1. The impact of random channel failures and their recovery on CRN supporting heterogeneous secondary traffic is investigated to obtain more realistic results of performance evaluation.
2. An in-depth dependability theory-based analysis for system availability and reliability is performed considering different dependability attributes, which could help to find means for providing services and achieving URC.
3. The joint use of channel reservation and retrial policy is proposed and tested to mitigate the forced termination and blocking of services.
4. The transient dynamics of the system is investigated to capture the short-term behavior of the CRN which portrays more practical queueing scenarios.

The rest of the chapter is structured as follows. In Section 5.2, an overview of the related work is given. In Section 5.3, we describe the system model together with the assumptions. Section 5.4 presents the proposed dynamic spectrum access (DSA) scheme. An extensive analytical model based on multi-dimensional continuous-time Markov chain (CTMC) is given in Section 5.5. Following this, in Section 5.6, we obtain the solution for the transient probabilities and derive dependability-oriented performance indices of the system. In Section 5.7, we present and discuss the main numerical results and thereafter conclude the chapter in Section 5.8.

5.2 Related Work

Most of the previous works on CRN are focused on aspects such as DSA and spectrum sensing (see e.g., Politis et al. [238]; Jiao et al. [239]). Few studies exist in the literature which explore the significance of channel failures when evaluating the performance of CRNs. However, most of those examined system-centric performance analysis instead of the analysis from the perspectives of dependability theory. With the focus on the topology of CRN, Sun et al. [240] investigated the spreading of random failures. While therein correlations among failures are studied, the analysis of failure statistics is confined. A retransmission-based secondary access mechanism was proposed by Hamza and Aïssa [241] to improve the performance of PU and SU services by taking into account the event of transmission failures. Therein, the SU throughput maximization was the main objective of authors however, the forced termination of ongoing transmissions is not considered. Rather, Rodríguez-Estrello et al. [242] investigated forced termination and blocking probability analysis by taking into account both resource insufficiency and link unreliability, wherein interrupted and blocked services are lost. However, in practice, interrupted and blocked users may retry again. From this point of view, appropriate retrial queueing models might be useful for cognitive systems.

As discussed in Chapter 1, retrials are characterized by the feature that a user who is unable to obtain service, joins an orbit (a fictitious queue) and make repeated attempts for service. Recently, retrial queueing models in CRN are receiving increasing attention which are more appropriate to characterize the impersistent and impatient behavior of SUs. These include the models proposed by Dudin et al. [71], AlQahtani [243], Nemouchi and Sztrik [244] and Zhang et al. [245]. However, amongst them just Dudin et al. [71] have taken (only) reneging, and AlQahtani [243] and Zhang et al. [245] have taken (only) balking phenomenon into account.

To improve spectrum utilization in CRNs, some researchers have also emphasized on channel reservation mechanisms. A channel reservation scheme for PUs was developed by Chakraborty and Misra [246], where a certain number of channels are reserved for PUs. However, by employing this policy, forced terminations of SUs may not be decreased substantially. Note however that none of the above studies is persued from a dependability perspective while performing reliability or availability analysis.

Recently, to investigate the performance from dependability theory perspectives, a DSA scheme with channel reservation was proposed by Balapuwaduge et al. [247] who later modified their work [248] to investigate the heterogeneity of failures. Nevertheless, none of them have dealt with the heterogeneity of SUs, while heterogeneity of users is a valuable feature as each arriving SU carries a distinct job and may differ in delay sensitivity. Neither do they investigate the transient dynamics of the system. Despite the

Table 5.1: Related work comparison among various Markov process-based models.

Reference	Channel Failure	Channel Reservation	Retrans-mission	Heteroge-neous SUs	Dependability Perspective	Time Domain
Dudin et al. [71]	✗	✓(S)	✓(R)	✓	✗	✗
Hamza and Aïssa [241]	✓(HM)	✗	✓	✗	✗	✗
Rodríguez-Estrello et al. [242]	✓(HM)	✗	✗	✗	✗	✗
Nemouchi and Sztrik [244]	✓(HM)	✗	✓	✗	✗	✗
Balapuwaduge et al. [247]	✓(HM)	✓(S)	✗	✗	✓	✗
Balapuwaduge et al. [248]	✓(HT)	✓(S)	✗	✗	✓	✗
Balapuwaduge et al. [102]	✗	✗	✗	✗	✓	✓
Chu et al. [249]	✗	✓(S)	✗	✓	✗	✗
Falcão et al. [250]	✗	✓(M)	✗	✓	✗	✗
Chapter 5	✓(HM)	✓(S)	✓(B+R)	✓	✓	✓
Chapter 6	✓(HT)	✓(M)	✗	✓	✓	✓

relevance of transient analysis for communication networks, the literature on it is sparse due to the fact that transient state queuing analysis pursued so far quickly becomes intractable. Another work by Balapuwaduge et al. [102] focused on investigating reliable communication from dependability theory's perspectives in time domain analysis, however, the considered reliability metrics therein are distinct from the metrics defined in this chapter.

Thereby, considering the aspects not fully covered by the aforementioned studies, we deal with the transient analysis of a multi-channel CRN considering different dependability attributes, whose definitions conform with the International Telecommunication Union (ITU) recommendations [251]. The main interest of the present work is to perform realistic reliability and availability analysis of error-prone CRNs. More specifically, this chapter evaluates the impact of random channel failures on the CRN system supporting heterogeneous traffic, in terms of dependability measures and investigates the advantages of joint use of channel reservation and retrial phenomenon on such CRNs' performance in time domain.

The literature on some key Markov process-based analytical models is summarized with comparison in Table 5.1. In the table, the "✓" symbol indicates that the analytical model supports the corresponding feature while the "✗" symbol indicates that the analytical model does not support the corresponding feature. Therein, the notations HM and HT denote channel failures of homogeneous and heterogeneous type, respectively. The notations S and M indicate channel reservation as single-level and multi-level, respectively. Also, B and R refer to balking and reneging phenomenon, if taken into account with retransmission.

5.3 Network Scenario and Assumptions

We consider a centralized CRN architecture that allocates spectrum resources for both PUs and SUs. In our analysis, we consider heterogeneous SU traffic types, i.e. low priority traffic or elastic traffic such as video streaming and file transfer, and high priority traffic or real-time traffic such as video conference and VoIP. Herein, the abbreviations ESU and RSU denote elastic and real-time traffic, respectively whereas for conciseness, the term SU denotes both SU services. The total licensed spectrum band in the CRN is divided into $M \in \mathbb{Z}^+$ equal bandwidth channels. Each channel is subject to different kinds of failures that could interrupt ongoing transmissions. As an essential measure, in order to avoid forced terminations, out of these M channels, a small number of channels, R, are reserved.

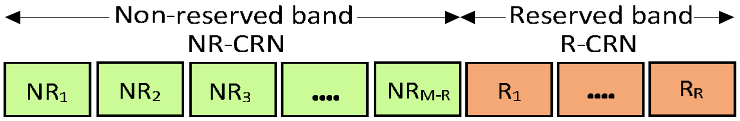

Figure 5.1: Reserved (R-CRN) and non-reserved (NR-CRN) channel assignment where NR_i, $i \in \{1, 2, ..., M - R\}$ represents the channel in the NR-CRN and R_j, $j \in \{1, 2, ..., R\}$ represents the reserved channels for forcibly terminated services in the R-CRN.

However, the reservation of channels for ongoing interrupted services will record an increase in the blocking probability as a consequence. Moreover, due to under-utilization of channels, the reservation will degrade the overall capacity. To reduce this capacity loss and blocking for new user requests, we set an upper bound on R as $R \leq \lfloor \frac{M}{A} \rfloor$ where the parameter A limits the value of R and $A > 1$. The set of reserved channels is denoted by R-CRN whereas that of non-reserved channels is denoted as NR-CRN, as depicted in Fig. 5.1. These reserved channels can only be accessed by the interrupted RSUs and PUs as a result of channel failures or the RSUs that are preempted upon PU arrivals. The reason for applying such restriction is to maintain a higher retainability level for established connections, which is one of the main key performance indicators (KPIs) in future wireless systems. Furthermore, the RSU services have priority over the ESU services in case of preemption of a SU service upon PU arrival i.e. the ongoing ESUs are subject to termination prior to the RSUs. In order to protect the blocked and preempted/interrupted ESU services by the preemptive priority mechanism and channel failures, an infinite retrial orbit is employed for later retrial transmission. Moreover, to develop the analytical model presented in Section 5.5, we made the following assumptions as the basis.

- The arrival of users follow Poisson processes with rates λ_{PU}, λ_{SE} and λ_{SR} for PU, ESU and RSU services respectively. And the service times for PU, ESU, and RSU services are exponentially distributed with corresponding service rates μ_{PU}, μ_{SE} and μ_{SR} respectively.

- The channel failure is generated by an exponential distribution with rate α per channel in both NR-CRN and R-CRN. Also, we assume that both occupied and idle channels are vulnerable to failures.

- As soon as a channel fails, the failed channel undergoes for repair immediately. The repair time of a failed channel in both NR-CRN and R-CRN is assumed to have an exponential distribution, with a repair rate per channel β.

- For ESU, a blocked service due to lack of capacity or system failure either enters the retrial orbit with probability b or balks the system with complementary probability $\bar{b} = (1 - b)$. While the users staying in the orbit, some impatient users may depart from the system when they make service attempts and finds no channel available. This queueing phenomenon is referred to as reneging. That is, if the retrial customer finds all the channels in the NR-CRN busy or failed, then it either returns to the orbit with probability r or reneges the system with probability $\bar{r} = (1-r)$. Moreover, an interrupted service due to channel failure or a preempted service upon PU arrival joins the orbit with probability θ or leaves the system with probability $(1 - \theta)$. Note that θ is used to represent the degree of impatience of ESU services.

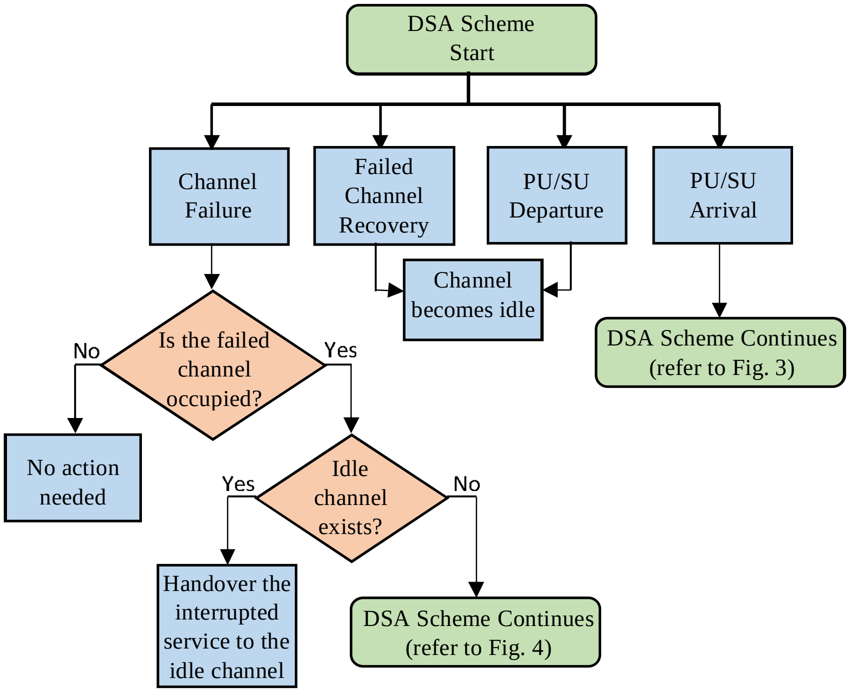

Figure 5.2: Flow chart illustration of the proposed DSA scheme in part.

- The classical retrial policy described by Phung-Duc [252], wherein the retrial rate is proportional to the number of users in the orbit is taken into account with truncation. The reason being that though the orbit is of infinite capacity, the number of users utilizing the system is finite for most of the applications. The random time of successive repeated attempts made by every orbiting ESU is exponentially distributed with parameter γ. The retrial rate being proportional to the number of ESUs in the orbit is given by $\gamma_n = min(n, N)\gamma$, where N denotes the maximum number of orbiting customers allowed to make retrials.

- The inter-arrival times, service times, inter-retrial times, and repair times are independent of each other.

- The spectrum adaptation and sensing latency is negligible in contrast with the duration between two consecutive events. Herein, the events indicate PUs' and SUs' arrivals or departures.

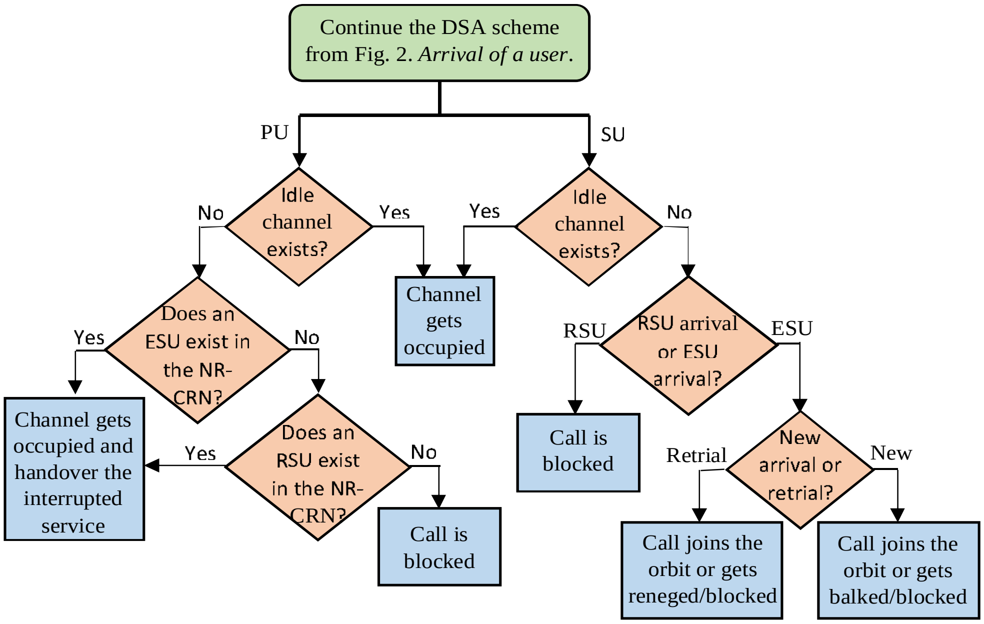

Figure 5.3: Flow chart illustration of the proposed DSA scheme when an user arrived.

5.4 The Proposed Dynamic Spectrum Access Scheme

In this section, we provide details of the channel access procedure adopted upon the occurrence of six events. The DSA scheme with those six events is partly illustrated in a flow chart in Fig. 5.2.

1) PU Arrival: If a new PU arrives and finds a vacant channel available in the NR-CRN, the arriving PU can commence transmission on that vacant channel. In the case where all the channels are occupied in the NR-CRN while at least one channel is occupied by an ESU, then one of those ESUs is preempted and that channel is re-allocated to the newly arriving PU, as indicated in Fig. 5.3. The preempted ESU performs handover to another idle channel in NR-CRN, if available, otherwise, it either leaves the system with probability $(1 - \theta)$ or joins the orbit with probability θ for later retrial. Moreover, if the arriving PU finds all the channels occupied by PUs or RSUs in the NR-CRN, then one of the RSUs in the NR-CRN is preempted. The preempted RSU is allowed to access the R-CRN, where the channels are reserved for those ongoing PU and RSU services which get interrupted and do not find any other channel in the NR-CRN. Accordingly, the preempted RSU performs spectrum handover to a vacant channel in the whole CRN, if available, otherwise, it is forced to terminate. As mentioned earlier, the reason for the above mentioned priority mechanism is to preserve ongoing RSUs' transmissions prior to that of the ESUs. However, if the entire NR-CRN is occupied by PUs and/or failed, then a new PU request gets blocked.

2) SU Arrival: When a new SU (either ESU or RSU) arrives, the system allocates a vacant channel to the newly arriving SU from the NR-CRN, if it exists. Otherwise, the new RSU request is blocked whereas the new ESU request either joins the retrial orbit with probability b or balks from the system with probability $(1 - b)$, as presented in Fig. 5.3. Moreover, there is no provision of exploring the channel availability in the R-CRN for a new SU. Since PUs being of highest priority over the channels in NR-CRN, hence ongoing PU services cannot be preempted by the newly arrived SU. Further, when an ESU service in the orbit retries to access the system, the ESU repeats the same process as a newly arriving ESU, however with the different (retrial) rate. That is, if upon retrial attempt, the ESU service finds no channel available for its transmission, then it rejoins the orbit with probability r or reneges the system with probability $(1 - r)$.

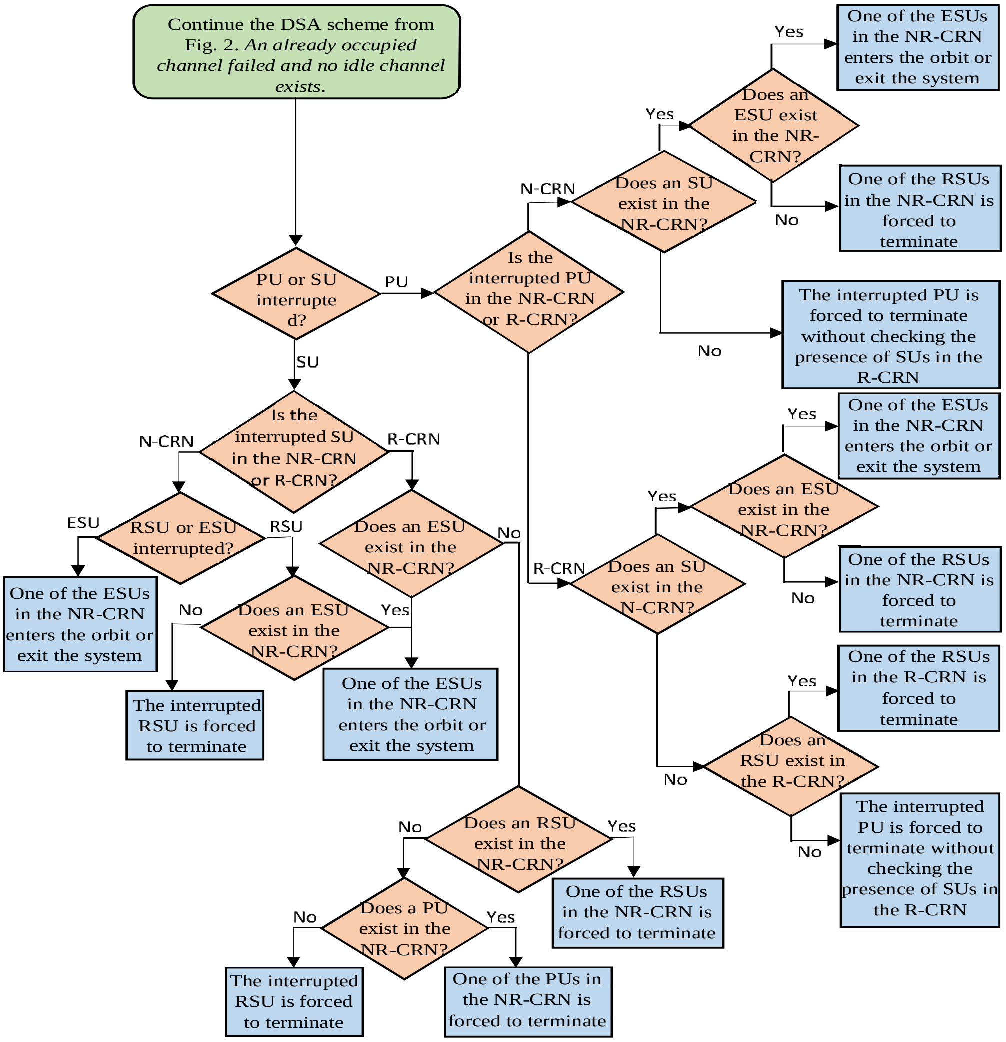

Figure 5.4: Flow chart illustration of the proposed DSA scheme when an occupied channel failed.

3) PU/SU Departure: A departure event of a PU or a SU service from the NR-CRN or the R-CRN merely creates a vacant channel, as depicted in Fig. 5.2. And after a departure of any service from the system, no handover actions are performed. The reason is as follows. Upon a user departure from the NR-CRN, if an ongoing RSU in the R-CRN would have handed over to the NR-CRN, the channel occupancy of the NR-CRN would increase, which, in turn, would increase the blocking probability of SUs. In addition, such a handover process would result in additional control traffic in the system and additional complexity in our CTMC model.

4) Channel Failure: When a vacant channel in the NR-CRN (R-CRN) fails, the number of channels available in the NR-CRN (R-CRN) is decreased by one. It follows that the total number of available channels in the system get accordingly decreased by one. On the other hand, if an occupied channel in the R-CRN or NR-CRN fails, the interrupted user may perform a spectrum handover procedure, as illustrated in Fig. 5.4. This is performed based on the priority levels assigned to PUs and SUs in the NR-CRN and R-CRN. In the proposed scheme, the ongoing PUs in the R-CRN have the highest priority whereas the RSUs in the R-CRN are regarded with the second highest priority. Furthermore, the PU, RSU and ESU services in the NR-CRN are considered as the third, fourth and fifth priority classes respectively. Let $PL(s)$ denote the priority level assigned to a service type, s. Accordingly, $PL(PU_{R-CRN}) > PL(RSU_{R-CRN}) > PL(PU_{NR-CRN}) > PL(RSU_{NR-CRN}) > PL(ESU_{NR-CRN})$ represents the priority level assignment for PU and SU services. In consequence, upon a channel failure, the interrupted PU services in the NR-CRN cannot preempt the RSU services in the reserved band, being of lower priority than the RSUs in the R-CRN.

5) Channel Repair: A channel is said to be repaired when a failed channel is re-established to its normal conditions and ready to serve the users. However, no spectrum handover is performed when a failed channel is repaired, as indicated in Fig. 5.2.

5.5 Markov Chain Modeling

We develop an analytical model utilizing a CTMC with discrete states for the proposed DSA. Let S denote the set of feasible states and $\boldsymbol{x} = (i_n, j_n^R, j_n^E, i_r, j_r^R, f_n, f_r, m)$ represents a state of the CTMC model where i_n, j_n^R and j_n^E denotes the number of in-service PUs, RSUs and ESUs in the NR-CRN, i_r, and j_r^R denote the number of in-service PUs and RSUs in the R-CRN respectively, f_n and f_r denote the number of failed channels in the NR-CRN and R-CRN respectively, and m denotes the number of ESUs in the retrial orbit.

It follows that the total number of occupied plus failed channels in the NR-CRN and in the R-CRN for a given state $\boldsymbol{x}$, denoted by $B_n(\boldsymbol{x})$ and $B_r(\boldsymbol{x})$ respectively, are given by $B_n(\boldsymbol{x}) = i_n + j_n^R + j_n^E + f_n$ and $B_r(\boldsymbol{x}) = i_r + j_r^R + f_r$. Furthermore, denote the sum of occupied plus failed channels in the whole CRN as $B(\boldsymbol{x})$, i.e., $B(\boldsymbol{x}) = B_n(\boldsymbol{x}) + B_r(\boldsymbol{x})$. Thus, the number of vacant channels in state $\boldsymbol{x}$ can be computed as $M - B(\boldsymbol{x})$. Table 5.2 list the state transition rates associated with the above mentioned events under certain conditions. Therein, notations AR and DP indicate an arrival and a departure event, respectively. RSU, ESU and PU service in the NR-CRN are denoted as RSU_N, ESU_N and PU_N, respectively. RSU and PU service in the R-CRN are denoted as RSU_R and PU_R, respectively. Also, e_k denotes an eight element vector whose k^{th} component is equal to 1 while the other components are equal to 0.

Therein, all possible transitions from a generic state $\boldsymbol{x}$ are in accordance with those six events illustrated in the proposed DSA scheme in Section 5.4 on the state space

$$\begin{aligned}S = \{(i_n, j_n^R, j_n^E, i_r, j_r^R, f_n, f_r, m) | 0 \le i_n \le M - R, 0 \le j_n^R \le M - R - i_n,\\ 0 \le j_n^E \le (M - R) - i_n - j_n^R, 0 \le i_r \le R, 0 \le j_r^R \le R - i_r,\\ 0 \le f_n \le (M - R) - i_n - j_n^R - j_n^E, 0 \le f_r \le R - i_r - j_r^R, m \ge 0\}.\end{aligned}$$

For instance, transitions from a generic state $\boldsymbol{x}$ are described as follows.

- Transition to $\boldsymbol{x} + \boldsymbol{e}_1$. This happens when a PU arrives at the system and occupies a vacant channel in the NR-CRN, if exist any i.e. if $B_n(\boldsymbol{x}) < M - R$.

- Transition to $\boldsymbol{x} + \boldsymbol{e_1} - \boldsymbol{e_2} + \boldsymbol{e_5}$. This happens when an arriving PU finds all the channels occupied in the NR-CRN with no ESU in service ($B_n(\boldsymbol{x}) = M - R; j_n^E = 0$) while at least one channel is occupied by an RSU ($j_n^R > 0$). One of those RSUs is preempted by the PU and the RSU performs handover to a vacant channel in the R-CRN ($B_r(\boldsymbol{x}) < R$), thereby increasing i_n by 1, decreasing j_n^R by 1 and increasing

j_n^E by 1.

- Transition to $\boldsymbol{x} - \boldsymbol{e_1} + \boldsymbol{e_4} + \boldsymbol{e_6}$. This happens when in the NR-CRN, a channel occupied by PU ($i_n > 0$) fails, increasing f_n by 1. While no vacant channel is available in the NR-CRN ($B_n(\boldsymbol{x}) = M - R$), the interrupted PU accesses a vacant channel in the R-CRN ($B_r(\boldsymbol{x}) < R$), thereby decreasing i_n by 1 and increasing i_r by 1.

Along the similar lines, the transitions and the transition rates are provided in Table 5.2, which are then used to formulate the transition rate matrix Q.

Table 5.2: Transitions form a generic state $\boldsymbol{x} = (i_n, j_n^R, j_n^E, i_r, j_r^R, f_n, f_r, m)$ of the proposed DSA scheme upon PU/RSU/ESU events and channel failures and repairs.

S.No.	Event	Dest. State	Trans. rate	Conditions
1.	PU AR. A vacant channel exists in the NR-CRN.	$\boldsymbol{x} + \boldsymbol{e_1}$	λ_{PU}	$B_n(\boldsymbol{x}) < M - R$
2.	PU AR. No vacant channels exist in the NR-CRN. An ESU_N enters the retrial orbit.	$\boldsymbol{x} + \boldsymbol{e_1} - \boldsymbol{e_3} + \boldsymbol{e_8}$	$\theta\lambda_{PU}$	$B_n(\boldsymbol{x}) = M - R; j_n^E > 0$
3.	PU AR. No vacant channels exist in the NR-CRN. An ESU_N gets terminate.	$\boldsymbol{x} + \boldsymbol{e_1} - \boldsymbol{e_3}$	$(1-\theta)\lambda_{PU}$	$B_n(\boldsymbol{x}) = M - R; j_n^E > 0$
4.	PU AR. No vacant channels exist in the NR-CRN. An RSU_N performs handover to the R-CRN.	$\boldsymbol{x} + \boldsymbol{e_1} - \boldsymbol{e_2} + \boldsymbol{e_5}$	λ_{PU}	$B_n(\boldsymbol{x}) = M - R; j_n^E = 0; j_n^R > 0; B_r(\boldsymbol{x}) < R$
5.	PU AR. An RSU_N is forced to terminate.	$\boldsymbol{x} + \boldsymbol{e_1} - \boldsymbol{e_2}$	λ_{PU}	$B(\boldsymbol{x}) = M; j_n^E = 0; j_n^R > 0$
6.	PU DP from the NR-CRN.	$\boldsymbol{x} - \boldsymbol{e_1}$	$i_n\mu_{PU}$	$i_n > 0$
7.	PU DP from the R-CRN.	$\boldsymbol{x} - \boldsymbol{e_4}$	$i_r\mu_{PU}$	$i_r > 0$

8.	ESU AR. A vacant channel exists in the NR-CRN.	$\boldsymbol{x}+\boldsymbol{e_3}$	λ_{SE}	$B_n(\boldsymbol{x}) < M - R$
9.	RSU AR. A vacant channel exists in the NR-CRN.	$\boldsymbol{x}+\boldsymbol{e_2}$	λ_{SR}	$B_n(\boldsymbol{x}) < M - R$
10.	ESU DP from the NR-CRN.	$\boldsymbol{x}-\boldsymbol{e_3}$	$j_n^E\mu_{SE}$	$j_n^E > 0$
11.	RSU DP from the NR-CRN.	$\boldsymbol{x}-\boldsymbol{e_2}$	$j_n^R\mu_{SR}$	$j_n^R > 0$
12.	RSU DP from the R-CRN.	$\boldsymbol{x}-\boldsymbol{e_5}$	$j_r^R\mu_{SR}$	$j_r^R > 0$
13.	Idle channel failure in NR-CRN.	$\boldsymbol{x}+\boldsymbol{e_6}$	$(M - R - B_n(\boldsymbol{x}))\alpha$	$B_n(\boldsymbol{x}) < M - R$
14.	Idle channel failure in R-CRN.	$\boldsymbol{x}+\boldsymbol{e_7}$	$(R - B_r(\boldsymbol{x}))\alpha$	$B_r(\boldsymbol{x}) < R$
15.	A failed channel in NR-CRN is repaired.	$\boldsymbol{x}-\boldsymbol{e_6}$	$f_n\beta$	$f_n > 0$
16.	A failed channel in R-CRN is repaired.	$\boldsymbol{x}-\boldsymbol{e_7}$	$f_r\beta$	$f_r > 0$
17.	An occupied channel in NR-CRN fails. No vacant channels exist in the CRN. An RSU_N is forced to terminate.	$\boldsymbol{x}-\boldsymbol{e_2}+\boldsymbol{e_6}$	$(M - R - f_n)\alpha$	$B(\boldsymbol{x}) = M; j_n^R > 0; j_n^E = 0$
18.	An occupied channel in R-CRN fails. No vacant channels exist in the CRN. An RSU_N is forced to terminate.	$\boldsymbol{x}-\boldsymbol{e_2}+\boldsymbol{e_7}$	$(R - f_r)\alpha$	$B(\boldsymbol{x}) = M; j_n^R > 0; j_n^E = 0$
19.	An occupied channel in NR-CRN fails. No vacant channels exist in the CRN. An RSU_R is forced to terminate.	$\boldsymbol{x}-\boldsymbol{e_5}+\boldsymbol{e_6}$	$(M - R - f_n)\alpha$	$B(\boldsymbol{x}) = M; B_n(\boldsymbol{x}) - f_n = 0; j_r^R > 0$

20.	An occupied channel in R-CRN fails. No vacant channels exist in the CRN. An RSU_R is forced to terminate.	$\boldsymbol{x}-\boldsymbol{e_5}+\boldsymbol{e_7}$	$(R-f_r)\alpha$	$B(\boldsymbol{x}) = M; B_n(\boldsymbol{x}) - f_n = 0; j_r^R > 0$
21.	An occupied channel in NR-CRN fails. No vacant channels exist in the CRN. An PU_N is forced to terminate.	$\boldsymbol{x}-\boldsymbol{e_1}+\boldsymbol{e_6}$	$(M-R-f_n)\alpha$	$B(\boldsymbol{x}) = M; i_n > 0; j_n^E = j_n^R = 0$
22.	An occupied channel in R-CRN fails. No vacant channels exist in the CRN. An PU_N is forced to terminate.	$\boldsymbol{x}-\boldsymbol{e_1}+\boldsymbol{e_7}$	$(R-f_r)\alpha$	$B(\boldsymbol{x}) = M; i_n > 0; j_n^E = j_n^R = 0$
23.	An occupied channel in NR-CRN fails. No vacant channels exist in the CRN. An PU_R is forced to terminate.	$\boldsymbol{x}-\boldsymbol{e_4}+\boldsymbol{e_6}$	$(M-R-f_n)\alpha$	$B(\boldsymbol{x}) = M; i_r > 0; j_n^E = j_n^R = j_r^R = 0$
24.	An occupied channel in R-CRN fails. No vacant channels exist in the CRN. An PU_R is forced to terminate.	$\boldsymbol{x}-\boldsymbol{e_4}+\boldsymbol{e_7}$	$(R-f_r)\alpha$	$B(\boldsymbol{x}) = M; i_r > 0; j_n^E = j_n^R = j_r^R = 0$
25.	An occupied channel in NR-CRN fails. No vacant channels exist in the CRN. An ESU_N enters the retrial orbit.	$\boldsymbol{x}-\boldsymbol{e_3}+\boldsymbol{e_6}+\boldsymbol{e_8}$	$(M-R-f_n)\theta\alpha$	$B(\boldsymbol{x}) = M; j_n^E > 0$
26.	An occupied channel in R-CRN fails. No vacant channels exist in the CRN. An ESU_N enters the retrial orbit.	$\boldsymbol{x}-\boldsymbol{e_3}+\boldsymbol{e_7}+\boldsymbol{e_8}$	$(R-f_r)\theta\alpha$	$B(\boldsymbol{x}) = M; j_n^E > 0$

27.	An occupied channel fails in NR-CRN. No vacant channels exist in the CRN. An ESU_N gets terminate.	$\boldsymbol{x}-\boldsymbol{e_3}+\boldsymbol{e_6}$	$(1-\theta)(M-R-f_n)\alpha$	$B(\boldsymbol{x})=M; j_n^E>0$
28.	An occupied channel fails in R-CRN. No vacant channels exist in the CRN. An ESU_N gets terminate.	$\boldsymbol{x}-\boldsymbol{e_3}+\boldsymbol{e_7}$	$(1-\theta)(R-f_r)\alpha$	$B(\boldsymbol{x})=M; j_n^E>0$
29.	An occupied channel fails in the NR-CRN. A vacant channel exists in the NR-CRN.	$\boldsymbol{x}+\boldsymbol{e_6}$	$(M-R-f_n)\alpha$	$B_n(\boldsymbol{x})<M-R; B_n(\boldsymbol{x})>f_n$
30.	An occupied channel fails in the R-CRN. A vacant channel exists in the R-CRN.	$\boldsymbol{x}+\boldsymbol{e_7}$	$(R-f_r)\alpha$	$B_r(\boldsymbol{x})<R; B_r(\boldsymbol{x})>f_r$
31.	A PU occupied channel fails in the NR-CRN. The interrupted PU accesses a channel in R-CRN.	$\boldsymbol{x}-\boldsymbol{e_1}+\boldsymbol{e_4}+\boldsymbol{e_6}$	$i_n\alpha$	$B_n(\boldsymbol{x})=M-R; i_n>0; B_r(\boldsymbol{x})<R$
32.	An RSU occupied channel fails in the NR-CRN. The interrupted RSU accesses a channel in R-CRN.	$\boldsymbol{x}-\boldsymbol{e_2}+\boldsymbol{e_5}+\boldsymbol{e_6}$	$j_n^R\alpha$	$B_n(\boldsymbol{x})=M-R; j_n^R>0; B_r(\boldsymbol{x})<R$
33.	A PU occupied channel fails in the R-CRN. The interrupted PU accesses a channel in NR-CRN.	$\boldsymbol{x}+\boldsymbol{e_1}-\boldsymbol{e_4}+\boldsymbol{e_7}$	$i_r\alpha$	$B_n(\boldsymbol{x})<M-R; i_r>0; B_r(\boldsymbol{x})=R$
34.	An RSU occupied channel fails in the R-CRN. The interrupted RSU accesses a channel in NR-CRN.	$\boldsymbol{x}+\boldsymbol{e_2}-\boldsymbol{e_5}+\boldsymbol{e_7}$	$j_r^R\alpha$	$B_n(\boldsymbol{x})<M-R; j_r^R>0; B_r(\boldsymbol{x})=R$

35.	An ESU occupied channel fails in the NR-CRN. The interrupted ESU enters the retrial orbit.	$\boldsymbol{x} - \boldsymbol{e}_3 + \boldsymbol{e}_6 + \boldsymbol{e}_8$	$\alpha\theta j_n^E$	$B_n(\boldsymbol{x}) = M - R; j_n^E > 0; B_r(\boldsymbol{x}) < R$
36.	An ESU occupied channel fails in the NR-CRN. The interrupted ESU gets terminate.	$\boldsymbol{x} - \boldsymbol{e}_3 + \boldsymbol{e}_6$	$(1-\theta)\alpha j_n^E$	$B_n(\boldsymbol{x}) = M - R; j_n^E > 0; B_r(\boldsymbol{x}) < R$
37.	An ESU in the orbit retries successfully.	$\boldsymbol{x} + \boldsymbol{e}_3 - \boldsymbol{e}_8$	$min\{m, N\}\gamma$	$B_n(\boldsymbol{x}) < M - R; m > 0$
38.	An ESU in the orbit retries. No vacant channels exist in the NR-CRN. The ESU exit the system.	$\boldsymbol{x} - \boldsymbol{e}_8$	$min\{m, N\}\gamma(1-r)$	$B_n(\boldsymbol{x}) = M - R; m > 0$
39.	ESU AR. No vacant channels exist in the NR-CRN. The ESU enters the retrial orbit.	$\boldsymbol{x} + \boldsymbol{e}_8$	$b\lambda_{SE}$	$B_n(\boldsymbol{x}) = M - R$

5.6 Performance Analysis

In this section, we consider the transient analysis of the CTMC under study followed by the computation of performance metrics. The size of the state space of the Markov chain makes a matrix exponential technique computationally unfeasible for the transient solution. To cope with such state spaces, we rely on the uniformization technique for the transient solution of the Markov chain to assess the performance both fast and accurately, as suggested by Pulungan and Hermanns [253].

5.6.1 Transient Analysis: Uniformization

Let $\pi_{\boldsymbol{x}}(t)$ denote the probability of the system being in state $\boldsymbol{x}$ at time t. Using the generator matrix Q, the dynamic behavior of the CTMC can be described by the Kolmogorov differential equation in the matrix form given by

$$\frac{d\Pi(t)}{dt} = \Pi(t)Q. \tag{5.1}$$

Then, the transient probability vector, $\Pi(t)$ constituted by the transient probabilities $\pi_{\boldsymbol{x}}(t)$ can be obtained as

$$\Pi(t) = \Pi(0)e^{Qt}, \tag{5.2}$$

with $\Pi(0)$ being the initial probability vector. As discussed in Chapter 2, to obtain the transient solution of a CTMC includes evaluating a system of first order linear differential equations or, computation of a matrix exponential series directly. However, according to Jiao et al. [254], due to the high computational cost, the direct computation of the matrix exponential is not regarded as a practical method for solving dependability models. Alternatively, this chapter resort to the uniformization method presented by Van Dijk et al. [255], because of its higher accuracy and efficient computation. This approach utilizes the transition probability matrix P of the uniformized Markov chain obtained by $P = I + Q/\Delta$, where $\Delta \geq max_i|q_{ii}|$ is referred to as the uniformization rate, and q_{ii} denotes the diagonal elements of Q. Using this, the probability vector is evaluated as

$$\Pi(t) = \sum_{k=0}^{\infty} \Pi(0)\frac{(\Delta t)^k}{k!}e^{-\Delta t}P^k, \tag{5.3}$$

by truncating the summation at level K with the error formula

$$\epsilon = 1 - \sum_{k=0}^{K} \frac{(\Delta t)^k}{k!}e^{-\Delta t}. \tag{5.4}$$

Note that the parameter t here provides a way to analyze the time domain in the proposed CTMC.

5.6.2 Transient Performance Metrics

The validity of any CTMC model can be best deciphered in terms of its performance metrics. According to Mendis et al. [235], when investigating specific QoS measures, dependability theory provides a more systematic methodology for performance analysis.

To validate the efficiency of the system, we derive mathematical expressions of some dependability-oriented transient performance metrics:

A. Capacity

The capacity of a service in a CRN is defined as the rate of service completions. That is, the average number of service completions per unit time. We let $\rho_{PU}(t)$, $\rho_{RSU}(t)$, and $\rho_{ESU}(t)$ be the capacity of PU, RSU and ESU services respectively, at time t. Then

$$\rho_{PU}(t) = \sum_{\boldsymbol{x} \in S} (i_n + i_r)\mu_{PU}\pi_{\boldsymbol{x}}(t), \tag{5.5}$$

$$\rho_{RSU}(t) = \sum_{\boldsymbol{x} \in S} (j_n^R + j_r^R)\mu_{SR}\pi_{\boldsymbol{x}}(t), \tag{5.6}$$

$$\rho_{ESU}(t) = \sum_{\boldsymbol{x} \in S} j_n^E \mu_{SE}\pi_{\boldsymbol{x}}(t). \tag{5.7}$$

B. Channel Availability

In a CRN, once all the channels are occupied at an arrival instant, a newly arrived user is blocked and the CRN is said to be unavailable for new user requests. Therefore, channel availability based performance measurements are important for both PU and SU services while performing channel access in a CRN, as suggested by Amich et al. [256]. The transient channel availability for PU or SU services is defined as the probability of allocating a channel to a new PU or SU arrival without being blocked at time t.

A newly arrived PU service is blocked when all the operational channels in the NR-CRN are occupied by PUs. On the other hand, a new SU service is blocked in the case where all the operational channels in the NR-CRN are occupied by PUs or/and SUs. Let $A_{PU}(t)$ and $A_{SU}(t)$ be the channel availability of PU and SU services at time t, respectively. We obtain

$$A_{SU}(t) = 1 - \sum_{\substack{\boldsymbol{x} \in S \\ B_n(\boldsymbol{x}) = M-R}} \pi_{\boldsymbol{x}}(t), \tag{5.8}$$

$$A_{PU}(t) = 1 - \sum_{\substack{\boldsymbol{x} \in S \\ i_n + f_n = M - R}} \pi_{\boldsymbol{x}}(t). \tag{5.9}$$

Accordingly, the blocking probabilities of both PU and SU services at time t, denoted as $P^B_{PU}(t)$ and $P^B_{SU}(t)$ respectively, are expressed

$$P^B_{PU}(t) = 1 - A_{PU}(t) \quad \text{and} \tag{5.10}$$

$$P^B_{SU}(t) = 1 - A_{SU}(t). \tag{5.11}$$

C. Retainability

The retainability of a service defines the probability that a connection, once established, will operate within the specified transmission quality without any interruption. Thus, mathematically, it can be expressed as

$$\theta(t) = 1 - P_F(t), \tag{5.12}$$

where $P_F(t)$ is the forced termination probability of that service at time t.

As mentioned earlier, the forced termination probability refers to the probability that an ongoing service is forced to terminate before the completion of its communication. Note that an ongoing SU service may get terminated owing to a channel failure or upon a PU arrival, when no other channel is available in the CRN. The *transient forced termination probability of RSUs*, $P_F^{RSU}(t)$, can be expressed as the ratio of mean forced termination rate of RSUs at time t to the mean admitted RSU rate at time t, $\Lambda_{RSU}(t) = A_{SU}(t)\lambda_{SR}$. Correspondingly, the forced termination probability of RSUs at time t is given by

$$P_F^{RSU}(t) = \frac{\Big(R_{RSU}(t) + R'_{RSU}(t)\Big)}{\Lambda_{RSU}(t)}, \tag{5.13}$$

where $R_{RSU}(t)$ and $R'_{RSU}(t)$ denote the forced termination rate of SUs at time t because of PU arrivals and channel failures, respectively. Then we have

$$R_{RSU}(t) = \lambda_{PU} \sum_{\substack{\boldsymbol{x} \in S \\ B(\boldsymbol{x})=M;\ j_n^E=0;\ j_n^R>0}} \pi_{\boldsymbol{x}}(t), \tag{5.14}$$

$$R'_{RSU}(t) = \alpha \sum_{\substack{\boldsymbol{x} \in S \\ B(\boldsymbol{x})=M; \\ (j_n^E=0;\ j_n^R>0) \text{ or} \\ (B_n(\boldsymbol{x})=f_n;\ j_r^R>0)}} (M - f)\pi_{\boldsymbol{x}}(t). \tag{5.15}$$

Likewise, the *transient forced termination probability of ESUs*, $P_F^{ESU}(t)$, refers to the probability of termination of an ongoing ESU service due to a channel failure or a PU arrival, when no other channel is available in the NR-CRN and the interrupted service leaves the system with probability $(1-\theta)$. It is obtained as

$$P_F^{ESU}(t) = \frac{\Big(R_{ESU}(t) + R'_{ESU}(t)\Big)}{\Lambda_{ESU}(t)}, \tag{5.16}$$

where $\Lambda_{ESU}(t) = A_{SU}(t)\lambda_{SE}$, and $R_{ESU}(t)$ and $R'_{ESU}(t)$, respectively are computed as

$$\begin{aligned} R_{ESU}(t) &= \lambda_{PU}(1-\theta) \sum_{\substack{\boldsymbol{x} \in S \\ B(\boldsymbol{x})=M;\ j_n^E>0}} \pi_{\boldsymbol{x}}(t), \\ R'_{ESU}(t) &= \alpha(1-\theta)\big(\Phi_1(t) + \Phi_2(t)\big), \end{aligned} \tag{5.17}$$

where

$$\Phi_1(t) = \sum_{\substack{\boldsymbol{x} \in S \\ B_n(\boldsymbol{x})=M-R;\ j_n^E>0}} j_n^E \pi_{\boldsymbol{x}}(t), \tag{5.18}$$

$$\Phi_2(t) = \sum_{\substack{\boldsymbol{x} \in S \\ B(\boldsymbol{x})=M;\ j_n^E>0}} (M - f)\pi_{\boldsymbol{x}}(t). \tag{5.19}$$

Along the similar lines, the *transient forced termination probability of PUs* due to channel failures, $P_F^{PU}(t)$, can be calculated as

$$P_F^{PU}(t) = R'_{PU}(t)/\Lambda_{PU}(t), \tag{5.20}$$

where $R'_{PU}(t)$ and $\Lambda_{PU}(t)$ are given by

$$R'_{PU}(t) = \alpha \sum_{\substack{\boldsymbol{x} \in S \\ B(\boldsymbol{x})=M; \\ j_n^E=j_n^R=0; \\ (i_n>0) \text{ or } (i_r>0)}} (M - f)\pi_{\boldsymbol{x}}(t). \tag{5.21}$$

and $\Lambda_{PU}(t) = A_{PU}(t)\lambda_{PU}$, respectively.

D. Network Unserviceable Probability (NUP)

To evaluate the overall satisfaction of the network from a user's point of view, NUP is an important metric to be determined. According to Lee [257], the service can be accomplished only if the network provides a connection when and as long as required. More specifically, for successful completion of a service, the service should neither be blocked upon its arrival nor be terminated before its completion. Accordingly, the transient NUP of a service, $Q(t)$, is defined as the probability that a service cannot be completed successfully at time t. It is calculated as the ratio between the rate of service completions and the rate of arrivals at time t. Therefore, the transient NUP for RSU services, $Q_{RSU}(t)$, is given by

$$\begin{aligned} Q_{RSU}(t) &= 1 - \big(\text{Prob. of completion of an RSU service}\big), \\ &= 1 - \frac{\lambda_{SR}\big(1 - P_{SU}^B(t)\big)\big(1 - P_{RSU}^F(t)\big)}{\lambda_{SR}}, \\ &= P_{SU}^B(t) + P_{RSU}^F(t) - P_{SU}^B(t)P_{RSU}^F(t). \end{aligned} \tag{5.22}$$

Similarly, the NUP for ESU and PU services at time t, denoted as $Q_{ESU}(t)$ and $Q_{PU}(t)$, respectively, is given by

$$Q_{ESU}(t) = P_{SU}^B(t) + P_{ESU}^F(t) - P_{SU}^B(t)P_{ESU}^F(t), \tag{5.23}$$

$$Q_{PU}(t) = P_{PU}^B(t) + P_{PU}^F(t) - P_{PU}^B(t)P_{PU}^F(t). \tag{5.24}$$

E. Mean Orbit Size

The expected number of users in the retrial orbit at time t, is obtained as

$$E[N(t)] = \sum_{\boldsymbol{x} \in S} m\pi_{\boldsymbol{x}}(t). \tag{5.25}$$

F. Loss Rate due to Impatience

Using the concept of Ancker Jr and Gafarian [258,259], we can obtain the average balking rate $B.R.(t)$, the average reneging rate $R.R.(t)$ and the average rate $L.R.(t)$ of user loss because of impatient behavior as follows

$$\begin{aligned} B.R.(t) &= \lambda_{SE}(1-b) \sum_{\substack{\boldsymbol{x} \in S \\ B_n(\boldsymbol{x})=M-R}} \pi_{\boldsymbol{x}}(t), \\ R.R.(t) &= \gamma(1-r) \sum_{\substack{\boldsymbol{x} \in S \\ B_n(\boldsymbol{x})=M-R}} min\{m, N\}\pi_{\boldsymbol{x}}(t), \\ L.R.(t) &= B.R.(t) + R.R.(t). \end{aligned} \tag{5.26}$$

5.7 Numerical Results and Discussion

This section presents the numerical results to assess our previous analysis. It is worth mentioning that the computation of transient probabilities by the suggested uniformization method will portray more realistic scenario of concerned CTMC model for which closed form analytical results are not possible to derive.

The numerical results are obtained by developing code in MATLAB to validate the applicability and tractability of the proposed computational approach in the CTMC model under study. The values of the network configuration are listed in Table 5.3 unless otherwise stated and $\epsilon = 10^{-5}$. The first state of set S is assumed as $\boldsymbol{x} = (0,0,0,0,0,0,0,0)$, i.e., a state in which all channels are vacant. Accordingly, $\Pi(0) = \{1,0,0,\cdots,0\}$ in Table 5.3 indicates that the probability of being in state in which whole spectrum is available at $t = 0$ is 1. In the figures, we derive the main performance measures as a function of the elapsed time (in time units) since the origin. Note that the curves representing *No Channel Reservation* are obtained by configuring the number of channels allocated to R-CRN, $R = 0$. In view of the fact that channel failures do not occur frequently, it is required to target a low failure rate in contrast with the arrival rates of PUs and SUs. The channel failure rate is taken as 0.05 failure per unit time i.e. $\alpha = 0.05$ by default for the results illustrated here. For instance with the PU arrival rate as 5 flows per time unit, on average a channel failure occurs every 100 PU arrivals, which is a reasonable configuration according to Balapuwaduge et al. [248]. On the other hand, once channel failure occurs, in order to maintain a sufficiently high level of satisfaction the channel should be repaired at a much higher rate. Thus, a repair rate of $\beta = 1.0$ per unit time is taken for our numerical experiment.

Table 5.3: Summary of symbols in the CTMC and value configuration

Symbol	Description	Default Value
M	Total number of channels in CRN	6
R	Number of reserved channels in R-CRN	2
A	Parameter which limits R	$2-4$
N	Maximum no. of allowable retrial customers	5
λ_{PU}	PU arrival rate	5 services per unit of time
$\lambda_{SR}, \lambda_{SE}$	RSU and ESU arrival rates	2.5 services per unit of time
μ_{PU}	PU service rate per channel	2 services per unit of time
μ_{SR}, μ_{SE}	RSU and ESU service rates per channel	1 service per unit of time
$\Pi(0)$	Initial Condition	$\{1, 0, 0, \cdots, 0\}$
α	Failure rate of channels	0.05 failures per unit of time
β	Repair rate per channel	1.0 repairs per unit of time
θ	Retrial probability for interrupted ESU	0.7
$\bar{b}$	Balking probability for blocked ESU	0.3
γ	ESU retrial rate	1 service per unit of time
$\bar{r}$	Reneging probability for blocked ESU	0.2

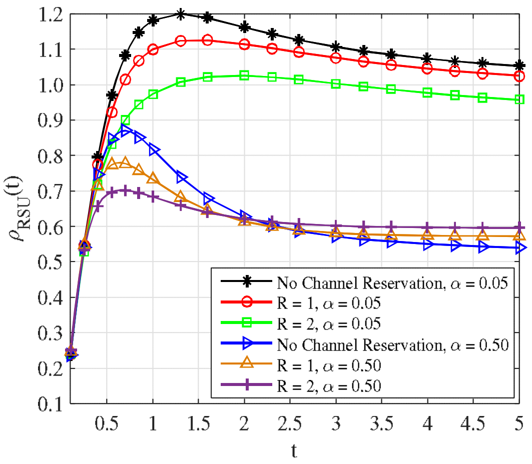

Figure 5.5: Capacity achieved by RSUs as a function of time when channel reservation is applied.

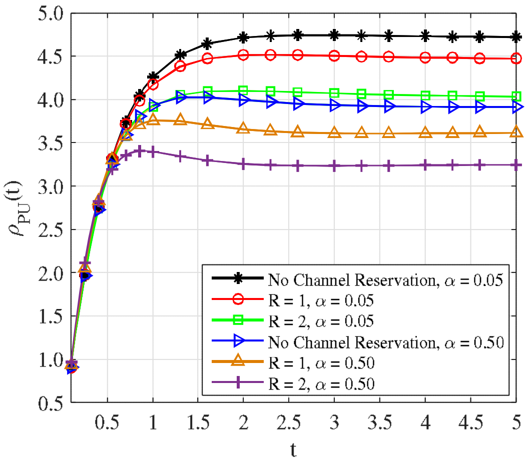

Figure 5.6: Capacity achieved by PUs as a function of time when channel reservation is applied.

A. Capacity

Figs. 5.5 and 5.6 represent the capacity of RSU and PU services as a function of time respectively, under different values of R and α. From Fig. 5.5, for all scenarios, we notice that the capacity of RSU initially increases with time, but eventually decreases and shows a steady behavior. As is to be expected, with a higher channel failure rate, the capacity loss increases because of the lack of idle channels and the effect is noticeable for reasonably larger values of t. Furthermore, it is observed that enabling channel reservation degrades the service capacity. However, the situation changes for larger values of t and with a higher α.

With smaller values of t and lower α, an RSU service is less likely to be interrupted and accordingly, a large portion of the reserved spectrum remains underutilized. This results in degradation of the service capacity as compared with the case of no channel reservation. On the other hand, with larger α, as time grows, channels are more likely to fail and some of the interrupted services are provided channel access opportunities in the reserved band until completion. Consequently, the channel reservation scheme shows better performance compared with the CRN without channel reservation in terms of RSU capacity, however, the effect doesn't agree for smaller values of t. Therefore, this

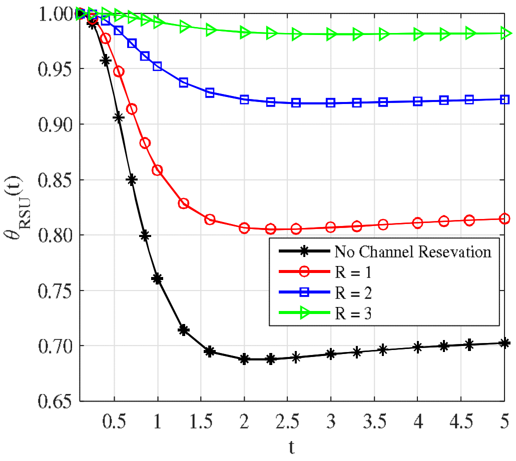

Figure 5.7: Retainability of RSU services as a function of time when channel reservation is applied.

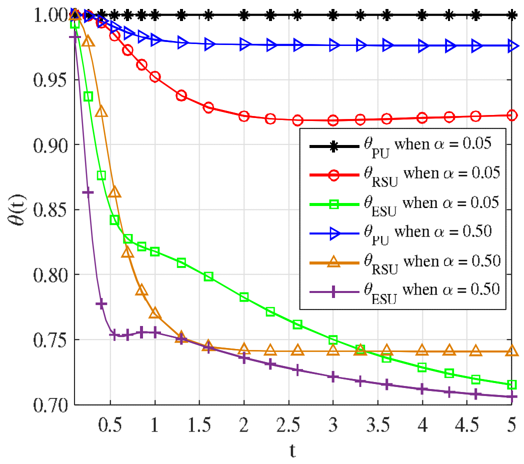

Figure 5.8: Retainability of SU and PU services as a function of time and channel failure rate.

result indicates a need for investigation of the network conditions which needs channel reservation as well as the conditions under which the channel reservation is not required.

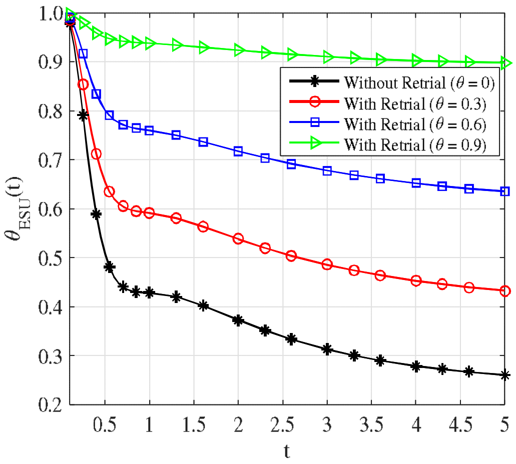

Figure 5.9: Retainability of ESU services as a function of time when retrial is permitted.

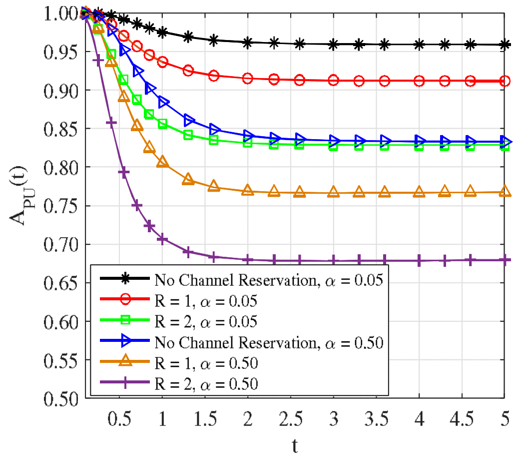

Figure 5.10: Channel Availability for PUs as a function of time when channel reservation is applied.

Fig. 5.6 shows the capacity of PUs as t varies. As shown in those curves, with a larger value of t, the capacity of PUs first increases, and then, it shows a steady behavior. Again, we observed that with higher channel failure rate, the PU capacity decreases due to the lack of idle channels. Furthermore, the PU capacity decreases due to channel reservation