Chapter 4

General Repair Distribution for Imperfection

"Science is beautiful gift to humanity;
We should not distort it."

A. P. J. Abdul Kalam

4.1 Introduction

With the advent of the latest technologies, the internet of things (IoT) becomes a great boon for human civilization. It requires various independent working systems like computing system, communication system, power supply system, etc. Power supply system is more prone to failure in nature among these and necessary to be fault-tolerant. The power generation failure causes power interruption, which affects adversely on the efficiency, quality, and output of the system. The critical fault-tolerant machining system recurrently must use redundancy if it is required to meet extremely high availability requirement levels. The fault-tolerant machining system can be modeled mathematically as a machine repair problem (MRP). The major task of the monitoring system is to detect the faults in the redundant machining system, isolation, reconfiguration to prevent a failed unit from adversely affecting the system,s ongoing performance. The notable research works on Markovian modeling of machine repair problem have been done in past (*cf.* Haque and Armstrong [59]; Shekhar et al. [159]). Redundancy revelations an important role in increasing the availability of the machining systems. Passive redundancy with spare units in repairable machining systems has been examined extensively in the past (*cf.* Kumar and Agarwal [104]; Huang and Ke [64]). However, almost all researches in machine repair problem consider perfect coverage and perfect switching of the available spare unit on failure of the operating unit, which is prone to breakdown.

When the breakdown of the operating unit is observed, there exists the possibility of replacing it with the available spare unit so that the machining system can operate in spite of the unexpected failure of the operating unit. The switching failure during the transition of the spare unit to the operational mode in case of replacement of a failed operating unit in a machining system cannot be ignored. Few researchers studied machine repair problems with switching failure of the spare unit in the past. (*cf.* Lewis [116]; Jain et al. [82]; Liou [124]). Kuo and Ke [111] computed steady-state availability of a repairable machining system with spare unit switching failure. Using the Probabilistic Global Search Lausanne (PGSL) method in cost analysis, an optimal number of spare units for machine repair systems with spare unit switching failure was determined by Ke et al. [98]. Shekhar et al. [153] used the concept of switching failure and geometric reneging in machine interference problem with spare units provisioning for performance modeling and reliability analysis. Yang and Tsao [192] considered the repairable system with multiple working vacations and retrial orbit and dealt with various reliability characteristics.

The likelihood that monitoring system,s tasks are accomplished correctly is called fault coverage or simply perfect coverage. These tasks can seldom be done with perfect certainty *i.e.* there is always some chance that at least one of the tasks, identification, isolation, or reconfiguration, is not done correctly. The machining system that is subject to some level of uncertainty in the monitoring process is framed as MRP with imperfect fault coverage (*cf.* Pham [139]; Moustafa [131]). In the machining system, faults may numerous in type and system analysts sometimes unable to identify them or to locate them i.e. the faults are not appropriately covered and hence unable to recover them. When the operating unit fails and covers perfectly, it is replaced with an available spare unit with some switchover time delay. Otherwise, if it is not covered perfectly, it is cleared from the system through the reboot process. The reboot is very instantaneous, so that the occurrence of any other event is rare. Reboot delay and switching delay hinder the functionality of the machining system for a moment (*cf.* Wang and Chen [177]; Jain [72]). Shekhar et al. [154] evaluated the mean time-to-failure and availability of redundant machine repair problems with switching failure and reboot delay. Recently, Shekhar et al. [155] dealt with the reliability analysis of the fault-tolerant multi-component machining system having multi-warm spares and reboot provisioning. Manglik and Ram [127] discussed the coverage factor of various types of failures in the multi-component repairable machining system.

Stochastic processes with discrete states in continuous time can be transformed into Markov processes by either the use of regeneration point, or Erlang,s method with division into fictitious stages, or the inclusion of the sufficient supplementary variable. Among these, the well-known method of including supplementary variables, the remaining or elapsed service time, for determining the queue size distribution explicitly is more justifiable. It is observed that the resulting equations simplify considerably when some arbitrary distributions associated with the process have rational Laplace transforms (*cf.* Cox [32]; Hokstad [60]). Gupta and Rao [58]; [57] used a supplementary variable, the remaining service time, and a recursive method firstly to obtain the steady-state probability distribution of the number of down units at arbitrary time epoch of a machine interference problem with spares. Wang et al. [181] used a recursive method and supplementary variable technique to develop steady-state analytic solutions of an $M/G/1$ machine repair problem with multiple imperfect coverages. Using the supplementary variable technique, Jiang et al. [87] obtained the distribution for the stationary queue at an arbitrary epoch for the $M/G/1$ queue in a multi-phase random environment with disasters.

In this chapter, a machine repair problem with two operating units and the provisioning of the spare unit is studied. The practical example of the studied machining system can be observed in the power supply system having two identical generators with the facility of one spare generator of the same capacity. All generators are under the monitor of an automatic switching system that manages the switchover process, the reboot process, activation of generation by self. It also alarms the repair facility on perfect coverage of the fault. The objective of this chapter is to provide an alternative solution technique, using the remaining repair time as the supplementary variable, to compute an explicit expression for steady-state probabilities for the machine repair problem that are used to obtain the various reliability measures of the machining system. The method is recursive and can be used for any repair time distribution such as Markovian (M), Erlangian (Er_n), deterministic (D), uniform ($U(a,b)$), generalized Erlang (GE_n) or hyperexponential (HE_n) etc. These distributions have a different coefficient of variation and represent a different type of repair architect like single or multi-phase repair, different types of repair to a different type of unit as per its requirement, etc. The mere input required for computation of state probabilities is the Laplace-Stieltjes transform (LST) of the repair time distribution.

There are many research studies on stochastic models of machine repair problems with realistic environments in recent literature base. Only a few of these studies focus on repairable machining systems with a supplementary variable technique that used the remaining repair time as a supplementary variable. On the survey base, it is noticed that no research article considers the different types of failures, breakdown, imperfect coverage, switching failure simultaneously in two operating units machining system with spare unit provisioning. Uncertainty in the failure is one of the important issues in the decision criterion. At present, works on the Markovian machine repairing system with imperfect coverage, switching failure, and reboot delay mainly focus on the exponential repair time distribution. However, from a hands-on perspective, the repair time can be an arbitrary random variable. This research gap gives the motivation to investigate a machine repairing system with imperfect coverage and switching failure and arbitrary distributed repair times. The purpose of this research is threefold and outlined as follows:

(i) to examine the machine repairing system with imperfect coverage of failure, reboot & switchover delay, and switching failure, in which the repair time follows the general distribution.

(ii) to propose a step-wise recursive solution procedure to compute the steady-state probability distribution of the number of failed units in the machining system, which are used to compute various system performance measures and reliability measures.

(iii) to perform a comparative analysis of the different repair time distributions and present numerical simulation in detail.

In this chapter, the discussion is made on the types of failure which may hinder the performance of the power supply system in terms of availability. The rest of the chapter is organized as follows Section (4.2) presents a detailed model description with assumptions and notations. In section (4.3), the recursive method is presented for the computation of the steady-state probabilities and availability of the repairable machining system. Different types of repair time distribution are used in section (4.4) and an explicit expression for state probabilities are derived. Parametric sensitivity has been discussed in section (4.5). Conclusions are drawn and future scope is given in section (4.6).

4.2 Model Description

For the modeling, we consider the requirement of a 10 MW power supply in the service system and assume that available generators have 5 MW electricity generation capacity. For that purpose, there are two primary active generators (operating units) in the system with the provision of one spare generator (spare unit) to produce reliable and uninterrupted power supply for the service system. All primary active generators and spare generator are under the care of one repairman and automatic monitoring device to detect failure or to provide immediate repair. The studied model is visualized as machine repair problem (MRP) with two operating units, one spare unit, and a single repairman. For developing the mathematical model, the following assumptions and notations are considered:

- The running times of the units between breakdowns have an exponential distribution with mean $1/\lambda$.

- The warm spare unit is also prone to failure in an inactive state too with an assumption that its lifetime has an exponential distribution with mean $1/\nu$ $(0 < \nu < \lambda)$.
- Automatic monitoring device detects the failure of the unit(s) with coverage probability c. On successful coverage, the failed unit is promptly supplanted by an available spare unit with exponentially distributed switchover time having the meantime $1/\sigma$. The switchover may be unsuccessful with failure probability p.
- Whenever failure of the unit(s) is not covered successfully, an unsafe failure state of the machining system, the failed unit is cleared from the machining system by a reboot process. Reboot delay is assumed to exponentially distributed with parameter β $(\beta >> \mu, \lambda)$. The reboot process is so fast so that the possibility of the occurrence of any other event is very rare.
- In automatic monitoring device, for uninterrupted in the functioning of the machining system, the available spare unit immediately switches in place of the failed operating unit with switching failure probability q.
- The failed unit is repairable and is immediately sent to a single repairman for repair based on a first-come, first-served ($FCFS$) discipline. The repair times are identically and independently distributed random variables (iidrv's) having a probability density function $b(v)$, distribution function $B(v)$ and mean repair time b_1. Once a unit is repaired, it is as good as new.
- The power generation system fails when the remaining electricity generation capacity is less than the required 10 MW, which is defined as a safe failure.
- All events are independent of the state of all the other events.

4.3 Steady-State Probabilities and Availability

For the modeling and computational purpose, the following supplementary variable: $V \equiv$ remaining repair time for the failed units under repair is used. The state of the system at time t is given by:
$M(t) \equiv$ number of active operating units in the machining system at time t,
$N(t) \equiv$ number of active spare unit in the machining system at time t,
$V(t) \equiv$ remaining repair time for the failed units under repair at time t,
and

$I(t) \equiv$ state of the machining system at time t
where

$$I(t) = \begin{cases} 0; & \text{if the machining system is in a safe failure state} \\ 1; & \text{if the machining system is in an unsafe failure state} \end{cases}$$

For $v \geq 0$, $t \geq 0$, let us define the following state probabilities:
$P_{2,1}(v,t) = \text{Prob}\{M(t) = 2, N(t) = 1, I(t) = 0,\ v \leq V(t) \leq v + dv\}$
$P_{2,0}(v,t) = \text{Prob}\{M(t) = 2, N(t) = 0, I(t) = 0,\ v \leq V(t) \leq v + dv\}$
$P_{1,0}(v,t) = \text{Prob}\{M(t) = 1, N(t) = 0, I(t) = 0,\ v \leq V(t) \leq v + dv\}$
$Q_{1,1}(v,t) = \text{Prob}\{M(t) = 1, N(t) = 1, I(t) = 1,\ v \leq V(t) \leq v + dv\}$
$R_O(v,t) = \text{Prob}\{$imperfect coverage of the failed operating unit,
$I(t) = 1,\ v \leq V(t) \leq v + dv\}$
$R_S(v,t) = \text{Prob}\{$imperfect coverage of the failed spare unit,
$I(t) = 1,\ v \leq V(t) \leq v + dv\}$
Hence,

$$P_{2,1}(t) = \int_0^\infty P_{2,1}(v,t)\,dv; \quad P_{2,0}(t) = \int_0^\infty P_{2,0}(v,t)\,dv; \quad P_{1,0}(t) = \int_0^\infty P_{1,0}(v,t)\,dv$$

$$Q_{1,1}(t) = \int_0^\infty Q_{1,1}(v,t)\,dv; \quad R_O(t) = \int_0^\infty R_O(v,t)\,dv \ \text{ and } \ R_S(t) = \int_0^\infty R_S(v,t)\,dv$$

Using the above-mentioned assumptions and notations, the following state transition diagram is developed and depicted in Fig. (4.1).

Now, relating the states of the $M/G/1$ fault-tolerant machining system with imperfect coverage, reboot delay, switchover delay, switching failure and spare provisioning at time t and $t + dt$ in Fig. (4.1), the following forward differential equations are obtained on balancing the inflow and outflow of the rates:

$$\frac{dP_{2,1}(t)}{dt} = -(2\lambda + \nu)P_{2,1}(t) + P_{2,0}(0,t) \tag{4.1}$$

$$\left(\frac{\partial}{\partial t} - \frac{\partial}{\partial v}\right)P_{2,0}(v,t) = -2\lambda P_{2,0}(v,t) + \nu c P_{2,1}(t) + b(v)P_{1,0}(0,t) + \beta R_S(t) \\ + (1-p)\sigma Q_{1,1}(t) \tag{4.2}$$

$$\left(\frac{\partial}{\partial t} - \frac{\partial}{\partial v}\right)P_{1,0}(v,t) = 2\lambda P_{2,0}(v,t) + 2\lambda q P_{2,1}(t) + P\sigma Q_{1,1}(t) \tag{4.3}$$

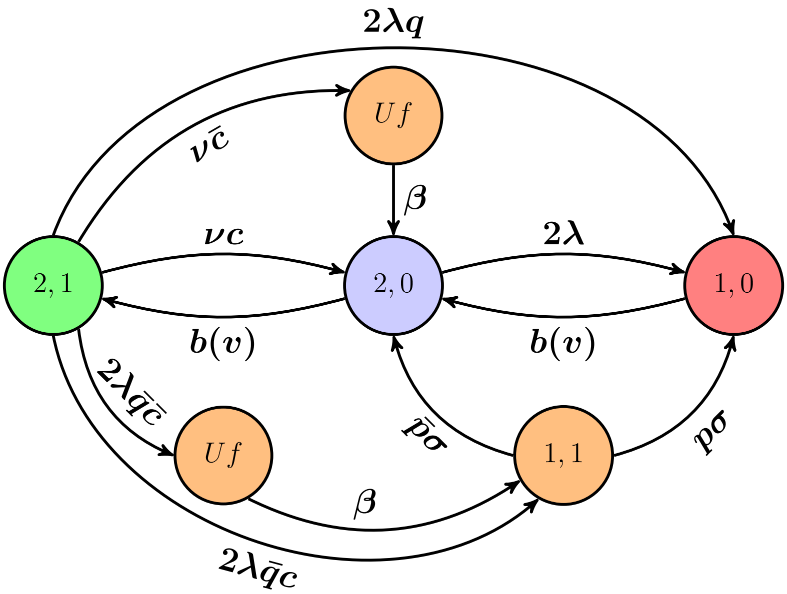

Figure 4.1: The state transition diagram

$$\frac{dQ_{1,1}(t)}{dt} = -\sigma Q_{1,1}(t) + 2\lambda(1-q)cP_{2,1}(t) + \beta R_O(t) \tag{4.4}$$

$$\frac{dR_O(t)}{dt} = -\beta R_O(t) + 2\lambda(1-q)(1-c)P_{2,1}(t) \tag{4.5}$$

$$\frac{dR_S(t)}{dt} = -\beta R_S(t) + \nu(1-c)P_{2,1}(t) \tag{4.6}$$

In steady-state (as $t \to \infty$), the state probabilities are defined as follows

$$P_{2,1} = \lim_{t\to\infty} P_{2,1}(t), \quad P_{2,0} = \lim_{t\to\infty} P_{2,0}(t), \quad P_{1,0} = \lim_{t\to\infty} P_{1,0}(t),$$
$$Q_{1,1} = \lim_{t\to\infty} Q_{1,1}(t), \quad R_O = \lim_{t\to\infty} R_O(t), \quad R_S = \lim_{t\to\infty} R_S(t),$$
$$P_{2,1}(v) = \lim_{t\to\infty} P_{2,1}(v,t), \quad P_{2,0}(v) = \lim_{t\to\infty} P_{2,0}(v,t), \quad P_{1,1}(v) = \lim_{t\to\infty} P_{1,1}(v,t),$$
$$Q_{1,1}(v) = \lim_{t\to\infty} Q_{1,1}(v,t), \quad R_O(v) = \lim_{t\to\infty} R_O(v,t), \quad R_S(v) = \lim_{t\to\infty} R_S(v,t)$$

Further, the following relations are also defined as

$$P_{2,1}(v) = b(v)P_{2,1} \tag{4.7}$$

$$Q_{1,1}(v) = b(v)Q_{1,1} \tag{4.8}$$

$$R_O(v) = b(v)R_O \tag{4.9}$$

$$R_S(v) = b(v)R_S \tag{4.10}$$

From Eq^n's. (4.1)-(4.10) and defined steady-state probabilities, the following steady-state equations are obtained:

$$-(2\lambda + \nu)P_{2,1} + P_{2,0}(0) = 0 \tag{4.11}$$

$$-\frac{d}{dv}P_{2,0}(v) = -2\lambda P_{2,0}(v) + \nu c\, b(v)P_{1,0}(0) + \beta b(v)R_S + (1-p)\sigma b(v)Q_{1,1} \tag{4.12}$$

$$-\frac{d}{dv}P_{1,0}(v) = 2\lambda P_{2,0}(v) + 2\lambda q\, b(v)P_{2,1} + p\sigma b(v)Q_{1,1} \tag{4.13}$$

$$-\sigma Q_{1,1} + 2\lambda(1-q)cP_{2,1} + \beta R_O = 0 \tag{4.14}$$

$$-\beta R_O + 2\lambda(1-q)(1-c)P_{2,1} = 0 \tag{4.15}$$

$$-\beta R_S + \nu(1-c)P_{2,1} = 0 \tag{4.16}$$

From Eq^n's. (4.11), (4.15), and (4.16), the following results are obtained

$$P_{2,0}(0) = (2\lambda + \nu)P_{2,1} \tag{4.17}$$

$$R_O = \frac{2\lambda(1-q)(1-c)}{\beta}P_{2,1} \tag{4.18}$$

$$R_S = \frac{\nu(1-c)}{\beta}P_{2,1} \tag{4.19}$$

Hence, from Eq^n. (4.14), we get

$$\begin{aligned} Q_{1,1} &= \frac{2\lambda(1-q)cP_{2,1} + \beta R_O}{\sigma} \\ &= \frac{2\lambda(1-q)}{\sigma}P_{2,1} \end{aligned} \tag{4.20}$$

Further, define Laplace transform in term of Laplace variable s for probability density function $b(v)$ of repair times and state probabilities as follows

$$\widetilde{B}(s) = \int_0^\infty e^{-sv} dB(v) = \int_0^\infty e^{-sv} b(v) dv$$

$$\widetilde{P}_{2,0}(s) = \int_0^\infty e^{-sv} P_{2,0}(v) dv$$

$$P_{2,0} = \widetilde{P}_{2,0}(0) = \int_0^\infty P_{2,0}(v) dv$$

$$\int_0^\infty e^{-sv} \frac{d}{dv} P_{2,0}(v) dv = s\widetilde{P}_{2,0}(s) - P_{2,0}(0)$$

and

$$\widetilde{P}_{1,0}(s) = \int_0^\infty e^{-su} P_{1,0}(v) dv$$

$$P_{1,0} = \widetilde{P}_{1,0}(0) = \int_0^\infty P_{1,0}(v) dv$$

$$\int_0^\infty e^{-sv} \frac{d}{dv} P_{1,0}(v) dv = s\widetilde{P}_{1,0}(s) - P_{1,0}(0)$$

Taking the Laplace-Stieltjes (LST) on both sides of Eq^n's. (4.12)-(4.13), on simplification, the resultant equations are

$$(2\lambda - s)\widetilde{P}_{2,0}(s) = (\nu c P_{2,1} + P_{1,0}(0) + \beta R_S + (1-p)\sigma Q_{1,1})\widetilde{B}(s) - P_{2,0}(0) \quad (4.21)$$

$$-s\widetilde{P}_{1,0}(s) = 2\lambda \widetilde{P}_{2,0}(s) + 2\lambda q \widetilde{B}(s) P_{2,1} + p\sigma \widetilde{B}(s) Q_{1,1} - P_{1,0}(0) \quad (4.22)$$

Now, a recursive method is developed to get the explicit expression $\widetilde{P}_{2,0}(0)$ and $\widetilde{P}_{1,0}(0)$ in term of $P_{2,1}$ as follows Substituting Eq^n's. (4.17), (4.19), and (4.20) into Eq^n's. (4.21), and setting $s = 2\lambda$ in Eq^n's. (4.21), it follows

$$P_{1,0}(0) = \frac{2\lambda + \nu - (2\lambda(1-p)(1-q) + \nu)\widetilde{B}(2\lambda)}{\widetilde{B}(2\lambda)} P_{2,1} \quad (4.23)$$

Substituting Eq^n's (4.20) and (4.23) into Eq^n's (4.22), and setting $s = 0$ in Eq^n's (4.22), we have

$$\widetilde{P}_{2,0}(0) = \frac{(2\lambda + \nu)\left(1 - \widetilde{B}(2\lambda)\right)}{2\lambda \widetilde{B}(2\lambda)} P_{2,1} \quad (4.24)$$

Differentiate Eq^n. (4.21) with respect to s and set $s = 0$ and put

$$b_1 = -\widetilde{B}^{(1)}(0) = -\left(\frac{\partial \widetilde{B}(s)}{\partial s}\right)_{s=0}$$

in result. From the resulting expression, the value of $\widetilde{P}_{2,0}^{(1)}(0)$ is derived in term of $P_{2,1}$ using the expressions in Eq^n's. (4.19), (4.20), (4.23), and (4.24).

$$\widetilde{P}_{2,0}^{(1)}(0) = \frac{(2\lambda+\nu)\left(1-2\lambda b_1-\widetilde{B}(2\lambda)\right)}{4\lambda^2\widetilde{B}(2\lambda)}P_{2,1} \tag{4.25}$$

Similarly, on differentiating Eq^n's. (4.22) with respect to s and setting $s=0$ in result and using Eq^n's. (4.20), (4.23), and (4.25) we have

$$\widetilde{P}_{1,0}(0) = \frac{(2\lambda+\nu)\left(\widetilde{B}(2\lambda)-1-2\lambda b_1\right)-4\lambda^2 b_1\left(p(1-q)+q\right)\widetilde{B}(2\lambda)}{2\lambda\widetilde{B}(2\lambda)}P_{2,1} \tag{4.26}$$

Since $\widetilde{P}_{2,0}(0)$, $\widetilde{P}_{1,0}(0)$, $Q_{1,1}$, R_O, and R_S are in term of $P_{2,1}$, on substituting Eq^n's. (4.18), (4.19), (4.20), (4.24), and (4.26) into the probability normalizing condition

$$P_{2,1}+\widetilde{P}_{2,0}(0)+\widetilde{P}_{1,0}(0)+Q_{1,1}+R_O+R_S=1 \tag{4.27}$$

the expression for state probability $P_{2,1}$ is as follows

$$P_{2,1} = \frac{\beta\sigma\widetilde{B}(2\lambda)}{D_1} \tag{4.28}$$

where

$$\begin{aligned} D_1 = &\left(\bar{q}\left(2\lambda\left(\beta+\bar{c}\sigma\right)-2b_1\beta\lambda\sigma\bar{p}\right)+\sigma\left(\beta\left(2b_1\lambda+1\right)+\nu\bar{c}\right)\right)\widetilde{B}(2\lambda) \\ &+b_1\beta\sigma\left(2\lambda+\nu\right) \end{aligned}$$

After obtaining the explicit expression for state probability $P_{2,1}$, the explicit expression for remaining state probabilities are obtained as follows

$$\widetilde{P}_{2,0}(0) = \frac{\beta\sigma(2\lambda+\nu)(1-\widetilde{B}(2\lambda))}{2\lambda D_1} \tag{4.29}$$

$$\widetilde{P}_{1,0}(0) = \frac{\left(4\lambda^2 b_1\left(q+(1-q)p\right)\widetilde{B}(2\lambda)+(2\lambda+\nu)\left(2b_1\lambda+\widetilde{B}(2\lambda)-1\right)\right)\beta\sigma}{2\lambda D_1} \tag{4.30}$$

$$Q_{1,1} = \frac{2\lambda(1-q)\beta\widetilde{B}(2\lambda)}{D_1} \tag{4.31}$$

$$R_O = \frac{2\lambda\sigma(1-c)(1-q)\widetilde{B}(2\lambda)}{D_1} \tag{4.32}$$

$$R_S = \frac{\sigma(1-c)\,\nu\widetilde{B}(2\lambda)}{D_1} \tag{4.33}$$

where $\bar{c}, \bar{p}$, and $\bar{q}$ are complementary probability of c, p, and q respectively. Since, there is demand of 10 MW power supply, atleast two operating units should function properly. Hence, the availability of the machining system is defined as

$$Av = P_{2,1} + \widetilde{P}_{2,0}(0)$$

and its explicit expression can be derived as follows

$$Av = \frac{\left(2\lambda + \nu\left(1-\widetilde{B}(2\lambda)\right)\right)\sigma\beta}{2\lambda D_1} \tag{4.34}$$

For the arbitrary distribution of the repair time, the explicit expression of state probabilities in Eq^n's. (4.28)-(4.33) and availability in Eq^n. (4.34) are derived. For standard repair time distribution, there is no need to repeat the recursive procedure completely for the derivation of expression of state probabilities and corresponding availability of the machining system. The Laplace-Stieltjes transform (LST) of the repair time distribution is merely required.

An recursive algorithm for computing steady-state probabilities is given as follows

Recursive algorithm

Step 1. Initialize $P_{2,1} = 1$.

Step 2. Compute $P_{2,0}(0)$ using the Eq^n. (4.17).

Step 3. Compute R_O and R_S using the Eq^n's. (4.18) and (4.19).

Step 4. Compute $Q_{1,1}$ using the result obtained in step 3 and the Eq^n. (4.20).

Step 5. Compute $P_{1,0}(0)$ from the Eq^n (4.23).

Step 6. Evaluate $\widetilde{P}_{2,0}(0)$ using the Eq^n's. (4.24).

Step 7. Determine the explicit expression for $\widetilde{P}^{(1)}_{2,0}(0)$ from Eq^n. (4.25).

Step 8. Compute $\widetilde{P}_{1,0}(0)$ using Eq^n. (4.26).

Step 9. Using the normalizing condition in Eq^n. (4.27) and Eq^n. (4.28), compute the value $P_{2,1}$.

Step 10. Using the value of $P_{2,1}$ from step 9 and Eq^n's. (4.29)-(4.33), evaluate all state probabilities $\widetilde{P}_{2,0}(0)$, $\widetilde{P}_{1,0}(0)$, $Q_{1,1}$, R_O, and R_S.

Step 11. Compute the availability of the machining system (Av) from the Eq^n's. (4.34).

4.4 Special Cases

Based on the flow of the solution procedure described in the previous section (4.3), the explicit expression for state probabilities and the availability of the machining system can easily be derived for different continuous distributions of service times. For the described recursive method, a Laplace-Stieltjes transform (LST) of the repair time distribution is merely required. The explicit expression of state probability $P_{2,1}$ and the availability of the machining system Av are presented for exponential (M), n-stage Erlangian (Er_n) and Deterministic (D) distribution.

Case 1: Exponential Distribution. The repair times follow an exponential distribution with mean rate μ. It is member of gamma distribution. The Laplace-Stieltjes transform of probability density function $b(v) = \mu e^{-\mu v}$ is given by

$$\widetilde{B}(s) = \frac{\mu}{s+\mu}$$

Hence, $b_1 = -\widetilde{B}^{(1)}(0) = \frac{1}{\mu}$. Succeeding the recursive algorithm discussed in previous section, step wise expression are as follows:

Step 1. $P_{2,1} = 1$
Step 2. $P_{2,0}(0) = 2\lambda + \nu$
Step 3. $R_O = \dfrac{2(1-q)(1-c)\lambda}{\beta}$, $R_S = \dfrac{\nu(1-c)}{\beta}$
Step 4. $Q_{1,1} = \dfrac{2\lambda(1-q)}{\sigma}$
Step 5. $P_{1,0}(0) = \dfrac{2\lambda\left[\{(1-q)p+q\}\mu + \nu + 2\lambda\right]}{\mu}$
Step 6. $\widetilde{P}_{2,0}(0) = \dfrac{2\lambda+\nu}{\mu}$
Step 7. $\widetilde{P}^{(1)}_{2,0}(0) = \dfrac{-2\lambda-\nu}{\mu^2}$
Step 8. $\widetilde{P}_{1,0}(0) = \dfrac{2\lambda\left[\{(1-q)p+q\}\mu + \nu + 2\lambda\right]}{\mu^2}$
Step 9.

$$P_{2,1} = \left[\frac{\beta\mu^2\sigma}{2\bar{q}\lambda\,\mu\left((\bar{c}\mu - \beta\,\bar{p})\,\sigma + \beta\,\mu\right) + \sigma\left((\mu+2\lambda)\,(\mu+\nu+2\lambda)\,\beta + \mu^2\nu\,\bar{c}\right)}\right] \tag{4.35}$$

Step 10.

$$\widetilde{P}_{2,0}(0) = \left[\frac{\beta\mu\sigma(2\lambda+\nu)}{2\bar{q}\lambda\,\mu\,((\bar{c}\mu-\beta\,\bar{p})\,\sigma+\beta\,\mu)+\sigma\,((\mu+2\lambda)\,(\mu+\nu+2\lambda)\,\beta+\mu^2\nu\,\bar{c})}\right]$$

$$\widetilde{P}_{1,0}(0) = \left[\frac{2\lambda\beta\sigma\,[\{(1-p)q+p\}\mu+\nu+2\lambda]}{2\bar{q}\lambda\,\mu\,((\bar{c}\mu-\beta\,\bar{p})\,\sigma+\beta\,\mu)+\sigma\,((\mu+2\lambda)\,(\mu+\nu+2\lambda)\,\beta+\mu^2\nu\,\bar{c})}\right]$$

$$Q_{1,1} = \left[\frac{2\mu^2\beta\lambda(1-q)}{2\bar{q}\lambda\,\mu\,((\bar{c}\mu-\beta\,\bar{p})\,\sigma+\beta\,\mu)+\sigma\,((\mu+2\lambda)\,(\mu+\nu+2\lambda)\,\beta+\mu^2\nu\,\bar{c})}\right]$$

$$R_O = \left[\frac{2\mu^2\sigma\lambda(1-q)(1-c)}{2\bar{q}\lambda\,\mu\,((\bar{c}\mu-\beta\,\bar{p})\,\sigma+\beta\,\mu)+\sigma\,((\mu+2\lambda)\,(\mu+\nu+2\lambda)\,\beta+\mu^2\nu\,\bar{c})}\right]$$

$$R_S = \left[\frac{\mu^2\sigma\nu(1-c)}{2\bar{q}\lambda\,\mu\,((\bar{c}\mu-\beta\,\bar{p})\,\sigma+\beta\,\mu)+\sigma\,((\mu+2\lambda)\,(\mu+\nu+2\lambda)\,\beta+\mu^2\nu\,\bar{c})}\right]$$

Step 11.

$$Av = \left[\frac{\sigma\beta\mu\,(2\lambda+\mu+\nu)}{2\bar{q}\lambda\,\mu\,((\bar{c}\mu-\beta\,\bar{p})\,\sigma+\beta\,\mu)+\sigma\,((\mu+2\lambda)\,(\mu+\nu+2\lambda)\,\beta+\mu^2\nu\,\bar{c})}\right] \tag{4.36}$$

Case 2: n-Stage Erlangian Distribution. The repair time of failed units has a n-stage Erlang distribution with shape parameter n and rate μ. It is also a member of gamma distribution and the sum of n independent exponential variables with mean $1/\mu$ each *i.e.* repair is done in n stages with mean repair rate μ. In this case, we have

$$\widetilde{B}(s) = \left(\frac{n\mu}{s+n\mu}\right)^n$$

and $b_1 = -\widetilde{B}^{(1)}(0) = \frac{1}{\mu}$. For the explicit expression for state probability P_{21} and the availability of the machining system Av, the following step wise results are obtained:

Step 1. $P_{2,1} = 1$

Step 2. $P_{2,0}(0) = 2\lambda + \nu$

Step 3. $R_O = \dfrac{2(1-q)(1-c)\lambda}{\beta}$, $R_S = \dfrac{\nu(1-c)}{\beta}$

Step 4. $Q_{1,1} = \dfrac{2\lambda(1-q)}{\sigma}$

Step 5. $P_{1,0}(0) = \dfrac{2\lambda+\nu}{\left(\frac{r\mu}{\mu r+2\lambda}\right)^r} + 2(1-q)(p-1)\lambda - \nu$

Step 6. $\widetilde{P}_{2,0}(0) = \dfrac{1}{2}\dfrac{(2\lambda+\nu)\left(1-\left(\frac{r\mu}{\mu r+2\lambda}\right)^r\right)}{\lambda\left(\frac{r\mu}{\mu r+2\lambda}\right)^r}$

Step 7. $\widetilde{P}^{(1)}_{2,0}(0) = \dfrac{1}{4}\dfrac{(2\lambda+\nu)\left(1-\left(\frac{r\mu}{\mu r+2\lambda}\right)^r-\frac{2\lambda}{\mu}\right)}{\lambda^2\left(\frac{r\mu}{\mu r+2\lambda}\right)^r}$

Step 8. $\widetilde{P}_{1,0}(0) = \dfrac{1}{2}\dfrac{(2\lambda+\nu)(\mu-2\lambda)-\left[4\{(1-q)p+q\}\lambda^2+2\lambda\mu+\mu\nu\right]\left(\frac{r\mu}{\mu r+2\lambda}\right)^r}{\lambda\mu\left(\frac{r\mu}{\mu r+2\lambda}\right)^r}$

Step 9.

$$P_{2,1} = \frac{\left(\frac{\mu n}{\mu n+2\lambda}\right)^n\beta\mu\sigma}{D_2} \tag{4.37}$$

where

$$\begin{aligned}D_2 =& 2\bar{q}\left(\frac{\mu n}{\mu n+2\lambda}\right)^n\lambda\left\{(\bar{c}\mu-\beta\bar{p})\,\sigma+\beta\mu\right\}\\ &+\sigma\left(\left\{(\mu+2\lambda)\,\beta+\bar{c}\mu\nu\right\}\left(\frac{\mu n}{\mu n+2\lambda}\right)^n+\beta\,(2\lambda+\nu)\right)\end{aligned}$$

Step 10.

$$\widetilde{P}_{2,0}(0) = \frac{1}{4}\frac{\mu\beta\sigma(2\lambda+\nu)\left(1-\left(\frac{r\mu}{r\mu+2\lambda}\right)^r\right)}{D_2}$$

$$\widetilde{P}_{1,0}(0) = \frac{\left[\sigma\beta\left\{((1-p)q+p)\,\lambda^2+\frac{1}{2}\lambda\mu+\frac{1}{4}\mu\nu\right\}\left(\frac{r\mu}{r\mu+2\lambda}\right)^r-\frac{1}{4}(2\lambda+\nu)(\mu-2\lambda)\right]}{D_2}$$

$$Q_{1,1} = \frac{2\lambda\beta\mu(1-q)\left(\frac{r\mu}{r\mu+2\lambda}\right)^r}{D_2}, \quad R_O = \frac{2\lambda\mu^2\sigma(1-q)(1-c)}{D_2}$$

$$R_S = \frac{\nu\sigma\mu(1-c)\left(\frac{r\mu}{r\mu+2\lambda}\right)^r}{D_2}$$

Step 11.

$$Av = \frac{\sigma\beta\left(2\lambda+\left(1-\left(\frac{\mu n}{\mu n+2\lambda}\right)^{n}\right)\nu\right)\mu}{2\lambda D_2} \tag{4.38}$$

Case 3: Deterministic Distribution. Also known as degenerate distribution and takes only single value. The repair time has a deterministic distribution with mean rate μ. In this case, LST of the corresponding PDF is as follows

$$\widetilde{B}(s) = e^{-\left(\frac{s}{\mu}\right)}$$

and $b_1 = -\widetilde{B}^{(1)}(0) = \frac{1}{\mu}$. The step wise recursive expression as per algorithm discussed in previous section are as follows

Step 1. $P_{2,1} = 1.$
Step 2. $P_{2,0}(0) = 2\lambda + \nu$
Step 3. $R_O = \dfrac{2(1-q)(1-c)\lambda}{\beta}$, $R_S = \dfrac{\nu(1-c)}{\beta}$
Step 4. $Q_{1,1} = \dfrac{2\lambda(1-q)}{\sigma}$
Step 5. $P_{1,0} = \left[\dfrac{(2\lambda+\nu)}{e^{-\frac{2\lambda}{\mu}}} - 2(1-q)(1-p)\lambda - \nu\right]$
Step 6. $\widetilde{P}_{2,0}(0) = \dfrac{(2\lambda+\nu)\left(1-e^{-\frac{2\lambda}{\mu}}\right)}{2\lambda\, e^{-\frac{2\lambda}{\mu}}}$
Step 7. $\widetilde{P}^{(1)}_{2,0}(0) = \dfrac{(2\lambda+\nu)\left(1-\frac{2\lambda}{\mu}-e^{-\frac{2\lambda}{\mu}}\right)}{4\lambda^2 e^{\frac{2\lambda}{\mu}}}$
Step 8. $\widetilde{P}^{(1)}_{1,0}(0) = \dfrac{\left\{(4(1-q)p+4q)\lambda^2+2\lambda\mu+\mu\nu\right\}e^{-\frac{2\lambda}{\mu}} - (2\lambda+\nu)(2\lambda+\mu)}{2\lambda\mu e^{-\frac{2\lambda}{\mu}}}$
Step 9.

$$P_{2,1} = \frac{\mathrm{e}^{-\frac{2\lambda}{\mu}}\beta\,\mu\,\sigma}{D_3} \tag{4.39}$$

where

$$\begin{aligned} D_3 =& 2\bar{q}\mathrm{e}^{-\frac{2\lambda}{\mu}}\lambda\,((\bar{c}\mu-\beta\,\bar{p})\,\sigma+\beta\,\mu) \\ &+\sigma\left(((\mu+2\lambda)\,\beta+\bar{c}\mu\,\nu)\,\mathrm{e}^{-\frac{2\lambda}{\mu}}+\beta\,(2\lambda+\nu)\right) \end{aligned}$$

Step 10.

$$\widetilde{P}_{2,0}(0) = \frac{\frac{1}{4}\left(\mu\beta\sigma(2\lambda+\nu)\left(1-e^{-\frac{2\lambda}{\mu}}\right)\right)}{D_3}$$

$$\widetilde{P}_{1,0}(0) = \frac{\left(\left(((1-p)q+p)\lambda^2+\frac{\lambda\mu}{2}+\frac{\nu\mu}{4}\right)e^{-\frac{2\lambda}{\mu}}-\frac{1}{4}(2\lambda+\nu)(\mu-2\lambda)\right)\sigma\beta}{D_3}$$

$$Q_{1,1} = \frac{2\lambda(1-q)\beta\mu e^{-\frac{2\lambda}{\mu}}}{D_3}, \quad R_O = \frac{2\lambda(1-q)(1-c)\mu^2\sigma}{D_3}, \quad R_S = \frac{\nu(1-c)\sigma\mu e^{-\frac{2\lambda}{\mu}}}{D_3}$$

Step 11.

$$Av = \frac{\sigma\beta\left(2\lambda+\left(1-\mathrm{e}^{-2\frac{\lambda}{\mu}}\right)\nu\right)\mu}{2\lambda D_3} \tag{4.40}$$

Besides the state probability and availability of the machining system for these standard repair time distribution, exponential distribution (M), n-stage Erlangian distribution (Er_n), and Deterministic (D), for some more standard continuous distribution, the numerical derivations are presented in next section (4.5) since algebraic expressions are very complex and tedious to express.

4.5 Numerical Result

In this section, the availability of the two-operating unit power system with spare unit provisioning has been dealt extensively with different repair time distribution numerically. For numerical simulation purpose, the values of the governing parameters are fixed as follows $\lambda = 0.5$, $\mu = 25$, $\beta = 75$, $\sigma = 50$, $q = 0.7$, $c = 0.8$, $p = 0.9$.

For the comparison and illustration purpose, following six different repair time distributions are considered from different distribution families. For study persistence, we set $b_1 = 0.04$ and fix the corresponding parameter(s) as follows

Exponential distribution (M): $\mu = 25$

$n-$stage Erlang distribution (Er_n): $n = 3$, $\mu = 25$

Deterministic distribution (D): $\mu = 25$

Uniform distribution ($U(a,b)$): $a = 0.02$, $b = 0.06$

Generalized Erlangian distribution (GE_n): $n = 4$, $\mu_1 = 60$, $\mu_2 = 100$, $\mu_3 = 120$, $\mu_4 = 200$

Hyperexponential distribution (HE_n): $n = 2$, $\mu_1 = 15$, $\mu_2 = 30$, $\alpha_1 = 0.2$, $\alpha_2 = 0.8$

For the illustration of the proposed methodology using a supplementary variable, Table (4.1) comprises the results of steady-state probabilities and availability of the machining system for six different repair time distributions and their fix parameter(s) value. From Table (4.1) it is clearly inferred the order of the availability of the machining system as

$$Av_D > Av_{U(a,b)} > Av_{GE_4} > Av_{Er_3} > Av_M > Av_{HE_2}$$

As per requirement of service facility, a system designer can choose appropriate repair time distribution.

Figs. (4.2)-(4.9) or Tables (4.2)-(4.9) summarize the variation in the value of state probability $P_{2,1}$ and availability of the machining system Av on varying the value of governing parameter μ, λ, ν, β, σ, p, c, and q respectively. It is clearly noticeable from Fig. (4.2) or Table (4.2) that $P_{2,1}$ and Av of the machining system increase on increasing the repair rate μ and the order of the availability is $Av_D > Av_{Er_3} > Av_M$. Deterministic repair time distribution is recommended for designing the service system but practically service in phases is more suitable.

Fig. (4.3) or Table (4.3) depicts the variation of the $P_{2,1}$ and Av with respect to the failure rate of an operating unit λ. Both performance indices are decreasing on increasing the value of λ, which is the expected trend and validate the current modeling and methodology. Significant discrimination of repair time distribution is observed for the higher value of λ. Similar observations are depicted in Fig. (4.4) or Table (4.4) in which the variation of $P_{2,1}$ and Av are plotted with respect to the failure rate of the spare unit ν. For the high order of availability and the initial state of the machining system, it is necessary to take measures to avoid the failure of the spare unit in the inactive state.

Fig. (4.5) or Table (4.5) summarizes the variability of the $P_{2,1}$ and Av with respect to reboot rate β. Slight improvement in the value of Av is observed by increasing the rate of the reboot process. It prompts that the reboot process is a necessary action to avoid the hindrance due to the failure of the unit. Fig. (4.6) or Table (4.6) depicts the changeability of the state probability and availability the switchover rate σ. For a high value of σ, higher availability and a higher probability of the initial state is observed. This implies that the automatic switchover of the spare unit in place of the failed unit should be prompt.

Figs. (4.7)-(4.9) or Tables (4.7)-(4.9) demonstrate the variation in the value of state probability and availability of the machining system with respect to probabilities p, c and q. From these results, one can judge how these probabilities are significant in the analysis of the service system and prompt the action to maintain the high grade of service. The overall observations justify that for the better service facility, preventive measures like provision of spare units are necessary, and working conditions of operating units and spare units should be proper to avoid frequent failure. Corrective measures like prompt and perfect switchover, prompt and fast repair, etc. should have opted. System analysts may opt for any repair policy as per the convenience of the design of the system and analyze the benefits in terms of availability. The suitable corrective and preventive action (CAPA) eliminate causes of non-conformities or other undesirable situations.

4.6 Conclusion

In this chapter, six different repair time distributions are analyzed for a spare provisioning machine repair problem with imperfect coverage, reboot delay, switchover delay, and switching failure. The recursive method is developed for deriving steady-state probabilities of the machining system using supplementary variable V, a remaining repair time of the failed units. The explicit expression for the steady-state probabilities and the availability of the machining system are also derived for different repair time distribution algebraically and numerically as a particular case. The extensive numerical discussion has also been included to give a quick insight into the characteristics of different repair time distributions of various distribution families. The ranking of the repair time distribution based on the availability of the machining system having the same mean repair time, is also presented. The implication of the present studied model and methodology for a power supply system is also done. The recursive method developed in this chapter works efficiently for the machine repair problem having a large number of units or any other queueing problems with any repair time distribution. Using this technique, one can explore more general queueing problems with state-dependent arrival and service processes in the future. Researchers can extend the present model for pressure coefficient in service, the catastrophic effect for the system, mixed spare provisioning, degraded failure, etc. The present study would be beneficial to the system manager in deciding due to the readily available explicit expression for state probabilities and precise recursive methods for determining them, even having no knowledge of mathematical modeling. The present study is pivotal for deciding effective corrective and preventive action by investigating the root cause of failure and quality management system (QMS).

Table 4.1: State probabilities and availability of the machining system

Distribution	M	Er_3	D	$U(a,b)$	GE_4	HE_2
Parameter(s)	$\mu = 25$	$\mu = 25$	$\mu = 25$	$a = 0.02$ $b = 0.06$	$\mu_1 = 60$ $\mu_2 = 100$ $\mu_3 = 120$ $\mu_4 = 200$	$\alpha_1 = 0.2$ $\alpha_2 = 0.8$ $\mu_1 = 15$ $\mu_2 = 30$
P_{21}	0.9123644	0.9123430	0.9123320	0.9123347	0.9123417	0.9123711
P_{20}	0.0437935	0.0443790	0.0446796	0.0446037	0.0444134	0.0436102
P_{10}	0.0371515	0.0365876	0.0362980	0.0363711	0.0365544	0.0373280
Q_{11}	0.0054742	0.0054741	0.0054740	0.0054740	0.0054741	0.0054742
R_O	0.0007299	0.0007299	0.0007299	0.0007299	0.0007299	0.0007299
R_S	0.0004866	0.0004866	0.0004866	0.0004866	0.0004866	0.0004866
Av	0.9561579	0.9567219	0.9570115	0.9569385	0.9567551	0.9559813

Table 4.2: Performance indices corresponding to Fig. (4.2)

Indices	Distribution	μ								
		24	26	28	30	32	34	36	38	40
$P_{2,1}$	M	0.9118	0.9120	0.9121	0.9122	0.9123	0.9124	0.9124	0.9125	0.9125
	Er_3	0.9118	0.9119	0.9121	0.9122	0.9123	0.9123	0.9124	0.9125	0.9125
	D	0.9118	0.9119	0.9121	0.9122	0.9123	0.9123	0.9124	0.9125	0.9125
Av	M	0.9556	0.9557	0.9559	0.9560	0.9561	0.9562	0.9562	0.9563	0.9564
	Er_3	0.9561	0.9563	0.9564	0.9565	0.9566	0.9567	0.9568	0.9569	0.9569
	D	0.9564	0.9566	0.9567	0.9568	0.9569	0.9570	0.9571	0.9571	0.9572

Table 4.3: Performance indices corresponding to Fig. (4.3)

Indices	Distribution	λ								
		0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
$P_{2,1}$	M	0.9749	0.9667	0.9586	0.9507	0.9428	0.9351	0.9274	0.9198	0.9124
	Er_3	0.9749	0.9667	0.9586	0.9507	0.9428	0.9350	0.9274	0.9198	0.9123
	D	0.9749	0.9667	0.9586	0.9507	0.9428	0.9350	0.9274	0.9198	0.9123
	$U(a,b)$	0.9749	0.9667	0.9586	0.9507	0.9428	0.9350	0.9274	0.9198	0.9123
	GE_4	0.9749	0.9667	0.9586	0.9507	0.9428	0.9350	0.9274	0.9198	0.9123
	HE_2	0.9749	0.9667	0.9586	0.9507	0.9428	0.9351	0.9274	0.9198	0.9124
Av	M	0.9905	0.9860	0.9816	0.9773	0.9730	0.9687	0.9645	0.9603	0.9562
	Er_3	0.9905	0.9861	0.9818	0.9775	0.9732	0.9690	0.9649	0.9608	0.9567
	D	0.9905	0.9861	0.9818	0.9775	0.9733	0.9692	0.9651	0.9610	0.9570
	$U(a,b)$	0.9905	0.9861	0.9818	0.9775	0.9733	0.9691	0.9650	0.9610	0.9569
	GE_4	0.9905	0.9861	0.9818	0.9775	0.9732	0.9690	0.9649	0.9608	0.9568
	HE_2	0.9905	0.9860	0.9816	0.9772	0.9729	0.9686	0.9644	0.9602	0.9560

Table 4.4: Performance indices corresponding to Fig. (4.4)

Indices	Distribution	ν								
		0.050	0.075	0.100	0.125	0.150	0.175	0.200	0.225	0.250
$P_{2,1}$	M	0.9179	0.9170	0.9161	0.9151	0.9142	0.9133	0.9124	0.9114	0.9105
	Er_3	0.9179	0.9170	0.9160	0.9151	0.9142	0.9133	0.9123	0.9114	0.9105
	D	0.9179	0.9170	0.9160	0.9151	0.9142	0.9133	0.9123	0.9114	0.9105
	$U(a,b)$	0.9179	0.9170	0.9160	0.9151	0.9142	0.9133	0.9123	0.9114	0.9105
	GE_4	0.9179	0.9170	0.9160	0.9151	0.9142	0.9133	0.9123	0.9114	0.9105
	HE_2	0.9179	0.9170	0.9161	0.9151	0.9142	0.9133	0.9124	0.9115	0.9105
Av	M	0.9565	0.9564	0.9564	0.9563	0.9563	0.9562	0.9562	0.9561	0.9561
	Er_3	0.9570	0.9569	0.9569	0.9568	0.9568	0.9568	0.9567	0.9567	0.9566
	D	0.9572	0.9572	0.9572	0.9571	0.9571	0.9570	0.9570	0.9570	0.9569
	$U(a,b)$	0.9572	0.9571	0.9571	0.9571	0.9570	0.9570	0.9569	0.9569	0.9569
	GE_4	0.9570	0.9570	0.9569	0.9569	0.9568	0.9568	0.9568	0.9567	0.9567
	HE_2	0.9563	0.9563	0.9562	0.9562	0.9561	0.9560	0.9560	0.9559	0.9559

Table 4.5: Performance indices corresponding to Fig. (4.5)

Indices	Distribution	β								
		50	55	60	65	70	75	80	85	90
$P_{2,1}$	M	0.9118	0.9120	0.9121	0.9122	0.9123	0.9124	0.9124	0.9125	0.9125
	Er_3	0.9118	0.9119	0.9121	0.9122	0.9123	0.9123	0.9124	0.9125	0.9125
	D	0.9118	0.9119	0.9121	0.9122	0.9123	0.9123	0.9124	0.9125	0.9125
	$U(a,b)$	0.9118	0.9119	0.9121	0.9122	0.9123	0.9123	0.9124	0.9125	0.9125
	GE_4	0.9118	0.9119	0.9121	0.9122	0.9123	0.9123	0.9124	0.9125	0.9125
	HE_2	0.9118	0.9120	0.9121	0.9122	0.9123	0.9124	0.9124	0.9125	0.9126
Av	M	0.9556	0.9557	0.9559	0.9560	0.9561	0.9562	0.9562	0.9563	0.9564
	Er_3	0.9561	0.9563	0.9564	0.9565	0.9566	0.9567	0.9568	0.9569	0.9569
	D	0.9564	0.9566	0.9567	0.9568	0.9569	0.9570	0.9571	0.9571	0.9572
	$U(a,b)$	0.9564	0.9565	0.9566	0.9568	0.9569	0.9569	0.9570	0.9571	0.9571
	GE_4	0.9562	0.9563	0.9565	0.9566	0.9567	0.9568	0.9568	0.9569	0.9569
	HE_2	0.9554	0.9556	0.9557	0.9558	0.9559	0.9560	0.9561	0.9561	0.9562

Table 4.6: Performance indices corresponding to Fig. (4.6)

Indices	Distribution	σ								
		30	35	40	45	50	55	60	65	70
$P_{2,1}$	M	0.9090	0.9102	0.9111	0.9118	0.9124	0.9128	0.9132	0.9135	0.9138
	Er_3	0.9090	0.9102	0.9111	0.9118	0.9123	0.9128	0.9132	0.9135	0.9138
	D	0.9090	0.9102	0.9111	0.9118	0.9123	0.9128	0.9132	0.9135	0.9138
	$U(a,b)$	0.9090	0.9102	0.9111	0.9118	0.9123	0.9128	0.9132	0.9135	0.9138
	GE_4	0.9090	0.9102	0.9111	0.9118	0.9123	0.9128	0.9132	0.9135	0.9138
	HE_2	0.9091	0.9102	0.9111	0.9118	0.9124	0.9128	0.9132	0.9135	0.9138
Av	M	0.9527	0.9539	0.9549	0.9556	0.9562	0.9566	0.9570	0.9574	0.9577
	Er_3	0.9532	0.9545	0.9554	0.9561	0.9567	0.9572	0.9576	0.9579	0.9582
	D	0.9535	0.9548	0.9557	0.9564	0.9570	0.9575	0.9579	0.9582	0.9585
	$U(a,b)$	0.9535	0.9547	0.9556	0.9564	0.9569	0.9574	0.9578	0.9581	0.9584
	GE_4	0.9533	0.9545	0.9554	0.9562	0.9568	0.9572	0.9576	0.9580	0.9583
	HE_2	0.9525	0.9537	0.9547	0.9554	0.9560	0.9565	0.9569	0.9572	0.9575

Table 4.7: Performance indices corresponding to Fig. (4.7)

Indices	Distribution	p								
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$P_{2,1}$	M	0.9204	0.9194	0.9184	0.9174	0.9164	0.9154	0.9144	0.9134	0.9124
	Er_3	0.9204	0.9194	0.9184	0.9174	0.9164	0.9153	0.9143	0.9133	0.9123
	D	0.9204	0.9194	0.9184	0.9174	0.9163	0.9153	0.9143	0.9133	0.9123
	$U(a,b)$	0.9204	0.9194	0.9184	0.9174	0.9163	0.9153	0.9143	0.9133	0.9123
	GE_4	0.9204	0.9194	0.9184	0.9174	0.9164	0.9153	0.9143	0.9133	0.9123
	HE_2	0.9204	0.9194	0.9184	0.9174	0.9164	0.9154	0.9144	0.9134	0.9124
Av	M	0.9646	0.9635	0.9625	0.9614	0.9604	0.9593	0.9583	0.9572	0.9562
	Er_3	0.9652	0.9641	0.9630	0.9620	0.9609	0.9599	0.9588	0.9578	0.9567
	D	0.9655	0.9644	0.9633	0.9623	0.9612	0.9602	0.9591	0.9581	0.9570
	$U(a,b)$	0.9654	0.9643	0.9633	0.9622	0.9611	0.9601	0.9590	0.9580	0.9569
	GE_4	0.9652	0.9641	0.9631	0.9620	0.9610	0.9599	0.9589	0.9578	0.9568
	HE_2	0.9644	0.9634	0.9623	0.9612	0.9602	0.9591	0.9581	0.9570	0.9560

Table 4.8: Performance indices corresponding to Fig. (4.8)

Indices	Distribution	c								
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$P_{2,1}$	M	0.9085	0.9090	0.9096	0.9101	0.9107	0.9113	0.9118	0.9124	0.9129
	Er_3	0.9085	0.9090	0.9096	0.9101	0.9107	0.9112	0.9118	0.9123	0.9129
	D	0.9085	0.9090	0.9096	0.9101	0.9107	0.9112	0.9118	0.9123	0.9129
	$U(a,b)$	0.9085	0.9090	0.9096	0.9101	0.9107	0.9112	0.9118	0.9123	0.9129
	GE_4	0.9085	0.9090	0.9096	0.9101	0.9107	0.9112	0.9118	0.9123	0.9129
	HE_2	0.9085	0.9091	0.9096	0.9102	0.9107	0.9113	0.9118	0.9124	0.9129
Av	M	0.9521	0.9527	0.9533	0.9538	0.9544	0.9550	0.9556	0.9562	0.9567
	Er_3	0.9527	0.9532	0.9538	0.9544	0.9550	0.9556	0.9561	0.9567	0.9573
	D	0.9530	0.9535	0.9541	0.9547	0.9553	0.9558	0.9564	0.9570	0.9576
	$U(a,b)$	0.9529	0.9535	0.9540	0.9546	0.9552	0.9558	0.9564	0.9569	0.9575
	GE_4	0.9527	0.9533	0.9539	0.9544	0.9550	0.9556	0.9562	0.9568	0.9573
	HE_2	0.9519	0.9525	0.9531	0.9537	0.9542	0.9548	0.9554	0.9560	0.9566

Table 4.9: Performance indices corresponding to Fig. (4.9)

Indices	Distribution	q								
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$P_{2,1}$	M	0.9031	0.9047	0.9062	0.9077	0.9093	0.9108	0.9124	0.9139	0.9155
	Er_3	0.9031	0.9046	0.9062	0.9077	0.9092	0.9108	0.9123	0.9139	0.9155
	D	0.9031	0.9046	0.9062	0.9077	0.9092	0.9108	0.9123	0.9139	0.9155
	$U(a,b)$	0.9031	0.9046	0.9062	0.9077	0.9092	0.9108	0.9123	0.9139	0.9155
	GE_4	0.9031	0.9046	0.9062	0.9077	0.9092	0.9108	0.9123	0.9139	0.9155
	HE_2	0.9031	0.9047	0.9062	0.9077	0.9093	0.9108	0.9124	0.9139	0.9155
Av	M	0.9465	0.9481	0.9497	0.9513	0.9529	0.9545	0.9562	0.9578	0.9594
	Er_3	0.9470	0.9486	0.9502	0.9519	0.9535	0.9551	0.9567	0.9584	0.9600
	D	0.9473	0.9489	0.9505	0.9521	0.9538	0.9554	0.9570	0.9586	0.9603
	$U(a,b)$	0.9473	0.9489	0.9505	0.9521	0.9537	0.9553	0.9569	0.9586	0.9602
	GE_4	0.9471	0.9487	0.9503	0.9519	0.9535	0.9551	0.9568	0.9584	0.9600
	HE_2	0.9463	0.9479	0.9495	0.9511	0.9527	0.9544	0.9560	0.9576	0.9592

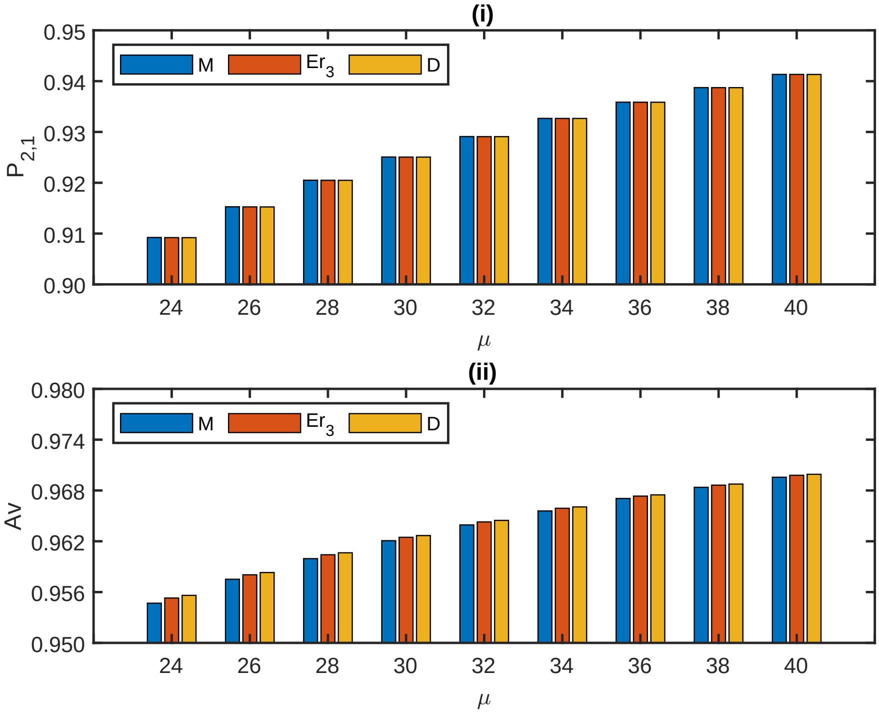

Figure 4.2: State probability $P_{2,1}$ and availability of the machining system Av wrt μ for different repair time distribution

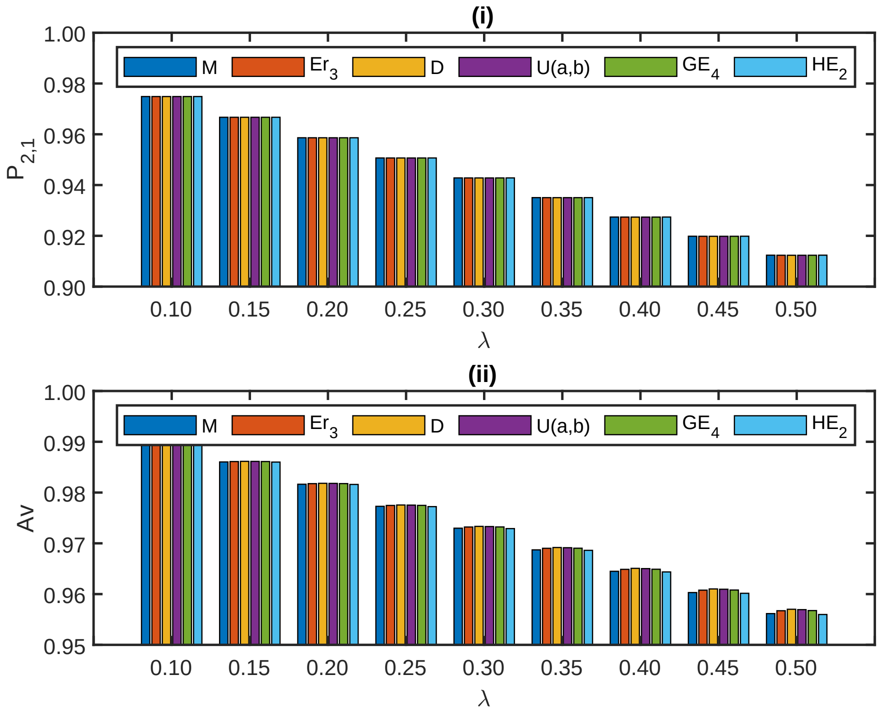

Figure 4.3: State probability $P_{2,1}$ and availability of the machining system Av wrt λ for different repair time distribution

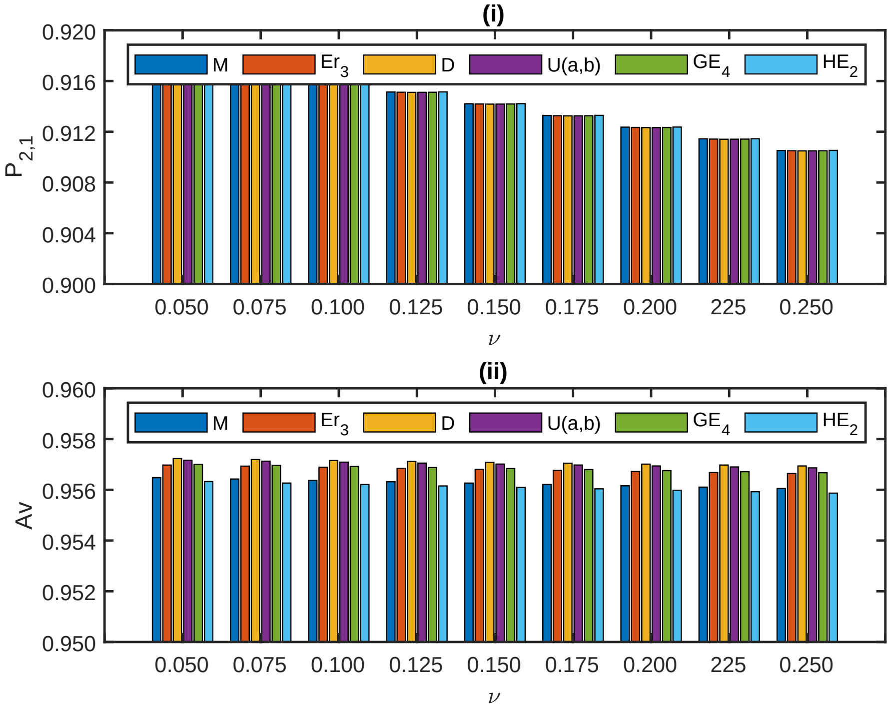

Figure 4.4: State probability $P_{2,1}$ and availability of the machining system Av wrt ν for different repair time distribution

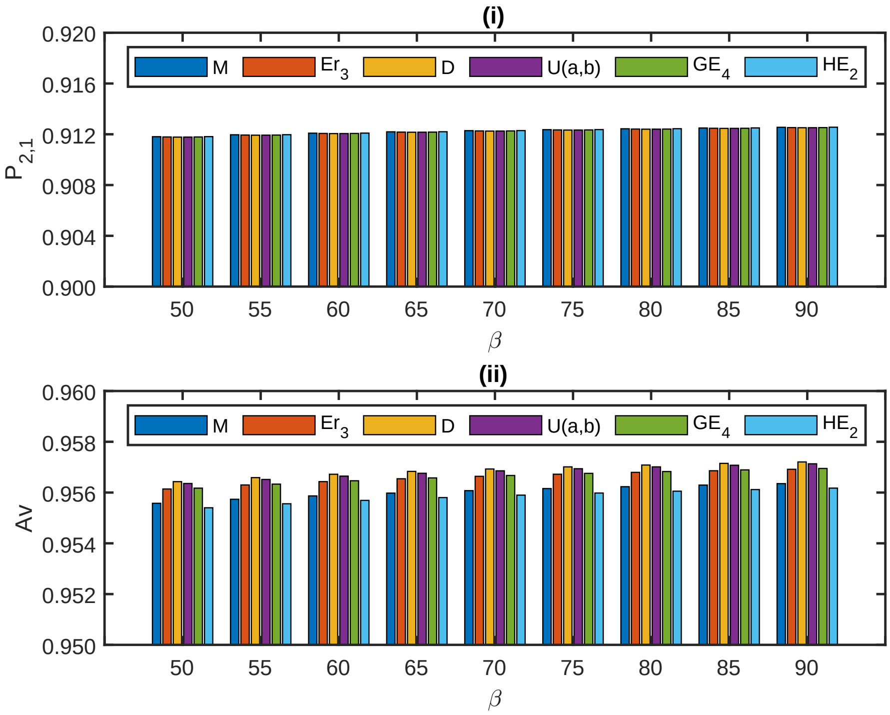

Figure 4.5: State probability $P_{2,1}$ and availability of the machining system Av wrt β for different repair time distribution

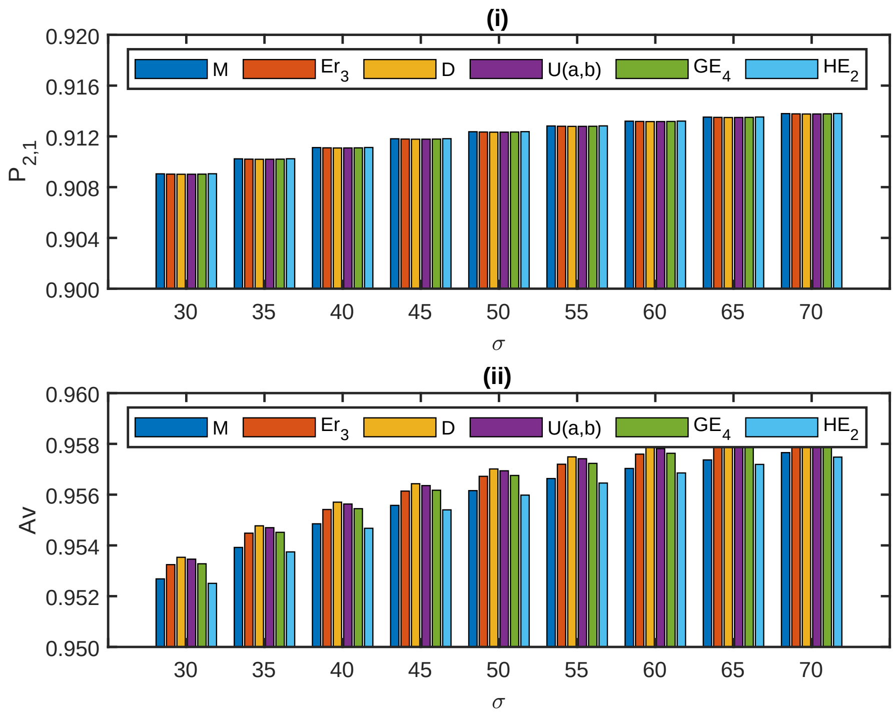

Figure 4.6: State probability $P_{2,1}$ and availability of the machining system Av wrt σ for different repair time distribution

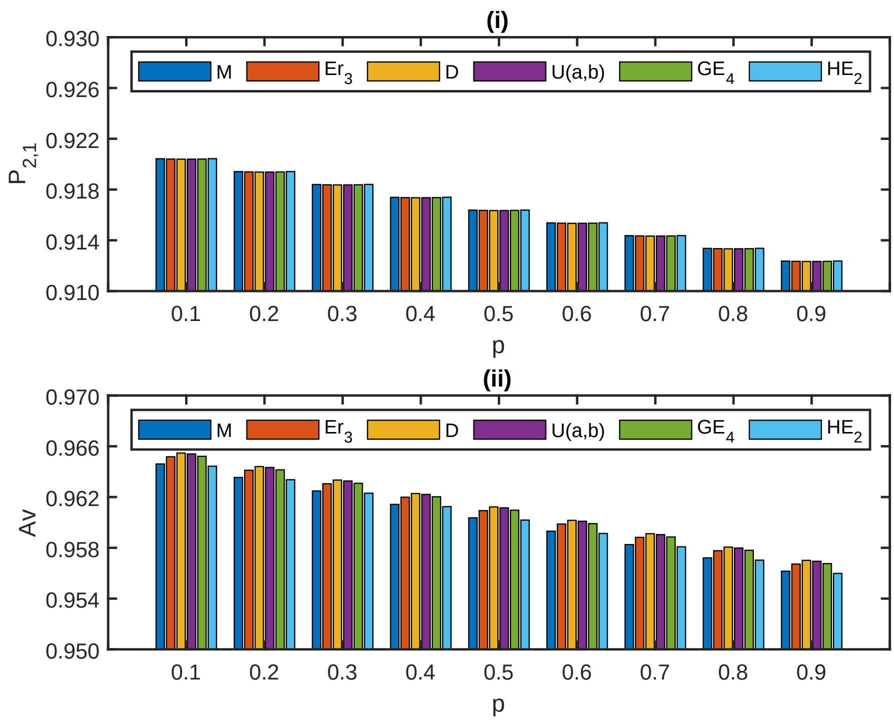

Figure 4.7: State probability $P_{2,1}$ and availability of the machining system Av wrt p for different repair time distribution

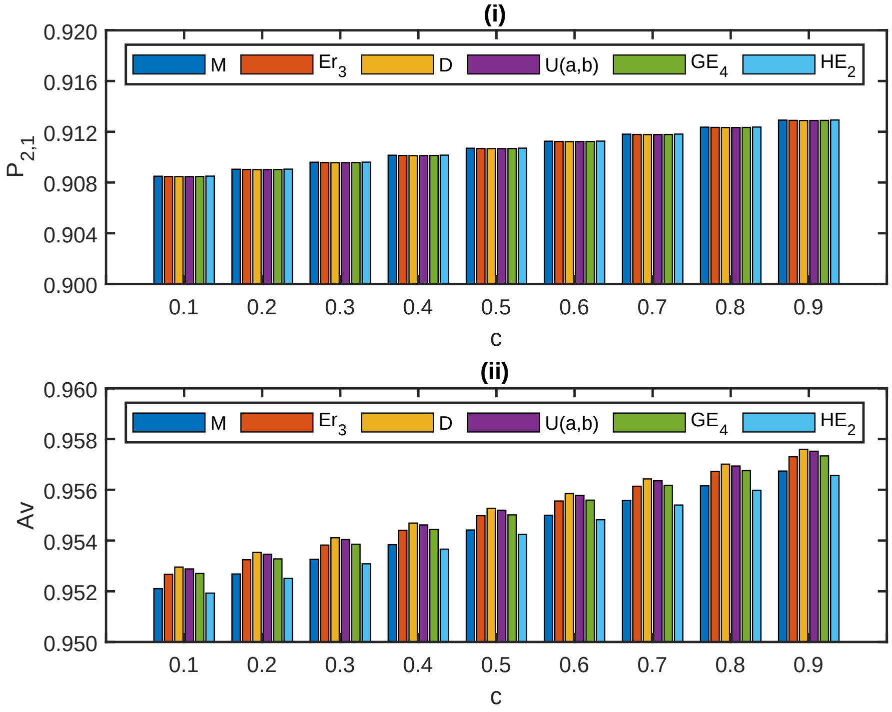

Figure 4.8: State probability $P_{2,1}$ and availability of the machining system Av wrt c for different repair time distribution

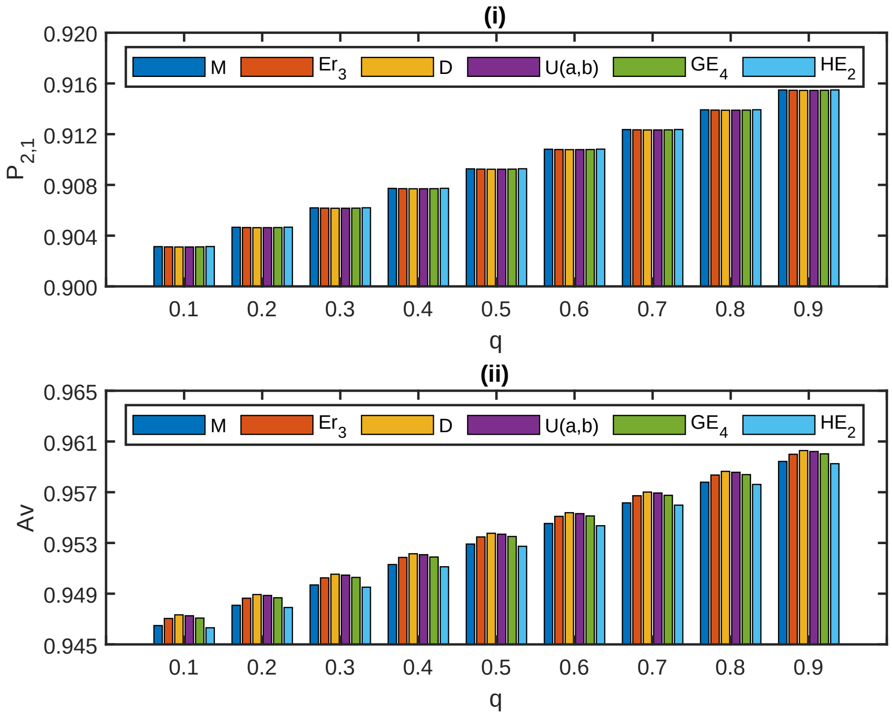

Figure 4.9: State probability $P_{2,1}$ and availability of the machining system Av wrt q for different repair time distribution