Chapter 1

General Introduction

> *"Although engineers want always to make everything better, they cannot make anything perfect. This basic characteristic flaw of the products of the profession's practitioners is what drives change and makes achievement a process rather than simply a goal".*
>
> *Henry Petroski*

1.1 Motivation

The development in science and technology has made today's engineering systems, including interrelated hardware and software units, more useful, sophisticated, powerful, high-tech, and complex as well. Each system comprises of operating units, has unique features and structures, and hence, system or unit failure at random and repair are having a more substantial effect on the efficiency of the system directly and economy and society indirectly. The malfunction due to degraded performance or complete failure of the system or units may lead to decreased reliability and mismanagement over the period, which governs the loss of time, money, or even sometimes life also. Random failure of a unit in the machining system enhances the production loss, manufacturing delay, high expected cost, long runtime. The foremost objective of the thesis is to optimize and control the function of the system over time to meet expectation and satisfaction at a minimum long-run expected cost.

The reliability of the machining system is qualitative property refers to the ability of proper and satisfactory intended functioning over the period. Reliability theory deals with the laws of the occurrence of failures of the system or units at random.

The increasing level of sophistication in the state-of-the-art associated processes, design, and function which implies that reliability issues will not only endure to exist but are likely to require ever more complicated modeling and sophisticated solutions for which there is need to access system performance measures effectively. For an assessment, the systems require state-of-the-art mathematical modeling with precise analysis. Consequently, the study of the reliability characteristics is vital for all stages from development to application, including design, manufacture, distribution, or/and operation. Statistical and stochastic models are suitable to evaluate system performance measures based on unit indices, optimal system design, incurred expected cost, etc. The in-depth review and analysis are required for the usefulness of the models and prediction purposes. The study on the issues related to reliability prediction addresses life testing, structural reliability, machine maintenance problems, and replacement problems.

Because today, the technological system without reliability, availability, maintainability, and safety (RAMS) is unthinkable, and the subject for study is progressing. Nowadays, thousands of millions are being disbursed on research to produce sophisticated techniques, quality-of-service (QoS), state-of-the-art design, just-in-time (JIT) maintenance, etc. for the machining system.

In the present thesis, The primary focus is on developing various reliability models of the unpredictable machining system having multi-unit, which are also prone to random failure along with discriminate failure processes, repair strategies, and governing policies. The main emphases have been made on modeling, the methodology used, analysis for increasing their usefulness in real-time problems. For each studied model, we extensively discuss realistic model description, analytical or numerical solution, reliability characteristics or performance indices, optimal governing parameters or threshold, and systematic interpretation or future scope. The exact optimal solution of such a problem is complicated and in many cases, impossible to obtain.

The need for spare units arises if the consequences of a failure or disruption of the operating unit are not acceptable. The types of machining system in which the need arises seem to be limitless. The spare unit may be hardware or software in nature, may be homogeneous or heterogeneous, may be in-house or third-party in availability. Majorly, this includes power backup, additional storage, extra units, alternative arrangements, etc. depending on the machining system or requirements. There are four critical reasons for the provisioning of spare units: safety, security, financial loss, and data loss.

Safety: The system, organization, process where there is a risk to life or health, for example, air traffic control, aviation ground lighting, medical equipment/emergency in hospitals, nuclear plants, oil explorations/refineries, etc.

Security: Security of the system against vandalism and espionage such as in attack area lighting, communication systems, military installations, etc.

Financial loss: The system where critical industrial processes and high costs involved such as in manufacturing system, production system, financial institutions, etc.

Data loss: The system where data and its storage are essential, and data may be catastrophic and irretrievable, for example, the data processing system, long-term laboratory type of testing or experiment, etc.

The content of the thesis is state-of-the-art for future study on reliability models by a research fellow, system designers, system analysts, and practicing engineer. It includes various well-studied system structures which imitate many real-life problems of the different area. It also addresses reliability evaluation, optimal system design, comprehensive algorithm, mathematical tools, and cited references. The unique features of this thesis on optimal system reliability modeling include

- complex analysis including in-depth literature survey and background knowledge on efficiency comparison
- Markov chain imbeddable structures
- Powerful tools for the development of optimal invariant design
- Application in understanding the behavior and advantages of the system, predictive, preventive or corrective measures

1.2 Machining System

The machining system can be defined technically as a set of operating units, each having a specified function, that works in a synchronized way to get the required quantity of products of the required quality, in the required time, by the best and more economical way. Machining system performs a sequence of operations that transform material from a given to the desired form for which three distinct stages: system planning, operation, and control comprise. The four factors mentioned above namely quantity, quality, time and cost encompass the machining system and require special attention and critical research.

The study of the machining system is of the little use unless it is used to obtain quality products and services at minimum cost, i.e., the cost analysis and reliability analysis are over rider factor in all machining system, and the alternative is limited to by the constraints of physical feasibility. The performance modeling of the machining system starts with the functional design, which must satisfy the need of customers, and the design for production. The system analyst must also take into account the cost incurred in the flow of jobs through the plant, cost of storage of in-process inventories, cost of maintenance and redundancy, etc.

The judgment of availability/reliability level, a proper replacement policy, maintenance policy, procedure, and schedules are some of the managerial responsibilities which should be taken into consideration for the appropriate functioning of the machining system. Since the problem of breakdowns and reserves can be seriously reflected in halts in the machining system, state-of-the-art design of repair facility is required for smooth production planning and control.

The industrial revolution in the UK after the second world war found the techniques of Operations Research and Queueing Theory of great use in the solution of many critical issues of machining problems. The application of these techniques provides a scientific basis to deal with machines and their management, which includes the art and science of planning, directing, and controlling the operating units, resources, and human efforts so that the established objective of machining system may be attained in accordance with accepted policies. Machines and plants, which breakdown, cause interference, reduce productivity, and increase manufacturing in lousy conditions. Fault units provide the terrible work, some of which may have to be scrapped and increase the incurred cost.

Just-in-time maintenance is concerned with the breakdown, which includes repair and/or replacement of the systems and units to maintain the availability of the machining system. Its function is to keep the operating units in satisfactory operating condition at minimum cost. Good maintenance also aims to prevent breakdowns before they occur. The strategy of maintenance is classified into two groups:

- **Preventive Maintenance:** It is observed with preventing the trouble before it occurs.
- **Corrective Maintenance:** It is concerned with corrective the trouble that has occurred.

1.3 Basic Layout of Machining System

Queueing theory is an application of Probability Theory and Operations Research dealing with the lack of balance of time between prospective customer's generation and service, both at irregular intervals. The rapid and spectacular developments and progress of the queueing modeling indicate its tremendous potentialities in promoting and sustaining industrial as well as economic growth and stability. In this thesis, the analysis have been done the machining units, their capabilities, and limitations, as well as some of the integration issues via the queue-theoretic approach. There are basic structures of waiting line problems involved in machining systems in general which can be classified as follows

(i) Single channel, single phase

(ii) Multiple channels, single phase

(iii) Single channel, multiple phases

(iv) Multiple channels, multiple phases

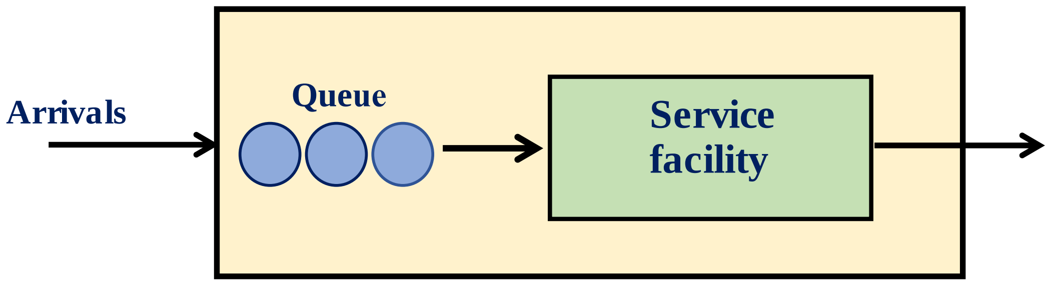

Figure 1.1: Single-channel, single-phase

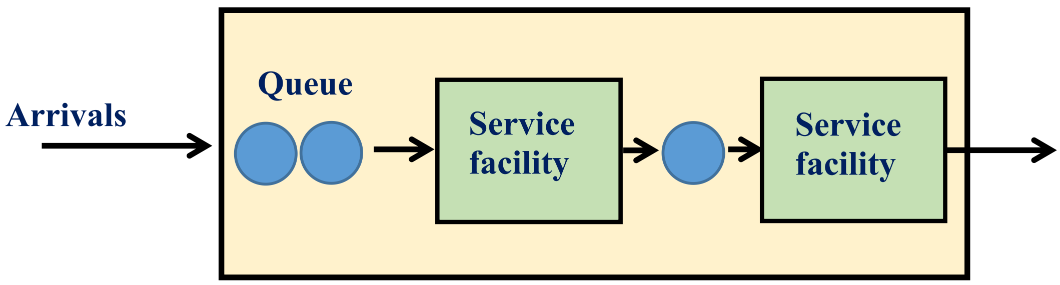

Figure 1.2: Single-channel, multi-phase

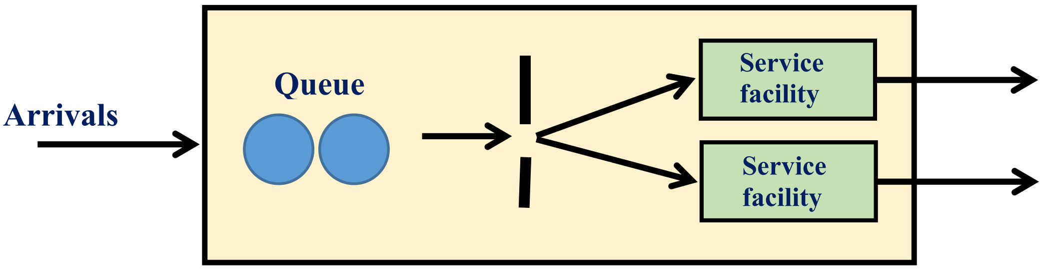

Figure 1.3: Multiple-channel, single-phase

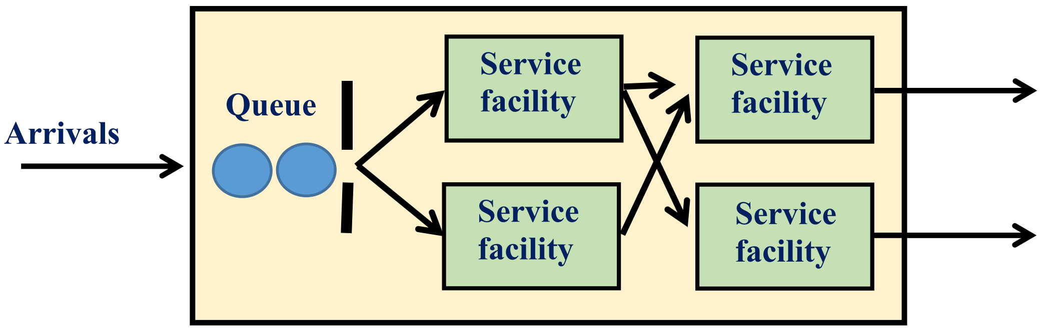

Figure 1.4: Multiple-channel, multiple-phase

Figs. (1.1)-(1.4) depict the basic layout and flow of failed units as a queueing problem. For example, to get N finished products, manufacturing must require four different operations A, B, C, D. In Fig. (1.5), each operating unit is dedicated to one operation, and sequential operations are required in tandem arrangement of these operating units. In the tandem system, if the operating units have heterogeneous efficiency and characteristics, the system results in unit starve or block. A starved operating unit is one that must remain idle because of the proceeding unit that can not supply its output. Similarly, a blocked operating unit is forced to be idle because it has no place to dispatch. Operating unit failure at random and unpredictable repair time would also lead to a starve-block situation.

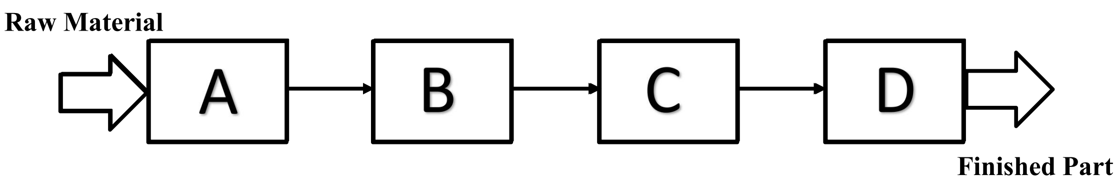

Figure 1.5: Tandem machining system

To help smooth out this hinderance or variation, the facility of temporary storage between operating units, referred to as a buffer, as shown in Fig. (1.6) is the necessary arrangement. In general, the buffer allows operating units to continue working while another unit undergoes schedule maintenance or repair.

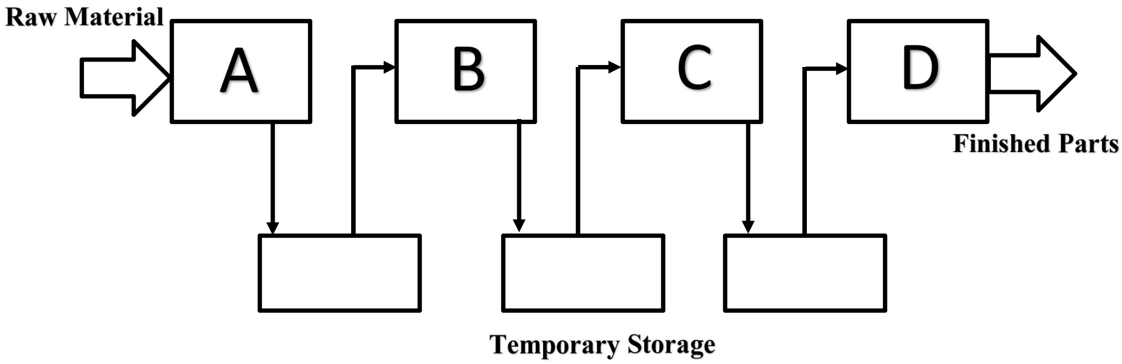

Figure 1.6: Machining system with buffer

The state-of-the-art machining system has the provision of spare units and tools that permit automatic changing and replacement, which makes them capable of performing a similar operation. The provisioning of spare units also maintains the continuous functioning of the system even though the operating unit faces random failure/repair. It is worth-nothing that the provision of the spare units increases the availability and reliability of the system up to some extent and also increases the incurred expected total cost.

Fig. (1.7) Illustrates the machining system wherein the failures and repairs are decoupled in the sense that if a unit fails, it does not hinder the production of the other. The need for a repair facility can not be avoided in handling and operating the machining system. The repairman not only repairs the failed units but also helps in replacing the spare units, routing the sequence, etc. The concept of the additional repairman is new practice of well-defined system to maintain the backlog

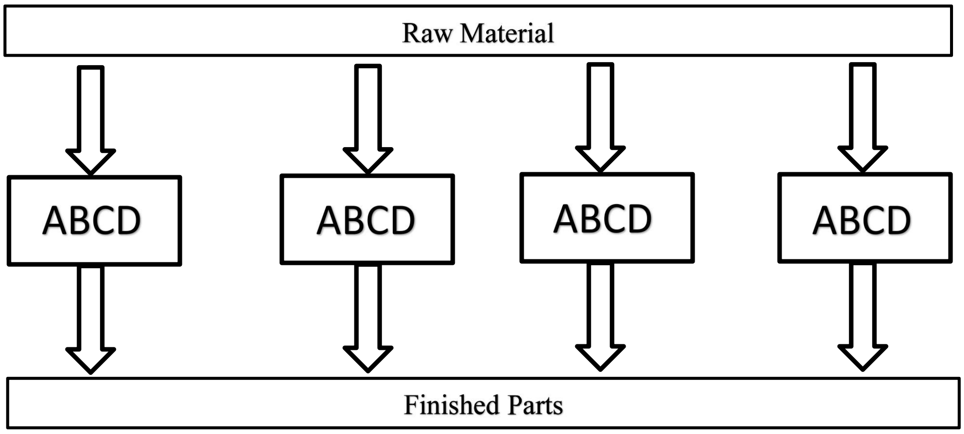

Figure 1.7: Machine system with spares

The main desirable characteristics of the machining system are flexibility, the rapid response of customer requests, qualities, and varieties of products. Fig. (1.8) shows the essential features of the manufacturing system.

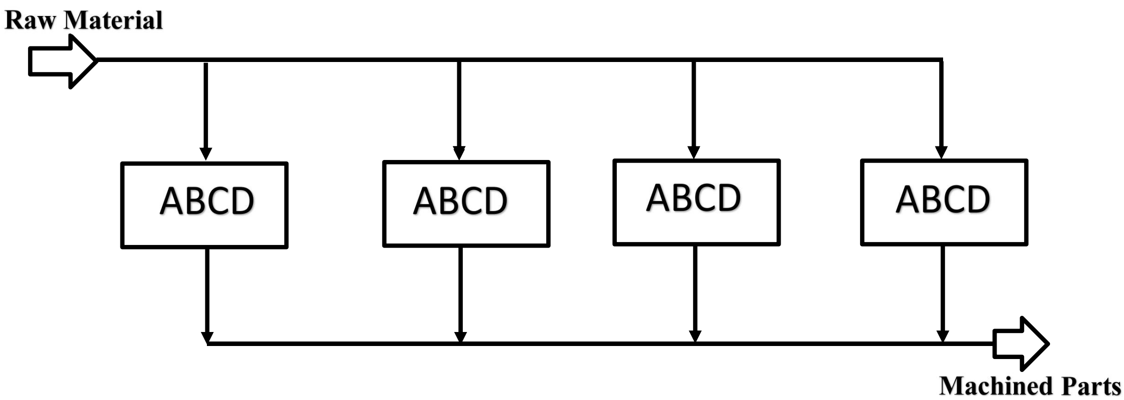

Figure 1.8: A flexible manufacturing system

1.4 Machining System in Modernization

Modernization is a socio-economic progressive transition from traditional to advanced technological machining systems. With the advent of high-tech automation, we are adapting the modernization paradigm for prompt and reliable services, quality products, and social progress.

Industry 4.0

It is the subclass of the fourth industrial revolution that apprehensions industry problems and developments. The fourth industrial revolution comprehends areas that are generally not classified as an industry, such as smart cities for case in point. Industry

4.0 alludes to the notion of organizations/factories wherein operating units are augmented with wireless connectivity and sensors, coupled to a system that can monitor the entire production procedure and make peculiar decisions.

Internet of Things (IoT)

It is an arrangement of interrelated computing devices, mechanical and computerized units, objects, or people that are characterized with unique identifiers (UIDs) and the ability to transfer information over a network system without individual-to-individual or individual-to-device physical interaction.

Robotics

Robotics deals with the state-of-the-art design, development, function, and use of electro-mechanical devices having artificial intelligence, as well as computer systems for their control, sensory feedback, and information processing. These technologies are used to develop machines that can supernumerary for humans and replicate human actions.

Flexible manufacturing system

A flexible manufacturing system (FMS) is another worldview of a manufacturing system wherein there is some amount of flexibility that permits the system to respond in case of vagaries, which may be predicted or unpredicted. This flexibility is categorized as routing and machine flexibility. The routing flexibility includes the system's efficiency to be changed to produce new product types, and the capacity to vary the sequence of operations executed on an item. Whereas, machine flexibility includes the ability to use multiple operating units to perform a similar process on a product, and also the system's capabilities to ingest with large-scale changes, such as in volume, capacity, or capability.

Additive manufacturing

Additive Manufacturing alludes to a procedure by which digital 3D design data is used to develop the item in layers by depositing material. It describes a specialized production technique that is distinct from conventional methods of material removal from a solid block. A variety of plastics, metals, and composite materials may be used.

Sustainable manufacturing

It is the conception of produced products through economically-sound processes that minimize negative environmental impacts while conserving vitality and natural resources.

Renewable and clean production

Renewable production generates items from sustainable sources with little or no pollution or global warming emissions. The clean production industry includes numerous economic activities and is expected to continue to rise rapidly in the future. There are vast economic opportunities for the organizations and countries that invent, manufacture, and export clean production technologies. Clean and renewable production sources provide a cleaner, healthier environment. Clean production sources are a better deal if developed thoughtfully.

Cloud manufacturing

Cloud manufacturing (CMfg) is a novel manufacturing paradigm industrialized from current standing progressive manufacturing models, e.g., ASP, AM, NM, MGrid. It also enterprises information technologies under the sustenance of cloud computing, Internet of Things (IoT), virtualization and service-oriented technologies, and cutting-edge computing technologies. It renovates manufacturing resources and manufacturing competencies into manufacturing services, which can be accomplished and worked in an intelligent and integrated way to empower the full sharing and to circulate of manufacturing resources and manufacturing competences. Cloud manufacturing delivers cheap, safe, reliable, high quality, and on-demand manufacturing services for the whole lifecycle of production.

Cloud data management

Cloud data management is an approach to accomplish data crosswise over cloud platforms, either with or instead of in-house storage. The cloud is valuable as a gathered information level fiasco recuperation, reinforcement, and long-term archiving.

1.5 Machine Repair Problem

Machine repairing is a typical example of a finite source queueing model, where the operating units are calling the population of prospective customers, an arrival corresponds to a unit breakdown, and the repair crews are the servers. We consider a machine repair problem (finite population) consists of identical operating units and repair crews. The duration of repair timing is identically exponential random variate with mean $1/\mu$, and the time that an operating unit remains working follows an exponential distribution with failure rate λ. Let P_n be the steady-state probability that there are n failed units in the system. The state transition diagram for the continuous-time Markov chain (CTMC) involved in machine repair model is depicted in Fig. (1.9)

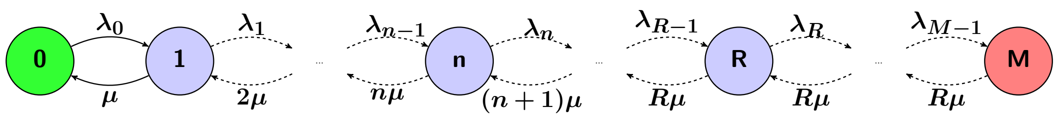

Figure 1.9: State transition diagram of machining system

The state-dependent failure and repair rates are denoted by λ_n and μ_n respectively, are given by

$$\lambda_n = \begin{cases} (M-n)\lambda, & \text{for } 0 \leq n < M \\ 0, & otherwise \end{cases} \tag{1.1}$$

and

$$\mu_n = \begin{cases} n\mu, & \text{for } 1 \leq n < R \\ R\mu, & \text{for } R \leq n \leq M \end{cases} \tag{1.2}$$

The steady-state probability P_n is obtained for this model by solving the Chapman-Kolmogorov equations which governs the model using the transition failure and repair rates in Eq^n's. (1.1) & (1.2). Therefore, the queue-size distribution can be obtained by using product-type solution as follows

$$P_n = \begin{cases} \binom{M}{n} \rho^n P_0, & \text{for } 1 \leq n < R \\ \\ \binom{M}{n} \frac{n!}{R^{n-R}R!} \rho^n P_0, & \text{for } R \leq n \leq M \end{cases} \tag{1.3}$$

where, $\rho = \frac{\lambda}{\mu}$, and P_0 can be determined using normalizing condition of probabilities as follows

$$P_0 = \left(1 + \sum_{n=1}^{R} \binom{M}{n} \left(\frac{\lambda}{\mu}\right)^n + \sum_{n=R+1}^{M} \frac{n!}{R^{n-R}R!} \left(\frac{\lambda}{\mu}\right)^n \right)^{-1} \tag{1.4}$$

To make the system functioning smoothly, the repair of the failed units has to be done efficiently and timely. Thus, machine repair is a significant feature of all the machining systems. Sometimes it so happens that there is overcrowding, and it takes a long to the last one to get checked up. The delay in repair may lead to long queues or waiting lines and blocking in the services. However, it is quite impossible to predict correctly when and how failed units will arrive and how much time will failed units take to get repaired. These decisions are often unpredictable in nature. To overcome these problems, system designer has to increase the service facility, but at the same time, this will also lead to an increase in the service cost. On the other hand, providing less service facility would decrease the service cost but cause a long waiting line. The degraded repair will cause an excessive waiting and result in high downtime of failed units. In some sense excessive waiting is costly; it may be a social cost, loss of customer, loss of production, etc. Therefore, the main objective is to attain stability between the cost of service and the cost of waiting.

1.6 Redundant Machining System

Redundancy is the facility of more than one unit to perform a required function in the system. The redundancy of units has many applications to increase system reliability with some additional cost. Redundancy may classify as active or passive, depending on functioning nature. The active redundant unit presents in parallel to the operating unit and is always working, whereas passive redundant unit becomes the operating unit when any unit in operation is not available due to random failure or some other reasons. The passive redundancy is commonly known as a spare unit or standby unit. The machining system having the facility of redundancy for the operating unit is called a redundant machining system. Depending on failure characteristics, passive redundancy is categorized into three types: hot, warm, and cold

- **Hot Redundancy** In the hot redundancy, the spare unit is subjected to failure as the same as an operating unit in an inactive state.
- **Warm Redundancy** In the warm redundancy, the spare unit is subjected to failure with some lesser rate as compared to the operating unit in the inactive state.
- **Cold Redundancy** In the cold redundancy, the spare unit is not subjected to failure until it is in an inactive state.

The provision of active and passive redundancy improves the efficiency of the machining systems at some additional cost. The classification of the redundancy is depicted in Fig. (1.10).

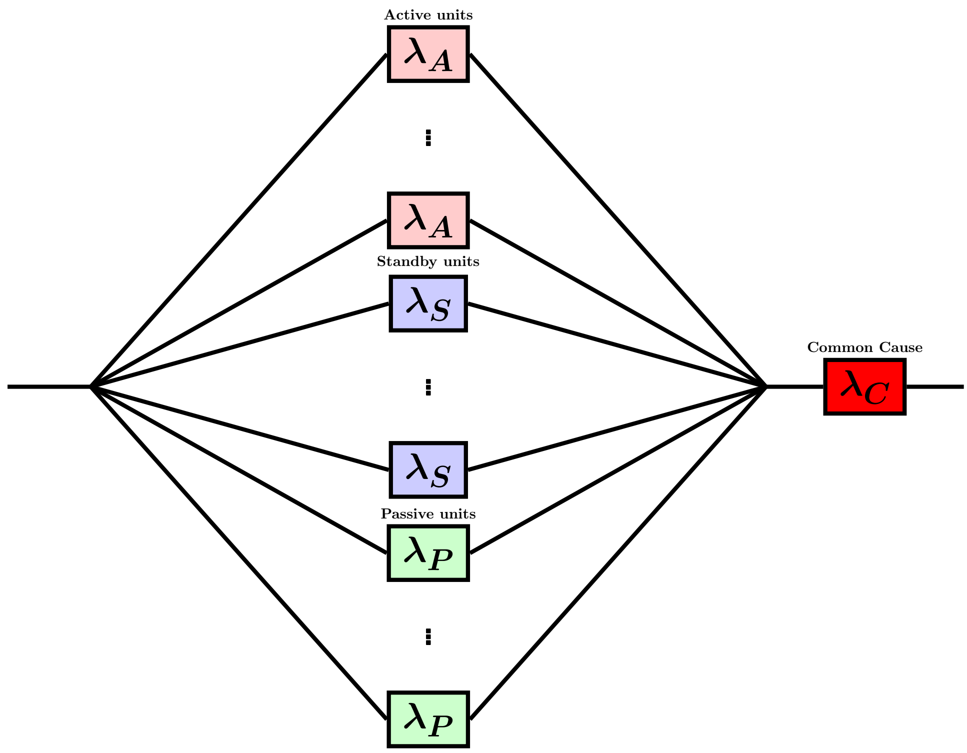

Figure 1.10: Classification of redundancy

For the redundant machining system having M operating units and S_1 cold spare units, the birth rate is

$$\lambda_n = \begin{cases} M\lambda\,; & 0 \le n < S_1 \\ (M+S_1-n)\,\lambda\,; & S_1 \le n < M+S \\ 0\,; & \text{elsewhere} \end{cases} \tag{1.5}$$

Similarly, if there are S_2 warm spare units in the system, and $\nu; 0 < \nu < \lambda$ be failure rate of spare unit in inactive state, the birth rate is figured as

$$\lambda_n = \begin{cases} M\lambda + S_2\nu\,; & 0 \le n < S_2 \\ (M+S_2-n)\,\lambda\,; & S_2 \le n < M+S_2 \\ 0\,; & \text{elsewhere} \end{cases} \tag{1.6}$$

For S_3 hot spare units with a failure rate as same as the failure of the operating unit, the birth rate is characterized as

$$\lambda_n = \begin{cases} (M+S_3)\,\lambda\,; & 0 \le n < M+S_3 \\ 0\,; & \text{elsewhere} \end{cases} \tag{1.7}$$

The layout design for the machining system of M operating units along with provisioning of the mix of S_1 cold spare, S_2 warm spare and S_3 warm spare units under the care of repair facility having R repairmen is shown in the Fig. (1.11).

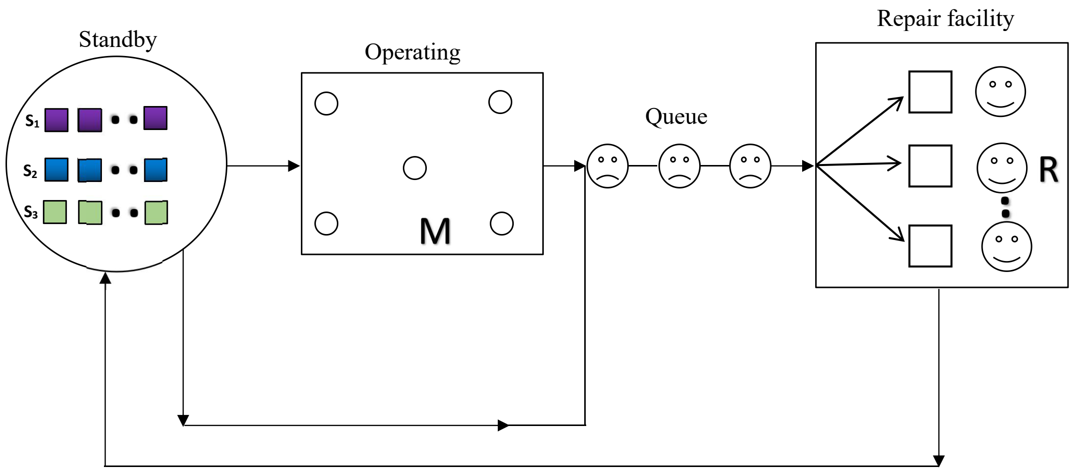

Figure 1.11: Machine repair problem with spares

Hence, the birth rate is designated as

$$\lambda_n = \begin{cases} (M+S_3)\lambda + S_2 \nu; & 0 \le n < S_1 \\ (M+S_3)\lambda + (S_1+S_2-n)\nu; & S_1 \le n < S_1+S_2 \\ (M+S_1+S_2+S_3-n)\lambda; & S_1+S_2 \le n < M+S_1+S_2+S_3 \\ 0; & \text{elsewhere} \end{cases} \tag{1.8}$$

For the provision of R repairmen in service facility, the death rate for modeling is formulated as follows

$$\mu_n = \begin{cases} n\mu; & 1 \le n < R \\ R\mu; & R \le n \le M+S_1+S_2+S_3 \end{cases} \tag{1.9}$$

1.7 Basic Elements of Queueing System

A queueing system or waiting line model contains input streams of either single prospective customer/failed unit or batch of customers which wait for getting the required service if the server is busy; otherwise, receive intended service if the server is idle. The service facility comprises one or more servers, which are arranged in different specifications or designs and select the waiting customer for rendering the necessary service following some pre-specified service discipline or policies. The queueing system also includes the customer whose service is in progress.

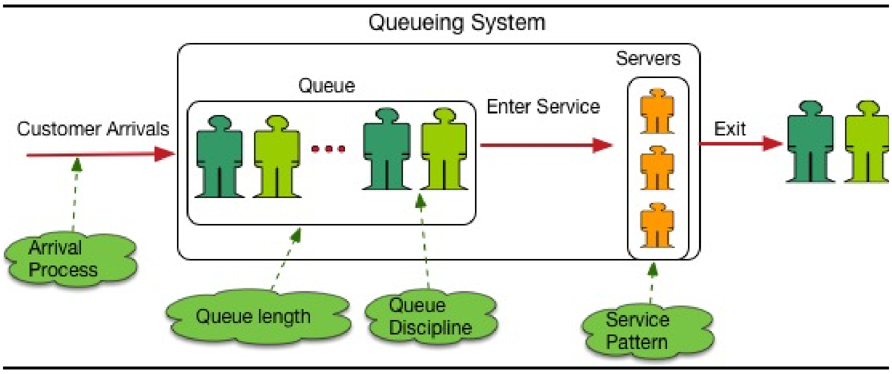

Figure 1.12: Queueing system

Fig. (1.12) depicts the flow of the customers in the queueing system. Different types of queueing systems are mainly categorized by the following primary factors.

- Input characteristic
- Service mechanism
- Queue discipline and customer behavior
- Service channels

The brief description of basic elements are underlined as follows

Input characteristic: It specifies the probability law governing the arrival statistics of the customers at the servers at time $t_1, t_2, ..., t_m$ where $t_k < t_{k+1}$ and $k = 1, 2, .., m$. Let $\tau_m = t_{m+1} - t_m$ denotes the inter-arrival time between the $(m+1)^{th}$ and m^{th} customers. The input characteristic is specified by the probability distribution of the sequence of arrival instants t_m and the sequence of inter-arrival times τ_m. The notation $N(t)$ indicates the number of customers in the system at an arbitrary time t.

Service mechanism: The sequence of service times s_m specifies the service mechanism where s_n represents the time required to serve the n^{th} customer. The successive duration s_m are statistically independent of one another and also of the sequence of time inter-arrivals τ_m. Fig. (1.13) depicts the arrival and departure time of the customer in the queue.

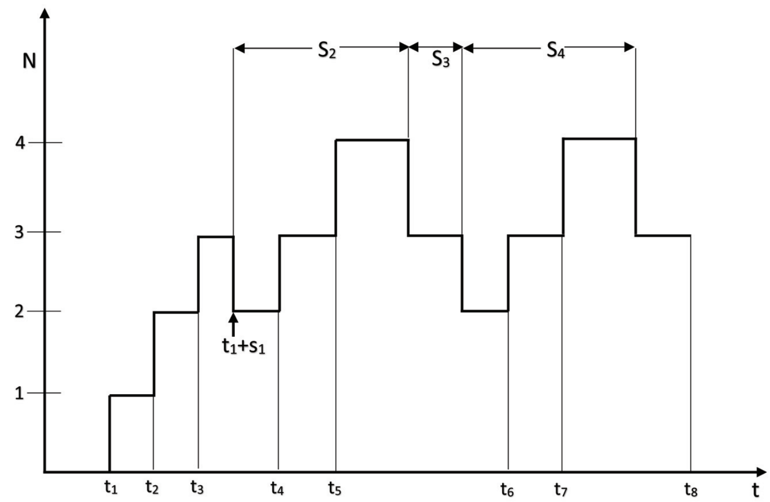

Figure 1.13: Arrivals and departures of customers at the queue

Queue discipline and customer behavior: The rule by which the waiting customers are selected for service is known as queue discipline. The usual discipline is *first come, first serve* (FCFS). However, other forms of service discipline may be *last in, first out* (LIFO), *service in random order* (SIRO), *priority*, *processor sharing* (PS), etc. Some governing specifications of design for service facility and customer pattern can also be figured like controlled/uncontrolled arrivals, impatience behavior of the customers, threshold policies, etc. At the epoch of a busy service facility, the arriving customer may choose to wait for service, or may immediately decide not to join the queue, i.e., *balking*. If a customer joins the queue, becomes impatient after waiting for some time seeing the long queue, and he decides to leave the queue, this behavior of the customer is known as *reneging*. In *jockeying*, customer switches from one queue to another queue in the hope of early service or due to some other reasons.

Service channels: The queueing system may have one or more than one service channel to provide service at the same or different rates to the arriving customers, i.e., homogeneous or heterogeneous. Also, the system may have either a limited (finite) or unlimited (infinite) capacity for holding waiting customers. Also, the population size of the customers may be finite or infinite.

1.8 Some Important Process

In this section, we extant a brief description of some vital discrete-time and continuous-time process which are valuable for studies in queueing theory in detail

1.8.1 Stochastic Process

Let $N(t)$ be a random variable signifying the state of a system at time t for $t \geq 0$. Stochastic process is defined as the collection of such random variable, i.e., $\{N(t), t \in T\}$ where T indicate the domain of time t.

Discrete-time Stochastic Process

The stochastic process with T as a discrete set, i.e., $T = \{0, 1, 2, \ldots\}$.

Continuous-time Stochastic Process

The stochastic process with T as a continuous set, i.e., $T = \{t | t \geq 0\}$.

State Space

The collection of all possible values of $N(t); t \in T$ is called state space of the stochastic process.

1.8.2 Counting Process

A continuous-time stochastic process $\{N(t), t \geq 0\}$ is a counting process if $N(t)$ satisfies

(i) $N(t) \geq 0$

(ii) $N(t)$ is the integer valued

(iii) $N(t)$ is a nondecreasing function of t, and

(iv) for $s < t$, $N(t) - N(s)$ represents the number of events that occur in the interval $(s, t]$.

1.8.3 Markov Process

A stochastic process is termed as a Markov process if it satisfies Markovian property, i.e., stochastic behavior of the process in which the future is only dependent on the present state but independent of the past progress. Mathematically, it is expressed as

$$P\{X(t_n + s) \leq x \mid X(t_1) = x_1, X(t_2) = x_2, \ldots, X(t_n) = x_n\}$$
$$= P\{X(t_n + s) \leq x \mid X(t_n) = x_n\}, s > 0; \quad 0 \leq t_1 < t_2 < \ldots < t_n$$

1.8.4 Markov Chain

If the state space S is discrete, i.e., finite or countable infinite set and whose index set is $T = (0, 1, 2, ...)$, the Markov process is called the Markov chain.

Continuous-Time Markov Chain

Consider a continuous-time stochastic process $\{N(t), t \geq 0\}$ with discrete states $0, 1, 2, ...$ it is known as a continuous-time Markov Chain if the following condition is satisfied

$$P(N(t+h) = j \mid N(0) = i, N(x) = i_x, 0 \leq x < t) = P(N(t+h) = j \mid N(t) = i) \forall h \geq 0$$

Discrete-Time Markov Chain

A discrete-time stochastic process $\{N(t), t = 0, 1, 2, \ldots\}$ with discrete states $0, 1, \ldots$ is called Markov chain if the equation

$$\begin{aligned} P(N(t+1) = j|N(0) = i_0, N(1) = i_1, N(2) = i_2, ..., N(t) = i) \\ = P(N(t+1) = j|N(t) = i) = P_{ij}(t) \end{aligned}$$

is satisfied for all possible states of $i_0, i_1, i_2, ...i_{t-1}, i, j$ and $t \geq 0$. $P_{ij}(t)$ is known as the transition probability for the process from the state i at time t to state j at $t+1$.

1.8.5 Poisson Process

A counting process $\{N(t), t \geq 0\}$ is called Poission process with the parameter $\lambda > 0$ if the following conditions are satisfied

(i) $N(0) = 0$

(ii) $\{N(t), t \geq 0\}$ has stationary and independent increments

(iii) $P(N(t + \Delta t) - N(t) = 1) = \lambda(t)\Delta t + o(\Delta t)$, and

(iv) $P(N(t + \Delta t - N(t)) > 1) = o(\Delta t)$

In a Poisson process, the number of event in any interval of the length Δt follows the Poisson distribution with parameter $\lambda \Delta t$, i.e., for all $t \geq 0$ and $\Delta t > 0$

$$P\{N(t+\Delta t) - N(t) = n\} = P(N(\Delta t) = n) = \frac{\lambda \Delta t}{n!} e^{-\lambda \Delta t}; \quad n = 0, 1, 2, ... \quad (1.10)$$

1.8.6 Renewal Process

Let $\{X_t\}$ be independent identically distributed (i.i.d) non-negative random variables, $X_t \sim F(t)$ an arbitrary distribution.Then, the counting process

$$N(t) = \max\{ n \mid S_n = X_1 + X_2 + X_3 + ... + X_n < t\}$$

is called renewal process. The mean number of events $m(t)$ on $(0,t)$ is called the renewal function

$$E[N(t)] = m(t)$$

Renewal process generalizes the Poisson process by allowing the inter-occerance time between two successive events to be independent and identically distributed (iid) random variable having an arbitrary distribution.

1.8.7 Birth and Death Process

A continuous-time Markov chain $\{N(t), t \geq 0\}$ with discrete states $0, 1, ...$ and homogeneous transition rate matrix $\mathbf{V}_{ij}$ is called a birth and death process if $v_{ij} = 0$ for all i and j such that $|i - j| > 1$.

1.8.8 Chapman-Kolmogorov Equation

Using the Markov property of the process, the Chapman-Kolmogorov equation gives multi-step transition probability from state i to state j over all possible k values and is expressed by

$$P_{ij}(t+s) = \sum_{k=0}^{\infty} P_{ik}(t) P_{kj}(s) \tag{1.11}$$

This equation describes that in order to move from state i to state j in time t, $X(t)$ moves to state k in time t and then from k to j in the remaining time s.

1.8.9 Stationary State

In a stationary state, the system is independent of the initial conditions. Let $P_n(t)$ be the probability that there are n events at time t in the system, then for this state, solution will be obtained by letting $t \to \infty$, i.e., $\lim_{t\to\infty} P_n(t) = P_n$. This state is also referred to as time-independent, intransient, homogeneous in time, steady-state, or limiting state also. It is useful for analyzing the stable or long-run machining system.

1.8.10 Non-Stationary State

The non-stationary state is equivalently termed as transient state, time-dependent, or non-limiting state. In this case, the operating characteristics of the queueing system are dependent on time.This state occurs at the beginning of the operation of the system. It is useful for analyzing the new-designing or developing state of the machining system.

1.9 Some Standard Probability Distribution

Let E be a random experiment and S a sample space associated with the experiment. A function X assigning to every element $s \in S$, a real number, $X(s)$, is called a *random variable*. There are two types of random variable discrete random variable & continuous random variable which are characterized by probability density function $f(x)$ and cumulative distribution function $F(x)$. In addition to unique moment generating function $m_X(t)$, every random variable X has mean $E(X)$ and variance $V(X)$. The random variable characterizes the event of random failure, repair time, etc. in the machining system. Some standard distributions are described as follows.

Degenerate Distribution

A discrete random variable X is degenerate variate with parameter $c\,(c \in (-\infty, \infty))$ if its density is

$$f(x) = \begin{cases} 1; & x = c \\ 0; & x \neq c \end{cases}$$

For degenerate random variate, we have

$$\text{E(X)} = c \quad \text{and} \quad \text{V(X)} = 0$$

Geometric Distribution

A discrete random variable X is said to follow a geometric distribution with parameter $p\,(0 < p < 1)$ if its density is given by

$$f(x) = \begin{cases} (1-p)^{x-1}p; & x = 1, 2, 3, \ldots \\ 0; & \text{elsewhere} \end{cases}$$

Then,

$$E(X) = \frac{1}{p} \quad \text{and} \quad V(X) = \frac{q}{p^2}. q = 1 - p$$

Binomial Distribution

A discrete random variable X has a binomial distribution with parameters n, $(n \in I^+)$ and $p\,(0 < p < 1)$ if its density is given by

$$f(x) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x}; & x = 0,1,2,3,\ldots,n \\ 0; & \text{otherwise} \end{cases}$$

Then, we have

$$E(X) = np \quad \text{and} \quad V(X) = npq,\, q = 1-p$$

Poisson Distribution

A discrete random variable X is said to have a Poisson distribution with parameter $k\,(k > 0)$ if density function f is given by

$$f(x) = \begin{cases} \frac{e^{-k} k^x}{x!}; & x = 0,1,2,3,\ldots \\ 0; & \text{otherwise} \end{cases}$$

For the Poisson variate,

$$E(X) = k \quad \text{and} \quad V(X) = k$$

Gamma Distribution

A continuous random variable X with density

$$f(x) = \begin{cases} \dfrac{\lambda^k}{\Gamma(k)} x^{k-1} e^{-\lambda x}; & x > 0 \\ 0; & x < 0 \end{cases}$$

is said to have gamma distribution with parameters $\lambda\,(\lambda > 0)$ and $k\,(k > 0)$. So, we have

$$\text{E(X)} = \frac{k}{\lambda} \quad \text{and} \quad \text{V(X)} = \frac{k}{\lambda^2}$$

Exponential Distribution

A continuous random variable X with density,

$$f(x) = \begin{cases} \lambda e^{-\lambda x}; & x > 0 \\ 0; & x < 0 \end{cases}$$

is said to have exponential distribution with parameter $\lambda\,(\lambda > 0)$. Then,

$$\mathrm{E(X)} = \frac{1}{\lambda} \quad \text{and} \quad \mathrm{V(X)} = \frac{1}{\lambda^2}$$

Uniform Distribution

A continuous random variable X assuming all values in the interval $[a,b]$,where both a and b are finite, with density

$$f(x) = \begin{cases} \dfrac{1}{b-a}; & a < x < b \\ 0; & elsewhere \end{cases}$$

A uniform variate X over the interval $[a,b]$ has

$$\mathrm{E(X)} = \frac{a+b}{2} \quad \text{and} \quad \mathrm{V(X)} = \frac{(b-a)^2}{12}$$

Erlang Distribution

A continuous random variable X with density

$$f(x) = \frac{\lambda^k x^{k-1} e^{-\lambda x}}{\Gamma(k)}; \quad x > 0$$

where $k\,(k = 1,2,3,...)$ is the shape parameter and $\lambda\,(\lambda > 0)$ is the rate parameter, is said to have Erlang distribution. Hence,

$$\mathrm{E(X)} = \frac{k}{\lambda} \quad \text{and} \quad \mathrm{V(X)} = \frac{k}{\lambda^2}$$

Hyperexponential Distribution

A continuous random variable X is said to follow k-stage hyperexponential distribution if its density is given by

$$f(x) = \sum_{i=1}^{k} a_i \lambda_i e^{-\lambda_i x}; \quad x \geq 0; 0 \leq a_i \leq 1, \text{ such that} \sum_{i=1}^{k} a_i = 1$$

where $k = 1,2,3,...$ and $\lambda_i > 0 \,\forall\, i$. So, mean and variance of X are given by

$$E(X) = \sum_{i=1}^{k} \frac{a_i}{\lambda_i} \quad \text{and} \quad V(X) = 2\sum_{i=1}^{k} \frac{a_i}{\lambda_i^2} - \left(\sum_{i=1}^{k} \frac{a_i}{\lambda_i}\right)^2$$

respectively. For this distribution, standard deviation exceeds the mean in general except for degenerate case when all λ_i's are equal.

Hypoexponential or Generalized Erlang Distribution

A continuous random variable X is said to have hypoexponential or generalized Erlang distribution it its probability density function with parameters $\lambda_1, \lambda_2, ..., \lambda_k (\lambda_i > 0 \forall i)$ is given by

$$f(x) = \sum_{i=1}^{k} \lambda_i e^{-\lambda_i x} \left(\prod_{j=1, j \neq i}^{k} \frac{\lambda_j}{\lambda_j - \lambda_i} \right); \quad x \geq 0$$

The coefficient of variation is less than 1, i.e., standard deviation is less than mean. The mean and variance are respectively as follows

$$E(X) = \sum_{i=1}^{k} \frac{1}{\lambda_i} \quad \text{and} \quad V(X) = \sum_{i=1}^{k} \frac{1}{\lambda_i^2}$$

Normal Distribution

A continuous random variable X with density

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}; \quad -\infty < x < \infty$$

is said to have normal distribution with parameters $\mu\,(-\infty < \mu < \infty)$ and $\sigma\,(\sigma > 0)$. The mean and variance of X are respectively given by

$$E(X) = \mu \quad \text{and} \quad V(X) = \sigma^2$$

If $\mu = 0$ and $\sigma^2 = 1$, X is known as standard normal variate.

1.10 Solution Techniques

The queueing models involved in the modeling of the machining system can be solved by using several analytical and numerical techniques to obtain the stationary or non-stationary queue-size distribution. The results are also useful for sensitivity and optimal analysis of the machining system. In this section, we outline some techniques that are used to solve the queueing models investigated in the present thesis.

Let $\{X(t), t \geq 0\}$ be a finite state continuous-time Markov chain with state space t. Let $\mathbf{Q} = (r_{mn})$ be a generator matrix where $r_{mn}, m \neq n$ represents the transition rate to state n from state m and diagonal element $r_{mn} = -r_{mn} = -\sum_{m=n} r_{mn}$. Now assume that k be the number of non-zero entries in $\mathbf{Q}$ and the unconditional probability of the continuous-time Markov chain at time t in state m is $P_m(t)$. Then, transition state is represented by the row vector $\mathbf{P}(t)$. Now Chapman-Kolmogorov difference equation governing the behavior of continuous-time Markov chain is

$$\frac{d\mathbf{P}(t)}{dt} = \mathbf{Q}(t)\mathbf{P}(t); \quad \mathbf{P}(0) = \mathbf{P}_0 \tag{1.12}$$

where $\mathbf{P}_0$ denote the initial probability vector of the continuous-time Markov chain, $\mathbf{P}(t)$ is transient-state probability vector.

The solution of Eq^n. (1.12) provides non-stationary queue-size distribution of the queue. For the stationary solution, the Eq^n. (1.12) is deduced to system of linear equation on limiting $t \to \infty$ as

$$\mathbf{QP} = 0 \tag{1.13}$$

The stationary solution also satisfies the normalizing condition of the probability

$$\mathbf{e}^T\mathbf{P} = 1 \tag{1.14}$$

1.10.1 Transient Solution Method

Analytically, Eq^n. (1.12) gives

$$\mathbf{P}(t) = \mathbf{P}(0)e^{\int \mathbf{Q}(t)\,dt} = \mathbf{P}(0)e^{\mathbf{P}t} \tag{1.15}$$

where $e^{\mathbf{P}t}$, exponential matrix is defined by Taylor series

$$e^{\mathbf{P}t} = \sum_{i=0}^{\infty} \frac{(\mathbf{P}t)^i}{i!} \tag{1.16}$$

This method is significantly advantageous overimplicit ODE method.

1.10.2 Runge-Kutta Method

The governing system of differential equations for machining system with initial condition Eq^n. (1.12) can be re-written as

$$\frac{d\mathbf{P}(t)}{dt} = f(t,\mathbf{P}); \quad \mathbf{P}(t_0) = \mathbf{P}_0 \tag{1.17}$$

When the function f and the data t_0, P_0 are known, according to Runge-Kutta procedure, we have

$$\mathbf{P}_{n+1} = \mathbf{P}_n + \frac{1}{6}(\mathbf{k}_1 + 2\mathbf{k}_2 + 2\mathbf{k}_3 + \mathbf{k}_4),$$
$$t_{n+1} = t_n + h$$

for n = 0, 1, 2, 3, ..., using

$$\mathbf{k}_1 = h\,f(t_n, \mathbf{P}_n),$$
$$\mathbf{k}_2 = h\,f\left(t_n + \frac{h}{2}, \mathbf{P}_n + \frac{\mathbf{k}_1}{2}\right),$$
$$\mathbf{k}_3 = h\,f\left(t_n + \frac{h}{2}, \mathbf{P}_n + \frac{\mathbf{k}_2}{2}\right),$$
$$\mathbf{k}_4 = h\,f(t_n + h, \mathbf{P}_n + \mathbf{k}_3).$$

Here $\mathbf{P}_{n+1}$ is the fourth order approximation of $\mathbf{P}(t_{n+1})$, and the next value $(\mathbf{P}_{n+1})$ is determined from the current value $(\mathbf{P}_n)$ plus the weighted-average of four increments, where each increment is the product of the size of the interval, h, and a projected slope quantified by function f on the right-hand side of the differential equation.

1.10.3 Supplementary Variable Technique

Non-Markovian queues can be solved by practicing of the embedded Markov chain or by employing the supplementary variable method. The supplementary variable technique facilitates transforming a continuous-time non-Markovian process to the Markovian process by adding additional variable(s) in state-space called the supplementary variable. The supplementary variable is continuous and makes the resulting process a continuous state space and continuous-time Markov process. Resultant Markovian based modeling is computationally easy to solve. For the queueing model with either general interarrival time (GI) or general service time (G) wherein a pair of variable, $N(t)$ defining the state of the system at time t and $X(t)$ representing the expanded time of the customers, are used. The expanded time may be elapsed or remaining time. Therefore, non-Markovian process $\{N(t)\}$ changed to the Markovian process $\{N(t), X(t)\}$, which can be solved on employing the Lagrangian method.

1.10.4 Matrix Inversion Method

The primary method for solving the general system of a linear equation Eq^n. (1.13) is called the matrix inversion method. Matrix inversion method mainly consists of

two parts: Forward elimination and Backward elimination
Forward Elimination: Step-by-step reduction of the system size yielding either a degenerate equation with no solution (which indicates the system has no solution) or an equivalent more straightforward system of equations in triangular or echelon form.
Backward Elimination: Step-by-step back-substitution to calculate the solution of the simpler system.

1.10.5 Matrix Analytic Method

The matrix analytic method is a procedure to determine the stationary probability distribution of a Markov chain which has a reiterating structure after some point and a unbounded state space in no more than one dimension. Such models are often designated as $M/G/1$ type Markov chains because they can figure transitions in an $M/G/1$ queueing model. The method is a more intricate form of the matrix geometric method and is the classical solution technique for $M/G/1$ chains.

A stochastic matrix of an $M/G/1$ type is one of the form

$$\mathbf{Q} = \begin{pmatrix} \mathbf{B}_0 & \mathbf{B}_1 & \mathbf{B}_2 & \mathbf{B}_3 & \cdots \\ \mathbf{A}_0 & \mathbf{A}_1 & \mathbf{A}_2 & \mathbf{A}_3 & \cdots \\ \mathbf{0} & \mathbf{A}_0 & \mathbf{A}_1 & \mathbf{A}_2 & \cdots \\ \mathbf{0} & \mathbf{0} & \mathbf{A}_0 & \mathbf{A}_1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

where $\mathbf{B}_i$ and $\mathbf{A}_i$ are $k \times k$ matrices, if $\mathbf{Q}$ is irreducible and positive recurrent then the stationary queue-size distribution is specified by the solution to the equations

$$\mathbf{QP} = \mathbf{P} \quad \text{and} \quad \mathbf{e}^T\mathbf{P} = 1 \tag{1.18}$$

where $\mathbf{e}$ epitomizes a vector of suitable dimension with all values equal to 1. Matching the dimensional structure of $\mathbf{Q}$, $\mathbf{P}$ is partitioned to $\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{P}_3$, ...To calculate these probabilities, the column stochastic matrix $\mathbf{G}$ is computed such that

$$\mathbf{G} = \sum_{i=0}^{\infty} \mathbf{G}^i \mathbf{A}_i$$

G is called the auxiliary matrix. Matrices are defined

$$\bar{\mathbf{A}}_{i+1} = \sum_{j=i+1}^{\infty} \mathbf{G}^{j-i-1}\mathbf{A}_j \tag{1.19}$$

$$\bar{\mathbf{B}}_i = \sum_{j=i}^{\infty} \mathbf{G}^{j-i}\mathbf{B}_j \tag{1.20}$$

then $\mathbf{P}_0$ is found by solving

$$\bar{\mathbf{B}}_0\mathbf{P}_0 = \mathbf{P}_0$$

$$\left(\mathbf{e}^T + \mathbf{e}^T\left(\mathbf{I} - \sum_{i=1}^{\infty}\bar{\mathbf{A}}_i\right)^{-1}\sum_{i=1}^{\infty}\bar{\mathbf{B}}_i\right)\mathbf{P}_0 = 1$$

and hence,

$$\mathbf{P}_i = (\mathbf{I} - \bar{\mathbf{A}}_1)^{-1}\left[\bar{\mathbf{B}}_{i+1}\mathbf{P}_0 + \sum_{j=1}^{i-1}\bar{\mathbf{A}}_{i+1-j}\mathbf{P}_j\right], i \geq 1$$

1.10.6 Successive Over Relaxation

In numerical linear algebra, the technique of successive over relaxation (SOR) is a modified scheme of the Gauss-Seidel procedure for determining the solution of a linear system of equations, ensuing in faster convergence. A analogous method can be used for any slowly converging iterative procedure.

Assumed a system of n linear equations with n unknown **P** derived from $Eq^n.'s$ (1.13) and (1.13) omitting the redundant equation such that

$$\mathbf{QP} = \mathbf{b} \tag{1.21}$$

where **Q** is coefficient matrix, **P** is column vector of unknowns, and **b** is column vector of right hand sides of the system of equation.

$$\mathbf{Q} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}, \quad \mathbf{P} = \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_n \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

Then, **Q** can be disintegrated into a diagonal matrix D, and strictly lower and upper triangular matrices L and U such that

$$\mathbf{Q} = \mathbf{D} + \mathbf{L} + \mathbf{U}$$

The system of linear equations $\mathbf{QP} = \mathbf{b}$ can be restructured as

$$(\mathbf{D} + \omega\mathbf{L})\mathbf{P} = \omega\mathbf{b} - [\omega\mathbf{U} + (\omega - 1)\mathbf{D}]\mathbf{P}$$

for a constant $\omega > 1$, referred as the relaxation factor. The scheme of successive over relaxation is an iterative procedure that solves the left hand side of this expression for $\mathbf{P}$, using previous value for $\mathbf{P}$ on the right hand side. Analytically, this can be transcribed as

$$\mathbf{P}^{(k+1)} = (\mathbf{D} + \omega\mathbf{L})^{-1}\left(\omega\mathbf{b} - [\omega\mathbf{U} + (\omega - 1)\mathbf{D}]\mathbf{P}^{(k)}\right) = \mathbf{L}_w\mathbf{P}^{(k)} + \mathbf{c}$$

where $\mathbf{P}^{(k)}$ is the k^{th} approximation or iteration of $\mathbf{P}$ and $\mathbf{P}^{(k+1)}$ is the next or $k+1$ iteration of $\mathbf{P}$. However, by captivating advantage of the triangular form of $(\mathbf{D} + \omega\mathbf{L})$, the elements of $\mathbf{P}^{(k+1)}$ can be computed sequentially using forward substitution

$$\mathbf{P}_i^{(k+1)} = (1 - \omega)\mathbf{P}_i^{(k)} + \frac{\omega}{\mathbf{a}_{ii}}\left(\mathbf{b}_i - \sum_{j<i}\mathbf{a}_{ij}\mathbf{P}_j^{(k+1)} - \sum_{j>i}\mathbf{a}_{ij}\mathbf{P}_j^{(k)}\right), \quad i = 1, 2, \ldots, n$$

1.10.7 Eigenvalue and Eigenvector

Let $\mathbf{Q}$ be any square matrix. A scalar λ is referred as an eigenvalue of $\mathbf{Q}$ if there exists a non-zero (column) vector $\mathbf{P}$ such that

$$\mathbf{QP} = \lambda\mathbf{P} \tag{1.22}$$

Any vector satisfying the Eq^n.(1.22) is called an eigenvector of $\mathbf{Q}$ corresponding to the eigenvalue λ.

1.10.8 Laplace Transform

Assume $f(t)$ be a real-valued function of real variable t, defined for $t > 0$. Let s be a variable that assume to be real, and consider the function $\bar{F}(s)$ defined by

$$\bar{F}(s) = \int_0^\infty e^{-st} f(t)dt \tag{1.23}$$

for all values of s for which this integral exists.The function $\bar{F}(s) = L\{f(t)\}$ by the integral is called the Laplace transform of the function $f(t)$. With the help of Laplace transform, the system of differential equations with initial conditions is transform into system of linear equations which is computationally easy to solve.

1.11 Performance Characteristics of the Machining System

Performance characteristics are the epitome of the modeling for the analysis of the system understudies. It may be classified as queueing characteristics and reliability characteristics since both are worthy in the framework of analysis of the real-time systems. Performance characteristics are derived in terms of parameters, rates, thresholds, state of the machining system, etc.

Consider the machining system comprises of M operating units and S spare units having state-dependent breakdown rates and repair rates λ_n and μ_n, respectively. The state transition diagram of the machine repair problem is depicted in Fig. (1.14).

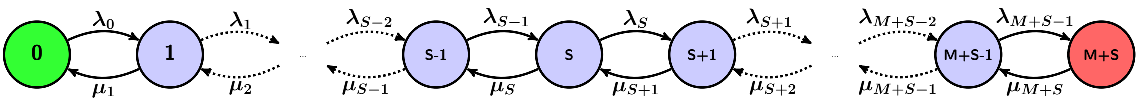

Figure 1.14: State transition diagram for MRP with standby

1.11.1 Queueing Characteristics

The state of the machining system, in general, characterized in terms of the expected number of failed or working units in the system, which define various queueing characteristics as follows

Expected number of failed units

Let $N(t)$ be a number of failed unit in the system at time t. The expected number of the failed unit in the system at time t is defined as

$$E_N(t) = \sum_{n=0}^{M+S} n P_n(t) \tag{1.24}$$

Throughput of the system

The throughput of the system demarcated as the mean number of repaired units at time t and is expected as

$$\tau(t) = \sum_{n=1}^{M+S} \mu_n P_n(t) \tag{1.25}$$

Expected number of operating units

Expected number of the operating units in the system at time t is expressed as

$$E_O(t) = \sum_{n=0}^{M} (M-n)P_n(t) \tag{1.26}$$

Expected number of spare units

Expected number of spare unit in the system at time t is defined as

$$E_S(t) = \sum_{n=0}^{S} (S-n)P_n(t) \tag{1.27}$$

Machine availability

Machine availability at time t is defined as the proportion of the expected number of working units in the system at time t out of the total number of units available in the system initially

$$MA(T) = 1 - \frac{E_N(t)}{M+S} \tag{1.28}$$

Expected delay time

The expected delay time is defined as the ratio of the expected number of failed units in the system and the throughput of the system at time t.

$$E_D(t) = \frac{E_N(t)}{\tau(t)} \tag{1.29}$$

Effective failure rate

Effective failure rate of the unit is defined as

$$\lambda_{eff} = \sum_{n=0}^{M+S-1} \lambda_n P_n(t) \tag{1.30}$$

Expected waiting time

Expected waiting time of the failed unit for being repaired is defined as

$$E_W(t) = \frac{E_N(t)}{\lambda_{eff}} \tag{1.31}$$

Probability that server is Idle

It is defined as the probability that there is no failed unit in the system at time t and is expressed as

$$P_I(t) = P(N(t) = 0) = P_0(t) \tag{1.32}$$

Probability that server is busy

It is defined as probability that there is at least one failed unit in the system at time t and is expressed as

$$P_B(t) = 1 - P_I(t) \tag{1.33}$$

1.11.2 Reliability Characteristics

Reliability characteristics encompass not only units and systems but also the technical, operational, management activities involved in the machining system. For defining various reliability characteristics, we consider the machining system comprises with M operating units and S spare units with state-dependent rates λ_n and μ_n respectively for breakdown and repair. The state transition diagram for the machine interference problem is depicted in Fig. (1.15) wherein system gets failed if all available operating units and spare units fail.

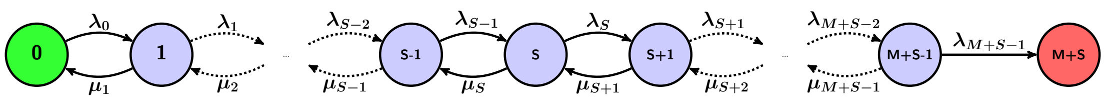

Figure 1.15: State transition diagram for system failure

Reliability

Let T be the time-to-failure of the system T is the continuous random variable characterized by probability density function $f(t)$ and cumulative distribution function $F(t)$. Reliability of a system or unit at time t is defined to be the probability that system or unit will perform an intended function without failure over the interval $(0,t)$, a specific period, under certain stated operating conditions. It is the probability of a nonfailure over time. Mathematically,

$$R(t) = Pr\{T \geq t\} = \int_t^{\infty} f(u)\,du = 1 - F(t) \tag{1.34}$$

where $R(t) \geq 0$, $R(0) = 1$, and $\lim_{t\to\infty} R(t) = 0$. $F(t)$ is also known as unreliability. Reliability $R(t)$ is non-increasing function of t satisfies $\frac{dR(t)}{dt} = -f(t)$

Failure frequency

It measures the number of times the system fail in unit interval of time. It is defined as

$$FF(t) = \lambda_{M+S-1} P_{M+S-1}(t) \tag{1.35}$$

Mean time-to-failure ($MTTF$)

Mean time-to-failure ($MTTF$) describes the expected time-to-failure for a non-repairable system. $MTTF$ can be mathematically calculated as

$$MTTF = \int_0^\infty uf(u)du = \int_0^\infty R(s)ds \tag{1.36}$$

System availability

The availability of the system is defined as the probability that a unit or system is carrying out its required function at a given point of time when used under stated operating conditions. Mathematically, it is expressed as

$$Av = 1 - P_{M+S}(t) \tag{1.37}$$

1.12 Different Arrangements in the Machining System

For the seek of high reliability, less expected total cost, better performance of the machining system, different arrangements of units, alternative policies are required to study in detail. For that purpose, we summarize some common arrangements in brief.

1.12.1 Redundancy

Redundancy is a facility of the existence of more than the required units for performing the intended function.

Passive redundancy

The redundancy wherein the alternative units of performing the function are inoperative until required and are switched on to working upon failure of the operating unit.

Active redundancy

The redundancy wherein all redundant units are functioning simultaneously rather than being switched on when required.

Mixed spare redundancy

The redundancy wherein the alternative unit of performing the function are of different types in failure characteristics, i.e., the mix of hot, warm, and cold spare units.

1.12.2 $K-$out-of$-M$:G system

The system which is functioning if at least K of its M units are operative. According to this arrangement, the maximum of $M-K$ units is allowed to fail before the system fails.

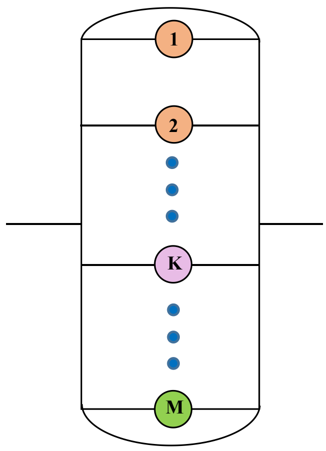

Figure 1.16: $K-$out of$-M$:G system

1.12.3 $K-$out-of$-M$:F System

The system which is not functioning when at least K of its M units are inoperative. In this arrangement, the maximum of K units is allowed to fail before the system fails.

1.12.4 (m,M) Machining System

Suppose a system comprises M identical and independent units of performing the function in normal conditions. For the successful operation of the system in short mode, at least m units should be operative, such system knows as (m,M) machining system. This arrangement allows the maximum $L=M-m+1$ units failure.

1.12.5 N-Policy

N policy is the repair-controlled policy in which the repairman initiates the repair process when there is an accumulation of N failed units in the system and continues until the system is empty. It refers to one of the different vacation policy also.

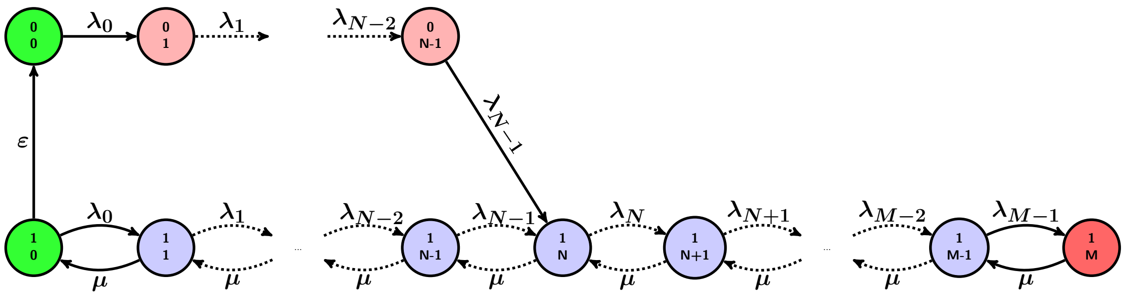

Figure 1.17: State transition diagram of MRP with N-policy

1.12.6 T-Policy

In T policy, repairman takes a vacation of t time unit after completion of each busy period.

1.12.7 D-Policy

D policy is also a repair-controlled policy in which repairman activates for repair when the accumulative services time of failed units in the system exceed threshold D.

1.12.8 F-Policy

F policy is a controlled joining policy in which the caretaker of the failed unit is not permissible to join the system if the system's state achieves the capacity of the system K and continues forbidden until there is only F failed units remain in the system.

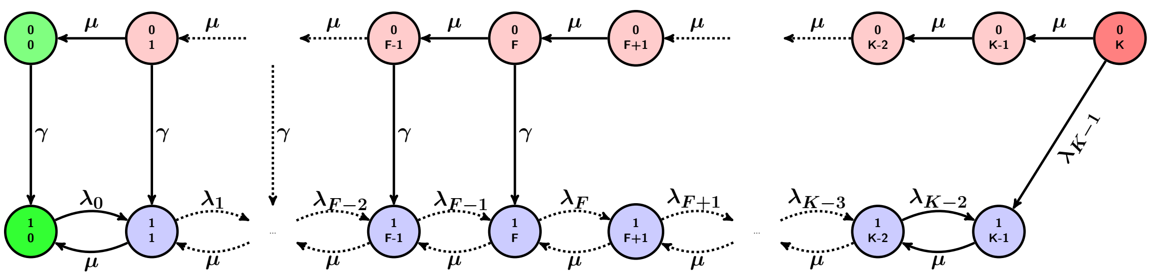

Figure 1.18: State transition diagram of MRP with F-policy

1.12.9 Series System

It is an arrangement of the multi-units such that the system is said to be working if and only if all the units in the system are operative. The layout of the series system is depicted in Fig. (1.19)

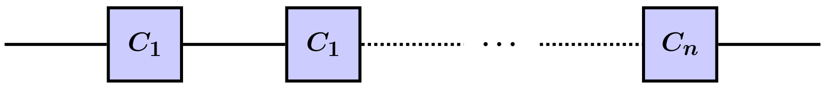

Figure 1.19: Multi-units series system

Let X_i be time-to-failure of i^{th} unit with reliability $R_i(t)$ and Y_s be the time-to-failure of the system having n units. Hence, Y_s is defined as

$$Y_s = min\{X_1, X_2, ..., X_n\}$$

and therefore, the reliability of the series system is given as

$$R_s(t) = R_1(t)R_2(t)...R_n(t)$$

1.12.10 Parallel System

It is a configuration of the multi-units such that the system is said to be working iff at least one unit in the system is operative. The layout of the parallel system is pictured in Fig. (1.20).

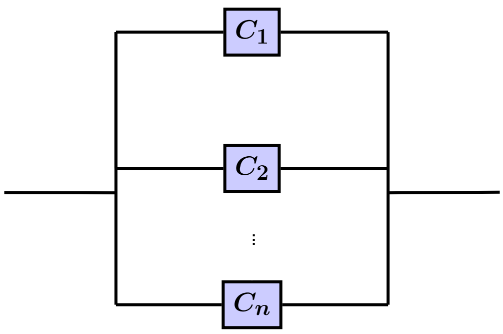

Figure 1.20: Multi-units parallel system

Hence, the time-to-failure Y_s of the system which has n units with time-to-failure X_i and reliability $R_i(t)$ is characterized as

$$Y_s = max\{X_1, X_2, ..., X_n\}$$

and hence, the reliability of the parallel system is given as

$$R_s(t) = 1 - (1 - R_1(t))\,(1 - R_2(t))\ldots(1 - R_n(t))$$

1.13 Fuzzy Number and Fuzzy Arithmetic

Let U be the universe of discourse, $U = \{u_1, u_2, \ldots, u_n\}$. A fuzzy set $\widetilde{A}$ on U is a set of ordered pairs

$\{(u_1, \eta_{\widetilde{A}}(u_1)), (u_2, \eta_{\widetilde{A}}(u_2)), \ldots, (u_n, \eta_{\widetilde{A}}(u_n))\}$

where $\eta_{\widetilde{A}} : U \to [0,1]$ is the membership function of $\widetilde{A}$ and $\eta_{\widetilde{A}}(u_i)$ indicates the grade of membership of u_i in $\widetilde{A}$.

Definition 1.13.1. *A fuzzy set $\widetilde{A}$ on the universe of discourse U is convex if and only if $\forall u_i, u_j \in U$, $\eta_{\widetilde{A}}(\beta u_i + (1-\beta)u_j) \geq \min(\eta_{\widetilde{A}}(u_i), \eta_{\widetilde{A}}(u_j))$, where $\beta \in [0,1]$.*

Definition 1.13.2. *A fuzzy set $\widetilde{A}$ on the universe of discourse U is classified as a normal fuzzy set if $\exists u_i \in U$, $\eta_{\widetilde{A}}(u_i) = 1$.*

Definition 1.13.3. *A fuzzy number is a fuzzy set on the universe of discourse U when it is both convex and normal.*

Definition 1.13.4. *The α-cut $\widetilde{A}_\alpha$ of the fuzzy set $\widetilde{A}$ on the universe of discourse U is defined as $\widetilde{A}_\alpha = \{u_i \,|\, \eta_{\widetilde{A}}(u_i) \geq \alpha, u_i \in U\}$, where $\alpha \in [0,1]$. It is crisp set in nature. If $\alpha_2 \geq \alpha_1$, then $a_1^{(\alpha_2)} \geq a_1^{(\alpha_1)}$ and $a_2^{(\alpha_1)} \geq a_2^{(\alpha_2)}$ i.e. $\left[a_1^{(\alpha_1)}, a_2^{(\alpha_1)}\right] \supseteq \left[a_1^{(\alpha_2)}, a_2^{(\alpha_2)}\right]$ or $\widetilde{A}_{\alpha_1} \supseteq \widetilde{A}_{\alpha_2}$. By decomposition theorem, fuzzy number is union of all α-cuts, i.e., $\widetilde{A} = \bigcup_{\alpha} \widetilde{A}_\alpha; \alpha \in [0,1]$.*

Let $\widetilde{A}$ and $\widetilde{B}$ be two fuzzy numbers on the universe of discourse U with the membership functions $\eta_{\widetilde{A}}$ and $\eta_{\widetilde{B}}$ respectively, where $\eta_{\widetilde{A}} : U \to [0,1]$ and $\eta_{\widetilde{B}} : U \to [0,1]$ and let x and y be two real numbers in U. Using Zadeh's extension principle, the fuzzy number arithmetic operation $\odot$ (fuzzy number addition $\oplus$, fuzzy number subtraction $\ominus$, fuzzy number multiplication $\otimes$, fuzzy number division $\oslash$) is defined as follows

$$\eta_{\widetilde{A}\odot\widetilde{B}}(z) = \bigvee_{z=x.y} (\eta_{\widetilde{A}}(x) \wedge \eta_{\widetilde{B}}(y)) \tag{1.38}$$

where '.' is addition(+), subtraction(-), multiplication($\times$), or division(/) for real numbers if fuzzy arithmetic operation $\odot$ is either fuzzy number addition $\oplus$, fuzzy number subtraction $\ominus$, fuzzy number multiplication $\otimes$, or fuzzy number division $\oslash$ respectively.

A fuzzy number $\widetilde{A}$ on the universe of discourse U may be characterized by a trapezoidal distribution function parametrized by a quadruplet (a,b,c,d). The membership grade function of the fuzzy number $\widetilde{A}$ is defined as

$$\eta_{\widetilde{A}}(u) = \begin{cases} 0; & u < a \\ \frac{u-a}{b-a}; & a \leq u \leq b \\ 1, & b \leq u \leq c \\ \frac{d-u}{d-c}; & c \leq u \leq d \\ 0; & u > d \end{cases} \tag{1.39}$$

Let $\widetilde{A}$ and $\widetilde{B}$ be two trapezoidal fuzzy numbers parametrized by the quadruplet (a_1,b_1,c_1,d_1) and (a_2,b_2,c_2,d_2) respectively, where $a_1 \leq a_2$, $b_1 \leq b_2$, $c_1 \leq c_2$ and $d_1 \leq d_2$. The α-cut of corresponding trapezoidal fuzzy numbers respectively are given by

$$\begin{aligned} \widetilde{A}_\alpha &= [(b_1-a_1)\alpha + a_1, -(d_1-c_1)\alpha + d_1] \\ \widetilde{B}_\alpha &= [(b_2-a_2)\alpha + a_2, -(d_2-c_2)\alpha + d_2];\ \alpha \in [0,1] \end{aligned} \tag{1.40}$$

The resultant α-cut is crisp interval. Using α-cut approach, the fuzzy arithmetic operations of the trapezoidal fuzzy numbers $\widetilde{A}$ and $\widetilde{B}$ can be expressed as interval arithmetic on α-cuts. Let $I_1 = [l_1,u_1], I_2 = [l_2,u_2]$ are two crisp intervals. The basic interval arithmetic operations are defined as follows

$$\begin{aligned} I_1 \oplus I_2 &= [l_1,u_1] \oplus [l_2,u_2] = [l_1+l_2, u_1+u_2] \\ I_1 \ominus I_2 &= [l_1,u_1] \ominus [l_2,u_2] = [l_1-u_2, u_1-l_2] \\ I_1 \otimes I_2 &= [l_1,u_1] \otimes [l_2,u_2] = [\{l_1l_2 \wedge l_1u_2 \wedge u_1l_2 \wedge u_1u_2\}, \{l_1l_2 \vee l_1u_2 \vee u_1l_2 \vee u_1u_2\}] \\ I_1 \oslash I_2 &= [l_1,u_1] \oslash [l_2,u_2] = \left[\left\{\frac{l_1}{l_2} \wedge \frac{l_1}{u_2} \wedge \frac{u_1}{l_2} \wedge \frac{u_1}{u_2}\right\}, \left\{\frac{l_1}{l_2} \vee \frac{l_1}{u_2} \vee \frac{u_1}{l_2} \vee \frac{u_1}{u_2}\right\}\right] \end{aligned} \tag{1.41}$$

For example, let $\widetilde{A}$ and $\widetilde{B}$ be two trapezoidal fuzzy numbers, where $\widetilde{A} = (1,2,4,6)$ and $\widetilde{B} = (2,5,6,8)$. The α-cuts are $\widetilde{A}_\alpha = [\alpha+1, 6-2\alpha]$, $\widetilde{B}_\alpha = [3\alpha+2, 8-2\alpha]$. Then, based on above defined interval arithmetic Eq^n(1.41), we have

$$\widetilde{A}_\alpha \oplus \widetilde{B}_\alpha = [\alpha+1, 6-2\alpha] \oplus [3\alpha+2, 8-2\alpha] = [4\alpha+3, 14-4\alpha]$$

$$\widetilde{A}_\alpha \ominus \widetilde{B}_\alpha = [\alpha+1, 6-2\alpha] \ominus [3\alpha+2, 8-2\alpha] = [3\alpha-7, 4-5\alpha]$$

$$\begin{aligned}
\widetilde{A}_\alpha \otimes \widetilde{B}_\alpha &= [\alpha+1, 6-2\alpha] \otimes [3\alpha+2, 8-2\alpha] \\
&= [\min\{(\alpha+2)(3\alpha+2), (6-2\alpha)(8-2\alpha), (\alpha+1)(8-2\alpha), (6-2\alpha)(3\alpha+2)\}, \\
&\quad\quad \max\{(\alpha+2)(3\alpha+2), (6-2\alpha)(8-2\alpha), (\alpha+1)(8-2\alpha), (6-2\alpha)(3\alpha+2)\}] \\
&= [3\alpha^2+5\alpha+2, 4\alpha^2-28\alpha+48]
\end{aligned}$$

and

$$\begin{aligned}
\widetilde{A}_\alpha \oslash \widetilde{B}_\alpha &= [\alpha+1, 6-2\alpha] \oslash [3\alpha+2, 8-2\alpha] \\
&= \left[\min\left\{\frac{\alpha+1}{3\alpha+2}, \frac{\alpha+1}{8-2\alpha}, \frac{6-2\alpha}{3\alpha+2}, \frac{6-2\alpha}{8-2\alpha}\right\},\right. \\
&\quad\quad \left.\max\left\{\frac{\alpha+1}{3\alpha+2}, \frac{\alpha+1}{8-2\alpha}, \frac{6-2\alpha}{3\alpha+2}, \frac{6-2\alpha}{8-2\alpha}\right\}\right] \\
&= \left[\frac{\alpha+1}{8-2\alpha}, \frac{6-2\alpha}{3\alpha+2}\right]
\end{aligned}$$

Hence, we have the membership of $\widetilde{A} \oplus \widetilde{B}$

$$\widetilde{A} \oplus \widetilde{B} = \bigcup_\alpha \left(\widetilde{A} \oplus \widetilde{B}\right)_\alpha = \bigcup_\alpha \left(\widetilde{A}_\alpha \oplus \widetilde{B}_\alpha\right) = \begin{cases} 0; & u < 3 \\ \frac{u-3}{4}; & 3 \le u \le 7 \\ 1; & 7 \le u \le 10 \\ \frac{14-u}{4}; & 10 \le u \le 14 \\ 0; & u > 14 \end{cases} = (3, 7, 10, 14)$$

Similarly, the membership function for $\widetilde{A} \ominus \widetilde{B}$, $\widetilde{A} \otimes \widetilde{B}$, $\widetilde{A} \oslash \widetilde{B}$ respectively are as follows

$$\widetilde{A} \ominus \widetilde{B} = \bigcup_\alpha \left(\widetilde{A} \ominus \widetilde{B}\right)_\alpha = \bigcup_\alpha \left(\widetilde{A}_\alpha \ominus \widetilde{B}_\alpha\right) = \begin{cases} 0; & u < -7 \\ \frac{u+7}{3}; & -7 \le u \le -4 \\ 1, & -4 \le u \le -1 \\ \frac{4-u}{5}; & -1 \le u \le 4 \\ 0; & u > 4 \end{cases} = (-7, -4, -1, 4)$$

$$\widetilde{A}\otimes\widetilde{B}=\bigcup_{\alpha}\left(\widetilde{A}\otimes\widetilde{B}\right)_{\alpha}=\bigcup_{\alpha}\left(\widetilde{A}_{\alpha}\otimes\widetilde{B}_{\alpha}\right)=\begin{cases}0; & u<2\\ \frac{\sqrt{12u+1}-5}{6}; & 2\leq u\leq 10\\ 1; & 10\leq u\leq 24\\ \frac{7-\sqrt{u+1}}{2}, & 24\leq u\leq 48\\ 0; & u>48\end{cases}$$

and

$$\widetilde{A}\oslash\widetilde{B}=\bigcup_{\alpha}\left(\widetilde{A}\oslash\widetilde{B}\right)_{\alpha}=\bigcup_{\alpha}\left(\widetilde{A}_{\alpha}\oslash\widetilde{B}_{\alpha}\right)=\begin{cases}0; & u<\frac{1}{8}\\ \frac{8u-1}{2u+1}; & \frac{1}{8}\leq u\leq\frac{1}{3}\\ 1; & \frac{1}{3}\leq u\leq\frac{4}{5}\\ \frac{6-2u}{3u+2}; & \frac{4}{5}\leq u\leq 3\\ 0; & u>3\end{cases}$$

The membership function corresponding to the result for fuzzy arithmetic operation for fuzzy number is depicted in the Fig. (1.21).

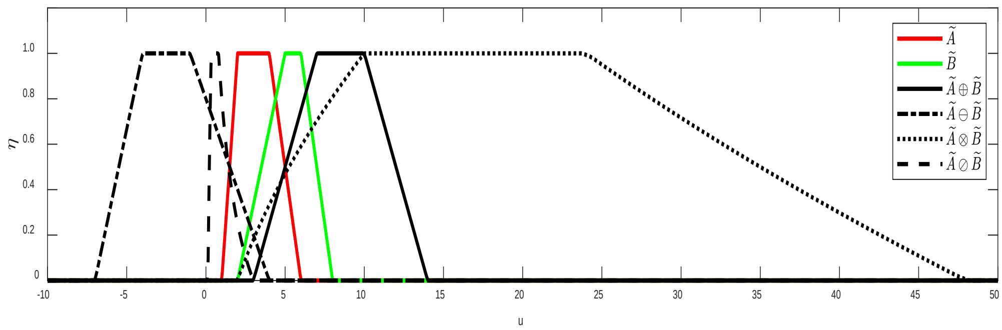

Figure 1.21: Fuzzy arithmetic

In general, fuzzy number addition $\oplus$ and fuzzy number subtraction $\ominus$ of two trapezoidal fuzzy numbers are the trapezoidal fuzzy number. It is not true generally for fuzzy multiplication $\otimes$ and fuzzy division $\oslash$. The resultants are fuzzy sets.

1.14 Review of literature

The machining systems have permeated all areas of our lives in diverse activities. The machining systems are becoming ever more complex as technological advances permit ever-faster performance. As time progresses, an operating unit becomes prone to failure. The failure of units may result in loss of production, money, goodwill, etc.. Provisioning of spare units and corrective maintenance provided by the repair facility resume the perfect operation of the machining system again. In the modern context, failure and repair are coupled events in a typical machining system.

1.14.1 Literature on Queueing theory of Historical Importance

Queueing theory gets its inception fame with the sequence of paper on telecommunication by Danish Engineer, A. K. Erlang. His Impressive contribution to tele-traffic has stimulated and continue to stimulate an enormous volume of work on queueing models. Technical analysis of the queueing system grew considerably with the advent of Operation Research in the late 1940s and early 1950s. During the past, the literature on the subject has grown tremendously. Applications have extended into several areas. Interesting and fruitful interactions between theoretical structures and practical applications have led to the rapid development of the subject.

Several researchers have contributed a great deal towards its theoretical development, while professionals have made equally significant contributions. Besides Erlang, two other most notable pioneers and contributions are Pollaczek in the thirties through the sixties and Takacs for last six decades. The first textbook on the subject, *Queue, Inventories, and Maintenance* was written in 1959 by Morse [14]. Saaty [148] wrote his famous book *Elements of Queueing Theory with Apllications* in 1961, Kleinrock [103] completed his book *Queueing System* in 1976. The new significance of queueing theory was also done by Borel, Kendall, Khintchine, and Kolmogorov. Extensive work has been done on the queueing systems in different frameworks for many years. The literature on queues has been quite vast. Here, we restrict ourselves to the development of queueing models, which are closely related to our investigations on prediction of performance of machining system.

Worthy and notable works have been done by several researchers on the performance modeling of machine repair problems (MRP). Now, we focus on a brief survey of the past literature in the areas. Plam [137] considered the single server machine interference problem with Poisson input and exponential service time distribution. Phipps [140], Naor [132], Morse [130], Jaiswal [86], Ferdinand [42], Osaki [135], Maritas [128] studied machine repair problems and suggested various measures of interest from theoretical as well as application viewpoints.

1.14.2 Literature on Machine Repair Problem with Spare

The spare provisioning is an important aspect of have reliable machining systems. Taylor and Jackson [166] conceptualized first time the MRP with cold spare units provisioning. It is noticed that in the past, many researchers have worked on MRP with the cold spare (*cf.* Schouten et al. [168]; Wang [174]). There are some notable works on MRP with warm spares also (*cf.* Albright [4]; Sivazlian and Wang [162], Wang [170], Wang and Sivazlian [172]) present in literature. Some other practitioners incorporated the mixed type of spares in MRPs (*cf.* Sztrik [164], Wang [175], Wang [173], Wang and Kuo [180], Rao and Gupta [147], Jain and Baghel [69]). The pioneer works for modeling of spare units in machining system can be found in the contribution of the following researchers in Table (1.1).

Table 1.1: Contributions on MRP with spares

Authors	**Year**	**Key Feature**	**Methodology**
Madhu Jain [72]	2013	Machine repair system, Markovian model, Priority queue, Transient, Mixed standbys, Queue size, Availability	Runge-Kutta method, Neuro-fuzzy
Madhu Jain, Chandra Shekhar and Shalini Shukla [85]	2013	Markovian queue, Machine repair, Warm spares, Switching failure, Common cause failure, Partial breakdown	Matrix method
Chongquan Zhong and Haibo Jin [207]	2014	Cold standby, Semi-Markov process, Preventive maintenance	Laplace transform
Wei Huang, James Loman,Thomas Song [66]	2015	Reliability modeling, Warm standby, Active turned into standby, Exponential distribution, Product expansion,Monte Carlo simulation	Analytical solutions
Baoliang Liu, Lirong Cui, Yanqing Wen and Jingyuan Shen[125]	2015	Reliability, Phase-types distribution, Cold spare, Multiple Vacations, Vacation interruption	Matrix method
Jau Chuan Ke, Tzu Hsin Liu, Dong Yuh Yang [98]	2016	Optimization, Probabilistic global search, Lausanne method, Sensitivity analysis	Supplementary variable technique

Ching Chang Kuo and Jau Chuan Ke [111]	2016	Switching failure, Standby, Unreliable server, General repair, Availability	Supplementary variable technique
M. Sadeghi and E. Roghanian [149]	2017	Markov process, Switching mechanisms, Reliability, MTTF, Steady-state, Availability	Matrix method
Chen Wu Lin [28]	2018	Machine repair problem, Recovery policy, Warm Spare, Reliability, Retrial queue, Sensitivity analysis	Cramer's rule method
Kamlesh Kumar, Madhu Jain, and Chandra Shekhar [108]	2019	Machine repair, Threshold F-policy, Warm standbys, Two heterogeneous servers, Queue size	Cramer's rule method

1.14.3 Contributions in Machine Repair Problem with Switching Failure and Common Cause Failure

Switching failure

The spare unit is used for keeping the uninterrupted functioning of the redundant machining system. The spare unit automatically becomes operative as soon as there is a failure of the operating unit. The automatic switching is possible only when the spare unit switches successfully and efficiently. If the spare unit fails in the switching process, the system is not able to replace the failed units. Before the provisioning of spare units in any machining system, it is necessary to ensure that the spare unit switches over to the primary system promptly in place of failed units for uninterrupted functioning of the system. But, due to some reasons such as poor automation and mishandling, sometimes, it has been observed that switching fails with a probability q, known as a switching failure of spare unit. This process continues for all available standbys until a successful switching or exhaust of all available standby in the pool occurs.

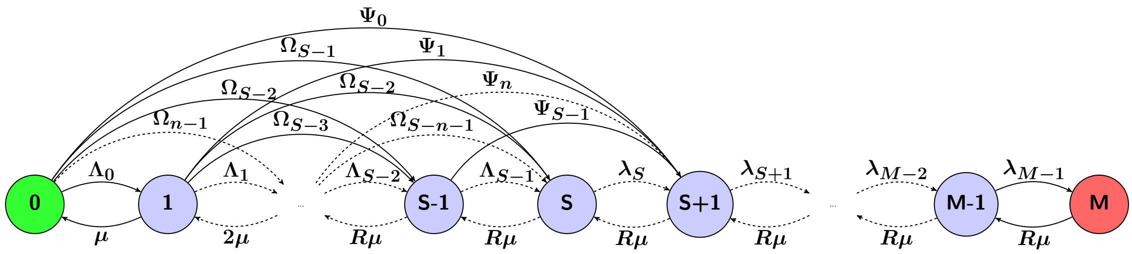

Figure 1.22: State transition diagram of MRP with switching failure
$(\Omega_n = M\lambda\,\bar{q}q^n, \Psi_n = M\lambda q^{S-n}, \Lambda_n = M\lambda\bar{q} + (S-n)\nu)$

Common cause failure

In some cases, the system fails due to the simultaneous failure of one or more units due to common factors. These types of failures which are caused by some common causes are called common cause failures and result in substantial economic losses. Common cause failures may arise due to general power supply failure, environmental conditions (e.g., earthquake, flood, etc.), general maintenance problems, etc. Such failures are rarely confrontation in the machining system, including manufacturing systems, transport systems, communication networks, etc., but lead to huge losses. State transition diagram for model of machine repair in which each unit can fail individually with failure rate λ as well as system can fail with common cause failure rate $\lambda_c(\lambda_c \ll \lambda)$.

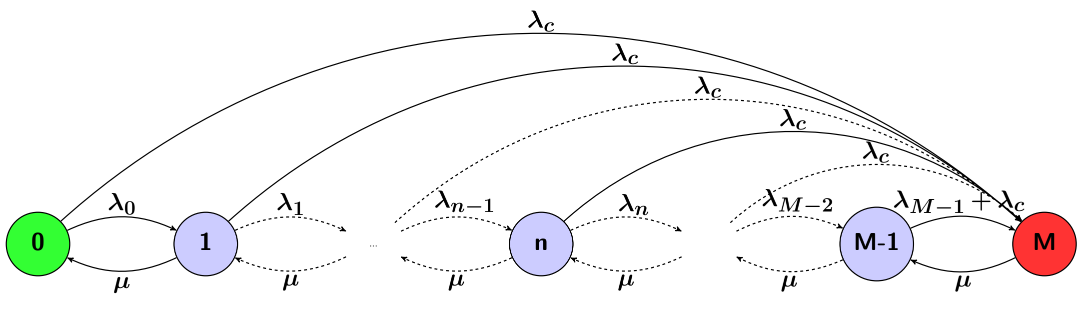

Figure 1.23: State transition diagram of MRP with common cause failure

The remarkable work done on switching failure and common cause failure is compiled in Table (1.2)

Table 1.2: Contributions in MRP with switching failure & common cause failure

Authors	**Year**	**Key Feature**	**Methodology**
Jyh Bin Ke, Jyh Wei Chen and Kuo Hsiung Wang [67]	2011	Mean time-to-failure, Reboot delay, Reliability, Sensitivity, Switching failure	Cramer rule method
Kuo Hsiung Wang, Tseng Chang Yen and Yu Chiang Fang [184]	2012	Availability, Imperfect coverage, Warm spare units	Supplementary variable
Madhu Jain, Chandra Shekhar and Shalini Shukla[85]	2013	Markovian queue, Machine repair, Warm spares, Switching failure, Common cause failure, Partial breakdown	Matrix method
Madhu Jain, Ritu Gupta[73]	2014	System availability, Reliability, Markov model, Human error, Common cause failure	Runge-Kutta method
Ying Lin Hsu, Jau Chuan Ke, Tzu Hsin Liu, and Chia Huang Wu[63]	2014	Machine availability, Reboot delay, Repair pressure coefficient, Switching failures	Probabilistic global search Lausanne method
Madhu Jain [68]	2016	Redundant system, Mixed spares, Imperfect repair, Transient probabilities, Availability, Delayed repair, Reboot, Switching failure	Runge-Kutta method
Chandra Shekhar, Madhu Jain, Ather Aziz Raina, and Rajesh Prasad Mishra [153]	2017	Machine repair, Spare provisioning, Geometric reneging, Switching failure, Reliability, $MTTF$	Runge-Kutta method

Siamak Alizadeh and Srinivas Sriramula [5]	2018	Markov chains, Safety instrumented systems, Safety related systems, Common cause failure, Process demand, Hazardous event frequency	Matrix method
Jiang Cheng, Yinghui Tang and Miaomiao Yu [29]	2019	Micro-inverters, Common cause failure, Solar energy generating system, Reliability, Switch failure	Laplace - Stieltjes transform (LST)

1.14.4 Contributions in Machine Repair Problem with Imperfect Coverage and Reboot Delay

A redundant system with spare units provisioning should include some means by which units can be detected, isolated, and reconfigured in the event of failures; This process is often called redundancy management with perfect coverage. In real-world systems, redundancy management tasks can rarely be done with certainty, and as a result, such the system gets hinder and requires some efficient corrective measures. The system is said to be subject to Imperfect coverage. The corrective measures may include a reboot or recovery process, which increases the delay time. Rebooting is a process by which a hinder machining system is restarted intentionally to remove the unpredictable fault. The reboot can be either *hard* wherein the system power is physically resprouted and returned, allowing the initial boot of the unit, or a *soft*, where the system restarts without the need to interrupt power. The reboot process is a quick process that extracts the faulty unit and reconfigures the system.

Some notable researches in past on imperfect coverage and reboot delay are summarized in the following Table (1.3)

Table 1.3: Contributions in MRP with imperfect coverage & reboot delay

Authors	Year	Key Feature	Methodology
Jau Chuan Ke, Ying Lin Hsu, Tzu Hsin Liu, and Zhe George Zhang [93]	2013	Imperfect coverage, Machine availability, Reboot delay, Service pressure	Probabilistic global search Lausanne method

Madhu Jain and Ritu Gupta [74]	2013	Reliability, Multiple vacations, Imperfect fault coverage, Optimal replacement policy	Supplementary variable technique
Kuo Hsiung Wang, Tseng Chang Yen and Jen Ju Jian [185]	2013	Imperfect coverage, Mean time-to-system failure, Reboot delay, Service pressure condition, Reliability, Sensitivity analysis	Cramer rule method
Jau Chuan Ke and Tzu Hsin Liu [96]	2014	Availability, Imperfect coverage, Reboot delay, Sensitivity analysis	Supplementary variable technique
Kuo Hsiung Wang, Je Hung Su, Dong Yuh Yang [181]	2014	Cost optimization, Imperfect coverage, Sensitivity analysis	Newton-quasi method
Madhu Jain, Rakesh Kumar Meena [76]	2017	Fault tolerant system, Server vacation, Machine repair, Queue length, Imperfect coverage, Reboot, Unreliable server	Runge-Kutta method
Chandra Shekhar, Madhu Jain, Ather Aziz Raina, Javid Iqbal[155]	2018	Reliability, Fault tolerant system, Active redundancy, Reboot, Recovery delay, Common cause failure	Cramer rule method
Tseng Chang Yen, Kuo Hsiung Wang [198]	2018	Availability, Comparisons, Imperfect coverage, Reliability, spare switching failures	Matrix method
Ching Chang Kuo, Jau Chuan Ke [110]	2019	Series system, Reboot delay, Unreliable server, Repair	Supplementary variables technique

1.14.5 Fuzzy Machine Repair Problem

Fuzzy sets and fuzzy logic is extremely useful to many real-time problems involved in research and development of technological-social-economic process, including engineering problem (mechanical, civil, chemical, electrical, aerospace, biomedical, agricultural, computer, environmental, industrial, geological and mechatronics), computer software development, natural science (Mathematics, Biology, Chemistry, and Physics), medical science, social science (economics, management, political science, and psychology), and public policy. Fuzzy set and fuzzy logic are the appropriate tools to translate the vagueness or uncertainty due to linguistic errors, experimental errors, etc. It is well-defined with a membership grade function.

Fuzzy logic has been used most successfully in many fields, such as image processing, control systems engineering systems, power engineering, industrial automation, robotics, consumer electronics, and optimization. Fuzzy queues are also more useful and realistic than commonly used crisp queues. Therefore, fuzzy queues with vague and uncertain information are much more valuable and practical than commonly used crisp queues.

To deal with uncertain information in decision making, Zadeh [199] introduced the concept of fuzziness. The idea of fuzzy logic has been applied in various frameworks by different researchers. Li and Lee [120] investigated the analytical result for two individual queues $M/F/1/\infty$ and $FM/FM/1/\infty$ where F denotes the fuzzy time, and FM indicates the exponential distribution with fuzzified parameter. Nobel research works on fuzzified machining problems are available in the following research publications in Table (1.4)

Table 1.4: Contributions in MRP with fuzzy parameters

Authors	**Year**	**Key Feature**	**Methodology**
Chiang Kao, Chang Chung Li, Shih Pin Chen [91]	1999	Queueing theory, Parametric prograamming, Membership functions	α-cut approach
Shih Pin Chen [24]	2004	Fuzzy set, Finite capacity, Membership function	α-cut approach
Shih Pin Chen [27]	2006	Machine interference problem; Fuzzy sets, Mathematical programming	α-cut approach
Shih Pin Chen [26]	2006	Machine repair, Machine interference, Fuzzy sets, Non-linear programming	α-cut approach
Jau Chuan Ke, Hsin I. Huang, Chuen Horng Lin [94]	2006	Fuzzy sets, Multiple vacation, Parametric nonlinear programming	α-cut approach

Harish Garg [48]	2013	Reliability, Availability optimization, Fuzzy confidence interval	PSO, Fuzzy methodology
Chandra Shekhar, Madhu Jain, Siddharth Bhatia [154]	2014	Availability, Membership function, Machine repair problem, Cold standby, Reboot delay	α-cut approach
Madhu Jain, Rakesh Kumar Meena [76]	2017	Threshold policy, Vacation, Machine repair, Neuro-fuzzy inference system	Runge-Kutta method
Sudeep Singh Sanga, Madhu Jain [151]	2019	Double orbits Retrial, fuzzy queue α-cut Parametric non-linear programing	Genetic algorithm

1.14.6 Machine Repair Problem with Vacation

In any real-time framework, when there is no failed unit in the system, it is always a better policy to administer a leave of the server to reduce the cost of service. There are many types of vacation strategy; Some common and important vacations are as follows

- **N-policy**: The server initiates rendering service only when there are N failed units in the system and continues until all failed units get repaired.
- **Multiple vacation**: On repairing all failed units, the server takes leave for a random period. At the end of the vacation, if it finds no waiting failed unit, it takes another vacation of random period, otherwise initiates to serve the failed units.
- **Single vacation**: At the end of the vacation, if it finds no waiting failed unit, it stays idle in the system until the failed unit arrives.
- **Working vacation**: During vacation, the server may also function for the system remotely at a lesser repair rate. It is known as a working vacation. Working vacation helps failed units or systems to reduce waiting time and increase system efficiency. Working vacation may be multiple or single.
- **Vacation interruption**: Sometimes on vacation, failed units experience long waiting delay. To diminish the long waiting delay, the system uses a unique

strategy to call server for rendering the repair before the scheduled end of vacation time and is referred as vacation interruption. The policy for vacation interruption may be waiting time of failed units or the number of failed units in the system.

In this thesis, we investigate working vacation and vacation interruption in MRP. From inception to development of working vacation and its interruption is studied extensively in past. Some of published articles are tabulated in the following Table (1.5).

Table 1.5: Contributions in MRP with working vacation & vacation interruption

Authors	Year	Key Feature	Methodology
Kuo Hsiung Wang, Wei Lun Chen and Dong Yuh Yang [176]	2009	Cost Optimization, Working vacation	Newton's method
Mian Zhang and Zhengtng Hou [202]	2010	Working vacations, Vacation interruption, $M/G/1$ queue	Method of supplementary variable
Mian Zhang and Zhengtng Hou [201]	2011	Markovian arrival precess, Vacation interruption, Working vacations	Censoring technique, matrix-analytic method
Charan Jeet Singh, Madhu Jain and Binay Kumar [160]	2012	State dependent, Queue, Arbitrary service time, Vacation, Average queue	Supplementary variable
Baoliang Liu, Lirong Cui, Yanqing Wen and Jingyuan Shen [125]	2015	Reliability, Phase-type distribution, Multiple vacations, Working vacations, Vacation interruptions	Matrix-analytic method
Doo Ho Lee and Bo Keun Kim [114]	2015	$M/G/1$ queue, Single working vacation, Vacation interruption, Sojourn time	Laplace- Stieltjes transform
Kaili Li, Jinting Wang, Yanjia Ren and Jingwei Chang [119]	2016	Queueing,Working vacation, Vacation interruptions, Equilibrium strategies, Stationary distribution	Transient solution

P. Rajadurai, M.C. Saravanarajan and V.M. Chandrasekaran [145]	2018	Retrial queues, Feedback, Working vacation, Balking, Server breakdown	Supplementary variable method
P. Rajadurai, M.C. Saravanarajan and V.M. Chandrasekaran [144]	2018	$G-$queue, Working vacations, Vacation interruption	Supplementary variable technique
Wojciech M. Kempa and Martyna Kobielnik [102]	2018	Finite buffer, Integral equations, Queue-size distribution, Transient state, Working vacation	Laplace transforms
Dong Yuh Yang and Chih Lung Tsao[192]	2019	MTTF, Reliability, Retrial, Steady-state availability, Working vacation	Cramer's rule

1.14.7 Machine Repair Problem with Newton-quasi Method

Finite-dimensional non-linear problems are generally solved by iteration. In 1959, Davidon [33] disuse on the minimization problem and Brocken [17] for systems of equations introduced new techniques that were iterative, quite unlike to any other method in use at the time. These papers have undergone a large amount of research in the late sixties and early seventies. Fletcher and Powell [43] presented significant revisions and explanations of Davidon's work and gave rise to a new class of algorithms, called Newton-quasi, variable metric, variance, secant, update or modification methods. Applications of Newton-quasi method in machine repair problem and queueing problem have been sought in the work of following contributors in Table (1.6).

Table 1.6: Contributions in MRP with Newton-quasi method

Authors	Year	Key Feature	Methodology
Kuo Hsiung Wang and Dong Yuh Yang [183]	2009	F-policy, Optimization, Startup, Server breakdowns	Matrix analytical method, Newton-quasi method
Kuo Hsiung Wang, Wei Lun Chen and Dong Yuh Yang [176]	2009	Cost Optimization, Working vacation	Newton's method

Chia Huang Wu and Jau Chuan Ke [186]	2010	Cost, Balk, Renege	Matrix analytic approach, Newton-quasi method
Dong Yuh Yang, Kuo Hsiung Wang and Chia Huang Wu [193]	2010	F-policy, Optimization, Sensitivity analysis, Working vacation	Newton-quasi method
Jau Chuan Ke, Chia Huang Wu and Wen Lea Pearn [93]	2011	Bernoulli vacation schedule, Single vacation policy	Matrix analytic approach, Newton-quasi method
Jau Chuan Ke, Ying Lin Hsu, Tzu Hsin Liu and Zhe George Zhang [187]	2013	Imperfect coverage, Machine availability, Reboot delay, Service pressure	Probabilistic global search Lausanne method
Chia Huang Wu, Wen Chiung Lee, Jau Chuan Ke and Tzu Hsin Liu [97]	2014	Cost, Controllable repair policy	Particle swarm optimization, Newton-quasi method
Jau Chuan Ke, Tzu Hsin Liu and Chia Huang Wu [195]	2015	Multiple heterogeneous repairmen, Profit	Probabilistic global search Lausanne, Newton-quasi method
Dong Yuh Yang and Ying Yi Wu [195]	2017	Optimization, Reneging, Working breakdown	Newton-quasi method, Runge-Kutta method
Fu Min Chang, Tzu Hsin Liu and Jau Chuan Ke [22]	2018	Queueing, Feedback, Impatience, Optimization, Unreliable-server	Newton-quasi (QN) method

1.14.8 Machine Repair Problem with Supplementary Variable

The supplementary variable technique is also one of the basic techniques that have been applied to solving queueing problems. The researchers have made a lot of contributions to predict the performance of queueing systems in different frameworks using this technique. The supplementary variable technique was introduced by Cox [32]. Keilson and Kooharian [101] studied time-dependent queueing processes using supplementary variable technique. The major contributions on queueing and MRP via supplementary variable technique are also cited in research articles by Hokstad[60], Lee [115], etc.

Choi and Park [30] studied $M/G/1$ retrial queue with the Bernoulli feedback schedule by considering supplementary variable technique. A $G/M(a,b)/1$ queue with server vacation using supplementary variable technique was given by Choi and Han [31]. Mittler and Kern [129] studied machine interference problems with generally distributed failure, repair, and walking times. Wang and Kuo [179] used the recursive method using the supplementary variable technique wherein the supplementary variable is the remaining service time. The recent works for the development of supplementary variable technique with elapsed or remaining time in MRP or queueing problems are tabulated in the Table (1.7)

Table 1.7: Contributions in MRP with supplementary variable method

Authors	Year	Key Feature	Methodology
Hirokazu Ozaki and Atsushi Kara [136]	2012	Shared protection systems, Probability distribution, Survival function, MTTF	Supplementary variable method
Vijay Vir Singh, Mangey Ram and Dilip Kumar Rawal[161]	2013	Controller failure, Maintenance model, MTTF	Supplementary variable method
Shan Gao, Jinting Wang, Wei Wayne Li[46]	2014	Retrial queue, Working vacation, Vacation interruption, Conditional stochastic decomposition	Supplementary variable method
Ching Chang Kuo and Jau Chuan Ke [111]	2016	Switching failure, Standby, Unreliable server, General repair, Availability	Supplementary variable technique
Jau Chuan Ke, Tzu Hsin Liu, Dong Yuh Yang [98]	2016	Optimization, Probabilistic global search Lausanne method	Supplementary variable technique
S. Pradhan and U.C. Gupta [142]	2017	Batch-arrival, Batch-service, Queueing, Joint distribution, Multiple roots	Supplementary variable
Dong Yuh Yang and Ya Dun Chang [191]	2018	Busy period, Cost optimization, Retrial, Machine availability	Supplementary variable technique

Madhu Jain, Sandeep Kaur and Parminder Singh [75]	2019	Non-Markovian, Unreliable server, Vacation,Queue length, Bulk input	Supplementary variable technique
Ching Chang Kuo and Jau Chuan Ke [110]	2019	Series system,Imperfect coverage, Reboot delay, Unreliable server, Repair	Supplementary variable technique
Madhu Jain, Rakesh Kumar Meena and Pankaj Kumar [77]	2019	Maintenance, Availability, Imperfect recovery, Vacation, Reboot delay	Supplementary variable technique