Chapter 5

Fuzzy Analysis of Unreliable Repair Facility

"All human things are subject to decay, and when fate summons, Monarchs must obey"

Mac Flecknoe

5.1 Introduction

Many physical and biological phenomena can be signified in terms of an assemblage of individuals, customers or data governed by the competition of the two basic random mechanisms of birth and death like neutron multiplication, nuclear collision cascades, epidemics and ecology, bacterial growth, genetics, telecommunications, computer networks, broadcasting, etc. In the day-to-day, it is observed that the machining system gets congested, and the service delay in the machining system increases like communication delay, processing delay, transmission delay, queueing delay, retransmission delay, etc. The knowledge of the relationship between congestion and delay is essential for designing the congestion control machining system.

Queueing theory provides the theoretical platform to understand the relationship between birth, death, and waiting delay systematically. Queues are ubiquitous. The classical queueing model $M/M/1$ refers to a single server system having negative exponential arrivals and service time with infinite capacity and infinite source (*cf.*Kleinrock [103]; Gross et al. [55]). This is the most fundamental queueing model and the most widely used system for the analysis and understanding of the practicality of congestion or waiting line problems. The $M/M/1$ is a good approximation for a large number of queueing systems in service-related applications. Vijayashree and Janani [169] obtained an explicit expression for the time-dependent system size probabilities of the single server Markovian queueing model using Laplace transform and generating function techniques. But, this solution cannot handle the dynamic behavior of governing parameters.

The idea of the catastrophe occurring at haphazard, leading to obliteration of all customers, units, or data there in the machining system and the temporary inactivation of the service facilities until a new arrival of the customer, unit, or data, is not unusual in many service systems (Ammar [8]). This phenomenon is prominent in the presence of unwanted customers, units, or viruses having the characteristic to remove all the regular customers, units, or data in the machining system or may occur from outside the machining system from another service facility. It has enormous applications in many areas, especially in computer and communication systems, industries, biosciences, and population genetics. In the real-world, catastrophes appear in numerous situations in practice, for instance, in the production line, in the service industries, in the health care system, in population genetics, in the transportation network, in the telecommunication industry, etc. Artalejo [9] gave a comprehensive survey on a queueing system with a catastrophe. The effect of a catastrophe on services in many applications was studied in recent research articles (*cf.* Crescenzo et al. [37]; Atencia and Moreno [11]; Kumar et al. [106]; Kumar et al. [107]). Kumar [109] developed a stochastic queueing model for a catastrophe cum restoration queueing system with correlated batch arrivals and general service time distribution. Baumann and Sandmann [15] studied for the multi-dimensional Markov chains or generator matrix with block tridiagonal transition probability involved in a queueing system with random catastrophe. Various time-dependent performance measures were computed for a varying catastrophe intensity cum restorative Markovian queueing model with finite capacity by Bura and Kumar [20]. All researchers considered static queueing optimization problem in which characteristic would not change over time. It is generally seldom in practical purpose.

Dimon and Economon [38] derived the explicit expressions and computational schemes for various performance descriptors of the single server queue with the catastrophe that occurred according to a Poisson process and types of reneging. Goswami [54] obtained the steady-state probabilities using the displacement operator method for discrete-time queueing systems with two heterogeneous servers subject to catastrophe. Including the non-stationary birth-death process with catastrophe in the queueing systems $M(t)/M(t)/1$ and $M(t)/M(t)/\infty$, various applications were provided by Giorno et al. [53]. Mulatier [34] studied for critical catastrophe in a nuclear reactor by investigating the spatial behavior of the fluctuations in confined geometry. Dharmraja et al. [36] used jump-diffusion approximations of the continuous-time

Ehrenfest model that defined over the integers and subjected to catastrophe occurring at a constant rate, which led to the Ornstein-Uhlenbeck process with catastrophe. Dharmaraja and Kumar [35] obtained explicit time-dependent probabilities of system size for a Markovian queueing system with heterogeneous servers and catastrophes. There was a wide gap that all researches and catastrophe considered its rate of occurrence static and derived steady-state or transient expressions for associated performance measures. No research article has been found in the literature for its dynamic behavior or its fuzzy analysis.

In a modern automatic congestion control machining system, an intelligent system is required to interface between increasing demands of performance, product quality, material consumption, hierarchical information, scheduling, supervision, fault detection, diagnosis, etc. The fuzzy system is a powerful tool to understand partly incomplete, vague, and imprecise information in the high-level automatic control machining system. Specifically, in many practical applications, the statistical information of arrival or service of the customers, units, or data may be obtained subjectively or more suitably described by linguistic terms such as fast, moderate, or slow, rather than probabilistic in the automatic control machining system. The arrival rate of the customer, unit, or data and service rate of the server or machine cannot be adjudged exactly. Fuzzy queues are much more realistic than commonly used crisp queues to analyze the machining system when parameters are described in terms of possibility rather than probability. $FM/FM/1$ denotes a single-machine (server) queueing system with fuzzified inter-arrival time and service time. The inter-occurrence time of catastrophe is also fuzzified to make the studied queueing system more possibilistic. To deal with imprecise information in the making decision, Bellman and Zadeh [16] and Zadeh [200] introduced the concept of fuzziness. Fuzzy queues involved in different real-life systems have been discussed by several researchers (*cf.* Li and Lee [120]; Buckley [19]; Negi and Lee [133]; Kao et al. [91]). Buckley [18] gave a survey on the fuzzy queue in detail. Performance measures of fuzzy queues having a fuzzy random event like arrival, service, catastrophe is fuzzy as well. Among these researches in literature, no research article has been found on a fuzzy catastrophe.

To conserve the fuzziness of input information completely, the fuzzy performance measure should also be described by a membership function rather than a crisp value. A mathematical programming approach is used to derive the membership function of the system performance measures of the queueing system with fuzzy parameters. The underlying idea is to apply the concept of α-cut and Zadeh's extension principle (*cf.* Zadeh [200]; Yager [188]; Zimmermann [208]; Chen [27]). Kurano et al. [112] found Pareto optimal policy for maximizing the infinite horizon fuzzy expected discounted reward overall stationary policies under some partial order. On the basis of Zadeh's extension principle, mixed-integer nonlinear programs, which parameterized by the possibility level α, was formulated by Chen [25] to compute the lower and upper bounds of the minimal expected total cost per unit time at α and described the membership function of the minimal expected total cost per unit time of the fuzzy objective value. Pardo and Fuente [138] expressed the standardized profit functions as membership functions, which completely kept the uncertainty of the initial information when the service time was fuzzy with different levels of possibility ranging from the pessimist case to the optimistic. Ahmed and Sardar [1] proposed an approximate but convenient method for solving fuzzy linear programming with a fuzzy non-negative technical coefficient and without using the ranking functions. Gupta et al. [56] considered the allocation problem of repairable components for a parallel-series system as a fuzzy multi-objective nonlinear programming optimization problem. Murthy [3] developed a fuzzy programming model with quadratic membership functions for the solution of a multi-objective problem. Recently, Garg and Ansha [49]; Garg [51]; Garg [47]; Garg [50] highlighted new applications of fuzzy theory and developed various types of new fuzzy numbers and their arithmetic. Inspiring from previous findings, the methodology is developed to determine the membership function of fuzzy performance measures of the studied machining system in a fuzzy environment.

The classical queueing models with catastrophes are comprehensively deliberated as mentioned in the literature review; however, there is no work on fuzzy queue with fuzzy catastrophe. On this observation in literature, the state-of-the-art fuzzy queue $FM/FM/1$ is proposed with a fuzzy catastrophe in this chapter, i.e. the novelty of the present chapter is to study stochastic process with the fuzzy environment in a systematic manner for a better insight of dynamic behavior of the system. The proposed model deals with the dynamic behavior of catastrophe and other events. It describes the model's uncertainty and subjective ambiguity in a better fashion. The proposed methodology covers a wide range of sensitivity or strategic planning. Along with this line, the purpose in current study is to investigate a pair of mathematical programs which are formulated to calculate the lower and upper bounds of the α-cut of the fuzzy machining system performance measures. The membership functions of the fuzzy machining system performance measures are derived numerically by enumerating different values of α. The advantage of the present study is threefold: (1) System designers and learners can understand the importance of the fuzzy environment to deal with dynamics of service system, engineering system, etc. and their modeling. (2) The results directly provide a sensitive range of the studied machining system, which may help system analysts to identify crucial parameters and their rates. (3) Research in new front can also be enhanced by learning the proposed methodology.

The description of a classical $M/M/1$ machining system with a catastrophe and its performance indices is presented in section (5.2). In section (5.3), the key idea of the procedure to determine fuzzy parameter patterns is delineated using parametric non-linear programming. An illustrative example is discussed in detail in the next section (5.4). Extensive numerical computations of the system characteristics with MATLAB have been performed to show the role played by the involved fuzzy arrival, fuzzy departure, and fuzzy catastrophe in section (5.5). Finally, current and future directions of research and application of the classical model are discussed in the last section (5.6).

5.2 Model Description

The single machine (server) classical $M/M/1$ model is considered with catastrophe having infinite capacity and infinite source of prospective customers, units, or data. The inter-arrival time of the customer follows an exponential distribution with parameter λ and service time for the customer by the machine (server) also follows an exponential distribution with parameter μ. The random catastrophe leads to force to abandon all waiting customers, units, or data in the machining system instantly and makes the service facility inoperative until the new customer, unit, or data arrives follow Poisson process. The inter-occurrence time of catastrophe follows an exponential distribution with parameter γ. The state transition diagram of the governing model is depicted in Fig. (5.1)

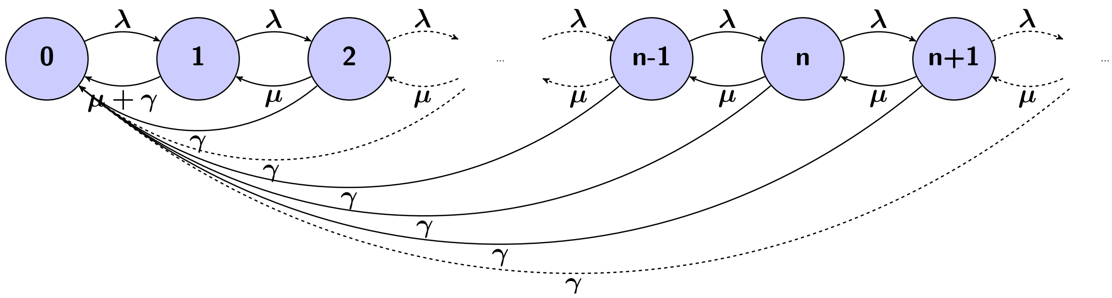

Figure 5.1: The State transition diagram

Let state of the system $N(t)$ be defined as number of the customers, units or data in the machining system at any instant t. Hence, $P_n(t)$ represents the probability that there are n customers, units or data in the machining system at time t. Using birth and death process and balancing the inflow and outflow rates, the Chapman-Kolmogorov differential-difference equations of the governing model are given as follows

$$\begin{aligned} \frac{dP_0(t)}{dt} &= -\lambda P_0(t) + \mu P_1(t) + \gamma(1 - P_0(t)) \\ \frac{dP_n(t)}{dt} &= -(\gamma + \lambda + \mu) P_n(t) + \lambda P_{n-1}(t) + \mu P_{n+1}(t); \quad n = 1,2,3.... \end{aligned} \tag{5.1}$$

with initial condition that there is no customer, unit, or data in the machining system at time $t = 0$, i.e., $P_0(0) = 1$ and $P_n(0) = 0, n = 1,2,\ldots$. Using product type solution method (Kumar and Arivudainambi [105]), the obtained steady-state probabilities $P_n; n = 0,1,2,3,\ldots$ under the normalizing condition

$$\sum_{n=0}^{\infty} P_n = 1$$

is as follows

$$P_n = (1-\rho)\rho^n; n = 0,1,2,3,\ldots \tag{5.2}$$

where

$$\rho = \frac{\left[(\lambda + \gamma + \mu) - \sqrt{(\lambda + \gamma)^2 + \mu^2 + 2\mu\gamma - 2\lambda\mu}\right]}{2\mu} \tag{5.3}$$

Using above derived steady-state probabilities $P_n; n = 0,1,2,3,...$ and Little formula (Gross et al. [55]), the following system performance characteristics are developed as follows

- The expected number of the customers, units or data in the machining system (L)

$$L = \frac{\left[\lambda + \gamma + \mu - \sqrt{(\lambda + \gamma)^2 + \mu^2 + 2\mu\gamma - 2\lambda\mu}\right]}{\left[\mu - \lambda - \gamma + \sqrt{(\lambda + \gamma)^2 + \mu^2 + 2\mu\gamma - 2\lambda\mu}\right]} \tag{5.4}$$

- The expected number of the customers, units or data in the queue (L_q)

$$L_q = \frac{\left[\lambda + \gamma + \mu - \sqrt{\gamma^2 + 2\gamma\lambda + 2\mu\gamma + \lambda^2 - 2\lambda\mu + \mu^2}\right]^2}{2\mu\left[\mu - \lambda - \gamma + \sqrt{\gamma^2 + 2\gamma\lambda + 2\mu\gamma + \lambda^2 - 2\lambda\mu + \mu^2}\right]} \tag{5.5}$$

- The expected waiting time of the customers, units or data in the machining system (W)

$$W = \frac{\left[\lambda+\gamma+\mu-\sqrt{\gamma^2+2\gamma\lambda+2\mu\gamma+\lambda^2-2\lambda\mu+\mu^2}\right]}{\lambda\left[\mu-\lambda-\gamma+\sqrt{\gamma^2+2\gamma\lambda+2\mu\gamma+\lambda^2-2\lambda\mu+\mu^2}\right]} \tag{5.6}$$

- The expected waiting time of the customers, units or data in the queue (W_q)

$$W_q = \frac{\left[\lambda+\gamma+\mu-\sqrt{\gamma^2+2\gamma\lambda+2\mu\gamma+\lambda^2-2\lambda\mu+\mu^2}\right]^2}{2\lambda\mu\left[\mu-\lambda-\gamma+\sqrt{\gamma^2+2\gamma\lambda+2\mu\gamma+\lambda^2-2\lambda\mu+\mu^2}\right]} \tag{5.7}$$

- The probability that there is no customer, unit or data in the machining system (P_0)

$$P_0 = \frac{\left[\mu-\lambda-\gamma+\sqrt{\gamma^2+2\gamma\lambda+2\mu\gamma+\lambda^2-2\lambda\mu+\mu^2}\right]}{2\mu} \tag{5.8}$$

- The variance of the number of the customers, units or data in the machining system (σ^2)

$$\sigma^2 = \frac{2\mu\left[\lambda+\gamma+\mu-\sqrt{\gamma^2+2\gamma\lambda+2\mu\gamma+\lambda^2-2\lambda\mu+\mu^2}\right]}{\left[\mu-\lambda-\gamma+\sqrt{\gamma^2+2\gamma\lambda+2\mu\gamma+\lambda^2-2\lambda\mu+\mu^2}\right]^2} \tag{5.9}$$

5.3 The Fuzzy System and Parametric NLPs

To increase the applicability of the studied classical $M/M/1$ machining system with catastrophe, the fuzzy description of the system parameters are allowed. Suppose the catastrophe rate of the machining system γ, arrival rate of the customer, unit, or data λ, and the service rate of the arrived customer, unit, or data μ are approximately known and can be exemplified by the fuzzy sets $\widetilde{\gamma}, \widetilde{\lambda}$, and $\widetilde{\mu}$ respectively. Let $\eta_{\widetilde{\gamma}}(w), \eta_{\widetilde{\lambda}}(x)$, and $\eta_{\widetilde{\mu}}(y)$ symbolize the membership functions of $\widetilde{\gamma}, \widetilde{\lambda}$, and $\widetilde{\mu}$ respectively, then the fuzzy sets are defined as

$$\begin{aligned}
\widetilde{\gamma} &= \{(w, \eta_{\widetilde{\gamma}}(w)) \mid w \in W\} \\
\widetilde{\lambda} &= \{(x, \eta_{\widetilde{\lambda}}(x)) \mid x \in X\} \\
\widetilde{\mu} &= \{(y, \eta_{\widetilde{\mu}}(y)) \mid y \in Y\}
\end{aligned} \tag{5.10}$$

where W, X, and Y are the crisp universal sets of the catastrophe rate of the machining system, arrival rate of the customer, unit, or data, and the service rate of the waiting customer, unit, or data respectively. Let $f(w,x,y)$ denotes the desired system characteristic of interest of study *i.e.* expected number of the customers, units, or data in the machining system (L), expected number of the customers, units or data in the queue (L_q), expected waiting time of the customers, units or data in the machining system (W), expected waiting time of the customers, units, or data in the queue (W_q), Probability that there is no customer, unit, or data in the machining system (P_0) and variance of the state of the system (σ^2). Since $\widetilde{\gamma}, \widetilde{\lambda}$, and $\widetilde{\mu}$ are fuzzy numbers, $f(\widetilde{\gamma}, \widetilde{\lambda}, \widetilde{\mu})$ is also a fuzzy number. Following Zadeh's extension principle (*cf.* Zadeh[200]; Parde[141]), the membership function of the desired machining system characteristic $F = f(\widetilde{\gamma}, \widetilde{\lambda}, \widetilde{\mu})$ is defined as

$$\begin{aligned} \eta_{f(\widetilde{\gamma},\widetilde{\lambda},\widetilde{\mu})}(z) &= \sup_{\Omega} \min\{\, \eta_{\widetilde{\gamma}}(w), \eta_{\widetilde{\lambda}}(x), \eta_{\widetilde{\mu}}(y) \mid z = f(w,x,y)\,\} \\ s.t.\ \ \Omega &= \{w \in W, x \in X, y \in Y \mid w > 0, x > 0, y > 0\}. \end{aligned} \tag{5.11}$$

The membership function in Eq^n. (5.11) is not in the understandable form for practical use and it is very hard to imagine its shape. Parametric non-linear programs (NLPs), mathematical programming techniques, are developed to find the α-cuts of $f(\widetilde{\gamma}, \widetilde{\lambda}, \widetilde{\mu})$ based on the extension principle.

To re-express the membership function $\eta_{f(\widetilde{\gamma},\widetilde{\lambda},\widetilde{\mu})}(z)$ of the desired machining system characteristics $\widetilde{F}$ in the comprehensible and customary form, Zadeh's approach is employed that depends on α-cuts of $\widetilde{F}$. The α-cuts of $\widetilde{\gamma}, \widetilde{\lambda}$, and $\widetilde{\mu}$ as crisp intervals are demarcated as follows

$$\begin{aligned} \gamma(\alpha) &= [w_\alpha^L, w_\alpha^U] = \left[\min_{w\in W}\{w \mid \eta_{\widetilde{\gamma}}(w) \geq \alpha\}, \max_{w\in W}\{w \mid \eta_{\widetilde{\gamma}}(w) \geq \alpha\}\right] \\ \lambda(\alpha) &= [x_\alpha^L, x_\alpha^U] = \left[\min_{x\in X}\{x \mid \eta_{\widetilde{\lambda}}(x) \geq \alpha\}, \max_{x\in X}\{x \mid \eta_{\widetilde{\lambda}}(x) \geq \alpha\}\right] \\ \mu(\alpha) &= [y_\alpha^L, y_\alpha^U] = \left[\min_{y\in Y}\{y \mid \eta_{\widetilde{\mu}}(y) \geq \alpha\}, \max_{y\in Y}\{y \mid \eta_{\widetilde{\mu}}(y) \geq \alpha\}\right] \end{aligned} \tag{5.12}$$

The constant catastrophe rate of the machining system, arrival rate of the customer, unit, or data, and the service rate of the waiting customer, unit, or data by a machine shown as intervals when the membership function are no less than a given possibility level α. Therefore, the bounds of these intervals can be designated as functions of α as

$$w_\alpha^L = \min \eta_{\widetilde{\gamma}}^{-1}(\alpha),\ w_\alpha^U = \max \eta_{\widetilde{\gamma}}^{-1}(\alpha)$$

$$x^L_\alpha = \min \eta^{-1}_{\widetilde{\lambda}}(\alpha), x^U_\alpha = \max \eta^{-1}_{\widetilde{\lambda}}(\alpha)$$

$$y^L_\alpha = \min \eta^{-1}_{\widetilde{\mu}}(\alpha), x^U_\alpha = \max \eta^{-1}_{\widetilde{\mu}}(\alpha)$$

Since the membership function defined in Eq^n (5.11) is parametrized by α, the α-cuts of $\widetilde{F}$ is used to construct its membership function.

To derive the membership function $\eta_{\widetilde{F}}(z) = \min(\eta_{\widetilde{\gamma}}(w), \eta_{\widetilde{\lambda}}(x)$, and $\eta_{\widetilde{\mu}}(y))$ using Zadeh's extension principle, it is essential that at least one of the following cases to be hold such that $z = f(x,y,z)$ satisfies $\eta_{\widetilde{F}}(z) = \alpha$.

$$\text{Case(i)}: \left(\eta_{\widetilde{\gamma}}(w) = \alpha, \eta_{\widetilde{\lambda}}(x) \geq \alpha, \eta_{\widetilde{\mu}}(y) \geq \alpha\right)$$
$$\text{Case(ii)}: \left(\eta_{\widetilde{\gamma}}(w) \geq \alpha, \eta_{\widetilde{\lambda}}(x) = \alpha, \eta_{\widetilde{\mu}}(y) \geq \alpha\right)$$
$$\text{Case(iii)}: \left(\eta_{\widetilde{\gamma}}(w) \geq \alpha, \eta_{\widetilde{\lambda}}(x) \geq \alpha, \eta_{\widetilde{\mu}}(y) = \alpha\right)$$

This can be consummated using parametric NLP techniques. The NLPs to find the lower and upper bounds of the α-cut of $\eta_{\widetilde{F}}(z)$ for the respective cases (i)-(iii) are

$$(F)^{L_i}_\alpha = \min_\Omega f(x,y,z) \tag{5.13a}$$
$$(F)^{U_i}_\alpha = \max_\Omega f(x,y,z); i = 1,2,3 \tag{5.13b}$$

The definition of $\gamma(\alpha), \lambda(\alpha)$, and $\mu(\alpha)$ in Eq^n. (5.12) implies that $w \in \gamma(\alpha), x \in \lambda(\alpha)$, and $y \in \mu(\alpha)$ can be substituted with $w \in [w^L_\alpha, w^U_\alpha], x \in [x^L_\alpha, x^U_\alpha]$, and $y \in [y^L_\alpha, y^U_\alpha]$ respectively. For given $0 < \alpha_2 < \alpha_1 \leq 1$, we have $[w^L_{\alpha_1}, w^U_{\alpha_1}] \subseteq [w^L_{\alpha_2}, w^U_{\alpha_2}]$, $[x^L_{\alpha_1}, x^U_{\alpha_1}] \subseteq [x^L_{\alpha_2}, x^U_{\alpha_2}]$, and $[y^L_{\alpha_1}, y^U_{\alpha_1}] \subseteq [y^L_{\alpha_2}, y^U_{\alpha_2}]$ (*cf.* Zimmermann[208]). This also implies that the α-cuts form a nested structure with respect to α, hence, the lower bounds Eq^n. (5.13-a) have the same smallest element and the upper bounds Eq^n. (5.13-b) have the same largest element. To evaluate the membership function $\eta_{\widetilde{F}}(z)$, it suffices to discover the left and right shape functions of $\eta_{\widetilde{F}}(z)$, which is corresponding to finding the lower bound $(F)^L_\alpha$ and upper bound $(F)^U_\alpha$ of the α-cuts of the $\widetilde{F}$, which can be amended as

$$\begin{aligned}(F)^L_\alpha &= \min_\Omega f(x,y,z) \\ &\quad s.t.\ w^L_\alpha \leq w \leq w^U_\alpha, x^L_\alpha \leq x \leq x^U_\alpha \text{ and } y^L_\alpha \leq y \leq y^U_\alpha\end{aligned} \tag{5.14a}$$

$$\begin{aligned}(F)^U_\alpha &= \max_\Omega f(x,y,z) \\ &\quad s.t.\ w^L_\alpha \leq w \leq w^U_\alpha, x^L_\alpha \leq x \leq x^U_\alpha \text{ and } y^L_\alpha \leq y \leq y^U_\alpha\end{aligned} \tag{5.14b}$$

At least one of w, x, or y must touch the boundaries of their α-cuts to satisfy

$\eta_{\widetilde{F}}(z) = \alpha$. This mathematical model is a conventional mathematical programs with boundary constraints and advances itself to the methodical study of how the optimal solutions change with $w^L_\alpha, w^U_\alpha, x^L_\alpha, x^U_\alpha, y^L_\alpha$, and y^U_α as α varies over (0,1]. The optimality modelling is a special case of parametric NLPs (Gal [45]).

The crisp intervals $[(F)^L_\alpha, (F)^U_\alpha]$ derived in Eq^n. (5.14) represents the α-cuts of $\widetilde{F}$. Hence for $0 < \alpha_2 < \alpha_1 \leq 1$, we have $(F)^L_{\alpha_1} \geq (F)^L_{\alpha_2}$ and $(F)^U_{\alpha_1} \leq (F)^U_{\alpha_2}$ due to $\widetilde{F}$'s convexity (*cf.* Zimmermann [208]). In other words, $(F)^L_\alpha$ increases and $(F)^U_\alpha$ decreases as α increases. Consequently, the membership function $\eta_{\widetilde{F}}(z)$ can be found from Eq^n. (5.14) in systematical steps.

If both $(F)^L_\alpha$ and $(F)^U_\alpha$ are invertible with respect to α, then a left shape function $L(z) = [(F)^L_\alpha]^{-1}$ and a right shape function $R(z) = [(F)^U_\alpha]^{-1}$ can be expressed, from which the membership function $\eta_{\widetilde{F}}(z)$ is structured as

$$\eta_{\widetilde{F}}(z) = \begin{cases} L(z); & (F)^L_{\alpha=0} \leq z \leq (F)^L_{\alpha=1} \\ 1; & (F)^L_{\alpha=1} \leq z \leq (F)^U_{\alpha=1} \\ R(z); & (F)^U_{\alpha=1} \leq z \leq (F)^U_{\alpha=0} \end{cases} \tag{5.15}$$

For the complex cases, the values of $(F)^L_\alpha$ and $(F)^U_\alpha$ cannot be solved analytically. This implies that an explicit-form of membership function for $\widetilde{F}$ cannot be derived. Hence, the numerical solutions for $(F)^L_\alpha$ and $(F)^U_\alpha$ at different possibility levels α can be collected to structure the approximate shapes of $L(z)$ and $R(z)$. That is, the set of intervals $\{[(F)^L_\alpha, (F)^U_\alpha] \mid \alpha \in [0,1]\}$ shows the shape of $\eta_{\widetilde{F}}$, although the exact function is not known explicitly.

Since the anticipated machining system characteristics of the studied fuzzy queue with the fuzzy catastrophe are designated by membership functions, the values preserve all the fuzziness involved in the catastrophe rate of the machining system, the arrival rate of the customer, unit, or data and the service rate of a waiting customer, unit, or data by a machine (server). From the application point of view, the system analyst may seek a crisp concrete value for a certain system characteristic rather than a fuzzy set (*cf.* Yager [188]). For this purpose, the fuzzy values are defuzzified for system characteristics using Yager's raking index method. Since Yager's method possesses the property of area compensation, this method is adopted to transform the fuzzy values of the system characteristics into crisp value. Suitable values of the desired machining system characteristics are calculated as

$$O(E[\Lambda]) = \int_0^1 \frac{(E[\Lambda])^L_\alpha + (E[\Lambda])^U_\alpha}{2}\, d\alpha \tag{5.16}$$

where $E[\Lambda]$ is a convex fuzzy number and $\left((E[\Lambda])^L_\alpha, (E[\Lambda])^U_\alpha\right)$ is the α-cut of $E[\Lambda]$.

Note that this method is a robust ranking technique that possess the properties of compensation, linearity and additivity.

5.4 Numerical Example

To understand the complete procedure of the proposed method to derive membership functions of various desired machining system characteristics of the fuzzy queue with fuzzy catastrophe, a realistic example is illustrated in this section. Let the catastrophe rate of the machining system, arrival rate of the customer, unit, or data, and service rate of the waiting customer, unit or data by machine (server) are trapezoidal fuzzy numbers and represented by

$$\widetilde{\gamma} = [0.005, 0.010, 0.015, 0.020]$$
$$\widetilde{\lambda} = [0.2, 0.3, 0.4, 0.5]$$
$$\widetilde{\mu} = [0.5, 0.6, 0.7, 0.8]$$

First, it is easy to find analogy of Eq^n. (5.12) as $[w_\alpha^L, w_\alpha^U] = [0.005 + 0.005\alpha, 0.020 - 0.005\alpha]$, $[x_\alpha^L, x_\alpha^U] = [0.2 + 0.1\alpha, 0.5 - 0.1\alpha]$, and $[y_\alpha^L, y_\alpha^U] = [0.5 + 0.1\alpha, 0.8 - 0.1\alpha]$. From the previous experiences about the machining systems, it is obvious that when $w = w_\alpha^L$, $x = x_\alpha^U$, and $y = y_\alpha^L$ the desired machining system characteristics of fuzzy queue like L, L_q, W, W_q, and σ^2 attain their maximum value, and when $w = w_\alpha^U$, $x = x_\alpha^L$, and $y = y_\alpha^U$ they attain their minimum value. The probability that there is no customer, unit, or data in the system P_0 attains its maximum and minimum value when $w = w_\alpha^U$, $x = x_\alpha^L$ & $y = y_\alpha^U$ and $w = w_\alpha^L$, $x = x_\alpha^U$ & $y = y_\alpha^L$ respectively. Analogous to Eq^n. (5.14)(a)-(b), the α-cuts of fuzzy system characteristics of fuzzy queue $\widetilde{L}$, $\widetilde{L_q}$, $\widetilde{W}$, $\widetilde{W_q}$, $\widetilde{P_0}$, and $\widetilde{\sigma^2}$ given by Eq^n. (5.17)-(5.22) respectively. Obviously, the inverse function of these α-cuts must exist analytically with respect to α for obtaining the membership function of these machining system characteristics which analogy to Eq^n. (5.15), otherwise, one can opt numerical approach for the same.

$$[L]_{\alpha}^{L}=\frac{\left[\frac{51}{50}+\frac{39\,\alpha}{200}-\frac{\sqrt{16016-88\,\alpha-79\,\alpha^{2}}}{200}\right]}{\left[\frac{29}{50}+\frac{\alpha}{200}+\frac{\sqrt{16016-88\,\alpha-79\,\alpha^{2}}}{200}\right]} \tag{5.17a}$$

$$[L]_{\alpha}^{U}=\frac{\left[\frac{201}{200}+\frac{\alpha}{200}-\frac{\sqrt{1601\,\alpha^{2}+402\,\alpha+401}}{200}\right]}{\left[\frac{39\,\alpha}{200}-\frac{1}{200}+\frac{\sqrt{1601\,\alpha^{2}+402\,\alpha+401}}{200}\right]} \tag{5.17b}$$

$$[L_q]_{\alpha}^{L}=\frac{\left[\frac{51}{50}+\frac{39\,\alpha}{200}-\sqrt{\frac{1001}{2500}-\frac{11\,\alpha}{5000}-\frac{79\,\alpha^{2}}{40000}}\right]^{2}}{2\left(\frac{4}{5}+\frac{\alpha}{10}\right)\left[\frac{29}{50}+\frac{\alpha}{200}+\sqrt{\frac{1001}{2500}-\frac{11\,\alpha}{5000}-\frac{79\,\alpha^{2}}{40000}}\right]} \tag{5.18a}$$

$$[L_q]_{\alpha}^{U}=\frac{\left[\frac{201}{200}+\frac{\alpha}{200}-\sqrt{\frac{401}{40000}+\frac{201\,\alpha}{20000}+\frac{1601\,\alpha^{2}}{40000}}\right]^{2}}{2\left(\frac{1}{2}+\frac{\alpha}{10}\right)\left[\frac{39\,\alpha}{200}-\frac{1}{200}+\sqrt{\frac{401}{40000}+\frac{201\,\alpha}{20000}+\frac{1601\,\alpha^{2}}{40000}}\right]} \tag{5.18b}$$

$$[W]_{\alpha}^{L}=\frac{\left[\frac{51}{50}+\frac{39\,\alpha}{200}-\frac{\sqrt{16016-88\,\alpha-79\,\alpha^{2}}}{200}\right]}{\left(\frac{1}{5}+\frac{\alpha}{10}\right)\left[\frac{29}{50}+\frac{\alpha}{200}+\frac{\sqrt{16016-88\,\alpha-79\,\alpha^{2}}}{200}\right]} \tag{5.19a}$$

$$[W]_{\alpha}^{U}=\frac{\left[\frac{201}{200}+\frac{\alpha}{200}-\frac{\sqrt{1601\,\alpha^{2}+402\,\alpha+401}}{200}\right]}{\left(\frac{1}{2}-\frac{\alpha}{10}\right)\left[\frac{39\,\alpha}{200}-\frac{1}{200}+\frac{\sqrt{1601\,\alpha^{2}+402\,\alpha+401}}{200}\right]} \tag{5.19b}$$

$$[W_q]_\alpha^L = \frac{\left[\frac{51}{50}+\frac{39\,\alpha}{200}-\sqrt{\frac{1001}{2500}-\frac{11\,\alpha}{5000}-\frac{79\,\alpha^2}{40000}}\right]^2}{2\left(\frac{4}{5}+\frac{\alpha}{10}\right)\left(\frac{1}{5}+\frac{\alpha}{10}\right)\left[\frac{29}{50}+\frac{\alpha}{200}+\sqrt{\frac{1001}{2500}-\frac{11\,\alpha}{5000}-\frac{79\,\alpha^2}{40000}}\right]} \tag{5.20a}$$

$$[W_q]_\alpha^U = \frac{2\left[\frac{201}{200}+\frac{\;}{\alpha}200-\frac{\sqrt{1601\,\alpha^2+402\,\alpha+401}}{200}\right]^2}{\left(1-\frac{\alpha^2}{25}\right)\left[\frac{39\,\alpha}{200}-\frac{1}{200}+\frac{\sqrt{1601\,\alpha^2+402\,\alpha+401}}{200}\right]} \tag{5.20b}$$

$$[P_0]_\alpha^L = \frac{1}{\left(\frac{4}{5}+\frac{\alpha}{10}\right)}\left[\frac{29}{100}+\frac{\alpha}{400}+\frac{\sqrt{16016-88\,\alpha-79\,\alpha^2}}{400}\right] \tag{5.21a}$$

$$[P_0]_\alpha^U = \frac{1}{\left(\frac{1}{2}+\frac{\alpha}{10}\right)}\left[\frac{39\,\alpha}{400}-\frac{1}{400}+\frac{\sqrt{1601\,\alpha^2+402\,\alpha+401}}{400}\right] \tag{5.21b}$$

$$[\sigma^2]_\alpha^L = \frac{2\left(\frac{4}{5}+\frac{\alpha}{10}\right)\left[\frac{51}{50}+\frac{39\,\alpha}{200}-\frac{\sqrt{16016-88\,\alpha-79\,\alpha^2}}{200}\right]}{\left[\frac{29}{50}+\frac{\alpha}{200}+\frac{\sqrt{16016-88\,\alpha-79\,\alpha^2}}{200}\right]^2} \tag{5.22a}$$

$$[\sigma^2]_\alpha^U = \frac{2\left(\frac{1}{2}+\frac{\alpha}{10}\right)\left[\frac{201}{200}+\frac{\alpha}{200}-\frac{\sqrt{1601\,\alpha^2+402\,\alpha+401}}{200}\right]}{\left[\frac{39\,\alpha}{200}-\frac{1}{200}+\frac{\sqrt{1601\,\alpha^2+402\,\alpha+401}}{200}\right]^2} \tag{5.22b}$$

The membership function of the various desired fuzzy machining system characteristics of the fuzzy queue $\widetilde{L}$, $\widetilde{L_q}$, $\widetilde{W}$, $\widetilde{W_q}$, $\widetilde{P_0}$, and $\widetilde{\sigma^2}$ are derived as follows in Eq^n. (5.23)-(5.28) from above α-cuts since they all are invertible. These algebraic expression of membership functions also represent the shape function of the corresponding fuzzy system characteristics. Figs. (5.2)-(5.7) depict the shape of the membership function of the desired fuzzy system characteristics $\widetilde{L}$, $\widetilde{L_q}$, $\widetilde{W}$, $\widetilde{W_q}$, $\widetilde{P_0}$, and $\widetilde{\sigma^2}$ respectively of the studied fuzzy queue with fuzzy parameters $\widetilde{\gamma}$, $\widetilde{\lambda}$, and $\widetilde{\mu}$ along with two notable informative characteristics: support and core. The overall shape turn out as expected.

$$\eta_{\widetilde{L}}(z)=\begin{cases} L_{\widetilde{L}}(z); & \frac{523}{1638}\le z\le\frac{1120}{2323} \\ 1; & \frac{1120}{2323}\le z\le\frac{2125}{1209} \\ R_{\widetilde{L}}(z); & \frac{2125}{1209}\le z\le\frac{761}{80} \end{cases} \tag{5.23}$$

where,

$$L_{\widetilde{L}}(z)=\frac{4\left(z^2+31z-10\right)}{(z^2+z+20)}$$
$$R_{\widetilde{L}}(z)=\frac{\left(-z^2-z+100\right)}{(z^2+41z+20)}$$

$$\eta_{\widetilde{L_q}}(z)=\begin{cases} L_{\widetilde{L_q}}(z); & \frac{220}{2847}\le z\le\frac{367}{2340} \\ 1; & \frac{367}{2340}\le z\le\frac{1127}{1006} \\ R_{\widetilde{L_q}}(z); & \frac{1127}{1006}\le z\le\frac{5199}{604} \end{cases} \tag{5.24}$$

where,

$$L_{\widetilde{L_q}}(z)=\frac{4\left[-10z^2+284z-200+15\sqrt{4z^4+100z^3+777z^2+1764z}\right]}{[20z^2+59z+400]}$$
$$R_{\widetilde{L_q}}(z)=\frac{\left[-60z^2-2161z-2000+20\sqrt{4z^4+428z^3+12257z^2+42436z}\right]}{[20z^2+821z-400]}$$

$$\eta_{\widetilde{W}}(z)=\begin{cases} L_{\widetilde{W}}(z); & \frac{902}{565}\le z\le\frac{2258}{1405} \\ 1; & \frac{2258}{1405}\le z\le\frac{2843}{647} \\ R_{\widetilde{W}}(z); & \frac{2843}{647}\le z\le\frac{761}{40} \end{cases} \tag{5.25}$$

where,

$$L_{\widetilde{W}}(z) = \frac{\left[z-5+\sqrt{9z^2+1230z-1975}\right]}{z}$$
$$R_{\widetilde{W}}(z) = \frac{\left[2z+205-\sqrt{9z^2+830z+40025}\right]}{z}$$

$$\eta_{\widetilde{W_q}}(z) = \begin{cases} L_{\widetilde{W_q}}(z); & \frac{1100}{2847} \le z \le \frac{367}{702} \\ 1; & \frac{367}{702} \le z \le \frac{2417}{863} \\ R_{\widetilde{W_q}}(z); & \frac{2417}{863} \le z \le \frac{5199}{302} \end{cases} \tag{5.26}$$

where,

$$L_{\widetilde{W_q}}(z) = \left[\frac{Q}{6z} + \frac{\left(432z^2+15048z-20519\right)}{6zQ} - \frac{(12z+59)}{6z}\right]$$
$$R_{\widetilde{W_q}}(z) = \left[-\frac{(2z-821)}{6z} + \left(1+I\sqrt{3}\right)\left(\frac{P}{12z} - \frac{19\left(16z^2+1192z+36739\right)}{12zP}\right)\right]$$

$$\eta_{\widetilde{P_0}}(z) = \begin{cases} L_{\widetilde{P_0}}(z); & \frac{1638}{2161} \le z \le \frac{2323}{3443} \\ 1; & \frac{2323}{3443} \le z \le \frac{2417}{863} \\ R_{\widetilde{P_0}}(z); & \frac{2417}{863} \le z \le \frac{5199}{302} \end{cases} \tag{5.27}$$

where,

$$L_{\widetilde{P_0}}(z) = \frac{4\left(-40z^2+29z+1\right)}{(20z^2-z+1)}$$
$$R_{\widetilde{P_0}}(z) = \frac{\left(-100z^2-z+1\right)}{(20z^2-39z-1)}$$

$$\eta_{\widetilde{\sigma^2}}(z) = \begin{cases} L_{\widetilde{\sigma^2}}(z); & \frac{1543}{3663} \le z \le \frac{671}{939} \\ 1; & \frac{671}{939} \le z \le \frac{3136}{647} \\ R_{\widetilde{\sigma^2}}(z); & \frac{3136}{647} \le z \le \frac{9999}{100} \end{cases} \tag{5.28}$$

where,

$$L_{\widetilde{\sigma^2}}(z) = \frac{\left[4z - 100 + 60\sqrt{4z+1}\right]}{(z+20)}$$

$$R_{\widetilde{\sigma^2}}(z) = \frac{(z-100)\left[-z + 20\sqrt{4z+1}\right]}{[z^2 - 1600z - 400]}$$

and

$$P = \sqrt[3]{1792z^3 + 317760z^2 + 24R + 26739012z + 582943661}$$

$$Q = \sqrt[3]{(819072z^2 + 24S - 1331748z + 1918621)}$$

$$R = \sqrt{\begin{pmatrix} -43200z^6 - 8924064z^5 - 806458908z^4 - 37101519060z^3 \\ -751870882275z^2 - 3353823679500z - 529530750000 \end{pmatrix}}$$

$$S = \sqrt{\begin{pmatrix} -139968z^6 - 14626656z^5 + 675169668z^4 - 8313831900z^3 \\ +31788168525z^2 - 41870119500z + 21389250000 \end{pmatrix}}$$

The crisp intervals for the fuzzy machining system characteristics $\widetilde{L}$, $\widetilde{L_q}$, $\widetilde{W}$, $\widetilde{W_q}$, $\widetilde{P_0}$, and $\widetilde{\sigma^2}$ at different probabilistic level α can also be determined from the respective equation of membership functions Eq^n. (5.23)-(5.28) or the respective Figs. (5.2)-(5.7). Fig. (5.2) prompts two vital information for the expected number of the customers, units, or data in the machining system (L). First, the support of $\widetilde{L}$ ranges from 0.3193 to 9.5125. Though the expected number of the customers, units, or data in the machining system (L) is fuzzy, this observation indicates that it is impossible for its values to fall below 0.3193 or exceed 9.5125. Second, the core of $\widetilde{L}$, α-cut at $\alpha = 1$, contains the values from 0.7049 to 1.7577, which are the most possible values for the L. The similar results are also observed from Fig. (5.3) for the expected number of the customers, units, or data in the queue (L_q) with support ranges from 0.0773 to 8.6076 and core which ranges from 0.2915 to 1.1203.

Fig. (5.4) sketches the shape of the membership function of $\widetilde{W}$, fuzzy expected waiting time of the customer, unit, or data in the machining system. For the possibility level $\alpha = 0$, the range of the W is approximately $[1.5965, 19.0250]$. This indicates that the expected waiting time of the customer, unit, or data in the machining system cannot exceed 19.0250 or fall below 1.5965. For the possibility level $\alpha = 1$, the range of W is approximately $[2.3498, 4.3941]$, which indicates that it is possible that the W falls in this interval, although it is imprecise. Figs. (5.5) -(5.7) depict the vital information for the expected waiting time of the customer, unit, or data in the queue (W_q), the probability that there is no customer, unit, or data in the machining system (P_0) and variance of the state of the machining system (σ^2) respectively as shape function along with corresponding support and core. Overall, in a nutshell, it is observed that the proposed method of NLPs of fuzzy systems in a studied machining system is giving vital information precisely for the status of the customer (unit or data), machine (server), and machining system. The present range of machining system parameters γ, λ, and μ illustrate low expected waiting time as estimated.

5.5 Numerical Result

The tractability and implementation of the suggested method are demonstrated by computing the various machining system characteristics with respect to a diverse set of fuzzy numbers of system parameters. The trends of machining system performance characteristics based on the numerical experiments have been displayed graphically. By conducting a numerical experiment, the sensitivity of machining system characteristics by varying values of different parameters is explored in Table (5.1).

Figs. (5.8)-(5.13) and corresponding Table (5.2)-(5.7) summarize the results of sensitivity and depict the shape of membership function of machining system characteristics namely expected number of the customers, units or data in the machining system (L), expected number of the customers, units, or data in the queue (L_q), expected waiting time of the customer, unit or data in the machining system (W), expected waiting time of the customer, unit, or data in the queue (W_q), probability that there is no customer, unit, or data in the machining system (P_0) and variance of the state of the machining system (σ^2) respectively with respect to governing system parameters like fuzzy catastrophe rate of the machining system $(\widetilde{\gamma})$, fuzzy arrival rate of the customer, unit, or data $(\widetilde{\lambda})$ and fuzzy service rate of the machine (server) $(\widetilde{\mu})$. For Figs. (5.8)-(5.13) and Table (5.2)-(5.7), the default trapezoidal fuzzy number of machining system parameters are fixed as in Table (5.1). From Fig. (5.8), it is clear that the shape of membership function of the expected number of customers, units, or data in the machining system (L) is analogous for all different fuzzy numbers corresponding to $\widetilde{\gamma}$, $\widetilde{\lambda}$, and $\widetilde{\mu}$. The informative values for support and core are summarized in the corresponding Table (5.2). Fig. (5.8) and Table (5.2) illustrate that how better service rate and restricted arrival rate improve the expected number of the customers, units, or data in the machining system. Fig. (5.9) and Table (5.3) summarize the observation for membership function of expected number of customers, units, or data in the queue (L_q). The rough shape of membership function $\eta_{\widetilde{L_q}}$ looks alike for a different set of fuzzy numbers for fuzzy parameters. From both the extreme levels of possibility $\alpha = 0$ and $\alpha = 1$, it is clear that the value of the expected number of the customers, units, or data waiting in the queue can be made lower with maintaining good services facility. Figs. (5.10)-(5.11) and corresponding Tables (5.4)-(5.5) comprises the variability in shape and characteristics of membership function of $\widetilde{W}$ and $\widetilde{W_q}$ respectively. For possibility level $\alpha = 0$, wide different extreme value of

$$\widetilde{W}\Big\{(19.0250, 19.0250, 4.7154, 8.2328); (19.0250, 8.0000, 19.0250, 8.0000); \\ (19.0250, 19.0250, 17.2848, 15.3605)\Big\}$$

and

$$\widetilde{W_q}\Big\{(17.2152, 17.2152, 2.7625, 6.3151); (17.2152, 6.4000, 17.2152, 6.4000); \\ (17.2152, 17.2152, 15.4922, 13.5909)\Big\}$$

are observed for different fuzzy numbers of fuzzy system parameters $\widetilde{\lambda}$, $\widetilde{\mu}$, and $\widetilde{\gamma}$ respectively. This indicates that the machining system characteristics will not exceed these limits. Figs. (5.12) shows that membership functions of probability that there is no customer, unit, or data in the machining system look-alike for different sets of fuzzy numbers of machining system parameters. This seems that $\eta_{\widetilde{P_0}}$ is the trapezoidal fuzzy number for system parameters. The corresponding Table (5.6) for support and core indicates a wide range of possible value of P_0 for a different set of fuzzy numbers. The membership function for the fuzzy variance of the state of the machining system is depicted in Figs. (5.13), and corresponding support and core are tabulated in Table (5.7). Figs. (5.13) and Table (5.7) prompt a very narrow range and less variability in most possible value of variance. Thus, this sensitivity illustrates that the analysis results provide vital information to system managers in the decision-making. If the manager specifies the range of the expected value of machining system characteristics with possibility value of arrival of the customer, unit, or data and catastrophe, the desired service rate range can also be readily determined.

In a comparison of findings in the literature, a wide range of uncertainty and ambiguity are covered in illustration and numerical experimentation. The current study provides the more sensitivity of parameters in inter- and intra- values all parameters. The analytic results, graphical depiction, and tabular secularization are provided simultaneously to enrich a glance. The derived results also provide the platform for the new extension in the machining system, queueing system and other optimization problems as well in simpler fashion. The current findings enrich the literature of fuzzy theory and queueing theory for a new type of problem with a better methodology for future research.

5.6 Conclusion

The extended classical $M/M/1$ queueing system for fuzzy parameters and fuzzy catastrophe are represented more accurately. The derived analytic result and proposed process are more useful for system designers. In this work, the state-of-the-art procedure is presented to derive membership function of various machining system performance indices of the fuzzy queue with the fuzzy catastrophe, which might arrive either from outside the machining system or from some other service station. Specifically, a catastrophe is an event that occurs at random times and produces the instant clearing of the machining system from customers, units, or data, i.e., abandoning all services. So that each catastrophe event causes a jump of the process from the current state to zero states. In present study, we have nonetheless used the α-cut approach to translate the fuzzy problem into a conventional crisp interval problem along with an illustrative example. The key ingredient for our study is the sensitivity of machining system characteristics of the fuzzy queue with fuzzy parameters besides the proposed method. The proposed model retains the essential aspects of the classical queueing model under a fuzzy environment, and as such, it offers valuable insights for randomness, vagueness, or fuzziness into the behavior of membership function of the machining system characteristics. The modeling and analysis of machining systems with catastrophes may be used to study the migration processes with catastrophes and computer networks with virus infections or a reset order.

This chapter can be extended with the amendment of varying catastrophe intensity in destroying a different number of the customers, units, or data at a time or restoration, which governed with a fuzzy number. Different types of fuzzy numbers (*cf.* Garg and Ansha [49]; Garg [51]; Garg [50]) can also experiment for the definition of uncertainty and linguistic ambiguity in the broadening aspect for fuzzy parameters. Some variant of real-time customer's behavior and just-in-time service quality can also be studied in the fuzzy environment using the proposed methodology and non-linear program. This methodology can also depict uncertainty and ambiguity behavior of other optimization problems and management problems like inventory, reliability, supply chain, etc.

Table 5.1: The trapezoidal fuzzy number of the fuzzy machining system parameters

	$\widetilde{\lambda}_i$	$\widetilde{\mu}_i$	$\widetilde{\gamma}_i$
i=1	[0.2,0.3,0.4,0.5]	[0.5,0.6,0.7,0.8]	[0.005,0.010,0.015,0.020]
i=2	[0.50,0.55,0.60,0.65]	[0.6,0.7,0.8,0.9]	[0.0050,0.0055,0.0060,0.0065]
i=3	[0.15,0.20,0.25,0.30]	[0.50,0.55,0.60,0.65]	[0.0060,0.0085,0.0110,0.0135]
i=4	[0.1,0.2,0.3,0.4]	[0.6,0.8,1.0,1.2]	[0.0075,0.0125,0.0175,0.0225]

Table 5.2: The support and core of the fuzzy expected number of the customers, units, or data in the machining system $(\widetilde{L})$

		i=1	i=2	i=3	i=4
$\widetilde{\lambda}_i$	Support	[0.3193, 9.5125]	[0.0648, 9.5125]	[0.2224, 1.4146]	[0.1384, 3.2931]
	Core	[0.7049, 1.7577]	[0.3841, 1.2829]	[0.3841, 0.6815]	[0.3841, 0.9393]
$\widetilde{\mu}_i$	Support	[0.3193, 9.5125]	[0.2757, 4.0000]	[0.4181, 9.5125]	[0.1953, 4.0000]
	Core	[0.7049, 1.7577]	[0.5730, 1.2407]	[0.9127, 2.1980]	[0.4160, 0.9534]
$\widetilde{\gamma}_i$	Support	[0.3193, 9.5125]	[0.3286, 9.5125]	[0.3237, 8.6424]	[0.3176, 7.6803]
	Core	[0.7049, 1.7577]	[0.7310, 1.8544]	[0.7162, 1.7881]	[0.6981, 1.7103]

Table 5.3: The support and core of the fuzzy expected number of the customers, units, or data in the queue $(\widetilde{L_q})$

		i=1	i=2	i=3	i=4
$\widetilde{\lambda_i}$	Support	[0.0773, 8.6076]	[0.0039, 8.6076]	[0.0405, 0.8288]	[0.0168, 2.5260]
	Core	[0.2915, 1.1203]	[0.1066, 0.7209]	[0.1066, 0.2762]	[0.1066, 0.4549]
$\widetilde{\mu_i}$	Support	[0.0773, 8.6076]	[0.0596, 3.2000]	[0.1233, 8.6076]	[0.0319, 3.2000]
	Core	[0.2915, 1.1203]	[0.2087, 0.6870]	[0.4355, 1.5107]	[0.1222, 0.4654]
$\widetilde{\gamma_i}$	Support	[0.0773, 8.6076]	[0.0813, 8.6076]	[0.0792, 7.7461]	[0.0766, 6.7955]
	Core	[0.2915, 1.1203]	[0.3087, 1.2048]	[0.2989, 1.1468]	[0.2870, 1.0793]

Table 5.4: The support and core of the fuzzy expected waiting time of the customer, unit, or data in the machining system $(\widetilde{W})$

		i=1	i=2	i=3	i=4
$\widetilde{\lambda_i}$	Support	[1.5965, 19.0250]	[1.2965, 19.0250]	[1.4827, 4.7154]	[1.3836, 8.2328]
	Core	[2.3498, 4.3941]	[1.9203, 3.6653]	[1.9203, 2.7262]	[1.9203, 3.1309]
$\widetilde{\mu_i}$	Support	[1.5965, 19.0250]	[1.3783, 8.0000]	[2.0905, 19.0250]	[0.9767, 8.0000]
	Core	[2.3498, 4.3941]	[1.9099, 3.1017]	[3.0424, 5.4951]	[1.3865, 2.3836]
$\widetilde{\gamma_i}$	Support	[1.5965, 19.0250]	[1.6430, 19.0250]	[1.6185, 17.2848]	[1.5882, 15.3605]
	Core	[2.3498, 4.3941]	[2.4367, 4.6361]	[2.3873, 4.4703]	[2.3271, 4.2757]

Table 5.5: The support and core of the fuzzy expected waiting time of the customer, unit, or data in the queue $(\widetilde{W_q})$

		i=1	i=2	i=3	i=4
$\widetilde{\lambda_i}$	Support	[0.3864, 17.2152]	[0.0789, 17.2152]	[0.2698, 2.7625]	[0.1682, 6.3151]
	Core	[0.9715, 2.8007]	[0.5328, 2.0597]	[0.5328, 1.1049]	[0.5328, 1.5165]
$\widetilde{\mu_i}$	Support	[0.3864, 17.2152]	[0.2979, 6.4000]	[0.6163, 17.2152]	[0.1596, 6.4000]
	Core	[0.9715, 2.8007]	[0.6957, 1.7174]	[1.4518, 3.7768]	[0.4073, 1.1634]
$\widetilde{\gamma_i}$	Support	[0.3864, 17.2152]	[0.4064, 17.2152]	[0.3958, 15.4922]	[0.3829, 13.5909]
	Core	[0.9715, 2.8007]	[1.0290, 3.0119]	[0.9963, 2.8670]	[0.9567, 2.6981]

Table 5.6: The support and core of the fuzzy probability that there is no customer, unit, or data in the machining system $(\widetilde{P_0})$

		i=1	i=2	i=3	i=4
$\widetilde{\lambda_i}$	Support	[0.0951, 0.7580]	[0.0951, 0.9391]	[0.4141, 0.8181]	[0.2329, 0.8785]
	Core	[0.3626, 0.5865]	[0.4380, 0.7225]	[0.5947, 0.7225]	[0.5157, 0.7225]
$\widetilde{\mu_i}$	Support	[0.0951, 0.7580]	[0.2000, 0.7839]	[0.0951, 0.7052]	[0.2000, 0.8366]
	Core	[0.3626, 0.5865]	[0.4463, 0.6357]	[0.3127, 0.5228]	[0.5119, 0.7062]
$\widetilde{\gamma_i}$	Support	[0.0951, 0.7580]	[0.0951, 0.7527]	[0.1037, 0.7555]	[0.1152, 0.7589]
	Core	[0.3626, 0.5865]	[0.3503, 0.5777]	[0.3587, 0.5827]	[0.3690, 0.5889]

Table 5.7: The support and core of the fuzzy variance of the state of the machining system $(\widetilde{\sigma^2})$

		i=1	i=2	i=3	i=4
$\widetilde{\lambda}_i$	Support	[0.4212, 100.0000]	[0.0690, 100.0000]	[0.2719, 3.4157]	[0.1575, 14.1377]
	Core	[1.2019, 4.8470]	[0.5316, 2.9286]	[0.5316, 1.1460]	[0.5316, 1.8215]
$\widetilde{\mu}_i$	Support	[0.4212, 100.0000]	[0.3517, 20.0000]	[0.5929, 100.0000]	[0.2335, 20.0000]
	Core	[1.2019, 4.8470]	[0.9012, 2.7799]	[1.7458, 7.0294]	[0.5890, 1.8625]
$\widetilde{\gamma}_i$	Support	[0.4212, 100.0000]	[0.4366, 100.0000]	[0.4285, 83.3333]	[0.4185, 66.6667]
	Core	[1.2019, 4.8470]	[1.2654, 5.2934]	[1.2291, 4.9855]	[1.1855, 4.6354]

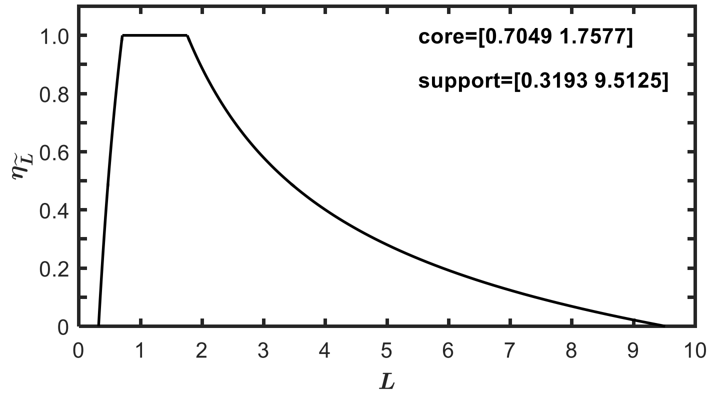

Figure 5.2: The membership function $(\eta_{\tilde{L}})$ of the fuzzy expected number of the customers,units, or data in the machining system (L)

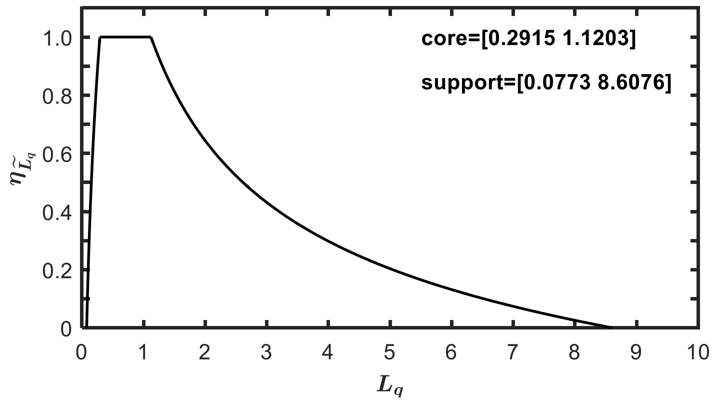

Figure 5.3: The membership function $(\eta_{\tilde{L_q}})$ of the fuzzy expected number of the customers, units, or data in the queue (L_q)

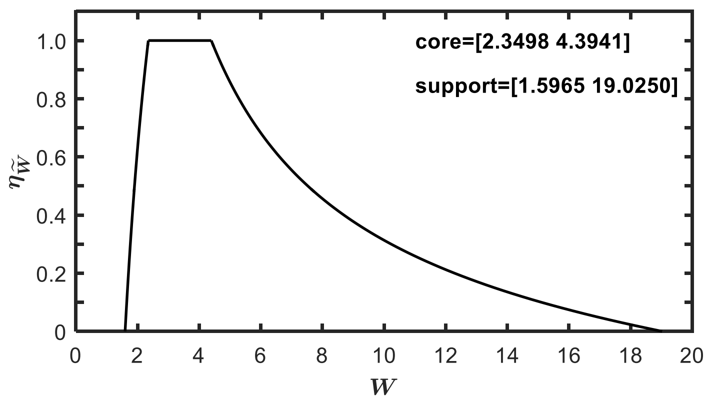

Figure 5.4: The membership function $(\eta_{\tilde{W}})$ of the fuzzy expected waiting time of the customer, unit, or data in the machining system (W)

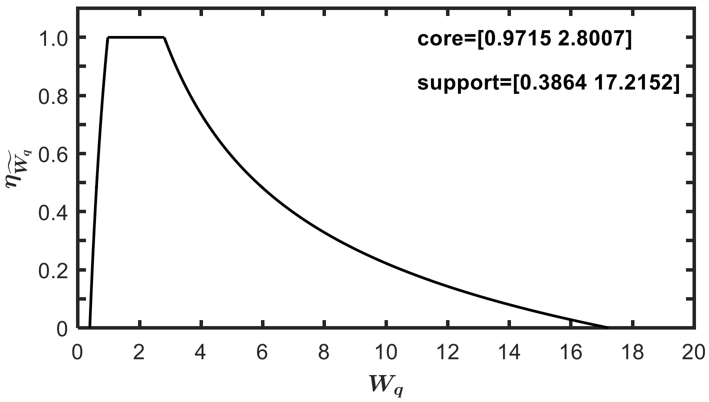

Figure 5.5: The membership function $(\eta_{\widetilde{W_q}})$ of the fuzzy expected waiting time of the customer, unit, or data in the queue (W_q)

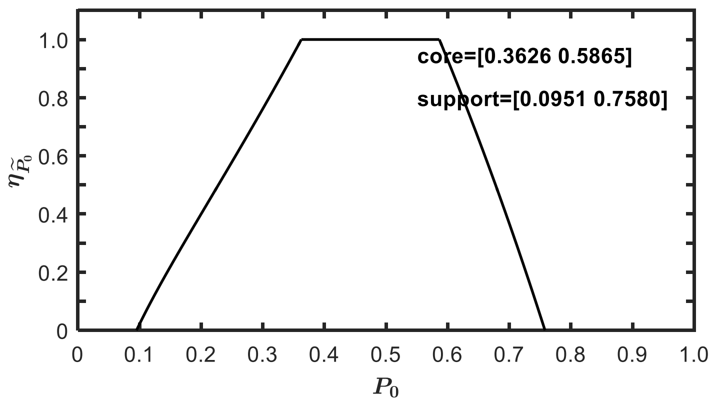

Figure 5.6: The membership function $(\eta_{\widetilde{P_0}})$ of the fuzzy probability that there is no customer, unit, or data in the machining system (P_0)

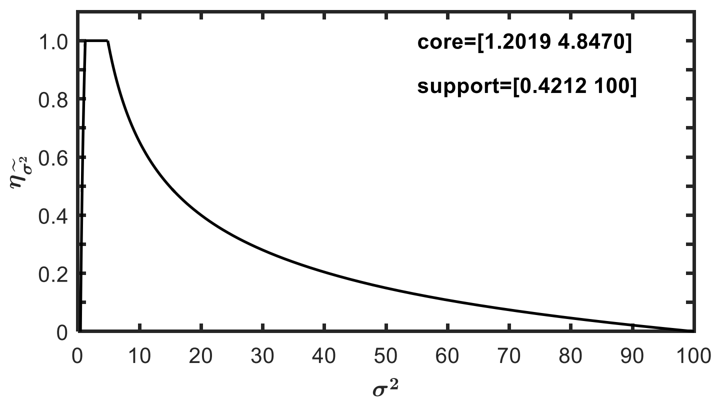

Figure 5.7: The membership function $(\eta_{\widetilde{\sigma^2}})$ of the fuzzy variance of the state of the machining system (σ^2)

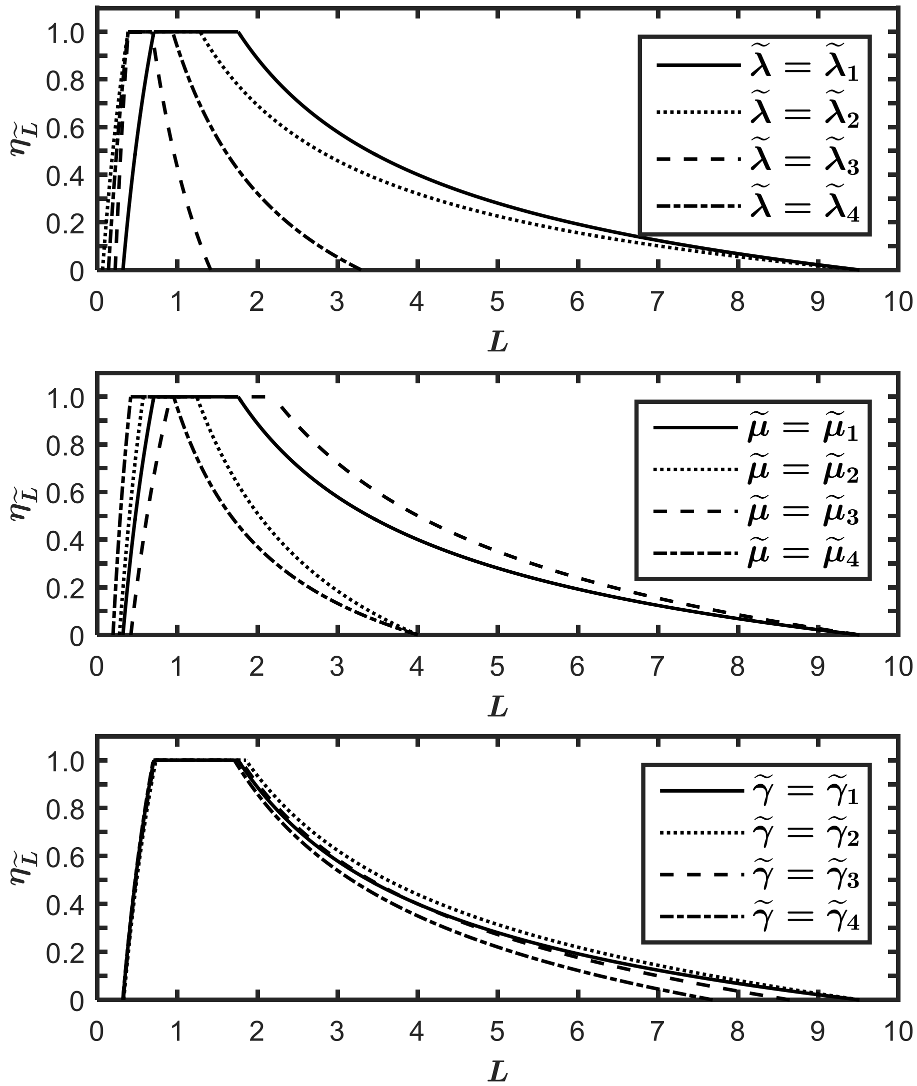

Figure 5.8: The sensitivity of the membership function $(\eta_{\widetilde{L}})$ wrt to fuzzy machining system parameters

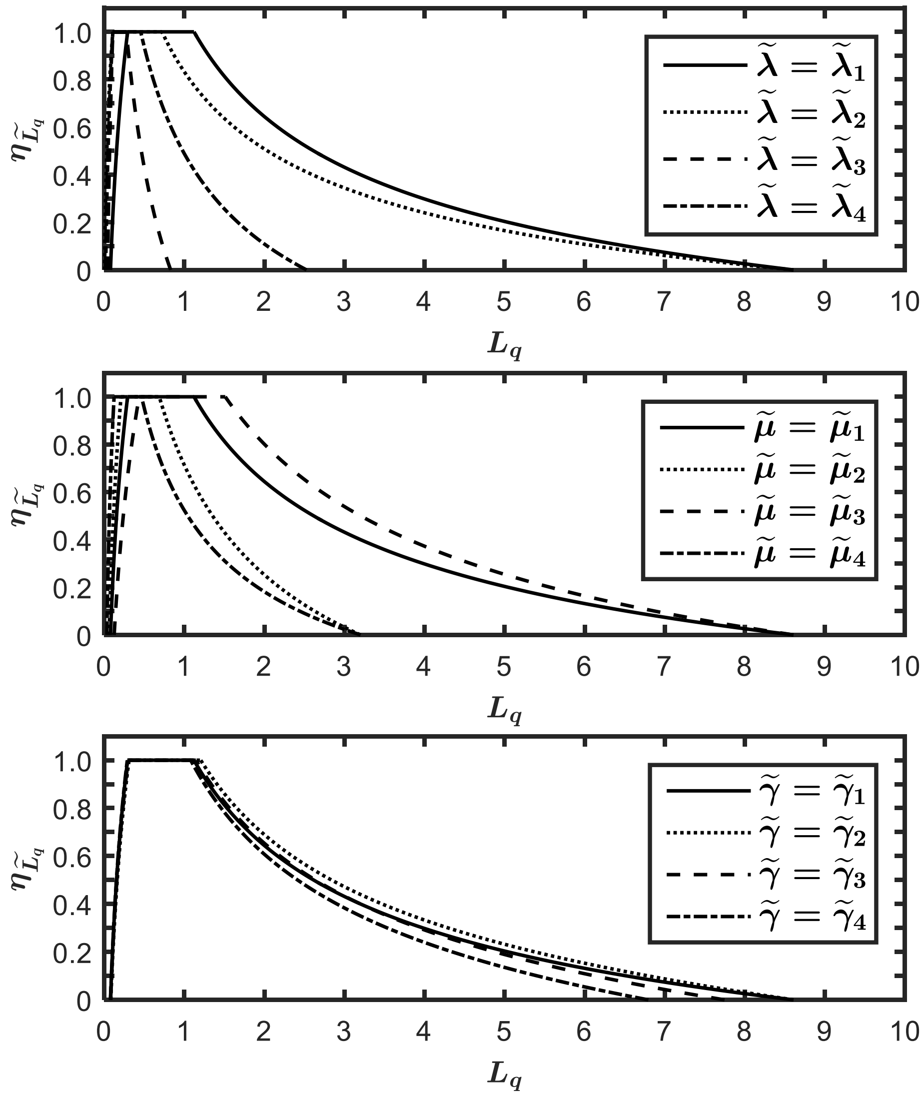

Figure 5.9: The sensitivity of the membership function $(\eta_{\widetilde{L_q}})$ wrt to fuzzy machining system parameters

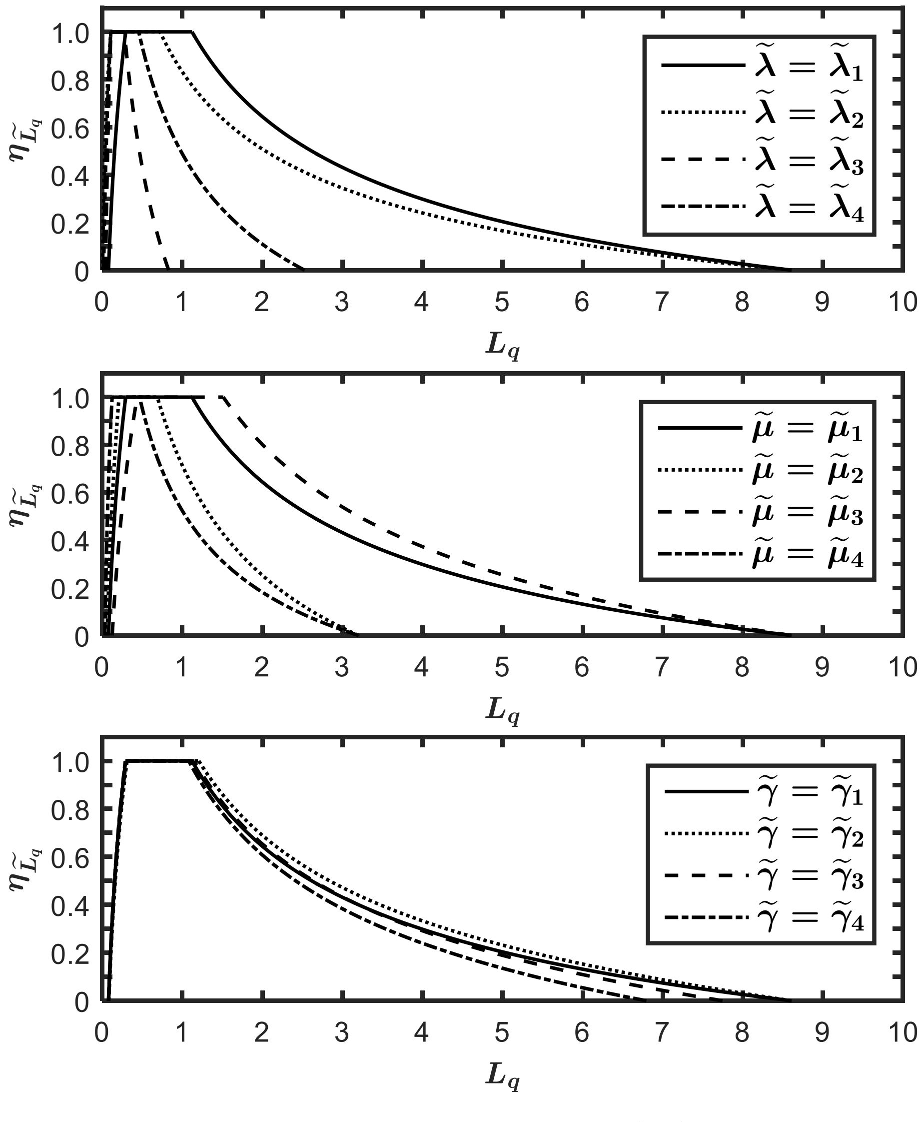

Figure 5.10: The sensitivity of the membership function $(\eta_{\widetilde{W}})$ wrt to fuzzy machining system parameters

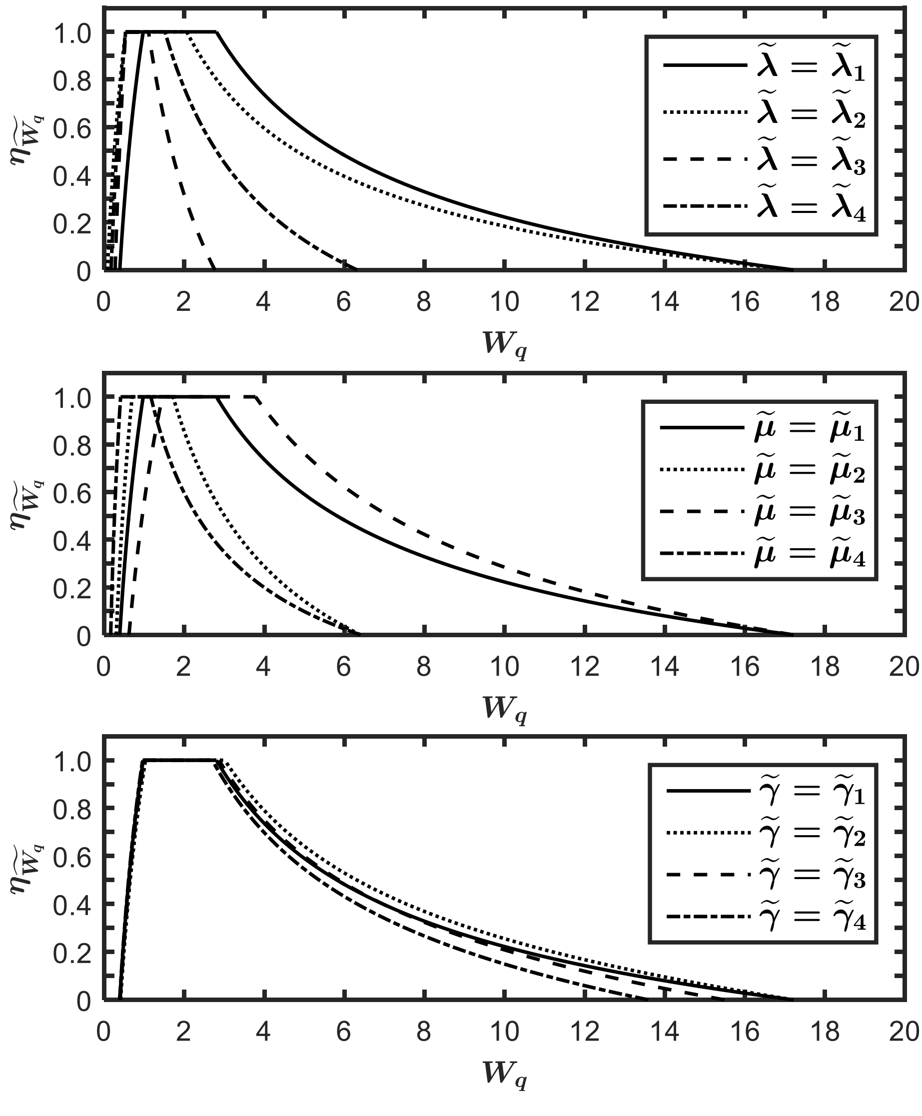

Figure 5.11: The sensitivity of the membership function $(\eta_{\widetilde{W_q}})$ wrt to fuzzy machining system parameters

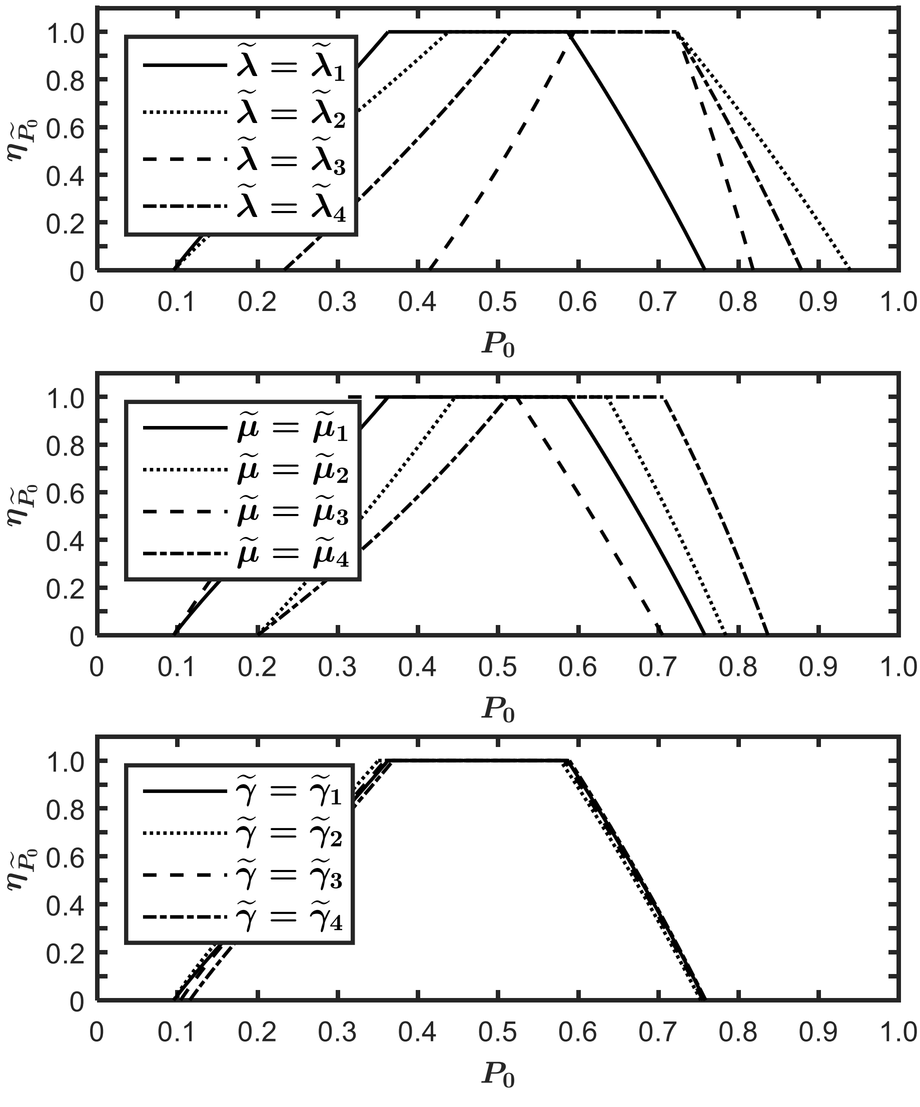

Figure 5.12: The sensitivity of the membership function $(\eta_{\widetilde{P}_0})$ wrt to fuzzy machining system parameters

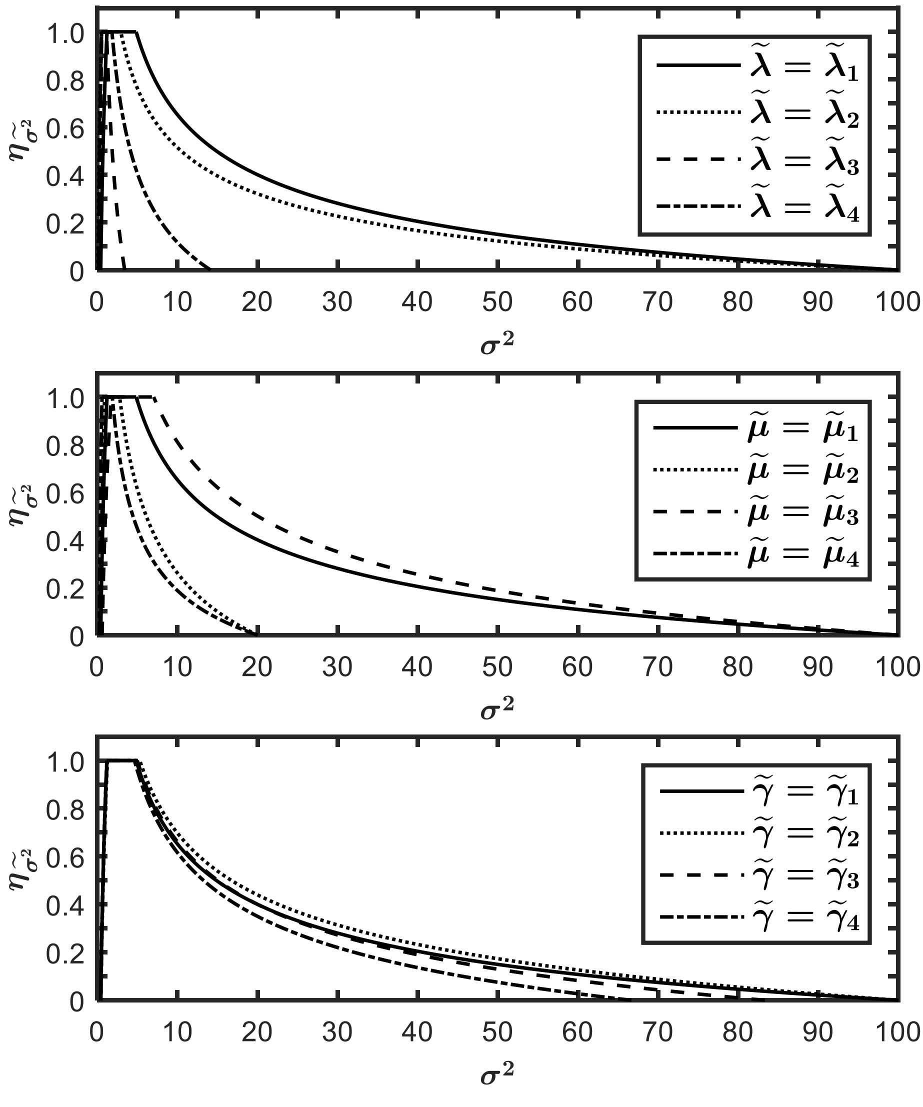

Figure 5.13: The sensitivity of the membership function $(\eta_{\widetilde{\sigma^2}})$ wrt to fuzzy machining system parameters