CHAPTER

6 Inventory Modelling for high technology multi-generation products under limited warehouse storage space

Warehousing facilities play a crucial role in the entire supply chain (Autry et al. 2005). The integration of inventories with the logistics of warehouses and transportation can generate a source of competitive advantage for a firm (Parast & Spillan, 2014). The utilization of all the resources of a firm, which also includes the storage space, is important to create and sustain the competitive advantage (Barney, 1991). Since the replenishment of the raw materials or the finished goods happen in discrete lots, while consumptions happen continuously, some inventories need to be stored to meet the demand before the next replenishment. Many a time, the sub-optimal planning of inventory policies can also be discovered. One of the possible reasons, for this, is not giving adequate attention to warehousing space in the planning of inventories, which leads to the under-utilization of space or renting of costlier space from external agents.

The earlier two chapters have taken into account the influence of the trade credits and selling price, both of which are marketing mix variables. However, it is not only the marketing mix but also the resource constraints that can also influence the inventory norms. The constrained resource may be space, funds, labor, etc. In this chapter[8], the primary discussion agenda is how the warehousing space constraints influence the inventory decisions for the multi-generational products. The efficiencies of the overall supply chain of such products can be increased immensely if the storage space constraints are given due consideration while formulating the inventory policies. While the ordering costs per unit will fall with a higher lot size, the inventory carrying costs may increase non-linearly after the storage space of the captive facilities is fully utilized.

[8] This chapter is based on the following research paper.
Nagpal, G. & Chanda, U. (2021). "P-Model of Inventory Optimization for high technology multi-generation products under limited warehouse storage space", *International Journal of Applied Management Science* (Revision Submitted)

Inventory optimization under warehousing constraints for the single-item scenarios is a rich work in the literature. A few of the pioneering research studies on linking inventory norms with the storage space have been done by Homer (1966), Evans (1967), Page & Paul (1976), Zoller (1977), and by Buffa & Reynolds (1979). The demand functions being worked upon by the existing research has been very varied, with a few studies considering the constant demand, while most of them considering the variable demand. When it comes to the studies on variable demand, some of them have considered time-dependent demand, while many have considered stock-dependent demand or credit-linked demand. While a few have used the stochastic demand pattern, most of the studies have used a deterministic demand function. While most of the studies have been done on the EOQ Model. Some researchers also considered the uncertainty of demand and used fuzzy logic to lay down the inventory optimization models (Maiti, 2008 and Rong et al. 2008). Palanivel et al. (2016) considered the impact of inflation and shortages while formulating the inventory model for deteriorating items with stock dependent demand. Flamand (2017) also worked on inventory optimization for the product assortment in the context of shelf space dynamics in a retail setup. Shaikh et al. (2019) used particle swarm optimization to solve a two warehouse model with interval-valued inventory cost but only for single item scenario.

Table6.1 summarizes the prominent research studies that have been done on inventory optimization under storage space constraints in case of multiple item scenarios. It can be inferred that there has been only one study for the technology products by Kumar & Chanda (2018). Although this study is very enriching and a pioneer in the inventory modeling for diffusion dependent demand under storage space constraints, but it has considered only the single generation technology products.

Table 6.1. Existing Literature on the Multi-Item inventory Modelling under storage space constraints

Thesis	Deterioration	Technology product	JRP for multiple items	Demand substitution	multi-generation
Atabaki & Mohammadi (2017)	-	-	☑	-	-
Carvajal et al. (2020)	-	-	☑	-	-
Chakraborty et al. (2020)	☑	-	☑	-	-
Erlebacher (2000)	-	-	☑	-	-
Federgruen et al. (2018)	-	-	☑	-	-
Flamand et al. (2017)	-	-	☑	☑	-
Gutierrez et al. (2013)	-	-	☑	-	-
Hausman et al. (1998)	-	-	☑	-	-
Hoque et al. (2006)	-	-	☑	-	-
Hubner & Schnaal (2016)	-	-	☑	☑	-
Hubner & Schnaal (2017)	-	-	☑	☑	-

Hubner et al. (2015)	-	-	☑	☑	-
Jing & Mu (2020)	-	-	☑	☑	-
Kumar & Chanda (2018)	☑	☑	-	-	-
Moon et al. (2011)	-	-	☑	-	-
Palencia et al. (2019)	-	-	☑	-	-
Yao (2010)	-	-	☑	-	-

Legends: JRP: Joint Replenishment Problem, ☑: Yes, -: No

It is evident from the Table6.1 that not much of work has been done on the inventory modeling for multi-generational technology products under warehousing constraints. Therefore, this research study aims to formulate and test (with numerical illustration) a model for optimizing the inventory policies for innovation products under storage space constraints. This research work puts forward the inventory optimization model for the high technology multi-generation products under the situation of limited warehouse storage space. It is assumed that the manufacturer has its own space which has a lesser opportunity cost of usage as compared to another rented space, which carries higher charges on per unit area basis. Therefore, the manufacturer utilizes the own storage space for the period where his own space is sufficient to keep the inventories. When its in-house space is not sufficient to store all the inventories, it stores the excess inventories at the rented space. While the research work has been done earlier on various demand patterns but there is no research present on the inventory optimization under storage space constraints for the generations of innovative products. This chapter discusses an inventory optimization model for hi-technology products under storage space constraints using competition-substitution between two generations. Few theorems are also discussed on the dynamics of inventory decisions with the change in innovation effect, imitation effect, with the rent of the leased location and with the storage capacity of own location. The chapter also performs numerical illustrations to show the behavior of the proposed model.

6.1. Inventory Modelling Framework

The assumptions and notations for the proposed model have been outlined in sections 6.1.1 and 6.1.2 respectively, which will be further used for inventory modeling in further subsections.

6.1.1. Notations of the Model

O fixed cost of ordering per order for any shipment exclusive of the product-specific costs (j = 1,2)

O_j fixed product-specific ordering cost per order for j^{th} generation product (j = 1,2)

C_j inventory holding expense (as a % of cost) for j^{th} generation product (j = 1,2)

M_j the size of market potential for j^{th} generation product (j = 1,2)

I_j the inventory carrying cost as % of the product cost in the own warehouse for j^{th} generation product (j = 1,2)

I_{j1} the inventory carrying cost as % of the product cost in the rented warehouse for j^{th} generation product (j = 1,2)

$\lambda_1(t)$ and $\lambda_1'(t)$ demand rate at time $'t'$ of 1^{st} generation product for $t \leq \tau$ and $t \geq \tau$ respectively

$\lambda_2(t)$ demand rate at time $'t'$ of 2^{nd} generation product

p_j coefficient of innovation for j^{th} generation product (j = 1,2)

q_j coefficient of imitation for j^{th} generation product (j = 1,2)

τ be the time at which the second-generation product is introduced

$\emptyset(t)$ is the conditional probability of a prospective adopter (who has not yet adopted the product till time t) adopting the product in time (t, $t+\Delta t$)

$F_j(t)$ is the cumulative fraction of adopters of ith generation product till time t.

$f_j(t)$ is the fraction of adopters of ith generation product at time t.

ξ is the storage capacity of the first warehouse

P_j is the price of jth generation product

y_m is a binary variable, that is zero, if the first generation product has been discontinued till the mth planning horizon, and 1 if the first generation product is still in business in the mth planning horizon

$z_{i,m,n}$ is a binary variable, that is 1, if the rented warehouse is used in the ith replenishment cycle of mth planning horizon with n cycles (in case of single generation product scenario), else, 0

$Z_{i,m,n}$ is a binary variable, that is 1, if the rented warehouse is used in the ith replenishment cycle of mth planning horizon with n cycles (in case of two generations product scenario), else, 0

$TC_{m,n}$ is the total cost of ordering and holding in mth planning horizon with n replenishment cycles

TCM_m is the total contribution margin, i.e. Revenue, net off material cost for the goods sold in the m_{th} planning horizon

$TP_{m,n}$ is the total profit incurred from the goods sold in the mth planning horizon with n replenishment cycles

ρ is the length of the planning horizon for which the inter-replenishment time interval is fixed

6.1.2. Assumptions of the Model

The following are the assumptions used in the Model:

- The supply gets replenished instantaneously
- There is zero safety stock in the procurement of inventories
- The entire demand at any time has to be met unless the product has been discontinued.
- The rate of demand is influenced by the innovation effect as well as the imitation effect. The rate of demand is influenced by the innovation diffusion process and follow the assumptions as discussed in section 3.1.3 and can be given as follows:

$$\lambda_1(t) = M_1 f_1(t)\ for\ t < \tau$$

$$\lambda_1'(t) = M_1 f_1(t) - M_1 f_1(t) F_2(t) \; for \; t > \tau$$

$$\lambda_2(t) = M_2 f_2(t) + M_1 f_1(t) F_2(t)$$

- The price of the product is deterministic and constant
- Trade shipments for both the generation products are consolidated together, and therefore, the frequency of replenishment is the same for both of them.
- The fixed ordering cost has the product-non-specific component and the product-specific component.
- The number of replenishment cycles and thus, the length of each replenishment cycle is fixed for each planning horizon at the beginning of that period.

There are two warehouses for storage, the rented one and the inhouse one. Since the rented one has higher storage costs per unit time, no material is used from the inhouse store unless the rented store is vacant. The capacity of the inhouse warehouse is limited. In this subsection, the demand model for two successive technology generation products will be discussed. A single generation model framework shall also be discussed, and later it will be extended for two-generation products.

6.1.3. **Inventory Modeling Framework**

The framework uses the Bass Model (1969) that is most commonly used to derive the demand for technology products. The consumption of these products has two components: the innovation influence (mass media or advertising) and the imitation influence (feedback sharing by the existing adopters). The equations for the adoption at any time instant and the cumulative adoption have already been mentioned in Section 3.1.3 and 3.1.4.

6.1.3.1. **Cost model for a single generation** $(t \leq \tau)$

Since there is no deterioration of the product (as per the assumption), the consumption of inventory takes place on account of the demand usage only. Therefore, the demand rate is the rate of change in inventory.

$$\lambda_j(t) = -\frac{d\big(I_j(t)\big)}{dt} \; ; \; (j = 1,2) \tag{6.1}$$

If time t lies in the ith replenishment cycle of the mth planning horizon, in which a total of n replenishments are done, then inventory at time t is the demand of the product from time t till the end of the corresponding replenishment cycle.

Let $t_{(i-1),m,n} = \left\{(m-1) + \frac{(i-1)}{n}\right\}\rho$ be the time at which the ith replenishment cycle begins and that $t_{i,m,n} = \left\{(m-1) + \frac{(i)}{n}\right\}\rho$ is the time at which it ends. This is because the model of zero buffer stock or zero safety stock is being considered here. Thus, it can be said that inventory at time t is:

$$I_1(t) = \int_{t=t}^{t=t_{i,m,n}} \lambda_1(t)\, dt \tag{6.2}$$

The illustration of the above concept is given in Figure6.1.

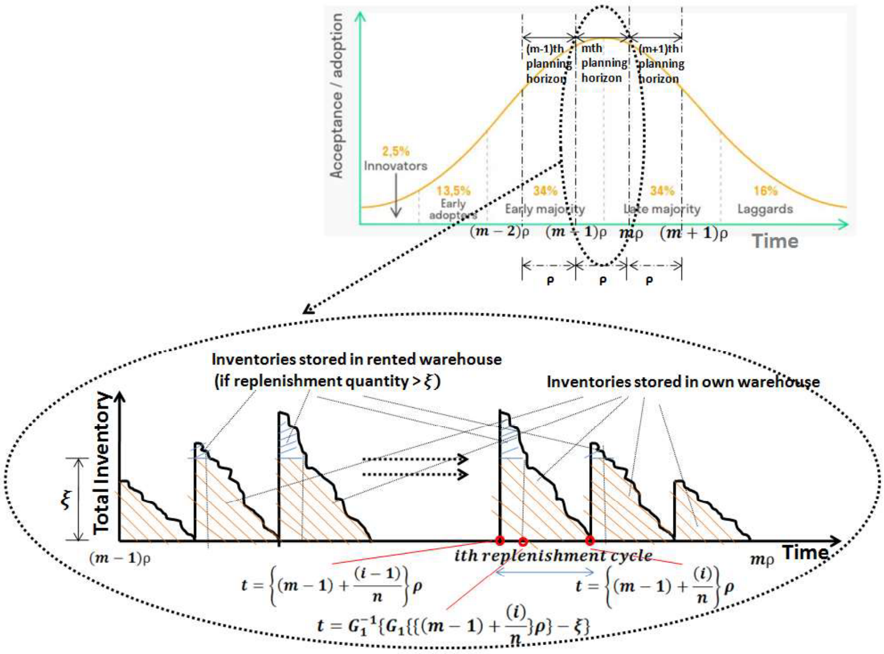

Figure6.1. The P Model of inventory management before the launch of the second generation under two warehouse scenario (Source: Self compilation by the author)

If the demand in any replenishment cycle is more than ξ, then the rented warehouse has to be used. So, it can be said that the second warehouse is used ith replenishment cycle of mth planning horizon if

$$G_1\{t_{i,m,n}\} - G_1\{t_{(i-1),m,n}\} > \xi \tag{6.3}$$

Let there exist a binary variable z_{im} where

$z_{im} = 1$if $G_1\{t_{i,m,n}\} - G_1\{t_{(i-1),m.n}\} - \xi > 0$,

And $z_{i,m,n} = 0$ otherwise.

If the second warehouse is used in the ith replenishment cycle, the material in it will get consumed when the remaining demand in that replenishment cycle reduces to ξ. From this time onwards, the material of the first warehouse can be consumed.

Let $p_{i,m,n}$ be the point of time from which the inventories at the inhouse warehouse start getting consumed. This instant of time can be written as

$p_{i,m,n} = G_1^{-1}\{t_{i,m,n}\} - \xi\}\ if\ z_{i,m,n} = 1, and$

$= t_{(i-1),m,n} if\ z_{i,m,n} = 0$

The inventory carrying cost from time

The Total Cost in the mth planning horizon with n replenishment cycles can be stated as the sum of ordering costs and the inventory carrying costs

$$TC_{m,n} = n(O + O_1) + \left[\sum_{i=1}^{i=n} \int_{t_{(i-1),m,n}}^{t_{i,m,n}} I_1(t)dt\right] + (I_{11} - I_1)C_1 \left[\sum_{i=1}^{i=n} \int_{t_{(i-1),m,n}}^{p_{i,m,n}} I_1(t)dt\right] - I_1 C_1 (\xi) \sum_{i=1}^{i=n} (p_{i,m,n} - t_{(i-1),m,n}) \quad (6.4)$$

The contribution margin (or, the revenue net off basic procurement cost) per unit in the mth planning horizon is given by

$$TCM_m = y_m \int_{t=(m-1)\rho}^{t=m\rho} (P_{01} - C1)\lambda_1(t)\, dt \quad (6.5)$$

The profit margin in the mth planning horizon is given by

$$TP_{m,n} = [TCM_m - TC_{m,n}] \quad (6.6)$$

The objective function is to the value of n for which $TC_{m,n}$ is minimum since TCM_m is constant for a given value of m.

Min. $TC_{m,n}$

Subject to the constraints n is a positive integer

In the above formulation, n is the only decision variable.

6.1.3.2. Cost Model for two generations

After the launch of the next generation, the inventories of both the products are kept together in the warehouses. Thus, let there be another function $G_3(t) = G_1(t) + G_2(t)$ (6.7)

where, $G_3(t)$ denotes the cumulative demand of a product till time t.

After the second generation product has been introduced, the organizations need to re-work the inventory norms for the next planning horizon. For ease, let it be assumed that the second generation product gets introduced exactly at the time the inventories of the earlier generation product have got exhausted. Figure6.2 shows how the inventory behaves post the launch of the second generation.

The mth planning horizon after the launch of the second generation starts at $t = \tau + (m-1)\rho$ and ends at $t = \tau + m\rho$.

Now, the time at the beginning of the ith replenishment cycle in a period of m replenishments is given by

$$T_{(i-1),m,n} = \tau + \left\{(m-1) + \frac{(i-1)}{n}\right\}\rho$$

$$T_{i,m,n} = \tau + \left\{(m-1) + \frac{i}{n}\right\}\rho$$

The second warehouse has to be used if the total demand of both the products in any replenishment cycle is greater than ξ. So, it can be said that the second warehouse is used ith replenishment cycle of mth planning horizon if

$$G_3\{T_{i,m,n}\} - G_3\{T_{(i-1),m,n}\} > \xi \quad (6.8)$$

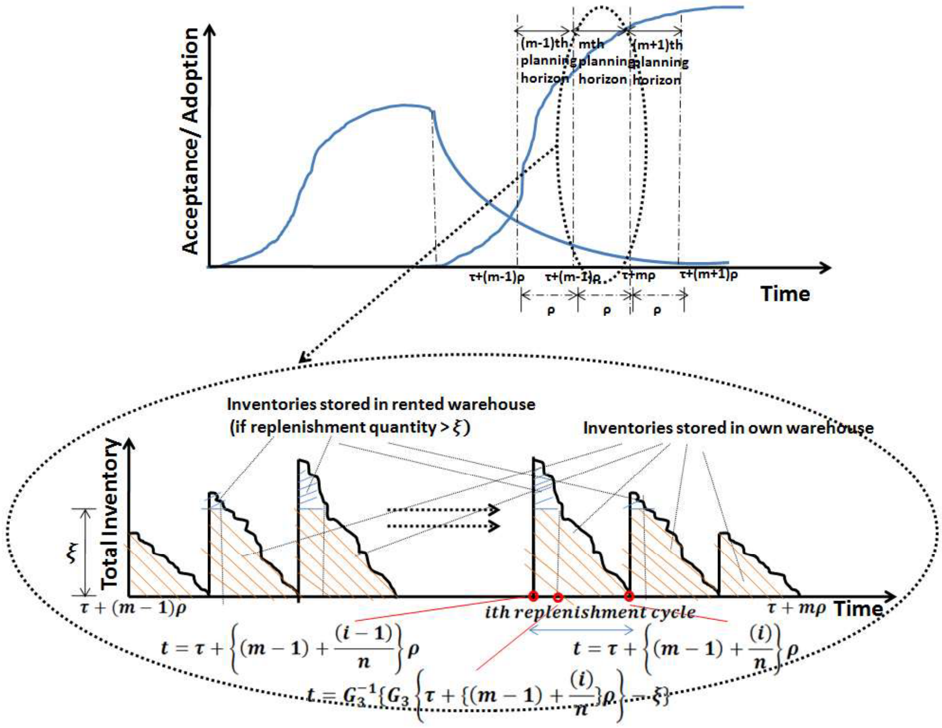

Figure6.2. The P Model of inventory management in the mth planning horizon with n replenishments before the launch of the first generation (Source: Self compilation by the author)

Let there exist a binary variable $Z_{i,m,n}$ where

$Z_{i,m,n} = 1$ if $G_3\{T_{i,m,n}\} - G_3\{T_{(i-1),m,n}\} - \xi > 0$,

And $Z_{i,m,n} = 0$ otherwise.

If the second warehouse is used in the ith replenishment cycle, the material in it will get consumed when the remaining demand in that replenishment cycle reduces to ξ. This instant of time till which it is used can be given as

$P_{i,m,n} = G_3^{-1}\{T_{i,m,n}\} - \xi\}\ if\ Z_{i,m,n} = 1, and$

$= T_{(i-1),m,n} if\ Z_{i,m,n} = 0$

$$TC_{m,n} = n(O + y_m O_1 + O_2) + y_m C_1\{\left[I_1 \sum_{i=1}^{i=n} \int_{T_{(i-1),m,n}}^{T_{i,m,n}} I_1(t)dt\right] + (I_{11} - I_1)\left[\sum_{i=1}^{i=n} \int_{T_{(i-1),m,n}}^{P_{i,m,n}} I_1(t)dt\right] - I_1(\xi)\sum_{i=1}^{i=n}(P_{i,m,n} - T_{(i-1),m,n})\} + C_2\{\left[I_2 \sum_{i=1}^{i=n} \int_{T_{(i-1),m,n}}^{T_{i,m,n}} I_2(t)dt\right] + (I_{21} - I_2)\left[\sum_{i=1}^{i=n} \int_{T_{(i-1),m,n}}^{P_{i,m,n}} I_2(t)dt\right] - I_2(\xi)\sum_{i=1}^{i=n}(P_{i,m,n} - T_{(i-1),m,n})\} \quad (6.9)$$

The contribution margin in the mth planning horizon is given by

$$TCM_m = y_m \int_{t=\tau+(m-1)\rho}^{t=\tau+m\rho} [P_{01} - C1]\lambda_1'(t)\,dt + \int_{t=\tau+(m-1)\rho}^{t=\tau+m} [P_{02} - C2]\,\lambda_2(t)\,dt \quad (6.10)$$

The profit margin in the mth planning horizon is given by $TP_{m,n} = TCM_{m,n} - TC_{m,n}$

The objective function is to minimize $TC_{m,n}$ since $TCM_{m,n}$ is constant for a given value of *m*.
Subject to the constraints n is a positive integer
And y_m is a binary variable, that is zero, if the first generation product has been discontinued till the mth planning horizon, and 1 if the first generation product is still in business in the mth planning horizon.
In the above formulation, there are two decision variables n and y_m.
Since our objective is to minimize the cost function $TC_{m,n}$hence the necessary conditions for minimizing $TC_{m,n}$ are $\frac{\partial[TC_{m,n}]}{\partial n} = 0$ with the sufficient condition $\frac{\partial^2(TC_{m,n})}{\partial n^2} > 0$

Depending on the path of ordering costs per unit time, purchase costs per unit time, and inventory holding costs per unit time, the following theorems are being proposed:

Theorem 1: *With the increase in the rent of the rented warehouse, the optimal number of the replenishment cycles increases in the intermediate planning horizons, while does not get influenced in the initial or later periods of the planning horizon.*
Proof: See Appendix 6.A.

Theorem 2: *As the values of innovation and imitation coefficients rise, the dependence on rented warehouse becomes more uneven with time; the optimal number of replenishment cycles rises in the initial stages of the product life cycle, and declines in the later stages.*
Proof: See Appendix 6.B.

Theorem 3: *The higher the rent premium charged for the rented warehouse, the earlier it makes sense to discontinue the earlier generation products.*
Proof: See Appendix 6.C.

Theorem 4: *If the rent charged by the rented warehouse crosses a certain threshold, the optimal service level may fall to less than 100%, unless the dynamic pricing is used to charge the customer higher for the higher cost-to-serve. And the higher will be this fall during the peak stages of the product life cycle.*
Proof: See Appendix 6.D.

Theorem 5: The higher the amount of capacity deficit of the first warehouse, and the higher the optimal number of replenishments in the planning horizon.
Proof: See Appendix 6.E.

In the next subsection, the solution procedure to find the optimal value of replenishment frequency is discussed. Since the cost function as defined in equation (6.4) and (6.9) is highly non-linear, hence finding an analytical solution for the problem is difficult. The problem is solved numerically under given parameter values. Once the value of optimal replenishment frequency in each planning horizon is known, the replenishment quantities can also be worked out using the demand equations.

6.1.4. Solution procedure

The solution procedure to find the optimal solutions can be summarized in the following algorithm

Step 1: Enter the base values of all model parameters such as per-unit costs, coefficients of innovation and imitation, potential market sizes, time to the introduction of second-generation products, etc. for each generation independently.

Step 2: Compute the total cost at all possible values of replenishment frequency for the given value of τ using equation (6.4) and (6.9).

Step 3: Select the appropriate value of replenishment frequency where the first derivative of the total cost is zero and the second derivative is positive.

Step 4: Finally, compute the value of replenishment lot sizes using the L.H.S. part of the equations (6.3) and (6.8) respectively.

6.1.5. Numerical illustration

Now, the working of the model was tested after assigning suitable values to the parameters as mentioned below.

$I_1 = .15, I_2 = .15, p_1 = .5, q_1 = 2.5, p_2 = .6, q_2 = 4.0,\ I_{11} = .15,\ I_{12} = .24, P_1 = INR\ 3500,$
$P_2 = INR\ 4500, C_1 = INR\ 1500, C_2 = INR\ 2200, \tau = 0.5,\ \rho = 0.5,\ \ \xi = 10000, M_1 = 100000,$
$M_2 = 120000, A = INR\ 750000, A_1 = INR\ 250000, A_2 = INR\ 250000$

First, the single generation model was tested. On deploying the model over Matlab, the following results can be obtained as shown in Table6.2.

Table6.2. Determination of Optimal number of replenishments for different planning horizons in single generation scenario (All costs and revenues in Mn currency)

	m=1			*m=2*			*m=3*		
n	***HC***	***OC***	***TC***	***HC***	***OC***	***TC***	***HC***	***OC***	***TC***
1	3.53	1.00	4.53	3.75	1.00	4.75	1.49	1.00	***2.49***
2	1.63	2.00	***3.63***	1.83	2.00	***3.83***	0.62	2.00	2.62
3	1.35	3.00	4.35	1.69	3.00	4.69	0.38	3.00	3.38
Rev	127.9			136.87			62.05		

TC%	1.27%	1.33%	2.39%

Legends: HC: Holding Costs, OC: Ordering Costs, TC: Total Replenishments Costs

As shown, the optimal number of replenishments is higher in the early stages of the product life cycle and declines in the later stages of the product life cycle. This is because the higher (or lower) volumes in the initial stages of the product life cycle cause the inventories to increase. The only way to reduce inventory carrying costs is to increase the frequency of replenishment. With similar reasoning, it can be easily understood why the number of optimal replenishments declines in the later stages of the product life cycle. Then, the model has been tested for the time post the launch of the second-generation product. The following results were obtained as shown in Table6.3.

Table6.3. Determination of Optimal number of replenishments for different planning horizons in two generations scenario (All costs and revenues in Mn currency)

	m=1, ym=1				*m=2, ym=1*				*m=3, ym=1*			
n	***HC1***	***HC2***	***OC***	***TC***	***HC1***	***HC2***	***OC***	***TC***	***HC1***	***HC2***	***OC***	***TC***
1	3.8	9.9	1.25	15.0	1.1	7.6	1.25	9.9	0.0	1.1	1.25	***2.4***
2	2.4	3.9	2.50	***8.8***	0.6	3.0	2.50	***6.2***	0.0	0.6	2.50	3.1
3	4.4	3.1	3.75	11.2	1.1	2.3	3.75	7.1	0.0	0.4	3.75	4.2
Revenue	411.0				267.2				54.2			
TC %	2.14%				2.31%				5.78%			

Legends: HC: Holding Costs, OC: Ordering Costs, TC: Total Replenishments Costs

It is generally observed that with the launch of the second-generation product, the replenishment costs as a % of the revenues rise. This is because the second-generation product is an enhanced version of the earlier product, and results in the addition of volumes and therefore, higher utilization of the rented warehouse, which shoots up the weighted average holding cost. It can also be observed that the sales of the first generation product decline sharply after the launch of the second-generation product.

6.2. Major academic and business implications of the proposed EOQ models

This research study illustrated how to integrate the inventory decisions for innovation diffusion dependent demand of multi-generation products with the warehousing capacity constraints. Thus, it would not be incorrect to say that it can help in establishing a sound collaboration between the inventory managers and the logistics managers for the technology products, increasing the efficiencies of the overall value chain. The study also showed how the optimal service level may not be 100% if the rent premium of the rented warehouse is above a certain level. The dynamics of the rent and the amount of capacity constraint were also linked with the optimal number of replenishments in each planning time horizon and with the exit timing of the earlier generation products. This research brings to the surface the crucial importance of space constraints and the rentals in the planning of the inventories. The following are the key learnings that this study has to offer to the inventory practitioners:

a) The rental premium on the rented warehouse and the higher capacity constraint of the inhouse warehouse tend to
   I. jack up the optimal number of replenishment cycles
   II. expedite the exit of the earlier generation products from the market
   III. increase the costs at the optimal point
b) The higher rental premium on the rented warehouse can reduce the optimal service levels unless the dynamic pricing is adopted to cover the higher cost-to-serve.

Warehousing managers and inventory managers need to work in tight collaboration with each other to maximize the utility of their systems to the organizational system as a whole.

The chapters till now have discussed the framework for integrating the inventory policies of multi-generational technology products with the warehousing space constarints, trade credits and price elasticity of demand. In all these chapters, the deterministic business environment was considered. But in real life, it can be observed that the business paramters are not certain in nature but possess a certain degree of impreciseness. The next chapter is going to discuss how the impreciseness can be handled in inventory modelling for technology generation products.

Appendix 6

A. Theorem 6.1: *With the increase in the rent of the rented warehouse, the optimal number of the replenishment cycles increases in the intermediate planning horizons, while does not get influenced in the initial or later periods of the planning horizon.*

Proof: The higher rent premium charged by the external space provider causes the inventory carrying cost per unit to rise if the second warehouse is being utilized. Therefore, the inventory reduction becomes more paramount, and if the 100% service level has to be met, the only way to reduce the inventory is to compress the length of replenishment cycles and increase the replenishment frequency. However, the above argument holds good only when there is a high dependency on the rented warehouse which is in the peak stage of the product life cycle and hence the intermediate planning horizons.

The illustration of this concept is given below in figure 6.A.1. In the figures that follow, the TC signifies the sum of holding costs and ordering costs; the OC signifies the ordering costs, and the HC signifies the holding costs. TCM stands for the Total Contribution Margin, i.e. the Revenue net off the basic cost of material; and TP signifies the total profit after all expenses.

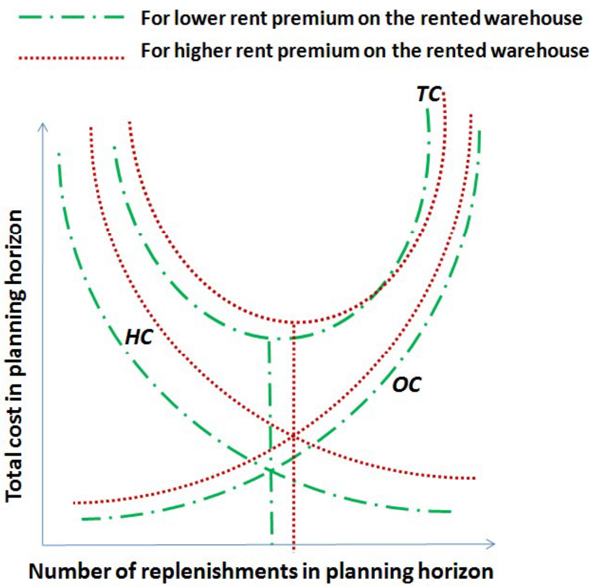

Figure6.A.1. Influence of increase in rent of the second warehouse on the optimal replenishment frequency

B. Theorem 6.2: As the values of innovation and imitation coefficients rise, the dependence on rented warehouse becomes more uneven with time; the optimal number of replenishment cycles rises in the initial stages of the product life cycle, and declines in the later stages.

Proof: With the higher coefficients of innovation and imitation, the diffusion of the product is faster, leading to higher volumes, and thus higher dependence on the rented warehouse in the initial stages of the life cycle, and vice versa in the later stages of the product life cycle. The higher dependence on the rented warehouse in the initial stages spurs the inventory carrying costs, and makes it a compelling need for the inventories to be reduced, and therefore the replenishment frequency to be increased. Figure6.B.1 depicts this phenomenon.

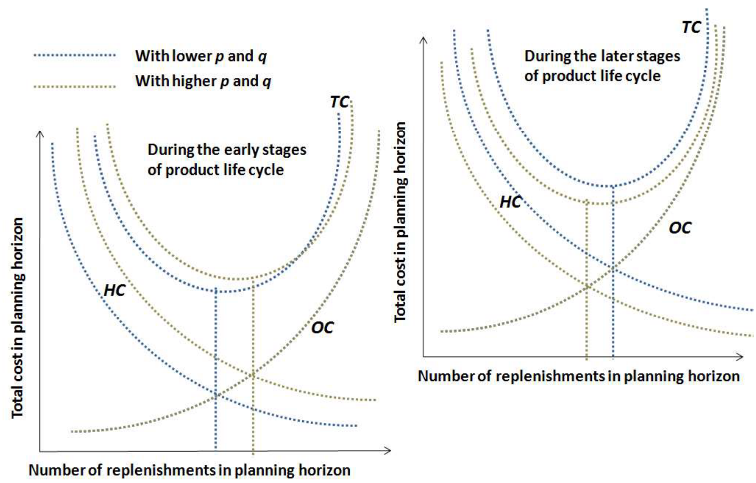

Figure6.B.1. Influence of the coefficient of innovation and imitation on the optimal number of replenishments

C. Theorem 6.3: *The higher the rent premium charged for the rented warehouse, the earlier it makes sense to discontinue the earlier generation products.*

Proof: As shown in figure6.C.1, this refers to the later stages of the first generation product when the profit margins are already wafer-thin due to insignificant volumes. The first generation needs to be continued only as long as its revenue net of the inventory carrying costs and basic purchase costs are significant enough to meet the product-specific ordering costs. The higher rental charges cause the former figure to decline, and be overpowered by the product-specific ordering costs much earlier, and therefore, expedites the phase-put of the later generation product.

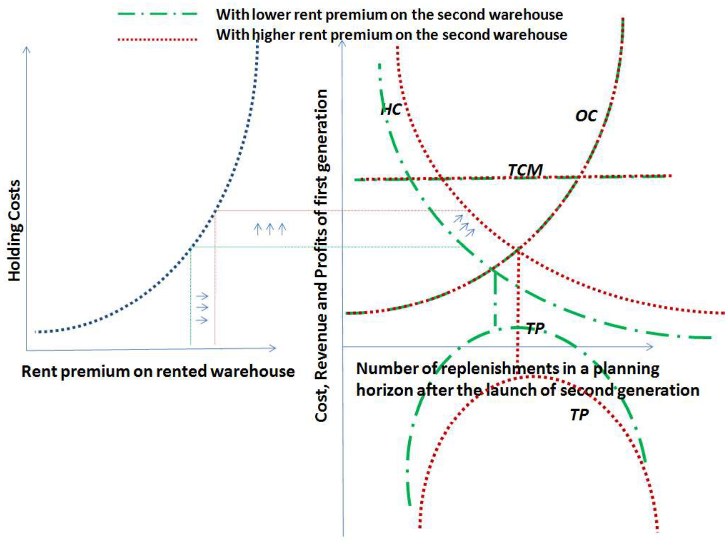

Figure6.C.1. Influence of the rent of rented warehouse on the profitability of the first-generation product in the later stages of the product life cycle

D. Theorem 6.4. *If the rent charged by the rented warehouse crosses a certain threshold, the optimal service level may fall to less than 100%, unless the dynamic pricing is used to charge the customer higher for the higher cost-to-serve. And the higher will be this fall during the peak stages of the product life cycle.*

Proof: As the rent charged by the rented warehouse increases, the average TC per unit item increases if $z_{i,m,n} = 1$ $(before\ the\ lanch\ of\ the\ second\ generation)$ and $Z_{i,m,n} = 1$ $(after\ the\ launch\ of\ the\ second\ generation)$. During the peak stages of the product life cycle, the demand is expected to be higher, and hence, the use of the second warehouse becomes almost indispensable. With the increase in rent beyond a threshold limit, the TC per unit exceeds the contribution margin per unit, thereby, resulting in the losses. However, if the service level is reduced to a point where the total demand to be met can be stored in the inhouse warehouse, the optimal profit will increase. This is illustrated in Figure 6.D.1.

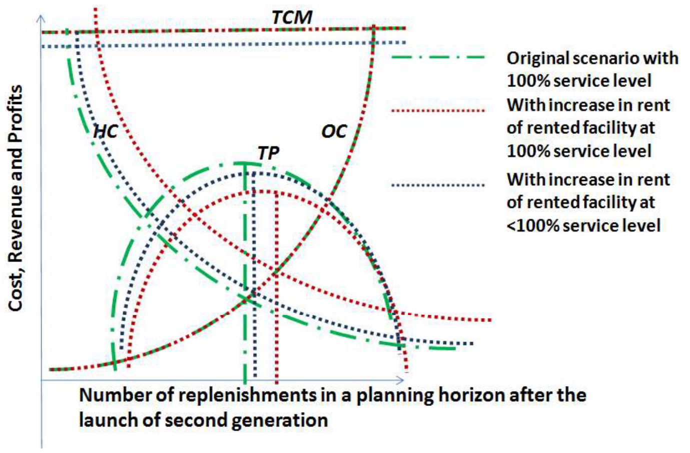

Figure6.D.1. The influence of the rent premium of the rented warehouse on the optimal service level

E. Theorem 6.5. *The higher the amount of capacity deficit of the first warehouse, and the higher the optimal number of replenishments in the planning horizon.*

Proof: With the higher capacity deficit of the own warehouse, the per-unit inventory carrying costs rise to higher levels due to higher dependency on the rented warehouse that carries a rent premium over the in-house warehouse. This tends to push the inventory costs upwards, thereby, bringing the optimal number of replenishments upwards. The graphical illustration of this phenomenon is shown here in Figure6.E.1.

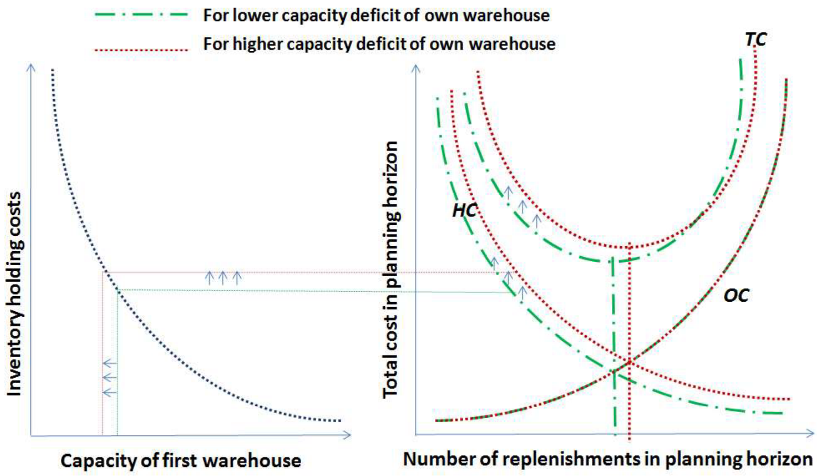

Figure6.E.1. Influence of the capacity deficit in the first warehouse on the optimal number of replenishments