CHAPTER 5

OPTIMAL CYCLE LENGTH AND NUMBER OF INSPECTIONS IN A DETERIORATING PRODUCTION PROCESS WITH INVESTMENT ON SETUP COST REDUCTION AND QUALITY IMPROVEMENT

This chapter provides the analytical model and numerical examples to determine simultaneously optimal production run length and inspection schedules in a deteriorating production process under the investment in both setup cost reduction and quality improvement. Also the analytical model and numerical case studies to determine optimal cycle length using both time-varying lot sizes and common cycle approach, and the number of inspections considering process inspection and restoration under the investment in both setup cost reduction and quality improvement are presented. The managerial implications based on the results are presented.

5.1 Optimal Cycle Length and Number of Inspections in a Deteriorating Production Process with Investment in Setup Cost Reduction and Quality Improvement

Kim et al., (2001) presented an economic production quantity model to determine the optimal production run length and inspection schedules simultaneously in a deteriorating production process. The production process is subject to a random deterioration from the in-control state to the out-of-control state and, thus, produces some proportion of defective items. They assumed that one inspection at the end of each production run must be done to ensure that the process is in the in-control state at the beginning of the next production cycle. Hou (2007) considered an economic production quantity model with imperfect production processes, in which the setup cost and process quality are functions of capital investment. In real production environment, the defective items are produced due to imperfect production processes. The defective items must be rejected, repaired and reworked, and thus substantial costs incur. Also, in practice, setup cost can be controlled and reduced through various efforts such as worker training, procedural changes and specialized equipment acquisition. Freimer et al., (2006) considered two kinds of investments in process improvements: (i) reducing setup costs and (ii) improving process quality. They have considered a production system of a single item on a single machine which produces some (time-varying) fraction of defective items.

This research extends the work of Kim et al., (2001) to derive the optimal production run length and inspection schedule by optimizing the (i) investment on setup cost reduction, (ii) quality improvement and (iii) both setup cost reduction and quality improvement. In this work it is assumed that the relationship between setup cost reduction (or process quality improvement) and capital investments can be described by the logarithmic investment function. Setup cost reduction is also considered an important element of successful implementation of JIT production. While the setup cost will be fixed in the short term, it will tend to decrease in the long term because of the possibility of investment in specialized equipment. However, substantial capital expenditure required for the installation of new equipment or production process gives rise to high interest charges and depreciation cost. The product can be produced more efficiently using a specialized equipment or production process that substantially reduces the production setup time and setup cost. It is thus economical to produce in smaller batches there by reducing the inventory holding cost. Therefore, for attaining production system efficiency, reduced lot sizes alone are not sufficient, unless accompanied by corresponding setup cost reduction and quality improvement. The remainder of the section is organized as given below.

The analytical models for optimal production run length and number of inspections in a deteriorating production process with investment in (i) setup cost reduction, (ii) quality improvement, and (iii) both setup cost reduction and quality improvement are provided. Algorithms are presented to determine the optimal production run length, optimal number of inspections, optimal setup cost, and percentage of defective items for optimal quality investment. Solution procedure and numerical example are described to determine the optimal production run length, the optimal setup cost and the optimal process quality.

The symbols used in this model are defined as follows:

D demand rate (units per unit time)

i_K fractional opportunity cost of capital investment in setup cost reduction per unit time

i_α fractional opportunity cost of capital investment in process quality improvement per unit time

K_o original setup cost for each production run ($ per setup)

K setup cost for each production run ($ per setup)

h inventory holding cost per unit time ($ per unit per unit time)

M_K capital investment in setup cost reduction

M_α capital investment in process quality improvement

N number of inspections undertaken during each production run

P production rate per unit time (units per unit time), P > D

r restoration cost of the process from the out-of-control state to the in-control state ($ per restoration)

s cost incurred by producing a defective item ($ per unit)

T production run time (time)

v inspection cost ($ per inspection)

α_o original percentage of defective items produced when the process is in the out-of control state

α percentage of defective items produced when the process is in the out-of control state

μ deterioration rate of the production process

The assumptions used in the development of mathematical model are (Lee and Rosenblatt, 1987 and Kim et al., 2001):

1. The production process is in-control state at the beginning of a production cycle. The production process may shift from the in-control state to the out-of control state during a production run.
2. When the production process is in the in-control state, it only produces perfect items, and when the process shifts to the out-of control state, a fixed portion of produced items are defective.
3. During a production run, inspections to monitor the state of production process are undertaken N times including an inspection at the end of production run.
4. As soon as the process is detected to be in the out-of-control state by inspection, it is restored to the in-control state instantaneously.
5. The elapsed time until shift is exponentially distributed.

5.1.1 Optimal cycle length and number of inspections in a deteriorating production process

The analysis for setup cost reduction, quality improvement, and both setup cost reduction and quality improvement are presented.

5.1.1.1 Model for setup cost reduction in a deteriorating production process

The mathematical formulation for optimization of setup cost reduction is as follows:

Rosenblatt and Lee (1986) presented an economic production quantity (EPQ) model which determines an approximated optimal production run length in deteriorating production process with fixed setup cost and quality. The total cost presented by Rosenblatt and Lee (1986) is a sum of setup cost, holding cost and rework cost, and is expressed as

$$TRC = \frac{KD}{PT} + \frac{h(P-D)T}{2} + s\alpha D - \frac{s\alpha D}{\mu T}\left[1-\exp(-\mu T)\right] \tag{5.1}$$

Further, Lee and Rosenblatt (1987) presented the average cost per unit time as the sum of setup cost, holding cost, inspection cost, restoration cost and cost incurred by defective items. The average cost per unit time as a function of number of inspections and production run length is:

$$C(N,T) = \frac{KD}{PT} + \frac{h(P-D)T}{2} + s\alpha D + \frac{DN}{PT}\left[v + \left(r - \frac{s\alpha P}{\mu}\right)\left\{1-\exp\left(-\frac{\mu T}{N}\right)\right\}\right] \tag{5.2}$$

Freimer et al. (2005) presented results for optimal run length and expected total cost of a production system assumed to produce some time varying proportion of defective parts considering the opportunity to invest in reducing setup cost and improving process quality.

A logarithmic investment function M_K required to reduce the setup cost from K_o to $K=K_o\,(1-\delta)$. In order to reduce the cost of production setup, investment is required in improving the production process. Considering an opportunity cost of $i_K M_K$ is charged per unit time, the total cost function is (Freimer et al., (2005)):

$$C(K,N,T) = i_K M_K + \frac{KD}{PT} + \frac{h(P-D)T}{2} + s\alpha D + \frac{DN}{PT}\left[v + \left(r - \frac{s\alpha P}{\mu}\right)\left\{1-\exp\left(-\frac{\mu T}{N}\right)\right\}\right] \tag{5.3}$$

The relationship between the investment function M_K and setup cost K is described as,

$$M_K = A_K - B_K \ln(K) \tag{5.4}$$

since $M_{K_o} = (A_K - B_K \ln(K_o)) = 0$; $\quad A_K = B_K \ln(K_o)$ (5.5)

Substituting A_K in Eq. (5.5) in Eq. (5,4) and considering $K = K_o(1-\delta_K)$, results in:

$$B_K = -\frac{M_K}{\ln(1-\delta_K)} \tag{5.6}$$

where A_K and B_K are constants.

Differentiating Eq. (5.3) with respect to setup cost yields,

$$\frac{\partial C(K,N,T)}{\partial K} = -\frac{i_K B_K}{K^*} + \frac{D}{PT} = 0 \tag{5.7}$$

Simplifying Eq. (5.7) results in the optimal setup cost as:

$$K^* = \frac{i_K B_K PT}{D} \tag{5.8}$$

From Eqs. (5.4), (5.5) and (5.8), investment in setup cost reduction is possible when $K_o > K^*$ or $\frac{K_o D}{i_K B_K PT} > 1$. Therefore investment in setup cost reduction is appropriate for high value of initial setup cost (K_o), high demand rate (D), low fractional opportunity cost of capital per unit time (i_K), small production run time (T).

Differentiating Eq. (5.3) with respect to the number of inspections undertaken (N) yields,

$$\frac{\partial C(K^*,N,T)}{\partial N} = \frac{D}{PT}\left[\left(v + r - \frac{s\alpha P}{\mu}\right) - \left(r - \frac{s\alpha P}{\mu}\right)\left(1 + \frac{\mu T}{N^*}\right)\exp\left(-\frac{\mu T}{N^*}\right)\right] = 0 \tag{5.9}$$

$$\lim_{N\to 0}\frac{\partial C(K^*,N,T)}{\partial N} = \frac{D}{PT}\left(v + r - \frac{s\alpha P}{\mu}\right) \tag{5.10}$$

$$\lim_{N\to \infty}\frac{\partial C(K^*,N,T)}{\partial N} = \frac{Dv}{PT} \tag{5.11}$$

$$\frac{\partial^2 C(K^*,N,T)}{\partial N^2} = \frac{D\mu^2 T}{PN^3}\left(\frac{s\alpha P}{\mu} - r\right)\exp\left(-\frac{\mu T}{N^*}\right) \tag{5.12}$$

If $\frac{s\alpha P}{\mu} > v + r$, then $\lim_{N\to 0}\frac{\partial C(K^*,N,T)}{\partial N} < 0$ and $\lim_{N\to\infty}\frac{\partial C(K^*,N,T)}{\partial N} > 0$, and also $\frac{\partial^2 C(K^*,N,T)}{\partial N^2} > 0$ for all N and hence an unique optimal solution satisfying

$\frac{\partial C(K^*, N, T)}{\partial N} = 0$ exists. If $\frac{s\alpha P}{\mu} \le v + r$, then $\lim_{N\to 0} \frac{\partial C(K^*, N, T)}{\partial N} \ge 0$ and $\lim_{N\to\infty} \frac{\partial C(K^*, N, T)}{\partial N} > 0$,

which implies $C(K^*, N, T)$ is a strictly increasing function of *N*, for all $N \ge 1$ and hence *N=1*.

Differentiating Eq. (5.3) with respect to production run time (T) yields,

$$\frac{\partial C(K^*, N^*, T)}{\partial T} = -\frac{K^* D}{PT^2} + \frac{h(P-D)}{2} - \frac{DN^*}{PT^2}\left[\left(v + r - \frac{s\alpha P}{\mu}\right) - \left(r - \frac{s\alpha P}{\mu}\right)\left(1 + \frac{\mu T}{N^*}\right)\exp\left(-\frac{\mu T}{N^*}\right)\right] = 0 \tag{5.13}$$

where K^* is substituted from Eq. (5.8). Further differentiating Eq. (5.13) with respect to production run time (T) yields,

$$\frac{\partial}{\partial T}\left(T^2 \frac{\partial C(K^*, N^*, T)}{\partial T}\right) = -i_K B_K + hT(P-D) + \frac{\mu^2 TD}{PN^*}\left(\frac{s\alpha P}{\mu} - r\right)\exp\left(-\frac{\mu T}{N^*}\right) \tag{5.14}$$

$$\lim_{T\to 0}\left(T^2 \frac{\partial C(K^*, N^*, T)}{\partial T}\right) = -\frac{DN^* v}{P} < 0 \tag{5.15}$$

$$\lim_{T\to\infty}\left(T^2 \frac{\partial C(K^*, N^*, T)}{\partial T}\right) = \infty \tag{5.16}$$

From Eqs. (5.15)-(5.16), the optimal solution satisfying $\frac{\partial C(K^*, N^*, T)}{\partial T} = 0$ also exists. If $\frac{\partial}{\partial T}\left(T^2 \frac{\partial C(K^*, N^*, T)}{\partial T}\right) > 0$ from Eq. (5.14), which implies T^* can be uniquely determined for $\left(T^2 \frac{\partial C(K^*, N^*, T)}{\partial T}\right) = 0$.

Substituting $T=T^*$ in Eq. (5.9), and using Eq. (5.13), results in

$$T^* = \frac{2 i_K B_K}{h(P-D)} \tag{5.17}$$

The solution algorithm for this model is as follows:

Step 1: Determine T^* from Eq. (5.17).

Step 2: If $\frac{s\alpha P}{\mu} \leq v + r$, then $N^*=1$. Otherwise, substitute the value of T^* from Eq. (5.17) in Eq. (5.9) to find N^* numerically.

Step 3:

(i) If $N^* < 1$, then choose $N^*=1$. Determine K^* (1) from Eq. (5.8). If $K^* \leq K_o$, then optimal setup cost is taken as K^*, otherwise, set $K^* = K_o$, and no capital investment for setup cost reduction is required. Find $T^*(1)$ from Eq. (5.13) numerically till convergence is obtained.

(ii) If $N^* > 1$, then for $J^* < N^* < J^*+1$ (where J is a positive integer). Determine $K^*(J)$ and $K^*(J+1)$ from Eq. (5.8). If $K^* \leq K_o$, then optimal setup cost is taken as K^*, otherwise, set $K^* = K_o$. Find $T^*(J)$ and $T^*(J+1)$ from Eq. (5.13) numerically till convergence is obtained.

The convergence criteria for algorithm is: $\left| \frac{\frac{\partial C(K^*, N^*, T)}{\partial T}}{\frac{\partial^2 C(K^*, N^*, T)}{\partial T^2}} \right| \leq 10^{-3}$

Step 4:

(i) If $N^* < 1$, then compute $C^{**}(K^{**}(T^{**}), N^{**}=1, T^{**}(N^{**}=1))$ from Eq. (5.3).

(ii) If $N^* > 1$, then for $J^* < N^* < J^*+1$ (where J is a positive integer), then compute $C(K^*(J), J, T^*(J))$ and $C(K^*(J+1), J+1, T^*(J+1))$ from Eq. (5.3) respectively, and choose the one with smaller value $(C^{**}(K^{**}(T^{**}), N^{**}, T^{**}(N^{**})))$ as the optimum solution.

5.1.1.2 Model for quality improvement in a deteriorating production process

The classical economic production quantity (EPQ) model assumes that the production facility is failure free and all the items produced are of perfect quality, and that quality level is fixed at an optimal level. However, in real production environment, it can often be observed that the product quality is not always perfect and usually depends on the state of production process. Moreover, it has been evidenced that just-in-time production is based on the belief that product quality can be improved through various efforts.

Although the production process begins by producing a lot in the in-control state, it may become out-of-control producing defective items. Thus defective items produced due to imperfect production process incur substantial costs. Substantial investment in improving the quality of production process results in reduction of the cost incurred on defective items produced in a process. A logarithmic investment function M_α is required to reduce the percentage of defective items produced from α_o to $\alpha=\alpha_o\,(1-\delta_\alpha)$. Considering an opportunity cost of $i_\alpha M_\alpha$ is charged per unit time, the total cost function is (Freimer et al., (2005)):

$$C(\alpha,N,T)=i_\alpha M_\alpha+\frac{KD}{PT}+\frac{h(P-D)T}{2}+s\alpha D+\frac{DN}{PT}\left[v+\left(r-\frac{s\alpha P}{\mu}\right)\left\{1-\exp\left(-\frac{\mu T}{N}\right)\right\}\right] \tag{5.18}$$

The relation between the investment function for quality improvement M_α and the percentage of defective items when the process is in out-of-control state α is described by;

$$M_\alpha=A_\alpha-B_\alpha\ln(\alpha) \tag{5.19}$$

Since $M_{\alpha_o}=(A_\alpha-B_\alpha\ln(\alpha_o))=0$; $\quad A_\alpha=B_\alpha\ln(\alpha_o)$ (5.20)

Substituting A_α from Eq. (5.20) in Eq. (5.19), and considering $\alpha=\alpha_o(1-\delta_\alpha)$, results in:

$$B_\alpha=-\frac{M_\alpha}{\ln(1-\delta_\alpha)} \tag{5.21}$$

where A_α and B_α are constants

Differentiating Eq. (5.18) with respect to the percentage of defective items produced (α) yields,

$$\frac{\partial C(\alpha,N,T)}{\partial\alpha}=-\frac{i_\alpha B_\alpha}{\alpha^*}+sD-\frac{sDN}{\mu T}\left\{1-\exp\left(-\frac{\mu T}{N}\right)\right\}=0 \tag{5.22}$$

Simplifying Eq. (5.22) results in:

$$\alpha^*=\frac{i_\alpha B_\alpha}{sD\left(1-\frac{N}{\mu T}\left\{1-\exp\left(-\frac{\mu T}{N}\right)\right\}\right)} \tag{5.23}$$

From Eqs. (5.19), (5.20) and (5.23), investment in quality improvement is possible when $\alpha_o>\alpha^*$ or $\frac{\alpha_o sD}{i_\alpha B_\alpha}\left(1-\frac{N}{\mu T}\left\{1-\exp\left(-\frac{\mu T}{N}\right)\right\}\right)>1$. Therefore investment in quality improvement is appropriate for high value percentage of defective items produced when the process is in the out-of-control state (α_o), high cost incurred for producing a defective item (s), high demand rate (D), low fractional opportunity cost of capital per unit time (i_α), more production run time (T) and less number of inspections undertaken during each production run (N).

Differentiating Eq. (5.18) with respect to the number of inspections undertaken (N) yields,

$$\frac{\partial C(\alpha^*, N, T)}{\partial N} = \frac{D}{PT}\left[\left(v + r - \frac{s\alpha^* P}{\mu}\right) - \left(r - \frac{s\alpha^* P}{\mu}\right)\left(1 + \frac{\mu T}{N^*}\right)\exp\left(-\frac{\mu T}{N^*}\right)\right] = 0 \qquad (5.24)$$

The conditions for determination of optimal value of N^* are given in Eqs. (5.10) – (5.12).

Differentiating Eq. (5.18) with respect to production run time (T) yields,

$$\frac{\partial C(\alpha^*, N^*, T)}{\partial T} = -\frac{KD}{PT^2} + \frac{h(P-D)}{2} - \frac{DN^*}{PT^2}\left[\left(v + r - \frac{s\alpha^* P}{\mu}\right) - \left(r - \frac{s\alpha^* P}{\mu}\right)\left(1 + \frac{\mu T}{N^*}\right)\exp\left(-\frac{\mu T}{N^*}\right)\right] = 0 \qquad (5.25)$$

$$\frac{\partial}{\partial T}\left(T^2 \frac{\partial C(\alpha^*, N^*, T)}{\partial T}\right) = hT(P-D) + \frac{\mu^2 TD}{PN^*}\left(\frac{s\alpha P}{\mu} - r\right)\exp\left(-\frac{\mu T}{N^*}\right) \qquad (5.26)$$

$$\lim_{T\to 0}\left(T^2 \frac{\partial C(\alpha^*, N^*, T)}{\partial T}\right) = -\frac{D(K + N^* v)}{P} < 0 \qquad (5.27)$$

$$\lim_{T\to \infty}\left(T^2 \frac{\partial C(\alpha^*, N^*, T)}{\partial T}\right) = \infty \qquad (5.28)$$

From Eqs. (5.27)-(5.28), the optimal solution satisfying $\frac{\partial C(\alpha^*, N^*, T)}{\partial T} = 0$ also exists. If $\frac{\partial}{\partial T}\left(T^2 \frac{\partial C(\alpha^*, N^*, T)}{\partial T}\right) > 0$ from Eq. (5.26), which implies T^* can be uniquely determined for $\left(T^2 \frac{\partial C(\alpha^*, N^*, T)}{\partial T}\right) = 0$.

Substituting Eq. (5.24) in Eq. (5.25), results in

$$T^* = \sqrt{\frac{2KD}{Ph(P-D)}} \qquad (5.29)$$

The solution algorithm for this model is as follows:

Step 1: Calculate the value of T^* from Eq. (5.29) and initialize N^*.

Step 2: Substitute the value of T^* from Eq. (5.29) and N^* to find α^* using Eq. (5.23).

Step 3: Compute N^* from Eq. (5.24) using T^* from Eq. (5.29), α^* from Eq. (5.23). If $\alpha^* \le \alpha_o$, then optimal quality is taken as α^*, otherwise, set $\alpha^* = \alpha_o$, and no capital investment for quality improvement is required. Using the current value of N^*, repeat the steps 2 and 3 till the convergence in the value of N^* is obtained.

The convergence criteria for algorithm is: $\left| \dfrac{\dfrac{\partial C(\alpha^*, N, T)}{\partial N}}{\dfrac{\partial^2 C(\alpha^*, N, T)}{\partial N^2}} \right| \le 10^{-3}$

Step 4:

(i) If $N^* < 1$, then choose $N^*=1$.

(a) Substitute the value of and N^* in Eq. (5.23) to find α^*.

(b) If $\alpha^* \le \alpha_o$, then optimal quality is taken as α^*, otherwise, set $\alpha^* = \alpha_o$, and no capital investment for quality improvement is required.

(c) Compute T^* from Eq. (5.25) using N^*, α^* from Eq. (5.23).

(d) Repeat the steps (a) and (b) till the convergence in the value of T* is obtained.

(ii) If $N^* > 1$, then for $J^* < N^* < J^*+1$ (where J is a positive integer),

(ii) Substitute the value of J^* and J^*+1 in Eq. (5.23) to find $\alpha^*(J)$ and $\alpha^*(J+1)$.

(iii) If $\alpha^* \le \alpha_o$, then optimal quality is taken as α^*, otherwise, set $\alpha^* = \alpha_o$.

(iiii) Compute $T^*(J)$ and $T^*(J+1)$ from Eq. (5.25) numerically, using J^* and J^*+1, $\alpha^*(J)$ and $\alpha^*(J+1)$ from Eq. (5.23).

(iiv) Repeat the steps (e) and (g) till the convergence in the value of $T^*(J)$ and $T^*(J+1)$ are obtained.

The convergence criteria for algorithm is: $\left| \dfrac{\dfrac{\partial C(\alpha^*, N^*, T)}{\partial T}}{\dfrac{\partial^2 C(\alpha^*, N^*, T)}{\partial T^2}} \right| \le 10^{-3}$

Step 5:

(i) If $N^* < 1$, then compute $C^{**}(\alpha^{**}(N^{**}=1), N^{**}=1, T^{**}(N^{**}=1))$ from Eq. (5.18).

(ii) If $N^* > 1$, then for $J^* < N^* < J^*+1$ (where J is a positive integer), then compute $C(\alpha^*(J), J, T^*(J))$ and $C(\alpha^*(J+1), J+1, T^*(J+1))$ from Eq. (5.18) respectively, and choose the one with smaller value $(C^{**}(\alpha^{**}(N), N^{**}, T^{**}(N^{**})))$ as the optimum solution.

5.1.1.3 Model for both setup cost reduction and quality improvement in a deteriorating production process

Considering opportunity costs of $i_K M_K$ and $i_\alpha M_\alpha$ charged per unit time to yield the optimal setup cost and optimal quality, the total cost function is:

$$C(K,\alpha,N,T) = i_K M_K + i_\alpha M_\alpha + \frac{KD}{PT} + \frac{h(P-D)T}{2} + s\alpha D + \frac{DN}{PT}\left[v + \left(r - \frac{s\alpha P}{\mu}\right)\left\{1 - \exp\left(-\frac{\mu T}{N}\right)\right\}\right] \quad (5.30)$$

The partial derivatives of Eq. (5.30) with respect to K and α are given by;

$$\frac{\partial C(K,\alpha,N,T)}{\partial K} = -\frac{i_K B_K}{K^*} + \frac{D}{PT} = 0 \quad (5.31)$$

$$\frac{\partial C(K,\alpha,N,T)}{\partial \alpha} = -\frac{i_\alpha B_\alpha}{\alpha^*} + sD - \frac{sDN}{\mu T}\left\{1 - \exp\left(-\frac{\mu T}{N}\right)\right\} = 0 \quad (5.32)$$

Simplifying Eqs. (5.31) - (5.32) yields the optimal setup cost and optimal quality as given by Eq. (5.8) and Eq. (5.23) respectively.

Eq. (5.30) is convex in K and α, since $\frac{\partial^2 C(K,\alpha,N,T)}{\partial K^2} = \frac{i_K B_K}{K^{*2}} > 0$, $\frac{\partial^2 C(K,\alpha,N,T)}{\partial \alpha^2} = \frac{i_\alpha B_\alpha}{\alpha^{*2}} > 0$, $\frac{\partial^2 C(K,\alpha,N,T)}{\partial K \partial \alpha} = 0$, and the determinant of the Hessain's matrix can be calculated as; $\frac{\partial^2 C(K,\alpha,N,T)}{\partial K^2}\frac{\partial^2 C(K,\alpha,N,T)}{\partial \alpha^2} - \left(\frac{\partial^2 C(K,\alpha,N,T)}{\partial K \partial \alpha}\right)^2 = \frac{i_K B_K}{K^{*2}}\frac{i_\alpha B_\alpha}{\alpha^{*2}} > 0$.

The partial derivative of Eq. (5.30) with respect to the number of inspections undertaken (N) is given by Eq. (5.24) and the conditions for determination of optimal value of N^* are given in Eqs. (5.10) – (5.12).

The partial derivative of Eq. (5.30) with respect to the production run time (T) is same as that given by

$$\frac{\partial C(K^*,\alpha^*,N^*,T)}{\partial T}$$
$$= -\frac{K^*D}{PT^2} + \frac{h(P-D)}{2} - \frac{DN^*}{PT^2}\left[\left(v + r - \frac{s\alpha^*P}{\mu}\right) - \left(r - \frac{s\alpha^*P}{\mu}\right)\left(1 + \frac{\mu T}{N^*}\right)\exp\left(-\frac{\mu T}{N^*}\right)\right] = 0 \quad (5.33)$$

where K^* and α^* are substituted from Eqs. (5.8) and (5.23). The conditions for determination of optimal value of T^* are given in Eqs. (5.15) – (5.16).

The solution algorithm is as follows:

Step 1: Calculate the value of T^* from Eq. (5.17) and initialize N^*.

Step 2: Substitute the value of T^* from Eq. (5.17) and N^* to find α^* using Eq. (5.23).

Step 3: Compute N^* from Eq. (5.24) using T^* from Eq. (5.17), α^* from Eq. (5.23). If $\alpha^* \le \alpha_o$, then optimal quality is taken as α^*, otherwise, set $\alpha^* = \alpha_o$, and no capital investment for quality improvement is required. Using the current value of N^*, repeat the steps 2 and 3 till the convergence in the value of N^* is obtained.

The convergence criteria for algorithm is: $\left|\frac{\frac{\partial C(\alpha^*,N,T)}{\partial N}}{\frac{\partial^2 C(\alpha^*,N,T)}{\partial N^2}}\right| \le 10^{-3}$

Step 4:

(i) If $N^* < 1$, then choose $N^*=1$.

(a) Using the values of N^*, find $\alpha^*(1)$ in Eq. (5.23) and $K^*(1)$ from Eq. (5.8).

(b) If $K^* \le K_o$, then optimal setup cost is taken as K^*, otherwise, set $K^* = K_o$, and no capital investment for setup cost reduction is required. Similarly, if $\alpha^* \le \alpha_o$, then optimal quality is taken as α^*, otherwise, set $\alpha^* = \alpha_o$, and no capital investment for quality improvement is required.

(c) Compute $T^*(1)$ from Eq. (5.13).

(d) Repeat the steps (a) to (c) till the convergence in the value of $T^*(1)$ is obtained.

(ii) If $N^* > 1$, then for $J^* < N^* < J^*+1$ (where J is a positive integer),

(e) use the values of J^* and J^*+1 in Eq. (5.8) to determine $K^*(J)$ and $K^*(J+1)$ and in Eq. (5.23) to find $\alpha^*(J)$ and $\alpha^*(J+1)$.

(f) If $K^* \le K_o$, then optimal setup cost is taken as K^*, otherwise, set $K^* = K_o$. Similarly, if $\alpha^* \le \alpha_o$, then optimal quality is taken as α^*, otherwise, set $\alpha^* = \alpha_o$.

(g) Compute $T^*(J)$ and $T^*(J+1)$ from Eq. (5.13) numerically.

(h) Repeat the steps (e) to (g) till the convergence in the value of $T^*(J)$ and $T^*(J+1)$ are obtained.

The convergence criteria for algorithm is: $\left| \dfrac{\dfrac{\partial C(K^*, N^*, T)}{\partial T}}{\dfrac{\partial^2 C(K^*, N^*, T)}{\partial T^2}} \right| \le 10^{-3}$

Step 5:

(i) If $N^* < 1$, then compute $C^{**}(K^{**}(T^{**}(N^{**}=1)), \alpha^{**}(N^{**}=1, T^{**}(N^{**}=1)), N^{**}=1, T^{**}(N^{**}=1))$ from Eq. (5.30).

(ii) If $N^* > 1$, then for $J^* < N^* < J^*+1$ (where J is a positive integer), then compute $C(K^*(T^*(J)), \alpha^*(J, T^*(J)), J, T^*(J))$ and $C(K^*(T^*(J+1)), \alpha^*(J+1, T^*(J+1)), J+1, T^*(J+1))$ from Eq. (5.30) respectively, and choose the one with smaller value $(C^{**}(K^{**}(T^{**}(N^{**})), \alpha^{**}(N, T^{**}(N^{**})), N^{**}, T^{**}(N^{**})))$ as the optimum solution.

5.1.2 Numerical example

5.1.2.1 Numerical example for setup cost reduction model

The following data used by Kim et al., (2001) is considered in this study. P=40, D=30, K_o=50, h=0.1, s=10, α_o=0.05, i_k=0.01, M_k=100, δ_k=0.25.

Table 5.1 shows the optimal number of inspections, the optimal production run length, and the minimum average cost of the proposed model and the solution given by Kim et al., (2001), and the percentage difference of the production run length and the minimum average cost between the two solutions. Table 5.1 show that considering setup cost reduction reduces the optimal number of inspections. The impact of setup cost reduction is more at lower values of restoration cost of the process from the out-of-control state to in-control state (r) for a given range of values of deterioration rate of the production process (μ) and inspection cost (v).

When low values of restoration cost of the production process (*r*) and inspection cost (*v*), with increase in μ from 0.1 to 0.5, the present method predicts optimal number of inspections less than or equal that predicted by Kim et al., (2001) without considering setup cost reduction. However at higher values of *r* and *v* considered in this study for μ from 0.1 to 0.5, the optimal number of inspections predicted by the present method agrees with that obtained from Kim et al. (2001), which shows no influence of investment in setup cost reduction.

With the investment in setup cost reduction, there is a significant reduction in optimal production run length (T^{**}) at low and intermediate values of restoration cost (*r*) and inspection cost (*v*) for all values of deterioration rate (μ) in comparison to without investment in setup cost reduction. However at high values of restoration cost (*r*) and inspection cost (*v*), there is a marginal reduction in the optimal production run length (T^{**}) with investment in setup cost reduction. For all the parameters considered in this study the optimal production run length (T^{**}) reduces with investment in setup cost reduction.

The average cost per unit time $C^{**}(K^{**}(N^{**}), N^{**}, T^{**}(N^{**}))$ is slightly reduced with investment in setup cost reduction (Table 5.1). The influence of setup cost reduction on the average cost per unit time is more at (i) μ=0.1, *r*=10 and *v*=10 as compared to μ=0.5, *r*=10 and *v*=10, and (ii) μ=0.1, *r*=10 and *v*=10 as compared to μ=0.1, *r*=180 and *v*=10. This indicates decrease in reduction of average cost function with investment on setup cost reduction as: (i) deterioration of the production process (μ) increases from 0.1 to 0.5 for a given restoration cost (*r*) and inspection cost (*v*), (ii) restoration cost (*r*) increases for a given deterioration of the production process (μ) and inspection cost (*v*).

Table 5.1 Comparison of the optimal solution with and without investment in setup cost reduction

			Kim et al.(2001)			Setup cost reduction				% Difference with Kim et al.(2001)	
μ	r	v	J^{**}	T^{**}	C^{**}	K^{**}	J^{**}	T^{**}	C^{**}	T^{**}	C^{**}
0.1	10	10	2	8.25	13.81	33.07	2	7.13	13.60	-15.64	-1.57
		20	2	9.41	15.51	28.12	1	6.07	15.30	-55.10	-1.38
	20	10	2	8.33	14.43	33.56	2	7.24	14.23	-15.04	-1.43
		20	2	9.50	16.11	28.72	1	6.20	15.86	-53.30	-1.58
	30	10	2	8.41	15.04	34.07	2	7.35	14.86	-14.41	-1.24
		20	2	9.60	16.71	29.36	1	6.33	16.42	-51.59	-1.78
	60	10	2	8.67	16.87	35.70	2	7.70	16.73	-12.53	-0.83
		20	1	8.04	18.33	31.44	1	6.78	18.07	-18.53	-1.46
	120	10	1	8.15	20.49	30.71	1	6.63	20.23	-22.98	-1.30
		20	1	8.85	21.38	36.61	1	7.90	21.26	-12.04	-0.57
	180	10	1	9.11	23.51	37.48	1	8.09	23.42	-12.66	-0.38
		20	1	9.86	24.30	43.47	1	9.38	24.28	-5.14	-0.09
0.2	10	10	3	8.61	16.00	28.72	2	6.20	15.86	-38.94	-0.88
		20	2	8.73	18.09	37.07	2	8.00	17.97	-9.14	-0.66
	20	10	3	8.81	17.14	30.02	2	6.48	16.97	-36.04	-1.00
		20	2	8.98	19.09	38.55	2	8.32	19.00	-7.97	-0.48
	30	10	3	9.02	18.27	31.44	2	6.78	18.07	-32.98	-1.12
		20	2	9.24	20.07	27.91	1	6.02	19.90	-53.44	-0.85
	60	10	2	8.85	21.38	36.61	2	7.90	21.26	-12.04	-0.57
		20	1	8.61	22.54	34.51	1	7.45	22.38	-15.63	-0.71
	90	10	1	9.05	23.80	36.84	1	7.95	23.70	-13.87	-0.40
		20	1	9.81	24.60	43.04	1	9.29	24.57	-5.63	-0.11

Table 5.1 Comparison of the optimal solution with and without investment in setup cost reduction (contd….)

			Kim et al.(2001)			Setup cost reduction				% Difference with Kim et al.(2001)	
μ	r	v	J^{**}	T^{**}	C^{**}	K^{**}	J^{**}	T^{**}	C^{**}	T^{**}	C^{**}
0.3	10	10	3	8.28	17.72	33.85	3	7.30	17.52	-13.38	-1.16
		20	2	8.53	19.99	35.60	2	7.68	19.83	-11.05	-0.79
	20	10	3	8.65	19.23	29.18	2	6.30	19.08	-37.41	-0.77
		20	2	8.97	21.24	27.02	1	5.83	21.14	-53.83	-0.45
	30	10	3	9.06	20.73	31.94	2	6.89	20.51	-31.49	-1.05
		20	1	8.15	22.44	30.57	1	6.60	22.17	-23.58	-1.24
	50	10	1	8.48	23.19	32.00	1	6.91	22.98	-22.81	-0.90
		20	1	9.27	24.03	38.90	1	8.39	23.96	-10.46	-0.28
	60	10	1	9.08	23.98	36.93	1	7.97	23.88	-13.94	-0.40
		20	1	9.85	24.77	43.29	1	9.34	24.75	-5.46	-0.08
0.4	10	10	4	8.98	19.09	32.88	3	7.09	18.89	-26.59	-1.05
		20	2	8.58	21.42	35.57	2	7.67	21.27	-11.81	-0.70
	20	10	3	8.73	20.92	29.88	2	6.45	20.76	-35.41	-0.78
		20	2	9.24	22.82	30.11	1	6.50	22.59	-42.22	-1.02
	30	10	2	8.61	22.54	34.51	2	7.45	22.38	-15.63	-0.71
		20	1	8.88	23.71	35.63	1	7.69	23.59	-15.53	-0.52
	40	10	1	8.78	23.69	34.17	1	7.37	23.54	-19.08	-0.65
		20	1	9.57	24.50	41.09	1	8.87	24.46	-7.94	-0.16
0.5	10	10	4	8.93	20.25	32.67	3	7.05	20.05	-26.68	-0.98
		20	2	8.78	22.52	36.51	2	7.88	22.40	-11.45	-0.54
	20	10	3	8.97	22.28	31.77	2	6.86	22.08	-30.85	-0.91
		20	1	8.78	23.68	34.38	1	7.42	23.54	-18.36	-0.61
	30	10	1	8.72	23.67	33.43	1	7.21	23.51	-20.91	-0.68
		20	1	9.53	24.49	40.67	1	8.78	24.45	-8.60	-0.18

5.1.2.2 Numerical example for quality improvement model

Apart from the data used in the previous example on the optimization of setup cost reduction, the following data are used to obtain the numerical results of optimization of quality improvement: i_α=0.01, M_α=100, δ_α=0.25.

Table 5.2 shows the optimal quality, optimal number of inspections, the optimal production run length, and the minimum average cost of the due to investment on quality improvement. Table 5.2 also shows the percentage difference of the production run length and the minimum average cost between the present solution and that given by Kim et al., (2001) with no investment in process quality improvement. Table 5.2 shows that for deterioration rate (μ) of 0.1, inspection cost of (v) of 10 and restoration cost of the process (r) of 10, 20, 30, and 60 respectively, there is no need for investment in quality improvement. The optimal percentage of defective items produced decrease with (i) increase in deterioration rate of the production process (μ) from 0.1 to 0.5 and (ii) increase in restoration cost of the process from out-of-control state to in-control state (r) (for various parameters of μ=0.1 to 0.5).

The optimal number of inspections are unity for μ=0.3 to 0.5, for all the parameters of r and v considered in this study. The optimal number of inspections decrease to unity for μ=0.1 and 0.2, with increase in r, and also the optimal number of inspections decrease or remain same with increase in v for a given r for μ=0.1 and 0.2.

With investment in quality improvement, the optimal production run length (T^{**}) (i) increases with increase in μ from 0.1 to 0.5, for a given set of values of r and v, (ii) increases with increase in r, for a given set of μ and v, and (iii) increases with v, for a given μ and r. With increase in the parameters μ and r, the optimal production run length is higher due to investment in quality improvement as compared to without investment in quality (Kim et al., 2001). This indicates that investment in quality improvement results in reduction in the number of inspections and increase in the optimal production run length with (i) increase in deterioration rate of the production process, (ii) increase in the restoration cost of the process, (iii) increase in the inspection cost.

The average cost per unit time is reduced with investment in quality improvement (Table 5.2). The influence of investment in quality improvement on the reduction in average cost per unit time is more at (i) with increase in μ from 0.1 to 0.5 for a given r and v , (ii) with increase in restoration cost of the process (r) for a given μ and v, and (iii) with increase in inspection cost (v) for a given value of μ and r (the influence of inspection cost is negligible for higher values of μ and r).

Table 5.2 Comparison of the optimal solution with and without investment in quality improvement

			Quality improvement				% Difference with Kim et al.(2001)	
μ	r	v	α^{**}	J^{**}	T^{**}	C^{**}	T^{**}	C^{**}
0.1	10	10	0.05	2	8.25	13.81	0.00	0.00
		20	0.05	2	9.41	15.51	0.00	0.00
	20	10	0.05	2	8.33	14.43	0.00	0.00
		20	0.0364	1	8.21	16.10	-15.71	-0.06
	30	10	0.05	2	8.41	15.04	0.00	0.00
		20	0.0359	1	8.37	16.61	-14.70	-0.60
	60	10	0.05	2	8.67	16.87	0.00	0.00
		20	0.0343	1	8.89	18.11	9.56	-1.21
	120	10	0.0334	1	9.23	20.25	11.70	-1.19
		20	0.0312	1	10.16	21.02	12.89	-1.71
	180	10	0.0298	1	10.81	23.09	15.73	-1.82
		20	0.0281	1	11.76	23.76	16.16	-2.27
0.2	10	10	0.0481	3	8.68	16.00	0.81	0.00
		20	0.0218	1	8.85	17.42	1.36	-3.85
	20	10	0.0354	2	8.54	17.11	-3.16	-0.18
		20	0.0212	1	9.29	18.11	3.34	-5.41
	30	10	0.0217	1	8.90	17.98	-1.35	-1.61
		20	0.0207	1	9.75	18.78	5.23	-6.87
	60	10	0.0200	1	10.42	19.97	15.07	-7.06
		20	0.0192	1	11.23	20.67	23.33	-9.05
	90	10	0.0186	1	12.03	21.77	24.77	-9.32
		20	0.0181	1	12.78	22.37	23.24	-9.97

Table 5.2 Comparison of the optimal solution with and without investment in quality improvement (Contd…)

			Quality improvement				% Difference with Kim et al.(2001)	
μ	r	v	α^{**}	J^{**}	T^{**}	C^{**}	T^{**}	C^{**}
0.3	10	10	0.0180	1	8.64	17.36	4.17	-2.07
		20	0.0173	1	9.47	18.18	9.93	-9.96
	20	10	0.0175	1	9.28	18.13	6.79	-6.07
		20	0.0169	1	10.09	18.90	11.10	-12.38
	30	10	0.0170	1	9.94	18.87	8.85	-9.86
		20	0.0165	1	10.72	19.60	23.97	-14.49
	50	10	0.0162	1	11.23	20.23	24.49	-14.63
		20	0.0159	1	11.95	20.88	22.43	-15.09
	60	10	0.0159	1	11.86	20.86	23.44	-14.96
		20	0.0157	1	12.54	21.48	21.45	-15.32
0.4	10	10	0.0158	1	9.07	17.77	0.99	-7.43
		20	0.0154	1	9.88	18.57	13.16	-15.35
	20	10	0.0154	1	9.80	18.55	10.92	-12.78
		20	0.0151	1	10.57	19.29	12.58	-18.30
	30	10	0.0151	1	10.52	19.27	18.16	-16.97
		20	0.0149	1	11.24	19.97	21.00	-18.73
	40	10	0.0149	1	11.19	19.96	21.54	-18.69
		20	0.0146	1	11.88	20.60	19.44	-18.93
0.5	10	10	0.0147	1	9.35	18.01	4.49	-12.44
		20	0.0144	1	10.14	18.78	13.41	-19.91
	20	10	0.0144	1	10.11	18.78	11.28	-18.64
		20	0.0142	1	10.84	19.49	19.00	-21.50
	30	10	0.0142	1	10.82	19.49	19.41	-21.45
		20	0.0140	1	11.51	20.16	17.20	-21.48

5.1.2.3 Numerical example for both setup cost reduction and quality improvement model

The data used in this study is same as those used in the previous examples for optimization of setup cost reduction and optimization of quality improvement.

Table 5.3 shows the optimal solution with investment in both setup cost reduction and quality improvement. Table 5.3 also shows the calculation of percentage difference of the production run length and the minimum average cost without investment given by Kim et al., (2001). Table

5.3 shows that for all values of deterioration rate of the production process (μ=0.1 to 0.5) and the corresponding higher values of restoration cost (r) and inspection cost (v) considered in this study, such as for μ=0.1, (r, v)= (180, 10), (180, 20) and for μ=0.2, (r, v)= (60, 20), (90, 10), (90, 20) respectively, there is no requirement for investment in setup cost reduction and investment is only needed for quality improvement. Similarly for deterioration rate of the process (μ) of 0.1 and inspection cost (v) of 10 considered in this study, the investment in quality improvement is not required for restoration cost values (r) of 10, 20, 30 and 60 respectively.
The results obtained for optimal number of inspections for all the parameters of μ, r and v considered in this case is same as those obtained in the previous example for optimal investment in quality improvement.

With investment in both setup cost reduction and quality improvement, the optimal production run length (T^{**}) (i) increases with increase in μ from 0.1 to 0.5, for a given set of values of r and v, (ii) increases with increase in r, for a given μ and r and (iii) increases with v for a given set of μ from 0.2 to 0.5 and v from 10 to 20. The decrease in the production run length (T^{**}) with increase in inspection cost (v) for low values of deterioration rate (μ) and restoration cost (r) is due to the predominance of investment in setup cost reduction over investment in quality improvement.

The optimal production run length is lower due to investment in both setup reduction and quality improvement as compared to without investment (Kim et al., 2001), for values of deterioration rate of process (μ) from 0.1 to 0.2 and low value of restoration cost. The optimal production run length is higher due to investment in both setup reduction and quality improvement as compared to without investment (Kim et al., 2001), for values of deterioration rate of process (μ) from 0.3 to 0.5 and high value of restoration cost.

The average cost per unit time is reduced with investment in both setup cost reduction and quality improvement (Table 5.3). The influence of investment in both setup cost reduction and quality improvement on the reduction in average cost per unit time increases as compared to without investment (Kim et al., 2001) (i) with increase in deterioration rate (μ) from 0.1 to 0.5

for a given r and v, (ii) with increase in r for a given μ from 0.2 to 0.5 and v, and (iii) with increase in v from 10 to 20, for a given value of μ and r.

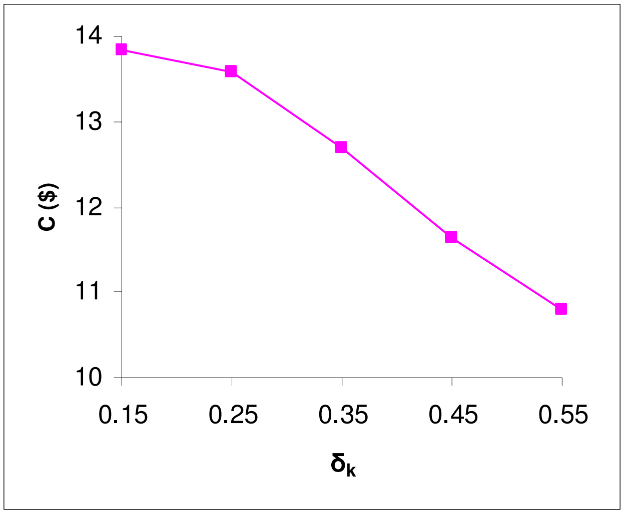

Fig. 5.1 Variation of C with δ_k

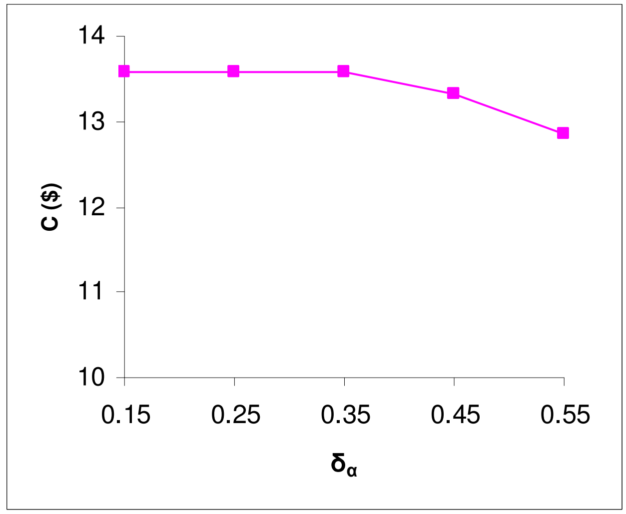

Fig. 5.2 Variation of C with δ_α

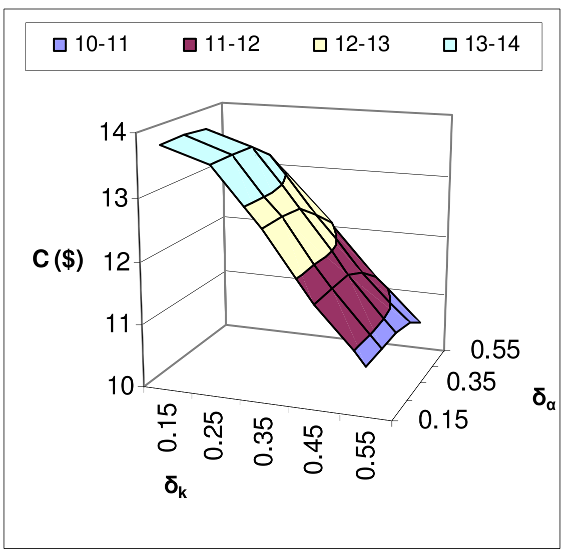

Fig. 5.3 Variation of C with δ_k and δ_α

Figures 5.1 – 5.3 show the variation of cost with investment in both setup cost reduction and quality improvement. Figures 5.1 shows that the total cost (C) decreases with increase in δ_k (percentage reduction in the setup cost), while Fig. 5.2 shows that total cost (C) initially remains constant and then decreases with increase in δ_α (percentage reduction in the defective items produced). As shown in Fig. 5.2 no investment in quality improvement is made for δ_α values of 0.15, 0.25 and 0.35 respectively as the total cost remained unchanged. Smaller values of δ_k and δ_α indicates higher values of B_k (rate of increase in dollar investment per fraction of reduction in setup cost) and B_α (rate of increase in dollar investment per fraction of reduction in non-conforming units) respectively. The total cost function (C) increases with increase in the values of B_k and B_α (or decrease in the values of δ_k and δ_α respectively). Figure 5.3 shows the variation of total cost (C) with both δ_k and δ_α. Figure 5.3 shows the variation of total cost (C) with both δ_k and δ_α. For the data considered in the analysis, the decrease in total cost (C) with increase in δ_k is significant as compared to decrease in total cost (C) with increase in δ_α.

Table 5.3 Optimal solution with investment in both setup cost reduction and quality improvement and its comparison without investment

			Both setup cost reduction and quality improvement					% Difference	
μ	r	v	K^{**}	α^{**}	J^{**}	T^{**}	C^{**}	T^{**}	C^{**}
0.1	10	10	33.07	0.0500	2	7.13	13.60	-15.71	-1.54
		20	29.47	0.0446	1	6.36	15.28	-47.96	-1.51
	20	10	33.56	0.0500	2	7.24	14.23	-15.06	-1.41
		20	30.46	0.0434	1	6.57	15.84	-44.60	-1.70
	30	10	34.07	0.0500	2	7.35	14.86	-14.42	-1.21
		20	31.53	0.0422	1	6.80	16.38	-41.18	-2.01
	60	10	35.71	0.0500	2	7.70	16.73	-12.60	-0.84
		20	35.19	0.0387	1	7.59	17.99	-5.93	-1.89
	120	10	35.89	0.0381	1	7.74	20.15	-5.30	-1.69
		20	44.96	0.0322	1	9.70	21.01	8.76	-1.76
	180	10	50.00	0.0298	1	10.81	23.09	15.73	-1.82
		20	50.00	0.0281	1	11.76	23.76	16.16	-2.27
0.2	10	10	30.46	0.0434	2	6.57	15.84	-31.05	-1.01
		20	35.08	0.0239	1	7.57	17.29	-15.32	-4.63
	20	10	32.66	0.0410	2	7.05	16.92	-24.96	-1.30
		20	38.51	0.0226	1	8.31	18.04	-8.06	-5.82
	30	10	34.05	0.0243	1	7.35	17.84	-22.72	-2.41
		20	42.13	0.0215	1	9.09	18.75	-1.65	-7.04
	60	10	47.11	0.0202	1	10.16	19.97	12.89	-7.06
		20	50.00	0.0192	1	11.23	20.67	23.33	-9.05
	90	10	50.00	0.0186	1	12.03	21.77	24.77	-9.32
		20	50.00	0.0181	1	12.78	22.37	23.24	-9.97

Table 5.3 Optimal solution with investment in both setup cost reduction and quality improvement and its comparison without investment (contd..)

			Both setup cost reduction and quality improvement					% Difference	
μ	r	v	K^{**}	α^{**}	J^{**}	T^{**}	C^{**}	T^{**}	C^{**}
0.3	10	10	31.97	0.0201	1	6.90	17.17	-20.00	-3.20
		20	40.06	0.0180	1	8.64	18.13	1.27	-10.26
	20	10	37.96	0.0185	1	8.19	18.06	-5.62	-6.48
		20	44.93	0.0172	1	9.69	18.90	7.43	-12.38
	30	10	43.52	0.0174	1	9.39	18.85	3.51	-9.97
		20	49.52	0.0165	1	10.69	19.60	23.76	-14.49
	50	10	50.00	0.0162	1	11.23	20.23	24.49	-14.63
		20	50.00	0.0159	1	11.95	20.88	22.43	-15.09
	60	10	50.00	0.0159	1	11.86	20.86	23.44	-14.96
		20	50.00	0.0157	1	12.54	21.48	21.45	-15.32
0.4	10	10	36.24	0.0167	1	7.82	17.67	-14.83	-8.04
		20	43.34	0.0157	1	9.35	18.54	8.24	-15.53
	20	10	42.60	0.0158	1	9.19	18.52	5.01	-12.96
		20	48.48	0.0152	1	10.46	19.29	11.66	-18.30
	30	10	48.05	0.0152	1	10.37	19.27	16.97	-16.97
		20	50.00	0.0149	1	11.24	19.97	21.00	-18.73
	40	10	50.00	0.0149	1	11.20	19.96	21.61	-18.69
		20	50.00	0.0146	1	11.88	20.61	19.44	-18.87
0.5	10	10	38.89	0.0151	1	8.39	17.95	-6.44	-12.81
		20	45.33	0.0145	1	9.78	18.77	10.22	-19.98
	20	10	45.07	0.0146	1	9.72	18.77	7.72	-18.70
		20	50.00	0.0142	1	10.84	19.49	19.00	-21.50
	30	10	50.00	0.0142	1	10.82	19.49	19.41	-21.45
		20	50.00	0.0140	1	11.51	20.16	17.20	-21.48

5.2 Optimal Cycle Length and Number of Inspections in a Deteriorating Production Processes with Investment in Setup Cost Reduction and Quality Improvement

Moon et al., (2002) studied the imperfect production processes having significant changeovers between the products. The mathematical models are developed using both the common cycle approach and the time-varying lot sizes approach, taking into account the effects of imperfect quality and process restoration. Common Cycle approach restricts all the products' cycle times to equal length. Time-varying lot sizes approach allows different lot sizes for any given products during a cyclic schedule. The constraint of setup time and total production time per cycle equal to cycle length are to be satisfied. The production of each product would be in batches and the issue of batching arises because the system usually incurs a setup cost and/or a setup time when the machine switches from one product to another. The setup cost and setup time depend only on the item going into production.

In this research, Moon et al., (2002) model is extended to consider an investment in setup cost reduction and quality improvement. The optimal cycle length, the number of inspections, setup cost and process quality are determined considering process inspection and restoration using both time-varying lot sizes and common cycle approach. In the imperfect process with inspection and restoration the process is inspected at regular intervals of time during the production of each product and if the system is found to be 'out-of-control', necessary actions are taken to restore it to the 'in-control' state.

The remainder of the section is organized as follows. Basic assumptions and notations are described. The mathematical models on (i) the imperfect process and (ii) the imperfect process with inspection and restoration, with capital investments to reduce setup cost and quality improvement are developed first using time varying lot sizes approach and then using common cycle approach. The algorithms to determine the expected total cost are described. In the subsequent section on numerical case studies, examples to show the optimal solutions of the time-varying lot sizes approach and the common cycle approach are provided.

The following notation is used in developing the models

i	item index, $i=1,2,\ldots\ldots\ldots, m$
A_i	setup costs, $i=1,2,\ldots\ldots\ldots, m$
d_i	constant demand rates ($d_i < p_i$), $i=1,2,\ldots\ldots\ldots, m$
h_i	holding costs, $i=1,2,\ldots\ldots\ldots, m$
i_a, i_α	fraction of amortization investment cost under setup reduction and quality improvement
n_i	number of inspections for item i, $i=1,2,\ldots\ldots\ldots, m$
p_i	constant production rates for item i, $i=1,2,\ldots\ldots\ldots, m$
r_0, r_1	cost parameters for process restoration cost functions
s_i	setup times, $i=1,2,\ldots\ldots\ldots, m$

t_i	length of the production run for items, $i=1,2,\ldots\ldots\ldots, m$
T_i	cycle length for items, $i=1,2,\ldots\ldots\ldots, m$
u_i	constant cost for producing defective items, $i=1,2,\ldots\ldots\ldots, m$
v_i	inspection costs, $i=1,2,\ldots\ldots\ldots, m$
α_i	constant fraction of nonc-conforming items, $i=1,2,\ldots\ldots\ldots, m$
ρ_i	d_i/p_i, $i=1,2,\ldots\ldots\ldots, m$

The following basic assumptions are used to formulate the mathematical model

1. Multiple items compete for the use of single facility
2. Demand rates, production rates, setup costs, holding costs, process inspection, and restoration costs for all items are known constants.
3. No backlogging of demand is permitted
4. Production capacity is sufficient to meet the total demand.
5. At the start of the production cycle, for each product the production process is always in an in-control state and perfect items are produced. Once the production process shifts to an out-of-control state, the shift cannot be detected until the end of production cycle, and the process continues production, and a fixed proportion of produced items are defective.
6. The elapsed time until the production process shifts, θ_i, is assumed to be exponential distributed for item *i*.
7. All defective items produced are detected after the production cycle is over, and rework cost for defective items will be incurred.
8. The process is brought back to the in-control state with each setup.
9. Following Porteus (1986), continuous relationship between the amount of investment and the setup cost reduction/quality improvement is considered. Sarker and Coates (1997) investigated the case of finite number of opportunities for setup cost reduction investment which is a more realistic situation. The commonly used setup reduction functions in the literature are exponential, linear, convex parabola, and concave parabola. The algorithms developed in this work are applicable to any setup cost reduction/quality improvement function as long as it is convex and once differentiable (Banerjee et al., 1996). The relationship between the setup cost reduction and capital investment can be described by the logarithmic investment function. That is setup cost A_i and the capital investment

function ϕ_{ai} can be stated as $\phi_{ai} = a_i \ln\left(\frac{A_{0i}}{A_i}\right)$ for $0 \le A_i \le A_{oi}$, $i = 1,2,......,m$. Similarly, the relationship between process quality α_i, and the capital investment in process quality improvement, $\phi_{\alpha i}$ can be stated as $\phi_{\alpha i} = b_i \ln\left(\frac{\alpha_{0i}}{\alpha_i}\right)$ for $0 \le \alpha_i \le \alpha_{oi}$, $i = 1,2,......,m$, where $\frac{1}{a_i}$ and $\frac{1}{b_i}$ are the fraction of the reduction in A_i and α_i per dollar increase in investment, respectively.

5.2.1 Optimal cycle length and number of inspections in a deteriorating production processes under time-varying lot sizes approach and common cycle approach

The models for investment in setup cost reduction and quality improvement in a deteriorating production processes under both time-varying lot sizes approach and common cycle approach are presented.

5.2.1.1 Model for investment in setup cost reduction and quality improvement in a deteriorating production processes under time-varying lot sizes approach

The total relevant cost in an economic production batch size model as the sum of setup cost and inventory carrying cost is

$$TRC = \sum_{i=1}^{m}\left[\frac{A_i}{T_i} + \frac{h_i d_i (1-\rho_i) T_i}{2}\right] \tag{5.34}$$

Moon et al., (2002) presented the imperfect process model under time-varying lot sizes models to determine the optimal cycle length. It is assumed that once a shift occurs the process stays in the 'out-of-control' state until the setup of the next production. Let t be the elapsed time for which the process remains in the 'in-control' state before a shift occurs. The number of non-conforming items produced while processing the ith product is

$$N_i = \int_0^{t_i} \alpha_i p_i (t_i - t)\frac{e^{-t/\theta_i}}{\theta_i} dt = \alpha_i p_i \left(t_i - \theta_i + \theta_i e^{-t_i/\theta_i}\right) \tag{5.35}$$

For mathematical simplicity, Moon et al., (2002) used McLaurin series approximation $e^{-t_i/\theta_i} \approx 1 - t_i/\theta_i + 0.5(t_i/\theta_i)^2$ to obtain the number of non-conforming items as

$$N_i = \frac{\alpha_i p_i t_i^2}{2\theta_i} \quad \text{where } t_i = \frac{d_i T_i}{p_i} \tag{5.36}$$

Therefore, the expected quality related cost per unit time due to the production of non-conforming units is

$$QC = \sum_{i=1}^{m} \frac{u_i}{T_i} N_i = \sum_{i=1}^{m} \frac{u_i \alpha_i d_i^2 T_i}{2 p_i \theta_i} \tag{5.37}$$

The expected total annual cost per unit time of their model includes setup cost, holding cost and quality related cost. The objective function is to minimize the expected total cost per unit time

$$\text{Min } \sum_{i=1}^{m} \left[\frac{A_i}{T_i} + (H_i + Q_i) T_i \right] \tag{5.38}$$

where $H_i = \frac{h_i d_i (1 - \rho_i)}{2}$, $Q_i = \frac{u_i \alpha_i \rho_i d_i}{2\theta_i}$, and the constraint for the expected total cost per unit time in Eq. (5.38) is that the sum of the ratios of setup time for the product to the cycle length for the product must be less than or equal to $\kappa = 1 - \sum_{i=1}^{m} \rho_i$, and is given as

$$\sum_{i=1}^{m} \frac{s_i}{T_i} \le \kappa, \qquad T_i \ge 0, \qquad i = 1,2,\dots\dots,m \tag{5.39}$$

In the present study, the setup cost and quality level are considered to be decision variables. Capital investments allocated to reduce setup cost and improve the quality level and hence we should include amortized investment cost. The total annual cost per unit time of the system is composed of setup cost, holding cost, quality related cost and the amortized total capital cost $(i_a \phi_{ai} + i_\alpha \phi_{\alpha i})$ and is given as follows

$$\text{Min } C_i = \sum_{i=1}^{m} \left[\frac{A_i}{T_i} + (H_i + Q_i) T_i + i_a a_i \ln\left(\frac{A_{oi}}{A_i} \right) + i_\alpha b_i \ln\left(\frac{\alpha_{oi}}{\alpha_i} \right) \right] \tag{5.40}$$

subject to upper bound of the equality constraint of Eq. (5.39) given as

$$\sum_{i=1}^{m} \frac{s_i}{T_i} = \kappa, \qquad T_i \ge 0, \qquad i = 1,2,\dots\dots,m \tag{5.41}$$

The Lagrangian function *L* of the above optimization problem in Eq. (5.40) with the constraint in Eq. (5.41) is

$$L=\sum_{i=1}^{m}\left[\frac{A_i}{T_i}+(H_i+Q_i)T_i+i_a a_i\ln\left(\frac{A_{oi}}{A_i}\right)+i_\alpha b_i\ln\left(\frac{\alpha_{oi}}{\alpha_i}\right)\right]+\lambda_1\left(\sum_{i=1}^{m}\frac{s_i}{T_i}-\kappa\right) \tag{5.42}$$

where λ_1 in Eq. (5.42) is the Lagarange multiplier corresponding to the constraint set in Eq. (5.41).

The Karush-Kuhn-Tucker (KKT) conditions for the minimization of Lagrangian function in Eq. (5.42) give,

$$T_i^*=\sqrt{\frac{A_i+\lambda_1 s_i}{H_i+Q_i}} \qquad i=1,2,.......,m \tag{5.43}$$

$$A_i^*=i_a a_i T_i \qquad i=1,2,.......,m \tag{5.44}$$

$$\alpha_i^*=\frac{2i_\alpha b_i\theta_i}{u_i\rho_i d_i T_i} \qquad i=1,2,.......,m \tag{5.45}$$

To determine the optimal values of T_i^*, A_i^*, α_i^*, the following algorithm is used.

1. Initialize $\sqrt{\lambda_1}=\sum_{i=1}^{m}\frac{\sqrt{s_i(H_i+Q_i)}}{\kappa}$, A_{i0} and α_{i0} to the values given in the example data.
2. Determine T_i from Eq. (5.43) and solve $\sum_{i=1}^{m}s_i\sqrt{\frac{H_i+Q_i}{A_i+\lambda_1 s_i}}=\kappa$ for $i=1,2,.......,m$ to determine the converged value of λ_1 using Newton-Raphson iterative procedure.
3. Determine A_i for $0\le A_i\le A_{oi}$, $i=1,2,.......,m$ and α_i for $0\le\alpha_i\le\alpha_{oi}$, $i=1,2,.......,m$ from Eqs. (5.44) and (5.45) using T_i from Eq. (5.43) based on the converged value of λ_1.
4. Repeat steps 2 and 3 till convergence in the value of A_i and α_i is obtained.

The convergence criteria for algorithm is: $\left|\sum_{i=1}^{m}A_{i,new}-\sum_{i=1}^{m}A_{i,old}\right|\le 10^{-3}$ and $\left|\sum_{i=1}^{m}\alpha_{i,new}-\sum_{i=1}^{m}\alpha_{i,old}\right|\le 10^{-3}$

5. Compute C_i from Eq. (5.40)

5.2.1.2 Model for investment in setup cost reduction and quality improvement in a deteriorating production processes under common cycle approach

In the common cycle (CC) approach ($T_1 = T_2 = \ldots\ldots\ldots = T_m = T$), the above expression for the expected total cost in Eq. (5.40) can be written as

$$\text{Min } \frac{1}{T}\sum_{i=1}^{m} A_i + \left(\sum_{i=1}^{m} H_i + \sum_{i=1}^{m} Q_i\right)T + \sum_{i=1}^{m}\left[i_a a_i \ln\left(\frac{A_{oi}}{A_i}\right) + i_\alpha b_i \ln\left(\frac{\alpha_{oi}}{\alpha_i}\right)\right] \quad (5.46)$$

subject to upper bound of the equality constraint of Eq. (5.39) given as

$$\frac{1}{\kappa}\sum_{i=1}^{m} s_i = T \quad (5.47)$$

The equality constraint in Eq. (5.47) shows that time devoted to changeover $\sum_{i=1}^{m} s_i$ is equal to the time available for machine setups κT.

Minimization of the expected total cost function in Eq. (5.46) yield,

$$T^* = \sqrt{\frac{\sum_{i=1}^{m} A_i}{\sum_{i=1}^{m} H_i + \sum_{i=1}^{m} Q_i}} \qquad i = 1,2,\ldots\ldots\ldots,m \quad (5.48)$$

$$A_i^* = i_a a_i T \qquad i = 1,2,\ldots\ldots\ldots,m \quad (5.49)$$

$$\alpha_i^* = \frac{2 i_\alpha b_i \theta_i}{u_i \rho_i d_i T} \qquad i = 1,2,\ldots\ldots\ldots,m \quad (5.50)$$

To determine the optimal values of T, A_i^*, α_i^*, the following algorithm is used.

1. Initialize A_{i0} and α_{i0} to the values given in the example data
2. Determine T from Eq. (5.48) and the optimal cycle length is maximum of {T, $\frac{1}{\kappa}\sum_{i=1}^{m} s_i$ }.
3. Determine A_i for $0 \le A_i \le A_{oi}$, $i = 1,2,\ldots\ldots\ldots,m$ and α_i for $0 \le \alpha_i \le \alpha_{oi}$, $i = 1,2,\ldots\ldots\ldots,m$ from Eqs. (5.49) and (5.50) using the value of T from step 2.
4. Repeat steps 2 and 3 till convergence in the value of A_i and α_i is obtained.

The convergence criteria for algorithm is: $\left|\sum_{i=1}^{m} A_{i,new} - \sum_{i=1}^{m} A_{i,old}\right| \le 10^{-3}$ and $\left|\sum_{i=1}^{m} \alpha_{i,new} - \sum_{i=1}^{m} \alpha_{i,old}\right| \le 10^{-3}$

5. Compute expected total cost from Eq. (5.46).

5.2.1.3 Model for investment in setup cost reduction and quality improvement in a deteriorating production processes with inspection and restoration under time-varying lot sizes approach

Moon et al., (2002) also presented imperfect process model with inspection and restoration under time-varying lot sizes to determine the optimal cycle length and optimal number of inspections. The process inspection cost carried out during production of item i is equal to $\sum_{i=1}^{m} n_i v_i$ and the expected quality cost per unit time is equal to $\sum_{i=1}^{m} \frac{T_i}{n_i} Q_i$. Moon et al., (2002) assumed that restoration cost to be a linear function of its detection delay and restoration time is negligible, and obtained the expected restoration cost per unit time as $\sum_{i=1}^{m} \frac{r_0 d_i}{p_i \theta_i} + \sum_{i=1}^{m} \frac{R_i T_i}{n_i}$, where $R_i = (r_1 \theta_i - r_0)\frac{d_i^2}{2 p_i^2 \theta_i^2}$. The objective function on the expected total annual cost per unit time of their model includes setup cost, holding cost, quality related cost, inspection cost and restoration cost and is

$$\text{Min } \sum_{i=1}^{m} \left[\frac{A_i + n_i v_i}{T_i} + \left(H_i + \frac{Q_i + R_i}{n_i} \right) T_i + \frac{r_0 d_i}{p_i \theta_i} \right] \tag{5.51}$$

subject to the constraint in Eq. (5.39).

The Lagrangian of the optimization problem given in Eq. (5.51) with the constraint set in Eq. (5.39) is written as

$$L = \sum_{i=1}^{m} \left[\frac{A_i + n_i v_i}{T_i} + \left(H_i + \frac{Q_i + R_i}{n_i} \right) T_i + \frac{r_0 d_i}{p_i \theta_i} \right] + \lambda_2 \left(\sum_{i=1}^{m} \frac{s_i}{T_i} - \kappa \right) \tag{5.52}$$

where λ_2 is the Lagarange multiplier corresponding to the constraint set in Eq. (5.39).

In the present study, considering the setup cost and quality level as decision variables, the total annual cost per unit time of the system composed of setup cost, holding cost, quality related cost, inspection cost and restoration cost and the amortized total capital cost $\left(i_a \phi_{ai} + i_\alpha \phi_{\alpha i}\right)$ is written as

$$\text{Min } C_i = \sum_{i=1}^{m} \left[\frac{A_i + n_i v_i}{T_i} + \left(H_i + \frac{Q_i + R_i}{n_i} \right) T_i + \frac{r_0 d_i}{p_i \theta_i} + i a_i \ln\left(\frac{A_{oi}}{A_i} \right) + i b_i \ln\left(\frac{\alpha_{oi}}{\alpha_i} \right) \right] \tag{5.53}$$

subject to equality constraint of Eq. (5.41).

The Lagrangian function L of the above optimization problem in Eq. (5.53) with the constraint in Eq. (5.41) is

$$L = \sum_{i=1}^{m} \left[\frac{A_i + n_i v_i}{T_i} + \left(H_i + \frac{Q_i + R_i}{n_i} \right) T_i + \frac{r_0 d_i}{p_i \theta_i} + i_a a_i \ln\left(\frac{A_{oi}}{A_i} \right) + i_\alpha b_i \ln\left(\frac{\alpha_{oi}}{\alpha_i} \right) \right] + \lambda_2 \left(\sum_{i=1}^{m} \frac{s_i}{T_i} - \kappa \right) \tag{5.54}$$

where λ_2 in Eq. (5.54) is the Lagarange multiplier corresponding to the constraint set in Eq. (5.41).

Using the objective function in Eq. (5.53) and the constraints in Eq. (5.42) and applying the Karush-Kuhn-Tucker (KKT) conditions for the minimum of the associated Lagrangian function in Eq. (17), yield

$$T_i^* = \sqrt{\frac{A_i + n_i v_i + \lambda_2 s_i}{H_i + \frac{Q_i + R_i}{n_i}}} \qquad i = 1,2,\ldots\ldots, m \tag{5.55}$$

$$n_i = \sqrt{\frac{(A_i + \lambda_2 s_i)(Q_i + R_i)}{v_i H_i}}, \qquad i = 1,2,\ldots\ldots, m \tag{5.56}$$

$$A_i^* = i_a a_i T_i \qquad i = 1,2,\ldots\ldots, m \tag{5.57}$$

$$\alpha_i^* = \frac{2 i_\alpha b_i n_i \theta_i}{u_i \rho_i d_i T_i} \qquad i = 1,2,\ldots\ldots, m \tag{5.58}$$

To determine the optimal values of $n_i^*, T_i^*, A_i^*, \alpha_i^*$, the following algorithm is used.

1. Initialize $\sqrt{\lambda_2} = \sum_{i=1}^{m} \frac{\sqrt{s_i\left(H_i + \frac{Q_i + R_i}{n_i}\right)}}{\kappa}$, n_i to unity, A_{i0} and α_{i0} to the values given in the example data.
2. Determine T_i from Eq. (5.55) and solve $\sum_{i=1}^{m} s_i \sqrt{\frac{H_i + \frac{Q_i + R_i}{n_i}}{A_i + n_i v_i + \lambda_2 s_i}} = \kappa$ for $i = 1,2,.......,m$ to determine the converged value of λ_2 using Newton-Raphson iterative procedure. The proof of convergence of λ_2 is shown in the Appendix C.1.
3. Determine n_i using the converged value of λ_2
4. Repeat steps 2 and 3 till convergence in the value of n_i is obtained.

The convergence criteria for algorithm is: $\left|\sum_{i=1}^{m} n_{i,new} - \sum_{i=1}^{m} n_{i,old}\right| \leq 10^{-3}$

5. Determine A_i for $0 \leq A_i \leq A_{oi}$, $i = 1,2,.......,m$ and α_i for $0 \leq \alpha_i \leq \alpha_{oi}$, $i = 1,2,.......,m$ from Eqs. (5.56) - (5.58) using the converged values of n_i and λ_2.
6. Repeat steps 2 to 5 till convergence in the values of A_i and α_i are obtained.

The convergence criteria for algorithm is: $\left|\sum_{i=1}^{m} A_{i,new} - \sum_{i=1}^{m} A_{i,old}\right| \leq 10^{-3}$ and $\left|\sum_{i=1}^{m} \alpha_{i,new} - \sum_{i=1}^{m} \alpha_{i,old}\right| \leq 10^{-3}$

7.
   a. If $n_i \leq 1$, $i = 1,2,.......,m$, then choose $n_i = 1$ for $i = 1,2,.......,m$. Using *A(n_i)* for $0 \leq A_i \leq A_{oi}$, $i = 1,2,.......,m$ and $\alpha(n_i)$ for $0 \leq \alpha_i \leq \alpha_{oi}$, $i = 1,2,.......,m$ from Eqs. (5.56) - (5.58), and solve Eq. (5.55) to determine the converged value of $T(n_i)$ for $i = 1,2,.......,m$ using Newton-Raphson iterative procedure. The proof of convergence of $T(n_i)$ is shown in the Appendix C.2.

b. If $n_i > 1$, $i = 1,2,......., m$, then choose $j_i < n_i < j_i + 1$ for $i = 1,2,......., m$, where *j* is a positive integer. Using *A(j_i) and A(j_i+1)* for $0 \le A_i \le A_{oi}$, $i = 1,2,......., m$ and *α(j_i) and α(j_i+1)* for $0 \le \alpha_i \le \alpha_{oi}$, $i = 1,2,......., m$ from Eqs. (5.56) - (5.58), and solve Eq. (5.55) to determine the converged value of *T(j_i) and T(j_i+1)* for $i = 1,2,......., m$ using Newton-Raphson iterative procedure.

8.

a. If $n_i \le 1$, $i = 1,2,......., m$, then compute $C(n_i)$ for $i = 1,2,......., m$ from Eq. (5.53).

b. If $n_i > 1$, $i = 1,2,......., m$, then choose $j_i < n_i < j_i + 1$ for $i = 1,2,......., m$, then compute $C(j_i)$ and $C(j_i+1)$ for $i = 1,2,......., m$ respectively from Eq. (5.53) and choose the one with smaller value of sum as the optimal solution.

5.2.1.4 Model for investment in setup cost reduction and quality improvement in a deteriorating production processes with inspection and restoration under common cycle approach

In the common cycle (CC) approach, the above expression for the expected total cost in Eq. (5.53) can be written as

$$\text{Min } \frac{1}{T}\sum_{i=1}^{m}(A_i + n_i v_i) + T\sum_{i=1}^{m} H_i + T\sum_{i=1}^{m}\left(\frac{Q_i + R_i}{n_i}\right) + \sum_{i=1}^{m}\left(\frac{r_0 d_i}{p_i \theta_i}\right) + \sum_{i=1}^{m}\left[ia_i \ln\left(\frac{A_{oi}}{A_i}\right) + ib_i \ln\left(\frac{\alpha_{oi}}{\alpha_i}\right)\right] \tag{5.59}$$

subject to the equality constraint of Eq. (5.47).

Minimization of Eq. (5.59) yield,

$$T^* = \sqrt{\frac{\sum_{i=1}^{m} A_i + \sum_{i=1}^{m} n_i v_i}{\sum_{i=1}^{m} H_i + \sum_{i=1}^{m}\frac{R_i + Q_i}{n_i}}} \qquad i = 1,2,......., m \tag{5.60}$$

$$n_i^* = T\sqrt{\frac{R_i + Q_i}{v_i}} \qquad i = 1,2,......., m \tag{5.61}$$

$$A_i^* = i_a a_i T \qquad i = 1,2,......., m \tag{5.62}$$

$$\alpha_i^* = \frac{2i_\alpha b_i \theta_i n_i}{u_i \rho_i d_i T} \qquad i = 1,2,.......,m \qquad (5.63)$$

To determine the optimal values of $n_i^*, T, A_i^*, \alpha_i^*$, the following algorithm is used.

1. Initialize n_i to unity, A_{i0} and α_{i0} to the values given in the example data.
2. Determine *T* from Eq. (5.60) and optimal cycle length is maximum of {T, $\frac{1}{\kappa}\sum_{i=1}^{m} s_i$ }.
3. Determine n_i using the value of *T* from step 2 above.
4. Repeat steps 2 and 3 till convergence in the value of n_i is obtained.
5. Determine A_i for $0 \le A_i \le A_{oi}$, $i = 1,2,.......,m$ and α_i for $0 \le \alpha_i \le \alpha_{oi}$, $i = 1,2,.......,m$ from Eqs. (5.56) - (5.58) using the converged values of n_i.
6. Repeat steps 2 to 5 till convergence in the values of A_i and α_i are obtained.
7.
   a. If $n_i \le 1$, $i = 1,2,.......,m$, then choose $n_i = 1$ for $i = 1,2,.......,m$. Using *A(n_i)* for $0 \le A_i \le A_{oi}$, $i = 1,2,.......,m$ and $\alpha(n_i)$ for $0 \le \alpha_i \le \alpha_{oi}$, $i = 1,2,.......,m$ from Eqs. (5.62) - (5.63), determine *T* from Eq. (5.60) numerically till convergence is obtained. The optimal cycle length is maximum of {T, $\frac{1}{\kappa}\sum_{i=1}^{m} s_i$ }.
   b. If $n_i > 1$, $i = 1,2,.......,m$, then choose $j_i < n_i < j_i + 1$ for $i = 1,2,.......,m$, where *j* is a positive integer. Using $A(j_i)$ *and* $A(j_i+1)$ for $0 \le A_i \le A_{oi}$, $i = 1,2,.......,m$ and $\alpha(j_i)$ *and* $\alpha(j_i+1)$ for $0 \le \alpha_i \le \alpha_{oi}$, $i = 1,2,.......,m$ from Eqs. (5.62) - (5.63), determine *T(j) and T(j+1)* from Eq. (5.60) numerically till convergence is obtained. The optimal cycle length is maximum of {T, $\frac{1}{\kappa}\sum_{i=1}^{m} s_i$ }.
8.
   a. If $n_i \le 1$, $i = 1,2,.......,m$, then compute expected total cost $C(n_i)$ for $i = 1,2,.......,m$ from Eq. (5.59).
   b. If $n_i > 1$, $i = 1,2,.......,m$, then choose $j_i < n_i < j_i + 1$ for $i = 1,2,.......,m$, then compute $C(j_i)$ and $C(j_i+1)$ for $i = 1,2,.......,m$ respectively from Eq. (5.59) and choose the one with smaller value of sum as the optimal expected total cost.

Table 5.4 Example data (r_o=\$10, r_1=\$0.1, i_k= i_α=0.1, a_i =1000, b_i =30)

Item	A_i (\$)	θ_i (y)	α_i	p_i (units/y)	d_i (units/y)	u_i (\$)	h_i (\$)	s_i (y)	v_i (\$)
1	125	1.2	0.20	5000	1850	30	12.50	0.00068	3
2	100	0.5	0.25	3500	1150	200	87.50	0.00171	3
3	110	0.8	0.30	3000	800	50	21.25	0.00091	3

Table 5.5 Example data (r_o=\$10, r_1=\$0.2, i_k= i_α=0.1, a_i =40, b_i =30)

Item	A_i (\$)	θ_i (days)	α_i	p_i (units/day)	d_i (units/day)	u_i (\$)	h_i (\$)	s_i (days)	v_i (\$)
1	75	10	0.20	1550	300	8	0.5	0.05	2
2	90	12	0.25	1890	400	5	0.4	0.08	2
3	50	15	0.30	1415	250	10	0.8	0.06	2
4	100	25	0.20	1260	300	12	1.0	0.05	2
5	80	8	0.15	1625	200	6	0.6	0.15	2

Table 5.6 Example data (r_o=\$150, r_1=\$10, i_k= i_α=0.1, a_i =40, b_i =30)

Item	A_i (\$)	θ_i (days)	α_i	p_i (units/day)	d_i (units/day)	u_i (\$)	h_i (\$)	s_i (days)	v_i (\$)
1	3000	80	0.12	133	20	10	0.0461	4.0	4
2	1800	75	0.08	300	24	15	0.0312	2.4	4
3	3600	140	0.10	266	30	25	0.0651	4.8	4
4	1500	90	0.06	146	36	16	0.1180	2.0	4
5	6000	210	0.15	532	40	20	0.1190	4.0	4
6	30000	112	0.05	373	50	30	0.0847	8.0	4

Table 5.7 Example data (r_o=\$10, r_1=\$2, i_k= i_α=0.1, a_i =40, b_i =30)

Item	A_i (\$)	θ_i (days)	α_i	p_i (units/day)	d_i (units/day)	u_i (\$)	h_i (\$)	s_i (days)	v_i (\$)
1	15	12.5	0.04	3075	400	0.6500	0.000130	0.125	2
2	20	8.0	0.07	8000	400	0.1775	0.000355	0.125	2
3	30	6.5	0.10	9500	800	0.1275	0.000255	0.250	3
4	10	15.0	0.03	7500	750	1.0000	0.000200	0.125	2
5	110	14.0	0.15	2000	80	2.7850	0.005570	0.500	3
6	50	18.0	0.08	6015	80	0.2675	0.000535	0.250	2
7	310	10.0	0.14	2400	104	1.5000	0.003000	1.000	3
8	130	9.0	0.10	1300	340	3.2900	0.000580	0.500	6
9	200	16.0	0.05	2000	340	0.9000	0.001800	0.700	4
10	24	20.0	0.12	15038	400	0.0400	0.000080	0.125	2

5.2.2 Numerical case studies

Tables 5.4 – 5.7 show the data chosen by Moon et al. (2002). The time varying optimal solution and the expected total cost for imperfect process model are given in Table 5.8.

Table 5.8 Optimal solutions and comparisons for imperfect process model

Example Data	Order periods Expected total cost	Order periods, set up cost Expected total cost	Order periods, quality level Expected total cost	Order periods, setup cost, quality level Expected total cost
	Without investment (Moon et al., 2002)	Investment in setup cost reduction	Investment in quality improvement	Investment in both setup cost reduction and quality improvement
I	T_1=0.1453 y, 34.87 days T_2=0.0707 y, 16.96 days T_3=0.1546 y, 37.11 days ETC=$ 9289.36	T_1=0.1191 y, 28.59 days, T_2=0.0753 y, 18.07 days, T_3=0.1434 y, 34.41 days, A_1 = $11.91, A_2 = $7.53, A_3 = $14.34 ETC=$ 7216.31	T_1=0.1446 y, 34.71 days, T_2=0.0709 y, 17.01 days, T_3=0.1534 y, 36.82 days, α_1 = 0.0024, α_2 = 0.0006 α_3 = 0.0029 ETC=$ 7450.65	T_1=0.1105 y, 26.51 days, T_2=0.0777 y, 18.66 days, T_3=0.1376 y, 33.02 days A_1 = $11.05, A_2 = $7.78, A_3 = $13.76 α_1 = 0.0032, α_2 = 0.0005, α_3 = 0.0033 ETC=$ 5348.45
II	T_1=5.71 days, T_2=7.06 days, T_3=5.37 days, T_4=4.27 days, T_5=10.73 days ETC=$ 2461.82	T_1=5.69 days, T_2=7.06 days, T_3=5.39 days, T_4=4.23 days, T_5=10.80 days A_1 = $22.75, A_2 = $28.24 A_3 = $21.58, A_4 = $16.90 A_5 = $43.20 ETC=$ 2438.00	T_1=5.77 days, T_2=7.13 days, T_3=5.38 days, T_4=4.23 days, T_5=10.62 days α_1 = 0.0224, α_2 = 0.0239 α_3 = 0.0378, α_4 = 0.0413 α_5 = 0.0306 ETC=$ 2395.10	T_1=5.76 days, T_2=7.13 days, T_3=5.41 days, T_4=4.19 days, T_5=10.69 days A_1 = $23.04, A_2 = $28.53, A_3 = $21.63, A_4 = $16.76, A_5 = $42.76 α_1 = 0.0224, α_2 = 0.038, α_3 = 0.0377, α_4 = 0.0418, α_5 = 0.0304 ETC=$ 2371.26

Table 5.9 Optimal solutions and comparisons for imperfect process model using common cycle (CC) approach

Example Data	Order periods, Expected total cost	Order periods, set up cost, Expected total cost	Order periods, quality level, Expected total cost	Order periods, setup cost, quality level, Expected total cost
	Without investment (Moon et al., 2002)	Investment in setup cost reduction	Investment in quality improvement	Investment in both setup cost reduction and quality improvement
I	T=0.0949 y, 22.78 days ETC=$ 10164.86	T=0.0949 y, 22.78 days, A = $9.4932 ETC=$ 7674.23	T=0.0949 y, 22.78 days, α_1 = 0.0037, α_2 = 0.0004, α_3 = 0.0047 ETC=$ 8071.62	T=0.0949 y, 22.78 days, A = $9.4932 α_1 = 0.0037, α_2 = 0.0004, α_3 = 0.0047 ETC=$ 5580.99
II	T=6.85 days ETC=$ 2735.28	T=6.85 days, A = $27.39 ETC=$ 2718.25	T=6.85 days, α_1 = 0.0189, α_2 = 0.0248, α_3 = 0.0298, α_4 = 0.0256, α_5 = 0.0475 ETC=$ 2655.65	T=6.85 days, A = $27.39 α_1 = 0.0189, α_2 = 0.0248, α_3 = 0.0298, α_4 = 0.0256, α_5 = 0.0475 ETC=$ 2638.63

For the data given in Table 5.4, the expected total cost using the investment in both setup cost reduction and quality improvement is $5348.45 which is lower than the expected total cost without investment ($9289.36). Using investment in both setup cost reduction and quality improvement for the data given in Table 5.4, the setup cost and fraction of non-conforming items are reduced to A_1 = $11.05, A_2 = $7.78, A_3 = $13.76, and α_1 = 0.0032, α_2 = 0.0005, α_3 = 0.0033 respectively. The order periods considering investment in both setup cost reduction and quality improvement for the data given in Table 5.4 are T_1=0.1105 year, T_2=0.0777 year, and T_3=0.1376 year. The imperfect process model using the data given in Table 5.5 also shows a reduction in the expected total cost from $ 2461.82 without investment to $ 2371.26 with investment in both setup cost reduction and quality improvement.

The common cycle solution of imperfect process model using the data given in Table 5.4 and 5.5 are given in Table 5.9. For the data given in Table 5.4, the optimal cycle length is taken as

0.0949 year and the expected annual total cost is obtained as $ 10,164.86. Considering the investment in both setup cost reduction and quality improvement for the data given in Table 5.4, the expected annual total cost is obtained as $ 5580.99, and the setup cost and fraction of non-conforming items are reduced to A = \$9.4932, $\alpha_1 = 0.0037$, $\alpha_2 = 0.0004$, $\alpha_3 = 0.0047$ respectively. The common cycle solution of imperfect process model using the data given in Table 5.5 also shows a reduction in the expected total cost from $ 2735.28 without investment to $ 2638.63 with investment in both setup cost reduction and quality improvement.

The time varying optimal solution and the expected total cost for imperfect process model with inspection and restoration using the data given in Table 5.4 and 5.5 are given in Table 5.10. For the data given in Table 5.4, the expected total cost using the investment in both setup cost reduction and quality improvement is $ 5485.19 which is lower than the expected total cost without investment ($ 8185.97). Using investment in both setup cost reduction and quality improvement for the data given in Table 5.4, the number of inspections are n_1=1, n_2=1, n_3=1, which are lower than the number of inspections without investment (n_1=3, n_2=6, n_3=4). The imperfect process model using the data given in Table 5.5 also shows a reduction in the expected total cost from $ 2378.06 without investment to $ 2353.64 with investment in both setup cost reduction and quality improvement.

Table 5.10 Optimal solutions and comparisons for imperfect process model with inspection and restoration

Example Data	Order periods, number of inspections Expected total cost	Order periods, number of inspections and set up cost Expected total cost	Order periods, number of inspections and quality level Expected total cost	Order periods, number of inspections, setup cost and quality level Expected total cost
	Without investment (Moon et al., 2002)	Investment in setup cost reduction	Investment in quality improvement	Investment in both setup cost reduction and quality improvement
I	T_1=0.1448 y, 34.75 days, T_2=0.0708 y, 16.99 days, T_3=0.1536 y, 36.86 days, n_1=3, n_2=6, n_3=4 ETC=$ 8185.97	T_1=0.1079 y, 25.91 days, T_2=0.0775 y, 18.61 days, T_3=0.1357 y, 32.56 days, n_1=2, n_2=6, n_3=3 A_1 = \$10.81, A_2 = \$7.75, A_3 = \$13.58 ETC=$ 6054.92	T_1=0.1488 y, 35.70 days, T_2=0.0739 y, 17.73 days, T_3=0.1580 y, 37.91 days, n_1=3, n_2=5, n_3=3 α_1 = 0.0071, α_2 = 0.0027, α_3 = 0.0085 ETC=$ 7830.72	T_1=0.1123 y, 26.96 days, T_2=0.0782 y, 18.78 days, T_3=0.1393 y, 33.43 days, n_1=1, n_2=1, n_3=1 A_1 = \$11.22, A_2 = \$7.82, A_3 = \$13.92 α_1 = 0.0031, α_2 = 0.0005, α_3 = 0.0032 ETC=$ 5485.19
II	T_1=5.78 days, T_2=7.13 days, T_3=5.38 days, T_4=4.23 days, T_5=10.61 days n_1=9, n_2=11, n_3=8, n_4=6, n_5=9 ETC=$ 2378.06	T_1=5.76 days, T_2=7.13 days, T_3=5.41 days, T_4=4.19 days, T_5=10.68 days, n_1=8, n_2=10, n_3=8, n_4=5, n_5=8 A_1 = \$23.06, A_2 = \$28.52 A_3 = \$21.63, A_4 = \$16.74 A_5 = \$42.72, ETC=$ 2353.42	T_1=5.78 days, T_2=7.13 days, T_3=5.38 days, T_4=4.23 days, T_5=10.61 days n_1=8, n_2=10, n_3=8, n_4=5, n_5=8 α_1 = 0.1788, α_2 = 0.2386 α_3 = 0.30, α_4 = 0.20 α_5 = 0.15 ETC=$ 2377.48	T_1=5.76 days, T_2=7.13 days, T_3=5.41 days, T_4=4.19 days, T_5=10.68 days n_1=8, n_2=10, n_3=8, n_4=5, n_5=8 A_1 = \$23.06, A_2 = \$28.52, A_3 = \$21.63, A_4 = \$16.74, A_5 = \$42.72 α_1 = 0.1793, α_2 = 0.2385, α_3 = 0.30, α_4 = 0.20, α_5 = 0.15 ETC=$ 2353.64

Using investment in both setup cost reduction and quality improvement for the data given in Table 5.5, the number of inspections are n_1=8, n_2=10, n_3=8, n_4=5, n_5=8, which are lower than the number of inspections without investment (n_1=9, n_2=11, n_3=8, n_4=6, n_5=9). Using investment in both setup cost reduction and quality improvement for the example II, the fraction of non-conforming items are reduced to α_1 = 0.1793, α_2 = 0.2385, α_3 = 0.30, α_4 = 0.20, α_5 = 0.15

respectively. It is shown that no investment is required for quality improvement for items 3 to 5 respectively.

Table 5.11 Optimal solutions and comparisons for imperfect process model with inspection and restoration using common cycle (CC) approach

Example Data	Order periods, Expected total cost	Order periods, set up cost, Expected total cost	Order periods, quality level, Expected total cost	Order periods, setup cost, quality level, Expected total cost
	Without investment (Moon et al., 2002)	Investment in setup cost reduction	Investment in quality improvement	Investment in both setup cost reduction and quality improvement
I	T=0.0949 y, 22.78 days, n_1=2, n_2=7, n_3=2 ETC=$ 8811.58	T=0.0949 y, 22.78 days, n_1=2, n_2=7, n_3=2 A = $9.4932 ETC=$ 6308.30	T=0.0949 y, 22.78 days, n_1=1, n_2=1, n_3=1 α_1 = 0.0037, α_2 = 0.0004, α_3 = 0.0047 ETC=$ 8167.15	T=0.0949 y, 22.78 days, n_1=1, n_2=1, n_3=1 A = $9.4932 α_1 = 0.0037, α_2 = 0.0004, α_3 = 0.0047 ETC=$ 5715.12
II	T=6.85 days n_1=2, n_2=2, n_3=2, n_4=1, n_5=1 ETC=$ 2692.25	T=6.85 days, n_1=11, n_2=11, n_3=11, n_4=9, n_5=6 A = $27.39 ETC=$ 2619.65	T=6.85 days, n_1=11, n_2=11, n_3=11, n_4=9, n_5=6 α_1 = 0.20, α_2 = 0.25, α_3 = 0.30, α_4 = 0.20, α_5 = 0.15 ETC=$ 2636.68	T=6.85 days, n_1=11, n_2=11, n_3=11, n_4=9, n_5=6 A = $27.39 α_1 = 0.20, α_2 = 0.25, α_3 = 0.30, α_4 = 0.20, α_5 = 0.15 ETC=$ 2619.65

The common cycle solution of imperfect process model with inspection and restoration using the data given in Table 5.4 to 5.7 are given in Table 5.11. For the data given in Table 5.4 without investment, the expected annual total cost is obtained as $ 8811.58 and the number of inspections are n_1=2, n_2=7, n_3=2. Considering the investment in both setup cost reduction and quality improvement for the data given in Table 5.4, the expected annual total cost is obtained as $ 5715.12, and the number of inspections are n_1=1, n_2=1, n_3=1 respectively. The common cycle solution of imperfect process model using the data given in Table 5.5 also shows a reduction in the expected total cost from $ 2692.25 without investment to $ 2619.65 with investment in both setup cost reduction and quality improvement. It is shown that no investment is required for quality improvement for all the items.

Table 5.12 shows the comparison of results of the expected total cost for the data given in Table 5.4 to 5.7 considering the investment in setup cost reduction and quality improvement. Data given in Table 5.6 is chosen to consider a situation where proportion of time available for setups is high and data given in Table 5.7 is applicable to a situation where as many as 10 items are to be produced in a single machine. The numerical study for the data given in Table 5.4 to 5.7 in Table 5.12 shows reduction in the expected total cost with investment in both setup cost reduction and quality improvement for (i) imperfect process model and (ii) imperfect process model with inspection and restoration.

Table 5.12 A comparative study of the expected total average cost

		Example Data			
		I	II	III	IV
Imperfect process model without investment (Moon et al. 2002)		9289.36	2461.82	1190.79	120.49
Imperfect process model with investment in	setup cost reduction	7216.31	2438.00	910.68	119.81
	quality improvement	7450.74	2395.10	1181.86	88.22
	both setup cost reduction and quality improvement	5348.45	2371.26	909.17	88.04
Imperfect process model with inspection and restoration without investment (Moon et al. 2002)		8185.97	2378.06	1166.74	72.99
Imperfect process model with inspection and restoration with investment in	setup cost reduction	6054.92	2353.42	893.06	72.59
	quality improvement	7830.72	2377.48	1166.51	72.82
	both setup cost reduction and quality improvement	5485.19	2353.64	893.06	72.59

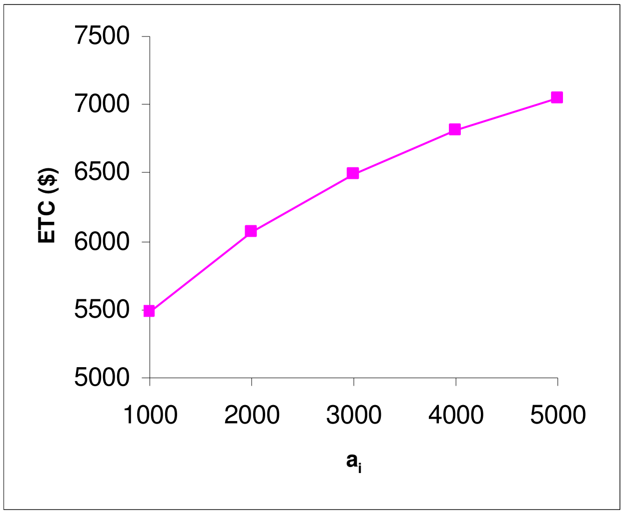

Fig. 5.4 Variation of ETC with a_i

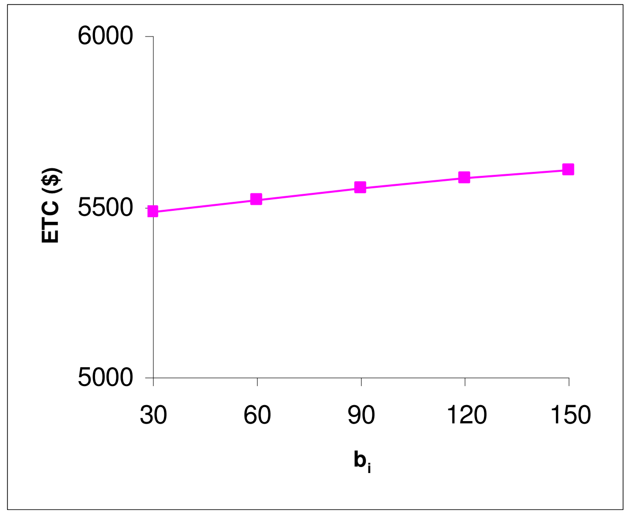

Fig. 5.5 Variation of ETC with b_i

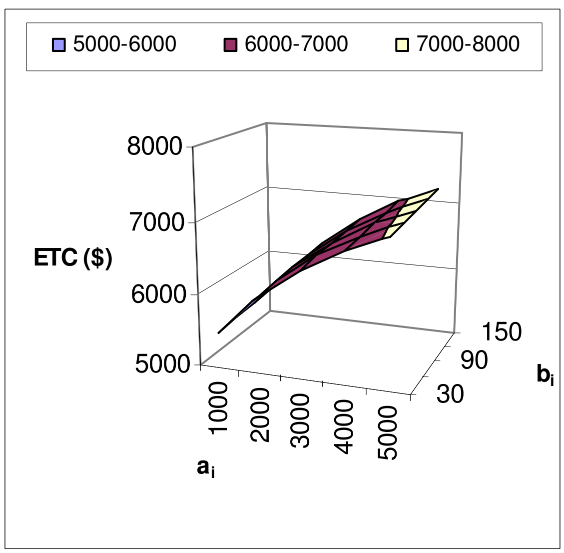

Fig. 5.6 Variation of ETC with a_i and b_i

Figures 5.4 – 5.6 show the variation of expected total cost for imperfect process model with inspection and restoration using the data given in Table 5.4. Figures 5.4 and 5.5 show the expected total cost (ETC) increases with increase in a_i (rate of increase in dollar investment per fraction of reduction in setup costs, (A_i)) and b_i (rate of increase in dollar investment per fraction of reduction in non-conforming units, (α_i)) respectively. Figures 5.4 - 5.5 show that increase in

ETC with increase in a_i is more as compared to increase in ETC with increase in b_i, due to the higher magnitude of a_i. Figure 5.6 shows the variation of ETC with both a_i and b_i respectively.

Figures 5.4 and 5.5 show the expected total cost (ETC) increases with increase in a_i (rate of increase in dollar investment per fraction of reduction in setup costs, (A_i)) and b_i (rate of increase in dollar investment per fraction of reduction in non-conforming units, (α_i)) respectively. Figures 5.4 - 5.5 show that increase in ETC with increase in a_i is more as compared to increase in ETC with increase in b_i, due to the higher magnitude of a_i. Figure 5.6 shows the variation of ETC with both a_i and b_i respectively. For the data considered in the analysis, the increase in expected total cost (ETC) with increase in a_i is significant as compared to increase in expected total cost (ETC) with increase in b_i.

5.3 Managerial Implications

The managerial implications are as follows:

- Investment in setup cost reduction is appropriate for high value of initial setup cost (K_o), high demand rate (D), low fractional opportunity cost of capital per unit time (i_K), small production run time (T). From Eqs. (5.4), (5.5) and (5.8), investment in setup cost reduction is possible when $K_o > K^*$ or $\dfrac{K_o D}{i_K B_K PT} > 1$.
- Investment in quality improvement is appropriate for high value of percentage of defective items produced when the process is in the out-of-control state (α_o), high cost incurred for producing a defective item (s), high demand rate (D), low fractional opportunity cost of capital per unit time (i_α), more production run time (T) and less number of inspections undertaken during each production run (N). From Eqs. (5.19), (5.20) and (5.23), investment in quality improvement is possible when $\alpha_o > \alpha^*$ or $\dfrac{\alpha_o sD}{i_\alpha B_\alpha}\left(1-\dfrac{N}{\mu T}\left\{1-\exp\left(-\dfrac{\mu T}{N}\right)\right\}\right) > 1$.
- Under the scenario of investment in both setup cost reduction and quality improvement, investment in quality improvement but not in setup cost reduction is recommended under conditions of high restoration cost and high inspection cost of the process. Similarly under this scenario of investment in both setup cost reduction and quality improvement,

investment in setup cost reduction but not in quality improvement is recommended for low value of deterioration rate of the process and low value of restoration cost. Table 5.3 shows that for all values of deterioration rate of the production process (μ=0.1 to 0.5) and the corresponding higher values of restoration cost (r) and inspection cost (v) considered in this study, investment is only needed for quality improvement and not in setup cost reduction is required for μ=0.1, (r, v)= (180, 10), (180, 20) and for μ=0.2, (r, v)= (60, 20), (90, 10), (90, 20) respectively. Similarly for deterioration rate of the process (μ) of 0.1 and inspection cost (v) of 10 considered in this study, the investment in setup cost reduction but not in quality improvement is required for restoration cost values (r) of 10, 20, 30 and 60 respectively.

- Reduction in the expected total cost results with investment in both setup cost reduction and quality improvement for all the cases considered in this study viz., (i) imperfect process model under time varying approach, (ii) imperfect process model under common cycle approach, (iii) imperfect process model with inspection and restoration under time varying approach, and (iv) imperfect process model with inspection and restoration under common cycle approach. The numerical study also shows (i) reduction in the expected total cost of time-varying lot sizes approach over common cycle approach, (ii) reduction in the number of inspections with investment in setup cost reduction and quality improvement. The results of expected total cost obtained based on the Example data I are as follows. As shown in Table 5.8 for optimal solutions of imperfect process model under time varying approach, the expected total cost with investment in both setup cost reduction and quality improvement is \$ 5348.45, while without investment (Moon et al., 2002) is \$ 9289.36. Table 5.9 shows that for optimal solutions of imperfect process model using common cycle (CC) approach, the expected total cost with investment in both setup cost reduction and quality improvement is \$ 5580.99, while without investment (Moon et al., 2002) is \$ 10164.86. Table 5.10 shows that for optimal solutions of imperfect process model with inspection and restoration under time varying approach, the expected total cost with investment in both setup cost reduction and quality improvement is \$ 5485.19, while without investment (Moon et al., 2002) is \$ 8185.97. As shown in Table 5.11 for optimal solutions of imperfect process model with inspection and restoration using common cycle (CC) approach the expected total cost with investment in both setup cost reduction and quality improvement is \$ 5715.12, while without investment (Moon et al., 2002) is \$ 8811.58. Table 5.10 shows that for optimal solutions of imperfect process model with inspection and restoration under time varying approach, the number of inspections for the three items with investment in both setup cost reduction and quality improvement are 1, 1, and 1 respectively, while without investment (Moon et al., 2002) are 3, 6, and 4 respectively. As shown in Table 5.11 for optimal solutions of imperfect process model with inspection and restoration using common cycle (CC) approach the number of inspections for the three items with investment in both setup cost reduction and quality improvement are 1, 1, and 1 respectively, while without investment (Moon et al., 2002) are 2, 7, and 2 respectively.