Chapter 1

Introduction

The realization of a new novel state of matter in a quantum system by any controlled method comprises a great interest in research that connects various subfields of condensed matter and different branches of physics. For this goal, three ideas are currently investigated. One idea is to generate a nontrivial phase of matter through the exploitation of some intrinsic properties of the material by experimentally designed fabrication techniques [1–3] or the same can be done in some artificial materials [4–12]. For example, the topologically insulating materials can be created by exploiting the large intrinsic spin-orbit coupling [1]. The other idea is to apply a static mechanical deformation to a solid state system, which can induce an effective gauge field in the system to generate novel properties [5, 13–16]. Finally, the third idea is to synthetically generate nontrivial exotic properties in a material by external periodic driving [17–20]. The periodic driving can be realized by an electromagnetic field like high intense laser or by periodic mechanical driving. The periodic driving can also generate effective gauge structures and induces some exotic properties in the system. This induced gauge field enters into the effective 'static' Hamiltonian corresponding to the time-periodic driven Hamiltonian that captures the essential characteristics of the driven system. Following the third idea, one can generate the desired Hamiltonian from an initial static Hamiltonian by a properly tailored driving protocol. Also, many of the times, the driving schemes allow one to explore the physical situation, which remains unreachable by the other two ideas. This thesis is going to explore certain aspects of the periodically driven classical Hamiltonian systems, as well as its quantized version.

The Hamiltonian systems undergoing periodic driving are studied extensively in the different branches of physics for the last many decades as these systems are occurred very naturally and also undergo a remarkable alteration at the long-time dynamical evolution [21–27]. This class of systems has also been investigated in the context of quenching dynamics. Some of the periodically driven quantum systems show the property of dynamical localization (DL) where energy never exceeds a maximum limit. This is a pure quantum mechanical effect which cannot be observed in its classical counterpart. Driven two-level system [28,29], quantum kicked rotor [30,31] and quantum Kapitza pendulum [32,33] are some of the systems which show the dynamical localization. Periodically delta kicked driving is a very special class of periodic driving scheme which has been investigated extensively to study classical and quantum chaos [34]. This special driving scheme simplifies the study of the classical as well as the quantum dynamics reasonably. Many of the times, this simplification helps to understand the dynamics of the system by analytical means. Moreover, this driving scheme has been realized experimentally by ultra-short laser pulses. This thesis is focussed on the study of different systems under this driving scheme.

The main aim of our research is to compute the effective time independent Hamiltonian for the periodically driven quantum systems. In general, the exact calculation of the effective time independent Hamiltonian is not possible. However, certain perturbation theories facilitate us to compute the effective Hamiltonians approximately using a suitably chosen perturbative parameter. The large frequency of the periodic driving is one such suitable parameter. In this thesis, all periodically driven systems are studied perturbatively at the large frequency limit. Moreover, these perturbation theories work very well for the systems with finite dimensional Hilbert space. Therefore, we have selected periodically driven systems with finite dimensional Hilbert space.

1.1 Floquet theory

Any generic periodically driven systems can always be studied under the Floquet theory. Therefore, these systems are also known as the Floquet systems in the literature. Applying this theory, in principle, one can mathematically derive the effective time independent Hamiltonian or the *Floquet Hamiltonian* of the corresponding time-periodic Hamiltonian $H(t)$, where $H(t+T) = H(t)$ and T is the driving period. This pure mathematical fact has recently motivated scientists to design different periodic driving protocols to synthesize desired effective static Hamiltonian. As a consequence, a new term, "*Floquet engineering*", has recently been coinage and getting recognition in the community to describe this research direction. Even though the Floquet theory ensures the existence of a Floquet Hamiltonian for every time-periodic Hamiltonian, but in practice, it is a very big challenge to design a driving scheme to achieve the targeted exotic properties in the system. Some of the very standard driving schemes are linearly or circularly polarized sinusoidal field [35–38], optimally designed repeated finite width pulse sequences for every time-period [27], and also delta-function kind ultrashort pulse sequences [39, 40].

The periodically driven systems are generally analyzed at two frequency regimes: the slow and the fast driving. The driving is called slow when the frequency of the external driving is lower than any characteristic frequency of the undriven system. In this case, the system almost adiabatically follows the instantaneous Hamiltonian. On the other hand, in the fast regime, the system feels an effective static potential depending on the driving amplitude. Our study is mostly restricted in the high frequency regime.

The Floquet theory is similar to the Bloch theory of the solid state physics, but it is defined in the time domain. According to the Floquet theory, time evolution operator corresponding to a time-periodic Hamiltonian can always be written as:

$$U(t_i \to t_f) = e^{-iG[t_f]} e^{-iH_{\text{eff}}(t_f - t_i)} e^{iG[t_i]}, \tag{1.1}$$

where $G[t] = G[t+T]$ is an explicitly time dependent as well as time-periodic Hermitian operator and H_{eff} is the time independent Floquet Hamiltonian or the effective Hamiltonian. The explicit time dependence of the operator is denoted by the square bracket $[\bullet]$. The time dependent operator $G[t_i]$ is determined by how we switch on the driving at the initial time t_i. Similarly, the way we switch off the driving at the final time t_f is captured by the operator $G[t_f]$. If we define the dynamics of the system stroboscopically, that is at time intervals $t_f = t_i + nT$ where n are positive integers, then the two operators $G[t_f]$ and $G[t_i]$ become equal where nT is defined as the stroboscopic time, measured in units of the driving period T. For such cases, the full evolution operator is equivalent to the evolution of the system generated by the static Hamiltonian H_{eff}, i.e.,

$$U(t_i \to t_f = t_i + nT) = e^{-iG[t_i]} e^{-iH_{\text{eff}} nT} e^{iG[t_i]} \tag{1.2}$$

Even though the above expression is exact, it is not possible to derive the Floquet

Hamiltonian exactly except for some exceptional cases. One has to adopt some perturbation scheme(s) to evaluate H_{eff} [27,41–48]. In the case of the fast driving or the high frequency regime, the Floquet Hamiltonian can be obtained perturbatively where the inverse of the driving frequency ω^{-1} becomes perturbation parameter. The high driving frequency means the corresponding energy $\hbar\omega$ is larger than any characteristic energy scale of the system. Therefore, this driving does not couple with the slow degrees of freedom of the system resonantly. That results in the renormalized and dressed of lower-energy Hamiltonian.

In many cases, at the high frequency limit, the Floquet Hamiltonian is simply the time average of Hamiltonian, $1/T\int_0^T H(t)\,dt$. However, there are certain interesting cases where the Floquet Hamiltonian can not be defined by the time average of the driven Hamiltonian and that too even at the limit of infinite frequency. These cases are very interesting because these may lead to some counter intuitive behaviors. One such well-known example is the observed dynamical stabilization in the Kapitza pendulum [32]. This happens naturally when the driving amplitude is proportional to a power of the driving frequency. Recently, this physical situation has been realized experimentally in cold atoms set up while studying the Harper-Hofstadter Hamiltonian [49–52] and the Haldane Chern insulator [53].

As we mentioned earlier, we are particularly interested in the δ-kicked driving protocol. In this protocol, the Hamiltonian of the system undergoes periodic driving in the form of the Dirac δ-function. For this type of driving, the time-evolution operator, defined between two consecutive kicks, also known as the Floquet operator, can be written as a product of two unitary operators: one comes from the time independent static part of the Hamiltonian and the other from the δ-kicked part. Because of this product form, the analytical and numerical study of this kind of driven system becomes simpler. One can now study the quantum dynamics of the system stroboscopically after every time period T easily. Moreover, a repetitive application of the Floquet operator n times on the initial quantum state, one will obtain the quantum state at time $t = nT$. In this thesis we studied two types of δ-kicked driving: (i) the single kicked case, the system experiences only a single δ-kick within the period T, and (ii) the double kicked case, the system experiences two kicks within a single time period. For the double kicked case, the second kick can be used to tune the effect of the first kick. The Floquet analysis of the single kicked case is done extensively in the last few years [40, 54]. Though the double kicked driving case is studied in some non-integrable cases [55,56], but the Floquet analysis of these systems is still pending.

The double kicked systems are attractive because previous studies have shown that if the polarity of the two δ-kicks are opposite, then this gives interesting spectral properties [55–58]. It has also been observed that the double kicked systems behave drastically different from its single kicked counterpart [55–57]. Moreover, the double kicked systems have already been identified as one of the methods to create a synthetic magnetic field by shaking specially designed quantum systems like cold atomic gases trapped in optical lattice [59]. In this thesis, we have considered two physical systems for this class of driving protocols: one is the Double Kicked Top (DKT), and another is the double kicked hard-core bosons (DKHCB) on a 1D lattice.

A double kicked system with opposite polarity was first studied in the kicked rotor model [55, 56]. This model is physically realizable in cold atoms loaded on optical lattice [31, 60–63]. This system shows a beautiful butterfly like spectrum which has similarity with the well known Hofstadter butterfly [64]. In Ref. [57], it is shown that the quasienergy spectrum of a class of driven $SU(2)$ systems also displays a butterfly pattern with the multifractal property. The multifractal property characterizes highly critical spectra. The double kicked top system is one example of such a class of systems. This model is physically realizable in a driven two-mode Bose-Einstein condensate (BEC) [65–68]. This system is an entirely different class of critical driven systems. The eigenstates of the Flquet operator also reveals the critical behavior via their fractal properties [58].

We have mentioned earlier that the DKT system is studied in some earlier publications. However, this model is never studied from the Floquet theory perspective. This prompted us to analyze this interesting system extensively. This study has shown that the dynamics and spectrum of the double kicked systems are fundamentally different from its single kicked counterpart. Therefore, we have studied the dynamics of another system under a double kicked driving protocol. We consider hard core bosons (HCBs) under the presence of double δ-kicks with opposite polarity within one time period. The single kicked HCB (SKHCB) has already been studied [40]. This study observed the dynamical localization, which was reflected in the decay of the current in the system. However, in the case of the double kicked HCB (DKHCB), a survival of current flow through the system is observed. We have chosen this model because this is one of the special cases for which we can study it analytically exact way without going into any perturbation scheme.

We have also studied a couple kicked top (CKT) model [69]. This is a single kicked system, but two nonintegrable chaotic systems are coupled with an interaction [69]. This system is interesting because, unlike the single top, its effective Hamiltonian is nonintegrable. We have found chaos in the classical dynamics of this system. At a particular limit, the effective Hamiltonian of the CKT becomes the well known Feingold-Peres (FP) model [70, 71]. Recently, the FP model has got prominence once again because its spectrum shows a so-called *non-standard* symmetries [72] with respect to the Altland-Zirnbauer tenfold symmetry classification of quantum systems [73]. This symmetry classification is the extension of the *standard* Wigner-Dyson's threefold classification. The effective Hamiltonian of the CKT system can be considered as a *generalized* version of the FP model, as it has an extra parameter to control the dynamics as well as the spectrum of the system.

1.2 Organization of the Thesis

Including the present introductory chapter, this thesis has total *six* chapters, which are followed by *four* appendices. Here is a brief description of the chapters:

Chapter 2: The periodically driven systems are studied under the Floquet theory. A brief discussion of the Floquet theory is presented in this chapter. The most important outcome of the Floquet theory is the effective time independent Hamiltonian. This effective Hamiltonian describes dynamics, which is equivalent to the dynamics governed by the periodically driven Hamiltonian. In general, it is not possible to compute the effective Hamiltonian exactly by any analytical means, and therefore one has to employ some perturbation theory. We discuss two such perturbation theories.

Chapter 3: A double kicked top system having exotic *butterfly* like fractal property in its spectrum is studied. The effective time independent Hamiltonian is calculated using a perturbation method. A comparative study of the effective Hamiltonian energy eigenvalues and the quasienergies of the Floquet time evolution operator is performed. A detailed discussion on the fractal and multifractal properties of the energy eigenvalues and the eigenvectors is presented. The self-similar property of the spectrum butterfly is investigated from a number theoretical perspective using the Farey sequence.

Chapter 4: Dynamics of a chain of hard core bosons experiencing an onsite staggered potential is studied under a scheme of single and double δ-kick driving. This study focusses on the properties of the current flow through the system and the work done by the driving on the system, at the asymptotic limit, when the driving frequency is very high. The saturated current and work done are investigated for different values of the kicking strength and the initial current in the system. The investigation reveals that, for the double kicked case, the current in the system survives at the high frequency limit, whereas the system under the single δ-kick driving experiences dynamical localization. The survival of the current is determined by the time interval between the two kicks. This also leads to three different phases of the system: the superfluid phase, the bosonic Mott insulator phase, and the intermediate of these two phases. A light-cone like spreading of the density of the particles in the real space is also observed.

Chapter 5: The couple kicked top system is studied from the Floquet perspective. Unlike the single kicked top case, the classical limit of the effective Hamiltonian of this system shows chaos. At the quantum mechanical level, the energy spectrum of the effective Hamiltonian shows some nonstandard symmetries, which leads to the nonstandard statistical properties in the spectrum.

Chapter 6: Finally, the thesis is concluded and immediate future scope is discussed.