Abstract

This thesis aims to investigate the interplay between the properties of algebraic structures (viz. groups and semigroups) and their associated graphs. The study of graphs associated to groups and semigroups have been extensively studied, because they have valuable applications and are related to automata theory. The main concerns of this thesis is to study cyclic graph, enhanced power graph and commuting graph associated with groups and semigroups.

After ascertaining the structure of the cyclic graph $\Gamma(S)$ of a semigroup S, this thesis classifies the semigroup S such that $\Gamma(S)$ is complete, bipartite, tree, acyclic, regular etc. Various numerical parameters namely: chromatic number, clique number and independence number of $\Gamma(S)$ are investigated. For a finite group G, the minimum degree, independence number and matching number of the enhanced power graph $\mathcal{P}_e(G)$ of group G are investigated. Then they were determined them when G is a finite abelian p-group, dihedral group, semidihedral group, dicyclic group, U_{6n} or V_{8n}. If G is any of these groups, it is shown that $\mathcal{P}_e(G)$ is perfect and then obtained its strong metric dimension. Additionally, an expression for the independence number of $\mathcal{P}_e(G)$ for any finite abelian group G is determined. Moving forward, this thesis reveals the basic properties including planarity, independence number, minimum degree of the enhanced power graph $\mathcal{P}_e(S)$ of a semigroup S. It is ascertained that the number of connected components in $\mathcal{P}_e(S)$ is equal to the number of idempotents of S. Also, a semigroup S has been constructed such that the chromatic number of $\mathcal{P}_e(S)$ is uncountable. It is shown that the edge connectivity

and the minimum degree of the commuting graph $\Delta(G)$ of a group G are equal. This thesis also established the detour properties, Laplacian spectrum, metric dimension and resolving polynomials of the commuting graph of semidihedral group. After investigation of various graph-theoretic parameters, the automorphism group and endomorphism monoid of the commuting graph of Brandt semigroups are obtained. A class of inverse semigroups is also ascertained such that its commuting graph is Hamiltonian. Finally, this thesis classifies finite semigroups such that the pair of graphs, viz. $\Gamma(S), \mathcal{P}_e(S)$ and $\Delta(S)$ are equal.