma Q)^2}} = \frac{1}{\sqrt{1 + \left(\dfrac{\Sigma Q}{\Sigma P}\right)^2}} \qquad (116)$$

and it is necessary merely to sum up separately the "in-phase" and "reactive" components of the several circuits to obtain the true power and reactive power of the supply circuit.

Example 25. *Parallel Circuits with Loads of Different Power Factors.*—Consider four different circuits connected in parallel, the kilovolt-ampere loads and power factors being as given in the second and third columns of the accompanying table. Assume the true power values to be measured in the direction of the vector of reference, and calculate for each load the values of the component vectors, namely:

Active component, P = Kv.-a. $\times \cos\theta$
Reactive component, Q = Kv.-a. $\times \sin\theta$

These values are recorded in columns 4 and 5 of the table.

Load number	Kilovolt-amperes	Power factor, $\cos\theta$	Active component, P = kv.-a. × $\cos\theta$	Reactive component, Q = kv.-a. × $\sin\theta$
1	100	0.5 (lag)	50	−86.6
2	120	0.8 (lag)	96	−72
3	80	1.0	80	0
4	60	0.0 (lead)	0	+60
			$\Sigma P = 226$	$\Sigma Q = -98.6$

If these component values are plotted as indicated in Fig. 105, it is easily seen that the total power in watts is $OA = \Sigma P$, while the reactive power is $AB = \Sigma Q$. The power factor of the total load is, therefore, the cosine of the angle of which the tangent is

$$\tan\theta_s = \frac{\Sigma Q}{\Sigma P} = -\frac{98.6}{226} = -0.436$$

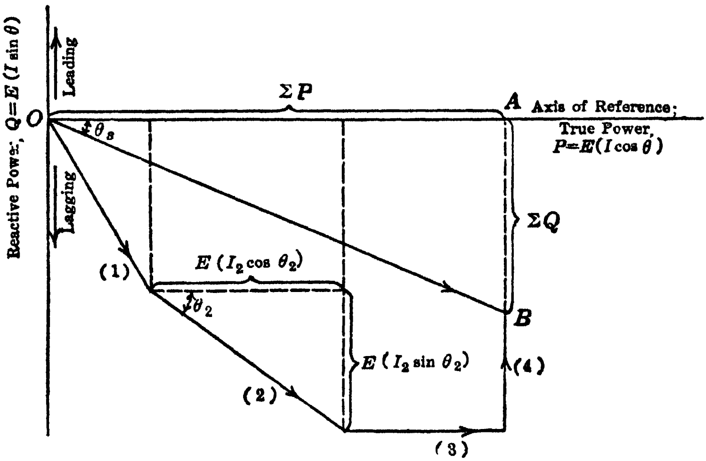


FIG. 105.—Vector diagram illustrating Example 25. Several loads in parallel

whence, from trigonometric tables, $\cos\theta_s = 0.916$ *lagging* (because $\tan\theta_s$ is negative). If trigonometric tables are not available,

$$\cos\theta_s = \frac{1}{\sqrt{1 + \tan^2\theta_s}} = \frac{1}{\sqrt{1 + (0.436)^2}} = 0.916,$$ and the total kilovolt-amperes are $\frac{226}{0.916} = 247$.

It is not necessary to draw the vector diagram as in Fig. 105 which merely serves to show how the vector OB, which represents the total kilovolt-amperes of the supply circuit, is equal to $\sqrt{(\Sigma P)^2 + (\Sigma Q)^2}$.

CHAPTER X

VOLTAGE CONTROL—ELECTRICAL CALCULATIONS FOR LONG TRANSMISSION LINES

93. Distinction between Line Regulation and Voltage Drop.—The inherent regulation of short lines was discussed in Art. 84, Chap. IX (refer also to Example 23, p. 251). If V = pressure at the sending end and E = pressure at the receiving end of a transmission line, the percentage drop of pressure is

$$\text{Per cent pressure drop} = \frac{V - E}{E} \times 100$$

and this is also the line *regulation* when the effects of line capacity are so small as to be negligible. On long high-voltage lines, there

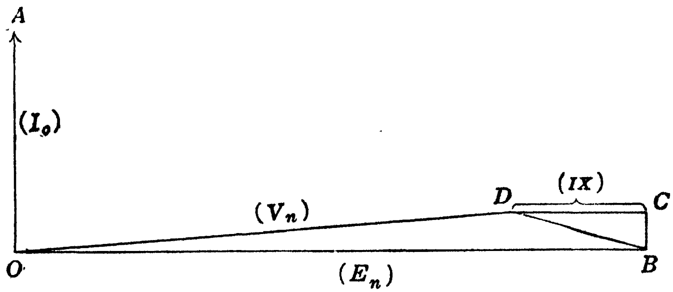


FIG. 106.—Vector diagram showing pressure rise at end of long unloaded line.

is an appreciable component of the total current which *leads* the voltage by one-quarter period, and as the phase of the e.m.f. induced by this current component *lags* one-quarter period behind the current, it will be in phase with the e.m.f. which produces the charging current. Thus *less* pressure will be required at the sending end of the line than would be necessary if the capacitance of the line were negligible. The well-known effect of a pressure *rise* at the receiving end of a long transmission line on open circuit, or when the load is very light, is clearly illustrated by Fig. 106. In this vector diagram, the pressure at the receiving end is represented by the vector OB, while OA, drawn at right angles to OB—in the forward direction—is the capacity current as calculated by formula (104). It is assumed that the load is entirely disconnected, and the current I_c is the total current on the line. The voltage component required at the generating end to overcome ohmic resistance is BC, in phase with I_c, and the component required to balance the e.m.f. of self-induction, as calculated by formula (96), is CD, drawn 90 deg. *in advance* of OA. The pressure required at the generating end is OD, which may be *smaller* than OB. It is true that the capacity has been assumed to be concentrated at the receiving end of the line; but with distributed capacity, the same effect of a *rise* in pressure as the distance from the generating end increases will occur. It will be seen that this is due to the e.m.f. of self-induction of the charging current being in phase with the impressed voltage. If the lines were without inductance, there could be no pressure rise.

The effect of capacity on the line when the distant end is closed on the load will depend upon the amount and nature of the load. If the load is heavy and largely inductive, the current put into the line at the generating end will be *less* than the load current, and the I^2R losses will, therefore, be smaller than if the line were without capacity.

Regulation is defined as the change of pressure at the receiving end when the load is thrown off (the supply voltage remaining constant); the regulation of a long high-voltage transmission line will, therefore, usually be greater than the pressure drop.

The rise of pressure at the end of a long transmission line is independent of the size and spacing of the wires. It may be calculated approximately as follows:

In Fig. 106, the IR drop (CB) due to the charging current I_c may be neglected, as it has no appreciable effect on the pressure rise ($E_n - V_n$) which, therefore, can be considered as being equal to the induced volts DC. By formula (104) the charging current is

$$I_c = 2\pi f E_{na} C_{mf.} L \times 10^{-6}$$

where $C_{mf.}$ is the capacity between wire and neutral plane in microfarads per mile, L is the length of the line in miles and E_{na} is the average value of the voltage between line and neutral plane. The induced volts are

$$DC = (IX) = 2\pi f \mathrm{L}\left(\frac{I_c}{2}\right)L$$

where **L** = the inductance (coefficient of self-induction) per mile (see formula (110) of Art. 89), and $\frac{I_c}{2}$ is the *average* value of the charging current, which falls off from its maximum value of I_c at the sending end of the line to zero at the receiving end. Substituting for I_c its value in terms of line capacity,

$$(IX) = \frac{1}{2}(2\pi f)^2 E_{na}(C_{mf.}\,\mathbf{L})L^2 \times 10^{-6}$$

Using for the quantity $(C_{mf.}\mathbf{L})$, an approximate average value of $\frac{1}{33,000}$ which is slightly less than that given by formula (112), the pressure rise (conductor to neutral) of the unloaded line is approximately

$$DC = (IX) = (E_n - V_n) = 6E_{na}f^2L^2 \times 10^{-10}$$

This pressure rise, expressed as a percentage of the average line voltage, is

$$\left.\begin{array}{l}\text{Per cent rise of pressure}\\ \text{due to permittance and}\\ \text{inductance of line}\end{array}\right\} = 6f^2L^2 \times 10^{-8} \qquad (117)$$

The average pressure on a long transmission line operating on open circuit is greater than half the sum of the sending-end and the receiving-end voltages, but the error introduced by putting $E_{na} = \frac{1}{2}(E_n + V_n)$ is very small, and this leads to the formula

$$E = V\left[\frac{1 + 3\left(\frac{fL}{100,000}\right)^2}{1 - 3\left(\frac{fL}{100,000}\right)^2}\right] \qquad (118)$$

which gives the relation between the voltage (E) at the receiving end and the voltage (V) at the sending end of an unloaded line with a degree of accuracy sufficient for practical purposes.

The *regulation,* on the sine-wave assumption, is equal to *percentage line drop* + $6f^2L^2 \times 10^{-8}$, but the last term is negligible unless the distance of transmission (L) is great.

As an example of the application of formula (117), the percentage line drop in Example 24 of the preceding chapter was 17.5 with the specified load connected to the receiving end of the line. The percentage *rise* of pressure at the end of the unloaded line, due to the fact that the charging current remains even when the load is disconnected, is approximately $6 \times (60)^2 \times (100)^2 \times$

$10^{-8} = 2.16$, whence the *regulation* is $17.5 + 2.16 =$ (say) 20 per cent.

94. Control of Voltage on Transmission Lines.—The pressure drop at the receiving end of a transmission line may be compensated for by raising the voltage at the generating end as the load increases. There are obvious disadvantages to such a method of operation, and it is better, if possible, to regulate the voltage at the point, or points, where constant pressure is required. This is especially true of transmission lines on which there are substations or branch lines at intermediate points.

Transformers, or auto-transformers, provided with taps on the windings and suitable switches by which the winding ratio may be altered, would be economical and generally desirable if the adjustment of voltage with variation of load could be effected without large sudden variations of pressure and without interruption of the supply circuit. A dial switch with comparatively few contacts may be combined with an induction-type regulator by which sudden changes in the voltage may be avoided; but there are also on the market devices to regulate the voltage of transmission lines by changing the transformer winding ratio without disconnecting the transformer from service or interrupting the load. These devices usually involve the principle of transferring the load from one to another of two parallel circuits within the ratio-adjusting apparatus during the short interval of time required for shifting the contact-making switches. Carefully designed mechanism to operate the ratio adjuster and open or close the circuit breakers in the proper sequence is a necessary feature of these devices.[1]

Voltage control is also obtainable by the induction type of regulator (both single phase and polyphase), which may be operated by hand or automatically, and provides a smooth variation of voltage. These regulators are connected as "boosters" with their secondary windings in series with the line. Synchronous generators, either independently driven, or motor driven by power from the line itself, may also be used as voltage boosters; but a further and very important means of regulating the voltage of transmission lines is by means of any apparatus or device which will control the *power factor* of the load. The manner in which

[1] A description of installations using the method of changing transformer taps under load for regulating the voltage will be found in a paper by H. C. Albrecht in the *Jour.* A.I.E.E., December, 1925.

changes of power factor affect the receiving-end voltage will be explained in a later article.

95. Effect of Boosting Voltage at Intervals along a Transmission Line.—If a long transmission line, insulated for a maximum working pressure of, say, 100,000 volts, can be worked as a 100,000-volt line at all times through its entire length, it will be more efficient than if only a portion of it is working as a 100,000-volt transmission while portions farther from the generating end are working at (say) 80,000 volts. By installing boosters along the line to maintain the pressure at or near the maximum working value, whatever the load may be, economies may frequently be effected. It is true that the energy put into the line at intermediate points cannot be cheaper—and, indeed, is usually more costly—than the energy supplied to the line at the generating end; but the booster system allows of the pressure being kept

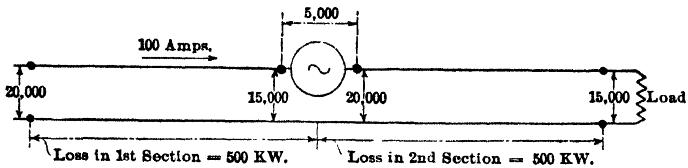


FIG. 107.—Method of maintaining pressure on long line.

up all along the line, thus effecting economy; provided always that the losses in the boosters themselves, their maintenance, and the necessary allowances for interest and depreciation do not counterbalance the saving.

As an example, consider a single-phase line conveying a current of 100 amp. at an initial pressure of 20,000 volts. Suppose the drop in pressure in the whole length of line to be as great as 10,000 volts; this will leave only 10,000 volts at the receiving end.

The power put into the line at the generating end = 2,000 kw.

The loss in the line = 1,000 kw.

The power available at the receiving end = 1,000 kw.

Hence, *efficiency of line* = 50 per cent.

Now imagine a booster to be introduced at a point halfway along the line. This booster may be considered as a suitably insulated alternator of 500-kw. capacity, capable of generating 100 amp. at 5,000 volts, the arrangement being as shown in Fig. 107. The drop in the first section of the line is, as before, 5,000 volts; and the drop in the second section is evidently similar—

namely, 5,000 volts—which means that the total amount of power dissipated in the line is the same as it was before the booster was introduced. But by providing this booster at the middle point of the line, it has been possible to raise the pressure at this point up to the initial value of 20,000 volts, with the result that 15,000 volts (1,500 kw.) are available at the receiving end. The additional power available for useful purposes has, of course, cost something to produce; but the point to be noted is this: By keeping up the pressure, it has been possible to transmit a greater amount of energy to the receiving end of the line *without increasing the losses in the conductors.* If, for the sake of simplicity, the losses in the booster are neglected, the line efficiency is arrived at thus:

Power supplied to the line = 2,000 + 500 = 2,500 kw.
Power lost in the line = 1,000 kw.
Power available at receiving end = 1,500 kw.

$$\text{Line efficiency} = \frac{1{,}500}{2{,}500} = 60 \text{ per cent.}$$

Boosters may be arranged to take their power from the generating end of the line; that is to say, they may take the form of

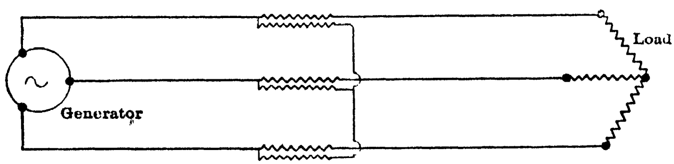


FIG. 108.—Transformers connected as "boosters" on transmission line.

variable-ratio transformers, with hand or automatic regulation, connected up as indicated in Fig. 108. Transformers so connected will provide the additional volts at the cost of a corresponding loss of current.

96. Control of Power Factor.—The advantages of operating alternating machinery and systems on unity power factor, when possible, are so well known that it will not be necessary to discuss the matter here. The line losses alone—as shown in Art. 9, Chap. I—are inversely proportional to the *square* of the power factor. Thus, if the total I^2R loss in the line were 200 kw. with a power factor of 0.707 (by no means an impossible figure in practice), this loss would be reduced to 100 kw. if the *same total power* could be transmitted at unity power factor.

The control of power factor is obtained by balancing any excess of inductive reactance with condensive reactance, or *vice versa.* With a varying load at the end of a long transmission line, the power factor at any given point on the line is continually changing, even if the power factor of the load remains constant; and the most convenient means of providing the reactance necessary to maintain a constant and improved power factor is to install synchronous machinery which can be made to draw leading or lagging currents from the line by over- or under-exciting the field magnets.

Bearing in mind that power-factor control is quite as important, if not more important, than voltage control so far as the losses and efficiency of transmission are concerned, the arrangement shown diagrammatically in Fig. 109 should be preferable to

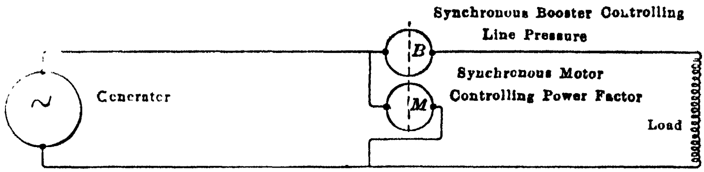


Fig. 109.—Control of voltage and power factor by means of synchronous machinery.

the arrangements of Figs. 107 and 108. Here B is a synchronous generator connected as a "booster" and provided with field regulation in order to control the *voltage.* It is direct coupled to the synchronous motor M, which is provided with field regulation in order to control the *power factor.*

Causes and Correction of Low Power Factor.—Lagging current components lower the power factor, and since all machinery or apparatus of which the magnetic field is produced by alternating currents drawn from the main system must have current components lagging 90 deg. behind the impressed e.m.f., it follows that every piece of apparatus of this type connected to the supply mains tends to lower the power factor of the system.

Transformers, induction motors, and all magnetic devices operated by alternating currents draw from the system lagging currents which increase the total current necessary to provide a given maximum amount of power, and, further, have the objectionable characteristic of demagnetizing the fields of the a.c. generators. The magnetizing component of the total current in a well-designed static transformer is but a small percentage of the total current; but in the induction motor the magnetizing current, even at full load, is appreciable, owing to the reluctance of the necessary air clearance between stator and rotor, which calls for a large number of magnetizing ampere-turns on the stator. The wattless current component in an induction motor may be between 25 and 30 per cent of the total current in the stator windings.

The greatly increased d.c. excitation of the generators because of lagging current components in the system may be thought of as supplying the magnetomotive force for those machines which are connected to the mains and require excitation, but are not provided with d.c. magnetizing windings. Thus, when the power factor of the load is low, the currents in the generator field windings may be considered as producing not only the flux necessary to generate the e.m.f. in the generator, but also as supplying the magnetomotive force which, by the agency of the wattless current, is transmitted to the machines and apparatus which require excitation but are not provided with d.c. field windings.

If the induced e.m.fs. which produce lagging current components are balanced by equal but opposite e.m.fs. which tend to produce leading current components, the power factor of the system may be unity, notwithstanding the low power factor of much of the apparatus connected to the lines. This suggests the use of electrostatic condensers to draw leading currents from the mains.

They may be used on small installations if they can be built of sufficient capacity at a reasonable cost; but they have not the advantage of synchronous machinery in which the amount and even the direction of the "reactive" power drawn from the transmission line may be regulated by varying the field excitation.

Synchronous motors (as in motor-generator sets and rotary converters, where a continuous-current load is required) and synchronous condensers, the latter being merely "idle" motors, may all be used for power-factor correction.

It is well known that a synchronous motor when under-excited will draw from the a.c. mains lagging components of current which will provide the ampere-turns of excitation to maintain the flux in the air gap at the required amount. It, therefore, follows that if the d.c. ampere-turns are increased beyond what will produce the correct strength of field to provide the necessary back e.m.f., a *leading* current component will be drawn from the supply mains to demagnetize the field and keep it of the required strength.

A small additional kilovolt-ampere capacity will render any synchronous motor available for this service. If there are no synchronous motors forming part of the load of a power system, synchronous condensers may be provided of sufficient capacity to supply, with over-excited fields, the e.m.f. necessary to annul the lagging components of current in the system. In such a case it is important to study the economics of the situation to be certain that the expenditure on such machinery will be justified on economic grounds.

It is not always necessary to bring the power factor up to unity, as a power factor in the neighborhood of 0.9 is not usually very objectionable. The very low power factors are, however, uneconomical and undesirable. The rotary converter—which is, of course, available only where continuous currents are needed—has a high efficiency and serves admirably to correct the power factor of the system.

A large wattless current may be needed to change the power factor from 0.9 to unity, but only about one-quarter of this amount will change the power factor from 0.8 to 0.9. That is why it may not always be desirable to aim at improving the power factor beyond about 0.9.

97. Maintaining Constant Voltage by Control of Power Factor. Figure 110 is similar to the fundamental regulation diagram Fig 102, except that the components of the total pressure drop which are due, respectively, to the resistance and the reactance of the line have been drawn separately for the "in-phase" and "wattless" components of the total line current. Thus, if E_n, I, and $\cos\theta$ stand, respectively, for the "star" voltage, the load current, and the power factor at the receiving end, the IR and IX drops due to the "in-phase" component (ON) of the current are MB and HM respectively. Consider, now, a short line of negligible capacity, and connect at the receiving end, in parallel with the load of power factor $\cos\theta$, synchronous reactors capable of providing, by variation of field excitation, any desired amount of "wattless" current. Let the vector OK of Fig. 110 represent the amount of (leading) "wattless" current supplied by this phase-modifying machinery; then I_s is the total current in the line, and $I_a = NS = OK + NA$[1] is the reactive component of the line current. The effect of this

[1] The quantity $NA = I \sin\theta$, being *negative* when the power factor of the load is *lagging*.

component of the line current upon the voltage regulation is indicated by the triangle HGD wherein $GH = I_aR$ is the resistance drop, and $DG = I_aX$ is the reactance drop. The point D is the end of the vector $OD = V_n$ which is the required voltage at the sending end of the line. In order to have constant voltage at both ends of the line, it follows that if $OB = E_n$ is the constant voltage (line to neutral) at the receiving end, the point D of the vector representing the voltage (line to neutral) at the sending end, must always fall on the circle of radius $OD = V_n$, whatever may be the load connected across the line at the receiver end. The diagram of Fig. 110 shows the relation between the various voltage vectors for a total load equal to $3E_nI \cos \theta$ watts.

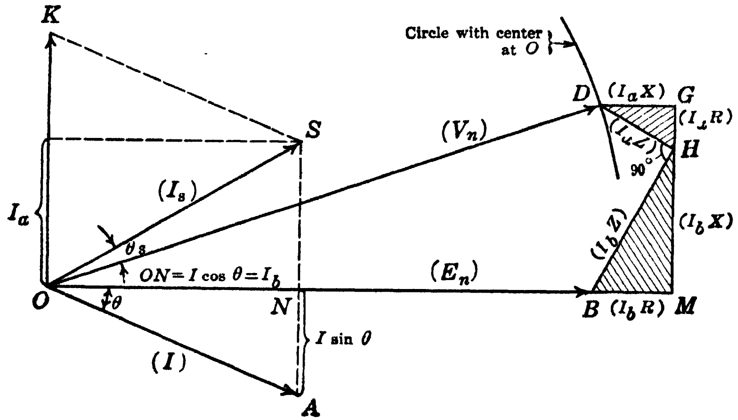


FIG. 110.—Vector diagram illustrating effect of reactors in maintaining constant voltage.

The current component in phase with the voltage is $ON = I_b = I \cos \theta$, and the hypotenuse HB of the triangle BMH is the impedance drop due to this component of the total current. In order that the point D shall fall on the circle of radius $OD = V_n$, it is necessary that a reactive component (I_a) of the total current be provided of such a value that the impedance drop due to this component shall be proportional to the length of the vector DH. The impedance per conductor is $Z = \sqrt{R^2 + X^2}$, where R and X stand, respectively, for the resistance and reactance of one wire of the transmission line,[1] and the reactive component

[1] In preceding chapters the symbols R and X have usually stood for the resistance in ohms *per mile* of conductor.

of the line current must, therefore, be $I_a = \frac{\text{length } DH}{Z}$. The reactive current to be supplied by phase-modifying synchronous reactors connected across the line at the receiving end is $OK = I_a - I \sin \theta$, wherein $\sin \theta$ is *negative* when the load power factor is *lagging* as shown in Fig. 110.

The above explanation and diagram illustrate the principle of constant-voltage transmission obtained by controlling the power factor to prevent changes in the voltage notwithstanding variations in the load at the receiving end of the line. By drawing the vector diagram of Fig. 110 accurately to scale, the length DH can be measured and the required kilovolt-ampere capacity of synchronous condensers calculated therefrom. This takes up very little time and is generally preferable to the use of formulas. The length DH divided by the impedance of the conductor is the "wattless" component I_a of the line current. The total kilovolt-ampere capacity of the synchronous machinery necessary to maintain constant voltage, assuming $OA = I$ to be the maximum amount of current required to supply the load, will be

$$\left.\begin{array}{r}\text{Kilovolt-ampere capacity}\\ \text{of synchronous reactors}\end{array}\right\} = \frac{3E_n(I_a - I \sin \theta)}{1{,}000} \qquad (119)$$

It is necessary to bear in mind that $\sin \theta$ is a *negative* quantity when the power factor of the load ($\cos \theta$) is *lagging*. When the quantity as calculated by formula (119) is *negative*, this indicates that *a lagging* component of current must be supplied by the synchronous reactors.

98. Circle Diagram for Constant-potential Lines.—The circle diagram of Fig. 110, which shows the relation between *voltage* vectors, may be modified with advantage to represent *current* vectors. For a given current component I_b which, when multiplied by $3E_n$, gives the true watts taken by the load on a three-phase transmission, it is desired to calculate the "reactive" component I_a of the total line current which will maintain the condition of constant voltage at both ends of the line. If the vectors of Fig. 110 are divided by the impedance $Z = \sqrt{R^2 + X^2}$, the lines BH and HD will represent the "active" and "reactive" components of the line current, respectively. By turning around the diagram so that the (horizontal) axis of reference is the line BH of Fig. 110 the modified circle diagram of Fig. 111, is obtained. Note that the triangles OCP and NOM

are similar, whence the coordinates of the center (C) of the circle are seen to be

$$m = PC = -\frac{E_n R}{Z^2} \tag{120}$$

$$n = OP = \frac{E_n X}{Z^2} \tag{121}$$

The radius of the circle (sending-end voltage) is

$$r = CS = \frac{V_n}{Z} \tag{122}$$

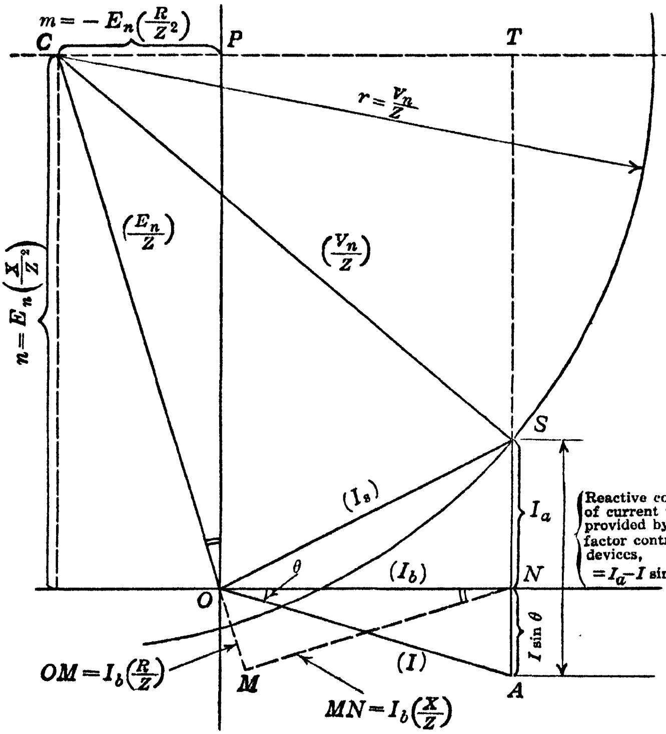


FIG. 111.—Diagram of current vectors for transmission at constant voltage.

If it is desired to calculate the amount of the necessary reactive current component without drawing the diagram to scale, write

$$\begin{aligned} I_a = NS &= NT - ST \\ &= NT - \sqrt{(CS)^2 - (CT)^2} \\ &= n - \sqrt{r^2 - (I_b - m)^2} \end{aligned} \tag{123}$$

where n, r, and m have the values as given above, and $I_b = I \cos \theta$. (A *positive* value for I_a indicates a *leading* reactive current component as shown in Fig. 111.)

The line current is $I_s = \sqrt{I_a^2 + I_b^2}$

The line losses, in kilowatts, are $\frac{3(I_a^2 + I_b^2)R}{1,000}$, and the line efficiency[1] is

$$\eta = \frac{\text{output}}{\text{input}} = \frac{E_n I_b}{E_n I_b + (I_a^2 + I_b^2)R}$$

or

$$\eta = 1 - \frac{(I_a^2 + I_b^2)R}{E_n I_b + (I_a^2 + I_b^2)R} \tag{124}$$

Example 26. *Constant-voltage Transmission.*—Assume transmission line data as follows:

Distance of transmission = 20 miles
System; three-phase (three conductors)
Frequency, $f = 60$
Conductors; copper cable No. 0000
Equivalent spacing, $d = 60$ in.
Voltage at the sending end, $V = 41,000$
Voltage at the receiving end, $E = 40,000$

It is desired to determine (*a*) the kilovolt-ampere capacity of synchronous phase-modifiers at the receiving end to maintain constant voltage, and (*b*) the line efficiency, under the following load conditions,

1. 12,000 kw. with power factor 0.85 (lagging)
2. 2,000 kw. with power factor 0.75 (lagging)

These loads are supposed to include the transformers at the receiving end of the line. The effect of charging current in this short line is small and will be neglected. The temperature of the line conductors is assumed to be 68°F.

The "star" voltages (line to neutral) are

$$\text{At the sending end, } V_n = \frac{41,000}{\sqrt{3}} = 23,700$$

$$\text{At the receiving end, } E_n = \frac{40,000}{\sqrt{3}} = 23,100$$

The "in-phase" component of the load current is

$$\text{For load 1, } I_b = \frac{12,000,000}{3 \times 23,100} = 173 \text{ amp.}$$

$$\text{For load 2, } I_b = 28.9 \text{ amp.}$$

[1] See Art. 15, Chap. II.

The quantities needed for the solution of equation (123) are obtained as follows:

From wire table on page 81, $R = 0.272 \times 20 = 5.44$ ohms. By formula (96), page 244, or chart on page 243,

$$X = 0.69 \times 20 = 13.8 \text{ ohms}$$

Therefore

$$Z = \sqrt{(5.44)^2 + (13.8)^2} = 14.82 \text{ ohms}$$

By formula (120),

$$m = -\frac{23{,}100 \times 5.44}{(14.82)^2} = -572$$

By formula (121),

$$n = \frac{23{,}100 \times 13.8}{(14.82)^2} = 1{,}450$$

By formula (122),

$$r = \frac{23{,}700}{14.82} = 1{,}600 \text{ (nearly)}$$

By formula (123), the required reactive component of the line current for the load condition 1 is

$$I_a = 1{,}450 - \sqrt{(1{,}600)^2 - (173 + 572)^2} = 35 \text{ amp.}$$

which, being positive, is a *leading* reactive current component. The (lagging) reactive current component of the load is

$$I \sin \theta = I \cos \theta \times \tan \theta = I_b \tan \theta = 173\ (-0.62)$$
$$= -107 \text{ amp. (approximately)}$$

Therefore, by formula (119), the necessary kilovolt-ampere output of the synchronous reactors, connected in parallel with the load, is

$$\frac{3 \times 23{,}100(35 + 107)}{1{,}000} = 9{,}850 \text{ kv.-a. (leading)}$$

By formula (124), the line efficiency is

$$1 - \frac{[(35)^2 + (173)^2]5.44}{23{,}100 \times 173 + [(35)^2 + (173)^2]5.44} = 0.959$$

For the load condition 2,

$$I_a = 1{,}450 - \sqrt{(1{,}600)^2 - (28.9 + 572)^2}$$
$$= -31 \text{ amp. (a } lagging \text{ reactive current component)}$$

Proceeding in the same manner as when calculating the conditions for load 1,

$$\left.\begin{array}{l}\text{Reactive kilovolt-amperes to be provided}\\ \quad\text{by synchronous reactors}\end{array}\right\} = 380 \text{ kv.-a. (lagging)}$$

$$\text{Line efficiency} = 0.9856$$

In these calculations, the power losses in the phase-modifying machines have been neglected. The efficiency of synchronous condensers is high, the losses being usually below 4 per cent of the kilovolt-ampere rating for large units of about 2,000 kv.-a. size, increasing to 5 per cent for 1,000 kv.-a. units, and 6 per cent for 500 kv.-a. units.

It may be well to point out that Example 26 serves merely to illustrate how the required kilovolt-ampere capacity of synchronous reactors may be calculated; but whether or not it would pay to install such machinery rather than use other means of maintaining constant voltage at the receiving end of this short line is an economic problem which can be solved without much difficulty if the annual costs—not only of the necessary apparatus or machinery, but also of the power wasted in the transmission line and regulating devices—are known.

When making calculations for longer transmission lines in which the electrostatic capacity between conductors is not negligible, it is obvious that, since the charging current due to the capacity of the line is a leading reactive component of the total current, less leading current will have to be provided by the power-factor-correcting machinery. The problem is essentially similar to the one solved in the preceding numerical example, but the voltage, current, and power factor at the point where the synchronous condensers are to be installed must first be determined as explained in Art. 91, Chap. IX, and the additional reactive component of current (whether leading or lagging) required to maintain constant voltage at the sending end of the section must then be calculated.

By installing synchronous reactors of the required kilovolt-ampere capacity at intervals of about 100 miles on a long transmission line, it is possible to operate the entire line at practically uniform voltage regardless of the amount and power factor of the load.

When a demand for power exists at intermediate points on the line it is often possible to install synchronous motors which may be used to control the power factor (and voltage); but at the end of the line, synchronous reactors are usually preferable.

In the case of tie lines interconnecting generating stations, the voltage is usually approximately the same at both ends, and the synchronous condensers, by controlling the power factor at one end of the line, serve to distribute the load in any desired manner between the two stations.

The importance of voltage regulation by the aid of synchronous machinery which alters the power factor according to the load should not be over-emphasized, and alternative methods should always be considered and costs compared; but the use of synchronous reactors properly located and operated, may effect great savings on large high-voltage a.c. systems, as their extensive use on the lines of the Southern California Edison Company seems to indicate. It should not be overlooked that the synchronous machinery needs only to take care of the *changes* which occur between minimum load and maximum load conditions. An appreciable voltage drop in transmission at light loads, when the power factor is usually low because of the relatively greater importance of transformer exciting current components, tends to reduce the *difference* between the full-load and the light-load drop of pressure which has to be taken care of by the voltage-regulating devices.

99. Electrical Calculations for Long Transmission Lines.—In Art. 91, Chap. IX, formulas were developed for calculating the conditions at the sending end of a transmission line in terms of the conditions at the receiving end, on the assumption that the whole of the line capacity was shunted across the load. It was further shown, by dividing a long line into a number of sections, each with a condenser of proportionately smaller capacity connected across the receiving end of the section, how the conditions at the sending end of the line could readily be calculated with an accuracy sufficient for practical purposes. It is necessary merely to consider the number of such sections to be increased without limit, in order to derive formulas which represent exactly the conditions in a long transmission line with constant resistance, inductance, and capacity per unit length of the line.

The diagram Fig. 112 represents a single wire of a long transmission line in connection with which the constants and other quantities are defined as follows:

E_n = voltage (line to neutral) at the receiving end.
V_n = voltage (line to neutral) at the sending end.

E = voltage (line to neutral) at any point P situated l miles from the receiving end.
I = amperes of current at the point P.
I_r = amperes of current per conductor at the receiving end.
I_s = amperes of current per conductor at the sending end.
R = resistance per mile of conductor (ohms).
X = reactance per mile of conductor (ohms).
Z = impedance per mile of conductor (ohms), $Z = R + jX$.
G = shunted leakage conductance (line to neutral) per mile (mhos)[1] being the "active" component of the *admittance.*
B = shunted capacity susceptance (line to neutral) per mile (mhos), being the "reactive" component of the *admittance.*
Y = shunted admittance (line to neutral) per mile (mhos), $Y = G + jB$.
L = length of transmission line (miles).

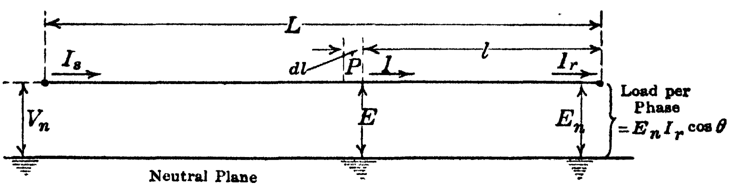


FIG. 112.—Diagram representing single conductor of long transmission line.

At any point P on the line at a distance l from the receiving end, consider an element of length dl. The difference in potential between the two ends of this section is

$$d\dot{E} = \dot{I}Zdl$$

whence

$$\frac{d\dot{E}}{dl} = \dot{I}Z \tag{125}$$

in which Z is a constant.

The change in the current is

$$d\dot{I} = \dot{E}Ydl$$

whence

$$\frac{d\dot{I}}{dl} = \dot{E}Y \tag{126}$$

[1] This is the conductance of the current paths over insulator surfaces and also through the air when the voltage is so high that the losses due to *corona* are appreciable. These current losses are very small on practical lines under normal working conditions, and may usually be neglected.

in which Y is a constant. Differentiating with respect to l,

$$\frac{d^2\dot{I}}{dl^2} = Y\frac{d\dot{E}}{dl} \tag{127}$$

Substituting (125) in (127),

$$\frac{d^2\dot{I}}{dl^2} = (YZ)\dot{I} \tag{128}$$

Proceeding in a similar manner for change of voltage, the corresponding differential equation is found to be

$$\frac{d^2\dot{E}}{dl^2} = (YZ)\dot{E} \tag{129}$$

These are the two fundamental equations for long transmission lines with uniformly distributed resistance, inductance, capacity, and leakage. The solution of equation (128) or (129) is given in any book on differential equations, and also in several textbooks on the theory of alternating currents.[1] The following forms of the solution are convenient in connection with power transmission line calculations. When the voltage and current conditions at the receiving end are known:

$$\dot{V}_n = \dot{E}_n \cosh L\sqrt{YZ} + \dot{I}_r\sqrt{\frac{Z}{Y}} \sinh L\sqrt{YZ} \tag{130}$$

$$\dot{I}_s = \dot{I}_r \cosh L\sqrt{YZ} + \dot{E}_n\sqrt{\frac{Y}{Z}} \sinh L\sqrt{YZ} \tag{131}$$

When the assumed or known conditions are at the sending end of the line, the equations take the form

$$\dot{E}_n = \dot{V}_n \cosh L\sqrt{YZ} - \dot{I}_s\sqrt{\frac{Z}{Y}} \sinh L\sqrt{YZ} \tag{132}$$

$$\dot{I}_r = \dot{I}_s \cosh L\sqrt{YZ} - \dot{V}_n\sqrt{\frac{Y}{Z}} \sinh L\sqrt{YZ} \tag{133}$$

Although these formulas are simple in appearance, it should be noted that Y and Z are complex quantities with both real and imaginary components, and the hyperbolic sines and cosines in the formulas cannot be read directly from tables of hyperbolic functions. The following article will explain how these equations may be solved, both with and without the aid of tables of hyper-

[1] See MAGNUSSON, C. E. "Alternating Currents," McGraw-Hill Book Company, Inc.; also PERNOT, F. E., "Electrical Phenomena in Parallel Conductors," John Wiley & Sons, Inc.; and STEINMETZ, C. P., "Transient Phenomena," McGraw-Hill Book Company, Inc.

bolic functions; but if the reader has any difficulty in using the so-called exact formulas for determining the conditions at one end of a transmission line in terms of the given or assumed conditions at the other end, he should use the method of Art. 91, Chap. IX, which, as previously mentioned, gives results of which the accuracy is quite sufficient for practical purposes.

100. Solution of Exact Equations for Long Lines. Transformation Formulas.—Hyperbolic sines or cosines may be expressed in exponential form, or in the form of a convergent series. Thus

$$\left.\begin{aligned}\sinh u &= \frac{1}{2}(\epsilon^{u} - \epsilon^{-u})\\ \cosh u &= \frac{1}{2}(\epsilon^{u} + \epsilon^{-u})\end{aligned}\right\} \qquad (134)$$

But

$$\epsilon^{u} = 1 + u + \frac{u^2}{1\cdot 2} + \frac{u^3}{1\cdot 2\cdot 3} + \frac{u^4}{1\cdot 2\cdot 3\cdot 4} + \cdots \qquad (135)$$

whence

$$\sinh u = u + \frac{u^3}{\lfloor 3} + \frac{u^5}{\lfloor 5} + \frac{u^7}{\lfloor 7} + \cdots \qquad (136)$$

$$\cosh u = 1 + \frac{u^2}{\lfloor 2} + \frac{u^4}{\lfloor 4} + \frac{u^6}{\lfloor 6} + \cdots \qquad (137)$$

By using the symbols K_A, K_B, and K_C for the three constants in equations (130) to (133), and making the proper substitutions,

$$K_A = \cosh L\sqrt{YZ} = 1 + \frac{(L^2YZ)}{2} + \frac{(L^2YZ)^2}{24} + \frac{(L^2YZ)^3}{720} + \cdots \qquad (138)$$

$$K_B = \sqrt{\frac{Z}{Y}}\sinh L\sqrt{YZ} = LZ\left[1 + \frac{(L^2YZ)}{6} + \frac{(L^2YZ)^2}{120} + \frac{(L^2YZ)^3}{5{,}040} + \cdots\right] \qquad (139)$$

$$K_C = \sqrt{\frac{Y}{Z}}\sinh L\sqrt{YZ} = LY\left[1 + \frac{(L^2YZ)}{6} + \frac{(L^2YZ)^2}{120} + \frac{(L^2YZ)^3}{5{,}040} + \cdots \right. \qquad (140)$$

The numerical values of these expressions for the constants K_A, K_B, and K_C could easily be calculated either by referring to tables of exponentials or hyperbolic functions, or by using the convergent series and including as many terms as might be required, *if Y and Z were simple numbers.* But, in a.c. transmission problems, both Y and Z are complex quantities, a fact which adds considerably to the labor of solving problems by means of the formulas (130) to (133).

Hyperbolic Functions of Complex Quantities.—Since Y and Z are vector quantities, they may be expressed by complex numbers. Thus, the quantity $L\sqrt{YZ}$, which is known as the *propagation constant* or *attenuation factor* of the line, may be written

$$L\sqrt{YZ} = L\sqrt{(G + jB)(R + jX)} \tag{141}$$

and $\sqrt{\frac{Z}{Y}}$, which is known as the *surge impedance* of the line, may be written

$$\sqrt{\frac{Z}{Y}} = \sqrt{(R + jX) \div (G + jB)} \tag{142}$$

The square root of the product of two vector quantities is also a vector quantity, and it will be necessary, therefore, to obtain the hyperbolic sine and cosine of a vector quantity which may be expressed either in terms of rectangular coordinates as $(u + jv)$ or in terms of polar coordinates as $r\underline{/\theta}$, where r is the length or size of the vector and θ is the angle between the vector and the axis of reference, being frequently referred to as the "slope" of the vector. (In order to convert degrees into radians, multiply by π and divide by 180.)

Although tables of complex hyperbolic functions are available,[1] they should preferably be used only by those who have a thorough understanding of their mathematical basis and are familiar with their use. Moreover, since an enormous number of values would have to be included in order to cover all possible combinations of the components u and v in the expression $(u + jv)$, it is usually necessary to resort to interpolation which is not only tedious, but likely to introduce errors when attempted by those who have not thoroughly familiarized themselves with the methods by constant use of the tables.

Since any one accustomed to the ordinary trigonometric tables can also use tables of hyperbolic functions of simple

[1] KENNELLY, A. E., "Tables of Complex Hyperbolic and Circular Functions," Harvard University Press; also COHEN'S "Formulæ and Tables for the Calculation of Alternating Current Problems," (Table XX), McGraw-Hill Book Company, Inc. This table is taken from MILLER, W. E., "Transmission Line Problems," *Gen. Elec. Rev., Supplement,* May, 1910.

numbers,[1] it is generally advisable to express the hyperbolic sines and cosines of complex quantities in the following forms:

$$\left.\begin{aligned}\sinh\,(u \pm jv) &= \sinh\,u\,\cos\,v \pm j\,\cosh\,u\,\sin\,v\\ \cosh\,(u \pm jv) &= \cosh\,u\,\cos\,v \pm j\,\sinh\,u\,\sin\,v\end{aligned}\right\}\quad(143)$$

Before illustrating the use of the "exact" formulas for transmission lines by means of numerical examples, a few words about the handling of vector quantities (including j terms) may not be out of place. A term preceded by the symbol j is a vector component at right angles to the axis of reference, *i.e.*, to the "real" component of the complex quantity. When adding or subtracting vector quantities, it is necessary, therefore, to keep "real"

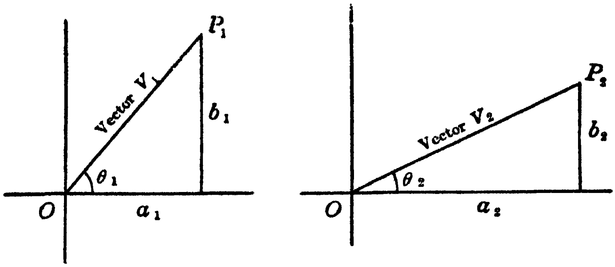


FIG. 113.—Two vector quantities in the same plane.

terms and "imaginary" or j terms separate. Thus, the summation of the two vector quatities of Fig. 113 is carried out as follows:

$$\begin{aligned}\dot{V}_1 &= +a_1 + jb_1\\ +\dot{V}_2 &= +a_2 + jb_2\\ \hline \text{sum }(\dot{V}_1 + \dot{V}_2) &= (a_1 + a_2) + j(b_1 + b_2)\end{aligned}$$

The simple rules of algebra apply to the multiplication and division of vector quantities, bearing in mind that $j = \sqrt{-1}$. For performing these operations, however, it is generally preferable to express the vector quantities in polar coordinates. Thus, the *size* of the vector $\dot{V}_1$ is the length $OP_1 = \sqrt{a_1^2 + b_1^2}$, and its *slope* is $\theta_1 = \tan^{-1}\left(\frac{b_1}{a_1}\right)$, whence, in polar form,

$$\dot{V}_1 = V_1 \underline{/\theta_1}$$

[1] A table of real hyperbolic functions—that is to say, functions in which the component jv in the complex quantity $(u + jv)$ is equal to zero—is included in KENNELLY's "Tables of Complex Hyperbolic and Circular Functions." A very complete table of the logarithms of real hyperbolic functions is included in PERNOT's "Electrical Phenomena in Parallel Conductors." Values and logarithms of hyperbolic functions are given in "Logarithmic and Trigonometric Tables," The Macmillan Company.

Similarly,

$$\dot{V}_2 = V_2 \angle \theta_2$$

To obtain the product of one plane vector by another, *multiply* their *sizes* and *add* their *slopes*. Thus, the product of $\dot{V}_1$ and $\dot{V}_2$, expressed in polar coordinates, is

$$(V_1 \times V_2)\angle \theta_1 + \theta_2$$

where $$V_1 = \sqrt{a_1^2 + b_1^2}$$

and $$V_2 = \sqrt{a_2^2 + b_2^2}$$

Applying this rule to the process of involution,

$$(\dot{V}_1 \dot{V}_2)^n = (V_1 \times V_2)^n \angle n(\theta_1 + \theta_2)$$

To divide one vector by another, *divide* their *sizes* and *subtract* their *slopes*. Thus, the ratio of $\dot{V}_1$ to $\dot{V}_2$, expressed in polar coordinates, is

$$\left(\frac{V_1}{V_2}\right)\angle \theta_1 - \theta_2$$

Example 27. Electrical Calculations Using Formulas Based on Uniformly Distributed Capacity.—It is proposed to use the same data as in Example 24, Chap. IX, in order that the results obtained by using the exact formulas may be compared with the results of calculations based on the assumption of the total line capacity concentrated at a point midway between the sending and the receiving ends of the transmission line. The known quantities are

Three-phase transmission at 60 cycles, $f = 60$
Length of line, $L = 100$ miles,
Load = 22,000 kv.-a
$E_n = 50{,}810$ volts (wire to neutral at the receiving end)
$I_r = 144.5$ amp. (current per conductor at the receiving end)
$\cos\theta = 0.8$ lagging (power factor of load)
$R = 0.272$ ohm
$X = 0.762$ ohm
$B = 5.6 \times 10^{-6}$ mhos
(R, X, B: per mile of single conductor)

In order to calculate the sending-end voltage ($\dot{V}_n$) and current ($\dot{I}_s$), use the formulas (130) and (131) which may be written:

$$\dot{V}_n = K_A\dot{E}_n + K_B\dot{I}_r \qquad (130a)$$

and

$$\dot{I}_s = K_A\dot{I}_r + K_c\dot{E}_n \qquad (131a)$$

where K_A, K_B, and K_C are complex quantities of which the values are given by formulas (138), (139), and (140). It will, therefore, be necessary to calculate the line impedance Z, the shunted admittance Y, and the product YZ. The impedance in ohms per mile is

$$Z = R + jX$$
$$= 0.272 + j0.762$$

which, in terms of polar coordinates becomes

$$Z = \sqrt{(0.272)^2 + (0.762)^2}\,\underline{/\tan^{-1}\frac{0.762}{0.272}}$$
$$= 0.809\underline{/70° \; 20'} \qquad (a)$$

The shunted admittance in mhos per mile is

$$Y = G + jB$$

but since the leakage conductance G is so small as to be negligible, put $G = 0$, whence

$$Y = 0 + jB = +j5.6 \times 10^{-6}$$

In terms of polar coordinates,

$$Y = 5.6 \times 10^{-6}\underline{|90°} \qquad (b)$$

The product of (a) and (b) is

$$YZ = (0.809 \times 5.6 \times 10^{-6})\underline{/70° \; 20' + 90°}$$
$$= 4.53 \times 10^{-6}\underline{\diagdown 160° \; 20'} \qquad (c)$$

The *propagation constant* of the line is

$$L\sqrt{YZ} = 100\sqrt{4.53 \times 10^{-6}}\underline{/\tfrac{1}{2}(160° \; 20')}$$
$$= 0.213\underline{/80° \; 10'} \qquad (d)$$

which, in complex notation becomes

$$L\sqrt{YZ} = 0.213 \cos 80° \; 10' + j0.213 \sin 80° \; 10'$$
$$= 0.0364 + j0.21 \qquad (e)$$

The *surge impedance* is

$$\sqrt{\frac{Z}{Y}} = \sqrt{\frac{0.809\underline{/70° \; 20'}}{5.6 \times 10^{-6}\underline{|90}}}$$
$$= \sqrt{0.1445 \times 10^6\underline{\diagdown 19° \; 40'}}$$
$$= 380\underline{\diagdown 9° \; 50'} \qquad (f)$$

Using the formulas (143) to calculate the hyperbolic sine and cosine of $L\sqrt{YZ}$

$$K_A = \cosh L\sqrt{YZ} = \cosh\,(0.0364 + j0.21)$$
$$= (\cosh 0.0364)(\cos 0.21) + j(\sinh 0.0364)(\sin 0.21)$$

Note that 0.21 radian $= \dfrac{0.21 \times 180}{\pi} = 12°$

$$
\begin{aligned}
\text{whence } K_A &= (1.0007 \times 0.978) + j(0.0364 \times 0.208) \\
&= 0.979 + j0.00757 \qquad (g) \\
&= 0.979 \underline{/0° \; 27'}
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
\sinh L\sqrt{YZ} &= (\sinh 0.0346)(\cos 0.21) + j(\cosh 0.0364)(\sin 0.21) \\
&= 0.0356 + j0.208 \\
&= 0.211 \underline{/80° \; 17'} \qquad (h)
\end{aligned}
$$

whence, by formula (139),

$$
\begin{aligned}
K_B &= (380 \times 0.211) \underline{/-9° \; 50' + 80° \; 17'} \\
&= 80.2 \underline{/70° \; 27'} \qquad (i)
\end{aligned}
$$

Also, since $$\sqrt{\frac{Y}{Z}} = \frac{1}{\sqrt{\frac{Z}{Y}}}$$

by formula (140),

$$
\begin{aligned}
K_C &= \frac{0.211 \underline{/80° \; 17'}}{380 \overline{\diagdown 9° \; 50'}} \\
&= 0.000555 \underline{\diagdown 90° \; 7'} \qquad (j)
\end{aligned}
$$

Solving for the vectors $\dot{V}_n$ and $\dot{I}_s$ by formulas (130*a*), and (131*a*), and using the direction of the vector E_n as the axis of reference,

$$\dot{E}_n = 50{,}810 \underline{/0° \; 0'}$$

and

$$
\begin{aligned}
\dot{I}_r &= 144.5 \underline{/\cos^{-1} 0.8 \text{ lagging}} \\
&= 144.5 \overline{\diagdown 36° \; 53'}
\end{aligned}
$$

whence, by (130*a*),

$$
\begin{aligned}
\dot{V}_n &= [(0.979 \times 50{,}810) \underline{/0° \; 27'}] + [(80.2 \times 144.5) \underline{/70° \; 27' - 36° \; 53''} \\
&= 49{,}700 \underline{/0° \; 27'} + 11{,}580 \underline{/33° \; 34'} \\
&= [49{,}700 \cos 0° \; 27' + 11{,}580 \cos 33° \; 34'] + j[49{,}700 \sin 0° \; 27' + 11{,}580 \sin 33° \; 34'] \\
&= 59{,}350 + j6{,}810
\end{aligned}
$$

The angle ("slope") of the vector $\dot{V}_n$ is $\tan^{-1} \dfrac{6{,}810}{59{,}350} = 6° \; 33$ in advance of the receiving-end voltage $\dot{E}_n$.

The length ("size") of $\dot{V}_n$ is $59{,}350 + \dfrac{(6{,}810)^2}{2 \times 59{,}350} =$ 59,740 volts[1] (*k*)

[1] Calculated by the convenient approximate formula for solving $\sqrt{a^2 + b^2}$ when b is small in relation to a. See footnote on p. 89.

The current in the line at sending end, by formula (131a), is

$$\dot{I}_s = [(0.979 \times 144.5)\underline{|0° 27 - 36° 53}] + [(0.000555 \times 50{,}810)\underline{|90° 7'}]$$

$$= 114 - j55.9$$

The "slope" of the vector $\dot{I}_s$ is $\tan^{-1} \dfrac{-55.9}{114} = 26° 6'$ lagging behind the receiving-end voltage.

The "size" of the vector $\dot{I}_s$ is $\dfrac{114}{\cos 26° 6'} = 127$ amp.

The relative positions of the vectors representing sending-end and receiving-end conditions are shown in the diagram Fig. 114,

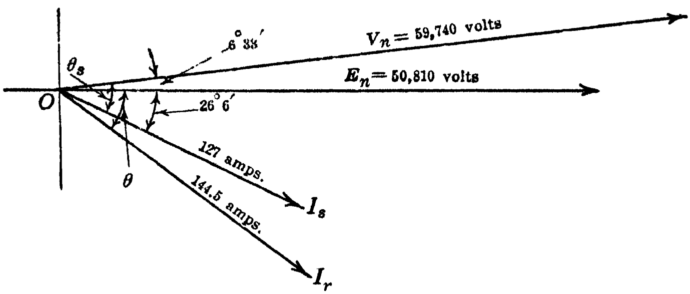


FIG. 114.—Vector diagram illustrating Example 27.

from which it is seen that the power factor at the sending end of the line is

$$\cos \theta_s = \cos (6° 33' + 26° 6') = 0.842 \text{ (lagging)} \qquad (l)$$

The line losses will be the difference between the power put in at the sending end and the power taken out at the receiving end; thus,

Kilowatts per phase at the sending end =

$$V_n I_s \cos \theta_s = 59.74 \times 127 \times 0.842 = 6{,}380$$

Kilowatts per phase of the receiving end =

$$E_n I_r \cos \theta = 50.81 \times 144.5 \times 0.8 = 5{,}870$$

$$\text{Difference} = I^2R \text{ loss per conductor} = 510 \text{ kw.}$$

whence the total full-load transmission losses are

$$510 \times 3 = 1530 \text{ kw.} \qquad (m)$$

The line drop per phase is

59,740 − 50,810 = 8,930 volts, or 17.55 per cent of the receiving-end voltage (n)

Comparing the results as given by (*k*), (*l*), (*m*), and (*n*) with the solutions obtained by the approximate method of Example 24, it is seen that there is no appreciable difference in the sending-end voltage and, therefore, in the percentage line drop. The actual line losses are 1.8 per cent greater than as indicated by the approximate calculations, but both methods of calculation give practically the same value for the power factor at the sending end of the line. This close agreement between the two methods of calculation was to be expected since, as previously mentioned, the use of the "exact" hyperbolic or equivalent formulas is an unnecessary refinement except in the case of very long transmission lines.

Example 28. *Calculation of Constants K_A, K_B, and K_C without Reference to Tables of Hyperbolic Functions.*—By using two or more of the terms in the convergent series, the constants as given by formulas (138), (139), and (140) may be calculated without using tables of hyperbolic functions. As an example, calculate K_B, using the data of the preceding Example 27.

By formula (139),

$$K_B = LZ\left[1 + \frac{(L^2YZ)}{6} + \frac{(L^2YZ)^2}{120} + \cdots\right]$$

The length of the transmission line is $L = 100$, whence

from (*a*) of Example 27, $LZ = 80.9\underline{/70^\circ\ 20'}$

from (*c*) of Example 27, $L^2YZ = 0.0453\ \overline{\backslash}\ \underline{160^\circ\ 20'}$

Computing first the quantity in brackets by which the impedance LZ has to be multiplied:

First term

(expressed in complex notation) $= +1.00000 + j0.00000$

Second term:

$$\frac{L^2YZ}{6} = 0.00755\ \backslash\underline{160^\circ\,20'} = -0.00711 + j0.00254$$

Third term:

$$\frac{(L^2YZ)^2}{120} = \frac{(0.0453)^2}{120}\ \underline{/2(160^\circ\ 20')}$$

$$= 0.0000171\ \backslash\underline{39^\circ\ 20'} = +0.00001 - j0.00001$$

Total $= +0.9929 + j0.00253$

The "slope" of this vector quantity is

$$\tan^{-1}\frac{253}{99{,}290} = 0^\circ\ 8'$$

and since the "size" of this vector is approximately the same as the "real" component,

$$K_B = (80.9 \times 0.993)\underline{/70° \, 20' + 0° \, 8'}$$
$$= 80.3 \underline{/70° \, 28'}$$

which checks as closely with the value calculated by the hyperbolic formulas as should be expected, seeing that the multiplications and divisions have been made on a 10-in. slide rule. It will be seen that the third term, involving the *square* of (YZ) is negligible, although it is usually desirable to include this term when making calculations on lines of 200 miles or more in length. Only in very rare instances is it necessary to include the fourth term, involving the *cube* of (YZ), and then only when calculating the constant K_A (formula (138)) for which the series converges less rapidly than for the other two constants.

Example 29. *Open-circuit Conditions on Long Transmission Line.*—Given a 60-cycle three-phase transmission line 300 miles long consisting of three copper conductors of 500,000 circular mils cross-section, with equivalent spacing of 16 ft. and a pressure of 88,000 volts between wire and neutral at sending end of line; calculate

1. The voltage (line to neutral) at the receiving end, under open-circuit conditions.
2. The current per wire at sending end of line.
3. The I^2R losses per conductor.

The known quantities are

Conductors, 500,000 circular mils stranded copper
Spacing, d = 17 ft. = 192 in.
Length of line, L = 300 miles
Frequency, f = 60

From table on page 81, the diameter of the conductors is found to be $2r$ = 0.813 in., and the resistance of one conductor is

$$(RL) = 0.115 \times 300 = 34.5 \text{ ohms}$$

From chart on page 243, for $a' = \frac{16}{0.813}$ and f = 60,

$$(XL) = 233 \text{ ohms}$$

By formula (109) the capacity (wire to neutral) is

$$C_m = \frac{0.0388}{\log\left(\frac{192}{0.406}\right)} = 0.0145 \text{ mf. per mile,}$$

whence the capacity susceptance (one conductor to neutral) is

$$(BL) = 2\pi \times 60 \times 0.0145 \times 10^{-6} \times 300 = 1.641 \times 10^{-3} \text{ mho}$$

Referring to the formulas (130) and (131), and putting $I_r = 0$ because the circuit is open at the receiving end,

$$\dot{E}_n = \frac{\dot{V}_n}{\cosh L\sqrt{YZ}}$$

and

$$\dot{I}_s = \dot{E}_n \sqrt{\frac{Y}{Z}} \sinh \sqrt{YZ}$$

The calculation of the required line constants is carried out as explained in Examples 27 and 28, either with the aid of tables of hyperbolic functions or by using the convergent series; the results are given below:

$$\text{The attenuation factor, } L\sqrt{YZ} = 0.04524 + j0.6195$$

$$\cosh L\sqrt{YZ} = 0.815 + j0.027$$

$$\sinh L\sqrt{YZ} = 0.037 + j0.581$$

$$= 0.582\underline{/86^\circ\ 21'}$$

The surge admittance $\sqrt{\frac{Y}{Z}}$ is the reciprocal of the surge impedance and may, if desired, be written

$$\sqrt{\frac{Y}{Z}} = \frac{1}{\sqrt{\frac{Z}{Y}}} = \frac{1}{378 - j27.5}$$

$$= \frac{1}{379\overline{\diagdown 4^\circ\ 10'}}$$

As an illustration of how the division of complex numbers may be carried out without first converting numerator and denominator into polar form,

$$\dot{E}_n = \frac{V_n}{\cosh L\sqrt{YZ}} = \frac{88{,}000 + j0.000}{0.815 + j0.027}$$

In order to eliminate the j term from the denominator, multiply both numerator and denominator by $0.815 - j0.027$; thus

$$\dot{E}_n = \frac{71{,}700 - j2{,}380}{(0.815)^2 + (0.027)^2} = 107{,}600 - j3{,}570$$

which, in polar form, becomes

$$\dot{E}_n = 107{,}700\overline{\diagdown 1^\circ\ 54'} \text{ (lagging behind } \dot{V}_n)$$

The charging current at the generating end is

$$\dot{I}_s = \dot{E}_n \frac{1}{\sqrt{\frac{Z}{Y}}} \sinh L\sqrt{YZ}$$

$$= \frac{(107{,}700 \diagdown 1^\circ\ 54')(0.582 \angle 86^\circ 21')}{379 \diagdown 4^\circ\ 10'}$$

$$= 165 \angle 88^\circ\ 37' (\text{leading } \dot{V}_n)$$

The power factor at the sending end of line is $\cos \theta_s = \cos 88^\circ\ 37' = 0.2414$, and the line losses per conductor are, therefore,

$$88 \times 165 \times 0.2414 = 350 \text{ kw.}$$

101. Approximate Formulas for Calculating Open-circuit Conditions on Long Lines.—In Art. 93, the relation between sending-end and receiving-end voltage was calculated on the assumption that the drop of pressure due to conductor resistance is negligible, and that the rise of voltage due to e.m.f. induced by the capacity current is directly proportional to the distance from the sending end. With the further assumption that the flux of induction inside the material of the conductor is negligible in comparison with the magnetic flux which surrounds the conductor, the receiving-end voltage was found to be

$$E_n = V_n \left[\frac{1 + 3\left(\frac{fL}{100{,}000}\right)^2}{1 - 3\left(\frac{fL}{100{,}000}\right)^2} \right] \tag{118}$$

where E_n and V_n are the voltages (line to neutral) at the receiving and the sending ends, respectively, f is the frequency, and L is the length of the line in miles.

Amperes of Charging Current at the Sending End.—The rise of voltage from V_n to E_n is generally as indicated on Fig. 115, and the current I_s at the sending end of the line is the product of BL and the *average* voltage, which would be $\frac{1}{2}(V_n + E_n)$ if the dotted straight line were the correct representation of the voltage changes along the line; but it is actually more nearly equal to $\frac{1}{3}(V_n + 2E_n)$. Thus

$$I_s = \frac{1}{3}(V_n + 2E_n)(BL) \text{ amp.} \tag{144}$$

which is the approximate value of the charging current at the sending end of the line, on the assumption of a sine wave of e.m.f.

Losses Due to Charging Current.—The I^2R losses, in watts per conductor, due to the charging current in a line which is open at the receiving end, are equal to the *average value of the square of the charging current* multiplied by the line resistance. If the voltage were constant at all points of the transmission line, the charging current would decrease in direct proportion to the distance from the sending end. Thus, at a distance x miles from the receiving end, the current would be $I_s \frac{x}{L}$ and the average value of the square of the charging current would be

$$\frac{1}{L}\int_0^L \left(I_s \frac{x}{L}\right)^2 = \frac{1}{3}I_s^2$$

Actually, the current falls off generally as indicated in Fig. 115, and the average value of the square of the current will be some-

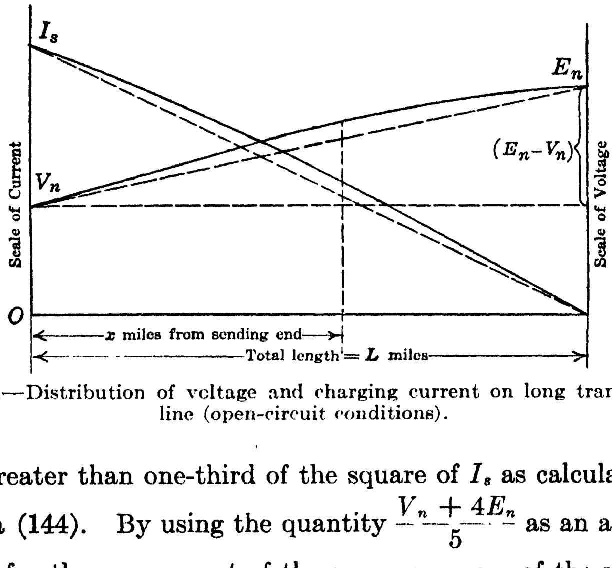


FIG. 115.—Distribution of voltage and charging current on long transmission line (open-circuit conditions).

what greater than one-third of the square of I_s as calculated by formula (144). By using the quantity $\frac{V_n + 4E_n}{5}$ as an approximation for the square root of the average square of the voltage, the following result is obtained for the approximate I^2R loss, in kilowatts per conductor of an open-circuited transmission line,

$$\begin{array}{c}\text{Kilowatt loss}\\ \text{per conductor}\end{array} = \frac{RL}{3{,}000}\left[\left(\frac{V_n + 4E_n}{5}\right)BL\right]^2 \qquad (145)$$

The approximate formulas (118), (144), and (145) will yield results of an accuracy sufficient for practical purposes in calculating the effects of capacity on lines up to 400 miles in length, which is considerably longer than a single section, without branches or tappings, of any high-tension line now in operation or likely to be erected in the near future.

Example 30. *Application of Approximate Formulas to Calculation of Open-circuit Conditions on Long High-voltage Lines.*—Given the following particulars relating to a three-phase high-voltage transmission,[1] calculate (*a*) the voltage at the sending end, (*b*) the current at the sending end, and (*c*) the I^2R loss per conductor due to the charging current.

Length of line, $L = 400$ miles

Conductors, 636,000 circular mils stranded aluminum

Equivalent spacing, $d = 17$ ft.

Resistance per conductor, $RL = 58.8$ ohms

Capacity susceptance (conductor to neutral) $BL = 2.212 \times 10^{-3}$ mhos

Pressure, line to neutral, at the receiving end, $E_n = 80{,}830$ volts

Frequency, $f = 60$

Calculations:

$$3\left(\frac{fL}{100{,}000}\right)^2 = 3\left(\frac{60 \times 400}{100{,}000}\right)^2 = 0.173$$

By (118), $$V_n = \left(\frac{1 - 0.173}{1 + 0.173}\right) \times 80{,}830 = 57{,}000 \text{ volts} \qquad (a)$$

By (144), $$I_s = \frac{57{,}000 + 2(80{,}830)}{3{,}000} \times 2.212 = 161 \text{ amp.} \qquad (b)$$

By (145), $$\text{Kw.} = \frac{58.8}{3{,}000}\left[\left(\frac{57{,}000 + 4(80{,}830)}{5}\right)\frac{2.212}{1{,}000}\right]^2 = 555 \quad (c)$$

The solutions as worked out by the "exact" method, using the hyperbolic formulas are

(*a*) $V_n = 54{,}494$ volts

(*b*) $I_s = 158.3$ amp.

(*c*) I^2R loss $= 545.8$ kw. per conductor

With shorter lines, the errors introduced by using the approximate formulas become smaller. For the 300-mile line of Example 29, the results as calculated by the approximate formulas are $E_n = 107{,}000$; $I_s = 165$; kilowatts per conductor $= 330$. With

[1] Data and exact solutions of problem are taken from NESBIT's, "Electrical Characteristics of Transmission Circuits," Westinghouse Electric and Manufacturing Company.

the aid of a slide rule, all three calculations can be made within a quarter of an hour, and the results do not differ appreciably from those obtained by using the hyperbolic formulas. The solutions as worked out in Example 29 are $E_n = 107{,}700$; $I_s = 165$; kilowatts per conductor = 350

The engineer who has a problem to solve is interested in getting results with a reasonable degree of accuracy in the shortest possible time and by using the simplest methods available to accomplish his purpose. The mathematician is interested in obtaining exact solutions to problems in connection with which he has usually been compelled to make sundry assumptions which may or may not invalidate the practical value of the "exact" solutions he obtains. In the hands of the mathematician, the hyperbolic transmission line formulas furnish an elegant and almost perfect means of predetermining the electrical conditions in circuits consisting of long parallel wires, and there are many problems which cannot properly be solved by other methods. The engineer who is familiar with the meaning and use of the hyperbolic formulas may prefer to use them in determining the pressure, current, and power factor under various assumed load conditions in line sections over 200 miles long; but, generally speaking, he is wasting time. An engineer who is not familiar with the meaning and use of the hyperbolic formulas should most decidedly avoid them, not merely because of the labor involved, but also because of the greater probability of errors. The main reason why the application of the hyperbolic formulas is tedious and apt to introduce errors is the lack of complete tables giving values for the attenuation factor, surge impedance, and hyperbolic functions of complex numbers without the necessity of interpolation. It is true that a great many graphical methods are available, but unless curves and charts are used constantly, they will not appreciably shorten the time required for the computations.

Whether or not the "exact" or approximate methods of calculation are used, the assumption of a sine wave of e.m.f. is generally made, and since the actual e.m.f. wave will usually cause the value of the charging current to be from 5 to 15 per cent greater than if both e.m.f. and current were sinusoidal, the practical advantage, if any, of carrying out the calculations by the "exact" method to include five or six significant figures is not obvious. If mathematical refinements, purporting to give results with an error not exceeding a small fraction of 1 per cent, are to be used, *all* known conditions likely to affect the results should be taken into account. Yet it is customary to assume the amount of the leakage currents between wires to be zero, not only because this is much smaller than the charging current, but also because it is difficult to estimate. Leakage through corona depends to some extent upon weather conditions and to a very large extent upon the voltage, while leakage over insulators varies between wide limits according to the surface condition of the porcelain. It is partly because the so-called exact formulas cannot take account of these variations that it is customary to assume the entire absence of leakage currents.

This book is written for engineers, that is to say for those who have developed or who are developing what is known as "engineering judgment," and this includes a knowledge of the degree of accuracy which is desirable in engineering calculations, or the relation between the probable error due to the method of calculation and the probable inaccuracy of the given or assumed data upon which the calculations are based. The engineer who is without the mathematical knowledge and practice which would enable him to use hyperbolic formulas for long lines advantageously need not despair of being able to design an entirely satisfactory high-voltage transmission line and to predetermine its performance under normal operating conditions with the needful degree of accuracy.

102. Circle Diagram for Constant-potential Transmission Taking Account of Distributed Capacity.—In Art. 98, the circle diagram (Fig. 111) was developed. It gives the relation between current (or power) vectors necessary to maintain constant voltage at both ends of a line when the effects of the charging current are so small as to be negligible. Following the same method as in Art. 98, circle diagrams for long lines, based upon the "exact" hyperbolic formulas, may be drawn, and these are often useful since it is easy to read from them the reactive amperes (or kilovolt-amperes) which must be provided by synchronous phase-modifying machinery. As an illustration of how this may be done, Fig. 116 shows the development of a circle diagram which gives the relation between "active" and "reactive" components of the current at the receiving end of the line, in order that constant voltage may be maintained at the sending end of the line. In this particular diagram, the constant voltage E at the receiving end has the same value as the constant voltage V at the sending end, but this equality is not necessary.

The "exact" formula giving voltage at the sending end of a transmission line in terms of voltage and current at the receiving end (as given in Art. 100) is

$$\dot{V}_n = K_A \dot{E}_n + K_B \dot{I}_r \qquad (130a)$$

where

$$\left.\begin{aligned} K_A &= \cosh L\sqrt{YZ} \\ &= 1 + \frac{(L^2YZ)}{2} + \frac{(L^2YZ)^2}{24} + \dots \end{aligned}\right\} \qquad (138)$$

and

$$\left.\begin{aligned} K_B &= \sqrt{\frac{Z}{Y}} \sinh L\sqrt{YZ} \\ &= LZ\left[1 + \frac{(L^2YZ)}{6} + \frac{(L^2YZ)^2}{120} + \dots\right] \end{aligned}\right\} \qquad (139)$$

This relation is indicted in Fig. 116 where the vector $OD = V_n$ is seen to be equal to the sum of the two vectors $OO' = (K_A E_n)$ and $O'D = (I_r Z_1)$. The symbol Z_1 has been used in place of the constant K_B in order to indicate clearly that this multiplier is simply a fictitious impedance which, in short lines, does not differ appreciably from the impedance LZ, but of which the value has to be determined by formula (139) when using the "exact" method of calculation on lines several hundred miles long.

In order to study the "real" and "imaginary" components of the load current $\dot{I}_r$, express the voltage vectors of the diagram Fig. 116 in terms of their "in phase" and "j" components, the axis of reference being $OB = \dot{E}_n$. Thus

$$\dot{V}_n = E_b + jE_a + (I_b + jI_a)(R_1 + jX_1) \qquad (146)$$

The symbols in this equation have the following meaning:

$\dot{V}_n$ = voltage (line to neutral) at sending end of line;

E_b and E_a = the "real" and "imaginary" components, respectively, of the vector $(K_A E_n)$ which is the voltage that would be required at the sending end of the line to produce E_n volts (line to neutral) at the receiving end under open-circuit conditions (*i.e.*, when the current I_r at the receiving end is equal to zero);

I_b and I_a = the "real" and "imaginary" components of the receiving-end current I_r;

R_1 and X_1 = the fictitious resistance and reactance, respectively, per conductor, being the components of the fictitious impedance $Z_1 = K_B$.

Multiplying out equation (146) and rearranging the terms,

$$\dot{V}_n = (E_b + I_bR_1 - I_aX_1) + j(E_a + I_bX_1 + I_aR_1)$$

as indicated graphically in Fig. 116. If all the vectors in this diagram are divided by Z_1, the result is a circle diagram of *current* vectors in which the length HD divided by Z_1 will be a measure of the reactive component I_a of current necessary at the receiving end to maintain constant voltage when the "in-phase" component of the load current is $I_b = \frac{\text{length } O'H}{Z_1}$

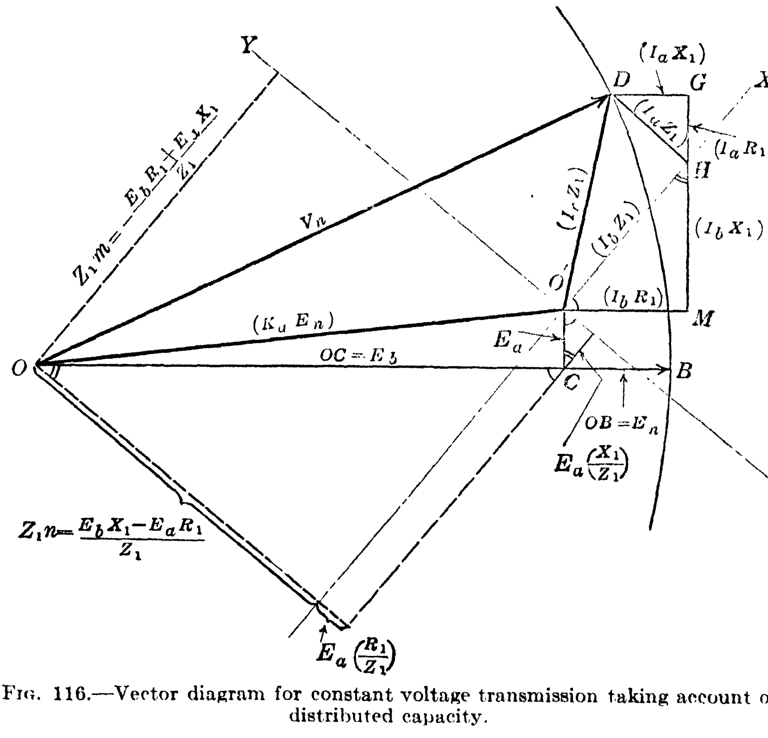


FIG. 116.—Vector diagram for constant voltage transmission taking account of distributed capacity.

The procedure necessary to transform the vector diagram of Fig. 116 into a more convenient diagram with $O'X$ as the horizontal axis of reference and $O'Y$ as the vertical axis is similar to that which was explained in Art. 98 when deriving Fig. 111 from Fig. 110. The position of the center of the circle relative to the new axes is easily located by noting the similarity of the right-angled triangles of Fig. 116 in which equal angles have been marked. When all the vectors of the diagram have been divided by Z_1, the resulting circle diagram will be generally similar to Fig. 111 except that the radius of the circle and the location of its center are determined by the following quantities:

$$m = PC \text{ (in Fig. 111)} = -\frac{E_b R_1 + E_a X_1}{R_1^2 + X_1^2} \tag{120a}$$

$$n = OP \text{ (in Fig. 111)} = \frac{E_b X_1 - E_a R_1}{R_1^2 + X_1^2} \tag{121a}$$

$$r = CS \text{ (in Fig. 111)} = \frac{V_n}{\sqrt{R_1^2 + X_1^2}} \tag{122a}$$

If it is desired to express the quantities as read off the circle diagram in terms of kilowatts or kilovolt-amperes of a three-phase transmission, it is necessary merely to multiply the quantities indicating amperes of current by $\frac{3E_n}{1{,}000}$.

CHAPTER XI

CORONA—ABNORMAL PRESSURE RISES—LIGHTNING ARRESTERS

103. Formation of Corona and Accompanying Losses of Power.—When the pressure on an overhead transmission system exceeds a certain critical value depending upon the spacing and diameter of the wires, there will appear on the surface of the conductors a halo-like glow to which the name "corona" has been given. Apart from this luminous effect, the appearance of the corona is accompanied by a certain loss of power proportional to the frequency and the square of the amount by which the pressure between conductors exceeds a certain value known as the disruptive critical voltage. If the distance between outgoing and return conductors is comparatively small (less than fifteen times the diameter of the wire) there will be a spark-over when the disruptive critical voltage is reached; but with the greater separation such as occurs on practical high-tension transmission lines, the effect of the high potential at the conductor surface is to break down the resistance of the air in the immediate neighborhood of the conductor surface. A luminous cylindrical coating of air, acting as a conductor of electricity, is thus formed, the diameter of which will depend on the amount by which the actual value of the applied potential difference between wires exceeds the disruptive critical value of the potential difference. The result is equivalent to an increase of the diameter of the conductors, thus raising the value of the voltage necessary to break down new concentric layers of surrounding air, until it is approximately equal to the voltage impressed on the wires.

Suppose that a cylindrical wire of radius r is surrounded by a concentric metal cylinder of internal radius R and that there is a potential difference of E volts between the rod and the cylinder; then, by formula (86) of Art. 73, Chap. VIII, the potential gradient at the surface of the wire will be

$$G = \frac{E}{r \log_e \frac{R}{r}} \qquad (86)$$

The corresponding formula for parallel wires is

$$G = \frac{E_n}{\log_\epsilon \left(\frac{d}{r}\right)} \tag{147}$$

where d is the distance between centers of wires and E_n is the pressure, in volts, between wire and neutral plane.

The breakdown gradient is about 30 kv. per centimeter, and it follows, therefore, that the voltage which will cause this critical gradient at the surface of the wire is

$$E_{o\,\text{max.}} = 30r \log_\epsilon \frac{d}{r} \text{ kv.}$$

where d and r must be in centimeters.

This voltage is not high enough to produce a visible corona, because immediately beyond the surface of the conductor and at any greater distance, the voltage gradient is *less* than 30 kv. per centimeter.

Mr. Peek[1] has found that, in order to produce visible corona, the voltage must be raised until the breakdown gradient of 30 kv. occurs at a distance equal to $0.301\sqrt{r}$ cm. beyond the surface of the wire. This leads to the following working formulas which include certain correcting factors which take account of atmospheric conditions and the surface condition of the conductor. The *disruptive critical voltage*, being the r.m.s. value of the pressure measured beween wire and neutral, is

$$E_o = m_o\, 21.1\, \delta r \log_\epsilon \frac{d}{r} \text{ kv.} \tag{148}$$

in which r = radius of conductor in centimeters.

d = distance between centers of the outgoing and return (parallel) conductors, in centimeters.

m_o = a factor depending upon the surface condition of the conductor,

= 1 for polished wires,

= 0.98 to 0.93 for roughened or weathered wires,

= 0.87 to 0.83 for stranded cables (average = 0.85).

δ = density of the air referred to the density at 25°C. and 76 cm. of barometric pressure.

$$= \frac{b}{76} \times \frac{273 + 25}{273 + t} = \frac{3.92b}{273 + t}$$

[1] Peek, F. W., Jr., "Dielectric Phenomena," McGraw-Hill Book Company, Inc.

where b is the barometric pressure in centimeters of mercury, and t is the temperature in degrees Centigrade. If b is expressed in inches of mercury and t is the temperature in degrees Fahrenheit. the density factor becomes

$$\delta = \frac{17.9b}{459 + t}$$

(Note that when $b = 29.9$ and $t = 77°$, $\delta = 1$.)

As a guide in estimating the average pressure at high altitudes, the following figures may be used:

Elevation.	sea level,	$b = 29.9$
	2,000 ft.,	$b = 27.6$
	4,000 ft.,	$b = 25.6$
	6,000 ft.,	$b = 23.7$
	8,000 ft.,	$b = 22.0$
	10,000 ft.,	$b = 20.4$
	12,000 ft.,	$b = 18.9$

The luminosity, or visible halo of light surrounding the conductor, does not occur until a higher pressure has been reached, the increase over the critical disruptive voltage being dependent upon the diameter of the conductor. Mr. Peek's formula for the *visual critical voltage* (kilovolts to neutral) is

$$E_v = m_v 21.1\delta r\left(1 + \frac{0.301}{\sqrt{\delta r}}\right)\log_\epsilon \frac{d}{r} \qquad (149)$$

where the surface factor m_v has the same value as m_0 for wires, and may be taken at 0.82 for a decided visible corona on seven-strand cables. The notation is otherwise as above.

Spark-over Voltage.—When the voltage is very high and the separation between wires is small, a disruptive spark may pass between the wires. The voltage at which this occurs is called the spark-over voltage, and the approximate formula proposed by Peek is

$$E_s = 21.1r\left(1 + \frac{0.01\frac{d}{r}}{\sqrt{r}}\right)\log_\epsilon \frac{d}{r} \qquad (150)$$

where E_s is the r.m.s. value, in kilovolts, of the spark-over pressure between wire and neutral. The sine wave of e.m.f. is assumed and the *maximum* voltage values are, therefore, obtained by multiplying by $\sqrt{2}$ the results obtained from these formulas.

Loss of Power Due to Corona.—The losses which occur through corona are not different in kind from I^2R losses; but since they occur whenever a visual corona appears and may reach high values if the line voltage appreciably exceeds the voltage at which corona is first formed, the importance of designing high-tension lines so as to avoid excessive corona formation is evident.

The loss of power due to corona formation is approximately proportional to the frequency (within the usual commercial range), and to the square of the excess of line voltage over the disruptive critical voltage. The current passing from the wires into the air on an alternating system is an energy current in phase with the pressure. The formula proposed by Mr. Peek for calculating these energy losses is:

$$\left.\begin{matrix}\text{Kilowatt loss per mile}\\ \text{of single conductor}\end{matrix}\right\} = \frac{390}{\delta}(f + 25)\sqrt{\frac{r}{d}}(E_n - E_o)^2 10^{-5} \quad (151)$$

where f = frequency

E_n = transmission pressure, conductor to neutral (kv.)

E_o = disruptive critical pressure (formula (148)) which, when r and d are in *inches* and *common logarithms* are used in place of the hyperbolic logarithms of the preceding voltage formulas, becomes

$$E_o = 123 m_o \delta r \log_{10}\left(\frac{d}{r}\right) \text{ kv. to neutral} \quad (148a)$$

The approximate power loss under storm conditions is obtained by taking E_o as 80 per cent of its (virtual) value as calculated by formula (148) or (148*a*).

Example 31. *Losses Due to Corona.*—Consider a 40-mile section of a three-phase, 60-cycle transmission line consisting of No. 1 seven-strand conductors spaced 6 ft. apart. Assume the average elevation of this section of the line to be 2,000 ft. and the maximum summer temperature to be 95°F. Calculate the permissible operating voltage, given that the losses due to ionization (corona losses) shall not exceed 300 kw.

In order to calculate the disruptive critical voltage by formula (148*a*), the following quantities are required:

From the wire table on page 81; $r = 0.164$

$$\frac{d}{r} = \frac{72}{0.164} = 438 \text{ and } \log\frac{d}{r} = 2.641$$

$$m_o = 0.85$$

$$\delta = \frac{17.9b}{459 + t}$$

where b has an average value of about 27.6 as given in the table on page 309.

Therefore

$$\delta = \frac{17.9 \times 27.6}{459 + 95} = 0.892$$

whence

$$E_o = 123 \times 0.85 \times 0.892 \times 0.164 \times 2.64 = 40.4 \text{ kv.}$$

By formula (151), the losses per mile are

$$\frac{390}{0.892}(60 + 25)\sqrt{\frac{0.164}{72}}(E_n - 40.4)^2 10^{-5} = 0.01775(E_n - 40.4)^2$$

The permissible losses per mile of single conductor are $\frac{300}{3 \times 40} = 2.5$ kw.

Therefore

$$(E_n - 40.4) = \sqrt{\frac{2.5}{0.01775}}$$

whence $E_n = 52.3$ kv. (line to neutral), or a voltage of 90,600 between wires.

104. Corona Considered as "Safety Valve" for Relief of Excess Voltages.—Since the rate at which energy will pass between lines or between lines and ground, as indicated by formula (151), is seen to be proportional to the *square* of the excess of pressure over the disruptive critical voltage, a small increase of pressure may lead to a very greatly increased dissipation of energy through the air. This property of the corona suggests the possibility of working high-voltage transmission lines at a normal pressure in the neighborhood of the critical disruptive voltage where the loss would be inappreciable. An extra-high-voltage discharge, due either to atmospheric lightning, or to internal causes, would then be largely dissipated in the corona itself. This may, to some extent, account for the fact that fewer lightning troubles are experienced on the very-high-voltage transmissions than on the lower-voltage lines. The insulation of the conductors being such as to withstand, without breakdown, pressures considerably in excess of the disruptive critical voltage of the corona, a large amount of oscillating energy can be dissipated in the air before the voltage rises to such a value as to pierce or shatter insulators or damage apparatus connected to the line. On the other hand, too much reliance should not be placed on the corona as a means of dissipating large amounts of suddenly impressed energy;

because lightning and similar disturbances, being to a great extent local, must discharge their power locally, and the corona losses over a *short* section of the transmission line cannot under any circumstances be very great.[1]

Generally speaking, a safe and economical voltage at which to operate high-voltage lines is up to, but not above, the disruptive critical voltage as calculated by formula (148) or (148*a*) for fair weather conditions at the highest summer temperature. This will lead to some power loss due to corona during storms, especially in hot weather. In the winter, the loss will be less.

With the present tendency toward longer transmission distances and higher voltages, the losses due to corona may be excessive unless conductors of large diameter are used. For this reason, aluminum cables, with or without steel core, have been used in place of copper; but there is now on the market a hollow conductor consisting of one or more layers of copper wire spiraled around a light supporting core of copper of which the cross-section is similar to that of an I beam, but it is twisted so that the web part shall support the wire strands at frequent intervals.

Although the increased diameter of conductor which may be necessary to avoid corona losses will necessarily increase the capacity and, therefore, the charging current between wires, it must not be overlooked that when corona does form on a wire of small diameter, the ionization of the air in the immediate neighborhood of the conductor has much the same effect as if the diameter were actually increased, and this leads to increased permittance between wires and, therefore, to a larger charging current.[2]

105. Transient Phenomena. Abnormal Voltages.—A complete discussion of transient electrical phenomena caused by lightning, switching operations, arcing grounds, or any accidental disturbance of the normal steady-state operating conditions is beyond the scope of this book. Indeed, a complete analysis of the causes and resulting phenomena which lead to occasional

[1] The possibilities of designing a line or a section of line so that the formation of corona may be utilized as a protection against lightning and similar disturbances, are fully discussed in a Paper by J. B. WHITEHEAD in the *Trans.* A.I.E.E., vol. 43, p. 1172, 1924.

[2] This effect is discussed by C. F. HARDING in his paper "Corona Losses between Wires of Extra High Voltages," *Trans.* A.I.E.E., vol. 43, p. 1182, 1924.

interruptions of service on electric transmission lines is not possible with the present limited understanding of the subject. By making certain assumptions as to wave shapes and the electrical conditions of the transmission system at the time when the disturbances occur, it is possible to predetermine the effects due to sudden changes in the current or voltage; but the formulas are long and not easily applied by those not thoroughly familiar with the mathematics involved. In the case of an actual transmission system with uniformly distributed inductance and capacity, modified by the connection of substation loads or interconnecting lines, the correct predetermination of transient phenomena is impossible, mainly because the exact nature of the initial disturbance—as in the case of lightning discharges—is unknown. By the collection and analysis of data showing the relative value of the many protective devices that have been tried, it is possible to determine whether or not the installation of certain types of lightning arresters and other protective devices is likely to prove effective in preventing damage to apparatus and in maintaining continuity of service.

No attempt will be made to deal exhaustively with the problems of lightning protection, but what follows will be helpful in indicating the nature of these problems and the means at present adopted to minimize the destructive effects of abnormal pressure rises. For a complete study of the principles underlying transient electrical phenomena, the reader is referred to authorities such as Steinmetz[1] and some of the more recent writings which have appeared from time to time in the technical journals.

The relation existing between the voltage and the current of any transient electrical disturbance occurring in a circuit depends upon the relation between the magnetic flux linkages per unit current—or the *inductance*—and the *permittance* or electrostatic *capacity*.

Consider a circuit in which there is alternating or oscillating energy which is not utilized by any form of receiving apparatus and is not dissipated in the form of heat through the ohmic resistance of the conductors, or through "dielectric hysteresis" or corona; it is obvious that, at the instant when the current wave passes through zero value, the whole of the energy must be stored in the electrostatic field, and similarly, at the instant when

[1] "Transient Electric Phenomena and Oscillations;" also FRANKLIN, W. S., "Electric Waves."

the pressure wave passes through zero value, the whole of the oscillating energy must be stored in the electromagnetic field. Moreover, so long as the interchange of energy from one form to the other continues without diminution of amount, *these two quantities must be exactly equal.* This conception of the oscillations of energy in a circuit having negligible resistance, but appreciable inductance and capacity, is fundamental, and it will be examined in further detail with a view to arriving at a definite relation between the amplitudes of the voltage and current waves.

Energy Stored in Magnetic Field.—Since the engineer usually prefers to think of volts and amperes, the product of which represents power (watts) or the rate at which work is being done, it may be said that the energy stored in a magnetic field during a short interval of time dt seconds is $ei \times dt$ watt-seconds or *joules.* In this connection the voltage e is the e.m.f. developed in a conductor carrying i amp. when a change in current di causes a change of flux $d\Phi$ in the short interval of time dt. Thus, in considering the flux linking with a circuit of one turn (a transmission line conductor) in a medium (air) of constant permeability, it is possible to write

$$e = \frac{d\Phi}{dt} = \mathbf{L}\frac{di}{dt}$$

whence

$$\left.\begin{array}{l}\text{Energy stored in magnetic field during the}\\ \quad\text{interval of time } dt\end{array}\right\} = \mathbf{L}\frac{di}{dt} \times i \times dt$$

$$= \mathbf{L}i \times di$$

and since the current grows from zero to its maximum value in a quarter of a period,

$$\left.\begin{array}{l}\text{Energy stored in magnetic field}\\ \quad\text{during one quarter period}\end{array}\right\} = \mathbf{L}\int_0^{I_{max.}} i \times di$$

$$= \tfrac{1}{2}\mathbf{L}I^2_{max.} \text{ joules} \qquad (152)$$

It is easy to show in a similar manner that the energy stored in the dielectric circuit in one quarter period while the value of e grows from zero to $E_{max.}$ is $\frac{1}{2}CE^2_{max.}$ where C is the electrostatic capacity of the circuit, or portion of circuit considered.

Thus, in the case of a pure, undamped, oscillation, when no energy is supplied from the outside to the circuit, or by the circuit to the outside, it follows that

$$\mathbf{L}I^2 = CE^2$$

whence

$$\frac{E}{I} = \sqrt{\frac{\mathbf{L}}{C}} \tag{153}$$

where $\mathbf{L}$ is expressed in henrys and C in farads. The quantity $\sqrt{\frac{\mathbf{L}}{C}}$ is thus seen to be of the nature of an impedance, and it may be expressed in ohms. It is generally called the *surge impedance*[1] or the *natural impedance* of the circuit or portion of the circuit considered. It denotes the ratio $\frac{\text{volts}}{\text{amperes}}$ of the oscillating energy, being the impedance offered to a traveling wave of which the frequency is determined by the line constants $\mathbf{L}$ and C.

In the case of an overhead transmission line, the approximate value per mile of the inductance between one conductor and neutral, as given in Art. 89, Chap. IX is

$$\textit{External inductance } \mathbf{L} = 0.000741 \log \frac{d}{r} \text{ henrys}$$

and the approximate capacity per mile as given by formula (109) is

$$C = \frac{0.0388}{\log \frac{d}{r}} \times 10^{-6} \text{ farads}$$

whence the *surge impedance* of an overhead transmission line is approximately

$$\sqrt{\frac{\mathbf{L}}{C}} = 138 \log \frac{d}{r} \text{ ohms} \tag{154}$$

In practical overhead work, the limiting values for the ratio $\frac{d}{r}$ will probably be 800 and 50; which, when inserted in formula (154), show that the "natural impedance" of an overhead transmission line must lie between 400 and 230, or, to be well on the safe side, between, say, 500 and 200 ohms.

[1] The *surge impedance*, as explained in Art. 100, Chap. X, is really the quantity $\sqrt{\frac{Z}{Y}}$.

But $\frac{Z}{Y} = \frac{R + jX}{G + jB}$, and since it is assumed that R is negligibly small in relation to X, and G negligible in relation to B, it follows that $\frac{Z}{Y} = \frac{X}{B} = \frac{2\pi f\mathbf{L}}{2\pi fC} = \frac{\mathbf{L}}{C}$.

A knowledge of this quantity renders it possible to determine the maximum value of any surge pressures that can possibly occur on the line due to the sudden interruption of the current. Thus, if the "natural impedance" is 300 ohms, and the instantaneous value of the current at the crest of the wave is 200, the surge pressure, however suddenly the current is interrupted, cannot possibly exceed $200 \times 300 = 60{,}000$ volts; because this is the maximum value of the pressure wave necessary to store in the electric field the whole of the energy stored in the magnetic field at the moment when the current was interrupted. It is safe to say that, on a practical transmission line, the surge pressure is never likely to exceed 200 times the current in amperes; but, with heavy currents, this may well be sufficient to break down insulation and cause considerable damage to power plant. It must not be overlooked that it is often more difficult to handle heavy currents at comparatively low pressures than small currents at very high pressures. When the current is large, the opening of switch or fuse on full load, or an accident causing a break in the circuit, with or without the formation of an arc across the gap, may lead to insulation troubles on many widely separated parts of the system; but on a high-pressure system, even if the current were as large, the insulation is frequently so good that it will withstand without injury the stress imposed on it by the highest possible value of the surge pressure.

In underground cables, the capacity is much larger compared with the inductance than in overhead systems, and the surge impedance $\sqrt{\frac{L}{C}}$ has then a smaller value, which may be about one-tenth of the value for overhead lines; but the transformers connected to transmission systems will always have a surge impedance considerably higher than that of the line itself.

The effect of the ohmic *resistance* in series with the inductive and condensive *reactances* of a circuit is to damp out the oscillations by dissipating the energy in the form of I^2R losses. With a sufficiently high value of resistance in the circuit, surges or oscillations of energy cannot take place.

106. Line Disturbances Caused by Switching Operations and Arcing "Grounds."—It is hardly necessary to add anything to what has already been said in order to emphasize the possible danger of suddenly switching a source of electrical energy on or off a long transmission line. Unfortunately the calculations of the probable surges or oscillations are not easily made, and, moreover, accurate data concerning the characteristics of the various circuits and apparatus connected to the system are rarely available. It follows that the engineer cannot predetermine accurately what will happen under the different probable or possible conditions of operation; but a general understanding of the principles underlying the creation of energy surges in a system of electric conductors will enable him to avoid obvious mistakes in the design and operation of a particular transmission scheme.

Suppose a flash-over to ground occurs at one of the insulators on a high-voltage transmission line with E_n volts between line and neutral (ground). If the grounding occurs at the instant of time when the voltage has reached its crest value ($\sqrt{2}\,E_n$ in the case of a sine wave of e.m.f.), high-frequency traveling waves will propagate in both directions from the flash-over point, with a maximum current value of $\sqrt{2}E_n\sqrt{\frac{C}{L}}$. The location of the flash-over, and the line impedance between this point and the source of supply, will determine the maximum possible amount of the power current which will flow into the short-circuit. Suppose that this power current is I amp. and that it is interrupted when its instantaneous value is $\sqrt{2}I$; then the maximum transient high-frequency voltage that can be induced—and superimposed upon the normal-frequency voltage—is $\sqrt{2}I\sqrt{\frac{L}{C}}$. These high-frequency oscillating transients, starting out in both directions from the point where the original disturbance occurs, will have a gradually decreasing amplitude owing to the damping effect of the line resistance and leakage over insulators or through the air (corona); but if these wave trains of oscillating energy reach the end of the line without greatly diminished amplitude, they will be "reflected," and the reflected waves, meeting the outgoing waves, may cause considerable amplification of the original disturbance.

There is also danger of abnormally high voltages due to surges at the points where there is a change in the constants of the circuit. Thus, if a transformer is connected across the ends of a long overhead transmission line, there will be a rise of pressure when a traveling wave arrives at this point because the *surge* *impedance* $\left(\sqrt{\frac{L}{C}}\right)$ of the transformer winding may be between 2,000 and 4,000 ohms, which is very much higher than that of the line itself (about 400 ohms as previously explained). For this reason the end turns of the transformer primaries must be specially insulated to withstand much higher voltages between turns than the remainder of the winding.

In the case of a change from underground to overhead transmission, a surge originating in the cable will produce a rise in pressure at the junction with the overhead line, while the contrary will occur (*i.e.*, the voltage will be reduced) if the surge is started in the overhead line and passes into the cable system of which the surge impedance will always be *smaller* than that of the overhead transmission.

With the good insulation provided on modern high-voltage systems, it is doubtful if the interruption of the current by opening switches under load is likely to cause serious voltage disturbances, except in the case of air-break switches where a long arc may be formed and suddenly interrupted—as for instance by a draught of air—when the current is of considerable value. Oil-break switches almost invariably open the circuit at the instant when the current is passing through zero value.

Although the mathematical solution of hypothetical problems based on the assumptions of uniformly distributed circuit constants and simple harmonic wave forms is a comparatively easy matter, the practical conditions with the mixed networks of large distributing systems are such that the ultimate effects of a disturbance on such systems cannot be accurately predicted even if it were possible to determine the exact character of the initial disturbance.

107. Lightning.—The foregoing considerations do not take into account the effects of lightning, either by direct stroke or by induction, because in such cases a pressure from an outside source is impressed upon the circuit, and the potential of these atmospheric charges may be tens of times greater than any surge voltage due to a redistribution of the energy stored in the circuit itself.

Although the understanding of lightning phenomena is still far from complete, it is generally agreed that a single stroke of lightning is of short duration, frequently not exceeding the onethousandth part of a second. If an overhead conductor receives a direct stroke of lightning, the potential value of the lightning charge is generally so enormously in excess of the working pressure on the conductors that the lightning leaps over the insulators down the pole to ground. Any charge on the line, which is not sufficiently high in potential above ground to jump over the insulators, will travel along the line in both directions until it is grounded through a lightning arrester or dissipated as I^2R losses in the conductors. If the resistance of an arrester or the path through which a discharge occurs were zero, the current passing would be a maximum. If C is the capacity in farads and $\mathbf{L}$ the inductance in henrys of unit length of line, then $\sqrt{\frac{\mathbf{L}}{C}}$ is the surge impedance of the circuit; and the maximum possible value of the current will be $I_{\max.} = E \div \sqrt{\frac{\mathbf{L}}{C}}$, where E is the impressed voltage, which may be considered as something less than the pressure which will cause a flash-over at the insulators.

The intense concentration of lightning disturbances is the cause of the difficulties experienced in protecting transmission lines by means of lightning arresters; experience tends to show that an arrester does not adequately protect apparatus at a greater distance than 500 ft.; yet it is unusual to find arresters on a transmission line at closer intervals than 2,000 ft.

Disturbances are most likely to occur on exposed heights and on open wet lowlands; special attention should, therefore, be paid to lightning protection at such places.

Although the quantity of electricity in a lightning flash may not be very great, the short duration of the flash accounts for currents which are probably of the order of 10,000 to 50,000 amp.

Apart from the effects of atmospheric electricity, it is necessary to guard against the abnormal pressure rises that will occur on long transmission lines through any cause, such as switching operations or an intermittent "ground." Overvoltages up to 40 per cent in excess of the normal line voltage can be produced by switching in a long line. High-frequency impulses or surges are set up, which, in the special case of an arcing ground, may give rise to a destructive series of surges, a state of things which will continue until the fault is removed. An arrester which may be suitable for dealing with transitory lightning effects may be quite inadequate to dissipate the charges built up by such continual surges.

108. Protection of Overhead Systems against Accumulations of High-potential Static Charges.—Under this heading the ordinary lightning rod and grounded guard wire will be briefly dealt with. If no guard wire is used, lightning rods, in order to be efficient, should be provided at frequent intervals along the line. They may be fixed to every pole or tower, but in any case they should not be spaced farther apart than 300 to 400 ft. unless the spacing of the supporting poles or towers has to be greater than this for economic reasons. It is especially important to provide them on the poles or towers in exposed positions, such as hill tops. They should project from 3 to 6 ft. or more above the topmost wire. A convenient form of lightning rod is a length of galvanized angle iron bolted to the pole top or forming an extension to the structure of a steel tower. Long lines have been worked satisfactorily for extended periods without lightning rods or guard wires, but these are extra-high-pressure transmissions which, on account of the better insulation throughout, are always less liable to trouble from lightning than the lower-voltage systems.

Although engineers are still divided in opinion as to the value of the protection afforded by overhead grounded guard wires carried the whole length of the line above the conductors, it is now generally recognized that this method of protection is efficient for comparatively low-voltage circuits carried on steel poles. The objections to the guard wire are the additional cost and the possibility of the wire breaking and falling across the conductors below, thus causing an interruption to continuous working. Trouble due to this cause is, however, exceedingly rare.

If the grounded wire above the conductors could be replaced by a cylinder entirely surrounding the power circuit, complete protection would be afforded against induced charges from thunder clouds. Indeed the condition would then be equivalent to an underground system of transmission. The protection afforded by one or two wires strung above the conductors and grounded at frequent intervals is, however, only partial, and the extra cost of the installation of such guard wires may not be justified on economic grounds. The present-day tendency is to omit them from wood-pole lines; but when the ground potential has already been carried to a higher elevation by the use of steel poles or towers, there appears to be no doubt that additional protection may be provided by grounded guard wires. It is probable that there is an economic advantage in providing grounded guard wires on rural or farm lines on the four-wire three-phase system with well-grounded neutral when the wires are carried on steel poles, but the trend in all recent overhead lines is to omit the ground wire. If the saving of cost effected by this omission were always applied to improving the insulation of the line generally, even when the operating voltage is comparatively low, there is little doubt that the ground wire might usually be omitted, although there will always be special cases or situations in which its installation will be an advantage. It is well known that extra-high-voltage transmission lines are less affected by lightning disturbances than the lower-voltage lines because of the higher insulation to ground.

Relieving Conductors of High-potential "Static."—By directing a stream of water from the nozzle of a grounded metal pipe on to the high-tension conductors, a high-resistance non-inductive path to ground is provided for the extra-high-potential charges on the line; but there will be very little leakage of power current. It is claimed that arresters constructed on this principle have been found useful in practice; but the employment of jets of water has its objections. It is usual to put the jets in action only at times when electric storms are pending; and the reliance on the "human element" renders the apparatus less valuable than an equally efficient device which is always ready to act. The chief function of the water jet is to prevent the building up of static pressures on the line caused by the contact of dust, snow, or rain-drops, blown against or falling upon an insulated line of considerable length, or by variations in the potential of the atmosphere at different parts of a line traversing hilly country.

Since other means, such as highly inductive resistances, or the grounding of the neutral point of transformers connected to the line, are available for preventing the accumulation of static charges on an overhead line, the advantages claimed for water jets are not obvious.

Probably the best way to provide a discharge path for gradually accumulated static charges on the line is to have the high-tension windings of the transformers star-connected, with the neutral point grounded, preferably through a resistance. Since this question of grounded or ungrounded neutral is still subject to discussion, the present trend will be briefly referred to in the following article.

109. Grounded versus Isolated Transmission Systems.—Whether or not it is advisable on three-phase transmissions to use the star connection with grounded neutral, or a system, either star or delta, without any connection to ground, is not a matter of very great importance; and since no theoretical conclusions based on general principles have been arrived at, the engineer is compelled to consider each paticular case on its own merits and be guided by practical results obtained under similar conditions.

With a view to eliminating the third harmonic and its multiples, and so obtaining as nearly as possible a sine-wave of e.m.f., the generators are usually Y connected, a practice which has the further advantage that the neutral point can be readily grounded if desired. The low-tension windings of the transformers, both at generating and receiving ends of the line are generally delta connected; but so far as the high-tension windings are concerned, these may be star at both ends, or delta at both ends, or star at one end and delta at the other. Then again, the neutral point of a high-tension system may be connected to ground either directly or through a resistance, the results being by no means the same in the two cases.

The object of grounding the neutral of a high-tension system is mainly to protect the insulation from abnormally high pressures which might aggravate the trouble in the event of a ground occurring on one wire, and so lead to serious interruption of service. It is, in fact, the question of line insulation considered in connection with continuity of service which is generally the determining factor in deciding whether or not the neutral shall be grounded, and whether the grounding, if adopted, shall be with or without the intervention of a resistance.

In England, it has always been considered good practice to ground the neutral point through a resistance, but both in the United States and on the continent of Europe it has, in the past, been customary to operate with the neutral point insulated. One of the main troubles with the ungrounded system is the fault known as an arcing ground or intermittent earth, which sets up high-frequency surges and not infrequently leads to a shutdown. The trend is now toward the grounded star connection which practical experience has shown to be generally preferable to the insulated system, especially in avoiding dangerous surging conditions. There is still a good deal of discussion regarding the desirability of a resistance or inductance, or both, in the connection between the neutral point and ground, but there is undoubted danger in the direct connection to ground because of the very large currents which may occur in the event of one of the phases being accidentally grounded.

110. Protection from High-frequency Disturbances—Static Condensers.—A non-inductive low-resistance direct connection to ground can obviously not be made on a high-tension a.c. overhead transmission line; but a path to ground may be provided either through a highly inductive choke coil, or through a condenser, or both, without the necessity of providing a spark gap in series. The inductive resistance may easily be designed to pass only an inappreciable current of normal or higher frequency, and it will, therefore, be useless for affording relief in the case of high-frequency surges; but it is capable of relieving the line of slowly accumulated static charges. The condenser, however, acts as an almost perfect insulator so far as direct currents are concerned; but it is pervious to high-frequency currents, and a suitably designed condenser, or rather battery of condensers, connected between line and ground without the intervention of any spark gap, is certainly an ideal device for dealing with the very-high-frequency oscillations that accompany lightning phenomena.

When a thundercloud passes over a transmission line, it will induce on the line an electrostatic charge of polarity opposite to that of the cloud. When the cloud is suddenly discharged, either to a neighboring cloud or to ground, the accumulated static charge will form two traveling waves moving in both directions from the point of disturbance. The potential which will be superimposed on the normal working voltage will depend upon the potential of the cloud and the height of the conductors above ground. The induced voltage at the instant of lightning discharge is not likely to exceed about 6,000 volts for each foot of elevation above ground level,[1] which explains why the flash-over of insulators on 220,000-volt transmission lines, due to lightning, is comparatively rare. On lower voltage lines, flash-overs will occur or lightning arresters will discharge.

[1] The potential gradient between cloud and ground just before a thundercloud discharges by direct lightning stroke to ground will be very much higher than this; probably in the neighborhood of 100,000 volts per foot.

Consider, now, the more common occurrence of a lightning discharge which raises the potential of the overhead circuit only a few thousand volts, an amount which will not be sufficient to cause a flash-over at the insulators or the discharge of any spark-gap type of lightning arrester. Traveling waves will, however, start out in both directions from the point of disturbance, with a frequency of many thousands of cycles per second. It is now generally understood that high-frequency traveling waves may break down the insulation of generators or transformers even when the voltage of the induced charges is small as compared with the normal operating voltage. The trouble is that the wave is short, and the point of zero potential may be only a few hundred feet behind the point of maximum potential. If, therefore, a traveling wave of this nature enters a piece of electrical machinery such as a generator or transformer, the full difference of potential, which may amount to only a few thousand volts, may be applied across adjacent layers of the coil winding, thus causing a puncture and ultimate breakdown of the insulation, even if the apparatus as a whole is insulated to withstand pressures of 100,000 to 200,000 volts to ground. As a protection against trouble of this sort from high-frequency induced charges, the condenser appears to offer a good solution.

It must not be understood from these notes on the uses of condensers as lightning arresters that the discharge is diverted to ground through the condenser and so dissipated, much as energy would be dissipated in a resistance; because the condenser cannot absorb or dissipate any but the smallest percentage of the energy passing through it. The energy is necessarily redelivered to the line from which it originally came, and is ultimately dissipated through the ohmic resistance of the conductors. The function of the condenser is, in fact, somewhat analogous to that of an air chamber on a water pipe in which the rate of flow is subject to sudden variations.

In view of the fact that the condenser merely *shunts* the high-frequency oscillations and so prevents damage to the apparatus to be protected, but returns nearly all this energy to the line where it ultimately dies out owing to the conductor resistance, it would seem advisable to provide some small amount of resistance in series with the condensers, even at the risk of slightly higher surge pressures across the apparatus to be protected. The intelligent combination of condensers, reactance coils, and resistances may be expected to afford good protection; but there is always a danger of resonance effects at certain critical wave frequencies.

Example 32. *Condensers for Protection against High-frequency Surges.*—The frequencies most likely to build up destructive voltages are between 30,000 and 100,000 cycles per second. Suppose that it is desired to afford protection against traveling waves of a frequency of 60,000 cycles. Assume a power station operating three phase at 33,000 volts, 60 cycles, with a current of 50 amp. per conductor. The protection afforded consists of three condensers each of capacity C_{mf} microfarads connected between line and ground, in series with a resistance of R ohms. It is desirable that the condenser should pass a current equal to the full-load current (*i.e.*, 50 amp.) when the surge frequency is 60,000 which has been assumed to be the frequency at which the protective devices are to function most effectively. When a non-inductive resistance is provided in the connection between the condenser and ground, it is customary to design it so that the voltage drop across the resistance shall be at least as great as that across the condenser when the high-frequency discharge occurs. Since these two voltage drops are in quadrature, each being equal to the full voltage to ground divided by $\sqrt{2}$,

$$R = \frac{\frac{1}{\sqrt{2}}\left(\frac{33,000}{\sqrt{3}}\right)}{50} = 270 \text{ ohms}$$

and since the condensive reactance is to have the same value,

$$\frac{1}{2\pi f C} = R$$

or

$$\frac{10^6}{2\pi \times 60,000 \times C_{\text{mf.}}} = 270$$

whence

$$C_{\text{mf.}} = 0.0098 \quad \text{mf.}$$

Although this combination of resistance and condenser will pass 50 amp. of current to ground at normal operating voltage when the surge frequency is 60,000, the current passing under normal working conditions will be negligible. Thus, when the frequency is 60, the condensive reactance will be 1,000 times as great as when the frequency is 60,000, namely 270,000 ohms per condenser. The 270 ohms of non-inductive resistance in series with this is negligible, whence the current per phase which passes to ground under normal operating conditions will be approximately $\frac{33,000}{\sqrt{3} \times 270,000} = 0.07$ amp.

Owing to the shorter distances of transmission in Europe, means of protection different from those used in America may be necessary, but apart from their high cost and difficulties of manufacture, there are probably many situations where condensers would be beneficial. In Germany where the electrolytic type of lightning arrester is not used, a large number of installations are protected by condensers. The types used include the Mosciki tube condenser (glass tubes coated with silver backed by a copper deposit) made in two sizes having capacities of 0.002 and 0.005 mf., respectively, and suitable for working pressures of 15,000 volts per tube. The Meirowsky condenser is also made in tubular form, but the glass of the Mosciki tube is replaced by impregnated paper. The Dubilier condensers are made with mica as the dielectric; they have proved efficient in practice and are more compact than the tubular designs.

Since capacity between line and ground, in combination with a non-inductive resistance—which may be additional or merely that of the line itself—is effectual in dissipating the energy of high-frequency oscillations induced by atmospheric disturbances, it would seem that the condenser formed by the overhead conductors and ground should be of some use in this connection. The effects of line capacitance are, however, largely counteracted by the line inductance; but since a condenser of appreciable capacity is obtained from a comparatively short length of insulated underground cable, it would appear to be advantageous —as a protection against induced high-frequency surges—to lead the current from an overhead transmission into generating stations and substations through a length of, say, 200 to 300 ft. of underground cable. Cf course this may not be possible in the case of very high pressures because of the prohibitive cost of the cable, but as a matter of fact any form of condenser becomes very costly when designed for use on high-voltage systems. The effect of an underground cable on a surge traveling along an overhead line has already been referred to in Art. 106. The greater capacity (per unit length) of the cable is such that its surge impedance $\sqrt{\frac{L}{C}}$ would be of the order of 40 ohms, compared with about 400 ohms for the overhead line. Some laboratory tests made in Germany in 1921[1] appear to indicate that when condensers are used for surge protection, switching operations may cause surge voltages with steep wave fronts which are more dangerous to apparatus connected to the line than the surges produced under similar conditions when condensers are not used. There is no doubt that complete protection against abnormal pressure rises is not possible at present, but each system should be studied as a separate problem, and so-called protective apparatus installed only when the additional cost and complication appear to be justified on economic grounds.

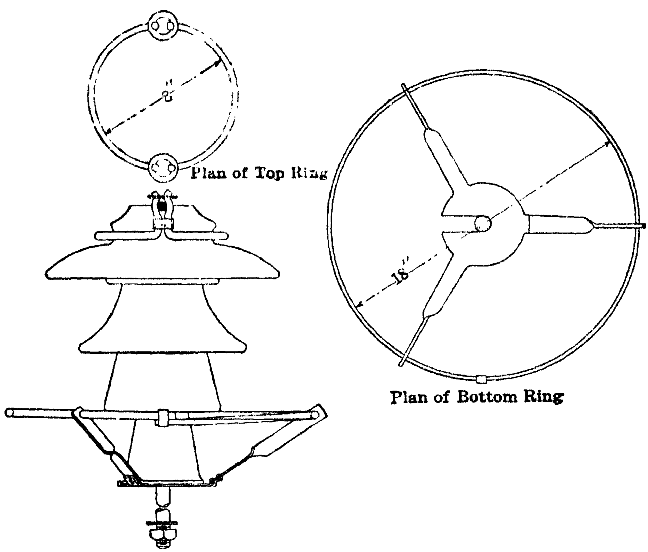


FIG. 117.—Arcing ring on pin type insulator.

111. Protection of Insulators from Power Arcs.—As a means of protecting insulators from flash-over caused by lightning, or the power arc following a high-potential discharge, the "arcing rings" first introduced by L. C. Nicholson may be mentioned. These rings, which are grounded, are placed in such a position as to take the arc and hold it at a sufficient distance from the porcelain of the insulator to prevent cracking or breakage by heat. The illustration Fig. 117 shows the arrangement of the grounded arcing rings attached to a pin-type insulator made by the Locke Insulator Manufacturing Company. It is not claimed that these

[1] PREHM, W., *Elektrotech. Zeit.*, p. 395, Apr. 21, 1921.

rings will protect an insulator against a direct lightning stroke; but their utility on high-pressure lines transmitting large amounts of power has been proved without doubt.

Similar attachments are frequently provided on the suspension type of insulator. When a grading shield[1] is provided, the arc will usually strike upward from the edges of the shield without damaging the porcelain of the units.

112. Horn-gap Arresters.—Nearly all lightning arresters are designed on the principle of one or more spark gaps between the conductors and ground, the air space being so adjusted that the normal difference of potential between the line and ground is insufficient to jump the gap; but abnormally high pressures will break down the insulation of the gap and so find a path to ground before the pressure is sufficiently high to damage the insulation of the line or the apparatus connected thereto.

The ordinary horn-gap arrester is so well known that it requires no detailed description. When the potential rises to such a value that it can jump the gap at the base of the curved wires, the power arc will follow the discharge, but owing partly to the upward tendency of the heated air and mainly to the magnetic field produced by the current itself, the arc is driven upward toward the ends of the "horns" where, after being sufficiently drawn out in length, it is finally ruptured. The horn gap is not effective when set to discharge at pressures below 13,000 volts, because with a small gap (less than 1 in.) the arc may not rise and break properly. The usual settings for horn gaps are as follows: The voltages in the table are the r.m.s. values on the sine-wave assumption, and they must not be used except as a rough indication of the probable gap between horns:

Gap, inches	Working voltage across gap	Spark-over voltage, approximate
1	21,000	30,000
1½	29,000	41,000
2	34,000	48,000
3	44,000	63,000
4	52,000	74,000
6	65,000	93,000
8	77,000	110,000
10	88,000	126,000
12	98,000	140,000

[1] Refer to Art. 76, p. 225, and Fig. 80.

A non-inductive resistance should be connected in the ground wire from the horn arrester. An ordinary wooden barrel filled with water, with a connecting plate at the bottom and the upper terminal carried about 6 in. below the surface of the water, makes an effective resistance. If no resistance is provided in the ground connection, the momentary discharge of the power current may be excessive, dangerous surges may be set up in the line, and there is the possibility of synchronous machines being thrown out of step.

It is necessary to bear in mind that no lightning arrester which will discharge only when the potential rises appreciably (say 30 per cent) above the operating voltage can protect apparatus connected to the lines against high-frequency (steep wave front) surges when the voltage of such traveling waves is *less* than that which will cause the arrester to discharge. This point was referred to in Art. 110 when discussing the function of condensers in connection with high-frequency low-voltage disturbances.

Another serious disadvantage to the ordinary horn-gap arrester is the likelihood of an intermittent arc setting up surges and high-potential disturbances which may lead to more trouble than the original cause of the spark-over. Fairly satisfactory results have been obtained by providing a number of horn gaps on a high-tension transmission and "grading" these, by adjusting some of them to discharge with a very small rise of pressure through a high resistance; while other sets would have larger gaps and lower resistances in series; the very largest gap being such as to break down only rarely, under exceptionally high pressures, and this should have a very low resistance in series but may with advantage be protected by a fuse.

The amount and nature of the resistance in the ground connection from a spark gap are important considerations. The writer has had experience with the ordinary water-barrel type of resistance on 22,000-volt circuits, and he believes this to be satisfactory if proper attention is paid to possible losses of water by leakage and evaporation.[1] This type of resistance is unsuitable in districts where thunderstorms are likely to occur when the temperature is below the freezing point of water.

[1] Refer also to article by H. M. King in *Elec. World*, vol. 79, p. 586, Mar. 25, 1922.

An improvement on the somewhat crude water-barrel resistance is the type of water resistance described by C. E. Bennett in the *Electrical World* (Vol. 75, p. 365), Feb. 14, 1920.

The amount of the non-inductive resistance in the ground connection varies considerably in practice, depending upon the voltage of the system and the position of the arrester in relation to the source of power. Generally speaking, it is advisable to make this resistance at least equal to the surge impedance $\left(\sqrt{\frac{L}{C}}\right)$ of the line or, say, 400 to 500 ohms; but, in order to avoid the passage (and interruption) of heavy currents, resistances between 1,000 and 1,500 ohms are occasionally used, while resistances no greater than one-quarter of the surge impedance are operating satisfactorily on some of the lower-voltage installations in Europe.

For the reasons stated above, namely that spark-gap arresters do not check all high-frequency surges and are likely to set up surges at the instant of discharge, they have been compared unfavorably with the very much more costly, but by no means perfect, electrolytic arresters with their ideal discharge characteristics. Nevertheless, experience has shown that horn arresters, *if intelligently installed and properly connected and adjusted*, are capable of affording useful protection on circuits up to 33,000 volts and even occasionally on systems of considerably higher voltage.

113. Methods of Grounding. Clearance between Arresters.—The ground wire from lightning rod, guard wire, or arrester, on high-tension transmission lines, should be as short and straight as possible. The ground plate should have a reasonably large surface, but the material is of little importance, except that it is not wise to bury aluminum wires in the ground because of possible electrolytic action. Galvanized iron is a good material. If the ground contact is made with one or more iron pipes buried or driven into the ground, these pipes may be from ¾ to 1½ in. in diameter, and a good connection should be made to the *top* of the pipe, as the inductive effect of an iron tube surrounding the ground wire might be considerable if a connection were made only at the bottom of the pipe. One or more pipes 8 to 10 ft. long, driven into the ground with 6 to 12 in. projecting above, will generally be found more effectual than buried plates. A ground of very low resistance is not essential on a high-tension system, and, generally speaking, the special forms of ground plate made of perforated copper, designed to hold, or to be in contact with, crushed charcoal, are unnecessary. Copper sulphate in the soil surrounding the grounding rods or plates is helpful in reducing the ground resistance.

Spacing of Lightning Arresters.—A reasonable distance must be allowed between the live metal parts of arresters placed side by side; the following limiting distances are suggested for guidance in installing lightning arresters, such as those of the horn-gap type where large arcs may be formed and blown or drawn from one element to another. These distances may be reduced if suitable partitions are provided between the arresters.

Potential Difference, Volts	Separation, Inches
11,000	24
22,000	32
33,000	42
44,000	50
66,000	66
88,000	84
110,000	108

114. Grounding through Inductive Reactance. Petersen Coil. As a means of preventing the destructive effects of intermittent grounds by limiting to a very small value the power current which will pass from the high-tension line to ground, Prof. W. Petersen has suggested the use of a choke coil in place of a resistance for grounding the neutral point. The amount of this reactance must be a function of the capacity of the transmission line, as will be understood by referring to Fig. 118. If No. 1 conductor is grounded owing to a flash-over, the current through the reactance will be $I_x = \frac{E_n}{\omega \mathrm{L}}$ where L is the inductance of the earth coil. This current will lag behind the voltage E_n (grounded phase to neutral point) by a quarter period. The total charging current between the overhead conductors and ground is the (vectorial) sum of the charging currents I_1 and I_2, which is a current one-quarter period *in advance* of the voltage E_n. The total current passing from phase 1 through the arc to ground is $I_s = \dot{I}_x + (\dot{I}_1 + \dot{I}_2)$, and it follows that this current will be zero if the lagging component (I_x) can be made equal to the leading component $(\dot{I}_1 + \dot{I}_2)$. This is the principle of the Petersen coil; actually there will be a small energy component of the total current which will never be exactly zero, and moreover it is necessary to pass sufficient current to operate circuit-breaker relays, the reactance of the coil being usually adjusted to pass a current of 20 to 30 amp. through the arc. The Petersen coil is usually in the form of an oil-immersed water-cooled reactance without iron core; it is provided with a number of taps so that it may be adjusted according to the electrostatic capacity of the circuit to which it is connected.

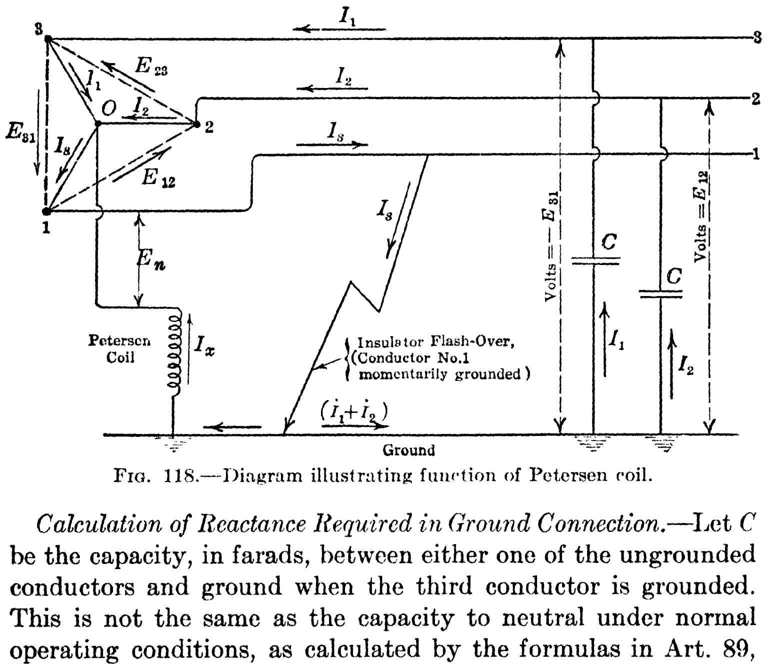


FIG. 118.—Diagram illustrating function of Petersen coil.

Calculation of Reactance Required in Ground Connection.—Let C be the capacity, in farads, between either one of the ungrounded conductors and ground when the third conductor is grounded. This is not the same as the capacity to neutral under normal operating conditions, as calculated by the formulas in Art. 89, Chap. IX. The value of C is found in practice to be about 75 per cent of the capacity between any one wire and neutral. Referring to Fig. 118, it will be seen that the current through the equivalent condenser of capacity C, connected beween conductor 2 and ground, will be $I_2 = \omega C E_{12}$ where E_{12} is the voltage between the (grounded) conductor 1 and conductor 2. The voltage E_{12} is $\sqrt{3}E_n$ where E_n is the star voltage (line to neutral). Similarly, but noting the phase relation of the voltages,

$$I_1 = \omega C(-E_{31})$$

whence $\dot{I}_1 + \dot{I}_2 = \omega C(\dot{E}_{12} - \dot{E}_{31})$. The *difference* between two equal vector quantities when the phase difference is 120 deg. is the same as the *sum* of two equal quantities with a phase difference of 60 deg., whence

$$\dot{I}_1 + \dot{I}_2 = \omega C\sqrt{3}E$$
$$= 3\omega C E_n$$

where E and E_n stand, respectively, for the voltages between conductors and between conductor and neutral.

In order to obtain minimum current through the arc, the reactance of the earth coil should, therefore, be such that

$$3\omega C E_n = I_x = \frac{E_n}{\omega L}$$

whence

$$L = \frac{1}{3\omega^2 C} \qquad (155)$$

If preferred, this relation may be expressed in terms of the normal charging current to neutral; thus, by formula (104), p. 257,

$$I_c = \omega\left(\sqrt{3}E_n\right)\left(\frac{4}{3}C\right)$$

and putting $X = \omega\mathbf{L}$ for the reactance of the coil,

$$X = \frac{4E_n}{3\sqrt{3}I_c} \text{ ohms (approximately)} \qquad (156)$$

The reactance X, or the inductance $\mathbf{L}$, is calculated for the frequency of the power arc, *i.e.*, the normal operating frequency of the system.

In America, where the trend has lately been toward the grounded neutral without resistance or reactance in the ground connection, the Petersen coil finds little application, but it has met with considerable success in Europe.

115. Multiple-gap Arrester with Non-arcing Cylinders.—In this type of arrester there are many air gaps in series between the line and ground. No single gap is greater than 1/32 or 1/16 in., and it occurs between the adjacent surfaces of small cylinders made of a so-called "non-arcing" metal as used in the earlier types of arrester designed by A. J. Wurtz. The number of gaps in series depends upon the working voltage of the line, and the last of the metal cylinders is connected to ground (or to one of the return conductors, as the case may be) through a non-inductive resistance, which may, with advantage, be shunted by a fuse in series with a spark gap. Sometimes a portion of this resistance is bridged by a number of spark gaps, all as shown in the diagram Fig. 119. These shunted gaps act as a sort of bypass for heavy discharges, the amount of the series resistance through which all discharges have to pass being comparatively small. The multiple-gap arrester is essentially a device for use on a.c. circuits, the principle of its action being as follows: There is a certain electrostatic capacity between consecutive cylinders and between each one of these cylinders and ground; and the potential gradient is considerably greater at the high-voltage end of the arrester with the result that when the total voltage across the arrester reaches a certain critical value, the breakdown occurs between the first and second cylinders. The second cylinder is then connected to the first by an arc, so that its potential rises accordingly, until a breakdown occurs between the second and third cylinders; and so on. The line current then follows the discharge and in so doing tends to produce a uniform fall of potential along the line of cylinders, with the result that the maximum potential difference between cylinders is considerably less than that required for the initial breakdown, and the power arc is ruptured as the current passes through zero value. When a breakdown occurs between two cylinders, the potential of the lower cylinder of the series will depend upon the quantity of electricity which passes to it from the more highly charged cylinder. The initial current is really a capacity current, and it will, therefore, be greater at the higher frequencies; but by a scientific proportioning of the shunted resistance, a very satisfactory arrester of this type can be made for use on circuits up to about 13,000 volts; it is less efficient on higher voltages, but is actually used on 20,000-volt, and even 35,000-volt transmission lines.

One reason why the multiple-gap arrester is not satisfactory on systems of very high voltage is that the necessary increase in the number of gaps to prevent arcing over by the line voltage alone is out of all proportion to the increase in voltage. There is also much uncertainty as to the number of gaps required, which will depend on the position of the arrester in relation to surrounding grounded objects. With the ground potential brought very near to the arrester, the potential gradient at the end near the line frequently becomes high enough to ionize the air between the cylinders, thus carrying the line potential to lower cylinders, until the remaining gaps are so few that a discharge occurs. In order to obtain the more equal division of the total potential difference and so allow of a reduction in the total number of gaps, such as would be obtained by removing the whole arrester to a considerable distance from grounded objects, a metal guard plate or shield is sometimes placed near the gaps at the high-potential end of the arrester and connected to the line wire as indicated in Fig. 119. The theory of the potential distribution over the

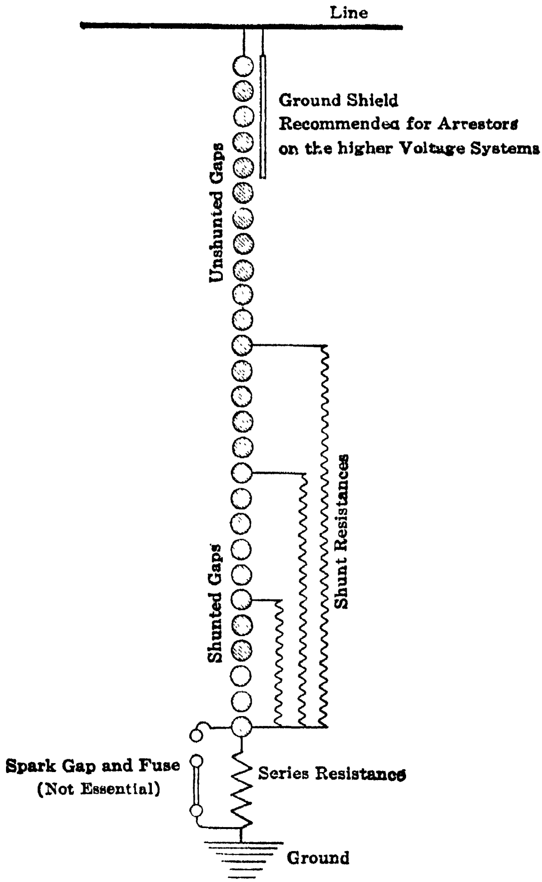


FIG. 119.—Diagram of multiple-gap arrester.

string of insulated metal cylinders in the multiple-gap arrester need not be discussed here, because the problem has already been considered in connection with the suspension type of insulator (refer to Art. 76, Chap. VIII). It is evident that if the electrostatic capacity between the consecutive elements of the arrester could be made large compared with the capacity to ground, a more uniform drop of potential over the series of elements would result, thus rendering this type of arrester more suitable for the higher pressures.

A special form of multiple-gap arrester is the compression-chamber arrester in which the pressure of the gases caused by the heat of the discharge is helpful in extinguishing the power arc.

116. Spark-gap Arresters with Circuit Breakers or Resetting Fuses.—If the resistance in series with a gap arrester is very small, a good path is provided to ground for taking a very heavy

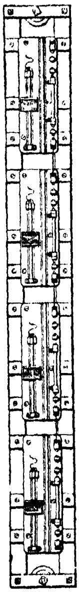


FIG. 120.—Garton-Daniels arrester for 10,000-volt circuit.

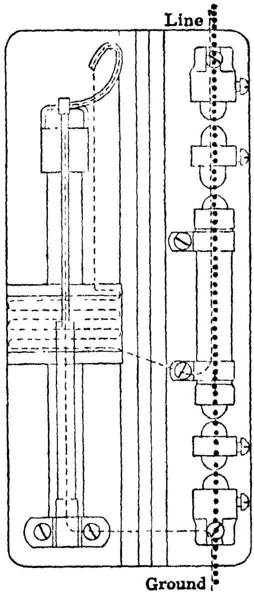


FIG. 121.—Diagram of Garton-Daniels arrester.

discharge; but there will be a large flow of power current in the arc following the discharge. This current may be interrupted by connecting some self-acting device, such as a fuse or automatic circuit breaker in the ground connection; and arresters, whether of the horn type or with any other kind of spark gap, are sometimes provided with fuses so arranged that when one fuse blows, the dropping of a lever or an equivalent device automatically inserts another fuse, so that the system is not left unprotected. Even without automatic replacement, if a number of gaps with fuses are connected in parallel, it will generally be found that one discharge will not blow all the fuses, and that during the passage of a single storm, the line will be adequately protected.

In the Garton-Daniels arrester, for use on a.c. circuits up to 20,000 volts, the principle of the multiple gap is combined with a simple type of automatic circuit breaker connected as a shunt to some of the spark gaps, in order that the discharge path for the lightning shall remain unaltered even during the operation of the arrester. The arrester is built up of several unit parts connected in series; each unit being rated for 3,300 volts. The illustration Fig. 120 shows a complete single-phase arrester for 10,000 volts. On a 20,000-volt circuit, there would be eight units in series, the total air-gap distance being $1\frac{1}{8}$ in., with a series resistance averaging 3,800 ohms. The diagram Fig. 121 shows a single unit of the Garton-Daniels arrester. The discharge follows the straight path through the two sets of air gaps and the resistance rod, as indicated by the round dots. The power current following the discharge will, after passing through the two upper gaps and the resistance rod, be shunted by the low-resistance winding of the circuit breaker; and if this following current is too heavy to be ruptured by the combined action of these two gaps and the resistance rod, the iron armature of the circuit breaker will be lifted by the action of the solenoid, thus throwing the two lower spark gaps in series, and extinguishing the arc.

117. Autovalve Arrester.—A recent form of multiple-gap arrester is known as the autovalve arrester. It includes certain features which are not present in the type which consists of a series of metal cylinders as described in Art. 115. When a charge of electricity is released on an overhead transmission system by the discharge of the thundercloud by which the charge is induced, it may be necessary to provide a path to ground capable of carrying a very large current for a short interval of time. The multiple-gap type of arrester with "non-arcing" metal cylinders will not do this satisfactorily, but the autovalve arrester provides a large area for the discharge between a number of semiconducting discs[1] separated by small air gaps. A column of resistance discs is constructed by stacking a large number of discs with an air gap of about 0.004 in. between them, this clearance being maintained by means of mica spacing rings which support the edges of the discs. When a potential difference slightly in excess

[1] Made by compressing a mixture of clay, corborundum, lamp black, etc., in suitable proportions.

of 350 volts per gap is reached, there will be a *glow discharge* in the air gaps between the discs. In this manner and by providing the necessary surface area to the discs, very large currents may be discharged for short periods of time. A distinctive feature of very great advantage of this arrester over other types of air-gap arresters is the fact that when the voltage drops to normal, the high resistance of the air gaps—which had been greatly reduced during the glow discharge—is again restored, thus shutting off the power current which otherwise would pass from line to ground. This arrester is, therefore, of the "valve" type, its behavior being generally similar to that of the electrolytic and oxide-film types referred to in the following articles.

118. Aluminum-cell Arrester.—When two aluminum electrodes are immersed in a suitable electrolyte, an insulating film of hydroxide of aluminum is formed on the surface of the metal; this effectually prevents the passage of any appreciable amount of current until a certain critical voltage is reached, when the film breaks down and the current is limited only by the resistance of the electrolyte. On lowering the voltage, the film is re-formed and the flow of current again limited to a very small amount.

With alternating currents, the critical potential difference per pair of plates is about 350 volts, and the practical construction of lightning arresters on this principle consists in stacking a large number of cone-shaped aluminum plates one within the other, with suitable separating washers of insulating material between them. In this manner a column is formed of a large number of cells in series, capable of withstanding high voltages. The whole is enclosed in a case containing oil, which improves the insulation and prevents the evaporation of the electrolyte which fills the spaces between adjacent trays within a short distance of the edge.

If cells built up in this manner are connected directly between line and ground, there will be an appreciable current passing through them, which is partly a leakage current, but chiefly a capacity current. It is, therefore, customary to insert a spark gap, usually of the horn type, in series with the aluminum-cell arrester, the gap being set to break down with a pressure slightly in excess of the normal working voltage.

Although the film of hydroxide is formed on the plates at the factory before the arresters are installed, it is necessary to maintain it by periodic "charging" of the cells, this being done by closing, or nearly closing, the spark gap in series, so as to put the full line pressure across the arrester. It is generally recommended that this be done once every day.

The effect of "charging" the cells, by passing an alternating current through the arrester, is to coat the electrodes with a thin insulating film of hydroxide of aluminum, the thickness of which is sufficient to withstand the line voltage. When this film is punctured by an overvoltage, the current passing through the many minute punctures re-forms the insulating film and thus prevents the passage of any but capacity and leakage currents until the applied pressure again exceeds the critical voltage. This particular type of arrester is, therefore, well adapted for the protection of large high-voltage a.c. systems, and in practice it has given excellent service. Its chief disadvantages are its high cost, the attention it requires in the matter of periodic charging, the impossibility of testing its condition while in use, and the necessary spark gap which may be the cause of destructive oscillations in the system. It is true that, on high-pressure systems, the surges set up by spark gaps in series with the cells are unlikely to cause serious trouble, but this suggests the possibility of using simpler and less costly devices. If the aluminum-cell arrester could be so modified that it could be permanently connected between line and ground without a spark gap in series, and if the necessity for daily "charging" could be dispensed with, this type of protective device would be ideal for use on high-voltage a.c. systems.

119. Oxide-film Lightning Arrester.—What is known as the oxide-film arrester has two particular operating advantages in that it has no liquid electrolyte and it does not require daily "charging." The electrodes of this arrester are circular discs of sherardized iron held apart by a porcelain ring, the active material between the plates being lead peroxide (PbO_2). This material is a fairly good conductor of electricity, but the effect of heat caused by an electric discharge is to convert it into a lower oxide (PbO), which is an insulator. Under the action of the line voltage the growth of this non-conducting film at the surface of the electrodes continues until it is thick enough to check the flow of current, the electromotive force across each pair of electrodes being from 250 to 300 volts. The number of cells in series is such that only a very small leakage current passes through the arrester when subjected to normal line pressure. Nevertheless it is found necessary to provide a spark gap in series, which is one of the disadvantages which it shares with almost every other type of lightning arrester. When an overvoltage occurs, *i.e.*, when the spark gap discharges, the insulating film of oxide is punctured and the discharge takes place through the lead peroxide, but the puncture holes are sealed up by the passage of the current. The oxide-film arrester can be used on high-voltage a.c. circuits in places where there are no attendants and where certain other types of equipment could, therefore, not be used.

120. Choke Coils.—When a lightning arrester is connected between line and ground in or near generating or sub-stations for the purpose of providing a path to ground for high-frequency surges, an inductive reactance is placed in series with the apparatus to be protected. This reactance must not be so great as to cause a serious drop in pressure when carrying the normal line current, neither must it be so small as to allow the induced charges traveling along the line to pass through it rather than jump the air gap of the lightning arrester. This reactance usually takes the form of an air-insulated coil of copper wire or rod, supported at each end on a suitable insulator. The "hour-glass" form of coil, in which the diameter of the turns increases from the center toward both ends, is mechanically stiffer than a cylindrical coil, and any arc that might be started between adjacent turns has a greater tendency to clear itself. The air space between turns is usually from 1/4 to 3/8 in. It has sometimes been argued that, except for the drop of pressure under working conditions and the higher cost, there is no objection to installing very large choke coils having a high inductance. This argument is, however, incorrect, except for the special case in which some protection against surges is provided on the machine side of the reactance in addition to the lightning arresters on the line side. A high reactance may be quite satisfactory if it is merely intended to hold back high-frequency currents traveling along the line; but surges may originate near the generators or transformers due to switching operations or other causes, and a very high reactance between the electrical plant and the line will tend to aggravate the effect of comparatively low-frequency surges which might otherwise be dissipated in the line, or even through the lightning arrester. In fact, choke coils should be designed with due regard to the apparatus they are intended to protect, with a view to avoiding the building up of high voltages at the terminals of the generating plant in the event of surges being set up in or near the plant itself. When the lightning arrester discharges, it does not follow that high-frequency waves do not find their way through the choke coil to the machines; but the inductance of the choke coil will lower the frequency of such waves, or, in other words, will reduce the steepness of the wave front to such an extent that the insulation of the machines will not be injured. The first few turns of a transformer or generator winding will act as a choke coil and usually prevent damage to the turns farther removed from the terminals; but they are likely themselves to suffer injury, as the charge will leap across the insulation and so get to ground. If it is assumed that the reactance of the first six turns of a transformer winding is sufficient to afford protection to the seventh and subsequent turns of the winding, then a choke coil having a reactance equal to that of the six turns of transformer winding will afford the necessary protection to the transformer. A higher reactance in series is unnecessary and may be dangerous.

As an example of what appears to be generally sufficient to afford reasonable protection to modern machinery, about 25 turns of copper rod wound into a coil 10 in. in diameter may be used on voltages from 10,000 to 25,000, while for pressures of the order of 100,000 volts, two such coils would be connected in series. The diameter of the copper rod would depend upon the current to be carried; but it is best to have it large enough in all cases to be self-supporting, although coils wound on insulating frames, with separating pieces between turns, are not necessarily objectionable.

121. Concluding Remarks on Lightning Protection.—The best means to adopt for the protection of any particular line or portion of a line against lightning disturbances is still largely a matter of conjecture, but by the exercise of sound judgment, an experienced engineer should be able to provide reasonable protection against discontinuity of service during atmospheric disturbances. There are many devices to choose from, each of which has a particular field of usefulness. It is probable that, in a few years' time, the additional information on this subject, which is continually being accumulated, will lead to uniformity in the protective arrangements adopted under the various conditions arising in practice. In the meanwhile, however, a careful record of all accidents due to lightning or abnormal pressure rises should be kept in connection with each power system with overhead transmission, as this will generally lead, after careful investigation, to certain amplifications or modifications of the existing protective arrangements such as to prevent the repetition of similar accidents. In this manner, very fair protection can be afforded at the present day to almost any overhead transmission system; but it is doubtful if it will ever be possible to protect apparatus against a direct lightning stroke. Damage to machinery due to this cause is, however, very rare.

In regard to the protection of the line itself, it is obvious that protective devices, however complete or perfect they may be, provided at the two ends of a long transmission, afford no protection to the insulators along the line. The frequently grounded guard wire would appear to be a good protection to a line; but here again the engineer must use his judgment, because certain portions of a line may require far more protection than other portions, and even if the cost of guard-wire protection be considered excessive for the entire length of a long-distance transmission, it may yet be a decided advantage to provide guard-wire protection near the generating and transforming stations and on those parts of the line most likely to be affected by atmospheric disturbances.

In some cases, it may be wise to improve the insulation and to raise the voltage at which a "spill-over" will occur; while under other circumstances it might be better to provide an easy path for a discharge over insulators, by means of suitably disposed arcing rings or equivalent arrangement. P. H. Thomas once explained the matter of line insulation by making use of a very simple analogy. Where a discharge strikes the line, a wave starts and the potential of this wave will be such as can be allowed by the line itself; the energy of the discharge is limited by the static capacity of the line and the voltage at which a "spill-over" will occur at the insulators. The energy of the traveling waves "grows less and less as they proceed. This action may be likened to the formation of a wave in a long, narrow trough with high sides containing water and normally less than half full, by sudden flooding of the trough by a large quantity of water at some particular point; the excess water spills over and escapes from the trough at the point of the flooding, but there is still a wave started in each direction as high as the sides of the trough will permit; this passes along until the end is reached or the energy of the wave is gradually dissipated. It makes no difference how much water is thrown into the trough, there can be a wave only as high as the sides will permit."

One point that is sometimes overlooked is the effect of the line *current* on pressure disturbances. The disturbances that are set up by switching operations or by power arcs following a lightning discharge will be far more serious when the current is large than when it is small. This is one reason why extra-high-tension transmissions suffer less from lightning disturbances than moderate-voltage systems on which the current is often larger. It is hardly an exaggeration to say that the handling of heavy currents on long-distance transmissions presents more engineering difficulties than insulation probelms on the high-voltage schemes.

Very-high-voltage transmission lines do, indeed, operate satisfactorily without lightning protection, especially when working at pressures near the critical voltage of the corona formation, and some relief to high-pressure energy is afforded by the corona itself.

It will be gathered from the foregoing remarks that not only the pressure of transmission, but the amount of power transmitted, is an important factor in the problem of lightning protection.

It is possible that the near future may see some developments in the matter of facilitating the dissipation of high-frequency energy in the line itself, with the object of rapidly limiting the amplitude of the traveling waves and the distance from the center of disturbance at which their effects can be of practical account. It is obvious that what is required is a line that will transmit, without undue loss, the power currents at normal frequency, and yet afford means for the rapid dissipation of high-frequency energy. Apart from the property peculiar to the corona, which leads to the dissipation of energy on over-voltages, there is a property common to all metallic circuits which leads to the more rapid dissipation of high-frequency than of low-frequency energy. The so-called "skin effect" which apparently increases the resistance of a conductor carrying alternating or fluctuating currents, owing to the forcing of the current toward the outside portions of the wire at the higher frequencies, is clearly of value in limiting the distance over which high-frequency disturbances are propagated. By covering the conductor with a thin layer of high-resistance metal, astonishing results can be obtained. Experiments made with wires having a coating of nickel only 0.07 mm. thick showed that the resistance offered to currents of 300,000 cycles per second was four times the resistance offered by the same wires without the coating of high-resistance metal. This was referred to by Gino Campos at the Turin International Electrical Congress of 1911.

Much may be accomplished by a careful study of local conditions and by the intelligent selection of such protective apparatus as may be available. On the other hand, it is not improbable that the coming years will see fewer rather than more protective devices than are now used. Careful design of the line as a whole, with the provision of suitable choke coils at the ends, or the special insulation of the end turns of the transformer windings to withstand very much higher pressures between turns than the remainder of the high-tension winding, will frequently be all that is necessary. Even if the installation of costly apparatus may slightly decrease the risk of damage by lightning, the economic aspects of the problem should be carefully considered before deciding to provide the additional protection.

For comparatively low voltages the use of condensers appears to be justified; but they are always costly, and it is questionable whether the money invested in them might not with advantage be spent on improved insulation, especially in the immediate vicinity of the terminal apparatus.

It must not be forgotten that nearly all existing types of protective devices momentarily connect the system to ground across what may be termed explosive distances in air. Such devices—as explained in preceding articles—are always apt to cause more trouble than the particular disturbance they are intended to prevent. Instead of inviting an "act of God" by the installation of more or less ingenious high-priced apparatus of uncertain protective value, it is beginning to be recognized that economy and better service conditions may be obtained by the intelligent reinforcing of insulation where this will be most effectual, and by the provision of carefully designed preventive resistances, reactances, or condensers, which do not lead to disruptive discharge when abnormal voltages are induced on the line, and are not dependent upon the momentary grounding of the system to relieve it of the abnormal transient voltages.

It is now customary to omit lightning arresters from high-pressure (220,000-volt) overhead transmission systems, and operate with star-connected transformers, the neutral being either solidly grounded, as in America, or grounded through a properly designed resistance or reactance, with or without a spark gap in series, as in Europe.

122. Continuity of Service. Localizing Faults.—Electrical troubles on overhead lines may be due to faulty insulators, or they may have their origin in lightning or switching operations causing high-frequency oscillations and abnormally high voltages, leading to fracture of insulators or breakdown of machinery. Flash-over of suspension-type insulator units has frequently been caused by droppings from large birds, eagles, hawks, and even owls, roosting or perching above the insulators. The provision of saw-tooth guards or pointed rods designed to encourage the birds to seek other roosting places appears to be the solution of this particular trouble.[1]

Troubles are more likely to be due to mechanical defects, or mechanical injuries sometimes difficult to foresee and guard against. Trees may fall across the line, land slides may occur and overturn supports, or severe floods may wash away pole foundations; and against such possibilities the engineer must, by the exercise of judgment and foresight, endeavor to protect his work. The width of the right of way should depend upon the height of trees and should be so wide that the tallest tree cannot fall across the wires; poles and towers should, if possible, be kept away from the sides of steep hills where the nature of the ground suggests the possibility of falling stones or of land slides, and the possibilities resulting from floods must be reckoned with. Other causes of mechanical failure are storms of exceptional violence, either with or without a heavy coating of ice on the conductors. When strong winds blow across ice-coated wires, the danger is not only that the wires themselves may break, but also that the resulting horizontal loading on the poles or towers may be great enough to break or overturn them.

Faulty mechanical design of the line as a whole, and improper supervision or inspection during construction, will account for many preventable interruptions to service, but even with the best and most substantial construction there is always the possibility of trouble arising at some place on a transmission line of considerable length.

Assuming the design and construction of the overhead transmission system to be in accordance with sound engineering practice, continuity of service will still be dependent upon

[1] See *Elec. World*, p. 611, Mar. 21, 1925, and p. 1058, Nov. 21, 1925.

thorough and periodic inspection and the maintenance of the line in proper condition. Attention to these matters is the duty of the operating and field engineers, and it is not proposed to discuss the subject in detail in these pages. The practice of working on live circuits—thus effecting replacements and carrying out repairs without disconnecting the line—is becoming more and more common.[1] Methods of detecting defective or leaky insulators were briefly referred to in Art. 78, Chap. VIII.[2]

123. Interference between Power and Telephone Lines.—Although the question of interference between power lines and neighboring telephone lines is a very important one, it is also a difficult one to settle satisfactorily or even discuss adequately in a book dealing primarily with the design of high-tension transmission lines. The problem is of particular interest to the telephone engineer who will no doubt ultimately find a satisfactory remedy for the very real troubles which are likely to occur—especially when abnormal conditions lead to unbalancing of the power load—when telephone wires run parallel to a.c. power lines for a considerable distance.

It is a comparatively easy matter to calculate the flux of induction which the current in the power conductors will set up in the loop formed by the telephone wires, and by carefully planned and frequent transpositions, this effect can be greatly reduced if not entirely overcome; but the electrostatic effects are probably of greater importance and are less easily remedied.

Insulation of Telephone Lines.—It has only lately been realized that one of the essential requirements for telephone lines strung on the same supports as, or very close to, high-pressure power conductors, is high insulation. This good insulation is necessary to prevent puncture of the insulators when high potentials are induced on the telephone wires at times of abnormal conditions—such as intermittent short-circuits, or lightning disturbances—on the power lines. As an example of good practice in this respect, the Georgia Railway and Power Company

[1] See *Elec. World*, vol. 79, p. 501, Oct. 1, 1921; also vol. 75, p. 1140, May 15, 1920; vol. 74, p. 993, Nov. 29, 1919; and vol. 88, p. 707, Oct. 2, 1926.

[2] For further information dealing with this particular phase of the general problem of maintaining satisfactory continuous operation, the reader should refer to the following articles, all of which have appeared in the *Elec. World:* Aug. 8, 1925; ·ILER, G. A., (testing stick); Apr. 12, 1924; BENNETT, C. E. (aislometer); June 16, 1923, DOBLE, F. C. (general, with theory and description of methods); Oct. 24, 1914 (telephone receiver).

has provided insulators suitable for a working pressure of 27,000 volts to carry the telephone wires which parallel their 110,000-volt power lines.

Electrostatic Induction.—The dielectric field due to the alternating voltage of the power conductors induces a varying charge upon the neighboring telephone wires. If each wire of the telephone line were at an equal average distance from each conductor of the power line, there would be no difference of potential created between the two sides of the telephone receiver, and there should be no buzzing, etc. due to this cause. In other words, with adequate, properly worked out transpositions, the capacity currents passing between the power line and the telephone line will not pass through the telephone receiver. But even if the electrostatic flux is at all times of the same kind and amount for both wires of a telephone circuit, this does not prevent the telephone circuit as a whole being subject to alternating pressures relative to ground, and these pressures may reach high values, depending upon the voltage of the power line and the proximity of the two (parallel) circuits. If the telephone circuit is grounded, the charging current passing between the power conductors and the telephone line will find its way to ground by flowing *along* the telephone wires, and since this current may amount to several amperes, trouble is almost certain to occur unless what are known as "drainage coils" are provided. If a choke coil with an iron core—such as the primary of an ordinary lighting transformer—is connected across the two wires of the telephone circuit and then has its middle point connected to ground, the electrostatic charge will be "drained" off the line without interfering with the operation of the telephone. The telephone line paralleling the 110,000-volt transmission of the Georgia Railway and Power Company is provided with 15- and 25-kw. standard 2,300-volt distribution transformers (with open secondaries) for this purpose.

When the telephone wires are carried on the same poles as the power conductors, the induced potential difference between line and ground may amount to thousands of volts. In addition to draining coils, high insulation of the telephone line is, therefore, required, including specially insulated transformers, platforms, etc. for the protection of men working on the telephone line.

Since the important range of frequency in telephone conversation is between 200 and 1,200 cycles per second, electric waves of frequencies between these values are particularly to be avoided. This suggests the desirability of eliminating the higher harmonics from the power circuit so far as possible by striving for a sine-wave of e.m.f. and current. Another reason why the higher harmonics are objectionable is the fact that the magnitude of the inductive effect on parallel circuits is proportional to the frequency.

Magnetic Induction.—Referring to Fig. 122, the voltage induced by the single-phase power circuit AB in the loop formed by the wires C and D of the parallel telephone circuit may be calculated as follows if it is assumed that there are no transpositions and that the current wave is a pure sine curve.

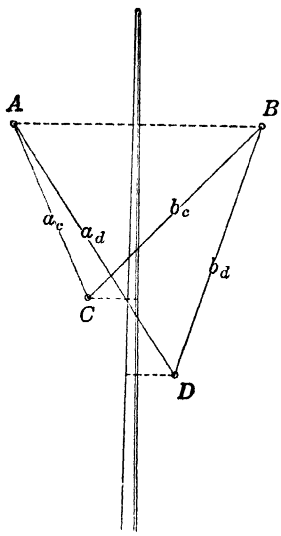


FIG. 122.—Telephone wires on same pole as single-phase power circuit.

By formula 92 (Art. 82) the flux in the loop CD due to the conductor A carrying a current I is

$$\Phi_A = \frac{2Il}{10}\left[\log_\epsilon\left(\frac{a_d}{r}\right) - \log_\epsilon\left(\frac{a_c}{r}\right)\right]$$

and the flux due to the current $-I$ in the conductor B is

$$\Phi_B = -\frac{2Il}{10}\left[\log_\epsilon\left(\frac{b_d}{r}\right) - \log_\epsilon\left(\frac{b_c}{r}\right)\right]$$

the total flux being

$$\Phi = \frac{2Il}{10}\log_\epsilon\left(\frac{a_d b_c}{a_c b_d}\right) \qquad (157)$$

By making the substitution and alterations as in obtaining formula (94),

$$\left.\begin{array}{l}\text{Volts induced per mile run of the}\\ \text{two parallel circuits}\end{array}\right\} = 0.00466fI\log\left(\frac{a_d b_c}{a_c b_d}\right) \quad (158)$$

Since the value of log 1 is zero, there will be no flux linkages with the telephone circuit when the condition $a_d b_c = a_c b_d$ is satisfied (see Fig. 122). This would be the case in either of the arrangements shown in Fig. 123. It is true that these arrangements refer to a telephone line running parallel to a single-phase power circuit, and that in any case transpositions would probably be necessary in practice; at the same time, attention should be given to the relative positions of power and telephone lines, apart from the question of transpositions. Practical methods of transposing both power and telephone wires will not be considered here, as the number of possible conditions to be remedied is almost unlimited; but once the principle is understood, some means of obtaining the required result can generally be found. Transpositions on the high-tension power lines should be avoided as far as possible, and it should not be necessary to transpose the power conductors of a high-voltage transmission line at more frequent intervals than every mile.

Although formulas (157) and (158) were worked out, for simplicity, by taking the case of a single-phase transmission, the inductive effects of any number of power conductors may be

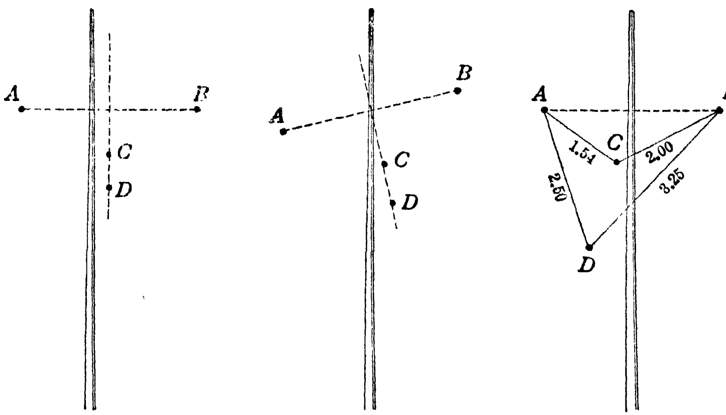


FIG. 123.—Arrangement of wires to avoid magnetic induction in telephone circuit.

calculated in a similar manner, proper attention being paid not only to the magnitude but also to the direction, or phase relation, of the currents in the several conductors (refer to Art. 85, Chap. IX).

The fundamental frequency f occurs in the formulas, because pure sine-waves are assumed; but it should be pointed out that disturbances due to voltages and currents of the fundamental frequency have little effect on the telephone, the chief trouble being due to the fact that in practice the pure sine-waves are not obtained, and the higher harmonics, even when of small magnitude, are likely to produce noises in so sensitive an instrument as the telephone receiver, which may render conversation impossible.

It is not proposed to discuss here the protective apparatus as used on telephone circuits paralleling power lines, partly because the subject is somewhat beyond the scope of this book, but also because the troubles of the telephone engineer in this connection have not been entirely overcome, and he is still working on the problem. By the cooperation of power and telephone engineers, much may be and has already been accomplished. It is unreasonable and uneconomical to look for the final solution of these interference problems in the wide separation between power and telephone circuits; but, on the other hand, with the growth of radio communication and the possibility of using the power conductors themselves for the transmission of telephonic messages, there will probably be less need in the future for carrying a metallic telephone circuit parallel to the power lines.

CHAPTER XII

TRANSMISSION OF ENERGY BY CONTINUOUS CURRENTS

124. Historical.—The first attempts at transmitting energy electrically were made with continuous currents. In the year 1880, or thereabouts, Gustave Cabanellas anticipated the Thury system by proposing to transmit energy by using a continuous current of constant value obtained from a number of dynamos connected in series. In 1883 Marcel Deprez exhibited at Munich two Gramme dynamos, specially wound for 2,400 volts, transmitting about 5 kw. over a distance of 8 km. In 1887 about 130 kw. were transmitted from Creil to Paris (about 53 km.), using continuous current at 10,000 volts. Later installations, including one of 1,000 kw. from Saint-Owen to Paris and another near Genoa (27 km. at 12,000 volts), were in successful operation until the year 1900 and probably later.[1] At this time it was generally thought that long-distance transmissions at high pressures would, in the future, be by continuous currents, and in 1902 R. Thury, working on the suggestion originally made by Cabanellas, developed his constant-current series system and transmitted 150 amp. at a maximum of 27,000 volts from St. Maurice to Lausanne.

Following the work of Nicola Tesla, and the practical transmission by three-phase currents by Dolivo Dobrowolski at the Frankfort Exposition in 1891, and also the pioneer work of S. Z. de Ferranti who transmitted underground by single-phase current at 10,000 volts from Deptford to London, the possibilities of alternating currents and the ease with which the pressure could be raised or lowered by static transformers led to the rapid use of polyphase currents for long-distance transmission of energy.

125. Present Status and Future Prospects of A.C. Transmission.—Of the present status of power transmission by alternating

[1] A complete list of the earlier d.c. transmissions using generators in series will be found in the paper by J. S. Highfield in the (British) *Jour. Inst. Elec. Eng.*, vol. 51, p. 640, 1913.

currents, little need be added to what has already been said, since the preceding chapters have dealt mainly with the theory and principles underlying present-day methods of electrical transmission nearly all of which—especially in America—is by means of polyphase alternating currents. An enormous amount of literature treating of electrical transmission from both the theoretical and operating points of view has been published during recent years, and some of the many articles and papers should be referred to in order to realize what has been accomplished, including the latest 220,000-volt long-distance transmissions.[1]

It is important to bear in mind that 220,000-volt transmission lines can be economical only when large amounts of energy have to be transmitted over long distances. In the future, higher pressures will probably be used. Assuming that it should ever be desirable to transmit so great a distance as 1,000 miles from a single generating station, line pressures of the order of 1,000,000 volts might be justified, and the feasibility of using voltages considerably higher than those of the present day has been demonstrated by tests recently conducted by the General Electric Company at its Pittsfield works. In order to avoid excessive losses due to corona, hollow conductors of large diameter would be necessary in connection with very high voltages. The use of higher pressures for the transmission of greater amounts of energy over greater distances is not likely to stand in the way of future developments of three-phase transmission systems; the obstacles to very rapid development are chiefly due to the fact that the current is alternating and not uni-directional, thus introducing certain difficulties due to the effects of inductance and capacity. These difficulties are not always easy to foresee, and they require for their correction or control additional apparatus which is always costly, frequently complex, and occasionally unreliable.

[1] It is difficult to make selections from the large amount of material available, but a good summary of what has been accomplished by the Southern California Edison Company transmitting at 220,000 volts from their Big Creek hydroelectric plants near Los Angeles, will be found in *Trans.* A.I.E.E., vol. 43, p. 1222, 1924. See also particulars of the Pitt River and Hat Creek developments of the Pacific Gas and Electric Company in articles by F. G. BAUM and S. BARFOED in the *Elec. World*, Jan. 27 and Feb. 3, 1923. As an example of a modern 220,000-volt transmission in Pennsylvania, see the description of the Wallenpaupack hydroelectric development of the Pennsylvania Power and Light Company by A. E. SILVER and A. C. CLOUGHER in the *Elec. World*, July 24, 1926.

One of the questions to which engineers have recently been giving attention is what has been referred to as the stability of transmission systems, a term which has not been very clearly defined, but which includes something more than the power limitations under normal operating conditions. Stability involves many factors besides the electrical constants of the line itself. For instance, if an electrical disturbance occurs due to any cause, the ability of the system to regain equilibrium, that is to say, to damp out or check any tendency on the part of machines to swing out of step, will depend upon a great many factors such, for instance, as the amount and power factor of the load at the instant the disturbance occurs, the location, capacity, and characteristics of synchronous reactors, and the nature of the regulation in the field windings of generators, whether quick or slow in correcting changes in voltage.

The mathematical solution of problems involving the stability of a.c. transmission systems is possible when all the conditions and necessary data are known; but on account of the complexity of modern interconnected systems, it is almost impossible to predict exactly what will happen when the normal conditions of operation are suddenly disturbed. A valuable collection of half a dozen papers bearing on this aspect of future developments in power transmission by alternating currents will be found in *Transactions* A.I.E.E.[1] Perusal of these papers will convince the reader that the determination of the maximum load which a large transmission system may safely carry is not a simple matter, and that although the doctors may successfully diagnose a case that is comparatively free from complications, they are not all agreed as to the most desirable remedy or as to its efficacy when applied. The reason why considerable attention has been given of late years to the question of stability is that, with the increasing magnitude and high cost of modern high-voltage long-distance transmission systems, it becomes important to transmit as large an amount of energy as the system is capable of transmitting without running too great a risk of endangering the quality or continuity of the service.

[1] Vol. 43, pp. 1–103, 1924. Refer also to the following more recent papers: CLARKE, EDITH, "Steady-state Stability in Transmission Systems," *Jour.* A.I.E.E., April, 1926; SHIRLEY, O. E., "Stability Characteristics of Alternators" and NICKEL, C. A., and LAWTON, F. L., "An Investigation of Transmission Line Power Limits," both papers in *Jour.* A.I.E.E., September, 1926.

The remainder of this chapter will be devoted to a brief investigation of what has been accomplished and what may yet be done in the matter of long-distance transmission by means of continuous currents.

126. General Description of the Thury System.—In the Thury system of electric power transmission by continuous currents, the current is constant in value and the pressure is made to vary with the load. All the generators and all the motors are connected in series on the one wire, which may be in the form of a closed loop serving a wide area, or it may consist merely of the

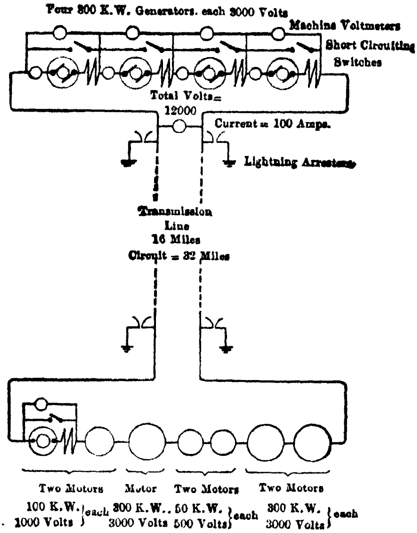


FIG. 124.—Diagram of connections—Thury series system.

outgoing and returning wires between a power station containing all the generators, and one or more substations, with motors, at the end of a direct transmission. The diagram Fig. 124 shows a typical arrangement of machines for a small installation on the Thury system. In this example there are four generators at one end of the line and seven motors of various capacities at the other end of the line, all the machines being connected in series and provided with short-circuiting switches. The connections are, however, so simple that the diagram is self-explanatory. The voltage at the terminals of any one dynamo is limited, because the necessity for a commutator renders it impossible to wind a continuous-current machine for so high a pressure as may be obtained from a.c. machines. The limiting pressure per commutator on the existing Thury systems is 5,000 volts, this being on the machines of the Metropolitan Electric Supply Company of London: the lowest is 1,300 on a small two-generator plant in Russia.

In order to obtain the high pressures required for economical transmission over long distances, it is necessary to connect many generator units in series; the difficulty of insulation between machines and ground being overcome by mounting the dynamos and motors on insulators and providing an insulated coupling between the electric generators and the prime movers. An insulating floor is also provided.

When a machine, whether generator or motor, is not in use, it is short-circuited through a switch provided for this purpose. As the motor load varies, generators are switched in or out of circuit, thus varying the total voltage. When it is required to switch in an additional generator, the machine is brought up to speed until it gives the proper line current before the short-circuiting switch is opened to throw the machine in series with the line. To start up a motor, the short-circuiting switch is opened when the brushes are in the position of zero torque. The brush rocker is then gradually moved round, and the motor, starting from rest, increases in speed until the brushes are in the required position; the actual speed, for any particular position of the brushes, being dependent upon the load.

The motors may be distributed anywhere along the line, either on the premises of private users of power, where they may be directly coupled to the machinery to be driven, or in substations, coupled to constant-pressure electric generators giving a secondary supply for lighting and power purposes. In most of the Thury undertakings in Europe, this secondary supply is three-phase alternating current.

A series-wound dynamo machine, with a current of constant value passing through field-magnet and armature windings, is essentially a constant-torque machine. In the case of a motor, if the load is decreased, the motor will increase in speed and tend to "run away"; with increase of load, the motor will slow down, and in time come to a standstill. In regard to the generators, the ideal prime mover is one which will give a constant torque, such as a steam engine with fixed cut-off and constant steam pressure; and a single generator so driven would be practically self-regulating and maintain constant current regardless of load, as the speed and, therefore, the pressure at terminals would vary in accordance with the motor load. The generators are, however, usually driven by prime movers which are far from fulfilling the ideal conditions. Most of the Thury stations are driven by water turbines, which are most efficient as constant-speed machines; while the maximum torque at low speeds is generally about twice the torque under conditions of highest efficiency at normal speed.

Just as various devices are provided, when working on the parallel system, to maintain constant pressure of supply, so in the series system it is necessary to provide regulating devices to maintain a constant current. Regulators controlled by the main current or by a definite fraction of the main current passing through a solenoid can be made to act on mechanism designed to vary the speed of the prime movers. This method is quite practicable, but where the type of engine, such as a water wheel under constant head, is not suited to variable-speed running, the machines may be run at constant speed, and the automatic device made to alter the magnetic field cut by the armature conductors, this being the only alternative means of varying the voltage generated. The alteration of the effective magnetic flux may be accomplished:

1. By shunting a portion of the main current so that it shall not all pass through the field winding.
2. By shifting the position of the brushes on the commutator.

A combination of both methods appears to give satisfactory results. The method (1) alone is likely to lead to sparking troubles because of the relatively greater armature reaction due to the weakening of the field; and, in practice, it is found inadvisable to shunt more than one-third of the total current. It is, of course, understood that the large variations of voltage are obtained by connecting more or fewer generators in series on the line.

The motors, whether connected directly to the machinery of mills or factories or used for driving sub-generators of the constant-pressure type, are usually required to run at constant speed. Their regulation is effected by a small centrifugal governor which rocks the brushes by acting on intermediate mechanism driven by the motor itself. The reversal of a motor is most simply effected by shifting the brushes round through the no-voltage position until the current reverses in the armature coils.

A short-circuit on a motor merely removes that portion of the total load from the system, and the regulators on the generators will readjust the pressure accordingly. If a short-circuit occurs on a generator, the prime mover may be protected from the shock by a slipping coupling, which is commonly provided. If, owing to the failure of a prime mover, a generator tends to reverse and be driven as a motor, it may be short-circuited by a switch that can very easily be made to operate automatically on reversal of current.

127. Straight Long-distance Transmission by Continuous Currents.—Although high-pressure direct current may be used on the loop system with any number of motors or motor substations distributed along the line—and, if desired, with any number of generating stations at suitable points on the loop—it will generally be found that a parallel constant-pressure system is preferable for covering a large industrial area, the simple reason being that, with the series system having a load more or less uniformly distributed along the loop, the system is a high-tension transmission at the start only, since the required voltage decreases with the distance from the generating plant. It is true that the cost of the insulation may, therefore, be less than for a system on which the pressure is high throughout, but that can be said of any low-tension system. The point is that, in the case of the series loop serving a wide district, with power taken off at intervals along the line, the average pressure at which power is supplied to the motors or substations is only about half that which is supplied to the line where it leaves the power station. It must not be concluded that the Thury system is not well adapted to supplying several motor substations. It is an easy matter, as previously mentioned, to connect any number of motors in series on the line, but in order to get the full benefit of the series system these substations should all serve a comparatively small district at the distant end of the transmission line.

Apart from these considerations, and notwithstanding the advantages of a long-distance straight transmission, the Thury system would appear to be admirably adapted to serve as a link between otherwise isolated power plants in an industrial or thickly populated district of considerable area. By means of this system there is not the slightest difficulty in putting in series a number of generating stations on the one power line, and stations supplying alternating current of various voltages and frequencies can thus be linked together with the greatest ease and simplicity.

128. Insulation of Line when Carrying Continuous Currents.—The question of sparking distances and the behavior of insulating materials when subjected to continuous-current pressures of high values is of the greatest importance when considering the relative values of the Thury system and the more common three-phase high-tension transmission. On the assumption of the theoretical sine-wave, the maximum instantaneous value of an alternating e.m.f. is $\sqrt{2}$ times the root-mean-square value, and comparisons between a.c. and d.c. transmission are usually made on this basis, which makes the allowable continuous-current pressure to ground or between wires, for the same insulation and spacing, $\sqrt{2}$ times the working pressure of an a.c. system. The ratio should, however, be based on experimental data, and, with a view to obtaining definite and conclusive information on this point, Mr. Thury conducted some years ago a very complete set of comparative tests with high voltages, both continuous and alternating. The results of these tests are probably more favorable to the a.c. systems than would have been the case had they been conducted on existing high-pressure power transmission systems, because the experimental alternator used in the tests gave a rather flat-topped e.m.f. wave without any irregularities. The tests conducted to determine the comparative pressures at which various insulating materials would be punctured all tend to show that, with continuous currents, something more than twice the alternating pressure is required to puncture the insulation; and, in regard to sparking distances, the d.c. voltage necessary to spark over a given distance is, on the average, double the a.c. voltage. In fact, this very complete series of tests seems to indicate that any existing transmission line designed for a definite maximum working pressure with alternating currents is capable of being used to transmit continuous currents at twice this pressure. It is also interesting to note that insulators which become hot when subjected to high a.c. voltages remain cool when tested with continuous currents. In fact, the leakage losses on the Thury transmissions are small. The total leakage loss over about 3,000 insulators on the St.

Maurice-Lausanne transmission (a distance of 35 miles), even in damp weather, is something of the order of 900 watts.

It is usual to employ two insulated wires for d.c. high-pressure transmission, but under certain conditions it might be quite satisfactory to use the earth as the return conductor. The arrangement with two wires and the entire electric circuit insulated from earth is usual for pressures up to 25,000 volts. It has the advantage over any grounded system that any point on the circuit may become grounded without causing a stoppage, and repairs can readily be carried out by temporarily grounding two more points, one on each side of the fault. The facility and safety with which repairs on the high-tension system can be carried out by grounding the point where the work is being done is another advantage of this arrangement.

If a ground connection is made at both ends of the two-wire transmission, the ground wire being so situated as to balance the load as well as possible, an arrangement equivalent to the ordinary three-wire system is obtained. The pressure between wires may then safely be doubled because the potential difference between any one wire and earth can never exceed half the maximum pressure of transmission. On the other hand, some of the advantages of the non-grounded system are lost.

A d.c. transmission to any economic distance by means of a single wire, using the earth as the return conductor, is by no means an impossible scheme. The ground resistance is practically zero, the loss of pressure being almost entirely in the immediate neighborhood of the grounding plates. Tests made on the St. Maurice-Luasanne line (35 miles) gave a total ground resistance of 0.5 ohm. Continuous currents of the order of 100 amp. returning through the earth do not appear to be objectionable in any way. By taking the ground connections to a considerable depth below the surface, the current density at ground level would everywhere be so small that interference with opposing interests would hardly be possible.[1]

[1] Lausanne, Switzerland, was supplied continuously for more than a year from the St. Maurice station through a single conductor, the return being through the earth. The current was 150 amp.; iron ground plates were used, and it was found that, after the first layer of oxide was formed, oxidization was very slow. The total resistance of the ground connections was about 1.6 ohms, and there was no trouble with neighboring telephone lines.

In London, J. S. Highfield of the Metropolitan Electric Supply Company tried the experiment of putting the return cable of his 90-amp. Thury

129. Relative Cost of Conductors: Continuous Current and Three-phase Transmissions.—In order to study the relative costs of conductor material required for the series d.c. system and the more common three-phase a.c. transmission, a basis of comparison is necessary, and the following assumptions will be made:

1. Same distance of transmission; no tapping of current at intermediate points.

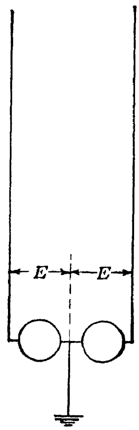


FIG. 125.

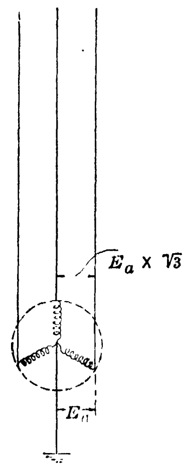


FIG. 126.

FIGS. 125 and 126.—Comparison of voltages on direct-current and three-phase systems.

2. Same total amount of power transmitted.
3. Same power loss in conductors (losses due to leakage and capacity of lines are neglected).
4. Same insulation used on both systems.

This last condition is practically equivalent to stating that the maximum value of the voltage shall be the same. It is proposed to consider the following four conditions:

a. Same maximum pressure above ground; the d.c. voltage being $\sqrt{2}$ times the a.c. voltage (sine-wave assumed).

$$\text{Ratio } \frac{E_a}{E} = \frac{1}{\sqrt{2}} \qquad (159)$$

transmission in parallel with the ground, and it was found that 30 amp. still returned through the cable, with 60 amp. through the ground. Very low ground resistance may be obtained by burying the ground plates a considerable distance below the surface.

where E and E_a stand, respectively, for the continuous and alternating voltages to ground (see Figs. 125 and 126).

b. Same as (*a*); but d.c. voltage double the alternating voltage.

$$\text{Ratio } \frac{E_a}{E} = \frac{1}{2} \tag{160}$$

c. Same pressure between wires; the allowable d.c. pressure being $\sqrt{2}$ times the a.c. pressure.

$$\frac{\sqrt{3}E_a}{2E} = \frac{1}{\sqrt{2}}, \text{ or ratio } \frac{E_a}{E} = \frac{\sqrt{2}}{\sqrt{3}} \tag{161}$$

d. Same as (*c*); but d.c. pressure double the a.c. pressure.

$$\frac{\sqrt{3}E_a}{2E} = \frac{1}{2}, \text{ or ratio } \frac{E_a}{E} = \frac{1}{\sqrt{3}} \tag{162}$$

To satisfy the condition of equal total power, the equation is

$$2E \times I = 3E_aI_a \cos\theta \tag{163}$$

and for equal line losses,

$$2I^2R = 3I_a^2R_a \tag{164}$$

where I is the current per conductor in the d.c. transmission, and R the resistance per mile of single conductor; while I_a and R_a are the corresponding quantities for the three-phase transmission.

In either system the total weight (and cost) of the conductors is proportional to $\frac{\text{number of conductors}}{\text{resistance of each conductor}}$ which gives the relation,

$$\frac{\text{Cost of conductors, d.c. system}}{\text{Cost of conductors, three-phase system}} = \frac{2R_a}{3R} \tag{165}$$

but R_a can be expressed in terms of R thus:
By formula (164)

$$R_a = \frac{2I^2R}{3I_a^2} \tag{166}$$

and by formula (163)

$$I = \frac{3E_aI_a\cos\theta}{2E}$$

or

$$I^2 = \frac{9E_a^2I_a^2\cos^2\theta}{4E^2}$$

which, when put for I^2 in formula (166), gives

$$R_a = \frac{3E_a^2 \cos^2 \theta \times R}{2E^2} \tag{167}$$

Thus the equation (165) becomes

$$\frac{\text{Cost of conductors, d.c. system}}{\text{Cost of conductors, three-phase system}} = \frac{E_a^2}{E^2} \cos^2 \theta \tag{165a}$$

Assuming the very common value of 0.8 for the power factor of the three-phase system, the numerical ratio for the four conditions previously stated would be:

a. For same maximum pressure to ground, with sine-wave assumption,

$$\frac{\text{D.c. cost}}{\text{A.c. cost}} = \frac{\cos^2 \theta}{2} = 0.32$$

b. Same as (*a*), but allowable d.c. pressure assumed to be *double* the a.c. pressure,

$$\frac{\text{D.c. cost}}{\text{A.c. cost}} = \frac{\cos^2 \theta}{4} = 0.16$$

c. For same maximum pressure between wires, with sine-wave assumed,

$$\frac{\text{D.c. cost}}{\text{A.c. cost}} = \frac{2}{3} \cos^2 \theta = 0.426$$

d. Same as (*c*), but allowable d.c. pressure assumed double the a.c. pressure,

$$\frac{\text{D.c. cost}}{\text{A.c. cost}} = \frac{1}{3} \cos^2 \theta = 0.213$$

The transmission line, apart from the cost of conductors, would be cheaper for the d.c. than for the three-phase scheme because there are fewer insulators required and only two instead of three conductors to string; and if a grounded guard wire is erected above the conductors, it is more convenient to arrange this over the two d.c. conductors than over the three a.c. wires, and it would not necessitate the same total height of tower. The important saving is, however, in the conductors themselves. Taking the figure most favorable to the d.c. scheme (*b*), the a.c. conductors to transmit the same power with the same loss would cost six and a quarter times as much as if d.c. transmission were used, and even under the assumption (*c*), most favorable to the three-phase scheme, the cost would still be 2.35 times the cost of the d.c. conductors. For the purpose of getting out preliminary estimates, it is certainly safe to assume that, if the power factor of the three-phase load may be taken as 0.8, the cost of conductors on a long-distance d.c. transmission would be only one-quarter of the cost of conductors with the a.c. scheme on the assumption of equal I^2R losses.

If a ground return were used on a straight long-distance transmission—a perfectly feasible arrangement—the cost of copper for the same total I^2R loss would be only one quarter of the cost of a two-wire transmission, since the loss in the ground return would be negligible. The cost of copper would then be only about one-sixteenth of the cost of copper on the equivalent three-phase transmission, a point which suggests that the Thury system is worthy of more serious consideration than it has so far received outside of Europe. It is not suggested that comparison on the basis of equal line losses is necessarily correct or justifiable on economic grounds; but this does not render the above comparisons less interesting.

130. Concluding Remarks on the D.C. Series System.—As an indication of what has been done in Europe since the introduction of the Thury system more than a quarter of a century ago, it may be stated that there are at present about 16 separate transmissions in operation, in Switzerland, Italy, France, Hungary, Spain, Russia, and England. The shortest length of loop is 12.4 miles (Batoum, Russia), with a line pressure of 2,600 volts. The longest is 248 miles (124 miles straight transmission), this being the Moutiers-Lyons line at a maximum pressure of 100,000 volts, the current being 150 amp.

In England, the d.c. series system has been adopted by the Metropolitan Electric Supply Company of London on its Western section. The plant has been in operation since March, 1911, and given entire satisfaction. The current is about 100 amp. but can be varied from 70 to 120 amp. without causing trouble through sparking on the commutators. The commutators measure 5 ft. in diameter and 6¾ in. in length. They have 1,439 segments and run sparklessly at 5,000 volts between brushes. The machines have six poles and only two sets of brushes.

An interesting account of the Thury system, by William Baum, with brief descriptions of the important European plants, will be found on page 1026 of the *General Electric Review* of November, 1915.[1]

[1] See also article by Dr. Louis Bell in *Elec. World*, p. 361, Feb. 14, 1914.

As an example of what might be done at the present time in the way of d.c. transmission on a large scale, it is clear that no difficulty need be experienced in building dynamos of a large size with 5,000 volts on one commutator. Assuming a current of 300 amp., which would probably be transmitted by two conductors connected in parallel, the output of each machine would be 1,500 kw. and two of these might be coupled to one prime mover. With two commutators per machine, the output would be 3,000 kw., and with four commutators, 6,000 kw. per unit. Six machines in series, each with four commutators, would have a total output of 36,000 kw. at 120,000 volts. There would be practically no new or experimental engineering work in connection with such a scheme.

Electrical engineers on the American continent are rather inclined to the belief that when energy has to be transmitted from one place to another, the one and only course open to them is to adopt the three-phase a.c. system. It is not suggested that at the present time this may not, in the majority of cases, be the best system available; but undoubtedly there are conditions under which the continuous-current series system would prove more economical and reliable. Of course, first cost of plant and operating charges have to be taken into account when comparing different systems, and the most satisfactory way of doing this is to reduce all estimated costs to the common basis of annual charges. The cost of the d.c. generators must be set against the combined cost of alternators and exciters and step-up transformers with all intermediate switchgear.

In this connection the writer cannot refrain from quoting a paragraph which appeared in one of the leading articles in the *Electrical World* of New York, in which reference is made to the fact that transmission by continuous currents has received considerable attention in Europe.

Any engineer who wanders through one of the large Thury stations and then calls to mind the usual long concrete catacombs bristling with high-tension insulators and filled with dozens of oil switches, scores of disconnecting switches, webbed with hundreds of feet of high-tension leads, and spattered with automatic cutouts, will stop and think a bit before he complacently sniffs at high-tension d.c. transmission.

In regard to reliability it is true that, on the Thury system, the generators have not the protection against lightning disturbances which the step-up transformers afford to the alternators on high-tension three-phase systems, and where thunderstorms are prevalent, this must not be overlooked, as the cost of protective apparatus may prove excessive. In this connection it is interesting to note that the charges of electricity in the upper atmosphere are always positive, and the negative wire will, therefore, tend to draw a lightning discharge away from the positive wire or grounded guard wire, but to how great an extent this would affect the proper disposition of the wires it is difficult to say.

The following is a brief summary of the important points in favor of, and unfavorable to, the employment of continuous currents on the series system for the purpose of transmitting energy at comparatively high voltages from one place to another.

Advantages of the D.C. Series System

1. The power factor is unity—a fact which alone accounts for considerable reduction of transmission losses.

2. Higher pressures can be used than with alternating current, the conditions, as shown by actual tests, being more favorable to d.c. transmission than is generally supposed. Without any alteration to insulation or spacing of wires, approximately double the working pressure can be used if direct current is substituted for alternating current. Moreover, the insulation is subjected to the maximum pressure only at times of full load, whereas on the parallel system the insulation is subject to the full electrical stress at all times.

3. There is no loss of energy through "dielectric hysteresis" in the body of insulating materials.

4. The necessity for two wires only, in place of three, effects a saving in the number of insulators required and allows cheaper line construction.

5. Where it is necessary to transmit power by underground cables, continuous currents have great advantages over alternating currents. Single-core cables can be made to work with continuous currents at 100,000 volts. By using two such cables and grounding the middle point of the system, it is, therefore, quite feasible to transmit underground at 200,000 volts.

6. The practicable distance of transmission, especially when the whole or a part is underground, is greater than with alternating currents.

head d.c. lines, and the ease with which highly inductive choke coils can be introduced on a d.c. system, without opposing any obstacle (except ohmic resistance) to the passage of the line current, tends toward the attainment of increased safety.

The use of underground cables, which are particularly suitable for d.c. transmission, would dispose of the possibility of interruption from lightning; or a comparatively short length of underground cable in the neighborhood of the generating stations and substations would probably afford satisfactory protection. With very high voltages, requiring line insulators capable of withstanding pressures of the order of 300,000 or 400,000 volts without arc-over, disturbances due to lightning would probably be as rare as they are on existing 220,000-volt a.c. transmission lines.

131. Commutation of High-pressure Alternating Currents. The Transverter.—In order to transform alternating currents into continuous currents, it is not necessary to use a motor-generator set or a rotary converter, because a commutator, driven at synchronous speed by a small synchronous motor, may be used to effect the transformation. If desired, the commutator may be stationary in which case the brushes would have to revolve; but a device of this nature always involves the rotation at synchronous speed of either brushes or commutator. The idea is not new; machines which operated on this principle were proposed many years ago by Hutin and Leblanc, Arnold and La Cour, and others. Difficulties in obtaining satisfactory commutation led to the abandonment of this type of machine.

In 1924, a machine named the Transverter was exhibited at the British Empire Exposition at Wembly (London). This was rated at 2,000 kw. to deliver 20 amp. at 100,000 volts direct current. It appears to be a development of the Arnold and La Cour synchronous converter, the most noticeable difference being the construction by which a number of commutators are connected in series. Although no new principle appears to be involved, the inventors, W. E. Highfield and J. E. Calverly, and the manufacturers, The English Electric Company, Ltd., are to be congratulated upon the production of a well-designed practical machine which appears to operate satisfactorily. The test of time and the performance of larger machines are necessary to determine whether or not the device will prove efficient and economical for the transmission of energy over long distances; but commutation troubles—especially with revolving brushes—

may prove insurmountable, and the high cost of the machine, due partly to the large number of connections between transformer coils and commutator segments, may be an obstacle to its adoption.

From what has been said, it is obvious that the transverter is reversible; the unit at the sending end of the d.c. transmission line would transform the alternating current into continuous current on the high-tension side of the step-up transformers, while the machine at the receiving end would convert the continuous current back into alternating current. The transmission would be by constant-pressure continuous current instead of constant-current continuous current as in the Thury system.

The machine at Wembly was provided with 10 commutators connected in series, each commutator being connected to a 36-phase system by means of tappings to the high-tension windings of transformers.

A complete illustrated description of the Transverter will be found in *Engineering* (London) May 2, 1924. Refer also to the *Electrician* (pp. 567, 573), May 9, 1924.

132. Looking Ahead. Future Possibilities of D.C. Transmission.— Although the Thury system of constant-current transmission appears to give satisfaction where it is in use, the fact that it is not more generally adopted seems to indicate that transmission of energy on a large scale by continuous currents must be looked for along other lines. The advantages of d.c. transmission being fairly obvious and generally recognized, it is safe to predict that, within 10, 20, or 30 years, long-distance transmission by high-voltage continuous currents will be an established fact, and this notwithstanding the enormous and rapidly increasing capital investment in three-phase high-tension transmission lines and the so-called superpower networks.

At the close of 1919, Dr. Dolivo-Dobrowolski, the great pioneer in the three-phase a.c. field, championed d.c. high-tension transmission as the system of the future. At the present day many scientists and engineers are working on the problem which, when solved, will make it possible to transmit with a given weight (and cost) of copper about three times the kilowatts that can safely be transmitted by means of alternating currents.

It is probable that the final solution will not involve the use of dynamo-electric machines provided with commutators, or of devices including mechanically driven commutators, but very rapid progress has been made of late years in the development of the mercury arc rectifier which, at the present day, is suitable for d.c. railway operation at 4,000 volts.

Some authorities think that hot-cathode tubes are not likely to be the solution of d.c. long-distance transmission,[1] but the work done by Dr. Hall of the General Electric Company suggests great possibilities for the high-voltage kenotrons. The solution suggested by D. C. Prince[2] involves the use of inverted electronic rectifiers or pilotrons, the d.c. current of the transmission line being re-converted into three-phase alternating current by inverted kenotrons, a synchronous motor being used to obtain the desired frequency and also to provide the "wattless" or reactive component of the power.

Continuous currents are of great advantage in the transmission of energy by underground or submarine cables, and the following chapter will deal briefly with the problems of underground electric power transmission.

[1] OSSANNA, J., *Elektrotech. Zeit.*, pp. 1061–1063, Aug., 17, 1922.
[2] *Gen. Elec. Rev.*, vol. 28, p. 676, 1925.

CHAPTER XIII

TRANSMISSION OF ENERGY BY UNDERGROUND CABLES

133. Introductory.—The principal use of underground electric cables is in connection with distributing systems in cities, at comparatively low pressures; but they are also used, to a limited extent, for the transmission of energy at fairly high voltages. In this chapter, underground cables will be considered chiefly in relation to straight long-distance transmission of energy, and what follows will, therefore, treat mainly of power cables for the transmission of energy at high voltages. No attempt will be made to deal with the historical aspect of the subject, or with the practical considerations connected with the handling, laying, and jointing of underground cables; neither will space permit even of the briefest treatment of subjects such as manufacturers' tests and the location of faults in cables, including protective devices designed to isolate or cut out faulty sections of a transmission system; but the construction engineer or student desiring more information than can be contained in a single chapter is referred to the many excellent publications on the manufacture and use of high-voltage cables.[1]

Notwithstanding the higher cost of underground transmission, it replaces overhead transmission, for comparatively short distances, in many cases where the latter system is not suitable. High-tension underground cables are used in populous districts where overhead construction is not permissible or advisable. The underground cable is not subject to damage by wind, ice, or thunderstorms, and the danger to life is obviously reduced by placing the high-voltage conductors underground. The unsightly appearance of poles and overhead conductors in the neighborhood of cities is another reason for putting wires underground, not-

[1] Beaver, C. J., "Insulated Electric Cables," Ernest Benn, Ltd., London; Del Mar, W. A. "Electric Cables," McGraw-Hill Book Company, Inc.; Meyer, E. B., "Underground Transmission and Distribution," McGraw-Hill Book Company, Inc.

withstanding the increase of cost. This consideration has more weight in Europe than in America and partly accounts for the fact that Europe is, and always has been, somewhat in advance of America in the design and manufacture of underground electric cables. The shorter distance of transmission, resulting in lower economical pressures on the Old World systems, is another reason why underground power cables are more extensively used in Europe than on the American continent. The writer has, therefore, no hesitation in using data and other information referring mainly to modern British practice, especially since he has been able to secure the collaboration of C. J. Beaver, chief engineer to W. T. Glover and Company of Manchester, England, who has kindly furnished much useful advice and material for this chapter.

Even when it is inadvisable to transmit by underground cable the whole distance of transmission, sections of the line passing through populous districts may be put underground, while overhead conductors are used in the open country. Also, as mentioned in the preceding chapter, a short length of underground cable is desirable as a protection against high-pressure surges, at each end of a long overhead transmission line, provided the voltage is not so high as to render the cost prohibitive.

Another use for insulated cables is as feeders on electric railway systems. In the trunk-line electrification of the New York Central Railroad, there are no less than 1,600,000 duct-ft. of conduit for insulated feeder cable. Some of these ducts are of tile, but iron pipe is also used.

The Thury system of transmission by continuous currents, which was explained in Chap. XII, lends itself to the extensive use of underground cables. Smaller and cheaper cables may be used for the same voltage and energy transmitted on a d.c. than on an a.c. transmission. J. S. Highfield has shown[1] that if the overall diameter of a 100,000-volt single-core lead-covered cable for alternating currents is 3.27 in., the equivalent cable for 100,000-volt continuous current would have an outside diameter of only 1.75 in. It is true that the comparison is between the r.m.s. values and not the maximum values of the voltage; but it serves to show that a considerable reduction of size and, therefore, of cost is effected when cables are used on d.c. instead of a.c. systems.

[1] Discussion of C. J. Beaver's paper on "Cables," *Jour. Inst. Elec. Eng.*, vol. 53, Dec. 15, 1914 and Mar. 1, 1915.

During the last few years very rapid progress has been made in the improvement of high-voltage insulated power cables. Considerable advance has been made in raising the voltage limitations of commercial cables, but still greater headway has been made in the reduction of dielectric losses.

134. Submarine Power Cables.—When transmitting electric energy across water which cannot be spanned by overhead conductors, the insulated cable becomes a necessity. Two examples of submarine power transmission occur in San Francisco, where a three-phase 12,000-volt cable 3½ miles long has been laid across the bay, and two 11,000-volt cables have been laid across the Golden Gate The carrying out of the latter project is described in the *Electrical World* (Vol. 67, p. 532), Mar. 4, 1916. The greatest depth of water at this point is 210 ft. The cables are three-core, each 13,500 ft. long, the cores being insulated with rubber (inside) and varnished cambric (outside). Impregnated jute is used as a filler. The lead sheath is 5⁄32 in. thick, and the armoring consists of 42 No. 4 B.w.g. galvanized-steel wires. The deep-water portion of the cable has cores of 250,000 circular mils section, with an outside diameter of 4 in., and a weight of 19 lb. per foot. At the shore ends, the cores are of 350,000 circular mils section, with an overall diameter of 4½ in. and a weight per foot of 22 lb. More recently, the Great Western Power Company has laid an 8-mile length of 11,000-volt three-phase cable under San Francisco Bay, a description of which will be found in the *Electrical World* of Feb. 17, 1923. The same power company has also laid two three-conductor 44,000-volt submarine cables across the Napa River at Napa, Calif. These cables have been in operation since August, 1925.

A three-phase cable for 26,000 volts, was laid about 17 years ago by the Shawinigan Water and Power Company at Three Rivers, Quebec. In the more recent installation at St. Louis (Union Electric Light and Power Company) a number of three-conductor submarine cables are in successful operation at 35,000 volts.

A number of islands off the Norwegian coast are connected by 22,000-volt submarine cables which supply power to the islands from the generating station at Aalesund, the distance between islands varying from 1,600 to 10,000 ft. The radial thickness of the paper insulation is about ½ in., and the lead covering is 5⁄32 in. thick.

In Germany there is the Stralsund channel project transmitting three-phase power at 15,000 volts from the generating station at Stralsund to the island of Rügen, the total length of submarine cable being 5,600 ft. Another example occurs in the straits between the mainland and the island of Fehmern, where two 3,300-ft. lengths of lead-covered, iron-wire-armored, paper-insulated 11,000-volt cables are laid under water.

A submarine power transmission of considerable interest is the international cable between Sweden and Denmark, which was put into operation in December, 1915. It is 3.4 miles long and connects a point near Palsjö in Sweden with Marienlyst in Denmark. The cable is three-core with impregnated paper insulation suitable for a working pressure of 35,000 volts (test pressure 87,500 volts). The cross-section of each core is 0.108 sq. in. and steel wire armoring is applied over the lead sheath. The greatest depth of the Oresund at the point traversed by the cable does not exceed 125 ft. The cable was laid in nine lengths of about 2,000 ft. A new cable was laid in 1922 for a working pressure of 50,000 volts.

What is probably the longest submarine power cable in the world was laid a few years ago and transmits power at 11,000 volts three-phase from Niihama to Shisakajima, Shikaku, Japan. A description of this cable will be found in the *Electrical World* of Mar. 31, 1923.

125. Voltage Limitations of Underground Power Cables.—Transmission and distribution by underground cables with three-phase alternating current at 30,000 and 35,000 volts is not uncommon. Single-conductor cables for 44,000 volts have been, for many years, in operation in America. A still higher pressure, namely 66,000 volts, is used to transmit power in a subway across the city of Cleveland; there are two three-phase circuits each capable of transmitting about 30,000 kw. a distance of 8¼ miles. In Italy, cables were built for working pressures up to 80,000 volts 5 or 6 years ago.

The Philadelphia Electric Company has installed single-conductor cable on two underground lines rated at 75,000 volts, three-phase. Each line is about 20,000 ft. long, the conductors being 750,000 circular mils and insulated with $^{30}/_{32}$ in. of impregnated paper.

Recent advances in the design and manufacture of high-voltage power cables, by permitting higher temperatures of the dielectric with comparatively small dielectric loss, have raised the permissible voltage gradient in the insulation,[1] and in connection with the design of cables for very much higher pressures than those now in use, maximum voltage gradients (under normal operating conditions) of 50,000 and even 52,000 volts per centimeter are being considered. For reasons explained in Chap. VIII (see formula (86), p. 204), a large diameter of conductor is desirable, and constructions involving hollow-core conductors are being tried. With a permissible voltage gradient of 50,000 volts per centimeter, single-core cables to carry 500 or 600 amp. at about 90,000 volts could probably be constructed with an external diameter not exceeding 4 in.

An interesting modern development is the proposal to transmit underground at 132,000 volts in Chicago. The scheme is by no means impossible, and experimental lengths of 130,000-volt power cables were in operation in Europe in 1924.[2]

While discussing the possibilities of constructing insulated cables for extra-high voltages, the advantages of d.c. transmission may be again referred to. With a dielectric thickness not exceeding ½ in., single-core cables can transmit continuous currents at 100,000 volts, and such cables are in operation in Europe. Some interesting information regarding the dielectric strength of various insulating materials when subjected to continuous and alternating voltages is given in the *General Electric Review* (p. 645), September, 1923. Some tests made abroad on cables give an average of 2.5 for the ratio $\frac{\text{d.c. pressure}}{\text{(r.m.s.) a.c. pressure}}$ for the same disruptive effect. When kenotron tests are made on cables, it is customary to use a d.c. test pressure equal to 2.4 times the specified a.c. test pressure.

136. Types and Construction of Power Cables.—For the transmission of three-phase alternating current, three-core cables will probably be economical for pressures up to about 66,000 volts. When the pressure exceeds this figure, a separate single-core cable should preferably be used for each phase. These separate cables of an extra-high-tension three-phase transmission should be symmetrically arranged with a small amount of insulation between the lead sheath and ground. Steel-armored single-core cables

[1] Refer to Arts. 72 and 73 of Chap. VIII.

[2] Refer to article by PHILIP TORCHIO in the *Elec. World*, Mar. 21, 1925.

are usually inadmissible for alternating currents because of the inductive effects.

The lead sheathing referred to is a necessity because impregnated paper is practically the only material at present available for the insulation of extra-high-tension underground cables. It

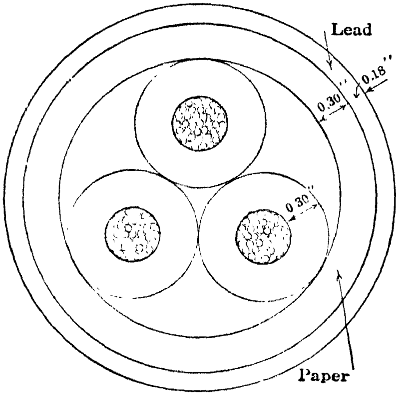
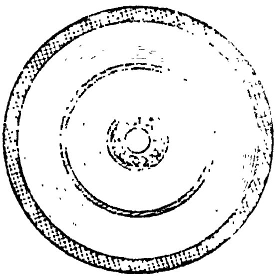


FIG. 127.—Three-core lead-covered cable.



		Radial thickness.	Radius.
Conductor.—	Inner lead tube—bore 0.27 in.	0.060 in.	0.195 in.
	Copper wires 19/15 S.W.G.	0.072 "	0.267 "
	Outer lead tube	0.050 "	0.317 "
Dielectric.—	First paper layer	0.545 "	0.862 "
	Lead inter-sheath	0.050 "	0.912 "
	Second paper layer	0.565 "	1.477 "
Outer sheath.—	Lead covering	0.160 "	1.637 "
	Complete diameter 3.27 in.		

FIG. 128.—Section through single-phase 173 kv. paper-insulated cable, with lead "inter-sheath."

is somewhat hygroscopic in character, and the protection of the lead covering is, therefore, provided. Figure 127 shows a section through a three-core paper-insulated underground cable, while Fig. 128 shows one of three separate cables designed for a working pressure of 173,000 volts between phases.[1] This cable is provided

[1] From C. J. BEAVER's paper on "Cables" in the *Jour. Inst. Elec. Eng.*, vol. 53, Dec. 15, 1914.

with a so-called "intersheath" of lead, the purpose of which is to improve the design by reducing the potential gradient at the surface of the conductor. The theory of this method will be discussed later.

The "clover leaf" or sector type of three-core cable is illustrated in Fig. 129. This design permits of a smaller overall diameter for a given voltage and so leads to a saving in cost; but it is rarely used for pressures exceeding 30,000 volts. The sector shape of cross-section is obtained by specially stranding wires of suitably varied diameters, or by rolling or hammering a circular strand to the desired shape. This construction was

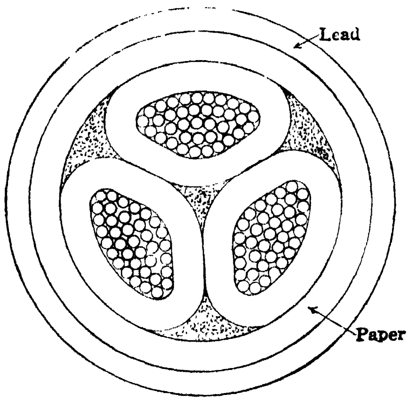


FIG. 129.—Three-core cable with shaped cores.

introduced in Europe many years before it was adopted in America; but British practice does not favor the use of sector-shaped cores in three-phase cables for working pressures exceeding 11,000 or 12,000 volts.

Concentric cables are used for single-phase transmissions and are, therefore, practically confined to railway work. Figure 130 is a section through such a cable, the return conductor being in the form of a layer of segmental strips laid over the insulation surrounding the inner core. When this outer conductor is grounded —as it usually is—the arrangement is very satisfactory and efficient. Concentric cables are simple to construct, and even when used with alternating currents, they can be armored and, therefore, buried directly in the ground, thus facilitating the dissipation of heat which is easily carried from the outer conductor to the lead sheath and armor.

There are two methods of manufacturing paper-insulated cables. In both methods a roll of paper is cut into narrow ribbons which are lapped on the copper conductor in successive layers until the required thickness is attained. In one method the paper is put on dry and the whole cable is immersed in the insulating oil. In the other, the impregnation is effected by passing the paper, *before* it is cut into ribbons, through a bath of the insulating oil. This last method, which was evolved and has been used for the last quarter of a century by W. T. Glover and Company of Manchester, England, appears to have the advantage of more certain and thorough impregnation of the insulation. One

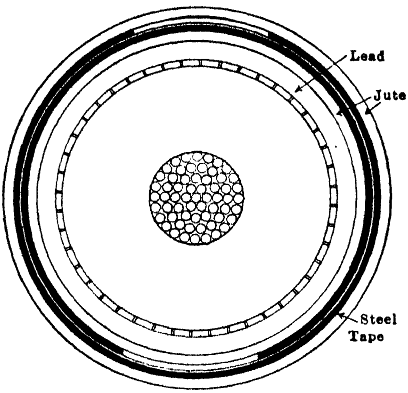


FIG. 130.—Section through concentric cable for single-phase transmission.

objection to insulating the conductors with paper which has been previously impregnated is that small air pockets are likely to occur between the lappings of the paper strip. By means of recently developed machinery, the strip of perfectly prepared impregnated paper is now applied under the surface of molten compound, thus insuring the total exclusion of air between layers.

With paper insulation the lead sheath must be very carefully applied to insure the absence of flaws and "pinholes" through which moisture might be admitted. The lead sheathing is applied by passing the insulated cable through dies in a hydraulic press which forces hot lead in a semifluid condition around the slowly moving cable, thus forming a closely fitting lead cylinder on the outside of the insulation. It is the practice of one English factory to apply a hydraulic test to the finished cables in order to detect the presence of imperfections in the lead covering, by forcing water into the dielectric so that its presence may be detected before the cable leaves the factory. It is partly in order to facilitate such testing that the same firm employs a metallic test sheath[1] enclosed in the cable, and lightly insulated from the lead covering. This lead sheath is also useful for maintenance tests, fault localization, the detection of incipient faults on live cables, and for use in connection with special protective devices.[2]

Insulating Materials Other Than Impregnated Paper.—The reason why reference has not previously been made to vulcanized rubber and vulcanized bitumen as materials for cable insulation is not that these are not in general use, but because they are not so suitable as paper for extra-high-tension power cables. Vulcanized bitumen as an insulator appears to have met with greater success in Europe than in America. It is a substance of which the physical properties are somewhat similar to those of vulcanized rubber. Cables insulated with vulcanized bitumen (without lead sheathing) have a special field of utility in mining work, where they are used not only as feeders to carry the electric energy down the shaft, but also as distributors under ground.

A type of insulation which is largely used in America is varnished cambric, or varnished cloth. The prepared cloth is applied to the conductor in the form of tape, a thin layer of a non-hardening viscous filler being applied between the layers to insure flexibility by permitting relative movement of the layers. Since it is customary to provide a lead sheath over power cables insulated with varnished cambric, there is no economic advantage in using this material as a substitute for paper.

137. Methods of Laying Underground Cables.—The present-day methods of laying underground cables may be classified under the headings:

1. Laid direct in the ground (armored cables).
2. Drawn into weldless steel tubes, stoneware or tile ducts, or fiber ducts in concrete.

The first-mentioned system is usually adopted for cross-country runs; the second, in congested areas where streets cannot be disturbed and where facilities for adding to the number of cables are desired.

[1] British patent 22355/12, Beaver and Claremont.

[2] British patent 13681/17 Beaver, Richards, and Claremont. Particulars of the test sheath cable and its potentialities will be found in *The Electrician* (London), July 5, 1918.

When power cables are laid direct in the ground, they are protected with only a lead sheath and steel-tape armoring. A cushion of tarred jute is placed over the lead sheath and over this is wound a layer of steel tape, about 1/32 in. thick, in an open spiral, leaving approximately 3/16 in. clearance between the convolutions. A second layer of steel tape is wound over the first so as to cover the unprotected spaces. Finally a heavy covering of tarred jute is wound over the armoring. It is customary to place a wooden board over the cable to act as a warning to workmen and so protect the cable in the event of the ground surface being disturbed after the cable has been laid.

The drawing-in system is the one most commonly adopted at the present time. The ducts for the cables may be tile or fiber set in concrete, or they may consist of wrought-iron pipes.

Tile or stoneware conduits can be supplied as single-way or multiple-way ducts. The opening is usually 3½ in. square when used for distributing purposes in or near cities. The inside corners are well rounded, and the outside dimensions are approximately as follows:

Single-way duct	5 by 5 by 18 in. long
Two-way duct	5 by 9 by 24 in. long
Three-way duct	5 by 13 by 24 in. long
Four-way duct	9 by 9 by 36 in. long
Six-way duct	9 by 13 by 36 in. long

These ducts are set in concrete forming a wall about 3 in. thick on all four sides.

Fiber conduits are light in weight and easy to handle. They are cylindrical pipes made of wood pulp saturated with an asphalt or bituminous compound containing a small amount of creosote, and are usually supplied in 5-ft. lengths.

Iron pipes are useful when slight bends to clear obstructions are of frequent occurrence. These can be supplied in 20-ft. lengths with threaded ends and couplings.

In all cases, the internal width of the duct should allow not less than ½-in. clearance for the largest cable to be drawn into it.

Manholes must be provided at intervals of 300 to 500 ft., the latter distance being rarely exceeded, as it represents the practical length of cable of large size which can be drawn into an underground duct without injury. The manholes, if required merely for drawing-in purposes and for joints (apart from the installation of transformers or other apparatus), should measure about 7 by 6 by 6 ft. high. If the construction permits of an arched roof, the walls might be about 5 ft. high with a total head-room at center of about 7 ft. The floor should be above the sewer level, and proper arrangements provided for drainage and ventilation. If transformers are placed in manholes, a space of about 4.5 cu. ft. per kilovolt-ampere should be provided.

138. Cable Terminals. Junction with Overhead Lines.—The connection between overhead wires and underground cables must

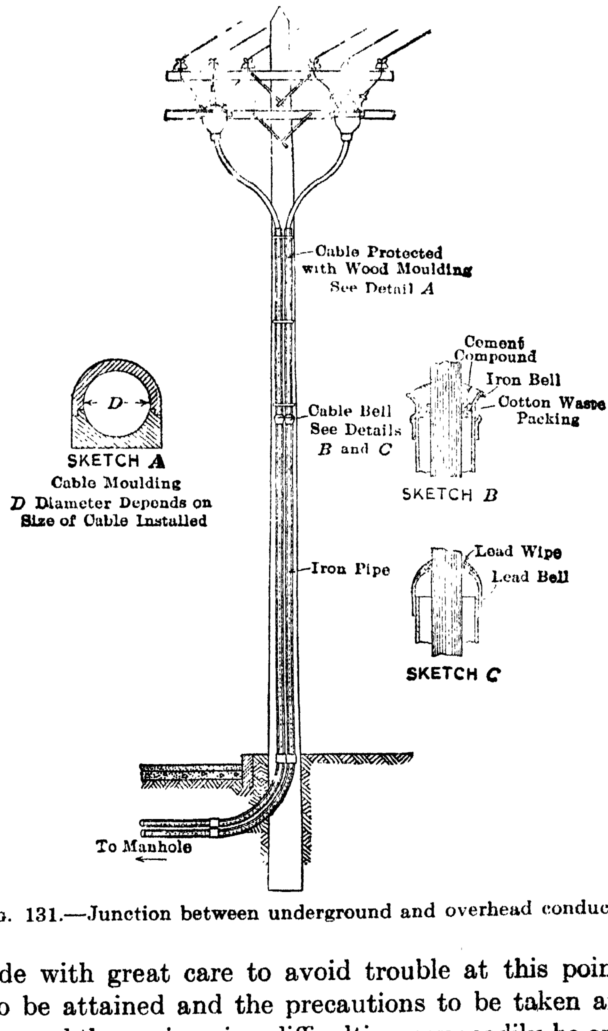


Fig. 131.—Junction between underground and overhead conductors.

be made with great care to avoid trouble at this point. The ends to be attained and the precautions to be taken are fairly obvious, and the engineering difficulties may readily be overcome.

Each particular case should be considered as a special problem, and the proper steps taken to provide adequate insulation and prevent deterioration or damage by water owing to improperly sealed joints.

Figure 131 shows the terminal pole of an overhead line and the methods employed for the protection of the cable. It is reproduced by kind permission of E. B. Meyer and his publishers

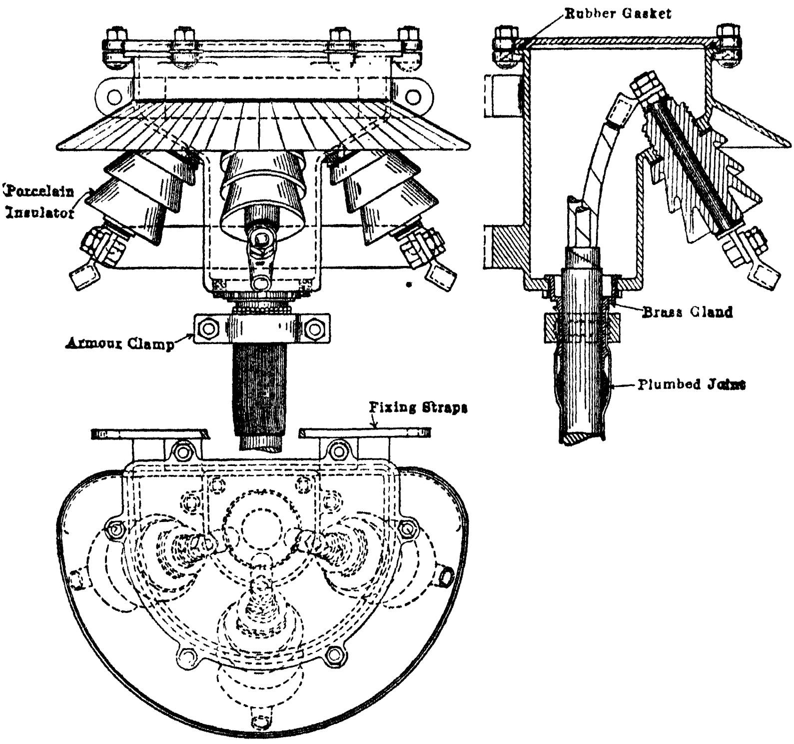


FIG. 132.—Detail of cable ends at junction with overhead line.

from "Underground Transmission and Distribution." The design of the actual terminals or "potheads," as they are sometimes named, varies considerably. One form—suitable for a working pressure of 11,000 volts, three phase—is shown in Fig. 132 which has been reproduced from a drawing kindly supplied by W. T. Glover and Company, Ltd., Manchester, England.

The pothead assembly illustrated by Fig. 133 is reproduced by permission from an article by G. H. Hagar in the *Electrical World* of Feb. 27, 1926. It shows the design adopted for the terminals of the 44,000-volts submarine cables across the Napa River, Calif., referred to in Art. 134.

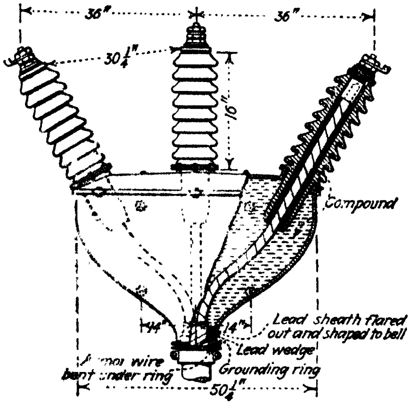


FIG. 133.—Pothead assembly for 44,000-volt submarine cable.

139. Design of Cables.—The theory of the *potential gradient* at the surface of a wire of radius r surrounded by a concentric metallic cylinder of internal radius R has already been discussed in Art. 73, Chap. VIII, in connection with insulating bushings. With a dielectric of constant specific inductive capacity throughout the total thickness, the maximum potential gradient, which occurs at the surface of the inner conductor, is given by formula (86):

$$G = \frac{E}{r \log_e \frac{R}{r}} \text{ volts per centimeter} \qquad (86)$$

where E stands for the potential difference between the conductor and the outside metallic sheath, in volts, and the dimensions are expressed in centimeters. This formula is, therefore, applicable to single-core cables with lead sheath, and to the concentric type of two-conductor cables.

The breakdown gradient of paper-insulated cables is about 200 kv. (r.m.s. alternating) per centimeter, but the maximum voltage gradient (at the surface of the conductor) which will break down the insulation depends upon the diameter of the conductor. The formula suggested by Peek for the gradient *at the surface of the conductor* which will cause breakdown of insulation on a single-conductor cable insulated with impregnated paper is

$$G_{\text{max.}} = 100\left(1 + \frac{1.1}{\sqrt{r}}\right) \text{ kv. per centimeter} \qquad (168)$$

where r is the radius of the conductor in centimeters. It is not advisable to submit the finished cable to test pressures which will cause the potential gradient at the surface of the conductor to exceed 150 kv. per centimeter,[1] and since the test pressure may be as high as 2½ times the working pressure, the maximum working stress will be of the order of 50 kv. per centimeter. It is quite possible to design practical paper-insulated cables for working stresses of 60 to 80 kv. per cm.; but owing to the fact that dielectric heating is likely to be excessive with these high gradients, about 50 kv. per cm. is a reasonable maximum at the present time. All the voltages here referred to are r.m.s. values, and not peak values.

The probable breakdown gradient and, therefore, the safe working pressure of a high-voltage insulated cable in service will depend upon many factors besides the test pressure it is able to withstand at the manufacturers' works. The time during which the insulation is subjected to electric stress has a great deal to do with the breakdown voltage, and the temperature of the insulation, whether determined by losses in the copper or in the dielectric, or an external source of heat, will also have a very appreciable effect upon the ability of cables to withstand high voltages. Other factors, including the normal frequency and the frequency of transient waves or surges, also the amount of bending which the cable has been subjected to before and during laying, will have an effect upon the voltage which will cause breakdown of the insulation. It follows that the mere application of a formula with constants based on the average of tests made under particular conditions will not insure reliability under service conditions. Notwithstanding the great advances that have recently been made, there is still much to be learned before the satisfactory performance of extra-high-tension cables under the usual operating conditions can be guaranteed.

A summarized record of our present knowledge regarding the performance of insulation under electrical stress has been published

[1] Test pressures of 200 kv. per centimeter and even higher are used by some cable makers in Europe.

by the Subcommittee on Wires and Cables of the A.I.E.E. Standards Committee,[1] and if the reader will refer to this, he will understand that it is not possible, in a single chapter, to do justice to so large and important a subject as the insulation of electric conductors for the underground transmission of power.

The *capacity* of a single-core cable per centimeter of length is the reciprocal of the elastance as given by formula (84), whence

$$\text{Capacity per centimeter} = \frac{2\pi Kk}{\log_\epsilon \frac{R}{r}} \text{ farads} \qquad (169)$$

where k is the relative inductive capacity or *dielectric constant* of the insulating material, the value of k for air being unity. The numerical value of K, as given in Art. 72, Chap. VIII, is 8.84×10^{-14}, and since there are 161,000 cm. in a mile, the capacity per mile of single-core cable, in microfarads, is

$$C_m = \frac{161{,}000 \times 2\pi \times 8.84 \times 10^6 \times k}{10^{14} \times \log_\epsilon \left(\frac{R}{r}\right)}$$

$$= 0.0895 \frac{k}{\log_\epsilon \left(\frac{R}{r}\right)}$$

$$= 0.0388 \frac{k}{\log_{10} \left(\frac{R}{r}\right)} \qquad (170)$$

Approximate values of k which apply to materials used for cable insulation are:

$k = 3.3$ for impregnated paper.
$k = 4.2$ for varnished cambric.
$k = 4.5$ for vulcanized bitumen.
$k = 3$ to 6 for vulcanized india rubber.

The *permittivity*, or relative specific inductive capacity (k) will depend somewhat upon the temperature of the dielectric, the effect of the higher temperatures in *increasing* the value of k being greatest for varnished-cambric insulation. The above values are averages for normal working temperatures.

The formula for the *insulation resistance* of a single-core cable is obtained as follows:

[1] This summary, brought up to date (1924), is given in the Appendix VIII of DEL MAR'S "Electric Cables."

Let ρ stand for the resistivity, or specific resistance, of the dielectric, in megohms. It is the electrical resistance of 1 centimeter cube of the cable insulation. Considering the insulation resistance of a length of cable l cm. long to be the sum of the resistances of successive concentric layers of insulation,

$$dR_i = \rho \frac{dx}{2\pi x l}$$

where dx is the length, and $(2\pi x l)$ the area, of a cylindrical element of radius x and length l, whence

$$R_i = \frac{\rho}{2\pi l}\int_r^R \frac{dx}{x}$$

$$= \frac{\rho}{2\pi l}\log_\epsilon\left(\frac{R}{r}\right) \qquad (171)$$

Let R_i stand for the insulation resistance, in megohms, of 1 mile of cable; then $l = 161{,}000$ cm.; and by converting the natural into common logarithms,

$$R_i \text{ (of 1 mile)} = 2.28 \times 10^{-6} \times \rho \log_{10}\left(\frac{R}{r}\right) \text{ megohms} \qquad (172)$$

Some average values of ρ are:

For impregnated paper, $\rho = 8 \times 10^8$

For varnished cambric, $\rho = 3 \times 10^8$ to 5×10^8

For vulcanized bitumen, $\rho = 2.4 \times 10^8$

For vulcanized india rubber, $\rho = \begin{cases} 4.8 \times 10^8 \\ \text{to} \\ 1.2 \times 10^{10} \end{cases}$

Temperature has a marked effect on the specific resistance of cable insulation, which decreases rapidly with increasing temperatures. This effect is particularly noticeable with paper insulation. Vulcanized india rubber is the material with the best temperature characteristics.

The *reactance* of a single-conductor cable without steel armoring would be calculated as for bare wires (refer to Art. 82, Chap. IX) and will depend upon the position of the return conductor. Single cables carrying alternating currents should not be armored or drawn into iron pipes.

The formulas for use with three-core cables are not so easily developed as those for single-core cables, and they are largely empirical, especially when the shape of the cores departs from the true circular section. Tables giving the approximate inductance and capacity of three-core cables may be found in the electrical engineering handbooks and in makers' catalogues.[1]

The reactive voltage drop per mile of single conductor of a three-core cable may be calculated by means of formula (96) of Art. 82, Chap. IX, which is correct for the usual frequencies, even when the distance (d) between centers of cores is as small in comparison with the radius (r) of the conductor as in three-core underground cables.

140. Economical Core Diameter of High-pressure Cables.—For a given voltage and constant diameter over the insulation of a single-core cable, there is a definite diameter of the core which will cause the potential gradient at the conductor surface to be a minimum. A small core will allow of a greater thickness of insulation; but on the other hand the smaller radius of curvature will tend to increase the stress; while the effect of too large a diameter of core is also to cause an increase in the stress through reduction of the total thickness of insulation. Formula (86) may be written

$$r \log_\epsilon \left(\frac{R}{r}\right) = \frac{E}{G}$$

and if it is assumed that both R and G are constant, the greatest maximum permissible voltage E will be obtained when the quantity $r \log_\epsilon \left(\frac{R}{r}\right)$ is a maximum, a condition which is satisfied when $\log_\epsilon \left(\frac{R}{r}\right) = 1$, or $R = 2.718r$. Assuming this ideal requirement to be fulfilled, even if the inner core has to be made hollow in order to economize conductor material, it follows that

$$r = \frac{E}{G} \tag{173}$$

from which an approximate value of economic conductor radius can be obtained. This radius will generally (in high-voltage cables) be found to be greater than that required to provide the necessary cross-section for current-carrying purposes, and this accounts for the fact that aluminum may prove to be more economical than copper as a conductor in high-tension underground cables. The 30,000-volt cables for the electrification of the Dessau-Bitterfeld railway in Germany from the power station

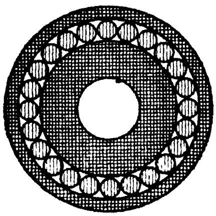
[1] Useful tables will also be found in the chapter on Cable Characteristics in WILLIAM NESBIT'S "Electrical Characteristics of Transmission Circuits," Westinghouse Electric and Manufacturing Company.

at Muldenstein have cores of stranded aluminum 0.512 in. in diameter (0.157 sq. in. in cross-section) covered with impregnated paper of a radial thickness of 0.512 in. The lead sheath is 0.138 in. thick and is covered with a layer of jute, the overall diameter of the cable being 2.05 in. By formula (86) the maximum gradient is

$$G = \frac{30,000}{2.54 \times 0.256 \log_\epsilon \left(\frac{0.512 + 0.256}{0.256}\right)}$$

$$= 42,000 \text{ volts per centimeter}$$

If stranded copper had been used (without a hollow, or non-metallic core), the diameter of the core of equivalent current-carrying capacity would have been about 3/8 in. which, with the same thickness of insulation, would cause the maximum gradient to be about 48,000 volts per centimeter.



Inner lead tube..........0.256 in. bore, or 0.140 in. radial.
Conductor..............24/0.082 in.
Outer lead sheath........0.05 in. radial.
Complete diameter........0.80 in.

FIG. 134.—Specially constructed core for e.h.t. underground cables.

The application of formula (86) to the calculation of the maximum potential gradient in cables with stranded wire cores, gives only approximate results. The comparatively small radius of the individual wires will bring about a concentration of stress appreciably greater than what would be indicated by formula (86.) One method of providing a smooth cylindrical surface to the cable core is to sheath over the stranded copper core with a thin wall of lead. Figure 134 shows the complete core (without insulation) of a high-pressure cable as designed by a British cable factory.[1] In this case the calculated "economic" radius for a maximum gradient of 50,000 volts (r.m.s.) is 0.8 in., and since the conductor is of sufficient carrying capacity when made up of 24 copper wires 0.082 in. in diameter, these wires are disposed around

[1] British patent 20549/14.

a hollow lead core of the necessary diameter, as shown in the illustration.

141. Grading of Cable Insulation.—The potential gradient at any distance x from the center of a single-core cylindrical cable is given by formula (83) of Art. 73, Chap. VIII, as

$$G = \frac{\Psi}{2\pi x K k}$$

For a given voltage and total capacity of the cable, which determine the total dielectric flux Ψ, it follows that if k is also constant, the gradient will have a value inversely proportional to the distance x from the center of the core. With a view to reducing the outside diameter of the cable, the value of k should change in successive layers of insulation in order to maintain G as nearly as possible at the maximum permissible value throughout the thickness of the insulation. In other words, if $k \propto \frac{1}{x}$, the potential gradient will be constant at all parts of the dielectric. By varying the nature of insulation in adjacent layers, a practical approximation to this condition can be obtained, especially with rubber, which can be made in varying qualities having markedly different permittivities. It cannot be said, however, that the grading of cables by this method, *i.e.*, by controlling the *capacities* of successive thicknesses of insulation, has met with much success in practice.

Another method of grading cables, which might be described as the conducting-layer method, gives promise of coming to the front as soon as very-high-voltage cables are in greater demand than they are at present. The principle is somewhat similar to that of the condenser type of insulating bushing, except that the intermediate metallic cylinders are all of the same length and the potential gradient is controlled by "anchoring" the voltage of these metallic intersheaths. The term "voltage grading" might be used to distinguish this method from "capacity grading." The insulation is the same throughout the total thickness, but being divided into two or more sections by means of metallic cylinders, each section can be made to take its proper share of the total potential difference by applying a definite voltage from an outside source to the intermediate metallic sheaths. Various methods of accomplishing the necessary distribution and "anchoring" of potential for both d.c and a.c. cables have been patented.[1]

[1] British patents 27858/08 and 27859/08.

The most obvious way of obtaining the required voltage control on an a.c. system is to take tappings from the high-voltage side of the power transformers for connection to the intermediate sheaths.

Example 33. *Design of Single-phase Concentric Extra-high-tension Power Cable.*—Let the working pressure be 100 kv. (alternating) between the inner and outer conductors. The further assumption will be made that the maximum stress must not exceed 40 kv. (r.m.s.) per centimeter. This is a low value, and 50 kv. would probably be permissible; but the lower figure has been chosen in order to provide a large factor of safety and keep the dielectric loss (and heating) in the neighborhood of its lowest practical limit.

Consider first the case of a cable without either "voltage" or "capacity" grading. By formula (173)

$$r = \frac{E}{G} = \frac{100}{40} = 2.5 \text{ cm.}$$

If a solid core of stranded cable were used, this would correspond to a cross-section of about 2¼ sq. in., which would certainly be in excess of the requirements, and a hollow core, constructed as shown in Fig. 134, should be adopted.

Solving for R in formula (86),

$$\log_e\left(\frac{R}{2.5}\right) = \frac{100}{40 \times 2.5} = 1$$

whence

$$\frac{R}{2.5} = 2.718, \text{ and } R = 6.8 \text{ cm.}$$

The dimensions of the cable, in inches, would be approximately:

Diameter over outer conductor = 5.56 in.
Diameter over lead sheath = 5.93 in.
Diameter over armor and jute = 6.6 in.

Consider now the alternative of intersheath or voltage grading. Assuming the total of 100 kv. to be divided into 40 kv. between inner conductor and intersheath, and 60 kv. between intersheath and outer conductor, the radius of the inner conductor, as calculated by formula (173), will now be

$$r = \frac{40}{40} = 1 \text{ cm.}$$

To calculate the radius over the insulation between the intersheath and the inner conductor,

$$\log_\epsilon\left(\frac{R}{1}\right) = \frac{40}{40 \times 1} = 1$$

whence

$$R = 2.718 \text{ cm.}$$

The radial thickness of the lead intersheath would be about 0.05 in., or 0.127 cm., whence the outside radius of the intersheath is $r' = 2.718 + 0.127 = 2.845$ cm. Considering this as the core of a cable with 60 kv. across the total thickness of insulation, and the same maximum voltage gradient as before, the radius over the insulation is

$$\log_\epsilon\left(\frac{R'}{2.845}\right) = \frac{60}{40 \times 2.845} = 0.527$$

whence

$$\frac{R'}{2.845} = 1.695 \text{ and } R' = 4.82 \text{ cm.}$$

The approximate dimensions of this cable, in inches, would be as follows:

Diameter over outer conductor	= 3.93 in.
Diameter over lead sheath	= 4.33 in.
Diameter over armor and jute	= 5.00 in.

In order to realize the advantage of this method of grading large high-voltage cables, these figures should be compared with those previously calculated for the cable without voltage grading.

It will be interesting to calculate the charging currents per mile of this cable. Assuming the specific-inductive capacity of the paper insulation to be $k = 3.3$ as given on page 385, the capacity per mile between core and intersheath, by formula (170), is $C_{\text{mf.}} = \dfrac{0.0388 \times 3.3}{\log_{10} 2.718} = 0.295$ mf. If the frequency is 25 cycles per second, the charging current, on the sine-wave assumption, will be

$$I_c = 2\pi \times 25 \times 0.295 \times 40{,}000 \times 10^{-6} = 1.86 \text{ amp.}$$

Similarly, between the intersheath and the outer conductor,

$$C_{\text{mf.}} = \frac{0.0388 \times 3.3}{\log 1.695} = 0.56$$

and

$$I_c = 2\pi \times 25 \times 0.56 \times 60{,}000 \times 10^{-6}$$
$$= 5.3 \text{ amp.}$$

The difference between these two values of capacity current must obviously be carried by the intersheath, and for every mile of cable the values of the charging current would be as follows:

In the core	1.86 amp.
In the intersheath	3.44 amp.
In the outer conductor	5.30 amp.

Thus, if the cable were 10 miles long, the current in the intersheath—if fed from one end only—would be 34.4 amp., which might be excessive. This is a point which must not be overlooked in the design and installation of intersheath cables.[1]

142. Capacity and Charging Current of Three-core Cables.—Mathematical formulas for calculating the electrostatic capacities of three-core cables are complicated and not very reliable owing to certain assumptions having to be made which are not always satisfied in a practical cable. The figures relating to capacities of cables, as furnished by cable makers, are, therefore, based on test data obtained from the finished cable.

The capacity between two parallel overhead wires, in microfarads per mile, is

$$C_{\text{mf.}} \text{ (between wires)} = \frac{0.0194}{\log \frac{d}{r}} \tag{174}$$

and the charging current (r.m.s. value) per mile of cable, on the sine-wave assumption, is

$$I_c = 2\pi f C_{\text{mf.}} E \times 10^{-6} \times L \text{ amp.} \tag{175}$$

where E is the (r.m.s.) voltage between wires, and L the distance of transmission in miles, while d and r in formula (174) stand, respectively, for the distance between centers of the conductors and the radius of the conductor cross-section.

If three wires occupy the corners of an equilateral triangle, the capacity as measured between wires is still given by formula (174), and the capacity between wire and neutral will be just twice this value, or

$$C_{\text{mf.}} \text{ (to neutral)} = \frac{0.0388}{\log \frac{d}{r}}$$

[1] The manner in which the intersheath carries a certain portion of the capacity current and so relieves the inner layers of the insulation of the heating due to this portion of the total charging current is clearly brought out in the paper by A. M. TAYLOR on "Underground Cables," *Jour.* (British) *Inst. Elec. Eng.*, vol. 61, p. 224, February, 1923.

as given by formula (109) Art. 89, Chap. IX. The voltage across this imaginary condenser is now $\frac{E}{\sqrt{3}}$ instead of $\frac{E}{2}$ as it would be in the case of a single-phase transmission, and the charging current per wire of the three-phase system is, therefore,

$$I_c = 2\pi f \frac{E}{\sqrt{3}} C_{\text{mf.}} \times 10^{-6} \times L \text{ amp.} \qquad (176)$$

In this formula, the capacity $C_{\text{mf.}}$ to neutral is just twice as great as the capacity between wires as given by formula (174), whence it follows that the charging current per wire of the three-phase system is $\frac{2}{\sqrt{3}}$ times as great as that of the single-phase system with the same spacing and line voltage.

In overhead systems, the capacity to ground is generally negligible; but in a cable transmission with or without lead

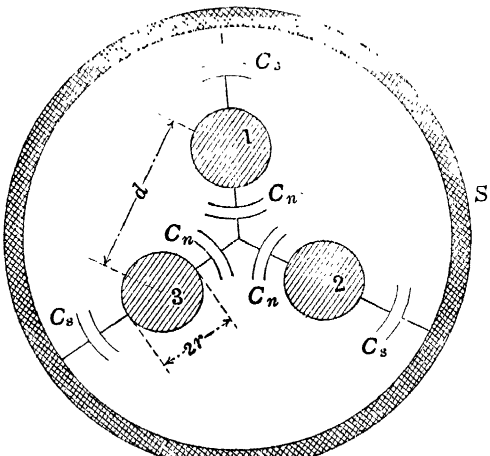


FIG. 135.—Diagrammatic representation of electrostatic capacities in three-core cable.

sheath, the condenser formed by the comparatively small thickness of insulation between conductor and ground must be reckoned with. Figure 135 is a section through a three-core high-tension cable with lead sheath. The capacity of each of the imaginary condensers shown in the diagram cannot be measured directly; but certain measurements can be made on the finished cable, from which the necessary data may be obtained. Let C_{ne}

stand for the effective equivalent capacity per conductor to neutral for 1 mile of cable. Then, since the voltage to neutral is $\frac{E}{\sqrt{3}}$, the charging current per conductor, on the sine-wave assumption, is

$$I_c = 2\pi f \frac{E}{\sqrt{3}} C_{ne} \times 10^{-6} \times L \text{ amp.} \tag{177}$$

if C_{ne} is expressed in microfarads.

Two measurements of capacity can readily be made on the finished cable:

a. Between one core (1) and the two remaining cores and the lead sheath, all connected together (2, 3, *S*).

b. Between any two cores, as (1) and (2).

In terms of the imaginary capacities C_s and C_n of Fig. 135,

$$\text{Capacity } (a) = C_s + \frac{1}{\frac{1}{C_n} + \frac{1}{2C_n}} = C_s + \frac{2}{3} C_n \tag{178}$$

and

$$\text{Capacity } (b) = \frac{1}{2}(C_s + C_n) \tag{179}$$

There is no constant ratio between wire to wire capacity and the wire to sheath capacity; but, generally speaking, the capacity (*b*) is from 58 to 66 per cent of the capacity (*a*). The most usual value is 60 per cent, whence, by equating (179) with 0.6 times (178), it follows that $C_s = C_n$.

The standard test for electrostatic capacity is between one core and the two remaining cores grounded to the sheath, namely, (*a*) as expressed by formula (178), and since the total capacity (C_{ne}) per core to neutral is obviously

$$C_{ne} = C_s + C_n \tag{180}$$

it follows that the approximate ratio of these two capacities is,

$$\frac{\text{Effective equivalent capacity, core to neutral } (C_{ne})}{\text{Measured capacity, one core to the remaining cores and sheath } (a)}$$

$$= \frac{(1 + 1) \times 3}{3 + 2} = 1.2 \tag{181}$$

The relation between the effective equivalent capacity (C_{ne}), conductor to neutral, and the measured capacity (*b*) between any two of the three cores is obtained by substituting for ($C_s + C_n$) in formula (179) the value given by formula (180), whence

$$C_{ne} = 2(b) \tag{182}$$

The measured capacity (*a*) for three-core paper-insulated cables designed for a working pressure of 10,000 volts will be about 0.4 mf. per mile for a $^{37}\!/_{0.083}$ in. cable, and 0.26 for a $^{19}\!/_{0.058}$ in. cable. With shaped instead of circular cores, the capacity is slightly greater, being from 1.08 to 1.1 times the capacity with circular cores of the same cross-section.[1]

Calculation of Capacity of Three-core Cables.—Although measured values of cable capacities are usually obtainable from manufacturers, it may be necessary to determine approximately the capacity of a cable for a special purpose before it has been made. It would seem at first sight from Fig. 135 and from the fact that C_s is found to be approximately equal to C_n that C_{ne} would be obtained by merely *doubling* the capacity to neutral as calculated by the usual transmission line formulas (see Art. 89, Chap. IX). It should be observed, however, that the proximity of the sheath will modify the distribution of the electrostatic flux between core and core, and the imaginary capacity C_n of Fig. 135 is not what it would be if the lead sheath were removed and replaced by a considerable extra thickness of insulation.

It is found in practice that the capacity of a three-core cable with shaped cores, having the same thickness of insulation between core and core as between core and sheath, is about the same as the capacity of a single-core cable having the same conductor cross-section and the same thickness of insulation between core and sheath. On this assumption, formula (170), Art. 139, can be used for predetermining the probable capacity of a three-core cable.

The value of R in formula (170) is taken as the radius of the conductor plus the thickness of insulation between cores, or between core and sheath. If these thicknesses are not exactly the same, the mean of the two thicknesses is taken. If the conductor is not circular in cross-section, the dimension r in the formula is the radius of a circular core of the same cross-section as the actual conductor.

As an example, let the equivalent diameter of each core of a three-core paper-insulated cable be 0.58 in., with insulation between cores 0.38 in. thick, and between core and sheath, 0.30 in.

[1] Values of capacitance and susceptance for three-core paper-insulated cables, calculated by the formulas in ALEXANDER RUSSELL'S "Alternating Currents," will be found in NESBIT'S "Electrical Characteristics of Transmission Circuits," Westinghouse Electric and Manufacturing Company.

thick. Assuming the specific inductive capacity of the insulation to be 3.3,

$$k = 3.3$$
$$r = 0.29$$

and

$$R = 0.29 + \left(\frac{0.38 + 0.3}{2}\right) = 0.63$$

whence,

$$\text{Capacity } (a) = \frac{0.0388 \times 3.3}{\log\left(\frac{0.63}{0.29}\right)} = 0.38 \text{ mf. per mile}$$

The approximate value of the capacity to neutral, for shaped cores, will, therefore, be

$$C_{ns} = 0.38 \times 1.2 = 0.456$$

and for cores of circular cross-section,

$$C_{ne} = \frac{0.456}{1.09} = 0.418 \text{ mf. per mile}$$

The results obtained by the above method are approximate only, but more accurate methods for predetermining the capacitance and, therefore, the probable charging current of three-core cables are available. Two valuable papers treating of the dielectric field in multiple-core cables have lately been published. The first, by D. M. Simons,[1] explains the use of the *geometric factor* which, in multiple-core cables, corresponds to the quantity $\log\left(\frac{R}{r}\right)$ which appears in the formulas for single-core cables. The second paper, by R. W. Atkinson,[2] gives results of experiments to determine the geometric factor for power cables with both round and sector-shaped cores, and to calculate the maximum stress in the dielectric of such cables.

143. Losses in Underground Cables.—In addition to the I^2R losses in the conductors, which can easily be calculated, some loss occurs in the dielectric of an underground cable. The ohmic resistance of the insulation being very high, the losses, when a high-tension cable is used on a continuous-current circuit, are very small; but with alternating currents there is a further loss due to dielectric hysteresis, which is proportional to the frequency of alternation of the electrostatic field.

[1] *Trans.* A.I.E.E., p. 600, 1923.
[2] *Trans.* A.I.E.E., p. 966, 1924.

The total charging current in a cable may be considered as made up of two components, one being the true capacity current of which the phase is exactly 90 deg. in advance of the impressed e.m.f., the other being the "energy" component in phase with the e.m.f. This last component is relatively small, being due to what is known as dielectric hysteresis and not to the ohmic resistance of the dielectric, the effect of which is usually negligible. The dielectric loss is equal to the product of the voltage and the "energy" or in-phase component of the charging current. Thus

$$\text{Watts lost} = \text{e.m.f.} \times \text{charging current} \times \cos \varphi \qquad (183)$$

where $\cos \varphi$ stands for the power factor of the cable. This will usually be about 0.012 for paper-insulated power cables, at ordinary working temperatures; but, with high temperatures, it is very much greater. As a rough indication of the manner in which the power factor (and therefore the dielectric loss) increases with temperature, the following figures may be useful; they refer to paper-insulated cables.

Temperature of Insulation, Degrees Centigrade	Power Factor, Per Cent
50	1.1 to 1.6
60	1.2 to 1.8
70	1.6 to 2.4
80	2 to 3

As an example of how the losses in a cable may be calculated, the data of the numerical example in Art. 142 may be used. The calculated capacity per core to neutral was $C_{ne} = 0.418$ mf. per mile. Assuming the voltage between wires to be 13,200, the frequency 50, and the distance of transmission 10 miles, the charging current per core, by formula (177), will be

$$I_c = 2\pi \times 50 \times \frac{13{,}200 \times 0.418}{\sqrt{3}} \times 10^{-6} \times 10$$
$$= 10 \text{ amp.}$$

whence the total dielectric loss is seen to be

$$W = 3 \times \left(\frac{13{,}200}{\sqrt{3}}\right) \times 10 \times 0.012 \times 10^{-3}$$
$$= 2.75 \text{ kw.}$$

The *apparent* power required is, however, $\frac{2.75}{0.012} = 229$ kv.-a., which is an indication of the size of generator required to keep the full voltage on the line when the receiving end is disconnected from the load.

144. Temperature Rise of Insulated Cables.—The layers of insulating material close to the conductors will be hotter than those near the surface of the cable; but the difference in temperature between the conductor and the external surface is not easily predetermined. It will depend upon the nature and thickness of the insulation, and also upon the type of cable, *i.e.*, whether two- or three-core, or concentric. The difference of temperature between the core and the external sheath of a fully loaded power cable will usually be between 10 and 25°C.; but the actual temperature of the insulation at the hottest parts will be determined not only by the rate at which the heat can be conducted from the cores to the surface through the insulation, but also by the rate at which it can be radiated or conducted from the outside surface of the cable.

The best conditions for cooling will usually occur with submarine cables; but armored cables buried direct in certain kinds of soil are also capable of dissipating large amounts of energy. When many cables are laid close together in multiple-way ducts, it is not possible to consider each cable independently of the others, since the temperature of the duct will depend upon the total cooling surface and the total amount of energy lost in all the cables. The problem of calculating temperature rise is thus seen to be a difficult one, and indeed more data must be accumulated before reliable empirical formulas will be available. Very little can be said here that will be of service to the engineer in determining exactly what will be the safe current for a given cable laid in a particular manner; but it is important to keep the temperature of the insulation within certain limits which cannot be exceeded without injuring the cable or leading to a greatly increased dielectric loss which aggravates the trouble and leads to rapid deterioration of the insulation. This limit may be set at about 85°C. for paper-insulated cables.

Temperatures above 135°C. are likely to char the insulation, but any temperature above 100°C. is to be avoided; there is always the possibility of expansion, with the formation of voids in which vapor may ionize. The allowable working temperature is usually calculated by the formula: Temperature in degrees Centigrade = 85 − E, where E stands for the working pressure in kilovolts. It has been found that paper-insulated cables are not injured by temperatures up to about 100°C. provided the *time* during which the cable works at the extra-high temperature is short, say 1 or 2 hours. It is not safe to operate continuously at temperatures above 85°C. In other words, the permissible maximum temperature is a function of the load factor.

For pressures up to 20,000 volts, the dielectric loss in three-phase high-grade paper cables is so small as to be negligible; and the permissible I^2R loss in the conductors will depend (1) upon the rate at which heat can be conducted through the insulation from the conductor to the outside surface, and (2) upon the facilities afforded for the cooling of the outside surface of the cable.

It is possible to run the current densities in three-core 20,000-volt power cables up to 1,000 amp. per square inch in cables of 0.25-sq. in. core section, and even 1,500 amp. per square inch if the core is not more than 0.1 sq. in. cross-section. It is rarely safe to allow the lead sheath to reach a temperature greater than 40°C.; but no hard-and-fast rule can be laid down in this connection. When the price of copper is high, it is important to load cables up to the safe limit. Research work is being carried on with a view to furnishing additional information on the heating of underground cables of different types and under different conditions of laying.

A simple case, which admits of calculation without a large amount of empirical data, is that of the single-core or concentric cable. Thus, if it is desired to calculate the difference in temperature between the core and external sheath of such a cable, the procedure would be similar to the method followed in calculating the ohmic resistance, except that the "heat-conductivity" of the dielectric would take the place of the electrical conductivity.

Example 34. *Temperature Rise of Insulated Cable.*—Following the method outlined in Art. 139 when developing an expression for the insulation resistance, consider a lead-covered single-core paper-insulated cable of core diameter $2r = 0.9$ in. and diameter over the insulation of $2R = 1.9$ in. The heat conductivity of the insulation must be determined experimentally, but it will be assumed that it is $k = 0.0025$. This coefficient may be defined as the number of watts that will be conducted through each square centimeter of a slab of insulating material 1 cm. thick when the difference of temperature is 1°C.

The cross-section of this conductor will be about 0.5 sq. in., the resistance being 0.1 ohm per mile, and if a current density of 1,300 amp. per square inch is assumed, the power loss per mile will be $0.1 \times (650)^2 = 42{,}250$ watts.

Considering a cylinder of the dielectric of thickness dx at a radius x from the center of the core, the difference of temperature between the two sides of this layer of insulation will be

$$dt = \frac{W dx}{k(2\pi x l)}$$

where l is the length in which the loss of W watts occurs (in this example $l = 161{,}000$ cm.). Thus

$$\begin{aligned} t &= \frac{W}{2\pi l k}\int_r^R \frac{dx}{x} \\ &= \frac{W}{2\pi l k}\log_e\left(\frac{R}{r}\right) \\ &= \frac{42{,}250}{2\pi \times 161{,}000 \times 0.0025}\log_e\left(\frac{1.9}{0.9}\right) \\ &= 12.5°\text{C.} \end{aligned}$$

If the cable were suspended in air, the difference of temperature between the lead sheath and the surrounding air could be calculated by assuming about 0.0012 watt to be radiated from each square centimeter of surface per degree Centigrade difference of temperature. The outside diameter of this cable—if there is no jute or steel armoring over the lead—will be about 2.15 in., and the rise in temperature of the lead sheath will, therefore, be

$$\frac{42{,}250}{\pi \times 2.15 \times 2.54 \times 161{,}000 \times 0.0012} = 12.7°$$

The total temperature rise of the copper on this basis is, therefore, 12.5 + 12.7 = 25.2°C.; but since the cable is not likely to be suspended in air, correction coefficients derived experimentally would have to be applied in order to determine the probable temperature rise under practical conditions. A further correction would have to be made if the cable is provided with an outer covering of jute.

Similar calculations for multiple-core cables can be made by using the experimentally determined *geometric factor* which was referred to in connection with the capacity of three-core cables.[1]

Since the final temperature rise of a cable under given conditions will be very nearly proportional to the square of the current,

$$I^2 = Kt$$

[1] Refer to paper by D. M. Simons, *Trans. A.I.E.E.*, p. 608, 1923.

and if the temperature rise t_1 is known for a given current I_1, the value of the constant K can be determined, and the temperature rise with any other current I_2 will be approximately

$$t_2 = \frac{I_2^2}{K}$$

The curves of Fig. 136 show not only the final temperature rise attained by three-core paper-insulated power cables drawn in iron pipes underground, but also the time required to bring about any particular rise in temperature. The vertical scale indicates degrees Fahrenheit above an initial temperature of 57° while the horizontal scale gives the hours during which the current has been on the cable. The cross-section of each core of this cable is 0.25 sq. in., and the tests were made with current densities

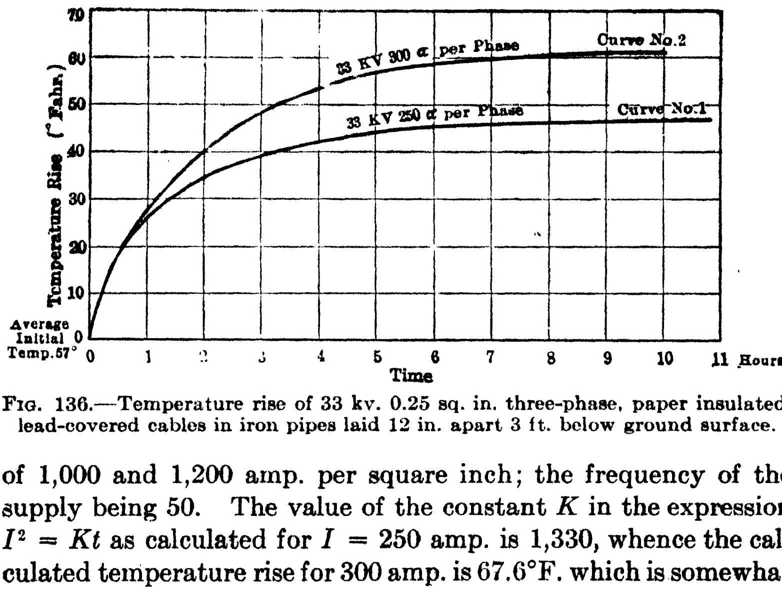


Fig. 136.—Temperature rise of 33 kv. 0.25 sq. in. three-phase, paper insulated, lead-covered cables in iron pipes laid 12 in. apart 3 ft. below ground surface.

of 1,000 and 1,200 amp. per square inch; the frequency of the supply being 50. The value of the constant K in the expression $I^2 = Kt$ as calculated for $I = 250$ amp. is 1,330, whence the calculated temperature rise for 300 amp. is 67.6°F. which is somewhat higher than the observed rise of 62° as one would expect it to be.

No appreciable rise of temperature was observed with the full working voltage on the cable without load. In other words, the dielectric losses were not of such magnitude as to add appreciably to the heating caused by the I^2R losses in the conductors.

145. Reliability of Cable Systems. Joints. Electrolysis.—Apart from mechanical injury, which must be guarded against by giving attention to the method of laying and to the handling of the cables during their installation, trouble can usually be traced (1) to poor joints, (2) to general or local overheating, and (3) to electrolysis.

Joints.—Even when the trouble is caused by overheating rather than initial weakness of insulation, this frequently occurs in the neighborhood of poorly made joints. A considerable amount of skill is necessary in making satisfactory joints on high-tension underground cables, and the difficulty experienced in obtaining skilled and reliable workmen has led to the development of designs employing special insulating spacers and accurately made metal sleeves or bridge pieces, which depend less upon the skill and experience of the jointer than the older methods involving paper or tape wrappings.

Careful bonding of the lead sheath is also a matter of considerable importance if electrolytic troubles are to be avoided, and this is provided for in the design illustrated by Fig. 137[1] which shows a joint box, for 33,000-volt three-core cables. The special jointing ferrules are designed with curved surfaces so shaped as to avoid concentration of stress, and thoroughly vitrified unglazed porcelain spacers are used between the cables in order to eliminate wrappings of tape or paper. The cast lead sleeve forming the bond between the lead sheathings is made in three parts to facilitate assembly. A high grade of insulating compound is poured inside the lead sleeve forming the bond between the lead sheathings, while bitumen may be used in the space between the lead and the outer case of cast iron.

An interesting article on high-tension cable joints appeared in the *Electrical World* (vol. 85, p. 1313), June 20, 1925. Figures 138 and 139 are reproduced, by permission, from this article. The former shows a joint of British design (W. T. Henley's Telegraph Works Company, Ltd.) for 35,000-volt three-phase cables installed in Chicago. Figure 139 is a joint of Italian design (Società Italiana Pirelli) for single conductors used on 46,000-volt three-phase system in New York.

An illustrated description by H. L. Wallan of the joints used by the Cleveland Electric Illuminating Company on their 66,000-volt, three-phase underground transmission, using single-core cables, will be found in the *Electrical World* of Nov. 24, 1923. A large number of these joints have now been in use for over 2 years.

[1] Design of W. T. Glover and Company, Manchester, England.

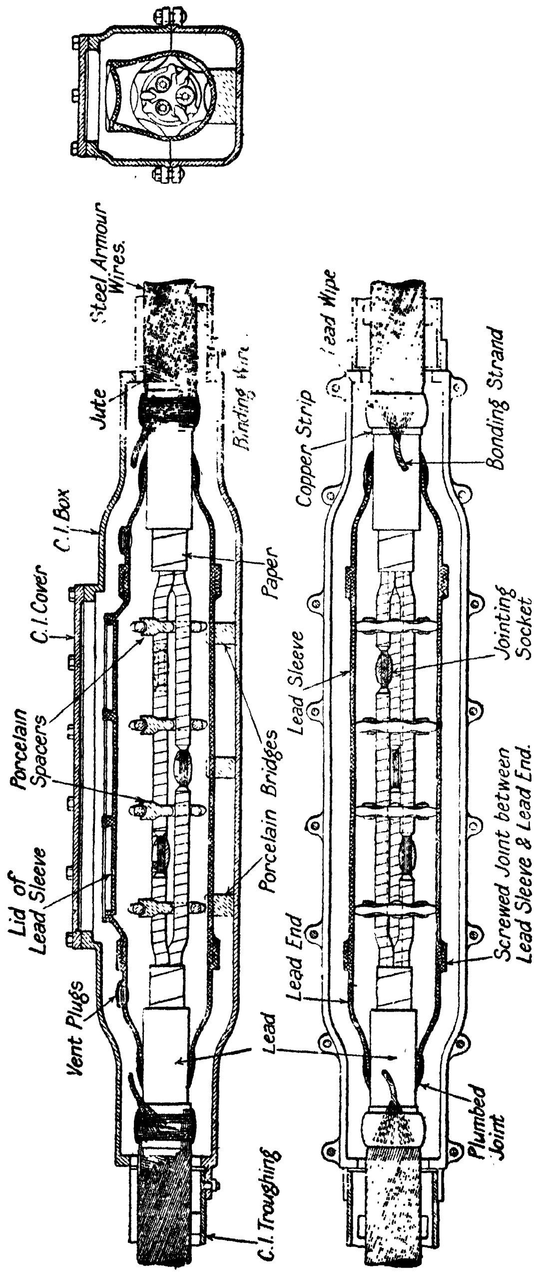


FIG. 137.—Joint for 33 kv. three-core cables.

Overheating.—Apart from the damage done to the insulation by very high temperatures, it is generally true that the dielectric loss increases with the temperature, and this is especially notice-

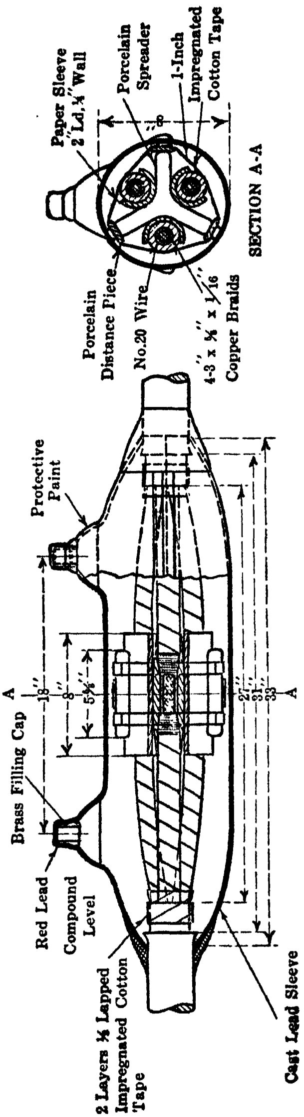


FIG. 138.—Joint for 35,000-volt three-core cable.

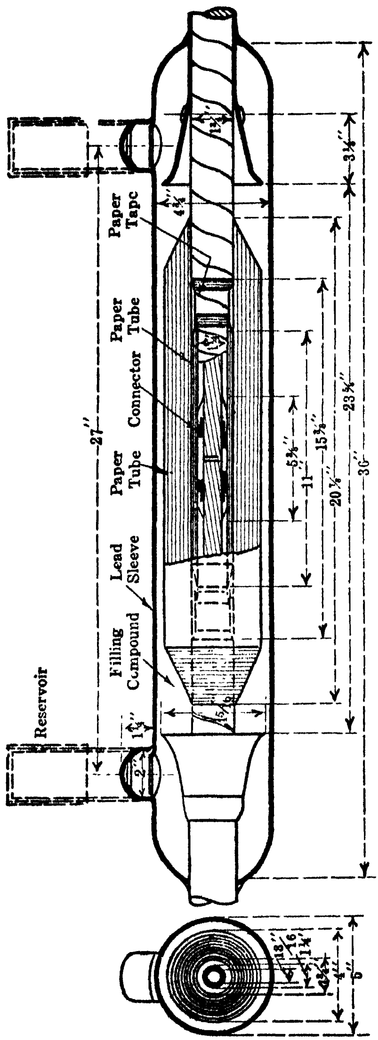


FIG. 139.—Joint for single cable of 46,000-volt three-phase system.

able with paper-insulated cables. It is nevertheless important to operate underground cables at a reasonably high current density; otherwise the interest on capital expenditure will be greatly in excess of the annual cost of the losses, and the system will be economically unsatisfactory. The engineer is, therefore, faced with a problem of considerable magnitude if underground transmission is to be adopted on a larger scale than at present. General or local overheating of cables is perhaps the chief cause of service interruptions on underground systems; yet a liberal cross-section of conductor can hardly be considered as the proper solution of the problem. Attention to all the causes that may lead to local overheating is very important; but the matter is not one that lends itself to lengthy discussion in these pages.

Electrolysis.—Chemical action, unassisted by electric currents, very rarely injures the lead sheath of underground cables; but electrolytic corrosion of the lead covering, resulting in perforations and damage to the insulation by the admission of moisture, is not uncommon. An underground cable laid within a few feet of an electric railway or car line, is liable to electrolysis; but in countries, such as England, where rules governing the maximum permissible potential difference between current-carrying rails and ground are rigidly enforced, trouble due to this cause is very rare. Perfect bonding of the lead sheath and also of the steel armoring, if provided, will be helpful in preventing trouble which otherwise might be experienced.

Leakage of current from the cables themselves, due to careless work and inefficient sealing at points where connections are made is another cause of electrolytic corrosion in underground distributing systems.

Electrolysis of lead and iron buried in the ground is usually caused by stray currents from electrically operated railroads or street-car lines which use continuous currents. The effects of alternating currents of frequencies between 15 and 60 cycles is practically negligible, being least with the highest frequency.

The actual amount of metal carried away from the surface of the anode (usually lead or iron) will depend not only upon the surface exposed and the density of the stray currents which cause the corrosion; but also on the nature of the soil and whether it is usually moist or dry. A continuous flow of 1 amp. from the lead sheathing of an underground cable will, in the course of a year, remove about 74 lb. of lead which will be carried away in the form of salts in solution.

The reader who desires to investigate more fully the subject of electrolytic corrosion is referred to the important contribution by McCollum and Ahlborn.[1] Their paper includes references to the work of previous investigators and brings out very clearly the importance of reversing the direction of the currents through the ground, even if the period of such reversal is measurable in hours or days. The shifting of loads on electric railway systems in cities usually produces large areas, called neutral zones, where the polarity of underground pipes reverses at more or less frequent intervals. Even when such intervals of reversal range from several minutes to 1 or 2 hours, the corrosion due to electrolysis is found to be less than would be expected if attributed to the average amount of current discharged from the pipes into the earth. It would seem as if the reversal of current actually causes metal to be redeposited on the corroded portions, even after the pipes have acted as anodes during a considerable lapse of time. The redeposited metal will probably have little effect in strengthening the pipe mechanically; but it will serve as an anode surface during the succeeding period of current discharge, and thus protect the uncorroded metal beneath, which otherwise would have been attacked. This action, due to comparatively slow reverals of current, will be interfered with by circulation of the electrolyte, and the action of air (oxygen and carbon dioxide) on the corroded metal.

Attempts to reduce the effects of electrolysis by means of insulating wrappings, varnishes, etc. on the outside of the lead sheath have not been successful; indeed there is always the possibility that when a discharge of current does occur, this will be localized, with the result that failure at some particular point may occur sooner than if the leakage had been distributed over a larger area of the lead covering. A commonly adopted means of mitigating the effects of electrolysis consists in providing metallic connections known as drainage cables or bonds from a number of points on the lead sheathing of the power cable to the return rail or negative bus of the street railway or other system which is the cause of the trouble.

[1] McCollum, B., and Ahlborn, G. H., "Influence of Frequency of Alternating or Infrequently Reversed Current on Electrolytic Corrosion," *Technological Paper* 72, Bureau of Standards, Washington.

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