Transform and Intercept Models for Bivariate Cascade High Sigma Manufacturing Processes

THESIS

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By

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CERTIFICATE

This is to certify that the thesis entitled ***"Transform and Intercept Models for Bivariate Cascade High Sigma Manufacturing Processes***" submitted by **Srinivasan Lakshminarasimhan** ID No **2003PHXF402** for award of Ph.D. Degree of the Institute embodies original work done by him under my supervision.

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Abstract

When two manufacturing process are serially dependant or cascade in nature, regression based cause selecting control charts pioneered by Zhang in 1984 are adopted for process monitoring. The objective is to identify the assignable causes of variation in the downstream process on account of an assignable cause variation in the upstream process. This unique characteristic of cause selecting control charts make it distinctly different from other multivariate process monitoring control charts.

In literature, cause selecting control charts have been discussed for 3 sigma manufacturing processes. Shewhart type control charts are not adoptable to processes with metric above 3 sigma, due to rare occurrence of defects. The type of control charts that is adaptable for high sigma process is a time between event control charts known as cumulative count of conforming items between two nonconforming items control chart.

In this work a high sigma cause selecting control chart has been designed for application of cascade process, thus upgrading the utility of cause selecting control charts from 3 sigma to high sigma. For brevity a two stage high sigma manufacturing process has been considered in this work. The defect counts in the high sigma processes above 3 sigma follow a geometric distribution. A new power transformation has been proposed to convert geometric data into normal form. A design flow detailing the stage wise procedure to draw the control chart has been established. The design methodology has been demonstrated using data from a high sigma pin manufacturing process. This is the

first contribution of this work. The lower control limit is the action limit for high sigma control charts. In control charts for geometric distributions negative lower control limits have been encountered. It is established that the proposed power transform performs better than traditional transforms in this regard. This is the second contribution.

As the cause selecting control chart methodology is based on regression models, the model issues and the presence of intercept term assumes importance. Unlike other designs of control charts the regression model not only affects the chart performance indices but also the chart statistic. This issue has been studied in detail and the impact of assuming an intercept model as no intercept model and a no intercept model as an intercept model have been studied. Its impact on type II error and chart performance is also studied in detail. This is the third contribution of this work. A modified design flow incorporating the intercept model issues has been designed. This is the fourth contribution of this work. The methodology has been demonstrated using data from precision pin manufacturing.

As the transforms have been established in this work, the need arose as to measure the effectiveness of this transformation by studying the impact of third and fourth moments of normality, namely skewness and kurtosis and their impact on lower control limit and type II error have been researched and results documented. This is the fifth contribution of this work. As six sigma processes are scarce the data, methodology and results have been validated through a data set from literature.

It has been established that the proposed power transform in this work is superior to the power transform proposed by Nelson (1994) in its ability to achieve near zero lower control limit in high sigma process situations. As proposed by Box and Cox (1964), traditionally optimization of transform is done with sum of square of residuals. For the first time the optimization of the transform has been done with focus on the application requirement namely, the ability to achieve a positive lower control limit in the case of high sigma geometric data. The discussion done in this work on intercept model errors in case of cause selecting control charts is opening up a new avenue in research and practice.

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List of Symbols

Symbols	Description
α	Probability of type I error.
α'	Type I error probability of joint control charts
N	Quality characteristics
p	Probability function
X	Upstream process, Regressor process
Y	Downstream process, Response process
p	Probability of nonconforming items
β	Probability of type II error
σ	Standard deviation
μ	Process mean
n	number of observations
q	Probability of failure occurrence
‘F’	Failure
‘S’	Success
β_0	Regression coefficient for intercept
β_1	Regression coefficient for regressor
ε	Error term
t	Student statistic
$\hat{\sigma}$	Estimated value of standard deviation
S_{xx}	Corrected sum of squares of X
R	Coefficient of determination
AR(1)	Time series model
$\hat{Y}$	Estimated or fitted value of response variable
Z	Transformed value
B	Negative binomial distribution
k	Control limit constant or Shift magnitude
p	Fraction defective
c	Number of defects
ℓ	Quantity of products in each sample
u	Number of defects per quantity of products in a sample
λ	Defect rate
λ	Transformed value

List of Symbols

Symbols	Description
Z_n	Cause selecting data
‘r’	increase of transform
X_i	Regressor variable at the ‘*i*’ th position
Y_i	Response variable at the ‘*i*’th position
φ	Standard normal distribution function
L	Constant
n	sample size
k	Shift magnitude
S_e	Standard error of estimate
S_B	Bounded
S_L	Long normal
S_U	Unbounded
SS_{RES}	Residual sum of squares
μ_ε	Mean of residuals
R^2	Co-efficient of determination
$\overline{Y}$	Average of Y

List of Abbreviations

Abbreviations	Description
CS	Cause selecting control chart
CCC	Cumulative count of conforming items control charts
HYP	High yield processes
SPC	Statistical process control
PCR	Principal component analysis
CL	Central line
UCL	Upper control limit
LCL	Lower control limit
DPMO	Defects per million opportunities
ppm	Parts per million
USL	Upper specification limit
LSL	Lower specification limit
mR_{res}	Moving range of residuals
mR	Moving range
ARL	Average run length
SSP	Six sigma process
EWMA	Exponentially weighted moving average
ln	Natural logarithm
CUSUM	Cumulative sum
MS_R	Mean square of residuals
MS_{RES}	Mean square of residuals

Chapter 1
Introduction

1.1 Introduction

Manufacturing Engineers' desire for reducing process variations finds expression in their continuous efforts in monitoring the process. This comprises of predicting the future behavior of the process by analyzing its past behavior. The past and current behavior of the process is known as 'voice of the processes'. It was Walter Shewhart who first devised an effective and simple way to define the voice of the process and he called it as 'control charts'.

1.2 Manufacturing Variations

Manufacturing variations constitute the biggest threat to the engineers. Variations in production processes are inevitable. They occur owing to wrong setting of machines, operator errors, faulty inputs and process capability limitations. The positive thing about process variation is that they follow a distribution and hence are understandable and controllable. The variations that occur due to men, machines, materials, method, measurement and environment are known as variations due to assignable causes as their root cause can fit into one of these reasons. Another type of variation, known as the common cause variations are the background noise arising out of inherent process capability limitations.

Manufacturing processes generally comprise of many process steps. A typical manufactured product will have its journey through many production processes and supporting activities like transport, storage, and inspection. Such a manufacturing process scenario can be depicted as follows in figure 1.1 :

Symbol	**Process**	**Quality Characteristic**
○	**Metal Drawing**	**Drawn Diameter, Drawn Length**
⇩	**Transport**	**----**
○	**Metal Stamping**	**Width, Depth**
⇩	**Transport**	**----**
▽	**Storage**	**----**
⇩	**Transport**	**----**

Figure 1.1. Process Flow Diagram

A manufacturing process step can be defined as an activity which adds some the value to the product. The 'process' is capable of transforming physical, chemical or other categories of the product's character. This desired value addition is measured in terms of 'critical to quality' characteristics. As each process step may have one or more critical to quality characteristic, the quality of products in its totality is measured by many such critical to quality characteristics and not by a single quality characteristic.

The above process sequence has two production process steps and many supporting steps like storage, transport. Only the ‘process’ steps with value addition in terms of improvement in size, shape or character are considered for control of quality. The quality of the end product will be determined by

Drawn Diameter

Drawn Length

Width

Depth.

Production processes are expected to deliver the desired value addition ‘with in specification’. In engineering parlance, specification means satisfying the customer requirements. It will be in the form of numerals. As it is not possible to achieve all the time, 100 % exact specification, ‘tolerances’ have been built into the specification to accommodate the possible ‘variations’ in practice. These variations are departure from specifications which normally may not affect the functional requirement of the product. A typical specification may be read as ‘Drawn Diameter = 1.65 ± 0.01 mm’. If a product falls within the range of 1.64 mm to 1.66 mm, it is said to satisfy the specification and it is acceptable.

1.3 Limitations of Multivariate Process Monitoring

Manufactured products may have many quality characteristics. From customers’ view point the total quality encompassing all the characteristics are of importance. When there are several process steps in a production line, in order to monitor the critical to quality characteristics of these steps together, a suitable process monitoring tool is required. Control charts are the time tested process monitoring tools for this purpose. When there are more than one process step and many quality characteristics, it is possible to run one control chart for each characteristic. But the distortion increases with many quality variables running with individual control charts which are to be monitored. Montgomery (2001) explains this distortion. If there are N statistically independent quality characteristics for a product and if an $\overline{X}$ chart with P (type I error) = α is maintained for each quality characteristic, the true probability of type I error for the joint procedure is,

$$\alpha' = 1-(1-\alpha)^{N} \quad (1.1)$$

and the probability that all p means will simultaneously plot inside their control limits when the process is in control is

$$P\{\text{all p means plot in control}\} = (1-\alpha)^{N} \quad (1.2)$$

Thus even for small values of N, the distortion in values will be severe. Thus monitoring two or more variables independently can be misleading. For a 3 sigma quality the probability that a point will fall outside control limit is 0.0027. If two independent charts are used to monitor quality, the joint probability that a data point will fall outside the control limits is (0.0027)*(0.0027) = 0.00000729. Likewise, the probability that a data point will fall within control limits for a control chart for one variable or attribute is, 0.9973 and the joint probability if the two individual control charts are employed is

0.99460 and hence the type I error probability and the probability of a point falling within limits, will be far below the value envisaged.

Woodall and Montgomery (1999), and Woodall (2000) explain that the performance of multivariate control charts in detecting process disturbances tend to deteriorate as the number of monitored variances increase. In multivariate process monitoring one is encountered with large data sets, autocorrelation, missing data, nonalignment of sampling points and many other problems which render the application complicated. Further multivariate process monitoring techniques as applied to control charting have profusely used principal components , partial least squares and latent structure methods. These are the reduction techniques used to detect the vital few characteristics to be monitored among the trivial many. These rendered the control charts to operate in a limited subspace where it is difficult to understand to which sub process or characteristic is out of control.

From practical point of view the multivariate control chart fails to identify as to which quality parameter or attribute or which process step is going out of control. This is overcome in the case of cause selecting control charts. It identifies the process step due to which an out of control situation has arisen.

1.4 Cause Selecting Control Charts

Cause- selecting control charts dealt with by Constable et al. (1987), Zhang (1989), Wade and Woodall (1993), Shu and Tsung (2000,2003), Shu et al. (2004, 2005) are employed in the case of a cascading process. In the case of a bivariate process steps X and Y, the assignable cause variation in upstream process X termed as specific quality is determined by a Shewhart type control chart for X. Likewise the assignable cause variation for downstream process Y can be determined from a Shewhart type control chart for Y. The total quality that is the quality variation in downstream process Y due to upstream processes X is determined by a joint reading of Shewhart type control chart for X and a residual control chart for Y. The residuals are the difference between the data Y and the fitted data $\hat{Y}$.

This chart was christened as cause selecting control chart. The cause selecting control charts were used to detect two kinds of quality, namely, total quality and specific quality. The specific quality refers to the assignable cause variations in the regressor process X (upstream process) and the response process Y (down stream process). These can be identified by two separate Shewhart type control chart for process X and Y.

The total quality refers to the process variation in process Y on account of a variation in process X. This is identified by a residuals control chart for process Y, which is known as cause selecting control chart (CS). The evolving decision matrix for specific and total quality is detailed as follows in table 1.1 :

Table 1.1 Cause Selecting Control Chart- Two Chart Decision Matrix

Sl.No	Shewhart Type Control Chart for Process X	Cause Selecting Chart for Process $\hat{Y}$	Interpretation
1	Signal	Signal	Both process out of control
2	No signal	Signal	Process Y out of control
3	Signal	No Signal	Process X out of control
4	No Signal	No Signal	Both process in control

To demonstrate the methodology of CS a worked example is detailed hereunder. The data of machined dimensions of two cascade processes have been simulated. The data and workings have been shown below in table 1.2:

Table 1.2 Demonstration Data Set for a Bivariate Cascade Process

Sl.No	X	Y	$\hat{Y}$	$Y-\hat{Y}$
1	1.56	1.76	1.89	–0.13
2	2.08	1.76	1.78	–0.02
3	2.21	1.51	1.76	–0.24
4	1.82	1.64	1.84	–0.20
5	1.69	2.14	1.87	0.28
6	2.34	1.76	1.73	0.04
7	2.16	2.02	1.77	0.25
8	2.21	1.51	1.76	–0.24
9	1.95	1.51	1.81	–0.30
10	1.69	1.64	1.87	–0.23
11	1.56	2.02	1.89	0.12
12	2.21	1.64	1.76	–0.12
13	2.34	1.64	1.73	–0.09
14	2.34	2.02	1.73	0.29
15	2.34	2.02	1.73	0.29
16	2.08	1.89	1.78	0.11
17	1.56	2.14	1.89	0.25
18	2.21	1.89	1.76	0.13
19	2.08	2.14	1.78	0.36
20	1.95	1.51	1.81	–0.30

Column 2 shows the machined data set X and column 3 shows machined data set Y. The relationship between X and Y may be linear or non linear. Any non linear physical relationship can be approximated into linearity. According to Montgomery, Peck and Vining (2006) in all applications of regression, the regression equation is an approximation to the functional relationship between the variables of interest. The said functional relationship may be of physical, chemical or mechanical in nature. Such types of models are known as mechanistic models. These models are linear, non-linear or complex in nature. These may be empirically approximated as a linear regression model. If the underlying process has more complex relationship, this has to be transformed into an approximation linear regression model. With the advent of computer programs any non linear process data may be easily approximated into a linear model. Microsoft excel spreadsheet does this job and no special programs are needed. This work confines to linear models of which non-linear data is a special case. The relationship between the upstream process X and down stream process Y can be explained by the scatter diagram shown below in figure 1.2. The plot is made of process variable X in column 1 and Y in column 3 of table 1.2.

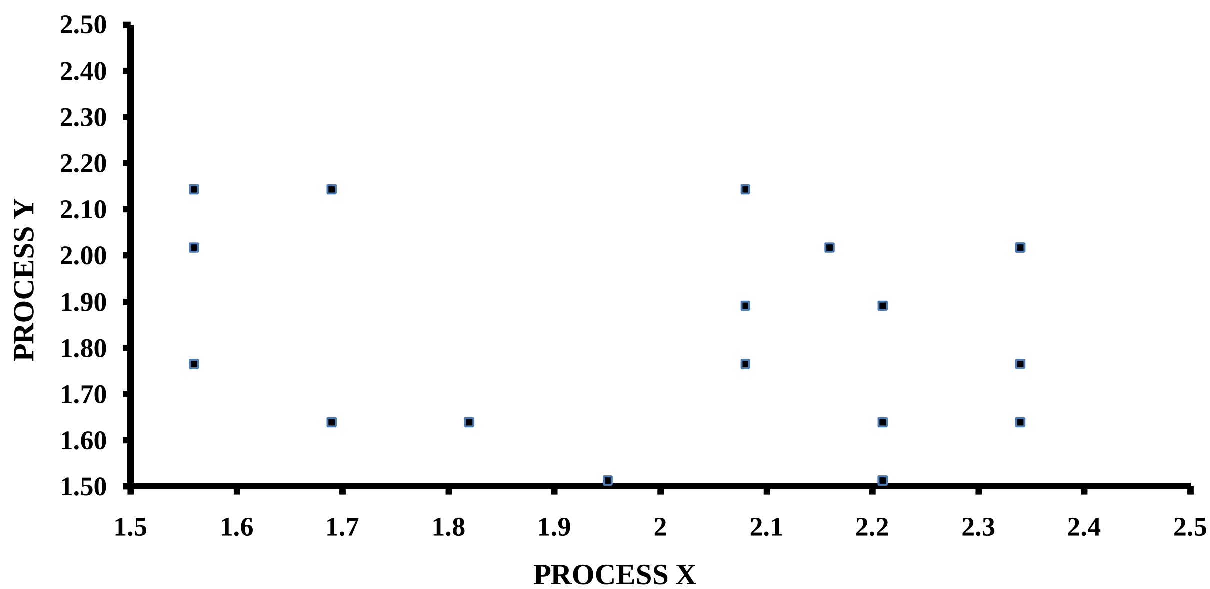


Figure 1.2 Scatter Plot of Process Variables X and Y

From the scatter plot the non linear relationship of the data sets X and Y are evident. If the data sets X and Y are fitted into a straight line $\hat{Y}$ = 2.2.-0.20 X', the fitted points of the response variable $\hat{Y}$ can be calculated. It is summarized in column $\hat{Y}$ in table 1.2. The relationship of 'X' the regressor variable adjusted against a covariate 'Y' is depicted by the regression equation $\hat{Y}$ =2.2– 0.20 X'. It can be seen that the relationship of X and $\hat{Y}$ is linear as shown in scatter diagram figure 1.3. The plot is made of process variable X in column 1 and $\hat{Y}$ in column 4 of table 1.2.

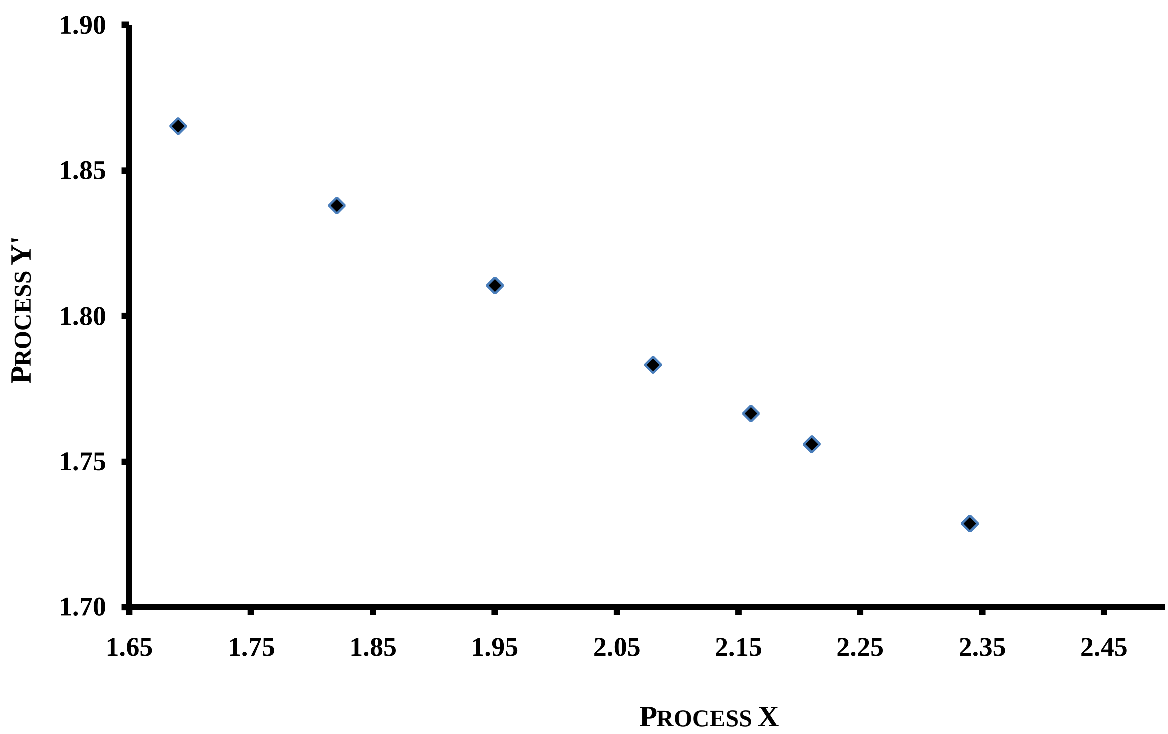


Figure 1.3 Scatter Plot of Process Variables X and $\hat{\mathbf{Y}}$

Constable et al. (1987) proposed that this empirical relationship can be utilized to find out the assignable cause in a downstream process due to an upstream process. The requirements proposed by him are that the two processes are cascade or serially dependent in nature.

Two charts have been proposed to find out a non conforming situation due to

a) Upstream process.
b) Down Stream Process.
c) Down Stream Process due to Upstream Process.

The first chart is a Shewhart type control chart drawn for process 'X' of data set in table 1.1 is shown in figure 1.4.

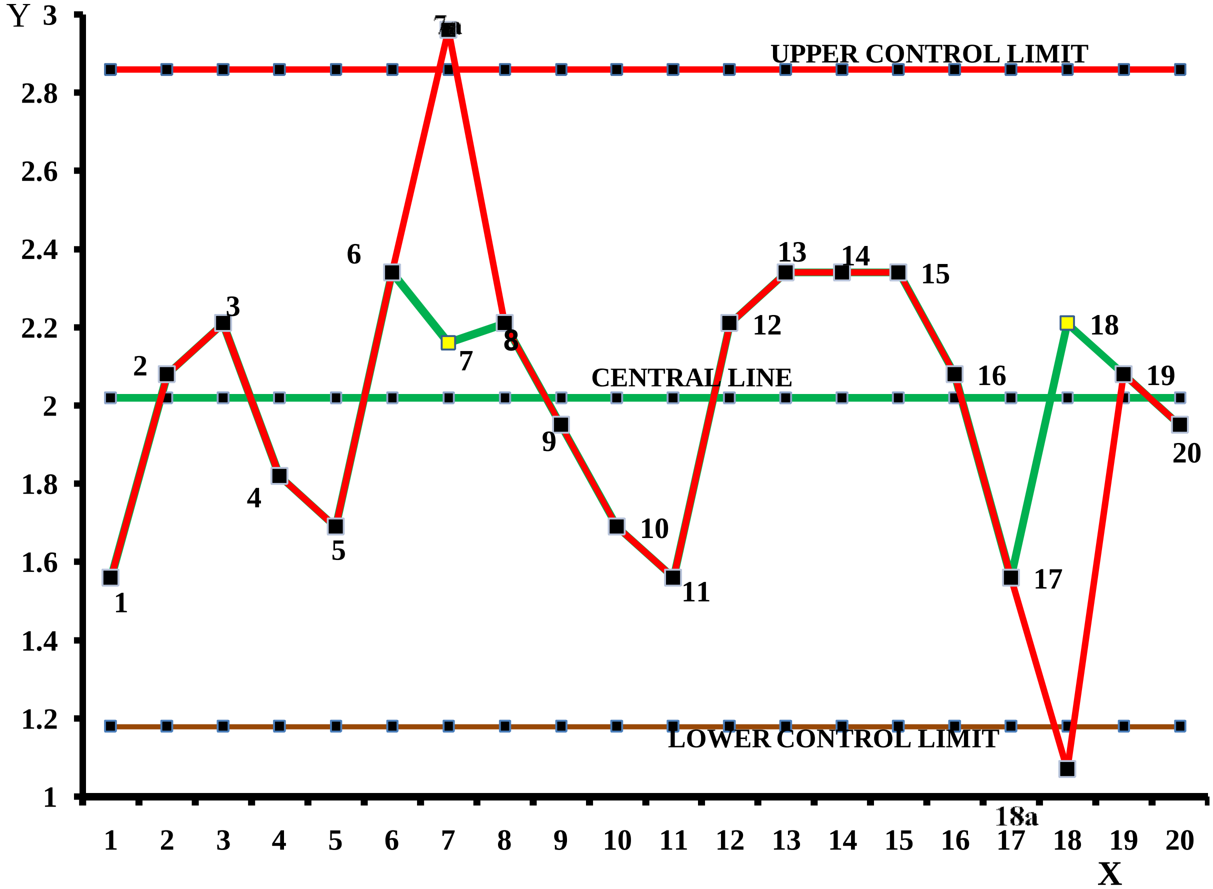


Figure 1.4 Shewhart Control Chart for Process X

Gitlow and Oppenheim (1991) in their work have dealt with residuals and Shewharts control chart. The regression residuals carry with them the process information and are a good candidate as control chart process statistic. The second chart in the CS concept is a regression residuals control chart. Referring to table 1.2, the residuals of the fitted and natural covariate Y and $\hat{\mathbf{Y}}$ is shown as column Y – $\hat{\mathbf{Y}}$. The control chart of this data shown in table 1.1, known as the cause selecting chart is shown in figure 1.5.

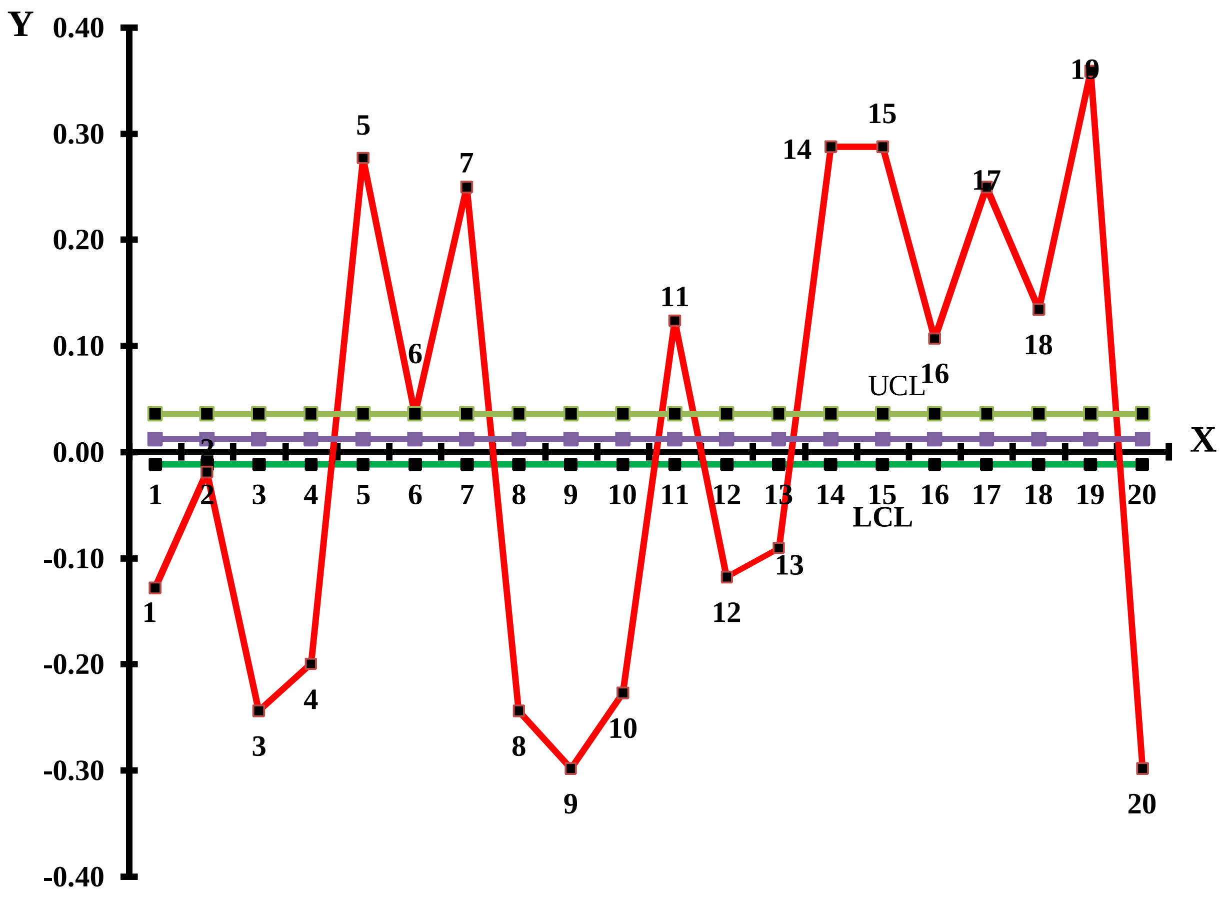


Figure 1.5 Residuals (Cause Selecting) Control Chart

The interpretation of process situation referring to table 1.1 and table 1.2 are detailed in table 1.3.

Table 1.3 Process Decision for Cause Selecting Control Chart

Data Point	Shewhart Type Control Chart for X	Residuals Cause Selecting Control Chart for Y' Residuals	Diagnosis
1	In Control	Out of Control	Process Y Out of Control
2	In Control	In Control	Processes X,Y In Control
3	In Control	Out of Control	Process Y Out of Control
4	In Control	Out of Control	Process Y Out of Control
5	In Control	Out of Control	Process Y Out of Control
6	In Control	In Control	Processes X,Y In Control
7	In Control	Out of Control	Process Y Out of Control
8	In Control	Out of Control	Process Y Out of Control
9	In Control	Out of Control	Process Y Out of Control
10	In Control	Out of Control	Process Y Out of Control
11	In Control	Out of Control	Process Y Out of Control
12	In Control	Out of Control	Process Y Out of Control
13	In Control	Out of Control	Process Y Out of Control
14	In Control	Out of Control	Process Y Out of Control
15	In Control	Out of Control	Process Y Out of Control
16	In Control	Out of Control	Process Y Out of Control
17	In Control	Out of Control	Process Y Out of Control
18	In Control	Out of Control	Process Y Out of Control
19	In Control	Out of Control	Process Y Out of Control
20	In Control	Out of Control	Process Y Out of Control

It can be seen from above chart that in data point 2 and 6, both the processes are in control and in all other data points the down stream process is out of control. For analogy referring to figure 1.4 if chart statistic 7a and 18a are out of control for the upstream process, the diagnosis will be for 7a process X will be out of control and for 18a process X,Y are out of control. If there is a data point 7a for which process X is out of control the diagnosis interpretation will be that process ‘X’ is out of control. If there is a data point

18a for which both ‘X’ and ‘Y’ are out of control the interpretation is that both the process are out of control. This scenario is detailed in table 1.4.

Table 1.4 Second Diagnosis Table

Data Point	Shewhart Type Control Chart for X	Residuals Cause Selecting Control Chart for $\hat{\mathbf{Y}}$ Residuals	Diagnosis
7a	Out of Control	In Control	Process X Out of Control
18a	Out of Control	Out of Control	Process X,Y Out of Control

The concepts explained are confined to 3 sigma process variables. There has been very many discussions on ‘variables control chart’ and ‘attributes control chart’. The variables are direct measures and attributes are ‘transformed’ measures of variables. This is due to the fact that the attributes like counts are obtained by inspection of products. Hence the basic measurements are variables. As there are limitations in building control charts for variables, in certain industrial situations attributes charts have been advocated. The issue is not whether it is a variables control chart or attributes control chart but whether it is able to effectively depict an out of control situation or not.

This work is a pioneering work which enhances the capability of cause selecting control charts to high sigma processes.

1.5 High Sigma Control Charts

The cause selecting control charts are limited to the 3 sigma process monitoring. The conventional control charts are not adaptable to high sigma processes. By high sigma process we refer to any production process above 3 sigma metric. The defective are rare in processes operating above 3 sigma metric.

The traditional control charts which worked for the 3 sigma status are built based on the assumption that the process data characteristics are normally distributed. With low count levels occurrence as detailed in table 1.5, for high sigma process, the normality assumption cannot be valid. Further impractical sample size and statistical axioms limit the use of 3 sigma compliant control charts for process sigma above 3.

Table 1.5. Defect Occurrence Time and Process Speed

Sl. No	Process Speed(Units per minute)	Time to get 3.4 (per million) defects in Days at the rate of 8 hour Shift Per Day	Time to get 66800 (per million) defects in Days at the rate of 8 hour Shift Per Day
		(6 sigma process)	(3 sigma process)
1	10	61	14
2	20	31	7
3	40	15	3
4	80	8	2
5	100	6	1

The effectiveness of a manufacturing engineer lies in detecting the assignable cause variation as quickly as possible and in taking counter measures to stop further occurrence of such defectives. This objective can never be achieved due to rare occurrence of defects in a high sigma process. In practice, the time taken for obtaining this sample size for various process speeds is given in table 1.5. Hence complacency may set in the shop floor due to such rare occurrences due to advertent lack of attention. This may result in slide down of process sigma from six to three rapidly.

Calvin (1983), Goh (1987), Kaminski et al. (1992), Xie and Goh (1992), Glushkovsky (1994) proposed a cumulative count of conforming items between two non conforming items (CCC) chart based on geometric distribution. In their own words 'Geometric distribution is a common distribution in practice although control charts based on such a distribution have not been widely studied'. When the poisson distribution is not suitable for a particular process, geometric distribution can be a good alternative. It is important to be aware of potential problems with traditional control charts and use different alternatives when necessary. They determined that as many types of measurement data follow geometric distribution, it is the distribution for the monitoring of high-yield processes based on the cumulative count of conforming items. When the process is improved, it is easy to switch to the control chart for a cumulative count of conforming items and this measurement follows geometric distribution. The setting up of CCC charts is similar to and as simple as a Shewhart's control chart.

1.6 High Sigma CCC Control Charts

The setting up of cumulative conforming count (CCC) charts is similar to and as simple as a Shewhart's control chart. Let n be the no. of items observed before one nonconforming item occurs (n^{th} item). If probability of a nonconforming item is 'p' then the central line of the control chart,

$$CL = \frac{1}{p} \tag{1.3}$$

Suppose the acceptable false alarm probability is ‘α’ the UCL and LCL are designed as

$$UCL = \frac{\ln\left(\frac{\alpha}{2}\right)}{\ln\left(1-p\right)} \tag{1.4}$$

$$LCL = \frac{\ln\left(\frac{1-\alpha}{2}\right)}{\ln\left(1-p\right)} \tag{1.5}$$

The CCC charts assume a fairly prior knowledge of ‘p’ and ‘α’. The points below lower control limits (LCL) are taken as process deterioration signals. The points above upper control limits (UCL) are taken as a sign of process improvement. The key issue discussed by them is that a process is presumed to be in control until a nonconforming item appears. As the process CCC is having the parameter of geometric distribution, there is a requirement to convert them to normal form. The transforms have been considered as a good option for this conversion process. The charting procedure is explained through a flow chart illustrated in figure 1.6.

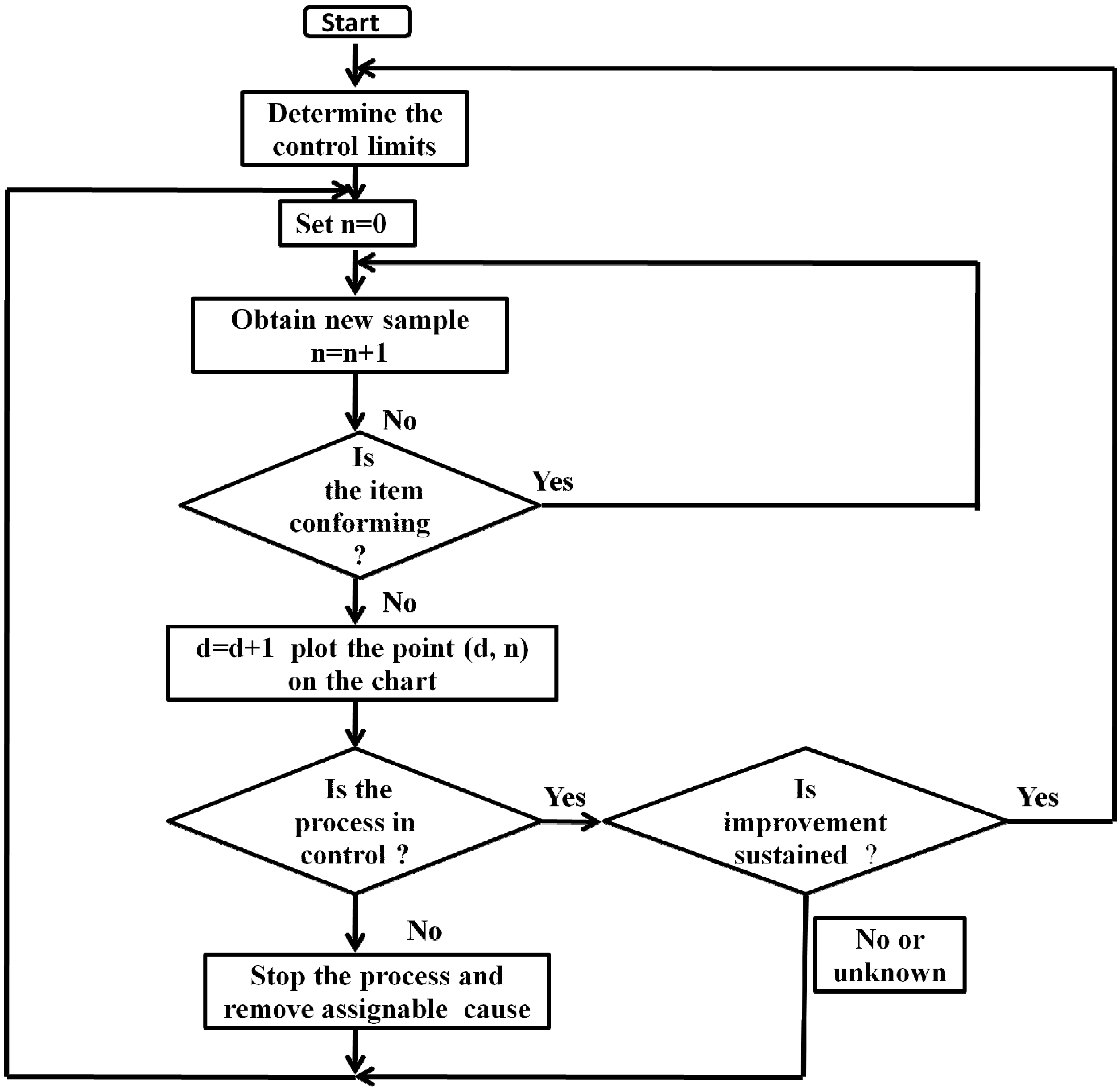


Figure 1.6. A Charting Procedure for CCC Chart.

1.7 Background of Research

All manufacturing systems consist of more than one production stage. The quality of the product is determined by many quality variables and attributes assignable to the production stages. This requires monitoring of multiple quality variables and attributes. A multivariate quality control chart in current literature are unable to pinpoint as to which stage of production process are the variable or attribute is out of control. Regression based cause selecting control chart owing to Zhang et al. (1989) is able to detect the quality nonconformance due to current process step and the previous process step. They are applicable to serially dependent or cascade process. The cause selecting control charts are limited to three sigma processes in current literature.

Globalization and economic boom have compelled continuous improvement of product quality. The manufacturing plants have to aim at ‘world class’ quality and work towards metrics like ‘six sigma’. According to Goh and Xie (2003), in six sigma practice in ‘control phase’ for want of appropriate control chart, only 3 sigma control charts are practiced in six sigma training and practice environ. There have been many contributions from research fraternity in the area of a control chart for six sigma processes. It is a powerful process monitoring tool for high quality, high sigma environ. In this work design and analysis of a control chart for high sigma bivariate linear cascade process has been achieved.

1.8 Objectives of Research

Within limits of overall objective of enhancing the cause selecting control chart methodology to six sigma or high sigma production processes,

i. To design a power transform for conversion of geometric form of high sigma count data to normal form to enable control chart construction.

ii. To ensure that the designed transform enables a near positive LCL.

iii. To incorporate appropriate check mechanisms in the control chart design for accommodating the intercept model errors.

iv. To study and record the influence of model errors on type II error performance.

v. To study the extent of normality achieved with skewness and kurtosis as metrics and to study their influence on LCL and type II error.

1.9 Methodology Adopted

i. Power transform route is adopted for conversion of geometric form of process data to normal data. The superiority of this transform in comparison with logirathmic and double square root transforms are established. Its effectiveness is tested through a case study involving two process scenarios. The first scenario involves the regressor process having higher average run length. The second scenario involves the response process having higher average run length.

ii. As the intercept model error affects the chart statistic their influence on type II error and ARL are studied. The established results are demonstrated through two case studies having an intercept model data and a no intercept model data and validated through data from literature.

iii. Skewness and kurtosis have been considered as measures of normality. Their influence on LCL and type II error has been studied and recorded.

The extent to which normality is achieved is measured with two case studies of process data having an intercept model data and a no intercept model data.

iv. A schematic flow showing the objectives, the issues emanated and faced, tackled and work done are shown.

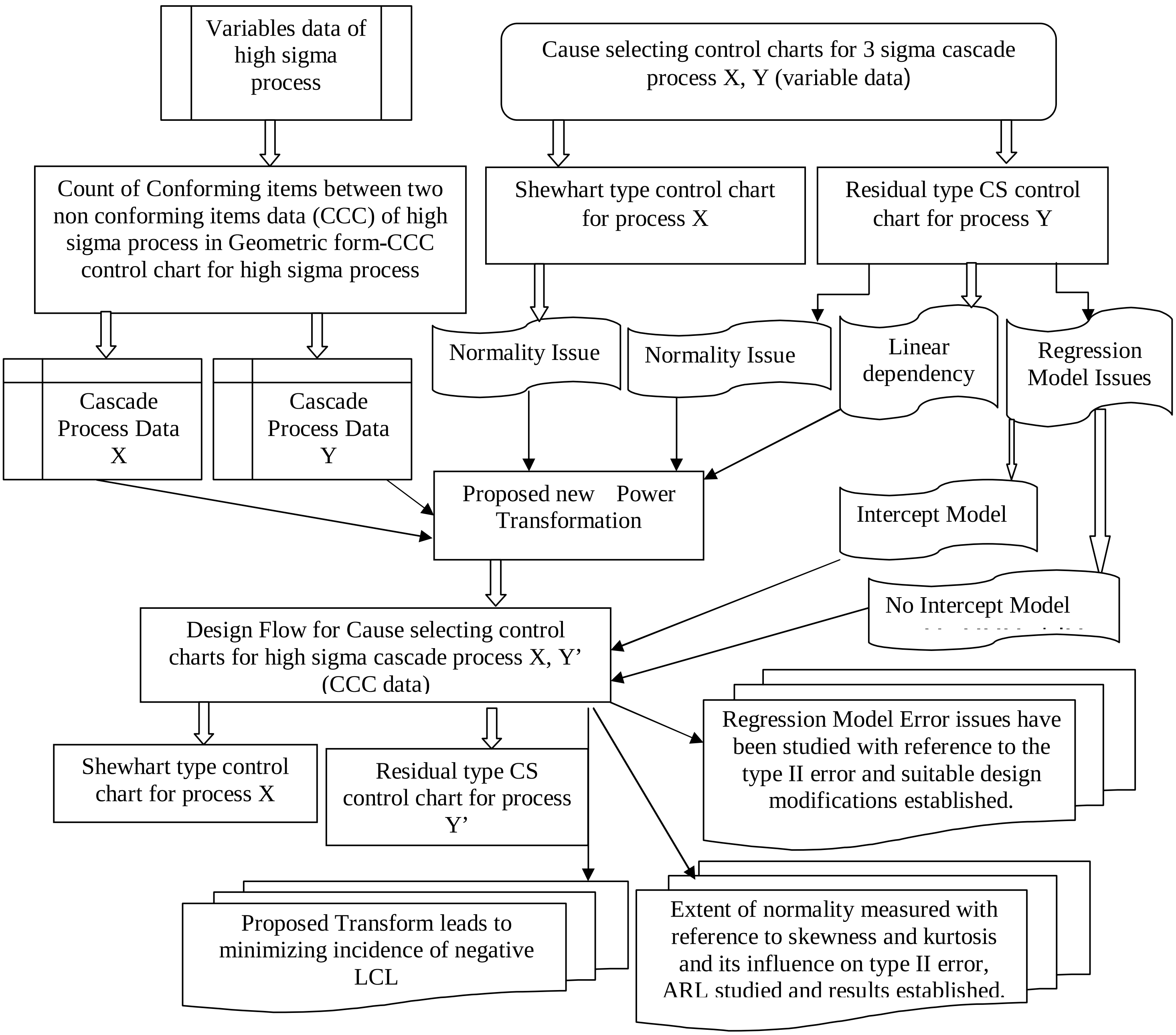


Figure 1.7 Methodology Adopted - Schematic Flow

1.10 Scope and Limitations of the Study

Cause selecting control charts have been proposed for 3 sigma cascade processes. Scope of this work is to enhance the capabiliy of cause selecting control charts to high sigma processes.

This involves first transforming the high sigma data in the form of cumulative counts of conforming products between two non conforming products following geometric distribution into normal form. A high sigma cause selecting control chart design flow has to be proposed.

Secondly as the regression concept is involved, the intercept model issues have to be addressed. Unlike conventional control charts this model error may affect chart statistic. Suitable design modifications are proposed to accomodate intercept model errors. The impact of chart performance on model errors have to be studied.

Thirdly the the extent of transformation to normality has to be measured.Its impac on chart performance indices have to be studied. Conventionally the first and second moments of normality namely the mean and standard deviation are used to ‘describe’ a normal distributon. As they lack inferential properties, the third and forth moments namely the skewness and kurtosis have been adopted to measure the extent of normality and its impact on type II error.

Control chart performance is measured by type I, α (alpha) error or false alarm and type II, β (beta) error or missed signals. We studied the extent of normality as seen from skewness and kurtosis with reference to beta error. A false alarm may be attended but a missed signal may go unattended . In high sigma processes due to scarcity of defects the missed signals are more harmful compared to false alrms and hence the emphasis on beta error.

This work is confined to high sigma bivariate manufacturing processes with linearly dependent cascade properties.

1.11 Organisation of Thesis

This thesis is organized into eight chapters. The first chapter is the introduction. This chapter details the basics of manufacturing variations, limitations of multivariate process monitoring, basics of cause selecting control charts, high sigma control charts, background of the work, objectives of the study, methodology adopted, and scope and limitations of the study and organization of the thesis.

The second chapter details the basic concepts relevant to this work. In this chapter the basics of high yield manufacturing processes, Motorola's six sigma concept, inherent and sustained process capability have been detailed. Details of long term dynamic process variations, basics of normal distribution, binomial and geometric distributions have all been enunciated. The rudiments of control charts and regression analysis which are the key stones of this work have also been detailed.

The third chapter details the review of literature on regression control charts, cause selecting control charts, control chart design methodology, control charts for high sigma process and transformations. It also details the culmination of literature review namely identification of research gap, motivation and drivers for research.

In the fourth chapter a discussion on probability limits and its limitations for high sigma cause selecting control charts has been written. The limitations of 'k' times sigma limits and limitations of traditional control charts for high sigma process have also been detailed. A discussion on transformations is given. A new power transform has been proposed for high sigma process cause selecting control chart. A design flow of charting procedure has been given in detail.

In the fifth chapter regression model error issues, and relative performance of the intercept model based cause- selecting control charts have been enunciated. The suitable design changes have also been established. A detailed design flow modification has been detailed. The model errors and respective changes in performance indices namely type II error and average run length have been studied and results detailed.

In the sixth chapter discussions on optimization of transforms, extent of normality with skewness and kurtosis as indices has been given in detail. The extent of normality have been researched within the compass of intercept model errors and its influence on type II error and average run length have been studied and results recorded

The seventh chapter discusses the issues of economic design, intercept models and lower control limit with particular reference to the results of this work. For the purpose of data and methodology validation, a data set from current literature has been demonstrated in detail. That the stochastic behavior and run rules are non-issues within limits of this work have been explained.

The eighth chapter details the concluding remarks. It also lists the areas for future research.

To sum up this work resulted in the enhancement of application of capability of cause selecting control charts from 3 sigma metric to high sigma for linearly and serially dependant (cascade) bivariate manufacturing processes. To achieve this, a new power transform has been proposed. Intercept model errors have been discussed. The extent of normality has been studied with reference to control chart performance indices.

List of publications and references have been given at the end of the chapters.

1.12 Conclusion

That the cause selecting control chart methodology was limited to 3 sigma process and not adaptable to six sigma process is a research gap. Motivated by this a design for bivariate high sigma cause- selecting control chart is proposed in this work. In doing so the issues like normality characteristic of control chart methodology and intercept model issues relating to regression models were addressed. In the next chapter the basic concepts relevant to this research have been explained.

Chapter 2
Basic Concepts

2.1 Introduction

In this chapter the basic concepts relevant to this work have been enunciated. In a short compass, the basics of high yield manufacturing processes, six sigma concept, inherent and sustained process capability, normal distribution, binomial distribution, geometric distribution, control charts and regression analysis concepts have been explained.

2.2 High yield Manufacturing Process

The High Yield Processes (HYP) by definition, are those processes working above the metric of 3 sigma. A process working on 4.0 sigma metric will yield 6210 defects per million and for Six Sigma Process (SSP) it is 3.4 defects per million. A 3 sigma process will yield 66810 defects per million. The goal of Six Sigma Process (SSP) is to reduce the variability of any process as compared to the process limits to a point where there is room for 1.5 standard deviation move, accounting for the natural variability of the process. Such a case is referred to as a six sigma level of quality, wherein not more than 3.4 defects per million opportunities will fall outside the limits. The defect count level for a normal process and six sigma process has been tabled in table 2.1and table 2.2.

Table 2.1. Defect Counts for a Centered Normal Distribution

Specification Limit ± Sigma	Percentage of Conforming Product	Defective ppm
1	68.27	317300
2	95.5	45500
3	99.73	2700
4	99.9937	63
5	99.99994	0.5
6	99.9999998	0.002

Table 2.2. Defect Counts for a 1.5 Sigma Shifted Distribution

Specification Limit ± Sigma	Percentage of Conforming Product	Defective ppm
1	30.23	697700
2	69.1	308700
3	93.32	66810
4	99.379	6210
5	99.9767	233
6	99.99966	3.4

2.3 Motorola's Six Sigma Concept

Motorola are the pioneers in theorizing the six sigma quality followed by the Allied Signal (Honeywell) and the General Electric. The six sigma concept itself can be defined in two ways. It is the six sigma for the whole organization (define, measure, analyze, improve, control) and six sigma at the machine output level. Confining ourselves to six sigma for the process or machine output, the process capability study assumes importance. Motorola defined that a six sigma process with an intuitive mean shift of 1.5 sigma in the long range will produce 3.4 defects per million outputs. The implied meaning is that the mean will be dynamic and not stable. A graphical depiction of a six sigma process with 1.5 sigma shift is shown in figure 2.1.

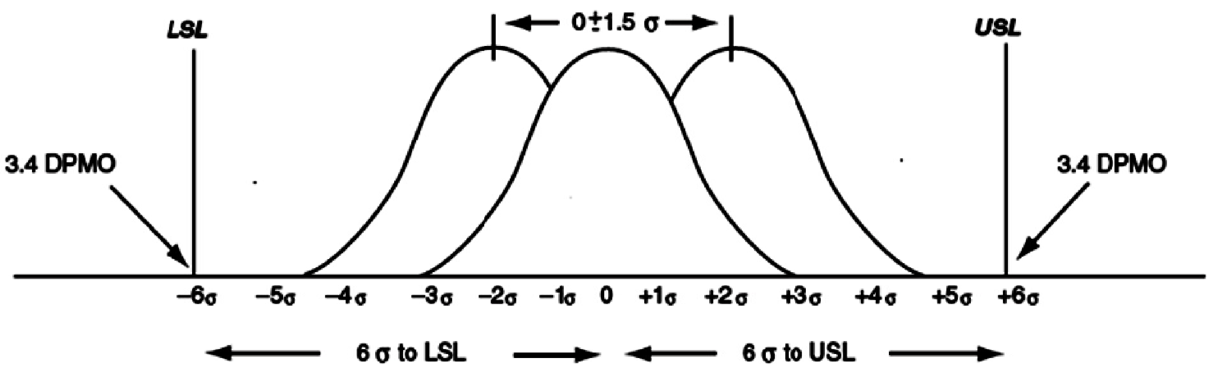


Figure 2.1 '1.5' Sigma Process Shift

Harry (2003) enunciated in detail about the statistical constructs and theoretical basics of six sigma process design and monitoring methodology.

2.3.1 Inherent and Sustained Capability of Process

There are two underlying principles regarding the data in a process capability. The capability indices are effective only if these two assumptions hold true. These are the statistical stability and normality. These variations can also be classified as the inherent process variation and total process variations. Inherent process variations are the variations caused only by common causes and is used to represent the true process capability and potential. It is often estimated from control charts after verifying statistical stability. The long term indices or the performance indices use the total process variance instead of the sample process variance. The process stability index is defined as the ratio between the variance of the sample and the total process variance. The characteristic of inherent and sustained process variation is shown in figure 2.2.

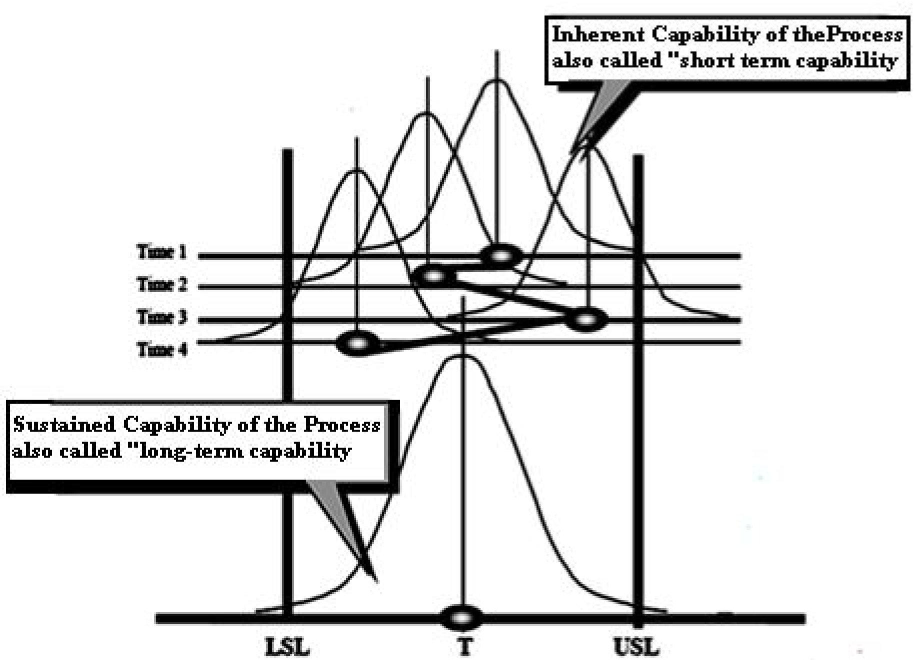


Figure 2.2.Inherent and Sustained Process Capability (Honeywell Corporation)

2.3.2 Long Term dynamic Process Variation

Six sigma actually translates to about 2 defects per billion opportunities, and 3.4 defects per million opportunities, which really corresponds to a sigma value of 4.5. Motorola has determined, through years of process and data collection, that processes vary and drift over time - what they call the Long-Term Dynamic Mean Variation. This variation typically falls between 1.4 and 1.6 and this is where from the difference of ‘1.5’ sigma come from. Figure 2.3 depicts the long term and short term process variation.

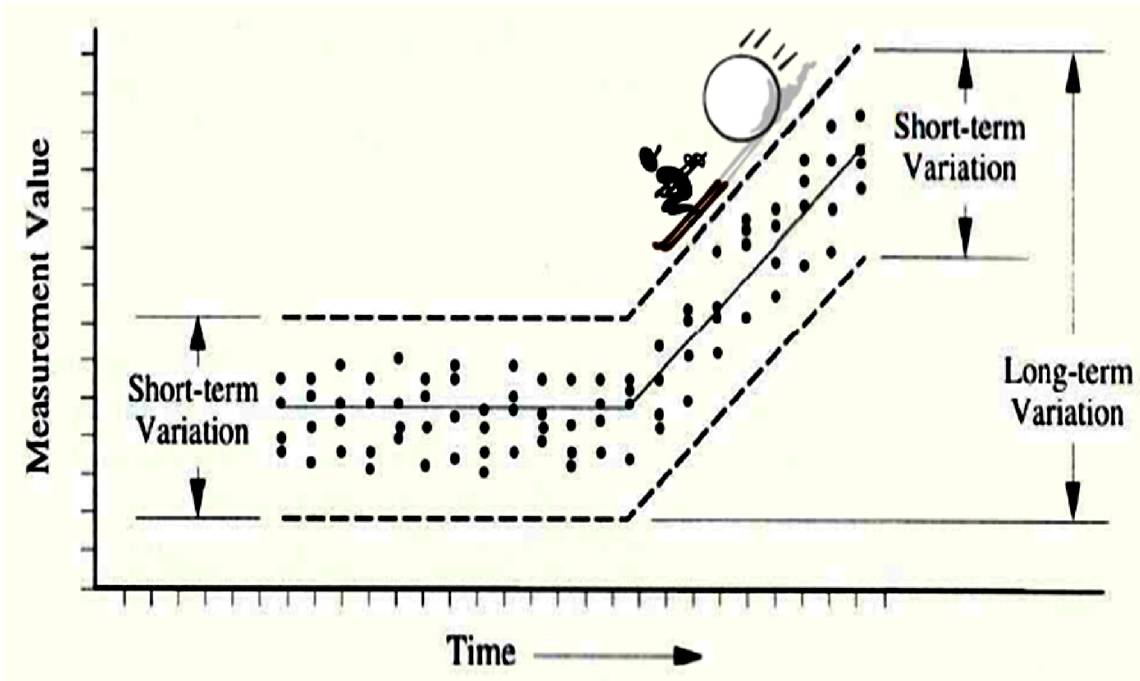


Figure 2.3. Long Term and Short Term Process Variation (Honeywell Corporation)

2.4 Normal Distribution

The normal distributions are very important class of statistical distributions. All normal distributions are symmetric and have bell-shaped density curves with a single peak as shown in figure 2.4. To speak specifically of any normal distribution, two quantities have to be specified. The mean 'μ' where the peak of the density occurs, and the standard deviation 'σ', which indicates the spread or girth of the bell curve. (The Greek symbol μ is pronounced as 'Mu' and the Greek symbol σ is pronounced sig-ma.)

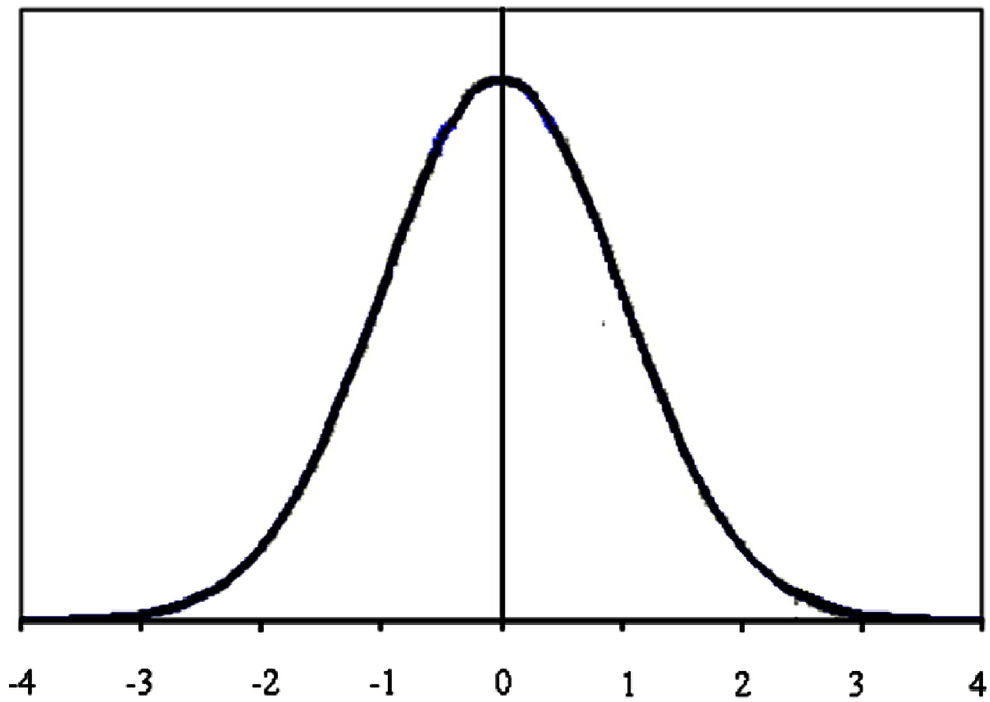


Figure 2.4. Bell Shaped Normal Curve (PDF)

All normal density curves satisfy the following property which is often referred to as the normal density. This can be actually specified by means of an equation. The height of the density at any value X has the following properties :

68% of the observations fall within 1 standard deviation of the mean, that is, between and μ-σ and $\mu + \sigma$.

95% of the observations fall within 2 standard deviations of the mean, that is, between

μ - 2 σ and $\mu + 2\sigma$.

99.7% of the observations fall within 3 standard deviations of the mean, that is, between

μ - 3 σ and $\mu + 3\sigma$

Thus, for a normal distribution, almost all values lie within 3 standard deviations of the mean.

2.5 Binomial Distribution

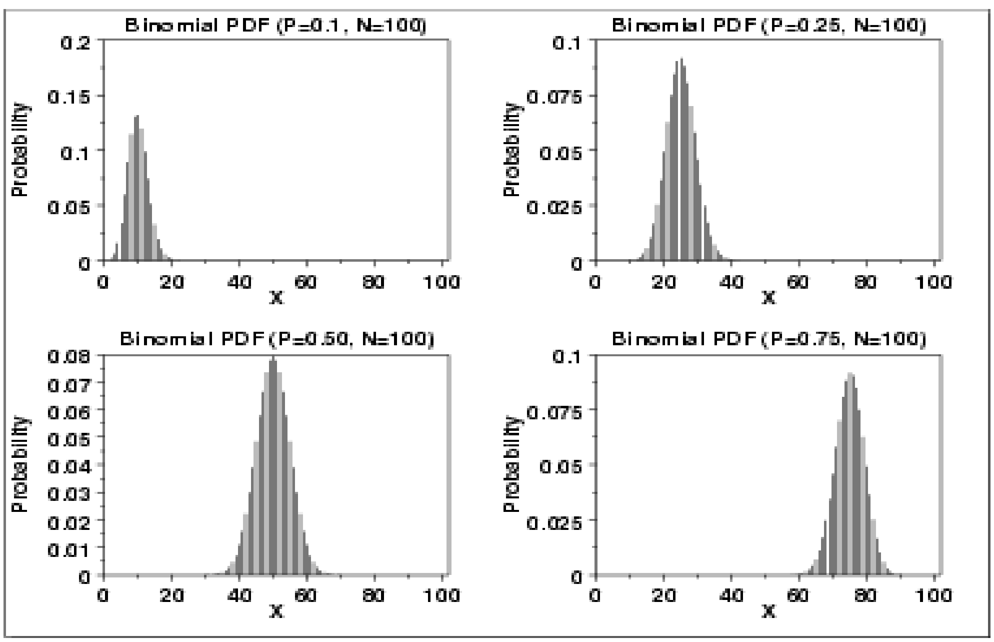


Figure 2.5. PDF of Binomial Distribution (www.itl.nist.gov)

The binomial distribution describes the possible number of times particular event will occur in a sequence of observations. The event is termed as binary; it may or may not occur. The binomial distribution is used when a researcher is interested in the occurrence of an event, not in its magnitude. For instance, in an attribute type inspection it is checked whether the product 'pass' or 'fail' and not the measure of deviation. The binomial distribution is defined by the number of observations, 'n', and the probability of occurrence, which is denoted by 'p'.

2.6 Geometric Distribution

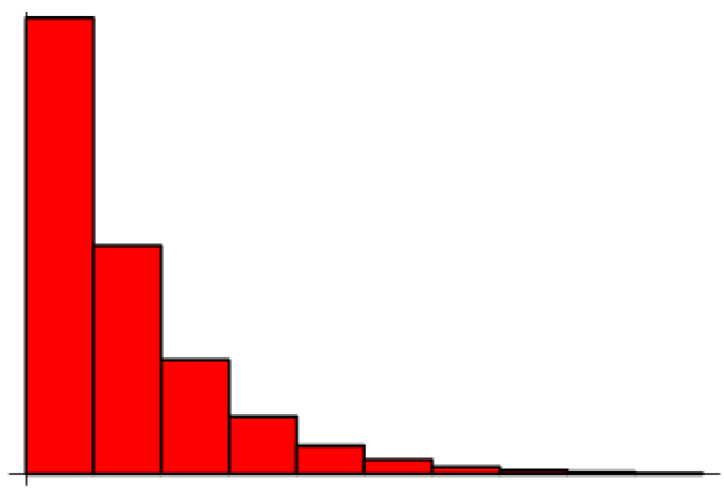


Figure 2.6. PDF of Geometric Distribution

To obtain a binomial random variable, we observe a sequence of 'n' Bernoulli trials and counted the number of successes. If the number of Bernoulli trials is not fixed in advance but continue to observe the sequence of Bernoulli trials until a certain number of 'n' successes occur, the random variable of interest is the number of trials needed to observe the 'n'th success.

Let us consider the first case of the instance when 'n =1'. Consider a sequence of Bernoulli trials with probability 'p' of success and 'q' is the probability of failure. This sequence is observed until the first success occurs. Let X denotes the trial number on which the first success occurs.

For example, if F and S represent failure and success, respectively, and the sequence starts with F, F, F, S… then X =4. Moreover, because the trials are independent, the probability of such sequence is

$$P(X=4) = (q)\,(q)\,(q)\,(p) = q^3 p = (1-p)^3 p. \quad (2.1)$$

In general, the pdf f(x) =P(X=x), of X is given by $f(x) = (1-p)^{x-1} p$, x=1,2,..., because there must be x -1 failures before the first success that occurs on trail x. We say that X has a geometric distribution. It is a case of binomial distribution, where the sample size is large relative to population size.

2.7 Poisson Distribution

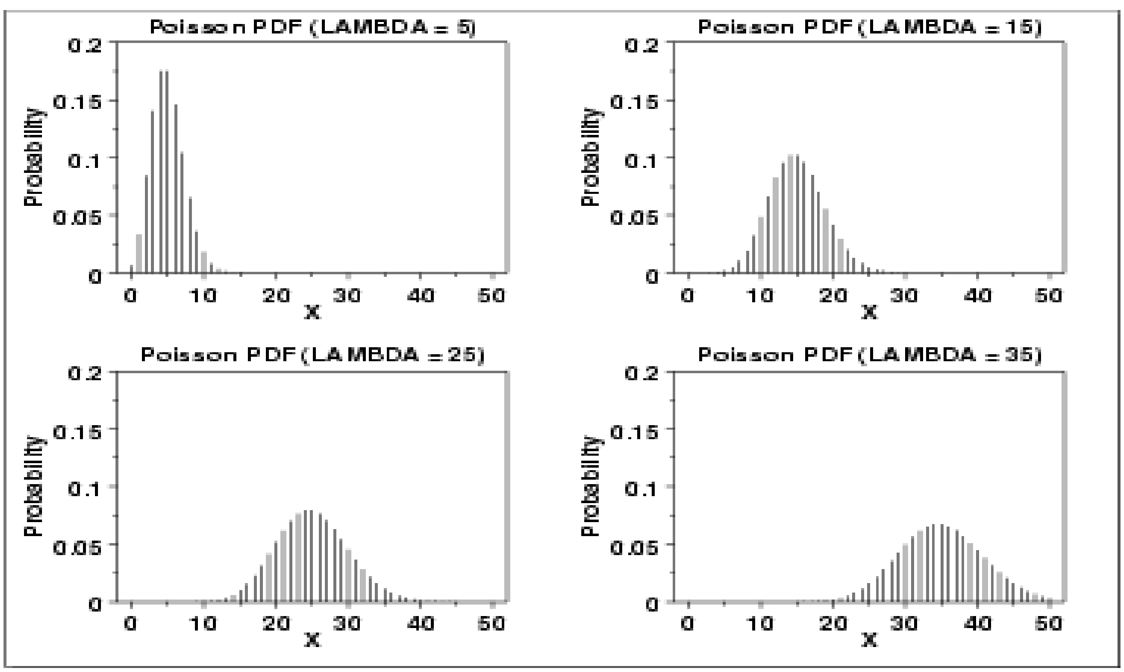


Figure 2.7. PDF of Poisson Distribution (www.itl.nist.gov)

The Poisson distribution is used to model the number of events occurring within a given time interval. When many quality characteristics are to be measured in a product 'C' charts and 'U' charts are deployed. They are modeled after a poisson distribution.

2.8 Control Charts

A systematic approach used to understand that the process is producing without much variability is known as the Statistical Process Control (SPC). The control charts are important tools of the SPC, which is the running record of the job. The primary function of the control chart is to detect causes of variation that are due to some assignable reasons. The random or common cause variability is inherent to the capability of the process. Random variation cannot be eliminated without modifying the process. The system itself must be changed to improve its capability. Assignable cause variability represents a change in the process in a systematic way. This change can be attributed to some identifiable cause or causes that are not an inherent part of the process and can, therefore, be eliminated. The characteristics of causes are shown in table 2.3. Juran (1998), Wheeler (1992, 1993), and Montgomery (2001) detail the causes and the quality journey in detail.

Table 2.3. Natural and Assignable Causes

Natural Causes	Assignable causes
Inherent to process	Exogenous to process
Random	Not random
Cannot be controlled	Controllable
Cannot be prevented	Preventable
Examples	Examples
environment	tool wear
accuracy of measurements	poor setting
capability of machines	poor maintenance

It is worth pointing out that SPC, as a technique is actually process evaluation, rather than control, since it does not directly control the process. It helps determine whether a process is in statistical control and flags out-of-control conditions when they occur. A typical control chart is shown in figure 2.8.

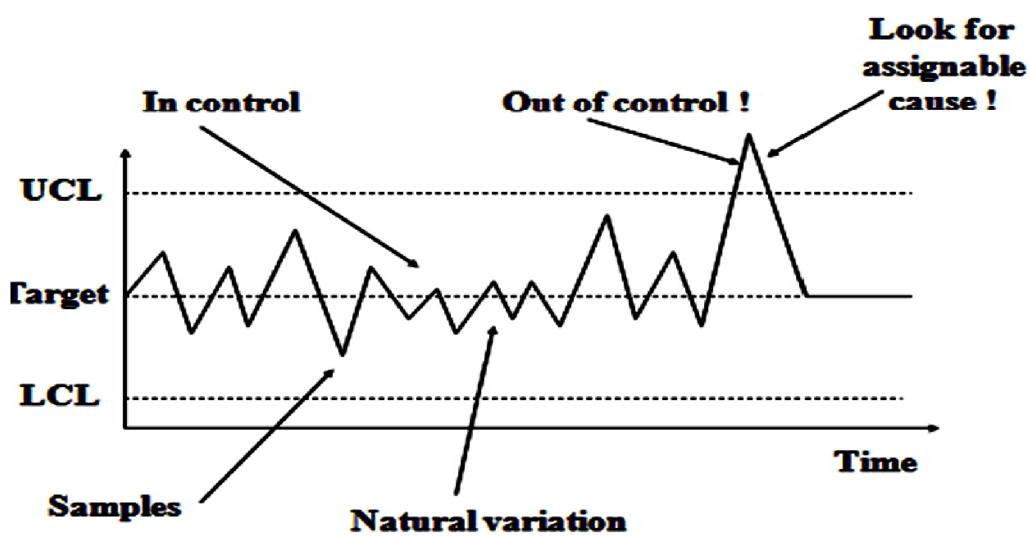


Figure 2.8. Control Chart for Averages

The target or process mean is known as central line (CL) of control charts. The upper control limit (UCL) and lower control limit (LCL) are action limits. Any point falling outside the limits indicates an out of control situation.

Traditional control charts have two objectives. The factory production personnel use it as a monitoring and control mechanism for process and expect it to signal deterioration of process parameters. The management looks at it as a tool for flagging an opportunity for *kaizen* or continuous improvement. It is being used as an important means to develop and sustain improvement in process.

Vermaat et al.(2003) Mast and Roes (2004), Trip and Wieringa (2006), gave a detailed account of robust individuals control chart. Shewhart's (1986) first designed a control chart for averages. This type of control chart is a running record of the job. It records variations due to assignable causes like men, machines, materials, methods and measurement. A typical variation in process data is shown in figure2.9.

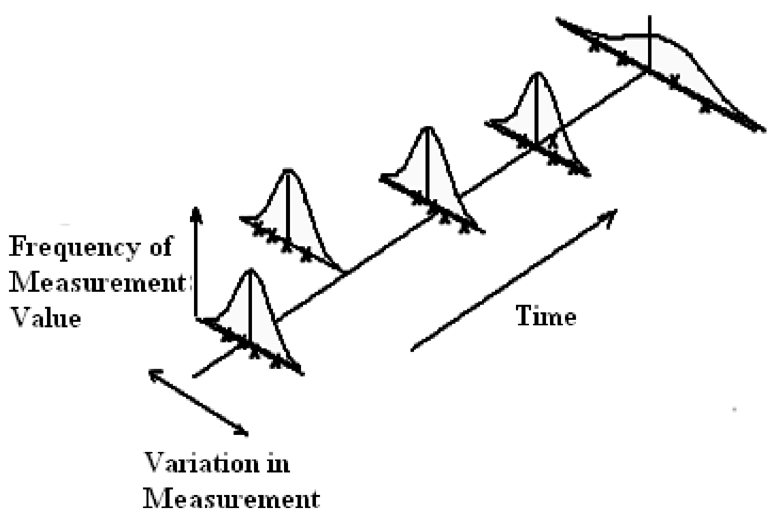


Figure 2.9. Typical Process Variations

2.9 Regression Analysis

The regression control chart of Mandel (1969) was a pioneering work in this field of regression control charts. It combined the power of control chart technique and the least squares regression analysis. Mandel designed a regression control chart which has a trended control limits and central line. The mean shift was measured in terms of the standard error of estimate of the regression line, instead of the conventional standard deviation. In all its aspects it resembled the tool wear type control chart. The regression analysis and its link to analysis of variance were dealt with by Schilling (1974a, 1974b).

Gitlow and Oppenheim (1991) detailed the residual analysis with Shewhart's control chart. They showed that the error term or the residuals distinctly represent the unexplained variations when drawn as a control chart. The basic assumptions on which the concept of regression hinges are:

i. The model errors have mean zero and constant variance, and are uncorrelated.
ii. The model errors have a normal distribution- this assumption is made in order to conduct hypotheses tests and construct confidence intervals. Under this assumption, the errors are independent.
iii. The form of the model, including the specification of the regressors, is correct.

The general least square regression model owing to Montgomery et al. (2006) is,

$$Y = \beta_0 + \beta_1 X_1 + \text{- - - - - - - - - - - - - - -} + \beta_{\boldsymbol{n}} X_{\boldsymbol{n}} + \varepsilon \qquad (2.2)$$

and for a bivariate cascade process it can be simplified as a parsimonious model,

$$Y = \beta_0 + \beta_1 X + \varepsilon \qquad (2.3)$$

Where β_0 is the intercept term and β_1............ β_{n} is the slope,

ε is the error term,

X is the regressor process (upstream process) and

Y is the response process (downstream process).

2.10 Conclusion

This research work is on cause selecting control charts for high sigma process. The basic ideas embracing this work have been presented in this chapter. In the next chapter review of relevant literature undertaken is detailed.

Chapter 3
Literature Review

3.1 Introduction

In current literature, cause selecting control charts have been discussed only for 3 sigma process. Cause-selecting control chart for high sigma serially interdependent processes has been developed in this work. In this chapter the research publications in current literature on cause selecting control charts and high sigma process control charts are reviewed.

3.2 Regression Control Charts

Cause selecting control charts originate from the regression control charts of Mandel (1969). He combined the theory of linear regression and control chart methodology to develop a process monitoring system for postal management application. The Mandel's control chart differed from typical control chart in several respects.

It was designed to control a varying (rather than constant) average. He tried to establish that in the postal delivery process the postman man hours (X) a dependant (regressor) variable serially dependant on an independent (response) variable, namely volume of post (Y). For his control chart the central line is the regression line by equation 3.1.

$$Y=\beta_0 + \beta_1 X + \varepsilon. \qquad (3.1.)$$

Further the control limits are parallel to the central line. His control chart involved cumbersome calculations as compared to the conventional control charts. For standard deviation 'standard error of estimate' (S_e) is used. He placed the control limits at a distance of ± 2 S_e. Mandel's control chart is the first known simple form of a bivariate control chart using regression models. He was motivated by earlier works of DiPaola (1945), Jackson (1956), Wallis and Roberts (1956), Weis (1957). Mandel's control chart is shown in figure 3.1.

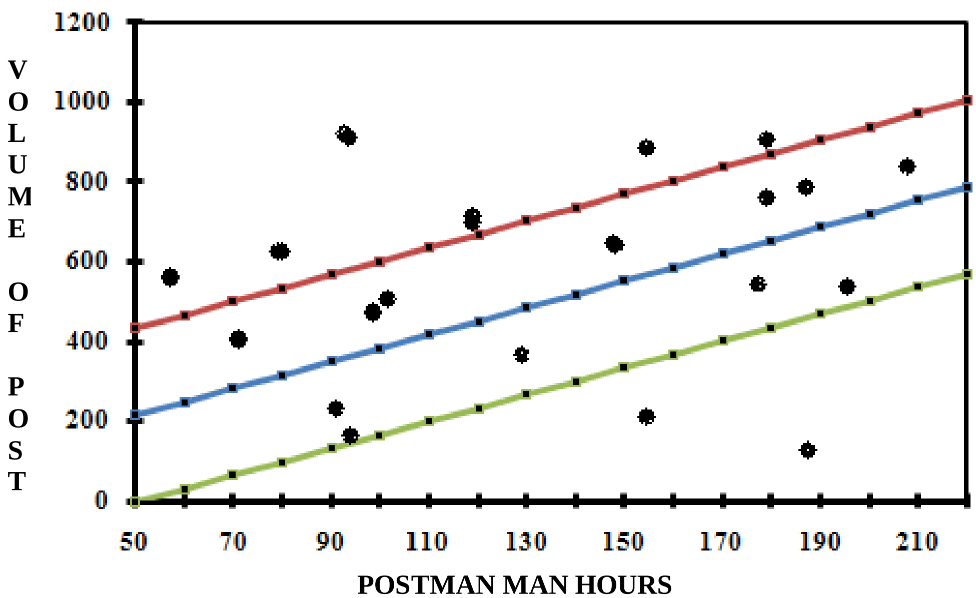


Figure 3.1. Mandel's Regression Control Chart – A Model

3.3 Cause Selecting Control Charts

Zhang (1984), Constable et al. (1987) gave the descriptive details of the concept of cause-selecting control chart. In their work, the terms for 'total quality' and 'specific quality' were defined. A two stage production process was considered. It was assumed that the process was having cascade effect. Cascade effect means that the independent quality characteristic of the first process will tell upon the quality of the dependent characteristic of the second process. As the customer is concerned only with the overall quality or total quality, a multivariate process monitoring system christened as cause selecting control chart system was developed.

They defined ‘specific quality’ as the assignable cause variation in the first and second processes. This was measured by two different Shewhart type control charts each for the processes X and Y. The cascade effects or serial dependence rendered such a process, a candidate for adopting regression analysis for what the authors called diagnosis. By using the word ‘diagnosis’ the authors explained the unique feature of this type of control chart system from traditional control charts. The traditional control chart only signals the occurrence of a nonconforming product. The cause selecting control chart systems goes a step further and diagnoses the root cause for the non-conformance in the downstream process due to an upstream process. This is termed as ‘total quality’ and is monitored by a residuals control chart for process ‘Y’.

Zhang’s (1989) diagnostics inference encompasses three control charts. They are the two assignable Shewhart type control charts for the upstream process X and downstream process Y and a residuals’ type control chart for process Y. This residual type control chart is known as cause selecting control chart. All manufactured products pass through several process steps. There is an ‘overall quality’ due to prior operations and ’specific quality’ due to current operation.

According to him the Shewhart’s control chart can be used to control specific quality of an independent operation but it seldom discriminates between the quality imparted by prior operation and that imparted by the current operation.

The cause selecting control charts discriminate between overall quality due to previous process steps and the current process steps put together. He defined overall quality as quality due to preceding sub process and specific quality due to current sub-process. If X represents quality of the previous process step and Y represents the quality of the current process step and if X and Y are paired and further if Y depends on X then the cause selecting value = $Y - \hat{Y}$ and $\hat{Y}$ is calculated on the basis of the relationship between the independent variable X and dependant variable Y by the method of curve fitting. The chart for residual $Y - \hat{Y}$ is known as the cause selecting control chart. Zhang's diagnostic matrix is detailed in table 3.1.

Table 3.1. Zhang's Cause Selecting Diagnostic Matrix

Process X	Process Y	Process Y Cause Selecting Process	Diagnosis
Signal	Signal	Signal	Process X,Y out of control
No Signal	Signal	Signal	Process Y out of control
Signal	Signal	No Signal	Process X out of control
No Signal	No Signal	Signal	Process Y out of control

Likewise in the downstream process there can be two kinds of variations. They are the variations occurring in downstream process themselves and those resulting from variations in the upstream process. These are defined by Zhang as controllable assignable causes and controllable assignable Cause due to an uncontrollable assignable cause. One more theory proposed by Zhang in his work was the diagnosis of two kinds of process capability which is beyond the scope of our work and hence not detailed.

Zhang (1989) extended the above idea of diagnostics systems to 5 scenarios and the emerging diagnostic matrix is summarised in the table 3.2.

Table 3. 2. Zhang's Modified Cause Selecting Diagnostic Matrix

Process X	Process Y	Process Y Cause Selecting Process	Diagnosis		
			Controllable Assignable Cause	Un Controllable Assignable Cause	Controllable Assignable Cause due to an un controllable assignable cause
Signal	Signal	No signal	Yes	yes	-
Signal	No Signal	Signal	yes	-	yes
Signal	Signal	Signal	yes	yes	yes
No Signal	Signal	Signal	-	yes	yes
No Signal	No Signal	No Signal	-	-	-

Thus in this work Zhang (1989) enlarged the scope of success to diagnose an assignable cause variation in the downstream process both due to upstream process and such of those variations occurring solely in the downstream process. As the product passes through the downstream process, the variation it contains due to upstream process is uncontrollable at that stage and hence the terminology 'uncontrollable assignable causes'.

Wade and Woodall (1993) in their work reviewed the cause selecting chart methodology. They further modified the diagnostic decision rules and simplified it. Their objective was also multivariate process monitoring in nature. They aimed at determining the non-conformance in a succeeding process due to a preceding process. They explained about the redundancy of the assignable control chart of process Y and showed that it has no role in the diagnostic process. Their modified diagnostic chart is shown in table 3. 3.

Table 3. 3. Wade and Woodall Cause Selecting Diagnostic Matrix

Process X	Process Y Cause Selecting Process	Diagnosis
Signal	Signal	Process X,Y out of control
No Signal	Signal	Process Y out of control
Signal	No Signal	Process X out of control
Signal	Signal	Process X, Y in control

They have also established that for this type of diagnosis, cause selecting control chart is superior compared to Hoteling's T^2 control chart.

3.4 Model Issues

Shu and Tsung (2000) in their work discussed about the phase 1 and phase 2 stages of control chart design. They used the prediction limits of the variable $\hat{Y}$ defined as in equation 3.1 and equation 3.2 as the upper and lower control limits to overcome the parameter estimation errors in the regression model respectively.

$$\text{Upper prediction limit} = \text{UCL}= t_{\alpha/2, n-2}\ \hat{\sigma} \sqrt{(1 + \frac{1}{n} + \frac{(x_0 - \widehat{x})^2}{s_{xx}})} \qquad (3.2)$$

$$\text{Lower prediction limit}=\text{LCL}= - t_{\alpha/2, n-2}\ \hat{\sigma} \sqrt{(1 + \frac{1}{n} + \frac{(x_0 - \widehat{x})^2}{s_{xx}})} \qquad (3.3)$$

Where t is the student statistic and, $\hat{\sigma}$ is the estimate of standard deviation, S_{xx} is the cumulative sum of squares of X.

Shu and Tsung (2003) in their work also dealt with model estimation of parameters. They used the method of fault signatures which are time varying to estimate the model parameters. In this regard they proposed the cumulative score and triggered cumulative score control charts to improve the model estimation and in turn the chart performance.

3.4.1 Economic Design

Ingman and Lipnik (2000), Chou et al. (2000, 2001), Kobayashi (2003), in their research considered the economic model for the control chart design. Yang and Chen (2003) considered two dependant process steps obeying. Weibull distribution and cost model and loss function. Quality loss is the loss due to quality variations. Taguchi's (1979), Wu et al. (2004) loss function is the Quadratic loss function sufficiently representing the quality loss suffered by theory. Using this, cost model had been developed and cost control chart has been constructed. By using economic theory they showed that type I error probability can be improved. The economic design is a separate theory and is a different statistical design.

3.4.2 Multicollinearity

Shu et al. (2004a) compared the control chart using the prediction limits and using principal component regression (PCR). They established that the prediction limits show better results in type I error estimates compared to PCR. They considered the canonical form of PCR. This was necessitated due to their model being multivariate cause selecting control chart (and not bivariate). For such problems it is necessary to combat multicollinearity or correlation between regressors. The PCR models effectively counter the multicollinearity.

Shu et al. (2005) compared the design of cause selecting control charts using model estimation error issues. They discussed in detail the effects of estimation of individual model parameters. The aim of their work was to quantify the effect of parameter estimation based on the fitness of the model which will improve the cause selecting control chart performance. They used the sample correlation coefficient as a performance measure to achieve their objective. According to Weisberg (1985) the sample correlation coefficient is the square root of coefficient of determination R, which is defined as

$$R^2 = \sum_{i=1}^{n} (\hat{Y}_i - \bar{Y})^2 / \sum_{i=1}^{n} (Y_i - \bar{Y})^2 \quad (3.4)$$

Where, $\hat{Y}_i$ is the fitted values of Y, $\bar{Y}$ is the mean of values of Y and Y_i is the i^{th} reading of Y,and for the no intercept model, it is

$$R_0^2 = 1 - \sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2 / \sum_{i=1}^{n} (Y_i - \bar{Y})^2 \quad (3.5)$$

Yang and Yang (2006) discussed various types of control charts for determining the controllable assignable cause and uncontrollable assignable cause. They proposed residual type Shewhart's control chart and least square regression residuals control chart. They considered these options based on the assumption that the data from the process are correlated and follow AR (1) time series model.

Yang et al. (2007) in their work considered the measurement error in control chart design. They used an exponentially weighed moving average control chart in the place of regular Shewhart type control chart for the regressor process. They determined that the presence of measurement errors may seriously affect the ability of the proposed control charts. The increase in such errors was shown to decrease the probability of detecting assignable causes. They discussed a EWMA control chart for the first step and Shewhart type control chart for the second step. This work emphasized the requirement of calibration of the measurement systems employed in production processes.

3.5 Control Chart for High Sigma Process

The 'six sigma process', 'high yield process', 'high quality process', 'high sigma process', 'near zero defect process' are all the terms which mean an industrial production process whose output quality is above three sigma. The short term variation in production processes and corresponding defects in parts per million (ppm) at various sigma levels are given below. Motorola Corporation introduced a shift factor of 1.5 sigma to the above values. When certain product deliverable is referred to as six sigma, it never means that the production process that made the product will render only 3.4 defects per million production units.

Giving effect to 1.5 sigma long term process variation the high sigma processes will yield the defect ppm as shown in table 3.4:

Table 3.4. Long Term Sigma Metric

Specification Limit	Percentage	Defective ppm
3 sigma	93.32	66810
4 sigma	99.3790	6210
5 sigma	99.9767	233
6 sigma	99.99966	3.4

Goh and Xie (2003) explains that as the performance of a process is calibrated in defects per million opportunities (DPMO), statistical monitoring and control are usually effected via conventional sample attributes such as np or p where n is the sample size and p is the fraction nonconforming of the process. They further stated that the use of such control charts is an integral part of six sigma training for black belts and green belts and discussions are invariably confined to basic Shewhart's principles. As scarcely any defective or nonconforming items can be found even with large sample size, for high sigma processes Shewhart's type text book control charts are not adequate. Thus Shewhart's control charts have been designed on the basis of 'economical means' of controlling quality. This economic philosophy worked until 3 sigma had been the industry norm. When Motorola's initiative of six sigma process (SSP) came into existence, each defective count mattered.

Calvin (1983) and Goh (1987) proposed a cumulative count of conforming (CCC) chart based on geometric distribution. Jones and Champ (2002) also worked about phase I control chart for time between events. In their own words 'Geometric distribution is a common distribution in practice although control charts based on such a distribution have not been widely studied'. When the Poisson distribution is not suitable for a particular process, geometric distribution can be a good alternative. It is important to be aware of potential problems with traditional control charts and use different alternatives when necessary.

They determined that as many types of measurement data follow geometric distribution, it is a distribution for the monitoring of high-yield processes based on the cumulative count of conforming items. When the process is improved, it is easy to switch over to the control chart for a cumulative count of conforming items and this measurement follows geometric distribution. The setting up of CCC charts is similar to and as simple as a Shewhart's control chart. Let n be the no. of items observed before one nonconforming item occurs (n^{th} item). If probability of a nonconforming item is 'p' then the central line of the control chart,

$$CL = \frac{1}{p} \tag{3.6}$$

And suppose the acceptable false alarm probability is 'α' the UCL and LCL are designed as

$$UCL = \frac{\ln\left(\frac{\alpha}{2}\right)}{\ln(1-p)} \tag{3.7}$$

$$LCL = \frac{\ln\left(\frac{1-\alpha}{2}\right)}{\ln(1-p)} \tag{3.8}$$

The CCC charts assumes a fairly prior knowledge of 'p' and 'α'. The points below LCL are taken as process deterioration signals. The points above UCL are taken as a sign of process improvement. The key issue discussed by them is that a process is presumed to be in control until a nonconforming item appears.

Xie et al. (2002) is a comprehensive compendium on high sigma process monitoring.

3.6 Transformations

Nelson (1994) advocated that a Weibull distribution with a shape parameter of 3.6 skewness and kurtosis of 2.72 is comparable to a normal distribution. Further according to him, the required transformation is simply to raise the inter event count to the power of 1/3.6. Nelson's control chart gave this transform to convert data that fit into an individuals control chart design. According to him, when Y_i is the data point Y at time interval *i* and X_i is the observed data and when,

$$Y_i = X_i \quad \text{where } `i' = 1, 2, 3\ldots\ldots\ldots n \tag{3.9}$$

the design procedure for the control chart for actual data is as follows:

$$\text{Upper Control Limit (UCL)} = \sum (y_i / n) + 2.66\ \text{mR} \tag{3.10}$$

$$\text{Lower Control Limit (LCL)} = \sum (y_i / n) - 2.66\ \text{mR} \tag{3.11}$$

$$\text{and Central Line (CL)} = \sum y_i / n \tag{3.12}$$

and mR is the moving range of Y_i

For interpretation of this control chart, he gave the rule that unlike the 'p' chart, the points above the UCL show improvement in quality and points below the LCL show the deterioration in quality. Thus, this chart gives a signal when any point Y_i is less than the lower control limit as depicted in the equation below,

$$Y_i < Y_i - 2.66\ \text{mR} \tag{3.13}$$

Any point above the upper control limit $Y_i < Y_i + 2.66$ mR is considered as a process improvement. According to McCool And Motley (1998) when the proportion of nonconforming product is extremely small, akin to a high sigma process 3 sigma control charts on the 0.2777^{th} power of X which is the number of items sampled until a nonconforming item is found renders the transformed variable approximately normal in distribution. They studied both this transformation and natural logarithmic transformation and established that both the procedures are identical. They further studied and hypothesized that when using probability limits this power transformation is more preferable when compared to log transformation. They compared the average run length of both the transformations and held that they are identical and using moving range has an implicit dependence on normality and hence it is proposed to use control limits based on power transformation. Another transform that is used in high yield processes is the logarithmic transform Z = ln (X).

McCool and Motley (1998) compared both the transforms in their work and derived their ARL. They concluded that, both the schemes have identical ARLs and further Y has long out of control ARL and Z has short in control ARL. They established that Nelsons' transform forms an approximate normal distribution and shown in figure 3.2. Logarithmic transform is shown in figure 3.3. In figure 3.2 and figure 3.3, X is the observed value and f(Z) is the transformed value.

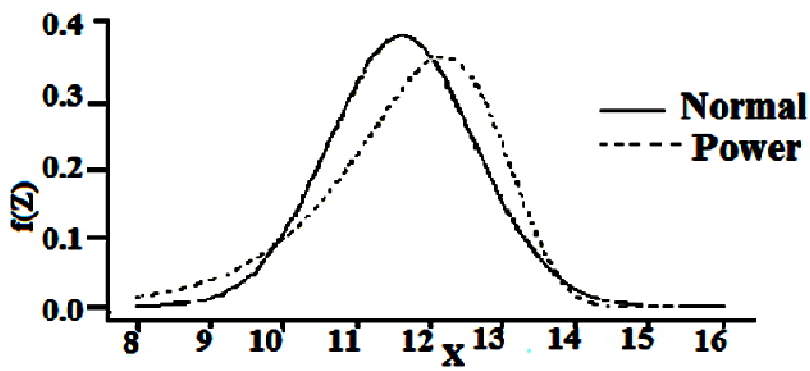


Figure 3.2. Probability Density Function of $Z = X^{0.2777}$

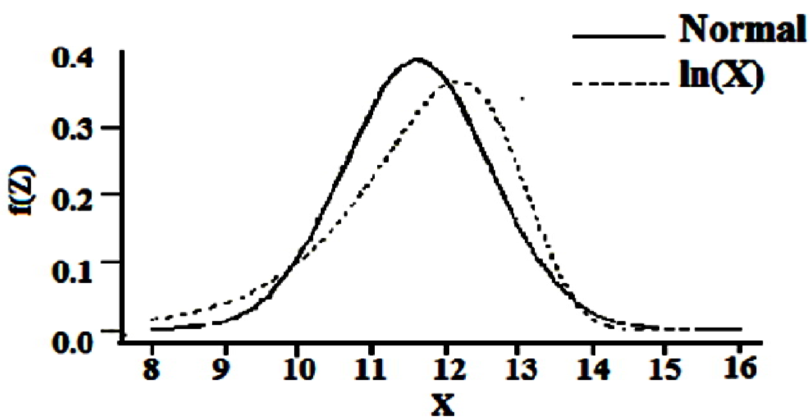


Figure 3.3. Probability Density Function of Z = ln(X)

There is gamut of research publications, and some of them reviewed have been summarized. Lucas (1985) detailed a CUSUM type control chart for high sigma process. Lucas (1989) discussed about low count process control charts. Bourke (1991) designed a run length control chart for high sigma process. Kaminsky et al. (1992) worked on geometric distribution control charts. Goh (1994) discussed in detail the practical issues involving high sigma control charts for process monitoring. Chan and Goh (1997) discussed a high yield process monitoring technique.

Chan et al. (2000) gave the design for a cumulative quality control chart. Sun and Zhang (2000) discussed about cumulative count of conforming control chart (CCC). Xie et al. (2000a) discussed about an optimal setting for control limits for geometric distribution for a high yield process. Xie et al. (2000) discussed about data transformation for high yield geometrically distributed quality characteristics.

Chang and Gan (2001), Chan et al. (2002) discussed about a CUSUM chart for high yield process monitoring. Kuralmani et al. (2002) discussed about a conditional decision procedure for a CCC chart for high yield process. Xie and Goh (1997) discussed about the probability limits for high yield process.

3.7 Motivation

The research issues and publications have been studied and summarised in table 3.5. It can be seen that the CS has been studied and developed for 3 sigma processes only. The basic charts and the important issues like model estimation have not been addressed for 6 sigma processes. Here it is meant that 6 sigma processes are ‘High Sigma Processes’ above 3 sigma metric.

Table 3.5 Research Gap Matrix

Sl.No	Research Issue	Sigma Level	Authors
1	Chart Basics	3	Constable et al.(1987), Zhang(1989), Wade and Woodall(1993)
		6	
2	Process Capability	3	Zhang(1989)
		6	
3	Model Estimation	3	Shu and Tsung(2000), Shu et al.(2004)
		6	
4	Stochastic Process	3	Shu and Tsung(2000), Fen Yang and Ming Yang(2006)
		6	
5	Economic Design	3	Yang and Chen(2003)
		6	
6	Parameter Estimation	3	Shu et al.(2005)
		6	
7	Measurement Errors	3	Yang et al .(2007)
		6	

The research gap in the cause selecting control chart theory for high sigma application is the motivation behind this work. A cause selecting control chart for high sigma bivariate cascade process monitoring with CCC data is proposed. For brevity only a bivariate case was considered. With the advent of computer and software, a bivariate case methodology can be easily applied to a multivariate case. For accomplishing this task the concept of transformation and the CCC chart methodology were applied.

3.8 Drivers

In order to accomplish the objectives stated it was necessary to address the issues involving cause selecting control charts and high sigma control charts. This involves the issues concerning the regression models and control charts theory for high sigma process. The CCC chart is characterized by geometric form of data set.

The typical issues arising in these high sigma control chart are enunciated here under:

i. The data sets are not in the form of normal distribution.
ii. Negative lower control limits have been frequently encountered.
iii. In contrast to the Shewhart's control chart the data point above upper control limit depicts process improvement. Any data point below the lower control limit is termed as process deterioration. They are listed in table 3.6.

Table 3.6 Basic Issues and Assumptions

Sl.No	Character	Assumptions		
		Control Chart	Regression Models	High Sigma Process Control Charts
1	Parameter	Normality Assumption	Normality Assumption	Geometric Distribution
2	Data	Independent	Independent	Independent
3	Model	Normal distribution	Regression Models	Geometric Distribution

In this research work the high sigma cause selecting control chart has been designed. The intercept model issues and chart performance issues have been dealt with.

3.9 Conclusion

From the literature review we conclude that the cause-selecting control chart is a special kind of multivariate control chart. It has superior inferential properties over multivariate control charts. Cause selecting control charts are applied to manufacturing processes with cascade or serially dependant properties. So far these control charts have been proposed for 3 sigma process monitoring. In this work this has been upgraded to high sigma process monitoring. The next chapter details the proposal of a new power transform for enhancing the cause selecting control chart application to high sigma process.

Chapter 4
Transforms for High Sigma Cascade Process

4.1 Introduction

An important assumption with which the control charts are built is that the data are normally distributed. Likewise the basis on which the linear regression models are formed is the assumption that the residuals or error terms are normally distributed. In this work for high sigma environ, a cause selecting control charts which work on the basis of regression models and Shewhart type control charts is proposed. Hence it is obligatory to satisfy both these assumptions for high sigma applications in which the count data are geometrically distributed. A new transform to convert geometric data into normal data is proposed.

4.2 Normality Assumptions

The statistical process control techniques in general and control charts in particular works on the basis of the assumption that the process data is normally distributed. Yourstone and Zimmer (1992) showed that significant departure from normality of data will adversely affect the inferences. It may be recalled that the design and performance criteria for control chart is the run length. This is a function of the parameter of the data set. Hence the importance of assuming normality need not be over emphasized. In the case of a process data set, if a function of the quality characteristic data follows a normal distribution, then it is feasible to work with the function of the data instead of basic data and resolve the normality issue. However such functions must be easy to find out and ready to apply. If cumbersome iterations and calculations are involved, the simplicity and elegance of the control chart procedure will be lost. This will make the control chart inaccessible for the production engineer.

The assumption that the data is normally distributed was first introduced when control charts constants were derived. These values were computed using normal probability distribution. Irving Burr (1967) examined 26 types of distributions which are non normal. Such studies concluded that while the distribution of measurements does affect the percentage of points falling within and outside the control limits; the variation is not appreciable compared to a normal distribution. From this argument while it may sound practical to use any distribution for control charts, using the normal distribution will have some distinct advantages. The normal distribution will facilitate equal control limits thus lending it to hypothesis testing. Further using predominantly skewed distributions like geometric data one will be handicapped with unequal control limits. Such distributions also lead profusely to negative lower control limits. This issue is very much reduced in the case of normal distribution. Thirdly all the model analysis exercises are justified by using normality assumption. Our study of linear regression model has the underlying assumption of normally distributed error terms or residuals. Hence the justification of the need to transform geometrically distributed high sigma count data to normal distribution.

4.3 Limitations of Traditional Control Charts for High Sigma Process

Traditionally for monitoring count type attribute data Shewharts' type 'p'chart, 'c' chart, and 'u' chart were used. As it is well known, the object of using such charts is the desire to monitor the process at low cost as it is very easy to obtain count data. In the case of near zero defect high sigma environment special problems are encountered as the defect counts are in terms of parts per million. These are discussed in the following paragraphs.

4.3.1 'P' Charts

It is known from Xie et al. (2000) and Kaminsky et al. (1992) that the defect counts follow a geometric distribution (a special case of binomial distribution). The normal approximation of such data qualifies them as a candidate for drawing a control chart. In case of 'p' charts when the process approaches near zero defect level, the 'p' or 'np' charts may not satisfy the normal distribution approximation unless the sample size is very large. For its normal approximation to be valid and the above requirement of large sample size to be valid as otherwise, false alarm rate,

$$P(\text{False Alarm}) = 1 - P(LCL \leq X \leq UCL) \tag{4.1}$$

Where p, n are the parameters of the binomial distribution, and

$$\text{Upper Control Limit} = p + 3\sqrt{\frac{p(1-p)}{n}} \tag{4.2}$$

$$\text{Central Line} = p \tag{4.3}$$

$$\text{Lower Control Limit} = p - 3\sqrt{\frac{p(1-p)}{n}} \tag{4.4}$$

As there is scarcity of defects in such high quality process this poses a serious practical problem. For Shewhart's control chart to be valid 'np' should normally be above 5. When p is equal to 3.4 X 10^{-6}, the sample size 'n' will be fairly very large equivalent to 2, 94,000 samples which is impractical and very difficult to obtain.

4.3.2 'C' Charts

A product is said to be defective if there is a non conformance in any of the critical to quality characteristics. The characteristic in which the non conformance is noted is a defect. For analogy variation in diameter is a defect that renders the product defective. The 'p' chart (for fraction defectives) is used whenever the sample size is variable. When the sample size is constant, the np charts are deployed. The 'c' chart is used for number of 'defects'. For constructing a c chart, samples of fixed size 'ℓ' (Quantity of products in each sample) is collected and the number of defects 'c' in each sample is recorded. The control limits are designed as follows:

$$\text{Central Line } \bar{C} = \frac{\text{Number of defects in all samples}}{\text{Total number of samples}} \quad (4.5)$$

$$\text{Lower Control Limit} = \bar{c} - 3\sqrt{\bar{c}} \quad (4.6)$$

$$\text{Upper Control Limit} = \bar{c} + 3\sqrt{\bar{c}} \quad (4.7)$$

4.3.3 'U' Charts

For manufacturing processes where products are produced continuously in which it is impractical to fix the sample size ℓ at each time a control chart for 'u' = c/ℓ can be used. The mean of 'u' = λ and its variance λ/ℓ where $\lambda > 0$ is the defect rate. The control limits of u carts are:

$$\text{Central Line} = \bar{u} \quad (4.8)$$

$$\text{Lower control limit} = \bar{u} - 3\sqrt{\frac{\bar{U}}{\ell}} \quad (4.9)$$

$$\text{Upper control limit} = \bar{u} + 3\sqrt{\frac{\bar{U}}{\ell}} \quad (4.10)$$

According to Chan et al. (2000), the 'c' charts and 'u' charts are unstable as the smaller values of 'ℓ' will result in frequent false alarms and very large values of 'ℓ' will result in few signals which may be missed. Process mean = $\lambda \times \ell$ is large enough, so that Poisson random variable can be approximated as a normal random variable. For high sigma processes defect rates are low and is of the order of parts per million. For analogy, When $\lambda < 0.01$ and when $\ell \leq 500$ which is a case of high sigma process,

$$c < 3\sqrt{\bar{c}} \tag{4.11}$$

$$\bar{u} < 3\sqrt{\frac{\bar{u}}{\ell}} \tag{4.12}$$

In such cases, both lower control limits,

$$c - 3\sqrt{\bar{c}} \tag{4.13}$$

$$\bar{u} - 3\sqrt{\frac{\bar{u}}{\ell}} \tag{4.14}$$

will be negative. It is customary to keep the lower control limits as zero in such cases. In practice, the negative control limits have no relevance as the number of products can never be negative, it also cannot be zero. The performance indices of a Shewhart type ‘c’ chart and ‘u’ chart can never be accomplished. Further the object of continuous improvement of the process cannot be accomplished using a negative value or zero as control limits.

According to Chan et al. (2000) when $\bar{c} < 0.09$, which easily occurs for a process with low λ, we have

$$= \bar{c} + 3\sqrt{\bar{c}} \qquad < 1 \tag{4.15}$$

$$= \bar{u} + 3\sqrt{\frac{\bar{u}}{\ell}} \tag{4.16}$$

$$= \left(\frac{\bar{c} + 3\sqrt{\bar{c}}}{\ell}\right) < \frac{1}{\ell} \tag{4.17}$$

and the upper control limits of the c chart and the u chart are less than 1 and $\frac{1}{\ell}$, respectively. In this case, every occurrence of a single defect in any sample will give c = 1 and u = $\frac{1}{\ell}$, and these values exceed the upper control limit of the ‘c’ chart and ‘u’ chart respectively,which indicates that the process is out of control. This is obviosly an overaction to the occurrence of defects in a stable process with low λ,because in such a process,defects will occur occassionally no matter how small the λ is and as far as $\lambda \neq 0$.

In view of the foregoing shortcomings, the poisson distribution based control charts rendered themselves unsuitable for high sigma process monitoring

4.4 Probability Limits

The Shewhart’s control charts are characterized by ‘k’ times, mean or moving range. Such ‘k’ times type control charts pose problems in the case of high sigma processes where the defect counts are of the order of parts per million (ppm). These types of charts may require very large and impractical number of samples to signal an opportunity for process improvement. One alternative, proposed in current literature is to use probability limits instead of ‘k’ times limits. In this chapter issues involved in using probability limits for high sigma control charts in general and high sigma cause selecting control charts in particular have been discussed.

It is important to be aware of potential problems with traditional control charts and use different alternatives when necessary. Geometric distribution is a common distribution in practice although control charts based on such a distribution have not been widely studied. When the Poisson distribution is not suitable for a particular process, geometric distribution can be a good alternative.

As many types of measurement data follow geometric distribution, it is a distribution for the monitoring of high-yield processes, based on the cumulative count of conforming items. When the process is improved, it is easy to switch to the control chart for a cumulative count of conforming items and this measurement follows geometric distribution. The setting up of cumulative count of conforming chart (CCC) chart is similar to and as simple as a Shewhart's control chart.

The CCC chart assumes a fairly prior knowledge of 'p' and 'α'. The points below LCL are taken as process deterioration signals. The points above UCL are taken as a sign of process improvement. The key issue is that a process is presumed to be in control until a nonconforming item appears.

4.5 Limitations of 'k' - Sigma Limits

If the observation X is geometrically distributed, then we have

$$P(X=x) = p\,(1-p)^{x-1}\ ,\ x= 1,2,\ldots\ldots,i \qquad (4.18)$$

If we have a subgroup of size n, then the total number of counts in the subgroup,

$X = X_1 + X_2 + \ldots\ldots\ldots + X_n$, has a negative binomial distribution.

Assuming that p is known and n is fixed, control limits can easily be obtained. Let B = X – n, the negative binomial distribution can be written in this form,

$$P\,[B = B] \;=\; \binom{n+z-1}{n-1} p^n (1-p)^z \;, \; z = 0,1,2,\ldots\ldots.. \qquad (4.19)$$

The expected value and variance of Z are given by

$$E\,[B] = \frac{n(1-p)}{p} \;, \qquad (4.20)$$

and

$$Var\,(B) = \frac{n(1-p)}{p2} \qquad (4.21)$$

respectively. In general, based on the conventional idea of k-sigma, that is control limits for B computed as the k times the standard deviation, the following control limits are obtained:

$$UCL_k = \frac{n(1-p)}{p} + k\sqrt{\frac{n(1-p)}{p2}} \quad \text{and} \qquad (4.22)$$

$$LCL_k = \frac{n(1-p)}{p} - k\sqrt{\frac{n(1-p)}{p2}} \qquad (4.23)$$

Because the k-sigma idea is based on normal approximation, the sample size has to be large in order that the probability of false alarm to be equal to the case in the conventional Shewhart charts. Stated alternatively, it is well known that for a negative binomial distribution to be approximated by a normal distribution, np has to be large. When p is small, the sample size n has to be very large.

The problem with LCL is a more serious one. In fact, LCL can almost never be greater than 0. In order for LCL to be positive, we have to have

$$\frac{n(1-p)}{p} > k\sqrt{\frac{n(1-p)}{p2}} \tag{4.24}$$

The above inequality can be simplified to be

$$n\,(1\text{-}p) > k^2 \tag{4.25}$$

$$p < 1 - \frac{k^2}{n} \tag{4.26}$$

Suppose that k = 3, and n = 5, which are the standard values commonly used, the value of ‘p’ should be, $p < -0.8$, and this can never happen. Also, when the inspection is continuous, n =1, so that even with k=1, value of ‘p’ should be $p < 0$ which can never be valid.

To sum up using ‘k’ times control limits the traditional control charts will be impractical for high sigma processes. They can only be adaptable to three sigma processes. The second issue is that the LCL can never be greater than zero. This is another issue encountered. Kaminsky et al. (1992), Chan and Goh (1997), Xie and Goh (1997), Chou et al. (1998), Xie et al. (1999),Xie et al. (2000), Kuralmani et al. (2002) , Liu et al. (2005) dealt with the use of probability limits for high sigma control charts.

Further the issue of negative LCL has been resolved by using exact probability limits. When there is prior knowledge of probability indices, exact probability limits have been proposed. If probability of a nonconforming item is ‘p’ then the central line of the control chart

$$CL = 1/p, \tag{4.27}$$

and suppose the acceptable false alarm probability is ‘α’ the UCL and LCL are designed as,

$$UCL = \frac{\ln\left(-\frac{\alpha}{2}\right)}{\ln(1-p)} \tag{4.28}$$

$$LCL = \frac{\ln\left(1-\frac{\alpha}{2}\right)}{\ln(1-p)} \tag{4.29}$$

These limits when used for the known indices, fraction non conforming ‘p’, false alarm probability ‘α’, and the control limits are summarized as per table 4.1.

Table 4.1. Control Limits Based on Exact Probability

Fraction non Conforming 'p'	α	CL	UCL	LCL
0.002	0.01	500	2647	3
0.0015	0.02	667	3068	7
0.001	0.05	1000	3687	25
0.0005	0.0027	2000	13212	3

It can be seen from the table 4.1 that the control limits are unequal. The unequal control limits restricts the basic character of control chart theory, namely,

i. Hypothesis testing

ii. Application of run rules

McCool and Motley (1998) studied the power transform of Nelson (1994) and logarithmic transform and probability based control limits. For known probability they suggested the following equalities for arriving at the control limits

$$\text{LCL} = \left[\frac{-\ln(1-\alpha)}{p}\right]^{0.277} \tag{4.30}$$

$$\text{UCL} = \left[\frac{-\ln(\alpha)}{p}\right]^{0.277} \tag{4.31}$$

Using the above relationships, the control limits for known probability is shown in table 4.2. As LCL is the action limit for high sigma count data only LCL is determined

Table 4.2. Control Limits Based on Known Probability

p	α	LCL
0.0020	0.0100	0.223
0.0015	0.0200	0.271
0.0010	0.0500	0.354
0.0005	0.0027	0.154

4.6 Limitations of Probability Limits

The cause selecting control charts work on the basis of Shewharts type control charts and a cause selecting chart based on residuals. As they work on the basis of regression equations, the probability limits independent of process statistic cannot be used. It may be recalled that the regression model is,

$$Y = \beta_0 + \beta_1 X + \varepsilon \tag{4.32}$$

the estimation of model in any control charting method decides the control limit and chart performance. In case of the regression based cause selecting control charts the model depicted in equation 4.32 decides not only the control limits, but also the charting statistic. The bivariate regression model shown above has,

X the regressor variable

Y the response variable

β_0 the intercept

β_1 co-efficient of regressor X.

ε residual or error term

In the cause selecting control chart methodology a Shewhart type control chart for variables X and Y and residuals control charts are drawn. The residuals chart is known as the cause selecting control chart which decides the quality variation in process Y on account of process X. The probability based control limits being common to all types of charts will not be viable for phase II operation, where the control limits are freezed. In the case of high sigma control charts the probability based limits render the advantage of a positive lower control limit which is the action limit. In the case of residuals control chart the probability based limits may not be realistic as the process data controls not only the chart performance but also the chart statistic. This disadvantage renders the probability limits inappropriate for any regression model based control chart.

With prior knowledge of false alarm probability and fraction nonconforming percentage, probability limits have been shown to be useful for overcoming negative LCL. In the case of high sigma process, as the process statistic is of geometric distribution, which is negatively skewed, the use of probability limits was advocated. However the unequal control limits restricts the use of high sigma control charts for application of run rules and hypothesis testing. As the probability limits are independent of the process statistic, probability limits cannot be readily applied to regression based cascade process monitoring. Another important impediment in using the probability limits for high sigma cause selecting control charts is the intercept model issue. As the model alters the chart statistic, probability control limits cannot be used. In this work a solution is suggested for overcoming this issue through data transformation.

4.7 Transforms Basics

The production process data recorded at intervals form the ingredients of any control chart. The general requirement for such a control chart is that the data is normally distributed. In real process situation there may be instances where it is not practical to satisfy this requirement. When control chart theory is applied to cause selecting control chart this condition is all the more necessary for the least square regression analysis to be valid. This traditional condition is insisted in the case of control chart theory for hypothesis testing. In the case of residual analysis the normality assumption facilitates the testing of hypothesis for arriving at the maximum likelihood estimate.

From the point of view of control charts the need for normal data is important, as it facilitates equal control limits and application of run rules for sensitizing the chart performance. Focusing the attention on the problem of designing a cause selecting control chart for high sigma process the data transformation should follow a normal distribution and it should remedy certain inherent difficulties encountered with the original parameter. It is needless to mention that it should facilitate the implied advantages of a normal distribution. When trying to choose an appropriate transformation only already tried transformations are laded upon.

4.7.1 Box Cox Transformation

Box and Cox (1964) contributed a detailed work on transformations. They discussed about the transformation for a dependant variable and the method of its optimization. Their method of optimization was to consider the following test of hypothesis to find out the maximum likelihood estimate.

$$\widehat{Y^{\lambda}} = \left\{\frac{Y^{\lambda}-1}{\lambda\dot{Y}^{\lambda-1}}\right\} \qquad \lambda \neq 0 \tag{4.33}$$

$$\widehat{Y^{\lambda}} = \widehat{Y} \ln (y) \qquad \lambda = 0 \tag{4.34}$$

4.7.2 Johnson's Curves

Johnson (1949) developed three families of distribution of a variable X which are easily transformed to a standard normal distribution. These distributions are labeled S_B , S_L and S_U. The subscripts B, L, U refer to X being bounded, bounded from below or long normal and unbounded. Farnum (1996) also studied this issue. The type of transformation and underlying conditions are listed in table 4.3.

Table 4.3. Johnson's System of Transformation

Johnson Family	Transformation	Parameter Conditions	X conditions
S_B	$Z=\gamma+\eta \ell n\left(\frac{X-\epsilon}{\lambda+\epsilon-X}\right)$	$\eta, \lambda > 0, -\infty < \gamma < \infty, -\infty < \epsilon < \infty$	$\epsilon < X < \epsilon + \lambda$
S_L	$Z=\gamma+\eta \ell n(X - \epsilon)$	$\eta > 0, -\infty < \gamma < \infty, -\infty < \epsilon < \infty$	$X > \epsilon$
S_U	$Z=\gamma+\eta \sinh^{-1}\left(\frac{X-\epsilon}{\lambda}\right)$	Same as S_B	$-\infty < X < \infty$

Detailed discussions of Johnson (1949) can be found in Slifker and Shapiro (1980), Bowman and Shenton (1983), and Stuart and Ord (1987), Hahn and Shapiro (1994), Johnson, Kotz and Balakrishnan (1994),

The objective of transform is to achieve normality and linearity. In the context of regression models the transform can be used to perform variance stabilization of the regressor, and to make the correlated variables of the regressor into independent data. Further the colinearity between two regressor variables can be overcome with transforms. Transforms can also be deployed for response variables. In a bivariate high sigma process count of conforming items between two nonconforming items is the data set collected. It is in the form of a geometric distribution. For converting the geometric distribution into normal form a transform is required. For cause selecting control charts for such applications a transform is proposed in this work

4.8 Geometric Distribution of High Sigma Data

Manufactured products are subjected to two types of inspection. The variable inspection is carried out on dimensional data and attribute inspection is carried out on binary ‘pass or fail’ basis. Dimensional characteristic inspection can be summarized as ‘within specification or out of specification’ style of attribute information. When critical pats are inspected and binary if a ‘pass or fail’ criterion is applied, the data modeled is Binomial distribution.

In the case of mass produced products, the economical way of inspection is the sampling process. As 100 % inspection is not feasible, a representative sample ‘n’ from a population is taken. This sample is inspected and the number of parts which fail to meet the specification known as the fraction of non conforming product, ‘p’ is calculated. When ‘n’ is very large compared to the population, say of the order of 10 % or more, such binomial distribution is known as Geometric distribution.

As the process sigma level increased in tandem with manufacturing technology and automation, the inspection systems have also improved. Manual inspection has given way to ‘online’ and automated inspection. This has opened the flood gates of product and process information, online in ‘real time’. Hence the logic and principles of ‘sampling’ and ‘piece to piece’ inspection has also changed. In high sigma manufacturing like the six sigma process, count of conforming items between two non conforming items is the characteristic that forms the building blocks of the control chart. This is nothing but the average run length. In such a case, 100 % of the population is sampled and the resulting parameter is a geometric distribution.

The geometric distribution for count of conforming items between two non conforming items has been dealt with by Kaminsky et. al. (1992), Xie et.al. (2000), Sun and Zhang (2000), Ranjan et al. (2003) and others. The geometric distributions being negatively skewed have the characteristic of negative lower control limits when organized into a control chart. Xie et al. (2000) proposed double square root transforms and logarithmic transforms for overcoming this difficulty. The logarithmic transform is characterized by the equation,

$$X\ (\text{transformed}) = \ln(X) \qquad (4.35)$$

The double square root transformation is akin to power transformation except that the value of power is $\frac{1}{4}$ i.e. .25.

$$X\ (\text{transformed}) = X^{0.25} \quad (4.36)$$

According to Nelson (1994) when the fraction nonconforming is small to the order of parts per million (ppm), the data can be transformed to $\frac{1}{3.6^{\text{th}}}$ power (0.277) for converting it into a normal distribution. It is of the form,

$$X\ (\text{transformed}) = X^{0.277} \quad (4.37)$$

The upper control limits (UCL) and lower control limits (LCL) were calculated by him as follows:

$$UCL = \text{Process Mean} + 2.66\ mR \quad (4.38)$$

$$CL = \text{Process} \quad (4.39)$$

$$LCL = \text{Process Mean} - 2.66\ mR \quad (4.40)$$

Any point above UCL is considered a process improvement and any point below LCL is deemed as process deterioration. While building a chart for a univariate process, using transforms will render the LCL positive as per condition:

$$\text{Process mean} > 3\ \text{SIGMA} \quad (4.41)$$

While using 3 sigma limits,

$$LCL = \text{Process Mean} - 3\ \text{SIGMA} \quad (4.42)$$

and for LCL to be positive,

$$\text{Process mean} > 3\ \text{Sigma} \quad (4.43)$$

4.9 Transforms for Residuals Control Chart

As the cause selecting control chart is a residuals chart, the difficulty arose in using the conventional transforms like power, logarithmic and double square root transforms. The fitted regression line process data for the dependent process chart is normally higher than the native dependant process data. Hence the resultant residuals have smaller mean and it is difficult to overcome the problem of negative LCL and the equation 5.26 and equation 5.28 are difficult to satisfy.

For analogy we have studied a precision pin manufacture cycle, shown in figure 4.1 comprising of metal drawing and metal forming processes. Two situations have been studied. Motivated by Xie et al. (2000) data two types of process situations are considered. At 50ppm the high sigma process count of conforming chart gives the highest run length of 500 for a percentage defective of 60. For a percentage defective of 200 the run length is 180. The process data is listed in table 4.4.

Table 4.4. Count of Conforming Items Data for Pin Manufacture

Data No.	1	2	3	4	5	6	7	8	9	10
Cold Drawing process	401	456	743	543	358	407	831	409	642	515
Cold Forming Process	332	31	113	306	257	58	53	259	211	301

Table 4.4. Count of Conforming Items Data for Pin Manufacture (continued)

Data No.	11	12	13	14	15	16	17	18	19	20
Cold Drawing process	662	354	433	364	793	620	751	352	323	323
Cold Forming Process	129	321	58	303	127	172	269	149	44	130

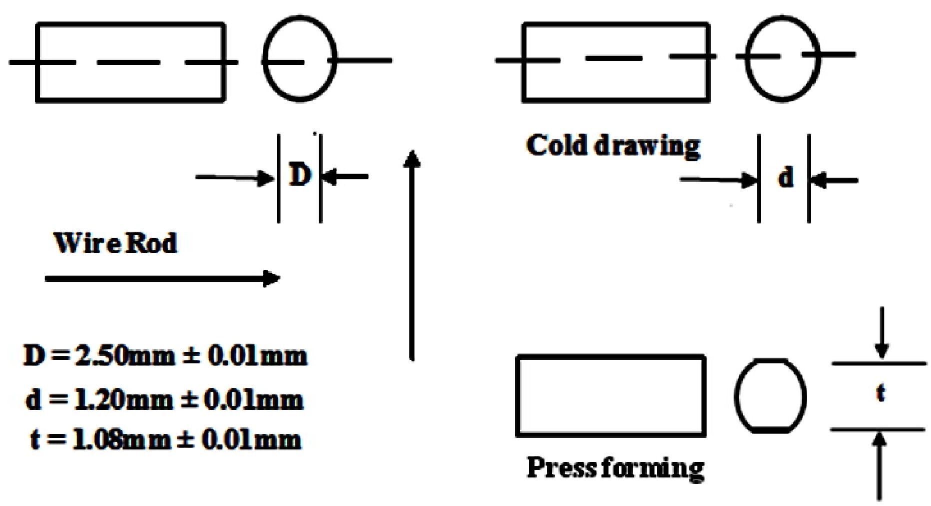


Figure 4.1. High Sigma Bivariate Production Process.

4.10 Design Flow A schema of flow diagram depicting the above process steps is shown in figure 4.2.

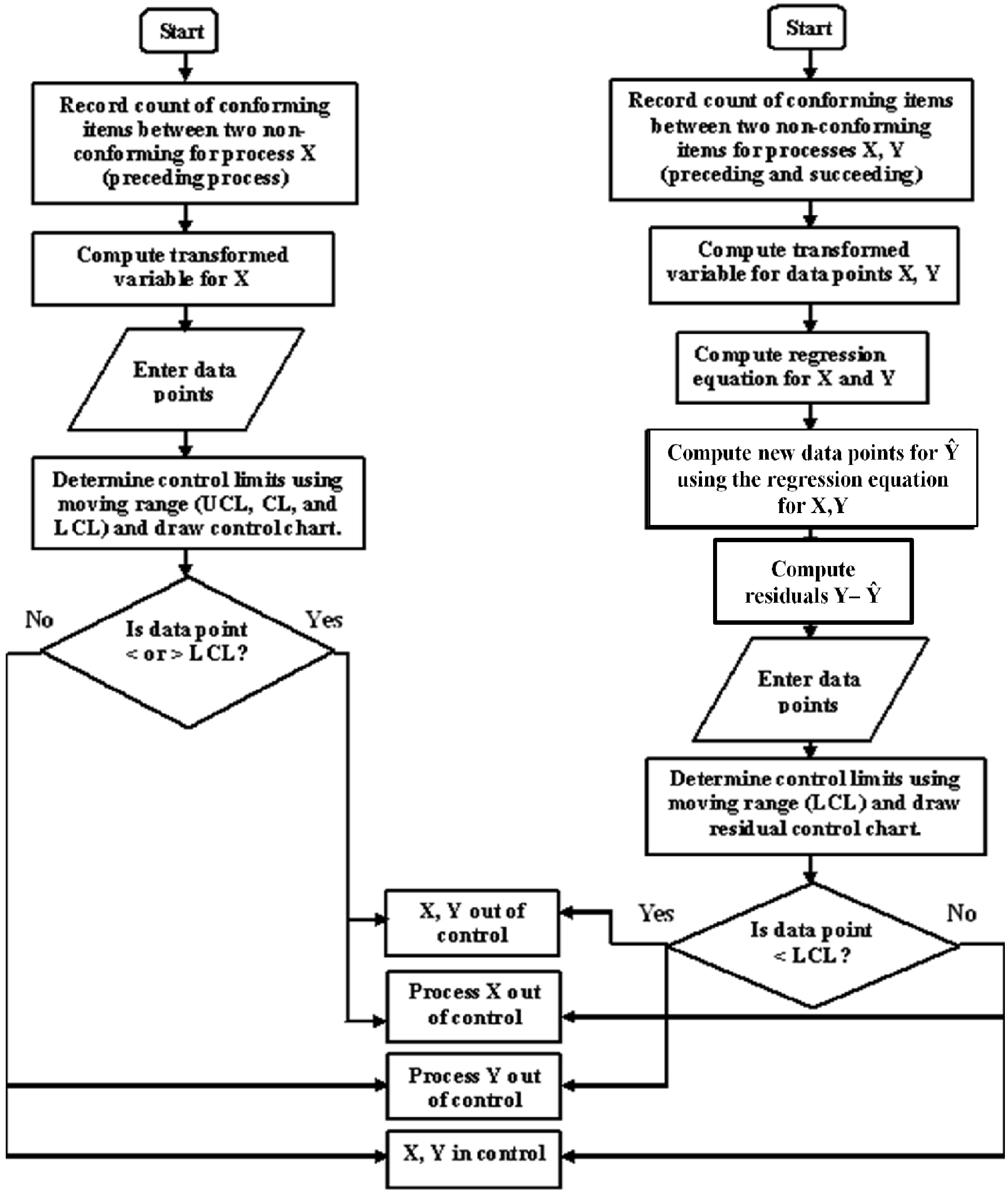


Figure 4.2. Design Flow

The brief procedure for building a cause selecting control chart for a bivariate high sigma process is enunciated hereunder:

i. Collect data on count of conforming items between two non conforming items for the two processes.

ii. Using power transforms convert process data for getting a positive LCL.

iii. Using the above transformed values, construct a Shewhart type control chart X with moving range.

iv. As a next step the cause selecting control chart is designed. The data X, Y are transformed and linear regression line has been fitted in the form

$$Y = a + b X \tag{4.44}$$

v. Using the transformed value Y it is possible to get the value $\hat{Y}$ applying the fitted regression line equation.

vi. Now the residue Y-$\hat{Y}$ is used to construct a residual control chart known as cause selecting control chart using, Cause selecting data

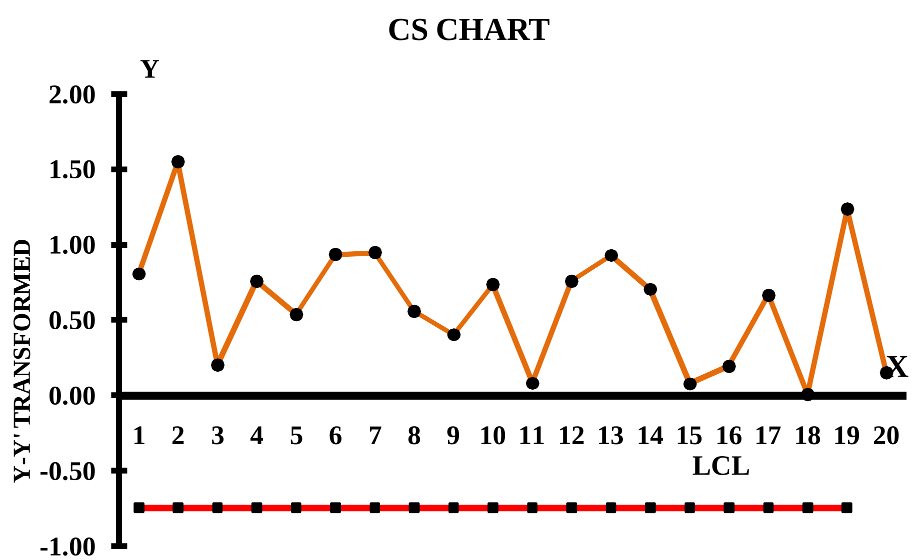
$$Z_n = Y_n - \hat{Y}_n \quad \text{where 'n' is } 1,2,3 \ldots\ldots\ldots n \tag{4.45}$$

Two types of transforms namely double square root transform and logarithmic transforms are considered following equations 4.35 and 4.36. The process data which is geometrically distributed has been transformed .The resulting scenario are discussed. In scenario 1, the metal drawing process having an average run length of 500 and metal forming having average run length of 180 and the resulting LCL is shown in table 4.5.

Table 4.5. Lower Control Limit Data I with Transforms

Transform	Regression equation	LCL
Logarithmic	Y = 5.64 - 0.10 X	–0.744
Double Square Root	Y = 4.12 - 0.13 X	–0.561

The cause selecting control chart for this scenario using logarithmic transform is shown in figure 4.3. The transformed data is shown in table 4.6



Sl. No	Residuals Y-Y'	LCL	Sl. No.	Residuals Y-Y'	LCL	Sl. No	Residuals Y-Y'	LCL	Sl. No	Residuals Y-Y'	LCL
1	0.81	-0.74	6	0.94	-0.74	11	0.08	-0.74	16	0.20	-0.74
2	1.55	-0.74	7	0.95	-0.74	12	0.76	-0.74	17	0.67	-0.74
3	0.20	-0.74	8	0.56	-0.74	13	0.93	-0.74	18	0.01	-0.74
4	0.76	-0.74	9	0.41	-0.74	14	0.70	-0.74	19	1.24	-0.74
5	0.54	-0.74	10	0.74	-0.74	15	0.08	-0.74	20	0.15	-0.74

Figure 4.3. Cause Selecting Control Chart –Logarithmic Transform

Table 4.6 Transformed Process Data for Pin Manufacture

Data No.	Cold Drawing	Cold Forming	Logarithmic Transform for Cold Drawing	DSR for Cold Drawing	Logarithmic Transform for Cold Forming	DSR for Cold Forming
1	332	401	5.805	4.269	5.994	4.475
2	31	456	3.434	2.36	6.122	4.621
3	113	743	4.727	3.26	6.611	5.221
4	306	543	5.724	4.182	6.297	4.827
5	257	358	5.549	4.004	5.881	4.35
6	58	407	4.06	2.76	6.009	4.492
7	53	831	3.97	2.698	6.723	5.369
8	259	409	5.557	4.012	6.014	4.497
9	211	642	5.352	3.811	6.465	5.034
10	301	515	5.707	4.165	6.244	4.764
11	129	662	4.86	3.37	6.495	5.072
12	321	354	5.771	4.233	5.869	4.338
13	58	433	4.06	2.76	6.071	4.562
14	303	364	5.714	4.172	5.897	4.368
15	127	793	4.844	3.357	6.676	5.307
16	172	620	5.147	3.621	6.43	4.99
17	269	751	5.595	4.05	6.621	5.235
18	149	352	5.004	3.494	5.864	4.331
19	44	323	3.784	2.576	5.778	4.239
20	130	323	4.868	3.377	5.778	4.239

In scenario 2, the metal drawing process having an average run length of 180 and metal forming process having an average run length of 500, is studied and the resulting scenario is shown in table 4.7.

Table 4.7. Lower Control Limit Data II with Transforms

Transform	Regression equation	LCL
Logarithmic	Y = 6.29-0.02 X	–0.219
Double Square Root	Y =3.25+0.39 X	–0.509

We can infer from table 4.5 and table 4.7 that the cause selecting control chart for residuals is not letting itself to satisfy the relationship described in equation 4.36 and equation 4.37, the issue of negative LCL requires further investigation.

4.11 Proposed Transform

The count of conforming items between two non conforming items forms a skewed geometric distribution. This has the disadvantage of negative lower control limit. In decision making process for these charts, any point above the UCL is considered a process improvement and any point below LCL is considered process deterioration. There cannot be a negative number of products and the negative control limits are impractical. To overcome this many researchers have proposed probability based control limits. But they are not equal with reference to the Center line of the control chart. Hence from practical view point, design and application of run rules and interpretation are difficult. Further direct depiction of meaning of data is also lost. Consideration of using transforms was thought of as an alternate to overcome the negative LCL.

In the foregoing discussion it is shown that the logarithmic and double square root transformation are having limitations for residual control chart type design of cause selecting control charts. Hence various transforms have been tried out and the resulting LCL for proposed transform is listed in table 4.8. The transform is of the general form:

$$X\ (\text{transformed}) = X^{1/r} \text{ where } r > 2 \qquad (4.46)$$

A study is conducted as to the applicability of the appropriate value for 'r' which will render the positive value for LCL. Various values of 'r' are considered. Simulation was done for 2 cases of the process situation. Situation 1 comprises of the cold drawing process having better run length compared to cold forming process. It is seen that the transform,

$$X\ (\text{transformed}) = X^{1/100} \text{ leads to a positive LCL} \qquad (4.47)$$

The transformed data is shown in table 4.8 below.

Table 4.8. Transformed Data of Process With Proposed Transform

Data No.	Cold Drawing	Cold Forming	Proposed Transform for Cold Drawing	Proposed Transform Cold forming
1	332	401	1.06	1.062
2	31	456	1.035	1.063
3	113	743	1.048	1.068
4	306	543	1.059	1.065
5	257	358	1.057	1.061
6	58	407	1.041	1.062
7	53	831	1.041	1.07
8	259	409	1.057	1.062
9	211	642	1.055	1.067
10	301	515	1.059	1.064
11	129	662	1.05	1.067
12	321	354	1.059	1.06
13	58	433	1.041	1.063
14	303	364	1.059	1.061
15	127	793	1.05	1.069
16	172	620	1.053	1.066
17	269	751	1.058	1.068
18	149	352	1.051	1.06
19	44	323	1.039	1.059
20	130	323	1.05	1.059

The LCL value with various transforms and its sensitivity is shown in table 4.9 and table 4.10 respectively for the two process scenarios. The first scenario is the situation in which the upstream process having higher ARL and the second scenario is the situation in which the downstream process is having higher ARL.

Table 4.9. Transformed Data with Proposed Transform – Process Situation 1

r	Transform	Cold Drawin g	Cold Forming	LCL Process Situation I	Sensitivity	
					Δr/r	ΔLCL/ LCL
4	0.25	200	60	-1.4	0	0
6	0.167	200	60	-0.6	50	57
8	0.125	200	60	-0.36	100	74
10	0.1	200	60	-0.25	150	82
50	0.02	200	60	-0.02	1150	99
60	0.017	200	60	-0.004	1400	100
70	0.014	200	60	-0.003	1650	100
80	0.013	200	60	-0.003	1900	100
90	0.011	200	60	-0.003	2150	100
100	0.01	200	60	-0.002	2400	100

Table 4.10 Transformed Data with Proposed Transform - Process Situation 2

r	Transform	Cold Drawin g	Cold Forming	LCL Process Situation II	Sensitivity	
					Δr/r	ΔLCL/ LCL
4	0.25	60	200	-0.561	0	0
6	0.167	60	200	-0.256	50	54
8	0.125	60	200	-0.161	100	71
10	0.1	60	200	-0.115	150	80
50	0.02	60	200	-0.016	1150	97
60	0.017	60	200	-0.013	1400	98
70	0.014	60	200	-0.011	1650	98
80	0.013	60	200	-0.01	1900	98
90	0.011	60	200	-0.009	2150	98
100	0.01	60	200	-0.008	2400	99

Situation 2 comprises of the succeeding process, namely the cold forming process having better ARL. Less healthy preceding process and healthier succeeding process showed that the transform given in equation (4.47) gives best results.

The resulting characteristic is shown in figure 4.4. It can be seen that as the 'r' value increases the probability of getting a positive LCL increases.

As the value of 'r' increases or the value of transform decreases the LCL approaches zero. In both the cases optimum results are seen to be obtained when transform is of the form $\frac{1}{100^{th}}$ power of the characteristic as seen in figure 4.4.

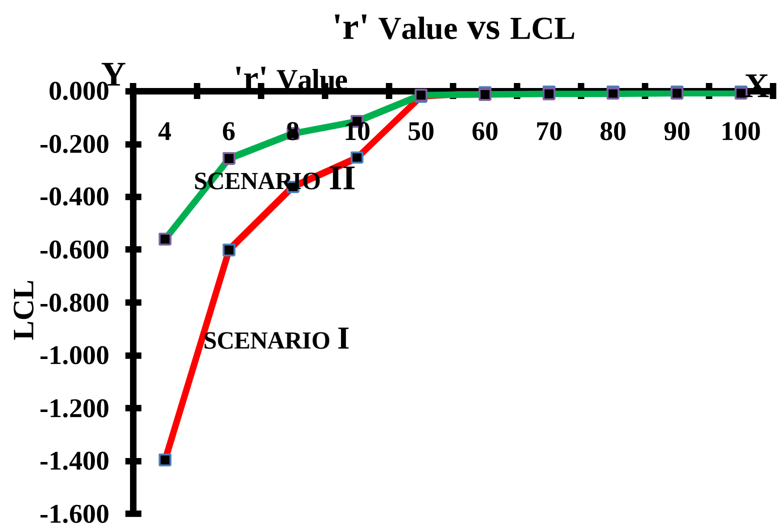


Figure 4.4. 'r' value –LCL Relationship

Figure 4.5 shows a normal distribution of data converted from geometrically distributed process data.

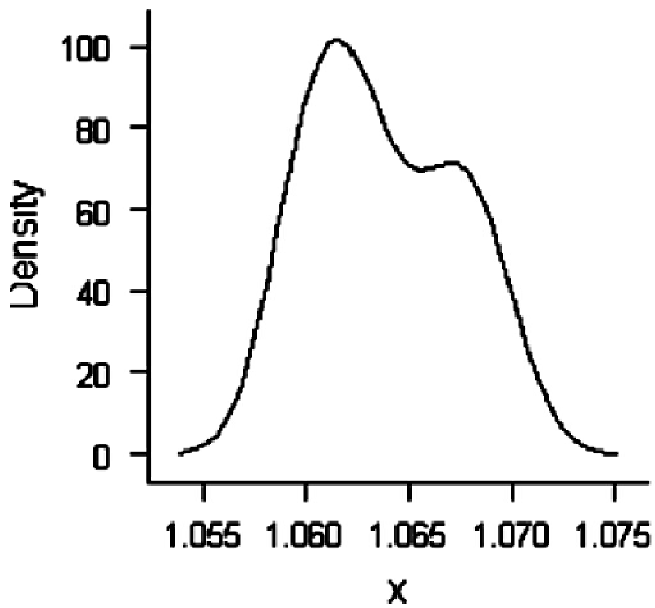
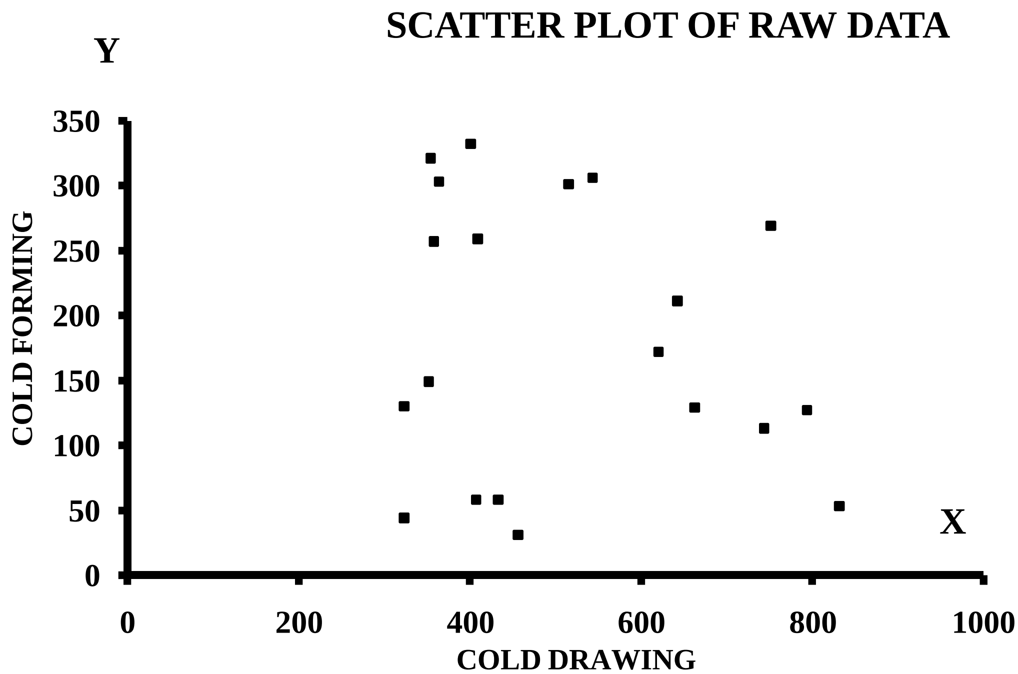


Figure 4.5. Normal Distribution of Residual Values

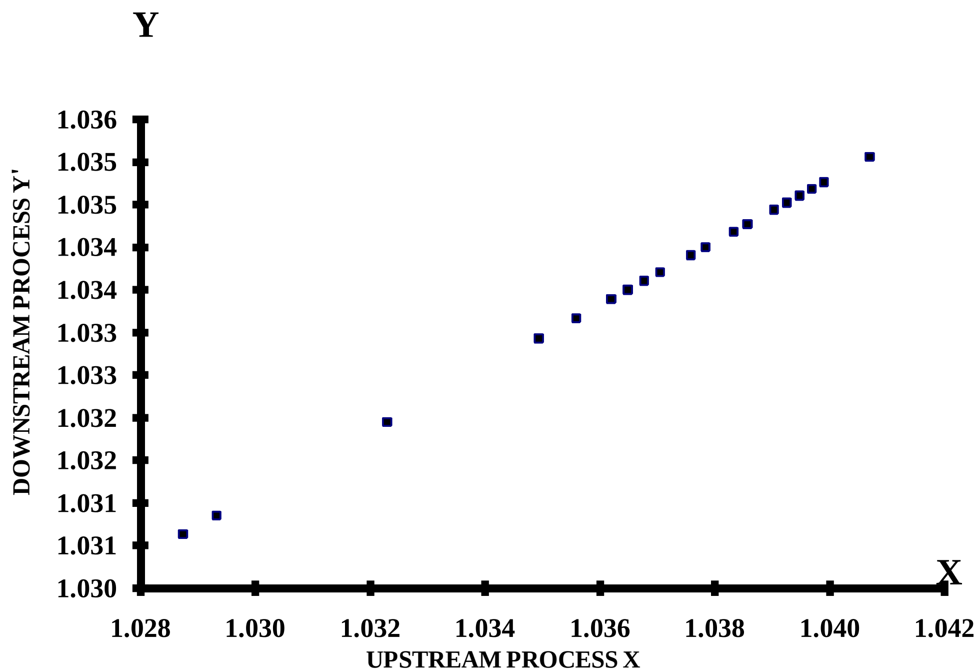
Thus the transform proposed and shown in equation 5.32 has enabled conversion of geometric data to normal data and rendered a positive LCL. This is the first and second contribution of this work.

The scatter plot of raw data of cold drawing and forming processes is shown in figure 4.6. The scatter chart of transformed data for cold drawing and residual data are shown in figure 4.7. It can be seen that while the raw data fails to form a regression line, the transformed data set of the two processes form a regression line. This vindicates the effectiveness of the new transform designed in terms of its ability to form a linear regression model.



SL No	X	Y	SL No	X	Y	SL No	X	Y	SL No	X	Y
1	401	332	**6**	407	58	**11**	662	129	**16**	620	172
2	456	31	**7**	831	53	**12**	354	321	**17**	751	269
3	743	113	**8**	409	259	**13**	433	58	**18**	352	149
4	543	306	**9**	642	211	**14**	364	303	**19**	323	44
5	358	257	**10**	515	301	**15**	793	127	**20**	323	130

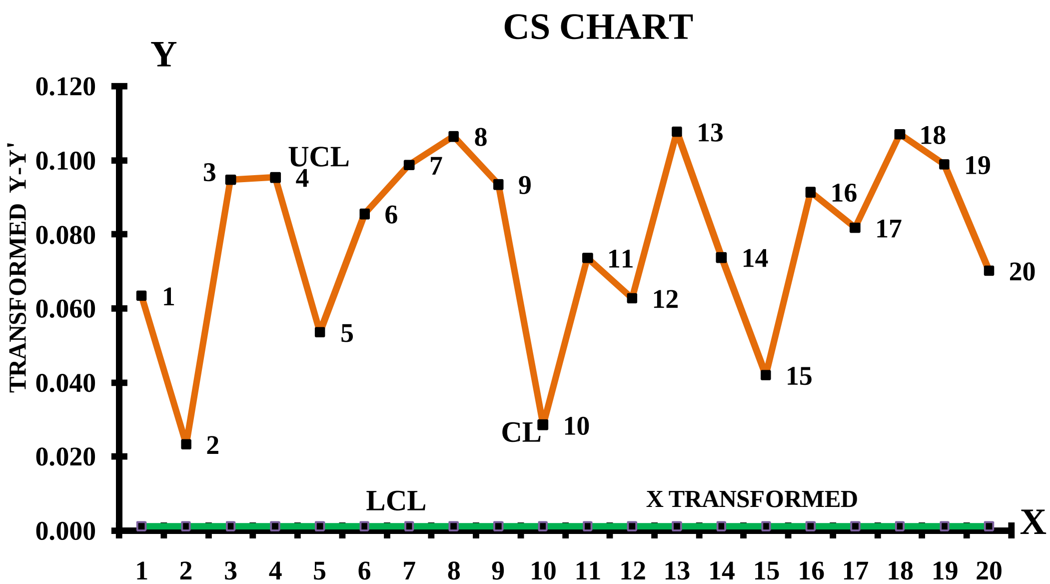
Figure 4.6. Scatter Plot of Raw Data



SL No	X	Y	SL No	X	Y	SL No.	X	Y	SL No.	X	Y
1	1.029	1.031	6	1.037	1.034	11	1.039	1.035	16	1.038	1.034
2	1.029	1.031	7	1.036	1.033	12	1.039	1.035	17	1.039	1.034
3	1.032	1.032	8	1.036	1.033	13	1.041	1.035	18	1.038	1.034
4	1.035	1.033	9	1.038	1.034	14	1.038	1.034	19	1.04	1.035
5	1.036	1.033	10	1.039	1.034	15	1.036	1.033	20	1.036	1.033

Figure 4.7 Regression Line of Transformed Process Data

The cause selecting control chart with positive grid lines is shown in figure 4.8. using the proposed transform.



Sl. No	Residual Y-Y'	LCL	Sl. No	Residual Y-Y'	LCL	Sl. No.	Residual Y-Y'	LCL	Sl. No.	Residual Y-Y'	LCL
1	0.064	0.001	6	0.086	0.001	11	0.074	0.001	16	0.092	0.001
2	0.023	0.001	7	0.099	0.001	12	0.063	0.001	17	0.082	0.001
3	0.095	0.001	8	0.107	0.001	13	0.108	0.001	18	0.107	0.001
4	0.095	0.001	9	0.094	0.001	14	0.074	0.001	19	0.099	0.001
5	0.054	0.001	10	0.029	0.001	15	0.042	0.001	20	0.07	0.001

Figure 4.8. Cause Selecting Control Chart With Proposed Transform

4.12 Conclusion

For upgrading cause-selecting control charts from 3 sigma to six sigma compliant the issue of converting geometric data into normal data has to be sorted out. A power transform has been proposed to meet this requirement. A detailed design flow has been drawn with explanatory numerical examples. A positive LCL is also achieved. The next chapter details the intercept model error issues.

Chapter 5

Intercept Model Errors

5.1 Introduction

In traditional control chart methodology the parameter that is followed by the data set is important from the point of view of the control limits and chart efficiency. In least square regression based control chart methodology, the model not only affects the chart efficiency but also the statistic being charted. In this chapter the regression model with or without intercept term and their bearing on the chart efficiency is discussed. The design of a two stage cause selecting (CS) for high sigma cascade process to overcome the model uncertainties due to intercept term is also discussed. The influence of these scenarios on type II error (missed signals). Average run length (ARL) of residuals type CS has been studied and inferences made.

5.2 Least Square Regression

It may be recalled that the general least square regression model is,

$$Y = \beta_0 + \beta_1 X_1 + \text{- - - - - - - - - - - - - - -} + \beta_n X_n + \varepsilon \qquad (5.1)$$

And for a bivariate cascade process it can be simplified as,

$$Y = \beta_0 + \beta_1 X + \varepsilon \qquad (5.2)$$

Where β_0 is the intercept term and $\beta_1 \ldots\ldots\ldots\ldots \beta_n$. is the slope, ε is the error term.

There are many industrial applications in which, the intercept term 'β_0' is zero. That is when the regressor 'X' = 0, response 'Y' is also zero. This implies that when process 'X' is not taking place process 'Y' cannot happen. For analogy, in case of a cold drawing process (upstream process) and press forming process (down stream process), unless cold drawing to ‘size’ takes pace, cold forming cannot be carried out. Instances that resemble no intercept in a regression model can be defined as,

$$Y = \beta_1 X + \varepsilon \quad (5.3)$$

Least square regression analysis forms the basis of cause selecting control charts. It involves fitting of regression model and establishing a straight line relationship between regressor X and response Y. The estimates for coefficients namely intercept $\beta_{0,}$ slope β_1, and error term ε is calculated. Further confidence intervals for these constants and the prediction intervals can be deduced.

The traditional control chart method involves determination of the parameters of the process data and estimation of the parameter metrics. For the generally assumed normal distribution the metrics are the mean and standard deviation. In traditional and current research the issue of the effects of parameter estimation has been discussed for almost all variants of control chart.

In the case of traditional control charts the effects of parameter estimation will affect the control limits and run length performance and associated indices. This will not in any way influence or alter the process statistic being charted .In case of CS which is a residuals’ control chart working on the basis of regression models, the estimation of standard deviation or moving range affects the control limits. The estimation of coefficients β_0 and β_1 affects the chart statistic unlike the traditional control charts.

Shu and Tsung (2000) and Shu et al. (2004, 2005) dealt with the estimate of coefficients β_0 and β_1 and the resultant effect on 3 sigma control chart performance indices. This may not be applicable to high sigma process where the count of conforming items (CCC) between two non conforming items is being used as chart statistic. Further before estimating the coefficients the intercept model has to be estimated. This has not been addressed. Hawkins (1993) addressed these issues of regression adjustment for variables.

In this work the idea of CCC has been extended to CS for a bi-variate high sigma process. Adopting an intercept model (equation 5.2) in the place of a no intercept model (equation 5.3) and adopting model (equation 5.3) in the place of model (equation 5.2) will have detrimental effects on the process statistic and the performance indices.

5.3 High Sigma Cause Selecting Control Charts

In current literature the CS has been discussed for 3 sigma process which is the normal industry norm. It may be recalled that a 3 sigma process will have process rejects up to 67000 per million opportunities. For a manufacturing operation to be world class, it has to aim at 6 sigma metric, which yields only 3.4 defects per million opportunities. As the defects are of the order of parts per million the monitoring tools used for 3 sigma are not adaptable to 6 sigma process. The ‘count of conforming items’ control chart (CCC) which uses the terminal nonconformity sequence as the statistic, is the appropriate one for high sigma process. This statistic follows the geometric distribution. The CCC chart or the count of conforming control chart was discussed by Calvin (1983), Goh(1987), Kaminski et al. (1992), Nelson(1994), Xie (2000). Lakshminarasimhan and Kannan (2007) dealt with the design of CS for a bi-variate high sigma process. They used a power transform,

$$X = X^{1/100} \quad (5.4)$$

to convert the geometric data into a normal form. This also addressed the issue of the negative lower control limits, as defined by the requirement that,

$$\mu_e > mR_e \quad , \quad (5.5)$$

where μ_e is the mean of residuals and mR_e is the moving range of the residuals 'ε'.

5.4 Intercept and No Intercept Models

Regression models serve the purpose of arriving at the linear relationship between the characteristic of upstream process and downstream process. Every care must be taken in estimating the coefficients. The intercept β_0 is influenced by near origin values. The slope β_1 is influenced by remote values of the regressor. The remote values or the outliers distort the relationship. The evaluation of the coefficients of the regression model and the error term are shown in figure 5.1.

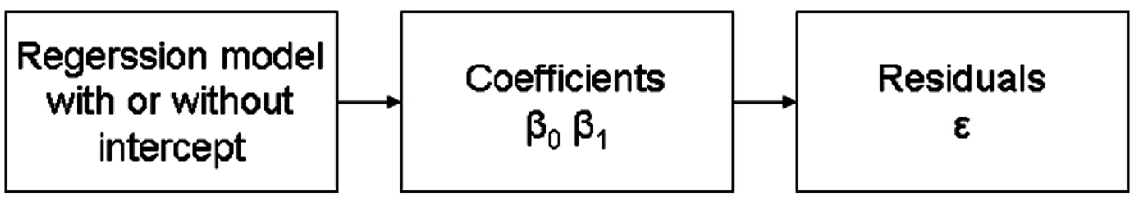


Figure 5.1. Evolution of Regression Model

In regression analysis the relationship between the regressor X and the response Y will not be the same with model uncertainties. Assume a data cluster which is far away from the origin. In such a case, assuming that it is a no intercept model and if it is forcibly fitted through the origin, the emerging fit will be a complex model. In such a situation the absence of intercept which is in reality present in the model may distort both the model and the regression analysis. Such a model is depicted in figure 5.2.

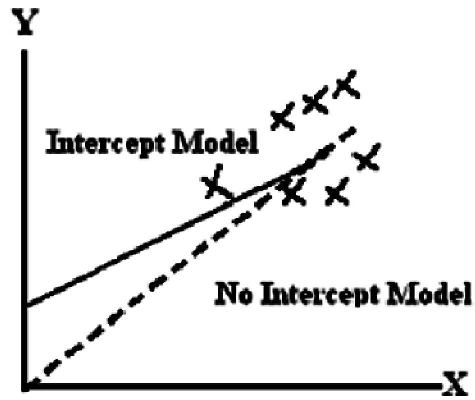


Figure 5.2. No Intercept Model Forcibly Fitted as Intercept Model

For such a fitted curve, the model near the origin will be more complex and may require many more polynomials to truthfully depict the model. Similarly a situation may arise in which a no intercept model may be forcibly fit as an intercept model. This is shown in figure 5.3.

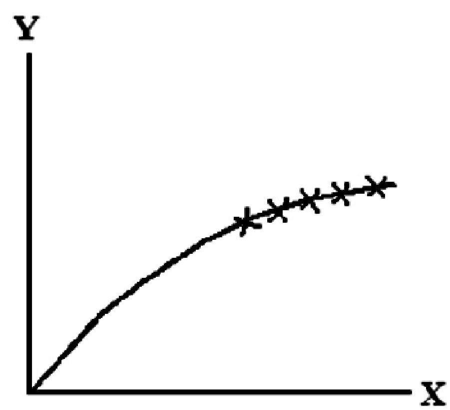


Figure 5.3. An Intercept Model Forcibly Fitted as No Intercept Model

The inclusion of intercept term which does not exist will distort the regression analysis. Montgomery et al. (2003), Draper and Smith (2005) details about regression model theory and applications.

Shu and Tsung (2000), Shu et al. (2004, 2005), discuss the model uncertainties and estimation errors for the coefficients $\beta_{0,}$ $\beta_{1.}$ It was suggested to use the prediction intervals for the coefficients as control limits to overcome model errors and uncertainties.

It may be recalled that the errors are normally and independently distributed as N (0, σ^2). Under such statistical assumption it is possible to construct 100(1-α) percent confidence interval for β_1 for a no intercept model. The lower confidence interval is,

$$\hat{\beta}_1 - t_{\alpha/2\,,\,n-1} \sqrt{MS_R} \,/ \sqrt{\sum x_i^{\,2}} \qquad (5.6\ )$$

Where $\boldsymbol{i}$ is 1, 2, 3………… n. $\hat{\beta}_1$ is the fitted regressor coefficient, $t_{\alpha/2\,,\,n-1}$ is the student statistic MS_R is the mean square of residuals and $x_i^{\,2}$ is the $\boldsymbol{i}$ th term of X. The 100 (1- α) percent lower confidence intervals for an intercept model is

$$\hat{\beta}_1 - t_{\alpha/2\,,\,n-2}\, Se(\hat{\beta}_1) \qquad (5.7)$$

where $Se(\hat{\beta}_1)$ is the estimated standard error, and it is,

$$Se(\hat{\beta}_1)=\sqrt{MS_R/S_{xx}} \qquad (5.8)$$

A regression model with a no intercept term will end up with a negative lower confidence interval, as it has to be less than the origin 0. That is the origin will find a place between the lower and upper confidence intervals. For a regression model with an intercept term, the lower and upper confidence interval will have positive value as it will not contain the origin. This fortifies the need for exact identification of the presence of the intercept β_0 in the first place, before proceeding with the estimation of intercept β_0 and slope β_1 .Unless the existence or otherwise of intercept β_0 is determined the estimation of β_0 and β_1 will yield erroneous regression models.

There have been many prescriptions for metric for determining the identity of intercept or no intercept model. Eisenhauer (2003), Hahn (1997) dealt with extensively this issue in their research work. Mullet (1976) dealt with wrong signs of regression coefficients. The co-efficient of determination R^2 is one of the metrics advocated. R^2 indicates the proportion of initial variation for Y for a no intercept model. The existence of intercept is recognized by the comparison of coefficient of determination for intercept and no intercept models. For the intercept model,

$$R^2 = \sum_{i=1}^{n} (\hat{Y}_i - \bar{Y})^2 / \sum_{i=1}^{n} (Y_i - \bar{Y})^2 \quad (5.9)$$

Where, $\hat{Y}_i$ is the fitted values of Y, $\bar{Y}$ is the mean of values of Y and Y_i is the ***i*** th reading of Y, and for the no intercept model, it is

$$R_0^2 = 1 - \sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2 / \sum_{i=1}^{n} (Y_i - \bar{Y})^2 \quad (5.10)$$

As these measures are based on identities, it is argued that they cannot be specific about the presence of intercept term. In instances, where there is a poor fit, variation will be more about $\hat{Y}_i$ than around $\bar{Y}$ Then R^2 may be negative due to the fact that the variation around fitted regression may exceed the variation around the mean. Hence

$$\sum_{i=1}^{n} (\hat{Y}_i - \bar{Y})^2 > \sum_{i=1}^{n} (Y_i - \bar{Y})^2 \quad (5.11)$$

This will result in a negative coefficient of determination. Likewise the F statistic which is as a metric will also be negative. Hence it is argued that the R^2 value for intercept and no intercept models are not comparable. Further for the no intercept model the numerator of F ratio is calculated from sum of square of predicted values around the origin. This is calculated around the mean for the intercept model and hence not comparable.

The Mean Square of residuals (MS_{Res}) is considered the best metric for the determination of the presence of intercept. The model with lower $MS._{res}$ will be the best fit for the model. As stated the other criteria is the negative lower confidence interval in residual analysis to confirm the 'no intercept' model as the best fit.

5.5 Modified Design flow

A detailed discussion on CS design flow can be referred to Lakshminarasimhan and Kannan (2007). Design modifications have been made to this design to incorporate the issue of origin in the regression model.

The procedure involves following steps:

i. Fit intercept model
ii. Examine whether equation (5.7) above is less than zero. If it is less than zero then the best fit is no intercept model.
iii. Further the mean square of residuals will be the minimum for the model to be determined. These two bases are the determinants for the model building.
iv. Design the CS chart with intercept or no intercept model.
v. The modified design flow is shown in figure 5.4. The modifications have been identified in dotted line.

To explicitly demonstrate the design methodology, two numerical examples have been explained. The data have been collected from a bivariate cold drawing X and metal forming Y processes. The process details are shown in diagram 5.5.

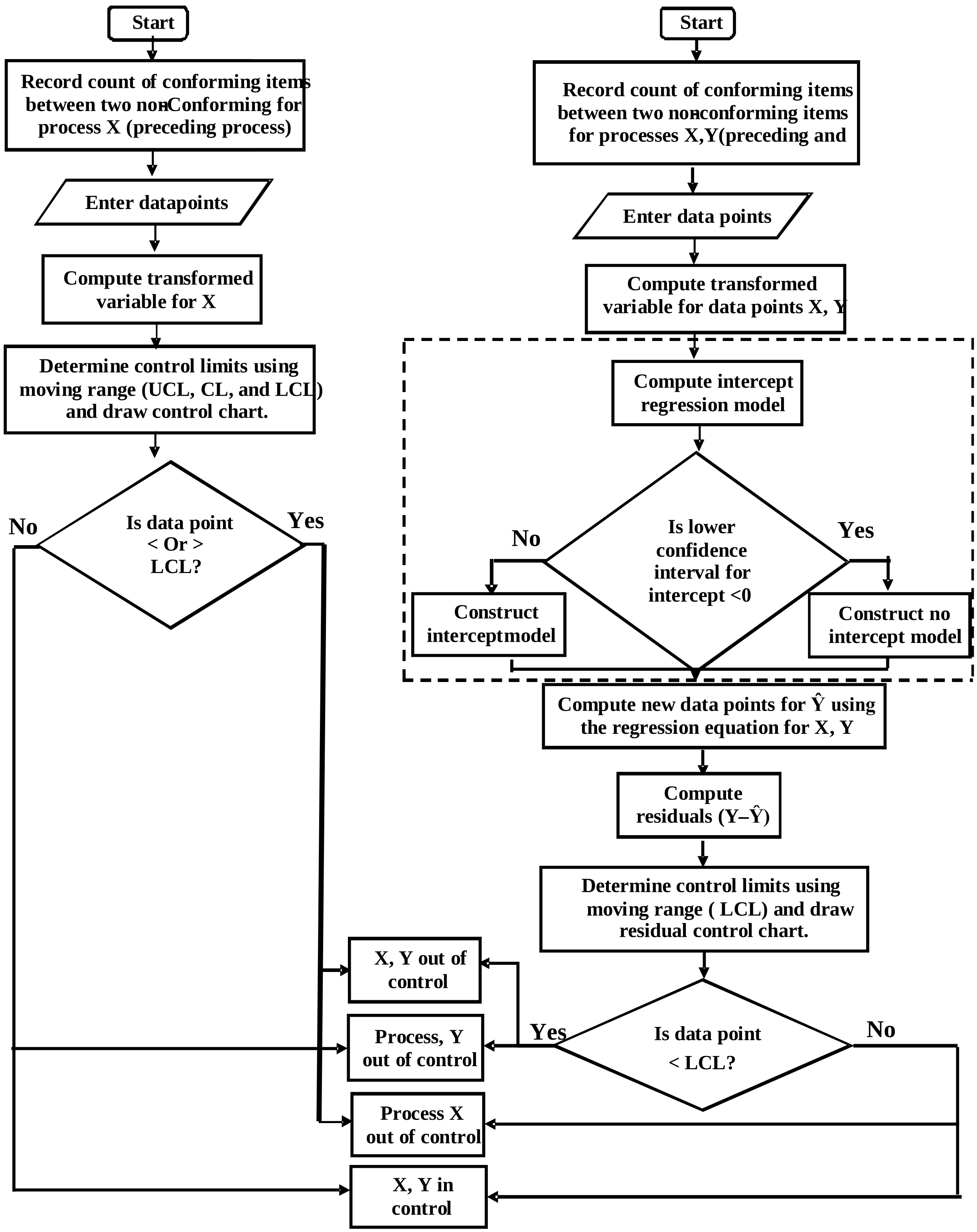


Figure 5.4. Modified Design Flow

5.6 Intercept Model Case Study

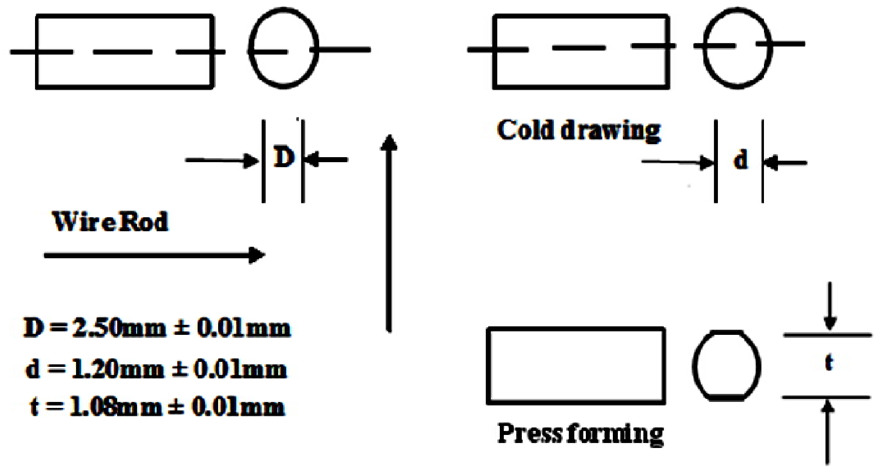


Figure 5.5. High Sigma Process

Data has been collected from a high sigma pin manufacturing process, and it was attempted to construct a cause selecting control chart. Following the design flow enunciated in the previous section, the intercept and no intercept models have been analyzed and appropriate model has been chosen. Then the Shewhart type control chart for process X and the residual control chart for process Y can be constructed.

The data set 1 contains the basic counts of conforming items between two nonconforming items. This data is in geometric form. The data has been transformed to normal form by adopting a transform in equation 5.4. Table 5.1 details the data set showing count of conforming items (CCC) and its transformed values for cold drawing and cold forming process.

Table 5.1. Data set 1 – Intercept Model Data

Basic Data			Transformed Data	
Data No	Cold Drawing CCC	Metal Forming CCC	Cold Drawing	Metal forming
1	47	46	1.0393	1.0390
2	17	29	1.0287	1.0342
3	36	46	1.0365	1.0390
4	49	41	1.0397	1.0378
5	58	48	1.0414	1.0395
6	39	40	1.0373	1.0376
7	59	45	1.0416	1.0388
8	55	38	1.0409	1.0370
9	24	50	1.0323	1.0399
10	46	43	1.0390	1.0383
11	24	22	1.0323	1.0314
12	36	26	1.0365	1.0331
13	36	27	1.0365	1.0335
14	17	31	1.0287	1.0349
15	19	50	1.0299	1.0399
16	40	44	1.0376	1.0386
17	50	38	1.0399	1.0370
18	25	39	1.0327	1.0373
19	12	33	1.0252	1.0356
20	32	43	1.0353	1.0383

The intercept model statistic was first evolved for this data set and is summarized in table 5.2.

Table 5.2. Intercept Model Regression Statistics for Data Set 1

Coefficients	Coefficient Values	Lower confidence interval @ 95%	Upper confidence interval @ 95%	Mean Square for Residuals
Intercept	0.8476063	0.602097617	1.09311501	0.000014500
X variable	0.1829355	-0.05413995	0.420010934	0.000005525

It can be inferred that the lower confidence interval for intercept term is 0.6 which is above zero and hence the intercept model is the best fit for this data set. For the purpose of study the no intercept model regression statistic has also been worked out and tabled as under in table 5.3

Table 5.3. No Intercept Model Regression Statistics for Data Set 1

Coefficients	Coefficient Values	Lower confidence interval @ 95%	Upper confidence interval @ 95%	Mean Square for Residuals
Intercept	Not Applicable	Not Applicable	Not Applicable	Not Applicable
X Variable	1.001418138	0.999370213	1.003466063	0.0000205

Comparing the mean square of residuals, we can confirm that the intercept model having lower MS_{res} and is the best fit for the data set. Two types of errors have been envisaged in interpreting the control charts. The first type of error is, understanding a noise as signal and the second type is, understanding a signal as noise. By filtering out the noise the control chart minimizes the number of times one interprets a noise as if it were a signal ('α' risk) and the number of times one misses a signal ('β' risk). The average run length (ARL) is the common performance indices for a control chart. According to Montgomery (2001)

$$\beta = \varphi (L - k\sqrt{n}) - \varphi (-L - k\sqrt{n}), \quad (5.12)$$

where φ denotes the standard normal distribution function

L = a constant to determine control limit

n = sample size

k = shift magnitude

$\alpha = 1 - \beta$ and

$$ARL = 1/\alpha \quad \text{or} \quad 1/(1-\beta)$$

The 'β' risk and ARL for the data set is calculated and the operation characteristic curves and ARL graph for the intercept and no intercept models are constructed. Table 5.4 and table 5.5 give the numerical values. The sensitivity has also been calculated.

Table 5.4. β, ARL Values for Intercept Model Fitted as Intercept Model (Data Set 1)

Position	Shift Magnitude	% of Shift Magnitude	β	Sensitivity Δ β / β	ARL	Sensitivity Δ ARL/ ARL
Mean	0.51	20	0.9835	0	61	0
	1.03	40	0.9483	4	19	68
	1.54	60	0.8686	12	8	87
	2.05	80	0.7291	26	4	94
LCL	2.57	100	0.5359	46	2	96

Table 5.5. β, ARL Values for Intercept Model Fitted as No Intercept Model (Data Set 1)

Position	Shift Magnitude	% of Shift Magnitude	β	Sensitivity Δ β / β	ARL	Sensitivity Δ ARL/ ARL
Mean	0.55	20	0.9819	0	55	0
	1.1	40	0.9405	4	17	70
	1.65	60	0.8438	14	6	88
	2.2	80	0.6772	31	3	94
LCL	2.75	100	0.4641	53	2	97

The operational characteristic curves for based on the two models are shown in the figure 5.6.

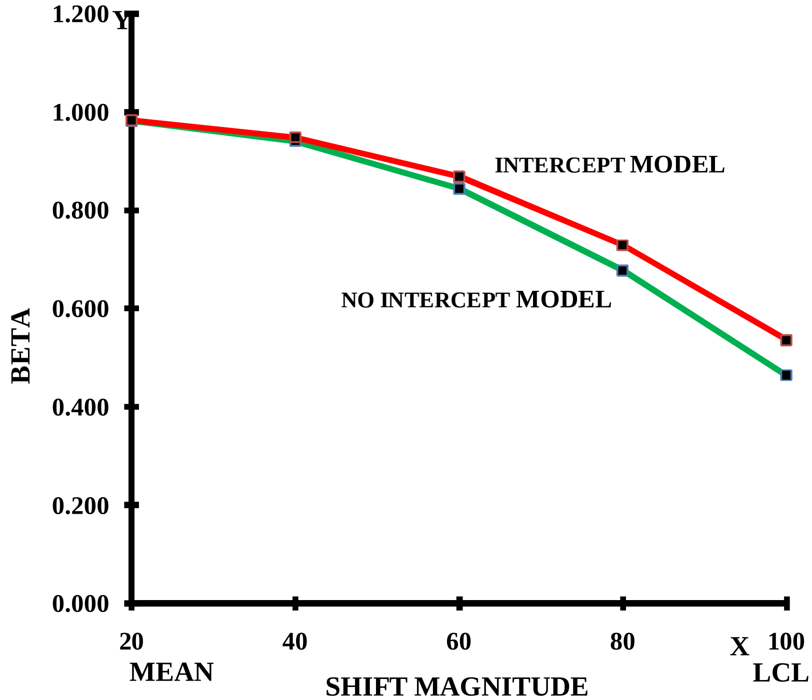


Figure 5.6 Operational Characteristic Curves for Intercept Model Data with Model Error
(Shift Magnitude Percentage from Mean to LCL)

The data set 1 above enunciated is an intercept model. As can be seen from the OC curve, when fitted as a no intercept model, the β risk near the lower control limit will appear to be less than the actual. The β risk has not changed appreciably at the process mean. From 40 % of shift from process mean, it starts showing a lower value than the actual. The magnitude increases as it approaches LCL. The ARL values for the two charts have been compared and depicted in the figure 5.7.

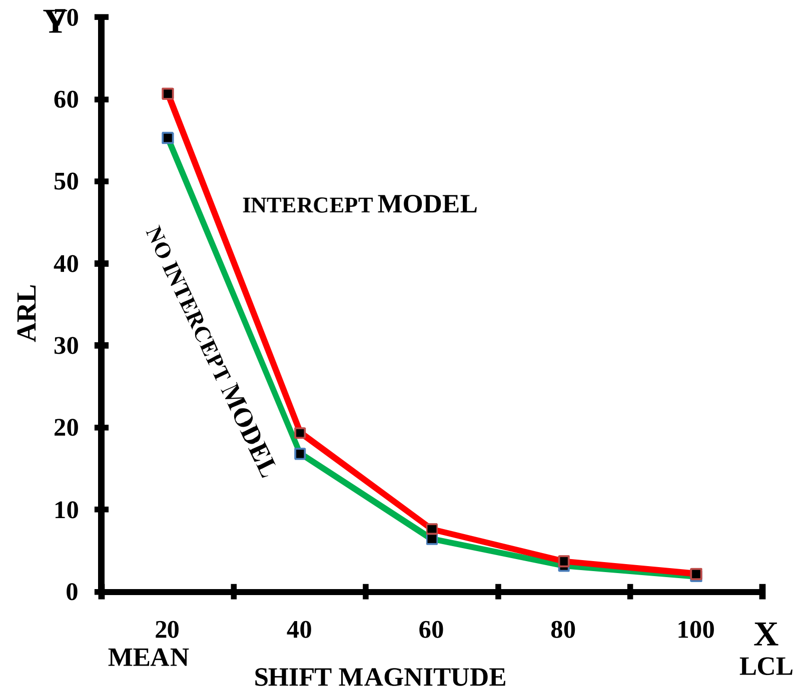


Figure 5.7. ARL Values for Intercept Model Data with Model Error
(Shift Magnitude Percentage from Mean to LCL)

This being an intercept model, if when depicted as a no intercept model may show a lower ARL value near process means. At the decision point which is the LCL there is no significant change in ARL value.

5.7 No Intercept Model Case Study

Another data set has been chosen to show the behavior of no intercept model forced fitted as intercept model. Its ‘β’ risk and ARL values have been calculated and analysis evolved. Table 5.6 shows the basic and transformed data set. Tables 5.7 and 5.8 show ‘β’ and ARL values for Intercept and No intercept model calculations.

Table 5.6 Data set 2 – No Intercept Model Data

Basic Data			Transformed Data	
Data No	Cold Drawing CCC	Metal Forming CCC	Cold Drawing	Metal forming
1	47	78	1.039	1.045
2	21	35	1.031	1.036
3	22	36	1.031	1.036
4	65	107	1.043	1.048
6	44	56	1.039	1.041
7	38	49	1.037	1.040
8	16	21	1.028	1.031
9	31	40	1.035	1.038
10	32	41	1.035	1.038
11	58	75	1.041	1.044
12	42	50	1.038	1.040
13	59	71	1.042	1.044
14	45	54	1.039	1.041
15	42	60	1.038	1.042
16	65	87	1.043	1.046
17	26	35	1.033	1.036
18	53	81	1.041	1.045
19	63	96	1.042	1.047
20	16	19	1.028	1.030

Table 5.7. β, ARL Values for No Intercept Model as Intercept Model (Data Set 2)

Position	Shift Magnitude	% of Shift Magnitude	β	Sensitivity Δ β / β	ARL	Sensitivity Δ ARL/ ARL
Mean	0.26	20	0.9911	0	112	0
	0.51	40	0.9840	1	63	44
	0.77	60	0.9713	2	35	69
	1.02	80	0.9494	4	20	82
LCL	1.28	100	0.9192	7	12	89

Table 5.8. β, ARL Values for No Intercept as No Intercept Model. (Data Set 2)

Position	Shift Magnitude	% of Shift Magnitude	β	Sensitivity Δ β / β	ARL	Sensitivity Δ ARL/ ARL
Mean	0.56	20	0.990	0	97	0
	1.13	40	0.982	1	54	44
	1.69	60	0.965	3	28	71
	2.25	80	0.937	5	16	84
LCL	2.82	100	0.894	10	9	90

Table 5.9. No Intercept Model as Intercept Model Regression Statistics for Data Set 2

Coefficients	Coefficient values	Lower confidence interval@ 95 %	Upper confidence interval @ 95%	Mean Square for residuals
Intercept	-0.01	-0.143	0.1234	0.000442000
X variable	1.0127	0.8842	1.1412	0.000001609

Table 5.10 No Intercept Model as No Intercept Model Regression Statistics for Data set 2

Coefficients	Coefficient Values	Lower confidence interval @ 95%	Upper confidence interval @ 95%	Mean Square for Residuals
Intercept	Not Applicable	Not Applicable	Not Applicable	Not Applicable
X Variable	1.00314886	1.002591366	1.003706355	0.000001527

From the tables 5.9 and 5.10 it is seen that the MS_{res} is lower for the no intercept model. Further the intercept model statistic contains the lower confidence interval less than zero. (-0.1431) indicating that y intercept (0) lies between the lower and upper confidence intervals. Hence the no intercept model is the best fit for this data set. The Operational Characteristic curves and the ARL graph for the no intercept model and its wrong fit as intercept model are shown in figures 5.8and 5.9.

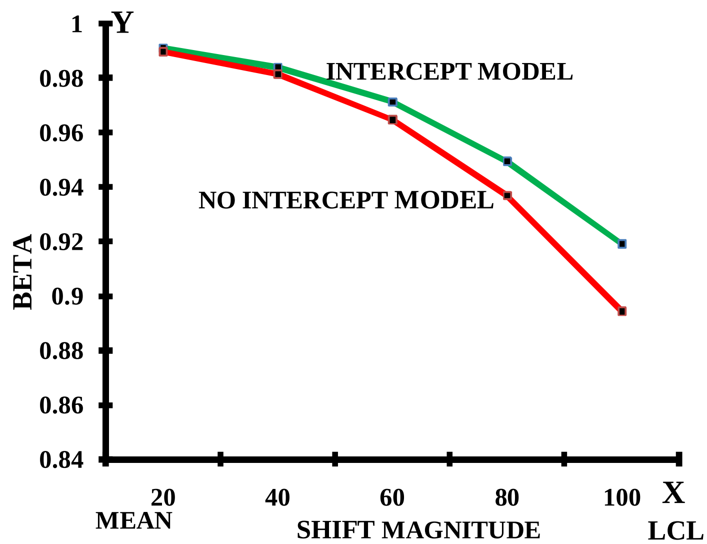


Figure 5.8. Operational Characteristic Curves for No Intercept Data with Model Error

(Shift Magnitude Percentage from Mean to LCL)

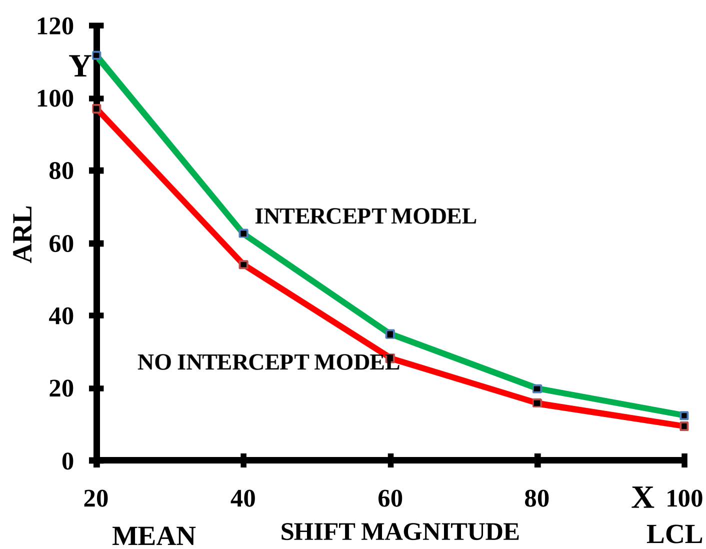


Figure 5.9. ARL Curves for No Intercept Data with Model Error

(Shift Magnitude Percentage from Mean to LCL)

From the above curves it can be inferred that, when a no intercept model is force fitted as intercept model, 'β' risk estimate will be higher than normal at the LCL which is the decision point. The difference is not appreciable near the mean but tends to increase as it approaches LCL. The ARL estimate will be higher than normal at Mean and LCL. The difference is wider at process mean and tends to decrease as it approached the LCL.

Figures 5.6, 5.7, 5.8 and 5.9 show the intercept model issues and their influence on type II error and average run length. Suitable design modifications have been shown in design flow figure 5.4. These are the third and fourth contribution by this work

5.8 Conclusion

Regression based process monitoring requires the determination of intercept term in the first place, so that the process data is arrived at with accuracy and without model errors. For 3 sigma cause-selecting control charts Shu and Tsung (2004) and Shu et al. (2004, 2005) proposed a method of using prediction interval, and principal component regression to obviate the model errors. In this chapter a design methodology is proposed for high sigma bi-variate cause-selecting control charts. As the regression model will alter the chart statistic the influence of model errors on chart performance have been studied and suitable design criteria incorporated in the design flow. The next chapter details the normality issue and its influence on chart performance.

Chapter 6

Normality and Chart Performance

6.1 Introduction

Optimizations of such transforms are normally done by determining the maximum likely hood estimate (MLE). In this work instead of MLE the skewness and kurtosis of the normally transformed data set and its influence on Lower Control Limit (LCL) and type II error have been studied. In case of a high sigma cause selecting control chart, the lower control limit is the action limit. For high sigma processes, as the defect counts seldom occur, the missed signals or the Type II error is important compared to Type I error or false signals. A false signal will draw frequent attention to process and is only important for avoidance from the perspective of economics of quality monitoring. A missed signal when neglected may degrade the high sigma level of the process and hence its importance over the Type I error. In this chapter, the influence of skewness on LCL and the influence of kurtosis on the Type II error have been studied for various values of power transformations and inferences have been made.

6.2 Maximum Likelihood Estimate

Box and Cox (1964), discussed about the transformation for a dependant variable and the method of its optimization. Their method of optimization was to consider the following test of hypothesis to find out the maximum likelihood estimate.

$$\widehat{Y^{\lambda}} = \left\{\frac{Y^{\lambda}-1}{\lambda \hat{Y}^{\lambda-1}}\right\} \qquad \lambda \neq 0 \tag{6.1}$$

$$\widehat{Y^{\lambda}} = \hat{Y} \ \ln y \qquad \lambda = 0 \tag{6.2}$$

where λ is the transform. The maximum likelihood estimate (MLE) of λ corresponds to the value of λ for which the residual sum of square (SS_{RES}) is minimum. The value of λ is determined by fitting the regression model for various values of λ and plotting the residual sum of squares SS_{RES} versus λ. The value of λ that minimizes the SS_{RES} is the MLE of λ. It can be further optimized by using a finer mesh of λ values. This is demonstrated in figure 6.1.

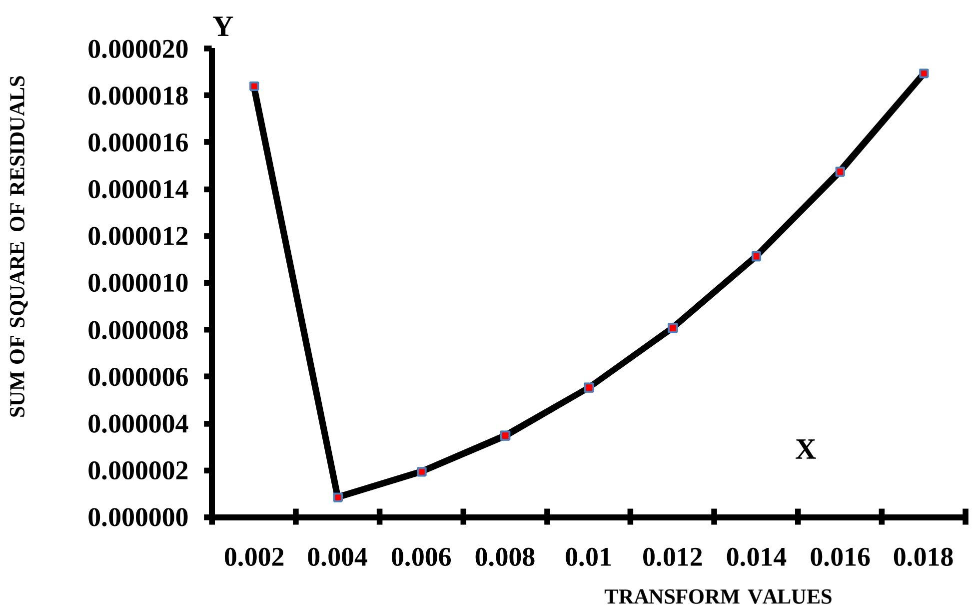


Figure 6.1 Transform Values and Sum of Square of Residuals

From the above chart, it is seen that the optimum power transformation value which has the lowest sum of squares of residuals is '0.004'. The theoretical optimization by MLE above stated may fulfill a mathematical axiom. But in practical context, the desired objectives are to be satisfied. That is the conversion of geometric data into normal data and its influence on the performance indices of cause selecting control charts. It is inferred that for the finer mesh of transformations have no qualitative or quantitative influences on skewness, kurtosis and performance of cause selecting control charts.

6.3 Normality Issue

The issue of normality of observations and testing of normality has been dealt with by Burr (1967), Shapiro and Francia (1972), Hopkins and Weeks (1990), Ramsey and Ramsey (1990), Yourstone and Zimmer (1992) Chou et al. (1998), and many other authors. Some researchers have a strong view that normality assumptions are not practical. While the research fraternity is keen to test the extent of normality many control charts in practice, work without testing as to their meeting the normality assumptions.

The important characteristics of normality are related in general to the first four moments. John Tukey (1977) supported the proposition that the first two moments namely the mean and standard deviation may not effectively describe the nature of the distribution. He also emphasized that normality will have consequential effect on accuracy of inferences like type I and type II errors. All these authors highlight one important point. That is, the normality assumption is important to the extent of its influence on type I and type II errors.

Brown (1997) discussed about skewness statistic of normal distribution as nearly equivalent to zero. He clarified that a skewness statistic of -0.01819 would be an acceptable value of a normally accepted data set. It can be said to be a negatively skewed data. Likewise a skewness value with a positive sign just above zero may demonstrate a positively skewed distribution. According to him normal distributions produce a kurtosis statistic of about zero. A kurtosis statistic of .09581 for a mesokurtic (normally high distribution) would be an acceptable value. If the kurtosis statistic departs further from zero a positive value indicates a leptokurtic (too tall) normal distribution. A negative value indicates possibility of a platykurtic (too flat or even concave) distribution.

Glass and Hopkins (1984), explained that with a small sample for leptokurtic distribution, the type I error will be smaller than the theoretical value. For platykurtic distribution it will be slightly greater. Further the issue is, testing the extent to which normality assumption is met. It is not for concluding whether the assumption of normality is met at all. The consequential issue is the extent of the influence of normality assumption on the metrics of performance of the control chart.

To sum up the skewness and kurtosis are the indices in addition to the mean and standard deviation to help one to identify whether the desired level of normality has been met. The extent by which the metrics of normality departs due to chance and sporadic causes in the real world process data and their bearing on the indices of performance of the high sigma cause selecting control chart is the matter of concern and study within the compass of this work. There have been many metrics of normality described in research literature. Many of the tests of statistics are descriptive in nature, that is, they are honed for best describing the normal distribution. Few of them are supportive for making inferences. Not influenced by the inertia of precedence, in this work skewness and kurtosis and their influence on the LCL and type II error have been studied and documented. This is explained by two case studies described below.

6.4 Case Study I – Intercept Model High Sigma Process Data

The study of a pin manufacturing process data is considered. It is a cascade process with two steps namely cold drawing and cold forming. The cold drawing process is a regressor process and the forming process is the response process. The regressor variable is adjusted for the response covariate and the evolving regression line is used for the cause selecting control chart set.

A set of intercept model data has been collected. The count of conforming items between two non conforming items has been taken as data set. It is in the geometric form. As enunciated in Lakshminarasimhan and Kannan (2007), it has been converted into normal data form using a power transformation (0.01). The data collected has an intercept term in it as depicted the equation 6.3.

$$Y = \beta_0 + \beta_1 X + \varepsilon \qquad (6.3)$$

Table 6.1 details the ‘intercept model’ data set. The raw data set in geometric form and transformed data set in normal form have been listed.

Table 6.1. Data With an Intercept Term

Data No	Geometric Data		Normal Data	
	Regressor Process	Response Process	Transformed Regressor Process	Transformed Response process
	X	Y	X'	Y'
1	47	46	1.039	1.039
2	17	29	1.029	1.034
3	36	46	1.036	1.039
4	49	41	1.04	1.038
5	58	48	1.041	1.039
6	39	40	1.037	1.038
7	59	45	1.042	1.039
8	55	38	1.041	1.037
9	24	50	1.032	1.04
10	46	43	1.039	1.038
11	24	22	1.032	1.031
12	36	26	1.036	1.033
13	36	27	1.036	1.034
14	17	31	1.029	1.035
15	19	50	1.03	1.04
16	40	44	1.038	1.039
17	50	38	1.04	1.037
18	25	39	1.033	1.037
19	12	33	1.025	1.036
20	32	43	1.035	1.038

Referring equation 6.3, 'β_0' is the intercept term and 'β_1' is the regressor coefficient. The normal distribution formed by the residual of the response variable $Y - \hat{Y}$. is given in figure 6.2 when the intercept model data has been fitted as intercept model. In normal industrial practice it is always assumed that the regression model has an intercept term.

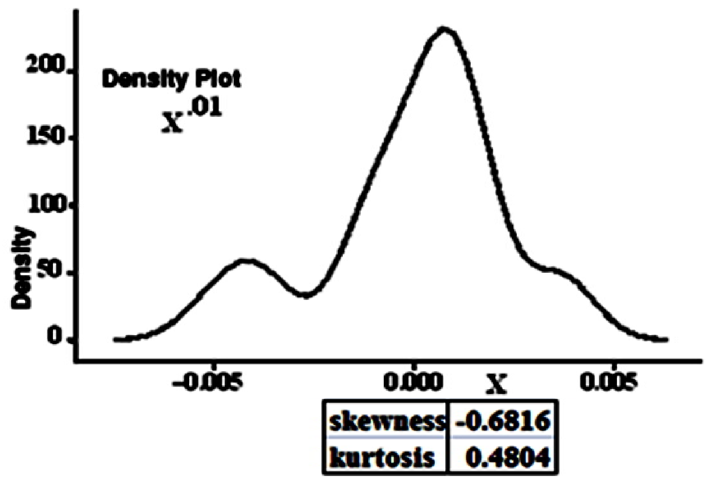


Figure 6.2. Normal Probability Distribution for Intercept Model Assumed as Intercept Model

As per customary practice, while arriving at the most likely estimate of the transform, the values shown in table 6.2 have been considered.

Table .6.2. Transform Values

Finer Mesh Transforms	0.002	0.004	0.006	0.008	0.01	0.012	0.014	0.016	0.018
Higher Value Transforms	0.2		0.4			0.6		0.8	

The table shows a finer value set and a higher value set of transformations.

6.4.1 Scenario I- Intercept Model Assumed as Intercept Model

In instances where an intercept model data is rightly assumed as intercept model data the influence of skewness on the lower control limit and the influence of kurtosis on type II error have been studied and explained in this section.

For the finer value of transforms, the skewness and kurtosis are almost constant in all the scenarios in this work. The skewness of the normal probability distribution for the finer value of transforms and its influence on lower control limit of residuals control chart has been shown in table 6.3 and the graph shown in figure 6.3.

Table 6.3 Finer Transforms Values and Skewness, Lower Control Limits

Transform	Skewness(S)	LCL	Sensitivity %	
			Δs/s	ΔLCL/LCL
0.002	-0.69	-0.001	0	0
0.004	-0.69	-0.002	0	102
0.006	-0.68	-0.003	0	205
0.008	-0.68	-0.005	1	309
0.01	-0.68	-0.006	1	415
0.012	-0.68	-0.010	1	778
0.014	-0.68	-0.008	1	632
0.016	-0.68	-0.010	1	742
0.018	-0.68	-0.011	1	854

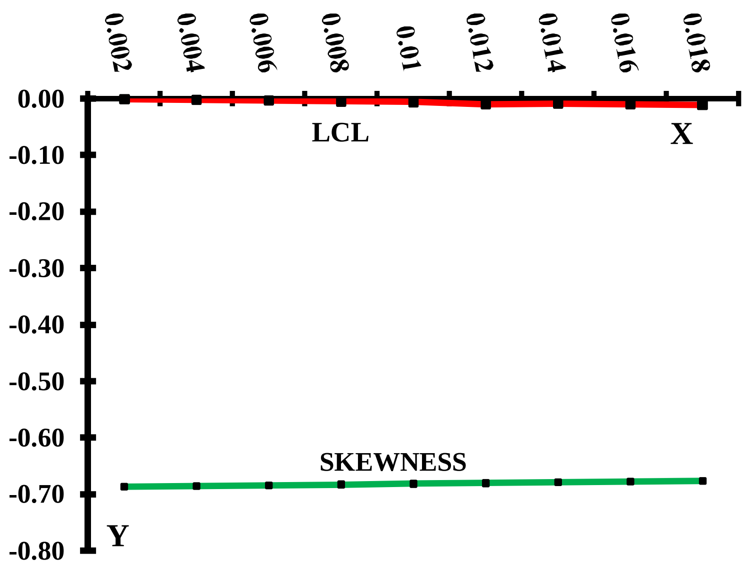


Figure 6.3. Skewness and LCL for Intercept Model Assumed as Intercept Model (Finer Mesh of Transforms)

For the finer mesh of transforms, the skewness of the transformed normal data is almost constant and will have no effect on the lower control limit. For the transformation values 0.200, 0.400, 0.600, 0.800 the behavior of skewness and its influence on lower control limit is shown in table 6.4 and in the illustration is shown in figure 6.4.

Table 6.4. Transform Values, Skewness and LCL

Transform	Skewness(S)	LCL	Sensitivity %	
			Δs/s	ΔLCL/LCL
0.2	-0.56	-0.23	0	0
0.4	-0.45	-0.98	21	316
0.6	-0.33	-3.04	41	1196
0.8	-0.22	-8.45	60	3497

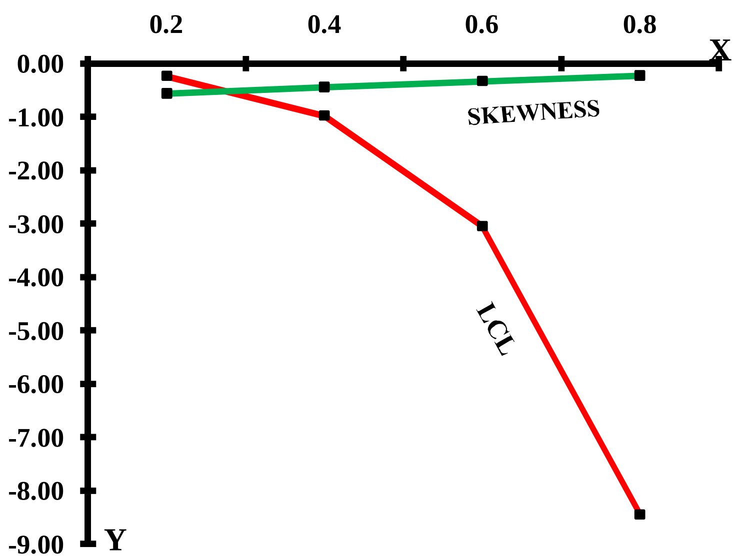


Figure 6.4. Skewness and LCL for Intercept Model Assumed as Intercept Model

It can be seen from the above illustration that as the power transform value increases the skewness of normal probability distribution increases and the lower control limit value decreases.

Likewise the kurtosis and its influence on beta error have also been studied and results recorded. It may be recalled that as type II error or the missed signals are more important in high sigma manufacturing environ kurtosis and its influence on missed signals have been studied and results recorded. The data for finer value of transforms is shown in table 6.5 and the illustration is shown in figure 6.5.

Table 6.5. Transforms Values, Kurtosis and Beta Error
(Finer Value of Transforms)

Transform	Kurtosis(k)	BETA β	Sensitivity %	
			Δk/k	Δβ/β
0.002	0.49	0.54		
0.004	0.49	0.54	0	0
0.006	0.48	0.54	1	0
0.008	0.48	0.54	1	0
0.01	0.48	0.54	1	0
0.012	0.48	0.54	2	0
0.014	0.48	0.54	2	0
0.016	0.48	0.54	2	0
0.018	0.47	0.54	3	0

It can be seen that for finer value of transforms the influence of kurtosis on the type II or beta error are as shown is not appreciable as shown in figure 6.5 below.

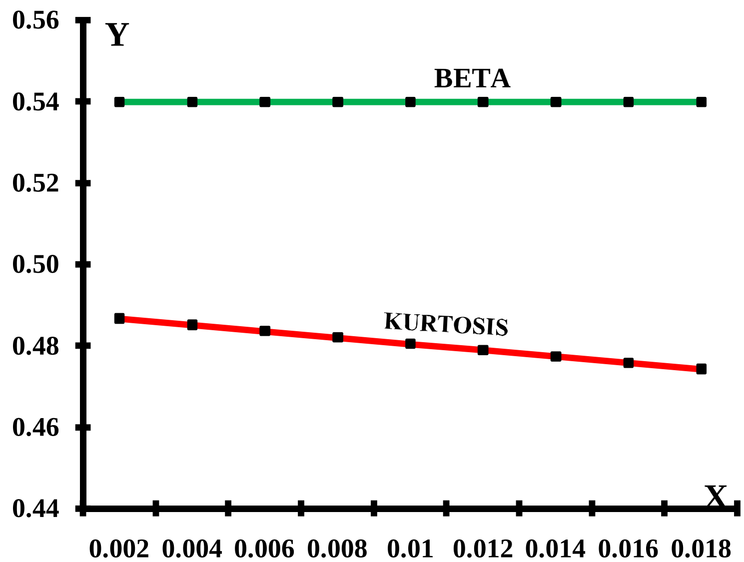


Figure 6.5. Transforms Values, Kurtosis and Beta Error
(Finer Value of Transforms)

For regular value of transforms the influence of kurtosis on beta error has been worked out detailed in table 6.6 and illustration 6.6.

Table 6.6. Transforms Values, Kurtosis and Beta Error

Transform	Kurtosis(k)	BETA β	Sensitivity %	
			Δk/k	Δβ/β
0.2	0.34	0.52	0	0
0.4	0.22	0.49	37	5
0.6	0.11	0.47	68	8
0.8	0.02	0.42	95	18

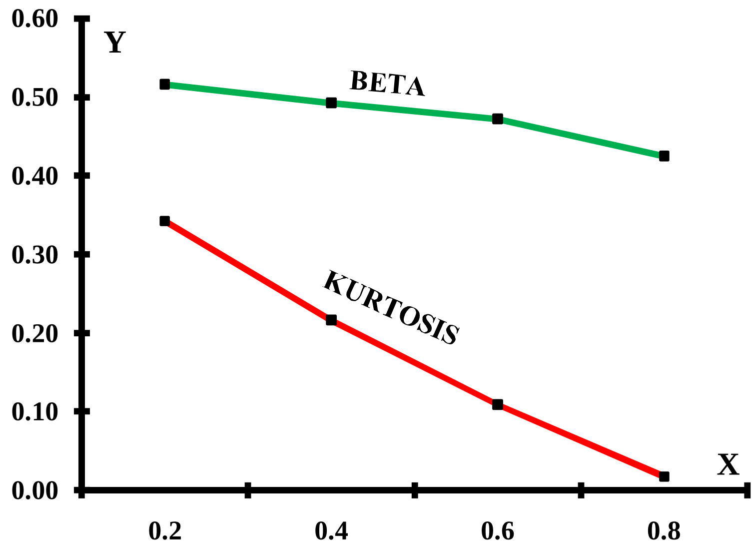


Figure 6.6. Transforms Values, Kurtosis and Beta Error
(Finer Value of Transforms)

6.4.2 Scenario II- Intercept Model Assumed as No Intercept Model:

In instances where an intercept model data is wrongly assumed as no intercept model data the influence of skewness on the lower control limit and the influence of kurtosis on type II error have also been studied and detailed in the forthcoming tables and illustrations. The data have been summarized in tables 6.7, 6.8, 6.9, 6.10. The illustrations detailing the behavior of data have been shown in figures 6.7, 6.8, 6.9, 6.10.

Table 6.7. Transforms (Finer) and Behavior of Skewness on Lower Control Limit

Transform	Skewness(S)	LCL	Sensitivity %	
			Δs/s	ΔLCL/LCL
0.002	0.725	-0.002	0	0
0.004	0.724	-0.005	0	-101
0.006	0.723	-0.007	0	-204
0.008	0.722	-0.010	0	-308
0.01	0.719	-0.023	1	-850
0.012	0.722	-0.012	0	-414
0.014	0.721	-0.015	1	-521
0.016	0.720	-0.018	1	-629
0.018	0.719	-0.020	1	-739

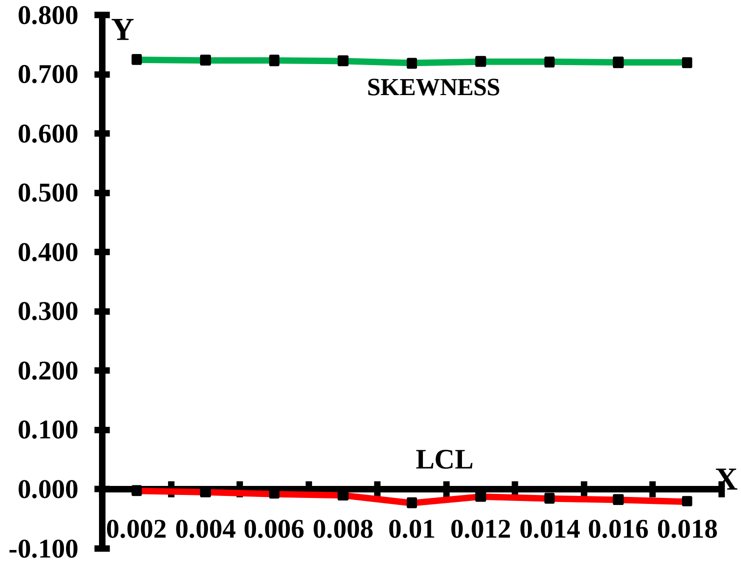


Figure 6.7. Transforms (Finer) and Behavior of Skewness on Lower Control Limit

Table 6.8. Transforms and Behavior of Skewness on Lower Control Limit

Transform	Skewness(S)	LCL	Sensitivity %	
			Δs/s	ΔLCL/LCL
0.2	0.65	-0.47	0	0
0.4	0.59	-1.83	10	289
0.6	0.54	-5.29	17	1025
0.8	0.50	-13.54	23	2776

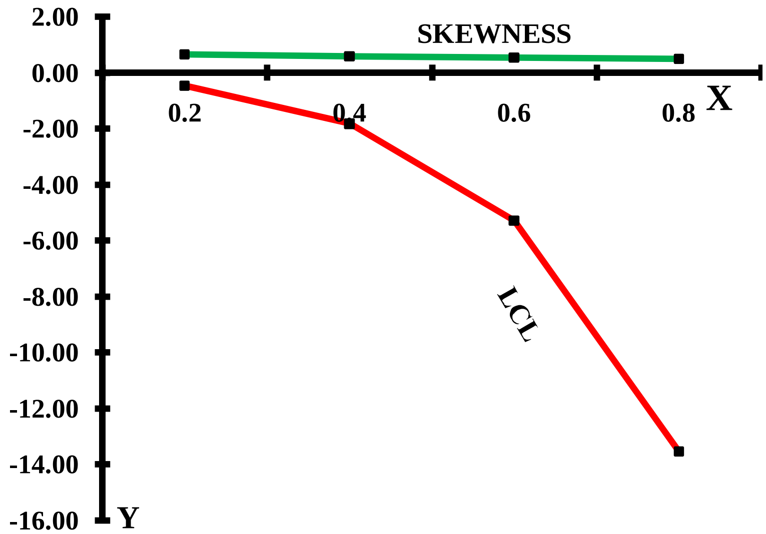


Figure 6.8. Transforms and Behavior of Skewness on Lower Control Limit

When an intercept model data is assumed as no intercept model data the influence of kurtosis on type II error are listed in the forthcoming tables.

Table 6.9. Transforms Values, Kurtosis and Beta Error (Finer Value of Transforms)

Transform	Kurtosis(k)	BETA β	Sensitivity %	
			Δk/k	Δβ/β
0.002	-0.654	0.464	0	0
0.004	-0.656	0.464	0	0
0.006	-0.658	0.464	0	0
0.008	-0.659	0.464	1	0
0.01	-0.667	0.464	2	0
0.012	-0.661	0.464	1	0
0.014	-0.662	0.464	1	0
0.016	-0.664	0.464	1	0
0.018	-0.666	0.464	2	0

The illustrations showing the behavior of skewness on the lower control limit and kurtosis on the beta error are shown in the following illustrations.

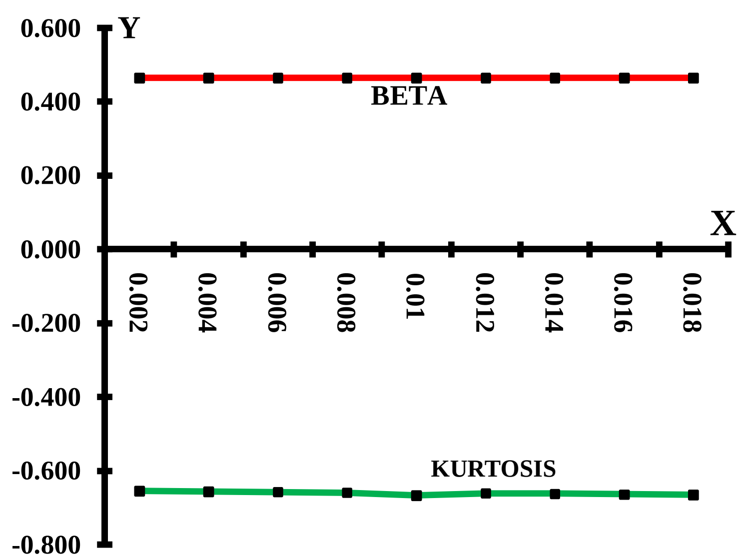


Figure 6.9. Transforms Values, Kurtosis and Beta Error (Finer Value of Transforms)

Table 6.10. Transforms Values, Kurtosis and Beta Error

Transform	Kurtosis(k)	BETA β	Sensitivity %	
			Δk/k	Δβ/β
0.2	-0.780	0.460	0	0
0.4	-0.842	0.456	8	1
0.6	-0.845	0.456	8	1
0.8	-0.798	0.468	2	2

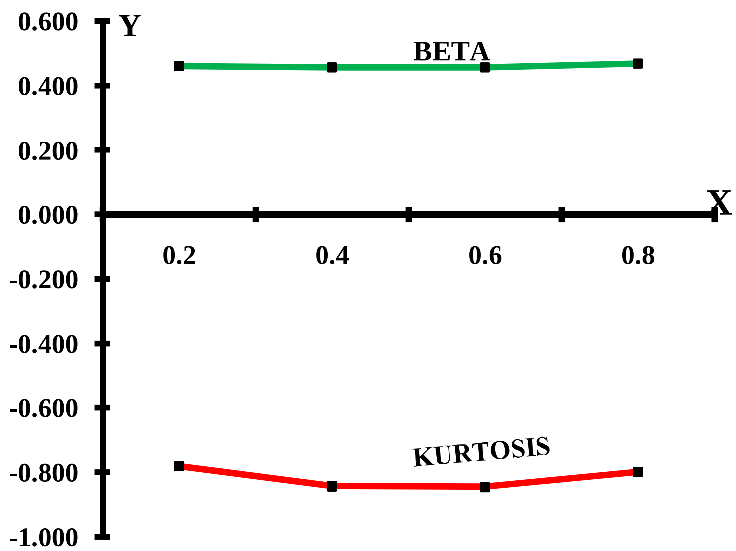


Figure 6.10. Transforms Values, Kurtosis and Beta Error

6.5 Case Study II – No Intercept Model High Sigma Process Data

In practical situations it is likely that the data from a cascade linearly dependent process may not have intercept term. An illustration from a no intercept model data set is considered. The data has been collected from a high sigma process with cascade property. It is identified that the nature of the process is no intercept model data by virtue of the fact that the lower confidance interval of the intercept term is negative. The data collected and transformed values are shown in table 6.11

Table 6.11. No Intercept Model High Sigma Process Data

Data No	Geometric Data		Normal Data	
	Regressor Process ‘X’	Response Process 'Y'	Transformed Regressor Process 'X'	Transformed Response Process 'Y'
1	47	78	1.039	1.045
2	21	35	1.031	1.036
3	22	36	1.031	1.036
4	65	107	1.043	1.048
5	61	78	1.042	1.045
6	44	56	1.039	1.041
7	38	49	1.037	1.04
8	16	21	1.028	1.031
9	31	40	1.035	1.038
10	32	41	1.035	1.038
11	58	75	1.041	1.044
12	42	50	1.038	1.04
13	59	71	1.042	1.044
14	45	54	1.039	1.041
15	42	60	1.038	1.042
16	65	87	1.043	1.046
17	26	35	1.033	1.036
18	53	81	1.041	1.045
19	63	96	1.042	1.047
20	16	19	1.028	1.03

The influence of skewness on the lower control limit and kurtosis on type II error has been studied in the context of two scenarios. The first scenario is the case where a no intercept model data is assumed wrongly as intercept model data and the second scenario is the case where a no intercept model data is rightly assumed as no intercept model data. The results of the work have been documented in the following tables and figures.

6.5.1 Scenario I- No Intercept Model Assumed as Intercept Model

Tables 6.12, 6.13, 6.14, 6.15 depict the data for the scenario where a no intercept model data is wrongly assumed as intercept model data. The figures 6.11, 6.12, 6.13, 6.14 show the respective graphs.

Table 6.12. Transforms (Finer) and Influence of Skewness on Lower Control Limit

Transform	Skewness(S)	LCL	Sensitivity %	
			Δs/s	ΔLCL/LCL
0.002	0.622	-0.0003	0	0
0.004	0.622	-0.0006	0.0	102
0.006	0.622	-0.0009	0.1	206
0.008	0.622	-0.0012	0.1	311
0.01	0.621	-0.0029	0.2	862
0.012	0.621	-0.0016	0.2	419
0.014	0.621	-0.0019	0.2	529
0.016	0.621	-0.0022	0.2	637
0.018	0.621	-0.0025	0.2	750

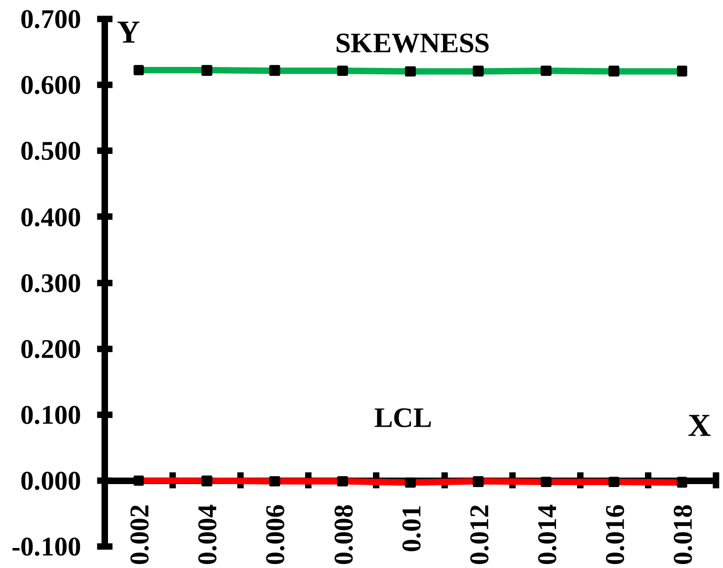


Figure 6.11. Transforms (Finer) and Influence of Skewness on Lower Control Limit

Table 6.13. Transforms and Influence of Skewness on Lower Control Limit

Transform	Skewness(S)	LCL	Sensitivity %	
			Δs/s	ΔLCL/LCL
0.2	0.609	-0.071	0	0
0.4	0.609	-0.357	0	401
0.6	0.619	-1.340	2	1781
0.8	0.637	-4.449	5	6149

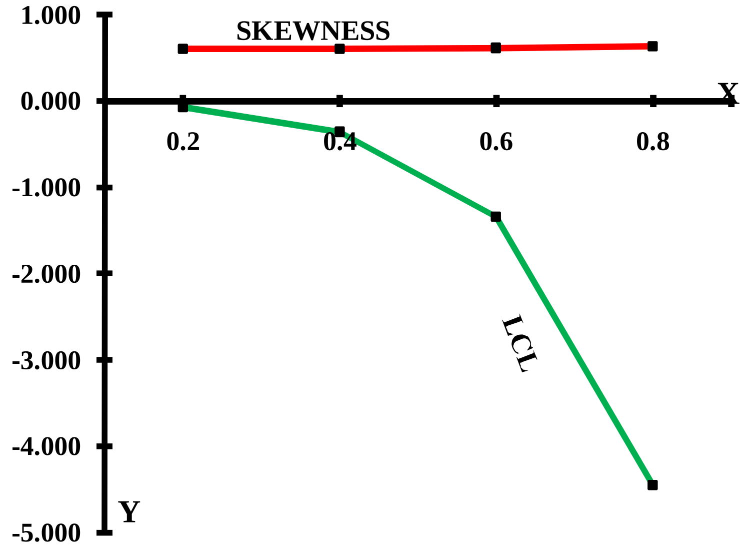


Figure 6.12. Transforms and Influence of Skewness on Lower Control Limit

Table 6.14. Transforms (finer) and Influence of Kurtosis on Beta Error

Transform	Kurtosis (k)	BETA β	Sensitivity %	
			Δk/k	Δβ/β
0.002	-1.042	0.919	0	0
0.004	-1.043	0.919	0.11	0.001
0.006	-1.045	0.919	0.23	0.000
0.008	-1.046	0.919	0.34	0.000
0.01	-1.051	0.919	0.86	0.000
0.012	-1.047	0.919	0.45	0.000
0.014	-1.048	0.919	0.55	0.000
0.016	-1.049	0.919	0.66	0.000
0.018	-1.050	0.919	0.76	0.000

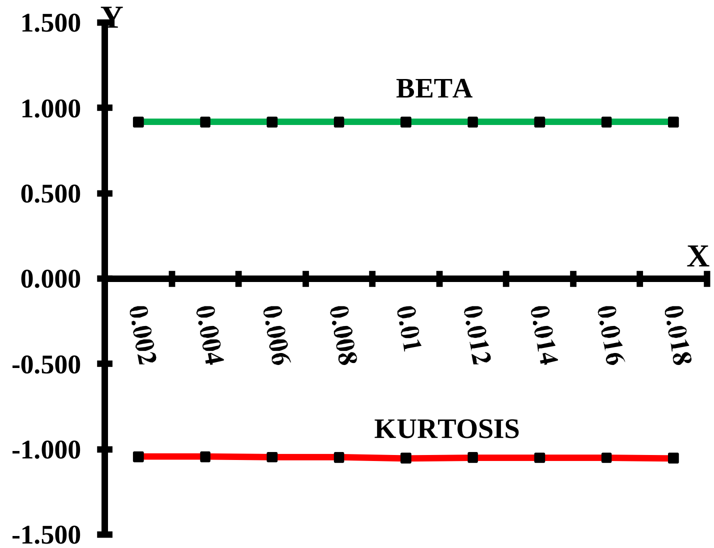


Figure 6.13.Transforms (finer) and Influence of Kurtosis on Beta Error

Table 6.15. Transforms and Influence of Kurtosis on Beta Error

Transform	Kurtosis (k)	BETA β	Sensitivity %	
			Δk/k	Δβ/β
0.2	-1.075	0.908	0	0
0.4	-0.971	0.885	10	3
0.6	-0.780	0.848	27	7
0.8	-0.545	0.824	49	9

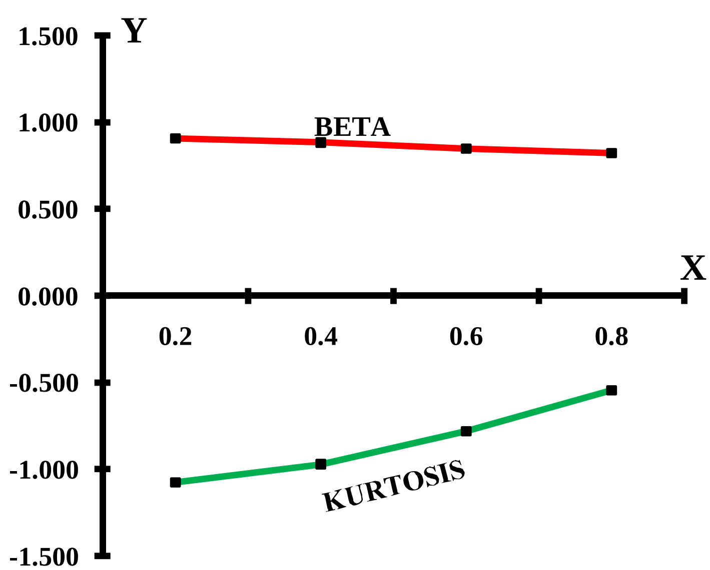


Figure 6.14. Transforms and Influence of Kurtosis on Beta Error

6.5.2 Scenario II - No Intercept Model Assumed as No Intercept Model

In this scenario the instances where a no intercept model data rightly assumed as no intercept model data has been considered. The data so researched have been detailed hereunder as tables 6.16, 6.17, 6.18, 6.19 along with graphs 6.15, 6.16, 6.17, 6.18.

Table 6.16. Transforms (finer) and Influence of Skewness on Lower Control Limit

Transform	Skewness(S)	LCL	Sensitivity %	
			Δs/s	ΔLCL/LCL
0.002	0.609	0.000	0	0
0.004	0.609	-0.001	0.00	101
0.006	0.609	-0.001	0.01	205
0.008	0.609	-0.001	0.02	310
0.01	0.609	-0.003	0.05	860
0.012	0.609	-0.002	0.02	561
0.014	0.609	-0.002	0.03	524
0.016	0.609	-0.002	0.03	634
0.018	0.609	-0.003	0.04	746

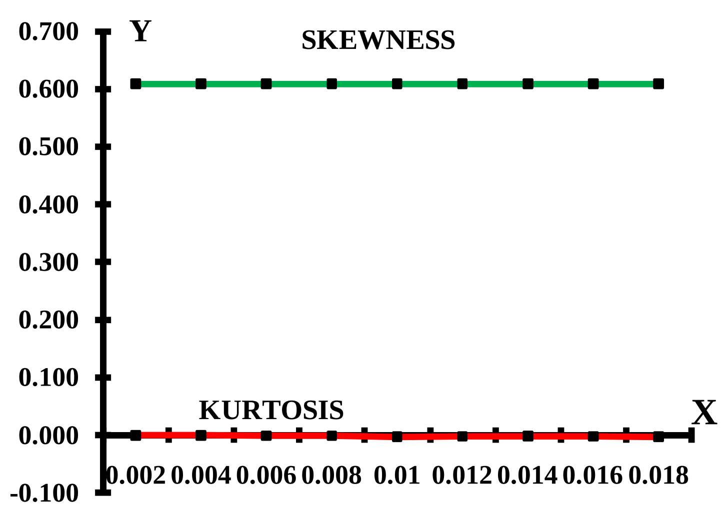


Figure 6.15. Transforms (finer) and Iinfluence of Skewness on Lower Control Limit

Table 6.17. Transforms and Influence of Skewness on Lower Control Limit

Transform	Skewness(S)	LCL	Sensitivity %	
			Δs/s	ΔLCL/LCL
0.2	0.62	-0.07	0	0
0.4	0.65	-0.36	5	401
0.6	0.70	-1.35	13	1786
0.8	0.76	-4.49	22	6184

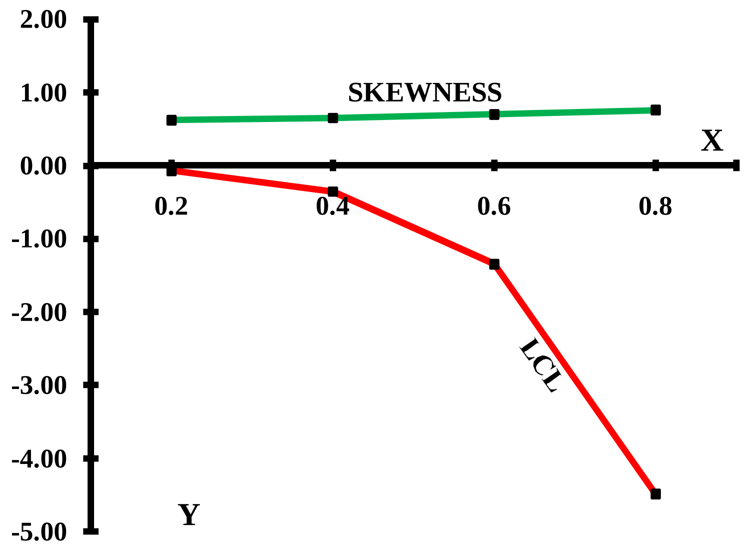


Figure 6.16. Transforms and Influence of Skewness on Lower Control Limit

Table 6.18. Transforms Values (Finer), Kurtosis and Beta Error

Transform	Kurtosis(k)	BETAβ	Sensitivity %	
			Δk/k	Δβ/β
0.002	-1.077	0.910	0	0
0.004	-1.078	0.910	0.085	0.0000
0.006	-1.079	0.910	0.167	0.0011
0.008	-1.080	0.910	0.248	0.0000
0.01	-1.084	0.910	0.624	0.0000
0.012	-1.081	0.910	0.327	0.0044
0.014	-1.081	0.910	0.404	0.0000
0.016	-1.082	0.911	0.479	0.1770
0.018	-1.083	0.908	0.553	0.1803

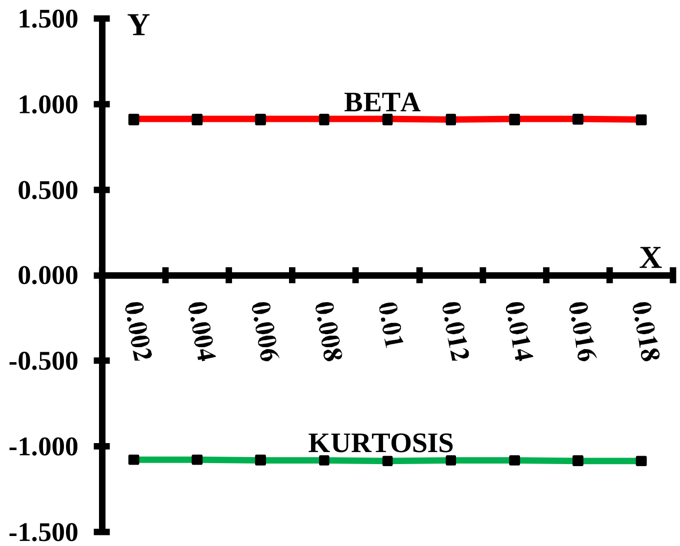


Figure 6.17. Transforms Values (finer), Kurtosis and Beta Error

Table 6.19. Transforms Values, Kurtosis and Beta Error

Transform	Kurtosis(k)	BETAβ	Sensitivity %	
			Δk/k	Δβ/β
0.2	-1.077	0.901	0	0
0.4	-0.925	0.877	14	3
0.6	-0.670	0.839	38	7
0.8	-0.360	0.816	67	9

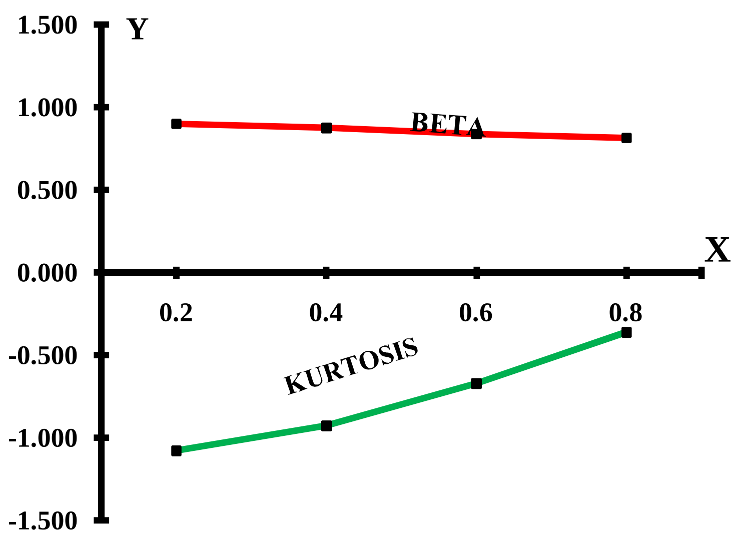


Figure 6.18. Transforms Values, Kurtosis and Beta Error

It can be seen that as the kurtosis decreases the beta error also decreases. It can be seen from above that for the finer mesh transform values, skewness has no effect on LCL and kurtosis has also had no effect on beta error. While the skewness has remained stable for transforms 0.2 to0.8, the LCL tends to decrease. Kurtosis, for transforms from 0.2 to 0.8 decrease and also type II error as transform value increases.

6.6. Discussions

The cause selecting control charts comprised of a Shewhart type control chart for the upstream process and a residual type control chart termed in current literature as cause selecting control chart. The action signals from both the charts enables the production engineer to find out whether there is a defect in the downstream process on account of a defect earlier occurred in the upstream process. A power transform is proposed in this work. . The power transform converted the geometric form of high sigma data into normal form. In research literature usually the metric for assessing the extent to which normality is achieved is measured by descriptive statistical indices. In this work two deterministic indices of normality, namely the third and fourth moments of the normal distribution known as skewness and kurtosis have been studied in relation to their response on LCL and type II error. The results are listed below in table 6.20.

Table 6.20 Summary of Results – Influence of Skewness on LCL and Kurtosis on Type II Error

Sl. No	Scenario	Skewness	LCL	Kurtosis	Type II Error
1	Intercept model data assumed as Intercept Model data.	Increases from -0.564 to0.225	Decreases from -0.235 to -0.8446	Decreases from 0.340 to .018	Decreases from 0.516 to 0.456
2	Intercept model data assumed as no Intercept Model data.	Decreases from 0.653 to 0.502	Decreases from - 0.471 to -13.538	Marginally decreases from -0.780 to -798	Marginally increases from 0.460 to 0.468
3	No intercept model data assumed as Intercept Model data	Increases from 0.609 to 0.637	Decreases from - 0.071 to -4.449	Increases from -1.075 to -0.545	Decreases from 0.908 to 0.824
4	No intercept model data assumed as No Intercept Model data	Increases from 0.620 to 0.756	Decreases from -0.071 to -4.491	Increases from -1.077 to -0.360	Decreases from 0.901 to 0.816

The influence of skewness on LCL and kurtosis on type II has been studied with reference to the following four scenarios.

i. An intercept model data treated as intercept model data

ii. An intercept model data mistreated as no intercept model data

iii. A no intercept model data mistreated as intercept model data

iv. A no intercept model data treated as no intercept model data

The study of the influence of skewness and kurtosis on LCL and type II error in the backdrop of model errors for the transform proposed and inferences made is the fifth contribution of this work.

6.7 Conclusion

Normality is a key requirement in both control chart methodology and in regression analysis. The issue is not the complete transformation to normality but to the extent to which the normality is achieved. In many real life shop floor problems, the data may far deviate from the expectations of theoretical parameters. Hence in this work the extent of normality is measured by third and fourth moments of the normal distribution, namely, skewness and kurtosis. The influence of skewness on LCL and kurtosis on type II error has been studied and results recorded. In the next chapter results and related issues have been discussed. The methodology and results have been proved using data from literature.

Chapter 7

Results and Discussions

7.1 Introduction

Cause selecting control chart of residual type for high sigma process has the lower control limit (LCL) as the action limit. This is akin to the regular high sigma control charts which use the count of conforming products between two non conforming products as the process data. In this work a new power transform is proposed. The effectiveness of power transform with respect to LCL has been addressed. Secondly the intercept model issue which has a bearing on the chart statistic and the resulting performance of the chart has been studied. Thirdly the skewness and kurtosis which are the third and fourth moments of the normal distribution (transform) has been studied with reference to their influence on the LCL and type II error.

7.2 Typical application

The application of this control chart proposed in this work is many and can be directly applied to critical process situations.Manufacturing apart this control chart may be readily applied to environmental pollution control applications, wherein near zero defects results are expected.

For analogy, consider the case of chrome plating effluent proposed treatment plant. In this process the chromium effluent is hexavalent. The treatment process involves reducing the hexavalent chromium to trivalent chromium as the first stage and alkali reduction and chromium precipitation of trivalent chromium as the second stage. Even though by theory of chemistry, the effluent treatment is an irreversible process, human errors of dosage, wrong estimation of incoming effluent, insufficient treatment time and many other factors contribute to bad treatment. In order to control such a process it is imperative to have a cause selecting control chart for process X, being the reduction of hexavalent chromium process as regressor and process Y ,the alkali reduction process as response, the covariant renders itself a fit candidate for the high sigma cause selecting control chart application.

7.3 Economic Design

Ever since Shewhart designed the first control chart on the basis of ‘statistical stability’ of the process some more design criteria have been advocated by researchers. The control charts were designed on the criteria

i. Statistical design.
ii. Economic designs.
iii. Heuristic designs.

All the three design criteria are water tight compartments having their own drivers. In this work statistical regression model based design is taken up.

According to Woodall and Montgomery (1999) as the economically designed charts have too poor statistical property and are in general incapable of detecting process improvement, the use of statistical constraints is recommended as proposed by Sanega (1989)

7.4 Lower Control Limit

The traditional control charts have two action limits namely the upper control limit (UCL) and lower control limits (LCL). Any point falling out side the UCL and LCL are diagnosed as an action signal showing an out of control situation. In case of high sigma control charts with count data any data point falling below the LCL is diagnosed as a signal. All points above the UCL are considered as a process improvement. Thus the LCL is the only action limit for the high sigma control chart.

In the case of count of conforming items control chart the process data is in the geometric form. As it is a negatively skewed distribution, more often negative LCL has been encountered. The first issue addressed in this work is the design of a power transformation for converting the geometric form of data into normal form of data.

In this work two process situations comprising of cold drawing and cold forming process have been considered. Two process scenarios have been analyzed for a pin manufacturing process and the relationship between the LCL and the transforms are shown in figures 7.1 and 7.2 and the data are shown in table 7.1 and table 7.2.

i. Cold forming (upstream process) having higher average run length.

ii. Cold drawing (downstream process) having higher average run length.

Table 7.1. Transform Value and Their Relative LCL Values for Scenario 1

TRANSFORMATIONS				
Transform values 'r'	1/r	Proposed transform	Double Square Root Transform	Logarithmic Transform
4	0.25	-1.395	-0.561	-0.744
6	0.16	-0.6	-0.561	-0.744
8	0.12	-0.362	-0.561	-0.744
10	0.10	-0.251	-0.561	-0.744
50	0.02	-0.019	-0.561	-0.744
100	0.01	0	-0.561	-0.744

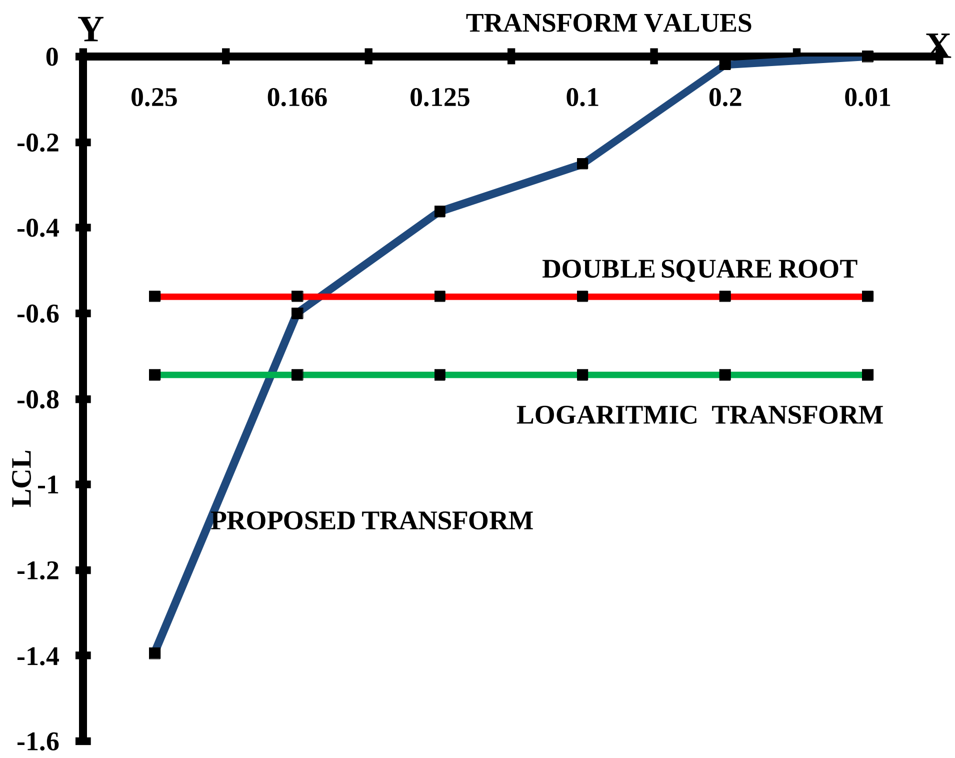


Figure 7.1. Transform Value and Relative Performance of Transforms

Scenario 1

Table 7.2. Transform Value and Their Relative LCL Values for Scenario II

		Transformations		
Transform values 'r'	1/r	Proposed transform	Double Square Root Transform	Logarithmic Transform
4	0.25	-0.561	-0.509	-0.219
6	0.16	-0.256	-0.509	-0.219
8	0.12	-0.161	-0.509	-0.219
10	0.10	-0.115	-0.509	-0.219
50	0.02	-0.016	-0.509	-0.219
100	0.01	-0.008	-0.509	-0.219

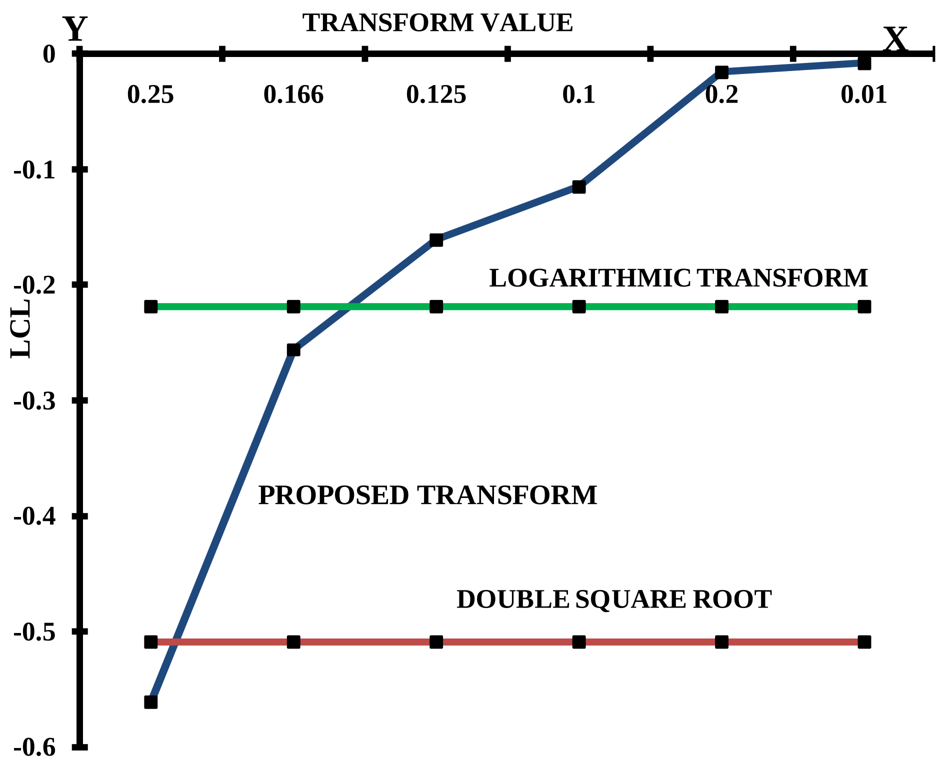


Figure 7.2 Transform Value and Relative Performance of Transforms Scenario II

It can be inferred from above two scenarios that irrespective of process status the transform proposed in this work has out performed the conventional logarithmic and double square root transforms. For the proposed ‘0.01’ power transform value the best near zero value of LCL has been obtained. In scenario 1 the double square root transform out performs the logarithmic transform and in scenario 2, the logarithmic transform out performs the double square root transform. In both the scenarios, the proposed power transform outweigh the conventional transforms.

7.5 Intercept Model Issue

Regression Models form the basis of cause selecting control charts for high sigma process. In the traditional control charts such parameter issues affect only the chart performance indices and not the chart statistic. The CS being a residual type chart based on the regression model, the coefficients of the model are affected by the model errors pertaining to the intercept term. In turn this affects the CS chart statistic .In earlier works the estimation errors pertaining to the slope and intercept coefficients have been addressed for 3 sigma control charts. They assume the presence of an intercept term.

The presence or otherwise of the intercept term has to be established first, before the estimation of coefficients. Model estimate of the intercept term for high sigma process is discussed in this work. The design flow for a cause selecting control chart for a high sigma cascade process has been established. The design concept is explained through numerical examples. It is shown that when a regression model with intercept term has been force fitted as a model with no intercept term, Type II error is deflated. The extent of deflation is more at lower control limit and less at process mean. The average run length is more deflated at the process mean and is having no significant effect near lower control limit. This is evident from figures 5.6 and 5.7.

It has also been established that when a data set with a no intercept model is force fitted as intercept model, the Type II error is inflated at the lower control limit and is not having any significance near the process mean. In such a scenario, the average run length is also inflated near the mean and lower control limit. The inflation is more in magnitude at the process mean compared to that at the lower control limit. This is shown in figures 5.8 and 5.9.

7.6 Third and Fourth Moments of Normality

Traditionally the transformations are optimized by maximum likelihood estimate. It is evident from the figure 6.1 that transform 0.004 is the optimum power transform. The object of power transform in this work is to convert the geometric data of high sigma process to normal data. The extent of normality is gauged by skewness and kurtosis. Skewness is a measure of symmetry, or more precisely, the lack of symmetry. A distribution, or data set, is symmetric if it looks the same to the left and right of the center point. The formula for skewness is,

$$skewness = \frac{\sum_{i=1}^{N}(Y_i - Y)^3}{(N-1)s^3} \tag{7.1}$$

Kurtosis is a measure of whether the data are peaked or flat relative to a normal distribution. That is, data sets with high kurtosis tend to have a distinct peak near the mean, decline rather rapidly, and have heavy tails. Data sets with low kurtosis tend to have a flat top near the mean rather than a sharp peak. A uniform distribution would be the extreme case.

$$kurtosis = \frac{\sum_{i=1}^{N}(Y_i - \bar{Y})^4}{(N-1)s^4} \tag{7.2}$$

In this work skewness and kurtosis is calculated using excel spread sheet.

It is thus demonstrated that the finer value of transformations have no change in skewness or kurtosis. In turn they also have no influence on LCL and type II error respectively. This is demonstrated in figures 6.3 and 6.5. For values of transform from 0.2 to 0.8 it is shown that the LCL has a bearing on skewness and type II error varies according to kurtosis. A summary of results obtained in this work is given in table 6.20.

It can be observed that skewness shift to the negative or positive side creates a negative shift in LCL. Kurtosis shifting to positive or negative side creates a decrease in type II error.

When there is a model error, for intercept model data, skewness is inflated and LCL is deflated and for no intercept model data skewness is deflated LCL is inflated.

When there is a model error, for no intercept model data, kurtosis is marginally deflated and type II error is deflated and for no intercept model data kurtosis is marginally inflated and type II error is marginally deflated.

It can be observed from above information that shift of skewness to the negative or positive side creates a negative shift in LCL. Kurtosis shifting to positive or negative side creates a decrease in type II error.

When there is a model error, for intercept model data, skewness is inflated and LCL is deflated and for no intercept model data skewness is deflated LCL is inflated.

When there is a model error, for no intercept model data, kurtosis is marginally deflated and type II error is deflated and for no intercept model data kurtosis is marginally inflated and type II error is marginally deflated.

When an intercept model data is construed as intercept model data the increase in the value of the transform decreases the negative skewness towards zero. This causes the decrease in value of negative lower control limit. This also leads to the distribution becoming from leptokurtic to mesokurtic. This decreases type II error probability.

When the intercept model is assumed as no intercept model there is a decrease in positive skewness and decrease in negative LCL. There is no appreciable change in kurtosis or type II error probability.

When a no intercept model data is assumed as intercept model data the positive skewness increases creating a decrease in negative LCL.

When a no intercept model data is assumed rightly as a no intercept model data, the negative LCL further moves to negative side. In both the cases the distribution transforms itself from platykurtic to mesokurtic. This decreases the type II error probability. In this work the influence of skewness and kurtosis on the high sigma cause selecting control chart performance indices has been studied. It has been established that for finer mesh of transformed values there is no significant change in both parameter indices and chart performance indices.

The traditional method of optimizing the transform is to identify the transform value for which the residual sum of squares of the error terms is the minimum. In other words it is called the maximum likelihood estimate (MLE).This paper establishes a new type of transform optimization for high sigma cause selecting control charts by studying the influence of skewness and kurtosis which are the third and fourth moments of the normal distribution. The study encompasses the intercept model error of the regression models which are the key stones of the cause selecting control charts.

7.7 Six Sigma Process Data

Six sigma concept was pioneered by Motorola corporation, Allied Signal (Honeywell), General Electric, Microsoft are the other companies which have qualified themselves in the concept of six sigma at the organization level. The control charts are deployed in the control phase. According to Goh and Xie (2003) even today only 3 sigma control charts are preached in the pedagogy for green belts and black belts. Hence there is a due need for a control chart that can work above 3 sigma. As the above mentioned companies are having electronic and software background and as there are no published information by them on the control chart data, in the high quality environ it is not possible to obtain process data.

One more source for the required data is to use data from research publications. As this work is a pioneer in its nature of designing a control chart for high sigma cause selecting control chart, it was not possible to obtain information for a bivariate cascade process. Further an overview of high sigma process show that there are no published data available for a high sigma cascade process.

7.8 Data Validation

In this work same two sets of data have been used to demonstrate the effectiveness of the proposed design, performance indices of the chart and the transform. As enunciated in the forgone sections from literature or from field it is impossible to get six sigma data as few handful organizations have been certified for six sigma concept. Further many of the pioneers of six sigma concepts are basically electronics based units. No data could be obtained from them for a cascade process.

Data validation is the art of ensuring that the body of data confirms to the assumptions made. It is also to ensure that the proposed concept is also validated. As this work is a first work on high sigma cause selecting control chart, in order to prove the concepts proposed and results derived data from published research in the area of high sigma cumulative counts of conforming data is used. The validation of the results of this research is further demonstrated using two sets of data from published research in the following sections.

7.9 Data from Literature

Mandel (1969) used post office mail delivery data to establish his ‘regression’ control chart. He used the data on mail processing hours and mail volume. Constable et al. (1987) used a bivariate process data from tetramicin pharmaceutical works to prove his ‘cause electing control chart’. Zhang (1989) used data from an antibiotic factory to substantiate his work on cause electing control carts. Wade and Woodall (1993) used process data for a automotive component earlier used in another research work. Shu and Tsung (2000) gave the theoretical frame work for using the prediction limits as the control limits. No numerical data was used. Yang and Chen (2003) in their work on cause selecting control chart for two failure mechanisms used data from a cotton yarn factory to substantiate their work. Shu and Tsung (2003) enunciated theoretical frame work on behavior of cause selecting control charts. Shu et al. (2004) used simulated samples of 50 to prove their discussion on model uncertainties for cause selecting control charts. Shu et al. (2004, 2005) again used simulated data in their publication on regular and EWMA control chart for residuals. Yang and Yang (2006) used golden thin film process bivariate data.

All these research publications used one time data from isolated processes to substantiate the theory proposed. These data sets belong to three sigma process situations. No data validation is demonstrated. This work enhances the cause selecting control chart theory to high sigma production processes. As there are no six sigma productions system data available in public domain which contains linear cascade property, two data sets of CCC data available from literature are used to validate the design methodology and also the data.

7.9.1 Xie et al (2002) Data Set

A set of geometrically distributed data was adopted from Xie et al. (2002) shown in table 7.3. The data set contained 100 observations of count of conforming items between two non conforming items.

Table 7.3 CCC Geometric Data – Xie et al. (2002)

Sl.No	Process X	Process Y	Sl.No.	Process X	Process Y
1	227	409	26	678	2196
2	2269	4845	27	2088	1494
3	1193	4809	28	1720	1906
4	4106	504	29	1656	548
5	154	257	30	201	987
6	12198	702	31	3705	6216
7	201	4298	32	4042	704
8	9612	1320	33	716	6477
9	4045	1845	34	2010	233
10	678	4641	35	402	855
11	2088	2815	36	539	188
12	1720	903	37	2665	4133
13	5562	755	38	1711	780
14	4042	565	39	1602	315
15	716	973	40	71	1425
16	2010	2555	41	546	580
17	402	1822	42	655	957
18	539	4324	43	2065	1443
19	8465	1140	44	286	3880
20	2269	109	45	1385	1357
21	1193	1981	46	354	234
22	4106	387	47	934	1836
23	154	3268	48	3539	7984
24	2011	2666	49	1671	110
25	4045	5498	50	3955	128

This has been assumed as data from two different processes having 50 data each. The assumption is that process performance at two different intervals can be assumed as two different processes. The data is summarized in table 7.4. As the linear or non linear relationship between the processes can be approximated into a linear relationship, it is assumed that the two processes are linearly dependent cascade processes. Owing to the design criteria proposed by Lakshminarasimhan and Kannan (2008) the regression statistic of the data set have been determined and shown in table 7.4.

Table 7.4 Intercept Model Regression Statistic

Coefficients	Value	Lower confidence interval @ 95%	Upper confidence interval @ 95%	Mean Square for Residuals
Intercept	1.11895289	0.81675489	1.421150889	0.000014000
X Variable 1	-0.04241789	-0.323691741	0.23885596	0.000151343

Table 7.5 shows the regression statistic for no intercept model data set.

Table 7.5 No Intercept Model Regression Statistic

Coefficients	Value	Lower confidence interval @ 95%	Upper confidence interval @ 95%	Mean Square for Residuals
Intercept	0	Not Applicable	Not Applicable	Not Applicable
X Variable 1	0.998989075	0.994261344	1.003716805	0.000319441

As the mean square of residuals is lower in for the intercept model data set we can infer that the data set is of an intercept model. Further for the intercept model statistic shown in table 7.5 the lower confidence intervals of the intercept is positive. This implies that the regression model has an intercept term.

The control charts are judged by the false alarm of type I error and missed signals or type II error. In the case of high sigma processes as the defects are scarce missed signals deserve attention compared to false alarms. This is because lack of attention to the missed signals will result in rapid slide down of the process sigma from six or more.

The β or type II risks computed for the data set for the following scenarios:

Scenario I : This intercept model data set assumed as intercept model.

Scenario II : This intercept model data set assumed a no intercept model.

Table 7.6 details the scenario I calculations and table 7.7 shows the scenario II. The sensitivity of the change in β from mean to LCL and the incremental changes in ARL have all been computed and detailed.

Table 7.6 Scenario I – Intercept Model Data Fitted as Intercept Model Xie et al. (2002)

Position	Shift magnitude	% Shift Magnitude	β	Sensitivity Δβ / β	ARL	Sensitivity ΔARL/ARL
Mean	0.60	20	0.980	0	49	0
	1.20	40	0.927	5	14	72
	1.80	60	0.805	18	5	90
	2.40	80	0.603	38	3	95
LCL	3.00	100	0.367	63	2	97

Table 7.7 Scenario II – Intercept Model Data Fitted as No Intercept Model Xie et al. (2002)

Position	Shift magnitude	% Shift Magnitude	β	Sensitivity Δβ/β	ARL	Sensitivity ΔARL/ARL
Mean	0.65	20	0.977	0	44	0
	1.30	40	0.913	7	11	74
	1.95	60	0.761	22	4	90
	2.60	80	0.524	46	2	95
LCL	3.25	100	0.278	72	1	97

The respective characteristic diagrams are shown in the figures 7.3 and 7.4. From the two curves the following is inferred. When the intercept model data mistakenly assumed as no intercept model data the type II or beta error is deflated. The deflation is more at the LCL compare to that at the process s mean. For this model error the ARL is deflated significantly at the process mean. There is no noticeable change at the LCL.

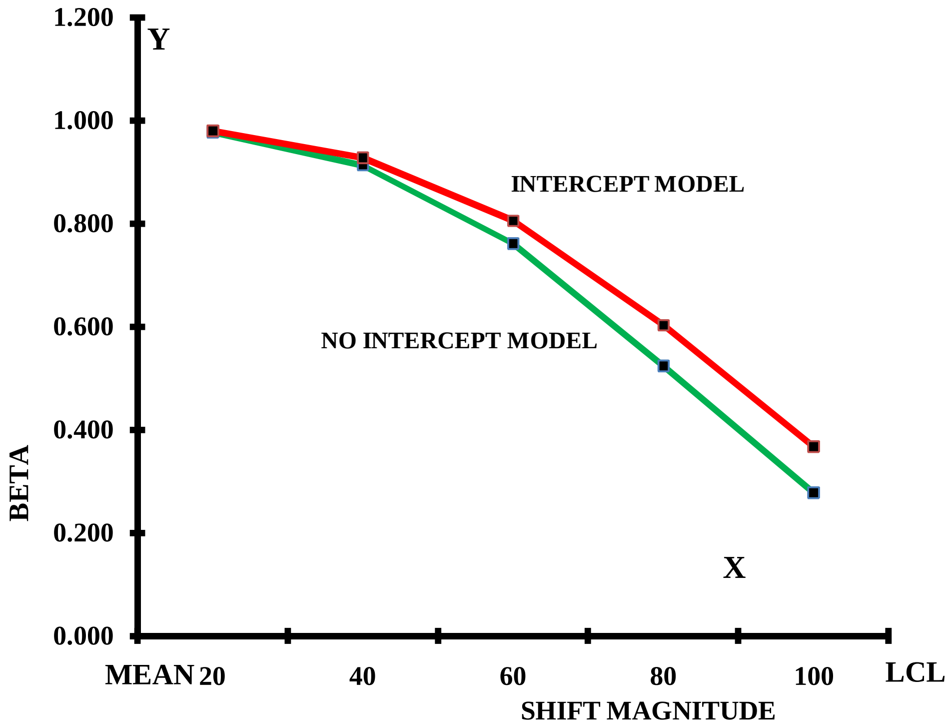


Figure 7.3 Shift Magnitude vs Beta Error Xie et al. (2002)

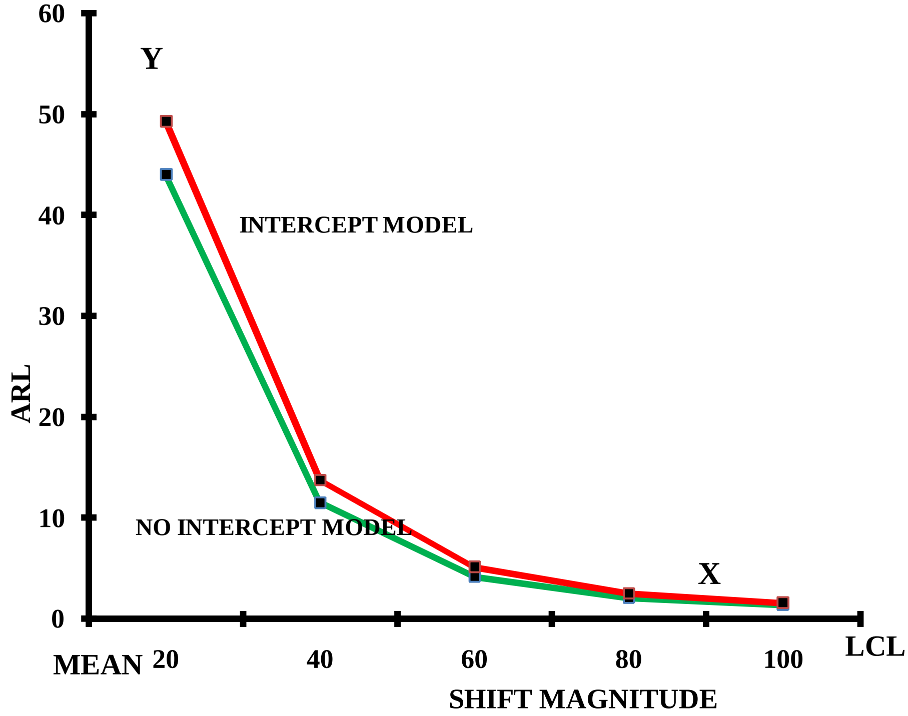


Figure 7.4 Shift Magnitude vs ARL Xie et al. (2002)

For the data set of Xie et al. (2002) extent of normality achieved have been checked for various transform values. The data set is an intercept model data set. When there is no model error, for various transform values the skewness and the respective LCL values have been calculated and listed in table 7.8. Figure 7.5 shows the graph of this data. As the transform value increases the skewness increases with corresponding decrease in LCL.

Table 7.8 Skewness and LCL – Xie et al. Data (2002)

Scenario I -Intercept model data fitted as intercept model

Transform	Skewness (S)	LCL	Sensitivity %	
			ΔS/S	ΔLCL/LCL
0.2	0.0587	-2.8277	0	0
0.4	0.4209	-24.2431	617	757
0.6	0.7447	-160.5086	1168	5576
0.8	1.0355	-971.1766	1664	34246

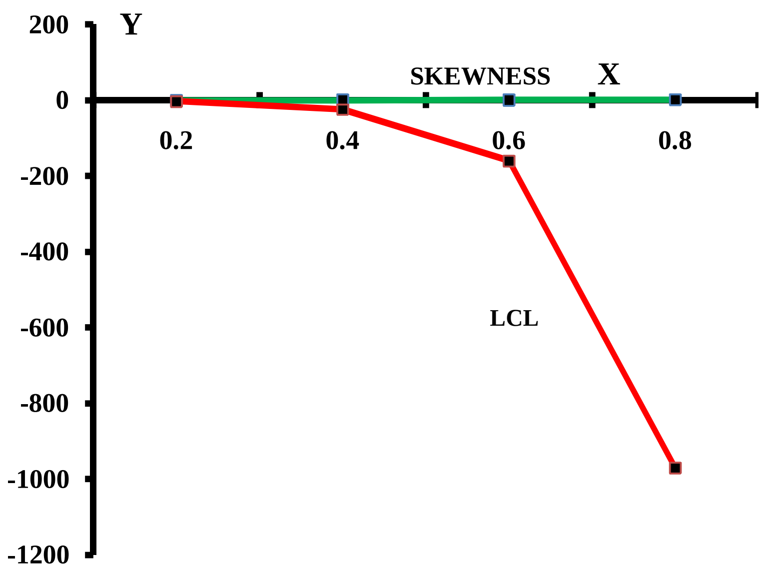


Figure 7.5 Skewness and LCL – Xie et al. Data (2002)

Scenario I – Intercept model data fitted as intercept model

As already established in this work, for finer mesh of transforms, the skewness has no appreciable impact on LCL.

For this data set the effect of kurtosis on type II error has also been studied. The data for various transform values have been summarized in table 7.9. For increase in transform value, increase in kurtosis results in marginal increase in beta error as can be seen from figure 7.6.

Table 7.9 Kurtosis and Beta Error – Xie et al. Data (2002)

Scenario I – Intercept Model Data Fitted as Intercept Model

Transform	Kurtosis(K)	Beta β	Sensitivity %	
			ΔK/K	Δβ/β
0.2	-0.8180	0.3446	0	0
0.4	-0.6757	0.3409	-17	-1
0.6	-0.2720	0.3520	-67	2
0.8	0.3306	0.3707	-140	8

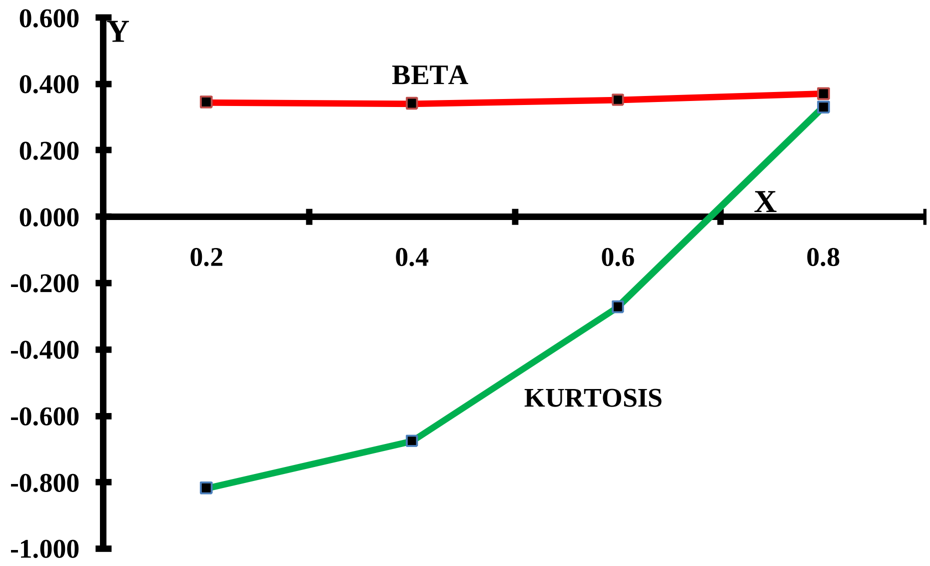


Figure 7.6 Kurtosis and Beta Error – Xie et al. Data (2002)

Scenario I – Intercept Model Data Fitted as Intercept Model

Assuming that there is model error and that Xie et al. Data (2002) which is an intercept model data is erroneously assumed as no intercept model, the skewness and kurtosis values for various transform values have been calculated and its influence on the LCL and Beta error have been studied. These data have been listed in tables 7.10, 7.11 and graphs are shown in figures 7.7 and 7.8.

Table 7.10 Skewness and LCL – Xie et al. Data (2002)(with model error)

Scenario II – Intercept Model Data Fitted as No Intercept Model

Transform	Skewness (S)	LCL	Sensitivity %	
			ΔS/S	ΔLCL/LCL
0.2	-0.12847	-4.10702	0	0
0.4	-0.16030	-31.86507	-25	-676
0.6	-0.07450	-193.70039	42	-4616
0.8	0.14820	-1084.92653	215	-26316

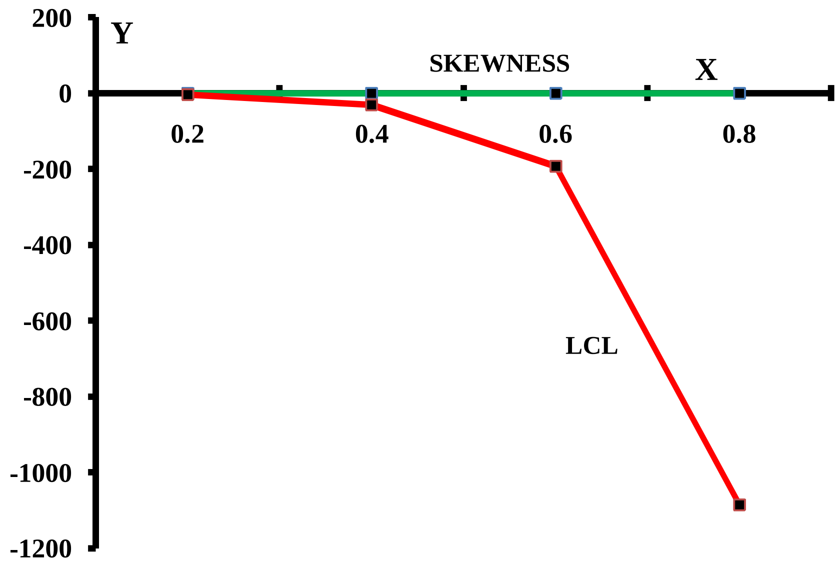


Figure 7.7 Skewness and LCL – Xie et al. Data (2002)(with model error)

Scenario II – Intercept Model Data Fitted as No Intercept Model

Table 7.11 Kurtosis and Beta Error – Xie et al. Data (2002)(with model error)

Scenario II – Intercept Model Data Fitted as No Intercept Model

Transform	Kurtosis(K)	Beta β	Sensitivity %	
			ΔK/K	Δβ/β
0.2	-0.776	0.278	0	0
0.4	-0.711	0.274	-8	-1
0.6	-0.554	0.271	-29	-2
0.8	-0.330	0.281	-58	1

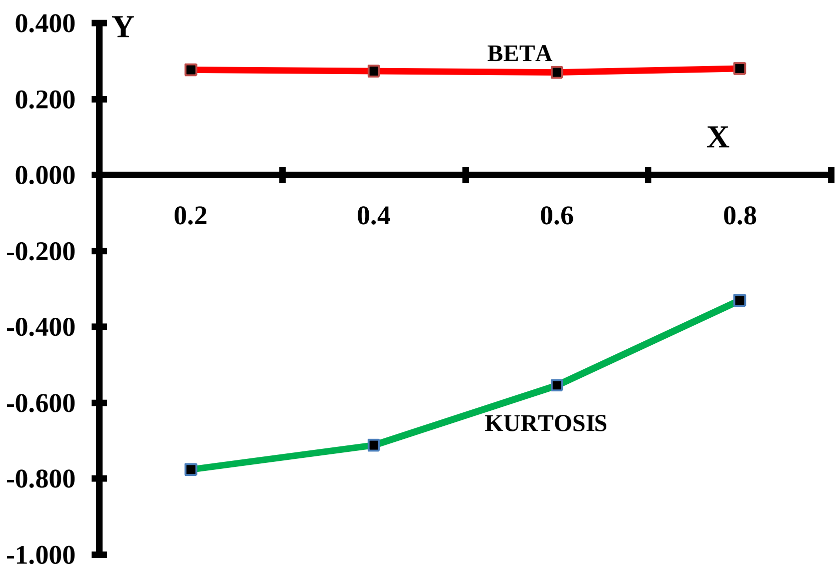


Figure 7.8 Kurtosis and Beta Error – Xie et al. Data (2002)(with model error)

Scenario II – Intercept Model Data Fitted as No Intercept Model

As the transform value increases the skewness increases with corresponding decrease in LCL. For increase in transform value, increase in kurtosis results in marginal increase in beta error.

The data and workings above described vindicate the results of our work established. They are summarized as follows in table 7.12.

Table 7.12.Summary of results - Xie et al. Data (2002)

Sl. No	Scenario	Skewness	LCL	Kurtosis	Type II Error
1	Intercept model data assumed as Intercept Model data.	Increases From0.0564 to 1.0355	Decreases from -0.282 to -971.18	Increases from -0.82 to 0.331	Increases from 0.345 to 0..372
2	Intercept model data assumed as no Intercept Model data.	Increases from -0.128 to 0.15	Decreases from – 4.11 to -1084.93	Decreases from -0.776 to -.330	Marginally increases from 0.278 to 0.281

The extent of normality as measured by third and fourth moments of the normal distribution namely skewness and kurtosis yields the following observations:

When this data (an intercept model data) is construed as intercept model (when there is no model error) the increase in the value of the transform increases the skewness. This causes the decrease in value of negative lower control limit. For this data set as the kurtosis increases marginally and beta error also increases. This unidirectional change is already established in this work.

When this data (an intercept model data) is construed as no intercept model (when there is model error) the increase in the value of the transform increases the skewness. This causes the decrease in value of negative lower control limit. For this data set as the kurtosis increases marginally and beta error also increases. This unidirectional change is already established in this work. All these points establish the fact that the increase in transform value will render it impossible to achieve transformation.

For the Xie et al. (2000) data from literature, the performance of various transforms have been studied and listed in table 7.13.

Table 7.13 Transforms Comparison – Xie et al. (2002) Data

Transform	Type of Transform		
	Power	Double Square Root	Logarithmic
0.01	0.745	-5.071	-3.403
0.2	-2.828	-5.071	-3.403
0.4	-24.243	-5.071	-3.403
0.6	-160.509	-5.071	-3.403
0.8	-971.177	-5.071	-3.403

The pictorial representation of this data is shown in figure 7.9.

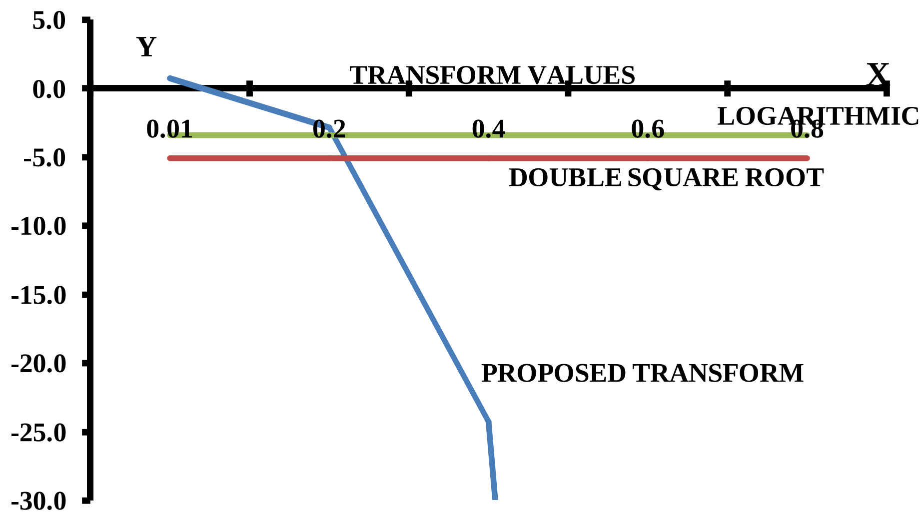


Figure 7.9 Xie et al. Data – Comparison of transforms

We can infer from above graph that the power transform proposed in this work namely '0.01' out performs Double Square Root and Logarithmic transforms.

7.10 Stochastic Behavior

The assumption that the process data are independent is common with the control chart theory and the regression analysis. The cause selecting control chart method encompasses both these ideologies. The run rules are riders for a control chart to enhance its performance. The relevance or otherwise of these issues to this research work is discussed in this section.

The CS is mainly discussed based on a simple linear regression model. However, the linear regression model may sometimes restrict the applications of the CS because it is insufficient to capture the stochastic nature of the output. First, the process may exhibit process dynamics or inertia. The linear regression model cannot characterize the dynamics between the input measure X and output measure Y. Furthermore, when data are collected at high frequency the disturbance to the current process step is more likely to be auto correlated instead of being independent. Hence it is imperative to account for autocorrelation if any in the design of high sigma cause selecting control chart for cascade manufacturing processes. This issue is discussed in this chapter.

In practice, the correlation of data is due to a tool failure, tool wear, machine setting issues and breakdowns. Machines and production systems also exhibit inertia. The feed back corrections affected many time fails to produce desired results due to such inertial forces. Unless this is accounted for, the control charts produce false alarms.

Yang and Yang (2006) studied an approach for controlling two cascade process steps with auto correlated observations. They modeled the observations of X as an AR (1) model data and observation Y as a co-variate transfer function of X. They concluded that the cause-selecting chart performs even with correlated data, as compared to the traditional control charts.

Crowder (1989) established the use of exponentially weighed moving average (EWMA) control chart performs well for forecasting purpose of future data. Crowder (1987) detailed a computer program and formulae for EWMA control chart.

Montgomery and Masterangelo (1991) discussed the forecasting features of EWMA chart with the presence of autocorrelation in data.

Chang (1991) used the principal component method for process quality that is characterized by two or three correlated variables.

Stimson and Masterangelo (1996) discussed about the effect of correlation for serially dependant process data. One important observation of their work is that the defect counts are in general following a binomial distribution. When induced by correlation the parameter switches to geometric distribution.

Rungner’s (2002) works discussed about the inertial elements that govern the independent and auto correlated data and the residual control chart method. To summarize the issue of correlation the questions that arise are as follows. When the whole data set is depicting a binomial distribution with patches of geometric data the adaptation of a remedy will lead to over correction of independent attribute process. The work on correlated variables or attributes, detailed in the foregoing discussion is applicable to 3 sigma process metric as default.

For 3 sigma process, charting data are the actual variables which may display correlation due to assignable causes. In case of high sigma process the count data is of attribute type and may not have correlation. In fact the underlying principle of CCC chart is that when ARL or CCC tends to decrease, the process is assumed to be shifted from target. Likewise it is impractical to assume that increase in CCC depicts a process shift. This is due to the underlying assumption that any point falling above the LCL which is an action limit is a process improvement according to Nelson (1994)

In other words the issue of the process sigma metric has not been accounted for. In the case of a high sigma process above 3 sigma with its scarcity of defect counts, the issue of correlation does not arise in the attribute of count of conforming items between two nonconforming items. This work is confined to the regression based high sigma cause-selecting control chart for a bivariate process. Montgomery et al. (2003) Draper and Smith (2005) details the correlation in regression analysis as the concept of ‘co linearity’. By definition co linearity is correlation between two regressor variables. As this work is confined to a bi-variate case with one regressor and one response variable, the issue of correlation in one variation stream and a resultant effect on the other variation stream does not arise.

Nelson (1980, 1998) defined the mean square successive difference test as a measure of independence of observations. It is average of square of moving ranges M and statistic M is,

$$M = \frac{\sum_{i=1}^{n-1}(x_{i+1} - x_i)^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2} \qquad (7.3)$$

It is used to detect non randomness in a series of observations. The ‘M’ statistic for data set is shown in table 7.14.

Table 7.14 ‘M’ Statistic

Model of Data	Variable	'M'
Intercept	Regressor X	0.243
Intercept	Response Y	0.072
No intercept	Regressor X	0.273
No intercept	Response Y	0.335

7.11 Run Rules

Weindling et al. (1970), Wheeler (1983, 1992, 1993), Nelson (1984), Davis and Woodall (1988), Palm (1990) Walker et al. (1991), Adke and Hong (1997) Clement (1991) dealt in details the signal rates of Shewhart’s control chart and the associated run rules. The control charts work on the basis of control limits. The control limits filter the noise factors and keep the performance indices of type I and type II errors at the designed level. In order to make the control charts sensitive to small shift patterns run rules are advocated. The other options are to increase the sample size and to tinker with control limits with resultant patch work in type I and type II errors. It may be recalled that the control limits are action limits. In the case of cause-selecting control charts with CCC data the lower control limit is the only action limit. Hence with only one action limit and for high sigma process data with its scarcity of points below lower control limits , the question of sensitizing run rules does not arise.

7.12 Conclusion

The normal probability distribution curves for finer mesh of transforms and regular value of transforms are shown in appendix 1. The influence of skewness and kurtosis on LCL and type II error is detailed in figures shown in appendix 2. That the stochastic behavior and run rules are beyond the requirements and scope of this work has been detailed in this chapter. The contributions of this work are listed in conclusion chapter.

Chapter 8

Conclusion

8.1 Introduction

All manufactured products are outcome of many stages of production processes. The customer is interested in the quality of all the characteristics of many stages of production processes together. This requires multistage process monitoring. In practical applications due to complicated statistics, multivariate control charts find little use in the shop floor. Cause selecting control charts work on the principle of least square regression for shop floor type control charts for multivariate process monitoring. In this work cause selecting control charts have been upgraded for high sigma applications. The specific contributions of this research are summarized in this chapter.

8.2 Specific Contributions

Cause selecting control charts are process monitoring tools for serially dependant or cascade manufacturing process. These control charts were first published by Zhang (1984). In current literature these charts are limited to application of 3 sigma process monitoring. In this work the capability of cause selecting control charts have been enhanced for application for bivariate high sigma environments. The specific outcomes are the following:

8.2.1 Power Transform

The high sigma process data is the count of conforming items between two nonconforming items. It is in the form of geometric distribution. For adaptation to high sigma process a transformation is required for conversion to normality. It has been shown that the traditional transformations like double square root transformation and logarithmic transformation are inadequate for the required adaptation. A new power transform has been proposed in this work.

The adequacy or otherwise of the proposed transform is judged by achieving a near zero or positive lower control limit. In high sigma control chart the LCL is the action limit. The geometric distribution being skewed negatively will always yield a negative LCL. But in practical shop floor work there cannot be any fractional or negative number of products and hence the desire for positive LCL. It is shown that the proposed power transform is superior to traditional transforms in this regard.

8.2.2 Intercept models

The cause selecting control charts work on the basis of least square regression principle. The distinct difference of this model is that it can exist with or without an intercept term. The shop floor process data may be depicted at times with or without an intercept term in a regression model. There can be practical instances in which a thread cutting process in a lathe is preceded by ‘turning to size’ process. The data from this process will be an intercept model data. In certain instances the first process may not have an impact on the second process. The data from this kind of production line may depict a no intercept model. In general the regression models have always assumed to contain an intercept term. This assumption will affect chart statistic unlike the traditional control charts. This model error issue has been studied with reference to type II error and suitable design changes have been incorporated in the design flow diagram as shown in figure 6.4. This will further enhance the capability of cause selecting control charts and make it fool proof in shop floor applications. Even though this contribution is a corollary of this work on high sigma environ, this also enhances the 3 sigma cause selecting control charts. The regression model being common to all applications of control charts this added advantage has resulted from this work.

8.2.3 Skewness and Kurtosis

The control chart methodology and regression analysis, both have an underlying assumption that the data from process follow a normal distribution. Traditionally the first two moments of normal distribution namely mean and standard deviation are considered for analyzing the distribution. As these are only descriptive indices, in this work, the third and fourth moments namely skewness and kurtosis which are having inferential qualities have been studied to further analyze the effectiveness of the transform. The influence of skewness and kurtosis with reference to LCL and type II error has been studied and recorded. This will render use as a guide and reference for the shop floor in designing and analyzing and interpreting the high sigma cause selecting control charts. This is a new treatment and revelation that will be used as resource that will percolate the use of cause selecting control chart for high sigma applications.

8.3 Scope for Further Work

In this research work a special type of multivariate control chart known as cause selecting control charts hither to applied to 3 sigma processes has been enhanced to high sigma processes. Though the results are focused on high sigma process environment, they are adaptable to 3 sigma environment also.

Specifically further works can be carried out in,

i. Adaptability of the proposed transform to high sigma Shewhart type control chart.
ii. Study of cause selecting control chart applications for 3 sigma and high sigma process in practical situations of production processes in parallel or combination of serial and parallel configurations.
iii. Study of multicollinearity issues in multiple regression in cascade high sigma process with multiple production stages.
iv. Measurement errors and behavior of cause selecting control charts for high sigma cascade processes of bivariate and multivariate nature.

8.4 Conclusion

This work has enhanced the use of cause selecting control chart for high sigma applications. For brevity a bivariate process has been considered. With the advent of computer software all the results recorded can be directly applied to multivariate cause selecting control charts.

A new power transform which enhanced capability over traditional logarithmic and double square root transform has been established. This will be an antidote to avoid negative LCL. The issues of intercept model errors and its influence on type II error ans ARL have been studied and design modifications incorporated. The influences of skewness and kurtosis on LCL and type II error have been studied and results recorded. This will aid the choice and selection of transforms for different shop floor high sigma environment applications.

It has been established that the proposed transform is superior over logarithmic transform, double square root transform and Nelson (1994) transform in terms of obtaining a positive lower control limit. For the first time the intercept model issues and its impact on control chart design has been discussed for cause selecting control charts. A new method of optimization of transform based on high sigma application requirement has been proposed. After Box and Cox (1964), this is a new method proposed for optimization.

Though advocated for high sigma application all these contributions are also adaptable to and will enhance the three sigma cause selecting control chart performance and high sigma Shewhart type control chart performance.

List of Publications

International Journals

1. Lakshminarasimhan, S. and Kannan, S.M. (2007) 'Transforms for cause-selecting control chart for a bivariate high-sigma process', *International Journal of Six Sigma and Competitive Advantage*, Vol. 3, No. 3, pp.266-281.
2. Lakshminarasimhan, S. and Kannan, S.M. 'Intercept Model Errors and Design of High Sigma Cause Selecting Control Charts'. *International Journal of Process Management and Bench Marking.* (in print – Vol. 2, Issue 4 - 2008)
3. Lakshminarasimhan, S. and Kannan, S.M. 'Influence of Skewness and Kurtosis on High Sigma Cause Selecting Control Chart Performance for Bivariate Cascade Processes'. *International Journal of Quality and Productivity Management.* (accepted for publication in Vol.3, Issue 4 - 2009)

International Conferences

1. Lakshminarasimhan. and Kannan,S.M. Process Monitoring Tool for Detecting Signals from Stationary White Noise for Six Sigma Manufacturing Process., Proceedings of the International Conference on Global Manufacturing and Innovation - July 27-29, 2006 – 1, Coimbatore Institute of Technology, Coimbatore.
2. Lakshminarasimhan, S. and Kannan, S.M. Process Monitoring Scheme for Tool wear for a Six Sigma Manufacturing Process, Proceedings of third international

conference on logistics and supply chain management, August 2006, PSG College of Technology, Coimbatore.

National Conferences

1. Kannan, S.M. Shilpa, M. and Lakshminarasimhan, S. Quality Improvement in Grinding Process using Design of Experiments, National conference on challenges in achieving global quality, CAGQ May 2004 , Thiagarajar College of Engineering, Madurai. pp 83-87
2. Kannan,S.M. Shilpa, M. Lakshminarasimhan, S. Sampling plan design for ensuring average outgoing quality, All India manufacturing technology, design and research conference, December 2004, Vellore Institute of Technology, Vellore, Tamil Nadu)
3. Kannan, S.M. Lakshminarasimhan, S. Janakiram, V.V.R. Product development using Mechatronic system National Conference on Product Development with Mechatronic Systems for Global Quality, PMGQ 2005, Thiagarajar College of Engineering, Madurai. Pp. 23-25.
4. Kannan,S.M. Lakshminarasimhan,S. Anbumeenakshi, S. Quality improvement in Machining Process Using Principal Component Analysis ,National Conference on Product Development with Mechatronic Systems for Global Quality, PMGQ 2005, Thiagarajar College of Engineering, Madurai. Pp. 123-126.
5. Kannan, S.M. Shilpa M. and Lakshminarasimhan, S. Parametric Design for Grooving Operation using Orthogonal Arrays – A case study National Conference

on Product Development with Mechatronic Systems for Global Quality, PMGQ 2005, Thiagarajar College of Engineering, Madurai. Pp. 489-493.

6. Lakshminarasimhan, S. and Kannan, Process Monitoring and Improvement using ANOM to achieve Product Life Cycle Economics in Manufacturing, National conference on design for product life cycle , Feb, 2006, BITS, Pilani.

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