CHAPTER 6

OPTIMAL BATCH SIZE IN A SINGLE-STAGE IMPERFECT PRODUCTION SYSTEM WITH INSPECTION ERRORS AND OPTIMAL NUMBER OF KANBANS IN A MULTI-STAGE JIT PRODUCTION-DELIVERY SYSTEM WITH REWORK CONSIDERATION

This chapter depicts the analytical model and numerical computation to determine the optimal batch size in a single-stage production system with rework consideration under different options of raw material ordering and finished goods delivery situations. The model and computational results to determine the optimal number of kanbans in a multi-stage JIT production-delivery system with rework consideration are presented. The managerial implications based on the results are summarized.

6.1 Optimal Batch Size in a Single-Stage Imperfect Production System due to Inspection Errors

Ojha et al., (2007) considered a manufacturing system which receives raw material from a supplier, processes it, and delivers it to the customer periodically. They determined the optimal raw material ordering quantity and finished product batch size such that the total cost for the system is minimized. Three different scenarios are considered, viz. (a) a single lot of raw material for multiple lot of finished product and delivery of the product in multiple installments, (b) a single lot of raw material for a multiple lot of finished product and delivery of the product in a single installment, and (c) lot-for-lot and delivery of finished product in single installment. Ben-Daya and Rahim (2003) modeled an imperfect multistage production system considering the (i) effect of inspection errors in screening non-conforming items at various stages and (ii) inspection and restoration of the processes at all stages. They have addressed the following important issues when dealing with imperfect multistage production processes: (i) Non-conforming items must be screened so that they are not passed to subsequent stages, (ii) While screening non-conforming items, good items may be incorrectly rejected and nonconforming items may be incorrectly accepted, and (iii) When the processes shift to an out of control state, production of non-conforming items must be detected.

This research work extends the Ojha et al., (2007) model for the three production – delivery policies incorporating the (i) effect of inspection errors that may be committed while screening defective items and (ii) inspection and restoration of the process as a mean of improving quality. The objective will be to minimize the total system cost in all three policies and to determine the optimal number of inspections and raw material ordering quantity.

The remainder of this section is organized as follows. The necessary notation and assumptions are stated. The determination of optimal batch size incorporating the expected quality cost is provided for all the three production delivery situations. The mathematical model incorporating process inspection and restoration costs in the all the three production delivery systems with defects and inspection errors is also presented. The solution procedure to determine (i) the production batch size and raw material ordering quantity, and (ii) the optimal number of inspections along with the optimal batch size and raw material ordering quantity are described. Numerical examples and sensitivity analysis of the parameters are also presented.

The notations used in this model are defined as follows:

A_f	setup cost for production ($ per batch)
A_r	ordering cost of raw material ($ per order)
C_i	cost of inspection ($ per unit)
C_l	cost of late delivery ($ per unit)
C	cost of raw material ($ per unit)
D_r	demand of raw material (units per cycle)
D_f	demand rate of the product (units per cycle)
E_1	probability of incorrectly rejecting a conforming item (type I error)
E_2	probability of incorrectly accepting a non-conforming item (type II error)
f_0	conversion factor of raw material ($f_0=D_f/D_r=Q_f/Q_r$)
i	yearly interest rate
L	time between successive shipments ($L=x/D_f$)
m	number of full shipment during production cycle time ($m=T/L$)
n	number of equivalent batches to a raw material supply
N_c	number of conforming items in a batch (units)

N_n number of non-conforming items in a batch (units)

N_r total number of items rejected in a batch (units)

N_{nc} total number of non-conforming items not screened in the inspection (units)

P Production rate (units per year)

Q_f production batch size (units)

Q_r ordering quantity of raw material (units)

$R(\tau)$ restoration cost, which is a function of detection delay; $R(\tau) = r_0 + r_1\tau$

s unit cost of producing a non conforming item

T production cycle time

t_1 production time of non-defective parts (years)

t_2 rework time of defective parts (years)

t_s setup time

V value added by the production process ($ per unit)

x shipment quantity to the customer

α percentage of defective items produced when the process is in the out-of control state

β fraction of items rejected on inspection of a batch V

β_1 fraction of bad quality items produced in a batch

η number of inspections of the process

π_1 cost of incorrectly rejecting a conforming item

π_2 cost of incorrectly accepting a non conforming item

τ detection delay, i.e. the time elapsed between the occurrence of a shift and the end of the production cycle

v inspection cost of the process

The assumptions used in the models are (Ojha et al., 2007 and Ben-Daya and Rahim, 2003):

1. the production rate is greater than the demand rate.
2. the process starts in the in-control state producing items of perfect quality and the process may shift to the out-of-control state after a time with a known probability distribution. In the out-of-control state, the process starts producing non-conforming items.
3. all the poor quality items can be reworked.
4. the demand is known and deterministic.

5. the product is delivered in equal quantities and at fixed intervals.
6. the product cannot be delivered until the whole lot is produced, reworked, and quality certified.
7. the rework rate is same as the production rate.
8. no defects are produced during rework process.
9. the setup time is negligible.
10. the number of undetected defectives passed onto the customer is negligible.

6.1.1 Model formulation for optimal batch size and number of inspections in a single-stage imperfect production system incorporating process inspection and restoration

The models formulated for optimal batch size and number of inspections in a single-stage imperfect production system are: (i) single purchase multiple delivery (SPMD) with inspection errors, (ii) single purchase multiple delivery (SPMD) with process inspection and restoration, (iii) single purchase single delivery (SPSD) with inspection errors, (iv) single purchase single delivery (SPSD) with inspection and restoration, (v) lot-for-lot (LFL) with inspection errors, (vi) lot-for-lot (LFL) with process inspection and restoration.

6.1.1.1 Single purchase multiple delivery (SPMD) with inspection errors

The SPMD policy is adopted when the customer requires the delivery to be made in small installments. Each batch of finished goods is inspected so that the non-conforming items can be removed from the batch and reworked. Non-conforming items at each level are removed and not passed to the following stage. The reworked items are again added to the finished goods lot for final dispatch.

Fig. 6.1 shows the consumption of raw material inventory, build-up of the finished goods inventory with the start of each production cycle, production of the non-defective end product, and rework of the defective items (Ojha et al., 2007). The raw material is consumed by the end of n^{th} cycle. The finished goods inventory starts to build up with start of each production cycle, and the production of non-defective end product occurs for at time t_1 at the rate of $P(1-\beta)$. After the production uptime t_1, the rework begins at rate P, continues for a time t_2, and reaches a maximum inventory of Q_f when all items are reworked. The delivery of the finished goods is made in small equal installments of x at an equal interval of time L. Each supply decreases the finished product inventory by x and the finished product inventory is consumed after m deliveries.

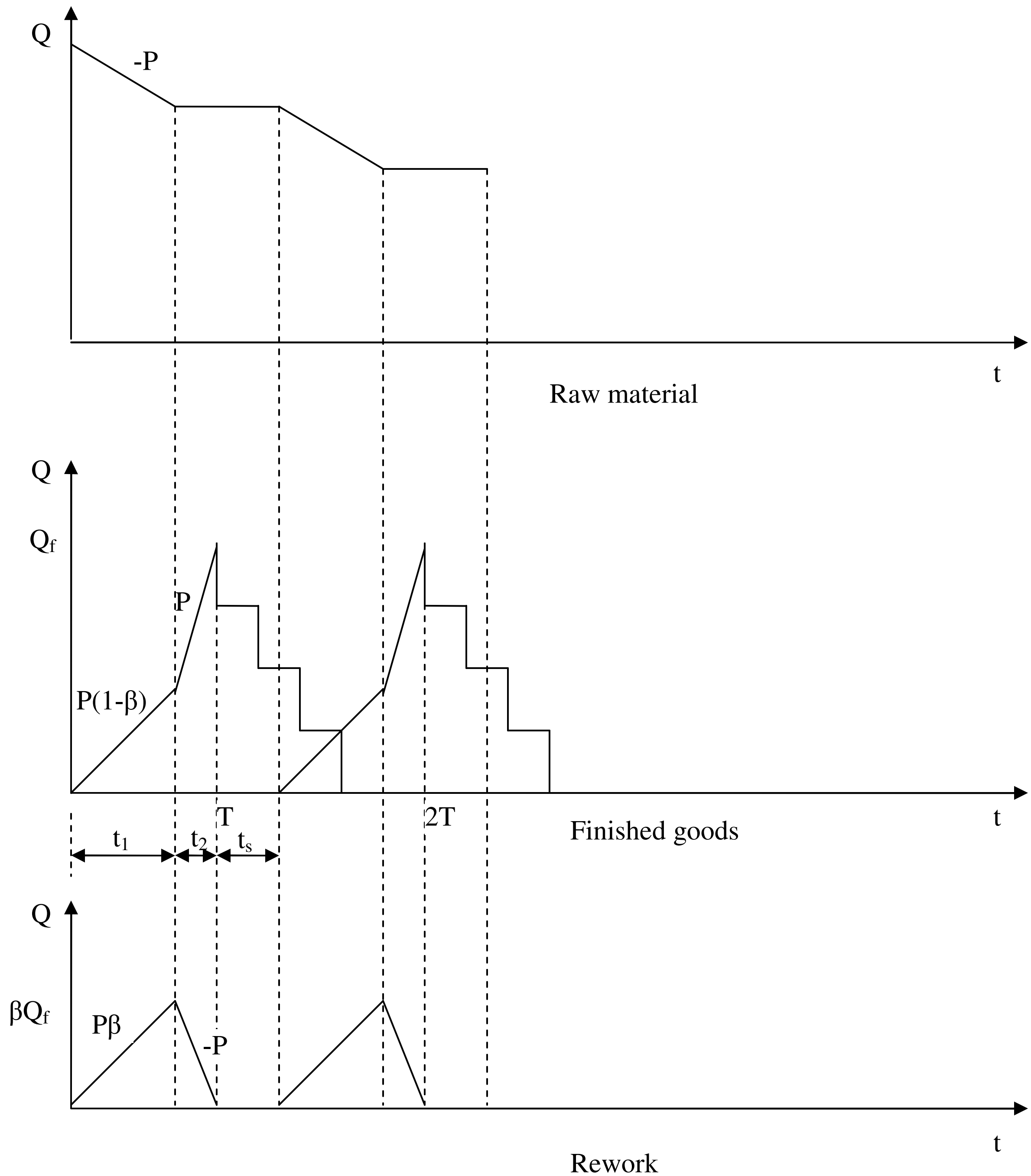

Fig. 6.1 Single purchase multiple delivery (SPMD)

The average raw material inventory is given by

$$\bar{I}_r = \frac{1}{T}\left[\left(\frac{1}{2}Q_r t_1 + Q_r T\right) + \left(\frac{1}{2}Q_r t_1 + Q_r 2T\right) + \ldots\ldots\ldots\left(\frac{1}{2}Q_r t_1 + Q_r (n-1)T\right)\right] = \frac{1}{T}\left[\frac{1}{2}nQ_r t_1 + \frac{1}{2}Q_r n(n-1)T\right] = \frac{Q_f}{Tf_0}\left[\frac{D_f}{P} + n - 1\right] \quad (6.1)$$

The inventory carrying cost of raw material is

$$I_R = \frac{iCQ_f}{2f_0}\left[\frac{D_f}{P} + n - 1\right] \quad (6.2)$$

The ordering cost of one lot of raw material is

$$A_R = \frac{D_r}{nQ_r}A_r = \frac{D_f}{nQ_f}A_r \quad (6.3)$$

The average finished product inventory which is delivered in small equal installments x is

$$\bar{I} = \frac{Q_f D_f}{2P}[1 + \beta(1-\beta)] + \frac{1}{mL}[x(m-1)L + x(m-2)L + \ldots\ldots + xL] = \frac{Q_f D_f}{2P}[1 + \beta(1-\beta)] + \frac{1}{2}x(m-1) \quad (6.4)$$

The total finished goods inventory carrying cost is

$$I_F = i(C+V)\left[\frac{Q_f D_f}{2P}[1 + \beta(1-\beta)] + \frac{1}{2}x(m-1)\right] \quad (6.5)$$

The processing cost of the raw material per batch quantity Q_f processed in every cycle is VQ_f. The total processing cost over the year is

$$K_F = (VQ_f)\frac{D_f}{Q_f} = VD_f \quad (6.6)$$

The proportion of the defective products in a cycle is β. The amount of the defective items produced in a cycle is βQ_f. The rework processing cost of the defectives per year is

$$K_{RE} = (V\beta Q_f)\frac{D_f}{Q_f} = V\beta D_f \quad (6.7)$$

The inventory of defective items produced per cycle is carried over for t_2 time. The rework inventory carrying cost is

$$I_{RE} = \frac{\beta Q_f}{2}i(C+V)\frac{D_f}{P} + \frac{\beta Q_f}{2}i(C+2V)\frac{\beta D_f}{P} = \frac{i\beta Q_f D_f}{2P}(C + V + \beta C + 2\beta V) \quad (6.8)$$

The total inspection cost per cycle is

$$C_I = Q_f C_i \tag{6.9}$$

The finished material are delivered late in each cycle by time t_2. The total late delivery per cycle is

$$C_L = t_2 C_l = \frac{\beta Q_f}{P} C_l \tag{6.10}$$

Also the finished products are not produced in time t_2. The cost per cycle to miss out an opportunity of producing more of the finished products is

$$C_{LP} = V\beta Q_f \tag{6.11}$$

The total cost is the sum of inventory carrying cost of raw material, ordering cost of raw material, setup cost, finished goods inventory carrying cost, processing cost, reworking processing cost of defective items, rework inventory carrying cost, inspection cost, late delivery cost and lost production cost is (Ojha et al., 2007):

$$TC(m,n) = \frac{iCQ_f}{2f_0}\left[\frac{D_f}{P} + n - 1\right] + \frac{D_f}{Q_f}\left(\frac{A_r}{n} + A_f\right) + i(C+V)\left[\frac{Q_f D_f}{2P}[1+\beta(1-\beta)] + \frac{1}{2}x(m-1)\right] + VD_f(1+\beta) + \frac{i\beta Q_f D_f}{2P}(C+V+\beta C+2\beta V) + Q_f\left(C_i + \frac{\beta C_l}{P} + V\beta\right) \tag{6.12}$$

A lot of Q_f conforming products are required for the final product. The number of items rejected from a lot is the sum of incorrectly rejecting a confirming item and correctly rejecting a non-conforming item. The conforming items in a batch are $(Q_f - N_n)$ and the number of non-conforming items in a batch are N_n are shown in Fig. 6.2. The number of incorrectly rejected conforming items are $(Q_f - N_n)E_1$ and the number of correctly accepted conforming items are $(Q_f - N_n)(1-E_1)$. The correctly rejected non-conforming items are $N_n(1-E_2)$ and the incorrectly accepted non-conforming items are $N_n E_2$.

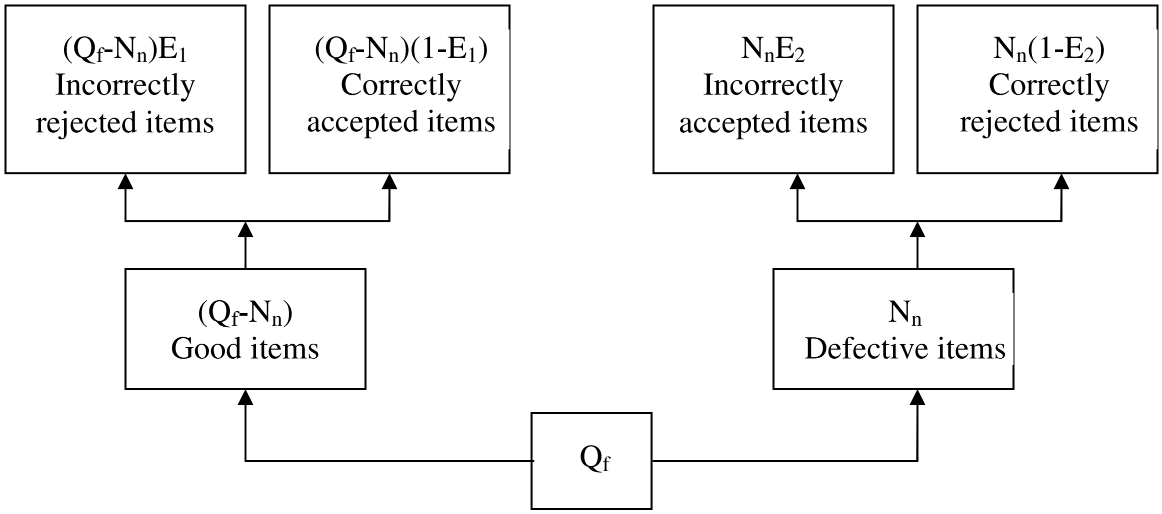

Fig. 6.2 Effect of inspection errors

Thus, the total number of items rejected in a batch is

$$N_r = N_n(1-E_2)+(Q_f - N_n)E_1 \qquad (6.13)$$

Therefore the effective rejection rate is $\beta = N_r / Q_f$.

The expected number of non-conforming items produced at any stage once the corresponding process shifts to an out of control state is given by (Ben-Daya and Rahim, 2003)

$$N_n = \int_0^{t_1} \alpha P(t_1 - t)\frac{e^{-t/\theta}}{\theta}dt = \alpha P\left(t_1 - \theta + \theta_1 e^{-t_1/\theta}\right) \qquad (6.14)$$

Using the approximation $e^{-t_1/\theta} \approx 1 - t_1/\theta + 0.5(t_1/\theta)^2$ by Mc Laurin series (Moon et al., 2002) and substituting in Eq. (6.14) the number of non-conforming items in a batch is

$$N_n = \frac{\alpha P t_1^2}{2\theta} = \frac{\alpha Q_f^2}{2P\theta} \qquad (6.15)$$

where $t_1 = Q_f / P$

From Eqs. (6.13) and (6.15), the effective rejection rate is

$$\beta = \frac{N_r}{Q_f} = \frac{\alpha Q_f}{2P\theta}(1-E_2)+\left(1-\frac{\alpha Q_f}{2P\theta}\right)E_1 \qquad (6.16)$$

The expected quality costs per unit time due to non-conforming items and inspection errors are given by (Ben-Daya and Rahim, 2003)

$$K_{QC} = \frac{D_f}{Q_f}\left(sN_n(1-E_2)+\pi_1\left(Q_f - N_n\right)E_1 + \pi_2 N_n E_2\right) =$$
$$D_f\left(\pi_1 E_1 + \left(s - sE_2 - \pi_1 E_1 + \pi_2 E_2\right)\frac{\alpha Q_f}{2P\theta}\right) \tag{6.17}$$

As $Q_f = mx$, the expected total cost from Eqs. (6.12), (6.16) and (6.17) is written as

$$TC(m,n) = \frac{iCmx}{2f_0}\left[\frac{D_f}{P}+n-1\right]+\frac{D_f}{mx}\left(\frac{A_r}{n}+A_f\right)+i(C+V)\left[\frac{mxD_f}{2P}[1+\beta(1-\beta)]+\frac{1}{2}x(m-1)\right]+$$
$$VD_f(1+\beta)+D_f\left(\pi_1 E_1+\left(s-sE_2-\pi_1 E_1+\pi_2 E_2\right)\frac{\alpha mx}{2P\theta}\right)+$$
$$\frac{i\beta mxD_f}{2P}\left(C+V+\beta C+2\beta V\right)+mx\left(C_i+\frac{\beta C_l}{P}+V\beta\right) \tag{6.18}$$

The total cost function is convex in both n and m. Optimal values of *n* and *m* are found by differentiating the total cost function with respect to n and m, and equating the resulting expression to zero. Differentiating Eq. (6.18) with respect to *n* and equating the resulting the expression to zero, the optimal value of *n* is obtained as;

$$n^* = \frac{1}{mx}\sqrt{\frac{2f_0 D_f A_r}{iC}} \tag{6.19}$$

Similarly, differentiating the Eq. (6.18) with respect to *m* and equating the resulting expression to zero yields;

$$\frac{\partial TC(m,n)}{\partial m} = \frac{iCx}{2f_0}\left[\frac{D_f}{P}+n-1\right]-\frac{D_f}{m^2 x}\left(\frac{A_r}{n}+A_f\right)+$$
$$i(C+V)\left[\frac{xD_f}{2P}[1+\beta(1-\beta)]+\frac{mxD_f}{2P}\frac{d\beta}{dm}(1-2\beta)+\frac{1}{2}x\right]+$$
$$VD_f\frac{d\beta}{dm}+D_f\left(s-sE_2-\pi_1 E_1+\pi_2 E_2\right)\frac{\alpha x}{2P\theta}+\frac{ixD_f\beta}{2P}\left(C+V+\beta C+2\beta V\right)+$$
$$\frac{imxD_f}{2P}\frac{d\beta}{dm}\left(C+V+2\beta C+4\beta V\right)+x\left(C_i+\frac{\beta C_l}{P}+V\beta\right)+mx\frac{d\beta}{dm}\left(\frac{C_l}{P}+V\right)=0 \tag{6.20}$$

where $\frac{d\beta}{dm} = \frac{\alpha x}{2P\theta}\left(1-E_1-E_2\right)$

The solution algorithm is as follows:

Step 1: Using the values of n^* from Eq. (6.19) and β from Eq. (6.16), solve Eq. (6.20) using Newton-Raphson iterative procedure till the convergence of m^* *and* n^* are obtained.

The convergence criteria for algorithm is: $\left| \dfrac{\dfrac{\partial TC(m,n)}{\partial m}}{\dfrac{\partial^2 TC(m,n)}{\partial m^2}} \right| \le 10^{-3}$

The convergence criteria for algorithm is: $\left| \dfrac{\dfrac{\partial TC(m,n,\eta)}{\partial m}}{\dfrac{\partial^2 TC(m,n,\eta)}{\partial m^2}} \right| \le 10^{-3}$

The convergence criteria for algorithm is: $\left| \dfrac{\dfrac{\partial TC(m,n,\eta)}{\partial \eta}}{\dfrac{\partial^2 TC(m,n,\eta)}{\partial \eta^2}} \right| \le 10^{-3}$

Step 2:

(i) If $n^* < 1$ then choose $n^*=1$.

   (a) If $m^* < 1$, then choose $m^*=1$. Determine β^* (1) from Eq. (6.16). Compute $TC^*(1,1)$ from Eq. (6.18).

   (b) If $m^* > 1$, then for $J_m^* < m^* < J_m^*+1$ (J_m is a positive integer), determine β^* (J_m^*) and β^* (J_m^*+1) from Eq. (6.16). Compute $TC^*(1, J_m^*)$ and $TC^*(1, J_m^*+1)$ from Eq. (6.18) and choose the one with smaller value as the optimal solution.

(ii) If $n^* > 1$, then choose $J_n^* < n^* < J_n^*+1$ (where J_n is a positive integer).

   (a) If $m^* < 1$, then choose $m^*=1$. Determine β^* (1) from Eq. (6.16). Compute $TC^*(J_n^*, 1)$ and $TC^*(J_n^*+1, 1)$ from Eq. (6.18) and choose the one with smaller value as the optimal solution.

   (b) If $m^* > 1$, and $J_m^* < m^* < J_m^*+1$ (where J_m is a positive integer), determine β^* (J_m^*) and β^* (J_m^*+1) from Eq. (6.16). Compute $TC^*(J_n^*, J_m^*)$, $TC^*(J_n^*+1, J_m^*)$, $TC^*(J_n^*, J_m^*+1)$ and $TC^*(J_n^*+1, J_m^*+1)$ from Eq. (6.18) and choose the one with smaller value as the optimal solution.

6.1.1.2 Single purchase multiple delivery (SPMD) with process inspection and restoration

Production process is inspected at regular intervals during a production run and if it is found to be out of control, necessary actions are taken to restore it to the in control state (Ben-Daya and Rahim, 2003). Lee and Rosenblatt (1989) determined the production and inspection schedules for a system in which cost of maintenance depends on detection delay, i.e., the number of periods in the out-of-control state. The optimum maintenance schedule is determined as a function of the cost of defective items, the cost of restoration and the mean time until the system is out of control. Scheduling inspections at regular intervals has been shown to be optimal in cases of exponentially distributed time to shift to the out of control state.

The expected number of non conforming items, $N_{i\eta}$ produced during inspection is given by

$$N_{i\eta} = \int_0^{t_1/\eta} \alpha P\left(\frac{t_1}{\eta} - t\right)\frac{e^{-t/\theta}}{\theta}dt = \alpha P\left(\frac{t_1}{\eta} - \theta + \theta e^{-t_1/(\eta\theta)}\right) \approx \frac{\alpha P}{2}\left(\frac{t_1}{\eta\theta}\right)^2 = \frac{\alpha Q_f^2}{2\eta^2 P\theta} \qquad (6.21)$$

The total expected number of non conforming items N_i, produced during a complete production run of the process is

$$N_i = \eta N_{i\eta} = \frac{\alpha Q_f^2}{2\eta P\theta} \qquad (6.22)$$

Thus, the effective rejection rate is

$$\beta_i = \frac{N_i}{Q_f} = \frac{\alpha Q_f}{2\eta P\theta}(1 - E_2) + \left(1 - \frac{\alpha Q_f}{2\eta P\theta}\right)E_1 \qquad (6.23)$$

Hence the expected quality costs per unit time due to non-conforming items and inspection errors are given by

$$K_{QCI} = \frac{D_f}{Q_f}\left(sN_i(1 - E_2) + \pi_1(Q_f - N_i)E_1 + \pi_2 N_i E_2\right) = D_f\left(\pi_1 E_1 + (s - sE_2 - \pi_1 E_1 + \pi_2 E_2)\frac{\alpha Q_f}{2\eta P\theta}\right) \qquad (6.24)$$

The inspection cost is given by

$$K_{IC} = \frac{D_f}{Q_f}\eta v \qquad (6.25)$$

It is assumed that the restoration cost depends on the detection delay, then the expected restoration cost during the inspection of the process is given by

$$K_{RC\eta} = \int_0^{t_1/\eta} \left[r_0 + r_1 \left(\frac{t_1}{\eta} - t \right) \right] \frac{e^{-t/\theta}}{\theta} dt = \left(r_0 + r_1 \frac{t_1}{\eta} - r_1 \theta \right) + (r_1 \theta - r_0) e^{-t_1/(\eta\theta)}$$

$$= (r_1 \theta - r_0) \frac{t_1^2}{2\eta^2 \theta^2} + \frac{r_0 t_1}{\eta\theta} = (r_1 \theta - r_0) \frac{Q_f^2}{2\eta^2 P^2 \theta^2} + \frac{r_0 Q_f}{\eta P \theta} \tag{6.26}$$

The expected restoration cost for the complete system is

$$K_{RC} = \frac{D_f}{Q_f} (\eta K_{RC\eta}) = \frac{Q_f D_f}{2\eta P^2 \theta^2} (r_1 \theta - r_0) + \frac{D_f r_0}{P\theta} \tag{6.27}$$

The expected total cost including inspection and restoration from Eqs. (6.12), (6.23), (6.24), (6.25) and (6.27) is written as

$$TC(m,n,\eta) = \frac{iCmx}{2f_0} \left[\frac{D_f}{P} + n - 1 \right] + \frac{D_f}{mx} \left(\frac{A_r}{n} + A_f \right) + i(C+V) \left[\frac{mxD_f}{2P} [1 + \beta_i (1 - \beta_i)] + \frac{1}{2} x(m-1) \right] +$$
$$VD_f (1 + \beta_i) + D_f \left(\pi_1 E_1 + (s - sE_2 - \pi_1 E_1 + \pi_2 E_2) \frac{\alpha mx}{2\eta P \theta} \right) + \frac{i\beta_i mxD_f}{2P} (C + V + \beta_i C + 2\beta_i V) +$$
$$mx \left(C_i + \frac{\beta_i C_l}{P} + V\beta_i \right) + \frac{D_f}{mx} \eta v + \frac{mxD_f}{2\eta P^2 \theta^2} (r_1 \theta - r_0) + \frac{D_f r_0}{P\theta} \tag{6.28}$$

Differentiating Eq. (6.18) with respect to *n* and equating the resulting the expression to zero, the optimal value of *n* is shown in Eq. (6.19).

Differentiating the Eq. (6.28) with respect to *m* and equating the resulting expression to zero yields;

$$\frac{\partial TC(m,n,\eta)}{\partial m} = \frac{iCx}{2f_0} \left[\frac{D_f}{P} + n - 1 \right] - \frac{D_f}{m^2 x} \left(\frac{A_r}{n} + A_f \right) + i(C+V) \left[\frac{xD_f}{2P} [1 + \beta_i (1 - \beta_i)] + \frac{1}{2} x \right] +$$
$$i(C+V) \frac{mxD_f}{2P} \left(\frac{\partial \beta_i}{\partial m} - 2\beta_i \frac{\partial \beta_i}{\partial m} \right) + VD_f \frac{\partial \beta_i}{\partial m} + D_f (s - sE_2 - \pi_1 E_1 + \pi_2 E_2) \frac{\alpha x}{2\eta P \theta} +$$
$$\frac{ixD_f}{2P} \left(\beta_i C + \beta_i V + \beta_i^2 C + 2\beta_i^2 V \right) + \frac{imxD_f}{2P} \left(\frac{\partial \beta_i}{\partial m} C + \frac{\partial \beta_i}{\partial m} V + 2\beta_i \frac{\partial \beta_i}{\partial m} C + 4\beta_i \frac{\partial \beta_i}{\partial m} V \right) +$$
$$x \left(C_i + \frac{\beta_i C_l}{P} + V\beta_i \right) + mx \left(\frac{C_l}{P} \frac{\partial \beta_i}{\partial m} + V \frac{\partial \beta_i}{\partial m} \right) - \frac{D_f}{m^2 x} \eta v + \frac{xD_f}{2\eta P^2 \theta^2} (r_1 \theta - r_0) = 0 \tag{6.29}$$

where $\frac{\partial \beta_i}{\partial m} = \frac{\alpha x}{2\eta P \theta} (1 - E_1 - E_2)$

Differentiating the Eq. (6.28) with respect to η and equating the resulting expression to zero yields;

$$\frac{\partial TC(m,n,\eta)}{\partial \eta} = i(C+V)\frac{mxD_f}{2P}\frac{\partial \beta_i}{\partial \eta}(1-2\beta_i)+VD_f\frac{\partial \beta_i}{\partial \eta}-D_f(s-sE_2-\pi_1E_1+\pi_2E_2)\frac{\alpha mx}{2\eta^2P\theta}+$$
$$\frac{imxD_f}{2P}\frac{\partial \beta_i}{\partial \eta}[(C+V)+2\beta_i(C+2V)]+mx\frac{\partial \beta_i}{\partial \eta}\left(\frac{C_l}{P}+V\right)+\frac{D_f}{mx}v-\frac{mxD_f}{2\eta^2P^2\theta^2}(r_1\theta-r_0)=0 \tag{6.30}$$

where $\frac{\partial \beta_i}{\partial \eta} = -\frac{\alpha mx}{2\eta^2P\theta}(1-E_1-E_2)$

The solution algorithm for this model is as follows:

Step 1: Using the values of n^* from Eq. (6.19) and β from Eq. (6.23), solve Eq. (6.29) using Newton-Raphson iterative procedure till the convergence of m^* *and* n^* are obtained.

Step 2: Repeat the step 1 using the current value of η^* obtained by solving Eq. (6.30) using Newton-Raphson iterative procedure, till the convergence of η^* is obtained.

Step 3:

(i) If $\eta^* < 1$ then choose $\eta^*=1$.

(a) If $n^* < 1$ then choose $n^*=1$.

1. If $m^* < 1$, then choose $m^*=1$. Determine β^* (1) from Eq. (6.23). Compute $TC^*(1,1,1)$ from Eq. (6.28).
2. If $m^* > 1$, then for $J_m^* < m^* < J_m^*+1$ (J_m is a positive integer), determine β^* (J_m^*) and β^* (J_m^*+1) from Eq. (6.23). Compute $TC^*(1, J_m^*, 1)$ and $TC^*(1, J_m^*+1, 1)$ from Eq. (6.28) and choose the one with smaller value as the optimal solution.

(b) If $n^* > 1$, then choose $J_n^* < n^* < J_n^*+1$ (where J_n is a positive integer).

1. If $m^* < 1$, then choose $m^*=1$. Determine β^* (1) from Eq. (6.23). Compute $TC^*(J_n^*, 1, 1)$ and $TC^*(J_n^*+1, 1, 1)$ from Eq. (6.28) and choose the one with smaller value as the optimal solution.
2. If $m^* > 1$, and $J_m^* < m^* < J_m^*+1$ (where J_m is a positive integer), determine β^* (J_m^*) and β^* (J_m^*+1) from Eq. (6.23). Compute $TC^*(J_n^*, J_m^*, 1)$, $TC^*(J_n^*+1, J_m^*, 1)$, $TC^*(J_n^*, J_m^*+1, 1)$ and $TC^*(J_n^*+1, J_m^*+1, 1)$

from Eq. (6.28) and choose the one with smaller value as the optimal solution.

(ii) If $\eta^* > 1$ then choose $J_\eta^* < n^* < J_\eta^*+1$ (where J_η is a positive integer)

(a) If $n^* < 1$ then choose $n^*=1$.

1. If $m^* < 1$, then choose $m^*=1$. Determine β^* (1) from Eq. (6.23). Compute $TC^*(1,1,J_\eta^*)$ and $TC^*(1,1, J_\eta^*+1)$ from Eq. (6.28) and choose the one with smaller value as the optimal solution.
2. If $m^* > 1$, then for $J_m^* < m^* < J_m^*+1$ (J_m is a positive integer), determine β^* (J_m^*) and β^* (J_m^*+1) from Eq. (6.23). Compute $TC^*(1, J_m^*, J_\eta^*)$, $TC^*(1, J_m^*+1, J_\eta^*)$, $TC^*(1, J_m^*, J_\eta^*+1)$ and $TC^*(1, J_m^*+1, J_\eta^*+1)$ from Eq. (6.28) and choose the one with smaller value as the optimal solution.

(b) If $n^* > 1$, then choose $J_n^* < n^* < J_n^*+1$ (where J_n is a positive integer).

1. If $m^* < 1$, then choose $m^*=1$. Determine β^* (1) from Eq. (6.23). Compute $TC^*(J_n^*,1, J_\eta^*)$, $TC^*(J_n^*+1, 1, J_\eta^*)$, $TC^*(J_n^*,1, J_\eta^*+1)$ and $TC^*(J_n^*+1, 1, J_\eta^*+1)$ from Eq. (6.28) and choose the one with smaller value as the optimal solution.
2. If $m^* > 1$, and $J_m^* < m^* < J_m^*+1$ (where J_m is a positive integer), determine β^* (J_m^*) and β^* (J_m^*+1) from Eq. (6.23). Compute $TC^*(J_n^*, J_m^*, J_\eta^*)$, $TC^*(J_n^*+1, J_m^*, J_\eta^*)$, $TC^*(J_n^*, J_m^*+1, J_\eta^*)$, $TC^*(J_n^*+1, J_m^*+1, J_\eta^*)$, $TC^*(J_n^*, J_m^*, J_\eta^*+1)$, $TC^*(J_n^*+1, J_m^*, J_\eta^*+1)$, $TC^*(J_n^*, J_m^*+1, J_\eta^*+1)$ and $TC^*(J_n^*+1, J_m^*+1, J_\eta^*+1)$ from Eq. (6.28) and choose the one with smaller value as the optimal solution.

6.1.1.3 Single purchase single delivery (SPSD, m=1) with inspection errors

The SPMD model reduces to SPSD when we substitute m by 1 in SPMD model as shown in Fig. 6.3. The behavior of the raw material and rework inventory in this model remains the same as the SPMD model, but the whole batch of finished goods manufactured in a production run is delivered immediately after production.

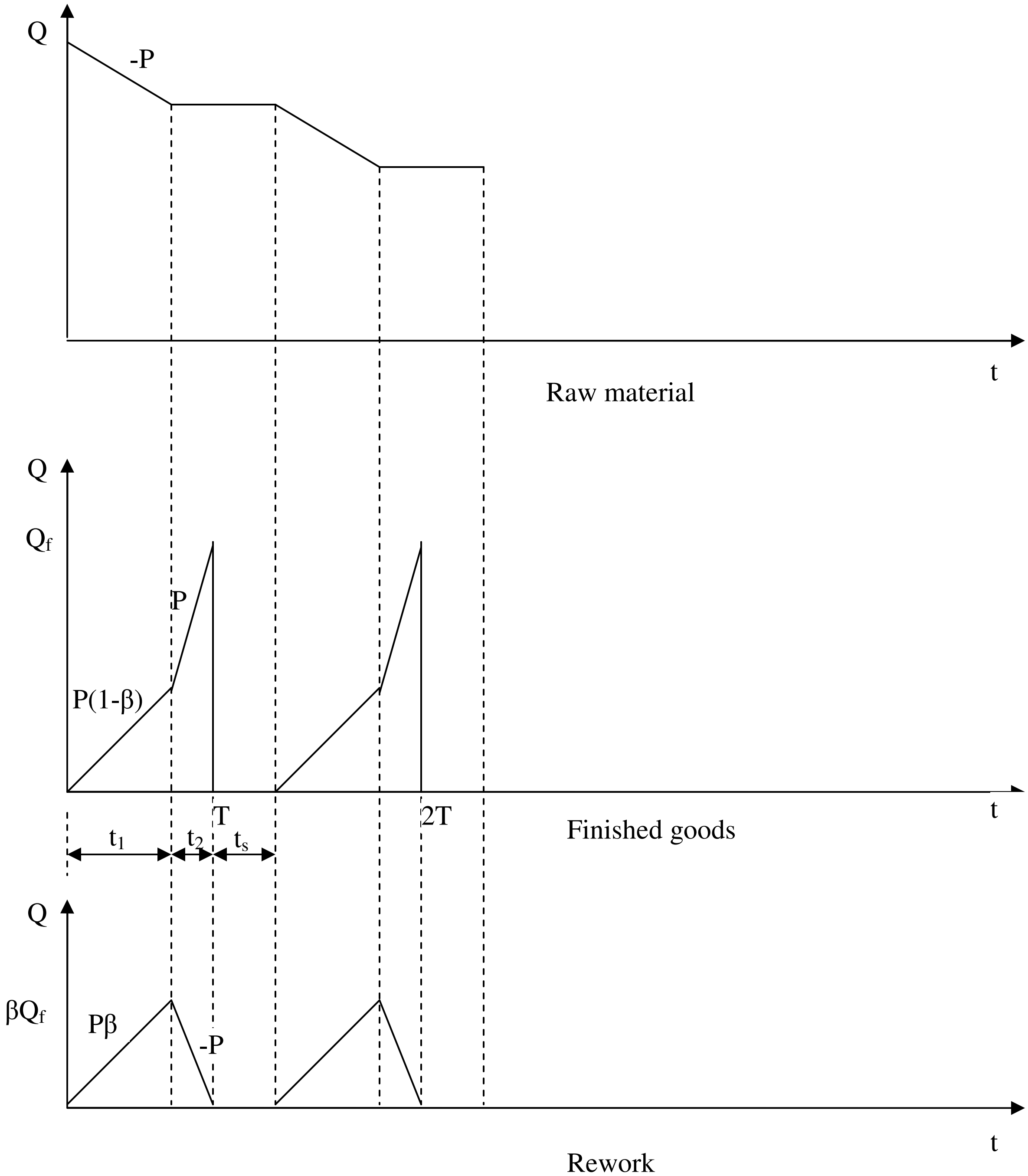

Fig. 6.3 Single purchase single delivery (SPSD)

Considering m=1 and x=Q_f in Eq. (6.18) yields,

$$TC(Q_f, n) = \frac{iCQ_f}{2f_0}\left[\frac{D_f}{P} + n - 1\right] + \frac{D_f}{Q_f}\left(\frac{A_r}{n} + A_f\right) + i(C+V)\frac{Q_f D_f}{2P}[1+\beta(1-\beta)] +$$
$$VD_f(1+\beta) + D_f\left(\pi_1 E_1 + (s - sE_2 - \pi_1 E_1 + \pi_2 E_2)\frac{\alpha Q_f}{2P\theta}\right) + \tag{6.31}$$
$$\frac{i\beta Q_f D_f}{2P}(C + V + \beta C + 2\beta V) + Q_f\left(C_i + \frac{\beta C_l}{P} + V\beta\right)$$

Differentiating Eq. (6.31) with respect to n and equating the resulting the expression to zero, the optimal value of n is obtained as;

$$n^* = \frac{1}{Q_f}\sqrt{\frac{2f_0 D_f A_r}{iC}} \tag{6.32}$$

Differentiating the Eq. (6.31) with respect to Q_f and equating the resulting expression to zero yields;

$$\frac{\partial TC(Q_f, n)}{\partial Q_f} = \frac{iC}{2f_0}\left[\frac{D_f}{P} + n - 1\right] - \frac{D_f}{Q_f^2}\left(\frac{A_r}{n} + A_f\right) + i(C+V)\frac{D_f}{2P}\left[[1+\beta(1-\beta)] + Q_f\frac{d\beta}{dQ_f}(1-2\beta)\right] +$$
$$VD_f\frac{d\beta}{dQ_f} + D_f(s - sE_2 - \pi_1 E_1 + \pi_2 E_2)\frac{\alpha}{2P\theta} + \frac{iD_f\beta}{2P}(C + V + \beta C + 2\beta V) +$$
$$\frac{iQ_f D_f}{2P}\frac{d\beta}{dQ_f}(C + V + 2\beta C + 4\beta V) + \left(C_i + \frac{\beta C_l}{P} + V\beta\right) + Q_f\frac{d\beta}{dQ_f}\left(\frac{C_l}{P} + V\right) = 0$$

(6.33)

where $\frac{d\beta}{dQ_f} = \frac{\alpha}{2P\theta}(1 - E_1 - E_2)$

6.1.1.4 Single purchase single delivery (SPSD, m=1) with process inspection and restoration

Considering m=1 and x=Q_f in Eq. (6.28) for the total cost including inspection and restoration yields,

$$TC(Q_f,n,\eta)=\frac{iCQ_f}{2f_0}\left[\frac{D_f}{P}+n-1\right]+\frac{D_f}{Q_f}\left(\frac{A_r}{n}+A_f\right)+i(C+V)\frac{Q_fD_f}{2P}[1+\beta_i(1-\beta_i)]+$$
$$VD_f(1+\beta_i)+D_f\left(\pi_1E_1+(s-sE_2-\pi_1E_1+\pi_2E_2)\frac{\alpha Q_f}{2\eta P\theta}\right)+\frac{i\beta_iQ_fD_f}{2P}(C+V+\beta_iC+2\beta_iV)+$$
$$Q_f\left(C_i+\frac{\beta_iC_l}{P}+V\beta_i\right)+\frac{D_f}{Q_f}\eta v+\frac{Q_fD_f}{2\eta P^2\theta^2}(r_1\theta-r_0)+\frac{D_fr_0}{P\theta}$$

(6.34)

Differentiating Eq. (6.31) with respect to n and equating the resulting the expression to zero, the optimal value of n is shown in Eq. (6.32).

Differentiating the Eq. (6.28) with respect to Q_f and equating the resulting expression to zero yields;

$$\frac{\partial TC(Q_f,n,\eta)}{\partial Q_f}=\frac{iC}{2f_0}\left[\frac{D_f}{P}+n-1\right]-\frac{D_f}{Q_f^2}\left(\frac{A_r}{n}+A_f\right)+i(C+V)\frac{D_f}{2P}[1+\beta_i(1-\beta_i)]+$$
$$i(C+V)\frac{Q_fD_f}{2P}\left(\frac{\partial\beta_i}{\partial Q_f}-2\beta_i\frac{\partial\beta_i}{\partial Q_f}\right)+VD_f\frac{\partial\beta_i}{\partial Q_f}+D_f(s-sE_2-\pi_1E_1+\pi_2E_2)\frac{\alpha}{2\eta P\theta}+$$
$$\frac{iD_f}{2P}(\beta_iC+\beta_iV+\beta_i^2C+2\beta_i^2V)+\frac{iQ_fD_f}{2P}\left(\frac{\partial\beta_i}{\partial Q_f}C+\frac{\partial\beta_i}{\partial Q_f}V+2\beta_i\frac{\partial\beta_i}{\partial Q_f}C+4\beta_i\frac{\partial\beta_i}{\partial Q_f}V\right)+$$
$$\left(C_i+\frac{\beta_iC_l}{P}+V\beta_i\right)+Q_f\left(\frac{C_l}{P}\frac{\partial\beta_i}{\partial Q_f}+V\frac{\partial\beta_i}{\partial Q_f}\right)-\frac{D_f}{Q_f^2}\eta v+\frac{D_f}{2\eta P^2\theta^2}(r_1\theta-r_0)=0$$

(6.35)

where $\frac{\partial\beta_i}{\partial Q_f}=\frac{\alpha}{2\eta P\theta}(1-E_1-E_2)$

Differentiating the Eq. (6.28) with respect to η and equating the resulting expression to zero yields;

$$\frac{\partial TC(Q_f,n,\eta)}{\partial\eta}=i(C+V)\frac{Q_fD_f}{2P}\left(\frac{\partial\beta_i}{\partial\eta}-2\beta_i\frac{\partial\beta_i}{\partial\eta}\right)+VD_f\frac{\partial\beta_i}{\partial\eta}-D_f(s-sE_2-\pi_1E_1+\pi_2E_2)\frac{\alpha Q_f}{2\eta^2P\theta}+$$
$$\frac{iQ_fD_f}{2P}\left[\frac{\partial\beta_i}{\partial\eta}(C+V)+2\beta_i\frac{\partial\beta_i}{\partial\eta}(C+2V)\right]+Q_f\frac{\partial\beta_i}{\partial\eta}\left(\frac{C_l}{P}+V\right)+\frac{D_f}{Q_f}v-\frac{Q_fD_f}{2\eta^2P^2\theta^2}(r_1\theta-r_0)=0$$

(6.36)

where $\frac{\partial\beta_i}{\partial\eta}=-\frac{\alpha Q_f}{2\eta^2P\theta}(1-E_1-E_2)$

6.1.1.5 Lot-for-lot (LFL, m=1,n=1) with inspection errors

The SPMD model reduces to LFL when both *m* and *n* are substituted by 1 in the SPMD model as shown in Fig. 6.4. In this policy, raw material ordering quantity is that required for one lot instead of multiple batches as in SPMD, and the whole production batch is delivered immediately after production. The rework inventory is similar to SPMD. Therefore we have m=1 ($x=Q_f$) and n=1.

Substituting $x=Q_f$, m=1, n=1 in Eq. (6.18), we have

$$TC(Q_f) = \frac{iCQ_f D_f}{2f_0 P} + \frac{D_f}{Q_f}(A_r + A_f) + i(C+V)\frac{Q_f D_f}{2P}[1+\beta(1-\beta)] + VD_f(1+\beta) + D_f\left(\pi_1 E_1 + (s - sE_2 - \pi_1 E_1 + \pi_2 E_2)\frac{\alpha Q_f}{2P\theta}\right) + \frac{i\beta Q_f D_f}{2P}(C + V + \beta C + 2\beta V) + Q_f\left(C_i + \frac{\beta C_l}{P} + V\beta\right) \tag{6.37}$$

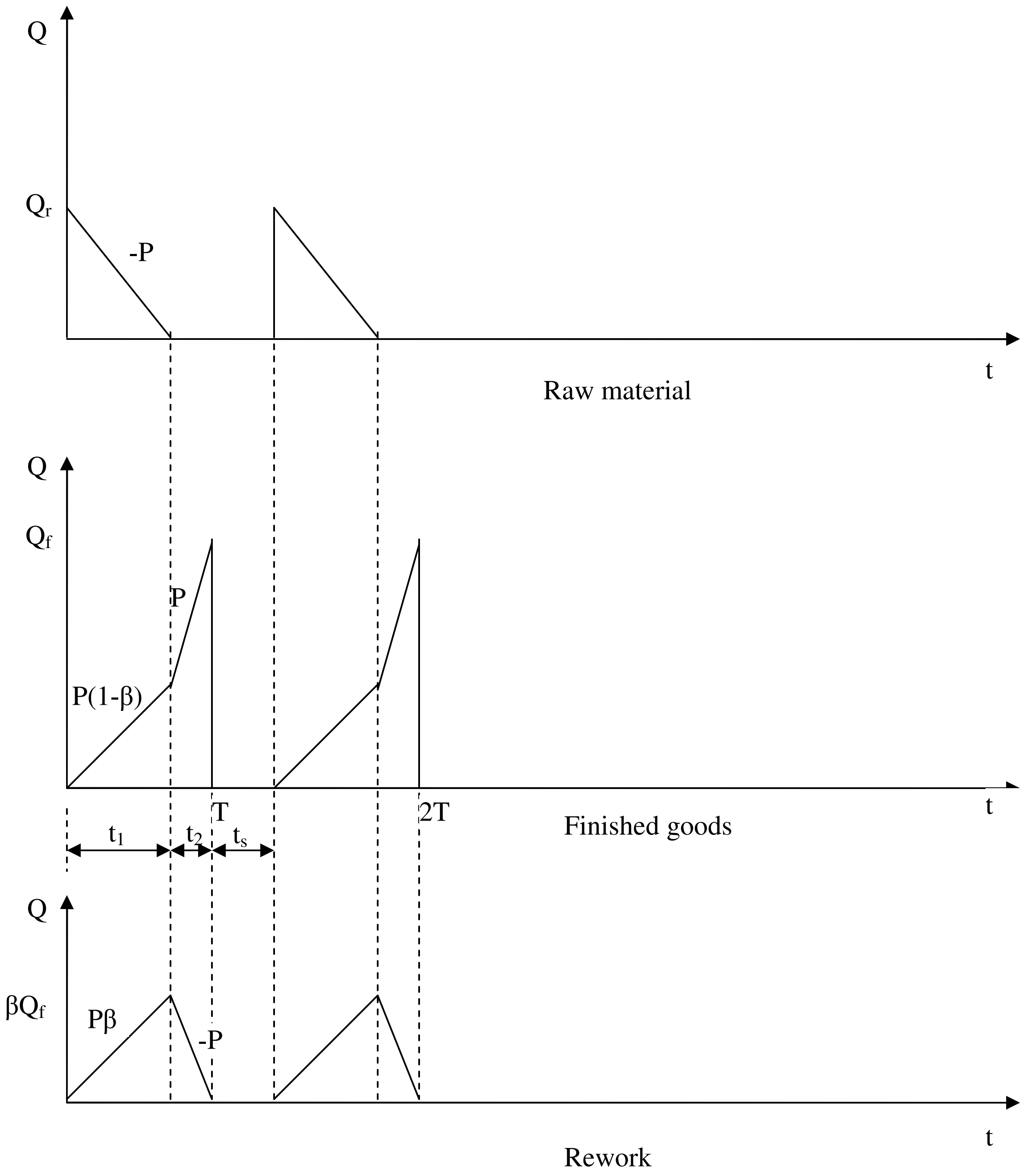

Fig. 6.4 Lot-for-Lot (LFL)

Differentiating the Eq. (6.37) with respect to Q_f and equating the resulting expression to zero yields;

$$\frac{\partial TC(Q_f)}{\partial Q_f}=\frac{iCD_f}{2f_0P}-\frac{D_f}{Q_f^2}\left(A_r+A_f\right)+i(C+V)\frac{D_f}{2P}\left[\left[1+\beta(1-\beta)\right]+Q_f\,\frac{d\beta}{dQ_f}(1-2\beta)\right]+$$
$$VD_f\,\frac{d\beta}{dQ_f}+D_f\left(s-sE_2-\pi_1E_1+\pi_2E_2\right)\frac{\alpha}{2P\theta}+\frac{iD_f\beta}{2P}\left(C+V+\beta C+2\beta V\right)+$$
$$\frac{iQ_fD_f}{2P}\frac{d\beta}{dQ_f}\left(C+V+2\beta C+4\beta V\right)+\left(C_i+\frac{\beta C_l}{P}+V\beta\right)+Q_f\,\frac{d\beta}{dQ_f}\left(\frac{C_l}{P}+V\right)=0$$

(6.38)

6.1.1.6 Lot-for-lot (LFL, m=1,n=1) with process inspection and restoration

Considering m=1, n=1 and x=Q_f in Eq. (6.28) for the total cost including inspection and restoration yields,

$$TC(Q_f,\eta)=\frac{iCQ_fD_f}{2f_0P}+\frac{D_f}{Q_f}\left(A_r+A_f\right)+i(C+V)\frac{Q_fD_f}{2P}\left[1+\beta_i(1-\beta_i)\right]+$$
$$VD_f\left(1+\beta_i\right)+D_f\left(\pi_1E_1+\left(s-sE_2-\pi_1E_1+\pi_2E_2\right)\frac{\alpha Q_f}{2\eta P\theta}\right)+\frac{i\beta_iQ_fD_f}{2P}\left(C+V+\beta_iC+2\beta_iV\right)+$$
$$Q_f\left(C_i+\frac{\beta_iC_l}{P}+V\beta_i\right)+\frac{D_f}{Q_f}\eta v+\frac{Q_fD_f}{2\eta P^2\theta^2}\left(r_1\theta-r_0\right)+\frac{D_fr_0}{P\theta}$$

(6.39)

Differentiating the Eq. (6.39) with respect to Q_f and equating the resulting expression to zero yields;

$$\frac{\partial TC(Q_f,\eta)}{\partial Q_f}=\frac{iCD_f}{2f_0P}-\frac{D_f}{Q_f^2}\left(A_r+A_f\right)+i(C+V)\frac{D_f}{2P}\left[1+\beta_i(1-\beta_i)\right]+$$
$$i(C+V)\frac{Q_fD_f}{2P}\left(\frac{\partial\beta_i}{\partial Q_f}-2\beta_i\,\frac{\partial\beta_i}{\partial Q_f}\right)+VD_f\,\frac{\partial\beta_i}{\partial Q_f}+D_f\left(s-sE_2-\pi_1E_1+\pi_2E_2\right)\frac{\alpha}{2\eta P\theta}+$$
$$\frac{iD_f}{2P}\left(\beta_iC+\beta_iV+\beta_i^{\,2}C+2\beta_i^{\,2}V\right)+\frac{iQ_fD_f}{2P}\left(\frac{\partial\beta_i}{\partial Q_f}C+\frac{\partial\beta_i}{\partial Q_f}V+2\beta_i\,\frac{\partial\beta_i}{\partial Q_f}C+4\beta_i\,\frac{\partial\beta_i}{\partial Q_f}V\right)+$$
$$\left(C_i+\frac{\beta_iC_l}{P}+V\beta_i\right)+Q_f\left(\frac{C_l}{P}\frac{\partial\beta_i}{\partial Q_f}+V\,\frac{\partial\beta_i}{\partial Q_f}\right)-\frac{D_f}{Q_f^2}\eta v+\frac{D_f}{2\eta P^2\theta^2}\left(r_1\theta-r_0\right)=0$$

(6.40)

Differentiating the Eq. (6.39) with respect to η and equating the resulting expression to zero results in Eq. (6.36).

6.1.2 Numerical computation

Consider a manufacturing system with the following data (Ojha et al., 2007 and Ben-Daya and Rahim, 2003): E_1=0.1, and E_2=0.1, A_f=\$50 per setup, A_r=\$150 per order, C=\$2 per unit, C_i=\$0.2 per unit, C_l=\$0.1 per unit time, D= 2000 units per year, f_0=0.95, i=0.18, P=3000 units per year, V=\$1 per unit, $\nu=0.05A_f$, $\alpha=0.05$, $\theta=0.05$, s=V, $\pi_1=0.5V$, $\pi_2=1.5V$, $r_0=0.15A_f$, $r_1=0.01A_f$.

Table 6.1 shows the optimal batch size and expected total cost for (i) imperfect production process with quality and rework, and the optimal number inspections, batch size and total cost for (ii) imperfect production process with quality and rework including process inspection and restoration for the following: (a) SPMD policy when the delivery size of finished product is x=20 units, (b) the SPSD policy, and (c) the LFL policy. The optimal batch size, ordering quantity and total cost increases for SPMD, SPSD and LFL policies by incorporating inspection and restoration of the process in the imperfect production process with quality and rework. The optimal number of inspections increases with increase in the optimal batch size and ordering quantity. The optimal expected total cost also reduces for SPMD and LFL policies for the given data by including process inspection and restoration.

Table 6.1 Optimal batch size and expected total cost

	Imperfect production process with quality and rework			Imperfect production process with quality and rework including process inspection and restoration		
	SPMD	SPSD	LFL	SPMD	SPSD	LFL
N	5	4	-	4	4	-
M	13	-	-	15	-	-
H	-	-	-	3	3	5
Q_f	260	293	528	300	335	571
Q_r	274	309	555	316	352	601
Expected Total Cost (ETC)	3512.78	3443.67	3765.00	3496.45	3496.30	3707.50

The optimal number of inspections, batch quantity and cost and are studied by varying the parameters such as the (i) fraction of nonconforming units (α), (ii) value added by the manufacturing process (V), (iii) type I inspection error (E_1), and (iv) type II inspection error (E_2). Table 6.2 and 6.3 show the influence of fraction of non-conforming units (α) on the optimal ordering quantity and optimal cost. It can be seen that as the fraction of non-conforming units increases, the optimal ordering quantity decreases, while the optimal expected total cost increases for SPMD, SPSD, and LFL policies. Incorporating process inspection and restoration increases optimal ordering quantity and reduces the optimal expected total cost for SPMD, SPSD, and LFL policies. This is owing to reduction in the expected number of non-conforming items produced incorporating process inspection and restoration. With increase in the fraction of non-conforming units (α) optimal number of inspections increase, while the optimal ordering quantity decreases and optimal expected total cost increases for SPMD, SPSD, and LFL policies. For the data considered the optimal number of inspections are lower for SPMD and SPSD policies, while optimal number of inspections are higher for LFL policy. For all the values of fraction of non-conforming units, the optimal ordering quantity is smaller for SPMD and SPSD, and larger for LFL.

Table 6.2 Effect of fraction of non-conforming units (α) with quality and rework

	Imperfect production process with quality and rework					
	SPMD		SPSD		LFL	
α	Q_r	ETC	Q_r	ETC	Q_r	ETC
0.05	274	3512.78	309	3443.67	555	3765.00
0.10	232	3667.44	247	3612.43	453	4090.68
0.25	168	4015.78	172	3977.98	324	4808.27
0.40	147	4287.87	140	4254.05	267	5352.84
0.50	126	4438.64	127	4411.56	243	5665.05

Table 6.3 Effect of fraction of non-conforming units (α) with quality and rework including inspection and restoration

	Imperfect production process with quality and rework including process inspection and restoration								
	SPMD			SPSD			LFL		
α	η	Q_r	ETC	η	Q_r	ETC	η	Q_r	ETC
0.05	3	316	3496.45	3	352	3496.30	5	601	3707.50
0.10	4	274	3581.50	4	279	3584.05	7	491	3826.40
0.25	5	189	3796.07	5	194	3791.10	9	352	4145.76
0.40	5	168	3918.75	5	158	3958.05	9	290	4417.38
0.50	5	147	4031.10	5	143	4055.59	9	263	4579.70

Table 6.4 and 6.5 show the impact of value added by the manufacturing process (V) on the SPMD, SPSD and LFL policies. The more the value added by the manufacturing process V, the higher will be the expected optimal expected total cost. With increase in the value added by the manufacturing process, the optimal batch size decreases while the expected total cost increases for the SPMD, SPSD and LFL policies. The optimal number of inspections increases with the increase in the value added by the manufacturing process for all the SPMD, SPSD and LFL policies. The optimal number of inspections are higher LFL policy while lower for SPMD and SPSD policies.

Table 6.4 Effect of value added by the manufacturing process (V) with quality and rework

	Imperfect production process with quality and rework					
	SPMD		SPSD		LFL	
V	Q_r	ETC	Q_r	ETC	Q_r	ETC
1.0	274	3512.78	309	3443.67	555	3765.00
4.0	168	10933.19	176	10857.67	334	11654.60
6.0	147	15784.22	146	15697.78	281	16728.37
8.0	126	20593.97	128	20503.56	248	21733.70

Table 6.5 Effect of value added by the manufacturing process (V) with quality and rework including inspection and restoration

	Imperfect production process with quality and rework including process inspection and restoration								
	SPMD			SPSD			LFL		
V	η	Q_r	ETC	η	Q_r	ETC	η	Q_r	ETC
1.0	3	316	3496.45	3	352	3496.30	5	601	3707.50
4.0	4	189	10801.96	4	196	10803.96	8	359	11266.94
6.0	4	168	15575.60	4	163	15599.50	8	301	16201.63
8.0	4	147	20337.76	4	142	20367.77	8	264	21092.79

Table 6.6 and 6.7 show the influence of type I inspection errors. As the inspection errors increase, the optimal ordering quantity decreases while the optimal expected total cost increases. Incorporating process inspection and rework increases the optimal ordering quantity for all the SPMD, SPSD and LFL policies. The optimal number of inspections decreases with increase in the type I inspection errors due the reduction in the ordering quantity.

Table 6.6 Effect of type I inspection error (E_1) with quality and rework

	Imperfect production process with quality and rework					
	SPMD		SPSD		LFL	
E_1	Q_r	ETC	Q_r	ETC	Q_r	ETC
0.10	274	3512.78	309	3443.67	555	3765.00
0.20	274	3834.16	300	3768.47	544	4104.95
0.50	253	4797.74	275	4735.28	512	5123.68

Table 6.7 Effect of type I inspection error (E_1) with quality and rework including inspection and restoration

	Imperfect production process with quality and rework including process inspection and restoration								
	SPMD			SPSD			LFL		
E_1	η	Q_r	ETC	η	Q_r	ETC	η	Q_r	ETC
0.10	3	316	3496.45	3	352	3496.30	5	601	3707.50
0.20	3	316	3827.85	3	340	3830.83	5	585	4077.41
0.50	2	274	4806.39	2	313	4816.46	3	545	5172.13

Table 6.8 and 6.9 show the influence of type II inspection errors. As the type II inspection errors increase, the optimal ordering quantity increases while the optimal expected total cost reduces.

Incorporating process inspection and rework increases the optimal ordering quantity for all the SPMD, SPSD and LFL policies.

Table 6.8 Effect of type II inspection error (E_2) with quality and rework

	Imperfect production process with quality and rework					
	SPMD		SPSD		LFL	
E_2	Q_r	ETC	Q_r	ETC	Q_r	ETC
0.10	274	3512.78	309	3443.67	555	3765.00
0.20	274	3506.90	313	3436.37	563	3749.65
0.50	274	3476.45	326	3414.17	589	3701.72

Table 6.9 Effect of type II inspection error (E_2) with quality and rework including inspection and restoration

	Imperfect production process with quality and rework including process inspection and restoration								
	SPMD			SPSD			LFL		
E_2	η	Q_r	ETC	η	Q_r	ETC	η	Q_r	ETC
0.10	3	316	3496.45	3	352	3496.30	5	601	3707.50
0.20	3	337	3487.13	3	357	3492.18	5	608	3701.02
0.50	3	337	3473.36	3	373	3479.24	5	633	3680.11

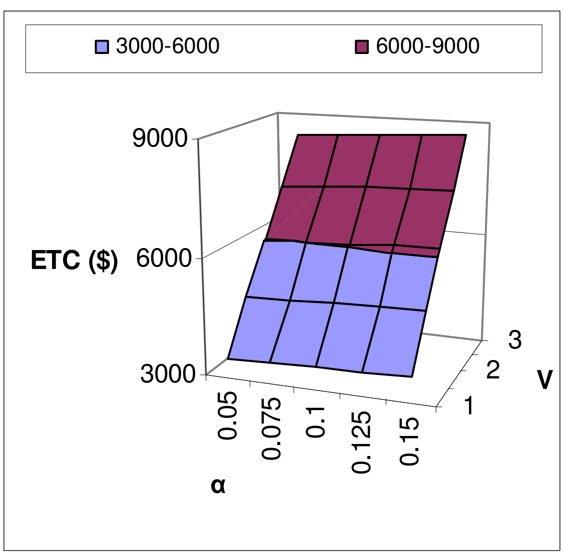

Fig. 6.5 Variation of ETC with α and V

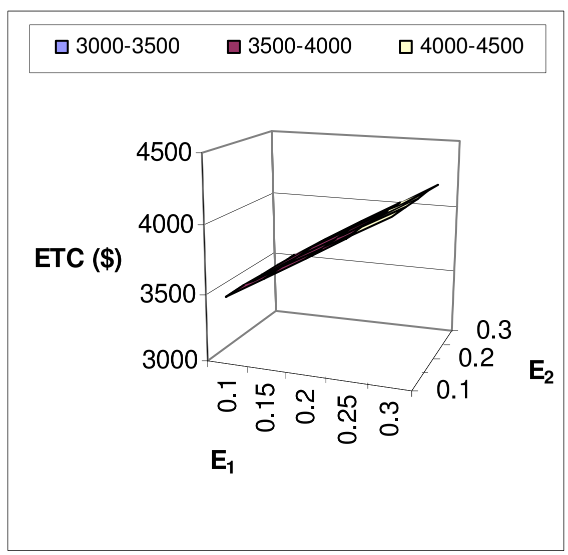

Fig. 6.6 Variation of ETC with E_1 and E_2

Figures 6.5 and 6.6 show the variation of expected total cost (ETC) for single purchase multiple delivery (SPMD) policy under quality and rework including inspection and restoration. Figure 6.5 shows the increase in expected total cost (ETC) with both fraction of nonconforming units (α) and value added by the manufacturing process (V) respectively. The increase in ETC with V is much higher compared to the increase in ETC with α. Figure 6.6 variation of (ETC) with both E_1 and E_2 respectively. For given value of E_2, ETC increases with increase in E_1, while for a given value of E_1, ETC decreases with increase in E_2.

6.2 Optimal Batch Size in a Single-Stage Imperfect Production-Delivery System with Rework

Sarker and Parija (1996) developed an ordering policy for raw materials, to meet the demands of a production facility which supplies a fixed quantity of finished products to outside buyers, at a fixed interval of time. They derived the cost function which consists of raw material and finished goods inventory. Their model assumes that a production facility purchases raw materials from outside suppliers and processes them to deliver a fixed quantity of finished products to a buyer at a fixed interval of time. A multi-order policy for procuring raw materials may lower inventory carrying cost because this policy results in a lower inventory carrying cost. Ideally, a manufacturer is expected both to synchronize production with the buyer's lumpy demand and to coordinate the ordering of raw materials with production schedules so that both raw materials and finished goods inventory are reduced. Sarker et al., (2008) developed models for the optimum batch quantity in a multi-stage system with rework process for two different operational policies: (i) the rework within the same cycle with no shortage and (ii) the rework done after N cycles, incurring shortages in each cycle. In the policy (i), an economic batch quantity model for defective items that are reworked within the same cycle is developed to avoid penalty cost incurred due to shortage. In the policy (ii), the defective items from each cycle are accumulated for a certain number of cycles before they are reworked. Some amount of shortage occurs in each cycle due to the production of defectives and hence a penalty cost is imposed for not correctly meeting the demand of good finished products after each cycle.

In this research, Sarker and Parija's (1996) model is extended by considering the reworking of defects in the same cycle and the total cost function is derived. The total cost function is minimized to determine the optimal batch size and production run length for a single-stage production system.

The remainder of the section is organized as follows. The assumptions and necessary notation are described. Mathematical model and numerical results for single-stage production-delivery system with rework are provided.

The notations used are defined as follows:

C_f	rework processing cost, $
D_f	demand of finished goods, units/year
D_r	demand of the raw materials, units/year
f	indicates a ratio of the quantity (or demand) of finished goods to the quantity (or demand) of raw materials required to produce that quantity of finished goods ($f=D_F/D_r = Q_f/Q_r$)
H_f	finished goods holding cost, $/unit-year
H_o	raw materials holding carrying cost, $/unit-year
$I(t)$	inventory level, units
K_o	ordering cost of the raw materials, $/order
K_s	manufacturing setup cost for each batch, $/batch
L_f	time between successive shipments of finished goods
m_f	number of full shipments of finished goods per cycle
n_o	number of orders for raw materials during uptime T_1 ($n_o=D_r/Q_o$)
P_f	production rate, units/year
Q_{avg}	average inventory of finished goods per cycle, units
Q_f	quantity of finished goods manufactured per setup, units/batch
Q_o	quantity of raw materials ordered each time, ($Q_o = Q_r/n$)
Q_r	quantity of raw materials required for each batch, ($Q_r = Q_f/f=nQ_o$)
Q_s	quantity of finished goods shipped
T	cycle time, ($T=Q_f/D_f=m_fL$)
T_1	quantity of goods manufactured during the production uptime ($T_1=Q_f/P_f=m_fL$)
x_f	fixed quantity of finished goods per shipment at a fixed interval of time, units/ shipment, ($x_f = Q_f/m=LD_f$)

β_f proportion of defectives produced in the stage

The assumptions in the model are

1. The model assumes that a production facility purchases raw materials from outside suppliers and processes them to deliver a fixed quantity of finished products to a buyer at a fixed interval of time.
2. The annual demand of this buyer is known and fixed.
3. The production rate of the facility is assumed to be greater than the demand rate so as to ensure no shortage of products due to insufficient production.
4. The raw material is nonperishable, and it is supplied instantaneously to the manufacturing facility.
5. Proportion of defective is constant in each cycle.
6. No product is scraped at any cycle.
7. No defectives are produced during the rework.
8. Inspection cost is ignored.
9. Cost of processing of jobs on the machine is ignored.

6.2.1 Model formulation for single-stage production-delivery system with rework

This model incorporates the rework in the production-delivery model considered by Sarker and Parija (2006). The behavior of raw material ordering and finished goods demand in this problem is different from the behavior in a traditional economic batch quantity model with continuous demand. A multi-order policy for procuring raw materials and fixed quantity of finished goods demand at regular intervals are considered. The total inventory costs of raw materials and finished goods in the production-delivery system are minimized to determine an (1) optimal raw materials order quantity, Q_o, procured during the uptime, T_1, (2) optimal finished goods quantity, Q_f, so that production with single setup meets the demand for that cycle, and (3) the rework of a defective item is done in the same cycle.

Raw materials are procured and converted to finished goods during the productive time, T_1. The manufacturer has a yearly demand of D_r units of raw materials in order to meet a yearly demand of D_f units of finished goods by buyers. The amount of raw materials ordered during any cycle should meet the requirements for production within that cycle. Raw materials must not be carried during the downtime period, (T_1, T), while finished goods may build up during the uptime period and deplete during the downtime period until reaching zero at the end of the cycle time, T.

Fig. 6.7 shows the raw material ordering quantity during uptime, T_1. The cost of raw material inventory is given by (Sarker and Parija, 1996)

$$TC_r = \frac{D_r K_0}{Q_o} + \frac{Q_o T_1 H_0}{2T} = \frac{D_r n_0 K_0}{Q_r} + \frac{Q_r D_f H_0}{2n_0 P_f} = \frac{D_f n_0 K_0}{Q_f} + \left(\frac{Q_r}{Q_f}\right)\frac{Q_f D_f H_0}{2n_0 P_f} = \frac{D_f n_0 K_0}{m_f x_f} + \frac{m_f x_f D_f H_0}{2 f n_0 P_f} \quad (6.41)$$

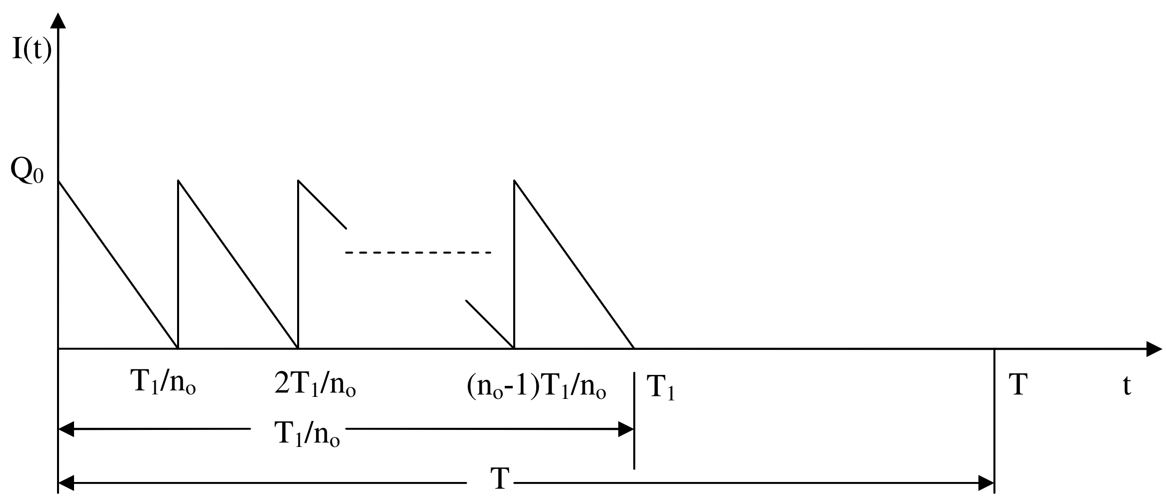

Fig. 6.7 Raw material inventory level in a single-stage production system

Fig. 6.8 shows the finished goods inventory. The derivation of average finished goods inventory is given in Appendix D.1.

$$TC_f = \frac{D_f K_s}{Q_f} + Q_{avg} H_f = \frac{D_f K_s}{m_f x_f} + \frac{1}{2}\left\{ m_f x_f \left(1 - \frac{D_f}{P_f}\right) + x_f \right\} H_f \quad (6.42)$$

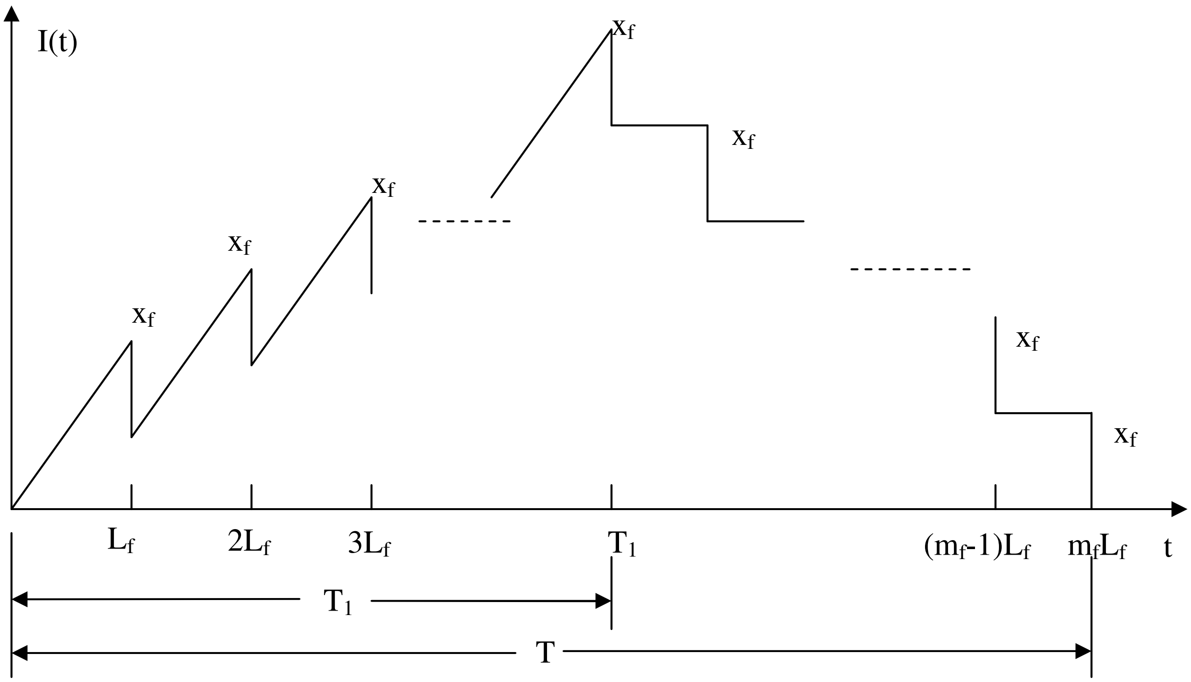

Fig. 6.8 Finished goods inventory level in a single-stage production system

The total cost is obtained as (Sarker and Parija,1996)

$$TC(m_f, n_0) = \frac{D_f(n_0K_0 + K_s)}{m_f x_f} + \frac{m_f x_f D_f H_0}{2fn_0P_f} + \frac{1}{2}\left\{m_f x_f\left(1 - \frac{D_f}{P_f}\right) + x_f\right\}H_f \qquad (6.43)$$

Defective items be produced in the stage are reworked within the same cycle at the same stage. Costs are involved in rework of defective products. When defective items are processed in a stage, the inventory of reworked items builds up, incurring carrying costs as well. The rework processing cost per cycle for $\beta_f Q_f$ units of defective items reworked in the stage can be written as $Q_f \beta_f C_f$. Hence reprocessing cost over the planning period is $Q_f(D_f/Q_f)\beta_f C_f = D_f \beta_f C_f$

The total cost considering the reworking of defects in the same cycle is obtained as

$$TC = \frac{D_r K_o}{Q_o} + I_{avg} H_o + \frac{D_f K_s}{Q_f} + D_f \beta_f C_f + Q_{avg} H_f \qquad (6.44)$$

Substituting $I_{avg} = \frac{Q_o T_1}{2T}$, $\frac{T_1}{T} = \frac{D_f}{P_f}$, $Q_o = \frac{Q_r}{n_o}$, $Q_f = m_f x_f$, $f = \frac{Q_f}{Q_r} = \frac{m_f x_f}{Q_r}$,

$\frac{D_r}{Q_r} = \frac{D_f}{Q_f} = \frac{D_f}{m_f x_f}$ and $Q_{avg} = \frac{1}{2}\left\{ m_f x_f \left(1 - \frac{D_f \left(1 + \beta_f - \beta_f^2\right)}{P_f} \right) + x_f \right\}$

The derivation of average finished goods inventory with rework is given in Appendix D.2.

The expression for the total cost is obtained as

$$TC(m_f, n_0) = \frac{D_f (n_0 K_0 + K_s)}{m_f x_f} + \frac{m_f x_f D_f H_0}{2 f n_0 P_f} + D_f \beta_f C_f + \frac{1}{2}\left\{ m_f x_f \left(1 - \frac{D_f \left(1 + \beta_f - \beta_f^2\right)}{P_f} \right) + x_f \right\} H_f \qquad (6.45)$$

The total cost function is convex as shown in Appendix D.3.

The optimal number orders of raw materials and the optimal number of shipments are

$$n_0^* = m_f x_f \sqrt{\frac{H_0}{2 P_f f K_0}} \qquad (6.46)$$

$$m_f^* = \frac{1}{x_f} \sqrt{\frac{2 D_f K_s}{\left(1 - \frac{D_f}{P_f}\left(1 + \beta_f - \beta_f^2\right) \right) H_f}} \qquad (6.47)$$

If $n_o > 1$ and $m_f > 1$ then choose $j < n_o < j+1$ and $j < m_f < j+1$ where j is a positive integer.

6.2.2 Computational results

Assume that the proportion of defective items β_f in the stage is 0.01 and the rework processing cost C_f in the stage is \$10.0. Table 6.10 provides the input data for a single-stage production system (Sarker and Parija, 2006). The comparison of the optimal results and modified results for a single-stage production system with rework consideration are shown in Tables 6.11 and 6.12 respectively. The optimal results for m_f, n_0, T_1, T, TC show a marginal increase with rework consideration in the single-stage production system. The modified results for m_f, n_0, T_1, T with and without rework consideration in the single-stage production system are similar except in few cases where they show marginal increase with rework consideration. For both the optimal and modified results, the total cost for rework in the system is high as the rework processing cost is also considered.

Table 6.10 Input data for a single-stage production system (*β_f=0.01, C_f=10*)

Problem	D_f	P_f	K_0	K_s	H_0	H_f	x_f	F
1	1200	2400	50	150	2	3	50	0.8
2	1500	1800	100	200	4	8	75	0.6
3	2400	3600	100	300	1	2	100	0.5
4	2400	6000	100	200	5	10	100	0.5
5	3600	7200	300	400	2	2	50	0.8
6	2000	2500	100	200	10	15	100	1.5
7	1000	1500	400	300	2	10	50	0.2
8	1200	2400	50	200	2	3	50	1.0
9	2500	3000	100	50	5	15	100	0.5
10	4000	5000	50	200	1	10	100	1.0

Table 6.11 Comparison of optimum results for a single-stage production system

S. No.	Sarker and Parija (1996)					Present work (with rework consideration)				
	m_f	n_0	T_1	T	TC	m_f	n_0	T_1	T	TC
1	9.79	1.58	0.204	0.408	1197	9.85	1.58	0.205	0.410	1314
2	8.94	2.89	0.373	0.447	2485	9.17	2.96	0.382	0.459	2613
3	14.70	2.45	0.408	0.612	1879	14.84	2.47	0.412	0.619	2110
4	4.00	1.15	0.066	0.166	4285	4.01	1.16	0.067	0.167	4518
5	33.90	1.29	0.235	0.471	3390	34.11	1.30	0.237	0.474	3742
6	5.16	1.88	0.206	0.258	3760	5.27	1.92	0.211	0.264	3929
7	8.48	1.22	0.283	0.424	3973	8.57	1.28	0.286	0.429	4060
8	11.30	1.63	0.235	0.471	1269	11.37	1.64	0.237	0.474	1388
9	3.16	1.29	0.105	0.126	3581	3.24	1.32	0.108	0.130	3812
10	8.94	1.26	0.179	0.223	2854	9.13	1.29	0.183	0.228	3219

Table 6.12 Comparison of modified results for a single-stage production system

S. No.	Sarker and Parija (1996)					Present work (with rework consideration)				
	m_f	n_0	T_1	T	TC	m_f	n_0	T_1	T	TC
1	10	2	0.208	0.417	1206	10	2	0.208	0.417	1323
2	9	3	0.375	0.450	2486	9	3	0.375	0.450	2614
3	14	2	0.388	0.583	1890	14	2	0.389	0.583	2121
4	4	1	0.067	0.167	4300	4	1	0.067	0.167	4532
5	33	1	0.229	0.458	3433	34	1	0.236	0.472	3796
6	5	2	0.200	0.250	3766	5	2	0.200	0.250	3937
7	8	1	0.267	0.400	4000	8	1	0.267	0.400	4087
8	12	2	0.250	0.500	1275	12	2	0.250	0.500	1391
9	3	1	0.100	0.120	3625	3	1	0.100	0.120	3856
10	8	1	0.160	0.200	2870	9	1	0.180	0.225	3236

6.3 Optimal Number of Kanbans in a Multi-Stage JIT Production-Delivery System with Rework Consideration

Wang and Sarker (2006) studied a multi-stage supply chain system linked by kanban mechanism that operates under a just-in-time delivery policy. They derived a cost function which consists of the cost of raw materials at the first stage, the cost of WIP in the intermediate stages, and the cost of finished goods at the last stage. The deliveries of raw materials from the suppliers, the work-in-process (WIP) in production stage, and the transshipments of finished goods to retailers are all controlled by the kanbans. The kanban is a practical tool for implementation of JIT delivery in supply chain operations. Kanbans are used as a means for production control and process improvement. In production control, kanbans tie different manufacturing processes and ensure that the delivery of necessary amounts of material and parts at the appropriate time and place. In process improvement, kanbans improves the operations in the production process with emphasis on reducing inventory costs. In a multi-stage production system, products move from one stage to the next stage, and every stage may yield a certain proportion of defective items. The non-reworked items become waste, creating additional costs for producers. Sarker et al., (2008) developed models for an optimal batch quantity for a manufacturing system that allows rework of defective items under two operational policies—reworking defectives within the same cycle and after N cycles. Their results show that the optimal quantity increases with defects in both policies.

In this research, Wang and Sarker's (2006) model is extended by considering the rework processing cost of defects per cycle for all stages and the total cost function is derived with an objective to determine the optimal batch quantity and number of kanbans in a multi-stage production system. The suppliers provide the manufacturers with the raw materials that are processed to manufacture end products that are finally shipped to retailers. The deliveries of raw materials from the suppliers, the work-in-process (WIP) in production stage, and the shipments of finished goods to retailers are all controlled by kanban mechanism.

The remainder of the section is organized as follows. The assumptions and notation are provided. Mathematical model formulation for multi-stage production-delivery system with rework controlled by kanban mechanism is described. Results for a multi-stage production-delivery controlled by kanban mechanism with rework consideration are described.

The notation used in this section are as follows

D	demand rate, units/year
f	ratio of the quantity of finished goods to the quantity of raw materials
H_o	holding cost of raw material inventory, \$/unit/year
H_i	holding cost of work-in-process inventory at stage i, \$/unit/year
i	an index of a production stage, $i=1,2,\ldots\ldots N+1$
$I(t)$	inventory level, units
K_o	setup (ordering) cost, \$/setup (order)
K_{is}	setup (manufacturing) cost at stage i, \$/ batch
K_{io}	setup (shipping) cost at stage i, \$/setup (ship)
m_i	number of shipments (kanbans) at stage i
m_{N+1}	number of shipments of finished goods at the last production system
n_o	number of orders of raw material inventory placed
P_1	production rate of the first production system, units/year
P_i	production rate of the ith production system, units/year
P_{N+1}	production rate of the last (N+1th) production system, units/year
Q_f	quantity of finished goods produced over a period, units/year
Q_0	quantity of raw material ordered each time, $Q_0=Q_r / n_0$

Q_{io} quantity of inventory per shipment at a fixed interval of time, $Q_{io}=Q_i/m_i$

T cycle time, year

T_{ui} uptime of the i^{th} production system, year

TC_f cost of finished goods inventory, \$/year

TC_r cost of raw material inventory, \$/year

TC_{wi} cost of work-in-process inventory at i^{th} production system, \$/year

TC total cost of multi-stage production system, \$/year

β_i proportion of defectives in stage i

The assumptions made in the analysis are

(1) The demands of the system are known

(2) The production rate of each stage is known and it is larger than the demand rate

(3) The total quantity of the products at each stage over a period are constant

(4) Shortage is not allowed at any stage

(5) The demand of raw material inventory for the products at the first stage is p_1, the production rate of stage 1. The orders arrive in lots on time when the orders are placed. Shortage is not allowed. So the input rate (replenishment) is considered infinite. The company orders raw materials in batches, i.e., EOQ is divided into a number of equal batches, n_o. When the production starts, the shipment (one batch) is set at a fixed interval during one period.

(6) The production at stage i is carried at a rate of p_i units/year. The parts produced by this stage are work-in-process inventories built-up before they are shipped. As the stock level reaches the lot size Q_{wi}, the parts are carried by containers from stage i to i+1. The semi-finished parts shipped to stage i from the preceding stage are input to this stage.

(7) The throughput of the plant N+1 is the finished goods of the N-stage production system. The total stock of the finished goods increases at a rate p_{N+1}.

(8) In a multi-stage system, defective items may be produced in each stage, and they are reworked within the same cycle at the same stage. Proportion of defective for a particular stage remains the same in each cycle for the whole planning period but may be different from that in other stages..

(9) No product is scraped at any cycle.

(10) No defectives are produced during the rework.

(11) Inspection cost is ignored.

(12) Cost of processing of jobs on the machine is ignored.

6.3.1 Model formulation for multi-stage production-delivery system with rework controlled by kanban mechanism

Defective units that are to be reworked at every stage are $\beta_i Q_f$ units. For all n stages the rework processing cost per cycle can be written as $Q_f \sum_{i=1}^{N+1} \beta_i C_i$. Hence rework processing cost over the planning period is $Q_f \sum_{i=1}^{N+1} (D/Q_f)\beta_i C_i = D\sum_{i=1}^{N+1} \beta_i C_i$

The batch size and the number of batches in each stage is related to the production batch size Q_f. The total quantity of products produced in each stage over a period T, Q_f, is assumed to be the same, i.e., $Q_f = m_i Q_{io}$.

6.3.1.1 Cost of raw material inventory

The raw material inventory arrives in lots on time when the order is placed. The raw materials are ordered in equal number of batches. It is assumed that the demand rate for the raw material inventory for the products at the first production stage is equal to that of the production rate of the first production stage. The raw material inventory model is illustrated in Fig. 6.9 (Wang and Sarker, 2006). The total raw material cost is written as (Wang and Sarker, 2006)

$$TC_r = \frac{D}{Q_f} n_o K_o + \frac{Q_f}{2} \frac{H_o}{n_o} \qquad (6.48)$$

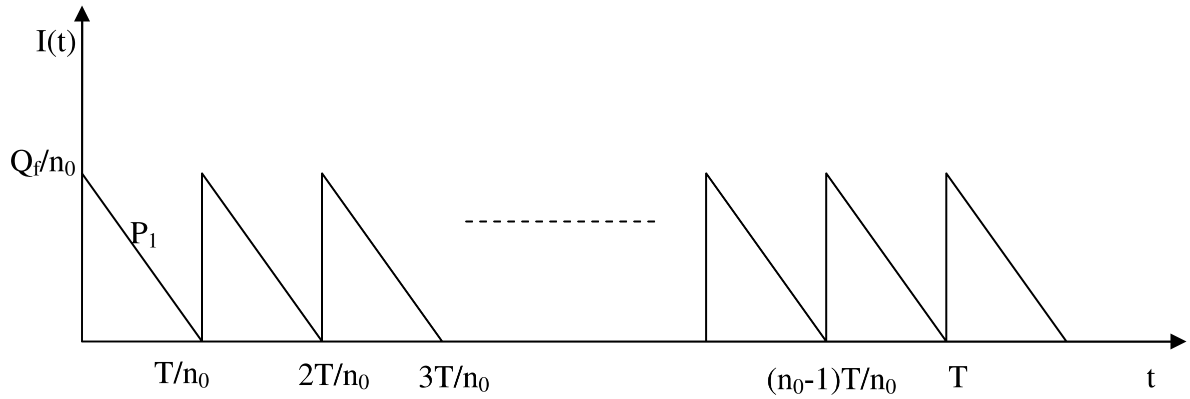

Fig. 6.9 Raw material inventory level in a multi-stage production system

6.3.1.2 Cost of work-in-process inventory

The production rate at the ith production system is carried at the rate of P_i. Since the semi-finished products are shipped in batches, the number of kanbans are K_i, or the batch size is Q_f/m_i. The level of work-in-process inventory at the ith production system is shown in Fig. 6.10.

The cost of work-in-process inventory is written as (Wang and Sarker, 2006)

$$TC_{wi} = \frac{D}{Q_f}\left[m_i K_{io} + K_{is}\right] + \frac{Q_f}{2}\left[H_i\left(1 - \frac{D}{P_i}\right) + \frac{H_i}{m_i}\right] \tag{6.49}$$

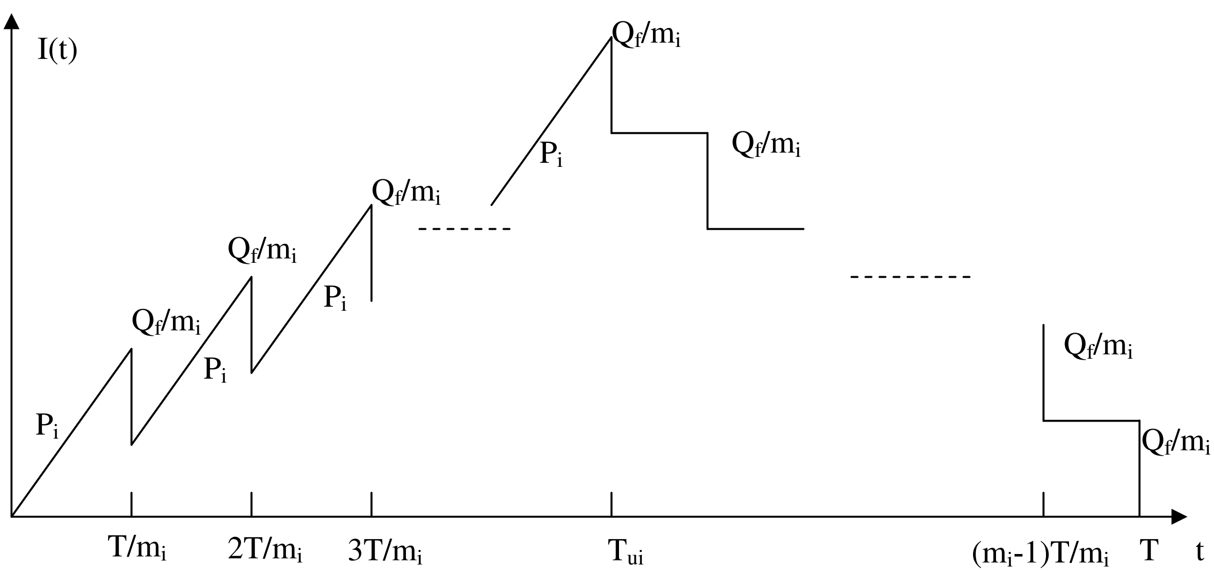

Fig. 6.10 Work-in-process inventory of an intermediate stage

6.3.1.3 Cost of finished goods inventory

The total cost of the finished goods inventory in the last stage (N+1[th] stage) of the production system can be written as (Wang and Sarker, 2006)

$$TC_f = \frac{D}{Q_f}\left[m_{N+1}K_{N+1,o} + K_{N+1,s}\right] + \frac{Q_f}{2}\left[H_{N+1}\left(1-\frac{D}{P_{N+1}}\right) + \frac{H_{N+1}}{m_{N+1}}\right] \tag{6.50}$$

6.3.1.4 Total cost of multi-stage production system

The total cost of multi-stage production-delivery system consists of (i) the cost of raw materials at the first stage, (ii) the cost of WIP in the intermediate stages, and (iii) cost of finished goods at the last stage, can be written as

$$TC\left(Q_f, n_1, m_1, \ldots, m_{N+1}\right) = \frac{D}{Q_f}\left[n_o K_o + \sum_{i=1}^{N+1} m_i K_{io} + \sum_{i=1}^{N+1} K_{is}\right] + \frac{Q_f}{2}\left[\frac{H_o}{n_o} + \sum_{i=1}^{N+1}\left\{H_i\left(1-\frac{D}{P_i}\right) + \frac{H_i}{m_i}\right\}\right] \tag{6.51}$$

6.3.1.5 Total cost of multi-stage production system with rework processing cost

Considering the rework processing cost, the total cost of multi-stage production-delivery system in Eq. (6.51) is written as

$$TC = \frac{DK_o}{Q_o} + I_o H_o + \sum_{i=1}^{N+1}\frac{DK_{io}}{Q_{io}} + \sum_{i=1}^{N+1}\frac{DK_{is}}{Q_i} + \sum_{i=1}^{N+1} D\beta_i C_i + \sum_{i=1}^{N+1} Q_{avg,i} H_i \tag{6.52}$$

Substituting $I_o = \frac{Q_o}{2}$, $Q_o = \frac{Q_r}{n_o}$, $Q_{io} = \frac{Q_i}{m_i}$ $Q_f = Q_i = m_i x_i$, $f = \frac{Q_f}{Q_r}$, $\frac{D_r}{Q_r} = \frac{D_i}{Q_i}$ and

$$Q_{avg,i} = \frac{Q_i}{2}\left\{\left(1-\frac{D_i\left(1+\beta_i-\beta_i^2\right)}{P_i}\right) + \frac{1}{m_i}\right\}$$

The expression for the total cost in Eq. (6.52) is obtained as

$$TC(Q_f, n_o, m_1, \ldots, m_{N+1}) = \frac{Dn_o fK_o}{Q_f} + \frac{Q_f H_o}{2fn_o} + \sum_{i=1}^{N+1} \frac{Dm_i K_{io}}{Q_f} + \sum_{i=1}^{N+1} \frac{DK_{is}}{Q_f}$$

$$+ \sum_{i=1}^{N+1} D\beta_i C_i + \sum_{i=1}^{N+1} \frac{Q_i}{2} H_i \left(1 - \frac{D(1+\beta_i-\beta_i^2)}{P_i}\right) + \sum_{i=1}^{N+1} \frac{Q_i}{2} \frac{H_i}{m_i} \quad (6.53)$$

Considering f=1. the total cost expression in Eq. (6.53) is modified as

$$TC(Q_f, n_o, m_1, \ldots, m_{N+1}) = \frac{D}{Q_f}\left[n_o K_o + \sum_{i=1}^{N+1} m_i K_{io} + \sum_{i=1}^{N+1} K_{is}\right] + \sum_{i=1}^{N+1} D\beta_i C_i$$

$$+ \frac{Q_f}{2}\left[\frac{H_o}{n_o} + \sum_{i=1}^{N+1} H_i \left(1 - \frac{D(1+\beta_i-\beta_i^2)}{P_i}\right) + \sum_{i=1}^{N+1} \frac{H_i}{m_i}\right] \quad (6.54)$$

If the integer restriction is not considered, the partial derivatives with respect to n_o, m_1, m_2,…., m_{N+1}, (where n_o, m_1,m_2,….,$m_{N+1} \geq 1$ and an integer) leads to

$$n_o^* = Q_f \sqrt{\frac{H_o}{2DK_o}} \text{ and } m_i^* = Q_f \sqrt{\frac{H_i}{2DK_{io}}} \quad (6.55)$$

Substituting the optimal values of n_o, m_1,m_2,….,m_{N+1} from Eq. (6.55) in Eq. (6.54), leads to

$$TC(Q_f, n_o, m_1, \ldots, m_{N+1}) = \sqrt{2DH_o K_o} + \sum_{i=1}^{N+1} \sqrt{2DH_i K_{io}} + \frac{1}{Q_f} \sum_{i=1}^{N+1} DK_{is} + \sum_{i=1}^{N+1} D\beta_i C_i$$

$$+ Q_f \sum_{i=1}^{N+1} \left\{ \frac{H_i}{2} \left(1 - \frac{D(1+\beta_i-\beta_i^2)}{P_i}\right) \right\} \quad (6.56)$$

The optimal production batch size Q_f is given by

$$Q_f^* = \sqrt{\frac{2D\sum_{i=1}^{N+1} K_{is}}{\sum_{i=1}^{N+1} \left\{ H_i \left(1 - \frac{D_i(1+\beta_i-\beta_i^2)}{P_i}\right) \right\}}} \quad (6.57)$$

Substituting the optimal value of production batch size Q from Eq. (6.57) in Eq. (6.56), leads to

$$TC(Q_f, n_o, m_1, \ldots, m_{N+1}) = \sqrt{2DH_o K_o} + \sum_{i=1}^{N+1} \sqrt{2DH_i K_{io}}$$

$$+ \sqrt{2D} \sqrt{\sum_{i=1}^{N+1} \left\{ H_i \left(1 - \frac{D_i(1+\beta_i-\beta_i^2)}{P_i}\right) \right\}} \sqrt{\sum_{i=1}^{N+1} K_{is}} + D\sum_{i=1}^{N+1} \beta_i C_i \quad (6.58)$$

6.3.2 Computational results

Assume that the proportion of defective items (β_i) in all stages is 0.01 and the rework processing cost (C_i) in all stages is \$10.0. Table 6.13 shows the input data for a multi-stage production system under kanban mechanism (Wang and Sarker, 2006). The comparison of optimal and modified results for a multi-stage production system under kanban mechanism with rework consideration are shown in Table 6.14. The optimal results of $n_0, m_1, ..., m_4, Q_f, TC$ show an increase with rework consideration. The modified results of $n_0, m_1, ..., m_4, Q_f$ are similar for with and without rework except in m_4 which shows an increase with rework consideration. The total cost for optimal and modified results increase with rework due to the rework processing cost consideration. Table 6.15 shows a comparison of all possible combinations of modified results for a multi-stage production system under kanban mechanism with rework consideration.

Figures 6.11 and 6.12 show the total cost (TC) increases with increase in proportion of defective items (β_i) and rework processing cost (C_i) respectively. Figure 6.13 shows the variation of total cost (TC) with both proportion of defective items (β_i) and rework processing cost (C_i). Total cost (TC) increases with increase in both β_i and C_i respectively. The increase in total cost (TC) with C_i is more at higher values of β_i.

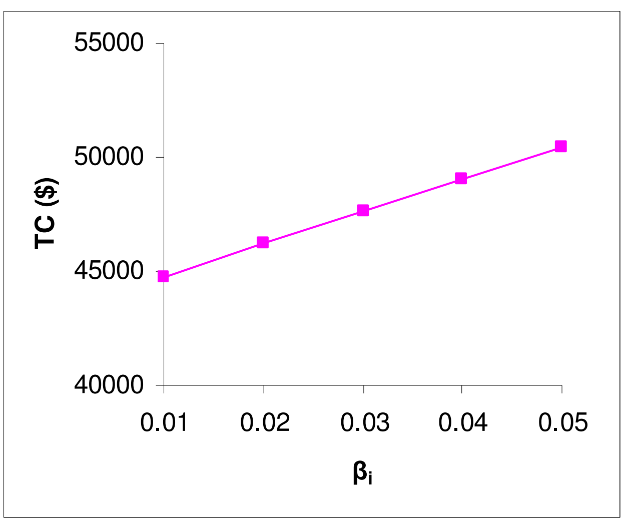

Fig. 6.11 Variation of TC with β_i

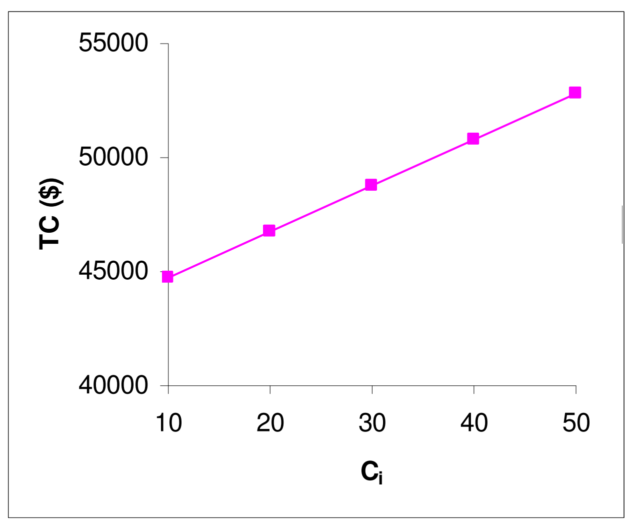

Fig. 6.12 Variation of TC with C_i

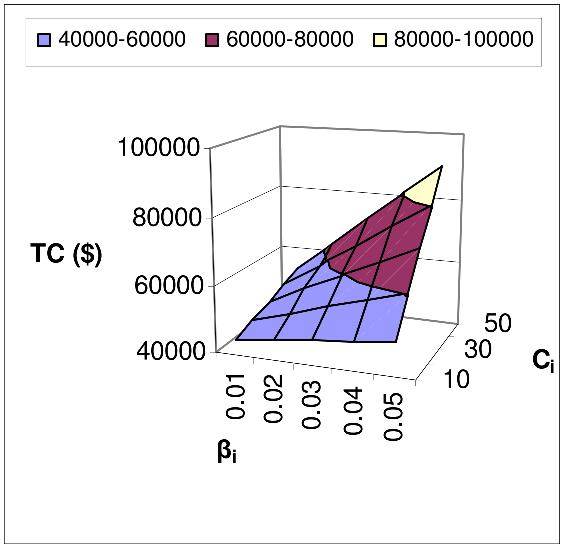

Fig. 6.13 Variation of TC with β_i and C_i

Table 6.13 Input data for a multi-stage production system under kanban mechanism (K_o=110, H_o=45, β_i=0.01, C_i=10 (i=1,2,3,4))

Demand of finished goods (units/year)	Production rate (units/year)	Setup/shipping cost ($/batch)	Setup cost ($/batch)	Holding cost ($/unit/year)
D=5000	P_1=5500	K_{1s}=300	K_{1o}=100	H_1 =30
	P_2=5600	K_{2s}=250	K_{2o}=80	H_2 =45
	P_3=6000	K_{3s}=300	K_{3o}=120	H_3 =25
	P_4=5500	K_{4s}=350	K_{4o}=100	H_4 =35

Table 6.14 Comparison of results for a multi-stage production system under kanban mechanism

	Wang and Sarker (2006)		Present work (with rework consideration)	
	Optimum results	Modified results	Optimum results	Modified results
n_0	5.74	6	5.98	6
m_1	4.92	5	5.12	5
m_2	6.73	7	7.02	7
m_3	4.10	4	4.27	4
m_4	5.31	5	5.54	6
Q_f	898	898	936	936
TC ($)	43277	43301	44732	44765

Table 6.15 Comparison of all possible modified results for a multi-stage production system under kanban mechanism

S. No.	Wang and Sarker (2006)							Present work (with rework consideration)						
	n_0	m_1	m_2	m_3	m_4	Q_f	TC ($)	n_0	m_1	m_2	m_3	m_4	Q_f	TC ($)
1	5.0	4.0	6.0	4.0	5.0	897.5	43512.5	5.0	5.0	7.0	4.0	5.0	935.6	44889.8
2	5.0	4.0	6.0	4.0	6.0	897.5	43546.1	5.0	5.0	7.0	4.0	6.0	935.6	44878.4
3	5.0	4.0	6.0	5.0	5.0	897.5	43620.1	5.0	5.0	7.0	5.0	5.0	935.6	44946.3
4	5.0	4.0	6.0	5.0	6.0	897.5	43653.6	5.0	5.0	7.0	5.0	6.0	935.6	44934.9
5	5.0	4.0	7.0	4.0	5.0	897.5	43477.4	5.0	5.0	8.0	4.0	5.0	935.6	44941.4
6	5.0	4.0	7.0	4.0	6.0	897.5	43510.9	5.0	5.0	8.0	4.0	6.0	935.6	44930.0
7	5.0	4.0	7.0	5.0	5.0	897.5	43584.9	5.0	5.0	8.0	5.0	5.0	935.6	44997.9
8	5.0	4.0	7.0	5.0	6.0	897.5	43618.5	5.0	5.0	8.0	5.0	6.0	935.6	44986.5
9	5.0	5.0	6.0	4.0	5.0	897.5	43396.5	5.0	6.0	7.0	4.0	5.0	935.6	44956.3
10	5.0	5.0	6.0	4.0	6.0	897.5	43430.0	5.0	6.0	7.0	4.0	6.0	935.6	44945.0
11	5.0	5.0	6.0	5.0	5.0	897.5	43504.0	5.0	6.0	7.0	5.0	5.0	935.6	45012.9
12	5.0	5.0	6.0	5.0	6.0	897.5	43537.6	5.0	6.0	7.0	5.0	6.0	935.6	45001.5
13	5.0	5.0	7.0	4.0	5.0	897.5	43361.3	5.0	6.0	8.0	4.0	5.0	935.6	45008.0
14	5.0	5.0	7.0	4.0	6.0	897.5	43394.9	5.0	6.0	8.0	4.0	6.0	935.6	44996.6
15	5.0	5.0	7.0	5.0	5.0	897.5	43468.9	5.0	6.0	8.0	5.0	5.0	935.6	45064.5
16	5.0	5.0	7.0	5.0	6.0	897.5	43502.5	5.0	6.0	8.0	5.0	6.0	935.6	45053.1
17	6.0	4.0	6.0	4.0	5.0	897.5	43452.2	6.0	5.0	7.0	4.0	5.0	935.6	44775.9
18	6.0	4.0	6.0	4.0	6.0	897.5	43485.7	**6.0**	**5.0**	**7.0**	**4.0**	**6.0**	**935.6**	**44764.5**
19	6.0	4.0	6.0	5.0	5.0	897.5	43559.8	6.0	5.0	7.0	5.0	5.0	935.6	44832.4
20	6.0	4.0	6.0	5.0	6.0	897.5	43593.3	6.0	5.0	7.0	5.0	6.0	935.6	44821.0
21	6.0	4.0	7.0	4.0	5.0	897.5	43417.0	6.0	5.0	8.0	4.0	5.0	935.6	44827.5
22	6.0	4.0	7.0	4.0	6.0	897.5	43450.6	6.0	5.0	8.0	4.0	6.0	935.6	44816.1
23	6.0	4.0	7.0	5.0	5.0	897.5	43524.6	6.0	5.0	8.0	5.0	5.0	935.6	44884.0
24	6.0	4.0	7.0	5.0	6.0	897.5	43558.2	6.0	5.0	8.0	5.0	6.0	935.6	44872.6
25	6.0	5.0	6.0	4.0	5.0	897.5	43336.1	6.0	6.0	7.0	4.0	5.0	935.6	44842.5
26	6.0	5.0	6.0	4.0	6.0	897.5	43369.7	6.0	6.0	7.0	4.0	6.0	935.6	44831.1
27	6.0	5.0	6.0	5.0	5.0	897.5	43443.7	6.0	6.0	7.0	5.0	5.0	935.6	44899.0
28	6.0	5.0	6.0	5.0	6.0	897.5	43477.3	6.0	6.0	7.0	5.0	6.0	935.6	44887.6
29	**6.0**	**5.0**	**7.0**	**4.0**	**5.0**	**897.5**	**43301.0**	6.0	6.0	8.0	4.0	5.0	935.6	44894.1
30	6.0	5.0	7.0	4.0	6.0	897.5	43334.6	6.0	6.0	8.0	4.0	6.0	935.6	44882.7
31	6.0	5.0	7.0	5.0	5.0	897.5	43408.6	6.0	6.0	8.0	5.0	5.0	935.6	44950.6
32	6.0	5.0	7.0	5.0	6.0	897.5	43442.1	6.0	6.0	8.0	5.0	6.0	935.6	44939.2

6.4 Managerial Implications

The managerial implications are as follows:

- For single purchase multiple delivery (SPMD), single purchase single delivery (SPSD), and for lot-for-lot (LFL) policies, with increase in the fraction of non-conforming units (α), and value added by the manufacturing process (V), the (i) optimal number of inspections increases, (ii) optimal ordering quantity decreases, and (iii) optimal expected total cost increases. As shown in Table 6.2 for imperfect production process with quality and rework of SPMD policy, with increase in the fraction of non-conforming units (α) from 0.05 to 0.50, the (i) optimal ordering quantity decreased from 274 to 126, and (ii) optimal expected total cost increased from $3512.78 to $ 4438.64. As shown in Table 6.3 for imperfect production process with quality and rework including process inspection and restoration of SPMD policy, with increase in the fraction of non-conforming units (α) from 0.05 to 0.50, the (i) optimal number of inspections increased from 3 to 5, (ii) optimal ordering quantity decreased from 316 to 147, and (iii) optimal expected total cost increased from $ 3496.45 to $ 4031.10. As shown in Table 6.2 for imperfect production process with quality and rework of SPMD policy, with increase in the value added by the manufacturing process (V) from 1.0 to 8.0, the (i) optimal ordering quantity decreased from 274 to 126, and (ii) optimal expected total cost increased from $3512.78 to $ 20593.97. As shown in Table 6.3 for imperfect production process with quality and rework including process inspection and restoration of SPMD policy, with increase in the value added by the manufacturing process (V) from 1.0 to 8.0, the (i) optimal number of inspections increased from 3 to 4, (ii) optimal ordering quantity decreased from 316 to 147, and (iii) optimal expected total cost increased from $ 3496.45 to $ 20593.97.
- In the model of optimal batch size in a single-stage imperfect production-delivery system with rework, wherein defective items are reworked within the same cycle, the total cost increase with defects due to rework processing cost consideration to compensate for the loss of planned products. As shown in Table 6.12 (S. No.1), the total cost with rework consideration is $ 1323, while the total cost without rework consideration (Sarker and Parija, 1996) is $ 1206. Also, in the model of optimal number of kanbans for multi-stage production-delivery system with rework, wherein defective items are reworked within the same cycle, the optimal quantity and total cost increase with defects due to rework

processing cost consideration. As shown in Table 6.14, the optimal quantity and total cost with rework consideration are 936 and $44765, while the optimal quantity and total cost without rework consideration (Wang and Sarker, 2006) are 898 and $43301.