# Appendix A

# Perturbation methods for calculation of $H_{\text{eff}}$

## A.1 Van Vleck expansion based method

We use the following identities in the expression of $H_{\text{eff}} = \sum_{0\leq n<\infty} \frac{1}{\omega^n} H^{(n)}$,

$$e^{iG} H e^{-iG} \;=\; H \;+\; i[G, H] \;-\; \frac{1}{2}\big[G, [G, H]\big] \;-\; \frac{i}{6}\,\big[G, \big[G, [G, H]\big]\big] \;+ ........ \tag{A.1}$$

and

$$\left(\frac{\partial}{\partial t} e^{iG}\right) e^{-iG} \;=\; i\left[\frac{\partial G}{\partial t}\right] - \frac{1}{2}\left[G, \frac{\partial G}{\partial t}\right] - \frac{i}{6}\left[G, \left[G, \frac{\partial G}{\partial t}\right]\right], \tag{A.2}$$

and consider terms upto $\mathcal{O}\big(\frac{1}{\omega^2}\big)$ which gives

$$\begin{aligned} H_{\text{eff}} = H_0 + V(t) + i\left[\frac{G^{(1)}}{\omega}, H\right] + i\left[\frac{G^{(2)}}{\omega}, H\right] - \frac{1}{2}\left[\frac{G^{(1)}}{\omega}\left[\frac{G^{(1)}}{\omega}, H\right]\right] - \frac{1}{\omega}\frac{\partial G^{(1)}}{\partial t} \\ - \frac{1}{\omega^2}\frac{\partial G^{(2)}}{\partial t} - \frac{1}{\omega^3}\frac{\partial G^{(3)}}{\partial t} - \frac{i}{2}\left[\frac{G^{(1)}}{\omega} + \frac{G^{(2)}}{\omega^2}, \frac{1}{\omega}\frac{\partial G^{(1)}}{\partial t} + \frac{1}{\omega^2}\frac{\partial G^{(2)}}{\partial t}\right] \\ + \frac{1}{6}\left[\frac{G^{(1)}}{\omega}, \left[\frac{G^{(1)}}{\omega}, \frac{1}{\omega}\frac{\partial G^{(1)}}{\partial t}\right]\right]. \end{aligned}$$

Here, $G^{(n)}$ is periodic $G^{(n)}(t+T) = G^{(n)}(t)$, and have zero mean over a time period T. Therefore $\langle G^{(n)} \rangle = 0$. At each order of perturbation, the time independent average is retained in $H_{\text{eff}}$ and $\hat{G}$ is designed to nullify the time dependent part. For order $\omega^0$

$$\begin{aligned} & H_0 + V(t) - \frac{1}{\omega}\frac{\partial G^{(1)}}{\partial t} \\ H_{\text{eff}}^{(0)} &= \left\langle H_0 + V(t) - \frac{1}{\omega}\frac{\partial G^{(1)}}{\partial t} \right\rangle \\ &= \langle H_0 \rangle + \langle V(t) \rangle - \left\langle \frac{1}{\omega}\frac{\partial G^{(1)}}{\partial t} \right\rangle \\ &= \frac{H_0}{T}\int_0^T dt + \frac{1}{T}\int_0^T V(t)dt - \frac{1}{\omega T}\int_0^T \frac{\partial G^{(1)}}{\partial t}dt. \end{aligned} \tag{A.3}$$

Potential $V(t)$ is periodic; $V(t+T) = V(t)$ and may be expanded in a Fourier series as

$$V(t) = V_0 + \sum_{1\leq n<\infty} V_n e^{in\omega t} + \sum_{1\leq<\infty} V_{-n} e^{-in\omega t}.$$

Similarly $G^1(t)$ can be expanded as Fourier series and has zero mean therefore $\frac{\partial G^{(1)}}{\partial t}$ also has zero mean. Therefore $H_{\text{eff}}^0 = H_0 + V_0$, where $V_0 = \frac{1}{T}\int_0^T V(t)dt$. Time dependent part can be written as

$$\begin{aligned} H_0 + V(t) - \frac{1}{\omega}\frac{\partial G^{(1)}}{\partial t} - \left\langle H_0 + V(t) - \frac{1}{\omega}\frac{\partial G^{(1)}}{\partial t} \right\rangle \\ = V(t) - V_0 - \frac{1}{\omega}\frac{\partial G^{(1)}}{\partial t}. \end{aligned} \tag{A.4}$$

By equating this time dependent part with zero, we have $V(t) = V_0 + \frac{1}{\omega}\frac{\partial G^{(1)}}{\partial t}$. Here $V(t) = V_0 + \sum\limits_{1\leq n<\infty} V_n e^{in\omega t} + \sum\limits_{1\leq n<\infty} V_{-n} e^{-in\omega t}$, therefore $\sum\limits_{1\leq n<\infty} V_n e^{in\omega t} + \sum\limits_{1\leq n<\infty} V_{-n} e^{-in\omega t} = \frac{1}{\omega}\frac{\partial G^{(1)}}{\partial t}$. As a result at order $\omega_0$

$$\begin{aligned} H_{\text{eff}}^0 &= H_0 + V_0 \\ G^{(1)} &= \frac{1}{i}\sum_n \frac{1}{n}\left(V_n e^{in\omega t} - V_{-n} e^{-in\omega t}\right). \end{aligned} \tag{A.5}$$

Similarly, for order $\mathcal{O}(\omega^{-1})$

$$H^1_{\text{eff}} = \sum_n \frac{1}{\omega n}[V_n, V_{-n}]$$

and

$$\begin{aligned} G^{(2)} = &\frac{1}{i}\sum_n \frac{1}{n^2}\Big(\big[V_n, H_0 + V_0\big]e^{in\omega t} - \text{h.c.}\Big) \\ &+ \frac{1}{2i}\sum_{n,m=1}^{\infty}\frac{1}{n(n+m)}\Big(\big[V_n, V_m\big]e^{i(n+m)\omega t} + \text{h.c.}\Big) \\ &+ \frac{1}{2i}\sum_{n\neq m=1}^{\infty}\frac{1}{n(n+m)}\Big(\big[V_n, V_{-m}\big]e^{i(n-m)\omega t} + \text{h.c.}\Big). \end{aligned} \tag{A.6}$$

Following the same procedure, finally we obtain

$$\begin{aligned} H_{\text{eff}} = H_0 + V_0 + \frac{1}{\omega}\sum_{n=1}^{\infty}\frac{1}{n}[V_n, V_{-n}] + \frac{1}{2\omega^2}\sum_{n=1}^{\infty}\Big(\Big[\big[V_n, H_0\big], V_{-n}\Big] + \text{h.c.}\Big) \\ + \frac{1}{3\omega^2}\sum_{n,m=1}^{\infty}\frac{1}{nm}\Big(\Big[V_n, \big[V_m, V_{-(n+m)}\big]\Big] - 2\Big[V_n, \big[V_{-n}, V_{(n-m)}\big]\Big] + \text{h.c.}\Big)...., \end{aligned} \tag{A.7}$$

and

$$\begin{aligned} G(t) = &\frac{1}{i\omega}\sum_{n=1}^{\infty}\frac{1}{n}\Big(V_n e^{in\omega t} - V_{-n}e^{-in\omega t}\Big) + \frac{1}{i\omega^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\Big([V_n, H_0 + V_0]e^{in\omega t} - \text{h.c.}\Big) \\ &+ \frac{1}{2i\omega^2}\sum_{n,m=1}^{\infty}\frac{1}{n(n+m)}\Big([V_n, V_m]e^{i(n+m)\omega t} - \text{h.c.}\Big) \\ &+ \frac{1}{2i\omega^2}\sum_{n\neq m=1}^{\infty}\frac{1}{n(n-m)}\Big([V_n, V_{-m}]e^{i(n-m)\omega t} - \text{h.c.}\Big)...... \end{aligned} \tag{A.8}$$

Now, we consider a system with periodic Dirac $\delta$ driving and calculate the effective time independent Hamiltonian for the same. The Hamiltonian for the corresponding periodically driven system can be written as: $H(t) = H_0 + V_0 \sum_n \delta(t - nT)$.

### A.1.1 General expression of effective Hamiltonian for a periodically driven system with Dirac $\delta$ kick

A time dependent Hamiltonian for a system with periodic $\delta$-function kick potential is written as

$$H(t) = H_0 + V(t), \tag{A.9}$$

where $V(t) = V \sum_n \delta(t - nT)$ can be expressed as Fourier series in terms of Fourier coefficients as $V(t) = V_0 + \sum_n (V_n\, e^{in\omega t} + V_{-n}\, e^{-in\omega t})$, where $V_0, V_n$ and $V_{-n}$ can be obtained as

$$\begin{aligned} V_0 &= \frac{1}{T}\int_0^T V(t)\, dt = \frac{V}{T}\int_0^T \delta(t - nT)\, dt = \frac{V}{T}, \\ V_n &= \frac{1}{T}\int_0^T e^{-in\omega t} V\delta(t - nT)\, dt = \frac{V}{T}e^{-in\omega T} = \frac{V}{T}, \\ V_{-n} &= \frac{1}{T}\int_0^T e^{in\omega t} V\delta(t - nT)\, dt = \frac{V}{T}e^{in\omega T} = \frac{V}{T}. \end{aligned} \tag{A.10}$$

Following the expression of $H_{\text{eff}}$ given in Eq. (A.7), and also from the above equation $[V_n, V_{-n}] = 0$ and $[[V_n, H_0], V_{-n}] = \frac{1}{T^2}[[V, H_0], V]$, the effective Hamiltonian for the $\delta$-kicked system becomes

$$H_{\text{eff}} = H_0 + \frac{V}{T} + \frac{1}{\omega^2 T^2}[[V, H_0], V] \left(\sum_{n=1}^{\infty} \frac{1}{n^2}\right) + \mathcal{O}\left(\frac{1}{\omega^3}\right), \tag{A.11}$$

where $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$. Finally we obtain the general expression of effective Hamiltonian $H_{\text{eff}}$ as

$$H_{\text{eff}} = H_0 + \frac{V}{T} + \frac{1}{24}[[V, H_0], V] + \mathcal{O}\left(\frac{1}{\omega^3}\right). \tag{A.12}$$

## A.2 Brillouin-Wigner Method

According to Brillouin-Wigner theory effective Hamiltonian can be obtained by $H_{\text{BW}} = \mathcal{P}\overline{H}\Omega_{\text{BW}}\mathcal{P}$, where $\mathcal{P}$ is the Projection operator and the wave operator $\Omega_{\text{BW}}$ is obtained by substituting the series of $1/\omega$ in $\Omega_{\text{BW}}$ as

$$\Omega_{\text{BW}} = \sum_{n=0}^{\infty} \Omega_{\text{BW}}^{(n)}, \tag{A.13}$$

where $\Omega_{\text{BW}}^{(n)}$ corresponds to $(1/\omega^n)$ coefficient in the iterative solution to $\Omega_{\text{BW}}$, in the recursion relation of $\Omega_{\text{BW}}$. Similarly, effective Hamiltonian can also be expanded in a series of $(1/\omega)$ as

$$H_{\text{BW}} = \sum_{n=0}^{\infty} H_{\text{BW}}^{(n)}, \tag{A.14}$$

and

$$H_{\text{BW}}^{(n)} = \mathcal{P}\overline{H}\Omega_{\text{BW}}^{(n)}\mathcal{P}. \tag{A.15}$$

Here $\overline{H} = (\mathcal{H} - \mathcal{M})$ plays the role of effective Hamiltonian in model space. Now, apply this method to the wave operator as

$$\sum_{n=0}^{\infty} \Omega_{\text{BW}}^{(n)} = \mathcal{P} + \sum_{n=0}^{\infty} \frac{\mathcal{Q}}{\mathcal{M}\omega}\overline{H}\Omega_{\text{BW}}^{(n)} + \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} \frac{\mathcal{Q}}{\mathcal{M}\omega}\Omega_{\text{BW}}^{(n)}\mathcal{P}\overline{H}\Omega_{\text{BW}}^{(n)}. \tag{A.16}$$

Now, by comparing the terms of same order of $(1/\omega)$, we obtain

$$\begin{aligned} \Omega_{\text{BW}}^{(0)} &= \mathcal{P} \\ \Omega_{\text{BW}}^{(1)} &= \frac{\mathcal{Q}}{\mathcal{M}\omega}\overline{H}\mathcal{P} \\ \Omega_{\text{BW}}^{(n+1)} &= \frac{\mathcal{Q}}{\mathcal{M}\omega}\overline{H}\Omega_{\text{BW}}^{(n)} - \sum_{m=0}^{\infty} \frac{\mathcal{Q}}{\mathcal{M}\omega}\Omega_{\text{BW}}^{(n)}\mathcal{P}\overline{H}\Omega_{\text{BW}}^{(n-m)} \end{aligned} \tag{A.17}$$

from $H^{(n)}_{\text{BW}} = \mathcal{P}\overline{H}\Omega^{(n)}_{\text{BW}}\mathcal{P}$, we obtain

$$
\begin{aligned}
H^{(0)}_{\text{BW}} &= \mathcal{P}\overline{H}\mathcal{P}\mathcal{P} = H_{0,0}, \\
H^{(1)}_{\text{BW}} &= \mathcal{P}\overline{H}\frac{\mathcal{Q}}{\mathcal{M}\omega}\overline{H}\mathcal{P}\mathcal{P} = \sum_{n_1 \neq 0} \frac{H_{0,n_1}\, H_{n_1,0}}{n_1\, \omega}, \\
H^{(2)}_{\text{BW}} &= \mathcal{P}\overline{H}\Omega^{(2)}_{\text{BW}}\mathcal{P} = \sum_{n_1, n_2 \neq 0} \left( \frac{H_{0,n_1}\, H_{n_1,n_2}\, H_{n_2,0}}{n_1\, n_2\, \omega} - \frac{H_{0,n_1}\, H_{n_1,0}\, H_{0,0}}{n_1^2\, \omega^2} \right), \\
H^{(n)}_{\text{BW}} &= \mathcal{P}\overline{H}\Omega^{(n)}_{\text{BW}}\mathcal{P}.
\end{aligned}
\tag{A.18}
$$

from above results we conclude that using the above expansion coefficients one can expand the effective Hamiltonian to various order of $(1/\omega)$. Thus for the high frequency range the $\omega^0$ order term for wave operator become, $\Omega^{(0)}_{\text{BW}} = \mathcal{P}$ and for effective Hamiltonian it becomes $H^{(0)}_{\text{BW}} = H_{0,0}$, which ensure that the eigen values of effective Hamiltonian are present in the first Brillouin zone, as the contribution from higher order terms in the series is very small to transport the eigen value to the higher photon number sectors.

# Appendix B

# Floquet operator of DKT model and Farey Sequence

## B.1 Compact form of Floquet Operator

Floquet time evolution operator for the DKT model can be written as

$$\mathcal{F} = \exp(-i\alpha J_x) \exp\left(-i\frac{\eta}{2j}J_z^2\right) \exp(-i\alpha J_x) \exp\left(i\frac{\eta}{2j}J_z^2\right). \tag{B.1}$$

Above Floquet operator is a product of four unitary operators which can also be written into compact form as

$$\begin{aligned}\mathcal{F} &= \exp(-i\alpha J_x) \exp\left(-i\frac{\eta}{2j}J_z^2\right)(1 - i\alpha J_x + ...) \exp\left(i\frac{\eta}{2j}J_z^2\right) \\ &= \exp(-i\alpha J_x)\left[1 - i\alpha \exp\left(-i\frac{\eta}{2j}J_z^2\right) J_x \exp\left(i\frac{\eta}{2j}J_z^2\right) + ....\right].\end{aligned}$$

Here $J_x = J_- + J_-$, where $J_-$ and $J_-$ are ladder operators therefore

$$\begin{aligned}
\mathcal{F} &= \exp(-i\alpha J_x)\Big[1 - i\alpha\langle m, s|e^{-i\frac{\eta}{2j}J_z^2}\Big(J_+ + J_-\Big)e^{i\frac{\eta}{2j}J_z^2}|m', s'\rangle + ...\Big] \\
&= \exp(-i\alpha J_x)\Big[1 - i\alpha e^{-i\frac{\eta}{2j}m^2}\langle m, s|\big(J_+ + J_-\big)|m', s'\rangle e^{i\frac{\eta}{2j}m'^2} + ...\Big] \\
&= \exp(-i\alpha J_x)\Big[1 - i\alpha e^{i\frac{\eta}{2j}(m'^2 - m^2)}\{C_{m'}\delta_{m,m'+1} + C_{m'}\delta_{m,m'-1}\} + ...\Big] \\
&= \exp(-i\alpha J_x)\Big[1 - i\alpha\{e^{i\frac{\eta}{j}(m'+1/2)}C_{m'}\delta_{m,m'+1} + \text{h.c.}\} + ...\Big] \\
&= \exp(-i\alpha J_x)\Big[1 - i\alpha\langle m, s|J_+ e^{-i\eta(2J_z+\mathbb{1})/2j} + \text{h.c.}|m', s'\rangle + ...\Big] \\
&= \exp(-i\alpha J_x)\Big[1 - i\alpha\{J_+ e^{i\eta(2J_z+\mathbb{1})/2j} + \text{h.c.}\} + ...\Big]
\end{aligned}$$

Finally we obtained the time evolution operator as a product of two unitary operators:

$$\mathcal{F} = \exp(-i\alpha J_x)\exp\big\{-i\alpha J_+ e^{i\eta(2J_z+\mathbb{1})/2j} + \text{h.c.}\big\}. \tag{B.2}$$

## B.2 Farey sequence

A Farey sequence $F_n$ is the set of rational numbers $p/q$ with p and q, with $0 < p < q < n$, ordered by size. Each Farey sequence start with value 0 , denoted by the fraction $\frac{0}{1}$, and ends with the value 1, denoted by the fraction $\frac{1}{1}$. If we have two fractions $\frac{a}{b}$ and $\frac{c}{d}$ with the properties that $\frac{a}{b} < \frac{c}{d}$ and $bc - qd = 1$. Then the fractions are known as Farey neighbours, they appear next to each other in some Farey sequence. The mediant of these two fractions is given by:

$$\frac{a}{b} \oplus \frac{c}{d} = \frac{a+c}{b+d}. \tag{B.3}$$

The Farey sequences of order 1 to 8 are given as:

$$
\begin{aligned}
F_1 &= \left\{\frac{0}{1}, \frac{1}{1}\right\} \\
F_2 &= \left\{\frac{0}{1}, \frac{1}{2}, \frac{1}{1}\right\} \\
F_3 &= \left\{\frac{0}{1}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{1}{1}\right\} \\
F_4 &= \left\{\frac{0}{1}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{1}{1}\right\} \\
F_5 &= \left\{\frac{0}{1}, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{1}{1}\right\} \\
F_6 &= \left\{\frac{0}{1}, \frac{1}{6}, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{1}{1}\right\} \\
F_7 &= \left\{\frac{0}{1}, \frac{1}{7}, \frac{1}{6}, \frac{1}{5}, \frac{1}{4}, \frac{2}{7}, \frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{1}{2}, \frac{4}{7}, \frac{3}{5}, \frac{2}{3}, \frac{5}{7}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \frac{1}{1}\right\} \\
F_8 &= \left\{\frac{0}{1}, \frac{1}{8}, \frac{1}{7}, \frac{1}{6}, \frac{1}{5}, \frac{1}{4}, \frac{2}{7}, \frac{1}{3}, \frac{3}{8}, \frac{2}{5}, \frac{3}{7}, \frac{1}{2}, \frac{4}{7}, \frac{3}{5}, \frac{5}{8}, \frac{2}{3}, \frac{5}{7}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \frac{7}{8}, \frac{1}{1}\right\}.
\end{aligned}
\tag{B.4}
$$

and so on

# Appendix C

# Dynamics of Periodically Driven HCB System

## C.1 Current operator in momentum space

The current operator in momentum space for a system of hard core bosons (HCB) for each $k$-mode is written as:

$$\begin{aligned}
\widehat{j} &= \frac{i}{L}\sum_{l=1}^{L}\Big(b_{l+1}^{\dagger}b_{l} \;-\; b_{l}^{\dagger}b_{l+1}\Big) \\
&= \frac{i}{L}\sum_{l}\sum_{kk'}\left[\frac{1}{L}e^{-ik'(l+1)}\tilde{b}_{k'}^{\dagger}e^{ikl}\tilde{b}_{k} \;-\; \frac{1}{L}e^{-ikl}\tilde{b}_{k}^{\dagger}e^{ik'(l+1)}\tilde{b}_{k'}\right] \\
&= \frac{i}{L}\sum_{kk'}\left[\sum_{l}\frac{1}{L}e^{i(k-k')l}e^{-ik'}\tilde{b}_{k'}^{\dagger}\tilde{b}_{k} \;-\; \sum_{l}\frac{1}{L}e^{-i(k-k')l}e^{ik}\tilde{b}_{k}^{\dagger}\tilde{b}_{k'}\right],
\end{aligned}$$

here in above equation $\frac{1}{L}\sum\limits_{kk'}e^{i(k-k')l} = \frac{1}{L}\sum\limits_{kk'}e^{-i(k-k')l} = \delta_{kk'}$, therefore

$$\widehat{j} = \frac{i}{L}\sum_{k}\left[e^{-ik}\tilde{b}_{k}^{\dagger}\tilde{b}_{k} - e^{ik}\tilde{b}_{k}^{\dagger}\tilde{b}_{k}\right] = \frac{i}{L}\sum_{k}\tilde{b}_{k}^{\dagger}\tilde{b}_{k}(e^{-ik} - e^{ik}) = \frac{2}{L}\sum_{k}\tilde{b}_{k}^{\dagger}\tilde{b}_{k}\sin k.$$

Now $k$ can be divided into two regions as:

$$\widehat{j} = \frac{2}{L}\left[\sum_{k=-\pi/2}^{\pi/2} \tilde{b}_k^{\dagger}\tilde{b}_k \sin k \;+\; \sum_{k=\pi/2}^{3\pi/2} \tilde{b}_k^{\dagger}\tilde{b}_k \sin k\right].$$

In the second term of RHS of above the equation we replace $k \to k+\pi$, then we have

$$\widehat{j} = \frac{2}{L}\left[\sum_{k=-\pi/2}^{\pi/2} \tilde{b}_k^{\dagger}\tilde{b}_k \sin k \;+\; \sum_{k=3\pi/2}^{5\pi/2} \tilde{b}_{k+\pi}^{\dagger}\tilde{b}_{k+\pi} \sin(k+\pi)\right],$$

here $k = 3\pi/2 = -\pi/2$ and $k = 5\pi/2 = \pi/2$, therefore

$$\begin{aligned}
\widehat{j} &= \frac{2}{L}\left[\sum_{k=-\pi/2}^{\pi/2} \tilde{b}_k^{\dagger}\tilde{b}_k \sin k \;-\; \sum_{k=-\pi/2}^{\pi/2} \tilde{b}_{k+\pi}^{\dagger}\tilde{b}_{k+\pi} \sin(k)\right] \\
&= \frac{2}{L}\sum_{k=-\pi/2}^{\pi/2}\left[\begin{pmatrix} \tilde{b}_k^{\dagger} & \tilde{b}_{k+\pi}^{\dagger} \end{pmatrix}\begin{pmatrix} \sin k & 0 \\ 0 & -\sin k \end{pmatrix}\begin{pmatrix} \tilde{b}_k \\ \tilde{b}_{k+\pi} \end{pmatrix}\right] \\
&= \sum_{k=-\pi/2}^{\pi/2}\left[\begin{pmatrix} \tilde{b}_k^{\dagger} & \tilde{b}_{k+\pi}^{\dagger} \end{pmatrix}\frac{2}{L}\sin k\sigma_z \begin{pmatrix} \tilde{b}_k \\ \tilde{b}_{k+\pi} \end{pmatrix}\right].
\end{aligned} \tag{C.1}$$

Here $\widehat{j}$ is current operator for all possible values of $k$ and $2/L \sin k\, \sigma_z$ is current operator for a defined value of $k$ and denoted as $\widehat{j}_k$.

## C.2 Floquet operator into $2 \times 2$ matrix form for SKHCB system.

Floquet operator $\mathcal{F}_k(T)$ can be written into $2 \times 2$ matrix form as: $\mathcal{F}_k(T) = e^{i2\cos k\,\sigma_z}\, e^{i\alpha\,\sigma_x} = e^{i\mu_k T(\vec{\sigma}.\hat{l})}$, which can be written as:

$$\begin{aligned}\mathcal{F}_k(T) &= \cos\mu_k T \,+\, i(\vec{\sigma}.\hat{l})\sin\mu_k T \\ &= \mathbb{1}\cos(2T\cos k)\cos\alpha \,+\, i(\sigma_x l_x \,+\, \sigma_y l_y \,+\, \sigma_z l_z)\sin(\mu_k T).\end{aligned}$$

Now by substituting the expression of $l_x, l_y$ and $l_z$ from Eq. (4.38)

$$\mathcal{F}_k(T) = \cos(2T\cos k)\cos\alpha + i\Big[\sigma_x \sin\alpha\cos(2T\cos k) - \sigma_y \sin(2T\cos k)\sin\alpha + \sigma_z \sin(2T\cos k)\cos\alpha\Big].$$

If $\sigma_x, \sigma_y$ and $\sigma_z$ can be written into matrix form then

$$\mathcal{F}_k(T) = \begin{bmatrix} \cos\alpha\, e^{i2T\cos k} & i\sin\alpha\, e^{i2T\cos k} \\ i\sin\alpha\, e^{-i2T\cos k} & \cos\alpha\, e^{-i2T\cos k} \end{bmatrix}. \tag{C.2}$$

Now let $\cos\alpha\, e^{i2T\cos k} = a$ and $i\sin\alpha\, e^{i2T\cos k} = b$, then Floquet time evolution operator can be written in $2 \times 2$ matrix form as

$$\mathcal{F}_k(T) = \begin{bmatrix} a & b \\ -b^* & a^* \end{bmatrix}. \tag{C.3}$$

## C.3 Current in SKHCB

Current flow throught the system is defined as

$$J(nT) = J(nT) = \frac{2}{L}\sum_k \sin k\langle\Psi_k(0)|\mathcal{F}_k^{n\dagger}\sigma_z\mathcal{F}_k^n|\Psi_k(0)\rangle \tag{C.4}$$

Here first we solve the central part as

$$\begin{aligned}
\mathcal{F}_k^{n\dagger}\sigma_z\mathcal{F}_k^n &= e^{i\mu_k nT(\vec{\sigma}.\hat{l})}\sigma_z e^{-i\mu_k nT(\vec{\sigma}.\hat{l})} \\
&= \left\{\mathbb{1}\cos(\mu_k nT) + i(\vec{\sigma}.\hat{l})\sin(\mu_k nT)\right\}\sigma_z\left\{\mathbb{1}\cos(\mu_k nT) - i(\vec{\sigma}.\hat{l})\sin(\mu_k nT)\right\} \\
&= \left[\sigma_z\cos(\mu_k nT) + i(\vec{\sigma}.\hat{l})\sigma_z\sin(\mu_k nT)\right]\left(\mathbb{1}\cos(\mu_k nT) - i(\vec{\sigma}.\hat{l})\sin(\mu_k nT)\right) \\
&= \sigma_z\cos^2(\mu_k nT) - i\sigma_z(\vec{\sigma}.\hat{l})\cos(\mu_k nT)\sin(\mu_k nT) \\
&\qquad + i(\vec{\sigma}.\hat{l})\sigma_z\sin(\mu_k nT)\cos(\mu_k nT) + (\vec{\sigma}.\hat{l})\sigma_z(\vec{\sigma}.\hat{l})\sin^2(\mu_k nT) \\
&= \sigma_z\cos^2(\mu_k nT) + (\vec{\sigma}.\hat{l})\sigma_z(\vec{\sigma}.\hat{l})\sin^2(\mu_k nT) - i\{\sigma_z(\vec{\sigma}.\hat{l}) - (\vec{\sigma}.\hat{l})\sigma_z\} \\
&\qquad\qquad \sin(\mu_k nT)\cos(\mu_k nT)
\end{aligned} \tag{C.5}$$

here $(\vec{\sigma}.\hat{l})\sigma_z(\vec{\sigma}.\hat{l}) = \sigma_z(l_z^2 - l_x^2 - l_y^2) + 2\sigma_x l_x l_z + 2\sigma_y l_y l_z$ and $\{\sigma_z(\vec{\sigma}.\hat{l}) - (\vec{\sigma}.\hat{l})\sigma_z\} = 2i\sigma_y l_x - 2i\sigma_x l_y$. Therefore

$$\begin{aligned}
\mathcal{F}_k^{n\dagger}\sigma_z\mathcal{F}_k^n &= \sigma_z\cos^2(\mu_k nT) + \{\sigma_z(l_z^2 - l_x^2 - l_y^2) + 2\sigma_x l_x l_z + 2\sigma_y l_y l_z\}\sin^2(\mu_k nT) - \\
&\quad i\{2i\sigma_y l_x - 2i\sigma_x l_y\}\sin(\mu_k nT)\cos(\mu_k nT) \\
&= \sigma_z\left[\cos^2(\mu_k nT) + (l_z^2 - l_x^2 - l_y^2)\sin^2(\mu_k nT)\right] \\
&\quad + \sigma_x\left[2l_x l_z\sin(\mu_k nT) - 2l_y\sin(\mu_k nT)\right] \\
&\quad + \sigma_y\left[2l_y l_z\sin^2(\mu_k nT) + 2l_x\sin(\mu_k nT)\cos(\mu_k nT)\right].
\end{aligned} \tag{C.6}$$

Here in the expression of current only $\sigma_z$ term appear because $|\Psi_k(0)\rangle$ is eigen sate of $H_k^{\nu}$, which is the eigen state of pseudospin operator $\sigma_z$. $\hat{l}$ is the unit vector therefore $l_x^2 + l_y^2 + l_z^2 = 1$. As a result current flow through the system of SKHCB is written as

$$\begin{aligned} J(nT) &= \frac{2}{L}\sum_k \left[\cos^2(\mu_k nT) + (l_z^2 - l_x^2 - l_y^2)\sin^2(\mu_k nT)\right] \langle\Psi_k(0)|\sigma_z|\Psi_k(0)\rangle \, \sin k \\ &= \frac{2}{L}\sum_k f(k)\langle\Psi_k(0)|\sigma_z|\Psi_k(0)\rangle \sin k, \end{aligned} \tag{C.7}$$

where $f(k) = [1 \, - \, 2(1 - l_z^2)\sin^2(\mu_k nT)]$.

## C.4 Floquet time evolution operator for DKHCB

According to the following identity for Pauli spin matrices, if:

$$e^{ia(\vec{\sigma}.\hat{n})} \, e^{ib(\vec{\sigma}.\hat{m})} = e^{ic(\vec{\sigma}.\hat{l})} \tag{C.8}$$

where $(a, b, c)$ are scalars and $(\hat{n}, \hat{m}, \hat{l})$ are unit vectors then $c$ and $\hat{l}$ can be found in terms of $a, b, \hat{n}$ and $\hat{m}$ using the following relation:

$$\begin{aligned} \cos c &= \cos a \, \cos b - \hat{n}.\hat{m} \, \sin a \, \sin b \\ \hat{l} &= \frac{1}{\sin q} \left(\hat{n}\sin a \, \cos b \, + \, \hat{m} \, \sin b \, \cos a \, - \, \hat{n}\times\hat{m} \, \sin a \, \sin b\right) \end{aligned} \tag{C.9}$$

Floquet operator for DKHCB system is written as:

$$\begin{aligned} &\mathcal{F}_k(T) = \exp[iT(1-\Delta)\sigma_z \cos k] \exp[i\alpha\sigma_x] \exp[i2T\Delta\sigma_z \cos k] \\ &\exp[-i\alpha\sigma_x] \exp[iT(1-\Delta)\sigma_z \cos k] \\ &= e^{iT(1-\Delta)\sigma_z \cos k} \, e^{i2T\Delta\cos k(\sigma_y \sin(2\alpha) + \sigma_z \cos(2\alpha))} \, e^{iT(1-\Delta)\sigma_z \cos k} \end{aligned} \tag{C.10}$$

To write Floquet operator in more compact form we consider first two terms for which $a = T(1-\Delta)\sigma_z \cos k$, $\hat{n} = z$, $b = 2T\Delta\cos k$, and $\hat{m} = \hat{y}\sin 2\alpha + \hat{z}\cos 2\alpha$ then according to above mention identity $e^{iT(1-\Delta)\sigma_z \cos k}\, e^{i2T\Delta\cos k(\sigma_y \sin(2\alpha)+\sigma_z\cos(2\alpha))} = e^{icT(\vec{\sigma}.\hat{l})}$, then $\cos c = \cos(T(1-\Delta)\cos k)\cos(2T\Delta\cos k) - \cos(2\alpha)\sin(T(1-\Delta)\cos k)\sin(2T\Delta\cos k)$ and vector $\hat{l}$ is written as:

$$\begin{aligned}
\hat{l} = \frac{1}{\sin c}\Big[&\hat{z}\sin(T(1-\Delta)\cos k)\cos(2T\Delta\cos k) + (\hat{y}\sin(2\alpha) + \hat{z}\cos 2\alpha) \\
&\sin(2T\Delta\cos k)\cos(T(1-\Delta)\cos k) - \{\hat{z}\times(\hat{y}\sin(2\alpha)+\hat{z}\cos(2\alpha))\} \\
&\sin(T(1-\Delta)\cos k)\sin(2T\Delta\cos k)\Big] \\
= \frac{1}{\sin c}\Big[&(\sin(2\alpha)\sin(T(1-\Delta)\cos k)\sin(2T\Delta\cos k))\hat{x} \\
&+ (\sin(2\alpha)\sin(2T\Delta\cos k)\cos(T(1-\Delta)\cos k))\hat{y} \\
&+ (\cos(2T\Delta\cos k)\sin(T(1-\Delta)\cos k) + \cos(2\alpha)\sin(2T\Delta\cos k)\cos(T(1-\Delta)\cos k))\hat{z}\Big].
\end{aligned}$$

Now, $\mathcal{F}_k(t)$ can be written as

$$\mathcal{F}_k(t) = e^{ic(\vec{\sigma}.\hat{l})}\, e^{iT(1-\Delta)\cos k\sigma_z} = e^{i\mu_k T(\vec{\sigma}.\hat{l})}, \tag{C.11}$$

then again using the identity

$$\cos(\mu_k T) = \cos c \cos(T(1-\Delta)\cos k) - (\hat{l}.\hat{z}) \sin c \sin(T(1-\Delta)\cos k)$$

$$= (\cos(T(1-\Delta)\cos k)\cos(2T\Delta\cos k) - \cos(2\alpha)\sin(T(1-\Delta)\cos k)\sin(2T\Delta\cos k))$$

$$\cos(T(1-\Delta)\cos k) - (\cos(2T\Delta\cos k)\sin(T(1-\Delta)\cos k) + \cos(2\alpha)\sin(2T\Delta\cos k)$$

$$\cos(T(1-\Delta)\cos k))\sin(T(1-\Delta)\cos k)$$

$$= \cos^2(T(1-\Delta)\cos k)\cos(2T\Delta\cos k) \ - \ \cos(2\alpha)\sin(T(1-\Delta)\cos k)$$

$$\cos(T(1-\Delta)\cos k)\sin(2T\Delta\cos k) \ - \ \cos(2T\Delta\cos k)\sin^2(T(1-\Delta)\cos k) -$$

$$\cos(2\alpha)\sin(2T\Delta\cos k)\cos(T(1-\Delta)\cos k)\sin(T(1-\Delta)\cos k)$$

$$= \cos(2T(1-\Delta)\cos k)\cos(2T\Delta\cos k) \ - \ \cos(2\alpha)\sin(2T(1-\Delta)\cos k)\sin(2T\Delta\cos k).$$

Using the relations $2\cos a \cos b = \cos(a+b) + \cos(a-b)$ and $2\sin a \sin b = \cos(a-b) - \cos(a+b)$ in the above equation we have:

$$\begin{aligned}\cos(\mu_k T) &= \frac{1}{2}\Bigg[\Bigg\{\cos(2\cos kT) + \cos\left(2\cos kT(1-2\Delta)\right)\Bigg\} \\ &\qquad - \cos 2\alpha \left\{\cos\left(2\cos kT(1-2\Delta)\right) - \cos(2\cos kT)\right\}\Bigg] \\ &= \frac{1}{2}\left[(1+\cos 2\alpha)\cos(2\cos kT) + (1-\cos 2\alpha)\cos(2\cos kT(1-2\Delta))\right].\end{aligned}$$

Finally,

$$\cos(\mu_k T) = \cos^2\alpha\cos(2T\cos k) \ + \ \sin^2\alpha\cos(2T\cos k(1-2\Delta)) \qquad \text{(C.12)}$$

and

$$\hat{l} = \frac{1}{\sin(\mu_k T)}\Bigg[\hat{l}\sin c\cos(T(1-\Delta)\cos k) + \hat{z}\cos c\sin(T(1-\Delta)\cos k) - (\hat{l}\times\hat{z})\sin c\,\sin(T(1-\Delta)\cos k)\Bigg]$$

$$= \frac{1}{\sin(\mu_k T)}\Bigg[\frac{1}{\sin c}\Bigg[(\sin(2\alpha)\sin(T(1-\Delta)\cos k)\sin(2T\Delta\cos k))\hat{x} + (\sin(2\alpha)\sin(2T\Delta\cos k)\cos(T(1-\Delta)\cos k))\hat{y} + (\cos(2T\Delta\cos k)\sin(T(1-\Delta)\cos k) + \cos(2\alpha)\sin(2T\Delta\cos k)\cos(T(1-\Delta)\cos k))\hat{z}\Bigg] \sin c\cos(T(1-\Delta)\cos k) + \Bigg\{(\cos(T(1-\Delta)\cos k)\cos(2T\Delta\cos k) - \cos(2\alpha)\sin(T(1-\Delta)\cos k)\sin(2T\Delta\cos k))\sin(T(1-\Delta)\cos k)\Bigg\}\hat{z} - \frac{1}{\sin c}\Bigg[(\sin(2\alpha)\sin(2T\Delta\cos k)\cos(T(1-\Delta)\cos k))\hat{x} - (\sin(2\alpha)\sin(T(1-\Delta)\cos k)\sin(2T\Delta\cos k))\hat{y}\Bigg]\sin c\sin(T(1-\Delta)\cos k)\Bigg].$$

Here $\hat{l}$ is a unit vector, therefore $\hat{l} = l_x\hat{x} + l_y\hat{y} + l_z\hat{z}$, and

$$\hat{l} = \frac{1}{\sin(\mu_k T)}\Bigg[\sin(2\alpha)\sin(T(1-\Delta)\cos k)\cos(T(1-\Delta)\cos k)\sin(2T\Delta\cos k) - \sin(2\alpha)\sin(T(1-\Delta)\cos k)\cos(T(1-\Delta)\cos k)\sin(2T\Delta\cos k)\Bigg]\hat{x} + \frac{1}{\sin(\mu_k T)}\Bigg[\sin(2\alpha)\sin(2T\Delta\cos k)\cos^2(T(1-\Delta)\cos k) + \sin(2\alpha)\sin(2T\Delta\cos k)\sin^2(T(1-\Delta)\cos k)\Bigg]\hat{y} + \frac{1}{\sin(\mu_k T)}\Bigg[\cos(2T\delta\cos k)\sin(2T(1-\Delta)\cos k) + \cos(2T(1-\Delta)\cos k)\cos(2\alpha)\sin(2T\Delta\cos k)\Bigg]\hat{z}.$$

From above expression $x, y, z$ component of $\hat{l}$ are

$$l_x = 0$$

$$l_y = \frac{\sin(2\alpha)\sin(2T\Delta\cos k)}{\left[1 - \{\cos^2\alpha\cos(2T\cos k) + \sin^2\alpha\cos 2T(1-2\Delta)\cos k\}^2\right]^{1/2}} \tag{C.13}$$

$$l_z = \frac{\cos^2\alpha\sin(2T\cos k) + \sin^2\alpha\sin(2T(1-2\Delta)\cos k)}{\left[1 - \{\cos^2\alpha\cos(2T\cos k) + \sin^2\alpha\cos(2T(1-2\Delta)\cos k)\}^2\right]^{1/2}}.$$

## C.5 Current flow in DKHCB

According to Floquet theory, $\mathcal{F}_k^n(T) = e^{-i\mu_k nT(\vec{\sigma}.\hat{l})}$, and current flow through double kicked HCB system is obtained as:

$$J(nT) = \frac{2}{L}\sin k\langle\Psi_k(0)|e^{i\mu_k nT(\vec{\sigma}.\hat{l})}\sigma_z e^{-i\mu_k nT(\vec{\sigma}.\hat{l})}|\Psi_k(0)\rangle. \tag{C.14}$$

First we calculate $\mathcal{F}_k^{n\dagger}\sigma_z\mathcal{F}_k^n$ as:

$$\begin{aligned}
\mathcal{F}_k^{n\dagger}\sigma_z\mathcal{F}_k^n &= e^{i\mu_k nT(\vec{\sigma}.\hat{l})}\sigma_z e^{-i\mu_k nT(\vec{\sigma}.\hat{l})} \\
&= \left\{\mathbb{1}\cos(\mu_k nT) + i(\vec{\sigma}.\hat{l})\sin(\mu_k nT)\right\}\sigma_z\left\{\mathbb{1}\cos(\mu_k nT) - i(\vec{\sigma}.\hat{l})\sin(\mu_k nT)\right\} \\
&= \left[\sigma_z\cos(\mu_k nT) + i(\vec{\sigma}.\hat{l})\sigma_z\sin(\mu_k nT)\right]\left(\mathbb{1}\cos(\mu_k nT) - i(\vec{\sigma}.\hat{l})\sin(\mu_k nT)\right) \\
&= \sigma_z\cos^2(\mu_k nT) - i\sigma_z(\vec{\sigma}.\hat{l})\cos(\mu_k nT)\sin(\mu_k nT) + i(\vec{\sigma}.\hat{l})\sigma_z\sin(\mu_k nT) \\
&\qquad \cos(\mu_k nT) + (\vec{\sigma}.\hat{l})\sigma_z(\vec{\sigma}.\hat{l})\sin^2(\mu_k nT) \\
&= \sigma_z\cos^2(\mu_k nT) + (\vec{\sigma}.\hat{l})\sigma_z(\vec{\sigma}.\hat{l})\sin^2(\mu_k nT) - i\{\sigma_z(\vec{\sigma}.\hat{l}) - (\vec{\sigma}.\hat{l})\sigma_z\} \\
&\qquad \sin(\mu_k nT)\cos(\mu_k nT),
\end{aligned}$$

here $(\vec{\sigma}.\hat{l})\sigma_z(\vec{\sigma}.\hat{l}) = (\sigma_x l_x + \sigma_y l_y + \sigma_z l_z)\{i\sigma_y l_x - i\sigma_x l_y + l_z\} = \sigma_z(l_z^2 - l_x^2 - l_y^2) + 2\sigma_x l_x l_z + 2\sigma_y l_y l_z$ and $\{\sigma_z(\vec{\sigma}.\hat{l}) - (\vec{\sigma}.\hat{l})\sigma_z\} = 2i\sigma_y l_x - 2i\sigma_x l_y$. Therefore

$$\begin{aligned}\mathcal{F}_k^{n\dagger}\sigma_z\mathcal{F}_k^n &= \sigma_z \cos^2(\mu_k nT) + \{\sigma_z(l_z^2 - l_x^2 - l_y^2) + 2\sigma_x l_x l_z + 2\sigma_y l_y l_z\}\sin^2(\mu_k nT) - \\ &\qquad i\{2i\sigma_y l_x - 2i\sigma_x l_y\}\sin(\mu_k nT)\cos(\mu_k nT) \\ &= \sigma_z\left[\cos^2(\mu_k nT) + (l_z^2 - l_x^2 - l_y^2)\sin^2(\mu_k nT)\right] \\ &\quad + \sigma_x\left[2l_x l_z \sin(\mu_k nT) - 2l_y \sin(\mu_k nT)\right] \\ &\quad + \sigma_y\left[2l_y l_z \sin^2(\mu_k nT) + 2l_x \sin(\mu_k nT)\cos(\mu_k nT)\right].\end{aligned}$$

As a result current

$$\begin{aligned}J(nT) &= \frac{2}{L}\sum_k \left[\langle\Psi_k(0)|\mathcal{F}_k^{n\dagger}\sigma_z\mathcal{F}_k^n|\Psi_k(0)\rangle\right]\sin k \\ &= \frac{2}{L}\sum_k \left[\cos^2(\mu_k nT) + (l_z^2 - l_x^2 - l_y^2)\sin^2(\mu_k nT)\right]\,\langle\Psi_k(0)|\sigma_z|\Psi_k(0)\rangle\,\sin k,\end{aligned}$$

here in the expression of current only $\sigma_z$ term appear because $|\Psi_k(0)\rangle$ is eigen sate of $H_k^\nu$, which is the eigen state of pseudo-spin half operator $\sigma_z$. $\hat{l}$ is the unit vector therefore $l_x^2 + l_y^2 + l_z^2 = 1$. Finally current flow through the hard core bosonic system under the presence of staggered on site potential in the form of double $\delta$-function kick within one time period is written as:

$$\begin{aligned}J(nT) &= \frac{2}{L}\sum_k \left[\cos^2(\mu_k nT) + \{l_z^2 - (1 - l_z^2)\}\sin^2(\mu_k nT)\right]\langle\Psi_k(0)|\sigma_z|\Psi_k(0)\rangle\sin k \\ &= \frac{2}{L}\sum_k \left[1 + 2(l_z^2 - 1)\sin^2(\mu_k nT)\right]\langle\Psi_k(0)|\sigma_z|\Psi_k(0)\rangle\sin k \\ &= \frac{2}{L}\sum_k f(k)\langle\Psi_k(0)|\sigma_z|\Psi_k(0)\rangle\sin k,\end{aligned} \tag{C.15}$$

here $f(k) = \left[1 + 2(l_z^2 - 1)\sin^2(\mu_k nT)\right]$.

# Appendix D

# Calculations for Coupled Kicked Top

## D.1 General formula for $H_{\text{eff}}$ for periodically driven system by BW method

According to BW method

$$
\begin{aligned}
H_{\text{BW}} &= \sum_{n=0}^{\infty} H_{\text{BW}}^{(n)} \\
H_{\text{BW}}^{(0)} &= H_{0,0} \\
H_{\text{BW}}^{(1)} &= \sum_{n_i \neq 0} \frac{H_{0,n_1} H_{n_1,0}}{n_1\, \omega} \\
H_{\text{BW}}^{(2)} &= \sum_{n_i \neq 0} \left( \frac{H_{0,n_1} H_{n_1,n_2} H_{n_2,0}}{n_1\, n_2\, \omega^2} - \frac{H_{0,n_1}\, H_{n_1,0}\, H_{0,0}}{n_1^2\, \omega^2} \right)
\end{aligned}
$$

where

$$H_{m,n} = \frac{1}{T} \int_0^T H(t)\, e^{i(m-n)\omega t} dt.$$

Here we consider a periodically kicked system with $\delta$-function kick. The time dependent Hamiltonian for such a system is written as $H(t) = H_0 + V \sum_n \delta(t - nT)$, where $H_0$ contain time independent part and potential $V(t) = V \sum_n \delta(t - nT)$ is time dependent which is periodic. For this system

$$H_{0,0} = \frac{1}{T} \int_0^T \left\{ H_0 + V \sum_n \delta(t - nT) \right\} dt = H_0 + \frac{V}{T}.$$

Therefore

$$H_{\rm BW}^{(0)} = H_0 + \frac{V}{T}. \tag{D.1}$$

For $H_{\rm BW}^{(1)}$ we first calculate $H_{0,n_1}$ and $H_{n_1,0}$ as:

$$H_{0,n_1} = \frac{1}{T} \int_0^T \left\{ H_0 + V \sum_n \delta(t - nT) \right\} e^{-in_1\omega t} dt = \frac{V}{T}$$
$$H_{n_1,0} = \frac{1}{T} \int_0^T \left\{ H_0 + V \sum_n \delta(t - nT) \right\} e^{in_1\omega t} dt = \frac{V}{T}.$$

Therefore

$$\begin{aligned} H_{\rm BW}^{(1)} &= \sum_{n_i \neq 0} \frac{H_{0,n_1} \, H_{n_1,0}}{n_1 \, \omega} \\ &= \left(\frac{V}{T}\right)^2 \frac{1}{\omega} \sum_{n_i \neq 0} \frac{1}{n_i} \\ &= \frac{V^2}{T^2 \omega} \left[ \sum_{n_i=-\infty}^{-1} \frac{1}{n_i} + \sum_{n_i=1}^{\infty} \frac{1}{n_i} \right] = 0. \end{aligned} \tag{D.2}$$

Here all the terms inside the bracket cancels each other. Now to calculate $H_{\rm BW}^{(2)}$ we first calculate $H_{n_1,n_2}$ and $H_{n_2,0}$, other terms for $H_{\rm BW}^{(2)}$ are already calculated. Here

$$H_{n_1,n_2} = \frac{1}{T} \int_0^T \left\{ H_0 + V \sum_n \delta(t - nT) \right\} e^{i(n_1 - n_2)\omega t} dt = H_0 \delta_{n_1,n_2} + \frac{V}{T}$$
$$H_{n_2,0} = \frac{1}{T} \int_0^T \left\{ H_0 + V \sum_n \delta(t - nT) \right\} e^{in_2\omega t} dt = \frac{V}{T}.$$

Therefore

$$
\begin{aligned}
H_{\mathrm{BW}}^{(2)} &= \sum_{n_i \neq 0} \frac{H_{0,n_1}\, H_{n_1,n_2}\, H_{n_2,0}}{n_1\, n_2\, \omega^2} - \sum_{n_i \neq 0} \frac{H_{0,n_1}\, H_{n_1,0}\, H_{0,0}}{n_1^2\, \omega^2} \\
&= \sum_{n_i \neq 0} \frac{(V/T)(H_0 \delta_{n_1,n_2} + \frac{V}{T})(V/T)}{n_1\, n_2\, \omega^2} - \sum_{n_i \neq 0} \frac{(V/T)^2\,(H_0 + \frac{V}{T})}{n_1^2\, \omega^2} \\
&= \frac{V H_0 V}{T^2} \frac{1}{\omega^2} \sum_{n_i \neq 0} \frac{\delta_{n_1,n_2}}{n_1\, n_2\, \omega^2} + \left(\frac{V}{T}\right)^3 \frac{1}{\omega^2} \sum_{n_i \neq 0} \frac{1}{n_1\, n_2} - \left(\frac{V^2 H_0}{T^2} + \left(\frac{V}{T}\right)^3\right) \frac{1}{\omega^2} \sum_{n_i \neq 0} \frac{1}{n_1^2} \\
&= \frac{1}{\omega^2 T^2} \left(V H_0 V - V^2 H_0\right) \sum_{n_i \neq 0} \frac{1}{n_1^2} \\
&= \frac{1}{12} \left(V H_0 V - V^2 H_0\right).
\end{aligned} \tag{D.3}
$$

By substituting Eqs. (D.1), (D.2), (D.3) in expression of BW method then we obtain

$$
H_{\mathrm{BW}} = H_0 + \frac{V}{T} + \frac{1}{12} \left(V H_0 V - V^2 H_0\right). \tag{D.4}
$$