CHAPTER 4

JIT INTEGRATED INVENTORY MODEL FOR A BUYER AND A VENDOR CONSIDERING THE IMPACT OF QUALITY IMPROVEMENT, SETUP COST AND LEAD TIME REDUCTIONS

This chapter presents the JIT integrated inventory models for a buyer and vendor with an objective to investigate the effect of quality improvement investment to determine the integrated optimal ordering policy. The effects of investments in setup cost reduction, quality improvement and lead time reduction on the JIT integrated inventory model are also analyzed. The numerical case studies and managerial implications based on the results obtained are also presented.

4.1 JIT Integrated Inventory Model for a Buyer and Vendor considering Quality Improvement

Banerjee and Kim (1995) have shown that a joint optimal integrated inventory replenishment policy, as opposed to independently derived policies for a buyer and a vendor, can result in significant economic benefits. They have presented a scenario involving a single buyer, a single vendor and a single product. The co-operative relationship between a buyer and a vendor, is indeed an important ingredient for the success of JIT. The practice of more frequent deliveries in small lot sizes based on a long-term, co-operative relationship between buyer and vendor must be mutually beneficial. Small lot sizes contribute to higher productivity in a firm through lower levels of inventory and scrap, high product quality, lower inspection costs and earlier detection of defects. Yang and Pan (2004) investigated a JIT model where a single vendor supplies a single buyer with a product by presenting an integrated inventory model that accounts for replenishment lead time reduction and quality improvement investment considerations. Quality improvement or lead-time reduction consideration in inventory models between the vendor and buyer is essential for implementing a just-in-time model.

Zhu et al., (2007) investigated the roles of buyer and vendor in quality improvement efforts in a supply chain. They analyzed a supply chain with a buyer and a vendor, where the buyer has the option to invest in the vendor's quality improvement. They showed that the buyer's involvement can have a significant impact on the profits of both parties and of the supply chain as a whole, and buyer cannot concede the responsibility of quality improvement to the vendor.

This research extends the work of Banerjee and Kim (1995) integrated vendor – buyer model to consider the impact of quality improvement investment to determine the integrated optimal ordering policy. The production process at the vendor can yield nonconforming units, which incur some quality costs, and therefore both vendor and buyer have the incentive to invest in vendor's quality improvement. In addition, quality-improvement decisions interact with operational decisions such as the buyer's order quantity and the supplier's production lot size. This work explores the independent buyer - vendor optimal policies and the impact of their investment in quality improvement. In this type of a JIT environment, it is important to find the optimal number of deliveries of the finished product to the buyer and raw materials to the vendor, such that both parties benefit. The determination of the optimal number of deliveries should be made based on the total relevant cost incurred by both buyer and vendor.

The remainder of the section is organized as follows. Notations and assumptions made in the analysis models are described. The quality improvement in an integrated inventory model with (i) joint investment, (ii) vendor investment, (iii) buyer investment, and (iv) both buyer and vendor investment are described. A numerical case study is presented with a comparison of results of integrated and decentralized quality improvement options. A discussion on vendor selection with quality and budgetary constraints is provided with results based on integrated and decentralized quality policies.

The notation used in the model are:

A_m raw material's ordering cost per order

A_r vendor's order processing and shipment cost per shipment

D demand for the product per unit of time

h_m vendor's raw material holding cost per unit per unit time

h_p vendor's finished goods inventory holding cost unit per unit time

h_r buyer's inventory holding cost unit per unit time

I(t) inventory level

M raw material's lot size factor

N vendor's production lot size factor

P vendor's production rate per unit of time

Q optimal lot size

S vendor's production setup cost per setup

i fractional opportunity cost of capital per unit time

θ probability that the process can go out of control

g cost of replacing a defective unit

The assumptions made in the analysis are (Banerjee and Kim (1995), Yang and Pan (2004) and Zhu et al., (2007):

(1) The demand rate, production rate and delivery lead time for the product is uniform and is deterministic.
(2) No backordering is allowed.
(3) The demand rate for the product on the customer is less than the production rate of the vendor.
(4) All the cost parameters are constant and known.
(5) The present model is followed by the work of Porteus (1986) to include the imperfect production process and the expected defective cost per year is given by $\frac{gNDQ\theta}{2}$, where *g* is the cost of replacing a defective unit for the vendor production batch size of *NQ*. The quality improvement and capital investment is: $q(\theta) = q \ln\left(\frac{\theta_o}{\theta}\right)$ for $0 \le \theta \le \theta_o$ (Yang and Pan, 2004), where θ_o is the current probability that the process can go out of control, and *q* is the percentage increase in $q(\theta)$ per decrease in θ. A smaller value of *q* lower capital investment for quality improvement per decrease in θ.
(6) If λg is the quality cost borne by the buyer and $(1-\lambda)g$ is the quality cost borne by the vendor, then the expected defective cost per year borne by the buyer is $\frac{\lambda gNDQ\theta}{2}$ and vendor is $\frac{(1-\lambda)gNDQ\theta}{2}$.

4.1.1 Integrated inventory model for a vendor and buyer

The buyer's total cost consists of the inventory holding cost and the purchase order processing and shipment cost due to multiple deliveries. The manufacturer's cost includes setup cost and holding cost of raw materials. The buyer wishes to receive lots of size Q at frequent intervals and the vendor produces a batch of NQ units. The raw materials are delivered in M equal lots for each production batch. It is assumed that the multiple input material deliveries are arranged in such a way that each succeeding delivery arrives at the time when the inventory from the previous delivery has just reached down to zero. Fig. 4.1 depicts the inventory time plots for single product for a buyer and vendor.

For the vendor, the average inventory can be evaluated as follows

$$\bar{I}_v = \frac{D}{NQ}\left\{\left[NQ\left(\frac{Q}{P}+(N-1)\frac{Q}{D}\right)-\frac{N^2Q^2}{2P}\right]-\left[\frac{Q}{D}\left(1+2+.......+(N-1)Q\right)\right]\right\} = \frac{Q}{2}\left(N\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right)=\frac{Q}{2}\left(\frac{D}{P}(2-N)+N-1\right) \tag{4.1}$$

Hence the total annual cost for the vendor is represented as the sum of raw materials ordering and holding cost, and the product setup and holding cost as

$$TC_s = \frac{DA_mM}{NQ}+\frac{Q}{2}h_m\frac{ND}{MP}+\frac{DS}{NQ}+\frac{Q}{2}h_p\left\{\frac{D}{P}(2-N)+N-1\right\} \tag{4.2}$$

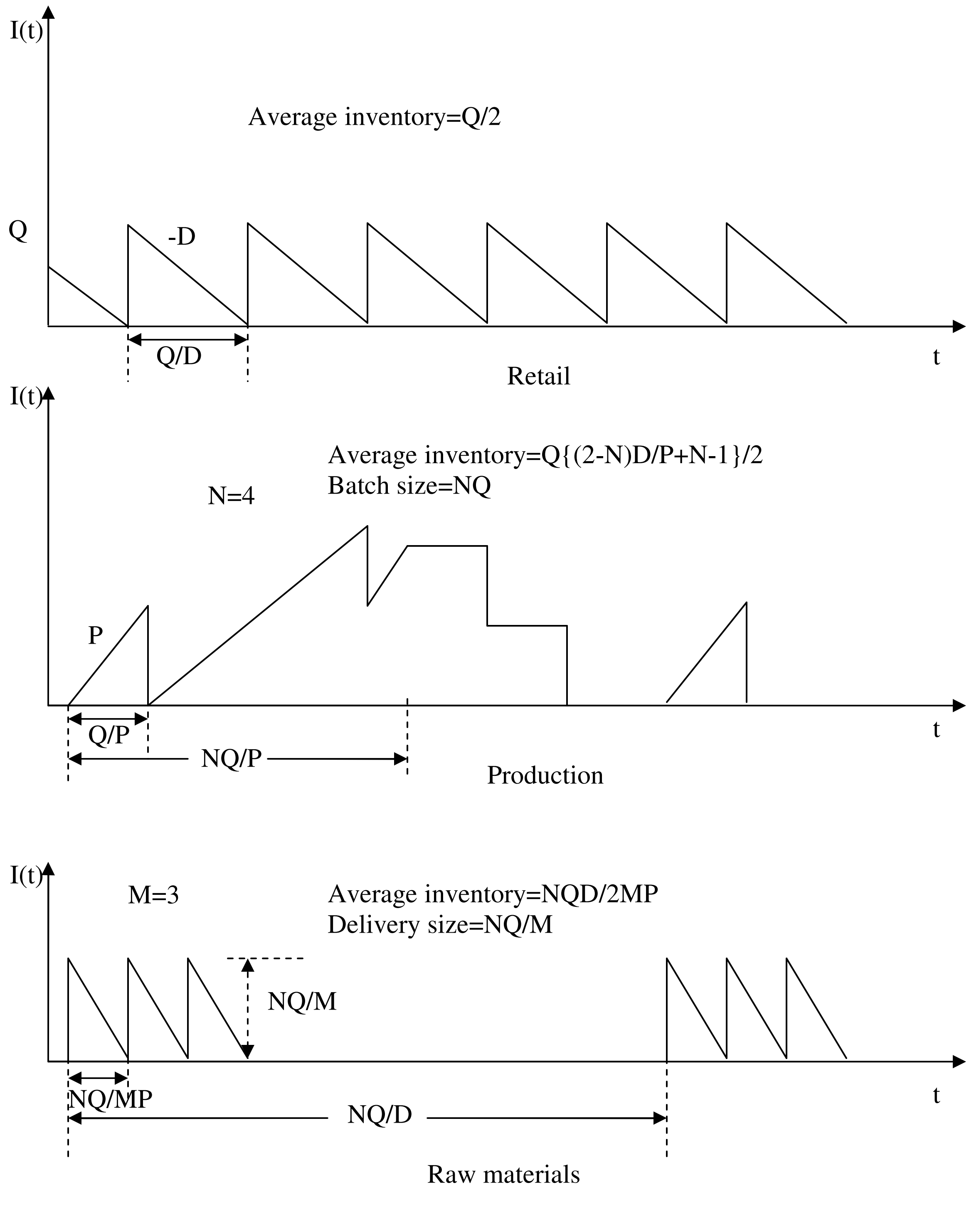

Fig. 4.1 Inventory pattern for raw materials, production and retail

The total annual cost for the buyer is represented as the sum of ordering cost and holding cost as

$$TC_b = \frac{DA_r}{Q} + \frac{Q}{2} h_r \tag{4.3}$$

The joint total cost for the vendor and buyer is given as

$$JTC = \frac{D}{Q}\left(A_r + \frac{S + A_m M}{N} \right) + \frac{Q}{2}\left[\left\{ \frac{D}{P}(2 - N) + N - 1 \right\} h_p + h_r + h_m \frac{ND}{MP} \right] \tag{4.4}$$

4.1.2 Quality improvement models

The quality improvement models considered are as follows: Quality improvement in (i) an integrated model with a joint investment, (ii) a decentralized model with vendor investment, (iii) a decentralized model with buyer investment, and (iv) in a decentralized model with both buyer and vendor investments

4.1.2.1 Quality improvement in an integrated model with joint investment

Under the assumption of imperfect quality, the total expected annual cost including the rework cost, for the vendor and buyer becomes:

$$TRC(M, N, Q, \theta) = \frac{\alpha}{Q} + Q\beta + iq \ln\left(\frac{\theta_o}{\theta} \right) \tag{4.5}$$

subject to the constraint $0 \le \theta \le \theta_o$, where,

$$\alpha = D\left(A_r + \frac{S + A_m M}{N} \right); \beta = \frac{1}{2}\left[\left\{ \frac{D}{P}(2 - N) + N - 1 \right\} h_p + h_r + h_m \frac{ND}{MP} + gND\theta \right] \tag{4.6}$$

The joint economic order quantity based on both the vendor's and buyer's total cost is

$$Q^* = \sqrt{\frac{\alpha}{\beta}} \tag{4.7}$$

Substituting the Q* in Eq. (4.7) in Eq. (4.5) results in

$$TRC(M, N, Q^*, \theta) = 2\sqrt{\alpha\beta} + iq \ln\left(\frac{\theta_o}{\theta} \right) \tag{4.8}$$

If Eq. (4.5) is minimized at θ = θ*,

$$\theta^* = \begin{cases} \theta_0 & \text{if } \theta_0 \le \dfrac{2iq}{gNDQ^*} \\ \dfrac{2iq}{gNDQ^*}, & \text{if } \theta_0 > \dfrac{2iq}{gNDQ^*} \end{cases} \tag{4.9}$$

Where Q* in Eq. (4.7) is substituted in Eq. (4.9) to determine θ*.

If Eq. (4.8) is minimized at N=N*,

$$TRC(M, N^*, Q^*, \theta) \le TRC(M, N^*+1, Q^*, \theta) \text{ and } TRC(M, N^*, Q^*, \theta) \le TRC(M, N^*-1, Q^*, \theta) \tag{4.10}$$

Using Eqs. (4.8) and (4.10), the computation for N* is

$$N^*(N^*-1) \le \frac{(S + A_m M)\left(h_r - h_p\left\{1 - \dfrac{2D}{P}\right\}\right)}{A_r\left[h_p\left(1 - \dfrac{D}{P}\right) + \dfrac{h_m D}{MP} + gD\theta\right]} \le N^*(N^*+1) \tag{4.11}$$

If Eq. (4.8) is minimized at M=M*,

$$TRC(M^*, N, Q^*, \theta) \le TRC(M^*+1, N, Q^*, \theta) \text{ and } TRC(M^*, N, Q^*, \theta) \le TRC(M^*-1, N, Q^*, \theta) \tag{4.12}$$

Using Eqs. (4.8) and (4.12), the computation for M* is

$$M^*(M^*-1) \le \frac{(A_r N + S)\dfrac{N h_m D}{P}}{A_m\left[\left\{\dfrac{D}{P}(2-N) + N - 1\right\}h_p + h_r + gND\theta\right]} \le M^*(M^*+1) \tag{4.13}$$

The proof of positive definiteness of Hessain matrix of $TRC(M, N, Q, \theta)$ is shown in Appendix B.1.

The solution algorithm is as follows:

Step 1: Initialize θ*= θ_o and M*=1

Step 2: Substitute the current value of M* for M in Eq. (4.11) to find N*

Step 3: Recalculate M* by using the current value of N* as N in Eq. (4.13)

Step 4: Repeat steps 2 and 3 until convergence occurs

Step 5: Use converged values of M* and N* in Eq. (4.7) to determine Q*.

Step 6: Recalculate θ* using converged values of M*, N* and Q* in Eq. (4.9).

Step 7: If $\theta_o \le \theta^*$, optimal values of M*, N* and Q* are found.

Step 8: If $\theta_o > \theta^*$, then steps 2 to 6 are repeated until convergence occurs. The convergence criteria for algorithm is that the new and old values of N^* and M^* are same. The convergence criteria for algorithm is: $\left|\dfrac{\theta^*_{\text{new}} - \theta^*_{\text{old}}}{\theta^*_{\text{old}}}\right| \le 10^{-4}$.

4.1.2.2 Quality improvement in a decentralized model with vendor investment

Under the assumption of imperfect quality, the total expected annual cost including the rework cost, for the vendor becomes:

$$TRC_s(M,N,Q,\theta) = D\left(\frac{S + A_m M}{NQ}\right) + \frac{Q}{2}\left[\left\{\frac{D}{P}(2-N) + N - 1\right\}h_p + h_m\frac{ND}{MP}\right] + \frac{gNDQ\theta}{2} + iq\ln\left(\frac{\theta_o}{\theta}\right) \tag{4.14}$$

subject to the constraint $0 \le \theta \le \theta_o$

The total annual cost for the buyer becomes:

$$TRC_b(Q) = \frac{DA_r}{Q} + \frac{Qh_r}{2} \tag{4.15}$$

The economic order quantity based on buyer's total cost is

$$Q^* = \sqrt{\frac{2DA_r}{h_r}} \tag{4.16}$$

If Eq. (4.14) is minimized at θ = θ*,

$$\theta^* = \begin{cases} \theta_0, & \text{if } \theta_0 \le \dfrac{iq}{gND}\sqrt{\dfrac{2h_r}{DA_r}} \\ \dfrac{iq}{gND}\sqrt{\dfrac{2h_r}{DA_r}}, & \text{if } \theta_0 > \dfrac{iq}{gND}\sqrt{\dfrac{2h_r}{DA_r}} \end{cases} \tag{4.17}$$

If Eq. (4.14) is minimized at N=N*,

$$TRC_s(M,N^*,Q^*,\theta) \le TRC_s(M,N^*+1,Q^*,\theta),\ TRC_s(M,N^*,Q^*,\theta) \le TRC_s(M,N^*-1,Q^*,\theta) \tag{4.18}$$

Using Eqs. (4.14) and (4.18), the computation for N* is

$$N^*\left(N^*-1\right)\le \frac{\left(S+A_m M\right)h_r}{A_r\left[h_p\left(1-\frac{D}{P}\right)+\frac{h_m D}{MP}+gD\theta\right]}\le N^*\left(N^*+1\right) \tag{4.19}$$

If Eq. (4.14) is minimized at M=M*,

$$TRC_s\left(M^*, N, Q^*, \theta\right)\le TRC_s\left(M^*+1, N, Q^*, \theta\right),\ \ TRC_s\left(M^*, N, Q^*, \theta\right)\le TRC_s\left(M^*-1, N, Q^*, \theta\right) \tag{4.20}$$

Using Eqs. (4.14) and (4.20), the computation for M* is

$$M^*\left(M^*-1\right)\le \frac{A_r N^2 h_m D}{A_m h_r P}\le M^*\left(M^*+1\right) \tag{4.21}$$

The proof of positive definiteness of Hessain matrix of $TRC_s\left(M, N, Q, \theta\right)$ is shown in Appendix B.2.

4.1.2.3 Quality improvement in a decentralized model with buyer investment

Under the assumption of imperfect quality, the total expected annual cost including the rework cost, for the buyer becomes:

$$TRC_b\left(Q,\theta\right)=\frac{DA_r}{Q}+\frac{Qh_r}{2}+\frac{gNDQ\theta}{2}+iq\ln\left(\frac{\theta_o}{\theta}\right) \tag{4.22}$$

subject to the constraint $0\le\theta\le\theta_o$

The economic order quantity based on buyer's total cost in Eq. (4.22) is given by

$$Q^*=\sqrt{\frac{2DA_r}{h_r+gND\theta}} \tag{4.23}$$

The total annual cost for the vendor becomes:

$$TRC_s\left(M, N, Q, \theta\right)=D\left(\frac{S+A_m M}{NQ}\right)+\frac{Q}{2}\left[\left\{\frac{D}{P}(2-N)+N-1\right\}h_p+h_m\frac{ND}{MP}\right] \tag{4.24}$$

If Eq. (4.24) is minimized at N=N*,

$$TRC_s\left(M, N^*, Q^*\right)\le TRC_s\left(M, N^*+1, Q^*\right) \text{ and } TRC_s\left(M, N^*, Q^*\right)\le TRC_s\left(M, N^*-1, Q^*\right) \tag{4.25}$$

Using Eqs. (4.24) and (4.25), the computation for N* is

$$N^*(N^*-1)\le \frac{(S+A_m M)h_r}{A_r\left[h_p\left(1-\frac{D}{P}\right)+\frac{h_m D}{MP}\right]}\le N^*(N^*+1) \tag{4.26}$$

The computation for minimization at M=M* is given by Eq. (4.21).

If Eq. (4.22) is minimized at θ = θ*, then θ* is given by

$$\theta^* = \begin{cases} \theta_0, & \text{if } \theta_0 \le \frac{iq}{gND}\sqrt{\frac{2(h_r+gND\theta)}{DA_r}} \\ \frac{iq}{gND}\sqrt{\frac{2(h_r+gND\theta)}{DA_r}}, & \text{if } \theta_0 > \frac{iq}{gND}\sqrt{\frac{2(h_r+gND\theta)}{DA_r}} \end{cases} \tag{4.27}$$

The proof of positive definiteness of Hessain matrix of $TRC_b(Q,\theta)$ is shown in Appendix B.3.

4.1.2.4 Quality improvement in a decentralized model with both buyer and vendor investment

Under the assumption of imperfect quality, the total expected annual cost including the rework cost, for the buyer becomes:

$$TRC_b(Q,\theta_b)=\frac{DA_r}{Q}+\frac{Qh_r}{2}+\frac{\lambda gNDQ\theta_b}{2}+iq\ln\left(\frac{\theta_o}{\theta_b}\right) \tag{4.28}$$

subject to the constraint $0\le\theta_b\le\theta_o$

The economic order quantity based on buyer's total cost in Eq. (4.28) is given by

$$Q^*=\sqrt{\frac{2DA_r}{h_r+\lambda gND\theta_b}} \tag{4.29}$$

If Eq. (4.28) is minimized at θ = θ*, then for the buyer,

$$\theta_b^* = \begin{cases} \theta_0 & \text{if } \theta_0 \le \frac{iq}{\lambda gND}\sqrt{\frac{2(h_r+\lambda gND\theta_b)}{DA_r}} \\ \frac{iq}{\lambda gND}\sqrt{\frac{2(h_r+\lambda gND\theta_b)}{DA_r}}, & \text{if } \theta_0 > \frac{iq}{\lambda gND}\sqrt{\frac{2(h_r+\lambda gND\theta_b)}{DA_r}} \end{cases} \tag{4.30}$$

Under the assumption of imperfect quality, the total expected annual cost including the rework cost, for the vendor becomes:

$$TRC_s(M,N,\theta)=D\left(\frac{S+A_m M}{NQ}\right)+\frac{Q}{2}\left[\left\{\frac{D}{P}(2-N)+N-1\right\}h_p+h_m\frac{ND}{MP}\right]+ \frac{(1-\lambda)gNDQ\theta}{2}+iq\ln\left(\frac{\theta_b}{\theta}\right) \tag{4.31}$$

subject to the constraint $0 \le \theta \le \theta_b$

If Eq. (4.31) is minimized at N=N*,

$$TRC_s\left(M, N^*, Q^*, \theta\right) \le TRC_s\left(M, N^*+1, Q^*, \theta\right), \;\; TRC_s\left(M, N^*, Q^*, \theta\right) \le TRC_s\left(M, N^*-1, Q^*, \theta\right) \tag{4.32}$$

Using Eqs. (4.31) and (4.32), the computation for N* is

$$N^*\left(N^*-1\right) \le \frac{\left(S + A_m M\right)h_r}{A_r\left[h_p\left(1-\frac{D}{P}\right)+\frac{h_m D}{MP}+\left(1-\lambda\right)gD\theta\right]} \le N^*\left(N^*+1\right) \tag{4.33}$$

The computation for minimization at M=M* is given by Eq. (4.21).

If Eq. (4.31) is minimized at θ = θ*, then for the vendor,

$$\theta^* = \begin{cases} \theta_b^* & \text{if } \theta_b^* \le \frac{iq}{\left(1-\lambda\right)gND}\sqrt{\frac{2\left(h_r + \lambda gND\theta_b^*\right)}{DA_r}} \\ \frac{iq}{\left(1-\lambda\right)gND}\sqrt{\frac{2\left(h_r + \lambda gND\theta_b^*\right)}{DA_r}}, & \text{if } \theta_b^* > \frac{iq}{\left(1-\lambda\right)gND}\sqrt{\frac{2\left(h_r + \lambda gND\theta_b^*\right)}{DA_r}} \end{cases} \tag{4.34}$$

The proof of positive definiteness of Hessain matrix of $TRC_b\left(Q,\theta\right)$ is shown in Appendix B.3.

4.1.3 Numerical case study

Using the data for a vendor and buyer (Banerjee and Kim, 1995) in Table 4.1, the joint optimal integrated policy (Banerjee and Kim, 1995) results in M*=1, N*=5, Q*= 140 units, yielding a total joint relevant cost of \$ 3924.3 per year. The vendor produces batch of 700 units and delivers to the buyer in a lot of 140 units. The vendor procures input materials in a lot size of 700 units. The results obtained considering the impact of quality improvement are shown in Table 4.1. Using the joint optimal integrated policy the vendor produces batch of 635 units and delivers to the buyer in a lot of 127 units. The vendor procures input materials in a lot size of 635 units. The joint optimal policy yields minimum total relevant cost (TRC) compared to decentralized vendor and/or buyer optimal policies. If the vendor independently makes all the investment in quality, then vendor procures input materials and produces a batch size of 672 units, and delivers in a lot of 112 units to the buyer. The delivery lot size to the buyer is based on the economic ordering quantity for the buyer. However, if the buyer makes the investment in quality, then the vendor produces a batch size of 432 units, and delivers in a lot of 72 units to the buyer. The total relevant cost ($5694.1) in buyer optimal quality investment is higher than the total relevant cost ($5353.8) in the vendor optimal investment in quality. In a decentralized optimal policy, when both buyer and vendor invest in the quality, the total relevant costs are higher than when buyer or vendor independently invests in quality. For the cases of λ=0.50 and λ=0.75, only the buyer invests in quality improvement. For the data given in Table 4.1, total relevant costs in a decentralized system are lower when the buyer makes a major proportion of investment in quality. For the cases of quality investments considered in the decentralized system, vendor procures and produces the batch size of 360 units for the cases of λ=0.25 and λ=0.50, and the vendor produces a batch size of 432 units for λ=0.75. Incorporating quality effects leads to a large rework cost which in turn leads to reduction in the production lot size.

Using the data for a vendor and buyer (Yang and Pan, 2004) in Table 4.2, the results obtained considering the impact of quality improvement are shown in Table 4.2. The joint optimal integrated policy (Banerjee and Kim, 1995) results in M*=2, N*=3, Q*= 128 units, yielding a total joint relevant cost of $ 1688.3 per year. The vendor produces batch of 384 units and delivers to the buyer in a lot of 128 units. The vendor procures input materials in a lot size of 192 units. The joint optimal integrated policy considering the impact of quality improvement results in a total relevant cost of $ 2154.2 per year which is lower than the total relevant costs incurred when the buyer and/or vendor makes the investment in quality. Using the example 2 considered in this study, no investment in quality is required under the joint optimal integrated policy and the vendor produces batch of 256 units and delivers to the buyer in a lot of 128 units and the vendor procures input materials in a batch of 256 units.

Table 4.1 Comparison of results of integrated and decentralized quality improvement (Banerjee and Kim, 1995)

Buyer: D= 5,000 units/year, h_r=\$ 8 per unit per year
Vendor: P=20,000 units/year, S=\$200 per setup, A_r=\$10 per shipment, A_m=\$25 per order, h_p=\$5 per unit per year, h_m=\$3 per unit per year.
i=0.1 /\$/ year, g=\$15 per defective unit, $q(\theta) = 4000\ln\left(\frac{0.0002}{\theta}\right)$

	Joint optimal integrated policy considering quality improvement	Vendor optimal policy considering quality improvement	Buyer optimal policy considering quality improvement	Decentralized optimal policy considering quality improvement		
				λ=0.25	λ=0.50	λ =0.75
M*	1	1	1	1	1	1
N*	5	6	6	5	5	6
θ^*_b	-	-	-	0.00011775	0.000058873	0.000032707
θ*	0.000016852	0.000015901	0.00002453	0.000039249	0.000058873	0.000032707
Q*	127	112	72	72	72	72
TRC, \$	5334.2	5353.8	5694.1	6260.5	6098.3	5712.4

The total relevant cost is lower when the vendor makes quality investment independently compared to buyer. Using the vendor optimal quality policy, the vendor procures and produces a batch of 300 units and delivers to the buyer in a lot of 100 units, which is the buyer economic ordering quantity. When the buyer makes an investment in quality, the vendor procures a lot of 108 units, produces a batch of 216 units and delivers to the buyer in a lot of 54 units. Under decentralized policy, no investment by both buyer and vendor for quality improvement are required. Optimal integer multiplier N is more likely to decrease when quality is considered due to large values of cost of replacing a defective unit and probability that the process can go out of control.

Table 4.2 Comparison of results of integrated and decentralized quality improvement (Yang and Pan, 2004)

Buyer: D= 1,000 units/year, h_r=\$ 5 per unit per year
Vendor: P=3200 units/year, S=\$200 per setup, A_r=\$25 per shipment, A_m=\$25 per order, h_p=\$4 per unit per year, h_m=\$3 per unit per year.
i=0.1 /\$/ year, g=\$15 per defective unit, $q(\theta)=4000\ln\left(\frac{0.0002}{\theta}\right)$

	Joint optimal integrated policy considering quality improvement	Vendor optimal policy considering quality improvement	Buyer optimal policy considering quality improvement	Decentralized optimal policy considering quality improvement		
				λ =0.25	λ=0.50	λ =0.75
M*	1	1	2	1	1	1
N*	2	3	4	3	3	3
θ^*_b	-	-	-	0.0002	0.0002	0.0002
θ*	0.0002	0.00017778	0.0002	0.0002	0.0002	0.0002
Q*	128	100	54	83	73	65
TRC, \$	2154.2	2175.2	2382.8	2182.5	2233.1	2301.5

4.1.3.1 Vendor selection

This section considers the case of a vendor with higher quality (or a lower probability that the process can go out of control) $q_1(\theta)=4000\ln\left(\frac{0.00002}{\theta}\right)$ as compared to $q(\theta)=4000\ln\left(\frac{0.0002}{\theta}\right)$. The results obtained are shown in Table 4.3. Table 4.3 shows lower total relevant costs of a vendor with high quality using the data of Banerjee and Kim (1995). The vendor procurement and production batch size, and delivery lot size to the buyer for the joint, vendor and buyer optimal policies are same in comparison with the results shown in Table 4.1. Total cost is minimum for the joint optimal integrated policy compared to the decentralized buyer and/or vendor optimal policies. No quality improvement investment is required in the case of decentralized buyer and vendor coordinated optimal quality policies.

Table 4.3 Comparison of results of vendor with high quality using integrated and decentralized quality policies (Banerjee and Kim, 1995)

	Joint optimal integrated policy considering quality improvement	Vendor optimal policy considering quality improvement	Buyer optimal policy considering quality improvement	Decentralized optimal policy considering quality improvement		
				λ =0.25	λ=0.50	λ =0.75
M*	1	1	1	1	1	1
N*	5	6	6	6	6	6
θ^*_b	-	-	-	0.00002	0.00002	0.00002
θ^*	0.000016852	0.000015901	0.00002	0.00002	0.00002	0.00002
Q*	127	112	77	99	89	82
TRC, $	4413.1	4432.8	4688.1	4454.0	4511.3	4593.0

The total relevant costs (shown in Table 4.4) of a vendor with high quality predicts no investment required for quality improvement. High quality vendor yields lower total costs as compared to the low quality vendor. The joint optimal policy yields minimum total cost compared to the buyer and/or vendor optimal quality policy. Using the joint optimal integrated policy for quality improvement, vendor procurement and production batch size, and delivery lot size to the buyer are high with the high quality vendor in comparison with the results shown in Table 4.4 for a low quality vendor.

Table 4.4 Comparison of results of vendor with high quality using integrated and decentralized quality policies (Yang and Pan, 2004)

	Joint optimal integrated policy considering quality improvement	Vendor optimal policy considering quality improvement	Buyer optimal policy considering quality improvement	Decentralized optimal policy considering quality improvement		
				λ =0.25	λ=0.50	λ =0.75
M*	2	1	2	1	1	1
N*	3	3	4	3	3	3
θ^*_b	-	-	-	0.00002	0.00002	0.00002
θ^*	0.00002	0.00002	0.00002	0.00002	0.00002	0.00002
Q*	124	100	90	99	96	94
TRC, $	1745.1	1773.1	1763.5	1778.5	1784.6	1791.1

The results for a vendor with lower cost rate for quality improvement $q_2(\theta) = 400\ln\left(\frac{0.0002}{\theta}\right)$ for the data given in Table 4.1 are shown in Table 4.5. Lower total relevant costs are associated with a vendor with lower cost rate for quality improvement under joint optimal integrated policy as well as buyer and/or vendor coordinated optimal policies. Also higher vendor procurement and production batch size are predicted with lower cost rate for quality improvement. For the case of buyer investment in quality for λ=0.50 and λ=0.75, the vendor is not required to make any further investment. Higher share of buyer investment in quality results in a lower total costs for the data given in Table 4.1. For λ=0.25, both buyer and vendor needs to make quality improvement investment.

Results shown in Table 4.6 yield lower total relevant costs for the vendor with lower cost rate for quality improvement. Joint optimal integrated policy gives minimum total cost as compared to decentralized buyer and/or vendor coordinated quality improvement policies. Also the vendor procurement and production batch size are higher under joint integrated optimal policy for the vendor with lower cost rate of quality.

Table 4.5 Comparison of results of vendor with lower cost rate for quality improvement (Banerjee and Kim, 1995)

	Joint optimal integrated policy considering quality improvement	Vendor optimal policy considering quality improvement	Buyer optimal policy considering quality improvement	Decentralized optimal policy considering quality improvement		
				λ=0.25	λ=0.50	λ=0.75
M*	1	1	1	1	1	1
N*	5	6	6	6	6	6
θ^*_b	-	-	-	0.0000066512	0.0000033256	0.0000022171
θ*	0.0000015377	0.0000015901	0.0000016628	0.0000022171	0.0000033256	0.0000022171
Q*	139	112	107	107	107	107
TRC, $	4159.2	4174.5	4190.4	4218.8	4202.6	4192.2

Table 4.6 Comparison of results of vendor with lower cost rate for quality improvement (Yang and Pan, 2004)

	Joint optimal integrated policy considering quality improvement	Vendor optimal policy considering quality improvement	Buyer optimal policy considering quality improvement	Decentralized optimal policy considering quality improvement		
				λ=0.25	λ=0.50	λ=0.75
M*	2	1	2	1	1	1
N*	3	3	4	3	3	3
θ^*_b	-	-	-	0.000077027	0.000038514	0.000025676
θ*	0.000014185	0.000017778	0.000014443	0.000025676	0.000038514	0.000025676
Q*	125	100	92	92	92	92
TRC, $	1834.7	1864.9	1848.8	1917.5	1901.3	1890.8

Under the decentralized scheme lower the total cost is associated with higher share of buyer investment in quality and there is no requirement for vendor investment (for λ=0.50 and λ=0.75).

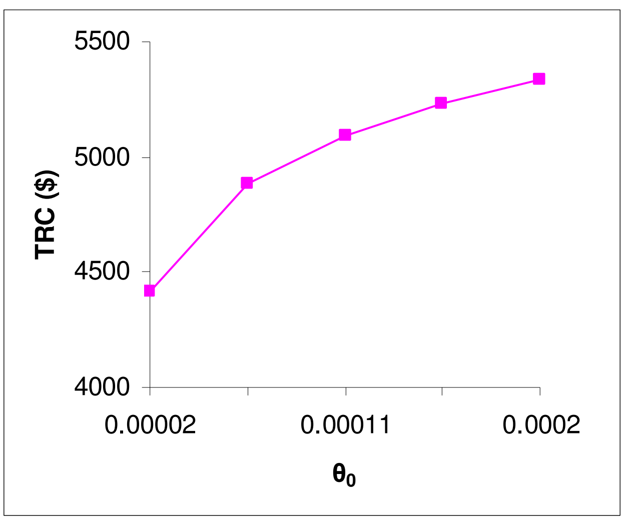

Fig. 4.2 Variation of TRC with θ_0

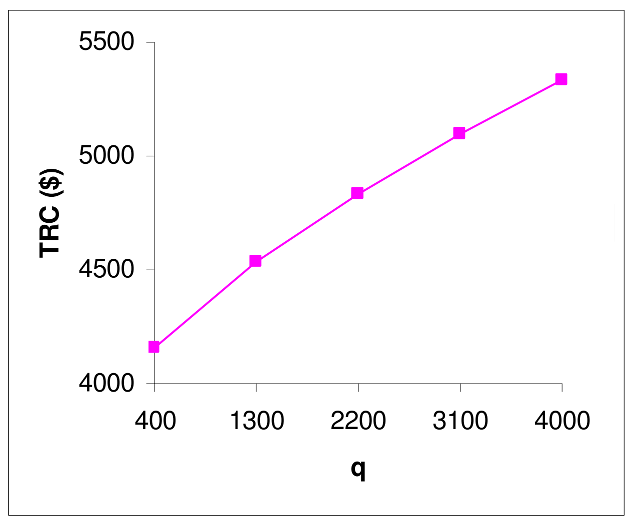

Fig. 4.3 Variation of TRC with q

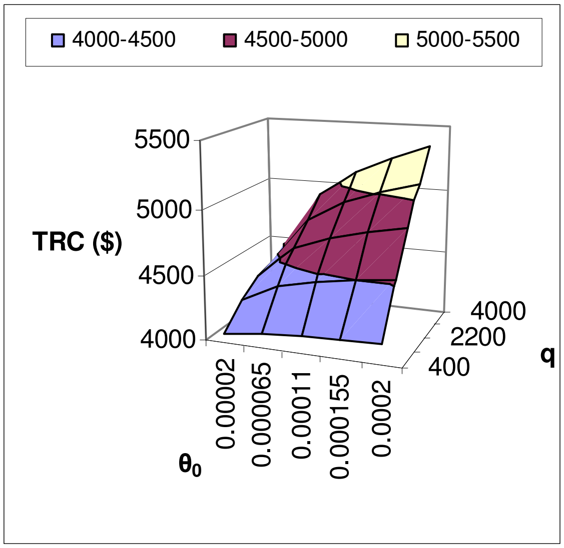

Fig. 4.4 Variation of TRC with θ_0 and q

Figures 4.2 – 4.4 show the variation of total relevant cost (TRC) for the joint optimal integrated policy considering quality improvement using the data given in Table 4.1. Figures 4.2 – 4.3 show the variation of TRC with θ_0 (current probability that the process can go out of control) and q (rate of increase in dollar investment per fraction of reduction in non-conforming units). As shown in Figs. 4.2 and 4.3, TRC increases with increase in θ_0 and q respectively. Lower value of θ_0 indicates a vendor with higher quality. Hence the TRC is lower in the case of a vendor with higher quality. Figure 4.4 shows the variation of TRC with both θ_0 and q.

Figure 4.4 shows the variation of TRC with both θ_0 and q. As shown in Fig. 4.4, for higher values of q (4000), the increase in TRC with θ_0 is higher as compared to lower values of q (400). Also for higher values of θ_0 (0.0002) the increase in TRC with q is higher as compared to lower values of θ_0 (0.00002).

4.1.3.2 Budgetary constraints

This section investigates the limit on the quality improvement due to constraints. Under the assumption of imperfect quality, the total expected annual cost including the rework cost, for the vendor and buyer given by Eq. (4.5) is subject to the constraint $\Delta\theta_o \le \theta \le \theta_o$, where $\Delta\theta_o = 0.00004$.

If Eq. (4.5) is minimized at $\theta = \theta^*$,

$$\theta^* = \begin{cases} \theta_0, & \text{if } \theta_0 < \dfrac{2iq}{gNDQ^*} \\ \dfrac{2iq}{gNDQ^*}, & \text{if } \theta_0 > \dfrac{2iq}{gNDQ^*} > \Delta\theta_0 \\ \Delta\theta_0, & \text{if } \dfrac{2iq}{gNDQ^*} < \Delta\theta_0 \end{cases} \tag{4.35}$$

Under the assumption of imperfect quality, the total expected annual cost including the rework cost, for the vendor is given by Eq. (4.14) subject to the constraint $\Delta\theta_o \le \theta \le \theta_o$

If Eq. (4.14) is minimized at $\theta = \theta^*$,

$$\theta^* = \begin{cases} \theta_0, & \text{if } \theta_0 < \dfrac{iq}{gND}\sqrt{\dfrac{2h_r}{DA_r}} \\ \dfrac{iq}{gND}\sqrt{\dfrac{2h_r}{DA_r}}, & \text{if } \theta_0 > \dfrac{iq}{gND}\sqrt{\dfrac{2h_r}{DA_r}} > \Delta\theta_0 \\ \Delta\theta_0, & \text{if } \dfrac{iq}{gND}\sqrt{\dfrac{2h_r}{DA_r}} < \Delta\theta_0 \end{cases} \tag{4.36}$$

Under the assumption of imperfect quality, the total expected annual cost including the rework cost, for the buyer is given by Eq. (4.22) subject to the constraint $\Delta\theta_o \le \theta \le \theta_o$

If Eq. (4.22) is minimized at $\theta = \theta^*$, then θ^* is given by

$$\theta^* = \begin{cases} \theta_0, & \text{if } \theta_0 < \dfrac{iq}{gND}\sqrt{\dfrac{2(h_r + gND\theta)}{DA_r}} \\ \dfrac{iq}{gND}\sqrt{\dfrac{2(h_r + gND\theta)}{DA_r}}, & \text{if } \theta_0 > \dfrac{iq}{gND}\sqrt{\dfrac{2(h_r + gND\theta)}{DA_r}} > \Delta\theta_0 \\ \Delta\theta_0, & \text{if } \dfrac{iq}{gND}\sqrt{\dfrac{2(h_r + gND\theta)}{DA_r}} < \Delta\theta_0 \end{cases} \tag{4.37}$$

Under the assumption of imperfect quality, in a decentralized model with both buyer and vendor investment under budgetary constraints, the total expected annual cost including the rework cost for the buyer is given by Eq. (4.28) subject to the constraint $\Delta\theta_o \le \theta_b \le \theta_o$, and the total expected annual cost including the rework cost for the vendor is given by Eq. (4.31) subject to the constraint $\Delta\theta_o \le \theta \le \theta_b$.

If Eq. (4.28) is minimized at $\theta = \theta_b^*$, then for the buyer,

$$\theta_b^* = \begin{cases} \theta_o, & \text{if } \theta_o \le \dfrac{iq}{\lambda gND}\sqrt{\dfrac{2(h_r + \lambda gND\theta_b)}{DA_r}} \\ \dfrac{iq}{\lambda gND}\sqrt{\dfrac{2(h_r + \lambda gND\theta_b)}{DA_r}}, & \text{if } \theta_o > \dfrac{iq}{\lambda gND}\sqrt{\dfrac{2(h_r + \lambda gND\theta_b)}{DA_r}} \ge \Delta\theta_0 \\ \Delta\theta_0, & \text{if } \dfrac{iq}{\lambda gND}\sqrt{\dfrac{2(h_r + \lambda gND\theta_b)}{DA_r}} < \Delta\theta_0 \end{cases} \qquad (4.38)$$

If Eq. (4.31) is minimized at θ = θ*, then for the vendor,

$$\theta^* = \begin{cases} \theta_b^* & \text{if } \theta_b^* \le \dfrac{iq}{(1-\lambda)gND}\sqrt{\dfrac{2(h_r + \lambda gND\theta_b^*)}{DA_r}} \\ \dfrac{iq}{(1-\lambda)gND}\sqrt{\dfrac{2(h_r + \lambda gND\theta_b^*)}{DA_r}}, & \text{if } \theta_b^* > \dfrac{iq}{(1-\lambda)gND}\sqrt{\dfrac{2(h_r + \lambda gND\theta_b^*)}{DA_r}} \ge \Delta\theta_0 \\ \Delta\theta_0, & \text{if } \dfrac{iq}{(1-\lambda)gND}\sqrt{\dfrac{2(h_r + \lambda gND\theta_b^*)}{DA_r}} < \Delta\theta_0 \end{cases}$$

(4.39)

Table 4.7 shows comparison of results with minimum quality level due to the budgetary constraints on the part of vendor not being able to bring the out of control probability function below a threshold value. Results indicate that threshold value of quality level is adopted due to constraint, for joint integrated, buyer and vendor optimal policies. For the data given in Table 4.1 considered in this study, the budgetary constraints yield a higher value of total relevant cost for all the cases of joint integration as well as buyer and vendor coordination. Under the joint optimal integrated policy with threshold quality for the data given in Table 4.1 yield, lower vendor procurement and production batch size, but higher buyer lot size.

Table 4.7 Comparison of results of vendor with budgetary constraints on the minimum quality using integrated and decentralized quality policies (Banerjee and Kim, 1995)

	Joint optimal integrated policy considering quality improvement	Vendor optimal policy considering quality improvement	Buyer optimal policy considering quality improvement	Decentralized optimal policy considering quality improvement		
				λ=0.25	λ=0.50	λ=0.75
M*	1	1	1	1	1	1
N*	4	5	6	5	5	6
θ^*_b	-	-	-	0.00011824	0.000058873	0.00004
θ^*	0.00004	0.00004	0.00004	0.00004	0.000058873	0.00004
Q*	137	112	62	72	72	68
TRC, $	5493.4	5507.2	6039.3	6270.6	6098.3	5848.2

Results with threshold on the quality for data of Yang and Pan, (2004) are same as those given in Table 4.4, as the optimal values of out of control probability variable are higher than the vendor's quality value.

4.2 JIT Integrated Inventory Model for a Buyer and Vendor considering Quality Improvement, Setup Cost and Lead Time Reductions

Yang and Pan (2004) presented a JIT integrated inventory model involving lead time and quality improvement investment. Their JIT integrated inventory model is useful particularly for JIT inventory systems where the vendor and the buyer form a strategic alliance for profit sharing. One of the major tasks of maintaining the competitive advantages of JIT production is to compress the lead time needed to perform activities associated with delivering high-quality products to customers. In the production environment, lead time is an important element in any inventory management system. Lead time can be reduced by an additional crashing cost, and hence is controllable.

This research extends the Yang and Pan (2004) just-in-time integrated inventory model with an objective to study the impact of investment in setup cost reduction to determine simultaneously the order quantity, setup cost, process quality level, number of deliveries and lead time. In a JIT environment a major thrust is to continually reduce set-up cost, it is likely that the implementation of integrated replenishment policy in conjunction with a set-up reduction program will result in smaller lot sizes. Furthermore, set-up cost reduction will also reduce the vendor's batch size, as well as the input material procurement lot sizes. It would be worthwhile to investigate the relationship between quality improvement, setup cost and lead-time reduction, and their impact on the lot size and inventory cost.

In the remainder of the section, notations and assumptions made in the model are provided. Quality improvement, setup cost and lead time reduction model is presented and a solution procedure is suggested for solving the proposed model. A numerical example is considered to illustrate the impact of setup cost reduction with capital investment.

The notations used in this model are:

A	purchaser's ordering cost per order
C_v	unit production cost paid by the vendor
C_p	unit purchase cost paid by the purchaser
D	average demand per year
g	cost of replacing a defective unit
i_s	fractional per unit time opportunity cost of capital for setup cost reduction
i_θ	fractional per unit time opportunity cost of capital for quality improvement
L	length of lead time
m	number of shipments delivered to the purchaser
P	production rate
Q	order quantity
r	annual inventory holding cost per $ invested in stocks
S	vendor's setup cost per setup
θ	probability of process being out of control

The assumptions made in the model are as follows

1. The product is manufactured with a finite production rate P, P>D.
2. The demand during the lead time L follows a normal distribution with mean μL and standard deviation $\sigma\sqrt{L}$. The reorder point (*ROP*) equals the sum of the expected demand during lead time and the safety stock, that is, the reorder point

$ROP = \mu L + k\sigma\sqrt{L}$, where k is known as the safety factor. Inventory is continuously reviewed and replenishments are made whenever the inventory level falls to the reorder point (*ROP*).

3. Based on the Liao and Shyu (1991) model, the replenishment lead time *L* has *n* mutually independent components, each of them has a different crashing cost for reduced lead time. These components are crashed one at a time starting with the one of least crashing cost per unit time and so on. The *i*th component has a minimum duration a_i and normal duration b_i, and a crashing cost per unit time c_i. Let $L_{\min} = \sum_{i=1}^{n} a_i \le L \le \sum_{i=1}^{n} b_i = L_{\max}$ and L_i be the length of the *i*th component of the lead time crashed to its minimum duration. Then L_i can be expressed as $L_i = \sum_{i=1}^{n} b_i - \sum_{j=1}^{i} (b_j - a_j),\ i = 1,2,..........,n$. Also let *R(L)* denote the lead time crashing cost per cycle for a given $L \in [L_i, L_{i-1}]$ and the crashing cost function is described by a piecewise linear function as $R(L) = c_i (L_{i-1} - L) + \sum_{j=1}^{i-1} c_j (b_j - a_j)$.

4. The process is assumed to be in control before beginning production of the lot. Once out of control, the process produces defective units and continues to do so until the entire lot is produced. While the vendor produces the item in the quantity *mQ*, the process can go out of control with a given probability θ each time another unit is produced. Porteus (1986) suggested the expected number of defective items in a run of size *mQ* can be evaluated as $m^2 Q^2 \theta / 2$ and then the expected defective cost per year is given by $gmQD\theta/2$.

5. The quality level and setup cost are considered to be decision variables, the control of quality level and setup cost are accompanied by varying the capital investment allocated to improve the quality level and reduce setup cost. The quality improvement and capital investment is represented by $\varphi_\theta = q_\theta \ln(\theta_0/\theta)$ for $0 < \theta < \theta_0$, where θ_0 is the current probability that the production process can go out of control and $1/q_\theta$ is the percentage decrease in θ per dollar increase in φ_θ. The relation between the setup cost and capital investment can be described as $\varphi_s = q_s \ln(S_0/S)$ for $0 \le S \le S_0$, where $1/q_s$ is the

percentage decrease in S per dollar increase in φ_s . In addition, as it takes investment to reduce setup cost and process quality improvement, we should include an amortized investment cost in the proposal model.

4.2.1 Quality improvement, setup cost and lead time reduction model

The objective of Yang and Pan (2004) integrated model is to minimize the sum of the ordering/setup cost, holding cost, quality improvement and lead time crashing cost by simultaneously determining the optimal values of *Q, m, θ,* and *L*, subject to the constraints $0 < \theta < \theta_0$. In the present work, setup cost reduction in incorporated in the Yang and Pan (2004) model and resulting expected total cost per year expressed in Eq. (4.40) is minimized by simultaneously determining the optimal values of *Q, m, θ, S,* and *L*, subject to the constraints $0 < \theta < \theta_0$ and $0 < S < S_0$.

$$TRC(Q, L, m, \theta, S) = \frac{D}{Q}\left[A + \frac{S}{m} + \left\{c_i(L_{i-1} - L) + \sum_{j=1}^{i-1} c_j(b_j - a_j)\right\}\right] +$$

$$\frac{Q}{2}\left[\left\{m\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P}\right\}rC_v + rC_p + gmD\theta\right] + rC_p k\sigma\sqrt{L} + i_\theta q_\theta \ln\frac{\theta_0}{\theta} + i_s q_s \ln\frac{S_0}{S} \quad (4.40)$$

The total relevant cost is minimized over *Q, m, θ, S,* and *L,* as follows:

However for a fixed values of *Q, θ,* and *S*, *TRC*(*Q,m,θ,S,L*) is concave in $L \in [L_i, L_{i-1}]$, because $\frac{\partial^2 TRC(Q, L, m, \theta, S)}{\partial L^2} = -\frac{1}{4} rC_p k\sigma L^{-\frac{3}{2}}$. Therefore for a fixed *Q, θ,* and *S*, the minimum joint total expected cost will occur at the end points of the interval $[L_i, L_{i-1}]$. On the other hand, for a fixed $L \in [L_i, L_{i-1}]$, the proof of positive definiteness of Hessain matrix of *TRC*(*Q,m,θ,S,L*) is shown in Appendix B.5. Thus to yield the optimal order quantity, process quality and setup cost, for a given value of $L \in [L_i, L_{i-1}]$, setting the partial derivatives of *TRC*(*Q,m,θ,S,L*) with respect to *Q, θ,* and *S* to zero, it follows that

$$Q^* = \sqrt{\frac{2D\left[A + \frac{S}{m} + \left\{c_i(L_{i-1} - L) + \sum_{j=1}^{i-1} c_j(b_j - a_j)\right\}\right]}{\left[\left\{m\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P}\right\}rC_v + rC_p + gmD\theta\right]}} \quad (4.41)$$

$$\theta^* = \frac{2i_\theta q_\theta}{gmDQ} \tag{4.42}$$

$$S^* = \frac{i_s q_s Qm}{D} \tag{4.43}$$

The optimum value of m from the conditions: $TRC(Q,L,m,\theta,S) \le TRC(Q,L,m-1,\theta,S)$ and $TRC(Q,L,m,\theta,S) \le TRC(Q,L,m+1,\theta,S)$, yields:

$$m^*\left(m^*-1\right) \le \frac{S\left[rC_p - \left(1-\frac{2D}{P}\right)rC_v\right]}{\left[A + \left\{c_i\left(L_{i-1}-L\right) + \sum_{j=1}^{i-1} c_j\left(b_j - a_j\right)\right\}\right]\left[rC_v\left(1-\frac{D}{P}\right) + gD\theta\right]} \le m^*\left(m^*+1\right) \tag{4.44}$$

The solution procedure to find the optimal values of *Q, m, θ, S, L* is as follows:

Step 1. For each L_i, $i=1,2,\ldots,n$, set $S_i=S_0$.

- (i) set $\theta_i=\theta_0$.
- (ii) substitute S_i and θ_i into Eq. (4.44) to find m_i
- (iii) use S_i, θ_i and m_i to compute Q_i using Eq. (4.41)
- (iv) use Q_i and m_i to determine θ_i from Eq. (4.42). If $\theta_i \ge \theta_0$, then set $\theta_i=\theta_0$.
- (v) repeat steps (ii) – (iv) till no change in the value of θ_i.
- (vi) use Q_i and m_i to determine S_i from Eq. (4.43). Check for the positive value of third principal minor of Hessain matrix given in Appendix B.5. If $S_i \ge S_0$, then set $S_i=S_0$.
- (vii) repeat steps (i) – (vi) till no change in the value of S_i.

Step 2. The solution found in step 1 is optimal for each L_i, $i=1,2,\ldots,n$. Use Eq. (4.40) to compute $TRC(Q_i^*,m_i^*,\theta_i^*,S_i^*,L_i^*)$ for $i=1,2,\ldots,n$.

Step 3. The optimal set of solutions $TRC(Q_s,m_s,\theta_s,S_s,L_s)$ are set to the minimum value of $TRC(Q_i^*,m_i^*,\theta_i^*,S_i^*,L_i^*)$ for $i=1,2,\ldots,n$.

4.2.2 Numerical computation

Consider an inventory system with the following characteristics (Yang and Pan, 2004): D=1000 units/year, P=3200 units/year, A=\$25/order, S=\$400/setup, C_p=\$25/unit, C_v=\$20/unit, r=0.2,

k=2.33, σ=7 unit/week, i_θ=i_s=0.1/\$/year, g=\$15 per defective unit, $\phi_\theta = 400\ln\left(\frac{0.0002}{\theta}\right)$, $\phi_s = 400\ln\left(\frac{400}{S}\right)$, and the lead time has three components with data shown in Table 4.8.

Applying the solution procedure yields the optimum lead time of L_s=42 days, optimum number of deliveries, m_s=1, optimal out of control probability, θ_s= 0.00005803, optimal setup cost, S_s=\$ 3.68 and optimal order quantity, Q_s=92 units (Table 4.9). The total relevant annual cost is \$1091.3. The proposed model is shown to provide a lower total inventory cost with smaller lot size (Table 4.10). Yang and Pan (2004) model which does not consider the investment in setup cost reduction results in larger lot size, more frequent delivery and higher quality compared to the present model.

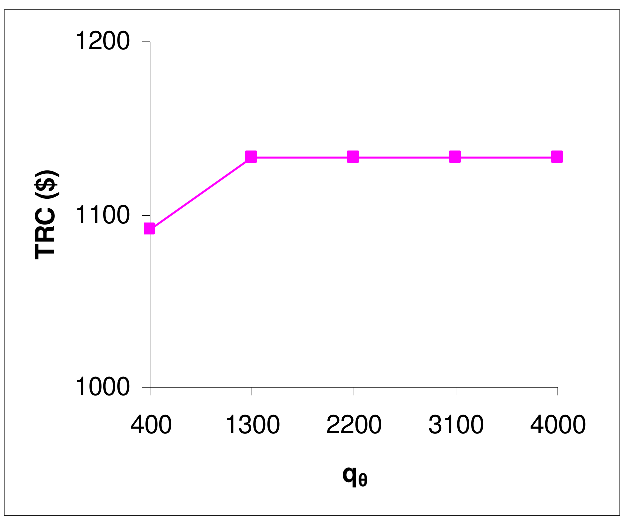

Fig. 4.5 Variation of TRC with q_θ

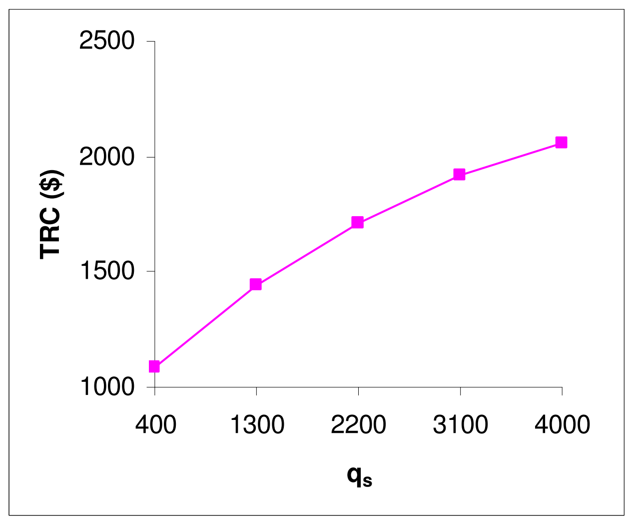

Fig. 4.6 Variation of TRC with q_s

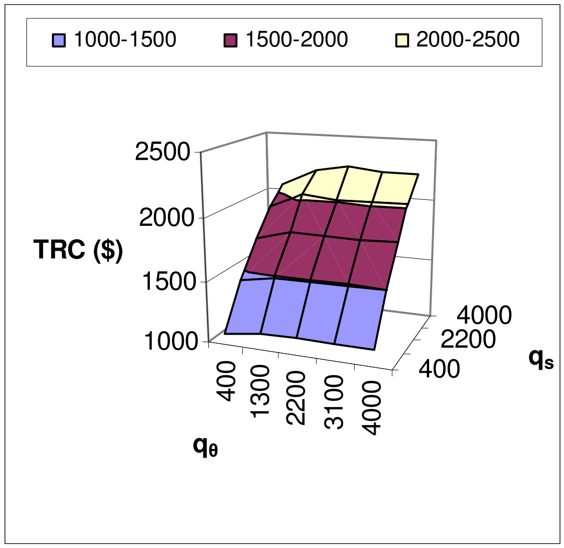

Fig. 4.7 Variation of TRC with q_θ and q_s

Figures 4.5 – 4.7 show the variation of total relevant cost considering investment in setup cost reduction, quality improvement and lead time reduction. Figures 4.5 – 4.6 show the variation of total relevant cost (TRC) with q_θ (rate of increase in dollar investment per fraction of reduction in non-conforming units) and q_s (rate of increase in dollar investment per fraction of reduction in setup cost). Figure 4.5 shows that TRC initially increases for increase in q_θ from 400 to 1300 and then TRC remains constant for further increase in q_θ from 1300 to 4000. This indicates that no investment in quality is required for the values of q_θ ranging from 1300 to 4000. Figure 4.6 shows that TRC increases with increase in q_s. Figure 4.7 shows the variation of TRC with both q_θ and q_s. For the data used in the Fig. 4.7, the TRC is higher for higher values of q_s.

Table 4.8 Lead time component data

Lead time component i	Normal duration b_i (days)	Minimum duration a_i (days)	Unit crashing cost c_i ($/day)
1	20	6	0.1
2	20	6	1.2
3	16	9	5.0

Table 4.9 Optimal values of quality improvement, setup cost and lead time reduction

i	L_i (days)	m_i	Q_i	θ_i	S_i ($)	TRC($)
0	56	1	89	0.00005963	3.58	1106.8
1	42	1	92	0.00005803	3.68	1091.3
2	28	1	118	0.00004536	4.70	1215.0
3	21	1	158	0.00003371	6.33	1447.0

Table 4.10 Summary of the comparison considering investment in setup cost reduction

	Traditional model (Yang and Pan, 2004)	Including crashing, frequent delivery, and quality investment (Yang and Pan, 2004)	Present work
Purchase order lot size	303	129	92
Lead time (days)	56	42	42
Number of deliveries	1	4	1
Probability of process being out of control	0.0002	0.000010336	0.00005803
Setup cost ($)	400	400	3.68
Total relevant cost per year ($)	3034.673	2273.359	1091.3

4.3 Managerial Implications

The managerial implications are as follows:

- The integrated inventory model is useful particularly for JIT inventory systems where the vendor and the buyer form a strategic alliance for profit sharing. The joint optimal policy yields minimum system wide costs as compared to decentralized vendor-buyer optimal policies. Buyer integration with high quality vendor is preferred due to lower total costs. High quality vendor yields lower total costs as compared to the low quality vendor due to lower expected defective costs per year and smaller investment for quality improvement. Limits on the quality improvement due to constraints yield a higher value of total relevant cost for all the cases of joint integration as well as buyer and vendor coordination. As shown in Table 4.1, the joint optimal integrated policy considering quality improvement yields a total relevant cost of $ 5334.2, as compared to the vendor and buyer optimal policy considering quality improvement which yields total relevant costs of $ 5353.8 and $ 5694.1 respectively. Under joint optimal integrated policy considering quality improvement, Table 4.3 shows the total relevant cost of \$ 4413.1 for a high quality vendor with $q_1(\theta) = 4000 \ln\left(\frac{0.00002}{\theta}\right)$, as compared to the total relevant cost of \$ 5334.2 shown in Table 4.1 for a vendor with quality $q(\theta) = 4000 \ln\left(\frac{0.0002}{\theta}\right)$. Under the joint optimal integrated policy considering quality improvement, Table 4.7 shows that for a vendor with budgetary constraints on the minimum quality, the total relevant cost is \$ 5493.4, which is higher than the total relevant cost of \$ 5334.2 shown in Table 4.1 for a vendor without budgetary constraints.

- Consideration of investment in setup cost reduction in the JIT integrated inventory model for a buyer and vendor with investments in both quality improvement and lead time reduction, leads to a lower total inventory cost with smaller lot size. As shown in Table 4.10, considering the Quality improvement, setup cost and lead time reduction, the purchase order lot size is 92 and total relevant cost per year is \$ 1091.3, while considering quality improvement and lead time reduction, the purchase order lot size is 129 and total relevant cost per year is \$ 2273.359.