Study on the Dynamics of Anisotropic Cosmological Models in a Modified Theory of Gravity

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Certificate

This is to certify that the thesis titled **Study on the Dynamics of Anisotropic Cosmological Models in a Modified Theory of Gravity** submitted by **Sankarsan Tarai, ID No. 2014PHXF0411H** for award of Ph.D. of the Institute embodies original work done by him under my supervision.

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Abstract

The present thesis is dedicated to study the modified theory of gravity, in particular the $f(R,T)$ gravitational theory. This modified theory of gravitation can be considered as a simple and important modifications or generalization of Einstein's General Relativity(GR) to explain the accelerated expansion of the universe. The main focus in this thesis is to construct cosmological models, which can be confronted with observations concerning late time cosmic speed up phenomena. This phenomena has been taken care by modifying the geometrical part of the field equations. A simple and general mathematical formalism has been presented in the thesis, so that the analysis and interpretation of the physical aspects of the cosmological model can be easily assessed. We have considered the specific form of $f(R,T)$ gravity such as $f(R,T) = \lambda R + \lambda T$ and $f(R,T) = R + 2f(T)$ in order to construct the cosmological models of the universe.

We have incorporated some degree of anisotropy in the spatial section of the model to take into account the possibility of cosmic anisotropic expansion, hence the models are presented with Bianchi type VI_h space-time. Within the formalism presented, the dynamical parameters of the universe are expressed in terms of the scale factor. We have discussed the models with power law cosmology, de Sitter space and hybrid scale factor. The deceleration parameter (DP) may evolve from a positive value in the early phase of cosmic evolution to negative value at late phase, which leads to the cosmic transition. Through this hybrid scale factor, the cosmic transit phenomenon has been simulated. Hence, the cosmological models presented in the thesis are either with constant DP or time varying DP.

Based on the modified gravity with power law and hybrid scale factor, we have reconstructed the anisotropic universe and the effect of anisotropy in the evolutionary behaviour of the parameters have been investigated. The effect of coupling parameter on the dynamical evolution of the universe is also assessed. The effect of viscous fluid, electromagnetic field on the dynamics of the universe are also studied. The viability of the cosmological models presented in the thesis are tested through the state finder pair and Om(z) diagnostic. In addition to this, the energy conditions, which are the geometrical diagnostic approach are also discussed. The models presented in the thesis allow any amount of anisotropy including the minimal one that resembles an almost isotropic universe.

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Notation

We have listed below some of the basic notations used in the thesis. We have made all attempts to keep it as standard as possible. In some places, we have used non standard notation because of the use of non metric connections. We have defined all the notaions used in the thesis at its first appearance.

g^{ij} : Lorentzian Metric

g : Determinant of g^{ij}

Γ^{λ}_{ij} : General Affine Connection (Christoffel Symbols)

$\{ {\lambda \atop ij} \}$: Levi-Civita connection

∇_i: Co-variant derivative w.r.t. Levi- Civita connection

(ij) : Symmetrization over the indices i and j

$[ij]$: Anti-symmetrization over the indices i and j

$R^{\lambda}_{\sigma ij}$: Riemann tensor

R_{ij} : Ricci tensor

R : Ricci scalar

S_M : Matter action

T_{ij} : Stress-energy tensor

Δ^{λ}_{ij} : Hyper-momentum

CHAPTER 1

Introduction

from the matter Lagrangian $\mathcal{L}_M$. Contracting (1.43), it gives relation between Ricci scalar R and T the trace of the energy momentum tensor as follows

$$f_R(R,T)R + 3\Box f_R(R,T) - 2f(R,T) = 8\pi T - f_T(R,T)T - f_T(R,T)\Theta. \tag{1.44}$$

The field equations are depend through Θ_{ab} due to the physical nature of the matter field. As it is depending on the matter field, we get several theoretical models corresponding to the different matter contribution for $f(R,T)$ gravity [76] models by specifying functional form of $f(R,T)$ as

$$f(R,T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R)f_3(T) \end{cases} \tag{1.45}$$

In this model, they have considered generalized gravity model($f(R)$ gravity model with an arbitrary coupling between matter and adding an extra term T, trace of energy momentum tensor coming from the matter Lagrangian $\mathcal{L}_{\mathcal{M}}$). They observed that the new matter and time dependent terms in gravitational field equations play the role of an effective cosmological constant in GR. They have seen that the equation of motion corresponding to the model show the presence of an extra force acting on test particles, and the motion is generally non geodesic and also obtained by using perihelion precession of mercury, an upper limit on the magnitude of the extra acceleration in the solar system.

Therefore, they predicted that $f(R,T)$ gravity model could lead to some major differences, as compared to the predictions of standard GR, or other generalized gravity models, in several problems of current interest such as cosmology, gravitational collapse or the generation of gravitational waves. It also, provide some specific signatures and effects, which could distinguish and discriminate between the various gravitational models.

1.5 Cosmological constant

The first physicist to consider a universe which exhibits an accelerated expansion was probably Willem de Sitter [113]. A de Sitter space is maximally symmetric, simply connected, Lorentizian manifold with constant positive curvature. It may be regarded as the Lorentizian analogue of n-sphere in n-dimensions. However, the de Sitter space- time is not a solution of the Einstein field equations, unless one adds a cosmological constant Λ to it, i.e. left side of the Einstein field equations. Though it is technically possible, this term was not included initially. The reason for not including this is that in order to arrive at the gravitational field equations, the left hand side of Einstein field equations has to be a second rank tensor constructed from the Ricci tensor and the metric, which divergence free. In fact, Einstein was the first to introduce the cosmological constant with an idea that it would allow him to derive a solution of the field equations describing a static universe (Einstein [114]). However, The idea of a static universe was immediately abandoned, when Hubble discovered that the universe is expanding and Einstein appears to have changed his mind about the cosmological constant. In any case, once the cosmological term is included in the Einstein equations, de Sitter space becomes a solution. Actually, the de Sitter metric can be brought into the form of the FLRW metric with the scale factor and the Hubble parameter given by

$$\mathcal{R}(t) = e^{Ht}, \tag{1.46}$$

$$H^2 = \frac{8\pi G}{3}\Lambda. \tag{1.47}$$

This is sometimes referred to as the de Sitter universe, which is expanding exponentially. The de Sitter solution is a vacuum solution. However, if we allow the cosmological term to be present in the field equations, the Friedmann equations will be modified as

$$\left(\frac{\dot{\mathcal{R}}}{\mathcal{R}}\right)^2 = \frac{8\pi G\rho + \Lambda}{3} - \frac{k^2}{\mathcal{R}^2}, \tag{1.48}$$

$$\frac{\ddot{\mathcal{R}}}{\mathcal{R}} = \frac{\Lambda}{3} - \frac{4\pi G}{3}(\rho + 3p). \tag{1.49}$$

From (1.49), it can be inferred that the universe has now entered a phase of accelerated expansion over the matter term on the right hand side. This is bound to happen since the value of the cosmological constant stays unchanged during the evolution, whereas the matter density decreases $\mathcal{R}^3$. In other words, the universe is bound to approach a de Sitter space asymptotically in time.

It has already been mentioned that there is absolutely no reason to discard the presence of a cosmological constant in the field equations from a gravitational and mathematical perspective. Nonetheless, it is also reasonable to assume that there should be a theoretical motivation for including it because there are numerous modifications that could be made to the left hand side of the gravitational field equation. It may still lead to a consistent theory from a mathematical perspective and we are not aware of any other theory that includes more than one fundamental constant. On the other hand, it is easy to see that the cosmological term can be moved to the right hand side of the field equations with the opposite sign and be regarded as some sort of matter term. It can then be put into the form of a stress-energy tensor $T_{ab} = diag(\Lambda, -\Lambda, -\Lambda, \Lambda)$. It can be noted that, the value of ω_{de} inferred from observations $\omega_{de} = -1.06^{+0.13}_{-0.08}$, which explains the success of the ΛCDM model.

Once the cosmological constant is considered to be a matter term, a natural explanation for it seems to arise: the cosmological constant represent the vacuum energy associated that empty space has a non-zero energy density. Actually, local Lorentz invariance implies that the expectation value of the stress energy tensor in vacuum is

$$T_{ab} = -\rho g_{ab}, \tag{1.50}$$

and ρ is generically non zero. In spite of any potential problems that it may have, it is still a remarkable fit to observational data while at the same time being elegantly simple.

1.6 Bianchi cosmologies

It was Friedmann [116], who has first investigated the most general non-static, homogeneous and isotropic space time described by Robertson-Walker metric

$$ds^2 = dt^2 - \mathcal{R}^2(t)\left(\frac{dr^2}{1-kr^2} + r^2d\theta^2 + sin^2\theta d\phi^2\right). \tag{1.51}$$

where, $\mathcal{R}(t)$ is the scale factor, $k = +1, 0, -1$ according to the universe is closed, flat or open respectively, with a suitable choice of r. Patridge and Wilkinson [117], Ehlers et al. [118] claimed that the present day universe is both spatially homogeneous and isotropic which can be well describe by FRW model. However there is a small magnetic field over cosmic distant scales and also there is evidence of a small amount of anisotropy (Sofue and Reich [119], Boughn et al. [120]). This recommended a departure from FRW metric at the early stage of evolution. Hence there is a need to search for a space time which can deal with the anisotropic universe. From the theoretical point of view, anisotropic universes have a greater generality than the isotropic models. The spatially homogeneous and anisotropic models known as the Bianchi models give a medium way between FRW and completely inhomogeneous and anisotropic universe. Bianchi type cosmology plays a major role in constructing cosmological models in modern cosmology. We have presented a brief discussion on Bianchi space times.

Luigi Bianchi [121] introduced the classifications of Bianchi universes. These are spatially homogeneous space times with three parameter isometric group acting on spatial slices. These can be described dynamically by the dynamical system with the difference that the state space variables S are independent of the spatial coordinates. Ellis and MacCullum [122] and Krasinski et al. [123] have classified the Bianchi cosmologies in two classes: class A and class B and then further divided into subgroups using the eigenvalues of the matrix. Then Taub [124] has introduced explicitly the nine types of Bianchi models, which are necessarily spatially homogeneous.

Collins and Hawking [125] suggested that out of these nine Bianchi types, Bianchi types I, V, VII_0, VII_h can tend towards isotropy at an arbitrarily large times. Therefore, it permits the formation of galaxies. Bianchi types I and VII_0 represent the generalized flat FRW models whereas Bianchi types V and VII_h represent the generalized open FRW model. The most general non-flat models are represented by Bianchi type II, VI_h, VII and IX space times. The metric forms of all Bianchi types are given below: Bianchi type-I metric:

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + C^2dz^2. \tag{1.52}$$

Locally Rotationally Symmetric Bianchi type-I metric:

$$ds^2 = -dt^2 + A^2dx^2 + B^2(dy^2 + dz^2). \tag{1.53}$$

Bianchi type-II metric:

$$ds^2 = -dt^2 + A^2(dx - zdy)^2 + B^2dy^2 + C^2dz^2. \tag{1.54}$$

Bianchi type-III metric:

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{-2\alpha x}dy^2 + C^2dz^2. \tag{1.55}$$

where α is a constant.
Bianchi type-IV metric:

$$ds^2 = -dt^2 + e^{-2z}[Adx^2 + (Az^2B)dy^2 + 2Azdxdy] + C^2dz^2. \tag{1.56}$$

Bianchi type-V metric:

$$ds^2 = -dt^2 + A^2dx^2 + e^{2\alpha x}(B^2dy^2 + C^2dz^2). \tag{1.57}$$

Bianchi type-VI_0 metric:

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{-2\alpha x}dy^2 + C^2e^{2\alpha x}dz^2. \tag{1.58}$$

Bianchi type-VII_0 metric:

$$ds^2 = -dt^2 + (Acos^2z + Bsin^2Z)dx^2 + (Asin^2z + Bcos^2z)dy^2 + (A+B)sin2zdxdy + Cdz^2. \quad (1.59)$$

Bianchi type-VIII metric:

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + (A^2sinh^2y + B^2cosh^2y)dz^2 - 2B^2coshydxdz. \quad (1.60)$$

Bianchi type-IX metric:

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + (B^2sin^2y + A^2cos^2y)dz^2 - 2A^2cosydxdz. \quad (1.61)$$

For the homogeneous Bianchi space time in all cases above A, B, C are the directional scale factors and functions of cosmic time t alone.

Bianchi type models or space-times are characterized by the kinematic quantities of the similar spatial hypersurface: bulk expansion rate H, trace free shear tensor $\sigma_{\alpha\beta}$ an auxiliary three-vector Ω_α that measure the rotation of the frame with respect to fermi-propagated one and a symmetric three-tensor $\eta_{\alpha\beta}$ which determine the internal geometry on the spatial hypersurface. This has been represented in Table 1.1:

Table 1.1: Bianchi model classification

Bianchi Type	**Physical Scales**				**Nature of the Model**
	h	n_1	n_2	n_3	
I	0	0	0	0	Abelian and Unimodular
II	0	1	0	0	Nilpotent and Unimodular
III	1	0	1	-1	Solvable and Not unimodular
IV	1	0	0	1	Solvable and Not unimodular
V	1	0	0	0	Solvable and Not unimodular
VI_0	0	1	-1	0	Solvable and Not unimodular
VI_h	h	0	1	-1	Solvable and Not unimodular
VII_0	0	1	1	0	Solvable and Not unimodular
VII_h	h	0	1	1	Solvable and Not unimodular
VIII	0	1	1	-1	Semisimple and Unimodular
IX	0	1	1	1	Semisimple and Unimodular

1.7 Energy conditions

Based on Raychaudhuri equation (Hawking and Ellis [126], Tahim et al. [127] the energy conditions are essentially describe the behaviour of a congruence of time like, space like or light like curves. Here, we have presented the time like and space like curves for which the Raychaudhuri equation can be written respectively as

$$R_{ij}U^iU^j + \frac{\theta^2}{3} + \sigma_{ij}\sigma^{ij} - \omega_{ij}\omega^{ij} + \frac{d\theta}{d\tau} = 0, \tag{1.62}$$

$$R_{ij}k^ik^j + \frac{\theta^2}{2} + \sigma_{ij}\sigma^{ij} - \omega_{ij}\omega^{ij} + \frac{d\theta}{d\lambda} = 0. \tag{1.63}$$

Where U^i and k^i are respectively time like and light like vectors tangent to the curves. θ is the scalar expansion which describes the expansion of volume. The positive parameters τ and λ are used to describe the curved of the congruence and the shear tensor σ_{ij} measures the distortion of the volume. ω_{ij} is the vorticity tensors that measures the rotation of the curves. The quadratic term in Raychaudhuri equation may be disregarded as the situation is of small distortions of the volume, without rotation. Then, the scalar expansion can be expressed as the function of the Ricci tensor as

$$\theta = -\tau R_{ij}U^iU^j = -\lambda R_{ij}k^ik^j. \tag{1.64}$$

The condition for attractive gravity is $\theta < 0$ imposing $R_{ij}U^iU^j > 0$ and $R_{ij}k^ik^j > 0$. These two conditions are called strong and null energy conditions respectively. The Null, Weak, Strong, Dominant energy conditions for a perfect fluid distribution can be respectively represented as

$$\begin{aligned} \textbf{NEC} &: \rho + p \geq 0, \\ \textbf{WEC} &: \rho \geq 0, \\ \textbf{SEC} &: \rho + 3p \geq 0, \\ \textbf{DEC} &: \rho - p \geq 0. \end{aligned}$$

1.8 Bianchi VI_h space time

In this thesis, we have considered Bianchi VI_h space-time and study the physical as well as dynamical behaviour of the universe using different matter fields. The Bianchi type-VI_h metric space-time can be represented as

$$ds^2 = dt^2 - A^2dx^2 - B^2e^{2x}dy^2 - C^2e^{2hx}dz^2, \tag{1.65}$$

where A,B,C are metric potentials with respect to time and $h = -1, 0, +1$ is a constant. To construct the Einstein field equations, we need to calculate the following useful tools as below,

1.8.1 Christoffel symbols

The Christoffel symbols can be calculated as

$$\Gamma^l_{jk} = g^{lm}\Gamma_{m,jk} = \frac{1}{2}g^{lm}\left(\frac{\partial g_{jm}}{\partial x^k} + \frac{\partial g_{km}}{\partial x^j} - \frac{\partial g_{jk}}{\partial x^m}\right)$$

The non-vanishing Christoffel symbol for Bianchi type VI_h can be calculated as

$$\Gamma^1_{14} = \frac{1}{2}g^{11}\left(\frac{\partial g_{11}}{\partial x^4} + \frac{\partial g_{41}}{\partial x^1} - \frac{\partial g_{14}}{\partial x^1}\right) = \frac{\dot{A}}{A}$$
$$\Gamma^1_{22} = \frac{1}{2}g^{11}\left(\frac{\partial g_{21}}{\partial x^2} + \frac{\partial g_{21}}{\partial x^2} - \frac{\partial g_{22}}{\partial x^1}\right) = -\frac{B^2}{A^2}e^{2x}$$
$$\Gamma^1_{33} = \frac{1}{2}g^{11}\left(\frac{\partial g_{31}}{\partial x^3} + \frac{\partial g_{31}}{\partial x^3} - \frac{\partial g_{33}}{\partial x^1}\right) = -\frac{hC^2}{A^2}e^{2hx}$$
$$\Gamma^2_{12} = \frac{1}{2}g^{22}\left(\frac{\partial g_{12}}{\partial x^2} + \frac{\partial g_{22}}{\partial x^1} - \frac{\partial g_{12}}{\partial x^2}\right) = 1$$
$$\Gamma^2_{24} = \frac{1}{2}g^{22}\left(\frac{\partial g_{22}}{\partial x^4} + \frac{\partial g_{42}}{\partial x^2} - \frac{\partial g_{24}}{\partial x^2}\right) = \frac{\dot{B}}{B}$$
$$\Gamma^3_{13} = \frac{1}{2}g^{33}\left(\frac{\partial g_{13}}{\partial x^3} + \frac{\partial g_{33}}{\partial x^1} - \frac{\partial g_{13}}{\partial x^3}\right) = h$$
$$\Gamma^3_{34} = \frac{1}{2}g^{33}\left(\frac{\partial g_{33}}{\partial x^4} + \frac{\partial g_{34}}{\partial x^3} - \frac{\partial g_{34}}{\partial x^3}\right) = \frac{\dot{C}}{C}$$
$$\Gamma^4_{11} = \frac{1}{2}g^{44}\left(\frac{\partial g_{14}}{\partial x^1} + \frac{\partial g_{14}}{\partial x^1} - \frac{\partial g_{11}}{\partial x^4}\right) = A\dot{A}$$
$$\Gamma^4_{22} = \frac{1}{2}g^{44}\left(\frac{\partial g_{24}}{\partial x^2} + \frac{\partial g_{24}}{\partial x^2} - \frac{\partial g_{22}}{\partial x^4}\right) = B\dot{B}e^{2x}$$
$$\Gamma^4_{33} = \frac{1}{2}g^{44}\left(\frac{\partial g_{34}}{\partial x^3} + \frac{\partial g_{34}}{\partial x^3} - \frac{\partial g_{33}}{\partial x^4}\right) = C\dot{C}e^{2hx}$$

An overdot over a field variable represents derivative with respect to t.

1.8.2 Ricci tensor and Ricci scalar

The Ricci tensors can be calculated as

$$R_{ij} = R^a_{ij}, a = \frac{\partial}{\partial x^j}\Gamma^b_{ib} - \frac{\partial}{\partial x^b}\Gamma^b_{ij} + \Gamma^a_{ib}\Gamma^b_{ja} - \Gamma^a_{ij}\Gamma^b_{ab}$$

The non-vanishing Ricci tensors are

$$\begin{aligned}
R_{11} &= \frac{\partial}{\partial x^1}\Gamma^b_{1b} - \frac{\partial}{\partial x^b}\Gamma^b_{11} + \Gamma^a_{1b}\Gamma^b_{1a} - \Gamma^a_{11}\Gamma^b_{ab} \\
&= -A\ddot{A} - \frac{A\dot{A}\dot{B}}{B} - \frac{A\dot{A}\dot{C}}{C} + h^2 + 1 \\
R_{14} &= \frac{\partial}{\partial x^4}\Gamma^b_{1b} - \frac{\partial}{\partial x^b}\Gamma^b_{14} + \Gamma^a_{1b}\Gamma^b_{4a} - \Gamma^a_{14}\Gamma^b_{ab} \\
&= \frac{\dot{B}}{B} + \frac{h\dot{C}}{C} \\
R_{22} &= \frac{\partial}{\partial x^2}\Gamma^b_{2b} - \frac{\partial}{\partial x^b}\Gamma^b_{22} + \Gamma^a_{2b}\Gamma^b_{2a} - \Gamma^a_{22}\Gamma^b_{ab} \\
&= -B\ddot{B}e^{2x} - \frac{B\dot{A}\dot{B}}{A}e^{2x} - \frac{B\dot{B}\dot{C}}{C}e^{2x} + he^{2x}\frac{B^2}{A^2} + e^{2x}\frac{B^2}{A^2} \\
R_{33} &= \frac{\partial}{\partial x^3}\Gamma^b_{3b} - \frac{\partial}{\partial x^b}\Gamma^b_{33} + \Gamma^a_{3b}\Gamma^b_{3a} - \Gamma^a_{33}\Gamma^b_{ab} \\
&= -C\ddot{C}e^{2hx} + \frac{h^2C^2}{A^2}e^{2hx} + \frac{hC^2}{A^2}e^{2hx} - \frac{C\dot{A}\dot{C}}{A}e^{2hx} - \frac{C\dot{B}\dot{C}}{B}e^{2hx} \\
R_{44} &= \frac{\partial}{\partial x^4}\Gamma^b_{4b} - \frac{\partial}{\partial x^b}\Gamma^b_{44} + \Gamma^a_{4b}\Gamma^b_{4a} - \Gamma^a_{44}\Gamma^b_{ab} \\
&= \frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C}
\end{aligned}$$

The Ricci scalar

$$R = \sum_{i,j=1}^{4} R_{ij}g^{ij} = R_{11}g^{11} + R_{22}g^{22} + R_{33}g^{33} + R_{44}g^{44}$$

can be calculated as

$$2\left(\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C}\right) + 2\left(\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC}\right) - \frac{2}{A^2}(1 + h + h^2)$$

1.8.3 Einstein tensor

The non-vanishing Einstein tensors for Bianchi VI_h can be obtained as

$$
\begin{aligned}
G_{ij} &= R_{ij} - \frac{1}{2}Rg^{ij} \\
G_{11} &= R_{11} - \frac{1}{2}Rg_{11} = A^2\left(\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{h}{A^2}\right) \\
G_{14} &= R_{14} - \frac{1}{2}Rg_{14} = \frac{\dot{B}}{B} + \frac{h\dot{C}}{C} - \frac{\dot{A}}{A}(1+h) \\
G_{22} &= R_{22} - \frac{1}{2}Rg_{22} = B^2\left(\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{h^2}{A^2}\right)e^{2x} \\
G_{33} &= R_{33} - \frac{1}{2}Rg_{33} = C^2\left(\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2}\right)e^{2hx} \\
G_{44} &= R_{44} - \frac{1}{2}Rg_{44} = -\frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{A}\dot{C}}{AC} + \frac{1+h+h^2}{A^2}
\end{aligned}
$$

1.9 Problems investigated

In this section, we have described the problem investigated and results obtained in the thesis. The main objective of the present work is to study the dynamical behaviour of anisotropic cosmological models of the universe using different matter fluids like perfect fluid, viscous fluid, cosmic string and magnetic field in Bianchi VI_h space time with different functional form of $f(R,T)$ theory of gravity. This work has a wide scope for the comparative study of anisotropic cosmological models in modified theories of gravity. We have presented our work into eight chapters including the Introduction as chapter 1.

In chapter 2, we have studied the dynamics of anisotropic universe in $f(R,T)$ gravity with a rescaled functional $f(R,T) = f(R) + f(T)$, where $f(R) = \lambda R, f(T) = \lambda T$, λ is a constant. These linear functions $f(R)$ and $f(T)$ rescale the modified gravity and generate the concept of time varying effective cosmological constants. In the first phase, we have employed the power law scale factor and in the second phase, the exponential scale factor has been used to construct the cosmological models. Here, the matter is considered to be of perfect fluid. For three cases of $h = -1, 0, +1$, cosmological models are constructed and their physical properties and behaviour of the models are presented.

In chapter 3, we have investigated the dynamical behaviour of the anisotropic universe with the functional $f(R,T) = \lambda R + \lambda T$. Three cosmological models are constructed using the power law expansion in Bianchi VI_h space time for three different values of $h = -1, 0, +1$. Here, the matter field has been considered as bulk viscous fluid. We have presented cosmological models for $h = -1, 0$ with bulk viscous fluid by incorporating the power law expansion scale factors. Also, we have shown the difficulty in obtaining a viable cosmological model for $h = +1$.

In chapter 4, we have reported the dynamical behaviour of Bianchi VI_h universe in the presence of one dimensional cosmic strings and quark matter. We have chosen the functional $f(R,T)$ as $f(R,T) = R + 2\Lambda_0 + 2\beta T$. We have derived a more general expression and investigated dynamics of universe concerning late time cosmic acceleration.

In chapter 5, we have considered the functional as a linear form of $f(R)$ and $f(T)$ respectively as $f(R) = \lambda R$ and $f(T) = \lambda T$. We have investigated the dynamical behaviour of the universe by incorporating additional anisotropy in the energy momentum tensor. The energy conditions of the model are also derived and analysed. Also, the effect of anisotropy on the dynamics of the universe is studied.

In chapter 6, we have investigated the $f(R,T)$ gravity in Bianchi VI_h space time filled with magnetized anisotropy matter content. The extended or $f(R,T)$ gravity induced by $f(R) = R$ and $f(T) = 2\Lambda_0 + 2\beta T$ are likely to experience the late time cosmic acceleration in a power law function. The effective cosmological constant and EoS parameters are derived and analysed. We have observed that due to the non-vanishing behavior of the scaling constant Λ and coupling constant β, the field equations of $f(R,T)$ gravity cannot reduce to Einstein field equations. The value of the anisotropy parameter k and the coupling constant β are constrained in such a way that, the behavior of the physical parameters aligned with the observational results. The presence of magnetic field in the field equations shows a substantial effect on the dynamical behavior initially but at late times the effect is minimal. The energy conditions are also derived and analyzed.

In chapter 7, the general formalism to investigate Bianchi VI_h universe has been derived in an extended theory of gravity. We have incorporated a one dimensional cosmic string with string density aligned along x-axis in the energy momentum tensor. We have considered the functional as $f(R,T) = R + 2\Lambda_0 + 2\beta T$ to derive the cosmological model. The scalar field reconstruction and some diagnostic approaches are incorporated to test the validity of the model. We have employed a hybrid scale factor that simulates a transition from a decelerated universe in recent past to an accelerated one to examine the effect of anisotropy in the cosmic evolution.

In chapter 8, the general formalism developed in chapter 7, has been investigated in hybrid scale factor. Cosmological models using hybrid scale factor has been developed by using the linear form of the functional $f(R,T) = \Lambda(R+T)$. Using the formalism developed, we have derived the general expressions of the dynamical features of an anisotropic universe using the hybrid scale factor.

1.10 Summary and conclusion

In this chapter, we have presented the basic introduction of Einstein's GR and the literature survey of alternative theories of gravity such as Brans Dicke theory, Scale covariant theory, Scale invariant theory, Barber's second self creation theory, Saez Ballester theory. Then we have presented a brief analysis on modified theories of gravity such as $f(R)$ gravity, $f(G)$ gravity, $f(T)$ gravity and $f(R,T)$ gravity. A brief discussions on cosmological constant, Bianchi cosmologies and energy conditions are also presented.The problem investigated and the results obtained are also presented. Though Einstein's GR has been successful in constructing cosmological models, but in recent years the cosmological models with modified or extended gravity gained momentum. We hope that the cosmological models obtained and presented in the thesis and its analysis definitely put some light in the context of the uncertainty prevailing in the studies of the late time cosmic acceleration.

CHAPTER 2

Dynamical Aspects of Bianchi VI_h Cosmological Models

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2.1 Introduction

Along the line of interest of incorporating some matter components in the action geometry, $f(R,T)$ theory has been proposed by Harko et al. [76] which, of late, has been an interesting framework to investigate accelerating models. Moreover, the reconstruction of arbitrary FRW cosmologies is possible by an appropriate choice of the functional $f(R,T)$. In recent history, many authors have investigated the astrophysical and cosmological implications of the $f(R,T)$ gravity with help of different matter. Myrzakulov [128] has studied the FLRW cosmology in $f(R,T)$ gravity taking the matter as perfect fluid. The behaviour of perfect fluid and massless scalar field is studied by Sharif and Zubair [129] for homogeneous and anisotropic Bianchi type I universe in $f(R,T)$ gravity. The stationary scenario between dark energy and dark matter is studied by Rudra [130] considering the matter field as perfect fluid in $f(R,T)$ theory of gravity. Alvarenga et al. [131] have studied the evolution of matter density perturbations in $f(R,T)$ gravity. Shabani and Farhoudi [132] have investigated the cosmological solutions of $f(R,T)$ modified theory of gravity for a perfect fluid in spatially FLRW universe through phase space analysis and the cosmological aspects of these theories are studied in [133]. Moraes et al. [134] have studied the hydrostatic equilibrium configuration of neutron stars and strange stars in $f(R,T)$ gravity. Houndjo and Piattella [135] have investigated a description of holographic dark energy model in term of reconstructed $f(R,T)$ gravity. Baffou et al. [136] have studied the evolution of the cosmological parameters, with ordinary matter and dark energy in the generalized $f(R,T)$ gravity and the cosmic late time acceleration is studied in mimetic $f(R,T)$ gravity with Lagrange multiplier [137]. Jamil et al. [138] have reconstructed some cosmological models for some specific forms of $f(R,T)$ in this modified gravity. Shamir et al. [139] obtained exact solution of anisotropic Bianchi type-I and type-V cosmological models whereas Chaubey and Shukla [140] have obtained a new class of Bianchi cosmological models using special law of variation of parameter. Using a decoupled form of $f(R,T)$ i.e. $f(R,T) = f(R) + f(T)$ for Bianchi type V universe. Ahmed and Pradhan [141] have studied the energy conditions of perfect fluid cosmological models. Mishra and Sahoo [142] have studied Bianchi type VI_h cosmological models assuming $f(R,T) = R+2f(T)$. They have obtained exact solutions to the modified field equations by assuming a specific anisotropic relation among the metric potential and the model constructed for $h = -1$ showing the accelerating behaviour. Samanta [143] has obtained exact solution of $f(R,T)$ gravitational field equations in Kantowski-Sachs space time and Shamir [144] has constructed some cosmological models in Bianchi type V space-time. In the frame work of this modified gravity, Mishra et al. [145] have presented the Einstein-Rosen non-static cosmological model with quadratic form of $f(R,T)$ gravity. Recently Yousaf et al. [146] have investigated the irregularity energy density factor in $f(R,T)$ theory responsible to disturb the stability of homogeneous universe. Also Yousaf et al. [147] have explored the evolutionary behaviour of compact objects.

In this chapter, we have presented cosmological models of the universe for the choice of the functional $f(R,T) = \lambda R + \lambda T$, where λ is a constant. We have incorporated two scale factors such as power law and exponential law to study the dynamics of the universe. This chapter is organised as: in section 2.2, we have derived the basic field equations of $f(R,T)$ gravity, in section 2.3, the dynamical parameters are derived and its behaviour are presented for $h = -1, 0, +1$ in three subsections respectively using the power law factor. In section 2.4, the physical behaviours of the model with power law has been discussed. In section 2.5, we have assumed the exponential scale factor and discussed the anisotropic behaviour of the universe for $h = -1, 0, +1$ in three subsections respectively. The derivation and behaviours of physical parameters along with energy conditions are presented in section 2.6. The conclusion of the chapter has been given at the end in section 2.7 .

2.2 Basic formalism

To study the dynamical behaviour of the model in the frame work of modified gravity, we consider spatially homogeneous and anisotropic Bianchi VI_h, (abbreviated as BVI_h) space time (1.65). To construct Einstein modified frame of field equation, we consider the action as given in eqn. (1.41). This frame is useful as long as it can be used to write the field equations of the corresponding gravity theory in a compact form. Now the stress energy tensor of the matter is defined as

$$T_{ij} = \frac{-2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L}_m)}{\delta g^{ij}}. \tag{2.1}$$

Assuming that Lagrangian density of the matter depends only on the metric tensor component g^{ij} and not on its derivatives, eqn. (2.1) reduces to

$$T_{ij} = g_{ij}\mathcal{L}_m - 2\frac{\partial \mathcal{L}_m}{\partial g^{ij}} \tag{2.2}$$

The matter source is a perfect fluid which can be expressed as

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij}. \tag{2.3}$$

where ρ and p are proper energy density and pressure of the fluid respectively and the matter Lagrangian can be taken as $\mathcal{L}_m = -p$. So the velocity in co-moving coordinates, which satisfy the condition $u_i u^j = 1$ and $u_i u^i = 0$. By varying the modified four dimensional Einsten-Hilbert action (1.41) with respect to the metric tensor component g^{ij}, the generalized Einstein field equations for the algebraic function $f(R,T) = f_1(R) + f_2(T)$ yields

$$\begin{aligned} f_R R_{ij} - \frac{1}{2} f(R) g_{ij} + (g_{ij}\Box - \nabla_i \nabla_j) f_R(R) &= 8\pi T_{ij} + f_T(T) T_{ij}, \\ &+ \left[p f_T(T) + \frac{1}{2} f(T) \right] g_{ij}. \end{aligned} \tag{2.4}$$

$f_R = \frac{\partial f(R)}{\partial R}$ and $f_T = \frac{\partial f(T)}{\partial T}$ are the partial differentiation of the respective functional with respect to their arguments. The functional $f(R,T)$ can be chosen arbitrarily to get viable cosmological models. The role of the particular choice of the functional $f(R,T) = \lambda(R+T)$, λ being a constant, can be well understood in getting such a simplified approach to a modified theory which does not contain any dark sector component but can be instrumental in providing viable accelerating models. With this choice of the functional, eqn. (2.4) reduces to

$$R_{ij} - \frac{1}{2} R g_{ij} = \left(\frac{8\pi + \lambda}{\lambda} \right) T_{ij} + \Lambda(T) g_{ij}. \tag{2.5}$$

Eqn. (2.5) can now be recast as the usual Einstein field equation, where $\Lambda(T) = p + \frac{1}{2}T$ can be identified with the cosmological constant that evolves with cosmic time. Using the Bianchi VI_h space-time (1.65), the $f(R,T)$ gravity field equations (2.5) can be explicitly written as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{h}{A^2} = \left(\frac{16\pi + 3\lambda}{2\lambda} \right) p - \frac{\rho}{2}, \tag{2.6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{h^2}{A^2} = \left(\frac{16\pi + 3\lambda}{2\lambda} \right) p - \frac{\rho}{2}, \tag{2.7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = \left(\frac{16\pi + 3\lambda}{2\lambda} \right) p - \frac{\rho}{2}, \tag{2.8}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{1 + h + h^2}{A^2} = -\left(\frac{16\pi + 3\lambda}{2\lambda} \right) \rho + \frac{p}{2}, \tag{2.9}$$

$$\frac{\dot{B}}{B} + h\frac{\dot{C}}{C} - (1+h)\frac{\dot{A}}{A} = 0. \tag{2.10}$$

An over dot on a field variable denotes differentiation with respect to time t. The constant exponent h decides the behaviour of the model and can take integral values such as -1, 0 and 1.

2.3 Dynamical features with power law

In this section, we have studied the dynamics of the background expansion of the models for assumed integral values($h=-1,0,1$) of the exponent h. The dynamics of these models can be studied if we change the metric potential into directional Hubble rates. Once the cosmic expansion behaviour is known, it becomes simpler to study the background cosmology of the diagonal BVI_h universe. Here, each value of h corresponds to a different cosmological model with different dynamical behaviour. In view of this, in the following, we discuss the dynamical features of the three possible models in the frame work of $f(R,T)$ theory. We define the directional Hubble parameters along different directions as

$$H_x=\frac{\dot{A}}{A}, \quad H_y=\frac{\dot{B}}{B}, \quad H_z=\frac{\dot{C}}{C}. \tag{2.11}$$

The mean Hubble parameter becomes $H=\frac{1}{3}(H_x+H_y+H_z)$. The field equations (2.6)- (2.10) can now be expressed as

$$\dot{H}_y+\dot{H}_z+H_y^2+H_z^2+H_yH_z-\frac{h}{A^2}=\alpha p-\frac{\rho}{2}, \tag{2.12}$$

$$\dot{H}_x+\dot{H}_z+H_x^2+H_z^2+H_xH_z-\frac{h^2}{A^2}=\alpha p-\frac{\rho}{2}, \tag{2.13}$$

$$\dot{H}_x+\dot{H}_y+H_x^2+H_y^2+H_xH_y-\frac{1}{A^2}=\alpha p-\frac{\rho}{2}, \tag{2.14}$$

$$H_xH_y+H_yH_z+H_zH_x-\frac{1+h+h^2}{A^2}=-\alpha\rho+\frac{p}{2}, \tag{2.15}$$

$$H_y+hH_z-(1+h)H_x=0, \tag{2.16}$$

$$\tag{2.17}$$

where $\alpha=\left(\frac{16\pi+3\lambda}{2\lambda}\right)$. With an algebraic manipulation of eqns. (2.12)- (2.16), the pressure p and rest energy density ρ can be obtained as

$$p = \frac{2}{(4\alpha^2-1)}\left[2\alpha\chi(H_x,H_y)-\xi(H_x,H_y,H_z,h)\right], \tag{2.18}$$

$$\rho = \frac{2}{(4\alpha^2-1)}\left[\chi(H_x,H_y)-2\alpha\xi(H_x,H_y,H_z,h)\right], \tag{2.19}$$

where,

$$\chi(H_x,H_y)=\dot{H}_x+\dot{H}_y+H_x^2+H_y^2+H_xH_y-\frac{1}{A^2}$$

$$\xi(H_x,H_y,H_z,h)=H_xH_y+H_yH_z+H_zH_x-\frac{1+h+h^2}{A^2}.$$

From eqns. (2.18) and (2.19), we obtain the equation of state parameter ($\omega = \frac{p}{\rho}$) and the effective cosmological constant ($\Lambda = \rho - p$) as

$$\omega = 2\alpha + \frac{(4\alpha^2 - 1)\xi(H_x, H_y, H_z, h)}{\chi(H_x, H_y) - 2\alpha\xi(H_x, H_y, H_z, h)}, \tag{2.20}$$

$$\Lambda = -\frac{\chi(H_x, H_y) + \xi(H_x, H_y, H_z, h)}{(2\alpha + 1)}. \tag{2.21}$$

The eqns. (2.18)-(2.21) provide the dynamical behaviour of the universe. However, the dynamics can only be assessed if the behaviour of these properties are known in terms of the directional Hubble rates for a given value of the exponent h. In other words, the formalism as described above, can help us to study a background cosmology for an assumed dynamics of the universe. In this circumstances to study the dynamical behaviour of the model, we consider the power law cosmology, where the cosmic expansion is governed through a volume scale factor of the form $v = t^m$, where m is an arbitrary positive constant usually determined from the background cosmology. Power law cosmology has been widely studied in recent times because of its functional simplicity and ability to provide a first hand information about the dynamics of the universe. For such an assumption, the radius scale factor can be $\mathcal{R} = (ABC)^{\frac{1}{3}} = t^{\frac{m}{3}}$.

2.3.1 Case I ($h = -1$)

A substitution of $h = -1$ in eqn. (2.16) yields, $H_y = H_z$, where the integration constant has been rescaled to unity. Assuming an anisotropic relationship $H_x = kH_y$, we can write the functionals $\chi(H_x, H_y)$ and $\xi(H_x, H_y, H_z, h)$ as

$$\begin{aligned}\chi(H_x, H_y) &= \dot{H}_x + \dot{H}_y + H_x^2 + H_y^2 + H_x H_y - \frac{1}{A^2} \\ &= (k+1)\dot{H}_y + (k^2 + k + 1)H_y^2 - \frac{1}{A^2},\end{aligned} \tag{2.22}$$

$$\begin{aligned}\xi(H_x, H_y, H_z, h) &= H_x H_y + H_y H_z + H_z H_x - \frac{1}{A^2} \\ &= (2k+1)H_y^2 - \frac{1}{A^2},\end{aligned} \tag{2.23}$$

where k is an arbitrary positive constant. For a power law cosmology, we have $H_x = \left(\frac{km}{k+2}\right)\frac{1}{t}$, $H_y = H_z = \left(\frac{m}{k+2}\right)\frac{1}{t}$. Consequently the directional scale factors are $A = t^{\frac{km}{k+2}}$ and $B = C = t^{\frac{m}{k+2}}$.

Thus we can have

$$\chi(t) = -m\left(\frac{k+1}{k+2}\right)\frac{1}{t^2} + (k^2+k+1)\left(\frac{m}{k+2}\right)^2\frac{1}{t^2} - \frac{1}{t^{\frac{2mk}{k+2}}}$$
$$= \left[\frac{m^2(k^2+k+1) - m(k+1)(k+2)}{(k+2)^2}\right]\frac{1}{t^2} - \frac{1}{t^{\frac{2mk}{k+2}}}, \tag{2.24}$$
$$\xi(t) = (2k+1)\left(\frac{m}{k+2}\right)^2\frac{1}{t^2} - \frac{1}{t^{\frac{2mk}{k+2}}}$$
$$= \left[\frac{(2k+1)m^2}{(k+2)^2}\right]\frac{1}{t^2} - \frac{1}{t^{\frac{2mk}{k+2}}}. \tag{2.25}$$

The dynamical behaviour of the model is decided from the behaviour of the EoS parameter ω and the effective cosmological constant Λ. However, these two parameters depend on the time varying nature of functionals $\chi(t)$ and $\xi(t)$ which in turn depend on the parameters m and k. If $mk > k+2$, the terms within the square brackets in eqns. (2.24) and (2.25) dominate at late times of cosmic evolution whereas the terms containing $t^{-\frac{2mk}{k+2}}$ dominate at early phase of cosmic evolution. Here, we intend to adopt a dimensional analysis to get some idea into the general behaviour of these functionals. Since m and k are two dimensionless constants, it appears that the dimensionality of the time dependent factors for a given functional should remain the same. In other words, we can have $m = 1 + \frac{2}{k}$ so that $\chi(t)$ and $\xi(t)$ become respectively

$$\chi(t) = \left(\frac{1-k^2}{k^2}\right)\frac{1}{t^2}, \tag{2.26}$$
$$\xi(t) = \left(\frac{1+2k-k^2}{k^2}\right)\frac{1}{t^2}. \tag{2.27}$$

The EoS parameter can give the ideal picture of our universe, which can be obtained from (2.20) as

$$\omega = 2\alpha + \frac{(4\alpha^2-1)\xi(H_x,H_y,H_z,h)}{\chi(H_x,H_y) - 2\alpha\xi(H_x,H_y,H_z,h)}$$
$$= 2\alpha + \frac{(4\alpha^2-1)\left(\frac{1+2k-k^2}{k^2}\right)\frac{1}{t^2}}{\left(\frac{1-k^2}{k^2}\right)\frac{1}{t^2} - 2\alpha\left(\frac{1+2k-k^2}{k^2}\right)\frac{1}{t^2}}$$
$$= 2\alpha + (4\alpha^2-1)\left[\frac{1+2k-k^2}{(1-2\alpha)(1-k^2)+4\alpha k}\right]. \tag{2.28}$$

We clearly get from the above expressions that, the EoS parameter ω is a constant quantity for a given value of scaling constant λ and the anisotropic parameter k. The cosmic acceleration of the universe can be characterized by the equation of state. More generally, the expansion of universe is accelerating when the EoS parameter should be negative with values less than $-\frac{1}{3}$ at late times. This behaviour will enable us to constrain the parameter k.

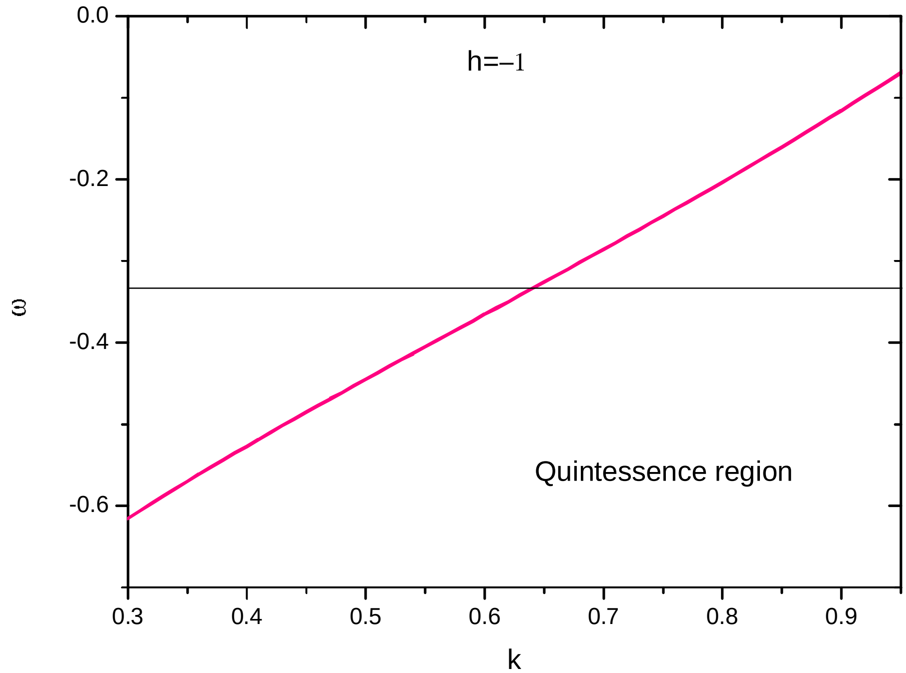

Figure 2.1: Variation of the EoS parameter with the parameter k for $h = -1$.

In Fig.2.1, we have shown the variation of ω as a function of k. Here we chose a negative value of the scaling constant λ i.e $\lambda = -\frac{8\pi}{8\pi+1}$. Accordingly the model parameter α is decided from the relation $\alpha = \left(\frac{16\pi+3\lambda}{2\lambda}\right)$. The reason behind this particular choice is to look at the present problem from the backdrop of GR where the modified gravity field equations (2.5) appears as the Einstein field equation with a time varying effective cosmological constant Λ. It can be inferred from the Fig.2.1 that, the EoS parameter ω increases almost linearly from a negative value for lower k to zero at higher k. It can be noted that, the present model will collapse at $k = 1$ and therefore, we restrict the values of k below 1. For $k \leq 0.64$, ω remains in the quintessence region. It may happen that, the results may be sensitive to the choice of the parameter α.

The effective time varying cosmological constant of the model is obtained from eqn. (2.21)

$$\begin{aligned}\Lambda(t) &= -\frac{\chi(H_x, H_y) + \xi(H_x, H_y, H_z, h)}{(2\alpha+1)} \\ &= -\frac{\left(\frac{1-k^2}{k^2}\right)\frac{1}{t^2} + \left(\frac{1+2k-k^2}{k^2}\right)\frac{1}{t^2}}{2\alpha+1} \\ &= \frac{2}{2\alpha+1}\left[\frac{k^2-k-1}{k^2}\right]\frac{1}{t^2}. \end{aligned} \tag{2.29}$$

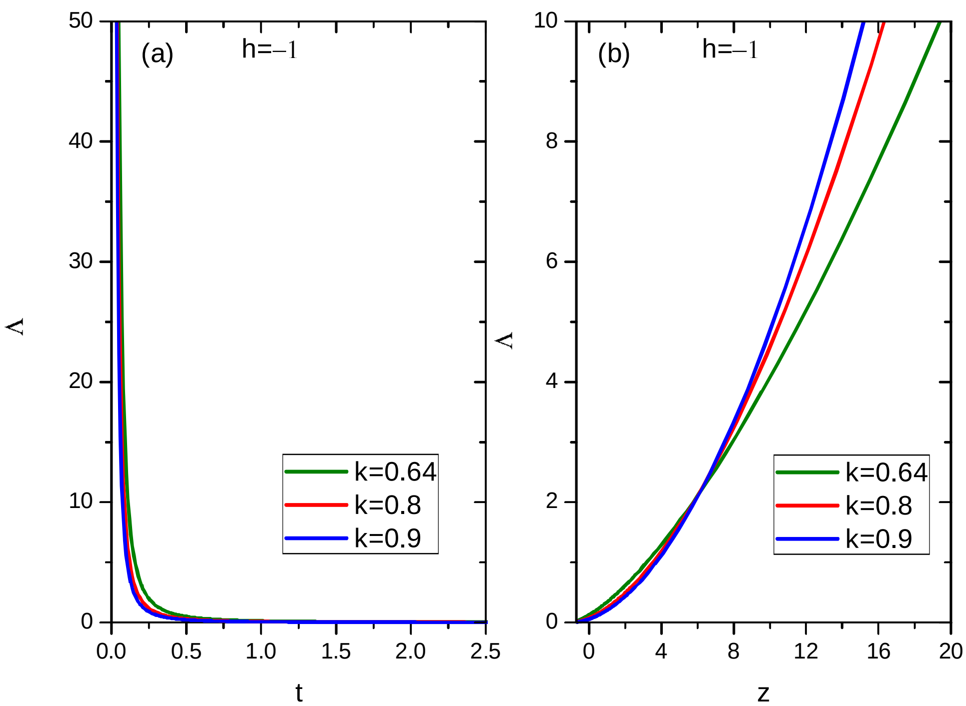

Figure 2.2: Evolution of the effective cosmological constant for three representative values of k in the model $h = -1$. (a) Effective cosmological constant is shown as a function of time. (b) Evolution of effective cosmological constant shown as a function of redshift.

Here, the effective cosmological constant is a time independent quantity and it decreases quadratically with cosmic time. In order to get viable cosmological models in conformity to recent observations, the cosmological constant should be dynamically varying from large positive values at an initial epoch to vanishingly null values at late times of cosmic evolution. Similarly, in Fig. 2.2, we have shown the dynamical variation of the effective cosmological constant for some representative values of k. As is required for an explanation to the late time cosmic acceleration, Λ varies from a large positive values in the beginning to vanishingly small values at late times. In the left panel of Fig.2.2(a), the evolution of Λ is shown as a function of cosmic time where as in the right panel its evolution is shown as function of redshift $z = \frac{1}{a} - 1$ with the scale factor at present epoch being unity in Fig.2.2(b). As it appears from the Fig. 2.2(a) and 2.2(b), the evolutionary behaviour of Λ is affected by the choice of the values of k. At remote past, curves of Λ with low values of k remain below the curves with higher values of k. However, at certain point of cosmic time corresponding to a redshift of $z = 6.2$, there occurs a reversal of the behaviour i.e curves of Λ with higher values of k remain below the curves with lower values of k. The choice of the model parameter α does not affect the general time varying trend of the effective cosmological constant.

2.3.2 Case-II ($h = 0$)

In this case, eqn.(2.16) reduces to $H_x = H_y$. An anisotropic relation $H_z = nH_y$ among the respective directional Hubble rates in the power law expansion of volume scale factor yields, $H_x = H_y = \left(\frac{m}{n+2}\right)\frac{1}{t}$ and $H_z = \left(\frac{mn}{n+2}\right)\frac{1}{t}$. The directional scale factors become $A = B = t^{\frac{m}{n+2}}$ and $C = t^{\frac{mn}{n+2}}$. Here n is a constant parameter. If $n = 1$, the model reduces to be isotropic. The functionals $\chi(t)$ and $\xi(t)$ for this model are obtained as

$$\begin{aligned}
\chi(t) &= 2\dot{H}_x + 3H_x^2 - \frac{1}{A^2} \\
&= -2\left(\frac{m}{n+2}\right)\frac{1}{t^2} + 3\left(\frac{m}{n+2}\right)^2\frac{1}{t^2} - \frac{1}{t^{\frac{2m}{n+2}}} \\
&= \left[\frac{3m^2 - 2m(n+2)}{(n+2)^2}\right]\frac{1}{t^2} - \frac{1}{t^{\frac{2m}{n+2}}}, \qquad (2.30)
\end{aligned}$$

$$\begin{aligned}
\xi(t) &= (2n+1)H_x^2 - \frac{1}{A^2} \\
&= \left(\frac{(2n+1)m^2}{(n+2)^2}\right)\frac{1}{t^2} - \frac{1}{t^{\frac{2m}{n+2}}}. \qquad (2.31)
\end{aligned}$$

The dimensional consistency of terms involved in the expressions of $\chi(t)$ and $\xi(t)$ constrains the exponent m to be $m = n + 2$. Using eqn. (2.30) and eqn. (2.31) in (2.20), the EoS parameter of the model is given as

$$\begin{aligned}\omega &= 2\alpha + \frac{(4\alpha^2-1)\xi(H_x,H_y,H_z,h)}{\chi(H_x,H_y)-2\alpha\xi(H_x,H_y,H_z,h)}\\ &= 2\alpha + \frac{(4\alpha^2-1)\left[\left(\frac{(2n+1)m^2}{(n+2)^2}\right)\frac{1}{t^2}-\frac{1}{t^{\frac{2m}{n+2}}}\right]}{\left[\frac{3m^2-2m(n+2)}{(n+2)^2}\right]\frac{1}{t^2}-\frac{1}{t^{\frac{2m}{n+2}}}-2\alpha\left[\left(\frac{(2n+1)m^2}{(n+2)^2}\right)\frac{1}{t^2}-\frac{1}{t^{\frac{2m}{n+2}}}\right]}\\ &= 2\alpha + \frac{(4\alpha^2-1)\left[(2n+1)m^2t^{\frac{2m}{n+2}}-(n+2)^2t^2\right]}{\left[3m^2-2m(n+2)-2\alpha(2n+1)m^2\right]t^{\frac{2m}{n+2}}+(2\alpha-1)(n+2)^2t^2}. \end{aligned} \tag{2.32}$$

Eqn.(2.32) will not give a clear picture of the universe, so we consider dimensional consistency terms to get a clear picture of the universe. Here, we have taken $m = n+2$. With this constraint, eqn. (2.32) becomes

$$\omega = \frac{1}{2\alpha}, \tag{2.33}$$

As in the previous model, the EoS parameter in the present model is also a constant quantity that depends on the scaling constant λ through α. In this model, we chose λ to assume a negative value so that α becomes negative. This puts ω in the negative domain. It is interesting to note here that, the EoS parameter is, in general, not affected by the choice of anisotropy in the model. In Fig. 2.3, we have plotted ω as a function of α in its negative domain. It is clear that, ω lies in the quintessence region (shaded portion in the plot) for the range $-1.5 \leq \alpha \leq -0.5$. For $\alpha > -0.5$, the EoS parameter enters into the phantom region. The behaviour of the effective cosmological constant of this model is obtained from eqn. (2.21) by using eqn. (2.30) and eqn. (2.31) as

$$\begin{aligned}\Lambda(t) &= -\frac{\chi(H_x,H_y)+\xi(H_x,H_y,H_z,h)}{(2\alpha+1)}\\ &= -\frac{\left[\frac{3m^2-2m(n+2)}{(n+2)^2}\right]\frac{1}{t^2}-\frac{1}{t^{\frac{2m}{n+2}}}+\left(\frac{(2n+1)m^2}{(n+2)^2}\right)\frac{1}{t^2}-\frac{1}{t^{\frac{2m}{n+2}}}}{(2\alpha+1)}\\ &= -\frac{2n}{(2\alpha+1)t^2}. \end{aligned} \tag{2.34}$$

It is certain from eqn. (2.34) that the effective cosmological constant can be positive for $\alpha < -0.5$. In other words, an accelerated expansion with positive cosmological constant in this model favours a quintessence phase. The time evolution of the effective cosmological constant is shown for different values of the anisotropic parameter n in Fig. 2.4. Here, the parameter α is chosen to be -1. A change in this value within the quintessence bound will result in a change in Λ without changing its general behaviour. Within the quintessence bound, higher value of

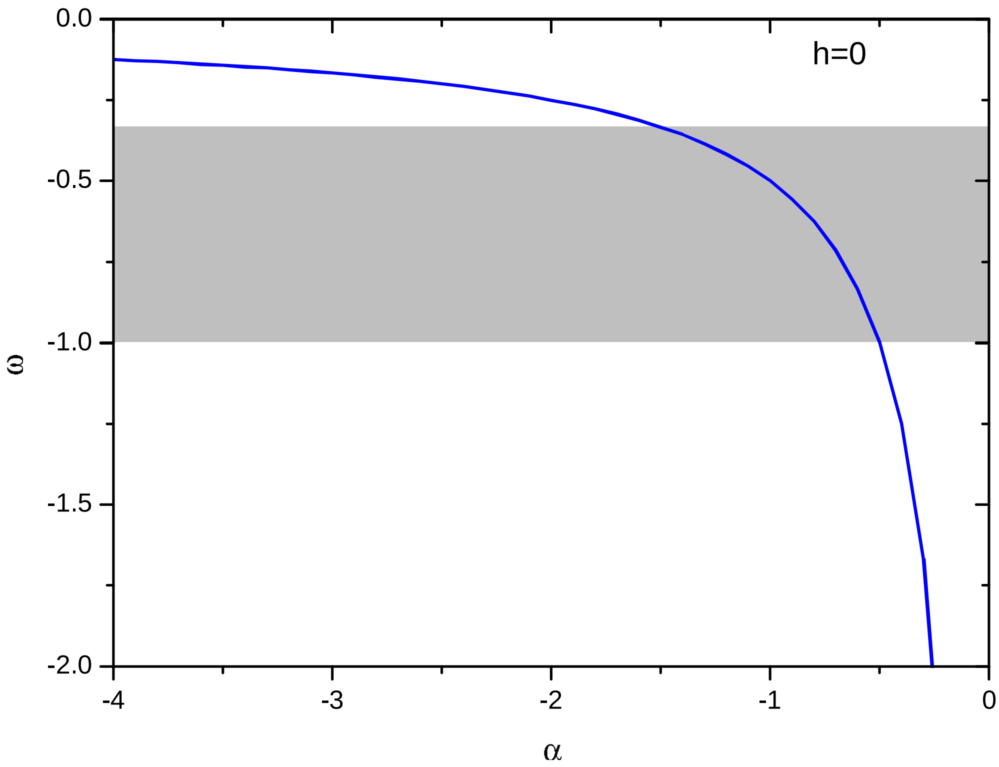

Figure 2.3: Variation of the EoS parameter with the parameter α for $h = 0$. The shaded portion denotes the quintessence region.

α will yield higher Λ. The values of n in Fig. 2.4 are chosen so as to get negative deceleration parameter. In order to satisfy this condition, n has to be constrained in the range $n > 1$. It is clear from the figure that, Λ decreases from large positive values to small positive values during the cosmic evolution and vanishes at late times. One interesting thing in the present model is that, even if the model favours a quintessence phase, the decrement in Λ is bit slower than that of the previous model with $h = -1$. As in the previous model, the behaviour of Λ is affected by the choice of the anisotropic parameter n. In order to assess the behaviour of the anisotropy dependence of Λ, we have shown its variation as a function of redshift z in the right panel of Fig. 2.4(b). In the remote past, the curves of Λ with higher values of n remain below the curves with lower values of n. However, there occurs a reversal in this behaviour at a red shift $z = 2.9$.

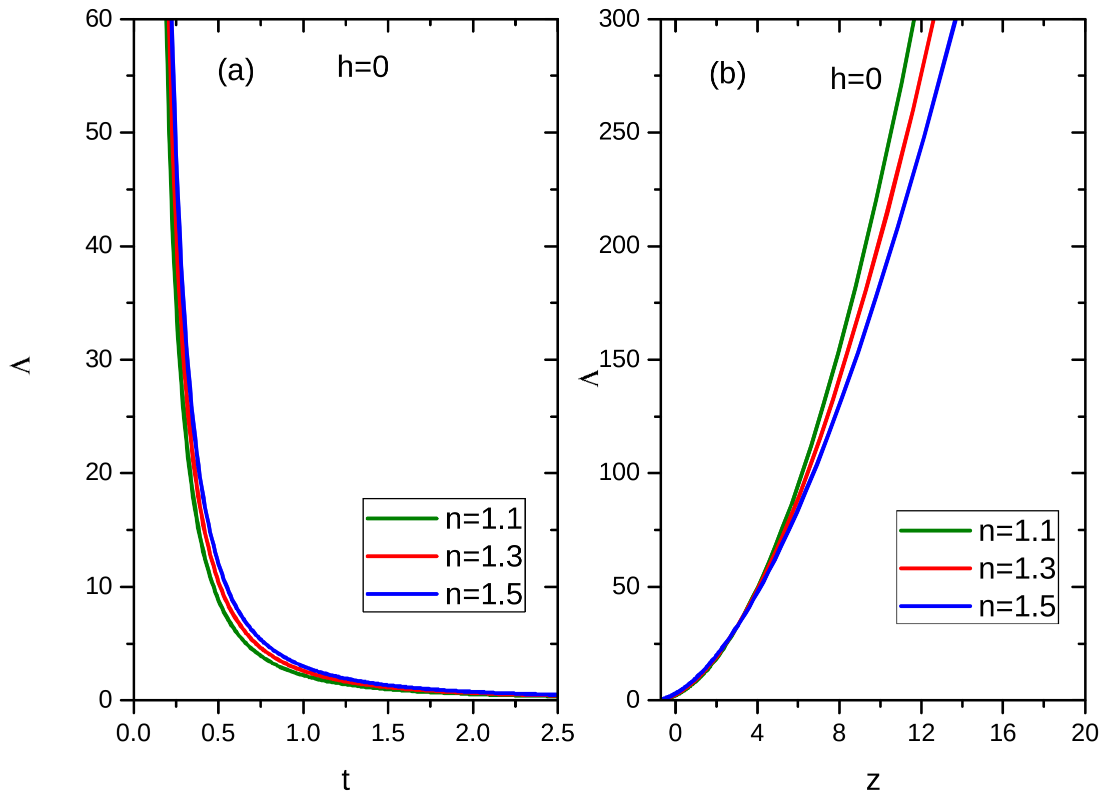

Figure 2.4: Evolution of the effective cosmological constant for three representative values of n in the model $h = 0$. (a) Effective cosmological constant is shown as a function of time. (b) Evolution of effective cosmological constant shown as a function of redshift. Here, we have considered a representative value $\alpha = -1$ so as to get positive values of the effective cosmological constant.

2.3.3 Case-III ($h = 1$)

In this model with $h = 1$, we obtain from eqn. (2.12) and (2.16) as

$$\frac{\dot{H}_x - \dot{H}_y}{H_x - H_y} + \theta = 0, \tag{2.35}$$

where θ is a constant. Eqn. (2.35) can be integrated for the power law cosmology to obtain

$$H_x = H_y + \frac{\epsilon}{t^m}. \tag{2.36}$$

Here the integration constant ϵ is related to the present day value of the directional Hubble parameters as $\epsilon = H_{x0} - H_{y0}$. For $h = 1$, eqn. (2.16) becomes $2H_x = H_y + H_z$ which implies $H_x = H$ and consequently

$$H_y = H - \frac{\epsilon}{t^m}, \qquad H_z = H + \frac{\epsilon}{t^m}. \tag{2.37}$$

One can note that, since the dimension of H is that of t^{-1}, ϵ has a dimension of t^{m-1}. The functionals $\chi(t)$ and $\xi(t)$ are obtained as

$$\chi(t) = 2\dot{H} + 3H^2 + \frac{\epsilon^2}{t^{2m}} - \frac{1}{t^{\frac{2m}{3}}}, \tag{2.38}$$

$$\xi(t) = 3H^2 - \frac{\epsilon^2}{t^{2m}} - \frac{3}{t^{\frac{2m}{3}}}. \tag{2.39}$$

From dimensional consistency, the parameter m can be constrained as $m = 3$. In order to get accelerating models, the deceleration parameter q should be negative which requires that m should be greater than 3. However, the dimensional analysis yields $q = 0$ for the present model. Also, interestingly the EoS parameter and the effective cosmological constant are obtained to be $\omega = 1$ and $\Lambda = 0$. Even though, a vanishing cosmological constant is acceptable, $\omega = 1$ may not be acceptable in the context of dark energy driven cosmic acceleration. In view of this, the BVI_1 model may not be in conformity with the present day observations.

2.4 Behaviour of physical parameters

In this section, we have presented behaviour of some physical parameters with the power law cosmology. The volume scale factor V and the average scale factor $\mathcal{R}$ can be defined respectively as

$$V = \sqrt{-g} = \sqrt{AB^2} = t^m, \tag{2.40}$$

$$\mathcal{R} = V^{\frac{1}{3}} = t^{\frac{m}{3}}. \tag{2.41}$$

It indicates that the spatial volume as well as the scale factor increases with increase in time and finally constant at infinite time. The Hubble parameter which is (i) a manageable two-dimensional phase space, (ii) reduction to the old linear redshift behaviour at low red shift, (iii) well behaved, bounded behaviour for high red shift, (iv) high accuracy in reconstructing many scalar field equations of state and the resulting distance-red shift relations,(v) good sensitivity to observational data, and (vi) simple physical interpretation can be defined as

$$H = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{c}\right) = \frac{m}{3t}. \tag{2.42}$$

The scalar expansion θ of the model is

$$\theta = 3H = \frac{m}{t}. \tag{2.43}$$

The deceleration parameter of the model is given by

$$q = -\frac{\mathcal{R}\ddot{\mathcal{R}}}{\dot{\mathcal{R}}^2} = -1 + \frac{3}{m}. \tag{2.44}$$

The deceleration parameter for power law expansion of the universe is constant. It is specified here that a positive q describes a decelerating universe where as a negative q describes an accelerating universe. For $m > 3$, the universe is accelerating and $m < 3$, the universe is decelerating. The shear scalar (σ^2) and the rate of anisotropy parameter of the model can be defined respectively as

$$\sigma^2 = \frac{1}{2}\left(\Sigma H_i - \frac{1}{3}\theta^2\right) = \frac{1}{2}\left(\frac{m^2(k^2+2)}{(k+2)^2} - \frac{m^2}{t^2}\right). \tag{2.45}$$

$$\mathcal{A} = \frac{1}{3}\Sigma\left(\frac{\Delta H_i}{H}\right)^2 = 0. \tag{2.46}$$

$\mathcal{A}$ is a measure of deviation from isotropic expansion. For an isotropic model, the rate of anisotropy, $\mathcal{A} = 0$. The state finder diagnostic pair (r, s) which provide us an idea about the geometrical nature of the model can be represented as,

$$r = \frac{\dddot{\mathcal{R}}}{\mathcal{R}H^3} = \frac{9}{m}\left(\frac{2}{m} - 1\right) + 1, \tag{2.47}$$

$$s = \frac{r-1}{3(q-\frac{1}{2})} = \frac{2}{m}. \tag{2.48}$$

These parameters are constants of cosmic time and depend only on the exponent m.

2.5 Dynamical features with exponential law

It is interesting to investigate cosmological models from the point of their compatibility with other cosmological and astrophysical data as well. It is also important to study the cosmological models of the universe filled with perfect fluid which is having EoS parametrization and investigating the compatibility of these models with the mechanical approach. We have considered the exponential expansion of volume factor in the form $V = e^{tm}$, where m is an arbitrary constant to be determined from the background cosmology. With this consideration the radius scale factor and deceleration parameter can be respectively obtained as $\mathcal{R} = (ABC)^{\frac{1}{3}} = e^{\frac{tm}{3}}$. The reason behind to carry such scale factor because, the universe transits from an decelerated expansion phase to a normal expansion phase through a contraction phase. As in the case of power law scale factor, in the following subsections, we have presented the cosmological models and its behaviour for the three values of $h = -1, 0, +1$

2.5.1 Case I ($h = -1$)

For exponential cosmology, the directional Hubble rate becomes $H_x = \frac{km}{k+2}$, $H_y = H_z = \frac{m}{k+2}$. Consequently the directional scale factors are $A = e^{\frac{tkm}{k+2}}$, $B = C = e^{\frac{tm}{k+2}}$. Then eqns. (2.22)-(2.23) becomes

$$\begin{aligned}\chi &= (k^2+k+1)\left(\frac{m}{k+2}\right)^2 - \frac{1}{e^{\frac{2ktm}{k+2}}}\\ &= \frac{m^2(k^2+k+1)}{(k+2)^2} - e^{\frac{-2ktm}{k+2}}, \end{aligned} \tag{2.49}$$

$$\begin{aligned}\xi &= (2k+1)\left(\frac{m}{k+2}\right)^2 - \frac{1}{e^{\frac{2ktm}{k+2}}}\\ &= \frac{m^2(2k+1)}{(k+2)^2} - e^{\frac{-2ktm}{k+2}}. \end{aligned} \tag{2.50}$$

The EoS parameter of the model is obtained from eqn. (2.20). But the clear picture of the EoS parameter will get after using eqn.(2.49) and eqn.(2.50) in eqn. (2.20) as

$$\begin{aligned}\omega &= 2\gamma + \frac{(4\gamma^2-1)\left(\frac{m^2(2k+1)}{(k+2)^2} - e^{\frac{-2ktm}{k+2}}\right)}{\frac{m^2(k^2+k+1)}{(k+2)^2} - e^{\frac{-2ktm}{k+2}} - 2\gamma\left(\frac{m^2(2k+1)}{(k+2)^2} - e^{\frac{-2ktm}{k+2}}\right)}\\ &= 2\gamma + \frac{(4\gamma^2-1)\left[(2k+1)m^2 - (k+2)^2 e^{-\frac{2ktm}{k+2}}\right]}{(3-4\gamma k-2\gamma)m^2 + (2\gamma-1)(k+2)^2 e^{-\frac{2ktm}{k+2}}}. \end{aligned} \tag{2.51}$$

Fig. 2.5 represents the graph between the EoS parameter and cosmic time. It is observed that, ω is asymptotically increases with the growth of cosmic time from a higher negative value. The effective cosmological constant (Λ) for $h = -1$ can be obtained from (2.21) as

$$\begin{aligned}\Lambda &= -\frac{\frac{m^2(k^2+k+1)}{(k+2)^2} - e^{\frac{-2ktm}{k+2}} + \frac{m^2(2k+1)}{(k+2)^2} - e^{\frac{-2ktm}{k+2}}}{2\gamma+1}\\ &= \frac{-(k^2+3k+2)m^2 + 2(k+2)^2 e^{\frac{-2ktm}{k+2}}}{(2\gamma+1)(k+2)^2}. \end{aligned} \tag{2.52}$$

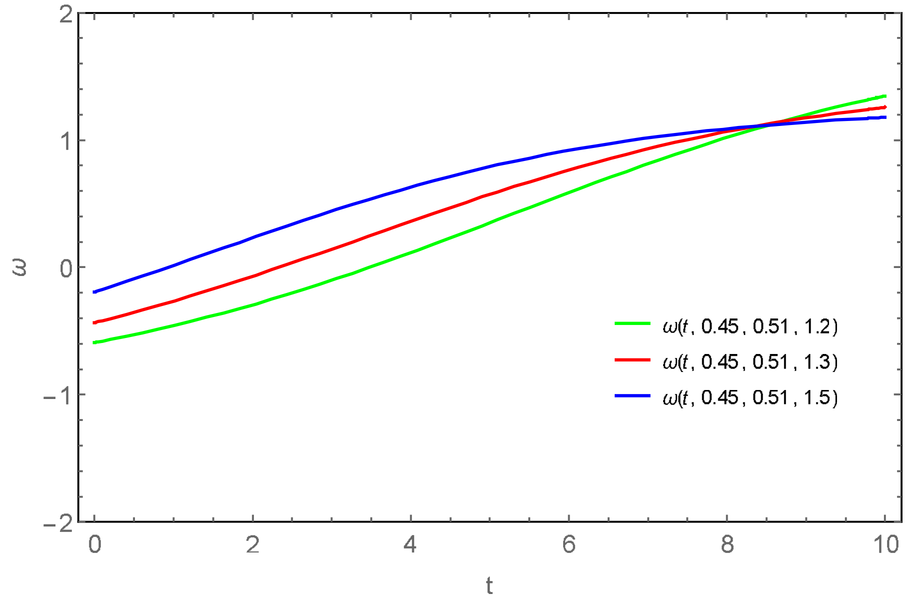

Figure 2.5: EoS parameter versus cosmic time t.

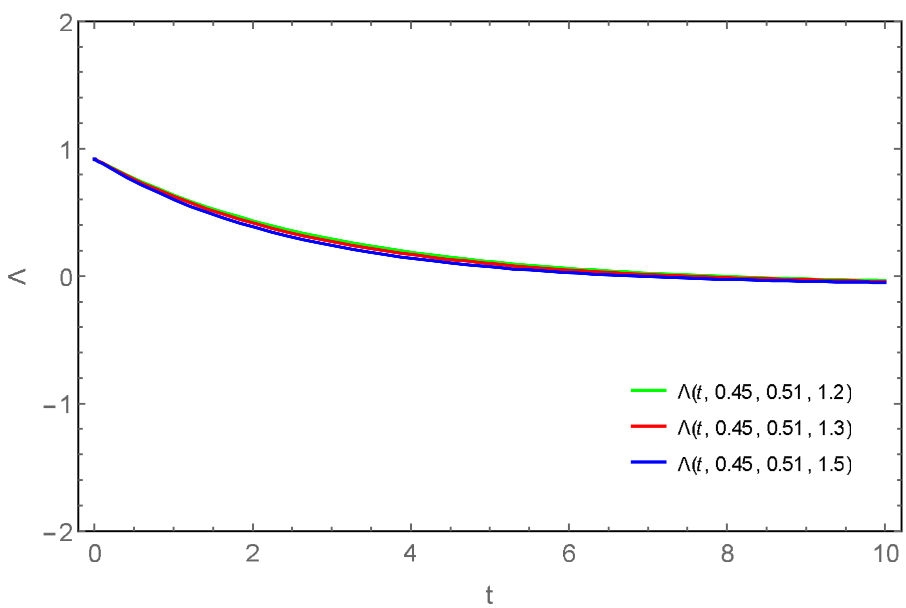

Figure 2.6: Effective cosmological constant versus cosmic time t.

In Fig. 2.6, it has been shown that Λ, the effective cosmological constant remains positive throughout the cosmic evolution with time dependent. At later epoch, the cosmological constant decreases with the growth of the cosmic time. Such behaviour is in consistent with ΛCDM model where a small positive value of cosmological constant is required to explain the

accelerated nature of the universe.

Since, the eqns. (2.12) - (2.16) are highly non-linear, an explicit relation between p and ρ could not be established. All the solutions were implicit in nature; hence it was difficult to study the behaviour of the universe. Therefore, we have studied the physical behaviour first by determining a general relationship between p and ρ with the help of ω. Then for the representative values of the parameters, the model will reduce to different physical states. When $m = 0.45$, $\gamma = 0.51$, $k = 1.362$, $t = 0.1$, the equation of state $\omega = -1/3$, which subsequently resulted $\rho + 3p = 0$. The behaviour of the universe is going to radiation era. In early stage of the big bang, most of the energy was in the form of radiation and that radiation was dominant on the expansion of the universe. Later, with cooling from the expansion the roles of mass and radiation changed and the universe entered a mass-dominated era. Recent, results are suggested that we have already entered an era dominated by dark energy, but examination of the roles of the mass and radiation are most important for understanding the early universe. Similarly, when $m = 0.45$, $\gamma = 0.51$, $k = 1.362$, $t = 7.25$, the EoS parameter becomes 1, which satisfies the stiff fluid model. However, with the same combination of values and for a very small t, the model reduces to false vacuum model.

2.5.2 Case II ($h = 0$)

For $h = 0$, the functionals χ and ξ with an exponential scale factor can be obtained as

$$\begin{aligned}\chi &= 2\dot{H}_y + 3H_y^2 - A^{-2}\\ &= 3\left(\frac{m}{k+2}\right)^2 - e^{\frac{-2tm}{k+2}}\\ &= \frac{3m^2}{(k+2)^2} - e^{\frac{-2tm}{k+2}}, \end{aligned} \tag{2.53}$$

$$\begin{aligned}\xi &= H_y^2 + 2H_yH_z - A^{-2}\\ &= \left(\frac{m}{k+2}\right)^2 + 2\left(\frac{m}{k+2}\right)\left(\frac{km}{k+2}\right) - e^{\frac{-2tm}{k+2}}\\ &= \frac{m^2(2k+1)}{(k+2)^2} - e^{\frac{-2tm}{k+2}}. \end{aligned} \tag{2.54}$$

As in the earlier case, here also the EoS parameter can be obtained from (2.20) as

$$\omega = 2\gamma + \frac{(4\gamma^2 - 1)\left[m^2(2k+1) - (k+2)^2 e^{-\frac{2tm}{k+2}}\right]}{(3 - 2\gamma - 4\gamma k)m^2 + (2\gamma - 1)(k+2)^2 e^{-\frac{2tm}{k+2}}}. \tag{2.55}$$

The graphical behaviour of the EoS parameter with the cosmic time t has been represented

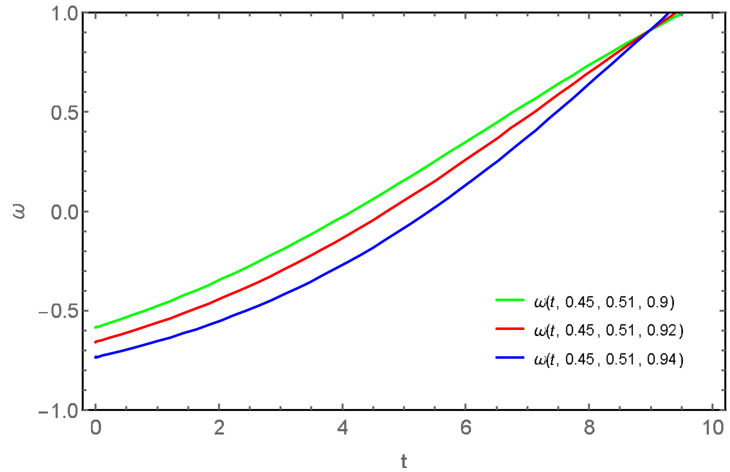

Figure 2.7: EoS parameter versus cosmic time t.

in Fig. 2.7. We have observed that with the increase in time, ω decreases and becomes flat at $\omega = -0.8$. It is worthy to note here that for different representative values of the parameter, the model gives different physical state. When $m = 0.45$, $\gamma = 0.51$, $k = 0.9$, $t = 1.9$, the EoS parameter becomes $-1/3$. This satisfies that the nature of the universe is in the radiation era. Whenever, the representatives values are, $m = 0.45$, $\gamma = 0.51$, $k = 0.9$, $t = 8.58$, the EoS parameter becomes 1, it satisfies the stiff fluid matter. In the same manner, we observed that with a very small time i.e. t the initial time, the physical state becomes a false vacuum model. We have also observed that ω is staying negative throughout the evolution for a particular value of $m = 2$ with time dependent, which is a good sign to explain the acceleration of the universe. With increase in time, ω enters into the quintessence region ($\omega < -1$). The effective cosmological constant for this model can be obtained from eqn. (2.21) as

$$\Lambda = \frac{-m^2(4+2k) + 2(k+2)^2 e^{\frac{-2tm}{k+2}}}{(k+2)^2(2\gamma+1)}. \tag{2.56}$$

We have plotted the effective cosmological constant with respect to the cosmic time in Fig. 2.8. The curve is lying entirely in the positive domain. It is clear from the graphical representation that Λ decreases from large positive values to small positive values during the cosmic evolution and vanishes subsequently.

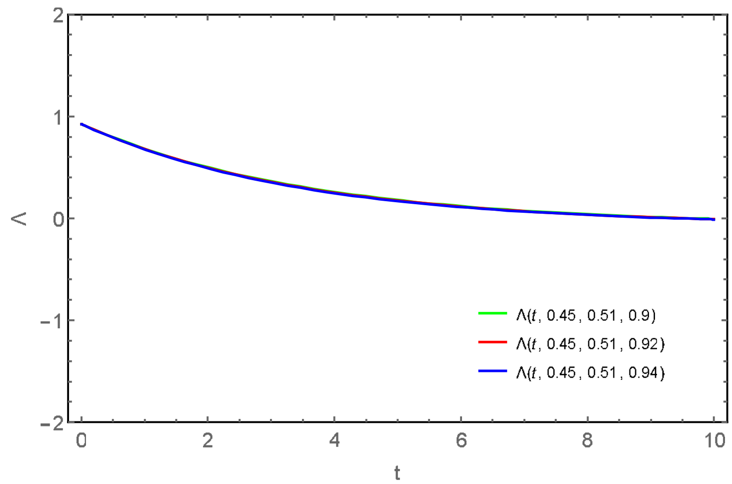

Figure 2.8: Effective cosmological constant versus cosmic time t.

2.5.3 Case III ($h = 1$)

For $h = 1$, we have obtained the following relation from eqns. (2.12)-(2.13),

$$\frac{\dot{H}_x - \dot{H}_y}{H_x - H_y} + \theta = 0. \tag{2.57}$$

Using the exponential cosmology, eqn. (2.57) gives $H_x - H_y = e^{\frac{tm}{3}+\epsilon}$. Here the integrating constant ϵ is related to the present day value of the directional Hubble parameter as $\epsilon = H_{x0} - H_{y0}$. Moreover, $H_x = H$ and consequently $H_y = H - e^{\frac{tm}{3}+\epsilon}$ and $H_z = H + e^{\frac{tm}{3}+\epsilon}$. The functionals $\chi(t)$ and $\xi(t)$ are obtained as

$$\chi(t) = 2\dot{H} + 3H^2 + e^{-6tm+2\epsilon} - e^{\frac{-2tm}{3}}. \tag{2.58}$$

$$\xi(t) = 3H^2 - e^{-6tm+2\epsilon} - 3e^{\frac{-2tm}{3}}. \tag{2.59}$$

The respective directional Hubble rates in the exponential law expansion of volume scale factors yields $H_x = m$, $H_y = m - e^{\frac{tm}{3}+\epsilon}$ and $H_z = m + e^{\frac{tm}{3}+\epsilon}$. The directional scale factors becomes $A = e^{\frac{tm}{3}}$, $B = e^{\frac{2ktm}{3(k+1)}}$ and $C = e^{\frac{2tm}{3(k+1)}}$. So, subsequently we get

$$\chi(t) = 3m^2 + e^{-6tm+2\epsilon} - e^{\frac{-2}{3}tm}. \tag{2.60}$$

$$\xi(t) = 3m^2 - e^{-6tm+2\epsilon} - 3e^{\frac{-2}{3}tm}. \tag{2.61}$$

The EoS parameter (ω) and the effective cosmological constants (Λ) calculated as

$$\omega = 2\gamma + \frac{(4\gamma^2 - 1)(3m^2 - e^{-6tm+2\epsilon} - 3e^{\frac{-2tm}{3}})}{3m^2(1-2\gamma) + (1+2\gamma)e^{-6tm+2\epsilon} - (1-6\gamma)e^{\frac{-2tm}{3}}} \tag{2.62}$$

$$\Lambda = \frac{-6m^2 + 4e^{\frac{-2tm}{3}}}{1+2\gamma}. \tag{2.63}$$

From the above expressions, it can be noted that both the EoS parameter and effective cosmological constant becomes constant at late time. The values of the parameter chosen in the previous case to show an accelerating model, in this case the EoS parameter behaves different, we have concluded that for $h = 1$, the model may not provide an accelerating universe. However, the positive effective cosmological constant will not have any role to play. Hence, a viable cosmological constant can not be obtained in this case.

2.6 Behaviour of physical parameters

We have presented here some of physical parameters of the model with the exponential scale factor. The volume scale factor of the model is given as

$$V = (ABC)^{\frac{1}{3}} = e^{tm/3}. \tag{2.64}$$

The deceleration of the model q can be given as

$$q = \frac{-\mathcal{R}\ddot{\mathcal{R}}}{\dot{\mathcal{R}}^2} = \frac{-e^{\frac{2tm}{3}}}{e^{\frac{2tm}{3}}} = -1. \tag{2.65}$$

The Hubble parameter H of the model is given as

$$H = \frac{1}{3}(H_x + 2H_y) = \frac{m}{3}. \tag{2.66}$$

The scalar expansion θ of the model is given as

$$\theta = 3H = m. \tag{2.67}$$

The shear scalar σ of the model is given as

$$\sigma^2 = \frac{1}{2}\left(\Sigma H_i - \frac{1}{3}\theta^2\right) = \frac{m^2}{3} - \frac{2km^2}{(k+2)^2}. \tag{2.68}$$

The rate of anisotropy parameter $\mathcal{A}$ is given as

$$\mathcal{A} = \frac{1}{3}\Sigma\left(\frac{\Delta H_i}{H}\right)^2 = \frac{4}{3}. \tag{2.69}$$

The state finder pair of the model is given as

$$r = \frac{\dddot{\mathcal{R}}}{\mathcal{R}H^3} = 1.$$

and

$$s = \frac{r-1}{3(q-\frac{1}{2})} = 0.$$

As expected, the deceleration parameter remains as a constant with value -1 leads to accelerating model. The scalar expansion depends on the exponent m and the rate of anisotropy found to be again a constant. The state finder diagnostic pair is well in agreement with the prescribed value.

2.7 Conclusion

The dynamics of the anisotropic universe is studied by using Bianchi type VI_h metric for a perfect fluid matter distribution in the frame work of $f(R,T)$ theory of gravity proposed by (Harko et al. [76]). In this work, we choose $f(R,T) = f(R) + f(T)$, where $f((R) = \lambda R$ and $f(T) = \lambda T$, where λ is a positive constant. We have examined three different models corresponding to three value of the metric parameter $h = -1, 0, 1$. The dynamics of the model are investigated for assumed power law expansion of the volume scale factor as well as exponential law expansion of the volume scale factor. We have adopted dimensional analysis method for both cases to study the physical parameters. In power law case, with $h = -1$ and $h = 0$, the behaviour of the cosmological constant is lying in positive axis throughout the evolution form large positive values at the beginning to small values of late times. The EoS parameter for both the models lying in negative domain throughout the evolution. With suitable choice of the model parameters both models favour quintessence phase ($-\frac{2}{3} \leq \omega \leq -\frac{1}{3}$). On the other hand, for exponential case, with same values of $h = -1, 0$, the cosmological constant is lying in positive domain.The EoS parameter of the model for both the cases are fluctuating, at beginning with small value ot time, the EoS parameter is staying in negative phase while with increase of time, it goes to positive phase at late time. However, for the third model with $h = 1$, viable cosmological model could not be attained for both cases. In general from first case, anisotropy affect the dynamics of the universe, where in second case it does not happen.

CHAPTER 3

Role of Bulk Viscous Fluid in Bianchi VI_h Universe with Modified Gravity

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3.1 Introduction

Cosmologists extensively studied the evolution of isotropic cosmological models with perfect fluid. However, it would be important to study cosmology with richer structure both geometrically and physically than the standard perfect fluid models. Bulk viscosity is such a dissipative phenomena that plays an important role in the study of accelerated expansion of the universe. Viscous fluid models have been used in an attempt to explain the observed highly isotropic matter distribution on the high entropy per baryon in the present state of the universe. The strong dissipation due to neutrino viscosity may considerably reduce the anisotropy of black body radiation. The viscosity mechanism in cosmology can explain the anomalously high entropy per baryon in the present universe. There have been considerable interests in cosmological models with bulk viscosity, since bulk viscosity leads to the accelerated expansion phase of the early universe, popularly known as the inflationary phase (Brevik and Timoshkin [148], Brevik et al. [149, 150]).

Motivated by the great success of cosmological constant as a simple and good candidate of dark energy, Harko et.al. [76] introduced a new generalized gravity model called as $f(R,T)$ theory along the line of interest of incorporating some matter components in the action geometry. Here, the Lagrangian is described by an arbitrary function of the Ricci scalar R and trace of the energy momentum tensor T. In this gravity, the cosmic acceleration may result either due to the diametrical contribution to the cosmic energy density or its dependency on matter contents. This theory can be used to examine several uses of current interest and may lead to some major differences; however of late it has been an interesting framework to investigate the accelerating models. Several authors have developed different ideas to study the nature of the universe in $f(R,T)$ gravity.

Belinski and Khalatnikov [151] studied the viscous fluid matter in Bianchi type I space-time. They have indicated that without removing the initial big bang singularity, the viscosity can affect the qualitative aspects of the solutions around the singularity. Baffou et al. [152] have studied FLRW universe using viscous generalized chaplygin gas with $f(R,T)$ gravity. Satish and Venkateswarlu [153] have obtained the Kaluza-Klein cosmological models filled with bulk viscous fluid in the framework of $f(R,T)$ gravity and Samanta et al. [154] discussed the bulk viscous fluid Kaluza Klein model with validity of the second law of thermodynamics and the generalized second law of thermodynamics in $f(R,T)$ gravity. Devnath [155] has studied bulk viscous FLRW cosmological model with isotropic fluid in $f(R,T)$ gravity. Fabris et al. [156]have developed a phenomenological model for dark energy based on a viscous dark fluid with considering the Eckart formalism. Saha [157] has obtained the Bianchi type I cosmological model with viscous fluid by considering a self consistent system of non-linear spinor and gravitational fields.

Singh and Srivastava [159] have described the effects of cosmic non-perfect fluid on the evolution of the universe in $f(R,T)$ gravity using a new Holographic dark energy model and also Srivastava and Singh [158], have studied new holographic dark energy model in modified gravity within the framework of flat FLRW model with bulk viscous matter content. Reddy et al. [160], have studied Kaluza-Klein cosmological model in the framework of $f(R,T)$ gravity by considering the matter fluid as the string and bulk viscous fluid. A spatial and anisotropic Bianchi type-III cosmological model is studied in the presence of bulk viscous fluid in $f(R,T)$ gravity in [161]. Naidu et al. [162], have investigated a spatially and anisotropic Bianchi type-V cosmological model in modified theory of gravity, where the source for energy momentum tensor is a bulk viscous fluid containing one dimensional cosmic string also Kantowski-Sachs space-time cosmological model is studied in frame work of $f(R,T)$ gravity in [163]. Singh and Kumar [164] have investigated the effect of bulk viscosity in $f(R,T)$ theory and suggested that inclusion of dissipative energy sources like bulk viscosity may be able to explain the early and late time accelerations of the universe.

As a sequel to our studies in the previous chapter on the dynamics of anisotropic universe, in this chapter, we have considered Bianchi type VI_h space time with the matter field in the form of viscous fluid.

3.2 Basic equations

By varying the modified four-dimensional Einstein-Hilbert action (1.41) with respect to the metric tensor components g^{ij}, the algebraic function $f(R,T)$ has been chosen as a sum of two independent functions $f(R,T) = f_1(R) + f_2(T)$. $f_1(R)$ depends on the curvature R whereas $f_2(T)$ is on the trace T [76]. Hence, the generalized Einstein field equations from(1.43) yields

$$f_R R_{ij} - \frac{1}{2} f(R) g_{ij} + \left(g_{ij}\Box - \nabla_i \nabla_j\right) f_R = 8\pi T_{ij} + f_T T_{ij} + \left[\bar{p} f_T + \frac{1}{2} f(T)\right] g_{ij}. \tag{3.1}$$

Here, $f_R = \frac{\partial f(R)}{\partial R}$ and $f_T = \frac{\partial f(T)}{\partial T}$. In order to frame a cosmological model, we assume the functional $f(R,T)$ in the form $f(R,T) = \lambda R + \lambda T$, subsequently the field equations (3.1), takes the form

$$R_{ij} - \frac{1}{2}Rg_{ij} = \left(1 + \frac{8\pi}{\lambda}\right)T_{ij} + \Lambda(T)g_{ij}. \tag{3.2}$$

where, $\Lambda(T) = \bar{p} + \frac{1}{2}T$ can be identified with the cosmological constant which is instead of being a pure constant evolves with cosmic time.

We are intending to study the dynamics of the anisotropic universe in the $f(R,T)$ gravity. We know that the standard FLRW universe is homogeneous and isotropic. Therefore, in order to address the small scale anisotropic nature of the universe, Bianchi space time is well considerable as it represents a globally hyperbolic spatially homogeneous, but not isotropic space time. So, we consider a Bianchi-type VI_h space-time (1.65), where the constant exponent h can be assumed values $-1, 0, 1$. The energy momentum tensor T_{ij} for the viscous fluid can be expressed as

$$T_{ij} = (\rho + \bar{p})u_i u_j - \bar{p}g_{ij}, \tag{3.3}$$

where ρ is the proper energy density and $\bar{p} = p - \zeta\theta$ is the viscous pressure and ζ is the bulk viscous coefficient. In the co-moving coordinate system, we have $u^i = (0,0,0,1)$. Also, $u^i = \delta^i_4$ which satisfies $g_{ij}u^i u^j = 1$ and $u^i x_i = 0$. With the co-moving coordinate system, the field equations (3.2) for the metric (1.65) and energy momentum tensor (3.3) can be obtained as,

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{h}{A^2} = \beta\bar{p} - \frac{\rho}{2}, \tag{3.4}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{h^2}{A^2} = \beta\bar{p} - \frac{\rho}{2}, \tag{3.5}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = \beta\bar{p} - \frac{\rho}{2}, \tag{3.6}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{1+h+h^2}{A^2} = -\beta\rho + \frac{\bar{p}}{2}, \tag{3.7}$$

$$\frac{\dot{B}}{B} + h\frac{\dot{C}}{C} - (1+h)\frac{\dot{A}}{A} = 0 \tag{3.8}$$

An over dot on the field variable A,B,C denotes the differentiation with respect to time t and $\beta = \left(\frac{3}{2} + \frac{8\pi}{\lambda}\right)$. In order to study the dynamical behaviour of the universe, we have redefined the above set of eqns. (3.4)- (3.8) in the form of Hubble rates along different direction (2.11) as

$$\dot{H}_y + \dot{H}_z + H_y^2 + H_z^2 + H_y H_z - \frac{h}{A^2} = \beta\bar{p} - \frac{\rho}{2}, \tag{3.9}$$

$$\dot{H}_x + \dot{H}_z + H_x^2 + H_z^2 + H_x H_z - \frac{h^2}{A^2} = \beta\bar{p} - \frac{\rho}{2}, \tag{3.10}$$

$$\dot{H}_x + \dot{H}_y + H_x^2 + H_y^2 + H_x H_y - \frac{1}{A^2} = \beta\bar{p} - \frac{\rho}{2}, \tag{3.11}$$

$$H_x H_y + H_y H_z + H_z H_x - \frac{1 + h + h^2}{A^2} = -\beta\rho + \frac{\bar{p}}{2}, \tag{3.12}$$

$$H_y + hH_z - (1 + h)H_x = 0, \tag{3.13}$$

The effect of both proper pressure and barotropic bulk viscous pressure can be defined as $\bar{p} = p - \zeta\theta$. From eqns. (3.9)- (3.13), a general expression based on directional Hubble parameter for the effective pressure $\bar{p}$ and rest energy density ρ can be established as,

$$\bar{p} = p - \zeta\theta = \frac{2}{1 - 4\beta^2}\left[\xi(H_x, H_y, H_z, h) - 2\beta\chi(H_x, H_y)\right], \tag{3.14}$$

$$\rho = \frac{2}{1 - 4\beta^2}\left[2\beta\xi(H_x, H_y, H_z, h) - \chi(H_x, H_y)\right], \tag{3.15}$$

where $\chi(H_x, H_y, H_z, h) = H_x H_y + H_y H_z + H_x H_z - \frac{(1+h+h^2)}{A^2}$ and $\xi(H_x, H_y) = \dot{H}_x + \dot{H}_y + H_x^2 + H_y^2 + H_x H_y - \frac{1}{A^2}$. Subsequently the effective EoS parameter $\omega_{eff} = \frac{\bar{p}}{\rho}$ and the effective cosmological constant Λ can be yielded from equations (3.14) and (3.15) as

$$\omega_{eff} = 2\beta + (1 - 4\beta^2)\frac{\chi(H_x, H_y, H_z, h)}{2\beta\xi(H_x, H_y, H_z, h) - \chi(H_x, H_y)}, \tag{3.16}$$

$$\Lambda_{eff} = -\frac{\left[\xi(H_x, H_y, H_z, h) + \chi(H_x, H_y)\right]}{1 + 2\beta}. \tag{3.17}$$

The bulk pressure $\bar{p}$, energy density ρ, EoS parameter ω_{eff} and effective cosmological constant Λ_{eff} will help in investigating the dynamical behaviour of the model. The understanding on the behaviour of the universe would be more appropriate if the properties of the parameters can be expressed in the form of Hubble rate. Because of the simplicity and ability to provide information about the dynamics of the universe, here we have considered the volumetric power law cosmic expansion in the form $v = t^m$, where m is an arbitrary constant calculated from the back ground cosmology. With this assumptions, in the subsequent section, we have developed the cosmological models in $f(R, T)$ gravity for the value of $h = -1, 0, +1$.

3.3 Cosmological models with power law

Each of the value of the exponent h in the field equations leads to a cosmological model with different dynamical behaviour. In the following subsections, we have presented the three cases:

3.3.1 Case-I ($h = -1$)

In this case, we observed that with a suitable absorption of integrating constant with metric potential, the Hubble rate in both y and z direction are same i.e. $H_y = H_z$. Moreover, to study the anisotropic nature of the space time, we assumed an anisotropic relationship between the directional Hubble parameter in the form $H_x = kH_y$, where k is a positive constant. Subsequently, the functionals $\chi(H_x, H_y)$ and $\xi(H_x, H_y, H_z, h)$ respectively reduced to $\chi = (k+1)\dot{H}_y + (k^2+k+1)H_y^2 - \frac{1}{A^2}$ and $\xi = (2k+1)H_y^2 - \frac{1}{A^2}$. For a power law cosmology, the directional Hubble parameters can be obtained as: $H_x = \left(\frac{km}{k+2}\right)\frac{1}{t}$, $H_y = H_z = \left(\frac{m}{k+2}\right)\frac{1}{t}$ and subsequently the directional scale factors provides $A = t^{\frac{km}{k+2}}$ and $B = C = t^{\frac{m}{k+2}}$. So, the functionals $\chi(H_x, H_y)$ and $\xi(H_x, H_y, H_z, h)$ takes the form

$$\chi = \left[\frac{m(1+k+k^2)}{(2+k)^2} - \frac{(1+k)}{(2+k)}\right]\frac{m}{t^2} - \frac{1}{t^{\frac{2mk}{2+k}}}, \tag{3.18}$$

$$\xi = \left[\frac{(1+2k)}{(2+k)^2}\right]\frac{m^2}{t^2} - \frac{1}{t^{\frac{2mk}{2+k}}}. \tag{3.19}$$

We know that the EoS parameter ω_{eff} and effective cosmological constant Λ_{eff} are defined in (3.16)- (3.17) depend on the functionals ξ and χ, which are functions of the cosmic time. So, using eqns. (3.18) and (3.19) in eqns. (3.16)-(3.17), respectively, we obtain

$$\begin{aligned}\omega_{eff} &= 2\beta + (1-4\beta^2)\frac{\xi(H_x, H_y, H_z, h)}{2\beta\xi(H_x, H_y, H_z, h) - \chi(H_x, H_y)} \\ &= 2\beta + \frac{\left[\frac{(1+2k)m^2}{(2+k)^2}\right]\frac{1}{t^2} - t^{\frac{-2km}{2+k}}}{\left[\frac{m(1+k+k^2)}{(2+k)^2} - \frac{(1+k)}{(2+k)} - 2\beta.\frac{(1+2k)m}{(2+k)^2}\right]\frac{m}{t^2} + (2\beta-1)t^{\frac{-2km}{2+k}}},\end{aligned} \tag{3.20}$$

$$\begin{aligned}\Lambda_{eff} &= -\frac{[\xi(H_x, H_y, H_z, h)) + \chi(H_x, H_y)]}{1+2\beta} \\ &= \frac{1}{1+2\beta}\left[\frac{-m(1+k+k^2)}{(2+k)^2} + \frac{(1+k)}{(2+k)} - \frac{m(1+2k)}{(2+k)^2}\right]\frac{m}{t^2} + 2t^{\frac{-2mk}{k+2}}.\end{aligned} \tag{3.21}$$

Since the functionals ξ and χ are essential for analysing the EoS parameter and effective cosmological constant, we are interested here to adopt a dimensional analysis on the dimensionless constant m and k as $m = 1 + \frac{2}{k}$. When $k = 1$, the model reduces to an isotropic one. So, $\chi(t)$ and $\xi(t)$ becomes $\chi(t) = \frac{(1-k^2)}{k^2}\frac{1}{t^2}$ and $\xi(t) = \left(\frac{1+2k-k^2}{k^2}\right)\frac{1}{t^2}$. Using this, we obtain from eqns. (3.20)-(3.21), the corresponding EoS parameter and effective cosmological constant as

$$\omega_{eff} = 2\beta + (1-4\beta^2)\left[\frac{k^2-2k-1}{(2\beta-1)(k^2-1)+4\beta k}\right], \tag{3.22}$$

$$\Lambda_{eff} = \frac{2}{1+2\beta}\left[1-\frac{k+1}{k^2}\right]\frac{1}{t^2}. \tag{3.23}$$

So, from eqn. (3.22), we can infer that for a given value of scaling constant $\mu = \frac{2\beta-3}{16\pi}$, $\beta = \frac{3}{2}+\frac{8\pi}{\mu}$, the EoS parameter is constant as the anisotropic parameter k is also a constant. It is also observed from eqn.(3.23) that, Λ_{eff}, decreases quadratically with the increase in cosmic time; of course with a given scaling constant. To frame a realistic cosmological model, we need to address the scaling constant and anisotropic parameter in such a way that the EoS parameter would be negative and would be less than $-1/3$ at late times. Moreover, in order to achieve a realistic cosmological model, the effective cosmological constant should be large at initial time and should vanish at late times. The same has been represented in Fig 3.1 and Fig 3.2 with an appropriate choice of cosmic time scale.

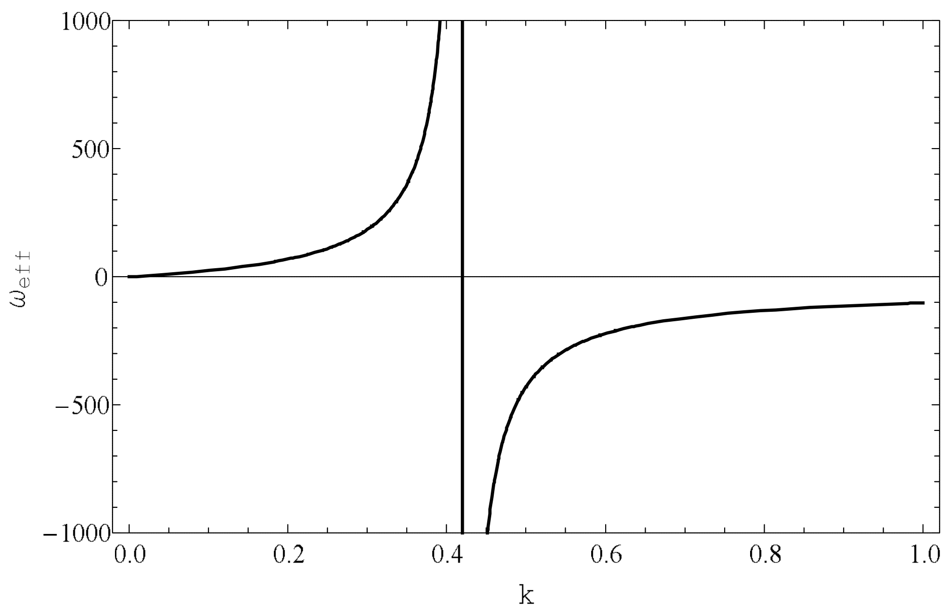

Figure 3.1: ω_{eff} versus k for $h = -1$.

From Fig 3.1, as indicated earlier, we have observed that ω_{eff} is a constant value for a given

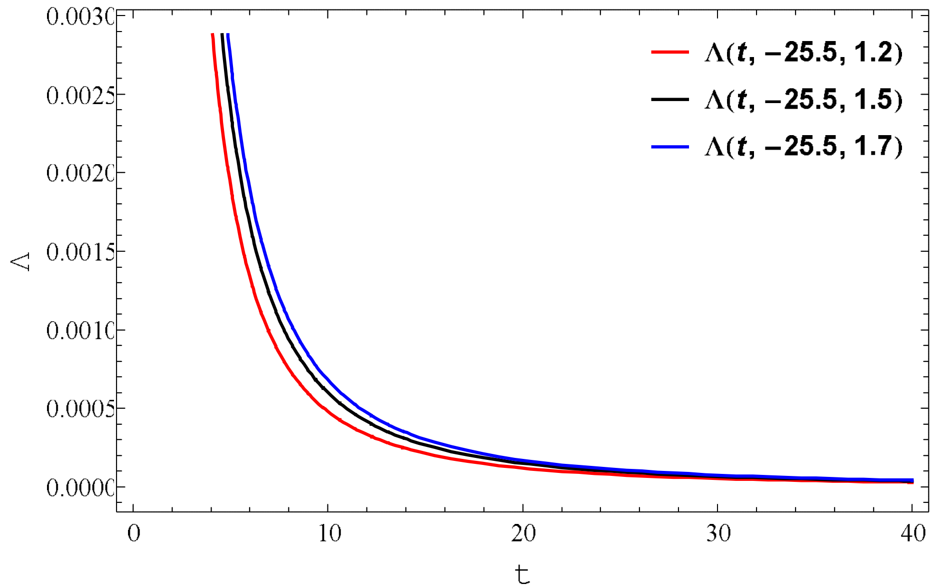

Figure 3.2: Effective cosmological constant versus t for $h = -1$.

value of β and assumed anisotropy parameter k. According to the observational data the ω_{eff} should stay in negative axis and less than $-\frac{1}{3}$. To stay in negative axis we chose a negative value of the model parameter ($\beta = -25.5$). We can observed from the Fig 3.1 that the EoS parameter increases nearly from a negative value for lower value of k to zero. As indicated earlier, in Fig 3.2, the effective cosmological constant varies from large positive values in early epoch to almost vanished at late time.

3.3.2 Case-II ($h = 0$)

Substituting the value of the exponent $h = 0$ in eqn. (3.13), we observed that the Hubble rate is same both in x and y directions. With an assumed anisotropy relation on the y and z direction in the form $H_z = rH_y$ leads the directional Hubble rate in power law expansion of volume scale factor as $H_x = H_y = \left(\frac{m}{r+2}\right)\frac{1}{t}$ and $H_z = \left(\frac{mr}{r+2}\right)\frac{1}{t}$. Thus the corresponding metric potentials are $A = B = t^{\frac{m}{r+2}}$ and $C = t^{\frac{mr}{r+2}}$. The functionals $\chi(t)$ and $\xi(t)$ for this model are

$$\chi(t) = \left[\frac{3m^2 - 2m(r+2)}{(r+2)^2}\right]\frac{1}{t^2} - \frac{1}{t^{\frac{2m}{r+2}}}, \tag{3.24}$$

$$\xi(t) = \left[\frac{(2r+1)m^2}{(r+2)^2}\right]\frac{1}{t^2} - \frac{1}{t^{\frac{2m}{r+2}}}. \tag{3.25}$$

The effective pressure and energy density from eqns. (3.14)-(3.15)

$$\bar{p} = \frac{2}{1-4\beta^2}\left[\xi(H_1, H_2, H_3, h) - 2\beta\chi(H_1, H_2)\right]$$
$$- \frac{2}{1-4\beta^2}\left[\frac{(2r+1)m^2 - 2\beta(3m^2 - 2m(r+2))}{(r+2)^2t^2} + (2\beta - 1)t^{\frac{-2m}{r+2}}\right], \tag{3.26}$$
$$\rho = \frac{2}{1-4\beta^2}\left[2\beta\xi(H_x, H_y, H_z, h) - \chi(H_x, H_y)\right]$$
$$= \frac{2}{1-4\beta^2}\left[\frac{2\beta(2r+1)m^2 - (3m^2 - 2m(r+2))}{(r+2)^2t^2} + (2\beta - 1)t^{\frac{-2m}{r+2}}\right]. \tag{3.27}$$

Using the dimensional consistency $m = r + 2$, in eqns. (3.26)-(3.27), we obtain

$$\bar{p} = \left(\frac{2}{1-4\beta^2}\right)\left(\frac{2r}{t^2}\right), \tag{3.28}$$
$$\rho = \left(\frac{2}{1-4\beta^2}\right)\left(\frac{4\beta r + 4\beta - 2}{t^2}\right). \tag{3.29}$$

Subsequently, the EoS parameter and the effective cosmological constant can be obtained

$$\omega_{eff} = 2\beta + (1-4\beta^2)\frac{\xi(H_x, H_y, H_z, h)}{2\beta\xi(H_x, H_y, H_z, h) - \chi(H_x, H_y)}$$
$$= 2\beta + (1-4\beta^2)\frac{(2r+1)m^2 - (r+2)^2t^{\frac{-2m+2(r+2)}{r+2}}}{2\beta(2r+1)m^2 - 3m^2 + 2m(r+2) + (2\beta-1)t^{\frac{-2m+2(r+2)}{r+2}}}, \tag{3.30}$$
$$\Lambda = -\frac{\xi(H_x, H_y, H_z, h) + \chi(H_x, H_y)}{1+2\beta}$$
$$= -\frac{(2r+1)m^2 + 3m^2 - 2m(r+2) - 2(r+2)^2t^{\frac{-2m+2(r+2)}{r+2}}}{(r+2)^2t^2(1+2\beta)}. \tag{3.31}$$

Using the dimensional consistency $m = r + 2$, in the above equations, we obtain

$$\omega_{eff} = \frac{1}{2\beta}, \tag{3.32}$$
$$\Lambda = \frac{-2r}{2\beta+1}. \tag{3.33}$$

As in the previous case, here also we have employed the dimensional consistency term $m = r+2$. In this case, we employed the same consideration in eqns. (3.26) - (3.27) and (3.30)-(3.31). As a result, the bulk viscous pressure remains in negative domain, the energy density remains in positive domain, ω_{eff}, which is a constant and the effective cosmological constant, Λ, is time varying. Again, to obtain an acceptable cosmological model, the scaling constant has been constrained to be negative, which ultimately assumed β to be negative. With this constraint, ω_{eff} would be in the negative domain and do not affect by the choice of anisotropy in the model, as there is no anisotropy term in the expression. The same has been represented in Fig. 3.3. The effective cosmological constant remains in the positive domain and decreases with increase in time (Fig. 3.4). It is important to note here that ω_{eff} lies in the quintessence region when the scaling parameter is $\leq -\frac{1}{2}$ and when it is more than -$\frac{1}{2}$, ω_{eff} enters into the phantom region.

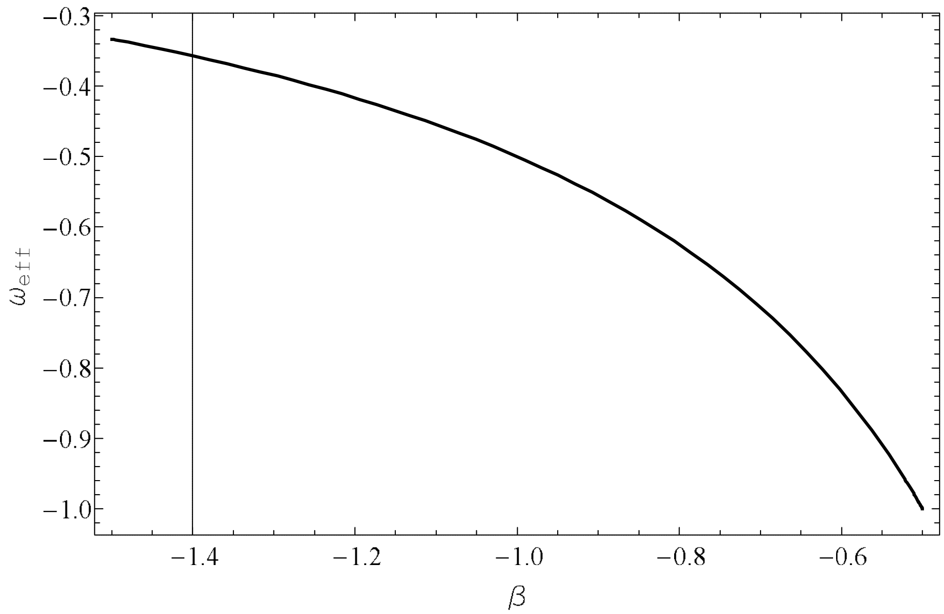

Figure 3.3: ω_{eff} versus β for $h = 0$.

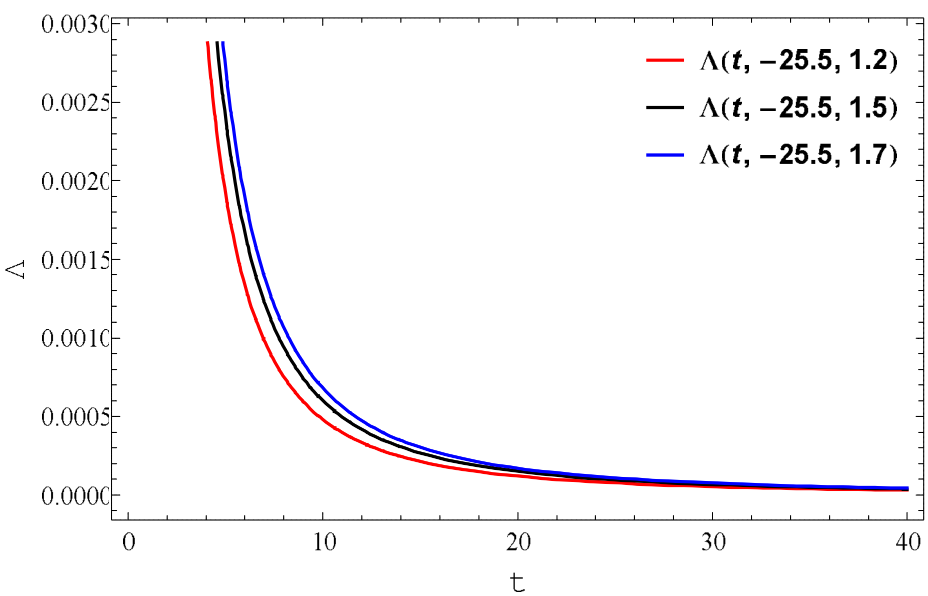

Figure 3.4: Effective cosmological constant versus t for $h = 0$.

3.3.3 Case-III ($h = 1$)

In this case, substituting $h = 1$, again in (3.13), we found that the change of Hubble rate in $x-$ direction is half of the sum of the Hubble rate in $y-$ and $z-$ directions. This leads to another important fact that the mean Hubble rate and the Hubble rate in $x-$ direction are same. As in the preceding section, here also, we have used the power law cosmology in the form $v = t^m$ and obtained the functional χ and ξ as

$$\chi(t) = 2\dot{H} + 3H^2 + \frac{\tau^2}{t^{2m}} - \frac{1}{t^{\frac{2m}{3}}}, \tag{3.34}$$

$$\xi(t) = 3H^2 - \frac{\tau^2}{t^{2m}} - \frac{3}{t^{\frac{2m}{3}}}. \tag{3.35}$$

Where τ is an integrating constant. In order to make the functionals dimensional consistent, the value of the exponent m should be 3. With this value of the exponent m, the deceleration parameter would not be negative, which in turn, does not provide an accelerating model. Moreover, w_{eff} found to be unity, which is not in agreement with the dark energy driven cosmic acceleration; though the effective cosmological constant vanishes. Therefore Bianchi type $VI_h(h = 1)$ space-time is not compatible in the study of present day accelerated expansion of the universe.

3.4 Physical parameters of the models

In this section, we have analysed the behaviour of the physical parameters of the cosmological models obtained in the previous section. The power law model studied is based on the fact that the growth of the scale factor ($\mathcal{R} \propto t^m$) depends on the exponent m. When m lies in the positive domain the observed universe is expanding whereas it contracts for a negative m. We know that the role of Hubble parameter and the deceleration parameter inscribed in the study of power law cosmology. We obtained both the parameters in the form $H = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = \frac{m}{3t}$ and $q = -\frac{\ddot{R}R}{\dot{R}^2} = -1 + \frac{3}{m}$. The deceleration parameter changes from positive to negative according to $m > 3$ and $m < 3$ and vanishes at $m = 3$. So, the late times describes an accelerating expansion, with the deceleration parameter being changed from positive to negative values depends on m. The Hubble parameter decreases with increase in time and may vanish at infinite future. The scalar expansion of the model is $\theta = \Sigma H_i = \frac{m}{t}$, which also indicates that it decreases with time and may vanish at late time. The shear scalar of the model, $\sigma^2 = \frac{m^2}{3t^2}$, and the average anisotropy parameter $\mathcal{A}$ is defined to be $\mathcal{A} = \frac{1}{3}\Sigma\left(\frac{\Delta H_i}{H}\right)^2$. The viscous coefficient ζ for $h = -1$ and for $h = 0$ can be respectively calculated as

$$\zeta(h=-1) = \frac{2}{4\beta^2 - 1}\left[\frac{(\gamma - 2\beta)(1-k^2) + (1-2\beta\gamma)(1+2k-k^2)}{k(k+2)}\right]\frac{1}{t}, \tag{3.36}$$

$$\zeta(h=0) = \frac{2}{4\beta^2 - 1}\left[\frac{(1-2\beta r)2r}{r+2}\right]\frac{1}{t}. \tag{3.37}$$

Fig. 3.5 and Fig. 3.6 respectively give the graphical representation of the viscous coefficient. For $h = -1$ and $h = 0$, it is observed that the coefficient remains positive throughout. Even if, for different representative value of the anisotropy parameter $k = 0.64, 0.8, 0.9$ in Fig. 3.5 and $r = 1.2, 1.5, 1.7$ in Fig. 3.6, the coefficient behave same. It is also observed that in both the cases the bulk viscous coefficient remains constant throughout. The state finder diagnostic pair that gives an impression on the geometrical nature of the model is found to be $r = \left(1 - \frac{3}{m}\right)\left(1 - \frac{6}{m}\right)$ and $s = \frac{2}{m}$. For a large value of the anisotropy relation m, the state finder pairs are having value $(1, 0)$.

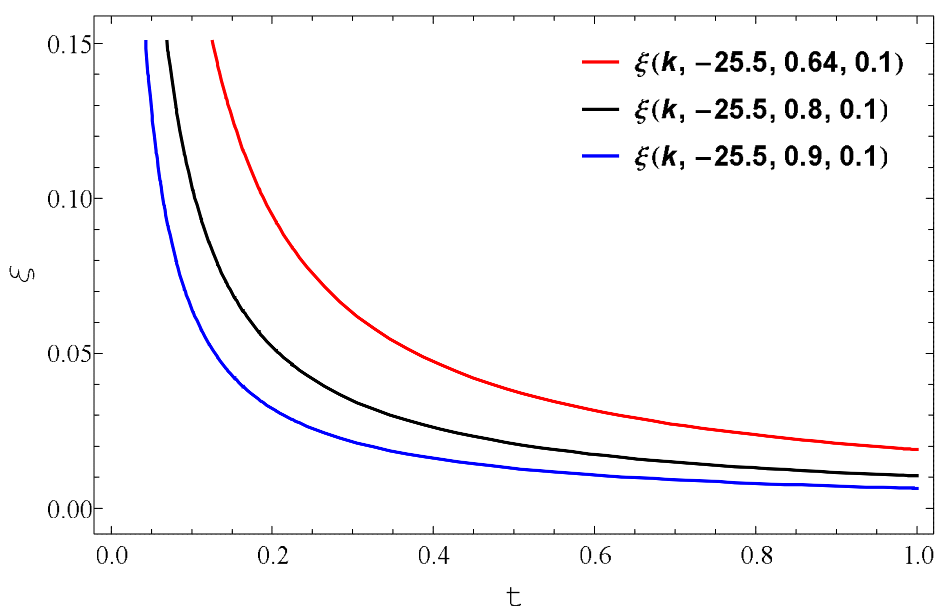

Figure 3.5: ζ versus cosmic time t for $h = -1$.

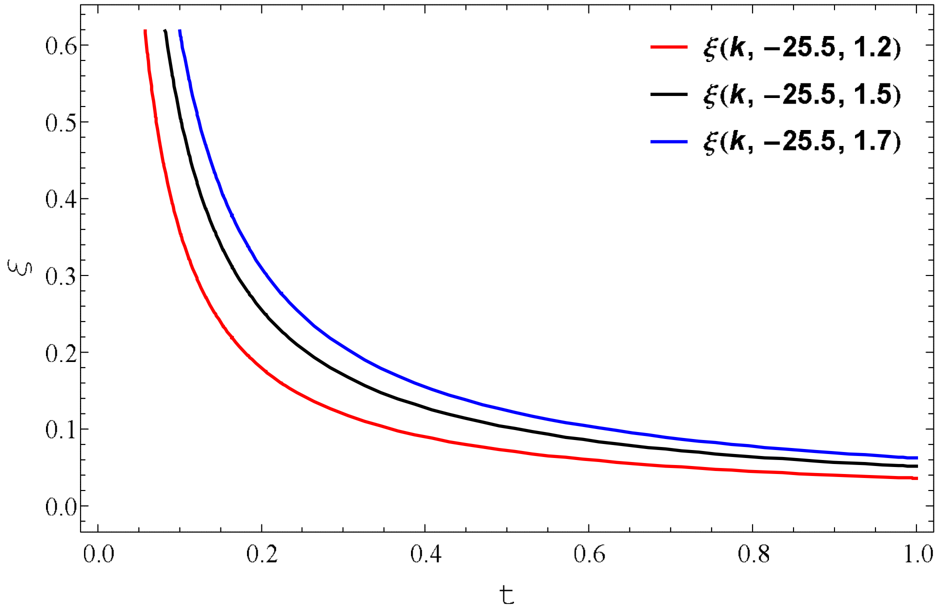

Figure 3.6: ζ versus cosmic time t for $h = 0$.

3.5 Conclusion

In this chapter, we have constructed the cosmological models of the universe in $f(R,T)$ gravity keeping the dimensional consistency at the background using power law scale factor. The linear functional $f(R) = \lambda R$ and $f(T) = \lambda T$ considered here generates the idea of a time varying effective cosmological constant. In both section for $h = -1$ and $h = 0$, we have obtained the cosmological models; however for $h = 1$, cosmological model could not be obtained. In the first two models obtained here, the effective cosmological constant start evolving from large positive value initially and subsequently become small at late times. This result is in accordance with the present observations on dark energy driven cosmic acceleration. For a large m, the state finder diagnostic pair having the value $(1,0)$, which is in agreement with the behaviour of ΛCDM model. The physical parameters are derived and analyzed.

CHAPTER 4

Cosmological Reconstruction in $f(R, T)$ gravity

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4.1 Introduction

Cosmological models are constructed in recent times to account for the predicted late time cosmic acceleration usually by incorporating possible DE candidates in the field equations or by modifying the geometrical part of the action. Amidst the debate that, whether dark energy exists or whether there really occurs a substantial cosmic acceleration (Riess et al. [10], Perlmutter et al. [12], Nielsen et al. [165]) researchers have devoted a lot of time in proposing different DE models. These models are also tested against the observational data accumulated over a long period of time. Some vector-tensor models are also proposed to explain the cosmic speed up phenomena without adopting these approaches. In these vector-tensor models, the presence of a vector field such as the electromagnetic field provides the necessary acceleration (Ferreira et al. [166], Jimenez and Maroto [167, 168], Jimenez et al. [169], Dale and Saez [170]).

Usually in GR, it is not possible to explain the late time cosmic acceleration without the assumption of additional dynamical degrees of freedom besides the tensor modes. Some scalar fields are considered as a solution to this. These scalar fields are usually ghost fields having negative kinetic energy, at least around at, cosmological or spherically symmetric backgrounds e.g. Boulware-Deser mode in massive gravity (Boulware and Deser [171]), bending mode in the self accelerating branch of Dvali-Gabadadze-Porrati model (Koyama [172], Sbisa [173],Gumrukcuoglu et al. [174]). Among all the constructed models to understand the cosmic speed up phenomena, geometrically modified gravity theories have attracted substantial research attention. In geometrically modified theories, instead of incorporating some additional matter fields (may be ghost scalar fields), the Einstein Hilbert action is modified considering some extra geometrical objects. These models thereby provide a ghost free and stable alternative to GR. In this context, Harko et al. [76] have proposed $f(R,T)$ gravity theory in which, the geometry part of the action has been modified in such a manner that, the usual Ricci scalar R in the action is replaced by a function $f(R,T)$ [76].

In the context of string theory cosmological models in $f(R,T)$ gravity, many researchers have studied the feature of the universe. Yadav [175] has searched the existence of Bianchi V string cosmological model in $f(R,T)$ gravity. In this work, the massive strings dominate the early universe but they do not survive for long term and finally disappear. Sharma and Singh [176] have investigated the Bianchi type II cosmological solutions of massive strings in the presence of magnetic field in the framework of $f(R,T)$ gravity with help of special law of variation for Hubble parameter. In the context of late time accelerating expansion of the universe, Aygun

[177] have investigated homogeneous and anisotropic Marder space-time with bulk viscous string matter distribution in $f(R,T)$ gravity. Zia et al. [178] have studied the anisotropic Bianchi type VI_0 dark energy cosmological transit models with string fluid source in $f(R,T)$ gravity. Pawar et al. [179] have studied Kaluza-Klein string cosmological model in framework of $f(R,T)$ gravity by using a time varying deceleration parameter.

In this chapter, we have constructed anisotropic cosmological models in $f(R,T)$ gravity. We have adopted a simple approach to the cosmic anisotropy to investigate the effect of anisotropy on cosmic anisotropy. In order to provide some anisotropic directional pressure, we have considered an anisotropic source along x-direction such as the presence of one dimensional cosmic string. The effect of the coupling constant in the determination of the cosmic evolution has been investigated. We organise the chapter as follows: in section 4.2, the basic equations concerning different properties of the universe are derived for Bianchi VI_h model in the framework of the modified $f(R,T)$ gravity. The dynamical features of the models are discussed in section 4.3. In section 4.4, the anisotropic universe with quark matter has been presented and the concluding remarks are given in section 4.5.

4.2 Basic equations for $f(R,T) = R + 2\Lambda_0 + 2\beta T$

In this case, we have considered a functional form of $f(R,T)$ in such way that the field equations in the modified gravity theory can be reduced to the usual field equations in GR under suitable substitution of model parameters. In this context, we have a popular choice, $f(R,T) = R+2\beta T$. However, we consider a time independent cosmological constant Λ_0 in the functional so that $f(R,T) = R + 2\Lambda_0 + 2\beta T$. Here β is a coupling constant. For this particular choice of the functional $f(R,T)$, the field equation in the modified theory of gravity (2.4) becomes,

$$R_{ij} - \frac{1}{2}Rg_{ij} = [8\pi + 2\beta]\, T_{ij} + \Lambda(T)g_{ij}. \tag{4.1}$$

Here $\Lambda(T) = (2p + T)\,\beta + \Lambda_0$ can be identified as the effective time dependent cosmological constant. If $\beta = 0$, the above modified field equation reduces to the Einstein field equation in GR with a cosmological constant Λ_0. It can be noted that, the effective cosmological constant $\Lambda(T)$ picks up its time dependence through the matter field. For a given matter field described through an energy momentum tensor, the effective cosmological constant can be expressed in terms of the matter components.

In this case, we consider the energy momentum tensor as

$$T_{ij} = (p + \rho)u_i u_j - p g_{ij} - \xi x_i x_j, \tag{4.2}$$

where $u^i u_i = -x^i x_i = 1$ and $u^i x_i = 0$. In a co-moving coordinate system, u^i is the four velocity vector and p is the proper isotropic pressure of the fluid. ρ is the energy density and ξ is the string tension density. The strings are considered to be one dimensional and thereby contribute to the anisotropic nature of the cosmic fluid. The direction of the cosmic strings is represented through x^i that are orthogonal to u^i. The field equations (4.1) of the modified $f(R,T)$ gravity theory, for Bianchi type VI_h space-time (1.65) with the energy momentum tensor eqn. (4.2) can be obtained as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{h}{A^2} = -\alpha(p - \xi) + \rho\beta + \Lambda_0, \tag{4.3}$$
$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{h^2}{A^2} = -\alpha p + +(\rho + \xi)\beta + \Lambda_0, \tag{4.4}$$
$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = -\alpha p + (\rho + \xi)\beta + \Lambda_0, \tag{4.5}$$
$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{1 + h + h^2}{A^2} = \alpha\rho - (p - \xi)\,\beta + \Lambda_0, \tag{4.6}$$
$$\frac{\dot{B}}{B} + h\frac{\dot{C}}{C} - (1 + h)\frac{\dot{A}}{A} = 0. \tag{4.7}$$

The above set of field equations for $h = -1$ can be written with respect to Hubble parameter as considered in (2.11). Moreover, eqn.(4.7) provides $H_y = H_z$. Then eqn.(4.3)-eqn.(4.6) becomes,

$$2\dot{H}_z + 3H_z^2 + \frac{1}{A^2} = -\alpha(p - \xi) + \rho\beta + \Lambda_0, \tag{4.8}$$
$$\dot{H}_x + \dot{H}_z + H_x^2 + H_z^2 + H_x H_z - \frac{1}{A^2} = -\alpha p + (\rho + \xi)\,\beta + \Lambda_0, \tag{4.9}$$
$$2H_x H_z + H_z^2 - \frac{1}{A^2} = \alpha\rho - (p - \xi)\,\beta + \Lambda_0. \tag{4.10}$$

with an algebraic manipulation, from the above field eqns.(4.8)-(4.10), we obtain the expressions for pressure, energy density and the string tension density as

$$p = \frac{1}{\alpha^2 - \beta^2}\left[(s_1 - s_2 + s_3)\,\beta - s_2\alpha + (\alpha - \beta)\,\Lambda_0\right], \tag{4.11}$$
$$\rho = \frac{1}{\alpha^2 - \beta^2}\left[s_3\alpha - s_1\beta - (\alpha - \beta)\,\Lambda_0\right], \tag{4.12}$$
$$\xi = \frac{s_1 - s_2}{\alpha - \beta}. \tag{4.13}$$

Consequently, the EoS parameter ω and the effective cosmological constant Λ can be expressed as

$$\omega = -1 + (\alpha+\beta)\,\frac{s_2 - s_3}{s_1\beta - s_3\alpha + (\alpha-\beta)\,\Lambda_0}, \tag{4.14}$$

$$\Lambda = \frac{\beta}{\alpha^2-\beta^2}\left[(s_2+s_3)\alpha - (2s_1 - s_2 + s_3)\beta - (\alpha+\beta)(s_2-s_1) - 2(\alpha-\beta)\Lambda_0\right] + \Lambda_0. \tag{4.15}$$

In the above equations, s_1, s_2 and s_3 are functions of the directional Hubble parameters and scale factor, as

$$s_1 = 2\dot{H}_z + 3H_z^2 + \frac{1}{A^2}, \tag{4.16}$$

$$s_2 = \dot{H}_x + \dot{H}_z + H_x^2 + H_z^2 + H_xH_z - \frac{1}{A^2}, \tag{4.17}$$

$$s_3 = 2H_xH_z + H_z^2 - \frac{1}{A^2}. \tag{4.18}$$

Eqns (4.11)-(4.15) describe the dynamical behaviour of the model. Once the evolutionary behaviour of the functions s_1, s_2 and s_3 are obtained from some assumed dynamics, the dynamical nature of the model can be studied easily and the modified gravity model can be reconstructed accordingly.

4.3 Dynamical parameters of the model

We intend to investigate the cosmic history through the assumption of an assumed dynamics concerning the late time cosmic acceleration. In view of this, we assume the scalar expansion be governed by an inverse function of cosmic time i.e. $\theta = (H_x + 2H_z) = \frac{m}{t}$ and also we assume that θ be proportional to the shear scalar $\sigma^2 = \frac{1}{2}\left(\sum H_i^2 - \frac{1}{3}\theta^2\right); i = x, y, z$. Consequently, $H_x = \left(\frac{km}{k+2}\right)\frac{1}{t}$, $H_y = H_z = \left(\frac{m}{k+2}\right)\frac{1}{t}$. The directional scale factors can be expressed as $A = t^{km/(k+2)}$, $B = C = t^{m/(k+2)}$. For such an assumption, the functions s_1, s_2 and s_3 reduce to

$$\begin{aligned}
s_1 &= 2\dot{H}_z + 3H_z^2 + \frac{1}{A^2}\\
&= -2\left(\frac{m}{k+2}\right)\frac{1}{t^2} + 3\left(\frac{m}{k+2}\right)^2\frac{1}{t^2} + \frac{1}{t^{\frac{2km}{k+2}}}\\
&= \left[\frac{3m^2 - 2(k+2)m}{(k+2)^2}\right]\frac{1}{t^2} + \frac{1}{t^{\frac{2km}{k+2}}},
\end{aligned} \tag{4.19}$$

$$\begin{aligned}
s_2 &= \dot{H}_x + \dot{H}_z + H_x^2 + H_z^2 + H_xH_z - \frac{1}{A^2}\\
&= -\left(\frac{km}{k+2}\right)\frac{1}{t^2} - \left(\frac{m}{k+2}\right)\frac{1}{t^2} + \left(\frac{km}{k+2}\right)^2\frac{1}{t^2} + \left(\frac{km}{k+2}\right)^2\frac{1}{t^2}\\
&+ \left(\frac{km^2}{(k+2)^2}\right)\frac{1}{t^2} - \frac{1}{t^{\frac{2km}{k+2}}}\\
&= \left[\frac{(k^2+k+1)m^2 - (k+1)(k+2)m}{(k+2)^2}\right]\frac{1}{t^2} - \frac{1}{t^{\frac{2km}{k+2}}},
\end{aligned} \tag{4.20}$$

$$\begin{aligned}
s_3 &= 2H_xH_z + H_z^2 - \frac{1}{A^2}\\
&= 2\left(\frac{km^2}{(k+2)^2}\right)\frac{1}{t^2} + \left(\frac{m}{k+2}\right)^2\frac{1}{t^2} - \frac{1}{t^{\frac{2km}{k+2}}}\\
&= \left[\frac{(2k+1)m^2}{(k+2)^2}\right]\frac{1}{t^2} - \frac{1}{t^{\frac{2km}{k+2}}}.
\end{aligned} \tag{4.21}$$

With the substitution of eqn.(4.19)-(4.21), the pressure, energy density and string tension density can be obtained as:

$$\begin{aligned}
p &= \frac{1}{\alpha^2 - \beta^2}\left[(s_1 - s_2 + s_3)\beta - s_2\alpha + (\alpha - \beta)\Lambda_0\right]\\
&= \frac{1}{(\alpha^2 - \beta^2)}\left[\left(\frac{\phi_1}{(k+2)^2}\right)\frac{1}{t^2} + \frac{(\alpha+\beta)}{t^{\frac{2km}{k+2}}} + (\alpha-\beta)\Lambda_0\right],
\end{aligned} \tag{4.22}$$

where $\phi_1 = m\{(k^2+k-2)\beta + (k^2+3k+2)\alpha\} - m^2\{(k^2-k-3)\beta - (k^2+k+1)\alpha\}$ is redefined constant.

$$\begin{aligned}
\rho &= \frac{1}{\alpha^2 - \beta^2}\left[s_3\alpha - s_1\beta - (\alpha - \beta)\Lambda_0\right],\\
&= \frac{1}{(\alpha^2 - \beta^2)}\left[\left(\frac{\phi_2}{(k+2)^2}\right)\frac{1}{t^2} + \frac{(\beta-\alpha)}{t^{\frac{2km}{k+2}}} - (\alpha-\beta)\Lambda_0\right],
\end{aligned} \tag{4.23}$$

where $\phi_2 = (2k+1)m^2\alpha - (3m^2 - 2km - 4m)\beta$ is redefined constant.

$$\begin{aligned}
\xi &= \frac{s_1 - s_2}{\alpha - \beta},\\
&= \frac{1}{(\alpha - \beta)}\left[\frac{(k-1)(m^2-m)}{(k+2)^2t^2} - \frac{2}{t^{\frac{2km}{k+2}}}\right].
\end{aligned} \tag{4.24}$$

These physical quantities evolve with the cosmic expansion. Their evolution is governed by two time dependent factors: one behaving like t^{-2} and the other behaving as $t^{-\frac{2km}{k+2}}$. Since m and k are positive quantities, the magnitude of the physical quantities (neglecting their sign) decrease monotonically with cosmic time. It is interesting to note that, ξ also decreases from a large value in the initial epoch to small values at late phase of cosmic evolution. This behaviour of ξ implies that, at the initial phase, more anisotropic components are required than at late phase. From eqns. (4.22)- (4.23), we obtain the EoS parameter $\omega = \frac{p}{\rho}$ and the effective cosmological constant respectively as

$$\begin{aligned}\omega &= -1 + (\alpha+\beta)\,\frac{s_2 - s_3}{s_1\beta - s_3\alpha + (\alpha-\beta)\,\Lambda_0},\\ &= -1 + (\alpha+\beta)\left[\frac{\phi_3}{\phi_4 + (\alpha-\beta)(k+2)^2\left\{\Lambda_0 t^2 - t^{2\left(\frac{k-km+2}{k+2}\right)}\right\}}\right], \qquad (4.25)\end{aligned}$$

where $\phi_3 = (k^2-2k)m^2 - (k^2+2k+3)m$ and $\phi_4 = (3m^2 - 2km - 4)\beta - (2k+1)m^2\alpha$ are redefined constants.

$$\begin{aligned}\Lambda &= \frac{\beta}{\alpha^2-\beta^2}\left[(s_2+s_3)\alpha - (2s_1 - s_2 + s_3)\beta - (\alpha+\beta)(s_2-s_1) - 2(\alpha-\beta)\Lambda_0\right] + \Lambda_0,\\ &= \frac{\beta}{(\alpha^2-\beta^2)}\left[\frac{\phi_5}{(k+2)^2t^2} - \frac{2(\alpha+\beta)}{t^{\frac{2km}{k+2}}} - 2(\alpha-\beta)\Lambda_0\right] - \frac{\phi_6}{(k+2)^2t^2}\\ &+ \frac{\beta}{(\alpha-\beta)t^{\frac{2km}{k+2}}} + \Lambda_0. \qquad (4.26)\end{aligned}$$

Where $\phi_5 = \{(k+1)\alpha + (k-3)\beta\}(m^2-m)$ and $\phi_6 = \frac{\beta(k-1)(m^2-m)}{(\alpha-\beta)}$ are some redefined constants.
The dynamical nature of the model can be assessed through the evolution of the EoS parameter ω. In Fig. 4.1, ω is plotted as function of red-shift for four different values of the coupling constant β namely $\beta = 0, 0.5, 1.0$ and 2.0. $\beta = 0$ refers to the case in GR. The anisotropic parameter is considered to be $k = 0.7$ and m is fixed from the observationally constrained value of deceleration parameter $q = -0.598$. For all the cases considered here, ω becomes a negative quantity and remains in the quintessence region throughout the period of evolution considered in the work. It decreases from some higher value at the beginning to low values at late times. However, at late phase of cosmic evolution, ω grows up a little bit which may be due to the anisotropic effect of cosmic strings.

The coupling constant β affects the dynamical behaviour of the EoS parameter. In order to understand the effect of β on ω, the EoS at the present epoch is plotted as a function of β in Fig. 4.2 for three different values of k. One can note that, ω increases with the increase in the

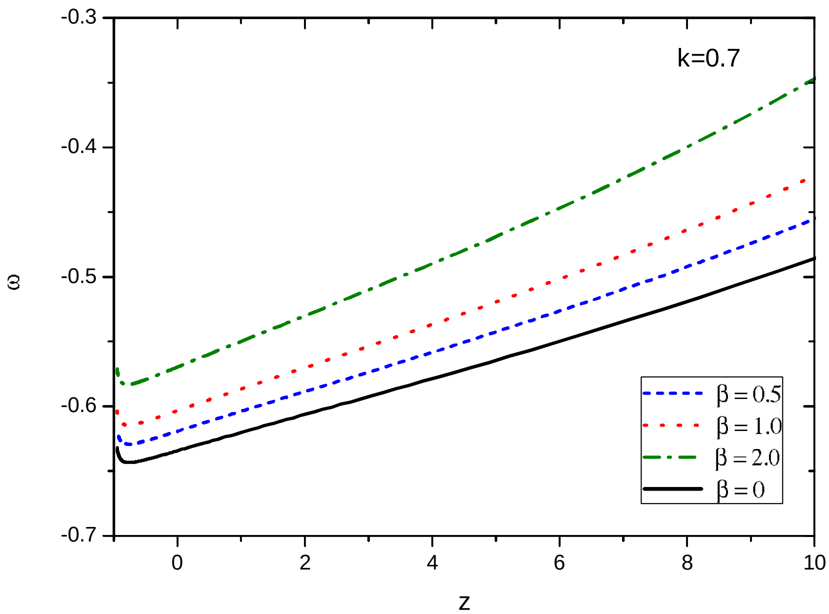

Figure 4.1: Dynamical evolution of the EoS parameter for different representative values of the coupling constant β.

value of the coupling constant. In view of the recent observations predicting an accelerating universe, the value of coupling constant β should have a lower value i.e. $\beta \leq 1$.

In Fig. 4.3, we have shown the effect of anisotropy on the EoS parameter. In the figure, we are assumed three representative values of the anisotropy i.e $k = 0.7, 0.8$ and 0.9 for a given coupling constant $\beta = 0.5$. Anisotropy brings a substantial change in the magnitude as well as the behaviour of the EoS parameter. There occurs a flipping behaviour of ω at a redshift $z_f \simeq 4$. At a cosmic time earlier to z_f, with the increase in the anisotropy of the model, ω assumes a higher value. In other words, prior to z_f, higher the value of k, higher is the ω. It displays an opposite behaviour at cosmic times later to z_f. Also, at the redshift z_f, curves corresponding to all k considered here cross each other. In general, the rate of evolution of the EoS parameter increases with the increase in the value of the anisotropic parameter.

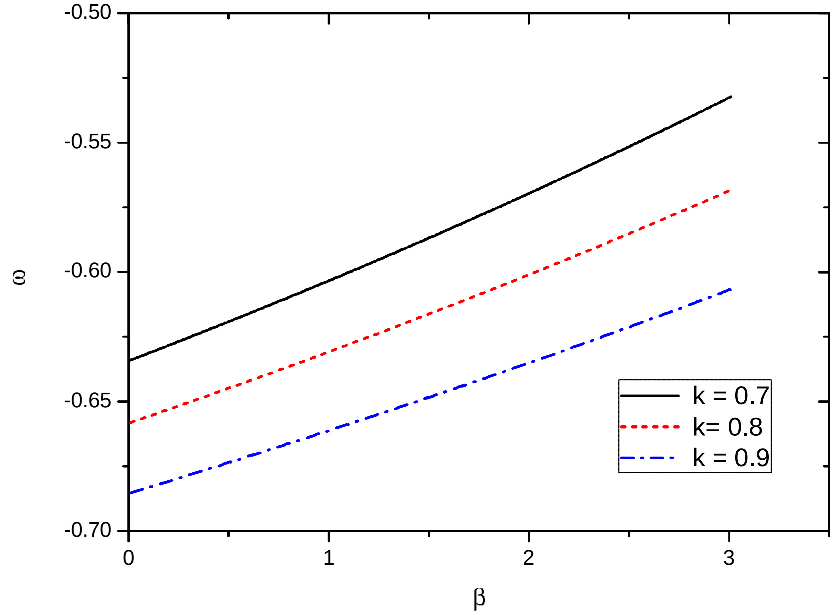

Figure 4.2: EoS parameter as function of the coupling constant at present epoch for a given anisotropic parameter.

4.4 Anisotropic universe with quark matter

It has been believed that, quarks and gluon did not yield to hadronization and resisted as a perfect fluid that spread over the universe. It may contribute to the accelerated expansion of the universe. Here, we have presented an anisotropic cosmological model with non-interacting quarks that may well be dealt as a Fermi gas with an equation of state given by (Kapusta [180], Aktas and Yilmaz [181]),

$$p_q = \frac{\rho_q}{3} - B_c, \tag{4.27}$$

where p_q is the quark pressure, ρ_q is the quark energy density and B_c is the bag constant. We assume that quarks exist along with one dimensional cosmic string without any interaction. The quark energy density can then be expressed as $\rho_q = \rho - \xi - B_c$. Going in the same manner as described in the previous section, we can have the expressions for the quark pressure and

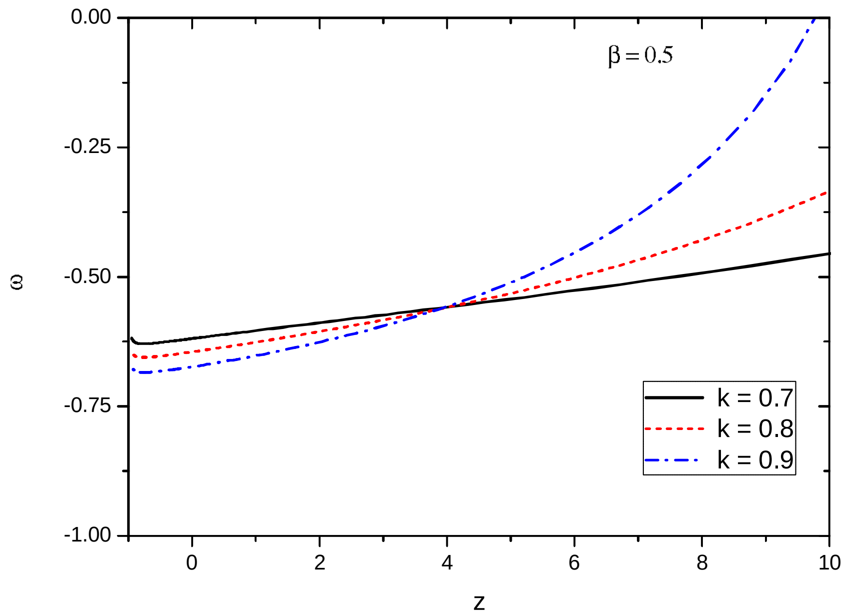

Figure 4.3: Effect of anisotropic parameter on the EoS parameter.

quark energy density as

$$\rho_q = \frac{1}{\alpha^2-\beta^2}\left[(\alpha+\beta)s_2+s_3\alpha-(\alpha+2\beta)s_1-(\alpha-\beta)\Lambda_0\right]-B_c, \tag{4.28}$$

$$p_q = \frac{1}{3(\alpha^2-\beta^2)}\left[(\alpha+\beta)s_2+s_3\alpha-(\alpha+2\beta)s_1-(\alpha-\beta)\Lambda_0\right]-\frac{4B_c}{3} \tag{4.29}$$

If we put $\beta=0$, the model reduces to that in GR with a cosmological constant. In that case, the above equations reduce to

$$\rho_q = \frac{1}{8\pi}\left[s_2+s_3-s_1-\Lambda_0\right]-B_c, \tag{4.30}$$

$$p_q = \frac{1}{24\pi}\left[s_2+s_3-s_1-\Lambda_0\right]-\frac{4B_c}{3}. \tag{4.31}$$

Substituting the expressions for s_1, s_2 and s_3 in eqns. (4.28) and (4.29), the quark matter energy density and quark pressure are obtained as

$$\rho_q = \frac{1}{\alpha^2-\beta^2}\left[\frac{\phi_7}{(k+2)^2}\frac{1}{t^2}+\frac{(\alpha+3\beta)}{t^{\frac{2km}{k+2}}}-(\alpha-\beta)\Lambda_0\right]-B_c, \tag{4.32}$$

$$p_q = \frac{1}{3(\alpha^2-\beta^2)}\left[\frac{\phi_7}{(k+2)^2}\frac{1}{t^2}+\frac{(\alpha+3\beta)}{t^{\frac{2km}{k+2}}}-(\alpha-\beta)\Lambda_0\right]-\frac{4B_c}{3}, \tag{4.33}$$

where $\phi_7 - \phi_2-(k-1)(m^2-m)(\alpha+\beta)$. For some reasonable value of the coupling parameter β and the anisotropic parameter k, the quark energy density and quark pressure decrease smoothly with the cosmic evolution. Bag constant certainly has a role to play at late times when the value of ρ_q and p_q are mostly dominated by this quantity.

4.5 Conclusion

In this chapter, we have investigated the dynamical behaviour of an anisotropic Bianchi type VI_h universe in the presence of one dimensional cosmic strings and quark matter. Anisotropic cosmological models are reconstructed for a power assumption of the scale factor in the frame work of $f(R,T)$ gravity. In the process of reconstruction and study of dynamical features of the model, we chose the functional $f(R,T)$ as $f(R,T) = R + 2\Lambda_0 + 2\beta T$. From some general expressions of the physical quantities, we derived the expression of the EoS parameter and the effective cosmological constant. The effects of anisotropy k and the coupling constant β are investigated. It is observed that, with an increase in the coupling constant the EoS parameter assumes a higher value. Anisotropy is observed to affect largely to the dynamics of the model. The EoS parameter undergoes an increased rate of growth with an increase in the anisotropy. We hope, the present study will definitely put some light in the context of the uncertainty prevailing in the studies of the late time cosmic phenomena.

CHAPTER 5

Effect of Anisotropy in the Cosmological Models

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5.1 Introduction

In the last three decades, the standard model of cosmology has gained a lot of research attention. The prime reason behind this is, its ability to address complex observational issues with a simple theoretical structure. The two major successes of this model are (*i*) the explanation of the observed light element abundances in early universe and (*ii*) the prediction of the relic CMB. Over a period of time, cosmological theories have come up with a number of novel predictions including the possible existence of topological defects, extra dimensions, inflation, relic non baryonic dark matter candidates.

Substantial progresses have been made in recent years in acquisition of cosmological data both in quality and quantity. This advancement obviously lowers the gap between theory and observation. Also it provides a window to understand the physics of the very early universe. In the past two decades, the supernovae cosmology project group and the high- z supernovae group have presented enough evidences with observations and theoretical justification that the universe is undergoing an accelerating expansion at the present epoch (Riess et al. [10], Schmidt et al. [11], Perlmutter et al. [12]). It is interesting to investigate anisotropic models in modified theories. Although there have been a lot of debate going on in the issue of cosmic anisotropy, anisotropic models can be more interesting in the sense that, they are quite general than the usual FRW models and may provide some interesting cosmological results (Antoniou and Perivolaropoulos [182], Mariano and Perivolaropoulos [183], Zhao et al. [184], Tripathy [185], Saadeh et al. [186]). Sharif and Zubair [187], have investigated Bianchi type cosmological models in $f(R,T)$ theory. Moreover, Shamir [188] has obtained the solutions of the LRS Bianchi type I space-time with the assumption of a relationship between the metric potentials of the space-time. Sharif and Nawazish [189] have investigated some cosmological models for Bianchi type I, III and Kantoskwi-Sachs space-time using Noether symmetry approach. Also,Sharif and Nawazish [190] have explored the Noether and Noether gauge symmetries of anisotropic cosmological model in $f(R,T)$ gravity. Shabani and Ziaie [191, 192] have studied the existence and stability of Einstein universe in $f(R,T)$ gravity. Zubair et al. [193] have analysed the stability of cylindrically symmetric objects with anisotropic fluids. Sahu et al. [194], have constructed accelerating cosmological models in a modified gravity theory. Sharif and Zubair [195] have investigated the energy conditions and stability of power law solutions in this modified gravity theory. Moraes et al. [196] have obtained the general solutions for static wormholes in $f(R,T)$ gravity and the physical and geometrical solutions are obtained using analytical approach.

In this chapter, we have presented the dynamical behaviour of the cosmological model with the functional in the form of $f(R,T) = \lambda(R+T)$ by incorporating source of anisotropic fluid in the energy momentum tensor. In section 5.2, the field equations in $f(R,T)$ gravity have been derived with the composition of energy density due to perfect fluid and anisotropic fluid. The dynamical features of the model have been presented in section 5.3. The cosmological model for $h = -1$ has been presented in section 5.4. In section 5.5, the physical behaviours of the model are discussed and energy conditions are presented in section 5.6. In section 5.7, the concluding remark is given.

5.2 Field equations for $f(R,T) = \lambda R + \lambda T$

With the choice of function $f(R,T) = \lambda(R+T)$, λ being a constant, the field equation for $f(R,T)$ gravity can be written as

$$R_{ij} - \frac{1}{2}Rg_{ij} = \left(\frac{8\pi + \lambda}{\lambda}\right)T_{ij} + \Lambda(T)g_{ij}, \tag{5.1}$$

where $\Lambda(T) = p + \frac{1}{2}T$ is an effective cosmological constant that depends on time. It picks up its evolutionary behaviour through the matter fields. It is worth to mention here that, the scaling factor λ cannot be zero as the model diverges for this value. Also, it is certain that, one cannot recover the corresponding field equations of GR by putting a value of λ by hand. However, as it can be seen from our discussion, we may obtain viable models by rescaling the GR equations through this parameter λ. An important feature of this model is that, the field equation appears to have the same form as that of GR with a time varying cosmological constant and a redefined Einstein constant ($\kappa = \frac{8\pi G}{c^4}$, G and c are respectively the Newtonian gravitational constant and speed of light in vacuum). We assume the energy momentum tensor as

$$T_{ij} = (p+\rho)u_iu_j - pg_{ij} - \rho_B x_i x_j, \tag{5.2}$$

where $u^iu_i = -x^ix_i = 1$ and $u^ix_i = 0$. In a co-moving coordinate system, u^i is the four velocity vector. x^i represents the direction of anisotropic fluid (here x-direction) and is orthogonal to u^i. ρ is the energy density and is composed of energy density due to the perfect fluid and anisotropic fluid ρ_B. The field equations (5.1) for Bianchi type VI_h space-time (1.65) and the energy momentum tensor (5.2) can be obtained as

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}-\frac{h}{A^2}=-\alpha(p-\rho_B)+\frac{\rho}{2}, \tag{5.3}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}-\frac{h^2}{A^2}=-\alpha p+\frac{1}{2}\left(\rho+\rho_B\right), \tag{5.4}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{1}{A^2}=-\alpha p+\frac{1}{2}\left(\rho+\rho_B\right), \tag{5.5}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{C}\dot{A}}{CA}-\frac{1+h+h^2}{A^2}=\alpha\rho-\frac{1}{2}\left(p-\rho_B\right), \tag{5.6}$$

$$\frac{\dot{B}}{B}+h\frac{\dot{C}}{C}-(1+h)\frac{\dot{A}}{A}=0. \tag{5.7}$$

where, $\alpha=\frac{16\pi+3\lambda}{2\lambda}$. In the set of above field equations, an overhead dot denotes time derivative.

5.3 Dynamical features of the model

In some recent papers, Tripathy et al. [197, 198] have calculated the energy and momentum of anisotropic BVI_h universes and have shown that the energy and momentum of such universes vanish for $h=-1$. If we assume that, the metric should envisages an isolated universe with null total energy and momentum, then the choice $h=-1$ is preferable to any other value of the exponent. In view of this, we assume this value of h and study the dynamics of the anisotropic universe in presence of anisotropic energy sources. The directional Hubble rates considered as in eqn.(2.11). Using this directional Hubble rates, the above set of equation (5.3)-(5.7) becomes as

$$\dot{H}_y+\dot{H}_z+H_y^2+H_z^2+H_yH_z-\frac{h}{A^2}=-\alpha(p-\rho_B)+\frac{\rho}{2}, \tag{5.8}$$

$$\dot{H}_x+\dot{H}_z+H_x^2+H_z^2+H_xH_z-\frac{h^2}{A^2}=-\alpha p+\frac{1}{2}\left(\rho+\rho_B\right), \tag{5.9}$$

$$\dot{H}_x+\dot{H}_y+H_x^2+H_y^2+H_xH_y-\frac{1}{A^2}=-\alpha p+\frac{1}{2}\left(\rho+\rho_B\right), \tag{5.10}$$

$$H_xH_y+H_yH_z+H_zH_x-\frac{1+h+h^2}{A^2}=-\alpha\rho+\frac{1}{2}\left(p-\rho_B\right), \tag{5.11}$$

$$H_y+hH_z-(1+h)H_x=0, \tag{5.12}$$

With $h=-1$, it is straightforward to get $H_y=H_z$ from (5.12). The mean Hubble parameter becomes, $H=\frac{\dot{\mathcal{R}}}{\mathcal{R}}=\frac{1}{3}(H_x+2H_z)$ where $\mathcal{R}$ is the mean scale factor of universe.

The set of field eqns. (5.8)-(5.11) can be reduced to

$$6(k+2)\dot{H}+27H^2+(k+2)^2\mathcal{R}^{-\left(\frac{6k}{k+2}\right)}=(k+2)^2\left[-\alpha(p-\rho_B)+\frac{\rho}{2}\right], \tag{5.13}$$

$$3(k^2+3k+2)\dot{H}+9(k^2+k+1)H^2-(k+2)^2\mathcal{R}^{-\left(\frac{6k}{k+2}\right)}=(k+2)^2\left[-\alpha p+\frac{1}{2}(\rho+\rho_B)\right], \tag{5.14}$$

$$9(2k+1)H^2-(k+2)^2\mathcal{R}^{-\left(\frac{6k}{k+2}\right)}=(k+2)^2\left[\alpha\rho-\frac{1}{2}(p-\rho_B)\right]. \tag{5.15}$$

Here, a linear anisotropic relation among the directional Hubble rates is assumed i.e. $H_x=kH_z$. Algebraic manipulation of the field equations (5.13)-(5.15) yields

$$p = -\frac{2}{1-4\alpha^2}[s_1(\mathcal{R})+s_3(\mathcal{R})-(1+2\alpha)s_2(\mathcal{R})], \tag{5.16}$$

$$\rho = \frac{2}{1-4\alpha^2}[s_1(\mathcal{R})-2\alpha s_3(\mathcal{R})], \tag{5.17}$$

$$\rho_B = \frac{2}{1-2\alpha}[s_2(\mathcal{R})-s_1(\mathcal{R})]. \tag{5.18}$$

Here,

$$s_1(\mathcal{R}) = \frac{1}{(k+2)^2}[6(k+2)\dot{H}+27H^2+(k+2)^2\mathcal{R}^{-\left(\frac{6k}{k+2}\right)}], \tag{5.19}$$

$$s_2(\mathcal{R}) = \frac{1}{(k+2)^2}[3(k^2+3k+2)\dot{H}+9(k^2+k+1)H^2-(k+2)^2\mathcal{R}^{-\left(\frac{6k}{k+2}\right)}], \tag{5.20}$$

$$s_3(\mathcal{R}) = \frac{1}{(k+2)^2}[9(2k+1)H^2-(k+2)^2\mathcal{R}^{-\left(\frac{6k}{k+2}\right)}]. \tag{5.21}$$

The EoS parameter $\omega=\frac{p}{\rho}$ and the effective cosmological constant Λ are obtained respectively

$$\omega = -1+(1+2\alpha)\left[\frac{s_2(\mathcal{R})-s_3(\mathcal{R})}{s_1(\mathcal{R})-2\alpha s_3(\mathcal{R})}\right], \tag{5.22}$$

$$\Lambda = \frac{1}{(1+2\alpha)}[s_1(\mathcal{R})+s_3(\mathcal{R})]. \tag{5.23}$$

The dynamical features of the model are decided by the physical quantities given in eqns (5.16)-(5.23). However these quantities depend on the mean scale factor. In view of this, we can study a background cosmology and associated dynamics if we presume the behaviour of the mean scale factor.

5.4 Model with power law cosmology

In the study of cosmic dynamics, power law cosmology has gained a lot of attention. Power law cosmology has emerged as an alternative to ΛCDM model and considers the evolution of classical fields that are coupled to the curvature of the space in such a way that their contribution to the energy density self-adjusts to cancel the vacuum energy (Dolgov [199]). The motivation for such a scenario comes from the fact that it does not encounter flatness and the horizon problem at all. Another interesting feature of these models is that they easily accommodate high redshift objects and hence alleviate the age problem. These models are also purged of the fine tuning problem (Dolgov et al. [200], Ford [201]). In power law expansion cosmology, the scale factor is assumed to grow as some power function of cosmic time i.e. $\mathcal{R} = t^{m/3}$, where m is a positive constant and can be constrained from observational data on the deceleration parameter (DP) or the jerk parameter (JP). The geometrical features such as Hubble parameter, DP and JP respectively become $H = \frac{m}{3t}$, $q = -1 + \frac{3}{m}$ and $j = \frac{9}{m}\left(\frac{2}{m} - 1\right) + 1$. Here, we have employed such a scale factor to investigate the background cosmology in the presumed modified gravity model. With such an assumption we obtain the pressure, energy density and density of anisotropic fluid source from (5.16)-(5.18) as

$$\begin{aligned}
p &= -\frac{2}{1-4\alpha^2}[s_1(\mathcal{R}) + s_3(\mathcal{R}) - (1+2\alpha)s_2(\mathcal{R})] \\
&= -\frac{2}{(1-4\alpha^2)}\left[\frac{1}{(k+2)^2}\left(6(k+2)\dot{H} + 27H^2 + \mathcal{R}^{-\left(\frac{6k}{k+2}\right)}\right)\right] \\
&- \frac{2}{(1-4\alpha^2)}\left[\frac{1}{(k+2)^2}\left(9(2k+1)H^2 - (k+2)^2\mathcal{R}^{-\left(\frac{6k}{k+2}\right)}\right)\right] \\
&+ \frac{2}{(1-4\alpha^2)}\left[\frac{(1+2\alpha)}{(k+2)^2}\left(3(k^2+3k+2)\dot{H} + 9(k^2+k+1)H^2 - (k+2)^2\mathcal{R}^{-\left(\frac{6k}{k+2}\right)}\right)\right] \\
&= -\frac{2}{(1-4\alpha^2)}\left[\frac{\phi_1 + 2\alpha\phi_2}{(k+2)^2}\right]\frac{1}{t^2} - \frac{2}{(1-2\alpha)}\frac{1}{t^{\frac{2km}{k+2}}},
\end{aligned} \tag{5.24}$$

$$\begin{aligned}
\rho &= \frac{2}{1-4\alpha^2}[s_1(\mathcal{R}) - 2\alpha s_3(\mathcal{R})] \\
&= \frac{2}{1-4\alpha^2}\left[\frac{1}{(k+2)^2}\left(6(k+2)\dot{H} + 27H^2 + (k+2)^2\mathcal{R}^{-\left(\frac{6k}{k+2}\right)}\right)\right] \\
&- \frac{4\alpha}{1-4\alpha^2}\left[\frac{1}{(k+2)^2}\left(9(2k+1)H^2 - (k+2)^2\mathcal{R}^{-\left(\frac{6k}{k+2}\right)}\right)\right] \\
&= \frac{2}{(1-4\alpha^2)}\left[\frac{\phi_3 - 2\alpha(2k+1)m^2}{(k+2)^2}\right]\frac{1}{t^2} + \frac{2}{(1-2\alpha)}\frac{1}{t^{\frac{2km}{k+2}}}, \qquad (5.25)\\
\rho_B &= \frac{2}{1-2\alpha}[s_2(\mathcal{R}) - s_1(\mathcal{R})] \\
&= \frac{2}{1-2\alpha}\left[\frac{1}{(k+2)^2}\left(3(k^2+3k+2)\dot{H} + 9(k^2+k+1)H^2 - (k+2)^2\mathcal{R}^{-\left(\frac{6k}{k+2}\right)}\right)\right] \\
&- \frac{2}{1-2\alpha}\left[\frac{1}{(k+2)^2}\left(6(k+2)\dot{H} + 27H^2 + (k+2)^2\mathcal{R}^{-\left(\frac{6k}{k+2}\right)}\right)\right] \\
&= \frac{2}{(1-2\alpha)}\left[\frac{\phi_4}{(k+2)^2}\right]\frac{1}{t^2} + \frac{4}{(1-2\alpha)}\frac{1}{t^{\frac{2km}{k+2}}}. \qquad (5.26)
\end{aligned}$$

where $\phi_1 = (k^2+k-2)m - (k^2-k-3)m^2$, $\phi_2 = (k^2+3k+2)m - (k^2+k+1)m^2$, $\phi_3 = 3m^2 - 2(k+2)m$ and $\phi_4 = (m^2-m)(2-k-k^2)$ are redefined constants that depend on the choice of the parameters m and k.

The EoS parameter ω and the effective cosmological constant are obtained from (5.22) and (5.23) as

$$\begin{aligned}
\omega &= -1 + (1+2\alpha)\left[\frac{s_2(\mathcal{R}) - s_3(\mathcal{R})}{s_1(\mathcal{R}) - 2\alpha s_3(\mathcal{R})}\right] \\
&= -1 + (1+2\alpha)\left[\frac{\left(3(k^2+3k+2)\dot{H} + 9(k^2-k)H^2\right)}{6(k+2)\dot{H} + 9(3-2\alpha(2k+1))H^2 + (1+2\alpha)(k+2)^2\mathcal{R}^{-\left(\frac{6k}{k+2}\right)}}\right] \\
&= -1 + (1+2\alpha)\left[\frac{(k^2-k)m^2 - (k^2+3k+2)m}{(3-2\alpha(2k+1))m^2 - (k+2)2m + (1+2\alpha)(k+2)^2 t^{\frac{2(k+2-km)}{k+2}}}\right], \qquad (5.27)\\
\Lambda &= \frac{1}{(1+2\alpha)}[s_1(\mathcal{R}) + s_3(\mathcal{R})] = \frac{1}{(k+2)(1+2\alpha)}\left[6\dot{H} + 18H^2\right] \\
&= \frac{2}{(1+2\alpha)}\left[\frac{m(m-1)}{k+2}\right]\frac{1}{t^2}, \qquad (5.28)
\end{aligned}$$

5.5 Physical behaviour of the model

In Fig. 5.1(a) and (b), we have shown respectively the behaviour of pressure and energy density as function of redshift $z = \frac{1}{\mathcal{R}} - 1$. The radius scale factor at the present epoch is considered to

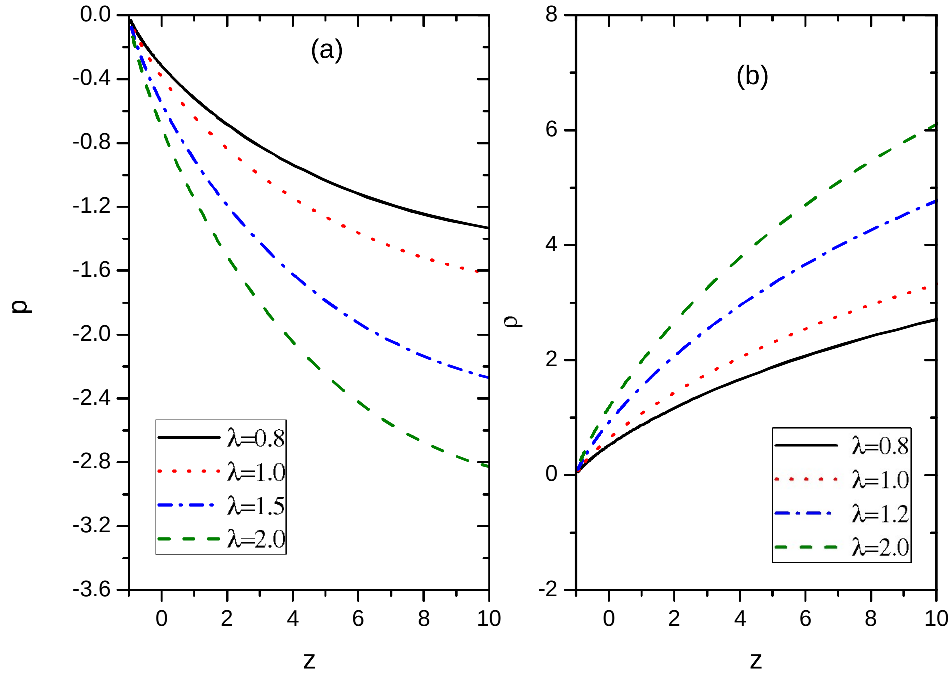

Figure 5.1: (a)Pressure as a function of redshift (b) Energy density as a function of redshift. Four different values of the scaling factor have been considered to assess its effect. Anisotropy parameter is assumed to be $k = 0.65$.

be 1. The behaviour of the physical quantities are shown for four different values of the scaling constant λ. The anisotropic parameter k can be considered as a free parameter in the work. However, we assume a representative value of anisotropy i.e $k = 0.65$ for plotting the figures. The exponent m has been constrained from the value of deceleration parameter, $q = -0.598$, obtained from an analysis of observational data (Montiel et al. [202]). In general, pressure is obtained to be negative in the whole range of redshift considered in the work. In fact, for a given value of λ, pressure increases from some large negative value at an early epoch to vanishingly small values at late times. The choice of the scaling parameter λ substantially affects the magnitude of pressure. At a given redshift, pressure assumes large negative values with an increase in λ. On the other hand, energy density remains in the positive domain and decreases to small values at late times in Fig.5.1(b). At a given redshift, energy density increases with an increase in λ. The evolutionary behaviour of the energy density of the anisotropic fluid source, ρ_B, is shown in Fig. 5.2. Its evolutionary behaviour is studied by assuming a fixed anisotropy

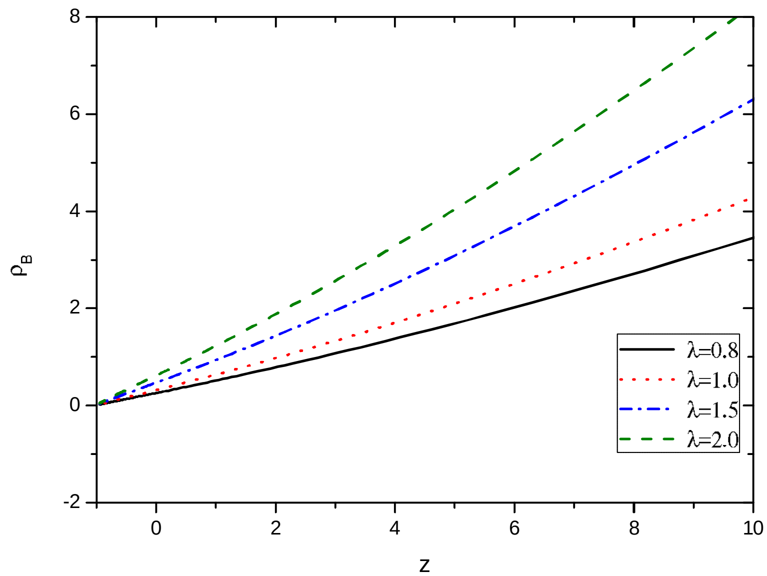

Figure 5.2: Evolution of energy density of the anisotropic fluid source for four different values of the scaling constant λ.

and an accelerating universe with $q = -0.598$. The magnitude of ρ_B is more at an initial epoch compared to late phase. At a late cosmic phase, ρ_B becomes negligible. The variation of ρ_B with different choices of λ is also shown in the figure for four representative values of λ. One can note that, ρ_B increases with an increase in λ at a given redshift. The scaling constant also decides the rate of decrement in the energy density of the anisotropic fluid source. Higher the value of λ, more is the rate of decrement (or slope) for ρ_B. It is evident from (5.27) that the EoS parameter ω evolves with time that depends on three parameters λ, m and k. In Fig. 5.3(a), the time evolution of ω is shown for four different values of λ. The anisotropy parameter is considered to be $k = 0.65$ and m is constrained from the deceleration parameter. ω remains in the negative domain favouring a quintessence phase. EoS parameter decreases with time. The choice of λ value affects the equation of state, mostly it affects the slope of the evolution curve. The slope increases for higher value of λ. ω evolves from $\omega = -1+(1+2\alpha)\left[\frac{(k^2-k)m-(k^2+3k+2)}{(3-2\alpha(2k+1))m-2(k+2)}\right]$ at the beginning to some higher negative values compared to this one at late phase of evolution. The value of ω at late times depends on the choice of λ. For the given anisotropic parameter i.e. $k = 0.65$, the value of ω in the present epoch becomes $-0.614, -0.611, -0.604$ and -0.597 corresponding to $\lambda = 0.8, 1.0, 1.5$ and 2.0 respectively.

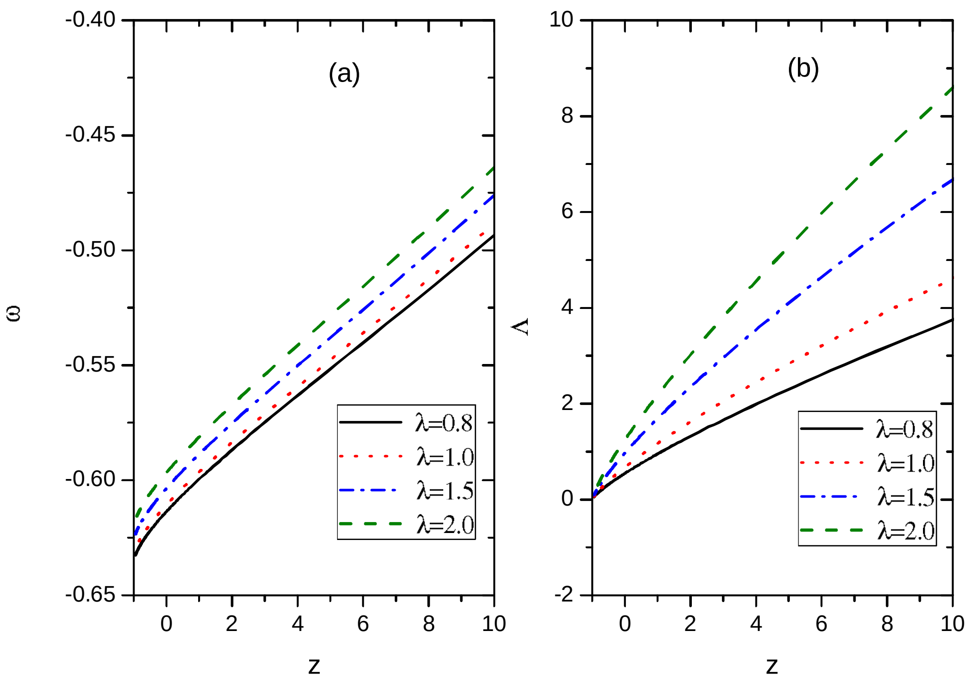

Figure 5.3: (a) Evolution of the EoS parameter. (b) Effective cosmological constant as a function of redshift.

The effective cosmological constant Λ decreases quadratically with time and depends on the parameters λ, m and k. In Fig. 5.3(b), Λ is shown as a function of redshift. It is a positive quantity for all the choices of λ. We have presented the modified gravity model in such a manner that, the theory behaves as that of GR with a time varying cosmological constant. In case of GR, the late time cosmic speed up phenomena, requires a positive non zero cosmological constant that should dynamically roll down to a value close to zero at late times. From our model, we obtain a similar behaviour of the effective cosmological constant. The evolutionary behaviour is more or less the same as that of the energy density of the anisotropic fluid ρ_B.

We have explored the effect of anisotropy on the dynamics of the universe as well as on the energy conditions. Here k is taken as a free parameter. Anisotropy substantially affects the dynamics as is evident from Fig. 5.4. In the figure, ω is plotted for three values of k namely $0.65, 0.8$ and 0.9. It is observed that, effect of anisotropy is more visible at early phase of evolution compared to that at late times. An increase in the value of k increases the value of ω at an early time and decreases the value at late times. In other words, model with higher k has less value of ω at late epoch. Interestingly all the curves with different values of anisotropy intersect at a particular redshift $z \sim 4.08$ when the EoS parameter is -0.562. At the present

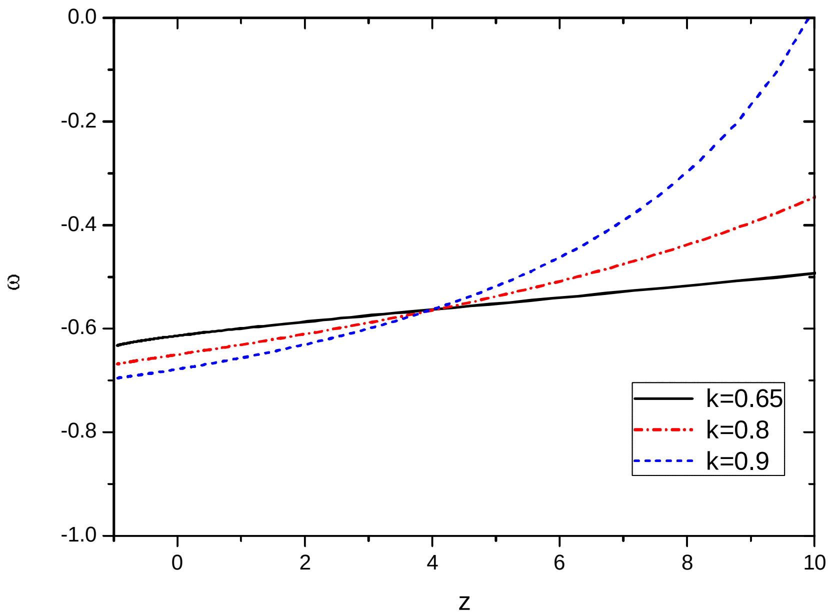

Figure 5.4: Effect of anisotropy parameter k on the EoS parameter.

epoch, the anisotropic effect on the equation of state is better understood through its values $\omega = -0.614, -0.65$ and -0.678 corresponding to $k = 0.65, 0.8, 0.9$. In order to assess the fact that, out of two parameters, λ and k, which one affects the EoS parameter to a greater extent, we have plotted the EoS parameter at the present epoch as a function of the scaling constant λ for four different values of k in Fig. 5.5. It is obvious from the plot that, for a given value of k, EoS parameter is marginally affected for a variation of λ within a suitable range. However, with an increase in k, the EoS parameter decreases substantially.

The energy density of anisotropic fluid ρ_B considered along x-direction is also affected by the variation in directional anisotropy rates in the same manner as that of ω. In Fig. 5.6(a), the effect of anisotropy on the anisotropic fluid density is shown for three representative values of k. One can note that, in the late phase the requirement of an anisotropic fluid is much less for higher anisotropy whereas at an initial cosmic phase, more anisotropic fluid is required to maintain a higher anisotropic expansion rate. However, all the curves of the anisotropic fluid energy density intersect at a redshift $z \sim 4.08$.

The dynamical features can also be assessed from a dimensional analysis of the quantities. In eqns. (5.24)-(5.26), the dynamical behaviour is governed by two terms: one depending on

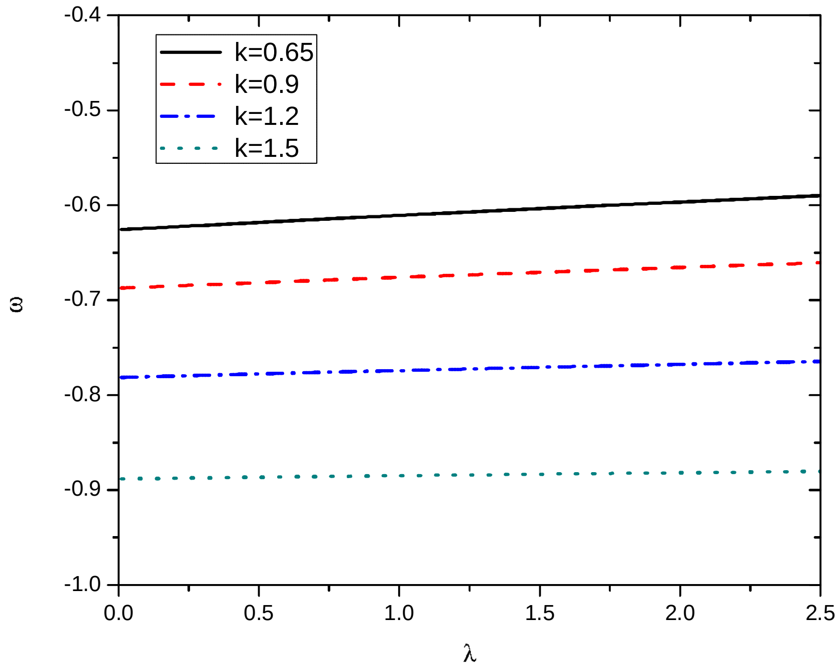

Figure 5.5: EoS parameter ω at the present epoch as a function of scaling constant λ. The effect of anisotropy is also shown for four representative values of k.

t^{-2} and other on $t^{-\frac{2km}{k+2}}$. Since the parameters m and k are dimensionless, we expect that the two terms appearing in these quantities must have the same dimensions as quantified. The parameter m is already constrained from the deceleration parameter. In view of the argument on the basis of qualitative dimensional analysis, we can put a constraint on k as $km = k + 2$. With this constraint on m and k, pressure, energy density and energy density for the anisotropic fluid reduce to

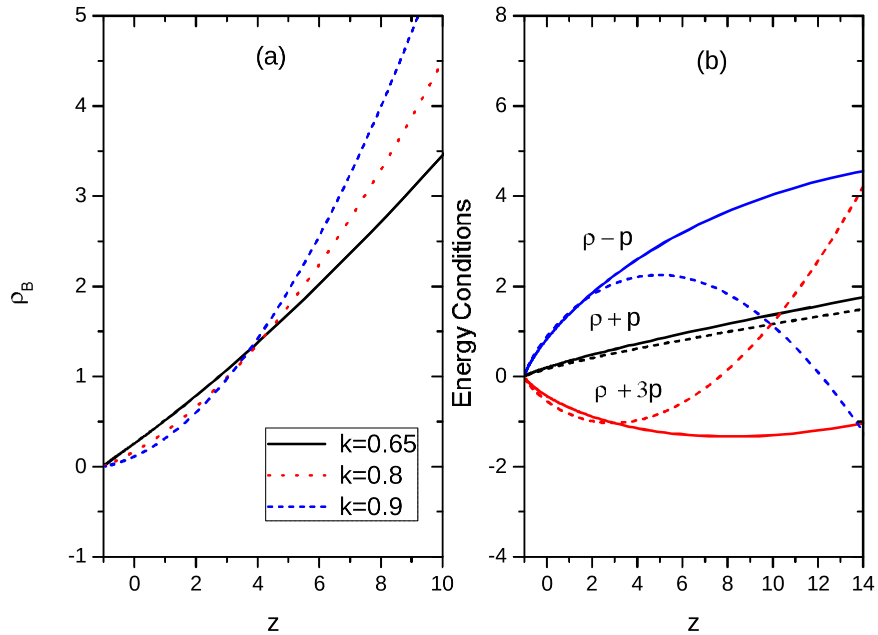

Figure 5.6: (a) Effect of anisotropy on the anisotropic fluid density (b) Effect of anisotropy on energy conditions. Solid curves in (b)refer to $k = 0.65$ and the dotted curves to $k = 0.9$ for the respective energy conditions.

$$
\begin{aligned}
p &= -\frac{2}{(1-4\alpha^2)}\left[\frac{\phi_1+2\alpha\phi_2}{(k+2)^2}\right]\frac{1}{t^2}-\frac{2}{(1-2\alpha)}\frac{1}{t^{\frac{2km}{k+2}}}\\
&= -\frac{2}{(1-4\alpha^2)}\left[\frac{(k^2+k-2)m-(k^2-k-2)m^2+2\alpha\left((k^2+3k+2)m-(k^2+k+1)m^2\right)}{(k+2)^2}\right]\frac{1}{t^2}\\
&-\frac{2}{(1-2\alpha)}\frac{1}{t^{\frac{2km}{k+2}}}\\
&= -\frac{1}{1-4\alpha^2}\left[\frac{1}{2}(3m^2-14m+3)-\alpha(m-1)^2+2(1+2\alpha)\right]\frac{1}{t^2}, \qquad (5.29)\\
\rho &= \frac{2}{(1-4\alpha^2)}\left[\frac{\phi_3-2\alpha(2k+1)m^2}{(k+2)^2}\right]\frac{1}{t^2}+\frac{2}{(1-2\alpha)}\frac{1}{t^{\frac{2km}{k+2}}}\\
&= \frac{2}{(1-4\alpha^2)}\left[\frac{3m^2-2(k+2)m-2\alpha(2k+1)m^2}{(k+2)^2}\right]\frac{1}{t^2}+\frac{2}{(1-2\alpha)}\frac{1}{t^{\frac{2km}{k+2}}}\\
&= \frac{2}{1-4\alpha^2}\left[(m-1)\left\{(3m-7)+2\alpha(m-3)\right\}+(1+2\alpha)\right]\frac{1}{t^2}, \qquad (5.30)\\
\rho_B &= \frac{2}{(1-2\alpha)}\left[\frac{\phi_4}{(k+2)^2}\right]\frac{1}{t^2}+\frac{4}{(1-2\alpha)}\frac{1}{t^{\frac{2km}{k+2}}}\\
&= \frac{2}{(1-2\alpha)}\left[\frac{(m^2-m)(2-k-k^2)}{(k+2)^2}\right]\frac{1}{t^2}+\frac{4}{(1-2\alpha)}\frac{1}{t^{\frac{2km}{k+2}}}\\
&- \frac{1}{1-2\alpha}\left[m(m-1)(m-3)+4\right]\frac{1}{t^2}. \qquad (5.31)
\end{aligned}
$$

For a given scaling constant λ, the magnitude of these quantities now quadratically decrease with time. Consequently, the EoS parameter becomes

$$\omega = -1 + \frac{(9m^2 + 54m + 33) + 2\alpha(3m^2 - 14m + 19)}{(12m^2 - 40m + 29) + 2\alpha(m^2 - 4m + 4)},$$

which is a non-evolving constant and depends only on λ and m. For $\lambda = 0.8$ and m as constrained from the deceleration parameter, we obtain $\omega = -0.577$. With the constraint from dimensional analysis, the effective cosmological constant becomes $\Lambda = \frac{(m-1)^2}{1+2\alpha}\frac{1}{t^2}$. The evolutionary behaviour of the effective cosmological constant remains the same as before. It decreases quadratically with time in the positive domain.

5.6 Energy condition of the model

Energy conditions put some additional constraints on the models (Carroll[203], Sharif et al.[204]). For a perfect fluid distribution the energy conditions are

Null Energy Condition(**NEC**) : $\rho + p \geq 0$,

Weak Energy Condition(**WEC**) : $\rho + p \geq 0, \quad \rho \geq 0$,

Strong Energy Condition(**SEC**) : $\rho + 3p \geq 0, \quad \rho + p \geq 0$,

Dominant Energy Condition(**DEC**) : $\rho \pm p \geq 0, \quad \rho \geq 0$

From eqns. (5.29)-(5.30), the energy conditions are obtained as

$$\textbf{NEC} : \rho + p = \frac{2[s_2(\mathcal{R}) - s_3(\mathcal{R})]}{1 - 2\alpha} \geq 0,$$
$$\textbf{WEC} : \rho = \frac{2}{1 - 2\alpha}[s_2(\mathcal{R}) - s_1(\mathcal{R})] \geq 0,$$
$$\textbf{SEC} : \rho + 3p = -\frac{2}{1 - 4\alpha^2}[s_1(\mathcal{R}) + (3 + 2\alpha)s_3(\mathcal{R}) - 3(1 + 2\alpha)s_2(\mathcal{R})] \geq 0,$$
$$\textbf{DEC} : \rho - p = \frac{2}{1 - 4\alpha^2}[2s_1(\mathcal{R}) - (1 + 2\alpha)s_2(\mathcal{R}) + (1 - 2\alpha)s_3(\mathcal{R})] \geq 0.$$

NEC is implied in all other energy conditions. Since in our calculation, we assume small values for the scaling constant λ, the factor $1 - 2\alpha$ is a negative quantity. Consequently, in order to satisfy the null energy condition and weak energy condition, we require $s_1(\mathcal{R}) > s_2(\mathcal{R})$ and $s_3(\mathcal{R}) > s_2(\mathcal{R})$. Dominant energy condition requires the inequality $s_2(\mathcal{R}) > \frac{2s_1(\mathcal{R})+(1-2\alpha)s_3(\mathcal{R})}{1+2\alpha}$ to be satisfied. For **SEC**, we need an extra condition $s_1(\mathcal{R}) > 6\alpha s_2(\mathcal{R})$. We have calculated the energy conditions within our formalism with a power law scale factor. These conditions can be expressed as

$$\begin{aligned}
\textbf{NEC}: \rho + p &- \frac{2}{1-2\alpha}\left[\frac{k(k-1)m^2-(k+1)(k+2)m}{(k+2)^2}\right]\frac{1}{t^2},\\
\textbf{SEC}: \rho + 3p &= \frac{2}{1-2\alpha}\left[\frac{3(k^2-k-2)m^2-(3k-1)(k+2)m}{(k+2)^2}\right]\frac{1}{t^2}\\
&+\frac{4\alpha}{1-2\alpha}\left[\frac{(3k^2-k+1)m^2-3(k+1)(k+2)m}{(k+2)^2}\right]\frac{1}{t^2}-\frac{4(1+\alpha)}{1-4\alpha^2}\frac{1}{t^{\frac{2km}{k+2}}},\\
\textbf{DEC}: \rho - p &= \frac{2}{1-4\alpha^2}\left[\frac{\left((k^2+3k+8)-4\alpha(k^2+k+1)\right)m^2+2(k^2+k-2)m}{(k+2)^2}\right]\frac{1}{t^2}\\
&+\frac{4}{1-2\alpha}\frac{1}{t^{\frac{2km}{k+2}}}.
\end{aligned}$$

The energy conditions as obtained are plotted in Fig. 5.7. The energy conditions in the figures are calculated for $k = 0.65$ and $\lambda = 0.8$. The present model satisfies all energy conditions except SEC. The behaviour of the energy conditions remain the same for different choices of the scaling constant λ. However, for large values of λ, say $\lambda = 20$ or more, the behaviour may change. Since we are interested in modified gravity models close to GR, we assume a small value of the scaling constant λ and in these range of λ, the behaviour of the energy condition remains almost the same as shown in the Fig. 5.7.

In Fig. 5.6(b) we have shown the effect of anisotropy on the energy conditions. We have considered two representative values 0.65 and 0.9 of the anisotropic parameter k for a given value of m and λ. As shown in the figure, the effect of anisotropy is dramatic. Anisotropy affects marginally to NEC. An increase in the value of k lowers the value of $\rho+p$ at late phase. However, at a late epoch, it is almost unaffected by the choice of k. Also, in case of SEC and DEC, the anisotropy has little affect during the late epoch. But at an initial epoch, an increase in the value of k decreases the value of $\rho-p$ and it goes down to the negative domain violating the dominant energy condition. With an increase in k, $\rho+3p$ increases into the positive domain for higher z values thereby enabling the model to satisfy the strong energy condition.

5.7 Conclusion

In this chapter, we have investigated the dynamical features of the cosmological models in a Bianchi type $VI_h(h = -1)$ space-time in presence of some anisotropic sources in $f(R,T)$ theory. It is observed that the choice of the scaling constant controlled the behaviour of EoS

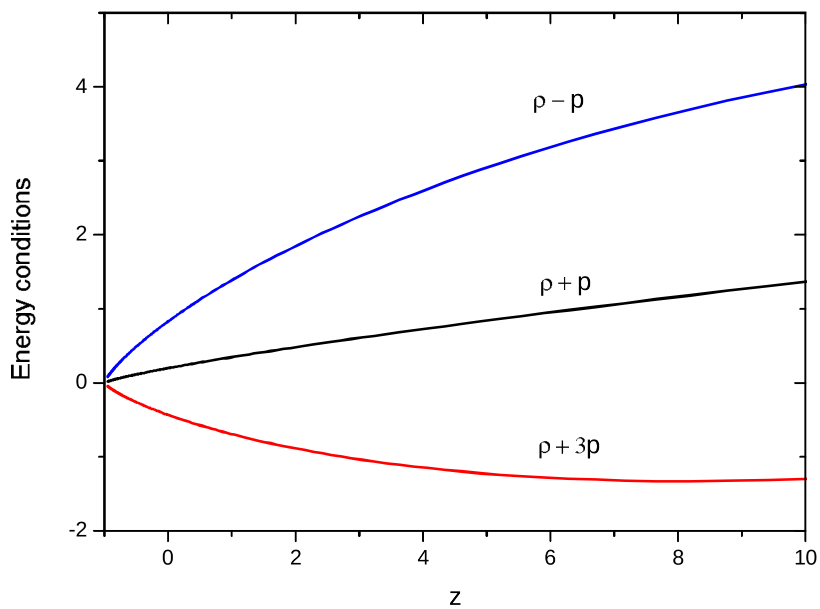

Figure 5.7: Energy conditions for the model shown for $\lambda = 0.8$ and $k = 0.65$.

parameter. The effective cosmological constant is time dependent and dynamically decreases with cosmic time irrespective of the choice of the scaling constant. The energy conditions which are few additional conditions for the matter content of the gravitational theory remain same for different values of the scaling constant; however for a higher value of the scaling constant, the behaviour may change. An increase in cosmic anisotropy within the present formalism substantially affect the energy conditions. It is also observed that more anisotropic fluid is required to maintain a higher anisotropic expansion rate at an initial cosmic phase.

CHAPTER 6

Role of Magnetic Field in the Cosmological Models with Extended Gravity

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6.1 Introduction

It is believed that magnetic fields exist in galactic and intergalactic spaces and therefore magnetic field may play an important role in the description of the cosmic dynamics and energy distribution (Maartens [205], Grasso and Rubinstein [206]). It can be seen that the large scale magnetic fields can be detected by observing their effects on the CMB radiation. Also, since the expansion rate will be different depending on the directions of the field lines, these fields would enhance anisotropies in the CMB. Mazharimousavi et al. [207] have obtained solution for modified gravity coupled with electromagnetic field. Bamba et al. [208] have demonstrated the logarithmic non minimal gravitational coupling of the electromagnetic theory in modified gravity. They have also mentioned that due to this effect, gravity may produce time variation of the fine structure constant which may increase with decrease of the curvature. Sharif and Yousaf [209] have analysed the role of electromagnetic field in $f(R)$ gravity. Saha [210] developed the system of interacting scalar and electromagnetic field within the framework of this theory and obtained an improved result for a standard Einstein-Hilbert model. Yousaf et al. [211] have investigated the electromagnetic field and modified gravity on the evolution of cylindrical compact object. Aktas and Aygun [212] have obtained the magnetized strange quark matter solution with cosmological constant. Agrawal and Pawar [213] have studied the magnetized domain wall and quark and strange quark matter in $f(R,T)$ gravity. Pradhan and Jaiswal [214] have studied Bianchi type V cosmological models in $f(R,T)$ gravity in the presence of magnetic field.

In this chapter, we have constructed an anisotropic Bianchi type VI_h cosmological model filled with magnetic field in $f(R,T)$ gravity (Harko et al. [76]). In section 6.2, we have developed the field equations of $f(R,T)$ gravity with the matter field in the form of electromagnetic field and presented the formalism for the dynamical parameters. In section 6.3, the cosmological model has been constructed with the power law cosmology. In section 6.4, the energy conditions and physical parameters of the the model are analysed. The conclusion is given in section 6.5

6.2 Field equations in the framework of $f(R,T)$ gravity

The gravitational action (1.41) for $f(R,T)$ gravity, which is the generalisations of Einstein GR has been considered. The matter Lagrangian can be considered either as $\mathcal{L}_m = -p$ or $\mathcal{L}_m = p$, however for the conciseness of expression, we will choose $\mathcal{L}_m = -p$. The energy momentum tensor for magnetic field can be defined as

$$T_{ij} = (p+\rho)u_i u_j - p g_{ij} + E_{ij}, \tag{6.1}$$

where $u^i x_i = 0$ and $u^i u_i = -x^i x_i = 1$. It is worthy to mention here that the co-moving coordinate system $u_i = \sqrt{g_{44}}(0,0,0,1)$ is the four velocity vector of the fluid. p, ρ respectively denote the pressure, energy density and E_{ij} is the part of the energy momentum tensor corresponding to the electromagnetic field and is defined as

$$E_{ij} = \frac{1}{4\pi}\left[g^{sp} f_{is} f_{jp} - \frac{1}{4} g_{ij} f_{sp} f^{sp}\right]. \tag{6.2}$$

Here, g_{ij} is the gravitational metric potential and f_{sp} is the electromagnetic field tensor. In order to neglect the effect of electric field, we assume an infinite electrical conductivity such that $f_{14} = f_{24} = f_{34} = 0$. Also, aligning the axis of the magnetic field along the axis of symmetry, we obtained $f_{12} = f_{13} = 0$, $f_{23} \neq 0$. So, the non-vanishing component of electromagnetic field tensor is only $f_{23} = -f_{32}$ and it can be represented as $f_{23} = -f_{32} = \mathcal{M}$, a constant, from the reference of Maxwell equations, which can be expressed as

$$F^{ij}_{;j} = 4\pi j^i, \tag{6.3}$$

where j^i represents the four-current. Now, the field equations for $f(R,T)$ gravity (2.4) for the choice of functional $f(R,T) = f(R) + f(T)$ can be reduced to

$$f_R R_{ij} - \frac{1}{2} f(R) g_{ij} + (g_{ij}\Box - \nabla_i \nabla_j) f_R(R) = 8\pi T_{ij} + f_T T_{ij} + \left[p f_T(T) + \frac{1}{2} f(T)\right] g_{ij}, \tag{6.4}$$

where f_R and f_T are partial differentiation with respect to the respective functions. We are interested to consider the functional form of $f(R,T)$ in the form $f(R,T) = R + 2\Lambda_0 + 2\beta T$, such that the field equations of $f(R,T)$ can be reduced to the field equations of Einstein GR. Here, Λ_0 and β are respectively the cosmological constant and coupling constant. With this, the field equations of $f(R,T)$ gravity can be obtained as

$$f_R R_{ij} - \frac{1}{2} f(R) g_{ij} = (8\pi + 2\beta) T_{ij} + \Lambda(T) g_{ij}, \tag{6.5}$$

where, $\Lambda(T) = (\rho - p)\beta + \Lambda_0$ be the time dependent cosmological constant and mostly controlled by the behaviour of pressure and energy density of the fluid. Now, in order to construct the cosmological model of the universe in frame of $f(R,T)$ gravity in the assumed functional form $f(R,T) = R + 2\Lambda_0 + 2\beta T$, we consider Bianchi type VI_h space-time (1.65). So, the field equations (6.5) with the space time (1.65) can be explicitly written as

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}-\frac{h}{A^2}=-(\alpha+\beta)p+\rho\beta-\alpha\mu+\Lambda_0 \tag{6.6}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}-\frac{h^2}{A^2}=-(\alpha+\beta)p+\rho\beta+\alpha\mu+\Lambda_0 \tag{6.7}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{1}{A^2}=-(\alpha+\beta)p+\rho\beta+\alpha\mu+\Lambda_0 \tag{6.8}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{C}\dot{A}}{CA}-\frac{1+h+h^2}{A^2}=(\alpha+\beta)\rho-\beta p-\alpha\mu+\Lambda_0 \tag{6.9}$$

$$\frac{\dot{B}}{B}+h\frac{\dot{C}}{C}-(1+h)\frac{\dot{A}}{A}=0 \tag{6.10}$$

An over dot represents ordinary derivative with respect to t and because of brevity, we take $h=-1$. Here $\alpha = 8\pi + 2\beta$ and $\mu = \frac{\mathcal{M}^2}{8\pi B^2C^2}$.

For $h=-1$, eqn.(6.10) reduces to $H_y = H_z$ with suitably absorbing the constant of integration. So the mean Hubble parameter becomes $H = \frac{\dot{\mathcal{R}}}{\mathcal{R}} = \frac{1}{3}(H_x + 2H_z)$. An anisotropic relation among the directional Hubble rates is assumed in the linear form as $H_x = kH_z$. With these considerations, the system of field equations (6.6)-(6.10) can be obtained in the form of scale factor as

$$\frac{1}{(k+2)^2}\left[6(k+2)\frac{\ddot{\mathcal{R}}}{\mathcal{R}}+3(5-2k)\frac{\dot{\mathcal{R}}^2}{\mathcal{R}^2}+(k+2)^2\mathcal{R}^{\frac{-6k}{k+2}}\right]=-(\alpha+\beta)p+\rho\beta-\alpha\mu+\Lambda_0, \tag{6.11}$$

$$\frac{1}{(k+2)^2}\left[3(k^2+3k+2)\frac{\ddot{\mathcal{R}}}{\mathcal{R}}+3(2k^2+1)\frac{\dot{\mathcal{R}}^2}{\mathcal{R}^2}-(k+2)^2\mathcal{R}^{\frac{-6k}{k+2}}\right]=-(\alpha+\beta)p+\rho\beta+\alpha\mu+\Lambda_0, \tag{6.12}$$

$$\frac{1}{(k+2)^2}\left[9(2k+1)\frac{\dot{\mathcal{R}}^2}{\mathcal{R}^2}-(k+2)^2\mathcal{R}^{\frac{-6k}{k+2}}\right]=(\alpha+\beta)\rho-\beta p-\alpha\mu+\Lambda_0, \tag{6.13}$$

If we represent the L.H.S. of the field equations (6.11)-(6.13) respectively as

$$S_1 = \frac{1}{(k+2)^2}\left(6(k+2)\frac{\ddot{\mathcal{R}}}{\mathcal{R}} + 3(5-2k)\frac{\dot{\mathcal{R}}^2}{\mathcal{R}^2} + (k+2)^2\mathcal{R}^{\frac{-6k}{k+2}}\right),$$

$$S_2 = \frac{1}{(k+2)^2}\left(3(k^2+3k+2)\frac{\ddot{\mathcal{R}}}{\mathcal{R}} + 3(2k^2+1)\frac{\dot{\mathcal{R}}^2}{\mathcal{R}^2} - (k+2)^2\mathcal{R}^{\frac{-6k}{k+2}}\right)$$

$$S_3 = \frac{1}{(k+2)^2}\left(9(2k+1)\frac{\dot{\mathcal{R}}^2}{\mathcal{R}^2} - (k+2)^2\mathcal{R}^{\frac{-6k}{k+2}}\right),$$

and with algebraic manipulations among the field eqns. (6.11)-(6.13), we can obtain the pressure (p),energy density (ρ) and magnetic energy density (μ) respectively as

$$p = \frac{-1}{2\alpha(\alpha+2\beta)}\left[(\alpha+2\beta)S_1 + \alpha S_2 - 2\beta S_3 - 2\alpha\Lambda_0\right], \tag{6.14}$$

$$\rho = \frac{1}{2\alpha(\alpha+2\beta)}\left[\alpha S_2 - (\alpha+2\beta)S_1 + 2(\alpha+\beta)S_3 - 2\alpha\Lambda_0\right], \tag{6.15}$$

$$\mu = \left(\frac{\alpha(1+\beta)+\beta}{2\alpha^2(\alpha+2\beta)}\right)S_2 - \frac{1}{2\alpha}S_1. \tag{6.16}$$

Moreover, using eqns. (6.14)-(6.15), we can express the EoS parameter and effective cosmological constant as

$$\omega = -1 + \frac{2(\alpha+2\beta)(S_3 - S_1)}{\alpha S_2 - (\alpha+2\beta)S_1 + 2(\alpha+\beta)S_3 - 2\alpha\Lambda_0}, \tag{6.17}$$

$$\Lambda = \frac{\beta}{\alpha}\left[(S_3 - S_1)\right] + \Lambda_0, \tag{6.18}$$

To study the accelerating behaviour of the universe, we will study eqns. (6.14)-(6.18), which may give us a clear picture about the cosmic expansion of the present universe. However, in order to understand the physical behaviour of these equations, we need to have a scale factor, so that the parameters can be expressed with respect to the cosmic time. Therefore, in this theme, we have considered the power law cosmology, where the cosmic expansion is governed through the scale factor of the form $\mathcal{R}(t) = t^{\frac{m}{3}}$, where m is an arbitrary constant. The motive behind choosing such a scale factor is because of its functional simplicity and provides some important information on the dynamical behaviour of the universe. Hence, in a very general way, we can mention, the solution obtained with power law cosmology presents the matter dominated phase and accelerating phase at the later stage. The deceleration parameter for the power law cosmology can be calculated as $q = -1 + \frac{3}{m}$. It is informative to mention here that the present value of deceleration parameter for the accelerating universe is $q = -0.81 \pm 0.14$.

Before we will move to find the behaviours of the expressions defined above, we will define some physical parameters of the universe that would be instrumental to study the behaviour of the universe. The scalar expansion θ, shear scalar σ^2 and the average anisotropy parameter $(\mathcal{A})$ can be respectively defined as $\theta = \Sigma H_i$; $\sigma^2 = \frac{1}{2}\left(\Sigma_i^2 H_i - \frac{1}{3}\theta^2\right)$, $\mathcal{A} = \frac{1}{3}\Sigma\left(\frac{\Delta H_i}{H}\right)^2$. From one of the basic dynamical equations of cosmology, we know $\frac{\ddot{\mathcal{R}}}{\mathcal{R}} = -\frac{4\pi G}{3}(\rho + 3p)$. The state finder pair (r, s) give the geometrical behaviour of the cosmological model which can be computed with the higher order derivatives of the scale factors as $r = \frac{\dddot{\mathcal{R}}}{\mathcal{R}H^3}$ and $s = \frac{r-1}{3(q-\frac{1}{2})}$. The pair can be expressed with respect to pressure and energy density as $r = 1 + \frac{9(\rho+p)}{2\rho}\frac{\dot{p}}{\dot{\rho}}$ and $s = \frac{\rho+p}{p}\frac{\dot{p}}{\dot{\rho}}$. Also the deceleration parameter can be expressed as $q = -\frac{\ddot{\mathcal{R}}}{\mathcal{R}H^2} = \frac{1}{2} + \frac{3p}{2\rho}$. It can be noted that the ΛCDM model with $\omega = -1$ leads to a constant state finder parameters as $(r, s) = (1, 0)$. This means that the ΛCDM model corresponds to a fixed point $(r = 1, s = 0)$ in the state finder rs plane [215]. So, with the power law scale factor, the pair can be obtained as $r = 1 - \frac{9}{m} + \frac{18}{m^2}$ and $s = \frac{(4-2m)}{m(1-m)}$. It is observed here that for $m = 2$, the (r, s) pair reducing to (1,0).

6.3 Dynamical behaviour with power law cosmology

With power law cosmology $\mathcal{R}(t) = t^{\frac{m}{3}}$, the directional Hubble parameters take the form $H_x = \left(\frac{km}{k+2}\right)\frac{1}{t}$ and $H_y = H_z = \left(\frac{m}{k+2}\right)\frac{1}{t}$. The metric potentials can be implied as $A = t^{\frac{km}{k+2}}$, $B = C = t^{\frac{m}{k+2}}$. The space-time becomes $ds^2 = dt^2 - t^{\frac{2km}{k+2}}dx^2 - t^{\frac{2m}{k+2}}(e^{2x}dy^2 + e^{-2x}dz^2)$ and the Einstein tensor can be redefined as

$$S_1 = \frac{1}{(k+2)^2}\left(6(k+2)\frac{\ddot{\mathcal{R}}}{\mathcal{R}} + 3(5-2k)\frac{\dot{\mathcal{R}}^2}{\mathcal{R}^2} + (k+2)^2\mathcal{R}^{\frac{-6k}{k+2}}\right)$$
$$= \left(\frac{9m^2 - 6m(k+2)}{3(k+2)^2}\right)\frac{1}{t^2} + \frac{1}{t^{\frac{2km}{k+2}}}, \tag{6.19}$$

$$S_2 = \frac{1}{(k+2)^2}\left(3(k^2+3k+2)\frac{\ddot{\mathcal{R}}}{\mathcal{R}} + 3(2k^2+1)\frac{\dot{\mathcal{R}}^2}{\mathcal{R}^2} - (k+2)^2\mathcal{R}^{\frac{-6k}{k+2}}\right)$$
$$= \left(\frac{3m^2(k^2+k+1) - 3m(k^2+3k+2)}{3(k+2)^2}\right)\frac{1}{t^2} - \frac{1}{t^{\frac{2km}{k+2}}}, \tag{6.20}$$

$$S_3 = \frac{1}{(k+2)^2}\left(9(2k+1)\frac{\dot{\mathcal{R}}^2}{\mathcal{R}^2} - (k+2)^2\mathcal{R}^{\frac{-6k}{k+2}}\right)$$
$$= \left(\frac{(2k+1)m^2}{(k+2)^2}\right)\frac{1}{t^2} - \frac{1}{t^{\frac{2km}{k+2}}}, \tag{6.21}$$

Now, the parameters (6.14)-(6.18) reduce to

$$
\begin{aligned}
p &= \frac{-1}{2\alpha(\alpha+2\beta)}\left[(\alpha+2\beta)S_1+\alpha S_2-2\beta S_3-2\alpha\Lambda_0\right]\\
&= \frac{-1}{2\alpha(\alpha+2\beta)}\left[\left(\frac{m^2[(k^2+k+4)\alpha+4\beta(1-k)]-m[(k^2+5k+6)\alpha+4\beta(k+2)]}{(k+2)^2t^2}\right)\right]\\
&- \frac{1}{2\alpha(\alpha+2\beta)}\left[4\beta t^{\frac{-2km}{k+2}}-2\alpha\Lambda_0\right], \qquad (6.22)\\
\rho &= \frac{1}{2\alpha(\alpha+2\beta)}\left[\alpha S_2-(\alpha+2\beta)S_1+2(\alpha+\beta)S_3-2\alpha\Lambda_0\right]\\
&= \frac{1}{2\alpha(\alpha+2\beta)}\left[\frac{m^2((k^2+5k)\alpha+4\beta(k-1))-m(\alpha(k^2+k-2)-4\beta(k+2))}{(k+2)^2t^2}-2\alpha\Lambda_0\right], \qquad (6.23)\\
\mu &= \left(\frac{\alpha(1+\beta)+\beta}{2\alpha^2(\alpha+2\beta)}\right)S_2-\frac{1}{2\alpha}S_1\\
&= \left[\left(\frac{\alpha(1+\beta)+\beta}{2\alpha^2(\alpha+2\beta)}\right)\left(\frac{m^2(k^2+k+1)-m(k^2+3k+2)}{(k+2)^2}\right)-\frac{1}{2\alpha}\left(\frac{3m^2-2m(k+2)}{(k+2)^2}\right)\right]\frac{1}{t^2}\\
&- \left(\frac{\alpha(1+\alpha+3\beta)+\beta}{2\alpha^2(\alpha+2\beta)}\right)t^{\frac{-2km}{k+2}}, \qquad (6.24)\\
\omega &= -1+\frac{2(\alpha+2\beta)(S_3-S_1)}{\alpha S_2-(\alpha+2\beta)S_1+2(\alpha+\beta)S_3-2\alpha\Lambda_0}\\
&= -1+\frac{m^2[4(\alpha+2\beta)(k-1)]+m[4(\alpha+2\beta)(k+2)]-4\beta(k+2)^2t^{\frac{2k(1-m)+4}{k+2}}}{m^2((k^2+5k)\alpha+4\beta(k-1))-m(\alpha(k^2+k-2)-4\beta(k+2))-2\alpha(k+2)^2t^2\Lambda_0}, \qquad (6.25)\\
\Lambda &= \frac{\beta}{\alpha}\left[(S_3-S_1)\right]+\Lambda_0\\
&= \frac{\beta}{\alpha}\left[\frac{(2k-2)m^2+2m(k+2)}{(k+2)^2t^2}-2t^{\frac{-2km}{k+2}}\right]+\Lambda_0. \qquad (6.26)
\end{aligned}
$$

We can now discuss the dynamical behaviour of the model from the above equations. In other words, we can say that the formalism as described above can help us to study the expansion rate of the universe for an assumed dynamics of the universe. In the above equations, all the physical quantities are expressed with respect to cosmic time; however, we can very much define a relation between the scale factor and the redshift as $1+z=\frac{1}{\mathcal{R}}$. Now, we can illustrate the physical parameters of the model graphically with respect to the redshift z. It is important to mention here that the recent observational results predicts a transition redshift of $0.4\leq z_t\leq 0.8$. We have expressed the physical quantities in the Planckian unit system ($c=G=h=1$), where c, G and h are the generic constants in the Einstein field equation of GR. Also 1 unit of cosmic time = 10 billion years.

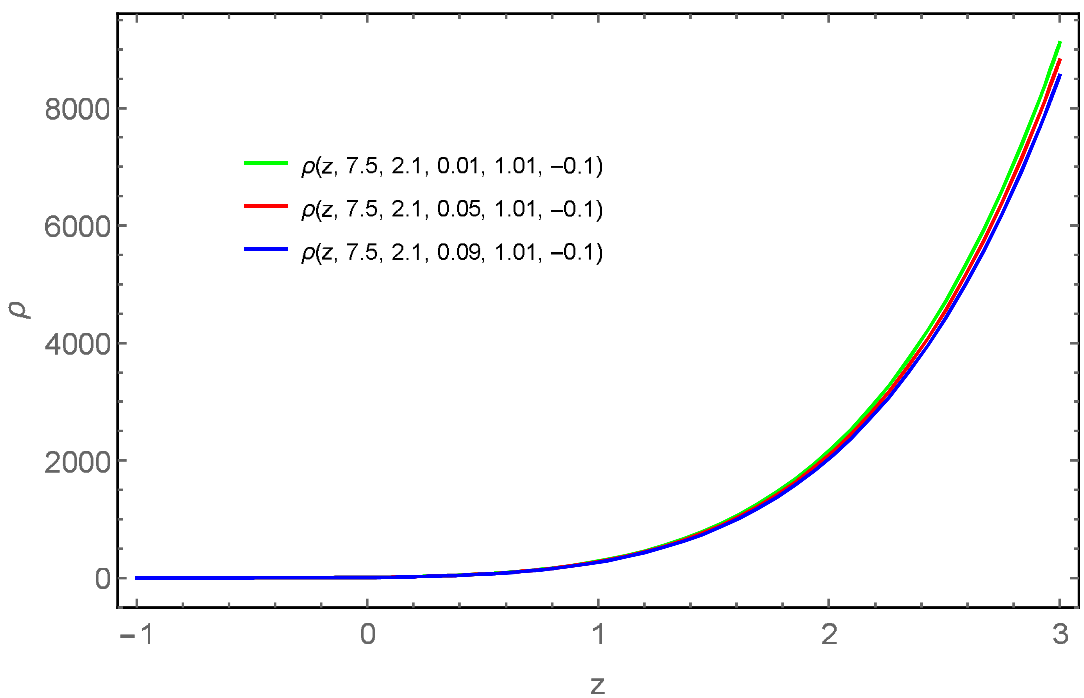

Figure 6.1: Energy Density (ρ) versus redshift(z) ($m = 7.5$,$\alpha = 2.1$, $\beta = 0.01$, $k = 1.01$, $\Lambda_0 = -0.1$)

We have chosen different representative value for the free parameters $m = 7.5$, $\alpha = 2.1$, $\beta = 0.01, 0.05, 0.09$, $k = 1.01$, $\Lambda_0 = -0.1$ in order to keep the redshift in the observed range. It is noticed that the deviation free anisotropic parameter k has the range $-1 < k < 1.01$ with three representative values of $\beta = 0.01, 0.05, 0.09$. For such formation, the behaviour of energy density (ρ) (Fig. 6.1) stay in the positive range and decrease to small values at the late times. The isotropic pressure(Fig. 6.2) remains negative for all these assumed values throughout the evolution as expected which in turn indicates the accelerated expansion of the universe. The concrete dynamical nature of the model can be observed through the evolution of the EoS parameter ω(Fig. 6.3). For all the above considered values of parameters, ω stays negative. Mostly it remains in the phantom region. However, it evolves from a phantom phase to a quintessence phase after crossing the phantom divide. It is clear from the figure that ω increases from some higher negative value at early time and overlaps with the Λ CDM model at late times. The time varying cosmological constant with the considered values of the parameters remains in negative domain. When α is considered to be negative, there is some deviation in cosmological constant, it stays negative for higher value of α at beginning then becomes positive

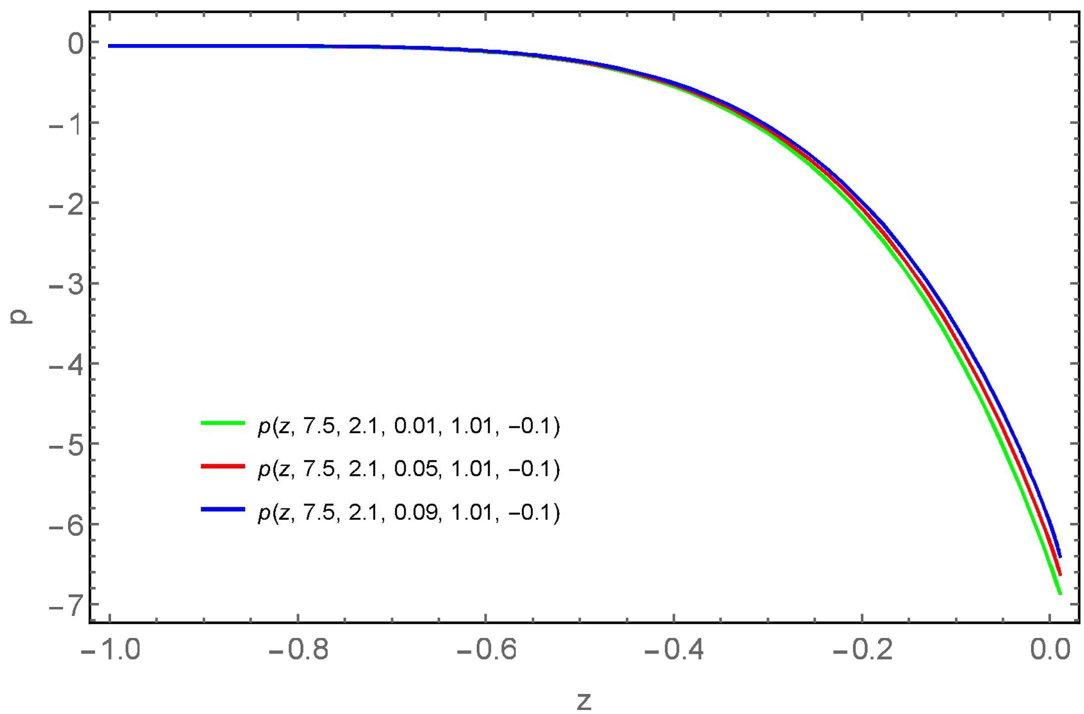

Figure 6.2: Pressure(p) versus redshift(z) ($m = 7.5$,$\alpha = 2.1$, $\beta = 0.01$, $k = 1.01$, $\Lambda_0 = -0.1$)

for low values of α at late time. The behaviour of magnetic field term μ is quite clear from Fig. 6.4. Though at the initial stage it has some role in the expansion history of the universe but at late times it does not show any significant effects to the dynamics of the universe.

6.4 Energy conditions of the model

The energy conditions are the co-ordinate invariants that incorporates the common characteristics shared by almost every matter field. As standard matter is assumed to satisfy the necessary energy conditions, so for a magnetic fluid distribution, the general inequalities of energy conditions are (i) Null Energy Condition (NEC): $\rho + p \geq 0$ (ii) Weak Energy Condition (WEC): $\rho + p \geq 0$, $\rho \geq 0$ (iii) Strong Energy Condition (SEC): $\rho + 3p \geq 0$, $\rho + p \geq 0$ (iv) Dominant Energy Condition (DEC): $\rho \pm p \geq 0$, $\rho \geq 0$. These conditions show that the violation of NEC leads to the violation of all other conditions(Raychaudhuri Equations). The energy conditions

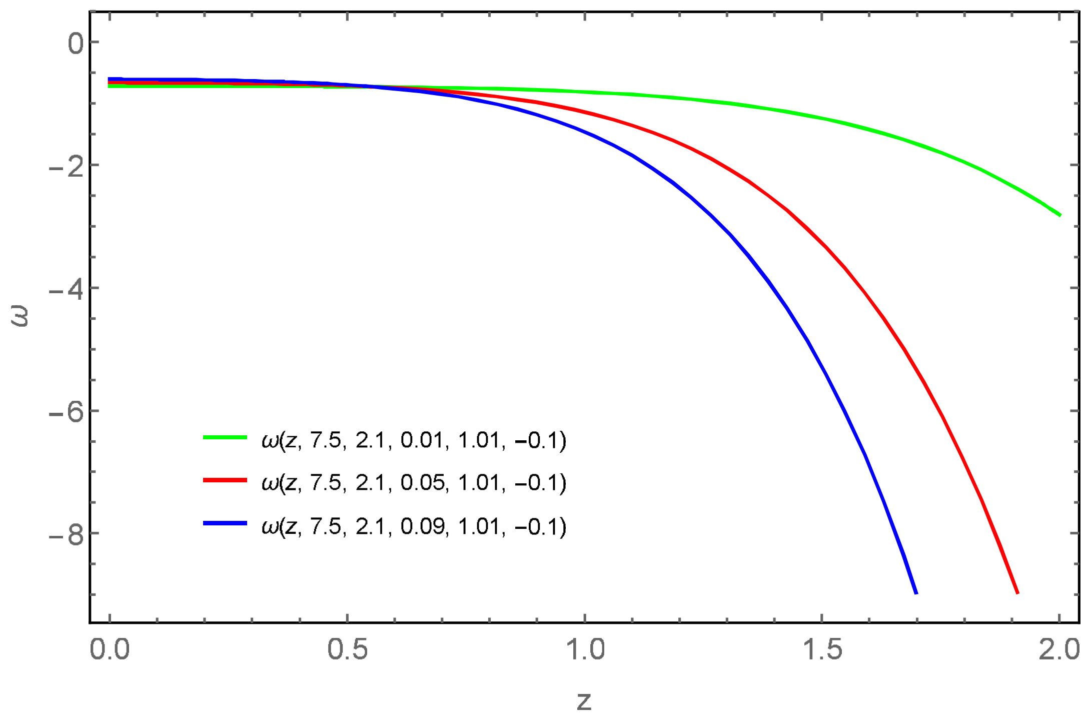

Figure 6.3: EoS parameter versus redshift (z) ($m = 7.5$,$\alpha = 2.1$, $\beta = 0.01$, $k = 1.01$, $\Lambda_0 = -0.1$)

for the cosmological model discussed here takes the following form

$$\begin{aligned}
\rho + p &= \frac{1}{\alpha(\alpha+2\beta)}\left[(\alpha+3\beta)S_3 - \alpha(\alpha+2\beta)S_1\right] \geq 0 \\
&= \frac{1}{2\alpha(\alpha+2\beta)}\left[\frac{m^2[4(\alpha+2\beta)(k-1)] + m[4(\alpha+2\beta)(k+2)]}{(k+2)^2t^2} - 4\beta t^{\frac{-2km}{k+2}}\right] \geq 0,
\end{aligned} \tag{6.27}$$

$$\begin{aligned}
\rho &= \frac{1}{2\alpha(\alpha+2\beta)}\left[\alpha S_2 - (\alpha+2\beta)S_1 + 2(\alpha+\beta)S_3 - 2\alpha\Lambda_0\right] \geq 0 \\
&= \frac{1}{2\alpha(\alpha+2\beta)}\left[\frac{m^2((k^2+5k)\alpha + 4\beta(k-1)) - m(\alpha(k^2+k-2) - 4\beta(k+2))}{(k+2)^2t^2} - 2\alpha\Lambda_0\right] \geq 0,
\end{aligned} \tag{6.28}$$

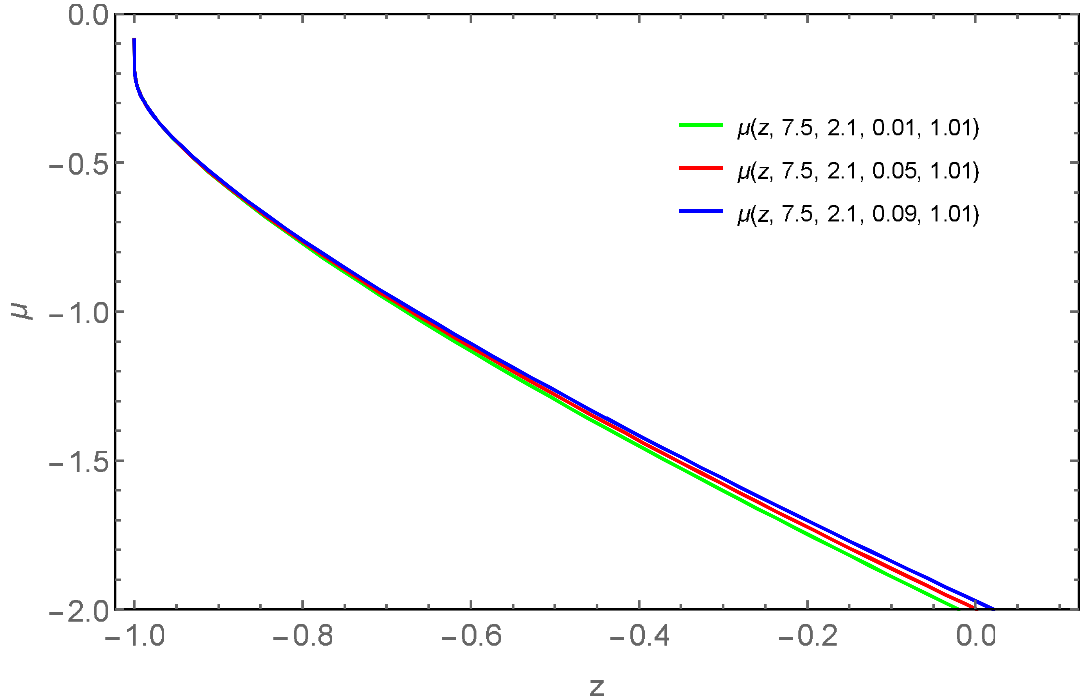

Figure 6.4: μ versus redshift(z) ($m = 7.5$,$\alpha = 2.1$, $\beta = 0.01$, $k = 1.01$, $\Lambda_0 = -0.1$)

$$\begin{aligned}
\rho + 3p &= \frac{1}{\alpha(\alpha+2\beta)}\left[-\alpha S_2 - 2(\alpha+2\beta)S_1 + (\alpha+3\beta)S_3 + 2\alpha\Lambda_0\right] \leq 0 \\
&= \frac{1}{\alpha(\alpha+2\beta)}\left[\frac{m^2[2\alpha(-k^2+k-6)+16\beta(k-1)] + m[2\alpha(k^2+7k+10)+10\beta(k+2)]}{(k+2)^2t^2}\right] \\
&+ \frac{1}{\alpha(\alpha+2\beta)}\left[-6\beta t^{\frac{-2km}{k+2}} + 2\alpha\Lambda_0\right] \leq 0, \qquad (6.29)\\
\rho - p &= \frac{1}{(\alpha+2\beta)}\left[S_2 + S_3 - 2\Lambda_0\right] \geq 0 \\
&= \frac{1}{(\alpha+2\beta)}\left[\frac{\alpha[(m^2-m)(k^2+3k+2)]}{(k+2)^2t^2} + 2\beta t^{\frac{-2km}{k+2}} - 2\Lambda_0\right] \geq 0. \qquad (6.30)
\end{aligned}$$

With the same representative values of the physical parameters, we have graphically represented the energy conditions of the model with power law cosmology (Fig. 6.5). We have considered the present observational value of deceleration parameter, jerk parameter and snap parameters as $q_0 = -0.81 \pm 0.14$, $r_0 = 2.16^{+0.81}_{-0.75}$ and $s_0 = -0.22^{+0.21}_{-0.19}$. It is clear from the figure that the present model is satisfying all the energy conditions except SEC.

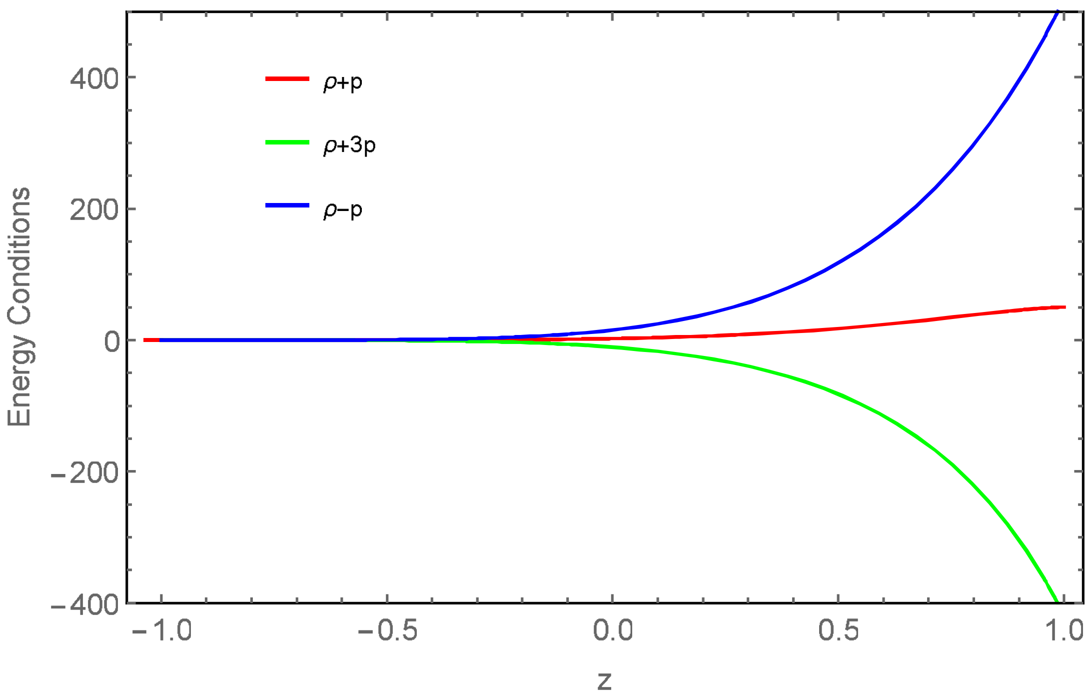

Figure 6.5: Energy conditions versus redshift(z) ($m = 7.5$,$\alpha = 2.1$, $\beta = 0.01$, $k = 1.01$, $\Lambda_0 = -0.1$)

6.5 Conclusion

In this chapter, we have studied the dynamical behaviour of Bianchi type $VI_h(h = -1)$ cosmological model in presence of magnetic field. We have developed a formalism to find the physical parameters from the field equations and with the power law assumption the cosmological model is constructed. We assume the functional in the form $f(R,T) = R + 2\Lambda_0 + 2\beta T$ to obtain the model in $f(R,T)$ gravity. The dynamical behaviour of the model is also investigated. We have observed that, the value of the anisotropy parameter k and the coupling constant β are played significant role to align the behaviour of the physical parameters with the observational results. The presence of magnetic field in the field equations shows a substantial effect on the dynamical behaviour initially but at late times the effect is minimal. The energy conditions in modified theory of gravity have a well defined physical motivation along with attractive nature of the gravity in GR. In this work all energy conditions are satisfied except SEC.

CHAPTER 7

Cosmological Models with a Hybrid Scale Factor-I

Contents of this chapter has been submitted for publication

7.1 Introduction

The most fascinating point in cosmological models on modified gravity is that generally the models are strongly constrained by cosmological observations. The most simple modification to GR has been suggested as $f(R)$ theory [216]. Recently the generalization of $f(R)$ gravity has been developed, which is known as $f(R,T)$ gravity [76]. Shabani and Farhoudi [133] obtained the cosmological parameters in terms of some defined dimensionless parameters that are used in constructing the dynamical equations of motion. Junior et al. [217] have reconstructed the ΛCDM model for $f(T,T)$ gravity. Velten and Carames [218] have challenged the viability of $f(R,T)$ as an alternative modification of gravity. Abbas and Ahmed [219] have formulated the exact solutions of the non-static anisotropic gravitating source in $f(R,T)$ gravity which may lead to expansion and collapse. Carvalho et al. [220] have shown the equilibrium configurations of white dwarfs in a modified gravity theory. Sharif and Nawazish [221] have determined the existence of noether symmetry isotropic universe in non-minimally coupled $f(R,T)$ gravity admitting minimal coupling with scalar field models. Yadav et al. [222] have studied the LRS Bianchi type I dark energy model in $f(R,T)$ gravity with hybrid law expansion. Esmaeilli [223] has studied the anisotropic behaviour of the Bianchi type I cosmological model in $f(R,T)$ gravity in the form of perfect fluid. The cosmological models are presented using power form of exponential function and hyperbolic form.

In section 7.2, we have presented the basic field equations in $f(R,T)$ gravity with some relevant quantities on the geometrical aspects of the proposed model. In section 7.3, the anisotropic nature of the space time has been discussed. The equation of state parameter and energy conditions are presented in section 7.4 and in section 7.5, the scalar field reconstruction is presented. Cosmological model with hybrid scale factor has been written in section 7.6. In section 7.7, we have given some diagnostic approach of the model and the physical parameters are discussed in section 7.8 and finally the conclusion at 7.9.

7.2 Basic field equations of $f(R,T) = R + 2\Lambda_0 + 2\beta T$

Within the scope of an extended gravity theory as proposed by Harko et al. [76], the Einstein-Hilbert action is given by (1.41), we expressed the Einstein field equation with consideration of the functional as $f(R,T) = R + 2\Lambda_0 + 2\beta T$ is

$$G_{\mu\nu} = \kappa T^{eff}_{\mu\nu}, \tag{7.1}$$

with

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}, \tag{7.2}$$

$$T^{eff}_{\mu\nu} = T_{\mu\nu} + \frac{\Lambda_{eff}(T)}{8\pi + 2\beta}\, g_{\mu\nu}. \tag{7.3}$$

Here we have a redefined Einstein matter-geometry coupling constant $\kappa = 8\pi + 2\beta$ and $\Lambda_{eff}(T) = (2p + T)\beta + \Lambda_0$. For $\beta = 0$, $\Lambda_{eff}(T)$ becomes the usual cosmological constant Λ_0 and the above field equation reduces to the Einstein field equations with a cosmological constant. For non-vanishing value of β, $\Lambda_{eff}(T)$ becomes a time dependent quantity.

We consider the universe to be filled with a cloud of one dimensional cosmic strings with string tension density ξ aligned along the x-axis. The energy-momentum tensor for such a fluid is given by,

$$T_{\mu\nu} = (p + \rho) u_\mu u_\nu - p g_{\mu\nu} - \xi x_\mu x_\nu, \tag{7.4}$$

with

$$u^\mu u_\mu = -x^\mu x_\mu = 1, \tag{7.5}$$

and

$$u^\mu x_\mu = 0. \tag{7.6}$$

ρ represents the energy density and is composed of the particle energy density ρ_p and the string tension density ξ, $\rho = \rho_p + \xi$.

We wish to investigate dynamical aspects of the universe in $f(R,T)$ theory as described above for an anisotropic Bianchi type VI_h (BVI_h) space-time given by (1.65).

Now, the field equations for BVI_h space-time with $h = -1$ in the extended theory of gravity can be written as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{1}{A^2} = -\alpha(p - \xi) + \rho\beta + \Lambda_0, \tag{7.7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2} = -\alpha p + (\rho + \xi)\beta + \Lambda_0, \tag{7.8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = -\alpha p + (\rho + \xi)\beta + \Lambda_0, \tag{7.9}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{1}{A^2} = \alpha\rho - (p - \xi)\beta + \Lambda_0, \tag{7.10}$$

$$\frac{\dot{B}}{B} = \frac{\dot{C}}{C}. \tag{7.11}$$

Here $\alpha = 8\pi + 3\beta$ and we denote the ordinary time derivatives as overhead dots. With $h = -1$, it is straightforward to get $H_y = H_z$ from (7.11). The mean Hubble parameter becomes, $H = \frac{\dot{\mathcal{R}}}{\mathcal{R}} = \frac{1}{3}(H_x + 2H_z)$ where a is the mean scale factor of universe. The set of field eqns. (7.7)-(7.11) can be reduced to

$$\frac{6}{(k+2)}\dot{H} + \frac{27}{(k+2)^2}H^2 + \frac{1}{A^2} = (k+2)^2\left[-\alpha(p-\xi) + \rho\beta + \Lambda_0\right], \tag{7.12}$$

$$\frac{3(k+1)}{(k+2)}\dot{H} + \frac{9(k^2+k+1)}{(k+2)^2}H^2 - \frac{1}{A^2} = (k+2)^2\left[\alpha p + (\rho+\xi)\beta + \Lambda_0\right], \tag{7.13}$$

$$\frac{9(2k+1)}{(k+2)^2}H^2 - \frac{1}{A^2} = (k+2)^2\left[\alpha\rho - (p-\xi)\,\beta + \Lambda_0\right]. \tag{7.14}$$

Some relevant quantities in the context of discussion of geometrical aspect of the model include

$$\text{Hubble rate:} \quad H = \frac{\dot{\mathcal{R}}}{\mathcal{R}} = \frac{1}{3}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right), \tag{7.15}$$

$$\text{Expansion scalar:} \quad \theta = u^l_{;l} = \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right), \tag{7.16}$$

$$\text{Deceleration parameter:} \quad q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right). \tag{7.17}$$

7.3 Anisotropic nature of the model

In this case, we have considered a spatially homogeneous and anisotropic BVI_h universe with different expansion rates along different spatial directions. The quantities that measures the departure from spatial isotropy are the Shear scalar σ^2 and the average anisotropy parameter $\mathcal{A}$ defined respectively as

$$\sigma^2 = \frac{1}{2}\left(\sum H_i^2 - \frac{1}{3}\theta^2\right) = \frac{1}{3}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^2, \tag{7.18}$$

$$\mathcal{A} = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\triangle H_i}{H}\right)^2. \tag{7.19}$$

$\triangle H_i = H_i - H$, where the directional Hubble rates are as in (2.11). In view of eq.(7.11), we have $H_y = H_z$. For isotropic models these quantities σ^2 and $\mathcal{A}$ identically vanish. From observational perspectives, the anisotropic nature of a model is usually quantified through the estimation of the amplitude of shear $\frac{\sigma}{H}$ at the present epoch. Using the data from differential microwave radiometers aboard the Cosmic Background Explorer (COBE), Bunn et al. have placed an upper limit to this quantity as $\left(\frac{\sigma}{H}\right)_0 < 3\times10^{-9}$ [224]. For the best case with $\Omega_0 = 1$, they have obtained $\left(\frac{\sigma}{H}\right)_{pl} \simeq 10^{-3} - 10^{-4}$ [224]. In that work Bunn et al. [224] have concluded that primordial anisotropy should have been fine tuned to be less than 10^{-3} of its natural value in the Planck era. Saadeh et al. used CMB temperature and polarisation data from Planck and obtained a tighter limit to the anisotropic expansion as $\left(\frac{\sigma}{H}\right)_0 < 4.7\times10^{-11}$ [186]. In view of these recent observational limits on cosmic shear and anisotropic expansion rates, we have adopted a simple approach in the present chapter and have assumed a proportional relationship between the amplitude of shear σ and Hubble rate. This assumption leads to an anisotropic relation among the directional Hubble rates $H_x = kH_y$. The parameter k takes care of the anisotropic feature of the model. Obviously, $k \neq 1$ provides an anisotropic model. For the Bianchi VI_h metric, we obtain the amplitude of shear expansion in the present epoch as

$$\left(\frac{\sigma}{H}\right)_0 = \sqrt{3}\left(\frac{k-1}{k+2}\right). \tag{7.20}$$

Even though tighter constraints on cosmic anisotropy are available in literature and evidences against the departure from global isotropy are being gathered, these observational analysis need to be fine tuned as the analysis are prior dependent (Saadeh et al. [225]). In view of this, we wish to construct some accelerating anisotropic models keeping enough room for any amount of cosmic anisotropy. However, we can set some constraints on the parameter k basing upon the observationally found upper bound on $\left(\frac{\sigma}{H}\right)_0$. While the bounds of Bunn et al. [224] constrain k as $k = 1.000000008$, that of Saadeh et al. [186] disfavours any classical finite departure from $k = 1$. However, in the present work, we consider $k = 1.0000814$ that provides $\left(\frac{\sigma}{H}\right)_0 = 4.7\times10^{-5}$, a result obtained in an earlier work (Mishra et al. [226]).

Within the formalism discussed here it is easy to show that $\mathcal{A} = \frac{2}{9}\left(\frac{\sigma}{H}\right)^2$. Consequently, the average anisotropic parameter in the present epoch can be calculated as $\mathcal{A}_0 = 4.91\times10^{-10}$. Since the large scale structure of the universe may show a departure from isotropy, the cosmic anisotropy can be estimated from Hemispherical asymmetries in the Hubble expansion. In a recent work, Kalus et al. [227] estimated the Hubble anisotropy of supernova type Ia Hubble diagrams at low redshifts ($z < 0.2$) as $\frac{\triangle H}{H} < 0.038$ [227]. Using the value of the anisotropy parameter k at the present epoch, we obtain the expansion asymmetry as $\frac{\triangle H}{H} = 0.814\times10^{-4}$. One can note that, the predicted anisotropy from our model is well within the observationally set up bounds (Campanelli et al. [228]).

7.4 EoS parameter and energy conditions

The presumed anisotropic relation among the directional Hubble rates has a simplified structure and within this formalism it can provide us a simple approach to study the cosmic dynamics. For a given anisotropic parameter k, the directional Hubble rates become $H_x = \left(\frac{3k}{k+2}\right) H$ and $H_y = H_z = \left(\frac{3}{k+2}\right) H$.

The physical properties of the model such as pressure, energy density and string tension density are obtained from the field equations (7.7)-(7.11) as

$$p = \left(\frac{1}{\alpha^2-\beta^2}\right)\left[\left(S_1(H)-S_2(H)+S_3(H)\right)\beta - S_2(H)\alpha\right] \tag{7.21}$$

$$+\left(\frac{1}{\alpha^2-\beta^2}\right)\left[\left(\alpha-\beta\right)\Lambda_0\right], \tag{7.22}$$

$$\rho = \left(\frac{1}{\alpha^2-\beta^2}\right)\left[S_3(H)\alpha - S_1(H)\beta - \left(\alpha-\beta\right)\Lambda_0\right], \tag{7.23}$$

$$\xi = \frac{S_1(H)-S_2(H)}{\alpha-\beta}, \tag{7.24}$$

where

$$S_1(H) = \frac{1}{(k+2)^2}\left[6(k+2)\dot{H} + 27H^2 + (k+2)^2\,\mathcal{R}^{-\left(\frac{6k}{k+2}\right)}\right], \tag{7.25}$$

$$S_2(H) = \frac{1}{(k+2)^2}\left[3(k^2+3k+2)\dot{H} + 9(k^2+k+1)H^2\right] - \mathcal{R}^{-\left(\frac{6k}{k+2}\right)}, \tag{7.26}$$

$$S_3(H) = \frac{1}{(k+2)^2}\left[9(2k+1)H^2 - (k+2)^2\,\mathcal{R}^{-\left(\frac{6k}{k+2}\right)}\right]. \tag{7.27}$$

Algebraic simplification of the above expressions yield

$$p = -\left(\frac{1}{\alpha^2-\beta^2}\right)\left[\phi_1(k,\beta)\dot{H} + \phi_2(k,\beta)H^2 - (\alpha+\beta)\,\mathcal{R}^{-\left(\frac{6k}{k+2}\right)}\right] - \frac{\Lambda_0}{(\alpha+\beta)}, \tag{7.28}$$

$$\rho = \left(\frac{1}{\alpha^2-\beta^2}\right)\left[\phi_3(k,\beta)\dot{H} + \phi_4(k,\beta)H^2 - (\alpha+\beta)\,\mathcal{R}^{-\left(\frac{6k}{k+2}\right)}\right] - \frac{\Lambda_0}{(\alpha+\beta)}, \tag{7.29}$$

$$\xi = \left(\frac{1}{\alpha-\beta}\right)\left[\phi_5(k)\left(\dot{H}+3H^2\right) + 2\,\mathcal{R}^{-\left(\frac{6k}{k+2}\right)}\right], \tag{7.30}$$

where

$$\phi_1(k,\beta) = \frac{3}{k+2}[(k+1)\alpha+(k-1)\beta], \tag{7.31}$$

$$\phi_2(k,\beta) = \left(\frac{3}{k+2}\right)^2[(k^2+k+1)\alpha+(k^2-k-3)\beta], \tag{7.32}$$

$$\phi_3(k,\beta) = -\frac{6\beta}{k+2}, \tag{7.33}$$

$$\phi_4(k,\beta) = \left(\frac{3}{k+2}\right)^2[(2k+1)\alpha-3\beta], \tag{7.34}$$

$$\phi_5(k) = \frac{3(1-k)}{k+2}. \tag{7.35}$$

It is interesting to note that for $\alpha+\beta=0$ i.e. for $\beta=-2\pi$ we have

$$\phi_1(k,\beta)=\phi_3(k,\beta) \quad \text{and} \quad \phi_2(k,\beta)=\phi_4(k,\beta), \tag{7.36}$$

and consequently in the limit $\beta\rightarrow -2\pi$,

$$p=-\rho. \tag{7.37}$$

In other words, within the scope of the present formalism, ΛCDM model with $p=-\rho$ can be recovered from the model for $\alpha+\beta=0$. Of course, overlapping of the present model with that of ΛCDM requires a negative coupling constant.

The EoS parameter is defined as the pressure to energy density ratio, $\omega=\frac{p}{\rho}$. For $\alpha\neq\pm\beta$, it is straightforward to obtain ω as

$$\omega=-1+(\alpha+\beta)\,\frac{S_2(H)-S_3(H)}{S_1(H)\beta-S_3(H)\alpha+(\alpha-\beta)\,\Lambda_0}. \tag{7.38}$$

As is obvious from the above expression, the dynamical behaviour of the EoS parameter depends on the parameters of the Hubble rate H and the coupling constant β. For any realistic cosmological model, the Hubble parameter is a decreasing function of time and therefore at late phase of cosmic evolution, we expect that, the functionals $S_2(H)$ and $S_3(H)$ will behave alike thereby cancelling each other at late times. Therefore, for any value of $\beta(\neq -2\pi, \neq -4\pi)$, the EoS parameter behaves as a cosmological constant ($\omega=-1$) at late epoch. However, at an early epoch, the Hubble rate assumes a very high value thereby pushes ω to a larger value.

In the limit $\beta\rightarrow 0$, the model reduces to that of GR and the EoS parameter becomes

$$\omega=-1+\frac{S_2(H)-S_3(H)}{\Lambda_0-S_3(H)}, \tag{7.39}$$

which becomes $\omega = -\frac{S_2(H)}{S_3(H)}$ in the absence of a cosmological constant Λ_0 term in the field equations. One can note that, similar conclusion on the dynamical evolution of ω as above may be derived for $\beta \to 0$. In other words, the dynamical behaviour of the EoS parameter will not be sensitive to the choice of the coupling constant at late times. All the trajectories of ω will behave alike at late phase of cosmic evolution. However, at an early epoch, the model will pass through different trajectories which may be β dependent.

Another dynamical parameter is the effective cosmological constant Λ_{eff} that appear in the equivalent Einstein Field equation for the extended gravity theory. Unlike the dynamical cosmological constant in GR, this effective cosmological constant depends on the matter field content such as the pressure and energy density. We can obtain Λ_{eff} as

$$\Lambda_{eff} = \left(\frac{\beta}{\alpha+\beta}\right)\left[(S_1(H)+S_3(H))-2\Lambda_0\right]+\Lambda_0. \tag{7.40}$$

In terms of the Hubble parameter, we may express Λ_{eff} as

$$\Lambda_{eff} = \frac{\beta}{\alpha+\beta}\left[\frac{6}{k+2}(\dot{H}+3H^2)-2\Lambda_0\right]+\Lambda_0. \tag{7.41}$$

For $\alpha+\beta \neq 0$, the magnitude of the effective cosmological constant decreases with the growth of cosmic time. The sign of this quantity will depend on the sign of β. Since at late times the contribution coming from the term $\dot{H}+3H^2$ is negligible, Λ_{eff} reduces to $\left(\frac{\alpha-\beta}{\alpha+\beta}\right)\Lambda_0$. Obviously as mentioned earlier, for a vanishing coupling constant β, it reduces to the usual time independent cosmological constant Λ_0.

Since energy conditions put some additional constraints on the viability of the models we wish to calculate the different energy conditions for the constructed model in the modified gravity theory. In our formalism, the energy conditions are obtained as

$$\begin{aligned}
\textbf{NEC} &: \rho+p = \frac{[S_3(H)-S_2(H)]}{\alpha-\beta} \geq 0,\\
\textbf{WEC} &: \rho = \left(\frac{1}{\alpha^2-\beta^2}\right)\left[S_3(H)\alpha - S_1(H)\beta - (\alpha-\beta)\Lambda_0\right] \geq 0,\\
\textbf{SEC} &: \rho+3p = \left(\frac{1}{\alpha^2-\beta^2}\right)\left[(S_3(H)-3S_2(H))\alpha + (2S_1(H)-3S_2(H)+3S_3(H))\beta\right]\\
&\quad + \frac{2\Lambda_0}{(\alpha+\beta)} \geq 0,\\
\textbf{DEC} &: \rho-p = \left(\frac{1}{\alpha^2-\beta^2}\right)\left[(S_2(H)+S_3(H))\alpha - (2S_1(H)-S_2(H)+S_3(H))\beta\right]\\
&\quad - \frac{2\Lambda_0}{(\alpha+\beta)} \geq 0,
\end{aligned}$$

These energy conditions are expressed in terms of the Hubble parameter as

$$\textbf{NEC}: \rho + p = \frac{-1}{\alpha+\beta}\left[\frac{k(k-1)}{(k+2)^2}9H^2 + \frac{k+1}{(k+2)}3\dot{H}\right], \tag{7.42}$$

$$\textbf{WEC}: \rho \geq 0, \tag{7.43}$$

$$\begin{aligned}\textbf{SEC}: \rho + 3p = &\frac{-1}{\alpha^2-\beta^2}\left[\left(\frac{3k^2+k+2}{(k+2)^2}9H^2 + \frac{k+1}{(k+2)}9\dot{H} - \frac{2}{A^2}\right)\alpha\right] \\ &- \frac{1}{\alpha^2-\beta^2}\left[\left(\frac{k^2-k-2}{(k+2)^2}27H^2 + \frac{3k-1}{(k+2)}3\dot{H} - \frac{2}{A^2}\right)\beta\right] - \frac{2\Lambda_0}{(\alpha+\beta)},\end{aligned} \tag{7.44}$$

$$\begin{aligned}\textbf{DEC}: \rho - p = &\frac{1}{\alpha^2-\beta^2}\left[\left(\frac{k^2+3k+2}{(k+2)^2}9H^2 + \frac{k+1}{(k+2)}3\dot{H} - \frac{2}{A^2}\right)\alpha\right] \\ &+ \frac{1}{\alpha^2-\beta^2}\left[\left(\frac{k^2-k-6}{(k+2)^2}9H^2 + \frac{k-3}{(k+2)}3\dot{H} - \frac{2}{A^2}\right)\beta\right] - \frac{2\Lambda_0}{(\alpha+\beta)}.\end{aligned} \tag{7.45}$$

We wish to present our model in such a manner that the WEC be satisfied through out the cosmic evolution. In order to achieve this, one has to take a balance between the parameters of the Hubble rates and the choice of the coupling constant β. Since at late times, our model overlaps with ΛCDM model, the NEC and DEC are satisfied atleast at late phase of cosmic evolution. On the other hand, the SEC condition is violated at late times even though there occurs some possibility that SEC be satisfied at an early epoch. In fact, a detailed analysis on these energy condition may be possible once the cosmic dynamics is fixed up from an assumed or derived Hubble rate.

7.5 Scalar field reconstruction

In GR, the late time cosmic acceleration phenomena is modelled usually through a scalar field ϕ which may either be quintessence like or phantom like with the EoS parameter being $\omega \geq -1$ or $\omega \leq -1$ respectively. The action for such cases is given by

$$S_\phi = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi} + \frac{\epsilon}{2}\partial_\mu\phi\,\partial^\mu\phi\, - V(\phi)\right], \tag{7.46}$$

where $\epsilon = +1$ for quintessence field and $\epsilon = -1$ for phantom field. $V(\phi)$ is the self-interacting potential of the scalar field. The scalar field dynamically rolls down the potential and thereby mediating for cosmic acceleration. In this work, we wish to draw a correspondence between the geometrically modified gravity theories discussed above with that of the scalar field cosmology and also wish to reconstruct the scalar field along with the scalar potential. In a flat Friedman background, the energy density and pressure are expressed by

$$\rho_\phi = \frac{\epsilon}{2}\dot{\phi}^2 + V(\phi), \tag{7.47}$$

$$p_\phi = \frac{\epsilon}{2}\dot{\phi}^2 - V(\phi). \tag{7.48}$$

A direct correspondence of our model with the scalar field yields

$$\dot{\phi}^2 = \frac{\epsilon}{\alpha-\beta}\left[S_3(H) - S_2(H)\right], \tag{7.49}$$

$$V(\phi) = \left(\frac{1}{\alpha^2-\beta^2}\right)\left[\left\{S_2(H)+S_3(H)\right\}\alpha - \left\{2S_1(H) - S_2(H) + S_3(H)\right\}\beta\right] - \frac{2\Lambda_0}{(\alpha+\beta)}. \tag{7.50}$$

Since the factor $S_3(H) - S_2(H)$ decreases with the cosmic evolution, we expect the magnitude of $\dot{\phi}$ to decrease with cosmic time. It is worth to mention here that the exact behaviour of the scalar field will be model dependent and can be investigated with some specific evolutionary behaviour of the Hubble rate.

7.6 Model with a hybrid scale factor

The formalism developed in this work can be used to investigate certain aspects of cosmic dynamics. One can note that all the dynamical properties are expressed in terms of the Hubble rate H. Therefore, if for a given dynamics the Hubble rate is known, then it becomes easy to track the evolution history. In view of this, we employ a hybrid scale factor (HSF) $\mathcal{R} = e^{at}t^b$ in the formalism. Here a and b are the model parameters and are constrained from different observational and physical basis. The reason behind the choice of such a scale factor is that it simulates a transition from a decelerated universe in recent past to an accelerated one. Moreover, the dynamical behaviour of HSF as predicted remains intermediate to that of the power law expansion and exponential expansion. The parameters of HSF have been constrained in some of our earlier works [229]. Transit redshift z_t is an important cosmological parameter which has been recently been constrained from an analysis of type Ia Supernova observation and Hubble parameter measurements as $z_t = 0.806$ (Jesus et al. [230], Farooq et al. [231]. In a recent work, we have constrained the parameters of HSF as $a = 0.695$ and $b = 0.085$ so as to obtain a transition redshift $z_t = 0.806$. The Hubble parameter for the HSF is given by $H = a + \frac{b}{t}$ so that the directional Hubble rates become $H_1 = \frac{3k}{k+2}\left(a + \frac{a}{b}\right)$ and $H_2 = H_3 = \frac{3}{k+2}\left(a + \frac{a}{b}\right)$.

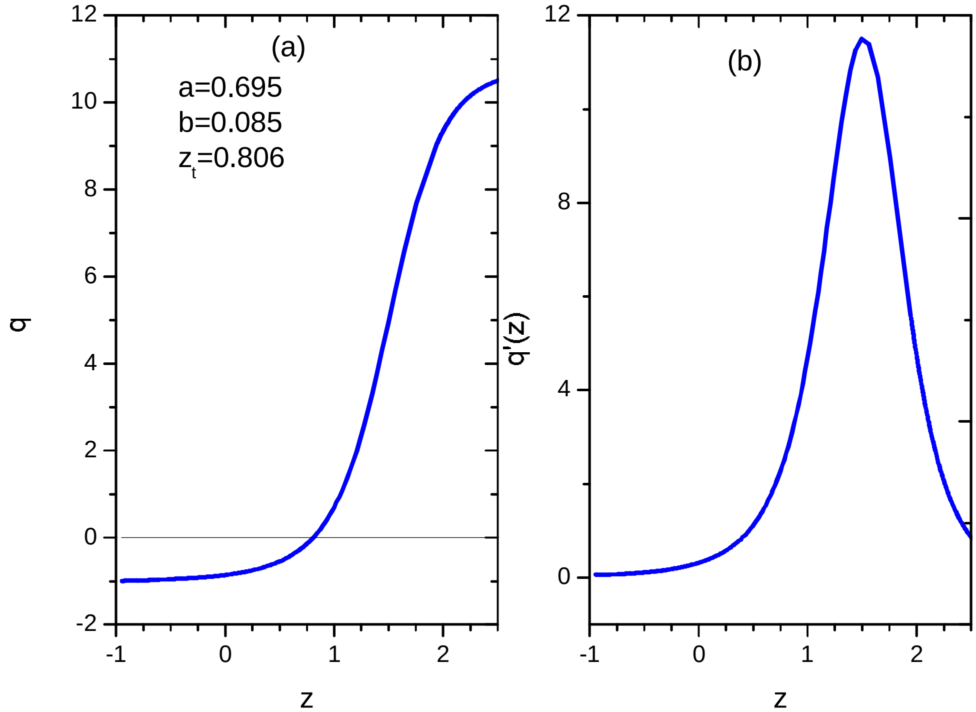

Figure 7.1: (a)Deceleration parameter for HSF showing the transition redshift (b) $q'(z)$ as a function of redshift. The model does not favour a slowing down at late phase.

The deceleration parameter for HSF is $q = -1 + \frac{b}{(at+b)^2}$. In Fig. 7.1(a), we have shown the deceleration parameter q which displays the signature flipping behaviour at a suitable transit redshift. Recently, analysis from a host of Hubble parameter measurements and type Ia Supernova observational data casts a doubt that, the universe has already reached the peak of its acceleration and may be we are currently witnessing a possible slowing down (Shefieloo et al. [232], Zhang and Xia [233]. Such a feature is investigated through the reconstruction of the slope of the deceleration parameter from observations. In order to check whether the HSF can predict such a feature we have plotted the function $q'(z) = \frac{dq}{dz}$ as a function of redshift $z = \frac{\mathcal{R}_0}{\mathcal{R}}$ in Fig.7.1(b). Here $\mathcal{R}_0$ is the scale factor at present epoch. The figure shows that there is no slowing down in cosmic acceleration at late phase of cosmic time. However, we find an interesting feature where $q'(z)$ peaks up at around $z = 1.5$. In order to have a quantitative idea about the deceleration parameter, we have listed some of its values at different epochs in Table-7.1.

In Fig.7.2, the dynamical aspect of the model is assessed through the plot of the EoS parameter

Table 7.1: Deceleration parameter at different epochs

epoch	z	q
Late phase	-0.9	-0.99
Present	0	-0.86
At transit	0.8	0
Early phase	1.5	11.5

as function of redshift. In the figure ω is shown for a fixed anisotropic parameter $k = 1.0000814$ and for three different values of the coupling constant β. In general, the EoS parameter decreases from an initial positive value to behave like a cosmological constant at late phase. The initial positive value depends on the choice of β. As expected, at late times, the model is insensitive to the choice of the coupling constant β. However at an early epoch, the EoS parameter evolves through different trajectories. The ω trajectory for low values of β remains in the top of others at early epoch. The β dependent splitting in these trajectories is visible at around $z = 1.5$. At the present epoch, the model predicts an EoS of $\omega = 0.89$ which is well within the observational constraints. In the figure, for a comparison, we have also shown the trajectories for two well known ω parametrizations such as CPL (Chevallier and Polarski [234], Linder [235]) and BA (Barboza and Alcaniz [236]) given by $\omega(z) = \omega_0 + \omega_a \frac{z}{1+z}$ and $\omega(z) = \omega_0 + \omega_0 \frac{z(1+z)}{1+z^2}$ respectively. It is clear the comparison that, in a time zone in the range $-0.25 \leq z \leq 0.5$, EoS from HSF is in close agreement with other models.

The effect of the anisotropic parameter k on the EoS is investigated in Fig.7.3. In the figure, we have shown the evolution of ω for a given coupling constant $\beta = -0.5$ for three representative values of k namely $k = 0.8, 1.0000814$ and 1.2. We note here that, we have considered a specific shear expansion to Hubble rate ratio in the present epoch within the observational limits and have constrained k to be 1.0000814. This value of k shows a very little departure from its isotropic value. Unlike that of the coupling constant, cosmic expansion anisotropy affects the cosmic dynamics both at an early epoch and at late times. The effect of cosmic anisotropy is almost symmetrical about $z = 1.15$. At epochs $z < 1.15$, higher the value of k, lower is the value of ω and at epochs $z \geq 1.5$, the EoS shows an opposite behaviour i.e higher the value of k, higher is the value of ω. A quantitative idea on the effect of the cosmic expansion anisotropy

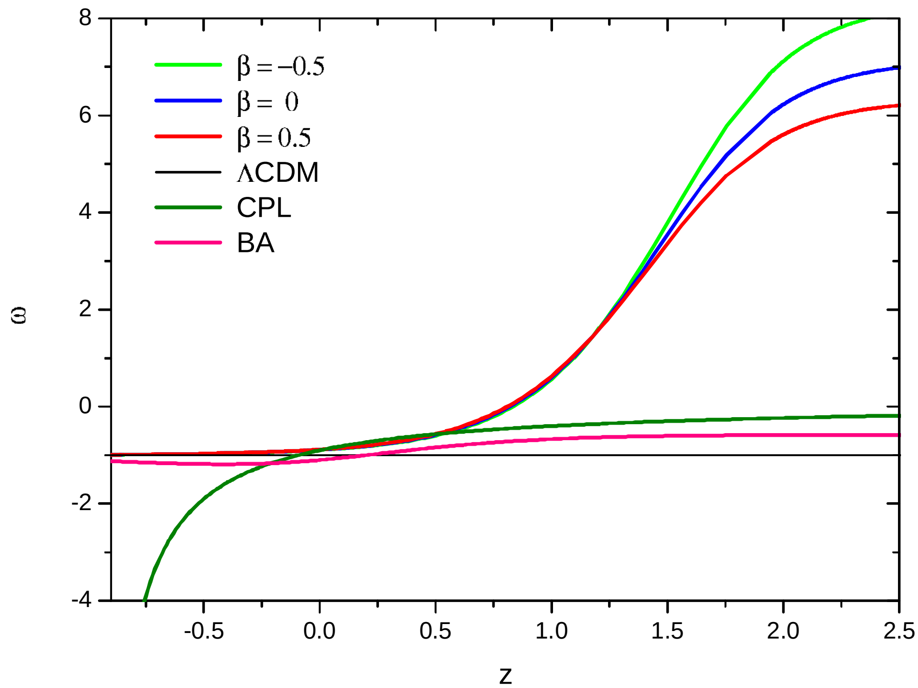

Figure 7.2: EoS parameter for three representative values of the coupling constant β. $k = 1.0000814$

on the EoS can be obtained from the values listed in Table-7.2.

In order to assess a simultaneous effect of the coupling constant β and the cosmological constant Λ_0 on the EoS parameter, we have shown the variation of ω at the present epoch with respect to β for four different values of Λ_0 and a given cosmic anisotropy $k = 1.0000814$ in Fig.7.4. The representative values of Λ_0 are considered as multiples of the present value of energy density ρ_0. One should note that, the present values of the energy density ρ_0 are also β dependent. ω increases with the increase in the coupling constant for a given value of Λ_0. Also for a given β, it increases with the increase in Λ_0. However, the net variation of ω with respect to β for the range of β considered in the figure for a given value of Λ_0 decreases with an increase in Λ_0. To get a quantitative view, in Table-7.3, the values of the EoS parameter at the present epoch are given for some representative values of the cosmological constant and the coupling constant.

In Fig.7.5, the quintessence like scalar fields are reconstructed from our model for three representative values of the coupling constant β. The anisotropy parameter k and the time independent cosmological constant Λ_0 are chosen to be 1.0000814 and ρ_0 respectively. As per expectation, the scalar field is found to decrease with the cosmic expansion. In Fig. 7.6, the evolution of the

Table 7.2: Variation of EoS parameter with anisotropy parameter

epoch	z	$k = 0.8$	$k = 1.0000814$	$k = 1.2$
Late phase	-0.9	-0.932	-0.994	-1.064
Present	0	-0.819	-0.892	-0.971
At transit	0.8	-0.012	-0.08	-0.152
Early phase	1.5	3.68	3.76	3.86

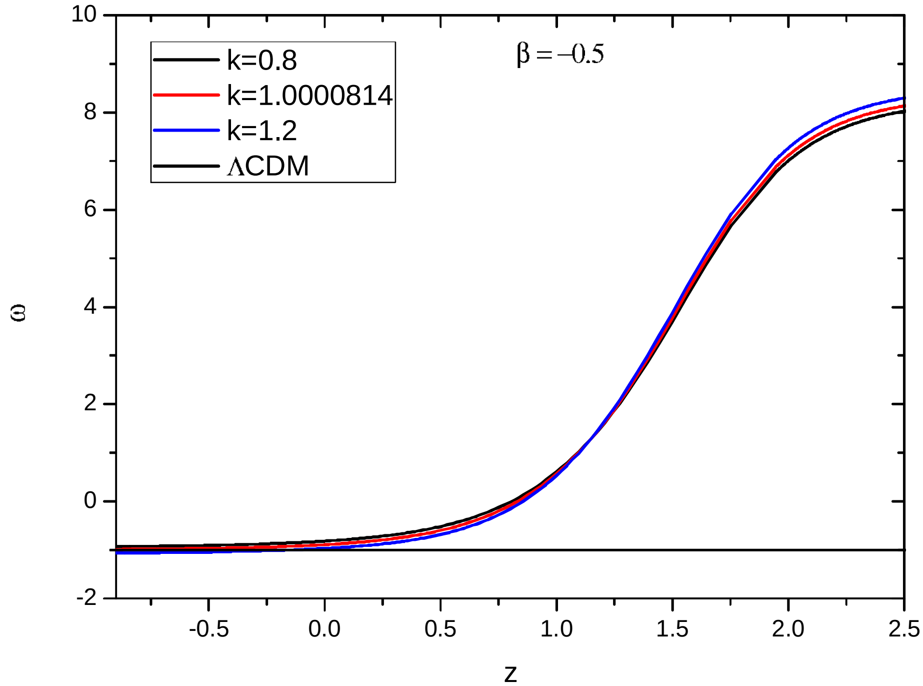

Figure 7.3: EoS parameter for three representative values of the anisotropy parameter k. $\beta = -0.5$

self-interacting potential for the quintessence like scalar filed is plotted. The self-interacting potential increases with cosmic expansion. The choice of the coupling constant β does not affect the general evolutionary behaviour of these two quantities. However with an increase in β value at a given epoch, the scalar field decreases and the potential increases.

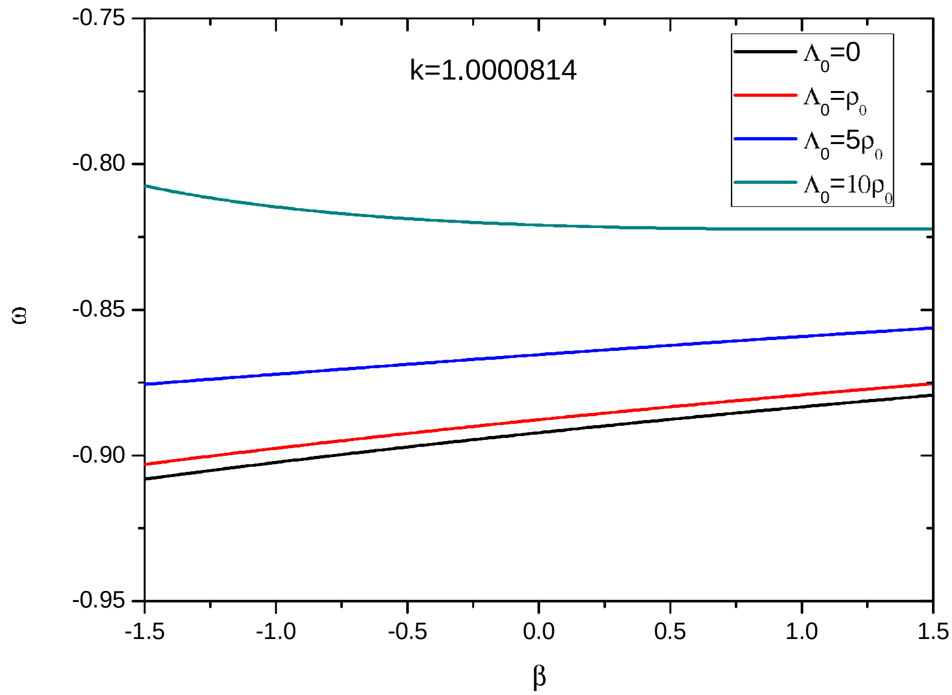

Figure 7.4: EoS parameter as function of coupling constant β for three representative values of the cosmological constant . $k = 1.0000814$

7.7 Diagnostic approach

There are two important diagnostic approaches used in literature. They are the determination of the state finder pair $\{j, s\}$ in the $j - s$ plane and the $Om(z)$ diagnostics. These geometrical diagnostic approaches are useful tools to distinguish different DE models. While the state finder pair involve third derivatives of the scale factor, the $Om(z)$ parameter involve only the first derivative of the scale factor appearing through the Hubble rate $H(z)$.

7.7.1 State finder diagnostic

State finder pairs provide a useful tool to distinguish DE models since they involve the third derivative of the scale factor. They are defined as

Table 7.3: EoS parameter at present epoch for different values of cosmological constant Λ_0

Λ_0	$\omega_0(\beta = -1)$	$\omega_0(\beta = -0.5)$	$\omega_0(\beta = 0)$	$\omega_0(\beta = 0.5)$	$\omega_0(\beta = 1)$
0	-0.902	-0.897	-0.892	-0.888	-0.883
ρ_0	-0.898	-0.892	-0.888	-0.883	-0.879
$5\rho_0$	-0.872	-0.869	-0.866	-0.862	-0.859
$10\rho_0$	-0.815	-0.819	-0.821	-0.822	-0.822

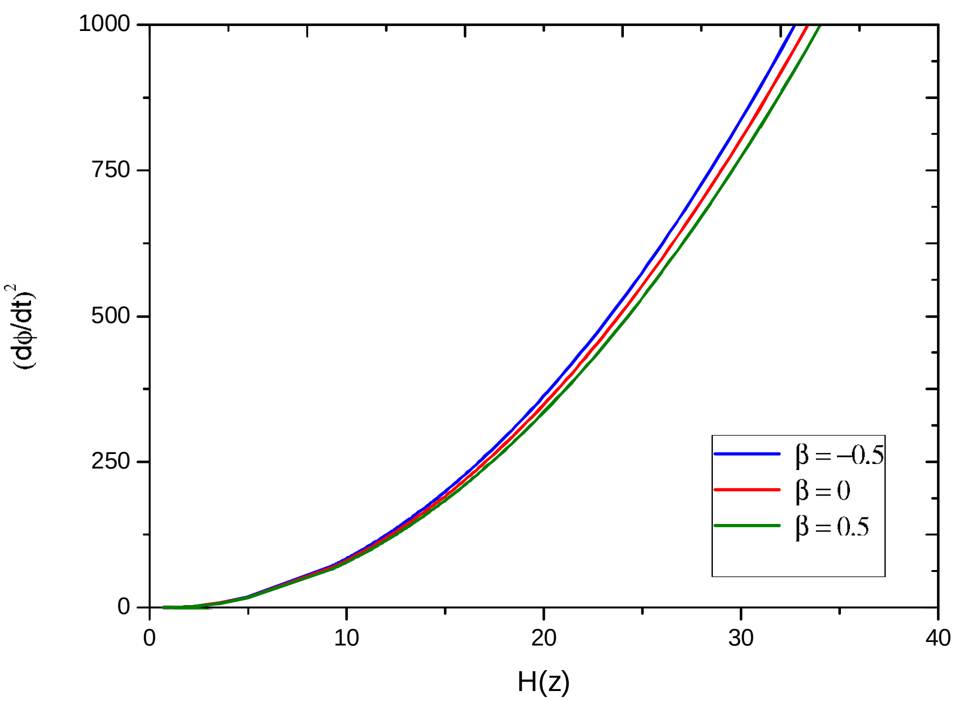

Figure 7.5: Squared Slope of Reconstructed Scalar field as a function of Hubble rate.

$$j = \frac{\dddot{\mathcal{R}}}{\mathcal{R}H^3} = \frac{\ddot{H}}{H^3} - (2+3q), \tag{7.51}$$

$$s = \frac{j-1}{3(q-0.5)}. \tag{7.52}$$

In our formalism, the deceleration parameter is a time varying quantity and therefore the state

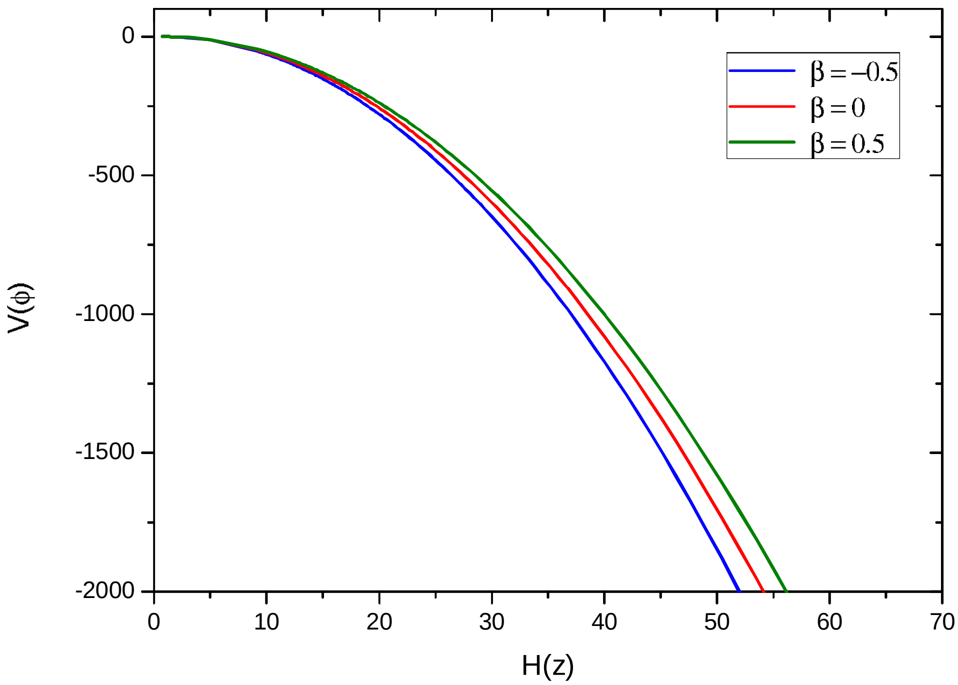

Figure 7.6: Scalar potential as a function of Hubble rate.

finder pair evolve with time. In Fig.7.7, the $j-s$ trajectory in the $j-s$ plane is shown for the HSF considered in this work. We observe that, our model evolves to overlap with the ΛCDM model in the present epoch.

7.7.2 $Om(z)$ diagnostic

Another geometric diagnostic methods is the $Om(z)$ diagnostic that involves first derivative of the scale factor and therefore becomes easier to apply to distinguish between different DE models (Sahni et al. [237]). The $Om(z)$ parameter is defined by

$$Om(z) = \frac{E^2(z) - 1}{(1+z)^3 - 1}, \tag{7.53}$$

where $E(z) = \frac{H(z)}{H_0}$ is the dimensionless Hubble parameter. Here H_0 is the Hubble rate at the present epoch. If $Om(z)$ becomes a constant quantity, the DE model is considered to be a cosmological constant model with $\omega = -1$. If this parameter increases with z with a positive

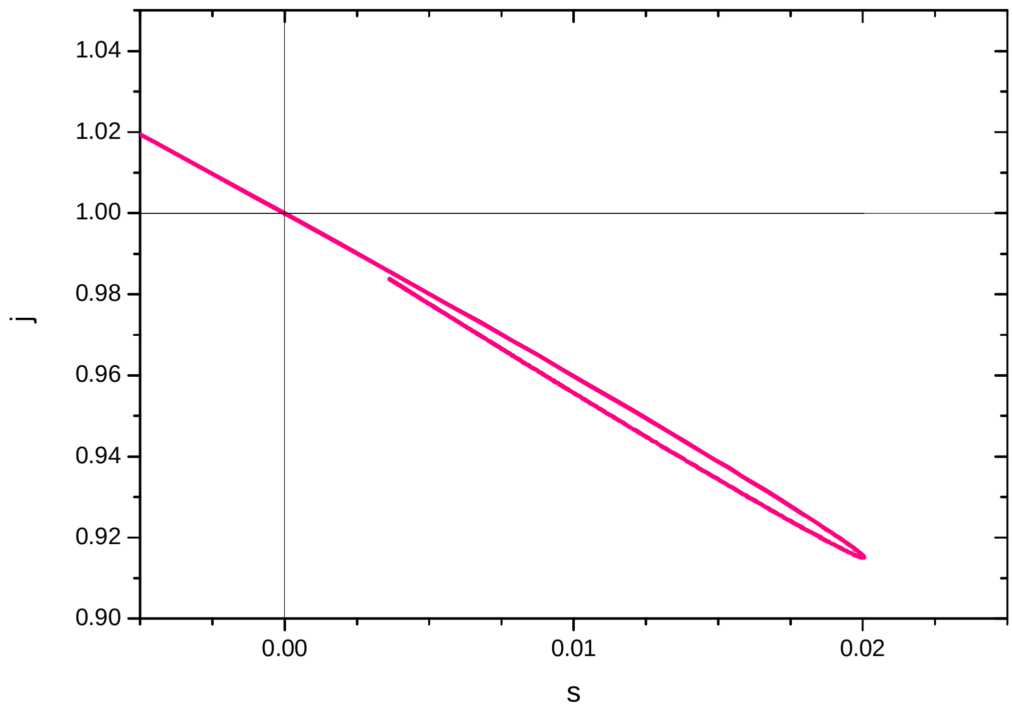

Figure 7.7: $j-s$ trajectory in the $j-s$ plane.

slope, the model can be a phantom model with $\omega < -1$. For a decreasing $Om(z)$ with negative slope, quintessence model are obtained ($\omega > -1$). In Fig.7.8, the $Om(z)$ parameter for HSF is shown as a function of redshift. It can be observed from the figure that, the model looks like a cosmological constant model for a substantial time zone in the recent past ($0 \leq z \leq 0.7$). Before this period, the model evolves as a phantom field.

7.8 Physical parameters of the model

Some relevant geometrical parameters such as the scalar expansion θ, shear scalar σ^2, average anisotropy parameter $\mathcal{A}$ are expressed as

$$\theta = 3H = (k+2)H_z, \tag{7.54}$$

$$\sigma^2 = \frac{1}{2}\left(\Sigma H_i^2 - \frac{1}{3}\theta^2\right) = \frac{1}{3}(k^2-2k+1)H_z^2, \tag{7.55}$$

$$\mathcal{A} - \frac{1}{3}\left(\frac{\Delta H_i}{H}\right)^2 - \frac{2}{3}\left(\frac{k-1}{k+2}\right)^2. \tag{7.56}$$

Of these geometrical parameters, θ and σ^2 depend on the cosmic dynamics whereas the average anisotropic parameter depends only on k.

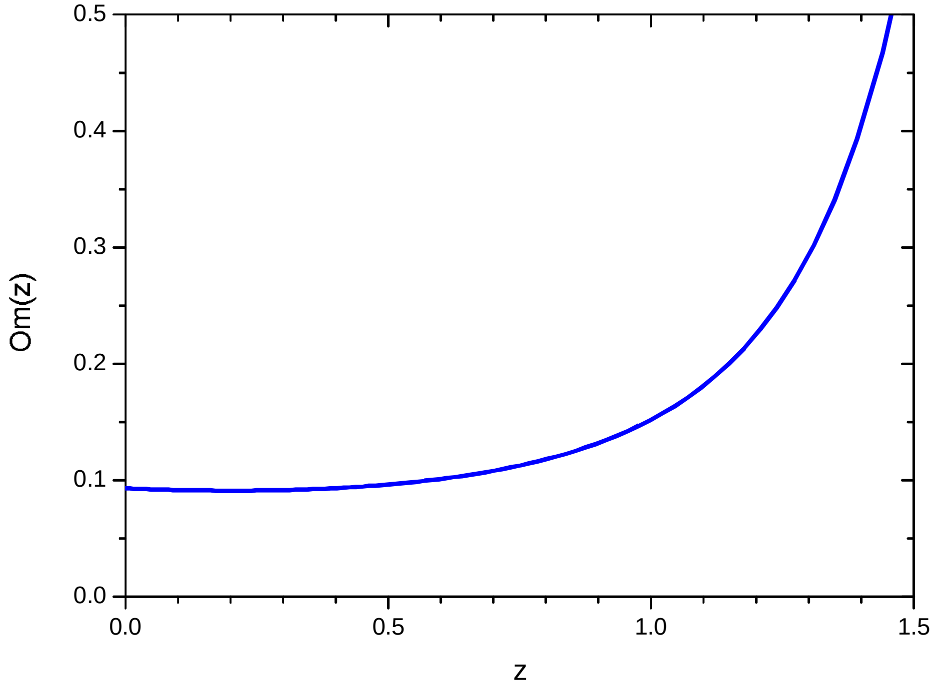

Figure 7.8: $Om(z)$ parameter.

Other dynamical parameters that depend on the higher derivatives of the scale factors are the deceleration parameter (DP) $q = -1 - \frac{\dot{H}}{H^2}$ and the jerk parameter $j = \frac{\dddot{\mathcal{R}}}{\mathcal{R}H^3}$. For the present model they are given by

$$q = -1 - \left(\frac{3}{k+2}\right)\frac{\dot{H}_z}{H_z^2}, \tag{7.57}$$

$$j = \left(\frac{3}{k+2}\right)^2 \frac{\ddot{H}_z}{H_z^3} - (2+3q). \tag{7.58}$$

One should note that, once the dynamical behaviour of H_z is known, then the evolutionary aspects of these two parameters can be well assessed.

7.9 Conclusion

In the present work, we have constructed a cosmological model in an extended theory of gravity by considering the functional $f(R,T) = R + 2\Lambda_0 + \beta T$, where Λ_0 is a constant. This model reduces to the usual GR equations with a cosmological constant in the limit of a vanishing coupling constant β. Investigation of dynamical features of universe in such an extended theory requires an involved calculation. In order to study certain dynamical cosmic aspects, we have adopted an interesting approach in the present work and obtained the expressions in a more general manner. Although the cosmological principle assuming a homogeneous and isotropic universe is a good approximation to the present universe, it is yet to be proven in high energy scales. In view of this, we have considered an anisotropic universe which is more general than the FRW model for our purpose. The anisotropic model we have constructed can be applicable to any amount of cosmic anisotropy. The anisotropic behaviour can be assessed through the value of the anisotropy parameter at the present epoch which has been constrained as $k = 1.0000814$. This value of cosmic anisotropy leads to $\left(\frac{\sigma}{H}\right)_0 = 4.7 \times 10^{-5}$. The expansion asymmetry from our model is obtained to be $\frac{\triangle H}{H} = 0.814 \times 10^{-4}$ which is in conformity with the observations.

A dynamically changing universe with a feature of early deceleration and late time cosmic acceleration is simulated through a hybrid scale factor. The parameters of the HSF are constrained from some physical basis to reproduce the transition redshift as obtained from different observational analysis. This HSF provides a good estimate of the deceleration parameter and the Hubble rate at the present epoch. Recently there has been a belief that, we are at the peak of the cosmic acceleration and the universe is now slowing down. We have investigated such a feature of the universe employing the HSF and obtained that there is no such slowing down in recent past or recent future.

The dynamical behaviour of the model is assessed through the calculation of the EoS parameter employing the HSF. The EoS parameter decreases from a positive value in an early phase to a value closer to -1 at late times. The behaviour of the EoS parameter is sensitive to the choice of the coupling constant at late times and all the trajectories of EoS parameter for different choices of the coupling parameter behave alike at late phase. However at an early phase, the trajectory splits into different β channels. Trajectory with low values of β lies in the top of all trajectories. Different diagnostic approaches have been adopted to analyse the viability of the present constructed model. At late phase, the model looks like a ΛCDM model for a substantial cosmic time zone. In the rest phase, it behaves as a quintessence field.

CHAPTER 8

Cosmological Models with a Hybrid Scale Factor-II

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8.1 Introduction:

Bianchi type models with anisotropic spatial sections are interesting in the sense that they are more general than the Friedman models. Even though there is a strong debate going on the viability of Bianchi type models [186], these models can be useful in the description of early inflationary phase and with suitable mechanism can be reduced to isotropic behaviour at late times. In the framework of $f(R)$ gravity, Shamir and co authors have investigated different aspects of Bianchi type models [238, 239, 240]. Momeni and Gholizade obtained some cylindrically symmetric solutions in this theory [241]. Singh et al. [242] have demonstrated the bouncing scenario in the frame work of $f(R,T)$ gravity. Nagpal et al. [243] have presented a $\Lambda(t)$ cosmological model obtained by a simple parametrization of the Hubble parameter in a flat FLRW space-time in $f(R,T)$ gravity. Moraes [244] obtained some exact solutions in higher dimensional space-time in $f(R,T)$ gravity. Zubair and Hassan [245] have reconstructed cosmological models for Bianchi type I, III and Kantowski-Sachs solutions.

The chapter is arranged as follow: in section 8.2, we have developed the basic field equations in the framework of $f(R,T)$ gravity and derived the relevant geometrical parameters. In section 8.3, the dynamics of the model is presented along with the energy conditions. The model is compared with some other observational aspects in section 8.4 and also shown the reduction to power law and exponential law. The conclusion is given in section 8.5.

8.2 Basic equations

In this section, we discuss briefly the formalism developed to investigate certain models in a minimally coupling $f(R,T)$ theory. We consider a Bianchi VI_h (BVI_h) space time (1.65) and study the anisotropic nature of the model. The matter field is considered through an energy momentum tensor

$$T_{ij} = (p+\rho)u_i u_j - p g_{ij} - \rho_B x_i x_j. \tag{8.1}$$

Here $u^i x_i = 0$ and $x^i x_i = -u^i u_i = -1$. In a co moving coordinate system, $u^i = \delta^i_0$ is the four velocity vector of the fluid. x^i represents the direction of anisotropic fluid (here x-direction) and is orthogonal to u^i. The energy density ρ is composed of energy density due to the perfect fluid and that due to an anisotropic fluid ρ_B. For the modified gravity model, we consider here a minimal coupling of geometry and curvature assuming $f(R,T) = f(R) + f(T)$.

A specific choice of $f(R,T) = \lambda(R+T)$ leads to the field equations (2.4) as

$$G_{ij} = \left(\frac{8\pi + \lambda}{\lambda}\right) T_{ij} + \Lambda(T) g_{ij}, \tag{8.2}$$

where λ is a non zero scaling factor that rescales the usual field equations in GR. The factor $\Lambda(T) = p + \frac{1}{2}T$ appearing in the field equation (8.2) may be identified with a time dependent effective cosmological constant. Here $\Lambda(T)$ depends on the matter content and helps in providing an acceleration. Even though, the field equations in (8.2) have the same mathematical form of GR with a time varying constant, it cannot be reduced to GR because of the non vanishing quantity λ. However, eqn.(8.2) is a rescaled generalisation of GR equations. The field equations (8.2) of the modified $f(R,T)$ gravity theory, for Bianchi type VI_h space-time(1.65) can be explicitly written as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{1}{A^2} = -\alpha\,(p - \rho_B) + \frac{\rho}{2}, \tag{8.3}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2} = -\alpha p + \left(\frac{\rho_B + \rho}{2}\right), \tag{8.4}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = -\alpha p + \left(\frac{\rho_B + \rho}{2}\right), \tag{8.5}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2} = \alpha\rho - \left(\frac{p - \rho_B}{2}\right), \tag{8.6}$$

$$\frac{\dot{B}}{B} = \frac{\dot{C}}{C}, \tag{8.7}$$

where $\alpha = \frac{16\pi + 3\lambda}{2\lambda}$ and ordinary time derivatives are denoted by overhead dots. The field equations eqn. (8.3)- eqn.(8.7) can be expressed in terms of H as

$$\frac{6}{(k+2)}\dot{H} + \frac{27}{(k+2)^2}H^2 + \frac{1}{A^2} = -\alpha(p - \rho_B) + \frac{\rho}{2}, \tag{8.8}$$

$$\frac{3(k+1)}{(k+2)}\dot{H} + \frac{9(k^2+k+1)}{(k+2)^2}H^2 - \frac{1}{A^2} = -\alpha p + \left(\frac{\rho + \rho_B}{2}\right), \tag{8.9}$$

$$\frac{9(2k+1)}{(k+2)^2}H^2 - \frac{1}{A^2} = \alpha\rho - \left(\frac{p - \rho_B}{2}\right). \tag{8.10}$$

Two different approaches can be adopted to get some viable cosmological models from the above field equations. Firstly, one can chose to assume a physically acceptable equation of state and then the dynamics of the universe is studied. On the other hand, basing upon the observationally chalked out dynamics of the universe, one may consider a presumed expansion behaviour and then the background cosmology is investigated. In the present formalism, we consider a presumed dynamical behaviour of the universe according to recent observational data and investigate the evolutionary behaviour.

8.3 Dynamics of the model

From the field equations (8.8)-(8.10), we will be able to obtain the expressions of the physical parameters of the model as

$$p = \frac{6}{(1-4\alpha^2)}\left[\frac{(k-1)+2\alpha(k+1)}{(k+2)}\frac{\ddot{\mathcal{R}}}{\mathcal{R}} + \frac{(2k^2-4k-7)+2\alpha(2k^2+1)}{(k+2)^2}\left(\frac{\dot{\mathcal{R}}}{\mathcal{R}}\right)^2\right] - \frac{2\mathcal{R}^{-\frac{6k}{k+2}}}{1-2\alpha}, \tag{8.11}$$

$$\rho = \frac{6}{(1-4\alpha^2)}\left[\frac{2}{k+2}\frac{\ddot{\mathcal{R}}}{\mathcal{R}} + \frac{(5-2k)-6\alpha(2k+1)}{(k+2)^2}\left(\frac{\dot{\mathcal{R}}}{\mathcal{R}}\right)^2\right] + \frac{2\mathcal{R}^{-\frac{6k}{k+2}}}{1-2\alpha}, \tag{8.12}$$

$$\rho_B = \frac{6}{(1-2\alpha)}\left[\left(\frac{k-1}{k+2}\right)\left(\frac{\ddot{\mathcal{R}}}{\mathcal{R}} + 2\frac{\dot{\mathcal{R}}^2}{\mathcal{R}^2}\right)\right] - \frac{4\mathcal{R}^{-\frac{6k}{k+2}}}{1-2\alpha}. \tag{8.13}$$

Other dynamical features of the model are the EoS parameter ω and the effective cosmological constant Λ. Using the scale factors, these parameters are obtained as

$$\omega = -1 + \left[\frac{(1+2\alpha)[3(k^2+3k+2)\frac{\ddot{\mathcal{R}}}{\mathcal{R}} + 6(k^2-3k-1)\frac{\dot{\mathcal{R}}^2}{\mathcal{R}^2}]}{6(k+2)\frac{\ddot{\mathcal{R}}}{\mathcal{R}} - 3(2k-5)\frac{\dot{\mathcal{R}}^2}{\mathcal{R}^2} + (k+2)^2\mathcal{R}^{-\frac{6k}{k+2}} - 2\alpha\left[9(2k+1)\frac{\dot{\mathcal{R}}^2}{\mathcal{R}^2} - (k+2)^2\mathcal{R}^{-\frac{6k}{k+2}}\right]}\right], \tag{8.14}$$

$$\Lambda = \frac{6}{(1+2\alpha)(k+2)}\left[\frac{\ddot{\mathcal{R}}}{\mathcal{R}} + 2\frac{\dot{\mathcal{R}}^2}{\mathcal{R}^2}\right]. \tag{8.15}$$

In the above equations, all the physical parameters are expressed in terms of the scale factor. Therefore, if the expansion history can be tracked by assuming a scale factor, then the background cosmology can be easily investigated. It is almost conclusive from different observational data that, the cosmic acceleration is a recent phenomena and there must have occurred a transition from deceleration to an accelerated one in recent past. One should note that a constant

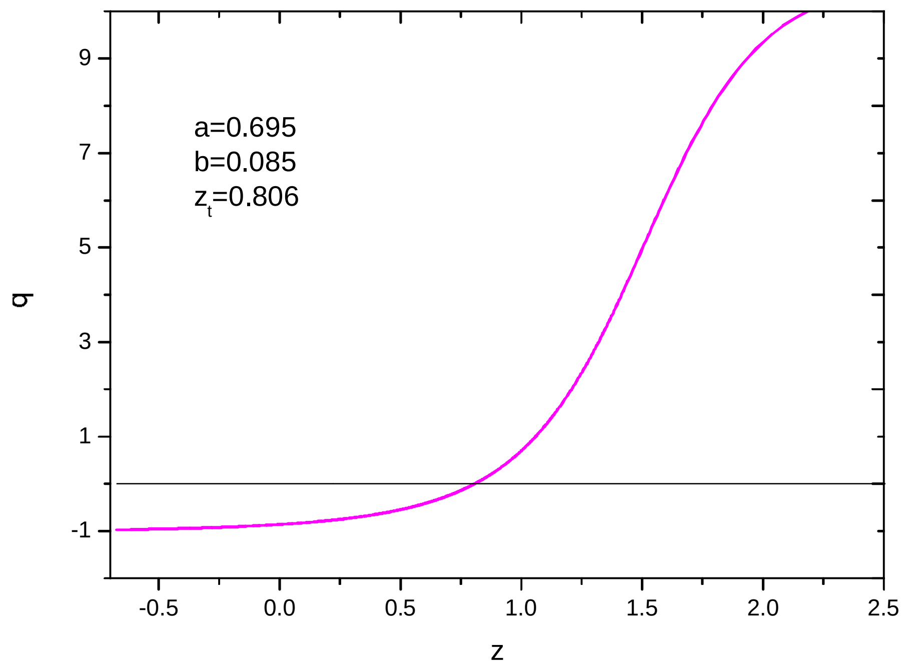

Figure 8.1: Deceleration parameter q for HSF.

deceleration parameter cannot explain such a characteristics of the expansion. In view of this a time varying DP is required to understand the present universe that can behave according to the early deceleration and late time acceleration. In other words, the deceleration parameter should be positive at some initial epoch and after a signature flipping at some point of time, it becomes negative to describe an accelerated universe. Such a deceleration parameter can be obtained by a hybrid scale factor (HSF), $\mathcal{R} = e^{at}t^b$, proposed in some earlier works [226, 229]. The time varying deceleration parameter as obtained from a HSF is given by $q = -1 + \frac{b}{(at+b)^2}$. It behaves as $q \simeq -1 + \frac{1}{b}$ when $t \to 0$ and as $t \to \infty$, it becomes $q \simeq -1$. The positive constant parameters of HSF, a and b can be constrained from the cosmic transit behaviour. Mishra and Tripathy [229] have constrained b from some physical and plausible arguments to be in a range $0 \leq b \leq \frac{1}{3}$. However, in the present work, we chose $a = 0.695$ and $b = 0.085$ so that it can predict a transition redshift of $z_t = 0.806$. This value of transition redshift has been obtained by Jesus et al. [230]. Similar results have also been obtained from an analysis of Hubble parameter data (Farooq et al. [231]). In Fig. 8.1, we have plotted the deceleration parameter obtained from the HSF that shows a transition at a redshift $z_t = 0.806$.

Assuming the HSF, we can express the directional Hubble parameters as, $H_x = \frac{3k}{k+2}(a+\frac{b}{t})$ and $H_y = H_z = \frac{3}{k+2}(a+\frac{b}{t})$. The mean Hubble rate becomes $H = a + \frac{b}{t}$. The directional scale factors can be obtained as $A = e^{\frac{akt}{k+2}} t^{\frac{bk}{k+2}}$ and $B = C = e^{\frac{at}{k+2}} t^{\frac{b}{k+2}}$. The kinematical parameters of the model with the presumed HSF are obtained as

$$\Theta = 3a + \frac{3b}{t}, \tag{8.16}$$

$$\sigma^2 = 3\left(\frac{k-1}{k+2}\right)^2 \left(a+\frac{b}{t}\right)^2. \tag{8.17}$$

It is now straight forward to obtain the expressions of the pressure p, energy density ρ and the energy density of the anisotropic fluid ρ_B from eqs. (8.11)-(8.12) using the HSF:

$$p = -\frac{6}{1-4\alpha^2}\left[\frac{b(3b\phi_2-\phi_1)+3at\phi_2(at+2b)}{(k+2)^2}\right]\frac{1}{t^2} - \frac{2}{1-2\alpha}e^{-\frac{6akt}{k+2}}t^{-\frac{6bk}{k+2}}, \tag{8.18}$$

$$\rho = \frac{6}{1-4\alpha^2}\left[\frac{3b^2\phi_3-2b(k+2)+3at\phi_3(at+2b)}{(k+2)^2}\right]\frac{1}{t^2} + \frac{2}{1-2\alpha}e^{-\frac{6akt}{k+2}}t^{-\frac{6bk}{k+2}}, \tag{8.19}$$

$$\rho_B = \frac{6(k-1)}{1-2\alpha}\left[\frac{b(3b-1)+3at(at+2b)}{k+2}\right]\frac{1}{t^2} - \frac{4}{1-2\alpha}e^{-\frac{6akt}{k+2}}t^{-\frac{6bk}{k+2}}. \tag{8.20}$$

In the above equations, we have redefined the constants as $\phi_1 = (2-k-k^2)-2\alpha(k^2+3k+2)$, $\phi_2 = (3+k-k^2)-2\alpha(k^2+k+1)$ and $\phi_3 = 3-2\alpha(2k+1)$. Consequently, the EoS parameter ω and Λ are obtained as

$$\omega = -1 + \left[\frac{3(1+2\alpha)\left(3b[3b(k^2-k)-(k^2+3k+2)]+9(k^2-k)at(at+2b)\right)}{3[3b^2\phi_3-2b(k+2)+3at\phi_3(at+2b)]+(1+2\alpha)(k+2)^2e^{-\frac{6akt}{k+2}}t^{\frac{2(k+2-3bk)}{k+2}}}\right], \tag{8.21}$$

$$\Lambda = \frac{6}{(k+2)(1+2\alpha)}\left[b(3b-1)+3at(at+2b)\right]\frac{1}{t^2}. \tag{8.22}$$

8.4 Comparison with CPL and BA

In order to assess the dynamical aspects of the model, we have plotted the EoS parameter ω as a function of redshift in Fig. 8.2. In this figures, we have fixed the anisotropic parameter as $k = 1.1$. The HSF parameters as constrained from recent transition redshift data are considered for plotting the figure. ω decreases from a positive value in the early phase to behave as a pure cosmological constant at late phase of cosmic time. The behaviour of the EoS parameter of

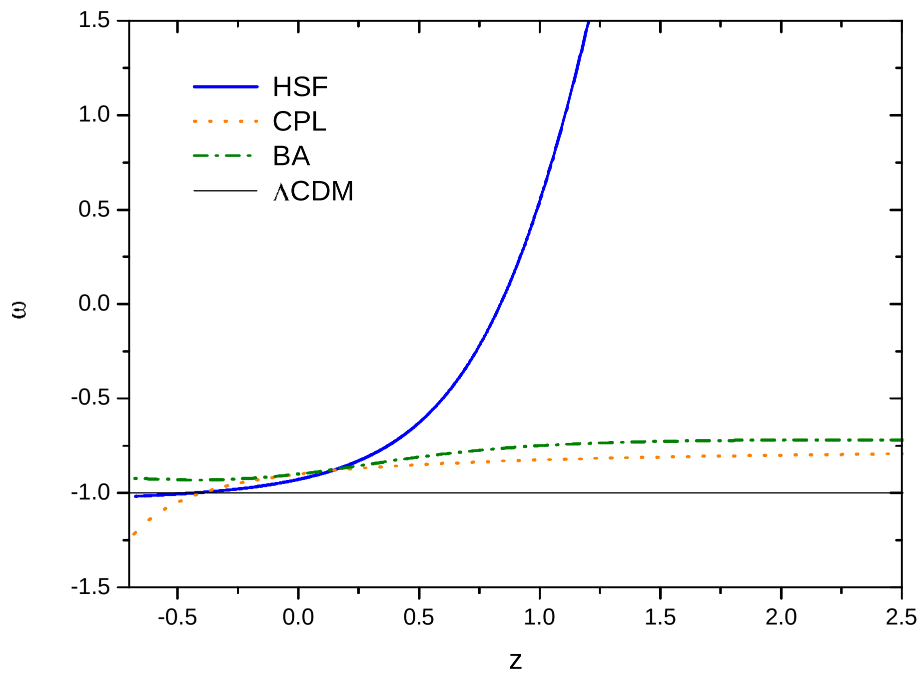

Figure 8.2: Evolutionary behaviour of ω of HSF. The EoS parameters as calculated from the CPL parametrization (dotted curve), BA parametrization (dash-dotted curve) and the prediction of ΛCDM model (solid black line) are shown for comparison.

HSF model has been compared with that of some well known EoS parametrizations such as the Chevallier-Polarski-Linder (CPL) (Chevallier and Polarski [234], Linder [235]) and Barboza-Alcaniz(BA) (Barboza and Alcaniz [236]) parametrizations given as

$$\textbf{CPL} : \quad \omega(z) = \omega_0 + \omega_a \frac{z}{1+z} \tag{8.23}$$

$$\textbf{BA} : \quad \omega(z) = \omega_0 + \omega_a \frac{z(1+z)}{1+z^2}, \tag{8.24}$$

where ω_0 and ω_a are constants. The redshift z is defined through $1+z = \frac{\mathcal{R}_0}{\mathcal{R}}$ where $\mathcal{R}_0$ is the scale factor at the present epoch.

In the low redshift region, predictions from these parametrizations are more or less the same as that of our model with HSF. However, at high redshift, EoS parameter from HSF rises with greater slope than these parametrizations. Since the HSF has a power law behaviour at early times and an exponential behaviour at late times, the same has been reflected in the figure. At late epoch, ω coincides with that of the ΛCDM model. At the present epoch, this model gives an $\omega = -0.929$ which is close to that of the ΛCDM model i.e $\omega = -1$ which is consistent with the observational bounds. It is to mention here that, the EoS from HSF, may have different trajectories if we constraint the HSF parameters from some other physical basis but at late times all those trajectories overlap with that of ΛCDM model.

The interesting feature of the dynamical properties of the model with HSF is that, the expressions as obtained above are more general than the power law or the exponential expansion. It is worth to mention here that, in most of the cosmological models, authors use either a power law scale factor or an exponential one. These two behaviours appear as the two extreme cases of HSF. In fact, the dynamical evolution track of the EoS parameter for HSF lies in between the two extreme tracks predicted by a power law and an exponential scale factor. From the set of the equations (8.18)-(8.22), we can always recover the relevant equations for the two extreme cases.

8.4.1 Case-I: $a = 0$

The power law behaviour can be recovered from HSF, if we consider $a = 0$ in the eqns. (8.18)-(8.22), so that we can obtain the required expressions for pressure, energy density, energy density of the anisotropic fluid, EoS parameter and the effective cosmological constant:

$$\begin{aligned}
p &= -\frac{6}{1-4\alpha^2}\left[\frac{b(3b\phi_2-\phi_1)+3at\phi_2(at+2b)}{(k+2)^2}\right]\frac{1}{t^2}-\frac{2}{1-2\alpha}e^{-\frac{6akt}{k+2}}t^{-\frac{6bk}{k+2}} \\
&= -\left[\frac{6b(3b\phi_2-\phi_1)}{(1-4\alpha^2)(k+2)^2}\right]\frac{1}{t^2}-\frac{2}{1-2\alpha}t^{-\frac{6bk}{k+2}}, \qquad (8.25)
\end{aligned}$$

$$\begin{aligned}
\rho &= \frac{6}{1-4\alpha^2}\left[\frac{3b^2\phi_3-2b(k+2)+3at\phi_3(at+2b)}{(k+2)^2}\right]\frac{1}{t^2}+\frac{2}{1-2\alpha}e^{-\frac{6akt}{k+2}}t^{-\frac{6bk}{k+2}} \\
&= \left[\frac{18b^2\phi_3-12b(k+2)}{(1-4\alpha^2)(k+2)^2}\right]\frac{1}{t^2}+\frac{2}{1-2\alpha}t^{-\frac{6bk}{k+2}}, \qquad (8.26)
\end{aligned}$$

$$\begin{aligned}
\rho_B &= \frac{6(k-1)}{1-2\alpha}\left[\frac{b(3b-1)+3at(at+2b)}{k+2}\right]\frac{1}{t^2}-\frac{4}{1-2\alpha}e^{-\frac{6akt}{k+2}}t^{-\frac{6bk}{k+2}} \\
&= \left[\frac{6b(3b-1)(k-1)}{(1-2\alpha)(k+2)}\right]\frac{1}{t^2}-\frac{4}{1-2\alpha}t^{-\frac{6bk}{k+2}}, \qquad (8.27)
\end{aligned}$$

$$\omega = -1 + \left[\frac{3(1+2\alpha)\left(3b[3b(k^2-k)-(k^2+3k+2)]+9(k^2-k)at(at+2b)\right)}{3[3b^2\phi_3-2b(k+2)+3at\phi_3(at+2b)]+(1+2\alpha)(k+2)^2e^{-\frac{6akt}{k+2}}t^{\frac{2(k+2-3bk)}{k+2}}}\right]$$
$$= -1+9(1+2\alpha)\left[\frac{b[3b(k^2-k)-(k^2+3k+2)]}{3[3b^2\phi_3-2b(k+2)]+(1+2\alpha)(k+2)^2t^{\frac{2(k+2-3bk)}{k+2}}}\right], \tag{8.28}$$

$$\Lambda = \frac{6}{(k+2)(1+2\alpha)}\left[b(3b-1)+3at(at+2b)\right]\frac{1}{t^2}$$
$$= \left[\frac{6b(3b-1))}{(k+2)(1+2\alpha)}\right]\frac{1}{t^2}. \tag{8.29}$$

A substitution of $b=\frac{m}{3}$ in the above equations recovers the assumption of a power law scale factor behaving like $\mathcal{R}=t^{\frac{m}{3}}$ as in chapter 5.

8.4.2 Case-II: b=0

If one considers $b=0$ in the HSF, de Sitter model with an exponential expansion can be achieved. In such a case the dynamical parameters of the model become

$$p = -\frac{6}{1-4\alpha^2}\left[\frac{b(3b\phi_2-\phi_1)+3at\phi_2(at+2b)}{(k+2)^2}\right]\frac{1}{t^2}-\frac{2}{1-2\alpha}e^{-\frac{6akt}{k+2}}t^{-\frac{6bk}{k+2}}$$
$$= -\frac{18a^2\phi_2}{(1-4\alpha^2)(k+2)^2}-\frac{2}{1-2\alpha}e^{-\frac{6akt}{k+2}}, \tag{8.30}$$

$$\rho = \frac{6}{1-4\alpha^2}\left[\frac{3b^2\phi_3-2b(k+2)+3at\phi_3(at+2b)}{(k+2)^2}\right]\frac{1}{t^2}+\frac{2}{1-2\alpha}e^{-\frac{6akt}{k+2}}t^{-\frac{6bk}{k+2}}$$
$$= \frac{18a^2\phi_3}{(1-4\alpha^2)(k+2)^2}+\frac{2}{1-2\alpha}e^{-\frac{6akt}{k+2}}, \tag{8.31}$$

$$\rho_B = \frac{6(k-1)}{1-2\alpha}\left[\frac{b(3b-1)+3at(at+2b)}{k+2}\right]\frac{1}{t^2}-\frac{4}{1-2\alpha}e^{-\frac{6akt}{k+2}}t^{-\frac{6bk}{k+2}}$$
$$= \frac{18a^2(k-1)}{(1-2\alpha)(k+2)}-\frac{4}{1-2\alpha}e^{-\frac{6akt}{k+2}}, \tag{8.32}$$

$$\omega = -1 + \left[\frac{3(1+2\alpha)\left(3b[3b(k^2-k)-(k^2+3k+2)]+9(k^2-k)at(at+2b)\right)}{3[3b^2\phi_3-2b(k+2)+3at\phi_3(at+2b)]+(1+2\alpha)(k+2)^2e^{-\frac{6akt}{k+2}}t^{\frac{2(k+2-3bk)}{k+2}}}\right]$$
$$= -1+\left[\frac{27(1+2\alpha)(k^2-k)a^2}{9a^2\phi_3+(1+2\alpha)(k+2)^2e^{-\frac{6akt}{k+2}}}\right], \tag{8.33}$$

$$\Lambda = \frac{6}{(k+2)(1+2\alpha)}\left[(3b-1)+3at(at+2b)\right]\frac{1}{t^2}$$
$$= \frac{18a^2}{(k+2)(1+2\alpha)}. \tag{8.34}$$

In the exponential model, ω increases from some higher negative value in the phantom domain in an initial epoch to behave like a cosmological constant at a late epoch. The effective cosmological constant becomes a time independent quantity for this model.

Different energy conditions like the Null energy condition (NEC) $(\rho + p \geq 0)$, Strong energy condition (SEC) $(\rho + 3p)$ and the Dominant energy condition (DEC) $(\rho - p)$ can also be investigated in the model with HSF. Different energy conditions as obtained in this model are

$$\rho + p = \frac{6}{1-4\alpha^2}\left[\frac{(1+2\alpha)(k+1)}{k+2}\frac{\ddot{R}}{R} + \frac{2(1+2\alpha)(k^2-3k-1)}{(k+2)^2}\left(\frac{\dot{R}}{R}\right)^2\right]$$
$$= \frac{6}{1-4\alpha^2}\left[\frac{3(\phi_3-\phi_2)(at+b)^2 - b(2k+4-\phi_1)}{(k+2)^2}\right]\frac{1}{t^2}, \tag{8.35}$$

$$\rho + 3p = \frac{6}{1-4\alpha^2}\left[\frac{3k-1+2\alpha(k+1)}{k+2}\frac{\ddot{R}}{R} + \frac{(6k^2-14k-16+6\alpha(2k^2-2k))}{(k+2)^2}\left(\frac{\dot{R}}{R}\right)^2\right]$$
$$- \frac{4}{1-2\alpha}R^{\frac{-6k}{k+2}}$$
$$= \frac{6}{1-4\alpha^2}\left[\frac{3(\phi_3-3\phi_2)(at+b)^2 - b(2k+4-3\phi_1)}{(k+2)^2}\right]\frac{1}{t^2} \tag{8.36}$$
$$- \frac{4}{1-2\alpha}e^{-\frac{6akt}{k+2}}t^{-\frac{6bk}{k+2}},$$

$$\rho - p = \frac{6}{1-4\alpha^2}\left[\frac{3-k-2\alpha(k+1)}{k+2}\frac{\ddot{R}}{R} + \frac{(-2k^2+2k+12)-2\alpha(2k^2+6k+4)}{(k+2)^2}\left(\frac{\dot{R}}{R}\right)^2\right]$$
$$+ \frac{4}{1-2\alpha}R^{\frac{-6k}{k+2}},$$
$$= \frac{6}{1-4\alpha^2}\left[\frac{3(\phi_3+\phi_2)(at+b)^2 - b(2k+4+\phi_1)}{(k+2)^2}\right]\frac{1}{t^2} \tag{8.37}$$
$$+ \frac{4}{1-2\alpha}e^{-\frac{6akt}{k+2}}t^{-\frac{6bk}{k+2}}.$$

The curves of the energy conditions will remain intermediate between the two extremes cases: $a = 0$ case and $b = 0$ case. It is to mention here that the energy conditions of the two extreme cases can well be recovered by using these extreme values of the HSF parameters.

8.5 Conclusion

In this chapter, we have developed a general formalism to investigate Bianchi VI_h universe in an extended gravity theory where the geometrical part of the action integral is modified. The Ricci Scalar R is replaced by a rescaled functional $\lambda(R+T)$ assuming the geometry to couple with a bit of matter minimally in two cases. The motivation behind this is to obtain a set of field equations that can look like the Einstein Field equations with a time varying cosmological constant. However, it is not possible to reduce the field equations to Einstein Field equations because of the non-vanishing nature of the scaling constant λ. Keeping in view of the recent observations predicting an accelerated universe at a late epoch that signals a possible transition from an initial state of deceleration, we employ a hybrid scale factor both cases. The HSF simulates a signature flipping behaviour of DP. The parameters of the HSF as have been constrained from some recent estimates of transition redshift. The HSF contains two factors, an exponential and a power law functions of the scale factor. While the power law factor dominates at an early cosmic epoch, the exponential part dominates at the late times.

Within the formalism developed here, we have derived the general expressions of the dynamical features of an anisotropic universe using the HSF. These expressions are more general in the sense that, the power law and exponential behaviour appear as the two extreme cases. In first case, the dynamical behaviour of the properties remain intermediate to these two extreme cases. In order to assess the dynamical aspects of the model, we have plotted the EoS parameter ω as a function of redshift and compared its behaviour with the CPL parametrization, BA parametrization and ΛCDM model. The EoS evolves with redshift and behaves as a pure cosmological constant at late phase of cosmic time. The rate of dynamical evolution is greatly affected with the change in the HSF parameters. However in the present work with parameters constrained from transition redshift, we obtain a quintessence like behaviour. This shows that the modified gravity theory reproduces quintessence phase of evolution. In second case, we have studied the anisotropic nature of the model using the hybrid law. Also, two important diagnostic approaches studied using HSW.

We have also calculated the energy conditions for both cases that can be suitably reduced to the results already obtained in some earlier works. Since the present approach with a hybrid scale factor appears to be more general than that with a power law or an exponential scale factor, the results of the present study may be more natural and be closer to observations.

CHAPTER 9

Summary and scope for future research

Before, concluding this thesis or discussing future perspective of the work presented here, let us attempt to summarize in this section some of the results presented so far. The motivation of this thesis with different type of modified gravity theory has been thoroughly discussed in chapter- 1. In chapter-2 to chapter-8, a number of specific models using Bianchi universe in $f(R,T)$ theory are introduced and the dynamical behaviour are focused on the cosmological and astrophysical aspects of this theory and on their viability.

As mentioned in the introduction, the $f(R,T)$ theory is introduced as tools of an action geometry that could help us to examine how much and in which ways one can deviate from GR. The prime interest is to construct cosmological models which can be confronted with recent observations concerning the late time cosmic speed up phenomenon without the need of DE. In fact, cosmological models in the framework of GR require either a cosmological constant or some scalar fields may be with an usual negative kinetic energy term in the Lagrangian which are called ghost fields. Cosmological constant is well known to be entangled with the fine tuning and cosmic coincidence problem. Therefore in the present thesis we have constructed some viable ghost free cosmological models in the framework of a class of modified gravity theory dubbed as the $f(R,T)$ theory where the Einstein-Hilbert contains terms of matter-geometry coupling in place of a Ricci Scalar and does not contain any additional dynamical degrees of freedom.

The late time cosmic speed up phenomenon is taken care of by the extra modification of the geometry part of the field equation. In all of our models, we have considered some specific forms of the functional $f(R,T)$ so that under suitable choices of the model parameters, the theory can be reduced to that of GR. Through out the thesis our emphasis is to present the formalism in a simplified and general manner so that, the inference on certain physical aspects of the model can be easily assessed. The results obtained from our models are confronted and compared with the recent observations. In most of the chapters of the thesis, we have presented the dynamical parameters of the universe in terms of the Hubble parameter or the scale factor. Therefore, it becomes easy to reach to a conclusive inference. In this approach, if a specific dynamical behaviour of the Hubble rate or the scale factor is known, then one can easily track the evolution history of the universe through the calculation of the EoS parameter and the DP. The viability of the constructed models are tested through certain diagnostic approaches such as the calculation of the state finder pair and the Om(z) diagnostic. In addition to the geometrical diagnostic approaches, the energy conditions also put some additional constraint on the models. We have discussed the energy conditions for all the constructed models and the dynamical evolution of the energy conditions for different choices of the model parameters have been discussed. In light of this, it is probably preferable to share that this theory is most advance theory to investigate the accelerating models without help of DE.

The present universe is observed mostly to be isotropic and homogeneous and can be well described by an FRW model. However, we have incorporated some degree of anisotropy in the spatial section of the model to take into account the possibility of cosmic anisotropic expansion as has been predicted in recent time. In view of this, the models are presented in the background of an anisotropic Bianchi VI_h metric. One interesting thing about our models is that, these models allow any amount of anisotropy possible including the minimal one resembling an almost isotropic universe. In this view, our model is more general than the FRW model and is interesting to investigate in the framework of modified gravity. The exponent h in the Bianchi VI_h metric can be either $1, 0, -1$. Considering these choices of the exponent, we have analysed the dynamical aspects of the models in chapter 2 and 3 and obtained that, a viable model can be achieved for the choice $h = -1, 0$. Tripathy and co workers have also obtained similar conclusion in favour of $h = -1$ based on the Tryon's conjecture advocating a null energy condition of isolated universe. In view of this, in rest of the works presented in the thesis, we concentrate only on this particular value of the exponent.

In general, the deceleration parameter is a dynamical quantity which may evolve from a positive value in the early phase of cosmic evolution to a negative value at late phase. This leads to a cosmic transition from a decelerated universe to an accelerated one. We have simulated a cosmic transit phenomenon through a hybrid scale factor. The hybrid scale factor is a product of a power law expansion scale factor and an exponential scale factor. The parameters of the hybrid scale factor have been constrained in the work from different physical basis. Also we have discussed under what situation, the model can either be reduced to a power law case or to a de Sitter space. Besides the consideration of a hybrid scale factor, we have considered some models for constant negative deceleration parameter simulated by a power law scale factor in chapter 2 to chapter 6.

We have reconstructed an anisotropic universe based on the modified gravity considering a power law expansion of the universe. The effect of the anisotropy on the evolutionary behaviour of the EoS parameter and the cosmological constant have been investigated for a simple extended gravity model with a cosmological constant where the functional is given by $f(R, T) = R + \Lambda_0 + \beta T$. The effect of coupling parameter on the dynamical evolution of the universe is assessed. An increase in the coupling parameter a higher value of the EoS parameter is obtained. Similarly, with an increase in the cosmic anisotropy the EoS parameter is found to increase. The work has been revisited with a hybrid scale factor. It is observed that the EoS parameter and the energy conditions are affected by the anisotropic parameter and model parameters. One interesting observation is that, with the increase in the anisotropy, the energy conditions are altogether altered. In otherwords, at the early phase, the model satisfies the SEC whereas for an increased anisotropy, at late there is a violation of SEC. In these models we have seen that, the models favour quintessence phase. We have reconstructed scalar fields from the constructed hybrid scale factor expansion in the frame work of the extended gravity and the reconstructed scalar fields slowly rolls from an early time to late phase of cosmic evolution. We have also investigated the effect of an one dimensional magnetic field on the cosmic dynamics in this extended gravity model. It is found that, the cosmic dynamics is greatly influence in presence of the magnetic field. Our models are compared with the well established models like CPL and BA.

In another choice of the functional i.e. $f(R,T) = \lambda(R+T)$, the model has a rescaled functional. Eventhough such a model can provide beautiful results concerning the late time cosmic speed up phenomenon it can not be reduced to GR. The interesting thing about this model is that, the rescaled functional provides a simplified structure of the field equations and can be elegant in reproducing results comparable to observations. The field equations are obtained in general formalism. In these models, the EoS parameter evolves in quintessence region to overlap with a cosmological constant at late times. Models with extra dissipative such as bulk viscous matter is considered in this model to assess their effect on the dynamics of the universe. It is observed that, the presence of bulk viscosity does not affect the dynamics greatly.

In future, the problem can be extended to study the dynamical behaviour of the cosmological models using the scale factors of trigonometric, logarithmic function and hyperbolic functions. Since two types of modified gravity models in the class of $f(R,T)$ gravity theory have obtained and the dynamical parameters are obtained in most general form in terms of the scale factor or the Hubble rate, there is enough scope in our models for investigation of bouncing universe, cyclic universe etc. The anisotropic behaviour can be further studied to check the possibility of getting a better range. In this thesis, we have only considered the Bianchi type VI_h space time, which is itself is an anisotropic expansion. Hence, other Bianchi type space-time both diagonal and non-diagonal can also be examined. The same formalism can be used to test the anisotropic behaviour with the isotropic FRW space-time to check whether anisotropic arises from isotropic background or not.

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List of Publications and Presentations

Publications

- B. Mishra, S. K. Tripathy, **Sankarsan Tarai**: Cosmological models with a hybrid scale factor in an extended gravity theory, *Mod. Phys. Lett. A*, **33**(9), 1850052 (2018).

- **Sankarsan Tarai**, B. Mishra: Dynamical aspects of the magnetized anisotropic cosmological model in extended gravity, *Eur. J. Phys. Plus*, **133**, 435 (2018).

- B. Mishra, **Sankarsan Tarai**, S. K. Tripathy: Dynamical features of anisotropic cosmological model, *Ind. J. Phys.*, **92**(9), 1199(2018).

- B. Mishra, **Sankarsan Tarai**, S. K. Tripathy: Anisotropic cosmological reconstruction in $f(R,T)$ gravity, *Mod. Phys Lett. A*, **33**(29), 1850170 (2018).

- B. Mishra, **Sankarsan Tarai**, S. K. J. Pacif: Dynamics of Bianchi VI_h universe with bulk viscous fluid in modified gravity, *Int. J. of Geom. Methods in Mod. Phys.*, **15**, 1850036 (2018).

- B. Mishra, **Sankarsan Tarai**, S. K. Tripathy: Dynamics of an Anisotropic Universe in $f(R,T)$ gravity, *Adv. High Energy Phys.*, **2016** , 8543560, (2016).

- B. Mishra, P. K. Sahoo, **Sankarsan Tarai**: Non-static Cosmological models in $f(R,T)$ gravity, *Astrophys. Space Sci.*, **359**, 15 (2015). **(Not included in the thesis)**

Presentation in Conferences and Workshop Attended

- Non-static Cosmological model in $f(R,T)$ gravity, *International Conference on General Relativity*, held at Department of Mathematics, Amravati University, Amravati during 25th-28th Nov., 2015.

- Attended workshop on *Cosmological and theoretical applications of exact solutions of Einsteins equations*, held at Centre for Theoretical Physics, JMI, New Delhi during 12th-23rd Feb., 2016.

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- Attended workshop on *Structure Formation in standard Cosmology*, held at Department of Mathematics, BITS-Pilani Hyderabad Campus during 19th -23rd Dec., 2016.

- Anisotropic cosmological reconstruction in $f(R,T)$ gravity, *National Workshop on Celebrating Centenary of Einstein's General Relativity-2017 : Hundred years with* Λ ,held at Department of Mathematics, University of Burdwan, Kolkata during 26th Jul.-1st Aug., 2017.

- Dynamics of anisotropic universe in $f(R,T)$ gravity with hybrid scale factor, *National conference on Differential Geometry and General Relativity*, held at Department of Mathematics, Sardar Patel University, Anand, Gujarat during 28th -29th Nov., 2017.

- Dynamical features of an anisotropic cosmological model, *30th meeting of the Indian Association of General Relativity and Gravitation*, held at Department of Physics, BITS-Pilani, Hyderabad Campus during 3rd-5th Jan., 2019.

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List of Publications and Presentations

Publications

- B. Mishra, S. K. Tripathy, **Sankarsan Tarai**: Cosmological models with a hybrid scale factor in an extended gravity theory, *Mod. Phys. Lett. A*, **33**(9), 1850052 (2018).

- **Sankarsan Tarai**, B. Mishra: Dynamical aspects of the magnetized anisotropic cosmological model in extended gravity, *Eur. J. Phys. Plus*, **133**, 435 (2018).

- B. Mishra, **Sankarsan Tarai**, S. K. Tripathy: Dynamical features of anisotropic cosmological model, *Ind. J. Phys.*, **92**(9), 1199(2018).

- B. Mishra, **Sankarsan Tarai**, S. K. Tripathy: Anisotropic cosmological reconstruction in $f(R,T)$ gravity, *Mod. Phys Lett. A*, **33**(29), 1850170 (2018).

- B. Mishra, **Sankarsan Tarai**, S. K. J. Pacif: Dynamics of Bianchi VI_h universe with bulk viscous fluid in modified gravity, *Int. J. of Geom. Methods in Mod. Phys.*, **15**, 1850036 (2018).

- B. Mishra, **Sankarsan Tarai**, S. K. Tripathy: Dynamics of an Anisotropic Universe in $f(R,T)$ gravity, *Adv. High Energy Phys.*, **2016** , 8543560, (2016).

- B. Mishra, P. K. Sahoo, **Sankarsan Tarai**: Non-static Cosmological models in $f(R,T)$ gravity, *Astrophys. Space Sci.*, **359**, 15 (2015). **(Not included in the thesis)**

Presentation in Conferences and Workshop Attended

- Non-static Cosmological model in $f(R,T)$ gravity, *International Conference on General Relativity*, held at Department of Mathematics, Amravati University, Amravati during 25th-28th Nov., 2015.

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- Anisotropic cosmological reconstruction in $f(R,T)$ gravity, *National Workshop on Celebrating Centenary of Einstein's General Relativity-2017 : Hundred years with* Λ ,held at Department of Mathematics, University of Burdwan, Kolkata during 26th Jul.-1st Aug., 2017.

- Dynamics of anisotropic universe in $f(R,T)$ gravity with hybrid scale factor, *National conference on Differential Geometry and General Relativity*, held at Department of Mathematics, Sardar Patel University, Anand, Gujarat during 28th -29th Nov., 2017.

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Study on the Dynamics of Anisotropic Cosmological Models in a Modified Theory of Gravity

THESIS

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by

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CHAPTER 9

Summary and scope for future research

Before, concluding this thesis or discussing future perspective of the work presented here, let us attempt to summarize in this section some of the results presented so far. The motivation of this thesis with different type of modified gravity theory has been thoroughly discussed in chapter- 1. In chapter-2 to chapter-8, a number of specific models using Bianchi universe in $f(R,T)$ theory are introduced and the dynamical behaviour are focused on the cosmological and astrophysical aspects of this theory and on their viability.

As mentioned in the introduction, the $f(R,T)$ theory is introduced as tools of an action geometry that could help us to examine how much and in which ways one can deviate from GR. The prime interest is to construct cosmological models which can be confronted with recent observations concerning the late time cosmic speed up phenomenon without the need of DE. In fact, cosmological models in the framework of GR require either a cosmological constant or some scalar fields may be with an usual negative kinetic energy term in the Lagrangian which are called ghost fields. Cosmological constant is well known to be entangled with the fine tuning and cosmic coincidence problem. Therefore in the present thesis we have constructed some viable ghost free cosmological models in the framework of a class of modified gravity theory dubbed as the $f(R,T)$ theory where the Einstein-Hilbert contains terms of matter-geometry coupling in place of a Ricci Scalar and does not contain any additional dynamical degrees of freedom.

The late time cosmic speed up phenomenon is taken care of by the extra modification of the geometry part of the field equation. In all of our models, we have considered some specific forms of the functional $f(R,T)$ so that under suitable choices of the model parameters, the theory can be reduced to that of GR. Through out the thesis our emphasis is to present the formalism in a simplified and general manner so that, the inference on certain physical aspects of the model can be easily assessed. The results obtained from our models are confronted and compared with the recent observations. In most of the chapters of the thesis, we have presented the dynamical parameters of the universe in terms of the Hubble parameter or the scale factor. Therefore, it becomes easy to reach to a conclusive inference. In this approach, if a specific dynamical behaviour of the Hubble rate or the scale factor is known, then one can easily track the evolution history of the universe through the calculation of the EoS parameter and the DP. The viability of the constructed models are tested through certain diagnostic approaches such as the calculation of the state finder pair and the Om(z) diagnostic. In addition to the geometrical diagnostic approaches, the energy conditions also put some additional constraint on the models. We have discussed the energy conditions for all the constructed models and the dynamical evolution of the energy conditions for different choices of the model parameters have been discussed. In light of this, it is probably preferable to share that this theory is most advance theory to investigate the accelerating models without help of DE.

The present universe is observed mostly to be isotropic and homogeneous and can be well described by an FRW model. However, we have incorporated some degree of anisotropy in the spatial section of the model to take into account the possibility of cosmic anisotropic expansion as has been predicted in recent time. In view of this, the models are presented in the background of an anisotropic Bianchi VI_h metric. One interesting thing about our models is that, these models allow any amount of anisotropy possible including the minimal one resembling an almost isotropic universe. In this view, our model is more general than the FRW model and is interesting to investigate in the framework of modified gravity. The exponent h in the Bianchi VI_h metric can be either $1, 0, -1$. Considering these choices of the exponent, we have analysed the dynamical aspects of the models in chapter 2 and 3 and obtained that, a viable model can be achieved for the choice $h = -1, 0$. Tripathy and co workers have also obtained similar conclusion in favour of $h = -1$ based on the Tryon's conjecture advocating a null energy condition of isolated universe. In view of this, in rest of the works presented in the thesis, we concentrate only on this particular value of the exponent.

In general, the deceleration parameter is a dynamical quantity which may evolve from a positive value in the early phase of cosmic evolution to a negative value at late phase. This leads to a cosmic transition from a decelerated universe to an accelerated one. We have simulated a cosmic transit phenomenon through a hybrid scale factor. The hybrid scale factor is a product of a power law expansion scale factor and an exponential scale factor. The parameters of the hybrid scale factor have been constrained in the work from different physical basis. Also we have discussed under what situation, the model can either be reduced to a power law case or to a de Sitter space. Besides the consideration of a hybrid scale factor, we have considered some models for constant negative deceleration parameter simulated by a power law scale factor in chapter 2 to chapter 6.

We have reconstructed an anisotropic universe based on the modified gravity considering a power law expansion of the universe. The effect of the anisotropy on the evolutionary behaviour of the EoS parameter and the cosmological constant have been investigated for a simple extended gravity model with a cosmological constant where the functional is given by $f(R, T) = R + \Lambda_0 + \beta T$. The effect of coupling parameter on the dynamical evolution of the universe is assessed. An increase in the coupling parameter a higher value of the EoS parameter is obtained. Similarly, with an increase in the cosmic anisotropy the EoS parameter is found to increase. The work has been revisited with a hybrid scale factor. It is observed that the EoS parameter and the energy conditions are affected by the anisotropic parameter and model parameters. One interesting observation is that, with the increase in the anisotropy, the energy conditions are altogether altered. In otherwords, at the early phase, the model satisfies the SEC whereas for an increased anisotropy, at late there is a violation of SEC. In these models we have seen that, the models favour quintessence phase. We have reconstructed scalar fields from the constructed hybrid scale factor expansion in the frame work of the extended gravity and the reconstructed scalar fields slowly rolls from an early time to late phase of cosmic evolution. We have also investigated the effect of an one dimensional magnetic field on the cosmic dynamics in this extended gravity model. It is found that, the cosmic dynamics is greatly influence in presence of the magnetic field. Our models are compared with the well established models like CPL and BA.

In another choice of the functional i.e. $f(R,T) = \lambda(R+T)$, the model has a rescaled functional. Eventhough such a model can provide beautiful results concerning the late time cosmic speed up phenomenon it can not be reduced to GR. The interesting thing about this model is that, the rescaled functional provides a simplified structure of the field equations and can be elegant in reproducing results comparable to observations. The field equations are obtained in general formalism. In these models, the EoS parameter evolves in quintessence region to overlap with a cosmological constant at late times. Models with extra dissipative such as bulk viscous matter is considered in this model to assess their effect on the dynamics of the universe. It is observed that, the presence of bulk viscosity does not affect the dynamics greatly.

In future, the problem can be extended to study the dynamical behaviour of the cosmological models using the scale factors of trigonometric, logarithmic function and hyperbolic functions. Since two types of modified gravity models in the class of $f(R,T)$ gravity theory have obtained and the dynamical parameters are obtained in most general form in terms of the scale factor or the Hubble rate, there is enough scope in our models for investigation of bouncing universe, cyclic universe etc. The anisotropic behaviour can be further studied to check the possibility of getting a better range. In this thesis, we have only considered the Bianchi type VI_h space time, which is itself is an anisotropic expansion. Hence, other Bianchi type space-time both diagonal and non-diagonal can also be examined. The same formalism can be used to test the anisotropic behaviour with the isotropic FRW space-time to check whether anisotropic arises from isotropic background or not.

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