Chapter 4

Concluding discussion and scope for future work

We wish to conclude by broadly outlining some possible directions for further exploration and study.

In Chapter 2, we constructed maps $\psi_{p,r}$ on the homology of the simplicial set $\mathcal{D}(E)$ of relatively flat connections: $\psi_{p,r} : H_r(\mathcal{D}(E)) \to H^{2p-r-1}(B, \mathbb{R}/\mathbb{Z})$. A possible direction of study is to explore if analogous maps can be constructed for the $G-$equivariant set-up.

Another question concerns families of structured bundles. Let $E \to B$ be a smooth vector bundle, and let $\mathcal{A}$ denote the space of all connections on it. Simons and Sullivan construct a certain equivalence relation on the set $\mathcal{A}$. The pairs $(E \to B, [\theta])$ where $[\theta]$ denotes the equivalence classe of the connection θ, are called 'structured vector bundles'. They construct a category of isomorphism classes of such structured vector bundles over a fixed base manifold B. Under the operation direct sum, this category becomes an additive category. The Grothendieck completion of this category is called the differential $K-$group of B, denoted by $\hat{K}(B)$.

We can fix a bundle $E \to B$ and consider *families* of structured bundles. It would be interesting to explore if fiber integration of differential $K-$theory could be employed to associate invariants to these families of structured bundles.

Yet another avenue of exploration could be to try to obtain an explicit description of projective and injective objects in the category $Fun(Man_{cpt}^{op}, Ab)$. This could enable us to verify the condition (2) in Proposition 44 and thus yield a complete answer to the Simons–Sullivan question about the uniqueness of differential $K-$theory functor.