Chapter 6

Fuzzy Analysis of Heterogeneous Repair Facilities

"When God pushes you to the edge of difficulty trust him fully because two things can happen. Either he'll catch you when you fall or he will teach you how to fly."

A. P. J. Abdul Kalam

6.1 Introduction

The objective of this chapter is to augment the state-of-the-art analysis of two-operating units, one-spare unit redundant repairable machining systems. Redundancy acts as an imperative measure for improving the reliability attributes of engineering systems. Redundancy is classified as active and passive redundancy, depending on its operating condition. In active redundancy, the redundant operating units are simultaneously working in parallel, and in passive or standby redundancy, the spare unit initiates its working only after the failure of the active operating unit at random epoch. The active redundancy and standby redundancy in repairable machining systems have been examined widely previously (*cf.* Kumar and Agarwal [104]; Yearout et al. [197]; Trivedi [167]; Haque and Armstrong [59]; Jain et al. [70]). Recently, Shekhar et al. [159] presented a detailed overview of queueing and reliability analysis of MRP with active and standby redundancy.

The increased system reliability characteristics can be accomplished by either enhancing the reliability of each operating unit in the active redundancy or adding redundant spare units. In active redundancy, all operating units function simultaneously, whereas, in standby redundancy, the redundant spare unit will be set into the function when the operating unit fails. There are three classifications of standby redundancy: cold, warm, and hot. In cold standby, the redundant unit does not fail before being set into the full function, whereas, in warm (hot) standby, the spare unit may fail while it is in the standby state with a failure rate less than (equal to) the failure rate of the operating unit. In standby redundancy, a switching mechanism is vital to detect the failed active unit and swap it with the standby unit if it is existing. This switching mechanism can be perfect, or imperfect at random. Many authors have contributed in the past on RAMS (reliability, availability, maintainability, and safety) analysis of the machining system in different frameworks (*cf.* Amiri and Taril [6]; Fazlollahtabar and Naini[41]; Salmasnia et al. [150]). Azimi et al. [12] integrated the DOE method and meta-heuristic algorithm to address the non-exponential redundancy allocation problem in series-parallel k-out-of-n systems with repairable components. Sadeghi and Roghanian [149] examined the warm standby repairable machining system with a switching failure mechanism for two cases: repairable and non-repairable switching failure mechanism. Aslam et al. [10] focused on studying three-component mixture of exponential, Rayleigh, Pareto, and Burr Type-XII distributions.

In the scenario of multiple repair facilities, if there is a noteworthy contrast in the working attributes of the repair facilities, the assumptions of identical repair facility may not be accurate. Major research works consider the homogeneous nature of multiple repair facilities, which means the same working or repair attributes that are seldom in practical worth. Multiple repair facilities, heterogeneous in nature, are exceedingly demandable for modeling purposes in the context of machine repair problems due to increasing in the multifaceted nature and constancy on the machining environment. At the instant of activation of a repair facility in multiple heterogeneous repair facilities systems, it follows a specific policy, and its modeling requires special state transition. A few of authors have research on multiple repair facilities, heterogeneous in nature, for the redundant repairable machining system under different assumptions (*cf.* Larsen and Agarawala [113]; Lin and Kumar [122]; Yamashiro and Yuasa [190]; Wang and Tie [182]; Jain et al. [81]). Among studies considering heterogeneous repair facility, Jain et al. [71] derived queue size distribution analytically by a recursive technique for active and standby redundancy in MRP with N policy and heterogeneous repairmen. Recently, Shekhar et al. [157] extended Jain et al. [71] problem for F-policy and determined optimal thresholds N and F for optimal cost.

Reported research has generally been apprehensive with obtaining measures of system effectiveness and optimum strategies. Within the context of customary machine repair problems (MRPs) alluded in the literature above, the inter-failure times and repair times are required to follow certain probability distributions with fixed parameters. This limit restricts the study for the rare problem of exactness. However, in many practical or real-time applications, the failure pattern and service pattern are more suitably designated either by subjectively in the etymological terms such as fast, moderate, slow, vagueness, uncertainty, or approximate estimation of parameters rather than by likelihood appropriation. Restated, the inter-failure rates and service rates are more possibilitic than probabilistic. The slackness with which the system measures are accounted is revealing of the vulnerability concerning these distributions. The versatile approach of defining vagueness, linguistic uncertainty, and estimation of parameters in fuzzy sets and their algebra persuade to develop an alternative method (*cf.* Zadeh [200]; Holloway [61]; Zimmermann [208]; Buckley et al. [18]). Zhang and Phillis [203] dealt with a parallel queueing system with two heterogeneous servers which are assigned customers in a fuzzy fashion. Jowers et al. [88] conceptualized the simulation of the continuous fuzzy system and broadened the platform for future research. Recently, Shekhar et al. [158] used parametric non-linear program for determining α-cut for various system performance characteristics, and hence the corresponding membership function.

If the usual crisp MRPs are extended to fuzzy MRPs, the governing reliability models become relatively user friendly in inferencing the decision in the context of risk and would have even more extensive applications (*cf.* Kao et al. [91]; Chen [27]; Ke et al. [94]; Huang et al. [65]). Ke et al. [92] used a parametric non-linear program for redundant MRP with imperfect coverage and determined the membership function for mean time-to-failure and availability of the system. Shekhar et al. [154] also applied a similar approach to determine the membership function of reliability characteristics for cold standby redundancy with switching failure and reboot. To broaden applications of reliability analysis in engineering, scientific, and/or managerial aspects, the procedure is proposed to compute membership function of reliability characteristics using fuzzy parameters for fuzzy MRP with heterogeneous multiple repair facilities.

Going for the objective of inferring and driving the membership functions of the reliability characteristic viz. mean time-to-failure ($MTTF$) and availability of the system for fuzzy MRPs, this chapter embraces the α-cut approach to decompose a fuzzy MRP to a family of crisp MRPs. As the α value differs, the parametric programming technique is applied to describe that family of crisp MRPs. The solutions from the parametric programs determine the membership functions of the reliability measures. To exhibit the legitimacy of the proposed approach; parametric non-linear program, the numerical outline has additionally given.

A case spurred by a real-life system is considered to exhibit the practical use and future scope of the proposed model and methodology. For instance, we encounter computing systems comprised of two processors for performing computational requests and accomplishing high reliability and execution. Such systems, in general, are supported with one standby processors to avoid any working hindrance to the random failure of any working processor. Since the system is of high performance, two permanent operators, automatic and/or manual, having different working attributes are employed to monitor continuously for any system interruption. Examples of such redundant machining systems are banking systems, electronic switching systems, a seat reservation systems et cetera.

There are numerous investigations on stochastic models within fuzzy milieus in a recent literature base. Just a few of these investigations concentrate on redundant repairable systems with fuzzy parameter patterns using parametric non-linear programming. Different from other models in previous studies, the current model provides (i) heterogeneous multiple repair facilities in MRP (ii) an appropriate estimation value from uncertain environments, and (iii) a correlation between fuzzy theory and the conventional method. This chapter is enriched with (i) concept of mathematical modeling of the problem involved in realistic machining system, (ii) mathematical concepts of Laplace transform, linear algebra, fuzzy sets and logic, non-linear program, probability theory, stochastic process, birth & death process, (iii) reliability characteristics like mean time-to-failure, availability, hazard rate function, reliability function, etc., and (iv) computer programming in MATLAB & MAPLE.

This chapter is composed of multiple sections as follows Section (6.2) presents a detailed model description with assumptions, notations, and mathematical formulation. In section (6.3), the derivation of the reliability characteristics of the repairable redundant machining system viz. mean time-to-failure and availability of the system is presented. In section (6.4), the conventional repairable system is extended in the fuzzy atmosphere and methodology for determining the corresponding membership functions for reliability characteristics are briefly discussed. In the next section (6.5), a mathematical non-linear programming approach is framed to determine the membership functions for the mean time-to-failure ($MTTF$) and availability of the system. For the validity of the proposed concept and methodology, a realistic illustrative example is presented, and numerical simulation for varied system parameters is done in section (6.6). Conclusions are inferred in section (6.7) along with the future scope of the chapter.

6.2 Model Description

For modeling the distinguishable multiple repair facilities within a redundant repairable machining system, some assumptions and notations are considered, and the associated governing equations for the state probabilities are developed. The structure of the studied system comprises two identical operating units that work independently and simultaneously (active redundancy) with the redundancy of a single warm spare unit under the supervision of two heterogeneous repairmen having distinguishable working attributes. Following are some notable assumptions and notations used for the detailed description of the system:

6.2.1 Failure Process

- The time-to-failure of the operating units follows an exponential distribution with rate parameter λ.
- The failed operating unit is immediately replaced, with negligible switchover time, by spare unit, if available. The switched spare unit has the same failure and operative characteristics as an operating unit on commencing operation.
- The spare unit, warm in nature, may fail before it is installed into the full operation and is continuously monitored by an automatic failure detection device. The time-to-failure of warm spare unit follows an exponential distribution with rate parameter $\nu\,(0 < \nu < \lambda)$.
- Operating or/and spare unit fail independent of the operating condition of the other units, and both are repairable. The failed unit is immediately sent to a repair facility. If any failed unit is undergoing repair, subsequently failed units must wait in a queue until any repairman is available.

6.2.2 Repair Process

- The time-to-repair of two heterogeneous repairmen is exponentially distributed with rate parameter μ and β $(\beta < \mu)$ respectively. The repair time is independent of the state of the system. The repairman with a faster repair rate is always preferred.
- When the system is empty, a failed unit is always repaired by the faster repairman.

- If a failed unit arrives at the system and finds that only one repairman is busy, the failed unit is assigned to the idle repairman immediately regardless of the rate of the repairman or how long the other repairman has been busy.
- When a repairman completes the repair, and there is a queue of failed units, the next failed unit in the waiting line is immediately allocated to the idle repairman; thus, the repairman can never be idle when there is a queue of waiting failed units.
- Once a failed unit is assigned to a repairman for repair, it remains with that repairman until its repairing is completed.
- Neither repairing jobs cannot be split and processed by both repairmen, nor a failed unit can be moved from the slower to the faster repairman.

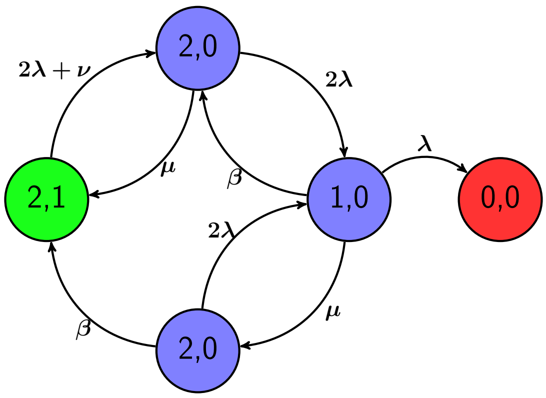

Figure 6.1: The state transition diagram for reliability analysis

Fig. (6.1) depicts the transition of states with associated rates of the governing redundant repairable machining system. The state of the system at time t is described by

$I(t) \equiv$ Number of operating units in the system.

$J(t) \equiv$ Number of spare unit in the system.

The stochastic process $\{I(t), J(t)\}$ represents the continuous time Markov chain (CTMC) on the state space $\S = \{(i, j) \mid i = 0, 1, 2 \text{ and } j = 0, 1\}$. Let P and Q represent the associated probabilities. Hence, the state probabilities are defined as follows

$P_{2,1}(t)$ =Probability that there are two operating units and one spare unit in the system at any instant t.

$P_{2,0}(t)$ =Probability that there are two operating units in the system and failed units are repaired by faster repairman at any instant t.

$Q_{2,0}(t)$=Probability that there are two operating units and failed units are repaired by slower repairman at any instant t.

$P_{1,0}(t)$ =Probability that there are one operating unit in the system at any instant t.

$P_{0,0}(t)$ =Probability that there is no operating unit in the system at instant t.

Assume that the machining system has two operating units and one spare unit initially *i.e.* in state $(2,1)$ so that the initial conditions are $P_{2,1}(0)=1$, $P_{2,0}(0)=0$, $Q_{2,0}(0)=0$, $P_{1,0}(0)=0$, and $P_{0,0}(0)=0$. Thus referring to state-transition rate diagram shown in Fig. (6.1), the states of the system satisfy the following set of Chapman-Kolmogorov differential-difference equations:

$$\frac{dP_{2,1}(t)}{dt} = -(2\lambda+\nu)P_{2,1}(t)+\beta Q_{2,0}(t)+\mu P_{2,0}(t) \tag{6.1}$$

$$\frac{dP_{2,0}(t)}{dt} = -(2\lambda+\mu)P_{2,0}(t)+\beta P_{1,0}(t)+(2\lambda+\nu)P_{2,1}(t) \tag{6.2}$$

$$\frac{dQ_{2,0}(t)}{dt} = -(2\lambda+\beta)Q_{2,0}(t)+\mu P_{1,0}(t) \tag{6.3}$$

$$\frac{dP_{1,0}(t)}{dt} = -(\lambda+\mu+\beta)P_{1,0}(t)+2\lambda P_{2,0}(t)+(\mu+\beta)P_{0,0}(t)+2\lambda Q_{2,0}(t) \tag{6.4}$$

$$\frac{dP_{0,0}(t)}{dt} = -(\mu+\beta)P_{0,0}(t)+\lambda P_{1,0}(t) \tag{6.5}$$

6.3 The Reliability Characteristics

6.3.1 Mean Time-to-Failure

The set of differential-difference Eq^n. (6.1)-(6.5) describe the governing model in the previous section can be solved using Laplace transformation to compute the transient-state probabilities. The definition for the Laplace transformation of the state probabilities and their derivatives are given as follows:

$$\widetilde{P}_{i,j}(s) = L\left(P_{i,j}(t)\right) = \int_0^{\infty} e^{-st} P_{i,j}(t)dt \qquad \forall \quad i,j$$
$$L\left(\frac{dP_{i,j}(t)}{dt}\right) = s\widetilde{P}_{i,j}(s) - P_{i,j}(0)$$

Using defined Laplace transform, the set of governing differential-difference Eq^n. (6.1)-(6.5) is transformed as set of linear Eq^n. (6.6)-(6.10) as follows

$$s\widetilde{P}_{2,1}(s) - 1 = -(2\lambda+\mu)\widetilde{P}_{2,1}(s) + \beta\widetilde{Q}_{2,0}(s) + \mu\widetilde{P}_{2,0}(s) \tag{6.6}$$
$$s\widetilde{P}_{2,0}(s) = -(2\lambda+\mu)\widetilde{P}_{2,0}(s) + \beta\widetilde{P}_{1,0}(s) + (2\lambda+\mu)\widetilde{P}_{2,1}(s) \tag{6.7}$$
$$s\widetilde{Q}_{2,0}(s) = -(2\lambda+\beta)\widetilde{Q}_{2,0}(s) + \mu\widetilde{P}_{1,0}(s) \tag{6.8}$$
$$s\widetilde{P}_{1,0}(s) = -(\lambda+\mu+\beta)\widetilde{P}_{1,0}(s) + 2\lambda\widetilde{P}_{2,0}(s) + (\mu+\beta)\widetilde{P}_{0,0}(s) + 2\lambda\widetilde{Q}_{2,0}(s) \tag{6.9}$$
$$s\widetilde{P}_{0,0}(s) = \lambda\widetilde{P}_{1,0}(s) \tag{6.10}$$

The above system of linear Eq^n. (6.6)-(6.10) can be solved to yield the state probabilities in the transformed form as follows

$$\widetilde{P}_{2,1}(s) = \frac{(s+\beta)\left(2\lambda^2 + (s+\mu)(s+5\lambda+\mu+\beta)\right) + 2\lambda^2(3s+2\lambda+\mu)}{D} \tag{6.11}$$

$$\widetilde{P}_{2,0}(s) = \frac{(2\lambda+\nu)\left(2\lambda^2 + (s+\beta)(s+3\lambda+\mu+\beta)\right)}{D} \tag{6.12}$$

$$\widetilde{Q}_{2,0}(s) = \frac{2\lambda\mu(2\lambda+\nu)}{D} \tag{6.13}$$

$$\widetilde{P}_{1,0}(s) = \frac{2\lambda(s+2\lambda+\beta)(2\lambda+\nu)}{D} \tag{6.14}$$

$$\widetilde{P}_{0,0}(s) = \frac{\lambda^2(s+2\lambda+\beta)(2\lambda+\nu)}{D} \tag{6.15}$$

where,

$$D = \Big(s\big(s+\lambda+\nu)(s+2\lambda+\beta)\big)\big(2(s+\lambda)^2+s(-s-\lambda+\mu+\beta)$$
$$+(s+\beta)\big)\times\big(2\lambda^3+s(\lambda+\mu)(s+5\lambda+\mu+\beta)\big)+\lambda^2(3s+2\lambda)+s\mu(-s+2\lambda)\Big)$$

On inverting $\widetilde{P}_{2,1}(s)$, $\widetilde{P}_{2,0}(s)$, $\widetilde{Q}_{2,0}(s)$, $\widetilde{P}_{1,0}(s)$ and $\widetilde{P}_{0,0}(s)$ to $P_{2,1}(t)$, $P_{2,0}(t)$, $Q_{2,0}(t)$, $P_{1,0}(t)$ and $P_{0,0}(t)$ respectively by Laplace inverse, the state probabilities of the machining system at time t are obtained. Suppose Z be the continuous random variable representing time-to-failure of the machining system and $P_{0,0}(t)$ represents the probability that there is no working unit in the system, *i.e.*, the system fails at or before time t. Thus, reliability function of the machining system can be expressed as:

$$R_Z(t) = 1 - P_{0,0}(t), \quad t \geq 0 \tag{6.16}$$

Using the theory of reliability, the failure density $Z(t)$ can be derived as:

$$Z(t) = -\frac{dR_z(t)}{dt} = -\frac{d(1-P_{0,0}(t))}{dt} = \frac{dP_{0,0}(t)}{dt} \tag{6.17}$$

The Laplace transform of the failure density in Eq^n. (6.17) can be written as $\widetilde{Z}(s) = s\widetilde{P}_{0,0}(s) - P_{0,0}(0)$. Hence, mean time-to-failure (T) of the machining system can be derived as:

$$T = -\frac{d\widetilde{Z}(s)}{ds}\bigg|_{s=0}$$
$$= \frac{2\lambda\,\left(8\lambda^2-\mu^2+6\beta(\lambda+\mu)+\beta^2\right)+(\nu+\mu)(2\lambda+\beta)(3\lambda+\mu+\beta)}{2\lambda^2(2\lambda+\beta)(2\lambda+\nu)} \tag{6.18}$$

6.3.2 Availability of the System

In this subsection, the availability behavior of the repairable redundant machining system is discussed by accomplishing a governing system of linear equations for the steady-state of the stochastic process shown in Fig. (6.2)

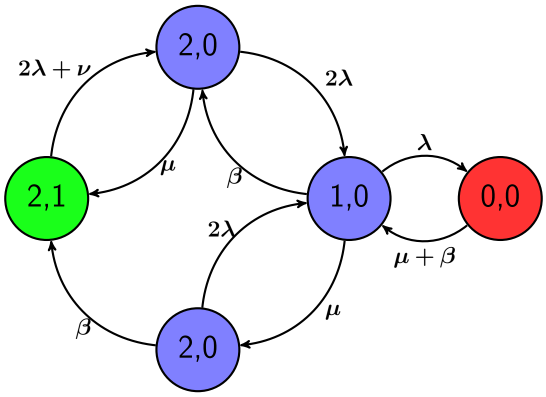

Figure 6.2: The state transition diagram for availability analysis

Hence, the Chapman-Kolmogorov balance equations in terms of steady-state probabilities are as follows

$$(2\lambda+\nu)P_{2,1} = \beta Q_{2,0} + \mu P_{2,0} \tag{6.19}$$
$$(2\lambda+\mu)P_{2,0} = \beta P_{1,0} + (2\lambda+\nu)P_{2,1} \tag{6.20}$$
$$(2\lambda+\beta)Q_{2,0} = \mu P_{1,0} \tag{6.21}$$
$$(\lambda+\mu+\beta)P_{1,0} = 2\lambda P_{2,0} + (\mu+\beta)P_{0,0} + 2\lambda Q_{2,0} \tag{6.22}$$
$$(\mu+\beta)P_{0,0} = \lambda P_{1,0} \tag{6.23}$$

The steady-state probabilities also satisfy the normalizing condition

$$P_{2,1} + P_{2,0} + Q_{2,0} + P_{1,0} + P_{0,0} = 1 \tag{6.24}$$

On solving the set of linear Eq^n. (6.19)-(6.23) for P_{21}, $P_{2,0}$, $Q_{2,0}$, $P_{1,0}$ and $P_{0,0}$ using the normalizing condition Eq^n. (6.24), we have

$$P_{2,1} = \frac{\mu\beta(4\lambda+\mu+\beta)(\mu+\beta)}{(2\lambda+\nu)(2\lambda+\beta)\left(\lambda^2+(\lambda+\mu+\beta)^2\right)+\mu\beta(\mu+\beta)(4\lambda+\mu+\beta)} \tag{6.25}$$

$$P_{2,0} = \frac{\beta(2\lambda+\nu)(2\lambda+\mu+\beta)(\mu+\beta)}{(2\lambda+\nu)(2\lambda+\beta)\left(\lambda^2+(\lambda+\mu+\beta)^2\right)+\mu\beta(\mu+\beta)(4\lambda+\mu+\beta)} \tag{6.26}$$

$$Q_{2,0} = \frac{2\lambda\mu(2\lambda+\nu)(\mu+\beta)}{(2\lambda+\nu)(2\lambda+\beta)\left(\lambda^2+(\lambda+\mu+\beta)^2\right)+\mu\beta(\mu+\beta)(4\lambda+\mu+\beta)} \tag{6.27}$$

$$P_{1,0} = \frac{2\lambda\,(2\lambda+\nu)(2\lambda+\beta)(\mu+\beta)}{(2\lambda+\nu)(2\lambda+\beta)\left(\lambda^2+(\lambda+\mu+\beta)^2\right)+\mu\beta(\mu+\beta)(4\lambda+\mu+\beta)} \tag{6.28}$$

$$P_{0,0} = \frac{2\lambda^2(2\lambda+\nu)(2\lambda+\beta)}{(2\lambda+\nu)(2\lambda+\beta)\left(\lambda^2+(\lambda+\mu+\beta)^2\right)+\mu\beta(\mu+\beta)(4\lambda+\mu+\beta)} \tag{6.29}$$

Thus, an explicit expression of the availability of the machining system (A) is computed as.

$$A = \frac{(\mu+\beta)((2\lambda+\nu)(2\lambda+\beta)(2\lambda+\mu+\beta)+\mu\beta(4\lambda+\mu+\beta))}{(2\lambda+\nu)(2\lambda+\beta)\left(\lambda^2+(\lambda+\mu+\beta)^2\right)+\mu\beta(\mu+\beta)(4\lambda+\mu+\beta)} \tag{6.30}$$

6.4 The Fuzzy Redundant Repairable System

The kernel of the present study is to encompass the efficacy of redundant repairable machining systems with heterogeneous repair facilities by incorporating linguistic vagueness, approximation in the estimation, uncertainty in intrinsic system parameters. In this section, the system parameters are allowed to follow fuzzy specifications to obtain a broad platform for the analysis of uncertainty and vagueness. The approximation, vagueness and uncertainty of failure rate of an operating units (λ), the failure rate of spare unit (ν), service rate of faster repairman (μ) and service rate of slower repairman (β) can be represented by the fuzzy sets $\widetilde{\lambda}$, $\widetilde{\nu}$, $\widetilde{\mu}$ and $\widetilde{\beta}$ respectively with associated respective membership functions $\eta_{\widetilde{\lambda}}(u)$, $\eta_{\widetilde{\nu}}(v)$, $\eta_{\widetilde{\mu}}(w)$ and $\eta_{\widetilde{\beta}}(x)$.

Hence, the following fuzzy sets represented as ordered pair are delineated

$$\widetilde{\lambda} = \{(u, \eta_{\widetilde{\lambda}}(u)) \mid u \in U\} \tag{6.31a}$$
$$\widetilde{\nu} = \{(v, \eta_{\widetilde{\nu}}(v)) \mid v \in V\} \tag{6.31b}$$
$$\widetilde{\mu} = \{(w, \eta_{\widetilde{\mu}}(w)) \mid w \in W\} \tag{6.31c}$$
$$\widetilde{\beta} = \{(x, \eta_{\widetilde{\beta}}(x)) \mid x \in X\} \tag{6.31d}$$

where U, V, W and X are the crisp universe of discourse of the system parameters namely the failure rate of an operating unit & spare unit and the service rate of a faster repairman & slower repairman respectively.

A fuzzy number, normal and convex fuzzy set, is viewed as an extension of a regular, real number in the sagacity of vagueness, uncertainty. It refers to a allied set of possible values instead of single certain value, where each possible value has its own weight between 0 and 1. Hence, fuzzy number is special case of a convex, normalized fuzzy set on the real line. Calculations with fuzzy numbers allow the incorporation of uncertainty on parameters, properties, geometry, initial conditions, etc. Let $r(u,v,w,x)$ denote the reliability characteristic of interest of studied redundant repairable machining system. Since $\widetilde{\lambda}$, $\widetilde{\nu}$, $\widetilde{\mu}$ and $\widetilde{\beta}$ are fuzzy numbers $\widetilde{r}(\widetilde{\lambda},\widetilde{\nu},\widetilde{\mu},\widetilde{\beta})$ is also a fuzzy number. Following Zadeh's extension principle, the membership function of the desired reliability characteristics $\widetilde{r}(\widetilde{\lambda},\widetilde{\nu},\widetilde{\mu},\widetilde{\beta})$ is derived as:

$$\eta_{\widetilde{r}(\widetilde{\lambda},\widetilde{\nu},\widetilde{\mu},\widetilde{\beta})}(z) = \sup_{\Omega} \min\{\, \eta_{\widetilde{\lambda}}(u), \eta_{\widetilde{\nu}}(v), \eta_{\widetilde{\mu}}(w), \eta_{\widetilde{\beta}}(x) \mid z = r(u,v,w,x)\,\} \tag{6.32}$$

For the studied redundant MRP with heterogeneous multiple repair facilities, the reliability characteristics r are derived as mean time-to-failure in Eq^n. (6.18) and availability of the system in Eq^n. (6.30). Thus, the membership grade functions $\eta_{\widetilde{T}}(z)$ for mean time-to-failure and $\eta_{\widetilde{A}}(z)$ for availability of the system respectively, are

$$\eta_{\widetilde{T}}(z) = \sup_{\Omega} \min\{\eta_{\widetilde{\lambda}}(u), \eta_{\widetilde{\nu}}(v), \eta_{\widetilde{\mu}}(w), \eta_{\widetilde{\beta}}(x) \mid z = r_T(u,v,w,x)\} \tag{6.33}$$

and

$$\eta_{\widetilde{A}}(z) = \sup_{\Omega} \min\{\eta_{\widetilde{\lambda}}(u), \eta_{\widetilde{\nu}}(v), \eta_{\widetilde{\mu}}(w), \eta_{\widetilde{\beta}}(x) \mid z = r_A(u,v,w,x)\} \tag{6.34}$$

where,

$$r_T(u,v,w,x) = \frac{(w+v)(2u+x)(x+3u+w) + 2u\left(8u^2 + 6ux - w^2 + wx + x^2\right)}{2u^2(2u+x)(2u+v)}$$

$$r_A(u,v,w,x) = \frac{(w+x)\,((2u+v)\,(2u+x)\,(w+x+2u)+xw\,(w+x+4u))}{(2u+v)\,(2u+x)\left(u^2+(w+x+u)^2\right)+xw\,(w+x)\,(w+x+4u)}$$

with

$$\Omega = \{u \in U, v \in V, w \in W, x \in X \mid u > 0, v > 0, w > 0, x > 0\}$$

The derived membership grade functions are not in explicit form and their practical applicability is very difficult. The imagination of membership's shapes and inference of any property is very typical and rarely judged. To overcome this difficulty, parametric non-linear programs based on the extension principle are employed and steps are described in the next section.

6.5 Parametric Non-Linear Programs

Parametric non-linear programs (NLPs), mathematical programming techniques, are generated to find the α-cuts of $\widetilde{r}(\widetilde{\lambda},\widetilde{v},\widetilde{\mu},\widetilde{\beta})$ based on the Zadeh's extension principle. To structure the membership function $\eta_{\widetilde{r}(\widetilde{\lambda},\widetilde{v},\widetilde{\mu},\widetilde{\beta})}(z)$ of the desired reliability characteristics $\widetilde{r}$ in the comprehensible and customary form, Zadeh's approach (*cf.* Zadeh [200]) espoused that depends on α-cuts of $\widetilde{r}$. The α-cuts of $\widetilde{\lambda}$, $\widetilde{v}$, $\widetilde{\mu}$, and $\widetilde{\beta}$ as different levels of crisp confidence intervals are demarcated as follows

$$\lambda(\alpha) = [u_\alpha^L, u_\alpha^U] = \left[\min_{u\in U}\{u \mid \eta_{\widetilde{\lambda}}(u) \geq \alpha\}, \max_{u\in U}\{u \mid \eta_{\widetilde{\lambda}}(u) \geq \alpha\}\right] \tag{6.35a}$$

$$v(\alpha) = [v_\alpha^L, v_\alpha^U] = \left[\min_{v\in V}\{v \mid \eta_{\widetilde{v}}(v) \geq \alpha\}, \max_{v\in V}\{v \mid \eta_{\widetilde{v}}(v) \geq \alpha\}\right] \tag{6.35b}$$

$$\mu(\alpha) = [w_\alpha^L, w_\alpha^U] = \left[\min_{w\in W}\{w \mid \eta_{\widetilde{\mu}}(w) \geq \alpha\}, \max_{w\in W}\{w \mid \eta_{\widetilde{\mu}}(w) \geq \alpha\}\right] \tag{6.35c}$$

$$\beta(\alpha) = [x_\alpha^L, x_\alpha^U] = \left[\min_{x\in X}\{x \mid \eta_{\widetilde{\beta}}(x) \geq \alpha\}, \max_{x\in X}\{x \mid \eta_{\widetilde{\beta}}(x) \geq \alpha\}\right] \tag{6.35d}$$

Therefore, the bounds of these crisp intervals can be designated as functions of α as: $u_\alpha^L = \min \eta_{\widetilde{\lambda}}^{-1}(\alpha)$, $u_\alpha^U = \max \eta_{\widetilde{\lambda}}^{-1}(\alpha)$, $v_\alpha^L = \min \eta_{\widetilde{v}}^{-1}(\alpha)$, $v_\alpha^U = \max \eta_{\widetilde{v}}^{-1}(\alpha)$, $w_\alpha^L = \min \eta_{\widetilde{\mu}}^{-1}(\alpha)$, $w_\alpha^U = \max \eta_{\widetilde{\mu}}^{-1}(\alpha)$, $x_\alpha^L = \min \eta_{\widetilde{\beta}}^{-1}(\alpha)$ and $x_\alpha^U = \max \eta_{\widetilde{\beta}}^{-1}(\alpha)$. Hence, $u \in \lambda(\alpha)$, $v \in v(\alpha)$, $w \in \mu(\alpha)$ and $x \in \beta(\alpha)$ can be replaced by $u \in [u_\alpha^L, u_\alpha^U]$, $v \in [v_\alpha^L, v_\alpha^U]$, $w \in [w_\alpha^L, w_\alpha^U]$ and $x \in [x_\alpha^L, x_\alpha^U]$ respectively. Consequently, the studied fuzzy redundant repairable MRP can be transformed to a family of conventional MRP with different α-cut sets.

To derive the membership function of reliability characteristics

$$\eta_{\widetilde{r}}(z) = \min(\eta_{\widetilde{\lambda}}(u), \eta_{\widetilde{v}}(v), \eta_{\widetilde{\mu}}(w), \eta_{\widetilde{\beta}}(x))$$

using Zadeh's extension principle, it is essential that at least one of the following cases to be hold such that $z = r(u,v,w,x)$ satisfies $\eta_{\tilde{r}}(z) = \alpha$:

$$\begin{aligned}
\text{Case (i)} &: \left(\eta_{\tilde{\lambda}}(u) = \alpha, \eta_{\tilde{\nu}}(v) \geq \alpha, \eta_{\tilde{\mu}}(w) \geq \alpha, \eta_{\tilde{\beta}}(x) \geq \alpha\right)\\
\text{Case (ii)} &: \left(\eta_{\tilde{\lambda}}(u) \geq \alpha, \eta_{\tilde{\nu}}(v) = \alpha, \eta_{\tilde{\mu}}(w) \geq \alpha, \eta_{\tilde{\beta}}(x) \geq \alpha\right)\\
\text{Case (iii)} &: \left(\eta_{\tilde{\lambda}}(u) \geq \alpha, \eta_{\tilde{\nu}}(v) \geq \alpha, \eta_{\tilde{\mu}}(w) = \alpha, \eta_{\tilde{\beta}}(x) \geq \alpha\right)\\
\text{Case (iv)} &: \left(\eta_{\tilde{\lambda}}(u) \geq \alpha, \eta_{\tilde{\nu}}(v) \geq \alpha, \eta_{\tilde{\mu}}(w) \geq \alpha, \eta_{\tilde{\beta}}(x) = \alpha\right)
\end{aligned}$$

This can be consummated using parametric NLP techniques. To compute the lower and upper bounds of the α-cut of $\eta_{\tilde{r}}(z)$ for the respective cases $(i)-(iv)$, the NLPs are

$$(r)_{\alpha}^{L_i} = \min_{\Omega} r(u,v,w,x) \tag{6.36a}$$

$$(r)_{\alpha}^{U_i} = \max_{\Omega} r(u,v,w,x)\,;\, i = 1,2,3,4 \tag{6.36b}$$

The definition of $\lambda(\alpha)$, $\nu(\alpha)$, $\mu(\alpha)$ and $\beta(\alpha)$ implies that $u \in \lambda(\alpha)$, $v \in \nu(\alpha)$, $w \in \mu(\alpha)$ and $x \in \beta(\alpha)$ can be substituted with $u \in [u_{\alpha}^{L}, u_{\alpha}^{U}]$, $v \in [v_{\alpha}^{L}, v_{\alpha}^{U}]$, $w \in [w_{\alpha}^{L}, w_{\alpha}^{U}]$ and $x \in [x_{\alpha}^{L}, x_{\alpha}^{U}]$ respectively. For given $0 < \alpha_2 < \alpha_1 \leq 1$, the following inclusion relation $[u_{\alpha_1}^{L}, u_{\alpha_1}^{U}] \subseteq [u_{\alpha_2}^{L}, u_{\alpha_2}^{U}]$, $[v_{\alpha_1}^{L}, v_{\alpha_1}^{U}] \subseteq [v_{\alpha_2}^{L}, v_{\alpha_2}^{U}]$, $[w_{\alpha_1}^{L}, w_{\alpha_1}^{U}] \subseteq [w_{\alpha_2}^{L}, w_{\alpha_2}^{U}]$ and $[x_{\alpha_1}^{L}, x_{\alpha_1}^{U}] \subseteq [x_{\alpha_2}^{L}, x_{\alpha_2}^{U}]$ is hold. This also entails that the α-cuts frame a nested structure with respect to α, hence, the lower bounds Eq^n. (6.36-a) and the upper bounds Eq^n. (6.36-b) have the same smallest and largest element respectively. To evaluate the membership function $\eta_{\tilde{r}}(z)$, it suffices to discover the left and right shape functions of $\eta_{\tilde{r}}(z)$, which is equivalent to finding the lower bound $(r)_{\alpha}^{L}$ and upper bound $(r)_{\alpha}^{U}$ of the α-cuts of the $\tilde{r}$, which can be amended as:

$$\begin{aligned}
(r)_{\alpha}^{L} &= \min_{\Omega} r(u,v,w,x)\\
&\quad s.t.\; u_{\alpha}^{L} \leq u \leq u_{\alpha}^{U},\, v_{\alpha}^{L} \leq v \leq v_{\alpha}^{U},\, w_{\alpha}^{L} \leq w \leq w_{\alpha}^{U} \text{ and } x_{\alpha}^{L} \leq x \leq x_{\alpha}^{U}
\end{aligned} \tag{6.37a}$$

$$\begin{aligned}
(r)_{\alpha}^{U} &= \max_{\Omega} r(u,v,w,x)\\
&\quad s.t.\; u_{\alpha}^{L} \leq u \leq u_{\alpha}^{U},\, v_{\alpha}^{L} \leq v \leq v_{\alpha}^{U},\, w_{\alpha}^{L} \leq w \leq w_{\alpha}^{U} \text{ and } x_{\alpha}^{L} \leq x \leq x_{\alpha}^{U}
\end{aligned} \tag{6.37b}$$

At least one of u, v, w or x must trace the limits of their α-cuts to palcate the condition $\eta_{\tilde{r}}(z) = \alpha$. This mathematical program is a conventional mathematical models with boundary constraints and proceeds itself to the systematic study of how the optimal solutions change with u_{α}^{L}, u_{α}^{U}, v_{α}^{L}, v_{α}^{U}, w_{α}^{L}, w_{α}^{U}, x_{α}^{L} and x_{α}^{U} as α differs over $(0,1]$. The optimal modeling is a special case of parametric NLPs. The crisp intervals $[(r)_{\alpha}^{L}, (r)_{\alpha}^{U}]$ derived in Eq^n. (6.37) represents the α-cuts of $\tilde{r}$. Hence for $0 < \alpha_2 <$

$\alpha_1 \leq 1$, we have $(r)^L_{\alpha_1} \geq (r)^L_{\alpha_2}$ and $(r)^U_{\alpha_1} \leq (r)^U_{\alpha_2}$ due to $\widetilde{r}$'s convexity in nature. In other words, $(r)^L_\alpha$ increases and $(r)^U_\alpha$ decreases as α increases. Consequently, the membership function $\eta_{\widetilde{r}}(z)$ can be found from Eq^n. (6.37) in systematical steps.

If both $(r)^L_\alpha$ and $(r)^U_\alpha$ are invertible with respect to α, then a left shape function $L(z) = [(r)^L_\alpha]^{-1}$ and a right shape function $R(z) = [(r)^U_\alpha]^{-1}$ can be expressed, from which the membership function $\eta_{\widetilde{r}}(z)$ is structured as:

$$\eta_{\widetilde{r}}(z) = \begin{cases} L(z), & (r)^L_{\alpha=0} \leq z \leq (r)^L_{\alpha=1} \\ 1, & (r)^L_{\alpha=1} \leq z \leq (r)^U_{\alpha=1} \\ R(z), & (r)^U_{\alpha=1} \leq z \leq (r)^U_{\alpha=0} \end{cases} \tag{6.38}$$

For the complex cases, the values of $(r)^L_\alpha$ and $(r)^U_\alpha$ cannot be solved analytically. This entails that an explicit-form of membership function for $\widetilde{r}$ cannot be formulated. Hence, the numerical values for $(r)^L_\alpha$ and $(r)^U_\alpha$ at different possibility levels α can be summarized to protray the approximate shapes of $L(z)$ and $R(z)$. That is, the set of intervals $\left\{[(r)^L_\alpha, (r)^U_\alpha] \mid \alpha \in [0,1]\right\}$ shows the shape of membership $\eta_{\widetilde{r}}$, although the exact membership function expression is not known explicitly. The membership function for mean time-to-failure and availability of the redundant repairable machining system can be determined in a similar manner by considering $r(u,v,w,x)$ as a respective explicit expression r_T and r_A. Since the redundant repairable machining system reliability characteristics are characterised by membership functions, the values preserve completely all the fuzziness, vagueness, uncertainty, etc. of the governing parameters like failure rate of an operating unit, the failure rate of a spare unit, the repair rate of faster repairman and the repair rate of a slower repairman. However, users or analysts may prefer in general a certain or definite single crisp value for a reliability characteristic rather than a fuzzy set. For seeking the required crisp value, the fuzzy values of the reliability characteristics are defuzzified using Yager,s ranking index method (*cf.* Yager [189]; Ghasemi et al. [52]). Since Yager,s method holds the property of area compensation, this strategy is embraced to change fuzzy values of reliability attributes into crisp ones. Appropriate estimations of system attributes are ascertained as

$$O(E[\Lambda]) = \int_0^1 (E[\Lambda])^L_\alpha + (E[\Lambda])^U_\alpha 2\, d\alpha \tag{6.39}$$

where $E[\Lambda]$ is a convex fuzzy number and $\left((E[\Lambda])^L_\alpha, (E[\Lambda])^U_\alpha\right)$ is the associated α-cut. Note that this method is a robust ranking technique that possesses the properties of compensation, linearity, and additivity (*cf.* Fortemps and Roubens [44]).

6.6 Illustrative Example and Numerical Results

A real-life example of computing system having redundancy of one spare processor (unit) is deliberated besides active redundancy of two operating processors (units) for emergence use at failure of any operating processors to demonstrate the practical usage of the proposed fuzzy repairable system and methodology under view of reliability theory. The computing system is monitored by two operators having heterogeneous caliber of rectifying the failed processor. For measuring efficiency and effectiveness of the machining system, the reliability characteristics: mean time-to-failure and availability of the system are most potential indices. Suppose the failure rate of operating/spare processors and the repair rate of faster/slower operator are fuzzy in nature to define the involved vagueness and uncertainty. The respective rate parameters which are trapezoidal fuzzy number are as follows $\widetilde{\lambda} = [0.6, 0.8, 1, 1.2]$, $\widetilde{v} = [0.1, 0.2, 0.3, 0.4]$, $\widetilde{\mu} = [3, 4, 5, 6]$ and $\widetilde{\beta} = [1, 1.5, 2, 2.5]$. For prespecified $\alpha (\alpha \in [0,1])$, defuzzification process has been executed to get crisp confidence interval. Consequently, crisp interval corresponding to given fuzzy number are as follows $\left[u_\alpha^L, u_\alpha^U\right] = [0.6 + 0.2\alpha, 1.2 - 0.2\alpha]$, $\left[v_\alpha^L, v_\alpha^U\right] = [0.1 + 0.1\alpha, 0.4 - 0.1\alpha]$, $\left[w_\alpha^L, w_\alpha^U\right] = [3 + \alpha, 6 - \alpha]$ and $\left[x_\alpha^L, x_\alpha^U\right] = [1 + 0.5\alpha, 2.5 - 0.5\alpha]$. Next, it is palpable to note that the reliability characteristics viz. mean time-to-failure and availability of the system attain their minimum value when $u = u_\alpha^U, v = v_\alpha^U, w = w_\alpha^L$ and $x = x_\alpha^L$ and their maximum value when $u = u_\alpha^L, v = v_\alpha^L, w = w_\alpha^U$ and $x = x_\alpha^U$.

6.6.1 The Fuzzy Mean Time-to-Failure

From the Eq^n. 6.37(a) and 6.37(b) for $r = r_T$, the left limit and right limit of crisp interval for the α-cut of fuzzy mean time-to-failure $(\widetilde{T})$ are derived as follows

$$(T)_\alpha^L = \frac{5}{4}\left(\frac{293\,\alpha^3 + 4048\,\alpha^2 + 21300\,\alpha + 120784}{(28 - 5\,\alpha)\,(34 + \alpha)\,(-6 + \alpha)^2}\right) \tag{6.40a}$$

$$(T)_\alpha^U = \frac{5}{4}\left(\frac{293\,\alpha^3 - 6975\,\alpha^2 + 60534\,\alpha - 282312}{(13 + 5\,\alpha)\,(-42 + \alpha)\,(3 + \alpha)^2}\right) \tag{6.40b}$$

Obviously the function $(T)_\alpha^U$ and $(T)_\alpha^L$ is invertible in nature, that yields the membership function:

$$\eta_{\widetilde{T}}(z) = \begin{cases} L(z); & \frac{37745}{8568} \le z \le \frac{5857}{644} \\ 1; & \frac{5857}{644} \le z \le \frac{285575}{11808} \\ R(z); & \frac{285575}{11808} \le z \le \frac{6535}{91} \end{cases} \tag{6.41}$$

The functional form of membership functions $L(z)$ and $R(z)$ are tedious to express. The graphical shape of the membership function is depicted in Fig. 6.3(i).

The fuzzy mean time-to-failure $\widetilde{T}$ has two noteworthy associate crisp quantities to be noted. First, at the possibility level $\alpha = 0$, the support of $\widetilde{T}$ ranges from 4.4053 to 66.1316; this demonstrates that, though the mean time-to-failure is fuzzy, it is incomprehensible for its values to fall below 4.4053 or surpass 66.1316. Second, the α-cut at plausibility level $\alpha = 1$ contains the values from 9.0947 to 24.9329, which are the most conceivable values for the mean time-to-failure.

6.6.2 The Fuzzy Availability of the System

Similarly, from the Eq^n. 6.37(a) & 6.37(b) for $r = r_A$, the α-cut of availability $\widetilde{A}$ are

$$(A)_\alpha^L = \frac{5\left(885\,\alpha^4 + 15164\,\alpha^3 + 110596\,\alpha^2 + 465856\,\alpha + 698624\right)}{4385\,\alpha^4 + 75164\,\alpha^3 + 572788\,\alpha^2 + 2196992\,\alpha + 3767296} \tag{6.42a}$$

$$(A)_\alpha^U = \frac{16545\left(295\,\alpha^4 - 8993\,\alpha^3 + 110250\,\alpha^2 - 662364\,\alpha + 1565352\right)}{4836655\,\alpha^4 - 147315577\,\alpha^3 + 1838937042\,\alpha^2 - 10914260000\,\alpha + 25942109976} \tag{6.42b}$$

It is apparent that the above functions are invertible. Hence, the membership function in shape form is

$$\eta_{\widetilde{A}}(z) = \begin{cases} L(z); & \frac{13645}{14716} \leq z \leq \frac{51645}{52933} \\ 1; & \frac{51645}{52933} \leq z \leq \frac{4155028575}{418172024} \\ R(z); & \frac{4155028575}{418172024} \leq z \leq \frac{108705}{108887} \end{cases} \tag{6.43}$$

The corresponding shape of the membership function of the fuzzy availability of the machining system is depicted in Fig. (6.3)(ii). From Fig. (6.3)(ii), for the possibility level $\alpha = 1$, the scope of the availability is approximately [0.9757, 0.9925], which demonstrate that it is irrefutably possible that the availability of the machining system falls in this interval, although it is imprecise. For the likelihood level $\alpha = 0$, the scope of the availability of the machining system is approximately $(0.9272, 0.9981)$. This signposts that the availability of the machining system cannot surpass 0.9981 or fall underneath 0.9272.

6.6.3 Numerical Result

For the tractability of the proposed non-linear program for computing the membership function of the reliability characteristics, the simulation of the machining system with a different set of fuzzy numbers of the governing system parameters is performed. Besides the default value of trapezoidal fuzzy number for fuzzy system parameters for illustrative example previously, some varied set of fuzzy numbers corresponding to fuzzy system parameters are considered for conducting the numerical simulation. These fuzzy numbers are tabulated in Table (6.1) and trapezoidal in shape. The varied shape of the membership functions are displayed in Figs. (6.4)-(6.7) and corresponding support and core are summarized in Tables (6.2)-(6.5).

From Fig. (6.4), it is noticeable that the shape of the membership function of mean time-to-failure and availability of the machining system is analogous for all different fuzzy number corresponding to the fuzzy failure rate of the operating unit. The informative values of support and core are summarized in Table (6.2). A higher possibility of the failure of operating units decreases the possibility of mean time-to-failure and availability of the machining system. It is obvious and validates the present modeling. Restrictive failure may increase the availability of the machining system that may be achieved with proper preventive and predictive maintenance.

Fig. (6.5) depicts the shape of the membership function of the reliability characteristics, namely mean time-to-failure and availability of the machining system for a different set of the fuzzy failure rate of the spare unit ($\widetilde{v}$). The corresponding support and core of the fuzzy set are tabulated in Table (6.3). It illustrates that failure of spare unit in the inactive state does not alter much in mean time-to-failure and availability of the machining system. But, it is necessary to vigilant for its state for a high grade of standby redundancy.

Fig. (6.6) portrays the shape of the membership function of mean time-to-failure and the availability of the machining system corresponding to the different fuzzy numbers which represent the vagueness of the service rate of a faster repairman. The corresponding most likely range of possible value of mean time-to-failure and availability of the machining system and most prevalence value are tabulated in Table (6.4) as support and core respectively. It prompts that the higher possible range of service rate increases the possibility of the availability of the machining system and mean time-to-failure. It is an obvious result. The support and core give quick insight to the system designer to set an appropriate service facility.

The similar kind of shape of membership function and observations for the fuzzy service rate of slower repairman are summarized in Fig. (6.7) and Table (6.5). This is also useful for an analyst to discover the appropriate design of the machining system.

These results correspond to the vagueness, uncertainty, assumptions, error in the estimation of system parameters to reliability characteristics viz. mean time-to-failure and availability of the machining system, and broaden the range of decision on knowing a sensitivity of different reliability characteristics. The support and core give the direct view of the maximum possible range and most likely range of desired systems. The current results and study directly benefit in system design and provide knowledge for the formulation and analysis of any real-time service system.

6.7 Conclusion

Redundant repairable MRP has been valuable in the machining framework. Moreover, a multiple-heterogeneous-repairmen repair facility in fuzzy MRPs is considerably more reasonable in many practical and realistic situations. This chapter applies the concepts of α-cut and Zadeh's extension principle to construct the membership function of mean time-to-failure and availability of the machining system using paired non-linear programming models. The presented methodology in the chapter can be extended similarly to derive membership functions of the corresponding fuzzy reliability and queueing measures. The α-cuts of the membership functions are evaluated, and their corresponding interval limits inverted to achieve explicit closed-form expressions for the system performance characteristics. Regardless the exact function is not known explicitly, the set of intervals $\{[r(u,v,w,x)^L_\alpha, r(u,v,w,x)^U_\alpha] \mid \alpha \in (0,1]\}$ can portray the shape of $\eta_{\widetilde{r}(\widetilde{\lambda},\widetilde{v},\widetilde{\mu},\widetilde{\beta})}(z)$. Applying fuzzy sets hypotheses for investigating fuzzy queueing problems can provide more information for administrative decision-making. It can easily be applied to the generalized MRP with any number of operating and spare units and non-Markovian MRPs. It can also be viewed for MRP with unreliable repairmen, switching failure of spare units, coverage failure, reboot & recovery, non-identical operating units, or catastrophe, etc.

Table 6.1: Fuzzy number for system parameters

	$i=1$	$i=2$	$i=3$	$i=4$
$\widetilde{\lambda}_i$	[0.6, 0.8, 1.0, 1.2]	[0.6, 0.7, 0.8, 0.9]	[0.4, 0.5, 0.6, 0.7]	[0.6, 0.9, 1.2, 1.5]
$\widetilde{v}_i$	[0.1, 0.2, 0.3, 0.4]	[0.10, 0.15, 0.20, 0.25]	[0.05, 0.15, 0.25, 0.35]	[0.10, 0.25, 0.40, 0.55]
$\widetilde{\mu}_i$	[3.0, 4.0, 5.0, 6.0]	[3.0, 3.5, 4.0, 4.5]	[1.0, 2.0, 3.0, 4.0]	[3.0, 5.0, 7.0, 9.0]
$\widetilde{\beta}_i$	[1.0, 1.5, 2.0, 2.5]	[1.25, 1.50, 1.75, 2.00]	[0.5, 1.0, 1.5, 2.0]	[0.50, 1.25, 2.00, 2.75]

Table 6.2: Support and core for reliability characteristics for varied $\widetilde{\lambda}$

$\widetilde{\lambda}_i$	[0.6, 0.8, 1.0, 1.2]	[0.6, 0.7, 0.8, 0.9]	[0.4, 0.5, 0.6, 0.7]	[0.6, 0.9, 1.2, 1.5]
$Support(\widetilde{T})$	(4.4053, 66.1316)	(7.6327, 66.1316)	(12.7891, 199.1635)	(2.9673, 66.1316)
$Core(\widetilde{T})$	[9.0947, 21.9329]	[14.9378, 30.5252]	[29.2829, 72.0000]	[6.1852, 16.4860]
$Support(\widetilde{A})$	(0.9272, 0.9981)	(0.9591, 0.9981)	(0.9764, 0.9994)	(0.8911, 0.9981)
$Core(\widetilde{A})$	[0.9757, 0.9925]	[0.9856, 0.9947]	[0.9929, 0.9978]	[0.9634, 0.9899]

Table 6.3: Support and Core for reliability characteristics for varied $\widetilde{v}$

$\widetilde{v}_i$	[0.1, 0.2, 0.3, 0.4]	[0.10, 0.15, 0.20, 0.25]	[0.05, 0.15, 0.25, 0.35]	[0.10, 0.25, 0.40, 0.55]
$Support(\widetilde{T})$	(4.4053, 66.1316)	(4.5053, 66.1316)	(4.4375, 68.2047)	(4.3155, 66.1316)
$Core(\widetilde{T})$	[9.0947, 21.9329]	[9.3149, 22.3497]	[9.2024, 22.3497]	[8.7981, 21.5386]
$Support(\widetilde{A})$	(0.9272, 0.9981)	(0.9284, 0.9981)	(0.9276, 0.9981)	(0.9262, 0.9981)
$Core(\widetilde{A})$	[0.9757, 0.9925]	[0.9762, 0.9927]	[0.9759, 0.9927]	[0.9749, 0.9924]

Table 6.4: Support and core for reliability characteristics for varied $\widetilde{\mu}$

$\widetilde{\mu}_i$	[3.0, 4.0, 5.0, 6.0]	[3.0, 3.5, 4.0, 4.5]	[1.0, 2.0, 3.0, 4.0]	[3.0, 5.0, 7.0, 9.0]
$Support(\widetilde{T})$	(4.4053, 66.1316)	(4.4053, 46.4114)	(2.6984, 40.5599)	(4.4053, 115.3173)
$Core(\widetilde{T})$	[9.0947, 21.9329]	[8.1304, 17.3804]	[5.5171, 13.3102]	[11.1630, 32.4846]
$Support(\widetilde{A})$	(0.9272, 0.9981)	(0.9272, 0.9966)	(0.7803, 0.9958)	(0.9272, 0.9992)
$Core(\widetilde{A})$	[0.9757, 0.9925]	[0.9699, 0.9889]	[0.9356, 0.9823]	[0.9834, 0.9962]

Table 6.5: Support and core for reliability characteristics for varied $\widetilde{\beta}$

$\widetilde{\beta}_i$	[1.0, 1.5, 2.0, 2.5]	[1.25, 1.50, 1.75, 2.00]	[0.5, 1.0, 1.5, 2.0]	[0.50, 1.25, 2.00, 2.75]
$Support(\widetilde{T})$	(4.4053, 66.1316)	(4.4053, 59.8932)	(3.8012, 59.8932)	(3.8012, 69.0288)
$Core(\widetilde{T})$	[9.0947, 21.9329]	[8.5230, 20.7200]	[7.9130, 19.4304]	[8.5230, 21.9329]
$Support(\widetilde{A})$	(0.9272, 0.9981)	(0.9272, 0.9977)	(0.9003, 0.9977)	(0.9003, 0.9982)
$Core(\widetilde{A})$	[0.9757, 0.9925]	[0.9724, 0.9917]	[0.9683, 0.9908]	[0.9724, 0.9925]

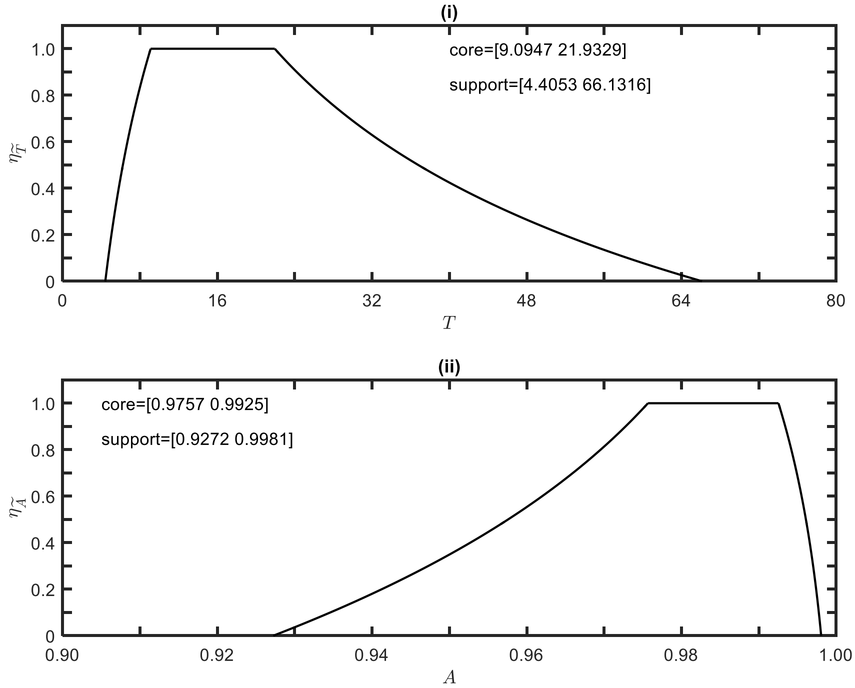

Figure 6.3: Membership grade function for reliability characteristics for illustrative example

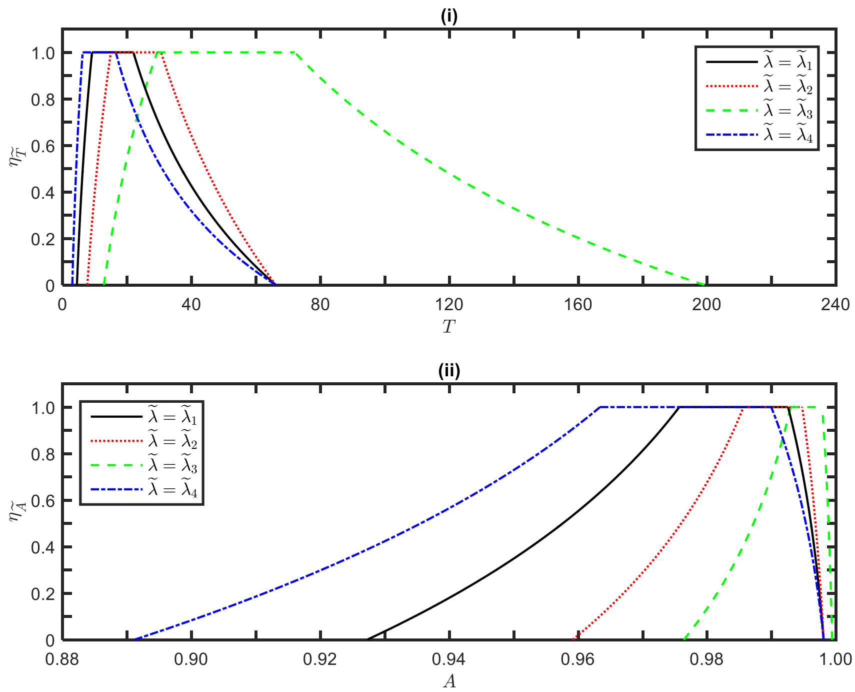

Figure 6.4: Membership grade function for reliability characteristics for varied $\widetilde{\lambda}$

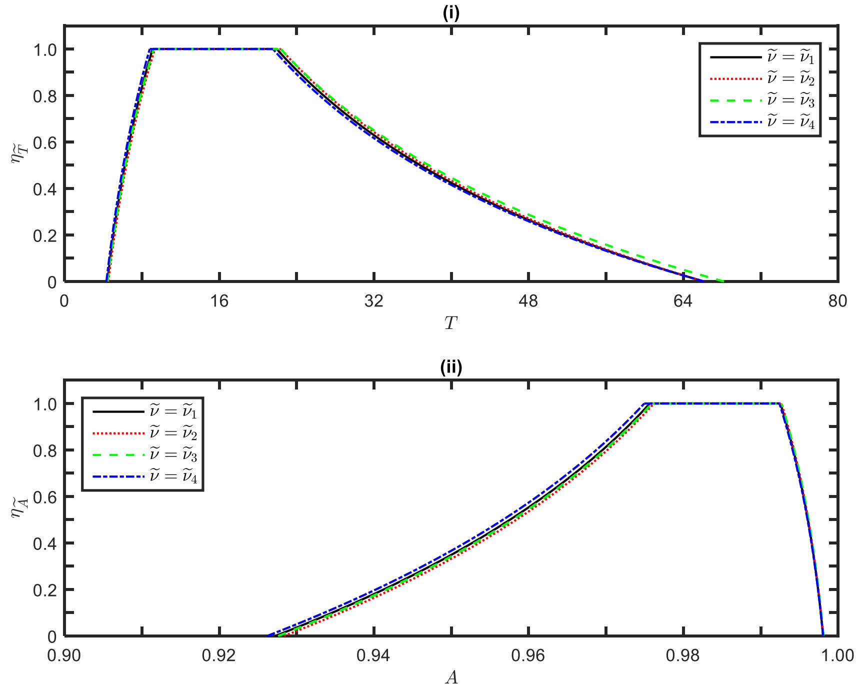

Figure 6.5: Membership grade function for reliability characteristics for varied $\widetilde{\nu}$

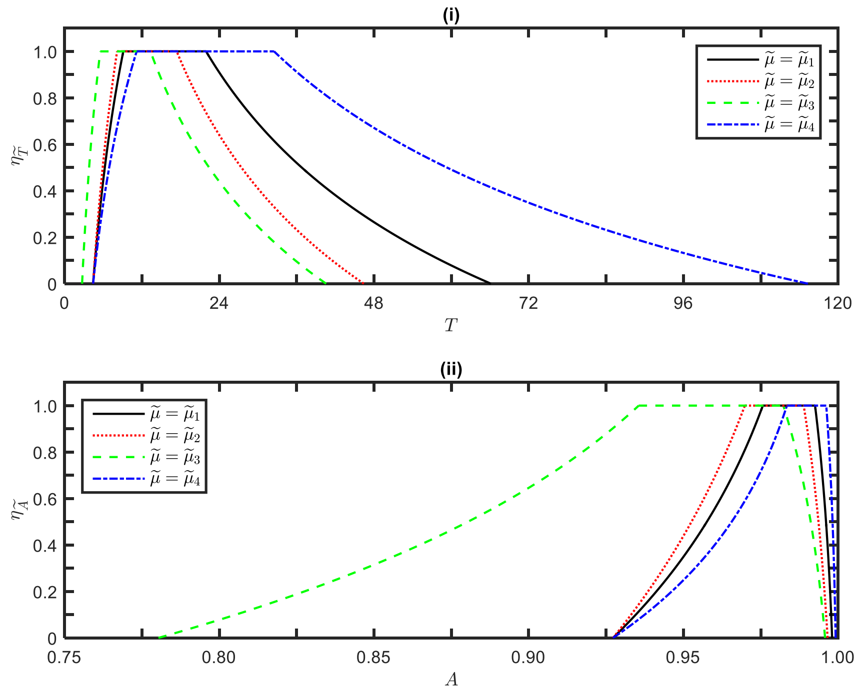

Figure 6.6: Membership grade function for reliability characteristics for varied $\widetilde{\mu}$

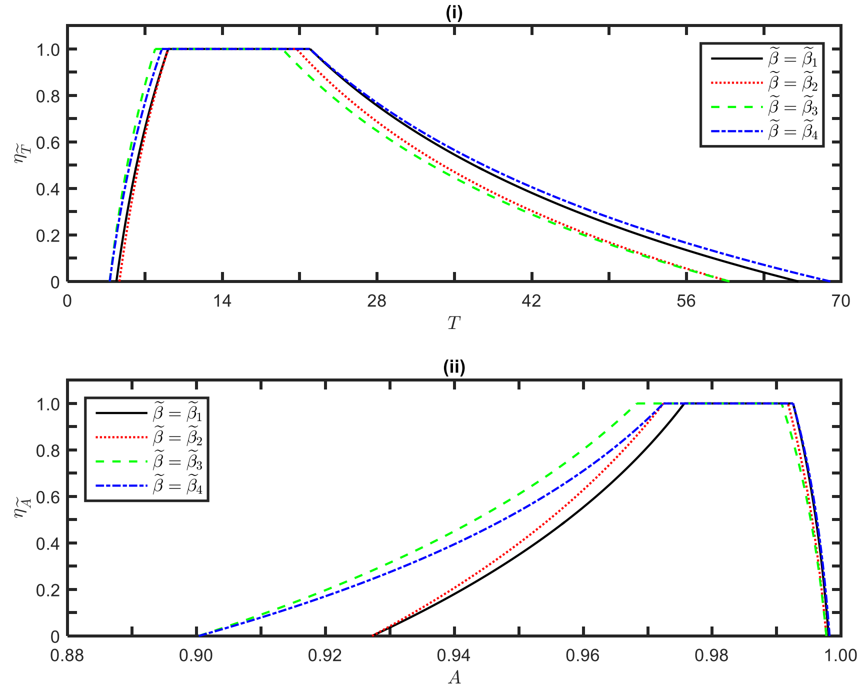

Figure 6.7: Membership grade function for reliability characteristics for varied $\widetilde{\beta}$