

STUDY OF HYDRODYNAMIC COEFFICIENTS
FOR ROUGH AND INCLINED CYLINDERS

THESIS

Submitted in partial fulfilment
of the requirements for the degree of
DOCTOR OF PHILOSOPHY

By

RAJIV GUPTA

Under the supervision
of Dr. S. GHOSHAL

BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE
PILANI (RAJASTHAN) INDIA

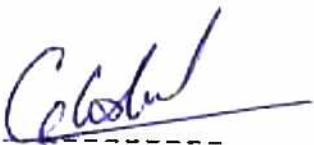
1995

BIRLA INSTITUTE OF TECHNOLOGY & SCIENCE
PILANI, RAJASTHAN

CERTIFICATE

This is to certify that the thesis entitled STUDY OF HYDRODYNAMIC COEFFICIENTS FOR ROUGH AND INCLINED CYLINDERS and submitted by Mr. RAJIV GUPTA ID.No. 88PHXF81Z for award of Ph.D. Degree of the institute, embodies original work done by him/her under my supervision.

Signature in full of
the Supervisor



Name in capital block
letters

(S. G. HOSHAL)

Designation

Associate Prof.

Date: 16/6/95

ACKNOWLEDGMENTS

I convey my sincere gratitude and thankfulness to my guide Dr.S.Ghoshal for introducing me to the area of offshore structures, all the guidance and continuous encouragement.

I thank all the faculty and staff members of the Hydraulics Laboratory at I.I.T., Bombay, where all the data used in my work has been collected.

I thank Dr.S.Venkateswaran, Director, BITS, Pilani, and other Administrative Deans for providing the necessary infrastructure and other facilities.

I am sincerely indebted to Professor I.J.Nagrath, Deputy Director, BITS, Pilani, for his keen interest in the work and constructive suggestions offered from time to time.

I wish to express my gratitude to Prof. H.S.Moondra, Dr. S.Kumar, and Dr.(Mrs.) A.Gupta, for thought provoking questions and critical comments.

I must thank Mr.J.P.Mishra, Dr.S.N.Sharan, Dr.C.B.Gupta and staff of Civil Engg. Group for their help in various forms without which in a new area like this it would have been very difficult for me to conduct the work.

Lastly, I must express my thanks to all who directly or indirectly contributed in the completion of this work.

Rajiv Gupta

NOMENCLATURE

a		WAVE AMPLITUDE
BBC		BOTTOM BOUNDARY CONDITION
C _w		WAVE CELOCITY
C _d		COEFFICIENT OF DRAG
C _L		COEFFICIENT OF LIFT
C _m		COEFFICIENT OF INERTIA
D		CYLINDER DIAMETER
d		STILL WATER DEPTH
DFSBC		DYNAMIC FREE SURFACE BOUNDARY CONDITION
F _h		TOTAL HORIZONTAL FORCE
F _t		TOTAL TRANSVERSE FORCE
g		ACCELERATION DUE TO GRAVITY
H		WAVE HEIGHT
k		AVERAGE HEIGHT OF ROUGHNESS PARTICLES
k		WAVE NUMBER
K-C		KEULEGAN - CARPENTER NUMBER
KFSBC		KINEMATIC FREE SURFACE BOUNDARY CONDITION
L		WAVE LENGTH
L _o		$2T^2 / 2\pi$
L _v		VERTICAL MEASUREMENT OF THE ORBIT SIZE OF THE WATER PARTICLE
p		PRESSURE AT ANY POINT

\mathbf{q}		VELOCITY VECTOR
Re		REYNOLDS NUMBER
SWL		STILL WATER LEVEL
T		WAVE PERIOD
t		TIME INSTANT
u		WATER PARTICLE VELOCITY IN X DIRECTION
u'		WATER PARTICLE ACCELERATION IN X DIRECTION
U_{max}		MAXIMUM HORIZONTAL WATER PARTICLE VELOCITY
w		VELOCITY COMPONENT IN Z DIRECTION
w'		WATER PARTICLE ACCELERATION IN Z DIRECTION
ω		ANGULAR WAVE FREQUENCY
ρ		MASS DENSITY OF WATER
π		3.14159
ϕ		VELOCITY POTENTIAL FUNCTION
θ_x		CYLINDER INCLINATION IN IN-LINE DIRECTION WITH RESPECT TO VERTICAL
θ_y		CYLINDER INCLINATION IN TRANSVERSE DIRECTION WITH RESPECT TO VERTICAL
η		ELEVATION OF FREE WATER SURFACE FROM THE STILL WATER DEPTH
ν		KINEMATIC VISCOSITY OF FLUID

ABSTRACT

In the recent past, offshore structures have gained an important role and considerable research is under progress for accurate evaluation of the hydrodynamic coefficients for the cylindrical members of the Jacket type offshore platforms.

The present work *studies* the effects of surface roughness, due to marine growth, and member inclination on the hydrodynamic coefficients after comparing the variation of water particle kinematics using Airy's linear and Stoke's fifth order wave theories.

The study is based on the experimental data obtained from a regular wave flume set-up and for Reynolds number (Re) varying from 11000 to 53000, the Keulegan-Carpenter number (K-C) varying from 4 to 14, and roughness parameter (k/D) varying from 0 (smooth) to 1/90. The in-line and lateral inclinations considered are $\pm 30^\circ$ and 20° respectively, from vertical.

The observations of the present study indicate that the surface roughness influences the inertia and lift coefficients to a large extent. The member orientation, on the other hand, influences all the three hydrodynamic coefficients.

CONTENTS

CHAPTER..	TITLE	PAGE
	ACKNOWLEDGMENTS	i
	NOMENCLATURE	iii
	ABSTRACT	v
	CONTENTS	vi
	LIST OF FIGURES	ix
CHAPTER 1	INTRODUCTION	1
CHAPTER 2	PERIODIC WAVES	10
2.1	INTRODUCTION	10
2.2	THEORY OF PERIODIC WAVES	10
2.2.1	BASIC FUNDAMENTAL EQUATIONS	10
2.2.2	AIRY'S THEORY (LINEAR WAVE THEORY OR SMALL AMPLITUDE THEORY)	14
2.2.3	STOKE'S FIFTH ORDER THEORY	17
2.3	COMPARISON OF THE RESULTS OBTAINED BY STOKE'S FIFTH ORDER THEORY AND AIRY'S THEORY	17
2.3.1	RESULTS	20
CHAPTER 3	DISCUSSIONS	23
3.1	INTRODUCTION	23
3.2	IN-LINE FORCE COEFFICIENTS (C_d AND C_m)	24
3.2.1	EFFECTS OF SURFACE ROUGHNESS	24
3.2.2	EFFECTS OF INCLINATION	26

3.2.3	DEPENDENCE UPON NON - DIMENSIONAL PARAMETERS (K-C AND Re)	30
3.3	LIFT COEFFICIENT (C_L)	31
3.3.1	EFFECTS OF ROUGHNESS AND INCLINATION	31
3.3.2	DEPENDENCE UPON NON - DIMENSIONAL PARAMETERS (K-C AND Re)	34
3.4	COMPARISON OF THE RESULTS OF 60MM AND 90MM DIAMETER CYLINDERS	35
3.4.1	RANGE OF Re AND K-C	35
3.4.2	COEFFICIENT OF INERTIA (C_m)	35
3.4.3	COEFFICIENT OF DRAG (C_d)	36
3.4.4	COEFFICIENT OF LIFT (C_L)	37
3.4.5	REASONS OF DIFFERENCE	39
3.5	DESIGN VALUES	39
CHAPTER 4	CONCLUSIONS	50
4.1	EFFECTS OF ROUGHNESS ON INCLINED CYLINDERS	51
4.1.1	THE COEFFICIENT OF DRAG (C_d)	51
4.1.2	THE COEFFICIENT OF INERTIA (C_m)	51
4.1.3	THE COEFFICIENT OF LIFT (C_L)	52
4.2	EFFECTS OF INCLINATION ON ROUGH CYLINDERS	52
4.2.1	THE COEFFICIENT OF DRAG (C_d)	52
4.2.2	THE COEFFICIENT OF INERTIA (C_m)	53
4.2.3	THE COEFFICIENT OF LIFT (C_L)	53
4.3	RELATIONS WITH Re AND K-C	54
4.4	EFFECTS OF CYLINDER DIAMETER	54

APPENDIX - A	SUMMARY OF PUBLISHED WORKS ON C_d , C_m AND C_L (1950 -1994)	56
APPENDIX - B	SOFTWARE LISTING I "Wv. Th."	60
APPENDIX - C	IN-PUT DATA FILE (for "Wv.Th.")	63
APPENDIX - D	OUT-PUT DATA FILE	70
APPENDIX - E	PLOTTING SOFTWARE DETAILS	77.Q
APPENDIX - F	GRAPHS OF C_d , C_m , AND C_L AGAINST K-C AND R_e	84
REFERENCES	117
BIBLIOGRAPHY	126

LIST OF FIGURES

Fig. No.	Title	Page
1.1	DEFINITION SKETCH FOR THE FORCES ACTING ON A CIRCULAR CYLINDRICAL MEMBER	2
2.1	DEFINITION SKETCH OF PROGRESSIVE WAVE TRAIN	11
2.2	WAVE AND FORCE PROFILE RECORDING	18
2.3	Re & $K-C$ vs. gT^2/H FOR CYLINDER OF 60mm DIAMETER	21
2.4	Re & $K-C$ vs. gT^2/H FOR CYLINDER OF 90mm DIAMETER	22
3.1	VARIATION OF FORCES FOR POSITIVE AND NEGATIVE IN-LINE INCLINATIONS	28
3.2	RELATIVE POSITIONS OF EDDIES FOR LATERALLY AND IN-LINE INCLINED CYLINDERS	32
3.3	C_m vs. Re FOR VARYING θ_x , FOR $\theta_y=0$, $k/D=0$ FOR 60mm AND 90mm DIAMETER CYLINDERS	41
3.4	C_a vs. Re FOR VARYING θ_y , FOR $\theta_x=0$, $k/D=0$ FOR 60mm AND 90mm DIAMETER CYLINDERS	42
3.5	C_a vs. Re FOR VARYING ROUGHNESS FOR $\theta_x=0$, $\theta_y=0$ FOR 60mm AND 90mm DIAMETER CYLINDERS	43
3.6	C_d vs. Re FOR VARYING θ_x , FOR $\theta_y=0$, $k/D=0$ FOR 60mm AND 90mm DIAMETER CYLINDERS	44
3.7	C_d vs. Re FOR VARYING θ_y , FOR $\theta_x=0$, $k/D=0$ FOR 60mm AND 90mm DIAMETER CYLINDERS	45
3.8	C_d vs. Re FOR VARYING ROUGHNESS FOR $\theta_x=0$, $\theta_y=0$ FOR 60mm AND 90mm DIAMETER CYLINDERS	46
3.9	C_L vs. Re FOR VARYING θ_x , FOR $\theta_y=0$, $k/D=0$ FOR 60mm AND 90mm DIAMETER CYLINDERS	47
3.10	C_L vs. Re FOR VARYING θ_y , FOR $\theta_x=0$, $k/D=0$ FOR 60mm AND 90mm DIAMETER CYLINDERS	48

3.11	C_L vs. Re FOR VARYING ROUGHNESS FOR $\theta_x=0$, $\theta_y=0$ FOR 60mm AND 90mm DIAMETER CYLINDERS	49
F-1	C_d vs. K-C & Re FOR VARYING ROUGHNESS FOR $\theta_x=0$, $\theta_y=0$	87
F-2	C_d vs. K-C & Re FOR VARYING ROUGHNESS FOR $\theta_x=7.2$, $\theta_y=0$	88
F-3	C_d vs. K-C & Re FOR VARYING ROUGHNESS FOR $\theta_x=-18.8$, $\theta_y=0$	89
F-4	C_d vs. K-C & Re FOR VARYING ROUGHNESS FOR $\theta_x=0$, $\theta_y=20$	90
F-5	C_d vs. K-C & Re FOR VARYING θ_x FOR $\theta_y=0$, $k/D=0$	91
F-6	C_d vs. K-C & Re FOR VARYING θ_x FOR $\theta_y=0$, $k/D=1/90$	92
F-7	C_d vs. K-C & Re FOR VARYING - θ_x FOR $\theta_y=0$, $k/D=0$	93
F-8	C_d vs. K-C & Re FOR VARYING - θ_x FOR $\theta_y=0$, $k/d=1/90$	94
F-9	C_d vs. K-C & Re FOR VARYING θ_y FOR $\theta_x=0$, $k/D=0$	95
F-10	C_d vs. K-C & Re FOR VARYING θ_y FOR $\theta_x=0$, $k/D=1/90$	96
F-11	C_m vs. K-C & Re FOR VARYING ROUGHNESS FOR $\theta_x=0$, $\theta_y=0$	97
F-12	C_m vs. K-C & Re FOR VARYING ROUGHNESS FOR $\theta_x=7.2$, $\theta_y=0$	98
F-13	C_m vs. K-C & Re FOR VARYING ROUGHNESS FOR $\theta_x=-18.8$, $\theta_y=0$	99
F-14	C_m vs. K-C & Re FOR VARYING ROUGHNESS FOR $\theta_x=0$, $\theta_y=20$	100
F-15	C_m vs. K-C & Re FOR VARYING θ_x FOR $\theta_y=0$, $k/D=0$	101
F-16	C_m vs. K-C & Re FOR VARYING θ_x FOR $\theta_y=0$, $k/D=1/90$	102
F-17	C_m vs. K-C & Re FOR VARYING - θ_x FOR $\theta_y=0$, $k/D=0$	103
F-18	C_m vs. K-C & Re FOR VARYING - θ_x FOR $\theta_y=0$, $k/D=1/90$	104
F-19	C_m vs. K-C & Re FOR VARYING θ_y FOR $\theta_x=0$, $k/D=0$	105
F-20	C_m vs. K-C & Re FOR VARYING θ_y FOR $\theta_x=0$, $k/D=1/90$	106
F-21	C_L vs. K-C & Re FOR VARYING ROUGHNESS FOR $\theta_x=0$, $\theta_y=0$	107
F-22	C_L vs. K-C & Re FOR VARYING ROUGHNESS FOR $\theta_x=7.2$, $\theta_y=0$	108

F-23	C _L vs. K-C & Re FOR VARYING ROUGHNESS FOR $\theta_x = -18.8$, $\theta_y = 0$	109
F-24	C _L vs. K-C & Re FOR VARYING ROUGHNESS FOR $\theta_x = 0$, $\theta_y = 20$	110
F-25	C _L vs. K-C & Re FOR VARYING θ_x FOR $\theta_y = 0$, $k/D = 0$	111
F-26	C _L vs. K-C & Re FOR VARYING θ_x FOR $\theta_y = 0$, $k/D = 1/90$	112
F-27	C _L vs. K-C & Re FOR VARYING - θ_x FOR $\theta_y = 0$, $k/D = 0$	113
F-28	C _L vs. K-C & Re FOR VARYING - θ_x FOR $\theta_y = 0$, $k/D = 1/90$	114
F-29	C _L vs. K-C & Re FOR VARYING θ_y FOR θ_x , $k/D = 0$	115
F-30	C _L vs. K-C & Re FOR VARYING θ_y FOR θ_x , $k/D = 1/90$	116

SECTION 1

INTRODUCTION

Offshore structures were first introduced in the Gulf of Mexico around 1945 for the purpose of oil and mineral exploration. Since then there has been a remarkable growth which manifested in the designs of various types of offshore platforms. Nowadays, offshore structures are being located in ever increasing depths and are being subjected to extremely hostile environmental conditions mainly due to ocean waves. Therefore, establishing the guiding factors for design of such structures has become important.

In ~~the~~ present ~~times~~, the widely used Jacket type platform, and its extensions, are sometimes as high as 300 meters from the mud-line and comprise of a framed structure supported by piles driven through its legs. During the last two decades, considerable work has been carried out in developing the design methodology to produce a functional, safe and economical structure. These structures are made of circular cylindrical members which are equally strong in all the directions so that they can withstand the wave forces from any direction, are cheaper and can be fabricated and installed with comparative ease.

These circular members are subjected to different types of loads. However, the present work concentrates only on the

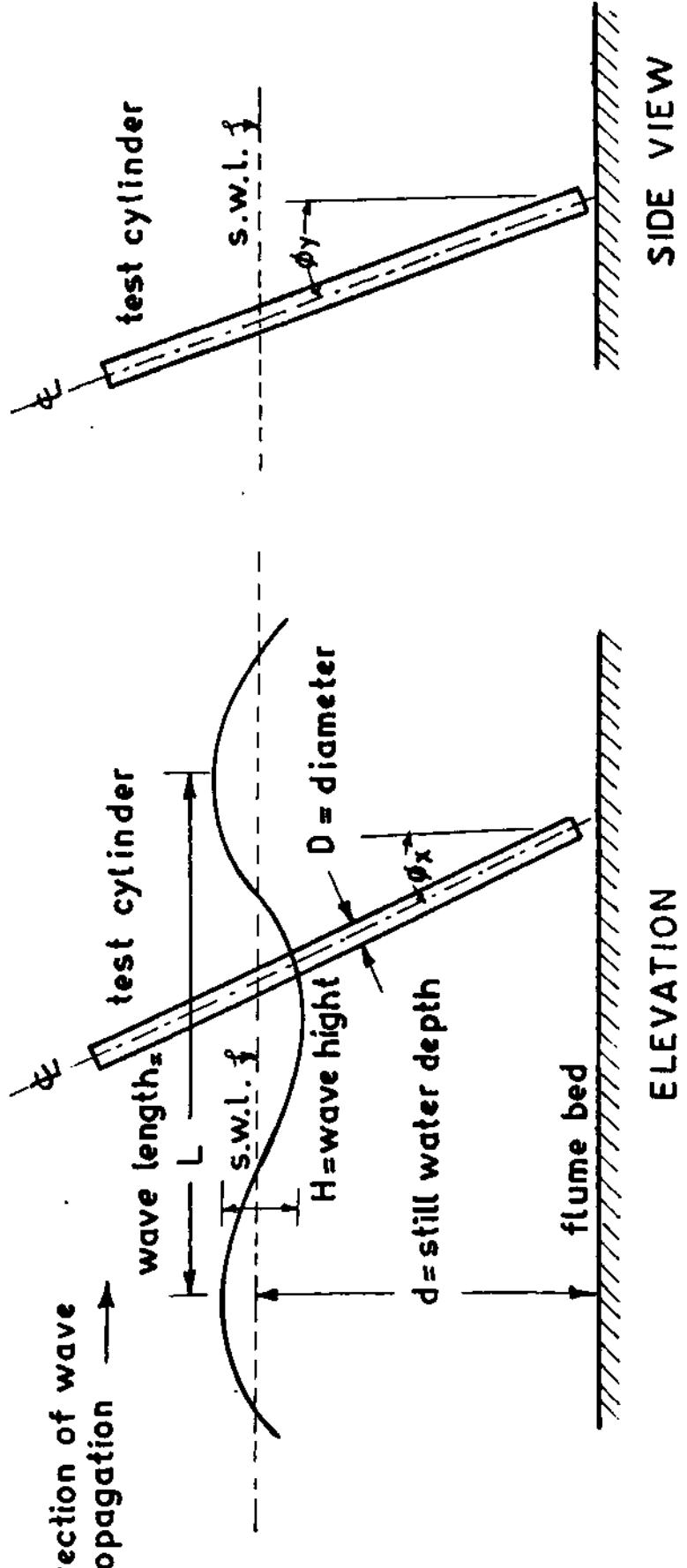
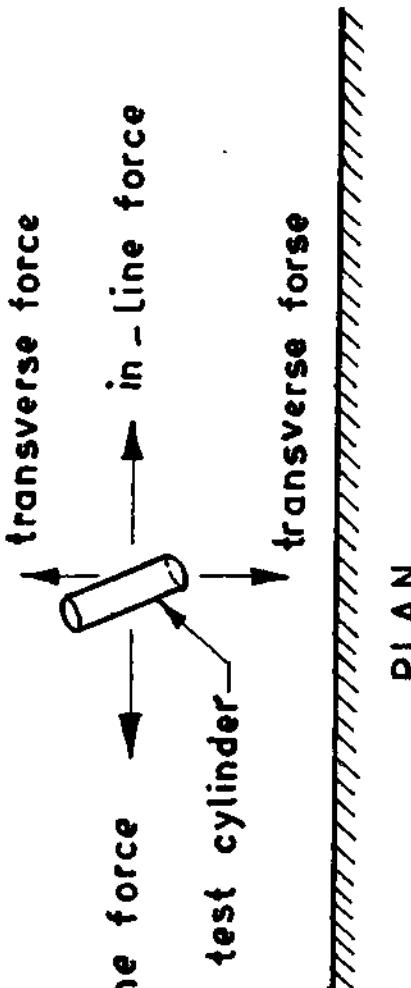


Fig. 1.1 Definition sketch for the forces acting on a circular cylindrical member

study of wave -induced forces which are least understood of all the forces which act on the circular members. The wave - induced forces may be divided into two parts, namely the in-line force and the transverse force. The first one is associated with the drag and inertial effects, linearly added together (Fig 1.1) while the second one is associated with the lift effects and is attributed mainly to the asymmetrical pattern of eddy shedding.

The in-line force for a smooth, vertical circular cylinder subjected to wave forces may be evaluated by the Morison Equation which needs the accurate evaluation of two hydrodynamic coefficients, namely the coefficient of drag (C_d) and the coefficient of inertia (C_m) as follows;

$$F_h = 0.5 \rho C_d D u |u| + 0.25 \rho C_m \pi D^2 u^2 \quad \dots \dots \dots \quad (1.1)$$

where

F_h = Horizontal force per unit length at any time "t"

C_d = Coefficient of drag

C_m = Coefficient of inertia

ρ = Mass density of the fluid

D = Diameter of cylinder

u = Local water particle velocity at any time "t"

u^2 = Local water particle acceleration at any time "t"

The transverse force may be evaluated by the equation,

$$F_t = 0.5 \rho C_L D u^2 \quad \dots \dots \dots \quad (1.2)$$

where

F_t = Transverse force per unit length at any time "t"

C_L = Coefficient of lift

To calculate the wave-induced forces, apart from the hydrodynamic coefficients, the water particle kinematics must also be determined which is evident from equations 1.1 and 1.2.

To obtain the water particle kinematics using a suitable wave theory and therefrom the values of the hydrodynamic coefficients, either field observations or laboratory tests under simulated environmental conditions with appropriate scaled models are conducted. In order to investigate the waves, a test set-up is used to measure the water surface elevations and other properties of the waves (Chakrabarti [8]).

Several wave theories have been proposed (ref. Wiegel [52]) to calculate the water particle kinematics with the help of wave height (H), wave period (T), and still water depth (d). However, the selection of a wave theory to determine these parameters is subjective since no single theory is found to predict satisfactorily all wave properties at all depths (ref. Sankar et al.[45]).

Ever since Morison et al. published their works on wave

induced forces, investigators (Chakrabarti [9], [10], [15]), Garrison [20], Jen [32], Sarpkaya [45], etc.) have been trying to establish the guiding factors to quantify these coefficients. The earlier works attempted to correlate these coefficients with non-dimensional parameters, like, the Reynolds number (Re). Due to wide scattering of points in certain cases, Re alone was not considered sufficient to study the variation of hydrodynamic coefficients and this led to the introduction of Keulegan Carpenter number ($K-C = u_{max}T/D$, u_{max} = max horizontal velocity of fluid particle, T = time period and D = diameter of cylinder). The importance of $K-C$ has been confirmed by various investigators such as Bidde [5], Chakrabarti [7], Hedeseth [28], and Sarpkaya [42] etc. Further attempts were made to introduce other parameters like Garrison's [21] 'Displacement ratio parameter' ($= 2a/D$, where a = amplitude and D = diameter of cylinder) and Stansmore's [48] 'Orbit size parameter' ($= L_v/D$, where L_v = particle orbit size and D = diameter of cylinder), whose importance are yet to be established thoroughly.

Significant work has been carried out to substitute the Morison equation, and to find out the different influencing factors so that the scatter in the plots between the coefficients and the non-dimensional parameters can be minimized and the design methodology can be specified.

Investigators like Chakrabarti et al. [16,17], Sarpkaya [43], Starsmore [48], Turum [50], Garrison [20] are still trying to correlate these two for circular cylinders. Summary of all these works has been presented in Appendix-A in tabular forms.

Investigations have further revealed that most of the submerged members do not remain perfectly smooth, but are roughened due to marine growth (ref. [9], [26] and [38]) around them. Moreover a majority of the structural members of the Jacket type platforms are inclined. Some investigators studied these new parameters separately, which are discussed below in brief.

Sarpkaya [44] performed the experiments with rough cylindrical models subjected to the oscillatory flow in a U-tube set-up covering a wide range of Reynolds number by varying the temperature. The relative roughness parameter k/D , (where k = average height of rough particles and D = diameter of the circular member) varied from 0.0012 to 0.02 in five equal steps. It was concluded by him that at higher range of $Re (>10^3)$, C_d increases while C_m decreases with the increase in surface roughness.

Garrison & Garrison et al [20,21] conducted some experiments by oscillating rough cylinders in otherwise still water. He covered the range of Re upto 2×10^6 . The relations of C_d and

C_m against Re obtained by him with the help of 'Displacement ratio' showed even wider scatter than that of Sarpkaya et al. [45].

Nath [39] in his experiments with rough cylinders observed that at roughness parameter of 0.02, the value of C_m is less than that for a smooth cylinder, while the value of C_m for the cylinder with roughness parameter 0.03 is more than that of a smooth cylinder. He could not offer any reason for this peculiar behaviour of the coefficient against roughness.

Chakrabarti [9] conducted the experiments in the wave flume with rough cylinders and concluded that C_d increases with the surface roughness but C_m remains unchanged with it.

Chakrabarti and Cotter [15] conducted experiments with bottom mounted cylinder for in-line inclinations of 0° , 30° , and 45° . A trend of decreasing value of C_m with increase in inclination was reported by them.

To summarize, a considerable amount of additional work is necessary to establish the relation between the hydrodynamic coefficients and the non-dimensional parameters, to calculate accurately the wave forces on rough and oblique members, using the Morison Equation.

In present work, the study of the behaviour of the

hydrodynamic coefficients for rough and inclined cylinders against Re and $K-C$ has been carried out and no experiments were conducted by the author. The data (Wave and Force Profiles Recordings, in all 98, and processed values of Hydrodynamic coefficients (C_d , C_m and C_L) and Non-Dimensional parameters namely Re and $K-C$), presented in tabular form) used in the present work was collected by Ghoshal [22] who conducted the experiments in a regular wave flume at Hydraulics Laboratory at I.I.T., Bombay. The author is thankful to the authorities, faculty and staff of The Dept. of Civil Engg., I.I.T., Bombay, for the provision of data.

In the present work, the results obtained by two theories, namely, Stoke's Fifth Order theory [22] and Airy's linear theory [ref. 45] are compared in chapter 2.

Apart from ref. [22], only present work covered the roughness and inclination of the circular cylinders simultaneously. However, it differs from the previous work (ref. [22]) in certain aspects which are listed below and a complete comparison is presented in chapter 3 along with the Results and Discussions of the present work.

The study by Ghoshal and Ghoshal et al. ([22] and [24]) for hydrodynamic coefficients against non-dimensional

parameters for inclined and rough cylinders of 60mm diameter, covered the range of Re , from 8000 to 36000, $K-C$ from 6 to 21 and the roughness parameter 0, 1/120 and 1/60. The variation of the inclination was considered to be $\pm 30^\circ$. The present study, for rough and inclined cylinders of 90mm diameter, covers a higher range of Re (upto 54000), lower range of $K-C$ (upto 4) and intermediate roughness parameters ($k/D = 0, 1/180$ and $1/90$). The range of in-line and transverse inclinations are same. For the purpose of easier interpretation and comparison, best-fit curves are drawn in each plot for each data set.

The results are presented in chapter 3 which is followed by the Conclusions in chapter 4.

CHAPTER 2

PERIODIC WAVES

2.1 Introduction

As mentioned in the previous chapter that to evaluate wave induced forces, using the Morison Equation, apart from the hydrodynamic coefficients, the velocity and acceleration of the water particles must be calculated by selecting a suitable wave theory after knowing or measuring the wave height (H), wave period (T) and still water depth (d) in the laboratory.

In the present chapter, the water particle kinematics obtained by Stoke's Fifth Order Theory (ref.[22]) and Linear Wave Theory are compared and the results are used to obtain the hydrodynamic coefficients.

2.2 Theory of Periodic Waves (Airy's theory).

2.2.1 Basic Fundamental Equations

Velocity Potential

Velocity potential (ϕ) is a scalar function of space and time such that its partial derivative with respect to any direction gives the component of velocity in the

direction of wave propagation

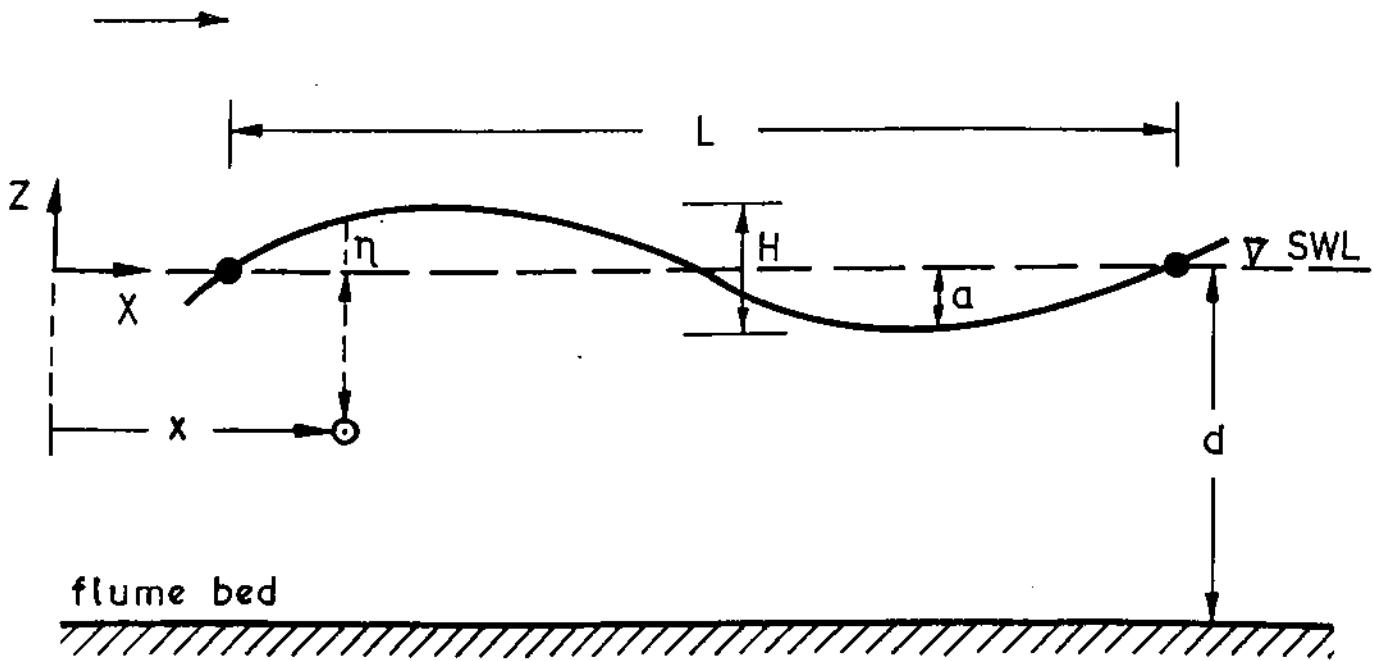


Fig. 2.1

Definition sketch for a progressive wave train
ref. [45] (Airy's theory).

corresponding direction.

According to Bernoulli's equation :

$$\frac{p}{\rho} + \frac{1}{2} q^2 + gz + \frac{\partial \phi}{\partial t} = 0 \quad \dots \dots \dots (2.1)$$

where

p = pressure at any point

ρ = mass density of the fluid

q = velocity vector

z = elevation from the still water level

g = acceleration due to gravity

t = time instant

ϕ = velocity potential

Consider a point $z = \eta$ on a free surface. Neglecting the velocity vector term (q^2) and applying Dynamic Free Surface Boundary Conditions (DFSBC) which implies pressure = 0 (atmospheric), we get

$$\rho \eta + \frac{\partial \phi}{\partial t} = 0 \quad \dots \dots \dots (2.2)$$

$$\eta = - \frac{1}{g} \left(\frac{\partial \phi}{\partial t} \right)_s \approx - \frac{1}{g} \left(\frac{\partial \phi}{\partial t} \right)_s = 0 \quad \dots \dots \dots (2.3)$$

Applying Kinematic Free Surface Boundary Condition (KFSBC) which implies that the particle on free surface remains on free surface, we get

$$w dt = d\eta \quad \dots \dots \dots (2.4)$$

or,

$$w = \frac{d\eta}{dt} \quad \dots \dots \dots \quad (2.5)$$

or,

$$\frac{\partial \theta}{\partial z} = \frac{\partial \eta}{\partial t} \quad \dots \dots \dots \quad (2.6)$$

Therefore, $\frac{d\eta}{dt} \approx \left(\frac{\partial \theta}{\partial z}\right)_z = 0 \text{ or } = n \quad \dots \dots \dots \quad (2.7)$

Equations (2.3) and (2.7) give,

$$\frac{\partial^2 \theta}{\partial t^2} + g \frac{\partial \theta}{\partial z} = 0 \quad \dots \dots \dots \quad (2.8)$$

Bottom Boundary Condition (BBC) states that at bed, the velocity in vertical direction will be zero which can be expressed as follow :

$$\frac{\partial \theta}{\partial z} = 0 \quad @ z = -d \quad \dots \dots \dots \quad (2.9)$$

Let $\theta = \theta(x, z, t)$
 $= f_x(x) \cdot f_z(z) \cdot f_t(t)$

Putting the expression in Laplace's equation ($\nabla^2 \theta = 0$), and applying BBC and DFSBC, we obtain the expression for θ . Once the expression for velocity potential is obtained, the velocity and acceleration of water particle can be found in desired direction by taking the derivative of θ in that direction.

2.2.2 Airy's Theory (Linear Wave Theory or Small Amplitude Theory.)

In analyzing a two dimensional wave train, two problems arise. The first one is due to the non-linearity of the free surface and the second one is due to the boundary conditions, specified at the free surface. To overcome these problems the theory assumes that the wave height is much smaller than both the wave length and the still water depth ($H \ll L, d$), so that the non linear terms which involve the product of wave height are negligible in comparison to other terms. Applying the boundary conditions, the Linear Wave Theory expresses the velocity potential (\emptyset) as follows :

$$\emptyset = \frac{gH}{2\omega} \frac{\cosh(k(z+d))}{\cosh(kd)} \sin(kx - \omega t) \dots\dots\dots(2.10)$$

wherein,

$\omega = 2\pi / T$ = angular wave frequency

$k = 2\pi / L$ = wave number

L = wave length

H = wave height

d = still water depth

Putting this value of \emptyset (equation 2.10) in equation (2.8), leads to the equation of the wave profile and linear dispersion relationship, respectively.

$$\eta = \frac{H}{2} \cos(kx - \omega t) \quad \dots \dots \dots \quad (2.11)$$

$$\omega^* = gk \tanh(kd) \quad \dots \dots \dots \quad (2.12)$$

If the reference frame moves with the same velocity as that of the wave, the waveform ' η ' will appear stationary, that is

$$kx - \omega t = \text{constant} \quad \dots \dots \dots \quad (2.13)$$

$$\text{therefore } kdx - \omega dt = 0. \quad \dots \dots \dots \quad (2.14)$$

$$\text{Hence } \frac{dx}{dt} = \text{wave celerity or speed} = \omega/k = L/T = C_0 \quad \dots \dots \dots \quad (2.15)$$

Substituting ω^* from equation (2.12) gives the value of C_0 .

$$C_0^* = (g/k) \tanh(kd) \quad \dots \dots \dots \quad (2.16)$$

$$\text{or } C_0 = ((gT/2\pi) \tanh(kd))^{1/2} \quad \dots \dots \dots \quad (2.17)$$

Using equation (2.17)

$$\theta = \frac{\pi H \cosh(k(d+z))}{k T \sinh k d} \sin(kx - \omega t) \quad \dots \dots \dots \quad (2.18)$$

The water depth (d) can be classified as deep, intermediate and shallow. According to Airy's theory, the criterion for the water depth is as follows:

- a) Shallow water waves $d/L < 1/20$; $0.0025 > d/gT^2$.

b) Intermediate depth waves $1/2 > d/L > 1/20$; $0.0025 < d/gT^2 < 0.08$.

c) Deep water waves $d/L > 1/2$; $d/gT^2 > 0.08$.

Equation (2.16) can be simplified in different water ranges as shown below :

(a) Deep water region ($d > L/2$)

$\sinh(k \cdot d) \approx e^{k \cdot d} / 2 \approx \cosh(k \cdot d)$, therefore celerity

$$C_o = gT/2\pi \quad \dots \dots \dots \quad (2.19)$$

$$\text{and } L_o = gT^2 / 2\pi \quad \dots \dots \dots \quad (2.20)$$

(b) Shallow water region ($d < L/20$)

$\sinh(k \cdot d) \approx k \cdot d$ and $\cosh(k \cdot d) \approx 1$, therefore celerity

$$C_o = (gd)^{1/2} \quad \dots \dots \dots \quad (2.21)$$

To obtain the wave length in shallow water we use:

$$\frac{d}{L_o} = \frac{d}{L} \tanh 2\pi \frac{d}{L} \quad \dots \dots \dots \quad (2.22)$$

From equation (2.18), expressions for velocities u and w can be obtained as:

$$u = \frac{\partial \theta}{\partial x} = \frac{\pi H \cosh(k(d+z))}{T \sinh(k \cdot d)} \cos(kx - \omega t) \quad \dots \dots \dots \quad (2.23)$$

$$w = \frac{\partial \theta}{\partial z} = \frac{\pi H \sinh(k(d+z))}{T \sinh(k \cdot d)} \sin(kx - \omega t) \quad \dots \dots \dots \quad (2.24)$$

Then taking the derivatives against time "t", we get the accelerations in x and z directions respectively :

$$u = \frac{\partial u}{\partial t} = \frac{2\pi^2 H \cosh(k(d+z))}{T^2 \sinh(kd)} \sin(kx - \omega t) \quad \dots \dots \dots (2.25)$$

$$w = \frac{\partial w}{\partial t} = \frac{-2\pi^2 H \sinh(k(d-z))}{T^2 \sinh(kd)} \cos(kx - \omega t) \quad \dots \dots \dots (2.26)$$

where u and w give accelerations at the free surface of the liquid in x and z directions respectively.

2.2.3 Stoke's Fifth Order Theory

Stoke's Fifth Order theory [47] gives the velocity as follows :

$$u = \frac{\partial \theta}{\partial x} = C(K_1 \cos \theta + 2K_2 \cos 2\theta + 3K_3 \cos 3\theta + 4K_4 \cos 4\theta + 5K_5 \cos 5\theta). \quad \dots \dots \dots (2.27)$$

The water particle acceleration, may be derived from equation (2.27), $\ddot{u} = \frac{\partial u}{\partial t}$

$$= \beta' (\bar{C})^2 (K_1 \sin \theta + 4K_2 \sin 2\theta + 9K_3 \sin 3\theta + 16K_4 \sin 4\theta + 25K_5 \sin 5\theta) \quad \dots \dots \dots (2.28)$$

The details of these constants are available in Skjelbreia and Handrickson [47].

2.3 Comparison Of The Results Obtained By Stoke's Fifth Order Wave Theory and Airy's Theory

In the present work, the accuracy of water particle kinematics is checked by comparing two wave theories,

Run # 3518 Date 7 Apr 1978 Plate diam .08 M Roughness 0 mm
 Phi X= 52.71 deg K-C= B, 943 Ra= 38746
 Wv Period= 2.02 sec Wave Ht= .2184 M Wv Length= 5.888 M Um_{max}= 3881 M/sec
 Cd(av) .1098 Cm(av) 1.018 CL(av) .007458

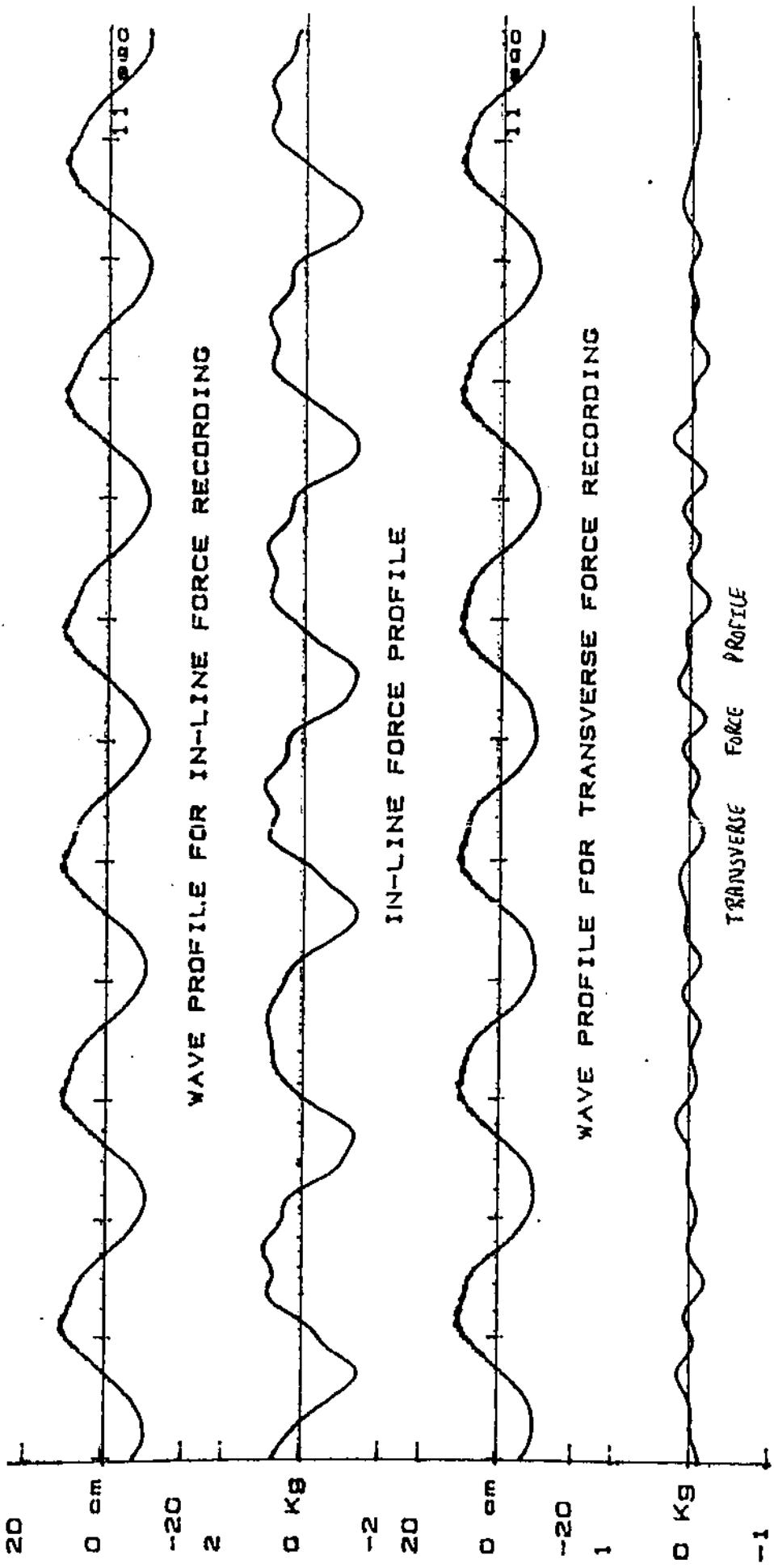


Fig. No. 2.2 Wave and Force Profiles Recordings.

mentioned above, for the same wave profile recordings.

The analog recordings of wave and force profiles (Fig. 2.2), in total ninety eight in number were available from the experimental work observations conducted by Ghoshal [22]. In this case Stoke's Fifth Order Wave Theory was used to calculate the water particle kinematics. From these recordings it can be observed that the range of time period (T) varied from 2 to 3 seconds, wave height (H) varied from 12 to 32 cms. The still water depth was however, kept constant at 1.2m.

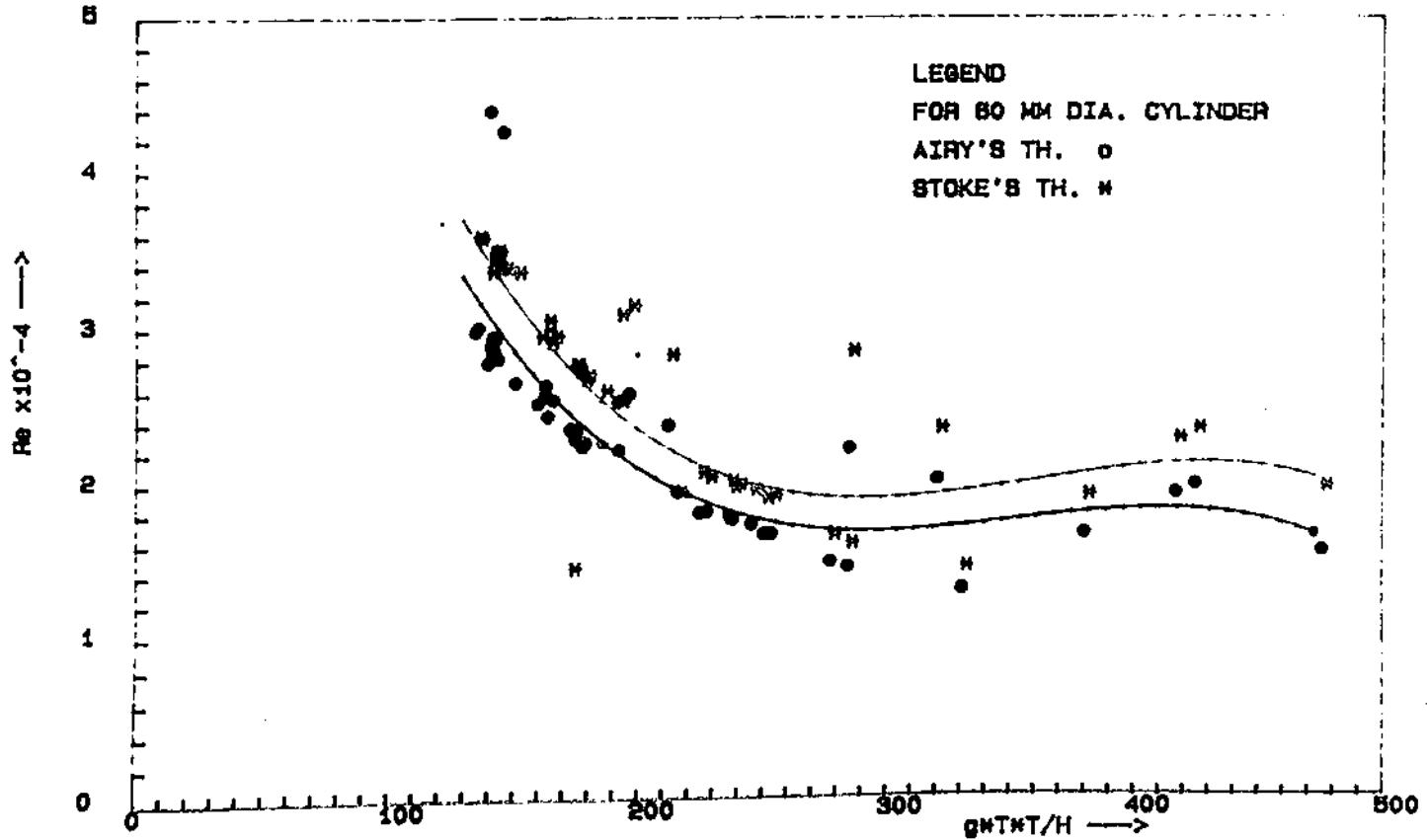
From each of the 98 recordings one complete wave cycle corresponding to phase angle form 0° to 360° had been selected. Each of these wave cycles were manually discretized to eight equal parts and the ordinates of each part was measured thereafter. These measured ordinates along with other necessary parameters (Diameter of cylinder (D), wave height (H), Time period (T) and still water depth (constant at 1.2m)) were used as the input data for the program "Wv.Th.", (presented in Appendix-B) which was developed for processing these input data to get the outputs, namely : gT^2/H , Re and K-C using Airy's theory. These outputs are presented in tabular form in the Appendix-D, and in graphical form at the end of this chapter (Fig. 2.3 and Fig. 2.4). A curve fitting technique was used to draw the weighted average lines. For the purpose of

graphical comparison the non-dimensional parameter gT^2/H was taken as the common parameter on x-axis. On y-axis, two different parameters Re and K-C were represented.

2.3.1 Results

From figures 2.3 and 2.4, it may be clearly observed that Stoke's Fifth Order Theory gives marginally higher values of Re and K-C as compared to the values obtained by Airy's Theory, for the identical gT^2/H value. It is primarily due to consideration of the non-linear terms in Stoke's Fifth Order Theory which are neglected in Airy's Theory. The results clearly verify the values of water particle kinematic obtained by Ghoshal [22] using Stoke's Fifth Order Theory. Therefore, the values obtained in Ghoshal [22] for hydrodynamic coefficients and non-dimensional parameters were used for the present study.

TH - 4797



K-C —>

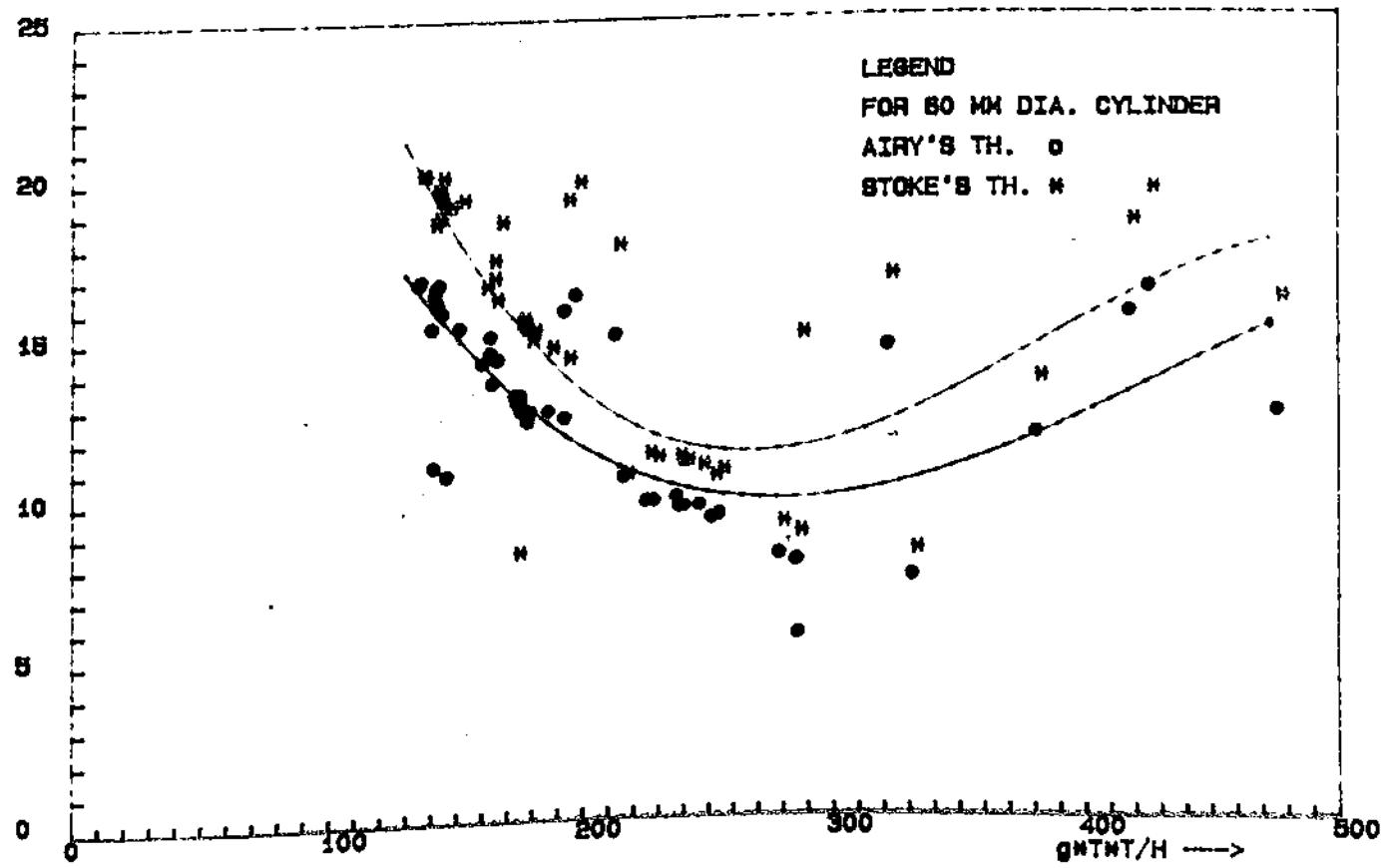


Fig. No. 2.3

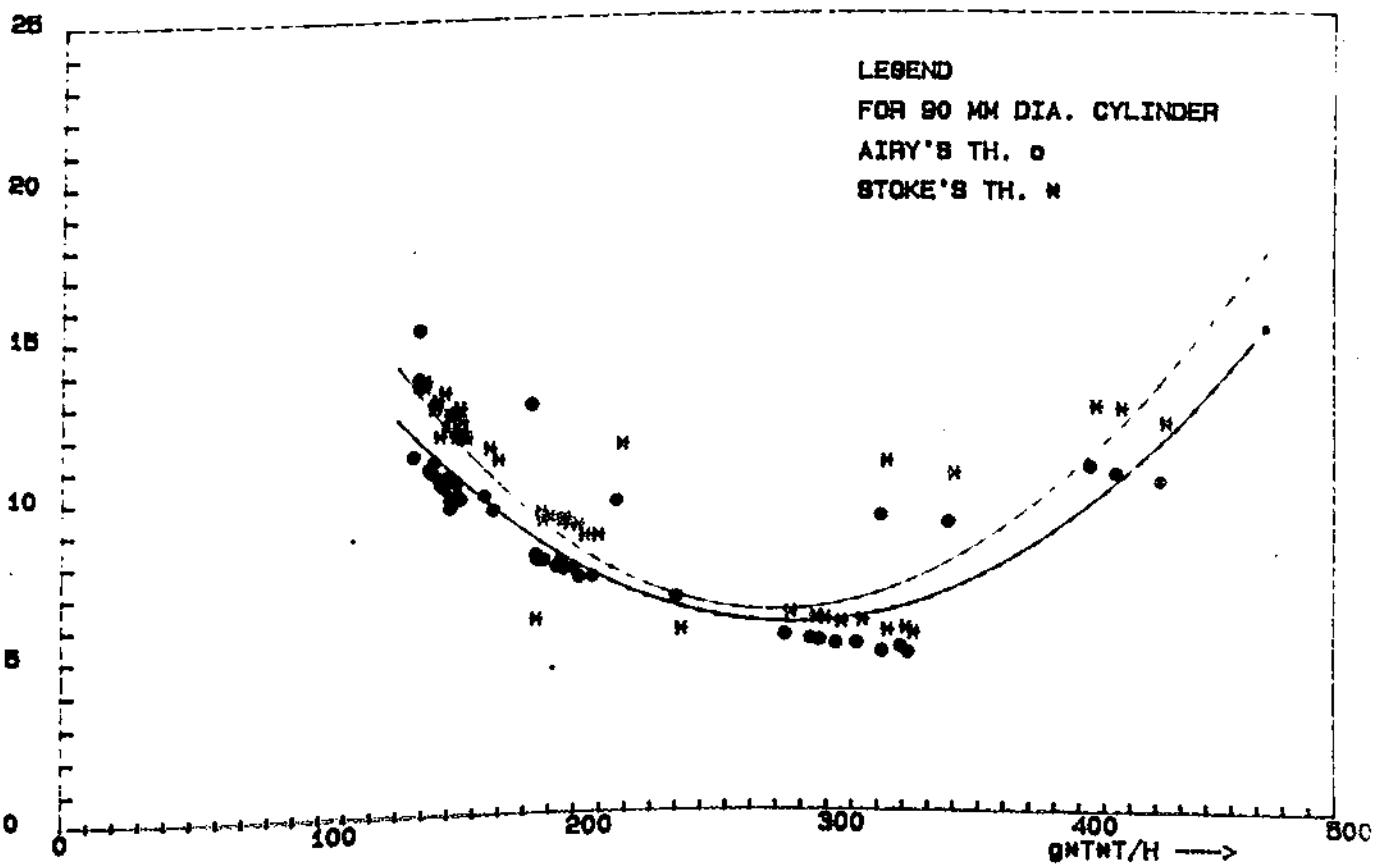
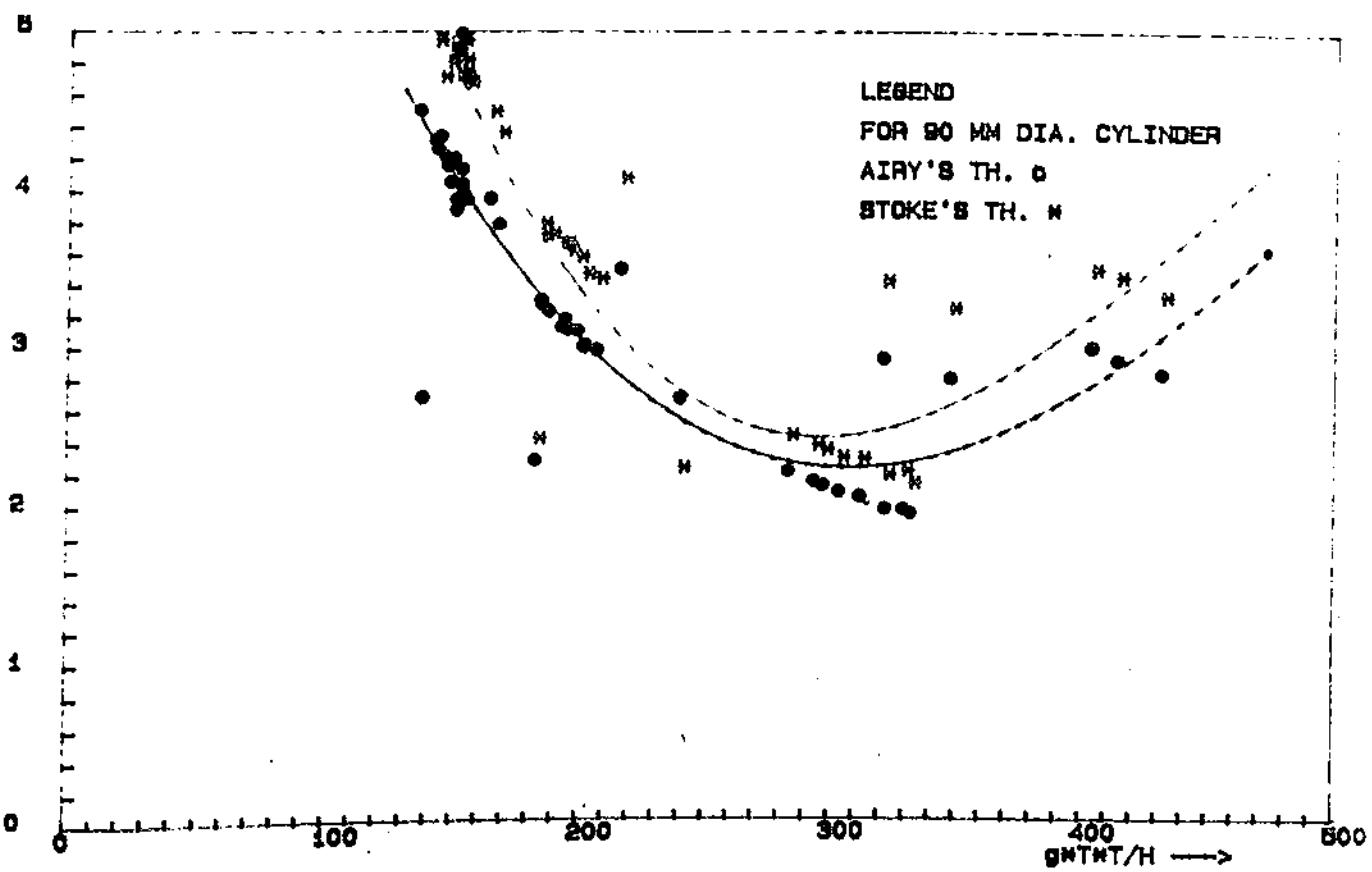


Fig. No. 2.4

CHAPTER 3

DISCUSSIONS

3.1 Introduction

Ghoshal [22] has analyzed the data collected experimentally for 60mm diameter inclined and rough cylinders using Stoke's Fifth Order theory and Fourier Series Data Reduction Technique. The present work includes the analysis of data collected by Ghoshal [22] for 90mm diameter cylinder in this chapter. The chapter also includes the effect of increase in diameter by comparing the results of 90mm diameter and 60mm diameter cylinders.

The data base (values of hydrodynamic coefficients and non-dimensional parameters) which had been used for this purpose was available in the Appendix - B of Ghoshal [22] where it was presented in a tabular form for various sets with boundary conditions i.e. cylinder diameter, in-line and lateral inclinations, and surface roughness parameter.

The variation of Drag, Inertia and Lift coefficients against Reynolds number (Re) and Keulegan - Carpenter number ($K-C$) are presented in Appendix-F from figures F-1 through F-30. In each figure for each set of data, a best-fit curve was superimposed over the data points.

3.2 In-line force coefficients (C_d and C_m)

3.2.1 Effects of Surface Roughness

For cylinders inclined in negative in-line direction, it is observed that the values of C_d are higher for smooth cylinders as compared to the rough cylinders (Fig. F-3). However, for subsequent increase in roughness, C_d is not influenced much by the surface roughness.

For cylinders with positive in-line inclinations the value of C_d is marginally higher for smooth cylinders as compared to the rough cylinders (Fig. F-2).

In the present work, C_d is not influenced by the surface roughness for vertical, and laterally inclined cylinders (Fig. F-1, F-4). Qualitatively, the results are agreeing with that of Nath [38]. The average value of C_d for vertical and laterally inclined cylinders is about 0.1, whereas for inclined (7.2°) cylinders, C_d varies from 0.5 to 0.1.

Hogben et al. [26] specified three ranges for relative surface roughness and indicated that for the intermediate range ($k/D = 0.002$ to 0.01), C_d is constant. In the present study the relative roughness parameters were, $1/90$ (≈ 0.01) and $1/180$ ($= 0.005$), which fall within the specified

intermediate range.

Within the experimental range, the cylinders might have behaved as hydrodynamically smooth as far as the drag force is concerned. For cylinders with negative in-line inclinations, the possible cause could be the hindrance in the flow field created by the cylinder, due to which the flow field changes and roughness plays its role but the subsequent increments in roughness do not effect the coefficient much.

Sarpkaya et al. [44] obtained the value of C_d for rough cylinder as around 1.8, whereas Nath [39] for almost identical surface roughness and other boundary conditions, obtained the value of C_d to be around 0.8. From Nath's work, it can be concluded that C_d and C_m are not significantly influenced by the surface roughness.

As far as the inertia coefficient is concerned, the inertia coefficient increases with the increase in surface roughness (Fig. F-12, F-13). This increase of C_m is most prominent for the initial change from smooth to rough but for subsequent increase in roughness, the change is marginal.

For laterally inclined (Fig. F-14) or for vertical (Fig. F-11) cylinders, the coefficient of inertia remains unaffected against any change in surface roughness. The observations by Nath [39] or Chakrabarti [9] are also identical with the present observations but not agreeable

with the results obtained by Sarpkaya [41].

3.2.2 Effects of Inclination

In the present study, C_d increases with the increase of positive in-line inclination (θ_x) (Fig. F-5, F-6) which is true for all cases of roughness. However, the change in magnitude is more pronounced when the cylinder achieves the first initial inclination (from 0° to 7.2°). For subsequent changes in inclination the change in magnitude is not so clearly distinguishable for smooth cylinders (Fig. F-5).

For cylinders with negative in-line inclinations (Fig. F-7, F-8), C_d increases uniformly for smooth cylinders (Fig. F-7). C_d is not effected much by the initial change of the angle of inclination from the vertical one, but for subsequent increase in inclination, the difference is pronounced for rough cylinders (Fig. F-8).

The value of C_d remains constant at around 0.075 for vertical cylinders and varies from 0.4 to 0.075 for inclination, $\theta_x = 7.2^\circ$. The value of C_d changes from 0.6 to 0.1 for inclination 18.7° and from 0.6 to 0.1 in case of 27.5° . These values increase marginally for rough cylinders for identical inclination angles.

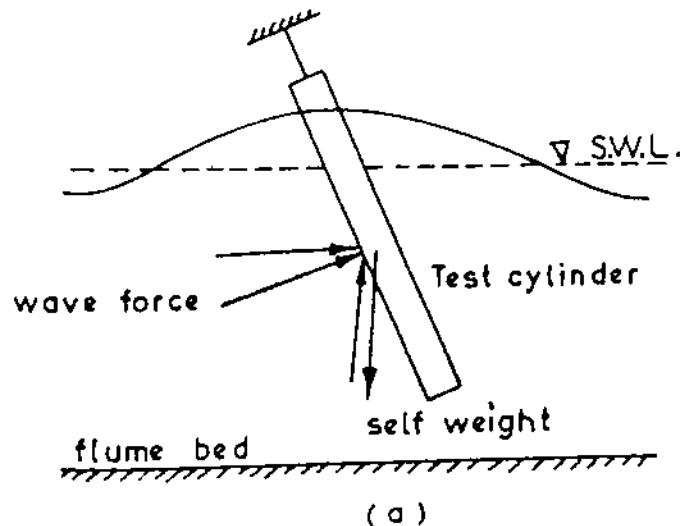
For cylinders with lateral inclination, C_d decreases marginally with increase in angle of inclination (Fig. F-9,

F-10) which is more pronounced in case of smooth cylinders as compared to rough cylinders. The results agree with that of Chakrabarti [12] who conducted the experiments with smooth cylinders.

As far as the coefficient of inertia is concerned, the effect of in-line inclination is not as clear as it is in case of coefficient of drag (Fig. F-15, F-16, F-17, F-18). For the initial change from its vertical position, that is for $\theta_x = 7.2^\circ$ (Fig. F-15), the magnitude of C_m is lower as compared to that of the vertical cylinder. For an intermediate angle like $\theta_x = 18.7^\circ$, the variation of C_m is almost identical as that of the vertical cylinder. Finally for an angle of inclination, like $\theta_x = 27.5^\circ$, the inertia coefficient is higher as compared to the vertical cylinder. This pattern persists for $Re < 45000$, beyond which the difference is not significant. Moreover, for cylinders with negative in-line inclinations the pattern is found to be more scattering (Fig. F-17, F-18). This scattering is reflected in relatively greater undulation of the best-fit curves, specially for cylinders with higher degree of inclinations like $\theta_x = -30.2^\circ$. The reason behind this behaviour may be attributed to the following phenomena.

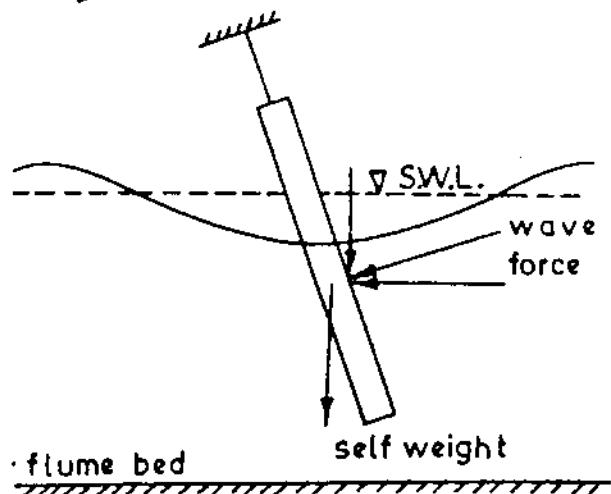
If we consider two extreme conditions of wave propagation, that is the passing of the crest and the passing

Direction of wave propagation



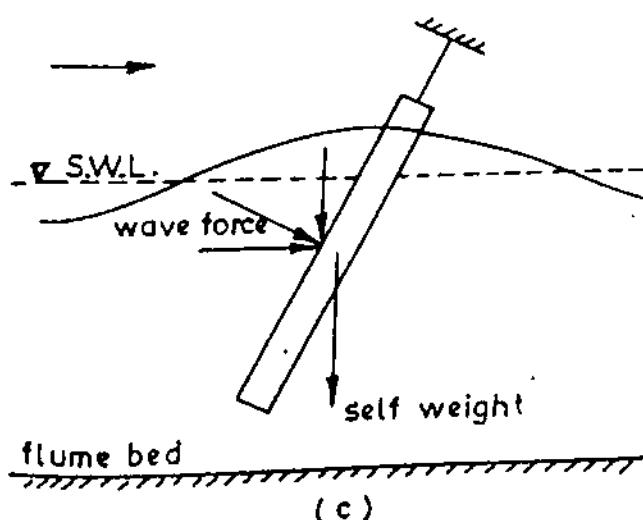
(a)

Wave crest passing the cylinder.
Maximum length of cylinder immersed
Maximum wave force.
Component of wave force acting
opposite to self weight.
Positive in-line inclination.



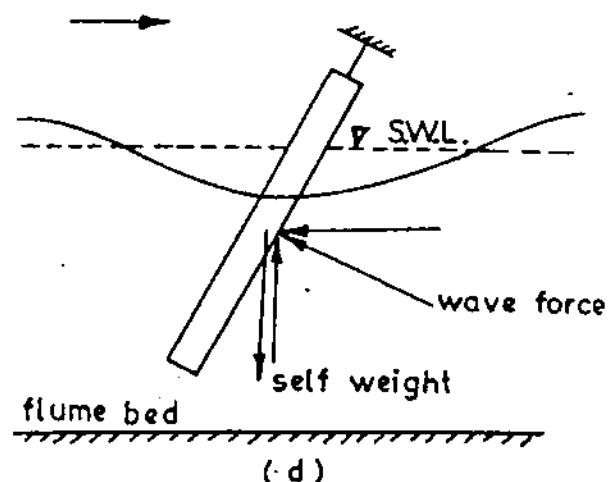
(b)

Wave trough passing the cylinder.
Minimum length of cylinder immersed
Minimum wave force.
Component of wave force acting
in direction of self weight.
Positive in-line inclination.



(c)

Wave trough passing the cylinder.
Maximum length of cylinder immersed
Maximum wave force.
Component of wave force acting in
direction of self weight.
Negative in-line inclination.



(d)

Wave crest passing the cylinder.
Minimum length of cylinder immersed
Minimum wave force.
Component of wave force acting
opposite to self weight.
Negative in-line inclination.

Fig.No. 3.1

Variation of forces for positive and negative in-line inclinations.

of the trough (Fig. 3.1), we can readily observe that the immersed length of the cylinder varies considerably for the condition when the resolved direction of the active wave force lies in the same direction as that of the self weight of the cylinder. This is also true for the opposite condition when the resolved direction of the active wave force lies in the opposite direction of that of the self weight of the test cylinder. Considering an immersed cylinder length in still water to be approximately 1.2m and an average wave height of 20cms, this variation is clearly reflected for the data sets with larger angles of in-line inclination. The interpretation of results become further difficult if the effects of the self excitation of the test cylinders, non-uniformity of the effect of eddy shedding throughout the length of the cylinder due to its in-line inclinations and complexity of the flow field around the boundary layers (ref. [21], [23], [26]) are taken into account.

The observations of the present study regarding the effect of in-line inclinations against the inertia coefficient do not agree with the results of Chakrabarti et al. [11,12,14 and 15] who observed a trend of decreasing value of C_m against the increasing angle of in-line inclinations. According to them for $K-C > 5$, the value of C_m for the 45° inclined cylinder is less than that for the 30° inclined cylinder and both of these are lower than that for

the vertical cylinder.

The coefficient of inertia is not significantly effected by the angle of the lateral orientation of the cylinder (Fig. F-19, F-20) which is true for all cases of surface roughness. This trend had already been indicated in case of the coefficient of drag and agrees with Chakrabarti [11,12,14,15].

3.2.3 Dependence upon Non-Dimensional parameters (K-C and Re)

Within the experimental range C_d seems to be dependent upon both Re and K-C whereas C_m seems to depend more on Re than upon K-C. For all types of in-line inclinations, C_d decreases with the increase of either Re or K-C. For vertical and laterally inclined cylinders, C_d remains more or less constant within the experimental range for any variation of Re and K-C.

The coefficient of inertia also decreases with the increase of Re or K-C for most of the cases but beyond the value of $Re \approx 45000$ or $K-C \approx 12$, an increase has been observed.

The relation of C_d against Re obtained in the present work, that is C_d decreases with increase in Re, agrees fairly well with the results of other investigators, such as Garrison [20], Sarpkaya [45, page 110], and others, but the

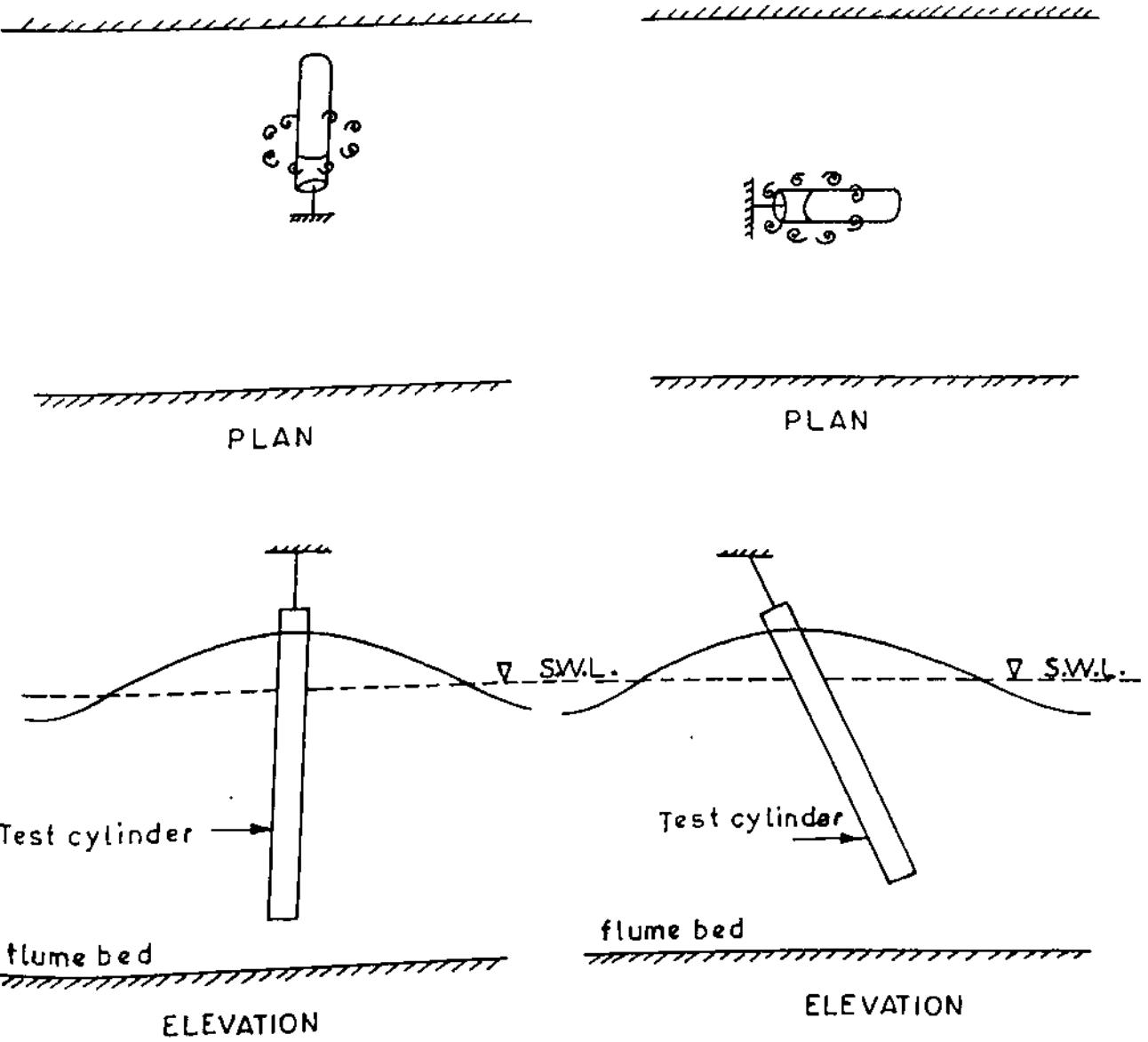
relation of C_L against Re does not agree with the results presented by other researchers, except Starsmore [48].

3.3 Lift coefficient (C_L)

3.3.1 Effects of Roughness and Inclination

Unlike the behaviour of the in-line force coefficients, which had been discussed in the previous section, the effect of surface roughness is extremely prominent against the variation of the lift coefficient (C_L). This may readily be observed by comparing the figures F-25 and F-26 where the first figure (F-25) is related to smooth cylinders with varying in-line positive inclinations and the second one (F-26) is related to the varying cylinder inclinations, but for a constant surface roughness ($k/D = 1/90$). The pronounced changes in the magnitude of C_L for these two cases indicate the influence of surface roughness against the lift coefficient for inclined cylinders. The identical trend may also be observed by comparing the figures F-27 and F-28, which depict the effects of negative in-line inclinations.

The reason behind this influence of surface roughness over the lift coefficient is the change in the behaviour of boundary layers resulting in a change in the formation and shedding of the vortices which are primary contributors to the lift force. In the present study it was observed that the



(a) Relative positions of eddies for a laterally inclined cylinder.

(b) Relative positions of eddies for a in-line inclined cylinder.

Fig. 3.2 Relative positions of eddies for a laterally inclined & in-line inclined cylinders.

coefficient of lift increases with the increase in the surface roughness, irrespective of its angular orientation, in-line as well as lateral. These observations do not agree with the results obtained by Sarpkaya et al. [45] who had concluded that the increase of the roughness parameter (k/D) decreases the magnitude of the C_L . The reason behind this disagreement may be attributed to the absence of any free surface and the lack of phase angle difference between the water particle velocity and acceleration pertaining to Sarpkaya's U-tube experimental set-up, both of which are present in the wave - flume experiments and having more similarity with the marine environment.

The effect of in-line inclination against the lift coefficient is not noticeable in the present study for both positive as well as negative in-line inclination (Fig. F-25, F-27) in case of smooth cylinders. However, for cylinders with non-smooth surfaces (Fig. F-26, F-28) there is some effect of the in-line orientation. In these cases, the first change of cylinder inclination with respect to vertical increases the magnitude of C_L , although subsequent increments in the angle of inclination either reduces the lift coefficient (Fig. F-26) or create marginal change in the relation of C_L against K-C (Fig. F-28)

Unlike the in-line one, a lateral inclination seems to have a more visible influence over the lift coefficient (Fig.

F-29, F-30). In this case, even for a smooth cylinder (Fig. F-29) the coefficient of lift increases by a large magnitude for the change of orientation from vertical to a lateral one. For a rough cylinder with lateral inclination (Fig. F-30), this behavioural pattern undergoes further changes with the inclination of a semi - peak at a relatively lower value of K-C (≈ 6.5). The reason behind this is the relative positions of vortices as indicated in Fig. 3.2. Eddies are always observed to form and travel by the side of the test cylinders rather than its front or back, related to the direction of wave propagation. Therefore the pressure fields inside the vortices are more predominant in case of laterally inclined cylinders than the in-line inclined one.

3.3.2 Dependence upon Non-dimensional Parameters (K-C and Re)

Within the experimental range, the coefficient of lift was observed to depend more on K-C than upon Re. Initially upto $K-C \approx 6$, C_L decreases with the increase in K-C. For further increment of K-C and upto $K-C \approx 12$, C_L increases steadily and attains a peak value at $K-C \approx 12$. Beyond this for most of the cases except for laterally inclined cylinders and the cylinders with negative in-line inclinations, C_L decreases with the increase in K-C. In general the present observations agree qualitatively with the results of Bidde [5], Chakrabarti [9] and Sarpkaya et al. [41,45].

3.4 Comparison of the results of 60mm and 90mm diameter cylinders

In this section, the observations of the present study is compared with that of Ghoshal [22]. For the purpose of better visualization, figures C-31 to C-39 of Appendix - C of Ghoshal [22] pertaining to the results using 60mm diameter cylinders had been redrawn and presented in figures 3.3 to 3.11 respectively in this document. It may be noted here that in the original figures of Ghoshal [22] only the data points were plotted whereas the weighted - average lines had been introduced at the time of redrawing them.

3.4.1 Range of Re and K-C

As far as the Reynolds number range is concerned, the present study is based from 11000 to 53000 whereas in case of Ghoshal [22] this was from 8000 to 35000. The Keulegan - Carpenter number, on the other hand, ranged from 4 to 14 in case of the present study, whereas for Ghoshal [22] it was from 6 to 21. Therefore the usefulness of the present study is primarily reflected at the behaviour of the hydrodynamic coefficients at a higher Reynolds number range.

3.4.2 Coefficient of Inertia (C_m)

Figures 3.3, 3.4 and 3.5 contain the details of plot of 90mm diameter and 60mm diameter cylinders for identical

boundary conditions, that is inclination and surface roughness.

In case of in-line inclined cylinders (Fig.No. 3.3), it may be observed that for identical value of Re and same angle of inclination, the magnitude of C_d is more for 90mm diameter cylinder as compared to that of 60mm diameter cylinder.

The nature of the weighted average lines for laterally inclined cylinders is different from those for vertical cylinders (Fig.No. 3.4).

From the weighted average lines of C_d for rough cylinders (Fig.No. 3.5) it may easily be concluded that within the experimental zone of roughness, C_d remains unaffected by it. This had also been confirmed in Ghoshal [22].

Apart from the difference of magnitude of the values of C_d for 60mm and 90mm diameter cylinders for identical values of Re and cylinder inclination, there is no noticeable difference between the present study and Ghoshal [22].

3.4.3 Coefficient of Drag (C_d)

Figures 3.6, 3.7 and 3.8 contain the details of plot of 90mm diameter and 60mm diameter cylinders for identical boundary conditions, that is inclination and surface

roughness.

In case of in-line inclined cylinders (Fig.No. 3.6) it may be observed that the saddle-points for 90mm diameter cylinders exist at $Re \approx 42000$ whereas for 60mm diameter cylinders it occurs at a lower value of Re , that is $Re \approx 30000$. Furthermore, the weighted average lines for 90mm diameter cylinders clearly indicates a tendency of increase of C_d for higher values of Re beyond $Re \approx 42000$ which was not so clearly visible in case of the plotting of figure C-34 of Ghoshal [22]. Moreover, it may also be observed that for the identical value of Re and for similar cylinder inclination, the magnitude of C_d for 90mm diameter cylinder is more as compared to that of 60mm diameter cylinder. This factor is more pronounced in case of $\theta_x = -18.8^\circ$ or for $\theta_x = -30.2^\circ$ than for the case of $\theta_x = 0^\circ$, that is for a vertical cylinder.

In case of laterally inclined cylinders and also for rough cylinders, there are considerable amount of overlappings between the weighted average lines for 60mm and 90mm diameter cylinders, indicating that there is no substantial difference of behaviour between these two cases.

3.4.4 Coefficient of Lift (C_L)

Figures 3.9, 3.10 and 3.11 contain the details of plot of 90mm diameter and 60mm diameter cylinders for identical

boundary conditions, that is inclination and surface roughness.

For cylinders with in-line inclination (Fig.No. 3.9) the present study yields the peak value of C_L for a Reynolds number of around 45000 while for Ghoshal [22] they may be observed at a Re around 25000. For both the cases, that is for both 60mm as well as 90mm diameter cylinders, there seems to be a decrease of C_L value for greater inclination with respect to that produced by a vertical cylinder. This decrease is clearly distinguishable in case of 60mm diameter cylinders (Fig.No. 3.9) which was not indicated in section 5.2.2.2 of Ghoshal [22].

For cylinders with lateral inclinations (Fig.No. 3.10), the magnitude of the lift coefficient is higher for laterally inclined cylinders with respect to the vertical cylinders. From figure 3.10 it may also be observed that this increase in the magnitude of the lift coefficient is more pronounced in case of 90mm diameter cylinders than that of 60mm diameter cylinders. Another interesting aspect of the behaviour of lift coefficient is the change in the nature of the curve for the laterally inclined cylinder with respect to the vertical cylinder. With reference to figure 3.11 related to varying degree of surface roughness it may be observed that although the peak value for any particular roughness parameter is almost identical for both 60mm and 90mm diameter cylinders,

however, in case of the later one, the occurrence of the peak value, that is the crest point of the curve, shifts towards a higher Reynolds number, as compared to that of the 60mm diameter cylinder. The most important observation for the present comparison is the initial decrease of C_L value for increasing Reynolds number up to $Re \approx 20000$ for the 90mm diameter cylinders. This initial decrease pattern is not at all noticeable in case of 60mm diameter cylinders. However, for both cases it is clearly indicated that the value of C_L increases with the increase in the roughness parameter.

3.4.5 Reasons of difference

From the above discussions it may be noted that the major difference in the behaviour between 60mm diameter (Ghoshal [22]) and 90mm diameter cylinders is the lack of overlapping of hydrodynamic coefficient values for identical values of Reynolds number. In most of the cases, a sort of "shift" was observed. This could be primarily attributed to the self excitation of the test cylinder which is partially dependent upon its self weight and are different in case of 60mm and 90mm diameter cylinders.

3.5 Design Values

The present study of the variation of hydrodynamic coefficients against Re and $K-C$ is limited to a maximum

Reynolds number of 53000 and Keulegan - Carpenter number of 14 which are lower as compared to those values obtained in the marine environment. Therefore the design values indicated in the graphical relation presented in Appendix - F may not be readily applicable for the wave forces generated at higher Reynolds numbers.

However, within the range covered in the present study, the design values of C_d , C_m and C_L for smooth vertical cylinders may be taken as 0.1, 4.0 and 0.01 respectively. For marine roughened or inclined cylinders, a suitable interpolation related to the corresponding roughness and orientation, using the graphical relations presented in Appendix - F, may be done.

41

COEFF. OF INERTIA →

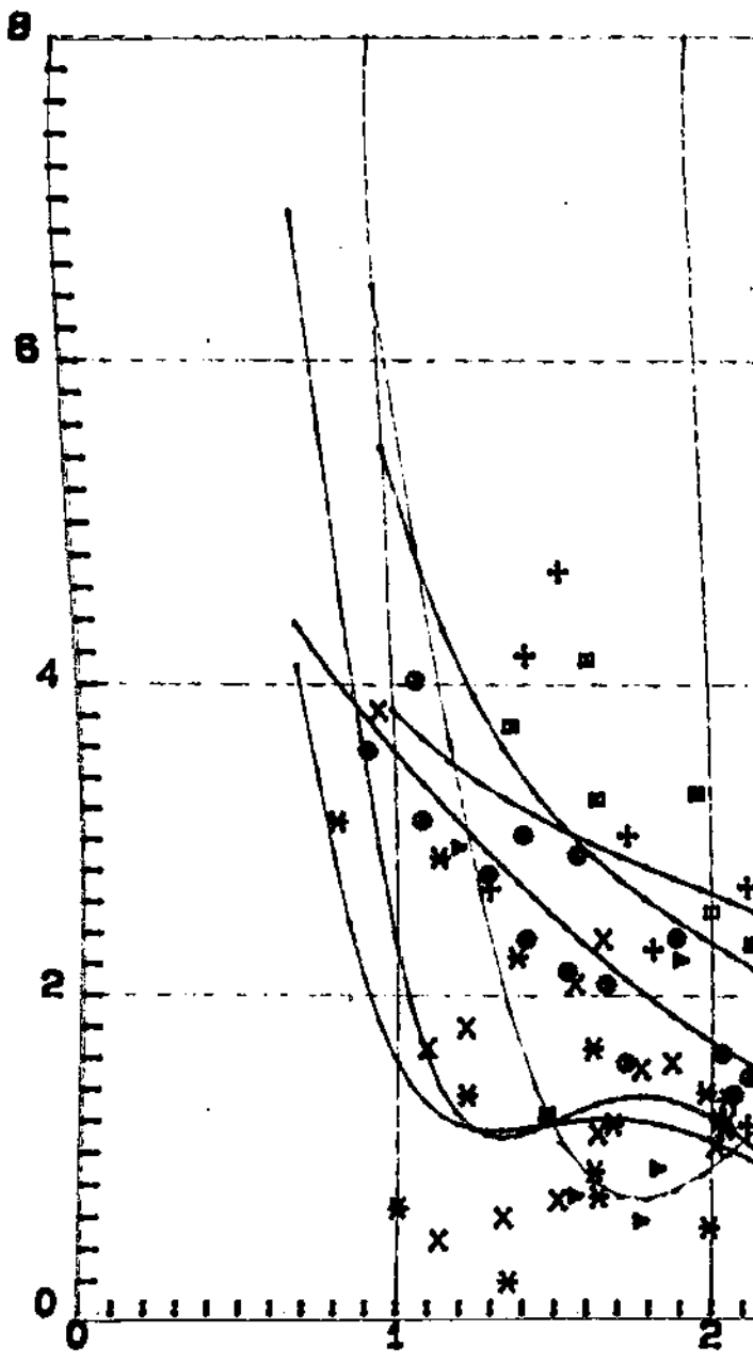
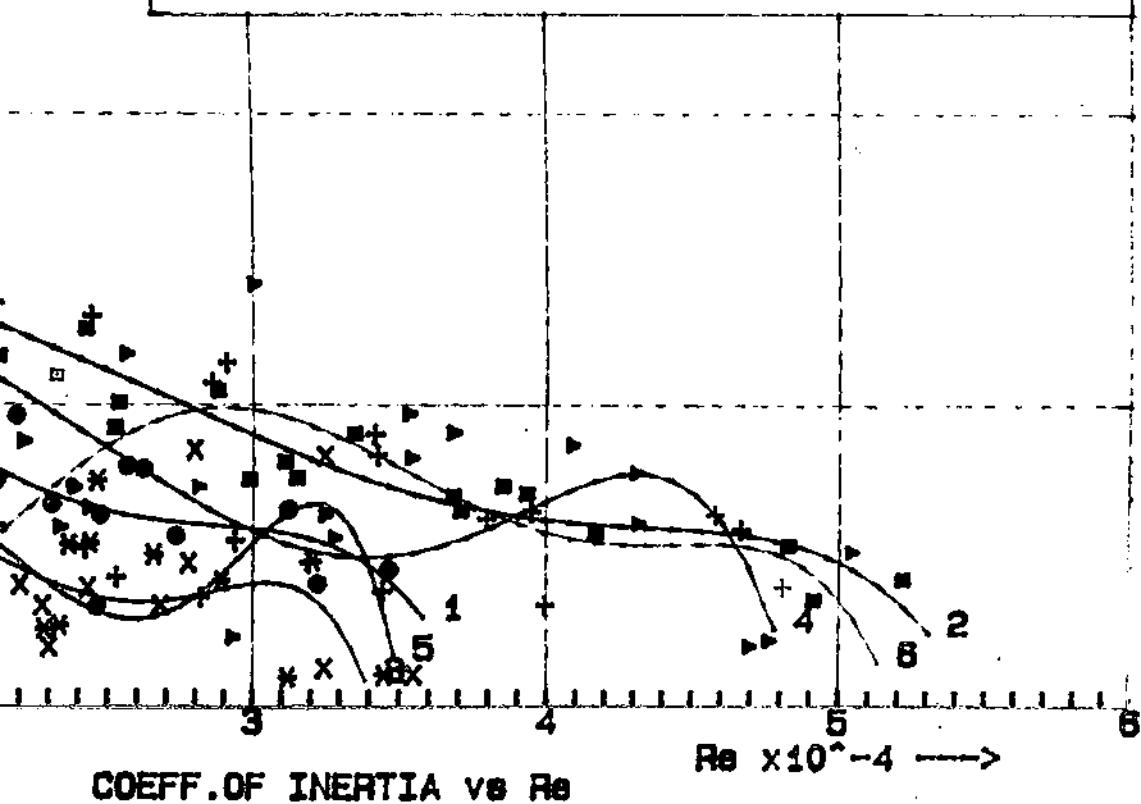
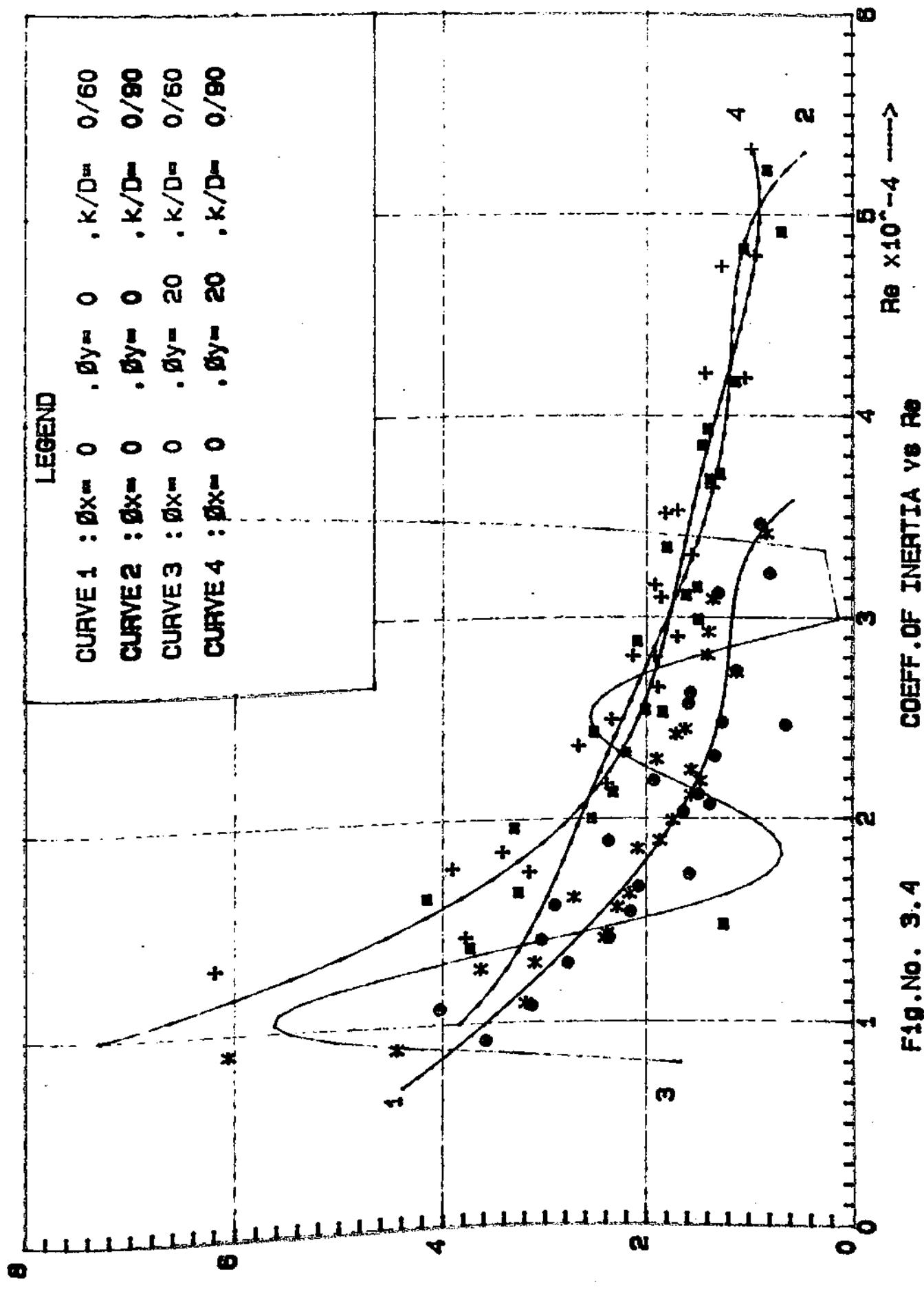


Fig. No. 3. 3

LEGEND

- CURVE 1 : $\theta_x = 0$, $\theta_y = 0$, $k/D = 0/60$
CURVE 2 : $\theta_x = 0$, $\theta_y = 0$, $k/D = 0/90$
CURVE 3 : $\theta_x = -18.8$, $\theta_y = 0$, $k/D = 0/60$
CURVE 4 : $\theta_x = -18.8$, $\theta_y = 0$, $k/D = 0/90$
CURVE 5 : $\theta_x = -30.2$, $\theta_y = 0$, $k/D = 0/60$
CURVE 6 : $\theta_x = -30.2$, $\theta_y = 0$, $k/D = 0/90$





COEFF. OF INERTIA

COEFF. OF INERTIA vs Re

Fig. No. 3.4

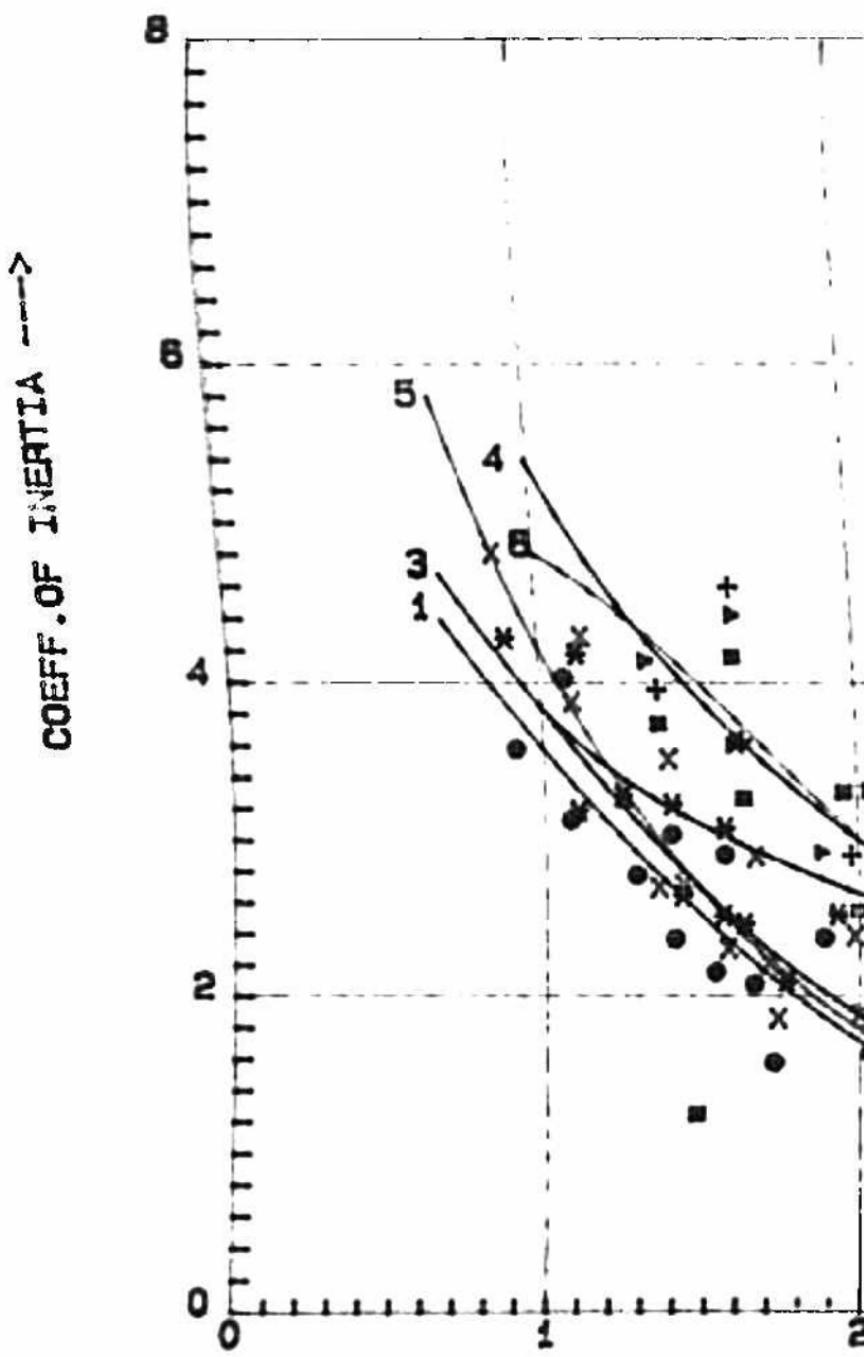


Fig.No. 3.5

LEGEND

- | | | |
|--------------------------|------------------|-----------------|
| CURVE 1 : $\theta_x = 0$ | , $\theta_y = 0$ | , $k/D = 0/60$ |
| CURVE 2 : $\theta_x = 0$ | , $\theta_y = 0$ | , $k/D = 0/80$ |
| CURVE 3 : $\theta_x = 0$ | , $\theta_y = 0$ | , $k/D = 1/120$ |
| CURVE 4 : $\theta_x = 0$ | , $\theta_y = 0$ | , $k/D = 1/180$ |
| CURVE 5 : $\theta_x = 0$ | , $\theta_y = 0$ | , $k/D = 1/60$ |
| CURVE 6 : $\theta_x = 0$ | , $\theta_y = 0$ | , $k/D = 1/90$ |

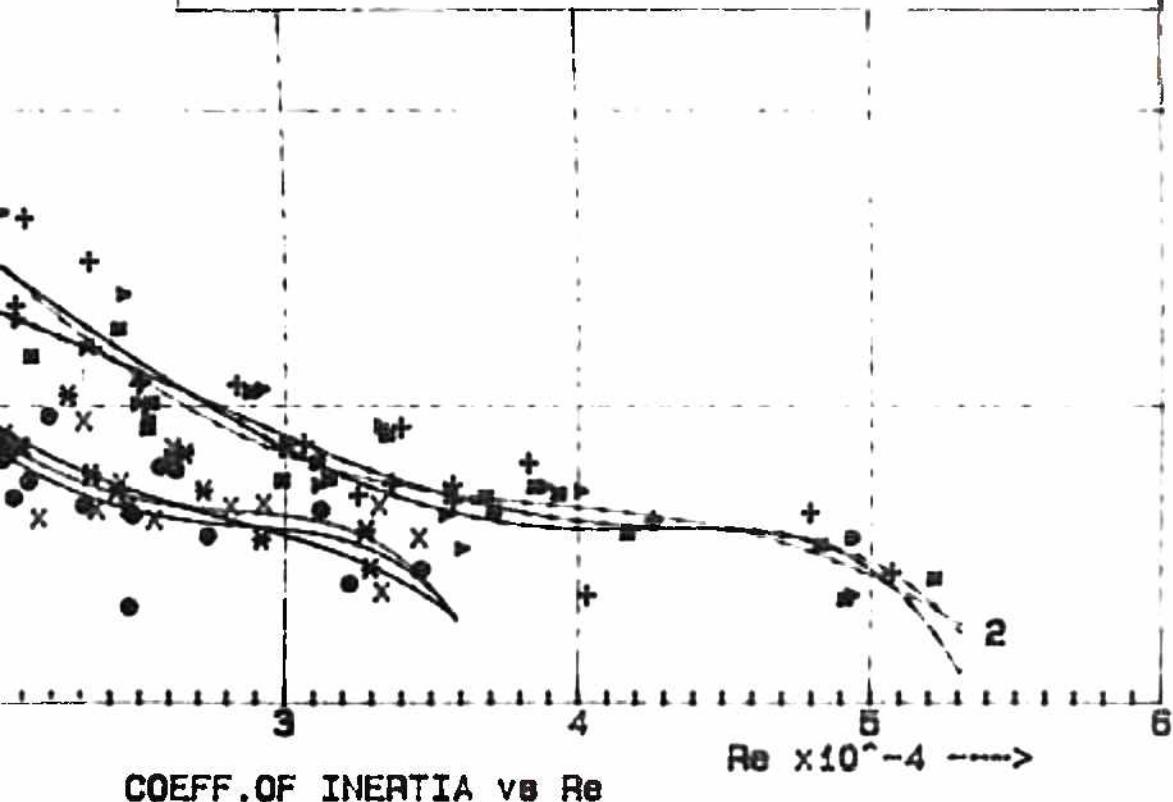
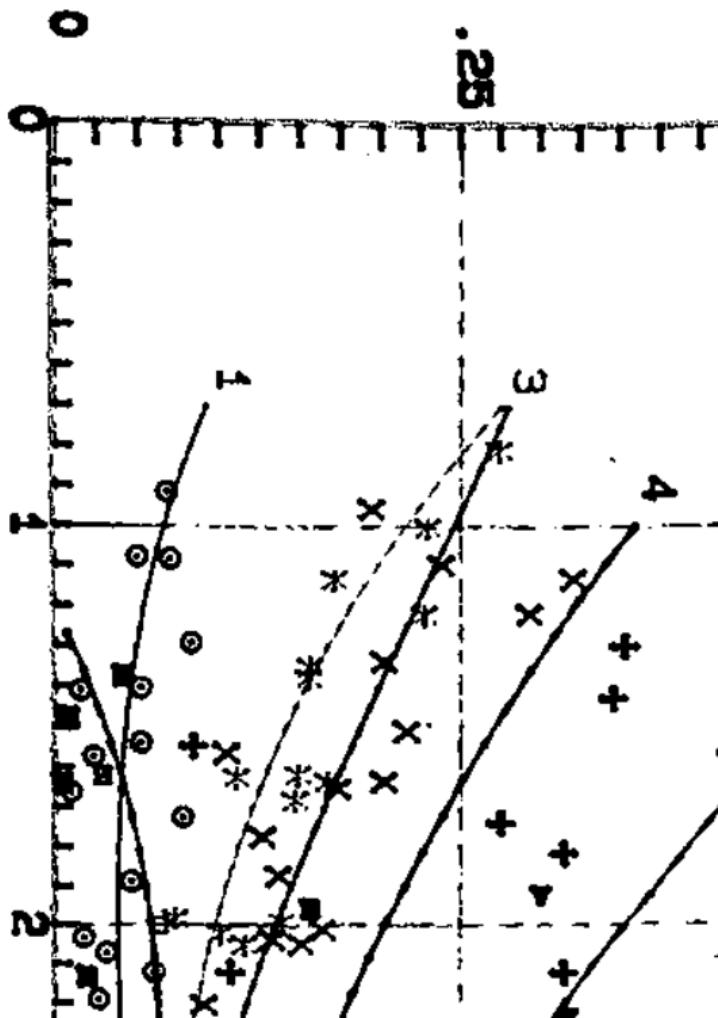
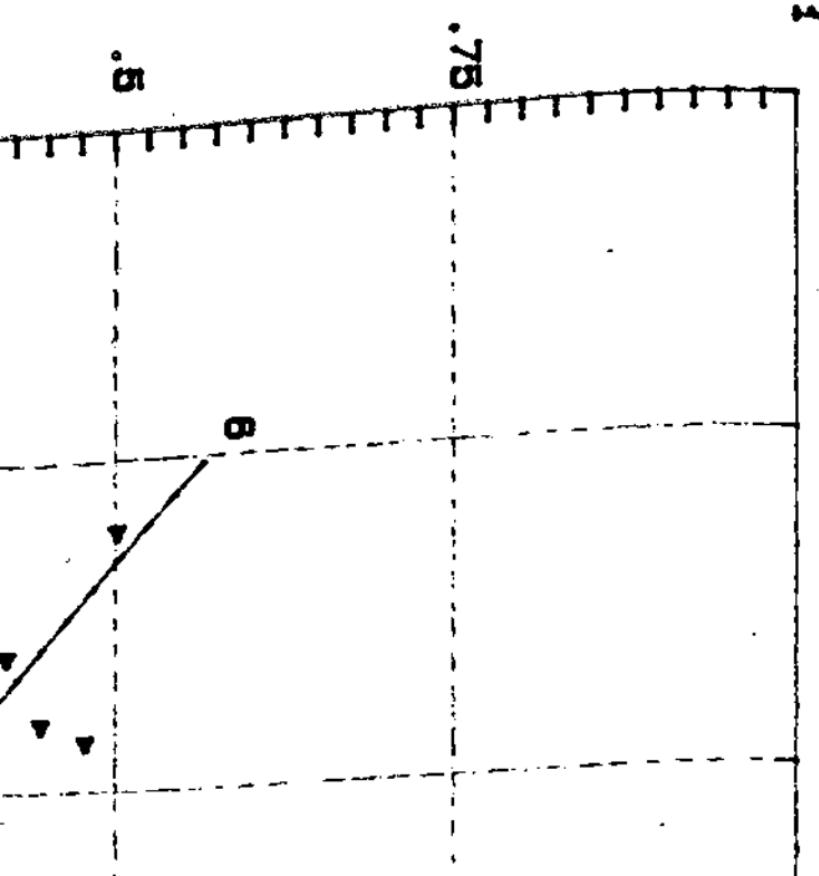


FIG. NO. 3.6

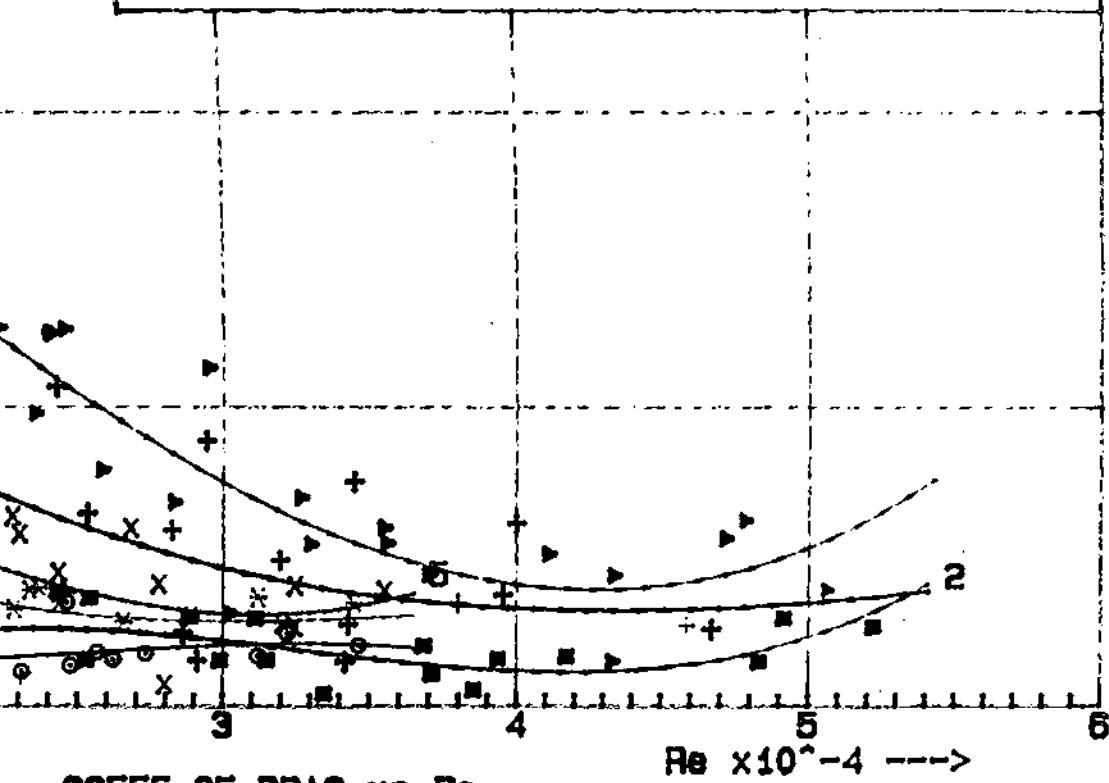


COEFF. OF DRAG →



LEGEND

- CURVE 1 : $\theta_x = 0$, $\theta_y = 0$, $k/D = 0/60$
- CURVE 2** : $\theta_x = 0$, $\theta_y = 0$, $k/D = 0/80$
- CURVE 3 : $\theta_x = -18.8$, $\theta_y = 0$, $k/D = 0/60$
- CURVE 4 : $\theta_x = -18.8$, $\theta_y = 0$, $k/D = 0/80$
- CURVE 5 : $\theta_x = -30.2$, $\theta_y = 0$, $k/D = 0/60$
- CURVE 6 : $\theta_x = -30.2$, $\theta_y = 0$, $k/D = 0/90$



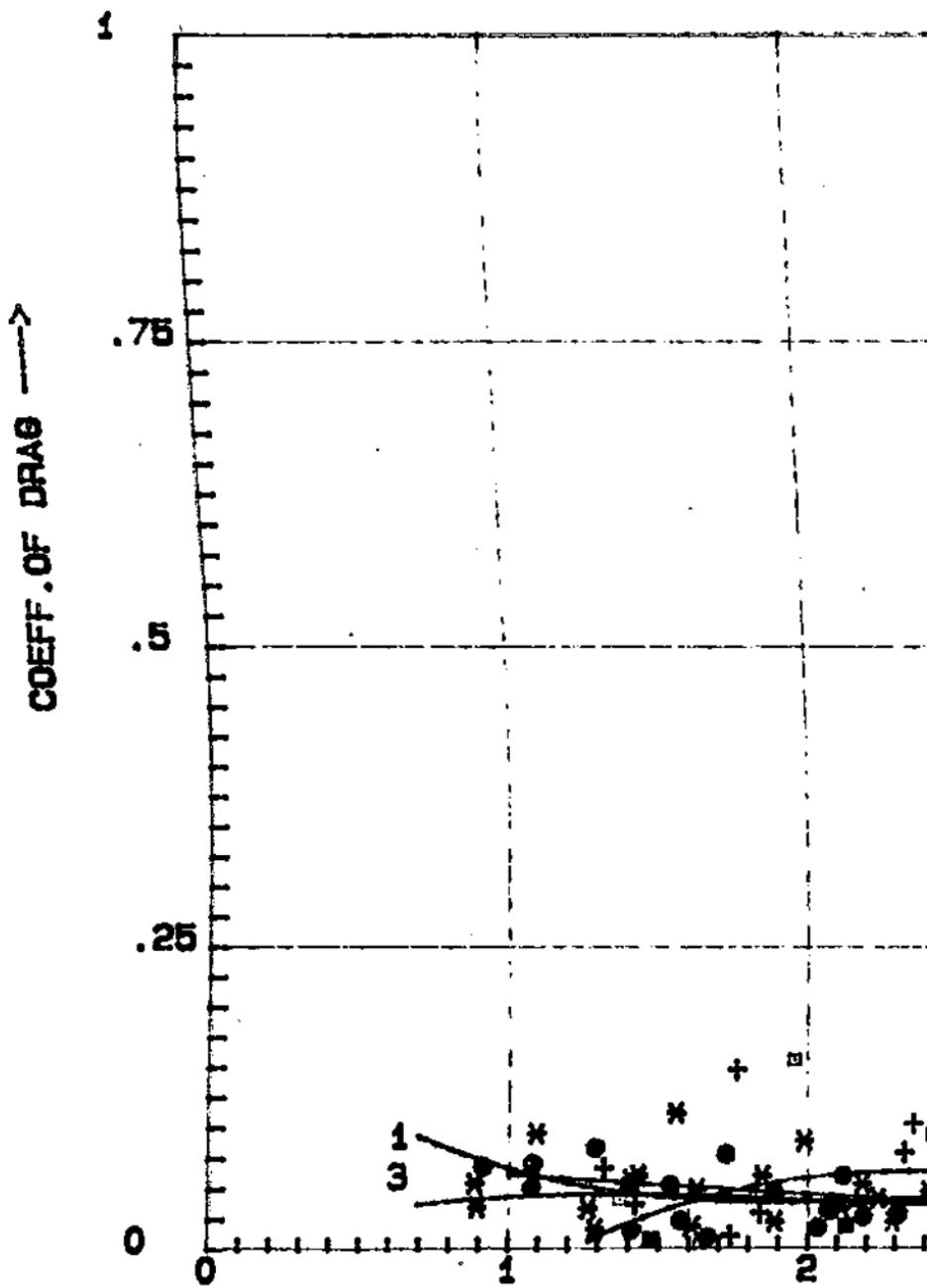
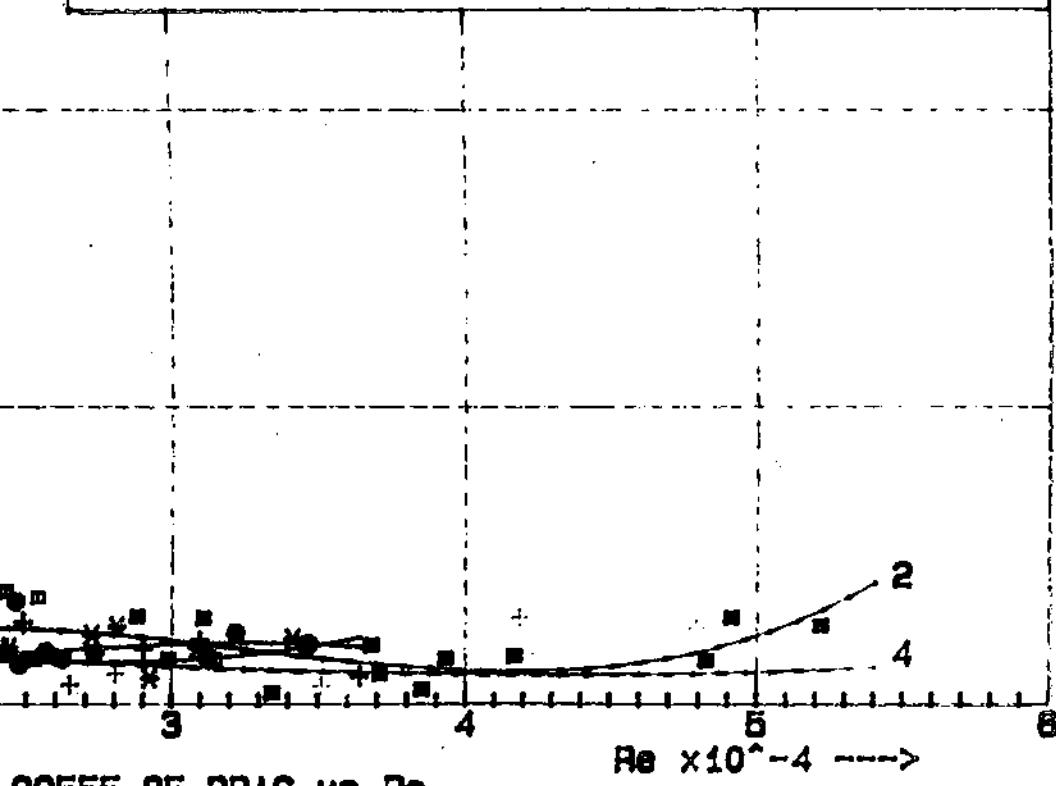


Fig.No. 3.7

LEGEND

- CURVE 1 : $\theta_x = 0$, $\theta_y = 0$, $k/D = 0/60$
CURVE 2 : $\theta_x = 0$, $\theta_y = 0$, $k/D = 0/80$
CURVE 3 : $\theta_x = 0$, $\theta_y = 20$, $k/D = 0/60$
CURVE 4 : $\theta_x = 0$, $\theta_y = 20$, $k/D = 0/80$



COEFF. OF DRAG vs Re

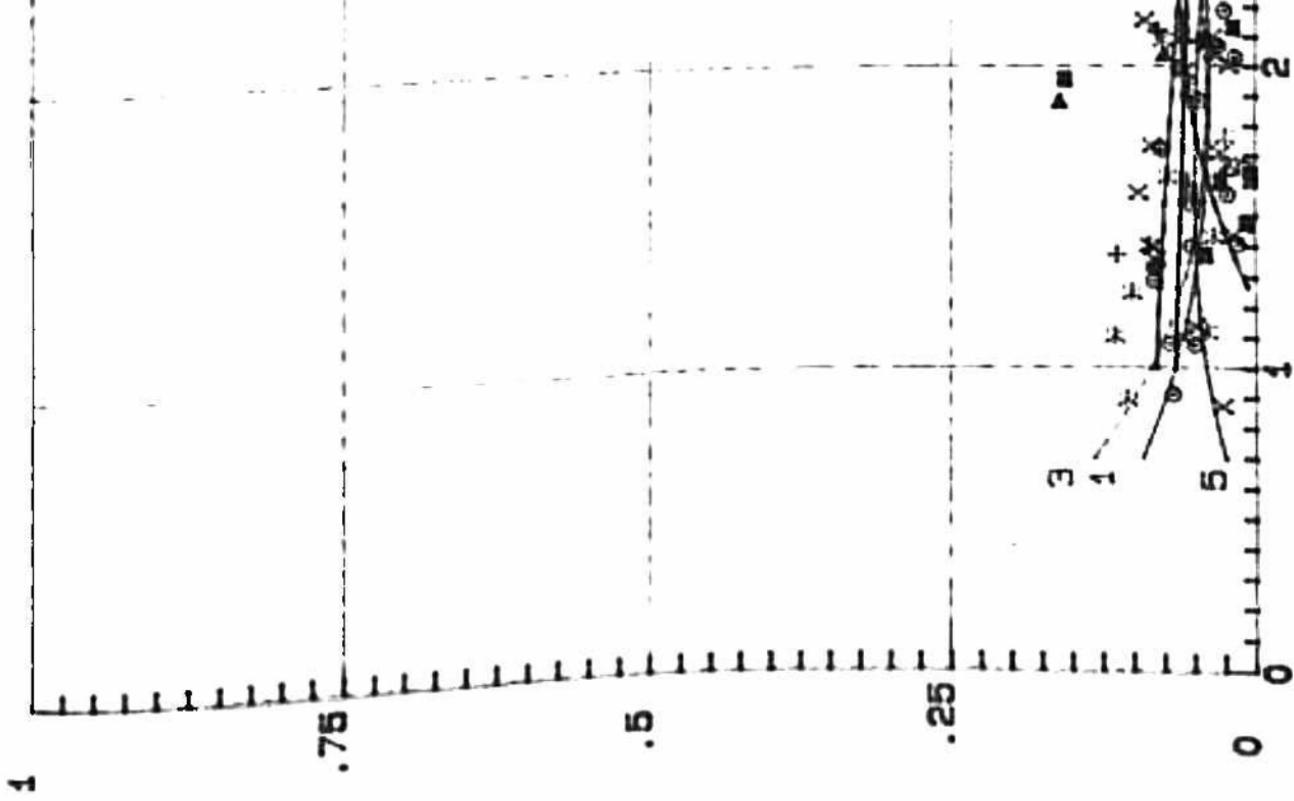
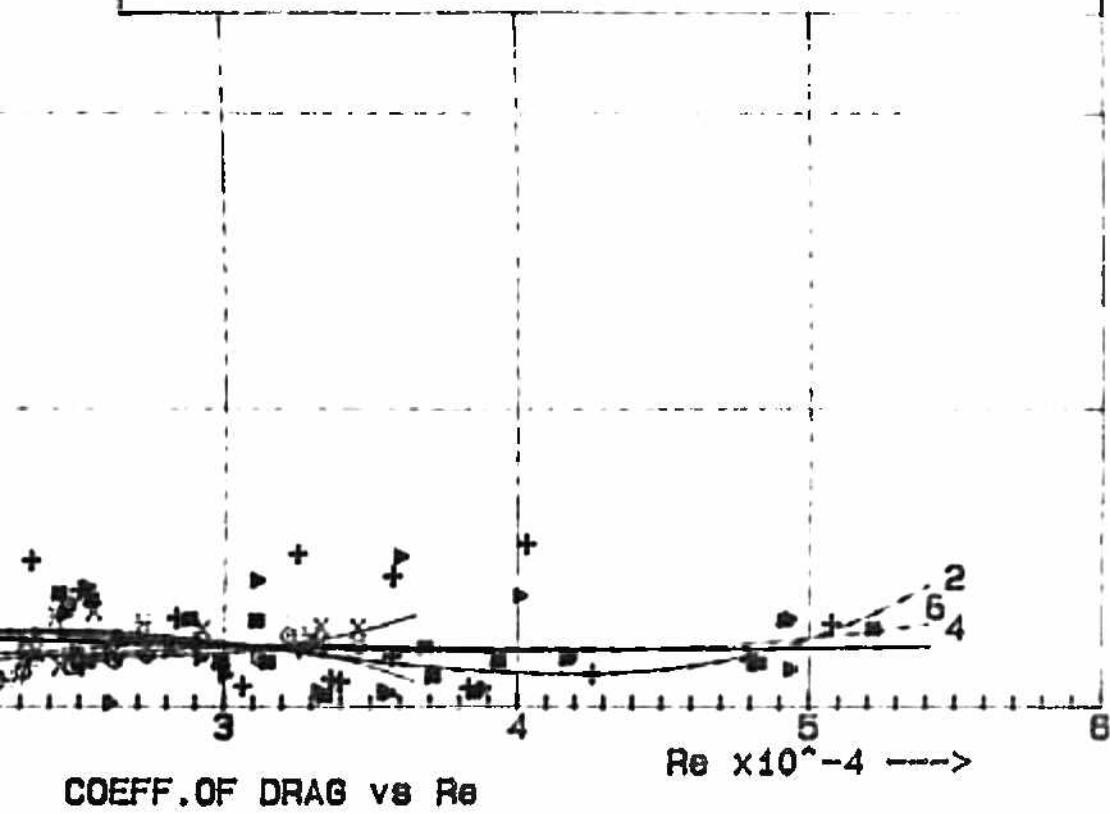


FIG. NO. 3.8

LEGEND

CURVE 1 : $\theta_x = 0$, $\theta_y = 0$, $K/D = 0/60$
CURVE 2 : $\theta_x = 0$, $\theta_y = 0$, $K/D = 0/90$
CURVE 3 : $\theta_x = 0$, $\theta_y = 0$, $K/D = 1/120$
CURVE 4 : $\theta_x = 0$, $\theta_y = 0$, $K/D = 1/180$
CURVE 5 : $\theta_x = 0$, $\theta_y = 0$, $K/D = 1/60$
CURVE 6 : $\theta_x = 0$, $\theta_y = 0$, $K/D = 1/90$



.4

LEGEND

- CURVE 1 : $\theta_x = 0$, $\theta_y = 0$, $k/D = 0/60$
CURVE 2 : $\theta_x = 0$, $\theta_y = 0$, $k/D = 0/90$
CURVE 3 : $\theta_x = -18.8$, $\theta_y = 0$, $k/D = 0/60$
CURVE 4 : $\theta_x = -18.8$, $\theta_y = 0$, $k/D = 0/90$
CURVE 5 : $\theta_x = -30.2$, $\theta_y = 0$, $k/D = 0/60$
CURVE 6 : $\theta_x = -30.2$, $\theta_y = 0$, $k/D = 0/90$

COEFF. OF LIFT

.075

.05

.025

0



Fig.No. 3.9 COEFF.OF LIFT vs Re

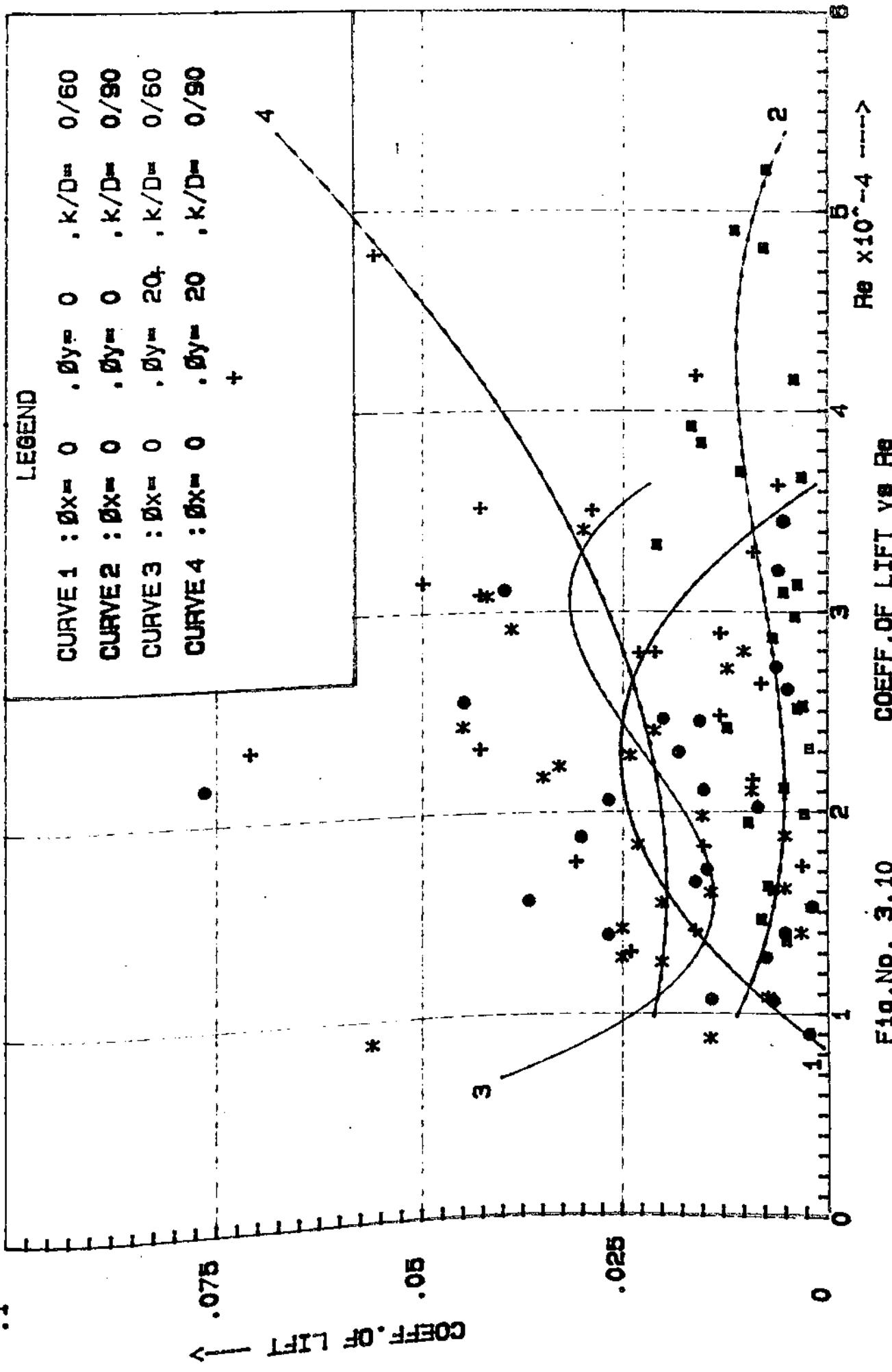


Fig. No. 3.10

Fig. No. 3.10

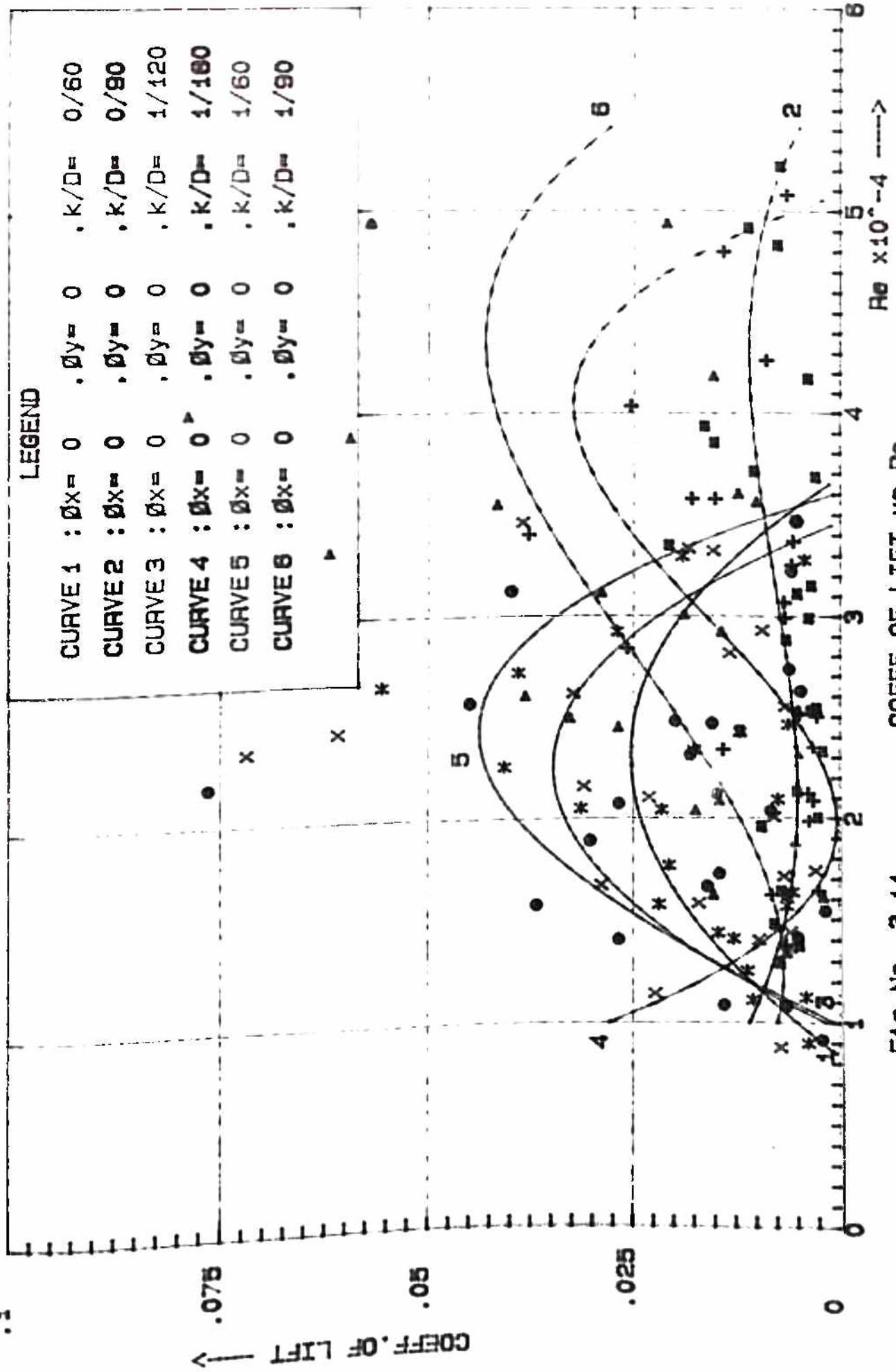


FIG. NO. 3.11

FIG. NO. 3.11

CHAPTER 4

CONCLUSIONS

The present work incorporates the effects of surface roughness, due to marine growth, and inclination on the hydrodynamic coefficients of cylindrical members of widely used Jacket type platforms.

The present study compared the results of wave particle kinematics obtained by two theories namely, Stoke's Fifth Order theory and Linear wave theory. It was observed that the former gives the higher values of wave particle kinematics for the identical situation.

The values of hydrodynamic coefficients and non-dimensional parameters, obtained with the help of Stoke's Fifth Order wave theory and Fourier Series Data Reduction Technique, presented in Appendix - B of Ghoshal [22] pertaining to 90mm diameter cylinders were used in the present work.

The present study covered the range of Reynolds number (Re) from 11000 to 53000 and that of Keulegan-Carpenter number (K-C) from 4 to 14. The in-line inclination varied from $+30^\circ$ to -30° whereas the transverse inclination was

limited to 20°. The roughness parameter (k/D) ranged from zero (smooth) to 1/90.

The following conclusions are drawn from the present study :

4.1 EFFECTS OF ROUGHNESS ON INCLINED CYLINDERS

4.1.1 The Coefficient of Drag (C_d)

The coefficient of drag decreases with increase in surface roughness of in-line inclined cylinder. The change is noticeable in case of cylinder with negative inclination (Fig. F-3) and is more prominent when the surface of cylinder is changed from smooth to rough. For cylinder with positive inclination, the change is marginally visible (Fig. F-2).

For vertical and laterally inclined cylinders (Fig. F-1, F-4), the variation of roughness fails to produce any noticeable effect in the magnitude of C_d .

4.1.2 The Coefficient of Inertia (C_m)

The behaviour of coefficient of inertia is just opposite to that of drag coefficient and it increases with increase in surface roughness of the in-line inclined cylinder. The change is noticeable in case of cylinder with negative inclination (Fig. F-13) and is more prominent when

the surface of cylinder is changed from smooth to rough. For cylinder with positive inclination, the change is marginally visible (Fig. F-12).

The inertia coefficient for the cylinder, which is laterally inclined (Fig. F-14) or vertical (Fig. F-11), remain unaffected against any change in the surface roughness.

4.1.3 The Coefficient of Lift (C_L)

The coefficient of lift increases with the increase in roughness for all types of cylinder inclination (Fig. F-22, F-23, F-24). The undulations are more in case of laterally inclined cylinder which is subjected to more fluctuations of eddy shedding as described in Section 3 (3.3.1) of the report.

4.2 EFFECTS OF INCLINATION ON ROUGH CYLINDERS

4.2.1 The Coefficient of Drag (C_d)

In case of cylinder with positive in-line inclinations (Fig. F-6), C_d increases with the rise in the degree of inclination, which is most prominent when the cylinder's position is shifted from vertical. For rough cylinder with negative in-line inclinations (Fig. F-8), the effect is opposite, that is, for the first shift from vertical, the change in magnitude of C_d is not clearly visible but for

subsequent increment in the inclination angle, this difference is more pronounced.

For laterally inclined cylinder (Fig. F-10), C_d decreases marginally by the change in the inclination angle for rough cylinder.

4.2.2 The Coefficient of Inertia (C_m)

For cylinder with positive in-line inclinations (Fig. F-16), the inertia coefficient, C_m , decreases with the initial change of inclination from vertical to an inclined one. For subsequent increments in the inclination angle the magnitude of the inertia coefficient increases. For negative in-line inclined cylinder (Fig. F-18), C_m increases with degree of inclination for cylinder with rough surface.

For cylinder with transverse inclination (Fig. F-20), C_m is not effected significantly by the angle of inclination.

4.2.3 The Coefficient of Lift (C_L)

For in-line inclined cylinder, the lift coefficient C_L is effected to a great extent (Fig. F-26, F-28). The first change of inclination with respect to vertical increases the magnitude of C_L , although subsequent increments in angle of inclination either reduce or create marginal change in the magnitude of the coefficient.

For laterally inclined cylinder, C_L increases with the angle of lateral inclination (Fig. F-30).

4.3 RELATIONS WITH Re AND $K-C$

The in-line coefficients either reduce or remain constant with increase in Re and $K-C$. In most of the cases, with increase in Re and $K-C$, C_L initially decreases upto $K-C \approx 6$ ($Re \approx 22000$); then it attains the peak value at $K-C \approx 12$ ($Re \approx 45000$) and again it decreases in the experimental range.

The present study indicates that the maximum changes in magnitudes of hydrodynamic coefficients take place in the range of 6 to 12 of $K-C$ and 22000 to 45000 to that of Re .

4.4 EFFECTS OF CYLINDER DIAMETER

A similar type of study was carried out by Ghoshal [22] pertaining to 60mm diameter rough and inclined cylinders, covering the range of Re from 8000 to 36000 and that of $K-C$ from 6 to 21. The in-line and transverse inclinations were same as that of present study, but roughness varied from 0 to 1/60. The two studies were compared which shows that the value of inertia coefficient is more for 90mm diameter cylinders as compared to 60mm diameter cylinders (Fig. 3.3, 3.4, 3.5). In case of C_d , either the values are same (Fig.

3.7, 3.8) or more (Fig. 3.6) for 90mm diameter cylinders as compared to 60mm diameter cylinders. Another important aspect of the comparison is shifting of saddle point in case of C_d from $Re \approx 30000$ (60mm diameter cylinder) to $Re \approx 42000$ (90mm diameter of cylinder).

The coefficient of lift, C_L , was found to be more in case of 60mm diameter cylinders. Apart from the initial decrease in magnitude of the coefficient, the shifting of peak value from $Re \approx 25000$ (Ghoshal [22]) to $Re \approx 45000$ was also observed in the present study (Fig. 3.9, 3.11).

Appendix - A

Summary of Published Works

on

Cd, Cm and CL (1950 - 1994)

TABLE I A.1
SUMMARY OF LITERATURE ON C_d AND C_m

AUTHOR	REF.	YEAR	BRIEF METHOD	RANGE OF C_d/C_m	REMARKS
MORRISON	26	1950	—	—	Pile design
KEULEGAN and CARPENTER	26	1958	FOURIER SERIES	C_d 1.0 - 2.2 C_m 0.8 - 2.7	Smooth horizontal pile in standing waves
LAIRD ET AL.	37	1960	—	C_d 1.3 - 1.4	Oscillating smooth vertical pile
ROSHKO	40	1961	—	C_d = 0.7	Wind tunnel at high Re values
ACHENBACH	1	1968	—	—	Wind tunnel tests
JEN	31	1968	—	C_m = 2.04	Smooth vertical pile in wave flume
ACHENBACH	2	1971	—	C_d 0.4 - 1.0	Rough pile in wind tunnel
BIDDE	5	1971	—	C_d 2.2 (max)	Smooth vertical pile in wave flume
SARPKAYA	45	1975	FOURIER SERIES	C_d 2.25 (max) C_m 2.25 (max)	U-tube experiments
SZECHENYI	49	1975	—	—	Rough pile in wind tunnel
SARPKAYA	41	1976	FOURIER SR. & LEAST SQ.	C_m 0.6 - 2.5 C_d 0.6 - 2.0	U-tube experiments
GARRISON ET AL.	21	1977	—	C_d 1.3 (max) C_m 0.65 (max)	Oscillating pile in still water
HOBGEN ET AL.	26	1977	—	—	An excellent review
SARPKAYA ET AL.	44	1977	—	C_d 0.7 - 1.9 C_m 1.0 - 2.0	Rough piles in U-tube

AUTHOR	REF.	YEAR	BRIEF METHOD	RANGE OF C_d/C_m	REMARKS
BEARMAN and GRAHAM	4	1979	—	C_d 2.0 (max) C_m 1.8 (max)	U-tube experiments
CHAKRABARTI	7	1980	—	C_d 1.8 C_m 2.0	Smooth cylinders
GARRISON	19	1980	—	C_d 0.2 - 2.0 C_m 0.2 - 0.9	Oscillating rough piles in still water
JEN	32	1980	—	—	Suggested new method
STARSMORE	48	1981	PEAK FORCE	C_d 0.3 - 1.5	Cylinder in ocean waves
CHAKRABARTI	9	1982	—	—	Rough vertical cylinders
KAPLAN ET AL.	33	1982	—	C_m 1.7 - 2.3	Inclined cylinder in ocean waves
SARPKAYA	43	1982	—	—	Rough piles in U-tube water channel
NATH	39	1982	—	C_d 0.3 - 1.8 C_m 0.4 - 2.0	Marine roughened cylinders
CHAKRABARTI ET AL.	15	1984	—	C_d 2.5 - 0.1 C_m 2.5 - 0.5	Inclined smooth cylinders
WILLIAM AN	53	1985	—	C_m 2 - 3	Integral equation method
CHAKRABARTI ET AL.	16	1986	—	—	Dynamic pressure
CHAKRABARTI ET AL.	17	1987	—	—	Total force coefficient for Inclined cylinders
GHOSHAL	22	1988	Fourier Sr.	—	Inclined & Rough cylinders
CHAKRABARTI ET AL.	18	1989	—	—	Modern testing facility
TDRUM	50	1989	—	—	Considered forces at surface zone
GARRISON	20	1990	FFT	C_m 2.0	Wake-Tank Test

TABLE : A.2
SUMMARY OF LITERATURE ON C_L

AUTHOR	REF.	YEAR	RANGE OF C_L	REMARKS
LAIRD ET AL.	37	1960	—	Observed Lift force for eddy shedding
LAIRD	34	1961	—	Concluded that Lift force is significant
LAIRD	35	1962	—	Ratio of drag and lift forces
BISHOP and HASSEN	6	1964	0.6	
BIDDE	5	1971	—	Observed eddy shedding pattern
LAIRD	36	1971	—	Observed eddy shedding pattern
SARPKAYA	45	1975	3.0	Conducted experiments in U-tube
CHAKRABARTI ET AL.	12	1976	—	Considered first 5 Fourier series coefficient
ISAACSON and MAULL	30	1976	1.5	Smooth vertical pile in wave flume
ZDRAVKOVICH and NAMORK	54	1977	—	Photographed eddy shedding pattern
BEARMAN and GRAHAM	4	1979	—	Observed large vibrations
ANGRILLI and COSSALTER	3	1982	—	Observed resonance
CHAKRABARTI	9	1982	2.0	Smooth vertical pile in wave flume
GHOSHAL	22	1988	—	Inclined & rough cylinders

Appendix - B

Software Listing I

"Wv. Th."

Appendix - B

PROGRAM "Wv.Th."

Tue Sep 25 23:19:17 IST 1990

```
*****  
#####file getting printed is.. /hdeg/r Gupta/r g1=====  
  
c THE PROGRAM UV.TH. TAKES THE INPUTS AS DIAMETER OF CYLINDER,  
c (i.e.60MM OR 90MM),WAVE HEIGHT (H), TIME PERIOD (T), AND THE  
c FREE SURFACE ELEVATION OF WATER PARTICLE FROM STILL WATER  
c DEPTH, WHICH IS TAKEN AFTER DISCRETIZING ONE CYCLE OF THE  
c WAVE PROFILE AT THE INTERVAL OF 45°. IT CALCULATES THE WATER  
c PARTICLE KINEMATICS WITH THE HELP OF "AIRY'S THEORY".IT ALSO  
c LISTS THE DIFFERENT PARAMETERS AS INDICATED IN THE OUTPUT  
c FILE.  
c dimension di(100), t(100), h(100), wl(100)  
c ,v(15),u(15), wla(100),z(10)  
c di IS THE DIAMETER OF CYLINDER  
c t IS THE TIME PERIOD  
c h IS THE WAVE HEIGHT  
c wl IS THE WAVE LENGTH OBTAINED BY STOKE'S FIFTH ORDER  
c THEORY (REF.[22])  
c v IS FOR VERTICAL VELOCITY  
c u IS FOR HORIZONTAL VELOCITY  
c wla IS THE WAVE LENGTH CALCULATED BY AIRY'S THEORY  
c z IS FOR THE ELEVATION OF FREE SURFACE WITH RESPECT TO STILL  
c WATER DEPTH  
c  
10 write(*,10)  
format('WV LT.(ST) WV.LT.(AR) DIFF. g*T*T/H V...*T/H  
      RE KC')  
read (*,n)  
do 110 i = 1,n  
write(*,*)i  
read(*,20)di(i),t(i),h(i),wl(i)  
write(*,20)di(i),wl(i),h(i),t(i)  
20 format(4(f9.4))  
w=1000.0  
vs=0.00097  
c w IS THE MASS DENSITY OF THE FLUID  
c vs IS THE DYNAMIC VISCOCITY OF THE FLUID
```

```

j=2
u(j)=0.1
do 90 angle = 0,360,45
read(*,30)z(angle)

c      angle INDICATES THE PHASE ANGLE FROM 0° TO 360° IN INTERVAL
c      OF 45°

30  format(f9.4)
    u(j-1) = u(j)
    j = j + 1
    d = 1.2
    theta=(angle)*(2.0*22.0)/(360.0*7.0)
    s=d-z(angle)
    a=((2*3.14/wl(i))
    do 50 i=0,20
    err1=((4*3.14*3.14)/(9.81*t(i)*t(i)*sinh(a*d))/cosh(a*d))
    err=abs(err1-a)
    if(err>0.00001)60,60,40
40  a=err1
50  continue
60  u(j) = ((3.14*h(i)*cosh(a*s)*cos(theta))/(t(i)*sinh(a*d)))
    v(j) = ((3.14*h(i)*sinh(a*s)*sin(theta))/(t(i)*sinh(a*d)))

c      v(j) AND u(j) ARE THE VERTICAL AND HORIZONTAL VELOCITIES OF
c      WATER PARTICLE BY AIRY'S THEORY.

    if (u(j-1)-u(j))70,70,80
70  u(j) = u(j-1)
80  u(j) = u(j)
90  continue
    wla(j)=(2*3.14)/(err1)
    diff=wl(i)-wla(j)
    g=9.81*t(i)*t(i)/h(i)
    vmax=u(j)*t(i)/h(i)
    re=(w*di(i)*u(j))/vs
    kc=(u(j)*t(i))/di(i)
    write (*,100)wl(i),wla(j),diff,g,vmax,re,kc,
c      diff INDICATES THE DIFFERENCE BETWEEN THE WAVE LENGTHS
c      OBTAINED BY STOKE'S FIFTH ORDER THEORY AND AIRY'S THEORY.
c      g IS USED FOR gT²/H
c      vmax IS USED FOR Vmax*T/H
c      re IS USED FOR REYNOLDS NUMBER
c      kc IS USED FOR KEULEGAN-CARPENTER NUMBER

100 format(7(f11.5,4x))
110 continue
stop
end

```

Appendix - C

In-put data file

(for "Wv.Th.")

Appendix - C

The section contains In-put data for the program "Wv.Th". In in-put file, first data indicates the graph no. from which the values in corresponding line is taken for different parameters. Second data of the line, represents the diameter of cylinder, which is taken as 60mm or 90mm; third, fourth and fifth data correspond to wavelength (L), wave height (H) and time period (T) respectively. The next nine values are the depth of water from the still water depth correspond to the phase angle 0° to 360° of each wave profile (in all 98) at the interval of 45° . These values were obtained after discretizing one complete cycle at eight equal intervals, each one corresponding to 45° , and equivalent depth was calculated which was the input values for the program. Applying the Linear wave theory, the kinematics of wave particles were calculated with the help of the program "Wv.Th." (presented in Appendix - B).

INPUT DATA FILE FOR COMPUTER PROGRAM "Wv.Th."

GRAPH, NO.	d,	w1,	H,	T,	Free surface elevation above the still water level corresponding to following phase angle (θ) obtained from the graphs (m.)										
					(m.)	(m.)	(sec.)	0°,	45°,	90°,	135°,	180°,	225°,	270°,	315°,
1	0.06,	5.777,	0.1734,	2.06,	.1000,	.0500,	00000,	-.0500,	-.0750,	-.0584,	-.0167,	.0334,	.1000		
2	0.06,	5.913,	0.2281,	2.08,	.1084,	.0500,	-.0334,	-.1000,	-.1250,	-.0500,	.0500,	.1000,	.1084		
3	0.06,	6.016,	0.3031,	2.09,	.1667,	.0834,	-.0500,	-.1167,	-.1334,	-.0917,	-.0334,	.0500,	.1667		
4	0.06,	5.937,	0.2224,	2.09,	.1000,	.0667,	-.0417,	-.1167,	-.1000,	-.0500,	.0500,	.1000,	.1000		
5	0.06,	6.184,	0.2984,	2.13,	.1834,	.1000,	-.0167,	-.1000,	-.1000,	-.0917,	-.0500,	.0500,	.1834		
6	0.06,	6.055,	0.3056,	2.10,	.1667,	.0667,	-.0667,	-.1334,	-.1167,	-.0584,	00000,	.1000,	.1667		
7	0.06,	9.402,	0.2088,	2.98,	.1334,	.0667,	-.0167,	-.0667,	-.0667,	-.0667,	00000,	.0334,	.1334		
8	0.06,	6.985,	0.2799,	2.34,	.1834,	.0834,	-.0417,	-.1000,	-.1167,	-.0750,	00000,	.0834,	.1834		
9	0.06,	5.897,	0.2325,	2.08,	.1167,	.0667,	-.0167,	-.0834,	-.1084,	-.0667,	.0167,	.1084,	.1167		
0	0.06,	5.853,	0.2927,	2.05,	.1667,	.1000,	-.0334,	-.1084,	-.1000,	-.0500,	.0167,	.1000,	.1667		
1	0.06,	8.340,	0.1818,	2.97,	.1167,	.0417,	-.0500,	-.0750,	-.0667,	-.0584,	.0167,	.0667,	.1167		
2	0.06,	7.957,	0.1757,	2.61,	.1000,	.0167,	-.0417,	-.0750,	-.0667,	-.0500,	00000,	.0667,	.1000		
13	0.06,	6.95,	0.2528,	2.34,	.1500,	.0167,	-.0750,	-.1167,	-.0917,	-.0667,	00000,	.0750,	.0500		
14	0.06,	5.983,	0.2581,	2.09,	.1167,	.0500,	-.0634,	-.1334,	-.1167,	-.0334,	.0050,	.1000,	.1167		
15	0.06,	6.085,	0.2677,	2.11,	.1167,	.0334,	-.0834,	-.1334,	-.0917,	-.0167,	.0417,	.1167,	.1167		
16	0.06,	5.981,	0.1288,	2.11,	.0667,	.0500,	.0167,	-.0334,	-.0667,	-.0584,	-.0334,	.0167,	.0667		
17	0.06,	6.002,	0.3012,	2.09,	.1667,	.1000,	-.0334,	-.1167,	-.1334,	-.0917,	00000,	.0500,	.1667		
18	0.06,	6.900,	0.2744,	2.32,	.1667,	.1000,	-.0334,	-.1167,	-.1334,	-.1167,	-.0917,	00000,	.0500		
19	0.06,	8.135,	0.2144,	2.65,	.1167,	.0334,	-.0500,	-.0667,	-.0834,	-.1000,	-.0500,	.0167,	.1167		

GRAPH,
NO. d, w₁, H, T, Free surface elevation above the still water level corresponding to following
phase angles (θ) obtained from the graphs (m.)

	(m.)	(m.)	(m.)	(sec.)	0°,	45°,	90°,	135°,	180°,	225°,	270°,	315°,	360°
20	0.06,	9.686,	0.2143,	3.05,	.1250,	.0750,	-.0167,	-.0667,	-.0834,	-.0667,	-.0334,	.0334,	.1250
21	0.06,	5.986,	0.2614,	2.09,	.1167,	.1000,	00000,	-.1167,	-.1167,	-.0834,	00000,	.0834,	.1167
22	0.06,	5.853,	0.2927,	2.05,	.1667,	.1500,	00000,	-.0834,	-.1167,	-.0667,	.0167,	.0750,	.1750
23	0.06,	5.576,	0.1387,	2.01,	.0584,	.0500,	-.0084,	-.0500,	-.0750,	-.0667,	00000,	.0334,	.0667
24	0.06,	5.595,	0.1437,	2.02,	.0667,	.0417,	00000,	-.0500,	-.0750,	-.0584,	00000,	.0334,	.0667
25	0.06,	5.683,	0.1771,	2.03,	.0834,	.0584,	-.0167,	-.0750,	-.0170,	-.0334,	.0334,	.0834,	.0917
26	0.06,	6.002,	0.3125,	2.08,	.1667,	.1000,	-.0417,	-.1334,	-.1334,	-.0667,	.0334,	.1334,	.0584
27	0.06,	5.668,	0.1794,	2.03,	.1000,	.0500,	-.0167,	-.0750,	-.0834,	-.0334,	.0500,	.0834,	.1000
28	0.06,	5.879,	0.2399,	2.07,	.1500,	.0834,	00000,	-.0584,	-.0834,	-.0917,	.0584,	.0500,	.1500
29	0.06,	5.983,	0.3135,	2.08,	.1667,	.1334,	-.0334,	-.1250,	-.1417,	-.0834,	.0084,	.1000,	.1667
30	0.06,	5.811,	0.1704,	2.07,	.0834,	.0750,	.0334,	-.0334,	-.0750,	-.0750,	-.0167,	.0500,	.0917
31	0.06,	5.865,	0.2382,	2.07,	.0834,	.0750,	.0334,	-.0334,	-.0750,	-.0750,	-.0167,	.0500,	.0917
32	0.06,	6.156,	0.3067,	2.12,	.1584,	.1167,	-.0250,	-.0500,	-.1334,	-.1417,	-.0834,	.0500,	.1584
33	0.06,	5.859,	0.2541,	2.06,	.1667,	.0834,	-.0250,	-.0834,	-.0917,	-.0750,	00000,	.0834,	.1667
34	0.06,	5.001,	0.3035,	2.09,	.1417,	.1250,	.0334,	-.0500,	-.1417,	-.1417,	-.0500,	.0500,	.1500
35	0.06,	5.783,	0.1758,	2.06,	.0834,	.0500,	-.0167,	-.0667,	-.0750,	-.0334,	.0167,	.0584,	.0834
36	0.06,	5.840,	0.2336,	2.06,	.1500,	.0834,	.0167,	-.0834,	-.0834,	-.0834,	-.0500,	.0334,	.1500
37	0.06,	6.022,	0.3046,	2.09,	.1500,	.0834,	00000,	-.0834,	-.1500,	-.1334,	00000,	.1167,	.1500
38	0.06,	5.698,	0.1888,	2.04,	.0917,	.0334,	-.0417,	-.0750,	-.0750,	00000,	.0417,	.0750,	.0917
39	0.06,	5.920,	0.2366,	2.08,	.1500,	.1000,	00000,	-.0667,	-.0834,	-.0667,	-.0334,	.0500,	.1500
40	0.06,	6.037,	0.2961,	2.10,	.1500,	.1334,	.0834,	-.0334,	-.1500,	-.1500,	-.0750,	.0334,	.1500

GRAPH,
NO. d, w}, H, T, Free surface elevation above the still water level corresponding to following
phase angles (θ) obtained from the graphs (m.)

	(a.)	(a.)	(b.)	(sec.)	0°,	45°,	90°,	135°,	180°,	225°,	270°,	315°,	360°
41	0.06,	5.808,	0.1271,	2.07,	.0667,	.0470,	-.0084,	-.0417,	-.0584,	-.0500,	00000,	.0417,	.0667
42	0.06,	5.850,	0.1672,	2.08,	.1000,	.0417,	-.0344,	-.0667,	-.0667,	-.0344,	.0167,	.0750,	.1000
43	0.06,	5.883,	0.2398,	2.07,	.1334,	.0500,	00000,	-.0500,	-.1000,	-.1000,	00000,	.0834,	.1334
44	0.06,	5.971,	0.2580,	2.09,	.1250,	.0584,	-.0334,	-.1334,	-.1334,	-.0834,	.0334,	.1000,	.1250
45	0.06,	5.998,	0.3008,	2.09,	.1667,	.0834,	-.0667,	-.1167,	-.1250,	-.0667,	.0334,	.1167,	.1667
46	0.06,	5.776,	0.1655,	2.06,	.1000,	.0334,	-.0334,	-.0667,	-.0667,	-.0334,	.0167,	.0667,	.0917
47	0.06,	5.933,	0.2419,	2.08,	.1417,	.0834,	00000,	-.0334,	-.0917,	-.1000,	00000,	.0667,	.1417,
48	0.06,	6.009,	0.2964,	2.09,	.1667,	.0834,	-.0667,	-.1500,	-.1167,	-.0500,	.0334,	.1250,	.1667
49	0.06,	5.695,	0.1714,	2.04,	.0750,	.0500,	-.0334,	-.0750,	-.0834,	-.0500,	.0167,	.0500,	.0750
50	0.06,	5.864,	0.2417,	2.07,	.1334,	.0834,	00000,	-.0417,	-.1000,	.1167,	.0500,	.0334,	.1334
51	0.06,	6.048,	0.3135,	2.09,	.1750,	.1167,	-.0167,	-.1500,	-.1500,	-.0917,	00000,	.0750,	.1667
52	0.06,	5.865,	0.2428,	2.07,	.1334,	.0917,	.0334,	-.0334,	-.0917,	-.1084,	-.0667,	.0334,	.1334
53	0.09,	6.057,	0.3039,	2.10,	.1667,	.0917,	-.0667,	-.1500,	00000,	.1000,	.1584,	.1334,	.0750
54	0.09,	6.126,	0.2909,	2.12,	.1750,	.0334,	-.0834,	-.1167,	-.1084,	-.0917,	00000,	.1000,	.1750
55	0.09,	7.986,	0.2080,	2.61,	.1334,	.0667,	-.0500,	-.0667,	-.0750,	-.0834,	-.0334,	.0417,	.1334
56	0.09,	9.411,	0.2101,	2.98,	.1334,	.0834,	00000,	-.0667,	-.0667,	-.0667,	-.0334,	.0334,	.1334
57	0.09,	6.891,	0.2440,	2.32,	.1334,	.0834,	-.0334,	-.1000,	-.1167,	-.0667,	00000,	.0584,	.1334
58	0.09,	5.669,	0.1306,	2.04,	.0667,	.0500,	-.0167,	-.0500,	-.0584,	-.0334,	.0084,	.0584,	.0667
59	0.09,	6.018,	0.2890,	2.09,	.1834,	.0834,	-.0250,	-.0917,	-.1084,	-.1167,	-.0334,	.0834,	.1834
60	0.09,	6.009,	0.2641,	2.10,	.1000,	.0917,	-.0167,	-.1250,	-.1334,	-.0667,	.0167,	.0917,	.1167
61	0.09,	5.800,	0.1348,	2.07,	.0750,	.0500,	00000,	-.0334,	-.0667,	-.0500,	00000,	.0334,	.0750

GRAPH, NO.	d,	wl,	H,	T,	Free surface elevation above the still water level corresponding to following phase angles (θ) obtained from the graphs (m.)								
					0°,	45°,	90°,	135°,	180°,	225°,	270°,	315°,	360°
(m.)	(m.)	(m.)	(sec.)										
62	0.09	5.982	0.2974	2.08	.1417	.1250	-.0334	-.1167	-.1417	.0824	.0500	.1167	.1417
63	0.09	5.881	0.2766	2.06	.1834	.1000	-.0334	-.1000	-.0834	-.1000	-.0334	.1084	.1834
64	0.09	5.666	0.2164	2.02	.1000	.0917	.0667	00000	-.0834	-.1084	-.0750	.0334	.1000
65	0.09	5.955	0.2906	2.08	.0417	.1167	-.0084	-.1167	.1500	-.1334	00000	.1167	.1417
66	0.09	8.136	0.3060	2.12	.1667	.0334	-.0834	-.1334	-.1334	-.0834	00000	.0667	.1667
67	0.09	5.727	0.2213	2.04	.1667	.0834	00000	-.0750	-.1167	-.1000	00000	.0667	.1084
68	0.09	5.667	0.1426	2.03	.0667	.0417	00000	-.0417	-.0667	-.0584	00000	.0417	.0667
69	0.09	6.028	0.2991	2.09	.1750	.0834	-.0667	-.1334	-.1167	-.0667	.0167	.0750	.1667
70	0.09	5.966	0.2782	2.08	.1417	.0834	00000	.0667	-.1167	-.1250	-.0584	.0667	.1417
71	0.09	5.822	0.2146	2.08	.1167	.0667	-.0500	-.0917	-.0834	-.0584	.0167	.0834	.1167
72	0.09	5.673	0.1391	2.04	.0667	.0500	00000	-.0500	-.0667	-.0500	00000	.0500	.0667
73	0.09	6.040	0.2833	2.10	.1334	.0334	-.0834	-.1500	-.1334	-.0667	.0334	.1167	.1334
74	0.09	5.938	0.2822	2.08	.1834	.1167	-.0250	-.0584	-.1000	-.1167	-.0167	.1167	.1834
75	0.09	5.939	0.2745	2.08	.1500	.0834	00000	-.0500	-.1000	.1084	-.1000	-.0667	.1500
76	0.09	5.824	0.2013	2.06	.1167	.0500	-.0417	-.0667	-.0834	-.0500	00000	.0834	.1167
77	0.09	5.755	0.1254	2.06	.0667	.0500	-.0084	-.0584	-.0667	-.0334	00000	.0500	.0667
78	0.09	5.986	0.3120	2.08	.1500	.1167	.0167	-.0834	-.1500	-.1167	-.0250	.1000	.1500
79	0.09	5.740	0.2176	2.04	.1167	.0667	-.0334	-.1000	-.0834	.0250	.0167	.0834	.1167
80	0.09	5.658	0.1415	2.03	.0667	.0500	00000	-.0500	-.0750	.0500	00000	.0500	.0667
81	0.09	5.966	0.2861	2.08	.1334	.0334	-.0750	-.1417	-.1334	-.0500	.0667	.1250	.1417
82	0.09	6.015	0.2869	2.09	.1500	.1000	00000	-.0500	-.1084	.1167	-.0500	-.0584	.1500

GRAPH, NO.	d,	v1,	H,	T,	Free surface elevation above the still water level corresponding to following phase angles (θ) obtained from the graphs (m.)									
					(m.)	(m.)	(sec.)	0°,	45°,	90°,	135°,	180°,	225°,	270°,
83	0.09,	5.830,	0.2093,	2.06,	.1167,	.0667,		-.0417,	-.0834,	-.0750,	-.0500,	.0167,	.0750,	.1167
84	0.09,	5.708,	0.1281,	2.05,	.0667,	.0584,		00000,	-.0417,	-.0667,	-.0417,	00000,	.0500,	.0667
85	0.09,	5.088,	0.3135,	2.10,	.1667,	.1000,		-.0167,	.1250,	-.1334,	-.0834,	00000,	.0584,	.1667
86	0.09,	9.418,	0.2032,	2.99,	.1167,	.1000,		00000,	-.0834,	-.0667,	-.0667,	-.0167,	.0500,	.1667
87	0.09,	5.918,	0.2816,	2.07,	.1417,	.1000,		.0667,	00000,	-.1000,	-.1334,	-.0750,	.0167,	.1417
88	0.09,	5.789,	0.2140,	2.05,	.1334,	.0917,		00000,	-.0584,	-.0834,	-.0667,	-.0167,	.0334,	.1334
89	0.09,	5.684,	0.1374,	2.04,	.0667,	.0417,		-.0084,	-.0584,	-.0667,	-.0417,	.0084,	.0500,	.0667
90	0.09,	8.125,	0.1981,	2.65,	.1167,	.0667,		-.0334,	-.0834,	-.0750,	-.0667,	-.0500,	.0500,	.0167
91	0.09,	5.919,	0.2751,	2.07,	.1417,	.1000,		.0334,	-.0584,	-.1000,	-.1667,	-.0750,	.0500,	.1417
92	0.09,	5.771,	0.2110,	2.05,	.1167,	.0667,		-.0167,	-.0834,	-.1000,	-.0667,	00000,	.0500,	.1667
93	0.09,	5.699,	0.1345,	2.04,	.0667,	.0334,		-.0167,	-.0500,	-.0667,	-.0334,	-.0084,	.0500,	.0667
94	0.09,	6.051,	0.3130,	2.10,	.1667,	.0667,		-.0500,	-.1500,	-.1500,	-.0667,	.0167,	.0917,	.1584
95	0.09,	9.322,	0.2128,	2.96,	.1000,	.0500,		-.0500,	-.1084,	-.0667,	-.0500,	.0084,	.0584,	.1000
96	0.09,	5.965,	0.2561,	2.09,	.1167,	.0834,		00000,	-.0834,	-.1334,	-.1334,	00000,	.0917,	.1167
97	0.09,	5.740,	0.2026,	2.04,	.1167,	.0750,		-.0334,	-.0834,	-.0750,	-.0500,	.0084,	.0667,	.1167
98	0.09,	5.713,	0.1297,	2.09,	.0667,	.0334,		-.0167,	-.0500,	-.0334,	.0334,	-.0667,	.0500,	.0667

Appendix - D

Out- put Data file

Appendix - D

The appendix contains Output data from the program Wv.Th. (listing is presented in Appendix-B).

The out-put data consists of the wave length calculated by Airy's theory (indicated by WV.LT.(Airy's)) and the difference between the wave lengths obtained by Stoke's Fifth Order Theory (input, but not shown in input-file to avoid the repetition) and Airy's theory, indicated by DIFF.(S-A). The out-put file also contains the values of $\alpha T^2/H$, $V_{max}T/H$, Re and K-C, in the same order. These values were used to plot the graphs, attached at the end of Section 2 of the report.

OUT-PUT DATA FROM THE COMPUTER PROGRAM "Wv.Th."

SL.NO.	WV LT. (Stoke's) (m.)	WV.LT. (Airy's) (m.)	DIFF. (S-A) (m.)	g*T*T*/H	VMAX*T/H	Re	K-C
1.	5.77700	5.78226	-0.00526	240.07912	3.31852	17278.47461	9.59051
2.	5.91300	5.91838	-0.00538	186.06743	3.33455	22619.26172	12.67684
3.	6.01600	6.02147	-0.00547	141.37598	3.20531	28753.44141	16.19218
4.	5.93700	5.94240	-0.00540	192.67563	3.36631	22157.56055	12.47778
5.	5.85300	5.85833	-0.00533	150.97353	3.10677	26597.22461	15.26459
6.	5.85800	5.86333	-0.00533	285.69632	3.44349	22270.51563	5.41393
7.	9.40200	9.41055	-0.00855	417.22571	4.46117	19334.94727	15.52489
8.	6.98500	6.99136	-0.00636	196.91008	3.49459	25856.05664	16.30224
9.	6.05500	6.06051	-0.00551	141.56445	3.21822	28968.70177	16.39146
10.	6.02300	6.02848	-0.00548	142.12624	3.20763	28622.30859	16.11834
11.	8.34000	8.34759	-0.00759	475.97931	4.11360	15575.38770	12.46420
12.	7.95700	7.96424	-0.00724	380.34549	4.01725	16727.81055	11.76383
13.	6.95000	6.95632	-0.00632	212.48274	3.56015	23790.82422	15.00011
14.	5.98300	5.98844	-0.00544	166.02502	3.33214	25453.33789	14.33376
15.	6.08500	6.09054	-0.00554	163.14943	3.36380	26398.28516	15.00816
16.	5.86500	5.87034	-0.00534	173.12549	3.24794	23564.94531	13.14335
17.	6.00200	6.00746	-0.00546	142.26779	3.20069	28531.96484	16.06746
18.	6.90000	6.90628	-0.00628	192.42473	3.46486	25349.05469	15.84598
19.	8.13500	8.14240	-0.00740	321.31873	4.04212	20228.66406	14.44383

SL.NO.	WV LT. (Stoke's) (m.)	WV.LT. (Airy's) (m.)	DIFF. (S-A) (m.)	g*T*T*/H	VMAX*T/H	Re	K-C
20.	9.68600	9.69481	-0.00881	425.84006	4.58017	19905.97852	16.35884
21.	5.98600	5.99145	-0.00545	163.92906	3.33307	25785.95508	14.52107
22.	5.85300	5.85833	-0.00533	140.84908	3.15166	27834.74023	15.37483
23.	5.57800	5.58308	-0.00507	285.74899	3.39263	14480.93164	7.84263
24.	5.59500	5.60009	-0.00509	278.55759	3.37024	14830.16504	8.07173
25.	5.68300	5.68817	-0.00517	228.26668	3.34191	18034.16992	9.86419
26.	6.02000	6.02548	-0.00548	135.81435	3.22875	30005.47656	16.81640
27.	5.66800	5.67316	-0.00516	225.34019	3.28633	17964.58984	9.82613
28.	5.87900	5.88435	-0.00535	175.21829	3.20590	22982.05078	12.81824
29.	5.98300	5.98844	-0.00544	135.38113	3.19442	29781.41016	16.69083
30.	5.81100	5.81629	-0.00529	246.68350	3.37873	17204.12109	9.59560
31.	5.86500	5.87034	-0.00534	176.46880	3.20140	22787.17969	12.70955
32.	6.15600	6.16160	-0.00560	143.07563	3.27344	29292.82031	16.73271
33.	6.00100	6.00646	-0.00546	163.83203	3.20036	24418.31445	13.55352
34.	6.02800	6.03348	-0.00549	141.18965	3.27657	29431.43164	16.57398
35.	5.78300	5.78826	-0.00526	236.80159	3.37064	17792.75000	9.87596
36.	5.84100	5.84631	-0.00531	178.20941	3.19370	22401.58789	12.43412
37.	6.02200	6.02748	-0.00548	140.67978	3.25204	43975.47266	11.00635
38.	5.89800	5.90337	-0.00537	216.23566	3.37918	19344.73633	10.63316
39.	5.92000	5.92539	-0.00539	179.38286	3.21909	22649.77148	12.69394
40.	6.03700	6.04249	-0.00549	146.10637	3.25690	42608.37109	10.71522

SL.NO.	WV LT. (Stoke's) (m.)	WV.LT. (Airy's) (m.)	DIFF. (S-A) (m.)	g*T*T*/H	VMAX*T/H	Re	K-C
41.	5.80800	5.81328	-0.00528	330.72278	3.42940	13024.86035	7.26462
42.	5.85000	5.85532	-0.00532	253.83961	3.34024	16608.50000	9.30814
43.	5.88300	5.88835	-0.00535	175.29135	3.25360	23314.30859	13.00356
44.	5.97100	5.97643	-0.00543	160.08939	3.30490	25235.47656	14.21108
45.	5.99800	6.00346	-0.00546	142.45699	3.19937	28482.31250	16.03950
46.	5.76600	5.77125	-0.00525	251.53909	3.34035	16599.75977	9.21379
47.	6.00900	6.01447	-0.00547	175.45259	3.27048	23526.83008	13.18548
48.	6.00900	6.01447	-0.00547	144.57172	3.20300	28097.56055	15.82283
49.	5.69500	5.70018	-0.00518	238.18726	3.37160	17522.52148	9.63155
50.	5.86400	5.86934	-0.00534	173.91339	3.24763	23455.91992	13.08254
51.	6.04800	6.05350	-0.00550	136.68600	3.21589	29838.15430	16.80302
52.	6.14500	6.15059	-0.00559	143.52332	3.22660	28920.79492	16.52020
53.	6.05700	6.06251	-0.00551	142.35635	3.21888	43220.22266	10.86908
54.	8.12600	8.13339	-0.00739	151.56433	3.91852	49888.47266	12.66551
55.	7.98600	7.99327	-0.00727	321.28220	3.95459	29241.19141	9.13950
56.	9.41100	9.41956	-0.00856	414.64413	4.46446	29204.48242	10.42203
57.	6.89100	6.89727	-0.00627	216.39896	3.58034	34937.88281	9.70669
58.	5.77700	5.78226	-0.00526	240.07912	3.42070	26715.79297	6.59055
59.	6.01880	6.02348	-0.00548	148.27356	3.16218	40570.36719	10.15411
60.	6.00900	6.01447	-0.00547	163.80954	3.38802	39533.59766	9.94197

SL.NO.	MV.LT. (Stoke's) (m.)	MV.LT. (Airy's) (m.)	DIFF. (S-A) (m.)	g*T*T*/H	VMAX*T/H	Re	K-C
61.	5.80000	5.80528	-0.00528	311.83136	3.40137	20511.51367	5.09449
62.	5.98200	5.98744	-0.00544	142.71010	3.26184	43272.35156	10.77855
63.	5.88100	5.88635	-0.00535	150.50513	3.11615	38821.68750	9.57697
64.	5.66600	5.67115	-0.00516	184.97563	3.28574	32659.55664	7.90039
65.	5.95500	5.96042	-0.00542	146.04948	3.25321	42171.08203	10.50424
66.	5.98100	5.98644	-0.00540	339.09238	3.47847	19701.21875	4.97808
67.	6.13600	6.14158	-0.00558	144.08517	3.24510	43459.41016	11.03332
68.	5.72700	5.73221	-0.00521	184.47942	3.27842	32997.90234	8.06126
69.	5.66700	5.67216	-0.00516	283.49249	3.39008	22095.51953	5.37139
70.	6.02800	6.03348	-0.00549	143.26666	3.20927	42613.52734	10.66548
71.	5.96600	5.97143	-0.00543	152.55925	3.25672	40415.23047	10.06688
72.	5.82200	5.82730	-0.00530	193.98750	3.28264	31729.05469	7.82728
73.	5.67300	5.67816	-0.00516	293.49600	3.39174	21458.05664	5.24212
74.	6.04000	6.04550	-0.00550	152.70772	3.30321	41346.04688	10.39776
75.	5.93800	5.94340	-0.00540	150.39682	3.13527	39467.46484	9.83081
76.	5.93900	5.94440	-0.00540	154.61560	3.22521	39491.91016	9.83690
77.	5.82400	5.82930	-0.00530	206.80437	3.28326	29768.16602	7.34355
78.	5.85900	5.86433	-0.00533	331.97540	3.44377	19450.72852	4.79833
79.	5.98600	5.99145	-0.00545	136.03200	3.24039	45098.20313	11.23335
80.	5.74000	5.74522	-0.00522	187.61624	3.25766	32240.77930	7.87630

SL.NO.	WV LT. (Stoke's) (m.)	WV.LT. (Airy's) (m.)	DIFF. (S-A) (m.)	g*T*T*/H	VMAX*T/H	R _e	K-C
81.	5.89700	5.90237	-0.00537	182.54616	3.30563	22855.63672	12.80931
82.	5.96600	5.97143	-0.00543	148.34666	3.27976	41856.97656	10.42600
83.	6.01500	6.02004	-0.00547	149.35886	3.30472	42091.00000	10.53470
84.	5.83000	5.83530	-0.00530	198.89977	3.28509	30968.49609	7.63966
85.	5.70800	5.71319	-0.00519	321.83081	3.40145	19721.05273	4.84140
86.	5.08800	5.09263	-0.00463	137.99712	2.90573	26831.96875	15.18242
87.	9.41800	9.42657	-0.00857	431.60623	4.49771	28360.55078	10.15483
88.	5.91800	5.92339	-0.00538	146.92369	3.24140	41567.26563	10.30406
89.	5.78900	5.79427	-0.00527	192.64731	3.22414	31228.02930	7.66629
90.	5.68400	5.68917	-0.00517	297.12732	3.39479	21214.85156	5.18271
91.	8.12500	8.13239	-0.00739	347.75735	4.03864	28012.05078	8.88950
92.	5.91900	5.92439	-0.00538	152.79851	3.24172	39973.01172	9.90886
93.	5.71100	5.71620	-0.00520	195.38638	3.24886	31026.37305	7.61678
94.	5.69900	5.70419	-0.00519	303.53378	3.39895	20792.54102	5.07954
95.	8.05100	8.05832	-0.00733	138.21756	3.92522	54282.46875	13.65103
96.	9.32200	9.33048	-0.00848	403.90649	4.49456	29980.10625	10.62713
97.	5.98500	5.99045	-0.00545	167.32159	3.33276	37891.18359	9.48355
98.	5.74000	5.74522	-0.00522	201.50688	3.25766	30018.30078	7.33336

Appendix - E

Plotting Software Details

Appendix - E

PROGRAM "PLOT"

The listing of the program "PLOT", presented in this section, first reads the values of C_d (drag coefficient), C_m (inertia coefficient), C_L (lift coefficient), K_C (Keulegan - Carpenter number) and Re (Reynolds number) which are stored in the file "Run.Dat" in the same sequence. The program also reads the values of Set No., D (diameter of cylinder), θ_x (angle of inclination in in-line direction), θ_y (angle of inclination in transverse direction) and k/D (surface roughness) from the file "Jet.Dat" in the same order. According to the chosen Set No. which specifies D , θ_x , θ_y and k/D , the program takes the corresponding values of chosen points between them i.e. Re vs C_d , K_C vs C_L etc.. While plotting the points, program takes care of the following points :

1. Choose the different symbol for each set.
2. Choose the different colour for each set.
3. Legend the graph according to the chosen parameters.
4. Legend the graph according to the chosen boundary conditions.
5. Draw the best-fit curve.

To draw the best-fit curve for each set, the program uses the Least square Fit, which is explained below with a set of values of $K-C$ and C_m for $\theta_1 = 27.5^\circ$, $\theta_2 = 0$, and $k/D = 0$. After drawing the curves of different degree of accuracy, it was observed that the third degree of accuracy is best fitted for C_d and C_l against non-dimensional parameters and fifth degree of accuracy for C_m . The graphs are presented in Appendix - F.

Least - Squares Fit to a Polynomial (Multiple Regression)

For an n^{th} degree polynomial, that is

$$y = a + bx + cx^2 + \dots + mx^n \quad \dots \dots \quad (E.1)$$

where a, b, c, \dots etc. are the constants and x_i and y_i represent two independent variables.

The method of least - squares requires that X^2 , to be minimized, where

$$X^2 = \sum (y_i / \sigma_i)^2 = \sum (1/\sigma_i^2)(y_i - a - bx_i - \dots - mx_i^n)^2 \quad \dots \dots \quad (E.2)$$

where

σ_i = standard deviation

The minimum value of X^2 can be determined by setting the derivatives of X^2 with respect to each of the constants equal to 0, which yields $n+1$ simultaneous equations. The equations can be rearranged to show the interaction of the

constants explicitly, in the following manner :

$$\begin{aligned}
 \Sigma y_i &= a\Sigma 1 + b\Sigma x_i + \dots + m\Sigma x_i^n \\
 \Sigma x_i y_i &= a\Sigma x_i + b\Sigma x_i^2 + \dots + m\Sigma x_i^{n+1} \\
 &\vdots \\
 \Sigma x_i^{n-1} y_i &= a\Sigma x_i^{n-1} + b\Sigma x_i^n + \dots + m\Sigma x_i^{2n-1} \\
 \Sigma x_i^n y_i &= a\Sigma x_i^n + b\Sigma x_i^{n+1} + \dots + m\Sigma x_i^{3n-1} \quad \dots\dots(E.3)
 \end{aligned}$$

The above equations can be written in matrix form as follows

$$\left[\begin{array}{c} \Sigma y_i \\ \Sigma x_i y_i \\ \vdots \\ \Sigma x_i^n y_i \end{array} \right] = \left[\begin{array}{cccc} \Sigma 1 & \Sigma x_i & \dots & \Sigma x_i^n \\ \Sigma x_i & \Sigma x_i^2 & \dots & \Sigma x_i^{n+1} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma x_i^{n-1} & \Sigma x_i^n & \dots & \Sigma x_i^{2n} \end{array} \right] \left[\begin{array}{c} a \\ b \\ \vdots \\ m \end{array} \right] \quad \dots\dots(E.4)$$

Then by using Gauss - Jordan elimination method and back substitution the values of constants a, b, \dots, m , can be determined. Once the optimum value of the coefficients are determined, the value of polynomial y_i can be obtained for the value of x_i and a best fit curve can be drawn for the plotted points.

To understand the method in a better fashion, let us take a set of readings for which values of x_i (K-C) and y_i

(C_m) are given below.

i	x_i	y_i
1	12.52	0.850
2	13.25	1.030
3	10.61	2.770
4	12.23	1.700
5	10.92	2.330
6	9.10	4.260
7	12.77	1.110
8	11.40	1.170
9	9.93	0.911
10	11.29	2.060
11	8.38	3.890
12	8.66	2.940
13	7.32	0.300
14	6.73	4.370
15	9.46	0.397
16	8.51	2.390
17	7.22	1.910
18	6.89	1.760
19	5.87	0.097
20	6.04	4.040
21	4.93	3.390
22	4.42	7.310
23	5.28	4.650
24	5.37	1.030

For the chosen set of the values, let us fit a third degree of polynomial.

$$y = a + bx + cx^2 + dx^3$$

Therefore, the corresponding matrix (E.4), for the third degree polynomial, will be

$$\begin{bmatrix} \Sigma y_i \\ \Sigma x_i y_i \\ \Sigma x_i^2 y_i \\ \Sigma x_i^3 y_i \end{bmatrix} = \begin{bmatrix} \Sigma 1 & \Sigma x_i & \Sigma x_i^2 & \Sigma x_i^3 \\ \Sigma x_i & \Sigma x_i^2 & \Sigma x_i^3 & \Sigma x_i^4 \\ \Sigma x_i^2 & \Sigma x_i^3 & \Sigma x_i^4 & \Sigma x_i^5 \\ \Sigma x_i^3 & \Sigma x_i^4 & \Sigma x_i^5 & \Sigma x_i^6 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

By substituting the values in the matrix, we get

$$\begin{bmatrix} 56.67 \\ 442.3 \\ 3830.9 \\ 36171.1 \end{bmatrix} = \begin{bmatrix} 24 & 209.1 & 1992.7 & 20382.5 \\ 209.1 & 1992.7 & 20382.5 & 219769 \\ 1992.7 & 20382.5 & 219769 & 2462449 \\ 20382.5 & 219769 & 2462449 & 2.8 \times 10^7 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

By applying first step of Gauss-Jordan elimination method, we get

$$\begin{bmatrix} 2.36 \\ 2.11 \\ 1.92 \\ 1.77 \end{bmatrix} = \begin{bmatrix} 1 & 8.7 & 83 & 849 \\ 1 & 9.53 & 97.48 & 1052 \\ 1 & 10.2 & 110.3 & 1235.7 \\ 1 & 10.78 & 120.8 & 1392.2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

Second step reduces the above matrix as

$$\begin{bmatrix} 2.36 \\ -0.3 \\ -0.289 \\ -0.283 \end{bmatrix} = \begin{bmatrix} 1 & 8.7 & 83 & 849 \\ 0 & 1 & 17.67 & 246.8 \\ 0 & 1 & 17.98 & 254.9 \\ 0 & 1 & 18.25 & 262.3 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

Which is further reduced as

$$\left[\begin{array}{c} 2.36 \\ -0.3 \\ 3.8 \times 10^{-2} \\ 0.030 \end{array} \right] = \left[\begin{array}{cccc} 1 & 8.7 & 83 & 849 \\ 0 & 1 & 17.67 & 246.8 \\ 0 & 0 & 1 & 26.51 \\ 0 & 0 & 1 & 26.71 \end{array} \right] \left[\begin{array}{c} a \\ b \\ c \\ d \end{array} \right]$$

The final form of the matrix will be

$$\left[\begin{array}{c} 2.36 \\ -0.3 \\ 3.8 \times 10^{-2} \\ 3.85 \times 10^{-2} \end{array} \right] = \left[\begin{array}{cccc} 1 & 8.7 & 83 & 849 \\ 0 & 1 & 17.67 & 246.8 \\ 0 & 0 & 1 & 26.51 \\ 0 & 0 & 0 & 1 \end{array} \right] \left[\begin{array}{c} a \\ b \\ c \\ d \end{array} \right]$$

Now by back substitution, we get the values of the constants d , c , b , and a .

$$d = -3.85 \times 10^{-2}$$

$$c = 3.80 \times 10^{-2} - 26.5d = 1.05$$

$$b = -0.3 - 17.67c - 246.8d = -9.5$$

$$a = 2.36 - 8.7b - 83c - 849d = 30.02$$

Substitute the values of constants in the third degree polynomial and get the values of y for the corresponding values of x . In the program "PLOT", the value of x is incremented by 0.5 and 900 for K-C and Re respectively.

i	x _i	y _i
1	4	6.44
2	5	4.11
3	6	2.74
4	7	2.10
5	8	1.97
6	9	2.10
7	10	2.28
8	11	2.26
9	12	1.81
10	13	0.71
11	14	-1.26

```

10 SCREEN 9 : COLOR 15,9: CLS
20 *****
30 ' DECLARING THE DIFFERENT ARRAYS OF SPECIFIED DIMENSIONS AND
   READING THE DATA SETS
40 *****
50 DIM KC(40,25),RE(40,25),CD(40,25),CM(40,25),CL(40,25),A(7,7)
60 DIM SET(40),IRUN(40),DIA(40),PHIX(40),PHIY(40),RFG(40),NSET(8),
   GRAPH(8),B(7), C(7)
70 INPUT "total set to read",TSET
80 LOCATE 5,10 : PRINT "Reading DATA set no. ="
90 *****
100 ' CREATING THE FILE JET.DAT TO STORE THE BOUNDARY CONDITIONS
110 *****
120 FOR ID=1 TO TSET
130 LOCATE 5,33: PRINT ID
140 OPEN "R",#2,"JET.DAT",24
150 FIELD #2,4 AS ZSET$,4 AS ZIRUN$,4 AS ZDIA$,4 AS ZPHIX$,4 AS
   ZPHIY$,4 AS ZRFG$
160 CODES%:=ID
170 GET #2,CODES%:CLOSE
180 SET(ID)=CVS(ZSET$):IRUN(ID)=CVS(ZIRUN$)
190 DIA(ID)=CVS(ZDIA$):PHIX(ID)=CVS(ZPHIX$)
200 PHIY(ID)=CVS(ZPHIY$):RFG(ID)=CVS(ZRFG$)
210 *****
220 ' CREATING THE FILE RUN.DAT TO STORE THE VALUES OF
   COEFFICIENTS AND NON-DIMENSIONAL PARAMETERS'
230 *****
240 FOR IR=1 TO IRUN(ID)
250 OPEN "R",#3,"RUN.DAT",20
260 FIELD #3,4 AS RKC$,4 AS RRE$,4 AS RCD$,4 AS RCM$,4 AS RCL$
270 CODER% =1R+100*ID
280 GET #3,CODER% :CLOSE
290 KC(ID,IR)=CVS(RKC$):RE(ID,IR)=CVS(RRE$)
300 CD(ID,IR)=CVS(RCD$):CM(ID,IR)=CVS(RCM$)
310 CL(ID,IR)=CVS(RCL$)
320 NEXT IR
330 NEXT ID
340 *****
350 ' COMMANDS TO PLOTTER FOR LEGENDS AND LABELS'
360 *****
370 OPEN "COM2:9600,n,,2" AS #1
380 PRINT #1,"in;SP1;"
390 CLS
400 LOCATE 5,10 : PRINT "O P T I O N S"
410 LOCATE 6,8:PRINT "----"
420 LOCATE 8,10:PRINT "1. Cd vs K-C":LOCATE 9,10:PRINT "2. Cm vs K-C"
430 LOCATE 10,10:PRINT "3. CL vs K-C":LOCATE 11,10:PRINT "4. Cd vs Re"
440 LOCATE 12,10:PRINT "5. Cm vs Re":LOCATE 13,10:PRINT "6. CL vs Re"
450 LOCATE 10,30:PRINT "ENTER your choice":LOCATE 10,47,1
460 INPUT VS :IF VS<1 OR VS>6 THEN GOTO 450
470 LOCATE 15,10:PRINT "HOW MANY SETS (max.=6)":LOCATE 15,34,1
480 INPUT TOSET :IF TOSET<1 OR TOSET>6 THEN GOTO 470
490 LOCATE 17,10:PRINT "ENTER SET NUMBER(S)"
500 FOR I=1 TO TOSET

```

```

510 LOCATE I+16,30,1
520 INPUT NSET(I)
530 NEXT I
540 BXS=1000:BYE=500:BYE=6500:BXE=10000
550 PRINT #1,"pu;pa";BXS;BYS;"pd;pa";BXE;BYS;BXE;BYE;BXS;BYE;
      BXS;BYS;CHR$(3)
560 TXS = 5000:TXE=10000:TYS=4000:TYE=6000
570 PRINT #1,"pu;pa";TXS;TYS;"pd;pa";TXE;TYS;TXE;TYE;TXS;TYE;
      TXS;TYS;CHR$(3)
580 IF VS>3 THEN GOTO 600
590 XOFST=3000:XVSTR=4000:VLNO=2:GOTO 610
600 XOFST=1500:XVSTR=2500:VLNO=5
610 FOR I = 1 TO VLNO
620 CXV=XVSTR+(I-1)*XOFST
630 PRINT #1,"pu;pa";CXV;BYS;"pd;"
640 IF CXV<TXS THEN PRINT #1,"pa";CXV;BYE;CHR$(3)
650 IF CXV>TXS THEN PRINT #1,"pa";CXV;TYS;"pu;pa";CXV;TYE;"pd;pa";
      CXV;BYE;CHR$(3)
660 NEXT I
670 YOFST=1500:YHSTR=2000:HLNO=3
680 FOR I=1 TO HLNO : CYH=YHSTR+(I-1)*YOFST
690 PRINT #1,"pu;pa";BXS;CYH;"pd;"
700 IF CYH<TYS THEN PRINT #1,"pa";BXE;CYH;CHR$(3)
710 IF CYH>TYS THEN PRINT #1,"pa";TXS;CYH;"pu;pa";TXE;CYH;"pd;pa";
      BXE;CYH;CHR$(3)
720 NEXT I
730 IF VS<4 THEN XTOF=600 ELSE XTOF=150
740 IF VS<4 THEN XTNO=14 ELSE XTNO=59
750 YTKE=BYS + 50
760 FOR I = 1 TO XTNO : XTK=BXS+XTOF*I
770 PRINT #1,"pu;pa";XTK;BYS;"pd;pa";XTK;YTKE;CHR$(3)
780 NEXT I
790 YTNO=39
800 YTOF=150
810 XTK = BXS+75
820 FOR I = 1 TO YTNO:YTK=BYS+YTOF +
830 PRINT #1,"pu;pa";BXS;YTK;"pd;pa";XTKE;YTK;CHR$(3)
840 NEXT I
850 YLEB=350 : IF VS<4 THEN IEND=15 ELSE IEND=6
860 IF VS<4 THEN ISTEP=5 ELSE ISTEP=1
870 IF VS<4 THEN HOF=600 ELSE HOF=1500
880 FOR I=0 TO IEND STEP ISTEP : XLEB=BXS-150+I*HOF
890 PRINT #1,"pu;pa";XLEB;YLEB;"1b" I;CHR$(3)
900 NEXT I
910 IF VS=1 OR VS=4 THEN XLEB=500
920 IF VS=2 OR VS=5 THEN XLEB=700
930 IF VS=3 OR VS=6 THEN XLEB=300
940 IF VS=2 OR VS=5 THEN IEND=40 ELSE IEND=20
950 IF VS=2 OR VS=5 THEN ISTEP=10 ELSE ISTEP=5
960 IF VS=2 OR VS=5 THEN TKYOF=150 ELSE TKYOF=300
970 FOR I = 0 TO IEND STEP ISTEP
980 IF VS=2 THEN K=I*.2:GOTO 1050
990 IF VS=5 THEN K=I*.2:GOTO 1050
1000 IF VS=1 THEN K=I*.05:GOTO 1050

```

```

1010 IF VS=4 THEN K=I*.05:GOTO 1050
1020 IF VS=3 THEN GOTO 1040
1030 IF VS=6 THEN GOTO 1040
1040 IF I=20 THEN K=.1 ELSE K=I*.005
1050 YLEB=BYS+I*TKYOF
1060 PRINT #1,"pu;pa";XLEB;YLEB;"1b" K;CHR$(3)
1070 NEXT I
1080 A$="COEFF.OF " :C$=" vs " :E$=" NO."
1090 IF VS=1 OR VS=4 THEN B$="DRAG"
1100 IF VS=2 OR VS=5 THEN B$="INERTIA"
1110 IF VS=3 OR VS=6 THEN B$="LIFT"
1120 IF VS<4 THEN D$="K-C" ELSE D$="Re"
1130 PRINT #1,"PU;PA 4000,25; 1b" A$ B$ C$ D$ E$ ;CHR$(3)
1140 PRINT #1,"pu;pa 2300,25;1b Fig.No.F- "CHR$(3)
1150 PRINT #1,"pu;pa 6500,5700; 1b LEGEND "CHR$(3)
1160 PRINT #1,"pu;pa 7000,175;1b"D$ E$ ;CHR$(3)
1170 IF VS<4 THEN GOTO 1190
1180 PRINT #1."1b x10^-4"CHR$(3)
1190 PRINT #1."1b --->"CHR$(3)
1200 PRINT #1."di0,1"CHR$(3)
1200 PRINT #1."pu;pa 225,3200;1b"A$ B$;CHR$(3)
1210 PRINT #1."1b --->"CHR$(3)
1220 PRINT #1."di1,0"CHR$(3)
1230 PRINT #1."di1,0"CHR$(3)
1240 INPUT"proceed?",JUNK$
1250 BEEP
1260 FOR IS=1 TO TOSET
1270 ID=NSET(IS)
1280 N=3
1290 FOR I=1 TO N+1
1300 FOR J=1 TO N+1
1310 A(I,J)=0
1320 NEXT J
1330 B(I)=0
1340 NEXT I
1350 FOR IR=1 TO IRUN(ID)
*****'COMMANDS FOR CHOOSING SCALE AND PLOTTER PEN'
1360 *****'*****'
1370 *****'
1380 *****'
1390 XOSE=1000
1400 YOSE=500
1410 IF VS<4 THEN MFACX=KC(ID,IR) ELSE MFACX=RE(ID,IR)
1420 IF VS<4 THEN MFY=600*KC(ID,IR) ELSE MFY=.15*RE(ID,IR)
1430 IF VS=1 OR VS=4 THEN MFACY=CD(ID,IR)
1440 IF VS=1 OR VS=4 THEN MFY=6000*CD(ID,IR)
1450 IF VS=2 OR VS=5 THEN MFACY=CM(ID,IR)
1460 IF VS=2 OR VS=5 THEN MFY=750*CM(ID,IR)
1470 IF VS=3 OR VS=6 THEN MFACY=CL(ID,IR)
1480 IF VS=3 OR VS=6 THEN MFY=60000!*CL(ID,IR)
1490 X1=XOSE+MFY
1500 Y1=YOSE+MFY

```

```

1510 FOR I=1 TO N+1
1520 FOR J=1 TO N+1
1530 A(I,J) = A(I,J) + (MFACX ^ (I+J-2))
1540 NEXT J
1550 B(I) = B(I) + MFACY*(MFACX^(I-1))
1560 NEXT I
1570 IF X1>10100 OR Y1>6500 GOTO 1900
1580 M1=2
1590 M2=3
1600 M3=4
1610 M4=5
1620 M5=6
1630 M6=1
1640 IF IS=1 THEN GOTO 1730 ELSE GOTO 1650
1650 IF IS=2 THEN GOTO 1750 ELSE GOTO 1660
1660 IF IS=3 THEN GOTO 1790 ELSE GOTO 1670
1670 IF IS=4 THEN GOTO 1820 ELSE GOTO 1680
1680 IF IS=5 THEN GOTO 1850 ELSE GOTO 1690
1690 IF IS=6 THEN GOTO 1880 ELSE GOTO 1900
1700 ****
1710 'COMMANDS FOR DRAWING SYMBOLS FOR DIFFERENT CHOSEN SETS'
1720 ****
1730 PRINT #1,"sp";M1;CHR$(3)
1740 PRINT #1,"pt0.1;pu";X1;Y1;"pd;ci 35,35;" :GOTO 1900
1750 L1=X1-30:L2=Y1-25
1760 PRINT #1,"sp";M2;CHR$3
1770 PRINT #1,"pt0.1;pu;pa";X1;Y1;"pd;pu"CHR$(3)
1780 PRINT #1,"pa";L1;L2;"er 60,50;" :GOTO 1900
1790 PRINT #1,"sp";M3;CHR$(3)
1800 PRINT #1,"pt0.1;pu;pa";X1;Y1;CHR$(3)
1810 PRINT #1,"lb*"CHR$(3):GOTO 1900
1820 PRINT #1,"sp";M4;CHR$(3)
1830 PRINT #1,"pt0.1;pu;pa";X1;Y1;CHR$(3)
1840 PRINT #1,"lb+"CHR$(3):GOTO 1900
1850 PRINT #1,"sp";M5;CHR$(3)
1860 PRINT #1,"pt0.1;pu;pa";X1;Y1;CHR$(3)
1870 PRINT #1,"lbx"CHR$(3):GOTO 1900
1880 PRINT #1,"sp";M6;CHR$(3)
1890 PRINT #1,"pt0.1;pu";X1;Y1;"pd;ci 35,120;" :GOTO 1900
1900 BEEP
1910 NEXT IR
1920 INPUT "proceed?",JUNK$
1930 ****
1940 'COMMANDS FOR GENERATING BEST FIT CURVE USING
    LEAST SQUARE METHOD'
1950 ****
1960 M=1
1970 P=0
1980 FOR K=1 TO N+1
1990 FOR I=M TO N+1
2000 T=A(I,M)

```

```

2010 FOR J=1 TO N+1
2020 A(I,J) =A(I,J)/T
2030 NEXT J
2040 B(I) = B(I)/T
2050 NEXT I
2060 M=M+1
2070 P=P+1
2080 IF P=N+1 GOTO 2150 ELSE GOTO 2090
2090 FOR I=M TO N+1
2100 FOR J=1 TO N+1
2110 A(I,J)=(A(I,J) - A(P,J))
2120 NEXT J
2130 B(I)=(B(I)-B(P))
2140 NEXT I
2150 NEXT K
2160 FOR I=N+1 TO 1 STEP -1
2170 R=0
2180 FOR J=N+1 TO 1 STEP -1
2190 IF J>I THEN GOTO 2200 ELSE GOTO 2210
2200 R=(A(I,J)*C(J))+R
2210 NEXT J
2220 IF I=N+1 THEN GOTO 2230 ELSE GOTO 2240
2230 C(I)=B(I):GOTO 2250
2240 C(I)= (B(I)-R)
2250 NEXT I
2260 FOR I=1 TO N+1
2270 NEXT I
2280 IF VS<4 THEN X7=4 ELSE X7=10000
2290 Y12=0
2300 FOR I=1 TO N+1
2310 Y12=Y12 +(C(I)*X7^(I-1))
2320 NEXT I
2330 IF VS=1 OR VS=4 THEN Y11=6000*Y12+YOSE
2340 IF VS=2 OR VS=5 THEN Y11=750*Y12+YOSE
2350 IF VS=3 OR VS=6 THEN Y11=60000!*Y12+YOSE
2360 X11=XOSE+1300
2370 IF VS<4 THEN X8=4 ELSE X8=10000
2380 PRINT #1,"pu"CHR$(3)
2390 FOR M=1 TO 50
2400 Y8=0
2410 FOR I=1 TO N+1
2420 Y8=Y8+(C(I)*X8^(I-1))
2430 NEXT I
2440 IF VS<4 THEN X9=XOSE+600*X8 ELSE X9=XOSE+.15*X8
2450 IF VS<4 THEN X8=X7+(M/5) ELSE X8=X7+(900*M)
2460 IF VS=1 OR VS=4 THEN Y9=6000*Y8+YOSE
2470 IF VS=2 OR VS=5 THEN Y9=750*Y8+YOSE
2480 IF VS=3 OR VS=6 THEN Y9=60000!*Y8+YOSE
2490 IF Y9<500 GOTO 2510
2500 PRINT #1,"pa";X9;Y9;"pd"CHR$(3)

```

```

2510 NEXT M
2520 *****
2530 'PRINTING THE BOUNDARY CONDITIONS AT PREDEFINED POINTS FOR THE
    CHOSEN SET'
2540 *****
2550 IF L=0 GOTO 2560 ELSE GOTO 2840
2560 PRINT #1,"pu;pa";X11;Y11;CHR$(3)
2570 X1=5200+1000*IS
2580 Y10=5200-(IS-1)*300
2590 PRINT #1,"pu;pa";6200;Y10;CHR$(3)
2600 PRINT #1,"lb:0x="CHR$(3)
2610 PRINT #1,"pu;pa";6200;Y10;CHR$(3)
2620 PRINT #1,"lb /"CHR$(3)
2630 PRINT #1,"pu;pa";7300;Y10;CHR$(3)
2640 PRINT #1,"lb,Oy="CHR$(3)
2650 PRINT #1,"pu;pa";7300;Y10;CHR$(3)
2660 PRINT #1,"lb /"CHR$(3)
2670 PRINT #1,"pu;pa";8400;Y10;CHR$(3)
2680 PRINT #1,"lb,k/D="CHR$(3)
2690 PRINT #1,"pu;pa";5200;Y10;CHR$(3)
2700 PRINT #1,"lb CURVE"CHR$(3)
2710 PRINT #1,"pu;pa";5800;Y10;CHR$(3)
2720 PRINT #1,"lb"IS;CHR$(3)
2730 PRINT #1,"pu,pa";6700;Y10;CHR$(3)
2740 PRINT #1,"lb"PHIX(ID);CHR$(3)
2750 PRINT #1,"pu,pa";7800;Y10;CHR$(3)
2760 PRINT #1,"lb"PHIY(ID);CHR$(3)
2770 IF RFG(ID)=1 THEN GOTO 2780 ELSE GOTO 2790
2780 IF DIA(ID)=60 THEN K$="1/60" ELSE K$="1/90"
2790 IF RFG(ID)=.5 THEN GOTO 2800 ELSE GOTO 2810
2800 IF DIA(ID)=60 THEN K$="1/120" ELSE K$="1/180"
2810 IF RFG(ID)=0 THEN K$="0" ELSE GOTO 2820
2820 PRINT #1,"pu,pa";9200;Y10;CHR$(3)
2830 PRINT #1,"lb" K$;CHR$(3)
2840 INPUT"proceed?",JUNK$
2850 NEXT IS
2860 PRINT #1,"pu;sp 0;"
2870 PRINT "DO U WANT TO PLOT MORE"
2880 INPUT L$:IF L$="y" GOTO 380
2890 PRINT #1,"pu;sp 0;":CLOSE #1: END

```

Appendix - F

Graphs of C_d , C_m and C_L

against

K_C and Re

Appendix - F

Values of the hydrodynamic coefficients, namely, coefficient of drag (C_d), coefficient of inertia (C_m) and coefficient of lift (C_L), and non-dimensional parameters, namely, Reynolds number (Re) and Keulegan-Carpenter number ($K-C$) were obtained from ref. [22], which were presented in tabular form.

The variations of C_d , C_m and C_L with respect to the parameters $K-C$ and Re are drawn for the 90mm diameter cylinders for varying, relative surface roughness ($k/D = 0, 1/180$ and $1/90$), angle of positive in-line inclination ($\theta_x = 0^\circ, 7.2^\circ, 18.8^\circ$ and 27.5°), angle of negative in-line inclination ($\theta_x = -18.8^\circ$ and 30.2°) and angle in lateral direction ($\theta_y = 20^\circ$). The graphs are presented in this section from Fig. F-1 to Fig. F-30. For each set, in each graph, a Best-fit curve is drawn. The graphs are arranged in following manner:

Parameter	Hydrodynamic coefficient	Variable	Figure numbers
K-C & Re	C_d	Roughness	F-1, F-2, F-3, F-4
		In-line inclination	F-5, F-6, F-7, F-8
		Lateral inclination	F-9, F-10
K-C & Re	C_m	Roughness	F-11, F-12, F-13, F-14
		In-line inclination	F-15, F-16, F-17, F-18
		Lateral inclination	F-19, F-20
K-C & Re	C_L	Roughness	F-21, F-22, F-23, F-24
		In-line inclination	F-25, F-26, F-27, F-28
		Lateral inclination	F-29, F-30

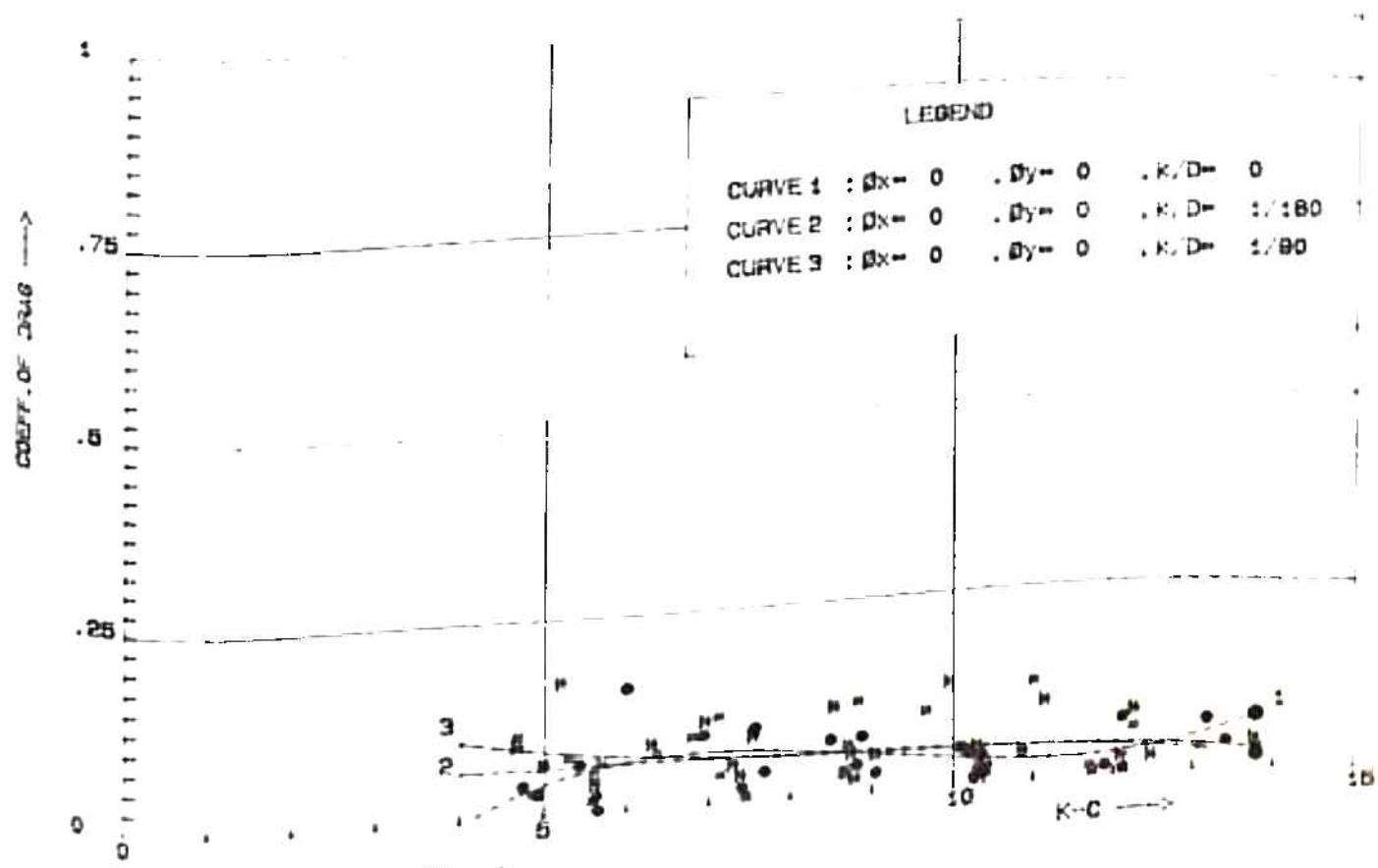
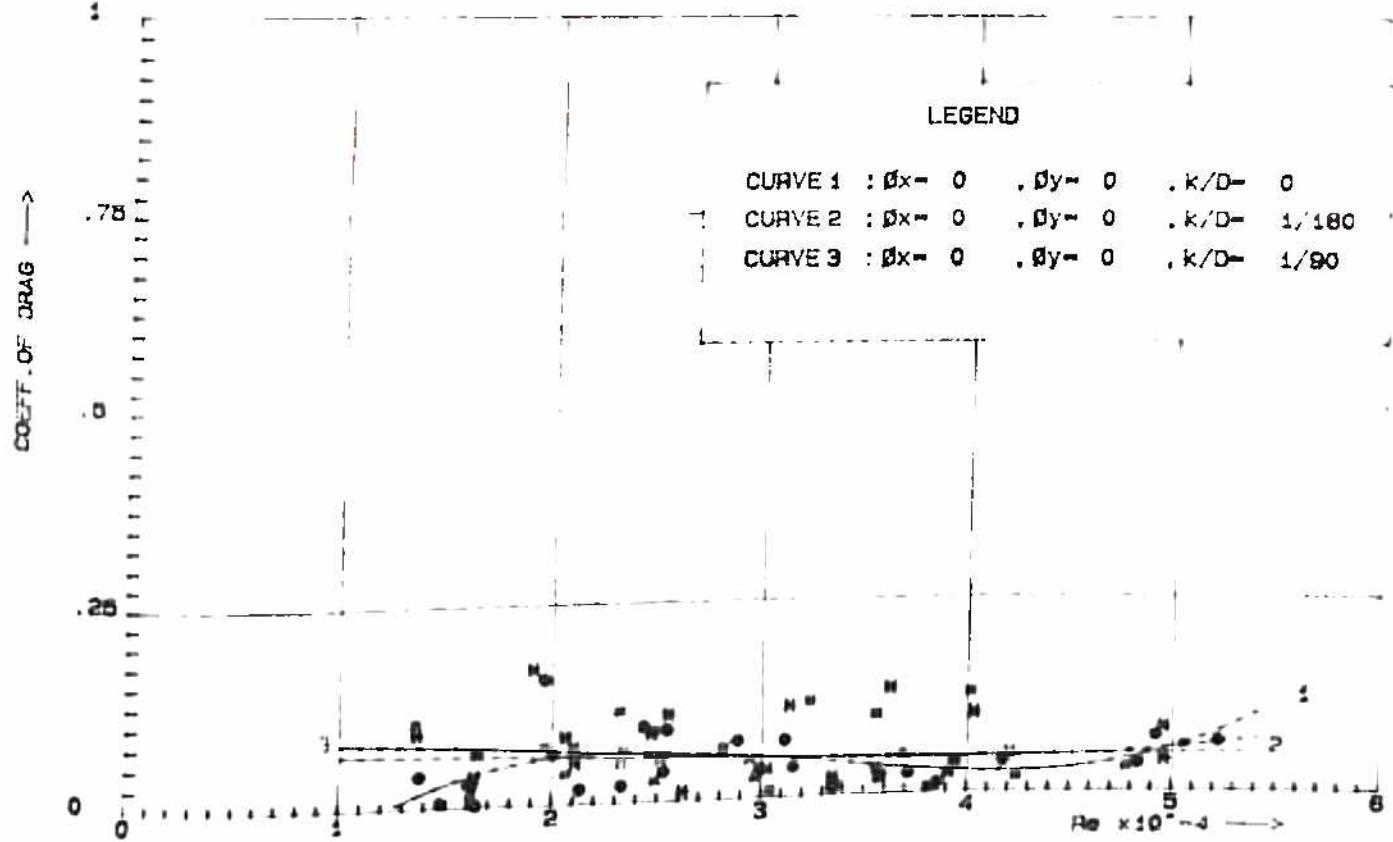


Fig. No. F - 1

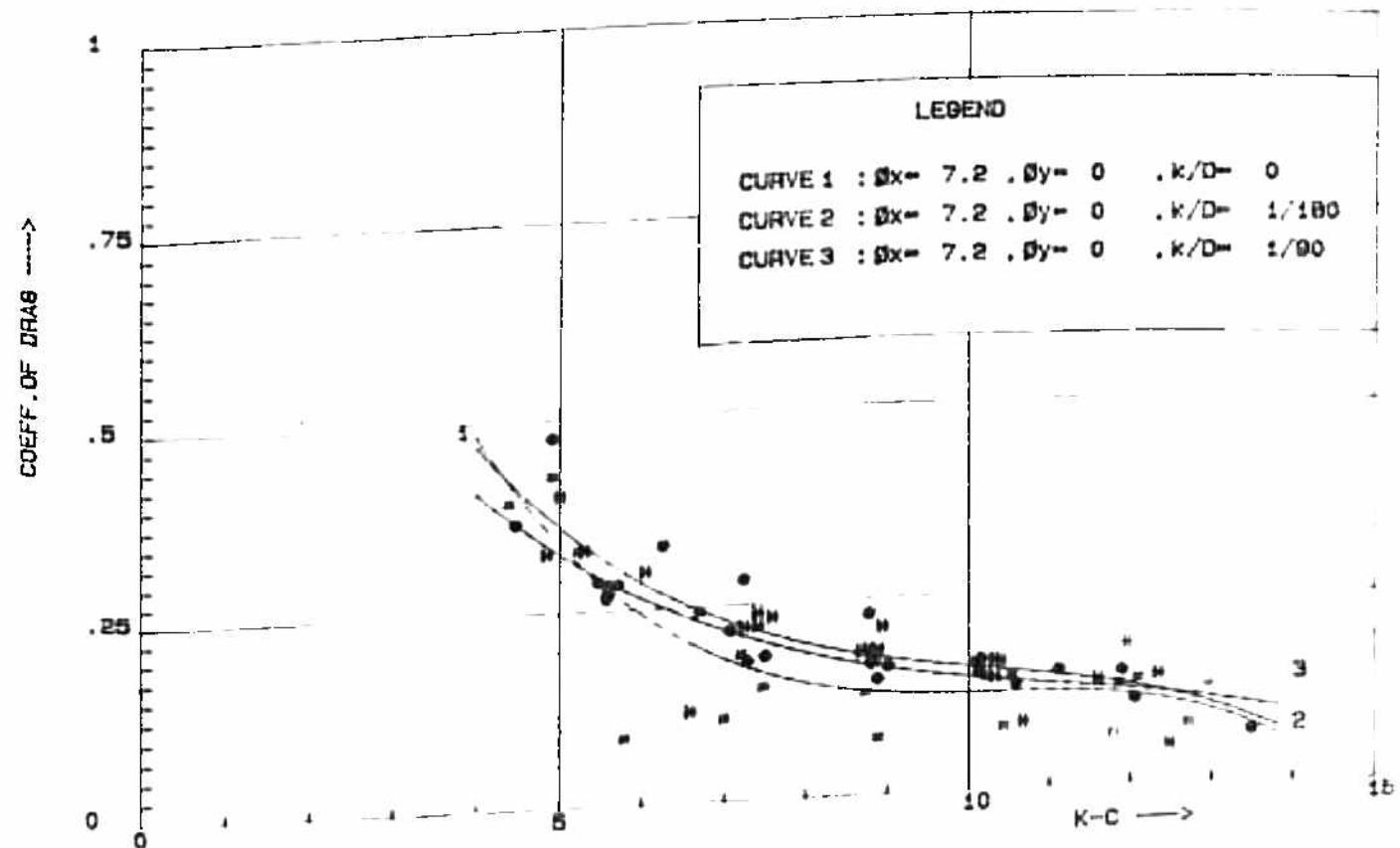
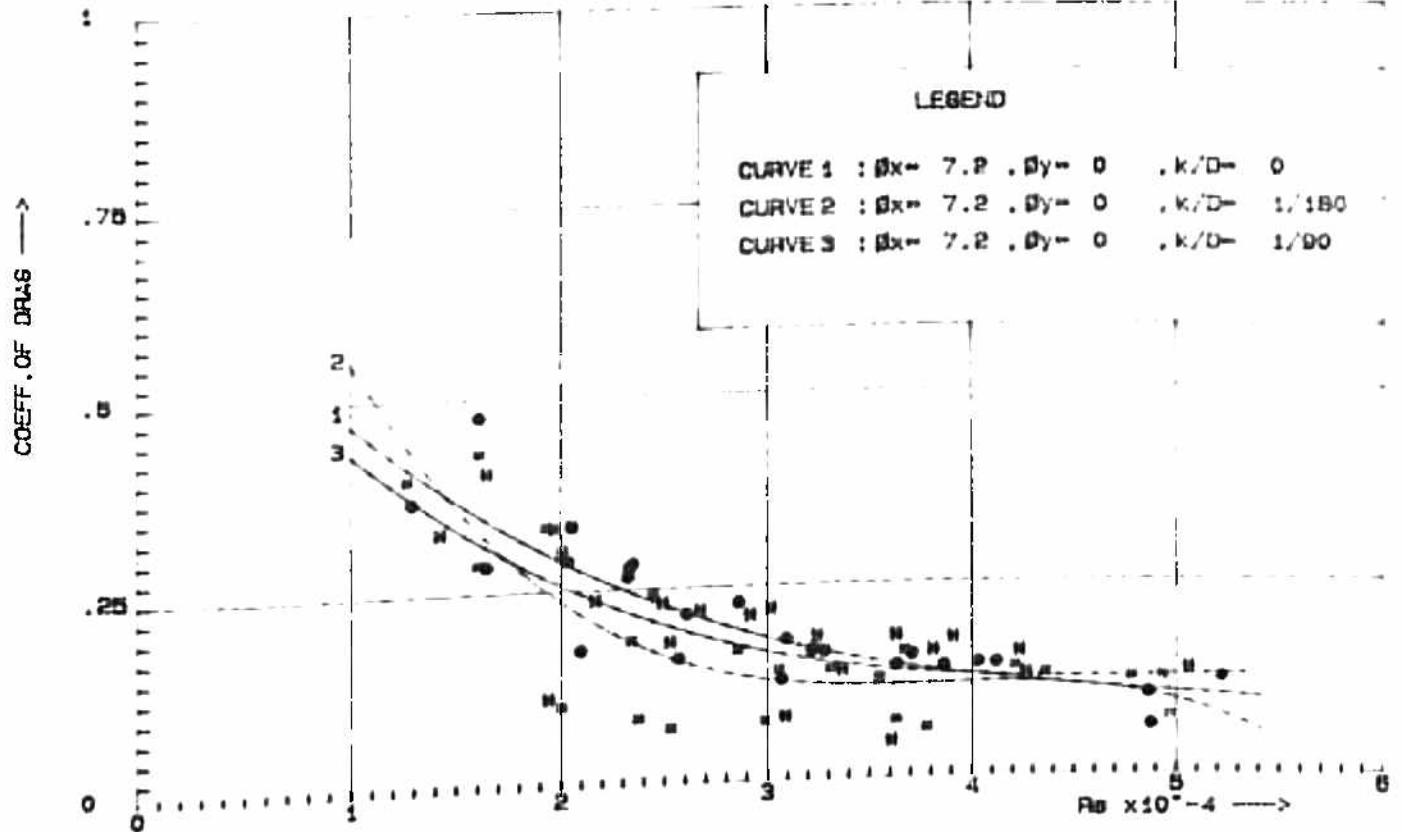


Fig. No. F - 2

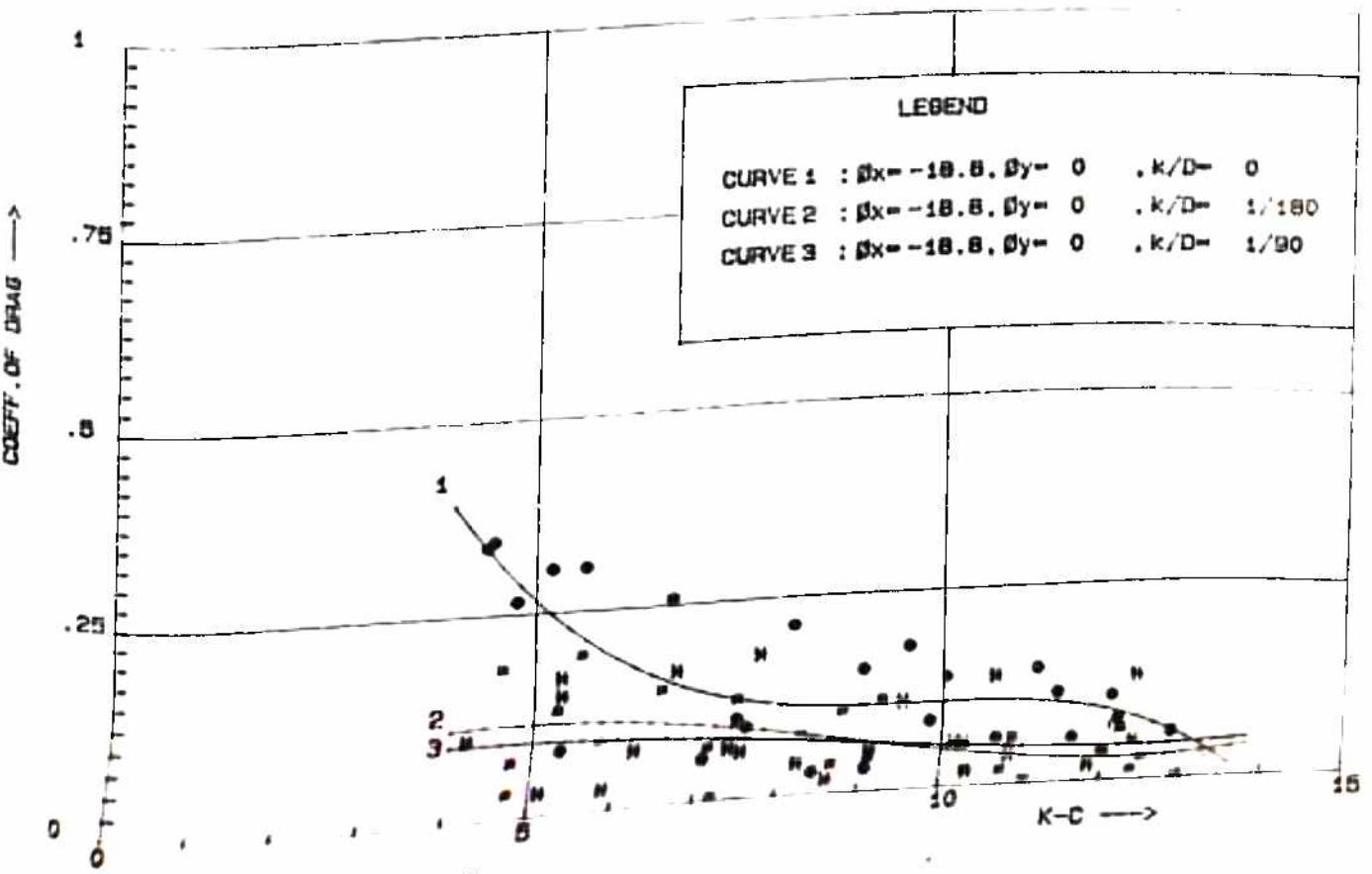
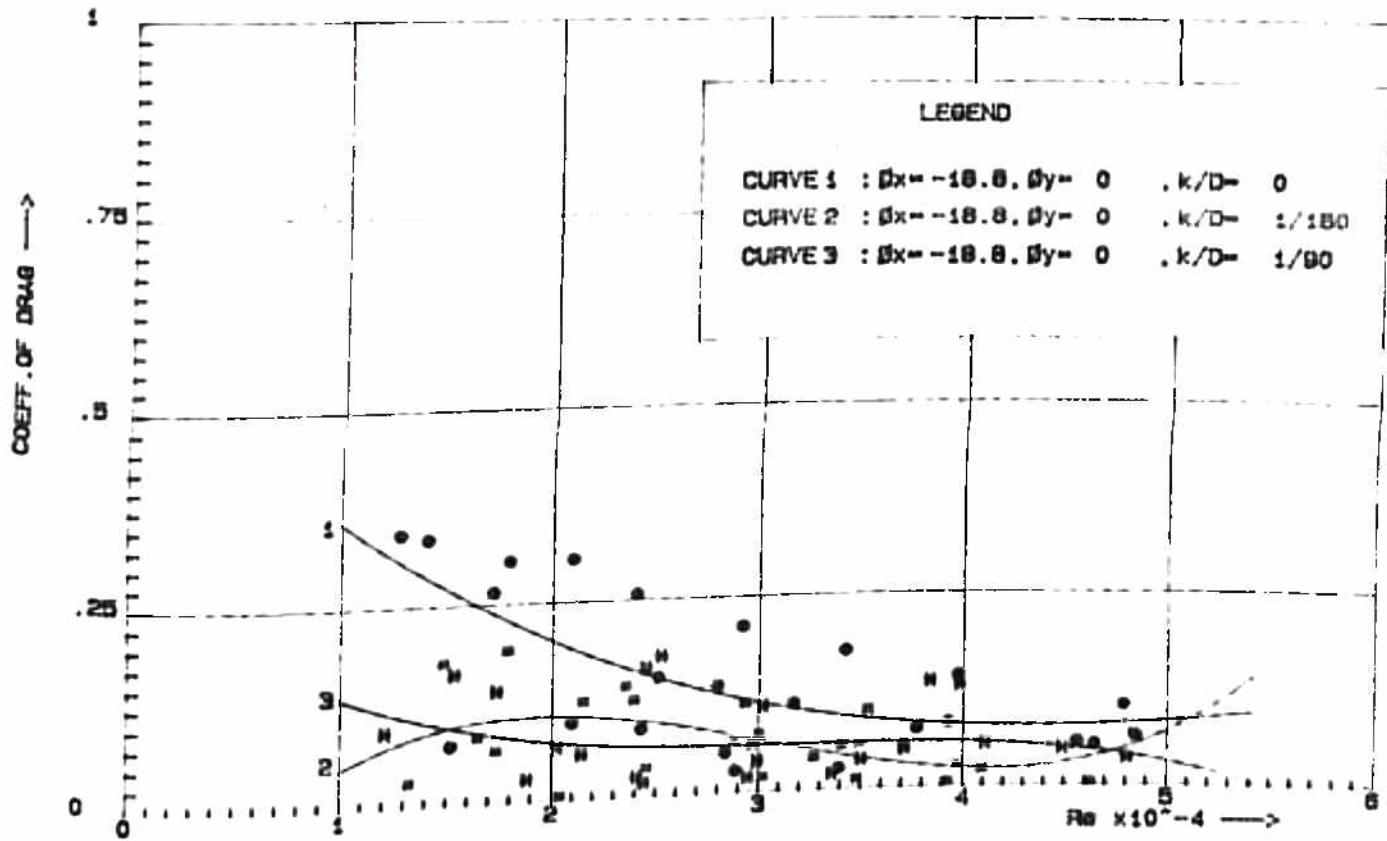


Fig. No. F - 3

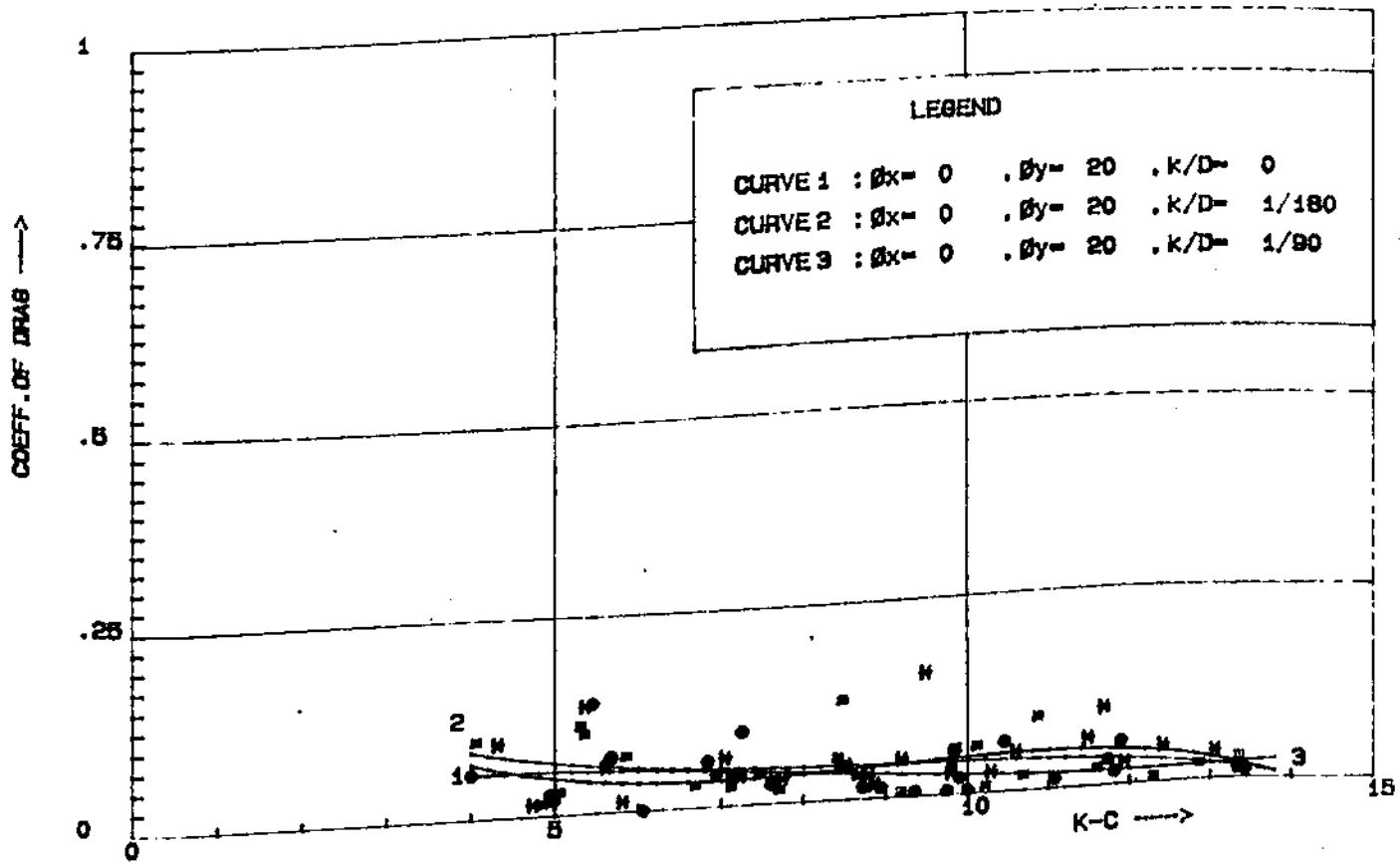
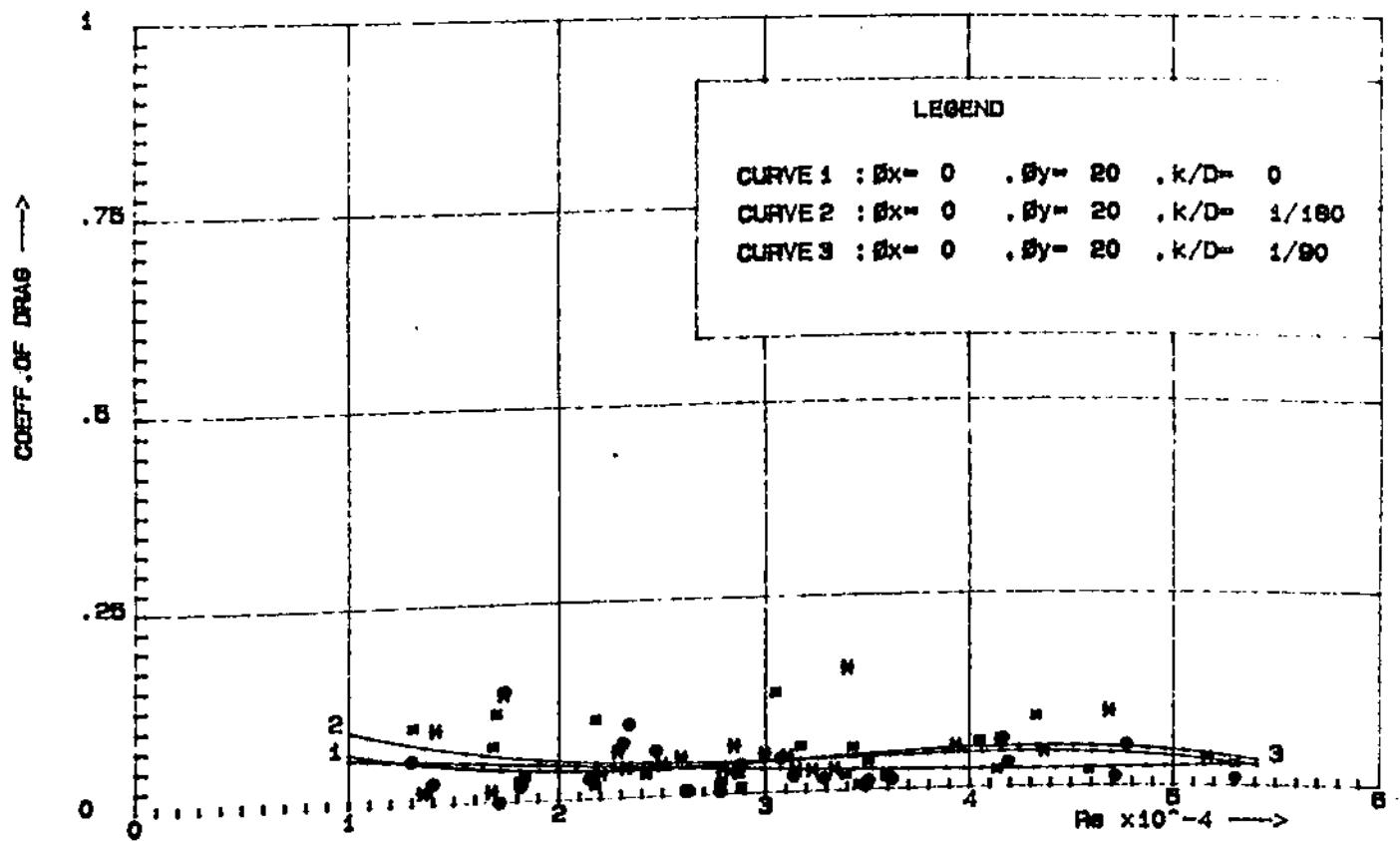


Fig. No. F - 4

COEFF. OF DRAG →

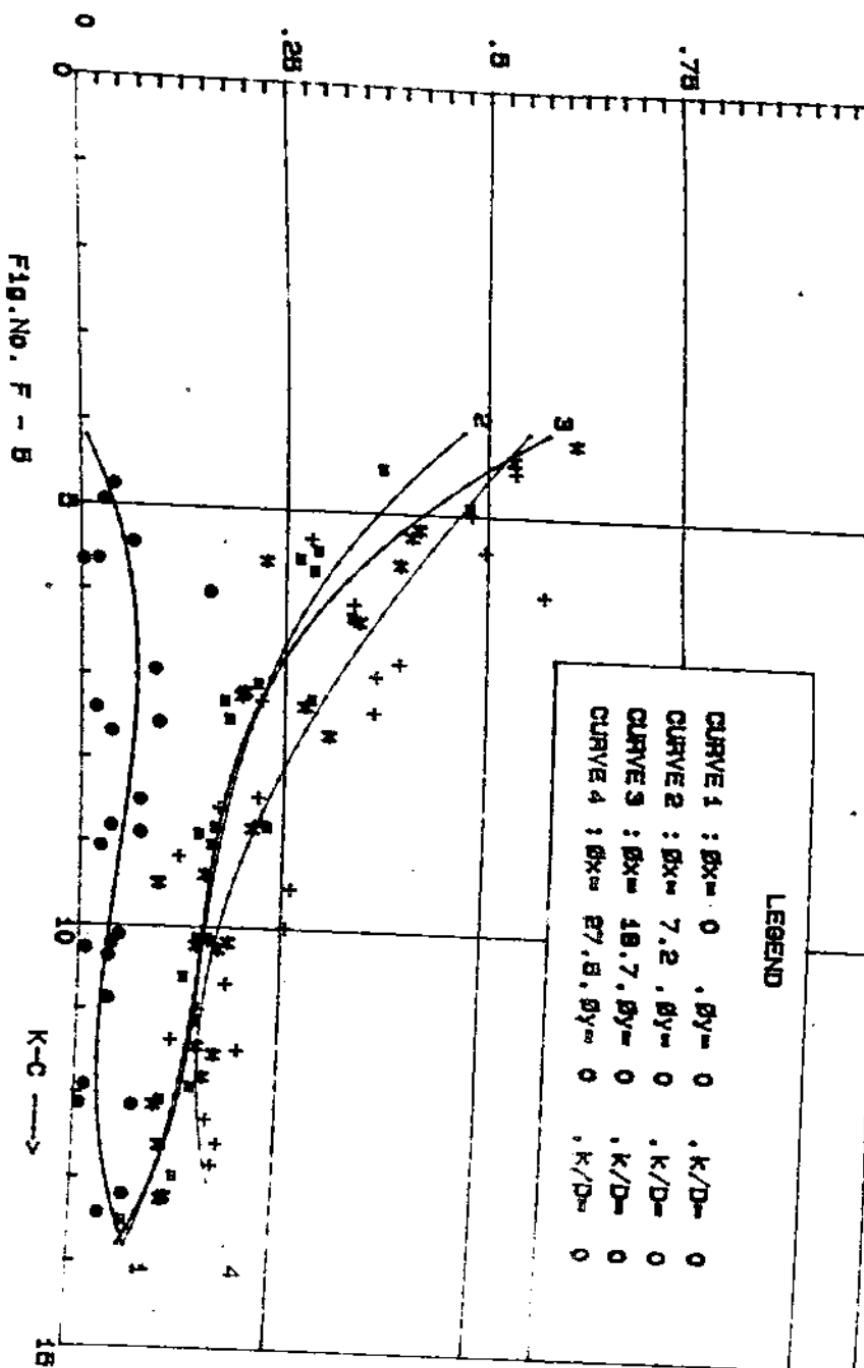
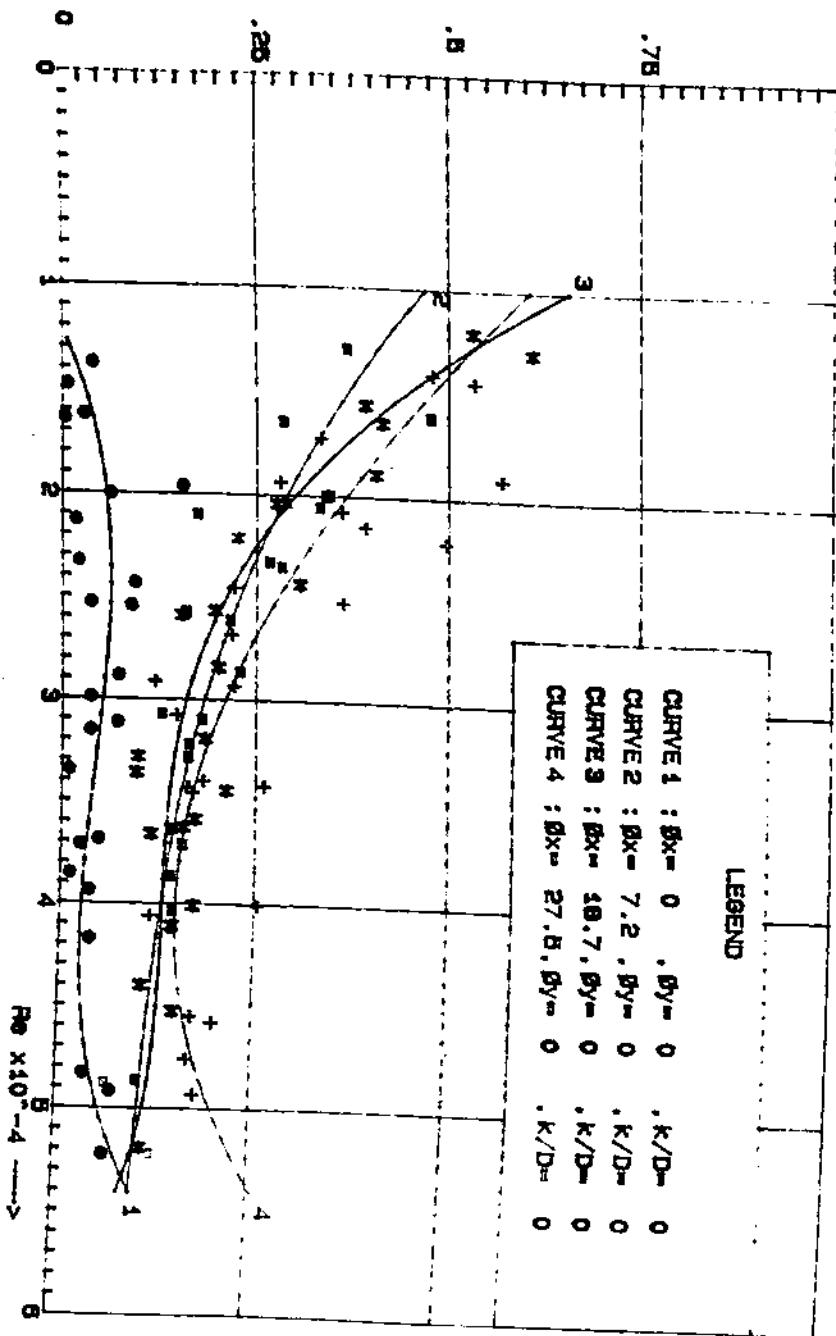


FIG. NO. F - B

COEFF. OF DRAG →



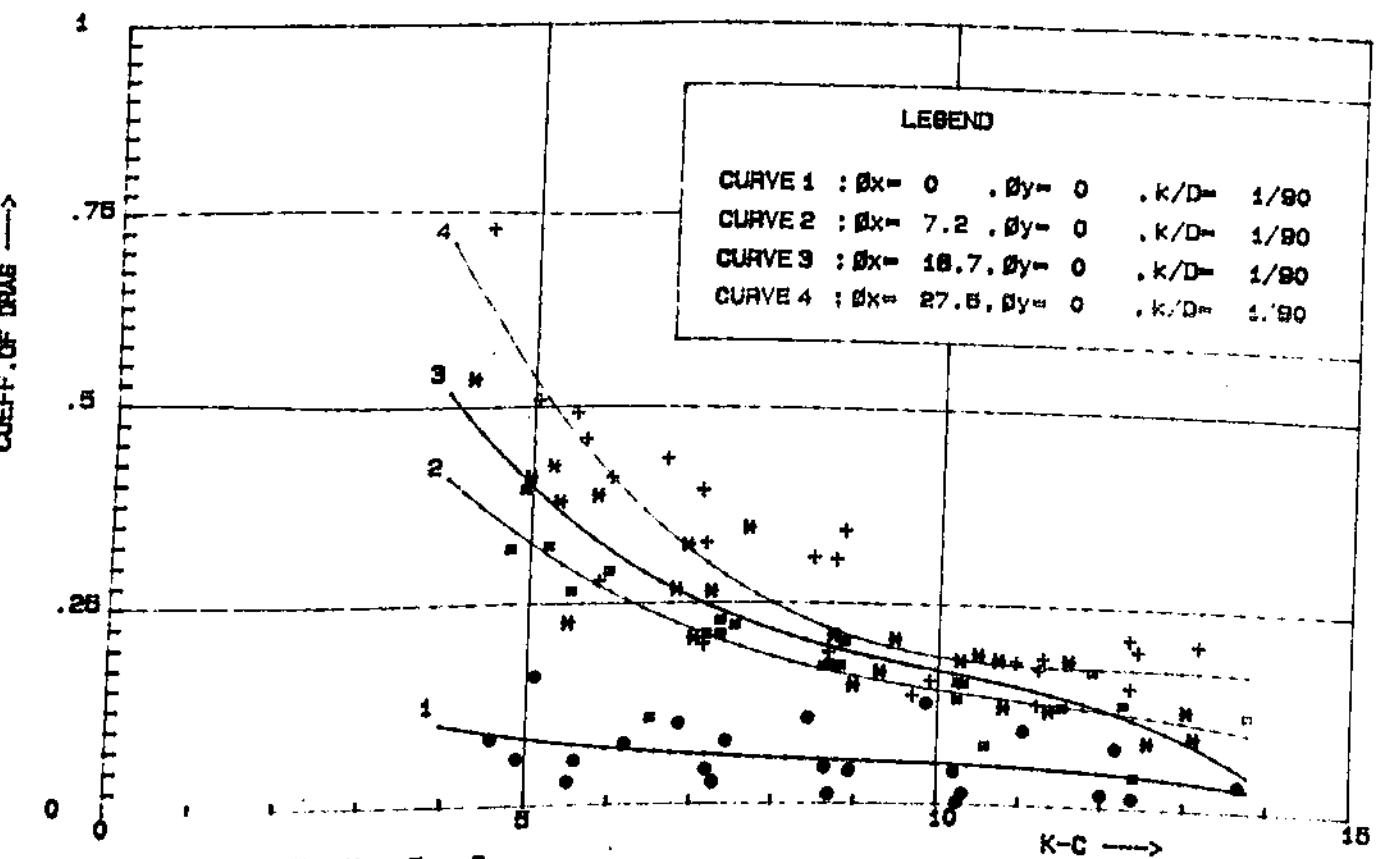
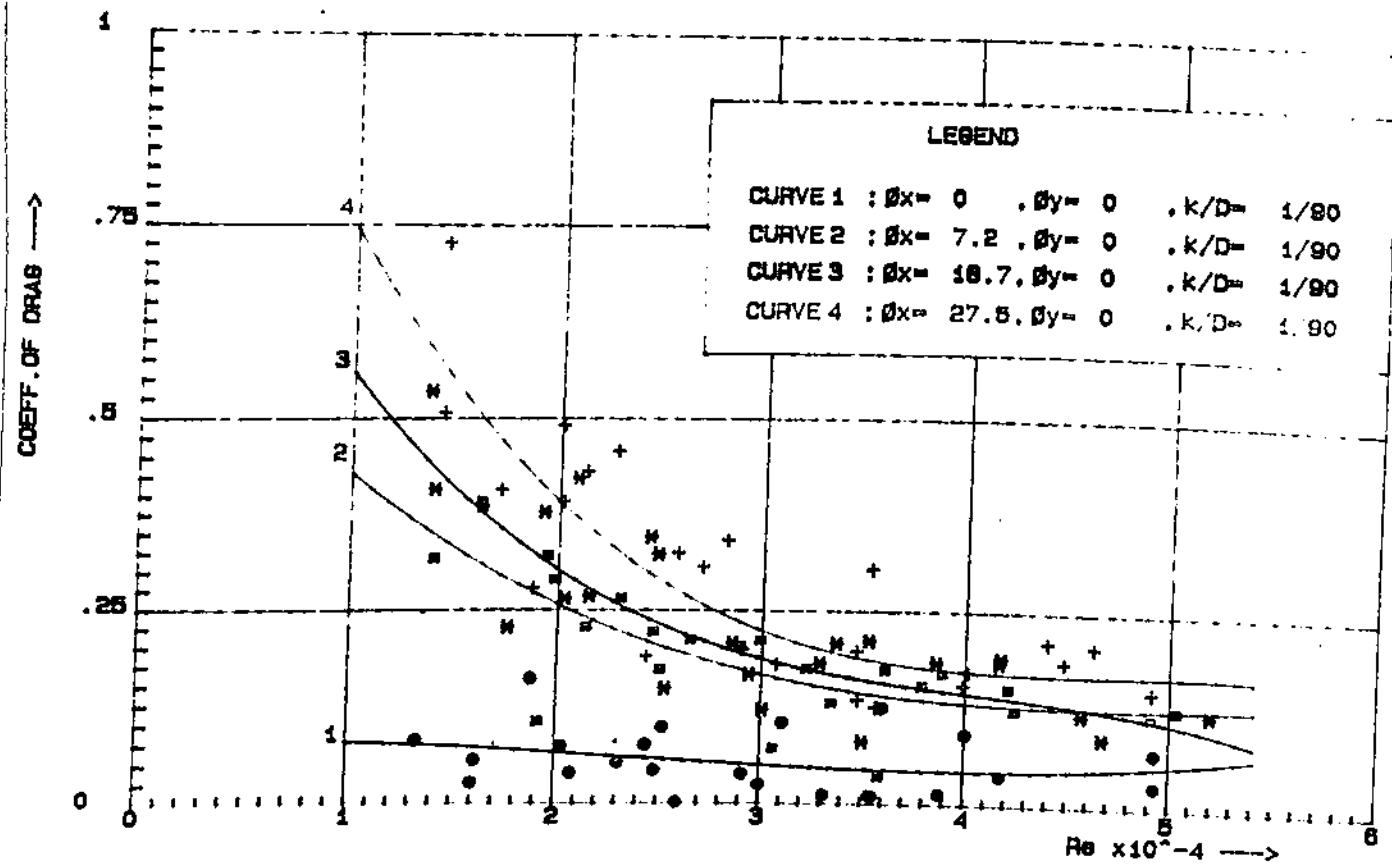


Fig. No. F - 6

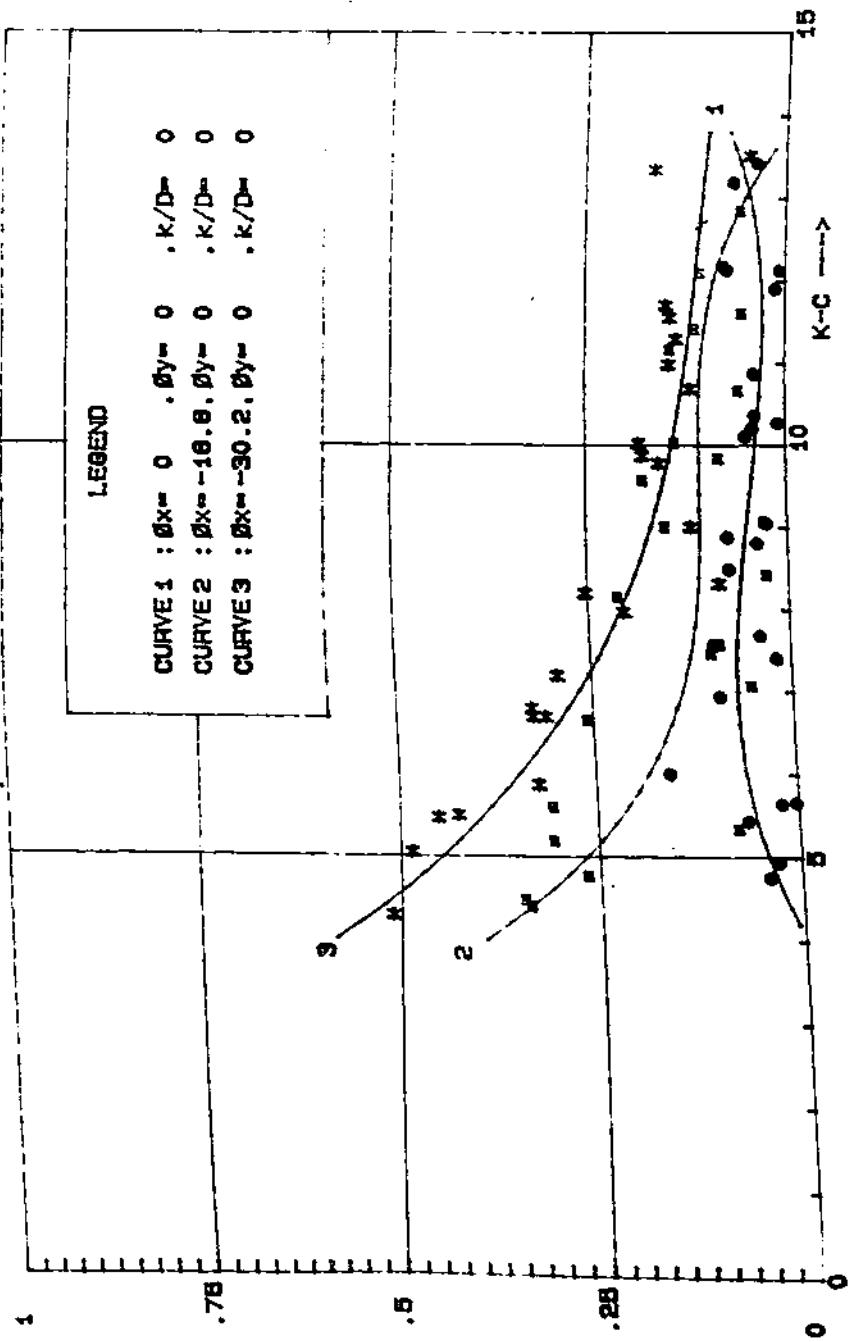
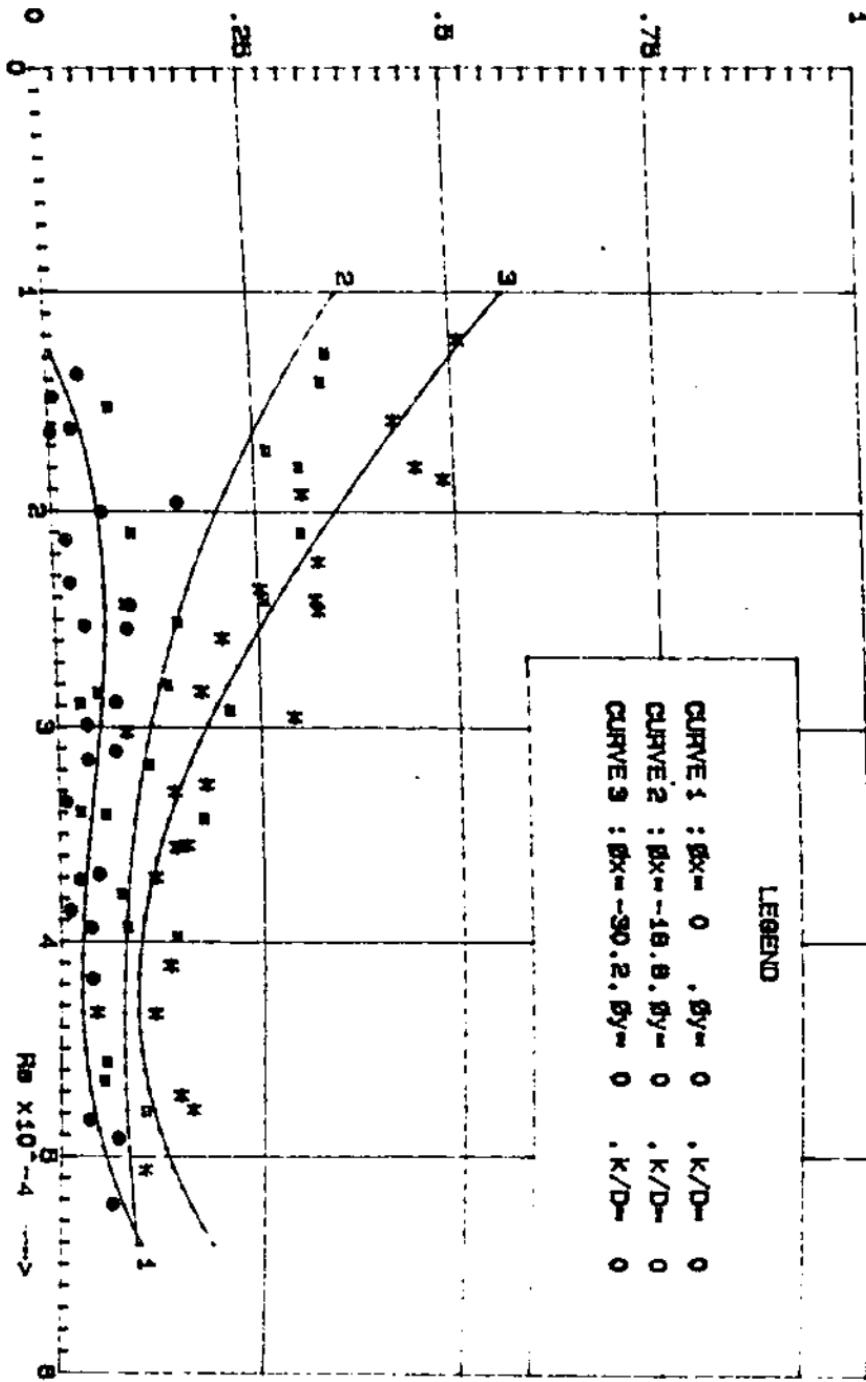


FIG. NO. F - 7

COEFF. OF DRAG →



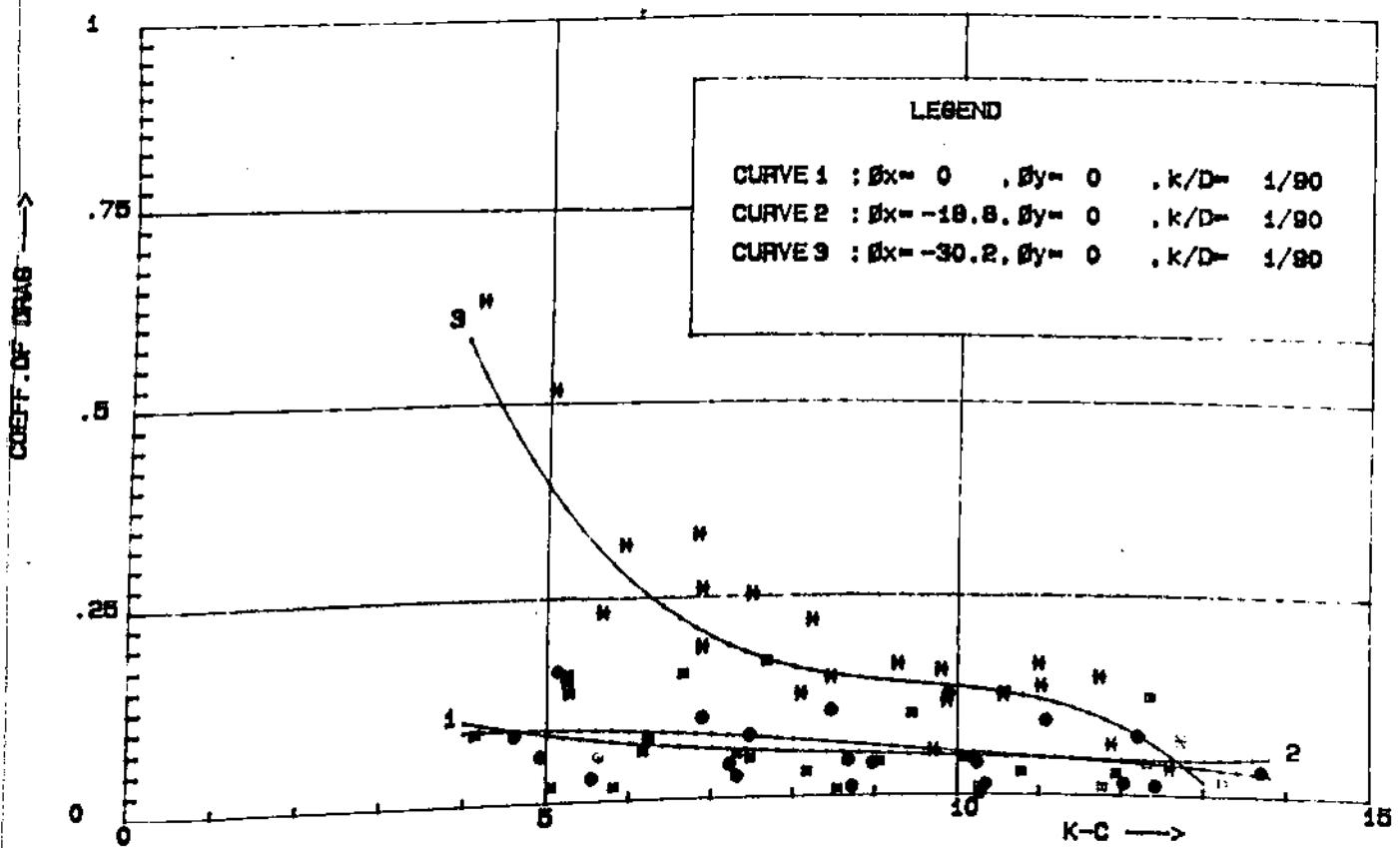
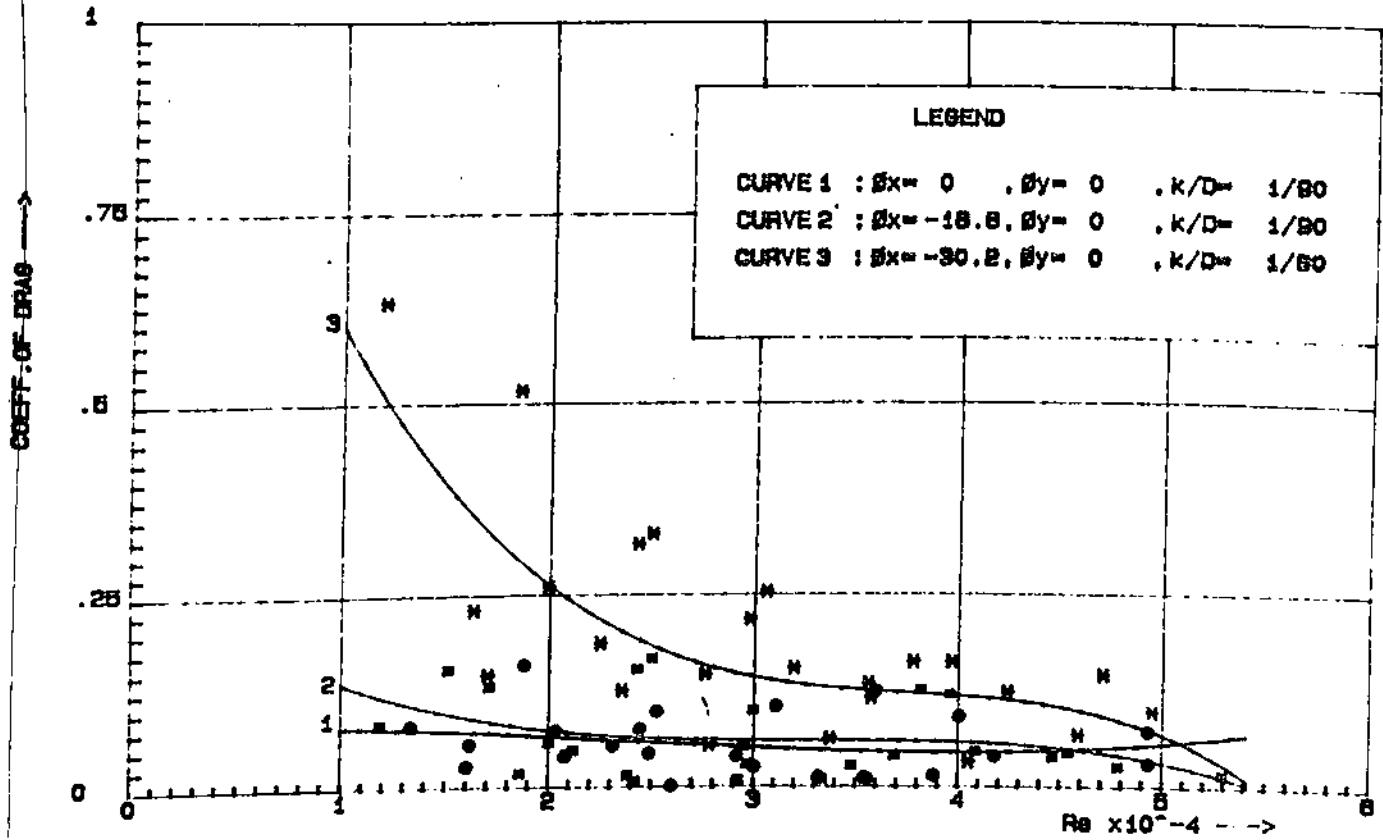


Fig.No. F - 8

COEFF. OF DRAG →

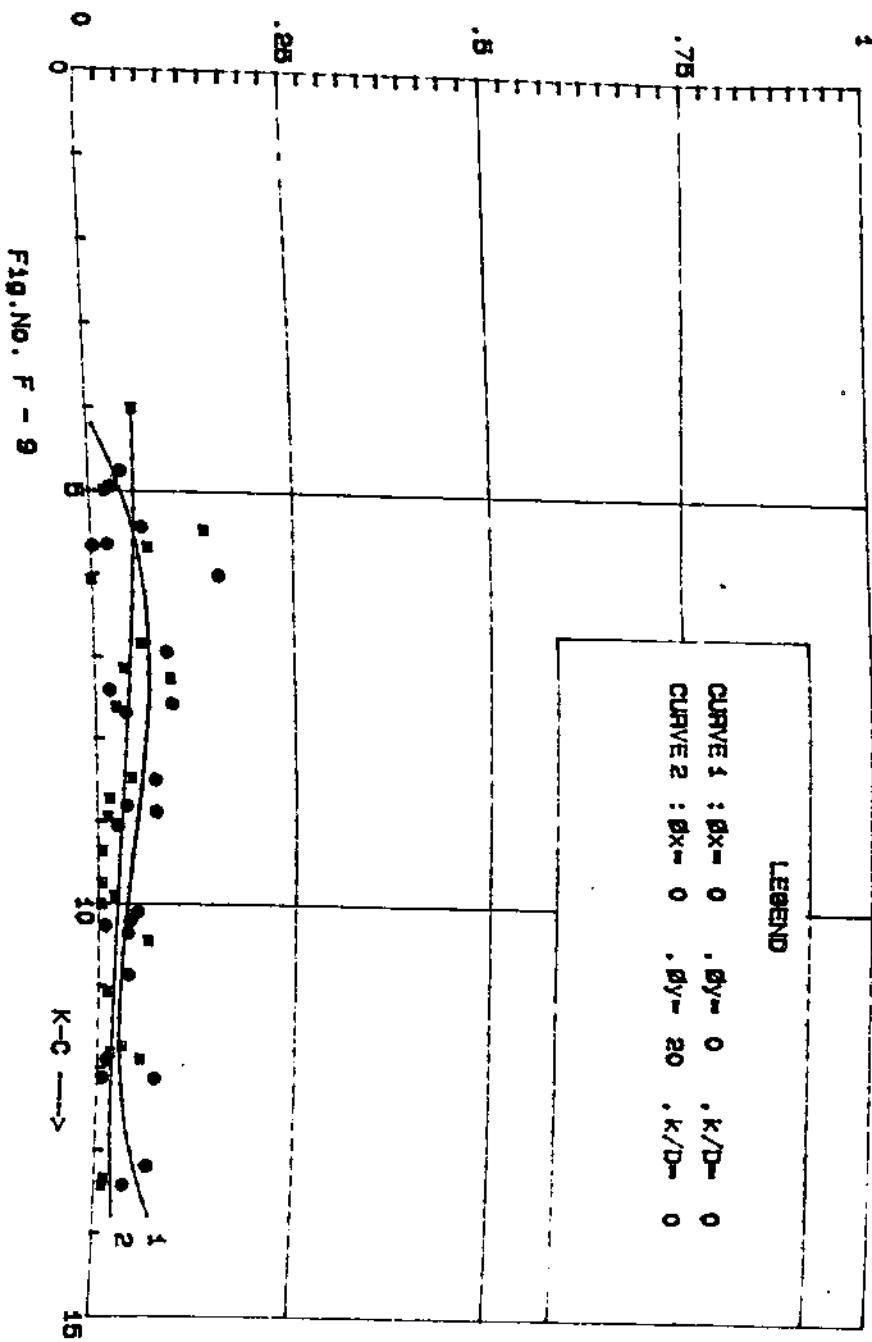
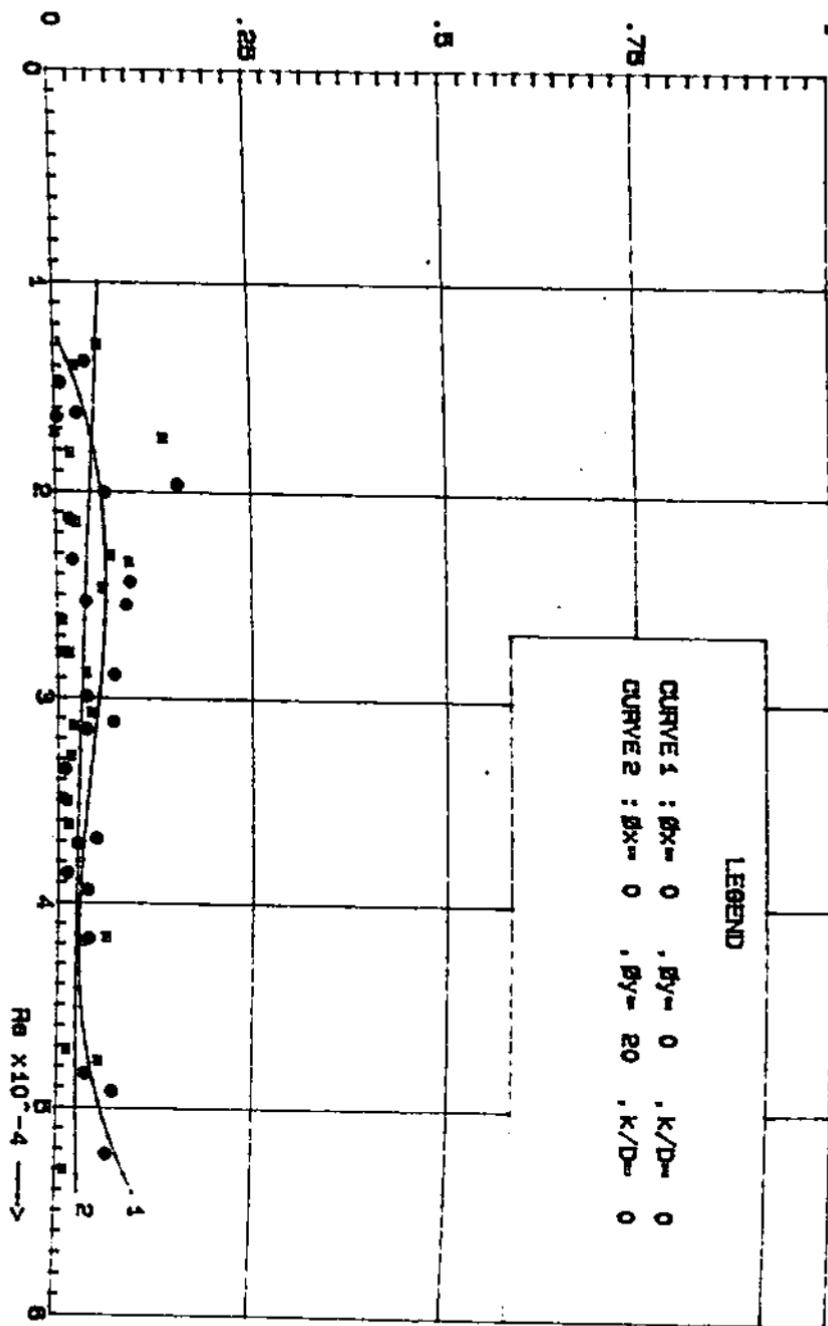
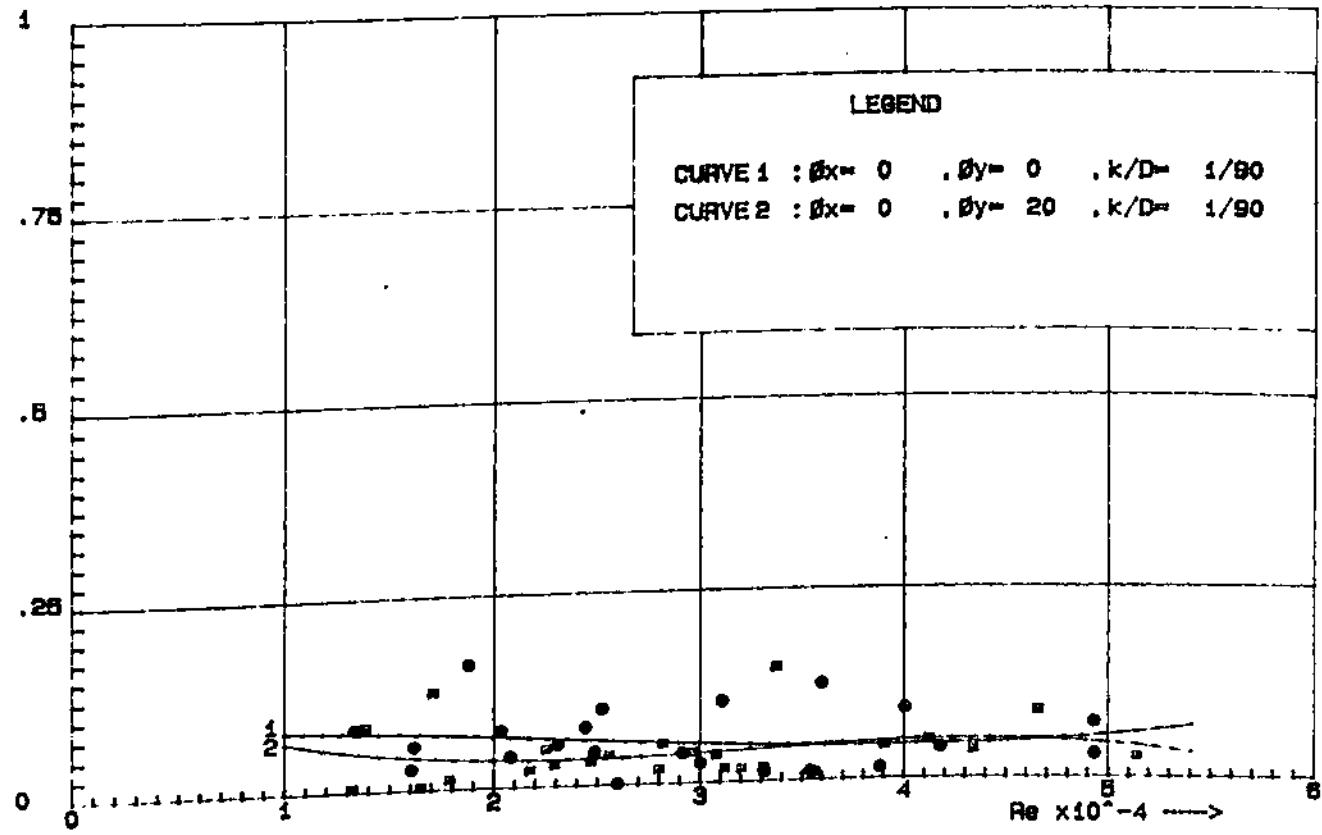


FIG. NO. F - 9

COEFF. OF DRAG →



COEFF. OF DRAG —



COEFF. OF DRAG —

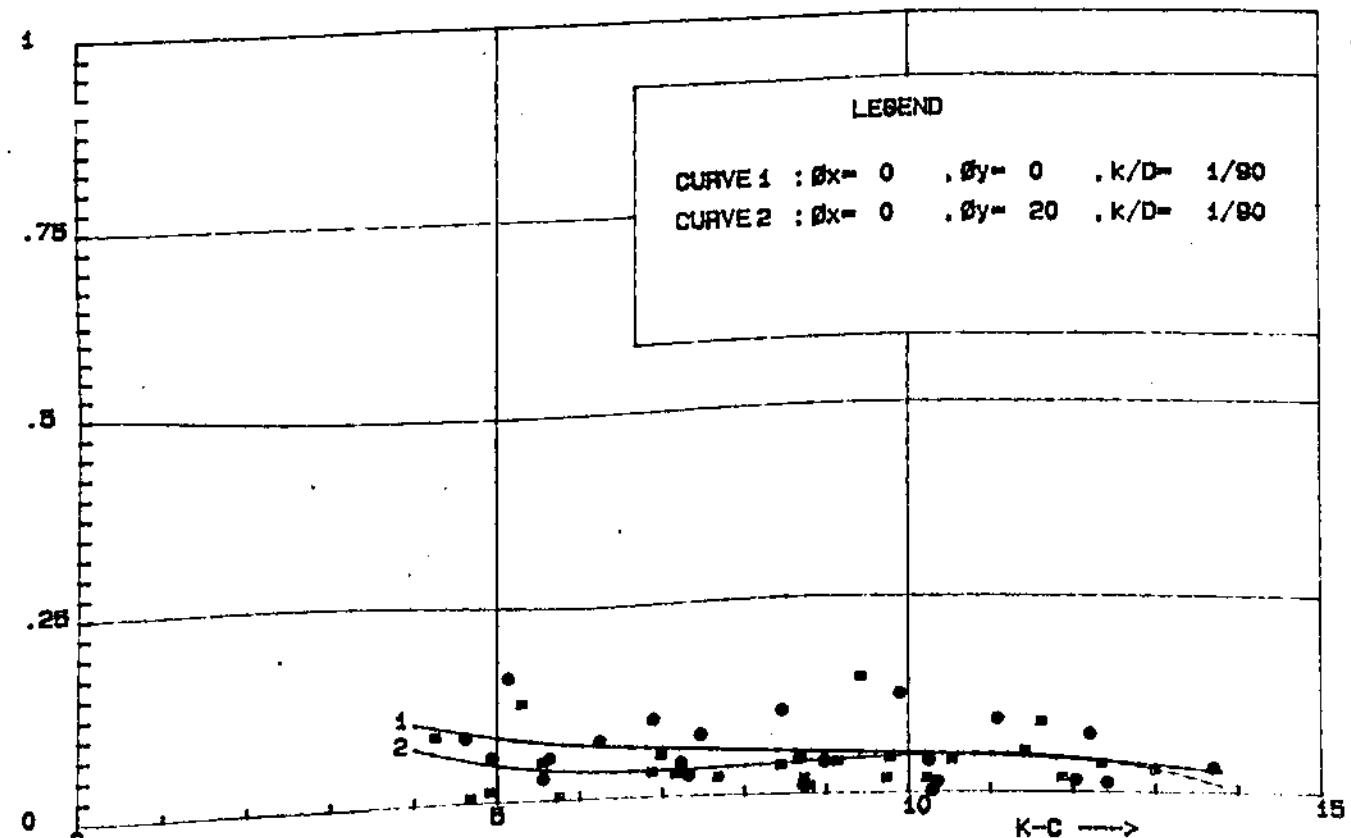
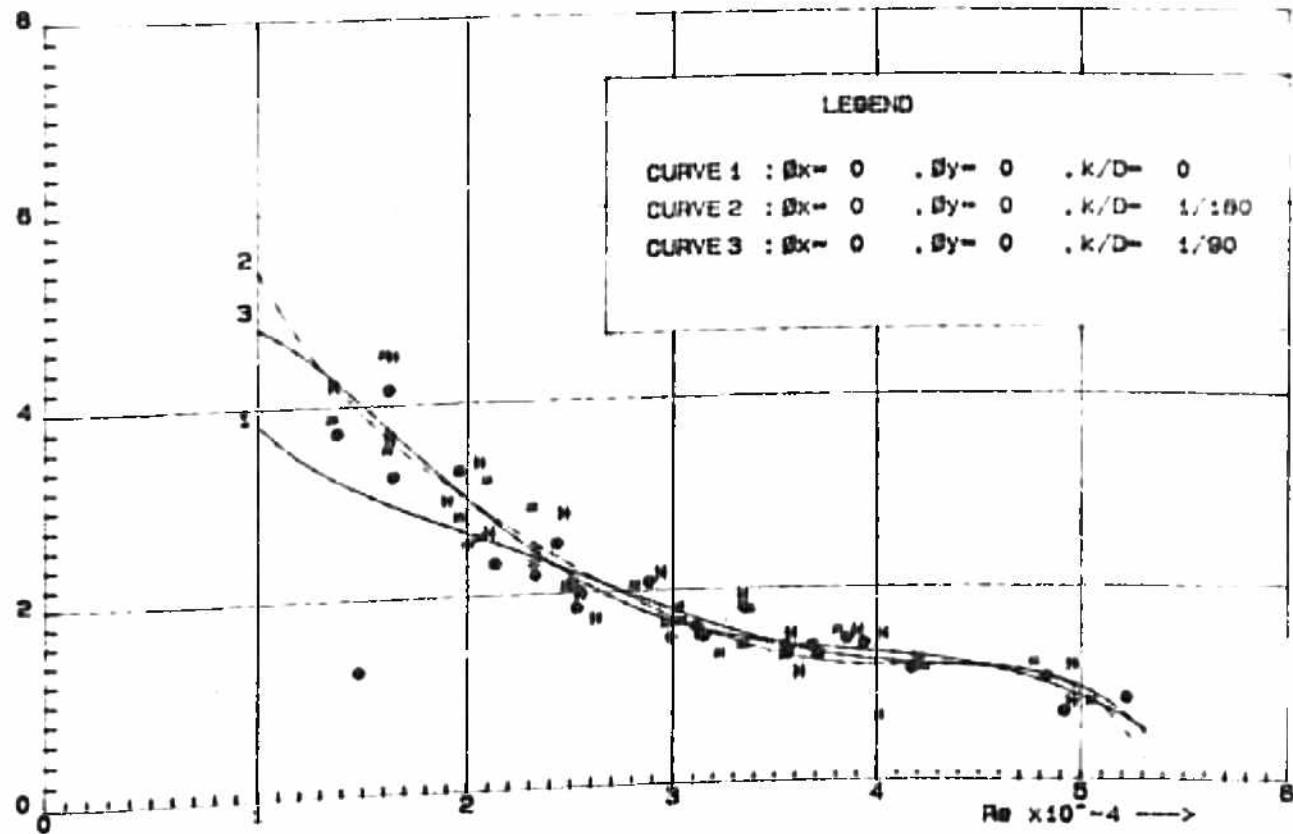


Fig. No. F - 10

COEFF. OF INERTIA —>



COEFF. OF INERTIA —>

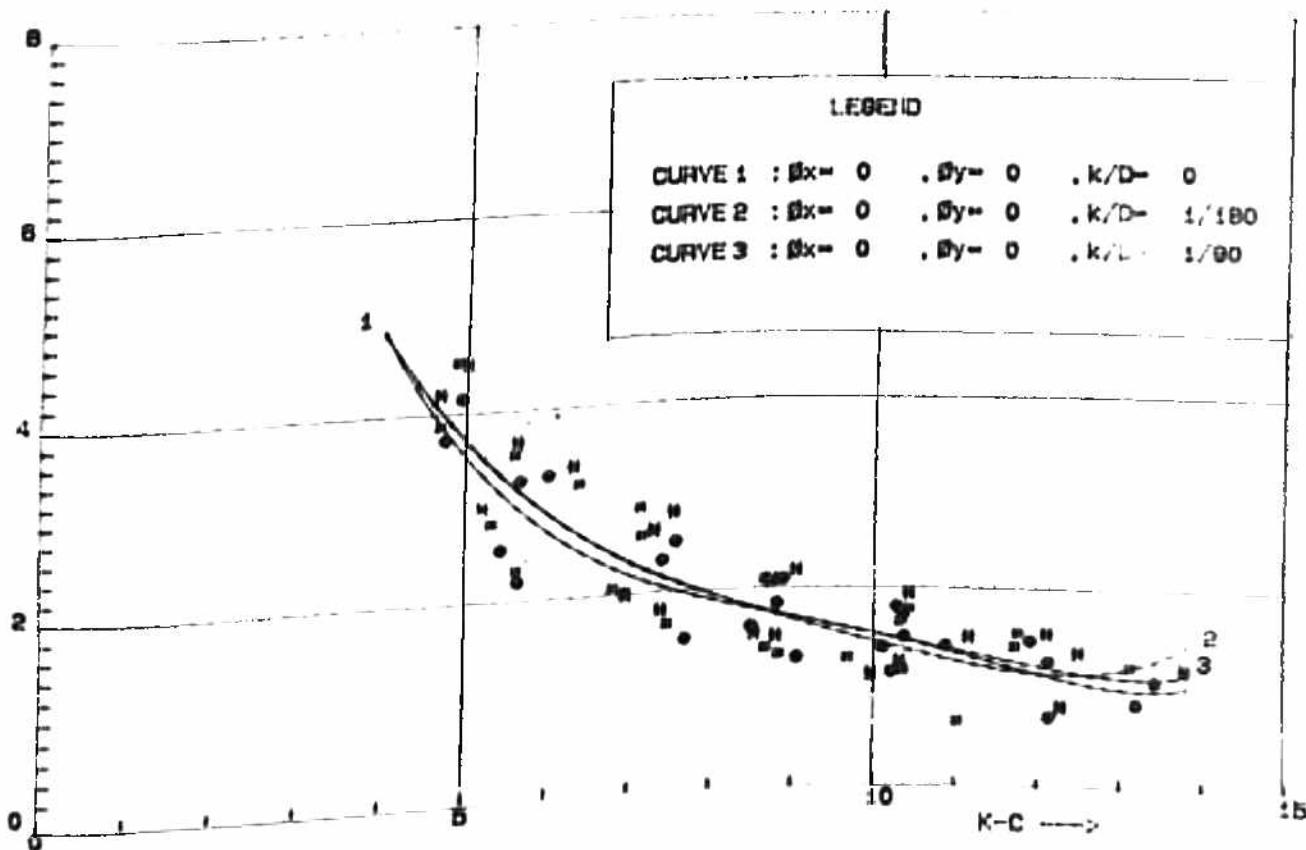
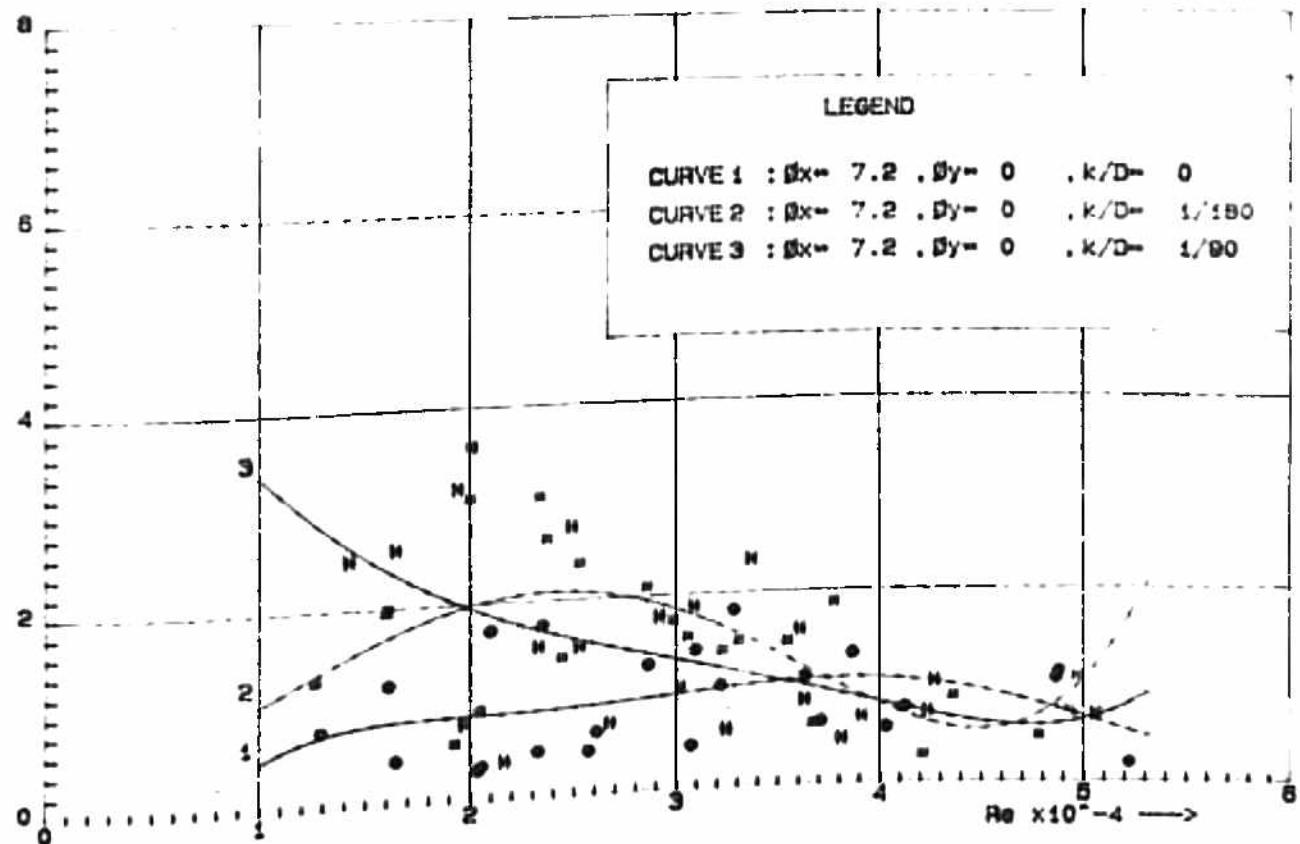


Fig.No. F - 11

COEFF. OF INERTIA —



COEFF. OF INERTIA —

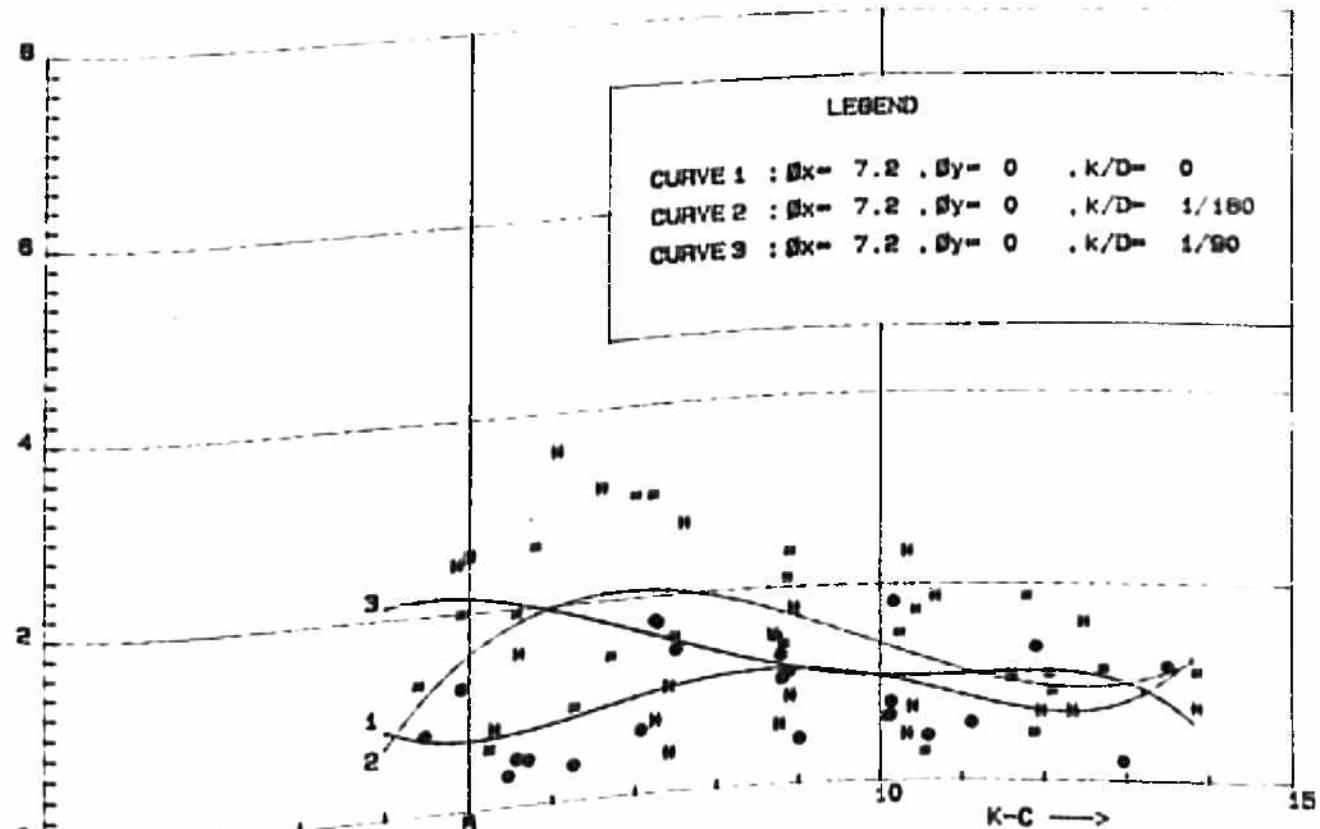
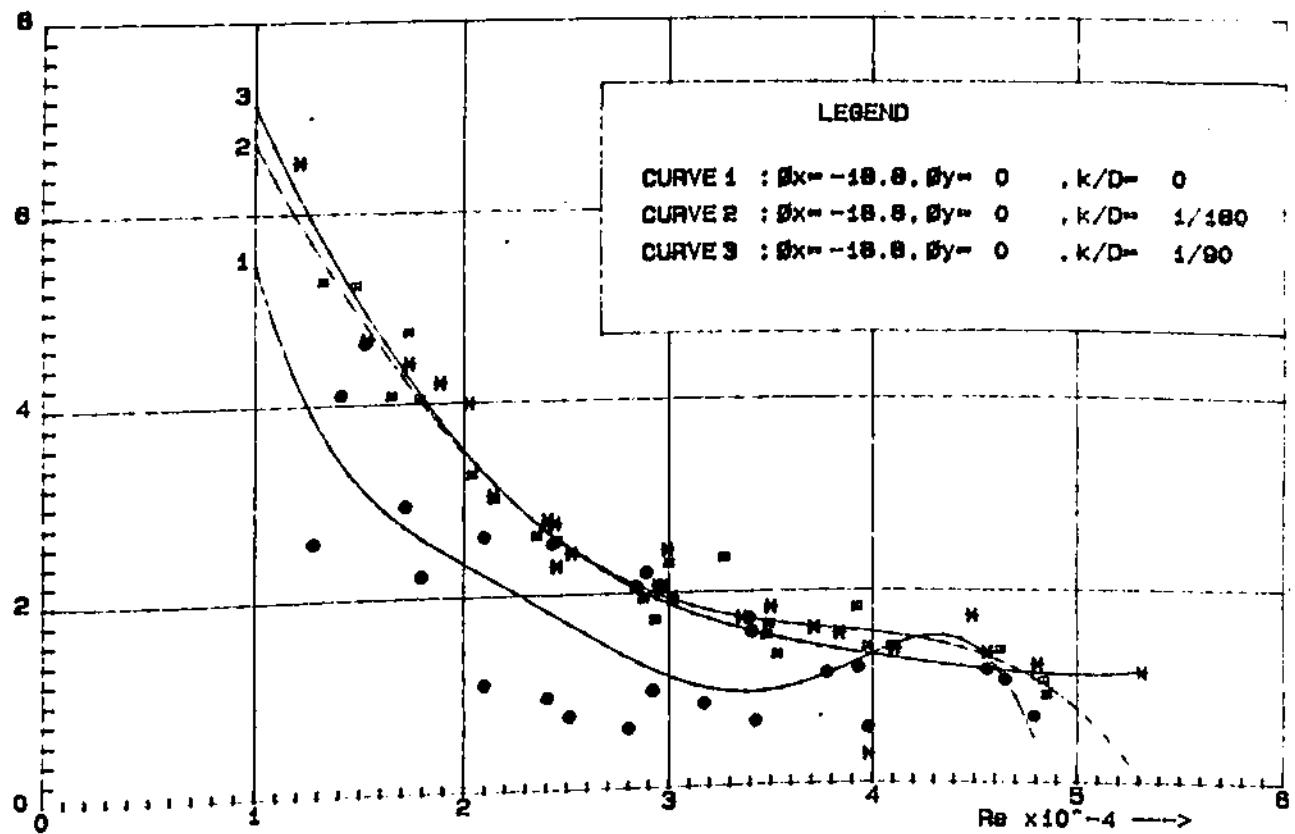


Fig. No. F - 12

COEFF. OF INERTIA \rightarrow



COEFF. OF INERTIA \rightarrow

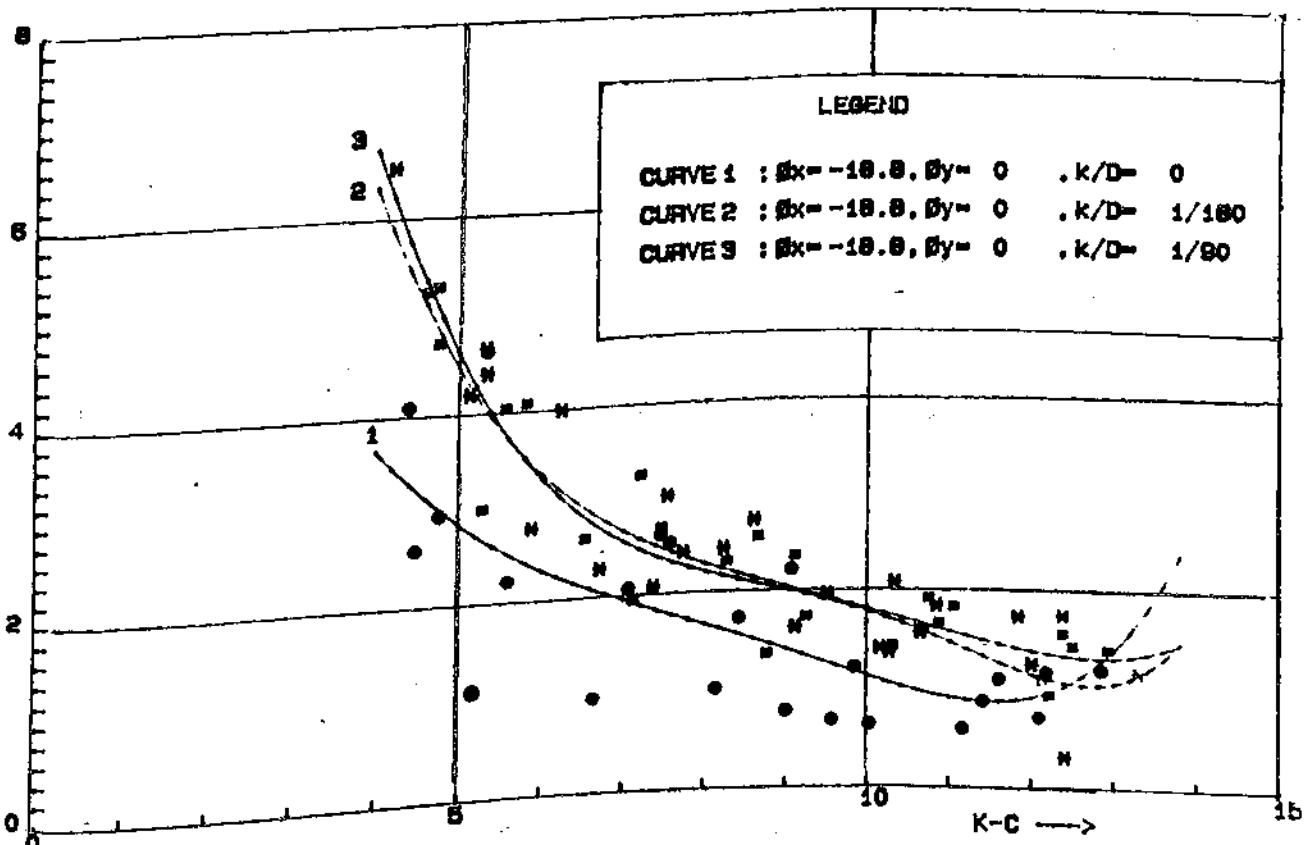


Fig. No. F - 13

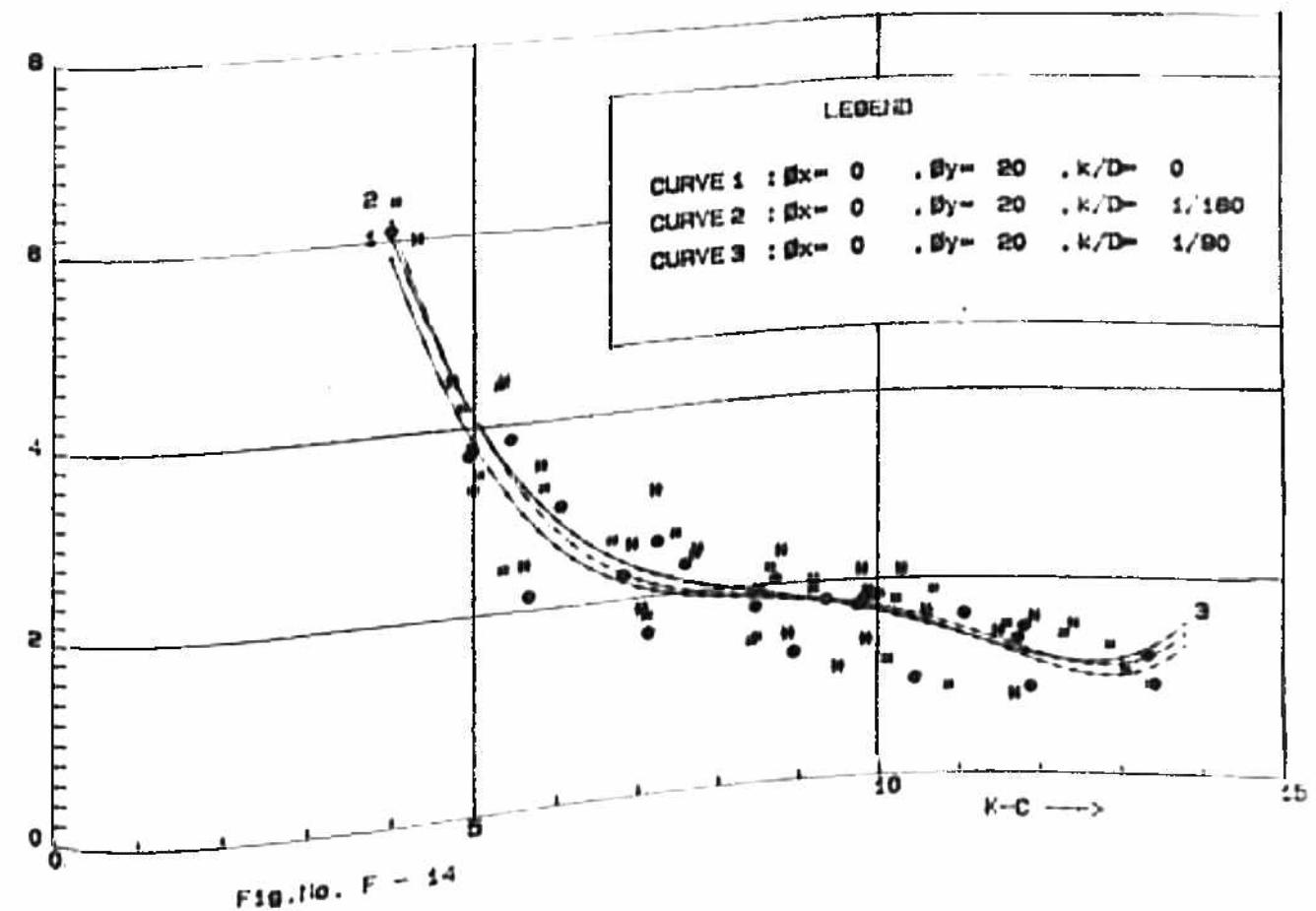
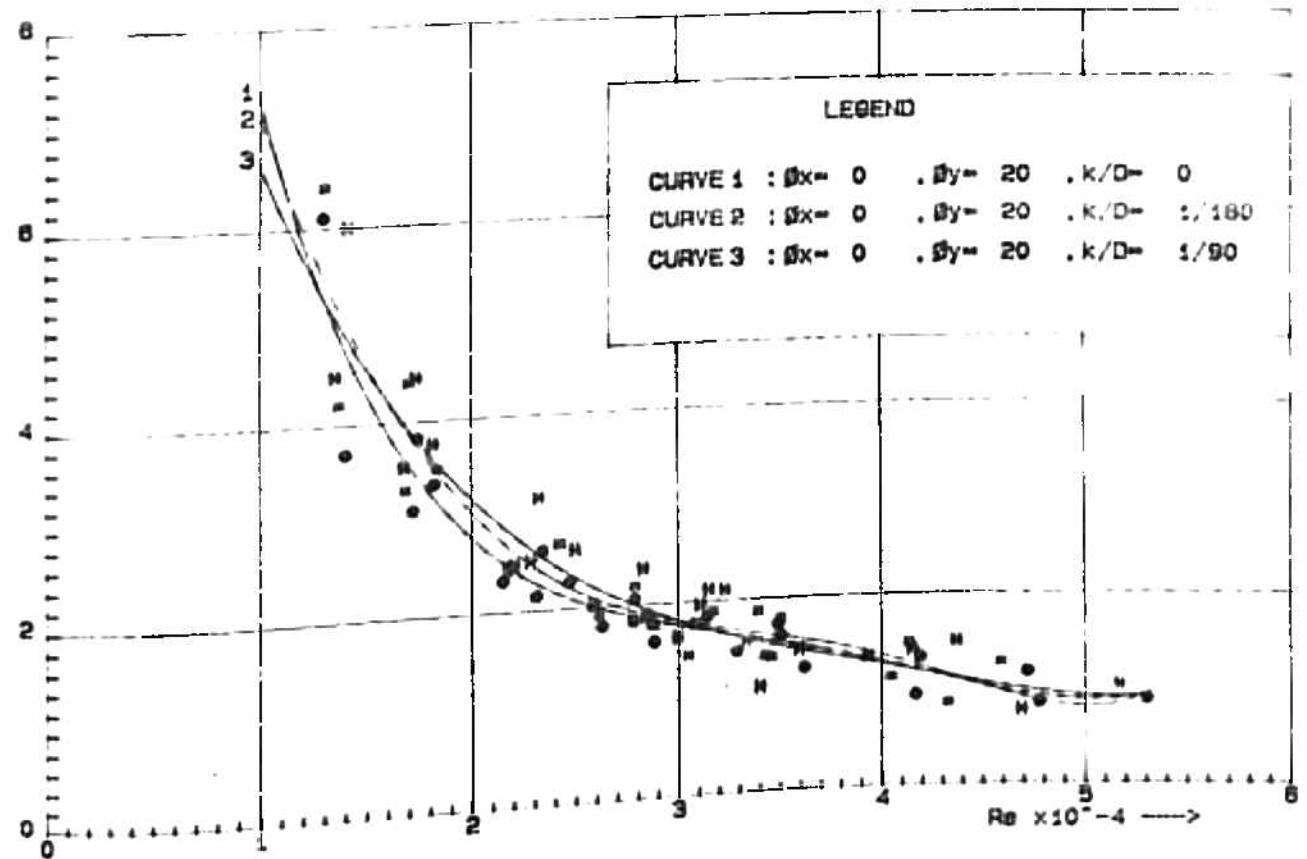
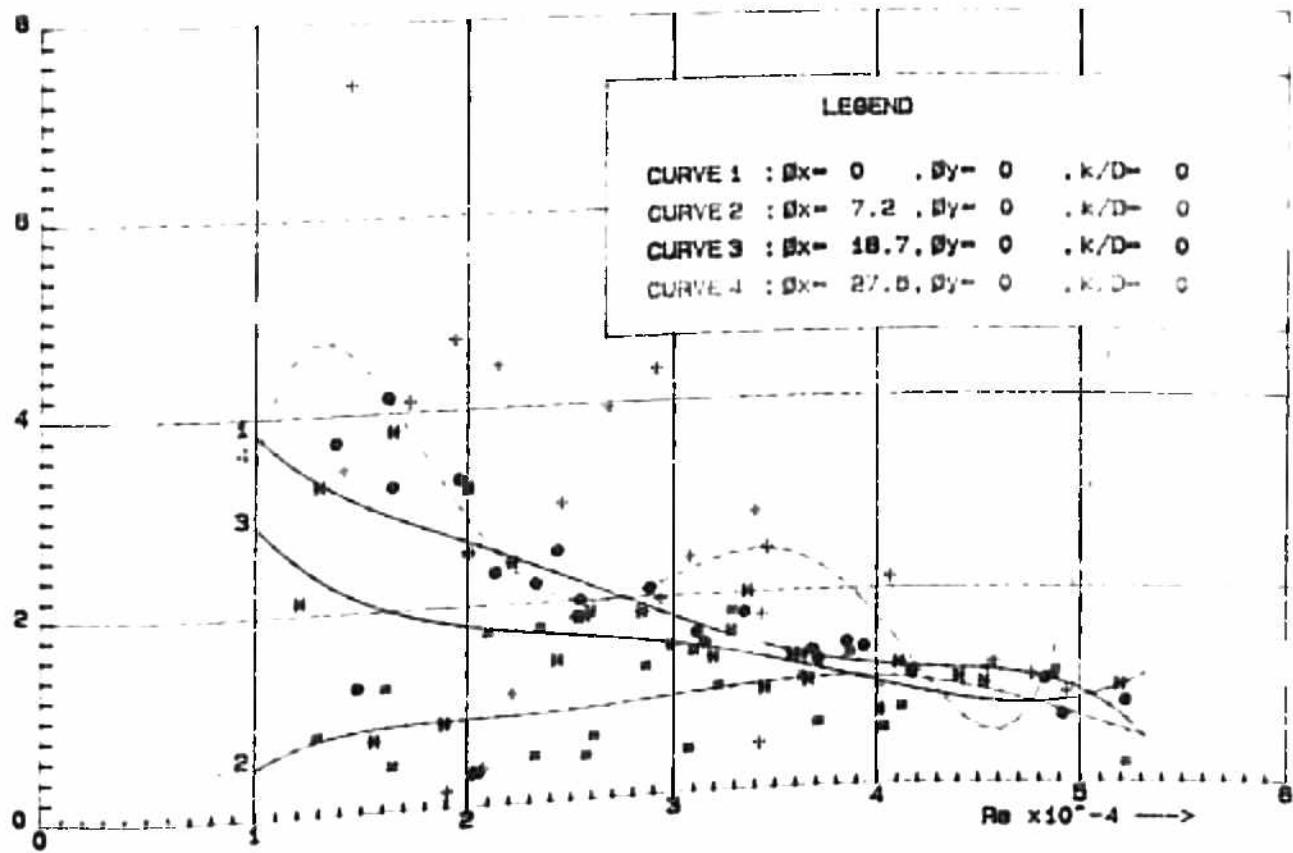


Fig. No. F - 14

COEFF. OF INERTIA →



COEFF. OF INERTIA →

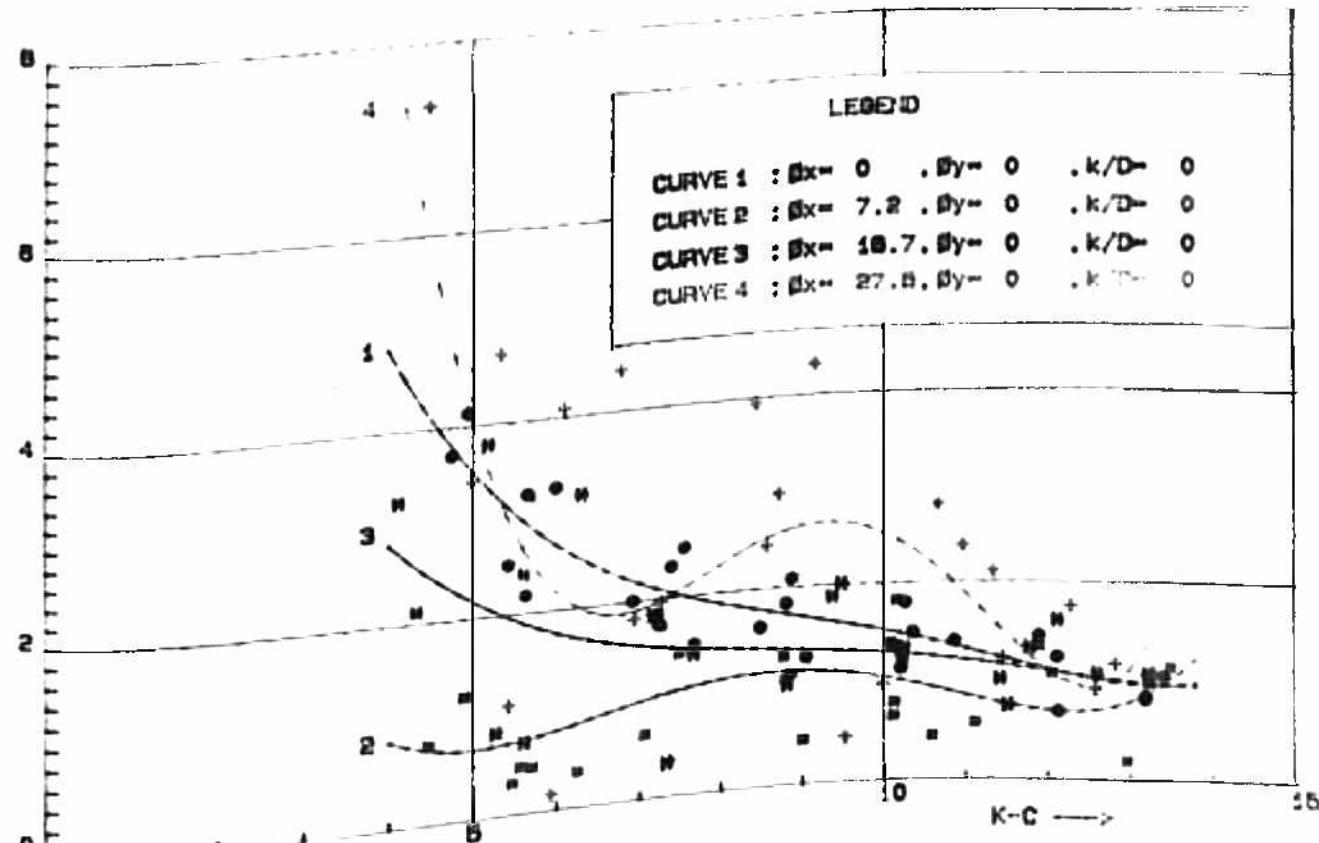


FIG. NO. F - 15

COEFF. OF INERTIA —→

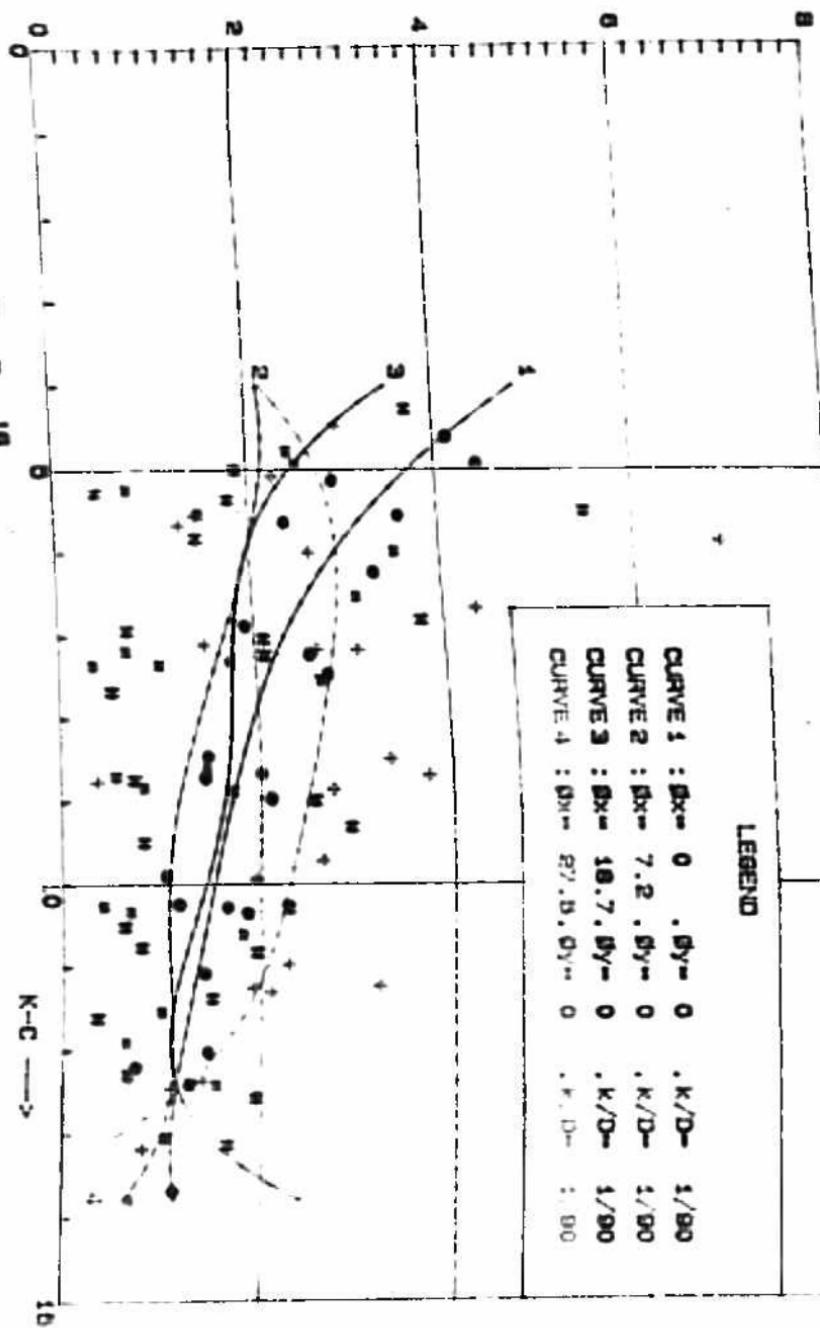
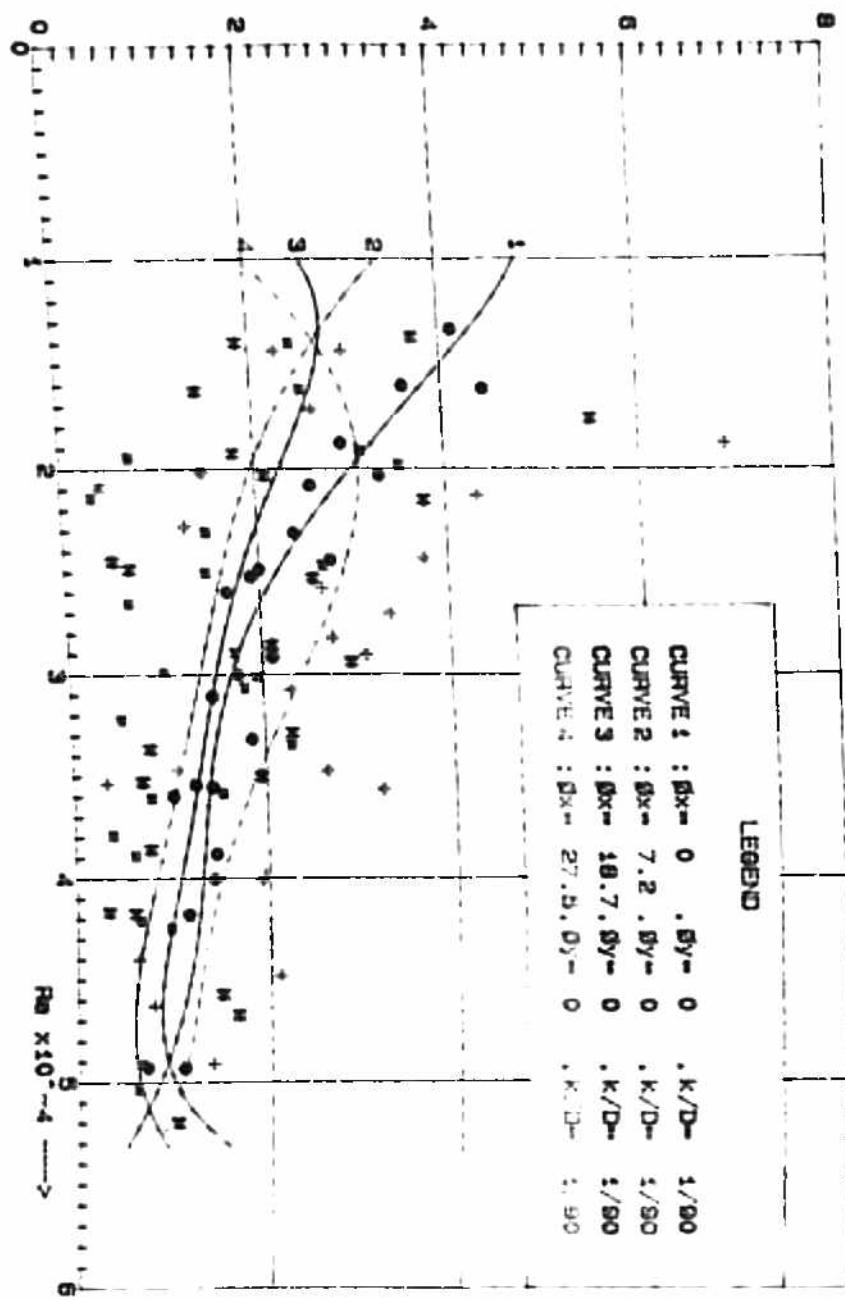
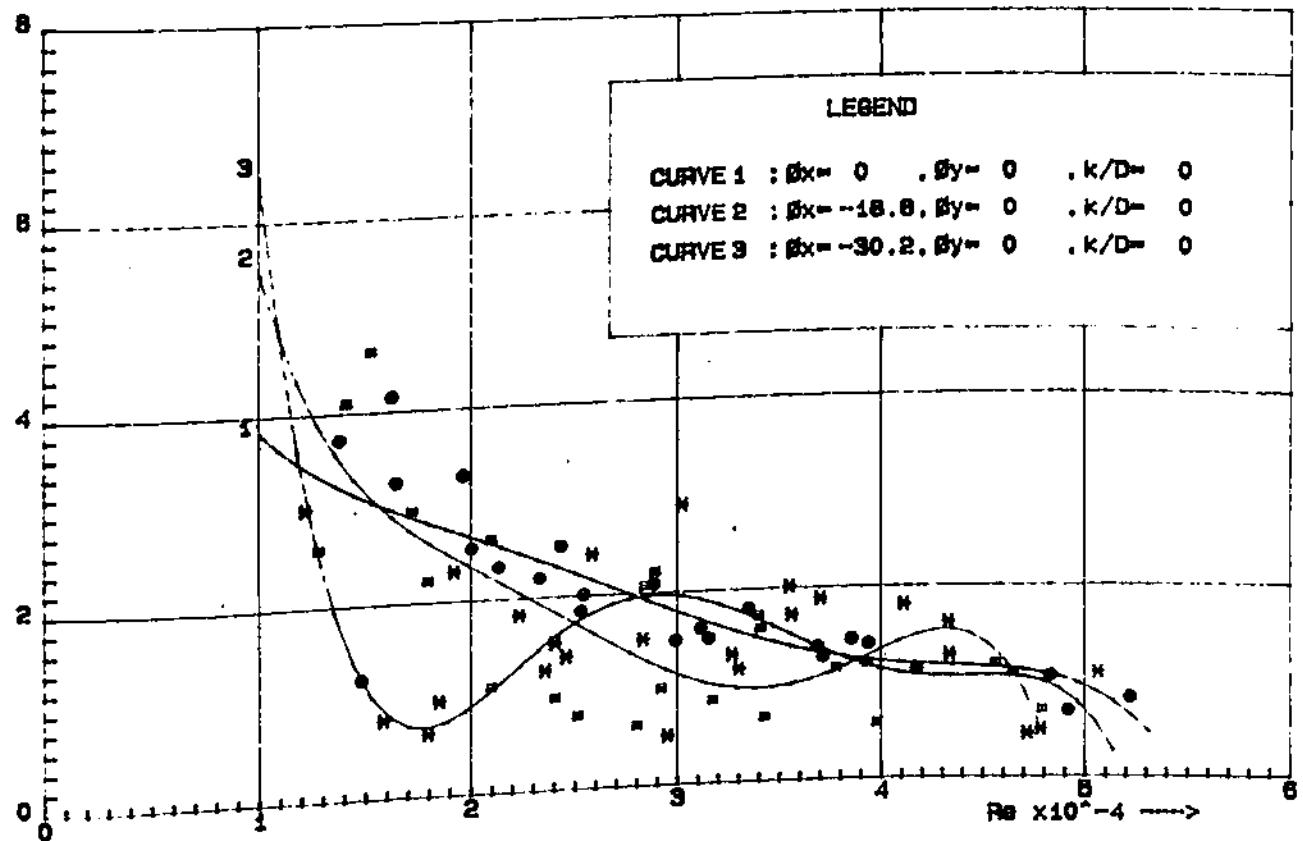


FIG. NO. F - 10

COEFF. OF INERTIA →





COEFF. OF INERTIA —>

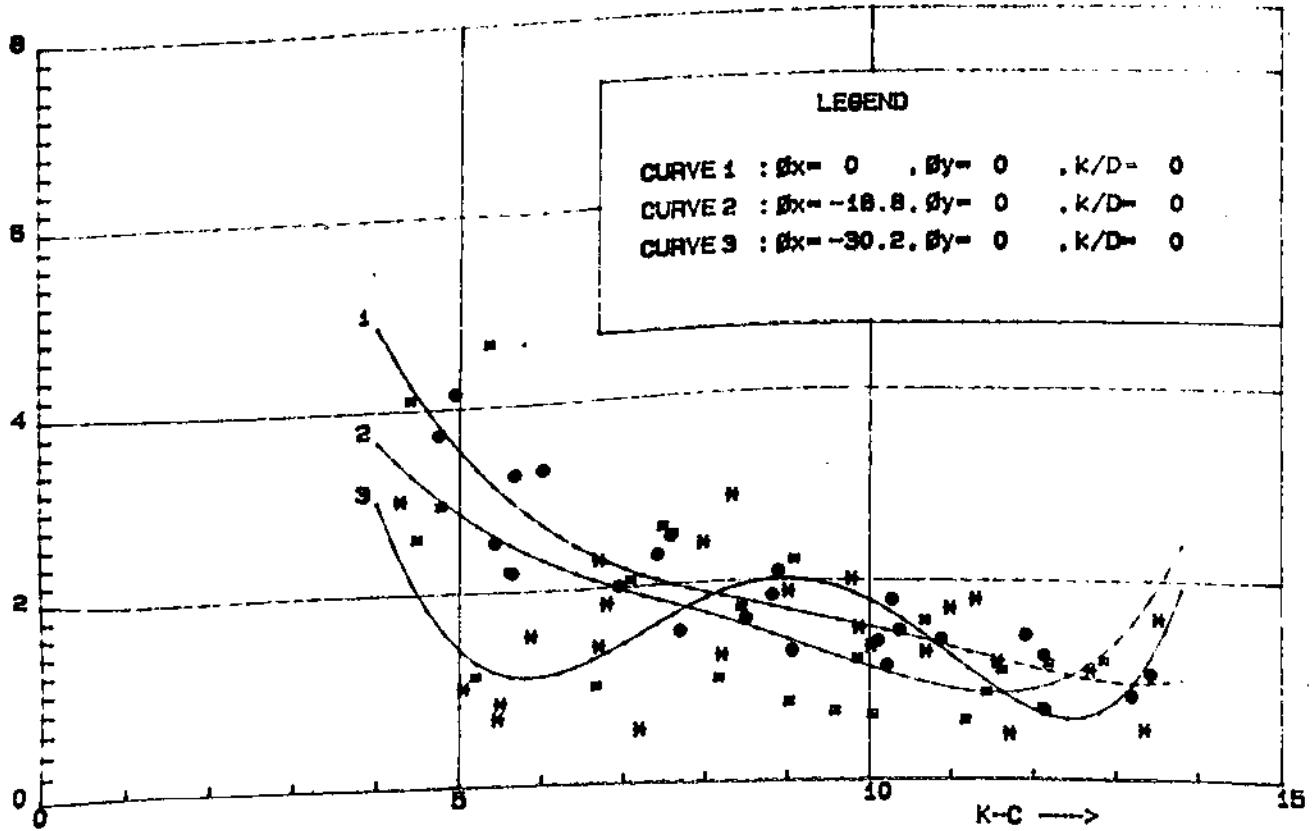
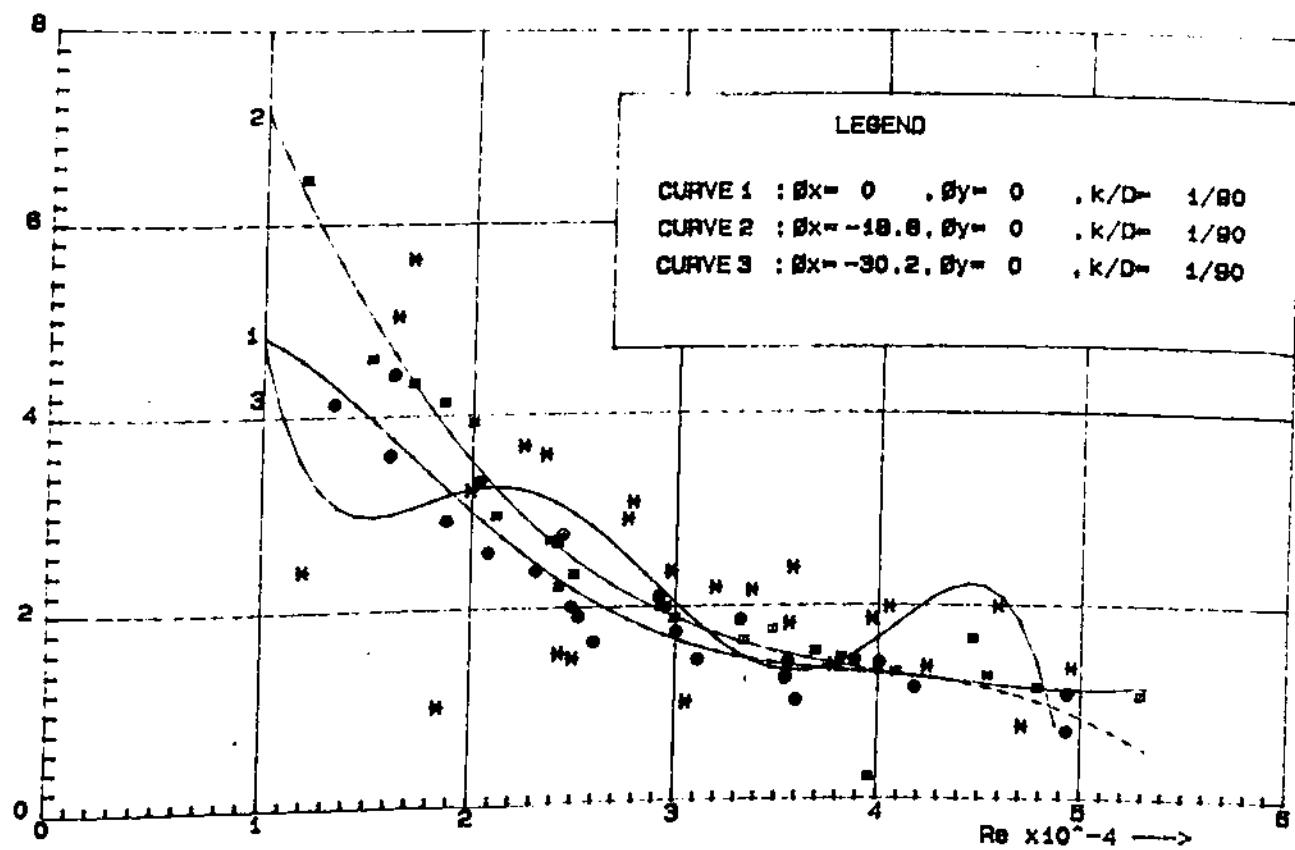


Fig.No. F - 17

COEFF. OF INERTIA →



COEFF. OF INERTIA →

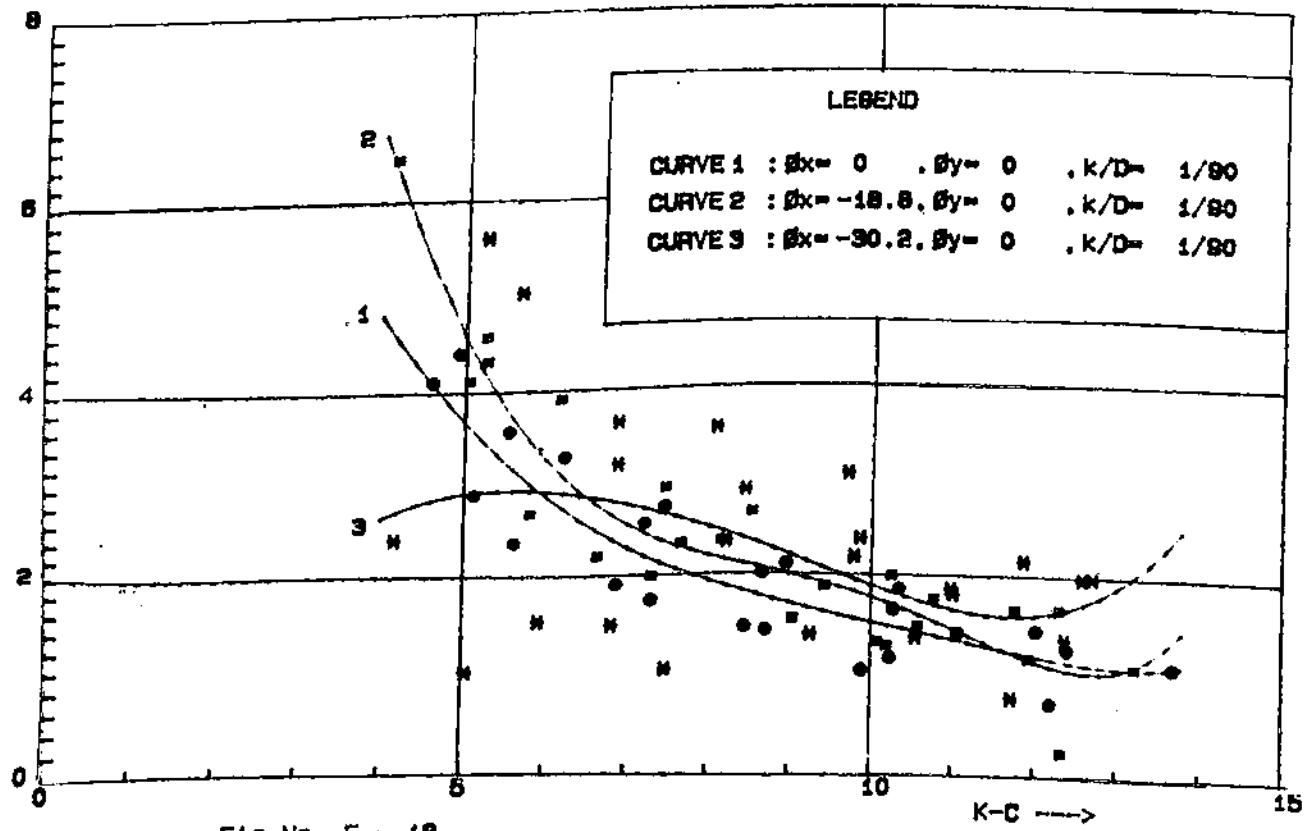
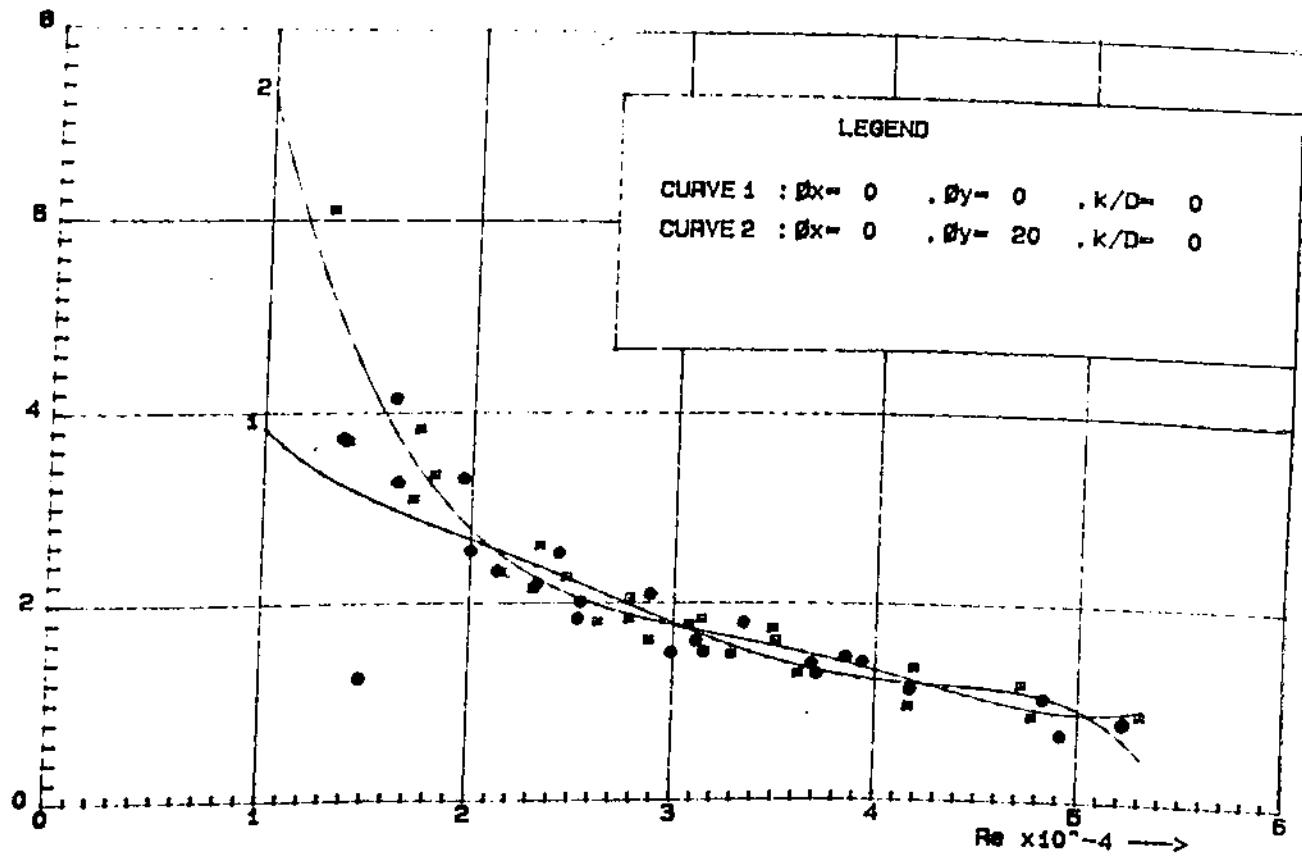


Fig.No. F - 18

COEFF. OF INERTIA →



COEFF. OF INERTIA →

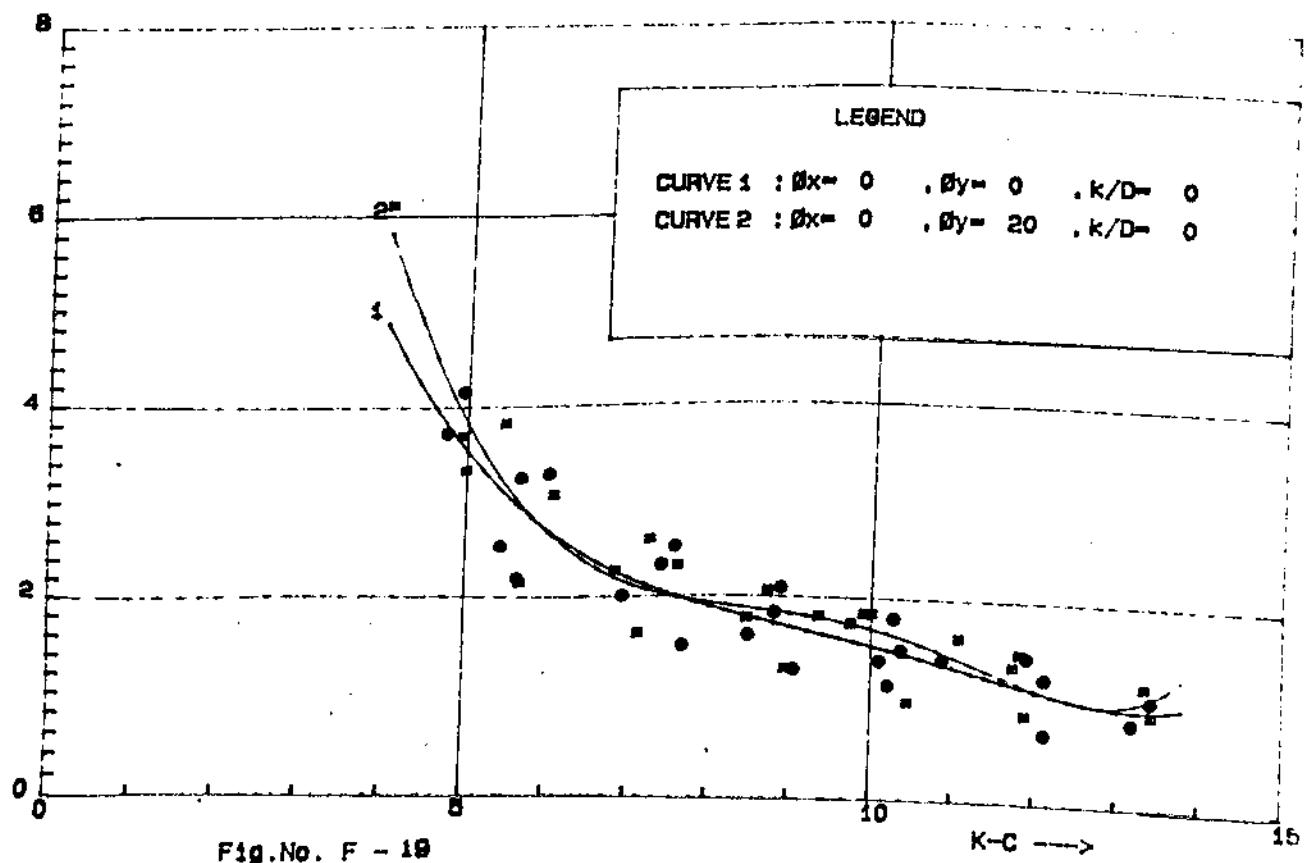
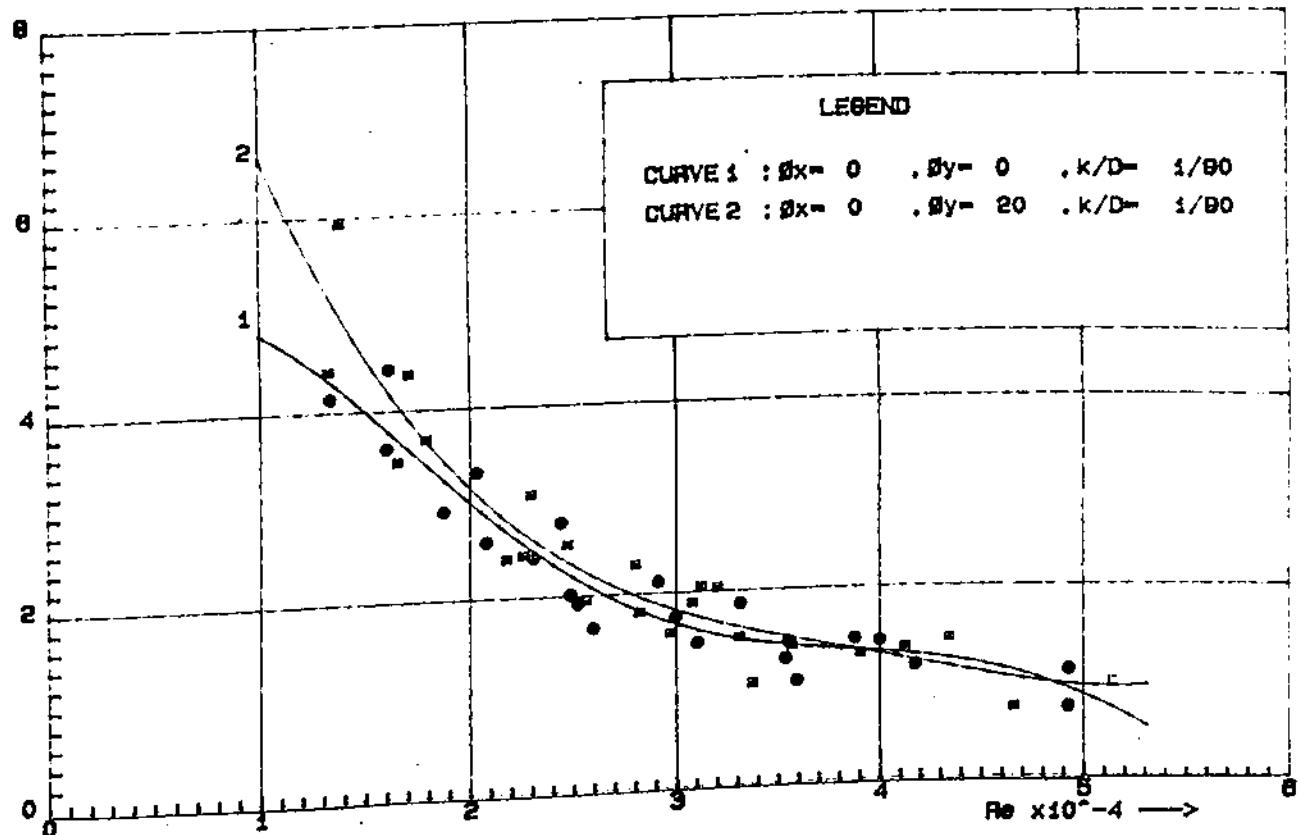


Fig. No. F - 19

COEFF. OF INERTIA →



COEFF. OF INERTIA →

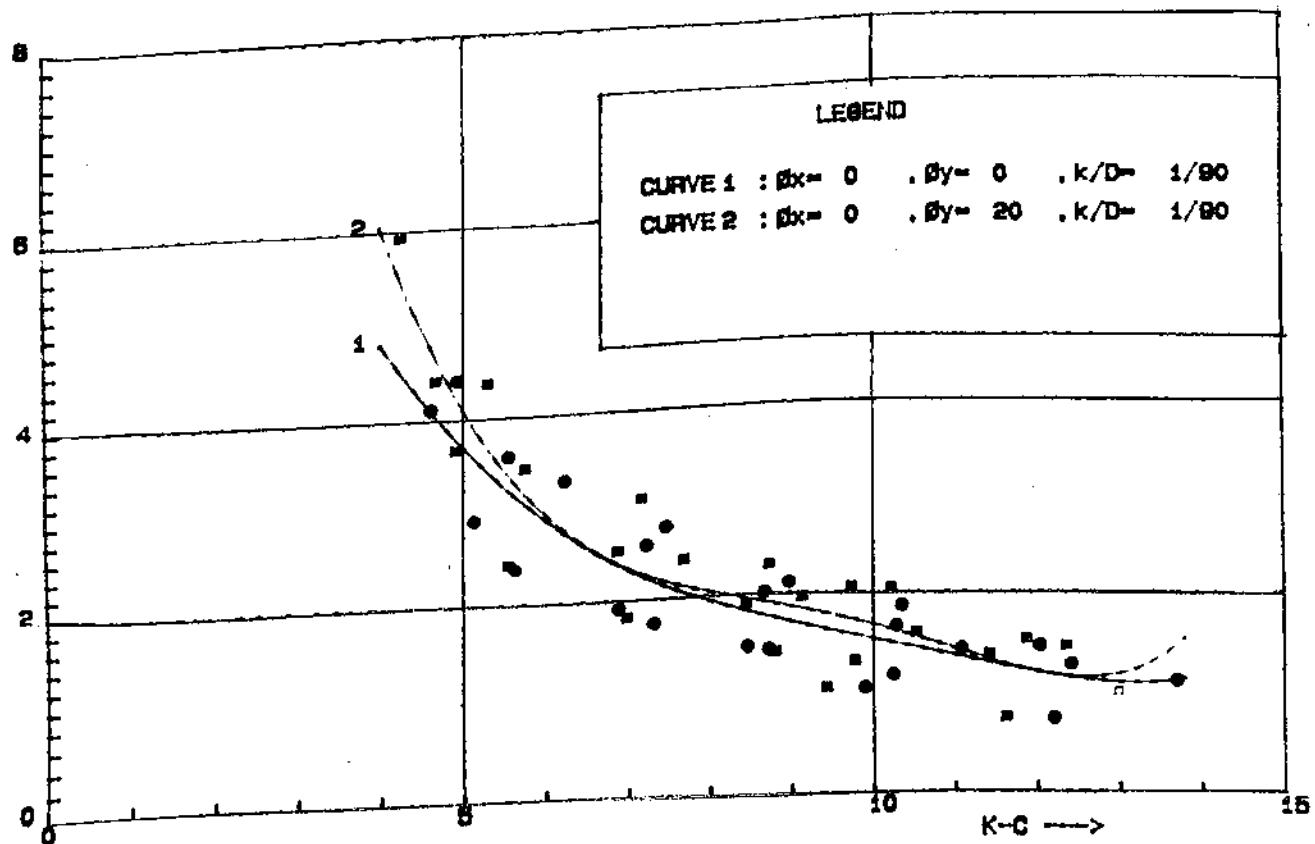


Fig. No. F - 20

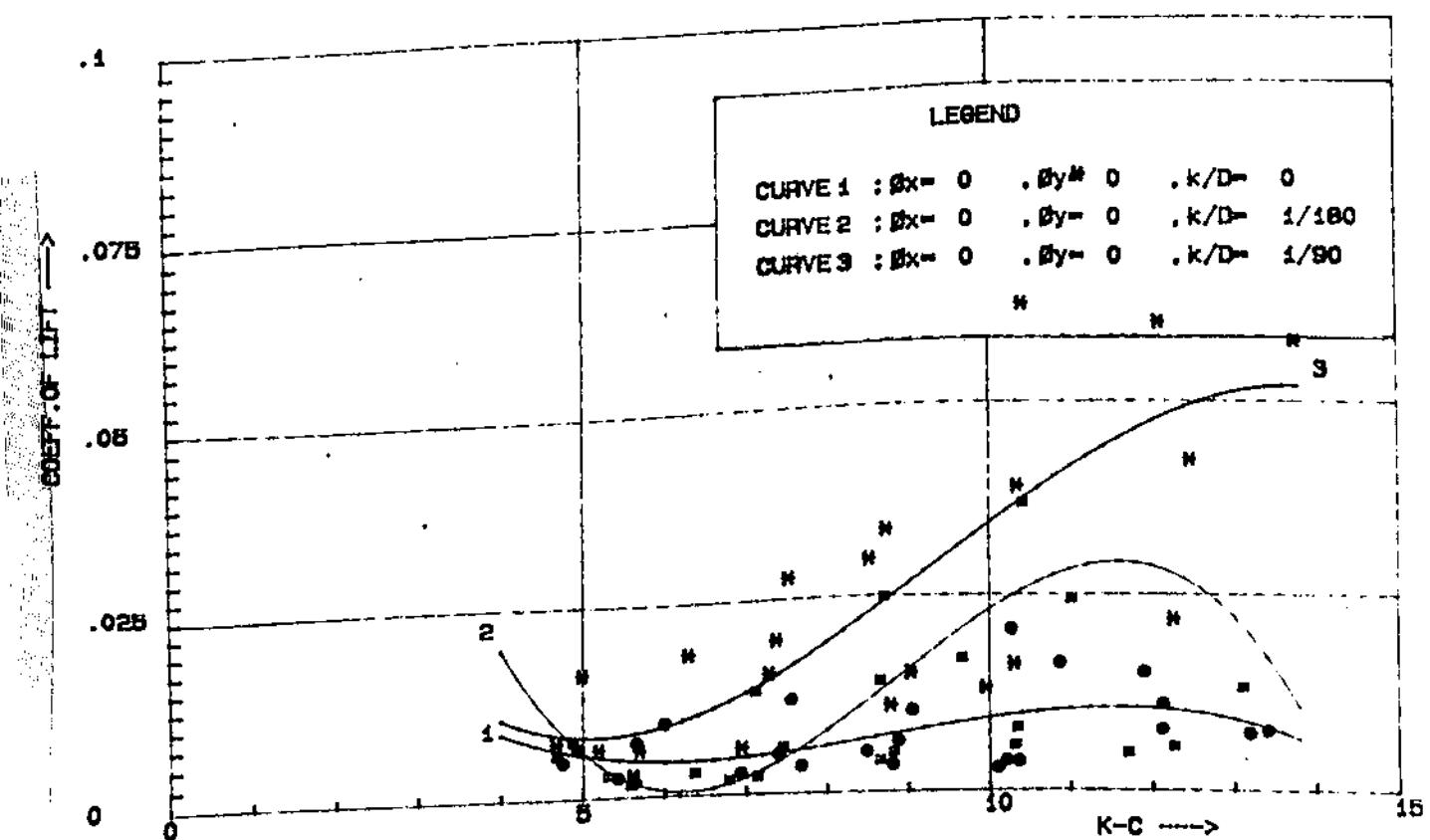
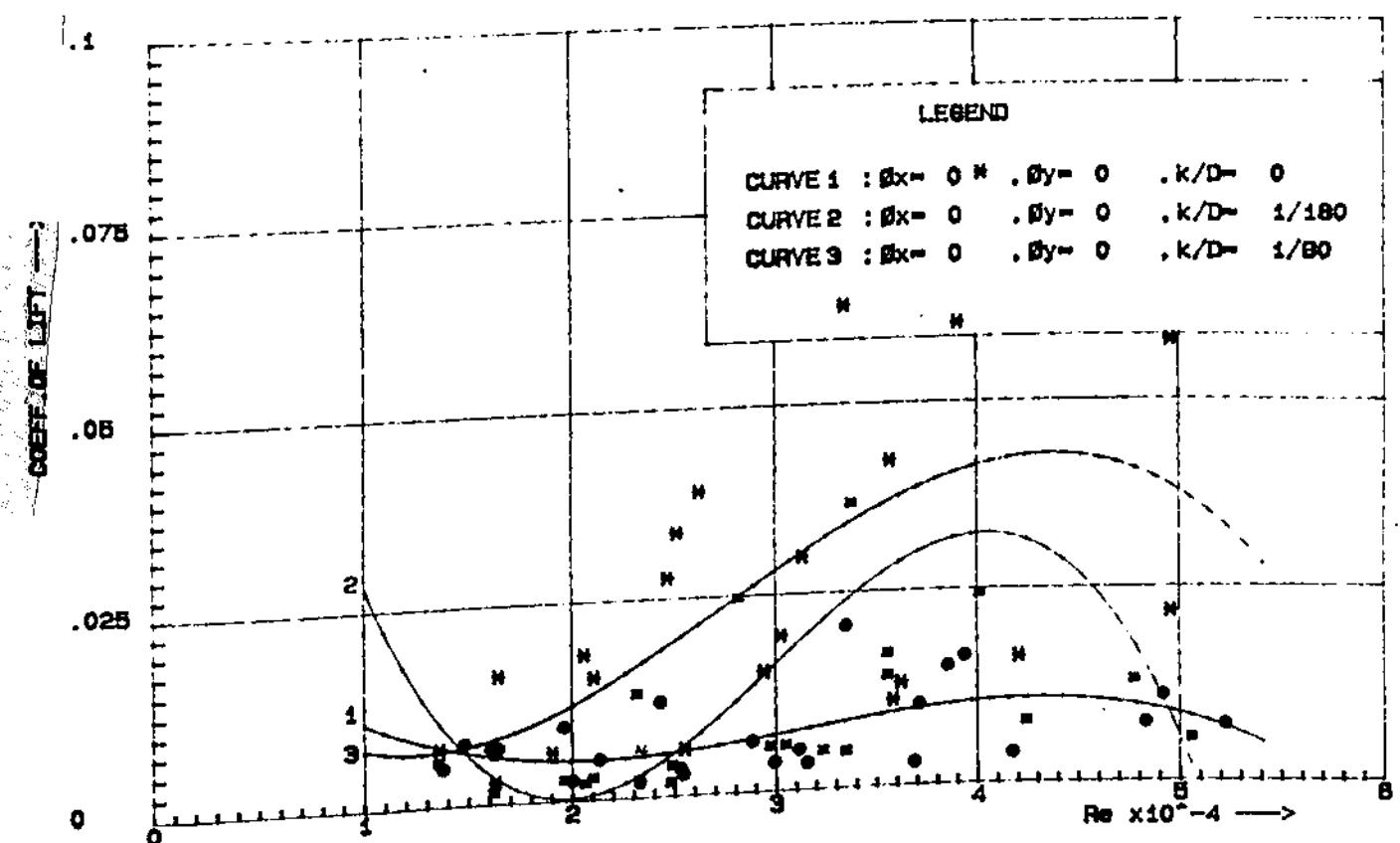
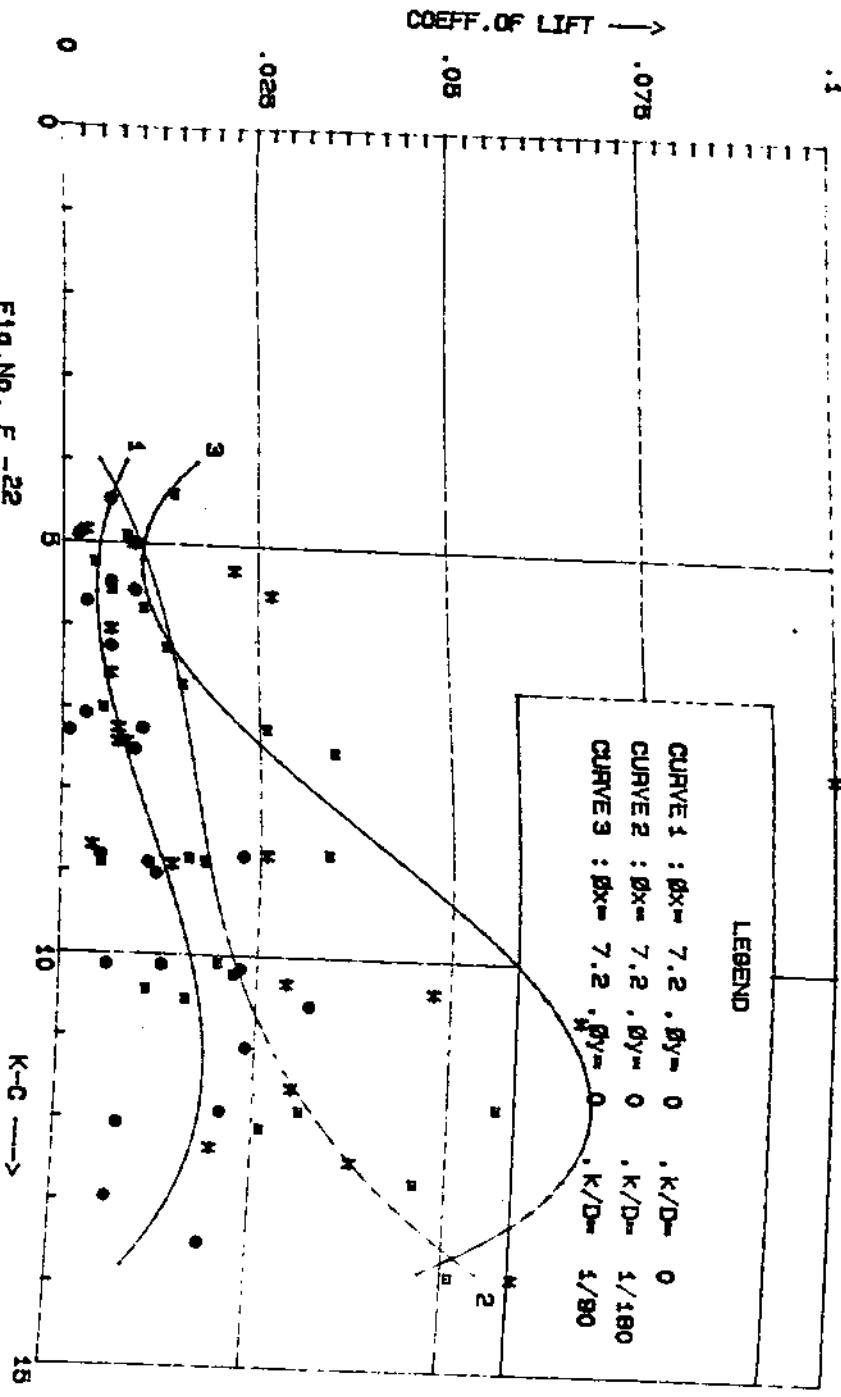
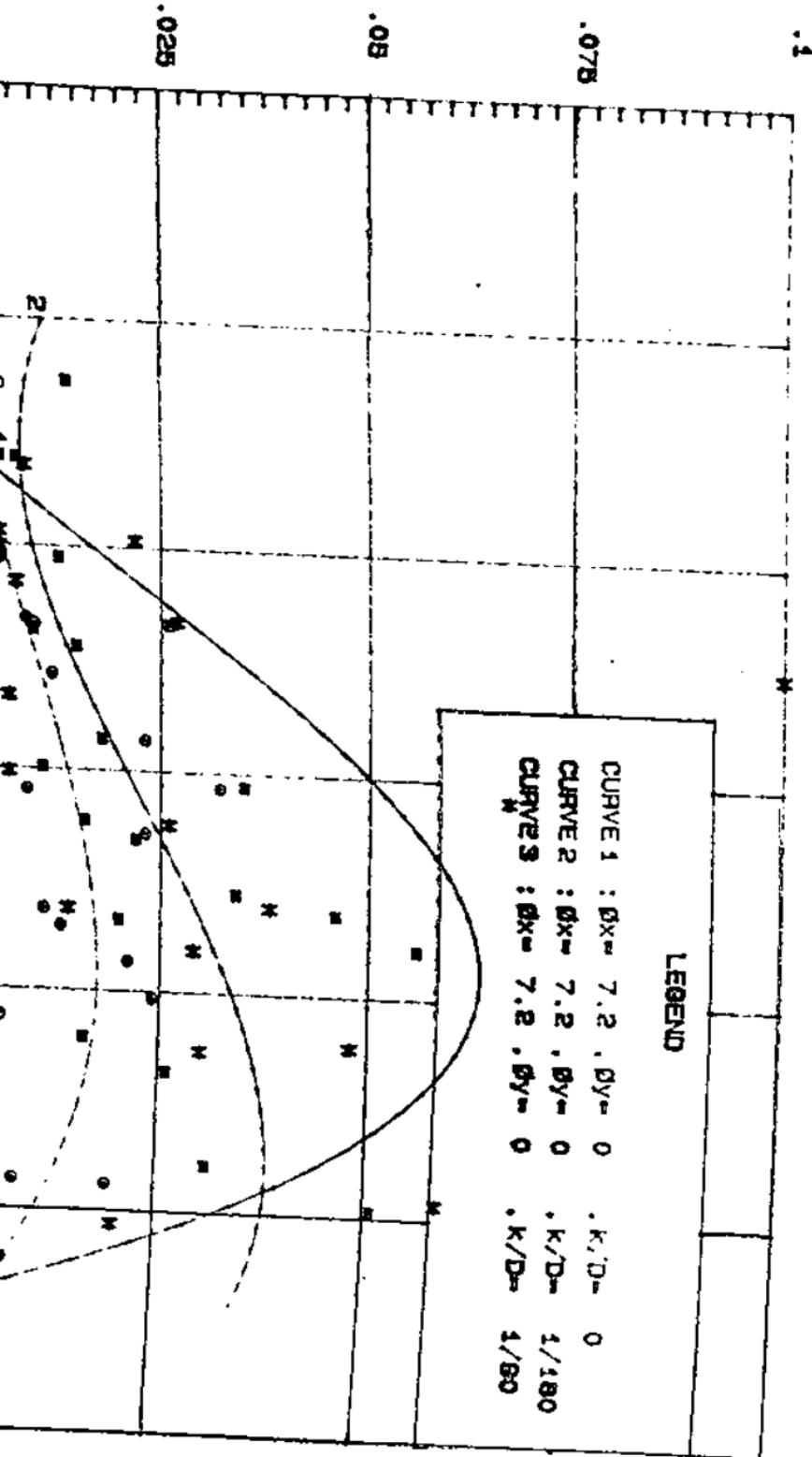


Fig.No. F - 21

FIG. NO. F - 22



COEFF. OF LIFT →



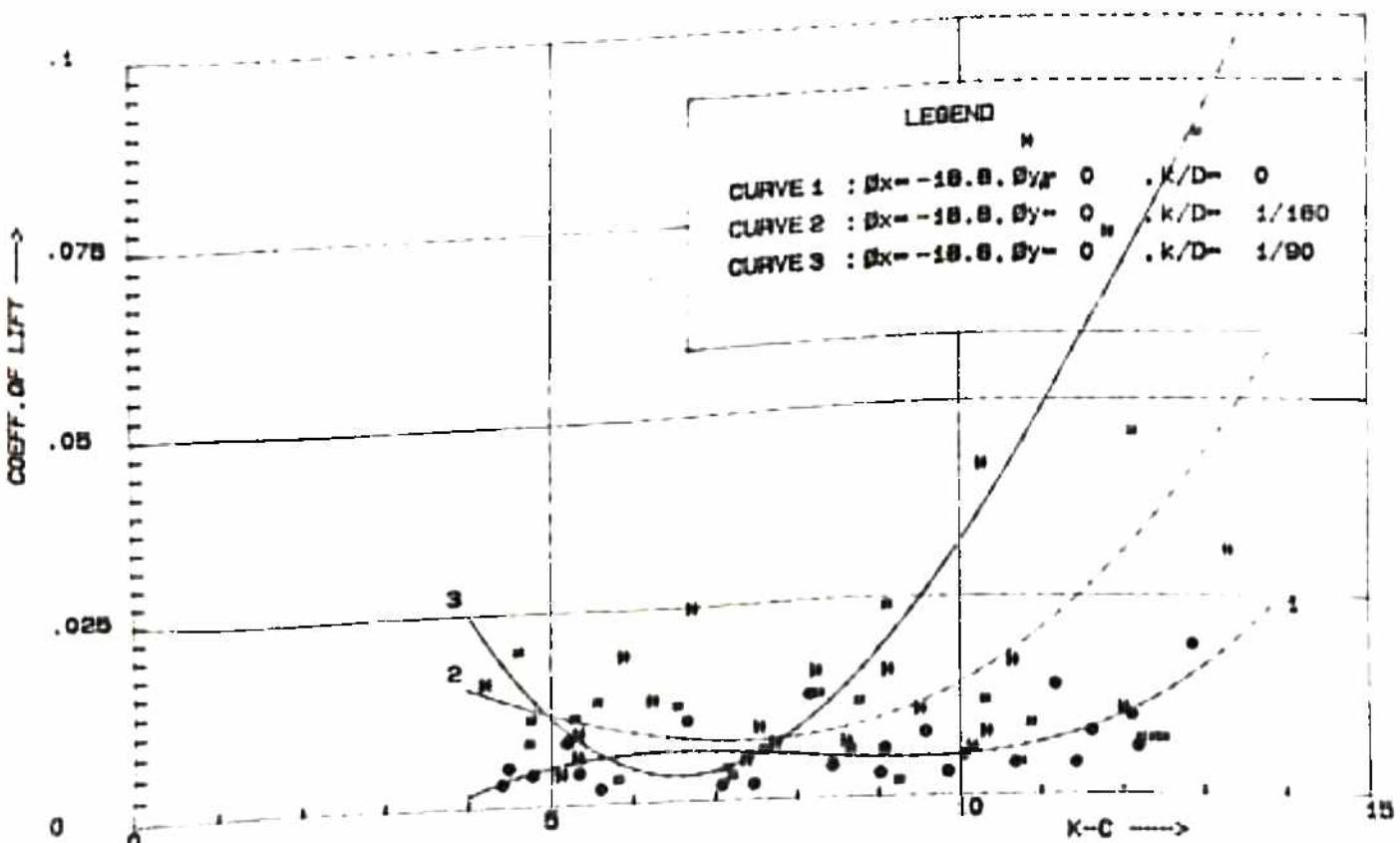
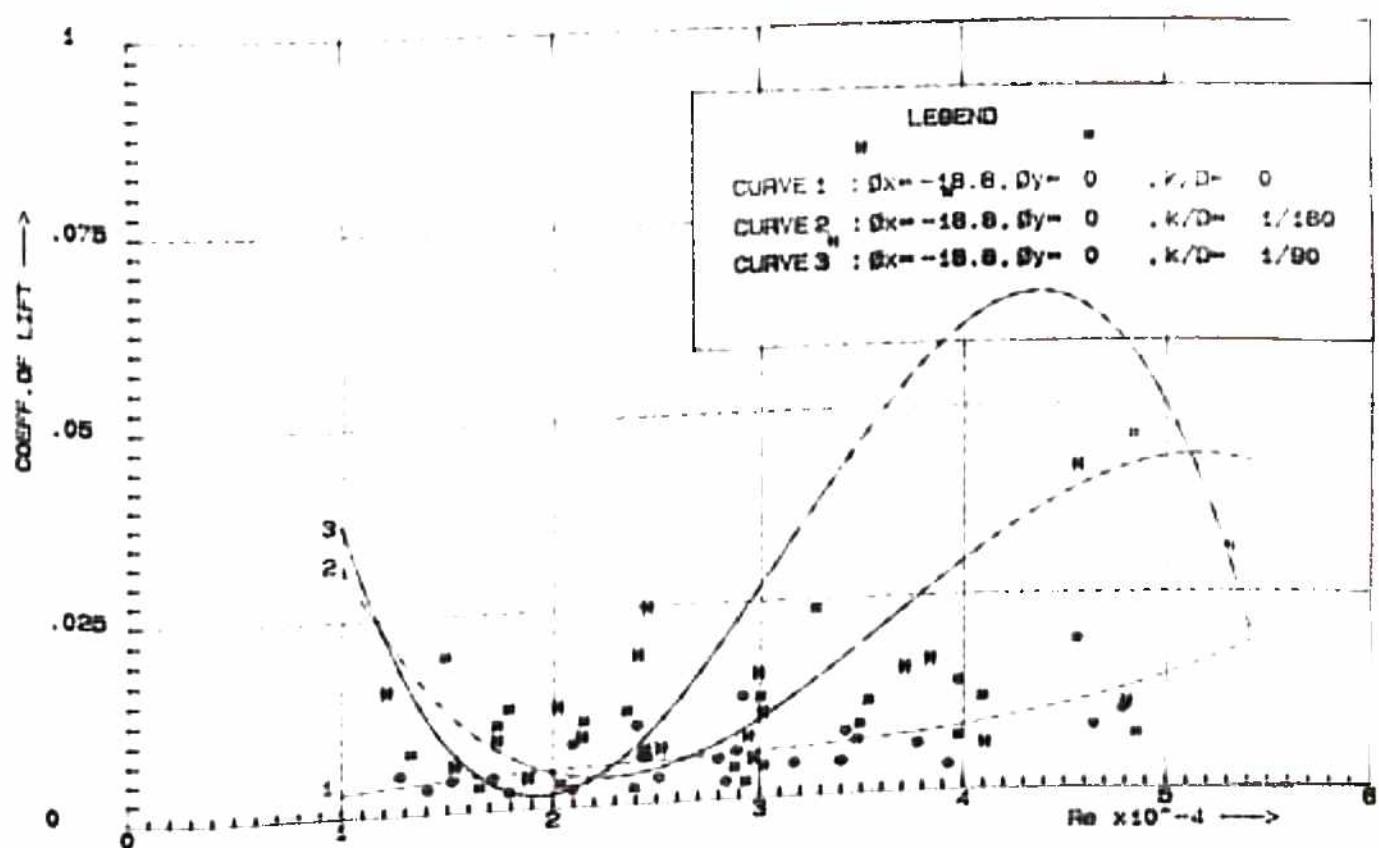


Fig.No. F -23

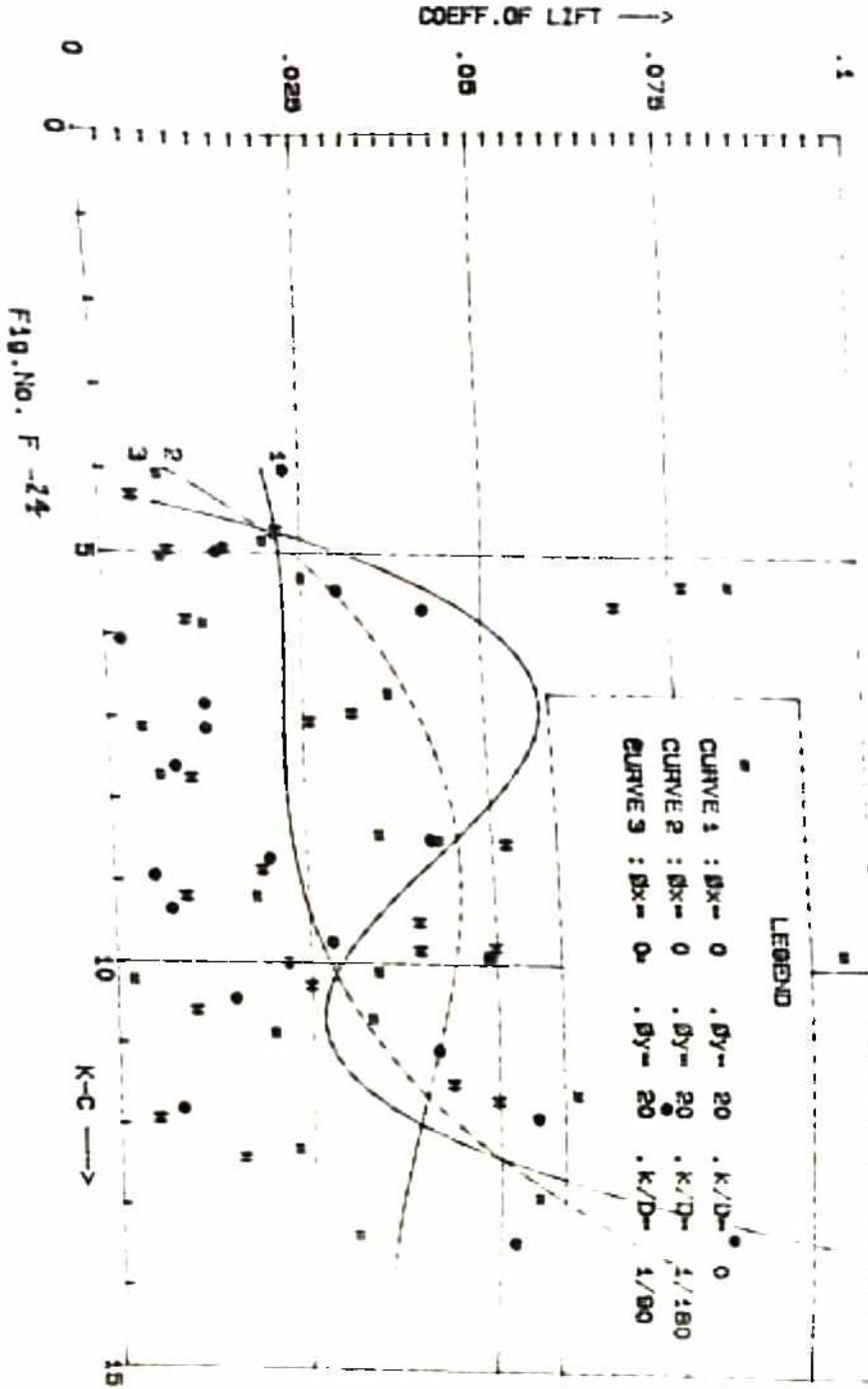
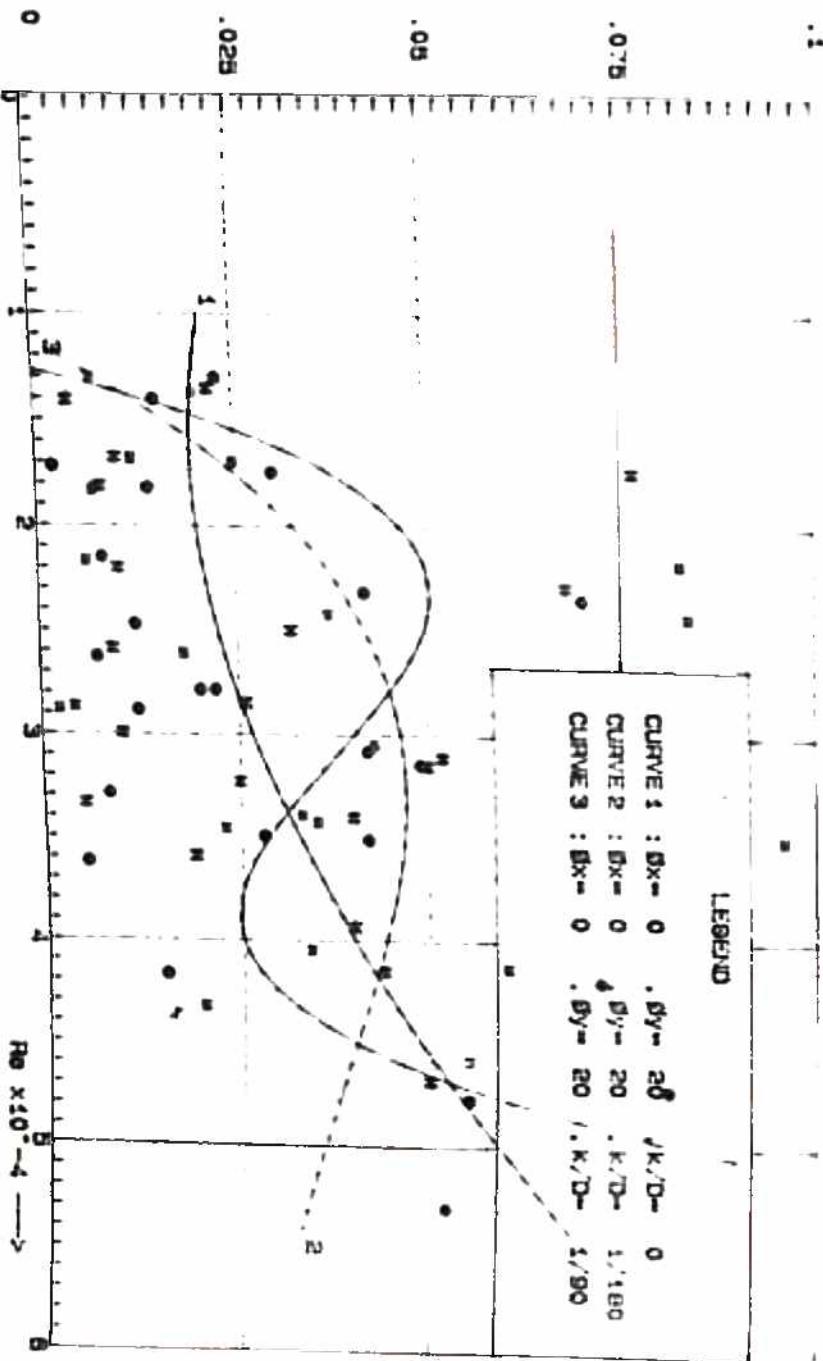


FIG. No. F - 24

COEFF. OF LIFT →



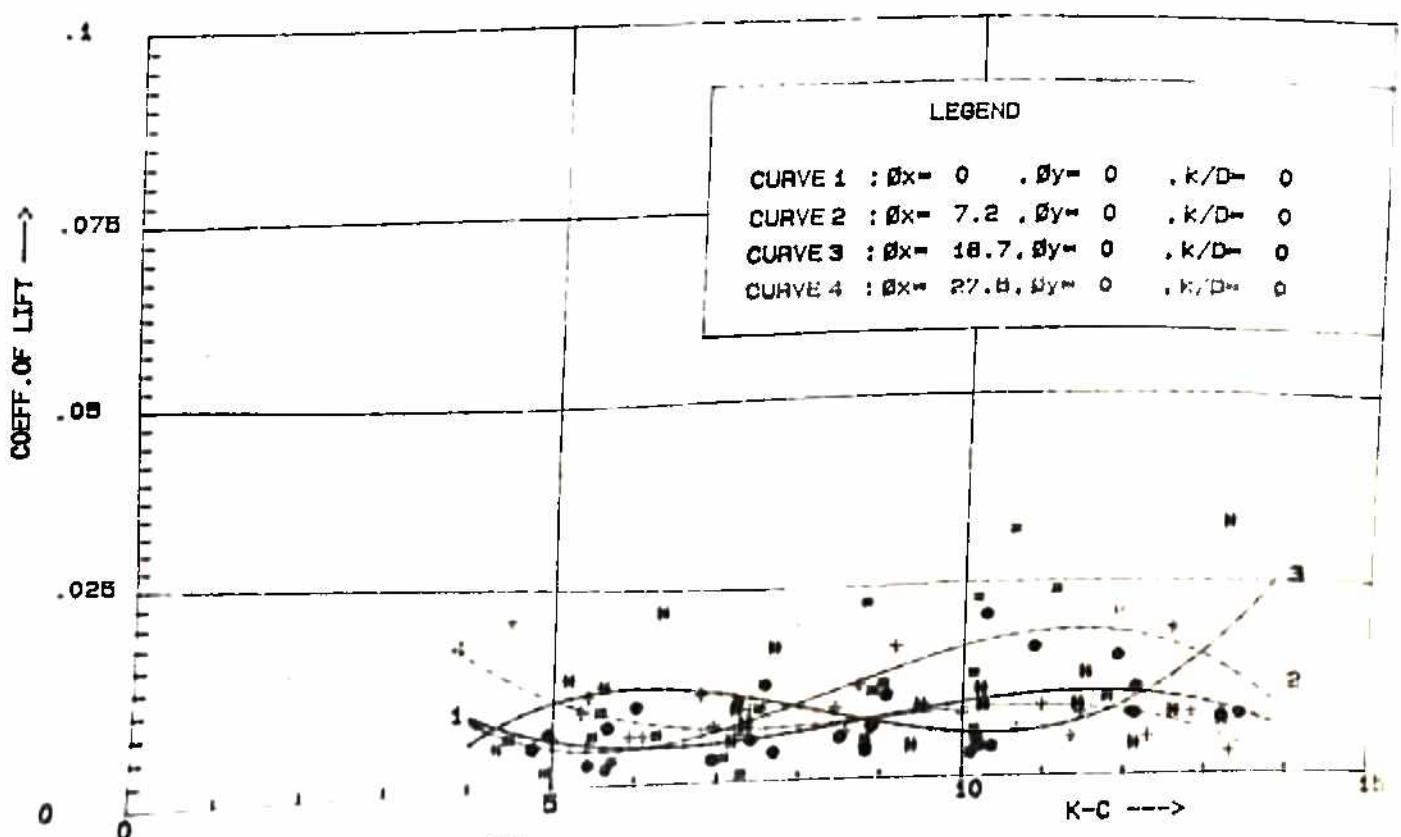
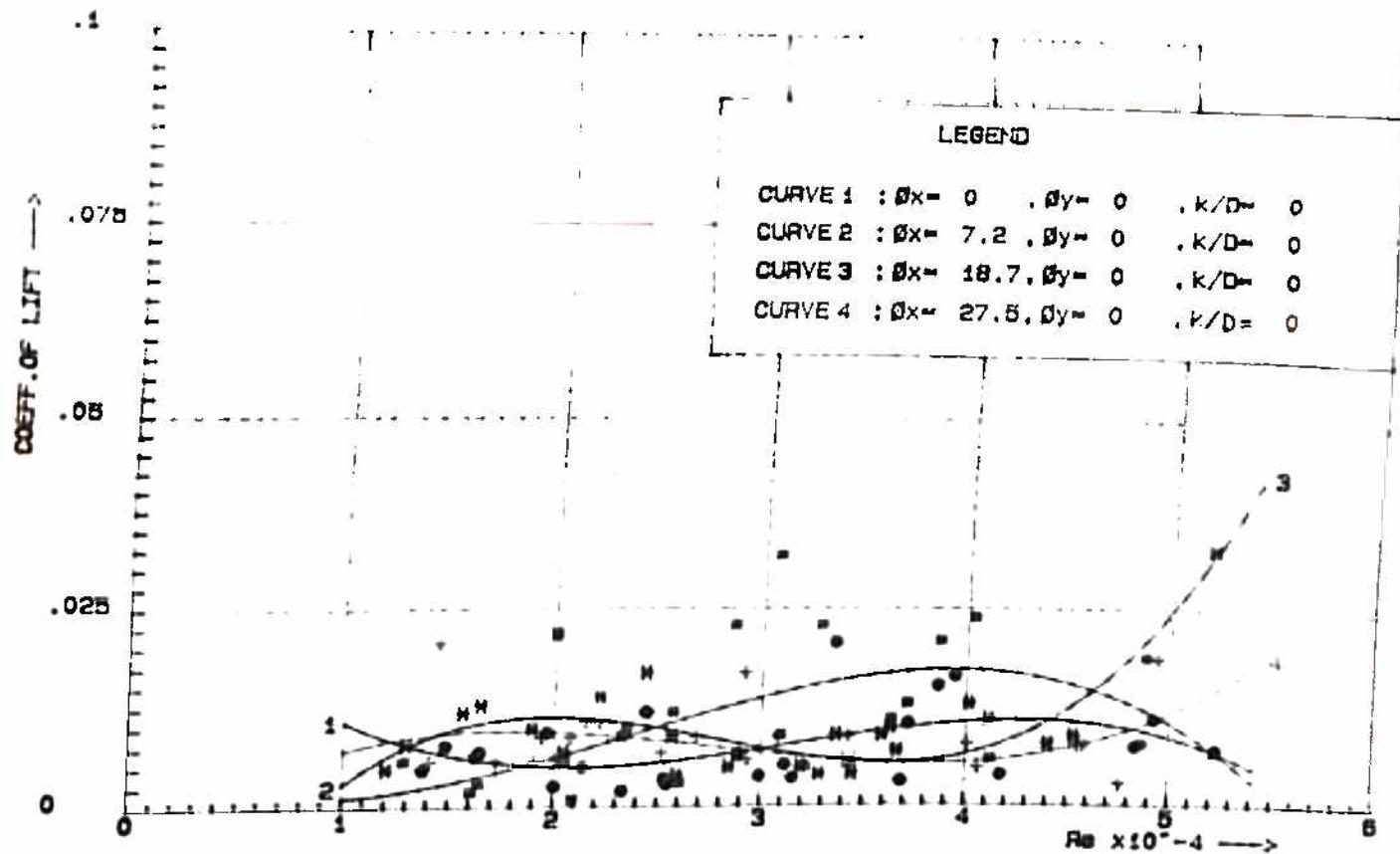


Fig. No. F - 25

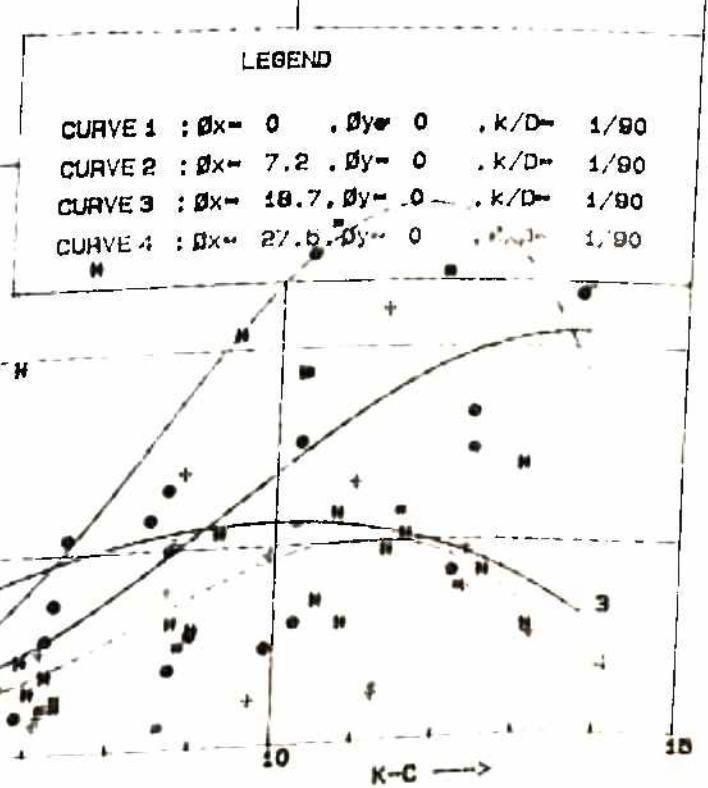
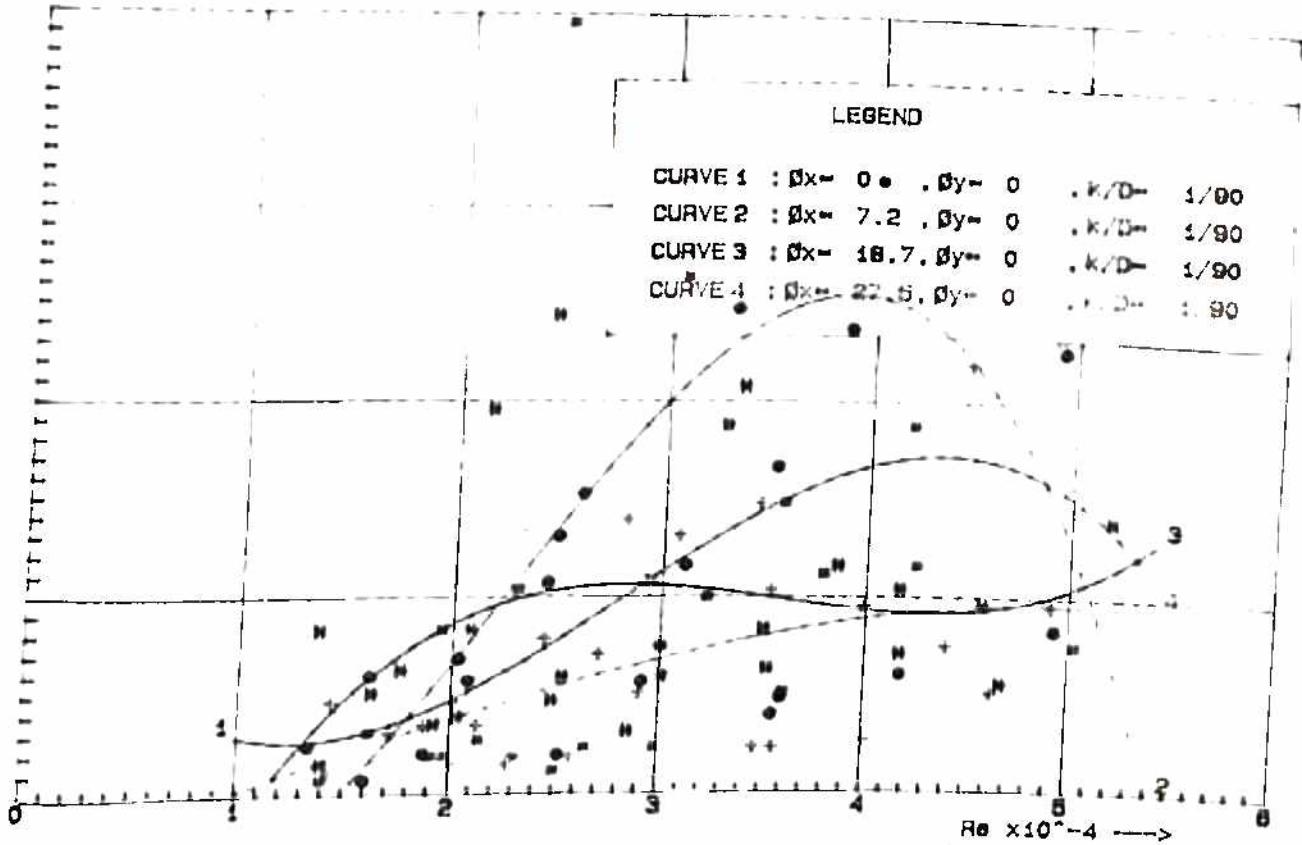


Fig. No. F - 2B

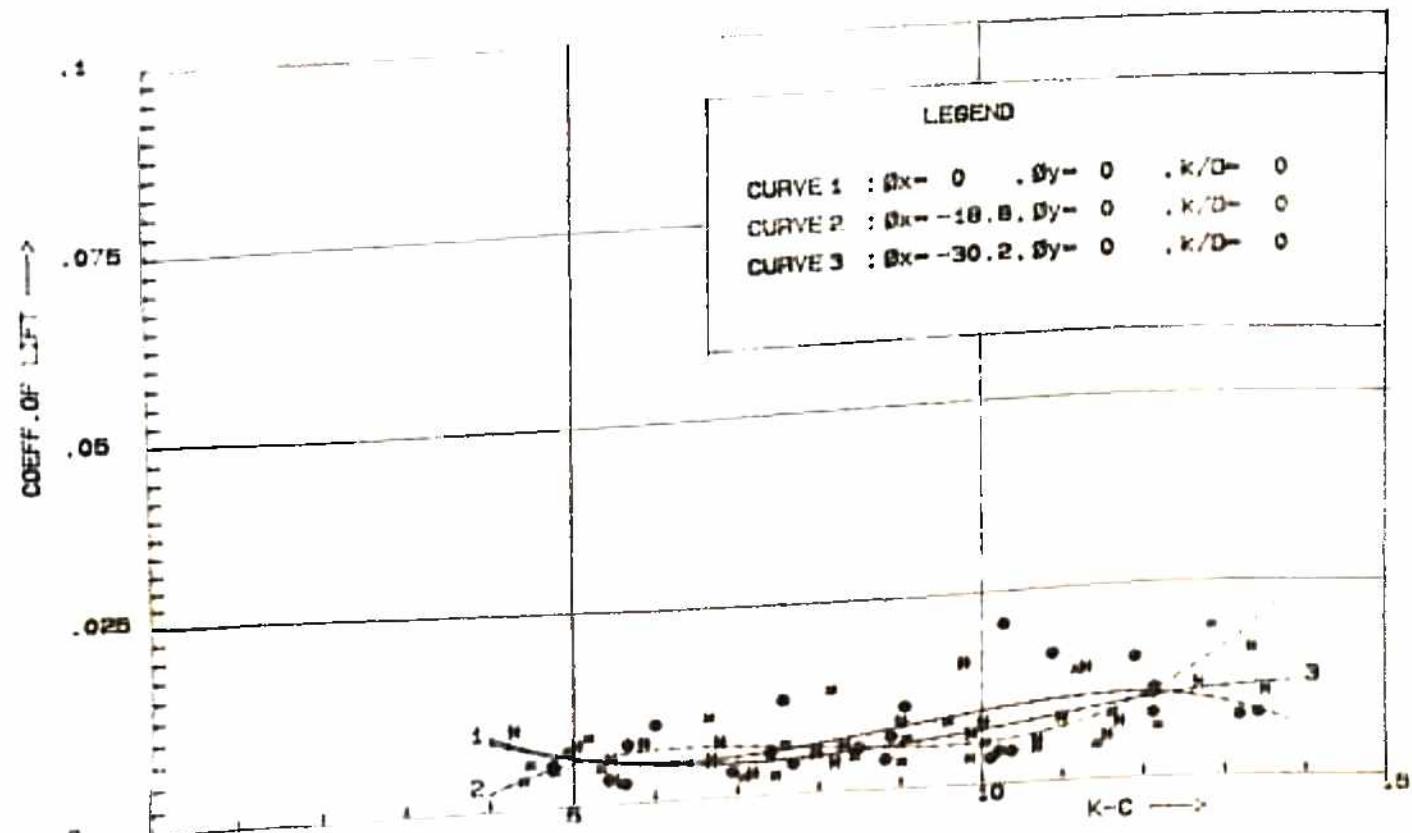
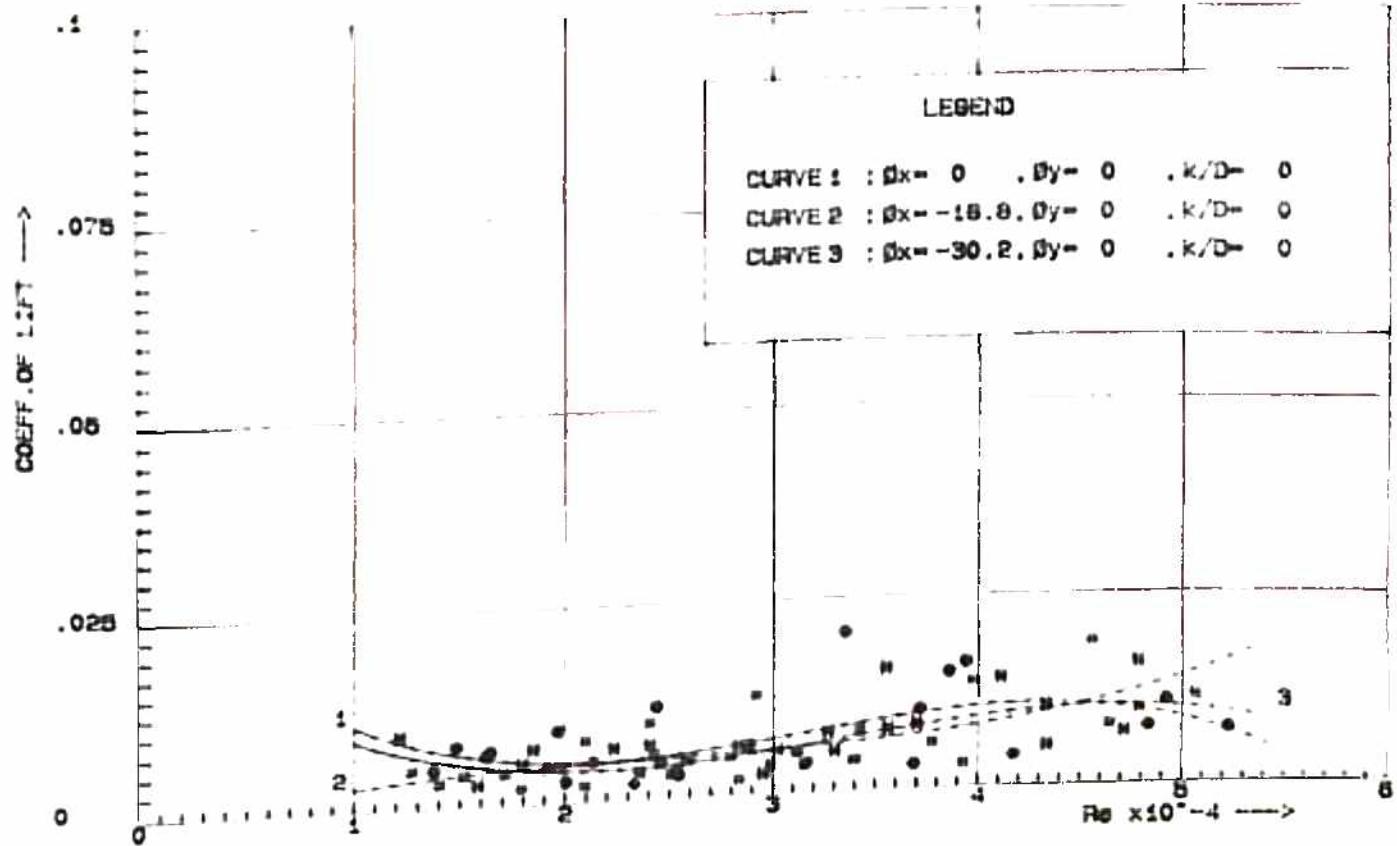
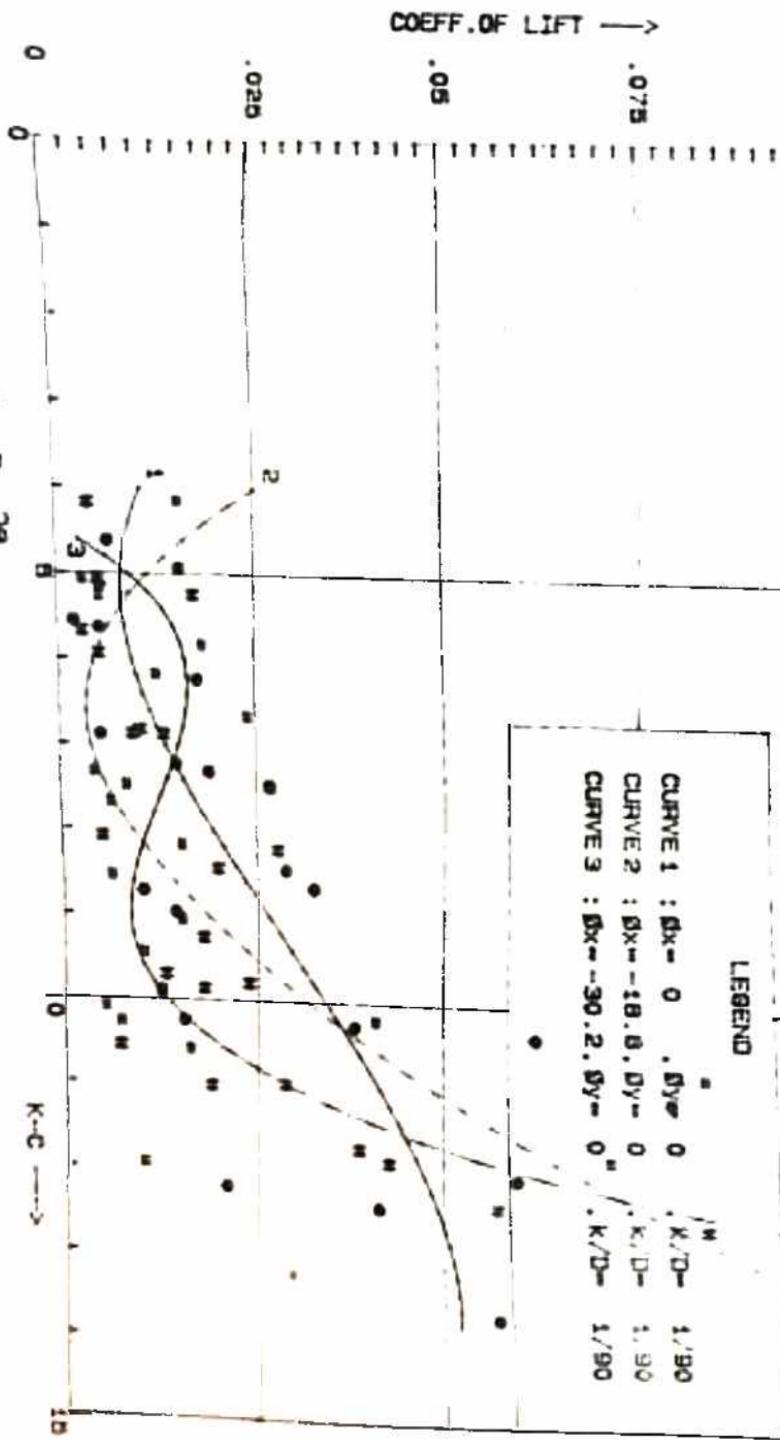
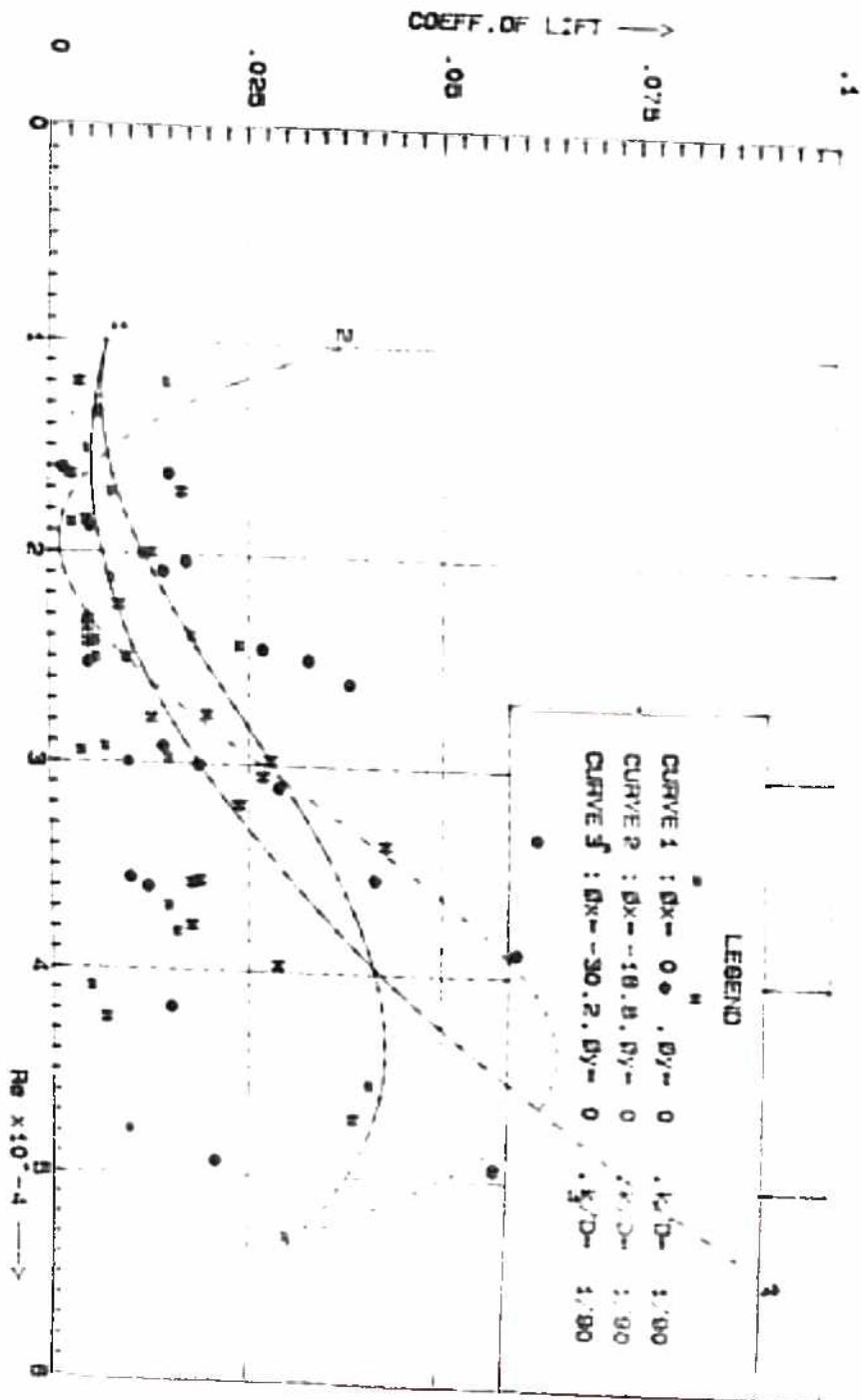
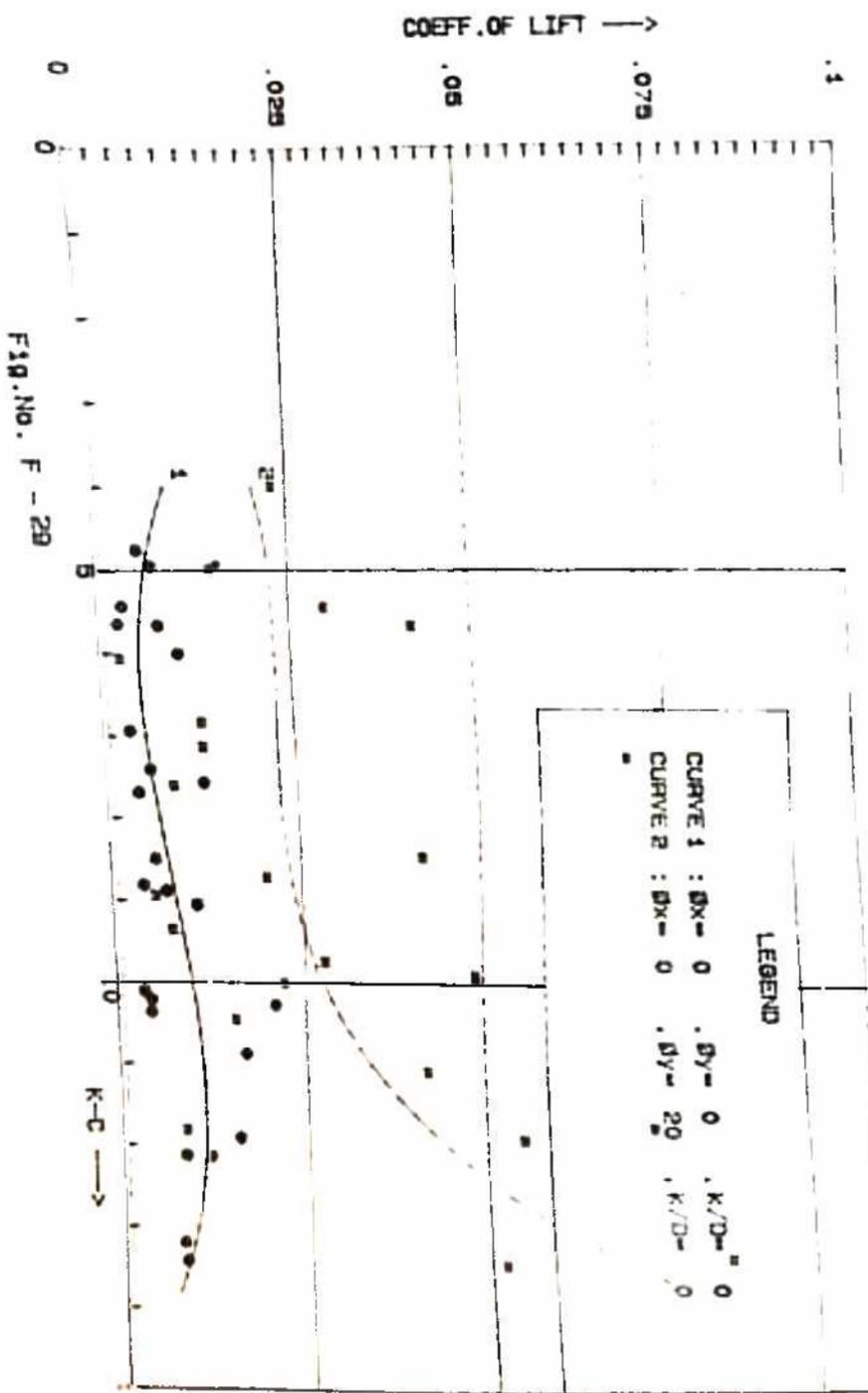


Fig.No. F - 27

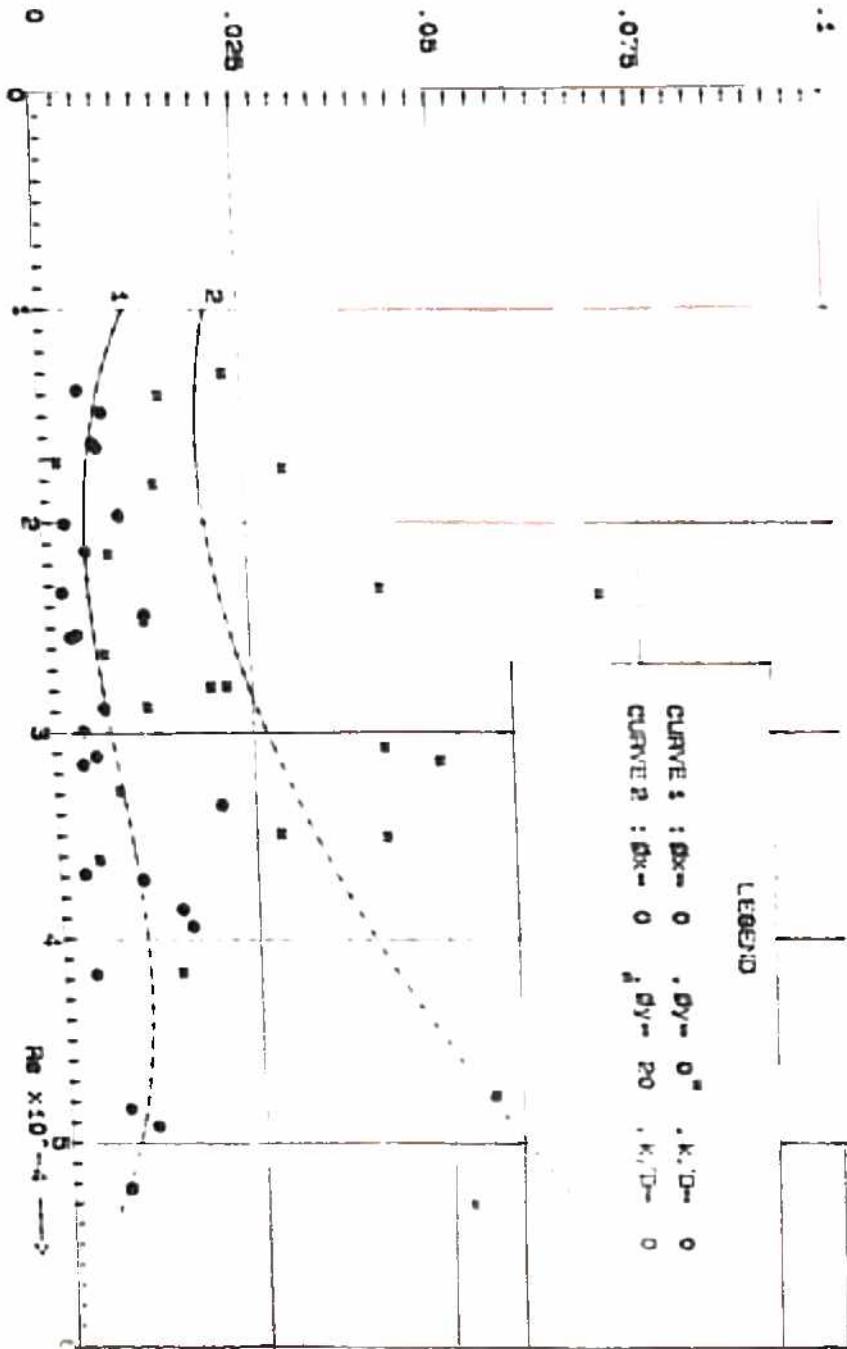
FIG. NO. F - 2B







COEFF. OF LOFT →



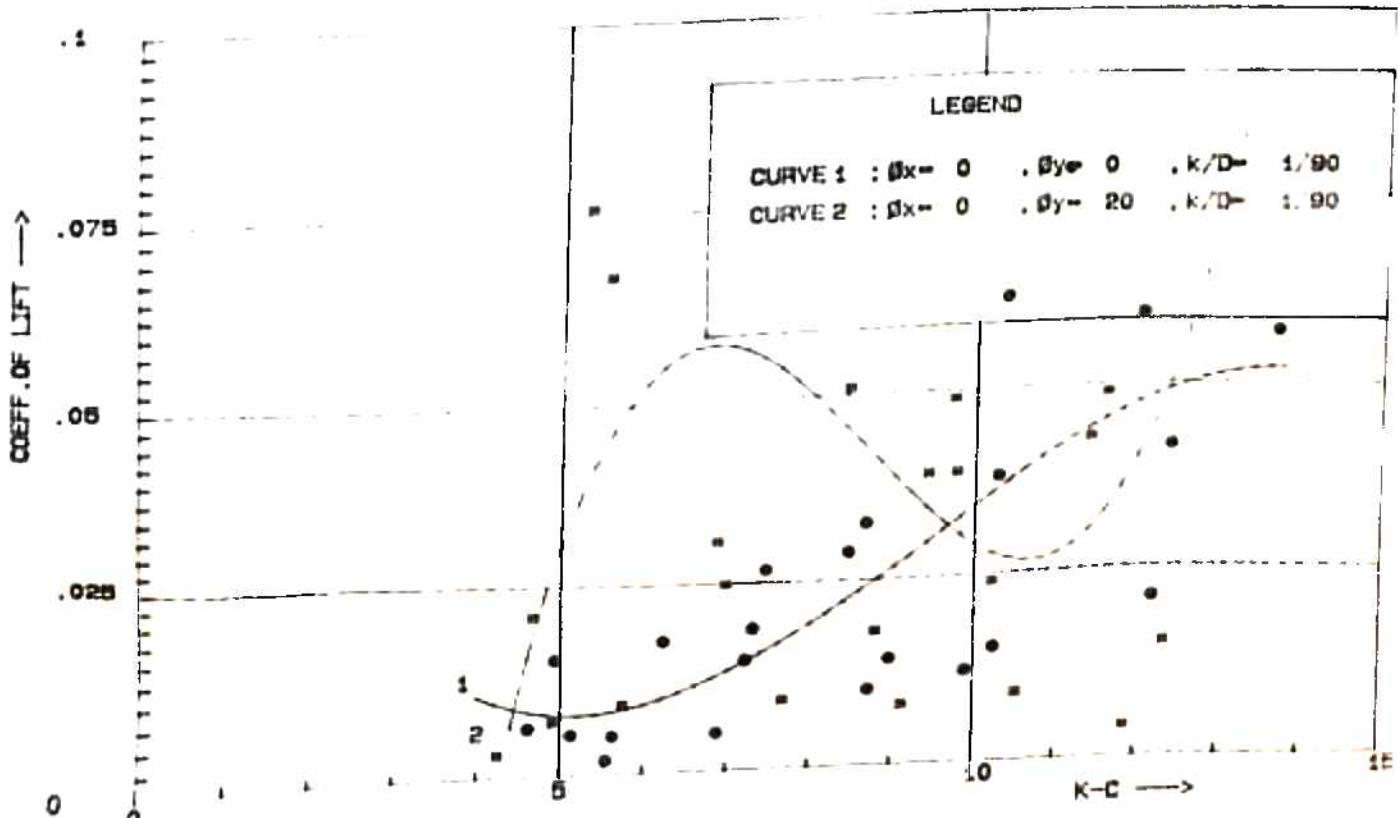
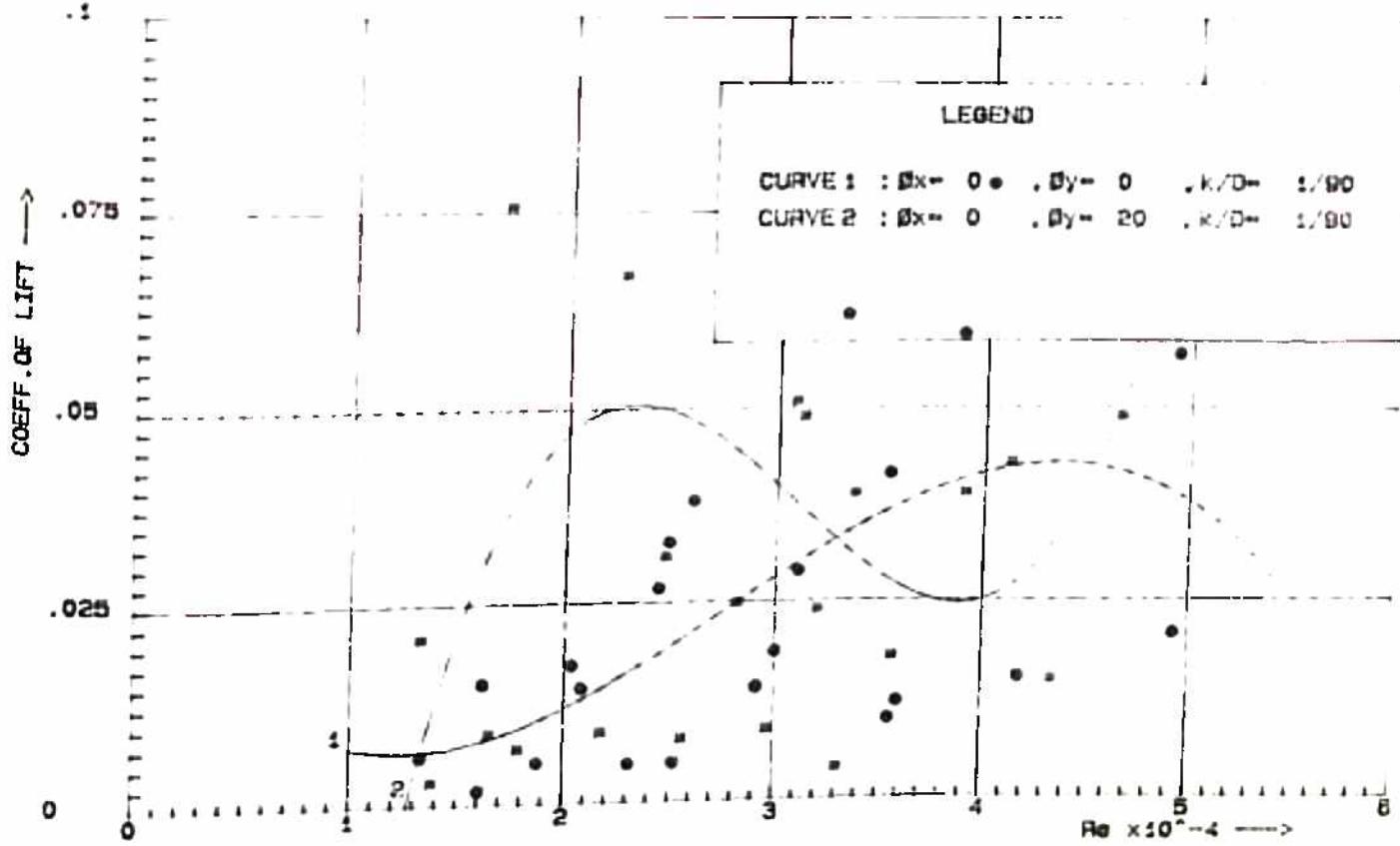


Fig. No. F - 30

References

Notations used in References and Bibliography

ASCE	American Society of Civil Engineers
ASME	American Society of Mechanical Engineers
BOSS	Behaviour of Off Shore Structures
Co	Company
Div.	Division
Engg.	Engineering
Hyd.	Hydraulics
I.I.T.	Indian Institute of Technology
Instrn	Institution
JFM	Journal of Fluid Mechanics
Jl	Journal
Mech.	Mechanics
No.	Number
OTC	Offshore Technology Conference
pp	pages
Proc.	Proceedings
Ser	Serial
Soc	Society
Trans	Transactions
Vol	Volume
Wat. etc.	Waterway, Port, Coastal, and Ocean Engineering

REFERENCES

1. Achenbach, E. 'Distribution of local pressure and skin friction around a circular cylinder in cross-flow upto $Re = 5 \times 10^6$ '. JFM, Vol 34, Part 4, pp 625 - 639, (1968).
2. Achenbach, E. 'Influence of surface roughness on the cross-flow around a circular cylinder'. JFM, Vol 46, Part 2, pp 321 - 325, (1971).
3. Angrilli, F.. and Cossalter, V. 'Transverse oscillations of a vertical piles in waves'. Jl. of Fluids Engg., ASME, Vol 104 No. 1, pp 46-53. (1982).
4. Bearman, P.W. and Graham, J.M.R. 'Hydrodynamic forces on cylindrical bodies in oscillatory flow'. BOSS 79, Vol 1, Paper 24, pp 309 - 322, (1979).
5. Bidde, D.D. 'Laboratory study of lift forces on circular piles'. Jl. of Wat. etc. Divn., ASCE, Vol 97 No. WW4, paper 8495, pp 595 - 614. (1971).
6. Bishop, R.E.D., and Hassan, A.Y. 'The lift and drag forces on a circular cylinder in a flowing fluid'. Proc. Royal Soc. London, Ser A, Vol 277, pp 32 - 50, (1964).
7. Chakrabarti, S.K. 'Inline forces on fixed vertical cylinder in waves'. Jl. of Wat. etc. Divn., ASCE, Vol 106, No. WW2, paper 15403, pp 491 - 493, (1980).
8. Chakrabarti, S.K. 'Laboratory generated waves and wave theories'. Jl. of Wat. etc. Divn., ASCE, Vol 106, No. WW3, paper 15644, pp 349 - 368, (1980).

9. Chakrabarti, S.K. 'Wave force coefficients for rough vertical cylinders'. Jl. of Wat. etc. Divn., ASCE, Vol 108, No. WW4, pp 445 - 455, (1982).
10. Chakrabarti, S.K. 'Hydrodynamic coefficients and depth parameter'. Jl. of Wat. etc. Divn., ASCE, Vol 111, No. WW3, paper 19395, pp 123 - 127, (1985).
11. Chakrabarti, S.K. Tam, W.A., and Wolbert, A.L. 'Wave Forces on a Randomly Oriented Tube'. 7th Annual OTC, paper 2190, pp 433 - 447, (1975).
12. Chakrabarti, S.K. Tam, W.A., and Wolbert, A.L. 'Total Forces on a Submerged Randomly Oriented Tube Due to Waves'. 8th Annual OTC, paper 2495, pp 725 - 740, (1976).
13. Chakrabarti, S.K. Tam, W.A., and Wolbert, A.L. 'Wave forces on a vertical circular cylinder'. Jl. of Wat. etc. Divn., ASCE, Vol 102, No. WW2, paper 12140, pp 203 - 221, (1976).
14. Chakrabarti, S.K. Wolbert, A.L., and Tam, W.A. 'Wave forces on inclined tubes'. Coastal Engg., 1, pp 149 - 165, (1977).
15. Chakrabarti, S.K. and Cotter, D.C. 'Wave force tests on vertical and inclined cylinders'. Jl. of Wat. etc. Divn., ASCE, Vol 110, No. WW1, paper 18599, pp 1 - 14, (1984).
16. Chakrabarti, S.K. Libby, A.R. and Kompare, D.J. 'Dynamic pressure around a vertical cylinder in waves'. 18th. OTC, OTC 5102, pp 193 - 199 (1986).
17. Chakrabarti, S.K. Armburst, S.F. 'Total force coefficients for inclined cylinders'. Jl. of Wat. etc. Divn., ASCE, Vol 113, paper 21686, July (1987).

18. Chakrabarti, S.K.
and
Brogren, E.E. 'Wave tank testing of Fluid-Structure Interaction'. Presented at the IAMV Testing Techniques Facilities Conference, June 1989, Washington, D.C., pp 567 - 574, (1989).
19. Garrison, C.J. 'A review of drag and inertia forces on circular cylinders'. 12th. OTC, Vol 2, OTC 3760, pp 205 - 218, (1980).
20. Garrison, C.J. 'Drag and inertia forces on a circular cylinder in Harmonic flow'. Jl. of Wat. etc. Divn. ASCE, Vol - 116, No WW2, paper 24421, pp 169 - 190, (1990).
21. Garrison, C.J.
Field, J.B.,
and May, M.D. 'Drag and inertia forces on a cylinder in periodic flow'. Jl. of Wat. etc. Divn. ASCE, Vol - 103, No WW2, paper 12913, pp 193 - 204, (1977).
22. Ghoshal, S. 'Hydrodynamic coefficients for inclined and rough cylinders'. Ph.D. Thesis, I.I.T., Bombay, India (1988).
23. Ghoshal, S. 'End effects of inclined cylinders in wave flume'. Proceedings of the 8th. Congress of the Asia and Pacific Divn. of the International Association for Hydraulic Research., Oct.,(1992).
24. Ghoshal, S.,
Deo, M.C., and
Narasimhan, S. 'Wave force coefficients for inclined rough cylinders'. Proceedings of ISOPE - 91, Edinburgh, Aug., (1991)
25. Ghoshal, S.
Deo, M.C., and
Narasimhan, S. 'Vortex shedding and transverse force around rough and inclined structural members'. International Seminar on Emerging Trends in Offshore Technology and Safety, New Delhi, India, Feb. (1992).

26. Hogben, N.,
Miller, B.L.,
Searle, J.W.,
and Ward, C.
'Estimation of fluid loading on off-shore structures'.
Proc. Instn. of Civil Engineers,
Vol 63, Part 2, paper 8029, pp 515 - 562, (1977).
27. Huang, M.C.
'Kinematics Prediction by Stokes and Fourier Wave Theories'.
Jl. of Wat. etc. ASCE, Vol 116, No. 1, paper 24306, pp 137 - 148, (1990)
28. Hudspeth, R.T.
'Importance of Parameter K'.
Journal of Hydraulic Engg., ASCE,
Vol 117, No. 12, paper 26460, Dec. (1991)
29. Hudspeth, R.T.,
Nath, J.H., and
Khare, P.
'Wave phase / amplitude effects on force coefficients'.
Jl. of Wat. etc. Divn., ASCE, Vol 114, No 1, paper 22095, pp 34 - 48, (1988).
30. Isaacson, M.D.S.Q.
and
Maull, D.J.
'Transverse forces on vertical cylinders in waves'.
Jl. of Wat. etc. Divn., ASCE, Vol 102, No WW1, paper 11934, pp 49 - 60, (1976).
31. Jen, Y.
'Laboratory study of inertia forces on a pile'.
Jl. of Wat. etc. Divn., ASCE, Vol 94, No WW1, paper 5806, pp 59 - 76 (1968).
32. Jen, Y.
'Wave force analysis - an alternative procedure'.
Jl. of Wat. etc. Divn., ASCE, Vol 106, No WW1, pp 117 - 121, (1980).
33. Kaplan, P.,
Jiang, C.W., &
Stritto, P.J.D.
'Force coefficient evaluation for offshore structure inclined members'.
BOSS'82, Vol 1, pp 373-381, (1982)
34. Laird, A.D.K.
'Eddy forces on rigid cylinders'.
Jl. of Wat. etc. Divn., ASCE, Vol 87, No WW4, paper 2991, pp 53 - 68, (1961).

35. Laird, A.D.K.
 'Water forces on flexible oscillating cylinders'.
 Jl. of Wat. etc. Divn., ASCE, Vol 88, No WW3, paper 3234, pp 125 - 137, (1962).
36. Laird, A.D.K.
 'Eddy formation behind circular cylinders'.
 Jl. of the Hyd. Divn., ASCE, Vol 97, No HY6, paper 8170, pp 763 - 775, (1971).
37. Laird, A.D.K.,
 Johnson, C.A.,
 and Walker, R.W.
 'Water eddy forces on oscillating cylinders'.
 Jl. of the Hyd. Divn., ASCE, Vol 86 No HY9, paper 2652, pp 43-54, (1980)
38. Nath, J.H.
 'Hydrodynamic coefficients for macro - roughness'.
 13th. OTC, OTC 3989, pp 337 - 356, (1981).
39. Nath, J.H.
 'Heavily roughened horizontal cylinders in wave'.
 Proc. conference on Behaviour of Offshore Structures. (1982).
40. Roshko, A.
 'Experiments on the flow past a circular cylinder at very high Reynolds number'.
 JFM, Vol 10, Part 3, pp 345 - 356, (1961).
41. Sarpkaya, T.
 'In-Line and Transverse Force on Cylinders in Oscillatory Flow at High Reynolds Numbers'.
 8th Annual OTC, paper 2533 pp 95 - 108, May 3-6, (1976).
42. Sarpkaya, T.
 'Hydroelastic Response of Flexibly-mounted cylinders in harmonic flow'.
 9th Annual OTC, paper 2897 pp 155 - 159, May 2-5, (1977).
43. Sarpkaya, T.
 'Wave forces on Inclined Smooth & Rough Circular cylinders'.
 14th Annual OTC, paper 4227 pp 731 - 736, May 3-6, (1982).

44. Sarpkaya, T.,
Collins, N.J.,
& Evans S.R.
'Wave forces on rough - walled
cylinders at high Reynolds number',
9th Annual OTC, paper 2533 pp
175 - 184, May 2-5, (1977).
45. Sarpkaya, T.
and
Isaacson, M.
'Mechanics of wave forces on off-
shore structures'.
Van Nostrand Reinhold Co, New York,
(1981).
46. Short term course on "Dynamics of offshore structures"
held at I.I.T, Bombay. Dec. 2 - 6
(1991).
47. Skjelbreia, L..
and
Hendrickson, J.
'Fifth order gravity wave theory'.
Coastal Engg., Ch 10, pp 184 - 196,
(1961).
48. Starsmore, N.
'Consistent drag and added mass coe-
fficients from full - scale data'.
13th. OTC, Vol 1, OTC 3990, pp 357
- 365, (1981).
49. Szechenyi, E.
'Supercritical Reynolds number simu-
lation for two dimensional flow
over circular cylinders'.
JFM, Vol 70, Part 3, pp 529 - 542,
(1975).
50. Torum, A.L.
'Wave forces of pile in surface
zone'.
Jl. of Wat. etc. Vol 115, No 4
paper 23716, pp 547 - 565, (1989).
51. Verley, R.L.P..
British, M..E.
Wolbert, A.L.
'Wave Induced Vibrations of Flexible
Cylinders'.
9th Annual OTC, May 2-5, paper 2190,
pp 167 - 174, (1977).
52. Wiegel, R.L.
'Oceanographical Engineering'.
Prentice - Hall.IIC / Englewood
Cliffs, N.J., (1964)
53. Williams, A.N.
'Wave forces on inclined circular
cylinders'.
Jl. of Wat. etc. Divn., ASCE, Vol
111, No WW3, (1985).

54. Zdravkovich, M.M
and
Namork, J.E. 'Formation and reversal of vortices
around circular cylinders subjected
to water waves'.
Jl. of Wat. etc. Divn., ASCE, Vol
103, No WW3, paper 13102, pp 378 -
383, (1977).

BIBLIOGRAPHY

1. Berniteas, M.M. 'Analysis of the hydrodynamic forces exerted on a harmonically oscillating circular cylinder in any direction θ with respect to a uniform current'. BOSS 79, Vol 1, Paper 25, pp 323 - 335, (1979).
2. Borgman, L.E. 'Wave forces on piling for narrow band spectra'. Jl. of Wat. etc. Divn., ASCE, Vol - 91, No. WW3, paper 4443, pp 65 - 90 (1965).
3. Bullock, G.N. 'Water particle velocities in regular waves'. Jl. of Wat. etc. Divn., ASCE, Vol 111, No 2, paper 19603, pp 189 - 199, (1985).
4. Chakrabarti, S.K. 'Wave forces on fixed offshore structures'. ASCE National Structural Engg. Convention, April 14 - 18, 1975, New Orleans, Louisiana, (1975).
5. Chakrabarti, S.K. 'Discussions on "Transverse forces on vertical cylinders in waves"'. Jl. of Wat. etc. Divn., ASCE, Vol 102, No. WW4, pp 491 - 493, (1976).
6. Chakrabarti, S.K. 'Transverse forces on vertical tube array in waves'. Jl. of Wat. etc. Divn., ASCE, Vol 108, No. WW1, paper 16842, pp 1 - 15, (1982).
7. Chu, Y. 'Orbital motion of water particles in oscillatory waves'. Jl. of Wat. etc. Divn., ASCE, Vol 109, No 2, paper 17930, pp 260 - 265, (1983).

8. Dattarri, J. 'Discussions on "Laboratory study of lift forces on circular piles.
Jl. of Wat. etc. Divn., ASCE, Vol 1,
No. WW3, pp 421 - 422, (1973).
9. Garrison, C.J. 'Comments on cross - flow principles
and Morison's equation'.
Jl. of Wat. etc. Divn., ASCE, Vol 3
No WW6, pp 1075 - 1079, (1985).
10. Ghoshal, S. 'Fluid-structure interaction'.
M.Tech. Dissertation, I.I.T.,
Bombay, (1984).
11. Laird, A.D.K. 'Flexibility in cylinder groups osc - illating in water'.
Jl. of Wat. etc. Divn., ASCE, Vol
92, No WW3, paper 4884, pp 69 - 85
(1966).
12. Laird, A.D.K.,
and Warren, R.P. 'Groups of vertical cylinders osc - illating in water'.
Jl. of the Engg. Mech. Divn., ASCE
Vol 89, No EM1, paper 3422, pp 25 -
35, (1963).
13. Malhotra, A.K. 'Ocean Science and Technology'.
National Book Trust, India,
2nd edition, (1990)
14. Naftay, A.M.,
and Wang, H.M. 'Probabilistic model of wave forces
on cylindrical pile'.
Jl. of Wat. etc. Divn., ASCE, Vol
109, No 2, paper 17976, pp 147 -
163, (1981).
15. Nortan, D.J.,
Heideman, J.C.,
and Mallard, W.W. 'Wind tunnel tests on inclined
circular cylinders'.
13th. OTC, Vol 4, OTC 4122, pp 67 -
75, (1981).
16. Priest, M.S. 'Shallow water wave action on a vert
ical cylinder'.
Jl. of Wat. etc. Divn., ASCE, Vol
88, No WW2, paper 3112, pp 1 - 9,
(1962).

17. Sarpkaya, T. 'Discussions on "Transverse forces on vertical cylinders in waves'. Jl. of Wat. etc. Divn., ASCE, Vol 102, No WW4, pp 493 - 495, (1976).
18. Sarpkaya, T. 'Discussion on "Comments on Cross-Flow principle and Morison equation'. Jl. of Wat. etc. Divn., ASCE, pp 1087, (1986).
19. Sarpkaya, T.
and
Rajabi, F. 'Hydrodynamic drag on bottom-mounted smooth and rough cylinders in periodic flow'. 11th. OTC, Vol 2, OTC 3761, pp 219 - 216, (1979).
20. Spring, B.H.,
and
Monkmeyer, P.L. 'Interaction of plane waves with vertical cylinders'. Coastal Engg., ASCE, Ch 107, pp 1828 - 1847, (1974).
21. Teng, C., and
Nath, J.H. 'Forces on horizontal cylinder towed in waves'. Jl. of Wat. etc. Divn., ASCE, Vol 111, No 6, paper 20172, pp 1022 - 1039, (1985).