Chapter 8

Unreliable Service and Vacation Interruption

8.1 Introduction

In a real-time scenario, due to the rapid advancement in science and engineering technologies, the redundant repairable machining systems have now become the essential requirement of daily life. There are many examples of machinery based service systems, to be specific, production systems, computer and communication systems, transportation and supply-chain management, flexible manufacturing systems, et cetera, which are cited as real-time industrial systems. The industrial systems advance with the updated digitization, technology, and become sophisticated and complex. The service quality and performance of any machining system is highly influenced by the variability of processing times, randomness of repair-times, amount of random failures, which are some of the common critical factors among all manufacturing and commercial industries. Machines are unreliable as their units and capability of working decrease significantly with the passes of time. It leads to unexpected failure of the machining system, costly repair of units, expensive replacement of units that cause the loss of production in any industrial management system. Therefore, the complexity of the fault-tolerant machining systems, as well as the costs caused by the unpredictable failure of their units, attract the attention of researchers and system analysts to maintain the market or acceptable value of any business industry. Industrial systems necessitate the high reliability and availability of the machining system. For the mathematical modeling of reliability-based machine repair problems through the queueing-theoretical approach, several research articles and texts (*cf.* [55], [29], [53], [192], [87], [193], [86]) have been presented by many of the researchers.

In the mid-twentieth century, the queueing-based telephonic communication problems developed by the Danish mathematician A. K. Erlang laid the framework for the development and implementation of queueing problems in real-time scenarios. Henceforth, during the last century, queueing problems remained more popular and emerged as the most prominent and active research area among scholars and researchers. However, because of the increasing complexity of many stochastic machining and service systems, classical queueing theory, which was once quite successful in modeling of telephone systems and other simple service systems, appears to be insufficient today. To overcome the constraints of classical queueing theory, the vacation queueing models were introduced and developed as the extension of the previous theory in the 1970s. In general, in the vacation queueing model, the server is allowed to take the vacation after the service completion instant rather than continuously waiting idly for the newly arriving customers. The provision of opting vacation by the server makes the queue-based service system more flexible in terms of the optimal operating conditions of the system with the minimum associated expected cost. Henceforth, the waiting line problems or merely the vacation queueing models fascinate considerable attention of system analysts, decision-makers, and researchers and become the active and popular area in the research and development (R&D) sector of service field. To analyze and study the vacation queueing models, the optimal operating policies in Markovian and non-Markovian environment has been used by many researchers and scientists (*cf.* [222], [22], [262], [177], [301], [171], [221], [88], [89], [212], [137], [233]).

In the past, the concept of queueing theory has been applied to many industrial problems with several basic queueing terminologies such as impatient, feedback, breakdown, batch arrival/service, the retrial of customers, and many more. In these models, generally, it is assumed that the service provided by the service provider is successful and satisfactory. However, this hypothesis may not always be correct in many real-time customer-service based management problems. Therefore, the new terminology, unreliable service, has been introduced in the queueing literature (*cf.* [205]) to ascertain whether the ongoing service of the customer has been completed satisfactorily or not. It can generally be happened because of some external interference, *i.e.* the service is interrupted neither by the server's fault nor by the unit's disorder.

In the forthcoming sections, we delineate the effect of working vacation and vacation interruption policy on some reliability characteristics of the redundant repairable machining system with unreliable service. For the comparison and stepwise understanding, the present study is demarcated in three-part (i) machine repair problem (MRP), (ii) MRP with working vacation and vacation interruption, and (iii) MRP with working vacation interruption with unreliable service. For the analysis purpose, the Chapman-Kolmogorov differential-difference equations are formulated. Next, to calculate the transient-state queue-size distribution, we employ the Runge-Kutta method of fourth-order since it is the arduous task to derive the closed-form expressions of the mean time-to-failure and reliability of the machining system analytically. For a better understanding from the mathematical point of view, the matrix structures using different levels and phases of the quasi-birth and death (QBD) process for all three developed models are also provided.

8.2 Machine Repair Problem (MRP)

In queueing literature, there are many basic models related to real-time Markovian modeling. To the specific, the machine repair model is the typical example of the finite population queueing model. In the machine repair model, units represent the population of prospective customers, the failure of the unit corresponds to the arriving customer, and the repairman who provides the repair to the failed units is known as the server. Subsequently, for the mathematical modeling, we deal with the finite population machine repair problem (MRP) consisting of M identical operating units and S warm standby units under the care of the single repairman. When the operating unit fails, it is immediately replaced by the available standby unit with the negligible switchover time. At the moment when the state of standby unit changes to the operating state, the failure characteristics of the standby unit become as same as the operating unit. For the normal working of the repairable machining system, there is a requirement of M operating units in the system. But the machining system can also continue functioning in degraded manner even when the number of operating units in the system is at least m. The (M,m) machining system operates in normal-short mode depending on the number of operating units in the system; therefore, the maximum $K = M+S-m+1$ units are allowed to fail. Moreover, we assume that the time-to-failure of the operating unit, as well as the standby unit, are independently and exponentially distributed random variates with parameter λ and $\nu\,(0<\nu<\lambda)$, respectively. Similarly, the time-to-repair the failed unit is also exponentially distributed with parameter μ_b. Henceforth, the state-dependent failure rate of units is represented as

$$\lambda_n = \begin{cases} M\lambda + (S-n)\nu; & 0 \le n < S \\ (M+S-n)\lambda; & S \le n < K \\ 0; & \text{otherwise} \end{cases}$$

In the last few decades, many of the research papers on performance characteristics of the fault-tolerant machining system have been published using the queuing-theoretic approach to deal MRP with essential and/or optional terminologies (*cf.* [26], [265], [245], [167], [97], [59], [127], [187], [104], [293], [217], [201], [72],

[286], [158], [128], [313], [102], [298], [207], [231]). More recently, the fault-tolerant machining system with the random failure events, common cause failure and imperfection is investigated by (*cf.* [229], [230]). For the comparative and optimal analysis, they provided the numerical simulation of several test experiments with different repair time distributions.

In this chapter, we present the reliability-based analysis of MRP with different machining variants. For that purpose, we define the states of the redundant machining system at time instant t using the fundamental law of Markov chain as

$$J(t) \equiv \text{State of the repairman/server}$$

$$N(t) \equiv \text{Number of failed units in the machining system}$$

Therefore, $X(t) = \{(J(t), N(t)); t \geq 0\}$ represents the continuous-time Markov chain (CTMC) on state space

$$\Theta \equiv \{(0,n);\ n = 0, 1, \cdots, K-1\} \cup \{K\}$$

where, K is the failure state of the machining system.

Hence, the Markov chain $\{X(t); t \geq 0\}$ is irreducible. Also, since the state space Θ is finite, the Markov chain is positive recurrent. To get the view of transitions between the precedence states of the machining system, the state transition diagram for the basic machine repair model is provided in Fig. 8.1.

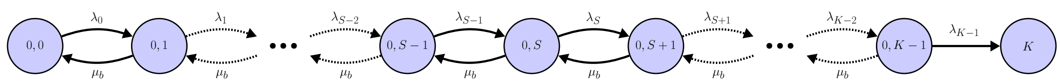

Figure 8.1: State transition diagram of the basic machine repair model.

Now, by balancing the input and output rate flows in Fig. 8.1 and using the fundamental law of quasi-birth and death (QBD) process, the governing Chapman-Kolmogorov differential-difference equations of the studied model are derived as follows

(i) When there is no failed unit in the machining system and the repairman is idle

$$\frac{dP_{0,0}(t)}{dt} = -\lambda_0 P_{0,0}(t) + \mu_b P_{0,1}(t) \tag{8.1}$$

(ii) When there are n failed units in the machining system and the repairman is busy in working mode

$$\frac{dP_{0,n}(t)}{dt} = -(\lambda_n + \mu_b)P_{0,n}(t) + \lambda_{n-1}P_{0,n-1}(t) + \mu_b P_{0,n+1}(t); \quad 1 \leq n \leq K-2 \tag{8.2}$$

(iii) When out of K, maximum allowed failed units, total $(K-1)$ units are failed

$$\frac{dP_{0,K-1}(t)}{dt} = -(\lambda_{K-1} + \mu_b)P_{0,K-1}(t) + \lambda_{K-2}P_{0,K-2}(t) \tag{8.3}$$

(iv) The state where the machining system fails completely

$$\frac{dP_K(t)}{dt} = \lambda P_{0,K-1}(t) \tag{8.4}$$

For employing the matrix method to determine the transient queue size distributions, the system of differential-difference equations (8.1)-(8.4) can be represented into the matrix form $D\mathbf{Y} = \mathbf{Q}_1\mathbf{Y}$, wherein, $\mathbf{Y}$ is the column vector of all time-dependent state probabilities of dimension $K+1$ and $D\mathbf{Y}$ is the derivative of the column vector $\mathbf{Y}$, and $\mathbf{Q}_1$ is the block-square transition matrix of order $K+1$ which is generated by using the tri-diagonal characteristics of matrix algebra. The block-structure of the transition rate matrix $\mathbf{Q}_1$ is partitioned as

$$\mathbf{Q}_1 = \begin{bmatrix} \mathbf{B}_{00}^1 & \mathbf{B}_0^1 & \mathbf{0} \\ \mathbf{C}_1^1 & \mathbf{A}_1^1 & \mathbf{B}_1^1 \\ \mathbf{0} & \mathbf{C}_2^1 & \mathbf{A}_2^1 \end{bmatrix}$$

where $\mathbf{B}_{00}^1$ is the scalar matrix and $\mathbf{B}_0^1$, $\mathbf{C}_1^1$ & $\mathbf{B}_1^1$ are the row and column vectors of order $K-1$, respectively. Similarly, $\mathbf{A}_1^1$ is the tri-diagonal square matrix of order $K-1$ and $\mathbf{C}_2^1$ & $\mathbf{A}_2^1$ are the zero vectors. The structures of these block sub-matrices are given as

$$\mathbf{B}_{00}^1 = [-\lambda_0]; \quad \mathbf{B}_0^1 = [\lambda_0, 0, 0, \cdots, 0]$$
$$\mathbf{B}_1^1 = [0, 0, \cdots, 0, \lambda_{K-1}]^T; \quad \mathbf{C}_1^1 = [\mu_b, 0, 0, \cdots, 0]^T$$

and

$$\mathbf{A}_1^1 = \begin{bmatrix} u_1^1 & v_1^1 & 0 & \cdots & 0 & 0 \\ w_2^1 & u_2^1 & v_2^1 & \cdots & 0 & 0 \\ 0 & w_3^1 & u_3^1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & u_{K-1}^1 & v_{K-1}^1 \\ 0 & 0 & 0 & \cdots & w_K^1 & u_K^1 \end{bmatrix}$$

where

$$u_n^1 = \begin{cases} -(\lambda_n + \mu_b); & 1 \le n \le K-1 \\ 0; & \text{otherwise} \end{cases}$$

$$v_n^1 = \begin{cases} \lambda_n; & 1 \le n \le K-2 \\ 0; & \text{otherwise} \end{cases}$$

$$w_n^1 = \begin{cases} \mu_b; & 2 \le n \le K-1 \\ 0; & \text{otherwise} \end{cases}$$

In the process of analyzing the efficiency and working quality of any service/machining system, performance measures play a vital role. These measures may either be qualitative or quantitative and helps the system engineers and decision-makers to rank the complex machining/service systems. The following are some essential queueing performance measures that necessarily be required to investigate the machine repair model.

- Expected number of failed units in the machining system at time t

$$E_N(t) = \sum_{n=0}^{K-1} nP_{0,n}(t) + KP_K(t) \tag{8.5}$$

- Probability that the repairman is idle at time t

$$P_I(t) = P_{0,0}(t) \tag{8.6}$$

- Failure frequency of the machining system at time t

$$FF(t) = \lambda_{K-1}P_{0,K-1}(t) \tag{8.7}$$

- Throughput of the machining system at time t

$$\tau_p(t) = \sum_{n=1}^{K-1} \mu_b P_{0,n}(t) \tag{8.8}$$

- Reliability of the machining system

$$R_Y(t) = 1 - P_K(t) \tag{8.9}$$

- Mean time-to-failure of the machining system

$$MTTF = \int_0^\infty R_Y(t)dt \tag{8.10}$$

The numerical simulation has been done in forthcoming section for the sensitivity analysis of above-mentioned performance characteristics of machine repair problem.

8.3 Working Vacation and Vacation Interruption

In the queueing literature, the vacation queueing models have emerged as intensive research topics in recent years. Though from the literature, it is observed that the existing queueing models mainly focused on maintenance optimization, but the reliability modeling where the repairman takes the sequence of vacations was less studied. The concept of working vacation (WV) policy was first conceptualized in 2002 by [222], inspired from the WDM optical access network using multiple wavelengths. In the redundant repairable machining systems, during the working vacation period, the repairman continues rendering the intended repair to the failed units rather than terminating the repair as on vacation in general. However, in real-time congestion problems, in spite of better than the complete vacation, the assumption of working vacation still seems more restrictive. Therefore, to overcome this limitation, in 2007 [170] proposed the vacation interruption (VI) policy for the single service provider in

Markovian environment. Under vacation interruption policy, during the working vacation period, if the repairman finds more failed units in the system waiting for repair than pre-specified threshold at the service completion instant, the repairman immediately terminates his vacation and resumes the regular working attribute. Several studies have been done in the context of parametric and optimal analysis of fault-tolerant machining system with working vacation and vacation interruption by many researchers (*cf.* [254], [78], [256], [273], [174], [172], [272], [214]).

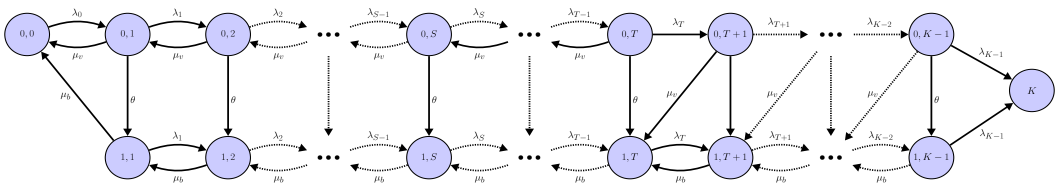

Figure 8.2: State-transition diagram of the machine repair model with working vacation and vacation interruption.

The basic assumptions considered previously for MRP queueing model are also supposed for the study of queueing model involved in MRP with working vacation (WV) and vacation interruption (VI). Besides these norms, some more assumptions are also considered. The repair time of the failed units in working vacation follow the exponential distribution with mean rate μ_v and the vacation time of the repairman also follow the exponential distribution with meantime $1/\theta$. We assume that the inter-failure times, repair times in both busy & WV states, and vacation times are mutually independent. Let, $J(t)$ denotes the state of the repairman at time t, and $N(t)$ represents the total number of failed units in the machining system at time t. Therefore, the possible states of the repairman are characterized as follows

$$J(t) = \begin{cases} 0; \text{ The repairman is in the working vacation period at time instant t} \\ 1; \text{ The repairman is in the normal busy period at time instant t} \end{cases}$$

Clearly, $\{J(t), N(t)\}$ for $t \geq 0$ is the continuous-time Markov chain (CTMC) with the state space

$$\Theta \equiv \{(0,0)\} \cup \{(j,n);\, j = 0,1 \;\&\; n = 1,2,\cdots,K-1\} \cup \{K\}$$

where, K is the state representing that the machining system fails completely.

Using the fundamental law of probability and balancing the transitions between adjacent states in Fig. 8.2 of the MRP with WV and VI policies, the governing differential-difference equations are developed as follows

(i) When there is no failed unit in the machining system and the repairman is on the working vacation

$$\frac{dP_{0,0}(t)}{dt} = -\lambda_0 P_{0,0}(t) + \mu_v P_{0,1}(t) + \mu_b P_{1,1}(t) \tag{8.11}$$

(ii) When there are n failed units in the machining system and the repairman is on working vacation

$$\frac{dP_{0,n}(t)}{dt} = -(\lambda_n + \mu_v + \theta)P_{0,n}(t) + \lambda_{n-1}P_{0,n-1}(t) + \mu_v P_{0,n+1}(t); \quad 1 \le n \le T-1 \tag{8.12}$$

(iii) States of the system at which the working vacation of the repairman is interrupted

$$\frac{dP_{0,n}(t)}{dt} = -(\lambda_n + \mu_v + \theta)P_{0,n}(t) + \lambda_{n-1}P_{0,n-1}(t); \quad T \le n \le K-1 \tag{8.13}$$

(iv) The state of the system after which the busy repairman takes the vacation

$$\frac{dP_{1,1}(t)}{dt} = -(\lambda_1 + \mu_b)P_{1,1}(t) + \mu_b P_{1,2}(t) + \theta P_{0,1}(t) \tag{8.14}$$

(iv) When there are n failed units in the machining system and the repairman is on regular busy mode

$$\frac{dP_{1,n}(t)}{dt} = -(\lambda_n + \mu_b)P_{1,n}(t) + \lambda_{n-1}P_{1,n-1}(t) + \mu_b P_{1,n+1}(t) + \theta P_{0,n}(t); \quad 2 \le n \le T-1 \tag{8.15}$$

$$\frac{dP_{1,n}(t)}{dt} = -(\lambda_n + \mu_b)P_{1,n}(t) + \lambda_{n-1}P_{1,n-1}(t) + \mu_b P_{1,n+1}(t) + \theta P_{0,n}(t) + \mu_v P_{0,n+1}(t); \quad T \le n \le K-2 \tag{8.16}$$

(v) When $(K-1)$: out-of-K, maximum allowed, units are failed in the machining system during the busy period of the repairman

$$\frac{dP_{1,K-1}(t)}{dt} = -(\lambda_{K-1} + \mu_b)P_{1,K-1}(t) + \lambda_{K-2}P_{1,K-2}(t) + \theta P_{0,K-1}(t) \tag{8.17}$$

(vi) The state of the machining system at which the system fails completely

$$\frac{dP_K(t)}{dt} = \lambda_{K-1}P_{0,K-1}(t) + \lambda_{K-1}P_{1,K-1}(t) \tag{8.18}$$

The generator matrix, denoted by $\mathbf{Q}_2$, is the composition of block metrics obtained by the corresponding transitions between adjacent states of the machining system. The structure of the generator matrix is expressed as follows

$$\mathbf{Q}_2 = \begin{bmatrix} \mathbf{B}_{00}^2 & \mathbf{B}_0^2 & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{C}_1^2 & \mathbf{A}_1^2 & \mathbf{B}_1^2 & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_2^2 & \mathbf{A}_2^2 & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{A}_{T-1}^2 & \mathbf{B}_{T-1}^2 & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{C}_2^2 & \mathbf{A}_T^2 & \mathbf{B}_T^2 & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{C}_3^2 & \mathbf{A}_{T+1}^2 & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{A}_{K-2}^2 & \mathbf{B}_{K-2}^2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{C}_3^2 & \mathbf{A}_{K-1}^2 & \mathbf{B}_{K-1}^2 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

where, the block matrix $\mathbf{B}_{00}^2$ is the scalar matrix and $\mathbf{B}_0^2$, $\mathbf{C}_1^2$ & $\mathbf{B}_{K-1}^2$ are the row and column vectors of order 2, respectively. The rest of all sub-block matrices are the square matrices of order 2. The vector and matrix representations of these matrices are viewed as

$$\mathbf{B}_{00}^2 = [-\lambda_0], \quad \mathbf{B}_0^2 = [\lambda_0, 0], \quad \mathbf{C}_1^2 = [\mu_v, \mu_b]^T, \quad \mathbf{B}_{K-1}^2 = [\lambda_{K-1}, \lambda_{K-1}]^T$$

$$\mathbf{A}_n^2 = \begin{bmatrix} -(\lambda_n + \mu_v + \theta) & \theta \\ 0 & -(\lambda_n + \mu_b) \end{bmatrix}; \quad 1 \le n \le K-1$$

$$\mathbf{B}_n^2 = \begin{bmatrix} \lambda_n & 0 \\ 0 & \lambda_n \end{bmatrix}; \quad 1 \le n \le K-2$$

$$\mathbf{C}_2^2 = \begin{bmatrix} \mu_v & 0 \\ 0 & \mu_b \end{bmatrix} \quad \& \quad \mathbf{C}_3^2 = \begin{bmatrix} 0 & \mu_v \\ 0 & \mu_b \end{bmatrix}$$

The closed-form expressions for the expected number of failed units in the machining system $E_N(t)$, probability that the repairman is on working vacation $P_{WV}(t)$, probability that the vacation of the repairman is interrupted $P_{VI}(t)$, reliability of the machining system $R_Y(t)$ et cetera are expressed in the following manner

- Expected number of failed units in the machining system at time t

$$E_N(t) = \sum_{j=0}^{1} \sum_{n=1}^{K-1} nP_{j,n}(t) + KP_K(t) \tag{8.19}$$

- Probability that the repairman is idle

$$P_I(t) = P_{0,0}(t) \tag{8.20}$$

- Probability that the repairman is in normal working mode

$$P_B(t) = \sum_{1}^{K-1} P_{1,n}(t) \tag{8.21}$$

- Probability that the repairman is on working vacation

$$P_{WV}(t) = \sum_{n=0}^{T} P_{0,n}(t) \tag{8.22}$$

- Probability that the vacation of the repairman is interrupted

$$P_{VI} = \sum_{n=T+1}^{K-1} P_{0,n}(t) \tag{8.23}$$

- Failure frequency of the machining system at time t

$$FF(t) = \lambda_{K-1} P_{0,K-1}(t) + \lambda_{K-1} P_{1,K-1}(t) \tag{8.24}$$

- Throughput of the machining system at time t

$$\tau_p(t) = \sum_{n=1}^{K-1} \mu_v P_{0,n}(t) + \sum_{n=1}^{K-1} \mu_b P_{1,n}(t) \tag{8.25}$$

- Reliability of the machining system

$$R_Y(t) = 1 - P_K(t) \tag{8.26}$$

- Mean time-to-failure of the machining system

$$MTTF = \int_0^{\infty} R_Y(t)dt \tag{8.27}$$

8.4 MRP with WV, VI and Unreliable Service

In this section, we choose the queueing terminology and assumptions as same as in previous sections along with the service failure. The random occurrence of the service failure is neither because of the server as it would appear in unreliable server queueing models (*cf.* [110], [111], [319], [179], [156], [42], [40], [120], [199]), nor by the units' disorder as it would be in several interruption models (*cf.* [284], [179], [155], [35], [303], [34], [282], [151], [240], [33]). The waiting and/or in-service failed unit do not abandon (due to the extreme need of repair) from the system, and we preserve the First Come First Serve (FCFS) protocol for repair. We assume that the random service failures occur due to external shocks, environmental forces. The failed units, for which primary repair remains incomplete, continues to strive for successful repair until it is entirely successful. Additionally, neither the repairman nor the caretaker knows whether the repair is successful or not until the service time is completed, and at that instant we hypothesize the quality check to take place, which determines that whether the service is completed & successful or not.

Let, $N(t)$ be the number of failed units in the machining system at time instant t, and $J(t)$ be the state of the repairman at time instant t. Then, there exist a total of four possible states of the repairman which are characterized as follows

$$J(t) = \begin{cases} 0; \text{ the repairman is in the WV period at time instant } t \\ 1; \text{ States which represents the check points during WV period} \\ 2; \text{ the repairman is in the busy period at time instant } t \\ 3; \text{ States which represents the check points during the busy period} \end{cases}$$

Then, $\{(J(t), N(t)); t \geq 0\}$ becomes the continuous-time Markov chain (CTMC) with state space

$$\Theta \equiv \{(0,0)\} \cup \{(j,n); j = 0,1,2,3 \text{ and } n = 1,2,\cdots,K-1\} \cup \{K\}$$

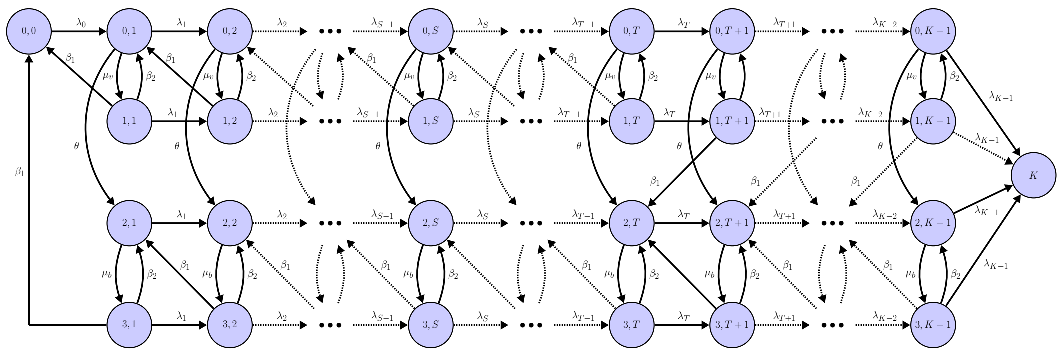

Figure 8.3: State transition diagram of the machine repair model with working vacation interruption and unreliable service.

Fig. 8.3 represents the state transition diagram of the MRP with multiple WV, VI policy, and unreliable service of the repairman. For mathematical modeling, repair times of the failed unit follow exponential distribution with parameter μ_v and μ_b when repairman is on vacation or busy respectively. After repair, the service is checked where checking time also follow exponential distribution with parameter β_1 and β_2 when repair is successful and not successful respectively. The governing set of differential-difference equations is given as follows

(i) State when the repairman is idle

$$\frac{dP_{0,0}(t)}{dt} = -\lambda_0 P_{0,0}(t) + \beta_1 P_{1,1}(t) + \beta_1 P_{3,1}(t) \tag{8.28}$$

(ii) When the repairman is in the working vacation state

$$\frac{dP_{0,n}(t)}{dt} = -(\lambda_n + \mu_v + \theta)P_{0,n}(t) + \lambda_{n-1}P_{0,n-1}(t) + \beta_1 P_{1,n+1}(t) + \beta_2 P_{1,n}(t); \quad 1 \leq n \leq T-1 \tag{8.29}$$

$$\frac{dP_{0,T}(t)}{dt} = -(\lambda_T + \mu_v + \theta)P_{0,T}(t) + \lambda_{T-1}P_{0,T-1}(t) + \beta_2 P_{1,T}(t) \tag{8.30}$$

(iii) When the working vacation of the repairman is interrupted

$$\frac{dP_{0,n}(t)}{dt} = -(\lambda_n + \mu_v + \theta)P_{0,n}(t) + \lambda_{n-1}P_{0,n-1}(t) + \beta_2 P_{1,n}(t); \quad T+1 \le n \le K-1 \tag{8.31}$$

(iv) Check points immediate after the repair is rendered during working vacation mode

$$\frac{dP_{1,1}(t)}{dt} = -(\lambda_1 + \beta_1 + \beta_2)P_{1,1}(t) + \mu_v P_{0,1}(t) \tag{8.32}$$

$$\frac{dP_{0,n}(t)}{dt} = -(\lambda_n + \beta_1 + \beta_2)P_{0,n}(t) + \lambda_{n-1}P_{1,n-1}(t) + \mu_v P_{0,n}(t); \; 2 \le n \le T \tag{8.33}$$

$$\frac{dP_{0,n}(t)}{dt} = -(\lambda_n + \beta_1 + \beta_2)P_{0,n}(t) + \lambda_{n-1}P_{1,n-1}(t) + \mu_v P_{0,n}(t); \quad T+1 \le n \le K-1 \tag{8.34}$$

(v) States representing that the repairman is on regular busy mode of his service

$$\frac{dP_{2,1}(t)}{dt} = -(\lambda_1 + \mu_b)P_{2,1}(t) + \theta P_{0,1}(t) + \beta_2 P_{3,1}(t) + \beta_1 P_{3,2}(t) \tag{8.35}$$

$$\frac{dP_{2,n}(t)}{dt} = -(\lambda_n + \mu_b)P_{2,n}(t) + \lambda_{n-1}P_{2,n-1}(t) + \theta P_{0,n}(t) + \beta_2 P_{3,n}(t) + \beta_1 P_{3,n+1}(t); \; 2 \le n \le T-1 \tag{8.36}$$

$$\frac{dP_{2,n}(t)}{dt} = -(\lambda_n + \mu_b)P_{2,n}(t) + \lambda_{n-1}P_{2,n-1}(t) + \theta P_{0,n}(t) + \beta_1 P_{1,n+1}(t) + \beta_2 P_{3,n}(t) + \beta_1 P_{3,n+1}(t); T \le n \le K-2 \tag{8.37}$$

$$\frac{dP_{2,K-1}(t)}{dt} = -(\lambda_{K-1} + \mu_b)P_{2,K-1}(t) + \lambda_{K-2}P_{2,K-2}(t) + \theta P_{0,K-1}(t) + \beta_2 P_{3,K-1}(t) \tag{8.38}$$

(vi) Check points immediate after the repair is rendered during regular busy period of the repairman

$$\frac{dP_{3,1}(t)}{dt} = -(\lambda_1 + \beta_1 + \beta_2)P_{3,n}(t) + \mu_b P_{2,1}(t) \tag{8.39}$$

$$\frac{dP_{3,n}(t)}{dt} = -(\lambda_n + \beta_1 + \beta_2)P_{3,n}(t) + \lambda_{n-1}P_{3,2}(t) + \mu_b P_{2,n}(t); \; 2 \le n \le K-1 \tag{8.40}$$

(vii) The state when the system fails completely

$$\frac{dP_K(t)}{dt} = \lambda_{K-1}P_{0,K-1}(t) + \lambda_{K-1}P_{1,K-1}(t) \tag{8.41}$$

Now, using the lexicographic sequence of the states of the machining system, the structure of the generator matrix is represented as

$$\mathbf{Q}_3 = \begin{bmatrix} \mathbf{B}_{00}^3 & \mathbf{B}_0^3 & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{C}_1^3 & \mathbf{A}_1^3 & \mathbf{B}_1^3 & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_2^3 & \mathbf{A}_2^3 & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{A}_{T-1}^3 & \mathbf{B}_{T-1}^3 & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{C}_2^3 & \mathbf{A}_T^3 & \mathbf{B}_T^3 & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{C}_3^3 & \mathbf{A}_{T+1}^3 & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{A}_{K-2}^3 & \mathbf{B}_{K-2}^3 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{C}_3^3 & \mathbf{A}_{K-1}^3 & \mathbf{B}_{K-1}^3 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

where, the block sub-vectors and matrices have following representation.

$$\mathbf{B}_{00}^3 = [-\lambda_0], \quad \mathbf{B}_0^3 = [\lambda_0, 0, 0, 0], \quad \mathbf{C}_1^3 = [0, \beta_1, 0, \beta_1]^T$$
$$\mathbf{B}_{K-1}^3 = [\lambda_{K-1}, \lambda_{K-1}, \lambda_{K-1}, \lambda_{K-1}]^T$$

$$\mathbf{A}_n^3 = \begin{bmatrix} -(\lambda_n+\mu_v+\theta) & \mu_v & \theta & 0 \\ \beta_2 & -(\lambda_n+\beta_1+\beta_2) & 0 & 0 \\ 0 & 0 & -(\lambda_n+\mu_b) & \mu_b \\ 0 & 0 & \beta_2 & -(\lambda_n+\beta_1+\beta_2) \end{bmatrix}; 1 \le n \le K-1$$

$$\mathbf{B}_n^3 = \begin{bmatrix} \lambda_n & 0 & 0 & 0 \\ 0 & \lambda_n & 0 & 0 \\ 0 & 0 & \lambda_n & 0 \\ 0 & 0 & 0 & \lambda_n \end{bmatrix}; \quad 1 \le n \le K-2$$

$$\mathbf{C}_2^3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \beta_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_1 & 0 \end{bmatrix}$$

and

$$\mathbf{C}_3^3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_1 & 0 \end{bmatrix}$$

To examine the performance of reliability characteristics, namely, reliability of the machining system, mean time-to-failure, and others, we delineate the closed-form expressions in terms of the transient-state probabilities of various states. Following are some critical performance measures of the machining system which are summarized as

- Expected number of failed units in the machining system at time t

$$E_N(t) = \sum_{j=0}^{3} \sum_{n=1}^{K-1} n P_{j,n}(t) + K P_K(t) \tag{8.42}$$

- Probability that the repairman is idle

$$P_I(t) = P_{0,0}(t) \tag{8.43}$$

- Probability that the repairman is in normal working mode

$$P_B(t) = \sum_{1}^{K-1} P_{2,n}(t) \tag{8.44}$$

- Probability that the repairman is on working vacation

$$P_{WV}(t) = \sum_{n=0}^{T} P_{0,n}(t) \tag{8.45}$$

- Probability that the vacation of the repairman is interrupted

$$P_{VI} = \sum_{n=T+1}^{K-1} P_{0,n}(t) \tag{8.46}$$

- Failure frequency of the machining system at time t

$$FF(t) = \sum_{j=0}^{3} \lambda_{K-1} P_{j,K-1}(t) \tag{8.47}$$

- Throughput of the machining system at time t

$$\tau_p(t) = \sum_{n=1}^{K-1} \mu_v P_{0,n}(t) + \sum_{n=1}^{K-1} \mu_b P_{1,n}(t) \tag{8.48}$$

- Reliability of the machining system

$$R_Y(t) = 1 - P_K(t) \tag{8.49}$$

- Mean time-to-failure of the machining system

$$MTTF = \int_0^\infty R_Y(t) dt \tag{8.50}$$

8.5 Special Cases

The studied models are advanced machine repair problems with many unique and emergent queueing terminologies. Relaxing one or more assumptions, our studied models match with the published models available in the existing literature. The special cases validate our modeling, methodology, and results. Some of them are as follows

Case 1: On considering the facility of cold standby units in place of warm standby units $\nu = 0$, and reliable service $\beta_1 = \mu_b$, $\beta_2 = 0$, our machine repair model results match with special case of MRP in article [181].
Case 2: If $K \to \infty$, $\beta_1 = \mu_b$, $\beta_2 = 0$, the studied queueing model gives similar results as in [171] for $M/M/1$ queue with working vacation (WV) and vacation interruption (VI).
Case 3: If $T = K \to \infty$, our model reduce to single server queue with working vacation and unreliable service (*cf.* [206]).
Case 4: If $T = K \to \infty$, $\theta \to \infty$, and $\mu_v = 0$, the model deduce to $M/M/1$ queue with unreliable service (*cf.* [205]).
Case 5: In the case of infinite capacity service system $K \to \infty$, on removing the assumptions of vacation interruption $T = K$ and unreliable service $\beta_1 \to \infty$, our model is equivalent to the model proposed by [222] in which author introduced the notion of working vacation in the single server queue.

8.6 Cost Analysis

In this section, we formulate the expected total cost function of the redundant repairable machining system to develop the cost optimization problem and calculate optimal system design parameters, which helps system analysts and engineers in decision making.

8.6.1 Steady-State Analysis

In this subsection, the steady-state analysis at equilibrium is performed to examine the optimal operating policy of the developed machine repair model with unreliable service and vacation interruption. In steady-state, *i.e.* in equilibrium ($t \to \infty$), the state probability distribution of the machining system is defined as follows

$$P_{0,0} = \lim_{t\to\infty} Pr\left[J(t) = 0, N(t) = 0\right]$$

$$P_{j,n} = \lim_{t\to\infty} Pr\left[J(t) = j, N(t) = n\right];\ j = 0,1,2,3\ \&\ n = 1,2,\cdots,K-1$$

and

$$P_K = \lim_{t\to\infty} P_K(t)$$

Now, using the state transition diagram in Fig. 8.3, the governing matrix formulation $\mathbf{Q}_3\mathbf{Y} = \mathbf{0}$ for the Markovian machine repair model with standbys provisioning, unreliable service, and vacation interruption is developed. The steady-state probability distribution can easily be computed under probability normalizing condition by employing the matrix method. Further, for the optimal analysis, the expected total cost function is also formulated in the next subsection using intrinsic performance measures, which incur some cost.

8.6.2 Cost Function

For the optimal analysis, the system design parameters, namely, μ_v (repair rate during WV) and μ_b (repair rate during the busy period), are taken into consideration. The main objective of our intuition is to exhibit the optimal repair rates, say μ_v^* & μ_b^* respectively for minimizing the incurred expected total cost in operating the redundant machining system. The system engineers and decision-makers have to identify the states of the machining system, which incur some costs. Following are some cost elements associated with different performance measures and states of the system, that are considered and defined as follows

$C_h \equiv$ Holding cost for each failed unit present in the machining system

$C_b \equiv$ Cost associated with the regular busy state of the repairman

$C_{wv} \equiv$ Cost associated with the working vacation busy state of the repairman

$C_{vi} \equiv$ Cost incurred with the vacation interruption of the repairman

$C_i \equiv$ Fixed cost for the idle state of the repairman

$C_1 \equiv$ Associated cost for providing the repair with rate μ_v

$C_2 \equiv$ Associated cost for providing the repair with rate μ_b

Using the concept of queueing-theoretic approach and the forementioned cost elements, we formulate the cost function as follows

$$TC(\mu_v, \mu_b) = C_h E(N) + C_b P_B + C_{wv} P_{WV} + C_{vi} P_{VI} + C_i P_I + C_1 \mu_b + C_2 \mu_v \quad (8.51)$$

The cost optimization (minimization) problem of the described model involved in MRP with WV, VI, and unreliable service can be represented mathematically as the unconstrained problem as follows

$$TC(\mu_v^*, \mu_b^*) = \min_{(\mu_v, \mu_b)} TC(\mu_v, \mu_b) \quad (8.52)$$

The expected total cost function is the implicit function of cost elements and performance measures, which depend on state-probabilities derived from the governing system of equations that are delineated in terms of rates. The expected total cost function is too complex to get optimal value via the theory of calculus since the first derivative is not evaluative directly, gradient method, and any other well-known optimization techniques. The direct-search method is the too time-taken computational technique to get any useful results. In the next section, we employ the metaheuristic technique, particle swarm optimization (PSO), which depends on the theory of survival of the fittest or nature-inspired behavior in swarm for existence.

8.6.3 Particle Swarm Optimization

For the optimal analysis using PSO algorithm, refer the section 1.10.3 and cite the system design parameters μ_v, μ_b and cost function TC in place of variables x_1, x_2 and objective function f, respectively.

8.7 Numerical Results and Discussion

The prime goal of the present chapter is to understand the qualitative and perceptible performance of the developed fault-tolerant machining system using several reliability-based performance measures. The expressions of some other queue-based performance measures are also provided for the straightforward comparative analysis. For validation of formulation and methodology, we establish numerical simulations through various numerical experiments for the three studied models.

Model 1: Machine repair model (MRP)

Model 2: MRP with working vacation (WV) and vacation interruption (VI)

Model 3: MRP with WV, VI, and unreliable service

For that purpose, we fix the default values of the system parameters as $M = 10$, $S = 5$, $m = 2$, $T = 8$, $\lambda = 0.3$, $\nu = 0.1$, $\mu_b = 3.0$, $\mu_v = 1.0$, $\theta = 6.0$, $\beta_1 = 5$, and $\beta_2 = 1$. To determine the state probability distribution numerically, we employ the Runge-Kutta method of fourth-order and develop the code in MATLAB (2018b) since it is not possible to derive the analytical expressions of governing state probabilities.

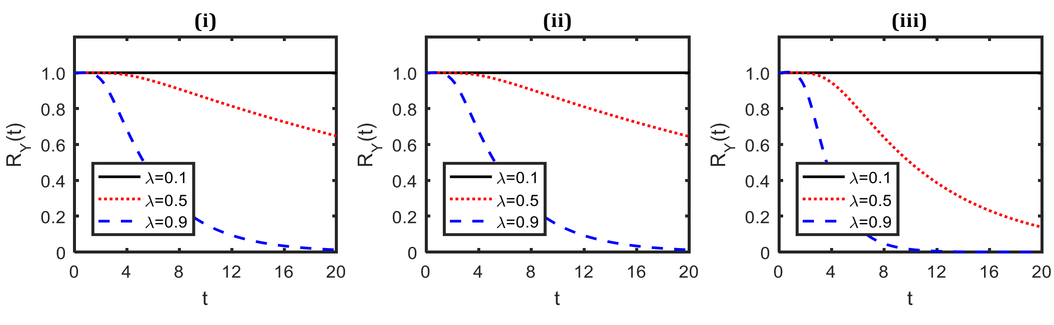

Figure 8.4: Variation of reliability of the machining system wrt failure rate of operating units λ for (i) Model 1, (ii) Model 2, and (iii) Model 3.

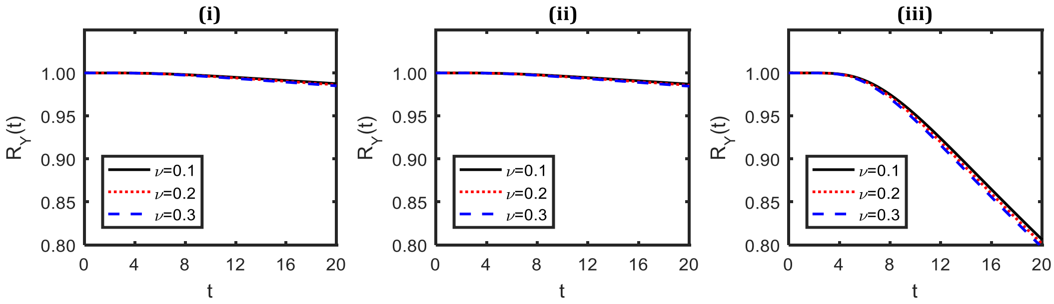

Figure 8.5: Variation of reliability of the machining system wrt failure rate of standby units ν for (i) Model 1, (ii) Model 2, and (iii) Model 3.

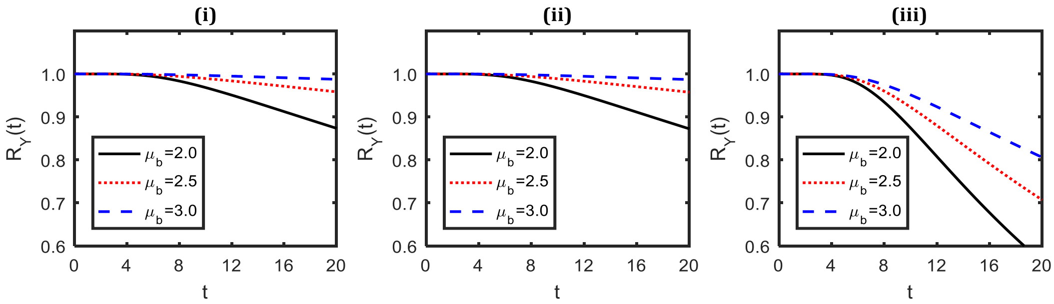

Figure 8.6: Variation of reliability of the machining system wrt repair rate μ_b for (i) Model 1, (ii) Model 2, and (iii) Model 3.

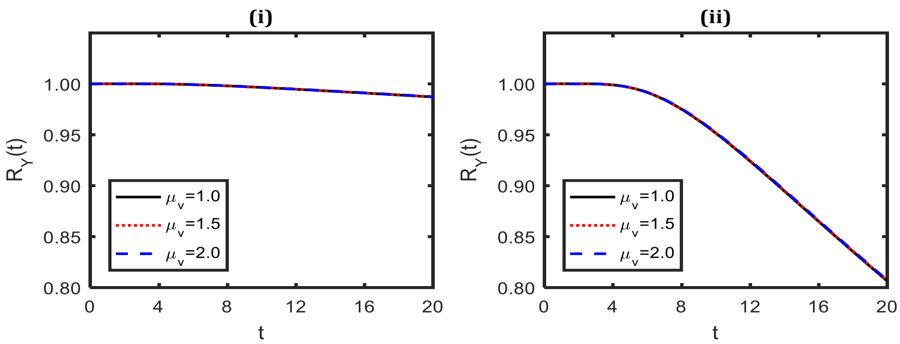

Figure 8.7: Variation of reliability of the machining system wrt repair rate μ_v for (i) Model 2, (ii) Model 3.

In Figs. 8.4–8.9, the variation of reliability of the machining system ($R_Y(t)$) is explored wrt the increasing values of different system parameters for all the developed models. From each figure, we observe that initially, the reliability of the machining system is constant, but after some time, it continuously decreases, which is the intuitively apparent result. We plot three different figures simultaneously to compare the findings of the studied models and to show the decrements in the reliability function with increasing values of time t. In Fig. 8.4 and Fig. 8.5, it is noted that the reliability of the machining system decreases with the increase in the failure rate of the operating units (λ) and standby unit (ν) respectively, which follow the obvious trend. This

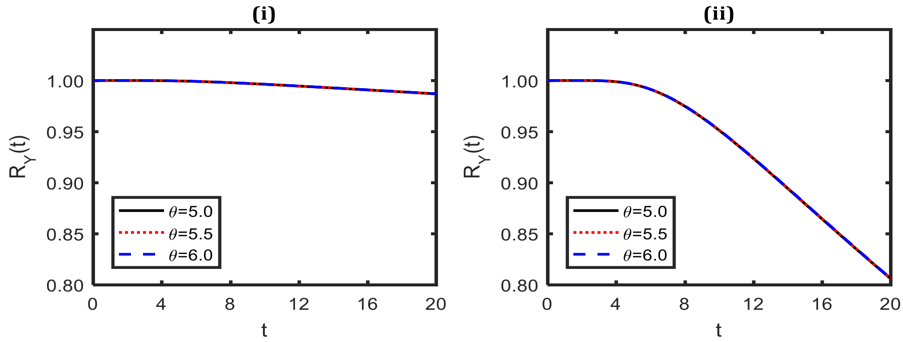

Figure 8.8: Variation of reliability of the machining system wrt vacation rate θ. for (i) Model 2, (ii) Model 3

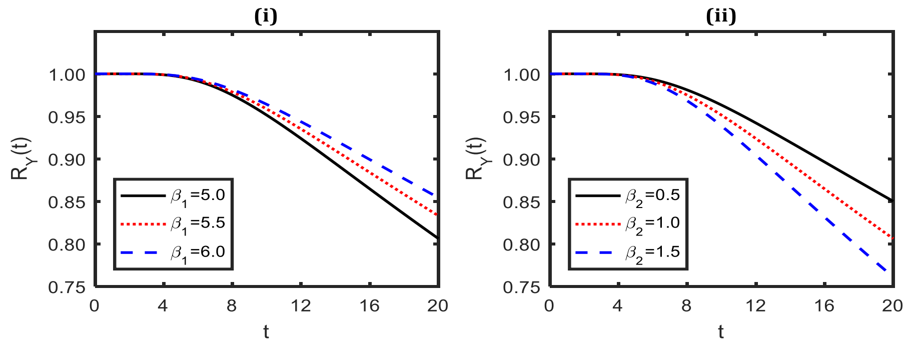

Figure 8.9: Variation of reliability of the machining system wrt (i) rate of successful service β_1, (ii) rate of unsuccessful service β_2 for Model 3.

also prompts that for higher values of λ, the reliability of the machining system decreases more rapidly. It is intuitively anticipated that the reliability of the machining system can be increased by maintaining the appropriate level of repair rate (μ_b). The results in Fig. 8.6 validate the hypothesis of the repair rate and demonstrate that the reliability of the machining system increases with the increasing value of μ_b. The effects of working vacations on the reliability of the machining system are depicted in Figs. 8.7–8.8. Fig. 8.9(i) shows that the high rate of successful service attempt increases the $R_Y(t)$, which is expected for any machining or service system. But, the reverse trend is depicted in Fig. 8.9(ii), which indicates that the cumulative value of unsuccessful attempts reduces the reliability of the machining system. It is recommended that proper preventive measures should opt to avoid the failure of the unit and optimal corrective maintenance strategies should be established to maintain the desired level of reliability at minimum expected cost.

The variation of the mean time-to-failure ($MTTF$) of the fault-tolerant machining system for different system parameter values is shown in Fig. 8.10–8.12. In each

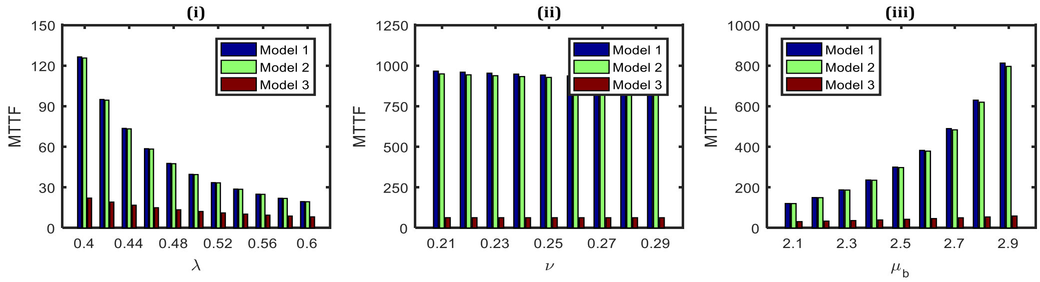

Figure 8.10: Effect of different system parameters on mean time-to-failure of the machining system in Model 1, 2, & 3.

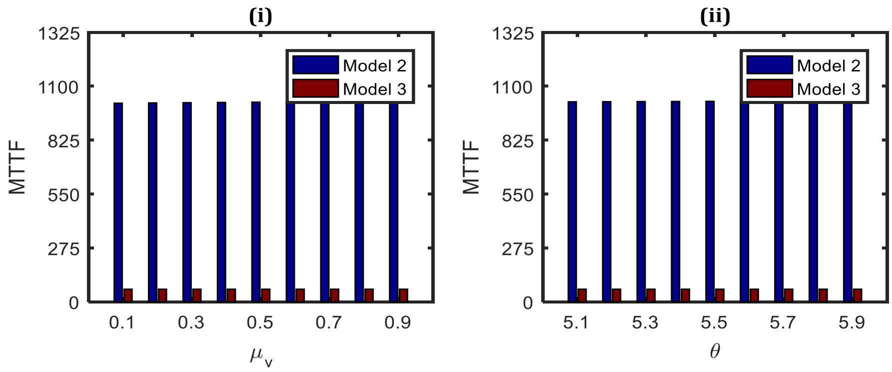

Figure 8.11: Effect of different system parameters on mean time-to-failure of the machining system in Model 2 & 3.

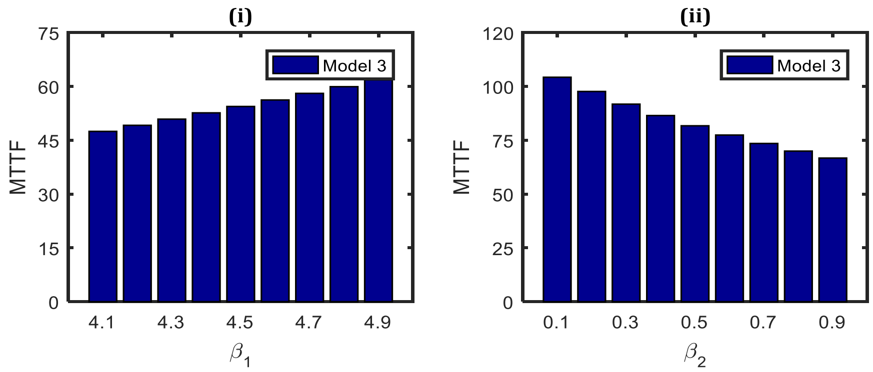

Figure 8.12: Effect of different system parameters on mean time-to-failure of the machining system in Model 3.

sub-figure, we portray comparative bar-graph for the studied model(s). The $MTTF$ decreases extensively for the higher values of failure rate (λ). It also decreases gradually wrt ν and β_2, and remains more or less constant wrt μ_v, θ. It also prompts that better corrective measures are always necessitate since the $MTTF$ is the increasing function for higher values of repair rate μ_b. Also, It appears that the value of $MTTF$ is very less for the third model, which includes the concept of unreliable service of the repairman. It prompts that perfect corrective measure is always important.

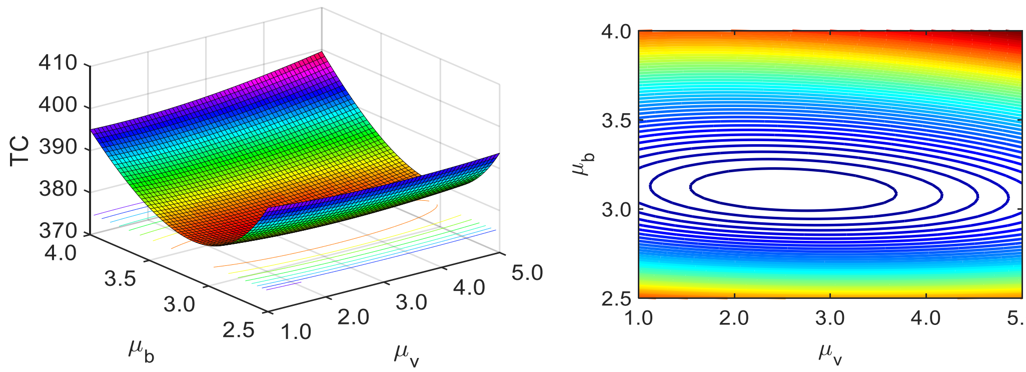

Figure 8.13: Surface plot of the expected total cost of the machining system wrt pair of system design parametrs.

For the optimal analysis in the steady-state condition, we delineate the surface plot and contour plot of the expected total cost function (eq^n(8.51)) in Fig. 8.13 for combined values of machining system design parameters μ_b and μ_v for the default system parameters value in Figs. 8.4–8.12 and unit cost elements as $C_h = 80$, $C_b = 30$, $C_{wv} = 20$, $C_{vi} = 20$, $C_i = 10$, $C_1 = 60$, and $C_2 = 7$ to validate its convex nature. The shape of these plots confirms that the developed expected total cost function is very much convex in nature.

To achieve the numerical solution of the governing cost optimization problem (eq^n(8.52)), we implement the swarm intelligence based global optimization technique, Particle Swarm Optimization (PSO). For that purpose, we fix the default values of system parameters as $M = 10$, $S = 5$, $m = 2$, $T = 8$, $\lambda = 0.1$, $\nu = 0.01$, $\theta = 3.0$, $\beta_1 = 3.0$, and $\beta_2 = 0.5$ along with the unit cost elements associated with performance measures and states of the machining system as $C_h = 80$, $C_b = 30$, $C_{wv} = 20$, $C_{vi} = 20$, $C_i = 10$, $C_1 = 60$, and $C_2 = 7$. We range the lower and upper bounds of both the decision variables μ_v and μ_b as (0 8]. The default values for parameters of PSO algorithm are set as $\kappa_1 = 2$, $\kappa_2 = 2$, and $\omega_2 = 0.5$. The random vectors φ_1 and φ_2 take the values of their elements between 0 and 1.

For the aforementioned default values of the system parameters, some selected generations of the PSO algorithm are provided in the feasible domain for the illustrative purpose in Fig. 8.14. With the help of these generations, we depict the optimal combination of decision parameters μ_v and μ_b along with the optimal expected cost of the redundant machining system. Because the PSO algorithm is the generation and agent-based stochastic optimization technique, we easily examine that in the first generation, all the search particles (solution points) are randomly distributed in the whole feasible domain. After that, as the generation passes, *i.e.* in generations 25, 50, and 100, they approach closer and closer to the converging results in a significant

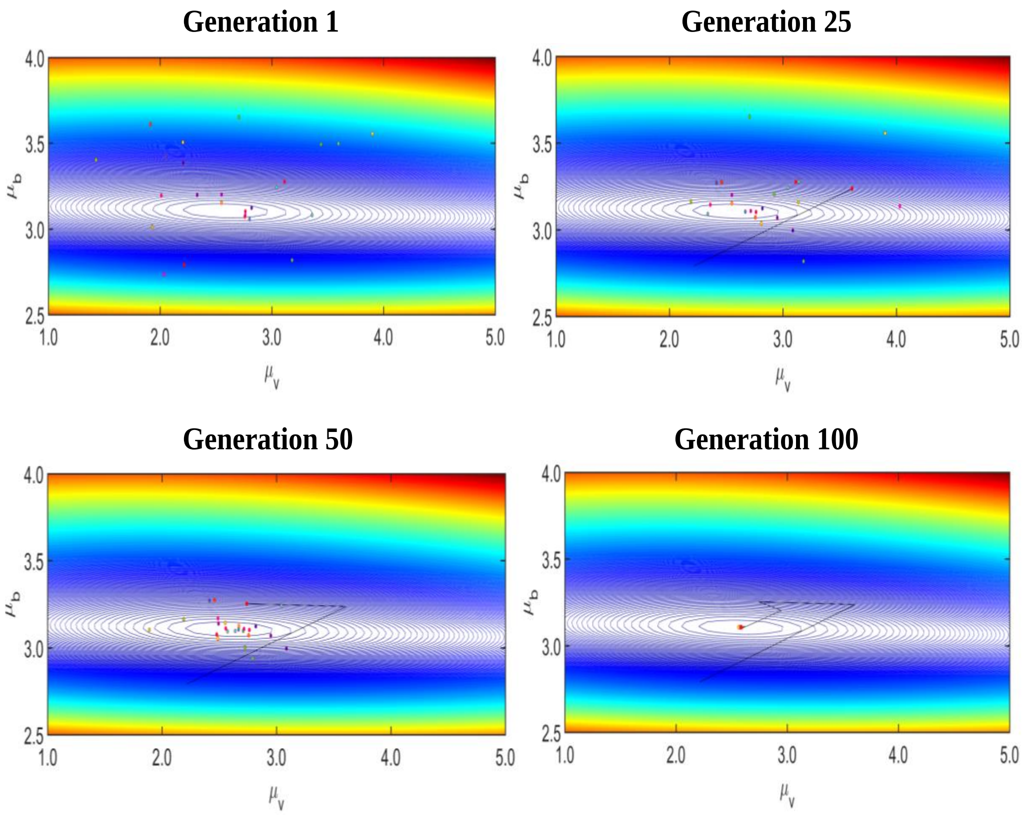

Figure 8.14: Several generations of the PSO algorithm in order to find the optimal pair (μ_v, μ_b).

manner by exploring and exploiting the whole feasible region and shows its capability to converge to the optimal results within the reasonable time interval. It implies the robustness of the PSO algorithm and confirms its applicability for all such numerical experiments. Using the PSO algorithm, we achieve the coordinates of the best particle as $[\mu_v^*, \mu_b^*] = [2.585448, 3.105217]$ along with the minimal expected cost of the machining system $TC^* = 378.073039$.

The numerical simulation for different combinations of system parameters and cost elements is performed by developing several test instances, and results are tabulated in Tables 8.1–8.3. For each test instance, we execute numerical experiments by employing the PSO algorithm for 50 search particles, 100 generations, and 20 runs. All the results obtained in each run of the PSO algorithm are mutually independent of each other. Besides, for the validating of the research findings and to show the robust nature of PSO algorithm, we utilize the concept of statistical parameters, namely mean ratio and the maximum ratio of the optimal cost (TC) among all runs of the PSO algorithm. From Tables 8.1–8.3, for all test instances, we examine that the mean-ratio $\left[\frac{TC}{TC^*}\right]$ and max ratio max $\left[\frac{TC}{TC^*}\right]$ ranges $[1.000000000000, 1.000012673892]$ and $[1.000000000000, 1.000038018489]$ respectively. It signifies that the searching characteristic of the PSO algorithm to move towards the best position.

Table 8.1 prompts clearly that for more operating units, better repair rates require. It allows the working vacation repair rate the little less, which signifies the non-working attitude of the server in vacation mode. The apparent result of the vacation interruption threshold is also revealed in Table 8.1, *i.e.* for high threshold value, high working vacation repair rate requires. Table 8.2 summarizes the optimal design specifications for different rates involved in the studied model. For higher failure rate of units, λ, ν, higher working repair rate is required. In the optimal design, machining system does not prefer working vacation repair. For long vacation time, lower working vacation repair rate is necessary since machining system stabilizes with time. The substantial effect of the unreliable service on the optimal design parameters is also clearly depicted. Table 8.3 tabulates the results of optimal design parameters with variation in the incurred cost for different states of the machining system. The results give glance for the design of the machining system under the constraints of resource or budget.

In a nutshell, we recommend following notable points from the studied models

- For the predictive maintenance, proper modeling, methodology, and analysis are required,
- For better preventive maintenance policy, the system designer should opt for regular maintenance check, redundancy under budget constraints, etc. so that, failure of units or redundant machining system can be delayed.
- For just-in-time corrective maintenance policy, the prompt repair facility should be made available with some budgetary constraints.

In short, the optimal design of the fault-tolerant machining system is required from installation to operation, from the operation to repair, and from repair to replacement.

8.8 Conclusion

In general, the permanent repair facility deteriorates the performance and service quality of any machining/service system due to exhaust work, wear or tear, more idleness, etc. To reduce the wastage of valuable resources, we use some critical queueing terminologies like working vacation, vacation interruption, and unreliable service of the repairman in our modeling and develop different models. To show the dynamical behavior of developed models and the comparative analysis among them, we use several concepts of reliability theory and queueing-theoretic approach. For that purpose, using the fundamental law of transition between adjacent states, the Chapman-Kolmogorov differential-difference equations are developed for each model, and corresponding matrix structures in terms of block-matrices are also provided. Moreover, to show the variability of reliability of the fault-tolerant machining system and mean time-to-failure, several plots are provided, and numerical simulation has been performed for the illustrative purpose.

With the observations of transient analysis, the research scientists, decision-makers, and engineers can conclude that the reliability of the machining system, mean time-to-failure, can be significantly improved by increasing the standby components and the repair rates of the repairman. As a conclusive remark, the findings of the reliability measures of the machining system reveal that the utilization of working vacation policy is more beneficial for the system analysts and engineers instead of employing unreliable service altogether. Numerical simulations and optimal analysis for multiple combinations of default data sets of system parameters and cost elements affirm that such queueing methodologies may be appropriate for many commercial and manufacturing industries. From the future perspective, one can extend this work to general and hyper-exponential service times rather than exponential repair times of the failed units.

Table 8.1: Optimal values of (μ_v^*, μ_b^*) along with minimum cost of the machining system TC^* using PSO algorithm for system thresholds.

(M,S,m,T)	μ_v^*	μ_b^*	TC^*	Mean $\left\{\frac{TC}{TC^*}\right\}$	Max $\left\{\frac{TC}{TC^*}\right\}$	CPU time
(10, 5, 2, 8)	2.585448	3.105217	378.073039	1.000000000281	1.000000000804	438.712
(12, 5, 2, 8)	2.013138	3.757219	457.574207	1.000000000034	1.000000000097	530.994
(14, 5, 2, 8)	0.435713	4.432288	545.939766	1.000000000015	1.000000000068	578.153
(10, 4, 2, 8)	2.387868	2.962138	367.620676	1.000000000457	1.000000001329	396.839
(10, 6, 2, 8)	2.706459	3.222075	386.475177	1.000000001501	1.000000004322	459.041
(10, 5, 3, 8)	2.585425	3.105148	378.071183	1.000000003709	1.000000011101	275.330
(10, 5, 4, 8)	2.585130	3.104612	378.055332	1.000000000686	1.000000002005	246.331
(10, 5, 2, 5)	2.539595	3.106588	378.053896	1.000000001082	1.000000001913	297.634
(10, 5, 2, 6)	2.572589	3.105605	378.067681	1.000000000182	1.000000000333	341.115

Table 8.2: Optimal values of (μ_v^*, μ_b^*) along with minimum cost of the machining system TC^* using PSO algorithm for system rates.

$(\lambda, \nu, \theta, \beta_1, \beta_2)$	μ_v^*	μ_b^*	TC^*	Mean $\{\frac{TC}{TC^*}\}$	Max $\{\frac{TC}{TC^*}\}$	CPU time
(0.10, 0.01, 3.0, 3.0, 0.5)	2.585448	3.105217	378.073039	1.000000000281	1.000000000804	438.712
(0.11, 0.01, 3.0, 3.0, 0.5)	2.350304	3.402750	415.536376	1.000000000574	1.000000001090	438.655
(0.12, 0.01, 3.0, 3.0, 0.5)	1.889981	3.696572	454.370538	1.000000000392	1.000000000948	311.395
(0.10, 0.03, 3.0, 3.0, 0.5)	2.071489	3.183738	394.941291	1.000000001246	1.000000002151	425.438
(0.10, 0.05, 3.0, 3.0, 0.5)	1.349724	3.245301	410.590482	1.000000000487	1.000000001337	527.494
(0.10, 0.01, 3.5, 3.0, 0.5)	1.160355	3.137190	377.035219	1.000000006007	1.000000010261	514.017
(0.10, 0.01, 3.25, 3.0, 0.5)	1.874644	3.122627	377.698759	1.000000000445	1.000000001233	317.015
(0.10, 0.01, 3.0, 2.5, 0.5)	1.219253	3.363463	420.133113	1.000012673892	1.000038018489	324.824
(0.10, 0.01, 3.0, 2.75, 0.5)	2.037311	3.224146	396.721953	1.000000000146	1.000000000425	320.377
(0.10, 0.01, 3.0, 3.0, 0.75)	2.125596	3.268074	393.522662	1.000000000043	1.000000000115	320.601
(0.10, 0.01, 3.0, 3.0, 1.0)	1.612174	3.424405	408.300535	1.000000000289	1.000000000472	353.761

Table 8.3: Optimal values of (μ_v^*, μ_b^*) along with minimum cost of the machining system TC^* using PSO algorithm for system incurred costs.

$(C_h, C_b, C_{wv}, C_{vi}, C_i, C_1, C_2)$	μ_v^*	μ_b^*	TC^*	Mean $\left\{\frac{TC}{TC^*}\right\}$	Max $\left\{\frac{TC}{TC^*}\right\}$	CPU time
(80, 30, 20, 20, 10, 60, 7)	2.585448	3.105217	378.073039	1.000000000281	1.000000000804	438.713
(85, 30, 20, 20, 10, 60, 7)	3.049352	3.154306	387.879518	1.000000000683	1.000000001908	385.228
(75, 30, 20, 20, 10, 60, 7)	2.082261	3.052302	367.882500	1.000000004205	1.000000012601	432.220
(80, 35, 20, 20, 10, 60, 7)	2.693672	3.112864	379.709848	1.000000001744	1.000000004899	352.091
(80, 40, 20, 20, 10, 60, 7)	2.800584	3.120475	381.332747	1.000000004705	1.000000013472	382.103
(80, 30, 25, 20, 10, 60, 7)	2.608424	3.103066	378.396733	1.000000000130	1.000000000197	345.326
(80, 30, 30, 20, 10, 60, 7)	2.631280	3.100929	378.719856	1.000000003277	1.000000012483	336.438
(80, 30, 20, 15, 10, 60, 7)	2.585449	3.105216	378.073036	1.000000000223	1.000000000517	354.679
(80, 30, 20, 25, 10, 60, 7)	2.585460	3.105217	378.07304	1.000000000005	1.000000000010	512.927
(80, 30, 20, 20, 5, 60, 7)	2.714117	3.110244	376.730988	1.000000000037	1.000000000107	289.431
(80, 30, 20, 25, 15, 60, 7)	2.455805	3.100195	379.399740	1.000000000450	1.000000001342	290.850
(80, 30, 20, 25, 10, 65, 7)	2.698389	3.025877	393.397104	1.000000000686	1.000000001058	290.158
(80, 30, 20, 25, 10, 70, 7)	2.801896	3.954549	408.345142	1.000000000524	1.000000001381	291.181
(80, 30, 20, 25, 10, 60, 8)	1.333921	3.125021	380.013859	1.000000000125	1.000000000365	289.993
(80, 30, 20, 25, 10, 60, 9)	0.263702	3.137975	380.801008	1.000000000000	1.000000000000	291.469