

## Abstract

This thesis addresses two problems. The first one concerns associating cohomological invariants to families of flat connections on a smooth vector bundle. Let  $E \rightarrow B$  be a smooth vector bundle of rank  $n$ , and let  $P \in I^p(GL(n, \mathbb{R}))$  be a  $GL(n, \mathbb{R})$ -invariant polynomial of degree  $p$  compatible with a universal integral characteristic class  $u \in H^{2p}(BGL(n, \mathbb{R}), \mathbb{Z})$ . Cheeger–Simons theory associates an invariant in  $H^{2p-1}(B, \mathbb{R}/\mathbb{Z})$  to any flat connection on this bundle. Generalizing this result, Jaya Iyer (*Lett in Math Phys*, 2016, 106 (1) pp. 131-146) constructed maps  $H_r(\mathcal{D}(E)) \rightarrow H^{2p-r-1}(B, \mathbb{R}/\mathbb{Z})$  for  $p > r+1$ . Here  $\mathcal{D}(E)$  is the simplicial set whose  $r$ -simplices are  $(r+1)$ -tuples of relatively flat connections. In this thesis, we construct such maps for the cases  $p < r$  and  $p > r+1$  using fiber integration of differential characters. We find that for  $p > r+1$  case, the invariants constructed here coincide with those obtained by Jaya Iyer, and in the  $p < r$  case the invariants are trivial. We further compare our construction with other results in the literature.

The second problem concerns uniqueness of differential cohomology theories. In arXiv:0810.4935, J.Simons and D.Sullivan construct a model of differential K-theory, and show that the differential K-theory functor fits into a hexagon diagram. They ask whether, like the case of differential characters, this hexagon diagram uniquely determines the differential K-theory functor. In this thesis, we provide a partial affirmative answer to their question: For any fixed compact manifold  $M$ , the differential K-theory groups  $\hat{K}(M)$  are uniquely determined by the Simons–Sullivan diagram up to an isomorphism compatible with the diagonal arrows of the hexagon diagram. We state a necessary and sufficient condition for an affirmative answer to the full question. This approach further yields an alternative proof of a weaker version of J.Simons and D.Sullivan’s results in *Journal of Topology*, 2008, 1:45–56 concerning axiomatization of differential characters. We further obtain a uniqueness result for generalised differential cohomology groups. The proofs here are based on the recent work *Proc. Math. Sci.* 129, 70(219) of R. Pawar.

This thesis is based on the two articles: 1. Mata,I. ‘Invariants of families of flat

connections using fiber integration of differential characters' *Lett. Math. Phys.* 110, 639–657 (2020), and 2. Mata, I. 'Uniqueness of differential characters and differential K-theory via homological algebra', arXiv:2005.02056 (accepted for publication in *J. Homotopy Relat. Struct.*)