Appendices

Appendix A

Chapter 2: Derivations and Proofs

A.1 Proof of Claim 1: Relay Selection Policy: Small Scale Fading with Path Loss

See that Γ_{R_1} and Γ_{R_2} are exponentially distributed with mean λ, and μ, respectively. Consider probability of the event $\Gamma_{R_1} > \Gamma_{R_2}$. It is given by [137]

$$\begin{aligned}\mathcal{P}(\Gamma_{R_1} > \Gamma_{R_2}) &= \int_0^\infty \mathcal{P}\left(\Gamma_{R_1} > t\right) \frac{1}{\mu} \exp\left(-\frac{t}{\mu}\right) dt, \\ &= \frac{1}{\mu} \int_0^\infty \exp\left(-\frac{t}{\lambda}\right) \exp\left(-\frac{t}{\mu}\right) dt, \\ &= \frac{\lambda}{\lambda + \mu}. \end{aligned} \tag{A.1.1}$$

Therefore, we get $\mathcal{P}(\Gamma_{R_1} > \Gamma_{R_2}) = \frac{\lambda}{\lambda+\mu} = \frac{\overline{\gamma}_{d_1 r_1}}{\overline{\gamma}_{d_1 r_1} + \overline{\gamma}_{d_1 r_2}}$. Furthermore, $\mathcal{P}(\Gamma_{R_2} > \Gamma_{R_1}) = 1 - \frac{\lambda}{\lambda+\mu} = \frac{\mu}{\lambda+\mu} = \frac{\overline{\gamma}_{d_1 r_2}}{\overline{\gamma}_{d_1 r_1} + \overline{\gamma}_{d_1 r_2}}$.

A.2 Proof of Claim 2: Relay Selection Policy: Large Scale Fading with Path Loss

Probability of the event $\mathcal{P}(\Psi_1 > \Psi_2)$ is given by

$$\mathcal{P}(10^{\frac{Y_1}{10}} > 10^{\frac{Y_2}{10}}) = \int_{-\infty}^{\infty} \mathcal{P}\left(Y_1 > y\right) p_{Y_2}(y)\, dy. \tag{A.2.1}$$

Since $\mathcal{P}\left(Y_1 > y\right)$ is the complementary cumulative distribution function (CCDF) of a Gaussian random variable, we have

$$\mathcal{P}\left(Y_1 > y\right) = \frac{1}{2}\text{erfc}\left(\frac{y - \mu_1}{\sigma_1 \sqrt{2}}\right).$$

Furthermore, the PDF of Y_2 is given by

$$p_{Y_2}(y) = \frac{1}{\sqrt{2\pi\sigma_2{}^2}} \exp\left(-\frac{(y-\mu_2)^2}{2\sigma_2{}^2}\right).$$

Substituting the CCDF and the PDF in (A.2.1), we get the desired expression for probability.

A.3 Proof of Claim 3: Relay Selection Policy: Small Scale Plus Large Scale Fading with Path Loss

Let $\frac{\Psi_2}{\Psi_1} \triangleq \Psi_3$ and $\frac{\gamma_{d_1r_1}}{\gamma_{d_1r_2}} \triangleq \gamma_3$. Probability of the event $\mathcal{P}(\Psi_1\gamma_{d_1r_1} > \Psi_2\gamma_{d_1r_2})$ is given by

$$\mathcal{P}(\frac{\Psi_2}{\Psi_1} < \frac{\gamma_{d_1r_1}}{\gamma_{d_1r_2}}) = \int_0^\infty \mathcal{P}\left(\Psi_3 < z\right) p_{\gamma_3}(z)\, dz. \tag{A.3.1}$$

We first determine the PDF of γ_3 from its CDF. The complementary CDF (CCDF) is given by

$$\begin{aligned}\mathcal{P}\left(\gamma_{d_1r_1} > \gamma_{d_1r_2} z\right) &= \frac{1}{\overline{\gamma}_{d_1r_2}} \int_0^\infty \exp\left(-\frac{yz}{\overline{\gamma}_{d_1r_1}}\right) \exp\left(-\frac{y}{\overline{\gamma}_{d_1r_2}}\right) dy,\\ &= \frac{\overline{\gamma}_{d_1r_1}}{z\overline{\gamma}_{d_1r_2} + \overline{\gamma}_{d_1r_1}}.\end{aligned} \tag{A.3.2}$$

Therefore, the CDF is given by

$$\mathcal{F}_{\gamma_3}(z) = \frac{z\overline{\gamma}_{d_1r_2}}{z\overline{\gamma}_{d_1r_2} + \overline{\gamma}_{d_1r_1}}. \tag{A.3.3}$$

By differentiating the CDF, we get the PDF. It is given by

$$p_{\gamma_3}(z) = \frac{\overline{\gamma}_{d_1r_1}\overline{\gamma}_{d_1r_2}}{\left(z\overline{\gamma}_{d_1r_2} + \overline{\gamma}_{d_1r_1}\right)^2}. \tag{A.3.4}$$

We now determine $\mathcal{P}\left(\Psi_3 < z\right)$. We have

$$\begin{aligned}\mathcal{P}\left(\Psi_2 < \Psi_1 z\right) &= \int_0^\infty \mathcal{P}\left(\Psi_2 < sz\right) p_{\Psi_1}(s)\, ds,\\ &= \frac{\kappa}{\sqrt{2\pi\sigma_1{}^2}} \int_0^\infty \mathcal{P}\left(Y_2 < \kappa \ln(sz)\right) \frac{1}{s} \exp\left(-\frac{(\kappa \ln s - \mu_1)^2}{2\sigma_1{}^2}\right) ds,\\ &= \frac{\kappa}{\sqrt{8\pi\sigma_1{}^2}} \int_0^\infty \operatorname{erf}\left(\frac{\kappa \ln(sz) - \mu_2}{\sigma_2\sqrt{2}}\right) \frac{1}{s} \exp\left(-\frac{(\kappa \ln s - \mu_1)^2}{2\sigma_1{}^2}\right) ds.\end{aligned} \tag{A.3.5}$$

Substituting (A.3.5), and (A.3.4) in (A.3.1), we get the desired expression for relay selection probability.

A.4 Proof of Result 1: FASER and its Upper Bound for MPSK Scheme.

Using Craig's formula for MPSK [152], the symbol error contribution of R_1-D_2 link and R_2-D_2 link can be expressed as

$$\begin{aligned}\text{FASER}_{MPSK} &= w_1 \frac{1}{\pi} \int_0^{\left(\frac{M-1}{M}\right)\pi} \mathbf{E}\big[\exp\left(-\mathcal{M}\Gamma_1 \csc^2\theta\right) d\theta\big] \\ &\quad + w_2 \frac{1}{\pi} \int_0^{\left(\frac{M-1}{M}\right)\pi} \mathbf{E}\big[\exp\left(-\mathcal{M}\Gamma_2 \csc^2\theta\right) d\theta\big]. \quad \text{(A.4.1)}\end{aligned}$$

After averaging over channel fading, we get

$$\begin{aligned}\text{FASER}_{MPSK} &= w_1 \frac{1}{\pi} \int_0^{\left(\frac{M-1}{M}\right)\pi} \left(1 + \mathcal{M}\overline{\Gamma}_1 \csc^2\theta\right)^{-1} d\theta \\ &\quad + w_2 \frac{1}{\pi} \int_0^{\left(\frac{M-1}{M}\right)\pi} \left(1 + \mathcal{M}\overline{\Gamma}_2 \csc^2\theta\right)^{-1} d\theta. \quad \text{(A.4.2)}\end{aligned}$$

To simplify further, we use the following [33, (9)].

$$\begin{aligned}\frac{1}{\pi} \int_0^{\left(\frac{M-1}{M}\right)\pi} \left(1 + \mathcal{M}\overline{\gamma} \csc^2\psi\right)^{-1} d\psi &= \frac{M-1}{M} \\ &\quad - \frac{1}{\sqrt{1+\frac{1}{\mathcal{M}\overline{\gamma}}}} \left(\frac{1}{2} + \frac{1}{\pi}\arctan\left(\sqrt{\frac{1-\mathcal{M}}{\mathcal{M}+\frac{1}{\overline{\gamma}}}}\right)\right). \quad \text{(A.4.3)}\end{aligned}$$

Using the above definite integral, FASER can be expressed as

$$\text{FASER}_{MPSK} = w_1 p_{\mathcal{E}_1} + w_2 p_{\mathcal{E}_2}, \quad \text{(A.4.4)}$$

where

$$p_{\mathcal{E}_1} = 1 - \left(\frac{1}{M} + \frac{\left(\frac{1}{2} + \frac{1}{\pi}\arctan\left(\sqrt{\frac{1-\mathcal{M}}{\mathcal{M}+\frac{1}{\overline{\Gamma}_1}}}\right)\right)}{\sqrt{1+\frac{1}{\mathcal{M}\overline{\Gamma}_1}}}\right), \quad \text{(A.4.5)}$$

$$p_{\mathcal{E}_2} = 1 - \left(\frac{1}{M} + \frac{\left(\frac{1}{2} + \frac{1}{\pi} \arctan \left(\sqrt{\frac{1-\mathcal{M}}{\mathcal{M} + \frac{1}{\overline{\Gamma}_2}}} \right) \right)}{\sqrt{1 + \frac{1}{\mathcal{M}\overline{\Gamma}_2}}} \right). \quad \text{(A.4.6)}$$

Using $w_1 + w_2 = 1$, and simplifying further, we get the expression in (2.2.11).

Furthermore, using the inequality $\sin^2\theta \leq 1$, FASER can be upper bounded as

$$\text{FASER}_{MPSK} \leq w_1 \frac{1}{\pi} \int_0^{\left(\frac{M-1}{M}\right)\pi} \left(1 + \mathcal{M}\overline{\Gamma}_1\right)^{-1} d\theta + w_2 \frac{1}{\pi} \int_0^{\left(\frac{M-1}{M}\right)\pi} \left(1 + \mathcal{M}\overline{\Gamma}_2\right)^{-1} d\theta. \quad \text{(A.4.7)}$$

After simplifying, we get

$$\text{FASER}_{MPSK} \leq w_1 \frac{M-1}{M} \left(\frac{1}{1 + \mathcal{M}\overline{\Gamma}_1} \right) + w_2 \frac{M-1}{M} \left(\frac{1}{1 + \mathcal{M}\overline{\Gamma}_2} \right), \quad \text{(A.4.8)}$$

which is the desired upper bound.

A.5 Proof of Result 2: FASER and its Upper Bound for MQAM Scheme.

The symbol error contribution of R_1-D_2 link and R_2-D_2 link is equal to [153]

$$\begin{aligned} \text{FASER}_{MQAM} = \; & \\ & w_1 \frac{4m}{\pi} \int_0^{\frac{\pi}{2}} \mathbf{E}\left[\exp\left(-m'\Gamma_1 \csc^2\theta\right)\right] d\theta - w_1 \frac{4m^2}{\pi} \int_0^{\frac{\pi}{4}} \mathbf{E}\left[\exp\left(-m'\Gamma_1 \csc^2\theta\right)\right] d\theta \\ & + w_2 \frac{4m}{\pi} \int_0^{\frac{\pi}{2}} \mathbf{E}\left[\exp\left(-m'\Gamma_2 \csc^2\theta\right)\right] d\theta - w_2 \frac{4m^2}{\pi} \int_0^{\frac{\pi}{4}} \mathbf{E}\left[\exp\left(-m'\Gamma_2 \csc^2\theta\right)\right] d\theta, \end{aligned} \quad \text{(A.5.1)}$$

where $m \triangleq 1 - \frac{1}{\sqrt{M}}$ and $m' \triangleq \frac{3}{2(M-1)}$. After averaging over channel fading, we get

$$\begin{aligned} \text{FASER}_{MQAM} = \; & \\ & w_1 \frac{4m}{\pi} \int_0^{\frac{\pi}{2}} \left(1 + m'\overline{\Gamma}_1 \csc^2\theta\right)^{-1} d\theta - w_1 \frac{4m^2}{\pi} \int_0^{\frac{\pi}{4}} \left(1 + m'\overline{\Gamma}_1 \csc^2\theta\right)^{-1} d\theta \\ & + w_2 \frac{4m}{\pi} \int_0^{\frac{\pi}{2}} \left(1 + m'\overline{\Gamma}_2 \csc^2\theta\right)^{-1} d\theta - w_2 \frac{4m^2}{\pi} \int_0^{\frac{\pi}{4}} \left(1 + m'\overline{\Gamma}_2 \csc^2\theta\right)^{-1} d\theta. \end{aligned} \quad \text{(A.5.2)}$$

The above sum of integrals can be expressed as

$$\text{FASER}_{MQAM} = \frac{4m}{\pi}\int_0^{\frac{\pi}{2}} \left[w_1 \left(1 + m'\overline{\Gamma}_1 \csc^2\theta\right)^{-1} + w_2 \left(1 + m'\overline{\Gamma}_2 \csc^2\theta\right)^{-1} \right] d\theta$$
$$- \frac{4m^2}{\pi}\int_0^{\frac{\pi}{4}} \left[w_1 \left(1 + m'\overline{\Gamma}_1 \csc^2\theta\right)^{-1} + w_2 \left(1 + m'\overline{\Gamma}_2 \csc^2\theta\right)^{-1} \right] d\theta. \quad \text{(A.5.3)}$$

Let $\mathcal{I}_1$ denote the first integral and let $\mathcal{I}_2$ denote the second integral. To simplify further, we use the following indefinite integral.

$$\int \left(1 + k\csc^2 y\right)^{-1} dy = y - \sqrt{\frac{k}{k+1}} \arctan\left(\frac{\tan y}{\sqrt{\frac{k}{k+1}}}\right) + \mathcal{C}, \quad \text{(A.5.4)}$$

where $\mathcal{C}$ is some real constant.

Using the above integral and substituting the limits, we can show that $\mathcal{I}_1$ simplifies to the following form.

$$\mathcal{I}_1 = 2m\left(1 - \left(w_1\sqrt{\frac{m'\overline{\Gamma}_1}{1 + m'\overline{\Gamma}_1}} + w_2\sqrt{\frac{m'\overline{\Gamma}_2}{1 + m'\overline{\Gamma}_2}}\right)\right). \quad \text{(A.5.5)}$$

Furthermore, we can show that $\mathcal{I}_2$ simplifies to the following form.

$$\mathcal{I}_2 = m^2 - \frac{4m^2}{\pi}\left[w_1\sqrt{\frac{m'\overline{\Gamma}_1}{1 + m'\overline{\Gamma}_1}} \cot^{-1}\left(\sqrt{\frac{m'\overline{\Gamma}_1}{1 + m'\overline{\Gamma}_1}}\right) \right.$$
$$\left. + w_2\sqrt{\frac{m'\overline{\Gamma}_2}{1 + m'\overline{\Gamma}_2}} \cot^{-1}\left(\sqrt{\frac{m'\overline{\Gamma}_2}{1 + m'\overline{\Gamma}_2}}\right) \right]. \quad \text{(A.5.6)}$$

Finally, $\mathcal{I}_1 - \mathcal{I}_2$ yields the required result.

Using the inequality $\sin^2\theta \leq 1$, upper bound of $FASER_{MQAM}$ can be derived by using (A.5.2) as,

$$\text{FASER}_{MQAM} = w_1\frac{4m}{\pi}\int_0^{\frac{\pi}{2}} \left(1 + m'\overline{\Gamma}_1\right)^{-1} d\theta - w_1\frac{4m^2}{\pi}\int_0^{\frac{\pi}{4}} \left(1 + m'\overline{\Gamma}_1\right)^{-1} d\theta$$
$$+ w_2\frac{4m}{\pi}\int_0^{\frac{\pi}{2}} \left(1 + m'\overline{\Gamma}_2\right)^{-1} d\theta - w_2\frac{4m^2}{\pi}\int_0^{\frac{\pi}{4}} \left(1 + m'\overline{\Gamma}_2\right)^{-1} d\theta. \quad \text{(A.5.7)}$$

After simplifying, the upper bound for $FASER_{MQAM}$ is,

$$\text{FASER}_{MQAM} \leq (2m - m^2)\left[\frac{w_1}{(1 + m'\overline{\Gamma}_1)} + \frac{w_2}{(1 + m'\overline{\Gamma}_2)}\right]. \tag{A.5.8}$$

A.6 Proof of Result 3: FASE Analysis

The FASE can be expressed as

$$\overline{\mathcal{S}}_\eta = w_1 \mathbf{E}\big[\log_2(1 + \Gamma_1)\big] + w_2 \mathbf{E}\big[\log_2(1 + \Gamma_2)\big]. \tag{A.6.1}$$

Unfolding the expectations, we get

$$\overline{\mathcal{S}}_\eta = w_1 \left(\frac{1}{\overline{\Gamma}_1}\int_0^\infty \log_2(1 + \gamma_1)\, e^{-\frac{\gamma_1}{\overline{\Gamma}_1}}\, d\gamma_1\right) + w_2 \left(\frac{1}{\overline{\Gamma}_2}\int_0^\infty \log_2(1 + \gamma_2)\, e^{-\frac{\gamma_2}{\overline{\Gamma}_2}}\, d\gamma_2\right). \tag{A.6.2}$$

To simplify further, we use the following [35].

$$\int_0^\infty \ln(1 + \gamma)\, e^{-\frac{\gamma}{\overline{\Gamma}}}\, d\gamma = \overline{\Gamma} \exp\left(\frac{1}{\overline{\Gamma}}\right) \text{Ei}\left(\frac{1}{\overline{\Gamma}}\right), \tag{A.6.3}$$

where $\text{Ei}(\cdot)$ is the exponential integral [154, (5.1.1)]. Using the above integral and simplifying further, we get the desired result.

Lower bound and upper bound: To derive the bounds, we use the following inequality [155].

$$\frac{1}{2}\ln\left(1 + \frac{2}{y}\right) \leq e^y \text{Ei}(y) \leq \ln\left(1 + \frac{1}{y}\right), y > 0. \tag{A.6.4}$$

Applying the above inequality in the exact FASE expression, we can obtain the lower and upper bounds. Furthermore, the upper bound can also be obtained using Jensen's inequality.

A.7 Proof of Result 4: Diversity Order Analysis

The analytical expression for exact FASER in (2.2.11) is complex. Therefore, in order to derive the diversity order, we use its upper bound, and lower bound. We first prove that the diversity order for MPSK upper bound is one in the scaling regime, in which the average SNRs $\overline{\Gamma}_1$, and $\overline{\Gamma}_2$, are very high and $\overline{\Gamma}_1 \approx \overline{\Gamma}_2$.

Diversity order of upper bound: In the scaling regime, the upper bound of FASER for MPSK is given by (2.2.12)

$$\text{FASER}_{\text{UB-MPSK}} = \frac{M-1}{M}\left(\frac{w_1}{1+\mathcal{M}\overline{\Gamma}_1} + \frac{w_2}{1+\mathcal{M}\overline{\Gamma}_2}\right) \approx \frac{M-1}{M}\left(\frac{w_1+w_2}{1+\mathcal{M}\overline{\Gamma}_1}\right).$$

It can be observed that $\text{FASER}_{\text{UB-MPSK}} \propto \frac{1}{\overline{\Gamma}_1}$. Thus, it can be stated that the diversity order of the FASER upper bound is one.

Diversity order of Lower bound: To derive an insightful lower bound on exact FASER, we use the following inequality. For $y > 0$, $\tan^{-1} y < y$. Using the above the inequality, the FASER can be lower bounded as

$$\begin{aligned}\text{FASER}_{\text{MPSK}} \geq \left(\frac{M-1}{M}\right) - \frac{1}{2}\left(\frac{w_1}{\sqrt{1+\frac{1}{\mathcal{M}\overline{\Gamma}_1}}} + \frac{w_2}{\sqrt{1+\frac{1}{\mathcal{M}\overline{\Gamma}_2}}}\right) \\ - \left(\frac{w_1}{\pi}\sqrt{\frac{1-\mathcal{M}}{\mathcal{M}}} + \frac{w_2}{\pi}\sqrt{\frac{1-\mathcal{M}}{\mathcal{M}}}\right) \triangleq \text{FASER}_{\text{LB-MPSK}}.\end{aligned} \quad \text{(A.7.1)}$$

For the scaling regime, using the fact that $w_2 + w_1 = 1$, we have

$$\text{FASER}_{\text{MPSK}} \geq \text{FASER}_{\text{LB-MPSK}} \approx \left(\frac{M-1}{M}\right) - \frac{1}{2}\left(\frac{1}{\sqrt{1+\frac{1}{\mathcal{M}\overline{\Gamma}_2}}} + \frac{1}{\pi}\sqrt{\frac{1-\mathcal{M}}{\mathcal{M}}}\right). \quad \text{(A.7.2)}$$

Let $M = 2$. We have $\mathcal{M} = \sin^2\frac{\pi}{M} = 1$. Further simplification yields the following.

$$\text{FASER}_{\text{LB-MPSK}} \approx \frac{1}{4\overline{\Gamma}_1}. \quad \text{(A.7.3)}$$

Therefore, the diversity order of the FASER lower bound is equal to 1.

Since the lower bound, and, the upper bound, both achieves diversity order of 1, d_{PRSP} is equal to one.

A.8 FASER in High SNR Regime for PRSP

In the scaling regime, assuming $\overline{\gamma}_1 = \overline{\gamma}_2 = \overline{\gamma}$, for equation (A.4.2). Furthermore in scaling regime we assume $\tilde{P}_r \to \infty$, hence we can neglect 1 in the denominator of both the terms of equation (A.4.2). The FASER expression for the scaling regime for

MPSK is given by

$$\begin{aligned}\text{FASER}^{\text{H}}_{\text{PRSP}} &= w_1 \, \frac{1}{\pi} \int_0^{\left(\frac{M-1}{M}\right)\pi} \frac{1}{\mathcal{M} \, \frac{\tilde{P}_r \, \bar{\gamma}}{\sigma_n^2} \, \csc^2\theta} \, d\theta + w_2 \, \frac{1}{\pi} \int_0^{\left(\frac{M-1}{M}\right)\pi} \frac{1}{\mathcal{M} \, \frac{\tilde{P}_r \, \bar{\gamma}}{\sigma_n^2} \, \csc^2\theta} \, d\theta, \\ &= \frac{w_1 + w_2}{\pi} \int_0^{\left(\frac{M-1}{M}\right)\pi} \frac{1}{\mathcal{M} \, \frac{\tilde{P}_r \, \bar{\gamma}}{\sigma_n^2} \, \csc^2\theta} \, d\theta.\end{aligned}$$

Since $w_1 + w_2 = 1$, we get

$$\text{FASER}^{\text{H}}_{\text{PRSP}} = \left[\frac{1}{\pi \, \mathcal{M} \frac{\tilde{P}_r \, \bar{\gamma}}{\sigma_n^2}}\right] \int_0^{\left(\frac{M-1}{M}\right)\pi} \sin^2\theta \, d\theta.$$

Let $K = \int_0^{\left(\frac{M-1}{M}\right)\pi} \sin^2\theta \, d\theta$. Therefore,

$$\text{FASER}^{\text{H}}_{\text{PRSP}} = \left[\frac{K}{\pi \, \mathcal{M}}\right] \left[\frac{1}{\overline{\overline{\Gamma}}}\right],$$

where $\overline{\Gamma}$ is the mean received SNR.

A.9 FASER in High SNR Regime for ORSP

To derive the expression for FASER for the system model considered, first, we determine the probability density function of the instantaneous SNR at the destination. Therefore, the end SNR at the destination node is given by

$$\Gamma_E = \frac{\tilde{P}_r \max\{\gamma_A, \gamma_B\}}{\sigma_n^2} = \frac{\tilde{P}_r Z}{\sigma_n^2},$$

where $\gamma_A = \min\{\gamma_{d_1 r_1}, \gamma_1\}$, $\gamma_B = \min\{\gamma_{d_1 r_2}, \gamma_2\}$, and $\tilde{P}_r$ denotes the power received at destination.

Using the fact that the instantaneous channel power gains are statistically independent, the complementary cumulative distribution function of γ_A is given by

$$F^c_{\gamma_A}(\gamma) = \mathcal{P}(\min\{\gamma_{d_1 r_1}, \gamma_1\} > \gamma), \tag{A.9.1}$$

$$= e^{-\left(\frac{1}{\overline{\gamma}_{d_1 r_1}} + \frac{1}{\overline{\gamma}_1}\right)\gamma}, \gamma \geq 0. \tag{A.9.2}$$

Therefore, the CCDF $F^c_{\gamma_A}(\gamma)$ is given by

$$F^c_{\gamma_A}(\gamma) = e^{-\left(\frac{1}{\overline{\gamma}_{d_1r_1}}+\frac{1}{\overline{\gamma}_1}\right)\gamma}. \tag{A.9.3}$$

Similarly, the CCDF of γ_B is given by

$$F^c_{\gamma_B}(\gamma) = e^{-\left(\frac{1}{\overline{\gamma}_{d_1r_2}}+\frac{1}{\overline{\gamma}_2}\right)\gamma}, \gamma \geq 0.$$

Now, we determine the PDF of Z. Using statistical independence, we have

$$\begin{aligned} F_Z(\gamma) &= \mathcal{P}(\max\{\gamma_A, \gamma_B\} \leq \gamma) \\ &= (1 - e^{-\overline{\gamma}_{1eff}\ \gamma})\ (1 - e^{-\overline{\gamma}_{2eff}\ \gamma}). \end{aligned} \tag{A.9.4}$$

where $\overline{\gamma}_{1eff} = \frac{1}{\overline{\gamma}_{d_1r_1}} + \frac{1}{\overline{\gamma}_1}$ and $\overline{\gamma}_{2eff} = \frac{1}{\overline{\gamma}_{d_1r_2}} + \frac{1}{\overline{\gamma}_2}$.

From the above equation, we can determine the probability density function. The pdf is given by

$$p_z(\gamma) = \overline{\gamma}_{1eff}\ e^{-\overline{\gamma}_{1eff}\ \gamma} + \overline{\gamma}_{2eff}\ e^{-\overline{\gamma}_{2eff}\ \gamma} - (\overline{\gamma}_{1eff} + \overline{\gamma}_{2eff})\ e^{-(\overline{\gamma}_{1eff}+\overline{\gamma}_{2eff})\ \gamma}.$$

Therefore, the FASER for the ORSP is given by

$$\begin{aligned} \text{FASER}_{\text{ORSP}} &= \mathbf{E}\left[\frac{1}{\pi}\int_0^{\left(\frac{M-1}{M}\right)\pi} \exp\left(-\mathcal{M}\Gamma_E \csc^2\theta\right)\, d\theta\right], \\ &= \mathbf{E}\left[\frac{1}{\pi}\int_0^{\left(\frac{M-1}{M}\right)\pi} \exp\left(-\mathcal{M}\frac{\tilde{P}_r Z}{\sigma_n^2}\csc^2\theta\right)\, d\theta\right], \end{aligned} \tag{A.9.5}$$

where $\mathcal{M} = \sin^2\left(\frac{\pi}{M}\right)$, and M is the modulation order. Furthermore, the average value of Z is given by

$$\mathbf{E}[Z] = \overline{Z} = \frac{1}{\overline{\gamma}_{1eff}} + \frac{1}{\overline{\gamma}_{2eff}} - \frac{1}{\overline{\gamma}_{1eff} + \overline{\gamma}_{2eff}}. \tag{A.9.6}$$

Therefore, solving (A.9.5), we get

$$\text{FASER}_{\text{ORSP}} = \frac{1}{\pi}\int_0^{\left(\frac{M-1}{M}\right)\pi} \frac{1}{1 + \mathcal{M}\ \frac{\tilde{P}_r\ \overline{Z}}{\sigma_n^2}\ \csc^2\theta}\, d\theta \tag{A.9.7}$$

Further, to derive FASER for the scaling regime, we assume $\tilde{P}_r \to \infty$, therefore we can neglect 1 in the denominator terms of above equation (A.9.7). We also assume, $\overline{\gamma}_{d_1r_1} = \overline{\gamma}_{d_1r_2} = \overline{\gamma}_1 = \overline{\gamma}_2 = \overline{\gamma}$. We get $\overline{\gamma}_{1eff} = \frac{2}{\overline{\gamma}}$ and $\overline{\gamma}_{2eff} = \frac{2}{\overline{\gamma}}$. It can be easily shown that $\overline{Z} = \frac{3\overline{\gamma}}{4}$.

In the scaling regime, the FASER is given by

$$\begin{aligned}
\text{FASER}^{\text{H}}_{\text{ORSP}} &= \left[\frac{1}{\pi \mathcal{M}\ \frac{\tilde{P}_r}{\sigma_n^2}}\right] \left[\frac{4}{3\overline{\gamma}}\right] \int_0^{\left(\frac{M-1}{M}\right)\pi} \sin^2\theta\ d\theta, \\
&= \left[\frac{4K}{3\ \pi\ \mathcal{M}}\right] \left[\frac{1}{\overline{\Gamma}^1}\right],
\end{aligned}$$

where $K = \int_0^{\left(\frac{M-1}{M}\right)\pi} \sin^2\theta\ d\theta$ and $\overline{\Gamma}$ is the average received SNR.

Appendix B

Chapter 3: Derivations and Proofs

B.1 Proof of Result 5: FASER Optimal Relaying Policy for MPSK

Consider the objective function in (3.2.4), which is convex in $\mathcal{G}_{\beta-\text{MPSK}}$. It is easy to verify that the constrained optimization problem is convex in $\mathcal{G}_{\beta-\text{MPSK}}$. To solve this FASER minimization problem, we can use Lagrange multiplier method.

Let $L_K(\mathcal{G}_{\beta-\text{MPSK}})$ denote the Lagrangian function given by

$$L_K(\mathcal{G}_{\beta-\text{MPSK}}) \triangleq \mathbf{E}\left(w_1 e^{-\mathcal{M}\Gamma_1} + w_2 e^{-\mathcal{M}\Gamma_2} + K\left(\mathcal{G}_{\beta-\text{MPSK}}\tilde{P}_r - P_{\text{th}}\right)\right). \qquad \text{(B.1.1)}$$

Now, equating the values of Γ_1 and Γ_2 from equation (3.2.1) in equation (B.1.1), we get

$$L_K(\mathcal{G}_{\beta-\text{MPSK}}) = \mathbf{E}\Bigg[w_1 \exp\left(-\frac{\mathcal{M}\mathcal{G}_{\beta-\text{MPSK}}\tilde{P}_r\gamma_1}{\sigma_n^2}\right) + w_2 \exp\left(-\frac{\mathcal{M}\mathcal{G}_{\beta-\text{MPSK}}\tilde{P}_r\gamma_2}{\sigma_n^2}\right) + K\left(\mathcal{G}_{\beta-\text{MPSK}}\tilde{P}_r - P_{\text{th}}\right)\Bigg]. \qquad \text{(B.1.2)}$$

In equation (B.1.2), by virtue of convexity, we can drop the expectation. Further, to obtain the optimum RGF $\mathcal{G}^*_{\beta-\text{MPSK}}$, we differentiate (B.1.2) with respect to $\mathcal{G}_{\beta-\text{MPSK}}$ and equate it to zero. By doing this, we get

$$w_1 \exp\left(-\frac{\mathcal{M}\mathcal{G}^*_{\beta-\text{MPSK}}\tilde{P}_r\gamma_1}{\sigma_n^2}\right)\left(\frac{\mathcal{M}\tilde{P}_r\gamma_1}{\sigma_n^2}\right) + w_2 \exp\left(-\frac{\mathcal{M}\mathcal{G}^*_{\beta-\text{MPSK}}\tilde{P}_r\gamma_2}{\sigma_n^2}\right)\left(\frac{\mathcal{M}\tilde{P}_r\gamma_2}{\sigma_n^2}\right) = K\tilde{P}_r. \qquad \text{(B.1.3)}$$

Let $c \triangleq \frac{\mathcal{M}\tilde{P}_r}{\sigma_n^2}$. Further simplification of (B.1.3) yields

$$w_1\gamma_1\, c\, \exp\left(-\, c\gamma_1\mathcal{G}^*_{\beta-\text{MPSK}}\right) + w_2\gamma_2\, c\, \exp\left(-\, c\gamma_2\mathcal{G}^*_{\beta-\text{MPSK}}\right) = K\tilde{P}_r. \qquad \text{(B.1.4)}$$

Furthermore, it is easy to verify that $\mathcal{G}^*_{\beta-\text{MPSK}} = 0$ when $w_1\gamma_1\ c + w_2\gamma_2\ c\ <\ K\tilde{P}_r$, which is the power conservation rule for the FASER optimal policy for MPSK.

B.2 Proof of Result 6: Exact FASER and its Upper Bound for MPSK

FASER optimal policy for MPSK provides optimal relay gain $\mathcal{G}^*_{\beta-\text{MPSK}}$. After unfolding the expectation, the exact expression for FASER for MPSK in terms of $\mathcal{G}^*_{\beta-\text{MPSK}}$ is given by

$$\begin{aligned}
\text{FASER}_{\text{MPSK}} = \frac{w_1}{\pi\overline{\gamma}_1\overline{\gamma}_2} \times \Bigg[&\int_{\theta=0}^{\left(\frac{M-1}{M}\right)\pi} \int_{\gamma_1=0}^{\infty} \int_{\gamma_2=0}^{\rho(\gamma_1)} e^{-\mathcal{W}\gamma_1} e^{\frac{-\gamma_1}{\overline{\gamma}_1}}\ e^{\frac{-\gamma_2}{\overline{\gamma}_2}}\ d\gamma_2\ d\gamma_1\ d\theta \\
&+ \int_{\theta=0}^{\left(\frac{M-1}{M}\right)\pi} \int_{\gamma_1=0}^{\infty} \int_{\gamma_2=\rho(\gamma_1)}^{\infty} e^{-\mathcal{W}\gamma_1} e^{\frac{-\gamma_1}{\overline{\gamma}_1}}\ e^{\frac{-\gamma_2}{\overline{\gamma}_2}}\ d\gamma_2\ d\gamma_1\ d\theta \Bigg] \\
+ \frac{w_2}{\pi\overline{\gamma}_1\overline{\gamma}_2} \Bigg[&\int_{\theta=0}^{\left(\frac{M-1}{M}\right)\pi} \int_{\gamma_1=0}^{\infty} \int_{\gamma_2=0}^{\rho(\gamma_1)} e^{-\mathcal{W}\gamma_2} e^{\frac{-\gamma_1}{\overline{\gamma}_1}}\ e^{\frac{-\gamma_2}{\overline{\gamma}_2}}\ d\gamma_2\ d\gamma_1\ d\theta \\
&+ \int_{\theta=0}^{\left(\frac{M-1}{M}\right)\pi} \int_{\gamma_1=0}^{\infty} \int_{\gamma_2=\rho(\gamma_1)}^{\infty} e^{-\mathcal{W}\gamma_2} e^{\frac{-\gamma_1}{\overline{\gamma}_1}}\ e^{\frac{-\gamma_2}{\overline{\gamma}_2}}\ d\gamma_2\ d\gamma_1\ d\theta \Bigg], \quad \text{(B.2.1)}
\end{aligned}$$

where $\mathcal{W} = c\ \mathcal{G}^*_{\beta-\text{MPSK}} \csc^2\theta$. From equation (3.2.7), we have $\mathcal{G}^*_{\beta-\text{MPSK}}$ is zero for $\gamma_2 < \rho(\gamma_1)$. Therefore, after simplification we get the expression for $\text{FASER}_{\text{MPSK}}$, which is given in (3.2.8).

Further, to obtain the upper bound of the FASER, we use $\sin^2\theta \leq 1$. Therefore, substituting $\sin^2\theta = 1$ in equation (3.2.8), we get

$$\begin{aligned}
&\text{FASER}_{\text{UB-MPSK}} = \\
&\frac{w_1}{\pi\overline{\gamma}_1\overline{\gamma}_2} \left[\frac{\pi\overline{\gamma}_1\overline{\gamma}_2(M-1)}{M}\mathbb{R} + \int_{\theta=0}^{\left(\frac{M-1}{M}\right)\pi} \int_{\gamma_1=0}^{\infty} \int_{\gamma_2=\rho(\gamma_1)}^{\infty} e^{-\mathbb{W}\gamma_1} e^{-\frac{\gamma_1}{\overline{\gamma}_1}}\ e^{-\frac{\gamma_2}{\overline{\gamma}_2}}\ d\gamma_2\ d\gamma_1\ d\theta \right] \\
&+ \frac{w_2}{\pi\overline{\gamma}_1\overline{\gamma}_2} \left[\frac{\pi\overline{\gamma}_1\overline{\gamma}_2(M-1)}{M}\mathbb{R} + \int_{\theta=0}^{\left(\frac{M-1}{M}\right)\pi} \int_{\gamma_1=0}^{\infty} \int_{\gamma_2=\rho(\gamma_1)}^{\infty} e^{-\mathbb{W}\gamma_2} e^{-\frac{\gamma_1}{\overline{\gamma}_1}}\ e^{-\frac{\gamma_2}{\overline{\gamma}_2}}\ d\gamma_2\ d\gamma_1\ d\theta \right],
\end{aligned} \quad \text{(B.2.2)}$$

where $\mathbb{W} = c\mathcal{G}^*_{\beta-\text{MPSK}}$. Further simplification of the above expressions gives us equation (3.2.9).

B.3 Proof for Optimality of FASER MPSK

To prove the optimality of FASER [36], let $f(x) \triangleq w_1 \exp(-\frac{\mathcal{M}\tilde{P}_r\gamma_1 x}{\sigma_n^2}) + w_2 \exp(-\frac{\mathcal{M}\tilde{P}_r\gamma_2 x}{\sigma_n^2})$ for $x \geq 0$. Further, for the policy whose relay gain is the function of $\mathcal{G}_{\beta-\text{MPSK}}$, we define the unconstrained Lagrangian function $L_{\mathcal{G}_{\beta-\text{MPSK}}}(\tilde{K})$ as

$$L_{\mathcal{G}_{\beta-\text{MPSK}}}(\tilde{K}) = \mathbf{E}[f(\mathcal{G}_{\beta-\text{MPSK}})] + \tilde{K}\ \mathbf{E}[\mathcal{G}_{\beta-\text{MPSK}}\tilde{P}_r], \tag{B.3.1}$$

for all $\tilde{K} \geq 0$. Further, consider a policy $\mathcal{G}_{\beta-\text{MPSK}}$ that specify the relay gain as

$$\tilde{\mathcal{G}}_{\beta-\text{MPSK}} = \arg\min_{x\geq 0}\{f(x) + \tilde{K}(x\tilde{P}_r)\}. \tag{B.3.2}$$

Note that $\tilde{\mathcal{G}}_{\beta-\text{MPSK}}$ is the function of channel power gain of γ_1 and γ_2. Further, for the above policy, let $K > 0$ denote the value of $\tilde{K}$ at which the above policy meets the power constraint in equation (3.2.5) with equality. Therefore, for this specific K value, let $\mathcal{G}^*_{\beta-\text{MPSK}}$ denote optimal relay gain function. Hence from equation (B.3.2) it can de deduced that $L_{\mathcal{G}^*_{\beta-\text{MPSK}}}(K) \leq L_{\mathcal{G}_{\beta-\text{MPSK}}}(\tilde{K})$. Substituting this in equation (B.3.1) and re-arranging. we get

$$\mathbf{E}[f(\mathcal{G}^*_{\beta-\text{MPSK}})] \leq \mathbf{E}[f(\mathcal{G}_{\beta-\text{MPSK}})] + \tilde{K}\ \mathbf{E}[\mathcal{G}_{\beta-\text{MPSK}}\tilde{P}_r - P_{\text{th}}]. \tag{B.3.3}$$

Further, we know that $\mathbf{E}[\mathcal{G}_{\beta-\text{MPSK}}\tilde{P}_r] \leq P_{\text{th}}$. Therefore, we can write equation (B.3.3) as

$$\mathbf{E}[f(\mathcal{G}^*_{\beta-\text{MPSK}})] < \mathbf{E}[f(\mathcal{G}_{\beta-\text{MPSK}})] \tag{B.3.4}$$

Therefore, $\mathcal{G}^*_{\beta-\text{MPSK}}$ satisfies the average relay transmit power constraint with equality and has the lowest possible FASER among all the feasible policies. Hence it is optimal.

B.4 Proof of Result 7: FASER Optimal Relaying Policy for MQAM

To derive FASER optimal policy for MQAM, we use the approach similar to that of MPSK. The objective function is convex. Therefore, we form the Lagrangian function, which given by

$$L_{K'}(\mathcal{G}_{\beta-\text{MQAM}}) \triangleq \mathbf{E}\left(w_1 e^{-m'\Gamma_1} + w_2 e^{-m'\Gamma_2} + K'\left(\mathcal{G}_{\beta-\text{MQAM}}\tilde{P}_r - P_{\text{th}}\right)\right). \tag{B.4.1}$$

Substituting Γ_1 and Γ_2 from equation (3.2.1) in equation (B.4.1), we get

$$L_{K'}\left(\mathcal{G}_{\beta-\text{MQAM}}\right) = \mathbf{E}\Bigg[w_1 \exp\left(-m^{'}\Gamma_{1,\mathcal{G}_{\beta-\text{MQAM}}}\right) + w_2 \exp\left(-m^{'}\Gamma_{2,\mathcal{G}_{\beta-\text{MQAM}}}\right) + K'\left(\mathcal{G}_{\beta-\text{MQAM}}\tilde{P}_r - P_{\text{th}}\right)\Bigg], \quad \text{(B.4.2)}$$

where $\Gamma_{1,\mathcal{G}_{\beta-\text{MQAM}}} = \frac{\mathcal{G}_{\beta-\text{MQAM}}\tilde{P}_r\gamma_1}{\sigma_n^2}$ and $\Gamma_{2,\mathcal{G}_{\beta-\text{MQAM}}} = \frac{\mathcal{G}_{\beta-\text{MQAM}}\tilde{P}_r\gamma_2}{\sigma_n^2}$.

By virtue of convexity, dropping the expectation in equation (B.4.2), differentiating it with respect to $\mathcal{G}_{\beta-\text{MQAM}}$ and at last equating the differentiated expression to zero, we get

$$w_1 \exp\left(-m^{'}\Gamma_{1,\mathcal{G}_{\beta-\text{MQAM}}}\right)\left(\frac{m^{'}\tilde{P}_r\gamma_1}{\sigma_n^2}\right) + w_2 \exp\left(-m^{'}\Gamma_{2,\mathcal{G}_{\beta-\text{MQAM}}}\right)\left(\frac{m^{'}\tilde{P}_r\gamma_2}{\sigma_n^2}\right) = K'\tilde{P}_r. \quad \text{(B.4.3)}$$

Let $c' \triangleq \frac{m^{'}\tilde{P}_r}{\sigma_n^2}$. We can express equation (B.4.3) as

$$w_1\gamma_1\, c' \exp\left(-\, c'\gamma_1\mathcal{G}^*_{\beta-\text{MQAM}}\right) + w_2\gamma_2\, c' \exp\left(-\, c'\gamma_2\mathcal{G}^*_{\beta-\text{MQAM}}\right) = K'\tilde{P}_r. \quad \text{(B.4.4)}$$

Furthermore, it is easy to verify that $\mathcal{G}^*_{\beta-\text{MQAM}} = 0$ when $w_1\gamma_1\, c' + w_2\gamma_2\, c' < K'\tilde{P}_r$, which is the power conservation rule for the FASER optimal policy for MQAM.

B.5 Proof of Result 8: Exact FASER and its Upper Bound (MQAM)

FASER optimal policy for MQAM provides optimal relay gain $\mathcal{G}^*_{\beta-\text{MQAM}}$. After unfolding the expectation, the exact expression for FASER for MQAM in terms of

$\mathcal{G}^*_{\beta-\text{MQAM}}$ is given by

$$
\begin{aligned}
\text{FASER}_{\text{MQAM}} = \; & \frac{4m\; w_1}{\pi\overline{\gamma}_1\overline{\gamma}_2}\Bigg[\int_{\theta=0}^{\frac{\pi}{2}}\int_{\gamma_1=0}^{\infty}\int_{\gamma_2=0}^{\rho'(\gamma_1)} \; e^{-\mathcal{Y}\gamma_1}e^{-\frac{\gamma_1}{\overline{\gamma}_1}} \; e^{-\frac{\gamma_2}{\overline{\gamma}_2}}\, d\gamma_2\, d\gamma_1\, d\theta \\
& + \int_{\theta=0}^{\frac{\pi}{2}}\int_{\gamma_1=0}^{\infty}\int_{\gamma_2=\rho'(\gamma_1)}^{\infty} \; e^{-\mathcal{Y}\gamma_1}e^{-\frac{\gamma_1}{\overline{\gamma}_1}} \; e^{-\frac{\gamma_2}{\overline{\gamma}_2}}\, d\gamma_2\, d\gamma_1\, d\theta\Bigg] \\
& - \frac{4m^2\; w_1}{\pi\overline{\gamma}_1\overline{\gamma}_2}\Bigg[\int_{\theta=0}^{\frac{\pi}{2}}\int_{\gamma_1=0}^{\infty}\int_{\gamma_2=0}^{\rho'(\gamma_1)} \; e^{-\mathcal{Y}\gamma_1}e^{-\frac{\gamma_1}{\overline{\gamma}_1}} \; e^{-\frac{\gamma_2}{\overline{\gamma}_2}}\, d\gamma_2\, d\gamma_1\, d\theta \\
& + \int_{\theta=0}^{\frac{\pi}{4}}\int_{\gamma_1=0}^{\infty}\int_{\gamma_2=\rho'(\gamma_1)}^{\infty} \; e^{-\mathcal{Y}\gamma_1}e^{-\frac{\gamma_1}{\overline{\gamma}_1}} \; e^{-\frac{\gamma_2}{\overline{\gamma}_2}}\; d\gamma_2\; d\gamma_1\; d\theta\;\Bigg] \\
& + \frac{4m\; w_2}{\pi\overline{\gamma}_1\overline{\gamma}_2}\Bigg[\int_{\theta=0}^{\frac{\pi}{2}}\int_{\gamma_1=0}^{\infty}\int_{\gamma_2=0}^{\rho'(\gamma_1)} \; e^{-\mathcal{Y}\gamma_2}e^{-\frac{\gamma_1}{\overline{\gamma}_1}} \; e^{-\frac{\gamma_2}{\overline{\gamma}_2}}\, d\gamma_2\, d\gamma_1\, d\theta \\
& + \int_{\theta=0}^{\frac{\pi}{2}}\int_{\gamma_1=0}^{\infty}\int_{\gamma_2=\rho'(\gamma_1)}^{\infty} \; e^{-\mathcal{Y}\gamma_2}e^{-\frac{\gamma_1}{\overline{\gamma}_1}} \; e^{-\frac{\gamma_2}{\overline{\gamma}_2}}\, d\gamma_2\, d\gamma_1\, d\theta\Bigg] \\
& - \frac{4m^2\; w_2}{\pi\overline{\gamma}_1\overline{\gamma}_2}\Bigg[\int_{\theta=0}^{\frac{\pi}{2}}\int_{\gamma_1=0}^{\infty}\int_{\gamma_2=0}^{\rho'(\gamma_1)} \; e^{-\mathcal{Y}\gamma_2}e^{-\frac{\gamma_1}{\overline{\gamma}_1}} \; e^{-\frac{\gamma_2}{\overline{\gamma}_2}}\, d\gamma_2\, d\gamma_1\, d\theta \\
& + \int_{\theta=0}^{\frac{\pi}{4}}\int_{\gamma_1=0}^{\infty}\int_{\gamma_2=\rho'(\gamma_1)}^{\infty} \; e^{-\mathcal{Y}\gamma_2}e^{-\frac{\gamma_1}{\overline{\gamma}_1}} \; e^{-\frac{\gamma_2}{\overline{\gamma}_2}}\; d\gamma_2\; d\gamma_1\; d\theta\;\Bigg], \qquad \text{(B.5.1)}
\end{aligned}
$$

where $m' = \frac{3}{2(M-1)}$ and $\mathcal{Y} = c'\; \mathcal{G}^*_{\beta-\text{MQAM}} \csc^2\theta$.

From equation (3.2.19), we have $\mathcal{G}^*_{\beta-\text{MQAM}} = 0$ for $\gamma_2 < \rho(\gamma_1)$. On further simplification, we get equation (3.2.20).

Further, to obtain the upper bound for FASER, we use the inequality $\sin^2\theta \leq 1$. Substituting $\sin^2\theta = 1$ in equation (3.2.20), we get

$$
\begin{aligned}
\text{FASER}_{\text{UB-MQAM}} = \; & \frac{4m^2}{\pi\overline{\gamma}_1\overline{\gamma}_2}\Bigg[\frac{w_1}{m}\Bigg[\mathcal{F}+\int_{\theta=0}^{\frac{\pi}{2}}\int_{\gamma_1=0}^{\infty}\int_{\gamma_2=\rho'(\gamma_1)}^{\infty} e^{-\mathbb{Y}\gamma_1}e^{-\frac{\gamma_1}{\overline{\gamma}_1}}e^{-\frac{\gamma_2}{\overline{\gamma}_2}}d\gamma_2\; d\gamma_1 d\theta\Bigg] \\
& -w_1\Bigg[\frac{\mathcal{F}}{2}+\int_{\theta=0}^{\frac{\pi}{4}}\int_{\gamma_1=0}^{\infty}\int_{\gamma_2=\rho'(\gamma_1)}^{\infty} e^{-\mathbb{Y}\gamma_1}e^{-\frac{\gamma_1}{\overline{\gamma}_1}}e^{-\frac{\gamma_2}{\overline{\gamma}_2}}d\gamma_2 d\gamma_1 d\theta\Bigg] \\
& +\frac{w_2}{m}\Bigg[\mathcal{F}+\int_{\theta=0}^{\frac{\pi}{2}}\int_{\gamma_1=0}^{\infty}\int_{\gamma_2=\rho'(\gamma_1)}^{\infty} e^{-\mathbb{Y}\gamma_2}e^{-\frac{\gamma_1}{\overline{\gamma}_1}}e^{-\frac{\gamma_2}{\overline{\gamma}_2}}d\gamma_2 d\gamma_1 d\theta\Bigg] \\
& -w_2\Bigg[\frac{\mathcal{F}}{2}+\int_{\theta=0}^{\frac{\pi}{4}}\int_{\gamma_1=0}^{\infty}\int_{\gamma_2=\rho'(\gamma_1)}^{\infty} e^{-\mathbb{Y}\gamma_2}e^{-\frac{\gamma_1}{\overline{\gamma}_1}}e^{-\frac{\gamma_2}{\overline{\gamma}_2}}d\gamma_2 d\gamma_1 d\theta\Bigg]\Bigg], \qquad \text{(B.5.2)}
\end{aligned}
$$

where $\mathbb{Y} = c'\; \mathcal{G}^*_{\beta-\text{MQAM}}$. On further simplification, we get the desired upper bound expression.

B.6 Proof of Result 9: FASE Optimal Relaying Policy

As the optimization problem stated for FASE is concave, this can be re-formulated as the following convex problem.

$$\min_{\mathcal{G}_\eta(\gamma_1,\gamma_2)} -\mathbf{E}\left[w_1 \log_2\left(1+\Gamma_1\right) + w_2 \log_2\left(1+\Gamma_2\right)\right], \tag{B.6.1}$$

$$\text{s.t. } \mathbf{E}\left[\mathcal{G}_\eta \tilde{P}_r\right] \leq P_{\text{th}}. \tag{B.6.2}$$

Clearly, the objective function in the above minimization problem is convex, we can again use Lagrange multiplier method to determine the optimal solution. The Lagrange function for the above optimization problem can be written as

$$L_{\mathcal{T}}\left(\mathcal{G}_\eta\right) = \mathbf{E}[-(\ w_1 \log_2(1+\Gamma_1) + w_2 \log_2(1+\Gamma_2)\ ) + \mathcal{T}(\mathcal{G}_\eta \tilde{P}_r - P_{\text{th}})], \tag{B.6.3}$$

where $\mathcal{T} > 0$. Substituting the value of Γ_1 and Γ_2 in equation (B.6.3), we get

$$L_{\mathcal{T}}\left(\mathcal{G}_\eta\right) = \mathbf{E}\left[-\left(\frac{w_1}{\ln(2)}\ln\left(1+\frac{\mathcal{G}_\eta \tilde{P}_r \gamma_1}{\sigma_n^2}\right)+\frac{w_2}{\ln(2)}\ln\left(1+\frac{\mathcal{G}_\eta \tilde{P}_r \gamma_2}{\sigma_n^2}\right)\right)+\mathcal{T}(\mathcal{G}_\eta \tilde{P}_r - P_{\text{th}})\right]. \tag{B.6.4}$$

Dropping the expectation and equating the derivative of the expression inside the expectation to zero, we get

$$\frac{w_1 \tilde{P}_r \gamma_1(\sigma_n^2 + \mathcal{G}_\eta \tilde{P}_r \gamma_2) + w_2 \tilde{P}_r \gamma_2(\sigma_n^2 + \mathcal{G}_\eta \tilde{P}_r \gamma_1)}{(\sigma_n^2 + \mathcal{G}_\eta \tilde{P}_r \gamma_1)(\sigma_n^2 + \mathcal{G}_\eta \tilde{P}_r \gamma_2)} = \mathcal{T}\tilde{P}_r \ln(2). \tag{B.6.5}$$

Simplifying the above equation further, we get a quadratic equation, which can be expressed as

$$\mathcal{A}{\mathcal{G}_\eta}^2 + \mathcal{B}\mathcal{G}_\eta + \mathcal{C} = 0, \tag{B.6.6}$$

where

$$\begin{aligned}
\mathcal{A} &\triangleq {\tilde{P}_r}^2 \gamma_1 \gamma_2 \mathcal{T} \ln(2), \\
\mathcal{B} &\triangleq \tilde{P}_r \gamma_1 \sigma_n^2 \mathcal{T} \ln(2) + \tilde{P}_r \gamma_2 \sigma_n^2 \mathcal{T} \ln(2) - \tilde{P}_r \gamma_1 \gamma_2 \\
\mathcal{C} &\triangleq \sigma_n^4 \mathcal{T} \ln(2) - w_1 \gamma_1 \sigma_n^2 - w_2 \gamma_2 \sigma_n^2.
\end{aligned}$$

The optimal solution is the unique positive root of the quadratic equation. Furthermore, it is easy to verify that $\mathcal{G}_\eta^* = 0$ for $w_1\gamma_1 + w_2\gamma_2 < \mathcal{T}\sigma_n^2 \ln(2)$, which is the power conservation rule for the FASE optimal policy.

B.7 Proof of Result 10: Exact FASE Expression and its Upper Bound

FASE optimal policy provides the optimal solution, that is, $\mathcal{G}_\eta^*$. Expanding the expectation, the FASE as a function of $\mathcal{G}_\eta^*$ can be written as the sum of two integral expressions:

$$\begin{aligned}\overline{\mathcal{S}}_\eta = \frac{1}{\bar{\gamma}_1\bar{\gamma}_2\ln(2)}\Bigg[& w_1 \int_{\gamma_1=0}^{\infty}\int_{\gamma_2=0}^{\mathcal{L}(\gamma_1)} \ln\left(1+\frac{\mathcal{G}_\eta^*\tilde{P}_r\gamma_1}{\sigma_n^2}\right) e^{-\frac{\gamma_1}{\bar{\gamma}_1}}\, e^{-\frac{\gamma_2}{\bar{\gamma}_2}}\, d\gamma_2\, d\gamma_1 \\ & + w_1 \int_{\gamma_1=0}^{\infty}\int_{\gamma_2=\mathcal{L}(\gamma_1)}^{\infty} \ln\left(1+\frac{\mathcal{G}_\eta^*\tilde{P}_r\gamma_1}{\sigma_n^2}\right) e^{-\frac{\gamma_1}{\bar{\gamma}_1}}\, e^{-\frac{\gamma_2}{\bar{\gamma}_2}}\, d\gamma_2\, d\gamma_1\Bigg] \\ & + w_2 \int_{\gamma_1=0}^{\infty}\int_{\gamma_2=0}^{\mathcal{L}(\gamma_1)} \ln\left(1+\frac{\mathcal{G}_\eta^*\tilde{P}_r\gamma_2}{\sigma_n^2}\right) e^{-\frac{\gamma_1}{\bar{\gamma}_1}}\, e^{-\frac{\gamma_2}{\bar{\gamma}_2}}\, d\gamma_2\, d\gamma_1 \\ & + w_2 \int_{\gamma_1=0}^{\infty}\int_{\gamma_2=\mathcal{L}(\gamma_1)}^{\infty} \ln\left(1+\frac{\mathcal{G}_\eta^*\tilde{P}_r\gamma_2}{\sigma_n^2}\right) e^{-\frac{\gamma_1}{\bar{\gamma}_1}}\, e^{-\frac{\gamma_2}{\bar{\gamma}_2}}\, d\gamma_2\, d\gamma_1\Bigg]. \end{aligned} \tag{B.7.1}$$

Since $\mathcal{G}_\eta$ is zero, when the condition in equation (3.2.25) is satisfied, after simplifying further, we get the desired expression in equation (3.2.26).

We now derive an upper bound for the FASE using Jensen's inequality. Applying the inequality in the exact FASE expression, we get

$$\begin{aligned}\overline{\mathcal{S}}_{\eta-\mathrm{UB}} = \frac{1}{\ln(2)}\Bigg[& w_1 \ln\Bigg(1+\frac{1}{\bar{\gamma}_1\bar{\gamma}_2}\int_{\gamma_1=0}^{\infty}\int_{\gamma_2=0}^{\mathcal{L}(\gamma_1)} \frac{\mathcal{G}_\eta^*\tilde{P}_r\gamma_1}{\sigma_n^2} e^{-\frac{\gamma_1}{\bar{\gamma}_1}}\, e^{-\frac{\gamma_2}{\bar{\gamma}_2}}\, d\gamma_2\, d\gamma_1 \\ & + \frac{1}{\bar{\gamma}_1\bar{\gamma}_2}\int_{\gamma_1=0}^{\infty}\int_{\gamma_2=\mathcal{L}(\gamma_1)}^{\infty} \frac{\mathcal{G}_\eta^*\tilde{P}_r\gamma_1}{\sigma_n^2} e^{-\frac{\gamma_1}{\bar{\gamma}_1}}\, e^{-\frac{\gamma_2}{\bar{\gamma}_2}}\, d\gamma_2\, d\gamma_1\Bigg)\Bigg] \\ & + \Bigg[w_2 \ln\Bigg(1+\frac{1}{\bar{\gamma}_1\bar{\gamma}_2}\int_{\gamma_1=0}^{\infty}\int_{\gamma_2=0}^{\mathcal{L}(\gamma_1)} \frac{\mathcal{G}_\eta^*\tilde{P}_r\gamma_2}{\sigma_n^2} e^{-\frac{\gamma_1}{\bar{\gamma}_1}}\, e^{-\frac{\gamma_2}{\bar{\gamma}_2}}\, d\gamma_2\, d\gamma_1 \\ & + \frac{1}{\bar{\gamma}_1\bar{\gamma}_2}\int_{\gamma_1=0}^{\infty}\int_{\gamma_2=\mathcal{L}(\gamma_1)}^{\infty} \frac{\mathcal{G}_\eta^*\tilde{P}_r\gamma_2}{\sigma_n^2} e^{-\frac{\gamma_1}{\bar{\gamma}_1}}\, e^{-\frac{\gamma_2}{\bar{\gamma}_2}}\, d\gamma_2\, d\gamma_1\Bigg)\Bigg]. \end{aligned} \tag{B.7.2}$$

Using the power conservation condition given by equation (3.2.25), the above expression can be simplified to obtain the desired upper bound expression in equation (3.2.27).

B.8 Proof of Result 11: FAEE Optimal Relaying Policy

To derive the FAEE optimal policy, we use a similar approach as that of FASE optimal policy. Since the FAEE optimization problem is also concave, we re-formulate it as a convex optimization problem as follows.

$$\min_{\mathcal{G}_{\mathcal{E}}(\gamma_1,\gamma_2)} -\mathbf{E}\left(\frac{w_1\,\log_2(1+\Gamma_1)+w_2\,\log_2(1+\Gamma_2)}{P_{d_1}+\mathcal{G}_{\mathcal{E}}(\gamma_1,\gamma_2)\;\tilde{P}_r+P_c}\right), \tag{B.8.1}$$

$$\text{s.t. } \mathbf{E}\left[\mathcal{G}_{\mathcal{E}}\;\tilde{P}_r\right]\leq P_{\text{th}}. \tag{B.8.2}$$

To derive the optimal policy, consider the following Lagrangian function:

$$L_{\mathcal{D}}(\mathcal{G}_{\mathcal{E}}) \;=\; \mathbf{E}\left(-\frac{w_1\,\log_2(1+\Gamma_1)+w_2\,\log_2(1+\Gamma_2)}{P_{d_1}+\mathcal{G}_{\mathcal{E}}\;\tilde{P}_r+P_c}+\mathcal{D}(\mathcal{G}_{\mathcal{E}}\;\;\tilde{P}_r\;-\;P_{\text{th}})\right). \tag{B.8.3}$$

Substituting the value of Γ_1 and Γ_2, dropping the expectation, differentiating it with respect to $\mathcal{G}_{\mathcal{E}}$ and then equating the resultant expression to zero, we get

$$w_1\left[\frac{\gamma_1}{\sigma_n^2+\mathcal{G}_{\mathcal{E}}\tilde{P}_r\gamma_1}-\frac{\ln\left(1+\frac{\mathcal{G}_{\mathcal{E}}\tilde{P}_r\gamma_1}{\sigma_n^2}\right)}{P_{d_1}+\mathcal{G}_{\mathcal{E}}\tilde{P}_r+P_c}\right]+w_2\left[\frac{\gamma_2}{\sigma_n^2+\mathcal{G}_{\mathcal{E}}\tilde{P}_r\gamma_2}-\frac{\ln\left(1+\frac{\mathcal{G}_{\mathcal{E}}\tilde{P}_r\gamma_2}{\sigma_n^2}\right)}{P_{d_1}+\mathcal{G}_{\mathcal{E}}\tilde{P}_r+P_c}\right]$$
$$=\mathcal{D}(P_{d_1}+\mathcal{G}_{\mathcal{E}}\tilde{P}_r+P_c)\ln(2). \tag{B.8.4}$$

The optimal solution is the unique positive root of the above transcendental equation. Furthermore, it is easy to verify that $\mathcal{G}_{\mathcal{E}}^*=0$ for $w_1\gamma_1+w_2\gamma_2<\mathcal{D}\;\sigma_n^2\;(P_{d_1}+P_c)\ln(2)$, which is the power conservation rule for the FAEE optimal policy.

B.9 Proof of Result 12: Exact FAEE Expression and its Upper Bound

The FAEE for the proposed model in terms of optimum RGF $\mathcal{G}_{\mathcal{E}}^*$ is given by

$$\overline{\mathcal{E}}=\mathbf{E}\left[\frac{w_1\,\log_2(1+\frac{\mathcal{G}_{\mathcal{E}}^*\;\tilde{P}_r\;\gamma_1}{\sigma_n^2})+w_2\,\log_2(1+\frac{\mathcal{G}_{\mathcal{E}}^*\;\tilde{P}_r\;\gamma_2}{\sigma_n^2})}{P_{d_1}+\mathcal{G}_{\mathcal{E}}^*\;\tilde{P}_r+P_c}\right]. \tag{B.9.1}$$

Further, unfolding the expectation, we get

$$\overline{\mathcal{E}} = \frac{1}{\overline{\gamma}_1\overline{\gamma}_2\ln(2)}\Bigg[w_1\int_{\gamma_1=0}^{\infty}\int_{\gamma_2=0}^{\mathbb{H}(\gamma_1)}\frac{\ln\Big(1+\frac{\mathcal{G}_{\mathcal{E}}^{*}\ \tilde{P}_r\ \gamma_1}{\sigma_n^2}\Big)}{P_{d_1}+\mathcal{G}_{\mathcal{E}}^{*}\tilde{P}_r+P_c}\ e^{-\frac{\gamma_1}{\overline{\gamma}_1}}\ e^{-\frac{\gamma_2}{\overline{\gamma}_2}}\mathrm{d}\gamma_2\ \mathrm{d}\gamma_1$$
$$+ w_1\int_{\gamma_1=0}^{\infty}\int_{\gamma_2=\mathbb{H}(\gamma_1)}^{\infty}\frac{\ln\Big(1+\frac{\mathcal{G}_{\mathcal{E}}^{*}\ \tilde{P}_r\ \gamma_1}{\sigma_n^2}\Big)}{P_{d_1}+\mathcal{G}_{\mathcal{E}}^{*}\tilde{P}_r+P_c}\ e^{-\frac{\gamma_1}{\overline{\gamma}_1}}\ e^{-\frac{\gamma_2}{\overline{\gamma}_2}}\mathrm{d}\gamma_2\ \mathrm{d}\gamma_1 \tag{B.9.2}$$
$$+ w_2\int_{\gamma_1=0}^{\infty}\int_{\gamma_2=0}^{\mathbb{H}(\gamma_1)}\frac{\ln\Big(1+\frac{\mathcal{G}_{\mathcal{E}}^{*}\ \tilde{P}_r\ \gamma_2}{\sigma_n^2}\Big)}{P_{d_1}+\mathcal{G}_{\mathcal{E}}^{*}\tilde{P}_r+P_c}\ e^{-\frac{\gamma_1}{\overline{\gamma}_1}}\ e^{-\frac{\gamma_2}{\overline{\gamma}_2}}\mathrm{d}\gamma_2\ \mathrm{d}\gamma_1$$
$$+ w_2\int_{\gamma_1=0}^{\infty}\int_{\gamma_2=\mathbb{H}(\gamma_1)}^{\infty}\frac{\ln\Big(1+\frac{\mathcal{G}_{\mathcal{E}}^{*}\ \tilde{P}_r\ \gamma_2}{\sigma_n^2}\Big)}{P_{d_1}+\mathcal{G}_{\mathcal{E}}^{*}\tilde{P}_r+P_c}\ e^{-\frac{\gamma_1}{\overline{\gamma}_1}}\ e^{-\frac{\gamma_2}{\overline{\gamma}_2}}\mathrm{d}\gamma_2\ \mathrm{d}\gamma_1\Bigg].$$

For $0 < \gamma_2 < \mathbb{H}(\gamma_1)$, we have $\mathcal{G}_{\mathcal{E}}^{*} = 0$. Applying this condition in the above equation (B.9.2), we get the simplified expression which is shown in equation (3.2.35).

To obtain the upper bound for the FAEE, we use the inequality $\ln(1+x) < x$ for $x > 0$. The upper bound of the equation (3.2.35) can be expressed as

$$\overline{\mathcal{E}}_{\mathrm{UB}} = \frac{1}{\overline{\gamma}_1\overline{\gamma}_2\ln(2)}\Bigg[w_1\int_{\gamma_1=0}^{\infty}\int_{\gamma_2=\mathbb{H}(\gamma_1)}^{\infty}\frac{\Big(\frac{\mathcal{G}_{\mathcal{E}}^{*}\ \tilde{P}_r\ \gamma_1}{\sigma_n^2}\Big)}{P_{d_1}+\mathcal{G}_{\mathcal{E}}^{*}\tilde{P}_r+P_c}\ e^{-\frac{\gamma_1}{\overline{\gamma}_1}}\ e^{-\frac{\gamma_2}{\overline{\gamma}_2}}\mathrm{d}\gamma_2\ \mathrm{d}\gamma_1 \tag{B.9.3}$$
$$+ w_2\int_{\gamma_1=0}^{\infty}\int_{\gamma_2=\mathbb{H}(\gamma_1)}^{\infty}\frac{\Big(\frac{\mathcal{G}_{\mathcal{E}}^{*}\ \tilde{P}_r\ \gamma_2}{\sigma_n^2}\Big)}{P_{d_1}+\mathcal{G}_{\mathcal{E}}^{*}\tilde{P}_r+P_c}\ e^{-\frac{\gamma_1}{\overline{\gamma}_1}}\ e^{-\frac{\gamma_2}{\overline{\gamma}_2}}\mathrm{d}\gamma_2\ \mathrm{d}\gamma_1\Bigg].$$

Appendix C

Chapter 4: Derivations and Proofs

C.1 Proof of Lemma 4.2.1: Statistical Average $\mathcal{EH}_{\max}$

For the simplified path loss and shadow fading model, the maximum energy harvested by the EHRNs is given by

$$\mathcal{EH}_{\max} = \max\{\mathcal{EH}_1, \dots \mathcal{EH}_n, \dots, \mathcal{EH}_L\}, \tag{C.1.1}$$

$$= \max\left\{\frac{\zeta_1\gamma_1}{\psi_1}, \dots, \frac{\zeta_n\gamma_n}{\psi_n}, \dots, \frac{\zeta_L\gamma_L}{\psi_L}\right\}, \tag{C.1.2}$$

where $\psi_1, \psi_2, \dots, \psi_n, \dots, \psi_L$ are i.i.d log-normal random variables, and $\gamma_1, \gamma_2, \dots, \gamma_L$ are exponential random variables. Note that all ψ_n and γ_n are statistically independent. The CDF of $\mathcal{EH}_{\max}$ is given by

$$\mathcal{P}(\mathcal{EH}_{\max} \leq y) = \mathcal{P}\left(\max\left\{\frac{\zeta_1\gamma_1}{\psi_1}, \dots, \frac{\zeta_n\gamma_n}{\psi_n}, \dots, \frac{\zeta_L\gamma_L}{\psi_L}\right\} \leq y\right). \tag{C.1.3}$$

By virtue of statistical independence, we have

$$\begin{aligned}\mathcal{P}(\mathcal{EH}_{\max} \leq y) = \mathcal{P}\left(\frac{\zeta_1}{\psi_1} \leq y\right)\mathcal{P}(\gamma_1 \leq y) \times \dots \\ \times \mathcal{P}\left(\frac{\zeta_n}{\psi_n} \leq y\right)\mathcal{P}(\gamma_n \leq y) \dots \times \mathcal{P}\left(\frac{\zeta_L}{\psi_L} \leq y\right)\mathcal{P}(\gamma_L \leq y).\end{aligned} \tag{C.1.4}$$

We have $\psi_n = 10^{\frac{Y_n}{10}}, n = 1, 2, \dots, L$, where $Y_n \sim \mathcal{N}(0, \sigma_n^2)$. Thus,

$$\mathcal{P}\left(\frac{\zeta_n}{\psi_n} \leq y\right)\mathcal{P}\left(\gamma_n \leq y\right) = Q\left(\frac{\epsilon}{\sigma_n}\ln\left(\frac{\zeta_n}{y}\right)\right)\left(1 - e^{-\frac{y}{\bar{\gamma}_n}}\right), \tag{C.1.5}$$

where $Q(\cdot)$ is the Gaussian Q−function and $\epsilon = \frac{10}{\ln 10}$.

Therefore, we have

$$\mathcal{P}(\mathcal{EH}_{\max} \leq y) = \prod_{n=1}^{L} Q\left(\frac{\epsilon}{\sigma_n}\ln\left(\frac{\zeta_n}{y}\right)\right)\left(1 - e^{-\frac{y}{\bar{\gamma}_n}}\right) \triangleq F_{\mathcal{EH}_{\max}}(y). \tag{C.1.6}$$

Thus, from the CCDF, we can compute $\overline{\mathcal{EH}}_{\max}$ using the following formula: $\overline{\mathcal{EH}}_{\max} = \int_0^\infty F^c_{\mathcal{EH}_{\max}}(y)\, dy$.

Suppose $d_1 \approx \ldots \approx d_n \approx \ldots d_L = d$, $\sigma_1 = \ldots = \sigma_n = \ldots = \sigma_L = \sigma$, and hence $\zeta_1 = \ldots = \zeta_n = \ldots = \zeta_L = \zeta$. Furthermore, $\overline{\gamma}_1 = \overline{\gamma}_2 = \ldots = \overline{\gamma}_L = \overline{\gamma}$. Therefore, by virtue of the i.i.d property, the cumulative distribution function (CDF) expression further simplifies to

$$F_{\mathcal{EH}_{\max}}(y) = \left(Q\left(\frac{\epsilon}{\sigma}\ln\left(\frac{\zeta}{y}\right)\right)\right)^L \left(1 - e^{-\frac{y}{\overline{\gamma}}}\right)^L, y > 0. \tag{C.1.7}$$

Since $\mathcal{EH}_{\max}$ is a non-negative random variable, the average energy harvested can be evaluated from the complimentary CDF (CCDF).

$$\overline{\mathcal{EH}}_{\max} = \int_0^\infty \left(1 - F_{\mathcal{EH}_{\max}}(y)\right) dy. \tag{C.1.8}$$

Using the fact that $Q(y) = \frac{1}{2}\mathrm{erfc}\left(\frac{y}{\sqrt{2}}\right)$, we get the desired result in terms of complementary error function.

$L = 1$ *scenario:* For the single EHRN scenario, we have $\overline{\mathcal{EH}} = \zeta \mathbf{E}\left[\frac{1}{\psi}\right]\overline{\gamma}$. Note that $Y \sim \mathcal{N}(0, \sigma^2)$. Since $\mathbf{E}\left[\frac{1}{\psi}\right] = \mathbf{E}\left[10^{-\frac{Y}{10}}\right] = \mathbf{E}\left[e^{-\frac{Y}{\epsilon}}\right] = e^{\frac{\sigma^2}{2\epsilon^2}}$, we get $\overline{\mathcal{EH}} = \zeta\overline{\gamma}e^{\frac{\sigma^2}{2\epsilon^2}}$.

C.2 Proof of Lemma 4.4.1: Link Outage Analysis

From the definition of link outage probability, we have

$$\begin{aligned} \mathcal{P}(\Gamma_D < \Gamma_{\text{th}}) = e^{-\frac{\gamma_0}{\overline{\gamma}_{r_{\mathbb{S}}d}}} \frac{L}{k\overline{\gamma}\,\overline{\gamma}_{r_{\mathbb{S}}d}} \int_{\gamma_0}^\infty \int_0^{\frac{\Gamma_{\text{th}}}{\mathcal{C}y}} \exp\left(-\frac{z}{k\overline{\gamma}}\right) \\ \times \left(1 - \exp\left(-\frac{z}{k\overline{\gamma}}\right)\right)^{L-1} \exp\left(-\frac{y}{\overline{\gamma}_{r_{\mathbb{S}}d}}\right) dz\, dy, \end{aligned} \tag{C.2.1}$$

where $\mathcal{C} = \frac{1}{\mathcal{D}^\nu \sigma_d^2}$.

Consider the inner integral. Using substitution $(1 - \exp(-\frac{z}{k\overline{\gamma}})) = t$, and simplifying further, we get

$$\int_0^{\frac{\Gamma_{\text{th}}}{\mathcal{C}y}} e^{-\frac{z}{k\overline{\gamma}}} \left(1 - e^{-\frac{z}{k\overline{\gamma}}}\right)^{L-1} dz = \frac{k\overline{\gamma}}{L}\left(1 - \exp\left(-\frac{a}{y}\right)\right)^L,$$

where $a = \frac{\Gamma_{\text{th}}}{k\mathcal{C}\overline{\gamma}}$, $b = \frac{1}{\overline{\gamma}_{r_{\mathbb{S}}d}}$. Substituting the above simplified expression and further simplification yields the desired expression for the outage probability.

We cannot simplify the integral further. However, for $L = 1$ and $\gamma_0 = 0$, the outage probability is given by

$$\frac{1}{\overline{\gamma}_{r_{\mathbb{S}}d}}\int_0^\infty \left(1-\exp\left(-\frac{a}{y}\right)\right) e^{-by}\,dy = 1 - b\int_0^\infty \exp\left(-\left(\frac{a}{y}+by\right)\right)\,dy.$$

Further simplification using the standard integral [156, (3.324.1)]

$$\int_0^\infty \exp\left(-\left(\frac{a}{y}+by\right)\right)\,dy = \sqrt{\frac{4a}{b}}\,\mathrm{K}_1\left(\sqrt{4ab}\right), \tag{C.2.2}$$

yields the desired expression for outage probability.

C.3 Proof of Result 13: Exact FASE

Substituting the expression for Γ_D given by equation (4.4.2) in equation (4.5.1), we get FASE as

$$\overline{\mathcal{S}} = \mathbf{E}\left[\log_2\left(1+\frac{\mathcal{A}\ \max\{\mathcal{EH}_1,\mathcal{EH}_2,\ldots,\mathcal{EH}_L\}\gamma_{r_{\mathbb{S}}d}}{\sigma_d^2\mathcal{D}^\nu}\right)\right]. \tag{C.3.1}$$

Since $\mathcal{EH}_n = k\gamma_n$, we have

$$\overline{\mathcal{S}} = \mathbf{E}\left[\log_2\left(1+\frac{\mathcal{A}\ k\ \max\{\gamma_1,\gamma_2,\ldots,\gamma_L\}\gamma_{r_{\mathbb{S}}d}}{\sigma_d^2\mathcal{D}^\nu}\right)\right]. \tag{C.3.2}$$

Let $\gamma_M \triangleq \max\{\gamma_1,\gamma_2,\ldots,\gamma_L\}$. The pdf of γ_M is given by [100]

$$p_{\gamma_M}(\gamma) = \frac{L}{\overline{\gamma}}e^{-\frac{\gamma}{\overline{\gamma}}}\left(1-e^{-\frac{\gamma}{\overline{\gamma}}}\right)^{L-1}, \gamma \geq 0 \tag{C.3.3}$$

Therefore, unfolding the expectation, equation (C.3.2) can be expressed as

$$\begin{aligned}\overline{\mathcal{S}} = &\frac{L}{\ln(2)\ \overline{\gamma}\ \overline{\gamma}_{r_{\mathbb{S}}d}}\int_{\gamma=0}^\infty\int_{\gamma_{r_{\mathbb{S}}d}=0}^\infty \ln\left(1+\frac{\mathcal{A}\ k\ \gamma\ \gamma_{r_{\mathbb{S}}d}}{\sigma_d^2\mathcal{D}^\nu}\right)\exp\left(-\frac{\gamma}{\overline{\gamma}}\right)\\ &\times\left(1-\exp\left(-\frac{\gamma}{\overline{\gamma}}\right)\right)^{L-1}\exp\left(-\frac{\gamma_{r_{\mathbb{S}}d}}{\overline{\gamma}_{r_{\mathbb{S}}d}}\right)\,\mathrm{d}\gamma_{r_{\mathbb{S}}d}\,\mathrm{d}\gamma.\end{aligned} \tag{C.3.4}$$

Further, considering the energy conservation rule, the above above expression becomes

$$
\begin{aligned}
\overline{\mathcal{S}} = \frac{L}{\ln(2)\ \overline{\gamma}\ \overline{\gamma}_{r_{\mathbb{S}}d}} \Bigg[& \int_{\gamma=0}^{\infty} \int_{\gamma_{r_{\mathbb{S}}d}=0}^{\gamma_0} \ln \left(1 + \frac{\mathcal{A}\ k\ \gamma\ \gamma_{r_{\mathbb{S}}d}}{\sigma_d^2 \mathcal{D}^{\nu}} \right) \exp\left(-\frac{\gamma}{\overline{\gamma}} \right) \\
& \times \left(1 - \exp\left(-\frac{\gamma}{\overline{\gamma}} \right) \right)^{L-1} \exp\left(-\frac{\gamma_{r_{\mathbb{S}}d}}{\overline{\gamma}_{r_{\mathbb{S}}d}} \right) \mathrm{d}\gamma_{r_{\mathbb{S}}d}\ \mathrm{d}\gamma \\
& + \int_{\gamma=0}^{\infty} \int_{\gamma_{r_{\mathbb{S}}d}=\gamma_0}^{\infty} \ln \left(1 + \frac{\mathcal{A}\ k\ \gamma\ \gamma_{r_{\mathbb{S}}d}}{\sigma_d^2 \mathcal{D}^{\nu}} \right) \exp\left(-\frac{\gamma}{\overline{\gamma}} \right) \\
& \times \left(1 - \exp\left(-\frac{\gamma}{\overline{\gamma}} \right) \right)^{L-1} \exp\left(-\frac{\gamma_{r_{\mathbb{S}}d}}{\overline{\gamma}_{r_{\mathbb{S}}d}} \right) \mathrm{d}\gamma_{r_{\mathbb{S}}d}\ \mathrm{d}\gamma \Bigg].
\end{aligned}
\tag{C.3.5}
$$

From the energy conservation rule, we have $\mathcal{A} = 0$ for $\gamma_{r_{\mathbb{S}}d} < \gamma_0$ and $\mathcal{A} = 1$ for $\gamma_{r_{\mathbb{S}}d} \geq \gamma_0$. Simplifying further, we get

$$
\begin{aligned}
\overline{\mathcal{S}} = \frac{L}{\ln(2)\ \overline{\gamma}\ \overline{\gamma}_{r_{\mathbb{S}}d}} & \int_{\gamma=0}^{\infty} \left[\exp\left(-\frac{\gamma}{\overline{\gamma}} \right) \left(1 - \exp\left(-\frac{\gamma}{\overline{\gamma}} \right) \right)^{L-1} \right. \\
& \times \left. \left[\int_{\gamma_{r_{\mathbb{S}}d}=\gamma_0}^{\infty} \ln \left(1 + \mathbb{C}_1 \gamma\ \gamma_{r_{\mathbb{S}}d} \right) \exp\left(-\frac{\gamma_{r_{\mathbb{S}}d}}{\overline{\gamma}_{r_{\mathbb{S}}d}} \right) \mathrm{d}\gamma_{r_{\mathbb{S}}d} \right] \right] \mathrm{d}\gamma,
\end{aligned}
\tag{C.3.6}
$$

where $\mathbb{C}_1 = \frac{k}{\sigma_d^2 D^{\nu}}$. Using the substitution $\gamma_{r_{\mathbb{S}}d} = \gamma_0 + u$ and simplifying the above inner integral with respect to u using the definite integral [156, (4.337.1)]

$$
\int_0^{\infty} e^{-\mu t} \ln(t + \lambda)\, dt = \frac{1}{\mu} \left(\ln(\lambda) + \mathrm{E}_1(\mu\lambda) e^{\mu\lambda} \right), \tag{C.3.7}
$$

we get the desired single integral expression for FASE in equation (4.5.2).

C.4 Proof of Result 14: Closed-Form FASE Upper Bound

To determine the expression for upper bound FASE, we first obtain the average SNR of $R_{\mathbb{S}} - D$ link, that is, $\mathbf{E}[\Gamma_D]$. Therefore, from equation (4.4.2), $\mathbf{E}[\Gamma_D]$ is given by

$$
\overline{\Gamma}_D = \mathbf{E}\left[\frac{\mathcal{A} \max\{\mathcal{EH}_1, \mathcal{EH}_2, \ldots, \mathcal{EH}_L\}\ \gamma_{r_{\mathbb{S}}d}}{\sigma_d^2 \mathcal{D}^{\nu}} \right]. \tag{C.4.1}
$$

Using statistical independence of $\mathcal{A}$, $\mathcal{EH}_{\max}$, and $\gamma_{r_{\mathbb{S}}d}$, we have

$$
\overline{\Gamma}_D = \frac{\mathbf{E}[\mathcal{A}]\ \mathbf{E}[\mathcal{EH}_{\max}]\ \mathbf{E}[\gamma_{r_{\mathbb{S}}d}]}{\sigma_d^2 \mathcal{D}^{\nu}}. \tag{C.4.2}
$$

It is easy to show that $\mathbf{E}[\mathcal{A}] = 1 - p_0$.

From equation (4.3.2), we see that $\mathcal{P}(\gamma_{r_{\mathbb{S}}d} \geq \gamma_0) = 1 - p_0 = \exp\left(-\frac{\gamma_0}{\overline{\gamma}_{r_{\mathbb{S}}d}}\right)$. Therefore, $\mathbf{E}[\mathcal{A}] = \exp\left(-\frac{\gamma_0}{\overline{\gamma}_{r_{\mathbb{S}}d}}\right)$. Further, as Rayleigh fading channel is considered for the $S - R_{\mathbb{S}}$ link, we have $\overline{\mathcal{EH}}_{\max} = k\overline{\gamma}\sum_{m=1}^{L}\frac{1}{m}$.

Substituting the expressions for $\mathbf{E}[\mathcal{A}]$, $\mathbf{E}[\mathcal{EH}_{\max}]$ and, $\mathbf{E}[\gamma_{r_{\mathbb{S}}d}]$, the expression for $\overline{\Gamma}_D$ is given by

$$\overline{\Gamma}_D = \frac{k\exp\left(-\frac{\gamma_0}{\overline{\gamma}_{r_{\mathbb{S}}d}}\right)\left(\sum_{m=1}^{L}\frac{1}{m}\right)\overline{\gamma}\;\overline{\gamma}_{r_{\mathbb{S}}d}}{\sigma_d^2 D^{\nu}}. \tag{C.4.3}$$

Lastly, substituting $\overline{\Gamma}_D$ expression in equation (4.5.3), we get the desired closed-form upper bound in equation (4.5.4).

C.5 Proof of Result 15: Asymptotic FASE

First, we consider the closed-form upper bound of FASE. For $\gamma_{r_{\mathbb{S}}d} \geq \gamma_0$, we have

$$\overline{\mathcal{S}} \leq \overline{\mathcal{S}}_{\text{UB}} = \log_2 e \times \log_e\left(1 + \mathbb{C}_1\mathbb{J}e^{-\frac{\gamma_0}{\overline{\gamma}_{r_{\mathbb{S}}d}}}\overline{\gamma}\;\overline{\gamma}_{r_{\mathbb{S}}d}\right), \tag{C.5.1}$$

where $\mathbb{J} = \sum_{m=1}^{L}\frac{1}{m}$.

We use the following inequality: for $y \geq 0$, $\ln(1+y) \leq y$. Using the inequality, we have

$$(\log_2 e)\;\log_e\left(1 + \mathbb{C}_1\mathbb{J}e^{-\frac{\gamma_0}{\overline{\gamma}_{r_{\mathbb{S}}d}}}\overline{\gamma}\;\overline{\gamma}_{r_{\mathbb{S}}d}\right) \leq (\log_2 e)\;\mathbb{C}_1\mathbb{J}e^{-\frac{\gamma_0}{\overline{\gamma}_{r_{\mathbb{S}}d}}}\overline{\gamma}\;\overline{\gamma}_{r_{\mathbb{S}}d}. \tag{C.5.2}$$

In the scaling regime, as $p_0 \to 0$, we have $\gamma_0 \to 0$. Therefore, in the scaling regime, $e^{-\frac{\gamma_0}{\overline{\gamma}_{r_{\mathbb{S}}d}}} \to 1$. Further simplification yields the designed expression for the asymptotic FASE.

C.6 Proof of Result 16: Exact FAEE

From equation (4.6.3), the average relay transmit power can be expressed as

$$\overline{P}_r = \mathbf{E}\left[\frac{\mathcal{A}\;\max\{\mathcal{EH}_1, \mathcal{EH}_2, \ldots, \mathcal{EH}_L\}}{\frac{(1-v)T}{2}}\right]. \tag{C.6.1}$$

By virtue of statistical independence of the random variables, $\overline{P}_r$ can be expressed as

$$\overline{P}_r = \frac{\mathbf{E}[\mathcal{A}]\;\mathbf{E}[\mathcal{EH}_{\max}]}{\frac{(1-v)T}{2}}. \tag{C.6.2}$$

Substituting $\mathbf{E}[\mathcal{A}] = \exp\left(-\frac{\gamma_0}{\overline{\gamma}_{r_{\mathbb{S}}d}}\right)$ and $\mathbf{E}[\mathcal{EH}_{\max}] = k\overline{\gamma}\mathbb{J}$, we get

$$\overline{P}_r = \frac{k\overline{\gamma}\mathbb{J}\exp\left(-\frac{\gamma_0}{\overline{\gamma}_{r_{\mathbb{S}}d}}\right)}{\frac{(1-\upsilon)T}{2}}. \tag{C.6.3}$$

Therefore, the average total power consumption is given by

$$\overline{P}_{\mathrm{T}} = \frac{(P_s + P_c)\frac{(1-\upsilon)T}{2} + k\overline{\gamma}\mathbb{J}\exp\left(-\frac{\gamma_0}{\overline{\gamma}_{r_{\mathbb{S}}d}}\right)}{\frac{(1-\upsilon)T}{2}}. \tag{C.6.4}$$

Further, substituting equation (4.5.2) and equation (C.6.4) in equation (4.6.1) we get (4.6.4).

C.7 Proof of Result 17: Closed-Form FAEE Upper Bound

It is straightforward to derive closed-form FAEE upper bound from the closed-form FASE upper bound $\overline{\mathcal{S}}_{\mathrm{UB}}$. Let $\overline{\mathcal{E}}_{\mathrm{UB}}$ denote the FAEE upper bound. We have

$$\overline{\mathcal{E}}_{\mathrm{UB}} = \frac{\overline{\mathcal{S}}_{\mathrm{UB}}}{\mathbf{E}(P_{\mathrm{T}})}. \tag{C.7.1}$$

Substituting the upper bound of FASE given in equation (4.5.4), we get

$$\overline{\mathcal{E}}_{\mathrm{UB}} = \frac{\log_2\left(1 + \mathbb{C}_1\mathbb{J}\exp\left(\frac{-\gamma_0}{\overline{\gamma}_{r_{\mathbb{S}}d}}\right)\overline{\gamma}\ \overline{\gamma}_{r_{\mathbb{S}}d}\right)\frac{(1-\upsilon)T}{2}}{(P_s + P_c)\frac{(1-\upsilon)T}{2} + k\overline{\gamma}\mathbb{J}\exp\left(-\frac{\gamma_0}{\overline{\gamma}_{r_{\mathbb{S}}d}}\right)}. \tag{C.7.2}$$

C.8 Proof of Result 18: Asymptotic FAEE Expression

We use the similar approach that we followed in deriving the expression for asymptotic FASE. First, we consider the closed-form upper bound of FAEE. For $\gamma_{r_{\mathbb{S}}d} \geq \gamma_0$, we have

$$\overline{\mathcal{E}} \leq \overline{\mathcal{E}}_{\mathrm{UB}} = \log_2 e \times \frac{\log_e\left(1 + \mathbb{C}_1\mathbb{J}\exp\left(\frac{-\gamma_0}{\overline{\gamma}_{r_{\mathbb{S}}d}}\right)\overline{\gamma}\ \overline{\gamma}_{r_{\mathbb{S}}d}\right)\frac{(1-\upsilon)T}{2}}{(P_s + P_c)\frac{(1-\upsilon)T}{2} + k\overline{\gamma}\mathbb{J}\exp\left(-\frac{\gamma_0}{\overline{\gamma}_{r_{\mathbb{S}}d}}\right)}, \tag{C.8.1}$$

where $\mathbb{J} = \sum_{m=1}^{L} \frac{1}{m}$.

We again use the inequality: For $y \geq 0$, $\ln(1+y) \leq y$. Applying the inequality, we have

$$\overline{\mathcal{E}}_{\text{UB}} \leq \log_2 e \times \frac{\left(\mathbb{C}_1 \mathbb{J} \exp\left(\frac{-\gamma_0}{\overline{\gamma}_{r_{\mathbb{S}}d}}\right) \overline{\gamma}\ \overline{\gamma}_{r_{\mathbb{S}}d}\right) \frac{(1-v)T}{2}}{(P_s + P_c)\frac{(1-v)T}{2} + k\overline{\gamma}\mathbb{J}\exp\left(-\frac{\gamma_0}{\overline{\gamma}_{r_{\mathbb{S}}d}}\right)}. \tag{C.8.2}$$

In the scaling regime, as $p_0 \to 0$, we have the threshold $\gamma_0 \to 0$. Therefore, in the scaling regime, we have $e^{-\frac{\gamma_0}{\overline{\gamma}_{r_{\mathbb{S}}d}}} \to 1$. Further simplification yields the desired expression for the asymptotic FAEE.

Appendix D

Chapter 5: Derivations and Proofs

D.1 Proof of Result 19: Optimal FASE Relaying Policy

We reformulate the optimization problem in equation (5.2.2) into a convex optimization problem.

$$\min_{\psi_{\mathcal{S}}} -\mathbf{E}\left[p_1 \log_2\left(1+\frac{\psi_{\mathcal{S}} P_r \gamma^s_{r_1 d}}{\sigma_d^2\; d^{\nu}}\right) + p_2 \log_2\left(1+\frac{\psi_{\mathcal{S}} P_r \gamma^s_{r_2 d}}{\sigma_d^2\; d^{\nu}}\right)\right], \tag{D.1.1}$$

$$\text{s.t. } \mathbf{E}\left[p_1 \frac{\psi_{\mathcal{S}} P_r \gamma^i_{r_1 d}}{\sigma_D^2\; D^{\nu}} + p_2 \frac{\psi_{\mathcal{S}} P_r \gamma^i_{r_2 d}}{\sigma_D^2\; D^{\nu}}\right] \leq I_{\text{th}}. \tag{D.1.2}$$

It can now be verified that the objective function shown in equation (D.1.1) is a convex function. Therefore, we use Lagrange multiplier method to obtain the optimum RGF $\psi^*_{\mathcal{S}}$. The Lagrangian function can be mathematically written as

$$\mathcal{L}_{\mathcal{M}}(\psi_{\mathcal{S}}) \triangleq \mathbf{E}\left[-\left(p_1 \log_2\left(1+\frac{\psi_{\mathcal{S}} P_r \gamma^s_{r_1 d}}{\sigma_d^2\; d^{\nu}}\right) + p_2 \log_2\left(1+\frac{\psi_{\mathcal{S}} P_r \gamma^s_{r_2 d}}{\sigma_d^2\; d^{\nu}}\right)\right) + \mathcal{M}\left(\frac{p_1 \psi_{\mathcal{S}} P_r \gamma^i_{r_1 d}}{\sigma_D^2\; D^{\nu}} + \frac{p_2 \psi_{\mathcal{S}} P_r \gamma^i_{r_2 d}}{\sigma_D^2\; D^{\nu}} - I_{\text{th}}\right)\right], \tag{D.1.3}$$

where $\mathcal{M} > 0$ is the Lagrange multiplier and is set such that it should satisfy the average interference constraint defined in equation (D.1.2). Further, to obtain $\psi^*_{\mathcal{S}}$, dropping the expectation (due to virtue of convexity) from equation (D.1.3), differentiating with respect to $\psi_{\mathcal{S}}$ and equating the simplified expression to zero, we get the quadratic equation presented in (5.2.6).

D.2 Proof of Result 20: Exact Optimal FASE Expression

We now use the optimized RGF and power conservation rule to derive the final expression for optimal FASE. For the proposed spectrally-efficient IC-PA-DAF policy, optimal FASE is given by

$$\overline{\mathcal{S}} = \mathbf{E}\left[p_1 \log_2\left(1+\frac{\psi^*_{\mathcal{S}} P_r \gamma^s_{r_1d}}{\sigma^2_d\ d^{\nu}}\right) + p_2 \log_2\left(1+\frac{\psi^*_{\mathcal{S}} P_r \gamma^s_{r_2d}}{\sigma^2_d\ d^{\nu}}\right)\right]. \tag{D.2.1}$$

Expanding the expectation, we get

$$\begin{aligned}\overline{\mathcal{S}} = \mathbb{X}\Bigg[\int_{\gamma^i_{r_2d}=0}^{\infty}\int_{\gamma^i_{r_1d}=0}^{\infty}\int_{\gamma^s_{r_1d}=0}^{\infty}\int_{\gamma^s_{r_2d}=0}^{\infty}\Bigg[&p_1 \log_2\left(1+\mathbb{Y}_1\gamma^s_{r_1d}\right)\\ &+ p_2 \log_2\left(1+\mathbb{Y}_1\gamma^s_{r_2d}\right)\Bigg]\mathbb{Z}\ \mathrm{d}\gamma^s_{r_2d}\ \mathrm{d}\gamma^s_{r_1d}\ \mathrm{d}\gamma^i_{r_1d}\ \mathrm{d}\gamma^i_{r_2d}\Bigg],\end{aligned} \tag{D.2.2}$$

where $\mathbb{X} = \frac{1}{\overline{\gamma}^s_{r_1d}\overline{\gamma}^s_{r_2d}\overline{\gamma}^i_{r_1d}\overline{\gamma}^i_{r_2d}}$, $\mathbb{Y}_1 = \frac{\psi^*_{\mathcal{S}} P_r}{\sigma^2_d\ d^{\nu}}$, and $\mathbb{Z} = e^{\frac{-\gamma^s_{r_1d}}{\overline{\gamma}^s_{r_1d}}} \times e^{\frac{-\gamma^s_{r_2d}}{\overline{\gamma}^s_{r_2d}}} \times e^{\frac{-\gamma^i_{r_1d}}{\overline{\gamma}^i_{r_1d}}} \times e^{\frac{-\gamma^i_{r_2d}}{\overline{\gamma}^i_{r_2d}}}$. Further, $\overline{\gamma}^s_{r_1d}$, $\overline{\gamma}^s_{r_2d}$, $\overline{\gamma}^i_{r_1d}$, and $\overline{\gamma}^i_{r_2d}$ is the average channel power gain of $\mathrm{R}^{\mathrm{S}}_1 - \mathrm{D}^{\mathrm{S}}_{\mathrm{Rx}}$ link, $\mathrm{R}^{\mathrm{S}}_2 - \mathrm{D}^{\mathrm{S}}_{\mathrm{Rx}}$ link, $\mathrm{R}^{\mathrm{S}}_1 - \mathrm{D}^{\mathrm{P}}_{\mathrm{Rx}}$ link and $\mathrm{R}^{\mathrm{S}}_2 - \mathrm{D}^{\mathrm{P}}_{\mathrm{Rx}}$ link respectively. Further applying power conservation rule, we get equation (5.2.8).

D.3 Proof of Result 21: Optimal FASE Upper Bound Expression

Applying the Jensen's inequality, the upper bound is given by

$$\overline{\mathcal{S}}_{\mathrm{UB}} = \left[p_1 \log_2\left(1+\mathbf{E}\left[\mathbb{Y}_1\gamma^s_{r_1d}\right]\right) + p_2 \log_2\left(1+\mathbf{E}\left[\mathbb{Y}_1\gamma^s_{r_2d}\right]\right)\right]. \tag{D.3.1}$$

Expanding the expectation, the above expression becomes

$$\begin{aligned}\overline{\mathcal{S}}_{\mathrm{UB}} =& p_1 \log_2\left(1+\left[\mathbb{X}\int_{\gamma^i_{r_2d}=0}^{\infty}\int_{\gamma^i_{r_1d}=0}^{\infty}\int_{\gamma^s_{r_1d}=0}^{\infty}\int_{\gamma^s_{r_2d}=0}^{\infty}\mathbb{Y}_1\gamma^s_{r_1d}\mathbb{Z}\ \mathrm{d}\gamma^s_{r_2d}\ \mathrm{d}\gamma^s_{r_1d}\ \mathrm{d}\gamma^i_{r_1d}\ \mathrm{d}\gamma^i_{r_2d}\right]\right)\\ &+ p_2 \log_2\left(1+\left[\mathbb{X}\int_{\gamma^i_{r_2d}=0}^{\infty}\int_{\gamma^i_{r_1d}=0}^{\infty}\int_{\gamma^s_{r_1d}=0}^{\infty}\int_{\gamma^s_{r_2d}=0}^{\infty}\mathbb{Y}_1\gamma^s_{r_2d}\mathbb{Z}\ \mathrm{d}\gamma^s_{r_2d}\ \mathrm{d}\gamma^s_{r_1d}\ \mathrm{d}\gamma^i_{r_1d}\ \mathrm{d}\gamma^i_{r_2d}\right]\right).\end{aligned} \tag{D.3.2}$$

Lastly, applying the power conservation rule, the upper bound expression for FASE is shown in equation (5.2.9).

D.4 Proof of Result 22: Optimal FAEE Relaying Policy

Converting the concave optimization function to convex function, we get

$$\min_{\psi_{\mathcal{E}}} -\mathbf{E}\left[\frac{p_1 \log_2\left(1+\frac{\psi_{\mathcal{E}} P_r \gamma^s_{r_1 d}}{\sigma_d^2\ d^{\nu}}\right) + p_2 \log_2\left(1+\frac{\psi_{\mathcal{E}} P_r \gamma^s_{r_2 d}}{\sigma_d^2\ d^{\nu}}\right)}{P_s + P_c + \psi_{\mathcal{E}} P_r}\right], \tag{D.4.1}$$

$$\text{s.t. } \mathbf{E}\left[p_1 \frac{\psi_{\mathcal{E}} P_r \gamma^i_{r_1 d}}{\sigma_D^2\ D^{\nu}} + p_2 \frac{\psi_{\mathcal{E}} P_r \gamma^i_{r_2 d}}{\sigma_D^2\ D^{\nu}}\right] \leq I_{\text{th}}. \tag{D.4.2}$$

It can be analyzed that the objective function in equation (D.4.1) is a convex function. Therefore, we will be using the convex optimizing technique (Lagrange multiplier method) to obtain optimum $\psi_{\mathcal{E}}$. The Lagrangian function for the optimization problem can be written as

$$\mathcal{L}_{\mathcal{M}'}(\psi_{\mathcal{E}}) \triangleq \mathbf{E}\Bigg[-\left(\frac{p_1 \log_2\left(1+\frac{\psi_{\mathcal{E}} P_r \gamma^s_{r_1 d}}{\sigma_d^2\ d^{\nu}}\right) + p_2 \log_2\left(1+\frac{\psi_{\mathcal{E}} P_r \gamma^s_{r_2 d}}{\sigma_d^2\ d^{\nu}}\right)}{P_{sc} + \psi_{\mathcal{E}} P_r}\right) + \mathcal{M}'\left(\frac{p_1 \psi_{\mathcal{E}} P_r \gamma^i_{r_1 d}}{\sigma_D^2\ D^{\nu}} + \frac{p_2 \psi_{\mathcal{E}} P_r \gamma^i_{r_2 d}}{\sigma_D^2\ D^{\nu}} - I_{\text{th}}\right)\Bigg], \tag{D.4.3}$$

where $P_{sc} = P_s + P_c$, $\mathcal{M}' > 0$ is the Lagrange multiplier and is set such that it should satisfy the average interference constraint defined in equation (D.4.2).

Further, to obtain $\psi^*_{\mathcal{E}}$, neglecting the expectation (due to virtue of convexity) from equation (D.4.3), differentiating it with respect to $\psi_{\mathcal{E}}$ and equating the simplified expression to zero, we get the equation (5.2.15).

D.5 Proof of Result 23: Exact Optimal FAEE Expression

The expression of optimal FAEE for the proposed model after expanding the expectation can be written as

$$\overline{\mathcal{E}}=\Bigg[\int_{\gamma^i_{r_2d}=0}^{\infty}\int_{\gamma^i_{r_1d}=0}^{\infty}\int_{\gamma^s_{r_1d}=0}^{\infty}\int_{\gamma^s_{r_2d}=0}^{\infty}\mathbb{X}\Bigg[\left(\frac{p_1\log_2(1+\mathbb{Y}_2\gamma^s_{r_1d})}{P_{sc}+\psi^*_{\mathcal{E}}P_r}\right) + \left(\frac{p_2\log_2\left(1+\mathbb{Y}_2\gamma^s_{r_2d}\right)}{P_{sc}+\psi^*_{\mathcal{E}}P_r}\right)\Bigg]\mathbb{Z}\,\mathrm{d}\gamma^s_{r_2d}\,\mathrm{d}\gamma^s_{r_1d}\,\mathrm{d}\gamma^i_{r_1d}\,\mathrm{d}\gamma^i_{r_2d}\Bigg], \quad \text{(D.5.1)}$$

where $\mathbb{Y}_2 = \frac{\psi^*_{\mathcal{E}}P_r}{\sigma^2_d\ d^{\nu}}$. We now use the optimized RGF and the power conservation rule ($\psi^*_{\mathcal{E}} = 0$ for $\gamma^s_{r_2d} < \beta'$) to derive the expression for optimal FAEE presented in (5.2.17).

D.6 Proof of Result 24: Optimal FAEE Upper Bound Expression

Using the inequality $\log_2(1+y) < y$ for $y > 0$, the upper bound expression for the optimal FAEE can be written as

$$\overline{\mathcal{E}}_{\mathrm{UB}} = \Bigg[\int_{\gamma^i_{r_2d}=0}^{\infty}\int_{\gamma^i_{r_1d}=0}^{\infty}\int_{\gamma^s_{r_1d}=0}^{\infty}\int_{\gamma^s_{r_2d}=0}^{\infty}\mathbb{X}\Bigg[\left(\frac{p_1\mathbb{Y}_2\gamma^s_{r_1d}}{P_{sc}+\psi^*_{\mathcal{E}}P_r}\right) + \left(\frac{p_2\mathbb{Y}_2\gamma^s_{r_2d}}{P_{sc}+\psi^*_{\mathcal{E}}P_r}\right)\Bigg]\mathbb{Z}\,\mathrm{d}\gamma^s_{r_2d}\,\mathrm{d}\gamma^s_{r_1d}\,\mathrm{d}\gamma^i_{r_1d}\,\mathrm{d}\gamma^i_{r_2d}\Bigg]. \quad \text{(D.6.1)}$$

Further, applying the power conservation rule, the upper bound optimized FAEE is expressed as shown in equation (5.2.18).

D.7 Proof of Result 25: Optimal Relay Gain Function for FASE

Since the optimization problem stated in (5.5.4) is concave, we re-formulate it into a convex optimization problem. Therefore the problem statement can be written as

$$\min_{\varphi_{\mathcal{S}}} -\mathbf{E}\left[\log_2(1 + C_1\ \varphi_{\mathcal{S}}\ \gamma^{\mathrm{s}}_{\max}\ \gamma^{\mathrm{s}}_{r_{\mathbb{S}}d}) \right], \tag{D.7.1}$$

$$\text{s.t. } \mathbf{E}\left[C_2\ \varphi_{\mathcal{S}}\ \gamma^{\mathrm{s}}_{\max}\gamma^{\mathrm{i}}_{r_{\mathbb{S}}d} \right] \leq I_{\mathrm{th}}. \tag{D.7.2}$$

It can be analyzed that the objective function written in equation (D.7.1) is convex. Therefore, to obtain the optimum solution, we use the Lagrange multiplier method. Hence, the Lagrange function for the optimization problem shown above can be written as

$$L_{\mathcal{M}}(\varphi_{\mathcal{S}}) = \mathbf{E}\left[-\log_2(1 + C_1\ \varphi_{\mathcal{S}}\ \gamma^{\mathrm{s}}_{\max}\ \gamma^{\mathrm{s}}_{r_{\mathbb{S}}d}) + \mathcal{M}(C_2\varphi_{\mathcal{S}}\gamma^{\mathrm{s}}_{\max}\gamma^{\mathrm{i}}_{r_{\mathbb{S}}d} - I_{\mathrm{th}}) \right], \tag{D.7.3}$$

where $\mathcal{M}$ is the Lagrange multiplier. Further, to obtain the optimum value of $\varphi_{\mathcal{S}}$, we differentiate the above equation with respect to $\varphi_{\mathcal{S}}$ after dropping the expectation (by virtue of convexity) and equating it to zero. Hence, we get

$$\begin{aligned} \frac{\partial}{\partial \varphi_{\mathcal{S}}}\left[-\log_2(1 + C_1\ \varphi_{\mathcal{S}}\ \gamma^{\mathrm{s}}_{\max}\ \gamma^{\mathrm{s}}_{r_{\mathbb{S}}d}) + \mathcal{M}(C_2\varphi_{\mathcal{S}}\gamma^{\mathrm{s}}_{\max}\gamma^{\mathrm{i}}_{r_{\mathbb{S}}d} - I_{\mathrm{th}}) \right] &= 0, \\ \frac{C_1\ \gamma^{\mathrm{s}}_{\max}\ \gamma^{\mathrm{s}}_{r_{\mathbb{S}}d}}{\ln(2)(1 + C_1\ \varphi_{\mathcal{S}}\ \gamma^{\mathrm{s}}_{\max}\ \gamma^{\mathrm{s}}_{r_{\mathbb{S}}d})} &= \mathcal{M}(C_2\gamma^{\mathrm{s}}_{\max}\gamma^{\mathrm{i}}_{r_{\mathbb{S}}d}). \end{aligned} \tag{D.7.4}$$

Further, rearranging the above equation, we get equation (5.5.6).

D.8 Proof of Result 26: Exact Expression for FASE

Expanding the expectation of the equation (5.5.3), we get

$$\begin{aligned} \overline{\mathcal{S}} = {} & \frac{L}{\overline{\gamma^{\mathrm{s}}}_{\max}\overline{\gamma^{\mathrm{i}}}_{r_{\mathbb{S}}d}\overline{\gamma^{\mathrm{s}}}_{r_{\mathbb{S}}d}\ln(2)} \int_{\gamma^{\mathrm{s}}_{\max}=0}^{\infty}\int_{\gamma^{\mathrm{i}}_{r_{\mathbb{S}}d}=0}^{\infty}\int_{\gamma^{\mathrm{s}}_{r_{\mathbb{S}}d}=0}^{\infty} \ln(1 + C_1\ \varphi^{*}_{\mathcal{S}}\ \gamma^{\mathrm{s}}_{\max}\ \gamma^{\mathrm{s}}_{r_{\mathbb{S}}d}) \exp\left(-\frac{\gamma^{\mathrm{s}}_{r_{\mathbb{S}}d}}{\overline{\gamma^{\mathrm{s}}}_{r_{\mathbb{S}}d}} \right) \\ & \times \exp\left(-\frac{\gamma^{\mathrm{i}}_{r_{\mathbb{S}}d}}{\overline{\gamma^{\mathrm{i}}}_{r_{\mathbb{S}}d}} \right) \exp\left(-\frac{\gamma^{\mathrm{s}}_{\max}}{\overline{\gamma^{\mathrm{s}}}_{\max}} \right) \left(1 - \exp\left(-\frac{\gamma^{\mathrm{s}}_{\max}}{\overline{\gamma^{\mathrm{s}}}_{\max}} \right) \right)^{L-1} \mathrm{d}\gamma^{\mathrm{s}}_{\max}\ \mathrm{d}\gamma^{\mathrm{i}}_{r_{\mathbb{S}}d}\ \mathrm{d}\gamma^{\mathrm{s}}_{r_{\mathbb{S}}d}. \end{aligned} \tag{D.8.1}$$

where, $\overline{\gamma^{\text{s}}}_{\max}$, $\overline{\gamma^{\text{s}}}_{r_{\mathbb{S}}d}$, and $\overline{\gamma^{\text{i}}}_{r_{\mathbb{S}}d}$ are the mean or average channel power gain of the $\text{S}^{\text{s}}_{\text{Tx}}-\text{R}^{\text{s}}{}_{\mathbb{S}}$ link, $\text{R}^{\text{s}}{}_{\mathbb{S}}-\text{D}^{\text{s}}_{\text{Rx}}$ link, and $\text{R}^{\text{s}}{}_{\mathbb{S}}-\text{D}^{\text{p}}_{\text{Rx}}$ link, respectively. Further, equating the optimal value of $\varphi_{\mathcal{S}}$ and applying the power conservation rule to the above equation, the optimal FASE expression can be written as

$$\overline{\mathcal{S}} = \frac{L}{\overline{\gamma^{\text{s}}}_{\max}\overline{\gamma^{\text{i}}}_{r_{\mathbb{S}}d}\overline{\gamma^{\text{s}}}_{r_{\mathbb{S}}d}\ln(2)}\int_{\gamma^{\text{i}}_{r_{\mathbb{S}}d}=0}^{\infty}\int_{\gamma^{\text{s}}_{r_{\mathbb{S}}d}=\rho(\gamma^{\text{i}}_{r_{\mathbb{S}}d})}^{\infty}\int_{\gamma^{\text{s}}_{\max}=0}^{\infty}\ln\left(\frac{C_1\gamma^{\text{s}}_{r_{\mathbb{S}}d}}{\mathcal{M}C_2\gamma^{\text{i}}_{r_{\mathbb{S}}d}\ln(2)}\right)\exp\left(-\frac{\gamma^{\text{s}}_{r_{\mathbb{S}}d}}{\overline{\gamma^{\text{s}}}_{r_{\mathbb{S}}d}}\right)$$
$$\times\exp\left(-\frac{\gamma^{\text{i}}_{r_{\mathbb{S}}d}}{\overline{\gamma^{\text{i}}}_{r_{\mathbb{S}}d}}\right)\exp\left(-\frac{\gamma^{\text{s}}_{\max}}{\overline{\gamma^{\text{s}}}_{\max}}\right)\left(1-\exp\left(-\frac{\gamma^{\text{s}}_{\max}}{\overline{\gamma^{\text{s}}}_{\max}}\right)\right)^{L-1}\text{d}\gamma^{\text{s}}_{\max}\,\text{d}\gamma^{\text{s}}_{r_{\mathbb{S}}d}\,\text{d}\gamma^{\text{i}}_{r_{\mathbb{S}}d}. \tag{D.8.2}$$

Assume, $1-\exp(-\frac{\gamma^{\text{s}}_{\max}}{\overline{\gamma^{\text{s}}}_{\max}}) = \text{u}$. We get

$$\overline{\mathcal{S}} = \frac{L}{\overline{\gamma^{\text{s}}}_{\max}\overline{\gamma^{\text{i}}}_{r_{\mathbb{S}}d}\overline{\gamma^{\text{s}}}_{r_{\mathbb{S}}d}\ln(2)}\int_{\gamma^{\text{i}}_{r_{\mathbb{S}}d}=0}^{\infty}\int_{\gamma^{\text{s}}_{r_{\mathbb{S}}d}=\rho(\gamma^{\text{i}}_{r_{\mathbb{S}}d})}^{\infty}\ln\left(\frac{C_1\gamma^{\text{s}}_{r_{\mathbb{S}}d}}{\mathcal{M}C_2\gamma^{\text{i}}_{r_{\mathbb{S}}d}\ln(2)}\right)\exp\left(-\frac{\gamma^{\text{s}}_{r_{\mathbb{S}}d}}{\overline{\gamma^{\text{s}}}_{r_{\mathbb{S}}d}}\right)$$
$$\times\exp\left(-\frac{\gamma^{\text{i}}_{r_{\mathbb{S}}d}}{\overline{\gamma^{\text{i}}}_{r_{\mathbb{S}}d}}\right)\left[\int_{\text{u}=0}^{1}\overline{\gamma^{\text{s}}}_{\max}\,\text{u}^{L-1}\text{du}\right]\text{d}\gamma^{\text{s}}_{r_{\mathbb{S}}d}\,\text{d}\gamma^{\text{i}}_{r_{\mathbb{S}}d}. \tag{D.8.3}$$

Applying power rule [156], $\left[\int_{\text{u}=0}^{1}\overline{\gamma^{\text{s}}}_{\max}\,\text{u}^{L-1}\text{du}\right] = \frac{\overline{\gamma^{\text{s}}}_{\max}}{L}$. Therefore, the above equation can be written as

$$\overline{\mathcal{S}} = \frac{1}{\overline{\gamma^{\text{i}}}_{r_{\mathbb{S}}d}\overline{\gamma^{\text{s}}}_{r_{\mathbb{S}}d}\ln(2)}\int_{\gamma^{\text{i}}_{r_{\mathbb{S}}d}=0}^{\infty}\left[\int_{\gamma^{\text{s}}_{r_{\mathbb{S}}d}=\rho(\gamma^{\text{i}}_{r_{\mathbb{S}}d})}^{\infty}\ln\left(\frac{C_1\gamma^{\text{s}}_{r_{\mathbb{S}}d}}{\mathcal{M}C_2\gamma^{\text{i}}_{r_{\mathbb{S}}d}\ln(2)}\right)\exp\left(-\frac{\gamma^{\text{s}}_{r_{\mathbb{S}}d}}{\overline{\gamma^{\text{s}}}_{r_{\mathbb{S}}d}}\right)\text{d}\gamma^{\text{s}}_{r_{\mathbb{S}}d}\right]$$
$$\times\exp\left(-\frac{\gamma^{\text{i}}_{r_{\mathbb{S}}d}}{\overline{\gamma^{\text{i}}}_{r_{\mathbb{S}}d}}\right)\text{d}\gamma^{\text{i}}_{r_{\mathbb{S}}d}. \tag{D.8.4}$$

Integrating the above equation by parts, we get

$$\overline{\mathcal{S}} = \frac{1}{\overline{\gamma^{\text{i}}}_{r_{\mathbb{S}}d}\ln(2)}\int_{\gamma^{\text{i}}_{r_{\mathbb{S}}d}=0}^{\infty}\ln\left(\frac{C_1\,\rho(\gamma^{\text{i}}_{r_{\mathbb{S}}d})}{\mathcal{M}C_2\gamma^{\text{i}}_{r_{\mathbb{S}}d}\ln(2)}\right)\exp\left(-\frac{\rho(\gamma^{\text{i}}_{r_{\mathbb{S}}d})}{\overline{\gamma^{\text{s}}}_{r_{\mathbb{S}}d}}\right)\exp\left(-\frac{\gamma^{\text{i}}_{r_{\mathbb{S}}d}}{\overline{\gamma^{\text{i}}}_{r_{\mathbb{S}}d}}\right)\text{d}\gamma^{\text{i}}_{r_{\mathbb{S}}d}$$
$$+\frac{1}{\overline{\gamma^{\text{i}}}_{r_{\mathbb{S}}d}\ln(2)}\int_{\gamma^{\text{i}}_{r_{\mathbb{S}}d}=0}^{\infty}\mathbf{E}_1\left(\frac{\rho(\gamma^{\text{i}}_{r_{\mathbb{S}}d})}{\overline{\gamma^{\text{s}}}_{r_{\mathbb{S}}d}}\right)\exp\left(-\frac{\gamma^{\text{i}}_{r_{\mathbb{S}}d}}{\overline{\gamma^{\text{i}}}_{r_{\mathbb{S}}d}}\right)\text{d}\gamma^{\text{i}}_{r_{\mathbb{S}}d}. \tag{D.8.5}$$

Applying the value of $\rho(\gamma^{\text{i}}_{r_{\mathbb{S}}d})$ from equation (5.5.7) in above equation can be written as

$$\overline{\mathcal{S}} = \frac{1}{\overline{\gamma^{\text{i}}}_{r_{\mathbb{S}}d}\ln(2)}\int_{\gamma^{\text{i}}_{r_{\mathbb{S}}d}=0}^{\infty}\mathbf{E}_1\left(\frac{\mathcal{M}C_2\ln(2)\gamma^{\text{i}}_{r_{\mathbb{S}}d}}{C_1\overline{\gamma^{\text{s}}}_{r_{\mathbb{S}}d}}\right)\exp\left(-\frac{\gamma^{\text{i}}_{r_{\mathbb{S}}d}}{\overline{\gamma^{\text{i}}}_{r_{\mathbb{S}}d}}\right)\text{d}\gamma^{\text{i}}_{r_{\mathbb{S}}d}. \tag{D.8.6}$$

Solving the above equation, we get

$$\overline{\mathcal{S}} = \frac{\mathcal{M}C_2}{C_1\overline{\gamma^{\text{s}}}_{r_{\mathbb{S}}d} + \mathcal{M}C_2\overline{\gamma^{\text{i}}}_{r_{\mathbb{S}}d}\ln(2)}\Bigg[-\overline{\gamma^{\text{i}}}_{r_{\mathbb{S}}d}\ \ln\Big(\frac{\mathcal{M}C_2\ln(2)}{C_1}\Big) + \frac{C_1\overline{\gamma^{\text{s}}}_{r_{\mathbb{S}}d}}{\mathcal{M}C_2\ln(2)}$$
$$\times \ln\Big(\frac{C_1}{\mathcal{M}C_2\ln(2)}\Big) + \Big(\frac{C_1\overline{\gamma^{\text{s}}}_{r_{\mathbb{S}}d}}{\mathcal{M}C_2\ln(2)} + \overline{\gamma^{\text{i}}}_{r_{\mathbb{S}}d}\Big) \times \Big[\ln\Big(\frac{\mathcal{M}C_2\ln(2)}{C_1\overline{\gamma^{\text{s}}}_{r_{\mathbb{S}}d}}\Big) + \ln(\overline{\gamma^{\text{s}}}_{r_{\mathbb{S}}d})$$
$$+ \ln\Big(1 + \frac{C_1\overline{\gamma^{\text{s}}}_{r_{\mathbb{S}}d}}{\mathcal{M}C_2\overline{\gamma^{\text{i}}}_{r_{\mathbb{S}}d}\ln(2)}\Big)\Big]\Bigg]. \tag{D.8.7}$$

Further simplifying the above equation, we get equation (5.5.8).

D.9 Proof of Result 27: Optimal Relay Gain Function for FAEE

Since the optimization problem stated in (5.5.12) is concave, we re-formulate it into a convex optimization problem. Therefore the problem statement can be written as

$$\min_{\varphi_{\mathcal{E}}} -\mathbf{E}\left[\frac{\log_2(1 + C_1\ \varphi_{\mathcal{E}}\ \gamma^{\text{s}}_{\max}\ \gamma^{\text{s}}_{r_{\mathbb{S}}d})}{(P_s + P_c) + C_3\ \varphi_{\mathcal{E}}\ \gamma^{\text{s}}_{\max}}\right], \tag{D.9.1}$$

$$\text{s.t. } \mathbf{E}\left[C_2\ \varphi_{\mathcal{E}}\ \gamma^{\text{s}}_{\max}\gamma^{\text{i}}_{r_{\mathbb{S}}d}\right] \leq I_{\text{th}}. \tag{D.9.2}$$

It can be analyzed that the objective function written in equation (D.9.1) is convex. Therefore, to obtain the optimum solution, we use the Lagrange multiplier method. Hence, the Lagrange function for the optimization problem shown above can be written as

$$L_{\mathcal{M}'}(\varphi_{\mathcal{E}}) = \mathbf{E}\left[-\frac{\log_2(1 + C_1\ \varphi_{\mathcal{E}}\ \gamma^{\text{s}}_{\max}\ \gamma^{\text{s}}_{r_{\mathbb{S}}d})}{(P_s + P_c) + C_3\ \varphi_{\mathcal{E}}\ \gamma^{\text{s}}_{\max}} + \mathcal{M}'(C_2\ \varphi_{\mathcal{E}}\ \gamma^{\text{s}}_{\max}\gamma^{\text{i}}_{r_{\mathbb{S}}d} - I_{\text{th}})\right]. \tag{D.9.3}$$

To obtain the optimum value of $\varphi_{\mathcal{E}}$, we drop the expectation of equation (D.9.3) (by virtue of convexity), differentiate it with respect to $\varphi_{\mathcal{E}}$, and equate it to zero, we get

$$\frac{\partial}{\partial\varphi_{\mathcal{E}}(\gamma^{\text{s}}_{\max}, \gamma^{\text{s}}_{r_{\mathbb{S}}d}, \gamma^{\text{i}}_{r_{\mathbb{S}}d})}\Bigg[-\frac{\log_2(1 + C_1\ \varphi_{\mathcal{E}}(\gamma^{\text{s}}_{\max}, \gamma^{\text{s}}_{r_{\mathbb{S}}d}, \gamma^{\text{i}}_{r_{\mathbb{S}}d})\ \gamma^{\text{s}}_{\max}\ \gamma^{\text{s}}_{r_{\mathbb{S}}d})}{(P_s + P_c) + C_3\varphi_{\mathcal{E}}(\gamma^{\text{s}}_{\max}, \gamma^{\text{s}}_{r_{\mathbb{S}}d}, \gamma^{\text{i}}_{r_{\mathbb{S}}d})\gamma^{\text{s}}_{\max}}$$
$$+ \mathcal{M}'(C_2\varphi_{\mathcal{E}}(\gamma^{\text{s}}_{\max}, \gamma^{\text{s}}_{r_{\mathbb{S}}d}, \gamma^{\text{i}}_{r_{\mathbb{S}}d})\gamma^{\text{s}}_{\max}\gamma^{\text{i}}_{r_{\mathbb{S}}d} - I_{\text{th}})\Bigg] = 0. \tag{D.9.4}$$

Further, re-arranging the above equation, we get equation (5.5.14).

D.10 Proof of Result 28: Exact Expression for FAEE

Expanding the expectation of the equation (5.5.11), we get

$$\overline{\mathcal{E}}=\frac{L}{\overline{\gamma}^{\mathrm{s}}{}_{\max}\overline{\gamma}^{\mathrm{i}}{}_{r_{\mathbb{s}}d}\overline{\gamma}^{\mathrm{s}}{}_{r_{\mathbb{s}}d}\ln(2)}\int_{\gamma^{\mathrm{s}}_{\max}=0}^{\infty}\int_{\gamma^{\mathrm{i}}_{r_{\mathbb{s}}d}=0}^{\infty}\int_{\gamma^{\mathrm{s}}_{r_{\mathbb{s}}d}=0}^{\infty}\frac{\ln(1+C_1\ \varphi_{\mathcal{E}}\ \gamma^{\mathrm{s}}_{\max}\ \gamma^{\mathrm{s}}_{r_{\mathbb{s}}d})}{(P_s+P_c)+C_3\varphi_{\mathcal{E}}\gamma^{\mathrm{s}}_{\max}}\exp\left(-\frac{\gamma^{\mathrm{s}}_{r_{\mathbb{s}}d}}{\overline{\gamma}^{\mathrm{s}}{}_{r_{\mathbb{s}}d}}\right)$$
$$\times\exp\left(-\frac{\gamma^{\mathrm{i}}_{r_{\mathbb{s}}d}}{\overline{\gamma}^{\mathrm{i}}{}_{r_{\mathbb{s}}d}}\right)\exp\left(-\frac{\gamma^{\mathrm{s}}_{\max}}{\overline{\gamma}^{\mathrm{s}}{}_{\max}}\right)\left(1-\exp\left(-\frac{\gamma^{\mathrm{s}}_{\max}}{\overline{\gamma}^{\mathrm{s}}{}_{\max}}\right)\right)^{L-1}\mathrm{d}\gamma^{\mathrm{s}}_{\max}\ \mathrm{d}\gamma^{\mathrm{i}}_{r_{\mathbb{s}}d}\ \mathrm{d}\gamma^{\mathrm{s}}_{r_{\mathbb{s}}d}. \tag{D.10.1}$$

Applying the optimal RGF and the power conservation rule, the above equation can be expressed as equation (5.5.16).

D.11 Proof of Result 29: Upper Bound Expression for FAEE

Applying the Jensen's inequality to the spectral efficiency term of equation (5.5.11), the upper bound of the FAEE can be written as

$$\overline{\mathcal{E}}_{\mathrm{UB}}=\frac{1}{\ln(2)}\ln\Bigg[1+\mathcal{R}\int_{\gamma^{\mathrm{s}}_{\max}=0}^{\infty}\int_{\gamma^{\mathrm{i}}_{r_{\mathbb{s}}d}=0}^{\infty}\int_{\gamma^{\mathrm{s}}_{r_{\mathbb{s}}d}=0}^{\infty}(C_1\ \varphi_{\mathcal{E}}\ \gamma^{\mathrm{s}}_{\max}\ \gamma^{\mathrm{s}}_{r_{\mathbb{s}}d})\exp\left(-\frac{\gamma^{\mathrm{s}}_{r_{\mathbb{s}}d}}{\overline{\gamma}^{\mathrm{s}}{}_{r_{\mathbb{s}}d}}\right)$$
$$\times\exp\left(-\frac{\gamma^{\mathrm{i}}_{r_{\mathbb{s}}d}}{\overline{\gamma}^{\mathrm{i}}{}_{r_{\mathbb{s}}d}}\right)\exp\left(-\frac{\gamma^{\mathrm{s}}_{\max}}{\overline{\gamma}^{\mathrm{s}}{}_{\max}}\right)\left(1-\exp\left(-\frac{\gamma^{\mathrm{s}}_{\max}}{\overline{\gamma}^{\mathrm{s}}{}_{\max}}\right)\right)^{L-1}\mathrm{d}\gamma^{\mathrm{s}}_{\max}\ \mathrm{d}\gamma^{\mathrm{i}}_{r_{\mathbb{s}}d}\ \mathrm{d}\gamma^{\mathrm{s}}_{r_{\mathbb{s}}d}\Bigg]$$
$$\times\Bigg[\mathcal{R}\int_{\gamma^{\mathrm{s}}_{\max}=0}^{\infty}\int_{\gamma^{\mathrm{i}}_{r_{\mathbb{s}}d}=0}^{\infty}\int_{\gamma^{\mathrm{s}}_{r_{\mathbb{s}}d}=0}^{\infty}\frac{1}{(P_s+P_c)+C_3\varphi_{\mathcal{E}}\gamma^{\mathrm{s}}_{\max}}\exp\left(-\frac{\gamma^{\mathrm{s}}_{r_{\mathbb{s}}d}}{\overline{\gamma}^{\mathrm{s}}{}_{r_{\mathbb{s}}d}}\right)$$
$$\times\exp\left(-\frac{\gamma^{\mathrm{i}}_{r_{\mathbb{s}}d}}{\overline{\gamma}^{\mathrm{i}}{}_{r_{\mathbb{s}}d}}\right)\exp\left(-\frac{\gamma^{\mathrm{s}}_{\max}}{\overline{\gamma}^{\mathrm{s}}{}_{\max}}\right)\left(1-\exp\left(-\frac{\gamma^{\mathrm{s}}_{\max}}{\overline{\gamma}^{\mathrm{s}}{}_{\max}}\right)\right)^{L-1}\mathrm{d}\gamma^{\mathrm{s}}_{\max}\ \mathrm{d}\gamma^{\mathrm{i}}_{r_{\mathbb{s}}d}\ \mathrm{d}\gamma^{\mathrm{s}}_{r_{\mathbb{s}}d}\Bigg], \tag{D.11.1}$$

where $\mathcal{R}=\frac{L}{\overline{\gamma}^{\mathrm{s}}{}_{\max}\overline{\gamma}^{\mathrm{i}}{}_{r_{\mathbb{s}}d}\overline{\gamma}^{\mathrm{s}}{}_{r_{\mathbb{s}}d}}$. Further, substituting the optimum $\varphi_{\mathcal{E}}$ and applying power conservation rule, the above equation can be expressed as (5.5.17).