A TEXT-BOOK OF

ENGINEERING DRAWING AND DESIGN.

PART I.—PRACTICAL GEOMETRY

A TEXT-BOOK OF

ENGINEERING DRAWING AND DESIGN:

INCLUDING

Practical Geometry, Plane and Solid, and Machine and Engine Drawing and Design.

BY

SIDNEY H. WELLS, C.B.E., WH.SC.,
A.M.INST.C.E., A.M.INST.MECH.E.,
LATE DIRECTOR GENERAL, DEPARTMENT OF TECHNICAL, INDUSTRIAL, AND COMMERCIAL EDUCATION, CAIRO, EGYPT, FORMERLY PRINCIPAL OF, AND HEAD OF THE ENGINEERING DEPARTMENT IN, THE BATTERSEA POLYTECHNIC, AND PREVIOUSLY TEACHER AT DULWICH COLLEGE.

With Numerous Illustrations, Examples and Test Questions, specially Intended for the Use of Students of Technical Schools and Colleges.

PART I.—PRACTICAL GEOMETRY.

Eleventh Edition.
(REPRINTED.)

LONDON:
CHARLES GRIFFIN & COMPANY, LIMITED;
42 DRURY LANE, W.C.2.

Printed in Great Britain by
Bell and Bain, Limited, Glasgow

PREFACE TO THE FIRST EDITION.

THIS book is intended for the use of Engineering students in schools and colleges, and as a text-book for examinations in which a knowledge of Practical Geometry and Machine Drawing is required.

The chief reason which has led to its preparation is that during the time I was engaged in teaching on the Engineering side of Dulwich College, and had charge of the classes in Geometrical and Mechanical Drawing, I found it impossible to obtain a book wherein the problems, or examples, were not accompanied by diagrams which the student could easily *copy*, without in the least knowing to what they referred. In Plane and Solid Geometry there was a lack of properly graduated questions, and such important parts of the subject as problems on loci, the construction of the useful plane curves and their practical application to cams and wheel-teeth, the interpenetration and development of simple solids, and isometric projection were only to be found in advanced books—although really more suited for elementary students than the troublesome problems on "points, lines, and planes" which usually precede them. In Machine Drawing and Design, again, the deficiency was still greater, there being no choice between sheets of Diagrams and Text-Books which, although they produce admirable copyists, are utterly devoid of any utility as regards *education* in design. The well-known treatise by Prof. Unwin is, of course, most excellent as an aid to design, but it does not profess to teach drawing, and is certainly not intended for elementary students.

Under these conditions, I found myself obliged to arrange questions so graduated in regard to sequence and difficulty as to be really helpful in "teaching" the subject, by bringing out important principles, by making clear mathematical relations, and by requiring the application of real thought and the knowledge gained in other classes and subjects. The setting of the questions was always preceded by a lesson in which, for geometrical drawing, typical problems were worked upon the blackboard and explained; and for machine drawing, the parts concerned were drawn separately or together, or illustrated by models, and the relations of, and reasons for, the shape and size of the different parts made clear. The book, then, has grown out of my own felt wants, and the effort to supply them by the questions and lecture-notes mentioned.

In Part I., I have included chapters on those parts of Practical Geometry already referred to as usually taken later, because I believe them to be essential to a good elementary course of Practical Geometry, and admirably suited for the ordinary engineering or technical student commencing without previous knowledge, and desiring to go on to an intelligent study of Machine Drawing and Design. I have also included some special cases of intersection, such as occur in metal plate work and in the drawing of some engine parts, in order that students may have no excuse for putting in the necessary curves by "guess work." Thus, in preparing Part I., I have steadily kept in view the work of Part II., from the conviction that Plane and Solid Geometry should always precede Machine Drawing, just as Arithmetic precedes Algebra.

In Part II., I have avoided dimensioning the illustrations except in rare cases, and have endeavoured to build up the subjects so that all primary and common parts are first explained and understood, such explanations not being repeated when the parts occur in connection with larger or complete designs. Such a method will, I am sure, commend itself to all true teachers. A student ought not to be told the sizes of bolts and nuts, or the diameter of flanges, or the details of stuffing boxes in drawing an engine cylinder, any more than we should expect to have to prove to him the truth of the triangle of forces, at each step in the graphical determination of the stresses in a roof-truss. But such an arrangement obviously requires that the examples be worked through in the order given, and especially is this so in the Sections on Engine Design, the examples in which have been intentionally arranged to show the interdependence of the different parts. I have throughout endeavoured to give the reasons for all features of the designs; when these are purely empirical or for workshop convenience, this is stated. My object will have been attained if I have made it impossible for a student to draw any part *without* having an intelligent reason for all he does.

It has certainly been my desire to make the book suitable for beginners, believing that the sizes and arrangements of simple common parts cannot be acquired too early, and I have therefore endeavoured to teach the principle of "drawing" as well as of "design." I have found some difficulty in deciding what terms and definitions to employ in order to make the book acceptable to the ordinary student and teacher, and yet free from unscientific and ambiguous expressions. Workshop terms are not fully satisfactory, for they vary with different localities, while scientific terms are often misused. I have restricted the word "compression" to represent a strain or change of form, and in other ways have adhered to the following notation:—

Stress		Strain
Pressure or Thrust	produces	Compression.
Tension		Extension.
Shearing Stress		Shear.

My thanks are especially due to Mr. F. W. Sanderson, M.A., Head-Master of Oundle Grammar Schools, who, during my association with him at Dulwich College, gave me much assistance in preparing the Examples, and who has been good enough to read the proof-sheets and to make many valuable suggestions, and to Professor Goodman, M. I. Mech. E., Assoc. M. Inst. C. E. of the Yorkshire College, Leeds, for much helpful counsel as to the arrangement of Part. II. I am indebted also to Mr. J. H. Wicksteed, M.I.C.E., M.I.Mech.E., for useful suggestions and drawings, and to Messrs. Allan & Co., Lambeth; Messrs.

Marshall & Sons, Gainsborough; Mr. W. Allchin, Globe Works, Northampton; The Kirkstall Forge Co.; The Globe Engineering Co.; Messrs. Schaffer & Budenberg of Manchester; and the Atkinson Gas Engine Co., for kindly supplying drawings for insertion in the text. Finally, I have to acknowledge the assistance received from the works of Professors Unwin and Ripper, and Mr. Henry Angel.

I shall be grateful to teachers and others who may use the book for information as to any errors which may have been overlooked.

SIDNEY H. WELLS.

BATTERSEA POLYTECHNIC, S.W.,
September, 1893.

NOTE TO THE ELEVENTH EDITION.

EDUCATION IN DESIGN was a chief factor in the preparation of this text-book for elementary students, and this Part I. on Practical Geometry has proved to be of the greatest value in leading up to an intelligent study of Machine Drawing and Design It has been adopted by teachers throughout the British Empire as well as in other countries. Many thousands of students have benefitted through its use and this further printing is offered in much confidence as the work has established itself as a standard for examination purposes.

C. G. & Co., LD.

CONTENTS OF PART I.

NOTE ON EXAMPLES.

S. and A. E. = Science and Art Department, Elementary.
S. and A. A. = ,, ,, Advanced.
S. and A. H. = ,, ,, Honours.
V. U. O. = Victoria University, Ordinary.
V. U. H. = ,, ,, Honours.

ENGINEERING DRAWING AND DESIGN.

PART I

PRACTICAL, PLANE, AND SOLID GEOMETRY.

SECTION I.

INTRODUCTION.

The following Exercises are intended for students using drawing instruments for the first time. All lines should be drawn with the T-square and set squares, and all divisions made with the dividers. Lines parallel to the long edges of the board should be drawn with the T-square, and lines at right angles with the set squares :—

EX. 1.—Draw a square of 3½" side, and divide it into small squares each of ½" side.

(Two adjacent sides of the square should be divided into seven equal parts, and lines drawn through the points parallel to the sides of the square.)

EX. 2.—Draw an oblong, sides 4" and 2½", and bisect each of the sides. Join the middle points of the sides to form a rhombus. Bisect the sides of this figure, and join the middle points, to form a second oblong. Again, bisect the sides of this oblong, and join the middle points to form a second rhombus. Try if the similar sides of the oblongs are parallel to each other, and also the sides of the rhombuses.

(A rhombus is a four-sided figure, having all its sides equal, but its angles not right angles.)

EX. 3.—Draw a circle of $3\frac{1}{2}''$ diameter. With the radius of the circle as distance, start from any point on the circumference, and step off distances round the circumference. *The radius should just step round the circumference six times.* Join the points together forming an equal six-sided polygon, known as a *hexagon.*

EX. 4.—Draw a line, A B, $3\frac{1}{4}''$ long. With A and B as centre, and the length of A B as radius, draw arcs cutting in C. Join C to A and B, then A B C will be an equilateral triangle. Find the middle point of each of the sides, and join to the opposite corner. These three lines will meet in a point Show, by drawing the circles, that this point is the centre of the inscribed and circumscribed circles of the triangle.*

(The inscribed circle is the circle touching the three sides, the circumscribed circle passes through the three corners.)

EX. 5.—Draw a circle $3\frac{1}{2}''$ diameter, and divide the circumference into eight equal parts. Join the points, forming a polygon having eight equal sides, known as an *octagon.*

(This is best done by drawing two diameters at right angles, and then two other diameters with the 45° set square, sloping right and left.)

EX. 6.—Draw a square of $2\frac{1}{2}''$ side. On each side of the square and outside it, construct an equilateral triangle. Draw the inscribed circle of each triangle, and also the inscribed circle of the square (find its centre by drawing the diagonals). Test your work by seeing if a circle drawn from the centre of the square passes through the centres of the triangles.

EX. 7.—Construct a square when the length of its diagonal is $4\frac{1}{2}''$.

(Draw a circle of this diameter and inscribe the square within it.)

EX. 8.—Draw a hexagon inside a circle of $2\frac{1}{2}''$ diameter. With each corner of the hexagon as centre, draw a circle of radius equal to half the side of the hexagon. Test your work by seeing if a circle drawn from the centre of the hexagon can be made to touch and include the six small circles.

(Find the centre of the hexagon by drawing two of its longest diagonals.)

EX. 9.—Using the 45° and 60° set squares, draw (*a*) a triangle, base 3″, base angles 45° and 60°; (*b*) an isosceles triangle, base $3\frac{1}{2}''$, base angles 45°; (*c*) a rhombus, sides $3\frac{1}{2}''$, acute angles 60°; (*d*) a parallelogram, sides 4″ and $2\frac{1}{2}''$, acute angles 45°.

* This point is only the centre of both circles when the triangle is equilateral.

SECTION II.

GEOMETRICAL CONSTRUCTIONS FOR LINES AND ANGLES.

THERE are a large number of simple problems which constantly occur in all kinds of mechanical drawing, such as the division of lines, arcs, and angles, the drawing of parallels and perpendiculars, which can generally be worked with the usual instruments, and without adopting any geometrical construction. But it often happens that such methods are not as convenient, or likely to be as accurate, as certain constructive methods based upon Euclid's *Elements*, of which the following are the most useful and important. They should, therefore, be remembered by the student, and adopted whenever special accuracy is desired:—

PROBLEM I. (Figs. 1, 2).—*To bisect a line, arc, or angle.*

Fig. 1.—Let A B be the given line or arc. With one end, A, as centre, and radius greater than half, A B, draw arcs on opposite

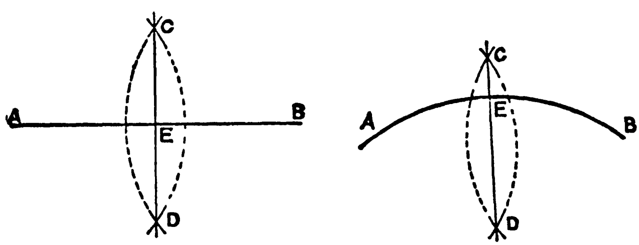


Fig. 1.

sides of A B. With the other end, B, as centre, and the same radius, draw arcs cutting the first arcs in C and D. Then the line joining C D will cut A B in its middle point, E, and will therefore bisect it.

(Note that only small arcs need be drawn, and that it is enough to simply mark the line or arc in the point, E, and not draw the whole line joining C D. It is evident that the radius of the arcs must exceed half A B, or the arcs will not cut.)

If the line or arc is to be divided into four, eight, or a greater number of equal parts, the same construction has simply to be repeated, treating the equal parts A E, E B, each in the same way as A B

Fig. 2.—Let A B C be any angle. With the meeting point or vertex, B, as centre, and any radius, draw an arc cutting the lines of the angle in D and E. Then, as before, with D and E as centres and radius greater than half D E, draw arcs to cut, as shown at F. Join B F. This line will bisect the angles, and any point on it will be equidistant from B D and B E. A repetition of this method will divide the angle into four, or eight equal parts.

(Note that this construction is only applicable when the required number of parts is even, and equal to some integral power of 2, as 2^2, 2^3, 2^4 . . . . = 4, 8, 16, &c.)

The following points should be carefully observed in connection with the division of lines, arcs, and angles:—

The exact point at which two lines meet is more accurately found when the angle between them is not less than about 10° or greater than 130°. This will be seen on reference to Fig. 3,

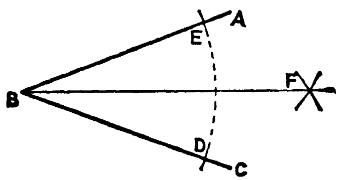


Fig. 2.

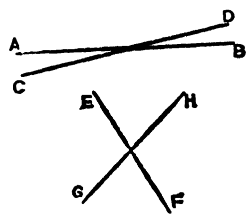


Fig. 3.

where the lines A B, C D give a bad meeting point, and the lines E F, G H a good one. It will be seen that, except when the angle is 90°, the lines are in contact for a length greater than their thickness; hence they do not give a decided point of intersection. The same remark applies to the intersection of arcs.

In bisecting angles it is necessary to obtain the bisecting point (F in Fig. 2) a good distance from the vertex, B, of the angle. If B and F are near together then the line drawn through them will most probably not fulfil the condition that any point in it shall be equidistant from the lines of the angle, except near the vertex. A little practice will soon convince the student of observing this and similar facts. Exactly in the same way, the bisection of a line, by the method just described, is likely to be more accurate when the radius of the arcs is considerably greater than half the line, than when it only slightly exceeds the half, as the former gives a clearly defined intersection point, and the latter a bad one. Other facts of this kind will be referred to in connection with later problems, but their intelligent study cannot be too soon, or too much, insisted upon.

EXAMPLES.

EX. 1.—Divide a line 3″ long into four equal parts by continued bisection.

EX. 2.—Draw an arc of $3\frac{1}{2}$″ radius and bisect it.

EX. 3.—Draw two lines meeting at any angle, and divide the angle into four equal parts by continued bisection.

EX. 4.—Construct a triangle, sides 5″, 4″, and 3″, and bisect each of its angles. The bisecting lines will meet at a point. Show, by drawing the circle, that this point is the centre of the inscribed circle of the triangle.

(Obtain the radius of the circle by drawing a perpendicular with set squares from its centre to one of the sides.)

EX. 5.—Draw an arc of $4\frac{1}{2}$″ radius, and divide it into four equal parts by continued bisection. Produce the bisecting lines *and notice that they pass through the centre of the arc.*

Perpendiculars to lines—PROBLEM II. (Figs. 4*a*, *b*, *c*, *d*).—*To draw a perpendicular to a given line, A B, from a given point, C.*

(*a*) (Fig. 4*a*).—*When the point is in the line and not near either end.*

With the given point C as centre, and any radius less than the shorter of C A or C B, cut the line A B in the points D and E, on

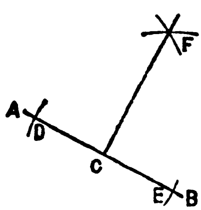


Fig. 4*a*.

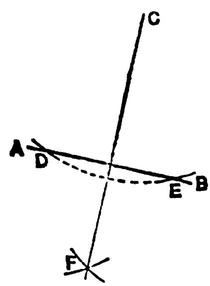


Fig. 4*b*.

opposite sides of C. With D and E as centre, and radius greater than half D E, draw arcs meeting on either side of the line, as shown in F. The line through F C is perpendicular to A B.

(*b*) (Fig. 4*b*).—*When the point is outside the line and not over either end.*

With the given point C as centre, describe an arc cutting the line in two points, D and E. With these points as centres, and with the same radius, draw arcs on the other side of A B, meeting in F. The line through C F is perpendicular to A B.

(*Euclid* i. 8, and Def. 10, for C D = C E, and F D = F E, and C F is common, therefore the angles made by A B and C F are right angles).

(*c*) (Fig. 4*c*).—*When the point is in the line, and near to, or at one end.*

With the given point C as centre, and any radius less than C A, draw an arc as shown, cutting C A in D. From D step off the same radius from D to E, and E to F. With E and F as centre, and any radius (for convenience the same as before), draw arcs cutting at G. The line through C G is perpendicular to A B.

(The angle D C E = 60°, also the angle E C F. But C G bisects the angle E C F making angle A C G = 60° + 30° = 90°).

(*d*) Fig. 4*d*).—*When the point is outside the line and over either end.*

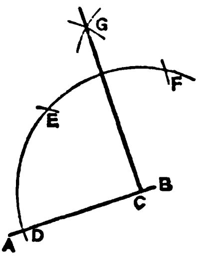


Fig. 4*c*.

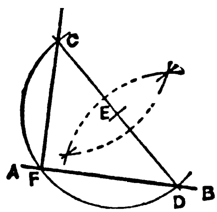


Fig. 4*d*.

Join the given point C to any convenient point D, near the further end of AB. Bisect C D in E, and with E as centre draw a semicircle passing through C and D, and cutting A B in F. The line through C F is perpendicular to A B.

(The angle in a semicircle is a right angle (*Euclid* iii., 31), and as C F D is an angle in a semicircle, C F is perpendicular to A B).

This construction can also be applied to *Case C*, Fig. 4*c*, by drawing any semicircle with centre E passing through the given point C, and cutting the line A B in a point D. Then by joining the points D E, and producing the line to cut the semicircle at a second point, G, a diameter will be drawn, and G will be the required point to join to C.

Notice that what has already been said about choosing the radii of the arcs so as to obtain sharp points of intersection applies equally to these problems, and that it is only necessary to draw part of the arcs through where the cutting point is likely to come. Also that in such an example as Fig. 4*b*, time is saved by using the same radius for the arcs, cutting at F as for the arc described from centre O, thus avoiding an alteration of the compasses.

EXAMPLES.

The following Examples are to be constructed geometrically:—

EX. 6.—Construct a square of 2½″ side.

EX. 7.—Construct an oblong, sides 3″ and 2″.

EX. 8.—Construct a triangle, sides 5″, 4″, 3½″, and draw from each corner a perpendicular to the opposite side.

EX. 9.—Construct a triangle, sides 5″, 4¼″, 2¾″. Bisect each of its sides, and through the points draw perpendiculars to the sides. These three lines will meet at a point. Show, by drawing the circle, that this point is the centre of the circle circumscribing the triangle.

EX. 10.—Mark any three points A, B, and C, not in the same straight line, and draw the circle passing through them.

(Find the centre by joining A to B and B to C, and bisect these lines by lines perpendicular to them; they will meet at the centre.)

EX. 11.—Draw any irregular triangle, and mark any point within it. From this point draw lines perpendicular to each of the sides.

Parallels to Lines—PROBLEM III. (Fig. 5).—*To draw a line parallel to the given line A B at a given distance away.*

Construct perpendiculars, A D, B C, from the ends of the line, or from any convenient points within it, and cut off a length on each equal to the given distance. Then through the two points draw a line which will be parallel to A B; or—

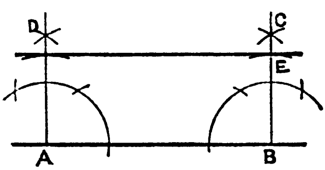


Fig. 5.

Draw a perpendicular from one end of the line as B C, cut off a length equal to the given distance, as B E, and through point E draw another perpendicular to B C, which will be parallel to A B.

As both these methods require the construction of two perpendiculars, there is no reason why they should not be equally accurate.

EXAMPLE.

EX. 12—Draw a rhombus, sides 3¼″, acute angles 45°. Then draw a second rhombus, parallel to and surrounding it, and ½″ away.

(Obtain the 45° by bisecting a right angle.)

Copying and Addition of Angles.—Similar straight-lined

figures are those which have their several angles equal, each to each, and the sides about the equal angles proportional to each other. Hence to draw one figure similar to another, it is necessary to know how to copy an angle, or, in other words, how to make an angle at a given point equal to a given angle. It is also convenient to be able, geometrically, to add angles together, as, for example, an angle of 135° can be found by adding angles of 90° and 45°; and 75°, by adding 60° and the fourth part of 60°.

PROBLEM IV. (Fig. 6).—*On a given line to draw a triangle having angles equal to those of a given triangle.*

Let A B C be the given triangle and D E the given line.

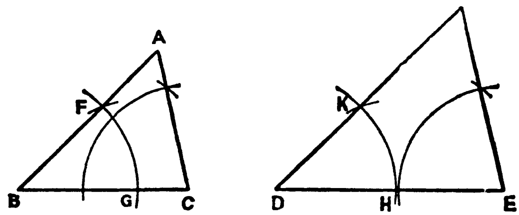


Fig. 6.

With centre B, draw an arc cutting B A, B C, in F and G. Draw a similar arc with the same radius, with centre D, cutting D E in H. Measure the chord, F G, in the compasses, and with H as centre set off the length of F G along the arc to K. Join D K and produce. Then the angle K D E is equal to the angle A B C. At E, make an angle equal to the angle at C or A in the same way, and the triangle will be complete. (*Euclid* vi., 4.)

(Equal angles in equal circles are subtended by equal chords.—*Euclid* iii., 26 and 29.)

Note.—If the triangles are large, accuracy will only be obtained by drawing arcs of large radius.

PROBLEM V. (Fig. 7).—*To add or subtract angles.*

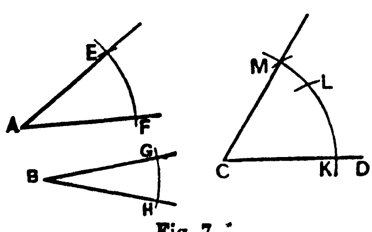


Fig 7.

Let A and B be two angles. It is required to make with the line C D an angle equal to A and B.

With A, B, and C as centres, draw arcs of the same radius, cutting the arms of A in E F and the arms of B in G H, and cutting the line C D in K. Take the length of the arc, E F, in the compasses and set it up from K to L. Similarly, measure the arc G H, and set it up from L to M. Join C M. Then the angle D C M is equal to the angles A and B.

EXAMPLES.

EX. 13.—Draw any irregular four-sided figure, no side less than $1\frac{1}{4}''$, and draw a second figure having equal angles and two of its sides $3''$ and $2\frac{1}{4}''$.

EX. 14.—Draw any irregular triangle and show, by adding the three angles together, that *the three angles of a triangle together equal two right angles.*

(Add two of the angles to the third angle, and the first and last lines should form one straight line.)

EX. 15.—Draw an isosceles triangle, base $3''$, vertical angle 45°.

(The sum of the base angles will be $180^\circ - 45^\circ = 135^\circ$, ∴ draw a line and a perpendicular to it, giving two right angles, bisect one of these right angles, thus giving an angle of $90^\circ + 45^\circ = 135^\circ$, then bisect this angle for one of the base angles.)

EX. 16.—Draw any line, A B, and mark a point, C, outside it. Through C draw a line parallel to A B.

(Join C by a line to any point D in A B, and from C draw a line making the same angle with C D as C D makes with A B.—*Euclid* i., 23.)

Construction of Angles and Protractors.—Lines may be readily drawn at different angles to a given line, by drawing a semicircle upon the line, and knowing that a semicircle contains 180°, dividing the semicircumference to obtain the desired angles. This method of setting off angles is much facilitated by remembering that *the radius of a circle steps round the circumference exactly six times*, and that if any two of these points next one another are joined to the centre by lines, the angle between the lines is 60°, for the whole angle at the centre is four right angles, or 360°, and the construction gives exactly one-sixth, or 60°. Hence a line may be quickly and accurately drawn at 60° to any given line as follows :—

Let A B be the given line, and let a line be required at 60° to A B, starting from the end A. With A as centre and any radius, draw an arc cutting A B in C, and from C set up the same radius to the point D, and join A D. Then the angle B A D is 60° (Fig. 8).

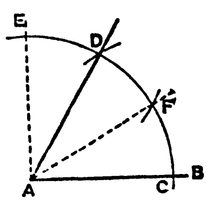


Fig. 8.

This construction suggests an easy method of trisecting a right angle; for if in Fig. 8 the lines A B, A E are at right angles, then the angle E A D is one third of a right angle, and by setting off the same radius as before from the point E to F, the angle B A F is made one-third of a right angle. Hence the lines A D and A F trisect the right angle.

The following angles are thus easily obtained:—30° by bisecting 60°; 75° = 60° + half of 30°; 120° by setting off two 60°, and 135° = 90° + 45° (half a right angle). Also 108° = $\frac{3}{5}$ of 180°, and is therefore found by drawing any semicircle, dividing it into five equal parts and joining the centre to the second division point from one end; the two angles thus formed will be 72° and 108°. To obtain 135° which = $\frac{3}{4}$ of 180°, divide the semicircle into four equal parts, and join to the first division from one end. These angles are important as being those of certain useful regular polygons, the construction of which will be described further on.

Protractors.—An extension of this method is employed to construct protractors, which enable angles of any degree of measurement to be set off. A semicircle of 6" diameter is drawn and its semicircumference is accurately divided into 180 equal

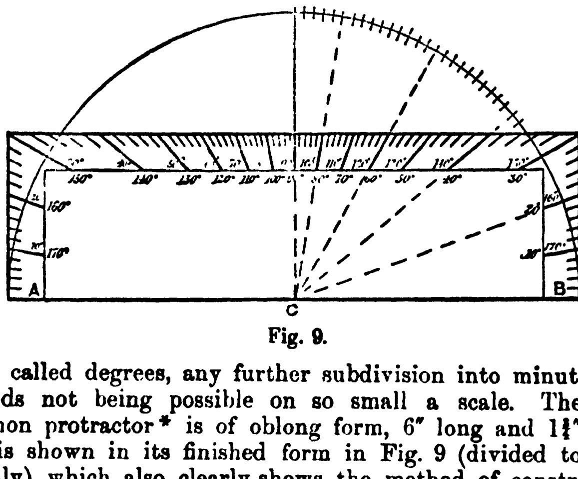


Fig. 9.

parts called degrees, any further subdivision into minutes and seconds not being possible on so small a scale. The most common protractor* is of oblong form, 6" long and 1¾" wide, and is shown in its finished form in Fig. 9 (divided to show 2° only), which also clearly shows the method of construction. Notice that the divisions are marked both ways to allow of using from either end, and that the lines showing each 10° are longer than the lines showing the smaller divisions. In constructing a protractor, the semicircumference should be divided by continual bisection as in Fig. 2, as repeatedly as possible, and then the dividers used to obtain the small divisions.

* This protractor is made of boxwood and its divisions are not usually very reliable. A more accurate instrument is the horn protractor, which is generally of semicircular form.

The protractor is used by placing the edge A B to coincide with the line from which the angle is to be drawn, and the middle point C against the point in the line from which the angle is to start.

EXAMPLES.

EX. 17.—Draw lines meeting at the following angles:—(*a*) 60°, (*b*) 75°, (*c*) 90°, (*d*) 105°, (*e*) 108°, (*f*) 120°, (*g*) 135°, (*h*) 140°.

EX. 18.—Construct a triangle base 2½″, one base angle 60°, verticle angle 45°.

EX. 19.—Construct a protractor 6″ × 2½″, to show divisions of 5°.

MISCELLANEOUS EXAMPLES.

(1) Draw a line, A B, 3″ long, and find three points beyond B through which A B would pass if produced.

(2) Draw a parallelogram, base 3″, diagonals 4¼″ and 6½″.

(The base, and half of each diagonal form a triangle.)

(3) Draw a line, A B, 3¼″ long, and produce it to a point C, so that B C shall be ¼ of A B.

(Divide A B into four by bisection, and add one piece on.)

(4) Draw two lines meeting at a point A at 135°, and bisect the angle, using only the parallel edges of a rule and pencil.

(Place one edge of rule coinciding with one arm of angle, and draw line along other edge, do the same with other arm of angle, the two lines drawn will meet in a point, which when joined to vertex bisects the angle.)

(5) Draw a circle of any diameter between 4″ and 6″, and find its centre (as though unknown) using only the parallel edge of a rule, a measuring rule and a pencil.

SECTION III.

DIVISION OF LINES AND CONSTRUCTION OF SCALES.

Division of Lines.—In the division of lines and angles by the method of bisection as explained in the preceding section, it was seen that the construction only applied to obtaining division into certain numbers of parts, and did not admit of general application. There are, however, other methods by which lines can be accurately divided into any desired number of equal parts, with which it is very necessary for the student to become familiar.

The most common method of division in practical mechanical drawing is known as "division by trial, or with dividers." Thus to divide a given line into any number of equal parts, the dividers are set to what the draughtsman considers to be approximately the right distance, and then, starting from one end this distance is stepped off along the line the required number of times. If the last step just reaches to the line end, the division is accurate, but if not, the dividers must be opened or closed until the equal division is obtained. Circles, arcs, and angles can be divided in the same way since although the dividers then really measure the length of chords, yet the arcs are proportional to them (*Euclid* iii., 28). For accuracy in division the use of dividers in a practised hand is more reliable than geometrical methods. It is, however, necessary to use some form of spring dividers, and to avoid making holes through the paper at each step.

The following is the geometrical construction for the division of lines into any required number of parts. It is based upon the properties of similar triangles, and is particularly useful, as permitting of division into fractional parts, or into parts proportional to a given ratio, or to the divisions of a given line.

PROBLEM VI. (Fig. 10*a*, *b*).—*To divide a line into any number of equal parts.*

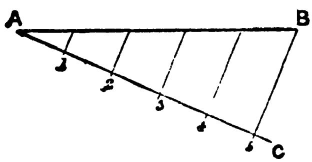


Fig. 10*a*.

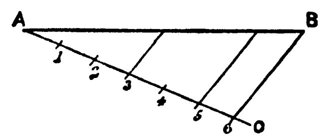


Fig. 10*b*.

Fig. 10*a*.—Let A B be the line to be divided into five equal parts. From one end, A, draw a line, A C, of any length, and at any angle to A B. Mark off upon this line five equal parts, as at 1, 2, 3, 4, 5. Join 5 to the end B, and through the points 1, 2, 3, 4, draw lines parallel to B 5, meeting A B as shown. Then A B is divided into five equal parts.

(The five triangles thus formed, each having A for a vertex, are similar; therefore, since A 5 is divided into five equal parts, A B is similarly divided (*Euclid* vi., 4). The equal parts set off down the line A 5 may be of any convenient length, but a little practice will show that the greatest accuracy is obtained when the angle B A C is small, as drawn, and the length is approximately equal to the fraction required of the given line.)

In order to divide a given line A B into three and a-half equal parts, it is only necessary to set off down the line corresponding to A C. $3\frac{1}{2} \times 2 = 7$ equal parts, and then draw parallels to B 7, through every other one to the given line.

By the same method a line can be divided into parts having a desired proportion to each other, or similarly to another divided line which may be either longer or shorter. Suppose we require to divide a given line A B (Fig. 10*b*) in the proportion of 3 : 2 : 1. Set off down the line A C, 3 + 2 + 1 = 6 equal parts of convenient length, and draw parallels as before from the points 3 and 5 to the line 6 B, then A B is divided into three parts in the required proportion. When the given line A B is to be divided proportionately to the divisions of another given line, it should be drawn from one end of the divided line, at an angle to it as before. Then by joining the ends of the two lines, and drawing parallels through the points in the divided line, the line will be divided similarly to the given divided line. Numerous useful problems in proportion can be worked in this way; the method is in fact a part of graphic arithmetic.

EXAMPLES.

EX. 1.—Divide a line 5″ long into four equal parts in three different ways.

EX. 2.—Draw a circle 4½″ diameter, and draw any diameter. Divide half the circle by continued bisection, and the other half with dividers, each into eight equal parts. Mark the points 1, 2, 3, ... to 16, and join the points 2 and 10; 7 and 15; 12 and 4. If accurate, these lines should pass through the centre of the circle.

EX. 3.—Draw two lines at any angle to each other and meeting, and divide the angle into three equal parts.

EX. 4.—Divide a line 7″ long into two parts, in the proportion of 1½ : 2.

EX. 5.—Divide a line 6″ long into three parts, in the proportion of 2 : 3 : 4.

EX. 6.—Find by construction the eighth part of 2·5″.

EX. 7.—Draw two lines at any angle, meeting at point A, and find a point, P, 2½″ from A, its distance from one line of the angle being twice its distance from the other line.

EX. 8.—Draw any angle and bisect it by using only a parallel rule and pencil.

EX. 9.—Draw a line, A B, 3″ long. Find by construction three points through which A B would pass if produced in a straight line.

EX. 10.—Draw any irregular five-sided figure, no side less than 1″. From it draw a second similar figure having its sides one and a-half times as long (see pp. 7, 8).

EX. 11.—Draw a line, A B, 4″ long, and mark three points in it, C D E. Then draw a second line $3\frac{1}{2}$″ long, and divide it proportionately to the divisions of the line A B.

Scales.—In most mechanical drawings, the objects represented are too large to be drawn full size, and are, therefore, drawn so that all parts are proportionately smaller. When this is done the drawing is said to be to scale. The ratio of the drawing to the object is decided beforehand, and generally varies with the size and nature of the object and the size of the paper. In machine drawings details of complicated parts are drawn to a larger scale than simple parts, while structures, such as roofs and bridges, plans of fields and buildings, are drawn to a small scale. The fraction which expresses the ratio of the drawing to the object it represents is called the "*representative fraction.*" Thus, suppose a drawing be made where a length of $1\frac{1}{2}$″ represents a length of 1 foot on the object. This is shown on the drawing by writing upon it, "*Scale* $1\frac{1}{2}$ *inches* = 1 *foot*," and as $1\frac{1}{2}'' = 12''$, the ratio is $\frac{1\frac{1}{2}}{12} = \frac{1}{8}$, therefore, the representative fraction is $\frac{1}{8}$. Hence the drawing might be marked, "*Scale* $\frac{1}{8}$ *of full size.*" The former method is, however, generally adopted, but the student should notice that the results are the same, and that a scale is described when either its representative fraction is given, or when the number of inches representing 1′ is stated. In a scale whose ratio is a fraction—that is, one where the drawing is made smaller than the object, the scale is said to be a "*reducing scale;*" but in the case of physical apparatus, clocks and watches, and other small mechanisms, the drawing requires to be larger than the object—that is, a length of 1″ on the object, is shown in the drawing by a length of probably 3″ or 6″. Hence, in the latter case, the drawing would be marked, "*Scale* 1 *foot* = 2 *inches,*" and the ratio would be $\frac{12}{2} = 6$, therefore the representative fraction is a whole number, and the scale is called an "*increasing scale.*" It is important to notice the different ways of stating an increasing or a decreasing scale.

Scales are constructed for the draughtsman's use, by dividing the edges of boxwood or ivory strips in a machine capable of working with great accuracy, and any ordinary scale is easily obtained. But it is necessary to be able to construct a scale, as a drawing has sometimes to be made to an unusual scale, or a machine-made scale may be unobtainable.

Before constructing a scale, it is necessary to know—1st, its size, or representative fraction; 2nd, the longest length it has to represent; 3rd, the different units of length it must show, as feet and inches, yards and feet, miles and furlongs.

It does not follow that in drawing a field 300′ long, say to a scale of $\frac{1}{100}$, the scale must be 3′ long, as that would be absurd. Scales are generally made 12″ or 18″ long, and longer lengths are taken off by marking off successive lengths.

The method of constructing a simple or *plain* scale is as follows:—

PROBLEM VII. (Fig. 11).—*To construct a scale where* $1\frac{1}{2}''$ = 1′ *long enough for* 6′, *to show feet and inches.*

Draw a straight line upon the paper of indefinite length, and

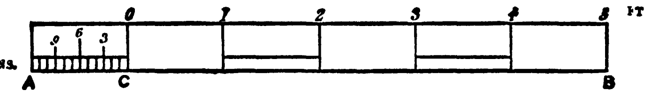


Fig. 11.

from one end, A, mark off a length, A B, equal to $1\frac{1}{2}'' \times 6'' = 9''$, since $1\frac{1}{2}''$ show 1′, and 6′ are to be shown.

Divide A B into six equal parts, then each part will represent 1′, as the whole length shows 6′.

Divide the first of these divisions, A C, into twelve equal parts, then each part will represent 1″.

Complete the scale in the manner shown in the figure. Notice that the divisions representing feet are carried to near the top line, that the 6″ division is somewhat shorter, the 3″ and 9″ divisions still shorter, the other inch divisions being shortest. This is done to better distinguish the different divisions and to make the important ones clearly seen. Notice also that a line is drawn through the top of the inch divisions, and repeated in alternate foot divisions. This is done to help in counting, a lined division and a plain division representing 2′. The bottom line A B is generally made dark as a finish.

Marking the scale is very important, and is generally wrongly done by beginners. What is desired is that the marking shall agree with the length taken off the scale, and this is only accomplished by marking as shown in the figure. The zero point is at C where the inch and foot divisions begin, and from that point inches are marked to the *left*, and feet to the *right*.

It is a common fault to mark the point C as 1′, this means that a length on the scale marked 2′ 3″ is really only 1′ 3″. An equally wrong result follows when the inch divisions are marked from A to C, beginning at A, then a length marked as 2′ 3″ is really only 1′ 9″.

It should be noticed that it is not necessary to further divide the scale. It is, therefore, waste labour to divide up the whole of the foot divisions into equal parts, although this is sometimes done in machine-made scales. The scale as drawn shows any length between 1″ and 6′.

The importance of accuracy in constructing a scale cannot be too strongly insisted upon. The same length taken from different parts of the scale should agree, otherwise the drawing made with the scales will be wrong, and all scales should be tested in this way. In showing inch divisions for a small scale this is very difficult, and the student will find that such small divisions can be made quite as accurately with the eye, as by using dividers. But accuracy is only obtainable with great care, and by using good instruments and hard pencils with fine points.

In setting off the total length of the scale, do not take a distance of say 1″ in the dividers and set this distance off repeatedly along the line until the right length is obtained. This cannot be accurate, as suppose the 1″ to be taken off the rule $\frac{1}{100}$″ too short or too long, a very probable error, then the whole line in the example given would be $\frac{1}{100} \times 9 = \cdot 09''$ short or long.

EXAMPLES.

EX. 12.—A line 2·5″ long is drawn to represent a length of 1′. What fraction is the line of the length it represents, and what length of line should be drawn to show lengths of 1″ and 5″? Divide the line to show inches and mark the divisions.

EX. 13.—Construct a scale, the representative fraction of which is $\frac{1}{300}$, reading yards and feet, long enough for 3 yards.

EX. 14.—The plan of a room, 41′ long and 28′ wide outside, is to be drawn upon a sheet of paper 22″ × 16″, leaving about 1″ border all round. Construct and mark the scale that should be used.

EX. 15.—Construct carefully the following scales, writing above each, its representative fraction, and marking clearly what the divisions represent:—

(*a*) Scale of $1\frac{1}{2}''$ = 1 foot, long enough for 6 feet, showing feet and inches.

(*b*) Scale of $\frac{3}{4}''$ = 1 foot, long enough for 8 feet, showing feet and inches.

(*c*) Scale of 1″ = 1 yard, long enough for 10 yards, showing yards and feet.

(*d*) Scale of $\frac{3}{4}''$ = 1 chain, long enough for 10 chains, showing chains and poles.

(e) Scale of 1 cm. = 10 cms., long enough for 1 metre, showing divisions of 10 cms. and c.metres (1 cm. = 0·39″).

EX. 16.—On a map 2·5 chains is represented by 1·5 inches. Draw a scale of feet for the map showing 500 feet, and divide it to show distances of 20 feet. What is the representative fraction of the scale?

EX. 17.—Construct a scale of $\frac{1}{20}$, long enough for 15 feet, showing feet and inches.

Diagonal Division and Diagonal Scales.—The number of equal parts into which it is possible to accurately divide a line by the methods previously described soon reaches a limit. It is, for example, difficult to show lengths of 1 inch on a scale where $\frac{1}{4}$ or $\frac{3}{8}$ inch = 1 foot, yet much smaller divisions than these are constantly required in scales for land measure, and on rules designed for measuring very small fractions of an inch such as $\frac{1}{100}$″ or $\frac{1}{200}$″.

The principle of diagonal division by which such small divisions may be accurately obtained, is as follows:—Suppose we require to show lengths of $\frac{1}{10}$ of the line A B (Fig. 12).

At one end, A, of the line draw a perpendicular of indefinite length, and mark along the perpendicular any ten equal lengths, starting from A, and ending at C. Join the last point to B, mark the points 1 . . . 9 as shown, and through the points 1 . . . 9, draw lines parallel to A B. Then all the small triangles, as C 3 F, C 5 E, and C 8 D are similar; and, therefore, since C 3 is $\frac{3}{10}$ of C A, so also is 3 F $\frac{3}{10}$ of A B, and so on with each of the triangles. Consequently the distance, 1 G, is 0·1 of A B, and if A B be 0·5″ long, then the length of 1 G = 0·05″ or $\frac{1}{20}$″.

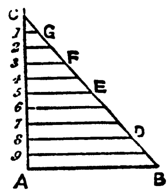


Fig. 12.

PROBLEM VIII. (Fig. 13).—*To construct a diagonal full size scale, 6″ long, to show inches, tenths of an inch, and hundredths of an inch.*

Draw the line A B 6″ long, and divide it into six equal parts to show inches, and divide the first of these divisions, A C, into ten equal parts to show $\frac{1}{10}$ inches. From the end, A, draw a perpendicular, and starting from A, mark off any ten equal lengths to the point E. Complete the oblong, E A B D, and through the points 1 . . . 9, draw lines parallel to A B, terminated by B D. From each of the points marking the inch division draw lines perpendicular to A B, and mark them 1″, 2″ . . . 5″, the division at C being 0. Join the points E . 9, and draw parallels to E . 9

through the points C. 1, 2, 3, . . . 8, in A C. The scale is now complete and can be used to measure any length between 0·01" and 6"; as, for example, the length between the two points, *x y*, is 4·26", and between the points, *m n*, is 3·14".

A scale of this construction is usually marked on one side of the common 6" boxwood protractor, and should form a part of every student's drawing outfit, as it is the only convenient and accurate method of measuring to the second place of decimals.

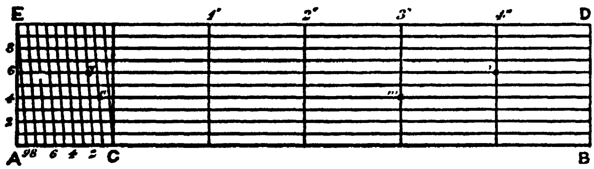


Fig. 13.

Diagonal scales, which are chiefly required for drawings of land and buildings, are constructed in this way. For example, if $1\frac{1}{4}'' = 1$ furlong, and a scale is required to show poles, the construction should be as follows:—(Fig. 13) Make A C $1\frac{1}{4}''$ long, and erect an indefinite perpendicular, A E, from the point A. Now as 10 chains = 1 furlong, and 4 poles = 1 chain, it will be best to divide A C into ten equal parts to show chains and to set four equal divisions up A E, and then finish as before.

Comparative Scales.—Comparative scales are those which enable different standards of length to be compared. Suppose a drawing is made in France to a scale of 2 cm. = 1 metre, or $\frac{1}{50}$ of full size, then it is convenient to be able to know what measure of yards and inches in the English standard corresponds to any given length on the French drawing, which has been drawn in metres and centimetres. To accomplish this a scale to English measure should be drawn having the same "*representative fraction.*" The usual method is to draw the English scale along one edge of the scale, and the French scale along the other edge. The comparison and conversion is then easily made.

EXAMPLES.

EX. 18.—Construct a full size diagonal scale 6" long, showing inches, $\frac{1}{10}$ inches, and $\frac{1}{100}$ inches.

EX. 19.—A distance of 11 miles 3 furlongs is shown on a map by $4\frac{1}{2}''$. Draw a scale for the map, showing furlongs by diagonal division. The scale to be long enough to measure 15 miles.

EX. 20.—A scale of yards and feet is drawn to a representative fraction of $\frac{1}{9}$. Construct a comparative scale of yards and metres, long enough for 2 metres, showing divisions of yards and feet, and of 10 centimetres.

SECTION IV.

CONSTRUCTION OF TRIANGLES— QUADRILATERALS—POLYGONS AND ELLIPSES.

THE working of the following problems will present no great difficulty to the student possessing a fair knowledge of Euclid's Elements, as they are simply a direct application of the principles of pure geometry. The questions on triangles and quadrilaterals are inserted because of their frequent occurrence in examination papers, and their general educational value, rather than for their practical use, which is somewhat limited, except in such work as plotting surveys of lands. The construction of the regular polygons and of ellipses is, however, of much importance, as polygonal and elliptical outlines are very common in engineering construction.

PROBLEM IX. (Fig. 14).—*To construct a triangle, knowing the perimeter, base, and one base angle.*

The perimeter of a figure is the total length of its sides or of its boundary. Thus the perimeter of a square of 2″ side is 8″, and of a circle is the length of its circumference.

Let the perimeter be $7\frac{1}{2}$″, base 3″, one base angle 45°. Draw the line A B 3″ long to represent the base and from the end, A, draw an indefinite line at 45° to A B. From this line cut off a part, A C, equal to the perimeter less the base—that is, $7\frac{1}{2}'' - 3 = 4\frac{1}{2}''$. Join C to B, and from B draw the line B D, cutting A C in D, so that the angle D B C = angle D C B. Then D A B is the required triangle.

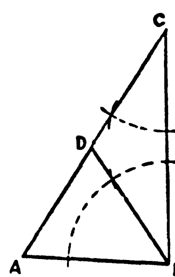


Fig. 14.

(B A + A C = given perimeter and D B = D C (*Euclid* I., 6), ∴ B A + A D + D B = given perimeter.)

PROBLEM X. (Fig. 15).—*To construct a triangle, knowing the base, altitude, and vertical angle.*

The altitude of a triangle is the perpendicular distance from the vertex to the base.

Let the base be 3½″, altitude 2½″, and vertical angle 40°.

Draw the line A B 3″ long to represent the base, bisect it at C, and draw a perpendicular, C D. Draw a line, E F, parallel to the base 2½″ away, then the vertex of the triangle must be in this line.

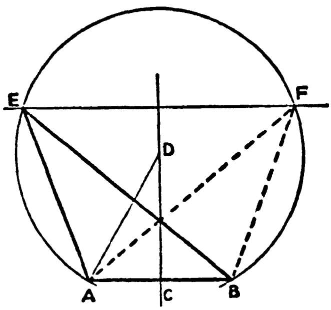


Fig. 15.

We now use the proof of *Euclid* iii., 20, which says "*the angle at the centre of a circle is double the angle at the circumference on the same base*," and we see that we ought to be able to draw a circle having A B for a chord, so that all angles contained in it shall be 40° (*Euclid* iii., 21), and to do this the angle at the centre must be 80°.

Therefore, from one end, A, of the base, draw a line making an angle with the base equal to one right angle less the required vertical angle—that is, 90° - 40° = 50°. Let this line cut C D in D. Then the angle A D C is 40°, and D is the centre of the required circle. With D as centre draw a circle passing through A and B, and cutting the line E F in E and F. Then either the triangle E A B or F A B is the required triangle.

EXAMPLES.

EX. 1.—Construct a triangle, sides 3¼″, 2⅜″, and 2″.

EX. 2.—Construct an isosceles triangle, base 2½″, vertical angle 40°.

(Find the measure of the base angles, knowing that the three angles equal two right angles.)

EX. 3.—Construct the following right angled triangles—(*a*) hypotenuse 5″, one side 2¼″; (*b*) hypotenuse 4½″, one acute angle 35°.

(The angle in a semicircle is a right angle, therefore draw a semicircle with the hypotenuse as diameter.)

EX. 4.—The distance from the centre of an equilateral triangle to the sides is 1¼″. Construct the triangle.

(Draw a circle of 1¼″ radius, and divide it into three equal parts, the sides of the triangle will be tangents to the circle through these points.)

EX. 5.—Construct a triangle, base 3½″, angles as 2 : 4 : 3.

(Draw a semicircle, and divide it into 2 + 4 + 3 = 9 equal parts, then joining the 3rd point from one end to centre, will give one angle, and the 2nd point will give another angle.)

EX. 6.—Construct a triangle with base, A B, 2¾″, angle, B A C = 50°, and side, B C, 2¼″. (S. & A. E., 1892.)

(Two triangles are possible, this being the ambiguous case of Trigonometry.)

EX. 7. Construct a triangle whose sides, *a b*, *b c*, *c a*, are 3¼″, 2⅜″, and 2″ respectively. On *a c* construct a second triangle *a d c*, whose vertical angle *a d c* is equal to the angle *a b c*, and the side *a d* 1⅝″. (S. & A. E., 1887.)

(The angles upon the same base and in the same segment of a circle are equal.)

EX. 8.—Two points, A B, are 3⅜ miles apart. Find the position of a point, P, so that P A is 1⅝ miles, and the angle A P B is 73°. (Scale 1″ = 1 mile.) (S. & A. E., 1886.)

(Use method of Prob. X.)

EX. 9.—Construct a triangle, perimeter 7″, one base angle 42°, altitude 2¼″.

(Adopt method of Prob. IX.)

EX. 10.—Construct an oblong, diagonal 6″, short sides 2¼″.

(Half the oblong is a right angled triangle, the diagonal being the hypotenuse.)

EX. 11.—Construct an oblong, diagonal 4½″, sides as 3 : 2.

(Draw any oblong sides as 3 . 2, and then a similar oblong having diagonals 4½″.)

EX. 12.—The line joining one corner of a square to the middle point of the opposite side is 4¼″. Draw the square.

(Draw any square and join one corner to centre of opposite side, make this line 4¼″ long, and draw a second square parallel to the first.)

EX. 13.—Draw a rhombus, longest diagonal 5½″, acute angles 50°.

(Diagonal is base of triangle of which each base angle is known.)

EX. 14.—Draw a quadrilateral A B C D, A B = 4¼″, angle A B C = 30°, B C = 5″, angle B C D = 95°, angle B A D = 110°

EX. 15.—The diagonals of a parallelogram are $2\frac{1}{2}''$ and $4\frac{1}{4}''$ long, they contain an angle of 61°. Construct the parallelogram.

(Half of each diagonal makes the sides of a triangle of which the vertical angle is known.)

Regular Polygons.—The geometrical construction of regular polygons depends upon Corollary I. of *Euclid* i., 32, which says that "*the interior angles of any straight lined figure together with four right angles are equal to twice as many right angles as the figure has sides.*"

The most common of the regular polygons used in engineering designs are the pentagon (five-sided), hexagon (six-sided), and octagon (eight-sided).

Pentagon.—Suppose we require to construct a pentagon of $2\frac{1}{4}''$ side. All the interior angles together with four right angles will equal $2 \times 5 \times 90 = 900°$, and, therefore, the interior angles will equal $900 - 360 = 540°$, and each interior angle will be $\frac{540}{5} = 108°$. Hence we could draw two lines $2\frac{1}{4}''$ long, meeting at an angle of 108°, and they would be two sides of the pentagon, and we could complete the pentagon by drawing other lines at 108° and $2\frac{1}{4}''$ long, until the figure closed. But this is a cumbersome method and would scarcely be accurate. The geometrical construction is as follows:—

PROBLEM XI. (Fig. 16).—*To construct a regular polygon on a given line.*

Let the polygon be a pentagon, and the given line be A B.

Produce A B, and with A, as centre, draw a semicircle of radius equal to the given line, A B.

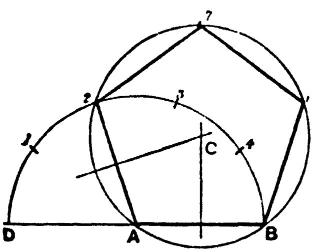


Fig. 16.

Divide this semicircle into the same number of equal parts as the sides in the required figure, in this case 5, and mark the points as shown. Join the centre, A, to the point, 2. Then A 2 is a second side of the pentagon. For A 2 = A B, and the angle 2 A B is $\frac{3}{5}$ of 180° = 108°, and hence, by always joining to the second point, counting from the opposite end of A B, we obtain for six divisions, $\frac{2}{3}$ of 180° = 120°, the angle of a hexagon; for nine divisions, $\frac{7}{9}$ of 180° = 140°, the angle of a nonagon.

Now a regular polygon can always be circumscribed by a circle, and hence, if we draw a circle containing the two sides, 2 A and A B, it will just contain the required polygon.

Draw the circle passing through 2 A B, having its centre at C (see Ex. 10, p. 7). With A B as distance start from B or 2, and step round the circle, marking the points 6 and 7, and complete the figure as shown.

It is difficult for beginners to finish these polygons accurately, the fault generally lies in a bad division of the semicircle, where a very small error makes a large difference in the result. This is the reason why polygons are seldom accurate when the angle 2 A B is set off with a protractor, as a slight error becomes multiplied as the polygon approaches completion.

Hexagon and Octagon.—Since the interior angles of the hexagon and octagon are respectively 120° and 135°, it is unnecessary in constructing either of these figures to divide the semicircle into either 6 or 8 equal parts. For the exterior angle ($\angle$ 2 A D of Fig. 16) is 60° for a hexagon and 45° for an octagon, it can, therefore, be easily found in the first case by taking the radius of the semicircle as distance and marking off from the point D to the point 2, thus making the angle 2 A D = 60°; and in the second case by bisecting the right angle, thus obtaining an angle of 45°.

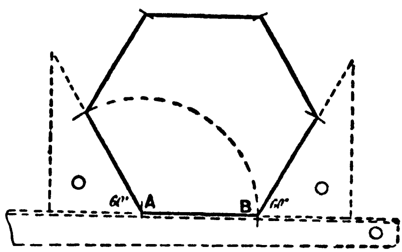


Fig. 17a.

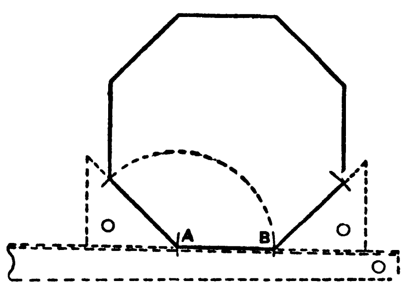


Fig. 17b.

The two set squares most commonly used are made with angles of 60° and 45°, and this enables a hexagon or an octagon to be very easily and accurately constructed, if the set squares are true. The method of using the set squares to draw these figures is shown in Fig. 17*a*, *b*.

In Fig. 17*a*, the hexagon is to be drawn on the line A B, and the set square is shown in position for drawing two other sides In Fig. 17*b*, the 45° set square is shown in position for obtaining two of the sloping sides of an octagon. It is easy to see how the figures may be completed in each case, and remembering that the hexagon is the most common of all plane figures drawn by the mechanical engineer, as nuts and bolt heads are hexagonal, it is very necessary for the student to know to what extent set squares may be used to assist in its construction.

EXAMPLES.

EX. 16.—Draw two lines 2″ long meeting at an angle of 108°, and consider them as two sides of a regular polygon. Complete the figure.

EX. 17.—Construct the following regular polygons:—(*a*) pentagon, 2″ side; (*b*) hexagon, 2″ side; (*c*) heptagon, 1·75″ side; (*d*) octagon, 1·5″ side; (*e*) nonagon, 1·5″ side.

EX. 18.—Draw a line A B, and take a point, P, outside it, 3¼″ away. Construct a pentagon to have one side in A B, and the opposite corner in P.

(Construct any pentagon, then copy this pentagon by parallels having its top corner in P, and mark the side opposite P, C D Join P through C and D to meet A B, the length they cut off on A B is the side of the required pentagon, then finish by drawing parallels.)

EX. 19.—Construct the regular polygon whose perimeter is 10″, and interior angles 135°.

(To do this by construction, a polygon of any length of side having interior angles of 135° should be drawn first, as this will tell the *number of sides*, then draw a similar polygon such that perimeter = 10″.)

The following should be drawn, using set squares :—

EX. 20.—Draw a hexagon 2″ side and an octagon 1¾″ side.

EX. 21.—Draw a hexagon and octagon, outside and inside circles of 3″ diameter.

EX. 22.—The longest diagonal of a hexagon is 4″ and of an octagon 5″. Draw the figures.

EX. 23.—The distance between the parallel side of a hexagon is 4¼″, and of an octagon 5″. Draw the figures.

PROBLEM XII (Fig. 18).—*To construct a regular polygon inside a given circle.*

Let the circle have the diameter, A B, and the required polygon be a heptagon. Divide the diameter of the given circle, A B, into the same number of equal parts as the sides in the required polygon; for a heptagon = 7, and mark as shown. With A and B, as centre, radius A B, describe arcs meeting in C. Join C through the point 2 to meet the circle on the other side of the diameter A B in the point D. Then the line A D is one side of the heptagon, and the figure can be completed by stepping off the length A D around the circle.

For this construction to be correct, in the case of a heptagon, the arc A D must evidently be $\frac{1}{7}$ of the circumference—that is, the semicircumference A B must be $3\frac{1}{2}$ times A D, or D B = $2\frac{1}{2}$ times D A. Now A C B is an equilateral triangle, therefore C A = C B, and the part A2 is $\frac{2}{7}$ of A B the diameter,

$$\therefore \frac{A2\,C}{B2\,C} = \frac{B2\,D}{A2\,D} = \frac{\text{arc }D\,B}{\text{arc }D\,A} = 2\tfrac{1}{2}.$$

The manner in which set squares may be used to draw a hexagon or octagon inside or outside a circle will be easily seen.

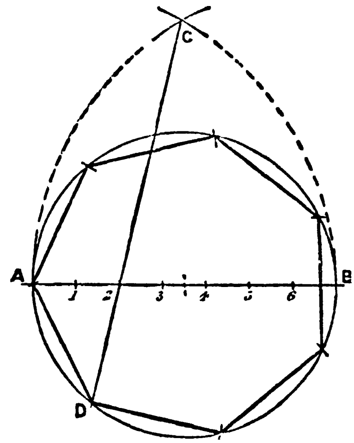


Fig. 18.

EXAMPLES.

EX. 24.—Construct an equilateral triangle, A B C, base A B, divide the base into 5 equal parts, and on the side of the base remote from C describe a semicircle. Draw a line from C through the second division point 2 counting from A, and meeting the semicircle in D. Measure the angles, B 2 D and A 2 D, and the chords, B D and A D, and show that

$$\frac{\angle B\,2\,D}{\angle A\,2\,D} = \frac{B\,D}{A\,D} = \frac{B2}{A2}$$

EX. 25.—Construct the following polygons in circles of 4″ diameter :—(*a*) pentagon, (*b*) hexagon, (*c*) heptagon, (*d*) octagon, (*e*) nonagon.

EX. 26.—Construct two hexagons having the same centre, length of sides $2\frac{1}{4}$″, the sides of one hexagon to make an angle of 30° with the sides of the other.

Construction of Ellipses.—The ellipse is the most common of a series of useful mathematical curves, often employed in architectural and engineering construction, many of which will be referred to in detail in Section VII. But its geometrical construction is given at this stage, because of its occurrence in other work, and the desirability that students should obtain an early knowledge of how it may be practically drawn.

The general method of construction adopted with all these curves, is to find a number of points through which it is known the curve must pass, and then to draw, by freehand or with the aid of "French curves," the curve passing through these points.

Arcs of circles cannot be used with any degree of accuracy.

The greater the number of points found, the more accurate the curve is likely to be, but the student should learn to exercise a wise discretion as to the exact number of points in particular cases.

As the curves are symmetrical, any error in drawing is easily detected.

The ellipse may be defined in many ways, but for the present we will take the following definition :—

"An ellipse is a closed curve traced out by a point moving in such a way, that the sum of its distances from two fixed points, called the foci, is always the same."

Thus, in Fig. 19, if F and F′ are the two fixed points or foci, and P the moving point, then if P moves so that at all times $PF + PF' =$ a constant, then the path of P is an ellipse.

Major Axis.—The line passing through the foci, and terminated by the curve, is called the "*major axis*" (A B in Fig. 19).

Minor Axis.—The line bisecting the major axis at right angles to it, and terminated by the curve, is called the "*minor axis*" (C D in Fig. 19).

The intersection of the axes is called the centre of the ellipse (O in Fig. 19).

Ordinates.—Lines parallel to the minor axis and terminated by the curve are called "*ordinates.*"

Since A and C (Fig. 19) are points in the ellipse, it follows that $AF + AF' = CF + CF'$; but $AF + AF' =$ major axis, A B; therefore, as C F and C F′ are equal, we have $CF =$ half major axis. Therefore,

The sum of the distances of any point in an ellipse from the foci is equal to the major axis.

The distance from either end of the minor axis, to either focus, is equal to half the major axis.

A circle may be regarded as an ellipse with its axes equal, and a straight line as an ellipse with its minor axis infinitely reduced.

There are several means of constructing an ellipse when the axes are known (or one axis and the foci, since the other axis is then easily found), the first of which is suggested by the above definition.

PROBLEM XIII. (Fig. 19).—*To construct an ellipse when the axes A B and C D are given.*

Method I. by "arcs of circles."—Find the foci, F and F′, by taking half the major axis as distance, either end of the minor

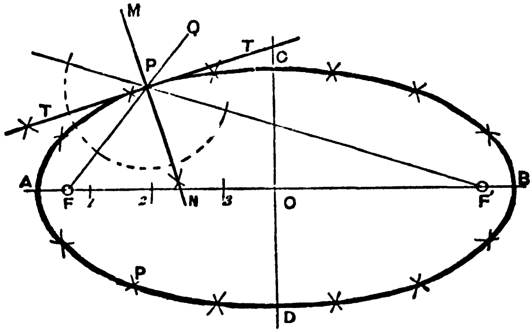


Fig. 19.

axis as centre, and cutting the major axis in these points. Mark any point 2 in the major axis A B, between the foci.

With A 2 as distance, F as centre, draw an arc.

With B 2 as distance, F′ as centre, draw an arc, cutting the first arc in P.

Then P F + P F′ = A 2 + B 2 = major axis; therefore P is a point in the ellipse.

In the same way by taking other points, 1, 3, &c., additional points can be found, and the curve drawn through them.

The points may be taken anywhere between the focus and centre, but are better when closest together nearest the focus.

Arcs can be drawn with the same radii on both sides of the major axis, and with both foci as centre, thus giving four points in the curve for each of the points, 1, 2, 3, . . .

This is a quick and accurate way of constructing an ellipse. It can also be applied for constructing other curves of a similar character, such as, for instance, where 2 P F + P F′ = a constant.

It is evident that the curve could be drawn mechanically. For let F P, P F′ be a continuous string, its ends being fixed at the foci. Then a pencil guided by the string, and keeping it tight, will describe an ellipse.

Method II. by "two circles" (Fig. 20).—Draw circles with centre, O, having the major and minor axes for diameter.

Take any point, 3, on the major axis circle, and join to the centre, cutting the minor axis circle in point 4.

Through the point (3) on the major axis circle, draw a line parallel to the minor axis.

Through the point (4) on the minor axis circle, draw a line parallel to the major axis, meeting the first line in *e*.

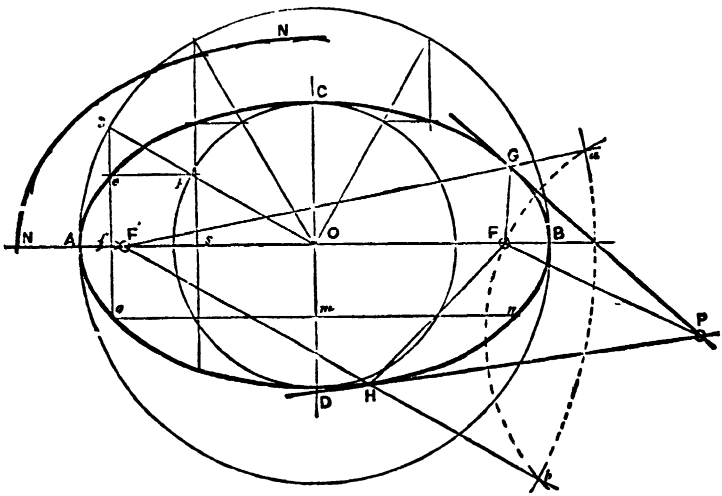


Fig. 20.

Then *e* is a point in the curve.

Repeating the construction with other points will enable the ellipse to be drawn.

Since the curve is symmetrical about its axes, points in it can be found, when one quarter has been constructed, by drawing ordinates from points in that quarter, and making them of equal length on both sides of the major axis. Thus, in the figure *e f* = *f g*, and similarly by drawing lines parallel to the major axis, so that *m n* = *g m*, the ellipse can be completed. It is better to draw one complete half of the ellipse before adopting this method.

Method III. by an "oblong" (Fig. 21).—Draw an oblong, A B C D, having the axes for diameters.

Divide half the major axis, A O, into any number; say, six equal parts and mark from A towards O, 1′, 2′, . . . 5′.

Divide the distance, A E (equals half minor axis), into the same number of equal parts, and mark from A towards E, 1, 2, 3, . . . 5. Join these points to the end, C, of the minor axis.

From the other end, D, of the minor axis, draw a line through point 3′ in A O, to meet the line C 3 in P.

Then P is a point in the curve, and by drawing other lines

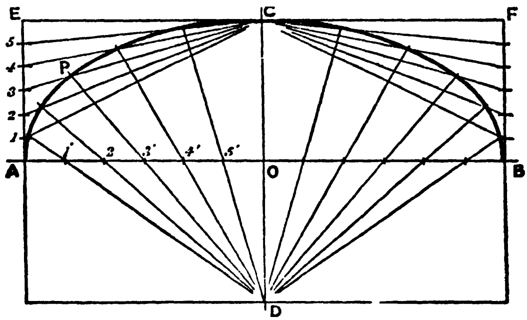


Fig. 21.

from D to meet the lines from C, the remaining points can be found.

(Note D 4′ meets C 4, D 5′ meets C 5, and so on.)

The ellipse is best completed by repeating this construction for the portion in C O B F, and then using the ordinate method, as described at the end of Method II.

Method IV. by "trammels" (Fig. 22).—A very convenient way

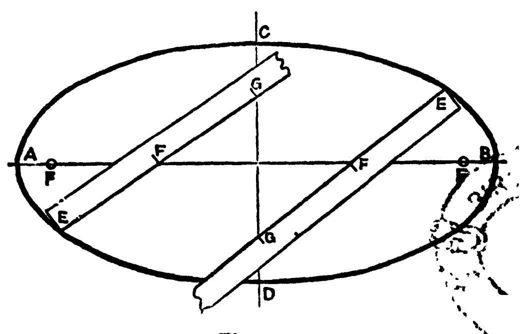


Fig. 22.

(known as the trammel method) of finding points in the curve of an ellipse, is as follows:—Mark off along the edge of a strip of paper, card, or wood, a distance, E F, equal to half the minor axis, and from the same end, a distance, E G, equal to half the major axis.

Place the strip in such a way that the minor axis point, F, is always on the major axis, and the major axis point, G, always on the minor axis, then the end, E, of the strip will be a point in the curve.

The strip is shown in position for two different points.

This is a common drawing office and workshop method, and as it readily gives any number of points, is very useful.

Tangents and Normals.—It is necessary to be able to draw the tangent and normal to an ellipse. A tangent to a curve is often of use as showing the direction of the curve. In an elliptical arch, the ends of the stones, or the radial members, would be normals to the curve.

"**The normal to an ellipse at any point in the curve bisects the angle between the lines joining that point to the foci. (These lines are called the focal distances.)**

"**The tangent at any point is at right angles to the normal at that point.**"

Thus to draw a normal and tangent at any point, P (Fig. 19), it is only necessary to join the point to the foci and bisect the angle F P F′ between the joining lines. This bisecting line M N is the normal, and a line at right angles, T T, is the tangent.

Or, produce F P to Q, then the tangent bisects the angle Q P′ F′, and the normal is perpendicular to it.

"**If two tangents are drawn to an ellipse from a point outside the curve, and the contact points are joined to a focus, then the angles between these lines and the line joining the focus to the point are equal.**"

Thus in (Fig. 20) the angles P F G and P F H are equal.

Hence to draw a tangent from a point outside the ellipse, it is necessary to adopt a construction making these angles equal. This can be done as follows :—

With the point P as centre, draw an arc passing through a focus, F. With the other focus, F′, as centre, and the major axis as distance, describe an arc, cutting the first arc in *a* and *b*.

Join *a* and *b* to the focus F′, cutting the curve in the points G and H. These are the contact points of the tangents from P.

In the triangles P *a* F′ and P *b* F′ the three similar sides are respectively equal, therefore the angles P F′ G and P F′ H are equal (*Euclid* i., 8), and, therefore, G and H must be the contact points of the tangents.

Parallels to an Ellipse.—A parallel to a curve is equidistant from it at all points. It has not necessarily the same mathematical properties as the curve to which it is parallel.

The curve used in constructing the arches of bridges is frequently a *parallel to an ellipse*, as this gives greater vertical clearance near the abutments than the true ellipse.

The parallel to an ellipse is most conveniently drawn by describing a large number of small radii of the required distance from points on the curve as centre; or it can be obtained by drawing a number of normals to the curve of the required length, and drawing the parallel curve through the ends of the normals.

The parallel curve, the curve, N N, to an ellipse, is shown partly constructed in Fig. 20. It is drawn touching the small arcs, described from points on the ellipse as centre, but might have been drawn by constructing a number of equal normals and drawing the curve through the ends.

A convenient practical way of finding whether a given curve is a true ellipse, is to draw lines representing the two axes, and mark the foci, *supposing the curve to be an ellipse.* Then measure the focal distances of a number of points and find if the sum is constant; or partly construct a true ellipse about the assumed axes and foci, when its difference from the given curve will show the error of the curve.

EXAMPLES.

EX 27.—Construct an ellipse, major axis 7″, minor axis 4″, by the following methods:—(*a*) "arcs of circles;" (*b*) "two circles;" (*c*) "oblong;" (*d*) "trammels." In each case draw a tangent and normal to the curve, from a point in the curve and from a point outside the curve.

EX. 28.—Work the following by drawing:—(*a*) major axis of an ellipse is 6″, minor axis 2½″, find the foci; (*b*) major axis 6½″, foci are 1″ from each end, find the minor axis; (*c*) minor axis is 3½″, foci are 4″ from the ends of the minor axis, find the major axis.

EX. 29.—Carefully draw an ellipse by two circles method, major axis 7″, minor axis 4″. Rub out all lines except the curve and the axis, and find the foci. Then take six different points in the curve and find the sum of the distances of each point from the foci.

EX. 30.—Construct a semi-ellipse, axis 4″ and 2″. Then draw a second parallel curve 2½″ away, and find if this curve is a true ellipse.

EX. 31.—Draw an ellipse, the distance between the foci being 2¼″, and the major axis 3″ long. (S. & A. E., 1891.)

EX. 32.—Two points, F and F′, 2″ apart, are the foci of, and P (2″ from F and ¾″ from F′) is a point on, an ellipse. Draw the curve. (S. & A. A., 1892.)

EX. 33.—Draw an ellipse inscribed in a parallelogram the sides of which are 4″ and 5″ long, and are inclined at 60° to one another. (V. U. Hon., 1889.)

SECTION V.

CIRCLES AND TANGENTS—AREAS—MISCELLANEOUS PROBLEMS.

The practical draughtsman does not usually adopt a geometrical construction to enable him to draw such circles and lines tangentially to each other as are required in ordinary mechanical drawings, since his own skill in draughtsmanship ensures sufficient accuracy. But instances often occur in solid geometry problems, as well as in mechanical drawings, where it is very advantageous, if not absolutely necessary, to be able to accurately determine the contact points of lines and circles, hence the student should render himself familiar with the chief geometrical constructions.

PROBLEM XIV. (Fig. 23).—*To draw a tangent to a circle, (a) from a point in the circumference, (b) from a point outside the circle.*

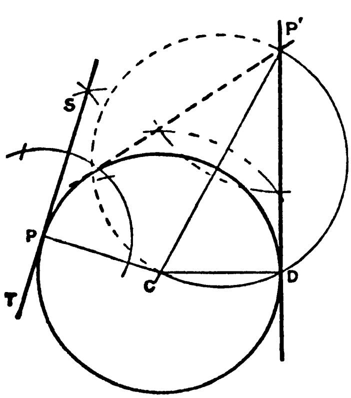


Fig. 23.

A tangent to a circle is a line which touches the circle, without cutting it. A tangent is at right angles to the radius passing through the point of contact.—*Euclid* iii., 18.

(*a*) Let P be the point in the circumference of the circle. Join P to the centre C, then the required tangent, S T, is the line drawn at right angles to the radius, P C.

(*b*) Let P′ be the point outside the circle. In order to draw the tangent correctly, we require to find its contact point, and knowing that the tangent is at right angles to the radius, we remember that "*the angle in a semicircle is a right angle.*" Therefore join the point P′ to the centre O, and on P′O describe a semicircle cutting the given circle in D. Then P′D is the required tangent, for it touches the circle at D, and is perpendicular to the radius, D O.

Notice that a second tangent can be drawn from P′ to touch the circle, as shown in the figure by dotted lines. It is easy to see that "*the two tangents are equal in length,*" a fact which should be remembered.

PROBLEM XV. (Fig. 24).—*In a given angle to inscribe a circle of given radius, and also to inscribe a second circle tangent to the first circle and to the angle.*

Let B A C be the given angle. The circle must evidently have its centre on the line bisecting the angle, therefore first bisect the angle B A C by the line A D.

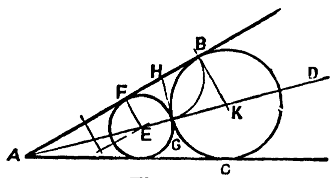


Fig. 24.

Draw a line parallel to A B, at a distance from it equal to the radius of the required circle cutting A D in E. (This is best done by first drawing a perpendicular to A B from any point in it, and making its length equal to the given radius.) E is the centre of the circle, and in order to accurately draw the circle, it is best to first draw a line from E perpendicular to A B or A C, to obtain the point of contact, F.

Next to draw a second circle touching the first and the sides of the angle. Draw the line G H from the point G, where the circle cuts A D, and perpendicular to A D. Then H G and H B will both be tangents to the required circle when it is drawn; therefore, if H B is made equal to H G, the point B so found will be the contact point of the required circle. A perpendicular, B K to A B, through the point B, will cut A D in K, which will be the centre of the required second circle.

PROBLEM XVI. (Fig. 25).—*To draw three circles of given radius in contact with each other.* (The method of this construction is useful in problems on spheres in contact.)

Let the circles be of 2″, 1½″, and 1″ radius.

Draw any straight line and draw any two of the circles (say of 2″ and 1½″ radius) touching each other, having their centres at A and B in the line.

Set off the radius of the third circle along the line, beyond the two circles, A and B to C and D, as shown. With A as centre, draw an arc passing through C, and with B as centre, draw an arc passing through D, meeting the first arc in E. Then E is centre of the third circle tangent to A and B, the contact points being found by joining E to A and B.

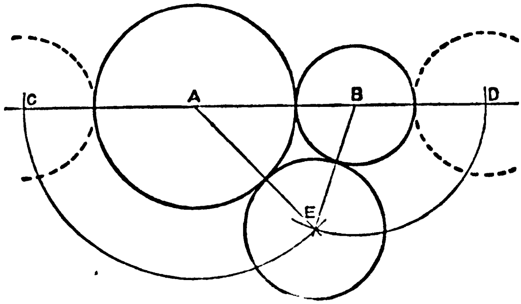


Fig. 25.

PROBLEM XVII. (Fig. 26).—*To draw all the tangents common to two circles of unequal size.*

(This problem is practically useful as representing the conditions of open and crossed driving belts connecting two pulleys.)

Let the circles have their centres at A and B, the larger circle having the centre, A.

Mark the radius of the smaller circle, B, inside the larger circle, A, as from the point C in the circumference to the point D

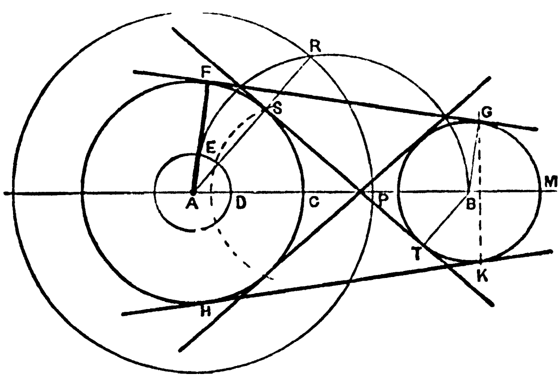


Fig. 26.

Then A D is the difference between the radii of the two circles. Describe a circle of radius, A D, centre A, and draw a semicircle on A B, cutting this circle in E. Join A E and produce to meet circle A at F, and from B draw a line parallel to A F, cutting circle B at G. Then F and G are the contact points of the common tangent to the two circles. The similar tangent, H K, on the opposite side is easily drawn by making arc C H = O F, and arc M K = M G.

The principle of this construction is that both the circles, A and B, have been equally reduced in size, the smaller, B, to a point, and the larger, A, to the circle A D E. Then the tangent from the point to the circle (tangent B E not drawn) is evidently parallel to the required tangent, F G.

A similar construction applies for the cross tangents, except that the larger circle, A, is to increase by an amount equal to the radius of the smaller circle, B, while the smaller circle, B, is reduced as before to a point. The dotted circle of radius, A P (O P = radius of circle B), cuts the semicircle on A B in R. Join A R, cutting the circle in S, and draw B T parallel to it. Then S and T are the contact points of one of the cross tangents, which tangent is evidently parallel to B R. The drawing of the other cross tangent will present no difficulty.

PROBLEM XVIII. (Fig. 27).—*To draw a continuous curve made up of circular arcs through a number of given points.*

Let the points be as marked, 1 . . . 7, the curve to be drawn through the points in the order of figuring.

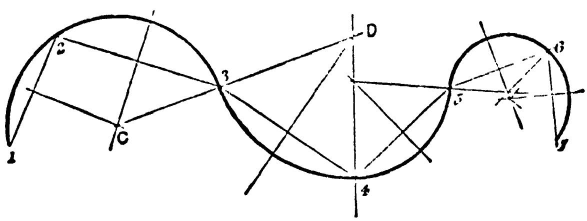


Fig. 27.

Draw the arc passing through the first three points, 1, 2, 3 (see Ex. 10, p. 7), the point C being the centre. The arc passing through the points 3 and 4 is to touch this arc at the point 3, and, therefore, the line joining C 3 must pass through the centre of the required circle. (*Euclid* iii., 11, 12. *When two circles touch, the line joining their centres passes through the point of contact*) Draw the line, O 3, and produce.

Next join the points 3, 4, and bisect the line 3, 4 by a perpendicular line cutting the line through O 3 in D. Then D is the centre of the arc passing through the points 3, 4.

Proceed in the same way for the remaining arcs, taking two points at a time. The construction for the centre of the arc through 4 and 5 is shown, the first step being to join the points 4 and D.

PROBLEM XIX. (Fig. 28).—*To inscribe a circle in a quadrilateral having two pairs of equal sides.*

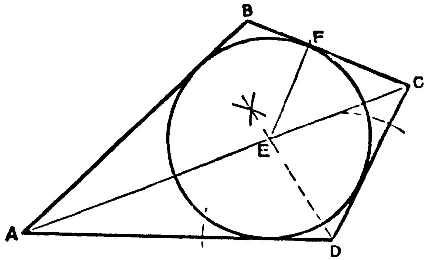


Fig. 28.

Let A B C D be the given figure, having A B = A D and B C = D C.

The line A C will bisect the angles B A D and B C D, and the centre of the required circle will, therefore, be upon this line.

To find the centre bisect either of the angles, A B C, by a line cutting A C in E. Then E is the centre of the circle touching the four sides of the figure, and its radius is best found by drawing a perpendicular, E F, from E, to either of the sides, as to B C.

PROBLEM XX. (Fig. 29).—*To describe a number of equal circles outside a given circle, each touching two of the circles and the given circle.*

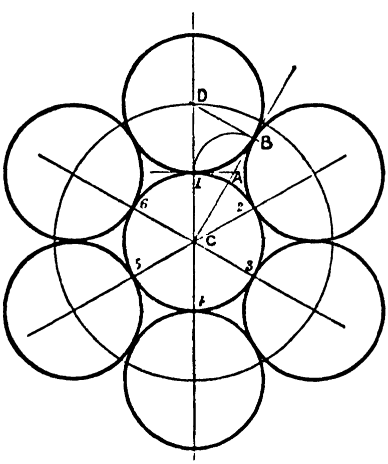


Fig. 29.

Let the given circle have the centre C, and let six equal circles be required.

Divide the circle into six equal parts, and draw radial lines as shown. Draw the line C B, bisecting the angle between C 1 and C 2. Then if the centres of the two circles are on the lines C 1 and C 2, the line C B will be tangent to both circles. Draw a line through point 1 perpendicular to C 1, meeting C B in A. Then A 1 and A B are tangents to the same circle, and must, therefore, be equal. Make A B equal to A 1 and B will be the contact point of the required circle with the line C B. Draw

B D perpendicular to C B, to meet C 1 in D. Then D is the required centre. The centres for the other circles are best found by drawing a circle centre C, radius, C D, to cut the radial lines.

EXAMPLES.

The following examples should be worked by geometrical constructions, and not guess work :—

EX. 1.—Draw a circle passing any three points, *a*, *b*, *c*, not in the same straight line.

EX. 2.—Draw an isosceles triangle, base 2¼″, height ¾″, and draw a circle passing through the three corners.

EX. 3.—Draw a circle 4″ diameter, and from a point in the circumference, and a point 1½″ away, draw tangents to the circle.

EX. 4.—Draw two lines of indefinite length, meeting at 30°, and draw two circles each touching each other and the two lines, the smaller circle to be ¾″ radius.

EX. 5.—Draw three circles of radii, 1″, 1¾″, and 2¼″, in contact with each other.

EX. 6.—Draw all the tangents common to the following circles :—(*a*) Circles, diameters 3¾″ and 2½″, centres 2¼″ apart; (*b*) circles 3½″ and 2¼″ diameters, centres 6″ apart.

EX. 7.—Mark any nine points, 1, 2, 3, 4, 5, 6, 7, 8, 9, arranged irregularly, as in Fig. 27, and draw a continuous curve passing through the points in the order 1 . . . 9.

EX. 8.—Draw a quadrilateral having two pairs of equal sides, long sides 4″, included angle 35°, short sides 1¾″, and inscribe a circle within the figure.

EX. 9.—Draw two equilateral triangles, sides 4″ long, in one describe three equal circles, each touching two sides and two of the circles, and in the other three equal circles, each touching one side and two circles.

(In the first case the triangle should be divided into three equal quadrilaterals having two pairs of equal sides, and in the second into three equal triangles.)

EX. 10.—Draw two parallel lines 3″ apart, and take any point, P, between them. Draw a circle to touch each of the lines and to pass through P.

EX. 11.—Describe a circle, A, of 1¼″ diameter, touching internally a circle, B, of 3½″ diameter, Describe a circle of 2″ diameter, touching both circles, A and B, the latter internally. (S. & A. E., 1892.)

EX. 12.—Draw a tangent to a circle from a point in the circumference, without using the centre of the circle.

EX. 13.—Describe a circle of ¾″ radius, and about it describe five equal circles, each touching two others and the original circle. (Woods & Forest, 1885.)

EX. 14.—Draw a line, A B, of indefinite length, and describe a circle of $\frac{1}{2}''$ radius touching it at A. Describe a second circle of $1''$ radius touching A B, and the first circle externally, and a third circle of $1\frac{1}{4}''$ radius touching A B, and the second circle externally. All three circles to be on the same side of A B. (Sandhurst, 1886.)

EX. 15.—Describe a circle in a quadrant of $2\frac{1}{2}''$ radius. In the circle describe an equilateral triangle. (Woolwich, 1881).

EX. 16.—Describe a series of circles, diameters $1''$, $1\frac{1}{4}''$, $1\frac{1}{2}''$, $1\frac{3}{4}''$, touching each other successively, and all touching a given line. (Woolwich, 1878.)

EX. 17.—Draw a circle $5''$ diameter, and in it describe four equal circles, each touching the first circle and two others.

Areas of Plane Figures.—The following typical examples of practical geometry problems on areas are inserted because of their educational value and general usefulness. It is often necessary to divide the plans of fields by fences into certain definite parts, or to compare the areas of irregular figures, or to construct certain figures of a required area, and as these all admit of an easy and accurate solution with the help of the drawing board, it is advisable for the student to become familiar with the methods employed

PROBLEM XXI.—*To construct an oblong equal in area to a given triangle.*

Construct the oblong on the same base as the triangle, and half the height, or on half the base and the same height. Then the oblong will contain an area equal to that of the triangle. (*Euclid* i., 41.)

PROBLEM XXII. (Fig. 30).—*To construct a square equal in area to a given oblong.*

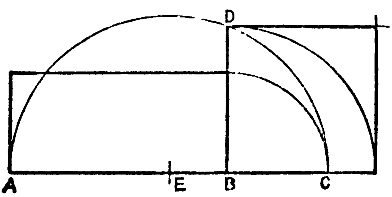


Fig. 30.

(If two lines, A C, D E, cut one another at right angles in a circle at a point B, the lines being terminated by the circumference, then the rectangle, A B, B C, made up of the segments of one is equal to the rectangle, D B, B E, made up of the segments of the other.—*Euclid* iii., 35.) But if one of the two lines, A C, passes through the centre of the circle, then it will bisect the other line, D E, and thus make the segments, D B, B E, equal to one another, and the rectangle becomes a square on D B or B E. Therefore, D B or B E is the side of the square equal in area to the rectangle, having sides equal to A B and B C. This construction is adopted for the present problem, although it is obvious that only half the circle need be drawn. Notice that the side of the required square is the "*mean proportional*" between the sides of the oblong—that is, $DB = \sqrt{AB \cdot BC}$.

Let the oblong have the sides, A B, B C. Draw the two sides end for end, making a continuous line, A B C, as shown. Bisect A C, the sum of the sides in the point E, and draw a semicircle, A D C. At B, the point where the two sides of the oblong join, draw a line, B D, perpendicular to A C, meeting the circle in D. Then the line B D is the side of the square, equal in area to the rectangle, A B, B C.

Notice that if any two of the three lines, A B, B C, or B D are known, the third can be found; hence, if one side of an oblong is known and the side of its equal square, the oblong can be constructed.

EXAMPLES.

EX. 18.—Draw any irregular triangle, no side less than 2½″, and construct a square equal to it in area.

(First reduce to an equal oblong and afterwards to a square.)

EX. 19.—Compare the areas of two triangles, each of 7″ perimeter, one equilateral, the other isosceles with a base 1½″ long, by drawing lines to represent the side of a square equal in area to each.

EX. 20.—A square of 3″ side is equal in area to an oblong, one side of which is 2½″ long. Construct the oblong.

PROBLEM XXIII. (Fig. 31).—*To reduce a given irregular figure to a triangle of equal area.*

The principle of this reduction depends upon the fact that triangles upon the same base and between the same parallels are equal (*Euclid*, i., 37), and the method consists of converting certain triangles, obtained from the figure, into other equal triangles, having one side in the same line as a side of the given figure—that is, parts are cut off the figure in some places while equal parts are added on in other places.

Let A B C D E F be the given figure. Produce A F to be the base of the required triangle. Then starting at the point A, join A C, and through B draw a parallel, B G, meeting F A in G. Join C G. Then A B C and A G C are equal triangles, being on the same base, A C, and between the same parallels, A C, B G, but the part H A C is common to both, therefore the part B H C (taken off the figure) is equal to the part G H A (added to the figure), and thus the five-sided figure, G C D E F, is equal to the six-sided figure, A B C D E F. Proceed in the same way, by joining D F, and drawing a parallel through E to K and joining D K, the figure has now only four sides. The construction is shown for the other sides, C D, D K, but notice that as D E F is a "*re-entrant angle*," the joining of D and F adds on a triangle to the original figure, the result is that the point K falls within the point F. Then the triangle C G M is equal to the figure A B C D E F.

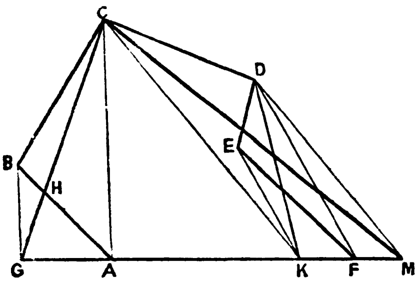


Fig 31.

PROBLEM XXIV. (Fig. 32)—*To construct a triangle equal in area to the sum or difference of two given triangles.*

Let A B C and D E F be the two given triangles. Draw a line, A G, in the triangle A B C perpendicular to B C to give

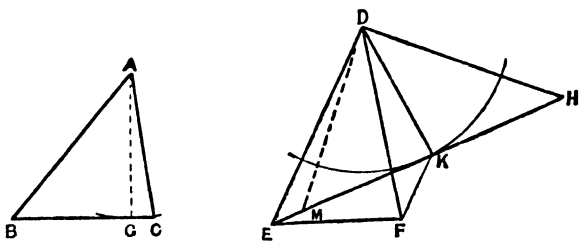


Fig. 32.

the altitude of the triangle. With D as centre radius, A G, draw an arc, and draw a line, E H, from E tangent to this arc, as shown Through F draw a line parallel to D E, meeting the line E H at K. Join D K. Then the triangle D E K is equal to the triangle D E F, because it is on the same base, D E, and between the same parallels, D E and F K, and its altitude is equal to the radius of the arc, which was made equal to the altitude of the triangle A B C. The triangles A B C and D E K can, therefore, be added together. Produce E K so that

K H = B C and join H to D, then the triangle D E H is equal to the sum of the two given triangles.

Similarly the triangle D E M is equal to the difference of the two triangles; K M being equal to B C.

Another way of working this problem is to draw the two triangles together, forming an irregular polygon, and reducing this to an equal triangle, as in Prob. xxiii.

It is clear from the above that the areas of different figures can be compared either by reducing each to an oblong, and finding the side of the square equal to each figure, or by reducing the figures to triangles, each having the same altitude, when the bases of the triangle will represent the area of the figures. These methods of geometrically comparing areas should be remembered.

EXAMPLES.

EX. 21—Draw an irregular five-sided and an irregular nine-sided figure, no side less than 1½", and reduce each figure to a triangle of equal area

EX. 22.—Construct triangles equal in area to the sum, and the difference, of the areas of an isosceles triangle base 4½", height 2", and an equilateral triangle of 3" side.

EX. 23.—Draw an irregular six-sided figure, no side less than 1¼", and anywhere inside it, draw an irregular four-sided figure, no side greater than 1". Then reduce the space between the two figures to a triangle of equal area.

EX. 24.—Construct the following figures, each having a perimeter of 9":—Equilateral triangle; isosceles triangle, base 2"; irregular triangle; square; oblong, short sides 1½"; irregular quadrilateral; pentagon; irregular five-sided figure; hexagon; irregular six-sided figure. Reduce each to a triangle of the same altitude, and draw lines the length of which shall represent the areas of the several figures.

(This exercise should show that with figures of equal perimeter, the greatest area is contained by the figure having the greatest number of equal sides.)

EX. 25.—Draw a triangle, sides, A B, 4", B C, 3", and C A, 3½". Then construct a second triangle, A B D, equal in area to A B C, having one side in A B, and the angle A B D twice the angle A B C.

EX. 26.—Draw an equilateral triangle 3" side, and construct a second triangle equal to it in area, having two of its sides 2¾" and 2½".

(First draw a triangle equal to the equilateral triangle, having one side 2¾", and then draw a triangle equal to this, having one side 2½", and the 2¾" side common.)

Division of Areas.—The geometrical division of plane areas generally depends upon the practical application of two of Euclid's Theorems, the first being that of equal triangles already referred to, and the second that "*the areas of similar figures are to one another as the squares on their similar sides*" (*Euclid* vi., 20).

PROBLEM XXV (Fig. 33).—*To divide a triangle into a number of equal parts by lines drawn through a given point in one of its sides.*

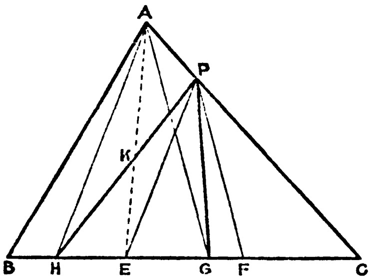


Fig. 33.

Let A B C be the triangle, and P the given point, and let the triangle require dividing into three equal parts.

Divide the base, B C, into three equal parts at the points E F. Join F to P, and through A draw a line A G parallel to F P, meeting the base, B C, in G. Join P G, then P G C is one-third of the whole triangle. Repeat the construction for the second division.

(If A E be joined, then A E B is one-third of the triangle, but the triangles H A E and H A P are equal, and have the part H A K common, therefore K H E, the part added on, is equal to K A P, the part taken away.)

PROBLEM XXVI. (Fig. 34).—*To divide a given irregular quadrilateral into two or more equal parts by lines drawn from one corner.*

Let A B C D be the given figure, and let it be required to divide it into two equal parts by lines drawn from A.

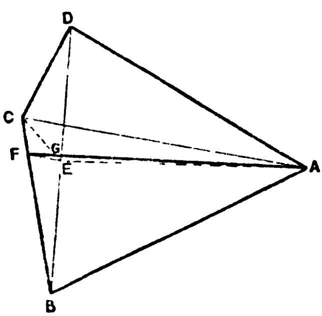


Fig. 34.

Join D B, and bisect it at E, and join E A, E C. Then the lines A E, E C divide the figure into two equal parts, for triangle D A E equals triangle B A E, also triangle D C E equals triangle B C E. Join A C, and through the point E draw E F parallel to A C, meeting B C in F. Join A F. Then A F divides the figure into two equal parts. For the triangles E C A and F C A are equal, being on same base and between same parallels, and the part G C A is common, therefore the triangle G C F added on is equal to the triangle G A E taken away.

By repeating the construction for the quadrilateral, A F C D, the figure can be further divided. The triangle A F B is divided by simply joining the point A to the division points on F B.

PROBLEM XXVII. (Fig. 35).—*To divide a triangle into two or more equal parts by lines parallel to one side.*

Let A B C be the triangle to be divided into two equal parts by lines parallel to B C.

On either of the other sides as A B, construct a semicircle and bisect it at D. With A as the centre, radius A D, draw an arc cutting the side A B in E, then the line E F parallel to B C divides the triangle into two equal parts.

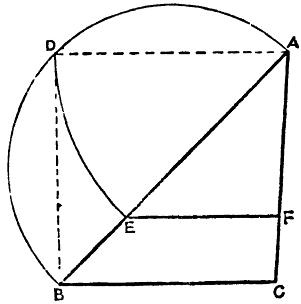


Fig. 35.

(For D A = D B, and angle B D A is a right angle, therefore square on A B equals twice the square on A D (*Euclid* i., 47), but A E (which equals A D) and A B are similar sides of similar triangles, and their areas are as the squares on these sides—that is, the triangle A E F is half the whole triangle A B C.)

EXAMPLES.

EX. 27.—Construct a square of $2\frac{1}{4}''$ side, and through one corner draw a line cutting off one-third of its area. (S. and A. E., 1886.)

EX. 28.—Construct a triangle A B C. A B = $2\frac{1}{4}''$, B C = $1\frac{3}{4}''$, C A = $2\frac{1}{2}''$. Mark a point D on the side A B, $1\frac{5}{8}''$ from A, and through D draw a line dividing the triangle into two equal parts. (S. and A. E., 1891.)

EX. 29.—The four sides of a quadrilateral, A B C D, are as follows:—A B = 4·5″, B C = 3″, C D = 4″, D A = 5″, and the diagonal A C = 6″. Draw the figure and divide it into five equal parts by lines drawn from A. (Woolwich, 1885.)

EX. 30.—Describe a circle of 2″ radius, and a second circle of two-thirds its area. (Woolwich, 1884.)

(Areas of circles are to one another as the squares on their diameters.)

EX. 31.—Construct an oblong, sides 4″ and $1\frac{1}{2}''$, and through the middle point of one long side draw lines dividing the oblong into three parts of equal area.

EX. 32.—Construct a triangle, sides $4\frac{1}{2}''$, 3″, $3\frac{1}{2}''$, and divide it into three parts of equal area by lines parallel to the shortest side.

EX. 33.—Draw any irregular six-sided figure, and divide it into five parts of equal area by lines drawn from the top corner.

(Reduce the figure to an equal triangle, vertex at the top corner, then divide base of triangle into seven equal parts, and join to vertex, transfer the points to the figure that falls outside it, by the principle of similar triangles.)

Miscellaneous Problems.—The following problems are inserted owing to their frequent occurrence in engineering drawing, and the value of the methods employed in admitting of extension to similar questions. It is not at all uncommon to require the meeting points of lines which meet off the paper, or to draw an arc of a circle when the centre is inaccessible.

PROBLEM XXVIII. (Fig. 36)—*To draw an arc of a circle through three given points, when the centre is inaccessible*

Let A B C be the three given points. Join A C, A B, and B C. With A and C as centres, and radius A C, draw arcs as shown, and produce A B and C B to meet the arcs in D and E respectively. Mark a number of small distances, E 1, 1 2, 2 3, &c., along the arc, *below* E, and equal distances, D 1′, 1′ 2′, 2′ 3′, &c., *above* D.

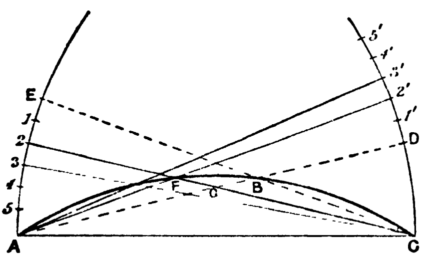


Fig. 36.

Join A to D 2′, and C to E 2 by lines cutting at the point F. Then F is a point in the required arc. Continue the construction for finding other points between A and B, and for points between B and C in a similar way by taking equal distances *below* D and *above* E.

(This method depends upon the fact that all angles in the same segment of a circle are equal (*Euclid* iii., 21). Since E 2 and D 2′ are equal arcs of equal circles, the angles D A 2′ and E C 2 are equal (*Euclid* iii., 27), also the angles A G F and C G B are equal, being opposite angles (*Euclid* i., 15), and, therefore, the third angle in each triangle, the angles A F C and A B C are equal.)

EXAMPLES.

EX. 34.—Draw a tangent to a circle from a point in the circumference without using the centre of the circle.

EX. 35.—Three points, A B C, are on the circumference of a circle, and are thus situated, A B = 3·25″, B C = 2·75″, A C = 5·5″. Draw the arc of the circle passing through the points without using the centre. (Woods and Forest, 1886).

EX. 36.—Draw an arc of a circle of large radius, and then find points in continuation of the arc in one direction without using the centre of the circle. In the same circle, equal angles are contained by equal segments.

(On the given arc draw any chord, A B, and draw an angle, A B C, in the segment. At any other point, D, in the arc, set off the length A C to E, and then set off the angle A C B at E, and the angle C A B at D, these two lines will meet at F, which will be a point in the arc, for the triangle D E F will be equal to triangle C A B.)

PROBLEM XXIX.—*To bisect the angle between two straight lines without using their meeting points.*

Draw a parallel to each line inside the angle at an equal distance from each. These two parallels will meet at a point if drawn at a sufficient distance from the given lines, and will make an angle equal to that between the given lines. Bisect this angle in the usual way, and the line will bisect the given angle.

EXAMPLES.

EX. 37.—Draw any two lines inclined to each other but not meeting, and bisect the angle between them, assuming the meeting point to be inaccessible.

EX. 38.—Draw any two lines at an angle to each other but not meeting, and find a point between them, such that its distance from the two lines shall be as 3 : 5, also a point outside the lines so that its distance shall be as 1 : 4.

PROBLEM XXX. (Fig. 37).—*To draw a line through a given point which shall meet at the intersection of two given inclined lines, when that intersection point is inaccessible.*

These conditions frequently occur in graphical solutions of roof stresses, when an inclined wind pressure is taken into account. In such a case the inclined resultant, R, of the downward forces is known, and the reaction at P which is vertical, that end being on rollers, also the point, Q, through which the other reaction must pass. Then, knowing that the directions of the three forces, P R Q, must pass through the same point, it is required to find the direction of the reaction Q.

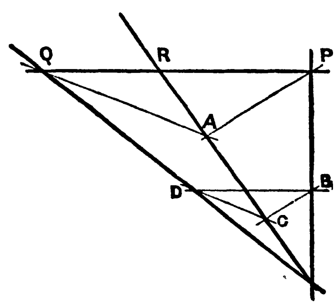


Fig. 37.

Let Q be the given point, and P B and R A the given lines.

Draw any triangle, P A Q, starting from the given point Q, and having angular points at A and Q

Then start from any other point, B, on the line P B, and draw a similar triangle, B C D, so that B D is parallel to P Q, B C to P A, and C D to A Q; thus, obtaining the point D, then the point D is in the line joining Q to the intersection of P and R. and the line Q D can therefore be drawn.

EXAMPLES.

EX. 39.—Draw any two lines inclined to each other but not meeting, and mark a point, P, outside and a point, Q, inside the lines. Then draw lines through P and Q to meet in the intersection of the given lines, without using that intersection.

EX. 40.—Draw any two lines approaching each other but not meeting, and mark a point, P, between them. Then draw a circle to pass through P, and to touch the two given lines, without using their meeting point.

(First draw the line bisecting the angle, then draw any circle not passing through P, and touching the lines; next draw the line passing through P, to meet at the intersection of the lines cutting the circle in A. Join A to the centre of the circle, and through P draw a parallel to this line, meeting the bisecting line of the angle in a point which is the centre of the required circle.)

SECTION VI.

GENERAL PROBLEMS ON LOCI—PATHS OF POINTS IN LINKWORK—CONSTRUCTION OF CAMS.

THIS section deals with what is perhaps the most important part of practical plane geometry, when regarded as a part of engineering drawing. There are very few mechanical engineers who do not frequently require to trace out the paths of certain points in mechanisms, such as engine valve gears and straight line or parallel motions, or to determine the form of grooved or curved plates called "cams," such that a certain desired motion may be produced. The geometrical constructions employed consists of finding a number of points in the particular path, or in the curve of the cam, and then drawing as smooth a freehand curve as possible through the points, and the student should remember that the number of points found is entirely a matter for individual decision, and should be settled by a consideration of each particular case. As a general rule, there are certain important parts of the curves, such as where the directions change, where it is advisable to find more points than at other parts. The first set of the following problems on lines and circles deals with conditions which cannot be met by the ordinary methods of constructive geometry, and should, therefore, be regarded as important.

It is customary to speak of the "*locus*" of a point rather than of the "*path*" of the point, locus being a mathematical term for the path in which any given point travels; as, for example, the locus of the centre of the crank pin of an ordinary engine is a circle, while the locus of a point on the piston is a straight line.

PROBLEM XXXI. (Fig. 38*a*).—*To draw the locus of the centres of circles, touching a given circle, and passing through a given point—that is, to draw a curve, every point on which shall be equidistant from the circumference of the given circle and from the given point.*

Let A be the given circle, and P the given point outside it. Join the point P to the centre of the circle by a line cutting the circle in B, and bisect the distance, B P, in C. Then C is evidently one point in the locus. Mark any point, 2, between C and B, and a point, 2′, between C and P, so that C 2 = C 2′. With P as centre, and radius, P 2, draw an arc, and with A as centre and radius, A 2′, draw a second arc cutting the first arc above and below the line A P at the points D and E. Then, as P 2 = B 2′, the points D and E are evidently points in the required locus. Other points in the curve are found by marking other equal distances, as C 3 and C 3′, on both sides of C, and proceeding as before. In drawing any circle from a point in the curve touching the given circle, and passing through the point P, the point should be joined to the centre of the circle and to P, to give the points of contact.

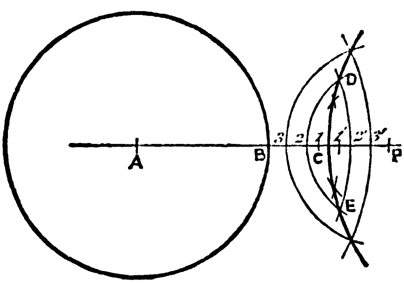


Fig. 38*a*.

Notice that it is only necessary to draw the arcs at about where they intersect, also that it is better to take the first one or two points very near to C.

PROBLEM XXXII. (Fig. 38*b*).—*To find the centre of the circles touching a given line and two given circles—that is, to find a point which shall be equidistant from the line and circles.*

Let the circles have the centres A and B, and let C D be the given line. It is necessary first to find a curve equidistant from the circumference of the two circles, and then a second curve equidistant from one of the circles and the given line. These two curves will intersect in a point, which will be equidistant from the line and the circles, and will, therefore, be the centre of the required circle.

First find points in the curve equidistant from the two circles, exactly as in the last problem, by drawing arcs centres A and B through the points on A B as shown. *Notice that when the two circles are of equal diameter, the curve is a straight line, and that when unequal, the curve bends towards the smaller circle. A point is a circle of indefinitely small radius.*

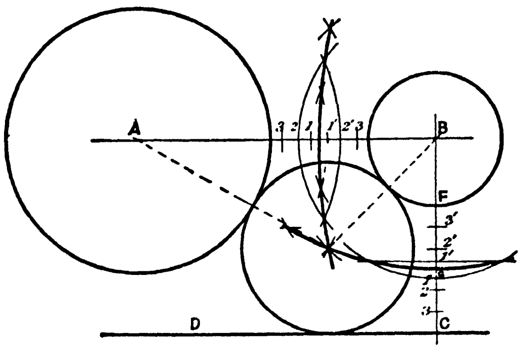


Fig. 38*b*.

Next draw the curve equidistant from the circle B and the given line C D. Draw a perpendicular, B C, from the centre of the circle to the line, and bisect the distance C F in G. Then G is evidently one point in the curve. Mark equal distances on either side and proceed as before, noticing that the points required are the intersections of *straight lines* through 1′, 2′, 3′, parallel to the given line, and *circles* through the points 1, 2, 3. drawn from the centre of the given circle.

EXAMPLES.

EX. 1.—Draw a straight line of indefinite length, and at any point, C, in it draw a perpendicular, C D, 1½″ long. Then draw a curve, such that all points on it shall be equidistant from the line and the point D.

(It will be seen later on that this curve is the mathematical curve known as a Parabola.)

EX. 2.—Describe a circle of 3″ diameter, and mark a point, P, outside the circumference 1″ away, and a point, Q, inside the circumference ¾″ away. Then draw curves equidistant from point P and the circle, and point Q and the circle.

EX. 3.—Draw two circles of 1½″ and ¾″ radius, centres 3″ apart, and a line parallel to the line joining the centres of the circles 2½″ away from the centre of the larger circle. Then find the centre of the circle touching the given circles and the given line.

EX. 4.—Draw any three circles of unequal diameters not touching or cutting, and find the centre of the circle touching all three circles externally.

EX. 5.—Draw any three circles of unequal diameters not touching or cutting, and find the centre of the circle touching all three circles and including them.

PROBLEM XXXIII. (Fig. 39).—*A pendulum of a given length swings uniformly through a given angle. A point uniformly descends the pendulum from the top to the bottom during one complete swing. Trace the locus of the point.*

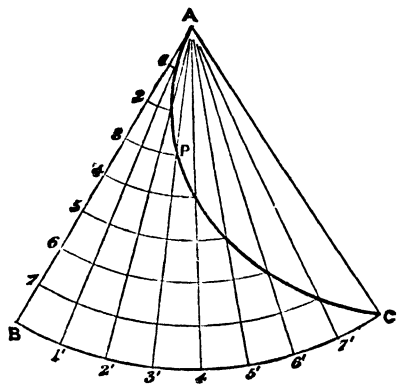


Fig. 39.

Let A B and A C represent the pendulum at beginning and end of the swing. Then the travel of the point from A towards B is the length A B, and of the pendulum bob the arc, B C, both uniformly and in the same time. Divide both travels into the same number of equal parts, say eight, and mark as shown. Draw the pendulum in the different positions, A 1′, A 2′, . . . A 7′. Then when the pendulum has reached the position A 3′ the point will have travelled the distance A 3, therefore, with centre A and radius A 3 draw an arc to meet the line A 3′ in the point, P, which will be one point in the locus of the point. Proceed in the same way for other points and draw the curve as shown.

PROBLEM XXXIV. (Fig. 40).—*To draw the path of a point in a link, one end of which moves in a circle while the other end moves in a straight line.*

(This is the combination of crank, connecting-rod, and guide bars, so common in steam engines.)

Let A B be the link, the end B moving in the circle of centre C, while the end A (the piston end) moves in the straight line through A C.

Divide the circle into any number of equal parts, and mark 1, 2, . . . 12, as shown. (Twelve is the most convenient number, as a quadrant can be divided into three by marking off the radius from each end.) With the length, A B, of the link as radius, and the points 1, 2, . . . 12 as centres, cut the line A C in the points 1′, 2′, 3′, . . . 12′, and join the points 2′, 2; 3′, 3, . . . thus drawing the link in each of the twelve positions. Then measure off from either end of the link in each position the distance from that end of the point, the locus of which is

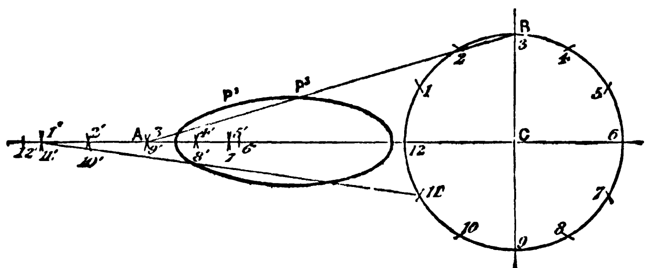


Fig. 40.

required, thus obtaining the points P^2, P^3, . . . through which the curve of the complete path is drawn.

It is interesting to note how the path of different points in the rod changes from a straight line at the guides (the end A), through oval curves of different degrees of convexity, until it reaches a circle at the crank end, B. Points in an extension of the rod beyond B, travel in oval paths, the long axes of which are at right angles to the line A C. Notice also that the piston end, A, does not move uniformly with an uniform movement of the crank.

PROBLEM XXXV. (Fig. 41).—*To trace the locus of a point in the linkwork known as Watt's Simple Parallel Motion.*

This linkwork consists of two links, A B and C D, pivoted at A and D, and having their other ends connected to a shorter link, B C, to a point P in which the piston-rod is attached. The link A B is the engine beam, and the link C D the radius bar. When the links are equal, the point P is in the middle of the short link, B C. This linkwork is also used on Richard's Engine Indicator

With centres A and D, and radii A B and D C respectively, draw arcs E B F and G C H. The end B must always move in the arc E B F and the end C in the arc G C H.

In problems of this kind it is always best to start by finding the limiting positions of the links. Suppose the link A B is moving upwards, then its limiting position is A E, when D C and C B are in one straight line. Find the point E by taking the sum of the lengths D C and C B as radius, from centre D. If the line A B now moves down, D C will continue to move upwards until the end C reaches the position G, where A B and

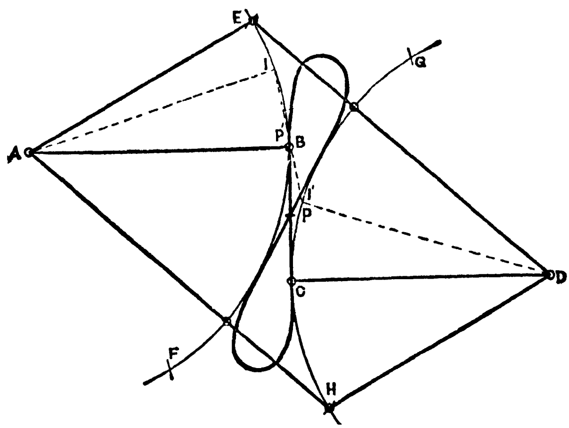


Fig. 41.

B C are in one straight line. In a similar way the limiting bottom positions F and H are found.

To find the path of P for the complete movement of the link-work, draw the links in a convenient number of different positions, and mark the position of P. For example, if link A B moves to A 1, then, with length of link B C as radius, and point 1 as centre, cut the locus of C (the arc H C G) in 1′, and join the points 1 and 1′, then mark the point P as P′. Notice that it is unnecessary to draw the links A B and C D, and that it is better to find a greater number of points at places where the curve changes in direction.

A very convenient method of obtaining points in the travel of the point P is to mark off the points B P C along the straight edge of a slip of paper, and the correct distances apart, and then if the paper be moved so that the point B is always on the circle E B F, and the point C on the circle G C H, the different positions of the point P can easily be marked. Care must be taken to move the points B and C in the right direction, since they do not both always move in the same direction. This mechanical method may be very accurate, and admits of useful extension to similar problems.

In the application of this linkwork to steam engines, the travel of the point, P, does not exceed that part which approximates to a straight line.

EXAMPLES.

EX. 6.—A pendulum, 4 feet long, is moved from rest, and makes one-half swing to the left and one complete swing (through 40°) to the right, while a fly travels from the top to the bottom. If the travels are uniform trace the path of the fly. (Scale 1" = 1 foot.)

EX. 7.—(*a*) A connecting-rod is 3′ 6″ long, the crank being 6″ long. Trace the paths of points, 1′ 3″, from each end of the rod during one complete revolution of the crank. (*b*) Mark on the line of travel of the piston end of the connecting-rod, the distances representing the travel of that end, while the crank pin end travels uniformly. (*c*) Work the same problem when the connecting-rod is 1′ 6″ long, and notice how the motion of the piston is affected by the length of the connecting-rod. (Scale $1\frac{1}{2}$″ = 1 foot.)

EX. 8.—A parallel motion consists of two arms, 4″ long, pivoted at their outer ends, and connected by a link 2″ long. In the central position the arms are parallel, and the link is inclined to them at 60°. Draw the complete path of (*a*) the central point of the link, (*b*) a point $\frac{1}{2}$″ from one end. (Vict. U. Hon., 1891.)

EX. 9.—Draw two lines, A P, C P, $2\frac{3}{4}$″ and $1\frac{1}{4}$″ long respectively, meeting at a point, P, so that angle A P B = 60°. Produce A P to B, and C P to D, so that A B = 4″, and C D = 2″. If A B and C D are links pivoted at A and C respectively, and P is a saddle which can travel along C D at two-fifths the speed it can move along A B, trace the locus of P. What is this curve? (S. & A. A., 1888.)

EX. 10.—Draw a rhombus A B C D, sides $1\frac{5}{8}$″ long, acute angles at B and D = 45°, and mark a point, P, in B C $\frac{1}{2}$″ from C. Draw a circle of $1\frac{13}{16}$″ diameter passing through C, such that the centre is on A C produced and beyond it. If the rhombus is a linkwork pivoted at A, trace the locus of the point P, when the joint, C, moves in the circumference of the circle. (S. & A. A., 1887.)

EX. 11.—A point O is $1\frac{3}{4}$″ from the centre of a circle of $\frac{3}{4}$″

radius. Determine the locus of the centres of circles bisecting the circumference of this circle and passing through the point O. (S. & A. A., 1891.)

(If the circles bisect the circumference of the given circle, they must pass through the ends of diameters. Therefore, by drawing a diameter in different positions, the centres of circles can be found, which would pass through the ends and the given point.)

Pantograph (Fig. 42).—This consists of an arangement of links in which two points move in similar paths, and is thus capable of application for reducing and enlarging any given pattern.

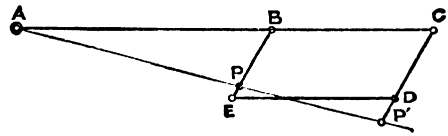


Fig. 42.

A C is a link pivoted at A, the links B E, E D, and D C form a parallelogram with the part B C. If a line be drawn from A through any point, P, in B E and produced to meet a point, P′, in C D or C D produced, then the points P and P′ trace out similar paths, the locus of P′ being the larger, for all positions of the linkage.

EX. 12.—Draw a pantograph as in Fig. 42 as follows:— A C = 4½″, B C = 1⅞″, B E = ⅞″, B P = ⅜″, angle A B E = 60°, and trace the locus of P′ when P moves in an equilateral triangle of 1″ side.

Watt's Double Parallel Motion (Fig. 43).—This consists of a simple parallel motion (Fig. 41) added to a pantograph (Fig. 42). The simple links A B, B C, and C D cause the point P

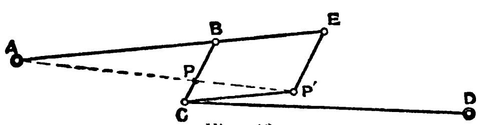


Fig. 43.

to move for a short distance in a straight line, while the pantograph links, A B E, B C, C P′, and P′ E, give a similar movement to the point P′. The link A E is half the engine beam, the piston-rod being attached to P′, and the pump-rod to P.

EX. 13.—Draw the linkwork of Watt's double parallel motion as in Fig. 43 as follows:—A E = 3¼″, A B = 2⅛″, B C = ¾″, angle A B C = 60°. Join the points A P′ to give P, then find length of C D knowing that $\frac{A\,B}{C\,D} = \frac{C\,P}{P\,B}$. Trace out the paths of P and P′ for all positions of the links.

Scott-Russell's Parallel Motion (Fig. 44).—This is the linkage applied to the "Grasshopper" type of side lever engine. A B is a link pivoted at A, and connected at B to the centre of the link P C, so that A B = B P = B C. Hence, B is the centre of a circle passing through P A C, having P C for a diameter, therefore if the point C move in the line A C, the point P will move in the line P A perpendicular to A C, since the angle P A C must always be a right angle (*Euclid* iii., 31).

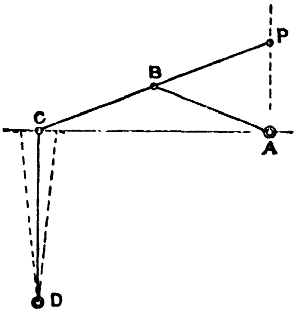


Fig. 44.

In actual practice it is more convenient to attach C to the end of a radius bar, C D, pivoted at D, as with a small movement the paths of C and P are sufficiently in straight lines for all practical purposes. The dotted lines show the travel of the radius bar.

EX. 14.—Draw the linkwork of Scott-Russell's parallel motion (Fig. 44) as follows:—A B = 1½", C D = 2", angle A B P = 45°, and trace out the path of P, while P moves in a line equal in length to twice P A.

Peaucellier's Straight Line Motion (Fig. 45).—This was invented by a French military officer in 1801, and consists of eight* links, so arranged that a certain point moves in a perfect

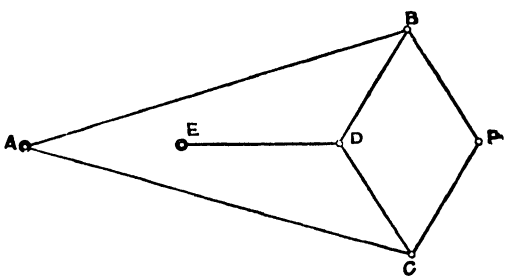


Fig. 45.

straight line for all possible positions of the linkwork. The links A B and A C are equal, and are pivoted together at A, their other ends are attached to two corners of an equal four-link frame, B D C P, of which the corner, D, is attached to one end

* In counting the number of links in any linkage, the fixed link, as A E in Fig. 45, is counted.

of a short link, D E, having its end, E, pivoted in the line A P, and such that A E = E D. Under these conditions the point P moves in a straight line, but if the distance A E is made greater or less than the length of the link E D, then the point P draws the arc of a circle having its centre to the right or the left of P respectively, the radius of which depends upon the ratio of A E to E D.

EX. 15.—Draw the linkwork of Peaucellier's straight line motion (Fig. 45) as follows:—A B = A C = 4¼″, A E = E D = 1⅝″, D B = D C = C P = P B = 1⅜″, and trace out the path of the point P for all possible positions of the links.

(Notice that the limiting positions are reached when the links E D, D B and E D, D C become a straight line.)

EX. 16.—Draw the linkwork of Peaucellier's straight line motion as in Fig. 45, but making A E = 1″ in the first case, and 2¼″ in the second case, and trace the path of P for each case.

Parallel Motions for Engine Indicators.—These comprise some important applications of straight line linkages, and afford very useful examples in drawing.

Thompson's Indicator (Fig. 46).—The end, A, of the link A B is attached to the piston of the indicator by a ball and socket joint, so that A moves in the vertical straight line shown dotted. C D and F E are swinging links pivoted to the indicator at D and F respectively. The pencil is fixed at P, and moves four times the distance of the piston travel, which is usually ¾″. The pencil link P B C is horizontal, and the link C D vertical, when the piston is at mid-stroke; and the points P A D are in one straight line for all positions of the movement.

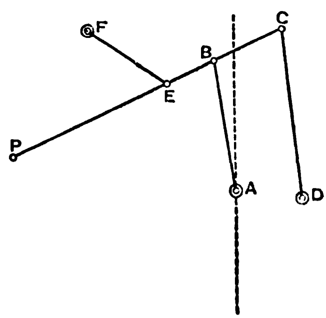


Fig. 46.

EX. 17.—Draw the linkwork of Thompson's indicator in position at bottom-stroke as follows:—P E = 1·81″, P C = 3 15″ = 4 times B C, B A = 1·34″, C D = 1·77″, E F = 0·98″, pivot D is 0·78″ to right of dotted line of piston travel, and pivot F is 1·45″ to left of same line; the pencil point P is 2·24″ to the left of the line and a distance equal to half the travel of the pencil below the pivot F. Find at least 6 points in the path of P for a travel of the point A of ¾″. Scale twice full size.

Crosby Indicator (Fig. 47).—The piston-rod is shown by

A B, and moves in a vertical straight line. A short link, B E, connects the piston-rod to the pencil link, P E F, and is connected at the point C to one end of a short swinging link, D C, pivoted at D to the indicator. The end, F, of the pencil link is attached to one end of a swinging link, F G, pivoted at G to the instrument, and the pencil is fixed at P. The piston-stroke is $\frac{3}{8}''$, and is multiplied six times.

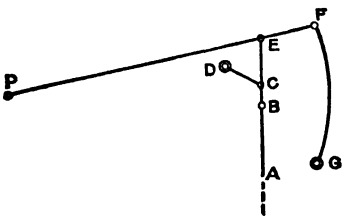


Fig. 47.

EX. 18.—Draw the link work of Crosby's indicator (Fig. 47) in position at half-stroke. Pencil link horizontal, fixed point G is $\frac{5}{8}''$ from piston-rod centre, fixed point D $\frac{7}{16}''$ from piston-rod centre and $1\frac{3}{16}''$ above G. B E = $\frac{3}{4}''$, B C = $\frac{1}{4}''$, G F = $1\frac{3}{4}''$, D C = $\frac{7}{16}''$, F P = $3\frac{3}{4}''$ = six times F E. Then draw path of P for a travel of the piston rod A B of $\frac{3}{8}''$. Scale twice full size.

Tabor Indicator (Fig. 48).—In this instrument the short link C D or E F of the Thompson or Crosby Indicator (Figs. 46 and 47) is dispensed with and replaced by a small pin, which is made to move in a curved slot cut in a small plate fixed to the instrument, the shape of which causes the pencil to move in a straight line. The piston-rod is shown by A B and is connected at B by a short link B C to a point C in the pencil-rod P E, the end E of the pencil-rod is attached to the end of a swinging link E F, pivoted at F. The slotted plate is shown at G, the pin D moving in the slot. The stroke of the piston is $\frac{5}{8}''$, and is multiplied five times. The points F B P are in the same straight line at all parts of the stroke.

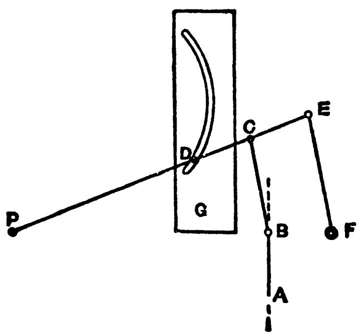


Fig. 48.

EX. 19.—Draw the link work of the Tabor indicator as in Fig. 48, when at bottom of stroke; the line F B P being horizontal. Length of E P = $3\frac{1}{8}''$ = five times E C, C D = E C, E F = $1\frac{1}{4}''$ and F is fixed $\frac{9}{16}''$ from centre of piston-rod. Links B C and E F are parallel. Trace out shape of curve in plate G, while P moves in a vertical straight line for $3\frac{1}{8}''$. Scale twice full size.

Link Work for Atkinsons' Differential Gas Engine (Fig. 49).—The object of this very ingenious arrangement of links to give two forward and two backward strokes of different lengths to the piston of the gas engine while the crank makes one complete revolution, thus giving one explosion per revolution. The crank shaft is shown at A, A B being the crank, the end B is connected by a link, B C E, at the point C to the end C of a swinging arm, C D, longer than the crank, and which, therefore, swings twice through a certain arc while the crank makes one revolution. The piston-rod, P E, is connected by a working joint to the piston at P, the opposite end being attached to a point E in the link B C E. As a result of this arrangement

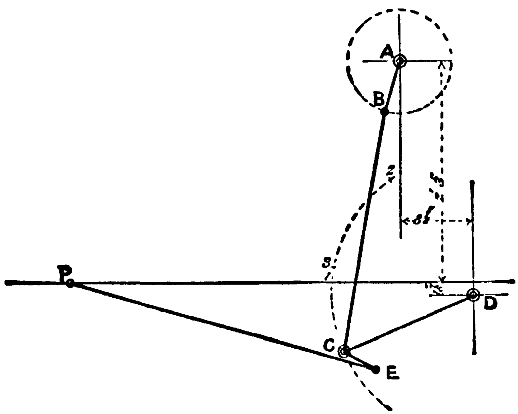


Fig. 49.

the piston moves in a horizontal line as follows, supposing it to start from the back stroke the crank A B being at about the position B′ :—

(1) Outwards for a short stroke while the crank moves through 59°, the link D C moving downwards, *the charging stroke ;*

(2) Inwards for a shorter stroke, while the crank moves through 76°, the link D C moving upwards, *the compression stroke ;*

(3) Outwards for a much longer stroke, while the crank moves through 92°, the link D C moving upwards, *the explosion stroke ;*

(4) Inwards for a stroke of the same length as (3), while the crank moves through 133°, the link D C moving downwards, *the exhaust stroke ;* the crank having thus made one complete revolution. The connecting-rod, P E, is inclined at 21° to the centre line of the movement of the piston, when at the limiting top and

bottom positions. These limiting positions are when the link B C and the crank A B are in the same straight line.

EX. 20.—Show the travel of the piston in an Atkinson's differential gas engine for an arrangement of links, as in Fig. 49. Take at least twelve different positions of the crank. Connecting-rod, P E = 3′, radius link, D C = 1′ 6″, crank 12⅞″ long, C B = 2′ 2⅜″, C E = 4″, angle B C E = 95°. Scale, 3″ = 1′.

Joy's Valve Gear (Fig. 50).—This arrangement affords a very useful example of the paths of points in linkwork, and is besides of great service in leading up to the drawing of valve diagrams. The slide valve is worked through a system of rods, which derive motion from a point in the connecting-rod, thus dispensing with the usual eccentrics and link motion. In Fig. 50 is shown the arrangement for a large vertical marine engine. P C is the piston-rod, C B the connecting-rod, and B A the crank. A link, D E, is attached at one end to a point D in the connecting-rod, and at the other end to the end E of a swinging link or radius-rod, E F, pivoted at the top end, F. From a point G in the link D E, a long link, G L, is carried to the front of the engine, and is connected at its end, L, through the link L M to the slide-rod M N, which moves in a parallel vertical line to the piston. A point, H, in this link is connected to one end of a radius rod, H K, the upper end of which is pivoted at K. Reversing is effected by attaching the link H K to a moving frame, which can swing about a pivot below K, so that H K can be moved through a sufficient arc to move the linkwork for reversing. The locus of the points E and H are, of course, circular arcs, and in drawing the linkwork it is usual to trace the complete paths of the points D, G, and L.

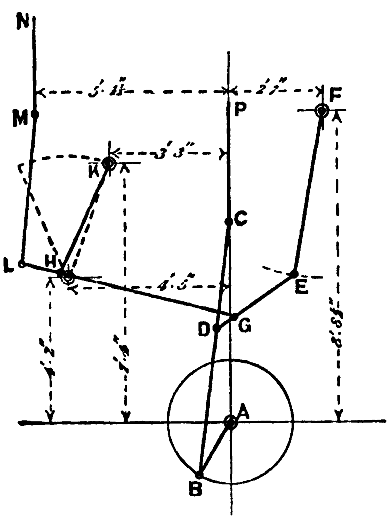


Fig 50.

EX. 21.—Draw the linkwork of Joy's valve gear, as in Fig. 50. Connecting-rod, C B = 7′ 2″, C D = 3′ 1″; crank, A B = 1′ 8½″; link F E = 4′ 8½″; link D E = 2′ 6″; D G = 8″; link

GL = 6′ 2″; LH = 1′ 1″; link LM = 4′ 3″; link HK = 3′ 5″. Trace out the complete paths of the points D, G, and L for one revolution of the crank. Scale, 1″ = 1′.

Ordinary Link Motion.—An example of the common form of link motion as used on locomotives, to enable the engine to be reversed and the travel of the slide valve to be altered while the parts are in motion, is illustrated in Fig. 50*b*.

A is the shaft centre, and A B the crank when at one of the dead centres. A C and A D are the centre lines of the two eccentric sheaves, and have connected to them in the usual way the eccentric-rods D E and C F. The ends, E F, of these rods are attached to the top and bottom of the curved link, E G F, which is capable of sliding up and down through guides at G, which guides form part of the slide valve rod, G H. This rod, G H, can only move to and fro along the line H G A, and receives the motion of the curved link at G. The link, therefore, swings about the point G as a centre, although it will be understood

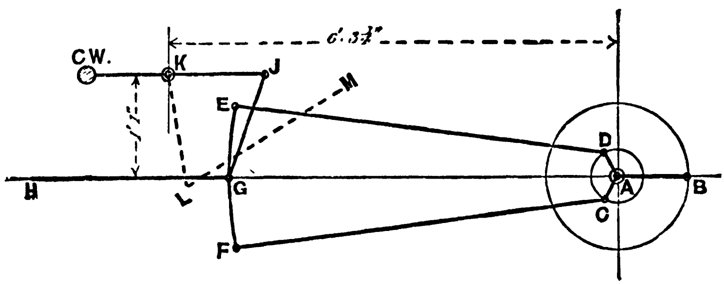


Fig. 50*b*.

that it can be so moved relatively to the valve-rod, as that any part of it can be brought opposite the rod. In the centre position shown, the rod H G will not receive any movement from the link, but if the link be moved down so as to make the points E and G coincide, then all the motion of the eccentric E D A will be transmitted to the valve-rod, and similarly for the eccentric, F C A, when the points F and G coincide. The movement of the link is effected by the rod G J fixed to the link at its centre point G, by a connection independent of that which affixes the link to the valve-rod, the end J, of this rod being connected to one end of the bellcrank lever J K L, which turns about the fixed point K as a centre. L M is the reversing rod which works the lever J K L, and C W is a counter-balance weight, but these are not material for the purposes of the drawing required. The eccentrics A D and A C are to be regarded simply as cranks, to which they are in every sense identical so far as the motion they transmit to the rods is concerned.

EX. 22. Draw the link motion in outline as shown in Fig. 50*b*. Scale $1\frac{1}{2}'' = 1'$. Lengths as follows:—A B = 1′, A D = A C = $3\frac{3}{4}''$ $\angle$ B A C = $\angle$ B A D = 115° C F = D E = 5′ $3\frac{1}{4}''$, radius of link, E G F = 5′ $3\frac{1}{4}''$, E F = 1′ 6″ (this is length of line joining E F), G J = 1′ 1″, J K = 1′ $3\frac{1}{2}''$. Trace out the paths of the ends E and F of the link, when in the central position shown, for one complete revolution of the crank, taking at least 10 points.

Cams.—A cam is usually either a plate with a curved edge, or containing a curved groove, the shape of which is arranged to impart a reciprocating linear movement to a given piece, while the cam itself receives uniform circular motion. Such pieces of mechanism are very common in general machinery, especially in sewing, weaving, and printing machines, and it is, therefore, very desirable for the engineering draughtsman to understand the principles of their design. The motion transmitted by a cam, although always linear, may be either uniform or variable, depending upon the shape of the cam; for example, a common form of lever punching machine is fitted with a cam, which gives to the punch an upward movement, a period of rest, and a downward movement, during each revolution of the driving shaft. It is important to remember that cams almost always receive a uniform circular motion, for on this fact the construction depends.

It was pointed out in connection with Prob. xxxiv. (Fig. 40) and Ex. 7, p. 52, that the combination of a crank and connecting-rod, one end of which moves in a straight line, does not give a uniform linear movement, for a uniform circular movement of the crank, but gives a motion which varies with the ratio of the connecting-rod length to the crank length. Hence any piece attached to this end, such as the piston, does not receive uniform linear motion. It is, however, easy to shape a cam which shall transmit a uniform linear motion, and it happens that the outline of such a cam coincides with a curve known in mathematics as the Archimedean Spiral, and we shall, therefore, first show how to construct such a spiral.

Spiral.—A spiral may be defined as a curve which approaches to, or recedes from, a certain fixed point called "the Pole." Each complete revolution of the curve is called a *"convolution,"* and hence a spiral may make any number of convolutions before reaching the pole. The line joining any point on the curve to the pole is called a *"radius vector."*

In an Archimedean Spiral the curve approaches the pole uniformly, so that the radii decrease in length uniformly with the increase of the angle passed through from the starting or initial line.

PROBLEM XXXVI. (Fig. 51).—*To construct an Archimedean Spiral of* $1\frac{1}{2}$ *convolutions, when the greatest radius is known.*

Let O be the pole and O A the greatest radius. Describe the circle having the pole as centre and O A as radius, and divide the circle into any number of equal parts, join each division point to the pole, and figure as shown. Then A may be regarded as a point which moves uniformly down the line A O, while A O rotates uniformly, and which reaches O when A O has made $1\frac{1}{2}$ revolutions. Therefore, divide A O into three equal parts, and mark the second division from O as B, then the point must reach B during one revolution. Divide A B into the same number of equal parts as the circle, and mark as shown. Then it is evident that when the line O A reaches the position O 3, the point will have moved down the line three divisions of A B, and so on for each position. Evidently, then, the best way of finding the locus of A is to draw arcs of circles, with the pole as centre, through each of the divisions of B A to the corresponding positions of the line O A, as shown in the figure, The second part of the curve is found in the same way by dividing the distance, B O, into six equal parts.

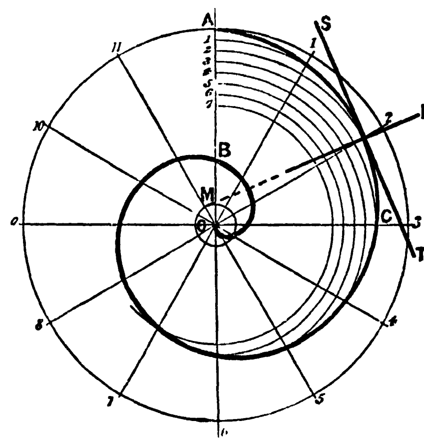


Fig. 51.

Normal and Tangent.—A normal to an Archimedean spiral is a tangent to the circle, having the pole for a centre, and a radius equal to "*the constant of the curve.*" This constant is found by dividing the difference between the lengths of any two radii by the circular measure of the angle between them. Therefore, in Fig. 51, measure O A and O C and find O A - O C. the angle between them is 90°, and its circular measure is $\frac{\pi}{2} = 1·57$, therefore the constant of the curve is $\frac{OA - OC}{1 \cdot 57} = a$. Draw the circle OM of radius equal to a, then MN a tangent to the circle is a normal to the curve, and a line, ST, at right angles is a tangent.

It is evident from the construction, that a cam shaped to the curve of the spiral would impart linear motion to a point uniform with the circular motion of tho cam. Cams are, therefore, made to this curve whenever such a motion is desired. Such a cam is shown in Fig. 52, and is known as the Heart Shape. It consists of two equal parts of an Archimedean Spiral reversed to each other, and symmetrical about the diameter, AOB, and thus gives a uniform rise and fall, through a distance equal to AO − BO—that is, equal to the greatest radius, minus the least radius, technically called the "*travel.*" If a semicircle be drawn with the pole O as centre, tangent to the curves of the cam, and replacing the lower half of the spirals as shown in Fig. 52, then it is evident that the point moved by the cam will remain at rest, while the circular part of the cam CDE is in contact with it, and will rise and fall uniformly when in contact with the curved parts CA and AE. This form of cam is the one already referred to as being used in punching machines, giving, as will be seen, a uniform rise for a quarter revolution of the shaft, a uniform fall for a quarter revolution of the shaft, and a period of rest for a half revolution of the shaft.

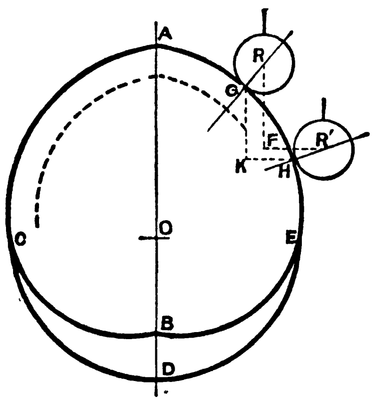


Fig. 52.

So far, cams have been designed to impart motion to a "*point,*" but it is obvious such a condition is impossible in practice. Cams invariably transmit their motion by making contact with a roller of convenient diameter, thus giving very smooth working. But the use of a roller necessitates an alteration in the shape of the cam from that already designed, the desire being that the *centre* of the roller shall move in the path of the cam as traced in Fig. 52. To ensure this, a curve must be drawn *parallel* to the original curve, inside it, at a distance away equal to the *radius* of the roller. This is best done, as in drawing the parallel to an ellipse (Fig. 20), by describing a large number of small arcs, with points on the original curve as centre, and with radius equal to the radius of the roller, and then drawing a smooth freehand curve, touching the arcs, as shown dotted in Fig. 52. Such a curve is called a parallel to the original curve, and is not necessarily a similar curve. The necessity for drawing this parallel curve can be seen in another way by referring to the figure. Let R and R′ be two positions of the roller, for we may suppose the cam fixed, and the roller to be moved around the cam. Then the travel of the roller is the vertical distance, R F, between the centres of R and R′. But if the rollers were the points G and H, the travel would be the distance G K, which is not equal to R F, as the normals G R and H R′ are not parallel.

In designing cams for practical use it is necessary to know the diameter of the shaft or rod on which the cam is fixed, the travel, and conditions of movement, the least amount of material beyond the shaft, and the radius of the roller. When these are known, the least radius of the *original* curve will be equal to the shaft radius + metal beyond shaft + roller radius, and the greatest radius will be equal to the least radius + the travel. Care must be taken that the curves of the cam do not rise or fall too suddenly, or the roller will jamb. Two or more cams can be fixed together, and be made to work rods so jointed, that a given point in the rods shall trace out almost any desired path, such, for example, as the outline of letters.

EXAMPLES.

EX. 23.—Draw the curve of an Archimedean Spiral of two convolutions, greatest radius 5″. Show by three examples that the length of the radii at different points vary with the angle passed through by the radius from the initial position.

EX. 24.—Work parts *b* and *c* of EX. 7, p. 52, then draw the curve of a cam to give an uniform rise and fall of 3″ to a roller 1″ diameter, during each revolution of the cam; least radius $2\frac{1}{4}$″.

EX. 25.—Draw the shape of a cam to give the following motion to a point:—first quarter of a revolution, point raised uniformly $3\frac{1}{2}$″; second quarter, point falls uniformly $3\frac{1}{2}$″, remainder of a revolution point remains at rest.

EX. 26.—Draw a cam which has to raise a valve at a uniform rate 6″ in two-fifth revolution, and lower it the same distance in one-fifth revolution. The valve remains at rest in the upper position for one-tenth revolution, and in the lower position for the rest of the revolution. Diameter of shaft 4″. Least metal around shaft 2″ Scale half full size. (Vic. B. Sc. Hon., 1889).

EX. 27.—Draw a cam to give the following motions. It revolves uniformly at a rate of two revolutions per minute, a tappet is to be raised 4″ at a uniform rate in 5 seconds, and allowed to remain in that position for 5 seconds; then allowed to drop 1″ and remain there another 5 seconds, again raised to 4″ for 10 seconds, and then allowed to drop suddenly to its original position, and remain there until again required to be raised. Diameter of shaft 3″, of roller on end of tappet $1\frac{1}{4}$″, least metal around shaft 2″. Scale half full size.

EX. 28.—Draw a line C A 3″ long, and from end A draw a line A B so that angle C A B = 150°, make A B = 5″ and A D = $2\frac{1}{2}$″. C is the centre of a shaft $1\frac{5}{8}$″ diameter, and A the centre of a roller $1\frac{1}{4}$″ diameter in its lowest position. The roller is moved by a cam on the shaft along the line A B as follows:—One-third of a revolution raised from A to D uniformly, one-sixth of revolution remains at rest, one-fourth of revolution raised uniformly from D to B, one-fourth of revolution falls back from B to A. Scale half size. (S. & A. H., 1887.)

SECTION VII.

CONSTRUCTION OF ELLIPSE, PARABOLA, HYPERBOLA, CYCLOIDAL CURVES AND INVOLUTES.

It is very important for the draughtsman to understand the construction, and some of the more useful properties of certain well-known mathematical curves, such as are frequently made use of in practical work.

These curves include the *ellipse*, *parabola*, and *hyperbola*, known as the conic sections, in consequence of their being derived from three different plane sections of a cone (see Figs. 86*a*, *b*) and used for the curves of arches, bridges, and roofs; the *cycloidal curves* used in constructing the teeth of wheels; and the *involute of a circle* used for the same purpose, and for the blades and guides of turbines.

These curves, as in the case of spirals and paths of points (Section vi.) can only be geometrically constructed by finding a number of points through which it is known the curve must pass, and then drawing the curve through these points by freehand or with the aid of French curves. Arcs of circles cannot be employed with any degree of accuracy; and as pointed out in previous examples, the greater the number of points found, the more accurate the curve, although for ordinary purposes it is usually sufficient to find the points not nearer than from $\frac{1}{2}''$ to $1''$ apart.

Ellipse, Parabola, and Hyperbola.*—Given a fixed straight line, and a fixed point, it is possible for another given point to move in three different ways with regard to its position from the fixed line and point. It can move, firstly, so that its distance from the line is always greater, in a constant ratio, than its distance from the point; secondly, so that its distance from the line shall be always equal to its distance from the point; and thirdly, so that its distance from the line is always less, in a constant ratio, than its distance from the point.

The three curves traced out under these different conditions are respectively, the *ellipse*, *parabola*, and *hyperbola*.

Hence we have for definitions of these curves—

Ellipse.—An ellipse is a curve traced out by a point moving in such a way that its distance from a fixed straight line is always greater than its distance from a fixed point, in a constant ratio.

Parabola.—A parabola is a curve traced out by a point moving in such a way that its distance from a fixed straight line is always equal to its distance from a fixed point.

Hyperbola.—An hyperbola is a curve traced out by a point moving in such a way that its distance from a fixed straight line is always less than its distance from a fixed point, in a constant ratio.

The fixed straight line is called the *directrix,* the fixed point the *focus,* and the line passing through the focus at right angles to the directrix, the *axis.* Lines at right angles to the axis terminated by the curve, are *ordinates.* The *vertex* of the curve is the point where the curve cuts the axis.

The ellipse is a closed curve, and has two directrices and two foci. The parabola is an open curve having one directrix and one focus. The hyperbola is an open double curve, having two directrices and foci.

These three curves can be constructed by an almost identical process, so that one example will suffice.

PROBLEM XXXVII. (Fig. 53*a*).—*To construct an ellipse when a directrix and a focus are given, also the vertex and axis.*

Let X Y be the directrix, F the focus, A the vertex, and the line through O A F the axis. What is required is to find a

*** For the common geometrical constructions of an ellipse see Section iv., Figs. 19, 20, 21, 22.**

number of points, P, so that $\frac{PF}{PT} = \frac{AF}{AO}$ where P F is the distance of P from the focus, and P T its distance from the directrix. This is conveniently done by making A O the hypotenuse and A F the base of a right angled triangle, A B O, where O B = A F and angle A B O is a right angle. Then produce the line O B, mark off any points as 1 . 2 . 3 . from A along the axis, and draw lines through each point parallel to A B meeting O B in the points 1′ 2′ 3′ . . Draw lines through each of the points in the axis perpendicular to the axis, from F as centre, with distance O 1′ cut the line through point 1 in the points 6 and 7, also from F with distance O 2′ cut the line through point 2 in the points 8 and 9, and so on for each succeeding line. Then the

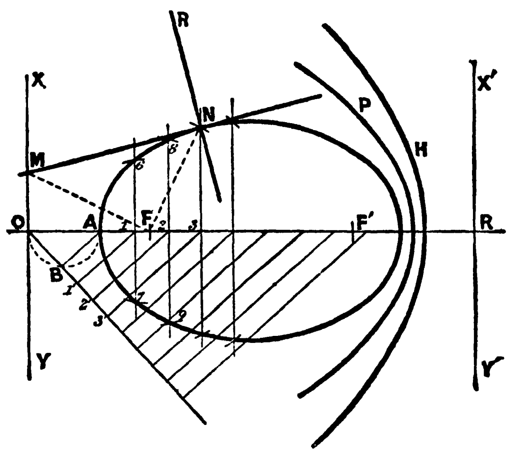


Fig 53*a*.

points 6 . 7 . 8 . 9 are in the curve of the required ellipse, for $\frac{6F}{6T} = \frac{8F}{8T} = \frac{BO}{AO} = \frac{AF}{AO}$ where 6 T and 8 T are the distances of points 6 and 8 from the directrix.

Continue this method until the curve is completed.

If the ratio of the distances from the focus and directrix is given (say $\frac{2}{3}$, so that A F = 2 and A O = 3), it is unnecessary to draw the triangles. Any three equal distances can be marked along the axis from O, as to O 3 and two of these distances taken as radius from F to cut the line drawn through 3 parallel to the directrix, and so on for each point. Exactly the same method is followed for the construction of the parabola and hyberbola.

The parabola is, however, more easily constructed by the method shown in Problem xxxi. (Fig. 38*a*) for drawing a curve equidistant from a point and a line, as we now know that curve to be a parabola. The curve which is equidistant from a straight line and the circumference of a circle (Fig. 38*a*) is also a parabola. The curve equidistant from the circumferences of two unequal circles is a hyperbola.

After completing the ellipse its second focus, F′, and its second directrix, X′Y′, can be found. In the right hand of Fig. 53*a* are shown a parabola, the curve P, and a hyperbola, the curve H, which are constructed together with the ellipse about the focus, F′, and the directrix, X′ Y′.

Tangents and Normals.—The usual methods of constructing tangents to an ellipse from points in the curve or outside it, have already been given in Section iv. (Figs. 19, 20). The rule which applies most conveniently to all three curves when constructed by the method just described, is the following:—"**If the tangent to an ellipse, parabola, or hyperbola be produced to meet the directrix, and the meeting point be joined to the focus, the angle made by this line, with the line joining the focus to the point of contact, is a right angle.**" Thus, in Fig. 53*a*, to draw a tangent at the point N, join N to the focus F, draw a line from the focus towards the directrix at right angles to the line N F, meeting the directrix in M, then the line N M is a tangent to the curve.

Normals.—**The normal to a curve at any point is at right angles to the tangent at that point.** Thus, in Fig. 53*a*, N R is a normal at N, being at right angles to the tangent N M.

EXAMPLES.

EX. 1.—A fixed point, F, is 2″ from a fixed straight line X Y. Find eight points in the path of a point P moving as follows:—(*a*) distance of P from fixed point to its distance from the fixed line to be as 3 to 4; (*b*) point P to be equidistant from fixed point and fixed line; (*c*) distance of P from fixed point to its distance from the fixed line to be as 4 to 3. Draw the curves and name them.

EX. 2.—The focus F of an ellipse is 1½″ from the directrix X Y, and the vertex of the curve is ½″ from the focus. Draw the ellipse, and draw a tangent and normal at any point in the curve.

EX. 3.—Construct a parabola (finding, at least, twelve points in the curve) when the distance of the focus from the directrix is 1″, and draw a tangent and normal at any point in the curve.

EX. 4.—Construct a hyperbola (finding at least twelve points in the curve), when the focus is $1\frac{1}{2}''$ from the directrix, and the vertex $1''$ from the focus. Draw a tangent and normal at any point in the curve.

Construction of a Rectangular Hyperbola.—A special case of a hyperbola is one in which the point moves in such a way that the product of its distance from two fixed lines at right angles is a constant. Such a curve is a rectangular hyperbola and is exceedingly useful, because it represents graphically the relation between the pressure and volume of a gas which expands according to Boyle's law (pressure × volume = a constant), a condition often met with in steam diagrams. A simple construction of such a curve is as follows :—

Fig. 53*b*. Let O V and O P be two lines at right angles, and such that distances along O V represent volume, while distances along O P represent pressure, and let A be a point in the curve,

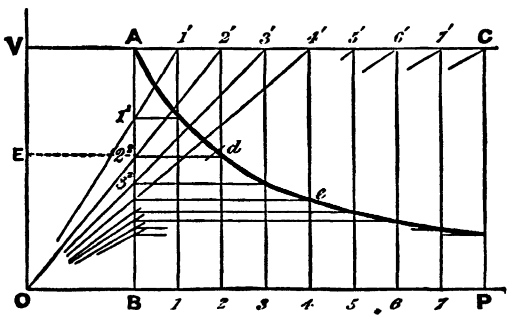


Fig. 53*b*.

which for ordinary practical problems will generally be the point from which the curve is required to start. Through A draw lines A B and A C parallel to O V and O P respectively, the line A C being produced as far as necessary since there is no limit to the curve. Mark any distances, equal or unequal, as 1, 2, 3 . . . along B P, and draw ordinates through each point, parallel to A B, to meet the line A C in the points 1′, 2′, 3′ . . . Join each of the top points 1′, 2′, 3′ . . to the point O, and mark the points where these lines cut the line A B, 1^2, 2^2, 3^2 . . . Through each of these points draw lines parallel to O P to meet the ordinate through the corresponding top point, thus 1^2 meets ordinate 1 1′, 3^2 meets ordinate 3 3′, &c., these points are points in the required curve. If this curve satisfies the condition required, then O B × B A = O 2 × 2 *d* = O 4 × 4 *e*, &c., and this we see is true, for V O 2 2′ is a parallelogram, of which the figures

V E 2^2 A and 2^2 B 2 *d* are complements about the diagonal O 2′, and are, therefore, equal (*Euclid* i., 43), while E O B 2^2 is common—that is, the figure V O B A = figure E O 2 *d*, and similarly for each of the remaining points.

The curve can evidently be produced backwards by a similar construction, so as to approach the line O V. But it will be evident that however much the curve be produced in either direction it can never touch the lines O V and O P, hence the lines may be said to be continually approaching the hyperbola, yet never touching it. Such lines are called "*asymptotes*," and when the asymptotes, as in this example, are at right angles to each other the curve is distinguished as a *rectangular hyperbola*.

EXAMPLES.

EX. 5.—Draw two lines, O V, O P, at right angles, intersecting at O, as in Fig. 53*b*, and mark a point A 1″ from O V and 2½″ from O P. Find at least seven points in the rectangular hyperbola drawn from A.

EX. 6.—Draw two lines, O V, O P, as in Ex. 5, and mark a point A ½″ away from O P and 3″ from O V. Draw the curve of a rectangular hyperbola from A towards O V to within ¼″ of O V.

Cycloidal or Rolling Curves.—There are three principal curves of this class, each being generated by a fixed point on the circumference of a circle, rolling in contact with a fixed line or circle in the same plane. These curves are all used in the construction of wheel teeth.

Cycloids.—A cycloid is the curve traced out by a fixed point on the circumference of a circle, rolling along a fixed *straight line*.

Epicycloids.—An epicycloid is the curve traced out by a fixed point on the circumference of a circle rolling round another *circle*, and *outside* it.

Hypocycloids.—A hypocycloid is the curve traced out by a fixed point on the circumference of a circle, rolling round another *circle*, and *inside* it.

Trochoids.—A trochoid is the curve traced out by a point rigidly fixed to a circle, within or without its circumference, as the circle rolls along a fixed straight line. When the fixed point is *without* the circumference, the curve is termed *superior*, and when *inside* the circumference, *inferior*. When the circle rolls round another circle, either outside or inside it, the curves are known as Epitrochoids and Hypotrochoids respectively.

The rolling circle is called the *generating circle*.

The fixed point is called the *generating point.*

The fixed line or circle is called the *directing line or circle.*

Evolutes and Involutes.—An *evolute* is the curve formed by the intersection of normals to a curve. An *involute* is the curve formed by drawing tangents to a curve, the length of each tangent being equal to the arc of the original curve from its point of contact to its intersection with the curve. It is the curve traced out by the end of a flexible thread unwrapped from the original curve. Tangents to an evolute are normals to an involute. Thus, in Fig. 54, a number of normals are drawn to the cycloid P S Q and the curves P R, R Q are drawn tangent to the normals; these curves are *evolutes.* Also, the cycloid P S Q is an *involute,* for, as will be seen later, it passes through the ends of tangents to the curves P R, R Q, each of which fulfils the condition that, if P be the intersection of the involute with the original curve, P′ a point in the involute, and E′ the contact point of the tangent with the original curve, then P′ E′ = length of arc P E′.

The *vertex* of the cycloid is at the point S; the points P and Q are called *cusps.*

PROBLEM XXXVIII. (Fig. 54).—*To construct a cycloid when the size of the generating circle is given.*

Let A B be the directing line, and P the generating point.

In one revolution of the circle P will reach a point, Q, on A B, so that the distance P Q equals the circumference of the circle

$$= (d \times \pi \text{ or } d \times \frac{22}{7} \text{ where } d = \text{diameter}).$$

C D is the locus or path of the centre of the circle for this revolution.

Divide C D into any number of equal parts, and mark as shown C, C^1, C^2, . . . C^7, &c., draw lines from these points to A B.

While C is moving to D, the point P moves round the circumference of the circle, therefore divide the circumference into the same number of equal parts, and mark the points 1, 2, 3, . . . 7.

To find the position of the generating point, at any position of the generating circle, proceed as follows:—If the generating circle move to the position C^1, the point P will have moved in the same time through the arc P 1. Draw the generating circle with centre C^1, and mark its contact point with the directing line E; from E mark off the distance P 1 along the circle to the point P^1, then P^1 is a point in the cycloid. Proceed in the same way for the other points until the curve is completed. Notice that only parts of the circles need be drawn.

Another way of finding the points is to draw lines parallel to the directing line through the division points 1, 2, 3, . . . of the generating circle in its first position, to meet the corresponding generating circle drawn from the centres C^1, C^2, C^3, . . . This is shown in the figure.

Tangents and Normals.—In all rolling curves the normal at any point passes through the corresponding point of contact of the generating circle with the directing line or circle. The tangent is at right angles to the normal.

To draw a tangent and normal at any point, O, in the curve (Fig. 54). With the point O as centre and the radius of the generating circle as distance, describe an arc cutting the line O D in N, and draw N M perpendicular to the directing line, meeting it in M. Then N is the centre of the generating circle

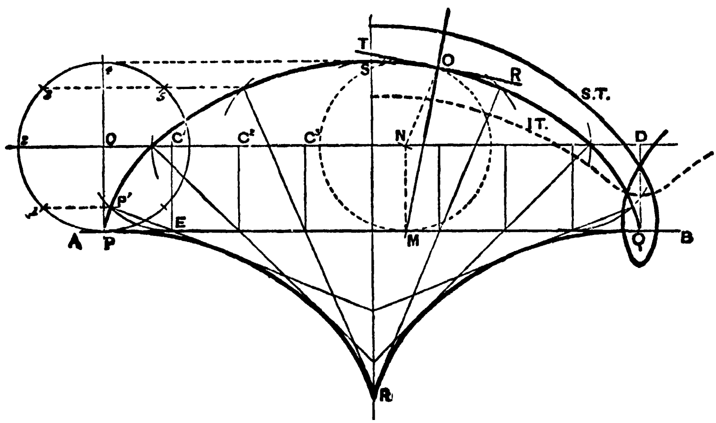


Fig. 54.

corresponding to the position O of the generating point, and M is its point of contact with the directing line; therefore the line through M O is a normal, and the line T R at right angles a tangent.

Evolutes of Cycloid.—Draw normals through each of the points found in constructing the curve, and produce them below the directing line. Then draw the curves Q R, R P tangent to the normals as shown. These curves are the evolutes.

The evolutes of a cycloid together make an equal cycloid. Thus the curves R Q and S Q are identical. This can be proved by cutting out the curve R Q in paper or card and applying to the curve S Q.

If a piece of thread be fixed at R, and wound round the curve of one of the evolutes as R P, so that the other end of the thread reaches to P, and then be slowly unwound from the curve, the end P, if the thread be kept tight, will trace out the cycloid P S Q. Hence the reason for this curve being called an "*involute.*" This arrangement forms what is known as an isochronous or equal timed pendulum, the pendulum bob being at one end of the thread P, the other end being fixed at R, curved guides being fixed in place of the evolutes. The time taken by the pendulum to swing through different arcs is then always the same, whatever be the length of the arc.

The cycloid has an important property in mechanics in that the evolutes R P or R Q are the curves of quickest descent from R to P or Q.

Trochoids.—The method of constructing the inferior and superior trochoids differs but little from the above, and should present no difficulty. Having found the new position of point P on the generating circles, having centres O_1, O_2, O_3, . . ., draw the radius through P in each case.

Then for the superior trochoid these radii must be produced the given distance, and their ends then represent points in the curve.

For the inferior trochoid points in the curve are obtained by marking along each radius from the centre the given distance.

These two curves are shown on the right hand of Fig. 54, the full-looped curve, S T, being the "*superior trochoid,*" and the dotted curve, I T, the "*inferior trochoid.*"

Tangents and normals to trochoids are drawn in a similar way as to cycloids, and the necessary construction will present no difficulty.

EXAMPLES.

EX. 7.—Describe a cycloid and its evolutes when the diameter of the generating circle is 5", and draw a normal and tangent at any point in the cycloid, not being one of the points found in constructing it. Then work the following:—(*a*) Show by cutting out a paper pattern that the curve of the evolute is a similar and equal cycloid; (*b*) show that the length of the normals from the directing line to the cycloid is equal to the length from the directing line to the evolute (note how this suggests an accurate way of finding points where the evolutes touch the normals); (*c*) measure the length of the cycloid, and show that it is eight times the radius of the generating circle; (*d*) find area between cycloid and directing line, and show that it is three times the area of generating circle; (*e*) find area between evolutes and directing line, and show that it is equal to the area of the generating circle.

EX. 8.—Draw the superior and inferior trochoids, when the diameter of the generating circle is 4″, the point for the superior curve being ½″ beyond the circumference and for the inferior curve ⅜″ within. Draw a normal and tangent to each curve at points not found in the construction.

Epicycloids. — PROBLEM XXXIX. (Fig. 55). — *To draw an epicycloid and its evolutes, given the directing and generating circles.*

Let the directing and generating circles have centres A and B respectively, P being the generating point. The construction is identical in principle with that of the cycloid, allowing only for the change from a directing line to a directing circle. But

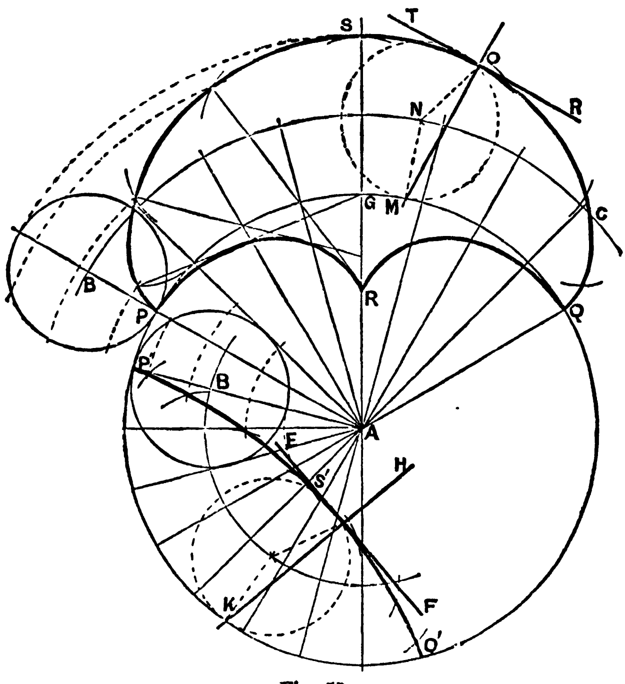


Fig. 55.

it is necessary to first find the position Q of the point P after one revolution of the generating circle. This is done by knowing that

$$\frac{\text{arc P Q}}{\text{circ. of directing circle}} = \frac{\text{angle P A Q}}{360^\circ},$$

and as the arc P Q equals the circumference of the generating circle, this becomes

$$\text{angle P A Q} = \frac{360 \times \text{rad. of generating circle}}{\text{rad. of directing circle}}.$$

Set off this angle. Draw the arc B C which is the locus of the centre of the generating circle, and, as before, divide it and the circle into the same number of equal parts, and then proceed as with the cycloid. The construction is clearly shown in the figure.

Normals and Tangents are drawn exactly as to a cycloid. Thus in the figure, N is the centre of the generating circle corresponding to the position O in the curve of the generating point, and M is the contact point of the rolling and directing circles. Then M O is a normal, and T R at right angles a tangent.

The evolutes are the curves P R, R Q, drawn tangent to the normals of the curve as before. They are similar curves to the original curve, P S Q, and are, therefore, epicycloids, but are not equal to the original curve. The hypocycloid and its evolutes are drawn in precisely the same way as the epicycloid, and do not, therefore, need separate explanation. In Fig. 55 the curve P′ S′ Q′ is the hypocycloid, the evolutes not being shown for want of space. They are, however, drawn touching the normals to the curve as before. E F is a tangent, and H K a normal. The generating circle rolls in the direction from P′ towards Q′.

Notice that both the epi- and hypocycloids are traced by the end of a thread unwound from the evolutes, as with a cycloid.

No difficulty should be experienced in drawing the epi- and hypotrochoids, as the construction is exactly similar.

EXAMPLES.

EX. 9.—Draw an epicycloid and its evolutes when the diameters of the directing and generating circles are 10″ and 4″ respectively, and draw a tangent and normal at any point in the curve not found in the construction. Show that the evolute is an epicycloid traced by a point on a circle of diameter equal to R G (Fig. 55) rolling on a circle of radius, A R.

EX. 10.—Draw a hypocycloid and its evolutes when the diameters of the directing and generating circles are 10″ and 3″ respectively, and draw a tangent and normal at any point in the curve not found in the construction.

EX. 11.—Show that when the diameter of the directing circle is twice the diameter of the generating circle, the hypocycloid is a straight line.

EX. 12.—Draw a hypotrochoid when the diameter of the directing circle is twice the diameter of the generating circle, and show that half the curve is a quadrant of an ellipse.

Involute of a Circle.—The involute of a circle is the curve traced out by the end of a piece of thread unwound from the circle, the thread being kept tight. The circle is then the evolute to this curve.

PROBLEM XL. (Fig. 56).—*To draw the involute of a circle.*

Let the circle have the centre C, and let P be the starting point of the curve or end of the supposed thread.

Let the thread be partly unwound, so that it assumes the position P^3 3.

It is evident P^3 3 must be a tangent to the circle, and be, therefore, at right angles to the radius C 3. Also P^3 3 must equal the length of the arc P 3. Then P^3 is a point in the involute.

If the arc P 3 be divided into a number of small parts, and the

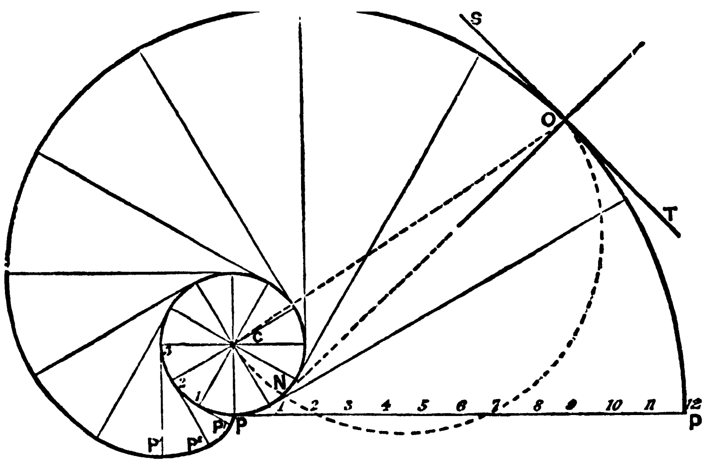


Fig. 56.

same number of parts be marked off from 3 to P^3, then the length P^3 3 may be assumed equal to the *chord* P 3 and P^3 be a point in the curve. But it is better to divide the circumference of the circle into, say, twelve equal parts, in which case the length of the tangent P^3 3 would be one-quarter of the circumference (which can be easily calculated), and each succeeding tangent would increase by one-twelfth of the circumference, if the circle is equally divided.

This construction is conveniently effected by drawing the tangent P, P^{12}, equal in length to the circumference of the circle, and dividing it into the same number of equal parts as the circle. The length of each tangent can then be taken from it, as, for example, $P^1 1 = P 1$, $P^2 2 = P 2$, &c.

Normals and Tangents.—Normals to the involute are tangents to the evolute, as in the cycloidal curves. Therefore, to draw a normal at any point O, it is only necessary to draw from that point a tangent to the circle. This is done by the method of Fig. 23, the point O is joined to C, and a semicircle is drawn upon it cutting the circle in the point N. Then the line N O is a normal, and the line S T at right angles through O is a tangent.

If in Fig. 56 we regard P, P^{12} as a straight line having one end touching the circle at P, then the involute is evidently the path of the end P^{12}, as the line rolls around the circle in an anti-clockwise direction. But as a line may be regarded as a circle of infinite radius, an involute is evidently an *epicycloid* having a rolling or generating circle of infinite radius. The involute has also the properties of an archimedean spiral, and if used as a cam would impart linear motion to a point uniform with the circular movement of the cam.

EXAMPLES.

EX. 13.—Draw the involute of a circle $2\frac{1}{2}''$ diameter, and draw a normal and tangent at any point in the curve not found when constructing it. Show that the radius at any point in the curve is proportional in length to the angle passed through by the radius from the starting point of the curve.

EX. 14.—Draw the curve traced out by the end of a straight line 3″ long as it rolls round the circumference of a circle 4″ diameter. (The curve is an involute.)

EX. 15.—Draw two circles of 5″ diameter in contact at a point P. From P draw part of an involute to each circle (about 2″ long), the curves for the two circles to be in opposite directions.

EX. 16.—Draw the curve traced by a point on a straight line which rolls on a semicircle of 3″ diameter. (Vict. Hon., 1892.)

SECTION VIII.

CONSTRUCTION OF CURVES FOR TEETH OF WHEELS.

THE most common and useful practical application of cycloidal and involute curves is to shape the teeth of geared wheels. The diameter and proportions of wheels for different speeds, and the number and sizes of the teeth, in order to transmit a required power is a question not of constructive geometry, but of machine design, and owing to its difficulty will not be dealt with in this book. The object of this section is merely to give the student a sufficient knowledge of the principle and method of shaping the teeth of wheels, as to fit him better for their complete design at a later stage. But in order to effect this the following general principles must be understood :—

When two toothed wheels are in gear it is most important that their relative velocity shall not vary during the revolution—that is, one wheel must not at one instant be moving 3 times as fast as the other, and at another instant only $2\frac{7}{8}$ times as fast. This fact is expressed in mechanics by saying that *the velocity ratio of the wheels must be constant at every part of the revolution.* When two simple circular discs transmit motion by the frictional contact of their rims, without slip, it is evident that their velocity ratio is constant and is equal to $\frac{R}{R'}$, where R and R′ are the radii of the two discs. Hence, with two toothed wheels in gear, the distance from the centre of each wheel to the point of contact with the tooth of the other wheel measured along the line joining the wheel centres, must be the same for each pair of teeth, otherwise, the velocity ratio will not be constant. It follows from this, that all these points of contact must lie on the circumferences of circles described from the centres of the wheels, and that if R and R′ be the radii of these circles, the velocity ratio is $\frac{R}{R'}$, and is precisely the same as if the wheels were replaced by two friction discs of radii, R, and R′. These circles are called the *pitch circles.** The diameter of a toothed wheel is the diameter of its pitch circle, and not its diameter outside or inside the teeth. The teeth must evidently be equally spaced around the pitch circle, the distance between the centre of one

* The pitch line or pitch circle in toothed gearing corresponds to the directing line or circle of the cycloids (pp. 69, 70).

tooth and the centre of the next tooth, *measuring along the circumference of the pitch circle or the pitch line*, being called the *pitch of the teeth*. It is easy to see that if any two of the three sizes, radius of pitch circle, pitch of teeth, and number of teeth are known, the third can be found. Also that the velocity ratio of the wheels equals

$$\frac{\text{radius of driver}}{\text{radius of follower}} \text{ or } \frac{\text{number of teeth in driver}}{\text{number of teeth in follower}},$$

and that either of these is equal to $\frac{\text{revolutions of follower}}{\text{revolutions of driver}}$. It is shown in text-books on mechanics that the conditions of constant velocity ratio for toothed wheels, as specified above, is only obtained when the normal to the two teeth at the point of contact is common to both, and that this condition is met by shaping the teeth to cycloidal or involute curves. It is also necessary that the teeth should roll smoothly when in contact, and not rub or grind, a condition which is also satisfied by using these

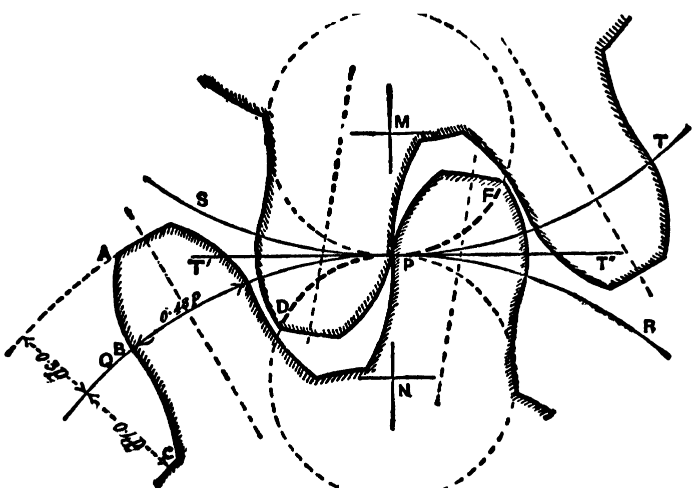


Fig. 57.

curves, for suppose a pinion (which is the name given to a small toothed wheel) is gearing with a rack as in Fig. 58, then we may suppose the rack to be fixed and the pinion to roll along it, and we see at once that a point on the pinion will describe a cycloidal path, so that if we wish to make the pinion leave the rack smoothly the teeth of the rack should be shaped to a cycloid which is exactly what is done in practice. A similar reasoning applies to the use of epi- and hypocycloids for the teeth of spur wheels.

In Fig. 57 part of two wheels are shown in gear, and sizes are marked on giving the usual proportions of the teeth, as taken from Professor Unwin's *Machine Design*, and which should be adhered to by students in working the examples of this section. They may be stated as follows:—Thickness of tooth along pitch circle 0·48 p, height outside pitch circle 0·3 p, depth inside pitch circle 0·4 p; where p = pitch of teeth, this gives a clearance between the teeth of 0·52 p. The pitch circles are marked Q P R and S P T, P being the pitch point. The *face* of the tooth is the part marked B A outside the pitch circle, and the *flank* of the tooth is the part marked B C inside the pitch circle. It is important to remember this distinction, as in working the *faces* of one wheel make contact with the *flanks* of the other wheel, and the *curves of the faces and flanks must be described with rolling circles of the same radius.*

The size of rolling circles used in drawing the curves for wheel teeth do not bear any fixed ratio to the size of the wheels, and vary with different makers. The size adopted in any particular case does not change the conditions of velocity ratio or smooth rolling, but only affects the thickness of tooth above and below the pitch circles. The first of the examples at the end of this section is intended to show the effect on the shape of the teeth of rolling circles of different diameters.

Rack and Pinion.—PROBLEM XLI. (Fig. 58).—*To draw the teeth of a rack and pinion in gear, knowing sizes of pitch and rolling circles and pitch of teeth.*

Draw the straight line Q P R to represent the pitch line of the rack, and from centre C draw the pitch circle S P T of the pinion, touching the pitch line of the rack in the point P, called the "*pitch point.*"

The faces of the rack teeth gear with the flanks of the pinion teeth, and these had better be considered first. If we decide to have radial flanks for the pinion, a usual construction, we know that they will be obtained by using a rolling circle of a diameter equal to the radius of the pinion, as this gives a hypocycloid which is a straight line. Therefore, draw a circle with centre A, and diameter equal to C P, and this will be the rolling circle for the faces of the rack teeth, which we know are to be cycloids. Then draw part of a cycloid, starting from the pitch point P, taking the pitch line Q P R of the rack for the directing line, and rolling the circle towards the right hand. The most convenient way of doing this, geometrically, since the complete curve is not required, is as follows:—Draw the locus of the rolling circle A 5′, and mark off four or five equal parts of short length, as 1′, 2′, . . 5′, and with each of these points as centre draw arcs of the rolling circle as shown. Mark the contact points of the rolling circles with the pitch line of the rack, as at 1, 2, 3, 4, 5. Then take the distance of the equal parts A 1′, 1′ 2′ (equal of course to P 1, 1 2, &c.) in the compasses, and mark off distances along each of the circles just drawn, from the points 1, 2, 3, 4, 5 to the left—that is, from 1 mark one distance, from

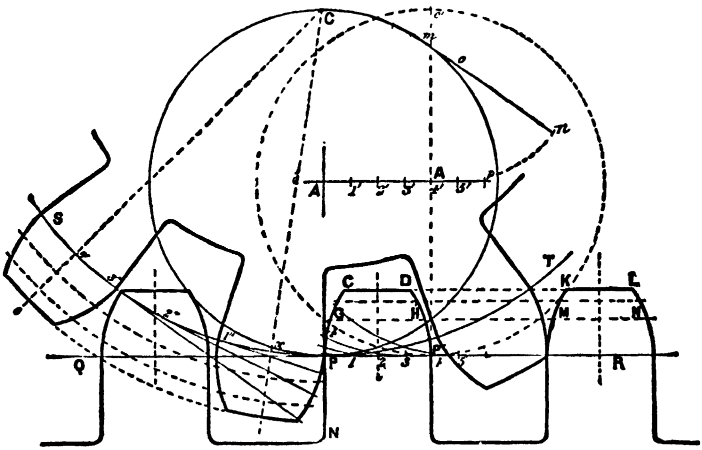


Fig. 58.

2 two distances, and so on, thus finding a sufficient number of points in the cycloid, through which the curve from P can be drawn. The radial line from P to A is the flank of the pinion tooth, in contact with the rack tooth at P.

Next consider the flanks of the rack teeth and the faces of the pinion teeth. Following a usual construction we will also make the rack teeth flanks *radial*—that is, they must be drawn perpendicular to the rack pitch line, as that may be regarded as the arc of a circle of infinite radius; the line P N, therefore, gives the flank of the rack tooth. But this line may be regarded as part of a cycloid traced by a generating or rolling circle of infinite radius, and we know, therefore, that we must take the rolling circle for the faces of the pinion teeth of infinite radius—that is, it must be a straight line. But the epicycloid traced by a line rolling round a circle is *an involute of the circle*, therefore the faces of the pinion teeth must be involutes of its *pitch circle*, S P T. Start from the point P and draw the part of an involute exactly as explained in Problem xl. (Fig. 56), and as seen in the figure, which shows the finding of three points by the tangents from the points 1″ 2″ 3″. This completes the curves of one side of the teeth of the pinion and rack in contact at P.

To finish the teeth as in the figure, set off the height of the tooth (0·3 p) above the pitch line of the rack, and outside the pitch circle of the pinion, also the depth of the tooth (0·4 p) below and inside the pitch line and circle respectively, and draw lines and circles to give the points and roots of the teeth, as shown by dotted lines. Next set off the thickness of the tooth (0·48 p) along the pitch line and circles, as at P 4 and P 1″, draw the centre lines of the teeth, mark off the pitch of the teeth along the pitch lines and circle, and draw the centre lines of the remaining teeth. The teeth are best completed by marking off distances on both sides of the centre lines at different positions, as, for example, K L = O D, M N = G H.

The flanks of the teeth should be slightly rounded at their junction with the roots.

Notice that the line Q P R is a common normal to the curves of the teeth in contact at the point P.

Setting off Pitch of Teeth.—It is important to be able to set off the pitch of the teeth correctly around the pitch circle, this can be done in one of three ways, as follows:—(*a*) Divide the pitch into a number of equal small parts, and set off the required number of parts from the centre of one tooth along the pitch circle to the centre of the other. (*b*) If N be the number of teeth in the wheel, then the angle at the centre made by the radii of two adjacent teeth $= \frac{360°}{N}$, that is, in Fig. 58, if x is the centre of one tooth, then by setting off the angle $x O y = \frac{360°}{N}$ gives the centre y of the next tooth. (*c*) In Fig 58, let $m n$ be a tangent to the pitch circle at m, make $m n$ = pitch of teeth, and mark a point o so that $m o = \frac{1}{4}$ of $m n$, then with o as centre radius, $o n$, describe an arc cutting the circle in p, then arc $m p$ $= m n =$ pitch.

Mechanical Method of Drawing Teeth.—A very convenient and, if carefully done, an accurate method of obtaining points in the curves of cycloidal teeth is as follows:—Draw upon a piece of transparent tracing paper a circle equal to the pitch circle, and draw a diameter as C A P (Fig. 58). To the right and left of P set off along the circumference a number of equal parts, and mark 1, 2, 3, . . Set off the same parts from P along the pitch line towards R, and mark as before 1, 2, 3, . . . Draw lines through each point perpendicular to the pitch line, then place the tracing paper over each line, so that the diameter C A P coincides with the line, and the point P with the point, and prick through the correspondingly marked point on the circle to the left. Thus if the tracing paper be placed so that the diameter C A P covers the line $c' 4' 4$, P coinciding with 4, then the point in the cycloid is at the point marked 4 on the circumference of the circle which is shown dotted. Repeat the process until sufficient points have been found.

This method may be equally well applied for epicycloidal and hypocycloidal teeth, and should be practised by the student.

Spur Wheels.—In drawing the teeth of spur wheels, the faces are made to epicycloidal and the flanks to hypocycloidal curves. Two spur wheels are shown in gear in Fig. 57, the pitch circles being Q P R and S P T, touching at P, and for convenience the rolling circles M and N are taken the same size. Start by rolling the circle M to the right *outside the circle* Q P R, and trace the curve for the face of tooth P, then roll the circle N to the left *inside the same circle* for the flank of the tooth. Similarly for the tooth on the other wheel, roll the circle N to the left *outside the circle* S P T for the face, and the circle M to the right *inside the circle* Q P R for the flank. Then complete the teeth as in the example of the rack and pinion.

Involute Teeth.—Wheels with involute teeth possess certain practical advantages as compared with cycloidal teeth due to the fact that the path of the points of contact of the teeth is a straight line, and not as with the latter a changing curve. This results in the angle which the direction of the pressure between the teeth (*called the angle of obliquity*) makes with the common normal at the point of contact being constant and not variable, as with cycloidal teeth. In consequence of these facts, wheels with involute teeth will work smoothly and regularly even if the distance between the centres be slightly altered, and they also exert a more uniform pressure on the axle bearings. Another advantage is that all involute teeth of the same pitch will gear together.

Turning to Fig. 57, the path of the points of contact between the teeth as the wheels revolve is that portion of the two rolling circles drawn in dark lines and marked D P F, and the angle between the tangent to this curve and the common tangent $T' T''$ is called the angle of obliquity, which alters with different points of contact. In Fig. 59 are shown some involute teeth in gear, and here the path of contact is the straight line

D P F, the angle of obliquity being the angle between this line of contact and the common normal T T′, this angle remaining constant for all positions of contact. The line of contact is a normal to the two curves at their point of contact.

The angle of obliquity for involute teeth can be decided before drawing the teeth, and is generally made equal to 15°.

PROBLEM XLII. (Fig. 59).—*To draw involute teeth for two spur wheels, knowing the pitch circles and the angle of obliquity.*

Draw the pitch circles touching at the point P, and the line T T′ through P at right angles to the line joining the centres. Next draw a line D P F through the point P making an angle of 15°, the given angle of obliquity with the line T T′. With the centres of the pitch circles draw circles (shown dotted in figure and marked B C) tangent to this line D P F. These are called "*base circles*," and are the circles of which involutes must be drawn for the shape of the teeth, in order that the path of contact may be in the line D P F, and the angle of obliquity 15°.

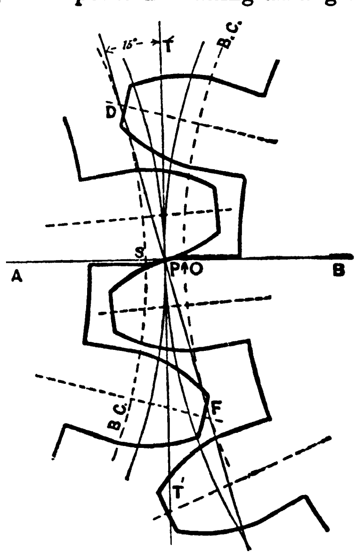


Fig. 59.

To draw the teeth of wheel A we see that D P is a tangent to the circle, and, therefore, if S be the starting point of the curve, so that it passes through P, then the arc D S must equal D P. Therefore, from D, the contact point of the circle B C with the line of contact D P F, set off D S = D P by either method (*a*) or (*c*) of setting off the pitch (p. 81). Then start from S, and draw the involute as before. To draw the tooth of wheel B, so that it shall be in contact at P, we see that F P is a tangent to B, and thus as before we make arc F O = F P, and start the involute from the point O. Since contact does not take place at any point of the teeth within the base circles, the flanks of the teeth from within the base circles are made radial—that is, the points where the curves start from the base circles are joined to the centres of the circles as shown, thus giving straight teeth from the base circle to the root circle.

The teeth are completed as in previous examples.

If the point circle for the tops of the teeth of wheel B cuts the line of contact D F in D, and the point circle of the teeth of wheel A cuts it in F, then D F is the length of the line of contact.

EXAMPLES.

EX. 1.—Draw an arc of a circle of 8″ radius and consider it as part of the pitch circle of a toothed wheel. Then draw completely four teeth as follows :—*a*, *b*, and *c* to have epicycloidal faces and hypocycloidal flanks, rolling circles, (*a*) 4″ diameter, (*b*) 2½″ diameter, (*c*) 5½″ diameter, (*d*) involute teeth radius of base circle 7·7″. To get sizes of teeth assume them as of 2″ pitch. (The object of this example is to show the effect of using rolling circles of different sizes, hence the teeth had better be drawn separate from each other, say about 3″ centres.)

* EX. 2.—Draw a rack and pinion showing four teeth on each as follows :—Pitch of teeth 2″, number of teeth in pinion twenty; pinion teeth to have involute faces and radial flanks, rack teeth to have cycloidal faces and radial or straight flanks.

* EX. 3.—Draw two equal spur wheels in gear showing five teeth in each. Pitch of teeth 2½″, number of teeth ten. Faces of teeth epicycloids, flanks hypocycloids. Rolling circles 1⅜″ radius.

* EX. 4.—Draw two spur wheels A and B in gear, showing five teeth in each. Involute teeth 2″ pitch, twelve teeth in wheel A, seventeen in wheel B, angle of obliquity 15°.

EX. 5.—The diameters of two spur wheels are 24″ and 36″, the pitch 2½″, and the path of contact a straight line at 75° to the line of centres. Draw a pair of teeth in contact of such length that two pairs of teeth may always be in contact. (Vict. Hon., 1891).

(Length of path of contact must be twice *normal pitch*, the normal pitch is distance from face of one tooth to face of next along line of contact; make a right angled triangle with hypotenuse equal to pitch, and base angle equal angle of obliquity—then base is normal pitch.)

* After drawing accurately three or four teeth on each wheel of Exs. 2, 3, 4, the student would do well to work as follows :—Cut the paper carefully round the teeth, leaving enough for the whole wheel, thus making a pattern of the wheels; and fix the wheels the right distance apart by sticking pins through their centres. Then place a sheet of paper beneath the teeth, and move the wheels as in actual working; prick through at the point of contact of the teeth upon the paper below, thus obtaining *the path of the points of contact*, compare the results with Figs. 57 and 59. Also draw the normals to the teeth at one or two different points of contact, and see if the normal at the point of contact of any two teeth is common to the curves of both teeth.

SECTION IX.

SOLID GEOMETRY.

Projection of Points, Lines, Surfaces, and Simple Solids.—Solid geometry or orthographical projection, as its name implies, deals with the drawing of solids, and enables the three dimensions of a solid *length*, *breadth*, and *thickness* to be shown upon a flat surface, such as a sheet of drawing paper. It must not be confounded with perspective, with which it has no connection, beyond the fact that both use many similar methods and terms, as perspective geometry depicts a solid as it appears to the eye to be, and shows its three dimensions in one drawing, whereas solid geometry depicts a solid as it really is, and requires at least two separate drawings to show its three dimensions. If, for example, we look down upon the top of a table, we see a view of the table which gives no idea of its height from the floor, but only shows its width and length, while if we look on one end of the table with our eyes on a level with the table top, we then see the height of the table, but cannot form any idea of its length. The principles of solid geometry recognise these facts, and suppose a solid to be looked at from different positions, and views to be drawn of its appearance from each position. These views have distinct names and are drawn in accordance with certain laws of projection, which it is very important should be clearly understood.

From the illustration of the table, we see that by looking at it from two different positions, we are able to show its three dimensions, length, width (or breadth), and height. The way in which solid geometry enables us to draw these two views is by supposing that the view of the table looking from above is drawn upon the floor underneath it, and that the view looking on the table end is drawn upon the wall at the further side of it. The floor and the wall are flat surfaces, called *planes*, mutually at right angles, and the way in which the views are depicted upon them, is to suppose that lines are drawn from each corner of the table perpendicular to the floor and wall, meeting these surfaces in points, which, if joined by lines, will form the two views of the table. Now, if we suppose the floor and wall to have been covered with a sheet of paper, and the drawings made upon it, and then the paper spread out flat, we should possess what is recognised as a drawing of the table, showing practically all its dimensions.

Plan, Elevation, Projections, Planes of Projection, Projectors.—The view of the table, and therefore of any solid, as seen from above is called its *plan*, and the view as seen from the end its *elevation*. The views of objects drawn on the principles of solid geometry are called its *projections*, the imaginary planes on which they are drawn are called the *planes of projection*, and the lines from the object to the planes are called *projectors*. We have seen that only two projections are necessary—one on the floor, a horizontal plane, the other on the wall, a vertical plane, and as these two planes are always required for the plan and elevation of a solid, they are termed the *horizontal plane of projection*, usually denoted by the capital letters HP, and the *vertical plane of projection*, usually abbreviated to V P. Evidently the two planes of projection intersect in a line, which, owing to the horizontal plane being supposed the plane of the ground, is called the *ground line*, and is generally denoted by the capital letters X Y.

All this will be clearly understood by reference to the following example.—In Fig. 60*a* is shown a representation of a simple

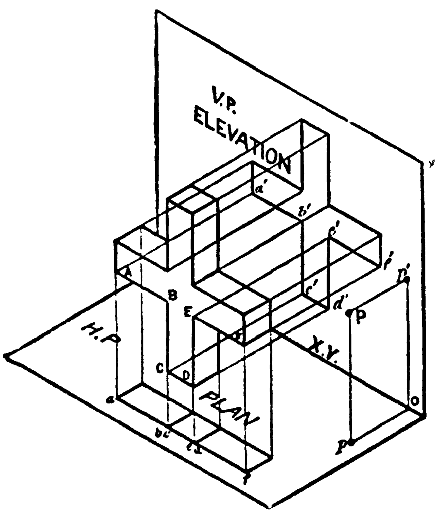


Fig. 60a.

solid, an equal armed cross made of square wood, with the horizontal and vertical planes of projection, having its plan and elevation drawn upon them Lines perpendicular to the H P are drawn through each corner of the solid, meeting the H P in the points marked *a*, *b*, *c*, *d* . . ., and similarly lines perpendicular to the V P are drawn through each corner, meeting the V P in the points marked *a'*, *b'*, *c'*, *d'* . . . To join these points in the right order we look at the solid, and see that A joins B, and that B joins C, and C joins D, and so on, and, therefore, by joining *a* to *b*, and *a'* to *b'*, &c., we obtain on the H P and V P a plan and an elevation of the solid. Notice that the plan really represents two faces of the cross, the upper and lower, which are similar, and, therefore, that each point in the plan shows at least two corners of the cross, and similarly with the elevation.

On the right hand of the cross is shown a point P with its plan *p*, and its elevation *p'*, the line P*p* being its projector to the H P, and the line P*p'* its projector to the V P. The plan and elevation of these projectors are drawn at *p o* and *p' o*, and it should be specially noticed that they make two lines, meeting on the ground line, each being perpendicular to it. The student should suppose that the plan and elevation of the projectors of the cross are drawn in this way, although in the figure they are omitted for the sake of clearness.

Now, imagine the V P to be turned upon the ground line as a hinge, away from the solid, as shown by the arrow, until it becomes horizontal and forms a continuation of the H P. We shall then have the representation of Fig. 60*b*, which is the usual solid geometry projection or drawing, showing a plan and elevation upon a single flat sheet of paper. With the aid of these two figures the student should now verify the following statements, all of which are important and should be remembered :—

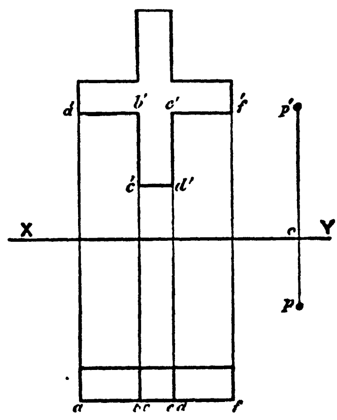


Fig. 60*b*.

(*a*) The plan is *below* the ground line, and the elevation *above* it.

(*b*) The plan and elevation of the same point are exactly one under the other, in a line perpendicular to the ground line, therefore the plan of a solid should be directly under the elevation.

(*c*) Heights above the H P are shown in the elevation.

(*d*) Distances in front of the V P are shown in the plan.

(*e*) The projectors are shown by lines, joining points in the plan and elevation, and perpendicular to the ground line.

(*f*) The elevation of a point in the H P, and the plan of a point in the V P, are shown by a point on the ground line, for if *p* and *p'* be these points, their elevation and plan are both shown by the point *o* (Fig. 60*b*). From this it follows that when a solid has one face or edge in the ground plane or H P, its elevation will begin from the ground line, and similarly if it has a face or edge in the V P its plan will also begin from the ground line.

We see from Fig. 60*b* that the plan and elevation are separated from one another, and that the distance between them depends only on the height of the solid above the H P and its distance in front of the V P.

In examples of solid geometry these distances may be given of any desired length, or may be left to the will of the student, in which case it is convenient to assume the solid, as standing on the H P and in front of the V P, as this gives an elevation starting from the ground line and a plan removed from it, thus separating the two drawings and adding to their clearness.

Marking Plans and Elevations.—It will have been noticed in Fig. 60*a* that each point in the solid is denoted by a capital letter, as A, B, C, while its plan is marked by the same letter in small type, as *a*, *b*, *c*, and its elevation by a similar letter with the addition of a dash, as *a'*, *b'*, *c'*. This is a convenient notation, usually adopted in solid geometry, and will be adhered to in all following examples.

A solid is bounded by surfaces, a surface by lines, and a line by points, and we shall, therefore, lead up to the projection of solids by examples dealing with points, lines, and surfaces.

In commencing solid geometry it will be found very helpful to make up a rough model of the planes of projection, and of the objects to be drawn. A book or instrument box opened at right angles very well represents the H P and V P, a drawing pin may represent a point, a pencil a line, and a set square or piece of card a surface or plane, while models of simple solids can be easily made. It is only in this way that the beginner can hope to gain an intelligent and useful knowledge of the subject, and be able to proceed with confidence to advanced problems and to machine drawing, where the objects to be drawn exist only as a mental picture, and where their positions relative to, and their projections upon, the planes of projection have to be vividly imagined before they can be represented upon the paper. All engineering draughtsmen use the results of the principles of solid geometry, although, as the student will see in due course, they appear to dispense with the actual use of projectors, ground line, planes, &c.

It will be seen from Fig. 61*a* that the H P and V P are carried on so as to extend on both sides of the ground line X Y. This is evidently correct, as a plane has no limit of either length or breadth. When thus regarded, the planes of projection are said to form four dihedral angles (angles formed by surfaces), and a point may be regarded as being in either one of the angles; as, for example, a point may be below the H P and behind the V P, and the position of its plan and elevation relative to the ground line are affected accordingly; but as this is a matter of theoretical rather than of practical importance, it will not be further considered, and reference will be made to the first dihedral angle only.

As the position of points, lines, and solids can only be stated as distances from the planes of projection, which, as we have seen, resolves itself into distances below and above the ground line, it is evident *that in all examples we must commence by drawing the ground line.* It should be noticed that when this is done the paper above the X Y represents the V P, and the paper below it the H P, and that if the paper be bent about the X Y, as a hinge, bringing the V P into a vertical plane, it will represent a model of the planes of projection.

The following points should be particularly observed:—

All construction lines, such as projectors, should be drawn as fine light lines, and the projections or plans and elevations of the line, figure, or solid being drawn, should be shown by dark lines.

Lines to represent the edges of a solid, not actually seen, owing to some part of the solid being between them and the eye of the observer, should be shown by dark *dotted* lines.

Projection of Points.—To show the projections of a point given its distance above the H P and in front of the V P, first draw the X Y, then through any point in it draw a perpendicular line to represent the projectors of the point, mark a point in this line above the X Y, equal to the height of the point above the H P, and a point in the line below the X Y, equal to the distance of the point in front of the V P. If the point is denoted by the letter P, its plan should be marked p and its elevation p'. When the distances are given in the question, it is better that they should be marked on the drawing, using dimension lines with arrow heads as in Fig. 50. The student should also aim at writing above the drawing a brief description of what the drawing represents (not a mere copy of the question), whether of a point, line, surface, or solid, and its special position relative to the planes of projection, as it is just as important to know exactly what position is indicated by a given drawing as to be able to make the drawing of a solid in a given position.

EXAMPLES.

EX. 1.—Draw the projections of the following points distinguishing the plan and elevation of each :—

(*a*) Point A in both planes.
(*b*) Point B in H P, 1¾″ in front of V P.
(*c*) Point C in V P, 1¾″ above H P.
(*d*) Point D, 1·8″ from both planes.
(*e*) Point E, 2·6″ above H P, 1·7″ in front of V P.
(*f*) Point F, 2·4″ below H P, 1·5″ behind V P.
(*g*) Point G, 2″ above H P, 1·9″ behind V P.
(*h*) Point H, 2¼″ below H P, 1·6″ in front of V P.

Projection of Lines.—Lines may be parallel to, perpendicular to, or inclined to either the H P or V P, and in some cases to both. Lines may also be contained by, or may lie in, either

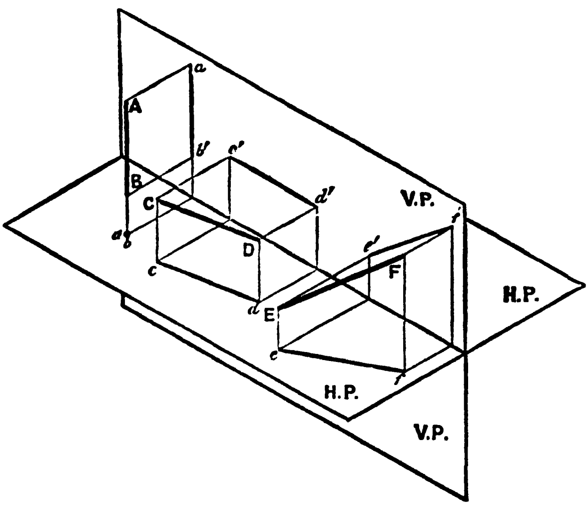


Fig. 61*a*.

or both of the planes. As the ends of a line are points, a line is spoken of as the line A B or C D, one letter being marked at each end, its plan is then marked *a b* or *c d*, and its elevation *a′ b′* or *c′ d′*. A line is fixed by stating its position relative to the planes of projection, both with regard to its inclination to them, and its distance from them, hence these conditions must be known before the projections of the line can be drawn.

Three lines, A B, C D, E F, each differently placed with regard to the planes of projection, together with their projectors, and their plans and elevations upon the H P and V P are represented in Fig. 61*a* in such a way as to show the principle of projection. In Fig. 61*b* the two planes are shown with the V P thrown down, thus forming one horizontal sheet, and showing the plans and elevations of the lines exactly as they should appear when drawn upon the paper. The positions of the lines are as follows:—

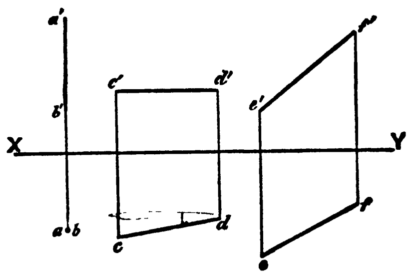


Fig. 61*b*.

A B is perpendicular to the H P and above it, parallel to the V P and in front of it.

C D is parallel to the H P and above it, inclined to the V P and in front of it.

E F is inclined to both planes, and removed from both.

With the help of these figures and of a model of the planes and a pencil to represent a line, the student should carefully verify the following statements:—

(*a*) **When a line is parallel to, or is contained by, the H P its plan is equal in length to the line itself.**

(*b*) **When a line is parallel to, or is contained by, the V P its elevation is equal in length to the line itself.**

(*c*) **When a line is parallel to both planes its plan and elevation are equal in length to the line itself.**

Therefore, when a line is parallel to, or is contained by, a plane, its projection upon that plane is a line equal in length to the line itself.

(*d*) **When a line is inclined to the H P its plan is shorter than the line itself.**

(*e*) **When a line is inclined to the V P its elevation is shorter than the line itself.**

(*f*) **When a line is inclined to both the H P and V P its plan and elevation are both shorter than the line itself.**

Therefore, when a line is inclined to a plane its projection upon that plane is a line, the length of which is less than the length of the line itself.

(*g*) **When a line is perpendicular to the H P its plan is a point.**

(*h*) **When a line is perpendicular to the V P its elevation is a point.**

Therefore, when a line is perpendicular to a plane its projection upon that plane is a point.

(*i*) **When a line is contained by both the H P and V P its plan and elevation coincide in the ground line.**

(*j*) **When a line is inclined to the H P and parallel to the V P its inclination is shown in the elevation.**

(*k*) **When a line is inclined to the V P and parallel to the H P its inclination is shown in the plan.**

Therefore, when a line is inclined to one of the planes of projection and parallel to the other, its inclination is shown upon the plane to which it is parallel. It will be seen later that when a line is inclined to both planes its inclination is not shown either in the plan or elevation.

The projections of a line should present no difficulty if it is remembered that the ends of the lines are points, whose projections can be found as already described. For if the plan and elevation of the points be joined, the joining lines will be the plan and elevation of the line having the points for its ends. Notice also, *that when a line is inclined to one of the planes, its projection upon the other plane must be drawn first.*

EXAMPLES.

EX. 2.—Draw the projections of a line 3½″ long, in the following positions, mark each end of the line in plan and elevation with letters, and mark the lengths and inclination of the lines. Write above each its position with regard to the H P and V P.

(*a*) Parallel to both planes and in both.

(*b*) Parallel to both planes and 1·6″ from each.

(*c*) Parallel to both planes, 1″ above H P, 2·3″ in front of V P.

(*d*) Parallel to both planes, 2″ above H P, 1·7″ in front of V P.

(*e*) Inclined 60° to H P, one end in H P; parallel to V P, 1·3″ in front.

(*f*) Inclined 45° to H P, one end 1·4″ above H P; parallel to V P, 1½″ in front.

(*g*) Inclined 60° to V P, one end in V P; parallel to H P, 1·3″ above.

(*h*) Inclined 45° to V P, one end 1″ in front of V P; parallel to H P and in H P.

(*i*) Parallel to V P and 1″ in front, its ends 1″ and 2½″ above H P. Show angle of inclination to H P.

(*j*) Parallel to H P and 1½″ above, its ends 1·3″ and 2·7″ in front of V P. Show angle of inclination to V P.

(*k*) Perpendicular to H P and in V P, one end in H P.
(*l*) Perpendicular to H P, one end $1\frac{1}{2}''$ above, $1''$ in front of V P.
(*m*) Perpendicular to V P, one end in V P, $1\frac{1}{4}''$ above H P.
(*n*) Perpendicular to V P and in H P, one end $1''$ in front of V P.

EX. 3.—The projectors of a line are $2\frac{1}{2}''$ apart, measuring along the X Y. The line is parallel to the V P, and $1\frac{1}{2}''$ in front, and is inclined at 60° to the H P, one end being in the V P. Draw the plan and elevation.

Traces of Lines.—When a line is inclined to a plane, it will evidently meet that plane if produced far enough. The *point* where the line meets the plane is called its *trace*. The horizontal trace, H T, of a line, is the intersection of the line with the H P, and the vertical trace, V T, its intersection with the V P.

If this definition is understood, no difficulty should be experienced in finding the traces of a line. When a line is parallel to a plane, it will, of course, have no trace upon that plane, but when it is inclined to a plane with one end in the plane, that point is its trace upon that plane, therefore, when a line is inclined to a plane without meeting it, it has only to be produced to meet the plane, and its trace will be the meeting point. Notice that the H T of a line must be in its plan, or the plan produced, and the V T of a line must be in its elevation, or the elevation produced. Thus, in Fig. 62*b*, the traces of the doubly inclined line A B are at the points marked H T and V T, the manner in which these traces are found being clear from the construction.

EXAMPLES.

EX. 4.—Draw the projections of a line as in Ex. 2, *e*, *f*, *k*, *l*, and find the H T of the line.

EX. 5.—Draw the projections of a line as in Ex. 2, *g*, *h*, *m*, *n*, and find the V T of the line.

EX. 6.—The end A of a line A B is $1''$ above the H P and $2\frac{1}{2}''$ in front of the V P, the end B is $2\frac{3}{4}''$ above the H P and $\frac{1}{2}''$ in front of V P, the projectors measuring along the X Y being $2\frac{1}{2}''$ apart. Draw the projections of the line, and find its H T and V T.

True lengths of Lines.—We have seen that when a line is inclined to both the H P and V P, its true length and inclinations are not seen, either in the plan or elevation, and we must now examine a method whereby the true length and inclination of the line can be ascertained. As there are conditions which occur in other problems of practical importance besides those of solid geometry, the student is advised to study closely the construction employed. Fig. 62*a* represents the H P and V P in position, with a line A B inclined to both, and its plan and elevation *a b*, *a′ b′*, the traces of the line being shown at H T and V T. If we fix our attention on A B, the line itself, and on *a b*, its plan, we can suppose that A B is held in position by the projectors A *a* and B *b*, which pass from the ends of the line itself to the ends of its plan, *perpendicular to the plan*. Suppose, further, that the line, its plan, and its projectors to the H P form a stiff frame, which can be turned about the plan of the line as a hinge, until it falls into the H P, as shown at A′ *a* and B′ *b*, then we have the true length of the line A B shown upon the H P at A′ B′, and we see at once that this true length is found by drawing perpendiculars from each end of the plan of the line, and making them equal in length to the distance of that end above the H P. A similar reasoning applies to the line and its elevation *a′b′*, together with its projectors to the V P, for if these be turned about the elevation *a′ b′* as a hinge, until they fall into the V P, we shall have the true length of the line shown in the elevation exactly as in the plan. The figure also shows that if the true length of the line in plan be produced, it will meet the H T, and similarly in the elevation (Fig. 62*b*), for by this construction we have produced the line to meet the two planes of projection, and we know the meeting points are the traces. The figure further shows that the real inclination of the line to the H P is the angle between the line itself, A B, and its plan *a b*, produced to meet at the H T, and that this is equal to the angle between the true length of the line A′ B′ and the plan *a b*—that is, the angle marked θ (theta);* and similarly the real inclination of the line to the V P is the angle between the line A B, and its elevation *a′ b′* produced to meet at the V T, and that this is equal to the angle marked φ (phi), between the lines A″ B″ and *a′ b′* (see Fig. 62*b*).

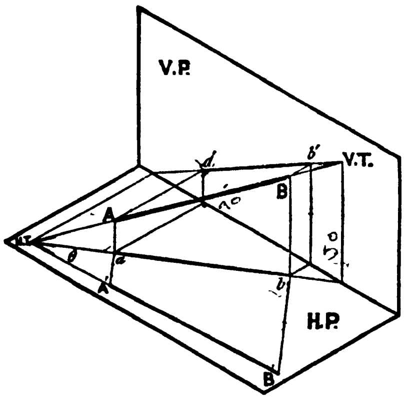


Fig. 62*a*.

* The construction for the true length of the line in the elevation is omitted in Fig. 62*a* for the sake of clearness.

PROBLEM XLIII. (Fig. 62*b*).—*Given the plan and elevation of a line to find its real length, its traces, and its inclination to the H P and V P.*

Let *a b a′ b′* be the given plan and elevation. From each end of the plan draw perpendicular lines equal in length to the height of the end above the H P, thus *a* A′ equals height of end A, and *b* B′ equals height of end B. Then A′ B′ is the true length of the line, and its inclination to the H P is shown by producing A′ B′ and *a b* to meet in the H T, thus making the angle marked *θ*. Proceed in the same way from the elevation *a′ b′*, obtaining the true length A″ B″, the inclination to the V P shown by the angle *φ* and the V T. As a test of accuracy see that the real lengths in plan and elevation are the same.

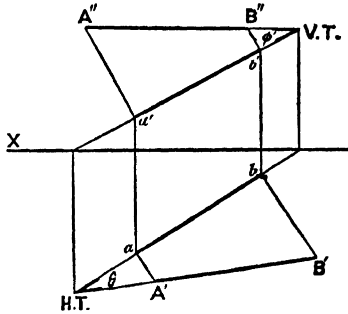


Fig. 62*b*.

EXAMPLES.

EX. 7.—Draw the projections of a line A B, as in Ex. 6, and find its real length and its inclination to the H P and V P.

EX. 8.—A is a point in the V P, 1¼″ above the H P. B is a point in the H P, 1¾″ from the V P. The real distance from A to B is 3″. Draw the plan and elevation of the line joining A and B. (S. & A. Elem., 1887.)

EX. 9.—A point 1·5″ from both planes of projection is distant 3·25″ from another point, 2·25″ from both planes of projection. Obtain the projections of the two points. (S. & A. E., 1888.)

EX. 10.—Three equal lines 1½″ long, A O, B O, C O, meet at a point O at equal angles with each other. Draw the plan of the lines when neither of them is parallel to the V P, and consider them as the plan of three equal rods, 3½″ long, forming a tripod stand standing in the H P, then draw the elevation of the rods, and find their inclination to the H P.

PROBLEM XLIV. (Fig. 63).—*To draw the projections of a line of given length inclined to both the H P and V P.*

Let the line be A B, and its inclination to the H P and V P be *θ* and *φ* respectively.

From any point, A, in the X Y draw a line A *b′* equal in length to the given line A B, and making an angle with the

X Y equal to the inclination of the line to the H P. Then A *b'* is the elevation and A *b* the plan of the line, supposing it to be in the V P. Let the end *b'* remain in the V P and the end A be moved away from it, then so long as the end A moves in the H P its path must be in the semicircle, having *b* for centre and *b* A for radius, while the further it moves away from the V P the greater inclination will it have to the V P, and the shorter will its elevation become. If, then, we can determine what length its elevation will be, when its inclination to the V P is φ, we can draw its elevation, knowing that the position of the end *b'* has not altered. The last problem enables us to do this for we saw then that when a line is inclined to the V P and has one end in the plane, its elevation, its real length, and the perpendicular from the end not in the plane, make a right angled triangle, of which the acute angle at the base is the angle of inclination to the V P. In the present case we know the hypotenuse of the triangle, the true length A *b'*, and the acute base angle, the inclination φ, and we can, therefore, find the length of the elevation of the line. This is shown at *b'* A', the angle A *b'* A being made equal to φ and the angle at A' a right angle (see Prob. xliii.) Make, therefore, *b' a'* equal to *b'* A' and this will be the elevation of the line, its plan can be found by making the projector *a' a* equal to A A', for this we know is the distance of the end A in front of the V P, or by dropping a projector from *a'* to meet the semicircle in *a*, then *a b* is the plan of the line.

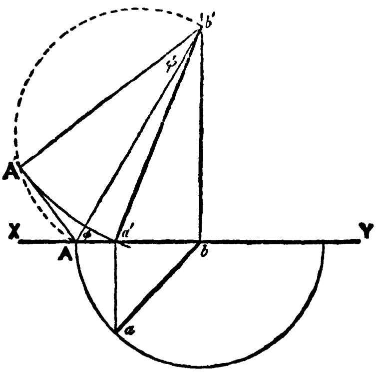


Fig. 63.

EXAMPLES.

EX. 11.—Draw the plan and elevation of a line A B 3½" long, inclined (*a*) 50° to the H P and 30° to the V P, (*b*) 25° to H P and 55° to V P.

EX. 12.—Draw the projections of a line C D 3" long, inclined 60° to the H P and 30° to the V P.

Projection of Plane Figures and Surfaces—Planes and

Traces of Planes.—Since plane figures and surfaces only possess two dimensions, length and breadth, it is only possible to draw their projections according to the principles of solid geometry, by supposing them to be contained by planes, the position of which relatively to the H P and V P of projection, coincides with the position of the figure. But in order that this principle may be appreciated, it is necessary to understand how planes are represented, since they are simply flat surfaces indefinite in extent, without thickness.

Planes.—If the student will take a model of the planes of projection, such as a book open at right angles, and a set square to represent a plane, he will see that the plane can be placed in many different positions relative to the H P and V P. It can, for example, be placed so that its surface is perpendicular to both planes and touching both, or touching one and removed from the other; or the plane may be inclined to either the H P or V P, and have its surface at the same time perpendicular to the other, or the set square can be so placed as that its surface is inclined to both planes. It does not follow that the set square, or the supposed plane, will necessarily meet the H P and V P in the position in which it is placed; but since a plane is indefinite in extent, it is evident that if produced far enough it will somewhere intersect the H P and V P, unless parallel to them, and that the intersections will be *lines*, making certain angles with the ground line, depending upon the position of the plane. These lines of intersections are called the *traces* of the plane—that is, "*the trace of a plane is its line of intersection with another plane.*" The line where a plane intersects the H P plane of projection is termed its "*horizontal trace,*" H T, and the line where it intersects the V P, its "*vertical trace,*" V T. Notice the distinction, that the *trace of a line is a point*, and the *trace of a plane a line.* From these considerations, as well as from the results of the little experiments first mentioned with the book and set square, we learn that *a plane can only be shown by its traces.*

Fig. 64 represents the traces of four planes in the only way in which they can be shown upon a flat sheet of paper, and with the help of a model the student should verify the following positions for the planes as shown:—

Plane A.—Perpendicular to both the H P and V P.

Plane B.—Perpendicular to H P, inclined to V P.

Plane C.—Inclined to H P, perpendicular to V P.

Plane D.—Inclined to both H P and V P.

It will be seen that the traces are produced beyond the **X Y**; this is the usual practice, and serves to show that the plane is not limited by the H P and V P.

With the help of Fig. 64, and of a model of the H P and V P, together with pieces of cardboard or set squares, to represent planes, the student should verify the following :—

(*a*) A plane parallel to the H P or V P has no H T or V T respectively; therefore, when a plane is parallel to another plane, it has no trace upon that plane. Also, a plane parallel to one of the planes of projection is perpendicular to the other plane.

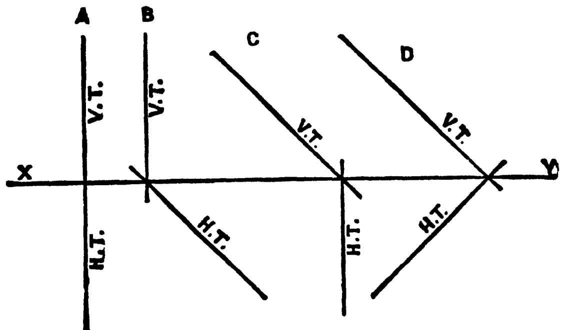


Fig. 64.

(*b*) When a plane is perpendicular to the H P its V T is a line perpendicular to the ground line; and when it is perpendicular to the V P, its H T is a line perpendicular to the ground line.

(*c*) When a plane is inclined to the H P, and perpendicular to the V P, its inclination is shown by the angle which its V T makes with the ground line; and similarly when a plane is inclined to the V P, and perpendicular to the H P, its inclination is shown by the angle which its H T makes with the ground line.

(*d*) When a plane is perpendicular to both the H P and V P, its traces form one straight line, perpendicular to the ground line.

(*e*) When a plane is inclined to both the H P and V P, its traces make angles with the ground line, which are not equal to the angles of inclination of the plane to the H P and V P.

(*f*) The H T and V T of a plane intersect in the ground line, whenever the plane has two traces, except in the case where the plane is inclined to both the H P and V P, so that the sum of its inclination equals 90°, when its traces are parallel to the ground line.

(*g*) Parallel planes have parallel traces.

It should be noticed that a number of planes may be arranged relatively to the H P and V P in such a way as to intersect each other. In such cases the lines of intersection are represented by their plans and elevations.

EXAMPLES.

EX. 13.—Show the following planes by their traces, and mark the traces as in Fig. 64 :—

(*a*) Plane perpendicular to both H P and V P.

(*b*) Plane inclined 45° to H P, perpendicular to V P.

(*c*) Plane perpendicular to H P, inclined 30° to V P.

(*d*) Plane perpendicular to H P, parallel to V P 1″ in front.

(*e*) Plane parallel to H P 1½″ above, perpendicular to V P.

(*f*) Plane inclined at any angle to both planes. (The method of drawing a plane inclined at given angles to the H P and V P is too advanced for insertion here.)

EX. 14.—Represent by their traces (*a*) two planes at right angles to each other and to the V P, one of them inclined at 40° to the H P. (*b*) Two parallel planes not at right angles to either plane of projection. (S. & A. E., 1886.)

The Projection of Lines contained by Planes.—Before proceeding to the projection of inclined plane figures or solids, it is necessary to understand the principles by which lines contained by given planes are projected. The student should first verify the following statements by using a pencil; a set square, or a piece of card to represent a plane, and a model of the planes :—

(*a*) When a line is contained by a plane its inclination cannot be greater than the inclination of the plane.

(*b*) When a line contained by a plane has the same inclination as the plane, it is perpendicular to the trace of the plane, hence the plan of a line lying in a plane inclined to the H P, and having the same inclination to the H P as the plane is perpendicular to the H T of the plane, and similarly the elevation of a line lying in a plane inclined to the V P, and having the same inclination to the V P as the plane is perpendicular to the V T of the plane.

(*c*) Horizontal lines lying in a plane have their plans parallel to the H T of the plane.

(*d*) Parallel lines lying in the same planes have parallel projections.

(*e*) A line may be contained by a plane and may be inclined at any angle between zero and the inclination of the plane.

PROBLEM XLV. (Fig. 65).—*Given the traces of a singly inclined plane to draw the projections of a line of given length contained by the plane, and having an inclination less than the angle of the plane.*

Let H T and V T be the traces of the given plane inclined at an angle θ to the H P, and let the angle of the required line A B be α, and be less than θ. At any point S in the V T draw a line S T making an angle α with X Y equal to its given inclination to the H P. If S T is longer than the required length A B, make S W equal to A B, and through W draw a line parallel to the V T, meeting the X Y in B′, and from B′ draw a line B′ a' parallel to S T meeting the V T in the point a'. Then a' B′ is the elevation of a line of the required length, and having the required inclination α to the H P, and a B is its plan, supposing the line to be in the V P. But the line a B has one end A as at a' in the plane,

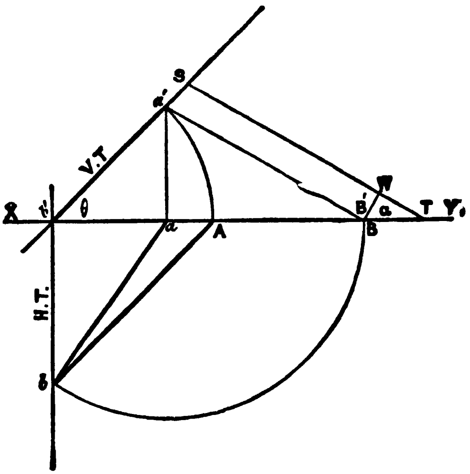


Fig. 65.

since that end is in the V T of the plane, and we know the length of its plan a B will not be altered so long as we do not alter its inclination to the H P.

Therefore, with the point a as centre, and the plan length a B as radius, draw an arc cutting the H T in the point b and join b to a, then $b\,a$ is the plan of the line lying in the plane and b' a' its elevation, because as the line has one end B in the H T and the other end A in the V P the line must lie wholly in the plane. If now the plane be turned into the H P about its H T as a hinge, the point a' will travel in the arc a' A drawn from b as a centre, and the line A b will be the true length of A B, and will, therefore, of course, be equal to a' B′.

EXAMPLES.

EX. 15.—Draw the plan and elevation of a line A B 3″ long inclined 30° to the H P lying in a plane inclined 45° to the H P.

EX. 16.—Draw the projections of two lines of any length meeting in a point and lying in a plane inclined 60°, one line being inclined 30° and the other 40°. Show also the real angle between the two lines.

EX. 17.—The H T of a vertical plane makes 35° with the X Y. Draw the elevation of a line lying in this plane inclined at 25° and passing through the X Y. (S. & A. E., 1891.)

Projection of Inclined Plane Figures.—We are now in a position to use planes in order to obtain the projections of plane figures, such as make up the faces of solids, when inclined to either the H P and V P. Their projections when contained by, or parallel to, either the H P and V P follows too naturally from the projection of lines to need explanation, as it is evident that the plan of a plane figure lying in the H P is the figure itself, its elevation being a line on the X Y, and *vice versâ* when contained by the vertical plane.

When a plane figure is inclined to one of the planes, say, a hexagon inclined at 30° to the H P, we suppose it to be contained by a plane making the same angle with the H P, and we see at once that if the plane containing the hexagon be thrown down into the H P, turning about its H T as a hinge, we shall then see the hexagon its true shape and size, and can, therefore, draw it in that position. Now let the plane with the hexagon be turned up again to its proper angle of inclination, we see that every corner of the hexagon describes an arc of a circle having the H T of the plane as a centre, and we can thus obtain the elevations of the corners of the hexagon when inclined, and from the elevations obtain the required plan, which we know will not be a regular hexagon because of its inclination. The modification demanded by a side of the hexagon being inclined at an angle differing from the inclination of the plane has already been explained in Problem xlv., Fig. 65.

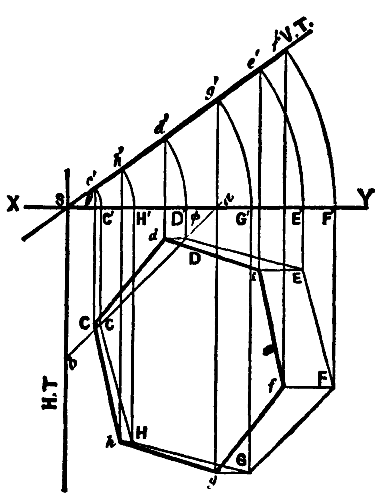


Fig. 66.

PROBLEM XLVI. (Fig. 66).—*To draw the plan and elevation of a hexagon when its plane is inclined to the H P, one side being inclined at a given angle to the V P.*

Draw the X Y and the traces of a plane, H T, V T, inclined at the given angle θ to the H P and perpendicular to the V P. Draw any line, *a b*, making an angle φ with the X Y equal to the angle of one side of the hexagon with the V P, and anywhere on this line construct the hexagon C D E F G H. Find the elevation of the hexagon (it will come upon the X Y as marked C, D, . . H′) With centre S (where the traces cut the ground line), throw up these points on the plane by drawing arcs through each meeting the V T in the points *c′*, *d′*, *e′*, *f′*, *g′*, *h′*, then this is the elevation of the hexagon when inclined. The points in the plan must evidently be directly underneath, and will also be in lines parallel to the X Y drawn through each of the points C, D, . H of the original hexagon. Mark these points, *c*, *d*, . . . *h*, and join them in the right order, thus obtaining the plan of the hexagon when inclined.

N.B.—The required plan and elevation should be made darker than the plan and elevation C, D, . . . H; C′, D′, . . . H′ as first drawn. The student will not probably find it necessary to mark all the points, except in beginning, and for difficult problems

PROBLEM XLVII. (Fig. 67).—*To draw the projections of a circle when its plane is inclined to the V P and perpendicular to the H P.*

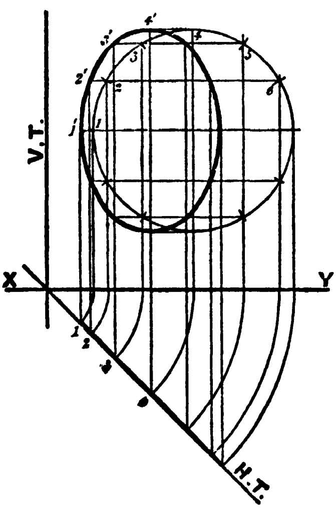


Fig. 67.

This problem is identical in principle with the last, but presents the additional difficulty that the circle has no corners which we can project as points, and thus obtain the boundary lines of the figure. We can, however, suppose it to have corners, or rather we can mark any definite points upon it, and treat these exactly as we treated the corner points of the hexagon. For convenience the points are taken at equal distances apart—that is, the circle is divided into a convenient number, eight or twelve equal parts. The problem will then represent no difficulty and can easily be followed from the figure. The elevation of the inclined circle is of course an ellipse, and must be drawn by freehand.

EXAMPLES.

EX. 18.—A hexagon of 1½″ side is inclined at 40° to the H P, one side being perpendicular to the V P. Draw its plan and elevation.

EX. 19.—A circle 3″ diameter has its plane inclined at 60° to the V P, its centre being 2½″ above the H P. Draw its plan and elevation.

EX. 20.—Draw the traces of a plane inclined at 30° to the H P and perpendicular to the V P, and at any point *a* in the H T draw a line *a b* 1½″ long, inclined at 20° to the H T. Consider *a b* as the plan of one side of an equilateral triangle lying in the plane, and draw its complete plan and elevation. (S. & A. E., 1886.)

(Find the plan of *a b*; when in the H P its length will be side of triangle. Draw the triangle, and transfer it to the plane.)

EX. 21.—Draw the plan of a hexagon of 1⅛″ side in any position, such that its plane is neither horizontal nor vertical. (S. & A. E., 1886.)

EX. 22.—The plan of a pair of compasses are two lines, each 3″ long, meeting at a point A, and including an angle of 30°. If the compass legs are actually 5″ long, determine the height of the joint above the H P.

EX. 23.—A regular hexagon of 1·25″ side has one side in the H P. The plane of the hexagon is vertical, and inclined at 43° to the V P. Draw the elevation of the hexagon. (S. & A. E., 1891.)

EX. 24.—Draw the traces of a plane inclined 35° to the H P and perpendicular to the V P, and draw the plan of an octagon of 1⅛″ side, lying in the given plane, and having one side in each plane of projection. (S. & A. E., 1892.)

EX. 25.—Draw two circles having the same centre of radii, ¾″ and 1¼″, and circumscribe the larger circle by a hexagon. Then draw an elevation of the figure when its plane is vertical and inclined at 40° to the V P, two sides of the hexagon to be vertical.

(This will represent the projection of one face of a hexagonal nut when its axis is inclined, which is a very common condition in machine drawing.)

Projection of Simple Solids.—Having already considered the projection of points, lines, and plane figures, which together make up a solid, we are now able to consider the drawing of solids themselves. Any difficulty attendant upon such projection is much simplified if the student will remember that almost all problems resolve themselves into, *firstly*, the projection of certain points, *secondly*, obtaining the projections of lines by joining the points; and *thirdly*, obtaining the projection of plane figures by joining the lines, thus giving the projections of a solid.

The simple solids are the **Cube, Prism, Pyramid, Sphere, Cylinder,** and **Cone,** which may be defined as follows:—

Cube.—A cube is a solid having six faces all equal squares (Fig. 68).

Prism.—A right prism is a solid having two equal and similar bases, and a number of equal oblong faces perpendicular to

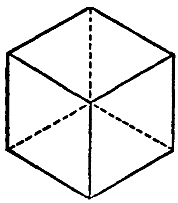


Fig. 68.

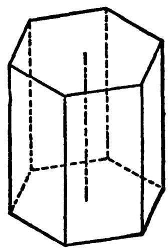


Fig. 69.

them. Prisms are distinguished according to the shape of their bases. Fig. 69 represents a *hexagonal* prism.

Pyramid.—A right pyramid is a solid having one base, and a number of equal triangular faces meeting in a point over the centre of the base. This point is called the apex. Pyramids

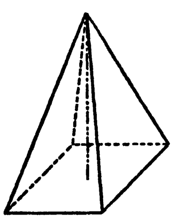


Fig. 70.

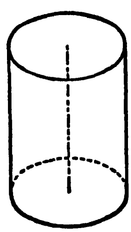


Fig. 71.

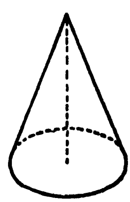


Fig. 72.

are distinguished according to the shape of their bases. Fig. 70 represents a *square* pyramid.

Spheres, cylinders, and cones are called solids of revolution, because they are generated by the revolution of certain plane figures about a fixed line, as an axis.

Spheres.—A sphere is a solid generated by the revolution of a semicircle about its diameter, as an axis. All points in the surface are equidistant from a point within the sphere, called the centre. All plane sections of a sphere are circles.

Cylinder.—A cylinder is a solid generated by the revolution of an oblong about one of its sides, as an axis. It has two equal circular bases, and may be regarded as a right prism having an infinite number of faces (Fig 71).

Cone.—A cone is a solid generated by the revolution of a right angled triangle about its perpendicular, as an axis. It has a circular base, and may be regarded as a pyramid having an infinite number of faces. The *vertex* of the triangle forms the *apex* of the cone (Fig. 72).

Axis.—The axis of a solid may be regarded as its central line. In a cube, the line joining the centre of any face to the centre of the opposite face, and in a sphere, any diameter may be called the axis. In a prism and cylinder the axis is the line joining the centres of the two bases, and in a pyramid and cone it is the line joining the apex to the centre of the base. The axis is shown by a dotted line in the figures.

A cube and sphere can be drawn if we know the length of side and the diameter respectively. In drawing the other four solids, we require to know the length of the axis, and the shape and size of the base.

The drawing of these solids may be rendered much easier and of much greater value to the student if he uses small models, which can be placed in different positions relative to the planes of projection. In this way a very complete conception of the solids may be acquired, which will prove of immense benefit in more advanced problems and especially in machine design, where the shape of so many common parts are identical with the simple solids just defined.

Very satisfactory models of these solids, except the sphere, can be made of stout paper, by first developing the surfaces of the solid then cutting the pattern to the pattern of the figure thus drawn, and afterwards folding upon the lines representing the edges of the solid, and gumming together.

Development of Surfaces.—By the development of a surface is understood the drawing upon a flat plane, such as a sheet of paper, the true shape of the complete surface, such that when the paper is cut to the figure thus obtained it would completely cover the solid of which it is the development. In practical engineering work, such as the construction of boilers, funnels, and in other iron and tin plate work, it is necessary to develop the surfaces of the structures before the plates of which they are made can be marked off, hence development is a very important part of engineering drawing.

Cube.—The development of a cube is a figure made up of six equal squares, and conveniently drawn as in Fig. 73, where the

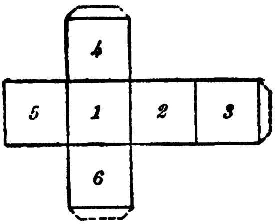


Fig. 73.

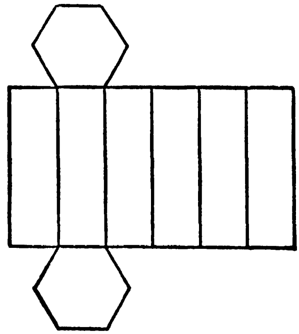


Fig. 74.

squares 1 and 3 form the top and bottom, when square 3 is the base, and squares 2, 4, 5, 6 the sides.

Parts are shown by dotted lines as extensions of squares 3, 4, and 6. These are merely strips to enable the sides to be joined together—that is, the slip on 3 is gummed to the top edge of 5, and the slips on 4 and 6 to the edges of square 3. Further strips may be added on the other sides if desired. The student should arrange to allow for similar strips in the development of the other solids.

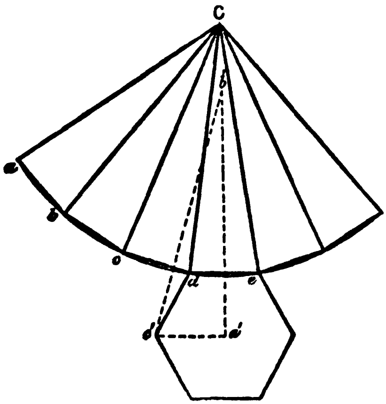


Fig. 75.

Prism.—The development of a prism is a figure made up of a number of equal oblongs and two equal figures representing the bases. Fig. 74 shows a convenient arrangement of the development of a hexagonal prism.

Pyramid.—The development of a pyramid is a figure made up of a number of equal triangles, and of one regular figure to represent the base. Fig. 75 shows the development of a hexagonal pyramid, obtained as follows:—Draw the base of the solid and a diagonal of the base, and then draw a right-angled triangle having the axis of the solid $a'\,b'$ as a perpendicular, and half the *diagonal* of the base of the solid $a'\,c'$ as a base, then the hypotenuse $b'\,c'$ is the length of the sloping edges of the solid. With the length of the sloping edges as radius, draw an arc of a circle from any centre as at C, mark off along this arc (as chords of the arc) the length of the sides of the base of the solid as *a b*, *b c*, *c d*, *d e*, and join *a* to *b*, *b* to *c*, and *c* as shown. This is the development of the faces of the pyramid. Draw the hexagon of the base on any of the lines as *d e* for a side, and the development will be complete.

Cylinder.—The development of a cylinder is made up of an oblong, the length of which is equal to the circumference of the base, and the height of which is equal to the axis, and of two circles equal in diameter to the bases. It is shown in Fig. 76.

Cone.—The development of a cone is made up of a segment of a circle, the radius of which is the hypotenuse of a right-

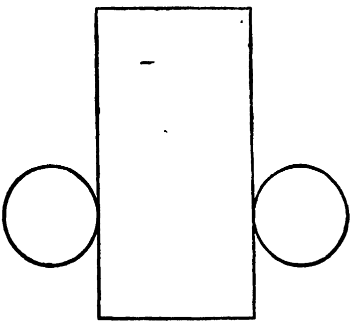


Fig. 76.

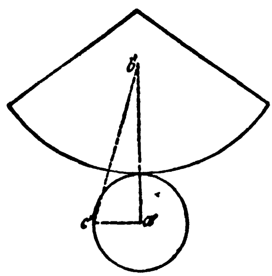


Fig. 77.

angled triangle, having the axis of the cone as a perpendicular, and the radius of the base, as a base, and the arc of which is equal to the circumference of the cone base; and of a circle, of diameter equal to the base. It is shown in Fig. 77. To obtain the radius of the arc draw a right-angled triangle, having the radius of the cone base $a'\,c'$ as a base, and the axis of the cone $a'\,b'$ as a perpendicular; then the hypotenuse $b'\,c'$ is the radius required. To set off the correct length around the arc (see p. 73).

EXAMPLES.

EX. 26.—Draw the development of the surfaces of the following solids:—(*a*) a cube, $2\frac{1}{2}''$ edge; (*b*) hexagonal prism, $1\frac{1}{8}''$ edge of base, axis $3''$; (*c*) square pyramid, $1\frac{3}{8}''$ edge of base, axis $3''$;

(*d*) cylinder base, 1½″ diameter, axis 3¼″; (*e*) cone base, 1½″ diameter, axis 3½″. Make models of the solids.

Plan and Elevations of Simple Solids.—The projections of the solids just developed, in simple positions, relative to the H P and V P, are shown in Fig. 78. The student should follow the drawings by placing the solids represented in the given positions, using a model of the planes as before.

Fig. 78 (*a*).—*A cube having one edge in H P, one face inclined at 30° to H P, and one face parallel to the V P 0·75″ in front.*

The elevation must be drawn first, as that shows the inclination and a true shape of one face.

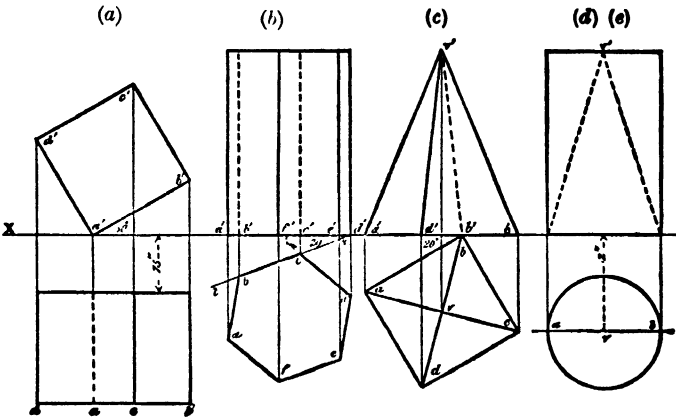


Fig. 78.

Fig. 78 (*b*).—*A hexagonal prism, having one base in H P, one face inclined at 20° to V P, nearest edge ½″ in front.*

First draw the lines *s t*, and the hexagon having one side *b c* in the line. Then draw a second X Y ½″ from the edge *c*.

Fig. 78 (*c*).—*A square pyramid with base in H P, one edge at 20° to V P and touching V P.*

Fig. 78 (*d*).—*A cylinder with one base in H P.*

Draw the plan first, and the diameter *a b* parallel to X Y, to give the correct points for the projection to the elevation.

Fig. 78 (*e*).—*A cone with one base in H P, axis 2″ in front of V P.*

Draw the plan first having its centre on the line *a b* parallel to the X Y. It is shown dotted in Fig. 78 (*d*). The projections of a sphere need no description, they are circles equal in diameter to the diameter of the sphere in all positions.

EXAMPLES.

Draw the plans and elevations of the following solids, marking each corner by a letter. Show on the drawings all sizes and angles given in the questions :—

EX. 27.—A cube, 2″ edge, in following positions :—(*a*) One face in H P, one face at 30° to V P, nearest edge 1″ in front. ; (*b*) two cubes, edges 2″ and 3″, stand one upon the other with their axes in one straight line, the edges of the base of the top cube making angles of 45° with the edges of the lower cube. The bottom cube stands in the H P, with one face at 45° to V P, nearest edge 1″ in front.

EX. 28.—A square prism, base 2″ edge, axis 3·5″, as follows:—(*a*) One face in H P, one base at 30° to V P, one edge of base in V P ; (*b*) one long edge in H P and perpendicular to V P, one face at 25° to H P, nearest base to V P 0·5″ in front.

EX. 29.—A hexagonal prism, base $1\frac{1}{4}$″ edge, axis $3\frac{1}{2}$″, as follows :—(*a*) One base in H P, one face parallel to V P, 1″ in front ; (*b*) one base in H P, one face perpendicular to V P, nearest edge 0·75″ in front.

EX. 30.—A pentagonal pyramid, base 1·8″ edge, axis $3\frac{1}{2}$″, as follows :—(*a*) Base in H P, one edge at 30° to V P, nearest corner 0·5″ in front ; (*b*) base in V P, axis 2″ above H P, one edge of base parallel to H P.

EX. 31.—A cylinder, base $2\frac{1}{2}$″ diameter, axis 3″, with a base in H P, axis 2″ in front of V P.

EX. 32.—A cone, base $2\frac{1}{2}$″ diameter, axis $3\frac{1}{2}$″, with base in V P, axis 2″ above H P.

Projection of Inclined Solids.—When a solid has a face or an edge inclined to either the horizontal or vertical planes, it is convenient to adopt the construction followed for inclined surfaces, by supposing that face or edge to lie in a plane having the required inclination. This is really the most important part of our work in solid geometry up to the present stage, when measured by the standard of practical usefulness, as it is a frequent occurrence in actual engineering drawing, to require the plan or elevation of singly inclined parts, such as hexagonal nuts and bolts, circular or elliptical covers, and of cylindrical outlines.

At a later period we shall explain other, and sometimes more convenient, methods of obtaining the projections of inclined solids, but as such methods largely involve the same principles as the following problems, it is very advisable that they should all be thoroughly understood.

PROBLEM XLVIII (Fig. 79).—*To draw the plan and elevation of a hexagonal prism, when one base is inclined to the ground.*

Suppose the base to lie on a plane having the given inclination θ to the H P. Draw the H T and V T of the plane, and obtain the plan and elevation of a hexagon equal to the base of the prism, when lying on this plane exactly as in Problem xlvi. Draw the plan in light lines, as it will evidently not be seen full. Complete the elevation of the prism by drawing a line perpendicular to the V T of the plane, and therefore, perpendicular to the plane itself, through each point in the elevation of the base. All the long edges of the prism are parallel to the V P, since the base is in a plane perpendicular to the V P, therefore draw lines parallel to the X Y, through each point in the plan of the base, and of indefinite length. Then having the elevation of the top base, g', h', i', j', k', l', its plan is obtained by drawing projectors from each point in the elevation to meet the line in the plan corresponding to the long edge, meeting at that point, as for example, the projector from h', where the long edge $a'h'$ meets this base, must be drawn to meet the plan of that edge, which is the line through the point a.

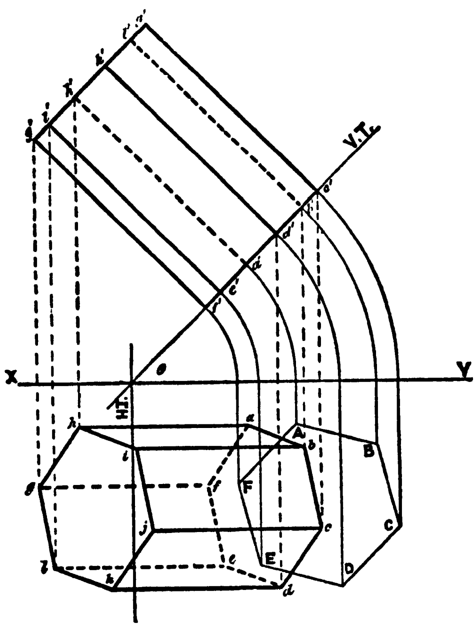


Fig. 79.

PROBLEM XLIX. (Fig. 80).—*To draw the plan and elevation of a cylinder when its axis is inclined to the V P.*

If the axis is inclined at an angle ϕ, then the base will be inclined at an angle of $90°-\phi$; therefore, draw the traces H T and V T of a plane perpendicular to the H P, and inclined at an angle of 90°−φ to the V P. Then proceed exactly as in Problem xlvii., Fig. 66, to obtain the plan and elevation of a circle equal to the base of the solid when lying in the plane. Draw lines through each point in the plan (only make the two outside lines dark), of length equal to the axis of the cylinder, and complete the plan. Draw lines parallel to the X Y through each point, 1, 2, 3, . . . in the elevation of the base, and from each of the points *a*, *b*, *c*, *d*, . . . in the plan of the front base of the cylinder draw projectors, to meet the elevation of the line in the plan which meets in that point, as, for example, through the point *c* where the line 3 *c* meets the plan of the front base of the solid, draw a projector to meet the elevation of the line in the point *c′* thus obtaining a point in the required elevation of the front base. By this construction we have really treated the lines 3 *c*, 5 *e*, . . . as though they were the long edges of a prism, a method that is always adopted for all circular and curved surfaces. It is convenient to speak of these imaginary edges or lines as "*stripes.*"

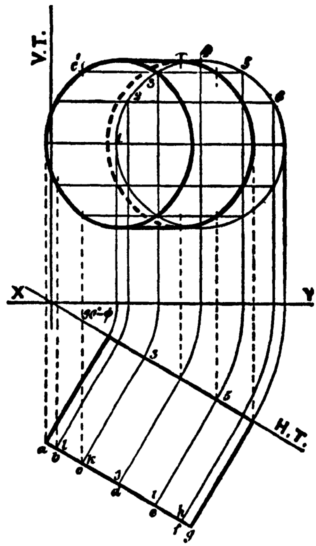


Fig. 80.

The student should now be able to proceed to the following examples, which have been specially selected as representing conditions that commonly occur in practical engineering drawings :—

EXAMPLES.

EX. 33.—Draw the plan and elevation of a hexagonal prism, edge of base 1¼″, height 3″, when its axis is inclined 60° to the ground, and two of its faces are perpendicular to the V P.

(The hexagon must be drawn with two sides perpendicular to X Y.)

EX. 34.—Draw the plan and elevation of a cylinder, base 2½″ diameter, 3″ high, when one base is inclined at 45° to the V P.

EX. 35.—A hexagonal prism, base 1¼″ edge, axis 1½″, stands centrally upon a circular block 3″ diameter and ½″ thick, both solids being pierced with a hole of 1¼" diameter. Draw a plan and elevation of the solids when the plane of the circular block is inclined at 35° to the ground, two faces of the hexagonal prism being parallel to the V P.

(This may be compared to an inclined hexagonal nut and washer.)

EX. 36.—An oblong block is 4½" long, 2" wide, and 1" thick. Two cylinders, 1½" diameter, 1" high, are fixed to the upper large face centrally, each ½" from the edge. Draw a plan of the solids when the plane of the lower face is inclined at 40° to the V P, the sides of the block being vertical.

(This represents the cap and bolt heads for a connecting-rod end.)

EX. 37.—A square bar, 2" square, 5" long, is pushed through the centre of a cylindrical block 1½" thick, 3½" diameter, so that the bar comes equally through the block on each side. Draw plan and elevation of the solid, when axis of bar is inclined 30° to H P and is parallel to V P, the sides of bar being at 45° to H P.

SECTION X.

THE PROJECTION OF ADDITIONAL PLANS AND ELEVATIONS—SECTIONS.

UP to the present stage we have only been concerned with the projection of the plan and elevation of solids upon the horizontal and vertical plans of projection—that is, we have obtained only one plan and one elevation of any given solid. But when solids become more complex in shape, as in the case of engine and machine parts, and most of the parts of practical engineering construction, it is not possible to show all the details of the parts in these two views, and it, therefore, becomes necessary to obtain other and additional views.

The manner of obtaining these views will be understood by the following illustration:—Suppose we stand looking straight upon the front end of a locomotive, we shall then see a view of the locomotive which shows us its height and width, but gives no idea of its length, but if we now walk round to the front or side of the locomotive we shall then see a view of it, which shows its height and length but gives no idea of its width, and again we could move to a position from which we looked on the corner of the locomotive, and we should then see a view which showed the height, length, and width, but without giving a correct idea of the real dimensions of the length or width, owing to their being inclined to the line of sight. If we supposed vertical planes to be placed behind the locomotive, and the views we see to be drawn upon them, we should thus obtain three different elevations, all of which it is important to notice have been found without altering the position of the solid, but by simply altering our position of observation with regard to it. The first view of the locomotive which we obtained by looking upon its end is called *an end elevation*, and the last view, that looking upon the front, *a front elevation*. We could of course have obtained a second end elevation by looking upon the back end of the locomotive, and for complex machine parts this is

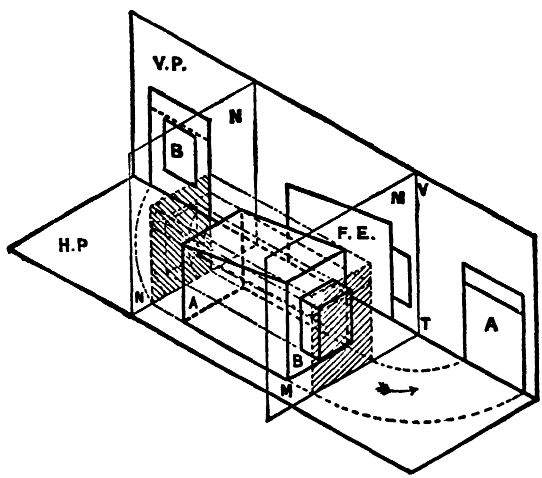


Fig. 80*a*.

usually done, although in most cases it is sufficient to draw one end elevation only.

It will be seen that, in order to measure the distance of a point or line from the ground line, it is necessary to look upon an end of the planes of projection, and to obtain an end elevation of them. This end elevation, so far as the planes of projection are concerned, will simply consist of two lines mutually perpendicular, meeting in a point which is the end elevation of the ground line. The end elevation of the point or line can then be drawn and its distance from the ground line readily obtained.

The principle of obtaining these additional elevations will be understood by reference to Fig. 80*a*, *b*, which represent the planes of projection in position with a solid standing in the H P of which we require to draw a front elevation and two end elevations. The projection of the front elevation F E needs no explanation. To obtain the elevation of end A, we suppose a vertical plane, M, placed on the other side of the solid, either touching the end B or removed from it, perpendicular to the lines of sight, and, therefore, in this case perpendicular to the V P. The view looking on the end A is then projected on this plane in the usual way, after which the plane is supposed to be turned upon its vertical trace, V T, as a hinge in the directior of the arrow away from the solid until it coincides with the V P, and the view from end A becomes projected upon the ordinary V P of projection on the right-hand side of the front elevation, as at A. For the elevation looking on end B a vertical plane, N, is placed on the further side of the solid, and then projected as before into the V P giving the view from end B upon the left-hand side of the front elevation. Notice that in each case all

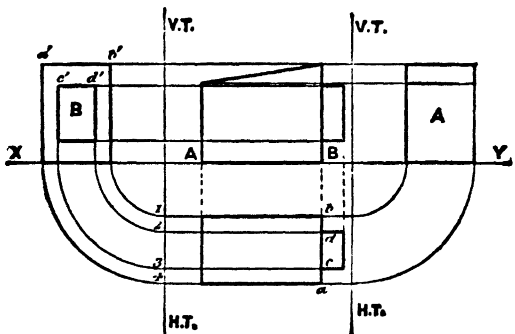


Fig. 80*b*.

points in the projection travel in arcs of circles having the V T of the planes as centre as the planes turn from their original position into the V P.

The plan and three elevations thus found are shown in Fig. 80*b* as they would be drawn upon the paper, and by the help of Fig. 80*a* should present no difficulty. With these two drawings before him the student should very carefully verify the following statements :—

(*a*) The end elevation looking upon the right-hand end of the solid is shown upon the left hand of the front elevation.

(*b*) The end elevation looking upon the left-hand end of the solid is shown upon the right hand of the front elevation.

(c) In an end elevation, that part of the drawing nearest to the front elevation represents the back portion of the solid, and that part of the drawing furthest from the front elevation represents the front portion of the solid.

These three conditions should be adhered to in all kinds of mechanical drawing whatsoever. It is remarkable what different customs prevail with regard to the position of the end elevation; many draughtsmen use either position without any regard to uniform methods, and as a result considerable confusion prevails.

At a later stage the student may find it apparently most convenient to put an end elevation next the end which it represents, rather than at the opposite end; but it is certainly not accurate.

The front elevation shows only *height* and *length;* the end elevation only *height* and *width*, and the plan only *length* and *width*. Hence we see that the three dimensions of a solid are shown in any two of the three views, and that if any two views are given the third can be obtained from them. For example, in Fig. 80*b*, lines are drawn from the front elevation to the end elevation, to give its dimensions in a vertical direction, while its dimensions in the other directions, as $a' b'$ and $c' d'$, are obtained from the plan, and are equal respectively to the sizes marked $a b$ and $c d$. The drawing of a view from others is a very important part of practical projection, and the student should notice that, although it may be desirable to work one or two problems by drawing the arcs marked 1, 2, 3, 4 in order to obtain an end elevation, it is better and quicker to adopt the method of taking the distance with the dividers direct from the plan.

Since the drawing of additional elevations requires the use of other vertical planes of projection, we see that other ground lines will be obtained where these planes intersect the H P. But this simply amounts to drawing a new ground line, and then obtaining a new elevation above this X Y in the usual way, knowing that the heights above the ground line are the same as in the first elevation. *This method is often referred to as an alteration of the ground line.* By exactly similar methods we may obtain additional plans from the first elevation, for we may suppose other horizontal planes to be placed in different positions relatively to the first elevation making new ground lines with the V P. In order to distinguish the different ground lines, they are marked as $X^1 Y^1$, $X^2 Y^2$, $X^3 Y^3$, &c.

PROBLEM L. (Fig. 81).—*Given a plan and elevation of a solid to obtain a second elevation on a given ground line $X^1 Y^1$ and a second plan on a given ground line $X^2 Y^2$.*

The plan and elevation of a simple solid are shown at P and E, it is required to draw a second elevation on $X^1 Y^1$ and a second plan on $X^2 Y^2$.

To obtain an *elevation* on $X^1 Y^1$ draw projectors through each point in the *plan* P perpendicular to $X^1 Y^1$, and mark off distances along each from $X^1 Y^1$ equal to the height of the point above the H P—that is, equal to the distance of the point above X Y in the first elevation E. The construction for one end is shown in the figure, the distance $e^2 d^2$ being equal to $e^1 d^1$, and $f^2 a^2$ equal to $f^1 a^1$, and so on for each point.

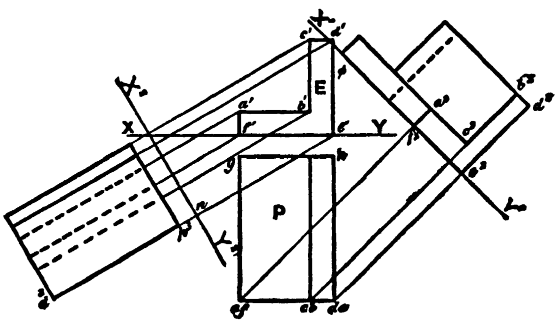


Fig. 81.

Some of the lines in this elevation will be dotted, and as this generally follows in the projection of inclined solids, it is convenient to always draw the parts of the solid shown by full lines before those parts shown by dotted lines—that is, first draw those parts nearest the point of observation, or furthest from the X Y.

To obtain a *plan* on $X^2 Y^2$, draw projectors perpendicular to $X^2 Y^2$ through each point of the first *elevation* E, and make the distance of each point in front of $X^2 Y^2$ equal to its distance in front of X Y, as, for example, the distance $n d^3$ equals the distance $e^1 d$, and $n h^3 = e^1 h$.

The construction of the last problem shows how the projection of solids in difficult positions may be simplified, for they can first be drawn in a simple position, and then by a suitable alteration of the ground line, they can be projected as required. For example, if we required an elevation of the block in the last problem, with its long edges inclined at 30° to the VP, we should first draw a plan and elevation, as at P and E, and then obtain a second elevation on a ground line XY′, drawn at an angle of 30° to the long edges of the solid in the plan P—that is, the angle φ would be 30°. But care must be taken to arrange the *first* plan and elevation so that they fulfil at least one of the required conditions; as, for example, suppose we require the plan and elevation of a hexagonal prism with its axis inclined 60° to the H P, and one face parallel to the V P, we should first draw a plan and elevation, with a base in the H P, and one edge of the base parallel to the V P—that is, to the X Y—thus giving a face parallel to V P, and satisfying one condition of the problem. The problem would then be completed by drawing a new ground line $X^2 Y^2$, making an angle of 60° with the axis of the solid in the elevation, and from this elevation obtain the required plan. If the solid had been first drawn without an edge of the base, parallel to the V P, no alteration of the X Y would have satisfied the conditions.

By an extension of this principle we can often obtain the plans or elevations of inclined solids in a simpler and quicker manner than by the method described in Problems xlviii. and xlix. An example is given in the following problem, from which it will be seen that the construction results in the required plan and elevation being obtained in a better, and in the more usual, position on the paper, than with the method of Problem l.

PROBLEM LI. (Fig. 82).—*To draw the plan and elevation of a solid made up of a square prism and a cylindrical block, when the axis of the block is inclined to the ground, and the faces of the prism make equal angles with the vertical plane.*

Draw the X Y and a line $s' t'$ inclined to it at the required inclination, θ, of the axis to the ground. At any part of the line draw a square A B C D equal to the base of the prism, having a diagonal A C on the line, and from the centre of the square draw a circle equal in diameter to the circular block. The square and circle then represent a view of the solid looking from above it in the direction of the axis. Draw the elevation of the solid as shown, commencing with the base of the circular block.

(A little difficulty will be found in making $e' f'$ equal to $f' g'$, but this can be avoided by drawing the line at any convenient position on the axis, and then drawing another X Y to pass through the point e'.)

The two drawings now made may represent the plan and elevation of the solid when the ground line is X′Y′. As the solid is not stated to be a given distance in front of the vertical plane, we may draw a line $s t$ to represent the plan of the axis at any convenient distance in front of the X Y. Then, to obtain the plan, proceed as follows:—Draw a projector from $a' c'$ to beyond the plan of the axis, the plan of a and c must be on this line, and a distance apart equal to the distance A C in the first view drawn, or what is better still for general application, make the distance of *a* and *c* on either side of the plan of the axis, equal to the distance *s′* A or *s′* C. The other points in the plan of the square prism can be obtained in the same way. To obtain the plan of the circular block, we must first *stripe* it as in Problem xlix., Fig. 80, therefore divide the circle centre *s′* into a convenient number of equal parts, and mark as shown 1, 2, 3, 4, 5, . . . 8. Draw the elevation of the stripes on the elevation of the block, and mark. Then to obtain the plan, draw projectors through the elevation of the points, as 2′ 4′, and, as before, mark off a distance along the projector from the point where it cuts the axis plan equal to the distance *t* 2 or *t* 4 in the first view drawn—that is, *p* 2 and *p* 4 equal *t* 2 or *t* 4. The completion of the plan needs no further description.

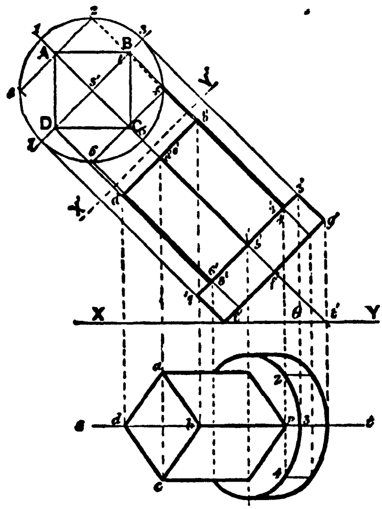


Fig. 82.

EXAMPLES.

EX. 1.—Draw the trace of a plane parallel to and 2½″ above the H P, and determine the projections of a point in this plane 3⅛″ from the ground line. (S. & A. E., 1886.)

EX. 2.—Two points *a b* on the ground line are 2½″ apart. A point P is 2¼″ from *a*, and 2⅜″ from *b*, and 1¼″ from the V P. Obtain its projections. (S. & A. Adv., 1886.)

EX. 3.—Draw the plan of a square prism, height 2¾″, side of base 1¼″, a diagonal being vertical. (S. & A. Adv., 1891.)

(By diagonal is meant a diagonal of the solid; first draw in a simple position and then alter the X Y.)

EX. 4.—Draw the plan of a cube of 2¾″ edge with a diagonal of the solid vertical.

EX. 5.—Draw the projections of a pentagonal pyramid, axis 3", edge of base 1¼", with its base in H P, and one edge of base perpendicular to V P. Then draw a second elevation when one sloping edge of the solid lies in the H P, and a second plan when a sloping edge of the solid is vertical.

EX. 6.—Draw the projections of a hexagonal prism, axis 3¼", edge of base 1¼" pierced with a central hole 1¼" diameter when standing with a base in the H P, and a face parallel to V P. Then draw a second plan of the solid upon X Y, making an angle of 45° with the elevation of the axis.

EX. 7.—A circular block is 3" diameter and 1" thick, and is pierced by a central square hole of 1¼" side. Draw a plan and elevation of the solid when the base is inclined 30° to the ground.

EX. 8.—An elliptical block is 3½" × 2" and 1" thick, draw its elevation when the plane of a base is inclined at 45° to the V P, the longest axis of the ellipse being vertical.

Sections.—Up to the present stage we have dealt only with solids, the form of which could be clearly seen by one or more views obtained by looking upon the outside of the solid from different positions. But it very frequently happens in practical engineering drawings that the parts to be drawn are hollow, and of a complex shape, which would not be clearly shown by dotted lines. In such cases we suppose a cut to be made completely or partly through the object, and that part of it between the eye of the observer and the plane of cutting to be taken away, the view of the remaining part thus showing the details of internal construction. The cut is termed a *section*, and the cutting plane a *section plane*, so that we speak of drawing *sectional views* or *sections*. In order to show what parts of the solid have been cut by the section plane, we cross-line the parts by a number of lines near together, conveniently drawn with the 45° or 60° set square, this cross-lining is called *sectioning*, and whenever a solid is made up of several parts, each part cut by a section is sectioned in a different direction to the part with which it is in contact, thus distinguishing the separate parts. Examples of sectioning will be seen in the second part of this book, although in machine drawings the sectioned parts are generally coloured, and not cross-lined.

In problems on solid geómetry, a section plane is shown by its trace upon the H P or V P, and is usually marked by the letters S T, meaning *trace of section plane*. The following examples will illustrate the method of obtaining the projection of sectional views:—

PROBLEM LII. (Fig. 83).—*A square block stands centrally* *upon a circular block, both solids being pierced by a cylindrical hole. To show the sections made by a vertical plane passing through the axis of the solids, and by a horizontal plane passing through the centre of the upper block.*

Let S T be the horizontal trace of the vertical section plane, and S′ T′ the vertical trace of the horizontal section plane.

The outline of the plan and elevation is not altered in any way by the fact of the two views being in section, and we, therefore, draw the outline of the plan and elevation in the ordinary manner. Dealing first with the vertical section, we see that its plan will simply be that part of the horizontal trace, S T, of the section plane between the boundary of the plan—that is, the line *a b*; also that the two lines in the elevation which represent the hole will be full lines, because the hole is fully revealed by the section, for in obtaining the elevation of the section, we are looking from the position of the arrow P, with the part of the solid between the arrow and the section plane removed. This is the only alteration in the elevation required by its being a section; but it should be noticed that, if the solids had formed one block, there would be no full line from *c′* to *d′* in the elevation, since the edge *c d* has been removed, we could, however, show a dotted line joining *c′ d′* as representing the back edge *e f*. All the parts of the solids cut by the section plane are section-lined in the elevation, the top block by lines sloping to the right, and the bottom block by lines sloping to the left.

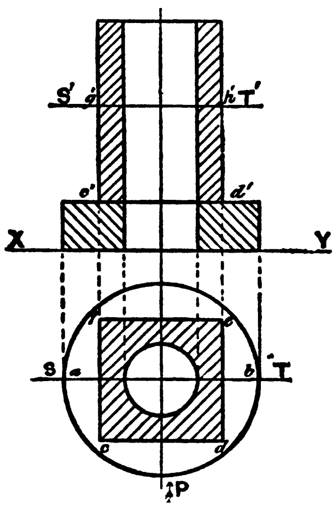


Fig. 83.

The elevation of the horizontal section is the line *g′ h′* and its plan, the sectioned plan of the square block, which should be lined in the same direction as the upper block in the elevation.

In practical engineering drawing the draughtsman does not concern himself with showing section planes, or with both the plan and elevation of a section. The elevation of Fig. 83 would be termed a sectional elevation on the line S T, or simply "*a* *section on* S T," and the plan, a sectional plan on the line S′ T′, or simply "*a section on* S′ T′."

PROBLEM LIII. (Fig. 84).—*To draw the plan and elevation of the sections of a hexagonal pyramid, and to find the true shape of the sections.*

Let S T be the H T of a vertical section plane, and S′ T′ the V T of a section plane inclined to the ground and perpendicular to the V P.

Mark the vertex of the solid *v* and the corners of the base *a*, *b*, *c*, *d*, *e*, *f* in both plan and elevation. Then to draw the vertical section made by the plane S T, draw projectors from each point 1, 2, 3, 4 of the section in plan to meet the elevation of the edges in the elevation, as, for example, the section plane cuts the sloping edge *v b* at the point 2, and, therefore, it must cut the elevation *v′ b′* of the same edge at the point 2′, found by drawing a projector from 2 to cut *v′ b′*. Proceeding in this way, we find the points 1′, 2′, 3′, 4′ in the elevation, and by joining in the right order we obtain the elevation of the section, which should be section-lined as shown.

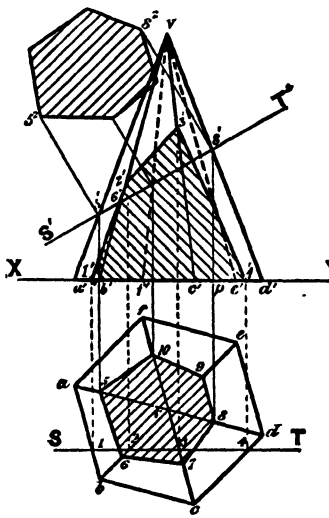


Fig. 84.

For the plan of the section on S′T′ we adopt the same method, as, for example, the section plane cuts the elevation *v′ b′* of the sloping edge *v b* at the point 6′, and, therefore, the plan of this point must be at the point 6, in the plan *v b* of the edge. found by drawing a projector through the point 6′. The plan of the section when completed is the irregular hexagon 5, 6, 7, 8, 9, 10, and should be section-lined as shown.

True Shape of Section.—We know from previous examples that the plan 5, 6, . . . 10 of the section on the line S′T′ cannot be the true shape of the section, because it is the plan of an inclined figure. We also know that a plane figure is only shown its true shape when it is projected upon a plane parallel to its own plane—that is, it must be looked at in a direction perpendicular to its own plane. The vertical section 1′, 2′, 3′, 4′ is the true shape of the section, as the V P of projection is parallel to the section plane S T. To draw the true shape of the section made by the inclined plane, we may regard the trace of the plane S′ T′ as a new X Y, and the line 5′ . . . 8′ as the elevation of the plane figure marked in plan 5, 6, . . . 10, of which we require a new plan on S′ T′ as a ground line. We then proceed as in the last series of examples, and draw projectors from each

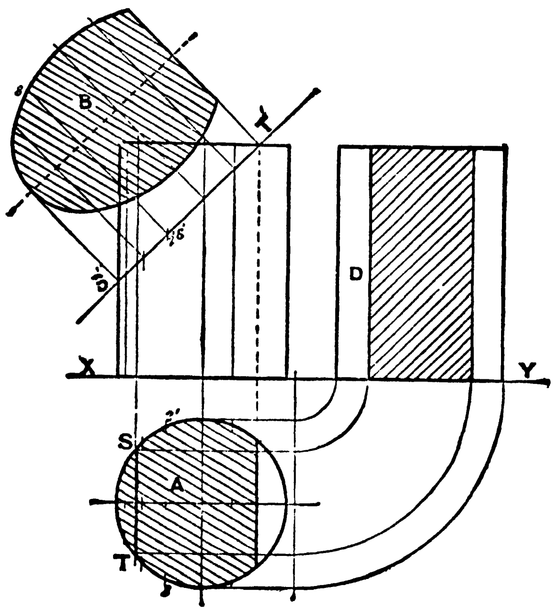


Fig. 85.

point 5′ . . . 10′ perpendicular to S′ T′, and set off along each its distance in front of the V P; thus $5^1 5^2$ is equal to $b' 5$ and $8^1 8^2$ to $p 8$, and so on. This is exactly the same as supposing the section plane to be turned into the V P of projection about its trace S′ T′ as a hinge, taking with it the outline of the section, and the projectors from each corner of the outline to the vertical plane.

It will be seen that a sectional view of a solid is of no service for the practical purpose of showing its construction and form, unless it shows the true shape of the section; hence, we do not find that engineering drawings generally contain either plans or elevations of inclined sections, but only the projections of their true shapes. The draughtsman chooses the projection of the sections in the positions most likely to add to the clearness of the drawing, and as a rule most sections on engineering drawings are either horizontal or vertical ones. But it often occurs that a section is taken through an inclined part, in which case the true shape of the section is required, and must be obtained on the principle of the last problem. It will be found that the true shape can often be drawn without first obtaining the plan or elevation of the section, but in many cases it is necessary to have either a part or the whole of the plan or elevation. The following are additional examples of plans, elevations, and true shapes of sections obtained in similar ways, of some simple solids which are of common application in practical construction:—

Cylinder (Fig. 85).—A is the plan and B the true shape of a section of the cylinder made by the inclined section plane S′ T′, and D the true shape of a section made by the vertical section plane S T. The view, D, might be termed a sectional end elevation. The cylinder must be striped as shown.

Cone (Figs. 86*a*, *b*).—Fig. 86*a*, A is the plan and B the true shape of a section of the cone made by the inclined section plane S T. In Fig. 86*b*, C is the plan and D the true shape of a section made by the plane $S^1 T^1$, and E is the plan and F the true shape of a section made by the plane $S^2 T^2$. The true shapes, B, D, and F, are the three conic sections—the ellipse, parabola, and hyperbola. The section of a cone by a plane, such as S T, which cuts all the positions of the generating line of the cone, is an ellipse, and is shown at B (Fig. 86*a*); the section by a plane, such as $S^1 T^1$, parallel to any one position of the generating line, is a parabola, and is shown at D (Fig. 86*b*); while the section by a plane, such as $S^2 T^2$, parallel to the axis of the cone, is a hyperbola, and is shown at F (Fig. 86*b*).

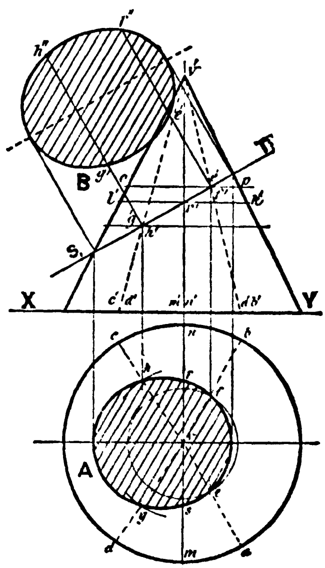


Fig. 86*a*.

The methods of obtaining the sections of a cone are as follows, and should be carefully mastered:—

Sections of Cone by Stripes (Fig 86*a*).—Divide the plan of the base into a number of equal parts, and join each point to the plan of the vertex. Imagine these lines to be stripes drawn down the cone, and draw their elevation. The lines representing the elevation of the stripes will cut the V T of the section plane in points, which are in the elevation of the section; therefore, the plan of each point is directly underneath its elevation, and upon the plan of the stripe whose elevation cuts the elevation of the section plane. Thus, in the figure, the dotted lines *v a*, *v b*, *v c*, *v d*, are the plans of 4 stripes, and *v′ a′*, *v′ b′* (which fall in the same

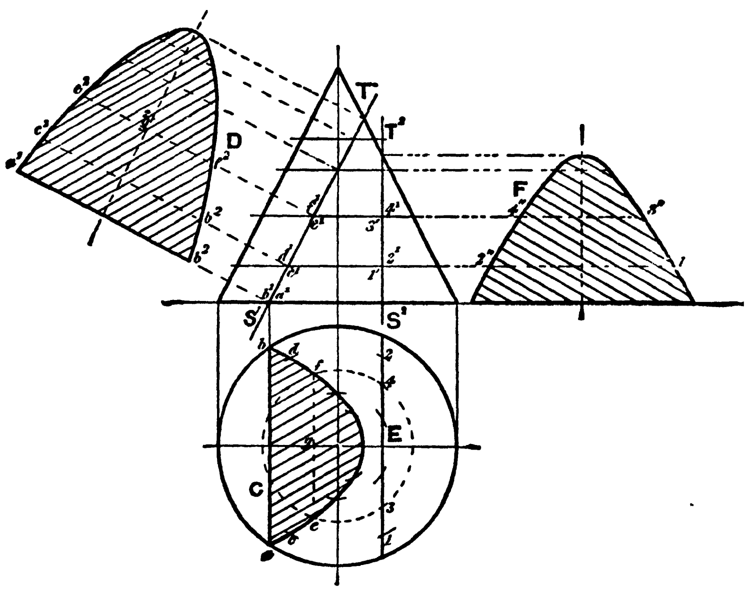


Fig. 86*b*

line), and *v′ c′*, *v′ d′* (which also fall in the same line) are their elevations. The stripe *v′ a′* cuts the line of the section *in front* at *e′*, and *at the back* at *f′*; therefore, a projector from *e′ f′* meets the plan of the stripes *v a* and *v b* in *e* and *f*, which are two points in the plan of the section. The same reasoning applies to the stripes *v c* and *v d*, which give two other points, *g* and *h*, in the plans. Thus, by taking a sufficient number of stripes the plan of the section can be drawn, and its true shape found as in Problem liii. But the method evidently fails for the stripes *v m* and *v n*, and to obtain the points *r* and *s* we have to proceed as follows :—

Sections of Cones by Cuts parallel to Base.—*All plane sections of a cone parallel to its base are circles.* If, then, we take any horizontal section of the cone in Fig. 86*a*, such as at the line *o p*, its plan will be a circle of diameter *o p*, and is shown drawn upon the plan of the cone. But the cut *o p* will pass through the point *e'* in the line of the section in front and the point *f'* at the back, so that a projector through the point *e' f'* will cut the circle in the two points *e* and *f*, which are evidently points in the plan of the section. Therefore, the distance *v r* or *v s* is equal to *r' k'* or *r' l'*. Any number of other points can be found by taking additional cuts at different heights. It is unnecessary to draw the whole of the circles. The sections of Fig. 86*b* are found in this way, for it is more convenient than the method of stripes.

It is more convenient to obtain the true shape of the section by drawing its centre as shown dotted (Figs. 85 and 86*a*, *b*), in any convenient position parallel to the trace of the section plane, and then mark off distances on each side of the centre line, the distances being taken from the centre line of the plan of the section to the extreme points of the section. Thus, in Fig. 86*b*, $g^2 e^2$ or $g^2 f^2$ in the true shape of the section D, is equal to *g e* or *g f* in the plan of the section C.

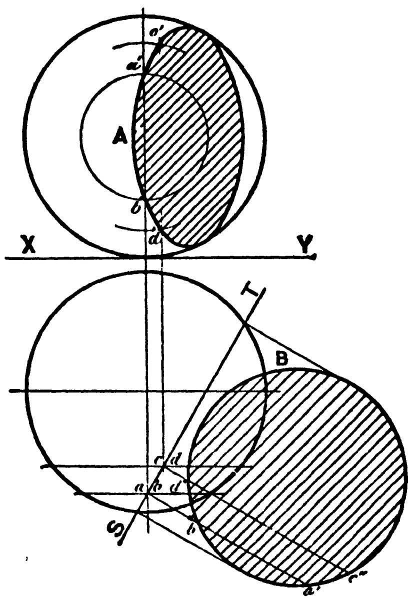


Fig. 87.

Sphere (Fig. 87).—A is the elevation and B the true shape of the section of a sphere made by the vertical plane whose horizontal trace is S T. The true shape of any plane section of a sphere is a circle, and its inclined projection an ellipse. The method is identical with the method of sections for a cone.

EXAMPLES.

EX. 9.—Draw a plan and two elevations of your drawing-instrument box, with the lid open, at an angle of 45° with the box, the end elevation to be in section. Scale, 6″ = 1′.

EX. 10.—A hexagonal right pyramid, side of base 1¼″, height 3¾″, stands on the H P. Draw the plan and make a section by a vertical plane, the H T of which is a line through one corner of the base, passing ⅜″ from the plan of the vertex. (S. & A. E., 1888.)

EX. 11.—A letter A is made of material ¼″ thick, it is 3″ high and 3″ wide at the base, the width of the material being ⅜″, and it stands in the H P parallel to the V P. Draw its plan and make an elevation on a line parallel to a diagonal of the rectangle at the top, and a sectional elevation on a line through the plan of a top corner and making 35° with the plan of the front face.

EX. 12.—A cone, 3″ high, where base is 2″ in diameter, has its axis horizontal. Draw an elevation on a plane inclined at 60° to the base, and a section of it by a horizontal plane ½″ above the axis. (Vict. Univ. Hon., 1890.)

EX. 13.—A hollow square block, 2″ outside edge, 1″ inside edge, 3″ long, stands with one base in H P and a vertical face at 30° to V P. Draw a plan and true shape of the section made by a plane inclined at 45° to the ground passing through the centre of the axis of the block.

EX. 14.—A hexagonal pyramid, 3″ axis, 1¼″ edge of base, lies with one triangular face in the H P, its axis being parallel to the V P. Draw its plan, and the plan and true shape of a section made by a plane inclined at 20° to the ground passing through the centre of the elevation of the axis.

EX. 15.—A cylinder, 2″ high, 2¼″ diameter of base, stands with one base in the H P. Draw plan and true shape of a section made by a plane inclined 30° degrees to the ground passing through the elevation of the axis at a point 1⅝″ from the base.

EX. 16.—A cylinder, 3″ high, 2½″ diameter of base, lies in the H P with a base at 60° to the V P. Draw its elevation, and the elevation of a section made by a vertical plane parallel to the V P cutting the plan of the axis ¼″ from one base.

EX. 17.—Draw the plans and true shapes of the three sections of a cone made by cutting planes, as in Fig. 86*a*, *b*. The cone to be drawn in each case 5″ high and 3″ diameter of base.

EX. 18.—Draw the elevation and true shape of the section of a sphere of 3½″ diameter made by a vertical plane inclined 45°

to the V P, and passing through the plan of the sphere $\frac{1}{2}''$ in front of its centre.

EX. 19.—A sphere of 4″ diameter rests on the H P, and the top quarter of the sphere is completely removed. Draw a plan of the remainder.

EX. 20.—A conical vessel open at the top is $4\frac{1}{2}''$ high, $3\frac{1}{2}''$ diameter outside at the bottom and 3″ diameter outside at the top, the thickness of the shell being $\frac{3}{8}''$. Draw its plan and elevation, the elevation to be in section, and the plan to show a horizontal section midway up the vessel.

Projection of Solids generated by the Revolution of Surfaces.—It has been pointed out that cylinders, cones, and spheres

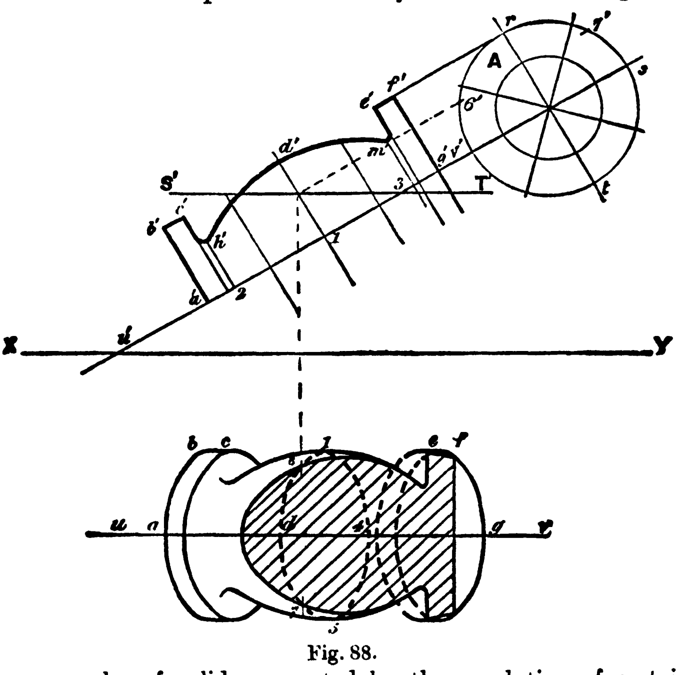


Fig. 88.

are examples of solids generated by the revolution of certain surfaces about a fixed axis. But the number of such solids of revolution is infinite, and as previous constructions do not apply except to simple cases, it is desirable to consider a more general example.

PROBLEM LIV. (Fig. 88).—*To draw the projections and section of a given solid of revolution.*

Let $u\,v$ be the plan and $u'\,v'$ the elevation of an axis, and $a'\,b$ . . . g' be the elevation of a surface revolving about $u'\,v'$. It is required to draw the plan of the solid as generated, and of the section made by a horizontal plane whose vertical trace is S′ T′.

The revolution of the points b', c', e', f' will generate circles lying in planes perpendicular to the axis $u'\,v'$, and, therefore, their plans can be drawn, as shown, exactly as in Problem xlvii. Find the points $h'\,d'\,m'$, so that d' shall be at the point of the curve furthest from the axis, and $h'\,m'$ at points in the curve nearest to the axis, then these points will also generate circles of radii equal to d' 1, h' 2, and m' 3 respectively, the plans of which can be found. The complete plan of the solid, so far as its outline is concerned, is then shown by the figure a, b, c, e, f, g, a. To obtain points in the plan of the section, we must proceed by taking cross-sections of the solid perpendicular to the axis, and then project these cross-sections upon the H P. For example, the true shape of the cross-section through d' 1 is shown in the figure marked A by the circle $r\,s\,t$, the section plane cutting this circle in the points 6′, 7′, the plan of these points is 6, 7, and give two points in the plan of the section. Other points are found in the same way, thus completing the sectional plan as shown. It is necessary to take cross-sections at all points where the direction of the curve changes.

EXAMPLES.

EX. 21.—A semi-ellipse axes $3\frac{1}{2}''$ and $2\frac{1}{4}''$ revolves about its major axis as an axis. The axis is inclined at 45° to H P and is parallel to V P. Draw the plan of the solid generated by the revolution of the semi-ellipse, and the plan of a section made by a horizontal plane passing through the centre of the axis.

EX. 22.—A line is parallel to the H P and inclined 35° to VP. A surface similar to that of Fig. 88 revolves about this line as an axis. Draw the elevation of the solid thus generated, and the elevation of the section made by a vertical plane parallel to the V P passing through the centre of the axis.

SECTION XI.

INTERPENETRATION AND DEVELOPMENTS OF SURFACES AND SOLIDS—SECTIONS OF SPECIAL SOLIDS—HELICES AND SCREW THREADS.

There are a number of problems of frequent occurrence in practical draughtsmanship which are best solved by the application of methods usually regarded as a part of solid geometry. Among such problems may be mentioned the drawing of an ordinary steam dome upon a cylindrical boiler, or of the semispherical ends of egg-ended boilers, and the finding of the true shape of the plates for such parts; the drawing of the contact lines of the cylindrical branches of cocks and valves with the main casing (see Fig. 166), and the drawing of the correct outline of such parts as at the junction of the crank-web and crankshaft, or the meeting of other flat and curved surfaces, as in connecting-rod ends and other similar parts.

These problems may generally be regarded as special cases of the interpenetration and development of surfaces and solids, as, for example, the steam and boiler may be treated as a case of the interpenetration of two cylinders, and the cock with its inlets and outlets as the interpenetration of a cylinder with a cone.

It will be understood that in the case of the steam dome and boiler, and of many similar examples, it is necessary to develop the true shape of surfaces in order that the plates may be so cut when flat, so that they shall join up correctly when bent to their required form.

PROBLEM LV. (Fig. 89).—*To draw the projections of the interpenetration of a horizontal and vertical cylinder and the development of their contact surfaces.*

Let A be the plan, B the end elevation, and C the side elevation of the cylinders.

The line of interpenetration is evidently shown in plan by the circle 1, 7, and in the end elevation B by the arc *d′ l′*. In order to find its side elevation on C, we imagine the vertical cylinder to have a number of stripes drawn upon it, and we then find the real length of each stripe from the top base of the cylinder to the point where it enters the horizontal cylinder. This is done as follows:—Divide the plan of the vertical cylinder, the circle on A, into a convenient number of equal parts, say 12, and mark as shown, 1, 2, . . . 12. Draw the elevation of the stripes in each of the elevations B and C, and mark the stripes 1′, 2′, . . . 12′ on B, and 1″, 2″, . . . 12″ on C. Care must be taken not to confuse the marking of the stripes in the two elevations, notice that the outside stripes 1′ and 7′, on the end elevation B, are the centre stripes on the side elevation C, while the centre stripes 4′ and 10′, on the end elevation, are the outside stripes on the side elevation. The stripes are correctly obtained on the end elevation B, by projecting from the plan,

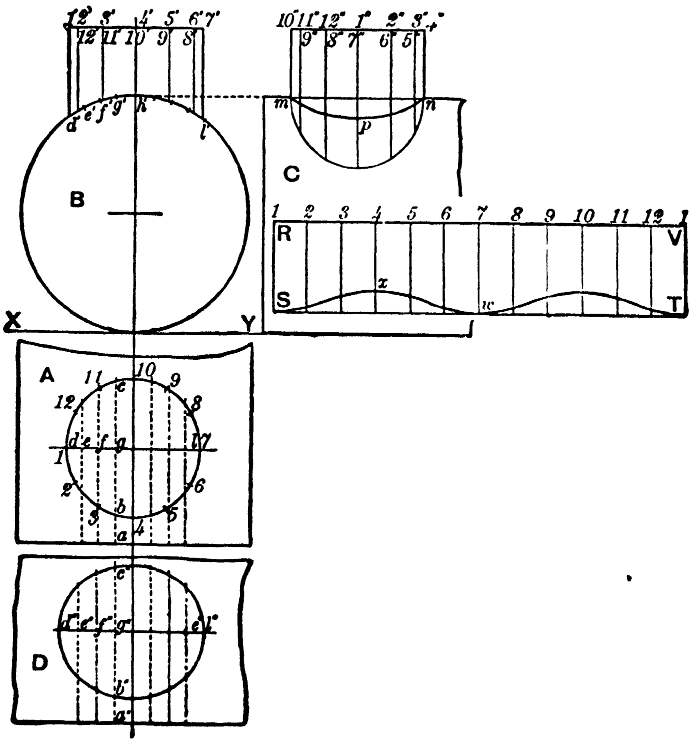


Fig. 89.

and, on the side elevation C, by drawing and dividing the semicircle on the line $m\,n$ as shown.

The real length of the stripes are shown in the end elevation B, therefore mark off on each stripe in the elevation C its real length as obtained from the end elevation, thus $1''p = 1'd'$ or

$7' l'$, and $4'' n$ and $10'' m = 4' h'$, and so on for each stripe. The curve $m p n$ drawn through the end of each stripe as thus found is the side elevation of the interpenetration. It will be noticed that it also represents the back half of the interpenetration.

To develop the surface of the vertical cylinder draw the oblong R S T V, the length of which R V equals the circumference of the cylinder base, and the height of which R S equals the length of the longest stripe on the cylinder, $1' d'$ or $7' l'$. Divide the oblong into the same number of equal parts as stripes on the vertical cylinder, and draw lines through each point. Mark the lines 1, 2, . . . 12, 1, as shown. These lines are the development of the stripes, the two end lines coinciding to form the stripe 1 when the oblong is bent to form the cylinder. Mark off the real length of each stripe as found from either of the elevations B or C, down the lines representing the stripes from the line R V as $4 x = 4' h'$, and draw the curve S W T as shown through the points thus found. The complete figure, R S W T V, is then the development of the vertical cylinder, supposing it simply to rest upon the horizontal cylinder, and is the shape to which a piece of paper must be cut, so that when it is bent to bring the edges R S and T V together, it shall exactly fit the horizontal cylinder.

We will now suppose the vertical cylinder to penetrate the horizontal one for a short distance and find the true shape of the hole of penetration in the surface of the horizontal cylinder. To do this we must stripe that part of the horizontal cylinder containing the hole, and then develop it with the stripes, the length of which between the extremities of the hole will enable us to find the true shape of the hole. Divide the arc $d' l'$ in the end elevation B into eight equal parts at the points $d', e', f', \ldots l'$. Consider these points as the elevation of the stripes, and draw their plan across the hole in the plan A as shown by the dotted lines $d, e, f, \ldots l$. Develop a part of the surface of the cylinder containing the hole, as in the Figure D, where the line $d'' l''$ is equal to the real length of the arc $d' l'$, and is found as shown on p. 73. Divide the line $d'' l''$ into eight equal parts, and draw the lines marked $e'', f'', g'', \ldots l''$ through each as shown dotted, these lines are the development of the horizontal stripes crossing the hole. The real lengths of each stripe between the extremities of the hole are shown by the length of the dotted lines crossing the hole in the plan A, thus $g'' b''$ or $g'' c'' = g b$ and $g c$, or measuring from the front of the cylinder $a'' b'' = a b$ and $a'' c'' = a c$. The closed curve drawn through the points thus found is the true shape of the hole.

The conditions of this problem are similar to the practical example of a steam boiler and dome, for the horizontal cylinder represents the boiler, and the vertical cylinder the dome. Then the development, R S W T V, is the shape for the plates of the dome before bending, neglecting the flange, and the development D shows the shape which the hole should be cut in the plates, so that when bent it shall be a circle.

EXAMPLES.

EX. 1.—Draw the curves of interpenetration of two cylinders each 3" diameter and 4" high. Axis of one horizontal, of the other vertical; both axes parallel to the V P.

EX. 2.—A horizontal cylinder, 4" diameter, 6" long, is interpenetrated by an oblong block 2½" wide, 2" thick, the sides of which are vertical. Draw the correct lines of interpenetration in the side elevation, and draw a development of the surface of the oblong block and of the hole in the cylinder. Height of block immaterial.

EX. 3.—A cylindrical boiler is 6' in diameter and has a cylindrical steam dome 2' 6" in diameter and 2' high. Draw three views of the arrangement and show the development of the plates of the steam dome and of the hole in the boiler shell. Scale 1" = 1'. (See Problem lv.)

EX. 4.—A right circular cylinder of 2¼" diameter penetrates another of 3" diameter, the axes being at right angles and passing ¼" from each other. Draw the projection of the curves of intersection on a plane parallel to the axes of the cylinders. (Vict. B. Sc. Hon., 1889.)

EX. 5.—Develop the surface of the cylinders in Ex. 4, the larger cylinder development to show the holes for the small cylinder. Cut out the figures and make a model of the cylinders in the given position.

Interpenetration of Cone and Cylinder.—PROBLEM LVI. (Fig. 90).—*To show the curve of intersection of a vertical cone and a horizontal cylinder in plan and elevation and to develop the surface of the cone.*

The cone is shown in the figure having its vertex marked v and v' in the plan and elevation respectively. The cylinder has the axis $a\,b$, and its diameter is such that it does not cut completely through the cone—that is, the diameter of the cylinder is less than the diameter of the section of the cone $a'\,b'$, which contains the axis of the cylinder. In order to save drawing only one-half of the cylinder is shown.

To obtain points in the intersection we take a number of horizontal cuts through both solids, and obtain from a plan of the section two points in the plan of the intersection. For example, if a horizontal section be taken at the line *c′ d′*, its plan will be the circle *g c h*, equal in diameter to *c′ d′* shown partly drawn upon the plan of the cone, and the oblong shown by the

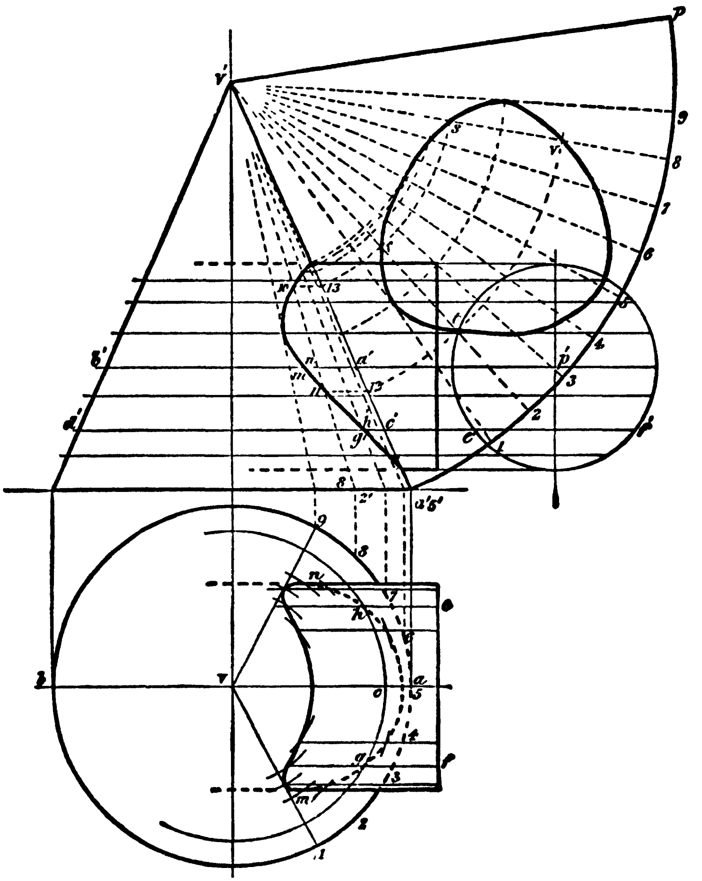


Fig. 90.

lines *g f*, *f e*, *e h*, which represents the section of the cylinder. The oblong and the circle cut at the two points *g* and *h*, which are evidently two points in the plan of the required intersection. The elevation of the points coincide at the point marked

$g'h'$, which is found by projecting from the plan of the points to the elevation of the line of section. The plan of the section of the cylinder is found by drawing its end elevation as shown by the circle. having the centre p', thus fe in the plan equals the distance $f'e'$ in the end elevation. By proceeding in this way for a number of horizontal sections sufficient points can be found through which to draw the curve showing the plan of the intersection, the lower half of the curve is of course dotted, the points $m\,n$, at which the dotted part begins, being found by taking a horizontal section through the axis of the cylinder.

Development.—Only one-half of the cone is developed in the figure to save drawing. Draw the sector $v'a'p$, centre v', and making the arc length radius $v'a'p$ equal to half the circumference of the cone base, as shown on p. 73. To develop the hole, we draw a number of stripes down the cone, over that part of it which contains the hole, then draw the stripes on the development $v'a'p$, and mark off on each the length contained between the extremities of the hole. Draw the lines $v\,1$ $v\,9$ in the plan, passing through the extremities of the curve of intersection, and divide the arc 1, 9 into any number—say, eight equal parts, draw the elevation of the stripes, and mark as shown. Then taking the stripe $v'2'$, we see that it crosses the hole at the points 10, 11, but the length 10 to 11 is not the true length of the line, because the whole stripe $v'2'$ is not parallel to the V P. To obtain the true length, draw lines parallel to the base, through the points to meet the line $v'5'$ in the points 12 and 13, then, as $v'5'$ is the real length of $v'2'$, so the distance 12 to 13 is the real length of the part 10, 11 of the stripe intercepted between the hole. Now draw the stripes upon the development of the cone surface, and figure each stripe; this is best done by drawing the middle line $v'5$ and marking off the other stripes on each side of it. Then an arc drawn through the point 13, with centre v' to cut the stripes $v'2$ and $v'8$, will give two points, $r\,s$, in the development of the hole, and an arc drawn through the point 12, to cut the same stripes, give two other points, t and v. By proceeding in this way the remaining points can be found, and the true shape of the hole drawn, as shown.

It is very instructive to develop the whole of the cone surface and then to make paper models of the solids, which will be found of great assistance in drawing the solids in more difficult positions, such as when the axis of the cylinder is inclined to the V P.

The above problem represents the condition of the casing and branches of ordinary steam- or water-cocks, as shown in Fig. 166, with the slight difference that only a frustrum of the cone is dealt with. To stripe the frustrum of a cone the circles of its top and bottom faces must be divided into the same number of equal parts.

EXAMPLES.

EX. 6.—A cone $3\frac{1}{2}''$ diameter of base, axis $4\frac{1}{2}''$ long, stands on the H P, and is completely penetrated by a cylinder $2''$ diameter and $4\frac{1}{2}''$ long. The axis of the cylinder is horizontal, parallel to the V P, and passes through the axis of the cone $1\frac{1}{2}''$ from the base. Draw the plan and elevation of both curves of intersection and the development of the cone surface. Make a paper model of the solids.

EX. 7.—Draw the plan and elevation of the solids in Ex. 6, when the cylinder is $3''$ diameter, and is inclined $30°$ to the V P, showing the curves of both intersections.

EX. 8.—A cone frustrum is $5''$ high, $3\frac{3}{8}''$ diameter at the bottom, and $4\frac{5}{8}''$ diameter at the top, it is completely penetrated by a horizontal cylinder $3''$ diameter, $5''$ long, the axis of which bisects the axis of the frustrum. Draw the plan and elevation of the solids, showing the curves of intersection.

Special Cases of Intersection.—The following examples illustrate problems of frequent occurrence in practical draughts-

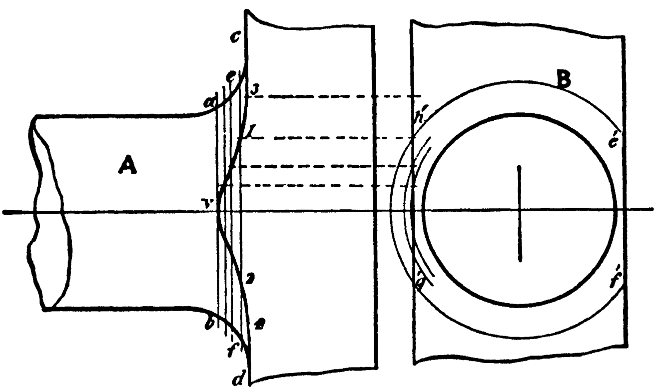


Fig. 91.

manship, and which should be mastered by all students proceeding to machine design:—We will suppose Fig. 91, A and B, to represent two elevations of a connecting-rod end of large size, although it equally well represents part of a crank shaft and crank web (see Fig. 196). A is a front elevation and B an end elevation, the conditions being that a round rod joins an oblong block, the rod increasing in diameter as it approaches the block, forming what is technically called a "fillet," and shown clearly in the view A. But as the width of the block is not as great as the *largest* diameter of the rod, the junction of the two solids produces a curve of the form shown in the front elevation, and it is to obtain this curve that the following geometrical construction is needed. Make a vertical section of the solids at the line *ef*, and draw the end view of the section on B, this end view will be the figure *e′ f′*, *g′ h′*, made up of the two parallel lines *e′ f′* and *g′ h′*, and the incomplete circle. The points *e′ f′* are evidently points on the surface of the oblong block, and by projecting across to the side elevation, give two points 1, 2 on line *ef* in the required curve. By taking other sections of the rod a sufficient

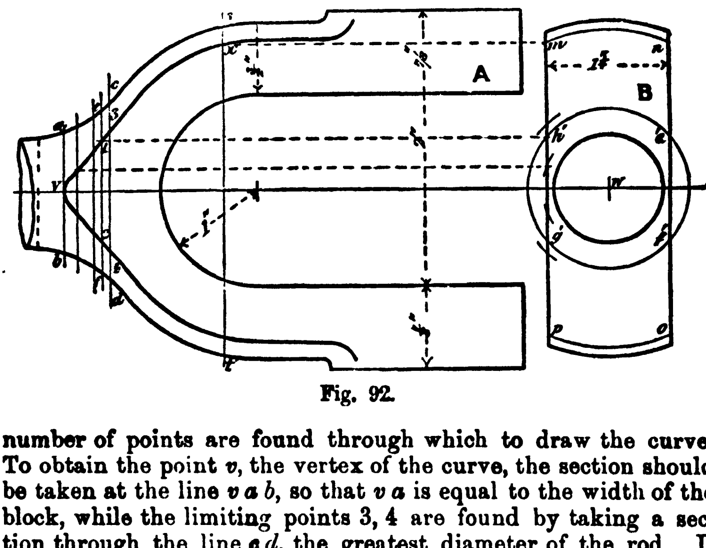


Fig. 92.

number of points are found through which to draw the curve. To obtain the point *v*, the vertex of the curve, the section should be taken at the line *v a b*, so that *v a* is equal to the width of the block, while the limiting points 3, 4 are found by taking a section through the line *c d*, the greatest diameter of the rod. If the width of the block is equal to the smallest diameter of the rod, the curve will form a point at *v*.

A second example is shown in Fig. 92, which represents a connecting-rod end such as used on the engine of Fig. 178. A is a front elevation and B a part end elevation, the problem being to find the inner curve on view A. We are at present only concerned with the geometry of the problem, so we may regard the solid as made up of a round rod or cylinder meeting with an increasing curve, a forked piece, the sides of which are flat surfaces, the top and bottom faces being turned cylindrical.

In the end view B, *m*, *n*, *o*, *p* is a section at the line *s t*, and as we know *m n* is part of a circle struck from *w* as centre, we obtain by a projector through *m* the point *x* in the curve. To obtain other points proceed exactly as in the last example, the same reading and lettering holding good for both.

EXAMPLES.

EX. 9.—Work the problem of Fig. 91 when the rod is 2" diameter, the block 2½" wide and 1¼" thick, the radius of the fillet between the rod and block being ⅜".

EX. 10.—Work problem of Fig. 92 to find the inner curve when sizes are as given in figure. Rod 1" diameter.

EX. 11.—A solid is made up of a cylinder 1¼" diameter, 3" long, which joins a sphere of 2½" diameter, by a fillet of ½" radius. Draw the plan of a horizontal section of the solid made by a plane parallel to the axis of the cylinder and ⅞" above.

Development of Spherical Surfaces.—A common type of steam boiler, known as "egg-ended," is constructed as a circular shell with spherical ends. Before bending the plates of which the ends are made they require to be cut to such a shape as that when bent they shall form part of the spherical end. The end is generally made up of four or six segments called "*gores*," the overlap of each required for rivetting together being allowed for in the development. In the following example the problem is treated as consisting simply of a cylinder with a spherical end, the semisphere being divided into six segments, and we shall show how to find the projection of the dividing or "contour" lines upon the front elevation of the solid, and the development of one segment or gore.

Fig. 93, A and B, represents part of the cylinder with a spherical end, A being the front elevation and B the end elevation. The semisphere is divided into six segments, as shown by the radial lines meeting at the centre *c'* of the end elevation. To draw the side elevation of the division lines *c' a'*, *c' b'*, we take a number of vertical sections of the semisphere by planes at right angles to the axis of the solid. The end elevations of such sections are circles, and each one gives by its intersection with the lines *c a*, *c b*, two points which can be projected across to the line of the section giving two points in the required curve. Thus a vertical section through the line *d e* gives for its end elevation a circle equal in diameter to *d e*, of which a part only, the arc *d' e'*, is shown drawn. This arc cuts the lines *c a*, *c b* in the points marked *f' g'*, and by drawing horizontal projectors through these two points to meet the section line $d\,e$, the two points f and g, which are points in the required curve, are obtained. It is better for the purposes of the development to take the sections, so that they equally divide the arc $c\,n$ and $c\,m$—that is, so that the parts $e\,h$, $h\,j$, &c., are equal.

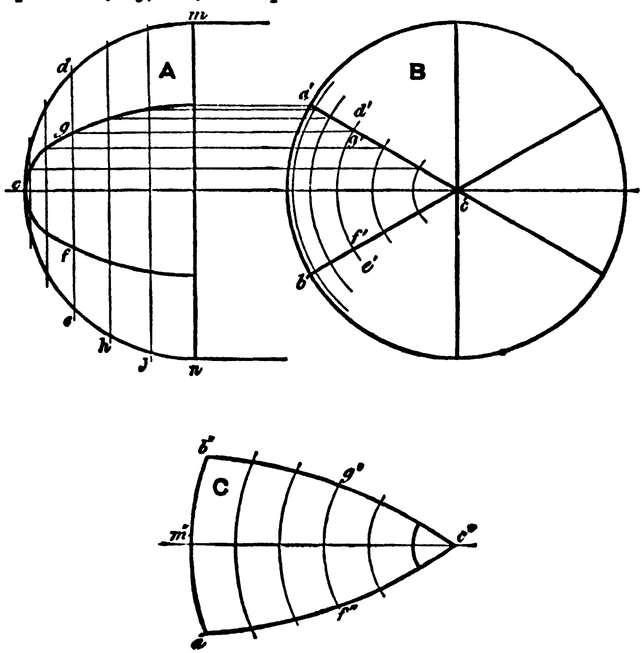


Fig. 93.

To develop one segment draw a line $c'' m''$ in the figure C equal in length to the half circumference of the semisphere—that is, equal to the arc $c\,m$ or $c\,n$, and divide the length $c'' m''$ into the same number of equal parts as the divisions on $c\,m$ or $c\,n$. Through each part draw arcs of circles with centre c'', for it is evident that the development of the arcs $a'\,b'$ $f'\,g'$ (Fig. B) will be arcs of circles, since $c'\,a' = c'\,b'$, and $c'\,f' = c'\,g'$, and so on for all similar contour lines. Make the length of each arc in Fig. C equal to the real length of the corresponding arc in Fig. B, thus $a''\,b'' = a'\,b'$, and $f''\,g'' = f'\,g'$. This is best done by stepping off the distance as a number of short chords, except for the outside arc $a''\,b''$, which can easily be calculated. The figure $c''\,a''\,b''$ is then the development of one-sixth of the semispherical surface.

EXAMPLES.

EX. 12.—A cylinder, 4″ diameter, 2″ long, has a spherical end which is divided into six segments. Draw a side and end elevation, and a development of one segment.

EX. 13.—Draw eight contour lines on a sphere of 4″ diameter.

EX. 14.—A sphere 3½″ diameter is penetrated by a square prism 2″ edge of base and 6″ long. The axis of the prism coincides with an axis of the sphere, the centre of the sphere being at the centre of the prisms. Draw a plan and elevation of the solids showing the curves of intersection when the axis of the prism is horizontal and parallel to the vertical plane, the sides of the prism being equally inclined to the paper. Develop one of the holes made in the surface of the sphere, and the surface of one part of the prism up to its contact with the sphere.

Projection of Helices and Screw Thread.—A helix may be defined as the curve traced out by a point moving round a cylindrical surface in such a way that its movement in the direction of the length of the cylinder shall be uniform with its movement around the surface of the cylinder. So that if a point starts from the base of a cylinder and moves in an upward direction and at the same time moving round the cylinder, so that when it has moved up, say ¼″, it shall have moved one-fourth the way round, and when it has moved ½″ up and 1″ up it shall have moved half round and wholly round, the path of the point would be a helix. The distance moved in the direction of the length of the cylinder during the complete revolution is called the *pitch* of the helix. Spiral staircases, spiral springs, and screw threads are generated by helices, but the latter example only will be explained in consideration of the great practical value of screw threads.

PROBLEM LVII. (Fig. 94).—*To draw a helix upon a given cylinder having a given pitch.*

Let *a′*, *b′*, *c′*, *d′* be the elevation of the cylinder and the circle of diameter *a b* its plan.

Make the distance *a′* 12′ equal to the given pitch. Then from the definition of a helix we know the curve must rise from *a′* to 12′ uniformly with its travel around the circumference of the circle which represents the plan of the cylinder. Therefore divide the circle and the pitch distance into the same number of equal parts, and mark as shown 1, 2, 11, and 1′, 2′, . . 6′. Draw projectors through each of the division points on the circle. Then when the point has moved round to 1, it must have moved up one-twelfth of its pitch, and it will, therefore, be on the horizontal line drawn through 1′, and at the point where this line cuts the projector through the point 1 on the circle, similarly the second point is where the projector through point 2 on the circle cuts the horizontal line drawn through the point 2′, and so on for the remaining points. The curve $a'\,e'\,12'$ is then a helix of one convolution.

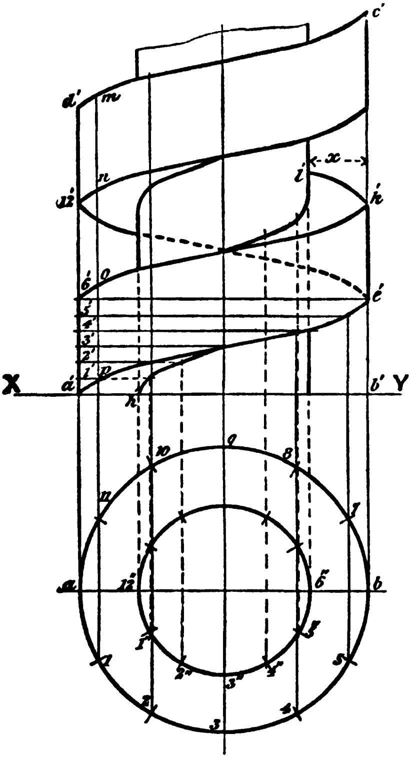


Fig. 94.

The figure also represents the geometrical projection of a square thread screw, for a further description of which see p. 166. The width of the thread $a'\,6'$ is half the pitch, therefore the curves beginning at 6′, 12′, and d' are each parallel to the half helix $a'\,e'$, and are half the pitch apart, and can be drawn by setting off distances of half the pitch along the projectors already drawn, starting from points on the curve $a'\,e'$—thus, $m\,n = n\,o = o\,p$. The depth of the thread is the distance marked x, therefore when the thread is cut, it leaves a cylinder of the diameter shown by the smaller circle in the plan. The inner edge of the thread is a helix of the same pitch drawn upon a cylinder of the diameter 6″ 12″, and is constructed in exactly the same manner as the helix representing the outer edge, the construction lines being shown dotted. Thus the curve starts from h' and rises to the line through 1′, while it has moved round one-twelfth of the circumference, and the first point in the curve is found by drawing a projector through the point 1″ on the circle to meet the horizontal line through the point 1′ in the elevation, and so on for the successive points. These inner curves disappear from sight at the centre of the screw, and are only seen on the top part of the thread from the centre to the limit of the cylinder, as seen in the figure. As these curves are similar, it is only necessary to obtain one in the manner just described, the others being conveniently found by setting off distances along the projectors. The parts of the curves, such as the one marked $k' l'$, are found by continuing the larger helix.

Helix upon a Conical Surface.—When a helix is traced upon a vertical cylinder, its plan is a circle, but when it is traced upon the surface of a cone it is continually approaching the axis, and, therefore, its plan is a spiral which uniformly approaches the point representing the plan of the cone vertex. The curve of a helix upon a cone is shown in plan and elevation in Fig. 95, the distance a' 12$'$ being the pitch. To obtain the curve, draw a number of stripes down the cone, and draw their plan and elevation as shown. Then after the first one-twelfth of its travel the point will be on the stripe v 1, and will have moved upwards from the starting point a' to the height of the horizontal line through 1$'$, its elevation will, therefore, be where the stripe v' 1$''$ cuts the line through 1$''$, and similarly for the second point, where the stripe v' 2$''$ cuts the line through 2$'$, and so on for each point. The plans of the points are found by drawing perpendiculars from the elevations to meet the plan of the corresponding stripe. For the stripes v 3, v 9 the method of sections must be adopted (see p. 125).

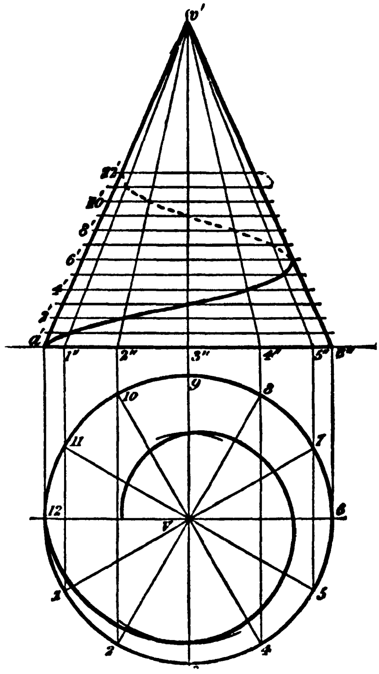


Fig. 95.

Spiral Springs.—When the material of the spring is of square section, it can be correctly drawn by adopting the construction of Fig. 94 for the square threaded screw, allowing for the absence of the solid cylindrical centre. The outer edge of the spring is a helix upon a cylinder equal to the outside diameter, and the inner edge a helix upon a cylinder equal to the inside diameter. With springs of circular section, a helix should be drawn upon a cylinder equal to the mean diameter of the inside and outside of the spring, which helix will represent the path of the centre of the material of which the spring is made, then a number of circles of diameter equal to the section of the spring should be drawn upon this helix as a centre, to give points for the lines of the spring

EXAMPLES.

EX. 15.—Draw a helix of one convolution upon a cylinder 3″ diameter, and develop the surface of the cylinder with the helix. Pitch of helix $1\frac{1}{2}$″.

EX. 16.—Show three threads of a square thread, outside diameter 3″, pitch 1″, depth $\frac{1}{2}$″.

EX. 17.—A square prism 4″ edge of base, 3″ high, is bored with a central hole and screwed internally with a square thread screw, $2\frac{1}{2}$″ diameter, $\frac{7}{8}$″ pitch, $\frac{7}{16}$″ deep. Show a vertical section through the centre of the prism when it stands with one base upon the paper.

EX. 18.—Draw a helix of one convolution upon a cone of $2\frac{1}{2}$″ diameter of base, 4″ high, and develop the cone surface with the helix. Pitch of helix 2″.

EX. 19.—A spiral spring is 2″ outside diameter, and is made of $\frac{3}{8}$″ round wire. Draw a length of the spring showing six coils, the pitch being $\frac{3}{4}$″. Show the two top coils in section, the section plane being vertical and passing through the centre of the spring.

SECTION XII.

ISOMETRIC PROJECTION.

The principles of isometric projection enable the three dimensions of a solid to be shown by one drawing, which, in appearance, is somewhat similar to a perspective representation, with the additional advantage that the actual sizes of the solid can be measured direct from the drawing.

If a cube be made to rest by one corner upon the paper, so that a diagonal of the solid is vertical, its plan will be represented by the drawing of Fig. 96. For the three top faces which meet in a solid right angle at A, are each equally inclined to the paper, therefore their plans are similar and equal figures, and for the same reason the length of the plans of all the edges are equal. It is also evident that the three lines A B, A C, and A D which represent the three edges of the solid right angle, make angles with each other of $\frac{360°}{3}$ or 120°, and that all other lines representing edges of the solid are parallel to one of these three lines. The figure is, therefore, very easily constructed, as the lines A C and A D make angles of 30° with the horizontal and 60° with the vertical, and can thus be drawn with the T square and 60° set square.

The above reasoning only strictly applies to oblong solids having solid right angles, but, as will be shown later on, the same construction can be very conveniently applied to irregular solids and solids with curved surfaces. For example, the drawings of the simple solids on pp. 104 and 105 are in isometric projection.

Referring again to the example of the cube in Fig. 96, it is evident that the length of the edges in the drawing, should not be equal to their real length, as they are all inclined to the

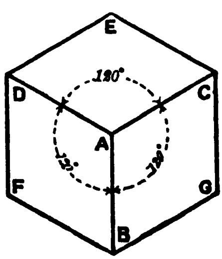


Fig. 96.

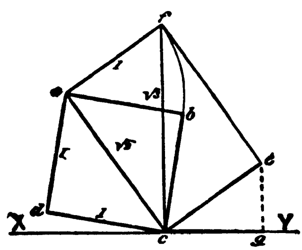


Fig. 97.

plane of the paper. The relation of their projected length to their real length can be seen on reference to Fig. 97; *a, b, c, d* is a face of the cube, and *a c* a diagonal of the face; if *a f* and *c e* are drawn at right angles to *a c*, and each made equal to the length of an edge of the cube, then the oblong *a, f, e, c* represents a section of the cube containing two diagonals of the solid, and *f c* is one of these diagonals. But in Fig. 96 this diagonal is supposed vertical therefore, draw X Y through *c* at right angles to *f c*, and the length *c g* is the projected plan of an edge of the cube. But if the cube edge *a d* or *d c* = 1, then *a c* = $\sqrt{2}$, and *f c* = $\sqrt{3}$; also, the triangles *e c g* and *c e f* are similar, therefore $ce : cg :: \sqrt{3} : \sqrt{2}$, hence, by constructing a right-angled triangle similar to the triangle fac, where the base is $\sqrt{2}$ and the perpendicular is 1, the hypotenuse will be $\sqrt{3}$, and real lengths along the hypotenuse, when projected upon the base, will give the isometric length.

The practical objection to this correct isometric projection is that it entails the use of an isometric scale, and that lengths of the object cannot be measured direct from the drawing. But if the cube in Fig. 96, or any other solid, is drawn its real size, the only alteration in the drawing is in its size, and not in its shape, and hence we see there can be no objection to making isometric projections the actual size of the objects they represent, thus dispensing with the use of an isometric scale, and making it possible to take measurements direct from the drawing. This arrangement is generally adopted in practice, and is adhered to in the following examples:—

It has been said, in referring to the drawing of the cube Fig. 96, that with oblong solids all lines are parallel to one of the three lines A B, A C, and A D.

These lines are termed the "ISOMETRIC AXES," and it is necessary in commencing any isometric projection to first set out these three lines.

We may now regard Fig. 96 not as the plan of a cube with a diagonal of the solid vertical, but as a drawing of a cube with one face lying upon the paper. On such a supposition the figure A C E D shows the top horizontal face, and the figures A D F B and A C G B vertical faces, so that in projecting a horizontal surface isometrically its length and breadth must be set off along the two sloping isometric axes A C and A D, while for a vertical surface, its length and breadth must be set off along the vertical axis A B, and one of the sloping axes A C or A D. It is important to remember this distinction.

Either surfaces or solids can be projected isometrically, and, as before stated, the construction can be extended to surfaces or solids not of oblong form, the method by which this is done will be clearly seen in the following examples, but it will be better understood by remembering that since the isometric axes represent lines at right angles only, the projection of figures containing other angles requires that they shall be surrounded by oblong figures, thus a circle is first enclosed in a square and a hexagon in an oblong.

PROBLEM LVIII. (Fig. 98*a*, *b*).—*To draw the isometric projections of a hollow square prism (a) with its axis vertical, (b) with its axis horizontal.*

Fig. 98*a*, draw the isometric axes *a b*, *a c*, *a d*, set off *a e* along *a c*, and *a f* along *a d*, equal in length to the edge of the prism base. Draw *f g* parallel to *a c* and *e g* parallel to *a d*, meeting at *g*, then the figure *a f g e* is the isometric projection of one outside square base of the prism. Set off $a\,1 = a\,2 = g\,3 = g\,4$, equal to the thickness of the sides of block, and draw lines as shown dotted to obtain the inside square 5, 6, 7, 8. Set off the length of the prism down the axis *a b* from *a* to *h*, and draw lines through *f* and *e* parallel to *a b*. Through the point *h* draw lines parallel to the other axes, as shown, thus completing the projection of the prism. Dotted lines representing the bottom base, can be drawn if desired.

Fig. 98*b*, to draw the prism with its axis horizontal, the square

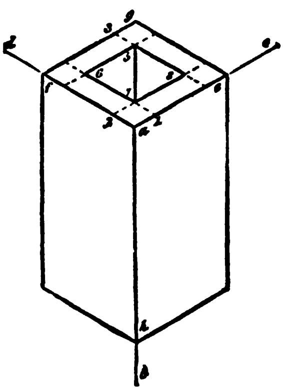


Fig. 98*a*

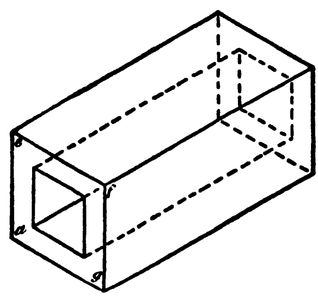


Fig. 98*b*.

representing its base must be drawn as a vertical face, and is thus shown at *a e f g*. The completion of the projection needs no further description.

EXAMPLES.

EX. 1.—Draw the isometric projection of an oblong, sides 3" and 2", when its plan is horizontal.

EX. 2.—A cube, 2½" edge, stands upon a square block 3¼" edge, 1" thick. Draw their isometric projection when the block stands upon the ground.

EX. 3.—Make an isometric projection of a wooden box 8" long, 6" deep, 4" broad outside, and having a flat lid opened through an angle of 120°, the thickness of the wood being ½" throughout. Scale, half full size. (Vict. Hon., 1892.)

EX. 4.—Two square timbers 9″ × 9″ are joined at right angles to each other by means of a tenon and mortise joint, the width of the mortise being 3″. Draw an isometric projection of the timbers ready for jointing but separate from each other. Scale, 3″ = 1′.

EX. 5.—Draw the isometric projection of a headed key 6″ long, taper $\frac{1}{8}$″ per foot, width $\frac{3}{4}$″, least thickness $\frac{5}{8}$″, thickness of head $1\frac{1}{4}$″. Use an isometric scale. (Vict. Hon., 1889.) See Fig. 133*b*.

PROBLEM LIX. (Fig. 99).—*To draw the insometric projection of a hexagonal prism, with its axis vertical.*

Draw the hexagon *a, b, c, d, e, f, g* (Fig. 99, A) representing the

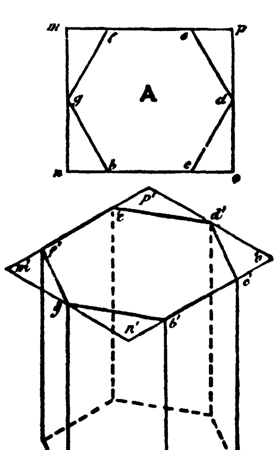


Fig. 99.

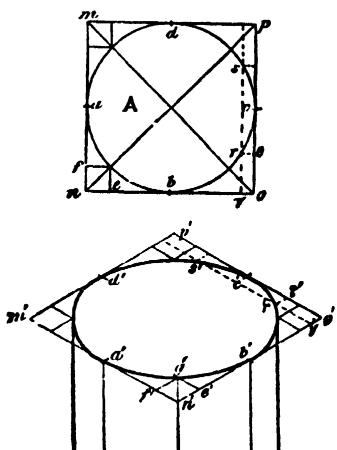


Fig. 100.

base of the prism, and surround it by the oblong *m, n, o, p.* Set off a long side of the oblong *n o* along one of the sloping insometric axis, and a short side *n m* along the other sloping axis, and complete the insometric projection of the oblong, as at *m′, n′, o′, p′*. Make *n′ b′* = *n b*, and *o′ c′* = *o c*, and by parallels mark the corresponding points *e′ f′* on the other side *m′ p′*; the points *g* and *d* are at the middle of *m n* and *o p*, therefore bisect *m′ n′* and *o′ p′* at *g′* and *d′*, and join as shown, thus obtaining the isometric projection of the hexagonal base. Draw lines through each corner of the hexagon parallel to the vertical axis, and equal in length to the height of the solid and complete as before.

EXAMPLES.

EX. 6.—Draw the isometric projection of a hexagon of $1\frac{1}{2}''$ edge, when its plane is vertical.

EX. 7.—Draw the isometric projection of a hexagonal prism, edge of base $1\frac{1}{4}''$, height $2''$, when its axis is vertical.

EX. 8.—Draw the isometric projection of a pentagonal pyramid edge of base $1\frac{1}{2}''$, axis $3\frac{1}{2}''$, and horizontal.

PROBLEM LX. (Fig. 100).—*To draw the isometric projection of a cylinder with its axis vertical.*

Draw the circle *a*, *b*, *c*, *d* (Fig A) and surround it by a square *m*, *n*, *o*, *p*, and project it isometrically as a horizontal surface, as shown at m', n', o', p'. Bisect the sides of the figure in the points a', b', c', d', thus obtaining four points in the projection of the circle. To obtain other points draw the diagonals of the square *m*, *n*, *o*, *p*, cutting the circle in four points, draw lines through these points parallel to the sides of the square, thus making four small squares. Draw the isometric projection of these squares in the corners of large squares, thus $n'e' = ne$ and $n'f' = nf$, thus giving the point g' as an additional point in the curve, and similarly for the other corners. A closed curve can then be drawn through the eight points, which represent the isometric projection of the circular base. Through each point in the curve draw lines parallel to the line $g'h'$, and make the length of each equal to the length of the cylinder, and complete the figure as shown.

The method adopted of finding additional points in the curve is one that should be carefully noticed, as its application is required for other curves. We could, of course, find still more points in the projection, although the figures would not then be squares, but oblongs, such, for example, as is shown by dotted lines for the points *r* and *s*, which are further from one side of the square than the other.

EXAMPLES.

EX. 9.—Draw the isometric projection of a circle of $3''$ diameter when its plane is horizontal.

EX. 10.—Draw the isometric projection of a right hexagonal prism, side of base $2''$, length $3''$, with a circular hole $1''$ in diameter bored through it at right angles to the axis. (Vict. Hon., 1890.)

EX. 11.—Draw the isometrical projection of a cross consisting of two cylindrical rods 1" in diameter and 6" and 4" long, intersecting axially and at right angles at the middle of their lengths. (Vict. Hon., 1891.)

EX. 12.—A bolt consists of a conical part 2" long, $\frac{3}{4}$" diameter at the end, and increasing to $1\frac{1}{4}$" at the head. The head is a square block, 2" × 2" and $\frac{3}{4}$" thick. Draw an isometric projection when the axis of the bolt is vertical, the head being at the top. (S. & A. A., 1887.)

EX. 13.—Draw the isometric projection of a 3" hexagonal nut and washer, showing the chamfers of the nuts; the washer to be $6\frac{1}{8}$" diameter and $\frac{3}{16}$" thick (see p. 169 and Fig. 117).

PROBLEM LXI. (Fig. 101).—*To draw the isometric projection of an irregular block, the plan and elevation of which is given.*

Let P be the plan and E the elevation of the block. Surround the elevation E by an oblong *m, n, o, p,* and draw the isometric

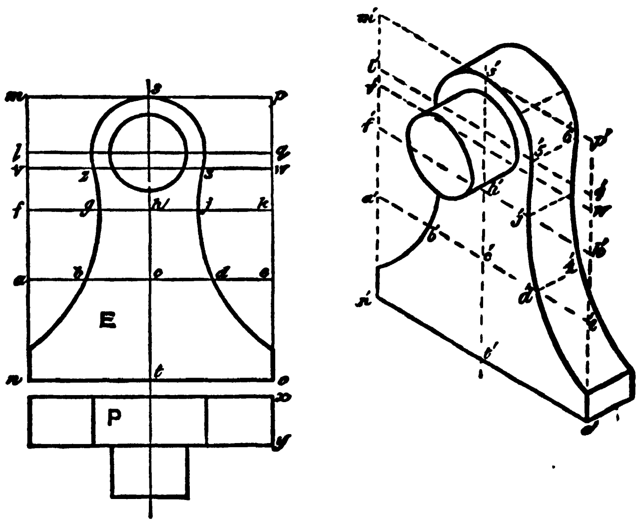


Fig. 101.

projection of the oblong in a vertical plane as *m', n', o', p'*, shown by dotted lines. Draw the centre line *s t* and *s' t'* of the oblong in each figure. Draw a number of horizontal lines across the oblong in Fig. E as *a e, f k* and *l q,* and project them isometrically as at *a' e', f' k', l' q',* so that *o' e' = o e, o' k' = o k,* and *o' q' = o q.* Then make *c' d'* and *c' b' = c d* and *c b* on the elevation, Fig. E, thus obtaining points *d'* and *b'* in the isometric projection of the curved edges. Take as many lines in this way as are necessary to give points through which to draw the curves, and let one line $l\,q$ pass through the centre of the top curve, and a line $v\,w$ through the points 2, 3, where the two curves meet, that is where the direction of the curve changes. The back curved edge is found by drawing lines sloping to the right at 30° equal in length to the thickness of the block, thus $d'\,4' = 5'\,6' = x\,y$. The cylinder and base will present no difficulty after previous Problems.

EXAMPLE.

EX. 14.—Work Problem lxi. (Fig. 101) taking any convenient and suitable sizes.

The student who wishes to acquire skill in isometric projection should work other and more difficult examples than those already given. Very good practice is gained by drawing the projection of such parts as an ordinary drawing office stool, a tee piece with three flanges for pipes, a simple shifting lathe head-stock, and a simple plummer block bearing (see Fig. 154*a*).

APPENDIX.

ADDITIONAL EXAMPLES.

THE following examples are taken by permission of the Board of Education and H.M. Stationery Office from the Examination Papers of the Board of Education in Practical, Plane, and Solid Geometry, 1897 to 1904.

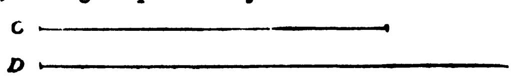
Questions marked * have diagrams attached to them.

SCALES.

1. The actual distance between two points is known to be 60 yards, but on a map the points are shown $4\frac{1}{2}$ inches apart. Draw a scale for the map, dividing and figuring it so as to be of practical use. Show 100 yards, and make the smallest division 10 yards. Write the representative fraction above the scale.

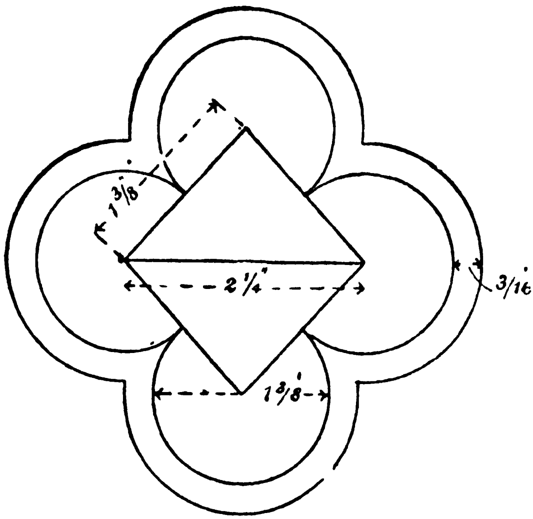
2. Draw a scale of 25 inches to 1 mile, reading up to 500 yards. No unit smaller than 10 yards need be shown. Draw a line representing 370 yards.

*3. The given line C represents a length of 11·3 feet. Construct a decimal scale of feet, and by its use measure and write down, as accurately as you can, the length represented by the line D.



CONSTRUCTION OF GEOMETRICAL FIGURES.

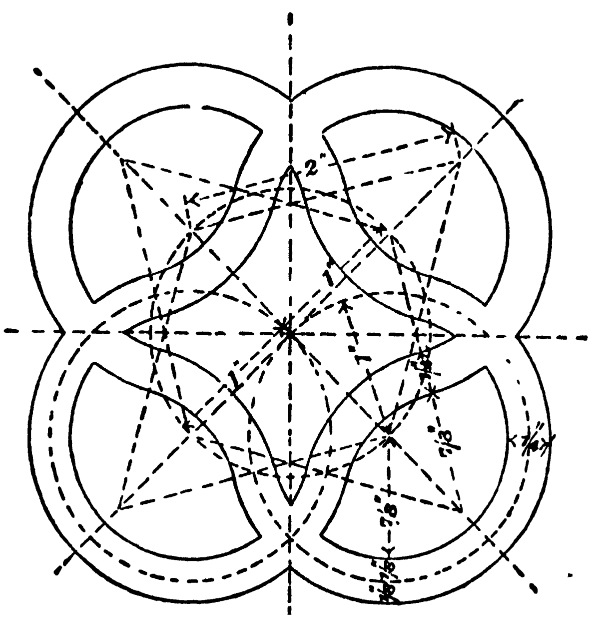
1. Draw a circle, radius 5 c.m. By stepping off with the dividers or otherwise, divide its circumference accurately into seven equal parts. From each point of division draw lines to every other point. (This exercise is to test your power of neat and accurate draughtsmanship).



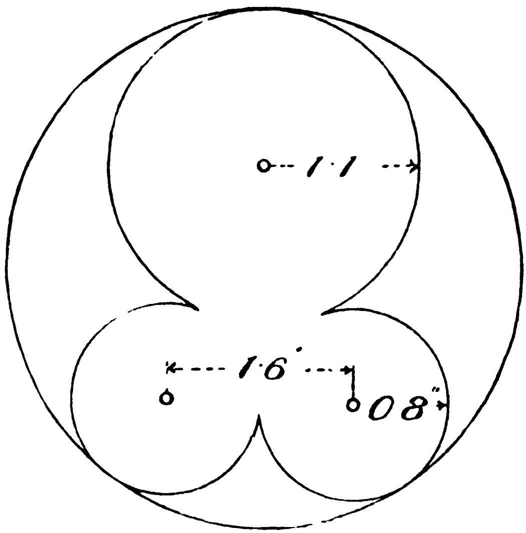
*2. Draw the figure from the dimensions given. It is *not* to be merely copied the same size as in the diagram.

*3. Draw the figure *to the dimensions indicated*. The dotted lines and circles indicate the construction and need not be reproduced. It will be seen that, for drawing the figure, only two radii are employed, namely of $\frac{7}{8}$ inch and $1\frac{1}{4}$ inches.

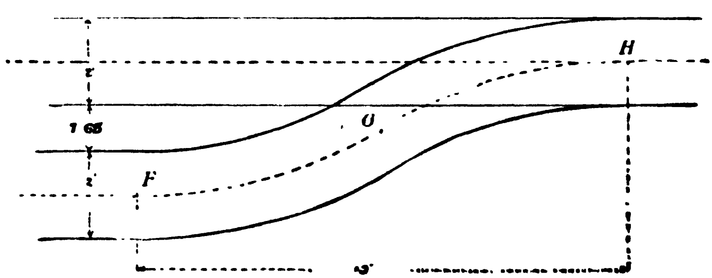
N.B.—No marks will be given for a mere reproduction of the figure.

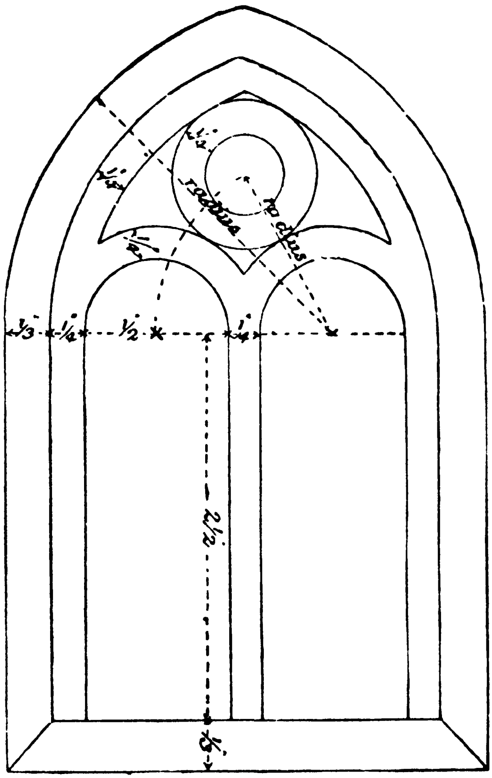


*4. Construct a figure made up of a trefoil of tangential arcs with the circumscribing circle, like the one shown, but to the dimensions given. Show all the construction lines clearly.

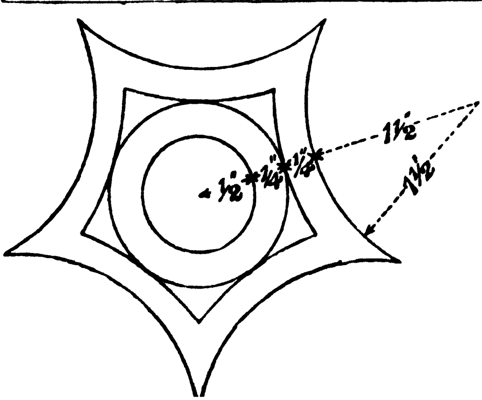


*5. The figure shows a junction of rails for small waggons, the lines drawn dotted being midway between the rails. Set out the figure to a scale of 1 centimetre to 1 foot, working to the given dimensions, and not copying the diagram. Show the construction for determining the centres of the circular arcs *FG*, *GH* which are of equal lengths and radii. Indicate on the drawing the radii to which the several portions of the curved rails must be bent.



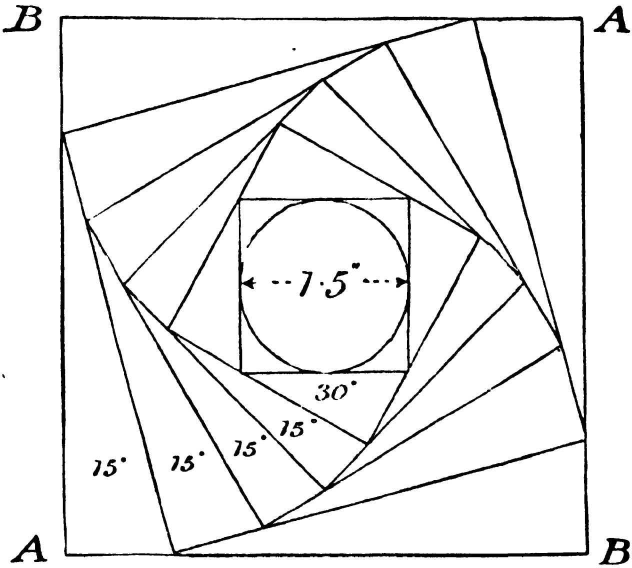


*6. Draw a figure similar to the given one, but using the dimensions marked on it. (No marks will be given for reproducing the diagram the same size.)

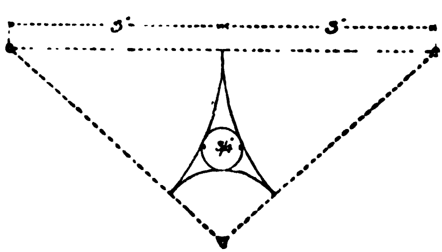


*7. Draw a figure similar to that shown in the diagram, but using the dimensions given in figures on it. (No marks will be given for mere reproduction of the diagram.)

*8. Describe a circle of ¾ inch radius. Then carefully draw in succession the series of circumscribing squares, so as to obtain a figure similar to the one shown. Then measure accurately and write down the lengths of the four sides *AB*, and the two diagonals *AA*, *BB*. *N.B.*—All the lines may be drawn by the use of the tee-square and the 45° and 60° set squares, without the employment of a protractor. (No credit will be given for a mere copy of the diagram.)



*9. Draw a figure like the one shown, but to the given dimensions. Show clearly all the construction lines. *N.B.*—No credit will be given for a mere copy of the diagram.



Construction and Measurement of Angles.

1. The tangent of an angle is 0·715, construct the angle. Determine its sine. The use of arithmetic is permitted.

2. The cosine of an angle is 0·610, construct the angle. Measure it in degrees and in radians. What is the tangent of the angle? The use of arithmetic is permitted.

3. Draw two lines OX, OY, at right angles. Mark a point P distance 5·25 inches from OY and 3·81 inches from OX. Join OP and measure the length of OP. Verify your answer by calculation. Measure the angle XOP in degrees; verify this result by first calculating the tangent of the angle XOP and then referring to a table of tangents of angles.

4. A tower is built on level ground and a flagstaff rises from its top. A person at a distance observes the angles of elevation of the top of the tower and the top of the flagstaff to be 18° and 27° respectively above the horizontal. He then walks 38·6 yards further off and finds the angle of elevation of the top of the flagstaff to be 18°. Determine and state the heights of the tower and flagstaff above the level of the eye of the observer. Use a scale of ½ inch to 10 yards

5. A surveyor is making a map on which he wishes to locate two inaccessible objects H and K situated towards the north. He lays off a base line DE, 20 chains or ¼ mile long, going due east. When stationed at D, he measures the angles EDK, KDH, by means of a sextant, and finds them to be 51° and 55°. When stationed at E the angles DEH and HEK measure 48° and 62°. Plot the points D, E, H, K to a scale of 8 inches to the mile. Measure the distance and direction of HK.

6. P and Q are two distant objects. A person stationed at a place A, observes the angle PAQ subtended by P and Q to be 31°. He then walks 320 yards in a direct line towards P, to a place B, and finds that the angle PBQ now subtended by P and Q is 44°. Find and measure the distance from A to Q, and the angle AQB that is subtended by A and B at Q, all four points being in one plane. Draw to a scale of ½ inch to 100 yards.

7. There are three places, O, A, B, on a level plane. A is 52 feet east and 263 feet south of O. B is 138 feet east and 217 feet north of O. Plot these points to a scale of 1 inch to 100 feet. Measure the distance apart of A and B. Measure and state the direction from A to B in degrees north of east.

8. A person starting from a place A walks 45 yards in a straight line eastwards. He then turns 60° to the left and walks 37 yards in the direction which he is now facing. He again turns to the left, through 110°, and goes a distance of 86 yards. Through what angle to the left must he turn in order to face his starting place A, and how far must he walk in order to get there? Measure the results. Use a scale ½ inch to 10 yards.

Construction of Triangles, Quadrilaterals and Polygons.

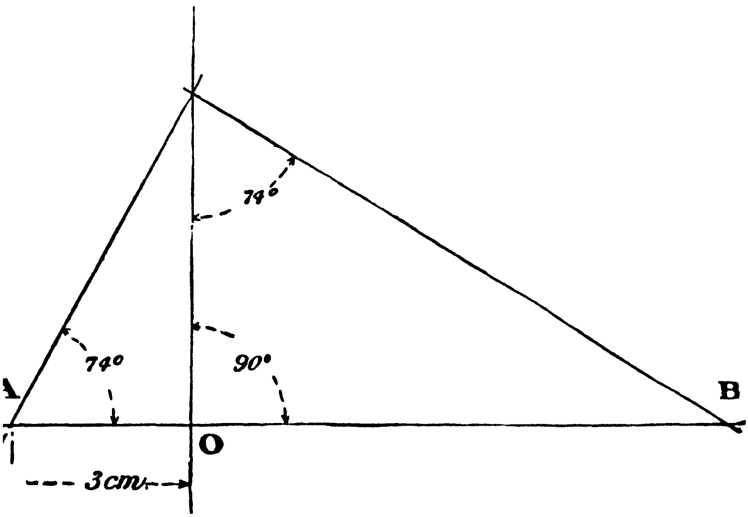
1. Construct a triangle ABC, having given:—Side $BC = 4·2''$; angle $ABC = 71°$; angle $BAC = 65·6°$. Measure the angle ACB, and the sides AB, AC.

2. A rectangle has one side 1¾ inches long, and a diagonal 2⅝ inches. Draw the rectangle, and enclose it in a square so that an angle of the rectangle is in each side of the square.

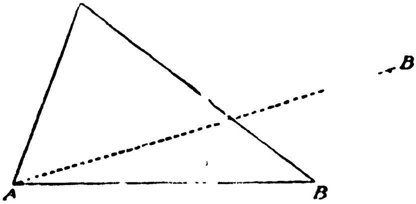
3. Draw a line AC $3\frac{1}{2}$ inches long. Through a point O in it, $1\frac{1}{2}$ inches from A, draw a line perpendicular to AC. On this line let two points B and D be on opposite sides of the line AC. Draw the quadrilateral $ABCD$ of which AC and BD are the diagonals, the angles ABC and ADC being 65° and 115° respectively.

4. Draw a square $ABCD$ of $2\frac{3}{4}$ inches side. Find a point P inside it, such that the angle APB may = 90°, and the angle CPD = 70°.

*5. Construct the figure accurately to the given data, and not by copying the diagram. Measure, in centimetres, the length of OB. Obtain tan 74° from the tables, square and multiply by 3. Compare this result with your previous measurement of OB.



*6. You are required to enlarge the given triangle, and also to alter its position, so that the side AB shall occupy the position AB'.



7. Construct a right-angled triangle ABC to the following data, C being the right angle :—

Base BC = 10 inches, base angle ABC = 13°.
Measure the height CA. Obtain tan 13° from the tables.
Calculate the height CA

Circles and Tangents.

1. Draw a circle of $\frac{3}{4}$ inch radius inscribed in a triangle whose sides are in the proportion of 4, 6, and 7. Indicate a point on the circumference of the circle, which shall be equi-distant from the two ends of the longest side of the triangle.

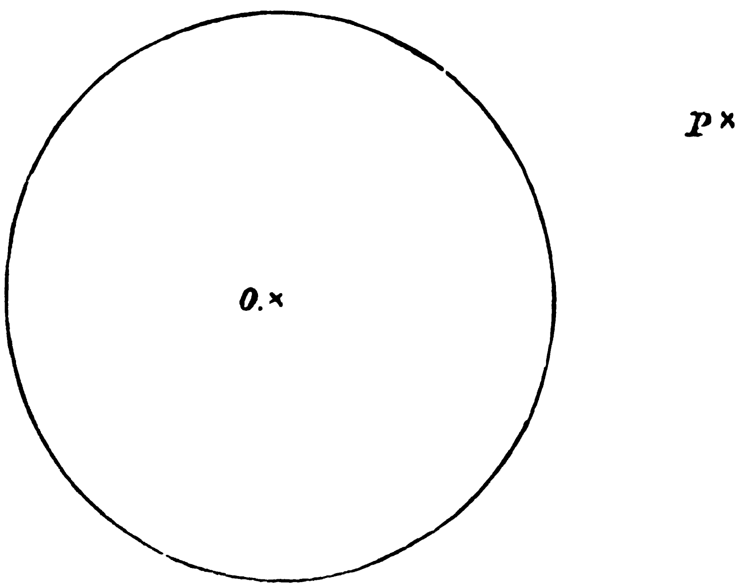
2. Inscribe a square in a circle of 2 inches radius. In the square draw four circles, each touching two other circles and one side of the square.

3. Draw an isosceles triangle TQP; $TQ = QP = 1$ inch; the base angles, each, equal to 30°. Q is on the circumference of a circle to which PT is tangent at the point P. Draw the circle.

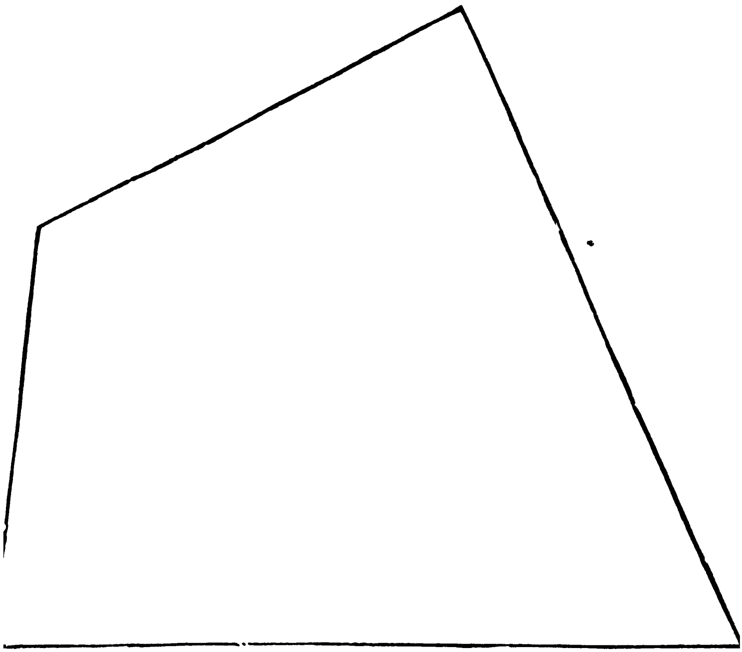
4. Describe a circle of $1\frac{1}{2}$ inches radius, and draw any diameter AB. From a point in AB produced, draw a tangent 2 inches long to the circle.

5. Draw a circle of 2 inches radius, and inscribe in it five equal circles each touching two of the others and also the circumference of the containing circle.

*6. From the point P draw a line cutting the given circle, centre O, so that the portion of the line intercepted within the circle may be 2 inches long.



*7. A board is of the shape and size given, the linear scale being 1 centimetre to 1 inch. It is required to cut from this board the greatest possible circular disc. Find, by construction, the centre of the disc. Draw the circle, and measure, to scale, the diameter of the disc.



Areas of Plane Figures.

1. From any angle of a regular pentagon of 2-inch sid draw a line dividing the pentagon into two parts, one of which will be twice as large as the other.

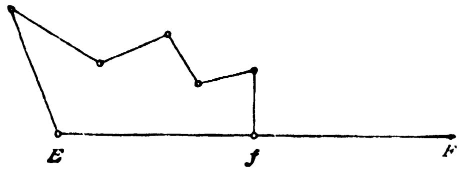
2. Draw two lines, AB, AC, making an angle of $40°$; take a point P on AB at 2 inches from A, and draw a line PD meeting AC at a point D, so that the triangle APD shall be equal in area to the difference of the squares of two lines, respectively 2 inches and 2½ inches in length.

3. Let $ABCD$ be an irregular quadrilateral figure, and O a point inside it. Construct the figure according to the following conditions:—

$AO = 2\frac{1}{4}$ inches. $BC = 3$ inches. $DO = 2\frac{1}{2}$ inches. $CD = CO \times 2$.
Angle $AOB = 60°$. $OAB = 60°$. $BOC = 90°$.

Draw a square equal in area to the quadrilateral.

*4. Having pricked off the given polygon, enlarge it to double size, *Ef* becoming *EF*. Then draw a rectangle with an area equal to that of the enlarged polygon and having *EF* for one side. (*For figure see next page*)

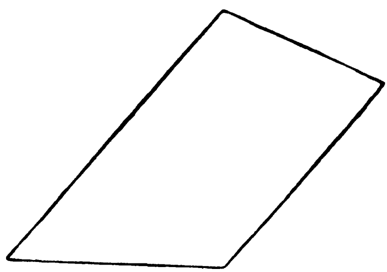


5. Draw a triangle ABC of which the base AB is $5\frac{1}{2}$ inches long, the vertical angle being 100° and the altitude $1\frac{1}{2}$ inches. On the base AB draw an isosceles triangle of which the area will be double that of the triangle ABC.

6. Draw an irregular pentagon $ABCDE$ from the following conditions, and reduce it to a triangle standing on the side AB (produced if necessary) and with D as vertex.

Side $AB = 1\frac{3}{4}''$; $BC = 2''$; $CD = 2\frac{1}{4}''$; $DE = 2\frac{1}{2}''$.
Angles $ABC = 120°$; $BCD = 80°$; $CDE = 125°$.

*7. Determine, in square inches, the area of the given quadrilateral. Any method may be employed, the use of arithmetic being allowed. Find also the area in square centimetres. The answers should be correct to within 1 per cent.



8. Draw a circle of 3·7 inches diameter. Divide it into three sectors which have their areas in the ratios of 4 : 5 : 6. Measure the lengths of the three arcs of the sectors.

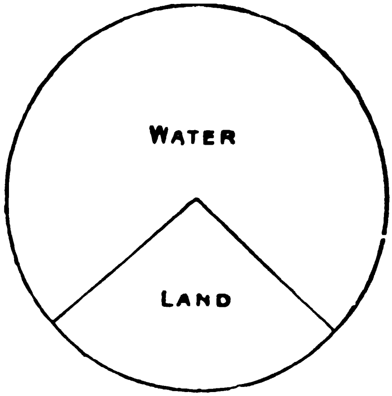
9. Draw a triangle ABB, base $BB = 10·8$ cm.; sides AB, $AB = 15·7$ cm. and 8·9 cm. Draw a line parallel to the base so as to bisect the sides in CC. Find in square inches the areas of the triangles ABB, ACC, and the area of the trapezoid $BBCC$. What are the ratios of these areas to one another?

10. Construct a triangle ABC to the following data:—

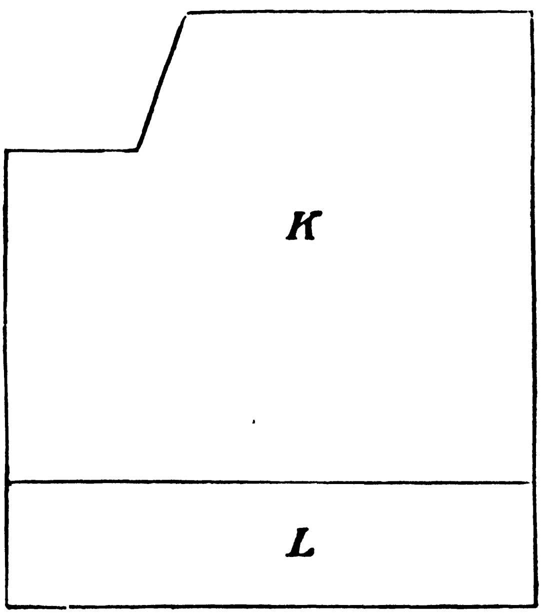
$$AB = 3·8 \text{ inches}; \; BC = 3·3 \text{ inches}, \sin ABC = 0·652.$$

Taking AB as the base draw the perpendicular from the vertex C, and measure the altitude of the triangle. Verify the result by calculation using the given line. What is the area of the triangle? What is the length of the base of a triangle of equal area, but having an altitude of 1·6 inches?

*11. The given circle is divided into two sectors, the areas of which represent the relative areas of land and water on the earth's surface. If the area of the land is 52 millions of square miles, what is the area of the water?

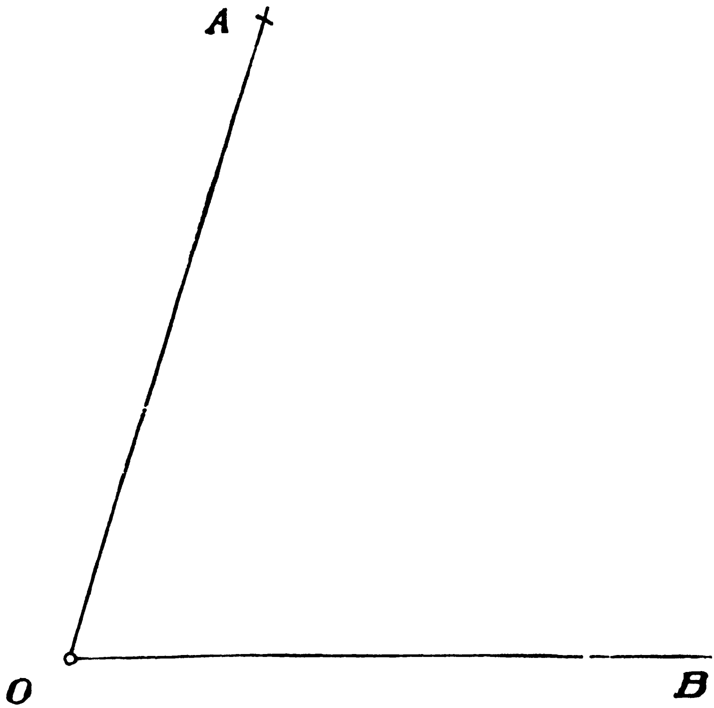


*12. The plan of a hall is given, scale ½ inch to 10 feet; *L* is the platform and *K* the body of the hall. Find the area of *K* in square feet. Calculate the sitting accommodation of *K*, 30 per cent. of its area being occupied by passages, and allowing one person to every 4 square feet of the remainder.



Problems on Measurement.

*1. Measure, as accurately as you can, the given angle AOB, in degrees and decimals of a degree. Measure also OA in inches and decimals of an inch. From A draw AM perpendicular to OB, and measure OM. By ordinary arithmetic divide OM by OA, and give the quotient.

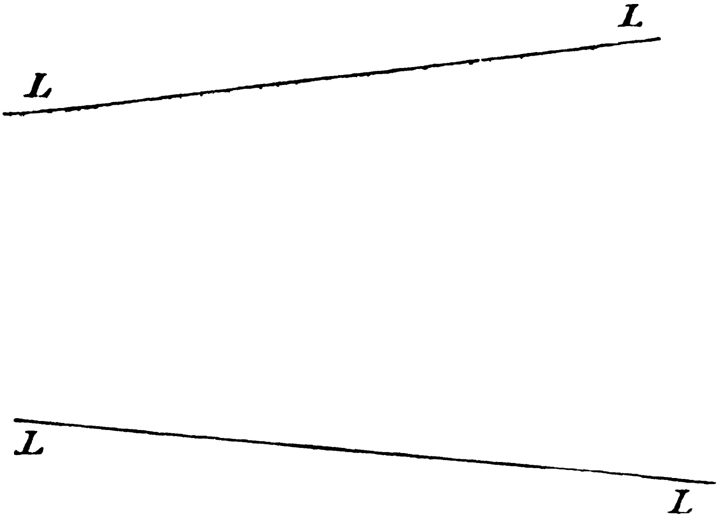


2. Draw a circle of 2·25 inches radius. In this circle inscribe a quadrilateral $ABCD$, having given

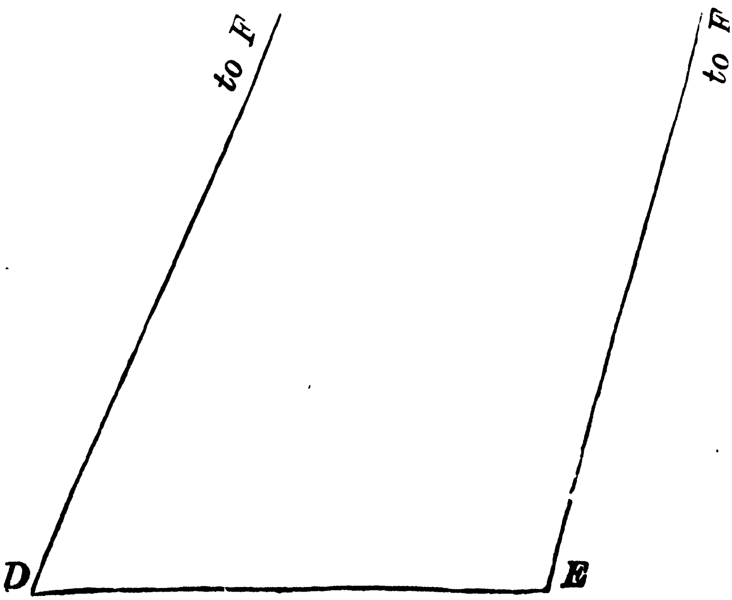
Sides $AB = 2{\cdot}87$ inches; $DC = 2{\cdot}5$ inches.
Angle $BCD = 76{\cdot}5^\circ$.

Measure, in degrees, the angle BAD. Draw the tangent to the circle at A. Join AC and measure the angles which AC makes with the tangent. Also measure the angles ABC and ADC.

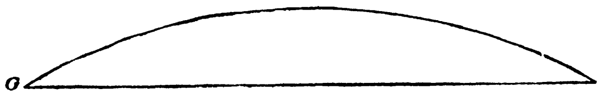
*3. Measure, in degrees, the angle between the two given lines LL, LL. With what accuracy do you consider you can measure a given angle in degrees by the appliances you possess? That is, state the greatest error to which you think your answer may be liable. What is the magnitude of the angles in radians? You may use arithmetic if you wish.



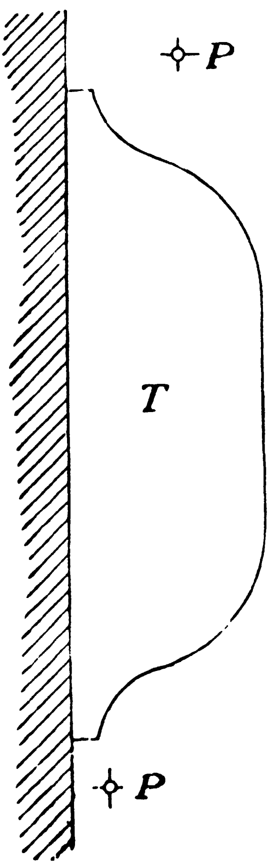
*4. A portion of a triangle is given, base DE, the vertex F being out of reach. By the aid of construction, measurement, and arithmetic determine the length of the side EF.



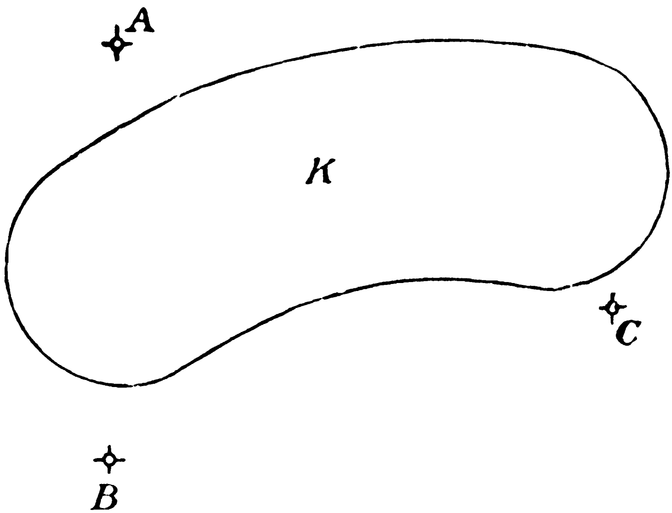
*5. A segment of a circle is given. Determine and measure in degrees, the angle contained by this segment. Mark a point P on the arc distant 0·2 inch from one end G; then draw accurately, and produce, the chord GP. Draw the tangent to the arc at the end G.



*6. In the given figure, P, P represent two points on the floor of a room. The distance apart of P, P is required; T is an obstacle preventing direct measurement. Suppose that by means of a chalked string and tape measure, or otherwise, you were able to draw straight lines and measure lengths on the floor of the room. Indicate clearly on your drawing what lines you would draw, and what measurements you would make, so that by plotting these to scale, or by easy calculation, or in any manner, the required distance between P and P could be ascertained.



*7. A, B, C are three points on the floor of a room. The shape of the triangle ABC is required; K is an obstacle preventing direct measurement. Having the same conditions and materials of the last question, set out in a scale drawing the shape of the triangle ABC. (A square is not available for drawing perpendicular lines on the floor, nor may you use a protractor).

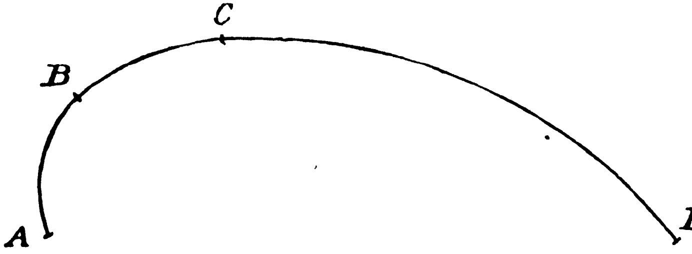


8. Draw a circle, diameter 4·2 inches. Mark a point A on its circumference, and with centre A draw a circle, 0·34 inch radius, cutting the first circle in B, B. Draw and produce the lines AB, AB. Measure the angle BAB. Your answer must be correct to within 0·5°, to ensure which you should employ a special method in order to find more accurately the directions of the lines AB.

*9. The given curve is made up of three circular arcs which join tangentially. Plot this figure to the following data:—

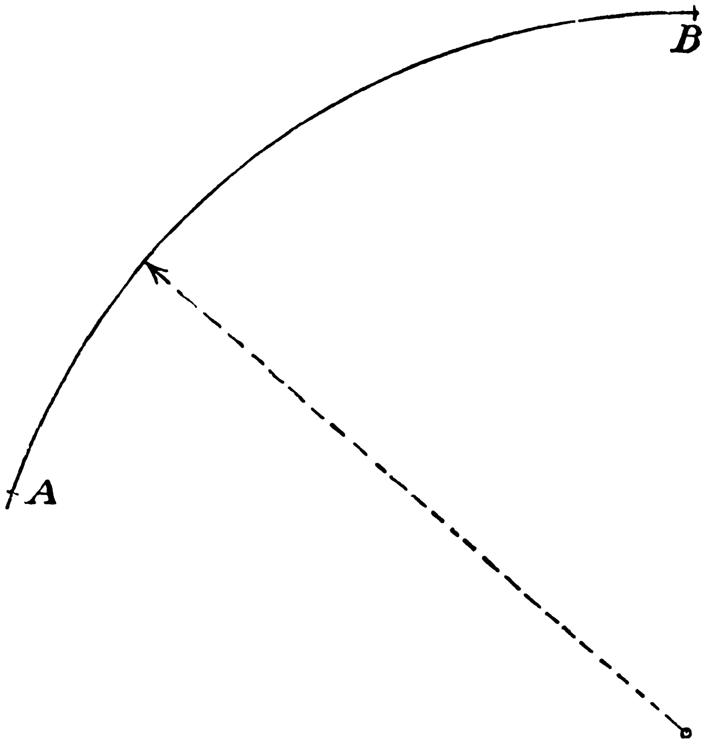
Radius of AB = 1 inch; angle subtended at centre = 80°.
Radius of BC = 2·25 inches; length of arc BC = 1·5 inches.
Length of arc CD = 4 inches; angle subtended at centre = 1 radian.

Your figure represents a path to the scale of 1 inch to 100 yards; measure the length of this path.

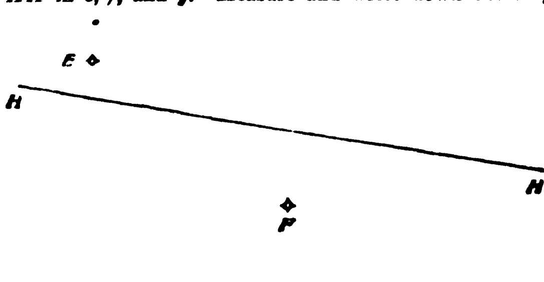


*10. The following construction is sometimes used for finding the approximate length of a circular arc:—Let AB be the arc. Draw the chord AB and produce AB to C making $BC = \frac{1}{2} AB$. Draw the tangent at B, and with centre C, radius CA cut this tangent in D. Then

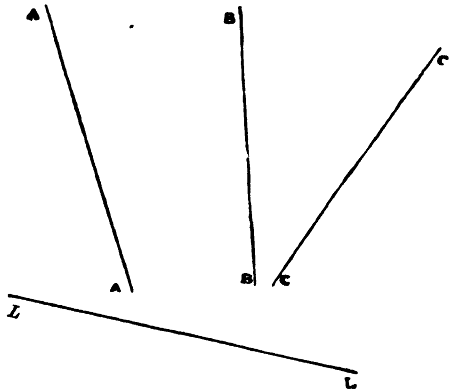
BD = arc BA nearly. Apply this construction to the given arc. Measure BD in centimetres. Also, by means of tracing paper and a pricker, or otherwise, find the exact length of the arc. Is BD too long or too short, and by what per cent.?



*11. Draw parallel lines through E, F, G, such that the line through F is equidistant from the lines through E and G. Let these parallels cut the line HH in e, f, and g. Measure and write down the lengths of ef and fg.

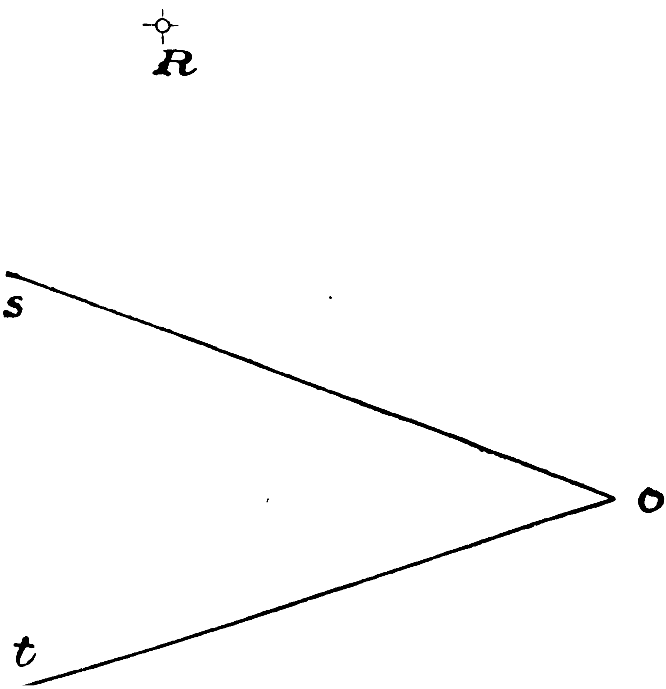


*12. Draw a line parallel to LL to cut the given lines AA, BB, CC, in a, b, c, so that ac shall be 1·6 ab. A locus may be used if desired.

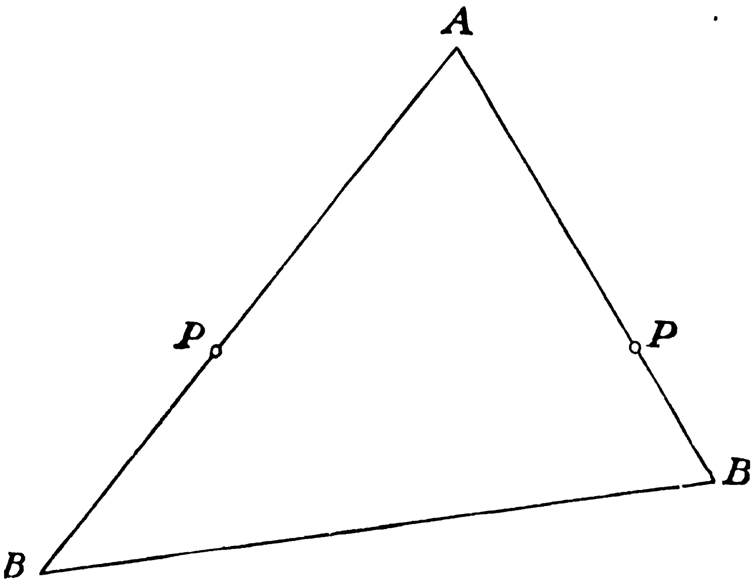


Loci.

*1. Through the given point R draw a straight line to cut the lines Os, Ot in S and T so that the ratio $RS : RT$ shall be 2 : 3. A locus may be used if desired.



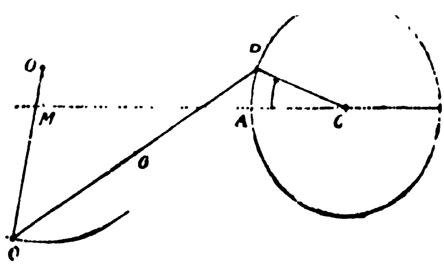
*2. The given triangle ABB moves so that the sides AB, AB always pass through the fixed points P, P. Trace the locus of the point A. What is this curve? Also plot the locus of one of the points B.

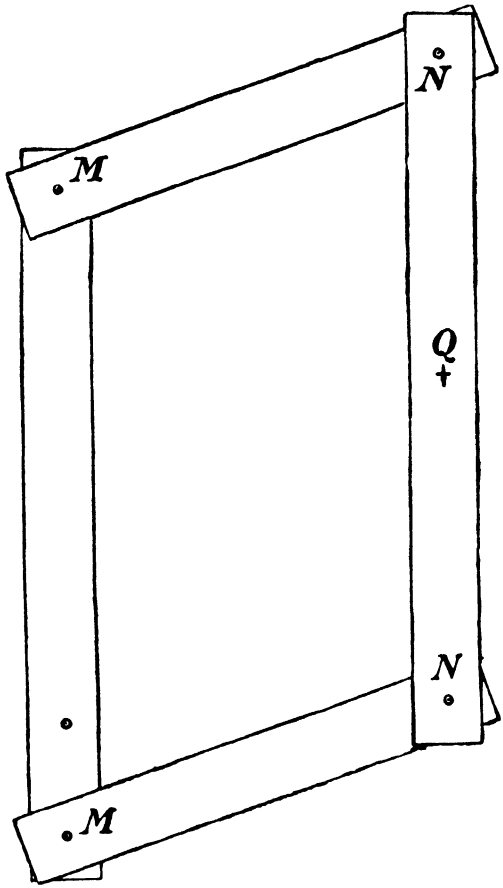


*3. PQ is a link one end P of which moves in a circular path, centre C, and the other end Q oscillates in a circular arc, centre O. The dimensions are:

$$CM = 6'\ 0''; \quad MO = 9''; \quad CMO = 90°.$$
$$CP = 2'\ 0''; \quad PQ = 6'\ 3''; \quad OQ = 3'\ 0''.$$
$$PG = GQ.$$

Plot the position of G when the angle $ACP = 0°$, and also when $ACP = 45°$. Scale 1 inch to 1 foot.





*4. Four strips of sheet celluloid or other material are pinned together at the ends as shown, the whole forming a jointed parallelogram. Suppose the strip *MM* to be fixed to the drawing board, and let the other strips be turned into successive positions, the two sides *MN* thus rotating about the points *M* as centres. Find the locus of *Q*, the middle point of the strip *NN*.

POINTS, LINES, PLANES.

1. Draw the projections aa', bb', cc' of the points A, B, and C.

A is 3 inches
B is 2 inches } above the horizontal plane of projection.
C is $1\frac{1}{2}$ inches

A is in the vertical plane of projection

$ab = bc = ac = 3\frac{1}{2}$ inches. $a'c' = 2\frac{1}{2}$ inches.

Letter each projection distinctly.

2. Show the projections of two points, P and Q, which are situated as follows:—P $\frac{1}{4}$ inch in front of the vertical plane of projection, and 2 inches

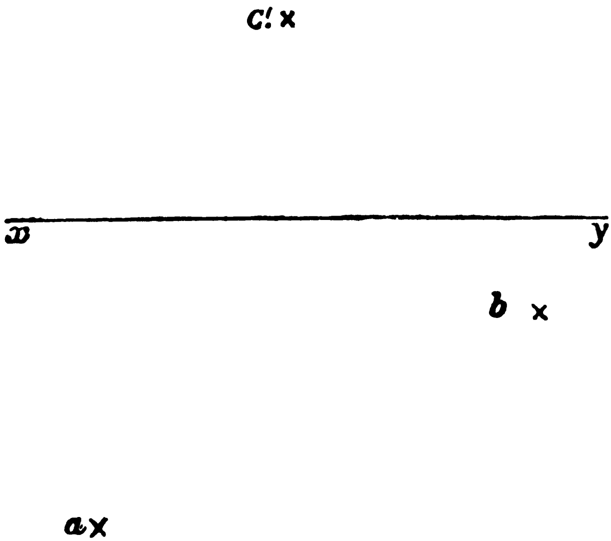
above the horizontal plane; Q 1¾ inches in front of the vertical plane, and ¾ inch above the horizontal plane. The real distance between the points P and Q is 3 inches.

3. A small object P is situated in a room at a distance of 17 inches from a side wall, 24 inches from an end wall, and 33 inches above the floor. Ascertain and measure the distance of P from the corner O of the room where these three mutually perpendicular planes meet. Scale, $\frac{1}{10}$th.

4. Draw the projections of a line, inclined at 30° to the horizontal plane. Its vertical trace is 1¾ inches above the horizontal plane of projection, and its horizontal trace is 2½ inches from the vertical plane of projection.

5. Draw two lines ab, ac, forming an angle of 60°. ab is inclined at 45°, ac at 30°, to the horizontal plane; and the point a is the plan of a point A 1½ inches above the horizontal plane. Find, on the two lines, lengths ab, ac, such that the triangle abc shall be the plan of an isosceles triangle, in which $AB = AC = 2$ inches.

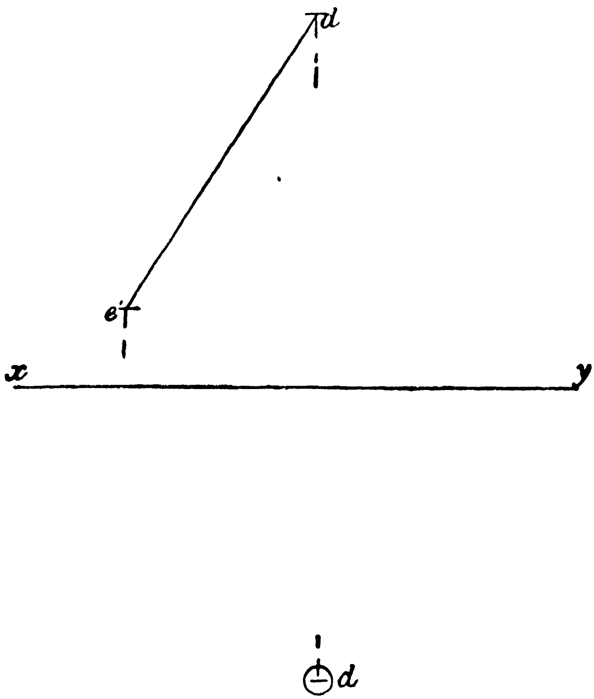
*6. a and b are two points in the horizontal plane, and c' a point in the vertical plane of projection. Draw the projections of the lines AC and BC, and determine their real length.



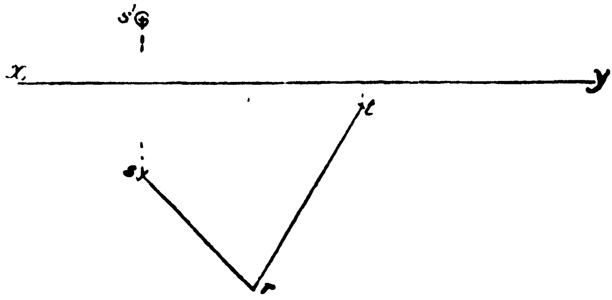
7. A line is 2·5 inches long, and its plan measures 2 inches; find and measure the inclination of the line to the horizontal plane. Find also and state how much higher one end is than the other. Draw the projections of this line when one end is in xy, and the plan makes 45° with xy.

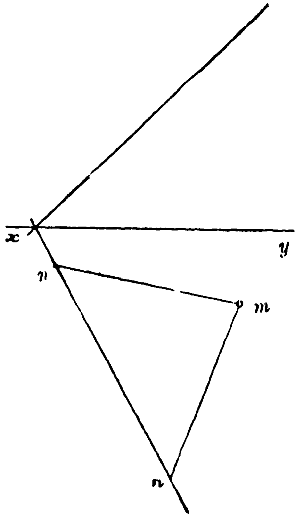
8. A person on the top of a tower 80 feet high, which rises from a horizontal plane, observes the angles of depression (below the horizon) of two objects H and K on the plane to be 14·3° and 25·5°; the directions of H and K from the tower being north and west respectively. Draw a plan, to a scale of 1 inch to 100 feet, showing the relative positions of the person and the two objects. Measure and state in feet the distance between H and K.

*9. The plan *d* and the elevation *d'* of a point *D* are given, from which a line *DE* is to be drawn making 45° with the horizontal plane. If *d'e'* is the elevation of the required line *DE*, draw and carefully letter the plan of it.



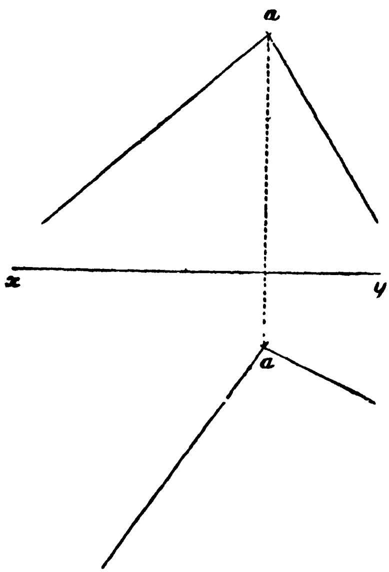
*10. *rs*, *rt* are the plans of two intersecting lines which are each really 2½ inches long. Taking *s'* as the elevation of the point of which *s* is the plan, draw an elevation of both lines, and find what the real angle is between the two lines.





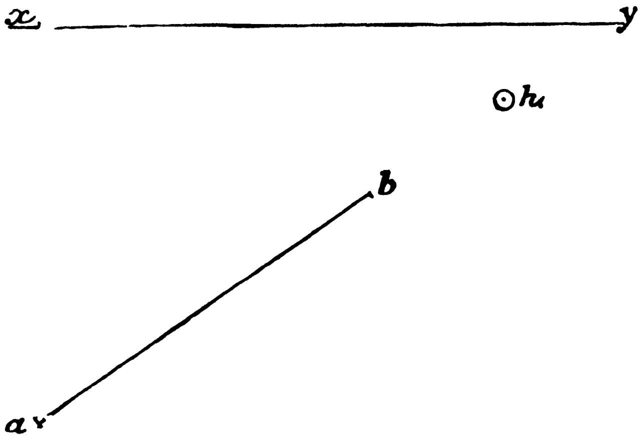
11. The horizontal and vertical traces of a plane make 40° and 55° respectively with the xy line. Find and measure the inclination of the plane to the horizontal. Find also and state in degrees the true angle between the traces of the plane.

***12.** The plans mn, mn are shown of two lines MN, MN which lie in the given plane. Draw the elevations of the lines.

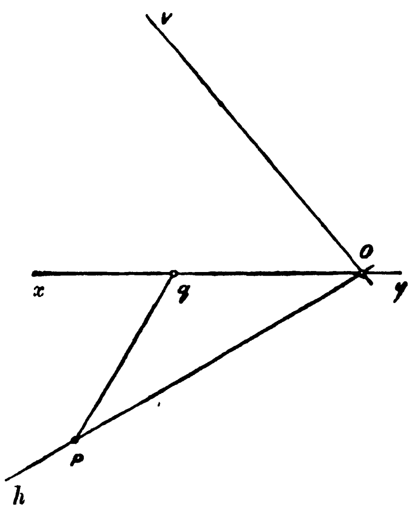


***13.** The figure shows two lines which meet at a point A in space. Determine and measure the real angle between the lines.

*14. *ab* is the plan of a line, and *h* is its horizontal trace when produced. If the line be inclined at 35° to the horizontal plane, draw its elevation on the given *xy* line.



*15. The plan *pq* is given of a line *PQ* which lies in the given plane *VOH*. Draw the elevation of *PQ*. Also determine the true shape of the triangle *POQ*.

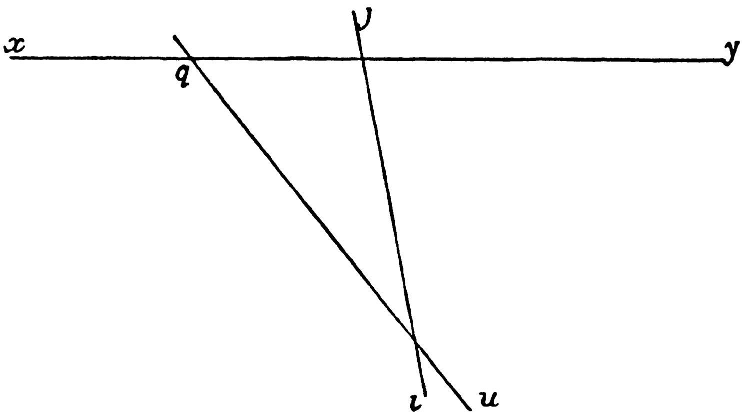


16. Assume an *xy* line, and draw the traces of a vertical plane *S*, making an angle of 38° with the vertical plane of projection. Find a point *P* lying in the plane *S*, 1½ inches from the vertical plane of projection,

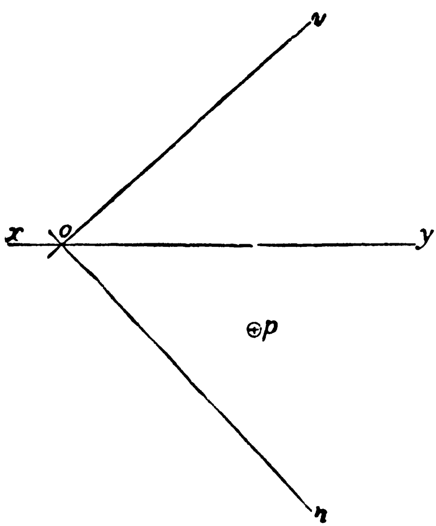
and 2 inches above the horizontal plane. Draw the projections of a line, lying in the plane S inclined at 60° to the horizontal plane and passing through P.

17. The plan of a line is perpendicular to the xy line, and its vertical and horizontal traces are, respectively, distant 1 inch and 2 inches from it. Draw a plane inclined at 45°, containing this line. Then draw a second plane, also containing the line, but perpendicular to the first plane.

*18. qu is the horizontal trace of a plane, which intersects another plane; the line of intersection of the two planes, of which ij is the plan, is

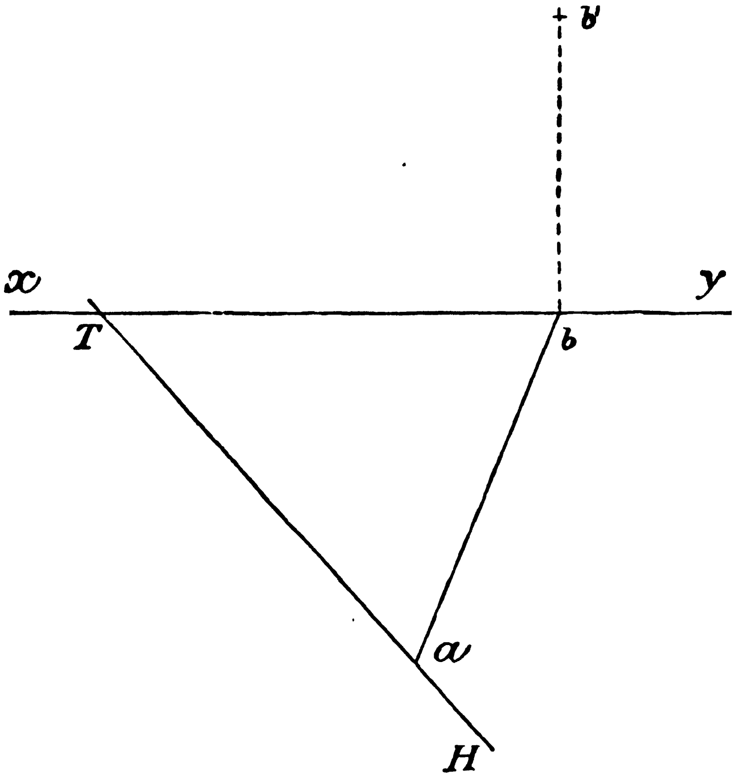


inclined at 45° to the horizontal plane. Draw the vertical trace of the first-mentioned plane, and also both traces of the other plane, when the real angle between the two planes is 110°.



*19. A point in the plane voh is represented in plan by p. From this point draw (in plan and elevation) two lines contained by the plane, one to have the same inclination to the horizontal plane as the given plane has, and the other an inclination of 25° to the horizontal plane.

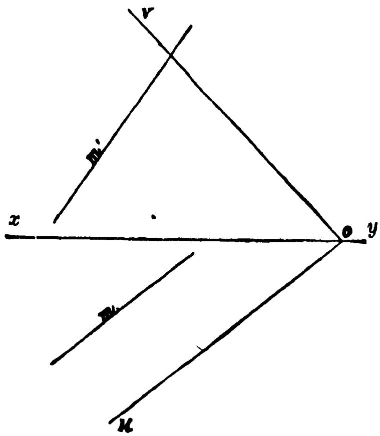
*20. *HT* is the horizontal trace of a plane. *AB* is a line in the plane. Complete the elevation of *AB*. Draw the vertical trace of the plane. Find and measure the inclination of the plane to the horizontal plane.



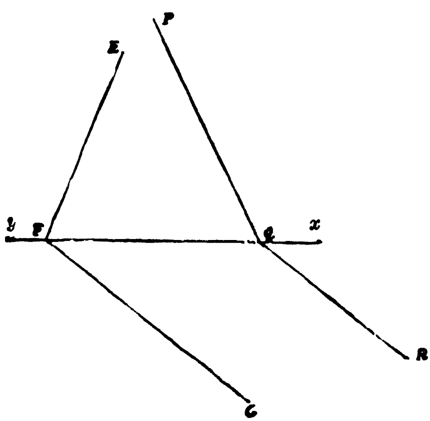
21. Draw two parallel planes, inclined at 52° to the horizontal plane. Their horizontal traces make an angle of 47° with the *xy* line, and the planes are ¾ inch apart, the distance being measured perpendicularly to their surfaces.

22. A horizontal line 1¾ inches above the horizontal plane makes an angle of 60° with the vertical plane of projection. Draw the traces of a plane containing the line and making an angle of 70° with the vertical plane.

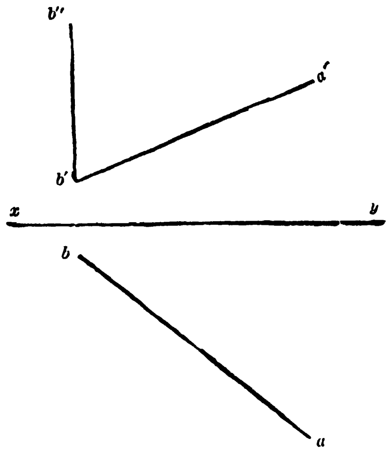
*23. Find the point where the line mm' meets the plane VOH.



*24. Find the intersection between the planes EFG and PQR.

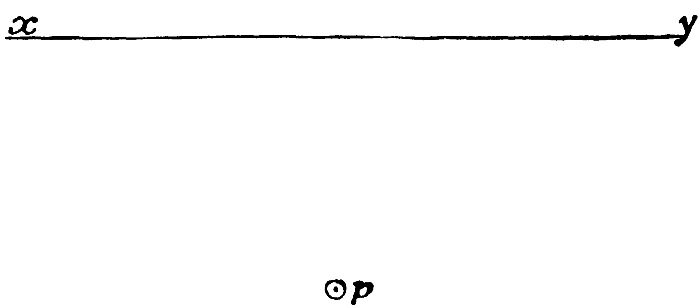


*25. Draw the traces of a plane containing the lines of which the projections b, $b'b''$, and ab, $a'b'$ are given.



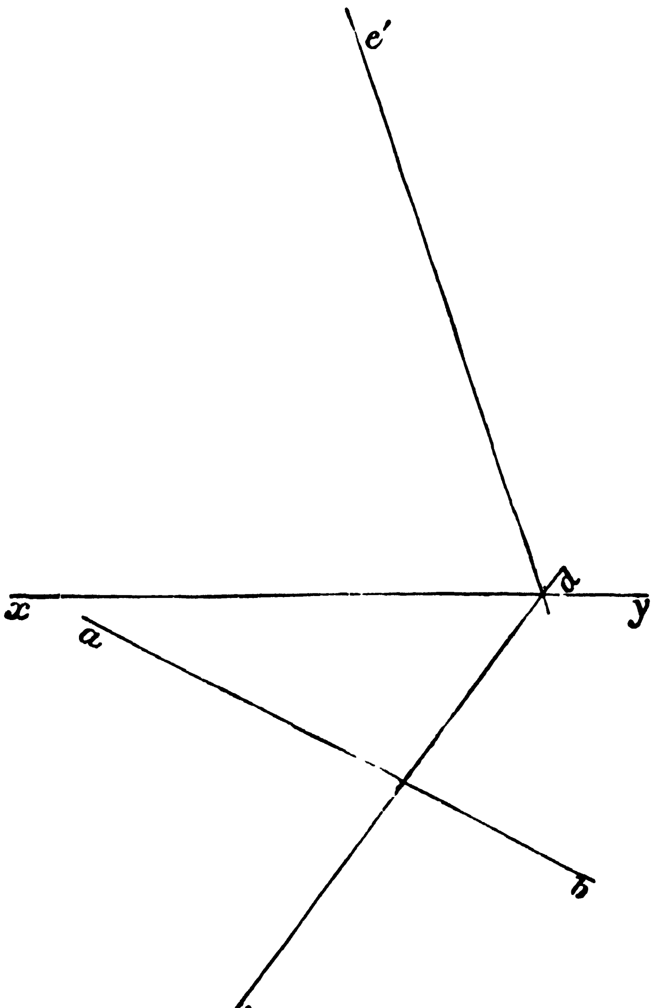
26. Find the real angle between the lines, the projections of which are given in Question 23.

*27. Draw two planes perpendicular to one another, and inclined to the horizontal plane at 45° and 60° respectively. The former plane is to be perpendicular to the vertical plane of projection, and the horizontal trace of each plane is to pass through the given point p.

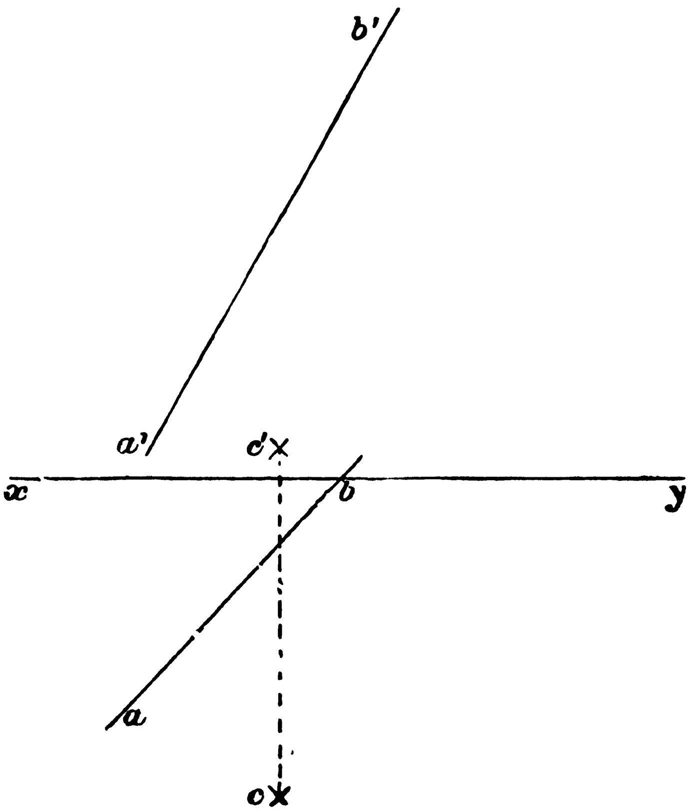


*28. In the plane $e'dc$ of Question 29 draw a line inclined at 45° to the horizontal plane, and on that line as base draw the plan of an equilateral triangle of 1½ inches side, contained in the given plane, its lowest angle touching the horizontal plane.

*29. *e′d*, *dc* are the traces of a plane. *ab* is the horizontal trace of a vertical plane. Find the elevation of the intersection of the two planes, supposing the point where the vertical traces meet to be beyond the limits of the paper.

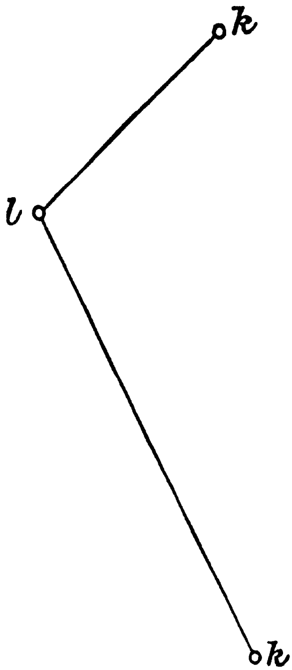


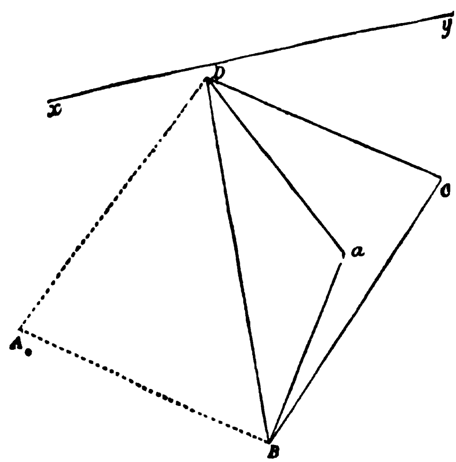
*30. ab, $a'b'$ is a given line, cc' a given point. Find the projections of a line drawn through the given point, meeting the given line at a point ½ an inch above the horizontal plane. Determine the traces of the plane containing the two lines.



PROJECTIONS OF PLANE FIGURES.

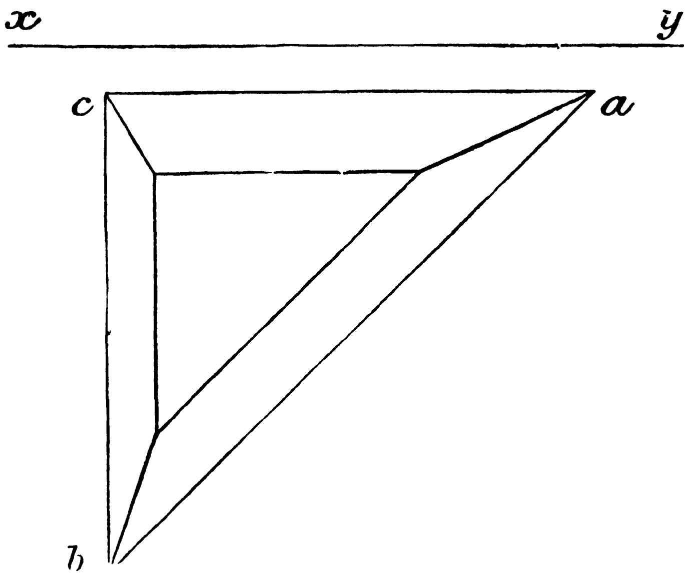
*1. The lines *lk*, *lk* are the plans of two sides *LK*, *LK* of a rectangle; complete the plan of the figure. If the diagonal *KK* be horizontal, find the lengths of the sides of the rectangle and the inclinations of its plane, measuring the results.



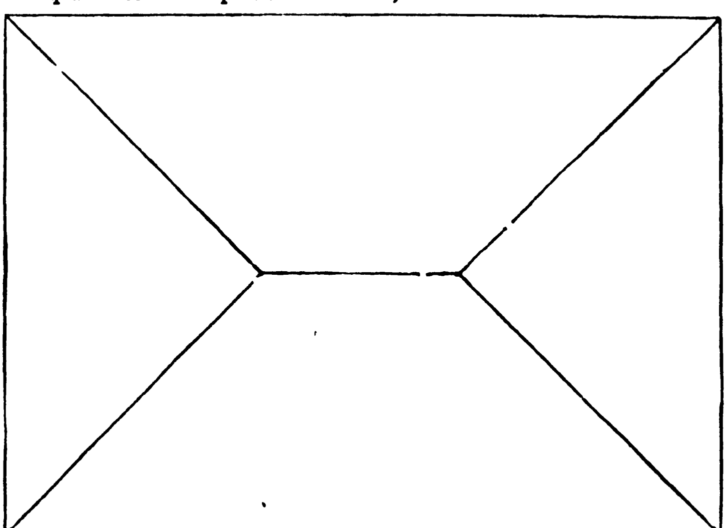


*2. A quadrilateral *ABCD* is cut out in sheet metal, and a model of a dihedral angle is made by bending the plate along the diagonal *BD*. A plan of the model is shown when resting on the face *BCD*. The original shape of the quadrilateral was $BCDA_0$. (*a*) Find the height of the corner *A* and index its plan *a* in inches. (*b*) Draw an elevation of the model on *xy*. (*c*) Find and measure the dihedral angle between the faces *BAD*, *BCD*.

*3. The plan is given of a thin 60° set square resting on its short edge *BC*; scale ½. (*a*) Determine the height of the corner *A* and index its plan *a* in inches. (*b*) Determine the length and inclination to the horizontal of the edge *AB*. (*c*) Draw an elevation of the set square on *xy*. Scale ½ as in the diagram.

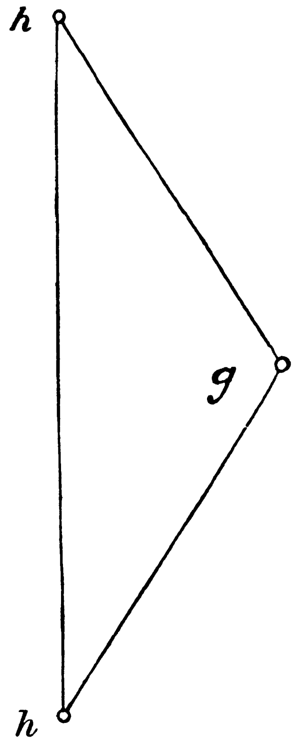


*4. The plan of the roof of a house is given, scale 1 inch to 10 feet. The surfaces are all inclined at 32° to the horizontal. Find the area in square feet of each portion of the roof, and state the total area.



5. Draw the plan and elevation of a square *ABCD* of 2·5 inches side when situated in any position such that no side is parallel to either plane of projection. Letter the corners of the square in both views.

*6. The triangle *hgh* is the plan of an isosceles right angled triangle *HGH*, the hypotenuse *HH* being in the horizontal plane. Determine and measure :—(*a*) The length of the side *HG*. (*b*) The height of *G* above the horizontal plane. (*c*) The inclination of the side *HG*. (*d*) The inclination of the plane *HGH*.

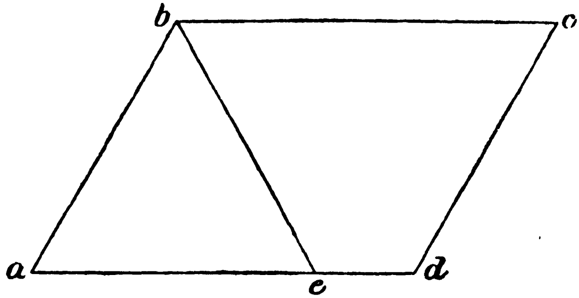


7. A vertical circle of 2 inches diameter rests on the ground with the plane of its surface perpendicular to both planes of projection. Draw the elevation of the circle when it has been turned about its vertical diameter through 30°

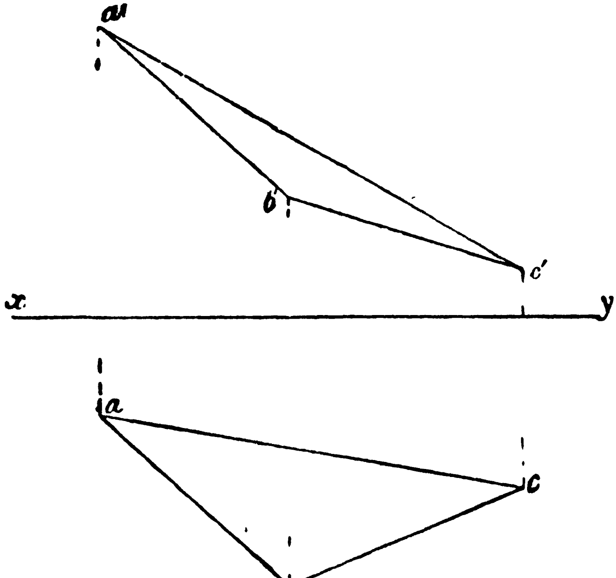
8. Draw a triangle *abc* with the following dimensions :—*ac* = *cb* = 1 inch, *ab* = 1⅗ inches. *abc* is the plan of an equilateral triangle, and *ab* is in the horizontal plane. Draw the elevation of the triangle, the *xy* line parallel to *bc*.

*9. *abcd* is the plan of a parallelogram ; *be* is a horizontal line, and the plane of the parallelogram is inclined at 25° to the horizontal plane.

Draw the elevation $a'b'c'd'$, the horizontal line *be* being 1 inch above the horizontal plane, and the *xy* line parallel to *bc* and *ad*.

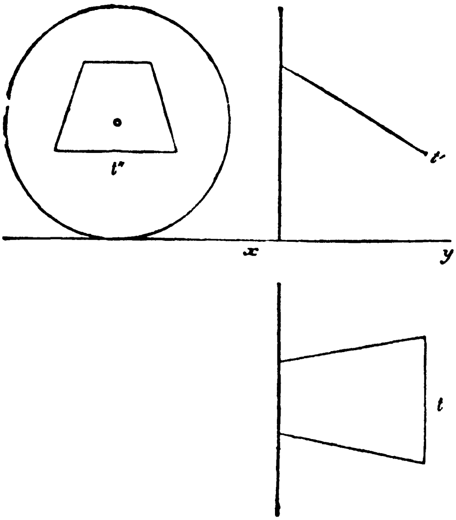


*10. The projections of a triangle *ABC* are given (*abc* in plan, and $a'b'c'$ in elevation); find the traces of the plane containing the triangle. Rotate the plane together with the triangle till they coincide with the *vertical* plane of projection, and thus show the true shape of the triangle *ABC*.



11. Show the true shape of a figure which lies in a plane inclined at 65°, its plan being a circle of 1½ inches diameter.

12. If one side (2½ inches long) of a square rest in a horizontal plane, and an adjacent side be inclined at 55° to the horizontal plane, write down, measured in degrees, the inclination of the diagonal to the horizontal plane.

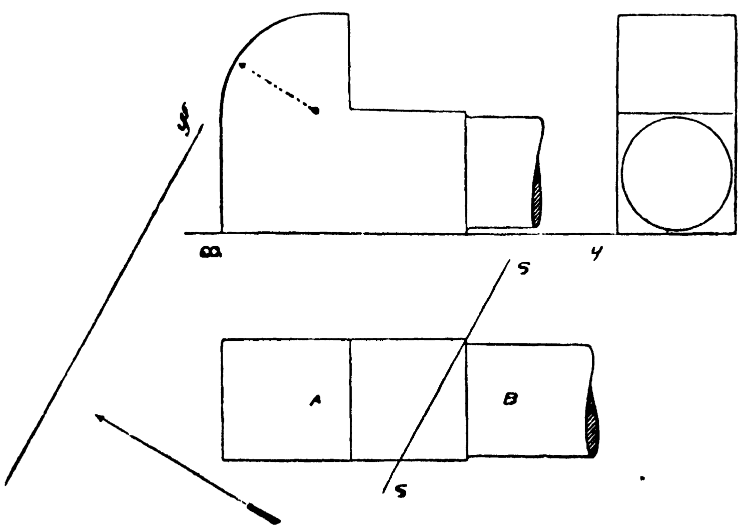


*13. The figure shows three views of a circular picture frame, which has been tilted off its foot T so that its plane is vertical. Draw the plan, and the elevation on xy, of the frame after it has been turned back into position with its foot on the ground.

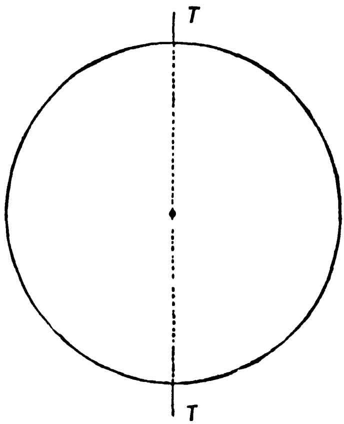
Projections and Sections of Solids.

1. The lengths of the edges of a rectangular prism are respectively 4, 3, and 2 inches. Determine and measure the length of a diagonal of the solid—that is, a line joining opposite corners.

*2. Three views are shown of a portion of a "hook bolt." A vertical section plane SS cuts the bolt into two parts A and B. Draw a sectional elevation of the part A on $x'y'$, the part B being supposed removed. In this view indicate by section lines the metal cut through by SS.

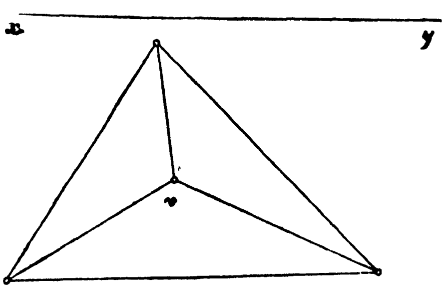


*3. A hemisphere is shown in plan resting with its flat face on the ground. Determine the plan of the section of the solid made by a plane inclined at 40°, which has *TT* for its horizontal trace.

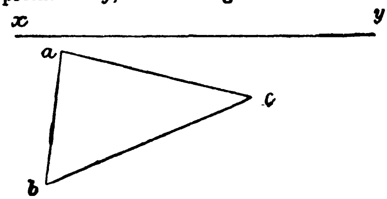


4. A tetrahedron, of which the edges are 1¾ inches long, has one edge horizontal on the ground, and a face containing that edge inclined at 35° to the horizontal plane. Draw the projections of the solid. Supposing the upper portion of the tetrahedron is cut off by a horizontal plane ⅞ inch above the ground, draw on the plan the resulting section, the part which is cut through being cross-lined.

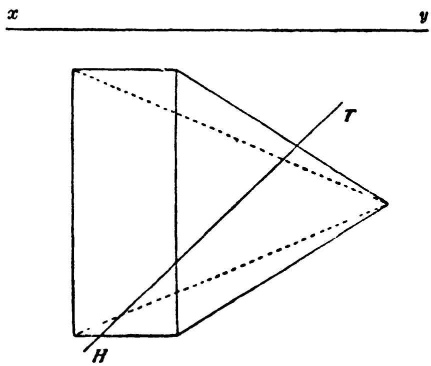
*5. The given figure is the plan of an irregular triangular pyramid resting with its base on the ground. Its vertex *v* is 3·9 inches high. Draw the elevation of the solid on the given *xy*. Show on the plan the shape of the section made by a horizontal plane 1·7 inches high.



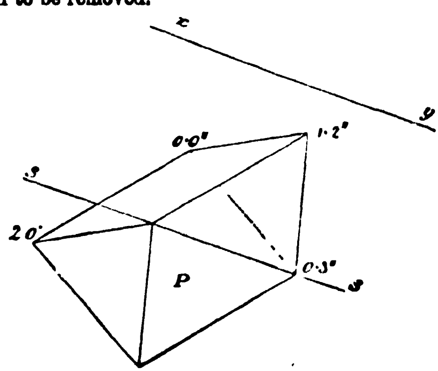
*6. *abc* is the plan of a prism standing upright, height 2·5 inches. Draw the plan of the prism after it has been tilted about the edge *BC* of the base until a rectangular face rests on the horizontal plane. And draw the elevation of the prism on xy, after tilting.



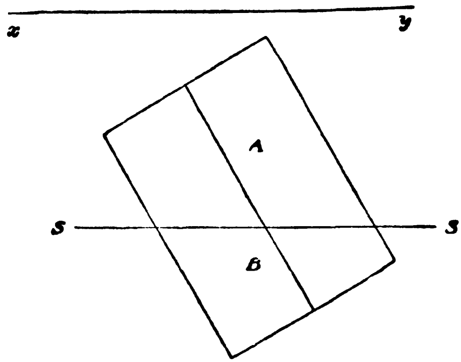
*7. The given figure is the plan of a square pyramid resting on a triangular face. Draw the elevation of the pyramid on xy. HT is the horizontal trace of a vertical plane cutting the pyramid. Find the elevation and also the true shape of the section.



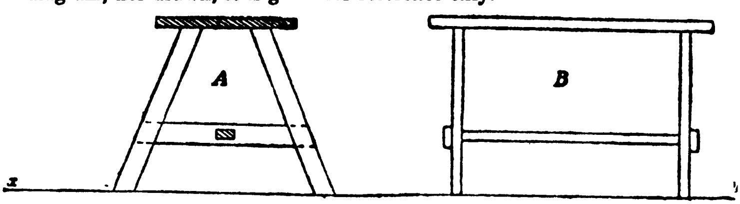
*8. The plan ot a riangular prism is given, the heights of four of its corners being marked. Write down the heights of the remaining two corners. The prism is cut by a vertical plane SS. Draw a sectional elevation on xy, the portion P of the prism in front of the section plane being supposed to be removed.

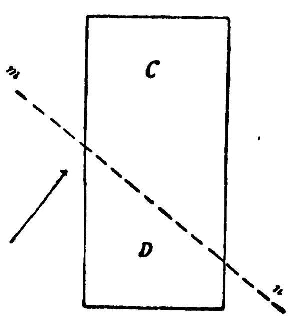


*9. The plan is given of a triangular prism with equilateral ends, which rests on one face. *SS* represents a vertical section plane, dividing the prism into two parts *A* and *B*. The latter being supposed removed, draw the sectional elevation of the part *A* on *xy*, indicating by diagonal lines the shape of the section.

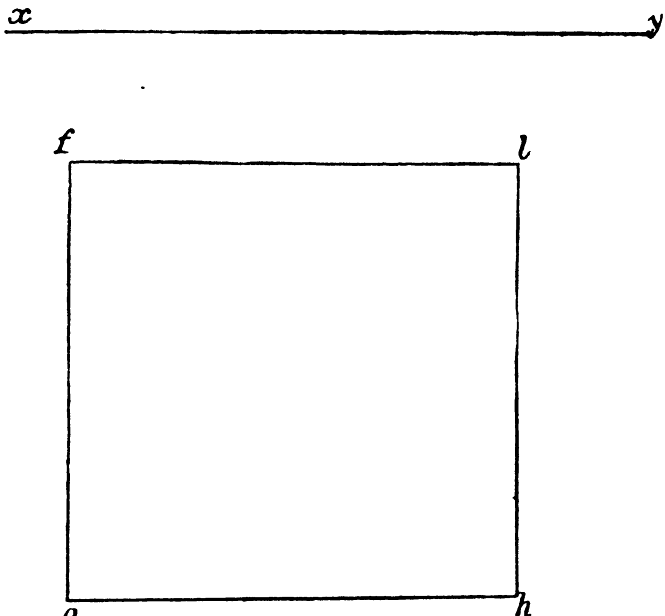


*10. *A* is the cross-section, and *B* one elevation of a wooden bench. Finish the incomplete plan *CD*. If the bench is cut into two portions, *C* and *D*, by a vertical plane, of which *mn* is the horizontal trace, then draw the sectional elevation of the portion *C* on a plane taken parallel to *mn*, looking in the direction of the arrow. Cross-line the parts actually cut through. *Note.*—The elevation *B* need neither be pricked off from the diagram, nor drawn, it is given for reference only.





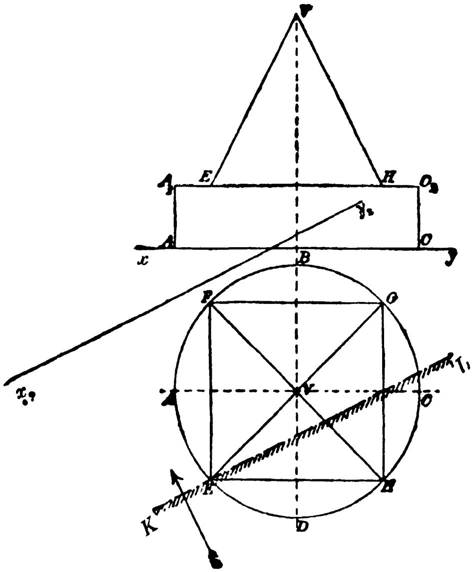
*11. The figure *fghl* represents the plan of a cylinder resting on the ground. Draw the elevation of the cylinder, and show the projections of a sphere of 1 inch *diameter* in contact with it on one side, and also the projections of a cube with an edge (½ inch long) touching the cylinder on the other side; both the sphere and cube are to be on the ground. Show carefully in plan the point and line of contact with the cylinder of the sphere and cube respectively. In plan the portions of the solids which are not seen are to be denoted by a dotted boundary line.



12. The base (2 inches radius) of a right cone, 2 inches high, rests on the horizontal plane. A sphere of 1¼ inches radius touches the cone at a point 2 inches from the apex. Draw the plan of the solids showing their point of contact.

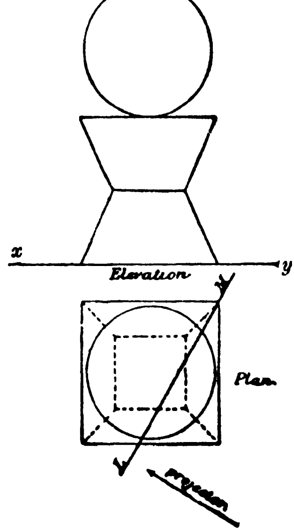
13. A pentagonal right pyramid (side of base 1½ inches, height 2½ inches) rests with one edge of the base in the horizontal plane, the base being tilted up till the highest point in it is 1 inch above the horizontal plane. Draw the elevation of the pyramid on an *xy* line parallel to the edge of the base resting on the horizontal plane, so that the base may be visible.

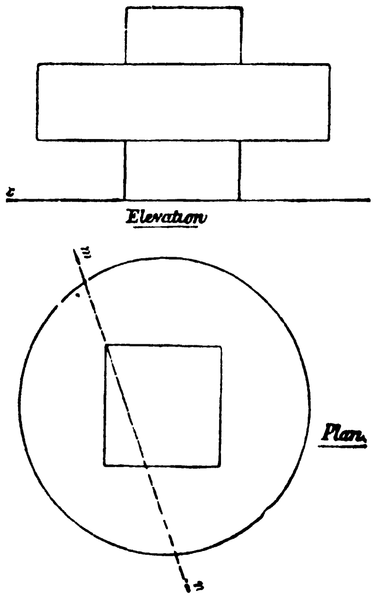
*14. The figure represents in plan and elevation a pyramid with a square base $EFGH$, and vertex V, standing on a cylinder, $ABCD$ in plan, height AA_2. The solids are intersected by a vertical plane whose trace is the shaded line KL. Show the intersection set up from a ground line x_2y_2, parallel to KL, and seen in the direction of the arrow. Show, also, the portions of the solids visible behind the plane of section. Shade the outline of the section.



15 Draw the plan of a regular tetrahedron, the apex of which is 2 inches above the base, which rests on the horizontal plane.

*16. Draw the sections of the given solids made by the vertical plane of which LM is the horizontal trace. Show also the elevation of the portions of the solids visible beyond the plane of section.





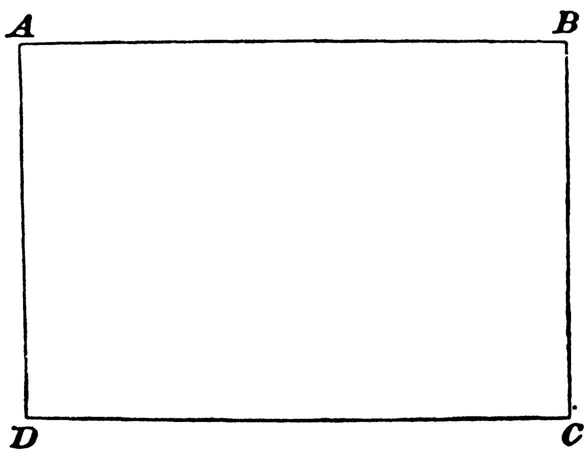
*17. The projections of a solid are given. Draw the true form of the section of it made by a vertical plane of which *mn* is the horizontal trace. Show also in elevation the portions of the solid seen behind the plane of section The edges of the section may be shaded.

18. Draw the projections of a regular octahedron of 2 inches edge, resting with one face in the horizontal plane. The *xy* line makes an angle of 25° with one edge of that face.

19. A right pentagonal prism (side of pentagon 2 inches) 4 inches long lies with one long edge in the horizontal plane, and one face containing that edge inclined at 30° to the horizontal plane. Draw its plan; also an elevation on a vertical plane, which makes an angle of 60° with each of the long edges.

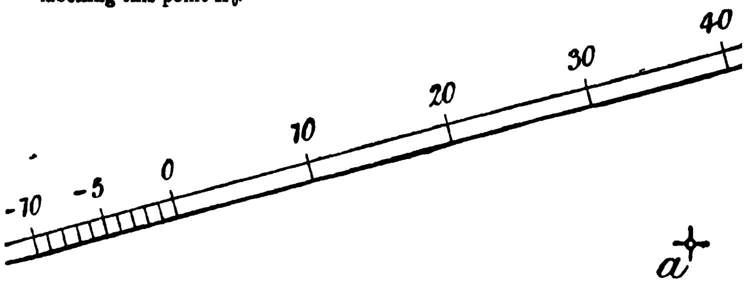
20. The axis of a right cylinder of 1 inch radius is inclined at 40° to the horizontal plane. Draw the true form of a horizontal section through it.

*21. The rectangle *ABCD* is the base of a pyramid, resting on the horizontal plane. If *V* be its vertex, the inclinations of the triangular sides to the horizontal plane are as follows:—*AVD* 50°, *AVB* 70°, *BVC* 55°. Complete the plan of the pyramid.

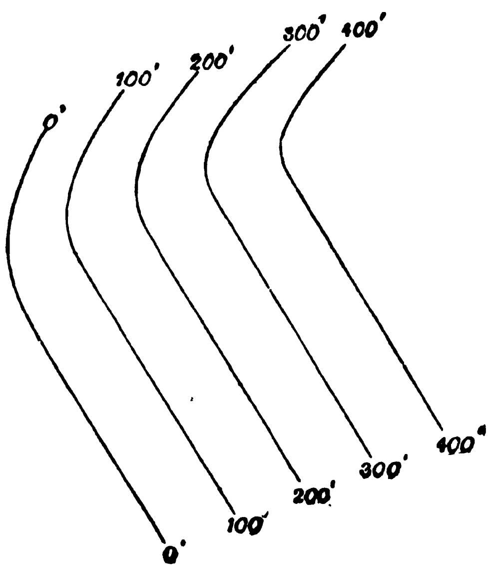


HORIZONTAL PROJECTION.

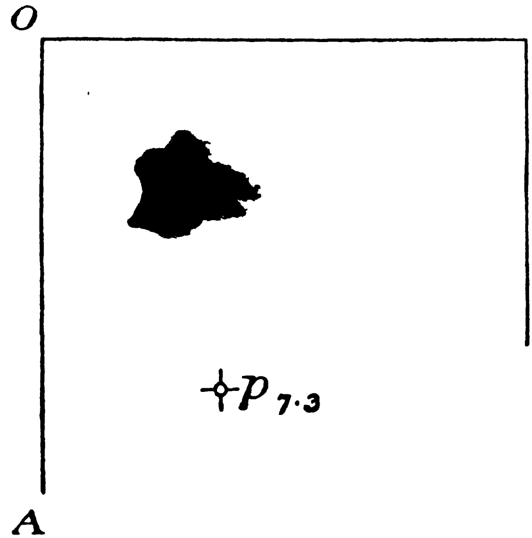
*1. A plane is represented by a scale of slope, the unit for heights being 0·1 inch. A point A in the plane is shown by its plan a :—(a) Draw the horizontal trace of the plane. Draw also the plan of a horizontal line lying in the plane and containing A (b) Measure the height of A and index its plan a. (c) If the plane were turned into the horizontal plane about its horizontal trace, show where the point A would be carried to, labelling this point A_0.



*2 The plan of a piece of ground is shown to a scale of ¼ inch to 100 feet. The form of the surface is indicated by horizontal sections or contour lines, at vertical intervals of 100 feet. A portion of the surface is seen to be plane. Draw the plan of any path of *steepest slope* up this plane. Find and measure in degrees the inclination of this path to the horizontal. Represent the plane by a scale of slope.

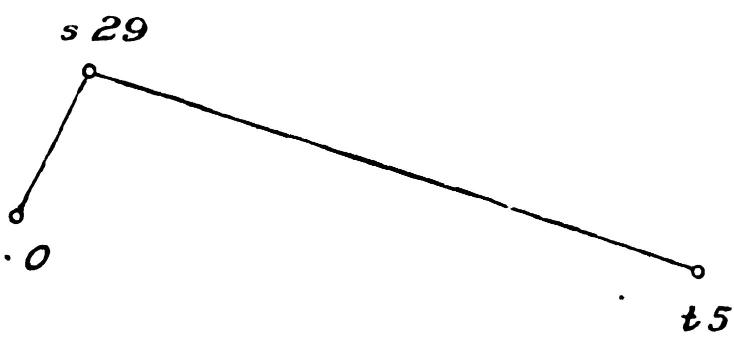


*3. The diagram shows the plan of a portion of a room drawn to a scale of 1 inch to 10 feet. O is one corner of the floor. P is a point 7·3 feet above the floor, shown by its figured plan $p_{7\cdot 3}$ (indexed in feet). Determine and measure in feet the distance of P from the corner O of the floor. Find also and measure the distance of P from the line OA on the floor.



4. On the face of a hill a path going North has an upward slope of 1 vertical in 5 horizontal. A path from the same point going East has an upward slope of 1 in 7. Determine and measure the direction (in degrees East of North), and the slope, of the steepest path up the hill.

*5. The figured plans of two lines *rs*, *st* are given. Determine the plan of the bisector of the angle *rst*. Show the point where this bisector meets the horizontal plane. Unit = 0·1 inch.



*6. The lines *PP*, *PQ*, whose plans are given, intersect one another The heights of the three points *P* are indexed. Determine and index the height of *Q*. Unit = 0·1 inch.