ELECTRICAL MEASUREMENTS IN THEORY AND APPLICATION

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Electrical Measurements in Theory and Application

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ELECTRICAL MEASUREMENTS IN THEORY AND APPLICAT

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PREFACE

This book is written for students who have had one year of college physics and desire further knowledge of electrical and magnetic matters. It can be used as a guide in laboratory work, but it is more than a laboratory manual. The principles involved are fully treated; the student is led to reason out the relationships rather than to memorize formulas; and he is urged to learn the facts from his own observations. To this end direct information is often replaced by suggestions as to how the knowledge can be obtained.

The time has certainly come when the electron theory of electrical phenomena should be presented to all students of physics and electrical engineering. Regarding the electron tubes used in radio communication, for instance, there is no doubt that the stream of electrons through the tube continues as an electron current through the connecting wires. Since this point of view has been taken throughout the book, the drifting of electrons along the circuit is called "an electron current," or simply "the current." It is hoped that this concept, which is in accordance with the ideas of modern physics, will add a concreteness to the subject that will prove helpful in the classroom. Where the direction of this current must be indicated, an arrow is placed alongside the circuit, pointing *up* the potential gradient, *i.e.*, from − to + potentials. This direction is also the direction of the electron current.

As in the previous editions, the simpler and more fundamental subjects are treated in the first chapters, and in the first part of each chapter; more difficult measurements and methods requiring more extended knowledge are reserved until the student has attained greater proficiency. The simple experiments in Chap. I on measuring current, resistance, electromotive force, and power by ammeter and voltmeter methods are intended to bring out the fundamental relations of electrical measurements with a minimum of apparatus to distract the student. It is hoped that students with little laboratory experience will build a correct and useful foundation in the use of electrical instruments, while those with college laboratory training can pass on to the advanced subjects and more complicated methods.

In revising the book for the fourth edition, the author has carefully examined each page and diagram in the light of classroom experience and has rewritten much of the text to make the subject clearer to the student reader and to bring it up to date. The author has relied on comments made by users of earlier editions in retaining topics that have proved most useful in their classes; he has omitted obsolete topics and has made many additions where teaching experience has indicated better methods and clearer explanations.

The meter-kilogram-second system is used in giving the definitions of the electrical and magnetic units. Since the unit of power in this system is the watt, the practical units of electrical quantities—ohm, volt, henry, etc.—fall naturally into the same system, thus making the definitions of the practical electrical units much simpler and more direct than heretofore.

The symbol for a voltaic cell has always been a long thin line drawn alongside a short thick line. A careful examination of the books published during the past half century shows that 50 years ago over 90 per cent of those using this symbol intended the long thin line to represent the negative side of the cell, perhaps because it looked like the long thin zinc rods that were then commonly used in primary batteries. Since then, decade by decade, there has been a steady shifting of the meaning ascribed to this symbol until now barely 10 per cent cling to the former use, and 90 per cent indicate the negative side of the battery by the short thick line, perhaps because it looks more like the – sign. For this reason the negative side of a battery is represented in the diagrams in this edition by the short thick line.

The measurement of capacitance and of inductance is usually made with alternating current. The methods are fully described and are illustrated with vector diagrams of the bridge networks. The bridge diagrams show telephone receivers used with 1,000-cycle-per-second current from electron-tube a-c generators, which is a satisfactory and inexpensive combination available to any laboratory. The methods are the same when more elaborate apparatus is used. With the advent of new forms of instruments, especially in the field of electronics, it is now comparatively easy for students to make accurate measurements of inductance and capacitance.

With the use of a newly developed triple-bladed switch, which is fully described, the difficult measurements of the hysteresis of steel are made as easily as other measurements with a ballistic galvanometer.

ARTHUR WHITMORE SMITH

ANN ARBOR, MICH.
February, 1948

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INTRODUCTION

GENERAL PRINCIPLES

1. Electrons.—The atoms of all substances are complicated structures, consisting of electrons and protons. The hydrogen atom consists of one electron, which is the smallest electrical charge (negative) that has ever been found, and a nucleus of one proton, which likewise is the smallest positive charge that has been isolated. The atoms of other elements contain a larger number of electrons and protons. In certain kinds of atoms some of the electrons can circulate from atom to atom, while in other atoms each electron is confined rather closely to its own atom. A substance composed of atoms of the former kind is an electrical conductor. If the atoms are of the latter kind, the substance is an insulator.

The electrons in a conductor are moving in every direction. When, in addition to this motion, there is a drift of the electrons along the conductor, this is called an "electron current."

2. Electronic Charge.—When a piece of glass is rubbed with silk, some of the electrons are removed from the glass and are added to the silk. Just how this is done is not yet fully understood, but that does not alter the fact. These electrons tend to return to their former state of equilibrium. The reason they do not do so at once is that it is almost impossible for them to move through silk or glass, whereas they can move readily through copper and other metals.

It has long been the custom to call this condition of the glass "positive" electrification, and the complementary condition of the silk "negative" electrification. In accordance with this established nomenclature, electrons are "negative."

Since an electron is negative, the addition of an electron to a neutral body will give it a negative charge. The addition of more electrons would increase this negative charge. The natural unit in which to express the magnitude of any charge would be the electron, but this is too small. The practical unit is defined in another way (see Art. 61).

The protons are not readily transferred from one body to another, and the usual method of obtaining a positive charge on a body is to remove some of its electrons. The body from which the electrons are taken will be left with a positive charge.

3. Electron Current.—Bodies in which the electrons can freely move from one part to another are called "electrical conductors." If electrons are added or removed at one point of a conductor, there is a movement of the electrons in the conductor to reestablish equilibrium. This flow of the electrons constitutes an electron current.

When a plate of zinc is placed in a solution of zinc sulphate, there is a strong tendency for zinc ions to go into solution, leaving electrons on the zinc plate. On the other hand, when a copper plate is placed in a copper sulphate solution, there is a great tendency for electrons in the plate to join the copper ions to form atoms of copper. The direction of these effects is noted here without stopping to consider why they are so. When these two arrangements are brought together, as in the Daniell cell, and the two plates are connected by a wire, the electrons on the zinc plate flow along the wire to the copper plate, where they meet the copper that is being deposited. In the solution, positive and negative ions travel between the electrodes. Thus there is a circulation of electrons through the circuit, the electrons in the wire drifting up the potential gradient toward the positive pole of the battery. Within the battery the energy of the chemical action at the electrodes is utilized in pulling the electrons down to the negative pole, whence they can flow up along the external circuit.

The number of electrons in a copper wire is many times greater than the number of atoms of copper. These electrons are in motion within the atoms, and a comparatively small drifting of a few of these electrons along the copper wire is sufficient to make an electron current. The wire becomes heated, and the space surrounding the wire becomes a magnetic field. When a wire carrying an electron current is grasped by the left hand, with the thumb pointing in the direction of the electron flow, the fingers point in the positive direction of the magnetic field.

4. Electric Current.—The appearance of a magnetic field around a wire has been known for well over a century (since Oersted's experiment, 1821) and has been ascribed to an electric current in the wire. The nature of this current was unknown, but a given direction was assigned to it in accordance with the magnetic field surrounding it. The current was said to flow in the direction that a right-handed screw advances when turned in the positive direction of the magnetic field.

It is now known that a current is a drifting of electrons along the wire, and since the electrons are negative in sign, they move in the opposite direction to that formerly assigned to the positive electric current.

In order to understand the language used in the older books, it is still useful to retain some of the old terms and to speak sometimes of the "positive direction" of an electric current. If the motion of the electrons is to be indicated, we can speak of the "electron current" without any confusion of terms. Since electrons are negative, either way of speaking results in the same meaning, but it is more in accord with modern ideas and the drifting of the electrons to forget the hypothetical direction of an "electric" current and to remember that the electrons leave the − side of a battery and drift up the potential gradient to the highest + position they can reach.

5. The Ampere.—As soon as one speaks of measuring a current, it is necessary to define the *unit* of current. A current has a magnetic effect around the wire carrying it, and when placed near a magnet or another current the wire experiences a push tending to move it sidewise across the magnetic field in which it finds itself (see Art. 239).

The unit of current is named an "ampere."[1] This name has long been used; it was officially adopted in 1881 by the Paris Congress of Electricians.

A current of 1 ampere is established as follows. Two parallel currents, I and I', in the same direction are urged toward each other with a force proportional to these currents. This apparent attraction is because each current is in the magnetic field of the other, and it tends to move directly across this field, *i.e.*, toward the other wire. When the wires are 1 meter apart in air (strictly, in vacuum) and the same current ($I = I'$) in each wire is large enough to give a push on each wire of 2×10^{-7} newton per meter length of the wire, this current is 1 ampere.

In framing the definition of any quantity, it is necessary to avoid using the name of any unit that has not been previously defined. Since in this chapter no electric or magnetic unit has thus far been defined, it is necessary to state the value of the ampere without mentioning any other electric or magnetic quantity. This is done as follows:

Definition.—One ampere is the current in each of two long parallel wires placed one meter apart in vacuum when the force per unit length acting on each wire is 2×10^{-7} *newton per meter.*

[1] André Marie Ampère (1775–1836), French physicist.

The number 2×10^{-7} occurs because of the units employed (meters, newtons) and the omission of the unnecessary reference to a magnet. This definition leads directly to the practical determination of a current in amperes by weighing on a sensitive balance the force between two coils when carrying this current, as is done with extreme accuracy at the National Bureau of Standards[1] (see Art.

6. A Loop of Current in a Magnetic Field.—When a loop of wire carrying a current is suspended in a magnetic field (*e.g.*, between the poles of a steel magnet) the current in one side of the loop will tend to move sidewise across the field, as explained above. On the other side of the loop the returning current is in the opposite direction, and this side of the loop will tend to move across the magnetic field in a direction opposite to that of the first side. The result is a turning effect on the loop as a whole, and the coil will turn until this torque due to the current is balanced by the torque of the suspension or spring holding the loop in place.

This turning of the coil is used to measure the amount of current in the coil, as in ammeters and galvanometers. In an ammeter the coil turns on pivots in jeweled bearings. In galvanometers the moving coil is suspended by a long, fine wire that is rolled out into a thin strip in order to give a very small opposing torque to the turning of the coil, thus making it possible for a very small current to produce an appreciable turning of the coil.

7. The Ohm.—The amount of current that flows in a given circuit is determined by the resistance of the circuit as much as by the source of electromotive force. The wire carrying the current becomes heated, and the greater the resistance, the more heat is given off each second for the same value of the current. This expenditure of energy in the wire is used to define the unit of resistance, which is called an "ohm."[2]

The rate at which energy is thus expended by a current of I amp in a resistance of R ohms (*i.e.*, the power) is

$$P = RI^2. \qquad \text{watts}$$

This relation shows at once just how much resistance is to be taken as 1 ohm. This name has long been used; it was reaffirmed by the Paris Congress of Electricians in 1881.

Definition.—One ohm is that amount of resistance in which the power expended is one watt when the current is one ampere.

[1] CURTIS, DRISCOLL, and CRITCHFIELD, *J. Research Nat. Bur. Standards*, vol. 28, p. 133, 1942.

[2] Georg Simon Ohm (1787–1854), German physicist.

It is not easy to measure directly a power of 1 watt, but this value for the ohm is established by accurate electrical measurements at the National Bureau of Standards.[1] From the definition of the watt (Art. 8) and of the self inductance of a coil (Arts. 416 and 417), the value of the latter can be computed as exactly as the dimensions of the coil can be measured.

In Art. 404 it is shown that the capacitance of a condenser can be measured in terms of resistances and the known frequency of an alternating current. Using the same condenser in Maxwell's method (Arts. 437 to 441) for the comparison of an inductance L with a capacitance C gives (when both L and C are known) the value of a resistance in ohms, as defined above, as accurately as the computed value of the inductance of the coil is known. (The uncertainty is about 20 parts in a million.)

8. How Much Is 1 Watt?—To answer this question it is necessary to make a brief survey of the units that are used in mechanics.

The meter is the standard of length. All other lengths, whether miles, inches, or centimeters, are calibrated in terms of the standard meter.

The kilogram is the fundamental standard of mass. All other units, whether pounds or grams, are calibrated in terms of the standard kilogram, which in the United States is kept at the National Bureau of Standards.

The Second.—The unit of time in physics, and for most measurements, is the mean solar second. Astronomers use the sidereal second, which is slightly shorter.

Derived Units.—From these three fundamental units, the unit of velocity is a meter per second, and the unit of acceleration is a *meter per second* per second. The force that will give this acceleration to a mass of one kilogram is called a "newton."[2] This name for the unit of force was adopted in 1938 by the International Electrotechnical Commission at their meeting in Torquay. A force of 1 newton will just about lift a weight of 102 g.

Power is expended when a force is applied to a moving body, as when a boy draws a sled or a horse pulls a cart. If the cart is stuck fast and cannot be moved, no power is expended upon it, although the force may be great. Power is measured by the product of the applied force and the velocity in the direction of this force. The power

[1] Curtis, H. L., C. Moon, and C. M. Sparks, *J. Research Nat. Bur. Standards*, vol. 16, pp. 1–82, 1936, and vol. 21, pp. 375–423, 1938.

[2] Sir Isaac Newton (1642–1727), English natural philosopher.

expended by a force of one newton when it is lifting a weight with a constant velocity of one meter per second is one *watt.*[1] This name for the unit of power was adopted by the International Congress of Electricians at Chicago in 1893.

Other Units.—In the illustration, just given, of a force lifting a weight, the power remains constant whether the operation lasts for a minute or an hour. But the amount of *energy* expended, or the *work* done, increases as time goes on. When the power is one watt, the energy expended each second is one *joule.*[2] This name for the unit of energy was also adopted by the International Congress of Electricians at Chicago in 1893.

A smaller unit of energy is sometimes used. This is the *erg,* which is 10^{-7} joule.

A smaller unit of force is the *dyne.* It is 10^{-5} newton.

9. Difference of Potential, Emf, Fall of Potential.—The difference of potential between two points is that difference in condition which produces an electron current from one point to the other as soon as they are connected by a conductor. Every battery or other electric generator possesses a certain ability to maintain a difference of potential between its terminals, and, therefore, the ability to drive a continuous current. This difference of potential produced by a cell or other generator, and which may be considered as the cause of the current, is called "electromotive force." It must be remembered that this quantity is not a force in the sense that this name is used in physics. In order to avoid using the word "force," it is commonly called "emf."

When a current flows through a conductor, there is a difference of potential between any two points on the conductor. This difference of potential is greater the farther apart the points are taken, and as the change is gradual, it is usually called a "fall of potential." It might also be called a "rise of potential," since, when *following* the electron current through a resistance, one is led to points that are more strongly + in potential. It can always be expressed by the formula RI, where R is the resistance of the conductor, or conductors, under consideration.

This apparent duplication of names may at first appear unnecessary, but the corresponding ideas are quite distinct, and the correct use of the proper term will add conciseness to one's thinking and speaking. Thus we have the emf of a battery; the fall of potential along a conductor; and the more general and broader term, "difference of poten-

[1] James Watt (1736–1819), Scottish engineer.

[2] James Prescott Joule (1818–1889), English physicist.

tial," which includes both of the above as well as some others for which no special names are used.

10. Ohm's Law.—*The electron current that flows through any conductor is directly proportional to the potential difference between its terminals.* This statement was first formulated in 1827 by Dr. Ohm, as the result of many experiments and measurements, and it is known as Ohm's law. It is usually written

$$V = RI \qquad \text{or} \qquad I = \frac{V}{R},$$

where V denotes the potential difference over the circuit through which is flowing the current I. The factor R is called the "resistance" of the conductor, and its value depends only upon the dimensions and material of the wire and its temperature. It is entirely independent of V and I.

This relation holds equally well whether the entire circuit is considered or whether only a portion of such circuit is taken. In the former case the law states that the current through the circuit is equal to the total emf in the circuit divided by the resistance of the entire circuit, including that of the battery and the connecting wires. When applied to a single conductor AB, the law states that the current flowing through the conductor is equal to the fall of potential between A and B divided by the resistance of this part of the circuit.

11. The Volt.—Now that the value of an ampere (Art. 5) and the value of an ohm (Art. 7) have been defined, it follows that the value to be used as 1 volt[1] is given by Ohm's law.

Definition.—*One volt is the difference of potential that, steadily applied to a conductor whose resistance is one ohm, will produce a current of one ampere.*

The name "volt" has long been used; it was reaffirmed by the Paris Congress of Electricians in 1881.

From the definition of the ohm it also follows that the power necessary to maintain a current of I amp under a difference of potential of E volts is

$$P = EI. \qquad \text{watts}$$

This leads to a second way of stating the definition for a volt. Since $E = P/I$, we can say,

One volt is the difference of potential that will maintain a current when the power expended is one watt per ampere.

[1] Alessandro Volta (1745–1827), Italian physicist.

12. The Point Principle. *Kirchhoff's First Law.*—When there is a steady electron current in a circuit, the number of electrons arriving each second at any point on the conductor is equal to the number leaving this point in the same time. This must be so, since there is no accumulation of electrons at any place in the circuit when the potential is unchanging. When two currents unite and flow together in a single conductor, the resultant current is the sum of the two currents for the same reason.

The point principle can be extended to any number of currents meeting in a common junction. If all currents flowing to the point are considered positive and all currents flowing from the point are considered negative, then when the currents have reached their steady values, the algebraic sum of all the currents meeting at the point must be zero, since it is not possible for electrons to accumulate indefinitely at any point. The same is true when the currents are not steady, provided that there is no capacitance at the point considered.

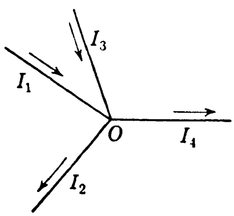


FIG. 1.—Currents meeting at O.

Thus in Fig. 1,

$$+I_1 - I_2 + I_3 - I_4 = 0.$$

This relation can be checked by noting the values of the three currents, I_1, I_2, and I_3, that are measured at B, Fig. 6.

13. Potential Differences.—Any hill may be called "up" or "down," according to the direction in which one is going. If, after traversing several paths, one returns to the starting point, there will have been as much downhill as uphill in the entire journey.

If going uphill is called "positive" and downhill is called "negative," the total height, up and down, that one has gone in the entire journey is zero when the journey ends at the starting point.

The word "level" is commonly used when speaking of points on the ground. Two points are at the same level when water will not flow from one to the other when it is free to do so. They are still at the same level when the water is absent.

The word "potential" is used to express the same idea in electrical phenomena. Two points are at the same potential when electrons will not flow from one to the other when they are free to do so. There is a difference of potential between two points when electrons will flow from one to the other if a conductor is provided. There will still be a difference of potential between these points if the conductor is absent.

14. The Circuit Principle. *Kirchhoff's Second Law.*—The differences in potential along an electrical circuit are due to the emfs of batteries or other sources and to the falls or rises of potential over resistances. If we imagine ourselves to start at a given point on an electrical circuit and to follow along the circuit until we again reach the starting point, we shall have found various ups and downs in potential, but *the sum of all the potential differences in the complete circuit will be zero.* This is so because we started and ended at the same point, and there can be no difference of potential between a given point and itself.

In tracing along a circuit, it makes no difference whether we follow the path of a current or not, but each potential difference that we meet *must be counted* with its *proper* sign. The poles of a battery are usually marked + and −, and in accordance with the usual meaning of these signs, the positive pole of a battery is at a higher potential than the negative pole. Therefore when we meet a battery of emf E and pass across it from the − side to the + side, we have gone up a potential difference of $+E$ volts. It makes no difference whether there is any current through this battery or not, unless we have also gone along a resistance, and then the same rule applies as for any other resistance. In the same way, when we are tracing the circuit across a battery from the + side to the − side, we are going down a potential difference, and now it is counted as $-E$ volts.

The same rule applies when we are passing along a resistance. When a wire of R ohms is connected to the two poles of a battery, the end that is joined to the + pole is at the higher potential. When we pass along R from the − end to the + end, we have gone up a potential difference of $+RI$ volts, where I denotes the amperes of current in the R ohms. When we pass along the circuit from the + end to the − end, the change in potential is counted as $-RI$.

In this case it was easy to tell which end of R is +, because it is connected to the + side of the battery. This also gives the rule for the signs of terms like RI when the directions of the electron flow is known. Since electrons in a conductor always move up a rise of potential, we see that when we are tracing along a circuit in the same direction that the electrons flow, the change in potential over a resistance R is $+ RI$. When we are tracing along R in a direction opposite to that in which the electrons are flowing, the change in potential traced over is $-RI$.

15. Illustration of the Circuit Principle.—Consider a circuit consisting of resistances and batteries in series, as shown in Fig. 2. An inspection of this circuit shows that the direction of the electron current is determined by the larger of the two batteries in spite of the other battery in the circuit, and this is shown by the arrow.

When we trace along this circuit in the counterclockwise direction from A, Fig. 2, toward B, we find a fall of potential of $-RI$ volts as we pass along R from its + end to the − end. In passing on beyond B, there is the further resistance r in the battery E and therefore a further fall of potential of $-rI$ volts. It often makes the problem simpler to show the internal resistance, as r, of a battery alongside the symbol, as E, for the emf of the battery; however there is no place inside the battery where we can use the E without also including the r.

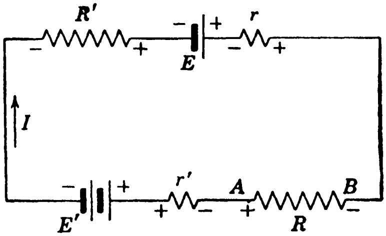


FIG. 2.—A series circuit of batteries and resistances to illustrate the Circuit Principle.

So then, in addition to tracing over r we also pass over the emf $-E$. This is counted negative, because in the direction we are going we step down from the + to the − side of the battery. Further tracing through R' shows that there is a fall of $-R'I$, and at the larger battery a rise of $+E'$ volts. In the latter battery there is also the fall of $-r'I$.

Equating the sum of all these potential differences equal to zero gives

$$-RI - rI - E - R'I + E' - r'I = 0$$

from A around to A.

16. Meaning of the Arrow.—The rise in potential in the circuit can be indicated on a diagram by means of an arrow placed alongside the circuit and pointing in the direction that is *up* the potential gradient. As explained in Art. 3, the direction of the electron flow is up the potential gradient. Therefore this arrow points in the direction of the current. Likewise, if the direction of the electron current is known from other considerations, an arrow pointing in this direction will also indicate the direction that is *up* the potential gradient.

17. Meaning of the Symbol I.—When the letter I is used in the diagram of an electrical circuit, it indicates the value of the current at the place where it is used. But when this symbol is used in connection with an arrow, it means the value of the current *in the direction of the arrow.* In the latter case the value of I may be a negative quantity if the arrow for any reason has been made to point down the potential gradient. Usually this is not a serious mistake and leads only to finding a negative value for the current that is denoted by I.

18. Further Illustration of the Circuit Principle.—In a network of conductors like the one shown in Fig. 3, let it be required to find the value of the current in the branch Q. Suppose that the value of each resistance is known except for r' and r''. The currents I' and I'', and E are known, but E' and E'' are not given.

There are three independent circuits in this diagram. Any other circuit would include only what is already included in these three. The simplest of these circuits that includes Q is *abdca*.

Before the equation for the potential differences in this circuit can be written, it is necessary to indicate the currents in each branch. Very likely the currents I_1 and I_2 in M and N are to the right, and arrows will be put on the diagram to indicate this.

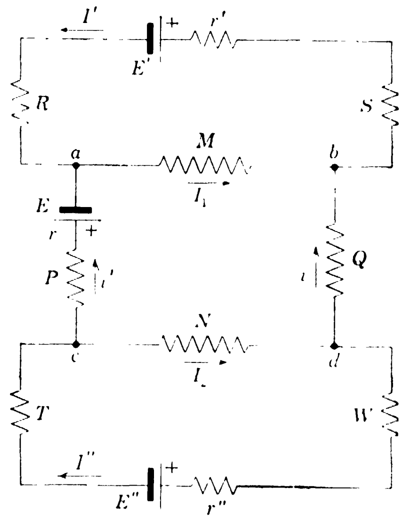


FIG. 3.—A network of three circuits. In the text problem the values of the currents i and i' are to be found.

The current in Q may be in either direction. So we shall set the arrow pointing from d to b, denoting by i the current *in this direction*. Similarly, in P the current from c to a is denoted by i'.

Now writing down the sum of all the potential differences that are encountered by a pencil point tracing along the circuit *abdca* gives

$$+MI_1 - Qi - NI_2 + Pi' + ri' - E = 0. \qquad (1)$$

19. Use of the Point Principle.—This equation (1) contains three unknown currents besides the one (i) whose value is to be found. These can be expressed in terms of the given currents I' and I''.

Applying the point principle at b gives

$$+I_1 + i - I' = 0, \qquad \text{and thus} \qquad I_1 = I' - i. \qquad (2)$$

At the point d,

$$+I_2 - i - I'' = 0, \qquad \text{and thus} \qquad I_2 = I'' + i. \qquad (3)$$

At the point c,

$$+I'' - i' - I_2 = 0.$$

Adding the last two equations gives

$$i' = -i. \tag{4}$$

20. The Completed Equation.—Substituting these values for the currents in Eq. (1) gives

$$+MI' - Mi - Qi - NI'' - Ni - Pi - ri - E = 0, \tag{5}$$

or

$$i = \frac{MI' - NI'' - E}{M + Q + N + P + r}, \tag{6}$$

which is the value of the current in Q in the direction shown by the arrow. Whether this is a positive or a negative number depends upon the relative values in the numerator of this fraction.

21. The Meaning of the Sign ±.—In algebra one might have an equation like

$$8x - x^2 = 15. \tag{7}$$

The solution of this equation by the rules of algebra gives

$$x = 4 \pm 1. \tag{8}$$

This means that either $x = 5$ or $x = 3$ will satisfy the relation shown in (7)—these two values and no others. Intermediate values like 4

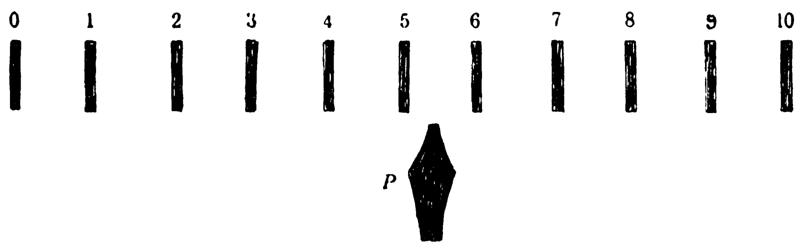


FIG. 4.—A centimeter scale and a pointer.

or 3½ will not do; exactly 3 and exactly 5 are the only numbers that are allowable.

The same sign is often used to express the result of a physical measurement, but in this case its meaning is quite different. Thus note the centimeter scale in Fig. 4. In the original diagram from which this was copied the lines were fine and distinct, and the millimeter divisions were clearly shown. The pointer P was sharp and its position on the scale could be read to within a fraction of a millimeter.

In the scale as shown in Fig. 4 there is some uncertainty as to the position of the pointer on the scale. For some purposes it is sufficient to say that the reading is 5 and to let it go at that. But if one wishes to make the reading as accurately as possible, it is evident that this is somewhat more than 5 and less than 5.5. On the millimeter scale (which is not visible in Fig. 4) it might be about 5.3, but how much uncertainty is there in this value? This is where the use of the sign ± comes in, and the reading is expressed as

$$x = 5.3 \pm c. \tag{9}$$

This does not mean, as in the case of Eq. (8), that the reading is either 5.3 + c or 5.3 − c, but it includes all the intermediate values as possible readings of the true value (as well as some larger and smaller values, as explained below).

22. The Proper Value for *c*.—The proper value to be given c in Eq. (9) requires the careful and experienced judgment of the observer. Evidently, taking a value like 0.5 and saying that the pointer in Fig. 4 is somewhere between 5.8 and 4.8 is not indicating the position of the pointer as closely as it can be read. On the other hand, to give c a value of 0.03 and say that the position of the pointer is between 5.33 and 5.27 is absurd. The chance is very small that the true value, if it were known, would be within this narrow range.

What, then, is the proper value for c? It should be large enough to give an *even* chance (50 per cent) that the true value is within the range that is indicated by $\pm c$. There is also an even chance that the true value is outside of this range, being either larger or smaller.

Thus if the observer of Fig. 4 reads the position of the pointer as 5.3 ± 0.1, he is saying that he considers it an even chance that the true value (if he could see the sharp pointer on the millimeter scale) is between 5.4 and 5.2. But he considers it *just as likely* that the value he is trying to measure is outside this range, either smaller or larger.

Every measurement made should have this indication of the degree of accuracy that the observer places upon it. Even with the original millimeter scale with the sharp pointer, where the reading of the millimeters could be made with certainty, all these considerations would apply to the estimated tenths of a millimeter in the value that is read on the scale.

23. Effect of an Uncertainty on the Result.—Oftentimes the value of a measurement is used in computing a final result. Whatever uncertainty there may be in this value is carried along and affects the final result. When this measurement enters into the final result as a simple factor (*i.e.*, with a power exponent of +1 or −1) the result will be uncertain by the same relative amount (in percentage) as the original measurement. When it enters as the square, the resulting uncertainty is twice that of the measurement. When the square root of the measurement is used, the final result is half as uncertain (in percentage) as the original measurement.

24. Several Uncertainties.—In most measurements the final result involves a number of factors, each of which is somewhat uncertain. It will be found that the relative uncertainty in one of these factors is larger than that of the others. This factor should be examined at the time of its measurement, and an effort should be made to determine its value with greater accuracy.

Since the uncertainty in estimating the amount of the largest uncertainty is always considerable (perhaps 50 per cent), it is not necessary to add the effects of smaller uncertainties in the other factors if they are definitely less than this.

When two factors with about equal relative uncertainties enter into the final result, either by division or by multiplication, the effect of both uncertainties is probably less than the sum of the two (in percentage) by as much as the hypotenuse of a right-angled triangle is less than the sum of its two sides.

CHAPTER I

AMMETER AND VOLTMETER METHODS

25. Hot-wire Ammeter.—When an electron current flows through a wire, one of the most noticeable effects is that the wire becomes warmed. The direct result of this is an increase in the length of the wire. It thus becomes possible to measure the value of the current in terms of the change in length of such a wire, and some ammeters are constructed on this principle.

26. The Portable Ammeter.—In addition to heating the wire, the current produces a magnetic effect in the surrounding space. This is manifest by its action upon a magnetic needle near it or by the force which the wire itself experiences when in the magnetic field of a magnet or another current. A current is not attracted by a magnet, but the wire carrying the current is urged *sidewise* across the magnetic flux (lines of force) near it.[1] From this it follows that a *loop* of wire carrying a current will tend to *turn* in a definite direction when it is in a magnetic field.

In the portable ammeter the current passes through a coil of many turns of fine wire wound on a rectangular form. This coil is mounted on jeweled bearings and can turn in the magnetic field between the poles of a strong permanent magnet. It is held in a definite position by a spiral spring at either end. When a current is passed through the coil, the wires parallel to the axis are urged sidewise across the magnetic flux between the poles of the permanent magnet, and the coil turns until the torque of the springs is sufficient to balance the couple due to the forces between the current and the magnetic field. A pointer attached to the coil moves over the scale and indicates the angle turned through; the scale is not graduated in degrees but in terms of the current required to produce the deflection. The scale is thus direct reading and gives the value of the current in amperes.

In ammeters for measuring large currents, a low-resistance shunt is placed in parallel with the moving coil to allow only a moderate

[1] This effect furnishes the basis for defining the value of the ampere (see Art. 191), and the precise relation between the current and the force acting upon the wire is fully worked out in Art. 239.

current through the latter. The scale is then graduated to read the value of the large total current through both the coil and its shunt. This arrangement is entirely similar in principle to the voltmeter and shunt described in Art. 48, the moving-coil system acting as a sensitive voltmeter.

27. Use of an Ammeter.—The portable ammeter is a good and accurate instrument for the measurement of electron current. As it is very delicate and sensitive, it must always be handled with care. Mechanical shocks or jars will injure the jeweled bearings, and too large a current through it will wrench the movable coil and bend the delicate pointer, even if the instrument is not burned out thereby.

When it is desired to use the ammeter for the measurement of current, it is connected in series with the rest of the circuit, and therefore the entire current passes through the instrument. Great care should always be exercised never to allow a larger current to flow through an ammeter than it is intended to carry. It is always best to have a key in the circuit, and while keeping the eye on the needle of the ammeter, to tap the key gently, thus closing it for a fraction of a second only. If the needle does not move very far, the key can be held down for a longer time. If it is then seen that the needle will remain on the scale, the key can be held down until the needle comes to rest. Behind the needle is a strip of mirror, and by placing the eye in such a position that the image of the needle is hidden by the needle itself, the error due to parallax in reading the scale can be avoided.

The scales of these instruments are graduated to read the current directly in amperes. Sometimes the pointer does not stand at the zero of the scale when no current is flowing. When this is the case, the position of rest should be carefully noted and the observed reoding corrected accordingly.

28. Laws of Electron Currents.—For this exercise, a dry cell, a coil of several ohms' resistance, a key, and the ammeter are connected in series, *i.e.*, one after the other to form a single and continuous circuit.

The electrons flow from the negative, or zinc, pole of the cell out into the external circuit. In order to read the ammeter properly, it should be connected into the circuit so that the electrons will enter it at the post marked − and leave the ammeter at the post marked +. Measure and record the value of the current at different points along this circuit, to determine whether the current has the same value throughout its path or whether it is smaller after passing through the resistances. Next add the remaining coils to the circuit, keeping them all in series, and note the value of the current at the same points as before. State in your own words the effect of adding resistance to the circuit.

Remove one of the coils and connect it in parallel with one of those still remaining in the circuit, *i.e.*, so that the current in the main circuit will divide, a part going through each of the two coils in parallel. Measure the part of the current through each coil; also the main current.

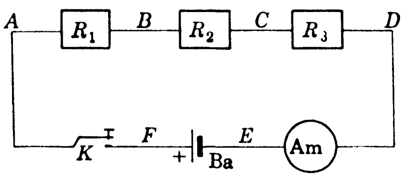


FIG. 5.—Resistances in series.

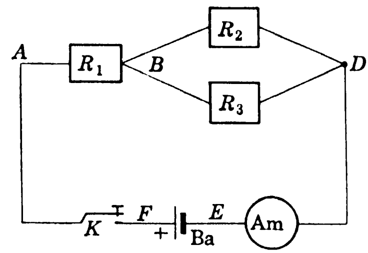


FIG. 6.—Resistances R_3 and R_2 in parallel.

Could you have foretold the value of the latter without measuring it? (See Art. 12.)

This exercise should show that the only thing in common to coils in *series* is the value of the *current* passing through them. For this reason an ammeter is always joined in series with the circuit in which the value of the current is desired. Record the readings of the ammeter in a neat tabular form similar to the following:

TO MEASURE THE CURRENT ALONG AN ELECTRICAL CIRCUIT

Ammeter zero	Ammeter reading	Current	Point at which current is measured

29. The Portable Voltmeter.—The construction of a voltmeter is the same as that of an ammeter, except that instead of a shunt in parallel with the moving coil, there is a high resistance in series with the coil. The current which will then flow through this instrument of high resistance depends upon the emf applied to its terminals, and the numbers written on the scale are not the values of the currents in the coil but are the corresponding values of the fall of potential over the resistance of the voltmeter. Therefore it is sometimes said that a voltmeter is really only a sensitive ammeter measuring the current through a fixed high resistance, while an ammeter is really only a sensitive voltmeter measuring the fall of potential over a low resistance.

30. Use of a Voltmeter.—The portable voltmeter is a good and accurate instrument for the measurement of a fall of potential. Like the ammeter, it is very delicate and sensitive and must always be handled with care. Mechanical shocks or jars will injure the jeweled bearings, and too large a current through it will wrench the movable coil and bend the delicate pointer, even if the emf is not much larger than that intended to be measured by the instrument.

Since a voltmeter is always used to measure the difference of potential between two points, it is not put into the circuit like an ammeter, but the two binding posts of the voltmeter are connected directly to the two points whose difference of potential is desired. The voltmeter thus forms a shunt circuit between the two points. There is a current through the voltmeter proportional to the difference of potential between the two points to which it is joined. This current passing through the movable coil of the instrument deflects the pointer over the scale, but the latter is graduated to read not the current but the number of volts between the two binding posts of the voltmeter. In some instruments a strip of mirror is placed below the needle. When the eye is so placed that the image of the needle is hidden by the needle itself, the reading can be taken without the error due to parallax. Sometimes the pointer does not stand at the zero of the scale when there is no current through the instrument. When this is the case, the position of rest should be carefully noted and the observed reading corrected accordingly.

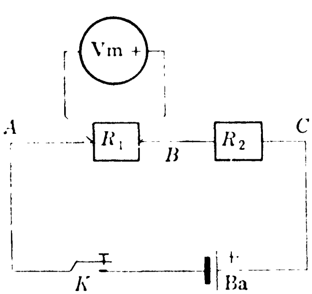


FIG. 7.—Connections for a voltmeter.

31. Fall of Potential in an Electrical Circuit.—For this exercise join a cell, two coils of several ohms' resistance, and a key in series. With the voltmeter measure the fall of potential over each coil, also over both together. Add a third coil and measure the fall of potential over each, also over all. Note where these last readings are the same as they were before and where they are different. Add a second cell and repeat the above readings. Note changes.

Join two of the coils in parallel, thus forming a divided circuit and allowing a part of the current to flow through each branch (see Fig. 6). Measure the fall of potential over each branch. Add a third coil in parallel with the other two and again measure the fall of potential over each.

This exercise should show, especially in connection with the preceding one, that the only thing common to several circuits in *parallel* is that each one has the same *fall of potential*. Hence a voltmeter is always joined in parallel with the coil over which the fall of potential is desired. The voltmeter indicates the fall of potential over itself; and if it forms one of the parallel circuits, the fall of potential over each one is the same as that indicated by the voltmeter.

Record the voltmeter readings as below:

FALL OF POTENTIAL IN AN ELECTRICAL CIRCUIT

Voltmeter		Fall of potential	Position of voltmeter
Zero	Reading		

32. Measurement of Resistance by Ammeter and Voltmeter. *First Method.*—To determine the resistance of a conductor, it is necessary only to measure with an ammeter the current flowing through it and to measure with a voltmeter the difference of potential between its terminals. In case the current is at all variable, the two instruments must be read at the same time, for Ohm's law applies only to simultaneous values of the current and voltage. The conductor, whose resistance R is to be measured, is joined in series with an ammeter Am, a key, a battery, and sufficient auxiliary resistance to keep the current from being too large. The electron current should leave the ammeter at the post marked +. Keeping the eye fixed on

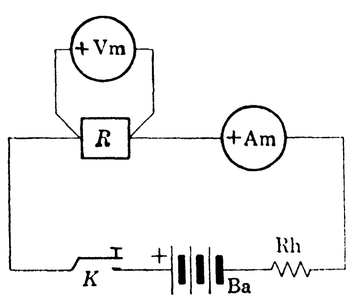


FIG. 8.—To measure a small resistance R.

the needle of the ammeter, close the key for a fraction of a second. If the deflection is in the right direction and is not too large, the key can be closed again and the value of the current read from the scale of the ammeter. Should the current be too large, the auxiliary resistance can be increased until the current is reduced to the desired value.

To measure the fall of potential over the conductor, its two terminals are joined to the binding posts of the voltmeter by means of additional wires. That terminal of the resistance at which the electron current leaves should be joined to the voltmeter post marked +. Close the key for an instant as before, keeping the eye on the voltmeter needle, and if the deflection is in the right direction and not too large, the reading of the voltmeter can be taken.

Now close the key and record simultaneous readings of the ammeter and the voltmeter. Do this several times, changing the current slightly by means of the auxiliary resistance before each set of readings. Compute the resistance of R from each set of readings by means of the formula

$$R = \frac{V}{I},$$

where V and I are the voltmeter and ammeter readings, corrected for the zero readings. The mean of these results will be the approximate value of the resistance.

A more exact value of R can be obtained by correcting the current as measured by the ammeter for the small current i that flows through the voltmeter. The current through R is, strictly, not I, but $I - i$. This, then, gives

$$R = \frac{V}{I - i}.$$

The value of the current i through the voltmeter can be computed. If the resistance of the voltmeter is S ohms,

$$i = \frac{V}{S}.$$

33. Measurement of Resistance by Ammeter and Voltmeter. *Second Method.*—This method differs from the first method by the position of the voltmeter. In the first method the ammeter measured both the current through R and the small current through the voltmeter, and therefore its readings were somewhat too high.

If the connections are made as shown in the figure, this error is avoided, as the current now passing through R is strictly the same as that measured by the ammeter. The voltmeter, however, now measures the fall of potential over both R and the ammeter, and therefore the resistances of both are measured together. The resistance of R is then found by subtracting the resistance of the ammeter from the measured amount. The formula then becomes

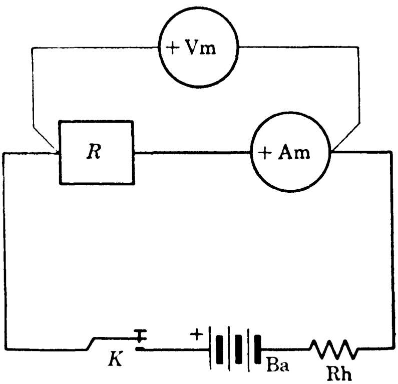


FIG. 9.—To measure a large resistance R.

$$R = \frac{V}{I} - A,$$

where A is the resistance of the ammeter.

This correction is easily made, and therefore this is the preferable method, except for very small resistances.

Measure the resistance of two coils and check results by also measuring them in series and in parallel. The measured resistances should be compared with the values computed from the formula—for series, $R = R_1 + R_2$, and for parallel,

$$R = \frac{R_1 R_2}{R_1 + R_2}.$$

Record the readings as follows:

TO MEASURE THE RESISTANCE OF..........

Coil measured	Ammeter reading	I	Voltmeter reading	V	Resistance	
					$\frac{V}{I}$	Corrected

34. Graphical Solution for Resistances in Parallel.—The equivalent resistance of two resistances in parallel is easily determined by a simple geometrical construction. Let two lines a and b be drawn at right angles to a base line PQ and at a convenient distance m apart. Let the lengths of these lines represent the two resistances on a convenient scale. Join the top of each line to the bottom of the other, as shown by the broken lines. From the intersection of these diagonals draw the line c, also perpendicular to the base line. The length of c gives the equivalent resistance of the parallel circuits on the same scale that was used in drawing a and b.

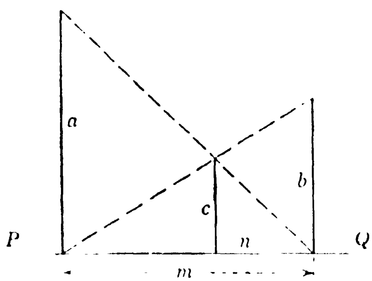


FIG. 10.—The resistance of a and b in parallel is shown by c.

This is easily shown. From similar triangles,

$$\frac{c}{a} = \frac{n}{m}$$

and also

$$\frac{c}{b} = \frac{m - n}{m} = 1 - \frac{n}{m} = 1 - \frac{c}{a},$$

from which

$$c = \frac{ab}{a + b}.$$

This diagram will thus show quickly the effect of connecting any two resistances in parallel.

35. Comparison of the Two Methods.—In each of the preceding methods for measuring resistance by means of an ammeter and a voltmeter, it is necessary to apply a correction to the observed readings in order to obtain the true value of the resistance being measured. It is the object of this section to inquire under what conditions these corrections are a minimum. In order to compare the two correction terms with each other, they will both be expressed in the form of factors by which the observed values are multiplied to obtain the true values.

In the first method the correction is applied to the ammeter reading, the true current through R being $I' - i$. The true value of the resistance is, then,

$$R = \frac{V'}{I' - i} = \frac{V'}{I' - \frac{V'}{S}} = \frac{V'}{I'\left(1 - \frac{V'}{I'S}\right)} = \frac{R'}{1 - \frac{R'}{S}} = R'\left(1 + \frac{R'}{S}\right)$$

approximately, where S is the resistance of the voltmeter and R' is written for V'/I', the uncorrected value of R.

In the second method the value of R is given by

$$R = \frac{V''}{I''} - A = R'' - A = R''\left(1 - \frac{A}{R''}\right),$$

where A is the resistance of the ammeter, and R'' is written for the uncorrected value of R in this method.

In the first method the correction factor is nearly unity when the resistance being measured is small. Other things being equal, a voltmeter having a high resistance S is better than a low-resistance instrument. In the second method the correction factor is near unity when the resistance being measured is large. The smaller the resistance of the ammeter, the better. Either method, thus corrected, should give the correct resistance of R.

36. To Find the Best Arrangement for Measuring Resistance with an Ammeter and a Voltmeter.—The division point between the two methods is at the resistance for which the correction factors R'/S and A/R'', are equal. For this resistance neither of the corrections is very large, and it is nearly true to write

$$R' = R'' = R$$

and

$$\frac{R}{S} = \frac{A}{R}.$$

The division point is, then, at

$$R = \sqrt{AS}.$$

This resistance is not a fixed value; it depends upon the resistances of the ammeter and the voltmeter that are used.

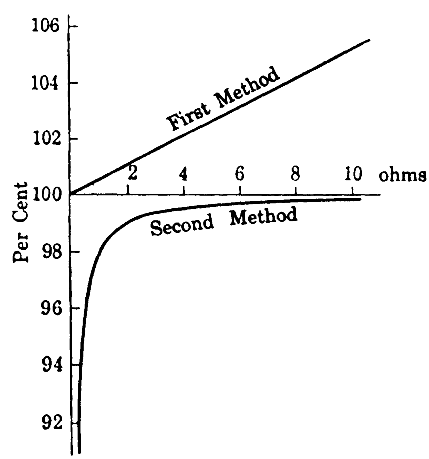


Fig. 11.—Showing the errors in the uncorrected values given by V/I in the two methods of measuring resistance.

For resistances smaller than this, the first method has the smaller correction. For larger resistances, the second method should be used. Whether the correction is applied or not, it should always be made as small as possible. If the proper method is selected, the correction will almost never be as large as 1 per cent.

The curves in Fig. 11 show the factor by which R' or R'' must be multiplied to give the true resistance when a 200-ohm voltmeter and a 0.02-ohm ammeter are used. It is seen that with a resistance of 2 ohms the corrections are equally large for each method, one giving uncorrected results that are 1 per cent too high, the other giving uncorrected results that are 1 per cent too low.

In measurements of this kind a larger error may be introduced by neglecting the temperature of the wire that is being measured. The resistance of a copper wire increases about 2 per cent for each 5° C rise in temperature, and unless the temperature of the wire is known more closely than 2° C, there is not much use in measuring the resistance more closely than 1 per cent.

37. Relation between Available Emf and Current.—The battery or emf to be examined is joined in series with a variable resistance, a key, and an ammeter, as shown in Fig. 12. By using various values of R, the current can be varied throughout its possible range, but R should not be reduced beyond the point that gives a current as large as can be measured with the ammeter. A voltmeter is joined in parallel with the resistance and key.

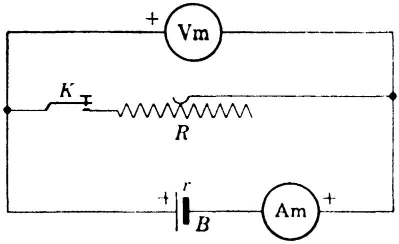


FIG. 12.—To measure the internal resistance of the battery B. Also to measure the available emf corresponding to the current drawn from the battery and the power expended in R.

When the key is closed, the current drawn from the cell is

$$I = \frac{E}{R' + r'},$$

where R' is the combined resistance of R and the voltmeter in parallel, and r' includes not only the resistance of the battery but also that of the ammeter. Clearing of fractions,

$$E = R'I + r'I.$$

The voltmeter measures $R'I$. Denoting this by E',

$$E = E' + r'I. \qquad (1)$$

The term $r'I$ is the fall of potential over the internal resistance of the battery (and the resistance of the ammeter if connected as in Fig. 12). E' $(= R'I)$ is the fall of potential over the entire external part of the circuit. Various names have been applied to this term E', such as "terminal emf," "terminal potential difference," "pole potential," "available emf," etc. It is that part of the total emf of the cell which is available for doing useful work. Its value, from Eq. (1), is

$$E' = E - Ir', \tag{2}$$

and from this it appears that the available emf is less as the current becomes larger.

The key should be kept closed as little as possible to avoid unnecessary polarization of the battery.

The observations can be recorded as follows:

RELATION BETWEEN AVAILABLE EMF AND CURRENT FOR A.....CELL

Ammeter		I, amperes	Voltmeter		E', volts
Zero	Reading		Zero	Reading	

38. Plotting the Curve.—From the values of E' and I a curve can be plotted, as in Fig. 13. The currents should have been chosen so that the plotted points will be well distributed along this curve. The curve shows that the available emf continually decreases as more current is drawn from the source, and it will decrease to zero if the current is sufficiently increased. This is true whether the current is supplied by a battery, a dynamo, or any other source, but it is not always safe to allow this current to flow if its value would be too large.

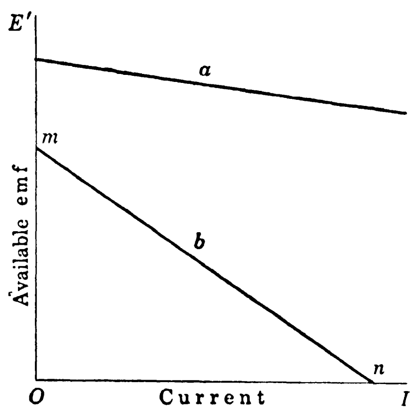


FIG. 13.—Relation between the available emf and the current drawn from the battery. Curve a shows a low resistance in the battery (a). Curve b indicates that the battery (b) has a high internal resistance (and also a smaller emf).

For the sake of comparison, curves should be drawn for different types of cells using at least one cell of low internal resistance. Determine from the curves the maximum current that each cell can furnish. Find one reason why an ammeter should not be joined to the poles of a cell, as is done with a voltmeter.

39. Maximum Current from a Battery.—The current from a given battery depends upon the resistance in the circuit, and by Ohm's law is

$$I = \frac{E}{R' + r'}.$$

The smaller the resistance of the circuit, the larger the current. The limit comes when the external resistance R has been reduced to zero, and then

$$I = \frac{E}{r'}$$

is the maximum current that can be drawn from the battery when it is connected with an ammeter as shown in Fig. 12.

The value of this maximum current is shown by the intercept on the horizontal axis in Fig. 13. For curve b this intercept On is shown in the figure. For curve a the value of the intercept must be estimated from the slope of the portion of the curve that is shown in the figure.

With a storage battery or a dynamo, where r is very small, this maximum current may be dangerously large. No attempt should ever be made to measure the current without first knowing approximately what its value may be, or else starting with considerable resistance R in the external circuit and cautiously reducing it while the ammeter is watched as the current increases. A single storage cell with internal resistance $r = 0.001$ ohm will supply over 1,000 amp on short circuit. If the ammeter is designed to measure as large a current as this, it will not be injured. But it would be rather hard on the cell and ruinous to an ammeter of smaller range.

40. Maximum Voltage.—The values of the available emf shown in Fig. 13 are given by the relation

$$E' = E - r'I.$$

For the particular point on this curve corresponding to $I = 0$, the ordinate is

$$E' = E.$$

Therefore the value of the full emf of the battery b is given by the intercept Om on the vertical axis in Fig. 13.

41. Internal Resistance of a Battery.—The internal resistance of a battery or other source of current is shown by the available emf curve, like a in Fig. 13, and its value is readily determined from the curve.

For one of the smaller values of the current I_1, the corresponding available emf is

$$E'_1 = E - r'I_1. \tag{3}$$

At a second point on this curve, where E'_2 and I_2 are the corresponding values,

$$E'_2 = E - r'I_2. \tag{4}$$

The unknown emf E of the battery can be eliminated by subtracting Eq. (4) from Eq. (3), giving

$$E'_1 - E'_2 = r'I_2 - r'I_1. \tag{5}$$

The internal resistance of the source, with its connections up to the points where the voltmeter, Fig. 12, is joined to the main circuit, has been denoted by r' in these equations, and solving Eq. (5) gives

$$r' = \frac{E'_1 - E'_2}{I_2 - I_1}. \tag{6}$$

Stated in words, this result is

$$\text{Resistance} = \frac{\text{change in potential difference}}{\text{change in current}},$$

which is a general statement of Ohm's law.

Thus by using two values of the available emf, corresponding to two widely different values of the current, it is not necessary to use Eq. (3) and measure the emf of the battery, which, as shown by Fig. 13, can hardly be done with a voltmeter unless r is very small.

Since the resistance r_a of the ammeter and other connections has been included in the value of r', this resistance must now be subtracted from the computed result to obtain the resistance of the battery alone. Thus

$$r = r' - r_a.$$

42. Slope of the Curve.—By looking at the steepness of curves like those shown in Fig. 13 one can tell which batteries have the greater internal resistance. The very steep curves correspond to batteries of large internal resistance, and the low-resistance batteries show curves that are nearly horizontal. When such a curve has been obtained for a given battery, the internal resistance is given by the negative slope of the curve, Eq. (6).

43. Measurement of a Resistance Containing an Emf.—The resistance of a circuit in which there is also an emf can be measured without the value of this emf being known. This problem might be the measurement of the internal resistance of a battery, as in the preceding articles, or it might be the determination of the resistance of a metallic circuit containing one or more emfs. If sufficient current can be drawn from the circuit, no other battery is required.

44. Useful Power from a Source.—Electrical power is measured by the product EI, where I denotes the current and E the fall of potential over the circuit in which the power is being expended. The unit of power is the watt, one watt being the product of one volt by one ampere.

When a battery is furnishing a current, the total power expended is supplied by the chemical reactions within the cell. Part of this power is expended in the external circuit, where it may be used in running motors or doing other useful work. The remainder is spent within the cell and goes only to warm the contents of the battery. In some cases the greater part of the energy is thus wasted within the cell.

The object of this experiment is to measure the power in the external circuit when various currents are flowing. The cell is joined in series with an ammeter and a resistance which will carry the largest current that may be used, as shown in Fig. 12. A voltmeter measures the fall of potential. Probably there will be found a point beyond which the useful power decreases even though the current is made larger. Since the current is proportional to the amount of chemicals used up in the cell, it will be well to express the results as a function of the current. This is best done by means of a curve, using values of the current for abscissas and the corresponding values of power for the ordinates.

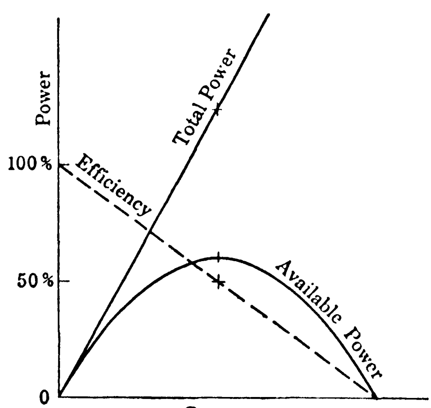


FIG. 14.—Power from a battery.

The curve showing the total power EI supplied by the cell can also be plotted. The ratio of the ordinates of these two curves, that is, the ratio of the useful power to the total power, gives the efficiency of the cell.

The efficiency can also be plotted as a curve on the same sheet with the other two curves.

Record the observations as follows:

USEFUL POWER FROM A..............CELL

$R = \frac{E'}{I}$	Ammeter		I, amperes	Voltmeter		E', volts	$W = E'I$, watts
	Zero	Reading		Zero	Reading		

Problem.—How much power is expended in an ammeter of 0.01 ohm resistance if it is connected to a storage battery of 10 volts and 0.01 ohm internal resistance? Why is it best not to try the actual experiment in the laboratory?

45. Maximum Amount of Available Power.—The amount of available power corresponding to a given current OC is shown graphically in Fig. 15 by the area of the rectangle $OABC$, whose sides are corresponding values of E' and I. When I is small, the area of this rectangle is small, increasing as I increases. After the mid-point is reached the relative decrease of E' is greater than the relative increase in I, and the area of the rectangle decreases to zero. Plotting the areas of the rectangles ($= W$) against the corresponding values of the current gives the curve of available power (Fig. 14).

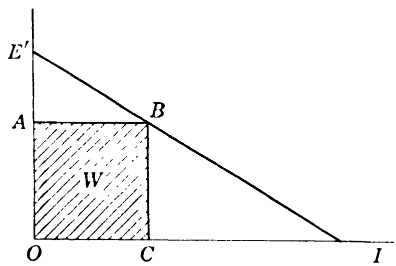


FIG. 15.—Showing the amount of available power W corresponding to a given current ($= OC$).

At the point where the power W is a maximum, the curve between W and I becomes parallel to the I axis, and in the notation of the calculus,

$$\frac{dW}{dI} = 0.$$

The relation between W and I is

$$W = E'I,$$

where both E' and I are variables. Substituting for E' its value in terms of I from Eq. (2), Art 37, gives, with the ammeter removed,

$$W = EI - I^2r,$$

and the derivative of this is

$$\frac{dW}{dI} = E - 2Ir.$$

For a maximum value of W, this expression is equal to zero, which is the case when the current is

$$I = \frac{E}{2r}.$$

When the current has this value, the battery is delivering more power than when the current is larger or smaller. Since for this current the resistance of the entire circuit is $2r$ and that of the battery is r, it follows that the resistance of the external circuit is also equal to r. Therefore the power *delivered* by the cell is a maximum when the external resistance is equal to the internal resistance. For power from a distant source delivered over a long line, all the resistance up to the point where the power is delivered should be counted as "internal" resistance.

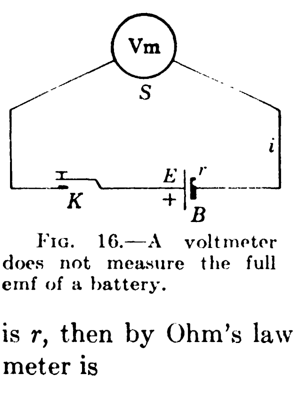


FIG. 16.—A voltmeter does not measure the full emf of a battery.

46. Why a Voltmeter Does Not Measure the Full Emf of a Battery.—When a cell B is joined to a voltmeter, the reading of the latter strictly does not give the emf of the cell. This discrepancy is small for cells of low internal resistance, but in many cases it cannot be neglected. If the resistance of the voltmeter is S and that of the cell is r, then by Ohm's law the small current through the cell and voltmeter is

$$i = \frac{E}{S + r}.$$

From this the emf of the cell is

$$E = Si + ri,$$

where Si is the fall of potential over the external circuit, in this case the voltmeter. Since a voltmeter measures only the fall of potential over its own resistance, the reading of the voltmeter is not E but Si, which is less than E by the amount ri. This term can be neglected when r is small, but not otherwise, unless i can be made very small.

In case r is known, a correction may be added to the voltmeter reading to obtain the value of E. If the above equation is written in the form

$$E = Si\left(1 + \frac{r}{S}\right) = E' + \frac{r}{S}E',$$

it is seen that the last term can be added to the voltmeter reading E' to obtain the full value of E.

47. How a Voltmeter Can Be Made to Read the Full Emf of a Battery.—In this method no current is taken from the battery, and therefore its full emf can be measured. The battery B is connected to a voltmeter as in Fig. 17, and then a second circuit is formed, consisting of the voltmeter, another battery Ba larger than B, and a variable resistance R.

Since there is no key in this second circuit, the voltmeter stands deflected and its reading V indicates the difference of potential between its terminals. When the key K is closed, the current through it may be either in one direction or in the opposite direction, or it may be zero, depending upon the value of V. If V is small, the current through K will be in the direction determined by the cell B, and the voltmeter will carry the combined current, $I + i$. If the potential difference V is large, it will cause a current to flow through the cell B in opposition to its emf E. In this case the main current I from the battery Ba divides at the voltmeter and flows through the two branches in parallel, just as though B were not present, except that the current in the lower branch is smaller than it would be with B absent.

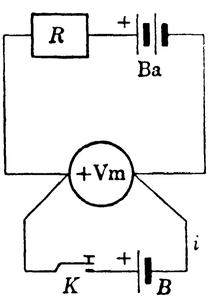


FIG. 17.—To measure the emf of B.

If i denotes the value of this reverse current through B, the voltmeter current is $I - i$. The change in the voltmeter current from I to $I + i$, or to $I - i$, when the key is closed, will cause a change in the voltmeter reading, and no other ammeter is needed to tell the direction of the current through B.

Since the current through B can be in either direction, there is one point when this current will be zero. This will be when the tendency for the current to flow through B and K, due to the potential difference V across the voltmeter, is just balanced by the tendency for the current to flow in the other direction due to E. For this case of no change in the voltmeter deflection when K is opened or closed, there is no current through B, and $E = V$. The value of E can now be read from the voltmeter.

Measure in this way the emf of several cells and compare the results obtained in each case with the readings of the voltmeter when it is used alone.

The readings may be recorded as follows:

Name of cell	Voltmeter readings			E
	Zero	Alone	With aux. battery	

48. Measurement of Current by a Voltmeter and Shunt.—When a current I flows through a resistance R, the fall of potential over R is $E = IR$, which is in accordance with Ohm's law and the definition of the term "fall of potential." In Art. 32 both E and I were measured and the value of R was then computed. When R is known, the experiment can be reversed and by measuring E the value of I can be computed. Thus a voltmeter may be used to measure currents in place of an ammeter. The arrangement is shown in Fig. 18.

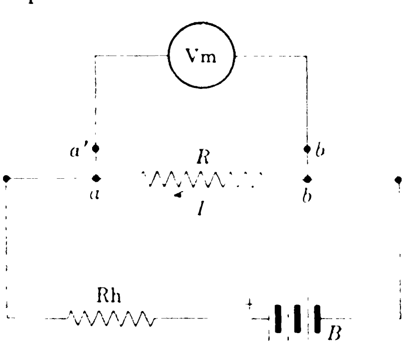


FIG. 18.—Measurement of current through *Rh* by a known resistance and a voltmeter.

Usually the resistance that would be suitable for this purpose is less than an ohm. Therefore any variation in the position of the voltmeter connections at a and b would cause considerable change in the resistance between these points. It is best to have these connections soldered fast and to let the voltmeter connections be made at the auxiliary points a' and b', where a little resistance, more or less, in the voltmeter circuit will be inappreciable.

Such shunts are often made having a resistance of 0.1, 0.01, 0.001 ohm or less. The current through the shunt is then 10, 100, 1,000 or more times the voltmeter reading. If the small current through the voltmeter is enough to be counted, it should be added to the current through the shunt to give the value of the total current that is measured by this arrangement (see also Fig. 98, and Arts. 151 and 196).

This principle is also used in the construction of ammeters for measuring large currents. The greater part of the current is carried by a shunt of low resistance, while the delicate moving coil carries only a small current and thus in reality acts as a sensitive voltmeter. The numbers on the scale, however, instead of reading volts are made to give the corresponding values of the currents passing through the instrument.

49. Potential Divider.—It is often necessary to use only a part of the given emf. This may be because a large emf is to be measured by a low-reading instrument, or it may be that a small fraction of a volt is required for use in some experiment. For this purpose, the source E is joined to a resistance AB that is large enough not to draw too much current from E or to become overheated itself. C is a sliding contact that can be moved from A to B, dividing the resistance in two parts, P and Q. The fall of potential between A and C is

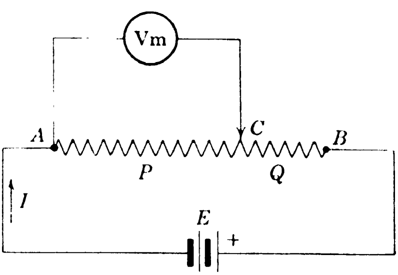


FIG. 19.—Showing the use of a potential divider AB.

$$v = PI,$$

and this value can be varied from zero to the full amount as desired by moving the sliding contact C.

A resistance AB that can be thus divided in various ratios is called a "potential divider." The emf E is divided into two parts, PI and QI, and by moving C, PI can be made any desired fraction of E (see Art. 51).

The same effect can be obtained by using two resistance boxes for P and Q. By changing the ratio of the resistances in P and Q, the fall of potential across AC can be varied.

50. To Use a Potential Divider.—Join the ends of a sliding rheostat to a constant emf. Connect a voltmeter to the sliding contact and to one end of the rheostat. Observe how the reading of the voltmeter varies as the sliding contact is moved from one end to the other. Note that moving the sliding contact does not affect the amount of current that is drawn from the battery, as would be the case when the rheostat is used as a variable resistance.

In the same way, the potential difference between A and C can be used wherever a fraction of the emf of E is desired.

51. To Compute the Ratio of a Potential Divider.—The arrangement in Fig. 20 shows the potential divider PQ connected across the battery B. The total fall of potential over $P + Q$ is V, and this is measured by the voltmeter Vm, but only the part PI is applied to the circuit of $G + R$.

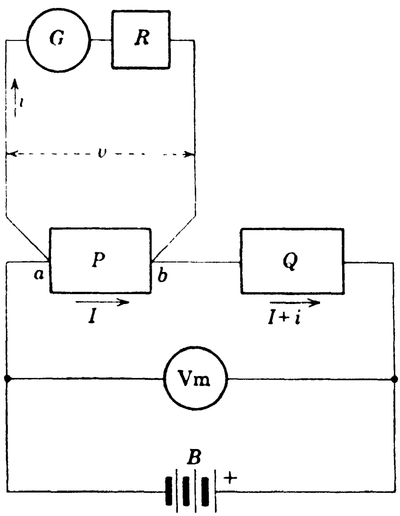


FIG. 20.—A small voltage v is obtained from the potential divider PQ.

Approximate Formula.—When the current i through $G + R$ is negligibly small compared with the current I through the potential divider, the voltage v across P, which is applied to the circuit $G + R$, is readily computed by the relation

$$v = PI = P\frac{V}{P + Q} = \frac{P}{P + Q}V.$$

This approximate value is often sufficiently accurate.

Exact Formula.—In case the exact value of v is required, it is necessary to consider the small current i. If this is comparable with the current through P, the correction will be appreciable. The combined resistance of P and $(G + R)$ in parallel is

$$p = \frac{P(G + R)}{P + (G + R)},$$

and this is the effective resistance between a and b, Fig. 20.

The voltage that is applied to the circuit $G + R$ is equal to the fall of potential between a and b, which is

$$v = p(I + i) = p\frac{V}{p + Q} = \frac{p}{p + Q}V,$$

where the value of p is computed by the formula given above.

52. Measurement of a High Resistance by a Voltmeter Alone.—This method, a modification of the ammeter and voltmeter method for the measurement of moderate resistances, is based on the fact that a voltmeter is really a very sensitive ammeter. It can be used as an ammeter whenever the high resistance in series with the moving coil will not interfere with the desired measurement of current.

The voltmeter is joined in series with a battery and the high resistance H to be measured. In this way the voltmeter serves as an ammeter to measure the current through the circuit. The reading V of the voltmeter gives the fall of potential over its own resistance S or, in symbols,

$$V = Si.$$

When the entire circuit is considered, the value of the current is given by the expression

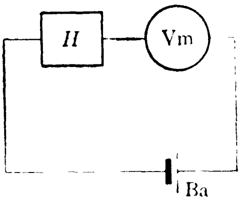


FIG. 21.—To measure the high resistance H.

$$i = \frac{E}{H + S}.$$

Eliminating i from these two equations and solving for H gives

$$H = S\frac{E - V}{V}.$$

The value of E, the emf of the battery, is easily measured by the same voltmeter by connecting it directly to the battery. A key arranged to short-circuit the high resistance will readily change the voltmeter from its position as an ammeter to that of a voltmeter.

When the emf of the source is too large to be measured by the voltmeter, or when for any other reason it cannot be determined, the resistance can still be measured. In the circuit shown in Fig. 21, the current is

$$i = \frac{V}{S} = \frac{E}{H + S}.$$

Now add a large known resistance R in series with H and the voltmeter. Then the current through the circuit will be

$$i' = \frac{V'}{S} = \frac{E}{H + S + R}.$$

Dividing one equation by the other gives

$$\frac{V}{V'} = \frac{H + S + R}{H + S},$$

and

$$H = \frac{V'(S + R) - VS}{V - V'}.$$

The observations can be recorded as follows:

To Measure the Resistance of..........

Vm. zero = Vm. resist. = ...ohms

Name of object measured	V	E	R

53. To Make a High-reading Voltmeter from a Low-reading Instrument.—Voltmeters are made in different ranges. Some will measure voltages over the range of 0 to 3 volts; others over the range 0 to 15 volts; or 0 to 150 volts, etc. In each of these instruments the moving system is the same and requires the same amount of current to operate it. The principle difference lies in the resistance of the voltmeter. If the resistance is 300 ohms, an emf of 3 volts will give a current of 0.01 amp through the voltmeter, and this is sufficient to deflect the movable pointer to the upper end of the scale. The range of this voltmeter would be 0 to 3 volts.

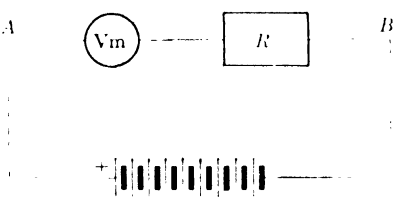


Fig. 22.—A large resistance R in series with a low-reading voltmeter Vm makes the combination AB into a high-reading voltmeter.

If the same instrument had a resistance of 1,500 ohms it would require 15 volts to give a current of 0.01 amp and deflect the pointer to the high end of the scale. The range would be 0 to 15 volts, and the instrument would be called a "15-volt voltmeter." If the resistance were increased to 15,000 ohms, the voltmeter could measure up to 150 volts.

Problems

1. If a 3-volt voltmeter has a resistance of 400 ohms, how much more resistance must be placed in series with the voltmeter in order to make it into a 15-volt voltmeter? *Ans.* 1,600 ohms.
2. When the 15-volt voltmeter of Prob. 1 is deflected to the high end of its scale, how many volts are distributed over the added resistance? *Ans.* 12 volts.
3. In Prob. 2, how many volts are applied to the original voltmeter? *Ans.* 3 volts.
4. How much current is required to deflect the 3-volt voltmeter of Prob. 1 to the high end of its scale? *Ans.* 0.0075 amp.

5. How much current is required to deflect the 15-volt voltmeter of Prob. 1 to the high end of its scale? *Ans.* 0.0075 amp.

6. Let the 3-volt voltmeter of Prob. 1 be connected to a 3-volt battery of negligible resistance, and let a shunt of 400 ohms be placed in parallel with the voltmeter. How much current is drawn from the battery? *Ans.* 0.015 amp.

7. In the arrangement of Prob. 6, how much current will flow through the voltmeter? *Ans.* 0.0075 amp.

8. In the arrangement of Prob. 6, what voltage will the voltmeter read? *Ans.* 3 volts.

9. Let the 400 ohms of Prob. 6 be placed in series with the voltmeter and the 3-volt battery. What will be the reading of the voltmeter? *Ans.* 1.5 volts.

54. To Make a High-reading Ammeter from a Low-reading Instrument.—An ammeter measures the current that passes through it. Such an instrument is placed in series with the circuit in which the value of the current is desired. The moving system and general appearance of an ammeter are like those of a voltmeter, and when the current to be measured is small it passes directly through the moving coil and causes the deflection of the pointer over the scale. Adding more resistance in series will reduce the value of the current, but the same value of the current will deflect the pointer to the same place on the scale of the ammeter.

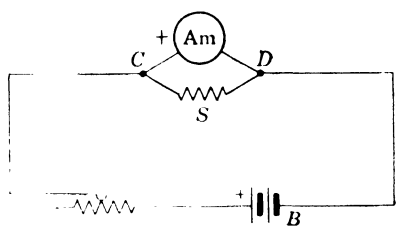


FIG. 23.—A small resistance S connected in parallel with a low-reading ammeter Am makes the combinations CD into a high-reading ammeter.

When it is desired to measure currents that are larger than are indicated by the upper end of the ammeter scale, it is necessary to arrange the circuit so that the ammeter itself measures only a certain fraction of the total current. Thus if a shunt of 0.05 ohm is placed in parallel with an ammeter whose resistance is 0.05 ohm, the current will divide, half passing through the ammeter and the other half through the shunt. The current measured by the ammeter should be multipled by 2 to give the value of the total current. This combination of an ammeter with a shunt of equal resistance makes an effective ammeter of twice the range of the ammeter alone. If a shunt of smaller resistance is used, a larger part of the current will pass through the shunt, and the range for the total current measured by the ammeter and shunt is increased. Ammeters for measuring large currents are thus provided with a built-in and permanent shunt of low resistance. The uncertain resistance of the contact connections of the shunt and ammeter will introduce uncertainty in the fraction of the current that will pass through the ammeter branch of the circuit, unless these connections are soldered permanently in a fixed position.

Problems

1. A certain ammeter has a range of 0 to 1.5 amp and a resistance of 0.05 ohm. When it is a part of a circuit of 10 ohms in which the current is 1.26 amp., what reading is to be expected on the scale of the ammeter? *Ans.* 1.26 amp.
2. When the ammeter of Prob. 1 is in series with a 100-ohm circuit where the current is 1.26 amp, what reading is to be expected on the scale of the ammeter? *Ans.* 1.26 amp.
3. When a shunt of 0.01 ohm is placed in parallel with the ammeter of Prob. 1, what reading is to be expected when the main current is 1.26 amp? *Ans.* 0.21 amp.
4. What is the range of the combined ammeter and shunt of Prob. 3? *Ans.* 0 to 9 amp.
5. What is the range of the combined ammeter and shunt of Prob. 3 when the ammeter branch of the combination has been increased by the addition of 0.16 ohm? *Ans.* 0 to 33 amp.

55. Time Test of a Cell.—The time test of a cell is designed to show how well the cell can maintain a current, how effective is the action of the depolarizer, and the rapidity and extent of the recovery of the cell when the current ceases. Such a test usually continues for an hour, and the results may best be shown graphically by means of curves. These curves should show the value of the emf of the cell at each instant during the test, the available emf, the current, and the internal resistance of the cell. After the current is maintained for an hour, the circuit is opened and the cell is allowed to recover. The curves should show the rate and extent of this recovery.

Of course it is evident that such a continued test may prove rather severe for cells intended for only a few minutes' use at one time. On the other hand, cells that make a good showing under the test might not be the best for long-continued intermittent service. Nevertheless, such a test gives the most information regarding the behavior of the cell that can be obtained in the short space of 2 hr.

The setup for making a time test is the same as that for measuring the internal resistance of the cell. The battery circuit remains closed all the time, however, except when it is opened for an instant to measure the emf of the cell. With the proper preparation beforehand it is not difficult to observe all the necessary data and record them in a neat and convenient form.

Readings should be taken once a minute. Practice in doing this should be obtained by measuring the emf of the cell before any current is drawn from it. When ready to commence the test proper, close the circuit through 5 or 10 ohms, or about as much resistance as the internal resistance of the cell, and as soon thereafter as possible take the first reading for the available emf. One minute later, open the circuit just long enough to take a reading of the emf of the cell. Thus every 2 min the available emf is recorded and on the intermediate minutes the value of the total emf is measured.

From these data two curves are plotted, one showing the variation in the emf of the cell during the test and the other showing the same with respect to the available emf. The internal resistance is computed at 5-min intervals, the values of E and E' being taken from the curves. The current is measured by the ammeter, or it may be computed from the values of R and the available emf.

After the first hour of the test, the battery key is changed so as to keep the circuit open, except for an instant each minute when it is closed long enough to read the voltmeter. Since the voltmeter draws some current from the cell, and as it is sometimes difficult to obtain simultaneous readings of the ammeter and voltmeter at the instant desired, more accurate results are possible when the measurements are made by the condenser and ballistic-galvanometer methods described in the next chapter. The keys can be worked by hand, and if proper care is exercised, good results may be expected. Better results may be obtained by using a special battery-testing key or a pendulum apparatus that will close and open the keys in precisely the same manner each time.

During the second hour the battery will recover more or less completely from the effects of polarization, and at the end of the 2-hr test the value of E should be about the same as at the beginning. The recovery curve may be plotted backward across the sheet containing the other curves, thus showing very clearly the extent of the recovery.

The observations may be recorded as below.

TIME TEST OF A.......... CELL

Time of day hour minute	E	E'	I	r

CHAPTER II

BALLISTIC-GALVANOMETER AND CONDENSER METHODS

56. A Voltmeter without Current.—The voltmeters discussed in Chap. I are essentially nothing but sensitive ammeters with a fixed resistance in series with the moving coil. Each one draws some current from the source whose emf is being measured. In most cases this is immaterial, but sometimes even a small current changes the emf that it is desired to measure.

In the present chapter we are to consider a voltmeter that draws no current and therefore is free from the latter objection. In a sense, then, this will be more like a real voltmeter, but a more important consideration is that the condenser methods do not change the potential differences that are being measured. It will also be possible to measure values of emf that do not exist long enough to read on a voltmeter.

This chapter might appropriately be combined with Chap. XIV, but it is introduced here because the manipulation of these methods is very simple, and with a little precaution there is not much danger of injuring valuable apparatus through the inexperience of the investigator. Articles 375 to 382 should be read for a more complete understanding of the use of condensers.

57. Transfer of Electrons.—When two insulated conductors A and B, Fig. 24, are connected together through a battery, a current of electrons will flow out of one conductor through the battery and into the other conductor until the difference of potential thus produced is sufficient to balance the emf of the battery. If the battery is increased, more electrons will flow from the first conductor to the other, and the only limit to the quantity of electrons that can thus be taken away from one conductor and added to another is the limit of the emf of the battery and the completeness of the insulation of the conductors.

When the conductors are near together, the electrons that are transferred from one conductor to the other are still so near to the place from which they started that, in effect, they partially neutralize the lack of electrons in the conductor they have left. This allows more electrons to be transferred from B to A before the potential difference between the conductors becomes equal to the emf of the battery.

An arrangement of conductors whereby a greater number of electrons can be transferred by a given emf than would be possible with the conductors separated is called a "condenser."

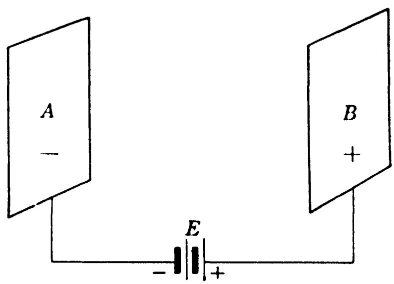


FIG. 24.—Electrons leave B and accumulate on A until the difference of potential has become as large as the emf of the battery.

58. Capacitance.—The use of this term has been growing in favor, and with this spelling it stands with resistance and inductance in expressing a property of a conductor. It means precisely the same as the older term "capacity," with the advantage that it does not suggest the misleading notion "all it can hold" (see Art. 62).

59. Condensers.—In order to make the capacitance as large as possible, condensers are constructed with broad sheets of tin foil placed as near as possible to other similar sheets. Actual contact is prevented by thin layers of mica, glass, or paraffined paper. Large capacitances are formed by building up alternate sheets of tin foil and dielectric, every other sheet of tin foil being connected to one terminal post and the intermediate ones to the other terminal (Fig. 25).

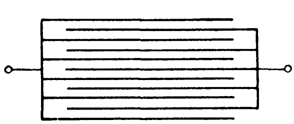


FIG. 25.—The tin foil plates of a condenser.

The best condensers and those intended for standards are made with thin sheets of mica as insulation between the sheets of tin foil. Very good condensers are made with paper insulation, the whole pressed firmly together and boiled in paraffin until all the air and moisture have been expelled, when the whole is allowed to solidify under pressure.

60. Meaning of a Charge Q in a Condenser.—A charge Q in a condenser means that there is a charge of $-Q$ coulombs on one set of plates and an equal, though positive, charge of $+Q$ coulombs on the other set of plates. The surplus of electrons on one conductor is sometimes called a "negative charge of electricity," and corresponding to this, the other conductor, which has lost the electrons, is said to possess a "positive charge of electricity." The discharge of the condenser consists of the electrons, $-Q$, flowing through the discharging circuit to the other set of plates and thus supplying the lack of electrons, $+Q$, thereon. As explained in Art. 3, this flow of electrons is an electron current, and the direction of the electron flow is to points of higher potential.

The positive charge, consisting of protons, is in the nuclei of the atoms, and these cannot move without taking the atoms with them. There is no flow of atoms along a copper wire.

61. A Coulomb. *The Unit of Quantity.*—When there is a current in a wire, the drifting of electrons along the wire carries a quantity of them past a given place in the circuit each second. The quantity Q thus transferred in t sec is

$$Q = It.$$

When the current is 1 amp, the quantity transferred across a given section of the circuit each second is 1 coulomb.[1] This name for the unit of quantity was adopted in 1881 by the Paris Congress of Electricians.

The definition of a coulomb of electrons follows at once from the definition of the ampere (Arts. 5 and 191).

Definition.—One coulomb of electrons is the quantity transferred by an ampere in one second.

This quantity is about 6.242×10^{18} electrons.

62. Capacitance of a Condenser.—The larger the area of tin foil in a condenser and the nearer the sheets are to each other, the greater will be the number of electrons transferred by a given emf E. If the electrons are measured in coulombs,[2] the quantity Q transferred from one set of plates to the other is

$$Q = CE,$$

where C denotes the capacitance of the condenser expressed in units of such magnitude as will make this a true equation.

The capacitance of a condenser, $C = Q/E$, is thus measured by the number of *coulombs per volt* required to charge it.

[1] Charles Augustin Coulomb (1736–1806), French natural philosopher.

[2] An electrical quantity of 1 electron is inconceivably small, yet its value has been accurately measured by various methods and found to be

$$e = 16.02 \times 10^{-20} \text{ coulomb}.$$

If ordinary electrical charges were measured by counting the number of electrons that are thus collected together, the resulting numbers would be inconveniently large. If the electrons constituting a negative charge of 1 coulomb were placed in a row, 40,000 in each millimeter, the line would reach from the earth to the sun.

Therefore, instead of counting the individual electrons, we shall count about 6.242×10^{18} of them at each clip and express the result in coulombs.

63. A Farad.—In the practical system of units the unit of capacitance is the *farad, which is the capacitance of a condenser that is charged to a potential difference of one volt by one coulomb.*

The farad[1] is far too large a capacitance for ordinary use, and it is customary to express capacitances in terms of a smaller unit, the *microfarad*, which, as its name indicates, is one-millionth of a farad.

Problem.—A 3-μf paraffined-paper condenser is about 1 ft square and 1 in. thick. How large a pile of such condensers would have a capacitance of 1 farad? A 30-ft cube?

64. Ballistic Galvanometer.—A ballistic galvanometer is one in which the moving system, whether coil or magnet, is made comparatively heavy and massive so that it will swing slowly. Such a galvanometer is designed to measure not steady currents, like an ammeter, but transient currents that may exist for only a very small fraction of a second. Indeed, the duration is so short that it is customary not to speak of them at all as currents but only to consider the total quantity of electrons that has passed.

Thus there are two reasons for having a slow-moving galvanometer. In the first place, it is most sensitive when in the position of rest and therefore should not turn appreciably from this position before all the electrons have been able to flow through the galvanometer and exert their full effect in turning the coil. In the second place, the coil gives one kick and then settles back to the position of rest again, and the only thing that can be measured is the maximum deflection which it attains. Therefore it must move slowly enough to enable one to read the deflection at the end of its swing.

65. Damping of a Galvanometer. *Critical Damping.*—After the galvanometer has given its deflection, the very fact that the moving system is massive and at the same time can move freely, which is essential for a good ballistic galvanometer, makes it very slow in coming to rest again. It will swing back and forth many times until its energy has been used up in friction against the air and in other ways, when it will finally settle down at rest. That the swings decrease at all is due to the *damping* of the motion, as this effect of friction and other factors is called.

If the damping is increased, as would be the case if the coil were surrounded with oil, the swings would decrease more rapidly and the coil would quickly come to rest. Such damping might be so great

[1] Michael Faraday (1791–1867), English physicist.

that there would be no swings, and the coil would slowly creep back to the position of rest, possibly taking longer to do so than when it is allowed to swing freely.

Thus it is seen that there is some intermediate value of the damping that would allow the coil to swing back to rest not too slowly and yet bring it to rest without its swinging past the position of rest. This value of the damping is the *critical damping*, and with this damping the coil is brought to rest in the minimum time.

The most convenient way to increase the damping of a ballistic galvanometer is to join its terminals by a low resistance, as shown at S, Fig. 26. Because of the motion of the coil in a strong magnetic field, an induced current will flow through S, and the supply of energy in the coil is quickly dissipated as heat by the electron current in the wire. Often S is adjusted to such a value as to give critical damping. The galvanometer can then be used once a minute, or oftener if desired.

The effect of the shunt in reducing the deflections is discussed in the next chapter.

66. Use of a Ballistic Galvanometer and Condenser.—When the poles of a battery are joined to the plates of a condenser, the condenser becomes charged, as explained above. The amount of this charge depends upon the emf E of the battery and the capacitance C of the condenser, being given by the relation

$$Q = CE.$$

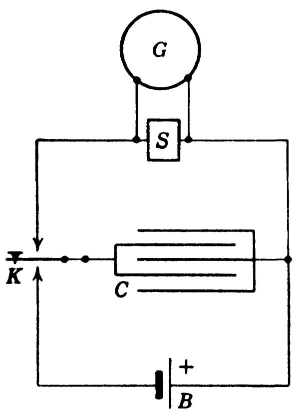


FIG. 26.—Use of a condenser.

When the condenser is connected to a ballistic galvanometer, the charge Q in the condenser passes through the galvanometer and produces a deflection d, the relation being

$$Q = kd,$$

where k is the constant of the galvanometer.

If this operation has been carefully arranged, so that there has been no leakage of the charge elsewhere, it is evident that the quantity that has been discharged through the galvanometer is the same quantity that was put into the condenser by the battery. That is,

$$CE = kd.$$

This is a very useful relation, since it can be used to determine the value of any one of the factors involved when the other three are known or can be measured.

The best arrangement for using a ballistic galvanometer with a condenser is shown in the figure, where G represents the galvanometer and C the condenser, with the battery at B. These are connected through the key as shown. *Note that the tongue of the key is connected to the condenser and to the condenser only.* This precaution is necessary in order that by no possibility can the battery ever be joined directly to the galvanometer. Arranged as shown, when the key is depressed, the condenser is joined to the battery and becomes charged. When the key is raised, the condenser is joined to the galvanometer, through which the charge passes, producing a deflection.

67. To Determine the Constant of a Ballistic Galvanometer.—For deflections that are not too large, the maximum swing is proportional to the quantity of electrons passed through the galvanometer, and the proportionality factor is called the "constant" of the galvanometer. It is determined by discharging through the galvanometer a known quantity and noting the resulting deflection. Then

$$Q = kd,$$

where k is the desired constant. If k is known, for a galvanometer, any other quantity of electrons can be measured by sending it through the galvanometer and noting the corresponding deflection.

The galvanometer, condenser, and battery are connected as shown in Fig. 26. The scale and telescope should be adjusted so that both the divisions and the numbers on the scale are distinctly seen in the telescope. The eyepiece must be focused on the cross hair of the telescope, which should appear very clearly defined. When focusing the telescope upon the image of the scale as seen reflected from the mirror of the galvanometer, one must remember that the image is not on the surface of the mirror but lies as far back of the mirror as the scale is in front of it.

After the setup has been tried and found to work correctly with an old dry cell for B, a standard cell whose emf is much more exactly known may be substituted for it. Ten independent determinations of d should be made, with the resultant mean value used in the computation for k.

The relation given in Art. 66 is

$$CE = kd.$$

In the present case the condenser used must be one of known capacitance. Likewise, E must be known as exactly as the value of k is desired. And d is measured. Hence the computed value of k is

$$k = \frac{CE}{d}.$$

If the value of C is given in microfarads, k will be expressed in *microcoulombs per scale division.* A microcoulomb is one-millionth of a coulomb and is the quantity of electrons that is represented by a current of one ampere flowing for one-millionth of a second.

The observations can be recorded as follows:

Galvanometer			Mean deflection at ____ meter	C	E	k
Zero	Reading	Deflection				
k for the scale 1 meter from the mirror =						

68. The Standard Value of k.—When the mirror of a galvanometer is turned through a given angle, the magnitude of the corresponding deflection depends upon the distance of the scale from the mirror, being larger for the greater distance. This makes little difference for a given galvanometer with a fixed scale, but for the comparison of different galvanometers or for the same galvanometer with the scale at different distances, the value of the constant k has little meaning unless this factor is taken into account. Usually this is done by correcting the observed value of k to the value it would have when the deflection is read on a scale at 1 meter from the mirror.

69. Comparison of Emfs by the Condenser Method.—The arrangement of a condenser with a ballistic galvanometer may be conveniently used to measure the emf of a battery or any other difference of potential. It thus serves as a voltmeter, with the advantage over the ordinary voltmeter that it measures the total emf of the battery, no matter what the internal resistance of the latter may be.

The setup is arranged as shown in Fig. 27. When the key is depressed, the condenser is charged, and when the key is raised, it is discharged through the galvanometer. It is immaterial whether the key works this way or whether the condenser is charged when the key is up and is discharged by depressing the key. It is absolutely necessary, however, that the tongue of the key be joined to the condenser, as is shown in Fig. 27.

When the battery is connected to the condenser, a current flows until the difference of potential between the two sets of condenser plates is equal to the emf of the battery. When this current is flowing, the available emf (see Art. 37) is only

$$E' = E - Ir;$$

but as the charge in the condenser increases, the current through the battery becomes less and less, and when it reaches zero,

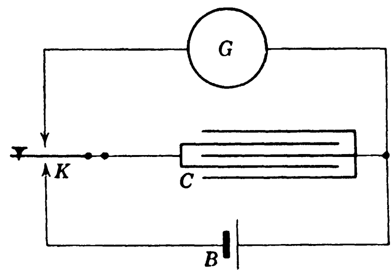


FIG. 27.—To measure the emf of B.

$$E' = E.$$

Therefore the final difference of potential impressed on the condenser is equal to the total emf of the battery, and thus this emf can be measured.

The final charge in the condenser is

$$Q = CE = kd.$$

Solving this for E gives

$$E = \frac{k}{C} d.$$

The factor k/C can be determined by using a cell of known emf E_s and observing the corresponding deflection d_s. Then

$$E_s = \frac{k}{C} d_s \qquad \text{or} \qquad \frac{k}{C} = \frac{E_s}{d_s},$$

so that, finally,

$$E = \frac{E_s}{d_s} d.$$

After this coefficient of d has been determined once and for all, the emf of any cell can be measured quickly and easily by observing the corresponding deflection of the galvanometer.

Inasmuch as the reading must be caught quickly at the end of the swing, it will be best to take several trials and to use the mean deflection for computing the value of E. The readings may be recorded as follows:

Name of cell	Galvanometer			Mean deflection	$\frac{k}{C}$	Emf of cell
	Zero	Reading	Deflection			

70. Taking the Reading.—The correct value of the deflection usually is not obtained on the first reading. The observer notes the end of the swing, perhaps to the nearest large division of the scale. Then on the second or third repetition of the deflection, knowing about where it will end, he takes the final reading to the nearest tenth of the smallest scale division.

It is better to read the position of rest (to the nearest tenth of a small scale division) than to try to set the scale on a given line, which usually results in other uncertainties. If the scale is moved at all, it should be by several divisions, so that the next observations of the zero point and the reading at the end of the swing will be taken on new parts of the scale and not be biased by an attempt to duplicate the first readings. The zero-point reading should be taken before the galvanometer is deflected. The difference between the readings at the end of the swing and at the zero point is called the "deflection."

71. Direction of Deflection.—The galvanometer should be deflected a few times, always in the same direction, before it is finally read. It is well to have the galvanometer critically damped (see Art. 65) so that it will not go past the position of rest, or zero point, on its return swing. The frame, wire, and even the silk insulation are all slightly magnetic, and when the coil is turned one way in the strong magnetic field of the permanent steel magnets of the galvanometer, the material of the coil is magnetized in that direction. This magnetization tends to hold the coil on that side of the zero point. The twisting of the suspension wire also has something of the same effect. With a sensitive galvanometer the coil does not return to exactly the same position of rest after it has been deflected in the opposite direction. After a few swings in the same direction, a condition is reached that can be duplicated for each successive deflection.

72. Accuracy of Measurement.—From an inspection of the equation $E = (k/C)d$, it is seen that the value of E that is computed by this formula depends directly upon the value of d. Therefore E cannot be determined any more exactly than d is known.

This means that the deflection should be determined as carefully as possible. The values of the capacitance of the condenser and the galvanometer shunt, if a shunt is used, should be so chosen that the deflection will be large enough to be measured accurately. For a deflection of 100 divisions, each division means 1 per cent of the whole. For a deflection of 10 divisions, each division is 10 per cent of the whole deflection, in which case the error due to the uncertainty in reading the deflection is very large. Usually a deflection of 100 to 200 small scale divisions (millimeters) will be found suitable. If it is too large, other uncertainties begin to appear.

When results better than approximate values are desired, several determinations of the deflection should be made, with the mean value used in computing the value of E. If the individual deflections vary among themselves, this average value will be better than any single one. But it should be noted that this average value has no meaning in decimal places beyond those estimated and recorded for each deflection.

The d that is used in Art. 69 to compute the value of E really should be the mean of, say, 1,000 or more deflections that might have been observed. The few deflections that are actually observed are only 5 or 10 taken at random out of this 1,000. How certain can one be that the average of these few values is equal to the mean of the 1,000 that might have been taken? Probably it is not any better than the average amount by which these few values differ from their mean. If this is f per cent, then the computed value of E will likewise be uncertain by f per cent by this cause alone, since this factor is used in the formula $E = (k/C)d$. (See Art. 69.)

73. Advantages of the Condenser Method.—One advantage of the condenser method is that it measures the full emf of the source, because no current is drawn and therefore there is no fall of potential in the resistance of the circuit. It is true that while the condenser is being charged, there is a current through the battery, and writing the equation of potential differences for the charging circuit, Fig. 27, gives

$$E - ri - Ri - V = 0,$$

where V is the voltage across the partly charged condenser and R

includes all the resistances between the condenser and the battery of emf E and internal resistance r.

At the start, the charging current i may be fairly large, but the larger it is, the quicker will it become zero as the condenser becomes charged. At the end, when $i = 0$, the value of V is

$$V = E,$$

and the final charge in the condenser is

$$Q = CV = CE,$$

whatever the resistance of the emf source.

If the resistance in the charging circuit is very large, it may be necessary to keep the condenser connected to the emf source for some time—long enough to charge the condenser fully. This time should be long enough that doubling the time will not increase the deflection of the galvanometer.

Caution.—The condenser used in these measurements should be one that gives only a negligible amount of residual discharge (see Art. 380).

Another advantage of this method is in the short time that the unknown emf needs to be available. Even with a moderate amount of resistance in the circuit, a very small fraction of a second is sufficient to charge the condenser (see Fig. 185). Thus the emf of a changing source can be measured at any given instant by connecting it to the condenser for this instant (see Fig. 151). This method requires a switch that will connect E to the condenser at the proper instant. If E is changing rapidly, the circuit resistance, $R + r$, must be small enough to allow the condenser to be fully charged during the instant it is connected to E. The condenser is then discharged through the galvanometer as usual.

74. Effect of Resistance in the Charging Circuit.—It has been shown above that the condenser, Fig. 27, receives its full charge in a very short time, even with some resistance in the battery circuit. Mathematically, it is true that the condenser never reaches its full charge; but as nearly as can be measured by reading the deflection of the ballistic galvanometer, the full charge is attained in a very short time.

How long is a "short time" for a given arrangement like Fig. 27? This question can be answered by observing the deflections when the key is closed on the battery side for different lengths of time, say for a fraction of a second and for 1, 2, or 5 sec. If doubling the time (with a good condenser) gives no larger deflection, then that time is long enough.

How much is "some resistance" in an arrangement like Fig. 27? This can be answered by adding various amounts, say 100, 1,000, 9,000, or more, ohms in the circuit between the battery and the condenser. If the addition of a given resistance in series with the battery gives no observable decrease in the deflection, then this amount of resistance will cause no trouble in measuring the emf of the battery.

Of course, these two questions are closely related. When the resistance in series with the battery is large enough to reduce the deflection by an observable amount, then the key should be held closed for a longer time in order to charge the condenser fully. An old dry cell may have enough internal resistance to require several seconds to charge the condenser.

75. Comparison of Capacitances by Direct Deflection.—The same arrangement described above and shown in Fig. 27 can be used exually well for the measurement of the capacitance of a condenser. It is necessary only to go through the experiment as before and to observe the galvanometer deflection with the first condenser, for which

$$CE = kd.$$

Now, replacing the condenser by another one but using the same battery and everything else the same as before, the relation becomes

$$C'E = kd',$$

where d' is the galvanometer deflection when the condenser of capacitance C' is used. Dividing the second equation by the first gives

$$C' = C\frac{d'}{d}.$$

If C is a known capacitance, then the value of C' can be determined as exactly as the flings d and d' can be measured. Each of these should be taken and recorded several times, with the mean values used in the computation.

76. Calibration of the Scale of a Ballistic Galvanometer. *Potential-divider Method.*—The quantity of electrons discharged through a ballistic galvanometer is measured by observing the deflection, or first throw, on the scale. The divisions on the scale are in millimeters or other equal spaces, and the number of scale divisions passed over by the index gives the magnitude of the deflection. Sometimes the scale is bent into the arc of a circle and the deflection measures 2θ, where θ is the angle turned through by the mirror on the moving coil. More often the scale is straight and the deflection is proportional to $\tan 2\theta$. Most galvanometers are designed to give deflections that, as nearly as possible, are proportional to the quantity of electrons that is discharged through them. Just how nearly this is the case can be determined by calibrating the scale.

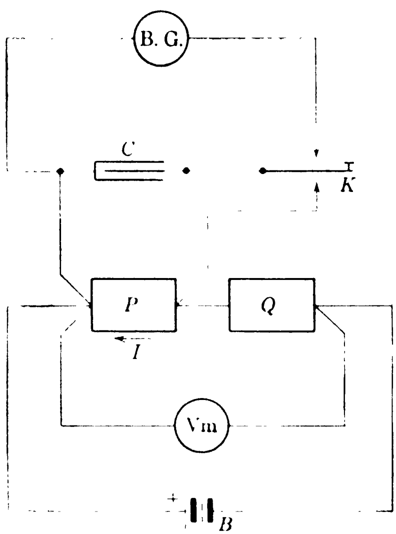


FIG. 28.—Calibration of the scale of a ballistic galvanometer.

The calibration consists in discharging quantities of different known amounts through the galvanometer and observing the corresponding deflections. The different charges can be obtained by charging a condenser to different potential differences by the arrangement shown in Fig. 28. The battery B is connected to the potential divider PQ, and the current is maintained at a fixed value I by keeping the total resistance of $P + Q$ equal to a constant amount. By varying P while keeping $P + Q$ constant, various voltages PI can be obtained for charging the condenser.

77. Plotting Values of k.—From the relation derived in Art. 66,

$$CE = CPI = kd, \tag{1}$$

from which

$$d = \frac{CI}{k} P. \tag{2}$$

This shows that the deflections as read on the scale should be proportional to the resistances in P when k, C and I are kept constant. However, it is very likely that k does not have the same value for deflections of various magnitudes. The arrangement of Fig. 28 enables one to examine this relationship. From Eq. (2), the value of k is

$$k = \frac{CPI}{d} = \frac{CPV}{d(P + Q)}, \tag{3}$$

where V is read on the voltmeter of Fig. 28. The computation for k will be somewhat simplified if $P + Q$ is kept at 1,000 times V.

Using a 2-volt battery for B and varying P by 100-ohm steps, the observer can note the deflections and compute the corresponding values of k. If these 20 values of k agree with each other as closely as the accuracy of reading the deflections would lead one to expect, they can be averaged to give the mean value of k for the galvanometer.

But if the values of k show a continuous change as the deflections are made larger, it is better to draw a curve with the values of k for ordinates and deflections for abscissas. Since the variation in k is small, the curve can be magnified by taking the point $k = 0$ at quite a distance below the bottom edge of the sheet on which the curve is drawn.

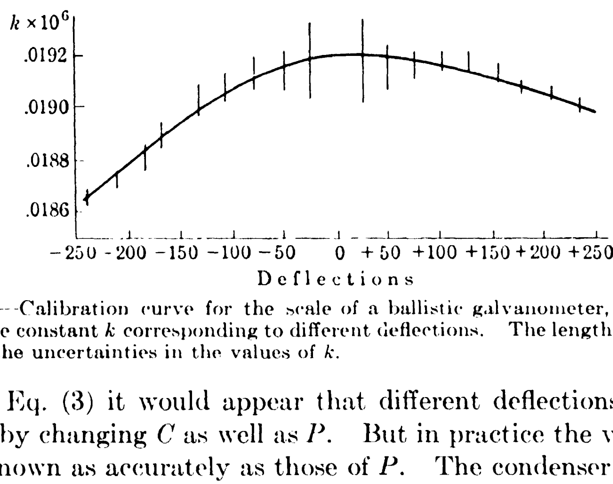


FIG. 29.—Calibration curve for the scale of a ballistic galvanometer, showing the values of the constant k corresponding to different deflections. The lengths of the short lines show the uncertainties in the values of k.

From Eq. (3) it would appear that different deflections could be obtained by changing C as well as P. But in practice the values of C are not known as accurately as those of P. The condenser should be kept constant at such a value that the changes in P will give deflections over the full length of the scale that is to be calibrated.

In case the value of k as determined by the use of a standard cell (Art. 67) does not lie on the curve of Fig. 29, the value that was used for V or for C in Eq. (3) should be modified so that the curve does pass through the point that was determined by the standard cell.

78. Internal Resistance of a Source by the Condenser Method. The condenser method offers a convenient and excellent means for determining the internal resistance of a cell, the principal advantages being the wide range of resistances that can be measured and the short time that the cell must be in use. In the ammeter-voltmeter method a considerable current must oftentimes be drawn from the cell and for a period long enough to read both instruments. Such readings seldom can be repeated, for, owing to polarization, the cell does not return to its original condition.

The setup for using the condenser method is shown in the figure. When K_2 is closed, there is a current through R and the cell, the value of which is

$$I = \frac{E}{R + r},$$

where r is the internal resistance of the battery. This gives

$$r = R\frac{E - E'}{E'}, \tag{4}$$

where E' is written for RI, the external fall of potential. If R is known, only E and E' remain to be measured.

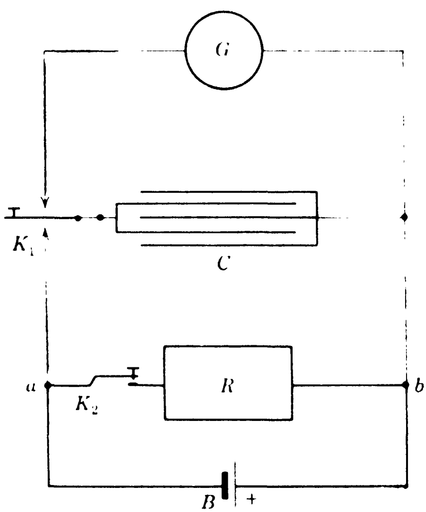


FIG. 30.—To measure the resistance of B.

When K_1 is depressed, the condenser is charged to the emf E of the cell. This emf is measured by the deflection of the galvanometer when K_1 is raised, the relation being

$$E = \frac{k}{C}d.$$

When a current is drawn from the cell by closing K_2, the potential difference between a and b is

$$E' = E - Ir = RI.$$

If the condenser is now charged from this potential difference, the relation is

$$E' = \frac{k}{C}d'.$$

Substituting these expressions in Eq. (4) gives

$$r = R\frac{d - d'}{d'}. \tag{5}$$

When this method is used, it is necessary to keep K_2 closed only long enough to depress and raise K_1. With skill, this interval can be reduced to less than a second when the keys are worked by hand. If conditions require that the current be drawn from the cell for a shorter time than this, a special testing key can be used.

79. Special Testing Switch.—An inexpensive switch that combines both K_1 and K_2 of Fig. 30 can be made from a double-pole, double-throw switch like the one shown in Fig. 31. The movable blade on one side is used for the tongue of K_1, and the two fixed jaws are the upper and lower contacts of K_1, Fig. 30.

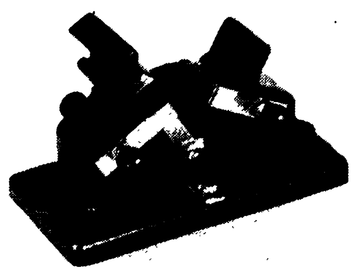


FIG. 31.—The blades of this double-pole, double-throw switch can be used for K_1 and K_2, Fig. 25, by shortening one clip. Care must be taken to have K_2 closed for only an instant. (*Courtesy of Leeds & Northrup Co.*)

The movable blade on the other side of the switch and one of the fixed jaws serve as K_2. In the switch as shown in Fig. 31, both blades enter their jaws together. This means that K_1 and K_2 are both closed downward (Fig. 30) at the same time. Shortening the jaw of K_1 by a few millimeters will allow K_2 to make its contact first and to remain closed until after the condenser contact is broken. This assures that E' is measured, not E.

For the measurement of E the connection to R is opened by the key K (Fig. 32) and only the condenser side of the switch is used. In using this style of switch as a condenser key, great care must be taken not to touch any metal part of the switch with the fingers and thus partially discharge the condenser.

80. Diagram of the Switch.—The connections that are made to the double switch of Fig. 31 are shown in Fig. 32, which is drawn similar to Fig. 30. H is the insulated handle that moves the blades of both K_1 and K_2. When in use, the blades are kept in the upper position, with the condenser connected to the galvanometer and its shunt S. Quickly throwing H down and back again charges and discharges the condenser. If K remains closed throughout this operation, the voltage E' across R from a to b, Fig. 30, is measured. When K is open, the galvanometer deflection measures the full emf E of the battery E.

81. The Largest Uncertainty.—It is seen in Eq. (5), above, that the value of r depends upon both d' and $d - d'$. When either one of these quantities is small, the percentage uncertainty in its measurement is large, and the value of r cannot be known to a better degree than the largest uncertainty in these factors. Therefore the resistance R should be set at a value that will make each of these quantities as large as possible.

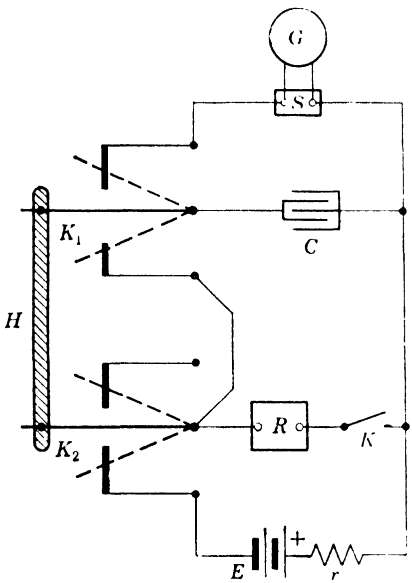


FIG. 32.—Diagram of the switch of Fig. 31, showing its connections when used in Fig. 30. Note the short clip of K_1.

Evidently both cannot be as large as d, since increasing one decreases the other. The uncertainty in $d - d'$ is due to the uncertainty in d as well as that in d'. Therefore, when these measurements are made, the value of R should be adjusted to give d' an intermediate value. This would be a value of R that will make d' between one-third and one-half as large as d. Thus neither d' nor $d - d'$ need be too small for accurate measurement.

Even with this optimum arrangement, the value of r is not determined any better than the largest uncertainty in d' or $d - d'$. The capacitance of the condenser should be sufficient to make these deflections as large as convenient.

82. Measurement of Insulation Resistance.—A ballistic galvanometer can be used to advantage in the measurement of extremely high resistances. Usually such resistances need not be determined with great accuracy, but the order of magnitude is required. Resistances up to a few megohms, for example, the insulation of the electric wiring of a building from the water pipes, can be measured by a voltmeter, as explained in Art. 52. Larger resistances, such as the insulation of the coils from the frame of an electric motor, can be measured by the ammeter-voltmeter method (Art. 33), with a sensitive galvanometer used in place of the ammeter.

But the current through the highest resistances, such as the insulation of electric cables or the resistance between the plates of a good condenser, is too small to be observed, even with a sensitive galvanometer. True, its value can be computed, but it is usually called a "leakage" current. It is then necessary to use one of the following methods.

83. Insulation Resistance by Leakage.—This method is used when the resistance to be measured is so large that the current which it is possible to pass through it is too small to be measured by a sensitive galvanometer. The method consists, in brief, in letting the current flow into a condenser for a sufficient time and then discharging the accumulated quantity through a ballistic galvanometer.

The setup is arranged as shown in Fig. 33, where R denotes the large resistance to be measured. A battery of sufficient emf E supplies the current that flows through R and gradually charges the condenser C when K is closed on a. The switch K can be a charge-and-discharge key, or it can be one side of a double-throw switch like the one shown in Fig. 31. When a sufficient charge has accumulated in the condenser, it is discharged through the galvanometer by throwing K from a to b. The swings of the galvanometer can be checked by a shunt S that is adjusted to give critical damping (Art. 65).

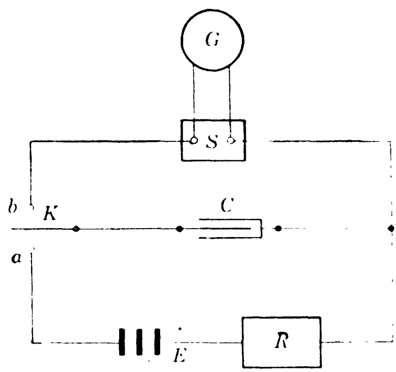


FIG. 33.—To measure a resistance R of many megohms.

At any instant during the time the current is flowing, the fall of potential over R is

$$RI = E - V, \tag{6}$$

where V is the difference of potential across the condenser.

In this method it is assumed that the condenser has considerable capacitance and that the condenser is discharged before V has reached an appreciable part of E. If the experiment is not worked in this way, the following discussion does not apply and the formula of Art. 85 should be used.

At the start, and as long as V can be neglected in comparison with E, the current through R is, from (6),

$$I = \frac{E}{R}. \tag{7}$$

If this current flows into the condenser for t sec, the accumulated charge is

$$q_x = It; \tag{8}$$

and when the condenser is discharged through the galvanometer, there is a deflection, or fling, of d scale divisions, such that

$$q_x = kd. \tag{9}$$

Thus the current is

$$I = \frac{kd}{t},$$

and, from (7),

$$R = \frac{E}{I} = \frac{tE}{kd}.$$

Determination of the Constant.—The "constant" k of the galvanometer can be determined by the method described in Art. 67, giving

$$k = \frac{E'C'}{d'},$$

where E', C', and d' denote the particular values used in this determination of the constant.

If the same battery is used in finding the constant as in the experiment proper, then $E = E'$, and the absolute value of the emf employed does not enter into the computation. The formula then becomes

$$R = \frac{t}{C'}\frac{d'}{d}.$$

84. Comparison of High Resistances.—If the value of one high resistance is known, other resistances of the same order of magnitude can be directly compared with it. This can be done without knowing the constant of the galvanometer or the capacitance of any condenser.

Suppose another resistance, R_1, is substituted for R in the setup shown by Fig. 33 and the current is allowed to leak through it into the condenser as before.

In the first case

$$R = \frac{tE}{kd}.$$

and now

$$R_1 = \frac{t_1E}{kd_1},$$

where t_1 and d_1 denote the observations made when the current was leaking through R_1.

Eliminating E and k by division gives

$$R_1 = R\frac{t_1 d}{t d_1},$$

where, as in the preceding article, it is supposed that V can be neglected in comparison with E. From Fig. 35 it appears that this approximate value of R_1 becomes more nearly correct as d and d_1 are more nearly equal to each other.

85. The More Exact Formula.—In Art. 83 an assumption was made that is not quite true. It was there supposed that the leakage current remains constant, as shown by curve I, Fig. 34. From this assumption it follows that the charge collected in the condenser increases directly with the time the current is flowing or $q_x = It$, as

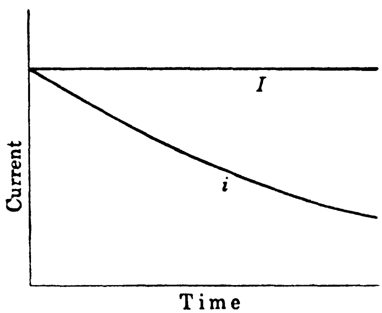


FIG. 34.—The current flowing into a condenser decreases with time, as shown by the curve i.

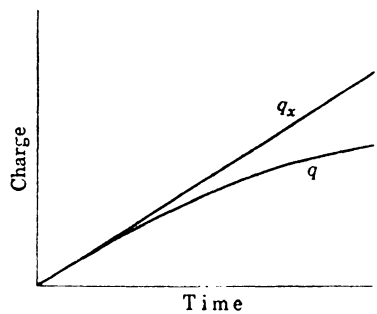


FIG. 35.—The charge in a condenser increases with time, but more and more slowly, as shown by curve q.

shown by curve q_x, Fig. 35. As a matter of fact, the current becomes less and less as the condenser becomes charged, as shown by curve i, Fig. 34. The corresponding charge collected in the condenser is shown by curve q, Fig. 35. At the beginning there is not much difference between q_x and q, and it is seen that the method above is approximately true when t is a small part of the time required to half-charge the condenser.

In order to make a satisfactory measurement, it is necessary to collect a sufficient quantity in the condenser to give a measurable deflection when it is discharged through the galvanometer. Likewise, the time allowed for the collection of this quantity cannot be determined with accuracy if it is too short. Either or both of these conditions may make it desirable to allow the current to flow for a longer period and thus render it no longer permissible to neglect the increasing value of V.

Taking these conditions into account leads to the following considerations:

The current flowing into the condenser at any instant is

$$i = \frac{E - V}{R},$$

or, in terms of q, the charge in the condenser,

$$i = \frac{dq}{dt} = \frac{E - \frac{q}{C}}{R},$$

since $q = CV$.

Separating the variables for integration puts this in the form

$$\frac{-dq}{Q - q} = \frac{-dt}{RC},$$

where Q is written for EC. This Q is the charge in the condenser, C, when it is joined directly to E. Integrating this equation gives

$$\log (Q - q) = \frac{-t}{RC} + K,$$

where K is the constant of integration.

At the start, when both t and q are zero, this equation gives

$$\log Q = K.$$

After t' sec, when q' has collected in the condenser,

$$\log (Q - q') = \frac{-t'}{RC} + K.$$

Subtracting the latter from the former gives the change during t' sec.

$$\log \frac{Q}{Q - q'} = \frac{t'}{RC}.$$

Hence the value of R is

$$R = \frac{t'}{C \log \frac{Q}{Q - q'}} = \frac{t'}{2.303\, C \log_{10} \frac{d_o}{d_o - d'}},$$

where d_o and d' are the deflections when the quantities Q and q' are discharged through the galvanometer.

86. Insulation Resistance of a Cable.—Let a measured length of the cable be coiled up and placed in a tank of water to give contact over the outside surface if there is not a lead covering. Each end of the cable should extend out of the water or sheath for some distance and must be kept thoroughly dry. A few turns of bare copper wire are tightly wound around the cable near the middle of this dry portion of the insulation and are connected to the common point between the condenser and the battery, as shown in Fig. 36. This guard wire serves to intercept any current that may be leaking along the surface of the insulation. Connection is made to the water by means of a wire of the same kind as that in the cable, to avoid voltaic effects. The wires connecting R to C and the key should be well insulated from the table and from all places where a charge can leak across. As far as possible, they should be stiff enough to stand in the air. Where necessary, they can be supported on blocks of sulfur.

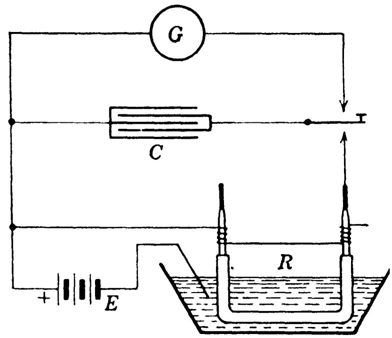


FIG. 36.—To measure the insulation resistance R of a length of cable.

The determination consists, then, in allowing the leakage current to flow into the condenser long enough (t' sec) to give a fair charge q', which, when discharged through the galvanometer, will give a deflection d'. When R is short-circuited and the condenser is charged directly from E, the charge is Q, giving a deflection d. For best results, q' should be about half of Q.

Owing to dielectric absorption of the charge, the first values of R will be too small. In reporting any value of R, the duration of the test should also be stated.

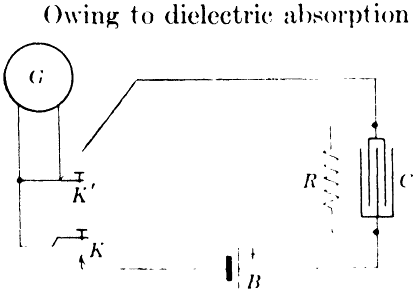


FIG. 37.—To measure the resistance of a condenser.

87. Insulation Resistance by Loss of Charge.—If the high resistance also has considerable capacitance, it will not be necessary to use a separate condenser. Thus if it is required to measure the insulation of a condenser or of a long cable, the arrangement will be as shown in Fig. 37, where CR represents the condenser of capacitance C and resistance R. When K is closed, with K' also closed, the condenser is charged to the full potential difference of the battery. When the key is opened, the charge Q begins to leak away through the high resistance R. At the start, and before the charge in the condenser has been appreciably reduced by the leakage, the current through R is

$$I = \frac{E}{R}.$$

This current will reduce the original charge in the condenser by the amount

$$q_x = It$$

in t sec, where t is not too great to consider the current constant during this interval. This loss of charge can be determined by recharging the condenser through the galvanometer to the original difference of potential. Then, as before,

$$q_x = kd, \qquad \text{and} \qquad I = \frac{kd}{t},$$

from which

$$R = \frac{E}{I} = \frac{Et}{kd} = \frac{t}{C'}\frac{d'}{d},$$

where E cancels out if the same battery is used to determine k.

88. A More Exact Formula for the Loss-of-charge Method.—As in the previous case, the current leaking out of C can be considered constant only as an approximation for a short portion of the discharge. For more precise results it will be necessary to take account of the decreasing value of this current.

As the charge in the condenser leaks away, the potential difference across the condenser decreases and finally becomes zero. Let V denote the value of this potential difference at the time t. The current flowing through R at this instant is $i = V/R$. Since this current is supplied from the charge in the condenser, it is measured by the rate of *decrease* of this charge,

or
$$i = -\frac{dq}{dt} = \frac{V}{R} = \frac{q}{CR},$$

since $V = q/C$.

Separating the variables for integration,

$$\frac{dq}{q} = \frac{-dt}{CR},$$

and performing the integration gives

$$\log q = \frac{-t}{CR} + B, \tag{10}$$

where q is the charge remaining in the condenser after the charge has been leaking out for t seconds.

At the start, when $t = 0$, $q = Q$. Putting these values in Eq. (10) gives, for the value of the constant of integration B,

$$\log Q = B.$$

If q' denotes the quantity that has leaked out of the condenser in t' sec, then

$$q = Q - q',$$

and

$$\log (Q - q') = \frac{-t}{CR} + \log Q.$$

Solving this for R gives

$$R = \frac{t'}{C \log \frac{Q}{Q - q'}} = \frac{t'}{2.303\, C \log_{10} \frac{d_o}{d_o - d'}},$$

where d_o and d' are the deflections when Q and q' are discharged through the galvanometer.

At the beginning of this test, the values of R will usually be too low because of the effect of "absorption," by which a part of the charge disappears. This reduces the charge in the condenser just as though it had leaked out. The true value of the resistance will be obtained only after several hours, in some cases several days; but if a first test is being made, it is well to determine the value of R at intervals of a few minutes. A curve plotted with the time of day for abscissas and the corresponding values of R for ordinates will show this variation and indicate the maximum value of the insulation resistance.

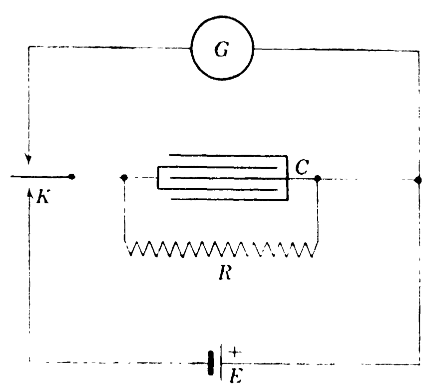


FIG. 38.—The charge in the condenser can leak away through the high resistance R.

A resistance not having any capacitance can be measured by this method by adding a condenser in parallel with it. But in such a case the arrangement shown in Fig. 33 would be preferable.

89. To Reduce the Effect of Absorption.—One difficulty in measuring the leakage resistance of a cable or a condenser arises from the absorption of the charge by the dielectric (see Art. 380). This may be avoided in part by first charging the condenser and allowing it to leak for a period of t_1 sec. Next, discharge the remaining charge q_1 through a ballistic galvanometer and observe the deflection d_1. Then recharge and allow the charge to leak away for a longer period t_2, say twice as long. For the first period,

$$\log q_1 = \frac{-t_1}{CR} + B,$$

and for the second,

$$\log q_2 = \frac{-t_2}{CR} + B.$$

The difference is

$$\log \frac{q_1}{q_2} = \frac{t_2 - t_1}{CR},$$

and

$$R = \frac{t_2 - t_1}{C \log \frac{q_1}{q_2}} = \frac{t_2 - t_1}{2.303\ C \log_{10} \frac{d_1}{d_2}}. \tag{11}$$

An electrometer can be used in place of the galvanometer, and it has the advantage that the loss of charge can be observed continuously. The curve of discharge can be plotted from readings taken at definite times. The effect of dielectric absorption of the charge is shown by the variation of this curve from the one plotted from Eq. (10). Any two points of this curve can be used in Eq. (11) to obtain the value of R.

90. The Quadrant Electrometer.—The quadrant electrometer is an instrument for measuring electromotive forces or potential differences without drawing any current from the source that is being measured. In this respect it is more like a condenser (Art. 59) than the ordinary voltmeter. It is better than the former in that it need not be discharged through a ballistic galvanometer; the voltage is measured by the steady deflection of the mirror as read on the scale.

The essential part of a quadrant electrometer is a small cylindrical box divided into four quadrants. Each quadrant is mounted on a pillar of amber or other good insulator, and the quadrants thus mounted are separated from each other by narrow air gaps (about 1 mm). Opposite quadrants are joined together by a light wire, and the two pairs of quadrants are connected to two binding posts, which also are well insulated from the case of the instrument. The object of this high insulation is to prevent the leaking away of any charges that may be given to the quadrants.

Within the hollow box formed by the quadrants is hung a large flat "needle" of thin aluminum or silvered paper. On the same axis

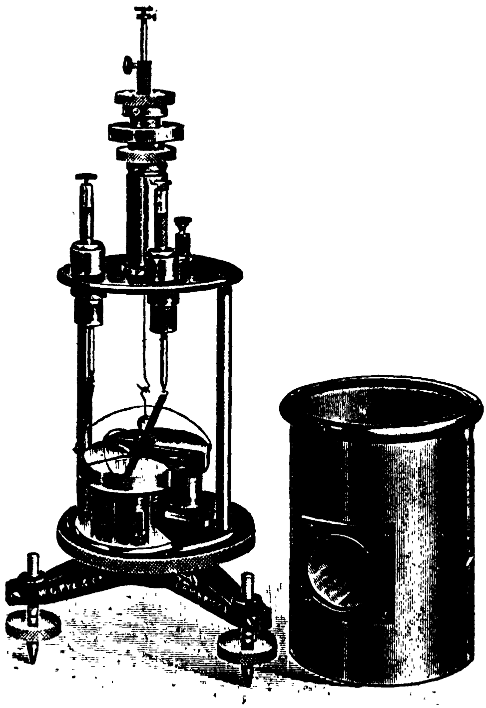


FIG. 39. A quadrant electrometer, with the outer case removed to show the quadrants and needle.

is fastened a small mirror, and the whole is suspended by a flat strip of phosphor bronze or by a silvered fiber of quartz, in the same manner as a galvanometer coil. The upper end of the suspension wire is attached to a well-insulated binding post for outside connections. The shape of this needle (horizontal plan) is such that as it turns there is little change in the size of the portion that is visible through the slits between the quadrants. There is thus no change in the "edge correction" due to the nonuniformity of the electrostatic field near the edge of the quadrant.

When in use, the needle of the electrometer is connected through a high resistance to a battery of many volts. In the Dolezalek form shown in Fig. 39 this should be about 80 volts. The other terminal of this battery should be connected to the metal case of the electrometer. It is better to have this grounded if there is no other ground on the battery circuit. The needle is thus given a charge, either positive or negative, but with the quadrants all at the same potential there is no deflection. When a difference of potential is applied to the two pairs of quadrants, the needle will turn into those quadrants having the greater difference from its own. For a potential difference that is not too large, the deflection is directly proportional to this difference of potential; it is also proportional to the difference between the potential of the needle and the average potential of the quadrants.[1]

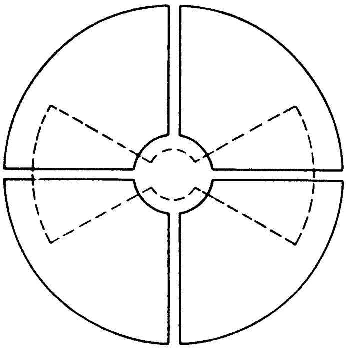


FIG. 40.—Top view of the quadrants of an electrometer. The zero position of the needle is shown by the dotted outline.

A common form of electrometer is shown in Fig. 39. At the center is seen the hollow brass box that is cut into four quadrants, with each quadrant mounted separately on its pillar of amber. In this figure two of the quadrants have been drawn forward to reveal the suspended needle that hangs freely within this quartered box. When in use, these two quadrants are replaced in their symmetrical position. A top view of the quadrants is seen in Fig. 40, and the outline of the needle is shown by the broken line.

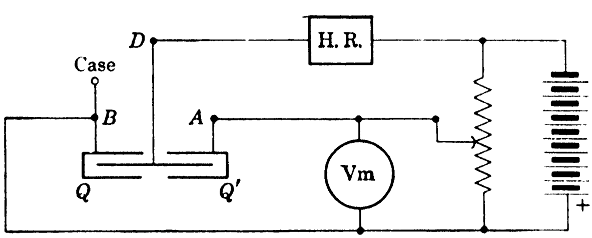


FIG. 41.—Showing the arrangement for giving a known potential difference to the quadrants QQ'.

91. Calibration of the Scale of a Quadrant Electrometer.—In order to determine the relation between the reading on the scale used with the quadrant electrometer and the corresponding potential difference between the quadrants, an arrangement such as that shown in Fig.

[1] See further, STARLING, S. G., "Electricity and Magnetism," 4th ed., p. 159.

41 can be used. *A* and *B* denote the binding posts that are connected with the quadrants which are shown in section by *Q′* and *Q*. *D* represents the binding post that is connected to the moving needle through the suspension fiber.

A potential divider of large resistance is connected across a battery of about 80 volts to give any desired part of this voltage between the quadrants. The value of this part of the voltage can be read from the voltmeter. The needle is connected through *D* to one terminal of the battery and thus is maintained at a constant potential above the case and one pair of quadrants. A high resistance is inserted in this line to avoid any accident if the needle should touch one of the quadrants.

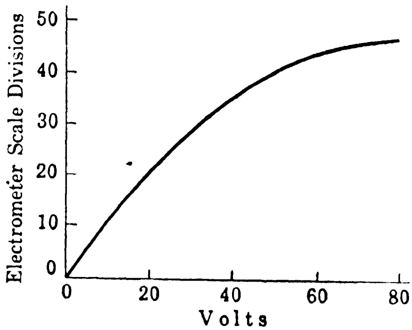


FIG. 42.—A calibration curve for a quadrant electrometer connected heterostatically.

The potential of the metal case of the electrometer is considered the zero from which the other potentials are counted. This arrangement of the connections is called "heterostatic," because the potentials of *A*, *B*, and *D* are all different. When the needle is connected to one pair of the quadrants instead of to the battery, the arrangement is called "idiostatic," because now the needle has the same potential as one pair of quadrants.

The calibration curve is obtained by setting the voltmeter at definite voltages about 5 volts apart and reading the corresponding steady deflections of the electrometer. The curve is plotted with the voltmeter readings as abscissas and the deflections as ordinates. For the smaller voltages this curve is nearly a straight line, but as the difference in potential between *D* and *A* becomes less, the deflection of the electrometer increases more slowly than at first.

92. Measurement of Emf with a Quadrant Electrometer.—In order to measure an unknown emf with a quadrant electrometer, it is connected to the quadrants *A* and *B* Fig. 41, with the needle still at 80 volts or whatever potential was used for the calibration curve. As soon as the needle settles down to a steady position, the deflection can be read, and the corresponding voltage is obtained from the calibration curve. No current is drawn from the source, and the measurement gives the full emf.

The difference of potential between the plates of a charged condenser can be measured in this way, since the condenser is not discharged by the electrometer.

93. Discharge Curve of a Condenser.—When the plates of a charged condenser are connected to the terminals of a high resistance, the surplus electrons on the negative plate flow through the resistance until the lack of electrons on the positive plate has been filled. This electron current usually lasts for only a small fraction of a second, because the charges on the plates are always very small fractions of a coulomb. However, if the resistance of the circuit is large enough, the current will be very small and the condenser will be discharged slowly. With a resistance of several hundred megohms the condenser may discharge slowly enough to be followed with an electrometer. It is never possible to insulate the condenser plates so completely that

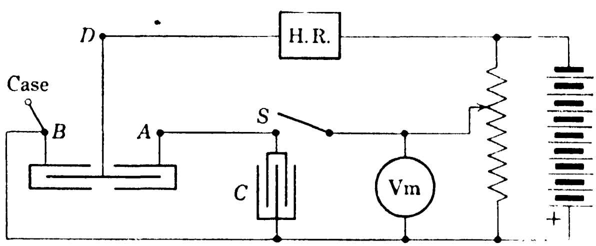


FIG. 43.—Showing the arrangement for observing the discharge of the condenser C through its own high-resistance insulation.

there is no leakage of charge from them, and this leakage of a paraffined-paper condenser may be enough to discharge the condenser in an hour or so without any other conductor across the condenser.

In order that the condenser shall not be discharged by the handling of the connections, it should be connected to the electrometer beforehand, as shown in Fig. 43. The insulation of a key at S is not sufficient for this purpose. The connection at S should be a flexible wire hooked lightly to the terminal of the condenser. When this connection is opened, only that part which is still joined to the battery circuit should be handled, and a wide air gap should be made, leaving the condenser connected to the electrometer. By reading the electrometer every minute or two and plotting the readings against the time of day, a curve is obtained that shows the manner of discharge of the condenser.

A look at the calibration curve will probably show that there is not much change in the deflection for voltages from 80 down to about 50. Therefore in observing the discharge of the condenser it will be better to start with the condenser charged to about 50 volts.

When the charge in a condenser leaks out through a high resistance R, the current is

$$i = -\frac{dq}{dt} = \frac{V}{R} = \frac{q}{CR},$$

as shown in Art. 88. The quantity q remaining in the condenser after the current has been flowing for t seconds is

$$q = \epsilon^{-\frac{t}{CR}+B} = \epsilon^B \epsilon^{-\frac{t}{CR}} = Q\epsilon^{-\frac{t}{CR}},$$

from Eq. (10), Art. 88, where Q is the maximum value of q.

The corresponding voltage across the condenser is

$$V = \frac{q}{C} = \frac{Q}{C}\epsilon^{-\frac{t}{CR}} = V_o\epsilon^{-\frac{t}{CR}},$$

and this is the equation of the curve shown in Fig. 44, when C and R are constants.

94. Measurement of a High Resistance by an Electrometer and a Condenser.—If the charge on the condenser considered in the pre-

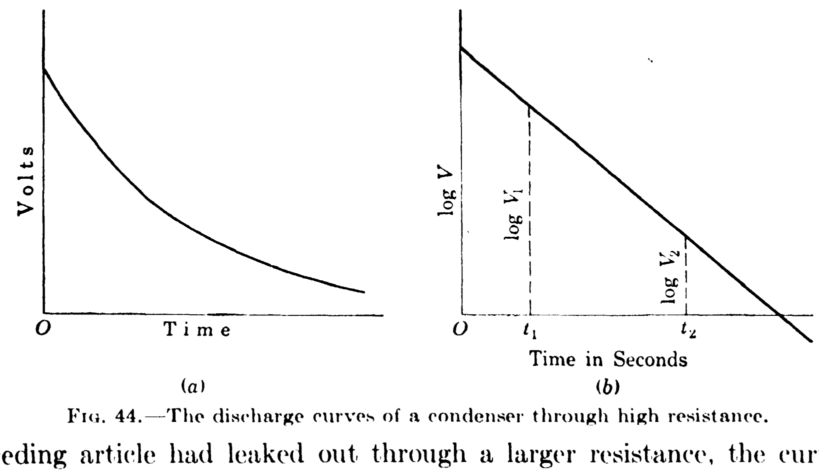


FIG. 44.—The discharge curves of a condenser through high resistance.

ceding article had leaked out through a larger resistance, the curve shown in Fig. 44 would have been less steep; and if the resistance had been smaller, the charge would have leaked out more rapidly. The form of the curve indicates, therefore, the resistance of the discharge circuit. Since it is difficult to draw a curve with accuracy, it is better to express this relationship as a straight line. This is done by plotting log V for the ordinates of the curve instead of V. Since $V = q/C$,

$$\log V = \log q - \log C = B - \frac{t}{CR} - \log C,$$

from Eq. (10), Art. 88. The constants B and $\log C$ can be eliminated from consideration by taking the difference between two values of $\log V$ that may be read from the straight line of Fig. 44(*b*). The values of these are

$$\log V_1 = B - \frac{t_1}{CR} - \log C$$

and

$$\log V_2 = B - \frac{t_2}{CR} - \log C,$$

giving a difference of

$$\log V_1 - \log V_2 = \frac{t_2 - t_1}{CR},$$

from which the value of R is given as

$$R = \frac{t_2 - t_1}{C(\log V_1 - \log V_2)}.$$

If common logarithms are used in plotting the curve of Fig. 44(*b*), the value of R is

$$R = \frac{t_2 - t_1}{2.303\, C\,(\log_{10} V_1 - \log_{10} V_2)} \qquad \text{ohms}$$

where C is the capacitance of the condenser in farads and $(t_2 - t_1)$ is the time in seconds during which the voltage across the condenser fell from V_1 to V_2 volts. If C is expressed in microfarads, the value of R will be given in megohms. The larger the resistance R, the less steep is the curve of Fig. 44.

Sometimes this curve of $\log V$ is not a straight line. This means that R has not remained constant during the time the curve was being taken. In this case the initial value of R can be obtained from the slope of the first part of the curve. The final value of R is given by using only the last part of the curve.

95. Accuracy of *R*.—The expression for R is the reciprocal of the negative slope of the log curve of Fig. 44(*b*). Therefore this curve should be plotted on scales that will make both $(t_2 - t_1)$ and $(\log V_1 - \log V_2)$ as large as possible on the plotted page. The value of R cannot be determined any better than these differences can be read from the curve. And the curve is no better than its agreement with the points through which it is drawn.

These considerations point out the place of the greatest uncertainty, and the more care should be taken to make this particular measurement with greater accuracy.

CHAPTER III

THE CURRENT GALVANOMETER

96. Two Types of Galvanometers.—A galvanometer is a delicate and sensitive instrument for the measurement of small electron currents. All galvanometers consist of two essential parts—a coil of wire through which can flow the current to be measured, and a permanent steel magnet. In some galvanometers the coil is comparatively large and is rigidly fixed to the frame of the instrument, while the magnet is a small piece of steel suspended lightly by a fiber of untwisted silk or of quartz. In other galvanometers the arrangement is reversed. The coil is made as light as possible and is suspended by a thin strip of phosphor bronze or gold between the poles of a large and strong magnet that often forms the body of the instrument. In either style the movable portion is made to turn as easily as possible, the amount of turning being measured by the mirror, scale, and telescope method.

There are two ways of using a galvanometer. A transient current, like the discharge of a condenser, will produce a fling or kick of the galvanometer, after which it will settle back to the original position. Evidently the only thing that can be measured in this case is the maximum fling. But if the current is steady, the galvanometer will settle down at a deflected position, and the deflection, as the distance of this position on the scale from the position of rest is called, measures the current.

97. Figure of Merit. (*a*) *Direct Deflection.*—Most galvanometers are so constructed that, for small angles at least, the deflection is directly proportional to the current. That is,

$$I = Fd.$$

The factor F is called the "figure of merit" of the galvanometer, and it is defined as the *current per scale division* (1 mm) that will deflect the galvanometer. The figure of merit of most galvanometers is smaller than one hundred-millionth of an ampere per millimeter.

Inasmuch as the deflection will vary with the distance of the scale from the mirror, this distance should be recorded with the other observations. The standard value of F is computed for a distance of 1 meter.

In order to determine the figure of merit, it is necessary to send a small known current through the galvanometer and observe the *steady* deflection it produces. The method can be understood by reference to Fig. 45. The galvanometer is joined in series with a battery, a large resistance, and a key. When the key is closed, the current through the circuit, and therefore through the galvanometer, is, by Ohm's law,

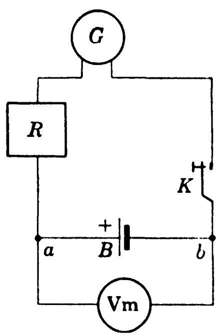


FIG. 45.—To determine the figure of merit of G.

$$i = \frac{E}{g + r + R},$$

where E denotes the emf of the battery, and g, r, R, the resistances of the galvanometer, battery, and R, respectively. If this current produces a steady deflection of d scale divisions (millimeters), the figure of merit is $F = i/d$.

By using a voltmeter to measure directly the fall of potential between a and b, the current i will be given by

$$i = \frac{V}{R + g},$$

where V is the voltmeter reading. Then

$$F = \frac{V}{(R + g)d}.$$

Thus, by the use of the voltmeter, the somewhat uncertain emf of the cell is replaced by the definite voltmeter reading, and the unknown resistance of the cell does not appear in the equations. Of course V and d must be simultaneous values, and it is understood that d is the *steady* deflection produced by the steady current i.

98. Figure of Merit. (*b*) *Potential-divider Method.*—With a sensitive galvanometer it is usually not possible to make R large enough to use the simple method. It is then most convenient to use a value for E that is only a small fraction of the emf of the cell. This can be done by the potential-divider method shown in Fig. 46. The fall of potential between a and b is now PI instead of E, where each factor P and I can be made as small as necessary. For most galvanometers it is convenient to make $P + Q = 1{,}000$ ohms, and R about 100,000 ohms.

The current from the battery divides at a, one part, i, going through the galvanometer, another part, I, through P, and the third part flowing through the voltmeter. The fall of potential from a to b across P is the same as through R and the galvanometer, or

$$PI = (R + g)i, \tag{1}$$

where g is the resistance of the galvanometer.

The current through Q is $I + i$, and the fall of potential from a to c, which is measured by the voltmeter, is

$$PI + Q(I + i) = V. \tag{2}$$

Eliminating I between Eqs. (1) and (2) and solving for i gives

$$i = \frac{PV}{(Q + P)(R + g) + PQ}.$$

The figure of merit is, then,

$$F = \frac{PV}{(P + Q)(R + g) + PQ} \cdot \frac{1}{d}.$$

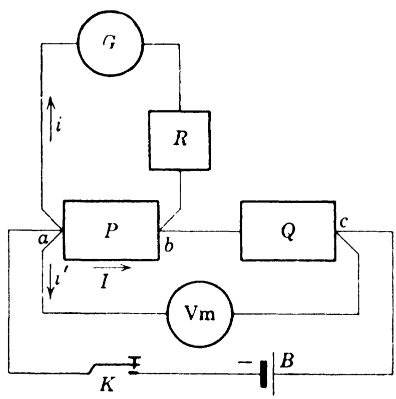


FIG. 46.—The potential divider PQ gives across ab a small fraction of the emf of the battery B.

It is evident that the computed value of F cannot be any more certain than are the measured values of V and of d.

It is always a wise precaution not to connect the battery into the circuit until after the rest of the setup is completed and has been carefully examined to make sure that no unintended connections have been made.

At the start, P should be set very much smaller than Q, perhaps $P = 1$, and $Q = 999$, so that the first deflection of the galvanometer will not be too large. When this has been tried, P can be increased enough to give a deflection of 100 or 200 mm. The observations can be recorded as follows:

TO DETERMINE THE FIGURE OF MERIT OF GALVANOMETER NO. . . .

Zero	Reading	Deflection	R	V	P	Q	F

When scale to mirror is 1 meter, F =

In case the values of F are nearly the same for large and small deflections they can all be averaged to give a mean value of F for the galvanometer.

But, when the values of F show a continous variation as the deflections become larger, it is better to show these values by a curve as in Fig. 47. Then the proper value of the figure of merit for any given deflection is read directly from the curve.

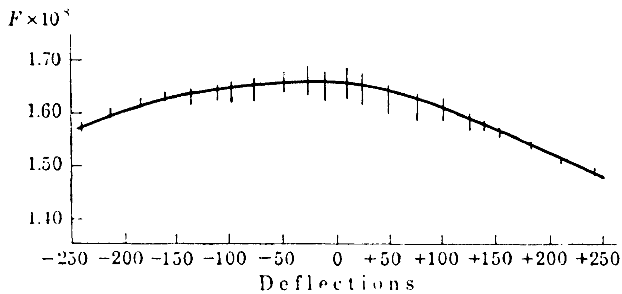


FIG. 47.—Calibration curve for a galvanometer scale, showing the values of the figure of merit corresponding to different deflections. The lengths of the short lines show the uncertainties in the values of F.

99. Current Sensitivity of a Galvanometer.—The figure of merit expresses the sensitivity of a galvanometer in the old way. The smaller the figure of merit, the more sensitive is the galvanometer. It is especially useful when the galvanometer is used to measure a small current, for then

$$i = Fd.$$

The more modern practice is to express the sensitivity as the deflection D that is (or would be) produced on a scale at 1 meter distance by a current of 1 microampere (= 0.000001 amp). This deflection is not observed directly, but is computed, like F, from the observations in the above experiment. This gives

$$D = \frac{1}{10^6 F} = \frac{(P + Q)(R + g) + PQ}{10^6 PV} d.$$

100. Megohm Sensitivity.—The sensitiveness of a galvanometer is often expressed in megohms (millions of ohms). This means the number of megohms that can be placed in series with the galvanometer to give a deflection of one scale division per volt of the applied emf. Evidently this number is the same as D. In some respects this is a better way of expressing the sensitivity. A value of D = 5,000, for example, is the result of a computation and could not be actually observed, while it would be possible to place 5,000 megohms in series with the galvanometer and obtain a deflection of 1 mm when 1 volt is applied.

101. Voltage Sensitivity.—For some purposes it is necessary to know the sensitiveness of a galvanometer to differences of potential applied to its terminals. This will depend largely upon the resistance of the galvanometer, as a small resistance will allow a larger current to flow. The voltage sensitivity is expressed in volts per scale division, and is given by the product of the figure of merit and the resistance of the galvanometer.

102. The Best Galvanometer.—The best galvanometer to use for a particular purpose depends upon the special requirements. When the same current is passed through several galvanometers in series, the one that gives the largest deflection is the most sensitive for current measurements. Probably this one could have been picked out from the fact that it has the highest resistance because it is wound with many turns of fine wire. If the same galvanometers are joined in parallel and connected to a difference of potential of a few microvolts, probably a very different galvanometer will now show the largest deflection. This one will have a low resistance, and it will be sensitive for voltage measurements, provided that the source can supply the current required.

For many purposes a galvanometer may be "too good." If the current to be measured is not extremely minute, the deflection may be too large to measure on the scale. Extreme sensitiveness is often obtained by using a very fine suspension fiber. This makes the period long, and the zero, or resting point, is more variable than it is with a stiffer suspension.

For most purposes a galvanometer with D about 100 scale divisions per microampere and g about 100 ohms is satisfactory. The suspension should be strong enough to withstand moderate jars. The moving parts should have sufficient clearance to allow the use of the instrument without too tedious a process of leveling it. The interior should be easily accessible.

103. Factor of Merit.—When the sensitiveness of a galvanometer is expressed in terms of the deflection D at a distance of 1,000 scale divisions, due to a current of 1 μamp, this sensitiveness can be increased by using a weaker suspension wire or by winding more turns of wire on the coil, without making any real improvement in the design of the galvanometer. In order to make a fair comparison of different galvanometers, it is necessary to take these differences into account.

To make this comparison it is usual to reduce the observed deflection D to what it would be if the period of the galvanometer were 10 sec and the resistance 1 ohm. It is found that D varies as T^2 and about as $g^{3/4}$, where T denotes the period of the galvanometer and g is its resistance.

The sensitiveness, thus modified, is called the "factor of merit" of the galvanometer. Thus

$$\text{Factor of merit} = \left(\frac{10}{T}\right)^2 \left(\frac{1}{g}\right)^{\frac{3}{4}} D.$$

The unit of this quantity is not named. The unit of a similar quantity in which the reference period is taken as one second has been called a "D'Arson."

104. The Common Shunt.—When the current to be measured by any instrument is larger than the range of the scale, the latter can be increased to almost any desired extent by placing a shunt in parallel with the instrument, as shown in Fig. 48. The shunt and instrument thus form two branches of a divided circuit, and the current through one branch is directly measured. When the current in this branch is known, the total current can be computed.

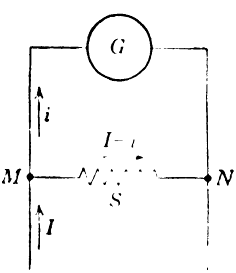


FIG. 48.—Use of a shunt.

Thus let G denote the galvanometer or ammeter, and S the shunt. The fall of potential through the galvanometer from M to N is equal to the fall of potential through the shunt between the same points, or

$$ig = (I - i)s,$$

where g denotes the resistance of the galvanometer and s that of the shunt. This gives for the value of the main current

$$I = i\frac{g + s}{s}.$$

105. The Multiplying Factor of a Shunt.—The factor $(g + s)/s$, by which the current measured by the galvanometer must be multiplied to give the total current in the main circuit, is called the "multiplying factor" of the shunt. In order that this factor may be expressed in convenient round numbers, 10, 100, 1,000, etc., it is necessary to have a series of shunts carefully adjusted to $\frac{1}{9}$, $\frac{1}{99}$, $\frac{1}{999}$, etc., of the resistance of the galvanometer. Such shunts will not have the same multiplying factor when used with a galvanometer of different resistance, and therefore they can be used advantageously only with the galvanometer for which they were made. Placing a shunt in parallel with a galvanometer reduces the total resistance of the circuit, and therefore the current measured by the galvanometer times the multiplying factor of the shunt does not give the value of the original current but the value of the new main current. Sometimes an extra resistance of $0.9g$, $0.99g$, or $0.999g$ is inserted to keep constant the total resistance of the circuit.

When the galvanometer is used ballistically, these shunt ratios are not the same as for steady currents, because of the varying amounts of damping produced by the different shunts.

106. The Universal Shunt.—The *universal shunt* is so called because it can be used with any galvanometer and its shunt ratios will be the same. This arrangement is shown in Fig. 49. The total resistance,

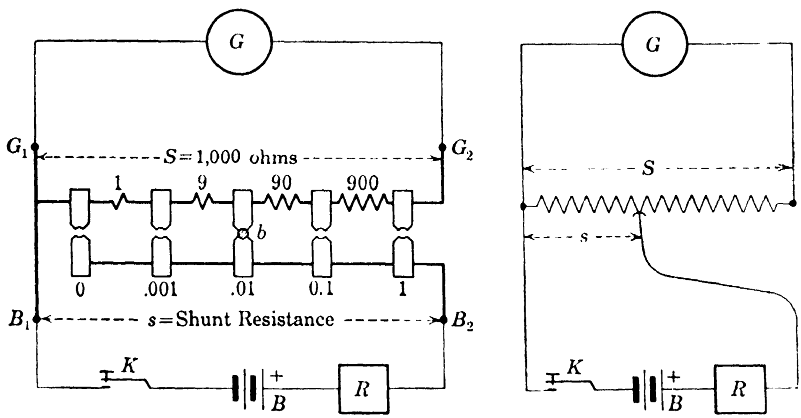


FIG. 49. Diagram illustrating the universal shunt.

shown in the figure as 1,000 ohms, is connected as a permanent shunt on the galvanometer. The current to be measured divides at b, most of it passing through the part s of the total resistance S. In the figure s is shown as 10 ohms, the remaining 990 ohms acting merely as resistance in series with the galvanometer.

The expression for the multiplying factor of a universal shunt may be derived as follows: Let g denote the resistance of the galvanometer, S the total resistance of the shunt, and s the portion of S corresponding to a common shunt that carries that part of the current not flowing through the galvanometer. The fall of potential over the galvanometer and the part of S that is in series with the galvanometer is equal to the fall of potential over the part s. Let I denote the value of the main current, and i the current through the galvanometer. Then

$$i[g + (S - s)] = (I - i)s,$$

or

$$I = i\frac{g + S}{s}.$$

For this form of shunt, therefore, the multiplying factor is inversely as s, since $(g + S)$ remains constant while s varies. For this reason, a shunt of this type can be used with different instruments, and the deflections of each will be proportional to the values of s for the same value of the main current. That is, the shunt will have the same series of multiplying factors for every galvanometer. The numbers 1, .1, .01, etc. that are usually seen on a universal shunt box indicate the relative values of the galvanometer current i for the same current I in the main line.

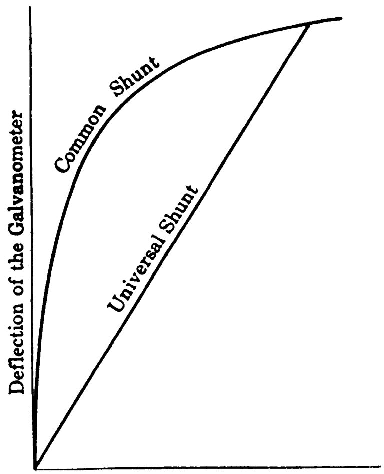


FIG. 50.—Shows the relation between the galvanometer deflection and the resistance in the shunt. With the same total current for each deflection, the straight-line curve for the universal shunt shows that the galvanometer deflection is proportional to the resistance in the shunt.

The curve for the common shunt shows the deflections of the same galvanometer with the same total current when common shunts of different resistances are used. For small values of the shunt, the deflections increase rapidly as the shunt is made larger. The larger deflections are not changed much by increasing the resistance of the shunt.

The advantage of the universal shunt is evident.

107. Arrangement of the Universal Shunt.—Any ordinary resistance box having a traveling plug for making a third connection at any intermediate point can be used as a universal shunt for any galvanometer, or two resistance boxes may be used when a shunt box as shown in Fig. 49 is not available. The shunt ratios are very accurate for all values, since all the coils are even ohms and can be adjusted much more precisely than with common shunts. Differences in temperature between the galvanometer and shunt produce no error, but these differences should remain constant while measurements are being made.

When a common shunt is used, the combined resistance of the shunt and the galvanometer is always less than that of the galvanometer. With a universal shunt (especially when set at the 0.1 ratio) the combined resistance of the shunt and the galvanometer may be much larger than the resistance of the galvanometer alone. In many kinds of work it is not essential that the resistance shall be constant or even known. Where it must be known, it can be determined for the galvanometer and its shunt combined as readily as for the galvanometer by itself.

Universal shunts are marked with the numbers 0.1, 0.01, and 0.001, implying the fractions of the current that they pass through the galvanometer. It is evident that when the universal shunt is used at the point marked 1, the galvanometer is in parallel with the large resistance of the entire shunt, and therefore the current in the galvanometer is not quite the full current on the main line. If S is several times g, this slight reduction in the sensitiveness is of small moment. The essential thing is that when the shunt is set at 0.1, 0.01, etc., the same total current will give deflections 0.1, 0.01, etc., as large as with the shunt set at 1. This arrangement of the galvanometer shunts is especially useful because a given series of shunts will have the same relative multiplying factors when used with any galvanometer. Since the damping is constant, the shunt ratios remain the same when the galvanometer is used ballistically.

108. To Measure the Multiplying Factor of a Shunt. *Test of a Shunt Box.*—The effect of a shunt in increasing the amount of current that can be measured by a galvanometer may be determined experimentally for the actual conditions of using the shunt.

Let us consider the case of a steady current I flowing through the circuit and dividing between the galvanometer and its shunt. The galvanometer will give a steady deflection d due to the current i through it. Then

$$i = Fd,$$

where F is the figure of merit of the galvanometer alone.

Without changing the deflection or the current in any way, let us, secondly, think of the combined galvanometer and shunt as a single instrument. The total current, I, passes through this instrument, and the deflection is d. Then

$$I = F'd,$$

where F' is the figure of merit of the combined galvanometer and shunt.

The multiplying factor m of a shunt is the ratio of the main current to the current through the galvanometer. This gives

$$m = \frac{I}{i} = \frac{F'd}{Fd} = \frac{F'}{F}.$$

Thus, by determining the values of F and F', the multiplying factor of a shunt can be obtained. Or if m is known, the value of F' can be computed ($= mF$). This measured value of m should be compared with the computed value, $m = (g + s)/s$, when possible.

In the same manner, all the shunts in the shunt box should be tested and the multiplying factor of each one determined. The results should be compared with the values stamped on the shunt box and also with the computed values as determined by the relative resistances of the galvanometer and the shunt.

When using the shunts of lowest resistance, it may be better to determine the figure of merit by the direct-deflection method (Art. 97). The scale deflection should be about the same for each shunt.

109. Resistance of a Galvanometer by Half Deflection. (*a*) *Resistance in Series.*—In some of the foregoing exercises it is necessary to know the resistance of the galvanometer as it has been used, either alone or combined with a shunt. When this is unknown, it can be determined with a fair degree of accuracy by the method of half deflection.

Let the galvanometer be connected to a source of potential difference that is small enough to keep the deflection on the scale with no additional resistance in series with the galvanometer. Then add enough resistance, R, in series with the galvanometer to make the deflection exactly one-half of its former value. This means that the current has been reduced to half its former value and therefore the resistance $R + g$ in the galvanometer circuit has now been made twice as much as when the galvanometer was used alone. That is,

$$R + g = 2g, \qquad \text{or} \qquad g = R.$$

To avoid the errors arising from thermal currents, etc., it is best to reverse the battery and repeat the measurements, taking the mean of the two results as the correct value of g. The resistance of a voltmeter can be measured by this method.

110. Resistance of a Galvanometer by Half Deflection. (*b*) *Resistance in Parallel.*—With a sensitive galvanometer the arrangement shown in Fig. 51 is used. The potential divider PQ allows a small part of the emf of the battery to be used in the galvanometer circuit, and the large resistance R keeps the galvanometer current small enough to give a fair deflection, say 100 or 200 mm. Then let a resistance box S be joined in parallel with the galvanometer and let the resistance in S be varied until the deflection is just half of its former value. The current is now divided between the galvanometer and its shunt, half of the original current flowing through the galvanometer and the rest through the shunt. If the main current is unchanged, this means that the current is equally divided between the galvanometer and its shunt. From this it follows that the resistance of the galvanometer is equal to the resistance of the shunt.

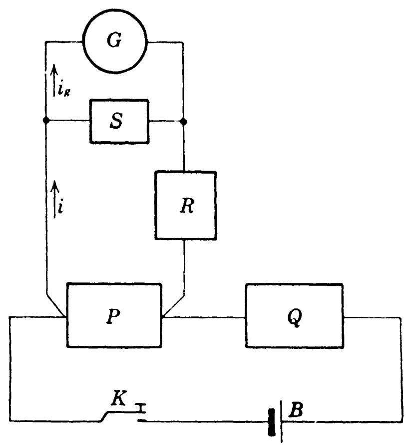


FIG. 51.—Resistance of a galvanometer. When $S = g$, $i_g = \frac{1}{2}i$.

It is true that the addition of the shunt has reduced the resistance of this portion of the circuit, but as S is small compared with the resistance R in this circuit, the current i in the second case, which is divided between the galvanometer and the shunt, will be larger than the current in the first case by less than can be read on the galvanometer scale. This point can be tested by placing the shunt resistance in series with R and noting whether it produces an appreciable effect in reducing the deflection.

Problem.—Let the student give a mathematical proof that $g = S$.

111. A More Satisfactory Method. (*c*) *Full Deflection.*—It is not always true that half the deflection of a galvanometer corresponds to half as much current. Naturally this casts some doubt on the accuracy of any half-deflection method. This difficulty can be avoided by a slight change in the method shown in Fig. 51. At the time the shunt S is added to the circuit, let the resistance P be doubled, keeping $(P + Q)$ the same as before. This doubles the current through R, and this doubled current is halved by S, thus giving the same current in G as before. The deflection is at the same point on the scale as before and no question arises regarding the relation of the two deflections.

112. Making an Ammeter.—An interesting way to use a galvanometer is to construct a direct-reading ammeter with the galvanometer as the moving element. The problem is to set up the galvanometer with a suitable shunt and other resistances, so that when a current of 1 amp (or other assigned amount) is passed through the arrangement, the galvanometer will stand deflected 100 divisions.

When a known resistance of 0.01 ohm is available, it makes a useful shunt to carry most of the current that is to be measured. This low resistance should have permanent current and potential terminals, as shown in Fig. 99. If the galvanometer is connected to the potential terminals, the current will divide between the two parallel circuits, most of it flowing through the low resistance of the shunt and a small part of it, i, flowing through the galvanometer.

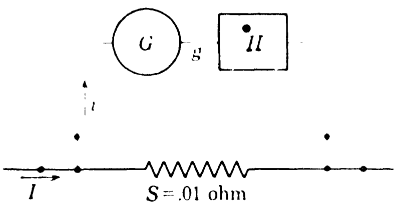


FIG. 52.—An ammeter built up with a galvanometer and shunt.

From the law of shunts (Art. 104),

$$I = \frac{g + s}{s} i = \frac{g + s}{s} Fd',$$

where d' is the deflection corresponding to the main current, I, and F is the figure of merit of the galvanometer (Art. 98).

Since F and s are already fixed and we wish to have d' equal to 100 divisions when I is 1 amp, the resistance of the galvanometer circuit should be

$$g = G + H = \frac{Is}{Fd'} - s = \left(\frac{1}{100F} - 1\right)s.$$

If the galvanometer resistance is less than this, it can be increased by adding in series sufficient resistance H, as in Fig. 52, to give the desired amount.

If F has been determined accurately, the galvanometer should show a deflection of 100 divisions when the main current is 1 amp. This can be tested by putting a good ammeter in the main line to measure I. If this check is satisfactory, it gives confidence that by changing H and s the galvanometer can be set to measure any other desired range of current.

113. To Make a Microammeter.—After an ammeter as shown in Fig. 52 has been built and is found to measure the main current I

as well as a good ammeter in the main line, an ammeter of any other range can be made with the same degree of accuracy.

Suppose it is desired to make an ammeter in which a main current of 1 μamp (0.000001 amp) will give a deflection of 100 divisions on the scale of the galvanometer. With this small current the value of the shunt S will need to be larger. Various values can be tried in the above formula for g to find resistances that are suitable and available to use for S and H, Fig. 52. When the ammeter is built, one must trust the correctness of this computation for the accuracy of the instrument.

114. To Make a Voltmeter.—A very excellent voltmeter can be made with a galvanometer and a high resistance of 100,000 ohms or more. The ideal voltmeter is an instrument that will measure emf without destroying or changing what it is trying to measure. Therefore, the less current that is drawn from the battery or other source, the better, since this will allow the available emf at the terminals to be nearer the value of the full emf.

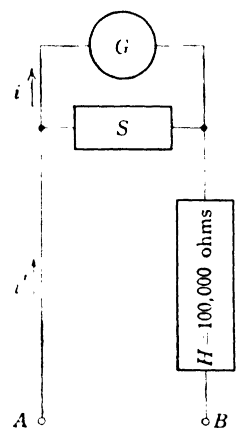


FIG. 53.—A voltmeter built up with a galvanometer and a high resistance.

Suppose the problem is to make a voltmeter that will read 100 divisions per volt. The arrangement of additional resistances depends upon the current sensitivity of the galvanometer, and this determines which of the following methods should be used.

(*a*) *By Series Resistance.*—The current through the galvanometer for a steady deflection of 100 divisions will be

$$i = 100F, \tag{3}$$

where F is the figure of merit (Art. 98) of the galvanometer. In order to obtain this current from a 1-volt source, the resistance of the galvanometer circuit must be

$$g' = H + g = \frac{V}{i} = \frac{1}{100F}, \tag{4}$$

where g denotes the resistance in the galvanometer itself and H is the resistance that is added in series with the galvanometer.

Solving this equation gives the value of H. When this amount of resistance is available, it can be added to the galvanometer as shown in Fig. 53 (with S omitted) to make a direct-reading voltmeter. If H

requires more resistance than is available, the following modification can be used.

(b) *By Shunt and Series Resistance.*—When the largest resistance that is available is still too small for H and would allow a current greater than $100F$ in the galvanometer, a shunt S can be connected in parallel with the galvanometer, as in Fig. 53. By the law of shunts (Art. 104),

$$i' = \frac{g + s}{s} i, \tag{5}$$

where i' denotes the value of the current in H, and i is the part of this current that passes through the galvanometer. The total resistance of this circuit is

$$H' = H + \frac{gs}{g + s}, \tag{6}$$

and the current for V volts between A and B (Fig. 53) is

$$i' = \frac{V}{H'} = \frac{V(g + s)}{Hg + Hs + gs}. \tag{7}$$

Equating these two values for i' from (5) and (7) gives

$$\frac{V(g + s)}{Hg + Hs + gs} = \frac{(g + s)i}{s}. \tag{8}$$

Solving (8) for the value of s gives

$$s = \frac{Hi}{V - (H + g)i} g. \tag{9}$$

If it is desired to have a galvanometer current of $i = 100F$ with $V = 1$ volt, then the shunt should be

$$s = \frac{100FH}{1 - 100F(H + g)} g \tag{10}$$

Of course s cannot be computed any more closely than g is known. Hence the importance of knowing g is evident. Since the galvanometer coil is of copper wire, which changes with temperature, the value of g should be determined at the time it is used.

If this value of s is a fraction of an ohm over an integer, and S (Fig. 53) can be varied only by 1-ohm steps, a little resistance can be added to g to make the value of s a whole number.

The voltmeter thus built can be compared with a standard voltmeter. When both voltmeters are in parallel with each other they should read the same.

To show the advantage of a high-resistance voltmeter, it can be used to measure the emfs of several old dry cells. After this is done, the same cells can be measured with the standard voltmeter. The latter will be found to give results that are too low, because of the internal resistance of the cells.

115. To Make a Voltmeter by Using a Microammeter.—If a microammeter has been constructed as described in Art. 113, a voltmeter is readily made by adding a resistance of 1 megohm in series with it. Strictly, the total resistance of this voltmeter should be only 1 megohm, but if the microammeter is less than 1,000 ohms, the extra resistance will reduce the deflection by less than can be read on the scale.

This voltmeter can be compared with a standard voltmeter. It should read 100 divisions when in parallel with the standard voltmeter at 1 volt. This check tests both the accuracy of building the microammeter and that of making the voltmeter.

Problem.—What should be the total resistance of this voltmeter so that 1 mv will give a deflection of 100 divisions?

CHAPTER IV

THE WHEATSTONE BRIDGE

116. The Wheatstone Bridge.—The Wheatstone bridge consists, essentially, of two circuits in parallel and through which an electron current can flow. Let these circuits be represented by ABD and ACD, Fig. 54, and let the currents through the two branches be denoted by I and I'. Since the fall of potential from A to D is the same whichever path is considered, there must be a point C on one circuit that has the same potential as any chosen point B on the other, and a galvanometer bridged across between these points will indicate zero deflection. When the current in the galvanometer is zero with currents in the other branches, the bridge is said to be balanced.

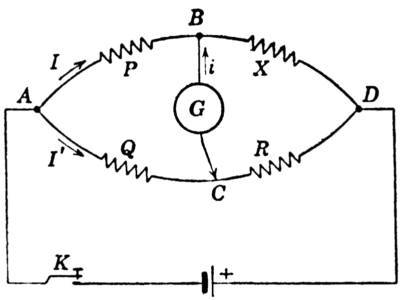


FIG. 54.—Principle of the Wheatstone bridge.

To find the relations that will give the balanced condition, let us write out the equations of potential differences for the circuits of the network shown in Fig. 54. Let the arrows indicate the direction that is *up* the potential gradient, *i.e.*, from the negative side of the battery towards the positive side. When the bridge is not quite balanced, let us take the case for which B is at a slightly higher potential than C. Then as our pencil point traces over the circuit $ABCA$, the potential differences it passes over are

$$+PI - gi - QI' = 0,$$

where g and i refer to the galvanometer.

Similarly, for the circuit $BDCB$,

$$+X(I + i) - R(I' - i) + gi = 0,$$

where the current in X is the sum of the currents in P and the galvanometer.

For the special case of $i = 0$, when a balance is obtained, the equations become

$$PI - QI' = 0$$

and

$$XI - RI' = 0.$$

Solving the first equation for I' and using this value in the second equation gives

$$\frac{X}{P} = \frac{R}{Q}$$

as the relationship of the resistances when the bridge is balanced. In the usual method of using the Wheatstone bridge, three of these resistances are known and the value of the fourth is easily computed from the above relation as soon as a balance is obtained.

117. The Slide-wire Bridge—Simple Method.—The Wheatstone bridge principle is used in several forms of apparatus for the measurement of resistance. The simplest of these is the slide-wire bridge as shown in Fig. 55. The unknown resistance that is to be measured is placed at X, while at R is the known resistance, usually a box of coils. The branch ACD consists of a single uniform wire, usually a meter[1] in length, stretched alongside or over a graduated scale.

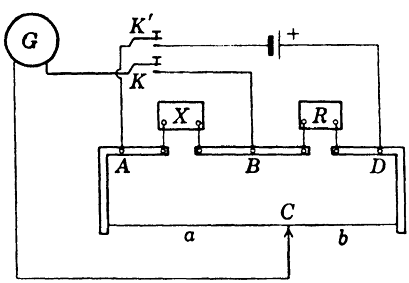


FIG. 55.—The simple slide-wire bridge.

This wire should be of a high-resistance alloy. The balance is obtained by moving the contact C along the wire until a point is found for which the deflection of the galvanometer is zero when K' and K are closed. This contact should not be scraped along the wire; it should always be raised, moved to the new point, and then gently but firmly pressed into contact with the wire. Neither should it be used for a key, as the continual tapping will dent the wire and destroy its uniformity. The two keys, K' and K, are combined into a single successive-contact key, often called a "Wheatstone bridge key," in which one motion of the hand will first close the battery key and then, after the currents have been established, will close the galvanometer circuit. The need of such a key is very evident when there is self inductance (Art. 414) in X.

[1] Because a meter scale of 1,000 numbered divisions is available.

When the point C has been located, we have, as in the preceding article,

$$xI' = apI,$$

where a is the length of the bridge wire from A to C, and p is the resistance per unit length of this wire. Thus ap is the resistance of this portion of the bridge wire, and x denotes the value of the resistance of X. Similarly,

$$RI' = bpI,$$

where b is the length of the bridge wire from C to D. In this discussion the resistances of the heavy straps are neglected and A and D are considered as being at the ends of the bridge wire. Dividing each member of the first equation by the corresponding member of the second equation gives

$$\frac{x}{R} = \frac{a}{b}.$$

Thus the ratio of the unknown resistance to R is given by the ratio of the two lengths into which the bridge wire is divided by the balance point.

This relation can be expressed in terms of a single length a by writing

$$b = c - a,$$

where c denotes the total length of the bridge wire. Then

$$x = R\frac{a}{c - a} = R\frac{a}{1{,}000 - a},$$

if the total length of the bridge wire is 1,000 mm.

Measure in this way the resistances of two or more coils. Also measure the same coils when joined in series and compare the result with the computed value,

$$R = R' + R''.$$

When two coils are joined in parallel, the measured resistance should fulfill the relation

$$\frac{1}{R} = \frac{1}{R'} + \frac{1}{R''}.$$

Problems

1. Exchange the positions of the battery and the galvanometer and then deduce the formula for x, as above.

2. Prove that for three resistances in series

$$R = R_1 + R_2 + R_3$$

and in parallel

$$R = \frac{R_1R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}.$$

3. Deduce the corresponding expressions for five resistances.

118. Calibration of the Slide-wire Bridge.—In deducing the formula for the slide-wire bridge, it was assumed that the bridge wire was divided into 1,000 parts of equal resistance and that the readings obtained from the scale corresponded to these divisions. To make sure that the scale readings do thus correspond to the bridge wire, it is necessary to calibrate the wire, *i.e.*, to determine experimentally which readings on the scale correspond to the 1,000 equal-resistance points on the wire.

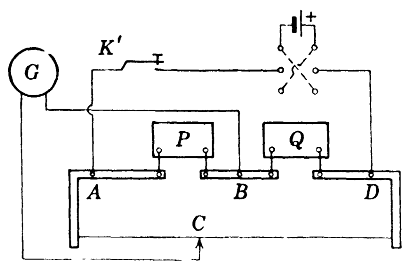


FIG. 56.—Calibration of the simple bridge.

Two well-adjusted resistance boxes are inserted in the two back openings of the bridge, and the battery and galvanometer are connected in the usual manner. Only the battery key is needed in this calibration, as good resistance boxes are noninductive. Closing the galvanometer key often gives a small deflection due to thermal emfs, and this deflection interferes with the determination of the balance point. It is better, therefore, to keep the galvanometer circuit closed and to observe the deflection when the battery circuit is opened and closed. If now, for example, 500 ohms are put in each box, the balance point will be at the middle of the bridge wire, and this should be at the point marked 500 on the scale. If it is not here but falls a distance f below 500, then f is the correction that must be added to the observed reading to obtain the true reading. Since there may be thermal currents in the galvanometer circuit, this value of f should be computed from the mean of two readings, one taken with the battery current direct and the other taken with the battery reversed.

In the same way the true location of the points 100, 200, 300, 400, 500, 600, 700, 800, and 900 can be found. It will be found convenient to keep the sum of P and Q always constant at 1,000 ohms.

Finally, a calibration curve is drawn, with the readings of the scale for abscissas and the corresponding corrections for ordinates. The corrections for any point on the scale can then be read directly from the curve. The corrected readings are thus expressed in thousandths of the total bridge wire, including all the resistance of straps, connections, etc., between the two points where the battery is attached.

CALIBRATION OF BRIDGE No. ....

P	Q	Scale readings			True reading $\frac{P}{P+Q}$	Cor.
		Battery direct	Battery reversed	Mean (m)	(n)	$(n-m)$

119. Double Method of Using the Slide-wire Bridge.—In the simple method above, only a single balance point was obtained, and the value of the unknown resistance was computed from the relation

$$\frac{x}{R} = \frac{a'}{1{,}000 - a'},$$

where a′ denotes the reading on the scale at the point of balance and is assumed to be the length of one portion of the bridge wire.

The measurement of resistance will be more precise if x and R are exchanged with each other, without, however, changing the value of either one, and a new balance point is determined. This second balance point will be, say, at a'' on the scale, and

$$\frac{x}{R} = \frac{1{,}000 - a''}{a''}.$$

Combining these two equations by the addition of proportions,

$$\frac{x}{R} = \frac{1{,}000 + (a' - a'')}{1{,}000 - (a' - a'')} = \frac{1{,}000 + d}{1{,}000 - d},$$

in which the actual values of a' and a'' do not appear, but only their difference. Thus all questions regarding the starting point of the scale or the wire are eliminated, and if d is small, any error made in its determination will have only a small effect upon the value of x as computed from this equation.

120. Advantages of the Double Method.—When it is desired to use the slide-wire bridge with some degree of precision, several precautions are necessary in order to avoid the principal errors. Prominent among these are the effects due to thermal emf in the galvanometer circuit, to balance which it is necessary to set the sliding contact on the wire at a point somewhat to one side of the true balance point to obtain zero deflection of the galvanometer. If the scale is displaced endwise with respect to the wire, or if the index from which the readings are taken is not exactly in line with the point at which contact is made on the wire, the effect is much the same.

Let a' denote the observed reading on the scale, and $a' + w$ the true balance point, where w denotes the displacement of the reading due to the causes noted above. The actual value of w is unknown, but it is constant in amount and sign, at least while one set of readings is being taken. For a balance of the bridge with x and R in the positions shown in Fig. 55, we have

$$\frac{x}{R} = \frac{a' + w}{c - (a' + w)},$$

where c is the total length of the bridge wire expressed in the same units as a' and w—usually in millimeters.

Interchanging x and R gives a new balance at a'', and

$$\frac{x}{R} = \frac{c - (a'' + w)}{a'' + w}.$$

Combining these two expressions by the addition of proportions gives

$$\frac{x}{R} = \frac{c + (a' - a'')}{c - (a' - a'')} = \frac{c + d}{c - d}.$$

It is seen that w has been eliminated by this double method and that the only measured quantity appearing in the final expression is d, the length of the wire between the two *observed* balance points. The value of c should be determined by the method given in Art. 141, as this may be greater than the meter of bridge wire because of the added resistance of the copper straps and the connections at each end of the wire. The "bridge wire" really includes all the resistance from A to D. However, if d is small, a slight uncertainty in the value of c will produce a negligible error in the computed value of x. This means that R should be taken as near to the value of x as is convenient.

121. The Best Position of Balance. (*a*) *Simple Bridge.*—The formula deduced in Art. 117 for the value of a resistance measured by the simple slide-wire bridge is

$$x = R\frac{a}{c - a}. \qquad (1)$$

Suppose that the value of a can be read to within $\pm h$ mm of its true value. This uncertainty will be the same at one part of the scale as at any other, but its effect in the formula depends upon the value of a. Using a value of a that is uncertain by $\pm h$ gives

$$R\frac{a \pm h}{c - (a \pm h)} = x \pm f, \qquad (2)$$

where f denotes the corresponding error introduced into x. It is required to find the value of a that will make the effect of $\pm h$ as small as possible.

122. The Total Error.—Subtracting (1) from (2) leaves

$$\pm f = R\left(\frac{a \pm h}{c - (a \pm h)} - \frac{a}{c - a}\right) = R\frac{\pm hc}{(c - a \mp h)(c - a)}. \qquad (3)$$

While the uncertainty h cannot be neglected in computing the value of x, in finding the small quantity f it is near enough to call $c \mp h = c$. Then

$$f = R\frac{hc}{(c - a)^2}. \qquad (4)$$

This value of f, given by (4), is the uncertainty in the computed value of x by (1) due to the uncertainty h in determining the proper value for a.

In the notation of the calculus, the same result is obtained. When h is small, f corresponds to dx, and h to da. Differentiating (1) gives

$$\frac{f}{h} = \frac{dx}{da} = R\frac{c}{(c - a)^2},$$

the same as before.

123. The Relative Error.—The relative error in the value of x as given by (1) is usually of greater importance than the actual error in ohms. It is evident that an error of 1 ohm in a total of 10 ohms is a very different thing from an error of 1 ohm in 1,000 ohms. The relative error is the ratio of the actual error to the total quantity measured. Thus from (1) and (4) the relative error e is

$$e = \frac{f}{x} = h\frac{c}{(c - a)a}.$$

From this it appears that even the relative error is not the same for the same error in reading but that it depends upon the value of a. If this expression is examined for a minimum value of e,

$$\frac{de}{da} = \frac{-c(c - 2a)}{[(c - a)a]^2} h = 0.$$

This is satisfied if

$$(c - 2a) = 0.$$

Thus, in reading the value of a, a given error (say 1 mm) will produce the least effect on the computed value of x when the balance point comes at the middle of the bridge wire.

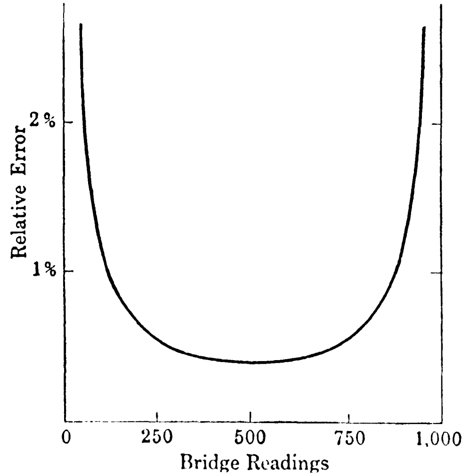


FIG. 57.—An uncertainty of 1 mm. in estimating the value of a produces less error in x when the balance point is near the middle of the bridge.

While this error is smallest at the middle of the bridge wire, it is not much greater when the balance falls at any point on the middle third of the scale. Figure 57 shows how rapidly this error increases when the balance point is near either end of the bridge wire.

124. The Best Position of Balance. (*b*) *Double Method.*—The above discussion applies to the simple slide-wire bridge. When the double method is used, the formula for the resistance being measured is

$$x = R\frac{1{,}000 + (a' - a'')}{1{,}000 - (a' - a'')},$$

as deduced in Art. 119.

If this is written as

$$x = R\frac{c + z}{c - z},$$

it can be shown in the same way that a given error in measuring z will have the least effect upon the computed value of x when $z = 0$, *i.e.*, when both a' and a'' are at the middle of the bridge wire. In this case

$$\frac{f}{h_z} = \frac{dx}{dz} = R\frac{2c}{(c - z)^2},$$

and

$$e = h_z\frac{2c}{(c - z)(c + z)} = \frac{2ch_z}{c^2 - z^2},$$

which evidently is a minimum when $z = 0$.

125. Sources of Error in Using the Slide-wire Bridge.—These may be summarized as

1. Errors in setting, due to
   a. Thermal currents.
   b. Contact maker not in line with index.
   c. Nonuniform wire or scale.
   d. Ends of wire and scale not coincident.
2. Errors in reading
   e. The position of balance.
   f. True value of R, loose plugs, etc.

The effect of a and b can be eliminated by using the double method, as explained in Art. 120.

The only way to avoid the effect of c or d is to calibrate the bridge wire and correct all readings, or to use the double method.

The error in reading the position of the index after a balance has been found is often greater than the uncertainty of the setting. In the preceding section it was shown that this error, which is about the same for all parts of the scale, has the least effect on the computed value of x when the reading is near the middle of the bridge wire.

The error in the resistance coils of a good box is very small. However, the value of R read from the box and used in computations may be very different from the actual resistance of the experiment. If some of the plugs are loose or if they make poor contact because of dirt or corrosion, the resistance may be considerably increased. Moreover, the resistance actually used in the bridge includes all the connections and lead wires used to join the box to the bridge. In the same way, the resistance measured includes the lead wires and connections.

If the apparent middle point of the bridge wire $[= \frac{1}{2}(a + a')]$ is not constant for different measurements it may lead to greater uncertainty than the errors mentioned above.

126. The Wheatstone Bridge Box.—In the slide-wire form of the Wheatstone bridge, the balance is obtained by locating a certain point

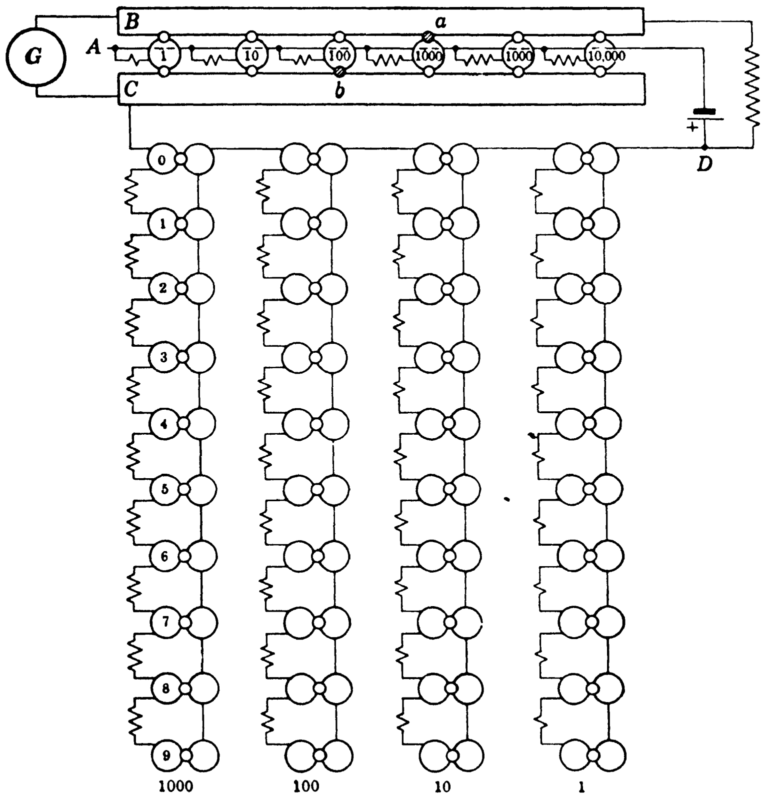


FIG. 58.—Diagram of a decade Wheatstone bridge box.

on the wire, and the accuracy of the measurement depends upon the accuracy with which the lengths of the two portions of the wire can be measured. In the Wheatstone bridge box the wire is replaced by a few accurately adjusted resistance coils. Thus while the number of ratios that can be employed is less than 10, the values of these few ratios are precise, even when the ratio is far from unity. The usual arrangement is to make P and Q (Fig. 54) the two ratio arms with the unknown resistance in X and to obtain the balance of the bridge by adjusting the resistance of R. The value of the unknown is then given by the usual relation

$$X = R\frac{P}{Q}$$

and is known as accurately as the value of R can be found.

In a common form of the Wheatstone bridge box, P and Q each contain 1-, 10-, 100-, and 1,000-ohm coils, thus giving ratios of 1,000, 100, 10, 1, 0.1, 0.01, and 0.001. The rheostat arm R can be varied by 1-ohm steps from 0 to 11,110 ohms. This gives a range of measurement of unknown resistances from 0.001 to 11,110,000 ohms.

A more convenient form is the decade bridge. The rheostat arm is arranged on the decade plan with one plug for each decade. The resistance in this arm is indicated by the position of the plugs, which always remain in the box. The ratio arms consist of a single series of coils of 1 ohm and 10, 100, 1,000, 1,000-, and 10,000-ohms, as shown in Fig. 59, and any coil can be used in either arm (but of course the same coil cannot be used in both arms at the same time). The connections not visible are clearly indicated by lines drawn on the top of the box. The different parts should be carefully compared with the diagram of Fig. 54, Art. 116, and the points A, B, C, D should be located before an attempt to use the box is made. The resistance to be measured is joined to the posts marked X. The battery and the galvanometer are connected as shown, with a key in each circuit—preferably a successive-contact key. When it is known that X is a noninductive resistance, it is better to omit the galvanometer key and thus avoid the troublesome deflections due to thermoelectric currents in the galvanometer circuit.

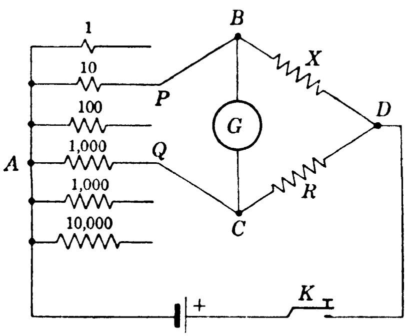


FIG. 59.—Showing the six resistance coils from which P and Q may be chosen.

127. To Use a Wheatstone Bridge Box.—When starting to obtain a balance, set each of the ratio arms at 1,000 ohms and determine an approximate value of the resistance. This is done by shunting the galvanometer with the smallest shunt available, and with R set at 1 ohm the keys are quickly tapped and the direction of the deflection is noted. The key should not be held down long enough to cause a large deflection, as the direction can be seen from a small one just as well and with less danger to the galvanometer. Next, R is set at 9,000 ohms and the key is tapped. Usually this deflection will be in the opposite direction. If it is not, try zero and infinity. Knowing that the value of X lies between 1 and 9,000, say, divide this range by next trying 100 ohms, and if X is less than this, try 10 ohms. Suppose X is between 10 and 100. This range is divided by trying 50, and so on until it is reduced to a single ohm. Let us suppose that X is found to be between 68 and 69 ohms. Then the ratio arms are changed so as to make R come to 6,800 or 6,900. The exact value for a balance is determined by continuing the same process and is found, say, when R is 6,874 ohms. This example then gives $X = 68.74$ ohms.

If the best balance, obtained with no shunt on the galvanometer or with a universal shunt set at 1.0, still gives some deflection, the next figure for X can be obtained by interpolation between two adjacent values of R, but this is not usually required. If greater accuracy is desired, it is necessary to make a second measurement with the battery current reversed through the bridge. This will reverse some of the errors, especially the effect of thermal currents in the galvanometer branch. The mean of these two measurements will then be nearer the true value of X than either one alone.

Measure the resistance of several coils and check the results by measuring their resistance when joined in series and parallel. If some of these coils have an iron core, notice the effect of first closing the galvanometer key and then closing the battery key. Remember that the formula for this method was deduced on the assumption that all the currents were steady and that there was no current through the galvanometer.

The bridge balances can be recorded as follows:

Object measured	P	Q	R	X	Temperature of X

128. The Per Cent Bridge.—A very useful and convenient form of the Wheatstone bridge is the arrangement shown in Fig. 60. When it is desired to compare coils that are nearly equal to each other, the ratio of the unknown resistance to the standard resistance is often more desired than the actual value of the unknown resistance. In the per cent bridge the coils are arranged so that the dial readings give directly the ratio of the unknown resistance to the resistance with which it is being compared.

As shown in Fig. 60, one of the ratio arms of the bridge is a fixed resistance of 1,000 ohms. The other ratio arm consists of a fixed resistance of 950 ohms in series with P, which is a dial resistance box of tens, units, and tenths of ohms. When x and R are equal, it will require 50 ohms in P to give a balance. Mark this setting of P as 100 per cent. If x is 1 per cent larger than R, it will require another 10 ohms in P, and this setting can be marked 101 per cent; 70 ohms would correspond to 102 per cent; etc. If the setting of P for a balance is 76.1 ohms, it indicates that x is 102.61 per cent of R. If the index on the first dial can be set to read "20" when it stands at 70 ohms, the last three figures, 261, can be read directly from the box. The range of this bridge is from 95 to 105 per cent. By increasing the ratio arms to 10,000 ohms each, the setting of P can be read to thousandths of 1 per cent. If lead wires are needed to connect x to the bridge, the lead wires for R should be made of the same resistance.

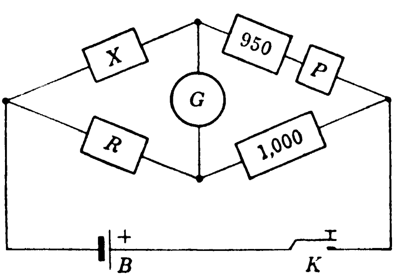


FIG. 60.—The direct-reading per cent bridge.

129. Location of Faults.—A *fault* on a telephone, electric-light, or other line is any trouble which impairs the insulation of the line or which interferes with the proper working of the line. The principal kinds of faults are named as follows:

A *ground* is an electrical connection more or less completed between the line and the earth. In the case of a cable any connection from one of the wires to the lead covering of the cable constitutes a ground.

A *cross* is an electrical connection between two wires.

An *open* is a break in the line.

In testing for a fault it is first necessary to determine in which of these classes the given trouble belongs. A testing circuit is made by connecting a battery in series with a voltmeter or other current indicator. The faulty line is then picked out from among the good ones by one of the methods outlined below.

Test for Grounds.—The test to find grounded wires can be made at any point along the line. The battery side of the testing circuit just described is connected to the ground, and the wire from the voltmeter side is brought into contact successively with each wire to be examined. When the grounded wire is reached, the battery circuit is completed through the earth, and this will be indicated by a deflection of the voltmeter.

Tests of this kind made with alternating current are often unreliable because of the capacitance of the line.

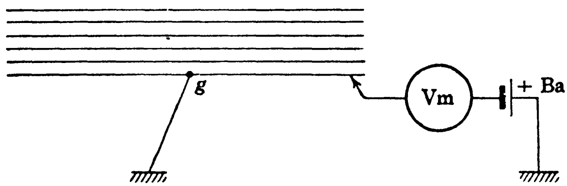


FIG. 61.—Picking out the grounded wire.

Test for Opens.—Before testing for opens, join the distant ends of the wires together and ground them. The test is applied at the near end, with the testing circuit used in the manner just described. As each wire is tried, the voltmeter will indicate the ground that has been placed on the other end, unless the line is open at some intermediate point. If the line is broken, the pointer of the voltmeter will remain at rest, showing that there is no electrical connection through that wire.

Test for Crosses.—In testing for crosses, the near ends of all the wires are connected together and to one end of the testing circuit.

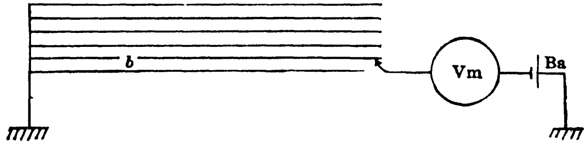


FIG. 62.—Picking out the broken line.

The wires are then disconnected one at a time, and the free end is joined to the other end of the testing circuit. A movement of the voltmeter pointer shows that the wire being tested is crossed with one of the other wires. If there is no indication of a cross, the wire is laid at one side and the test is continued with other wires until all the good wires have been removed and only crossed wires remain.

When there are several sets of crossed wires among those remaining, the near ends of all the crossed wires should be separated. Then connecting them one at a time to the battery side of the testing circuit, touch each of the other wires in succession with the wire from the voltmeter. A deflection indicates that the line touched is crossed with the one joined to the battery.

Of course the wires must not be connected at the other end or through any switchboard, as this would make another cross on each wire, and the voltmeter would show that each line is crossed with the others.

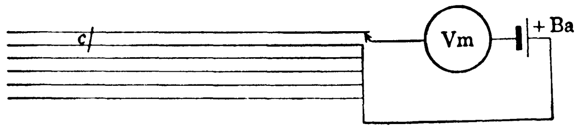


FIG. 63.—Picking out the crossed lines.

130. Methods for Locating Faults.—The usual methods for locating faults in a telephone line or cable are based on the principles of the simple slide-wire bridge. The following methods are illustrative examples of this kind of measurement and show the general mode of procedure. They are called "loop tests," because the wire being tested is joined to a good wire, thus forming a long loop out on one wire and back on the other.

If the line is open so that it cannot be used as one arm of a Wheatstone bridge, the position of the break can be located by comparing the capacitance of the line out to the break with the capacitance of a similar line whose length is known.

Sometimes the broken end of the wire falls across another wire, and the place can be found by locating the position of this cross between the wires.

131. The Murray Loop.—In the Murray loop method the grounded wire is joined to one end of a slide-wire bridge, as shown at A, Fig. 64. A second wire of the same length and resistance as the first, similar to it but free from faults, is joined at D to the other end of the bridge. These two wires are joined together at the far end of the line, thus forming the loop. This loop is divided into two portions by the fault at F, which in the figure is shown as a ground. These two parts form two arms of a Wheatstone bridge, the other two being formed by the bridge wire ACD. It is usually best to connect the galvanometer and battery in the position indicated, the battery connection at F being made through the earth.

Let C indicate the balance point. Let p denote the resistance per millimeter of the bridge wire and p' the resistance per unit length of the wire forming the loop. Then by the circuit principle for the circuit *CADC*, through the slide wire and the galvanometer, we can write

$$apI + gi - bpI' = 0 \tag{5}$$

For the case of balance, the galvanometer current i is zero, and so

$$apI = bpI' = (c - a)pI' \tag{6}$$

where c denotes the whole length of the bridge wire ($= a + b$), and a is the reading on the bridge scale, measured from the end connected to the faulty wire. Similarly,

$$dp'I = (2L - d)p'I', \tag{7}$$

from which

$$d = 2L\frac{a}{c}, \tag{8}$$

where d is the distance to the fault and L is the length of the faulty wire.

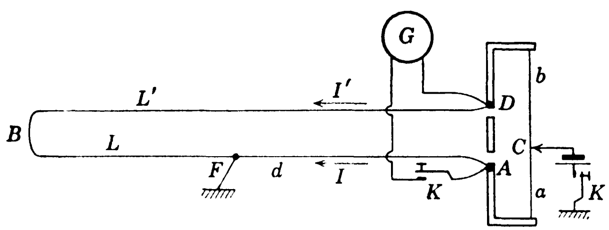


FIG. 64.—Locating the position of the ground F.

This determination can be checked by interchanging the good and the faulty wires in the arms of the bridge. This change requires a slight modification in the formula used in solving for d.

132. Accuracy of Measurement.—The value of d that is computed by (8) depends directly upon the measured value of a. When the location of the balance point is uncertain by $\pm h$, it gives

$$2L\frac{a \pm h}{c} = 2L\frac{a}{c} \pm 2L\frac{h}{c} = d \pm d', \tag{9}$$

where $d' = 2Lh/c$ is the resulting uncertainty in the value of d.

The uncertainty in the value of a may be (a) in finding the balance point or (b) in reading the scale after the balance is found. It may be due (c) to uncertainty at A or D regarding the spot where the galvanometer wire connects with the bridge circuit or (d) to the amount of uncounted resistance between A and the zero end of the bridge wire. A common cause that gives a displaced balance point is (*e*) thermal emf in the galvanometer circuit. When a is small, any error in reading the scale has a correspondingly large relative effect on its value.

Each of these points should be carefully examined with respect to its effect on d'. The largest effect should be made less, if possible. Sometimes better results are obtained when the positions of the galvanometer and the battery (Fig. 64) are exchanged with each other.

133. Fisher's Method.—It sometimes happens that no good wire like the grounded one can be obtained. It is still possible to locate the fault, provided that *two* good wires can be obtained, the only requisite being that they extend from the bridge to the far end of the faulty wire.

Make the connections as shown in Fig. 65, using one of the good wires, H, to complete the loop with the faulty wire. When the bat-

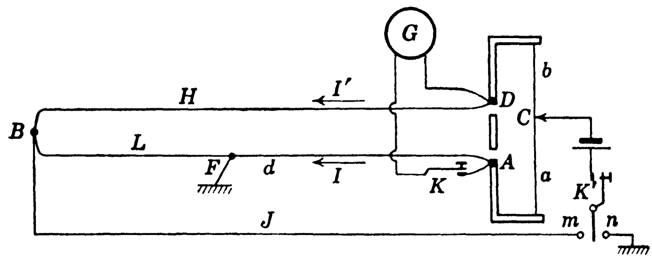


FIG. 65.—Finding the relative resistances of the two lines.

tery circuit is connected to the ground at n, a balance means that the equation of potential differences for the circuit $CADC$ is

$$apI_n + 0 - bpI'_n = 0, \tag{10}$$

with b and p having the same meaning as in Art. 131 and the subscript n indicating that the switch is closed at n. The arrows indicate the direction that is *up* the potential gradient. From this,

$$apI_n = bpI'_n. \tag{11}$$

For the circuit $AFBDA$ the equation is

$$dp'I_n - (L - d)p'I'_n - Hp''I'_n = 0, \tag{12}$$

which gives

$$dp'I_n = (Lp' - dp' + Hp'')I'_n. \tag{13}$$

Dividing (11) by (13), membe by member, gives

$$\frac{a}{dp'} = \frac{b}{Lp' - dp' + Hp''} = \frac{c}{Lp' + Hp''}. \tag{14}$$

Since the wires are different, the p' cannot be divided out of this equation, and it is necessary to compare the resistances of the wires L and H. This is readily done by closing the circuit at m, which in effect connects the battery to the point B on the loop. The good wire J is used for this connection. The ground at F will make no difference if there is not a second ground at some other place. The balance point will now be found at another reading a' on the slide wire.

For the case of a balance, which means no current in the galvanometer with K and K' both closed, the potential differences in the circuit $CADC$ are

$$a'pI_m - b'pI'_m = 0, \tag{15}$$

or

$$a'pI_m = b'pI'_m. \tag{16}$$

For the circuit $ABDA$ the equation is

$$Lp'I_m - Hp''I'_m = 0, \tag{17}$$

which gives

$$Lp'I_m = Hp''I'_m. \tag{18}$$

Dividing (16) by (18), member by member, gives

$$\frac{a'}{Lp'} = \frac{b'}{Hp''} = \frac{c}{Lp' + Hp''} \tag{19}$$

Since the third members of (14) and (19) are identical, the first members must be equal to each other, or

$$\frac{a}{dp'} = \frac{a'}{Lp'}. \tag{20}$$

The distance d to the ground at F is, then,

$$d = L\frac{a}{a'}, \tag{21}$$

where L is the full length of the faulty wire and d is the distance to the fault. Evidently d is given in the same units in which L is measured, irrespective of the units that are used for a and a'.

The laboratory observations can be recorded as follows:

Line	Fault	Loop	L	a	a'	d computed	d observed

134. Location of a Cross.—The methods for locating a cross are similar to those just given for locating a ground. The only difference is that instead of having the battery connected to F by means of the

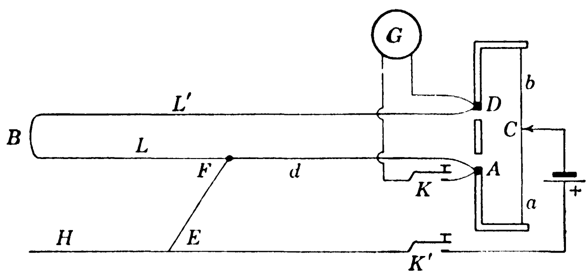


Fig. 66.—Locating the position of the cross EF.

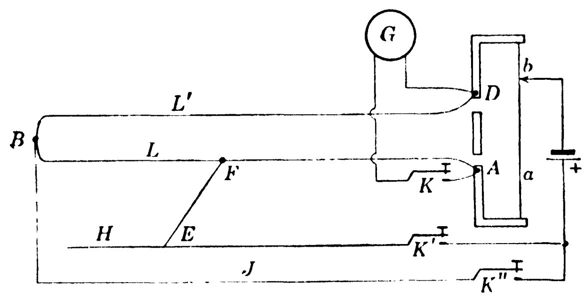


Fig. 67.—Locating the position of a cross when the wires are all unlike.

earth, as shown in Fig. 64, the connection is now made by means of the wire, which is crossed with the line AB. The balance point is found the same way as before, and the distance to the fault is computed by the formulas given above. Figure 66 shows the Murray loop for locating the cross EF between the lines L and H. Of course the location of the cross could have been made equally well by using line H in the bridge instead of L.

135. Location of a Cross. *Wires Unlike.*—In case the lines L and L' are unlike, it will be necessary to use another line, J, as shown in Fig. 67, and to obtain a second balance as in the preceding method (Art. 133). The distance to the cross is then given by the formula $d = L(a/a')$, derived as in Art. 133.

Problem. Draw the diamond-shaped diagram for Wheatstone bridge. Letter this diagram the same as in Fig. 67 to show the four arms of the bridge in each figure.

136. The Varley Loop Test.—It is often convenient to make the test for fault location by the use of a Wheatstone bridge box or a portable testing set, instead of using the slide-wire bridge. In the Varley loop method the far end of the grounded wire is connected to

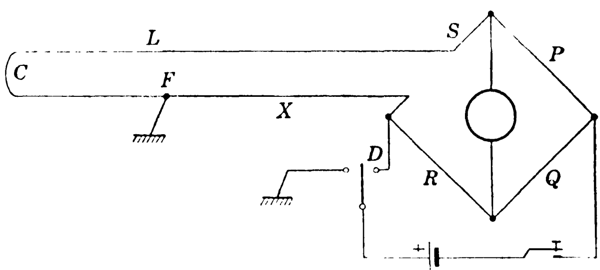


FIG. 68. Varley loop method for locating the ground at F, using a Wheatstone bridge box.

a good wire, and the circuit or loop thus formed is connected into the Wheatstone bridge as the unknown arm. This arrangement is shown in Fig. 68. The usual balance of the bridge gives the resistance S of this complete loop.

When the battery connection is changed from D to the ground, which, in effect, means that the battery is connected at F, and the bridge is again balanced, the relation is

$$\frac{S - X}{R + X} = \frac{P}{Q} = A,$$

where X is the resistance of the line from the bridge out to the ground at F, and A is written for the ratio of the arms P and Q. Usually A is 0.1 or 0.01. This gives

$$X = \frac{S - AR}{1 + A}.$$

When the number of ohms per meter or ohms per mile for this wire is known, the distance out to F is readily computed (by division). If both of the wires forming the loop are alike, the distance from the bridge end of the line to the ground at F is

$$d = \frac{X}{S} L,$$

where L is the total length of the loop. In practice this method is readily applied, as it requires only two balances of the bridge testing set, and a simple computation gives the distance, d.

In locating the position of a *cross* between two wires, the only difference in procedure is that the battery is connected to the line which is crossed with the one being tested, instead of being connected to the ground.

137. The Faultfinder.—Inasmuch as the meter slide-wire bridge is awkward to use except on a laboratory table, the essential parts are arranged in a compact box for field use.

Fig. 69.—Portable testing set.

One form of such a portable testing set is shown in Fig. 69. The ratio arms of the bridge are varied by turning the left-hand dial, and the other four dials control the known resistance R. The resistance to be measured is connected to the large binding posts at the front of the box. The battery and galvanometer are contained within the box.

This testing set is not only a portable Wheatstone bridge; by means of the knife switches at the left, the connections can be quickly changed for the Murray or Varley loop method.

138. Location of Opens.—When one of the lines is broken, it will not be possible to use it for one arm of a Wheatstone bridge. If the line is one of a pair, it may have sufficient capacitance to be measured. The simplest way is to charge one piece of the broken wire and then discharge it through a ballistic galvanometer. This deflection is compared with the deflection obtained when a known length of a similar wire is charged and discharged in the same way. Then

$$d = L \frac{d'}{d''}.$$

Usually a more exact determination can be made by the bridge method (Art. 398). Let *ac* and *ef* represent the two wires of a pair, of which *ac* is broken at the point *b*. A similar pair is shown by *mn* and *hj*. The line *mn* and the part *bc* of the broken line are joined to the noninductive resistances R' and R'', as shown. The other wires of these pairs, and the remainder of the broken wire, are joined to the

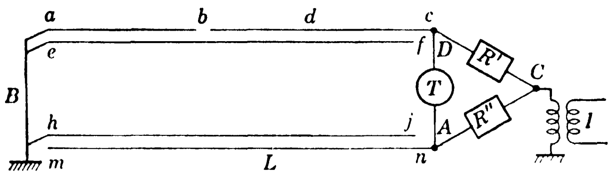


FIG. 70. Finding the distance out to the break at *b*.

ground. An induction coil *I* or some other source of alternating emf is used to charge the lines through the resistances. When the latter are adjusted to give a minimum sound in the telephone *T*,

$$d = L\frac{R''}{R'}.$$

139. Resistance of Electrolytes.—When there is a current through an electrolyte, it is accompanied by a separation of the substance in solution. The negative ions move in the same direction as the electron current, while the positive ions travel in the opposite direction, each being liberated at the electrodes. In general, this action causes polarization, which gives an extra emf in the circuit. In order, therefore, to measure the resistance of an electrolyte, it is necessary to employ an alternating current. This can be most readily obtained from an audio frequency oscillator (Fig. 175).

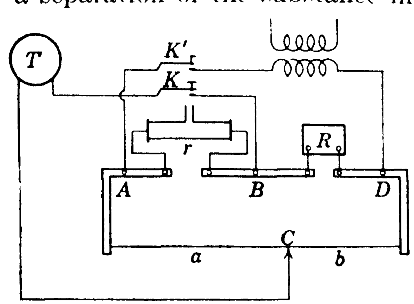


FIG. 71.—Resistance of an electrolyte.

The electrolyte is placed in a suitable cell and made the fourth arm of a Wheatstone bridge, the oscillator being used in place of the usual battery. The resistance of the electrolyte can then be determined by the Wheatstone bridge method in the usual way, and when the bridge is balanced,

$$r = R\frac{a}{b}.$$

Since an alternating current is employed, this balance can be found by means of a telephone receiver connected in the usual place for a galvanometer. For purposes of instruction, the best form of cell for holding the electrolyte is a cylindrical tube with a circular electrode closing each end. The resistance measured by the bridge is then the resistance of the electrolyte between the two electrodes; and if the resistance of this column of the electrolyte is known, the resistivity s of the solution can be calculated the same as for metallic conductors, or

$$s = r\frac{A}{L},$$

where A is the cross section of the tube containing the solution and L is the distance between the electrodes. In case the form of the cell is such that A and L cannot be measured, the value of A/L can be determined by using a solution for which s is known.

The conductivity of the solution, c, is the reciprocal of this, or

$$c = \frac{1}{s} = \frac{L}{rA}.$$

The resistance of an electrolyte, or, more strictly, its conductivity, depends upon the amount of the substance in solution—*i.e.*, upon the number of ions per cubic centimeter. Therefore, if we wish to compare the conductivities of different electrolytes, it is necessary to express the concentrations in terms of the number of ions per cubic centimeter. This is usually stated in terms of the number of gram molecules of substance that are dissolved in 1 liter of the solution. For the purpose of this experiment it is necessary to express the concentrations in terms of the number of gram molecules in 1 cu cm of the solution. The molecular conductivity μ of an electrolyte is then defined as the conductivity per gram molecule of salt contained in each cubic centimeter of solution.

$$\mu = \frac{c}{m} = \frac{1}{ms} = \frac{L}{mrA} = \frac{bL}{amRA},$$

where m = number of gram molecules in 1 cc of the solution.

The most interesting application of the conductivity of solutions is the knowledge it gives regarding the degree of dissociation of the dissolved substance. The conductivity of an electrolyte is due entirely to the ions it contains and is directly proportional to the number of ions per cubic centimeter. Most salts are completely dissociated in very dilute solutions, and therefore the molecular conductivity of such solutions is not increased by further dilution. Call this value μ_0. Then if μ denotes the molecular conductivity of a more concentrated solution of the same salt, the relative dissociation in the solution is

$$\alpha = \frac{\mu}{\mu_0}.$$

Express results by means of a curve, using values of μ for ordinates and the corresponding values of $1/m$, which is the number of cubic centimeters containing 1 gram molecule, as abscissas.

CHAPTER V

THE WHEATSTONE BRIDGE (*Continued*)

140. The Slide-wire Bridge with Extensions.—The measurement of resistances by the slide-wire bridge can be made with more precision by using a longer bridge wire. The uncertainty in locating the balance points probably will be about the same, but since the distance, $a' - a''$, between the two balance points is increased, the percentage error will be less.

As it would be inconvenient to have the apparatus much over a meter in length and as only the middle portion of the bridge wire is used in making careful measurements, the effective length of the bridge wire is increased by adding a resistance at each end. These extensions may consist of known lengths of wire similar to that used for the bridge wire—or any two equal resistances may be used and their equivalent lengths determined experimentally by the method shown below. The meter of wire provided with a scale then becomes only a short portion along the middle of the total length of the bridge wire. While this arrangement makes possible a greater precision of measurement, it also lessens the range of the bridge, as only those balances which fall on this limited section of the wire can be read.

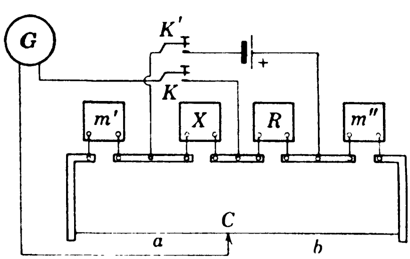


FIG. 72.—Bridge with extensions m' and m''.

The extensions are placed in the outside openings on the back of the bridge, between the ends of the bridge wire and the battery connections. They should be nearly equal. Let m' and m'' denote the number of millimeters of bridge wire having the same resistance as each extension, respectively, and let L denote the total length of the bridge wire including both of its extensions.

With the resistance X to be measured and the known resistance R in the middle openings of the bridge, as shown in Fig. 72, the first balance point is found. The reading on the scale at this point will be called a'. Then, as in Art. 117,

$$\frac{X}{R} = \frac{m' + a'}{m'' + b'} = \frac{m' + a'}{L - (m' + a')}. \tag{1}$$

Interchanging the positions of X and R, and calling the scale reading at the new balance point a'',

$$\frac{X}{R} = \frac{L - (m' + a'')}{m' + a''}. \tag{2}$$

And by the addition of proportions,

$$\frac{X}{R} = \frac{L + (a' - a'')}{L - (a' - a'')} = \frac{L + d}{L - d}. \tag{3}$$

It is evident that this arrangement reduces the range of the bridge, for only those values of X can be measured which are near enough equal to R to give balance points on the scale. But what is lost in range is more than made up in the greater precision of measurement.

Dividing out the fraction in Eq. (3) gives

$$\frac{X}{R} = 1 + 2\frac{d}{L} + 2\frac{d^2}{L^2} + 2\frac{d^3}{L^3} + . \tag{4}$$

All the terms after the second are negligible if d is small in comparison with L, so that

$$X = R + 2R\frac{d}{L} \text{ (approximately)}. \tag{5}$$

The only part of X, then, that is measured by the bridge is the second term, and a small error in it will only slightly affect the computed value of X.

141. To Find the Length of the Bridge Wire with Its Extensions. The total length of the bridge wire, including the extensions at each end, can be determined as follows: A good resistance box P is used in place of the unknown resistance X shown in the figure of the preceding section. Then with both extensions connected in the bridge, the values of P and R are adjusted to bring the balance point near one end of the scale. Let a' denote this scale reading, corrected if necessary by the calibration curve for this wire when used as a simple bridge. Then

$$\frac{P}{P + R} = \frac{m' + a'}{m' + c + m''} = \frac{m' + a'}{L},$$

where c denotes the original length and L the total length of the bridge wire.

Interchanging P and R, the balance falls near the other end of the wire, and

$$\frac{R}{P+R} = \frac{m' + a''}{L}.$$

By subtraction,

$$\frac{P-R}{P+R} = \frac{a' - a''}{L},$$

whence

$$L = \frac{P+R}{P-R}(a' - a'').$$

This method may also be used to determine whether there is any extra resistance in the straps and connections at the ends of the usual meter of bridge wire.

Notice that the value of L can be determined only as accurately as the distance d $(= a' - a'')$ can be measured. Therefore this distance should be as long as can be conveniently measured on the scale, say 85 or 90 cm. The reason for making d small in Art. 140 does not appear here.

Problem.—In calibrating a bridge wire it was found that for $P = 100$ and $Q = 900$ ohms, the balance point fell at 95 on the scale; while for $P = 900$ and $Q = 100$, the balance point was at 903. What is the effective length of the bridge wire?

Ans. $L = 1{,}010$ mm.

142. To Calibrate the Slide-wire Bridge with Extensions.—The formula deduced for this method in Eq. (5), above, works very well as long as X and R are nearly equal; but several errors may occur in its use, the principal of which are

1. Using a wrong value for L.
2. Neglecting all the terms containing d in powers higher than the first.
3. Errors in the determination of d, due to nonuniform bridge wire, scale errors, etc.

The method of calibration described below corrects for all these errors at once by finding a correction to be added to the observed value of d, which will give to $1 + (2d/L)$ the true value of X/R.

With the bridge set up as shown in Fig. 72, with the extensions in place, and two good resistance boxes, P and Q, in place of X and R, we have

$$P = Q\left(1 + \frac{2d'}{L}\right), \tag{6}$$

and solving for d' gives

$$d' = \frac{L}{2Q}(P - Q). \tag{7}$$

This is the value that d' must have in order that Eq. (6) shall give the correct values of the resistances being measured.

Starting with P and Q each 1,000 ohms, the value of d should be zero. Then increasing P by successive small steps, the corresponding observed values of d can be determined. These observed values of d will not agree with the values of d' computed from Eq. (7) above, and therefore if used in Eq. (6), they will not give the correct values for P. This is because of the errors noted above. It is therefore necessary to add to the observed length of bridge wire d a certain amount h such that

$$d + h = d',$$

and this corrected value, d', should be used in Eq. (5), above.

In the present case where P and Q are known, the values of d' are computed from Eq. (7), while the corresponding values of d are observed on the bridge wire. The differences give the values of h, and a calibration curve can be drawn, as shown, that will give the correction to be used at each point. The correction increases rapidly with d, owing to the increasing importance of the second error noted above.

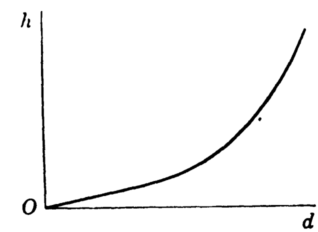


Fig. 73.—Calibration curve.

In case d is not zero when P and Q are nominally equal, it means that one is really a little larger than the other. Let d_0 denote this value of d. This can be reduced to zero by adjusting the value of Q; or, if it is more convenient, d_0 may be subtracted from each value of d throughout this calibration, as a sort of zero correction.

The readings may be recorded as below:

To Calibrate Bridge No. .... With Extensions No. .... and No. ....

P	Q	a'	a''	d $= a' - a''$	d' Eq. (7)	h $= d' - d$

143. Measurement of Resistance by Carey Foster's Method.— One of the most exact methods for comparing two resistances is the one devised by Carey Foster of England. The Wheatstone bridge is arranged in the manner used for the slide-wire bridge with extensions, except that now the extensions become the resistances to be compared.

Thus in Fig. 74 let S be the resistance that is to be compared with R. These two are placed in the bridge as shown, being connected together by the bridge wire. The other arms of the bridge, P and Q, become merely ratio coils and may have any value, although they must be nearly equal to each other in order that the balance may fall on the bridge wire, and for the most sensitive arrangement *all the arms* of a Wheatstone bridge should be nearly equal. For the greatest accuracy, R and S should each be in a constant-temperature oil bath.

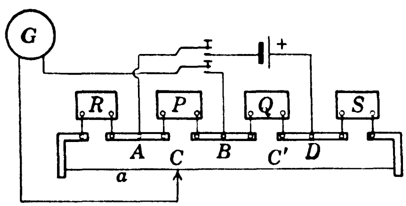


FIG. 74.—Carey Foster bridge, comparing R and S.

Let the balance point be found by moving the contact C along the wire until a point is reached for which there is no deflection of the galvanometer. Let a' be the scale reading at this point. It is immaterial whether this scale extends the entire length of the bridge wire or not.

Now let R and S be interchanged. This will make no difference in the total length of the extended bridge wire ACD. But if the resistance of S is less than that of R, the new balance point will not fall at the same point as before, for it will be necessary to add to S enough of the bridge wire to make the part AC the same as before. The resistance of AC in the first instance is

$$R + a'p = \frac{P}{P + Q} T.$$

After R and S have been exchanged, the resistance of AC' is

$$S + a''p = \frac{P}{P + Q} T,$$

where T denotes the total resistance of ACD. Equating these two expressions,

$$R + a'p = S + a''p,$$

and

$$S = R - (a'' - a')p,$$

where p is the resistance per unit length of the bridge wire.

144. Correction for Lead Wires or Other Connections.—If either S or R is connected to the bridge by wires or other connections, the resistance of these wires will be included in the measured values. If S and R denote the values of these resistances, exclusive of the lead wires, the last equation above should be written

$$S + s = R + r - (a'' - a')p,$$

where s and r denote the resistances of the connections.

If S and R can be removed or short-circuited, leaving only the connections, a second pair of balances will give

$$s = r - (a_2 - a_1)p.$$

Subtracting this from the first equation gives

$$\begin{aligned} S &= R - [(a'' - a') - (a_2 - a_1)]p \\ &= R - (d' - d_1)p, \end{aligned}$$

where d' and d_1 are the differences in the balance points in the two cases.

145. To Determine the Value of p.—The resistance per unit length of bridge wire can be readily determined by the same arrangement as shown above. Let R and S be two resistances that are normally equal. When the two balances are obtained,

$$S = R - (a'' - a')p,$$

as shown above.

Now let S be shunted by a rather large resistance P so as to give a small but definite change in this resistance. Then

$$\frac{SP}{S + P} = R - (a_2 - a_1)p,$$

where a_1 and a_2 are the new balance points. By subtraction,

$$S - \frac{SP}{S + P} = [(a_2 - a_1) - (a'' - a')]p,$$

and

$$p = \frac{S^2}{(S + P)(d_1 - d')}.$$

The resistances S and P should be well known, but extreme accuracy is not essential in order to obtain a fair degree of accuracy in the values of p.

146. The Ratio Box.—A more recent method for the exact comparison of two nearly equal resistances uses the ratio box shown in Fig. 75. The principle of this method is shown in Fig. 60 (Art. 128). The ratio box forms two arms of a Wheatstone bridge. When the bridge is balanced, the setting of the ratio box shows the magnitude of the unknown resistance in per cent of the standard. It can be used with resistors of the same nominal value, or in the ratio of 10:1. The

FIG. 75.—The ratio box. (*Courtesy of Leeds & Northrup Co.*)

range of the box shown in Fig. 75 is 99.000 to 101.110 per cent in steps of 0.001 per cent.

147. Comparison of Two Nearly Equal Resistances by Substitution.—The Wheatstone bridge box, described in Art. 126, is a fairly accurate instrument for the measurement of resistance. Its accuracy is limited by the accuracy of its own coils and the uncertainty in the connections to the unknown resistance. It is possible, however, to use such a box for measurements that are more accurate than its own coils.

Thus suppose that we wish to measure a resistance that is slightly less than 100 ohms and that there is available a standard 100-ohm coil whose resistance is accurately known. With the unknown resistance connected in the Wheatstone bridge, the latter is balanced as exactly as possible. If the best balance is not exact, the deflection of the galvanometer is carefully observed and recorded. Then without changing the bridge or moving any of the plugs, the standard resistance S is substituted in place of the unknown resistance X. If the key is now closed, the bridge will be found unbalanced, because, as we have supposed in this case, S is larger than X. When this is found to be the case, a high-resistance box R is connected in parallel with S and the resistance is adjusted until the galvanometer gives the same deflection as was observed for the best balance with X. Then the combined resistance of S and R must now be equal to X, or

$$X = \frac{SR}{S + R} = S\left(1 - \frac{S}{S + R}\right).$$

The actual values of the other arms of the bridge are thus immaterial.

An uncertainty in the resistance of the shunt R will affect only the value of the fraction $S/(S + R)$, which is small compared with 1. When X is nearly equal to S, thus requiring a large value of R to balance the bridge, the uncertainty due to R will be slight.

In case the unknown resistance is larger than the standard, the shunt should be applied to it. Then

$$S = \frac{XR}{X + R}, \qquad \text{and} \qquad X = \frac{SR}{R - S} = S\left(1 + \frac{S}{S + R}\right).$$

148. Precise Comparison of Two Resistances.—The last method leads directly to a method for the accurate comparison of two resistances that are nearly equal.

Let the two resistances to be compared form two arms, S and R, of a Wheatstone bridge. The other arms may be two resistances, A and B, of about the same nominal resistance as S. The more nearly $A = B = S$, the better, but the exact values do not enter in the measurement. The bridge is balanced by adjusting one or both of the high-resistance shunts, P and Q, that are in parallel with S and R, respectively. When R is not closely equal to S, the balance may be obtained by the use of only one of the shunts. If R is only slightly larger than S, the value of Q necessary to give a balance would be very high, possibly beyond the range of any available resistance box. In this case a moderate resistance can be used in P, making the resistance of this arm equal to

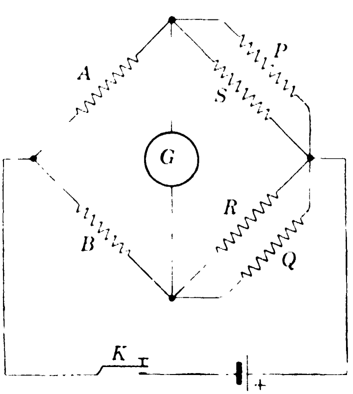


FIG. 76.—The balance is obtained by adjusting the shunts P and Q.

$$S_1 = \frac{SP}{S + P}.$$

It will then be possible to obtain a balance by adjusting Q, giving for the resistance of R and Q the value

$$R_1 = \frac{RQ}{R + Q}.$$

When the bridge is thus balanced we have

$$\frac{A}{B} = \frac{S_1}{R_1},$$

where the ratio A/B is a constant but unknown quantity nearly equal to unity.

For the next step the resistances A and B are interchanged with each other without altering the rest of the bridge. This, in general, will upset the balance of the bridge, but the latter can be restored by readjusting the shunts to new values, P' and Q'. We shall now have

$$\frac{A}{B} = \frac{R_2}{S_2},$$

where

$$R_2 = \frac{RQ'}{R + Q'} \qquad \text{and} \qquad S_2 = \frac{SP'}{S + P'}.$$

Equating this to the former expression for A/B eliminates this unknown ratio, giving

$$\frac{S_1}{R_1} = \frac{R_2}{S_2},$$

or

$$\frac{R}{S} = \sqrt{\frac{PP'}{QQ'}\frac{(R + Q)(R + Q')}{(S + P)(S + P')}}.$$

In computing the value of this factor it is permissible to use the nominal values of S and R, since these are added to the large resistances P and Q.

The advantages of this method are apparent when it is seen that all uncertainty in the ratio of the arms A and B is completely eliminated, and, since P and Q enter in both numerator and denominator, the uncertainties in their values will have only a slight effect on the result. Of course the usual precautions against thermal emfs and changes in temperature should be observed.

In some cases it is better to use the shunts on the ratio arms of the bridge, thus varying the ratio to equal R/S.

149. Measurement of Low Resistance.—It is often necessary to measure a small resistance, in the form of either a short length of metal rod or a coil of low resistance. The determination of the resistivity of a metal usually requires the accurate measurement of a bar of the material.

The usual forms of the Wheatstone bridge are not well adapted for the measurement of such very small resistances. This is because of the uncertain resistances of the contacts and connections by which the small resistance is joined into the bridge. If the measured resistance is small, it is evident that a small contact resistance may introduce a relatively large uncertainty in the computed result.

150. Millivoltmeter and Ammeter Method.—If a sensitive millivoltmeter is available, the resistance of a sample piece of wire, MN, Fig. 77, can be measured by the ammeter-voltmeter method. As shown in Fig. 77, the voltmeter measures the fall of potential between m and n. Since the main current does not pass through the contacts at m and n, the resistance of these contacts will not affect the fall of potential between m and n. These contact resistances are, in fact, a portion of the voltmeter circuit and therefore are negligible. The resistance measured by this method is, then, the resistance of that portion of the wire between m and n.

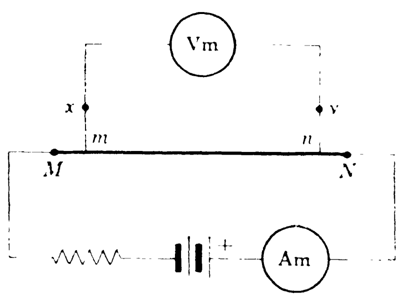


FIG. 77.—Measurement of mn by the ammeter-voltmeter method.

151. Four-terminal Resistances.—If it is desired to preserve this particular sample, mn (Fig. 77), as a fixed and constant resistance, the potential wires xm and yn should be soldered to the main wire MN at m and n. Then each time it is employed the same resistance mn will be used. The voltmeter is disconnected at x and y, and the current circuit is broken at M and N. All contact resistances in the measured portion of the wire are thus eliminated. Such fixed resistances are called "four-terminal resistances." Standards of low resistance are always thus provided with four terminals, two for current terminals and two for potential terminals.

152. Low-resistance Standard.—An external view of a standard of low resistance is shown in Fig. 78, and the diagram of connections is given in Fig. 79. AB is a heavy piece of resistance metal of uniform cross section and uniform resistance per unit of length; CD is another piece of resistance metal of smaller cross section, and the two are joined together by a heavy copper bar, AC, into which both are silver-soldered; LL are the current terminals and PP are the potential terminals. The resistance of AB between the marks 0 and 100 on the scale S is 0.001 ohm.

From the point 1 on the resistance CD to 0 on AB is also 0.001 ohm, from 2 to 0 is 0.002 and so on, and from 9 to 100 is the total

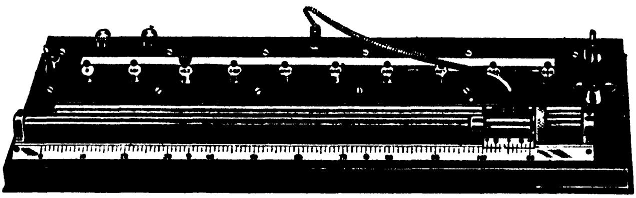


FIG. 78.—Adjustable low-resistance standard. (*Courtesy of Leeds & Northrup Co.*)

resistance of 0.01 ohm. The slider M moves along the resistance AB and its position is read on the scale S, which is subdivided into 100 equal parts and can be read by a vernier to thousandths. Subdivided in this way the resistance between the tap-off points PP may have any value from 0.000001 to 0.01 ohm by steps of 0.000001 ohm.

There is a binding post on the bar AC that makes it possible to use AB separately. On account of its large current-carrying capacity,

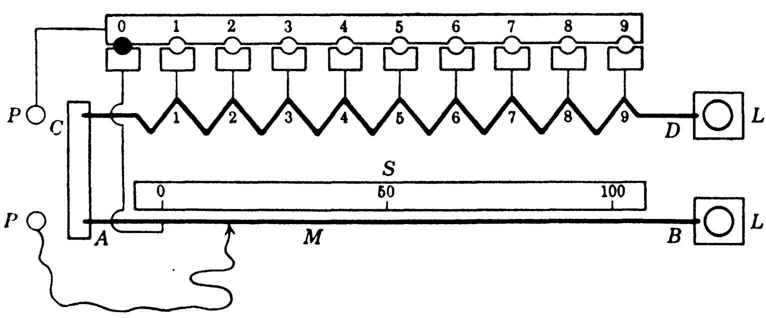


FIG. 79.—Diagram of a variable standard low resistance.

this bar is a very satisfactory standard resistance for measuring current by the potentiometer method.

153. The Kelvin Double Bridge.—While low resistances can be measured by the use of a millivoltmeter and and an ammeter, the accuracy of the determination is limited by the accuracy with which the deflections of the instruments can be read. The best and most precise method for the measurement of low resistances is that of the

Kelvin[1] double bridge, a modification of the Wheatstone bridge. As this is a null method, the results do not depend upon the readings of any instrument.

The general arrangement is shown in Fig. 80. The resistance to be measured is shown by X, with current terminals at M and N, and potential terminals at x and y. R is the standard low resistance with current terminals S and T and potential terminals v and z. R and X are joined at J with as good a connection as practicable, and together with P and Q they constitute the arms of a Wheastone bridge. In the ordinary Wheatstone bridge the galvanometer is connected between D, Fig. 80, and the junction of X and R. In the present case this junction is not a single point but consists of the resistance $mMJSs$ with the unknown amount of contact resistance at M and S, and it is not possible to connect the galvanometer at any desired point along this irregular resistance. However, by using a potential divider ab in parallel from m to s, the galvanometer can be connected at a point E that has the desired potential between that of m and that of s. The potential at E is varied by adjusting the ratio of the resistances a and b.

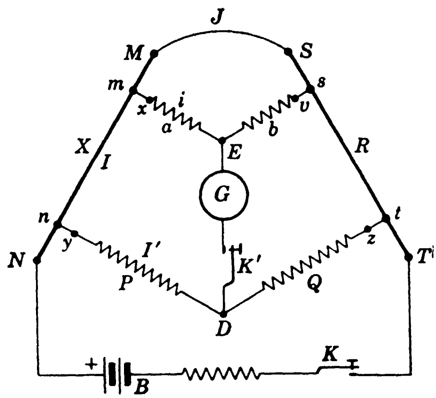


FIG. 80.—Diagram of the Kelvin double bridge.

Contact resistances at M and N do not enter as a part of X because they are not in the part being measured. At x and y the contact resistances are parts of the large resistances a and P.

The bridge can be balanced, giving zero deflection of the galvanometer, by changing the value of R or by varying the ratio of P to Q, whichever is the more convenient with the apparatus used.

Derivation of the Formula.—For this condition of balance, E and D will be at the same potential, and the fall of potential from n to D is equal to the fall from n to E. Writing out this equation gives

$$PI' = XI + ai.$$

[1] William Thomson, Lord Kelvin (1824–1907), British mathematician and physicist.

Similarly, for the branches Dt and Et,

$$QI' = RI + bi.$$

By division,

$$\frac{P}{Q} = \frac{X + a\dfrac{i}{I}}{R + b\dfrac{i}{I}} = \frac{X + ar}{R + br},$$

where the ratio i/I is denoted by r for convenience. Evidently r will be a small number, and its smallness will depend upon the smallness of the joint resistance J. Then

$$X = \frac{P}{Q}(R + br) - ar$$
$$= \frac{P}{Q}R + rb\left(\frac{P}{Q} - \frac{a}{b}\right).$$

If $P/Q = a/b$, this last term vanishes. In case the equality is not precise, the effect of this term is made less by making r as small as possible. This means that the connection MJS, Fig. 80, should have as low a resistance as possible.

154. Adjustment of the Ratios.—If the resistances that are used for a and b are not sufficiently accurate, the ratio a/b can be made actually equal to P/Q by slightly adjusting the value of a. Opening the joint at J makes a Wheatstone bridge that will balance when

$$\frac{P}{Q} = \frac{X + a}{R + b}.$$

X and R are usually small in comparison with a and b, and also are in the same ratio, so that the balance expressed by this equation can be made perfect by adjusting the value of a or b. In this way a/b can be made to equal P/Q, and then a, b, and J do not appear in the final value for X.

When P/Q is set at 10^k, the result is given by the direct reading of R with the decimal point moved k places.

155. Advantages of the Kelvin Bridge.—Low resistances can also be measured by the potentiometer method (see Art. 184). With the Kelvin bridge, however, an absolutely steady current is not essential and the result is given by a simple computation from a single direct reading.

If for any reason it is necessary to use long wires to connect the bridge to the potential terminals of the low resistance, it will probably be better to use the potentiometer method.

156. The Student's Kelvin Bridge.—The student's Kelvin bridge is a simplified instrument for making low-resistance measurements and for teaching the principles of these measurements. It consists

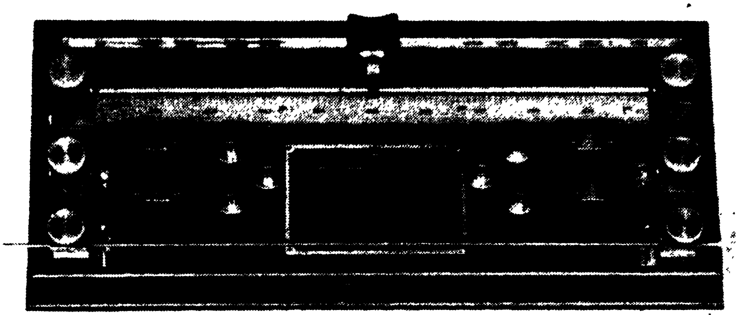


FIG. 81.—Student's Kelvin bridge. (*Courtesy of Leeds & Northrup Co.*)

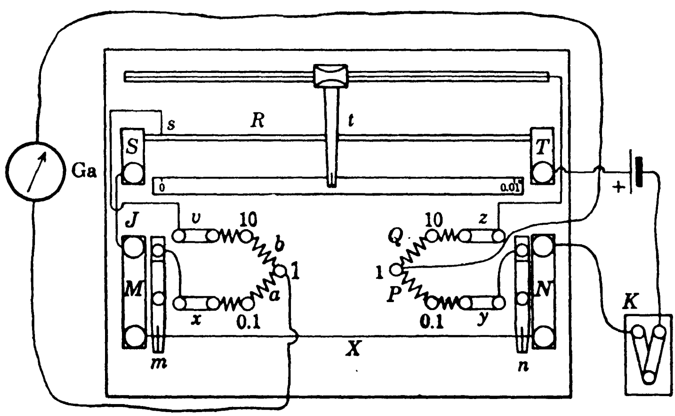


FIG. 82.—Diagram of connections in the student's Kelvin bridge.

of a 0.01-ohm calibrated variable standard of resistance, a double set of ratio coils with three different multiplying ratios, and a pair of current and potential contacts for the sample to be measured. It has, therefore, three ranges: 0 to 0.001 ohm, 0 to 0.01 ohm, and 0 to 0.1 ohm. It is designed for measuring samples about 18 in. long and up to 3/8 in. in diameter, but binding posts are provided for connecting to any four-terminal resistance.

As small a current should be used as will give the necessary sensitivity, and not more than 20 amp should be passed through the standard resistance. Excessive current will heat the wire and its resistance will be increased. The zero reading of the galvanometer should be observed after the galvanometer has been connected to the bridge, to eliminate the effect of any thermal emf that may be present. The ratio of P to Q can be changed by moving one of the galvanometer connections near the center of the bridge. The other galvanometer connection controls the ratio of a to b. Make both connections at the same value (marked 10, 1, or 0.1), close the battery circuit, and balance the galvanometer by moving the contact on the standard slide-wire resistance. The maximum accuracy is obtained when the contact is at the upper end of the slide wire, and therefore a ratio value should be used that will bring the balance point as high as possible. The value of the unknown resistance, X, is obtained from the formula

$$X = R\frac{P}{Q},$$

where R is the reading on the scale and the values of P/Q are stamped on the bridge.

The resistivity, ρ, of the metal is given by the relation

$$\rho = X\frac{A}{l}.$$

157. Definite Relation between Resistance and Temperature. When the resistance of a wire is measured at different temperatures, it is found to have different values. Usually the resistance increases as the temperature rises. At each definite temperature, however, there is a definite resistance for each wire, and if corresponding values of resistance and temperature are plotted, the result will be a smooth curve that is not far from a straight line.

In alloys the change in resistance is very much less than it is for the pure metals, and in at least one case the resistance actually decreases with an increase of temperature.

Let $LMNP$, Fig. 83, represent the relation between resistance and temperature for a given coil. Suppose the portion MN has been experimentally determined and plotted. If this limited part of the curve is nearly a straight line, the relation between the resistance R and the temperature T is easily expressed, as is shown below.

158. Supplemental Degrees for the Variation of Resistance with Temperature.—The straight line MN, Fig. 83, shows a linear relationship between the resistance and the temperature of a given metal. This relation is not a direct proportionality, but it can be obtained by adding to each value of the temperature a certain constant that is characteristic of the particular metal considered.

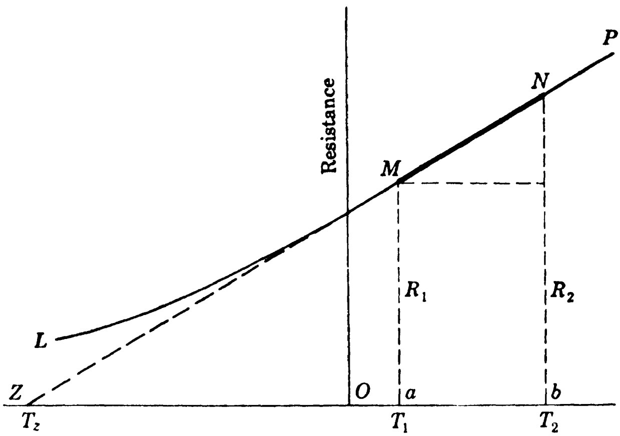


FIG. 83.—The relation between resistance and temperature. Over the observed range MN, the resistance is proportional to the temperature above the point Z.

Let R_1 and R_2 denote the resistances of a coil of metal wire at temperatures T_1 and T_2 on the centigrade scale. Then from the similar right-angled triangles ZaM and ZbN, Fig. 83,

$$\frac{R_1}{R_2} = \frac{T_1 + T_z}{T_2 + T_z}, \tag{8}$$

where the constant T_z is shown as ZO on the temperature axis. If this constant is to be added to T_1, it is evident that it must be expressed in the same units as T_1.

Solving Eq. (8) for the value of T_z gives

$$T_z = \frac{R_1T_2 - R_2T_1}{R_2 - R_1}. \tag{9}$$

Thus it is not necessary actually to extend the line back to Z, since this point is readily determined by computation.

These formulas take no account of the change in dimensions with change of temperature and therefore apply to the conductor as a whole.

159. Determination of the Supplemental Degrees.—In this experiment several coils of different metals are arranged in an oil bath where the temperature can be raised as desired. A very convenient arrangement is to use an electric water heater to hold the oil bath. With a suitable resistance in series this can be easily warmed and maintained at the desired temperatures. Starting at room temperature, carefully determine the resistance of each coil. A Wheatstone bridge box gives a convenient and accurate method for resistances having large temperature coefficients, like copper and iron. For alloys like German silver and manganin it is better to use a more delicate method, such as the Carey Foster bridge.

After the resistances of the coils have been obtained at room temperature, the bath is warmed 10 or 15° and when things have become steady at the new temperature the resistances are again measured. In the same way the resistances are determined at five or six different temperatures, and the results are plotted as a temperature-resistance curve for each coil.

The place on the temperature axis at which this portion of the curve points is determined by calculation, as shown above. In this computation it is best to use corresponding values of resistance and temperature as read from the curve and which therefore are less liable to error than are single observations.

Supplemental Degrees in the Range 0 to 100° C

Metal	T_z (centigrade)	Metal	T_z (centigrade)
Aluminum	236	Lead	243
Brass	556	Manganin, 0°–30°	40,000
Copper	234	Platinum	273
German silver	3,200	Silver	250
Gold	272	Tungsten	196
Iron (pure)	160	Zinc	249
Mercury	1,100	Platinoid	4,500

It is to be noted that these values of T_z imply nothing regarding the behavior of these metals at very low temperatures. This is merely a very simple way of expressing the slope of the temperature-resistance curve in the region where it has been studied.

160. Uncertainty.—Probably the largest uncertainty in Eq. (9) is in the value of the small change $R_2 - R_1$ in the resistance. This difference, as read from the curve MN, contains all the error of the measured resistances. In plotting the observed values, a scale for

R should be chosen that will bring M near the bottom of the graph sheet and N near the top.

The circumstances of the measurements will indicate the error in finding the temperature of the coil at the time its resistance is measured.

The observed values that are plotted for the curve MN will not lie exactly on a straight line. Therefore there will be some question as to the best location of the curve MN, especially as the plotted location of each point is more or less uncertain. Since the curve is a straight line, a ruler may be used to draw it. The observer must use his judgment as to how steep or how flat the line should be drawn. This variation in the position of the best line indicates the effect of all the uncertainties on the determination of $R_2 - R_1$.

161. Use of the Supplemental Degrees.—When the resistance R_1 of a conductor at a temperature T_1 is known, it is a simple matter to find its resistance R at any other temperature, T. Since ZMN is a straight line,

$$\frac{R_1}{T_1 + T_z} = \frac{R}{T + T_z}.$$

The proportion shown in this equation is very convenient and can be used in many ways. It is especially useful when the change in resistance is large.

162. Resistance Thermometer.—Oftentimes it is impossible to use a mercury thermometer in some situation where the temperature is desired. This may be an inaccessible part of some machine or a distant place in a factory. If a coil of copper or platinum wire can be used and its resistance measured, the temperature can be determined from the relation above.

One of the best methods for measuring temperatures that are either too high or too low for the use of mercury is to measure the resistance of a fine platinum wire and then read the temperature from the resistance-temperature curve. In fact, temperatures can be measured more exactly with a platinum resistance thermometer than with a mercury thermometer, even at ordinary temperatures.

163. Temperature Coefficient of Resistance.—The change of resistance with temperature is often expressed in another way. From

$$\frac{R_1}{T_1 + T_z} = \frac{R}{T + T_z}$$

it follows that

$$R = R_1 \frac{T + T_z}{T_1 + T_z}.$$

Clearing of fractions by performing this division gives

$$R = R_1\left[1 + \frac{1}{T_1 + T_z}(T - T_1)\right]$$
$$= R_1(1 + a_1 t),$$

where t is written for the temperature difference, $T - T_1$, and a_1 is written for $1/(T_1 + T_z)$. The coefficient a_1 is called the "temperature coefficient of resistance."

This formula is convenient to use in computing a small change in resistance corresponding to a small change t in temperature. Note that a_1 is the reciprocal of $(T_1 + T_z)$, where T_1 is the temperature at which the resistance of the conductor is R_1.

164. Second-degree Relation.—In case a straight line cannot represent the resistance of a coil with sufficient accuracy, a curve showing this relation can be used. In this case all first-degree equations are likewise insufficient and it is then necessary to use a second-degree equation to approximate more closely to the actual resistance-temperature curve. Usually this is written in the form

$$R = R_0(1 + at + bt^2),$$

where a and b are coefficients to be determined.

165. Temperature Coefficient of Copper.—The effect of a small amount of other elements alloyed with copper not only appears in the decreased conductivity of the metal; the temperature coefficient is also decreased in the same ratio. Hard-drawing the metal has a similar effect. The temperature coefficient of a sample of copper wire expressed in terms of the resistance at 20° C, is given by multiplying the number expressing the per cent conductivity by 0.00394.[1]

The table below gives a few values for copper furnished for electrical purposes and for the temperature range 10 to 100° C.

Per cent conductivity	a_{20}	Supplemental degrees (centigrade)
101	0.00398	231.3
100	0.00394	233.8
99	0.00390	236.4
98	0.00386	239.1
97	0.00382	241.8
96	0.00378	244.6

[1] *Bull. Nat. Bur. Standards*, vol. 7, p. 83, 1911.

Problems

1. What is the resistance at 100° C of a coil of copper wire that is 13.46 ohms at 30° C? The supplemental degrees for copper are 234° C.

   Ans. $R_{100} = 334/264 \times 13.46 = 17.02$ ohms.

2. What is the resistance at 100° C of a coil of copper wire that is 13.46 ohms at 30° C if $\alpha_{20} = 0.00394$?

   Solution.—First find the resistance at 20° C. Then, using this value, find the resistance at 100° C.

$$R_{100} = 13.46 \frac{1 + 0.00394\ (100 - 20)}{1 + 0.00394\ (30 - 20)} = 17.02 \text{ ohms.}$$

3. How much resistance is there in a coil of copper wire that increases 1 ohm per 1° C?

4. What is the resistance at 50° C of a coil of platinoid wire that is 40 ohms at 10° C? The supplemental degrees for platinoid are 4,500° C.

   Ans. 40.4 ohms

5. What is the resistance at 100° C of a coil of iron wire that is 10 ohms at 20° C?

   Ans. 14.44 ohms.

6. A certain electrical machine should not be allowed to run hotter than 70° C. The resistance of a copper coil embedded in the machine can be measured as the temperature rises. How large can this resistance be allowed to become if it is 15 ohms at 20° C? *Ans.* 17.95 ohms.

CHAPTER VI

POTENTIOMETER AND STANDARD-CELL METHODS

166. A Scale for Voltage.—When one wishes to measure an unknown quantity, like the height of a table or of a bush, it is necessary to have a standard scale to stand alongside the unknown in order to make the comparison. In somewhat the same way an unknown emf can be measured by comparing it with a scale of known emf. A simple way to make such a scale is shown in Fig. 84, where AD is a wire of high resistance and 2 meters in length. In this wire is a current I sufficient to make the rise in potential from A to D exactly 2 volts. Then each millimeter along this wire will correspond to 1 mv, and a meter scale alongside the wire will measure the difference of potential from A up to any selected point.

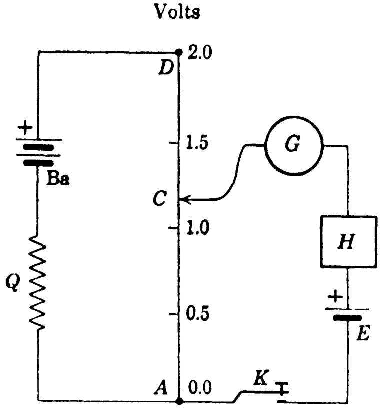


FIG. 84.—A scale of voltage to measure the emf of E.

To use this scale let the negative side of the unknown emf, shown at E, be connected to the zero end of the scale at A. Let the positive side be connected through a suitable galvanometer to a movable key that can make contact with the wire at any point. When this contact is made too high on the scale, the galvanometer will deflect; when it is made too low, the galvanometer will deflect in the opposite direction. At the critical point where the rise of potential along the wire is equal to E, there will be no current in the galvanometer, and the value of E can be read on the millimeter (or millivolt) scale.

This method is better than one (as with a voltmeter) that draws current from E, because in this measurement there is no uncertainty due to the unknown fall of potential in the internal resistance of the cell that is being measured. The true value of E can be measured as accurately as the scale can be read.

167. The Potentiometer Method.—The method for measuring a potential difference that was illustrated above is called the "potentio*meter* method," since it *measures* potential difference. One great advantage of this method is that no current is drawn from the emf that is being measured. Being a null method, it is capable of greater sensitiveness and precision than any method that depends upon the reading of the deflection of a galvanometer or other instrument.

In the illustration just given, the accuracy of the measurement is limited by the accuracy of reading the scale after the balance point has been found. By using two well-adjusted resistance boxes of a few thousand ohms each instead of the two sections of wire (in Fig. 84), the reading of the balance point can be made much closer, as is shown below.

168. The Resistance-box Potentiometer.—In the resistance-box potentiometer the wire AD of the preceding section is replaced by two similar and well-adjusted resistance boxes; and instead of actually moving C along the wire, resistance is transferred from AC to CD by changing the plugs while the total resistance is kept unchanged.

The arrangement is shown in Fig. 85. The cell to be measured is placed at E, where it is in series with a sensitive galvanometer, a high resistance H, a key, and a resistance box R. Through the last can be passed a small and constant current from the battery B. In Art. 47 this R was the voltmeter, and the *current* through it was varied until the fall of potential Ri just balanced the emf of the cell. In the potentiometer method the *current is kept constant* and R is varied. Furthermore, the galvanometer is a more sensitive indicator to show when the balance has been secured.

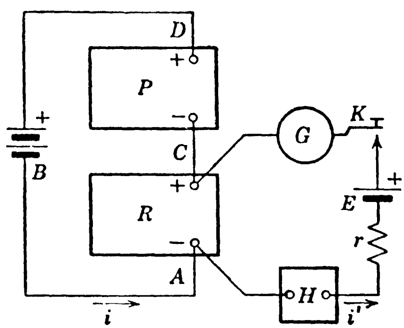


FIG. 85.—A potentiometer built up with resistance boxes R and P.

The rest of the setup consists of a constant emf B, for supplying the constant current, and a resistance box P, which should be identical with R for convenience. The resistance used in both P and R should be kept equal to the total amount of one box; and if the boxes are alike, it is very easy to keep the sum at this value.

After the arrangement is set up as shown in Fig. 85, having several thousand ohms in R and P before the battery B is connected, it is first tested with an old cell for E so as not to endanger a valuable standard by an accident or a wrong connection. If $R + P$ are kept at the total amount in one box, a balance should be obtained without much difficulty.

The high resistance H may be reduced after the galvanometer deflections have been brought down to a few divisions. This will allow a finer adjustment of R. When it is certain that everything is working correctly the cell to be measured may be substituted for the cell at E.

169. Derivation of the Formula.—For the general case of a slight current in the galvanometer circuit when K is closed, let i' denote the value of the electron current in the direction shown by the arrow. Then the various differences in potential in the galvanometer circuit would be as indicated by the + and − signs. The internal resistance of the cell is shown by itself, as the sign of the fall of potential through it is independent of the sign of E, since the current in this circuit may be in either direction. (See Art. 18.)

If we start at A and trace clockwise through the circuit of RGE back to A, the various potential differences passed over along this path are

$$+R(i - i') - Gi' - E - ri' - Hi' = 0. \tag{1}$$

For the particular case of zero deflection of the galvanometer, $i' = 0$, and Eq. (1) becomes

$$+Ri - E = 0. \tag{2}$$

Thus we see why it is that the internal resistance of the cell has no effect on the value of R when a balance is obtained.

After it is found that a balance can be obtained without taking too much current through E and the galvanometer, a standard cell of known emf E' can be used in the place of E. When the new balance has been obtained, by changing the resistance in R to R' we have, the same way as before,

$$E' = R'i. \tag{3}$$

Since both E' and R' are known, this equation determines the value of the current i as

$$i = \frac{E'}{R'}; \tag{4}$$

and if the current has been kept constant while R was varied, by also varying P so as to keep $(P + R)$ constant, and if the battery B has not changed, then this value of i can be used for the value of the current in Eq. (2), giving

$$E = E' \frac{R}{R'}, \tag{5}$$

which expresses the emf of the cell to be measured in terms of known quantities.

170. A Direct-reading Resistance-box Potentiometer.—When many measurements of emf are to be made with a potentiometer, the computations indicated in Eq. (5), above, may become very tedious. This work can be eliminated by adjusting the value of the constant current i to a simple round number. When

$$i = \frac{E'}{R'} = 0.00010000, \qquad \text{ampere}$$

the computation consists merely in moving the decimal point four places to the left. This adjustment can be made as follows:

With a cell E' of known emf (say 1.0183 volts), the resistance in box R, Fig. 86, is set to read this same number (say 10,183 ohms). This also fixes the resistance in box P, since $R + P$ is to be kept at some chosen and convenient value (say 19,999 ohms). This, in general, will not give a balance, because Ri is too large. Instead of changing R, as in Art. 168, reduce the current i by adding a third resistance Q, Fig. 86, in series with the main battery. By adjusting the resistance in Q to give zero deflection of the galvanometer, the current i is brought to the desired value of 0.00010000 amp, where the four zeros at the right are known and significant figures.

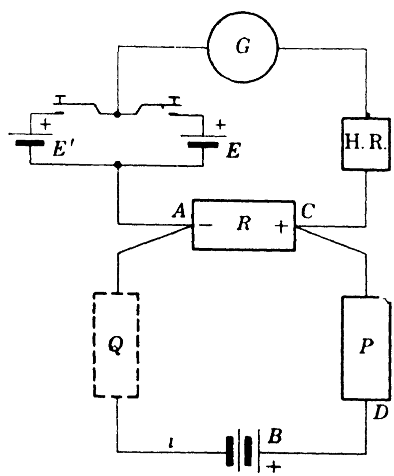


FIG. 86.—A direct-reading potentiometer built up with resistance boxes R, P, and Q.

With Q left at this value, the known cell E' is replaced by the unknown cell E, whose emf is desired, and a new balance is obtained by adjusting R and P, keeping their sum constant at the previously chosen constant amount (say 19,999 ohms).

The emf of this cell is easily computed from the relation

$$E = Ri = \frac{R}{10{,}000}, \qquad \text{volts}$$

where R is the resistance required for a balance.

Since the current in R and P is continually becoming less, its value as given in Eq. (4) should be determined near the time that it is to be used in Eq. (2) to obtain the result given in Eq. (5).

There should be no key in the circuit of PRQ and the main battery. If the battery B consists of one or two dry cells, the current i will be more nearly constant after it has been continuously flowing for a day or longer.

The readings can be arranged as below:

POTENTIOMETER MEASUREMENTS

Name of cell	R	P	Q	E

171. Standard Cells.—In all measurements of emf with the potentiometer it is necessary to have a known emf that can be used as explained in Art. 169 [E', Eq. (5)], to standardize the value of the current through the potentiometer. Such a known emf is furnished by a standard cell, which is a primary battery set up in accordance with definite specifications so that it will possess a definite emf. Such a cell is used as a standard of emf, and is never expected to furnish a current. Since the emf of a cell depends upon the materials used in its construction and not at all upon its size, standard cells are made small, both for economy of materials and for convenience in handling.

172. The Weston Normal Cell.—The Weston normal cell[1] has been the subject of much study and investigation, so that now it is possible for investigators in different parts of the world to set up such cells and know that they will have the same emf to within less than a 10-thousandth of a volt. In order to attain such accuracy, it is necessary that the cells be set up in strict accordance with the specifications, using only the purest materials.

[1] Edward Weston (1850–1936), Anglo-American inventor.

The cell is usually set up in an H-shaped glass vessel having dimensions of a few centimeters. At the bottom of one leg pure mercury is placed to form the positive electrode of the cell. Connection to this is made by a fine platinum wire sealed into the glass, the inner end being completely covered by the mercury. At the bottom of the other leg there is placed, similarly, some cadmium amalgam, which, when warm, can be poured in like the mercury and then hardens as it cools. A second platinum wire through the glass at the bottom of this leg makes electrical connection with the amalgam, which is the negative electrode of the cell. The electrolyte is a saturated solution of cadmium sulphate, containing crystals of this salt in order to keep the solution saturated at all times. The mercury electrode is protected from contamination by the cadmium in this solution by a thick layer of a paste consisting mainly of mercurous sulphate. Cadmium ions from the solution coming through this paste form cadmium sulphate, and only mercury ions pass on and come into contact with the mercury electrode. This paste is thus an efficient depolarizer.

The emf of a Weston normal cell that has been set up in accordance with the specifications is

$$E_t = 1.01830 - 0.0000406(t - 20° \text{ C}) - 0.00000095(t - 20° \text{ C})^2 + 0.00000001(t - 20° \text{ C})^3.$$

This temperature formula was recommended by the London International Conference on Electrical Units in 1908, and the value 1.01830 international volts was found by the International Scientific Committee after an exhaustive series of measurements.

173. The Weston Unsaturated Standard Cell.—The Weston normal cell described above is seldom used in ordinary electrical measurements. When it is kept at a constant temperature its emf is constant and definitely known, but it is not always convenient to maintain a constant-temperature chamber.

A considerable part of this temperature variation is due to the change in concentration as more or less of the crystals of cadmium sulphate dissolve. By omitting the crystals the electrolyte is unsaturated and remains at a constant concentration. Consequently, the emf changes very slightly with the temperature. The temperature coefficient is usually less than 0.000005 volt per degree centigrade and is almost invariably negative in sign. For most purposes the emf of the cell may be considered as constant for ordinary changes in room temperature. This modified form of cell is therefore more suitable for general laboratory use, although it is necessary to determine the emf of each cell that is made.

The construction of this Weston standard cell is illustrated in Fig. 87. The materials forming the electrodes are held in place by porcelain retainers provided with cotton packing. The cell is sealed to prevent leakage and evaporation.

To preserve the constancy of a standard cell, it should not be subjected to temperatures below 4° or above 40° C. If by chance it is subjected to extreme temperatures, it should be set aside for a month at a practically constant temperature. It will probably then be nearly back to its original emf.

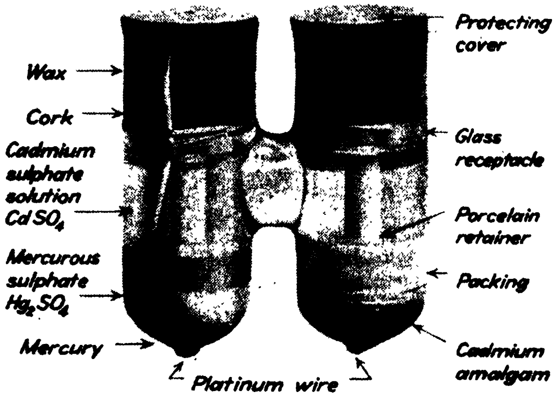


FIG. 87.—Construction of the Weston unsaturated-type standard cell. *(Courtesy of Weston Electrical Instrument Co.)*

174. Use of a Standard Cell.—No current greater than 0.0001 amp should ever be permitted to pass through a standard cell, and then only for a moment. A standard cell should never be connected to a voltmeter. The internal resistance is high, and therefore it would not be possible for it to furnish much current. In fact, any appreciable current drawn from the cell would polarize it somewhat, thereby decreasing its emf by an unknown amount and thus destroying the only value of the cell. The depolarizer tends to restore the emf to its original value, but the time required depends upon the amount of polarization.

Standard cells may be used to charge condensers to a known difference of potential, for in this case there is no steady current drawn from the cell, and the transient current is not sufficient to cause an appreciable polarization. When used in connection with a potentiometer, the cell should always be placed in series with a sensitive galvanometer.

If it is necessary to reduce the deflection, this should be done by means of a high resistance in series with the cell instead of by a shunt on the galvanometer, which would still allow the large current to flow through the standard cell. When using the galvanometer, the key should be tapped lightly and quickly so as to give a deflection of only a centimeter or two. This will indicate the direction of the current as clearly as a larger deflection and does not injure the standard cell.

175. The Student's Potentiometer.—For purposes of instruction the three-box potentiometer just described enables one to see clearly all parts of the setup, and, as has been shown, it is convenient to use. But if one has many measurements of emf to make, the mere changing

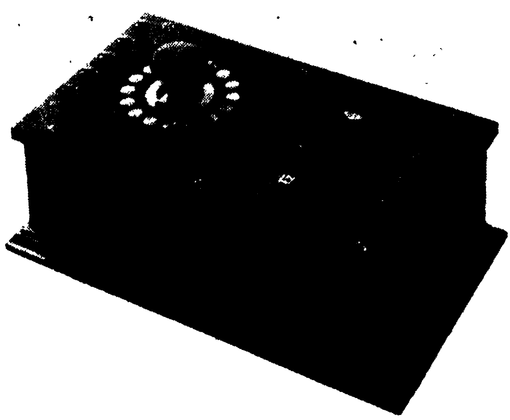


FIG. 88. Student's potentiometer. (*Courtesy of Leeds & Northrup Co.*)

of the resistances to obtain the balances becomes a laborious task. It is then desirable to have a quicker and simpler way of varying the resistances.

In the student's potentiometer shown in Fig. 88 and the diagram Fig. 89, the resistances corresponding to the slide wire of Fig. 84 are arranged in two dials. Moving the sliding contacts over these dials corresponds to moving the ends *A* and *C* of the galvanometer circuit along the slide wire of Fig. 84.

There are no moving or sliding contacts within the potentiometer, except the two contact points at *A* and *B* (see diagram, Fig. 89). All other connections are brazed or soldered. The moving contact points at *A* and *B* are in the galvanometer circuit and not in a calibrated resistance circuit, so that any variation in contact resistance at these points can cause no error in the measurements.

Connections from the calibrated resistors are brought out to ten binding posts, five on each end of the instrument. The five binding posts on the left of the instrument and the one in the central position on the right are the six used for potentiometer measurements. The other four binding posts at the right of the instrument are provided for making independent connections to the slide wire, either with or without end coils, so that the instrument may be used as part of a Wheatstone bridge circuit. For this purpose an additional scale 0 to 1,000 is provided on the slide wire.

To standardize the potentiometer against the standard cell, switch S (in Fig. 89) is closed to STD CELL, contacts A and B are set to corre-

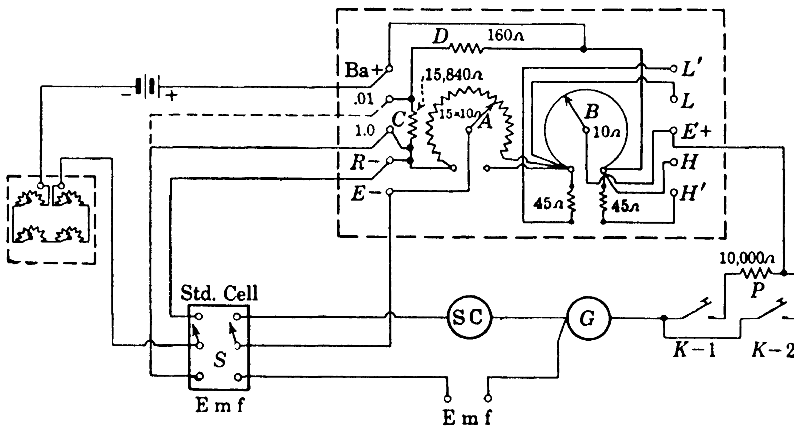


FIG. 89.—Diagram of the electrical circuits in the student's potentiometer.

spond to the certified cell voltage, and the rheostat is adjusted until galvanometer G shows a balance as a key is closed. The regulating rheostat is a four-dial resistance box, total 999.9 ohms, adjustable in steps of 0.1 ohm. With a 2- or 3-volt battery, the total circuit resistance will be 200 to 300 ohms and the four-dial rheostat regulates the current within ± 0.05 per cent or better. As the current is only 0.0101 amp, dry cells are a satisfactory source.

Two tapping keys are shown: one for closing the circuit through a 10,000-ohm resistor P, to aid in making the initial balance, and to protect galvanometer and standard cell; the other to close the circuit directly to obtain the final balance.

The range is changed by two shunt coils C and D, so proportioned that when connection is made to binding post 1, the current through A and B is 100 times that through C and D. When connection is made to post 0.01, coil C is in series with A and B and the current through this circuit is now $\frac{1}{100}$ of its former value. The total current remains the same.

176. The Type K-2 Potentiometer.—A more elaborate form of potentiometer is shown in Fig. 90, and the electrical connections are shown in Fig. 91. The essential part of the instrument consists of

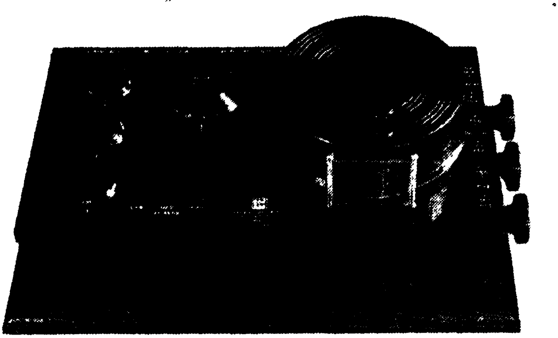


FIG. 90.—Type K-2 potentiometer. (*Courtesy of Leeds & Northrup Co.*)

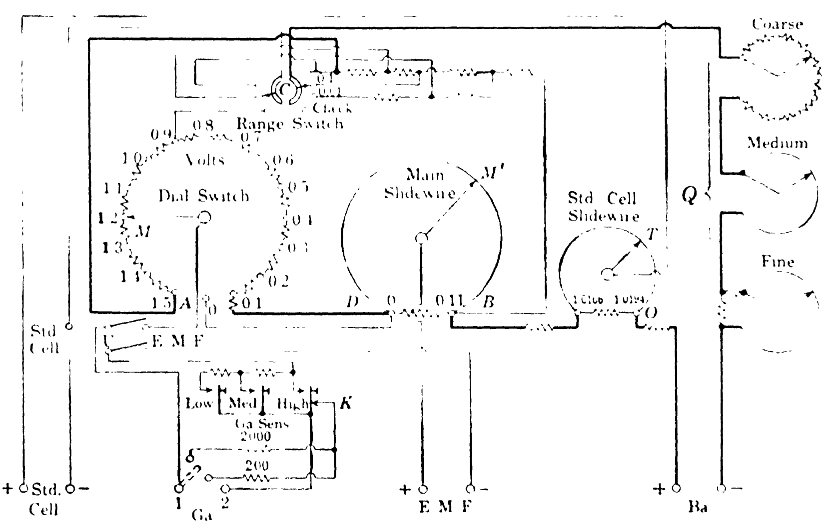


FIG. 91.—Diagram of the electrical circuits in the potentiometer shown in Fig. 90.

15 five-ohm coils *AD*. These are adjusted to equality to a high degree of accuracy, are connected in series, and have in series with them an extended wire *DB*, the resistance of which from 0 to 1,000 on its scale (the entire scale reading from 0 to 1,110) is also 5 ohms. A contact brush *M* can make contact on the studs in 5-ohm steps, and a contact point *M'* can make contact at any point on the extended wire *DB*.

Current from the battery *Ba* flows through these resistances, and by means of the regulating rheostat *Q* it is adjusted to be 0.02 amp. The fall of potential across any one of the coils *AD* is consequently 0.1 volt, and that across the extended wire *DB* is 0.11 volt. By placing the contact point *M′* at zero and moving the contact *M*, the fall of potential between *M* and *M′* may be varied by steps of 0.1 volt from 0 up to 1.5 volts. By moving the contact point *M′* along the wire, the fall of potential between *M* and *M′* may be varied in infinitesimal steps. In making measurements, the unknown emf, in series with a galvanometer is connected between the points *M* and *M′* in parallel with this part of the potentiometer circuit, as in Figs. 84 and 85. The contact points *M* and *M′* are adjusted until the galvanometer shows zero deflection; then the value of the unknown emf can be read from the position of the points *M* and *M′*. In the diagram, Fig. 91, the unknown emf is introduced at the binding posts marked EMF, and the galvanometer at the point *Ga*. When the key marked Low is depressed, the circuit is closed through a high resistance; when key Med is closed, the circuit is closed through a lower resistance, and when key High is closed, the circuit is closed through zero resistance. The purpose of keys Low and Med is to protect the galvanometer against excessive deflections when the opposing emfs are not approximately balanced. They should be used until the deflections have been reduced to a few scale divisions.

177. Quick Method of Applying the Standard Cell.—The known emf—in practice the standard cell—may be connected across *one* portion of *AO*, and the current may be adjusted as described above; the unknown potential difference may be balanced against the fall of potential in *another* portion of *AO*. The connections for accomplishing this with a single galvanometer are shown in Fig. 91. At the point 0.9 on the series of resistance *AD*, a wire is permanently attached that leads to one point of the double-throw switch *U*. The resistance between 0.9 and *B* corresponds with the emf of 1.01 volts, and between *B* and 1.0166 is a sufficient addition to make the resistance between 0.9 and this point correspond to an emf of 1.0166 volts. At this point there is a small circular slide wire with a movable contact point, *T*, through which connection to the standard cell is made. The slide wire has a resistance value that makes possible a practically continuous variation in the standard cell circuit of voltages from 1.0166 to 1.0194. This range corresponds with the variations among different cadmium cells.

This part of the potentiometer circuit is connected to the standard cell and galvanometer when the switch *U* is closed on the side marked

STD CELL. When thus connected to the standard cell and with T set at the value of the emf of the cell, the current through the potentiometer is adjusted by the rheostats Q until the galvanometer shows zero deflection. This makes the potentiometer direct reading in volts, and with the switch U on EMF, the brushes M and M' can be moved to the positions for a balance with the unknown emf.

When thus balanced on the unknown emf, the switch U can be quickly tipped to STD CELL to see that the adjustment of Q is still correct, then quickly back to EMF to see that the setting of M' is still right. If the galvanometer shows a balance for each position of U, the reading of MM' will give the emf of the unknown. A slight deflection calls for a slight readjustment of Q and a corresponding readjustment of M'.

178. Galvanometers for Use with a Potentiometer.—The function of a galvanometer in connection with a potentiometer is that of an indicator of absence of current and, consequently, of absence of potential difference at its terminals. It is essential, therefore, that it should respond readily by a deflection when a slight potential difference exists.

When the total resistance of the circuit containing the galvanometer is relatively low, a small unbalanced potential difference will produce a larger current than when the total resistance is high. In low-resistance circuits, as in ordinary voltage and current measurements, and in thermocouple work, the instrument should have a good microvolt sensitivity; while in high-resistance circuits, such as those met with in many of the potential measurements of physical chemistry, an instrument of high current sensitivity is desirable. At the same time, the internal resistance and external critical damping resistance, in the first instance, should be low; in the second, these values should be correspondingly higher.

179. A Direct-reading Potentiometer without a Standard Cell.—A direct-reading potentiometer that dispenses with both the fragile standard cell and the delicate galvanometer can be readily built up by using a voltmeter of suitable range to read the emf to be measured. The arrangement is very simple, and the measurements are as accurate as the voltmeter (calibrated) can be read.

The voltmeter is connected to the slider of a wire rheostat or potential divider so that it can be made to read at any point of its scale. The cell or other emf to be measured is placed at E, Fig. 92. When the key is closed, the potential differences in the upper circuit are

$$E - ir - V = 0,$$

where V is the voltage across the voltmeter.

When the key is open, the voltmeter stands deflected, owing to the current I flowing through it from the working battery. When the key is closed, the current through the voltmeter is $I \pm i$, the sign depending upon the relative values of E and V. Owing to this change in the voltmeter current, the pointer will move from V toward the value of E, and the potential divider should be varied to change V in the direction indicated by this motion of the pointer. By adjusting V until this motion is reduced to zero and therefore $i = 0$, we have

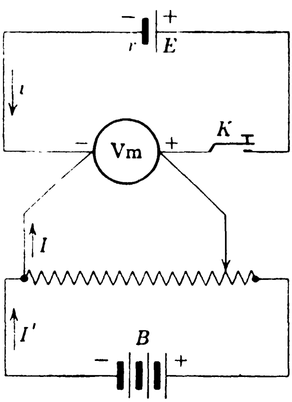


FIG. 92.—The voltmeter can be adjusted to read directly the emf of E.

$$E = V.$$

The voltmeter reading that remains unchanged by opening and closing the key therefore gives the value of the emf that is being measured. This voltmeter-potentiometer is especially useful in locations where laboratory facilities are not readily available.

180. Calibration of a Voltmeter by the Potentiometer Method.—A low-reading voltmeter can be readily calibrated by the potentiometer

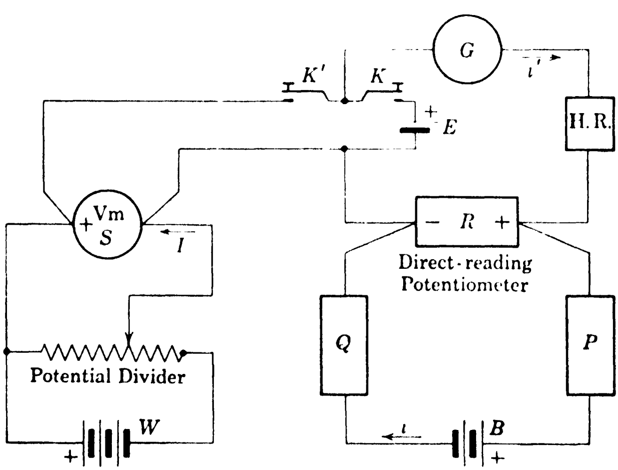


FIG. 93.—Calibration of the millivoltmeter Vm.

method described in Art. 168, above. The voltmeter is connected to the sliding part of a potential divider so that the pointer of the voltmeter can be brought to any desired place on the scale. The resistance of the potential divider should be a few hundred ohms, or enough to draw only a small current from the battery so that the voltage will remain fairly constant. The voltmeter, while still connected with its battery, is also inserted in the galvanometer circuit of the potentiometer in place of the unknown emf. There will thus be two electric circuits, each with its own battery; one battery, W, supplying the current through the potential divider and the voltmeter, while the potentiometer current through P and Q comes from the battery B. The galvanometer circuit connects these two circuits, and the current through it may be in either direction, or made zero by adjusting the resistance in R.

Writing the equation for the potential differences in the circuit through the voltmeter and the galvanometer, before a balance has been obtained, gives

$$S(I + i') + Gi' - R(i - i') = 0,$$

where S is the resistance of the voltmeter. For no current through the galvanometer this equation becomes

$$SI = R'i,$$

where R' is the value of R that gives the balance.

The value of the constant current i is determined by using a cell of known emf in the same manner as explained in Art. 169, Eq. (4). The voltmeter, with its battery, is removed from the galvanometer circuit by leaving K' open, and the standard cell is inserted in the same circuit by using the key K as shown in Fig. 93. The resistance in R is now adjusted to a new value R'' to give no current through the galvanometer. Then

$$E = R''i,$$

since by keeping $R + P$ constant, the current is the same as before. Therefore

$$SI = E\frac{R'}{R''},$$

and this is what the voltmeter should read. If the voltmeter reading is V, the correction to be applied is

$$c = E\frac{R'}{R''} - V.$$

181. The Calibration Curve.—When the values of the true voltages are plotted against the actual readings of the voltmeter, the resulting curve is so nearly a straight line that it is not very useful. Therefore the corrections c (Art. 180) are plotted against the voltmeter readings, as shown in Fig. 94. This makes it possible to use a magnified scale for the ordinates, and on this scale the small corrections appear large enough to be easily read. The plotted points are connected by the dotted line to aid the eye in noting the general trend of the corrections.

Since the corrections have been determined for only a few definite points on the scale of the voltmeter, the dotted line connecting two adjacent points has a limited significance. For example, in Fig. 94 the correction for 8.5 volts may not be midway between the values of the corrections for 8 and 9 volts. The dotted line shows that the correction for 7 volts is not midway between those for 6 and 8 volts. The curve shows in a general way that the correction to be applied to readings of the voltmeter between 3 and 8 volts is about +0.03 volt. Only at the points actually calibrated is the precise correction known. If corrections more accurate than these are required, they must be determined for more points, sometimes for each line on the voltmeter scale.

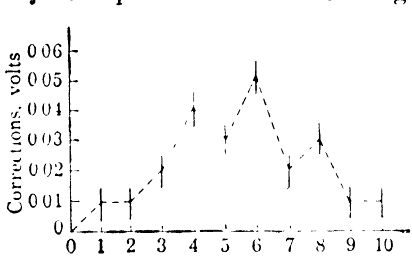


FIG. 94.—Voltmeter calibration curve. The uncertainties in the measured values of the corrections are indicated by the lengths of the short lines.

182. Uncertainty in the Correction.—Probably the largest uncertainty in the value of the voltmeter correction c is in reading the value of V on the scale of the voltmeter. If the voltmeter can be read to within 0.01 volt, then each value of c will be uncertain by that amount.

In plotting the calibration curve, the points should be made large enough to show this uncertainty. Thus if the uncertainty in V is 0.01 volt, the horizontal diameter of the dot will be 0.01 on the scale for V. The vertical diameter of the dot will likewise be 0.01 volt on the scale for c. In plotting a point on the magnified scale of Fig. 94, each round dot is drawn out into a short line that plainly shows the amount of uncertainty that has been ascribed to c.

In view of this, there is no use in questioning just where the dotted curve shall be drawn. Any place within the uncertainty in c is as good as the data warrant, and any curve that passes through any part of the short line goes through the dot. Perhaps the curve should be drawn with a brush that makes a line wide enough to cover these short lines.

183. Measurement of Voltages Greater Than about 2 Volts.—Inasmuch as most direct-reading potentiometers are designed to measure voltages up to 1.5 or 2, it is necessary to use a volt box when higher voltages are measured. The volt box is a particular form of potential divider connected to the large voltage to be measured. A definite fraction of this voltage is then measured by the potentiometer.

The connections of a volt box are shown in Fig. 95. Any voltage up to 230 can be applied to the total resistance of 40,000 ohms. The fall of potential over 400 ohms of this resistance can be measured with a potentiometer, and 100 times this value gives the voltage that is applied to the box.

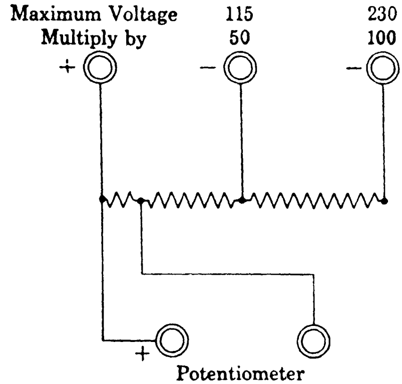


FIG. 95.—Diagram of a volt box.

It is evident that a small current is drawn from the source, and therefore a volt box can be used to measure only those voltages which are not altered by this current.

With the aid of such a volt box, a high-reading voltmeter can be calibrated with a low-reading potentiometer. By an increase in the resistance and the insulation, the range of a volt box can be extended to measure several thousand volts.

184. Comparison of Resistances by the Potentiometer.—One of the accurate methods of comparing two resistances, particularly when these are not very large, makes use of a potentiometer. The two resistances to be compared are joined together in series with a battery and sufficient other resistance to ensure a steady current. This current should not be large enough to change the resistances by heating them. Other things being equal, it is desirable to have the fall of potential over each resistance about 1 volt. Let the two resistances be denoted by T and S; then the fall of potential over each will be TI and SI, respectively.

The actual measurements are very simple. With the potentiometer set up as used for the comparison of emfs, measure the fall of potential over each resistance. Let the readings on the potentiometer be R' and R'', respectively. Then from the conditions of balance,

$$TI = R'i \qquad \text{and} \qquad SI = R''i,$$

where i denotes the current through the potentiometer resistances.

From this it follows at once that

$$T = S\frac{R'}{R''},$$

and this relation can be determined as accurately as the potentiometer measurements can be made.

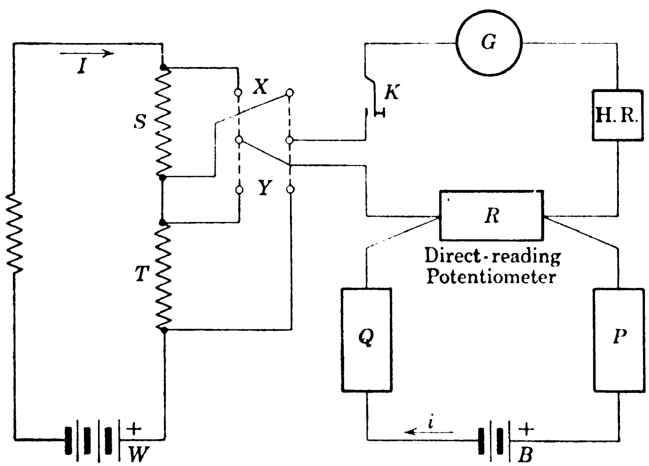


FIG. 96.—Comparison of the resistances S and T.

185. Direct Comparison of an Unknown Voltage with a Standard Cell.—It is not always necessary to have a potentiometer in order to use a standard cell for precise measurements. Often it is desirable to have a single balance give a direct comparison of the voltage measured and the emf of the standard cell. The two following methods illustrate this use of a standard cell.

186. Calibration of a Voltmeter. *Divided-potential Method.*—The voltmeter is joined to a storage battery or other source that will maintain the deflection steady at the point it is desired to calibrate. If the voltmeter is direct reading, it should read the difference of potential between its own binding posts. To calibrate the scale, then, it is necessary only to measure this same difference of potential by some precise method and to compare this measured value with the reading of the voltmeter.

In parallel with the voltmeter is joined a circuit consisting of two resistance boxes A and B; and in parallel with A is a circuit containing the galvanometer, a standard cell, a high resistance, and a key. It is best to have about 1,000 ohms in A and B together for each volt read by the voltmeter. A preliminary calculation will give the resistance that should be placed in each A and B to give a fall of potential over A about equal to the emf of the standard cell. When these approximate resistances have been placed in A and B, the key K can be quickly and cautiously tapped and the direction of the deflection noted. When a standard cell is used in this way, as little current as possible should be allowed to flow through it. Even a slight polarization will lower its emf by an unknown amount, and then it can no longer be called a "standard" cell. Furthermore, nothing is gained by deflecting the galvanometer "off the scale," as a deflection of a centimeter or two takes less time and is fully as definite as the larger deflection. The high resistance is inserted for this very protection of the cell and therefore it is better than a shunt on the galvanometer.

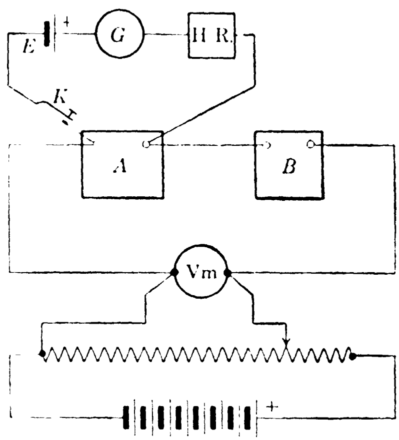


FIG. 97.—Calibration of the voltmeter Vm.

By adjusting the values of A and B, we can reduce the galvanometer deflection to zero. The high resistance can be short-circuited for the final balance if the deflections are small. When thus balanced, the potential differences in the circuit through the voltmeter and A and B are

$$Ai + Bi - SI = 0,$$

where SI is the fall of potential over the voltmeter. This is also what should be indicated on its scale.

Similarly, for the circuit through the standard cell,

$$Ai = E.$$

Eliminating i by division,

$$SI = E\frac{A + B}{A}.$$

If V is the reading of the voltmeter, then

$$V + c = SI,$$

and the correction to be applied at this point is

$$c = SI - V.$$

The computations can be greatly reduced if A is kept at the value of $1{,}000E$. Thus if the emf of the standard cell is 1.018 volts at the temperature of the room, A would then be set at 1,018 ohms and the adjustment would be made by changing B. The value of SI is then given directly by the sum of A and B, divided by 1,000.

Other points on the scale can be obtained by using a different number of cells in the main battery, or, if the voltmeter is low reading, by adding some resistance in series with the battery. If the battery cannot be divided into a sufficient number of steps, a useful method is to join a high-resistance rheostat across the battery terminals and to connect the voltmeter to one terminal and to the sliding contact. Any desired voltage can then be obtained up to the maximum of the battery by simply sliding the movable contact along the rheostat.

It is evident that this method cannot be used to calibrate points below the emf of the standard cell.

A calibration curve should be plotted, with voltmeter readings as abscissas and the corresponding corrections as ordinates. If there is a zero correction because the needle does not indicate zero correctly, such a correction should be made before computing the calibration corrections.

187. For a Quick and Approximate Calibration.—In case the voltmeter reading can be varied continuously or by very small steps, either by means of a sliding rheostat as shown in Fig. 97 or by means of a resistance box in series with the battery, it will be found more expeditious to set A and B at the values corresponding to a given voltage, and then by varying the rheostat to bring the voltmeter to this voltage, the balance being indicated by the galvanometer the same as before. The voltmeter reading should give this same voltage; if it does not, the discrepancy is the correction that must be applied to the voltmeter reading at this point.

In two respects this method does not meet the conditions for an accurate calibration. First, the points for which the corrections are found are not definite lines on the scale. Secondly, the position of the voltmeter pointer must be estimated to the nearest tenth of a scale division, and this cannot be done as accurately as when the pointer is brought into coincidence with a line of the scale.

CHAPTER VII

MEASUREMENT OF CURRENT

188. Action between Currents.—One of the fundamental experiments of Ampère showed that when two electron currents are placed near each other there are forces acting upon them. If the two wires carrying the currents are parallel to each other, and the currents are both in the same direction, then the forces on the wires tend to draw them together. If the currents are in opposite directions, then the wires are urged away from each other. It makes no difference whether the two currents are wholly distinct and separate currents or whether the same current flows out along one wire and returns on the other; the force on a given length of one wire is the same, for the same value of the current, in either case.

This action between two parallel currents is explained by saying that there is a field of magnetic flux around a current. Since this magnetic flux is the same on all sides of the current, there is no resultant force on the wire. But when a second current is placed in this field, it is urged to move in the direction that will increase the total amount of magnetic flux which is linked with the circuit of the second current. This action results in the apparent attraction or repulsion of currents in parallel wires.

189. Force on a Current.—The investigations of Ampère showed that the force F exerted upon a given length l of one of the parallel conductors is proportional to the magnitude of each current I and I', to the length l of the portion considered, and inversely proportional to the distance r between the conductors. This can be expressed by the relation

$$F = \frac{k\mu II'l}{r}, \qquad \text{newtons}$$

where k is the proportionality factor that depends upon the magnitude of the unit in which the values of I and I' are expressed, and μ is the magnetic permeability (see Art. 251) of the surrounding medium.

190. Force per Meter.—The relation above shows that the force on any given portion of the wire is proportional to the length of the portion considered. On the other hand, the force per meter, f, which is given by

$$f = \frac{dF}{dl} = \frac{k\mu II'}{r},$$

is a definite amount whatever length is considered. It depends only on the amount of current in the parallel wires and their distance apart. This relation supposes that the parallel conductors are long enough that an increase in their length in either direction will not increase the force per meter, f, exerted upon the section that is under consideration.

191. The Ampere.—The action between two parallel currents is used as the basis for defining the magnitude of unit electron current. Consider two long straight wires placed parallel to each other at a fixed distance of 1 meter. The effect of the surrounding medium is made definite by specifying that these wires are in vacuum. The effect is practically[1] the same if they are in air.

Let the current in one wire return in the other, and let this current be adjusted to the amount that will give a force per meter on each wire of 2×10^{-7} newton per meter. This amount of current is called an "ampere."

Definition.—One ampere is the amount of current in each of two long parallel wires placed one meter apart in vacuum, when there is 2×10^{-7} *newton per meter acting on each wire.*

192. Definition of 2 Amperes.—A current of 2 amp is not given by increasing the current in the parallel wires described above until the force is doubled. A little investigation will show that if the current is twice as large in each of the two wires, the force experienced by a given length of one wire is four times as great as before. Therefore let us consider that the current in one of the long parallel wires is maintained constant at the value of the ampere just defined. The current in the other wire is then measured by the force experienced by each meter of the second wire. When the force is doubled, the current in the second wire is 2 amp.

In one sense the conditions imposed by this definition of an ampere are ideal rather than practical. All portions of the cross section of the second wire are supposed to be in a region where the flux density is the same that it is at points 1 meter from the center of the first wire. If the cross section of the second wire is too large to meet this requirement with sufficient accuracy, the definition implies that this wire is in a uniform magnetic field where the flux density at every point is

[1] $\mu = 1.00000036$ for air at 1 atmosphere pressure.

equal to the flux density at points that are 1 meter from the center of the first wire.

193. The Determination of the Ampere.—It would be very inconvenient to arrange two long parallel wires for the measurement of current in the laboratory, but the wires can be wound into uniform helices and the force between them, when carrying a current, can be measured by a sensitive balance. Such measurements have been made with a high degree of accuracy at the National Bureau of Standards.[1]

While such a known current cannot be preserved, its value can be expressed in terms of the resistance of a standard ohm and the emf of a standard cell by using an arrangement similar to Fig. 100. Thus the value of the true ampere, as defined in Art. 191, is known and can be used in practical electrical measurements.[2]

194. Galvanometers.—The D'Arsonval type of galvanometer consists of a moving coil suspended between the poles of a permanent magnet something like the arrangement in an ammeter, but greater sensitiveness is secured by using a long, fine strip of phosphor bronze, or gold, for the suspension in place of the jeweled bearings. Such a galvanometer measures very small currents and is useful in measurements where the current is small or is made zero in the final adjustment. It can be used to measure larger currents by using a low-resistance shunt, as in the ammeter; and it will serve as a voltmeter when used in series with a high resistance (see Arts. 112 and 114).

Since the scale from which the deflection of the galvanometer is read usually is divided into millimeters, it will be necessary to calibrate it in order to read the value of the current. A setup for determining the figure of merit can be used (see Fig. 46, Art. 98), and deflections corresponding to different values of the current can be observed. A curve can then be plotted between deflections and currents, and from this curve the value of the current corresponding to any deflection can be read.

[1] Curtis, Driscoll, and Critchfield, *J. Research Nat. Bur. Standards*, vol. 28, p. 133, 1942.

[2] *The International Ampere.*—Formerly, before it was possible to make a precise determination of the ampere, the value of the ampere was expressed in terms of the amount of silver that would be deposited in a specified form of voltameter. The best value that could be established was that an ampere would deposit 0.001118 g of silver per second. By international agreement this amount of current was taken as the unit of current. It is now known that this "international ampere" is slightly too small, being equal to 0.999835 amp. Had not world-shaking events interfered, its use in electrical measurements was to have been discontinued in 1940. The date now set is Jan. 1, 1948.

195. Measurement of Current by Means of a Standard Cell.—The potentiometer method is convenient and accurate for the measurement of current. It depends only on the measurements of a resistance and a difference of potential, both of which can be determined with a very high degree of precision. In this method the current I is passed through a four-terminal resistance R of 1 ohm or less, the value of which is accurately known. The resulting fall of potential RI is then measured with the potentiometer in terms of a standard cell, the emf of which is expressed in terms of the Western normal cell, taken as 1.01830 volts at 20° C (see Art. 172). Since this value was determined from an exhaustive series of comparisons with the current balance, the measurement of a current in terms of a standard cell amounts to a measurement in terms of the absolute current balance (see Art. 193).

In the method described below, the measurements are expressed in terms of a standard cell, either by the regular potentiometer method (Art. 167ff.) or by a direct comparison similar to the method for calibrating a voltmeter (Art. 186).

196. Standard Resistances for Carrying Current.—When a low resistance is used, the question of contact resistance at the connections

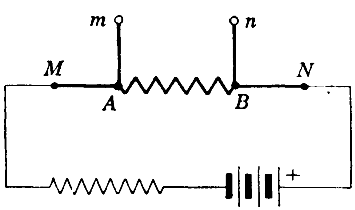


FIG. 98.—Diagram showing the four-terminal resistance AB, connected into the circuit.

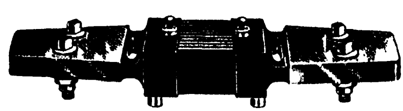


FIG. 99.—Current-carrying resistance; 0.00001 ohm for currents from 500 to 3,000 amperes.

becomes important and such a resistance is usually provided with potential terminals. This arrangement is shown diagrammatically in Fig. 98. MN is a low resistance with potential terminals permanently attached at A and B. When a current I is flowing through this resistance, the fall of potential between m and n is unaffected by any contact resistance at M or N. And if very little or no current is taken through m and n, resistance at these points has little or no effect on the measured fall of potential.

It is evident, then, that the only useful resistance in this arrangement is that of the main circuit between A and B, where the potential terminals are connected, and it is the resistance of this portion that is marked on standard resistances of this type. The potential terminals are clearly seen in Fig. 99.

These four-terminal resistances cannot be added in series or in parallel. They can be measured and used with a potentiometer or with the Kelvin bridge.

197. Calibration of an Ammeter.—*Potentiometer Method.* For the calibration of an ammeter, the instrument is joined in series with a standard resistance. The same current must therefore pass through them both. Its amount is determined by measuring the fall of potential over the standard resistance and comparing this value with the reading of the ammeter. The difference is the ammeter correction. The arrangement is shown in Fig. 100.

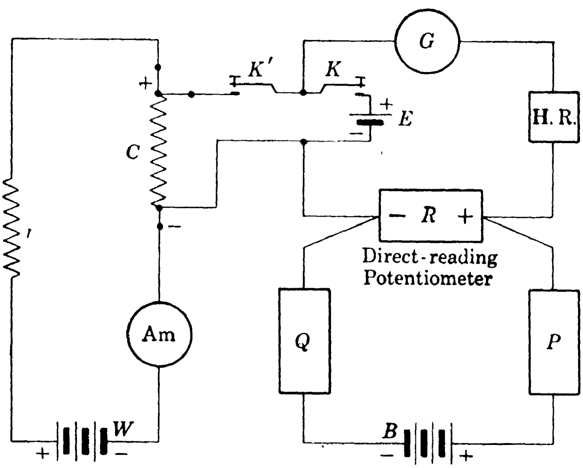


FIG. 100.—Calibration of the ammeter *Am*.

The ammeter is connected in series with a storage battery, a variable rheostat, and the known low resistance C, which should be a standard manganin resistance provided with permanent potential terminals. The fall of potential over the latter is measured by the potentiometer in the usual way. The best results are obtained when this is about 1, although the heat produced in the standard resistance, CI^2, should be kept below 1 watt. By means of the keys K' and K, either C or the standard cell E may be used in the galvanometer circuit. In the latter case, when a balance has been obtained by adjusting R until there is no deflection of the galvanometer,

$$E = R'i.$$

When the coil C carrying a current I has been substituted for the standard cell and a balance has been obtained by readjusting R to some new value, R'', we have

$$CI = R''i,$$

from which

$$I = \frac{E}{C}\frac{R''}{R'}.$$

This value for the current is compared with the ammeter reading I_A and the corresponding correction is

$$c = I - I_A.$$

A calibration curve should be drawn, using the observed ammeter readings as abscissas and the corresponding corrections for ordinates.

198. A Potentiometer Reading Directly in Amperes.—When the auxiliary resistance Q in the battery circuit, Fig. 100, is so adjusted that with the potentiometer balanced against the standard cell, the reading is $R' = 10{,}000E/C$, the potentiometer becomes direct reading in amperes, and the true value of the current flowing through the ammeter can be read directly from the setting of the potentiometer.

199. Calibration of an Ammeter. *Divided-potential Method.*—In this method the same arrangement that was used above, Art. 186, to calibrate the voltmeter is now used to measure the fall of potential over the standard resistance that carries the current to be measured.

The current from a storage battery flows in series through the ammeter, the standard resistance R, and an adjustable resistance r. In parallel with R is a circuit of much higher resistance, consisting of two well-adjusted resistance boxes. In parallel with one of these is the galvanometer with a standard cell and a high resistance of about 10,000 ohms to prevent too great a current through the standard cell.

Let the currents through the ammeter, standard resistance, B, and the galvanometer be denoted by I, I', i, and i', respectively. When the galvanometer shows zero deflection,

$$E_s = Ai.$$

At the same time, the circuit ABR gives

$$RI' = (A + B)i.$$

Solving for the current in R,

$$I' = \frac{E_s}{R}\frac{A + B}{A}.$$

Since $I = I' + i$, we have, finally,

$$I = \frac{E_s}{R} \cdot \frac{A + B + R}{A}.$$

It is evident that if A is kept at some round number, say 5,000, and the adjustment is made by varying B, the computations will be much simplified.

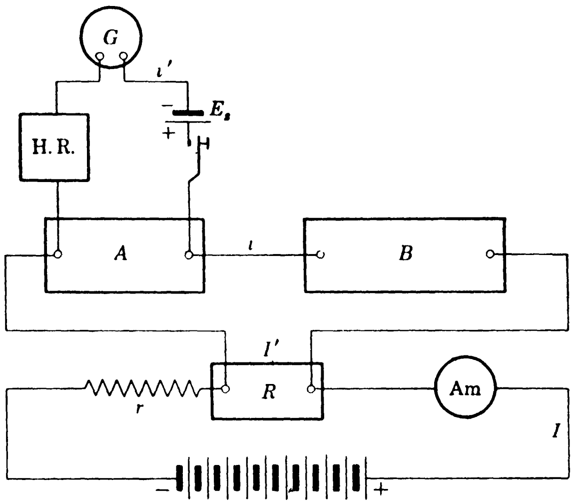


FIG. 101.—Calibration of the ammeter *Am.*

If the reading of the ammeter is I_A, then

$$I_A + c = I,$$

and the correction to be added to this reading is

$$c = I - I_A.$$

CALIBRATION OF AMMETER NO. . . .

Am. zero = E_s = at °C R =

Ammeter		Standard cell		Resistances		I	Correction
Reading	Corrected	Temp.	Emf	A	B		

200. The Kelvin Balance.—The Kelvin balance is an accurate semi-portable instrument for the measurement of current. It has six flat coils placed horizontally, through which the current passes in series. Two of these coils are carried on a balanced beam, one at either end, while above and below each of these movable coils is one of the fixed coils. The diagram shows the relative positions of the coils. The movable coil *cd* is shown in a vertical section through a diameter, while the broken lines indicate the magnetic field in the same plane owing to the fixed coils *ab* and *ef*. It will be seen that at *c* the field is horizontal and directed to the right. If the electron current in the coil *cd* is flowing out of the paper at *c*, then this portion of the circuit will be urged downward. At the other side of the coil the direction of the field is to the left, and the electron current at *d* is flowing into the paper. Hence this side of the coil will also be urged downward. The same is true for all parts of the coil *cd*, and therefore the coil as a whole is

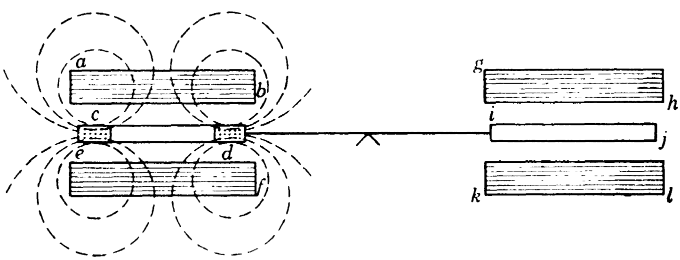


FIG. 102.—Coils of the Kelvin balance.

urged downward with a force proportional to the product of the current it carries and the density of the magnetic flux in which it moves (see Art. 188). The latter is proportional to the current in the fixed coils; therefore the downward pull is proportional to the square of the current through the coils. The action at the other end of the balance is the same, but the direction of the current through the coils is such that the movable coil *ij* is urged upward. Thus this effect is added to the other (see also Art. 239).

To restore the balance, a sliding weight is drawn along a graduated beam until the movable coils again stand in their original position midway between the fixed coils. This position is indicated by a short scale at the end of the beam, over which moves a pointer. The position of the weight on the beam is read from a scale of equal divisions, and, as shown above, this is proportional to the square of the current. To obtain the value of the current in amperes, the square root of this reading is multiplied by the constant corresponding to the particular weight used. There are four such weights, and the constants are 0.5, 1, 2, and 4, respectively, for most balances. In the centiampere balance the result will then be given in centiamperes.

One other matter must be noticed. When the sliding weight is placed on the beam at the zero end of the scale, it is necessary to place an equal counterpoise in the pan at the other end. If this does not establish a complete balance, a small brass flag carried by the moving system can be turned so as to throw more weight to one side or the other as may be required to restore the balance.

201. Calibration of a Kelvin Balance.—The weights that accompany a Kelvin balance are adjusted to give the correct value of the current when the corresponding constants are used. If for any reason it is desired to check the readings of the balance more closely, the scale can be calibrated by means of a potentiometer and a standard Weston cell. The arrangement is the same as that used for the calibration of an ammeter and shown in Fig. 100, above.

In case the corrections are nearly proportional to the readings on the uniform scale, it may be desirable to change the mass of the sliding weights.

202. The Electrodynamometer.—The e-lec″tro-dy″na-mom′-e-ter is an instrument for measuring currents. It consists essentially of two vertical coils, one fixed in place and the other free to turn about the vertical axis common to both coils. Sometimes the movable coil is outside the other, as in the Siemens type; in other forms the movable coil is within the fixed coil. In either case, when a current flows through the movable coil it tends to turn in the same manner as the coil of a D'Arsonval galvanometer. In the electrodynamometer the magnetic field is not due to a permanent steel magnet but is produced by the current flowing in the fixed coil. Thus the deflection depends upon the current I in the fixed coil as well as upon the current i in the movable coil; and the resulting deflection is given by

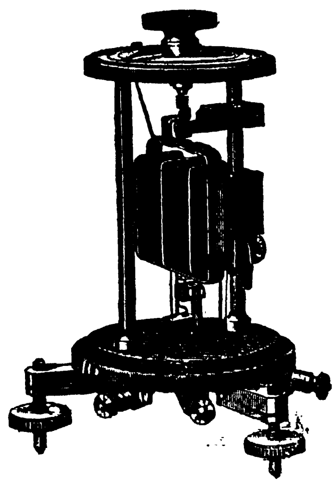


FIG. 103.—Siemens electrodynamometer.

$$iI = A^2D,$$

where A^2 is a constant including all the factors relating to the size and form of the coils, etc., and also including the restoring couple of the suspension.

If the same current flows through both coils in series,

$$I^2 = A^2D$$

and

$$I = A\sqrt{D}.$$

In the Siemens electrodynamometer the coil is brought back to its initial position by the torsion of a helical spring. D is the number of divisions of the scale that measures the amount of this torsion.

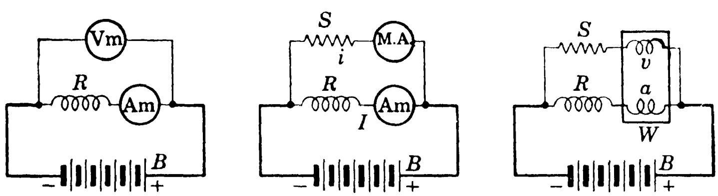
When a coil carrying a current is suspended in a magnetic field, *e.g.*, the earth's field, it tends to turn so as to add its magnetic field to the other. If the electrodynamometer is set in such a position that the earth's field is added to the field due to the current in the fixed coil of the instrument, the deflection will be increased by a corresponding amount. If the two fields were opposed to each other, the deflection would be lessened. This effect can be eliminated by turning the instrument so that the plane of the movable coil is east and west.

203. Calibration of an Electrodynamometer.—In order to use an electrodynamometer for the measurement of a current, it is necessary to know the value of the constant A, or, better, to have a calibration curve. Such a curve is obtained by joining the instrument in series with a calibrated ammeter or a Kelvin balance and observing corresponding readings on the two instruments when they are carrying the same current. The curve is carefully plotted, with currents used as ordinates and the corresponding deflections for abscissas. This procedure gives a horizontal parabola passing through the origin, and from this curve the value of the current corresponding to any deflection can be read. With such a curve, the dynamometer becomes a direct-reading ammeter. The deflection is independent of the direction of the current through the instrument, and therefore it can be used with alternating currents as well as with direct currents. Reversing the direction of the current through one coil only, however, will reverse the direction of the deflection.

CHAPTER VIII

MEASUREMENT OF POWER

204. The Measurement of Electrical Power.—This usually resolves itself into the simultaneous measurement of emf and current. As stated in Art. 8, the unit of power is the watt, and is the power expended by a current of one ampere under a potential difference of one volt. Article 44 showed a simple method for measuring the power expended in a circuit by the current from a battery, with an ammeter and a voltmeter used. A single instrument combining in itself the functions of both an ammeter and a voltmeter is called a "wattmeter." With such an instrument the power may be read directly from a single scale, in the same way that the current is read from the scale of an ammeter.



(a) (b) (c)

FIG. 104.—Measurement of power.

205. The Use of an Electrodynamometer for the Measurement of Power.—An ammeter and a voltmeter connected as shown in Fig. 104 (*a*) for the measurement of a resistance will give at the same time the power expended in R and Am. Let B denote the source of the current. The voltmeter Vm measures the fall of potential E between the terminals, while the ammeter Am gives the value of the current. The product $EI = W$ gives the power in watts.

This result can be expressed in a different form. If in place of a direct-reading voltmeter there had been a large resistance of S ohms in series with a milliammeter for measuring the current i through it, then

$$E = Si \qquad \text{and} \qquad W = SiI.$$

In this form it is seen that the measurement of power implies the product of two currents; and in Art. 202 it was seen that an electrodynamometer is an instrument for measuring the product of two currents. Therefore an electrodynamometer can be used as a wattmeter if it is connected into the circuit in the proper manner for this purpose.

Let R, Fig. 104 (c), be the circuit in which the power is to be measured. The low-resistance coil a of the wattmeter W is connected in series with R as was the ammeter of Fig. 104 (a). The other coil, v, is joined in series with a resistance of several hundred ohms to form a shunt circuit of high resistance, and this is connected in the place of the voltmeter to measure the fall of potential over R and a. Let i denote the value of the current through this shunt circuit, and S its resistance. The fall of potential is then Si, as in Fig. 104 (b). This current i through one coil of the instrument, together with the main current I through the other coil, will produce a deflection D proportional to the product of the two currents. From the equation of the electrodynamometer,

$$iI = A^2D,$$

where A is the same constant that was previously determined.

Since the power being expended in R is $W = SiI$, we now have

$$W = SiI = SA^2D.$$

If the constant of the instrument is known, then SA^2 becomes the factor for reducing the scale readings to watts. In case this factor is unity, as it can be made by adjusting the value of S, the wattmeter is said to be "direct reading."

It may be that the value of A is not known but that instead there is a calibration curve for the instrument when it is used as an electrodynamometer. In this case the value of A^2D can be obtained directly from the curve, for it is the square of the current I' that would produce the same deflection D.

Thus the power expended in R is

$$W = SI'^2,$$

where S is the resistance of the shunt circuit and I' is not any real current but the current that gave the same deflection when the instrument was used as an electrodynamometer. Its value can be obtained from the calibration curve.

206. The Portable Wattmeter.—The portable wattmeter is essentially a moving-coil electrodynamometer. The fixed coil that carries the main current is wound in two sections on a long cylindrical tube. Within this coil and midway between the two sections is the movable coil, which is wound with fine wire upon a short section of a cylindrical tube of somewhat smaller diameter than the fixed coil. In series with this movable coil is a high resistance of fine wire wound on a flat sheet of insulation to make it nearly noninductive. This coil and the high resistance serve as a voltmeter to measure the voltage factor of the power. The movable coil is supported on pivots so that it can readily turn about its vertical diameter. Attached to the movable coil is a long, light pointer that moves over a graduated scale.[1]

In the position of rest, the axis of the movable coil makes an angle of about 45° with the axis of the fixed coil. When deflected so that the pointer is at the middle of the scale, the two coils are at right angles. At the extreme end of the scale the coil stands at 45° on the other side of the symmetrical position. This gives a fairly uniform scale over its entire length. A spiral spring brings the coil to the zero position and provides the torque necessary to balance the electrodynamic couple due to the currents in the two coils.

In addition to the main fixed coil there is another fixed coil of fine wire having the same number of turns as the other, so that a current sent through one coil and back through the other will produce no magnetic field at the place of the movable coil. It is then possible to compensate for the effect of the shunt current passing through the series coil, for the shunt current can be led back through this second coil and thus be made to neutralize its action upon the movable coil. When it is desired not to use the compensation coil, it is replaced by an equal resistance, this connection being brought out to a third binding post.

In the wattmeter reading up to 150 watts, the resistance of the series coil is 0.3 ohm and that of the shunt circuit is 2,600 ohms. The compensation winding is about 3 ohms.

207. Comparison of a Wattmeter with an Ammeter and a Voltmeter.—The reading of a wattmeter can be compared with the power measured by an ammeter and a voltmeter, provided that the latter instruments are connected to measure precisely the same power as the wattmeter. This means, for the uncompensated wattmeter, that the current through the series coil of the wattmeter must be measured by the ammeter, and the voltmeter must be connected so as to measure the same fall of potential as the shunt coil of the wattmeter.

This is accomplished by the connections shown in Fig. 105. The power thus measured is not that expended in R alone but also includes the power expended in the voltmeter and in the shunt circuit of the

[1] See Fig. 248, Art. 476, for a picture of one form of wattmeter.

wattmeter. But since both instruments measure this same power, the reading of the wattmeter should agree with the product of the readings from the ammeter and voltmeter.

If the wattmeter is compensated so that the power measured by it does not include the power expended in its own shunt circuit, then the ammeter must be connected so as not to measure this shunt current. But as it is not possible to connect the ammeter and voltmeter so as not to measure the power expended in one or the other of them, the best arrangement will be to join the voltmeter in parallel with R, as shown in Fig. 106. The power measured by the ammeter and the voltmeter will be that expended in both R and the voltmeter; that

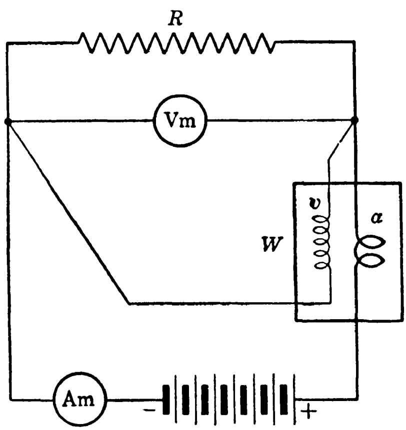


FIG. 105.—Comparison of a wattmeter, W, with an ammeter and a voltmeter.

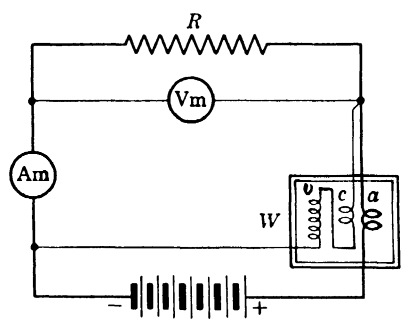


FIG. 106.—Connections for a compensated wattmeter.

measured by the wattmeter will be greater than this by the amount of power expended in the ammeter. The latter can be computed from the formula rI^2 and then added to the product of the readings from the ammeter and voltmeter. With this slight correction the wattmeter reading should equal VI.

208. Power Expended in a Rheostat. (*a*) *When Carrying a Constant Current.*—The object of this exercise is to give the student some personal experience in the measurement of power and in the careful use of a variable rheostat. In this first part a variable resistance is joined in series with a large emf and considerable other resistance, so that the variations in r will not materially change the value of the current through the circuit. If desired, this variable resistance may include an ammeter, and the current may be kept always at the same value.

The wattmeter is connected to r as shown in Fig. 107, and readings of the power expended in the rheostat are taken for the entire range of the resistance. If the values of the latter are not known, they can be measured by one of the methods previously given. A curve should be drawn, using the resistances in the rheostat as abscissas and the corresponding amounts of power as ordinates. The report should contain a discussion explaining why this curve comes out with the form it has.

(*b*) *When under Constant Voltage.*—In this part of the exercise the arrangement is much the same as before, except that the high emf is replaced by a few cells of a storage battery, and r is now the only resistance in the circuit. Starting with the largest values of r, take readings of the wattmeter and plot as before. It will not be safe to reduce r to zero, and readings should be continued only for current values that are not too large for the apparatus used. This portion of the report should give a discussion of what would probably happen if the rheostat were reduced to zero.

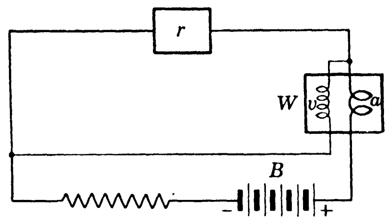


FIG. 107.—Measurement of the power expended in r.

209. Efficiency of Electric Lamps.—An interesting exercise and one that furnishes considerable information is the determination of the efficiency of various types of electric lamps. By means of a wattmeter, the amount of electrical energy supplied to the lamp is readily measured. The light that the lamp furnishes can be measured by a photometer. The efficiency of the lamp is expressed as the ratio of the candle power of the lamp to the electrical power expended, and is given as so many "candles per watt." Thus the efficiency is not an abstract ratio, as in most cases, because it is not possible to measure light in watts. But this does not prevent a satisfactory comparison of different lamps.

Any good form of photometer can be used to measure the candle power of the lamp being examined. If none is at hand, a simple form can be made by standing some object in front of a white screen. The standard light and the one being measured will each cast a shadow of the object on the screen, and by varying the distance of one of the lights from the screen, the intensities of the two shadows can be made equal. The intensities of the two lights are then to each other as the squares of their respective distances from the screen.

If a number of different kinds of lamps are available, it is instructive to measure the efficiency of each one. Tungsten lamps can be compared with carbon lamps, and lamps that have been in use for a long time can be compared with new lamps of the same kind. It is also interesting to determine the efficiency of the same lamp when burned at different voltages, and it is well to plot a curve with efficiencies as ordinates and the corresponding voltages for abscissas. Incidentally, the curve showing the variation of candle power with voltage can be plotted.

210. Measurement of Power in Terms of a Standard Cell.—In the preceding chapters there have been given methods for measuring either a current or a difference of potential in terms of the emf of a standard cell. By combining two of these methods it is possible to measure power in like manner, and this is especially useful when it is desired to know accurately the value of a given amount of power. For example, in some methods of calorimetry it is necessary to have a known amount of heat supplied. Often the actual amount is not essential, but whatever it is, it must be known to a high degree of accuracy. In such cases it is convenient to generate the heat by an electric current flowing through a resistance coil and then to measure the electrical power expended in terms of a standard cell of known emf. This means the measurement of both the current and the fall of potential, as the resistance of the coil usually cannot be accurately determined under the conditions of actual use.

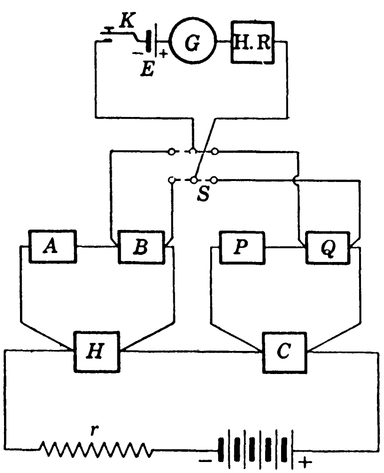


Fig. 108.—Measurement of power expended in H in terms of a standard cell E.

One convenient arrangement, which is capable of wide variation in the amount of power that can be measured, is shown in the figure. The heating coil in which the power is expended is denoted by H. In series with this is a coil C of sufficient current-carrying capacity not to be heated by the current through it. There is also a variable resistance r, by which the current can be brought to any desired value. The current through C is measured by the method for calibrating an ammeter (see Art. 199), and the fall of potential over H is measured by the method for calibrating a voltmeter (see Art. 186).

The standard cell is joined in series with a sensitive galvanometer and a high resistance. The circuit thus formed is connected to the middle of a double-throw switch S. One end of this switch is connected to B, which is a portion of a shunt around H. By adjusting A and B, the fall of potential over the latter can be made equal to the emf of the standard cell, as shown by no deflection of the galvanometer when the key K is closed. The total fall of potential over H is, then

$$V = \frac{A + B}{B} E_s.$$

In the same way there is a shunt circuit PQ in parallel with C, and the part Q is joined to the other end of the double-throw switch. When the switch is thrown to this side, P and Q can be adjusted to give no deflection of the galvanometer when the key is closed. This means that the fall of potential over Q has been made the same as the emf of the standard cell, and the total fall over C is computed as shown above for H. This, divided by the resistance C, gives the current through C as

$$I = \frac{E_s}{C} \frac{P + Q}{Q}.$$

A little consideration will show that the main current through the battery is larger than I by the amount of current that flows through the shunt circuit PQ, and the current through H is smaller than the main current by the amount of current that flows through the shunt AB. If Q is set equal to B, then, since the fall of potential over each is the same, the currents through these two shunts will be equal. Therefore the current through H will be equal to the current through C. The power expended in H is, then,

$$W = VI = \frac{E_s^2}{B^2} \frac{(A + B)(P + Q)}{C},$$

when the two shunt currents have thus been made equal.

211. Calibration of a Noncompensated Wattmeter.—The wattmeter in this case may be an electrodynamometer with two separate coils, or it may be the regular portable wattmeter used without the compensation coil. The series coil of the wattmeter is connected in series with a resistance H in which can be expended the power measured by the wattmeter. There is also in series a standard resistance C whose value is accurately known and which has sufficient current-carrying capacity not to be heated by the currents used in the calibration. A variable resistance r and a storage battery z complete the main circuit. The shunt coil ab of the wattmeter is connected in parallel with H. Two accurate resistances A and B are also connected in parallel with H. The power measured by the wattmeter is then the total power expended in H, the wattmeter shunt circuit, and the circuit consisting of A and B. This power is the product of the current through these three in parallel and the potential difference between m and n. Each of these quantities is determined by the potentiometer.

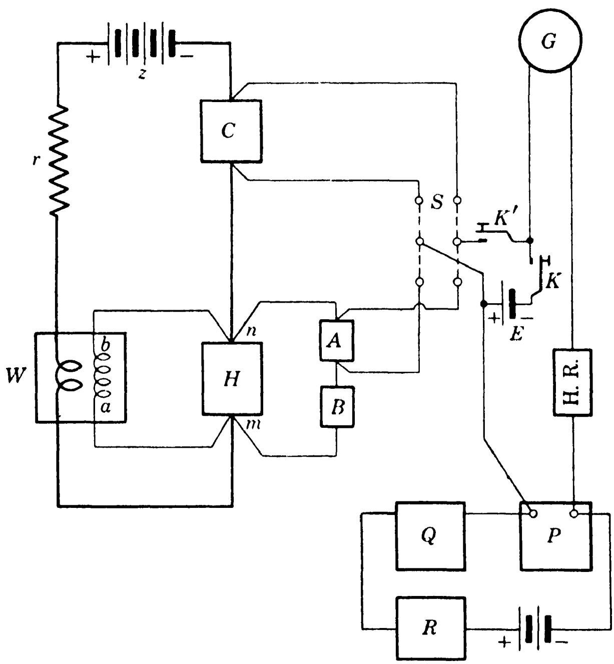


FIG. 109.—Calibration of the wattmeter W.

The potentiometer is represented by the three resistances P, Q, and R. Across P the galvanometer and standard cell are joined in the usual way. When P has been adjusted for a balance.

$$E = iP \qquad \text{or} \qquad i = \frac{E}{P},$$

where E is the emf of the standard cell and i is the steady constant current that is flowing through the main circuit of the potentiometer.

To determine the value of the current through the standard resistance, wires are brought from the terminals of C to the double-throw switch S. When this switch is thrown up and K' is closed, C is connected into the galvanometer circuit in the place of the standard cell. Readjusting the potentiometer for a new balance P' gives

$$IC = iP' = \frac{EP'}{P},$$

and hence

$$I = \frac{EP'}{CP}.$$

To determine the fall of potential over H, the double-throw switch is thrown down, thus connecting the resistance A into the galvanometer circuit. Let i' denote the value of the small shunt current flowing through A and B. Then, adjusting the potentiometer for a balance,

$$i'A = iP'' = \frac{EP''}{P},$$

where P'' is the new reading of the potentiometer. The fall of potential over both A and B is $(A + B)/A$ times as large, or

$$V = \frac{EP''}{P}\frac{A + B}{A},$$

and this is the same as the fall of potential over H.

The power measured by the wattmeter is, then,

$$W' = VI = \frac{E^2}{P^2}\frac{P'P''}{C}\frac{A + B}{A}.$$

If the reading of the wattmeter is W, the correction to be added to this reading is

$$c = W' - W.$$

Different readings of the wattmeter are secured by changing the current through H. A calibration curve can be plotted, with the readings of the wattmeter used as abscissas and the corresponding corrections used for ordinates.

212. Calibration of a Compensated Wattmeter.—For this purpose a potentiometer may be used, as in the preceding method, but as this instrument is not always at hand, a method using resistance boxes is given here. The principal difference between the compensated wattmeter and the uncompensated one is that the former measures only the power expended in the circuit to which it is attached, while the latter measures, in addition to this, the power expended in its own shunt circuit.

Thus let *W*, Fig. 110, represent the wattmeter connected in the circuit to measure the power expended in *H* and *C* together. *C* is a standard resistance for use in the measurement of the current, and *H* is sufficient other resistance to give the required amount of power. As the power expended in the shunt coil is not measured by the wattmeter, it should not be measured by the standard cell; therefore *C* is placed inside, next to *H*.

The calibration circuit consists of four resistance boxes *P*, *Q*, *R*, and *S*, joined in series with a battery of a few volts sufficient to maintain a small constant current through the circuit. This circuit is joined through the galvanometer to the standard cell and to the wattmeter circuit in three places, as shown, each connection being provided with a key. When they are finally balanced, no current flows through any of these connections.

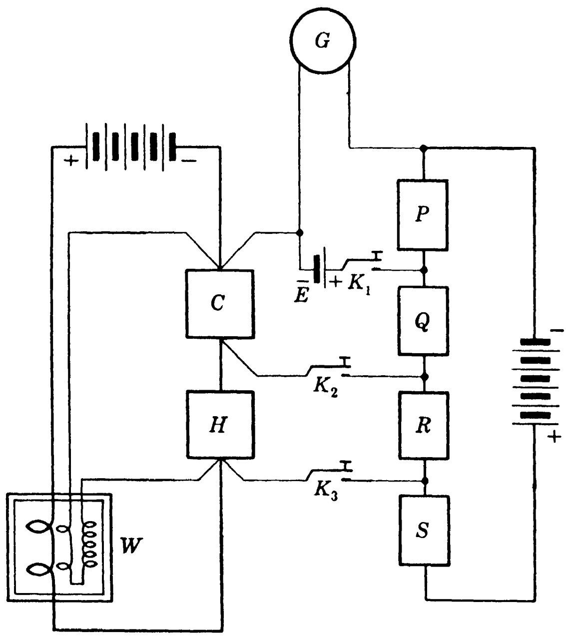


FIG. 110.—Calibration of the wattmeter *W*.

The measurements are made as follows: First, *P* is set at some convenient value, say 1,000 ohms, and some of the remaining resistances are then changed until there is no deflection of the galvanometer upon closing the key K_1. This fixes the total resistance of this circuit and it is thereafter kept constant at this amount. The fall of potential over *C* should be a little larger than the emf of the standard cell. A little more resistance added to P will then be required to give no deflection when the key K_2 is used. This resistance is added by varying Q and S, with their sum kept constant, until there is no deflection of the galvanometer upon closing the key K_2. This balance measures the value of the current through C, because

$$IC = i(P + Q) = (P + Q)\frac{E}{P},$$

and therefore

$$I = \frac{E}{C}\,\frac{P + Q}{P}.$$

This is the effective current actuating the wattmeter.

The fall of potential over CH is measured in the same way. This will be greater than that over C alone, and therefore for a balance it will require a resistance greater than $P + Q$. The resistance R is now varied, with $R + S$ kept constant, until there is no deflection of the galvanometer upon closing K_3. The fall of potential over CH is then the same as that over the three resistances P, Q, and R. That is,

$$V = i(P + Q + R) = (P + Q + R)\frac{E}{P}.$$

The power measured by the wattmeter is, then,

$$W' = VI = \frac{E^2(P + Q)(P + Q + R)}{P^2C}.$$

If the reading of the wattmeter is W, the correction to be added to this reading is

$$c = W' - W.$$

In case V is too large to be measured directly as shown here, a shunt AB may be placed around H as shown in the preceding method, and the potential fall over A alone may be measured. The total fall of potential over H is then readily computed. The addition of this shunt will make no difference in the wattmeter, since H with its shunt now replaces H alone, and the wattmeter measures whatever power is expended in either arrangement.

213. Calibration of a High-reading Wattmeter.—A high-reading wattmeter is one that measures large amounts of power. In calibrating such a wattmeter it is often impossible, and usually inconvenient, to expend sufficient power to bring the reading up to the high values indicated on the scale. But this is not necessary, for all that is required is that there shall be a large current through the series coil and a small current at high voltage through the shunt circuit. By using different batteries to supply these two currents, there need be no great expenditure of energy. As shown in the figure, the battery B'', consisting of one or two large storage-battery cells, supplies a large current through the series coil of the wattmeter and the low standard resistance C. The latter should be of such a value that the

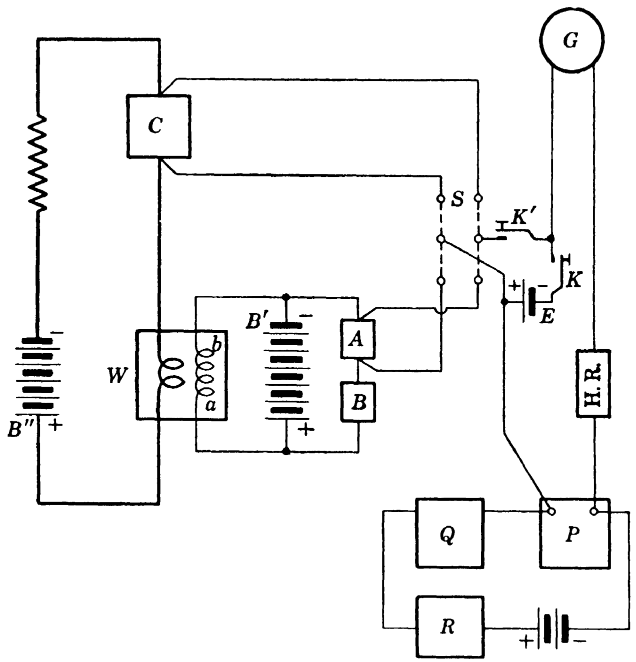


FIG. 111. Calibration of the high-reading wattmeter W.

fall of potential over it will be 1 volt or less, in order that this may be readily measured by the potentiometer. Since the resistance of the shunt circuit is large, it will require a large number of cells in the battery B', but these cells can be small, as only a small current will be needed. In parallel with this circuit is placed another high-resistance circuit AB, so divided that the fall of potential over the portion A shall be about 1 volt. The calibration is then the same as given above for the case of an uncompensated wattmeter, and the wattmeter should read the product of the current through C and the voltage across A and B.

If the reading of the wattmeter is W, the correction to be added to this reading is

$$c = VI - W.$$

In making this calibration it is best to have the currents through the wattmeter about the same as those that will be used when the instrument is measuring power.

213A. Equivalent Values of the International Units.—The units that are defined and used throughout this book are the absolute mks units based on the fundamental units of the meter, kilogram, and mean solar second.

Formerly it was not possible to establish these units in concrete form with the accuracy that is required in electrical measurements. Therefore, by international agreement, concrete standards were adopted which were as nearly as possible equal to the true values of the ohm and the ampere.

It is now possible for the National Bureau of Standards to establish the absolute values of the ohm and the ampere with a high degree of accuracy, and after Jan. 1, 1948 these absolute units will be used. The following table shows the values of the international units in terms of the absolute units to be adopted by the National Bureau of Standards.

1 international ohm	=	1.000495 absolute ohms
1 international volt	=	1.000330 absolute volts
1 international ampere	=	0.999835 absolute ampere
1 international coulomb	=	0.999835 absolute coulomb
1 international farad	=	0.999505 absolute farad
1 international henry	=	1.000495 absolute henrys
1 international watt	=	1.000165 absolute watts
1 international joule	=	1.000165 absolute joules

CHAPTER IX

ELECTRON TUBES

214. Electron Tubes.—As usually seen at the present time, an electron tube contains three essential parts. The *filament* is similar to the filament of an ordinary electric-light bulb but shorter and of less resistance, so that it is lighted by a low-voltage battery. In some tubes the tungsten filament contains a small amount of thorium, and an abundant emission of electrons is obtained at a temperature below red heat.

Surrounding the filament and a short distance from it is a thin metal sheath called the "*plate*." This is provided with an outside terminal. Together with the filament, it constitutes a two-electrode tube. In most tubes, however, a network of fine wires surrounds the filament about midway between the filament and the plate. From its form this is called the "*grid*." The grid is well insulated from both the filament and the plate, and it has its own outside terminal. Such a tube, called a "three-electrode electron tube," has the familiar four prongs projecting from its base for the external connections. In general appearance an electron tube resembles an electric-light bulb, and the pressure within the tube is reduced to a very good vacuum. Because of this fact these tubes are often called "vacuum tubes." See also Art. 222.

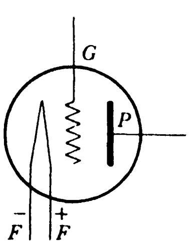


FIG. 112.—Diagram of an electron tube.

215. Diagram of an Electron Tube.—In the diagrammatic representation of an electron tube the arrangement shown in Fig. 112 is used. The filament is shown by the narrow loop *FF*, the + and − signs indicating the terminals that are to be connected to the battery which supplies the heating current. This battery, of a few cells, is commonly called the "A" battery.

The vertical line *P* at the right-hand side of the figure represents the plate, taken as being at right angles to the plane of the paper. In reality, of course, this sheath surrounds the filament equally on all sides.

The wavy line G represents the grid of fine wires between the filament and the plate. In an actual tube the connections to the plate and to the grid are placed in the base near the filament connections. In the figure these are shown at the side and top of the diagram to avoid confusion in the drawing. The circle around the whole indicates that these elements are enclosed within the glass bulb of the electron tube.

The potential of the plate is usually reckoned from the negative terminal of the filament, and the battery that supplies this voltage is commonly called the "B" battery. If a battery is used to give a definite potential to the grid, it is called the "C" battery.

216. Electron Emission.—It has been stated in earlier chapters that a current is a stream of electrons along the wire. Evidence of such a stream of electrons is afforded by the phenomena within an electron tube. When a hot filament is used in a vacuum tube with a cold electrode, it is possible to pass an electron current through the vacuum space from the hot filament to the cold electrode by an emf that maintains the filament a few volts more negative than the cold electrode. When both electrodes are cold, the vacuum makes a very good insulator.

It has been shown by experimental investigation that the particles passing through the vacuum from the hot filament to the cold electrode have the same mass and carry the same charge as the electrons that are found in many other phenomena. That is, each one has a mass that is only 1/1,838 of the mass of an atom of hydrogen, while the charge carried by each one is equal to the charge carried by each atom of hydrogen in electrolysis. The supply of these electrons seems to be inexhaustible and is vastly greater than could have been supplied by the filament itself. The electrons therefore must have been supplied to the filament by flowing in through the colder metal connections, and as there is no accumulation of electrons on the cold electrode in the vacuum tube, they must have been able to flow away over the connecting wires. Tested with an ammeter, these wires are found to carry an electron current, the direction of which is that taken by the electrons as they pass along the wire on their way back from the cold electrode to the hot filament. If the battery between the cold electrode and the hot filament is reversed, no current passes through the circuit, since no electrons can leave the surface of the cold electrode.

217. Effect of Temperature upon Electron Emission.—As already suggested, electrons are more readily emitted from the filament when it is hot than when cold. This relation between the electron current and the temperature can be investigated with an arrangement as shown in Fig. 113.

The filament is heated by a current from the A battery of a few storage cells. The current I can be varied by the rheostat Rh and measured by the ammeter Am. The temperature[1] of the filament may be taken as being proportional to $I^{\frac{1}{2}}$.

From the plate is a circuit through the B battery of 20 to 100 volts, a key, and the galvanometer, leading to the negative end of the filament. If the B battery is connected so as to drive electrons from the plate to the filament, no current flows, even with the filament

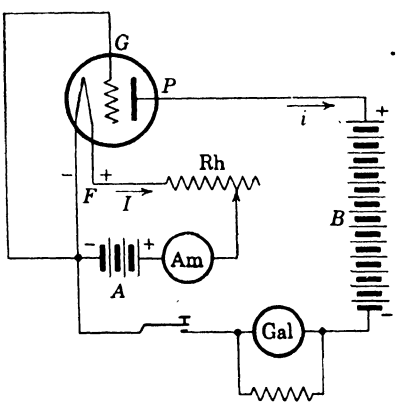


Fig. 113.—The plate current i is increased by increasing the temperature of the filament.

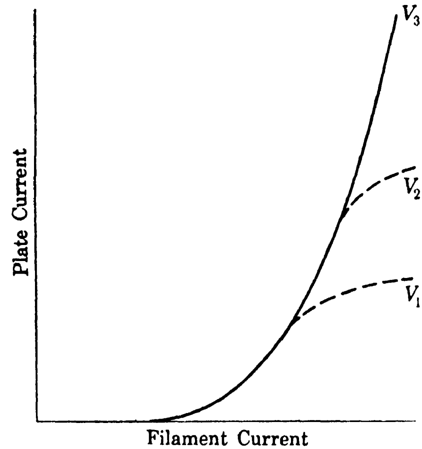


Fig. 114.—The plate current increases very rapidly as the filament-heating current is increased.

heated to incandescence. With the + end of B connected to the plate, a current is readily obtained, and it may be necessary to use a shunt with the galvanometer. In using 100 volts with a galvanometer, great care must be exercised to make sure that there will be no leakage current from any part of the battery to the galvanometer. The connection from the + terminal to the plate should be especially well insulated from the rest of the arrangement.

Keeping the voltage of the B battery constant, and starting with a small current through the filament from the A battery, the growth of the "plate current" i that flows through the tube as the filament is heated more and more can be observed. Since this current flows across the space from the filament to the plate, it is sometimes called the

[1] See *Gen. Elec. Rev.*, vol. 30, p. 312, 1927.

"space current." This relation can be shown by a curve between corresponding values of I and i (see Fig. 114).

Similar curves can be determined for several different values oı the B-battery voltage. If i seems to reach a maximum value as I is increased, it is not because of any lack of electrons but because the potential gradient[1] near the filament is not sufficient to overbalance the effect of the space charge.

218. The Space Charge.—As pointed out above, current does not pass through the tube when the plate is connected to the negative end of the B battery. If the negative potential of the plate is due to a static charge of electrons upon it, the effect is the same, and any electrons in the neighborhood are urged back towards the filament. The negative charge need not even be on a metal plate in order to produce this modification of the potential gradient between the plate and the filament; every electron in the space around the filament, whether stationary or moving in the current from the filament to the plate, produces something of this effect.

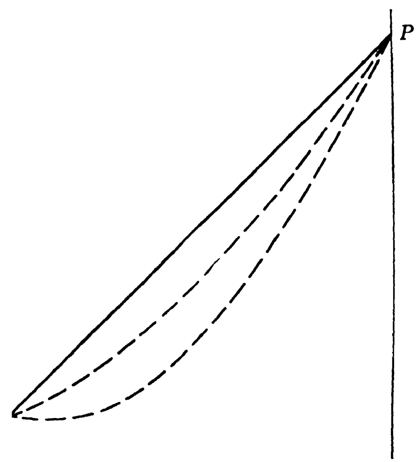


FIG. 115.—The potential gradient is uniform between two parallel plates. A space charge changes this, as is indicated by the dotted lines.

When the effect of the negative charge of the electrons in the space around the filament is considered, the electrons concerned are referred to collectively as the "space charge." The electron current is most concentrated near the filament, and it is here that the effect of the space charge is most marked. When there is a large emission of electrons, the space charge around the filament may be sufficient to act like a negative potential on the plate and prevent a further increase of the electron current from the filament.

When a difference of potential is applied to two parallel plates like P and F, Fig. 115, this can be represented as a uniform fall of potential across the space from one plate to the other. But a cloud of electrons in this space will lower the potential of this region so that the

[1] The term "potential gradient" denotes the fall of potential per centimeter, and it is measured in volts per centimeter.

actual fall of potential is no longer uniform but is more like the broken line in Fig. 115. The larger the number of electrons, the greater is this effect, until the condition is reached in which there is no rise in potential near F, and there would be no tendency for electrons to move from F towards P. This would prevent the continued increase of the current.

In the electron tube the case is similar to the parallel plates, but it is not as simple. The filament and sheath are more nearly like two concentric cylinders. The fall of potential across the space between two concentric cylinders is not uniform but logarithmic, as shown in Fig. 116. Since the surface of the filament is very small, the density

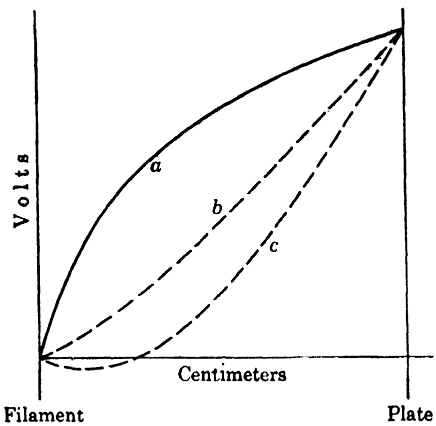


FIG. 116.—Most of the potential difference between a wire and a concentric cylinder is near the wire. But a space charge may greatly modify this distribution.

of the electrons near it is large. The effect of this large negative charge near the filament reduces the potential gradient in this region, as explained above, in spite of the large gradient of the normal distribution of the potential, and it may even reverse the direction of the fall of potential, as shown in Fig. 116.

A larger voltage for the B battery will give a steeper potential gradient throughout the space between the filament and the plate, and this will give sufficient rise of potential around the filament to carry a larger stream of electrons. This effect is shown in Fig. 114 by the larger final values of the plate current.

219. Relation of the Plate Current to the Plate Voltage.—The current through the plate circuit depends to some extent upon the voltage of the B battery. As this voltage is increased, the current becomes larger, until a point is reached where a further increase in voltage does not increase the current. The actual value of this saturation current depends upon the temperature of the filament. As the supply of electrons becomes more abundant, the final value of the current is larger, and the approach of the saturation value of the current is less marked. Figure 117 shows curves for several different temperatures of the filament. The normal current for this filament is 1.00 amp, and with this heating the saturation value of the plate current is reached with 80 volts. However, with a current of 0.80 amp through the filament, the plate current reaches its saturation value at 40 volts and does not increase as the voltage is further increased.

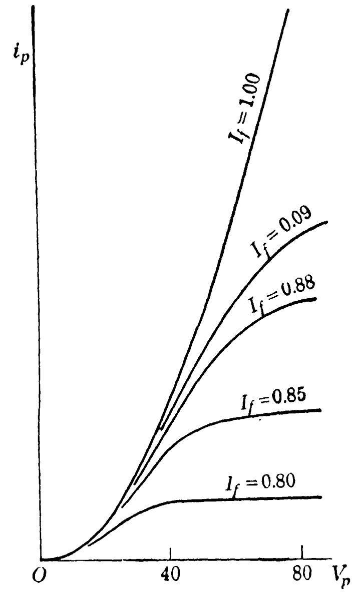


FIG. 117.—The relation between the plate current, i_p, and the emf of the "B" battery, V_p, for five different temperatures of the filament.

220. The Electron Tube as a Rectifier.—As indicated in Art. 216, the stream of electrons can pass through the tube in only one direction. When an alternating emf is used in the plate circuit instead of the B battery, there will be current through the circuit whenever the emf is in the proper direction, but there will be no current during the other half of the cycle. The current that flows, then, will be in one direction only, but it is not a steady current. It is more nearly like the positive loops of an alternating current separated by intervals of zero current. The English name of "valves" for these tubes indicates this rectifying property. This variable direct current is not desirable where a steady current is required, but it is suitable for some purposes and is often useful when no other source of direct current is available. By rectifying a high-voltage alternating current, it is possible to obtain a high-voltage d-c source. A modification known as the "tungar rectifier" is used for charging storage batteries.

221. The Grid.—The effect of the space charge around the filament in reducing the electron current has been noted in Art. 218. If a grid of fine wires is placed near the filament, the potential of this region can be varied at will. When the grid is given a negative potential with respect to the filament, the effect of the space charge is increased, and this may be carried so far as practically to stop the electron current. On the other hand, if the grid is made positive with respect to the filament, the effect of the space charge can be neutralized or even overbalanced, thus giving an increased potential gradient near the filament and a greater acceleration to the electrons in this region.

The electron tube with such a grid serves as a very useful relay in an electric circuit. By merely applying a voltage to the grid, involving very little expenditure of power, the current through the plate circuit can be varied considerably. Since this plate current is driven by the high-voltage B battery, the variation in power is correspondingly greater. The response of the plate current is very prompt and the tube thus acts as an inertialess relay.

222. Screen Grid.—Some electron tubes are provided with a second grid which is placed between the first grid and the plate and is maintained at a constant positive potential of about half that of the plate. While the plate is supposed to be maintained at a constant potential, yet when the current in the plate circuit is varying there are corresponding variations in the potential of the plate. These changes in the plate potential influence the potential of the first, or control, grid through electrostatic induction, and this reacts to magnify the change in the plate current. The purpose of the second grid is to screen the first grid from these effects of the plate and to leave it free to respond only to the potentials that are impressed on it from the grid circuit. The connection from the screen grid of the tube to the battery and thence to the ground (or to the filament) should be of low impedance, so that the charges on this grid can vary quickly when this is necessary to keep the potential constant.

223. Space-charge Grid.—Sometimes the two grids described in the preceding article are connected in a different manner. The grid nearest the filament is maintained at a small positive potential (perhaps connected to the positive end of the filament). It is used to neutralize some of the dense space charge near the filament and thus to allow the use of a lower plate voltage. The other grid is then used for the control grid.

224. Static Characteristic of a Three-electrode Tube.—The variation produced in the plate current by a change in the grid potential is shown by the curve *KGJ*, Fig. 118. The B battery is supposed to have maintained a constant voltage V_p in the plate circuit for all points on this curve. Such a curve, called the "static characteristic curve" for the electron tube, shows the steady value of the plate current for any given set of steady values of the grid and plate voltages.

While the tube is not ordinarily used in this manner, curves obtained in this way are useful in showing what effects may be expected in the tube when these voltage values are varied in any manner.

The arrangement for the experimental determination of the static characteristic curve is shown in Fig. 119. The A and B batteries are connected in the usual way. With a sliding-potential divider ED connected as shown in the figure, the grid can be maintained at any desired voltage. The potential of the grid is usually compared with the potential of the negative end of the filament, and this value is measured by the voltmeter Vm. For negative grid voltages, battery C is reversed. The current through the plate circuit is measured by the galvanometer or a milliammeter. Plotting corresponding values of the grid voltage V_g and the plate current i_p gives one of the curves shown in

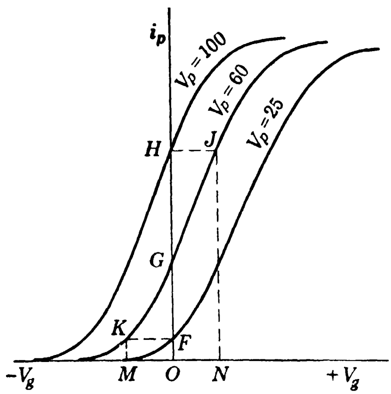


FIG. 118.—Characteristic curves of a three-electrode tube for different plate voltages.

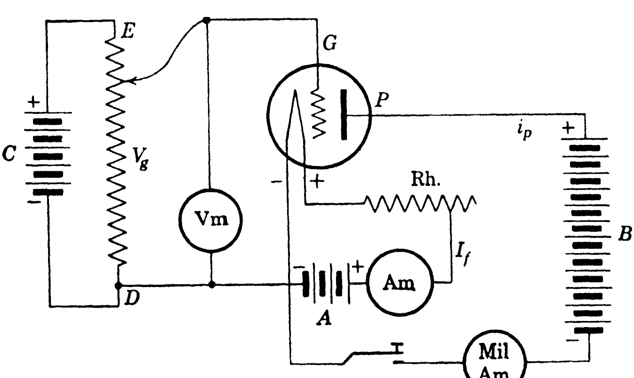


FIG. 119.—An arrangement for the determination of the characteristic curve of the electron tube.

Fig. 118. If the plate had been maintained at a different potential by using a different value for the B-battery voltage V_p, a similar characteristic curve would have been obtained but shifted to the right or left, as shown by the curves of Fig. 118.

225. The Surface Characteristic.—The total characteristic of an electron tube involves more factors than can be represented by a single curve or even by a series of such curves, although a person who is familiar with these curves can read much information from a set of curves like those shown in Fig. 118. It is not as easy, however, to see how the current increases through the values *OF*, *OG*, and *OH* when V_p is increased as it is to refer to the i_p-V_p curve of Fig. 117 for $I_f = 1.00$ amp. In fact, the latter curve may be considered as being drawn on a plane through *OH* and perpendicular to the plane of the paper in Fig. 118. If the entire series of both sets of mutually perpendicular curves were drawn, the result would be a surface. The curves shown in Figs. 117 and 118 are the intersections of this surface with different vertical planes.

Such a surface can be constructed in the form of a plaster model. A series of small holes are drilled in a smooth board 1 cm apart each way, dividing the board into squares. Each row of holes in one direction is marked with a value of V_p from zero to the maximum that is used. The rows of holes in the cross direction are marked with the values of V_g, with the zero line through the center of the board. Along one row of holes set a series of vertical wires of lengths to represent the values of i_p. The tops of these wires will make a curve like one of those in Fig. 117 or 118. When all the rows are filled with vertical wires, their tops will indicate the characteristic surface corresponding to the filament temperature that was maintained while these currents were being measured. To make the surface more apparent, the whole group of wires can be filled up with plaster of Paris, smoothed off even with the tops of the wires. The whole series of characteristic curves can be drawn on this surface by connecting the ends of the wires in each row for which V_p has a constant value. Such a surface is useful in giving a complete survey of the action of the tube with a given filament temperature. A series of such surfaces will represent the characteristics for different temperatures of the filament.

It will be found that a certain limited part of these surfaces contains the most desirable characteristics for the principal uses of an electron tube. In drawing a single characteristic curve, a person familiar with the general form of these surfaces will choose such values of V_p, V_g, and I_f as will place the curve through this most favorable region.

226. Voltage Amplification.—Since the B battery maintains the plate at a constant potential, at first sight there would seem to be no possibility of voltage amplification. One meaning that may be given to this term is illustrated in the curves of Fig. 118. With a plate voltage of 60 and with the grid connected to the negative end of the filament, the plate current of a few milliamperes is represented by *OG*. This current can be increased to *OH* by increasing the plate potential to 100 volts, or, without changing V_p, the current can be increased to this same value by increasing the potential of the grid by the amount *ON* (= 5 or 6 volts), which gives the value *JN* as the new plate current. In this case the application of a few volts on the grid has produced as great a change in the current as could be given by an additional 40 volts in the plate circuit.

In the same way the current could be reduced to *OF* by reducing the plate voltage to 25 volts. Or, without a change in the plate voltage, the current can be reduced to *KM* by giving the grid a negative potential of a few volts (= *OM*).

227. Amplification Factor.—It was shown above that a change ΔV_g in the grid potential is equivalent to a much larger change ΔV_p in the plate potential. This is expressed by the relation

$$a\Delta V_g = \Delta V_p$$

or

$$a = \frac{\Delta V_p}{\Delta V_g}.$$

This ratio *a* is called the "amplification factor" for the tube. Referred to the model of the surface characteristic, *a* is the ratio of the slope of the surface in the direction V_g to the slope in the direction of V_p. The value of *a* is not constant for all values of V_g, but it is nearly constant for the voltages commonly employed.

228. Determination of the Amplification Factor.— While the value of the amplification factor *a* can be obtained from the curves of Fig. 118 when these are accurately drawn to scale, this method gives its value for steady conditions only. Electron tubes are commonly used with rapid changes in voltage, and it is desirable to make a direct determination of *a* under conditions resembling those that obtain in the use of the tube. This can be done with the arrangement shown in Fig. 120, in which a small alternating voltage is introduced into the grid circuit and at the same time enough alternating voltage is added to the plate battery to keep the current from changing.

By means of the steady voltages from the batteries *A*, *B*, and *C*, the electron tube can be set at any desired point on the characteristic curve, or surface. A small alternating current can be passed through the noninductive resistances g and p. These resistances should not be large enough to increase materially the resistances of the grid and plate circuits, and for the same reason a low-resistance telephone receiver T should be used.

By tracing out the connections, it will be seen that when the alternating current i' is flowing from N to M, the fall of potential gi' is added to the voltage of the C battery. This tends to increase the plate current. At the same time, the fall of potential pi' is subtracted from the voltage of the B battery, and this tends to decrease the current. When these two effects are balanced, there is no change in the

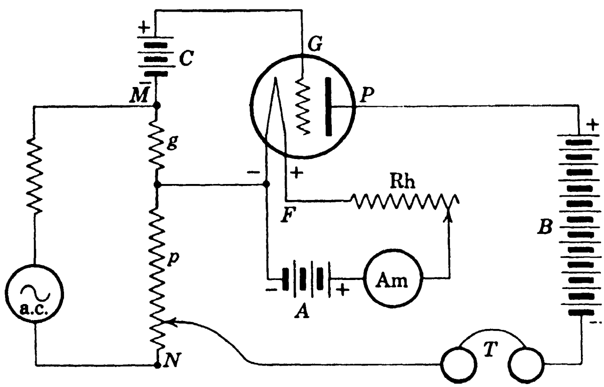


FIG. 120.—The arrangement for the direct determination of the amplification factor.

plate current, and no sound is heard in the telephone receiver T. The amplification factor is, then,

$$a = \frac{pi'}{gi'} = \frac{p}{g}$$

for this point on the characteristic curve.

By changing the voltage of one of the batteries, the value of a for the new conditions can be determined in the same way.

229. The Electron Tube as an Amplifier.—One of the important uses of an electron tube is that of a relay or amplifier in which the plate current is made to reproduce all the variations of the original current. Along the middle portion of the characteristic curve, the relation between V_g and i_p is very nearly linear. Therefore when V_g is made to vary in any given manner, the plate current i_p will vary in the same manner and reproduce the same changes. For example, if a weak telephone current is transformed to a higher voltage (and a still smaller current) and this voltage is applied to the grid of the electron tube, the plate current will respond with the same variations and much greater power.

This is illustrated in Fig. 121, where *EKFH* is a part of the straight portion of the static characteristic curve (see Fig. 118). With a proper amount of grid potential, the operating point of the tube can be set at *K* near the middle of this straight portion. When a varying voltage is added to this grid potential, the operating point of the tube

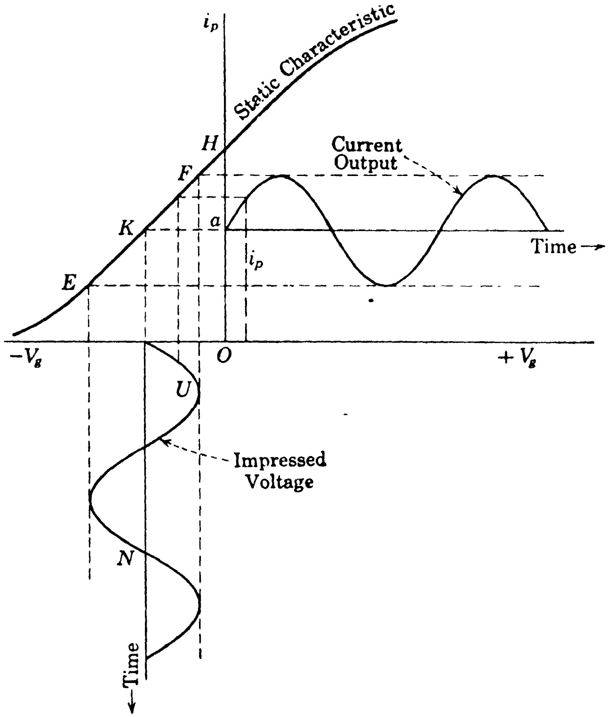


FIG. 121.—Showing how the wave form *UN* of the grid voltage is reproduced in the current output.

swings between *E* and *F*, Fig. 121, and the plate current varies in the same way as this varying voltage, since *EF* is a straight line.

Thus all the variations in the grid potential (slight power) are reproduced in the variations of the plate current (large power = $V_p i_p$). When this current is passed through the primary of a transformer, only the varying part is passed on by the secondary current. When the changes in V_g are large enough to extend beyond the linear part of the curve, the plate current is no longer proportional to the applied voltage, and the resulting wave form is distorted from the original. For a greatly distorted case see Fig. 122.

230. The Electron-tube Voltmeter.—One of the useful applications of the peculiar properties of an electron tube is that of the electron-tube voltmeter for measuring a-c voltage of any frequency.

The principle of this use of the tube is shown in Fig. 122. The curve *EKFG* is the lower part of the static characteristic curve for the tube (see Fig. 118). With the normal filament current and plate voltage, the plate current is *OG* with zero grid voltage. With a suitable grid voltage the operating point for the tube can be brought to *K*,

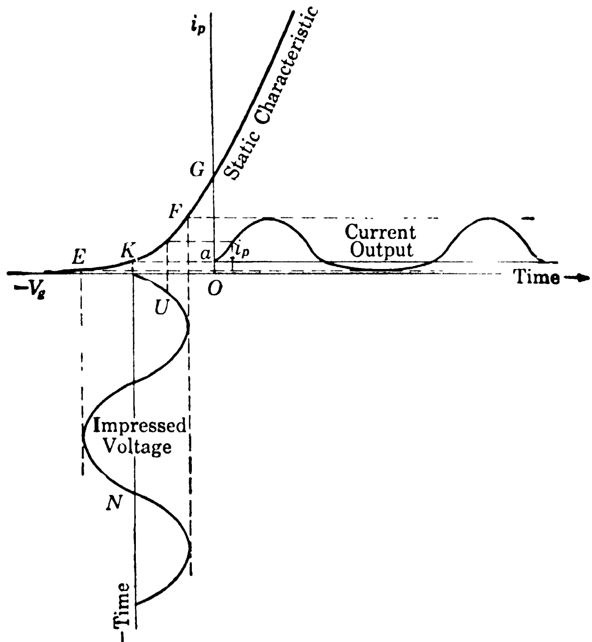


FIG. 122.—Showing how a sine wave grid voltage gives a distorted wave in the current output. This gives a larger plate current through the tube.

Fig. 122, on the curved part of its static characteristic curve, and the plate current can be reduced to *Oa*, which may be a tenth of its former value *OG*.

When an alternating voltage as shown by *UN* is added to the steady voltage in the grid circuit, it will cause corresponding changes in the plate current. The operating point for the tube will now swing between *E* and *F* with every alternation of the applied voltage. The plate current, instead of remaining at the steady value *Oa*, will rise and fall with every alternation but will not fall quite to zero. However, the increase in this current at *F* is much more than its decrease at E, and the ammeter will show a larger reading for i_p, as is evident from the curve for i_p shown in Fig. 122.

This *increase* in the ammeter reading depends on the value of the alternating voltage added in the grid circuit. A calibration curve can be drawn showing the relation between the readings of the ammeter and the corresponding values of the alternating voltage in the grid circuit. Or a new scale can be put on the ammeter to read directly these values of the alternating voltage. The ammeter is then called a "voltmeter."

Since this relation does not depend on frequency, the voltmeter can be calibrated as a low frequency (60 cycles) and used at higher frequencies.

231. Radio Communication.—Electron tubes are used extensively in radio communication both as detectors and as amplifiers, but it is beyond the design of this book to discuss the various circuits that are used. To understand how the results are obtained requires at least an elementary knowledge of alternating currents and the effects of inductance and capacitance upon such currents. These subjects are treated in the following chapters, and after the last page has been read it will be easier to understand the following article on the action of an electron tube as a generator of alternating currents. This topic is introduced here because the device may be employed as a source of high-frequency alternating current for use in some of the measurements of capacitance and inductance that are described in the following chapters.

232. The Electron Tube as an Alternating-current Generator. In studying the effects of inductance and capacitance and in using alternating currents for bridge measurements, it is desirable to have a frequency of about 1,000 cycles per sec. This is not readily obtained from alternating-current dynamos, but such a current can be obtained with the arrangement shown in Fig. 123. The grid of the electron tube is connected to the resonance circuit CL, which contains the proper amounts of inductance and capacitance to resonate freely at the desired frequency. When the resonance current is oscillating through C and L, the potential of the grid is raised and lowered with every surge of the charge in and out of the condenser. This will impress the same variations on the current in the plate circuit, but this current is still a direct current. In this circuit is placed the primary of a telephone transformer, the secondary of which becomes a source of alternating current of the frequency of the variations in the plate current. In order to maintain the oscillations in the resonance circuit CL, the plate current also passes through a coil Q that is inductively coupled with L, and in the sense that will tend to increase the oscillations in CL. For a 1,000-cycle current it has been found suitable to use the coils of a small, bell-ringing transformer for L and Q, taking the 110-volt winding for L and the 14-volt winding for Q. A small "grid" condenser of 0.005 μf is used for C. A 5-watt tube is satisfactory, and 200 volts can be used for the plate battery. Such a generator will begin to oscillate as soon as the circuits are closed and will continue as long as the proper filament current is maintained.

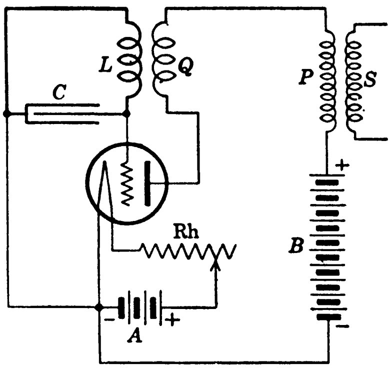


FIG. 123.—An electron-tube generator for supplying audio frequency alternating current from the terminals of S.

233. Constant-frequency Generator.—One of the disadvantages of the simple electron-tube generator described above is that the variations in the current taken from the output transformer PS react upon the oscillating circuit to change the frequency of the current. An arrangement[1] that can maintain a constant frequency is shown in Fig.

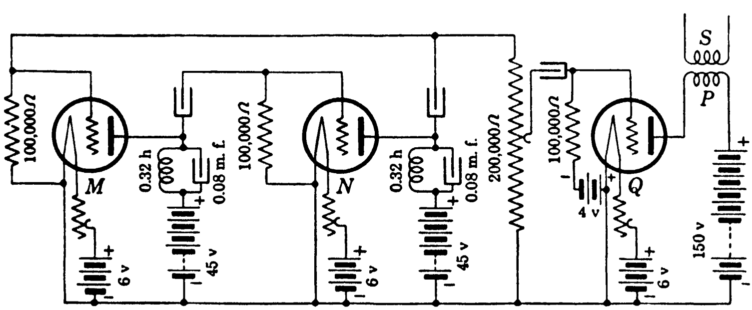


FIG. 124.—Alternating-current generator for a small current of 1,000 cycles per second.

124. There are two tubes, each similarly connected to the other. Any variation in the plate current of one tube is transmitted to the grid of the other tube. This in turn affects the plate current of the second tube, and this, somewhat magnified, is transmitted back to the grid of the first tube, thus increasing the original disturbance.

[1] GUNN, R., *J. Optical Soc. Am. and Rev. Sci. Inst.*, vol. 8, p. 545, 1924.

The frequency of oscillation is determined by the values of the self-inductance L and the capacitance C in the parallel resonance combinations in the plate circuits of the tubes M and N. The two parallel resonance combinations should be tuned to give the same frequency, that is, $L_1C_1 = L_2C_2$, and the resistance of these coils should be small.

When a source of constant frequency has been obtained, it is connected to the grid of the power tube Q. The plate current of Q is forced to vary with the frequency that is impressed on the grid, and passing through the primary P of a small transformer gives alternating current of this frequency in the secondary S.

One set of values that have been found to give satisfactory results is indicated on the diagram of Fig. 124. This gives an alternating current of about 1,000 cycles per sec, which is a useful tone when a telephone is used as the detecting instrument. The three filaments can be heated from the same 6-volt battery if desired. Likewise the various voltages for the plate circuits of the tubes can be taken from a single battery.

It is now possible to obtain from makers of electrical apparatus complete oscillators embodying the principles outlined above and ready to deliver an audio-frequency current when connected to the laboratory circuit of 60 cycles per sec (see also Art. 370).

CHAPTER X

THE MAGNETIC CIRCUIT

234. Magnetic Effect of an Electron Current.—Every current, whether small or large, has some magnetic effects associated with it. If the region around a wire carrying an electron current is investigated with the aid of a small compass needle, it will be found that although the needle is strongly acted upon, neither end is attracted toward the wire carrying the current. If all the other directive effects, such as the earth's magnetic field and the supports of the needle, can be eliminated, the needle will stand at right angles to the current. If the compass is moved slowly in the direction that is pointed out by the needle, it will be found to trace a path encircling the current and returning finally to the starting point. Any number of such paths can be traced by starting from different points in the neighborhood of the current.

If iron filings are sprinkled on a smooth sheet of paper held horizontally near a current in a vertical wire, and the paper is gently tapped, the particles of iron are seen to align themselves in the same directions as the lines that were traced by the compass needle. The iron filings thus give a visual picture of these paths.

235. Magnetic Flux.—The bundle of lines showing the various paths that can be traced around a magnet or current indicates a magnetic condition in the surrounding space. This is made evident when this region is investigated with a wire whose ends are connected to a ballistic galvanometer. When such a wire is moved across these lines, it causes a deflection of the galvanometer. The quickness with which the wire is moved makes no difference in the galvanometer deflection, but if the wire is moved across a greater area, the deflection is correspondingly increased.

Quantities that are thus measured with reference to an area are called "fluxes," and in this case the wire is said to have enclosed more of the "magnetic flux."

If the test wire is moved in the direction of these lines of flux, there is no deflection of the galvanometer, because there has been no change in the amount of magnetic flux enclosed by the wire.

If by any means whatsoever the magnetic flux enclosed by this loop of wire is made to change, there is a corresponding deflection of the galvanometer. Magnetic flux is detected, then, by the deflection of the galvanometer, and the magnitude of this deflection may be taken as an indication of the total change in the flux enclosed by the wire and its connections to the galvanometer. It does not matter whether the wire is moved across the flux or the flux is moved across the wire, or whether the flux changes in amount without any motion—the effect on the galvanometer is the same.

236. Induced Emf.—The deflection of the galvanometer shows that a transient current has passed through it. This current is due to an emf in the galvanometer circuit. That it is an emf and not something like a condenser discharge is shown when resistance is added in the galvanometer circuit. The deflection (corrected for the changed damping) is then decreased in proportion to the increase in the total resistance of the circuit, showing that the current has been reduced in accordance with Ohm's law.

Since the quickness with which the change in flux is made (provided the change is completed before the galvanometer has moved appreciably from its position of rest) does not change the deflection, it must be that when this change is made in a short time the current, and therefore the emf is correspondingly larger during this short interval. These considerations are expressed in the relation

$$e = \frac{d\Phi}{dt},$$

where e denotes the emf when the flux is changed by an amount $d\Phi$ during the time dt (*i.e.*, at the constant rate of $d\Phi/dt$ webers per sec).

237. The Weber.—Evidently the magnetic flux Φ should be measured in the proper units in order to make this relation a true equation without the addition of a proportionality factor. Since e is measured in volts and t in mean solar seconds, this equation fixes the amount of magnetic flux Φ that is to be taken as unity. This amount of flux is called a "weber."[1] When the flux that is linked with an electric circuit changes at the constant rate of 1 weber per sec, it produces an emf of 1 volt in that circuit. This leads to the following definition.

Definition.—One weber is that amount of magnetic flux which will induce an electromotive force of one volt in a wire that is linked with the flux at a constant rate of one weber per second.

[1] Wilhelm Eduard Weber (1804–1891), German physicist.

The name "weber" for this unit amount of magnetic flux was officially adopted by the International Electrotechnical Commission during their meeting at Brussels in 1935.

238. Action of Magnetic Flux on an Electron Current.—If we were to investigate the region around the current with the aid of another wire carrying a second current, we should find that the second wire experiences a force urging it sidewise across the magnetic flux. There is no force tending to move the wire along the magnetic flux in the direction traced by the moving compass needle.

Because of this force on a wire carrying a current, it will require work to push the wire across the magnetic flux, and the amount of this work will depend upon how much flux the current is pushed across. The magnitude of this force is computed in Art. 239.

239. Force on an Electron Current.—In the absence of a magnetic field, no power is required to move a current sidewise to itself. But when a current I is moved across magnetic flux Φ the power required is

$$P = Ie = I\frac{d\Phi}{dt} \qquad \text{watts}$$

because of the emf e induced in the wire by this rate of change in the amount of magnetic flux that is being linked with the current.

From the relation between force and power (power = force $\times$ velocity) the push f on the wire carrying the current is

$$f = \frac{P}{v} = \frac{P}{\frac{ds}{dt}} = \frac{I\frac{d\Phi}{dt}}{\frac{ds}{dt}} = I\frac{d\Phi}{ds} \qquad \text{newtons}$$

where $v = ds/dt$ is the velocity with which the wire is moved over the distance ds.

The value of this expression ($d\Phi/ds$) is seen if both numerator and denominator are multiplied by l, the length of the wire considered. Then

$$f = I\frac{l\,d\Phi}{l\,ds} = I\frac{l\,d\Phi}{dA} \qquad \text{newtons}$$

where $dA = lds$ is the area moved over by the wire. Since $d\Phi/dA$ is the density B (see Art. 254) of the flux (in webers per square meter) at the place occupied by the wire, this gives

$$f = BIl \qquad \text{newtons}$$

as the force experienced by l meters of wire carrying a current of I amp in a region where the flux density is B webers per square meter.

240. The Maxwell. *A Smaller Unit of Flux.*—A little consideration will show that a weber is a rather large amount of magnetic flux (because a second is so long). It is suitable to use with dynamos and motors or with large coils of wire. But the flux through a small magnet, or in an ordinary room due to the magnetism of the earth, is a very small fraction of 1 weber. In such cases it is useful to employ a smaller unit of flux. This smaller unit is called a "maxwell."[1] It is one 100 millionth (10^{-8}) of a weber. The name "maxwell" for this smaller unit has been in use for many years; it was officially reaffirmed in 1930 by the International Electrotechnical Commission at their meeting in Oslo.

Definition.—A maxwell is the amount of magnetic flux equal to a hundred-millionth (10^{-8}) *of a weber.*

241. Centimeters.—Corresponding to the maxwell for the smaller unit of flux, the smaller unit of length of a centimeter is usually employed in the discussion of magnetic quantities.

242. Representation of Magnetic Flux.—Since the magnetic flux is continuous throughout the magnetic circuit, such a circuit may be

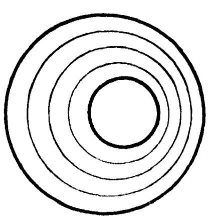


FIG. 125.—Magnetic flux in a magnetic circuit of wood or air. The three lines represent a magnetic flux of 3 maxwells due to a mmf of 12 ampere turns (See Fig. 127). The flux has a greater density where the circuit is narrow.

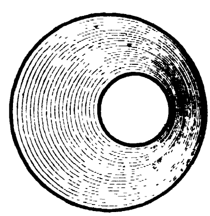


FIG. 126.—Magnetic flux in a circuit of iron. The 20 lines represent a magnetic flux of 20 maxwells due to a mmf of 12 ampere turns. (See Fig. 127.) In comparison with Fig. 125, this should show 2,000 maxwells, or more, but it is not possible actually to draw all of these lines in this small figure.

thought of as divided lengthwise into a number of parallel circuits so that there will be 1 maxwell of magnetic flux in each strand of this divided circuit. Such strands have been called "tubes of induction" or "lines of force." Perhaps it is better to speak of the "magnetic flux" and to think of these lines as merely a pictorial representation of the flux. Such representation is useful in showing how the flux passes continuously from one medium into another with no change in amount

[1] James Clerk Maxwell (1831–1879), British physicist.

at the boundary. The total flux in a given circuit can be shown by the total number of these strands or lines.

Figures 125 and 126 represent the magnetic flux in two circuits of different material but having the same size and subjected to the same magnetizing effect of the electron current. The total flux as well as the flux density is much greater in the iron ring. If the wire connected to a ballistic galvanometer could be made to cut across this flux, there would be a much greater deflection for the iron ring than for the wooden one. Since the wire cannot be moved through the iron, the same effect is obtained by reversing the direction of the flux while the wire remains stationary. Half of the galvanometer deflection then measures the flux.

243. Tubes of Magnetic Flux.—If the paths traced by the moving compass needle are indicated by lines drawn in space around the current, these lines will be close together in places where the flux is dense and farther apart in those portions of the magnetic field where the flux is less dense. But if this mode of representation is adopted, we must not fall into the error of thinking that in the space between two of these lines there would be no magnetic flux. To avoid this possibility the lines are sometimes thought of as "tubes" that are broad enough to just touch each other and leave no space between.

244. Magnetic Field.—The region occupied by these tubes of magnetic flux is called the "magnetic field." All of the space in which a current produces any magnetic effect is the magnetic field of the current.

245. The Magnetic Circuit.—When a wire is wound into a helix and this solenoid carries a current, all the paths traced by the compass needle or by the iron filings are found to pass through the inside of the solenoid from one end to the other, then out through the surrounding space to the starting points. It is therefore natural to associate the idea of a magnetic *circuit* with the phenomena of magnetism.

The magnetic flux is continuous and forms a closed circuit with itself. At one place it may be spread out over a large cross section and in another part of the circuit it may be confined within narrow limits, but the total amount of magnetic flux is the same for each cross section of the circuit.

Similar paths can be traced through the region around a permanent magnet, but in this case the compass needle cannot be carried through the magnet itself, and other methods of measurement are necessary to show that the paths are continuous through the magnetized steel. The whole sheaf of paths in any given case maps out the magnetic circuit for that case.

246. Magnetomotive Force.—The magnetic flux in the air around a wire carrying a current is due to the stream of electrons along the wire. In fact, there is such a strict proportionality between them that a current is usually measured by means of the magnetic flux associated with it. With a solenoid it is easily seen that the number of turns of wire is as important as the value of the current. A current of n amp through *one* turn of wire gives the same magnetic effect as 1 amp through n turns of wire that occupy the same place as the single turn of wire. The magnetic effect of a current is thus measured in "ampere turns," and every current has a circuit of at least one turn.

In the case of a steel magnet or a magnetized piece of iron, the magnetic flux is similarly due to the circulatory motion of electrons, but in this case the electrons are moving within the atoms of iron or spinning on their axes.

"Magnetomotive force" is the name that has been given to this magnetic influence of a current. It can be measured in ampere turns or in other units proportional to ampere turns. The effect of magnetized iron or steel can be expressed likewise in terms of the equivalent ampere turns. There can be no magnetic flux that is not associated with the corresponding ampere turns of electrons in a wire or within the atoms of a magnetized substance. Inasmuch as it is not possible to count the turns or to measure the current to determine the equivalent ampere turns of a piece of magnetized steel, it is customary to speak of the effect of the amperes of current in the actual turns of wire as the magnetomotive force, and to measure experimentally the relation of these ampere turns to the total magnetic flux in the circuit due to both the current and the magnetized steel.

247. The Ampere Turn.—The words "magnetomotive force" are often abbreviated to the initials mmf. The unit of magnetomotive force is called an "ampere turn," and it is the mmf due to an ampere in one turn of wire.

The magnetomotive force F due to a current in a coil of N turns is given by the relation

$$F = NI, \qquad \text{ampere turns,} \qquad (1)$$

where I denotes the value of the current in amperes.

248. Representation of Magnetomotive Force.—The turns of wire in a solenoid are counted along the length of the solenoid. Since magnetomotive force is measured by the number of ampere turns that are linked with the magnetic circuit, the mmf is measured likewise along the length of the magnetic circuit. Whether the ampere turns are uniformly distributed along the magnetic circuit or are bunched together over a short distance, the total mmf is the same amount. This mmf is distributed along the magnetic circuit in proportion to the reluctance of the various parts of the circuit,[1] similarly to the volts along an electric circuit. This distribution can be represented by a series of marks or arrowheads properly spaced along the length of the magnetic circuit, the total number of arrowheads being made equal to the total number of ampere turns. As shown in Fig. 127, this is quite different from the representation of magnetic flux, which is measured by the maxwells through the total cross section of the circuit.

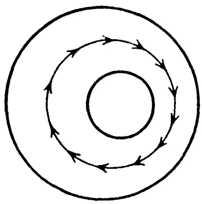


Fig. 127.—The arrows show the amount and distribution of the mmf for a circuit of either iron or wood. The mml depends solely upon the number of ampere turns of the magnetizing current, and is independent of the material of the magnetic circuit.

249. Arrows per Centimeter.—Where the arrows are close together in the representation of mmf (Fig. 127), there the magnetic intensity H is great; where they are far apart, H is less. The value of H at any point of the circuit is given by the number of arrows per centimeter at that point. This value may change abruptly where the circuit passes from one medium into another, as from iron into air, although a wire and ballistic galvanometer would show that the magnetic flux is the same in each medium (see Art. 257).

250. Magnetomotive Force in a Divided Circuit.—The mere fact that a magnetic circuit is divided into two parallel branches does not change the number of ampere turns that are linked with the total circuit. If the total mmf along a given divided circuit, as in Fig. 129, is 9 ampere turns, the mmf along each branch is 9 ampere turns also. Even if one branch is wholly in air and the other wholly in iron, the ampere turns around one branch are the same as the ampere turns linked with the other branch, and therefore the amount of mmf distributed along the length of each branch must be the same.

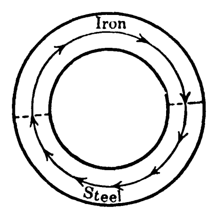


Fig. 128.—This shows the distribution of mmf in a circuit half of steel and half of the more permeable iron.

251. Relations in the Magnetic Circuit.—In a magnetic circuit like the ring shown in Fig. 129 but uniformly wound with N turns of wire instead of the three turns shown in the figure, the amount of flux ϕ, measured in maxwells, depends upon a number of factors. Primarily, ϕ depends on the current I and the number of turns of wire wound

[1] If the ampere turns are bunched at one place, the magnetic circuit may not be the same circuit that it was when these ampere turns were uniformly distributed.

on the ring. It also depends on the size of the ring, being greater in a ring of greater cross section A and inversely proportional to the length[1] l of the magnetic circuit for the same values of N and I. Experimental investigation also shows that the amount of flux depends upon the quality of the material of the ring. With a ring of iron the flux may be several hundred or even several thousand times as much as in a ring of wood or brass with all of the other factors the same. This help that is rendered by the electrons in the atoms of the material is expressed by the factor μ, which is called the "permeability" of the substance.

Collecting these factors together gives

$$\phi = IN\left(\frac{A}{l}\right)\mu k, \qquad (2)$$

where k is a proportionality factor depending upon the units in which the other quantities are measured. The factors in this expression can be rearranged in different ways to suit various purposes.

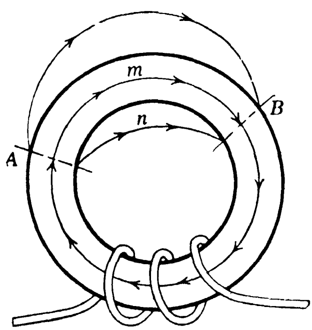


FIG. 129.—A ring of iron magnetized by three turns of current. The arrows indicate a mmf of 9 ampere turns, and show how this is distributed. Between A and B the mmf along one path is the same as along any other path.

252. Reluctance.—One way of looking at the magnetic circuit is similar to Ohm's law for the electric circuit. All the factors in Eq. (2) that relate to the size and material of the magnetic circuit may be collected together and written as

$$R = \frac{l}{\mu A}.$$

From analogy with the resistance in an electric circuit, R is called the "reluctance" of the magnetic circuit. A circuit that does not have a constant cross section may be considered as a series of circuits having lengths l_1, l_2, l_3, etc., and cross sections A_1, A_2, A_3, etc. The total reluctance is then

$$R = \frac{l_1}{\mu_1 A_1} + \frac{l_2}{\mu_2 A_2} + \frac{l_3}{\mu_3 A_3} + \text{etc.}$$

Iron in its various forms is one of the best conductors of magnetic flux. Most other substances are rather poor conductors, but there

[1] The ring is supposed to be thin in the radial direction, so that the mean circumference l is practically the same as the outer or the inner circumference.

are no insulators of magnetic flux.[1] Even when a magnetic circuit is made largely of iron, the magnetic flux does not follow the iron exclusively, but there is some flux that finds a path through the surrounding air. It is extremely difficult to build a circuit, even of soft iron, in which all the flux will follow the iron path. If two paths are open to the magnetic flux, it will divide just as an electron current would do between two wires in parallel. The total reluctance of a circuit includes the effect of the reluctance of such parallel leakage paths.

In iron and similar magnetic substances, there is a difference, too, between electric resistance and magnetic reluctance, inasmuch as the former is constant for all ranges of current, while the value of the reluctance of a magnetic circuit depends upon the amount of magnetic flux through it, being greater for very small or very large values of the magnetic flux than it is for moderate values.

253. Law of the Magnetic Circuit.—In the preceding article, all the factors in Eq. (2) relating to the size and material of the magnetic circuit were grouped together under the name of "reluctance." In a similar way, all the factors that relate to the magnetic effect of the current may be grouped together into the single quantity F. The factor k is usually considered as belonging to this group. Then

$$F = kNI = 0.4\pi NI, \tag{3}$$

and this constitutes a measure of the magnetomotive force (when expressed in gilberts, see Art. 265). This peculiar value of $k(= 0.4\pi)$ is not due to the magnetic circuit. It comes from the geometry of space.

The relation between the amount of magnetic flux ϕ and the applied mmf F can now be written

$$\phi = \frac{kNI}{l/\mu A} = \frac{F}{R},$$

or, in words,

$$\text{Magnetic flux} = \frac{\text{magnetomotive force}}{\text{reluctance}}.$$

This law is especially useful in solving problems that require the computation of the value of the magnetic flux in a given circuit. It can be applied to a complete circuit or to a limited portion of a magnetic circuit, and to circuits of any material, with one exception. For substances that show magnetic hysteresis, such as iron, steel,

[1] In somewhat the same way there are no insulators of the electric flux between the oppositely charged plates, *e.g.*, of a condenser.

nickel, etc., this law is applicable only when the substance is being magnetized by a mmf that is greater than any mmf that has been used since the substance was unmagnetized.

254. Magnetic Flux Density, B.—Another way of looking at Eq. (2), Art. 251, is to think of the magnetic relations in a given *material* rather than in a particular circuit of length l and cross section A. From the meaning of the term, the density of magnetic flux is given by the flux per unit area, where the measured area is normal to the direction of the flux. If we divide Eq. (2) by A, it becomes

$$\frac{\phi}{A} = \mu \frac{kNI}{l} = \mu \frac{F}{l}.$$

The quantity ϕ/A is the density of the magnetic flux, expressed in maxwells per square centimeter. This quantity is of sufficient importance to be measured in units of its own, which are called "gausses." The magnitude of the flux density is usually denoted by the symbol B, so that

$$B = \frac{\phi}{A}, \qquad \text{gausses}$$

when the flux is measured in maxwells and the area normal to the flux is in square centimeters.

If the flux is not uniformly distributed over the cross section A, the density at any place should be expressed as

$$B = \frac{d\phi}{dA} \qquad \text{gausses}$$

where $d\phi$ is the amount of flux that is distributed over the small area dA at the place under consideration.

255. The Gauss. *The Unit of Flux Density.*—From the meaning of the words "flux density" we have the following definition.

Definition.—A gauss is the density of magnetic flux when there is one maxwell per square centimeter, the area being measured normal to the direction of the flux.

The symbol B is commonly used to denote the value of the flux density. The name "gauss"[1] for this unit of flux density was officially adopted in 1930 by the International Electrotechnical Commission at their meeting in Oslo.

256. Components of Magnetic Flux Density.—The magnetic flux density at any point has a definite value and direction, which may be the resultant due to several effects. Conversely, any given value of

[1] Karl Friedrich Gauss (1777–1855), German mathematician.

the flux density can be divided up into components in any desired directions.

Sometimes we wish to know something about the value of the flux density in a given case, but both its direction and magnitude are unknown. If the component of this flux density in a chosen direction can be found, then three such components (say at right angles to each other) will fully determine both the direction and the magnitude of the resultant flux density.

257. Magnetic Intensity, H.—A given amount of magnetomotive force may be distributed over a magnetic circuit of considerable length, or it may be concentrated in a circuit of shorter length. In the latter case the magnetic effect is more intense than it was in the former case. The magnetomotive force per unit of length is called the "magnetic intensity." The magnitude of the magnetic intensity is usually denoted by the letter H, and its relation to the magnetomotive force F is

$$H = \frac{F}{l}.$$

When the magnetomotive force is not uniformly distributed along the length of the circuit, the magnetic intensity at any given place should be expressed as

$$H = \frac{dF}{dl},$$

where dF is the amount of magnetomotive force distributed over the short length dl at the place under consideration.

258. The Unit of Magnetic Intensity.—As shown in the preceding articles, magnetic intensity is measured in terms of magnetomotive force per unit of length. When the flux density is measured in webers per square meter, the appropriate unit for the magnetic intensity H is 1 ampere turn per linear meter. When centimeters are used, it would be 1 ampere turn per cm.

When the flux density is expressed in gausses, one could still express the corresponding value of the magnetic intensity in ampere turns per centimeter, but it is usual to employ a unit that is slightly smaller (about four-fifths as large). This smaller unit is called an "oersted"[1] and its value is defined below (Art. 260).

259. Relation between B and H.—The value of the flux density B at a given place in a magnetic circuit was shown in Art. 254 to be given

[1] Hans Christian Oersted (1777–1851), Danish physicist.

by the relation

$$B = \frac{\phi}{A} = \mu \frac{kNI}{l} \qquad \text{gausses}$$

In Art. 253 the magnetomotive force F in the circuit was written as

$$F = kNI, \qquad \text{gilberts}$$

and in Art. 257 the magnetic intensity H was expressed as

$$H = \frac{F}{l}. \qquad \text{oersteds}$$

Thus the expression for the flux density B becomes

$$B = \mu H. \qquad \text{gausses}$$

This relation is very generally used in expressing the magnetic properties of different substances, and it gives a ready means for determining the value of μ when both B and H can be measured.

260. The Oersted. *The Unit of Magnetic Intensity.*—The magnetizing effect of a current is due to the ampere turns of the current circuit, as shown in Art. 247. The magnetic intensity can be expressed in terms of ampere turns per centimeter, but the oersted is defined in a different manner. The relation between the value of the magnetic intensity H and the resulting value of the flux density B is

$$B = \mu H.$$

There is no reason why the permeability μ of any particular medium should have the value 1, but since the properties of vacuum do not depend upon any substance it was chosen as a medium having unit permeability. From this choice of unit permeability for vacuum, it follows that the magnetic intensity H must be measured in a unit of such size that $H = 1$ will give $B = 1$ gauss in vacuum. This amount of magnetic intensity was named an "oersted" by the International Electrotechnical Commission at Oslo in 1930.

Definition.—An oersted is the magnetic intensity that can produce in vacuum a flux density of one gauss.

The oersted is slightly smaller than a magnetic intensity of 1 ampere turn per centimeter, and therefore it takes a larger number of oersteds to measure a given magnetic intensity. The exact relation is

$$H = \frac{4\pi}{10} pI = 1.257 pI, \qquad \text{oersteds}$$

where pI is the number of ampere turns per centimeter.

261. The Gilbert. *The Unit of Magnetomotive Force.*—The value of the magnetomotive force F in a magnetic circuit is given by the relation between the value of the magnetic intensity H and that of the mmf. Since

$$H = \frac{dF}{dl} \tag{4}$$

where dF is the amount of magnetomotive force that is distributed along a length, dl, of the circuit, this gives

$$dF = Hdl, \tag{5}$$

and by integration,

$$F = \int Hdl. \tag{6}$$

When H is constant over a length of l cm, this becomes

$$F = Hl. \tag{7}$$

The unit of magnetomotive force is called a "gilbert."[1] This name has been used for many years and was officially adopted as the name of unit magnetomotive force by the International Electrotechnical Commission in 1930 at their meeting in Oslo. The value of a gilbert follows from the relation shown in Eq. (7), above.

Definition.—A gilbert is the amount of magnetomotive force over each centimeter of a magnetic circuit when the uniform magnetic intensity is one oersted.

The total mmf in a magnetic circuit is given by Eq. (6) when it is integrated along the entire circuit. In a circuit where the mmf is uniformly distributed along the entire length of the circuit, this integral becomes the simple product Hl.

262. The Relation between Gilberts and Ampere Turns.—Up to this point we have spoken of magnetomotive force as being measured by the number of ampere turns that are linked with the magnetic circuit. In the gilbert we have another unit for magnetomotive force, about four-fifths of an ampere turn. The reason for this unit is the rather arbitrary selection of unity for the permeability of empty space.

It is now possible to compute the exact relation between gilberts and ampere turns. In the definition of the ampere (Art. 191), the force per unit length on each of the two parallel wires 1 meter apart was specified as 2×10^{-7} newton per meter when the current in each

[1] William Gilbert (1544–1603), English scientist.

wire is 1 amp. The reason for this push on a wire is that it is in the magnetic field of the current in the other wire.

263. The Flux Density 1 Meter from an Ampere.—In this case it is possible to compute the value of the flux density B at a place 1 meter from a current. In Art. 239 it is shown that the force on a wire carrying I amp is

$$f = BIl, \qquad \text{newtons}$$

where l is the length (in meters) of wire in a region where the flux density is B webers per square meter. In the case of the parallel wires used in defining the ampere, $I = 1$, $l = 1$, and $f = 2 \times 10^{-7}$ newton. Therefore in the region occupied by the other wire, 1 meter from the first,

$$B = \frac{f}{Il} = 2 \times 10^{-7} \text{ weber per sq meter} = 2 \times 10^{-3} \text{ gauss.}$$

264. The Magnetic Intensity 1 Meter from an Ampere.—From the definition of an oersted (Art. 260), and since μ is taken as 1 in vacuum, it is seen that the magnetic intensity necessary to produce the above value of B is

$$H = 2 \times 10^{-3}. \qquad \text{oersted}$$

265. The Magnetomotive Force Due to an Ampere Turn.—Let us consider a circular path, each element of which is 1 meter from a long, straight wire carrying a current I. This path is 200π cm in length. The value of H at any place along this path is

$$H = 2 \times 10^{-3} \times I. \qquad \text{oersteds}$$

From the definition of the gilbert (Art. 261), the magnetomotive force along this path is

$$F = Hl = 2 \times 10^{-3} \times I \times 200\pi = 0.4\pi I. \qquad \text{gilberts}$$

Since the current must be in a closed circuit, this path is linked once with the circuit of the current. And the magnetomotive force along any other path (see Fig. 129) that is linked with this current will also have the same value. The magnetomotive force along a path that is linked N times with a current of I amp is

$$F = 0.4\pi NI. \qquad \text{gilberts}$$

266. The Magnetic Intensity within a Ring Solenoid.—When the N turns of wire are uniformly distributed along the full length l of a magnetic circuit, the magnetic intensity will have a constant value along the circuit. This is

$$H = \frac{F}{l} = \frac{0.4\pi NI}{l} = 0.4\pi pI, \qquad \text{oersteds}$$

where p is written for N/l, the turns per centimeter of the coils carrying the current.

In a circular-ring solenoid it is evident that the turns of wire are closer together (p larger) on the inner side of the ring than on the outer side. If the ring is flattened so that a considerable portion of its length is straight, the value of H will be more nearly constant for points in the middle of this straight part.

267. Permeability, μ.—In discussing the relations in the magnetic circuit in Art. 251, the factor μ was introduced to represent the influence of the medium upon the resulting magnitude of the magnetic flux. In article 259 this same factor remains in the relation between B and H. This factor is called the "magnetic permeability" of the medium, and even in a vacuum it is not an abstract number, because B and H do not become quantities of the same nature.

The permeability is always a positive quantity. For most substances its value is nearly 1. In a few materials it is slightly less than 1, and these substances are called "diamagnetic." Substances for which the permeability is slightly greater than 1 are called "paramagnetic." For ferromagnetic substances like iron, nickel, cobalt, etc., the permeability may reach a value of several thousands. The unit of permeability has no name. A magnetic circuit of large permeability will have a correspondingly small reluctance. The reciprocal of permeability is called "reluctivity."

268. Relative Permeability, μ'.—Another way in which the term "permeability" is sometimes used is in the comparison of the amounts of magnetic flux in two media that are subjected to the same magnetic intensity. For example, if the flux density is B_i in a certain circuit consisting of iron, and if the flux density is B_a for the same circuit when the iron has been replaced by air, then

$$\frac{B_i}{B_a} = \mu'$$

where μ' is the relative permeability of iron as compared with air.

The relative permeability, being the ratio of two similar quantities, is an abstract number. This is not the case with the permeability μ, which is the ratio of quantities, B and H, that are dissimilar in nature.

269. Intrinsic Magnetization.—The magnetic flux · through a magnetized circuit may be considered as the sum of two superimposed parts—one portion due to the current in the magnetizing solenoid and a second part due to the electronic mmf within the substance.

If the magnetizing solenoid were in vacuum, there would be some flux through it, and

$$B_0 = \mu_0 H, \tag{8}$$

where the subscripts refer to the values in vacuum. When a magnetic substance is within the solenoid, the total flux density is this B_0 *plus* the flux density B_I that is contributed by the substance. Thus

$$B = B_0 + B_I. \tag{9}$$

When H is made larger and larger, the value of B continues to increase due to the part B_0. The part B_I seems to reach a definite limit for each substance, which is reached when all of the movable electronic orbits, or whatever parts are free to turn, are oriented as nearly as possible into alignment with the applied mmf.

270. Intensity of Magnetization, J.—A conception of this quantity is obtained from the following considerations. If a bar of uniformly magnetized steel is placed in a magnetic field of intensity H, the maximum couple exerted by the field is

$$T = J \times \text{volume} \times H \tag{10}$$

or

$$J = \frac{T}{H \times \text{volume}}. \tag{11}$$

Hence the intensity of magnetization J for a given piece of material is given by the maximum couple per cubic centimeter that would be exerted by a unit magnetic field. This quantity J is called the "intensity of magnetization." It should be noted that this is not magnetic intensity H.

271. Susceptibility.—The susceptibility κ of a substance is the ratio of the intensity of magnetization J to the value of H necessary to produce it, or

$$\kappa = \frac{J}{H}. \tag{12}$$

There is a simple relation between κ and μ for a substance in a given magnetic condition. Substituting in Eq. (9), Art. 269, the values of the different flux densities with the aid of the relation $B_i = 4\pi J$ and Eqs. (8) and (12) gives

$$\mu H = \mu_0 H + 4\pi\kappa H$$

or

$$\mu = \mu_0 + 4\pi\kappa. \tag{13}$$

272. Effect of the Ends of a Bar.—When a closed iron ring is magnetized, the circuit is uniform and each part offers equal reluctance. This uniformity is disturbed if an air gap is cut across the ring. In the language of the magnetic circuit, it is said that the reluctance of this part of the circuit has been greatly increased by the introduction of the air gap in place of the iron. A considerable part of the applied mmf will be required across the air gap and only a small and unknown part will be distributed along the iron portion of the circuit, because of the greater permeability of iron. If the air gap is made larger, a still smaller part of the mmf is applied to the iron. This shows the difficulty of studying the magnetic properties of straight bars or of other forms of open magnetic circuits.

273. Electronic Mmf.—In the usual consideration of the subject of the magnetic flux through a medium, such as iron, it has been supposed that the large amount of flux through the circuit is due to the high permeability of the iron. The modern conception of atomic structure indicates that even the interior of atoms is as empty as the solar system, with a few electrons circulating in orbits through this space. Even in solid iron, then, there is little that can be considered as having the nature of a substance uniformly distributed and completely filling the space it occupies. And this leads to a modification of the former notion regarding the reason for the large flux through iron.

274. Domains.—In all that has been considered thus far regarding the magnetic flux in iron, the circuit has been treated as though it were a uniform continuous medium. When the atomic structure of iron is considered, it is far from a uniform medium. Formerly each atom was considered to be like a tiny magnet, and the magnetization of iron was supposed to consist in turning these tiny magnets in the direction of the magnetizing field. It was soon realized that the atoms of iron cannot turn bodily within the solid iron, and now the magnetic effect of an atom is thought of as due to the spin of some of the electrons within the atoms. Such a spinning electron has a definite amount of magnetic flux associated with it like that due to the current in a tiny circuit. The spinning electrons in adjacent atoms tend to align themselves parallel to each other, and this produces magnetic saturation of the iron. But the electron spins are not thus aligned with each other in the same direction throughout the whole volume of a block of iron—only in a small region or domain, perhaps 0.001 cm across. In adjacent domains the alignment is in different directions, thus forming tiny closed magnetic circuits within the iron.

When iron is magnetized by an external current, the individual electron spins are not deviated gradually more and more as the magnetizing current is increased, until they are finally aligned parallel to the magnetic field of the external current. Rather, a whole domain suddenly gives way and changes the direction of its magnetization. As the magnetizing current is increased, the direction of magnetization in domain after domain swings into line with the field of the applied current.

These sudden changes in the total flux through the iron as domain after domain suddenly falls into line can be heard as a crackling sound in a telephone connected through an amplifier to a coil of a few thousand turns of wire around the iron.

When the temperature is high enough (Curie point), the thermal vibration of the atoms destroys the orderly alignment of the electron spins, and iron loses its strong magnetic properties.

While the atoms of all elements have spinning electrons, in most cases the spins in opposite directions cancel the magnetic effect. Only in the atoms of the ferromagnetic elements are there electrons whose spin is not thus counterbalanced.[1]

[1] See further, *Electronic Engineering*, vol. 17, p. 142, 1944.

CHAPTER XI

MEASUREMENT OF MAGNETIC FLUX AND MMF

275. Measurement of Magnetic Flux.—The measurement of magnetic flux is based on the principle that there is an emf in a loop of wire whenever there is a change in the amount of flux that is linked with this loop. As this emf exists only while the flux is changing, it gives rise to a transient current, the total quantity in which can be measured by a ballistic galvanometer. If the loop or coil of wire is connected to the ballistic galvanometer while the magnetic flux is changing, there is a deflection of the galvanometer due to the passage of the electrons in the transient current through the circuit.

276. The Ballistic Galvanometer.—A ballistic galvanometer, in which the moving coil is massive and slow moving (Art. 64), is used to measure transient currents, or quantities of electrons, like the induced current in a secondary coil or the discharge of a condenser. The period of the moving coil of the galvanometer is very long in comparison with the duration of the induced current, and thus the coil remains practically at rest until the entire quantity has passed through it. It is therefore impossible to obtain a steady deflection, but the galvanometer coil gives a sudden throw and then settles back to the position of rest. The extent of this throw gives a measure of the quantity that has passed through the galvanometer; the greater the quantity, the greater the throw, the exact relation depending upon the galvanometer employed.

These galvanometers are of two general types: (1) The magnet may be fixed, and the coil suspended and movable; or (2) the magnet may be suspended and movable, and the coil fixed.

277. Effect of Damping.—Let θ_1, θ_2, θ_3, etc., be the successive deflections of the galvanometer to the right and left when it is allowed to swing freely after the discharge of the condenser through it. It is observed that each deflection is less than the one before it by a certain constant ratio, so that

$$\frac{\theta_1}{\theta_2} = \frac{\theta_2}{\theta_3} = \frac{\theta_3}{\theta_4} = \text{etc.} = f, \text{ say.}$$

If these deflections are laid off as ordinates, each one being erected at that point on the axis of abscissas corresponding to the time at which it was observed, they would appear as in Fig. 130. Time is reckoned from the discharge of the condenser and when the galvanometer coil begins to move. After half of a single period the first throw θ_1 is observed. The succeeding deflections follow at equal intervals of a whole single period. Knowing the value of f, any given deflection say θ_1, can be computed from the later readings, since

$$\theta_1 = \theta_2 f = \theta_3 f^2 = \theta_4 f^3 = \text{etc.},$$

where the exponent of f in each case is equal to the number of single periods between the desired and observed deflections.

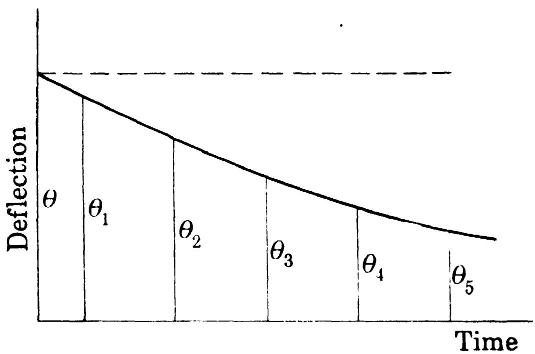


FIG. 130.— Damping of a galvanometer.

Had there been no damping, all these deflections would have been larger and each equal to θ, the value of which is

$$\theta = \theta_1 f^{\frac{1}{2}},$$

where the exponent of f is $\frac{1}{2}$ because the interval between θ and θ_1 is half of a single period.

Since

$$f = \frac{\theta_1}{\theta_2} = \frac{\theta_2}{\theta_3} = \frac{\theta_1 + \theta_2}{\theta_2 + \theta_3} = \frac{s_1}{s_2},$$

the value of f is seen to be given also by the ratio of two consecutive swings—a swing being the full amplitude from the turning point at the right to the turning point at the left.

Using this value of f gives

$$\theta = \theta_1 \left(\frac{s_1}{s_2}\right)^{\frac{1}{2}} = j\theta_1.$$

278. Closed-circuit Constant.—It thus appears from the above discussion that a given impulse may not always produce the same deflection θ_1; if the damping is made greater, the observed deflection is less. If k is the constant of the galvanometer without damping, the equation should be written

$$Q = kjd,$$

where jd is the deflection of the galvanometer, corrected for damping. When the galvanometer is connected to a closed circuit of comparatively low resistance R, the damping is very great, often so great that the damping factor cannot be determined in the usual way. But even so, it is found that the observed deflection is proportional to the quantity of electrons that passed through the galvanometer, and kj may be called the "closed-circuit constant."

279. Relation between the Galvanometer Deflection and the Change in Flux.—Writing out the sum of all the potential differences in the galvanometer circuit at a moment when the flux Φ that is linked with this circuit is changing at the rate of $d\Phi/dt$ (expressed in webers per second) gives

$$\frac{d\Phi}{dt} - Ri - L\frac{di}{dt} = 0, \qquad \text{volts} \tag{1}$$

where R and L denote the total resistance and self inductance of the entire circuit, including the galvanometer, and i is the value of the current at the moment for which the equation was written (see Art. 415).

Since a ballistic galvanometer measures only the total quantity Q of electrons that are passed through it before it has moved appreciably from its position of rest, where

$$Q = \int dq = \int idt, \tag{2}$$

it will be necessary to integrate Eq. (1) in order to eliminate the value of the current i and express Φ in terms of Q.

280. Integration of Equation (1).—Integrating Eq. (1) with respect to time gives

$$\int d\Phi - \int Ridt - \int Ldi = h, \tag{3}$$

where h is the constant of integration. Performing the operations that are indicated in Eq. (3) gives

$$\Phi - Rq - Li = h. \tag{4}$$

At a time just before there has been any change in Φ, and therefore when no charge q has passed through the galvanometer and there is no current in the galvanometer circuit, Eq. (4) becomes

$$\Phi_1 - 0 - 0 = h, \qquad (5)$$

where Φ_1 denotes the amount of magnetic flux that was linked with the galvanometer circuit. When the flux is changing from Φ_1 to Φ_2 there is a transient current in the circuit, giving a total quantity Q through the galvanometer.

A short time after the flux has become constant at the new value, Φ_2, the current in the galvanometer circuit will have died down to zero. At this time Eq. (4) will be

$$\Phi_2 - RQ - 0 = h. \qquad (6)$$

Using the value of h from Eq. (5),

$$\Phi_2 - \Phi_1 - RQ = 0, \qquad (7)$$

or, writing $\Delta\Phi$ for the change in flux, $\Phi_2 - \Phi_1$,

$$\Delta\Phi = RQ = Rkjd = wd, \qquad \text{webers} \qquad (8)$$

where w is the combined constant Rkj.

281. Measurement of Flux in Maxwells.—If it is desired to express the flux in maxwells, then (Art. 240)

$$\Delta\phi = \Delta\Phi 10^8 = Rkjd10^8 = cd. \qquad \text{maxwells} \qquad (9)$$

The constant c is called the "magnetic ballistic constant," and its value depends upon the resistance of the galvanometer circuit as well as upon k or j.

282. Flux Turns.—If the wire linked with the flux is wound up into a coil of n turns, and the flux ϕ passes through each of these turns, the galvanometer deflection will be n times as large, and we have

$$\Delta(\phi n) = cd. \qquad (10)$$

The product, ϕn, is called "flux turns." If the entire flux does not pass through all of the turns, the flux turns are the summation of the amounts of flux linked with each turn of the coil. This summation is denoted by (ϕn) maxwell turns.

Note that the deflection depends only upon the total change in the flux threading the coil, not at all upon the manner in which that change is produced.

283. Fluxmeter-galvanometer.—In the measurements of magnetic flux it is necessary to read the deflection of a galvanometer, and with the usual ballistic galvanometer it requires considerable expertness to read the scale just at the end of the deflection. A galvanometer having a high critical damping (Art. 65) resistance (70,000 ohms) is very greatly overdamped on a low-resistance circuit, and the free motion of the coil may be extremely slow. When magnetic flux is cutting the closed galvanometer circuit, the deflection is quick and responsive, and the galvanometer coil turns so as to cut out of the galvanometer circuit as many flux turns as are being cut into the circuit by the external flux through the test coil. When the flux ceases to change, the galvanometer coil stops moving and stands practically at rest. If the ending point of the deflection is near the natural resting point of the galvanometer, there is very little tendency for the scale reading to change after the flux has reached its steady value, and the observation is taken under stationary conditions.

284. Fluxmeters.—Portable instruments are made that will measure the change in magnetic flux by the deflection of a pointer moving over a scale graduated in maxwells. In appearance, one of these instruments resembles a sensitive voltmeter, but in principle it is simply an overdamped ballistic galvanometer with the moving element suspended by an almost torsionless suspension so that the pointer stands at any place on the scale. When flux is measured, the pointer moves quickly from one position to another.

285. To Measure the Flux in an Iron Ring.—From what has been said thus far it will be seen that the magnetic flux is not measured directly. It is only the *change* in the *flux turns* that affects the galvanometer and is measured by the deflection. The value of the total flux must be inferred from measurements of the change in flux. The change that is usually made in the flux through a magnetic circuit is that produced by reversing the magnetizing current. It is then assumed that the flux is reversed also and therefore that the corresponding *change* in flux is equal to twice the total value of the flux itself. This assumption is true if the residual magnetization due to larger currents has been removed from the iron by a suitable process of demagnetization (see Art. 291).

The experimental arrangement for measuring the magnetic flux in a closed-ring circuit is shown in Fig. 131. The iron ring is shown at Z, with the primary winding connected to the reversing switch S. If the ammeter reads the current on either side of zero, it should be placed between the ring and the reversing switch, where it will show the reversal of the current each time the switch is thrown. If the scale of the ammeter extends only on one side of the zero mark, the ammeter should be placed on the battery side of the reversing switch, in the part of the circuit where the current is in the same direction whether it is reversed through the winding on the ring or not. When S is thrown over, the current through the primary winding is reversed, thus changing the flux in the iron from ϕ in one direction to ϕ in the other direction—a change of 2ϕ. If the galvanometer remains connected to the test coil (key closed) while this change in flux occurs, there is a deflection d, and as shown before (Arts. 279 to 282),

Change in flux turns

$$= \Delta(\phi n) = 2\phi n = cd,$$

where n is the number of turns in the test coil. The amount of flux in the iron before reversal is

$$\phi = \frac{cd}{2n}.$$

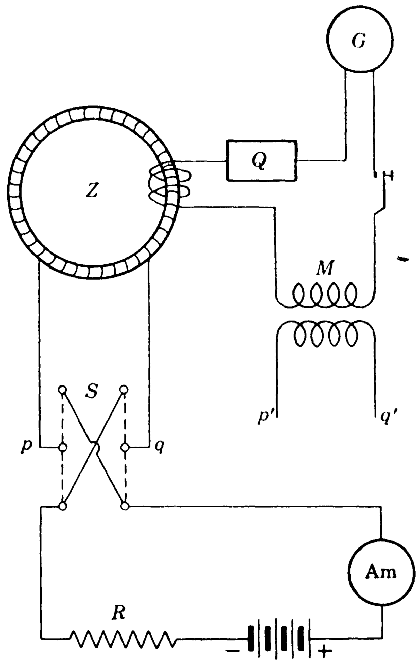


FIG. 131.—Arrangement for determining the magnetic flux in the ring Z.

286. Mmf Due to a Current.—The most convenient way to obtain a definite mmf in a magnetic circuit is to link the circuit with a number of turns of wire N carrying an electron current I, as indicated in Fig. 131. If it is desired to express the value of the mmf in ampere turns, this is given simply by NI. It is shown in Arts. 253 and 265 that the magnetomotive force F, expressed in gilberts, is

$$F = 1.257NI. \qquad \text{gilberts}$$

Since the value of F depends upon NI, it is evident that a small current in a coil of many turns will produce the same mmf as a large current in a coil of few turns.

287. Relation between Magnetic Flux and Mmf.—In the case of a magnetic circuit of iron or other ferromagnetic material, the magnetic flux is not linearly proportional to the mmf. This is shown by measuring the flux in the iron ring (Art. 285) that corresponds to different values of the magnetizing current and then plotting the results as a curve (Fig. 132).

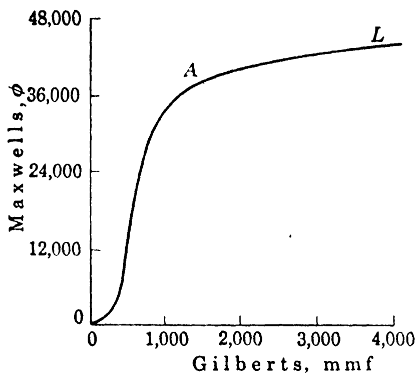


FIG. 132.—Magnetization curve for a circuit of tool steel, 50 cm in length and 3 sq cm in cross section.

It is seen that for small values the flux increases faster than the mmf, while for larger values the flux increases more and more slowly, although the curve never reaches a maximum, however much the mmf is increased.

The effect of previous magnetization should be removed (Art. 291) before these measurements of the magnetic flux are made.

288. The Reluctance of a Magnetic Circuit.—Another way of showing the relations in a magnetic circuit is to plot the values of the reluctance of the circuit corresponding to different values of the flux. This does not require any new measurements, since the values of the reluctance R are computed from the relation

$$R = \frac{F}{\phi},$$

and the same values of ϕ and F are used that were obtained before for the curve in Fig. 132. The reluctance is less for medium values of the flux than it is for small or large values. The minimum reluctance corresponds to a value of the flux of about half saturation.

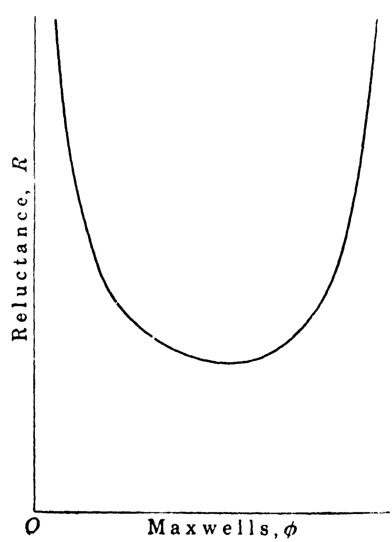


FIG. 133.—Showing the variation in the reluctance of a magnetic circuit as the flux is increased.

289. Demagnetization. *Hysteresis.*—As stated above, the reversal of the magnetizing current does not ensure a change in the value of the flux from ϕ in one direction to an equal amount in the opposite direction (giving a change of 2ϕ) unless the effect of any stronger magnetization has been removed from the iron. To obtain this essential demagnetization requires an understanding of the condition of the iron and the careful manipulation of the rather elaborate process of demagnetization.

When a ring of steel is magnetized by a current that starts at zero and steadily increases to a final value, the resulting flux density increases as shown by the curve in Fig. 137. But now suppose that when the point A on the curve of Fig. 137 has been reached, the current is *decreased* gradually to zero. Would the magnetization curve be retraced, or would a new curve be obtained? Investigation reveals that as the current is slowly decreased, the magnetization of the steel does not decrease to its former values, and when the current reaches zero a large amount of "residual" magnetization still remains. This decreasing magnetization is shown by the curve AD, Fig. 134. The ordinate OD shows the residual flux density when the current has been decreased to zero.

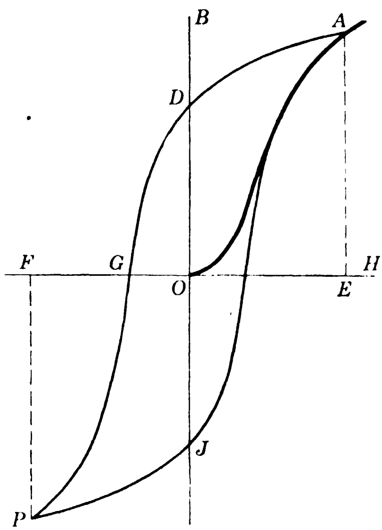


FIG. 134.—Normal hysteresis curve for steel.

It will even require the application of a reversed magnetic intensity, equal to OG, to reduce the magnetic flux density to zero. This negative value of H is called the "coercive force" of the steel. It has a large value for hard iron and steel but is small for soft iron and silicon-iron alloys. If the reversed H is increased to a value OF, equal and opposite to OE, the material is found to be magnetized as strongly as before but in the opposite direction, and it will hold this magnetization just as persistently as the other. If H is reduced to zero and again increased to OE, the flux density follows as shown by the curve PJA. This lagging of the values of B behind the corresponding changes in H is called "hysteresis," from a Greek word meaning "to lag behind." The complete curve as thus drawn between B and H is called a "hysteresis curve."

There is not just one hysteresis curve for a given kind of iron. The curve shown in Fig. 134 was traced by reducing the value of H

from the point A. A similar curve can be drawn starting from any other point on the curve OAL, Fig. 137, and a series of such curves are shown in Fig. 136. The upper tips of all these hysteresis loops rest upon the curve OAL; in fact, the standard method for finding the magnetization curve OAL is to determine the location of the tips of various hysteresis curves.

290. Unsymmetrical Hysteresis Curves.—In the case shown by Fig. 134 the magnetic intensity H was varied between equal positive and negative values, and the corresponding changes in the flux density resulted in the symmetrical curve shown there. Other ways of changing H would result in other forms of hysteresis curves. If the steel has been magnetized to the extent shown by the point A and then the current is gradually reduced to zero, the steel will be left at D. In a closed-ring circuit the steel will hold this magnetization for a long time.

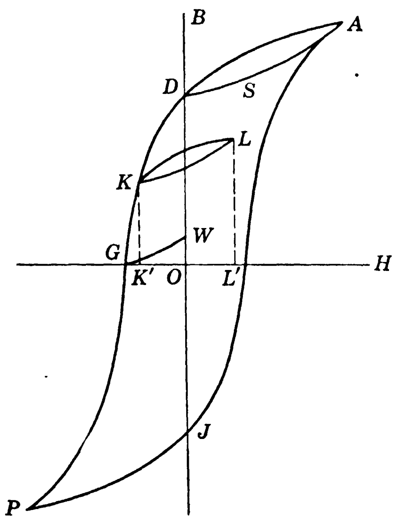


FIG. 135.—Unsymmetrical hysteresis loops.

If after H has been reduced to zero it is increased again in the same direction the steel will be carried back to A along a path like DSA, Fig. 135. By making and breaking the magnetizing current, the steel can be taken around the loop $DSAD$ as many times as desired, but the magnetization is never reversed or reduced to a value smaller than OD.

If when the steel is at D a small value of H is applied and varied between equal positive and negative values, the steel will be carried around a small loop like KL. No matter how many times this small value of H is reversed, the values of B are always in the same direction and merely vary between the values of KK' and LL', Fig. 135, and after many reversals the steel is magnetized nearly as strongly as before.

If the magnetic flux is reduced to zero by bringing the steel to a point like G, the steel is not demagnetized, for when H is reduced to zero, B increases to OW. By varying H between zero and OG, the steel can be carried around the small loop GW as often as desired.

Even if the steel should be carried to a point slightly below G, so that when H is reduced to zero the curve would come back to O, the steel would not be in the unmagnetized condition but would respond differently to a small negative value of H than to an equal positive value.

There is no limit to the number of curves that might be traced, and when a piece of steel or iron has been subjected to various unknown changes in H, there is no telling in what condition it may be left.

291. Demagnetization of Iron or Steel.—When a ring of iron or steel has been magnetized by a current, the material retains a large part of the magnetization after the current is stopped. This residual magnetization must be removed before the normal magnetic qualities of the material can be studied. As shown above, it is not easy to leave the iron unmagnetized when the current is removed. Simply stopping the current has little effect. Reversing the current leaves the iron as strongly magnetized in the other direction. But by carefully following the process of demagnetization that is outlined below it is possible to reduce the magnetization gradually and to leave the iron as nearly demagnetized as may be required.

When a current that is as large as any that has been used previously is reversed a few times in the primary winding of the ring, the iron is carried around a symmetrical hysteresis curve. If this current is reduced slightly and reversed a few more times, the first reversal will not carry the iron to an equal negative value of B, but after several reversals the iron will be carried around a symmetrical loop that is slightly smaller than the former loop. If the decrease in current has been too great, the resulting hysteresis loop will be unsymmetrical, as shown above.

Many reversals are desirable, but, having regard for the labor and time involved, 10 reversals are usually sufficient, especially if they are slow[1] enough to allow the eddy currents to die away after each reversal. This method requires smaller steps and slower reversals on the steep part of the B-H normal magnetization curve. The allowable speed of reversals can be determined as follows: Let t be the number of seconds between one reversal and the next. Then with the arrangement as in Fig. 139, let the current be reversed, and t seconds later close the galvanometer key K. If there is any sign of a deflection from the changing flux, t is too short, and the reversals should be made more slowly. Alternating currents are useless except for fine wires or very thin sheets of iron or steel.

[1] SMITH, A. W., "Demagnetization of Iron," *Phys. Rev.*, vol. 10, p. 284, 1917.

After the iron has been reduced from the first hysteresis curve to a symmetrical loop that is slightly smaller, the same process can be used again to bring the iron to another cycle that is still smaller. By repeating this process again and again, the magnetization can be gradually worked down to as small a value as may be required. Usually it is safe to reduce the current by steps of about 10 per cent of the

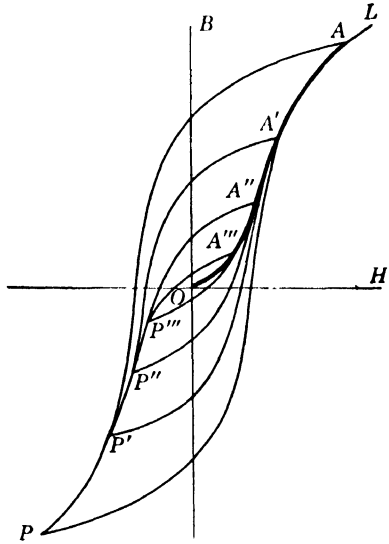


FIG. 136.—Showing a few of the hysteresis loops through which a bar of steel is carried when it is being demagnetized.

maximum value, until the point of maximum permeability of the iron has been reached, and then to use steps that will decrease the flux density by about 1,000 gausses at each step.

When reducing the current it is somewhat better to make the reductions alternately at opposite ends of the hysteresis loop, *i.e.*, first reduce the positive value of H, and next reduce H when it is negative, and the next time when it is positive, and so on. This helps to keep the decreasing loops symmetrical with respect to the axes of B and H.

CHAPTER XII

MAGNETIC TESTS OF IRON AND STEEL

292. B-H Curves.—When studying a given sample of iron or steel, we are usually not so much interested in the properties of the particular piece under investigation as in the specific properties of that grade of steel. The total amount of the magnetic flux, $\phi = BA$, depends upon both the quality of the steel and the dimensions of the sample studied. The factor that is characteristic of the material is the *flux density*

$$B = \frac{\phi}{A}, \qquad \text{gausses}$$

where the value of B is independent of the size of the magnetic circuit.

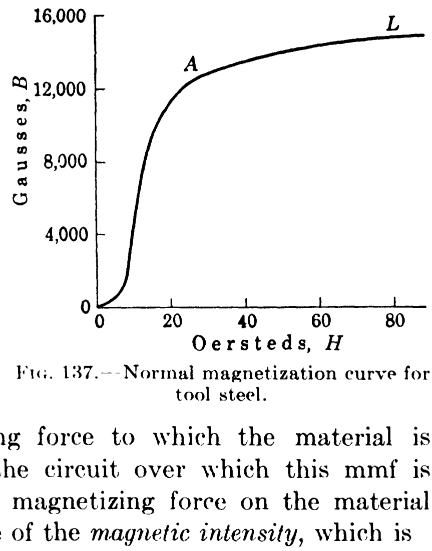


FIG. 137.—Normal magnetization curve for tool steel.

In the same way the total mmf does not give much information regarding the magnetizing force to which the material is subjected unless the length of the circuit over which this mmf is distributed is also known. The magnetizing force on the material is measured by the average value of the *magnetic intensity*, which is

$$H = \frac{\text{mmf}}{l}, \qquad \text{oersteds}$$

where l is the length of the magnetic circuit.

The relation between B and H for tool steel is shown in Fig. 137. The curve rises rapidly at first and then more slowly as the value of B becomes larger, like the curve of Fig. 132, but the form and size of the *B-H* curve is independent of the dimensions of the magnetic circuit in which this steel is used. The results of magnetic tests and measurements on different substances are therefore expressed in terms of B and H instead of by the total flux and mmf.

293. Normal Magnetization Curve.—If it were possible to obtain a ring or bar of iron that never had been near a current or any magnetized steel, the relation between B and H could be studied by gradually increasing H from zero to larger values and measuring the corresponding values of B. But after the iron has once been magnetized, it is necessary to demagnetize it before making measurements on the relation between B and H. The standard curve for showing the magnetization of iron is obtained after the material is thoroughly demagnetized. In order to locate a point on this curve, the current that magnetizes the iron is reversed many times until the iron is in a symmetrical cyclic condition corresponding to a hysteresis curve like one of those shown in Fig. 136. Then the galvanometer is read at the time of the next reversal of the current. The deflection measures the total height of the corresponding hysteresis loop, and half of this deflection measures the height of the tip of the loop above the H axis. A series of such points would locate the tips of the loops shown in Fig. 136, and the curve through these points is called the "normal magnetization curve" for the given substance. This curve is nearly like the one that would have been obtained by gradually increasing the magnetization the first time that the iron was ever magnetized.

294. Normal Magnetization Determination during Demagnetization.—It makes little difference in the normal magnetization curve whether the iron is first demagnetized down to nearly zero and the values of B are then determined for increasing values of H, or whether the points on the curve are determined in the opposite order. At any stage in the process of demagnetization, after many reversals with a given current I, the iron is magnetized as strongly as though the demagnetization had been completed and the current I then applied and reversed a few times. If the deflection of the galvanometer is read for the last reversal of current before the next smaller value is considered, it will measure the normal value of B corresponding to that value of the current. It is therefore possible to take the readings for a curve of normal magnetization during the process of demagnetization, and with the saving of considerable time and effort.

295. Permeameters.—The standard method for magnetic testing is the ring method, since a ring circuit is uniform throughout its length and there are no ends to introduce disturbing factors. Because of the difficulty of making rings and the labor of carefully winding each one, many other arrangements have been used in which the coils are wound on permanent forms that can be slipped over a bar or rod of iron. The various forms of permeameters have magnetic circuits that more or less satisfactorily approach the form of a ring circuit.

296. The Double-bar Permeameter.—In the double-bar permeameter shown in Fig. 138, the magnetic circuit consists of two rectangular bars, 1 by 3 cm in cross section and 36 cm in length. At each end these bars are connected by a flat block of Armco iron. The sides of the bars and the soft iron blocks are finished to a smooth surface to ensure good contact between them. The bars are held firmly against the iron block by a yoke provided with a screw clamp that does not mar the smooth surface of the bars when they are clamped together. The magnetic circuit is thus closed, as shown in Fig. 139, and consists almost entirely of the bars that are being tested.

As will be seen in Fig. 138, the bars thread through two brass tubes of rectangular section, slightly larger than the bars, mounted on end brackets and upon which are wound three layers of well-insulated wire.

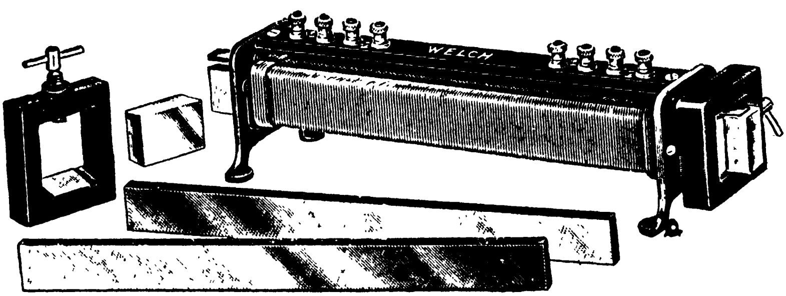


FIG. 138.—The Smith permeameter for obtaining the normal magnetization and hysteresis curves for iron or steel.

Considerable care is exercised to ensure the uniform spacing of the turns of wire in these coils, and the number of turns per centimeter is constant along the length of the greater part of each bar. A part of the magnetic circuit lies beyond the ends of the coils, where it would be practically impossible to wind the coils. The reluctance of this end portion of the magnetic circuit includes the reluctance of the iron block and the contact reluctance between the block and bars. These reluctances are made small by having the iron block short and wide. In addition there is the reluctance of a short length of the bars and an equivalent further length of the same material for the path of the flux as it turns to reach the side of the bar. This extra length is kept small by making the bars broad and thin. The total reluctance of the end portion is thus largely due to the material of the bars and is therefore a definite fraction of the reluctance of the entire circuit. This fraction is determined experimentally when the permeameter is designed, and the number of turns that are required for the end portions of the magnetic circuit are wound as extra turns at each end of both long coils.

The two long solenoids and the four extra end coils are all joined in series, forming a single electric circuit with the ends brought out to a pair of binding posts. The magnetic flux is measured by means of a small test coil of 20 turns, wound on the brass tubes before the current winding is put on. Ten turns are distributed over the middle third of each bar, thus giving an average of flux density measurement for both bars that is not affected very much by small irregularities at the ends of the bars.

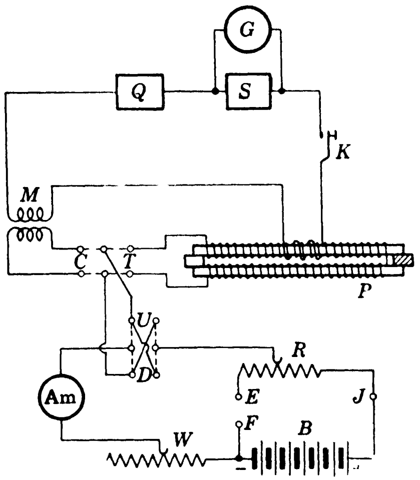


FIG. 139.—Normal magnetization of the iron bars in the permeameter P.

After one pair of bars are tested, the bars of another grade of iron or steel can be inserted easily and tested without disturbing the test coils or the solenoids that carry the magnetizing current.

297. Circuit Connections. *Using the Double-bar Permeameter.*—The experimental arrangement is shown in Fig. 139. The permeameter is shown at P with its primary winding connected to the double throw switch CT. When this switch is closed on T, connection is made to the reversing switch UD. The latter switch should be of the quick-reversing type (Fig. 140) that will change the current through P from its full value in one direction to the same value in the opposite direction before the galvanometer has moved more than a small part of its full deflection. By tracing the circuit through this switch it is seen that only one of the cross connections is in the circuit at one time, and this keeps the same resistance in the circuit whether the switch is closed at U or at D. A battery of 20 to 40 volts is desirable to swamp the emf induced in the primary of P and to compel a quick rise of the current after reversal. Two

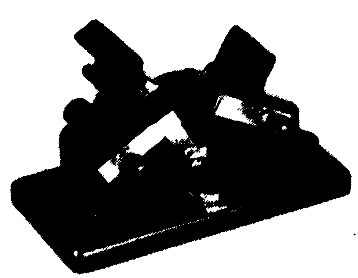


FIG. 140.—A simple and inexpensive switch for reversing a current quickly. (*Courtesy of Leeds & Northrup Co.*)

rheostats, W and R, are shown. These should be able to carry the maximum current that will be used, perhaps 2 or 3 amp, and should have perhaps 50 ohms each. When small currents are needed, a very large resistance is required for R, unless a small part of the battery can be used. By connecting E and F, the rheostat EJ becomes a potential divider across the battery, and the current through P can be made as small as required by moving the sliding contact toward E, with W kept large. Care should be taken to make sure that the resistance of EJ is large enough to be safely connected across the battery and that the current through the part R does not exceed the safe carrying capacity of the rheostat.

The potential divider is not desirable with large currents in P, as it puts an extra heating load on the rheostat EJ.

298. The Magnetic Ballistic Constant.—When a ballistic galvanometer is used to measure a change in magnetic flux of $\Delta(\phi n)$ flux turns, the relation is given by

$$\Delta(\phi n) = cd, \tag{1}$$

where the value of the constant c depends upon the resistance of the circuit in which the galvanometer is connected as well as upon the unit in which ϕ is expressed and the sensitivity of the galvanometer and its damping factor. Thus

$$c = Rkj10^8, \tag{2}$$

as shown in Art. 281, and its value can be changed by changing the resistance R of the galvanometer circuit.

A known amount of magnetic flux is most conveniently obtained by the use of a mutual inductance consisting of primary and secondary coils wound on a core of nonmagnetic material. The magnetic flux that is linked with the secondary coil, due to a current of I' amp in the primary coil, is

$$(\Phi n) = MI', \qquad \text{weber turns}$$

when the mutual inductance M is expressed in henrys and the flux is in webers.

If the measurements are to be made in terms of maxwells ϕ instead of in webers Φ, then

$$(\phi n) = (\Phi n)10^8 = MI'10^8, \qquad \text{maxwell turns}$$

and from Eq. (1), a reversal of the current I' will give a change in the flux turns in the secondary circuit of

$$2MI'10^8 = cd', \tag{3}$$

where d' is the deflection of the galvanometer in this secondary circuit when I' is reversed in the primary coil.

The magnetic ballistic constant c for this galvanometer when connected as a part of this particular circuit is, then,

$$c = \frac{2MI'10^8}{d'}. \tag{4}$$

299. To Measure the Flux Density in a Steel Bar.—When the magnetic flux that is to be measured is in a steel bar of A sq cm cross section with a flux density of B gausses, then $\phi = BA$ and Eq. (1) becomes

$$An\Delta B = cd, \tag{5}$$

where n can be written as a separate factor if the test coil is so arranged that all the flux BA is linked with each one of the n turns of the test coil in the galvanometer circuit, as in the permeameter shown in Fig. 138 or the ring in Fig. 131.

This relation enables one to measure the *change* in the flux density B by observing the corresponding deflection d of the galvanometer. Thus

$$\Delta B = \frac{cd}{An}. \tag{6}$$

If the change in B has been a reversal of the magnetic flux, then

$$B = \frac{1}{2}\Delta B = \frac{MI'10^8}{And'}d, \tag{7}$$

by inserting the value of c from Eq. (4).

If precise measurements of B are required, the scale of the galvanometer should be calibrated by determining the values of c corresponding to the various deflections that are used. For permalloy or other material that is easily magnetized, it is better to substitute an equal resistance for the secondary coil of P during the determination of the constant.

300. To Make the Galvanometer Direct Reading.—It is convenient and saves a lot of multiplication to have the combined factors in Eq. (7) equal to some round number, like 100. Then

$$B = 100d. \tag{8}$$

This means that the combined constants

$$\frac{MI'10^8}{And'} = 100,$$

or

$$d' = \frac{M10^8}{100An} I' \tag{9}$$

where d' is the deflection that the galvanometer should give when I' is reversed in the primary coil of the mutual inductance. The deflec-

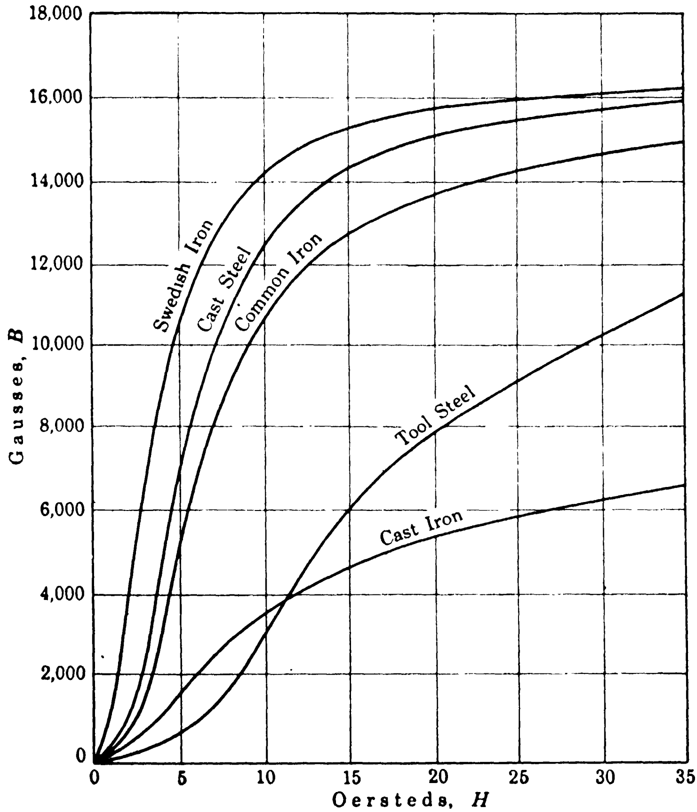


FIG. 141.—Normal magnetization curves.

tion of the galvanometer can be adjusted to this value by adjusting the resistance in the galvanometer circuit. When this adjustment has been completed,

$$B = 100d \qquad \text{gausses}$$

for any bar of A sq cm cross section.

Both the test coil of n turns and the secondary coil of the mutual inductance, or an equal resistance, should be kept in the galvanometer circuit. Any change in the resistance of this circuit will change this factor (= 100) which has been adjusted to the desired value.

301. The Magnetic Intensity, H.—The value of the magnetic intensity due to the current in the N turns of the primary winding is shown in Arts. 260 and 266 to be

$$H = \frac{4\pi NI}{10l} = 0.4\pi pI, \qquad \text{oersteds}$$

where I is the value of the current expressed in amperes, and p denotes the turns of wire per centimeter. This gives the average value of H along the middle portion of the iron bars. The value of the constant $4\pi/10l$ is sometimes stamped on the name plate of the permeameter.

302. The Ring Method.—While this is one of the older methods, it still remains standard for determining the magnetic properties of iron or other magnetic material, so that other methods are checked by comparing their results with those obtained by the ring method. This is because of the simplicity and uniformity of the magnetic circuit. The ring has no ends and there are no joints across which the flux must pass. When a ring is wound with a uniform solenoid of p turns per centimeter, the mmf uniformly is distributed along the circuit, and if the material is homogeneous, all the flux follows the iron path. Whether the sample of iron to be tested is in the form of a ring, with carefully wound primary and secondary windings, or is in the form of straight bars in the double-bar permeameter (Art. 296), the method of measurement is the same.

303. The Iron Ring.—The ring method was originally used with a ring of iron. This avoids the complications arising from joints or other irregularities in the magnetic circuit, and it is still the standard method when the steel to be tested can be obtained in the form of a ring.

Since the primary turns are closer together on the inside of the ring than on the outside, the magnetic intensity H will not be uniform across the iron but will be greater near the inner side. This is also seen from the relation $H = NI/5r$. For this reason it is best to use a wide, thin ring, shaped like a wagon tire, with a radial thickness less than one-tenth (preferably one-twentieth) of the radius of the ring.[1]

If this ring is tested as a sample of a larger piece, the material should be altered as little as possible in making the ring. It is better

[1] *Bull. Bur. of Standards*, vol. 5, p. 442, 1909.

to cut it out of a thick plate, but if it must be bent and welded, it should be worked as little as possible. If it is welded or cast, the outside scale should be taken off all around to about ⅛ in. deep, so that the measured cross section shall represent homogeneous material. As working in the lathe hardens the iron near the cut surfaces, the ring should be carefully annealed in a furnace from which air is excluded, by packing the ring in aluminum oxide.

If the ring is a new one that has not been magnetized since it was annealed, the smallest currents may be used first to determine the virgin *B-H* curve. In this case great care must be observed that at no time does the current exceed, even for an instant, the value being used at the time. If the iron has been previously magnetized, it must be thoroughly demagnetized before it is used (see 291).

304. Correction for the Flux through the Air.—The flux through the test coil consists mainly of the flux through the ring, but there is always some flux through the air space surrounding the iron and included within the test coil. Let the cross-sectional area of the iron be denoted by A sq cm and that of the surrounding air space by a sq cm. The area of the test coil is then $(A + a)$ sq cm and the flux through this coil is

$$\phi = BA + B_0a,$$

where B_0 denotes the value of the flux density in the air around the iron. The flux density in the iron is

$$B = \frac{\phi - B_0a}{A}.$$

The value of B_0 can be computed, since $B_0 = \mu_0H$, and μ_0 is very nearly unity for air. Usually this correction is negligible, but when high values of H are used, this correction should be subtracted from the measured values of ϕ as shown above.

In case there is considerable air space between the iron and the test coil, it is sometimes found convenient to place a small auxiliary test coil in this space. By making

$$B_0a'n' = B_0an,$$

where a' and n' refer to the auxiliary coil, the flux turns through this coil can be made to neutralize the above correction. When the two test coils are in series, the resultant measured by the galvanometer is

$$\phi n = BAn + B_0an - B_0a'n' = BAn,$$

and

$$B = \frac{\phi}{A}.$$

This correction is small when the iron bars fill nearly all the space within the test coil. When small bars are used in large test coils, this correction should be made.

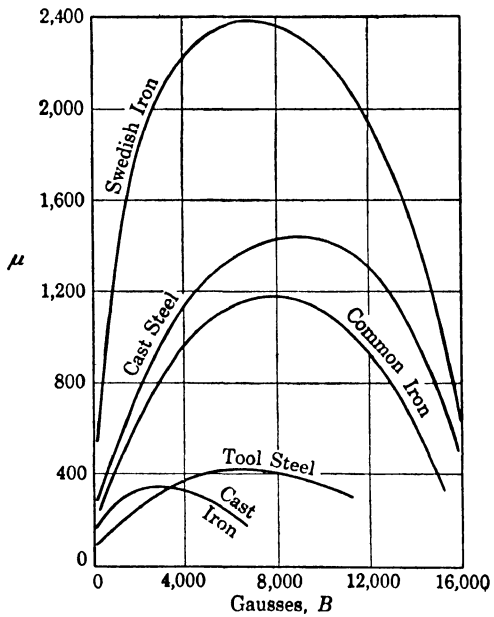


FIG. 142.—Permeability curves.

305. Permeability Curve.—The qualities of different grades of iron are well shown by the permeability curves, in which values of the permeability

$$\mu = \frac{B}{H}$$

are plotted as ordinates, with the corresponding values of B for abscissas. Figure 142 shows the permeability curves corresponding to the magnetization curves of Fig. 141. Note that the maxima of these curves correspond to the points where lines through the origin are tangent to the magnetization curves.

The permeability starts at a value of about 50 or 100 and, after passing the maximum, decreases rapidly and then slowly to a limiting value of unity. For the largest values of B that have been measured, μ is about 2.

306. Burrows's Double Bar and Yoke.—In the double-bar-and-yoke method as used at the Bureau of Standards, the effect of the extra reluctance in the joints and yokes is balanced by extra current windings around the bars close to the yokes. In case only one bar for testing is available, a second bar of similar material is used. There are then three magnetizing circuits, and the currents through these are adjusted to such values as will give a uniform magnetic flux throughout the length of the test bar.

The condition of uniformity along the test bar is determined by the use of two test coils. One of these is wound around the middle of the bar, under the primary winding, and each half of the other coil is

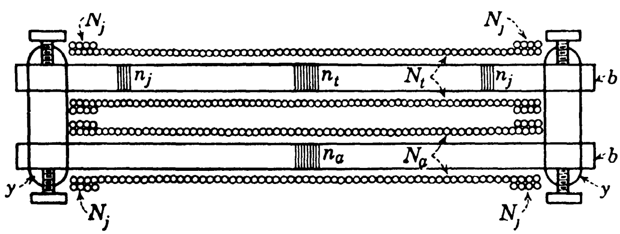


FIG. 143.—Burrows's double-bar-and-yoke method.

similarly placed about 1 in. from either end of the bar. When these coils are connected in series with the ballistic galvanometer and opposing each other, there will be no deflection if all the flux passing through the middle section of the bar continues in the bar to the ends. The current through the extra end windings is adjusted until the flux is thus uniform along the bar.

In case the auxiliary bar is of different material, the magnetizing current around it is adjusted until the flux through it is the same as that through the bar being tested. This equality is shown by a third test coil placed around the middle of the auxiliary bar, having the same number of turns and used in opposition to the main test coil around the bar being tested. When the flux has been made uniform throughout the circuit, its value is measured by the galvanometer deflection as in the ring method.

307. The Fahy Permeameter.—This is an arrangement of a yoke and the test bar in which all the mmf is supplied by a magnetizing coil around the yoke. This mmf is used in part over the yoke portion of the magnetic circuit and in part over the bar to be tested. The distinctive feature of this method is the measurement of the mmf acting on the bar by a magnetic-potential solenoid arranged in parallel with the bar. The deflection of a ballistic galvanometer connected to this solenoid measures the mmf when the latter is reversed.

Because of the uneven distribution of the magnetizing coils, the flux does not have the same value along the circuit. The average value of the flux through the bar is measured by the deflection of a ballistic galvanometer as in the method of reversals; for the test coil, a solenoid wound uniformly over the full length of the bar is used. This method has the same simplicity as the ring method in that it does not require any adjustment or compensation of the magnetizing current, and it has the further advantage that a single bar can be tested.

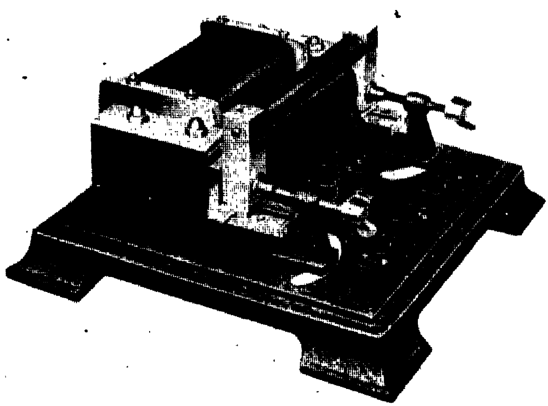


FIG. 144.—The Fahy permeameter, simplex type.

A general view of the permeameter is shown in Fig. 144, and the electrical circuits are shown in Fig. 145. The bar of steel to be tested is clamped against the flat ends of the U-shaped electromagnet that forms the larger part of this permeameter as seen in Fig. 144. The coil of this magnet is connected through the double-throw switch *TC*, Fig. 145, to the quick-reversing switch *UD*, like the one shown in Fig. 140. The magnetizing current is adjusted by the rheostat *W* and measured by the ammeter, although no computations are made from this value of the current. When the current is reversed, with K_1 closed, the galvanometer measures the value of the magnetic flux in the bar, in the same way as in the ring method. When K_2 instead of K_1 is closed, the galvanometer measures the flux through the *H* coil, which is proportional to the value of *H* in the steel bar.

By closing the double-throw switch at *C*, the mutual-inductance coil *M* can be used to give a known amount of flux turns to determine the magnetic ballistic constant of the galvanometer (Art. 298) or to make the galvanometer direct reading (Art. 300).

308. To Make the Galvanometer Direct Reading for B.—By setting the resistance in Q, Fig. 145, at the proper value, the galvanometer can be made to read directly the values of the flux density B in the bar of steel that is being tested. When the magnetizing current is reversed, the change in the flux turns in the B-test coil of n turns is

$$\Delta(\phi n) = 2BAn = cd.$$

The magnetic ballistic constant for this circuit through Q and the galvanometer is found by using the known mutual inductance of M henrys. Then, as in Art. 298,

$$c = \frac{2MI'10^8}{d'}.$$

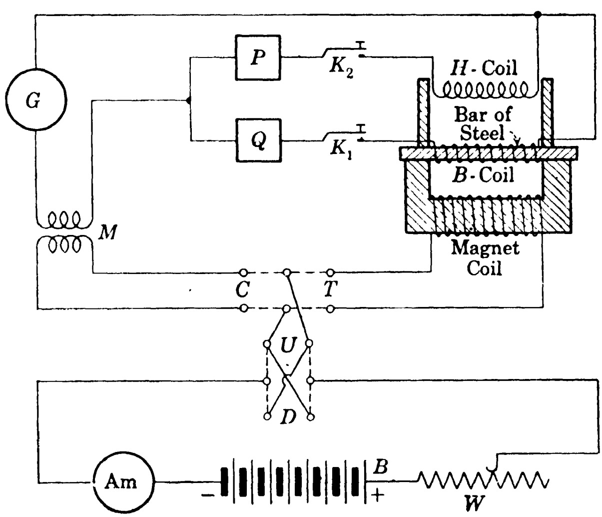


FIG. 145.—Connections for using the Fahy permeameter.

The galvanometer can be made direct reading for values of B, as was done in Art. 300. When this is done (by adjusting Q),

$$B = 100d$$

for all bars having this cross section of A sq cm.

In case the range in the resistance of Q is insufficient to bring d down to the desired value, a shunt can be used on the galvanometer. In any case it may be convenient to have the galvanometer critically damped (Arts. 65 and 278).

309. To Make the Galvanometer Direct Reading for H.—The change in the flux turns through the H coil, Fig. 145 (in air or bakelite), when the magnetizing current is reversed is

$$\Delta(\phi n)_H = 2B'A'n' = 2\mu HA'n'.$$

When this change in flux is measured by the galvanometer, the relation is

$$2\mu HA'n' = c'd,$$

where the constant c' is for the circuit through P and the galvanometer. Therefore c' is not the same as c for the circuit through Q, but its value is found in the same way (Art. 298) by using the mutual inductance M. Then

$$c' = \frac{2MI''10^8}{d''}$$

and

$$H = \frac{MI''10^8}{A'n'd''}\, d, \tag{30}$$

since μ = one for air.

If it is desired to have a deflection of $d = 100$ for $H = 10$ oersteds, say, the coefficient of d in Eq. (10) should be $1/10$. This is the case when P has been adjusted to give

$$d'' = 10\,\frac{MI''10^8}{A'n'}$$

for a reversal of I'' amp in the primary of coil M. The number of turns in the H coil times its effective area is given by the maker. (On one coil, $A'n' = 27{,}700$.) Then

$$H = \frac{d}{10}.$$

The total mmf measured by this H coil of length l is

$$F = Hl,$$

and since this coil is in parallel with the bar, the mmf along the length of the bar l_B is this same amount, and

$$F = H_B l_B.$$

In the manufacture of the permeameter the length of the H coil is carefully made the same as the length of the portion of the steel bar that lies between the ends of the electromagnet, so that $l = l_B$, and therefore the value of the magnetic intensity H_B in the steel bar is equal to the value H that is measured by the H coil.

310. The Hysteresis Curve.—The hysteresis curve for steel is shown by the loop *UNDJU* in Fig. 146. With each reversal of the magnetizing current the steel is carried around the hysteresis cycle, as explained in Fig. 134, but in order to trace the form of this curve it is necessary to be able to stop at various points along the curve by changing the magnetic intensity H in steps that are smaller than a complete reversal.

311. Standard Point of Reference.—Since only changes in B can be measured and not the full value of B itself, it is necessary to choose one point on the hysteresis curve as a point of reference and then measure the change in B when the steel is carried from any condition to this reference point. When the steel is in the form of thin sheets or fine wires, it makes little difference whether point N ($H = 0$) or point D, Fig. 146, is chosen for this reference point. But for solid bars of steel there are eddy currents in the body of the metal whenever B is changed, and the magnetizing effect of these eddy currents is added to the value of H. Thus B does not reach its final value until the eddy currents die away. Because of this effect it has been shown[1] that the change in B can be measured more accurately when the change ends with a large value of H. Therefore it is better to choose the lower tip, D, of the hysteresis curve as the point of reference for locating the other points of the curve.

312. To Locate Points on the Hysteresis Curve.—The location of point U at the upper tip of the hysteresis loop, Fig. 146, is readily made by reversing the current. The change in flux density for this reversal is $\Delta B' = 2B'$, and

$$B' = \tfrac{1}{2}\Delta B',$$

where B' denotes the value of the flux density for the tip of the loop.

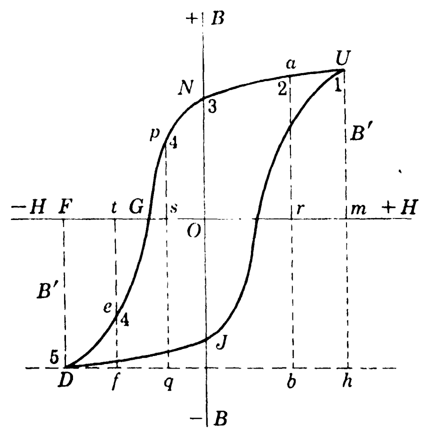


Fig. 146.—Hysteresis curve for hardened tool steel. When the switch (Fig. 148) is thrown down, the steel is carried along the curve *UaNpGeD*. When the switch is thrown up, the steel is carried along *DJU*.

When the current is reduced and the steel is brought to a point like a on the upper part of the curve, the value of B is reduced from Um to ar, Fig. 146. A reversal of this current will not reverse ar, but this value of B can be determined in another way. When H is changed

[1] Smith, A. W., "Time Lag in Magnetization," *Phys. Rev.*, vol. 9, p. 419, 1917.

from $+Or$ to $-OF$, the steel is carried along the curve from a to b with a change in B of $\Delta B = ab$. This can be measured by the deflection of the ballistic galvanometer, as shown in Art. 300.

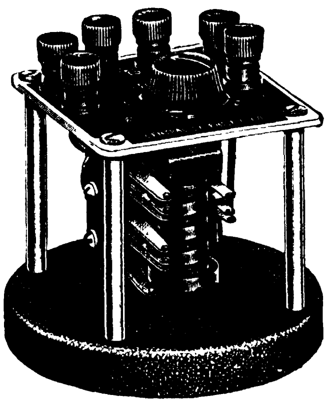


FIG. 147A.—Hysteresis switch. Turning the knob halfway round and back again changes the circuit through the steps that are shown in Fig. 148 when the blades are moved from *1* to *5* and back to *1*, and the steel is carried through the steps shown in Fig. 146. (*Courtesy of the Central Scientific Co.*)

Referring to Fig. 146, it is seen that

$$B = ar = ab - rb = ab - B', \quad (11)$$

since $rb = FD = Um = B'$. Thus for the point a,

$$B = \Delta B - B'. \quad (12)$$

313. Other Points on the Curve.—The same procedure gives the value of B for any other point along the curve. For a point like e, $\Delta B = ef$; and $ef - B'$ is a negative value for B ($= -te$), which is then plotted downward from the axis OH. The corresponding values of H are computed from the readings of the ammeter that measures the current in the magnetizing coil, as in Art. 301, and

$$H = 0.4\pi pI. \quad \text{oersteds}$$

The various changes in the magnetizing current that are required to make the steps described above are easily made with the aid of the switch shown in Fig. 147A or Fig. 147B described below.

314. The Three-bladed Hysteresis Switch.—As shown above, in order to measure the value of B for a point on the hysteresis curve it is necessary to make a quick change from this value of B to the maximum value at the lower tip of the hysteresis loop. It must be quick because the entire change must be completed in time to be fully measured by the deflection of the ballistic galvanometer. This can be done by using a triple-

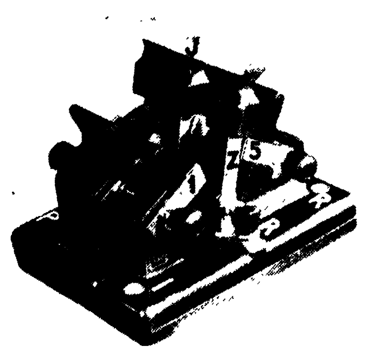


FIG. 147B.—Hysteresis triple-bladed switch. When the handle J is thrown *1* to *5* and back to *1*, the steel is carried through the steps shown in Fig. 146.

pole, double-throw switch like the one shown in Fig. 147B, the action of which is diagramed in Fig. 148.

The three blades of this switch are shown at V, Y, and Z. They are all fastened to the handle J and move together. When the blades are set in the position indicated by the dashed line 1, the current from the battery passes through the blade V to P and the permeameter, then back to P' and the blade Y, and then to Z and the ammeter. The current is a maximum, and the steel is at U, Fig. 146.

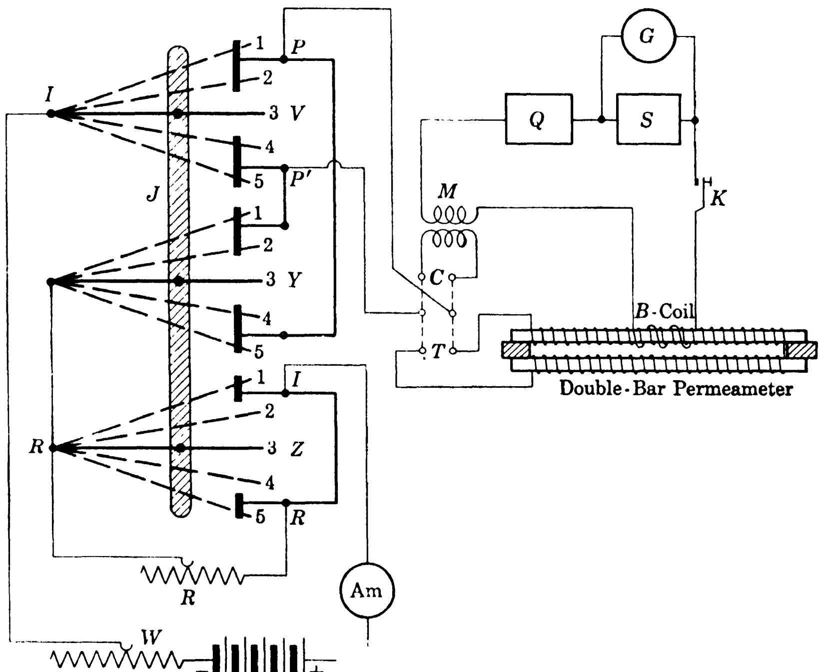


FIG. 148.—Diagram of the three-bladed switch of Fig. 147, showing the connections for obtaining a hysteresis curve with either a ring or the double-bar permeameter. The numbered dashed lines show the five positions of the switch blades V, Y, and Z. J is the plastic handle for moving the blades together.

Position 2.—When the blades are moved down to position 2, V and Y still make contact on their clips as before, but Z is now open. This adds the resistance R to the circuit, and with the smaller current the steel in the permeameter is brought to some point a, Fig. 146. The point a may be at any chosen place between U and N, depending upon the resistance in R.

Position 3.—By moving the switch to position 3, the circuit is opened and the current is zero. This position corresponds to point N on the hysteresis curve, Fig. 146.

Position 4.—When the switch is moved to position 4, the blades V and Y make contact on the other side of the switch, thus reversing the direction of the current in the coils of the permeameter. Z is still open, thus keeping R in the circuit, and the current has an intermediate value. The steel is brought to some point like p or e, Fig. 146, depending on the resistance in R.

Position 5.—When the switch is moved to position 5, the blades V and Y slide along their broad contacts and Z is closed. This short-circuits R, allowing the current to rise to its maximum negative value as set by the resistance in W, and the steel is brought to the lower tip of the hysteresis loop at D.

Positions 5 *to* 1.—When the switch is thrown back, the current passes through these changes in the inverse order and the steel is carried along the curve DJU, Fig. 146. One can follow the progress along the curve and observe these changes in the current by watching the ammeter while the switch is moved from contact to contact.

315. Construction of the Switch.—A double-pole, quick-throw switch as pictured in Fig. 140 is readily obtained. A third blade is added by sawing such a switch in two and mounting one of the halves alongside a complete switch with a rigid handle to move the three blades together, as photographed in Fig. 147B. In this picture the switch is shown in position 4. The middle blade Y is seen in contact with its clip, while Z is still open, because of its shortened clip 5. When the switch is thrown a little further to the right, Z is closed to short-circuit the resistance R, Fig. 148.

The original switch has a spring clicking device that holds the blades midway between the contacts when they are left in that position. By cutting additional notches the blades will remain in the positions indicated by 2 and 4, Fig. 148. It is thus possible to tell by the feel when the changes in current occur, without looking at the switch or the ammeter.

316. Hysteresis by Direct Deflection.—The experimental arrangement is shown in Fig. 148. The bars of the steel to be examined are shown in the double-bar permeameter. If the steel is in the form of a ring, as shown at Z in Fig. 131, this ring would stand in place of the permeameter. The primary winding is connected at T to one end of the double-pole, double-throw switch CT. When CT is closed at T this connection leads to the quick-reversing switch VY, which is shown in Fig. 147.

One of the principal difficulties in the determination of a hysteresis curve for a sample of steel has been in the complicated and laborious procedure that is usually required to carry the steel around the hysteresis cycle and to make possible a stop at any desired point. The reversing and short-circuiting switch shown in Fig. 147 reduces this labor to a minimum and avoids the danger of confusion in the manipulation of the keys, once the circuit has been properly connected.

A ballistic galvanometer *G* connected to a test coil of a few turns of wire around the steel will measure the change in the magnetic flux density *B* corresponding to any of the steps as the steel is carried around the hysteresis curve. In practice it is found that some of these changes in the flux density can be measured with less uncertainty than others. In particular, those steps that end at *D*, with a large value of the magnetic intensity *H*, can be measured with more certainty than the steps that end with small values of *H*.[1,2] Thus when the steel is at *a*, a single motion of the switch carries it to *D* and the galvanometer measures the change in flux density, which is shown by *ab*, Fig. 146. In the same way, when the steel is carried from *p* to *D* the galvanometer measures the value *pq*. The positions of the points *a* and *p* are determined by the value of the adjustable resistance *R*. When a different value of this resistance is used, these points fall at other places along the curve *UND*, and as many points as desired may be located by using different values of *R*. The readings for any point can be repeated as often as necessary, and the points along the curve may be taken in any order that is convenient. Usually there is less chance of confusion when one begins at *U*, Fig. 146, and takes the observations for points along the curve successively in decreasing order along *UaNpGeD*.

The corresponding values of *H* are computed from the readings of the ammeter in the battery circuit.

The steel may be left standing at any point on the hysteresis curve while the observations are recorded, but when the switch is next moved it must continue to carry the steel along the hysteresis curve in the same direction until the tip of the curve has been reached.

317. Drawing the Lower Side of the Hysteresis Loop.—Points on the lower branch of the hysteresis curve can be plotted by using the same values of *H* and *B* with the signs reversed, as the curve is symmetrical and the branch *DJU* is a repetition of the part *UND*.

An easier way is to lay a second piece of paper under the curve *UND* and fasten both together by two pins through *U* and *D*. Several points along the path of the curve are pricked through both papers

[1] EVERSHED and VIGNOLES, *Electrician*, vol. 29, p. 584, 1892.
[2] SMITH, A. W., "Time Lag in Magnetization," *Phys. Rev.*, vol. 9, p. 419, 1917.

with a needle point. The lower paper is then placed over the curve, with the points U and D interchanged, and fastened on the two pins. The intermediate points are then pricked through onto the other sheet, thus outlining the curve DJU.

318. Determination of the Change in Flux Density.—In this experiment the change in the magnetic flux density is not a reversal but a change like that from ar, Fig. 146, in one direction to rb in the other. This gives a total change of ab, which may be denoted by the symbol ΔB. The corresponding change in flux turns in the secondary circuit is ΔBAn, and this is measured by the galvanometer deflection, the relation being

$$\Delta BAn = cd. \tag{13}$$

The change in the flux density is, then,

$$\Delta B = \frac{c}{An} d \tag{14}$$

319. The Magnetic Ballistic Constant.—This constant may be determined by means of a known mutual inductance having its secondary coil in the galvanometer circuit, as in the previous methods.[1]

With a mutual inductance of M henrys, the flux turns that are linked with the galvanometer circuit are

$$(\Phi n') = MI', \qquad \text{weber turns}$$

when the current in the primary coil is I' amp. In this expression the flux is given in webers. If the number of maxwells is desired, the relation is

$$(\phi n') = MI'10^8, \qquad \text{maxwell turns}$$

Then, as in Eq. (13),

$$\Delta(\phi n') = M\Delta I'10^8 = cd', \tag{15}$$

[1] Inasmuch as no current is used for the permeameter when the constant of the galvanometer is being determined, the same battery, ammeter, and rheostats can be used to furnish the current I. To make this change it is only necessary to change the switch from T to C, Fig. 148.

Since the zero position of a sensitive D'Arsonval galvanometer depends upon the direction in which it was last deflected, all deflections should be in one direction only, and therefore the constant must be determined for deflections in this direction also. If greater accuracy is desired, the scale should be calibrated by determining the value of c for deflections throughout the range that will be used.

where d' is the deflection of the galvanometer when the current in the primary coil of the mutual inductance M is changed by $\Delta I'$ amp. This gives

$$c = \frac{M \Delta I' 10^8}{d'}. \tag{16}$$

320. To Make the Galvanometer Direct Reading.—Using the value of c from Eq. (16) in Eq. (14) gives

$$\Delta B = \frac{10^8 M \Delta I'}{And'} d. \tag{17}$$

It is often convenient to have the coefficient of d in Eq. (17) equal to some round number, say 200. In this case

$$\frac{10^8 M \Delta I'}{And'} = 200, \tag{18}$$

or

$$d' = \frac{10^8 M \Delta I'}{200An}. \tag{19}$$

This means that when the constant c is being determined [Art. 319, Eq. (16)], the resistance Q in series with the galvanometer (Fig. 148) should be adjusted to make the deflection d' equal to the value given by Eq. (19), above. Then

$$\Delta B = 200d. \tag{20}$$

By choosing a value for d' that is about the mean of the values that are expected for d in Eq. (14), the corresponding value for $\Delta I'$ can be computed from Eq. (19). When this change in the primary current is made, the galvanometer deflection should be this chosen value of d'. If it is not, the resistance Q of the galvanometer circuit is adjusted until the deflection is d' divisions when the primary current is changed by $\Delta I'$ amp.[1]

The secondary of this mutual inductance must remain a part of the galvanometer circuit, as shown at M, Fig. 148, since any change of the resistance of this circuit will change the value of the constant.

321. To Compute the Value of B.—When the resistance of the galvanometer circuit has been adjusted as in Art. 320 above, it means that the constant c in Eq. (14) has been made equal to $200An$, and therefore

$$\Delta B = 200d.$$

[1] See footnote on p. 376.

When the maximum current is reversed, the steel is carried from the upper tip of the hysteresis curve, Fig. 146, to the lower tip. In this case the value of B for the tip of the curve is

$$B' = \frac{\Delta B'}{2} = 100d',$$

where B' refers to the value of B for the tip of the curve.

For any other point, like a Fig. 146, the value of B is

$$B = \Delta B - B' = 200d - B'.$$

322. Corresponding Values of the Magnetic Intensity, H.—These are determined by the current, which may be read by an ammeter. Values of H may then be computed by the relation

$$H = 0.4\pi pI, \qquad \text{oersteds}$$

as shown in Arts. 253 and 260, where p denotes the number of turns per centimeter in the primary winding of the permeameter. These values are plotted along the axis OH.

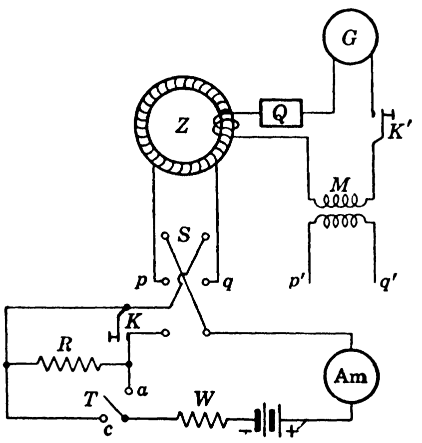


FIG. 149.—To determine the hysteresis curve for the ring Z.

323. Hysteresis by Direct Deflection. *Without Using a Special Switch.*—In case the special switch shown in Fig. 147 is not available, the same cycle can be performed with a simple reversing switch in place of the part VY, Fig. 148, in conjunction with a separate key to short-circuit R after each reversal of the current. However, it will require very strict and careful attention to the order in which the keys are worked to keep the iron from being carried through unsymmetrical loops that are not desired.

324. Energy Loss through Hysteresis.—Since it requires a reversed current to bring the magnetization of a ring or bar to zero, there is always a considerable loss of energy when a piece of iron is carried through a cycle of magnetic changes. This energy is represented by the area of the hysteresis loop, which is narrow for wrought iron, large and broad for cast iron.

The amount of energy thus transformed into heat can be determined as follows: A part of the work required to magnetize the iron is lost as heat in the magnetizing solenoid. This is the regular Ri^2 loss. Another part is used in maintaining the current against the induced emf due to the newly formed magnetic field. This induced emf is

$$e = -N\frac{d\phi}{dt} = -\frac{ANdB}{dt},$$

where A is the cross section of the iron and N is the number of turns in the magnetizing solenoid.

Since e is negative, *i.e.*, opposed to the current, the positive work required to change the current through one cycle of values (from $+i$ to $-i$ and back to $+i$) is

$$W_1 = \int eidt = \int \frac{ANidB}{dt}dt. = \int \frac{AHl}{4\pi}dB = \frac{\text{volume}}{4\pi}\int HdB,$$

since

$$Hl = 4\pi Ni, \qquad \text{and} \qquad Al = \text{the volume of the iron.}$$

For one complete cycle, $\int HdB$ is the area of the hysteresis curve, measured in the units of H and B. This area, then, gives the energy expended per cycle in each 12.57 cc of iron, and the energy W_1 expended in each cubic centimeter of the iron is

$$W_1 = \frac{1}{4\pi}\int HdB = \frac{\text{area of curve}}{4\pi}.$$

325. Hysteresis Coefficient.—When the iron is carried through a hysteresis cycle between larger positive and negative maximum values of B_{max}, the hysteresis curve is higher and a little broader, but the area does not increase quite as fast as $(B_{max})^2$. Experiment shows that over the usual range of the flux density in transformers and electrical apparatus, *i.e.*, from $B_{max} = 1{,}000$ to $B_{max} = 12{,}000$, the energy loss due to hysteresis is proportional to about the 1.6 power of B_{max}. For most practical purposes the empirical relation,

$$W_1 = \eta(B_{max})^{1.6}, \qquad \text{ergs per cycle per cubic centimeter}^1$$

gives the hysteresis loss over this range with sufficient accuracy, but above $B_{max} = 15{,}000$ the formula may be in error by 40 per cent.

[1] Articles 324 and 325 are expressed in cgs units.

The coefficient η is called the "coefficient of hysteresis," and its value indicates the magnetic qualities of the steel. The smaller this coefficient, the smaller is the heat due to hysteresis and the more suitable is the material for use in transformers, etc.

The value of the exponent of B_{max} varies between 1.4 and 1.8 for different materials and for different ranges of the flux density. If a different value of the exponent is used in the equation for W_1, above, it will require a different value for η; and since no value of the exponent will give the actual loss for any material over the entire range of B_{max}, the usual value of 1.6 is always taken in practice. The values of η are then comparable and serve as an index to the characteristics of the different materials. The range of values for this coefficient is shown in the table. (In this section, B_{max} denotes the flux density at the tip of the hysteresis curve.)

Hysteresis Coefficients for Various Steels

Material	η	Material	η
Best silicon steel	0.0006	Soft machine steel	0.009
Silicon steel sheet	0.0010	Cast steel	0.012
Soft iron wire	0.0020	Hard cast steel	0.025
Ordinary sheet iron	0.0040	Hard tungsten steel	0.058

326. Effect of Temperature.—When iron is warmed, it is usually more easily magnetized by small values of H, but it does not reach as high values of B for larger values of H. Above a red heat the values of B are not much larger than for other substances.

In general, the hysteresis loss of energy per cycle diminishes when the temperature increases.

327. Eddy-current Loss.—There is another cause of energy loss in iron besides that due to magnetic hysteresis. This is the heating by the electric currents that are induced in the solid metal by the changing magnetic flux. This loss is reduced by the use of thin sheets of iron or steel, but it cannot be wholly eliminated. The energy loss due to the eddy currents is proportional to the square of the frequency of reversals, and for high frequencies it may exceed the loss due to hysteresis.

The total loss per cubic centimeter can be expressed as

$$W_1 = af + bf^2,$$

where f denotes the frequency. If W_1 can be measured (by wattmeter or calorimeter) for two different frequencies, the constants, a and b, can be determined. The hysteresis loss af can then be compared with the loss bf^2 due to the eddy currents.

328. Permanent Magnets.—By looking at the hysteresis curve of Fig. 146, it is seen that the bar is still magnetized after the current is stopped. The magnetic flux through the bar completes its circuit through the heavy yoke, and outside the iron there is little to indicate that it is magnetized. When the bar is withdrawn from the yoke, it is in the same condition as before, but now the flux must complete its circuit through the air and there is plenty of evidence that the bar is magnetized. The bar itself is a permanent magnet, and the areas where the flux enters and leaves the iron are its poles.

329. Magnetic "Poles."—Inasmuch as magnetic "poles" appear only at surfaces where magnetic flux passes from one substance to another of different permeability, it should not be necessary to use the idea of poles at all. Every case of a piece of material turning or pointing in a certain direction is merely the effect of electronic currents tending to move across the magnetic flux in their neighborhood, in the same way that the coil of a galvanometer turns because a force of

$$dF = BIdl$$

is exerted on each element of the circuit dl, tending to make it move at right angles both to the direction of B and to its own length.

The idea of a magnetic pole is simple and it has long been used. The fundamental units of magnetism were defined in terms of a "unit magnetic pole," and the idea may still be useful in discussing the relations of magnetic quantities. The unit pole stands for a bundle of 4π maxwells radiating from a point (which never occurs), and the expression "unit pole" is a short way of describing such a bundle of magnetic flux.

CHAPTER XIII

ALTERNATING CURRENTS

330. Alternating Current.—An alternating current is the same as any other electron current, except that it flows in one direction for only a very short time; it then reverses and flows in the other direction for an equally short time. In ordinary lighting circuits there are from 100 to 300 such reversals each second. In some other cases there may be many millions of reversals each second. While the current is flowing in one direction it is the same as any other current of the same number of amperes. The only peculiarity of an alternating current is that it is continually being made to change. And just as a material body cannot change its velocity from one direction to the opposite without first slowing down to zero and then starting up in the other direction, so the current cannot instantly change from its full value in one direction to the full value in the other; it requires some time to die down to zero and then to build up in the opposite direction. It does not have time to build up very far before it must begin to decrease again, so there is never a time when the current is not changing in amount. In fact, the value of the current as it changes from one direction to the other and back again goes through the same variations as the velocity of a pendulum bob when swinging to and fro.

331. Tracing Alternating-current and Emf Curves.—In Chap. VII some methods were given for measuring the current flowing through a circuit. The same arrangements can be used to measure the value of an alternating current. Since the balance point would vary rapidly up and down the slide wire of the potentiometer, it will be necessary to add some mechanical device that will close the galvanometer circuit for only an instant at the particular time when the current has the value that it is desired to measure. As the current will have the same value 60 (say) times in each second, the galvanometer circuit can be closed 60 times each second, and this is often enough to produce a steady deflection of the galvanometer when the potentiometer is not balanced. Thus the actual setting is made as easily as when the current is steady.

Let AD represent the slide-wire potentiometer, S the resistance through which the alternating current is flowing, and M the instantaneous contact maker which closes the galvanometer circuit for a very short time once in each cycle. Let i denote the value of the alternating current at the instant the galvanometer circuit is closed by M. The fall of potential over S is then Si, and if C is moved so that the fall of potential over AC is the same as Si, there will be no deflection of the galvanometer. Therefore the distance AC is proportional to the current i.

Now let the contact maker M be turned a few degrees so as to close the circuit just a little later than before. The value of the current at this point will be somewhat different, and its value can be found by moving C along till a balance is again obtained. In this way the values of the current for the complete cycle can be determined, and a curve can be plotted to show how the current varies with the time.

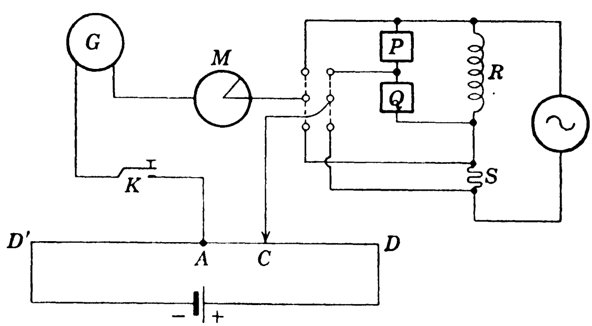


FIG. 150.—Potentiometer for tracing alternating-current curves.

When the current reverses and the fall of potential over S is in the other direction, C must be moved to the other side of A to find a balance. Therefore AD' is merely a second slide-wire potentiometer on which all negative values of the current can be measured. In this manner the values of the current at various instants corresponding to the different settings of M can be measured. These should extend far enough to complete at least one cycle, *i.e.*, until the readings begin to repeat themselves. If the dynamo that is generating the current has two poles, there will be one cycle for each revolution of the armature. If there are more poles, there will be as many cycles in each revolution as there are pairs of poles.

It is absolutely necessary that the instantaneous contact maker M close the galvanometer circuit at precisely the same point in each cycle. The contact maker is therefore placed on the dynamo shaft. There is no electrical connection between the two, but the contact maker will keep rigid step with the current. If the contact maker cannot be mounted on the dynamo that generates the alternating current, it can be driven by a synchronous motor which will keep in step with the current that is being traced.

The curve for the alternating emf can be traced in the same manner. Usually the full emf is too large to be measured directly by the potentiometer, but by the fall-of-potential method, as used in the method for calibrating a high-reading voltmeter, any small portion of this emf can be obtained. It is then only a matter of increasing the scale to get the curve of the full emf. When the current and emf curves are both drawn on the same sheet, the phase relation between them is clearly shown.

Sometimes it works better to use a condenser and ballistic galvanometer, as shown in Fig. 151, in place of the potentiometer shown in Fig. 150. When the condenser key is closed on b, the contact maker M renews the charge in the condenser at each revolution and the condenser

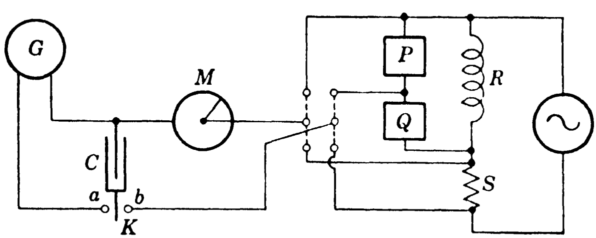


FIG. 151.—Condenser and ballistic galvanometer for tracing alternating-current curves.

is kept charged to the voltage that is being measured at this setting of M. When the key is closed on a, the condenser is discharged and the deflection of the galvanometer measures the voltage at this point on the curve being traced.

After tracing the curves for a current flowing through a noninductive resistance, it will be interesting to do the same for a coil having a considerable inductance. This should show the effect of inductance in making the current "lag behind the emf." If desired, the experiment can be further varied by connecting a condenser in the circuit and finding the effect it has upon the current and emf curves.

332. The Rosa Curve Tracer.—A convenient and compact form of the contact maker and potentiometer of Fig. 150 is embodied in the Rosa curve tracer shown in Fig. 152. The potentiometer wire DD', Fig. 150, is wound in the screw groove of the bakelite cylinder shown at the front of Fig. 152. The cylinder is turned by the handle at the right-hand end, and this moves the contact C along the wire. When the balance point is reached, the record is made by printing the point on the paper on the drum seen in the background. This process avoids the trouble of reading, recording, and plotting the observed values of the emf or current.

The disk that carries the contact maker is shown at the left-hand end of Fig. 152 and is driven by a synchronous motor. Lifting the handle shown in front not only prints the point through the typewriter ribbon in front of the paper; it also sets the movable brush M ahead a few degrees for the next determination and moves the paper-covered drum ahead by the proper amount for receiving the imprint of the next point.

The points may be plotted close together to form a practically continuous curve, or fewer may be printed at appreciable distances apart, depending upon the desire of the operator and the nature of the curve

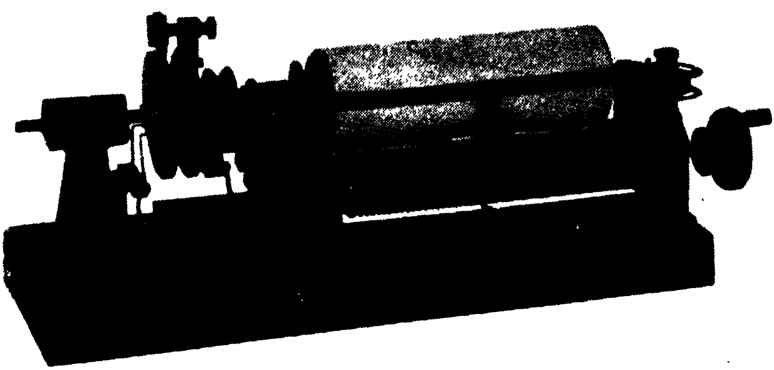


Fig. 152.—The new Rosa curve tracer.

being traced. The Rosa curve tracer is sensitive and accurate, yet at the same time it is very simple and easily operated.

333. Instantaneous Values of the Current.—When an alternating current flows through a circuit, there are induced other emfs besides that impressed by the dynamo; and these extra emfs often have considerable influence in determining what the resulting current shall be. As will be explained in Art. 415, because of the fact that the current is changing at the rate of di/dt amp per sec, there will be induced in the circuit an emf of $-L(di/dt)$ volts. In order to maintain the current i, the dynamo must furnish not only the emf Ri required by Ohm's law but in addition must supply an emf sufficient to counterbalance, at each instant, this induced emf. That is, the dynamo must supply an emf whose value at any instant is

$$e = Ri + L\frac{di}{dt}.$$

This is the general equation of any current varying in any manner whatsoever. Solving this equation for the current gives

$$i = \frac{e - L\dfrac{di}{dt}}{R},$$

which shows that the instantaneous value of the current is given by the usual form for Ohm's law—taking into account all the emfs in the circuit at that instant.

334. Measurement of an Alternating Current.— Instruments are seldom made to give the instantaneous value of the current, but they record some kind of an average value. Evidently, the arithmetical average of the values of a current that is negative as much as it is positive would be zero, and therefore such an instrument as a galvanometer or an ordinary ammeter would be useless for the measurement of an alternating current. But an electrodynamometer or the Kelvin balance measures the current equally well whichever way the current flows through it.

335. Relation of the Instrument Reading to the Instantaneous Values of the Current.—When the current through an ammeter has a definite steady value, the reading of the instrument depends upon this value, but when the current is continually changing, the effect on the moving system is some kind of average. As pointed out above, the average of an alternating current may be zero; but even for a rectified current, which flows always in a single direction however much its value may vary, an ordinary ammeter will never measure the maximum value, but it will indicate an average value. In the case of an electrodynamometer this average value is not the arithmetical average of the different instantaneous values of the current, because the effect on the instrument depends upon the *square* of the current. The reading of the electrodynamometer will therefore indicate the average, or the mean value, of the squares of the instantaneous values of the current.

The heating and power effects of an electric current are proportional to the square of the current, and therefore the mean square value measures the rate of heat production or the power developed by a current. Consequently, the average square of the current is the value usually required, and instruments giving these values find the greatest use. If the average value or the maximum value is required, either of these can be computed from the mean square value if the wave form is known. For a sine wave these relations are simple and can be derived as shown below.

336. Half-period Average Value of a Sinusoidal Current.—The usual alternating current follows closely the sine law, and its value at any instant is given by the equation

$$i = I \sin \omega t,$$

where I and ω are constants.[1] The average value of such a current throughout one period is zero. To find the average value of a sinusoidal current for the half period beginning with a zero value, it is necessary only to determine the area included between the curve and the axis of abscissas and to divide this area by the length of the base. Then

$$\text{Average ordinate} = \frac{\text{area}}{\text{base}} = \int_0^{\pi} \frac{ydx}{\pi}.$$

But for a sine curve,

$$y = a \sin x.$$

Hence,

$$\text{Average ordinate} = \frac{a}{\pi}\int_0^{\pi} \sin x dx = -\frac{a}{\pi}\Big[\cos x\Big]_0^{\pi} = \frac{2a}{\pi}$$
$$= 0.6366 \text{ of maximum ordinate.}$$

337. Effective, or Root Mean Square, Value of a Sinusoidal Current. Inasmuch as the heating and power effects of an electron current, as well as the dynamometer readings, depend upon the squares of the values of the current, it is more useful to know the average value of the square of the current than merely its average value. This can be found in the same way as before. Since the squares of negative quantities are positive, we are not limited to half a period but can extend the integration over the whole period.

$$\text{Mean square of } y = \int_0^{2\pi} \frac{y^2dx}{2\pi} = \frac{a^2}{2\pi}\int_0^{2\pi} \sin^2 x dx.$$

But $2 \sin^2 x = 1 - \cos 2x$, and

$$\int_0^{2\pi} \sin^2 x dx = \Big[\Big(\frac{x}{2} - \frac{\sin 2x}{4}\Big)\Big]_0^{2\pi} = \pi.$$

Hence the mean square of $y = a^2/2$, and the square root of the mean square value of the current is 0.707 of the maximum value.

Because this value of the current is the square root of the mean square value, it is sometimes called the "root mean square" or rms value.

[1] In case the current cannot be represented by a single term, it can always be expressed by a series of such terms..

Since this is the value that expresses the effect of the current in producing heat, light, or power, it is usually called the "effective value" of the current. These names also apply to alternating current of any wave form.

338. Definition of an Ampere of Alternating Current.—Since an alternating current has no steady value, it is meaningless to speak of an ampere of alternating current without having some definite convention or definition. Naturally, an ampere of alternating current should be able to produce as much heating as an ampere of direct current in the same resistance. Inasmuch as the power expended in an electric circuit is proportional to the square of the current, the square root of the average value of i^2 is of more practical importance than the average value of i. This root mean square or effective value of the current furnishes, then, the basis for defining the value of an alternating current.

The deflection of an electrodynamometer is proportional to i^2, and if the current is varying rapidly, the deflection measures the average value of i^2. Therefore if an electrodynamometer has been calibrated for direct current, it may be used to measure alternating current, and, by definition, *that amount of current which will give the same deflection as one ampere of direct current is called one ampere of alternating current.*

339. Alternating-current Instruments.—Alternating-current ammeters and voltmeters are calibrated to read the effective, or rms, value of the current because the power expended in a circuit depends upon the square of the current. The heat produced in a hot-wire ammeter is evidently proportional to the average square of the current; and the deflection of an electrodynamometer (Art. 202) likewise depends upon the average square of the current. Therefore, when such instruments are calibrated to measure direct currents, if they are used to measure alternating current they will indicate the square root of the average square of the instantaneous values of the current. This value is the "effective" value.

340. To Find the Emf That is Required to Maintain a Given Current in an Inductive Circuit.—Let the instantaneous values of the given alternating current be represented by the equation

$$i = I \sin \omega t, \tag{1}$$

where I denotes the maximum value of the current, and ω is a constant that makes ωt equal to 2π when the time t has become equal to that for one complete cycle of the current. It is required to find a similar expression for the emf that will maintain this current in a circuit containing both resistance and self inductance.

The general equation (see Art. 333) is

$$e = Ri + L\frac{di}{dt},$$

where Ri is the emf required to maintain the current through the ohmic resistance, and $L(di/dt)$ is the emf to balance the induced emf.

Substituting the above value of the current gives

$$e = RI \sin \omega t + L\omega I \cos \omega t. \tag{2}$$

A person who remembers a little trigonometry will recall that a sine and a cosine term can be added, giving a result that has the general form of

$$e = k \sin (\omega t + \alpha), \tag{3}$$

where k and α are constants depending on the values of R, L, and ω.

The values of k and α can be determined in terms of R, L, and ω by expanding $\sin (\omega t + \alpha)$ as follows:

$$\sin (\omega t + \alpha) = \cos \alpha \sin \omega t + \sin \alpha \cos \omega t \tag{4}$$

or, more generally,

$$k \sin (\omega t + \alpha) = k \cos \alpha \sin \omega t + k \sin \alpha \cos \omega t, \tag{5}$$

where k is any common multiplier.

The right-hand side of this equation is seen to be similar to the right-hand side of Eq. (2), and it is identical with it when k and a have such values that

$$k \cos \alpha = RI \qquad \text{and} \qquad k \sin \alpha = L\omega I, \tag{6}$$

i.e., when

$$k = I\sqrt{R^2 + L^2\omega^2}, \qquad \text{and} \qquad \alpha = \tan^{-1}\frac{L\omega}{R}. \tag{7}$$

If the right-hand members of Eqs. (2) and (5) are identical, the left-hand members must also be equal when k and α have these values. That is,

$$\begin{aligned} e &= k \sin (\omega t + \alpha) \\ &= I\sqrt{R^2 + L^2\omega^2} \sin (\omega t + \alpha). \end{aligned} \tag{8}$$

From this relation it appears that the *maximum* value of the impressed emf e is

$$E = I\sqrt{R^2 + L^2\omega^2}, \tag{9}$$

where I is the maximum value of the (sine-wave) current.

Problems

1. Draw the curve representing the current as given in Eq. (1) for at least two cycles. On the same axis draw the two components of the emf as given by Eq. (2), and by addition obtain the curve for e.
The following constants may be used: $R = 2$ ohms, $I = 1$ amp., $\omega = 400$, $L = 0.01$ henry.
2. Draw the curves showing the power expended by each of the two components of the emf. This should show sine curves of twice the frequency, one curve being wholly above the zero line. Prove that these curves are *sine* curves.

341. Graphical Representation.—The term $RI \sin \omega t$ in Eq. (2) may be represented by the curve $OA'A''$, Fig. 153, where the values of this term are plotted as ordinates against the corresponding values of ωt. The values of $\sin \omega t$ could be obtained from a table of natural sines, or the curve can be constructed graphically as shown in Fig. 153

Let the line OA be laid off, on a suitable scale, equal to RI, and consider this line in various positions around the fixed point O. Let ωt denote the angle between OA and the horizontal axis. For any chosen value of ωt, the distance of A above the horizontal axis is $RI \sin \omega t$, and this value of e can be graphically transferred to the curve, as shown by the horizontal dotted line AA', to locate the point A' corresponding to the chosen value of ωt. By taking values of ωt from O to 2π, the entire curve $OA'A''A'''$ can be plotted.

In a similar way the curve for the emf component $L\omega I \cos \omega t$ can be plotted. In this case the line OB is made equal to $L\omega I$ on the same scale that was used for OA, and the angle with the horizontal axis is $(90° + \omega t)$, since $\cos \omega t = \sin (90° + \omega t)$. For the value of ωt shown in Fig. 153, the position of B locates the point B' on the curve. In the same way the entire $B'B''B'''$ can be drawn.

342. The Resultant Curve.—By adding the ordinates $O'A'$ and $O'B'$ the point C' is located on the resultant curve, and $O'C'$ gives the value of e Eq. (2) for this value of ωt.

This resultant curve $C'C''C'''$ can also be constructed in the same way as the others. By treating OA and OB as vectors, their resultant is OC, Fig. 153, making an angle α with OA and an angle $(\omega t + \alpha)$ with the horizontal axis. For the value of ωt chosen in Fig. 153 the position of C locates C' on the curve. From the construction, C' must be the same point that was found by adding the ordinates of the other curves.

Thus the curve $C'C''C'''$ gives the values of e, Eq. (2), as time t goes on, repeating this cycle every time that ωt increases by 2π.

343. Comparison of the Two Diagrams.—In Fig. 153 there are two diagrams showing the graphical representation of Eq. (2). In (*b*) the curves show the values of e and its two components at each moment of the cycle. By measuring along the horizontal axis it is seen that $B'B''B'''$ reaches zero 90° sooner than $A'A''A'''$, and that $C'C''C'''$ is an angle α ahead of $A'A''A'''$.

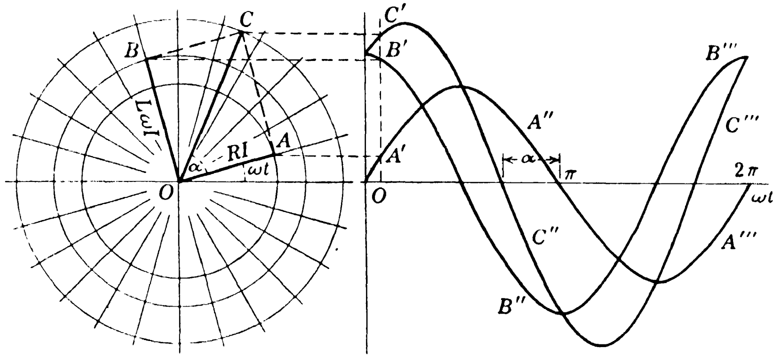


FIG. 153.—Graphical construction of sine wave curves representing alternating current and emf.

Perhaps these angles are more evident in the construction diagram (a). The maximum value of each emf is also clearly shown. Nearly everything that is shown by Eq. (2) and the curves in (b) is also shown in the construction diagram (a), and in many ways the latter is the simpler diagram.

344. The Emf Triangle.—Let us lift the rectangle $OACB$ out of the construction diagram of Fig. 153(a) and consider it by itself, as in Fig. 154. As shown above, this rectangle shows as much as the curves in (b). In fact, since $OB = AC$, the lower right-hand half of this rectangle is enough to show all that is given by the full rectangle. This is the emf triangle, Fig. 155, and its sides represent the maximum values of the emfs.

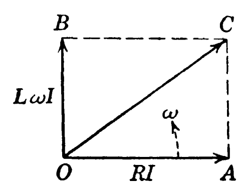


FIG. 154.—Addition of two harmonic quantities.

345. Triangle of Effective Values.—Since the effective value of a sine wave emf is 0.707 of its maximum value, a triangle similar to Fig. 154, but with each side 0.707 as long, will represent the relation between the various effective values of the emfs. The side OA represents the emf required to keep the current flowing through the resistance R. This is called the "active", or "power", component of the emf. The side OB represents the emf required to balance the induced emf. This is called the "reactive component" of the emf. The geometrical sum of these two gives the total emf that must be supplied to maintain the current, and it is called the "impressed emf." Since $OB = AC$, the same values are shown by the emf triangle, Fig. 155.

The angle of lag of the current behind the impressed emf is shown by the angle α. This may have any value from 0 to 90°, depending upon the amount of inductance in the circuit.

From the figure it is evident that the length of OC is

$$I\sqrt{R^2 + L^2\omega^2}$$

and it is ahead of OA by the angle $\alpha = \tan^{-1}\frac{L\omega}{R}$.

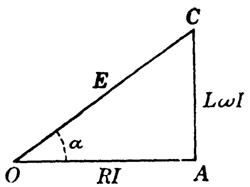


FIG. 155.—Emf triangle.

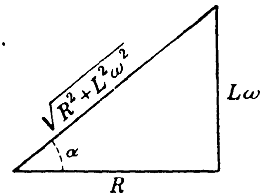


FIG. 156. Impedance triangle.

346. The Impedance Triangle.—If each side of Fig. 155 is divided by the current I, the result will be the triangle of resistance and impedance as shown in Fig. 156.

The side R denotes the resistance, but this is not necessarily the same as the d-c resistance. It is the factor which, multiplied by the square of the current, will give the amount of heat produced in the circuit.

The other side of the triangle, $L\omega$, is called the "reactance." It is also measured in ohms, but there is no loss of energy or production of heat because of it. The hypotenuse is called the "impedance."

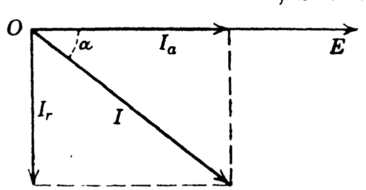


FIG. 157.—Relation between the emf and the components of the current.

347. Components of Current.—In what has been said above, the current has been the basis for reference, and the different components of emf have been considered in their relation to the current in the circuit. The same relations would have been obtained by starting from the impressed emf. The current then would be represented by two components; the active component I_a being in phase with the emf impressed on the entire circuit, and the reactive component I_r being in quadrature with this emf. The resultant of these two currents is I, Fig. 157, and is behind E by the phase angle, α, which is the same as shown in Fig. 155.

The fact that these figures are printed on the page at different angles does not affect the relation between E and I.

The reactive component I_r of the current is sometimes called the "wattless component," because on the average over the whole cycle no power is required to maintain it. The power that is expended to make the current increase during a part of the cycle is received back again when the current decreases in a later part of the cycle.

348. Effective Values.—In all these diagrams, I denotes the maximum value of the current. When it is so used, the result obtained for E will be the maximum value of the emf. If the ammeter reading is used for I, the same construction can be used and the result will be the voltmeter reading value of E. But in this case the diagram no longer has the significance that was given it in Fig. 154, and it becomes merely a graphical construction to reach a desired result.

349. Measurement of Impedance by Ammeter and Voltmeter. From what has just been said, it will be seen that the impedance of a circuit is merely the ratio of the impressed emf to the resulting current. To measure it, therefore, it is necessary only to measure the voltage and current in precisely the same way as the d-c resistance is measured by an ammeter and a voltmeter. The ratio gives the impedance Z.

$$Z = \frac{E}{I} = \text{impedance} = \sqrt{R^2 + L^2\omega^2}.$$

350. Self Inductance by the Impedance Method.—If the values of R and ω are known, this method gives a means for computing the self inductance L of the circuit. If the current passes through n cycles per sec, then $\omega = 2\pi n$.

The d-c value may be used for R if the wire is not too large and if there are no closed circuits, or masses of metal or iron in the vicinity.

In case heat is produced or energy is otherwise expended outside of R, it will be necessary to determine one other quantity before the impedance triangle can be drawn. This may take the form either of finding the angle of lag of the current behind the impressed emf, or of determining the equivalent resistance of the circuit *i.e.*, the noninductive resistance in which the same amount of power would be expended (see Art. 353).

351. Impedance and Angle of Lag by the Three-voltmeter Method.— In this method a noninductive resistance capable of carrying the current is placed in series with the impedance to be measured. Three voltmeter readings are taken, as nearly simultaneously as possible, to measure the voltage across the resistance and the impedance and across both together.

It is best to use three voltmeters, as shown in Fig. 158, but if only one is available it may be transferred quickly from one position to another. If the voltmeter shunts an appreciable current from the main circuit, two equivalent resistances should occupy the places of the missing voltmeters.

The voltmeter is readily transferred to the various positions by means of two double-throw switches S and T, as shown in Fig. 159, where Vm denotes the voltmeter, and U and W are resistances, each equal to the resistance of the voltmeter. With both switches thrown to the right, the voltmeter is placed across AC and measures the total voltage over both the impedance and the noninductive resistance. When both switches are thrown to the left, the voltmeter is across BC and measures the voltage over the impedance alone. When the switch S is thrown to the left and the switch T is to the right, the voltmeter is across AB and measures the voltage over the resistance R only. The resistances U and W are simultaneously transferred to the positions not occupied by the voltmeter.

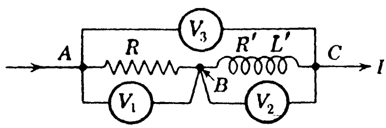


FIG. 158.—Three-voltmeter method.

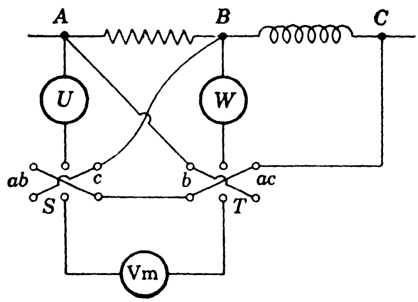


FIG. 159.—Switches for putting the voltmeter in three places.

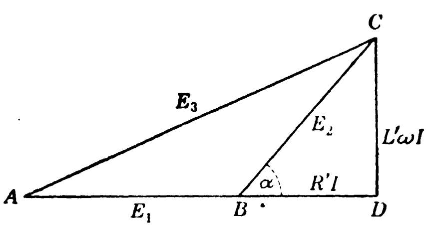


FIG. 160.—Determination of the angle of lag.

Comparing these voltmeter readings with Fig. 155 it will be seen that E_2, the reading of the voltmeter across BC, is the emf impressed upon the coil, and it would be the hypotenuse of the corresponding emf triangle if the other elements were known. In the same way, E_3 is the hypotenuse of the emf triangle for the entire circuit AC, while for the part AB, in which there is no inductance, the hypotenuse coincides with the base of the triangle and is measured by E_1.

Combining these three emfs gives the triangle ABC, Fig. 160. Extending the side AB until it meets the perpendicular from C gives the complete emf triangle for the entire circuit AC. The angle DBC gives the lag of the current behind E_2, and BDC is the emf triangle for the coil.

If the current is also known, either by direct measurement or by computation if R is known, the impedance of the coil is given by the relation

$$\text{Impedance} = \frac{E_2}{I}.$$

352. Determination of Effective Resistance.—Knowing the value of one side of the impedance triangle and the angle α, the other sides are readily constructed. The base of this triangle, R', gives the value of the *effective resistance* of the coil.

In a coil of fine wire, with no other conductors near, this resistance will be the same as that measured by a Wheatstone bridge using direct current. In case there is a thick piece of metal or other closed circuit in which the varying magnetic flux from the coil can induce an electron current, a larger value for R' will result. When the coil consists of thick wire, there will be some eddy current circulating in this metal, thus increasing the heating effect. This is equivalent to the current I flowing through an increased resistance, which appears in the increased value of R' in the impedance triangle.

353. Wattmeter Determination of Effective Resistance.—The effective resistance can also be determined by the use of a wattmeter. From the definition of resistance as the property of a circuit by which the electrical energy is transformed into heat, we have

$$W = RI^2.$$

The electrical power W can be measured with a wattmeter, and the current I with an a-c ammeter. Then

$$R = \frac{W}{I^2}.$$

The result of using a solid iron core within a coil of wire is greatly to increase this effective resistance over the ohmic resistance of the coil as measured by d-c methods. The inductance is likewise increased. If the core consists of a bundle of fine iron wire, the increase of the resistance is less, while the inductance is greater than with the solid core.

354. Inductive Circuits in Series.—When two inductive circuits are joined in series, the same current must of course flow through them both. But, in general, the emf over one will not be in phase with that over the other, and therefore the total emf required to maintain the current will be less than the sum of the two parts. This is readily seen from the figure. $A'B'C'$ represents the two inductive circuits in series, and the diagram ABC shows the emf triangles for each part and for the whole circuit. The triangle ANB is the emf triangle for the portion $A'B'$ and corresponds to Fig. 155 for the first part of the circuit.

Similarly, the emf triangle for the part $B'C'$ is shown by BMC, which is drawn in the position shown because the point B in each triangle represents the one point B' between the two parts of the circuit, and therefore it should occupy only one position on the diagram. The total emf over the entire circuit is given then by AC, while the emf triangle for the entire circuit is found by completing the triangle AKC.

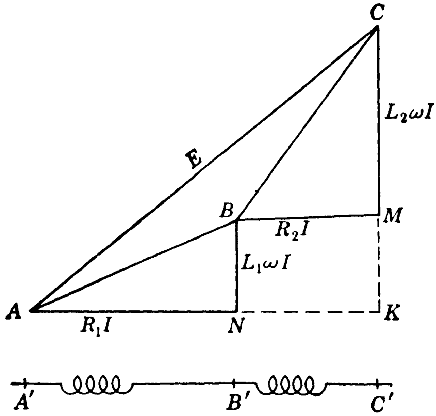


FIG. 161.—Two inductive circuits in series.

From this construction it is seen that the total resistance in the circuit is the sum of the resistances of each part; and the total inductance is the sum of the separate inductances. Of course the two parts are supposed to be far enough apart to avoid mutual induction between them. The angle of lag of the current behind the impressed emf is intermediate between the angles of lag in each part of the circuit considered separately, and is given by

$$\alpha = \tan^{-1}\frac{CK}{AK} = \tan^{-1}\frac{L'\omega}{R'}.$$

where R' and L' denote the total resistance and inductance in both circuits.

Problems

1. Given two circuits in series, with $R_1 = 18$ ohms, $L_1 = 0.01$ henry, $R_2 = 4$ ohms, $L_2 = 0.02$ henry, $\omega = 400$. Find the emf necessary to maintain 10 amp through the circuit; also the emf over each part.

Solution.—Draw to scale a figure similar to Fig. 161. Then use the same scale to measure AC, AB, and BC.

2. Given the same circuit as above. What is the value of the current when the impressed emf is 100 volts?

Solution.—Draw the line AC to represent the value of E. At A construct the angle $CAH = \tan^{-1} L'\omega/R'$, as shown by the dotted lines, Fig. 162. Extend AH to meet the perpendicular from C at K. Then $I = AK/R'$, where $R' = R_1 + R_2$.

Solution.—Second method. Assume a value I' for the current and find the corresponding value E' for the impressed emf, as above. Then E': $100::I':I$.

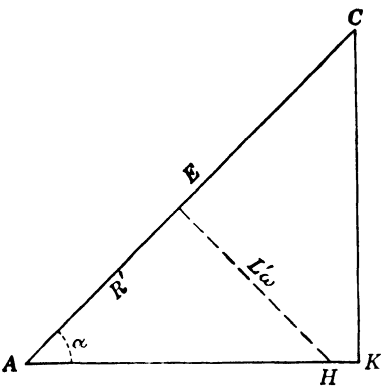


FIG. 162.—Construction of the angle of lag for an emf triangle.

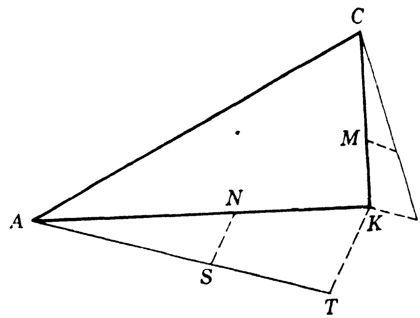


FIG. 163.—The line NS, drawn parallel to KT, divides AK in the same ratio as AT.

3. When the impressed emf is 200 volts, what is the emf over each part of the above circuit?

Solution.—Draw the triangle KAC as before. Divide the side AK in the ratio of the two resistances by laying off AS and ST to scale and then drawing SN parallel to TK. In the same way the side KC can be divided in the ratio of the two inductances. Through the points M and N draw lines parallel to CK and AK; the intersection of these lines locates the position of B, Fig. 161. The values of AB and BC can then be measured.

355. Inductive Circuits in Parallel.—With a divided circuit having two or more inductances in parallel, it is much more difficult to calculate what part of the current will pass through each branch, and graphical methods become more useful. The following example for two inductive circuits in parallel can of course be extended to as many parallel circuits as desired.

Since each branch will have the same impressed emf, the hypotenuse of each emf triangle will be identical. Let this be laid off to scale, as shown by AB, Fig. 164. Since each triangle is right-angled, it will be inscribed within a semicircle drawn on AB as a diameter. From the constants of the circuit the angle of lag in each branch can be determined as in Prob. 2 of Art. 354 above. The base, AN, of the first triangle can then be laid off, making this angle with AB. The intersection of AN with the semicircle locates the other corner of this triangle at N, and the line NB completes the other side. The value of the current is $I_1 = AN/R_1$, and is laid off in the direction AN.

Similarly, the triangle AMB is laid out for the other branch, and the value of the current is determined. The resultant current is the geometrical sum AI of these two components, and it lags behind the impressed emf by the angle HAB.

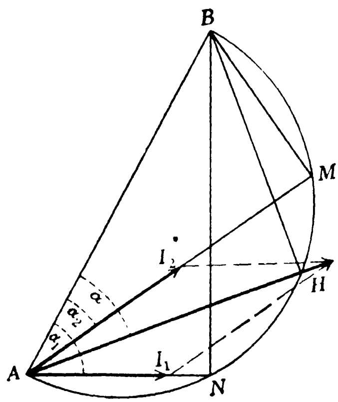


FIG. 164.—Inductive circuits in parallel.

The point H, where the line AI cuts the semicircle, is the right-angled corner of a new triangle AHB, which represents the resultant or equivalent single circuit which could replace the two parallel circuits. The emf AH is $R'I$, and HB is $L'\omega I$, where R' and L' denote the values of the resistance and the inductance, respectively, of this equivalent circuit. These values can be taken from the figure as accurately as the lines can be measured.

Problems

1. Given a coil with $R_1 = 22$ ohms and $L_1 = 0.03$ henry, in parallel with a second coil with $R_2 = 8$ ohms and $L_2 = 0.03$ henry. $E = 100$ volts, and $\omega = 400$. Find the values of the currents through each branch and in the main circuit; also the equivalent resistance and the equivalent inductance of the two coils in parallel, and the angle of lag of each current behind the impressed emf.
2. Same as Prob. 1, but with $L_2 = 0$.

356. The Emf Required to Maintain a Current in a Circuit Having Capacitance and Resistance.—In a circuit having a condenser in series with a resistance and an alternating emf, the relation at any instant is

$$e = Ri + \frac{q}{C}, \tag{10}$$

where q/C is the difference of potential across the condenser of capacitance C farads, when its charge is q coulombs.

If the current flowing through R, Fig. 165, and into the condenser is following the sine law, its value at any instant is

$$i = I \sin \omega t. \tag{11}$$

Then

$$q = \int idt = \int I \sin \omega t \, dt = -\frac{I}{\omega} \cos \omega t. \quad (12)$$

Putting this value in Eq. (10) gives

$$e = RI \sin \omega t - \frac{I}{C\omega} \cos \omega t. \quad (13)$$

Since

$$-\cos \omega t = +\sin (\omega t - 90°),$$

$$e = RI \sin \omega t + \frac{I}{C\omega} \sin (\omega t - 90°). \quad (14)$$

This shows that the last term in Eq. (14) represents an emf that is 90 electrical degrees behind the current.

Adding the two terms of Eq. (13), as in Art. 340, gives

$$e = k \sin (\omega t - \alpha), \quad (15)$$

where k and α are new constants depending on the values of C, R, and ω (see Art. 360).

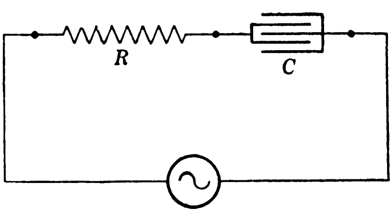


FIG. 165.—Capacitance and resistance in series.

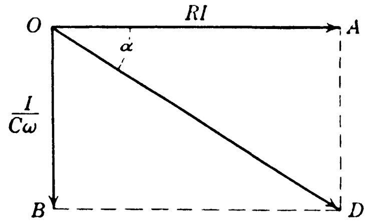


FIG. 166.—*OD* represents the resultant emf in a capacitance circuit.

357. Graphical Representation of the Emfs in a Circuit Having Resistance and Capacitance.—The two terms of Eq. (13) can be represented by two lines drawn at right angles, as was done in Art. 341. The result will be different, however, for after laying off OA, Fig. 166, equal to RI, the other side OB must be drawn *downward* in order to represent the negative term in Eq. (13). The geometrical sum of these two is

$$OD = E = I\sqrt{R^2 + \frac{1}{C^2\omega^2}},$$

and this is behind OA by the angle

$$\alpha = \tan^{-1} \frac{1}{RC\omega}.$$

The current is thus ahead of the impressed emf by the angle α.

358. The Triangle of Emfs.—The triangle OAD shows the relation of these emfs, and usually this part of Fig. 166 is all that need be drawn. This triangle of emfs corresponds to the similar relation shown in Fig. 155. OA is called the "active component" of the emf, and AD is the reactive component. The resultant of these, OD, is the impressed emf.

359. Impedance Triangle.—Dividing each side of the emf triangle by the value of the current I gives the corresponding impedance triangle.

360. The Maximum Value of the Emf.—Expanding the factor $\sin(\omega t - \alpha)$, of Eq. (15), Art. 356, gives

$$e = k \cos \alpha \sin \omega t - k \sin \alpha \cos \omega t. \qquad (16)$$

When this is compared with Eq. (13), it is seen that

$$k = I\sqrt{R^2 + \frac{1}{C^2\omega^2}} \qquad \text{and} \qquad \alpha = \tan^{-1}\frac{1}{RC\omega}. \qquad (17)$$

Hence, from Eq. (15),

$$e = I\sqrt{R^2 + \frac{1}{C^2\omega^2}} \sin\left(\omega t - \tan^{-1}\frac{1}{RC\omega}\right).$$

The maximum value of e is

$$E = I\sqrt{R^2 + \frac{1}{C^2\omega^2}}.$$

361. Condenser Current.—If the resistance of the circuit is zero, the current flowing into the condenser is

$$I = EC\omega,$$

where I and E may denote either the maximum values or the effective values of the current and emf.

362. Two Condenser Circuits in Series.—The graphical construction for two condenser circuits in series follows directly from the construction of Fig. 166. The triangle FKG, Fig. 168, represents the emfs in the first part, $F'G'$, of the circuit. The corresponding triangle for $G'H'$ will be drawn as shown by GLH. The total emf over the whole circuit is represented by the resultant FH. The two circuits thus act like a single circuit of resistance, $R = R_1 + R_2$, and capacitance C, where

$$\frac{1}{C\omega} = \frac{1}{C_1\omega} + \frac{1}{C_2\omega}.$$

Problems

1. Given the values of R_1, R_2, C_1, C_2, and ω, find by a graphical construction the emf required to maintain a current of I amp; also the emf across each part of the circuit.
2. Given the values of C, I, ω, and E, what resistance is in the circuit?

   Solution.—Lay off E as a diameter of a circle. Inscribe $I/C\omega$ and measure the value of RI.

3. Given C_1, R_1, E_1, E_2, E, and ω. Find C_2 and R_2.
4. Given C_1, R_1, C_2, R_2, E, and ω. Find the emf over each part of the circuit.
5. Same as Prob. 4, but with $R_1 = 0$.
6. Same as Prob. 4, but with C_2, Fig. 167, replaced with $R = 0$. Is this equivalent to $C_2 = 0$, or $C_2 = \infty$? Does this value of C_2 agree with the definition of unit capacitance in Art. 63?

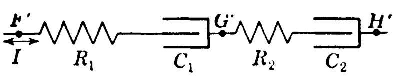


FIG. 167. Two capacitance circuits in series.

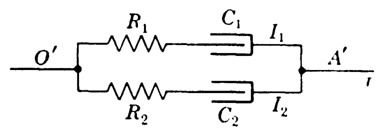


FIG. 169.—Two capacitance circuits in parallel.

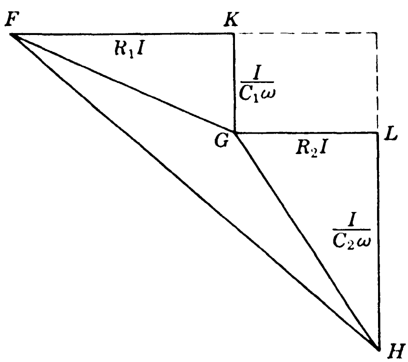


FIG. 168.—Emf diagram for two capacitance circuits in series.

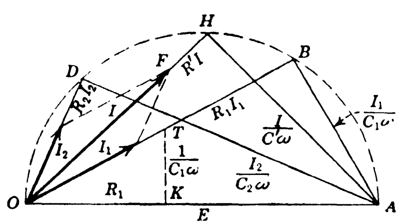


FIG. 170.—Emf and current diagram for two capacitance circuits in parallel.

363. Capacitance Circuits in Parallel.—The diagram for the graphical solution of two capacitance circuits in parallel is constructed as shown in Fig. 170. Since the two circuits are in parallel, each one will be subjected to the same impressed emf E. Let E be laid off to scale along a straight line, as OA, Fig. 170, and at O construct the angle AOB by laying off OK and KT proportional to R_1 and $1/C_1\omega$, respectively. Draw the straight line OT and extend it until it intersects at B the circle drawn on OA as a diameter. OBA is then the right-angled triangle representing the emfs in the first circuit, R_1, C_1, Fig. 169. The current I_1 in this branch will be in phase with OB, the active component of the emf, and it can be shown on this diagram as I_1, laid off to an appropriate scale along OB.

In the same way the emf triangle for the other circuit, R_2C_2, can be constructed within the same semicircle, as shown by ODA. The current in this branch of the circuit is shown by I_2.

364. Equivalent Capacitance and Resistance.—The single circuit that would be equivalent to the two parallel circuits of Fig. 169 is determined from the diagram of Fig. 170. The current through such an equivalent circuit is I, the resultant of I_1 and I_2, and this is shown by the diagonal of the parallelogram, OF. Extending OF to the circle gives OH, which is the active component of the emf in the equivalent circuit. The value of this emf is obtained by measuring OH with the same scale that was used in laying off OA. In the same way, the line HA gives the value of the reactive component of the emf.

Dividing the sides of this emf triangle OHA by the value of the resultant current I will give the corresponding impedance triangle, the sides of which are the resistance and the reactance of a single circuit that is equivalent to the two circuits in parallel.

Problems

1. Given the values of ω, C_1, R_1, C_2, and R_2 for two circuits in parallel, draw the graphical construction and find the values of the equivalent resistance and capacitance.

2. Same as Prob. 1, but with one of the circuits consisting of resistance only.

3. Same as Prob. 1, but with one of the circuits consisting of a condenser only.

Note.—Draw the emf triangle OAB with successively smaller values assigned to R_1, and observe the progressive changes as R_1 approaches zero.

4. A condenser of capacitance C is connected in parallel with a resistance R. Find the equivalent resistance and capacitance of the combination for a given value of ω.

5. In a circuit like that of Prob. 4, show how the resultant current changes in phase and amount as ω increases.

365. Circuits Having Resistance, Inductance, and Capacitance.—In a circuit having a condenser and a coil in series, there will be three parts in the total emf. The instantaneous value of the impressed emf is

$$e = Ri + L\frac{di}{dt} + \frac{q}{C}$$

$$= RI \sin \omega t + L\omega I \cos \omega t - \frac{I}{C\omega} \cos \omega t.$$

There will thus be three components to be laid off in the vector diagram, *viz.*, RI as in Fig. 155 or Fig. 166, with $L\omega I$ drawn 90° ahead of RI, as shown in Fig. 155, and $I/C\omega$ drawn 90° behind RI, as was done in Fig.

166. This is shown in Fig. 172. These three vectors can be combined by obtaining the resultant of any two and then combining this resultant with the third one. In Fig. 172 the $L\omega I$ and $I/C\omega$ are first combined, giving a resultant, $L\omega I - I/C\omega$, which is combined with RI giving the final resultant, OD. This is the maximum value of the impressed emf, and its value is given from Fig. 172 as

$$E = OD = I\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}.$$

In the case illustrated by Fig. 172, this resultant emf is ahead of the current by the angle AOD, and the circuit behaves like a slightly inductive circuit. Evidently, there is a wide range of possibilities in such a circuit, and the current may be ahead or behind the impressed emf according as $I/C\omega$ or $L\omega$ has the larger value.

366. Series Resonance in an Electrical Circuit.—In a circuit containing a condenser in series with a low-resistance coil of considerable inductance, the diagram of Fig. 172 will be modified by having OA very short. For large values of ω, the resultant, OD, will be nearly equal to $L\omega I$, while for small values of ω, this resultant emf is nearly the same as $I/C\omega$. At some intermediate value of ω we shall have

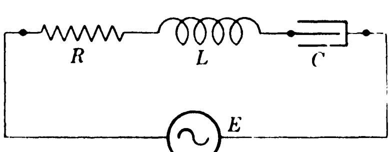


FIG. 171.—Resistance, inductance, and capacitance in series.

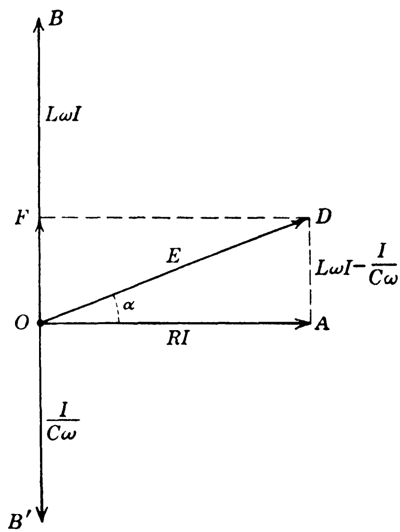


FIG. 172.—Emf diagram for a circuit containing resistance, inductance, and capacitance in series.

$$L\omega I = \frac{I}{C\omega},$$

and for this condition the resultant emf is only

$$E = RI.$$

Thus when R is small, a small emf will maintain a large current through the coil and condenser, with a correspondingly large emf across each. This does not imply the expenditure of much power in either the coil or the condenser, because the current is so far out of phase with the emf that the power factor for either part of the circuit (cos α, see Art. 477) is nearly zero. The circuit is said to be in resonance for this particular frequency n ($= \omega/2\pi$) of the impressed emf. For the entire circuit cos α is nearly unity (see Fig. 172), but E is small and the power is only RI^2.

367. Parallel Resonance.—Another arrangement that will give electrical resonance consists of a condenser in parallel with an inductance, as shown in Fig. 173. The emf diagram for this portion of the circuit is shown in Fig. 174. Let OA represent to scale the value of the impressed emf between O' and A'. This, then, is the hypotenuse of the emf triangle for the coil, the other sides of which are constructed with the aid of the broken-line circle. The current is in phase with R_1I_1 and is represented by the line OI_1.

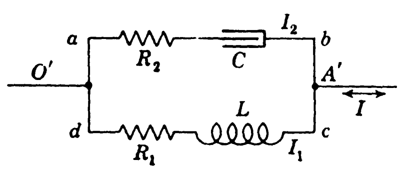


FIG. 173.—Parallel resonance. Alternating current for which $CL\omega^2 = 1$, cannot pass through this sifter when $R_1 + R_2 = 0$.

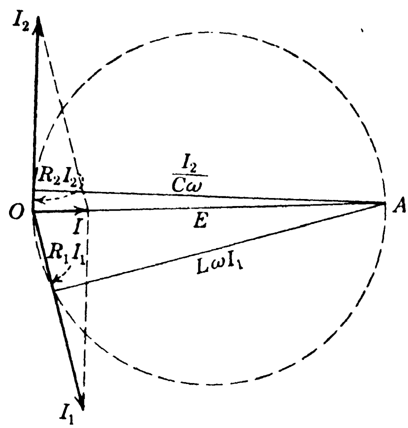


FIG. 174.—Parallel resonance with a capacitance C and an inductance L.

The same emf, E, is also impressed on the condenser, and the emf triangle for this part of the circuit is constructed in the upper half of the circle. A small resistance R_2 is indicated in series with the condenser, and the condenser current is in phase with R_2I_2. When $R_2 = 0$, this current is 90° ahead of E. The resultant current in the main line is shown by I.

When the frequency of the impressed emf is low, most of the current will flow through the coil. As the frequency increases, the condenser current increases. At the same time I_1 decreases, and this has a double effect. In the first place the resultant I comes nearer to OA; and in the second place, as R_1I_1 becomes less, it is more nearly in line with R_2I_2, which results in a very small value for I. Thus for a certain frequency the main current, I, is very small, while the current in the coil and condenser is large. This effect is called "parallel resonance." Since only a small current is in phase with the impressed emf, only a small amount of power is expended.

This combination of inductance and capacitance may be looked upon as an arrangement whereby a small main current is greatly multiplied by resonance to give a large current in the coil and condenser; or it may be considered as a *sifter* that will nearly stop current of this particular frequency while letting currents of all other frequencies pass through the main circuit.

The impedance of a circuit, tuned to parallel resonance, approaches infinity as R becomes zero. Or $I = 0$ when $R_1 + R_2 = 0$.

368. Frequency Sifters or Filters.—In making measurements of inductances and capacitances it is often essential that the a-c source shall supply a pure sine-wave current. In other words, the frequency of the current must have only a single value, that of the fundamental, with all of the harmonics absent. Even when the frequency does not enter in the measurement as an explicit factor, a definite value is better, since the inductance or the capacitance that is being measured may not have the same value for different frequencies.

By means of a wave-form filter or sifter, it is possible to pick out and greatly amplify a single frequency while suppressing the other frequencies that may be present. This is done by resonating circuits similar to *abcd*, Fig. 173, in which current of the desired frequency is selected and amplified while not amplifying current of the other frequencies. By means of an air-cored transformer, current of this frequency can be passed on to the bridge.

369. Sources of Alternating Current.—The alternating current used for power and lighting has a frequency of about 60 cycles per sec (in some localities perhaps twice this). In a-c measurements requiring a current of several amperes, the most practical source is such a power supply. For more refined methods a higher frequency is desirable, for two principal reasons. Inasmuch as the effects of inductance and capacitance are due to the terms $L\omega$ and $C\omega$, it is seen that these quantities are more prominent when ω has a large value. And if a telephone receiver is used to determine the balance of a bridge, the combination of ear and telephone is more sensitive to a frequency of about 1,000 cycles per sec ($\omega = 6{,}000$). For frequencies of 1,000,000 per sec, the effects of inductance and capacitance are greatly increased, but this frequency is inaudible and other detectors must be used.

In some instances specially designed dynamos have been built with a large number of poles to give a frequency of 1,000 or 2,000 cycles per sec.

370. Electron-tube Oscillator.—An electron tube can be made to set up oscillating currents having a frequency that is determined by the inductance and capacitance of the oscillating circuit. The frequency can thus be adjusted to almost any desired value, and such an arrangement has the advantage of being entirely silent in operation (see Fig. 124, Art. 233).

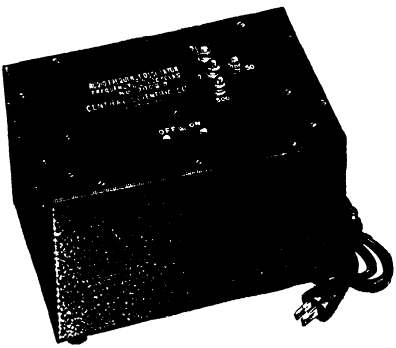


FIG. 175.—Audio-frequency oscillator. When the cord is connected to a 110-volt 60-cycle lighting circuit, a small amount of 1,000-cycle current is available at the binding posts on the top of the case. This is a sufficient source for one of the bridges described in the following chapters. (*Courtesy of the Central Scientific Co.*)

It is now possible to obtain in a compact and convenient form a complete and efficient electron-tube oscillator that may be operated by internal batteries or outside power from 60-cycle lines. Some of these are adjusted to give an output at 1,000 cycles per sec, and others can give a wide range of frequencies by the turning of a dial.

More detailed descriptions can be obtained from the makers of scientific and radio apparatus.

371. Alternating-current Detectors. *Telephone Receivers.*—The ordinary galvanometer cannot be used to indicate the balance of an a-c bridge, and most a-c galvanometers lack sufficient sensitiveness for this purpose. A telephone receiver is a very satisfactory instrument, although it does not indicate which way the bridge should be changed to approach nearer to the balanced condition. With a headset of high-grade Baldwin or Brandes receivers, a very small alternating current can be detected.

Copper Oxide Rectifier.—In situations where the use of a telephone receiver is not suitable, either because of the low or high frequency of the a-c source or the unavoidably noisy surroundings, it is often desirable to use a current galvanometer in series with a small copper oxide rectifier. This will allow only current in one direction to pass, and these unidirectional pulses of current will come often enough to give a steady deflection of the galvanometer. While this deflection will always be in the same direction, whichever way the bridge is out of balance, the bridge can be balanced as readily as with the telephone.

372. Cathode-ray Oscillograph.—A very interesting and useful indicator of small alternating currents is the cathode-ray oscillograph, shown in Fig. 176. When this instrument is connected to an a-c circuit, the wave form of the current is seen on the fluorescent screen of the tube. It can therefore be used in a bridge circuit in the place of phones, and the sine wave seen on the screen indicates the out-of-balance of the bridge.

When the bridge is balanced, the sine wave has zero amplitude and becomes a line along the axis. In the oscillograph shown in Fig. 176

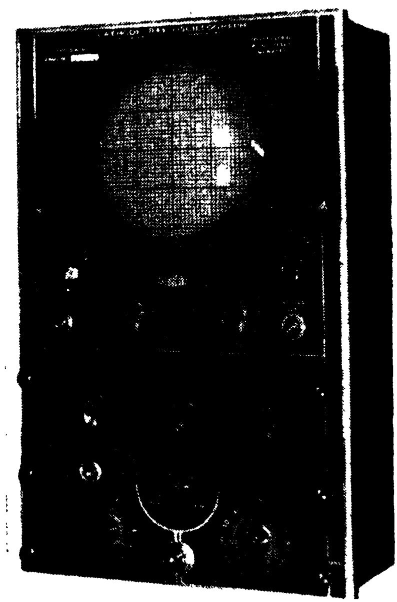


FIG. 176.—Cathode-ray Oscillograph. (*Courtesy of the Allen B. DuMont Laboratories, Inc.*)

there is a built-in amplifier that gives on the screen the wave form of a very small alternating voltage. It is therefore as sensitive an indicator of the bridge balance as telephones with two or three stages of amplification.

373. Electron-tube Amplifiers.—In Art. 229 it was shown that an electron tube can be arranged to amplify the variations in a current. By using such an arrangement between the bridge and the telephone the balance can be set much closer. It may even be desirable to use two amplifying tubes in series if the balance point gives silence in the receivers when used alone. With the two-stage amplifier used with the telephone receivers, a very definite setting of the bridge is possible. In making such a setting, the variable resistance, capacitance, or inductance should be continuously variable, and the setting should be varied back and forth from one side of the balance point to the other until the point of minimum sound is located. This cannot be done with the plug form of resistance or condenser.[1]

374. A Convenient Amplifier and Null Detector.—Sometimes telephone receivers cannot indicate the balance point of a bridge satisfactorily, either because other noise in the room makes it difficult to hear the weak sound at the balance point or because of the very small current in the telephone branch of the bridge. The latter condition calls for the amplification of this weak current.

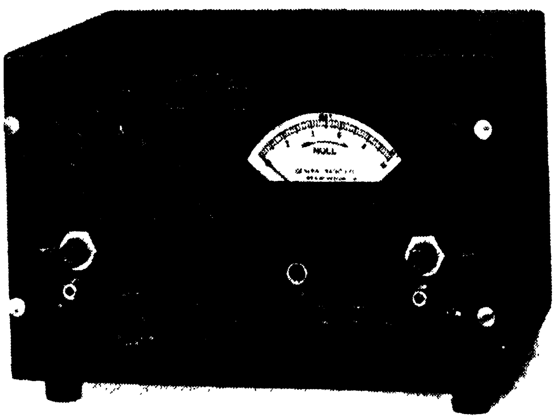


FIG. 177.—Amplifier and Null Detector. (*Courtesy of General Radio Co.*)

A useful combination of an amplifier with a visual null detector is shown in Fig. 177. This is a compact arrangement enclosed in a walnut case and operated from internal batteries. It is also possible to use power from an outside 60-cycle line. Built into the circuit is an electron-tube volmeter (Art. 230) that indicates the balance point of the bridge on the visible scale of the instrument. Phones can also be attached if desired.

[1] See further, TERRY, "Advanced Laboratory Practice in Electricity and Magnetism."

CHAPTER XIV

MEASUREMENT OF CAPACITANCE

375. Standard Condensers.—Mica condensers of a few tenths of a microfarad are often assembled in a box for convenient use. When the capacitances have been adjusted to the given values, such condensers serve as useful standards. Figure 178 shows the arrangement of the brass blocks on the hard-rubber top of a box containing four condensers. Each condenser is permanently connected to two of these

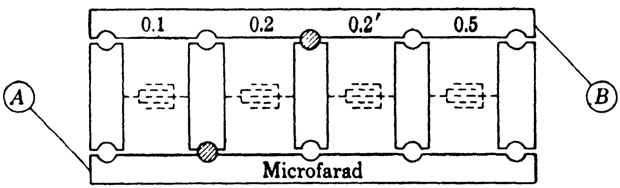


FIG. 178.—Showing how the condensers are connected to the blocks on the top of the case.

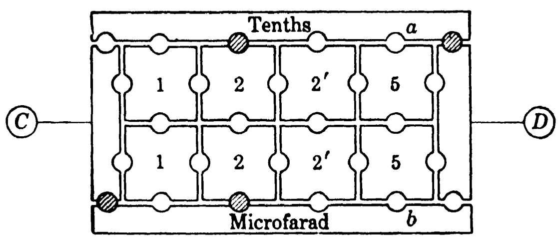


FIG. 179.—Showing the top of a standard condenser. The terminals of each section are connected to a pair of square blocks.

metal blocks. The longer blocks at either side are connection pieces to the binding posts A and B. By means of plugs, one or more of the condensers can be connected to these long blocks and thus to the binding posts. With two plugs inserted as shown in the figure, the condenser of 0.2 mf is connected to A and B. When the four condensers are all connected in parallel, the capacitance is 1 μf. When they are all used in series, the combined capacitance is $\frac{1}{22}$ μf. In this box the different condensers are permanently connected together.

The top of another condenser box is shown in Fig. 179. The terminals of each condenser are connected to an independent pair of square blocks, each of which is marked with the capacitance of the condenser. Thus, the two blocks marked "1" are the terminals of a condenser having a capacitance of 0.1 μf. The long bars are for connections. With four plugs inserted as shown in Fig. 179, the condenser having a capacitance of 0.2 μf is connected to the binding posts, C and D. By means of plugs, the four condensers can be placed in parallel or in series as desired. Thus, two additional plugs placed at a and b will join the 0.5-μf section in parallel with the 0.2 μf already connected, giving a capacitance of 0.7 μf connected to C and D.

376. Condensers in Parallel.—When two condensers are connected individually to the same emf, of course each one becomes charged to this difference of potential. The total charge is

$$Q = Q_1 + Q_2 = C_1E + C_2E = CE,$$

where Q_1 and Q_2 are the charges in each condenser. This combination acts, then, like a single condenser whose capacitance is

$$C = C_1 + C_2.$$

Hence

Law I.—The combined capacitance of several condensers in parallel is equal to the sum of the separate capacitances.

377. Condensers in Series.—If the condensers are joined in series, as shown in Fig. 181, it is evident that the difference of potential over

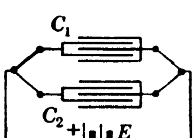


FIG. 180.—Condensers in parallel.

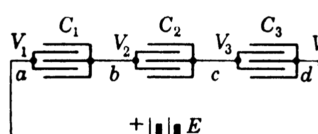


FIG. 181. Condensers in series.

each condenser is only a part of the total emf of the battery. Each of the condensers will have the same charge, for, as they are joined in series, all the electrons that leave one set of plates in C_2 must go to the adjoining set of plates in C_1, if the intermediate parts are well insulated. This is shown more fully below.

When a series of condensers are connected to a battery of emf $= E$, as shown in Fig. 181, electrons are urged along the battery circuit from a through the battery to d. Whatever may be the number or the capacitances of the condensers in series, this flow of electrons will continue until the potential difference between a and d has reached the full value of E.

The intermediate portions of the condensers consist of a series of insulated sections of conductors. In each of these sections the electrons will distribute themselves so as to make the whole section at a uniform potential. The large accumulation of electrons at d tends to make everything in that neighborhood at a negative potential, including the C_3 end of the section C_3C_2. In the attempt to keep this section at a level potential, the electrons will flow from the C_3 end to the C_2 end, thus leaving the C_3 end with a "positive" charge and making the total charge on both sets of plates in this condenser equal to zero.

In the next insulated section the electrons will likewise move so as to keep all parts at the same potential level, and this will result in the same number as before moving from C_2 to C_1. Thus there has been the same number of electrons moving along each section of the circuit, and each condenser has the same charge.[1]

The capacitance between each condenser as a whole and the surrounding conductors and walls of the room is usually insignificant. In case it is not, the charges due to such capacitances are added to those discussed above.

378. Equivalent Capacitance of Condensers in Series.—The potential differences across each condenser, Fig. 181, will be, respectively,

$$V_1 - V_2 = \frac{Q}{C_1}$$

$$V_2 - V_3 = \frac{Q}{C_2}$$

$$V_3 - V_4 = \frac{Q}{C_3}.$$

Adding these equations gives

$$V_1 - V_4 = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right) = \frac{Q}{C}.$$

Hence the equivalent capacitance C of the combination is given by the relation

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

[1] This condition is well illustrated by a water analogy, in which a long trough is divided into a number of sections by several transverse partitions. When the half-filled trough stands on the table, the water is at the same level throughout. But when one end of the trough is raised a few inches, the water must flow along each section of the trough in order to remain level. Evidently it cannot all be at the same level, and at each dam there is a difference of level. The sum of these differences is equal to the total difference in level between the two ends.

The resulting capacitance C is therefore smaller than the smallest capacitance in the series.

Law II.—When several condensers are connected in series, the joint capacitance is the reciprocal of the sum of the reciprocals of the several capacitances.

This is similar to the law for resistances in parallel.

379. Comparison of Capacitances by Direct Deflection.—When a condenser of capacitance C farads is charged to a potential difference of E volts, the quantity it contains is

$$Q = CE. \quad \text{coulombs} \tag{1}$$

If this quantity is discharged through a ballistic galvanometer, giving a fling of d mm as measured on the scale, we have

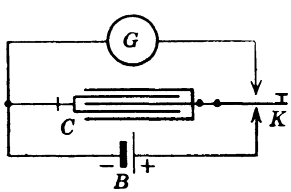


FIG. 182. — Condenser and ballistic galvanometer for the comparison of capacitances.

$$Q = kd. \tag{2}$$

Combining Eqs. (1) and (2) gives

$$C = \frac{k}{E} d. \tag{3}$$

Now if the same experiment is repeated, with the same battery and galvanometer but with another condenser of capacitance C_1, we have

$$C_1 = \frac{k}{E} d_1, \tag{4}$$

and from Eq. (3)

$$C_1 = C \frac{d_1}{d}. \tag{5}$$

If C is a known capacitance, then the value of C_1 can be determined as exactly as the flings d and d_1 can be measured. Each of these should be taken several times, with the mean values used in the computation.

380. Study of Residual Discharges.—When a condenser or a cable is charged for a long time and then discharged, it is nearly always found that the quantity of electricity obtained from the condenser on discharge is less than the total amount of the original charge. The remainder of the original charge is said to be "absorbed," meaning that this charge remains in the condenser after the plates have been brought to the same potential, but not specifying the manner in which it is retained. After a short interval of time, a portion of this absorbed charge is released and can be discharged by again joining the two plates of the condenser; this portion is termed a "residual discharge." Several such residual discharges can be obtained from ordinary condensers and a great many from a poor condenser.

This phenomenon depends upon the material composing the dielectric between the plates of the condenser, and is the more marked the greater the heterogeneity of the dielectric. In case the dielectric is air, or a sheet cut from a crystal of quartz or Iceland spar (*i.e.*, a homogeneous substance), there is no residual discharge. The charge which is thus absorbed and can be recovered later should not be confused with any leakage there may be through the condenser.

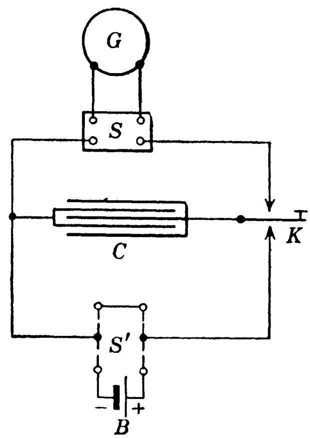


FIG. 183.—To measure residual discharges from the condenser C. If there is leakage across the switch S, the battery B should be disconnected after using.

To make a short study of residual discharges proceed as follows: Charge a paraffined-paper condenser from six or eight cells for 3 min. When ready to begin observations on residual discharges, disconnect the battery from the condenser and thoroughly discharge the latter by throwing the switch S' to the other side for 5 sec and then leaving it open. Wait 1 min and again discharge the condenser—this time through the galvanometer—and observe the deflection. The key K is supposed to remain closed on the lower contact shown in Fig. 183. Probably this would be the upper contact on the actual key. Pressing the key to the other contact joins the condenser to the galvanometer. Discharge the condenser through the galvanometer at 1-min intervals for 30 min. Express the results in the form of a curve, using time for abscissas and the sum of all previous discharges for ordinates. In some instances the sum of the residual discharges is greater than the original discharge. Repeat, charging the condenser for 15 min in order to observe the effect of the time of charging upon the amount of the residual charge. If there is any leakage across the switch S' the battery should be disconnected after use.

381. Charging a Condenser through a Resistance.—When a condenser is connected in series with a resistance and a battery, a current flows until the potential difference across the condenser equals the emf of the battery. The initial value of this current, as well as the time required to charge the condenser, depends upon the amount of resistance in the circuit.

Figure 184 shows such a circuit, where R represents the total resistance, including that of the battery. Writing down the potential differences in this circuit gives

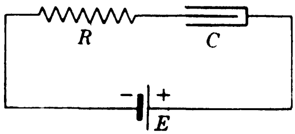


FIG. 184.—Charging a condenser through a resistance.

$$E - Ri - \frac{q}{C} = 0$$

or, expressing the current in terms of the charge by the relation $i = dq/dt$,

$$E - R\frac{dq}{dt} - \frac{q}{C} = 0.$$

In order to separate the variables, q and t, this can be cleared of fractions by multiplying by Cdt and then written in the form

$$\frac{-dq}{EC - q} = \frac{-dt}{RC}.$$

Integrating this gives

$$\log (EC - q) = -\frac{t}{RC} + B,$$

where B denotes the constant of integration. Rewriting this in the exponential form, we have

$$EC - q = \epsilon^B \epsilon^{-\frac{t}{RC}}.$$

If t is measured from the instant that the circuit is closed, when $q = 0$, then when $t = 0$, this equation becomes

$$EC = \epsilon^B,$$

and since this is a constant, it always has this value. We now have

$$q = Q - Q\epsilon^{-\frac{t}{RC}},$$

where Q is written for EC, this being the maximum value of q, and the final charge in the condenser.

Figure 185 shows the increase in the charge of a 1-μf condenser when it is charged through 1,250 ohms. A larger emf will give a larger final charge, but the same time will be required to bring the charge to half of its final value.

382. Time Constant of a Condenser Circuit.—As shown in Fig. 185, a condenser charges more and more slowly as time goes on. By definition, the rate of change in the charge measures the current flowing into the condenser. This current is large when the circuit is first closed and becomes less and less as the condenser becomes charged. At any given instant during the charging, Fig. 184, the current is

$$i = \frac{dq}{dt} = \frac{Q}{RC}\epsilon^{-\frac{t}{RC}} = \frac{E}{R}\epsilon^{-\frac{t}{RC}}.$$

At the start, when $t = 0$, the initial value of the current is

$$i_0 = \frac{E}{R}.$$

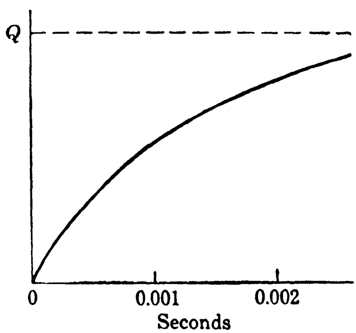


FIG. 185.—Increase of the charge in a 1-μf condenser when charged through 1,250 ohms.

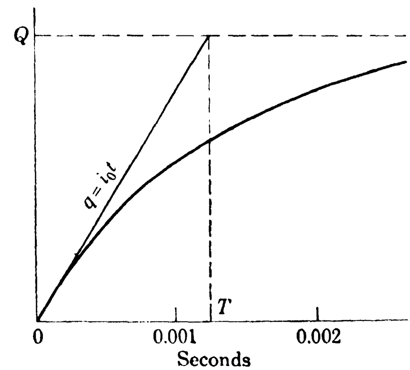


FIG. 186.—The time period OT is the *time constant* for a series circuit of R = 1250 ohms and C = 1 μf.

If the current remained at this value during the charging of the condenser, the condenser would be fully charged in T sec, where

$$Q = i_0 T,$$

or

$$T = \frac{Q}{i_0} = \frac{Q}{E/R} = RC.$$

Actually, when $t = T$ the coefficient of ϵ is -1, and the condenser lacks full charge by ϵ^{-1} of its final value.

This value of $t = T = RC$ is called the "time constant" of the circuit. It is a characteristic constant of the circuit and measures the rate of charging or discharging for any voltage applied to the circuit.

383. Bridge Method for Comparing Two Capacitances.—This is a null method and therefore capable of more exact measurements than the direct-deflection method. The two condensers are placed in two arms of a Wheatstone bridge setup, as shown in Fig. 187.

When the key is depressed, both condensers become charged to the full potential difference of E, and the points A, B, and F all come to the same potential. If no charge passes through the galvanometer, then C_1 is charged through R_1 and C_2 through R_2. During the very short interval that is required to charge the condensers there will be transient currents through R_1 and R_2, and perhaps in the galvanometer also.

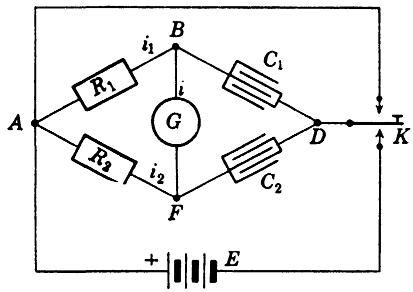


FIG. 187.—Capacitance bridge for comparing C_1 and C_2.

When the key is closed on the upper contact point, the condensers are discharged.

By working the charge-and-discharge key quickly, one may check the deflection of the galvanometer even when R_1 and R_2 are far from a balance. The galvanometer should be thus protected.

Writing the equation for the potential differences in the circuit $BAFB$ at any instant while the condensers are being charged gives

$$R_1 i_1 + L_1 \frac{di_1}{dt} - R_2 i_2 - L_2 \frac{di_2}{dt} + Gi + L \frac{di}{dt} = 0, \qquad (6)$$

where $L_1(di_1/dt)$, etc., are the emfs due to inductances in the corresponding branches. The left-hand side of E is taken as positive. If i_1 denotes the electric current flowing from A to B, the point A is at the higher potential. If the electron current flowing from B to A is denoted by i_1, A is still at the higher potential, and Eq. (6) is written the same in either case.

384. Integration of Equation (6).—The quantities appearing in Eq. (6) are not directly measurable. The ballistic galvanometer G measures the total quantity of electrons passing through it, and this quantity is expressed as

$$Q = \int dq = \int i\,dt,$$

where dq is the quantity transferred by the current i in the interval dt. This suggests that the ballistic galvanometer can give some information regarding the *integral* of the quantities appearing in Eq. (6).

The integration of this equation is indicated in the following manner, where dq_1 is written for i_1dt, etc.

$$\int R_1dq_1 + \int L_1di_1 - \int R_2dq_2 - \int L_2di_2 + \int Gdq + \int Ldi = H.$$

Performing the integration thus indicated gives the general equation

$$R_1q_1 + L_1i_1 - R_2q_2 - L_2i_2 + Gq + Li = H.$$

At the beginning, before any of the currents have had time to start, i, i_1, and i_2 are each equal to zero. The quantities q, q_1, and q_2 likewise are each equal to zero at the start. Therefore H, the constant of integration, has the value zero.

At the end, after things have reached a steady condition, i, i_1, and i_2 are zero again. Therefore, when this steady state has been reached, the general equation above reduces to

$$R_1Q_1 - R_2Q_2 + GQ = 0,$$

where Q_1 is the value of q_1 at this time and denotes the total quantity that has passed through R_1 after the closing of the key. As no currents are flowing, this value does not change further, however much longer the key may be held closed. In the same way, Q and Q_2 are the final values of q and q_2.

385. Conditions for a Balance.—It is to be noticed that zero deflection of the galvanometer does not mean no current through it, as there may be currents through it first in one direction and then in the other before the condensers are fully charged. But zero deflection means that as many electrons have passed through the galvanometer in one direction as in the other and that, taking account of signs, the total quantity through the galvanometer has been zero.[1] Hence when R_1 and R_2 are adjusted so that opening or closing K produces no deflection of the galvanometer, $Q = 0$.
Then

$$R_1Q_1 = R_2Q_2.$$

Also, since no charge passed through the galvanometer,

$$Q_1 = C_1E, \qquad \text{and} \qquad Q_2 = C_2E.$$

[1] Strictly true only if the magnetization of the galvanometer is not affected by the current and if the coil has not turned enough to be in a magnetic field of different flux density.

Substituting these values gives

$$C_1 = C_2 \frac{R_2}{R_1}.$$

The resistances R_1 and R_2 should be large, 5,000 ohms or more, so that the fall of potential produced by the small charging currents may be appreciable. While inductance in these arms does not affect the result, as seen by the preceding integration, a large amount of inductance may render it more difficult to determine whether the galvanometer deflection is zero or not.

Make several determinations of each unknown capacitance by using various values for R_1 and finding the corresponding values of R_2. Try R_1 = 10 ohms, 100 ohms, 1,000 ohms, 10,000 ohms. How much effect has a 1 per cent change in R_1? Check results by measuring the capacitance of the condensers when joined in series and in parallel.

The observations can be recorded in a form like the following:

To Measure the Capacitance of. . . . .

R_1	R_2	Uncertainty in finding the value of R_2	C_2	C_1	Uncertainty in C_1 due to the uncertainty in R_2

386. Potential Changes in the Bridge.—To understand what is taking place in the bridge circuits during the short interval in which currents are flowing, let us consider the changes in potential occurring at the points A, B, F, and D, Fig. 187. To be definite, let us suppose that the key K is on its upper contact, connecting D and A, and that the negative (right-hand) side of E is grounded, thus maintaining it at the potential of the earth.

Before K is depressed, the entire bridge is at a uniform potential equal to that of the positive terminal of the battery, E. When K is depressed, D drops at once to the potential of the earth (and that of the negative side of E), to which it is thus connected. At the first instant there are no charges in the condensers, and therefore no potential differences across them. Therefore the potentials of B and F will also drop with that of D, and the full emf of the battery is impressed upon R_1 and R_2. This determines the initial values of the currents through these resistances. As the condensers become charged, the currents fall to zero, as shown in Fig. 188. Since these currents flow into the condensers, the charges increase as shown in Fig. 189.

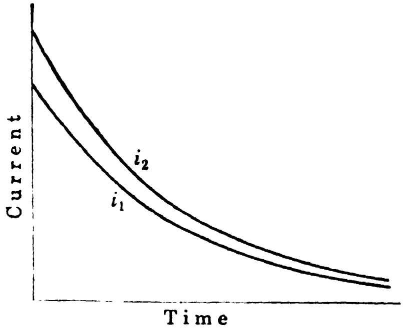


FIG. 188.—The currents through R_1 and R_2 decrease to zero in a short time.

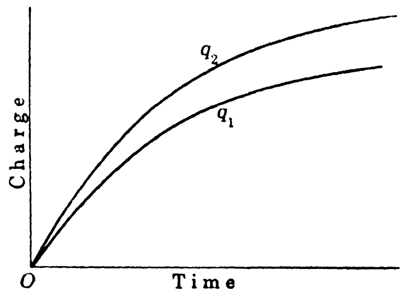


FIG. 189.—The charges in the condensers increase and finally reach constant values.

The potentials of the points B and F rise as the condensers become charged, because the potential of D remains constant and the difference of potential across each condenser is proportional to its charge (see Fig. 190). Since the galvanometer is connected across BF, zero deflection indicates that B and F are at the same potential (on the average), and thus for a balance the two curves must coincide (if they are exponential).

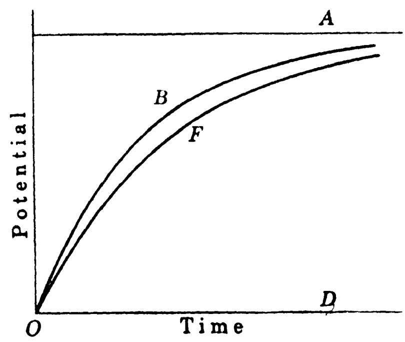


FIG 190.—These curves show the change in potential of the points A, B, F, D, in the bridge of Fig. 187 with D grounded, after the key is depressed.

Had any other point been selected as the grounded one, the curve corresponding to the grounded point would have been a straight line at zero potential, but the relative difference between the various curves would be the same as before.

387. The Bridge Method with Alternating Current.—The bridge method for comparing capacitances can be used with alternating or variable current, and with a telephone or other a-c detector in the place of the ballistic galvanometer that is shown in Fig. 187. The arrangement is then like Fig. 191, where the arms R_1 and R_2 are noninductive resistances. Silence in the telephone indicates that the *current* through it is zero. When a telephone is used for the current detector, the variable resistances R_1 and R_2 should be dial resistance boxes or some form in which the resistance can be varied continuously through the balance point. A sound in the telephone does not indicate whether R_2 is too large or too small for a balance, but R_2 can be varied in the direction that gives less sound until a minimum sound is obtained. It is often difficult to tell whether a faint sound is the minimum that can be obtained or not. The balance point can be located by varying R_2 back and forth from a value that is slightly too small to a value on the other side of the balance point that is just large enough to give the same loudness of sound as the smaller value, decreasing this range in R_2 until the sounds are just audible.

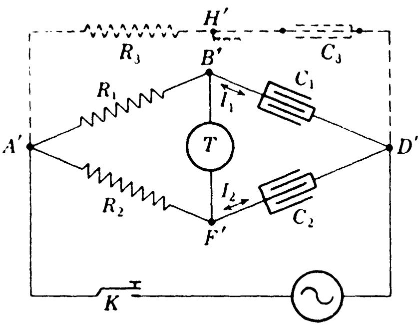


FIG. 191.—The capacitance bridge.

Since the balance is for *zero current* through the telephone branch of the bridge, the conditions for a balance are slightly different from the case with a ballistic galvanometer for which a balance indicates zero *quantity* through it. Since the arms R_1 and R_2 are noninductive,

$$R_1 i_1 = R_2 i_2 \tag{7}$$

for the case of a balance.

Likewise for the condenser arms of the bridge,

$$\frac{q_1}{C_1} = \frac{q_2}{C_2} \tag{8}$$

at each instant when there is no current through the telephone, if we let q_1, q_2 denote the instantaneous values of the charges in the condensers. Since $i_1 = (dq_1/dt)$ this can be written (by differentiation)

$$\frac{i_1}{C_1} = \frac{i_2}{C_2}. \tag{9}$$

Dividing Eq. (7) by Eq. (9) gives

$$R_1 C_1 = R_2 C_2 \tag{10}$$

as the condition for a balance. This gives for the value of the unknown condenser,

$$C_1 = C_2 \frac{R_2}{R_1}. \tag{11}$$

In the discussion above it has been supposed that the condenser arms of the bridge contain capacitance only. If either condenser shows residual charge (Art. 380) or if there is any resistance in either arm, it probably will be impossible to obtain complete silence in the telephone. The nearest balance that can be obtained may give only a minimum of sound. For a better balance it is necessary to use the modifications of the Wien bridge (Art. 392).

388. Grounding the Bridge.—When a current flows through a bridge the various parts of the apparatus are brought to different potentials, as shown in Art. 386. If these parts have any electrostatic capacitance between themselves and the surroundings, it will require the addition of corresponding charges thus to change the potentials. If any of these charges flow in through the telephone receivers, perhaps to charge the receivers themselves, there will not be silence at the correct balance point. Since the observer is usually connected to the earth, the parts of the bridge connected to the telephones should be at zero potential also. If F' were grounded, the potential of the telephone would remain at zero, due to the addition of sufficient charges at this point, but such additional charges are not desirable in the bridge.

The same effect can be obtained by joining another circuit, $A'H'D'$, in parallel with the bridge, as shown by the dotted part of Fig. 191. This circuit is made similar to the bridge by setting $R_3 = R_1C_1/C_3$ with the aid of the telephone receiver connected temporarily between B' and H' after the bridge is approximately balanced. Referring to Fig. 192 (for the balanced condition), note that there are three superimposed triangles, ABD, AFD, and AHD, with the points B, F, and H, all at the same potential. If any one of these points is grounded, the other two points will be at zero potential also. Thus by grounding the point H', Fig. 191, the points B' and F' are kept at zero potential without the addition of extra charges at these points.

Other bridges can be grounded in this manner[1] by a circuit of resistance, inductance, and capacitance similar to the bridge itself. In case it is desired to keep some other point of the bridge at zero potential, the corresponding point on the auxiliary circuit is grounded.

[1] This is a Wagner ground (see Art. 441).

389. Vector Diagram for the Capacitance Bridge.—The relationships among the resistances and the capacitances of the bridge, when used with sinusoidal alternating current, are clearly shown by a careful study of the corresponding vector diagram. The arrangement shown in Fig. 191, with a noninductive resistance in series with each condenser, constitutes two similar capacitance circuits in parallel with each other. The method for drawing the vector diagrams for these circuits when a sinusoidal alternating emf is applied was given in Art. 363 and Fig. 170.

The emf triangles for these circuits are shown in Fig. 192, where E represents the emf applied to the bridge. It is supposed that there are no resistances in the condenser arms of the bridge. In this case the points B and F represent the points B' and F' where the telephone is connected. Since B and F are some distance apart in Fig. 192, it means that a corresponding voltage is applied to the telephone circuit. But by increasing R_2, the point F will be moved along the circle towards B, and for the proper resistances, F and B can be made to coincide. There will then be no current through the telephone, and the bridge is said to be balanced.

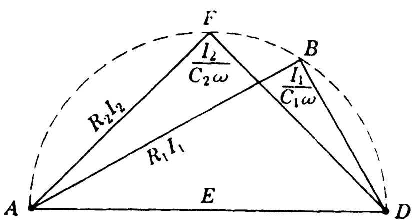


FIG. 192.—Emf diagram for the capacitance bridge. By decreasing R_1, B is brought nearer to F and for a balance the two triangles coincide.

In this balanced condition the triangles ABD and AFD have been made similar, so that

$$R_1I_1 : R_2I_2 = \frac{I_1}{C_1\omega} : \frac{I_2}{C_2\omega}$$

or

$$C_1 = C_2\frac{R_2}{R_1}.$$

From the figure it appears that a 1 per cent change in R_2 will change the position of F by a greater amount when AF and FD are equal than when they are widely different. Therefore, for greatest sensitiveness, R_2 should be about equal to $1/C_2\omega$, or about 150 ohms for $C = 1\mu$f and with 1,000 cycles per sec current.

390. The Distinction between a Condenser and a Capacitance.—In the discussion of the effect of capacitance in an electric circuit, it was shown (Art. 356) that the extra component of emf to maintain a current I, because of the capacitance C in the circuit, is $E' = I/\omega C$.

It was also shown that the phase relation between E' and I is 90°—not almost 90° but exactly that, as shown in Fig. 193.

But it is not possible to build an actual condenser in which this phase relation is fully 90°. In the best condensers it is almost 90°, and in poorer condensers it may lack this by several degrees, as shown in Fig. 194.

In order to compare Fig. 194 with Fig. 193, the triangle OFK is built out into a right-angled triangle by the dashed lines FH and HK, which may be considered as representing two components of the actual emf E_c across the condensers. The component HK is $I/\omega C$ because of the capacitance C of the condenser part of the circuit. The other component is not described so easily. It is the amount by

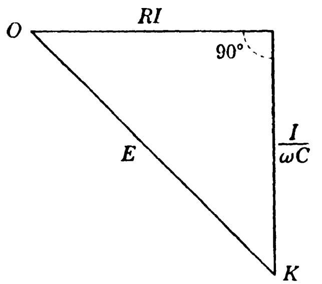


Fig. 193.—Phase relation between current (which is in phase with RI) and the component of emf due to capacitance.

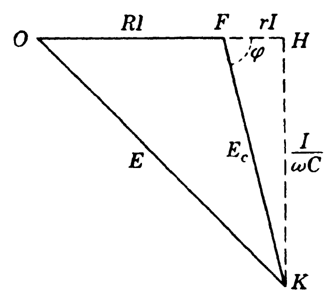


Fig. 194.—Phase relation between current (which is in phase with RI) and the emf E_c across a condenser in series with R.

which OF must be built up to equal OH. Therefore it is an emf of the same nature as RI and in phase with I. It can be represented by the expression rI, where r denotes some sort of fictitious resistance in series with R and the condenser. However, r is not a constant for a given condenser, as it varies inversely with the frequency of the current. If it is called a "resistance," there is no place in a condenser where FH can be measured without HK. However, the value of r for a given condition can be determined by indirect methods, as shown below.

391. Power Factor of a Condenser.—Since the power expended by a current is given by the expression

$$\text{Power} = EI \cos \varphi,$$

there is no power expended in a circuit because of its capacitance. As seen in Fig. 193, $\cos \varphi = \cos 90° = 0$.

In a condenser the power expended is

$$P = E_c I \cos \varphi,$$

and, as seen in Fig. 194, $\cos \phi$ is not zero. For angles nearly equal to 90° we can write

$$\cos \varphi = \cot \varphi = \frac{FH}{HK} = r\omega C.$$

This shows the importance of r. By determining its value, and knowing ωC, we can determine the value of the factor $\cos \varphi$.

The factor, $\cos \varphi$, is called the "power factor" of a condenser. As it is fairly constant for a given condenser, its value may be marked on the condenser as one of its characteristic constants.

392. Wien Capacitance Bridge. *Series Resistance of a Condenser.*—In the diagram of Fig. 192 it has been assumed that there is no resistance in the condenser arms of the bridge. Every condenser, however, in which there is any loss of energy when it is charged and discharged, behaves as though there were a small amount of resistance r in series with the condenser. The heat that is produced in the condenser is then represented by rI^2, where I is the effective value of the current through the condenser. The bridge of Fig. 191 should then be represented as shown in Fig. 195, and the corresponding vector diagrams are shown in Fig. 196. The small resistance r_1 acts as additional resistance in series with R_1, but it is not possible to connect the telephone across the capacitance alone. The diagrams for the condensers becomes triangles, *DMF* and *DNB*, with the telephone across *FB*.

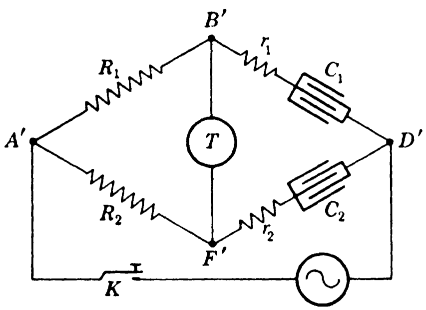


FIG. 195.—Wien capacitance bridge using series resistances in the condenser arms of the bridge.

It is now not sufficient to make *AMD* coincide with *AND*, but in addition r_1 or r_2 must be adjusted to bring F and B together. When this has been done, there are two sets of similar triangles in Fig. 196(*b*), from which

$$R_1 I_1 : R_2 I_2 = r_1 I_1 : r_2 I_2 = \frac{I_1}{C_1\omega} : \frac{I_2}{C_2\omega},$$

giving

$$C_1 = C_2 \frac{R_2}{R_1}$$

as before, but r_2 must be set equal to $r_1(C_1/C_2)$ in order to find the value of R_2 that gives silence in the telephone.

Inasmuch as the equivalent series resistance of a condenser cannot be varied, it is necessary to add a small amount of external resistance in series with C_1 or C_2 in order to obtain the value required for a balance. Usually this will be only a few ohms or a few tenths of an ohm. If it is not known which arm requires the additional resistance, it will be convenient to have a low-resistance box in series with each condenser, so that either one can be used.

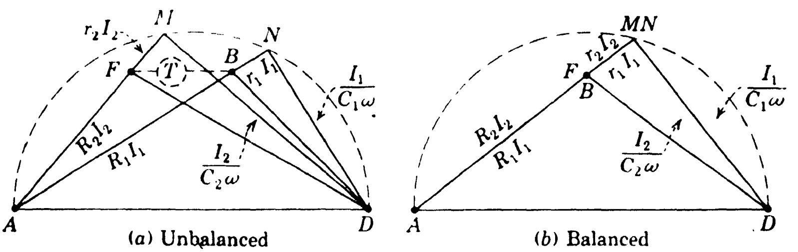


FIG. 196.—Vector diagrams for the Wien capacitance bridge having series resistances in the condenser arms.

If the sound in the telephone is too faint to guide in the final setting of the bridge, an electron-tube amplifier (Arts. 229 and 373) can be used to magnify the sound.

393. Modifications.—In the article above it is shown that a better balance of the capacitance bridge can be obtained by adding a small variable resistance in series with the standard condenser to match the power loss in the unknown condenser. This is satisfactory when only the capacitance of the unknown condenser is required. But when the value of the equivalent resistance or the power factor of the unknown condenser is required, it is necessary to know the value of this small added resistance. This is not easy, as the total value of this resistance is of the order of a small fraction of an ohm. Small known variable resistances are not usually available, and the unknown resistance of the contacts and connections of such a resistor may be an appreciable amount of the whole and lead to a corresponding uncertainty in the computed value of the equivalent resistance of the unknown condenser.

In this case better results can be obtained by one of the following methods.

394. Power Factor of a Condenser by a Resistance-shunted Bridge Method.—When power is absorbed in one of the condensers that are being compared by the bridge method, Fig. 187, the condenser current, instead of being in exact quadrature with the applied voltage, becomes slightly more nearly in phase with it. This means that the currents in the two resistance arms of the bridge, R_1 and R_2, Fig. 187, are not quite in phase with each other. This makes it impossible to obtain an exact balance by merely changing the ratio of R_1 and R_2.

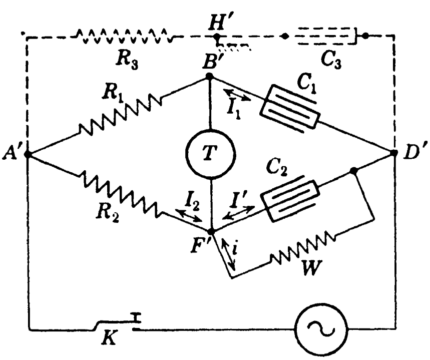


Fig. 197.—The resistance-shunted capacitance bridge.

The phase of the current in the other branch of the bridge can be changed by adding a high resistance W, Fig. 197, in parallel with the standard condenser. By varying the resistance in W, the power expended in the resistance-condenser combination can be made to correspond with that expended in the unknown condenser. This

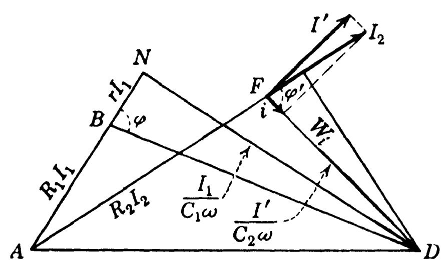


Fig. 198.—Diagram for the unbalanced condition of the resistance-shunted bridge.

changes the phase of I_2 to equal that of I_1, and the bridge can be balanced.

The vector diagram corresponding to the bridge of Fig. 197 is shown in Fig. 198. The diagram for the branch R_1 with the unknown condenser C_1 is drawn as in Fig. 196. In the other branch the current I_2 is the resultant of the current I' in the condenser C_2 and the current i in the resistance W. These two currents are at right angles to each other, as shown in Fig. 198. Varying the resistance W will change the value of i and therefore the angle φ'. Changing R_2 will change the relative lengths of AF and FD, Fig. 198. By varying first one and then the other, the point F can be brought to coincide with B, and the bridge is balanced. The angle φ now equals the angle φ'.

From the figure, the power factor of the unknown condenser C_1 is

$$\cos \varphi = \cos \varphi' = \frac{i}{I_2} = \frac{i}{I'} = \frac{E/W}{EC_2\omega} = \frac{1}{WC_2\omega},$$

since I_2 is very nearly the same as I' for this large value of W.

With this small value of $\cos \varphi$ the line ND is practically as long as BD, and so we also have

$$C_1 = C_2 \frac{R_2}{R_1},$$

as in the previous case.

395. Power Factor of a Condenser by a Capacitance-shunted Bridge Method.—If there is available a variable condenser of small

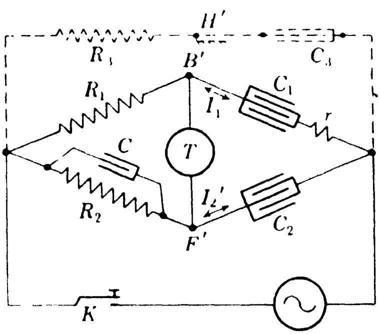


FIG. 199.—The capacitance-shunted bridge.

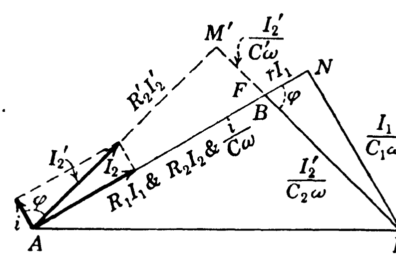


FIG. 200.—The diagram for the balanced condition of the capacitance-shunted bridge.

capacitance, *e.g.*, from zero to 0.001 μf, it is very convenient to use it as a shunt on one of the resistance arms of the capacitance bridge, as shown in Fig. 199. This will add another component to the current in C_2, and by adjusting this component, i, the emf across C_2 can be brought into phase with that across the condenser (r, C_1).

The vector diagram for the balanced condition is shown in Fig. 200. The part $ABND$ is the same as in Fig. 196, with r representing the equivalent series resistance within the condenser C_1.

When the standard condenser C_2 has no power loss, and $C = 0$, the diagram AFD for the lower branch of the bridge coincides with AND, and the bridge is not balanced. By increasing C, Fig, 199, a component i is added to I_2 to make up the total current I_2' in the standard condenser C_2. This will throw I_2' out of phase with I_1 and will bring F, Fig. 200, nearer to B. When F and B coincide, as shown in the diagram, the bridge is balanced.

Observe that at A, Fig. 200, i and I_2 combine into I_2', thus making the branch $A'F'$, Fig. 199, behave like a series circuit of R_2' and C', as shown by the dashed lines of the diagram.

The power factor of condenser C_1 is

$$\cos DBN = \cos \varphi = \cot \varphi = \frac{i}{I_2} = \frac{EC\omega}{E/R_2} = R_2C\omega,$$

and this can be determined as exactly as C is known.

396. Shielded Circuits.—In every bridge arrangement there is a small amount of capacitance between its conductors and the surroundings—the walls of the room, the hands of the operator, etc. A varying potential applied to these conductors means a varying charge and therefore extra currents through parts of the bridge. These extra currents cause a poor and false balance of the bridge. Their effect is somewhat reduced by the use of a Wagner ground, which keeps the potential of the telephone or other detector at the potential of the surrounding walls and thus reduces the charges on this part of the bridge.

To reduce still further the effect of this variable capacitance, each arm of the bridge and each connecting conductor should be individually enclosed within a well-grounded metal shield. This does not eliminate the capacitance to the surroundings (shields), but it makes it constant, and its effect is counted as a part of the circuit when that is measured and calibrated.

397. Substitution.—In measurements by the bridge method, the ratio between the known and unknown values is determined. When these values are nearly equal, it is often better to make one balance with the unknown condenser in the bridge and then a second balance with a known condenser standing in the place of the unknown, with the other arms of the bridge undisturbed.

For the first balance,

$$C_1 = C_2 \frac{R_2}{R_1}. \tag{12}$$

For the second balance,

$$C_s = C_2 \frac{R_2}{R_1 + R}, \tag{13}$$

where R is the resistance that must be added to R_1 for the second balance.

Dividing (12) by (13) gives

$$C_1 = C_s\left(1 + \frac{R}{R_1}\right), \tag{14}$$

where all uncertainty in the actual values of C_2 and R_2 disappears and the unchanged part of R_1 enters only in the small fraction R/R_1.

398. Comparison of Capacitances by the Series Capacitance Bridge. *Gott's Method.*—This is another bridge method and differs in arrangement from the preceding only by having the galvanometer and battery interchanged. In the bridge method (Art. 383) the balance is obtained for the conditions which exist *during* the charging or the discharging of the condensers. In the present method the capacitances of the condensers are compared after everything has reached the steady and permanent condition. Thus if one (or both) of the condensers has resistance in series with itself, this will not affect the value of the final charge in the condenser, and the capacitance is readily measured.

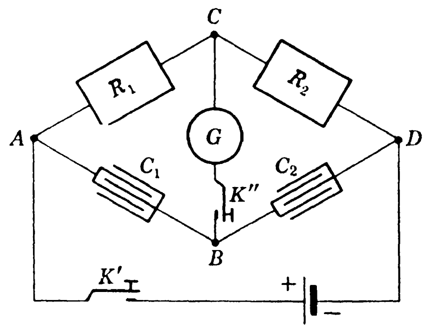


Fig. 201.—Gott's method.

The arrangement is shown in Fig. 201. When the battery key K' is closed, the two condensers in series are charged to the difference of potential between A and D. The point B, between the two condensers, has a potential intermediate between that of A and D. The measurement consists in adjusting R_1 and R_2 until C, on the upper circuit, has the same potential as B in precisely the same way as in the measurement of resistance by the Wheatstone bridge method. When this adjustment is correct, there will be no deflection of the galvanometer upon closing K''.

Since the condensers are joined in series, each one must contain the same charge. The point B is insulated as long as K'' remains open, and therefore whatever charge appears on one condenser must have come from the other one. Moreover, for a balance, the closing of K'' produces no deflection of the galvanometer, *i.e.*, there is no change in the charges on the conductors joined at B.

Writing out the equations for the potential differences in the circuits $BACB$ and $DBCD$ gives

$$\frac{q}{C_1} - R_1 i = 0 \quad \text{and} \quad \frac{q}{C_2} - R_2 i = 0,$$

from which

$$C_1 = C_2 \frac{R_2}{R_1}.$$

It should be noted that closing K'' brings B to the potential of C, whether there is a balance or not. A second closing of K'' can produce no deflection. It is therefore necessary to discharge the condensers completely after each closing of K''. This can be most quickly done by opening K' before opening K''.

If one of the condensers has considerable absorption or leakage, it will seriously influence the results, for after some of the charge has leaked away, it is no longer true to say that the charges in the two condensers are equal. The effect of this source of error is reduced by closing K'' as soon as possible after closing K'. When the resistances R_1 and R_2 are free from inductance and capacitance, it is allowable to omit K'' from Fig. 201 and to observe the deflections of the galvanometer when K' is closed or opened.

The observations can be recorded in the same form as used in Art. 385.

399. Potential Changes in the Series Capacitance Bridge.—In this arrangement all parts of the bridge are at the same potential before the battery key is closed. At the moment that this key is closed, the potential of the point A changes at once to a value above that of D by the potential difference at the battery. The points B and C rise as quickly to intermediate potentials, as shown in Fig. 202. When the galvanometer key is closed, the point B is brought to the same potential as C, and the amount of this change in B is shown by the deflection of the galvanometer.

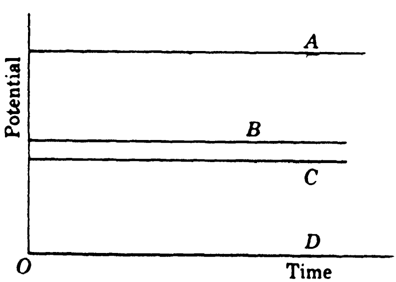


FIG. 202.—Showing the potentials of four points on the bridge of Fig. 201 (with D grounded) after closing the key. For a balance the curves for B and C coincide.

400. The Series Capacitance Bridge with Alternating Current.—The series capacitance bridge can be used with alternating current instead of the battery shown in Fig. 201 (Art. 398). In case one of the condensers shows much absorption (Art. 380), the alternating current may give a more definite balance point than can be obtained with direct current and a ballistic galvanometer. In this connection it is well to note that a different frequency of the alternating current may give a different balance point and that the "capacitance" of a condenser is the value which the measurements give under the particular conditions of charge and discharge that are used. One reason for a more definite balance point with alternating currents is that the periods of charging and discharging are very definite.

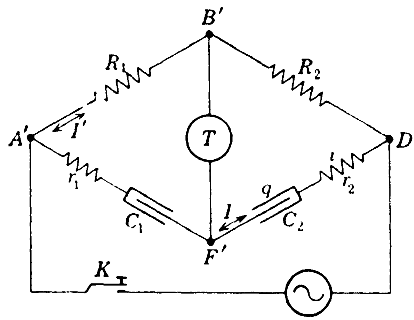


FIG. 203.—Series capacitance bridge for the comparison of capacitances. i denotes the instantaneous value of the current whose effective value is I, and q denotes the instantaneous value of the charge in the condenser.

The arrangement of the bridge is shown in Fig. 203. As explained above (Art. 392), there is, in effect, some resistance in series with each condenser that shows residual charge. This is represented by r_1 and r_2 and may be increased by the addition of a variable resistance to make adjustment possible.

For the case of a balance, with zero current through the telephone, the fall of potential between A' and B', Fig. 203, is equal to the potential difference between A' and F' *at each instant*. This means that

$$R_1i' - \frac{q}{C_1} - r_1i = 0,$$

where q and i are the instantaneous values of the charge and the current in the condenser.

At the same time,

$$R_2i' - \frac{q}{C_2} - r_2i = 0$$

for the other arms of the bridge. Eliminating i' between these equations leaves

$$R_1\frac{q}{C_2} + R_1r_2i = R_2\frac{q}{C_1} + R_2r_1i,$$

or, after collecting similar terms together,

$$\left(\frac{R_1}{C_2} - \frac{R_2}{C_1}\right) q + (R_1 r_2 - R_2 r_1) i = 0.$$

Since q and i may both be positive at the same time or may vary in any manner whatsoever, the only way that the sum of these two terms can always be equal to zero is for the coefficients of q and i to be each equal to zero. That is,

$$\frac{R_1}{C_2} - \frac{R_2}{C_1} = 0 \qquad \text{and} \qquad R_1 r_2 - R_2 r_1 = 0,$$

or

$$C_1 = C_2 \frac{R_2}{R_1} \qquad \text{and also} \qquad r_1 = r_2 \frac{R_1}{R_2}.$$

This means that two separate conditions must be fulfilled in order to have zero current in the telephone branch of the bridge. This is accomplished by adjusting R_1 to give a minimum sound, then adjusting r_1 for a better balance, and then readjusting R_1, and so on until the best silence is obtained. It will also give a better balance to use a Wagner grounding circuit (Art. 388), which in this case will consist of two resistances in the same ratio as R_1 and R_2. If the sound in the telephone is too weak, an amplifier (Art. 373) can be used in the telephone branch of the bridge.

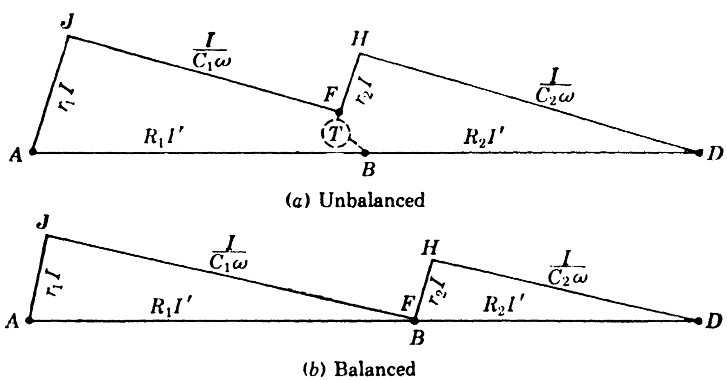


FIG. 204.—Vector diagrams for the series capacitance bridge. In order to make the diagrams more clear, the resistances r_1 and r_2 are shown much too large in comparison with R_1 and R_2.

401. Vector Diagrams for the Series Capacitance Bridge.—The vector diagrams for this bridge are shown in Fig. 204. The resistance arms of the bridge, $A'B'$ and $B'D'$, will give the straight line ABD.

The condenser arms give the triangles AJF and FHD. For a balance it is necessary to bring F to coincide with B, as shown in the diagram (b) for the balanced condition.

The resistances r_1 and r_2 include not only the resistances added in series with the condensers but also the equivalent series resistances that can represent the effect of absorption in the production of heat in the condensers.

From Fig. 204(b),

$$\frac{R_1 I'}{R_2 I'} = \frac{\frac{I}{C_1}}{\frac{I}{C_2}} = \frac{r_1 I}{r_2 I},$$

or

$$C_1 = C_2 \frac{R_2}{R_1}.$$

402. Comparison of Capacitances by the Method of Mixtures. This method was devised by Lord Kelvin to avoid some of the difficulties in the preceding methods. It is especially applicable to cases where the two capacitances are very dissimilar, *e.g.*, if the capacitance of a long cable is to be compared with that of a standard condenser. The method consists in charging the condensers to such potentials that each will contain the same quantity. They are then discharged, the one into the other, and the charges allowed to mix. If the charges are not equal, the difference will remain in the condensers and is later discharged through the galvanometer.

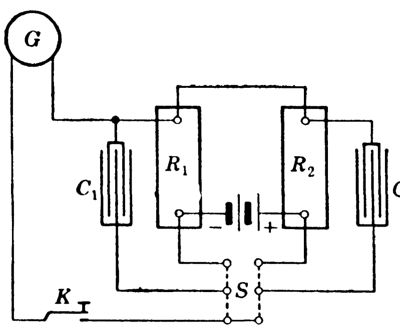


FIG. 205.—Method of mixtures.

The arrangements and connections are shown in Fig. 205. Two boxes of moderately high resistance, R_1 and R_2, are joined in series with a battery of a few cells. A small current i is allowed to flow continuously through R_1 and R_2, thus maintaining across R_1 a steady potential difference,

$$V_1 = R_1 i,$$

and across R_2 a potential difference,

$$V_2 = R_2 i.$$

The condenser to be measured is connected in parallel with R_1 through the double-pole, double-throw switch S. It is thus charged to the potential difference of R_1i volts and therefore receives a charge,

$$Q_1 = C_1R_1i.$$

At the same time, the known condenser is connected in parallel with R_2 by the other side of the switch. The charge it receives is

$$Q_2 = C_2R_2i.$$

By opening S, these condensers are disconnected from R_1 and R_2, but each condenser still retains its charge. When S is closed on the other side, each of the condensers discharges into the other, and the electrons constituting the negative charge of one condenser thus "mix" with the positive charge of the other condenser. If the charges in the two condensers are equal, one will just balance the other, and there will be no resultant charge in either condenser. In case the charge in one condenser is larger than that in the other, a part of this charge will be left after the "mixing," part of it being in each condenser.

Closing the tapping key K enables the condensers to be discharged through the ballistic galvanometer. If there is zero deflection, it shows that there was no charge remaining after the mixing, and therefore

$$Q_1 = Q_2.$$

This condition can be obtained by varying R_1, and therefore Q_1, until zero deflection is obtained. Then

$$C_1R_1i = C_2R_2i,$$

or

$$C_1 = C_2\frac{R_2}{R_1}.$$

403. Absolute Capacitance of a Condenser.—The comparison of one capacitance with another, as shown above, is a simple matter, but it is more difficult to measure accurately the capacitance of a condenser independently of any known capacitance. One method that has been used employs a tuning fork to operate the key K of Fig. 182. If the condenser can be fully charged and discharged n times each second, the rapid succession of charges through the galvanometer will produce a steady deflection equal to the deflection for an equivalent steady current of

$$i = nQ$$

and this can be measured as exactly as the deflection can be read. Then

$$Fd = nQ = nCE,$$

and

$$C = \frac{Fd}{nE},$$

where E is the emf of the charging battery and F is the figure of merit of the galvanometer.

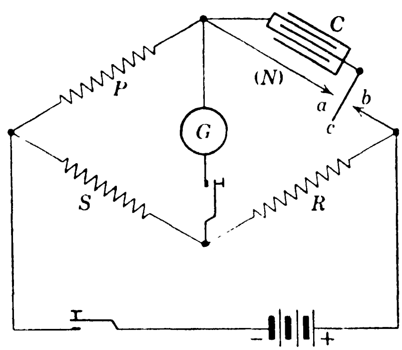


FIG. 206.—Absolute capacitance of a condenser. A motor-driven tuning fork connects c, first to a, and then to b, alternately, n times each second.

404. Absolute Capacitance by the Wheatstone Bridge Method. From the preceding method it appears that the condenser and its vibrating key are equivalent to a resistance of

$$N = \frac{E}{i} = \frac{1}{nC}.$$

It is, therefore, possible to use this arrangement as one arm of a Wheatstone bridge (Fig. 206) and obtain a steady balance in the usual way. To a first approximation,

$$C = \frac{1}{nN} = \frac{S}{nPR}.$$

A closer examination will show that the zero deflection of the galvanometer is due to the resultant action of a steady current in one direction and the intermittent charges in the other direction. Taking into account the entire effect, the complete formula is

$$C = \frac{S}{nPR}\left\{\frac{1 - \dfrac{S^2}{(S+R+r)(S+P+g)}}{\left(1 + \dfrac{Sr}{P(S+R+r)}\right)\left(1 + \dfrac{Sg}{R(S+P+g)}\right)}\right\}.$$

The proper choice of the resistances that are used is important. The use of storage batteries will make r small, and by setting

$$S/R = 100/100{,}000$$

or less, the factor in brackets can be made nearly equal to unity.

It is also essential that the condenser shall be fully charged each time the contact is closed, and this requires a low resistance in the charging circuit. With $S = 100$, $P = 1{,}000$, $g = 20$, $r = 6$, and $R = 100{,}000$, the condenser will reach its full charge in less than 0.001 sec.[1]

After a balance of the bridge is obtained, the condenser should be disconnected and a second balance obtained to determine the capacitance of the connections and other metal parts in this arm of the bridge. The change in capacitance is the capacitance of the condenser.

405. Constant Speed.—The value of n that enters in the formula above can be accurately determined from the speed of the commutator that makes the condenser connections. In the method as used at the Bureau of Standards, this commutator was driven by an electric motor that was hand controlled by a carbon rheostat in the armature circuit. The slightest variation in speed would deflect the galvanometer one way or the other. By controlling the speed of the driving motor so that the galvanometer is kept at or close to zero and by measuring on a chronograph the total number of revolutions for several minutes, the value of n can be found as accurately as may be required.

This combination of a Wheatstone bridge and condenser can be used as an auxiliary apparatus for indicating the speed of a motor in any situation where it is desired to hold the motor at a constant speed.

[1] For the complete discussion of this method and the description of the special commutator used to charge and discharge the condenser, see *Bull. Bur. Standards*, vol. 1, p. 153, 1905.

In a later and improved arrangement the contacts at a and b, Fig. 206, are made and broken in a vacuum to avoid sparking and corrosion. The moving tongue c is operated by a tuning fork driven by an alternating current of constant frequency. This frequency is maintained at 100 cycles per sec by a crystal oscillator. The value of n is thus accurately known in terms of mean solar seconds of standard time.

CHAPTER XV

INDUCTANCE

406. Electromagnetic Induction.—When the electron current through a circuit is started or stopped, or changed in any manner whatever, it is observed that other currents are set up in all the other closed circuits that are near the first one. If some of these circuits are not closed, the tendency to produce a current is present just the same, but as the circuits are open, no current results. In other words, an emf is induced in every conductor near a circuit in which the current is varying. If a current in the second circuit is varying, there will be a corresponding emf induced in the first circuit. This action is called "mutual induction."

When the primary current is alternating, the induced emf is alternating also and can be detected with a telephone receiver. When a direct current is stopped or started, the induced emf is transient and is present only as long as the current is changing. In the latter case a ballistic galvanometer would be more suitable than the telephone receiver for measuring the induced current.

407. Induced Electromotive Force.—Every electron current is associated with magnetic flux in the surrounding space. Any metallic circuit in this region will be linked with some of this magnetic flux.

As shown in Art. 236, any change in the amount Φ of flux thus linked with a circuit gives rise to an emf of

$$e = \frac{d\Phi}{dt} \qquad \text{volts} \tag{1}$$

in that circuit. There will be a corresponding current if the metallic circuit is closed.

408. Emf in a Coil.—In the preceding article the magnetic flux Φ was considered as linked with a loop of wire consisting of a single turn. If the flux is linked with n turns of a coil, there is an induced emf in each turn, and the total emf is

$$e = n\frac{d\Phi}{dt} \tag{2}$$

when the flux through the coil is made to change.

409. Flux Turns.—In case the same flux does not pass through each turn of the coil, the two factors Φ and n of the flux turns cannot be separated, and this quantity Φn must then be considered as a summation of the amounts of flux through each of the various turns of the coil. This summation can be denoted by (Φn), and then the value of the induced emf will be given by

$$e = \frac{d(\Phi n)}{dt}. \qquad \text{volts} \tag{3}$$

410. Meaning of Mutual Inductance.—The magnetic flux that is linked with the secondary circuit depends upon the current in the primary coil and the constants of the two coils. These constants include the number of turns of wire in each coil, the effective distance between the coils, their cross-sectional area, and their relative positions. For a given pair of coils in a fixed position, all these factors are constant, and all of them can be combined into a single resultant factor that may be denoted by M. The flux turns that are linked with the secondary circuit can then be expressed as

$$\Phi n = Mi, \tag{4}$$

and the emf induced in the secondary coil is

$$e = \frac{d(\Phi n)}{dt} = M\frac{di}{dt}, \tag{5}$$

where di/dt is the rate of change of the primary current.

The coefficient M is called the "coefficient of mutual induction," or simply the "mutual inductance" of the pair of coils.

411. The Henry. *The Unit of Mutual Inductance.*—The relation shown in Eq. (5), above, leads directly to the definition of the unit that should be used for the measurement of mutual inductance. This unit is called a "henry"[1] and is defined as follows.:

Definition.—One henry of mutual inductance is the inductance between two circuits so arranged that the induced emf in one of them is one volt whenever the current in the other circuit is changing at the rate of one ampere per second.

This name for the unit of inductance was adopted by the International Congress of Electricians at Chicago in 1893.

412. Secondary Definition of Mutual Inductance.—In Eq. (4), above, the mutual inductance is expressed in terms of the flux turns

[1] Joseph Henry (1797–1878), American physicist.

in the secondary coil. Solving this equation for M gives

$$M = \frac{\Phi n}{i}. \tag{6}$$

This equation leads to a secondary definition of M that is often useful, and it gives another view of mutual inductance. *The mutual inductance of a pair of coils is measured by the weber turns through the secondary per ampere in the primary.* This gives M in henrys.

413. Reason for a Negative Sign.—Strictly speaking, Eq. (5) should be written

$$-e = M\frac{di}{dt}, \tag{7}$$

although if we are looking only for the numerical values of e and M, the sign does not enter. The electromotive force due to a changing flux is as real when it is in one direction as another. But in case the direction of the induced current is considered, it is necessary to determine which direction shall be called "positive."

If the direction of the primary current is called "positive," the direction of the magnetic flux due to this current should also be called "positive." A current in the secondary coil that produces flux in the same direction would also be called "positive." An emf that causes this current would likewise be called "positive."

Experiment shows that the electromotive force induced in the secondary coil is in this positive direction when the positive primary current is *decreasing i.e.*, when the change in the primary current is expressed as $-di/dt$.

Hence, taking account of these signs, $e = M\ (-di/dt)$, or

$$-e = M\frac{di}{dt}. \qquad \text{volts}$$

414. Meaning of Self Inductance.—It is but a step from considering the action of a current on an adjacent circuit to the case of action upon the same circuit. If the second circuit above were included as a part of the primary circuit, then the induced emf would be in the same circuit as the inducing current, and in a direction opposed to any change in this current. Furthermore, this same kind of action would appear in every turn of the primary coil, the current in each turn inducing an emf in each of the other turns.

The total flux turns Φn that measure the linking of the magnetic flux with a circuit depend not only on the value of the current i but also on the number of turns of wire in the circuit, the area of these turns, their relative positions, and the distances from each other. These factors are constants for a given coil or circuit, and they all can be combined into a single constant L. Then

$$\Phi n = Li. \tag{8}$$

When there is a change in this current, there is induced in the circuit a corresponding emf of the amount given by

$$e = \frac{d(\Phi n)}{dt} = L\frac{di}{dt}. \tag{9}$$

The constant L is called the "coefficient of self induction," or simply the "self inductance" of the circuit.

415. The Negative Sign.—When the induced emf in a circuit is in the direction that will increase the emf which is diving the current, this induced emf is called "positive." Investigation shows that the induced emf has this positive direction when the current is decreasing, and its value can be expressed as

$$e = L\left(\frac{-di}{dt}\right) = -L\frac{di}{dt}. \tag{10}$$

416. The Henry of Self Inductance.—Since the induced emf is due to the same effect whether it is in one circuit or another, the practical unit of self inductance is called a "henry," as in the case of mutual inductance.

Equation (10) shows the value that should be taken for the unit of self inductance in order to avoid the introduction of a proportionality factor in this relation.

Definition.—One henry of self inductance is the inductance in a circuit so arranged that the induced emf in the circuit is one volt whenever the current in this circuit is changing at the rate of one ampere per second.

417. Secondary Definition of Self Inductance.—In Eq. (8), above, the value of the self inductance is expressed in terms of the flux that is linked with the circuit. Solving this equation for L gives

$$L = \frac{\Phi n}{i}. \tag{11}$$

This shows that the self inductance of a coil is measured by the number of weber turns linked with it when carrying a current of 1 amp.

For an inductance of 1 henry there is 1 weber turn when the current is 1 amp.

418. Effect of Iron in the Magnetic Circuit.—These definitions of inductance presuppose that μ is constant and that Φ is proportional to the primary current. This is the case in air and most other substances. When iron is near the coils, the inductance is no longer constant but is different for each value of the current. From Eqs. (9) and (8), above,

$$L \frac{di}{dt} = n \frac{d\Phi}{dt} = n \frac{d\Phi}{di} \frac{di}{dt}, \tag{12}$$

or

$$L = n \frac{d\Phi}{di}, \tag{13}$$

which might be called the "instantaneous value" of the inductance. When Φ is proportional to i, this reduces to the second definition given above, but this is not the case with iron in the magnetic circuit. In the magnetization curves for iron it is seen that Φ/I and $d\Phi/dI$ are not the same. In this case the value of L varies from point to point as the current changes, and the two forms of definitions give different values for the inductance for the same value of the current. Moreover, these values are not the same for a decreasing current as for an increasing current. The choice of the proper definition for any particular problem is decided by a consideration of the attending circumstances.[1]

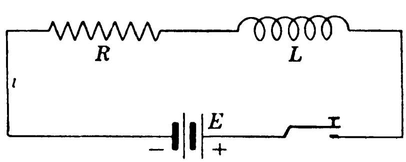


FIG. 207.—A circuit containing resistance and inductance.

419. Starting and Stopping a Current.—When a constant emf E is applied to an inductive circuit, Fig. 207, the current starts at the value zero and increases to its final steady value. While the current is thus increasing, there is induced in the same wire an emf, the value of which is

$$e = -L \frac{di}{dt}. \tag{14}$$

This emf is in the direction that will oppose the change in the current. For an increasing current, it is directed in opposition to the emf that is causing the current to flow. When the current is decreasing, this induced emf helps to maintain the current. It is then added to E, but it is still written as $-L(di/dt)$ because di is now negative and this makes the whole expression positive.

[1] STARLING, "Electricity and Magnetism," p. 302.

Let R denote the resistance and L the self inductance of a circuit to which E is applied. The sum of all the potential differences in the circuit is

$$E - Ri - L\frac{di}{dt} = 0. \tag{15}$$

This equation is often written in the form

$$E = Ri + L\frac{di}{dt}, \tag{16}$$

which shows that while a part, Ri, of the applied emf is effective in maintaining the current i, another part, $+L(di/dt)$, is required to make the current change. This equation is very important in all discussions of varying currents.

Value of the Current.—Solving Eq. (15) for the value of the current at any instant after the steady emf has been applied gives

$$i = \frac{E - L\frac{di}{dt}}{R}, \tag{17}$$

which is fully analogous to Ohm's law for a steady current.

At the start, when the current is changing most rapidly, the second term in the numerator is nearly equal to E. As the current approaches its final value, it changes more slowly, the induced emf becomes less and less, and finally the steady value of the current is $i = E/R$.

420. Dying Away of a Current.—Suppose that a steady current has been flowing in a circuit and that suddenly the battery, or other source of emf, is removed without breaking the circuit or in any way changing its resistance or inductance. Equation (16) now becomes

$$0 = Ri + L\frac{di}{dt}, \tag{18}$$

and the current thus left to itself dies away to zero at a rate that is shown by the following relations:

Rewriting (18) to separate the variables,

$$\frac{di}{i} = -\frac{R}{L}dt. \tag{19}$$

The integral of this is

$$\log i = -\frac{Rt}{L} + c, \tag{20}$$

or, in the exponential form,

$$i = \epsilon^c \epsilon^{-\frac{Rt}{L}} = C\epsilon^{-\frac{Rt}{L}}. \tag{21}$$

The value of the constant of integration C is given by the fact that at the start, when $t = 0$, the current had the value I. Putting these values in (21) gives $I = C$. Therefore (21) becomes

$$i = I\epsilon^{-\frac{Rt}{L}}. \tag{22}$$

From this equation it appears that it is the self inductance which prevents the current from falling to zero immediately, and the greater the self inductance, the more slowly will the current die away. If it is desired to state how rapidly the current decreases, it is necessary to state how long it takes for the current to fall to half value—or to some other definite fraction of its original amount—since the value of i from (22) will reach zero only after an infinite time.

When t has the value L/R, the current equals I/ϵ or $I/2.72$, and this interval in which the current falls to 0.368 of its original value is called the "time constant" of the circuit.

421. Beginning of a Current.—When a circuit containing an emf is closed, the current rises from zero to its final value at a rate depending upon the self inductance in the circuit. This relation can be found from the equation

$$E = Ri + L\frac{di}{dt}. \tag{23}$$

Separating the variables changes this to the form

$$\frac{-di}{\frac{E}{R} - i} = -\frac{R}{L}\,dt, \tag{24}$$

the integral of which is

$$\log\left(\frac{E}{R} - i\right) = -\frac{Rt}{L} + c, \tag{25}$$

or

$$i = \frac{E}{R} - C\epsilon^{-\frac{Rt}{L}}. \tag{26}$$

The constant of integration is determined by the condition that when $t = 0$, $i = 0$. Hence $C = E/R = I$, where I is the final value of the current.

Therefore

$$i = I - I\epsilon^{-\frac{Rt}{L}}. \tag{27}$$

Here also it is seen that it is the self inductance that keeps the current from rising suddenly to its full value as soon as the circuit is closed. The greater the self inductance in comparison with the resistance, the more slowly will the current rise, but it never quite reaches its maximum value. Therefore in comparing different circuits it is necessary to compare the periods taken for the currents to rise to half value—or to some other definite fraction of the final steady values. When $t = L/R$, the current lacks I/ϵ of its final value, and this period in which the current rises to 0.632 of its maximum value is called the "time constant" of the circuit.

Problems

1. Given a circuit in which the resistance is 10 ohms and the self inductance is 0.01 henry, draw a curve showing the rise of the current for the first 0.004 sec after applying a steady emf of 100 volts.
2. The above circuit is placed in parallel with a noninductive resistance of 10 ohms, and the current in the main line is 20 amp. Draw a curve showing how the current dies away in the parallel circuits when the main line switch is suddenly opened.

The logarithms that appear in the equations through the process of integration are of course not the common logarithms with the base 10 but the natural logarithms with the base $\epsilon = 2.718+$. The following table will be useful in finding the time at which the current will have a given fraction of its maximum value.

Example.—Suppose it is desired to find the time at which the current in Prob. 2 has fallen to three-tenths of its maximum value. In this case, then, $\epsilon^{-\frac{Rt}{L}} = 0.3$, and this value is found in the first column of the table. In the last column is given the value of the exponent, $-Rt/L = -1.204$, which is the natural logarithm of 0.3. Therefore

$$t = 1.204\frac{L}{R} = 0.000602 \text{ sec.}$$

This procedure is simpler than the computation to find the value of the current at an arbitrarily chosen time.

SHORT TABLE OF NATURAL LOGARITHMS

Number $\epsilon^{-\frac{Rt}{L}}$	Natural logarithms	
	Tabular value	Numerical value $-\frac{Rt}{L}$
0.90	9.895–10	−0.105
0.80	9.777–10	−0.223
0.70	9.643–10	−0.357
0.60	9.489–10	−0.511
0.50	9.307–10	−0.693
0.40	9.084–10	−0.916
0 30	8.796–10	−1.204
0.20	8.391–10	−1.609
0.10	7.697–10	−2.303
0.07	7.341–10	−2.659
0 04	6.781–10	−3.219
0.01	5.395–10	−4.605

422. Quantity Passing through the Secondary Circuit.—When the current is started or stopped in the primary coil of a mutual inductance, the emf in the secondary rises quickly to a maximum value and then subsides to zero. Even the maximum value is not constant for the same change in the primary current but depends upon how quickly the current is made to change. Likewise, the secondary current is not definite in either amount or duration. The one thing that is definite is the quantity of electrons Q which passes along the secondary circuit when a given change I is made in the primary current. This is the same whether the current is changed quickly or slowly.

Let e denote the instantaneous value of the emf induced in the secondary coil at the time t. Then, as shown above, the emf induced in the secondary coil is

$$e = M\frac{di'}{dt}. \tag{28}$$

Expressed in terms of the current i that flows in the secondary circuit, this is

$$e = Ri + L\frac{di}{dt}. \tag{29}$$

Equating these two values of e gives

$$M\frac{di'}{dt} = Ri + L\frac{di}{dt}. \tag{30}$$

Integrating this equation with respect to the time,

$$\int_t^{t'} Mdi' = \int_t^{t'} Ridt + \int_t^{t'} Ldi = R\int_{q=0}^{q=Q} dq + L\int_{i=0}^{i=0} di = RQ, \quad (31)$$

where the integration is extended from the time t before the primary current begins to flow, till the time t' when the primary current has reached its steady value I and the secondary current has died away to zero.

Therefore

$$MI = RQ, \quad (32)$$

or, solving for the value of Q,

$$Q = \frac{MI}{R}. \quad (33)$$

When I, the change in the primary current, is expressed in amperes, R in ohms, and M in henrys, then Q will be given in coulombs.

CHAPTER XVI

MEASUREMENT OF INDUCTANCE

423. Effects of Variable Currents.—The inductance of a circuit is not in evidence when steady currents are flowing. It is only when the current is started or stopped, or made to vary in some other way, that the effect of inductance is made manifest, and it is through the measurement of such effects that the value of an inductance can be determined.

The falls of potential due to resistance can be balanced in a Wheatstone bridge, and therefore such an arrangement is usually employed to measure the additional effect of inductance. For currents that vary in any manner whatsoever, the conditions for a balance can be found by analytic methods. For the special case of a sinusoidal current, the conditions for a balance are clearly and vividly shown by graphical methods, and a study of such diagrams gives a visual picture of the conditions that will give a balance in the bridge. Therefore both methods are given in this chapter, so that each may supplement the other and give a clearer understanding of what is being done when making a measurement of inductance.

424. Frequencies.—In most of the methods that are described in this chapter, a telephone receiver is used to detect a balance with 1,000-cycle current. This frequency gives the best tone to be heard in a telephone.

Oftentimes, when the bridge is nearly balanced, the telephone current becomes too small to be audible. In such cases it is desirable to amplify this current by the use of an electron tube, as shown in Fig. 217. This amplifier can be built up in the laboratory or obtained ready made from the makers of scientific or radio apparatus. Such apparatus is now available for any laboratory and would stand in the place of the simple telephone indicated in the diagrams.

The same methods and discussions apply when other frequencies are used. When a sensitive alternating-current galvanometer is available, current of 60-cycle (per sec) or other frequency may be used.

In the more elaborate forms of amplifiers the telephone receivers are replaced by a cathode-ray tube, and the wave form of the current is seen on the screen at the end of the tube. As a balance of the bridge is approached, this wave is reduced to a horizontal line (see Fig. 176).

425. Comparison of Two Self Inductances.—The self inductance of a coil can be measured in terms of the known self inductance of another coil by using alternating current in a Wheatstone bridge arrangement, as shown in Fig. 208. The two coils are placed in two adjacent arms of the bridge, with two dial resistance boxes for the other arms. A headset of telephone receivers is connected across from B to F instead of the galvanometer in the usual Wheatstone bridge. The points A and D are connected to a source of alternating current of audio frequency. If 1,000-cycle (per sec) alternating current is available (see Art. 370), it is useful for this purpose. The key K should be a knife switch that will remain closed and leave the hands free to make other adjustments.

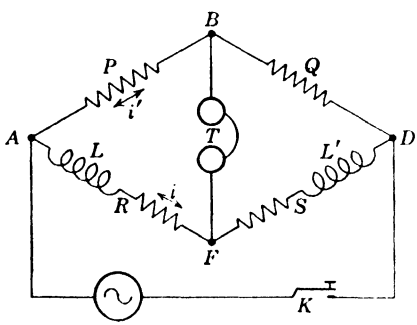


FIG. 208.—The bridge arrangement for measuring the inductance L' in terms of the known inductance L.

The inductance coils will necessarily have resistances of some values R and S, each or both of which can be increased by adding noninductive resistances in series with the coils. By listening at the telephone receivers while the resistances Q and S are varied, the values for a minimum sound can be determined. This adjustment requires a skillful procedure on the part of the operator in adjusting the bridge, because if either Q or S is not set at the correct value, no adjustment of the other can give silence in the telephones.

The process by which this double adjustment can be made is as follows: If the inductances of the coils are fixed values L and L', the resistance P can be set at about 1,000 ohms, with Q varied to give a first approximation to a minimum sound in the telephone. Probably this will be far from silence, because R and S are not set at the proper values. By varying one of these, say S, the sound can be reduced to a lower minimum. Now it will be possible to readjust Q to a sharper minimum, and then S can be readjusted. By successively varying Q and S, the correct values for each can soon be reached. If perfect silence is not attained, it is probably because of a lack of fineness in the adjustment of the resistances (see below).

426. Conditions for a Balance.—The relation between the inductances and the resistances when there is zero current through the telephone can be found as follows: Writing the instantaneous values of the potential differences in the circuit $ABFA$ at any instant when the currents are i and i' gives, for the case of a balance,

$$Pi' - Ri - L\frac{di}{dt} = 0. \tag{1}$$

Similarly, for the circuit $BDFB$,

$$Qi' - Si - L'\frac{di}{dt} = 0, \tag{2}$$

since there is no current from B to F, as is indicated by zero sound in the telephone.

Solving Eq. (1) for the value of di/dt gives

$$\frac{di}{dt} = \frac{P}{L}i' - \frac{R}{L}i, \tag{3}$$

and substituting this value in Eq. (2) gives

$$Qi' - Si - L'\frac{P}{L}i' + L'\frac{R}{L}i = 0, \tag{4}$$

or, by collecting like terms together,

$$\left(Q - L'\frac{P}{L}\right)i' + \left(L'\frac{R}{L} - S\right)i = 0. \tag{5}$$

Since i and i' may have any values at this instant, the only way Eq. (5) can be satisfied is when each coefficient is equal to zero, *i.e.*, when

$$Q = \frac{L'}{L}P \qquad \text{and} \qquad S = \frac{L'}{L}R. \tag{6}$$

Both of these conditions must be fulfilled in order to obtain silence in the telephones. The bridge can be balanced for varying currents only when *both* the resistances and the inductances are adjusted to the values that will satisfy the relations in Eqs. (6), which can be combined into the triple equation

$$\frac{L}{L'} = \frac{R}{S} = \frac{P}{Q}, \tag{7}$$

where R and S include the resistances of the inductance coils.

It is seen that the balance of the inductance bridge is independent of the particular way (di/dt) in which the currents are made to vary, and therefore any wave form of alternating current may be used. This is all on the supposition that there are no capacitances in the arms of the bridge and that L and L' are constants, *i.e.*, contain no iron or other magnetic substance that might cause the self inductance of the coil to vary as the iron is magnetized by the changing current. (Note that the hysteresis curves, Fig. 136, are not straight lines through the origin.)

From Eq. (7) it appears that the two inductive arms of the bridge must be *equally inductive*, or, in other words, the henrys per ohm must be the same for each arm.

427. Use of a Variable Self Inductance.—When a variable standard of self inductance is available, it can be used for the known inductance in the above method. With the standard set near the middle of its scale, the resistance ratio P/Q can be set at approximately the proper value (within 25 per cent) and the balance obtained by varying L. It will be also necessary to adjust R/S to the same value as P/Q. This can be done as before by alternately adjusting L and S. Sometimes the bridge can be balanced with steady direct current. Then when alternating current is used, only L need be varied to obtain the balance.

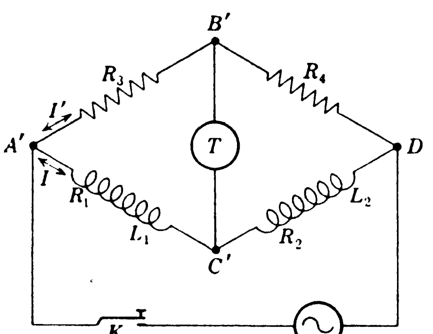
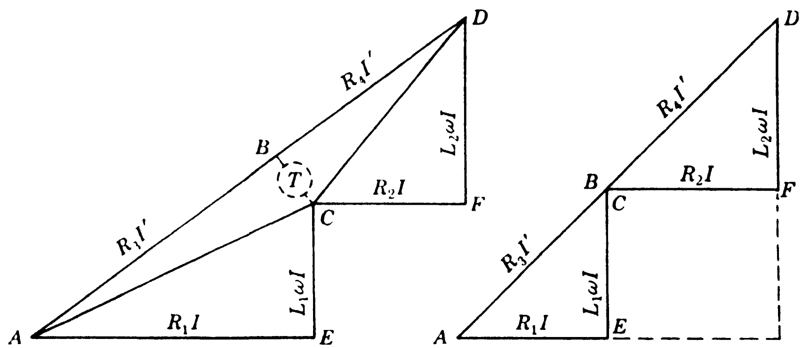


FIG. 209.—Inductance bridge with L_1 and L_2 in series.

428. The Vector Diagrams for the Self Inductance Bridge. *Coils in Series.*—If one can visualize the vector diagram and see what changes are required to balance a bridge, it is a great help in understanding the measurements that are made with alternating currents. First let us consider the case where the two inductances to be compared are connected in series, as indicated in Fig. 209. This is, in effect, an inductive circuit $A'C'D'$ in parallel with a noninductive circuit $A'B'D'$. The graphical representation of the inductive branch is shown by the two emf triangles, AEC and CFD, Fig. 210, which are lettered the same (without the primes) as the corresponding parts in Fig. 209.

The corresponding diagram for the noninductive circuit $A'B'D'$ reduces to a straight line which begins at A and ends at D, since this circuit is subjected to the same emf as $A'C'D'$. This is shown by ABD. The emf impressed on the telephone is shown by BC, provided the telephone current is negligible compared with the currents through the other parts of the bridge.

In order to obtain no current through the telephone, it is necessary that B and C coincide. By increasing the value of L_1, or decreasing R_1, the point C can be brought onto the line AD. Then, by varying R_3, the point B can be moved along AD until it coincides with C, as shown in Fig. 210(*b*). The line ABD still represents the upper circuit of the bridge, Fig. 209, and is not the same line as ACD but is a second and independent line in Fig. 210(*b*) as well as in Fig. 210(*a*). Thus



(*a*) Unbalanced (*b*) Balanced

FIG. 210.—Emf diagrams for the inductance bridge, with the coils in series.

ABD and ACD, Fig. 210(*b*), form a double line between A and D, and while R_3I' is not the hypotenuse of triangle AEC, it is equal to AC, and the two values can be used interchangeably. Then from similar triangles,

$$\frac{L_1}{L_2} = \frac{R_1}{R_2} = \frac{R_3}{R_4}.$$

If L_1 is variable, it is often convenient to set $R_1{:}R_2 = R_3{:}R_4$ by a direct-current balance, or otherwise, and then with alternating currents it is necessary only to vary L_1 to obtain a balance. But this is not necessary. By varying R_3 with alternating currents, B can be moved along AD until it is somewhere nearly opposite to C, Fig. 210(*a*). Then, by varying L_1, C is brought nearer to B. A further change of R_1 brings B nearer to C, and after a few such adjustments, B and C are brought together.

From the construction of the figure it is seen that the effect of a 1 per cent change in L_1 upon the position of C is greatest when C is midway between A and D. Therefore, for the greatest sensitiveness of the bridge, the impedances of the arms should be as nearly equal as practicable.

429. The Vector Diagrams for the Self Inductance Bridge. *Coils in Parallel.*—When the two inductances are placed in adjacent and parallel arms of the bridge, as shown in Fig. 211, there is a noninductive resistance in series with each coil. The diagram for the circuit $A'C'D'$ is shown by ACD, Fig. 212(*a*). The corresponding diagram for $A'B'D'$ is shown by the diagram ABD. For no emf on the telephone, B must be brought to coincide with C. This can be done as in the previous case, by first adjusting R_4, and then adjusting R_2 or L_2, until the minimum sound is given by the telephone. When

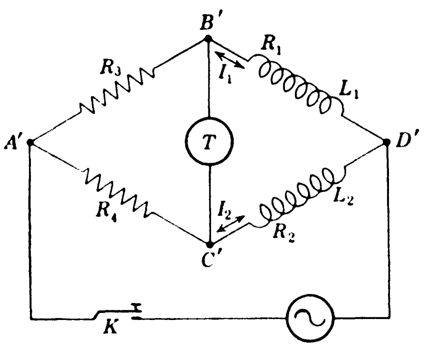


FIG. 211.—Inductance bridge with L_1 and L_2 in parallel.

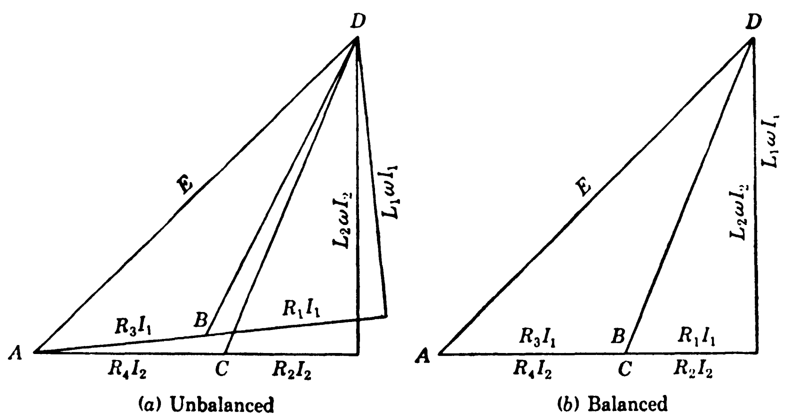


FIG. 212.—Emf diagrams for the inductance bridge, with the coils in parallel.

B is brought to C, the two diagrams coincide throughout, Fig. 212(*b*), and

$$\frac{L_1}{L_2} = \frac{R_1}{R_2} = \frac{R_3}{R_4}.$$

It is often necessary to add some resistance to R_1 or R_2 in order that the ratio of these resistances shall be the same as the ratio of the inductances, *i.e.*, so that the emf diagrams for the two inductances shall be similar triangles.

The most sensitive arrangement occurs, as before, when AC and CD are nearly equal.

430. Choice of Ratios.—It was shown above (Art. 426) that the inductance bridge is balanced when

$$\frac{L}{L'} = \frac{R}{S} = \frac{P}{Q},$$

and at first sight it might seem that any one of these ratios could be chosen, and then the other two ratios adjusted to this value. Usually, however, the resistances R and S are small compared with L and L' or with P and Q, and the ratio R/S is not a strong guide for adjusting the other ratios. When the inductances are fixed, it is of course necessary to take L/L' as the ratio to which the others are brought. When the known inductance is variable, the ratio P/Q can be set at some convenient value and each of the other ratios can be brought to this value.

431. Direct-current Balance.—When the resistances of the coils are wholly unknown, it is sometimes helpful to measure them by a d-c method (perhaps by using a galvanometer and a battery to balance the bridge), so that the relation $L/R = L'/S$ may be kept approximately correct while the preliminary adjustments of P and Q are being made. Then, when the bridge is nearly balanced, Q and S can be varied alternately to bring B and C, Fig. 210, nearer to each other, until silence in the telephones is attained.

432. Fine Adjustment of the Resistance Ratio.—It has been shown in Eq. (6), Art. 426, and also in Figs. 210 and 212, that it is necessary to have the resistances of the inductive coils in exactly the same ratio to each other as the inductances of the coils in order to balance the bridge and to give silence in the telephone. Usually these resistances are small. When they are adjusted by 1-ohm steps, this may mean steps of 5 or 10 per cent, and the nearest balance may thus be 3 or 5 per cent on one side of the true value. This makes it impossible to obtain complete silence in the telephone. To make an exact balance would require the use of tenths and hundredths and possibly thousandths of ohms in the adjustable resistance S (Fig. 213), and usually these small resistances are not available. However, this fine adjustment can be obtained by using a 1-ohm resistance U in parallel with a slide wire like the right-hand dial of the student's potentiometer

(Fig. 89). When the telephone circuit is connected to the moving contact F, it can, in effect, be moved across the 1 ohm in very fine gradations.

Usually P or Q will be over 1,000 ohms, and an adjustment to the nearest ohm is sufficient. When this is not the case, a fine adjustment can be made as described below (see Art. 440).

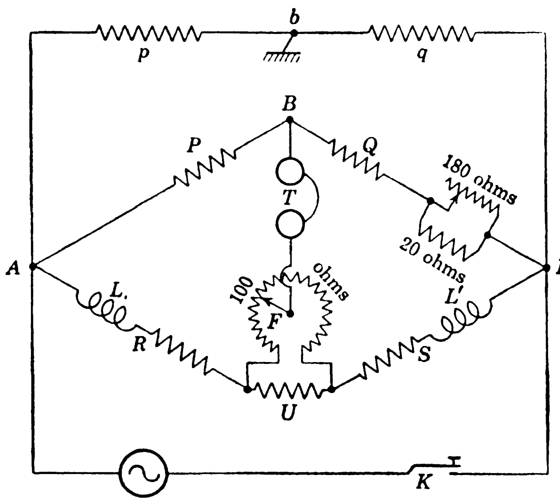


FIG. 213.—Showing the ground connection for an inductance bridge. Also the fine adjustment of the ratio R/S and for the resistance Q.

433. To Amplify the Telephone Current.—If the telephone current is too weak to clearly indicate the adjustment of P and Q, Fig. 213, when near the balance point, it can be amplified by the use of an electron tube connected as shown in Fig. 217. The primary of a small audio-frequency transformer is connected in place of the telephone circuit from B to F. The secondary of this transformer is connected to the grid of the electron tube and serves as a variable C potential (Art. 229). The telephones are placed in the plate circuit in series with the B battery of 45 volts. The current through the telephones is always in the same direction, but it will vary (see Fig. 121) with every change in the potential of the grid, and the weak alternating current from B to F will be reproduced in the variations of the larger current through the telephone.

434. Grounding the Telephone.—Each time the current changes in the inductance bridge, there is a corresponding change in the potentials of the various parts. If these parts have any capacitance, there is also a change in their electrostatic charges. The currents due to these varying charges are usually too small to produce a noticeable effect in the arms of the bridge, but if they pass through the telephone they may cause an audible hum and make it impossible to secure silence when the bridge is balanced. It is therefore desirable to have the telephone receivers remain at the potential of the earth, and this can be accomplished without actually connecting the telephone to the ground.

Let a potential divider be connected from A to D, Fig. 213, in parallel with the bridge, and let the point b divide it in two parts p and q, having the ratio P/Q. Then if b is grounded, B and the telephone will also be at the potential of the earth. If the arms Q and R of the bridge were interchanged, an inductance would be required in one part of the potential divider to make it similar to the path ABD (see also Art. 388).

Noise in the phones at balance. If the coils L or L' in the bridge have much magnetic leakage, there may be stray flux through the air that will give a hum in the phones even when the latter are not connected to the bridge. Phones when in use should be kept far from all coils carrying alternating current.

435. An Inductance Bridge.—An inductance bridge box is made by Leeds and Northrup. In general appearance it resembles the Wheatstone bridge box made by the same firm. The ratio arms are similar in the two boxes and each has a rheostat arm, but as the actual resistance in this arm of the inductance bridge is immaterial, the fine adjustment is made by two small circular rheostats, each having many steps. The ratio arms correspond to P and Q of Fig. 213, while the rheostat arm of variable resistance is connected between the two inductance coils, as shown in Fig. 214. By means of a plug key, the galvanometer connection C may be joined to either C' or C'', thus putting the resistance R in series with either R' or R'' as desired. The resistance balance is easily made by varying R, after which the inductance balance is obtained by varying L'. The value

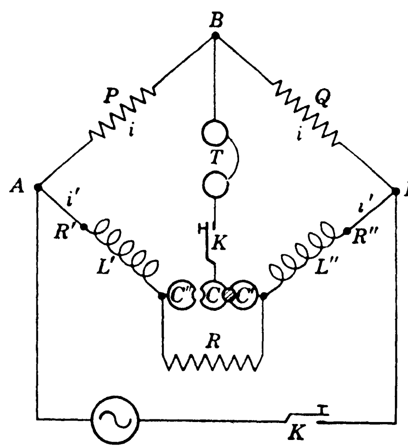


FIG. 214.—Diagram of the inductance bridge.

of the unknown inductance is, then,

$$L'' = \frac{Q}{P} L',$$

where Q/P is an integer power of 10.

436. Comparison of a Mutual Inductance with a Self Inductance.—The preceding method gives also a very satisfactory way of measuring the mutual inductance between two coils. Call the self inductances of the primary and secondary coils L_m and L_n, and their mutual inductance M. The emf induced in these coils when connected in series and carrying a current i is

$$L_m \frac{di}{dt} + M \frac{di}{dt} + L_n \frac{di}{dt} + M \frac{di}{dt} = (L_m + L_n + 2M) \frac{di}{dt}.$$

But the coefficient of di/dt is, by definition, the inductance of the circuit, and if the two coils thus joined together in series were made one arm of the bridge shown in Fig. 213, the inductance measured by that method would be

$$L_s = L_m + L_n + 2M. \tag{8}$$

If one of the coils were reversed, the emf induced in each coil by the mutual inductance of the other would be reversed also, and therefore the inductance measured by the bridge in this case would be

$$L_r = L_m + L_n - 2M. \tag{9}$$

Subtracting Eq. (9) from Eq. (8) gives

$$L_s - L_r = 4M.$$

Hence to determine the mutual inductance of a pair of coils it is necessary only to measure the self inductance of both together when joined in series—first with the coils direct and again with one of the coils reversed. One-fourth of the difference between the two inductances gives the value of the mutual inductance.

437. Measurement of a Self Inductance by Means of a Capacitance. *Maxwell's Method.*—In this method the self inductance to be measured is placed in one arm of a Wheatstone bridge, the other arms of which should be as free from inductance as possible. The effect of this inductance can be balanced by placing a condenser in parallel with

R, as shown in Fig. 215. With the condenser so placed, a part of the current through P will flow into the condenser, and therefore the fall of potential over R will be smaller than it would be without the condenser. If the capacitance of the condenser is of the proper amount, this latter effect will just equal the effect of the coil in reducing the fall of potential across S, and therefore the potentials of B' and F' will rise and fall together. In this case a telephone or other current detector will indicate no current through the branch, $B'F'$.

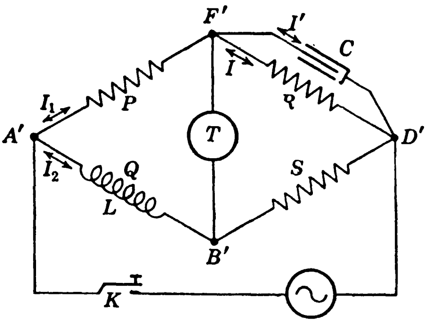


FIG. 215.—Comparison of a capacitance C with a self-inductance L.

When such a balance has been obtained, we can write for the circuit $B'A'F'B'$ at any one instant

$$Qi_2 + L\frac{di_2}{dt} - Pi_1 = 0, \tag{10}$$

and for $D'F'B'D'$,

$$Ri - Si_2 = 0, \tag{11}$$

where

$$i_1 = i + i' = i + \frac{dq}{dt} = i + \frac{d}{dt}(CRi) = i + CR\frac{di}{dt}, \tag{12}$$

since both the current, i, through R, and i' flowing into the condenser, are added together to make the current, i_1, through P.

Substituting in Eq. (10) the value of i_2 from Eq. (11) and i_1 from Eq. (12) gives

$$Q\frac{Ri}{S} + L\frac{R}{S}\frac{di}{dt} - P\left(i + CR\frac{di}{dt}\right) = 0, \tag{13}$$

or

$$(QR - PS)i + (LR - PCRS)\frac{di}{dt} = 0. \tag{14}$$

This equation shows that two conditions must be fulfilled to give a balance with varying currents. Not only must the resistances be adjusted to make

$$QR = PS, \tag{15}$$

but the self inductance of the arm AB must have the value

$$L = PSC. \tag{16}$$

438. Vector Diagram for the Inductance-capacitance Bridge.—The adjustments that are necessary to balance an inductance against a capacitance in the opposite arm of a bridge network may be more clearly understood by a study of the vector diagram for this arrangement.

In drawing the emf diagram it will be simplest to solve first the parallel circuits between F' and D', in Fig. 215 (see Art. 364). R' and

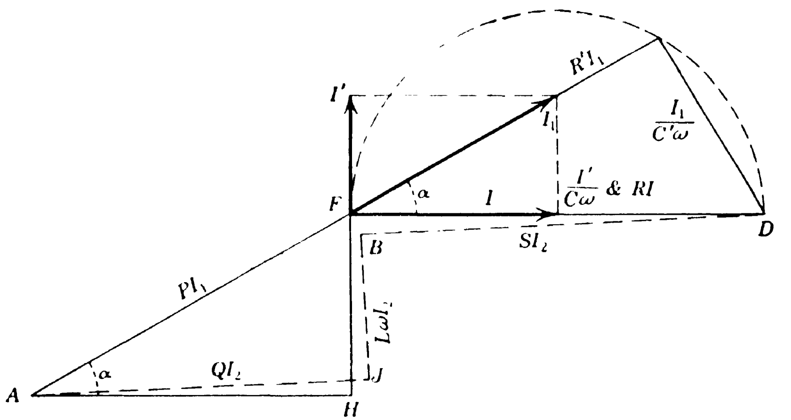


FIG. 216.—Emf diagram for the comparison of C and L. The dashed lines show the condition when the bridge is unbalanced.

C' denote the equivalent series resistance and capacitance of this arm of the bridge.

The currents I and I' through R and C are shown in Fig. 216 in the usual way, the impressed emf over the branch $F'D'$ being represented by FD. The resultant current, I_1, through this branch is ahead of the emf impressed on this part of the circuit by the angle α, where

$$\tan \alpha = \frac{I'}{I} = \frac{EC\omega}{\frac{E}{R}} = RC\omega. \tag{17}$$

Since the noninductive arm $A'F'$ carries this resultant current, I_1, the emf over this arm is in phase with I_1, as shown by AF.

The diagram for the inductive branch $A'B'D'$ presents no difficulties. In general, it would be as shown by the dashed lines ABD. For a balance of the bridge, L and Q must be varied to bring B into coincidence with F. This also means that J falls at H and the diagram is shown by the full lines $AHFD$. The phase angle HAF is equal to α and is given by

$$\tan HAF = \frac{L\omega}{Q}, \tag{18}$$

where L and Q are the values that give the balance.

Comparing Eq. (17) and Eq. (18),

$$RC\omega = \frac{L\omega}{Q},$$

or

$$L = RQC. \tag{19}$$

Equating equal parts from the two diagrams gives

$$QI_2 = PI_1 \cos \alpha \tag{20}$$

and

$$SI_2 = RI = RI_1 \cos \alpha. \tag{21}$$

Dividing one equation by the other gives

$$\frac{Q}{S} = \frac{P}{R} \tag{22}$$

as a second relation that exists when the bridge is balanced.

If Q is not accurately known, its value is given by Eq. (22), and then Eq. (19) becomes

$$L = PSC, \tag{23}$$

which is the relation that was found in Art. 437.

439. Adjustments to Balance a Self Inductance with a Capacitance.—Formerly it was a long and tedious process to measure a fixed self inductance by Maxwell's method. The bridge must be balanced as shown by Eq. (22), and then P must be adjusted to the value that will satisfy Eq. (23). This upsets the relation in Eq. (22), and the bridge must be rebalanced for each value of P that is used in Eq. (23).

With modern dial resistance boxes and audio-frequency alternating current, the problem is greatly simplified, and accurate measurements are readily made. Since no restrictions have been placed upon the manner in which the current varies, as expressed by di/dt, the relations derived in Art. 437 are true for alternating current. The diagrams of Fig. 216 represent the conditions when the alternating current is sinusoidal. The current may be obtained from an electron-tube generator (Art. 233) or from an audio-frequency oscillator (Art. 370) or other source. A frequency of 1,000 cycles per sec is desirable, especially when telephones are used for determining the balance point.

The arrangement of the apparatus is shown in Fig. 217. The coil L is made one arm of the bridge with the arm S a noninductive resistance of about the same value as the impedance of Q. The other arms, P and R, should be over 1,000 ohms each to allow an adjustment of many steps in balancing the bridge. The capacitance C should be of the order of magnitude that is indicated by Eq. (19). It is also desir-

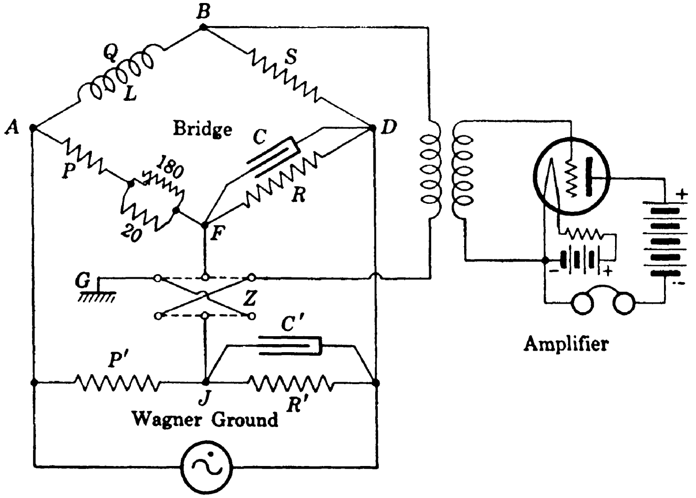


FIG. 217.—Details of the bridge arrangement for comparing L with C.

able to have tan α, Eq. (17), neither too large nor too small. The condition of Eq. (23) is fulfilled when P is adjusted to have the value

$$P = \frac{L}{CS}$$

and the condition of Eq. (22) shows that R must have the value

$$R = \frac{PS}{Q} = \frac{L}{CQ}.$$

The balance is obtained by adjusting first P and then R alternately until each has been brought to these values. A pair of headphones connected at B and F will indicate when the balance has been obtained. A closer balance can be made with the aid of an electron-tube amplifier in the telephone circuit as shown in Fig. 217. The phones should never be clamped over the ears before the sound has become barely audible.

440. Fine Adjustment of Resistance.—The final adjustment of P and R may require variations of hundredths, or even thousandths, of ohms to give complete silence in the phones. Even when such accuracy is not required for the value of L, it is necessary to make the resistance balance very precise in order to obtain a sharp and definite balance point. Such small resistances are not available usually, but the same adjustment can be obtained in another way with ordinary resistances. Let a part of P be a fixed resistance of 20 ohms in parallel with a dial resistance box set at 180 ohms. The resistance of this combination is 18 ohms. After the other part of P is adjusted to the nearest ohm, this shunt of 180 ohms can be varied to change the total resistance by very small steps. When complete silence has been attained, the resistance of this portion of P is computed by the formula $x = ab/(a + b)$.

441. The Wagner Ground.—The Wagner[1] grounding circuit can be made similar to either ABD or AFD. In Fig. 217 the grounding circuit AJD is shown similar to AFD. By means of the double-pole, double-throw reversing switch Z, the telephone can be connected at J while P' and R' are adjusted to balance against ABD. During this adjustment the circuit AFD serves as the grounding circuit with F connected to the ground. Usually this Wagner grounding circuit need be adjusted only to the nearest ohm. When the switch is thrown back and the telephone is connected at F again, the final adjustment of P can be made.

442. Calibration of a Variable Standard of Self Inductance.—The bridge arrangement described above is conveniently used in the calibration of a variable self inductance. When the resistance balance, $PS = QR$, has been attained, there is a value of L that will give a balance for each given value of C. By the use of a subdivided standard condenser, several values of C can be obtained, and corresponding to each one the true value of L that will give a balance is PSC. The reading L' of the variable inductance when it is adjusted to give zero current through the telephones should equal this value. If it does not, the correction at this point on the scale is

$$c = PSC - L'.$$

If more points are desired for the calibration, a slightly different value of PS can be used, and then the same values of C will require that the inductance be set at different values to balance the bridge.

[1] WAGNER, *Elektrotechnische Zeitschrift*, vol. 32, p. 1001, 1911.

443. Measurement of a Small Self Inductance.—In the measurement of the self inductance of a coil by the bridge method, the inductances and capacitances of the various parts of the bridge and connections enter as unknown factors. When L is small, this uncertainty may introduce considerable error. By the method given above (Art. 437), this error can be reduced. When the balance has been obtained, let the small inductance L' be added to L and placed so that there will be no mutual inductance between it and either of the original coils. A corresponding resistance can be removed from R to maintain the d-c balance. It will now require a larger value of P to give the a-c balance, and the computed value of the self inductance will be larger than the former value by the amount L'.

444. Anderson's Method for Comparing a Self Inductance with a Capacitance.—In Anderson's modification of Maxwell's method, the effect of the condenser is varied without disturbing the resistance balance. The setup differs from Maxwell's method by having a resistance r in series with the condenser and then by connecting the galvanometer at H', Fig. 218, instead of at F'. This makes no difference with the resistance balance, but the effect of the condenser can be varied by changing the value of r.

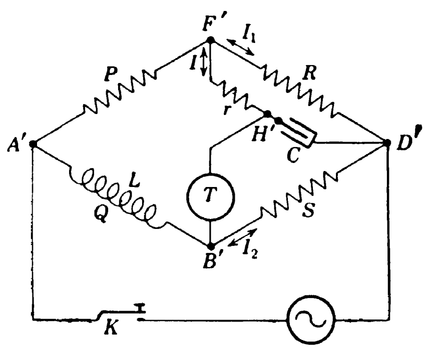


FIG. 218.—Anderson's method for comparing fixed values of C and L.

When r and R have been adjusted by varying first one and then the other until there is no current through the telephone branch, the potentials of B' and H' must always remain equal. Then

$$P(i_1 + i) + ri = Qi_2 + L\frac{di_2}{dt}, \tag{24}$$

where the currents are as shown in the figure. Similarly, for the other branches,

$$\frac{q}{C} = Si_2, \tag{25}$$

and

$$Ri_1 = ri + \frac{q}{C}. \tag{26}$$

Substituting in Eq. (24) the values of i_1 and i_2 from Eqs. (25) and (26) gives

$$\frac{P}{R}\left(ri + \frac{q}{C}\right) + (P + r)i = \frac{Q}{S}\frac{q}{C} + \frac{L}{SC}\frac{dq}{dt},$$

or

$$\left(\frac{P}{R} - \frac{Q}{S}\right)\frac{q}{C} + \left(\frac{Pr}{R} + P + r - \frac{L}{SC}\right)i = 0, \qquad (27)$$

since

$$\frac{dq}{dt} = i.$$

This equation can be satisfied when the coefficients of the variables, q and i, are each equal to zero, *i.e.*, when

$$PS = QR$$

and when

$$L = PSC + rC(S + Q), \qquad (28)$$

which reduces to Maxwell's value when $r = 0$.

In making a measurement of self inductance, the quantity PSC is set at a value that is slightly smaller than the estimated value of L. This leaves a relatively small quantity for the second term in Eq. (28), which is measured by the value of the dial resistance r that gives the balance with alternating current.

445. Vector Diagrams for Anderson's Method.—The vector diagrams for this case will show the changes that take place in the emfs of the bridge when the adjustments are being made.

The conditions in the unbalanced state are shown in Fig. 219 (*a*). The parallel branches between D' and F' give the emf triangles DHF and DF, with the currents I and I_1 as shown. Both of these currents flow through P, giving rise to PI and PI_1, the vector sum of which is AF. $AJBD$ represents the emfs along $A'B'D'$. By varying r, the line DH can be brought to coincide with DB. By varying S, BJ can be made to coincide with HE. When H and B are together, there is silence in the telephone.

When the bridge is balanced [Fig. 219(*b*)],

$$SI_2 = \frac{I}{C\omega} \qquad (29)$$

and

$$L\omega I_2 = rI + PI + (EJ). \qquad (30)$$

From similar triangles,

$$(EJ) = QI_2 \frac{rI}{I/C\omega} = QI_2 rC\omega, \tag{31}$$

and rewriting Eqs. (30) with the help of Eqs. (29) and (31) gives

$$L\omega I_2 = rSC\omega I_2 + PSC\omega I_2 + QrC\omega I_2,$$

from which

$$L = PSC + rC(S + Q).$$

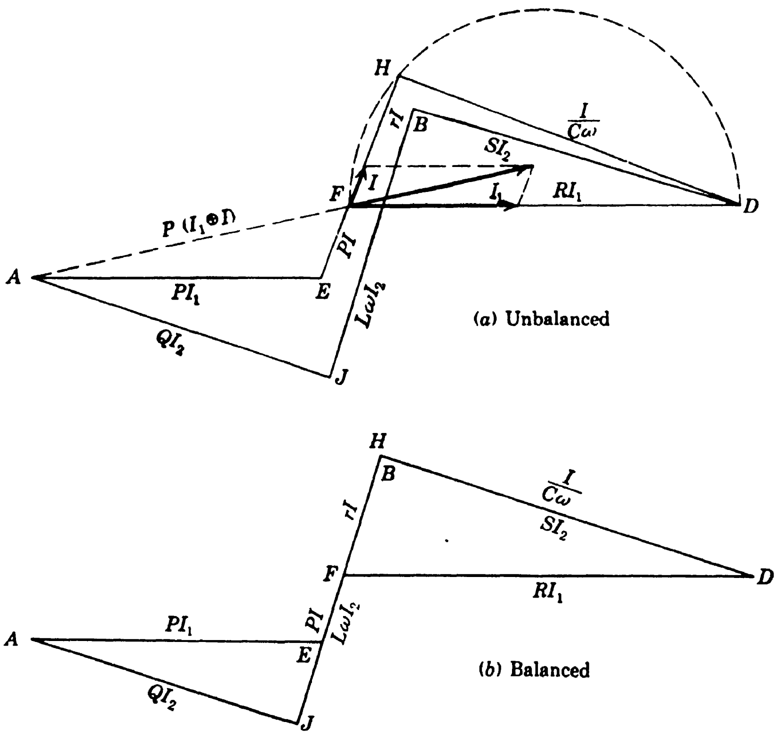


FIG. 219.—Emf diagrams for Anderson's method for comparing fixed values of C and L.

When $r = 0$, this arrangement becomes the same as for Maxwell's method, and B falls on F. For larger values of L, the addition of r makes it possible to bring H coincident with B. It is therefore desirable, to set PSC a little below the expected value of L and then adjust r to give the final balance.

446. The Emf of Mutual Inductance.—When there is an alternating current in one of the coils of a mutual inductance, there is an emf in the other coil. The value of this emf can be found from the following considerations.

Let the instantaneous value of the alternating current in the primary coil be written as

$$i = I' \sin \omega t.$$

The flux turns through the secondary coil due to this current will be

$$(\Phi n) = Mi,$$

where M denotes the value of the mutual inductance between the coils. The emf in the secondary coil is measured by the rate of change in this flux through it and is

$$e = \frac{d(\Phi n)}{dt} = M \frac{di}{dt},$$

or, using the value of i,

$$e = M \frac{di}{dt} = M\omega I' \cos \omega t,$$

The maximum value of this emf is $M\omega I'$, where I' is the maximum value of the primary current. The effective value is

$$E\ (\textit{effective}) = M\omega I,$$

where I is the effective value of the primary current.

In any circuit that may be connected to the secondary coil, this emf will produce a current in the same way as any other alternating emf. In most cases it is not necessary to refer to its relation to the primary current, except in its value of $M\omega I$. In a vector diagram containing both I and $M\omega I$, the vector $M\omega I$ would be drawn 90° ahead of I, the same as with $L\omega I$ in a similar diagram.

447. Direct Comparison of Two Mutual Inductances.—Mutual inductances are readily measured by comparing the emfs induced in their secondaries. If the current through the primary coils is direct, the induced emf will appear at the make and break of the circuit; while if alternating current is used, there will be an alternating emf in the secondary.

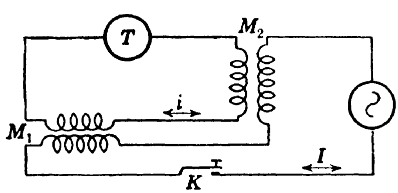


FIG. 220.—Comparison of two mutual inductances that can be adjusted to equality.

In the direct comparison of two mutual inductances, the two primaries are joined in series and connected to the source of current, preferably a low-voltage alternating circuit of audio frequency. The two secondaries are also joined in series, with the two induced emfs opposed to each other.

When the same current flows through the primary coils, the emfs induced in the secondaries are proportional to the mutual inductances of the coils. If the two secondaries are joined in series with a telephone, or other current detector, the effective[1] value of the secondary current is

$$i = \frac{M_1\omega I - M_2\omega I}{Z},$$

where Z is the total impedance[2] of the secondary circuit. If, now, the mutual inductance of one of the coils can be varied until the telephone shows by its silence that $i = 0$, we have

$$M_1 = M_2.$$

This method requires a variable standard of mutual inductance, and a telephone to indicate zero current in the secondary circuit. By turning the movable coil of the standard until the emf induced in it is just equal and opposite to that induced in the secondary of the other pair of coils, as shown by zero sound in the telephone, the value of the mutual inductance can be read directly from the standard.

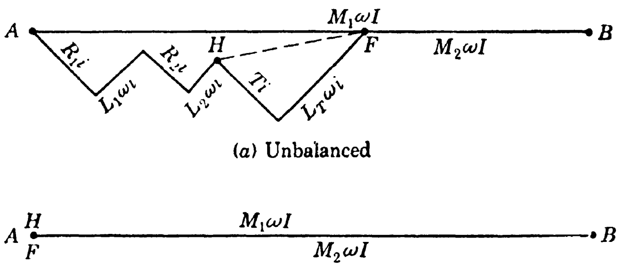
It is instructive to interchange the primary and secondary coils of one of the mutual inductances. It will be found that the value of the mutual inductance of this pair of coils is the same, whichever coil is used for the primary.

448. Vector Diagrams.—The vector diagrams for this method are shown in Fig. 221. In the unbalanced condition there is a current through the secondary circuit, and corresponding potential differences in each resistance and inductance. Across the telephone there is the voltage HF. When the mutual inductance M_2 is increased enough to give zero current in the telephone circuit, all the potential differences shown by AHF shrink to zero, and $M_1\,\omega I = M_2\,\omega I$.

It thus appears that only mutual inductances of the same value as the standard, that is, up to 10 mh, can be measured by this method. However, the range can be extended by adding to the variable standard another mutual inductance of fixed value. This is done by joining the primaries in series and also the secondaries, thus adding together the emfs induced in the two secondaries.

[1] See Arts. 240, 307, and 308.
[2] See Art. 313.

Because of the directness and simplicity of this method, it is less liable to errors than the more complicated arrangements. Care must be exercised that there is no mutual inductance between the primary of one pair of coils and the secondary of the other pair, by keeping them far apart and at right angles to each other.



(*a*) Unbalanced

(*b*) Balanced

FIG. 221.—Vector diagram for the comparison of two equal mutual inductances.

449. Compensation for the Impurity of a Mutual Inductance.—In the vector diagram of Fig. 221 (*b*), it is assumed that $M_1\omega I$ and $M_2\omega I$ are the only emfs in the secondary circuit and that at the balance point there should be absolute silence in the telephones in the secondary circuit. However, this is not always the case.

There is a certain amount of capacitance between the conductors that form the primary and secondary windings of a coil, and when there is a difference of potential between these conductors, a capacitance-current component is added to the primary current. This means that the primary currents in the two coils are not strictly in phase with each other, because a part of the current is used to charge this capacitance. Therefore $M_1\omega I_1$ is not exactly in opposite phase with $M_2\omega I_2$ and cannot entirely balance it.

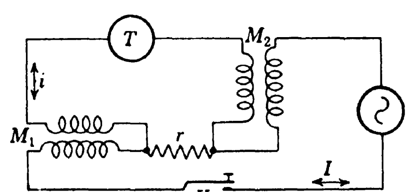


FIG. 222.—Compensation for the impurity of a mutual inductance.

In this case a better balance can be obtained by introducing into the secondary circuit a small emf which will differ in phase from $M_2\omega I_2$ by 90°. Such an emf is found across a small resistance in the primary circuit, as shown by *r* in Fig. 222. This resistance may be only a short length of wire in the primary circuit, depending upon the amount of the capacitance between the two windings. Joining the primary and secondary circuits as shown in Fig. 222 helps to reduce any large difference of potential between them.

450. Ballistic Galvanometer vs. Telephone.—The comparison of mutual inductances offers a good example of the peculiar distinctions between a telephone receiver and a ballistic galvanometer for indicating a condition of balance.

a. Using a Telephone and Varying Current.—In the discussion above it was supposed that a sinusoidal current was used, and therefore the emfs in the telephone circuit could be represented by a vector diagram. When the primary current is not sinusoidal but has any wave form, a balance can still be obtained. This is shown as follows:

Writing the instantaneous values of the potential differences for the complete secondary circuit when not in balance gives, at any instant t,

$$M'\frac{dI}{dt} - R'i - L'\frac{di}{dt} - Ti - L\frac{di}{dt} - M''\frac{dI}{dt} - R''i - L''\frac{di}{dt} = 0,$$

where T and L refer to the telephone.

Zero sound in the telephone indicates that there is no variation in the telephone current, and therefore $di/dt = 0$. Since the current in the coils is varying, the telephone current i must be zero, because $i = 0$ is the only amplitude for an alternating current when $di/dt = 0$. Substituting these values for i and di/dt in the equation above gives

$$M' = M''.$$

b. Using Direct Current and a Ballistic Galvanometer.—When a ballistic galvanometer is used to indicate the balance between the two mutual inductances, no information is obtained regarding the *current* in the secondary circuit. The deflection of the galvanometer measures the total quantity q that has passed through the galvanometer. In terms of the current i,

$$q = \int i dt.$$

Zero deflection indicates that the total quantity through the galvanometer in one direction is zero. An alternating current through the galvanometer gives zero deflection because there is as much quantity in one direction as in the other, giving a total of zero in one direction.

Integrating the above equation from the time the battery key is closed until the currents have become steady gives

$$M'I - R'q - Tq - M''I - R''q = 0,$$

where T denotes the resistance of the galvanometer, and q is written for $\int idt$.

For zero deflection of the ballistic galvanometer $q = 0$, and then

$$M' = M''.$$

Oftentimes this is a more sensitive method than using a telephone.

Problem.—Does it make any difference in the measured value of M' whether one of its coils or the other one is used as the primary?

451. Comparison of Two Unequal Mutual Inductances. *With Direct Current and a Ballistic Galvanometer.*—Let M be a pair of coils whose mutual inductance is known and M' another pair whose mutual inductance is desired. The primaries of the two coils are joined in series with a battery and key, the coils themselves being placed as far

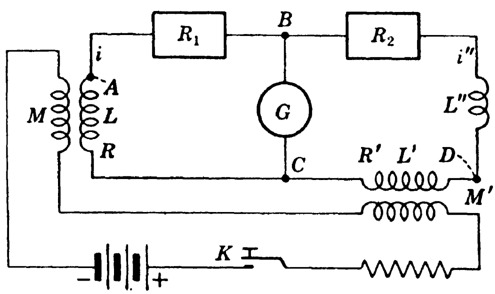


FIG. 223.—Comparison of two mutual inductances.

apart as possible and at right angles to each other, so that each secondary will be influenced only by its own primary. The two secondaries are also joined in series with two resistance boxes, and a galvanometer is connected across from B to C. This is not a Wheatstone bridge arrangement, for the two emfs in the two secondaries act together and the current flows through the four arms in series.

By properly adjusting R_1 and R_2, the galvanometer will give no deflection when K is opened or closed. Writing the instantaneous values of the potential differences for the circuit $CABC$ gives, at the instant t,

$$M\frac{dI}{dt} - Ri - L\frac{di}{dt} - R_1 i - Gi' - L_g\frac{di'}{dt} = 0,$$

and for $DCBD$

$$M'\frac{dI}{dt} - R'i'' - L'\frac{di''}{dt} + Gi' + L_g\frac{di'}{dt} - R_2 i'' - L''\frac{di''}{dt} = 0,$$

where L_g denotes the inductance of the galvanometer circuit.

Integrating each of these equations from the time when the key is first closed and $I = 0$, to the time when the primary current has reached its steady value $I = I_0$, gives

$$(R + R_1)q = MI_0, \quad \text{and} \quad (R' + R_2)q'' = M'I_0,$$

when q', the integrated current through the galvanometer, is zero. Dividing one equation by the other gives

$$\frac{M}{M'} = \frac{R + R_1}{R' + R_2},$$

since $q = q''$, when $q' = 0$.

In this method the galvanometer current may be zero, but in general it consists of two transient currents, each of which has an effect upon the galvanometer even when following one another in a short interval of time. This may produce an unsteadiness of the galvanometer and render an exact setting difficult, if not impossible. In order that no current should pass through the galvanometer, it is necessary that the potential difference between B and C shall remain zero for each instant while the primary current is changing, and this requires that the self inductance of each branch shall be proportional to the emf induced in that part of the circuit (see Fig. 225).

Usually this is not the case, but it is not difficult to add some self inductance L'' in the part of the circuit that is deficient and thus fulfill this condition. The galvanometer will then indicate a much closer balance, or it may be replaced by a telephone, with an alternating current used in the primaries as shown in Art. 452.

452. Comparison of Two Unequal Mutual Inductances. *With Alternating Current and a Telephone.*—When it is not possible to balance one mutual inductance against the other, as in the method of Art. 447, they can be compared by the arrangement shown in Fig. 224. The two secondaries are joined in series, helping each other, and connected to an external circuit consisting of two resistance boxes in series. The current through this circuit is due to the combined emfs of both coils.

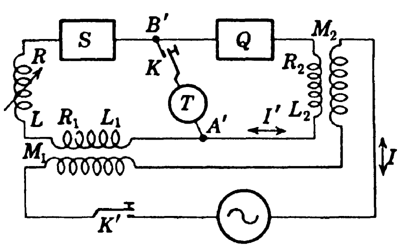
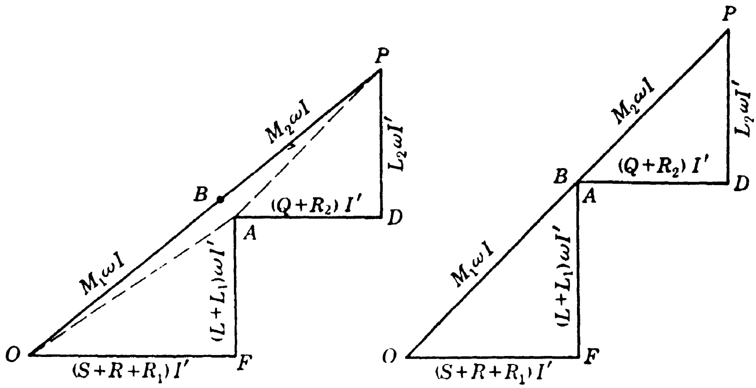


FIG. 224.—Comparison of two unequal mutual inductances. R_1 is the resistance in the secondary coil L_1. R_2 denotes the resistance in the secondary coil L_2.

This may also be thought of as one coil supplying the emf needed to keep the current flowing through its part of the circuit and the other emf being used to keep the current flowing through the rest of the circuit. If these portions of the circuit can be measured, the relative values of the emfs are determined.

453. The Vector Diagrams.—Since the two primaries are in series and carry the same current, the emfs induced in their secondaries will be in phase with each other. When K is open, there is only a single current through the secondary circuit. The emf diagram for the secondary circuit is then as shown in Fig. 225(*a*).

Starting at O, the vector $(S + R + R_1)I'$ is laid off horizontally to the right, where $(S + R + R_1)$ denotes the total resistance from B' to A' through the secondary coil L_1. Since $(L + L_1)\omega I'$ is 90° ahead of $(S + R + R_1)I'$, it is drawn as FA.



(*a*) Unbalanced (*b*) Balanced

FIG. 225.—Emf diagrams for two mutual inductances in series.

Likewise $(Q + R_2)I'$ is drawn parallel to OF as AD, where $(Q + R_2)$ denotes the total resistance in the secondary coil L_2 and the added resistance box Q. Then $L_2\omega I'$ is drawn as DP. These four vectors from O to P show the components of the emf that is required to maintain the current I' in the combined secondary circuit of the two coils. The emf available for this purpose is $M_1\omega I + M_2\omega I$. Therefore this is also drawn as OP.

The diagram for the unbalanced case shows that $M_1\omega I$ is not quite enough, either in phase or in amount, to maintain the current in its part of the circuit, lacking by the voltage shown as AB. The telephone will respond to this voltage. When S and L are adjusted to give zero sound in the phones, $M_1\omega I$ is just enough to maintain the current I' in its part of the circuit from B around to A, while $M_2\omega I$ will maintain the current through the rest of the circuit.

By varying S and L, the point A can be brought to coincide with B. For this case no current will flow through a telephone joined from A to B; and therefore this diagram, Fig. 225(b), corresponds to the arrangement when there is silence in the telephone. The two triangles are similar and give the relation

$$\frac{S + R + R_1}{Q + R_2} = \frac{M_1}{M_2} = \frac{L_1 + L}{L_2}.$$

If S and Q are very large resistances, a good balance can be obtained even when L_1 and L_2 are neglected, but in arranging the circuit it is best to fulfill this relation as nearly as possible. For the best balance a variable self inductance should be included in the secondary circuit and adjusted as may be found necessary to reduce the sound from the telephone.

454. Comparison of Two Unequal Mutual Inductances. *Divided-potential Method.*—A simple and convenient arrangement which is used at the Bureau of Standards is shown in Fig. 226. The two primaries are in series, and therefore each carries the same current. The emf induced in the secondary coil of the smaller mutual inductance M_1 is balanced against a part of the emf from the secondary coil of the larger mutual inductance M_2. This balance is not true at each instant, but it is an integrated balance and is shown by a ballistic galvanometer. One of the advantages of this method is that the resistance of the secondary coil in the smaller mutual inductance need not be known.

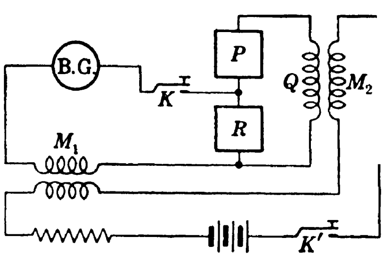


FIG. 226.—Comparison of unequal mutual inductances by the divided-potential method.

When the equations of potential differences in the two secondary circuits have been integrated, as was done in Art. 451, the ratio of the two mutual inductances is found to be

$$\frac{M_1}{M_2} = \frac{R}{P + Q + R},$$

where Q is the resistance of the secondary coil of M_2.

455. The Divided-potential Method Using Alternating Current.—When alternating current is used in place of the battery shown in Fig. 226 it is found impossible to obtain a balance. This is because the current in the circuit of RPQ is out of phase with the induced emf, due to the self inductance in the coil Q. By adding a self inductance L to R the emf over this part of the circuit can be advanced in phase. It is then possible to obtain a good balance against the voltage induced in the secondary coil of M_1, using the arrangement shown in Fig. 227.

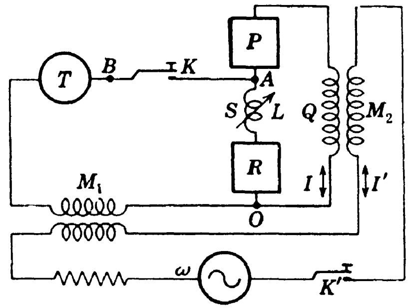


FIG. 227.—Comparison of unequal mutual inductances by the divided-potential method using alternating current.

456. Vector Diagrams for Comparing M_1 with M_2 by the Divided-potential Method.—The vector diagram for the circuit RPQ, Fig. 227, is shown in Fig. 228(a). The components of the emf required to maintain the current I are laid off in order, giving $ODAJH$. The total voltage OH is $M_2\omega I'$ induced in the secondary coil of the larger mutual inductance M_2.

The induced voltage $M_1\omega I'$ in the other coil is in phase with $M_2\omega I'$ since there is the same current I' in the primary coils. Therefore it is shown as OB, Fig. 228(a). In the unbalanced case this is not quite

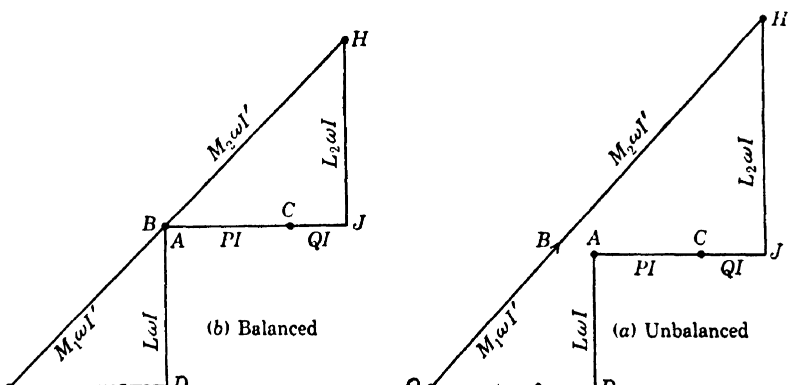


FIG. 228.—Vector diagrams for the divided-potential method.

the same as OA in the main circuit, the difference, BA, appearing across the key K, Fig. 227, when this is open. When K is closed, there will be current in the telephone T.

By changing L and R, the telephone can be brought to silence. This means that A has been moved to coincide with B, as is shown in

Fig. 228(*b*) for the balanced case. From similar triangles in the balanced condition we have

$$\frac{M_1}{M_2} = \frac{R + S}{R + S + P + Q}.$$

457. Measurement of Mutual Inductance with an Alternating-current Bridge.—The mutual inductance of two coils can be measured by the arrangement shown in Fig. 229. When variable inductometers of suitable range are available, the method is not as laborious as might appear, because each of the three balances is made independently.

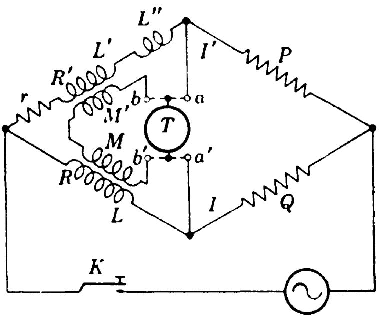


FIG. 229.—Measurement of a large mutual inductance M' with a small variable standard M.

The primary coils of the standard and of the inductance to be measured are connected in two arms of the bridge as shown. A variable self inductance L'' and a resistance r are included with the primary of the larger mutual inductance. The noninductive arms, P and Q, are set at the ratio approximately estimated for M'/M.

The first balance is made with direct currents by varying r until

$$\frac{R'}{R} = \frac{P}{Q},$$

where R and R' denote the total resistances in their respective arms of the bridge.

The second balance is made with 1,000-cycle alternating current by varying L'' until there is no current through the telephone. Then

$$\frac{L' + L''}{L} = \frac{P}{Q}.$$

(With sufficient skill and practice on the part of the operator, both of these balances can be obtained with the alternating current, by adjusting r and L'' alternately until complete silence is obtained.)

When thus balanced, the current will divide in the ratio

$$\frac{I'}{I} = \frac{Q}{P}.$$

The two secondaries are now connected in series with the telephone and in opposition to each other, and the standard mutual inductance is varied to give a balance. Then

$$M \frac{dI}{dt} = M' \frac{dI'}{dt}$$

and

$$M' = M \frac{P}{Q}.$$

458. Comparison of Mutual Inductances Using Direct Current. When variable inductances and alternating currents are not available, mutual inductances can be compared by the direct-current method

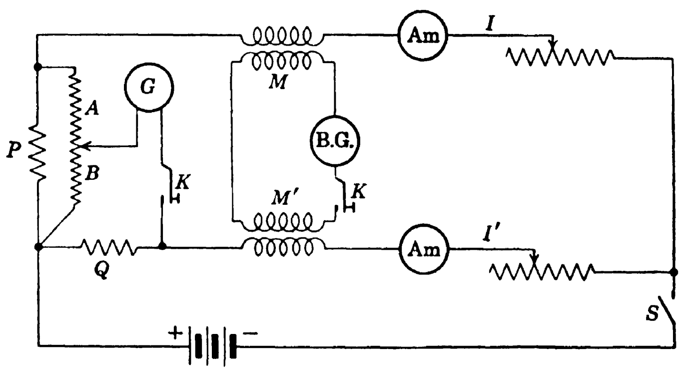


FIG. 230.—Comparison of two mutual inductances using direct currents.

shown in Fig. 230. Each primary coil is connected in a separate circuit with a rheostat for changing the current and an ammeter for reading the value of the current. The two secondaries are connected in opposition to each other and in series with a ballistic galvanometer or a fluxmeter. The currents I and I' are adjusted to values inversely proportional to the estimated values of the inductances, and the deflection of the galvanometer is observed when the switch S is opened. For zero deflection,

$$MI = M'I'$$

when these values of the currents are read from the ammeters just before the switch is opened.

For a more exact comparison of the currents, their ratio can be determined by the potential divider AB. P and Q are two accurate resistances of an ohm or two and capable of carrying the currents. In parallel with the larger fall of potential is placed the shunt circuit

AB, which may consist of two good resistance boxes. For zero deflection of G, the fall of potential over B will be equal to that over Q. Then

$$P(I - i) = (A + B)i$$

and

$$QI' = Bi,$$

which give

$$\frac{M}{M'} = \frac{I'}{I} = \frac{P}{Q}\frac{B}{A + B + P}.$$

459. Comparison of the Mutual Inductance between Two Coils with the Self Inductance of the Secondary.—A convenient method for measuring mutual inductance in terms of a self inductance is shown in Fig. 231. The self inductance is placed in one arm of the Wheatstone bridge, which ordinarily would make it impossible to obtain a balance because of the emf $L\omega I_1$, in this coil. In the present arrangement the main current passes through the other coil, and this induces an emf of $M\omega I$ in the first coil $A'C'$. In case $M\omega I$ is just sufficient to balance $L\omega I_1$, the effect of the latter is neutralized and the coil behaves like a noninductive resistance. The bridge can then be balanced without difficulty.

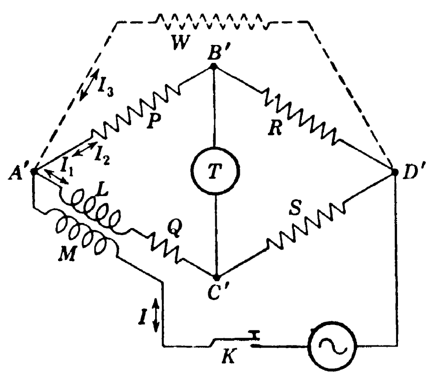
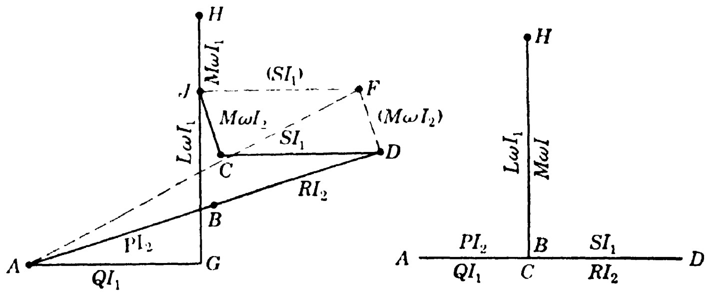


FIG. 231.—Comparison of the self inductance L, with the mutual inductance M.

460. Drawing the Vector Diagram.—The emf vector diagram for the circuit shown in Fig. 231 is shown in Fig. 232. Starting at A, the vector QI_1 is drawn for the resistance part of the bridge arm $A'C'$. The self inductance of this arm gives the vector $L\omega I_1$, which is drawn as GH, 90° ahead of QI_1. In addition there are other voltages in this coil due to the current in the primary of the mutual inductance M. Since these differ in phase, they are best considered separately.

A part of the current in the primary of M is I_1, the current in Q. Therefore the emf induced by this part of the primary current is $M\omega I_1$ and it will be in phase with $L\omega I_1$, or opposite to it, depending upon which end of the primary coil is connected to A'. Only the latter case will lead to a balance, and for this case $M\omega I_1$ is drawn downward from H to J. This leaves the other component, $M\omega I_2$, of induced emf in the arm $A'C'$, which is to be drawn at right angles to I_2. But neither I_2 nor PI_2 is yet drawn. So, leaving $M\omega I_2$ for the present, we can draw SI_1 as shown by the dotted line JF.

To complete the diagram from F back to A there remain the vectors $(P + R)I_2$ and $M\omega I_2$. These are at right angles to each other, and both together make FA. Therefore on FA as a hypotenuse the right-angled triangle FDA is constructed. This locates the point D, and now the heavy lines JC and CD can be drawn, putting $M\omega I_2$ and SI_1 in their proper places parallel to the dotted lines. The line AD is divided at B into PI_2 and RI_2.



(*a*) Unbalanced (*b*) Balanced

FIG. 232.—Emf diagrams for comparing L and M.

As M is increased, the point C moves nearer to B, until at the balance point they coincide and the diagram (*a*) changes into that of (*b*).

461. The Relation between M and L.—From Fig. 232(*b*), it is seen that the condition for a balance is

$$L\omega I_1 = M\omega I$$

in addition to the usual relation

$$\frac{P}{Q} = \frac{R}{S}.$$

When M can be varied continuously by moving one coil with respect to the other, the balance is easily obtained. The value of M giving this balance is, then,

$$M = L\frac{I_1}{I} = L\frac{R}{R + S}.$$

In case M cannot be varied, it is still possible to obtain a balance by changing the current, I, through the primary coil. Ordinarily, this current is the total bridge current, $I_1 + I_2$, but by the use of another resistance W in parallel with the bridge, the current I can be increased without unbalancing the bridge. Then

$$M = L\frac{I_1}{I} = L\frac{1}{1 + \frac{S}{R} + \frac{Q + S'}{W}},$$

from the laws of parallel circuits. In the diagrams of Fig. 232, $M\omega I_2$ will now be replaced by $M\omega(I_2 + I_3)$.

462. Measurement of a Mutual Inductance in Terms of a Known Capacitance. *Carey Foster's Method Using Direct Current.*—In this arrangement, shown in Fig. 233, one pair of coils is replaced by a condenser, and the transient current from the secondary coil S is balanced against the current that is charging the condenser C.

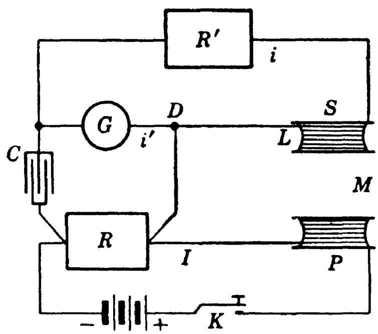


FIG. 233.—Comparison of a capacitance and a mutual inductance.

If the condenser were removed, there would be a current from S through R' and the galvanometer each time the key in the primary circuit is closed, and a current in the opposite direction when the key is opened. With the condenser in place, this current arrives at C just in time to charge the condenser, or to help charge it. The final charge in the condenser is

$$Q = C \times V = CRI, \tag{32}$$

where I is the final steady current through R. The amount of this charge is the same whether there is a current through the galvanometer or not. By adjusting the resistance in R', the current through it can be made too small, too large, or just sufficient to supply the charge in the condenser, the rest of the charge coming directly through the galvanometer. When there is no deflection of the galvanometer, the adjustment is complete, and the arrangement is said to be balanced.

Writing out the sum of the various potential differences in the circuit through $SR'G$ gives

$$M\frac{dI}{dt} - L\frac{di}{dt} - (S + R')i + Gi' + L_g\frac{di'}{dt} = 0. \tag{33}$$

Integrating this between the limits of time, t', just before K is closed, to t'', when the primary current has reached its steady value I, gives

$$MI - (S + R')Q' + Gq = 0, \tag{34}$$

where Q' is the total quantity that has passed through R', and q is the quantity through the galvanometer. Then $Q' + q = Q$, the total charge in the condenser. But if the galvanometer deflection is zero, then $q = 0$, and

$$MI = (S + R')Q = (S + R')CRI,$$

from Eq. (32). Hence

$$M = (S + R')CR.$$

It is to be noted that the galvanometer current is not required to be zero for a balance, and in fact it usually is not zero for each instant from t' to t''. Zero deflection merely indicates that the algebraic sum of the *quantities* passing through the galvanometer is zero. Nevertheless, the more nearly the galvanometer currents are equal to zero at each instant, the steadier will be the zero deflection at the balance.

463. Modification of the Carey Foster Method.—In Art. 464 and Figs. 234 and 235 it is shown that there can be a balance at each instant, with zero current through the galvanometer. This is accomplished by adding a resistance, N, in series with the condenser. Then for the condenser circuit, with the notation of Fig. 233,

$$Ni + \frac{q}{C} - R(I - i) = 0,$$

or by differentiation,

$$N\frac{di}{dt} + \frac{i}{C} - R\frac{dI}{dt} + R\frac{di}{dt} = 0. \tag{35}$$

For the circuit through S,

$$(R' + S)i + L\frac{di}{dt} - M\frac{dI}{dt} = 0. \tag{36}$$

Eliminating dI/dt from Eqs. (35) and (36) leaves

$$\left(\frac{N + R}{R} - \frac{L}{M}\right)\frac{di}{dt} + \left(\frac{1}{RC} - \frac{R' + S}{M}\right)i = 0.$$

This equation is satisfied for all values of i and di/dt when

$$\frac{N + R}{R} = \frac{L}{M}$$

and

$$\frac{1}{RC} = \frac{R' + S}{M},$$

or when

$$M = (R' + S)CR$$

and

$$N = R\frac{L - M}{M} = \frac{L}{(R' + S)C} - R.$$

The first of these conditions is the same as was found in the preceding article. The second condition gives the value of the resistance that must be placed in series with the condenser to give zero current through the galvanometer at all times. It is convenient to obtain the final balance by varying the value of the mutual inductance M.

464. Comparison of a Mutual Inductance with a Capacitance, Using Alternating Current.—One of the most satisfactory methods for comparing a mutual inductance with a standard condenser is a modification of the Carey Foster method so that the arrangement remains balanced at each instant. In this case alternating currents and a telephone receiver can be used. The principal change is in the addition of a resistance, N, in series with the condenser, as shown in Fig. 234.

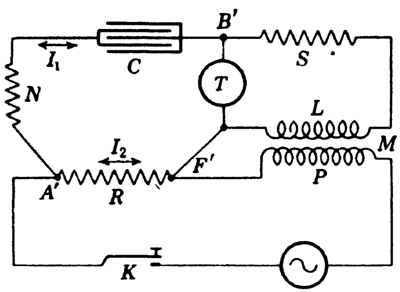


FIG. 234.—Comparison of a capacitance with a mutual inductance, using alternating current.

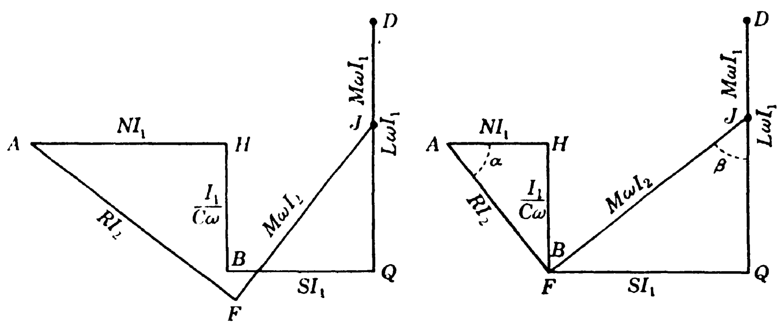
In this method the induced emf in the secondary of a pair of coils is made to supply the varying charge in the condenser. The main current divides at A', the part I_2 passing through R, and the other part, I_1, flowing through the condenser C, the resistances N and S, and the coil of self inductance L. The resistance of the secondary coil is included in the value of S, and N includes whatever equivalent resistance there may be in the condenser. At F' the currents unite and pass through P, the primary of the mutual inductance M.

The conditions in this circuit are shown by drawing the diagram for the various emfs, taking these in order along the path $A'B'SF'A'$.

Starting with N, and laying off the voltage NI_1 as shown in Fig. 235, the voltage over the condenser would be drawn as HB. The voltage over S, which includes the fall of potential in the secondary coil, is drawn parallel to NI_1, since these are in phase, and $L\omega I_1$, is 90° ahead, as shown by QD. There is also in this secondary coil the emf induced by the primary current. This emf is in two parts, which are not in phase with each other. The part of the current, I_1, that came through S and L continues on through the primary coil and induces in the secondary an emf of the amount

$$e = M\frac{di_1}{dt} = M\omega I_1' \cos \omega t.$$

where I_1' denotes the maximum value of the current I_1.



(a) Unbalanced (b) Balanced

FIG. 235.—Emf diagrams for the comparison of C and M.

The value of this emf is, then, $M\omega I_1$, and since it is a cosine term, it is either in the same phase as $L\omega I_1$ or directly opposite to $L\omega I_1$, according to the direction that I_1 flows through the primary coil.

The other component of the current, I_2, also flows through the primary coil and gives rise to an emf of $M\omega I_2$ in the secondary, which differs by 90° from RI_2. Since I_2 is behind I_1 by the angle α, $M\omega I_2$ is behind $M\omega I_1$ by the same angle.

Inasmuch as it is desired to obtain a balance, with no current through the telephone circuit, it will be necessary to connect the primary coil into the circuit in the direction that gives $M\omega I_1$ opposed to $L\omega I_1$, and $M\omega I_2$ an angle α behind, as shown in the figure by DJF. The current through R will be behind the condenser current I_1, and therefore RI_2 is represented by AF. As shown above, AFJ is a right angle. For zero emf across the telephone, it is seen that F must coincide with B. As N is decreased, the part $HBQJ$ (Fig. 235) increases while the right-angled lines JFA connect across from J to A, and F approaches B. As S is varied, B is moved along BQ and can be brought nearer to F. By adjustment of first one and then the other, B can be made to coincide with F, and the telephone shows this balance by silence.

When a balance is obtained, there are two similar triangles in the diagram, and from these,

$$\frac{M\omega I_2}{RI_2} = \frac{SI_1}{I_1/C\omega} = SC\omega = \frac{M\omega}{R}$$

or

$$M = SRC.$$

The value of N necessary for this balance can be determined as follows: From the diagram,

$$\frac{NI_1}{RI_2} = \frac{L\omega I_1 - M\omega I_1}{M\omega I_2}$$

and

$$N = R\frac{L - M}{M}.$$

If the rest of the circuit is well insulated, it may improve the balance to have the point F' grounded.

465. Measurement of the Power Factor and Capacitance of a Condenser by Comparison with a Mutual Inductance.[1]—Another method for comparing a capacitance with a mutual inductance is derived from the Carey Foster method. The positions of the telephone and the source of alternating current are exchanged, and by using a variable self inductance, a balance can be obtained without adding an auxiliary resistance in series with the condenser. While this arrangement can hardly be called a bridge method, the drawing of Fig. 236 has been made to resemble the familiar Wheatstone bridge in order to render simple some of the following explanation of its use.

The unknown condenser C, with its equivalent resistance represented as N, is made one arm of this network. Nothing else is put into this arm to introduce uncertain amounts of residual capacitance or inductance. A second arm, $B'H'$, is an adjustable noninductive resistance of R ohms. The circuit $A'J'H'$, analogous to the other two arms of a Wheatstone bridge, consists of a resistance S and a self inductance of $L + L'$ henrys. In order to continue the analogy of a

[1] Adapted from a paper by Arthur W. Smith, *Rev. Sci. Inst.*, vol. 4, p. 280, 1933.

Wheatstone bridge the telephone should be connected across from B' to a point J' on the branch $A'J'H'$. Of course it is not possible to locate in an actual coil such a point as J', which will have all of the resistance S of the coil in the part $A'J'$ and only inductance in the part $J'H'$, but the same effect can be obtained in another way. If $P'H'$ and $J'H'$ are two coils having a mutual inductance of M henrys equal to the self inductance L' of the coil $J'H'$, there will be induced in $P'H'$ an emf of $M\omega I$ volts, which is equal to the emf $L'\omega I$ that is induced in $J'H'$ due to the current I through $A'J'H'$. When there is no current through $P'H'$, the resistance of this coil has no effect, and the point P' has the same potential that J'would have if $J'H'$ had zero resistance. Therefore the telephone can be connected at P', and it will show by its silence when a balance has been obtained.

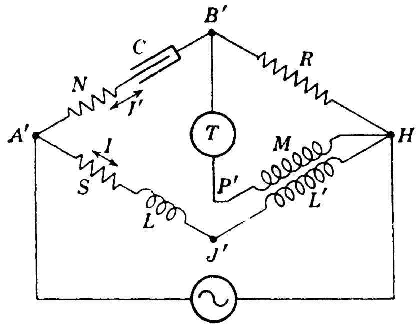


FIG. 236.—To measure the power factor and the capacitance of the condenser C.

For the sake of explanation, the two inductances L and L' have been treated as separate and distinct, although both are in series in the same circuit $A'J'H'$. The only necessary requirement is that L can be reduced fully to zero, and therefore L' must include the inductance of the primary coil of M, the inductances of the various parts of the rest of the circuit $A'J'H'$, and something more than the minimum inductance of the variable inductor L. All this, to an amount equal to M, is the L' in this discussion. Whatever inductance is over this amount is L. The zero position on the scale of this variable self inductance is obtained by using a condenser of zero (or known) power factor. For another condenser with a power factor larger than zero, the balance is obtained by increasing the value of L.

466. The Vector Diagrams.—The relations of the various emfs to each other are made clear by considering the vector diagram for the network shown in Fig. 236 when a harmonic alternating current passes through it. For the arm $A'B'$ there are the two component emfs NI' and $I'/C\omega$ at right angles to each other as shown in Fig. 237. For $B'H'$ the emf RI' is drawn parallel to NI'. The emfs in the branch $A'J'H'$ are shown by SI, $L\omega I$ and $L'\omega I$, Fig. 237. The additional emf in the coil $P'H'$ is $M\omega I$, and this is equal and opposite to $L'\omega I$. For zero current in the telephone, B and P must coincide as shown in Fig. 238. In this balanced condition,

$$NI' = L\omega I \tag{37}$$

and

$$RI' = M\omega I. \tag{38}$$

Therefore

$$N = \frac{LR}{M}. \tag{39}$$

Equating the vertical vectors of Fig. 238 gives

$$SI = \frac{I'}{C\omega}, \tag{40}$$

which combined with Eq. (38) gives

$$C = \frac{M}{RS} \tag{41}$$

for the capacitance of the condenser being measured.

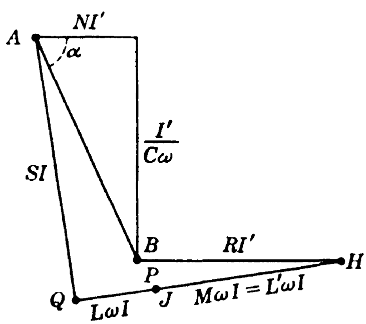


FIG. 237.—Vector diagram when the bridge shown in Fig. 236 is not quite balanced. The points B and P must coincide with each other to give silence in the telephone.

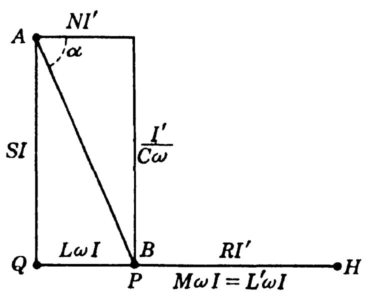


FIG. 238.—Vector diagram when the bridge shown in Fig. 236 is balanced. For a mica condenser QP is about 1/10,000 of QH.

467. Substitution Methods.—In each of the methods described above, the value of the unknown quantity is given in terms of a standard of the same kind, and certain resistances. If these resistances are accurate to only 1 part in 1,000, the final result cannot be more accurate than this amount, no matter how good the balance may be. However, by using a substitution method, it is possible to obtain results that are more accurate than the resistance boxes that are used in the measurement. Thus in the preceding article, the value of the unknown condenser is given by $C = M/RS$. Now let a second balance be made with a known condenser C', using the same M and S with all of the arrangement undisturbed. If C' is about the same value as C, the new balance can be attained by slight changes in R and L, say to $R + r$ and $L + l$. Then

$$C' = \frac{M}{(R + r)S} = C\,\frac{R}{R + r},$$

from Eq. (41), and

$$C = C'\,\frac{R + r}{R}.$$

The secret of increased accuracy lies in the manipulation of the resistance R. If the part of $R + r$ that is denoted by R consists of *precisely the same resistance coils* that were used for the first balance, then $R \equiv R$, even though the value of R may not be exactly what it is marked in the resistance box. Then

$$C = C' + C'\,\frac{r}{R},$$

where the first term is as good as the standard, and the second term is as uncertain as the resistances r and R. When r/R is a small fraction, this second term is only a small part of C, and the uncertainty in the exact value of the resistances used for r and R will have a correspondingly small effect on the total value of C.

Example: Suppose $M = 50$ mh; $S = 40$ ohms; and a balance is obtained when R is adjusted to the value that is marked 1,248.4 ohms. If the resistance box is uncertain by 0.1 per cent, this value of R is uncertain by 1.2 ohms due to the incorrect values of the resistance coils that are used for R. This gives

$$C = 1.0013\ \mu\text{f},$$

which is uncertain by 0.0010 μf, owing to the uncertainty in the value of R besides the uncertainties in the values of M and S.

Now let a condenser of known value, say $C' = 0.9945\ \mu$f, be substituted for C and a new balance obtained with, say, $R' = 1{,}265.8$ ohms, obtained by *adding* 17.4 ohms to the *identical coils* that were used for R. Then

$$\begin{aligned} C &= C' + C'\,\frac{17.4}{1{,}248.4} \\ &= 0.9945 + (0.01386 \pm 0.00002) = 1.0084\ \mu\text{f}. \end{aligned}$$

Thus the uncertainty of the resistance coils that are used for R does not affect the first five figures of this result, and the actual values of M and S do not enter provided they remain constant during the measurement of C.

Practically the same result would be attained if only the 1,200-ohm coils of R had kept unchanged and the 48.4 ohms had been changed to the 65.8 ohms.

468. Power Factor of a Condenser. *Substitution Method.*—The power factor of a condenser is given by cos α, Fig. 238, and since α is very nearly 90°, it is allowable to write

$$F = \cos \alpha = \cot \alpha = NC\omega.$$

Using the values of N and C from Eqs. (39) and (41) gives the simple expression

$$F = \frac{L\omega}{S}$$

for the power factor of the condenser. However, the value of L is difficult to determine both because it is small and also because of the uncertainty of the division point between L and L'. The latter difficulty is avoided in the method of substitution, Fig. 239.

Let a balance be first obtained with a standard condenser whose power factor, F_s, is known. Let this value of $L + L'$ be denoted by L_s. When the unknown condenser, C_1, is put in the place of the standard, C_2, it will be necessary to change R and L to restore the balance. Let L_x denote the new value of $L + L'$. Then the change in L has been

$$\Delta L = L_x - L_s, \qquad \text{henry}$$

and the value of ΔL can be accurately measured without knowing the zero point on the scale of L.

The power factor for the standard condenser is

$$F_s = \frac{(L_s - L')\omega}{S}.$$

and for the unknown condenser

$$F_x = \frac{(L_x - L')\omega}{S},$$

The difference in the power factors of the two condensers is

$$\Delta F = F_x - F_s = \frac{(L_x - L_s)\omega}{S} = \frac{\Delta L\omega}{S}.$$

This gives for the power factor of the unknown condenser

$$F_x = F_s + \frac{\Delta L\omega}{S}.$$

469. Calibration of the Variable Self Inductance.—When the variable self inductance L is an inductometer of several millihenrys it is not possible to read its scale close enough to determine ΔL with any accuracy. By attaching a long pointer, a meter or more in length, to the knob or handle that turns the movable coil of L, and letting the end of this pointer move over a millimeter scale, it is easy to make very fine readings of ΔL.

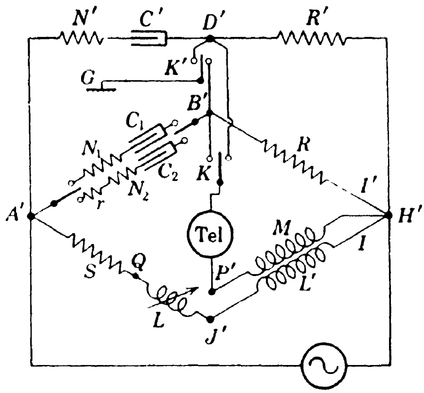


FIG. 239.—Electrical circuits of the alternating-current bridge for comparing the condensers N_1C_1 and N_2C_2. The Wagner ground circuit $A'D'H'$ is made similar to the bridge circuit $A'B'H'$, by adjusting R' when the switches K and K' are thrown to the right. The small resistance r is used only for the fine calibration of the scale of L.

Evidently this millimeter scale must be calibrated in terms of henrys before it can be used to measure ΔL. By adding a small known resistance r, Fig. 239, in series with the known condenser C_2, the power factor of this branch of the circuit will be increased from $N_2C_2\omega$ to $(r + N_2)C_2\omega$. This change in the power factor corresponds to a change in L of

$$\Delta L_r = \frac{S\Delta F}{\omega} = SrC_2.$$

If the end of the long pointer must be moved d mm along the millimeter scale to restore the balance when r is added, then each millimeter of this scale corresponds to

$$\frac{\Delta L_r}{d} = \frac{SrC_2}{d} \quad \text{henrys.}$$

470. Grounding the Circuit.—In order to obtain no sound in the telephone at the balance point it may be necessary to ground the circuit, as shown in Fig. 239. The terminals of the source of emf, $A'H'$, are connected to a third circuit, $N'C'R'$, which is grounded at D'

when the switch K' is closed to the left. This circuit is made similar to N_1C_1R by obtaining a balance with both K and K' closed to the right. The telephone is now connected at D', and R' can be adjusted for a balance.

When K and K' are thrown back to the left the telephone branch $B'P'$ will remain at zero potential. For a better balance the telephone current can be amplified as shown in Fig. 244.

471. Measurement of Frequency.—A simple and direct method for measuring the frequency of an alternating current is shown in Fig. 240(a). The primary of a variable mutual inductance M is joined in

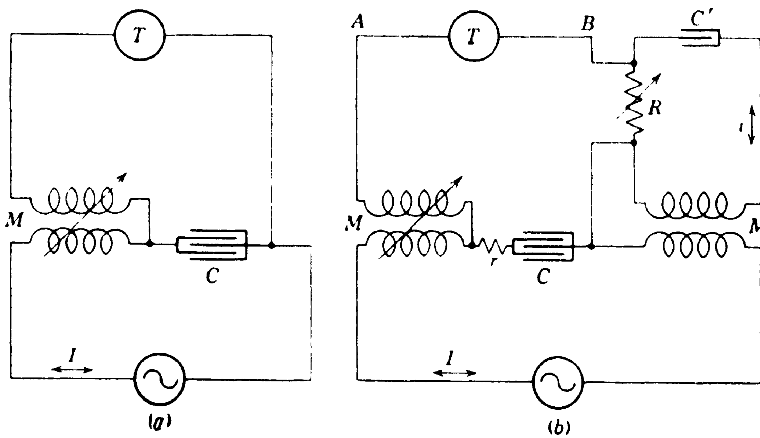


FIG. 240.—Arrangement for measuring the frequency of an alternating current.

series with a condenser C and connected to the source whose frequency is desired.

The secondary coil of the mutual inductance is joined in series with a telephone or other detector and connected to the terminals of the condenser. The emf across the condenser is $I/C\omega$, and when properly connected is in opposition to $M\omega I$ in the secondary coil. When M is adjusted to give minimum sound in the telephone,

$$M\omega I = \frac{I}{C\omega} \quad \text{and} \quad \omega = \frac{1}{\sqrt{MC}}.$$

Usually a balance can be obtained as sharply as the value of M can be read on the scale of the mutual inductance.

If there is a residual hum in the telephone, it is probably due to the power loss in the condenser between the two points where the telephone circuit is connected. This introduces an unbalanced emf rI into the telephone circuit. If a sharper balance is desired, another emf Ri can be introduced into the telephone circuit, as shown in Fig. 240(b). C' is a small capacitance and R is a dial resistance that can be varied to make Ri equal to rI.

The vector diagram of the emfs in the telephone circuit when not quite balanced is shown in Fig. 241. For a balance A and B are brought together by varying M and R.

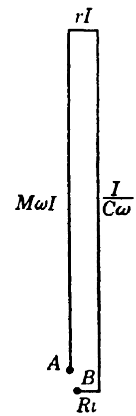


FIG. 241.—Vector diagram for Fig. 240 when not quite balanced.

472. A Frequency Bridge.—In most of the arrangements for measuring inductance and capacitance, the balance does not depend upon the frequency of the alternating current except insofar as the quantity being measured varies with the frequency. There are a number of bridge methods, however, in which the frequency is one of the quantities contributing to the balance. When the other factors are known, these methods may be used for the measurement of the frequency.

Thus in Maxwell's method an inductance in one arm is balanced by a capacitance in the opposite arm of the bridge. The balance could have been effected with the condenser in the same arm as the coil, as shown in Fig. 242, and with the other arms of noninductive resistances. This means that the inductance and the capacitance must be adjusted

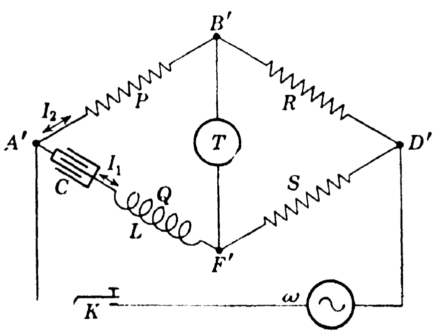


FIG. 242.—Frequency bridge.

to make the resultant effect of the first arm noninductive also. The diagrams for the unbalanced and the balanced conditions are shown in Fig. 243, and are self-explanatory.

For the balance

$$L\omega I_1 = \frac{I_1}{C\omega},$$

from which

$$\omega = \frac{1}{\sqrt{LC}},$$

and the bridge is not balanced for other frequencies. Therefore, with a complex wave form, the harmonics will still be heard in the telephone.

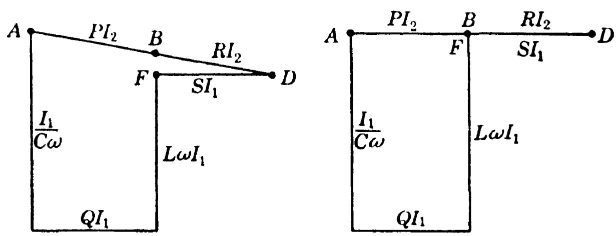
Since the frequency, n, is given by the relation

$$\omega = 2\pi n,$$

its value is

$$n = \frac{1}{2\pi\sqrt{LC}}.$$

A frequency bridge can thus be made with fixed resistances and a variable self inductance. In addition to the regular scale, the inductance can be calibrated to read directly the value of the frequency. By using a few fixed values of C, the use of the bridge can be extended to a wide range of frequencies.



(a) Unbalanced (b) Balanced

FIG. 243.—Emf diagrams for the frequency bridge.

473. Use of the Frequency Bridge.—In the measurement of the frequency of an actual source of alternating current, there are several modifications that may be made in the assembling of the apparatus to make the balance point of the bridge more sharp and definite. These are shown in Fig. 244.

a. Grounding the Telephone Circuit.—A Wagner ground circuit, consisting of two resistances, W and N, set in the same ratio as P and R, is usually essential.

b. Reducing the Higher Frequencies.—The original source often contains several frequencies which are higher harmonics of the fundamental frequency that is being measured. When the bridge is balanced for the fundamental frequency, it is not balanced for these higher frequencies, and this gives rise to a disturbing noise in the telephones that makes it difficult to determine when the balance for the fundamental has been obtained. By adding a large self inductance L''' in series with the source, the components of the current corresponding to the higher frequencies are reduced much more than the fundamental component. Even though the total current is less than before, a larger part of it has the fundamental frequency that is being measured, and the balance point can be located more definitely than before.

c. Amplifying the Fundamental Component of the Current.—A further advantage is obtained by using a wave-form filter (Arts. 367 and 368) between the bridge and the source. This consists of a condenser C' in parallel with a self inductance shown in two parts as L' and L''. By adjusting the value of L'' to give resonance for the fre-

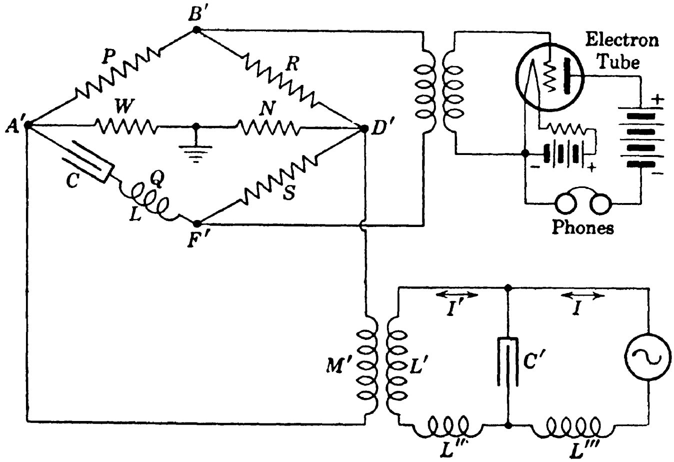


FIG. 244.—Arrangements for picking out and measuring the fundamental frequency of a complex wave form.

quency that is being measured, the component of the current having this frequency is amplified in the circuit of $C'L'L''$, while the components having other frequencies are not amplified. If L' is one coil of a mutual inductance, there will be induced in the other coil an emf of the same frequency as the current through L'. This emf can be applied to the bridge and its frequency can be measured. Since other frequencies have been largely suppressed, this balance should be very sharp and definite.

With the bridge slightly off balance, the value of L'' should be adjusted to give a maximum sound of the fundamental frequency.

d. Amplifying the Telephone Current.—In the process of suppressing the higher frequencies, the intensity of the current has been reduced, and probably the telephone current should be amplified by the use of a one-stage electron-tube amplifier, as shown in Fig. 244.

CHAPTER XVII

CALIBRATION OF ALTERNATING-CURRENT INSTRUMENTS

474. Alternating-current Ammeters.—In the usual d-c ammeter, the pointer moves up the scale to indicate the value of the current only when the current flows through the instrument in the proper direction. When the current is reversed, the moving coil tends to turn in the opposite direction, and in some instruments the scale is extended on

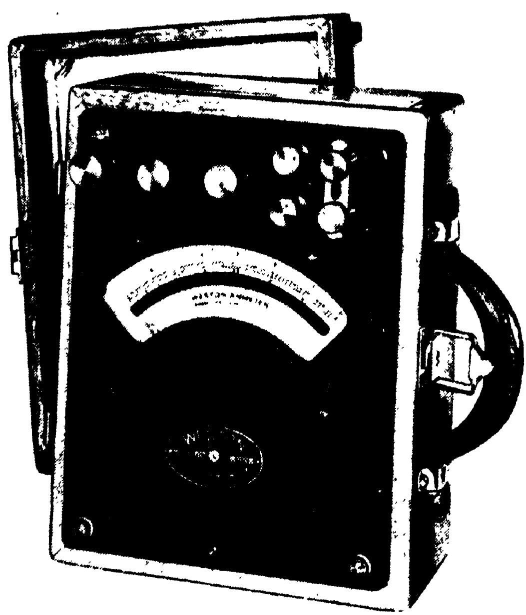


FIG. 245.—Direct-current and alternating-current ammeter. (*Courtesy of the Weston Electrical Instrument Co.*)

each side of zero so that current in either direction can be measured. When an alternating current is passed through such an instrument, the pointer tends to deflect first to one side and then to the other, but as it is not able to follow the reversals of the current, the resultant effect is merely a vibration of the pointer in some position near zero.

In other forms of instruments, such as the Kelvin balance and the electrodynamometer, the reading is independent of the direction in which the current flows through the instrument, and an alternating current will give a definite and steady reading. Of course this reading will not be the maximum value of the current, nor will it be the arithmetical mean of the instantaneous values. Since the deflection of an electrodynamometer depends upon I^2 (see Art. 202), the resultant deflection will be proportional to the mean square value of the current (see Art. 337). As pointed out above, this value represents the effect of an alternating current for heating, lighting, or power purposes, and an ampere of alternating current is defined (Art. 338) as that amount of alternating current which will deflect an electrodynamometer to the same reading as is given by 1 amp of direct current.

Precision ammeters constructed on the electrodynamometer principle (see Fig. 245) work perfectly well with direct current, and they can be calibrated by means of a standard cell and potentiometer (see Art. 197) to read the value of the current in amperes. When they are used with alternating currents, the reading will indicate the effective, or root mean square, value of the current to the same degree of accuracy.

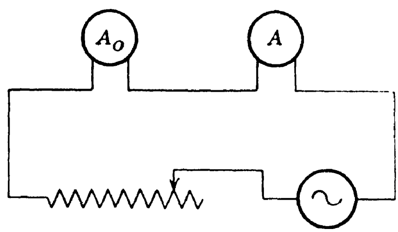


FIG. 246.—Comparison of the ammeter A with the standard ammeter A_0.

Alternating-current ammeters operating on the movable-iron principle do not read alike on alternating and direct currents. These ammeters are calibrated by a direct comparison with an electrodynamometer standard ammeter that has been checked against a standard cell with the potentiometer. The arrangement for this comparison is shown in Fig. 246.

475. Alternating-current Voltmeters.—Having established the value of an alternating-current ampere, the value of the a-c volt follows directly from Ohm's law. Voltmeters are made on the electrodynamometer principle similarly to the ammeters, and these can be calibrated with d-c voltage by the potentiometer-and-standard-cell method (see Arts. 180 and 183). They will then measure a-c voltage (root mean square values) in volts.

Alternating-current voltmeters operating on the movable-iron principle are calibrated by a direct comparison with a calibrated voltmeter of the electrodynamometer type. This comparison is made by connecting the two voltmeters in parallel to the same potential divider, as shown in Fig. 247, and observing simultaneous readings of the two instruments.

476. Alternating-current Wattmeters.—As shown in Art. 205, an electrodynamometer may be used for the measurement of power. If the high-resistance coil is used as a voltmeter to measure e, while the other coil carries the current i, the turning effect will be proportional to ei. Since the moving system has quite a bit of inertia, it will average

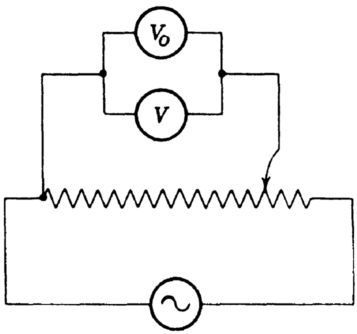


FIG. 247.—Comparison of the voltmeter V with the standard voltmeter V_o.

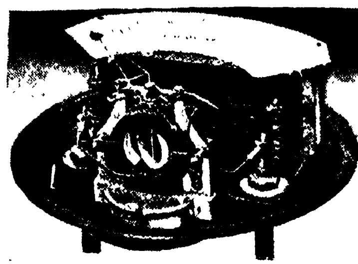


Fig. 248.—Interior of a wattmeter. Current through the fixed coils produces magnetic flux (perpendicular to the paper). Current in the movable coil tends to move across this flux, and the motion of the coil is shown by the attached pointer.

all the turning effects impressed upon it and take up a position corresponding to the average effect.

477. Alternating-current Power.—The power expended in an a-c circuit at any instant is

$$W = ei, \qquad \text{watts}$$

where e and i are the values of the emf and current at this instant. The average expenditure of power is therefore the average value of this product. The relation of this average power to the effective values of the current and voltage is readily obtained for the case of sine wave currents. For this case,

$$W = ei = [E_o \sin(\omega t + \alpha)](I_o \sin \omega t),$$

where E_o and I_o denote the maximum values of e and i.

Expanding this expression gives

$$W = E_o I_o (\sin^2 \omega t \cos \alpha + \sin \omega t \cos \omega t \sin \alpha).$$

For a complete cycle, there is a negative product, sin ωt cos ωt, to cancel each positive product, thus giving the last term an average value of zero. The average value of $\sin^2 \omega t$ for one cycle (see Art. 337) is

$$\frac{1}{2\pi}\int_0^{2\pi} \sin^2 \omega t \; d(\omega t) = \frac{\pi}{2\pi} = 0.5,$$

and therefore the average power is

$$\begin{aligned} W &= 0.5E_oI_o \cos \alpha \\ &= 0.707E_o \times 0.707I_o \cos \alpha \\ &= EI \cos \alpha, \end{aligned}$$

where E and I denote the effective values that would be indicated by a voltmeter and an ammeter. The factor cos α is called the "power factor" of the circuit.

478. Comparison of a Wattmeter with an Ammeter and a Voltmeter.—When power is expended in a noninductive circuit, the value of cos α is unity, and therefore

$$W = EI.$$

In this case the wattmeter can be compared directly with the ammeter and voltmeter, as was done in Art. 207 and Fig. 105.

If the resistance in which the power is expended can be accurately determined, the voltmeter may be omitted and the power computed by the relation

$$W = RI^2.$$

479. Calibration of a Wattmeter on an Inductive Circuit.—It is sometimes desirable to check the readings of a wattmeter when it measures the power in an inductive circuit and therefore when the voltage impressed on the shunt circuit of the wattmeter is not in phase with the current through the main circuit.

The power expended in an inductive circuit is, as shown above,

$$W = EI \cos \alpha,$$

and therefore it cannot be measured with an ammeter and a voltmeter. However if the inductive circuit has an a-c resistance (see Art. 352) that is unchanged from its d-c value, the power can be computed from

$$W = RI^2,$$

and only the ammeter reading will be required. If the wattmeter is compensated so as not to measure the power expended in its shunt circuit, the ammeter should be placed where it will not include this shunt current in the measured value of I.

480. Comparison of a Wattmeter with a Standard Electrodynamometer Wattmeter.—A wattmeter that is carefully and accurately constructed on the electrodynamometer principle can be calibrated by one of the d-c methods given in Chap. VIII. It will then measure a-c power also and can be used as a standard wattmeter. Other wattmeters can then be directly compared with this standard by connecting both instruments so that each will measure the same power. This arrangement is shown in Fig. 249.

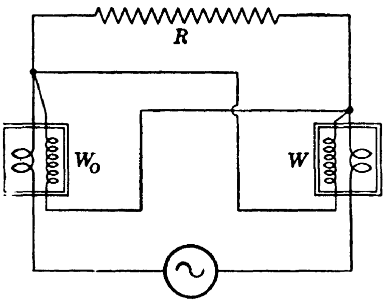


FIG. 249.—Comparison of the wattmeter W with the standard wattmeter W_o.

481. Calibration of a Watt-hour Meter.—A watt-hour meter measures the total energy that has passed through it, and this requires a combination of a wattmeter and a timekeeper. The latter usually means some kind of running mechanism, the speed of which at each instant is made proportional to the power being expended. A watt-hour meter can be calibrated by connecting it with a standard wattmeter so that the same current and voltage affects each, as shown in the arrangement of Fig. 249. By maintaining the power constant for a measured period of time, the total energy is given by the product of watts times hours, and this product should be recorded on the dials of the watt-hour meter. The time can be measured with a standard clock or a good watch. Usually it will not be necessary to continue the test long enough to observe the change in the dial readings, as the rotations of the faster moving disk can be counted for a period sufficient to determine the rate at which the meter is running.

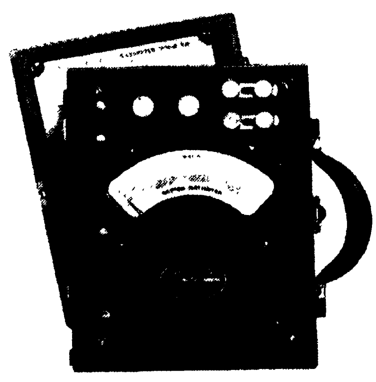


FIG. 250.—An electrodynamometer wattmeter. (*Courtesy of the Weston Electrical Instrument Co.*)

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