Chapter - 8

Concurrent Decision Approach for Software Component Selection

8.1 Overview

The present chapter attempts the development of concurrent decision approach for software component selection in both non-fuzzy as well as fuzzy environment. For non-fuzzy environment the approach uses *AHP* to determine attributes weights a priori. These weights are later used to identify best alternatives using *TOPSIS*. Similarly, for fuzzy environment *Fuzzy AHP* and *Fuzzy TOPSIS* are used to select software component. The comparison of both the approaches is mentioned by demonstrating case studies. The applicability of the developed approaches will help various stakeholders of the component based projects in taking key decisions for selecting components that will fit the project scope and constraints and also accommodate customer criteria/requirements. This chapter embarks upon the usages of decision techniques such as *AHP, Fuzzy AHP, TOPSIS* and *Fuzzy TOPSIS* for the effective selection of software components in designing *CBSS*. On the basis of the critical literature review (chapter 1) it is seen that a concurrent and integrative approach is needed to accomplish the task of software component selection. The current chapter is organized in the following manner: Section 8.2 provide the methodology for software component selection in non fuzzy environment. In section 8.3, a case study is considered to demonstrate the applicability of the approach. Section 8.4 provides the methodology for software component selection in fuzzy environment. In section 8.5, another case study is presented to show the applicability of the developed approach. Section 8.6 compares both the approaches and discusses the result obtained. In section 8.7, benefits of the developed methodologies are given. Finally section 8.8 provides concluding remarks of the chapter.

8.2 Methodology – Concurrent Decision Framework (*CDF*)

In *AHP* the number of pair-wise comparisons in a decision problem having '*m*' alternatives and '*n*' criteria is expressed by the following equation (Triantaphyllou, 1999):

$$\frac{n(n-1)}{2} + n\frac{m(m-1)}{2} \tag{8.1}$$

However, this becomes highly unmanageable if the criteria and alternatives are very large. Thus, a technique should be evolved which overcomes such shortcomings. In the chapter a concurrent decision approach is developed that comprises of *TOPSIS* along with *AHP*. This approach is more efficient as compared to existing decision approaches due to following reasons: it handles both tangible as well as intangible attributes; there is no limit in terms of number of criteria/attributes and/or alternatives; it utilizes concurrent engineering approach to execute non-overlapping steps thus minimizing overall selection time. The *TOPSIS* technique is based on the concept that the chosen alternative should have the shortest Euclidean distance from the positive ideal solution and farthest from the negative ideal solution. The positive ideal solution is a hypothetical solution for which all attribute values correspond to the maximum attribute values in the pool satisfying solutions; the negative ideal solution is a hypothetical solution for which all attribute values correspond to the minimum attribute values in the pool satisfying solutions.

In the chapter the process of software component selection during CBSS design phase is formulated as a multi-criteria decision making problem using the developed concurrent decision based approach. The *CDF* is based on the techniques of *AHP* and *TOPSIS* to restructure the complex domains composed of diverse internal and external factors in the software component selection process. Primarily there are two phases involved in the *CDF*, which is shown in Figure 8.1.

8.2.1 Phase I

In this phase the main concern is to identify various organizational and management factors that affect software component selection during CBSS design phase. The phase comprises following 4 steps:

Step 1: Planning and Objective

In this step the goal of the software component is identified as per stakeholders' point of view. For each domain the factors or attributes associated with software component will vary.

Step2: Domain scope

This step focuses on identifying domain scope, definition and requirements (based on software engineering activities such as: inception, elicitation and elaboration).

Step 3: Time scale and effort

This step focuses on setting up the milestones for accomplishing (pre)defined tasks and scenarios required for software component identification and selection.

Step 4: Assumptions and constraints

In this step the assumptions and constraints are identified for software component identification and selection. For the *CBSS* project; available infrastructure, support and environment are assessed. Various contract and legal issues are also discussed and identified. Budget and staff and/or evaluation team training issues are also sorted out.
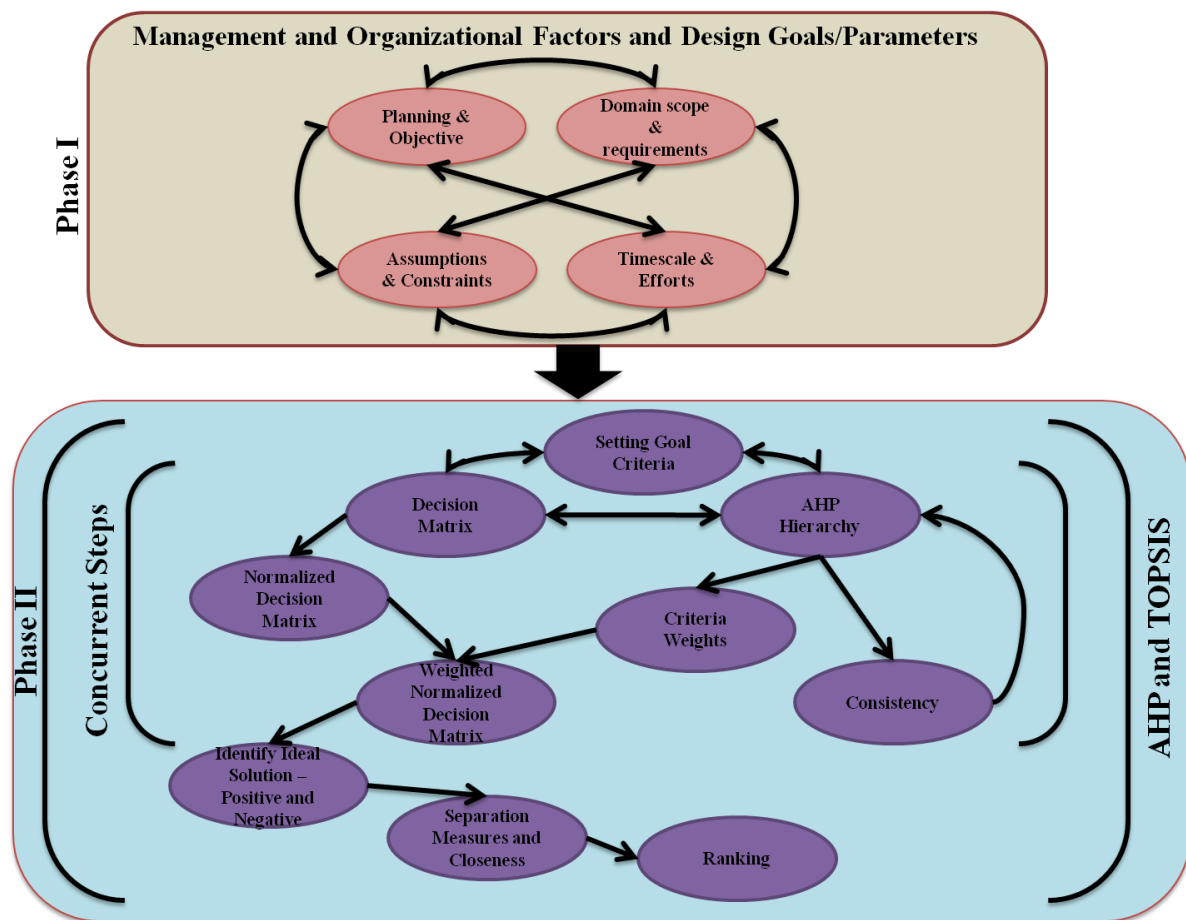

Figure 8.1 Concurrent decision framework

8.2.2 Phase II

In this phase, see Figure 8.1, the pair-wise comparison method of *AHP* is combined with the other steps of *TOPSIS* in concurrent fashion.

Step 1

This step produces a decision matrix of criteria and alternatives based on the information available about the problem. If the number of alternatives is m and the number of criteria is n, then the decision matrix having an order of $m \times n$ is represented as follows:

$$D_{m \times n} = \begin{bmatrix} a_{11} & a_{12} & ... & a_{1n} \\ a_{21} & a_{22} & ... & a_{2n} \\ ... & ... & ... & ... \\ a_{m1} & a_{m2} & ... & a_{mn} \end{bmatrix} \tag{8.2}$$

where an element a_{ij} of the decision matrix $D_{m \times n}$ represents the value of the ith alternative in terms of jth decision criteria.

Step 2

In this step the decision matrix is converted to a normalized decision matrix 'R' so that the scores obtained in different scales become comparable. An element r_{ij} of the normalized decision matrix R is calculated as follows:

$$r_{ij} = \frac{a_{ij}}{\left[\sum_{i=1}^{m} (a_{ij})^2 \right]^{0.5}} \tag{8.3}$$

Step 3

For each element of the hierarchy structure all the associated elements in low hierarchy are compared in pair wise comparison matrices as follows:

$$A = \begin{bmatrix} 1 & \dfrac{w_1}{w_2} & ... & \dfrac{w_1}{w_n} \\ \dfrac{w_2}{w_1} & 1 & ... & \dfrac{w_2}{w_n} \\ ... & ... & ... & ... \\ \dfrac{w_n}{w_1} & \dfrac{w_n}{w_2} & ... & 1 \end{bmatrix} \tag{8.4}$$

Where A = comparison pairwise matrix, w_1 = weight of element 1, w_2 = weight of element 2, w_n = weight of element n.

In order to determine the relative preferences for two elements of the hierarchy in matrix A, an underlying semantical scale (Saaty, 1996) is employed with values from 1 to 9 to rate relative importance (Table 8.1).

Preferences expressed in numeric variables	Preferences expressed in linguistic variables
1	Equal importance
3	Moderate importance
5	Strong importance
7	Very strong importance
9	Extreme importance
2,4,6,8	Intermediate values between adjacent scale values

Table 8.1 AHP Judgment scale (adopted from Saaty, 1996)

Some methods like eigen value method are used to calculate the relative weights of elements in each pair wise comparison matrix. The relative weights (W) of matrix A is obtained from following equation:

$$(A - \lambda_{max} I) \times W = 0 \qquad (8.5)$$

where λ_{max} = the biggest eigen value of matrix A and I = Identity matrix.

The principal eigen vector (λ_{max}) of the above matrix represents the relative weights of the decision criteria.

For a single expert judgment the normalized weight or importance of the ith criteria (W_i) is calculated as:

Divide each entry $A(a_{ij})$ by the total in its column.

$$a'_{ij} = \frac{a_{ij}}{\sum_{i=1}^{m} a_{ij}} \qquad (8.6)$$

Total each entry in row and take out the average.

$$N_i = \frac{\sum\limits_{j=1}^{n} a'_{ij}}{n} \qquad (8.7)$$

For group judgments the normalized weight or importance of the ith criteria (W_i) is determined by calculating the geometric mean of the ith row (GM_i) of the above matrix and then normalizing the geometric means of rows. This can be represented as follows:

$$GM_i = \left\{ \prod_{j=1}^{N} c_{ij} \right\}^{\frac{1}{N}} \text{ and } W_i = \frac{GM_i}{\sum_{i=1}^{N} GM_i} \qquad (8.8)$$

The consistency property of matrices is checked to ensure that the judgments of decision makers are consistent. The consistency index (*CI*) of a randomly generated reciprocal matrix shall be called to the random consistency index (*RI*), with reciprocals forced. An average *RI* for the matrices of order *N* from 1 to 15 was generated by using a sample size of 100. The last ratio that has to be calculated is consistency ratio (*CR*). Generally, if *CR* is less than 0.1, the judgments are consistent, so the derived weights can be used. Otherwise, the decision-maker has to reconsider the entries of pair-wise comparison matrix. To check the consistency in pair-wise comparison judgment, *CI* and *CR* are calculated using the following equations:

$$CI = \frac{\lambda_{max} - n}{n - 1} \qquad (8.9)$$

$$CR = \frac{CI}{RI} \qquad (8.10)$$

where *RI* is random consistency index and its value can be obtained from Table 8.2.

N	1	2	3	4	5	6	7	8	9
RI	0	0	0.58	0.9	1.12	1.24	1.3	1.41	1.45

Table 8.2 Random Consistency Index

Step 4

The weighted normalized matrix is obtained by multiplying each column of the normalized decision matrix *R* with the associated criteria weight corresponding to that column. Hence an element v_{ij} of weighted normalized matrix *V* is represented as follows:

$$v_{ij} = W_j \times r_{ij} \qquad (8.11)$$

196

Step 5

This step produces the positive ideal solution (A^+) and negative ideal solution (A^-) in the following manner:

$$A^+ = \left\{ \left(\max v_{ij} / j \in \tau \right), \left(\min v_{ij} / j \in \tau' \right) \, for \, i = 1,2,3,..,m \right\}$$
$$= \left\{ V_1^+,...,V_n^+ \right\}$$
(8.12)

$$A^- = \left\{ \left(\min v_{ij} / j \in \tau \right), \left(\max v_{ij} / j \in \tau' \right) \, for \, i = 1,2,3,..,m \right\}$$
$$= \left\{ V_1^-,...,V_n^- \right\}$$
(8.13)

where $\tau = \{ j = 1,2,...,n \}$, j is associated with the benefit or positive criteria, and $\tau' = \{ j = 1,2,...,n \}$, j is associated with the cost or negative criteria.

The most preferred one has the maximum value among the alternatives. Therefore, A^+ indicates the positive ideal solution. Similarly, A^- indicates the negative ideal solution.

Step 6

The N dimensional Euclidean distance method is applied, as shown in equation (8.14) and equation (8.15), to measure the separation distances of each alternative from the positive and negative ideal solution:

$$S_i^+ = \left\{ \sum_{j=1}^n \left(V_{ij} - V_j^+ \right)^2 \right\}^{0.5}, i = 1,2,...,m$$

(8.14)

and

$$S_i^- = \left\{ \sum_{j=1}^n \left(V_{ij} - V_j^- \right)^2 \right\}^{0.5}, i = 1,2,...,m$$

(8.15)

where, S_i^+ and S_i^- are the separation distances of alternative i from the positive ideal solution and negative ideal solution, respectively.

Step 7

In this step the relative closeness (C_i^*) value of each alternative with respect to the ideal solution is determined using equation (8.16). The value of C_i^* lies within the range from 0 to 1:

$$C_i^* = \frac{S_i^-}{\left(S_i^+ + S_i^-\right)} \qquad\qquad (8.16)$$

Step 8

At the final step, the preference order is ranked. All the alternatives are now arranged in a descending order according to the value of C_i^*. In *TOPSIS* method, the chosen alternative has the maximum value of C_i^* with the intention to minimize the distance from the ideal solution and to maximize the distance from the negative ideal solution. Finally, the best alternative is recommended to the *design team* to develop feasible design solution.

8.3 Case Study - CDF

One of the critical requirements for the project (Upadhyay et al., 2009) is to identify database server (software component) from the available pool of alternatives. Initially the list discovered had all available database servers from the reference of software components (Upadhyay et al., 2010). Later, on the basis of potential requirement - "*relational database should be ODBC compatible*", the first filtering is done and finally three prominent software components <A_1, A_2, A_3>– *ORACLE*, *DB2* and *Microsoft SQL Server* are identified. Based upon Phase I the pertinent attributes and their respective values for the alternatives are identified (Table 8.3). These form the decision matrix and *AHP* structure (Table 8.3 and Figure 8.2). The *AHP* structure so formed is used for the computation of weights for the attributes as priori. These weights are later used to generate weighted normalized matrix. The weight calculation is done using *AHP* expert grade method. For each pair-wise comparison matrix the judgments entered have *CR* less than **0.1**. This shows judgments are consistent.
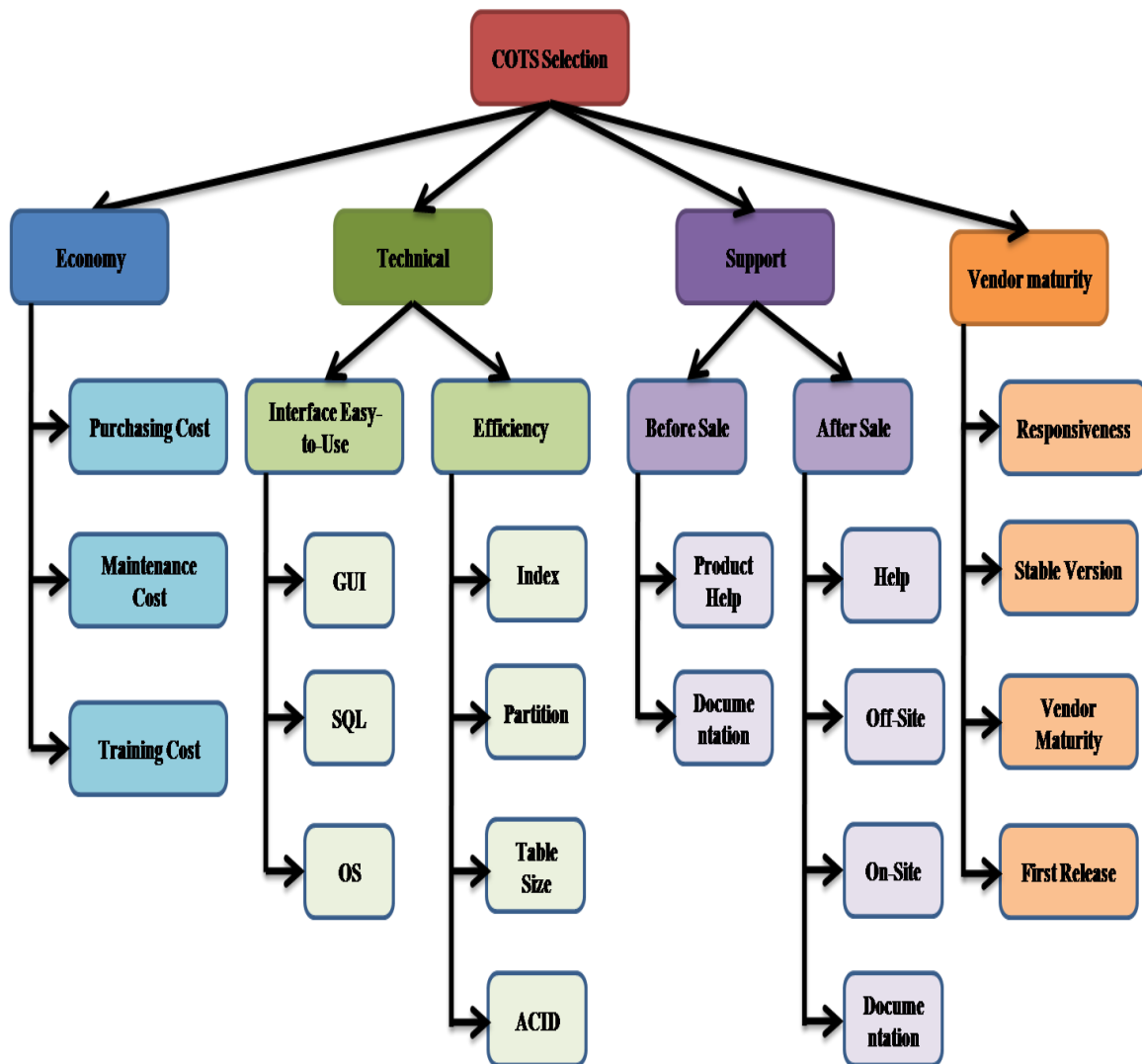
Figure 8.2 AHP hierarchical structure (considering criteria and attributes)

As the attributes identified in previous step contains both qualitative and quantitative values of varied measures, scales and ranges, so the normalization was done to put each value in the range of 0-1, Table 8.4, for easy computation.

Alternatives \ Attributes	Economy E_1	Purchasing cost $E_{1.1}$	Maintenance cost $E_{1.2}$	Training cost $E_{1.3}$	Technical T_1	Interface easy-to-use $T_{1.1}$	GUI $T_{1.1.1}$
A₁	8	25000	5000	15000	8	8	9
A₂	6	40000	5000	1000	7	6	5
A₃	7	19999	3000	1200	7	7	7
Scales	Point 1-10	$	$	$	Point 1-10	Point 1-10	Point 1-10

	SQL $T_{1.1.2}$	OS support $T_{1.1.3}$	Efficiency $T_{1.2}$	Table size $T_{1.2.1}$	Index $T_{1.2.2}$	ACID $T_{1.2.3}$	Partition $T_{1.2.4}$
A₁	7	10	8	512	10	10	10
A₂	8	10	7	500	10	10	10
A₃	7	4	6	524	10	10	5
Scales	Point 1-10	Point 1-10	Point 1-10	GB	Point 1-10	Point 1-10	Point 1-10

	Support S_1	After sale support $S_{1.1}$	Documentation $S_{1.1.1}$	Onsite $S_{1.1.2}$	Offsite $S_{1.1.3}$	Help $S_{1.1.4}$	Before sale support $S_{1.2}$
A₁	7	6	6	8	6	6	7
A₂	9	8	7	8	7	7	7
A₃	7	6	6	8	6	6	7
Scales	Point 1-10	Point 1-10	Point 1-10	Point 1-10	Point 1-10	Point 1-10	Point 1-10

	Help $S_{1.2.1}$	\Documentation $S_{1.2.2}$	Vendor maturity V_1	Responsiveness $V_{1.1}$	First release $V_{1.2}$	Stable version $V_{1.3}$	Market share $V_{1.4}$
A₁	8	7	7	7	8	7	6
A₂	8	6	9	8	8	8	9
A₃	8	6	7	7	8	7	4
Scales	Point 1-10	Point 1-10	Point 1-10	Point 1-10	Point 1-10	Point 1-10	Point 1-10

Table 8.3 Decision matrix, pertinent attributes values

Alternatives \ Attributes	E_1	$E_{1.1}$	$E_{1.2}$	$E_{1.3}$	T_1	$T_{1.1}$	$T_{1.1.1}$
A_1	0.655386	0.487954	0.650945	0.692636	0.628539	0.655386	0.722897
A_2	0.491539	0.780726	0.650945	0.461757	0.549972	0.491539	0.40161
A_3	0.573462	0.390343	0.390567	0.554109	0.549972	0.573462	0.562254

	$T_{1.1.2}$	$T_{1.1.3}$	$T_{1.2}$	$T_{1.2.1}$	$T_{1.2.2}$	$T_{1.2.3}$	$T_{1.2.4}$
A_1	0.549972	0.680414	0.655386	0.697247	0.57735	0.57735	0.666667
A_2	0.628539	0.680414	0.573462	0.068091	0.57735	0.57735	0.666667
A_3	0.549972	0.272166	0.491539	0.713589	0.57735	0.57735	0.333333

	S_1	$S_{1.1}$	$S_{1.1.1}$	$S_{1.1.2}$	$S_{1.1.3}$	$S_{1.1.4}$	$S_{1.2}$
A_1	0.523205	0.514496	0.545455	0.57735	0.545455	0.545455	0.57735
A_2	0.672692	0.685994	0.636364	0.57735	0.636364	0.636364	0.57735
A_3	0.523205	0.514496	0.545455	0.57735	0.545455	0.545455	0.57735

	$S_{1.2.1}$	$S_{1.2.2}$	V_1	$V_{1.1}$	$V_{1.2}$	$V_{1.3}$	$V_{1.4}$
A_1	0.57735	0.636364	0.523205	0.549972	0.57735	0.549972	0.520266
A_2	0.57735	0.545455	0.672692	0.628539	0.57735	0.628539	0.780399
A_3	0.57735	0.545455	0.523205	0.549972	0.57735	0.549972	0.346844

Table 8.4 Normalized decision matrix

The weights computed during *AHP* structure formation is used to generate weighted normalized decision matrix, Table 8.5. Using weighted normalized decision matrix, the ideal positive and negative solution were identified, Table 8.6. This matrix helps in identifying the maximum and minimum values for each pertinent attributes. These values are later used to identify Euclidean distance measures, Table 8.7. The distance measures are then used for ranking the alternatives, Table 8.7. The alternatives thus generated are arranged in descending order. In the list, alternative A_2 is given the highest preference based upon the

values and judgments provided. Thus on applying the concurrent decision based approach in this example, A_2 is selected as the best alternative for the project. However, it is to be noted here that the decision of selecting the best alternative purely depends upon the discretion of the design team. For instance, in this example, the design team may choose the other alternative in place of A_2 as the cost of A_2 is higher than that of the remaining ones.

Alternatives \ Attributes	E_1	$E_{1.1}$	$E_{1.2}$	$E_{1.3}$	T_1	$T_{1.1}$	$T_{1.1.1}$
A_1	0.106574	0.321388	0.120537	0.108177	0.373472	0.163846	0.440823
A_2	0.07993	0.514221	0.120537	0.072118	0.326788	0.122885	0.244902
A_3	0.093252	0.257098	0.072322	0.086542	0.326788	0.143366	0.342862
Weights	0.162612	0.658645	0.185173	0.156182	.59419	0.25	0.6098
	$T_{1.1.2}$	$T_{1.1.3}$	$T_{1.2}$	$T_{1.2.1}$	$T_{1.2.2}$	$T_{1.2.3}$	$T_{1.2.4}$
A_1	0.091036	0.152855	0.491539	0.145509	0.172841	0.137929	0.168733
A_2	0.104041	0.152855	0.430097	0.01421	0.172841	0.137929	0.168733
A_3	0.091036	0.061142	0.368654	0.148919	0.172841	0.137929	0.084367
Weights	0.165529	0.22465	0.75	0.20869	0.29937	0.2389	0.2531
	S_1	$S_{1.1}$	$S_{1.1.1}$	$S_{1.1.2}$	$S_{1.1.3}$	$S_{1.1.4}$	$S_{1.2}$
A_1	0.088844	0.428729	0.193636	0.14738	0.058585	0.153616	0.096227
A_2	0.114228	0.571639	0.225909	0.14738	0.068349	0.179219	0.096227
A_3	0.088844	0.428729	0.193636	0.14738	0.058585	0.153616	0.096227
Weights	0.169807	0.8333	0.355	0.25527	0.107405	0.28163	0.16667
	$S_{1.2.1}$	$S_{1.2.2}$	V_1	$V_{1.1}$	$V_{1.2}$	$V_{1.3}$	$V_{1.4}$
A_1	0.433013	0.159091	0.038399	0.130474	0.085563	0.114449	0.21141
A_2	0.433013	0.136364	0.04937	0.149113	0.085563	0.130799	0.317114
A_3	0.433013	0.136364	0.038399	0.130474	0.085563	0.114449	0.14094
Weights	0.75	0.25	0.073391	0.237237	0.1482	0.2081	0.406349

Table 8.5 Weighted normalized decision matrix

Ideal solution \ Attributes	E_1	$E_{1.1}$	$E_{1.2}$	$E_{1.3}$	T_1	$T_{1.1}$	$T_{1.1.1}$
A^+	0.106574	0.514221	0.120537	0.108177	0.373472	0.163846	0.440823
A^-	0.07993	0.257098	0.072322	0.072118	0.326788	0.122885	0.244902
	$T_{1.1.2}$	$T_{1.1.3}$	$T_{1.2}$	$T_{1.2.1}$	$T_{1.2.2}$	$T_{1.2.3}$	$T_{1.2.4}$
A^+	0.104041	0.152855	0.491539	0.148919	0.172841	0.137929	0.168733
A^-	0.091036	0.061142	0.368654	0.01421	0.172841	0.137929	0.084367
	S_1	$S_{1.1}$	$S_{1.1.1}$	$S_{1.1.2}$	$S_{1.1.3}$	$S_{1.1.4}$	$S_{1.2}$
A^+	0.114228	0.571639	0.225909	0.14738	0.068349	0.179219	0.096227
A^-	0.088844	0.428729	0.193636	0.14738	0.058585	0.153616	0.096227
	$S_{1.2.1}$	$S_{1.2.2}$	V_1	$V_{1.1}$	$V_{1.2}$	$V_{1.3}$	$V_{1.4}$
A^+	0.433013	0.159091	0.04937	0.149113	0.085563	0.130799	0.317114
A^-	0.433013	0.136364	0.038399	0.130474	0.085563	0.114449	0.14094

Table 8.6 Ideal positive (A^+) and negative (A^-) Solution

Measures \ Alternatives	A_1	A_2	A_3
Positive distance measure S^+	0.07213384	0.06669075	0.16722573
Positive distance measure S^-	0.10406203	0.14254299	0.0285478
Ranking C^* (A2>A1>A3)	0.59060427	0.68126196	0.14582054

Table 8.7 Measures and ranking

8.4 Methodology – Fuzzy Concurrent Decision Framework (*FCDF*)

In this section methodology for software component selection in a fuzzy environment is considered. The approach is based on the techniques of *FAHP*, *FMADM* and *CE* principles. Primarily there are four steps involved in the fuzzy-based method:

(1). Identification of design goals/parameters.

(2). Generation of hierarchical criteria.

(3). Pairwise significance comparison matrices and specification of fuzzy-weight criteria using *FAHP*.

(4). Fuzzy ranking of alternatives using *FTOPSIS*.

The framework, *FCDF*, of the developed fuzzy-based method is presented in Figure 8.3, and the details are presented in the following sub-sections.

A simple two phase structure - Phase I and Phase II, is associated with the *FDCF*, see Figure 8.3. Phase I deals with the identification of various organizational and management factors that affect designing of *CBSS* and is same as mentioned in sub-section 8.2.1. The description of phase II is as follows:

8.4.1   Phase II

Phase II comprises concurrent execution of *FTOPSIS* and *FAHP*. The pair-wise comparison method of *FAHP* is combined with the other steps of *FTOPSIS*. This phase comprises 9 steps. Step 1 to step 3 (for *FTOPSIS*) and step 4 (for *FAHP*) are performed concurrently.

Step 1

The relevant objective or goal, decision criteria and alternatives of the problem are identified in this step.
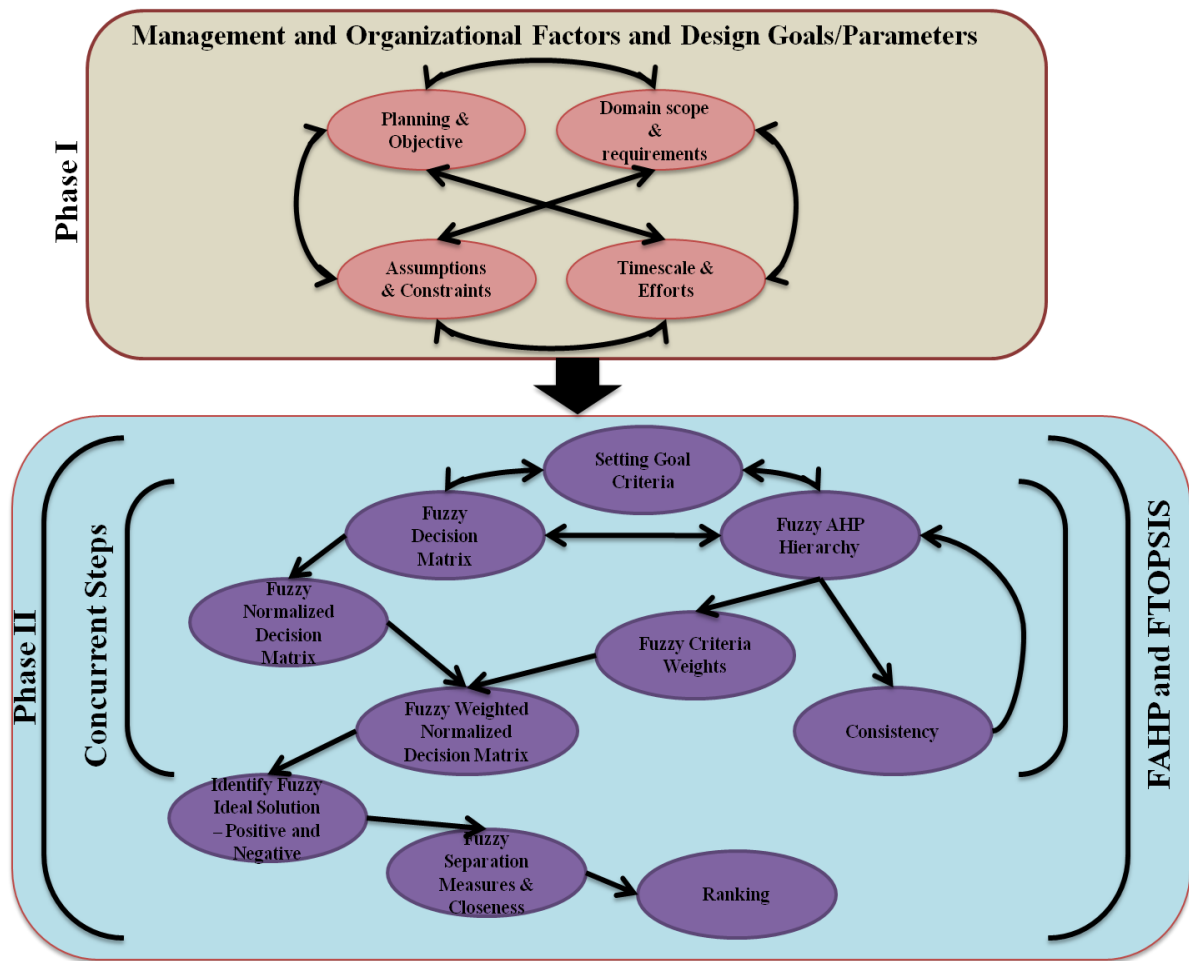
Figure 8.3 Concurrent decision framework for fuzzy environment, FCDF

Step 2

This step produces a decision matrix of criteria and alternatives based on the information available regarding the problem. If the number of alternatives is '*m*' and the number of criteria is '*n*', then the decision matrix having an order of $m \times n$ is represented as follows:

$$D_{m \times n} = \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \end{array} \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ . \\ . \\ . \\ A_m \end{array} \overset{\displaystyle C_1 \quad C_2 \quad C_3 \quad ... \quad C_n}{\left[\begin{array}{ccccc} a_{11} & a_{12} & a_{13} & ... & a_{1n} \\ a_{21} & a_{22} & a_{23} & ... & a_{2n} \\ . & . & . & ... & . \\ . & . & . & . & ... & . \\ . & . & . & . & ... & . \\ a_{m1} & a_{m2} & a_{m3} & ... & a_{mn} \end{array} \right]} \tag{8.17}$$

where an element a_{ij} of the decision matrix $D_{m \times n}$ can be represented as per the following cases:

Case 1: as the triangular fuzzy number (TFN) of the ith alternative in terms of jth decision criteria.

Case 2: Simple qualitative or quantitative value associated with i^{th} alternative in terms of jth decision criteria.

Step 3

In this step the decision matrix is converted into a normalized decision matrix so that the scores obtained in different scales become comparable. An element $\tilde{r}_{ij}$ of the normalized decision matrix $\tilde{R}$ is calculated depending upon the following cases:

$$\tilde{R} = [\tilde{r}_{ij}]_{m \times n} \tag{8.18}$$

Case 1: if a_{ij} in equation (8.17) is represented as TFN then $\tilde{r}_{ij}$ is calculated as:

$$\tilde{r}_{ij} = \left(\frac{x_{ij}}{c_j^*}, \frac{y_{ij}}{c_j^*}, \frac{z_{ij}}{c_j^*} \right), \ j\varepsilon B \tag{8.18a}$$

$$\tilde{r}_{ij} = \left(\frac{x_j^-}{z_{ij}}, \frac{y_j^-}{y_{ij}}, \frac{z_j^-}{x_{ij}} \right), \ j\varepsilon C \tag{8.18b}$$

$$c_j^* = \max_i z_{ij}, if \quad j\varepsilon B; \tag{8.18c}$$

$$x_j^- = \min_i x_{ij}, if \quad j\varepsilon C; \tag{8.18d}$$

where B and C are the set of benefit criteria and cost criteria respectively, and:

Case 2: if a_{ij} in equation (8.17) is represented as value on qualitative or quantitative scale then $\tilde{r}_{ij}$ is calculated as:

$$\tilde{r}_{ij} = \frac{a_{ij}}{\left[\sum_{i=1}^{M} (a_{ij})^2 \right]^{0.5}} \tag{8.18e}$$

Step 4

In the current work Chang's extent *FAHP* (Chang, 1996) is utilized. Let $X = \{x_1, x_2, x_3, \ldots, x_n\}$ an object set, and $G = \{g_1, g_2, g_3, \ldots, g_n\}$ be a goal set. According to the method, each object is taken and extent analysis for each goal performed respectively. Therefore, m extent analysis values for each object can be obtained, with the following signs:

$$\tilde{M}_{gi}^1, \tilde{M}_{gi}^2, \ldots, \tilde{M}_{gi}^m, \qquad i = 1, 2, \ldots, n, \qquad (8.19)$$

where $\tilde{M}_{gi}^1$ $(j = 1, 2\ldots, m)$ all are TFNs. The steps of Chang's extent analysis can be given as in the following sub-steps:

Step 4.1: The value of fuzzy synthetic extent with respect to the ith object is defined as

$$S_i = \sum_{j=1}^{m} \tilde{M}_{gi}^j \otimes \left[\sum_{i=1}^{n} \sum_{j=1}^{m} \tilde{M}_{gi}^j \right]^{-1} \qquad (8.20)$$

To obtain $\sum_{j=1}^{m} \tilde{M}_{gi}^j$, perform the fuzzy addition operation of m extent analysis values for a particular matrix such that:

$$\sum_{j=1}^{m} \tilde{M}_{gi}^j = \left(\sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j \right) \qquad (8.21)$$

and to obtain $\left[\sum_{j=1}^{n} \sum_{j=1}^{m} \tilde{M}_{gi}^j \right]^{-1}$, perform the fuzzy addition operation of $\tilde{M}_{gi}^j$ $(j = 1, 2, \ldots, m)$ values such that

$$\sum_{i=1}^{n} \sum_{j=1}^{m} \tilde{M}_{gi}^j = \left(\sum_{i=1}^{n} l_i, \sum_{i=1}^{n} m_i, \sum_{i=1}^{n} u_i \right) \qquad (8.22)$$

and then compute the inverse of the vector above, such that:

$$\left[\sum_{i=1}^{n} \sum_{j=1}^{m} \tilde{M}_{gi}^j \right]^{-1} = \left(\frac{1}{\sum_{i=1}^{n} u_i}, \frac{1}{\sum_{i=1}^{n} m_i}, \frac{1}{\sum_{i=1}^{n} l_i} \right) \qquad (8.23)$$

Step 4.2: As $\tilde{M}_1 = (l_1, m_1, u_1)$ and $\tilde{M}_2 = (l_2, m_2, u_2)$ are two triangular fuzzy numbers, the

degree of possibility of $\tilde{M}_2 = (l_2, m_2, u_2) \geq \tilde{M}_1 = (l_1, m_1, u_1)$ is defined as:

$$V\left(\tilde{M}_2 \geq \tilde{M}_1\right) = \sup_{y \geq x}\left[\min\left(\mu_{\tilde{M}_1}(x), \mu_{\tilde{M}_2}(y)\right)\right] \qquad (8.24)$$

and can be equivalently expressed as follows:

$$V\left(\tilde{M}_2 \geq \tilde{M}_1\right) = hgt(\tilde{M}_1 \cap \tilde{M}_2) = \mu_{\tilde{M}_2}(d) \qquad (8.25)$$

$$= \begin{cases} 1, & \text{if } m_2 \geq m_1 \\ 0, & \text{if } l_1 \geq u_2 \\ \dfrac{l_1 - u_2}{(m_2 - u_2) - (m_1 - l_1)}, & \text{otherwise} \end{cases} \qquad (8.26)$$

Step 4.3: The degree possibility for a convex fuzzy number to be greater than k convex fuzzy M_i ($i=1, 2,\ldots, n$) numbers can be defined by

$$V(\tilde{M} \geq \tilde{M}_1, \tilde{M}_2, ..., \tilde{M}_k) = V\left[\begin{array}{c}\left(\tilde{M} \geq \tilde{M}_1\right) \text{ and} \\ \\ (\tilde{M} \geq \tilde{M}_2) \text{ and...and } (\tilde{M} \geq \tilde{M}_k)\end{array}\right]$$

$$= \min \left(V(\tilde{M} \geq \tilde{M}_i)\right), \quad i = 1, 2, 3, ..., k \qquad (8.27)$$

Assume that $d\left(A_i\right) = \min V(S_i \geq S_k)$

for $k = 1, 2, ..., n; k \neq i$. Then the weight vector is given by

$$W' = (d'(A_1), d'(A_2), ..., d'(A_n))^T \qquad (8.28)$$

where $A_i = (i = 1, 2, ... n)$ are n elements.

208

Figure 8.4 illustrates equation (8.25) where d is the ordinate of the highest intersection point D between $\mu_{\tilde{M}_1}$ and $\mu_{\tilde{M}_2}$ to compare $\tilde{M}_1$ and $\tilde{M}_2$, we need both the values of $V\left(\tilde{M}_1 \geq \tilde{M}_2\right)$ and $V\left(\tilde{M}_2 \geq \tilde{M}_1\right)$.
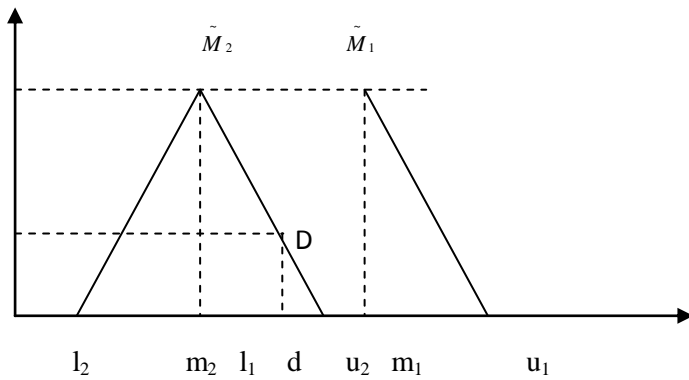
Figure 8.4 The intersection between $\tilde{M}_2$ and $\tilde{M}_1$ (adopted from Kahraman et al., 2004)

Step 4.4: Via normalization, the normalized weight vectors are

$$W = (d(A_1), d(A_2), \ldots\ldots, d(A_n))^T \tag{8.29}$$

W is a non-fuzzy number.

The consistency property of matrices is checked to ensure that the judgments of decision makers are consistent. To check the consistency in pair-wise comparison judgment, *CI* and *CR* are calculated using the equations, equation (8.9) and equation (8.10).

Step 5

The weighted normalized matrix is obtained by multiplying each column of the normalized decision matrix $\tilde{R}$ with the associated criteria weight (equation (8.28), equation (8.9) and equation (8.10)) corresponding to that column. Hence an element v_{ij} of weighted fuzzy normalized decision matrix $\tilde{V}$ is represented as follows:

$$\tilde{V} = \begin{bmatrix} \tilde{v}_{11} & \tilde{v}_{12} & \tilde{v}_{13} & ... & \tilde{v}_{1n} \\ \tilde{v}_{21} & \tilde{v}_{22} & \tilde{v}_{23} & ... & \tilde{v}_{2n} \\ \tilde{v}_{31} & \tilde{v}_{32} & \tilde{v}_{33} & ... & \tilde{v}_{3n} \\ . & . & . & . & . \\ . & . & . & . & . \\ \tilde{v}_{m1} & \tilde{v}_{m2} & \tilde{v}_{m3} & . & \tilde{v}_{mn} \end{bmatrix} = \begin{bmatrix} \tilde{w}_1\,\tilde{r}_{11} & \tilde{w}_2\,\tilde{r}_{12} & \tilde{w}_3\,\tilde{r}_{13} & ... & \tilde{w}_n\,\tilde{r}_{1n} \\ \tilde{w}_1\,\tilde{r}_{21} & \tilde{w}_2\,\tilde{r}_{22} & \tilde{w}_3\,\tilde{r}_{23} & ... & \tilde{w}_n\,\tilde{r}_{2n} \\ \tilde{w}_1\,\tilde{r}_{31} & \tilde{w}_2\,\tilde{r}_{32} & \tilde{w}_3\,\tilde{r}_{33} & ... & \tilde{w}_n\,\tilde{r}_{3n} \\ . & . & . & . & . \\ . & . & . & . & . \\ \tilde{w}_1\,\tilde{r}_{m1} & \tilde{w}_2\,\tilde{r}_{m2} & \tilde{w}_3\,\tilde{r}_{m3} & . & \tilde{w}_n\,\tilde{r}_{mn} \end{bmatrix} \tag{8.30}$$

Step 6

This step produces the fuzzy positive ideal solution (FPIS, F^+) and fuzzy negative ideal solution (FNIS, F^-) in the following manner:

$$A^+ = \left\{ \left(\max v_{ij} / j \in \tau \right), \left(\min v_{ij} / j \in \tau' \right) \; for \; i = 1,2,3,..,m \right\}$$
$$= \left\{ \tilde{V}_1^+,...,\tilde{V}_n^+ \right\} \tag{8.31}$$

$$A^- = \left\{ \left(\min v_{ij} / j \in \tau \right), \left(\max v_{ij} / j \in \tau' \right) \; for \; i = 1,2,3,..,m \right\}$$
$$= \left\{ \tilde{V}_1^-,...,\tilde{V}_n^- \right\} \tag{8.32}$$

where $\tau = \{ j = 1,2,...,n \}$ and j is associated with benefit or positive criteria, and $\tau' = \{ j' = 1,2,...,n \}$ and j' is associated with cost or negative criteria.

The most preferred one has the maximum value among the alternatives. Therefore, F^+ indicates the fuzzy positive ideal solution. Similarly, F^- indicates the fuzzy negative ideal solution.

Step 7

The N dimensional Euclidean distance method is applied, as shown in equation (8.33) and equation (8.34), to measure the separation distances of each alternative from the positive and negative ideal solution:

$$S_i^+ = \left\{ \sum_{j=1}^{N} d\left(\tilde{V}_{ij} - \tilde{V}_j^+ \right)^2 \right\}^{0.5} , i = 1,2,...,m \tag{8.33}$$

and

$$S_i^- = \left\{ \sum_{j=1}^{N} \left(\tilde{V}_{ij} - \tilde{V}_j^- \right)^2 \right\}^{0.5} \quad , i = 1, 2, ..., m \tag{8.34}$$

where S_i^+ and S_i^- are the separation distances of alternative i from the fuzzy positive ideal solution and fuzzy negative ideal solution, respectively.

Step 8

In this step the relative closeness (C_i^*) value of each alternative with respect to the ideal solution is determined using equation (8.35). The value of C_i^* lies within the range from 0 to 1:

$$C_i^* = \frac{S_i^-}{\left(S_i^+ + S_i^- \right)} \tag{8.35}$$

Step 9

At the final step, the preference order is ranked. All the alternatives are now arranged in a descending order according to the value of C_i^*. In *FTOPSIS* method, the chosen alternative has the maximum value of C_i^* with the intention to minimize the distance from the fuzzy ideal solution and to maximize the distance from the fuzzy negative ideal solution.

Finally, the best alternative is recommended to the *FCDF* teams to develop feasible design solution.

8.5 Case Study - FCDF

To check the appropriateness of the *FCDF* methodology the example mentioned in section 8.3 is used and the Table 8.3 is utilized as the base of the methodology. The pair-wise comparison matrix of criteria and relative weights/importance/priority of the criteria in the *FAHP* structure of alternatives is shown below. The membership function, detailed scale and pair wise comparison is shown from *Appendix D* and *Appendix E* respectively. The judgments entered have *CR* less than 0.1, thus the judgments are consistent.

Goal COTS selection: CR: 0.0483

Goal	Economy	Support	Technical	VendorMaturity	Priority
Economy	E	E	I-S	M	0.423
Support	E	E	I-M	M	0.476
Technical	S	M	E	S	1
VendorMaturity	I-M	I-M	I-S	E	0

Economy CR: 0.0280

Economy	MaintenanceCost	PurchasingCost	TrainingCost	Priority
MaintenanceCost	E	M	E	0.852
PurchasingCost	I-M	E	S	1
TrainingCost	I	I-S	E	0.08

Technical CI: 0.000

Technical	Efficiency	Interface	Priority
Efficiency	E	M	1
Interface	I-M	E	0.42

Efficiency CR: 0.0695

Efficiency	ACID	Index	Partition	TableSize	Priotiry
ACID	E	E	E	E	0.50
Index	E	E	M	E	1
Partition	E	I-M	E	M	0.906
TableSize	E	E	I-M	E	0.423

Interface CR: 0.090

Interface	GUI	OS	SQL	Priority
GUI	E	M	I-M	0.927
OS	I-M	E	E	0.494
SQL	M	E	E	1

Vendor Maturity CR: 0.0806

VendorMaturity	FirstRelease	MarketShare	Responsiveness	StableVersion	Priority
FirstRelease	E	I-M	E	I-M	0.359
MarketShare	M	E	E	M	1
Responsiveness	E	E	E	E	0.428
StableVersion	M	I-M	E	E	0.76

Support CR: 0.000

Support	AfterSale	BeforeSale	Priority
AfterSale	E	I-M	0.42
BeforeSale	M	E	1

After Sale CR: 0.0776

AfterSale	Documentation	Help	Off − Site	On − Site	Priority
Documentation	E	M	M	E	1
Help	I-M	E	M	M	0.967
Off − Site	I-M	I-M	E	I-M	0.448
On − Site	E	I-M	M	E	0.820

Before Sale CR: 0.000

BeforeSale	ProductHelp	Documentation	Priority
ProductHelp	E	M	1
Documentation	I-M	E	0.42

In order to illustrate step 4 calculations of section 8.4, Goal matrix calculation is used as an example for calculating the weight/priority as follows:

Goal	Economy	Support	Technical	VendorMaturity	Priority
Economy	(1,1,1)	(1,1,1)	(1/7,1/5,1/3)	(1,3,5)	
Support	(1,1,1)	(1,1,1)	(1/5,1/3,1)	(1,3,5)	
Technical	(3,5,7)	(1,3,5)	(1,1,1)	(3,5,7)	
VendorMaturity	(1/5,1/3,1)	(1/5,1/3,1)	(1/7,1/5,1/3)	(1,1,1)	

$S_E = (3.14, 5.2, 7.33) \otimes (1/38.6, 1/26.4, 1/15.88) = (0.81, 0.19, 0.45)$

$S_S = (3.2, 5.33, 8) \otimes (1/38.6, 1/26.4, 1/15.88) = (0.08, 0.20, 0.50)$

$S_T = (8, 14, 20) \otimes (1/38.6, 1/26.4, 1/15.88) = (0.20, 0.53, 1.25)$

$S_{VM} = (1.54, 1.86, 3.33) \otimes (1/38.6, 1/26.4, 1/15.88) = (0.03, 0.07, 0.20)$

These fuzzy values are compared by using equation (8.26):

$V (S_E \geq S_S) = 0.973$; $V (S_E \geq S_T) = 0.423$; $V (S_E \geq S_{VM}) = 1$;

$V (S_S \geq S_E) = 1$; $V (S_S \geq S_T) = 0.476$; $V (S_S \geq S_{VM}) = 1$;

$V (S_T \geq S_E) = 1$; $V (S_T \geq S_S) = 1$; $V (S_T \geq S_{VM}) = 1$;

$V(S_{VM} \geq S_E) = 0; \qquad V(S_{VM} \geq S_S) = 0.48; \quad V(S_{VM} \geq S_T) = 0;$

Then priority weights are calculated by using equation (8.27):

$d'(E) = \min(0.973, 0.423, 1) = 0.423; \qquad\qquad d'(S) = \min(1, 0.476, 1) = 0.476;$

$d'(T) = \min(1, 1, 1) = 1; \qquad\qquad d'(VM) = \min(0, 0.48, 0) = 0;$

Priority weights form W' = (0.423, 0.476, 1, 0) vector. After the normalization of these values priority weight with respect to main goal is calculated as (0.22, 0.25, 0.52, 0). Accordingly all the other matrices are computed and the overall result is shown in Table 8.8.

Criteria	C1	C2	C3	C4	C5	C6	C7
Weights	0.22	0.44	0.51	0.04	0.52	0.29	0.382
Criteria	C8	C9	C10	C11	C12	C13	C14
Weights	0.413	0.20	0.70	0.14	0.32	0.17	0.35
Criteria	C15	C16	C17	C18	C19	C20	C21
Weights	0.25	0.29	0.311	0.139	0.248	0.300	0.70
Criteria	C22	C23	C24	C25	C26	C27	C28
Weights	0.29	0.70	0	0.168	0.140	0.298	0.392

Table 8.8 Criteria weights

Once weights are calculated then Table 8.3 can be normalized using equation (8.18). The result of this along with respective criteria weights are shown in Table 8.9. The next step uses the fuzzy membership function discussed in Appendix D to transform Table 8.9 into Table 8.10 as explained by the following example. If the numeric rating is 0.60, then its fuzzy linguistic variable is ''M''. This transformation is also applied to the criteria/attribute weight for C1- C28 respectively. Then, the resulting fuzzy linguistic variables are show as Table 8.10. Programs mentioned in Appendix G and Appendix H is used to calculate weights for fuzzy *AHP* and computations for fuzzy *TOPSIS* respectively.

	C1	C2	C3	C4	C5	C6	C7
A1	0.655386	0.487954	0.650945	0.692636	0.628539	0.655386	0.722897
A2	0.491539	0.780726	0.650945	0.461757	0.549972	0.491539	0.40161
A3	0.573462	0.390343	0.390567	0.554109	0.549972	0.573462	0.562254
Weights	0.22	0.44	0.51	0.04	0.52	0.29	0.382
	C8	C9	C10	C11	C12	C13	C14
A1	0.549972	0.680414	0.655386	0.697247	0.57735	0.57735	0.666667
A2	0.628539	0.680414	0.573462	0.068091	0.57735	0.57735	0.666667
A3	0.549972	0.272166	0.491539	0.713589	0.57735	0.57735	0.333333
Weights	0.413	0.20	0.70	0.14	0.32	0.17	0.35
	C15	C16	C17	C18	C19	C20	C21
A1	0.523205	0.514496	0.545455	0.57735	0.545455	0.545455	0.57735
A2	0.672692	0.685994	0.636364	0.57735	0.636364	0.636364	0.57735
A3	0.523205	0.514496	0.545455	0.57735	0.545455	0.545455	0.57735
Weights	0.25	0.29	0.311	0.139	0.248	0.300	0.70
	C22	C23	C24	C25	C26	C27	C28
A1	0.57735	0.636364	0.523205	0.549972	0.57735	0.549972	0.520266
A2	0.57735	0.545455	0.672692	0.628539	0.57735	0.628539	0.780399
A3	0.57735	0.545455	0.523205	0.549972	0.57735	0.549972	0.346844
Weights	0.29	0.70	0	0.168	0.140	0.298	0.392

Table 8.9 Normalized decision matrix for fuzzy TOPSIS analysis

	C1	C2	C3	C4	C5	C6	C7
A1	H	L	H	H	H	H	H
A2	M	H	H	M	M	M	M
A3	M	L	L	M	M	M	M
Weights	L	M	M	VL	M	L	L
	C8	C9	C10	C11	C12	C13	C14
A1	M	H	H	H	M	M	H
A2	H	H	M	VL	M	M	H
A3	M	L	M	H	M	M	L
Weights	M	VL	H	VL	L	VL	L

Continued…

	C15	C16	C17	C18	C19	C20	C21
A1	M	M	M	M	M	M	M
A2	H	H	H	M	H	H	M
A3	M	M	M	M	M	M	M
Weights	L	L	L	VL	L	L	H
	C22	C23	C24	C25	C26	C27	C28
A1	M	H	M	M	M	M	M
A2	M	M	H	H	M	H	H
A3	M	M	M	M	M	M	L
Weights	L	H	VL	VL	VL	L	L

Table 8.10 Decision matrix using fuzzy linguistic variables

The fuzzy linguistic variable is then transformed into a fuzzy triangular membership function as shown in Table 8.11. This is the first step of the *FTOPSIS* analysis. The fuzzy attribute weight is also calculated in Table 8.11. The second step in the analysis is to find the weighted fuzzy decision matrix. Table 8.3 criteria weights are used to generate fuzzy weighted decision matrix as shown in Table 8.12. The multiplication is computed on the basis of multiple equations presented in Appendix C**.** According to Table 8.12, the elements V_{ij} are normalized as positive triangular fuzzy numbers and their ranges belong to the closed interval [0, 1]. Thus, we can define the fuzzy positive-ideal solution (FPIS, F^+) and the fuzzy negative-ideal solution (FNIS, F^-) as (1, 1, 1) and (0, 0, 0) for all elements. This is the third step of the *FTOPSIS* analysis. For the fourth step, the distance of each alternative from F^+ and F^- can be currently calculated using equations (26) and equation (27) respectively. The fifth step solves the similarities to an ideal solution by equation (28). The resulting *FTOPSIS* analyses are summarized in Table 8.13.

	C1	C2	C3	C4
A1	(0.55, 0.70, 0.85)	(0.15, 0.30, 0.45)	(0.55, 0.70, 0.85)	(0.55, 0.70, 0.85)
A2	(0.35, 0.50, 0.65)	(0.55, 0.70, 0.85)	(0.55, 0.70, 0.85)	(0.35, 0.50, 0.65)
A3	(0.35, 0.50, 0.65)	(0.15, 0.30, 0.45)	(0.15, 0.30, 0.45)	(0.35, 0.50, 0.65)
Weights	(0.15, 0.30, 0.45)	(0.35, 0.50, 0.65)	(0.35, 0.50, 0.65)	(0.00, 0.10, 0.25)
	C1	C2	C3	C4
A1	(0.55, 0.70, 0.85)	(0.15, 0.30, 0.45)	(0.55, 0.70, 0.85)	(0.55, 0.70, 0.85)

A2	(0.35, 0.50, 0.65)	(0.55, 0.70, 0.85)	(0.55, 0.70, 0.85)	(0.35, 0.50, 0.65)
A3	(0.35, 0.50, 0.65)	(0.15, 0.30, 0.45)	(0.15, 0.30, 0.45)	(0.35, 0.50, 0.65)
Weights	(0.15, 0.30, 0.45)	(0.35, 0.50, 0.65)	(0.35, 0.50, 0.65)	(0.00, 0.10, 0.25)
	C5	C6	C7	C8
A1	(0.55, 0.70, 0.85)	(0.55, 0.70, 0.85)	(0.55, 0.70, 0.85)	(0.35, 0.50, 0.65)
A2	(0.35, 0.50, 0.65)	(0.35, 0.50, 0.65)	(0.35, 0.50, 0.65)	(0.55, 0.70, 0.85)
A3	(0.35, 0.50, 0.65)	(0.35, 0.50, 0.65)	(0.35, 0.50, 0.65)	(0.35, 0.50, 0.65)
Weights	(0.35, 0.50, 0.65)	(0.15, 0.30, 0.45)	(0.15, 0.30, 0.45)	(0.35, 0.50, 0.65)
	C9	C10	C11	C12
A1	(0.55, 0.70, 0.85)	(0.55, 0.70, 0.85)	(0.55, 0.70, 0.85)	(0.35, 0.50, 0.65)
A2	(0.55, 0.70, 0.85)	(0.35, 0.50, 0.65)	(0.00, 0.10, 0.25)	(0.35, 0.50, 0.65)
A3	(0.15, 0.30, 0.45)	(0.35, 0.50, 0.65)	(0.55, 0.70, 0.85)	(0.35, 0.50, 0.65)
Weights	(0.00, 0.10, 0.25)	(0.55, 0.70, 0.85)	(0.00, 0.10, 0.25)	(0.15, 0.30, 0.45)
	C13	C14	C15	C16
A1	(0.35, 0.50, 0.65)	(0.55, 0.70, 0.85)	(0.35, 0.50, 0.65)	(0.35, 0.50, 0.65)
A2	(0.35, 0.50, 0.65)	(0.55, 0.70, 0.85)	(0.55, 0.70, 0.85)	(0.55, 0.70, 0.85)
A3	(0.35, 0.50, 0.65)	(0.15, 0.30, 0.45)	(0.35, 0.50, 0.65)	(0.35, 0.50, 0.65)
Weights	(0.00, 0.10, 0.25)	(0.15, 0.30, 0.45)	(0.15, 0.30, 0.45)	(0.15, 0.30, 0.45)
	C17	C18	C19	C20
A1	(0.35, 0.50, 0.65)	(0.35, 0.50, 0.65)	(0.35, 0.50, 0.65)	(0.35, 0.50, 0.65)
A2	(0.55, 0.70, 0.85)	(0.35, 0.50, 0.65)	(0.55, 0.70, 0.85)	(0.55, 0.70, 0.85)
A3	(0.35, 0.50, 0.65)	(0.35, 0.50, 0.65)	(0.35, 0.50, 0.65)	(0.35, 0.50, 0.65)
Weights	(0.15, 0.30, 0.45)	(0.00, 0.10, 0.25)	(0.15, 0.30, 0.45)	(0.15, 0.30, 0.45)
	C21	C22	C23	C24
A1	(0.35, 0.50, 0.65)	(0.35, 0.50, 0.65)	(0.55, 0.70, 0.85)	(0.35, 0.50, 0.65)
A2	(0.35, 0.50, 0.65)	(0.35, 0.50, 0.65)	(0.35, 0.50, 0.65)	(0.55, 0.70, 0.85)
A3	(0.35, 0.50, 0.65)	(0.35, 0.50, 0.65)	(0.35, 0.50, 0.65)	(0.35, 0.50, 0.65)
Weights	(0.55, 0.70, 0.85)	(0.15, 0.30, 0.45)	(0.55, 0.70, 0.85)	(0.00, 0.10, 0.25)
	C25	C26	C27	C28
A1	(0.35, 0.50, 0.65)	(0.35, 0.50, 0.65)	(0.35, 0.50, 0.65)	(0.35, 0.50, 0.65)
A2	(0.55, 0.70, 0.85)	(0.35, 0.50, 0.65)	(0.55, 0.70, 0.85)	(0.55, 0.70, 0.85)
A3	(0.35, 0.50, 0.65)	(0.35, 0.50, 0.65)	(0.35, 0.50, 0.65)	(0.15, 0.30, 0.45)
Weights	(0.00, 0.10, 0.25)	(0.00, 0.10, 0.25)	(0.15, 0.30, 0.45)	(0.15, 0.30, 0.45)

Table 8.11 Decision matrix and fuzzy attribute weights

	C1	C2	C3	C4
A1	(0.08, 0.21, 0.38)	(0.05, 0.15, 0.29)	(0.19, 0.35, 0.55)	(0.00, 0.07, 0.21)
A2	(0.05, 0.15, 0.29)	(0.19, 0.35, 0.55)	(0.19, 0.35, 0.55)	(0.00, 0.05, 0.16)
A3	(0.05, 0.15, 0.29)	(0.05, 0.15, 0.29)	(0.05, 0.15, 0.29)	(0.00, 0.05, 0.16)
	C5	C6	C7	C8
A1	(0.19, 0.35, 0.55)	(0.08, 0.21, 0.38)	(0.08, 0.21, 0.38)	(0.12, 0.25, 0.42)
A2	(0.12, 0.25, 0.42)	(0.05, 0.15, 0.29)	(0.05, 0.15, 0.29)	(0.19, 0.35, 0.55)
A3	(0.12, 0.25, 0.42)	(0.05, 0.15, 0.29)	(0.05, 0.15, 0.29)	(0.12, 0.25, 0.42)
	C9	C10	C11	C12
A1	(0.00, 0.07, 0.21)	(0.03, 0.49, 0.72)	(0.00, 0.07, 0.21)	(0.05, 0.15, 0.29)
A2	(0.00, 0.07, 0.21)	(0.19, 0.35, 0.55)	(0.00, 0.01, 0.06)	(0.05, 0.15, 0.29)
A3	(0.00, 0.03, 0.11)	(0.19, 0.35, 0.55)	(0.00, 0.07, 0.21)	(0.05, 0.15, 0.29)
	C13	C14	C15	C16
A1	(0.00, 0.05, 0.16)	(0.08, 0.21, 0.38)	(0.05, 0.15, 0.29)	(0.05, 0.15, 0.29)
A2	(0.00, 0.05, 0.16)	(0.08, 0.21, 0.38)	(0.08, 0.21, 0.38)	(0.08, 0.21, 0.38)
A3	(0.00, 0.05, 0.16)	(0.02, 0.09, 0.20)	(0.05, 0.15, 0.29)	(0.05, 0.15, 0.29)
	C17	C18	C19	C20
A1	(0.05, 0.15, 0.29)	(0.00, 0.05, 0.16)	(0.00, 0.05, 0.29)	(0.00, 0.05, 0.29)
A2	(0.08, 0.21, 0.38)	(0.00, 0.05, 0.16)	(0.08, 0.21, 0.38)	(0.08, 0.21, 0.38)
A3	(0.05, 0.15, 0.29)	(0.00, 0.05, 0.16)	(0.00, 0.05, 0.29)	(0.00, 0.05, 0.29)
	C21	C22	C23	C24
A1	(0.19, 0.35, 0.55)	(0.05, 0.15, 0.29)	(0.30, 0.49, 0.72)	(0.00, 0.05, 0.16)
A2	(0.19, 0.35, 0.55)	(0.05, 0.15, 0.29)	(0.19, 0.35, 0.55)	(0.00, 0.07, 0.21)
A3	(0.19, 0.35, 0.55)	(0.05, 0.15, 0.29)	(0.19, 0.35, 0.55)	(0.00, 0.05, 0.16)
	C25	C26	C27	C28
A1	(0.00, 0.05, 0.16)	(0.00, 0.05, 0.16)	(0.05, 0.15, 0.29)	(0.05, 0.15, 0.29)
A2	(0.00, 0.07, 0.21)	(0.00, 0.05, 0.16)	(0.08, 0.21, 0.38)	(0.08, 0.21, 0.38)
A3	(0.00, 0.05, 0.16)	(0.00, 0.05, 0.16)	(0.05, 0.15, 0.29)	(0.02, 0.09, 0.20)

Table 8.12 Fuzzy weighted decision matrix

	V_{i1}	V_{i2}	V_{i3}	V_{i4}
A1	(0.08, 0.21, 0.38)	(0.05, 0.15, 0.29)	(0.19, 0.35, 0.55)	(0.00, 0.07, 0.21)
A2	(0.05, 0.15, 0.29)	(0.19, 0.35, 0.55)	(0.19, 0.35, 0.55)	(0.00, 0.05, 0.16)
A3	(0.05, 0.15, 0.29)	(0.05, 0.15, 0.29)	(0.05, 0.15, 0.29)	(0.00, 0.05, 0.16)
A+	(1, 1, 1)	(1, 1, 1)	(1, 1, 1)	(1, 1, 1)
A-	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)
	V_{i5}	V_{i6}	V_{i7}	V_{i8}
A1	(0.19, 0.35, 0.55)	(0.08, 0.21, 0.38)	(0.08, 0.21, 0.38)	(0.12, 0.25, 0.42)
A2	(0.12, 0.25, 0.42)	(0.05, 0.15, 0.29)	(0.05, 0.15, 0.29)	(0.19, 0.35, 0.55)
A3	(0.12, 0.25, 0.42)	(0.05, 0.15, 0.29)	(0.05, 0.15, 0.29)	(0.12, 0.25, 0.42)
A+	(1, 1, 1)	(1, 1, 1)	(1, 1, 1)	(1, 1, 1)
A-	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)
	V_{i9}	V_{i10}	V_{i11}	V_{i12}
A1	(0.00, 0.07, 0.21)	(0.03, 0.49, 0.72)	(0.00, 0.07, 0.21)	(0.05, 0.15, 0.29)
A2	(0.00, 0.07, 0.21)	(0.19, 0.35, 0.55)	(0.00, 0.01, 0.06)	(0.05, 0.15, 0.29)
A3	(0.00, 0.03, 0.11)	(0.19, 0.35, 0.55)	(0.00, 0.07, 0.21)	(0.05, 0.15, 0.29)
A+	(1, 1, 1)	(1, 1, 1)	(1, 1, 1)	(1, 1, 1)
A-	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)
	V_{i13}	V_{i14}	V_{i15}	V_{i16}
A1	(0.00, 0.05, 0.16)	(0.08, 0.21, 0.38)	(0.05, 0.15, 0.29)	(0.05, 0.15, 0.29)
A2	(0.00, 0.05, 0.16)	(0.08, 0.21, 0.38)	(0.08, 0.21, 0.38)	(0.08, 0.21, 0.38)
A3	(0.00, 0.05, 0.16)	(0.02, 0.09, 0.20)	(0.05, 0.15, 0.29)	(0.05, 0.15, 0.29)
A+	(1, 1, 1)	(1, 1, 1)	(1, 1, 1)	(1, 1, 1)
A-	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)
	V_{i17}	V_{i18}	V_{i19}	V_{i20}
A1	(0.05, 0.15, 0.29)	(0.00, 0.05, 0.16)	(0.00, 0.05, 0.29)	(0.00, 0.05, 0.29)
A2	(0.08, 0.21, 0.38)	(0.00, 0.05, 0.16)	(0.08, 0.21, 0.38)	(0.08, 0.21, 0.38)
A3	(0.05, 0.15, 0.29)	(0.00, 0.05, 0.16)	(0.00, 0.05, 0.29)	(0.00, 0.05, 0.29)
A+	(1, 1, 1)	(1, 1, 1)	(1, 1, 1)	(1, 1, 1)
A-	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)

Continued…

	V_{i21}	V_{i22}	V_{i23}	V_{i24}
A1	(0.19, 0.35, 0.55)	(0.05, 0.15, 0.29)	(0.30, 0.49, 0.72)	(0.00, 0.05, 0.16)
A2	(0.19, 0.35, 0.55)	(0.05, 0.15, 0.29)	(0.19, 0.35, 0.55)	(0.00, 0.07, 0.21)
A3	(0.19, 0.35, 0.55)	(0.05, 0.15, 0.29)	(0.19, 0.35, 0.55)	(0.00, 0.05, 0.16)
A+	(1, 1, 1)	(1, 1, 1)	(1, 1, 1)	(1, 1, 1)
A-	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)
	V_{i25}	V_{i26}	V_{i27}	V_{i28}
A1	(0.00, 0.05, 0.16)	(0.00, 0.05, 0.16)	(0.05, 0.15, 0.29)	(0.05, 0.15, 0.29)
A2	(0.00, 0.07, 0.21)	(0.00, 0.05, 0.16)	(0.08, 0.21, 0.38)	(0.08, 0.21, 0.38)
A3	(0.00, 0.05, 0.16)	(0.00, 0.05, 0.16)	(0.05, 0.15, 0.29)	(0.02, 0.09, 0.20)
A+	(1, 1, 1)	(1, 1, 1)	(1, 1, 1)	(1, 1, 1)
A-	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)
weights	(0.2, 0.3, 0.5)	(0.2, 0.3, 0.5)	(0.2, 0.3, 0.5)	(0.2, 0.3, 0.5)
	S+	S-	C_i^*	
A1	22.95	6.25	0.21	
A2	22.6	6.44	0.22	
A3	31.10	13.60	0.30	

Table 8.13 Fuzzy TOPSIS analysis

The developed framework and aforementioned structured steps are found to be useful in evaluating and ranking software component effectively for the identified problem. Due to the highest value of C_i^* the alternative "A3" was recommended to the design team.

8.6 Discussion

It can be seen that the criteria that were identified during the software component selection process were mainly comprised of subjective/qualitative factors. Thus final result in software component selection varies. In fuzzy consideration, alternative "A3" got the highest rating though it was rated low in non fuzzy environment. The criteria in both the examples

were same but considering impreciseness in decision making candidate alternative decisions were different. This is so because most of the criteria were largely qualitative thus fuzzy methodological framework has a great impact on the overall result and decision making process. This leads to the conclusion that when high level of subjective/qualitative criteria are involved then concurrent fuzzy decision based methodology is best suited for COTS selection as it covers human impreciseness in taking decision under fuzzy environment. For quantitative analysis (criteria) either of the methodological frameworks can be used. A non fuzzy framework works fine for such kind of factors.

8.7 Usefulness of the Developed Frameworks

Different stakeholders in CBSS project e.g. customer, decision makers, modelers, analysts, designers, developers, quality analysts, domain experts, testers, marketing personnel and consultants can be benefited by the developed framework because of following reasons:

- The framework is flexible as it allows customization or tailoring and meeting various projects' requirements.
- The developed methodology is a powerful tool in the hands of the system analyst, designer, decision makers and developers.
- Using this and morphological chart/tree, the system analyst, decision makers and designer can generate alternative design solutions and select the optimum one.
- Similarly, this method can be exploited to improve quality and reduce cost and time-to-market in CBSS industry.
- It provides more concise, structural, and step-by-step activities for determining the best software component product by utilizing integrated/concurrent approach.
- The framework undertakes considerations of fuzzy environment.
- It involves group decision environment approach.
- The *FAHP* structure is helpful in establishing priorities for the criteria/sub-criteria.
- The *FTOPSIS* is used to rank alternatives.
- It is also possible to exploit the methodology to extend the useful product life in the CBSS market by making strategic changes in the software component. This framework gives a comprehensive knowledge to the designer and decision maker about CBSS design and helps in the selection of right software component at the right time and at the right cost from the global market.

8.8 Concluding Remarks

In this chapter, a concurrent approach to provide an effective decision for both non-fuzzy and fuzzy environment for the selection of components is presented. Selecting the 'right' software component during CBSS design phase is critical for the success of a component based software systems project. The technical experts, the economic experts, the evaluators, the stakeholders, and the design team have subjective opinions on the requirements, performance and capabilities of the software component and these often are imprecise and vague. The developed approach will help concerned stakeholders of the project in filtering out inappropriate components for the project.

In the next chapter, the conclusion and future work of the research thesis is mentioned. Step by step procedure for the implementation of the developed methodologies and framework is also presented.