RADIO-FREQUENCY ELECTRICAL MEASUREMENTS

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RADIO-FREQUENCY ELECTRICAL MEASUREMENTS

A Guide for Radio Engineering Laboratory Instruction

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This book is dedicated to
my fellow radio amateurs

W 9 Y H

PREFACE TO THE SECOND EDITION

In this revised edition are described the more important new methods and the important changes in standard methods. The new edition departs from the old in that exact instructions for the procedure to be followed in making the more difficult measurements are given. The author has tried to explain the physical principles and measurement procedure in essentially the same way that he has found necessary for the guidance of the students in his laboratory. Thus it is believed that this book will function as a working manual for a representative set of laboratory measurement experiments and that additional mimeographed instructions will not be needed.

No attempt has been made to write a comprehensive treatise; nevertheless, many important methods and techniques necessary in practical and research engineering are covered. Therefore the book should prove to be a useful, ready manual for the practicing engineer.

As the modern radio amateur acquires more experience, he finds it necessary to make a considerable number of quantitative measurements. This book may be used as a convenient manual and guide for the technically minded amateur also; and he should find no difficulty in following the instructions, even though he wishes to omit analyses and discussions of theory. Many nonmathematical explanations are given for the benefit of those who are always interested in *why* certain fundamentals are true.

HUGH A. BROWN.

URBANA, ILL.,
March, 1938.

PREFACE TO THE FIRST EDITION

Rapid progress is being made in the science and art of radio communication and in the laboratory methods of electrical measurement work at radio frequencies. This text is intended to present both the well-known methods of making certain measurements and some of the important advances recently made in the solution of radio-frequency measurement problems. Certain methods recently originated and developed by individual investigators are described in the text, and in such cases full credit is given to those responsible.

This text is prepared for the use of the student who has a knowledge of alternating-current phenomena and circuits equivalent to that usually acquired in the fourth year of the college curriculum in electrical engineering. Knowledge of the elementary principles of radio communication is also assumed. The book is also intended to serve as a ready manual for the use of the radio engineer, and the experienced amateur.

The method of treatment of the subject is perhaps a little different from that found in most treatises for laboratory use. First the theory and principles are derived and explained. Then follows a brief description of the necessary steps in the laboratory procedure or manipulation. Finally there is given a discussion of the precision attainable, precautions, and general merits of the method.

Thus, without giving detailed instructions about the number and kind of quantitative data that should be taken, etc., which the author feels may well be left to the judgment of the student, emphasis is put upon the explanation of principles that are fundamental and upon the underlying theory of each method of solution of a measurement problem, except in cases requiring a large amount of derivation, extra discussion, etc. In such cases references to well-known standard treatises are given. In the omission of some possible explicit directions it is hoped that the student, while enlightened in the fundamentals, may have the pleasure and benefit of thinking out the details of the problems for himself.

The author is deeply indebted to those who have given permission to use certain material, in either its original or slightly modified form. It was the author's intention to use published material only with permission, and if it appears anywhere that permission was not obtained on the author's part, it was due to oversight. Inestimable credit is due the many investigators and authorities whose work has made this book possible.

Acknowledgment is due Dr. Jacob Kunz of the Physics Department of the University of Illinois for his kindness in helping to clear up certain matters in the theoretical treatise of equivalent antenna constants given by Dr. J. Zenneck ("Wireless Telegraphy"), and used by the author; and acknowledgment is due Mrs. Carrie N. Brown, the author's wife, for her suggestions as to wording of the text material.

HUGH A. BROWN.

URBANA, ILL.,
August, 1931.

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ABBREVIATIONS FOR BIBLIOGRAPHY

Ann. de physique	*Annales de physique* (France)
Ann. der Physik	*Annalen der Physik* (Germany), formerly *Wiedemanns Ann.* (see below)
Bell Lab. Record	*Bell Laboratories Record* (U.S.A.)
Bell System Tech. Jour	*Bell System Technical Journal* (U.S.A.)
Bell Tel. Lab. Reprint	*Bell Telephone Laboratories Reprint*
Bull. de la Soc. des électriciens	*Bulletin de la Société des électriciens* (France)
Communication and Broadcast Engineering	(U.S.A.)
Compt. rend	*Comptes rendus* (France)
Electrotech. Ztsch	*Electrotechnischer Zeitschrift* (Germany)
Elec Communication	*Electrical Communication* (U.S.A.)
Elec. Eng	*Electrical Engineering* (U.S.A.)
Elec. World	*Electrical World* (U.S.A.)
Elektrische Nachrichten Technik	(Germany)
Experimental Wireless and Wireless Engineer	(England)
General Radio Experimenter	(U.S.A.)
Jahrbuch der drahtlosen Telegraphie	(Germany)
Jour. Am. Chem. Soc	*Journal, American Chemical Society*
Jour. A.I.E.E	*Journal, American Institute of Electrical Engineers;* this occasionally refers to the annual transactions
Jour. Am. Soc. Naval Eng	*Journal, American Society Naval Engineers*
Jour. Franklin Inst	*Journal, Franklin Institute* (U.S.A.)
Jour. I.E.E	*Journal, Institution of Electrical Engineers* (England)
Jour. Optical Soc. Am	*Journal, Optical Society of America*
L'Onde électrique	(France)
Nat. Bur. Standards Jour. Res	*National Bureau of Standards Journal of Research* (U.S.A.)
Phil. Mag	*Philosophical Magazine* (England)
Phil. Trans. Roy. Soc. (A)	*Philosophical Transactions Royal Society* (Series A) (England)
Proc. A.S.T.M	*Proceedings American Society of Testing Materials*

Proc. Physical. Soc. London	*Proceedings Physical Society of London*
Proc. I.R.E	*Proceedings Institute of Radio Engineers* (U.S.A.)
Phy. Rev	*Physical Review* (U.S.A.)
Physik. Ztsch	*Physikalischer Zeitschrift* (Germany)
Q.S.T	*The Q.S.T. Magazine* (U.S.A.)
Radio Eng	*Radio Engineering* (U.S.A.)
Radio News	(U.S.A.)
Radio Rev	*Radio Review* (England)
Rev. gén. élec	*Revue générale électrique* (France)
Rev. Sci. Instruments	*Review of Scientific Instruments* (U.S.A.)
R. C. A. Application Note	*Technical Notes, Radio Corporation of America*
Trans. A.I.E.E	*Transactions American Institute of Electrical Engineers*
Univ. of Ill. Eng. Exp. Sta. Bull	*University of Illinois Engineering Experiment Station Bulletin* (U.S.A.)
U. S. Bur. Standards (Rev. Circ.)	*United States Bureau of Standards Revised Circular*
U. S. Bur. Standards Sci. Paper	*United States Bureau of Standards Scientific Paper*
U. S. Bur. Standards Tech. Paper	*United States Bureau of Standards Technical Paper*
Wiedemanns Ann	*Wiedemanns Annalen der Physik* (Germany)
Wireless World and Radio Review	(England)
Ztsch. für. Phys	*Zeitschrift für Physik* (Germany)

RADIO-FREQUENCY ELECTRICAL MEASUREMENTS

CHAPTER I

MEASUREMENT OF CIRCUIT CONSTANTS

1. Introduction. *Scope of Measurement Problems.*—In order to describe the action of an electromagnetic system it is essential to know the relation between cause and effect. When a known voltage is applied between any two points, it is necessary to be able to predict the resultant currents and voltages in other parts of the system. The behavior of electromagnetic systems is completely predicted by Maxwell's equations in terms of electric and magnetic field intensity, charge, length, time, conductivity, dielectric constant, and magnetic permeability. His equations, however, do not readily lend themselves to the solution of problems involving systems of any but the simplest mathematical shapes. Because of the prohibitively complex mathematics necessary to solve problems involving nonanalytic forms, a simpler, approximate method of analysis has been developed which yields a high degree of accuracy when electromagnetic radiation is a negligible factor. This is the so-called "quasi-stationary" analysis in terms of resistance, inductance, and capacitance.

For fixed charges, the concepts of voltage and capacitance may be readily formed. Similarly, for steady transfers of charge, no electromagnetic radiation occurs, and the concepts of conduction current, resistance, and inductance may be formed. When time variations of current occur, however, a changing electromagnetic field is set up which propagates away from the source at a finite velocity, or radiates; and then the concepts formed for steady charges and steady transfers of charge immediately break down. From an engineering standpoint, however, the concepts so formed remain valid as long as the system considered is small compared with the electromagnetic wave length. Under such conditions, the addition of the concept of displacement current leads to a method of analysis that, in most cases, is completely satisfactory.

Even at very high frequencies such conditions are usually fulfilled in what are loosely referred to as *circuits*, and in transmission lines. Furthermore, special applications of the so-called *circuit parameters*—inductance, resistance, and capacitance—lead to convenient analyses of certain of the properties of systems designed specifically to radiate, such as radio antennas.[1]

Methods of measurement of the circuit parameters are therefore of the most fundamental importance and warrant considerable detailed discussion.

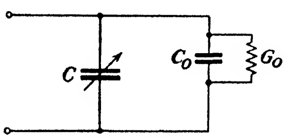
Impedance Standards.—In order to determine the magnitudes of circuit parameters, it is necessary to determine the relation between cause and effect. As an example, if a known sinusoidal voltage of known frequency be applied to the terminals of a two-terminal network and the resulting current measured, the impedance of the network will be given by the quotient of the voltage and current. From the laws relating the circuit parameters to impedance, the value of the parameter represented by the two-terminal network can then be computed. In order to determine the impedance completely, the voltage-current quotient must be known in both magnitude and phase. While it is sometimes possible to determine impedance experimentally from voltage and current measurements, such measurements are often extremely difficult to make with any accuracy because of the disturbing effects of the meters used. As a consequence, such determinations are usually made by comparing the unknown impedance in some manner with a known standard impedance.

[1] It should be appreciated that approximations are perhaps the essence of engineering. In applying science to obtain a practical solution to a problem, it is never economical, either of time or of money, to carry out a solution to a degree of refinement greater than necessary to predict the behavior in which one is interested. For steady currents, for instance, the simple application of Ohm's law relates voltage and current: for slowly varying currents, when radiation is negligible, the quasi-stationary analysis suffices; for rapidly varying currents, when radiation is appreciable, Maxwell's equations are applicable; as a further step, when the electromagnetic wave length becomes comparable to molecular dimensions, Maxwell's equations break down, and it is necessary to resort to the use of the quantum theory and wave mechanics.

The laws relating impedance to the circuit parameters and frequency are well known. If a pure, constant inductance L could be physically realized, for instance, the impedance at any frequency would be given simply by $j\omega L$; the impedance of a pure, constant resistance R would be R; and the impedance of a pure, constant capacitance C would be $1/j\omega C$. Such circuit elements would be ideal standards, since their impedances would be known explicitly at any frequency, both in magnitude and in phase. Unfortunately, circuit elements with such desirable characteristics cannot be made. A standard of any one of the three circuit parameters will, in practice, be contaminated by the presence of residual quantities of the other two. In addition, the parameters may change with frequency because of redistribution of current in conducting bodies (*skin effect*) and because of changes in their immediate electrical environment.

Ultimately, practical impedance standards usually revert to circuit elements whose performance is calculated at very low frequencies from dimensions. The standards desired at high frequencies are "carried up" in frequency from these primary standards by extrapolation. It is therefore highly desirable to choose impedance standards the behavior of which, as a function of frequency, is predictable with a good degree of accuracy. The disturbing effects of residual parameters in an impedance standard usually increase with frequency, and it is consequently of great importance to choose a physical circuit element in which residual parameters are as small as possible. Experience of many investigators in the field of high-frequency measurements has led to the widespread adoption of the variable air condenser as the circuit element best satisfying this criterion.

Variable Air Condenser as Impedance Standard.—In order to serve as a satisfactory impedance standard, a variable air condenser should be mechanically and electrically stable; it should be capable of being set to a calibrated scale precisely; and it should be very little affected by temperature and humidity. In addition, it should have low values of residual parameters, and it should be shielded so that changes in electrical environment will not appreciably affect its characteristics. Mechanical stability requires rugged construction; precision of setting requires good bearings and freedom from *backlash;* stability with temperature requires careful attention to the thermal coefficients of expansion of the various component parts; rapid increase of loss with increasing humidity may be minimized by proper choice of material for insulating supports. Residual inductance depends upon the geometry of the metallic structure; residual resistance, upon losses in the metallic structure and in the dielectric supports. In particular, the use of low power-factor insulating supports, placed in regions of low electric field intensity, is desirable in order to minimize residual resistance. Design of a precision-type variable air condenser must, of necessity, require compromises between desirable mechanical and electrical characteristics. Mechanical stability, for instance, is highest with large structures, wide spacing, sturdy supporting members, and rugged insulating members. Residual electrical parameters, on the other hand, are least for compact structures having small, closely spaced plates and a minimum of insulating material.



C = Air Capacitance
C_0 = Capacitance of Dielectric Supports
G_0 = Equivalent Conductance representing Dielectric Loss

FIG. 1.—Equivalent circuit of a variable condenser.

At low frequencies the only residual parameter that is of importance in a good air condenser is the dissipative component representing loss in the dielectric supports. In most variable air condensers the dielectric supports lie in an electric field which depends only upon the applied voltage and not upon the position of the rotor plates. The loss in the supports is therefore independent of the condenser settings. The dielectric supports may, in effect, be regarded as small imperfect condensers in parallel with the main capacitance between rotor and stator plates. The main capacitance, for the moment, may be regarded as loss-free, since there is little loss in the air dielectric and since the ohmic resistance of the metallic structure is very small compared with the capacitive reactance. The power factors of solid dielectric materials of the types used as insulators in air condensers have been found experimentally to be nearly independent of frequency. To a first degree of approximation, therefore, a variable air condenser may be considered as a pure variable capacitance in parallel with a small fixed capacitance of constant power factor. A simple equivalent circuit for such a condenser is depicted in Fig. 1. In this figure, the air capacitance C is shown shunted by a capacitance C_0 and conductance G_0, representing the imperfect capacitance of the dielectric supports. The power factor of the insulators is constant and equal to

$$\text{p.f.} = \frac{G_0}{\omega C_0}; \text{ nearly.}$$

and since C_0 is constant,

$$\frac{G_0}{\omega} = \text{constant.}$$

The effective conductance therefore varies directly with frequency. In the expression for the complex impedance of the circuit of Fig. 1, G_0^2 is very small compared with $\omega^2(C + C_0)^2$. Hence the effective series resistance is given approximately by

$$R = \frac{G_0}{\omega^2(C + C_0)^2}.$$

Since the quantity $G_0/\omega = R\omega(C + C_0)^2 =$ constant, it has been adopted in some quarters as the *figure of merit* for a variable air condenser. For a representative type of commercial precision condenser the figure of merit has a value of about 0.04×10^{-12}.

These precision-built variable condensers are satisfactory as standards at low frequencies. Precision calibration is usually made at 1,000 cycles, and at such frequencies, the condenser equivalent circuit of Fig. 1 is very nearly correct. The inductive reactance of the terminal leads and of the current paths along the rotor shaft and stationary plate supporting rods may be neglected. For a 1,000-mmf. condenser having a shaft 6 or 8 in. long, the inductive reactance becomes important at radio frequencies. While resistance losses on the plates and in the current paths to the plates may be neglected at low frequencies, these losses become important at higher radio frequencies owing to skin effect, and owing to lowered capacity reactance, hence the equivalent circuit of Fig. 1 is no longer correct. The inductive reactance of the current paths also becomes important at radio frequencies. Variable condenser impedance has been thoroughly investigated by Field and Sinclair.[1] They point out

[1] *Proc. I.R.E.*, Vol. 24, No. 2, February, 1936. In this paper, the measurement of the residual inductance and resistance is described, and valuable results are given.

that at radio frequencies, the variable condenser should properly be represented by the simplified equivalent circuit of Fig. 2. R is the effective resistance of the current paths along the rods and shafts supporting the plates and over the surface of the plates proper. L is the inductance of these current paths, C is the static capacitance, and G is a conductance due to losses in the insulating supports. The foregoing authors give as an example the following:

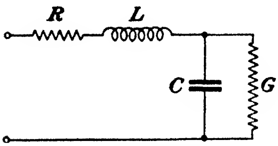


FIG. 2.—Simplified equivalent circuit of a variable condenser at radio frequencies.

L = 0.0604 microhenry (μh).
R = 0.017 ohm.
δC = 2.4 micromicrofarads (mmf.).
G = 0.210 micromhos.

The maximum capacitance of the condenser was 1,450 mmf. δC is the capacitance between the terminal leads. Of course, this particular condenser has a rather long shaft and is not always of a practical size for radio-frequency circuits, but the example serves to show what error will be made in the effective capacitance value if it is calibrated at 1,000 cycles, say, and then used at 1 and 6 Mc. frequencies. Taking the effective capacitance as $\frac{C}{1 - \omega^2 LC}$, as given in the paper referred to, the error in effective capacitance for C = 1,100-mmf. setting is small, below 1 per cent at 1 Mc.; but at 6 Mc., it becomes about 10 per cent. The total conductance of the condenser circuit of Fig. 2 is given in the foregoing paper as $(G + R\omega^2 C^2)$ with sufficient exactness, and the corresponding approximated impedance is given as

$$Z = R_e - j\frac{S_e}{\omega} = \left[R + G\left(\frac{S}{\omega}\right)^2\right] - j\left(\frac{S}{\omega} - \omega L\right), \qquad (1)$$

where S, the static elastance, is $1/C$. The error at 6 Mc. for the type of condenser used as the preceding example shows that it is very desirable to use calibrated condensers having much shorter current paths to the plates, smaller plate area, and closer plate spacing at frequencies well above 1 or 2 Mc. At higher radio frequencies, it is well to check the effective capacitance by a method that does not involve the use of standards at low frequencies, as described in Art. 7 of this chapter.

Inductance and Resistance Standards at Radio Frequencies.—It is not possible to construct an inductance coil that does not possess some self-capacitance.[1] However, useful inductance standards have been built which are wound on low loss forms and with separated turns to reduce the self-capacitance. The flux linkage in a coil may vary with frequency, owing to changes in current distribution with frequency; hence the true inductance will also vary. In general, this inductance decreases somewhat with increasing frequency, as does the inductance of a single wire or of a two-wire line. The practical radio-frequency equivalent circuit for a coil for all frequencies below its fundamental is represented as shown in Fig. 3, wherein the equivalent lumped self-capacitance C_0 is in parallel with the series connection of the true inductance and effective resistance at the particular frequency at which the coil is being used. Considering this parallel circuit as an apparent inductance and resistance between terminals A and B, the apparent values are

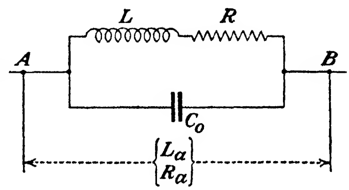


FIG. 3.—Equivalent circuit of a coil at radio frequencies.

$$R_a = \frac{R}{(1 - \omega^2 LC_0)^2},$$

$$L_a = \frac{L}{(1 - \omega^2 LC_0)}, \quad (2)$$

where R and L are the effective resistance and true inductance, respectively, at the given frequency $f = \omega/2\pi$.[2] Special construction is effective in reducing the change in L with frequency, and the value of C_0 to very small values.

Radio-frequency standard resistances in which the inductance and self-capacitance are reduced to very low values are of many

[1] For a discussion of self-capacitance of coils, see Art. 4 of this chapter.

[2] The fundamental frequency of a coil is that at which the coil with its self-capacitance forms a resonant circuit (see Art. 4), and at higher frequencies the inductance may even be negative. Equations (2) are given in a discussion of self-capacitance of coils in *U. S. Bur. Standards, Rev. Circ.* 74, p. 133.

types. The Ayrton-Perry, woven, and metallized types are widely used. The Ayrton-Perry resistance, illustrated in Fig. 20, Chap. VI, is a bifilar wound coil on a thin flat card form and arranged so as to reduce self-capacitance. The woven type consists of resistance wire in zigzag arrangement held in place by weaving into cloth or with ceramic materials.[1] The small-sized metallized resistor is used in some signal generator attenuators. With this device it is possible to obtain resistance values of 1,000 ohms or more with practically no self-capacitance effect and with very little inductive reactance, since the length of the resistor is not greater than ½ in. The residual impedances (due to inductance and capacitance effects) of resistance standards are serious limitations on their accuracy and reliability at radio frequencies. The wires in the Ayrton-Perry and woven types must be placed in close proximity to each other so as to reduce the flux, which would surround the wires and cause disturbing reactance. The wires really must be separated by thin silk insulation. If a No. 28 wire is used, a spacing between edges of parallel wires, equal to the wire diameter, allows disturbing flux to surround the resistance wires. Spacing the wires close together, using thin insulation, causes undesirable distributed capacitance effects, if the length of the resistance wire so wound is considerable, and the resistance is also considerable. As a result of these considerations, it has been found by experience that reliable higher radio-frequency Ayrton-Perry wound resistor units cannot be successfully made in values greater than about 10 ohms. The units making up the attenuation networks of some signal generators are of this type and value. For radio frequencies below 3 Mc., the units may be 100 ohms or so. When such units are placed in a *decade box*, the leads, switch blades, contact paths, etc., may easily reduce the allowable frequency for accuracy to 50 kc. or less. Probably the most reliable resistance unit at radio frequencies is a straight, very short, very small diameter high-resistance wire having zero temperature coefficient. For instance, a piece of resistance wire is described having a diameter of 0.0004 in. (0.4 mil), a length of 1/16 in., and a resist-

[1] The Ayrton-Perry type is described in "Electrical Engineer's Handbook," Communication and Electronics, Sec. 10, p. 17; see also Fig. 20, Chap. VI of this book; and the woven type in *Bell Lab. Record*, January, 1935, p. 136.

ance of 7 ohms.[1] The inductive reactance at 100 Mc. is 1 ohm, which added in quadrature to the resistance results in an impedance which is only 1 per cent greater than the resistance. The skin effect is entirely negligible. The inductance is, of course, calculated from the length and diameter of the wire.

2. Capacitance Measurement by Resonance. *Underlying Theory and Application.*—The e.m.f. applied to a series circuit consisting of resistance, inductance, and capacitance in series produces the maximum current when the natural frequency of this circuit is the same as the frequency of the impressed e.m.f. If the inducing or impressed e.m.f. has a constant amplitude, the natural frequency of the circuit in question is obtained from the well-known relation

$$\omega L = \frac{1}{\omega C},$$

where

$$\omega = 2\pi f.$$

Hence,

$$f = \frac{1}{2\pi\sqrt{LC}};$$

L and C are henrys and farads, respectively. If the circuit is excited by a damped sine wave of e.m.f., the resonant frequency will also depend upon the resistance R of the circuit, or

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}.$$

Since the undamped e.m.f. is easy to produce by means of an oscillating vacuum tube, the *damped-wave* excitation is not generally used.

Returning to the first case, resonance is observed in the circuit containing R, L, and C with the aid of a highly sensitive thermogalvanometer or by maximum plate or grid current change of the vacuum-tube oscillator which produces the undamped currents. A standard variable condenser is used for tuning purposes and is varied in value until maximum current is indicated. If an unknown capacitance is now connected in parallel with

[1] *General Radio Experimenter*, Vol. 12, No. 1, June, 1937.

the standard, the capacitance of the latter must be decreased until maximum current in the circuit is again obtained. This decrease in the value of the standard is equal to the value of the unknown capacitance.

The total capacitance for resonance C_r should not be more than equal to the full-scale value of C_s, the standard.

Before adding the unknown,

$$C_r = C_{s1}.$$

When unknown is added,

$$C_r = C_x + C_{s2}.$$

Equating, the unknown is

$$C_x = C_{s1} - C_{s2}. \tag{3}$$

This method of measurement is easily prepared for and permits rapid manipulation. For rapid determination of small capacitance it is recommended.

Apparatus for Measurement.—Figure 4 shows the arrangement of the necessary apparatus. A coupling coil, attached as shown,

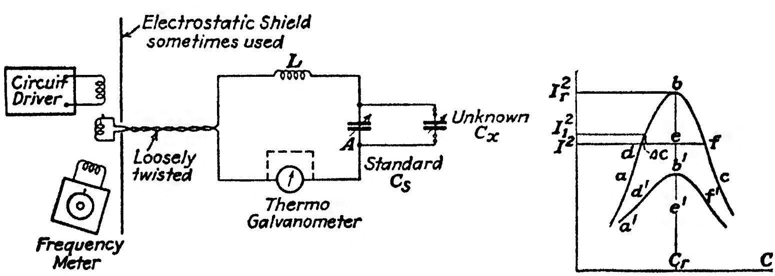


Fig. 4.—Apparatus arrangement and resonance curves for capacitance measurements by the resonance method.

is quite desirable, as it prevents possible excessive electrostatic effects of oscillator potentials on the condenser. Short, stiff leads from the standard variable condenser to the terminals of the unknown must be provided. These are adjusted in position so that the unknown capacitance may be connected in (after resonance is first effected with the standard), without adding lead wires to the circuit. This prevents errors due to capacitance of leads to the unknown. The resonance indicator shown in

Fig. 4 is a thermocouple-type milliammeter which should have a low resistance in order that the resonance may be sharp. A 100-milliamp.-range instrument of this type has a resistance of between 4 and 5 ohms, and such a value of resistance noticeably lowers the sharpness of resonance of a well-designed test circuit. A much better resonance indicator is formed by connecting a small coil of a few turns in series with a microammeter and the plate filament circuit of a type 199 vacuum tube having its grid connected to its plate. The tube is used merely as a rectifier. The small coil is then coupled to coil L in Fig. 4. This coupled resonance indicator introduces very little series loss resistance into the test circuit.

Suggestions for Manipulation.—In calibrating a variable condenser, the standard should be set at its maximum value, and the variable at its minimum. Then, after adjusting for resonance with the standard near its maximum value, the unknown at its minimum should be added. After readjusting the standard for resonance and obtaining the capacitance readings, the unknown may be increased, say, 10 divisions on the 100-division scale, resonance again adjusted, etc. In this way, calibration is rapidly carried out. If the unknown capacitance is larger than the standard, another condenser may be first measured and then connected in parallel with the standard before the unknown is added. Briefly, the standard is set near its maximum value with the above-mentioned auxiliary measured capacitance (this may be a variable condenser whose rotary plates are held firmly in one position) in parallel with it. The inductance L (Fig. 4) is reduced to regain resonance with the circuit driver, whose frequency is left unchanged. The unknown, whose capacitance is larger than that of the standard but less than the standard plus the auxiliary capacitance, is now added in parallel; the auxiliary is removed; and the standard is reduced until resonance is regained. The unknown is then equal to the value of the auxiliary plus reduction in the standard.

If the unknown is a variable condenser whose maximum value is greater than that of the standard but less than twice the maximum value of the latter, another procedure is possible which does not require a measured fixed auxiliary capacitance. The unknown is calibrated in steps, as explained, up to the value where the standard is reduced to zero. Then the standard is reset to near its maximum value; the total capacitance in the test circuit is now nearly equal to twice the maximum value of the standard, and the inductance L of the test circuit is now reduced to regain resonance. Then the unknown (condenser being calibrated) is now increased a few scale divisions, and the standard reduced to regain resonance again. The reduction in the standard is then equal to the increase in the unknown, and the capacitance of the latter at its last setting is the value at which this latter test was started (nearly the maximum value of the standard) plus the reduction in the standard. Procedure for the next higher setting of the unknown is made in like manner, and so on.

The degree of precision of measurement depends upon the exactness with which the point of resonance is obtained as well as upon the standard capacitance. For moderate precision, a thermogalvanometer of the 2½-in.-dial type and having a current-carrying capacity of about 100 milliamp. and a resistance of 4.50 ohms is adequate. This is inserted directly in the condenser circuit. If a sharper resonance is desired, the coupled resonance indicator is necessary.

When it is desired to obtain resonance more accurately, using a thermogalvanometer or milliammeter of some 5 to 20 ohms resistance, a portion of the resonance curve may be plotted as shown in Diagram *a*, Fig. 4. The resonant capacitance is not easily obtained with exactness because of a very small variation of I^2 in the region of resonance with small changes of capacitance. On the steep portions of the resonance curve, however, the changes in I^2 are large, as shown: hence if portion *abc* of the resonance curve is plotted, the value of resonant capacitance may be found by bisecting line *df* and projecting the point of bisection *e* down to the capacitance scale on the abscissa or taking the mean of readings *d* and *f*.

Another method of obtaining resonance consists of observing the value of capacitance, which results in a maximum change in the plate or grid currents of the oscillator or circuit driver which excites the condenser circuit. This method of resonance indication is recommended only in case a compensation circuit[1] is used to shunt the normal plate or grid current out of the indicating milliammeter in order to magnify the current variations. The

[1] See Chap. VI, Fig. 3. See also Figs. 6, 7 of this chapter.

reaction of the tuned circuit on the oscillator or circuit driver is small for loose coupling but sufficient to produce a minute change in the driver plate current, this change being greatest when the test circuit is in resonance with the driver frequency.

It is important to calibrate a condenser in the same manner in which it is to be used. If the unknown condenser has its rotary and stationary plates within a shielded container and has the rotary plates connected to the container, which is usually the case, it is the usual practice to ground point A, Fig. 4, when using such a condenser. In this case, the adjustments of the standard for resonance may be made by moving its rotor dial by hand. The standard, of course, should have its rotor grounded and shielded also. Under these conditions, there is little danger of the body capacitance of the operator affecting the results. On the other hand, if the unknown should consist of two fixed plates separated by air or a solid dielectric with no shielding container and is to be used with both its plates above ground potential, it is not exactly correct to measure its capacitance with the use of a standard condenser having shielded plates, in which the shield must be grounded. The best procedure is to measure the direct capacitance between the terminals of the unknown in the manner described; then the capacitance between each terminal of the unknown and ground may be measured separately, the condenser being placed as near as possible in the position in which it is to be used.

If hand capacitance disturbs the deflection of the resonance indicator in Fig. 4, it may be advisable to use an *incremental* or *step-by-step* method of adjusting C_s for resonance. This consists of changing the capacitance C_s by a small amount and then reading the deflection of the resonance indicator after removing the hand from the apparatus. Then another small adjustment is made, and results observed; the procedure is in several such steps until the reading of the resonance indicator is a maximum. Sometimes it is permissible to adjust C_s with a long handle, such as a wooden or glass rod, in which case deflections may be observed while varying the capacitance.

Degree of Precision Attainable.—When the foregoing precautions are carefully observed, the degree of precision of the measurement depends very largely upon the resonance curve, and it is assumed that point b, Fig. 4*a*, indicates true resonance.

This means that the mean of the readings d and f on the resonance curve gives the true value of C_r. If portions adb and cfb of the resonance curve are steep, the sharpness of resonance $\omega L/R$ or Q of the circuit is high. It is to be noted that when the sharpness of resonance is high, the power factor is very low and is equal to the reciprocal of that quantity. If the resistance R is so high that $\omega L/R$ is small, the resonance curve is somewhat as indicated by the lower curve $d'b'f'$.

For a truly symmetrical resonance curve the determination of e, the mean of d and f, will depend upon the sensitivity and scale of the current-squared resonance indicator. It may be that a change of 5 per cent in I^2 may not be noticed as C is varied. Assuming that the condenser scale can be read to a very fine degree with the aid of the Vernier or micrometer scale, let the noticeable 5 per cent change in I^2 correspond to a change in C of ΔC. Then for $C + \Delta C$ let the current squared be I_1^2. If the error ΔC in reading I^2 is made equally on both sides of the resonance, the correct value of Cr, Fig. 4*a*, will not be affected, but it is possible to read the two points as $d + \Delta C$ and $f + \Delta C$, whence the mean value will be $e + \Delta C$, and the error of measurement will be ΔC. If the resonance curve is like $d'b'f'$ in comparative slope at d' and f', the error $\Delta' C$ will be greater for the same noticed change in I^2. It is evident that a high Q circuit comprising R, L, and C is necessary for a sensitive adjustment of C. If the decrement is increased, the increase in the error ΔC may be calculated from a derived formula.[1] This has the approximate form

$$\Delta C = \frac{\delta^2 Cr}{\pi K_1}[2(1 - K_1) \pm \sqrt{3K_1 - 1}],$$

where $\delta = R/2fL = \pi/Q$, and $K_1 = I_1^2/I_r^2$. (See Fig. 4,*a*.)

The derived equation shows that the magnitude of this error varies inversely as the square of the circuit Q.

There is a possibility of obtaining a resonance curve that is unsymmetrical. Causes of an unsymmetrical resonance curve may be listed as follows:

1. Too great a change in variable capacitance.
2. High-loss resistance of the variable condensers.

[1] Brown, H. A., "Radio-Frequency Electrical Measurements," 1st ed., Chap. I, pp. 12–14.

3. Too close coupling to the circuit driver.
4. The presence of a strong low-order harmonic in the inducing e.m.f.

1. A resonance curve that is truly symmetrical on each side of resonance occurs only when plotted with I or I^2 as a function of the *elastance* $1/C$. When plotted against C, the capacitance and the resonance curve may be made symmetrical when certain conditions are imposed. To derive these, let it be supposed that I^2 is obtained for some value of $C = C_r + C'$, where C_r is the capacitance for resonance and C' is the change in C from resonance value C_r to obtain I^2. Now,

$$I^2 = \frac{E^2}{R^2 + \left[\omega L - \frac{1}{\omega(C_r + C')}\right]^2}.$$

But

$$\omega^2 = \frac{1}{C_r L}, \text{ or } L = \frac{1}{\omega^2 C_r}.$$

Substituting this relation and also the relation

$$Q = \frac{\omega L}{R} = \frac{1}{\omega R C_r},$$

where Q is for the test circuit at resonance,

$$I^2 = \left(\frac{E}{R}\right)^2 \frac{1}{1 + Q^2\left[\frac{\pm C'}{C_r \pm C'}\right]^2}. \tag{4}$$

It is noted that if C' is very small compared to C_r, the value of I^2 is the same whether C' is positive or negative. This means that the resonance curve is symmetrical about the resonance point (C_r and I_r^2) when $C' < C_r$. It may also be noted that at resonance, $I_r^2 = E^2/R^2$, and

$$\left(\frac{I}{I_r}\right)^2 = \frac{1}{1 + Q^2\left[\frac{(\pm C')}{C_r \pm C'}\right]^2}, \tag{5}$$

which is useful in estimating how much the current will decrease from the value at resonance for various changes of capacitance from that at resonance and for different values of the Q of the test circuit at resonance.

2. If the condensers C_x and C_s have high loss which varies with the capacitance settings, the peak of the resonance curve may be shifted with the changing total R of the circuit as C_s is varied. R_s is high when C_s is near its minimum, and C_s should be kept well above this value.

3. If the coupling to the circuit driver is too close, the unsymmetrical or even *double-humped* typical resonance curve of coupled circuits may result. A test for the effect of coupling is to observe the capacitance necessary to obtain I_r^2 while increasing the condenser capacitance through resonance and while decreasing through resonance. If the capacitance C_r for resonance is not the same for increasing and for decreasing capacitance, and if the current drops suddenly following a gradual rise, the coupling to the circuit driver is too close. The reaction of the test circuit upon the frequency of the circuit driver may shift the frequency of the latter considerably when the tune of the test circuit is slightly off resonance. A heterodyning oscillator may be placed at a considerable distance from the circuit driver; and if the pitch of the beat note in a telephone receiver connected in the plate circuit of this oscillator does not change when the test-circuit tuning approaches resonance, the circuit-driver frequency is not affected by any reaction. If a noticeable change occurs, the coupling of the pickup coil of the test circuit, Fig. 4, to the circuit driver should be decreased.

4. The presence of a strong lower-order harmonic in the inducing e.m.f. would tend to increase the effective value of the current in the circuit as the capacitance is decreased below the value of C_r for the fundamental frequency. The capacitance for resonance to the second harmonic is $C_r/4$. It is possible to calculate the shift in the apparent resonance point due to the effect of a second harmonic, but the complex analysis is hardly justified. It is more practical to test for this effect as follows: Reduce the capacitance of the standard condenser to one-fourth the value of C_r at the fundamental frequency. If no noticeable current-squared reading can be detected, the effect is practically *nil*, provided a very sensitive current indicator is used. If a small reading is obtained at $C_r/4$, increase C_s and observe the current-squared meter. If the deflection reduces to zero before

the region of the normal resonance curve is reached, the effect is *nil*. If it does not, it is possible that there will be some effect of shifting the point of I_r^2 to the left, Fig. 4,*a*. There is practically no possibility of such an effect with a well-designed circuit driver of sufficient power output capacity, with a tank circuit Q of 20 or higher.

In the past, certain investigators and writers have derived expressions for the correction necessary when the resonance-

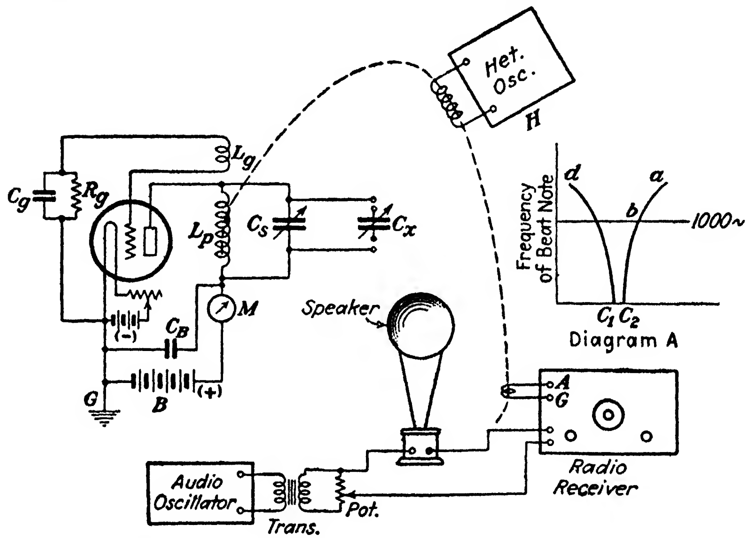


FIG. 5.—Test circuit for capacitance measurement by heterodyne tone.

curve peak is shifted, resulting in an unsymmetrical curve. They have also derived expressions for the probable error in the value of C_r when the coupling is close. However, there may be no certainty of results if the resonance curve is noticeably unsymmetrical, and it is essential first to take sufficient readings to examine the symmetry of the resonance curve before making actual measurements. A small dissymmetry at the lower ends of the resonance curve is allowable if none exists above $I_r^2/2$.

3. Capacitance Measurement by Heterodyne Tone. *Description of Method.*—The method depends upon the change of frequency in a vacuum-tube oscillator when the capacitance of the oscillatory, or "tank," circuit is changed. With reference to Fig. 5, the precision standard variable condenser C_s is part of the tank circuit of the vacuum-tube oscillator which has a grid coil L_g coupled to the plate-tank inductance coil L_p. This is a conventional inductive reversed feedback oscillator and will function easily at ordinary radio frequencies even though the tank-circuit capacitance is rather large, say 1,600 mmf. The heterodyning oscillator H is adjusted until its frequency is sufficiently close to that of the first oscillator so that the radio receiver gives the difference frequency, or beat note, between the two oscillators. This beat note may be easily adjusted to be equal in pitch to a note from a stable-frequency audio oscillator shown in the figure. The tone from the audio oscillator is injected into the receiver loud-speaker in the manner illustrated. When the beat-note frequency and that of the audio oscillator tone are nearly the same, there is a very slow pulsation in sound intensity; and when the pulsation is reduced to zero, the beat-note and audio-oscillator frequencies are exactly alike.

When an unknown capacitance C_x is added in parallel with the standard C_s, the zero beating effect is destroyed owing to the change in frequency of the oscillator. Then if C_s is reduced by an amount equal to C_x, the zero beating between audio notes is restored. As in the case of the capacitance by resonance method, this is a capacitance-difference operation, *i.e.*, decreasing a standard capacitance by a known amount when an unknown capacitance is added in order to restore certain initial conditions.

Apparatus and Circuit Arrangement.—The circuit may vary from that shown in Fig. 5, although the particular setup shown is recommended. A radio-frequency oscillator that beats on the heterodyning oscillator with an audio-frequency note may be substituted for the audio oscillator; this is known as the *three-frequency* method. The audio oscillator may drive a separate loud-speaker, if so desired, but the beating of the audio-oscillator tone and the beat note from the receiving-set speaker is usually not so noticeable to the ear, as with the arrangements shown in Fig. 5. It is important to connect a small coil of five or six turns to the antenna and ground terminals of the receiving set so that the latter will not pick up broadcast-station carrier waves or local disturbances. This pickup coil is sufficient for the reception of the carrier waves from the two oscillators placed a few feet away. One of these oscillator carrier waves may be much more intense than the other, and the oscillator responsible must be sufficiently removed so that it will not cause overloading of the radio receiver. If this occurs, a smooth variable-frequency beat note cannot be attained, and the zero-beat condition cannot be maintained.

Method of Manipulation.—As the capacitance is varied, two points will be found where the beat note appears and will match in tone the standard pitch. Referring to Diagram *A*, Fig. 5, as the capacitance of C_s or C_x is decreased the beat note in the receiver output will become audible at point *a* and will quickly decrease through the 1,000-cycle pitch at point *b* and to zero pitch at C_2. For most of the range $C_2 - C_1$ the beat frequency is too low to be audible, and for further reduction of capacitance the beat note begins and rises quickly to inaudibility at *d*. It is important to obtain the adjustment of equal tones by setting C_s for one point on Diagram *A* each time that C_x is added, say, point *b*. The following manipulation is therefore suggested: With stiff wire leads provided from C_s to the terminals of C_x, but not connected to the latter, set the heterodyne oscillator for a desired frequency, and tune the receiver to this carrier. A receiver having a beat oscillator is convenient for picking up an unmodulated carrier. Next start the vacuum-tube test circuit oscillator by increasing the coupling between L_g and L_p until the vacuum-tube plate current shows a marked decrease. It may be preferable to tune the receiver to the test-circuit oscillator which has been first set into operation with condenser C_s set at maximum value. Then the heterodyning oscillator may be tuned to produce a very high-pitched beat note. Then decrease C_s slowly until the pitch decreases to a point where it is in unison with that of the audio-oscillator tone. The unison adjustment can be obtained very closely by observing the beats between the two tones in the loud-speaker or even between a head set and a tuning fork. The unknown capacitance C_x is then added in parallel to C_s without disturbing the position of the leads, and C_s is decreased until point *b*, Diagram *A*, is again reached and the beat of the two notes is nearly zero. The grounded side of C_x may be connected permanently, the high-potential plate connection, only, being removed. When performing the manipulation, it will be found that body capacitance will change the beat-note frequency very greatly. It is important to make no changes in body position during the adjustments. It is well to ground point G, have the variable condensers so connected that the rotary plates are grounded through by-pass condenser C_B, and not remove the hand from the adjusting knob of C_s while making a measurement. Very loose coupling between the oscillators and receiving circuit is necessary, often 10 or 15 ft. separation. C_x must be completely moved from the position where it is to be connected when C_s is being first set for zero beating of the audio tones.

Theoretical Accuracy of Adjustment.—It will now be realized that the adjustment of C_s as described is very sensitive. Let it be assumed that the beat note and 1,000-cycle tone differ by ½ cycle per second, which is easily detected by the ear; then, the audio frequency of the receiver is $(1{,}000 \pm \frac{1}{2})$ cycles.

Let the radio frequency of the heterodyne oscillator equal 10^5 cycles. Imagine the capacitance C_s to be off the correct value by ΔC_s to produce an oscillation frequency in the receiver circuit of $(10^5 \pm \frac{1}{2})$ cycles.

Then, it may be easily shown that

$$\frac{C_s}{C_s + \Delta C_s} = \frac{(f + \Delta f)^2}{f^2} = \frac{(10^5 + 0.5)^2}{10^{10}} = 1.00001 \text{ (nearly)},$$

from which

$$\Delta C_s = 0.00001 C_s$$

to a close approximation. It is seen that if $C_s = 1{,}000$ micro-microfarads (mmf.) ΔC_s is equal to only $10^3 \times 0.00001 = 0.01$ mmf. Most standard condensers of the size assumed have accuracy of 1 mmf.; hence the degree of sensitiveness may be carried far beyond the accuracy of the standard.

A limitation on the accuracy of measurement lies in the constancy of pitch of the audio-oscillator tone. If the source is a tuning-fork oscillator of 1,000-cycle pitch, which remains within one cycle of the correct frequency, the probable error is very small and may be estimated as described.

The three oscillators described in the preceding paragraphs must be of the stable-frequency type; at least they should not drift in frequency over a short interval, say, three or four minutes. Some idea of the frequency stability of the oscillators may be obtained by setting the oscillators for a very slow intensity pulsation of the beat note and observing the variation in rate of this pulsation for a few minutes. A stabilized audio-frequency vacuum-tube oscillator is to be preferred to the tuning-fork type.[1] A good degree of precision may be obtained by eliminating the audio oscillator and reducing the standard capacitance C_s to zero beat with the heterodyning oscillator when the unknown is added. When doing this it is best to take the new setting of C_s as the mean of the values for which the audible beat does not cease and where it reoccurs as C_s is reduced.

If it be supposed that the frequency for zero-beat adjustment can vary 50 cycles without apparently disturbing the zero-beat condition, let $\omega^2 = 1/LC$ be the relation in the oscillator-tank circuit for a certain frequency $f = \omega/2\pi$. C represents the total capacitance. When the unknown is added and C_s reduced to apparent zero beat, the total capacitance may be $(C - \Delta C)$, resulting in a frequency $(f + \Delta f) = \omega'/2\pi$.

Then

$$\omega^2 = \frac{1}{LC}, \text{ and } (\omega')^2 = \frac{1}{L(C - \Delta C)}.$$

From these relations,

$$-\frac{\Delta C}{C} = \left[\left(\frac{f}{f + \Delta f}\right)^2 - 1\right]. \tag{6}$$

If f is assumed to be 10^6 cycles,

$$\frac{\Delta C}{C} = 1 - \left(\frac{10^6}{10^6 + 50}\right)^2 = 0.0001,$$

which indicates a possible error of 0.01 per cent. This is, of course, about ten times the precision of calibration of precision variable condensers.

4. Self-capacitance and True Inductance of a Coil. *Fundamental Considerations.*—Practically every coil used in radio circuits and apparatus has individual turns adjacent to each other forming electrostatic capacitances. These may be disposed in a complex manner but may be, in general, termed self-capacitance elements. If an e.m.f. is applied to the coil either by magnetic induction or by connections to the coil terminals, and the frequency increased, there will be a certain frequency for which resonance between the inductance and capacitance elements of

[1] HORTON, J. W., *Bell System Tech. Jour.*, Vol. 3, p. 521, July, 1924.

the coil will take place. This condition causes maximum circulating currents within the coil itself.

Earlier investigators considered that the long small-diameter solenoid-type coil could approximately be treated as a transmission line with distributed constants and showed that at the fundamental resonance frequency the current and potential would be distributed sinusoidally along the length of the coil. At this frequency, the input reactance

$$X = \sqrt{\frac{L_0}{C_0}} \tan \omega\sqrt{L_0 C_0}$$

is infinite, and this occurs when $\omega\sqrt{C_0 L_0} = \pi/2$. Here L_0 is the true inductance of the coil, and C_0 is a capacitance conceived in a manner similar to that for a parallel wire line. It was pointed out by these investigators[1] that a coil of this type is closely equivalent to an inductance L equal to the true inductance of the coil and shunted by a constant capacitance C_0, the equivalent self-capacitance of the coil. More recent investigations of the various types of coils in use at the present time show that at the fundamental resonant frequency the current and potential distributions are usually more complex than the simple sinusoidal modes previously mentioned. Experimental work has shown that at frequencies below fundamental resonance, the actual coil may be closely represented by the circuit of Fig. 6, where L is the true inductance of the coil, R the coil-conductor resistance, and C_0 the equivalent lumped self-capacitance of the coil.[2] Some investigators claim that even at frequencies of resonance, the error in assuming the circuit to be that of Fig. 6 is as low as 2 or 3 per cent. Since L is considered to be the flux linkage per ampere, $N\phi/10^8 I$, for constant current along the length of the coil, the small error mentioned would mean that, at resonance, the current distribution is not far from uniform. Considering the impedance Z_e of the equivalent coil circuit,

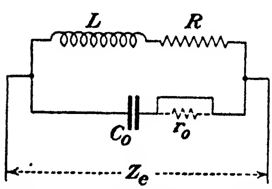


FIG. 6.—Equivalent circuit of a coil.

[1] MILLER, J. M., *Proc. I.R.E.*, Vol. 7, No. 3, pp. 299–326, 1919. Some bibliography of the work of others is given in this paper.

[2] See, for instance *U. S. Bur. Standards, Tech. Paper* 298, 1925.

$$Z_e = \frac{R}{[1 - \omega^2 C_0 L]^2 + \omega^2 C_0{}^2 R^2} + j\frac{\omega L[1 - \omega^2 C_0 L] - \omega C_0 R^2}{[1 - \omega^2 C_0 L]^2 + \omega^2 C_0{}^2 R^2} \quad (7)$$
$$= R_e + j\omega L_e.$$

For most coils with desirable values of Q, the terms $\omega^2 C^2 R^2$ and $\omega C R^2$ may be neglected, and

$$R_e = \frac{R}{(1 - \omega^2 C_0 L)^2}, \quad (8)$$
$$L_e = \frac{L}{1 - \omega^2 C_0 L}.$$

In addition to the effective resistance term R due to conductor loss at the operating frequency there is another loss-resistance term r_0, as a result of dielectric loss in the insulation between turns and in the coil form. This resistance would properly be shown in series with C_0 in Fig. 6. As this r_0 is usually quite small compared to R, it may be neglected. Special low-loss insulation is now in common use. For most radio-frequency circuits, coils with variable condensers are connected together and adjusted so as to form a resonant circuit and the frequency for resonance of the combination is sufficiently lower than the resonant frequency of the coil alone, so that the current in the coil is uniform throughout its length.

Methods and Procedure of Measuring Self-capacitance C_0.—*a*. If an e.m.f. is magnetically induced in a coil of true inductance L and self-capacitance C_0 to which a capacitance C is connected, as shown in Fig. 7, Diagram (*a*), conditions similar to a simple series circuit may be assumed as indicated in the figure. Resonance then occurs when

$$\omega^2 = \frac{1}{L(C + C_0)}$$

or

$$L(C + C_0) = \frac{1}{4\pi^2 f^2}. \quad (9)$$

Obviously, a graph of $(C + C_0)$ as a function of $1/f^2$, or of λ^2 (where λ is the wave length), will be a straight line when resonant frequencies for various values of C are found. This graph is shown also in Fig. 7. The axis of ordinates is erected at the point where $C = 0$, and the distance from this point to the intercept of the straight-line curve with the axis of abscissas is the value of C_0. Diagram (b) of Fig. 7 shows the arrangement for making the measurement. The circuit driver is loosely coupled to the inductance to be measured, and an accurate standard variable condenser C is connected to the coil. For C set at some value C_1, the frequency of the circuit driver is varied until a dip in the read-

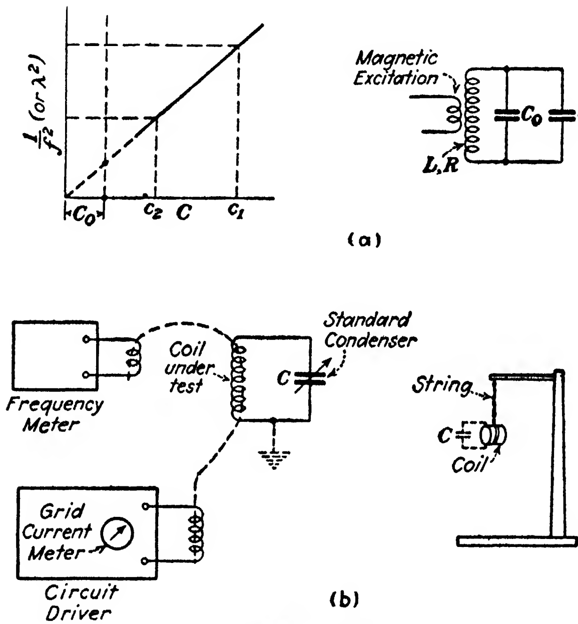


FIG. 7.—Test circuit and facilities for measurement of self capacitance.

ing of a direct-current milliammeter in the grid circuit of the circuit-driver vacuum tube is obtained. This dip indicates that maximum power is being absorbed by the test circuit and that the latter is then in resonance with the exciting frequency f_1. C is then reduced to value C_2 (see Diagram (a), Fig. 7), and the measurement repeated, resonant frequency f_2 being obtained. The circuit-driver frequencies f_1 and f_2 are of course obtained by using a frequency meter of a suitable type. For the first adjustment C_1 and f_1,

$$\omega_1^2 = \frac{1}{L(C_1 + C_0)}, \qquad \omega_1 = 2\pi f_1;$$

and for the second,

$$\omega_2^2 = \frac{1}{L(C_2 + C_0)}.$$

This gives

$$C_0 = \frac{f_2^2C_2 - f_1^2C_1}{f_1^2 - f_2^2}. \tag{10}$$

The true inductance L is then easily found, as indicated by one of the foregoing relations, when C_0 is known. If $C = 0$, $f_0 = 1/2\pi\sqrt{LC_0}$ and would be the natural fundamental frequency of the coil itself if the current distribution remained uniform at coil resonance, *i.e.*, the effective L unchanged. It is interesting to find this from the foregoing data and then find the actual frequency for resonance of the coil when C is removed. For most small coils, the difference is small. For long small-diameter single-layer solonoids, it may be considerable, owing to the nearly sinusoidal distribution of current in the coil for resonance with no externally added capacitance.

b. The self-capacitance may be also quickly determined by proceeding in a manner similar to that above, except that after f_1 is obtained corresponding to C_1, the frequency of the circuit driver is doubled, and C is then adjusted until resonance again obtains. In this case, the second relation is

$$4\omega_1^2 = \frac{1}{L(C_2 + C_0)},$$

and

$$C_0 = \frac{C_1 - 4C_2}{3}. \tag{11}$$

The expression for C_0 does not contain any frequency terms. After the circuit driver is adjusted to frequency f_1 for C_1 capacitance, a radio receiver incorporating a *beat oscillator* is tuned to zero beat with the second harmonic of the circuit-driver frequency. Circuit drivers using a class C vacuum-tube oscillator possesses a strong second harmonic in their outputs. Then the circuit-driver frequency is increased until zero beat in the receiver response is again obtained. The frequency is then $2f_1$.

c. Another method of determining self-capacitance consists of measuring the input reactance[1] or the input susceptance at the terminals of the coil. A radio-frequency bridge is desirable for this measurement. The expression for effective inductance in Eq. (8) can easily be transformed to

$$L_e = \frac{1}{\omega^2}\Delta s = \frac{1}{\omega^2}\left(\frac{1}{\frac{1}{\omega^2 L} - C_0}\right).$$

Then

$$\frac{1}{\Delta s} = \frac{1}{\omega^2 L} - C_0. \qquad (12)$$

Now, the effective input reactance at the coil terminals is

$$X_e = \omega L_e;$$

it follows the preceding relations that

$$\Delta s = \omega X_e. \qquad (12a)$$

Since L is practically constant for the usual radio frequencies, and C_0 may be considered constant, especially below resonance of the coil, $1/\Delta s$ is a direct function of $1/\omega^2$ with C_0 as the distance from the axis of $1/\omega^2$ to the intercept of the $1/\Delta s$ axis, as shown in Fig. 8. The input reactance is measured for various values of frequency and values of Δs calculated, and a plot is made as shown. C_0 is found graphically as shown. If input susceptances are measured, the procedure is as indicated in Fig. 8 also. Measurements must be made for values of ω not too near the value ω_0, or the curve will depart from a straight line.

When using method *a* or method *b*, very loose coupling with the circuit driver is necessary, and it should be made as loose as possible while still obtaining a noticeable change in grid current at resonance. Greater precision of measurement is obtained by using a compensating circuit, as described in Chap. III, page 157, to eliminate nearly all the normal plate current from a plate-circuit meter, and then observing the change in the small incremental plate current of the circuit driver. Much refinement is not justified in this measurement because of the effect of factors that may change with frequency, etc. It does, however, provide a

[1] This method was suggested to the author by Dr. D. B. Sinclair.

convenient means of determining the limits within which a coil may be expected to function with a given variable condenser range, as in radio-receiver tuned circuits and power-amplifier "tank" circuits. It may be desirable to test the coil with one side connected to ground and to the variable-standard condenser shield, as indicated in dotted lines in the test circuit in Diagram (*b*), Fig. 7. If it is desirable to obtain C_0 with the coil isolated as a

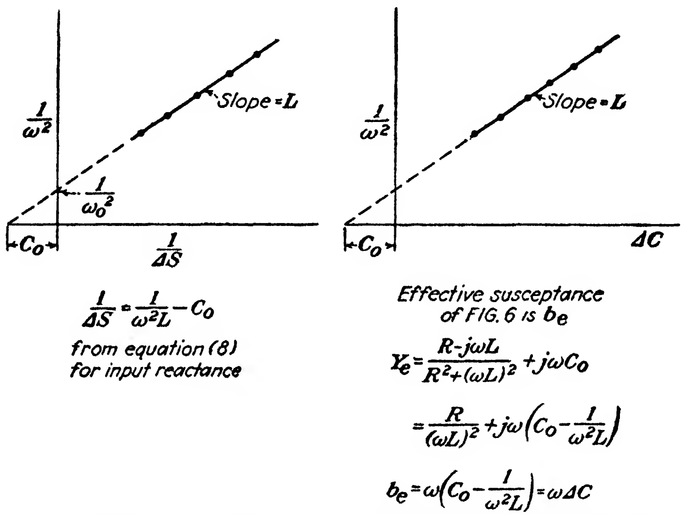


FIG. 8.—Straight-line curves (with C_0 intercepts) plotted for reactance and susceptance expressions for a coil with distributed capacitance C_0 and high values of Q.

scientific problem, the coil may be suspended by a string from a support like that shown. Then it is desirable to select two small mica dielectric fixed condensers to use for C_1 and C_2. When using method *c* it is important to work within the frequency limits for which the bridge is accurate. The radio-frequency bridge and its manipulation are described in the article following. The apparent or effective input reactance L_e [Eq. (7)] may be measured by a simple resonance method if the standard condenser is accurate at the frequency of measurement (see Introduction to this chapter). In this measurement, the calibrated standard variable condenser C_s is connected to the coil, with a thermomilliammeter in series for a resonance indicator. The coil is then loosely coupled to a circuit driver operating at a known frequency $f = \omega/2\pi$, and C_s is varied until resonance occurs for which $\omega L_e = 1/\omega C_s$. The measurement is repeated for various values of exciting frequency to obtain data for plotting $1/\Delta s$ versus $1/\omega^2$. Δs is calculated with the aid of Eq. (12*a*). A par-

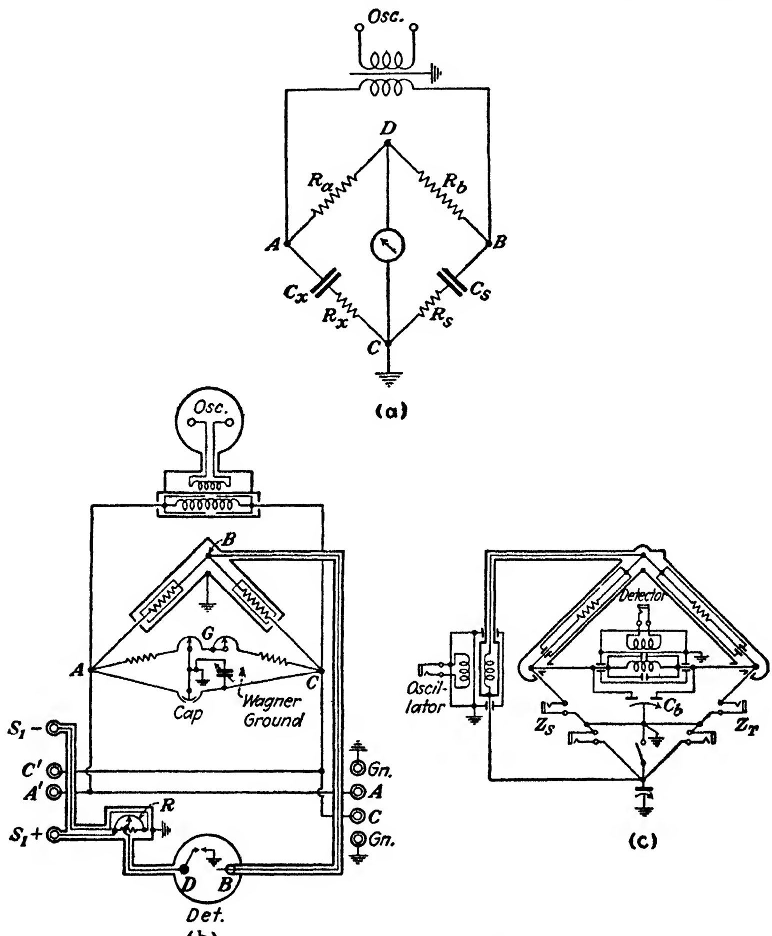


FIG. 9.—Schematic elementary and shielded types of radio frequency bridges.

allel substitution bridge method may be used to measure the input susceptance if it is desired to find C_0 from the plot between ΔC and $1/\omega^2$, as shown in Fig. 8.

5. Capacitance Measurement with Radio-frequency Bridges. *General Theory, Types of Bridges.*—The measurement of capacitance at radio frequencies by means of a bridge requires special considerations of its design. The ordinary unshielded bridge is unsuited for radio frequencies, owing to the variable stray admittances which may exist between the elements of the bridge and between these elements and the ground. By proper shielding of the elements, the admittances between terminals of the ratio arms and ground may be made a fixed value. This is also true of the stray admittances in the other arms. No attempt will be made to give a complete analysis of the capacitance bridge (De Sauty type) or of the more elaborate shielded impedance bridges. The modern radio-frequency bridges are based on the simple form. In Fig. 9, Diagram (*a*), is shown the elementary type. When balanced, the simple relations are

$$\frac{R_a}{R_b} = \frac{R_x}{R_s} = \frac{C_s}{C_x}. \tag{13}$$

The proof of Eq. (13) may be found in many textbooks of electrical measurements. To construct this bridge properly for radio frequencies, R_a and R_b must be high-frequency noninductive resistances and are made equal to simplify the balancing of the bridge for shunt-admittance effect, etc. It will be noted that the bridges have the junctions of the shielded capacitance leads grounded. It is pointed out by Field (see footnote 1, page 32) that it would be better to ground the junction of the ratio arms if it were not for the necessity of adding a variable standard resistance R_s (Fig. 9) in series with the standard condenser C_s in order to obtain a complete balance. The reason for this is given in the paper referred to. In Diagram (*b*) of the figure is shown a completely shielded bridge incorporating the well-known *Wagner-ground* feature, represented by the elements lettered *G* and *Cap.*[1] With this device the points *D* and *B* may both be brought to ground potential. This bridge has a maximum frequency range of 50 kc. In diagram (*c*) is shown a bridge whose range is 10 to 5,000 kc. It employs a balancing condenser C_b but does not use the complete Wagner-ground network.[2]

[1] This bridge is manufactured by Leeds and Northrup Company and is described in a paper by Behr and Williams, *Proc. I.R.E.*, Vol. 20, No. 6, June, 1932. The Wagner ground is described in this paper and elsewhere. For K. W. Wagner's original paper see *Electrotech. Ztsch.*, Vol. 32, pp. 1001–1002, 1911.

[2] YOUNG, C. H., "5-megacycle Impedance Bridge," *Bell Laboratories Record*, Vol. 15, No. 8, p. 261, April, 1937.

A useful, commercially available radio-frequency bridge is illustrated in Fig. 10 and deserves explanation.[1] The diagram is taken from the paper referred to. The shielding is not shown. The bridge uses equal resistance ratio arms R_A. It is a resistance-capacitance bridge, but for power-factor measurement it becomes a Schering bridge when C_P is varied to obtain a null balance, at which $PF_x = \omega C_P R_A = \omega C_x R_x$. R_s is set at zero. When it is

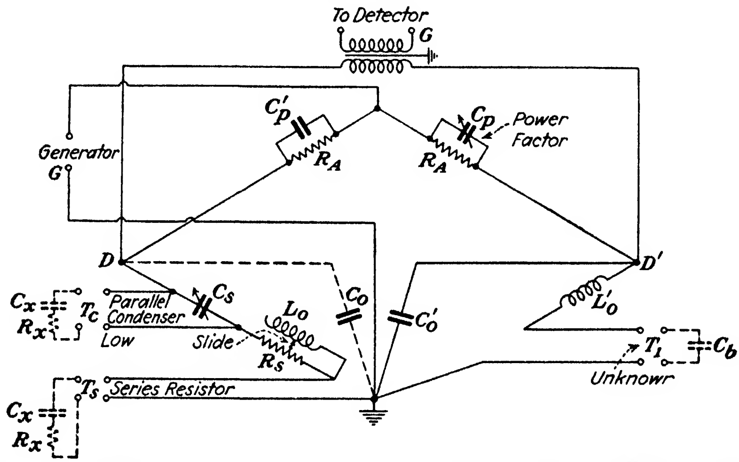


FIG. 10.—Radio-frequency capacitance—Schering bridge. (*General Radio Company, Cambridge, Mass.*)

used as a straight capacitance-resistance bridge, and the balance is obtained by varying C_s and R_s, C_P is left at its minimum value setting. Then, in order to have perfectly symmetrical equal ratio arms A, it is necessary to provide a fixed capacitance C_P' equal in value to the minimum capacitance of C_P.

The lower half of the bridge circuit below the diagonals DD' requires special design to make it function at frequencies as high as 5 Mc. When a capacitance C_x is to be measured, its equivalent series resistance R_x makes it impossible to obtain a perfect null balance by varying C_s, unless a noninductive variable resistance is provided in series with the standard condenser C_s. At radio frequencies there is an appreciable inductive reactance in a variable resistance of even the most carefully designed high-fre-

[1] See *General Radio Experimenter*, Vol. 8, No. 7, December, 1933, for more complete discussion. This bridge is manufactured by the General Radio Company, Cambridge, Mass. See Fig. 35 for a shielded high-voltage Schering bridge.

quency type. In this bridge, the problem of compensating for the effect of such reactance is solved by providing a variable resistance R_s combined with a small variable inductance L_0. As indicated in the figure, when the slide is moved so that R_s increases in value, the inductance L_0 is decreased in value; and when R_s decreases, L_0 increases. The net result of these combined variations is that the total inductive reactance remains constant, regardless of the value of R_s. It is to be noted that to accomplish this result the total inductance of L_0 must be equal to that of the maximum value of R_s. To compensate for the constant inductive reactance, an equal fixed inductive reactance L_0' is provided in the lower right-hand arm of the bridge. The standard capacitance C_s is so connected that there exists a capacitance from diagonal D to ground which is indicated in dotted lines by C_0. The effect of this would be to cause a serious error in capacitance measurements; hence a fixed capacitance C_0' equal to C_0 is connected from D' to ground.

Bridge Generators and Detectors.—In well-balanced bridgework, the device for indicating balance is usually termed a *detector.* For radio-frequency bridges it must employ actual *grid* or *plate* rectification and also considerable gain in order to be sensitive. The usual detector-amplifier combination with output meter may be used but is not always convenient to provide. About the most satisfactory combination of generator and detector is a modulated radio-frequency signal generator, such as described in Chap. VIII (pages 348–350), and a conventional all-wave radio receiver with loud-speaker as a detector. If the generator produces an unmodulated radio-frequency e.m.f., the receiver may conveniently incorporate the *beat oscillator.* This oscillator is designed to beat on the intermediate frequency, producing an audible note in the loud-speaker. A low-range rectifier-type alternating-current voltmeter may be connected across the voice coil of the loud-speaker input transformer to serve as visual null-balance indicator.

Methods of Measuring Capacitance.—The three methods to be explained are as follows:

1. Fundamental capacitance balance using standard and unknown.
2. Modified parallel substitution method.
3. Series substitution method.

1. The unknown capacitance C_x (Fig. 10) having a series loss resistance R_x is connected to terminals T_1, and C_s and R_s are varied

until balance is indicated by the indicating meter or telephone receiver associated with the detector. The capacitance of the unknown C_x is then equal to that of the standard C_s, since the bridge is the *unity-ratio* type.

2. The parallel substitution method is, in general, similar in principle to the resonance method of capacitance measurement (see Art. 2 of this chapter). The bridge is first balanced with C_s set at near its maximum value, and another convenient variable condenser C_b is connected to terminals T_1 (Fig 10). This merely acts as a *balancing condenser*, and with it the bridge is balanced. Terminals T_s are shorted. The unknown C_x is then connected to terminals T_c, being placed in parallel with C_s. Then C_s is reduced until bridge balance is restored, and the change in C_s is the capacitance of C_x. If C_x is a variable condenser, it may be set at its minimum value before being connected to T_c and then increased step by step as C_s is reduced in like steps, as explained in Art. 2.

A variable condenser having about twice the capacitance of C_s may be calibrated in the following manner: Procedure is first followed as explained above until the standard C_s has reduced to its zero scale value. Then the maximum value of C_x that may be thus measured is C_x', a value that is nearly one-half the maximum value of C_s. The next step is to set C_s to its maximum value while C_x is set to value C_x', both being connected in parallel as before. The bridge is then balanced by increasing C_b to the proper value. Then C_x is increased by some convenient increment ΔC_x, and C_s is reduced by an amount ΔC_s to restore bridge balance. If the new setting of C_x is now C_x'',

$$C_x'' = C_x' + \Delta C_x = C_x' + \Delta C_s. \tag{14}$$

In like manner, the remaining calibration points of C_x may be obtained.[1]

3. The unknown C_x is connected in series with the standard C_s in the series substitution method; this method applies to a limited range of capacitances to be measured. In Fig. 10, C_x is connected to terminals T_s, and C_b to T_1.

At balance the reactance X of C_s and C_x in series is

[1] This method is described by R. F. Field, *General Radio Experimenter*, Vol. 4, No. 8, January, 1930.

$$X = \frac{1}{\omega C_s'} + \frac{1}{\omega C_x}; \tag{14a}$$

and when C_x is removed, C_s must be changed to restore balance, and hence the same value of X; so that $X = 1/\omega C_s''$, and the expression for C_x is

$$C_x = \frac{C_s''C_s'}{C_s' - C_s''}. \tag{15}$$

This method must be used with caution, because placing a capacitance between R_s and ground places the capacitance of the shield of C_s and R_s to ground in shunt with C_x so that C_0' should be readjusted. It would be better to place C_x between C_s and diagonal D of the bridge shown in Fig. 10.

Sources of Errors in Measurement.—At radio frequencies, the proper compensation for shunt admittances of the elements of the bridge is of prime importance. When constructing a bridge in the laboratory, it is important to consult the literature referred to in this article.[1] It is pointed out that with the bridge described, a precision of about 1 per cent may be attained at a frequency of 1 Mc., and about 4 or 5 per cent at 5 Mc. These figures apply to the direct-reading bridge (Method 1). In the modified parallel substitution method (Method 2), somewhat better accuracy is obtained. The effect of inductance in the standard capacitance may be compensated somewhat by using the series substitution method (Method 3). In Eq. (1) of the Introduction to this chapter, it is noted that the effective reactance of a condenser is

$$X_e = \left[\frac{S}{\omega} - \omega L\right] = \left[\frac{1}{\omega C} - \omega L\right],$$

or

$$\frac{1}{\omega C_e} = \frac{1}{\omega C} - \omega L. \tag{16}$$

Now from Eq. (16), Eq. 14*a* takes the form

$$X = \frac{1}{\omega C_{ss}'} - \omega L + \frac{1}{\omega C_x};$$

[1] See also DYE and JONES, "A Radio Frequency Bridge for Impedance and Power Factor Measurements," *Jour. I.E.E.*, Vol. 7, No. 434, p. 169, February, 1933; FERGUSON and BARTLETT, "Measurement of Capacitance," Bell Tel. Lab., Reprint B-341, August, 1928; also Bibliography, *Proc. A.S.T.M.*, D150–36T, 1936.

then

$$\frac{1}{\omega C_x} = \frac{1}{\omega C_{ss}''} - \frac{1}{\omega C_{ss}'}.$$

C_{ss}' and C_{ss}'' are static capacitances of the standard C_s. Here the effect of the inductance L of the standard capacitance cancels out. Hence, Eq. (15) gives the effective capacitance at the testing frequency.

6. Measurement of Direct Interelectrode Capacitance. *Precision Requirements.*—It is often desirable to measure the direct capacitance between two electrodes of a thermionic vacuum tube quickly. As is well known, this cannot be done by connecting the electrodes in question to an ordinary capacitance-measuring device, because of the presence of the complex network of the measured capacitance associated with the other interelectrode capacitances. Methods have been devised by which the direct capacitance between any two electrodes can be measured as if the other electrode or electrodes were not present. For the case of the screen-grid types of tubes, the effective capacitance between grid and filament is extremely small and is difficult to measure accurately, especially because the introduction of the "magnesium getter" within the glass envelope results in a high-resistance path or paths being placed in shunt with the capacitance to be measured. Two methods of measurement applicable to ordinary two- and three-electrode tubes and to the lower capacitance grid types, respectively, will be described.

Method by Direct-capacitance Bridge.—A special form of capacitance bridge was described by Walsh,[1] in which electrode capacitances to be excluded from the measurement were connected across elements of the bridge so that their presence had no effect upon the capacitance to be measured. This form of bridge is shown in Fig. 11, which is taken from Walsh's paper. It is a form of the De Sauty capacitance bridge, the ratio arms being resistances R_1 and R_2. The bridge is shown arranged to measure the capacitance between the grid and plate of a three-electrode tube, this capacitance being denoted by C_{gp}. The plate-filament capacitance C_{pf} is shown connected across the primary of an output transformer used in this particular form of bridge. C_{pf} can have no possible effect upon the balance of the bridge provided

[1] *Proc. I.R.E.*, Vol. 16, No. 4, pp. 482–486, April, 1928.

it remains sufficiently small, so that its reactance is high, compared with that of the output transformer. C_s is a standard variable condenser which may be considered to have zero loss, and the resistance r_m is placed in series with C_s in order to obtain a fairly complete balance when C_{gf} is in shunt with R_1. At balance let the current flowing in C_s and C_{gp} branches be I, and let the current

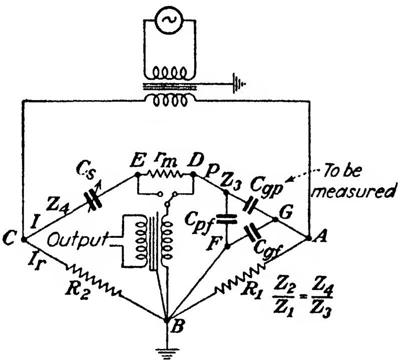


FIG. 11.—Circuit of direct electrode—capacitance bridge.

flowing in the ratio arms be I_r. The complex potential drops from C to D and from C to B, equal in magnitude and phase, give

$$I\left(r_m - j\frac{1}{\omega C_s}\right) = I_r R_2.$$

Likewise, the potential drops from A to D and from A to B, being equal, give

$$I\left(-j\frac{1}{\omega C_{gp}}\right) = I_r\left[\frac{1}{\left(\frac{1}{R_1} + j\omega C_{gf}\right)}\right].$$

Then dividing these equations eliminates the currents and gives the equality

$$\frac{R_2}{R_1} + j\omega C_{gf}R_2 = \frac{C_{gp}}{C_s} + j\omega C_{gp}r_m.$$

This equation is made up by two complex or vector expressions on each side of the equality sign; and if two such expressions having a complex quantity are equal, it is understood that the real terms on each side are equal to each other and also that the imaginary terms are equal to each other, or

$$\frac{R_2}{R_1} = \frac{C_{gp}}{C_s} \text{ and } \frac{C_{gf}}{C_{gp}} = \frac{r_m}{R_2},$$

from which

$$C_{gp} = C_s\frac{R_2}{R_1}. \tag{17}$$

It will be noted that from the relation of the imaginary terms and Eq. (17), the grid-filament capacitance may be at once determined, but the value of r_m need not be adjustable to a high degree of sensitiveness. It is possible to vary r_m considerably without a noticeable change in balance. This means that the grid-filament capacitance may be present across R_1 without affecting a ratio at balance which will give the direct capacitance C_{gp} when C_s and the ratio arms R_1 and R_2 are known.

It is quite important to arrange the bridge so that point B is grounded; this permits the grounding of the filament of the tube being measured. The capacitance from plate to grid is the most important to be measured because of its effect in producing regeneration, or feedback action, and is of the order of 10 mmf. for the small receiving and low-power amplifying tubes. The ratio arms may be adjusted so that larger values of variable standard capacitance may be used, but the most desirable condition for a bridge of this type is to have the ratio arms equal. It is convenient to place a high-grade tube socket across terminals D and A, and then place a known capacitance in shunt with the plate grid connection of the socket. The value of this known capacitance may be high enough so that the particular range of the variable standard condenser available may be used. With the socket and known capacitance in place, C_s is adjusted for balance, after which the tube to be measured is inserted in the socket and C_s increased until the balance is restored. The change in C_s is then the direct capacitance C_{gp} of the tube. The resistance r_m is adjusted to perfect the balance before inserting the tube, and after inserting r_m may be readjusted if necessary, but it will be found that little or no readjustment is needed. The input and output transformers of the bridge are of the balanced-winding, shielded, grounded-core type, and are quite necessary to reduce the possibility of spurious shunt admittances which affect the results. In this bridge, it was recommended that the junction of the ratio arms be grounded so that the measurement could be made with the filament grounded. The bridge was intended to function at audio frequencies also. As pointed out for the bridges of Figs. 9 and 10, it would be preferable to ground the junction of the capacitance arms, especially since there is a balancing resistance in series with C_s. However, the error is not serious if r_m (Fig. 11) is not too high and does not possess great physical proportions. If it is desired to make bridge measurements of direct electrode capacitance at radio frequencies below 5 Mc., the radio-frequency bridge of Fig. 10 should be used.

E. T. Hoch[1] describes a method of applying the Campbell-Colpitts bridge which is used for measurement of both the direct

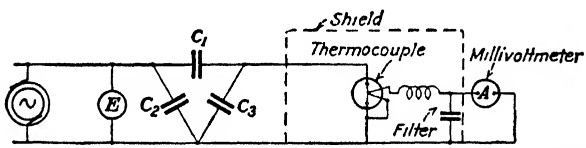


FIG. 12.—Fundamental circuit for measuring direct electrode capacitance by voltage-current method. (*After Loughren and Parker.*)

electrode capacitance and the conductance of vacuum tubes. This bridge is usually not available for the student, so the details of the theory and method are not given.

Direct Interelectrode Capacitance at Radio Frequencies.—A. V. Loughren and H. W. Parker[2] describe their modification of another method for measuring the direct electrode capacitance at radio frequencies, based upon the voltage-current method of measuring impedance. Figures 12 and 13 are taken from their paper and show, respectively, the fundamental circuit and the more elaborate arrangement for this measurement. Referring to Fig. 12, the electrode capacitances are shown connected in such a way that the capacitance C_1 to be measured is in series with the radio-frequency supply and the heater terminals of a thermojunction. One of the other electrode capacitances is in shunt with the supply circuit where it has no effect upon the amount of current flowing through the thermojunction, and the

[1] *Proc. I.R.E.*, Vol. 16, No. 4, pp. 487–493, 1928.
[2] *Proc. I.R.E.*, Vol. 17, No. 6, pp. 957–965, 1929.

other electrode capacitance is connected in shunt with the thermocouple heater so that its high reactance has practically no effect upon the total impedance of the circuit. Assuming that this capacitance has a value given for a certain tube, a calculation will show that its reactance at frequencies of the order of 500 kc. will have a negligible effect upon the total impedance of a 10- to 100-ohm thermocouple heater. The authors of the paper referred to state that this effect is negligible even for larger tubes with the water-cooled anode. Referring again to the figure, a thermovoltmeter or thermomilliammeter and standard resistance is used to measure the voltage impressed upon the circuit and the charging current,

$$I = 2\pi f C_1 E,$$

where f is the frequency, E is the effective impressed radio-frequency voltage, and C_1 is the direct electrode capacitance to

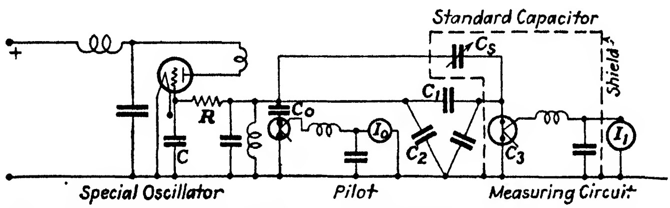


FIG. 13.—Actual measuring circuit arrangement based on Fig. 12.

be measured. It will be noticed that the millivoltmeter is connected to the thermojunction through a small low-pass filter so that any error, due to shunting of the radio-frequency current through the meter circuit, is prevented. The inductance of this filter must be of such a design as to present a high impedance to the radio-frequency e.m.f. compared to that of the heater circuit. Figure 13 shows a more elaborate circuit arrangement. There will exist a constant product of $2\pi f C_1 E$, and the constancy is indicated by the so-called *pilot circuit* made up of capacitor C_0 and current indicator I_0. This feature is not absolutely necessary if time can be taken to read and hold constant, manually, the value of E. It is not necessary to know f or E with this setup because the variable standard capacitance C_s is substituted for the direct capacitance and adjusted until the same current is indicated; this also eliminates the necessity for measuring the current accurately, because the capacitance of the standard may be measured to a considerably higher degree of precision than can be the radio-frequency voltage and current. The paper referred to shows two forms of a variable standard capacitor, the simplest of these being a metal rod, sliding in and out of a concentric metal tube, shown diagrammatically in Fig. 14. A further modification of the discussed setup is shown in this paper in which the thermo junction is replaced by a crystal

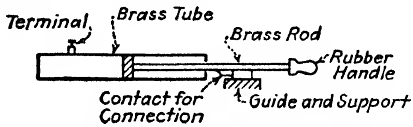


FIG. 14.—Standard capacitor C_s used in measuring circuit of Fig. 13.

detector, microammeter, and compensation circuit, resulting in a form of null-balance method.

7. Other Capacitance Measurements. *Measurement of Very Small Capacitance.*—The heterodyne-tone method (Art. 3) is very sensitive to small capacitances, and the degree of precision of measurement may be estimated from the precision and the scale characteristics of the standard condenser and the accuracy of the comparison audio tone. The change in frequency of the oscillator for an assumed percentage change in tank-circuit capacitance is readily calculated. It is not easy, however, to find capacitance standards that are accurate to more than 1 mmf.

Capacitance Measurement at High Radio Frequencies.—Since the conventional standard variable condenser is not reliable at radio frequencies at, say, 5 Mc. or more (see Introduction), it is possible to measure effective capacitance by measuring the current in, and the voltage across, the unknown capacitance. If the equivalent loss resistance of the condenser is low compared to the reactance, as it usually is, the voltage across the condenser will be practically equal to the reactance drop $I/\omega C_e$, where C_e is the effective capacitance, and I is the current. For high radio frequencies the current is accurately measured with a suitable high radio-frequency thermocouple (see Chap. VI). The voltage may be either known from a suitable radio-frequency voltage standard (Chap. VI) or measured with a special high radio-frequency vacuum-tube voltmeter using the acorn tube.[1]

[1] Such an instrument is described in Art. 2, Chap. VI, of this book. Another vacuum-tube voltmeter using the acorn tube, and within 3 per cent

A method which does not involve other standards requires the measurement of the resonant frequency of a concentric transmission line when the unknown effective capacitance is added in series. The input reactance of a line is given by the expression[1]

$$X_0 \cong -\sqrt{\frac{L_0}{C_0}} \cot \omega\sqrt{C_0L_0}, \text{ approximately.}$$

The geometric inductance and capacitance L_0 and C_0 of a concentric line can be calculated with good precision from the physical dimensions. Skin effect causes a small error in L_0, but this is not serious.[2] The input reactance X_0 as given above is zero at the fundamental frequency of resonance; this may be calculated and checked experimentally. At frequencies above but less than twice this value, the input reactance X_0 is positive. If a capacitance reactance X_c is placed in series with the input to the line, the resulting input reactance X_i will be zero if the two reactances X_0 and X_c are equal.

$$X_i = X_0 - X_c = 0.$$

Therefore the unknown capacitance may be placed in series with the input of the line and the exciting frequency varied between the fundamental resonance value and twice this value until the total reactance X_i is zero (this condition is indicated by maximum current).[3] Let this frequency be f_1 and $w_1 = 2\pi f_1$. The line-input reactance X_0 can then be calculated at frequency f_1, from

$$X_0 \cong \sqrt{\frac{L_0}{C_0}} \cot \omega_1\sqrt{C_0L_0}, \text{ approximately;}$$

and since

$$X_c = \frac{1}{\omega_1 C} = X_0,$$

the effective capacitance C at frequency f_1 is readily determined. The length of the testing concentric line must be so chosen that the series capacitance is the proper value to bring about the condi-

accuracy at 100 Mc., is described in *General Radio Experimenter*, Vol. 11, No. 12, May, 1937. It is manufactured by the General Radio Company.

[1] *Proc. I.R.E.*, Vol. 7, No. 3, p. 303.

[2] See Art. 6, Other Types of Attenuators, Chap. VI.

[3] Frequencies can be measured to any desired degree of precision, as described in Chap. II.

tion of zero total input reactance. This may require a number of experimental trials. Manipulation is facilitated by having a variable condenser as the unknown.

8. Inductance Measurement by Substitution Method. *Brief Description of Method and Procedure.*—This method is similar in principle to some of the previously described methods. For the range of frequencies and inductance values for which it is used, this method eliminates the inductance of the leads without the need for a standard inductance. The method is described by C. T. Burke,[1] and the setup is also used for the measurement of resistance. Figure 15 shows the diagram of connections and arrangement of apparatus. The *link-circuit* coupling to the

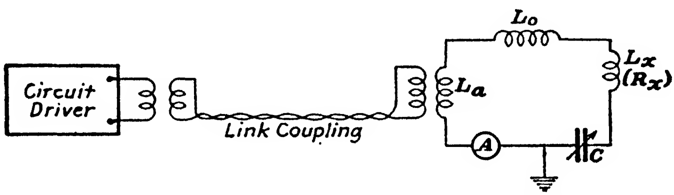


FIG. 15.—Circuit arrangement for measurement of inductance.

oscillator is useful to provide a measuring circuit isolated from the intense field of the oscillator. Thus the coil L_x may be introduced without danger of an e.m.f. being induced in it by direct influence of the oscillator. The coil to be measured has inductance L_x as indicated. The inductance of coil L_0 and the range of C are such that the circuit may be tuned to resonance with a certain excitation frequency, when L_x is in the circuit or when L_x is removed, by varying the standard condenser C. The ranges and values should also be such that C is near its maximum value when L_x is not in the circuit, and, when L_x is placed in the circuit, C should not be reduced to the lower end of its scale to regain resonance for a flexible test circuit. Instead of removing L_x from the test circuit, it is sometimes better to short-circuit it with a short-circuiting bar or small, porcelain-base switch. In this way, the shunt capacitance of the test circuit to ground and hence in shunt with C is not changed. This is important at high frequencies or if the coil has considerable physical size. It must be remembered that the measured value of L_x is the apparent inductance. For the settings of C for resonance

[1] *Trans. A.I.E.E.*, Vol. 46, p. 483.

with L_x in and out of the circuit it is easy to set up expressions from which

$$L_x = \frac{(C_2 - C_1)}{\omega^2 C_1 C_2}, \tag{18}$$

where C_1 and C_2 are the capacitances for resonance of the circuit with $f = \omega/2\pi$ for L_x in circuit and out, respectively. Features of some of the circuit and apparatus details such as the best kind of resonance indicator are discussed in Art. 2.

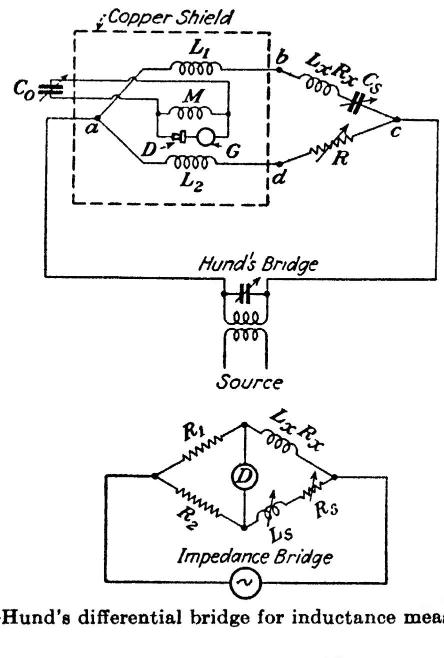


FIG. 16.—Hund's differential bridge for inductance measurement.

9. Bridge Methods of Inductance Measurement. *Hund's Differential System.*—August Hund[1] describes a method of measuring the inductance and resistance of a coil, and his apparatus is similar in principle to the resonance bridge described in Chap. II, Art. 8. The points of difference are best understood by referring to the diagram of connections as shown in Fig. 16. The inductances L_1 and L_2 take the place of the equal resistance arms of the resonance bridge. In place of the usual balance indicator, or *detector*, across the diagonals of the bridge, Hund uses a

[1] "Measurements at Radio Frequency," *Elec. World*, Vol. 84, p. 998, 1924.

coil M coupled magnetically to L_1 and L_2. These inductances are exactly alike except that they are wound in opposite directions and each has the same coefficient of coupling with M. When C_s is varied until L_x and C_s form a series resonant circuit at the applied frequency, the voltage drop across bc is due to R_x and the loss resistance r_c of C_s; R is varied until drop cd equals drop cb, so that the potential difference across bd is zero. If

$$L_1 = L_2,$$

then

$$R = (R_x + r_c)$$

for balance. Hence the induced e.m.fs. in the secondary coil M, due to the currents in L_1 and L_2, are equal and opposite. The galvanometer will then show zero deflection. C_0 is provided so that C_0 and M may be tuned to resonance with the test frequency when the bridge is not quite balanced. This makes for greater sensitivity of balance. The copper shield is provided around the inductance arms and M to eliminate the possibility of errors due to shunt admittances. The author referred to submits data in his paper to show that the accuracy of measurement is quite high—up to 250 kc. In using this bridge, care must be used in making and mounting the coils L_1, L_2, and M. Exact symmetry within the shield and between the coils must be maintained, or the apparent inductances of L_1 and L_2 will be different, causing an erroneous balance. L_1 and L_2 may be wound on the same form with M inside or outside; the author recommends a small number of turns for L_1 and L_2—say, 10 or 20—and about 100 to 200 turns for M. It is best to ground the shield and is also probably best to ground point c, allowing leads a, b, and d to come out through holes in the shield box. It is also best to substitute a vacuum-tube voltmeter for the detector-galvanometer circuit so that the coil M (Fig. 16) will not be shunted by a resistance, the latter being detrimental to sharp resonance unless it is very high in value. The resonance bridge, previously referred to, also provides a convenient and fairly precise method of measuring inductance at the lower radio frequencies. For this use it should also be suitably shielded.

The Shielded-impedance Bridge.—For measurements of inductance at the lower radio frequencies, and especially in the region of telephone carrier-current frequencies, a well-shielded and compensated impedance bridge is most desirable for rapid and accurate work. The well-known form of impedance bridge can easily be temporarily assembled and is as shown in the impedance-bridge diagram in Fig. 16. R_1 and R_2 are usually equal ratio arms, and R_s and L_s are standard variable resistance and inductance. It is evident that at balance

$$R_s = R_x$$

and

$$L_x = L_s.$$

A system of shielding, of course, improves the reliability of the bridge. If R_1 and R_2 only are shielded, as in the previous bridge, adjustments can be made on R_s and L_s in steps or small increments, removing the hand from the adjusting handles after each step adjustment and observing the approach to balance. Detectors for use with such a bridge are similar to those used on the radio-frequency bridges in Art. 5. The bridges shown in Figs. 9 and 10 should be used.

The impedance bridge of Shackleton and Ferguson[1] represents a very elaborately shielded and compensated form of impedance bridge used for frequencies below 100 kc. A very high degree of accuracy together with rapidity of manipulation is accomplished with this bridge in the measurement of inductance, capacitance, and resistance. It is not within the scope of this text to go into the details of the design of this special bridge, but the reader is referred to the paper mentioned above for a comprehensive discussion of its features.

10. Measurement of Mutual Inductance by Induced E.M.F. *General Principle and Procedure.*—When current at a frequency f flows in the primary winding of a mutual inductance, the e.m.f. induced in the open-circuited secondary is

$$E_s = 2\pi f M I_1.$$

Hence, a vacuum-tube voltmeter may be used to measure E_s at radio frequencies, and a thermomilliammeter and frequency meter may be used to measure the value of the current in the

[1] *Trans. A.I.E.E.*, Vol. 46, p. 519.

primary and its frequency, respectively. From these readings M may be calculated by formula above. For the lower radio frequencies this simple expedient is often sufficiently accurate.

The primary current I_1 should be a fairly pure sine wave so that wave-form errors in the vacuum-tube voltmeter are not present. The capacitance coupling between the two coils of a mutual inductance must be low so that the induced e.m.f. is due mostly to flux interlinkage. The capacitance coupling effect may be roughly observed by using the two coils of the mutual inductance as the two plates of a condenser in circuit with a milliammeter and the source at a desired frequency. If the current flow is appreciable, the secondary e.m.f. (as measured with the tube voltmeter) will not be correct. The result under these conditions is an *apparent* mutual inductance.

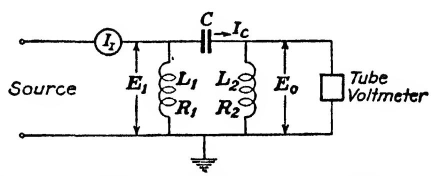


FIG. 17.—Equivalent circuits of mutual inductance with negligible resistance drop.

The effect of capacitance between the coils of a mutual inductance can be analyzed by assuming that the capacitance is concentrated between one end of the primary and one end of the secondary. Figure 17 shows the theoretical circuit. One end of the primary and of the secondary are connected together as shown, and the capacitance between the coils may be represented roughly by a capacitance C equal to one-third the geometric capacitance and connected to the free ends of the coils as shown in the figure. The basis upon which this one-third value is taken is the same as that derived by Morecroft for concentric coils.[1] If the resistance drops in the primary and secondary are negligible compared to the reactance drops, the analysis is simplified. When current I_1 flows in the primary, an e.m.f. equal to $-j\omega MI_1$ is induced in the secondary. At the same time, a current I_c flows through C and L_2 due to voltage $E_1 = j\omega L_1 I_1$. Then the current

[1] MORECROFT, J. H., "Principles of Radio Communication," 2d ed., pp. 232–233.

$$I_c = \frac{E_1}{j\left(\omega L_2 - \dfrac{1}{\omega C}\right)}.$$

The e.m.f. across the secondary due to I_c is vectorially

$$E_2' = E_1\left(\frac{Z_2}{Z_c + Z_2}\right) = E_1\left[\frac{j\omega L_2}{j\left(\omega L_2 - \dfrac{1}{\omega C}\right)}\right];$$

but

$$E_1 = I_1(j\omega L_1).$$

Hence,

$$E_2' = I_1\left[j\omega L_1\left(\frac{\omega L_2}{\omega L_2 - \dfrac{1}{\omega C}}\right)\right].$$

Let the e.m.f. induced in L_2 by flux linkage due to current I_1, the pure mutual effect, be

$$E_{2m} = (-j\omega M I_1).$$

The total e.m.f. induced in L_2 is the vector sum of the two components

$$E_2 = E_2' + E_{2m} \text{ (or } E_0 \text{ in Fig. 17)},$$

$$E_2 = I_1\left[j\omega L_1\left(\frac{\omega L_2}{\omega L_2 - \dfrac{1}{\omega C}}\right) - j\omega M\right]$$

$$= j\omega I_1\left[\frac{\omega L_1 L_2}{\omega L_2 - \dfrac{1}{\omega C}} - M\right]$$

$$= I_1(j\omega M_a),$$

where M_a is the coefficient of apparent mutual inductance; then the apparent mutual inductance

$$M_a = \frac{\omega L_1 L_2}{\omega L_2 - \dfrac{1}{\omega C}} - M. \tag{19}$$

Usually, the value of C is small, so that in comparison

$$\frac{1}{\omega C} >> \omega L_2, \tag{20}$$

and the fraction becomes

$$-\omega^2 L_1 L_2 C.$$

This is of the same sign as for M, and the apparent value M_a is greater than M, and also the increase is proportional to the square of the frequency. For high-resistance coils a very complex expression results which is of little practical value.

11. Mutual Inductance by Open and Closed Secondary. *Theory and Method.*—This method is most easily explained by

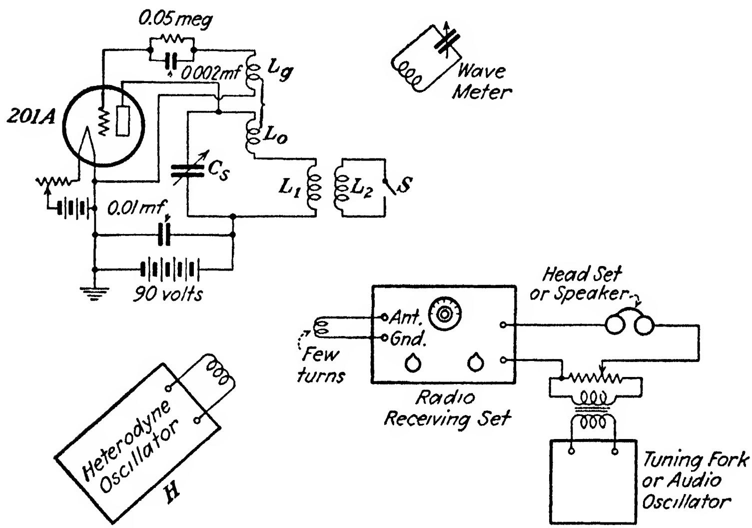


FIG. 18.—Circuit arrangement for measurement of mutual inductance and coupling coefficient by open and closed secondary.

referring to Fig. 18. The coils L_1, L_2 form the mutual inductance. L_1 is part of the tuned circuit of a plate-tuned oscillator. By means of standard condenser C_s, the wave from the heterodyning circuit driver H and that from the test oscillator are tuned into the receiving set, and the beat note is adjusted until it gives zero beat with the 1,000-cycle oscillator coupled to the plate circuit or simply with a tuning fork held near the ear. Then switch S is closed so that the induced current in L_2 changes the effective inductance of L_1, and C_s is varied until zero beat is again obtained. Referring to the figure, when switch S is open, condenser C_s is set to capacitance value C_1 as described above. Then

$$\omega^2 = \frac{1}{(L_1 + L_0)C_1} \tag{21}$$

for the zero beat. When switch S is closed, the equivalent inductance of L_1 becomes L_1', C_s is adjusted to value C_2 for zero beat as before, and

$$\omega^2 = \frac{1}{(L_1' + L_0)C_2}. \tag{22}$$

Eliminating L_0 between Eqs. (21) and (22),

$$L_1' = L_1 - \frac{(C_2 - C_1)}{\omega^2 C_2 C_1}. \tag{23}$$

It is well known that when current flows in the secondary of two coupled coils, the equivalent primary inductance is

$$L_1' = L_1 - \left(\frac{\omega M}{Z_2}\right)^2 L_2, \tag{24}$$

where M is the mutual inductance of L_1 and L_2.* From Eqs. (23) and (24), the decrease in L_1, when S is closed, is evidently

$$\frac{C_2 - C_1}{\omega^2 C_2 C_1} = \left(\frac{\omega M}{Z_2}\right)^2 L_2. \tag{25}$$

If the resistance of L_2 is negligible compared to the reactance at radio frequencies, $Z_2 = \omega L_2$, and

$$\frac{C_2 - C_1}{\omega^2 C_2 C_1} = \frac{M^2}{L_2} = \frac{K^2 L_1 L_2}{L_2}, \tag{26}$$

since

$$M = K\sqrt{L_1 L_2},$$

where K is defined as the coefficient of coupling. From Eq. (26),

$$M = \left[\frac{L_2(C_2 - C_1)}{\omega^2 C_2 C_1}\right]^{1/2} \tag{27}$$

and

$$K = \left(\frac{C_2 - C_1}{\omega^2 C_1 L_1 C_2}\right)^{1/2}. \tag{28}$$

* MORECROFT, J. H., "Principles of Radio Communication," 2d ed., p. 107.

In Fig. 18 is shown a *plate-tuned* oscillator, and it will usually be found that this type is more satisfactory than the tuned-grid circuit oscillator with inductive feedback from the plate circuit. If the latter type is used, it is best to connect the grid leak between grid and filament and the grid condenser in series with the grid coupling coil. The comparison audio tone is shown to be injected into the loud-speaker or head-set circuit. If this is not desired, the audio oscillator may drive a separate speaker near by, but it will be found that the beats in the audio tone will be much more intense if the comparison audio tone is injected into the receiving-set speaker as indicated in the figure. An alternative method is to use the dynatron oscillator[1] or other negative-resistance oscillator. Then L_1 and C_s are used as the tank, or oscillating circuit. This oscillator is substituted for the tuned-plate variety of Fig. 18, and the procedure is as previously described. From the values of C_s equal to C_1 and C_2 for the open and closed position of the short-circuiting switch S of the secondary coil L_2,

$$K = \left[\frac{C_2 - C_1}{C_2}\right]^{1/2} \tag{29}$$

The student may derive this expression.

The mutual inductance may then be found after L_1 and L_2 are measured. This method is very convenient. L_1 is quickly measured by tuning C_s to obtain zero beat with the heterodyne oscillator, the audio oscillator being shut off. Then the wave meter shown in the figure is adjusted, and as it approaches resonance with the dynatron oscillator it will react upon the latter and cause a change of frequency, bringing in a beat note. When the wave meter is in exact resonance with the dynatron tank circuit, its reaction upon the dynatron frequency is a pure resistance reaction; and this being small, the dynatron oscillator frequency returns to its original value, and zero beat with the heterodyne oscillator frequency is again obtained. It will be found that this is an extremely sensitive adjustment for resonance of the wave-meter circuit.[2] From the value of C_s of the dynatron tank circuit and the measured wave length, the value of L_1 may

[1] See Art. 22 of this chapter, Resistance of Oscillatory Circuit by Negative Resistance Measurement.

[2] AIKEN, C. B., *Proc. I.R.E.*, Vol. 16, No. 2, p. 125, 1928.

be calculated, the simple relation between wave length, frequency, and inductance and capacitance being used. After this, L_2 is measured by connecting it in place of L_1 and repeating the measurement. L_1 and L_2 may also be measured by a simple resonance method with some of the apparatus of Fig. 18. C_s and L_1 are connected together and coupled loosely to heterodyne oscillator H. C_s is varied until the dip in the grid current of the tube in H indicates resonance, and L_1 is then equal to $1/2\pi\sqrt{L_1C_s}$. L_2 is measured in like manner. Any good method of resonance indication with H is usable.

Test of Capacitance Effect.—The method just described is capable of high precision and is useful for very small coefficients of M or K. It is easy to measure the equivalent capacitance across L_1 due to the presence of L_2. The usual adjustment is made with S open, after which L_2 is removed. The change in C_s for zero beat with L_2 present, and not present, is the required capacitance, as in Chap. 1, Art. 3. If the presence of L_2 produces little effect on the adjustment of C in comparison with effect of closing switch S, the apparent value of mutual inductance M_a is very nearly the true value of M.

Precision.—For very loosely coupled coils the change in C_s needed to restore the zero beating of the audio tone and hence the oscillator frequency may be quite small. This change in C_s must be great enough to be read with a degree of certainty on the condenser scale. The first setting C_1 of C_s should be somewhere in the straight portion of its calibration curve. Then the change of capacitance, $C_2 - C_1$, may be estimated by the variation of capacitance with conveniently observed scale values. It is well to select a variable condenser that has a micrometer scale associated with its main scale. It is best to use values of C_1 or C_2 that will result in frequencies much lower than the fundamental resonance frequencies of the coils, to avoid errors due to nonuniform distribution of current in the coils.

12. Resistance by the Resistance-variation and Substitution Methods. *Theory and Procedure for the Resistance-variation Method.*—The test circuit for this method is shown in Fig. 19. It is coupled to a circuit driver by means of a small coupling coil L_c. The coupled resonance indicator consists of a type 199 tube with its grid tied to its plate to act as a rectifier, a by-pass condenser, a 3-volt filament battery, a coupling coil L', and a microammeter of 100- or 200-μa range. This instrument should be calibrated when coupled to a coil carrying known radio-frequency currents and a calibration chart made showing the manner in which the deflection varies with current. If the calibration shows the deflections to be nearly proportional to the square of the coupled current, the calibration curve may be drawn for arbitrary current values squared versus deflections of the microammeter. A precision, low-loss, variable condenser C

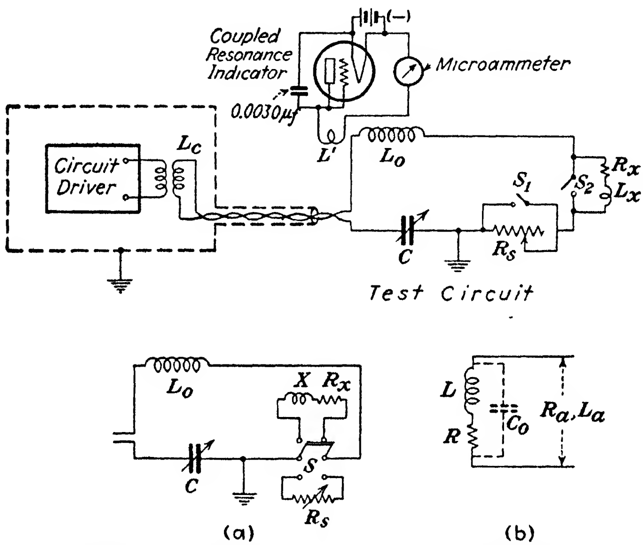


FIG. 19.—Arrangements for resistance measurement by resistance variation and substitution methods.

is connected with its rotary plates grounded, and a standard fixed or variable resistance R_s is connected as shown next to the ground connection. Switches S_1 and S_2 are closed, and condenser C is varied until maximum deflection of the microammeter of the coupled resonance indicator shows that resonance is obtained. Next, the standard resistance R_s is introduced by opening the short-circuiting switch S_1. This will reduce the current in the test circuit; and if R_s is variable, it may be adjusted until the new current is one-half the former value. Then the resistance of the test circuit is equal to this value of R_s. If R_s is not noninductive, C must be readjusted so as to maintain resonance, for at all times the current in the test circuit must be

$I = E/R$. If the standard resistance R_s is fixed in value or can be changed in only a few steps, let the current with S_1 closed be I_0, and with S_1 open and R_s in the circuit it will be I_1. Then, by Ohm's law, the resistance R_0 of the test circuit is

$$R_0 = R_s\left(\frac{I_1}{I_0 - I_1}\right). \tag{30}$$

The next step is to close S_1 and open S_2, introducing the unknown impedance whose resistance is to be measured. C is retuned for resonance, the current noted, and S_1 is again opened, introducing R_s', and the current again noted, readjusting C if necessary to maintain resonance. Now, the resistance of the test circuit plus that of the added unknown is found and is

$$R_0 + R_x = R_s'\left(\frac{I_3}{I_2 - I_3}\right), \tag{31}$$

where I_2 is the current with the unknown in the circuit and S_1 closed, and I_3 is the value for S_1 open. The measurement may easily be made for several frequencies of the circuit driver.

Theory and Procedure for the Substitution Method.—In this method, the standard resistance is substituted for the unknown. In Fig. 19, the test circuit is modified as shown in Diagram (*a*). The switch is first closed so as to place the impedance whose resistance R_x is to be measured into the test circuit. C is varied to obtain resonance by noting the maximum deflection of the microammeter. Then S is changed so as to substitute the variable standard resistance R_s for the unknown, C is then retuned for resonance, and R_s is varied until the current (and microammeter reading) is the same as previously. Then the resistance value of the standard is equal to the unknown. Since it is not required to know comparative current values, it is not necessary to calibrate the resonance indicator. It is also not necessary to take a reading for the test circuit alone as in the *half-deflection* or *resistance-variation* method. Several important precautions must be observed which will be discussed subsequently. In this method, the impedance of which the resistance is to be measured may be a coil, a condenser, or a resistance. It is more satisfactory for coils than for condensers; different methods will be described for the latter.

In case only a fixed standard resistance R_s is available, it is necessary only to calibrate the resonance indicator in values proportional to test-circuit current and then take a reading for the test circuit alone in addition to readings with the unknown and with the standard in the test circuit. The resistance of the unknown is

$$R_x = R_s \left[\frac{I_2(I_0 - I_1)}{I_1(I_0 - I_2)} \right], \tag{32}$$

where I_0, I_1, and I_2 are, respectively, the currents in the test circuit in amperes for the test circuit alone, unknown resistance in the circuit, and R_s substituted for the unknown. If the resonance indicator contains a rectifier-type microammeter, as shown in Fig. 19, a calibration curve is obtained between current in the test circuit and microammeter deflections D_0, D_1, D_2. It is not necessary to know the actual ampere values in the test circuit. Proportional values of current are required in Eq. (32), and these may be obtained from the calibration curve of the coupled resonance indicator.

Precautions and Precision with Both Methods.—For the resistance-variation method, the test circuit must have a low resistance and short low-resistance connecting leads. The coupled resonance indicator reflects a very small resistance into the test circuit if a microammeter is used in series with the coupling coil and rectifier tube. The standard resistance should have low distributed-capacitance effect at the frequencies of measurement. Further discussion of resistance standards is given in the Introduction to this chapter. For higher frequencies, fixed resistances made of straight or U-shaped wires are the best. These wires are of manganin or advance wire and must be sufficiently small so that the high-frequency resistance does not exceed the direct-current resistance by more than 1 or 2 per cent. These resistances should have a low temperature coefficient. Good designs for high-frequency standards include the *link resistor* and the Ayrton-Perry noninductive types.[1] The variable condenser C must be a high-grade low-loss type and well shielded. It must

[1] For link-resistor design see *U. S. Bur. Standards, Circ.* 74, p. 176. This resistor is a short piece of straight high-resistance wire held in a glass tube; several such units may be connected in series. The Ayrton-Perry bifilar wound resistance is shown in Fig. 20, Chap. VI, of this book.

have sufficient capacitance so that it is not set in the region of minimum capacitance, at which the equivalent series resistance is likely to be high for the test circuit. If the coil resistance is not greatly larger than that of the test-circuit resistance, it is necessary to take readings first for the test-circuit resistance alone. This means that when an unknown coil whose resistance is to be measured is added to the test-circuit condenser, C must be retuned for resonance, and this means that the selection of frequency and of L_0 must be such that the condenser-series equivalent resistance is negligible when the unknown is in the circuit and out of the circuit. This requirement means sufficiently large values of C such that the danger of nonuniform current distribution is obviated. It is easy to construct a test circuit having a resistance of a fraction of an ohm at broadcast frequencies; and as the adjustments are sensitive, accuracies of 1 per cent are attainable. It must be remembered that the resistance measured for the case of a coil is the apparent resistance; an equivalent lumped self-capacitance C_0 may be considered to be in shunt with the ordinary coil inductance. At (*b*), Fig. 19, is shown the inductance L in series with its true effective resistance R and C_0 across the terminals. This is equivalent to an apparent inductance L_a and resistance R_a. The apparent resistance is[1]

$$R_a = \frac{R}{(1 - \omega^2 L C_0)^2 + \omega^2 C_0^2 R^2}, \tag{33}$$

where $f = (\omega/2\pi)$ is the frequency, R the true resistance, L the nominal inductance, and C_0 the equivalent lumped self-capacitance. For a small tuning coil which will tune to resonate at 1,000 kc. with 300 mmf. capacitance, the inductance is $\frac{1}{12}$ millihenry, and the current is uniform in the coil because the frequency is very much lower than the natural frequency value. Let the self-capacitance C_0 be 5 mmf., to be conservative, and let the coil resistance R be 10 ohms. Then $\omega^2 L C_0 = 0.0166$, $\omega^2 C_0^2 R^2 = 9.93/10^8$, and $R_a = 1.01R$ very nearly; a small error is thus introduced. The natural frequency of this coil obtained from $\omega_0^2 = 1/LC_0$ is about 8 Mc., and near that frequency its apparent resistance is about 2,500 ohms. As long as R_a is not noticeably different from R, the method of measurement is

[1] *U. S. Bur. Standards, Circ.* 74, pp. 133–134.

reasonably accurate; but for frequencies nearer the fundamental frequency of the coil, the resistance R may be found approximately by measuring R_a in the manner described, measuring L and C_0 as described in Art. 4, and then using Eq. (33). Sometimes the testing voltage is induced in the turns of the coil whose resistance is to be measured. When this is done, corrections for the effect of distributed capacitance must be applied in a different manner.[1]

Most of the precautions to be observed with the first method must also be observed with the second. If the resistance being measured is that of a coil, the variable condenser C may be changed by an amount that will not cause an appreciable error. This has been explained in the preceding paragraph. For high radio-frequency work, it may not be possible to provide values of C that will give low series resistance in the condenser. For such high frequencies, the frequency-variation and negative-resistance methods, described later, are preferable. It is important to make sure that the coupling coil L_c is not too close to the circuit-driver tank coil, or the *drag-loop* effect will cause errors. To test for this, vary C and watch the meter of the resonance indicator. The deflection should come to a maximum and again decrease in a symmetrical manner. If it drops quickly after rising slowly, the coupling is too close.

13. Resistance by Reactance-variation and Frequency-variation Methods. *Theory of Reactance-variation Method.*—This method is based upon the dependence of the steepness of the resonance curve on the ratio of reactance of a circuit to its resistance. When the reactance is zero, the current is limited only by its resistance; and when the former is increased to a definite amount, and the change in current noted, the resistance may be determined. Let E be the e.m.f. (of frequency f) induced in a circuit whose reactance is varied by means of a variable capacitance C. At resonance the current $I_r = E/R$. R is the total circuit resistance. For this condition the total reactance

$$\left(\omega L - \frac{1}{\omega C_r}\right) = 0,$$

where L is the inductance of the circuit, and C_r is its capacitance

[1] See TERMAN, F. E., "Measurements in Radio Engineering," p. 80.

at resonance. Let the capacitance be changed to some value C_1, say, a value lower than C_r. Then

$$I_1 = \frac{E}{\sqrt{R^2 + \left(\frac{1}{\omega C_1} - \omega L\right)^2}}.$$

It is convenient to express the reactance as

$$\left(\frac{1}{\omega C_1} - \omega L\right),$$

or the derived expression for R will be negative; and this is correct because, by assumption, $1/\omega C_1$ is larger in value than ωL. If the two preceding expressions for I and I_r are divided into each other, and the quotient squared,

$$\left(\frac{I_r}{I_1}\right)^2 = \frac{R^2 + \left(\frac{1}{\omega C_1} - \omega L\right)^2}{R^2},$$

or

$$\frac{I_r^2 - I_1^2}{I_1^2} = \frac{1}{R^2}\left(\frac{1}{\omega C_1} - \omega L\right)^2.$$

But

$$\omega L = \frac{1}{\omega C_r};$$

substituting this and solving,

$$R = \frac{1}{\omega}\frac{C_r - C_1}{C_r C_1}\sqrt{\frac{I_1^2}{I_r^2 - I_1^2}}. \tag{34}$$

Hence, if the values of C_r, C_1, and ω and the values of current, or readings proportional to current squared, can be obtained, the resistance of the circuit may be determined. C_r is harder to determine owing to the zero slope of the resonance curve at resonance; hence steps will be taken to show how it can be eliminated. If the capacitance in the test circuit is increased to a value C_2 on the other side of the resonance point, it can be easily shown that

$$R = \frac{1}{\omega}\frac{C_2 - C_r}{C_2 C_r}\sqrt{\frac{I_2^2}{I_r^2 - I_2^2}}.$$

It is easy to adjust C_2 until $I_2 = I_1$ [Eq. (34)]. Then

$$R = \frac{1}{\omega}\left(\frac{C_2 - C_r}{C_2 C_r}\right)\sqrt{\frac{I_1^2}{I_r^2 - I_1^2}}. \tag{35}$$

C_r may now be eliminated using Eqs. (34) and (35), and the resulting expressions solved for the resistance gives

$$R = \frac{1}{2\omega}\frac{(C_2 - C_1)}{C_2 C_1}\sqrt{\frac{I_1^2}{I_r^2 - I_1^2}}. \tag{36}$$

Figure 20 shows the resonance curve with values of current squared plotted as ordinates. If C_1 and C_2 are adjusted until I_1^2

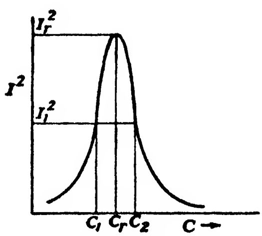


FIG. 20.—Resonance curve due to reactance variation.

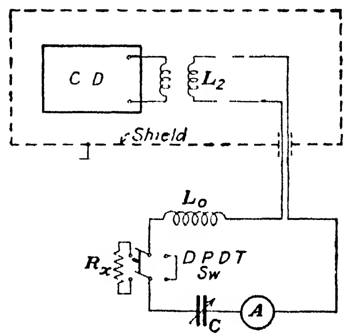


FIG. 21.—Circuit for measuring resistance, reactance-variation method.

is one-half the value of I_r^2, the radical term in Eq. (36) becomes equal to unity.

Manipulation.—Apparatus may be arranged as in Fig. 21, L_2 and L_0 being sufficient to make the test-circuit resonant with the circuit-driver frequency. The resistance R_x may be placed in the circuit by throwing the change-over switch to the left. With R_x out of the circuit, C is varied so as to obtain readings of I or I^2 to plot in the resonance curve. A thermogalvanometer with uniform scale will give readings approximately proportional to the current squared. From the readings obtained for C_2 and C_1, the resistance of the circuit may be determined, using Eq. (36), after which R_x is cut in, and the resistance of the circuit again determined. The value of R_x is then the difference of these two determinations.

Sources of Error, Precision.—Referring to Fig. 20, it is easily seen that, since the resonance curve has zero slope at C_r, the exact value of the latter is indefinite and would have to be estimated from several readings. However, I_r^2, being the maximum value, is determined with greater exactness; and also C_1 and C_2, being on a steep part of the curve, can be obtained with considerable exactness. It is well to take sufficient readings so that a portion of the resonance curve can be plotted to see if it is symmetrical. If it is not, abnormal conditions exist, such as too close coupling of L_2 and *C.D.*, temperature changes, too high a frequency causing variations in current distribution around the circuit, or a change of important magnitude in the condenser resistance as C is varied. This last possibility makes it imperative to have a high-grade condenser of the low-loss type so that its resistance changes very little with capacitance through a considerable portion of its range. It is also necessary to use the variable condenser in the upper region of its range, as the resistance of any condenser increases toward the point of minimum capacitance. Also, for good symmetry it is necessary that $\Delta C << C_r$, where $\Delta C = C_r - C_1 = C_2 - C_r$, as was shown in Art. 2, Eq. (4).

If the resistance of the test circuit and the unknown is low, a very small capacitance change will be the result; *i.e.*, $(C_2 - C_1)$ is quite small. Not only must the variable condenser C be accurately calibrated, but it must have quite an elaborate reading scale so that small changes in capacitance may be determined with good precision. One high-grade commercial variable condenser has a main scale of 25 divisions and a micrometer scale of 100 divisions on the shaft of a precision worm meshing into a worm wheel on the shaft of the rotary plates. This makes 2,500 scale divisions, and the backlash of the worm is very small so that a change of 1/2,500 of the total capacitance range may be read. For determining small changes $(C_2 - C_1)$, the condenser should be used in the range where the calibration curve is a *straight line*. Then the change of capacitance may be found by reading C_2 and finding C_1 by the change in scale reading and proportion with values given in the calibration table. The circuit resistance should always be low, and it may even be desirable to use the previously described microammeter and U.X. 199 rectifier or the input to a vacuum-tube voltmeter, in shunt with a small portion of

L_0 for reading the current. This device should be calibrated by placing it in a circuit loosely coupled to another in which there is a precision thermoammeter. The coupled resonance indicator is recommended because of its simplicity and the very low resistance that it adds to the test circuit. This method is fairly reliable at quite high frequencies, up to between 5,000 and 6,000 kc. It is worth while to test the method on one of the standard resistances referred to in the preceding section.

Theory and Procedure, Frequency-variation Method.—The frequency-variation method is quite old, but it has new importance. Radio-frequency voltage from the circuit driver is induced into the test circuit of Fig. 21, and when C is adjusted to obtain resonance the current is

$$I_r^2 = \frac{E^2}{R_t^2},$$

where R_t is the total series resistance of the circuit and the unknown R_x, and the circuit-driver frequency is f_r.

The circuit-driver frequency is set at a value $f_1 = \omega_1/2\pi$, and

$$I_1^2 = \frac{E^2}{R_t^2 + \left(\frac{1}{\omega_1 C} - \omega_1 L\right)^2},$$

where C and L are the effective capacitance and total apparent inductance, respectively, in series in the test circuit.

In this equation, the reactance term is shown as $\left(\frac{1}{\omega_1 C} - \omega_1 L\right)$ to obtain a positive value—when the frequency is lowered below resonance, the reactance of C exceeds that of L. Dividing the two expressions and introducing the resonance relation $\omega_r = 1/LC$,

$$\frac{I_r^2}{I_1^2} - 1 = \frac{L^2}{R_t^2} \frac{(\omega_r^2 - \omega_1^2)^2}{\omega_1^2}.$$

Then the circuit-driver frequency is increased to a value $f_2 = \omega_2/2\pi$ above f_r, a value at which $I_2 = I_1$, and

$$\frac{I_r^2}{I_2^2} - 1 = \frac{I_r^2}{I_1^2} - 1 = \frac{L^2}{R_t^2} \frac{(\omega_2^2 - \omega_r^2)^2}{\omega_2^2}.$$

From these two equations ω_r can be eliminated, and the result solved for R_t:

$$R_t = 2\pi L(f_2 - f_1)\sqrt{\frac{I_1^2}{I_r^2 - I_1^2}}. \tag{37}$$

If $I_r^2 = 2I_1^2$, the radical is unity, and

$$R_t = 2\pi L(f_2 - f_1) = \frac{f_2 - f_1}{2\pi f_r^2 C}. \tag{37a}$$

This is convenient when a calibrated condenser is used in the test circuit. If it is desired to obtain the Q of the circuit, it is not necessary to know L or C; and since the Q factor is $\omega_r L/R_t$, Eq. (37*a*) becomes

$$Q = \frac{\omega_r L}{R_t} = \frac{f_r}{f_2 - f_1}. \tag{38}$$

As in the resistance measurement by half-deflection and reactance-variation methods, the resistance of the test circuit can be found, and after that the resistance of the test circuit plus the unknown from which the latter is obtained. For measuring coils a convenient *Q meter* can be made by designing a test circuit so that its resistance is low compared with that of the coils to be tested, and in Eq. (38) R_t is practically that of the coil.

In this experiment, there is a problem of measuring the small change in frequency $f_2 - f_1$, for low resistances of test circuits and unknowns. Of course f_r may be measured with a good-quality wave meter. To measure $f_2 - f_1$, a heterodyning oscillator may be caused to beat on the circuit driver, producing an audio frequency which is heard in a radio receiving set. A good-quality stable audio oscillator, previously calibrated, has its output injected into the speaker or head-set circuit of the receiver. The circuit driver is set at f_2 for the required resonance indicator reading $\left(I_2^2 = \frac{I_r^2}{2}\right)$, and the audio oscillator is set at some convenient frequency f_a''. Then the heterodyning oscillator is tuned until the beat frequency produces a tone equal (or nearly so) to that produced by the calibrated audio oscillator, resulting in zero (or nearly zero) rate of pulsation of the intensity of the emitted sound. Then the circuit-driver frequency is changed to

f_1, for which $I_1^2 = I_r^2/2$, and the frequency of the calibrated audio oscillator is changed until the value f_a' is reached, at which zero-intensity pulsation (or nearly so) again obtains. Then $f_2 - f_1 = f_a'' - f_a'$.

Precision of Measurement.—Most of the precautions and requirements for good precision are indicated in the foregoing paragraphs. The resistance measured for a coil is again the apparent resistance, as in the previously described methods. If, however, L_0 and the test-circuit coupling coil is not used, and the circuit-driver voltage is induced into the coil under test, the resistance measured is the true resistance and not the apparent value. The coupled current indicator (Fig. 19) must then be used instead of A (Fig. 21) to measure the coil current, and this same current divides between the equivalent lumped distributed capacitance and the tuning condenser C, Fig. 21.

If this latter procedure is followed, the resistance of the test circuit (leads, condenser C, ammeter, etc.) cannot be separately determined, and the test circuit must be one of very low resistance if low-resistance coils are to be measured. The theory of this method is based on the expression $I_r = E/R$, where R is the resistance when the current is everywhere the same in value. If the frequency is set at a value near that for resonance in the coil to be measured, the current in the coil varies along the length of the coil. Then at the frequencies f_1 and f_2 the current distribution is radically different, and confusion results as to what the measured value will mean. If the frequency is such that the current in the coil is nearly uniform, there can be little question as to the correctness of the theory and method. This method possesses the outstanding advantage that a standard resistance is not needed, and its precision depends on the accuracy of frequency measurement. The accuracy is of course limited by the precision with which the capacitance or the inductance of the test circuit can be measured or is known. At frequencies of about 1 Mc. the commercial standard condensers are within 1 per cent in accuracy, but at higher radio frequencies special calibration of the condenser is necessary. This might be done by the series-substitution bridge or resonance method (page 33).

The resonance curve must be symmetrical about the resonant frequency f_r axis over the range through which the frequency is varied, *i.e.*, f_1 to f_2. If the change in frequency from resonance to f_1 or $f_2 = \Delta f$, then $\omega_1 = \omega_r - \Delta\omega$, and $\omega_2 = \omega_r + \Delta\omega$. The current in the test circuit is then

$$I^2 = \frac{E}{R_t^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \frac{E}{R_t^2 + \left[(\omega_r \pm \Delta\omega)L - \frac{1}{(\omega_r \pm \Delta\omega)C}\right]^2}$$

$$= \frac{E}{R_t^2 + \left[\frac{(\omega_r \pm \Delta\omega)^2 LC - 1}{(\omega_r \pm \Delta\omega)C}\right]^2}, \text{ and } \omega_r^2 = \frac{1}{LC} \quad (39)$$

When $\Delta\omega$ is small compared to ω_r, the current is the same for (+) or (−) values of $\Delta\omega$, and the curve is symmetrical over that range. Usually this is the case, as most coils and test circuits have a fairly high Q. If the resistance is high, the dissymmetry should be determined by calculation.

14. Loss Resistance of a Condenser, Substitution Method. *Description of Theory and Manipulation.*—This method, first described by Burke,[1] depends upon the principle that a condenser having a measurable equivalent series resistance due to power losses may be replaced by a standard condenser whose small low loss is constant and a precision standard variable resistance in a manner that will be described. In Diagram (*a*) of Fig. 22 is shown the test circuit, which is similar to that for resistance measurement by the same method. A very desirable type of variable condenser to use for this method is described in the Introduction to this chapter. When the circuit driver is adjusted to the desired testing frequency, the standard condenser C_s is varied to obtain resonance when the capacitance C_x, whose resistance is to be measured, is connected in parallel with C_s. The standard variable resistance R_s is reduced to zero value. For this state of affairs, Diagram (*b*) of the figure shows the capacitances together with their loss-resistance values. The total capacitance for resonance in the test circuit is $C_x + C_s'$ and $C_s' = C_i + C_v'$. The term C_i is the capacitance through the insulating supports of the stationary plates, and r is an equivalent series resistance due to loss in these supports. C_v' is the capacitance of the variable portion, *i.e.*, between the stationary and rotary plates. Now, the total capacitance is not exactly $C_v' + C_i + C_x$ because of r and R_x, as shown in the diagram, but

[1] *Trans. A.I.E.E.*, Vol. 46, p. 482, 1927.

it will be found that r is very small compared to the reactance of C_i, and it is assumed that C_x is a fairly good condenser at radio frequencies and $R_x << 1/\omega C_x$. In fact, the capacitances may be added numerically if the power factors of condensers in parallel are as high as 10 per cent. Now, at this adjustment (resonance with C_x connected) the current is I_0, and there will be a voltage E across the parallel condensers as shown.

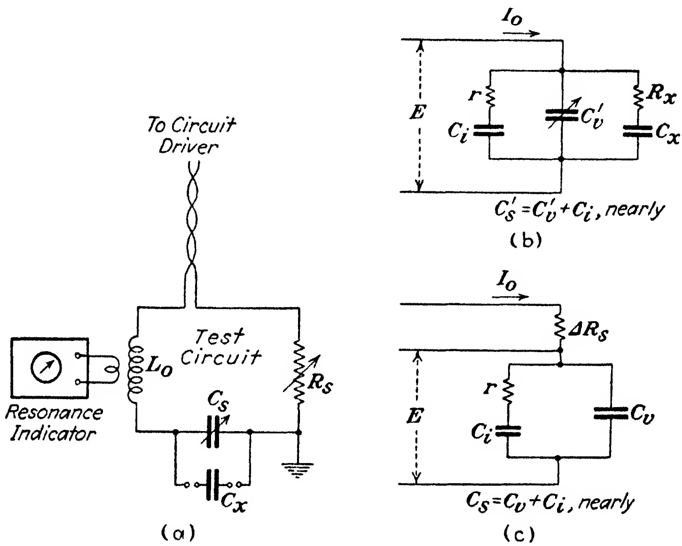


FIG. 22.—Circuit for measuring loss resistance of a condenser, substitution method.

The next step in the manipulation is to remove the capacitance C_x and readjust the standard condenser C_s to regain resonance in the circuit and then introduce a sufficient amount of standard resistance ΔR_s to regain the same value of current I_0 as for the case of R_x in the circuit. When C_x is removed, the current at resonance will be *greater* because of the removal of the power loss which was present in C_x. It will be realized that when resonance has been regained and the current I_0 has been reestablished, the voltage across the condensers is again equal to E because the total capacitance was the same as when C_x was in the circuit. It is now easy to derive an expression from which R_x may be determined. In Diagram (*b*), Fig. 22, the power consumed is

$$P' = (\omega C_i E)^2 r + (\omega C_x E)^2 R_x. \tag{40}$$

This is true because when the resistance of a condenser is low compared to its reactance, the current is ωCE, as the student may readily prove, and the power I^2R is then $(\omega CE)^2R$, where R is the equivalent series loss resistance. In Diagram (c), the voltage across the condensers is again E, as was pointed out, and the power consumed for ΔR_s added is

$$P = (\omega C_t E)^2 r + (\omega C_s E)^2 \Delta R_s, \tag{41}$$

because $I_0 = \omega(C_t + C_t)E$ and $C_s = C_t + C_v$. Now, it has been found that the power loss in the insulation of the precision-type condenser described[1] is constant regardless of the value of the variable portion C_v; hence r is always constant as C_v is varied. Then the first terms of the right-hand sides of Eqs. (40) and (41) are equal. The test circuit was in resonance for C_x in the circuit and out of the circuit, and in each case the current $I_0 = E_0/R_0$ for the entire circuit. Hence the test-circuit resistances were the same; with the same currents the powers consumed were the same; from Eqs. (40) and (41) $P = P'$, and

$$(\omega C_x E)^2 R_x = (\omega C_s E)^2 \Delta R_s,$$

or

$$R_x = \left(\frac{C_s}{C_x}\right)^2 \Delta R_s. \tag{42}$$

If C_s has zero loss, the equation for R_x is the same, and the proof is simple. If C_s has an equivalent series resistance in series with its variable portion that cannot be ignored but is known, a much more complex equation for R_x results and is so cumbersome as to be impracticable.

Current Indication, Shielding, Errors.—It will be realized that the current indicator in the substitution method must be very sensitive and reliable. The device should indicate the same value of current to a high degree of sensitiveness and precision. At the same time, it must not introduce high series resistance into the circuit. Since the condenser-loss resistances to be measured will often be less than 1 ohm, it is imperative that the test circuit have low resistance. If this is not the case, the addition of C_x with its attendant R_x may make little or no appreciable difference in the

[1] See Introduction to this chapter.

value of current at resonance. A copper rod inside a copper tube makes a low-resistance *link* between the pickup coil of the circuit driver and the test circuit. The inner rod is held inside the copper tube by isolantite disk insulators. The use of a higher powered circuit driver is often advisable in producing greater differences in current values.

It is best to shield the circuit driver and its coupled pickup coil to the link coupler. If the test circuit is shielded, there is danger of shunt admittances giving false values to R_x at 1,000 kc., or higher frequencies. Adjustments of tuning should be made either with long insulating handles or in an incremental manner, removing the body capacitance after each small change in C_s or R. Errors will result from the capacitance of leads also, unless they remain in one position. The lead from C_s to C_x (ungrounded side) should be a stiff wire which remains in place when one of its ends is removed a short distance away from the terminal of C_x. In this way, C_x may be disconnected, leaving the leads in place. A similar procedure holds when adding C_x to C_s.

15. Conductor Resistance and Condenser Resistance by Differential Thermometer. Calorimeter. *Description of Method.*—Calorimeter methods have been used to measure high-frequency resistance to some extent. They have an advantage over direct electrical methods, because resistance standards or zero-loss condensers are not required; however, the work of measurement is slow, and calorimeter errors are difficult to avoid. Fleming describes an interesting and practical method which does not require a measurement of the heat produced by flow of current in the resistance under consideration.[1] The method used for measuring the resistance of conductors is illustrated by the modified apparatus arrangement of Fig. 23. The wire W_1, whose resistance is required, is placed in the glass tube T_1, and a known high-frequency current passed through it by the means illustrated in the figure. An exactly similar wire W_2 is placed in tube T_2, and direct current passed through it, the value of this current being adjusted until the heat developed by it is equal to that developed by W_1. This condition of equilibrium is indicated when the small air bubble B in the differential

[1] FLEMING, J. A., "Principles of Electric Wave Telegraphy and Telephony," 3d ed., Chap. II, p. 143.

manometer *MN* is stationary at the middle of the scale. The liquid in the manometer tube may be colored water or alcohol, and the tube itself should be very small, practically of capillary size. The bubble may be formed by pouring the liquid in simultaneously through the two stopcocks. While in operation the bubble will move to the right if the current in W_1 should increase slightly. When conditions have become constant, and equilibrium is established, the I^2R loss in each of the wires is the

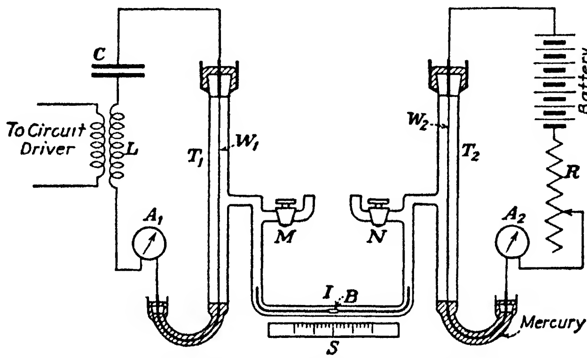


FIG. 23.—Apparatus and circuit arrangement for calorimeter method of measuring resistance. (*After Fleming.*)

same, and from the known $I_2^2R_2$ in W_2 the effective high-frequency resistance of W_1 is

$$R_1 = \frac{I_2^2R_2}{I_1^2}, \tag{43}$$

where R_2 is the direct-current resistance of W_2. To avoid errors due to inequalities in the two thermometer bulbs the run is repeated, passing the high-frequency current through W_2 and the direct current through W_1. The mean of the values thus measured is then obtained. The method may be extended to measure the loss resistance of a condenser in the manner described by Ramsey.[1] The thermometer bulbs are made large so as to hold the condenser; a similar condenser is placed in each bulb. The known radio-frequency current is passed through one condenser, and a known direct current is passed through a known resistance placed in the other thermometer bulb. The second condenser is placed in this bulb with the known resistance so as to

[1] RAMSEY, R. R., "Experimental Radio," 3d ed., p. 135.

equalize the thermal capacities. The procedure will be easily understood by the reader.

Precautions and Accuracy.—In performing the work as outlined for this method, considerable attention to detail is necessary for reasonably correct results. In addition to the usual precautions of exchanging containers, equal-size thermometer bulbs, etc., it is necessary to provide the best conditions for the electric circuit. If the condenser has small capacitance, the current required to produce an appreciable heating effect may result in too high a voltage drop through the condenser. A current of perhaps 0.63 amp. at 500 kc. in a 200 mmf. condenser requires 1,000 volts across the condenser. If the power factor is 0.63 per cent,

$$R = \frac{0.0063}{\omega C} = 10 \text{ ohms.}$$

The loss is $0.63^2 \times 10 = 4$ watts. A large-size condenser requires a large container, and, unless it were very well insulated, it would require, to cause sufficient expansion for easy reading of differences, some 2 to 4 watts to warm the air within 20 to 30 min. The voltage shown to exist may cause serious corona on the edges of the condenser plates at this frequency and may even spark over the enclosed air spaces or thin mica dielectric. A corona loss would result in considerable error. It is estimated from experiments in measuring resistance of wires that a change of 0.02 watt in the loss in one of the bulbs will cause the bubble in the manometer tube to move a noticeable amount. Then, if the dissipation in the resistance-carrying direct current is 4 watts, a precision of measurement of not greater than 0.5 per cent is the limit. This neglects other sources of error. A certain error of measurement of current results in twice as great an error in the measured value of resistance.

16. Bridge Measurements of Condenser-loss Resistance. *Condenser-loss Resistance.*—The resistance-capacitance bridge may be readily used to measure the equivalent series-loss resistance of a condenser if a loss-free variable-standard condenser and a standard variable known resistance could be provided. Then shielded equal-ratio arms could be provided, and at balance $R_s = R_x$ and $C_s = C_x$, R_s being in series with C_s in one arm of the bridge. Since practically there is no zero-loss standard condenser, this method is not practical, and the *parallel-substitution* method is used. One bridge setup for this purpose is shown in Fig. 24, wherein R_1 and R_2 are carefully matched equal-ratio arms. The bridge is balanced with connection P open, with R, a standard variable resistance, set at a value R'. C_s is adjusted to a suitable value, found by previous experience, and the balancing is done with *balancing condenser and resistance* C_b and R_b. These latter have nothing to do with the measurement itself, so that they are not calibrated and are not of high-precision construction. After balance is carefully obtained, the high-potential terminal of the unknown capacitance is connected to the stiff wire lead P, and balance is restored by readjusting C_s and changing R by the required amount. The resistance R_x of the capacitance C_x is then

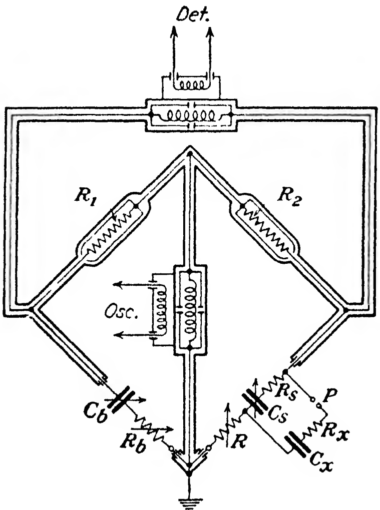


FIG. 24.—Shielded radio-frequency bridge for measuring circuit constants.

$$R_x = (R' - R)\left(\frac{C_s}{C_x}\right)^2, \qquad (44)$$

where $R' - R$ is the change in the standard resistance needed to restore balance, and C_x is the capacitance of the unknown or the decrease in C_s to restore balance when C_x was added in parallel. The proof is exactly like that for Eq. (42). Good precision is obtained with this method when R and C_s are known at the testing frequency with good precision.

It is not necessary to use the one type of bridge shown in Fig. 24. Some of the excellent commercial types shown in Figs. 9 and 10 are well adaptable to this measurement.

17. Coil-impedance Measurements with Radio-frequency Bridges.—The bridges previously discussed are often used to measure the effective reactance and resistance and the self-capacitance of coils. This latter is done by measuring the reactance of a coil or its susceptance by *series substitution* or by *parallel* *substitution* in the circuit, as mentioned by Field and Sinclair.[1] Detailed description of these methods will not be undertaken here, but it is readily appreciated that in parallel substitution, the unknown is added in parallel to the standard for susceptance measurements, and readjustments are made for balance. In series substitution, the unknown is added in series for reactance measurements (see Arts. 4 and 5). Great convenience is found in measuring resistance with bridges within the rated limits of frequency for accuracy of the bridge and the standard.

18. Ground and Ground-wire Resistance Measurements.—The simplest method of measuring ground resistance at frequencies below 200 kc. is to measure the resistance between parallel plates placed in the earth at the desired depth. At higher frequencies parallel wires buried in the ground may be used; the current is assumed to decrease exponentially along these electrode wires with distance from the input ends.[2] Ground resistance may also be calculated from the forward tilt of the radiated ground wave.[3] Ground-wire resistance and mutual impedance measurements are often required, and special methods are required for wires lying both above and below the surface of the earth.[4]

19. Measurement of Q of Coils. *Method of Measurement.*—The Q, or *Q-factor*, of a coil is now a well-known and useful quantity and is merely $\omega L/R$, the ratio of the reactance to the resistance of the coil. There are several ways of measuring this quantity without actually measuring the actual coil parameters L and R. For instance, in Art. 13, Eq. (38), it was shown that using the frequency-variation method for resistance measurement, the Q was found merely by finding the value of $\frac{f_r}{f_2 - f_1}$. This method is convenient if f_2 and f_1 are sufficiently far apart on the wavemeter or frequency-meter scale. If they are close together, their determined values by the absorption wavemeter is uncertain. If

[1] *Proc. I.R.E.*, Vol. 24, No. 2, February, 1936.

[2] STRUTT, M. J. O., "Elektrische Nachrichten Technik," Heft 10, Band 7, p. 387, 1930.

[3] BAILEY, A., S. W. DEAN, and W. T. WINTRINGHAM, *Proc. I.R.E.*, Vol. 16, No. 12, p. 1645, December, 1928. Ground-resistance measurement is more fully discussed by A. Hund, "High Frequency Measurements," pp. 290–298.

[4] *Bell System Tech. Jour.*, Vol. 10, p. 408, July, 1931; *ibid.*, Vol. 12, p. 162, April, 1933; *Physics*, Vol. 5, p. 35, January, 1934.

$Q = 100$, say $f_2 - f_1 = 10^4$ for $f_r = 10^6$, and finding of f_2 and f_1 with a wavemeter is easy. But if $Q = 10^3$, $f_r = 10^5$; then $f_2 - f_1 = 100$—only 100 cycles difference between f_2 and f_1—and heterodyne measurement methods and a secondary standard frequency may be necessary in order to measure f_1 and f_2 with sufficient accuracy to have a reasonably accurate value of $f_2 - f_1$.

The rise in voltage at resonance across the capacitance of a series circuit provides a convenient method of measuring Q. In Diagram (*a*) of Fig. 25, a voltage E is impressed on a circuit R, L, and C in series, and at resonance the current is E/R. The R and L are assumed to be that of a coil (these are *apparent values*), C having no appreciably comparable loss. The voltage E_c

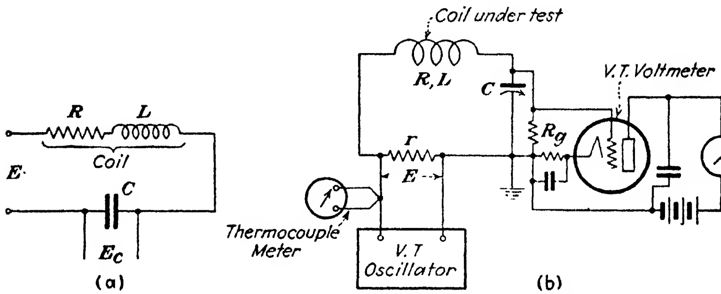


FIG. 25.—Test circuit for measuring Q of coils.

across C is $\frac{E}{R}X_c = \frac{E}{R\omega C} = \frac{E\omega L}{R}$, to an allowable approximation when $\frac{\omega L}{R}$ is greater than 10. Then $E_c/E = \omega L/R$, known as the resonant *rise* or *step-up*, is equal to the Q of the coil.

Convenient Arrangement for Measuring Q.—Diagram (*b*) of Fig. 25 shows how it is readily accomplished. Current measured by a thermocouple milliammeter passes through a small resistance wire of short length, good for high radio-frequency work, thus making it convenient to measure E, as explained above. The voltage E_c across a variable condenser C in the test circuit is measured by a vacuum-tube voltmeter when C is adjusted to obtain resonance in the test circuit. As the input capacitance of the tube voltmeter is appreciable, resonance should be obtained while the former is connected across the condenser. In a well-designed and convenient commercial form[1] of this test equipment, known as the *Q-meter*, the vacuum-tube voltmeter plate

[1] Manufactured by the Boonton Radio Corporation.

microammeter is calibrated directly in values of Q. Current obtained from a tube oscillator incorporated in the apparatus passes through a low resistance and thermomilliammeter which has a mark on its scale to indicate the current (and of course E) at which the main scale is calibrated in Q values. It should be noted that r in the test circuit [(b), Fig. 25] must be low compared to R (series-loss resistance of C is negligible in comparison). Then the current through R is small compared to that through r, and the thermocouple heater current is practically equal to the current through r.

20. Power Factor of Condensers by Potential Differences. *Theory and Manipulation.*—In a paper discussing the uses of the vacuum-tube voltmeter, E. B. Moullin[1] describes a method of measuring power factor of condensers by means of a vacuum-tube voltmeter and a simple bridge circuit. The tube voltmeter is used to measure the imperfection or component of unbalanced e.m.f. across the output diagonals of the bridge circuit. The bridge circuit is shown in Fig. 26. A radio-frequency e.m.f. E_i may be impressed on R_1 and R_2, equal noninductive resistances in series. The condenser C_x, having the unknown loss resistance of R_x, is in series with a variable-standard nearly zero-loss condenser C_s. A vacuum-tube voltmeter measures the e.m.f. E_0. When C_s is varied until it is nearly equal to C_x, the voltage E_0 will then be a minimum but will not reduce to zero, as the loss in C_x is greater than that in the standard; the loss in the latter is assumed to be zero. When this adjustment is made, it may be shown with the aid of vector diagrams or by quantitative relation of the various potential vectors that the power factor of the imperfect condenser is

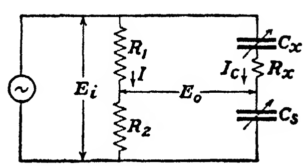


FIG. 26.—Circuit for measurement of condenser, loss resistance by potential drops.

$$\text{P.F. of } C_x = \frac{4E_0}{E_i}. \tag{45}$$

Limitations of Method.—This method is capable of producing fairly good results at the lower radio frequencies. To measure the input voltage—which must be of the order of 200 volts, for instance, at radio frequencies—a calibrated thermocouple may be

[1] *Jour. I.E.E. (London)*, Vol. 61, p. 302, 1923.

placed in series with R_1 or R_2. Then the value of that particular resistance must be adjusted so that, with the addition of the thermocouple heater resistance, the total will equal the value of the other resistance. If the resistance of the couple is r_t, and if the resistance with which it is connected is R',

$$R_1 = R' + r_t \qquad \text{and} \qquad R_1 = R_2.$$

A 1-megohm voltage divider and vacuum-tube voltmeter may also be used to measure E_i.

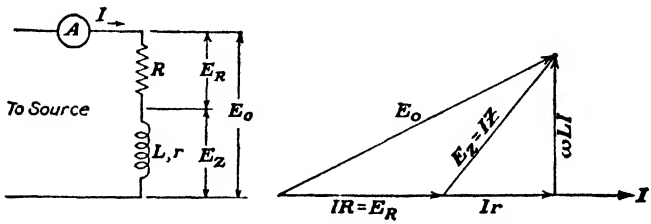


FIG. 27.—Circuit and potential vector diagram for three-voltmeter measurement of coil constants.

The voltage E_0 will be quite small. Let, for example, a condenser C_x have a power factor of 0.01 (1 per cent): If E_i equals 200 volts, then, from Eq. (45),

$$E_0 = \frac{0.01E_i}{4} = 0.5 \text{ volt};$$

hence the necessity of having E_i fairly large. C_x has a rather high power factor, somewhat like that of paper dielectric condensers. For high-grade condensers having a power factor of 0.005 per cent (0.00005), E_0 would be very small, indeed, to measure with a vacuum-tube voltmeter. Thus it is evident that this method may be used only with a fair success and then only for poorer grade condensers.

21. Resistance of Coils by Potential-difference Measurements. *Theory and Procedure.*—A very simple experiment in the measurement of resistance of coils at the lower radio frequencies may be made with the aid of a vacuum-tube voltmeter, standard resistance, and thermoammeter. This is really the well-known *three-voltmeter method*. The circuit connections are as shown in Fig. 27. The coil L with its attendant resistance r is connected in series with a known resistance R. A thermoammeter or other suitable device is used to measure the current flowing when the circuit is connected to a source. The voltage across the coil E_z and total voltage E_0 must be measured with a voltmeter that consumes practically no current, such as the electrostatic or the vacuum-tube voltmeter. The accuracy of the voltmeter reading may be checked by reading E_R, which is also IR. The vector diagram (Fig. 27) shows the addition of the various e.m.fs. For this diagram,

$$E_0^2 = (E_R + Ir)^2 + (\omega LI)^2,$$

and

$$E_z^2 = (Ir)^2 + (\omega LI)^2.$$

Solving the simultaneous equations for r,

$$r = \frac{(E_0^2 - E_z^2 - E_R^2)R}{2E_R^2}. \tag{46}$$

In this equation, E_R/R was substituted for I. If it is desired to use the current reading in place of E_R, Eq. (46) may be revised to suit. The inductance L of the coil may also be found by substitution of the value of r in one of the equations. The total resistance of a coil at some frequency may be thus measured and compared with the direct-current resistance. The frequency must be kept well below that of resonance of the coil with its distributed capacitance. Obviously, the degree of precision of this method depends upon the relative values of resistance and reactance in the circuit, as shown by the vector diagram.

22. Resistance of Oscillatory Circuit by Negative-resistance Measurement. *Theory of Method.*—A vacuum tube having two grids, especially the well-known screen-grid type 222, has, under certain conditions, negative resistance characteristics similar to those of the arc. Hull[1] pointed out these characteristics and showed that when a tuned circuit is connected in series with the plate circuit of the tube, oscillations will start when the negative resistance of the tube is equal to L/RC of the tuned circuit. In Fig. 28, Diagram (a), are shown characteristics of a screen-grid tube, or *tetrode*, of the 222 type. For a

[1] HULL, A. W., "The Dynatron," *Proc. I.R.E.*, Vol. 6, No. 1, pp. 5–35; see especially p. 17.

certain range of plate potential, the plate-current curve has a negative slope; in this range, the tube is said to have a *negative resistance*. If the negative resistance is $\Delta E_p/\Delta I_p$, it may reach high values when the control-grid bias is adjusted so that the slope of the curve is small. In Diagram (*b*) is shown how this negative resistance may vary with control-grid bias potential. For sufficiently negative bias, the negative-resistance value may be a megohm or more.

In Fig. 29 is shown the screen-grid tube, voltage supply, oscillating circuit *LRC*, and a special bridge circuit for measuring the negative resistance of the tube plate circuit. At first,

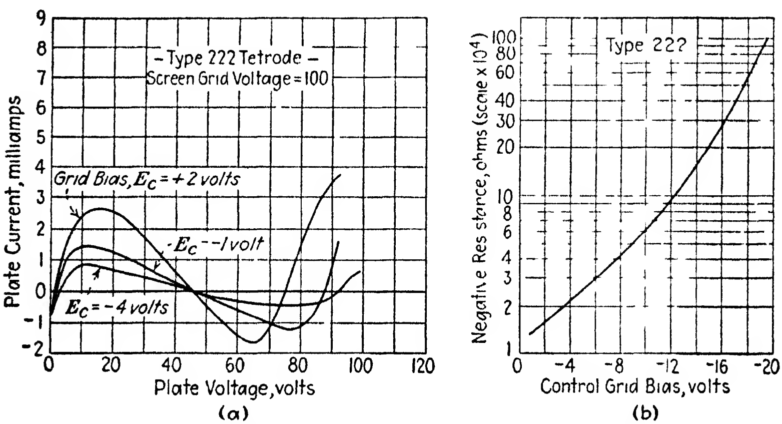


FIG. 28 —Characteristics of a type 222 screen-grid tube.

switch S_2 will be considered closed to eliminate the bridge circuit, and S_1 will be open to introduce the circuit whose resistance is to be measured. All the resistance R is to be considered in series with L, the condenser C to have no loss, as is valid when a low-loss condenser is connected in parallel with L having a Q of, say, 50 or 100. Now, when the plate voltage is adjusted by means of potentiometer P to a value lower than the screen voltage, and when in the region of negative resistance, preferably where the plate current is near zero [see Fig. 28, (*a*)], the control grid bias may be varied with potentiometer G until the negative resistance just becomes equal to the resistance of the parallel circuit at resonance, at which point oscillations will start. Hull gave the differential equation for potential drops and solved it for the oscillatory current which was a damped oscillatory-current equation. He showed that in order to maintain constant-amplitude oscillation it was necessary that

$$\bar{r} = -\frac{L}{RC}, \tag{47}$$

$\bar{r}$ being the negative resistance of the plate-filament circuit.

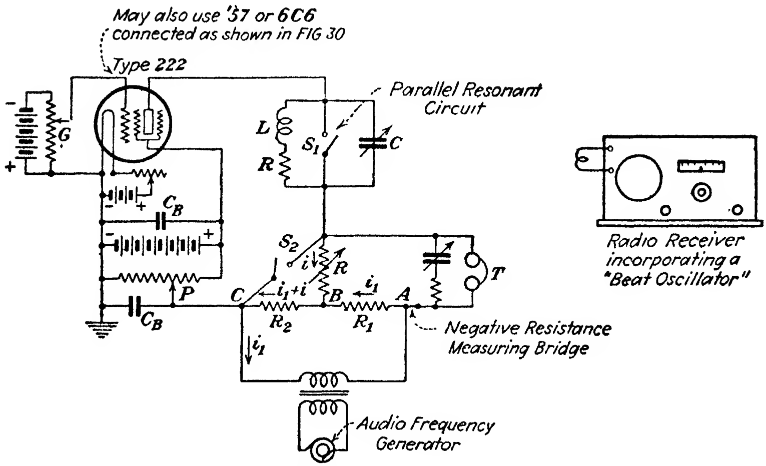


FIG. 29.—Circuit for measurement of parallel circuit resistance by negative-resistance method.

He also showed that the frequency of the oscillatory current is

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \left(\frac{R}{2L} - \frac{1}{2\bar{r}C}\right)^2}$$

and is practically

$$f = \frac{1}{2\pi\sqrt{LC}} \tag{48}$$

when the resonant oscillatory circuit has fairly low resistance or fairly high Q. It should be pointed out that the resistance of the parallel circuit L, R, C at resonance is L/RC when resonance is defined as the condition for unity power factor. It was pointed out that oscillations are maintained when the resistance of the tuned parallel circuit is equal to the negative-tube plate-circuit resistance.

The negative resistance is measured by opening S_2 and closing S_1, which short-circuits the oscillatory circuit. The circuit shown in Fig. 29 incorporates a Dingley bridge[1] because of its simplicity and adaptability to the measurement of a high negative resistance. When the audio-frequency generator is turned on, and adjusted to give a very few volts output, resistance R may be adjusted to a value that results in nearly zero sound in the telephone receiver. At a certain instant when the applied audio potential at A is positive and also positive at B with respect to C on the bridge circuit, it applies a positive increment to the plate potential of the tube. Now, owing to the negative resistance characteristic of the tube, this causes an increment of plate current to flow which is *opposite* to the voltage increment. This plate-current increment is indicated by i through R, and it of course has the frequency of the audio generator. Hence there will be currents in R and R_1 that will result in no audio voltage across the telephone receiver when R is set at the proper value, and

$$iR = i_1R_1.$$

This is the balanced condition. Taking potential drops through a closed loop,

$$(i_1 + i)R_2 + iR + (-\bar{r}i) = 0$$

results. From the two preceding equations, there readily follows

$$\bar{r} = R\frac{R_2}{R_1} + (R_2 + R) \tag{49}$$

at balance. Dingley suggests that if $R_2/R_1 = 99/1$, Eq. (49) becomes

$$\bar{r} = 100R + 99. \tag{50}$$

Manipulation.—The first step is to adjust the tube for negative resistance and very low plate current, as mentioned under Theory. Then G (Fig. 29) is varied until oscillations start and are maintained. Sometimes a slight change in plate current will show that oscillations are starting. A superheterodyne radio receiver incorporating a beat oscillator may be used to tune in to obtain a beat note from the dynatron oscillator. Then the potiometer G is again varied until the beat note disappears,

[1] *Proc. I.R.E.*, Vol. 19, No. 11, p. 1928.

which indicates that oscillations have ceased. Then the oscillatory circuit is short-circuited with S_1 and S_2 opened, and the measurement of the tube negative resistance may be carried out as indicated under Theory. Terman suggests a variable condenser and resistance in series placed across the telephone receivers to compensate for the effect of plate-circuit capacitance and thus give a more complete null balance.[1] He also describes a special-design shielded testing equipment. The negative resistance is due to secondary emission of electrons from the plate. This effect is also found in the older type 24-a indirectly heated cathode tubes which have the older design straight-side pear-shaped glass bulb. The modern type 24-a tubes have a specially

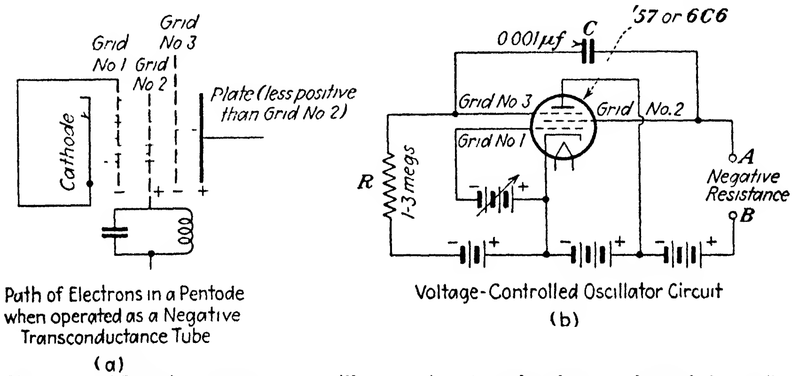


FIG. 30.—Negative-resistance oscillator using 3-grid tubes, and used for coil-resistance measurement.

treated plate which prevents secondary emission and cannot function as dynatrons. Recently a means was described for using one of the newer three-grid tubes[2] as a negative-resistance oscillator. By properly biasing grids 1 and 3 negatively, grid 2 to cathode circuit may acquire a negative-resistance characteristic, as indicated in Diagram (*a*), Fig. 30. If a tank circuit is placed in this circuit as shown in the figure, Diagram (*b*), the negative-resistance oscillator feature will be realized. The values of the biasing batteries may be found by experiment.

Precision of Method.—This method has been a favorite one for measuring coil resistances at radio frequencies up to 15 Mc.

[1] *Electronics*, Vol. 6, p. 340, December, 1933.

[2] R.C.A. Manufacturing Company, Radiotron Division, Application Note 45. For full treatment see E. W. Herold, *Proc. I.R.E.*, Vol. 23, No. 10, p. 1201.

At 20 to 30 Mc. the method breaks down, and it is believed that the electron transit time in vacuum tubes plays a part. No standard resistance with its uncertain values at radio frequencies is needed in this method. Uniform current distribution in the resistance being measured is assumed, and the equivalent lumped self-capacitance does not cause an error, because known capacitance values are not required. An accuracy of 1 per cent is attainable with careful manipulation. This method is satisfactory for coils, but for high noninductive resistance it is not so desirable.

23. Radio-frequency Measurement of Resistance and Impedance with Parallel-wire Line. *Fundamental Theoretical Principle.*—A method of measuring the complex impedance $(R + jX)$

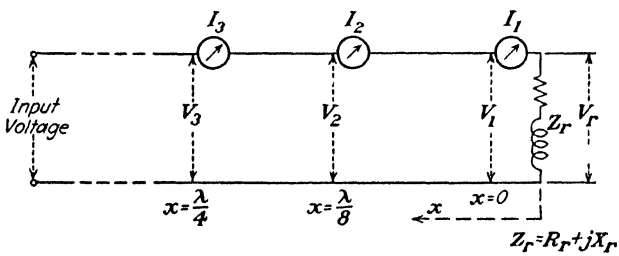


FIG. 31.—Test circuit for measurement of resistance and impedance $(R_r + jX_r)$ with parallel-wire line at radio frequencies.

with that of the well-known parallel line, or Lecher wires (see Art. 6, Chap. II), is described by Barrow.[1] The parallel-wire line-test circuit is shown in Fig. 31. At radio frequencies, the ordinary parallel-wire line shows the following characteristics:

$$r_w << \omega L, \qquad g << \omega C, \tag{51}$$

where r_w = wire resistance per unit length.
g = leakage conductance per unit length.
L = inductance per unit length.
C = capacitance per unit length.
$\omega = 2\pi f$ (f = frequency).

Under the foregoing conditions, the equations may be written for the voltage V_x between the wires at any distance x from the receiving end to which the unknown impedance $\dot{Z}_r$ is connected.

[1] "Measurement of Radio Frequency Impedance with Networks Simulating Lines," *Proc. I.R.E.*, Vol. 23, No. 7, p. 807, July, 1935 (see especially p. 808).

The same is true for equations for current I_x in the line. These equations are

$$V_x = V_r\left(\cos mx + j\frac{Z_0}{Z_r}\sin mx\right), \tag{52}$$

$$I_x = I_r\left(\cos mx + j\frac{Z_r}{Z_0}\sin mx\right),$$

where $m = \omega\sqrt{LC}$, and L and C are the effective geometrical line inductance and capacitance, respectively. $Z_0 = \sqrt{L/C}$ is the characteristic impedance of the line; V_r is the voltage at the receiving end. For spacings of ⅛ wave length ($\lambda/8$), as shown in Fig. 31, the voltages are, from Eq. (52),

$$\left.\begin{aligned} V_1 &= V_r \qquad \text{at } x = O, \\ V_2 &= \frac{V_r}{\sqrt{2}}\left(1 + j\frac{Z_0}{Z_r}\right) \qquad \text{at } x = \frac{\lambda}{8}, \\ V_3 &= jV_r\frac{Z_0}{Z_r} \qquad \text{at } x = \frac{\lambda}{4}. \end{aligned}\right\} \tag{53}$$

Z_0 and Z_r are, of course, complex. Now, let

$$\frac{Z_0}{Z_r} = c + jd. \tag{54}$$

Then, from Eq. (53), voltage ratios may be written as

$$B_1 = \left(\frac{V_2}{V_1}\right)^2 = \frac{1}{2}[(1 - d)^2 + c^2],$$

$$B_2 = \left(\frac{V_3}{V_1}\right)^2 = c^2 + d^2. \tag{55}$$

Solving Eqs. (55) for c and d,

$$\left.\begin{aligned} c &= B_1[(B_2 - B_1 + 1) - \tfrac{1}{4}(B_2 - 1)^2]^{1/2}, \\ d &= \tfrac{1}{2}(B_2 + 1) - B_1. \end{aligned}\right\} \tag{56}$$

Finally, from Eqs. (56) and (54) the values of the resistance and reactance components may be found and are

$$\left.\begin{aligned} R_r &= \frac{c}{c^2 + d^2}Z_0 = \left[a_2(a_1 - a_2 + 1) - \frac{1}{4}(a_1 - 1)^2\right]^{1/2} Z_0 \\ X_r &= \frac{-d}{c^2 + d^2}Z_0 = \left[a_2 - \frac{1}{2}(a_1 + 1)\right]Z_0, \end{aligned}\right\} \tag{57}$$

where

$$a_1 = \left(\frac{V_1}{V_3}\right)^2, \qquad a_2 = \left(\frac{V_2}{V_3}\right)^2.$$

Also, the current relations may be worked out from Eq. (53) at λ/8 spacings, and there results

$$\left.\begin{aligned} R_r &= [A_1(A_2 - A_1 + 1) - \tfrac{1}{4}(A_2 - 1)^2]^{\frac{1}{2}} Z_0, \\ X_r &= [\tfrac{1}{2}(A_2 + 1) - A_1] Z_0, \\ \text{where} \quad A_1 &= \left(\frac{I_2}{I_1}\right)^2, \qquad A_2 = \left(\frac{I_3}{I_1}\right)^2. \end{aligned}\right\} \qquad (58)$$

The preceding derivation was taken from Barrow's paper, referred to, except that one sign was changed.

Procedure, Precautions.—The measuring line must be excited by a suitable oscillator, and the latter must be sufficiently remote from the voltage- or current-measuring instruments to affect the latter directly. It is well to shield the oscillator and line-input coupling coil. If it is desired to measure voltages, a vacuum-tube voltmeter having a very low input capacitance must be used. The acorn-tube voltmeter described in Chap. VI is useful for this purpose. It is well to place a thermomilliammeter in the line at a current antinode, or *loop*, and note whether or not the current is seriously affected when the probe terminal of the voltmeter touches the wire at the points λ/8 and λ/4. If the current is not affected, it shows that the contact of the probe terminal does not affect the line-potential distribution. If the currents at the λ/8 spacings are to be measured, small compact-type thermomilliammeters mounted on low-capacitance insulators must be used. Long thin glass rods are best for this purpose. The reliability of these meters must be checked up at the test frequency used (see Art. 8, Chap. VI).

24. Other Methods of Resistance Measurement. *Willans' Method.*—This method of measuring resistance makes use of the reaction of a loosely coupled variable-tuned circuit containing the unknown resistance on the frequency of a vacuum-tube oscillator.[1] In Diagram (*a*), Fig. 32, the variable-tuned circuit with

[1] *Experimental Wireless and Wireless Engineer*, Vol. 2, p. 350, March, 1925.

unknown resistance in series is shown. When this circuit is in resonance with the oscillator frequency, it produces a purely resistance reaction on the oscillator-tank circuit, as is easily proved by elementary coupled-circuit theory. This will have no appreciable affect on the oscillator frequency if the coupling is loose, because the resistance reflected into the oscillator-tank circuit is very low.[1] When the coupled circuit is detuned, it will of course have a reactive reaction and change the oscillator frequency, the frequency change passing through a maximum

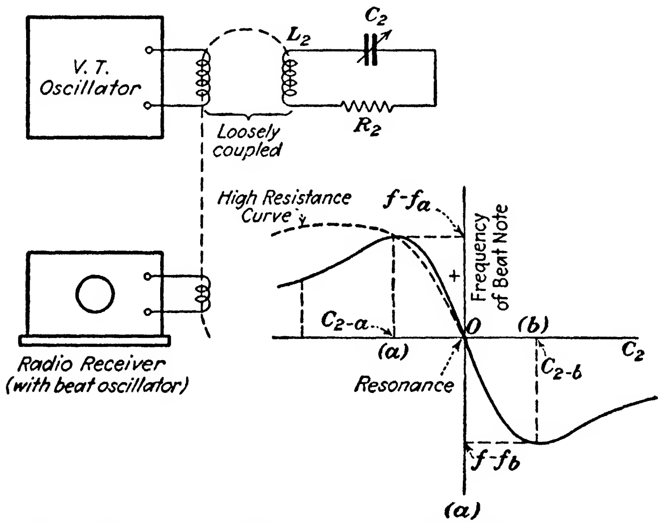


FIG. 32.—Test circuit for Willans' method of resistance measurement.

as the coupled-circuit tuning-condenser capacitance is decreased or increased from resonance.

The procedure in measurement will now be described, after which an outline proof of the method will be given.

For making the measurement, the apparatus is arranged as shown in Fig. 32. The unknown resistance and the resistance of the coupled circuit coil L_2 is to be considered as the resistance to be measured, and the variable condenser C_2 is assumed to have

[1] The oscillator frequency is $f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R_t}{r_p}}$, where r_p is the alternating-current plate resistance of the oscillator tube, and R_t is the tank-circuit resistance. R_t/r_p is negligible compared to $1/LC$ in tank circuits generally used.

zero-loss resistance in comparison. The vacuum-tube oscillator, which is of a stabilized type, is placed in operation; and the radio receiver, preferably with beat oscillator incorporated, is tuned to obtain zero beat with the emission from the oscillator. An ordinary broadcast receiver and a separate heterodyning oscillator may be substituted for the type shown. After zero beat is obtained, the coupled test circuit is placed at a considerable distance from the oscillator, and C_2 is varied until the zero-beat condition is disturbed and a tone is heard, due to the reactive reaction of the coupled circuit upon the oscillator frequency. C_2 is further adjusted until resonance occurs with the oscillator; this condition is determined by the restoration of the zero-beat condition. It will be found that the adjustment of C_2 for resonance and for the restoration of zero beat is quite sensitive. The next step is to change the value of C_2 from this setting until the beat note acquires its maximum frequency or pitch; the latter should not exceed a few hundred cycles, in order that the oscillator frequency ($f = \omega/2\pi$) may be assumed practically constant for the test. It will be found that there are two values of C_2 on either side of the zero-beat value for which maximum-frequency beat note is obtained, as is illustrated in Diagram (a) for the figure. This diagram also shows that when C_2 is set at its zero value, a low-pitch beat note should be obtained; and it is also important that as C_2 is varied the value for maximum-frequency beat note be not very sharply defined. Now, in the working equation,

$$R = \frac{1}{2\omega}\left(\frac{1}{C_{2-b}} - \frac{1}{C_{2-a}}\right), \tag{59}$$

where C_{2-b} and C_{2-a} are the values of C_2 for maximum beat-note frequency on each side of resonance [(a), Fig. 32]. These values of C_2 for maximum beat-note frequency are required. If the resistance of the coupled circuit is not too high, then the peaks of the beat-note curve may be determined with a good degree of precision with the aid of the ear; if desirable an additional variable audio-frequency oscillator may be provided and its tone matched with the apparent maximum beat-note frequency, the match being determined by zero pulsation or waxing and waning of the resulting sound. Then C_2 may be slightly varied back and forth, more accurately to determine the correct value for maximum beat-note frequency. The two values of C_2 thus obtained on each side of resonance, or zero beat, as indicated in Diagram (a), and the value of the oscillator frequency are all that are needed in the working equation. This gives the entire resistance of the coupled circuit $R_2L_2C_2$. If the resistance to be measured is then removed, another measurement can be made for the resistance of the remaining test circuit L_2C_2, and the resistance of the unknown is then the difference between these two measured values. If the unknown resistance is that of a coil, the coil itself can readily be used for L_2. The limitations of this method will be discussed following the theory of the method.

When the coupled circuit is in resonance, the oscillator frequency is practically $f = 1/2\pi\sqrt{L_1C_1}$ where L_1 and C_1 are for the oscillator-tank circuit. When C_2 (Fig. 32) is changed from the resonant value to a value C_2' on one side of resonance, the oscillator frequency is

$$f_a = \frac{1}{2\pi\sqrt{C_1(L_1 + L_1')}},$$

where L' is the change in effective oscillator-tank inductance due to the coupled circuit. From the foregoing,

$$\omega_a^2 = \frac{1}{C_1(L_1 + L_1')}, \qquad \omega_a = 2\pi f_a,$$

or

$$\omega_a = \frac{1}{C_1(\omega L_1 + \omega L_1')}, \tag{60}$$

where $\omega L_1'$ is the reactance reflected into the tank circuit by the coupled circuit. Interest lies in how much f_a will deviate from f, and Eq. (60) shows that the greater the value of L_1' the greater is the change. Again, from elementary coupled-circuit theory it is readily shown that

$$X_1' = \omega L_1' = \left(\frac{\omega M}{Z_2}\right)^2 X_2, \tag{61}$$

where M is the mutual inductance, and X_2 and Z_2 are the self-reactance and impedance of the secondary-coupled variable-tuned circuit in the figure. It will be found that for loose coupling the maximum beat frequency obtained [Diagram (*a*), Fig. 32] is only a few hundred cycles; therefore ω_a in Eq. (60) may be considered practically constant, since the testing frequency is well above a few hundred kilocycles. The greatest value of X_1', then, is practically dependent on X_2:

$$X_1' = (\omega M)^2 \frac{X^2}{R_2^2 + X_2^2}.$$

Differentiating X_1' with respect to X_2, and equating the derivative to zero, it is found that, for the maximum value of X_1',

$$R_2 = X_2 = \left(\omega L_2 - \frac{1}{\omega C_2}\right).$$

This will be the state of affairs at the maximum beat-frequency points *a* and *b* of Diagram (*a*), Fig. 32. At (*a*), C_2 is below the resonant value; the reactance of the coupled circuit is negative; it reflects a positive reactance into the oscillator-tank circuit, decreasing its frequency below the original value f. The beat frequency is $(f - f_a)$. Then

$$R_2 = \frac{1}{\omega C_{2-a}} - \omega L_2, \tag{62}$$

where C_{2-a} is the value of C_2 below resonance for maximum beat-note pitch. When C_2 is increased above resonance, the oscillator frequency is increased to a maximum value f_b; $f - f_b$ is now negative and is so plotted in Diagram (*a*), Fig. 32. Again, for maximum pitch of the beat note at (*b*),

$$R_2 = \omega L_2 - \frac{1}{\omega C_{2-b}}. \tag{63}$$

Here the inductive reactance L_2 is the greater term; hence the form of Eq. (63) compared to Eq. (62). From these two equations, it follows that

$$R = \frac{1}{2\omega}\left(\frac{1}{C_{2-b}} - \frac{1}{C_{2-a}}\right). \tag{64}$$

This method is actually capable of good precision. It was pointed out that ω (and the frequency) changes very little—a few hundred cycles in at least several hundred thousand—so the error is less than 0.1 per cent. C_2 is a precision-standard condenser calibrated accurately at low frequencies. It was pointed out in the introduction to this chapter that at radio frequencies the inductive reactance of the condenser cannot be neglected. The effective reactance of a condenser at radio frequencies is

$$X_c = \frac{1}{\omega C_e} = \frac{1}{\omega C_s} - \omega L.$$

Applying this to Eq. (64), it is seen that the constant radio-frequency reactance term cancels out, there is no error due to inductive reactance of the current paths, and the working equation holds with the given condenser-calibration data. The method fails for resistances so high that the peak of the beat-frequency curve cannot be found with certainty, as indicated by the dotted curve of Diagram (*a*) Fig. 32. It is necessary to make sure that L_2 is not used in the immediate neighborhood of its natural frequency of resonance.

Taylor's Method for High-resistance Measurement.—For high resistances such as 1,000 ohms or more, methods that depend upon resonance phenomena are not satisfactory because of the uncertainty with which resonance is detected. In the following method, the unknown high resistance is inserted as a shunt conductance G after resonance is obtained.[1] The test circuit is diagramed in Fig. 33.

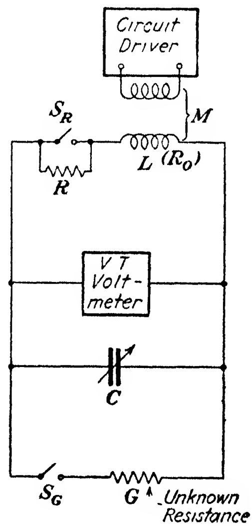


FIG. 33.—Test circuit for Taylor's method of resistance measurement.

A fairly low known resistance is connected so that it may be thrown in series with the coil L by means of switch S_R, and the unknown resistance having conductance G may be connected as shown with the aid of switch S_G. With S_R closed and S_G open, C is tuned to obtain resonance, indicated by maximum deflection of the vacuum-tube voltmeter, and the reading of the latter is V. S_R is now opened, C slightly readjusted if necessary, and the voltmeter reading V_R obtained. Lastly, both switches are closed, cutting out R and cutting in G; and V_G is obtained after

[1] TAYLOR, P. B., *Proc. I.R.E.*, Vol. 20, No. 11, p. 1802, November, 1932.

readjusting C slightly to insure resonance, owing to the possibility of G being reactive. The test-circuit resistances combine to give values that are

$$r_0 = R_0 + \frac{G_0}{\omega^2 C^2}$$

when V is obtained. Here R_0 is the resistance of L, and G_0 is the conductance of the voltmeter and condenser; when V_R is obtained, the circuit resistance is $r_0 + R$; and when V_G is obtained, the circuit resistance is

$$r_0 + \frac{G}{\omega^2 C^2}.$$

R_0 and G_0 can be eliminated by assuming

$$(R_0 + R)^2 < \omega^2 L^2,$$
$$(G_0 + G)^2 < \omega^2 C^2;$$

and the voltages will be

$$V = \frac{E}{r_0}\frac{1}{\omega C},$$
$$V_R = \left(\frac{E}{r_0 + R}\right)\frac{1}{C\omega},$$
$$V_G = \left(\frac{E}{r_0 + \frac{G}{\omega^2 C^2}}\right)\frac{1}{\omega C}.$$

Eliminating E and r_0 the solution for G is

$$G = \omega^2 C^2 R \frac{V_R}{V_G}\left(\frac{V - V_G}{V - V_R}\right). \tag{65}$$

This is the working equation for the conductance of the unknown. The best precision of measurement is obtained when

$$2V_R = 2V_G = V.$$

From this,

$$R = G\omega^2 L^2. \tag{66}$$

Then, using fixed inductance L, a series of known low resistances R may be provided suitable for the range of values of G to be measured. The upward range of conductance measured is limited mainly by the condition that G^2 must be small compared to ω^2C^2. The downward range is limited by the minimum known capacity C. The useful range with this method is between several thousand ohms and 1 megohm.

It is realized that there are several limitations due to the self-capacitance of L, capacitance of G, reactance of C, input capacitance of the tube voltmeter, and precision of the standards R. These latter are easily made with small-diameter high-resistance straight wires or with wire wound in the Ayrton-Perry, or woven, form. The precision and limitations of this method are fully treated in the paper referred to, as well as in the accompanying discussion of the paper, and will not be undertaken here. While the method is highly subject to limitations, it is one of the very few methods in existence for very high resistance. A three-voltmeter method previously described may also be possible for higher resistance if the current flowing in the test circuit could be measured with a suitable radio-frequency thermocouple. This current might be determined by the drop across a known impedance, as measured with the vacuum-tube voltmeter.

25. Insulation Power-factor and Loss-factor Measurement. *Definitions.*—The power factor of insulation or of insulation test samples is of course equal to the cosine of the phase angle between voltage impressed upon, and the alternating current flowing in the insulation placed between suitable metallic electrodes. Flat disk or plate electrodes are usually used for solid insulation testing, although tubular forms have also been used.

The loss factor, l.f., of insulation is defined[1] by the following equation,

$$\text{l.f.} = K \tan \delta = K \operatorname{cotan} \theta,$$

where K is the dielectric constant of the insulation material, δ is the phase difference (Diagram *c*, Fig. 34), and θ is the phase angle. It is known that the dielectric power loss in insulation is proportional to the loss factor, as defined above. When $\cos \theta$ is less than 0.1, and this is the case for all fairly good insulation, $\cos \theta$ is nearly equal to $\operatorname{cotan} \theta$, and

$$\text{l.f.} = K\ (\text{p.f.}), \tag{67}$$

[1] *Proc. A.S.T.M.*, Vol. 35; D150–35T, 1935.

where p.f. is the power factor. To obtain the dielectric constant, the capacitance C of the test sample with its electrodes is measured, after which the dielectric constant K is calculated from the formula

$$K = \frac{(C - C_e - C_g)t}{0.0885A}, \tag{68}$$

where t is the thickness of the flat sheet or disk test sample between electrodes in centimeters, and A is the contact area of either of the equal electrodes in square centimeters, C_e is the

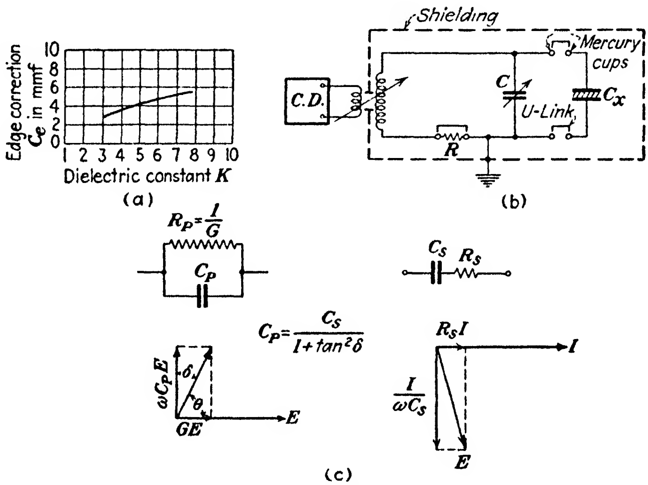


FIG. 34.—Relations and circuits in insulation testing.

edge correction for circular disk electrodes in micro-microfarads, C_g is a component of capacitance, or correction due to electrostatic flux lines passing from the high potential electrode to space and to ground. C is also in micro-microfarads. It will be noted that in Eq. (68) $\frac{0.0885A}{t}$ is the geometrical capacitance between the electrodes in micro-microfarads with dry air dielectric. The edge correction C_e is obtained with the aid of the curve of Diagram (a), Fig. 34.[1] It will be noted that the cor-

[1] This curve is taken from the *Proc. A.S.T.M.*, Vol. 36, Part I, p. 961, 1936. For details of the edge correction see *Bell System Tech. Jour.*, Vol. V, pp. 557–572, October, 1926; also *Proc. A.S.T.M.*, Vol. 36, Part II, 1936. Additional useful material on edge corrections is given by A. W. Barber, *Radio Eng.*, Vol. 17, No. 5, p. 26, May, 1937.

rection curve is for 4.5-in. diameter electrodes, and also that some knowledge of K is needed to apply the curve, hence K is obtained by the cut-and-try method. C_g can be estimated from the expression $C_g = 0.177D$, mmf., where D is the diameter of the circular ungrounded electrode in centimeters.

Measurement by Substitution in Resonant Circuit.—To determine the equivalent series resistance R from which the power factor $R\omega C$ may be calculated, a test circuit similar to that used for the measurement of resistance by the substitution method (see Diagram *a*, Fig. 19) may be used. However, C and R_s are interchanged in the test circuit, and the test sample takes the place of the unknown resistance R_x. C is then a practically loss-free standard variable air condenser. The procedure is described in Art. 12 of this chapter.

A method similar to that described in Art. 14 (Fig. 22) of this chapter is recommended in the A.S.T.M. standards (footnote 1 p. 88). The test circuit is shown in Diagram (*b*) of Fig. 34. The test sample C_x is connected in parallel with the practically loss-free variable air condenser C by means of pieces of bent-wire links inserted into small mercury cups, as indicated. The procedure is similar to that of Art. 14 for condenser-loss resistance measurement. Instead of using a variable resistance R, it is recommended that a series of differently valued fixed resistances be placed in the circuit (their terminals dipping into mercury cups) until the original current reading of ammeter A is approximately restored. The exact value of R can then be found by interpolation. Details of the test circuit are given in the standards referred to previously. The equivalent shunt capacitance of C_x is

$$C_p = \frac{C_2}{1 + \tan^2 \delta} - C_1, \tag{69}$$

where C_1 = the first resonant capacitance of C with C_x in the test circuit.

C_2 = the second resonant capacitance of C with C_x out of the test circuit.

δ = the dielectric loss angle or phase difference.

$C_p = \dfrac{C_x}{1 + \tan^2 \delta}$ for equivalent series and parallel imperfect capacitances, as shown in Diagram (*c*), Fig. 34

Unless tan δ exceeds 0.1, C_s and C_p are practically equal. The loss factor is obtained from the tangent of the loss angle,

$$\tan \delta = 2\pi f C_2 R_i \frac{C_2}{C_2 - C_1}, \tag{70}$$

where R_i is the value of R found by interpolation for equal current when C_x is removed. The power factor is practically equal to tan δ for good insulation. The above method is recommended for frequencies above 1.0 Mc. and has been used commercially at 10 Mc.

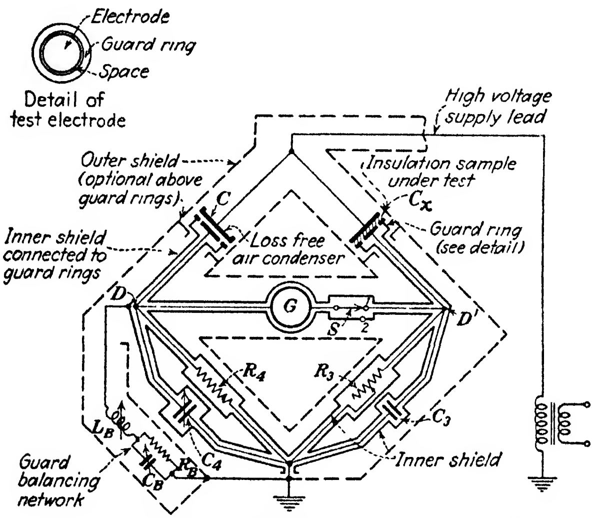


FIG. 35.—High-voltage Schering power-factor bridge.

Bridge Methods for Insulation Measurements.—The radio-frequency bridges of Figs. 9, 10, and 24 are well adapted to measurements of power factor and capacitance of insulation test samples at frequencies below 1.0 Mc. These are also adapted to similar measurements of liquid dielectric samples, especially oils. For this purpose, a suitable oil cell, such as described in the A.S.T.M. standards,[1] is desirable.

The A.S.T.M. standards describe a high-voltage Schering power-factor bridge, suitable for frequencies between 25 cycles and 1.0 Mc. with suitably designed ratio-arm resistances.[2] The bridge circuit with its double shielding is shown in Fig. 35. The

[1] *Loc. cit.*

[2] *Proc. A.S.T.M.*, Vol. 36, Part I, p. 971, 1936.

guard balancing network $C_B L_B R_B$ serves to bring the potential of the inner shields and guard rings around the test sample electrode to the same potential as the diagonals DD' of the bridge. This eliminates errors due to stray capacitances. To accomplish this, switch S is thrown from position 1 to position 2 alternately while C_4, R_3, C_B, and L_B are being varied until balance is obtained for S in either position. At balance the power factor of the test sample C_x is

$$\tan \delta = \cos \theta = \frac{2\pi f}{10^{12}}(R_4 C_4 - R_3 C_3), \tag{71}$$

where C_3 and C_4 are in micro-microfarads. When guard rings are used and shielding is provided around the high-potential electrode of C_x, no correction is necessary for edge effect, or for flux between the high-potential electrode and ground. Then the capacitance of the test sample C_x is

$$C_p = C\frac{R_4}{R_3} \tag{72}$$

Insulation Power Factor from Q Measurements.—The Q of a resonant circuit is $\frac{\omega L}{R} = \frac{1}{\omega CR}$ and the power factor of the condenser is of course $\omega CR = \frac{1}{Q}$. To measure the Q of an insulation sample, the "Q meter" of Fig. 25 is used, a standard coil being used instead of the indicated coil under test. The Q of the resulting *standard circuit* is then measured; let its value be Q_1. The insulation sample is placed between electrodes (the latter being connected in parallel with the variable condenser C of the Q meter) and another measurement made, giving a value Q_2. The Q of the sample is obtained from the equation[1]

$$Q_x = \frac{(\Delta C + C)Q_1 Q_2}{C_1(Q_1 - Q_2)} = \frac{1}{K\omega CR}, \tag{72}$$

where C_1 is the total capacitance of the resonant standard circuit.
C is the capacitance between the electrodes alone in air.
ΔC is the change in capacitance which takes place when the sample is placed between the electrodes.
Q_1 and Q_2 are as defined above.
K is the dielectric constant of the sample.

[1] BARBER, A. W., *Radio Eng.*, Vol. 17, No. 5, p. 26, May, 1937.

Then

$$K = \frac{(\Delta C + C)}{C}.$$

The paper referred to discusses the magnitude of the error resulting from small air spaces between the electrodes and sample and gives details of the construction of disk electrodes employing micrometers for precision work.

Methods Adapted to High Voltage and High Frequency.—The foregoing methods are well adapted to radio frequencies below 10 Mc., but the Q measurement method may be extended above this limit with care. At low voltages where these methods are practical, there is no ionization of the gaseous voids within the insulation, and the power factor will be lower than for higher potential gradients sufficient to cause ionization, resulting in additional power loss and hence higher power factors and loss factors.

Calorimeter methods are adaptable to high-voltage and high-frequency measurements, but they require great care and elaborate preparations. A very careful investigation of insulation loss factor has been made at potential gradients of from 2,200 to 10,300 volts per centimeter at frequencies in the neighborhood of 1,000 kc.[1] This investigation was made using both *transient* and *steady state* types of calorimeters. More recent investigations, using calorimeter methods, are in progress at frequencies of the order of 50 Mc.

[1] Race, H. H., and S. C. Leonard, *Elec. Eng.*, Vol. 55, No. 12, p. 1347, December, 1936. A valuable bibliography is given in this paper. The calorimeter method of Art. 15 may also be used with careful preparation.

CHAPTER II

MEASUREMENT OF FREQUENCY

1. Introduction. *Use of the Terms Frequency and Wave Length.*—In radio engineering, it has been almost universal practice in the past to deal with wave length rather than with frequency. This custom had its origin in the tendency to stress the concepts of the electromagnetic waves in space, paying little attention to the circuits and methods used in generating radiated waves. The reader is familiar with the relation between the wave length of a radiated wave, its velocity of propagation through space, and the frequency of the alternating current or potential in the radiating device, *viz.*,

$$v = \lambda f \text{ meters per second,}$$

where λ = wave length in meters.
v = velocity of the radiated wave.
f = frequency in cycles per second.

f is also the frequency of the alternating electric and magnetic fields in space. The frequency of the currents or potentials is more often considered in circuits containing lumped constants, and the term *frequency* is becoming more commonly used in place of the formerly used term *wave length.*

The older term *wavemeter,* therefore, is giving way to the term *frequency meter,* and the latter will be favored in the subsequent material of this chapter. Hence the term wavemeter or frequency meter will be understood to apply to a device comprising a variable condenser, one or more fixed inductances, and generally including some kind of resonance indicator. As is well known, this device will absorb maximum energy from a radiated or induction field when its capacitance and inductance are so proportioned that

$$f = \frac{v}{\lambda} = \frac{1}{2\pi\sqrt{LC}}. \tag{1}$$

L and C of the device are henrys' inductance and farads' capacitance, respectively. The maximum energy absorbed is accompanied by maximum current flow and maximum potential across the condenser. These latter phenomena are utilized for the so-called *resonance indication*. In all cases excepting that of direct measurements of distance between nodes of potential on parallel Lecher wires, the quantity directly measured is frequency. The wave length is then obtained for these measured frequencies by using the relation of Eq. (1) and taking the velocity v as 3×10^8 m. per second, so the commonly used value will introduce a slight error. Many interesting earlier types of wavemeters and frequency meters are to be found in the literature, but a review of these developments is not within the scope of this text. At the present time, the standard practice consists of constructing a wavemeter of the *absorption type* by providing a variable condenser with a suitable scale, a fixed inductance or set of inductances for extending the range, and a resonance indicator. This simple arrangement is connected as shown in Fig. 1. The resonance indicator may be a thermogalvanometer having not over 4 or 5 ohms resistance, but of course this means that some 100 milliamp. is necessary for full-scale deflection and that the power loss is considerable. A rectifier-type meter with a range of 1 milliamp. or less may be used by connecting it to a small pickup coil coupled closely to the wavemeter inductor. This uses very little power. A still better but more expensive method is to connect a vacuum-tube voltmeter input to the wavemeter circuit, and this latter is done in at least one modern commercial type. No space will be given to a description of the features of various wavemeters. The modern heterodyne frequency meter is displacing the absorption type for many uses, but the latter has the advantage of simplicity for some types of applications.

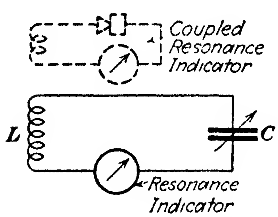


FIG. 1.—Simple wavemeter or frequency meter circuit.

Approximate and Precision Measurements.—The simplest measurement of the frequency of a current above 10,000 cycles consists of coupling the frequency meter or wavemeter magnetically to the field produced by this current and varying the natural period of the device by changing the value of the variable capacitance until *resonance* is indicated by the maximum deflection of the needle of a thermomilliammeter connected in series with L and C (which shows that the induced current in the frequency meter is a maximum). The values of capacitance and inductance for this condition of resonance may then be used in Eq. (1) to calculate the frequency of the inducing current. Generally, however, the frequency meter is a previously calibrated instrument with a curve furnished, showing the frequency in cycles or kilocycles per second (or wave length in meters) for each setting of the variable-condenser scale and fixed inductance used. A certain range of frequencies is covered by each inductance coil and the total range of the variable condenser. Without going into the details of high-grade frequency-meter construction, it should be noted that the variable condenser is generally of the straight-line-capacitance type, with semicircular plates so as to obtain the most rigid condenser construction. This means that the calibration curve will not be a straight line if frequency or wave length is plotted against uniform scale divisions of the condenser dial.

It will be found that the simple method of measurement outlined above does not permit of high precision. The exact resonance is somewhat difficult to obtain if the resistance of the device, including that of the resonance indicator, is high, this resulting in a flattened peak in the resonance curve [as shown in Fig. 2, curve (*b*)]. Obviously, if resonance curve (*a*) obtains, the accuracy of determination of resonance and hence of frequency is improved. It is well known that the sharpness of resonance of these curves is given by the expression $\omega L/R$, the Q of the wavemeter circuit. Even with the sharpest resonance possible in a frequency meter containing a thermomilliammeter, the resonance is not extremely sharp. The device also absorbs a quite appreciable amount of energy at resonance. Furthermore, if the source of the exciting field is a low-power device, such as a 1- to 5-watt tube oscillator, the coefficient of electromagnetic coupling with this source will be sufficiently great so that, owing to the reactions of coupled circuits, the presence of the frequency meter will change the frequency to be measured. A very low-range thermomilliammeter has too high a resistance to allow it to be used in a frequency meter; and in order that the resistance of the milliammeter may not be more than 5 ohms, the thermocouple must be one requiring 100 milliamp. for 5 or 6 millivolts potential across its junction. These considerations result in a practical average accuracy of not more than 0.25 per cent for this type of frequency meter so used. In order to make the resonance indication more exact, data for a resonance curve may be plotted for a few values on each side of resonance. The mean of two condenser settings for equal current indications on each side of resonance may be taken as indicating resonance. This is illustrated in Fig. 2, points *c* and *d*. The wavemeter condenser should have a Vernier scale or even a micrometer scale in order to obtain values on the steep portions of the

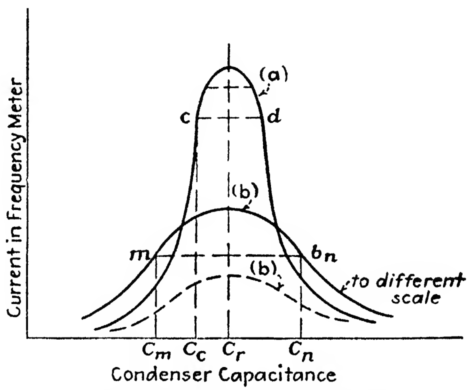


FIG. 2.—Resonance curves for different values of sharpness of resonance.

resonance curve. If the resistance of the wavemeter circuit is higher, giving a resonance curve such as the dotted curve (*b*) (Fig. 2), the precision of determination of two points on each side of resonance is lowered. This is illustrated with the full-line curve (*b*) (Fig. 2) which is dotted curve (*b*) drawn to an exaggerated scale. Here it is seen that points *m* and *n* are not so accurately found as are points *c* and *d*, and hence the resonant capacitance C_r is not so accurately determined as with a lower resistance wavemeter circuit. With the best design of wavemeters depending on absorption or reaction methods of resonance indication, the precision of measurement is about 0.1 per cent.

When these absorption wavemeters are tuned to resonance with a vacuum-tube oscillator or circuit driver, they produce a change of average plate and grid currents in the latter. This effect may often be used to indicate resonance and is useful in many cases. The maximum change occurs when the wavemeter is in resonance, because it is then that the maximum current flows in the wavemeter and the maximum resistance is reflected into the oscillating or tank circuit of the circuit driver. Tuning a wavemeter to resonance with a vacuum-tube oscillator also produces a reaction upon the oscillator frequency when the wavemeter circuit is not quite in exact resonance. In this state, it reflects a reactance into the oscillator-tank circuit, changing its frequency; and when it is in exact resonance, its reaction is a pure resistance reaction which, if the coupling of the wavemeter to the oscillator-tank circuit is not too close,[1] has negligible effect on the oscillator frequency.

The frequency of damped currents may be approximately measured if the damping is not too great. The natural logarithm of the ratio of two succeeding positive maxima of current is called the *logarithmic decrement* of the damped current and should not exceed 0.3 for a measurement of frequency within 3 per cent. Owing to this fact, and since most measurement problems deal with constant-amplitude currents, only the last division of this chapter will be devoted to the case of damped current frequencies.

2. Frequency Measurement with Heterodyne Frequency Meter. *Principle of Method.*—If e.m.fs. of different frequencies are impressed upon the same detector, the resulting current in the device will contain sum and difference frequency components. If one of these frequencies is variable, it may be adjusted until the difference-frequency component becomes so low as to be audible to the ear; and with further adjustment the difference frequency may be made less than 50 cycles, at which the ear can no longer hear it. This condition is that of *inaudible zero beat.* By means to be described later, the variable frequency may be adjusted to *exact* zero beat, *i.e.*, exactly the same as the comparing frequency. The exact zero-beat adjustment is not warranted in the method under discussion.

The heterodyne frequency meter is simply an electron-tube oscillator of suitable frequency stability, with a sufficiently

[1] This method of measuring frequency is described by C. B. Aiken, *Proc. I.R.E.*, Vol. 16, No. 2, p. 125. Also see H. A. Brown, "Radio Frequency Electrical Measurements," 1st ed., pp. 90 *ff*.

extended variable tank-condenser scale so that it may be calibrated to a certain degree of precision in frequency (or wave lengths). The output of this calibrated oscillator is caused to beat on the unknown frequency with zero beat as indicated by a sensitive detector amplifier or a radio-receiving set. The dial reading of the heterodyne frequency meter and the calibration curve then give the unknown frequency.

Procedure.—In Fig. 3 is shown the circuit arrangement for conducting the measuring procedure. The particular heterodyne frequency meter shown has a range of oscillation frequency as indicated, making it convenient for use in measuring frequencies

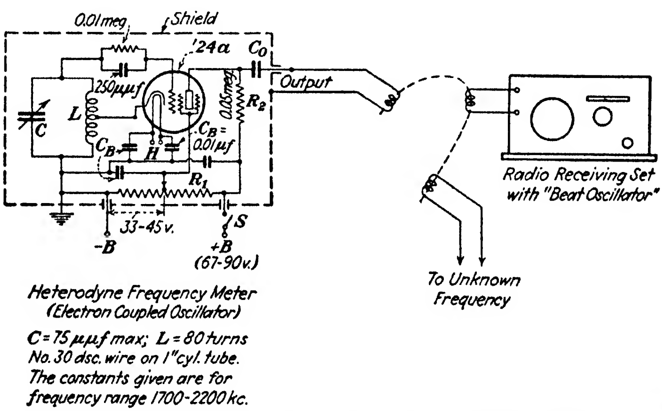


FIG. 3.—Electron-coupled heterodyne frequency meter and test circuits for measuring frequency.

in the amateur bands with the aid of the harmonics of its fundamental. This device and also the radio-receiving set and unknown frequency are connected to small coils of a few turns which are loosely coupled to each other. Connecting such a coil to the antenna and ground terminals of the receiving set makes the latter less liable to pick up unwanted radio waves. The radio receiver is first turned on, and its beat oscillator is also set into operation. This oscillator beats on the intermediate frequency of the radio receiver (which is assumed to be of the superheterodyne type). The receiver is then tuned until it responds with a beat-note signal to the unknown frequency. Then the beat oscillator of the receiver is turned off, and the heterodyne frequency meter is turned on, and its frequency is varied with the aid of condenser C until zero beat with the unknown frequency is noted in the receiver loud-speaker. The reading of the condenser scale is carefully noted, and the frequency is found from the calibration curve provided. It will be found that the frequency of the electron-coupled oscillator will drift until the cathode heater and tube are thoroughly warmed up. Hence it is well to have the heater turned on some time before the measurement is made and then close switch S to place the oscillator into operation when required. The frequency to be measured may be a harmonic of some frequency within the heterodyne frequency-meter range. In this case, the procedure is the same as before, and a harmonic of the heterodyne frequency-meter oscillator frequency is at zero beat with the unknown. The calibration of the receiver dial will show the unknown frequency approximately; hence the order of the harmonic that is beating on the unknown frequency is easily determined. If the unknown frequency to be measured is that of a distant radio station, it is sufficient to connect the receiver to an antenna, in which case the coupling coil shown connected to the receiver input is dispensed with. If the unknown frequency contains harmonics, the tuning of the receiver should be varied until it responds at the lowest frequency setting of the tuning dial that can be found, which will insure that it is responding to the fundamental of the unknown frequency. Of course, for measuring unknown frequencies below that of the heterodyne frequency-meter range, the receiver can be tuned to a known harmonic of the unknown. A detector and amplifier may be used in this experiment instead of a radio receiver, but it is less sensitive and will not prove to be so convenient. Many of the amateur receiving sets have beat oscillators provided, or they may be purchased cheaply and attached to a set. If such is not available, tuning to the unknown frequency may be accomplished by listening for a rushing sound when the unknown is tuned in or by modulating or interrupting the unknown rapidly with a switch or key as the receiver tune is varied. Care must be exercised to avoid overexcitation of the receiving set. This will cause humming or roaring in the loud-speaker, and if this occurs the coupling coils and apparatus should be separated until a clear beat note is obtained. The electron-coupled oscillator has the feature of being independent of its coupled load; *i.e.*, its frequency is not affected by coupling its output, provided the output coupling condenser, C_0, Fig. 3, is very small.

Precision of Measurement.—In the design of the electron-coupled heterodyne frequency meter, a high-grade straight-line frequency tank condenser C (Fig. 3) together with a large precision dial provided with a Vernier scale is desirable. The degree to which a frequency can be read depends upon the readable number of dial divisions, assuming that the calibration curve is accurate and is suitably plotted. In Fig. 4 are shown calibration curves of electron-coupled heterodyne frequency meters. Curve (a) is for the type illustrated in Fig. 3, having a straight-line frequency variable condenser, and curve (b) is for a type that has a straight-line capacitance tank condenser. In the case of the meter illustrated in Fig. 3, the voltage applied to the screen (which acts as the anode for the triode-oscillator combination) is derived from potentiometer resistance R_1, so that if the plate or screen voltages vary they will vary together. It has been shown that if they vary in the same proportion there is no change in oscillation frequency.[1] It is necessary to keep the heater voltage constant with the aid of a voltmeter and rheostat control.

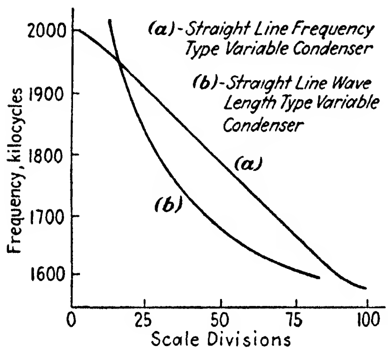


FIG. 4.—Calibration curves of electron-coupled heterodyne frequency meter using different types of variable tuning (tank) condenser.

3. Calibration of Heterodyne Frequency Meter. *Introduction.* Since the heterodyne frequency meter is in wide use to determine whether frequencies lie within a certain band, it is important to have available a method of calibrating this instrument that is precise and yet does not require a too complex assembly of apparatus. The temperature-controlled quartz-crystal piezo oscillator may be used to provide a known frequency, or the frequency of one of the well-known broadcast stations may be used as a standard. Regulations require broadcast-station frequencies to be accurate within 50 cycles, but actually the

[1] Dow, J. B., *Proc. I.R.E.*, Vol. 19, No. 12, p. 2095, 1931.

temperature-controlled quartz crystal maintains the frequency within 3 to 5 cycles per million.

Principle of Method.—A quartz-crystal piezo oscillator of conventional design gives quite a large number of harmonics of its fundamental frequency. By a method very similar to that of Art. 2, calibration may be carried out by adjusting the heterodyne frequency-meter frequency to zero beat with harmonics of the piezo oscillator whose fundamental is known, and whose harmonics are integral multiples thereof. This procedure is simple. The lowest frequency setting of the frequency-meter tank condenser is made, and the radio receiver with beat oscillator turned on is tuned to pick up this frequency. The heterodyne frequency-meter plate voltage is then shut off, the piezo oscillator turned on, and the receiver tune is altered sufficiently so as to pick up the nearest piezo-oscillator harmonic. The beat oscillator in the receiver is then turned off, the heterodyne frequency-meter plate voltage is again applied, and its tank condenser is varied until zero beat with the piezo-oscillator harmonic previously tuned in is obtained. The order of this harmonic may be determined from the calibration of the receiver dial (if it is accurate), or a circuit driver may be tuned to zero beat with the harmonic, and its wave length measured by an absorption wavemeter. The next higher harmonic of the piezo oscillator is then found with the receiver, the heterodyne frequency meter is tuned to zero beat with it, and so on, counting successive harmonics and noting the scale reading for each. If the precision piezo oscillator has a fundamental frequency of 100 kc., and the heterodyne frequency meter·being calibrated has an oscillation range of 1,500 to 2,200 kc. inclusive, only eight calibration-scale readings will be obtained corresponding to the eight harmonics of 100 kc. that may be tuned in. The number of calibration points may be doubled by providing a stable frequency oscillator to half the frequency of the piezo crystal standard. If this oscillator is of the class C type, a considerable range of harmonics is present; and for this an electron-coupled type at a frequency of 50 kc. will have ample harmonics in its output and is sufficiently stable.

Procedure.—Figure 5 shows the schematic arrangement of the test equipment. The 50-kc. stable frequency may be adjusted to 50 kc. ± 40 cycles (owing to audio limit in zero-beat adjustment) with the aid of an absorption wavemeter and the radio receiver. It is tuned to zero beat of its second harmonic with the 100-kc. oscillator, and measurement of its wave length with the wavemeter will show when it is at 50 kc. After this is done, the 50-kc. oscillator may be set at exactly 50 kc. by turning on the heterodyne frequency meter to be calibrated and adjusting it so that it beats on a harmonic of the piezo oscillator with a 200- to 400-cycle beat-note tone. Then the 50-kc. oscillator is turned on and adjusted until there is no pulsation in the 200- to 400-cycle

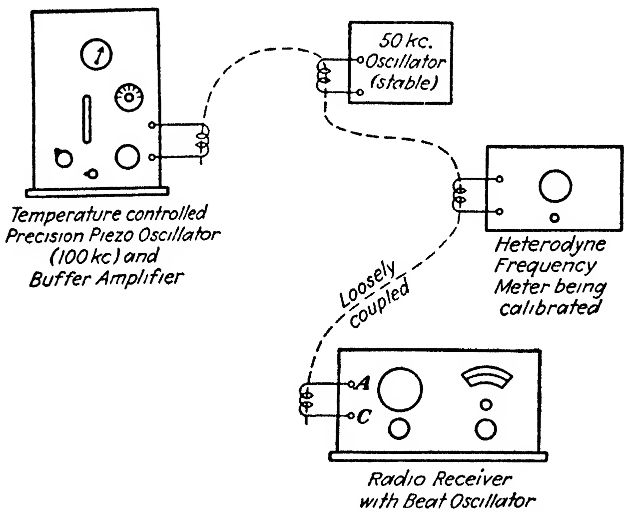


Fig. 5.—Arrangement of apparatus for calibrating a heterodyne frequency meter.

tone intensity. This means that the *n*th harmonic, say, of the piezo oscillator and the 2 *n*th harmonic of the 50-kc. oscillator are exactly alike in frequency, for there is no tone pulsation, and hence the 50-kc. oscillator is at exactly half the frequency of the piezo-electric standard. After this is done, the heterodyne frequency-meter condenser scale may be calibrated from the harmonics of the 50-kc. oscillator just as was described under Principle of Method.

If a broadcast station is to furnish the standard known frequency, the procedure is quite similar; in this procedure, the tenth to thirtieth harmonic of the 50-kc. oscillator is adjusted to zero beat with the broadcast-station frequency (500 to 1,500 kc.).

Precision of Measurement.—The heterodyne frequency meter may be set to within one cycle or less of the standard frequency if desired. This is not really justified, because the scale-reading limitations impose a much lower practical degree of precision. The student may determine the actual precision of calibration.

4. Precise Measurement of Frequency with Secondary-standard and 10-kc. Harmonic Generator. *Theory and Description.*—A secondary frequency standard may be defined as one that has been previously compared to an absolute or primary standard in order to determine its exact frequency. It is then operated under carefully controlled conditions in order to maintain the frequency very near to the previously determined value. The temperature-controlled quartz-crystal piezo oscillator makes a very satisfactory secondary standard for laboratory measurements. It should have an exact frequency of 50 or 100 kc., although a 1,000-kc. crystal is sometimes used.

A 10-kc. stabilized-frequency oscillator[1] may be used to over-excite an amplifier so as to produce a large number of harmonics. In this *harmonic amplifier*, the grid is biased beyond cutoff, and its operation as a class C amplifier results in pulses of plate current of such wave shape that some 200 harmonics may be detected with a radio-receiving set with beat oscillator. The oscillator and harmonic amplifier are shown diagrammatically in Fig. 6. If the 10-kc. oscillator-harmonic amplifier is adjusted so that one of its harmonics is at zero beat with the output frequency of the secondary-standard 50- or 100-kc. quartz-crystal oscillator, there will be produced a 10-kc. output with a series of harmonics thereof (20, 30, 40, 50, etc., kc.) which are accurate in frequency to the same degree as is the quartz-crystal frequency. If the unknown frequency is, say, 750 kc., it is at zero beat with the seventy-fifth harmonic of the 10-kc. source. If the unknown is 754 kc., there will be a 4-kc. beat-frequency note between the unknown and the seventy-fifth harmonic when a detector or receiving set picks up both frequencies. There will also be a 6-kc. beat note due to the seventy-sixth harmonic, but this will

[1] A stabilized-frequency oscillator based on the theory given by J. W. Horton (*Bell System Tech. Jour.*, Vol. 3, p. 521, July, 1934) is manufactured by the General Radio Company. For 10-kc. operation it is easily constructed in the laboratory. Another type which is simple to assemble is described by Ross Gunn, *Proc. I.R.E.*, Vol. 18, No. 9, p. 1566.

be less audible if the receiving set is tuned so as to receive 750 and 754 kc. with equal response. This 4-kc. beat note may be measured to determine its frequency to a good degree of precision with an audio-frequency bridge such as the Wien bridge, resonance bridge, or Hay bridge [1] It may also be measured by providing a calibrated audio-frequency oscillator and adjusting

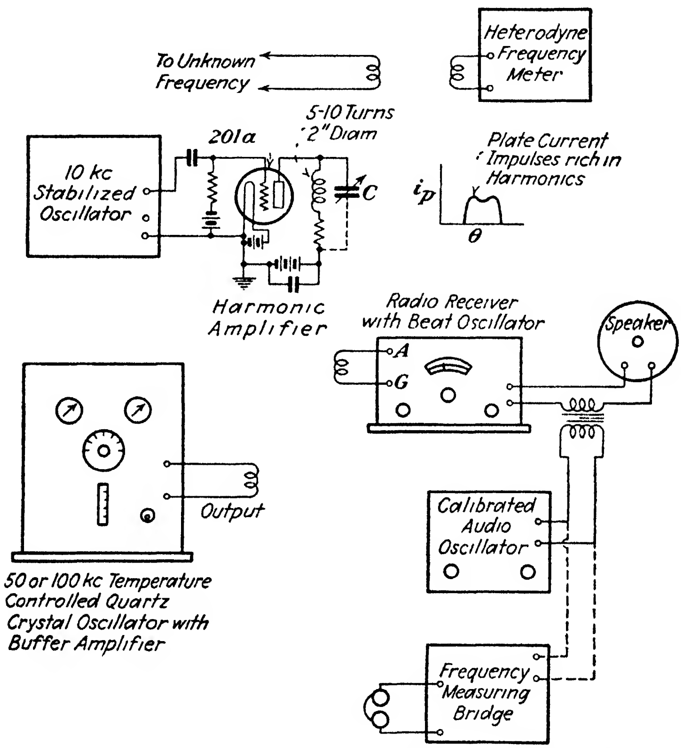


FIG. 6.—Test apparatus for measuring frequency by secondary-standard and 10-kc harmonic generator

its frequency until the pulsation in intensity with the audio frequency from the receiver is reduced to zero, when the two audio frequencies are alike and their value is obtained from the calibration curve of the audio oscillator.

[1] A frequency bridge marketed by the General Radio Co. incorporates the Wien circuit. The resonance and Wien bridges are described in Art. 8 of this chapter. For the Hay bridge in frequency measurement see *Proc. I.R.E.*, Vol. 17, p. 834, May, 1929.

Procedure in Making Precise Measurement.—The explanation of the procedure is somewhat lengthy, hence will be given in numbered steps.

1. The first and most important step is to adjust the 10-kc. oscillator to exact frequency. If the crystal oscillator is assumed to produce exactly 100 kc., the tenth harmonic of 10 kc. is exactly equal to the crystal frequency. The radio-receiving set is coupled sufficiently close to the 100-kc. crystal oscillator, the beat oscillator in the receiving set is turned on, and the tuning of the receiving set is varied until a harmonic of the crystal oscillator is picked up. The receiver beat oscillator is then turned off, the 10-kc. oscillator and harmonic amplifier are turned on, and after its temperature becomes constant the 10-kc. oscillator tuning is varied slightly until a beat note is heard in the receiver speaker. By further tuning this beat note is reduced to zero audibility, after which the beat oscillator is again turned on, and a beat note will appear which pulsates or flutters in intensity. Then the frequency of the 10-kc. oscillator is again varied, preferably with a trimmer condenser, until the intensity pulsation is reduced to zero. This means that an *n*th harmonic of the crystal oscillator and a 10*n*th harmonic of the 10-kc. generator are exactly alike in frequency, and hence the 10-kc. generator frequency is exactly one-tenth of the 100-kc. crystal oscillator frequency. This exact zero-beat condition should not vary over a considerable time if the measurement is to be precise. If a beat oscillator built into the radio receiver is not available, a separate heterodyning oscillator may be provided which beats on the input to the receiver and produces a suitable audio tone. It should be remembered that if both incoming frequencies are exactly alike, a separate heterodyning oscillator will beat on each of the radio frequencies with beat notes of exactly the *same audio frequency;* hence there will be no intensity pulsation of the audio frequency.

2. Next, the unknown frequency, which is assumed to be between 500 and 1,500 kc., is turned on; the radio receiver with its input coil coupled to the 10-kc. harmonic amplifier output and to the unknown frequency is turned on; and its tuning is varied until it picks up the unknown frequency and a harmonic of the 10 kc. simultaneously. These will produce an audible beat note of a frequency between 5,000 kc. and one that is too low to be heard.

3. To determine which harmonic of the 10-kc. output is beating with the unknown frequency, the unknown is turned off, and a calibrated heterodyne frequency meter with a range of 500 to 1,500 kc. is turned on and tuned to zero beat with the 10-kc. harmonic to which the radio receiver has already been tuned. From the calibration of the dial of this instrument the order of this harmonic is found.

4. The audio-frequency beat note produced by the unknown frequency and the 10-kc. harmonic is then measured with a precision audio-frequency bridge, shown in Fig. 6; or a calibrated audio oscillator, which has its output injected into the radio-receiver loud-speaker circuit, may be adjusted to zero beating with the received beat note. If the beat note is below audibility, the measurement of its frequency may be made by placing a milliammeter or microammeter in the detector plate circuit of the receiver and observing the rate of fluctuation of the meter needle. If it is too rapid to count, it may be done stroboscopically.

5. Again, let it be supposed that the 10-kc. harmonic upon which the unknown was beating was the seventy-fifth; its frequency is 750 kc. Let the measured value of the audio beat note be 4 kc. Then the unknown frequency may be either 746 or 754 kc. To determine which is correct, the calibrated heterodyne frequency meter may be used directly; or this instrument may be tuned to exact zero beat with the unknown frequency (with the aid of the receiver and beat oscillator), and the latter turned off. The heterodyne frequency meter is now beating on the seventy-fifth harmonic with a 4-kc. beat note. Then the frequency of the heterodyne frequency meter is slightly increased by Δf; and if the beat note increases in frequency, the unknown frequency is 754 kc., because

$$(754 + \Delta f) - 750 = 4 + \Delta f;$$

but if the beat note lowers in frequency, the unknown was 746 kc., because $750 - (746 + \Delta f) = (4 - \Delta f)$ kc.

Use of Interpolation Oscillator.—As previously noted, the unknown frequency (or a harmonic thereof) will be somewhere between the nth and the $(n + 1)$th harmonic of the 10-kc. oscillator. If it lies about halfway between these adjacent harmonics, the difference frequency is in the region of 5,000 cycles, and sometimes the latter is hard to measure with apparatus at hand. An auxiliary measuring instrument, known as an interpolation oscillator, is often used for the determination of the unknown frequency. The method is as follows: The interpolation oscillator incorporates a straight-line frequency variable-tuning or tank condenser of precision design and is equipped with a calibrated dial which may be read to a high degree of accuracy. This oscillator is now tuned to exact zero beat with the nth harmonic of the standard 10-kc. source (using the radio receiver with beat oscillator), and the dial reading is carefully noted. This oscillator is then tuned successively to exact zero beat with the unknown frequency and with the $n + 1$ harmonic, and dial readings noted. Since the oscillator-tank condenser is a high-precision straight-line frequency type, its calibration curve will be a straight line to a high degree of precision, and the value of the unknown frequency may thus be obtained by interpolation. For example, let the oscillator-dial reading be 60.00 for the fiftieth harmonic of the 10-kc. source; the dial reading for the unknown frequency may be 45.20; and the dial reading for the fifty-first harmonic is, say, 35.45. It is easily understood that the unknown frequency may now be obtained by simple proportion, thus:

$$\frac{10 \text{ kc.}}{24.55} = \frac{(\Delta_x) \text{ kc.}}{9.75},$$

where $24.55 = 60 - 35.45$ and $9.75 = 45.20 - 35.45$. From the foregoing,

$$\Delta_x = 3.97 + \text{kc.},$$

and the unknown frequency is 506.03 + kc. It should be noted that the interpolation oscillator variable condenser has a very small range, so that a comparatively large change in dial reading is necessary to change the frequency a small amount, such as 10 kc.

Procedure with Higher Unknown Frequencies.—The foregoing procedure is practically limited to frequencies below 1,500 kc., because harmonics above the 150th or 175th of the 10-kc. source are difficult to detect when the harmonic amplifier described above is used. For higher unknown frequencies it is merely necessary to tune a harmonic of the heterodyne frequency meter to zero beat with the unknown frequency. The heterodyne frequency meter is designed for 500- to 1,500-kc. frequency range. If it is the electron-coupled type, which is rich in harmonics, no difficulty will be found in tuning an unknown frequency to zero beat with its thirtieth harmonic or higher. By this means, the range of frequencies measurable by this method is great. The heterodyne frequency meter is readily calibrated on the 10-kc. harmonic sequence, in a manner that needs no explanation.

Controlled Multivibrator for 10-kc. Harmonic-sequence Output.—The *multivibrator* of Abraham and Block[1] is a device rich in harmonics, described by the authors as capable of obtaining more than 200 harmonics. Dye[2] found that its fundamental frequency may be controlled by an input voltage whose frequency may be changed as much as 4 per cent. In Fig. 7 is shown a

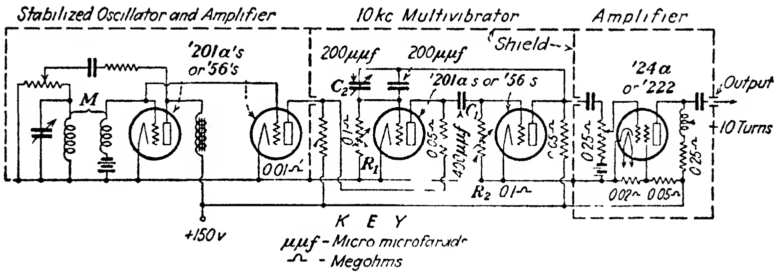


FIG. 7.—Stabilized amplifier—fundamental frequency controlled multivibrator circuits for production of 10 kc. and harmonic sequence.

10-kc. stabilized-frequency oscillator with its output injected into the plate circuit of a multivibrator whose fundamental frequency is nearly 10 kc. It will be noted that this latter device is merely a resistance-coupled amplifier whose output is fed back to the input, *i.e.*, to the grid of the first tube through coupling condensers C_2. The frequency at which a sort of feedback action and periodic steep wave-front disturbance or oscillation occurs is

$$\frac{1}{f} = C_2R_1 + C_1R_2, \text{ nearly;}$$

and for a 10-kc. oscillation the circuit constants of the device are shown in the diagram. It will be found that the fundamental

[1] *Ann. de physique*, Vol. 12, pp. 252 *ff.*, 1913. For a brief theory see Yatanabe, *Proc. I.R.E.*, Vol. 18, No. 2, p. 327, 1930.

[2] *Phil. Trans. Roy. Soc.* (*A*), Vol. 224, p. 259, 1914.

frequency of this oscillation will be the same as that of the 10-kc. oscillator; and if the frequency of the latter may be changed 4 or 5 per cent, the multivibrator frequency will change also; *i.e.*, the multivibrator is locked in step with the oscillator frequency. The multivibrator will generate a 10-kc. sequency of harmonics, some investigators claiming that some 400 harmonics may be obtained. The first hundred harmonics are quite intense, so are really better than those obtained from an ordinary class C amplifier. In using this device, the frequency of the 10-kc.

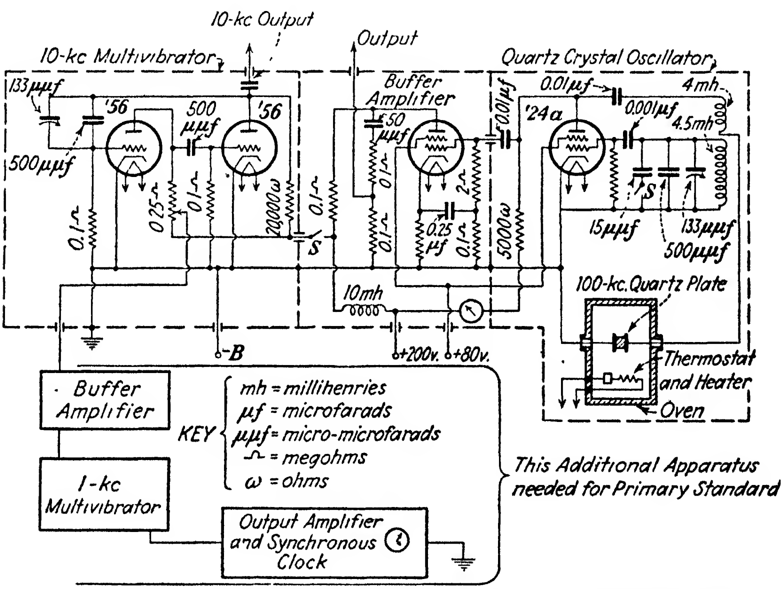


Fig. 8.—Circuit of quartz-crystal oscillator and synchronized multivibrator.

stabilized oscillator is varied slightly until the proper harmonic gives zero beat with the temperature-controlled crystal oscillator having a frequency of 50 or 100 kc. The procedure is then the same as previously described.

The 10-kc. oscillator may be eliminated if the crystal oscillator output is used to control the multivibrator. Clapp[1] found that if tube constants of the multivibrator were made unequal, an

[1] *Proc. I.R.E.*, Vol. 17, No. 2, pp. 252–271, 1929. For multivibrator design see V. J. Andrew, "Adjustment of the Multivibrator for Frequency Division," *Proc. I.R.E.*, Vol. 19, No. 11, p. 1911, November, 1931. Also, F. E. Terman, "Measurements in Radio Engineering," pp. 129–136.

injected frequency equal to a harmonic of the multivibrator will cause the frequency of oscillation of the latter to lock in so that the proper multivibrator harmonic and the injected frequency are always in phase. An arrangement of apparatus carrying out this purpose is shown in Fig. 8. This makes a secondary frequency standard having an accurately controlled 10-kc. output, with integral harmonics thereof. This is a somewhat more complicated setup and requires some skill in preliminary adjustment, but it simplifies the measurement procedure, because frequent checking of the 10-kc. frequency is unnecessary. In most power piezoelectric crystal oscillators, the quartz plate with its electrodes is connected between grid and filament of the associated vacuum tube. In this case, it functions in parallel or antiresonance because of the capacitance and tube-input capacitance, which are in parallel with the equivalent series *LR* and *C* of the quartz crystal itself. In the oscillator of Fig. 8, the quartz crystal is in the plate circuit and functions as a crystal resonator or filter at series resonance of its equivalent series *L* and *C*. In this circuit, the oscillation frequency is much less affected by the constants of the associated circuits and vacuum tubes.[1]

The secondary standard shown in Fig. 8 may be constructed in the laboratory, the usual precautions being observed about maintaining the quartz plate at a constant temperature with a fairly high degree of precision and also maintaining the oscillator at a fairly constant temperature. The test for control of the multivibrator by the crystal oscillator may be easily made. It is first necessary to adjust the multivibrator frequency to approximately 10 kc. with the crystal oscillator shut off. The best method of doing this is to couple the multivibrator fundamental output to a 10-kc. tuned circuit with a vacuum-tube voltmeter connected across the tune-circuit condenser. Maximum deflection, of course, indicates that the multivibrator fundamental has the desired frequency. A radio receiver, preferably incorporating a beat oscillator, is now coupled by means of small coupling coils to the multivibrator output, which can be obtained from the terminal labeled output on the buffer amplifier (Fig. 8). As the receiver tuning is varied, characteristic whistles every 10 kc. will

[1] This piezo oscillator is described in *General Radio Experimenter*, Vol. 2, No. 7, December, 1931. In recent designs the Colpitts type piezo oscillator is used.

indicate that the receiver is picking up the multivibrator harmonics, the whistles being produced by having the receiver beat oscillator turned on. If the variable coupling condenser (133 mmf.) is varied slightly from its setting for 10-kc. fundamental output, a series of varying whistles in the receiver will show that this variable condenser is varying the multivibrator frequency. This condenser is then left in its position for 10 kc., and a steady beat note will probably be heard in the receiver. Next, the quartz-crystal oscillator is turned on by closing a switch in its plate circuit, and the beat note will undergo a transient change and assume another character. If the multivibrator variable condenser then is varied, it will be found that no change in the receiver beat note takes place, showing that the crystal oscillator is holding the multivibrator fundamental and harmonics at a constant frequency despite the variable condenser changes. Varying the tank-circuit variable condenser of the quartz-crystal oscillator will, of course, produce a very slight variation in the multivibrator frequency and hence in the beat-note pitch. It is well to check the crystal oscillator thoroughly with a milliammeter in its plate circuit to see that it oscillates satisfactorily; *i.e.*, it should show a considerable decrease in plate current when the feedback coupling is brought up to the proper value. A telephone receiver should be connected to the buffer amplifier output to make sure that there are no spurious noises or hums in the oscillator and amplifier output; there should be no sound whatever in the telephone receivers. It is best to shield the three units of this equipment as indicated in Fig. 8 to prevent the possibility of feedback.

Radio Stations for Frequency Standards.—The received carrier wave of a radio station whose frequency is carefully controlled may be used in place of the temperature-controlled piezoelectric crystal oscillator. For this purpose, the oscillator and harmonic generator of Fig. 6 or 7 may be used, and the proper harmonic of the 10-kc. output adjusted to zero beat with the radio carrier wave by using the receiver with beat oscillator, as previously described. Broadcast stations are required to maintain their frequency within 50 cycles of the assigned frequency; and with the aid of constant-temperature quartz-crystal controlled transmission, with elaborate frequency deviation meters or checking equipment, they are mostly maintained within 3 to 4 cycles of the assigned frequency. They are on integral multiples of 10 kc., *i.e.*, 700, 710, 720, etc., so that a harmonic of a 10-kc. harmonic generator will fall on one of the broadcast carrier waves. Standard frequency transmissions are made from the U. S. Bureau of Standards Standard-Frequency Service station WWV, on 5, 10, and 15 Mc. They are maintained at these values to an extremely high degree of precision.

Precision of Measurement.—If it is assumed that steps will be taken to adjust to exact zero beat by the methods previously described in this chapter, measurements can be made to a high degree of precision, probably within 1 or 2 cycles of the reference standard. With the new low-temperature-coefficient quartz crystals, the coefficient is about 2 parts per million per degree centigrade. Hence if the temperature is held constant within 0.1°, the frequency deviation is found by actual measurement to be less than 1 part in 3 to 5 million over a considerable period of time.

5. Primary and Secondary Frequency Standards and Measuring Apparatus.—The design and operation of primary frequency standards requires a specialized technique and equipment. It is not intended to go into a detailed description of present-day equipment of this sort and the manipulation thereof. New developments in this field are being made frequently, so that a type now in use may be obsolete within a space of one or two years. The student may be interested in some of the past and present developments in this field.[1] A primary standard of frequency is fundamentally a high-precision quartz-crystal piezo oscillator driving a synchronous clock, and the frequency is measured by comparing time kept by this clock with standard time. Referring to Fig. 8, the output of the 10-kc. multivibrator (which is driven by the 100-kc. crystal oscillator) will drive a 1-kc. multivibrator by locking with its tenth harmonic. The 1-kc. output of this second harmonic is amplified and will then

[1] Hull, L. M., and J. K. Clapp, *Proc. I.R.E.*, Vol. 17, No. 2, p. 252, February, 1929; Clapp, J. K., *Proc. I.R.E.*, Vol. 18, p. 2003, December, 1930; Hall, E. L., V. E. Heaton, and E. G. Lapham, *Nat. Bur. Standards Jour. Res.*, Vol. 14, p. 85, 1935; Lapham, E. G., *Proc. I.R.E.*, Vol. 24, No. 11, p. 1495, November, 1936; "Measurement of Frequency at Radio Frequencies," *Gen. Radio Co. Bull.*; Andrew, V. J., *Communication and Broadcast Engineering*, Vol. 2, p. 21, May, 1935.

drive an electric clock designed to operate on a frequency of 1 kc. and keep standard time. With the additional multivibrator stage and 1-kc. clock indicated schematically, the system shown in Fig. 8 becomes a primary standard.

A secondary frequency standard is merely a precision constant-temperature quartz-crystal piezo oscillator driving a multivibrator or harmonic amplifier producing a series of highly accurate-frequency harmonic frequencies. The apparatus shown in detail in Fig. 8 constitutes a secondary standard.[1]

A convenient but somewhat lessened precision secondary standard is provided in the heterodyne frequency meter and calibrator recently announced by a well-known manufacturer.[2] This instrument is unique and deserves a brief description. It comprises a heterodyne frequency meter (see Art. 3), a piezo quartz-crystal oscillator for checking the calibration of the condenser dial of the former, and a sensitive detector amplifier in a single suitable cabinet or panel mounting. The heterodyne frequency meter has a range of fundamental frequency oscillation of 10,000 to 20,000 kc., using 10 coils in its oscillating or tank circuit. With each coil the variable tank condenser gives a range of 1,000 kc.; the dial of this condenser, which is of the straight-line frequency type, is calibrated in tenths of a megacycle; and the spaces between are divided into 10 parts. The coils are selected by a dial switch, and indicated switch points are in megacycles (Mc.). Hence, if the coil switch is at 14 and the condenser-dial reading is 0.452, the frequency generated is 14.452 Mc., 14,452 kc. The last digit is estimated. To measure an unknown frequency, the heterodyne frequency meter is tuned to zero beat with the unknown, and the frequency read on the dial. If the unknown frequency is below or above the fundamental frequency range of the instrument zero beat, adjustment is made between a harmonic of the unknown and the fundamental of the instrument or between the fundamentals of the unknown and a harmonic of the instrument, as the case may be. The quartz-crystal oscillator, called the piezoelectric calibrator, built

[1] The circuit, as shown with some refinements of the crystal temperature-control equipment (not shown), was kindly furnished by L. M. Craft, Collins Radio Company.

[2] General Radio Company, Cambridge, Mass. For full description see *General Radio Experimenter*, Vol. 11, Nos. 4 and 5, pp. 9–11, 1936.

into the instrument, has a low-temperature-coefficient quartz crystal; hence no temperature control is provided. The frequency of the piezoelectric calibrator is 1,000 kc. Figure 9 is a simple block diagram of the apparatus. To check the calibration of the condenser for any coil, the calibrator is turned on, and the condenser dial varied about its zero setting. As an example, with the 10-Mc. coil switched in, zero beat will be obtained between the tenth harmonic of the calibrator frequency and the heterodyne frequency meter at the zero setting of the condenser dial if its calibration is accurate and it is generating 10 Mc. at this setting. When the condenser-dial reading is set at 1.0, indicating a generated frequency of 11 Mc., zero beat should be

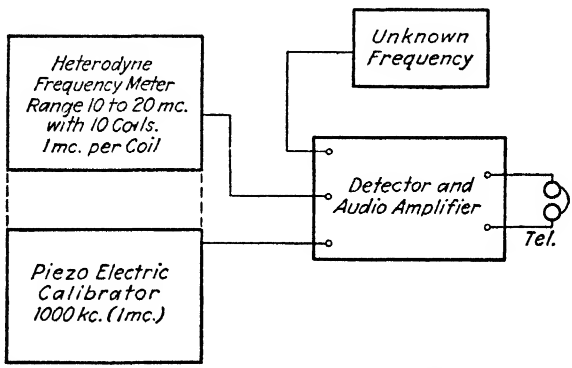


FIG. 9.—Diagram illustrating principle of the heterodyne frequency meter and piezoelectric calibrator.

obtained with the eleventh harmonic of the calibrator frequency if the calibration of the dial is accurate. If not, this reading will be slightly different from 1.0 for zero beat. The condenser-scale readings extend to 1.1 to provide for this possibility. This same *bracketing check* may be obtained for each coil-switch position. Additional calibration points may be obtained for certain settings of the condenser dial between the extreme positions of 0.1 and 1.0. These are the result of certain harmonics of the heterodyne frequency meter zero beating with certain harmonics of the calibrator for certain positions of the condenser dial. As an example, when the condenser dial is set at 0.5 with coil switch at 10, the frequency should be 10.5 Mc. with harmonics equal to 21, 42, 63, etc., Mc. These frequencies are also the twenty-first, ...-second, etc., harmonics of the piezoelectric calibrator frequency. Other intermediate zero beating points may be found. The beat notes between these harmonics will be of lower intensity than the beat notes between the heterodyne frequency-meter fundamental and calibrator tenth to twentieth harmonics, but many of the zero beats of harmonics may be used to check the scale calibration of the heterodyne frequency-meter condenser dial. It is claimed that this instrument gives an over-all error of measurement not exceeding 0.015 per cent at 10 Mc. and 0.01 per cent at 20 Mc. Checking points are obtained at multiples of ½, ⅓, ¼, and ⅕ Mc. over the dial range.

6. Measurement of Frequency with "Lecher Wires." *Principle of Method.*—It has been known for many years that stationary waves could be produced on wires in space and on parallel wires and that when conditions of excitation and wire length were right, nodes and antinodes of the potential and current on the wires were present. The various facts concerning the relations between frequency, wire length, velocity of wave propagation, and the electrical constants of the wires have been developed, measured, and analyzed by many famous investigators such as Heaviside, Hertz, Blondlot, Lecher, M. Abraham, MacDonald, Diesselhorst, and possibly others.[1] Therefore, the theoretical proof of the fact that the length of a wave may be measured by taking twice the distance between successive nodes will not be touched upon. A straight wire may be used on which to produce the waves, but it is more convenient to use two small wires in parallel and terminating on flat metal end plates; this arrangement is commonly called *Lecher wires*. This particular arrangement is due to Southworth[2] and is shown in Fig. 10. Two parallel wires about 2.5 to 4 cm. apart are mounted as shown. The plate *A* is divided so that a thermo milliammeter is connected across the gap between the edges as shown. At the other end, the wires pass through small holes in metal plate *B* and then over

[1] LECHER, E., "Eine Studie uber electrische Resonanzerscheinungen," *Wiedmanns Ann.*, Vol. 41, p. 850, 1888; BLONDLOT, R., "Sur un nouveau procédé pour transmettre des ondulations électriques le long de fils métalliques," *Compt. rend.*, Vol. 114, p. 283, 1892. A historical account may be found in "Principles of Electric Wave Telegraphy and Telephony," 3d ed., pp. 362–368, by J. A. Fleming. See also Hund, August, "Theory of Determination of Ultra Radio Frequencies by Standing Waves on Wires," *U. S. Bur. Standards, Sci. Paper* 491.

[2] *Radio Rev.*, Vol. 1, p. 583, 1919–1920; by permission of the publishers.

pulleys to a spool wound up with a spring, or to weights. This aids in changing the length of the active portion between the plates. A circuit driver is magnetically coupled at *A* or *B*. The large screen or shield shown in the figure is necessary to prevent the creation of standing waves on the wires to the left of plate *B* which, owing to coupling with wires between *A* and *B*, would cause a serious error in measurement. If reels or spools are provided so that the excess length of wires to the left of plate *B* is wound thereon, the large screen is unnecessary. When the frequency of the circuit driver is increased to a certain value,

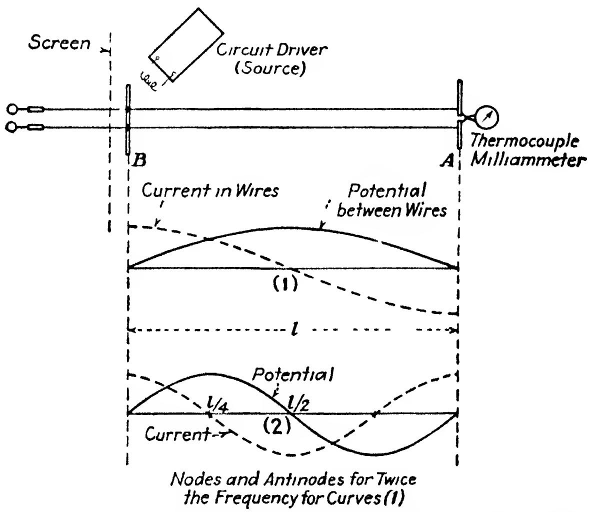


FIG. 10.—Southworth's Lecher-wire system for wave-length measurement.

the thermo milliammeter will show a sharp deflection and peak reading. When this occurs, there exist current antinodes (or points of maximum current) at the plates and a potential node on the wires halfway between the plates (curves 1). As has been proved by the investigators referred to, the distance between the plates for the condition of maximum deflection of the milliammeter is equal to one-half the wave length of the stationary wave on the wires. This assumes, of course, that the wires are fairly small, say No. 26 A.W.G.; are spaced as specified above; and have fairly good conductivity; copper is desirable.

The distance between plates may now be varied, and it will be noted that the *resonant length* is sharply defined. If the frequency of the circuit driver is doubled, it will be found that an additional potential node exists at the center (see curves 2, Fig. 10) where formerly potential nodes existed only at the plates. It is therefore important to know whether the length between the plates is one-half the wave length of the exciting source or some even submultiple of it. A quarter-wave-length standing wave can be induced on the wires only if the wires are open at one end (one plate removed). This makes it possible to have a potential antinode at the open end and a node at the plate at the other.

The form of Lecher-wire system having one end of the parallel wires open is preferable for close tuning, since it is not necessary to change the length of the parallel wires. A very practical measuring setup of this type is described by Takagishi[1] in his

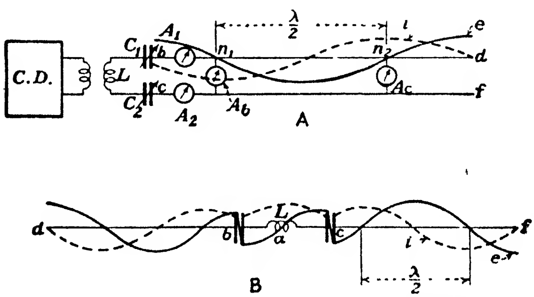


FIG. 11.—Conventional Lecher-wire system for wave-length (and frequency) measurement.

excellent paper on the analysis of possible errors of measurement with such a system.

The open-line measuring system is shown in Diagram A, Fig. 11. Two similar variable condensers C_1 and C_2 and coupling coil L are connected to one end of the wires as shown. C_1 and C_2 are varied until ammeters A_1 and A_2 indicate maximum current, indicating that the parallel-wire system is in resonance with the frequency of the circuit driver. The current readings should be the same, and two ammeters are preferably used so as to keep the system balanced in impedance. When the system is in resonance, the current and potential distribution between the wires may be somewhat as shown by the sine-wave curves of Diagram A. The longest wave length to which it will resonate

[1] *Proc. I.R.E.*, Vol. 18, No. 3, pp. 513–532, 1930.

will be that which provides only one node of potential in the neighborhood of points b, c, so that $bd = \lambda/4$. In the diagram, the exciting frequency is so high that stationary waves are produced which are sufficiently short to produce two potential nodes n_1 and n_2. The system may be in resonance with one or more potential nodes between the condensers C_1 and C_2 and the open end. The locations of the potential nodes will also be the locations of current antinodes. These points are located by placing a metallic link or bridge across the two wires and moving it along the wires until the current flow in the bridge is a maximum. The position for maximum current is the position of the node of potential and antinode, or loop of current. The bridge should consist of a small thermomilliammeter taking not more than 100 milliamp. and having two heavy short copper wires or rods attached to its terminals with which to bridge across the two parallel Lecher wires. It will be noted that when the bridge is placed across the wires at a point a little off resonance, it changes the readings of A_1 and A_2 and that as the bridge is moved through resonance the readings of A_1 and A_2 and of the milliammeter in the bridge pass through a maximum simultaneously. The conditions under which maximum current does not occur simultaneously in A_1 and A_2 are analyzed and discussed by Takagishi in the paper referred to. When the nodes n_1 and n_2, and others for still shorter waves, have been carefully located and marked, the distance between them and twice the distance between the last one (n_2, Fig. 11) and the open end may be averaged to obtain a more accurate measurement of half the wave length, after which the frequency may be determined, using the well-known relation. Diagram B of Fig. 11 is drawn to show how the potentials and currents may be distributed over the entire system. For clearness the two wires are drawn in line as shown, similar to a Hertzian lineal oscillator. If L is very small, and C_1 and C_2 very large, the fundamental wave length to which the wires will resonate is approximately four times the length ad of the parallel line. For the case of the fundamental, one potential node exists at a. The line will also resonate to the three, five, seven times and higher odd multiples of the fundamental frequency, corresponding to the above-described fundamental wave length. To find and measure distances between nodes it is, of course, necessary to excite the parallel-wire system at an odd multiple of the fundamental frequency. J. M. Miller[1] has given an excellent analysis of the simple parallel-wire line potential and current relations, proving the odd-multiple resonance characteristics. Returning to Diagram A, Fig. 11, if C_1 and C_2 are reduced in value, the fundamental is shortened to a lower limit of one-half the original value when C_1, $C_2 = 0$; this is the well-known shortening effect of a series condenser.[2] Thus this form of Lecher or parallel-wire system has a range of measurements without a change of length of wires which may be estimated. It will be noted that A_1, A_2 do not necessarily read the greatest current, as they are not always at a current antinode; they would always read the greatest current if placed at the middle point of L. However, as C_1 and C_2 are varied, the maximum readings of A_1, A_2 always indicate when the system is in resonance with an exciting frequency at either the fundamental or an odd multiple thereof

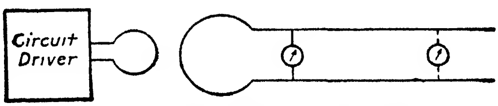


FIG. 12.—Dunmore and Engel Lecher-wire system.

In some cases where the exciting frequency is very high, it will be convenient or desirable to omit the tuning condensers C_1 and C_2 and use a single loop for L_1, as shown in Fig. 12. This system has a fixed fundamental and will resonate with two or more potential nodes to three times the fundamental frequency or to higher odd multiples. This form of Lecher wires is described by F. W. Dunmore and F. H. Engel.[3] They recommend that the thermo milliammeter in the bridge across the parallel wires be shunted with a No. 14 copper wire so as to reduce the resistance of the bridge, thus making the resonance indication sharper. They used the Lecher wires separated about 4 cm., whereas Takagishi used a separation of 40 cm. It is better not to exceed a separation of 10 cm., to avoid errors due to capacitance effects of the wires with surrounding objects, etc. For the setup of

[1] *Proc. I.R.E.*, Vol. 7, No. 3, pp. 300–326, 1919.

[2] See J. M. Miller's paper, *loc. cit.;* or J. H. Morecroft, "Principles of Radio Communication," 2d ed., p. 879.

[3] *Proc. I.R.E.*, Vol. 11, No. 5, p. 467.

Fig. 12, it is necessary to observe resonance by the reactions on the oscillator-load current, plate current, or grid current.

An interesting description of an earlier form of Lecher wires is given in "Wireless Telegraphy," by J. Zenneck.[1] One of the formulas for the velocity of propagation of the waves that has been developed by early investigators is given in the above-mentioned text on page 110. The formula is

$$V = \sqrt{\frac{1}{C_0L_0}}\left[1 - \frac{1}{8}\left(\frac{R_0}{\omega L_0}\right)^2\right] = \text{approx. } 3 \times 10^{10}\left[1 - \frac{1}{8}\left(\frac{R_0}{\omega L_0}\right)^2\right] \text{ cm./sec.,}$$

where R_0, L_0, and C_0 are resistance, inductance, and capacitance per unit length, respectively. The term $(R/\omega L_0)^2$ is usually small compared to one, and V reduces to

$$V = \sqrt{\frac{1}{C_0L_0}} = 3 \times 10^{10} \text{ cm./sec.,} \tag{2}$$

as given in approximate analyses. The student may use this equation to determine the effective resistance per unit length by increasing the wave length, calculating L_0, and using the foregoing equations.

There is an inherent source of error in the measurement of frequency by means of Lecher wires, due to the fact that the velocity of wave propagation along the length of the wires is decreased on account of the resistance of the wires. The correction needed to allow for this effect has been derived by Hund.[2] He points out that the distance l between nodes is equal to one-half the wave length to a high degree of accuracy. Hence the frequency is

$$f = \frac{v}{\lambda} = \frac{v}{2l},$$

where v is the actual velocity of propagation of the waves. The velocity of light is

$$v_0 = 2.998 \times 10^{10} \text{ cm./sec.;}$$

but v being less is

$$v = v_0(1 - \Delta);$$

[1] Pp. 110, 111.

[2] Hund, August, *Proc. I.R.E.*, Vol. 12, No. 6, pp. 817, 821, 1924.

hence

$$f = \frac{v_0}{2l}(1 - \Delta).$$

Substituting the value of v_0,

$$f = \frac{1.4991 \times 10^5}{l}(1 - \Delta).$$

The expression for Δ is given as

$$\Delta = \frac{\sqrt{r_0}}{(8 \log_\epsilon B)\sqrt{\omega\left[1 - \left(\frac{d}{a}\right)^2\right]}}.$$

a is the spacing of the parallel wires, and d is the wire diameter.

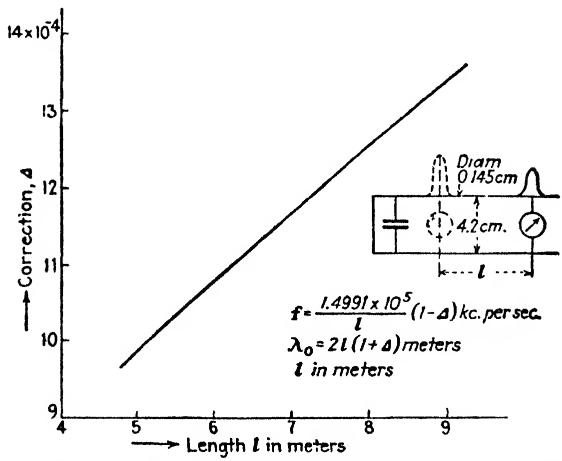


FIG. 13.—Correction curve for wave-length measurement with Lecher wires. (*After Hund.*)

r_0 is the direct-current resistance per centimeter length of the double line, expressed in c.g.s. electromagnetic units (10^9 e.m.u. = 1 ohm).

$$\omega = 2\pi f;$$
$$B = \frac{2a}{d},$$

when the spacing is large compared to the diameter. Figure 13, reproduced from the paper referred to, shows the variation of the correction factor for the parallel-wire system sketched on the curve sheet. The results show that if the correction is ignored; the measured values of l would give frequencies that are about 0.1 per cent too high. This result has been substantiated by experiment by the author of the paper.

Degree of Precision Obtainable, Precautions.—It will be found that the precision of adjustment for resonance is equal to that for a low-decrement tuned circuit, such as a high-grade frequency meter with coupled resonance indicator. However, it is well known that a high degree of precision is not obtainable unless elaborate precautions are taken and corrections made. According to the theory, the wires must be small, but small size results in higher resistance of the wires, changing the potential and current distribution from the ideal sinusoidal modes. At the higher frequencies, neighboring capacitances affect the distribution so as to reduce the accuracy of determination. An accuracy of 0.2 per cent is attained with longer waves (lower frequencies) and using corrections. The use of the plates at the ends terminates the two-wire line definitely; using a wire to connect across the ends would require a correction factor. Coupling the source to the loop or coil at one end introduces a small error due to distributed mutual inductance, and coupling it electrostatically to the center introduces an error due to neighboring capacitance, all affecting the current and potential distribution. With the Lecher wires open at one end and excited to produce several nodes so that a mean of the distances between the nodes may be obtained, Dunmore and Engel report a variation in measurements of only 1 mm. in 4.5 meters, and a high degree of precision readily attained. It is advisable to check the indications by calibrating the exciting circuit driver with the harmonic of a standard piezo oscillator. With a Lecher-wire system so checked, it is certainly very convenient to calibrate ultra-high-frequency wavemeters, etc.

Lecher-wire Measurement of Ultra-high Frequencies.—It was pointed out by Hoag[1] that at wave lengths less than 1 meter the usual method of determining nodes or antinodes on the parallel wires cannot be made sufficiently precise. In the paper referred to, a newly developed method of wave length determination is given, making it possible to measure wave length or fre-

[1] *Proc. I.R.E.*, Vol. 21, No. 1, p. 29, January, 1933.

quency to three significant figures. A later method is described by King[1] and will be briefly explained. If an oscillator is loosely coupled magnetically to the parallel Lecher wires, and the latter adjusted to a resonant length, it will be found that the current in a resonance indicator connected across one end of the parallel wires varies with the position of the oscillator, as the latter is moved in a direction parallel to the wires. The response of the resonance indicator is maximum when the oscillator is at a certain position, and at ¼ wave length from this point it will be a minimum. Points of minimum response of the resonance indicator

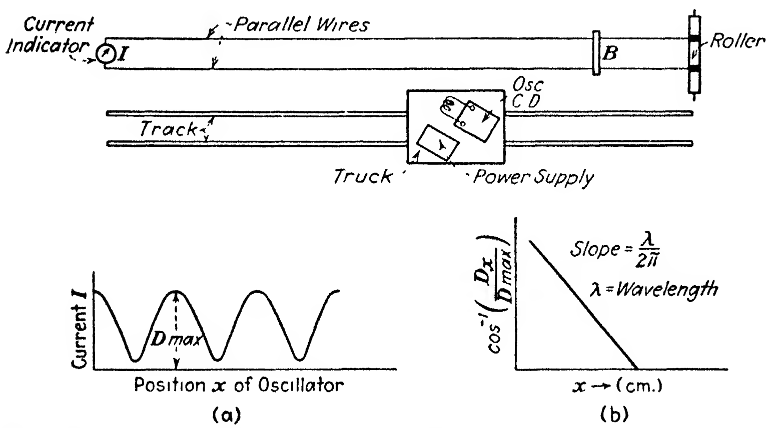


FIG. 13a.—Lecher wires and movable exciting oscillator for measurement of ultra-high radio frequencies, or ultra-short wave lengths.

are then ½ wave length apart. In Fig. 13a is shown the test apparatus. The oscillator, which is not too close to the parallel wires, is mounted on a small truck which may be moved along on a track to keep the former at a uniform distance from the wires. For a certain oscillator frequency, the length of the parallel wires is made a convenient value by taking up the unused wire on a metal roller mounted on an ordinary window-shade roller. A copper or brass bridge *B* is placed across the wires somewhere near the roller, as shown, and moved until maximum current is shown on the current indicator *I*. This tunes the parallel wires to resonance. The oscillator is now moved to a point where the deflection is a minimum, and from this point the oscillator is moved about a centimeter at a time, and the

[1] *Proc. I.R.E.*, Vol. 21, No. 8, August, 1933; see especially p. 1172.

comparative current indications are read. The resonance indicator must be calibrated to read comparative currents. If it is a thermocouple and galvanometer, or a rectifier and direct-current milliammeter, a calibration curve for current indications should be made. If the currents obtained for different positions of the oscillator are plotted, curve (*a*) of Fig. 13*a* will be obtained. Now, if the arc cosine of the ratio D_x/D_{max} is plotted against the position x, where D_x is the current at the corresponding position x, the slope of a straight line drawn through the points is $\lambda/2\pi$, where λ is the wave length of the source, or

$$\lambda = \frac{2\pi x}{\cos^{-1} D_x}.$$

This is shown by curve (*b*) of Fig. 13*a*. High accuracy is claimed for this method of measurement, with probable error of less than 2 or 3 per cent.

7. Determination of Frequency with Cathode-ray Tube. *Description of Method and Theory.*—L. M. Hull in 1921[1] described a very interesting method of comparing frequencies by applying potentials at the two frequencies to the deflector plates of a cathode-ray oscillograph tube. When the two frequencies are exactly alike, or when one is an exact multiple of the other, stationary Lissajou's figures are formed on the cathode-ray screen. If the relations stated above are not exact, there is a movement and change in the figure produced, the changes occurring more rapidly as the frequency relations separate. A simple analysis of observed phenomena will be made, following an explanation of the apparatus and circuits necessary.

Figure 14 shows the circuits and deflector platés of the cathode-ray tube. Circuit drivers 1 and 2 generate e.m.fs. of variable frequencies which are impressed upon the plates as shown, the plates *a* and *b* being grounded. The alternating voltage of oscillator *C.D.*1 will cause the spot *S* to move back and forth, tracing a horizontal line on the luminous screen of the tube. Voltage *C.D.*2 would cause a vertical line when acting alone. When both voltages at certain frequencies which are multiples of each other act simultaneously, complex Lissajous figures are formed. By means of these figures and their behavior, the fre-

[1] "The Cathode-ray Oscillograph and Its Application to Radio Work," *Proc. I.R.E.*, Vol. 9, No. 2, p. 130, 1921.

quency of $C.D.1$ may be compared to that of $C.D.2$. The following is a brief analysis of some of the conditions that may be produced to compare frequencies.

If the harmonic voltages of the circuit drivers are $A \sin \omega t$ and $B \sin (\omega t + \theta)$, the spot will be deflected proportionally to these e.m.fs., so that

$$x = A \sin \omega t, \tag{3}$$
$$y = B \sin (\omega t + \theta). \tag{4}$$

When the two e.m.fs. are exactly in phase and of the same fre-

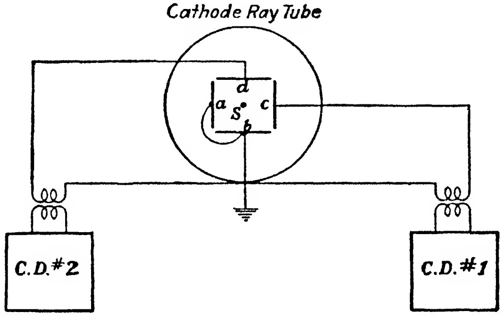


FIG. 14.—Elementary circuit for frequency measurement with cathode-ray tube.

quency, $\theta = 0$. Dividing the two equations,

$$\frac{x}{y} = \frac{A}{B}, \text{ or } y = \frac{B}{A}x.$$

This is a straight line of slope B/A. When $B = A$, the line is at an angle of 45 deg. to the horizontal. When $\theta = \pi/2$,

$$y = B \cos \omega t;$$

since

$$\cos \omega t = \sqrt{1 - \sin^2 \omega t}$$

and

$$\sin \omega t = \frac{x}{A} \text{ [from Eq. (3)]},$$

there results

$$y = B\sqrt{1 - \frac{x^2}{A^2}},$$

or

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1.$$

This is the equation of an ellipse of major and minor axes A and B in length. When $\theta = \pi$,

$$y = -\frac{B}{A}x.$$

Hence if one of the e.m.fs. changes its frequency slightly, so that it is soon opposite in phase to the other considered as steady, the figure on the cathode-ray tube screen will change from a straight line in one direction to an ellipse whose major and minor axes are along x and y, showing that the two e.m.fs. have become 90 deg. out of phase, as shown in Fig. 15. When the figure changes to a straight line at right angles to the first straight line, the e.m.fs. are 180 deg. out of phase. When the line again coincides with the line first noted (of slope $+B/A$), a complete cycle of phase change has been passed through. The phase differences considered above are one-fourth period apart. For small phase differences the nature of the figure may be determined. Rayleigh has given the equation of motion for any value of θ.[1] His equation may be derived as follows: Repeating the equations of motion of the spot,

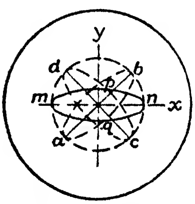


FIG. 15.—Ellipse patterns of Lissajou figures formed by two equal frequencies (see Fig. 14).

$$x = A \sin \omega t,$$
$$y = B \sin (\omega t + \theta),$$

θ being the phase angle of B. Now,

$$y = B(\sin \omega t \cos \theta + \cos \omega t \sin \theta)$$
$$= B(\sin \omega t \cos \theta + \sqrt{1 - \sin^2 \omega t}\,(\sin \theta),$$

since

$$\sin \omega t = \frac{x}{A},$$

$$\frac{y}{B} = \frac{x}{A} \cos \theta + \sqrt{1 - \frac{x^2}{A^2}}\,(\sin \theta).$$

[1] RAYLEIGH, LORD, "Theory of Sound," Vol. I, p. 26.

Squaring and letting

$$\cos^2 \theta + \sin^2 \theta = 1,$$

there results the form

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} - \frac{2xy}{AB} \cos \theta = \sin^2 \theta. \tag{5}$$

This is the general equation for the ellipses that may appear on the screen. The straight lines discussed above result from this equation when $\theta = 0$ or π. The path of the beam takes the forms shown in Fig. 15. For $\theta = 0$ the line *ab* will appear, and when θ has a small value the narrow ellipse with major axis on *ab* will form and gradually change to the ellipse *mpnq* (or circle *bdac*) with major and minor axes on the x and y axes when $\theta = \pi/2$. For $\theta = \pi$, *cd* appears; and for $\theta = 2\pi$, *ab* again appears. Therefore, if the two e.m.fs. are not quite in step over a period of time, a complete cycle of change in the figure on the screen shows one complete cycle lost or gained, and the number of complete changes per second is the difference in the two frequencies It is easy, then, to compare the average frequencies very accurately over a period of time, provided the beats are sufficiently slow so as to produce no additional pattern distortion. If the two e.m.fs. applied to the oscillograph deflector plates have frequencies that are integral multiples of each other, such as 1:3, 1:6, or 1:12, complex Lissajous figures are produced, and these figures change shape as the phase of one e.m.f. changes with respect to the other.[1] It is possible to compare frequencies by observing the shape and movement of the figures,[2] but space will be given to a method described by Kipping.[3]

If two e.m.fs. equal in frequency and amplitude but 90 deg. out of time phase with each other are applied to the two pairs of cathode-ray oscilloscope electrodes, the trace will be a circle, as previously explained. In Fig. 16, by applying an e.m.f. of frequency f_1 to a simple *RC* circuit, as shown, the two out-of-phase potentials will be produced across *R* and *C*. This is a

[1] WOOD, A. B., "Textbook of Sound," pp. 21–23, gives general mathematical relations and Lissajous figures.

[2] HAZEN and KENYON, "Primary Radio Standard—by Use of Cathode Ray Oscillograph," *U. S. Bur. Standards, Sci. Paper* 489.

[3] *Elec. Communication*, Vol. 3, No. 78, 1924.

phase-splitting circuit. A pair of plates P_2P_2' is connected across R, and the second pair P_1P_1' is connected across C. By varying R and C until the potentials across each are equal, a circular trace will be obtained. These are deflecting potentials, and the higher the anode potential the less will be the deflection of the electron beam because of its higher velocity. If, then, an e.m.f. E_2 of another frequency f_2 is added in series with the normal direct anode potential, the deflection will fluctuate as E_2 varies in value, and the trace will be a closed figure containing projections or cusps. The manner of inserting E_2 into the anode potential is shown in Fig. 16. The pattern diagrams of the figure

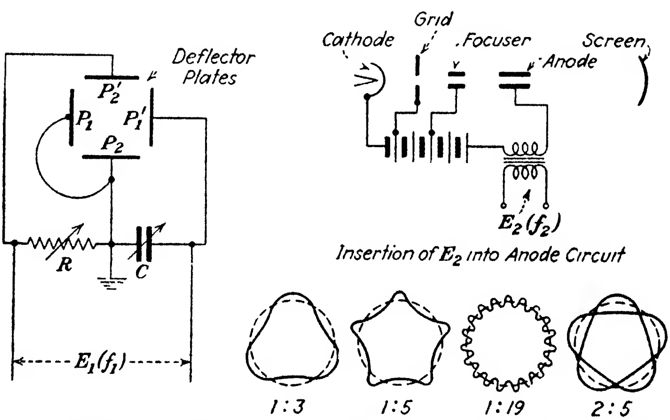


FIG. 16.—Kipping method of comparing frequencies with cathode-ray oscilloscope.

show cathode-beam traces when f_2 is made three, five, and nineteen times the frequency of E_1. When the ratio of f_2 to f_1 is 5:2, the figure is not a simple *gear-wheel* type. In each case, the number of cusps indicate the ratio of f_2 to f_1. If E_2 were made zero, the traces would be circles (shown dotted).

Manipulation.—The manipulation procedure has been suggested in the previous paragraph. Frequencies f_1 and f_2 should be stable so that a stationary trace may be obtained. If one frequency drifts slightly, causing a slow change in phase, the pattern will appear to revolve. If the ratio f_2 to f_1 is not integral, the trace may be very complex. If f_1 is a comparison standard, and f_2 a harmonic thereof produced by a tunable oscillator, the latter may be held exactly at *zero beat* with f_1 by varying the tuning of oscillator producing f_2, so that the gear-wheel trace, or pattern, is maintained stationary. To measure an unknown frequency, the oscillator is first tuned to zero beat with the unknown, and the frequency-scale reading of the oscillator (which is of the interpolation type) is read. Then the oscillator is tuned to the harmonics of the comparison standard f_1 just below and just above the unknown frequency, and the scale readings noted. These positions for integral harmonics will be indicated by the stationary gear-wheel patterns. Then the frequency of the uuknown may be found by interpolation between scale readings for the unknown and the next lower and next higher readings for the harmonics of f_1. It must be remembered that the

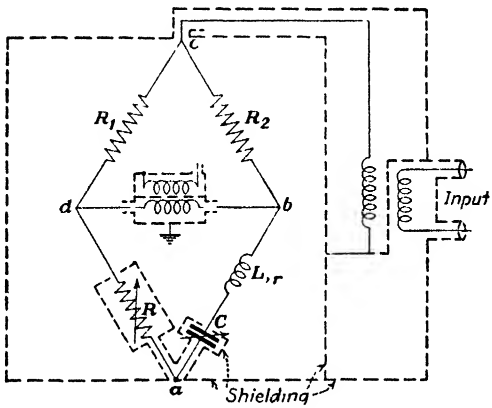


FIG. 17. - Resonance bridge for frequency measurement.

interpolation oscillator producing f_2 must have a linear scale for frequency.

8. Bridge Methods of Frequency Measurement. *The Resonance Bridge.*—For the lower radio frequencies, such as those used in carrier-current telephone circuits, measurements are conveniently made with what is known as a *resonance bridge.* Figure 17 shows how such a bridge is arranged and how it must be properly shielded. The arms R_1 and R_2 are fixed, and R is variable. When the unknown frequency is connected to the input, the capacitance C is adjusted until the arm ab is in resonance, although the inductance may be made variable, if convenient.

The only drop across arm ab is that due to the resistance r of ab and is in phase with the drop across R of arm ad. Then, if R is adjusted so that the resistance drops in the two arms *ab* and *ad* are equal, there is no output across *db*. This, then, is a frequency meter which gives zero response in a resonance indicator instead of maximum response as in the usual type. If the input e.m.f. contains harmonics of its fundamental frequency, the total residue of harmonic components is present across *db* after the fundamental has been balanced out. When it is used to determine this total residue, it is known as a *Belfils' bridge.* In frequency measurements, it is necessary to use electrostatically shielded input and output transformers for good precision. It is necessary to shield R from C and the other ele-

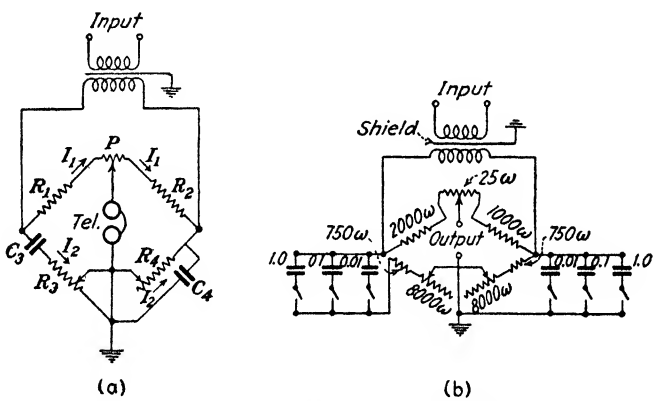


FIG. 18.—Wien Bridge used for measuring frequency.

ments due to varying-shunt admittances caused by changing R and C. A good discussion of a somewhat different shielding system for this bridge is given by J. G. Ferguson.[1]

For frequencies below 100 kc. this bridge is capable of high precision, as it is an easy matter to connect a detector amplifier to the output, making the balance very sensitive. It is, however, not so desirable in the measurement of e.m.fs. containing harmonics and cannot be used at all to measure the frequency of a harmonic component of an e.m.f. containing several harmonics such as that from a piezo oscillator. For the latter case the removal of one harmonic component by the *trap* effect of the bridge would change the output reading probably a very small amount.

[1] *Bell System Tech. Jour.,* Vol. 8, p. 560, July, 1929.

The Wien Bridge.—The Wien Bridge may also be used for measuring frequency, and in such work it has the advantage that it contains only resistance and capacitance units. The bridge circuit is shown in Fig. 18; the schematic arrangement, in Diagram (*a*). At balance, no current flows in the telephone receiver, and the currents I_1 and I_2 have paths as shown. The equations for potential drops are

$$I_1R_1 = I_2\left(R_3 - \frac{j}{\omega C_3}\right),$$

$$I_1R_2 = I_2\left(\frac{R_4}{1 + j\omega C_4R_4}\right).$$

The impedance of R_4 and C_4 in parallel is $\left(\frac{R_4}{1 + j\omega C_4R_4}\right)$. Dividing these two equations into each other,

$$R_4\frac{R_1}{R_2} = R_3 + \frac{R_4C_4}{C_3} + j\left(\omega R_3R_4C_4 - \frac{1}{\omega C_3}\right)$$

results. Equating real terms,

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} + \frac{C_4}{C_3}; \tag{6}$$

and from the imaginary terms,

$$\omega^2 = \frac{1}{R_3R_4C_3C_4}. \tag{7}$$

Now, if $R_3 = R_4 = R$, and $C_3 = C_4 = C$, Eq. (7) is simplified, and

$$f = \frac{1}{2\pi RC}, \tag{8}$$

since $\omega = 2\pi f$. However, in order to make this possible, Eq. (6) must also be satisfied, and this means that $R_1/R_2 = 2$, and a balance is always possible for some suitable frequency f when $R_3 = R_4$ and $C_3 = C_4$.

For frequency measurement, R_3 and R_4, equal variable resistances, are varied simultaneously until a null balance is obtained. In Diagram (*b*) of the figure, a practical form of bridge circuit is shown.[1] The 2:1 ratio arms are shown, as well as the other ratio arms. To provide increased range, capacitances of different values are simultaneously switched in or out of the circuit by convenient keys or other means. The range is from 20 to 20,000 cycles per second. An electrostatically shielded input transformer should be used; and with sensitive high-impedance telephone receivers, frequencies between 300 and 5,000 cycles may be measured with fair accuracy. With sensitive visual amplifier detectors, a precision of higher than 0.1 per cent is possible throughout a considerable portion of the bridge range. Owing to small inaccuracies of the resistance and capacitance units, the small 25-ohm potientiometer P is useful in perfecting balance but does not affect the accuracy. As in the case of the resonance bridge, this device is limited, to a certain extent, to pure sine-wave forms. The ear can discriminate against harmonics if the telephone receiver tends to resonate on the fundamental rather than the harmonics. With a visual detector, filtering or tuning must be provided in order to make the balance on the fundamental sufficiently sensitive when considerable harmonic components are present.

Other Frequency-measurement Devices.—Among other such apparatus may be mentioned the Hay bridge as developed by Soucy and De F. Bayly[2] and the electron-tube frequency meter of Guarnaschelli and Vecchiacchi.[3] The former has a maximum frequency limit of 5 kc. due to the fact that it has inductance in one of its arms, and the latter is reported to have an upper limit of 10 kc. These devices may be useful in measuring the frequencies of beat notes and in frequency calibration of beat-frequency oscillators. A commercial form of electron-tube frequency meter having a range of 0 to 5,000 cycles is available.[4]

9. Measurement of Frequencies of Damped Currents. *Theory of Damped-current Frequency Measurement.*—A damped oscillating current in any electrical circuit will produce in a coupled circuit an e.m.f. identical in frequency and damping to the

[1] A bridge of this type is manufactured by the General Radio Company. The diagrams were taken from the manufacturer's instruction pamphlet Form 300A.

[2] *Proc. I.R.E.*, Vol. 17, No. 5, May, 1929.

[3] *Proc. I.R.E.*, Vol. 19, No. 4, April, 1931.

[4] General Radio Company, Catalogue *J*, p. 55.

oscillating current. The form of induced e.m.f. in this coupled receiver circuit may be expressed as

$$E_m\epsilon^{-kt}\cos pt,$$

where E_m = maximum voltage.
k = damping coefficient of the induced e.m.f.
p = angular velocity of the harmonic component.
If the coupled circuit has constants R, L, and C, the damped induced e.m.f. will give rise to potential relations expressed as follows:

$$L\frac{di}{dt} + Ri + v = E_m\epsilon^{-kt}\cos pt,$$

where v is the voltage across C of the coupled circuit when there is no reaction on the source.

$$v = \frac{1}{C}\int idt.$$

Solutions and analyses will show what the frequency or frequencies of the current in the coupled circuit are. The coupled circuit will now be considered as a wavemeter or frequency-meter circuit; and it has been shown that when this circuit is in resonance with the exciting frequency $p/2\pi$, the current in the coupled frequency-meter circuit is a maximum. A good abstracted analysis of the solution of the foregoing equation is made by J. H. Morecroft,[1] in which he gives the solution of the equation for the current induced in the coupled circuit (the frequency-meter circuit for the present case) as

$$i = I_1\epsilon^{-kt}\sin(pt + \theta') + I_2\epsilon^{-kt}\sin(\omega t + \phi'),$$

showing that there are two damped components of current of frequencies $p/2\pi$ and $\omega/2\pi$. α is the damping $R/2L$ of the coupled circuit. When $\omega = p$, there is only one frequency and damping. Fleming and Morecroft abstract the classic work of V. Bjerknes[2] on damped-wave resonance and give Bjerknes'

[1] "Principles of Radio Communication," 2d ed., pp. 309–315.

[2] "Electrical Resonance," *Wiedmanns Ann.*, Vol. 55, p. 121, 1895. Also see analysis by J. A. Fleming, "Principles of Electric Wave Telegraphy," 2d ed., pp. 277–284.

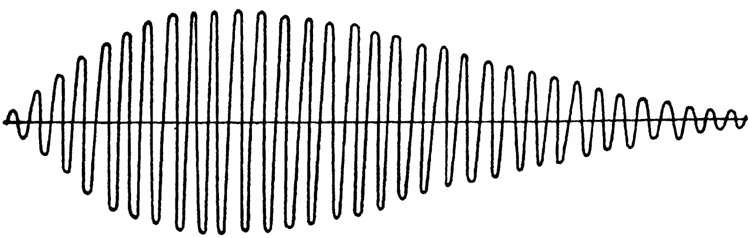
equation for the current as

$$i = mCM \cos (mt + \psi),$$

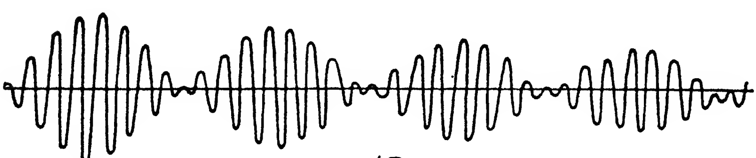
where

$$m = \frac{\omega + p}{2},$$

and M is a complex function. These authors show the form of the currents induced in the frequency meter or other coupled circuit for different relations for ω and p and for k and α. When



$\omega = P$



$\omega \neq P$

FIG. 19.—Damped-wave-current behavior with damped excitation. (*After Morecroft.*)

$\omega \neq \alpha$, the current takes a periodically pulsating form. Bjerknes and others have also evaluated the integrals of the preceding equations for the current and the effective value squared as

$$I^2 = \frac{E^2}{16L^2}\left(\frac{k + \alpha}{k\alpha}\right)\frac{1}{(p - \omega)^2 + (k + \alpha)^2}. \qquad (9)$$

Now, if the capacitance of the coupled frequency meter is varied, its natural frequency is varied, and the value of I induced will vary. Differentiating the foregoing equation with respect to ω and equating to zero gives for the maximum value of I^2

$$\omega = p. \qquad (10)$$

Therefore, maximum current in the frequency meter will indicate the setting for which its natural frequency will equal that of the damped current in the circuit exciting the frequency meter. It is of interest to note the wave forms of the induced currents in the frequency meter for resonance $\omega = p$ and for ω different from p. These are shown in Fig. 19.

Use of the Frequency Meter.—It has just been shown that the frequency meter may be used to measure frequencies of damped currents in a manner exactly similar to that used for undamped currents. It is important to prevent reactions of the frequency-meter circuit on the source, or the familiar double-humped resonance curve results; hence very loose coupling is essential. There are two commonly used methods of indicating maximum current in the frequency-meter circuit: one is by measurement of the current with a thermomilliammeter in the circuit; the other by connecting a crystal detector and telephone receivers in series across the condenser of the frequency meter. The first curve of Fig. 19 is called a *train of waves.* If the circuit in which these damped currents flow is set into the damped oscillation n_a times per second, there will be n_a wave trains per second; n_a may be at an audio-frequency rate. Then the *wave-train frequency,* or *group frequency,* of the oscillatory circuit will be heard in the telephone receiver, and maximum sound will indicate maximum current and resonance. Needless to say, the visual method of resonance indication gives greater precision. It is important to remember that placing the phones and detector in shunt with the frequency-meter condenser changes its calibration, especially at higher frequencies. It should be calibrated under conditions of actual use.

CHAPTER III

ANTENNA MEASUREMENTS

1. Introduction. *Measurable Constants of Antennas.*—In the previous chapters, the measurements of circuit constants, capacitance, inductance, and resistance are made on the basis of what is called *quasi-stationary* current; *i.e.*, the current flowing in the circuit is everywhere the same. In certain cases such as inductance coils or resistance coils, containing nonuniform self-capacitance, a quasi-stationary current is assumed, giving a value from the measurements termed the *apparent* value. Electromagnetic radiating devices known generally as *antennas* or *antennæ* are necessarily electromagnetic energy-absorbing and storing devices and in this way must be considered as possessing certain suitable circuit constants.

Owing to the fact that an antenna may acquire an electrostatic charge, it must have an *electrostatic capacitance;* owing to the current in the antenna during the period of changing charge, energy is stored in the magnetic field produced, and the device must possess *inductance;* and owing to the dissipation of energy in the form of heat in the antenna conductors, and the dielectrics and radiated energy during the charge and discharge cycle, the antenna must possess a total effective *resistance.*

When the current is different in every part of the antenna, and when the capacitance and inductance are distributed as in a transmission line, it is difficult to visualize a single measurable inductance, capacitance, and resistance. However, this may be done, and these measured constants are often valuable for the use of the radio engineer.

General Principles, Equivalent Values.—The maximum current flowing in the base, or current antinode, or "loop," of an antenna[1]

[1] For excellent material on the distribution of currents in antennas and open oscillators, see J. Zenneck, "Wireless Telegraphy," pp. 32–46; J. H. Morecroft, "Principles of Radio Communication," pp. 869–884. For mathematical analysis, see J. A. Fleming, "Principles of Electric Wave Telegraphy and Telephony," 3d ed., Chap. IV, pp. 337–376; also paper by J. M. Miller, *Proc. I.R.E.*, Vol. 7, No. 3, pp. 302–303, 1919.

may be squared and multiplied by a factor R_e to obtain the total power dissipation in the antenna circuit. This value of resistance includes components due to power losses of various kinds in the antenna circuit and the useful radiation-resistance component. Methods of measuring these will be discussed in this chapter.

Any sort of antenna structure or metallic line that possesses distributed circuit constants may exhibit *resonance* or anti-resonance at certain frequencies of excitation. It is not within the scope of this book to present the basic theory of antennas or lines and their related current- and potential-distribution values; however, a brief outline of elementary basic facts will aid in the later measurement-method theorems.

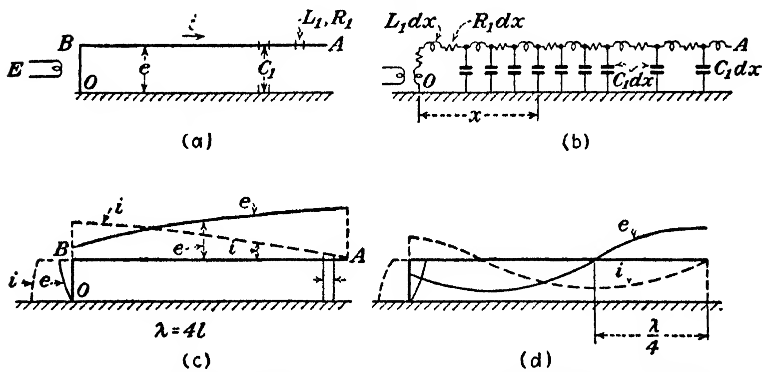


FIG. 1.—Current and potential distribution along a horizontal wire excited at the grounded end.

It has been shown in line and antenna theory that the current and potential distribution along the lineal dimension of the device is a function of the frequency of excitation and of the circuit constants per unit length. At certain critical frequencies, maximum effects occur. At (*a*) in Fig. 1 is shown a single wire line stretched parallel to a flat conducting plane and attached to it at 0. When this *parallel-wire earthed antenna* is magnetically excited by a source E at some frequency, a current will be found to vary along the line. When the frequency of E is increased from zero to a certain value, it will be found that the current will be a maximum at the grounded end or *base* 0, and the current will vary nearly sinusoidally from maximum at O to zero at the open end. At this frequency, the potential between the antenna wire and the earth will be found to increase sinusoidally, as shown in (*c*) (Fig. 1). In (*b*) of this figure, the geometrical ordinary line inductance and resistance are shown broken up into small portions along which the current is assumed to be constant. This length is dx, and the constants are L_1dx and R_1dx. Also the potential is constant along this length and the capacitance is C_1dx. The constants L_1, R_1, and C_1 are *per* unit length of wire in the antenna. This line parallel to earth is exactly equivalent to a parallel line of two metallic wires, and the same differential equations between current change and potential drop along the line hold for each type. When the frequency is at the lowest value which will result in maximum current at O, it is known as the *fundamental frequency* (or wave length) of the antenna. When the resistance is reasonably low, *i.e.*, $R_1dx << \omega L_1dx$ where $\omega = 2\pi f$, Miller[1] and others have deduced simple and convenient relations. At this fundamental frequency of resonance, equivalent values of inductance and capacitance are found to be related such that the fundamental frequency is

$$f = \frac{1}{2\pi\sqrt{L_eC_e}}, \qquad (1)$$

where

$$L_e = \frac{L_0}{2} = \frac{L_1l}{2}.$$

$$C_e = \frac{8}{\pi^2}C_0 = \frac{8}{\pi^2}C_1l. \qquad (2)$$

also,

$$R_e = \frac{R_0}{2} = \frac{R_1l}{2},$$

where l is the length of the antenna wire. From Eqs. (1) and (2) the fundamental frequency is related to L_0 and C_0 by

$$f = \frac{1}{4\sqrt{L_0C_0}}. \qquad (3)$$

Here it is seen that L_0 is the nominal inductance of the antenna wire, or of the wire line in the case of two parallel metallic wires. Also, the term C_0 is the normal geometric capacitance of the

[1] *Loc. cit.*, Introduction.

antenna to earth, and R_0 is the effective resistance of the wire. Therefore an equivalent series circuit may be conceived in which R_e, L_e, and C_e are placed, and at the fundamental of the antenna this series circuit is in resonance; and these equivalent values have a definite relation to the nominal low-frequency circuit constants. It must be remembered here that these relations have been derived on the basis of current at the base O of the antenna which is the *antinode*, or *current loop*, at the fundamental frequency. In addition to this, the current at the antinode is assumed to flow for the derivation of the equivalent constants L_e, C_e. These should then be considered as the values looking into the antenna circuit at the current antinode. This is the point at which most antenna measurements are made, as explained later.

Basic Types of Antennas.—The similarity of the wire parallel to earth to a parallel two-wire line has been pointed out. It is shown in line theory that changing the separation of the wires does not radically alter the fundamental frequency or seriously affect the potential and current distribution because of the reciprocal changes in capacitance and inductance per unit length. Furthermore, placing the wires at an angle with each other or even making this angle 180 deg. does not alter the relations and effects materially. This latter state of affairs is shown in (*a*), Fig. 2. Here OA is the same effective wire length as far as resonant fundamental wave length λ is in Fig. 1. This is the *lineal*, or the commonly called half-wave Hertz, antenna; and the equivalent circuit constants, as previously given for the grounded quarter-wave resonant type, have a similar meaning when considered at the current antinode O. If a wire of length OA is placed vertical and earthed, it is the Marconi quarter-wave type and similar to the type of Fig. 1, and the relations between length l and λ are as in (*b*), Fig. 2. In this case, L_1 and C_1 vary from point to point. The nominal values of L_0 and C_0 measured for the antenna are also different, but nevertheless Eqs. (1), (2), and (3) hold. For certain multiples of the fundamental frequencies, several quarter-wave distributions of potential and current will appear along all these and other types of antennas. It is not within the scope of this book to illustrate and discuss these. As a single illustration, the potential and current distributions are shown at (*d*) (Fig. 1) when the frequency reaches three times the fundamental frequency value. Maximum current will exist at 0 when the frequency is at the fundamental value and at *odd harmonics* thereof. Further discussion of this may be found in various texts.[1] Further consideration will be given this phenomenon in Measurement of Antenna Reactance.

2. Measuring Fundamental and Harmonic Frequencies of Antennas. *General Principle.*—This is a simple measurement procedure as seen in the Introduction. The fundamental resonant frequency of any antenna may be measured by placing a

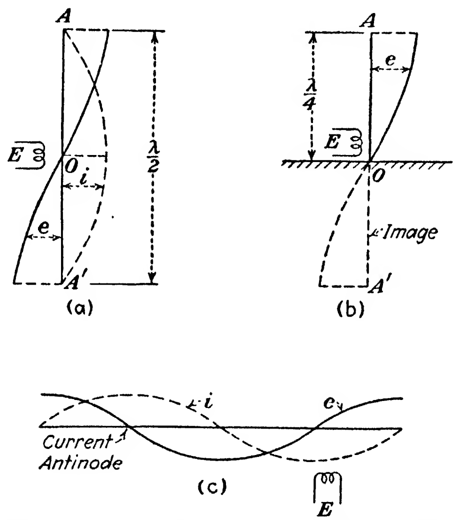


Fig. 2.—Potential and current distribution for half-wave dipole, and for quarter-wave earthed antennas.

current-resonance indicator at the current antinode and varying the frequency of a power-circuit driver (5 watts or more output capacity), which is loosely magnetically coupled to the antenna near its current antinode and potential node, until maximum current is indicated. Then the frequency of the circuit driver may be measured by one of several methods. When tuning a variable-frequency circuit driver to resonance with an open antenna, it will be found that the resonance indication is not so sharp as with small copper conductor coil and air dielectric capacitance circuits. In the case of the open radiating antenna, a radiation-resistance component is effectively introduced in series with the loss resist-

[1] *Op. cit.*, footnote, Introduction, reference to Miller's paper.

ance and the other equivalent series L_e and C_e of the antenna circuit. Hence the Q of the antenna circuit is lower than for *closed* R, L, C circuits. It is desirable, if possible, to vary the circuit-driver (or driving-oscillator) frequency, so that a resonance curve or a portion thereof may be obtained from which true resonance may be found with better precision. The procedure will be explained in the following paragraph.

Procedure for Earthed Antenna.—The test circuit is shown in Fig. 3, in which a thermomilliammeter is placed in series with the antenna at the point, or near the point, where it is earthed. The earth is always at zero potential so that at resonant frequencies a current loop or antinode will always be at the antenna-earth connection. The driving oscillator or circuit driver CD is very loosely magnetically coupled near the base, or *lead-in*, earth

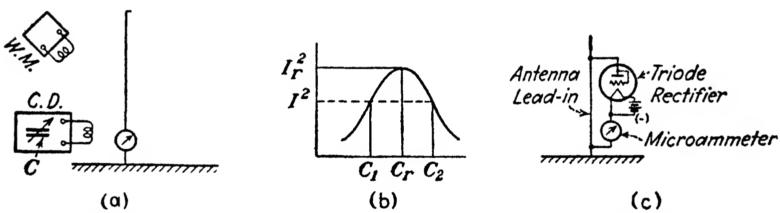


FIG. 3.—Measurement of antenna resonant frequencies, using resonance curves for higher precision.

connection, and its frequency varied until the antenna current-indicating device indicates resonance. Now, for increased precision of resonance setting the circuit driver may be provided with a straight-line frequency type of variable condenser in its tank circuit. Then changes of frequency are nearly directly proportional to the condenser-scale reading. A symmetrical resonance curve may be obtained as shown at (*b*) (Fig. 3), and from readings C_1 and C_2, C_r may be found by taking the mean. In doing this, C_1 and C_2 should be taken on a fairly steep portion of the curve but not too far apart, because it must be remembered that as the frequency is changed the voltage induced into the antenna is also changed, being $E_s = 2\pi f M I_1$, where f is frequency, M mutual inductance, and I_1 the circuit-driver tank current. If the percentage frequency change is small, the change in E_s is small also. It is important to see that the circuit-driver tank current I_1 does not change appreciably from C_1 to C_2 setting, and it may be necessary to control the oscillator-supply voltage or feedback excitation to maintain I_1 constant. In Fig. 4 are shown certain resonance curves obtained in the laboratory. Readings were obtained by varying the driver frequency, because the experiment was to be performed as it is in determining the fundamental of an antenna where the inductance and capacitance are constant. Curves were obtained when the output current of the circuit driver was allowed to decrease with the decreasing capacitance for increasing frequencies and when the driver current

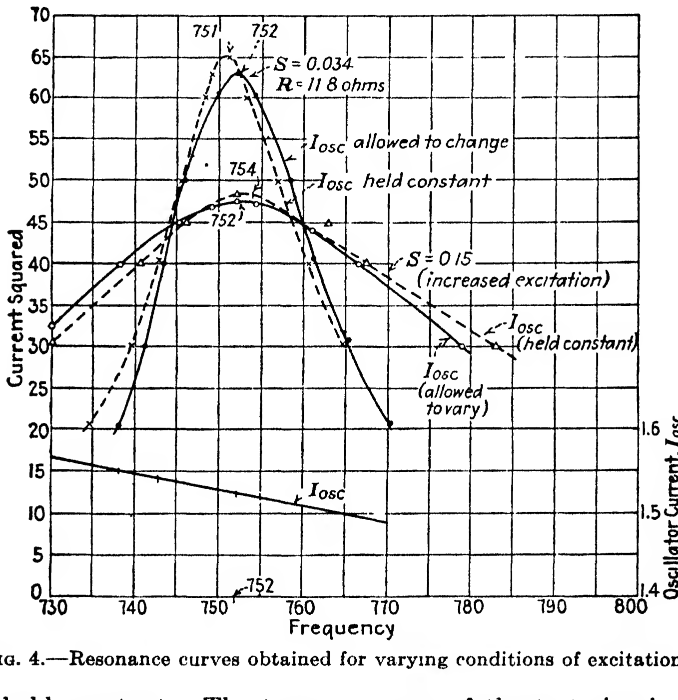


FIG. 4.—Resonance curves obtained for varying conditions of excitation.

was held constant. The true resonance of the test circuit was at 750 kc. to an accuracy of 0.10 per cent. The curves gave results differing from this by as much as 2 kc., or 0.25 per cent. S on the curve sheet of Fig. 4 indicates logarithmic decrement.

The ratio of the measured fundamental wave length λ_0 to its calculated value and the ratio of the length of the antenna including the lead-in and ground wire to the measured value of λ_0 may be determined. In many treatises on potential and current distribution on earthed antennas, both theoretical and experimental ratios between the length of the antenna and its fundamental wave length have been given. The simple approximate relation is given in many textbooks as 1:4 for simple vertical and horizontal antennas. It will be found that this ratio does not hold in practice and that for most types of earthed antennas, the ratio is 1:4+. The ratio given varies from 1:4.2 to 1:5+. When careful measurements of fundamental wave length and of antenna dimensions have been made, it is worth while to check the measured ratio against values given by various writers.[1] It is important to remember that the fundamental frequency or wave length of an earthed antenna depends not only on its length and form but upon the nature of the *earth plate*, comprising the conductivity of the soil at the surface, location of neighboring trees or buildings, diameter of the wire, etc. These and other factors account for the discrepancies in ratio given by different writers and investigators.

After the circuit-driver setting for resonance has been made, its frequency may be measured with an absorption wave meter or with a heterodyne frequency meter if greater precision is required. If it is desired to add no indicating meter in the antenna circuit because of the added resistance or slight resonant frequency-alteration effect, a resonance indicator employing a rectifier and microammeter may be used. These may be connected in series with a small coupling coil laid parallel to the antenna or even connected in shunt with a short length of antenna wire near the base, as shown at *c* (Fig. 3).

Procedure for Hertz, or Dipole, Antenna.—It is desirable to magnetically couple the circuit driver at the center of this type of antenna as shown at (*a*) (Fig. 2). The antenna wire AA' may, of course, be either vertical, and somewhat above the earth at its lower end, or horizontal, as is usually the case. If it is horizontal, and it is desired to measure the frequency with the antenna in position, a scaffold built up to the center of the antenna will not affect its frequency because it is near the potential antinode, and the circuit driver may be placed on this scaffold or other support.

If it is desired to measure frequencies of resonance at the various harmonics of the fundamental, it must be remembered that current antinodes at these frequencies are not always at the center.

[1] FLEMING, J. A., "Principles of Electric Wave Telegraphy," 3d ed., pp. 375–376, also pp. 647–651; MOULLIN, E. B., "Radio Frequency Measurements," p. 400, 1931 ed.: STERLING, G. E., "The Radio Manual," p. 53.

For this procedure the resonance indicator illustrated at (*c*) (Fig. 3) may be connected at the point where the current antinode is supposed to be. Sometimes a small neon-glow tube is placed at the end of the antenna wire, and at resonance on the fundamental or harmonics thereof there is a potential antinode at the end, and the neon-glow tube responds. The second-harmonic resonance on a Hertz antenna is shown at (*c*) (Fig. 2).

Procedure Using Feed Lines.—Various types of feed lines are in use for Hertz and other types of dipole antennas and antenna arrays. In Fig. 5 is shown a popular type of amateur Hertz-antenna feeding system which employs a nonresonant feed line. This is a twisted pair of rubber-insulated wires similar to con-

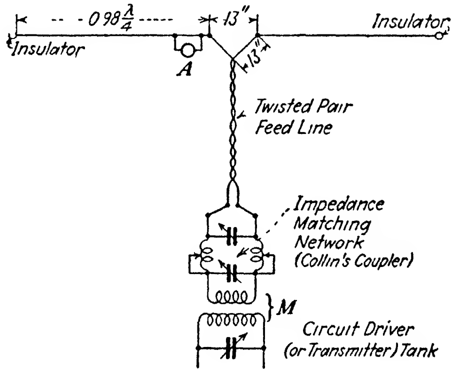


FIG. 5.—Hertz antenna with twisted-pair nonresonant feed line.

ventional No. 14 lamp drop cord. The characteristic impedance of this line is about 75 ohms. For measurement of frequency of the antenna with its feed line connected, it is best to omit the impedance matching network and connect the twisted pair to a small coil coupled to the transmitter tank circuit. An attached resonance indicator is shown at *A*; it should not be connected inside points at which the feed-line terminals are inserted. In this test circuit and procedure, care must be taken to prevent the antenna feed line and capacitance to ground from functioning as a quarter-wave antenna with a series *shortening* condenser. If the feed line is really nonresonant, it may be of any length, and resonance in the antenna will occur at the same frequency. Theoretically the input reactance of an antenna at the antinode of current is zero, and it should present a resistance reaction to the coupled circuit-driver tank when connecting switches are closed; hence the circuit-driver frequency will be the least affected for the condition of resonance in the antenna. This is also true if the circuit-driver tank is merely magnetically coupled to the antenna at the current antinode. To test the condition, a second oscillator, such as a heterodyne frequency meter, may be loosely coupled and tuned to zero beat with the circuit driver, using a radio receiver or detector amplifier. Then, if the circuit-driver frequency is equal to that needed for antenna resonance, disturbance of the zero-beat condition is least upon closing the

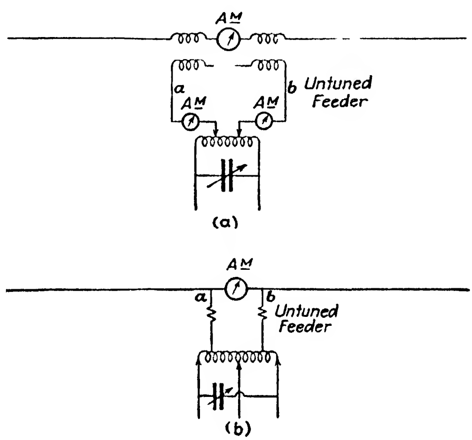


FIG. 6.—Use of "untuned" feeders to excite Hertz antennas for measurement of fundamental wave length

switches connecting the feed line to the circuit-driver tank. This test may also be made by bringing the circuit-driver tank near the antenna (in case it is magnetically coupled at the current antinode).

The untuned feeder system of current feed is shown in Fig. 6. Here the feeder line terminates in a small coupling coil or, better, is attached directly to a portion of the central part of the antenna wire, as shown, so that the impedance at the antenna end between *a* and *b* will be equal to what is known as the *surge*, or *characteristic, impedance* of the feeder line. The current and potential along the feeder line are then constant. For maximum current in the antenna loop and constant feeder-line potential, the frequency of the circuit driver is equal to the fundamental frequency of the antenna. It is necessary to make sure that the feeder line is absolutely untuned to be sure that it is not functioning as a coupled part of the Hertz antenna. It is very convenient to test the untuned feeder line by means of a neon-glow tube connected to each wire. If the glow is the same at any point along the feeder, it has the proper characteristic impedance at its terminals and is untuned with respect to the excitation frequency. To attain this condition, the spacing of the wires, the length of antenna wire between them at the antenna end (*ab*, Fig. 6), and the amount of resistance inserted at *ab* should be varied. It is desirable that the circuit driver or tube transmitter shall be the *push-pull* type, employing two tubes, so that a potential node will exist at the middle point of the tank or driver coil to which the

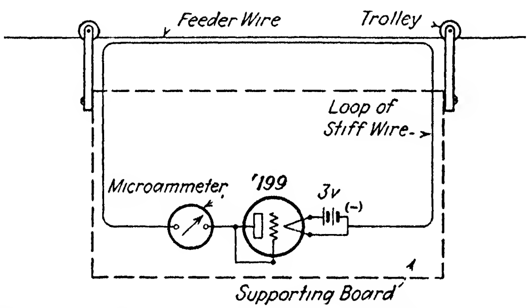


FIG. 7—Movable coupled current indicator to slide along antenna or feeder wire.

feeder-line taps are attached [Fig. 6, (*a*) and (*b*)]. The untuned parallel-wire feed-line condition can also be tested by a sensitive coupled current indicator such as a microammeter and small copper oxide rectifier or even a type 199 tube with two flashlight dry cells used as a rectifier. These are connected in series with a rectangular loop of wire which is held at a constant distance from the feed-line wire by means of small pulleys used as trolley wheels or by sliding hooks. This device is then moved along; and if the current is the same or nearly so at different points along the line, it is properly terminated and is now resonant. The device is shown in Fig. 7. Other arrangements involving thermogalvanometers or vacuum-tube voltmeters are possible.

Method for Voltage-feed Hertz Antennas.—A commonly used Hertz antenna employs a tuned quarter-wave parallel-wire line to provide the exciting potential at one end of the antenna wire. This arrangement is shown in Fig. 8 and is known as a *Zeppelin* *antenna.* An ammeter is often used in the middle of this type of antenna, but it is preferable to place a small milliammeter in shunt with a short length of the wire at its center. If the fundamentals of the antenna and of the parallel feeder line are alike, resonance will be indicated by maximum deflection of the thermoammeters A^M, as the frequency is varied, and this corresponds to the fundamental of the antenna. The fundamental of the parallel feed wire cannot be accurately determined from its dimensions; hence the most reliable method of measuring the fundamental of the Hertz antenna is to have a current indicator at its center and proceed by setting the driver to a certain frequency, a little below the calculated fundamental of the antenna. Then the condensers in the feed line are varied until maximum

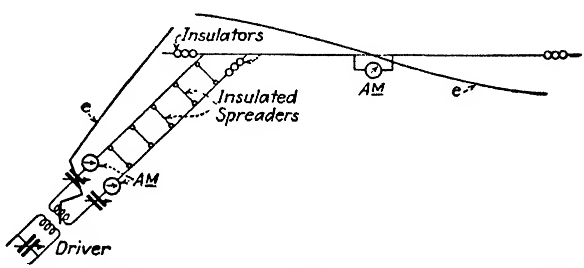


FIG. 8.—Diagram of "Zeppelin" feed Hertz antenna, showing potential distribution.

current in ammeters A^M is obtained and the reading of the thermomilliammeter at the center of the antenna is also noted. Then the driver frequency is changed to another value, and the process repeated. When the frequency of the driver is equal to that of the fundamental of the antenna, the greatest antenna current will be obtained when the condensers in the feed line are adjusted to resonance. The parallel wire of the feed line is usually made longer than a quarter wave length so that when the tuning condensers with their shortening effect are used, the total feed circuit has a quarter-wave potential distribution with the potential antinode at the end, connected to the Hertz antenna. Sometimes it is desired to have a longer feed line (this may be done) and the tuning adjusted so as to have three, five, seven, or any odd number of quarter waves in its potential distribution. When it is not possible or convenient to provide a resonance indicator at the center of the antenna proper, it may be convenient to provide a switch between the end of the antenna and the end of the Zeppelin feeder. With the switch open, the circuit driver or transmitter frequency is set at a value equal to the fundamental resonant wave length of the Hertz antenna wire in meters. The variable condensers in the feed line are then changed until the ammeters indicate resonance, which means that there is a quarter wave of potential distribution on a portion of the feed line and a voltage antinode at the far end, as shown in Fig. 8. The antenna of this feed system is voltage fed; *i.e.*, the source of excitation must be capable of providing a high alternating voltage at the proper frequency. Then the switch connecting one end of the feed line to the end of the antenna is closed; and if the reaction upon the circuit driver or transmitter frequency is very small or nearly zero, the antenna is in resonance with the driving frequency. This is true because the antenna at resonance presents an antiresonant noninductive impedance at its open end to the feed line, and the feed line when in resonance reflects zero reactance into the transmitter-tank circuit. The reactance at the current antinode is zero when the feed line is in resonance. This is discussed in Art. 4. J. J. Lamb[1] has given useful figures for the dimensions and adjustments for this type of antenna feed system.

When using a single-wire feed line, the measurement of fundamental frequency cannot be accomplished by placing a meter at the center of the antenna wire and varying the frequency of the circuit driver. When this antenna and feeder system is functioning properly, the single-wire feed line is actually terminated in its characteristic, or surge, impedance, hence has no standing wave present along its length. This means that the current in the feed wire and its potential are everywhere constant along its length. Furthermore, when the driver frequency is equal to the fundamental frequency of the antenna, the current in the antenna on each side of the feed-wire connection is the same, and only under the foregoing conditions. These facts were brought out in an excellent paper by W. L. Everitt and J. F. Byrne.[2] They also showed that the equal current condition will obtain, even though the connection of the feed wire to the

[1] "The Zepp, Facts and Figures for the Design of the Hertz Antenna with Two-wire Voltage Feed," *Q.S.T.*, Vol. 12, No. 9, pp. 33–36, September, 1928.

[2] "Single-wire Transmission Lines for Short Wave Antennas," *Proc. I.R.E.*, Vol. 17, No. 10, pp. 1840–1867.

antenna is not at the proper point to terminate the feed line in its surge impedance. This important point is analyzed in detail in the foregoing paper and so will not be taken up in this text. The manner of adjusting the antenna and feed wire and of measuring the fundamental of the Hertz antenna, briefly mentioned above, will be further clarified by referring to Fig. 9.

Diagram *A* shows the current distribution in the antenna when the wave length of the circuit driver is above the fundamental of the antenna. Hence ammeters placed in the antenna on each side of the junction and close to it will read unequally, as indicated by the current-distribution curve. If the driver wave length is too low, the inequality of the two meter readings will be reversed as in Diagram *B*; if the driver wave length (or frequency) is equal to the fundamental of the antenna, current distribution

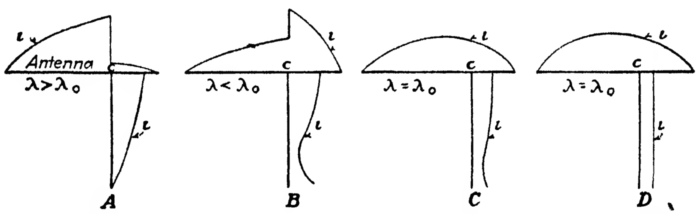


FIG. 9.—Current distribution in Hertz antennas with single-wire voltage feed.

will vary almost sinusoidally, and the two currents will be alike. The current distribution along the feed wire may still be non-uniform; and if the connection of the feed wire to the antenna is moved along the length of the latter to the proper point for the required terminating surge impedance, the current will be constant along the feed line, the antenna will be in resonance with the driver frequency, and the radiation from the feed wire will be zero, as pointed out by the authors referred to above. These conditions are shown by diagrams *C* and *D*, respectively. Figure 10 shows the setup for making this measurement. The ammeters *A* may also be milliammeters shunted across a short length of the antenna wire, the current indicator *a* being provided so that it may be slid along the feed line. It may be simply a flashlight bulb attached to small brass strips suitably constructed to slide on the wire. A long bamboo pole may be used to move *a* along the feed wire and to slide the connection of the feed wire along the antenna. After varying the circuit-driver frequency to obtain the fundamental of the antenna (when ammeters *A* read alike), the ammeters may be removed so that the feed-wire connection may be moved along the antenna wire until equal current distribution in the feed wire is found. Equal potential distribution also occurs along the feed wire under this latter condition, and it is possible to test, roughly, for equal potential by touching a neon-glow bulb to the feed wire. Here, also, the reaction of the feed line and its connected line is purely resistive when the connection of the feed line is made at the right point on the antenna and the driver frequency is equal to the resonant frequency of the antenna. As in the case of the other coupling systems discussed, this results in very little or no change in driver frequency.

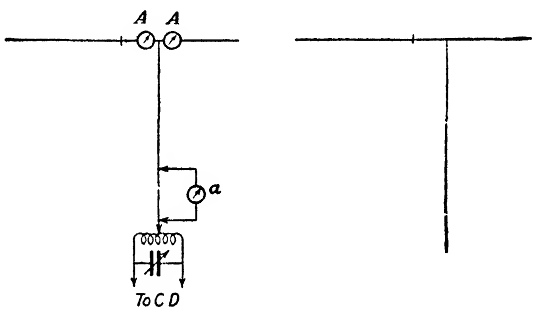


FIG. 10.—Circuit for measurement of Hertz antenna fundamental and adjustment of single feed-wire connection

Accuracy of Results—Theoretical Relations.—It may be assumed that the logarithmic decrement of most antennas is less than 0.2, so that the possibility of an error greater than 1 per cent in measurement of resonance as frequency is varied is remote. The results of measurements on earthed antennas are not easier to obtain but are usually more reliable than on the Hertz or dipole type of antenna, owing, of course, to the fact that a part of the former is always at ground potential.

The theoretical value of the ratio of fundamental wave length to the length of the Hertz antenna is variously given as from 2 to 2.53. A value of 2.1 has been used very often by many persons active in amateur work. Everitt and Byrne gave a concrete example of a Hertz antenna 18 meters in length whose fundamental was 37 meters, this being measured with the single feed wire in place. Rayleigh[1] showed by a discussion of con-

[1] RAYLEIGH, LORD, Paper, *Phil. Mag.*, Vol. 8, Ser. 6, p. 105, 1904.

temporary theoretical studies that the wave length of a straight rod-shaped oscillator approaches a value of twice the length of the rod when its cross section is very small, even though it be of cylindrical cross section and cut off square at the ends.

3. Capacitance and Inductance Measurement. *Principle of Method and Procedure.*—It was shown in Art. 1 of this chapter that the capacitance of an antenna is the capacitance per unit length times the length, or $C_0 = C_1 l$. This means, of course, that at low frequencies or when the frequency is zero, the potential along the antenna is constant; hence the capacitance C_0 may be measured by any low-frequency charge and discharge method, commutator, or other method. A capacitance bridge is very convenient for this purpose, and the setup for measurement is shown in Fig. 11. Owing to the higher resistance encountered in antennas than in most condensers, it is necessary to have a variable resistance in the standard condenser arm if a reasonable perfect balance is to be attained. It is generally interesting and often desirable to calculate the capacitance C_0 of an antenna from its dimensions before or after making measurements. L. W. Austin has given this subject a great deal of experimental study in reconciling the various theoretical capacitance formulas. He gives an approximate empirical formula[1] which is valuable, and also gives some bibliography of the more important theoretical formulas.

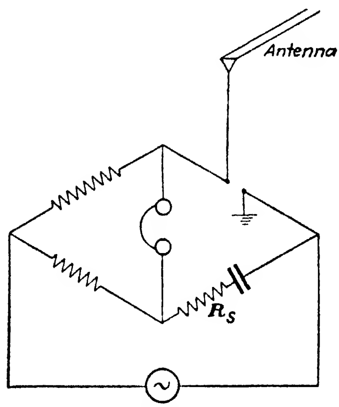


FIG. 11.—Bridge for measuring geometrical capacitance of antenna.

It was noted in the Introduction to this chapter that the effective capacitance near the fundamental frequency of an earthed or Marconi antenna is $\frac{8}{\pi^2}C_0$. For many actual antennas this relation does not hold; hence it is desirable to measure the effective capacitance at or near the antenna fundamental. One

[1] "Calculations of Antenna Capacity," *Proc. I.R.E.*, Vol. 8, No. 2, pp. 164–168. For other formulas see *U. S. Bur. Standards*, *Circ.* 74, 2d ed., p. 239.

method that has been used for many years is to measure the natural period of the antenna, first with no added inductance or capacitance, by one of the methods described in the preceding article. Then a capacitance four to five times as large as that of the antenna is inserted at the base of the antenna so as not to change the potential distribution, and the fundamental again is measured. For the antenna alone, resonance occurs when

$$\omega^2 = \frac{1}{C_e L_e},$$

where C_e and L_e are the effective capacitance and inductance of the antenna at the fundamental frequency $f = \omega/2\pi$. When the known series capacitance is added, the resonant frequency

$$f_S = \frac{\omega_S}{2\pi};$$

$$\omega_S{}^2 = \frac{1}{\left(\frac{C_S C_e}{C_S + C_e}\right)L_e}.$$

From these expressions,

$$C_e = C_S\left(\frac{f_S{}^2}{f^2} - 1\right). \qquad (4)$$

The effective inductance at the fundamental frequency may also be calculated from the values of ω and C_e. This is a better method of finding the effective inductance than by direct measurement, as will be explained in the following paragraph.

Precision of Measurement.—The potential distribution on a simple bent-wire earthed antenna is nearly as shown in Fig. 12. The effective capacitance due to this distribution is $\frac{8}{\pi^2}C_0$; and in changing from audio frequency to the fundamental, some thousand times or more, the effective capacitance has changed from $C_0(= C_1 l)$ to $\frac{8}{\pi^2}C_0$, or 19 per cent. The insertion of the series capacitance five times as great as C_e at the fundamental frequency alters the potential distribution about as shown by the dotted curve (Fig. 12). The resulting fundamental f_S as a proportion of f may quickly be estimated thus: Let $C_S = 5C_e$.

Then from Eq. (4),

$$C_e = 5C_S\left(\frac{f_S^2}{f^2} - 1\right), \tag{4a}$$

from which $f_S = 1.095f$, or a change of 9.5 per cent. The potential antinode will be moved back 9.5 per cent to the dotted line shown in the figure. The total electrostatic energy stored, and hence the effective capacitance, is thus changed by a negligible

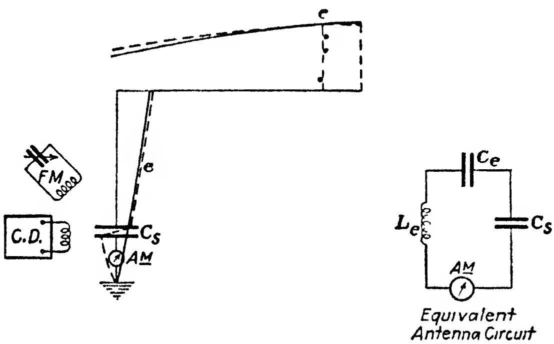


FIG. 12.—Circuit for measuring capacitance of an antenna. Dotted potential distribution due to series capacitance.

amount, as could be proved by integration under the curve

$$\int_0^l \tfrac{1}{2} C_1 l e^2 dx;$$

or by substituting in a differential equation[1] and evaluating. The change of resonant frequency of only 9.5 per cent when $C_S(= 5C_e)$ is added means that it is necessary to measure the resonant frequencies as accurately as possible in order to avoid an error in C_e of possibly as high as 10 per cent. The value of C_S to be inserted may be obtained by measuring the low-frequency capacitance $C_0 = C_1 l$ as described above or sometimes by calculation from antenna dimensions and using the relations previously worked out. To measure the effective inductance of a heavily loaded antenna in which $L_e = L_0/3$, large known inductances L_S and L_S' are inserted successively in series with the

[1] ZENNECK, J., "Wireless Telegraphy" (transl. A. E. Seelig), p. 410, note 40. Note refers to p. 41, line 6.

antenna, and the resonant frequency measured with each coil in series.

$$\omega_1^2 = \frac{1}{C_e(L_S + L_e)} \quad \text{and} \quad \omega_2^2 = \frac{1}{C_e(L_S' + L_e)},$$

from which

$$L_e = \frac{\omega_2^2 L_S' - \omega_1^2 L_S}{\omega_1^2 - \omega_2^2}, \tag{5}$$

where $\omega_1 = 2\pi f_1$ for L_S in series, and $\omega_2 = 2\pi f_2$ for L_S' in series with the antenna. Or L_0 may be determined from an inductance formula for a simple antenna[1] ($L_0 = L_1 l$). It is shown in *U. S. Bureau of Standards, Circular* 74, 2d ed. page 81, that if the ratio of L_s/L_0 is 1.0 or greater, the error in calculating the resonant frequency by taking $L_e = L_0/3$ is 0.7 per cent and smaller, as the ratio increases. Hence for an antenna loaded with an inductance greater than the calculated inductance L_0, the effective inductance may be taken as $L_0/3$. The student should consult the table given in the publication mentioned above for a special study of antenna measurements.

If a large inductance loading coil is placed in series with the antenna, the frequency of resonance is reasonably low, so that $L_e = L_0/3$; but if the loading becomes very small, $L_e = L_0/2$. There is an element of uncertainty in the measurement of the effective antenna inductance near the fundamental by adding a low-inductance loading coil to an antenna. This is pointed out on pages 85 and 86 of the circular referred to. Hence, it is more desirable to calculate the effective inductance at the antenna fundamental frequency from the measured effective capacitance at this frequency. It is interesting to compare the value of the effective inductance thus determined at the fundamental frequency of the antenna with the theoretically given value $L_0/2$, where $L_0 = L_1 l$, and L_1 is determined by calculation from a formula referred to in footnote 1.

[1] Morecroft, J. H., "Principles of Radio Communication," 2d ed., p. 208; *U. S. Bur. Standards, Circ.* 74, 2d ed., p. 247. G. W. O. Howe gives the inductance of a vertical earthed wire of length h and radius r as $L_1 = 2 \log_e \frac{h}{r}$ 0.10^{-9} henry per centimeter length.

4. Antenna Reactance, Resistance, and Impedance Measurement. *General Principles.*—The analogous parallel-line and antenna theoretical analyses result in equations for current and voltage distribution from which expressions for reactance or impedance may be obtained.[1] This reactance is practically equal to the impedance for most lines or antennas at radio frequencies and is the voltage at the input end divided by the input current.

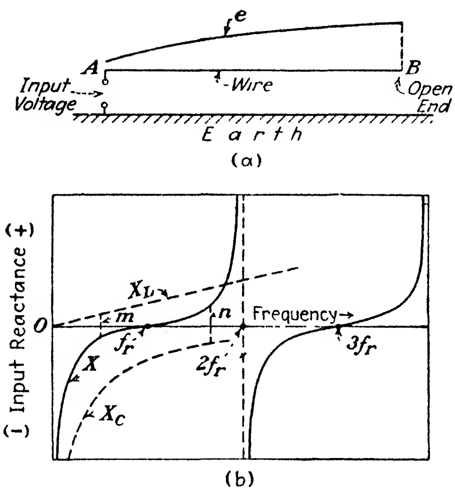


FIG. 13.—Antenna potential distribution at fundamental, and input-reactance variation with frequency of excitation.

For this type of circuit the input reactance is

$$X \cong -\sqrt{\frac{L_0}{C_0}} \cot\,(2\pi f\sqrt{C_0 L_0}), \text{ nearly.} \tag{6}$$

This reactance is conceived of as existing between the input terminals of the antenna wire shown at (*a*), Fig. 13. At (*b*) the fundamental frequency is f_r on the plot of the cotangent curve of Eq. (6), and at this point the input reactance is zero. This corresponds to $f = 1/4\sqrt{L_0 C_0}$ in Eq. (6) for which the cotangent is zero. The input impedance is the total resistance at this frequency and is the resistance between the input terminals. At a frequency $2f_r$ the input reactance is infinite, or nearly so when the resistance of the antenna is not high. It may be here noted that

[1] For a direct and understandable antenna analysis see J. M. Miller's paper, *loc. cit.* (footnote in Introduction to this chapter).

at $2f_r$ the antenna wire is a half-wave Hertz antenna tuned to its fundamental, and the input reactance at the center of the antenna wire would be zero. At $3f_r$ the reactance at input end A is again zero, and the potential and current distributions are as shown at (*d*) Fig. 1 (Chap. III). Of course the input reactance is zero for all odd multiples of the fundamental frequency. Impedance measurement at f_r, $3f_r$, etc., would give a measure of the resistance because of zero-input reactance; no appreciable reactance could be found at these frequencies.

At frequencies below f_r and between $2f_r$ and $3f_r$, the input reactance is negative; and for frequencies between f_r and $2f_r$ and between $3f_r$ and $4f_r$, it is positive. The problem to be considered is the measurement of input reactance at this range of frequencies. The method is to add a reactance of the opposite sign to the existing antenna reactance and equal to the latter so that the total reactance is zero. As a specific case, let it be required to measure the input reactance at a frequency m [see Diagram (*b*), Fig. 13], which is below the fundamental f_r. At this frequency, the antenna-input reactance is negative, and it would require a positive or inductive reactance to balance it to obtain zero-input reactance and hence resonance. An inductance coil placed in series with the input voltage and the input terminals will accomplish this end if the inductance were of such value that

$$X_L - X = 0.$$

The inductance coil has a reactance X_L, and the variation of this with frequency gives a straight-line curve of X_L versus frequency, as shown in the figure. By properly adjusting the series added inductance, a value is obtained where the ordinate to curve $X_L = -$*ordinate* to curve X, the antenna reactance. This adjusted value is shown at m, and the frequency corresponding to m is the frequency at which it is desired to measure the input reactance.

Similarly, if it is desired to measure the input reactance at frequency n [Diagram (*b*), Fig. 13], for which the antenna input reactance is positive, a capacitance is placed in series with the input terminals and varied until the reactance of the series condenser is equal to that of the antenna, as shown at n. The curve of series condenser reactance versus frequency is shown as X_c in the figure.

Procedure in Measurements.—The foregoing discussion indicates that a very simple measurement might be employed; a power-circuit driver may be loosely magnetically coupled near the antenna input terminals, and a known variable inductance placed in series with the input terminals for the case of measurement at frequencies for which the reactance is negative. Then the inductance is varied until a current indicator shows resonance, *i.e.*, zero total input reactance. The inductive reactance is then equal to the negative antenna reactance. Or the series or loading inductance may be set at various values successively, and excitation frequencies varied to obtain resonance for each case.

For frequencies giving a positive reactance, a variable known capacitance is placed in series with the input terminals, and this capacitance is varied until resonance occurs, at which value the input reactance equals the value of the series variable-condenser capacitance. For frequencies near $2f_r$ or $4f_r$, this reactance is very high, the condenser capacitance for resonance is very small, and the current antinode approaches the middle point of the antenna wire, resulting in small currents near the base at resonance.

Procedure by Reaction Method.—The preceding method is inconvenient because it is necessary to have a variable inductance, and means to calibrate it, at frequencies of measurement. The following method, due to Clapp,[1] is practical for antennas or lines at frequencies for which the input reactance is negative. For positive input reactance a modification is followed. The test circuit and apparatus are shown in Fig. 14. In Diagram (*a*), a shunt-feed magnetic feedback (reversed) oscillator is provided, and by means of switch S the antenna is connected in parallel with the tank condenser C_r. The oscillator is adjusted to the desired frequency for which the reactance is negative, and the reading of the plate-current meter is then reduced somewhat by shunting some of the current through R_p and P. This is done by adjusting potentiometer P so as to apply a portion of the filament potential to the meter through R_p. This is called a *compensation circuit*, and it reduces the meter current so that a lower range scale may be used and small current changes will be considerably magnified. R_p is necessary to make the device sensitive and may be from a few hundred to 1,000 ohms or more.

[1] *Proc. I.R.E.*, Vol. 18, No. 4, p. 571, 1930.

The oscillator is now exciting the antenna at a frequency for which the input reactance is negative. The switch S' is now closed, shorting meter M, and switch S is thrown so that the antenna is replaced by the substitution circuit R_sC_s. The reason for short-circuiting meter M while switch S is being changed is that when the oscillator load has been reduced, the plate current decreases considerably; and if the compensation circuit bridge is adjusted so that M is only a low-range reading, the plate-current range may damage the low-range meter. After S is thrown, C_s is adjusted until the coupled frequency meter indicates that the original frequency is again attained, and R_s is adjusted until the

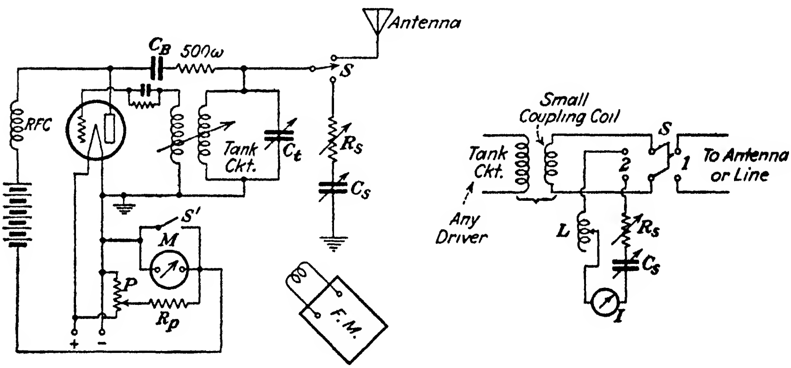


FIG. 14.—Apparatus and test circuit for measuring antenna and line input reactance and impedance.

plate-current increment ΔI_p flowing through M is the same as before S was changed. This means that the oscillator load is again the same and that R_s has the same value as that of the antenna. The impedance is then $\sqrt{R_s^2 + (1/\omega C_s)^2}$, where $\omega = 2\pi$ times the oscillator frequency. An alternative test circuit is shown at (*b*), Fig. 14, wherein a coupling coil is used, and the substitution circuit includes R_s, C_s, and L in series. The procedure is quite similar, except that for frequencies for negative antenna reactance the inductance L may be reduced to zero.

In the measurement of positive antenna or line reactance (at corresponding frequencies, see Fig. 13) variable inductance L in Diagram (*b*), Fig. 14, is sufficiently large so that the reactance of the substitution circuit is also positive; and when it is equal to that of the antenna or line, it produces the same reaction effect on the oscillator. The resistance of the substitution circuit is then varied until the meter M shows the same plate-current increment, as previously explained. The effective resistance of the substitution circuit may then be measured by the reactance-variation method. The inductive reactance of the substitution circuit may then be determined by tuning C_s to resonance, as indicated by substitution-circuit meter I, where $\omega L = 1/\omega C_s$. The frequency of the oscillator should be held at the original test value while this is done. The antenna impedance is then $\sqrt{R^2 + (\omega L)^2}$.

Precision Requirements.—This is usually not a precise measurement. It is usually worth while to provide a heterodyne frequency meter so that the oscillator-frequency changes of small magnitude can be easily detected by the disturbance of the zero-beat condition between the oscillator and frequency-meter output. If no detector amplifier is incorporated in the heterodyne frequency meter, the radio receiver with beat oscillator is convenient.

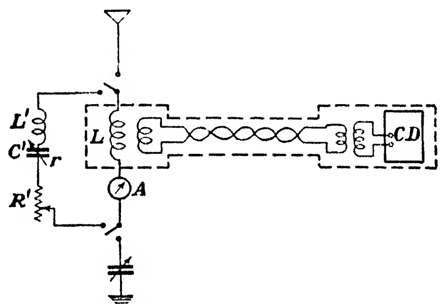


FIG. 15.—Circuit for measuring antenna effective resistance.

5. Other Methods for Earthed-antenna Total-resistance Measurement. *The Substitution Method.*—In 1912, L. W. Austin[1] described a method of measuring the total equivalent resistance of antennas which has been widely used today, except that undamped excitation has been substituted for the original buzzer excitation. The method is more desirable for antennas containing loading coils. Referring to Fig. 15, the antenna and ground lead may either be connected to the loading coil

[1] "Work of the U. S. Naval Radiotelegraphic Laboratory," *Jour. Am. Soc. Naval Eng.*, Vol. 24, p. 122, February, 1912.

(coupling coil) to be used in the antenna, or this coil may be connected to the substitution circuit $L'C'R'$. L' and C' should be equal to the effective value of the antenna inductance and capacitance C_e and L_e. With the switches first connected to the antenna, the frequency of the circuit driver is varied until resonance is obtained by maximum deflection of the resonance indicator A, and the value of the deflection noted. Next, the switches are changed so that R', C', L' are substituted for the antenna and ground, and R' is varied until the same deflection is obtained. If C' and L' are not correct for resonance when connections are changed to the substitution circuit, C' must be varied until maximum deflection is obtained, after which R' is adjusted until the current is the same as for resonance with the antenna in the circuit. R' usually contains inductance or distributed capacitance in a small amount, so it is usually necessary to vary C' for resonance again, after varying R' for approximately the same deflection as for the antenna resonance condition. C' and R' should be varied alternately in small steps until resonance and current equal to that mentioned above are both established in the substitution circuit. The value of R' will then be equal to the effective resistance of the antenna minus the resistance of the substitution circuit C', L', and connecting leads; when the antenna is connected, and resonance obtains,

$$I = \frac{E}{R_e + R_L + R_m};$$

when the substitution circuit is connected and adjusted,

$$I = \frac{E}{R' + r + R_L + R_m}.$$

R_m = resistance of A, and

$$R_e = R' + r,$$

where r is the resistance of C', L', and leads. A high-grade condenser may be used for C', and its resistance neglected if it is not near minimum capacitance position, but it is advisable to measure the resistance of L' plus leads by an approved method, such as described in Chap. I. In most cases, L' may be left in the circuit, and, if so, it becomes a part of coil L. This is sufficient for the measurement of antenna resistance only, and, when this is done,

$$R_e = R'.$$

The total equivalent inductance and capacitance cannot, of course, be determined accurately when L is eliminated from the circuit.

In arranging the apparatus for this method of measurement, it is important to shield the circuit driver, using a link circuit to couple to L and to make sure that there is no linkage of exciting field with L'. No mention has so far been made of the condenser in the ground lead of the antenna. When measuring the resistance of the loaded antenna, this condenser is not neces-

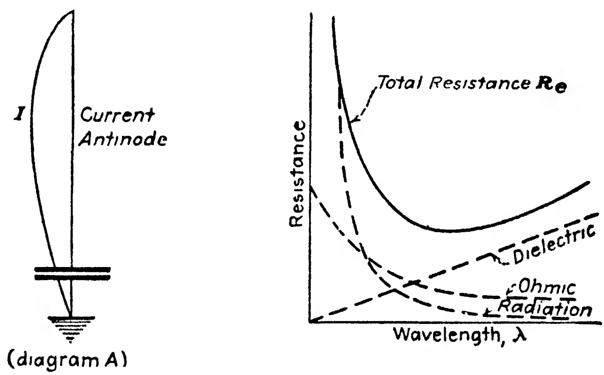


Fig 16 —Antenna component and total resistance variation with wave length, current distribution with small series capacitance

sary; but when the measurements are also to be made at frequencies above the fundamental of the antenna, it is necessary to use this condenser to tune to the higher frequencies. For such measurements it is necessary also to adjust C' to lower values. It is customary to measure the resistance for a range of frequencies varying from 30 to 50 per cent above the fundamental to much lower values, and it is also customary to plot results as variation of total effective resistance versus wave length of antenna excitation instead of frequency. Such a curve is shown in Fig. 16. The antenna series condenser theoretically allows measurements to be extended to 50 per cent below the fundamental wave length,[1] but actually it is difficult to go below 50

[1] Morecroft, J. H., "Principles of Radio Communication," 2d ed, p. 879.

per cent lower than the fundamental. For the longer wave lengths it is, of course, necessary to increase L. At the long wave lengths, the antenna constants are fixed in value and change to values indicated in the Introduction to this chapter when the wave length approaches the antenna fundamental; but at wave lengths below the fundamental they bear different relations to the zero-frequency values. At these wave lengths (frequencies *above* the fundamental frequency), the current antinode is at points along the antenna wire above the ground and moves farther up toward the open end as the wave length is lowered (see Diagram A, Fig. 16). At wave lengths not more than 20 per cent below the fundamental, the error made in assuming that maximum current at the base is not serious. This may be realized by a study of the results of calculations made by Stuart Ballantine.[1] Above this value the error, due to the decreasing current at the base, becomes serious, illustrating the extreme case of an antenna tuned to almost twice its fundamental frequency. To clarify the point: Suppose that with the aid of the series capacitance the antenna is in resonance with the driving frequency at nearly twice the fundamental frequency. The distribution of current is shown in Diagram A, Fig. 16. The current at the base is almost zero, so that R' placed in the substitution circuit would be extremely large for the same current. The current at the center of the vertical antenna is a definite value, although less than for the case of the lower antenna-fundamental frequency and current antinode at the base. Ballantine's results show that when the radiation component of the antenna total resistance is calculated on the basis of the value of the current at the antinode for the case of twice the fundamental frequency, the radiation component is over four times as great as at the fundamental frequency. Estimating the comparative values of the components due to skin effect and dielectric absorption, it is probable that the total resistance, on the basis of current value at the antinode, is about five times as great at twice the fundamental frequency as at the fundamental with the current antinode at the base of the antenna. The various components of the total antenna resistance curve of Fig. 16 are shown by the dotted curves. The sum of these gives the total effective resistance. As interest in this chapter centers

[1] *Proc. I.R.E.*, Vol. 12, No. 6, p. 831, December, 1924.

principally on the measurement of the total and radiation resistance, the component curves are not discussed. The student may find excellent discussions of the component curves in several texts.[1]

Half-deflection and Reactance-variation Methods.—The *half-deflection method* consists of inserting a variable known resistance into the base of the antenna until the current at the base is reduced one-half. The introduced resistance is then equal to the total effective antenna resistance. For this method the exciting frequency is first tuned to resonance with the antenna which may be either loaded or not. At resonance,

$$I = \frac{E}{R_e};$$

when resistance R' is added,

$$I' = \frac{E}{R_e + R'};$$

from which,

$$R_e = R'\left(\frac{I'}{I - I'}\right), \tag{7}$$

when $I' = I/2$, and $R_e = R'$. If a current-squared resonance indicator such as a thermogalvanometer with a uniform scale is used, it is best to obtain the comparative current values of the readings by actual calibration. In many such instruments, the uniform-scale readings are not exactly proportional to the squares of the current values. A theoretical disadvantage of this method lies in the change in the assumed distribution of current in the antenna when resistance is added in series. It is, however, a very convenient method to use when only approximate methods are desired. Since it is not necessary to reduce the current at resonance to exactly one-half if the comparative values of antenna current may be measured, it is well to use the link resistances for R', as these are inexpensive and reliable for radio frequencies.[2]

[1] *E.g.*, G. W. PIERCE, "Electric Oscillations and Electric Waves"; J. H. MORECROFT, "Principals of Radio Communication"; F. E. TERMAN, "Radio Engineering," 2d ed., p. 560.

[2] This is the resistance-variation method. Link resistances are mentioned in Art. 13, Chap. I. See also A.S.T.M. Standards, D150–36T, 1936.

Measurement of antenna resistance by the reactance-variation method is accomplished in a manner very similar to that described in Chap. I, Art. 13. The reactance may be varied by varying either a condenser in series with the antenna or the driving frequency. Owing to the high resistance and decrement of most antennas and the tendency for the resonance curve to be slightly shifted and unsymmetrical, as a result of high decrements or variable current distribution, this method is not so reliable as the others that have been described. It is, of course, convenient when high-grade radio-frequency standard resistances are not available.

6. Earthed-antenna Radiation Resistance and Effective Height. *Method by Standard Loop.*—The measurement of the radiation component of total antenna resistance is of considerable importance but, unfortunately, is fraught with difficulties. The description of one method will point out the uncertainty of measurement resulting from assumed variation and nonvariation of certain quantities. The concept of radiation resistance of a radiator of electromagnetic energy is based upon the actual power radiated, which may be determined theoretically in terms of the mean effective height of the antenna, the wave length, and the current at the antinode. Hence, the radiation resistance

$$R_2 = \frac{P_z}{I^2}$$

and has been derived by many writers. Their formulas differ in many respects. Some are very complex and contain integrals that cannot be evaluated in simple functions. Others have been reduced to simple forms for special forms of antennas or for assumed values of wave length, current distribution, etc. One form is

$$R_z = 160\pi^2\frac{(\alpha h)^2}{\lambda^2} \text{ ohms (Zenneck),}^{1} \qquad (8)$$

where α is the *form factor* due to the current distribution, h the total height, and λ the wave length. The *effective height* is αh.

[1] "Wireless Telegraphy," (transl. A. E. Seelig).

One formula for a single vertical-wire antenna takes the form

$$R_Z = 1{,}578\left(\frac{l}{\lambda}\right)^2 (0.33) \text{ ohms (Cutting),}^1 \qquad (8a)$$

where l is the length of the wire, and λ is the wave length, assumed to be considerably greater than the fundamental wave length of the antenna. Many able derivations and discussions of this formula are to be found in the literature, and the student is urged to become familiar with the theory as given in some of the previously mentioned references. Some derivations assume that the capacitance is concentrated at a point along the antenna above the earth and that a current of constant amplitude flows up the antenna to this *center of capacitance*, the height from the base to the center of capacitance, called the *equivalent*, or *effective*, *height* of the antenna. Some distinguish between the two terms, using the latter for the value when corrected for attenuation of waves, induction in near-by objects, etc.

One method of measuring the radiation resistance, which is termed the *method by standard loop*, consists in operating the antenna under test in conjunction with a loop antenna and measuring the currents in each, as well as their dimensions, from which the effective height may be calculated, using the well-known transmission formulas. It is convenient to use a coil or loop antenna as a transmitter, locating it about 1.5 or 2 wave lengths from the antenna to be measured. The former is excited with a fairly powerful oscillator, and the received current is measured at the base of the open earthed antenna with a calibrated thermocouple and millivoltmeter or with a low-range thermomilliammeter. For coil-to-antenna transmission, the complete formula is given as follows:[2]

$$I_r = \frac{376 N h_S h_r I_S}{\lambda d R} \sin \frac{\pi S}{\lambda}, \qquad (9)$$

[1] *Proc. I.R.E.*, Vol. 10, No. 2, pp. 129–136, 1922. Formulas are also given in this paper for L antennas and for loaded antennas.

[2] These and other transmission formulas are derived by J. H. Dellinger, "Radio Transmission," *Trans. A.I.E.E.*, Vol. 38, p. 1347, Part 2, 1919; also by J. H. Morecroft, "Principles of Radio Communication," 2d ed., pp. 855–860.

where h_s = height of transmitting coil in meters.
N = turns of transmitting coil.
S = width of transmitting coil in meters.
h_r = effective height of receiving earthed antenna (under test).
I_S = current in transmitting coil.
λ = wave length of transmission in meters.
d = distance between antenna and coil in meters.
R = total resistance of receiving antenna (under test).

When the term sine $\pi S/\lambda$ becomes very small, the angle may be substituted for the sine, and the formula becomes

$$I_r = \frac{1{,}184 h_S h_r S I_S N}{\lambda^2 d R}. \tag{10}$$

The terms in the equations are shown in Fig. 16*a*. If a power oscillator excites the coil transmitter, the currents in both coil

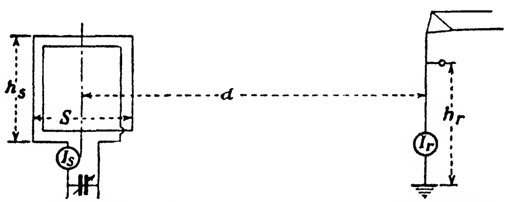


FIG. 16*a*.—Antennas with dimensions used in transmission equations.

antenna and earthed receiving antenna may be measured, and, using the physical dimensions as indicated, the effective height of the open earthed antenna may be obtained by substitution in Eq. (9) or (10). It is to be noted that the coil transmitting antenna is used in this procedure because the height h_S is the actual height of the coil in meters, so that the only unknown dimension is the effective height of the earthed antenna of which the value is desired. This assumes that R, the total resistance of the antenna under test, is known from previous measurement and that its resistance as a receiving antenna may be measured by the methods described previously, methods that, however, are theoretically for transmission measurements.

7. Hertz-antenna Resistance Measurement.—The total effective resistance of a Hertz, or rod-shaped dipole, antenna is the value that multiplied by the square of the current at the center (current antinode, or loop) gives the power input to the antenna.

Everitt and Byrne measured the power input[1] by immersing the power oscillator in an oil calorimeter. The power supplied to the antenna was then the power input to the oscillator minus the heat loss measured in the calorimeter. The antenna was fed with the single-wire transmission line, and the I^2R loss in the feeder was also deducted. The remainder was the power input to the antenna, and this, divided by the square of the current at the antinode of the Hertz antenna, gave its total resistance.

The radiation resistance of a Hertz antenna suspended in space is given by M. Abraham[2] as twice that of an earthed vertical-wire (half-dipole, or quarter-wave) antenna whose height is one-half the length of the Hertz antenna. He also calculates the radiation resistance of the former to be about 37 ohms, and, therefore, the Hertz antenna would have a value of 74 ohms, at its natural wave length $\lambda/\lambda_0 = 1$. Owing to the proximity of the antenna to earth, this is not correct, and a valuable curve showing the relation between the radiation resistance of a Hertz antenna and the ratio of height aboveground to total length of antenna is worked out by Everitt and Byrne, whose paper has just been cited. They show the measured radiation resistance to vary between 60 and 95 ohms for increasing ratios. For ratios of 1 and 1.5, the resistance is nearly 74 ohms. Since the measurement of resistance of a Hertz antenna involves the measurement of the radio-frequency power input to the antenna by some means such as described above, it is probably much more convenient for most users to determine it from the experimental curves mentioned above.

Procedure with Reaction Methods.—The total resistance of a Hertz antenna and its associated feed-line system may be readily measured by the reaction of the system on an oscillator, comparing this with the reaction of an adjustable substitution circuit on the same oscillator. When the reactions are alike, the constants of the antenna and of the substitution circuit are identical. For the case of a Zeppelin-fed antenna, the feeder is resonant, and its input reactance is zero. This feed system is

[1] See footnote 2, p. 148; power measurement described on p. 1866 of their paper.

[2] *Wiedmanns Ann.*, Vol. 66, p. 435; see also J. Zenneck, "Wireless Telegraphy," (transl. A. E. Seelig), pp. 40, 170.

shown in Fig. 8. Its reaction upon the tank circuit of a circuit driver or oscillator will therefore be a resistance reaction only. Figure 17 shows the essentials of the test circuit. The details of the oscillator and compensated plate-current meter may be found in Fig. 14. Switch S in Fig. 17 is used to substitute R_s for the feeder line. C_1 and C_2 are then used to retune to resonance with the oscillator frequency so that the test circuit coupled to the oscillator has zero reactance. R_s is then adjusted until the same plate-current increment (meter M, Fig. 14) is obtained. R_s is then equal to the resistance of the feeder plus that of the fed antenna. The antenna is then disconnected from the feeder, and another measurement is made to obtain the resistance of

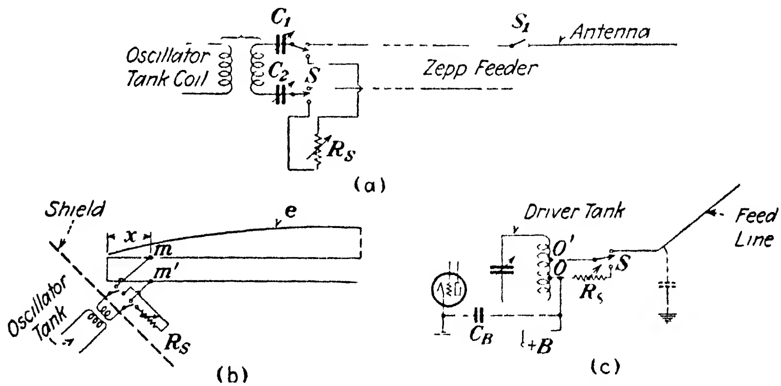


FIG. 17.—Test circuits for measuring input impedance of feeder lines.

the feeder alone. The difference gives the total resistance of the Hertz antenna. Since the reaction of the current at the antinode was made use of in this measurement, the resistance will of course be the value that when multiplied by the current at the antinode squared gives the total resistance. Hence the value obtained for the Hertz antenna itself is the value that is multiplied by the current at the antenna antinode squared to obtain the total power in the antenna circuit.

If a half-wave Hertz antenna may be opened at the center, a coupling coil and two variable condensers may be introduced to measure the antenna resistance by the reaction method, using the oscillator of Fig. 14. The test circuit and adjustments are the same as for measuring the input resistance of the resonant Zeppelin feed line. The arrangement of coupling coil and series variable condensers are similar to that shown in Fig. 8 for Zeppelin feed. The coupling coil and condensers should be so chosen as to bring the potential node as close to the condenser terminals as possible. Often it is desired to measure the resistance between two points on a resonant feed line, such as in (*b*), Fig. 17. A nonresonant feed line whose terminating characteristic impedance is known is to be connected to the resonant *stub feeder*, at a point where the input impedance to the latter is equal to the characteristic impedance of the nonresonant feed line. When the stub feeder is in resonance at its fundamental frequency, or odd harmonics thereof, the input impedance at any two points, such as *mm'* in (*b*), Fig. 17, is noninductive. Therefore if the reaction method is used, as assumed in the test circuit shown, the substitution circuit is a pure resistance, and the reaction upon the oscillator frequency should be the same for either position of the switch.

The voltmeter-ammeter method serves very well for measuring the input impedance, at two points on a line, such as *mm'* in (*b*), Fig. 17. The ratio of the voltage across *mm'* to the current input to these points from the driver coupling coil will give the resistance. If *mm'* is out near the open end of the line, a higher voltage is desired across *mm'*, and this may be easily provided by connecting a variable condenser across the driver coupling coil and tuning to parallel circuit resonance, or *antiresonance* of the driver coupling coil and variable condenser. A vacuum-tube voltmeter is well suited to measuring the potential across the antiresonant circuit which supplies current to the feed line because of the high input impedance of tube voltmeters.

When the Hertz antenna is fed with the single-wire feed line and is properly attached to the antenna wire, as explained in Art. 2, it constitutes a nonresonant feed-line wire, and the other side of this line is the earth and intervening space between the wire, earth, and antenna proper. When the feed line is connected to a point on the oscillator or driver tank coil, as shown in (*c*), Fig. 17, it has the effect of placing a resistance between this point and ground or between O and O' on the coil. This resistance is equal to the characteristic impedance of the single-wire feed line when the latter is properly adjusted. The simple test circuit is also shown in (*c*), Fig. 17. With the properly adjusted feed line, switch S is changed. R_s is varied until the reaction on the plate current (see Fig. 14 for the compensated plate-current meter) is the same as when the feed line is connected. For this condition, R_s is equal to the characteristic impedance of the single-wire feed line plus the resistance of the excited antenna. There should be no change in the driver frequency for either position of the switch S. There will, of course, be a change in frequency when S is open because of the change in effective inductance when there is no resistance or impedance externally connected between O and O'. At this point, it is well to remember that tap O' should be close to O, because if it is not, a considerable portion of the driver tank inductance will be shunted by the feed-line input impedance, which is resistive. In the operation of this type of antenna, the tank circuit of the final power amplifier is poorly tuned when the single-wire feed line is connected far from the radio-frequency ground-potential point. This results in excessive plate current because of low impedance of the tank circuit. If this feed system is used, the tap must be kept close to the grounded point, often less than one turn away from it.

8. Determination of Characteristic Impedance of a Line. *Principle Involved.*—The characteristic impedance of a transmission line is the impedance that when connected to the output end of a line will make it impossible to cause standing waves of potential and current to exist on the line. This means that the reflection phenomenon, which is the cause of standing waves, will be prevented by terminating the line in its *characteristic* impedance.[1] The fundamental equation for characteristic impedance of a line is $Z_0 = \frac{\sqrt{R_1 + j\omega L_1}}{\sqrt{g + j\omega C_1}}$. R_1 and L_1 are the series line resistance and inductance per unit length, g and C_1 are the conductance and capacitance between wires per unit length of line. For an ordinary, well insulated, line $R_1 < \omega L_1$ and $g < \omega C_1$. The characteristic impedance is then

$$Z_0 = \sqrt{\frac{L}{C}},$$

where L and C are the geometrical inductance and capacitance of the line, respectively. By substituting the equations for these

[1] For theory of transmission lines, see J. S. Weinbach, "Telephone Transmission," and many other books on telephony.

constants of a line in the preceding formula, it can be easily shown that for an open-wire transmission line

$$Z_0 = 276 \log_{10} \frac{2D}{d} \text{ (for 600 ohms } D = 98d), \qquad (11)$$

where D is the spacing between wires, and d the wire diameter, both in the same units. This equation holds when d is very small compared to D, so that there is no appreciable proximity effect. The ratio $\frac{d}{D}$ must not be greater than 0.05. For a concentric-tube transmission line, the expression for characteristic impedance is

$$Z_0 = 138 \log_{10} \frac{R}{r},$$

where R is the inner radius of the outer tube, and r is the radius of the inner rod or wire. With the aid of these formulas, the characteristic impedance may first be calculated in order to check measurements.

It is shown in transmission-line theory that the characteristic impedance is

$$Z_0 = \sqrt{Z_{ts} Z_{t0}}, \qquad (12)$$

where Z_{ts} is the input impedance with the far, or terminating, end short-circuited; and Z_{t0} is the input impedance with the terminating end open. The measurement of these impedances may be carried out in the usual manner, as discussed in Art. 4, but it has been pointed out that this may be done only at low radio frequencies for reasonably good results.[1] When a transmission line is infinitely long, its input impedance is complex and is known as its *characteristic impedance*. Therefore any finite line terminating in an impedance equal to the characteristic impedance of the line, if it were infinitely long, will behave as an infinite-length line, and its input impedance will be equal to the characteristic impedance of the corresponding infinite-length line. The complex input impedance and terminating impedance will then be exactly alike when the line is properly terminated to eliminate reflections and standing waves. For radio frequency these impedances will be non-inductive [Eq. (11)].

[1] *Proc. I.R.E.*, Vol. 15, No. 7, p. 612, 1927.

Procedure by Line-input Impedance-measurement Method.—The foregoing relations suggest a simple method of determining the characteristic impedance of a transmission line. The input impedance may be measured for various values of resistance connected to the far end of the line. When the terminating resistance is equal to the characteristic impedance of the line, the input impedance is equal to that connected to the far end; and, furthermore, the input impedance will then not have an inductive reaction on the driving oscillator (see Fig. 14) and the

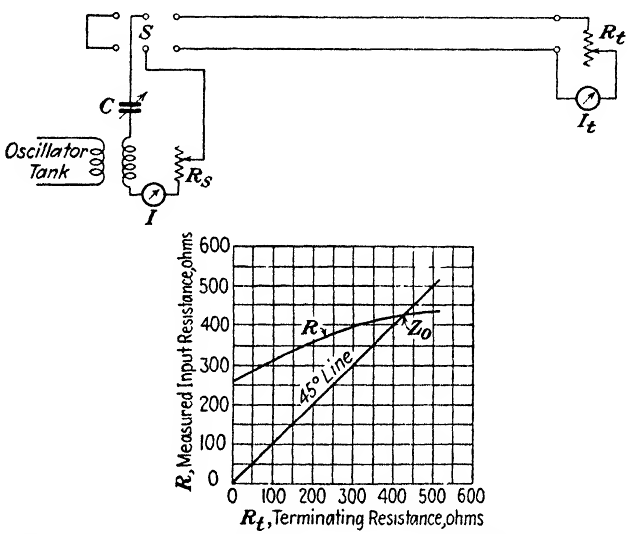


FIG. 18.—Test circuit for measuring characteristic impedance of a transmission line. (*After Boddie.*)

consequent change in oscillator frequency. A method that differs in minor details from that just briefly outlined is given by Boddie[1] and will be outlined. The test circuit given by the author referred to is shown in Fig. 18. Switch S is first closed to the left, R_s reduced to zero, and C varied until resonance is obtained as indicated by I. R_s is then increased until current I is reduced one-half, giving the resistance of the test circuit by the half-deflection method. S is then thrown to the right, R_t is set at about two-thirds of the calculated characteristic impedance, and the line-input *resistance* plus the test-circuit resistance is measured in a like manner. The difference between this

[1] See preceding footnote.

measured value and the resistance of the test circuit is the line-input resistance. R_t is then increased, and the line-input resistance again measured. This is repeated for values of R_t extending beyond the calculated characteristic impedance. The results are then plotted as shown in Fig. 18, equal scales being used for R and R_t. The 45-deg. line from the origin will then intersect the input resistance curve at a point whose ordinate is equal to the characteristic impedance Z_0 of the line.

This method is satisfactory if the frequency is not too high and R_s and R_t are fairly well known. It may be necessary to measure the values of R_s by the reactance-variation method. R_s may be made of very small, high-resistance, low-temperature-coefficient wire wound on noninductive forms and need be changed in only a few steps. The resistance is then measured by a resistance-variation method, as explained in Chap. 1. R_t also may be noninductively constructed and variable in steps.

Theory and Procedure with Roosenstein Method.—When very high radio frequencies are used, values of R_s and R_t are uncertain, and the current in the test circuit will not be constant at different points, so a method due to Roosenstein is recommended where the frequency is 20 Mc. or more.[1] The input reactance of a parallel-wire antenna or transmission line is given by Eq. (6); and for frequencies equal to

$$f = \frac{n}{4\sqrt{CL}}, \tag{13}$$

where $n = 1, 3, 5$, etc., or

$$f = \frac{n}{4(\sqrt{C_1L_1})l}, \tag{14}$$

where l is the length of the line, the input reactance is zero, and the line is in resonance with the exciting frequency. Here C and L are the geometrical capacitance and inductance of the line of total length l. If a portion of this line has a length equal to $\lambda/8$, its input reactance from Eq. (6) is

$$X = \sqrt{\frac{L}{C}} \cot 2\pi f\sqrt{C_1L_1}\,\frac{\lambda}{8}, \tag{15}$$

[1] *Proc. I.R.E.*, Vol. 19, No. 10, pp. 1859–1860, 1931.

where λ is the wave length, $L = \frac{\lambda L_1}{8}$ and $C = \frac{\lambda C_1}{8}$. C_1 and L_1 are the geometric capacitance and inductance, respectively, per unit length. The frequency f is set for resonance in a line of length l. Hence the value of resonant frequency as given by Eq. (14) should be substituted in Eq. (15), giving

$$X = \sqrt{\frac{L}{C}} \cot \frac{\pi\lambda n}{16l} (n = 1, 3, 5, \text{etc.}). \tag{16}$$

But for a transmission line open at the far end, it may resonate at frequencies that cause n quarter waves of potential or current distribution along the line: *i.e.*, at the lowest resonant frequency

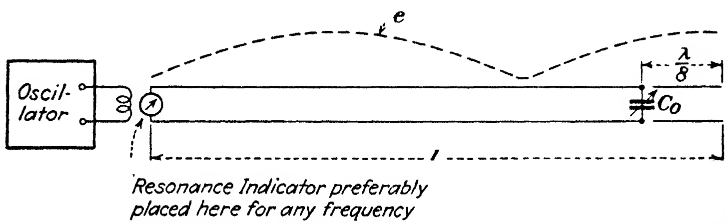


FIG. 19.—Test circuit for measurement of characteristic impedance of transmission line. (*After Roosenstein.*)

there is one one-quarter standing wave; at three times this frequency there are 3 one-quarter waves; etc. Hence

$$\lambda = \frac{4l}{n},$$

where n is the same as in Eqs. (13), (14), and (16). Substituting the preceding relation in Eq. (16) gives

$$X = \sqrt{\frac{L}{C}} \cot \frac{\pi}{4} = \sqrt{\frac{L}{C}}. \tag{17}$$

This gives the reactance at the input end of a transmission line whose length is one-eighth of the wave length of a line of length l in resonance at its fundamental frequency or an odd harmonic thereof.

The procedure is made clear with the aid of Fig. 19. The transmission line of length l is excited into resonance by setting the oscillator frequency at a suitable value which gives a maximum reading on the current indicator placed preferably at the center of the left end of the bridge, where there will always be a current antinode or loop at resonance at the fundamental frequency or any of its odd harmonics. The next step is to shorten the line by a length equal to $\lambda/8$, where λ is the wave length corresponding to the frequency of excitation for resonance. The shortening process will throw the line out of resonance, because a section whose input reactance is given by Eq. (17) has been removed. If a variable condenser is connected to the shortened end and adjusted until resonance again occurs, the reactance of the variable condenser will be equal to that at the input end of the short portion which was removed. Therefore, when resonance recurs,

$$\sqrt{\frac{L}{C}} = \frac{1}{\omega C_0}.$$

The expression $\sqrt{L/C}$ is that previously given for the characteristic impedance of a well-insulated radio-frequency transmission line. Therefore, when the variable condenser C_0 has been adjusted until resonance again occurs after the line has been shortened $\lambda/8$ m.,

$$Z_0 = \frac{1}{\omega C_0}, \tag{18}$$

the characteristic impedance of the transmission line. It should be remembered that the input reactance of the $\lambda/8$ portion is negative because the resonant frequency for the line of length l is considerably below the fundamental resonant frequency of the $\lambda/8$-length portion (see Fig. 13). Hence a variable condenser whose reactance is negative may be used to replace the removed portion. This method requires no resistances of known and noninductive value, and a high frequency calibrated condenser is all that is necessary in the way of known quantities besides the frequency. The potential distribution curve shown in Fig. 19 is for the third harmonic ($n = 3$) of the fundamental frequency.

9. Measurement of Attenuation Constant; Line Attenuation and Line Resistance.—A brief statement outlining the procedures of measurements will be given. Proofs of the measurement theorems will not be undertaken, because they involve a considerable portion of transmission-line theory which may be found in various texts.

Attenuation Constant of a Transmission Line.—The real values of currents at two points in a transmission line terminated in its characteristic impedance have the ratio

$$\frac{I_2}{I_1} = \epsilon^{-\alpha x},$$

where I_1 is the current at the input end, and I_2 is that at a point x distance from the input end. α is known as the *attenuation constant* of the line. From the foregoing,

$$\alpha = \frac{\log_{10} \frac{I_1}{I_2}}{0.4343x}. \tag{19}$$

For a radio-frequency transmission line, x may be taken in meters, and of course I_1 and I_2 are in amperes. The insertion of the meters to measure these currents is shown in Fig. 18.

Total Line Attenuation.—Sometimes radio-frequency non-resonant transmission lines are specified as having a total attenuation in decibels. The input power is $I_1^2R_0$, where R_0 is the input resistance and, of course, is the characteristic impedance of the line. The power output of the line is $I_1^2R_0$, because the line is terminated in its characteristic impedance, $Z_0 = R_0$, to be non-resonant. Hence the ratio of output to input is I_2^2/I_1^2. The attenuation in decibels is 10 $\log_{10} I_2^2/I_1^2$, or

$$db = 20 \log_{10} \frac{I_2}{I_1}. \tag{20}$$

The total attenuation of a line may be measured with a signal generator with attenuator, radio receiver, and output meter, as suggested by Thiessen.[1] With slight modification, the procedure is to connect a resistance R in series with the signal-generator output, as shown in Fig. 20; R should be adjusted such that $R + r_0 = Z_0$, the characteristic impedance of the line. A radio receiver having an input impedance about equal to the characteristic impedance of the line is then connected as shown, and the output meter indication noted for the modulated signal-generator output. Next, the line to be measured is interposed between the

[1] Described by C. C. Harris, *Proc. I.R.E.*, Vol. 24, No. 3, p. 425, March, 1936.

signal-generator attenuator output and the radio receiver, as shown below in the figure. The attenuator is then varied to increase the output until the output meter in the receiver again shows the original deflection. The line attenuation is then

$$db = 20 \log_{10} \frac{E_2}{E_1} \text{ decibels,} \tag{20a}$$

where E_1 and E_2 are the signal-generator output voltages for line out and in, respectively.[1]

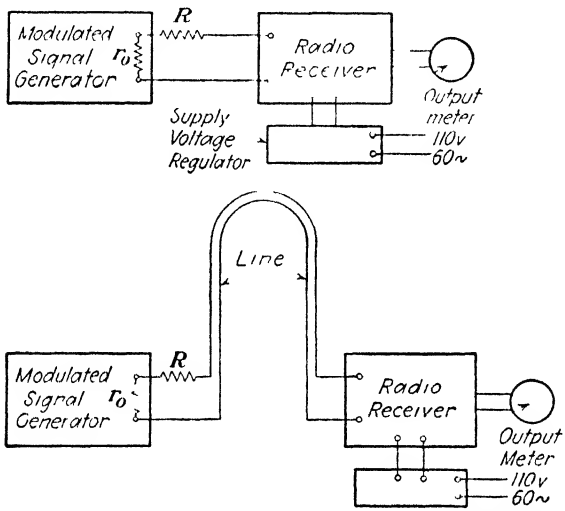


FIG. 20—Test arrangement for measuring total line attenuation. (*After Thiessen.*)

Measurement of Effective Line Resistance.—The ratio of the vector currents at a distance x and at the input end is

$$\frac{I_2}{I_1} = \epsilon^{-(\alpha+j\beta)x},$$

where $(\alpha + j\beta)$ is known as the *propagation constant* of the line. In line theory it is proved that

$$\alpha + j\beta = \sqrt{R_1 + j\omega L_1}\sqrt{g_1 + j\omega C_1} \tag{21}$$

where R_1 and L_1 are the effective resistance and inductance *per unit length*, respectively; and g_1 and C_1 are the conductance

[1] For discussion of decibel (db) measurement, see Chap. VI, Art. 10.

due to leakage and capacitance between wires *per unit length*, respectively. α is the attenuation constant which was also given by Eq. (20), and β is the wave-length constant. βx will give the angle that current I_2 will lag I_1. Using Eq. (21), it may be shown that for a well-insulated line with no dielectric loss in insulation between wires, and when the resistance is small compared to its inductive reactance, the attenuation constant is

$$\alpha = \frac{R_1}{2}\sqrt{\frac{C_1}{L_1}};$$

substituting $Z_0 = \sqrt{L/C} = \sqrt{L_1/C_1}$, and transposing,

$$R_1 = 2\alpha Z_0. \tag{22}$$

The effective resistance per meter, then, may be found by substituting the attenuation constant for the line of length x meters and the characteristic impedance, previously determined, in Eq. (22). This provides a convenient method by which the effective resistance of the line wire may be measured at the frequency of operation of the line.

CHAPTER IV

ELECTROMAGNETIC-WAVE MEASUREMENTS

1. Introduction. *Nature of Radiated Fields.*—The radiation of energy in the form of the electromagnetic wave results in the establishment of an *electromagnetic field intensity* in the medium through which the radiated energy is being propagated. The student will find interesting concepts of the mechanism of the electromagnetic wave in the ether, as well as classic mathematical analyses, in the numerous excellent treatises on electromagnetic waves and wireless telegraphy. This chapter deals with the problem of measuring the characteristics of wave propagation and of the available radiated energy at certain points in the field of influence of the radiator. It was said in early books that electricity is a mysterious agent known only by its effects, and this notion is particularly interesting in view of the technique required to obtain any noticeable effects of the existence of electromagnetic waves.

Useful Measurements.—The useful measurements are those which are made to determine the effects produced in receivers of electromagnetic waves. These very important devices take the various forms of receiving antennas which are so placed that they become equivalent to the load circuit electrically connected to a source of electric power. Potential differences and currents may be actually measured in the receiving antenna; hence it would be expected that the electromagnetic wave would create an impressed e.m.f. in the receiver (by means of electrostatic induction on the receiver antenna or by a change of the magnetic field linking its circuit). Maxwell and others proved that both magnetic field and electrostatic field intensities exist and that they are in time phase and in space quadrature. The measurement of the intensity of these fields is then of prime importance.

Starting with Maxwell's equations for the relations between magnetic and electrostatic field intensity, simple and useful relations between the field intensities and the various measurable quantities concerning the transmitting and receiving antennas have been derived. The effective value of the radiated magnetic field intensity at a distance d from an earthed transmitting antenna of effective height h_S and uniform sending current I_S is

$$H = \frac{2\pi h_S I_S}{10\lambda d} \text{ gausses,} \tag{1}$$

as given by Dellinger.[1] Also, the effective value of the radiated electric field is given as

$$\mathcal{E} = 300H = \frac{60\pi h_S I_S}{\lambda d} \text{ volts per centimeter.} \tag{2}$$

These expressions apply to a simple ideal flat-topped antenna; and the field intensities given by the foregoing equations are present at a distance d in a horizontal plane surrounding the transmitting antenna, theoretically at the surface of the earth. Expressions for the field intensities at a distance and an angle θ above the horizontal are much more complex, as given by Pierce.[2] Equations (1) and (2) show that the field intensities may be calculated if only the effective height, sending current at the base of the earthed transmitting antenna (current antinode), and wave length used are known. The effective height may be calculated or measured as explained in Chap. III. However, it is highly desirable to measure these field intensities directly. The current in a receiving antenna I_r at resonance may be expressed in a simple Ohm's-law relation:

$$I = \frac{E}{R} = \frac{h_r \mathcal{E}}{R} = \frac{300 h_r H}{R}, \tag{3}$$

where the receiving effective height times the electrostatic field intensity, measured in volts per meter, gives the total e.m.f. in

[1] DELLINGER, J. H., "Radio Transmission," *Trans. A.I.E.E.*, Vol. 38, Part 2, p. 1356, 1919. For an earthed vertical quarter-wave antenna

$$H = \frac{0.2\, I_S \cos\left(\frac{\pi}{2}\cos\theta\right)}{d_0 \sin\theta}$$

where d_0 is the distance from the earth terminal to point where H is taken and θ is the angle between the antenna wire and d_0.

[2] PIERCE, G. W., "Electric Oscillations and Electric Waves," p. 443.

the receiving antenna. The receiving effective height is ably discussed by Moullin and is shown to be equal to the transmitting effective height.[1]

While Eq. (3) holds for an earthed antenna, the case for a loop or coil receiving antenna is different. The induced e.m.f. in the coil for flux leakage ϕ at a frequency $\omega/2\pi$ is

$$E = \omega\phi \times 10^{-8} \text{ volt};$$

the flux linkage ϕ of the coil of N_r turns is

$$\phi = h_r L_r N_r H,$$

where L_r is the horizontal length of the coil (all dimensions in cms.).

$$I_r = \frac{\omega\phi}{10^8 R} = \frac{\omega h_r L_r N_r H}{10^8 R} = \frac{2\pi v}{\lambda}\frac{h_r L_r N_r H}{10^8 R},$$

where v, the wave velocity, is $3 \times 10^{10} = f\lambda$. The preceding equation then becomes

$$I_r = 600\pi\frac{h_r L_r H N_r}{\lambda R}. \tag{4}$$

This assumes that the dimensions of the coil antenna are small compared to the wave length and that the magnetic field intensity is uniform throughout the loop. Morecroft[2] finds the induced e.m.f. in the vertical wires of the coil antenna and obtains the expression

$$I_r = \frac{2h_r \mathcal{E} N_r}{R} \sin\frac{\pi S}{\lambda};$$

and when

$$\sin\frac{\pi S}{\lambda}$$

becomes very small, the sine equals the angle, and

$$I_r = \frac{2\pi S h_r \mathcal{E} N_r}{\lambda R}. \tag{5}$$

[1] MOULLIN, E. B., "Radio Frequency Measurements," pp. 441–444, 1931; see also CARSON, J. R. "Reciprocal Theorems in Radio Communication," *Proc. I.R.E.*, p. 952, Vol. 17, No. 6, 1929.

[2] "Principles of Radio Communication," 2d ed., p. 858.

Here S is the horizontal width of the vertical-planed coil and corresponds to L_r in Eq. (4). If desired, $\mathcal{E} = 300H$ may be substituted in the foregoing expression, making it similar to Eq. (4). Using the receiving earthed antenna, or coil antenna, it is possible to measure the magnetic field and electric field strengths by measuring the wave length, received current, resistance, dimensions, and receiving *effective height.* The general practice consists of introducing into the receiving antenna a known locally generated e.m.f., which will produce the same effect on a sensitive amplifier and detector as does the total e.m.f. induced in the receiving antenna. Now, the latter is $h_r\mathcal{E}$; and if the known locally generated e.m.f., for the same effect, is E, the two are equal, and

$$\mathcal{E} = \frac{E}{h_r} \text{ volts per meter.}$$

The electric field intensity is thus measured by a comparison method. It should be noted from Eq. (3) or (4) that it is also possible to produce a known magnetic field intensity H at the point of measurement and vary the value of this substituted field until the current in the testing or measuring antenna is the same as that produced by the unknown field to be measured. For equal response, the known and unknown are equal. The variable known field may be conveniently produced by a coil (transmitting) antenna, described in Chap. III, Art. 6. Following this general abstract of the principles involved in the measurement of field strengths, the description and procedure of a few representative methods will be given.

The Meter-ampere.—An examination of transmission formulas, whether from an open earthed transmitting antenna or from a transmitting coil antenna, shows that the received current in any sort of a receiving antenna is directly proportional to the current in the transmitting antenna and to the effective height of the latter. Therefore, the received current is

$$I_r = h_S I_S(F_0),$$

where h_S and I_S are effective height and maximum current of the transmitting antenna, respectively. The factor F_0 contains other terms including other dimensions of both transmitting and receiving antennas, the distance between them, and the wave length (or frequency). These will be assumed as constant, and the figure of merit of the radiated field strength, as regards its effect on the receptor, is expressed by the product $h_S I_S$, or, as it is commonly known, the *meter-ampere* values of the radiation of electromagnetic wave intensities. The effective height h_S is measured by methods described in Chap. III, and the current I_S is the current at the base of the open transmitting antenna, or the current in a coil transmitting antenna, assuming that the current is constant in all wires of the loop or coil.

2. Measurement of Field Strength by Coil-antenna Receptor. *Fundamental Principles.*—The principles underlying this method are almost identical with some of those discussed in the Introduction to this chapter. A few modifications that are used will be explained. The system about to be described was developed by the engineers of the Western Electric Company, and their technical papers describing the principles, apparatus, and procedure will be referred to presently. Referring to the development steps for Eq. (4), the e.m.f. induced in the coil antenna with its plane in the direction of propagation of the field H is

$$E = 2\pi f N_r h_r L_r H \times 10^{-8} \text{ volts.}$$

The product $h_r L_r$ is the area A, in square centimeters, of the opening in the coil antenna, then

$$E = 2\pi f N_r A H \times 10^{-8}.$$

The electric field is $\mathcal{E}' = 300H$ volts per centimeter, when H is measured in gausses. Substituting for H in the expression for E,

$$E = \tfrac{2}{3}\pi f A N_r \mathcal{E}' \times 10^{-10}.$$

It is customary to express the electric field strength in microvolts per meter, so the foregoing expression becomes

$$E = \tfrac{2}{3}\pi f A N_r \mathcal{E} \times 10^{-18} \text{ volt,} \qquad (6)$$

where $\mathcal{E}$ is the electric field intensity in microvolts per meter, A the area of the coil, and N_r the turns. The assumption is made that the e.m.f. per unit length of wire in the loop times the length of wire would give the voltage E, or that $\mathcal{E} h_r$ gives this value. In a paper by Bown, Englund, and Friis,[1] the authors mention

[1] BOWN, R., C. R. ENGLUND, and H. T. FRIIS, "Radio Transmission Measurements," *Proc. I.R.E.*, Vol. 11, No. 2, p. 122.

the fact of an error in measurements from this assumption but state that the probable error from their calculations is too small to warrant any correction. Then, from Eq. (6),

$$h_e = \tfrac{2}{3}\pi f A N \times 10^{-12} \text{ meters.} \qquad (7)^1$$

This is a form of the effective height of the receiving coil and may be determined by calculation. Then the voltage induced in the coil is determined by substituting a variable known locally generated voltage in the loop circuit until the same output is obtained from the amplifier detector connected to the coil as was obtained for the e.m.f. induced by the field to be measured. Then the electric field strength is

$$\mathcal{E} = \frac{E_m}{h_e} \text{ microvolts per meter,}$$

where E_m is in microvolts.

A modification for convenience is used as described in Jensen's paper referred to. From Eq. (6) the field strength is

$$\mathcal{E} = \frac{3E \times 10^{+18}}{2\pi f A N_r} \text{ microvolts per meter}$$
$$= k\frac{E}{f},$$

where E is measured in volts. To do this, a 1-ohm resistance is inserted in the coil antenna, and the current in this resistance is measured in amperes (or milliamperes). E in volts (or millivolts) is then numerically equal to the current. The constant k may be calculated or measured by testing in a known field strength. The variable condenser that is used to tune the coil antenna to the incoming frequency may be calibrated directly in frequency values. The description of two complete coil-antenna receptors, with comparison voltage generators, will now be outlined.

Description of Apparatus; Application of Principles; Procedure.—While some of the apparatus arrangement described by certain writers may at times seem unduly complex and require a great deal of preliminary calibration work, it is certain that these particular arrangements are the result of careful study and long

[1] JENSEN, AXEL G., "Portable Receiving Sets for Measuring Field Strengths at Broadcast Frequencies," *Proc. I.R.E.*, Vol. 14, No. 3, p. 343. Note that nomenclature is different from that in this book.

experience and therefore deserve careful study by the student or engineer inexperienced in this particular line of work. This description and discussion will, therefore, be devoted only to some of the field-strength measuring equipment that has actually proved successful in practice. (During the last fifteen years, many papers on the description of field-strength measuring apparatus have been written, and all are deserving of description. But the lack of space makes it imperative to limit this reproduction of the work to a very few representative types.) Bibliography on other types of equipment is to be found in the footnotes on pages 2 and 6 of the Bell Telephone Laboratories, Reprint B-316; and on pages 492 and 493 of the *A.I.E.E. Trans.*, Volume 46, p. 492, 1927.[1]

Description of Jensen Method.—An improved convenient type of field-strength measuring equipment is described by Jensen.[2] Its outstanding features will be briefly outlined. The author points out that the required values of inductance and resistance of the loop antenna are sources of difficulty, especially since the resistance was found to vary with the weather conditions. In order to eliminate the necessity of knowing the values of L and R for the loop, this author placed the input shunt in the middle of the loop coil, as had already been previously done by Bown, Englund, and Friis for long-wave measuring apparatus.[3] This connection of the 1-ohm resistance in the middle of the loop is shown in the schematic diagram of connections of Fig. 1. Current passing through this resistance thus introduces the comparison voltage at the middle of the loop, and the same step up of the e.m.f. across the tuning condenser due to resonance

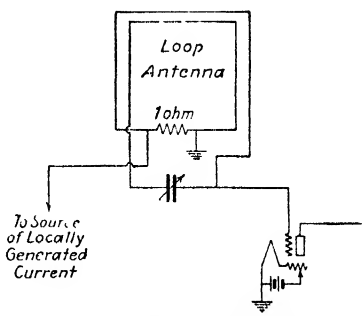


FIG. 1.—Diagram of receiving loop for field-strength measuring apparatus.

[1] For discussions on bibliography of methods used in England and continental Europe, see E. B. Moullin, "Radio Frequency Measurements," 1931 ed., Chap. X.

[2] JENSEN, AXEL G., "Portable Receiving Sets for Measuring Field Strengths," *Proc. I.R.E.*, Vol. 14, No. 3, pp. 333–344, 1926.

[3] *Proc. I.R.E.*, Vol. 11, No. 2, p. 126, 1923.

is present as for that resulting from the effect of the field to be measured. In this, as in other types of field-strength measuring equipment, it is desired to impress upon the amplifier receiver device an e.m.f. from the incoming wave to be measured, and a known e.m.f. from a local comparison oscillator or signal generator. In order to create the e.m.f. due to a wave passing by the loop or coil antenna, the latter is tuned to resonance with the frequency of undulation of the incoming wave. This is accomplished with a variable condenser connected across the loop. Now, the e.m.f. induced in the loop by the wave is induced in a series circuit comprising L and R of the loop, and C of the tuning condenser. At resonance the voltage across the condenser is greater than that induced in the loop turns, as it is in any series circuit at resonance, and the ratio of the voltage across the condenser to that actually induced in the loop is known as the *resonance step up*. Hence, when the local comparison and the field induced e.m.fs. are alike, the output indications of the second detector are alike, and Eq. (6) is used, from which the field intensity is

$$\mathcal{E} = \frac{3 \times 10^{12}}{2\pi A N_r} \cdot \frac{\alpha I}{f} = K \cdot \frac{\alpha I}{f} \tag{8}$$

in microvolts per meter, if $E = \alpha I$ millivolts, and the current I is the current in the 1-ohm resistance in milliamperes; and α is the attenuation ratio of the special potentiometer system used. A complete diagram of the apparatus arrangement and circuits used by Jensen is shown in Fig. 2. The current I is obtained by using a special potentiometer current divider labeled "Input Box" in the figure. The values of resistance units making up the potentiometer are shown also. Thermocouple $T.C.$, with millivoltmeter G, measures the input current I, from which the actual current $i = \alpha I$, flowing through the 1-ohm resistance at the output end, can be found for the value of α corresponding to the position of the tap-switch touching points between the resistance units. As the tap-switch contact is moved from left to right, beginning at the second tap the values of α are 0.0001, 0.0002, 0.0005, 0.001, 0.002, 0.005, 0.01, 0.02, 0.05, 0.1, 0.2, 0.5, and 1.0. This potentiometer must be carefully constructed for radio-frequency currents, and its construction is given by the author referred to. Figure 2 is reproduced from two diagrams of Jensen's paper. The extra-heavy double shielding of the local comparison generator is noted. The range of this equipment is given as from 20 to 30,000 microvolts per meter for frequencies of 500 to 1,200 kc. Above these frequencies the resistances of the

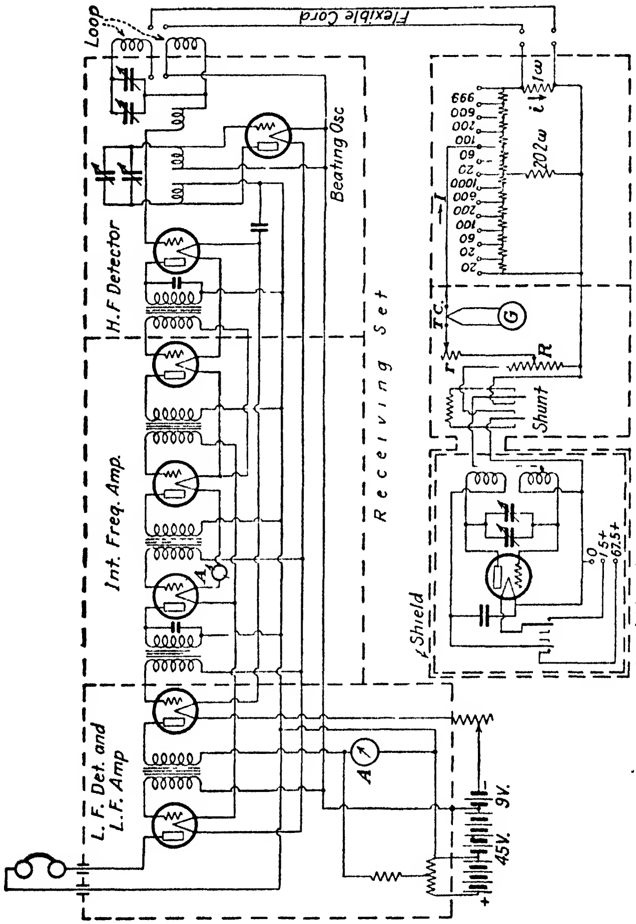


FIG. 2.—Field-strength measuring apparatus and circuits. (*After Jensen.*)

potentiometer become uncertain. Important points of detail of construction and precautions in arrangement and position of circuits cannot be given here.

The operation of this set is very simple and rapid. The signal is tuned in with the loop antenna properly oriented and the beating oscillator properly adjusted for maximum response; the level of the output is noted as indicated by a suitable microammeter. The local signal oscillator is turned on and adjusted to *the same frequency* as the incoming signal wave by zero beating, after which the loop is turned 90 deg. from its former position in order to cut out the e.m.f. induced in the loop by the field. The dial switch of the potentiometer is then adjusted until an equal output level is indicated, showing that the comparison voltage αI [Eq. (8)] is equal to the voltage induced in the loop by the electric field. The calibration constant K in Eq. (8) may be determined once for all by calculation, and the loop-tuning condenser may be calibrated directly in kilocycles by test. Then $\mathcal{E}$, the field intensity in microvolts per meter, is determined by applying the same equation.

In the manipulation of the apparatus of Fig. 2, the output is read by noting the current change in meter A in the second detector plate circuit when the input e.m.f. is applied. This change is very small and is magnified by using the compensation circuit, shown connected across the meter, to reduce the actual steady meter current to a small value. Meter A can then be a low-range milliammeter or a microammeter, and at normal input changes of a half or more of complete scale range of deflection will be obtained.

Modification of the Jensen Apparatus.—A glance at Fig. 2 will reveal that the local signal oscillator and input box really constitute the commonly known signal generator. These are readily available in commercial form and are guaranteed to give known output voltages from 1 microvolt to 0.1 volt or more for a wide range of radio frequencies. Using one of these should extend the range of the previously described apparatus to higher frequencies than those of the standard broadcast range for which it was originally designed. The resistance looking into the output terminals of many such commercial signal generators is from 5 to 50 ohms, and one should be preferably used that has a resistance of not over 10 ohms to insert into the middle of the receiving loop. Also, in Fig. 2 the receiving set is similar to the commercially available superheterodyne type. If such a receiver is used, the grid of the first radio-frequency amplifier tube is disconnected from the input tuned circuit and connected to the loop terminal. In doing this, care must be used not to change the effective bias voltage on the first tube. For such use a high-grade amateur receiver is admirably suited. It is housed in a metal cabinet which shields against magnetic feedback to the loop; it has a beat oscillator to aid in tuning in the wave when it is not modulated; and it often has a carrier-wave level indicator, known as an *R meter*. This latter takes the place of the usual output meter. If a diode detector is used, a suitable range milliammeter may be inserted in the diode circuit. The automatic volume control should be turned off, as may be done with merely a switch provided in such a receiver. A rectified symmetrically modulated carrier wave gives a current whose average value is constant and does not change for any degree of modulation up to 100 per cent. If the wave is overmodulated, this value will change. A schematic diagram similar to Fig. 1 showing the use of a standard

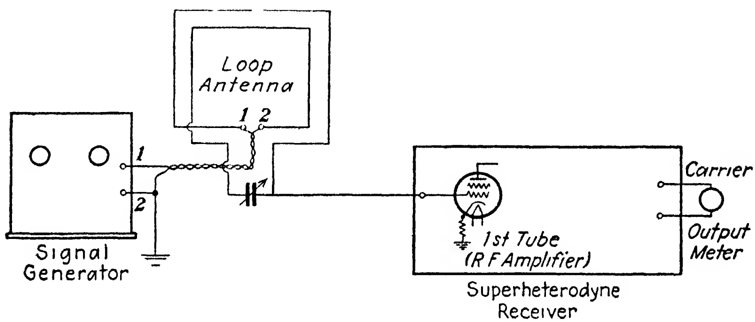


FIG. 3.—Schematic of modified Jensen-type field-strength measuring apparatus.

signal generator is shown in Fig. 3. Although commercial signal generators are usually well shielded, it is well to determine by experiment whether pickup in the receiver from the signal generator is serious. If so, it is often possible to shift the position of the latter to eliminate such pickup. The construction of a modern type of signal generator is given in Chap. VIII, Art. 3.

Fixed Inductively Coupled Attenuator Type and Its Use.—Recently a type of field-strength measuring equipment using a coupled-coil attenuator has been developed. The following description and explanation concern apparatus readily constructed, assembled, and calibrated in the laboratory and are also conveniently arrangeable for field work.[1] The method is best

[1] The author is indebted to W. E. Phillips, chief engineer of radio station WILL, University of Illinois, for this apparatus and method. It is somewhat similar to the R.C.A. equipment, and the intermediate-frequency attenuator and method of introducing local e.m.f. were first used by Friis and Bruce; their system will be described later.

understood by reference to the schematic diagram of Fig. 4. The local comparison voltage is obtained from the oscillator shown whose output can be varied by plate-battery circuit series variable resistance R_B. The three- or four-turn coil L_c is magnetically coupled to the oscillator-plate tank coil but electrostatically shielded from it. This coupling coil feeds the primary of the inductively coupled attenuator *A* whose secondary is inserted in series with the center of the receiving loop. Detail of attenuator *A* is shown at the right of the figure. A thermocouple *TC* is connected across the primary of the attenuator and serves to measure the voltage input when the current in the heater of

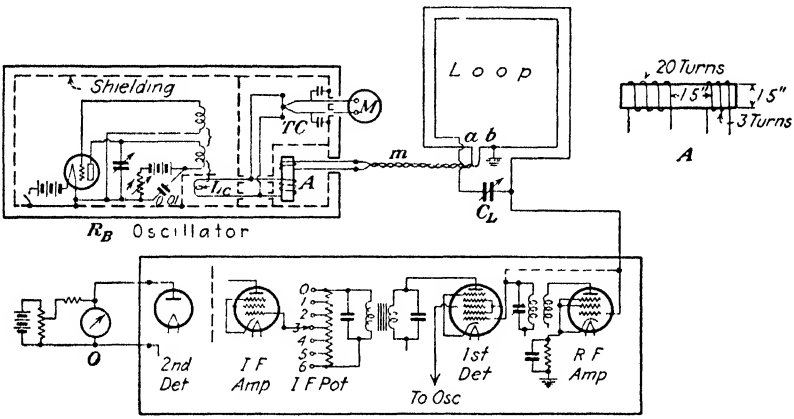


FIG 4.—Laboratory field-strength measuring apparatus.

T.C. is multiplied by its resistance. The middle of the loop is grounded, and one end is connected to the grid of the first tube of a syperheterodyne receiving set. A potentiometer with 6 to 10 steps with switch and contacts is connected as shown between the intermediate-frequency tuned circuit and the grid of the amplifying tube following. The resistance of this potentiometer should be about 0.2 meghoms. If a triode second detector is used, a plate-circuit milliammeter provided with compensating part of the plate current should be provided. For a diode detector a milliammeter (low range) is sufficient. The second detector and meter serves as a vacuum-tube voltmeter.

The calibration tests will now be explained. The only difficult determination is that of measuring the small secondary voltage of the attenuator A at radio frequencies. If a modern signal generator is available, the receiving-set input lead is disconnected from the loop terminal and connected to point ab in the loop, the receiver input being placed across the attenuator secondary at the end of the flexible, loosely twisted cable m. The oscillator is turned on, R_B adjusted for a desirable deflection of M, and the loop-tuning condenser C_L is varied until maximum deflection is noted on output meter O. The receiver input is then transferred to the signal-generator output, and the input voltage to the receiver, for the same receiver output as for connection to ab is found. This is the secondary voltage of attenuator A corrected for resonance step-up effect of the tuned-loop circuit. At the same time the primary input voltage to A is measured by the IR drop through thermocouple $T.C.$ For a certain range of radio frequencies, the ratio of primary input voltage of attenuator A to the secondary output voltage will be practically constant. For the attenuator design shown in the sketch A of Fig. 4 it was found to be about 110 for 550 to 1,500 kc., loop belonging to the Western Electric type 4B receiver being used. The primary input voltage $E_P = \omega L_P I_P$, practically, and the secondary induced voltage is $E_S = \omega M I_P \cdot E_P/E_S = L_P/M$, independent of frequency. At point ab, E_{ab} at resonance $= E_S \frac{\omega L}{R}$, where $\omega L/R$ is the secondary Q; ωL and R increase alike with frequency. This holds true until the frequency becomes so high that distributed capacitance effect in the coils of A are important. Spacing the turns slightly will reduce this effect.

The second step is the calibration of the potentiometer attenuator in the intermediate-frequency amplifier of the superheterodyne receiver. Attenuation ratios may be determined very readily. The first detector is a square-law weak-signal type, and amplitude of the difference frequency (between input and local oscillator frequencies) is directly proportional to the receiver-input value on the detector dynamic characteristic.[1] For this calibration the signal generator may again be used to obtain known inputs; and if one is not available, a near-by transmitting loop carrying a known variable current may also be used, using

[1] MORECROFT, J. H., "Principles of Radio Communication," 2d ed., pp. 565–567: also GLASGOW, R. S., "Principles of Radio Engineering," p. 370.

the receiving loop connected to the receiver. To measure the attenuation at point 1 on the intermediate-frequency potentiometer attenuator (Fig. 4), the switch point is first set to point O, and the input voltage adjusted to give a small reading of the second-detector output meter. The potentiometer switch is then changed to position 1, and the input voltage increased until the same output-meter reading is obtained. The attenuation ratio is then the ratio of the latter input voltage to the former, or it is the ratio of corresponding currents which produce the input voltages. In like manner the attenuation ratio for each potentiometer-switch position may be determined.

The next step is to determine the manner in which the deflection of the second-detector output meter O varies with input voltage. This will usually be found to be linear, but it is well to check this by varying the input voltage to the receiver in known proportions and observing the output-meter deflections. This second detector is really a vacuum-tube voltmeter, but it is not necessary to know its calibration in volts output versus volts input; merely relative values are needed.

Calibration is then complete, and the procedure in measuring the field strength of a received wave follows. The loop (antenna) is rotated so that its plane is parallel to the direction of the source of the wave so as to get the maximum induction in the loop. The loop-tuning condenser C_L (Fig. 4) is varied until resonance is obtained; the receiver is tuned to resonance; and the intermediate-frequency potentiometer attenuator is adjusted to give a sizable reading on the second-detector output meter O. The attenuation ratio of the intermediate-frequency potentiometer is α_1 for this adjustment. The loop is then rotated to a position at right angles to its first position, and the deflection of the output meter due to the incoming wave disappears. The oscillator is then turned on, and its frequency is adjusted until a deflection of the output meter indicates resonance with loop and receiver tune. The oscillator-output voltage will be in the neighborhood of, say, 30 milliamp. times 5 ohms thermocouple heater resistance divided by the attenuator A factor 110 and multiplied by the resonance stepup in the loop-tuned circuit. Thus a value of some 10 millivolts may appear at ab in the loop. This may be much larger than the voltage induced in the loop by the wave being measured; hence the potentiometer attenuator in the intermediate-frequency amplifier is adjusted until a readable deflection appears on the output meter. For this setting the attenuation ratio is α_2. If adjustments of oscillator output by means of R_B and of the potentiometer can be made so that the output-meter deflection is the same as for the incoming wave, the induced voltage in the loop is equal to the voltage at *ab* produced by the oscillator, but it is not necessary to have them alike, and it may not be possible to do so. If the deflections are different, the calibration curve of the second detector as an output voltmeter can be used to estimate the actual voltage induced in the loop by the incoming wave. When the loop is turned so as to receive the incoming wave, the output voltage delivered to the second detector is

$$e_1 = \frac{ke_x}{\alpha_1} \text{ microvolts,}$$

where e_x is the e.m.f. generated in the loop by the wave, and k is a constant. When the local oscillator is turned on, producing the known voltage e_c at *ab* in the loop (Fig. 4), the delivered voltage is

$$e_2 = \frac{ke_c}{\alpha_2} \text{ microvolts;}$$

e_c is determined from the oscillator output and constant of attenuator A, as explained previously. From the foregoing relations,

$$e_x = e_c \frac{\alpha_1}{\alpha_2} \frac{e_1}{e_2}.$$

The values of e_1 and e_2 are then found from the second-detector vacuum-tube voltmeter-calibration curve. If it is linear, e_1/e_2 is simply the ratio of the corresponding deflections. Using the preceding relation, the field strength in microvolts per meter is

$$\mathcal{E} = \frac{e_x}{h_e}, \tag{8a}$$

where h_e is the effective height of the *closed-loop* antenna in meters, as given in Eq. (7). It is well to remember that the same resonance step-up effect is present for both e_c and e_x when tuned to resonance in the case of each input voltage, so that no correction for this effect is necessary. There is no likelihood of the constant k changing or of the linear relation between the input voltage and intermediate-frequency output of the second detector being disturbed. It has been found that the input voltage may be increased to 1 volt without destroying this law of linearity.[1] There are many possible modifications of the apparatus described. If it is desired to use a complex attenuator which will attenuate the intermediate-frequency output in predetermined ratios, a constant-impedance type must be used so as not to vary the plate-circuit load on any of the tubes in the receiver, as this will change the amplification. If such an attenuator is used, it is well to calibrate its attenuation ratio in the manner previously described.

Fundamental Field-strength Measurement Apparatus for Very High Frequencies.—At radio frequencies above 1,500 kc., the production of an accurate local comparison e.m.f. becomes a perplexing problem; in fact it is a practical impossibility to design a known standard resistance, free from spurious shunt admittances. It is also difficult to eliminate the high-frequency pickup from the local comparison generator so that the comparison readings are due only to the drop across the 1-ohm resistance. An excellent paper describing the fundamental principles and development of a form of field-strength measuring apparatus, which functions at frequencies of from 1 to 30 Mc., was prepared by H. T. Friis and E. Bruce[2] of the Bell Telephone Laboratories.

While it is not within the scope of this book to describe fully the final measurement equipment, a very brief abstract of the principle involved will be given; it is taken in modified form from the paper referred to above. Figure 5, also taken in modified form from the paper, shows the principle of introduction of either the unknown signal e.m.f. E_x from the loop or the much more intense local comparison e.m.f. E_0 and the output of the beating oscillator E_B. The intermediate-frequency detector, usually called the *first detector*, is shown having an output voltage E_I. This detector tube is also calibrated as a vacuum-tube voltmeter,

[1] *Proc. I.R.E.*, Vol. 14, No. 4, p. 515, 1926.

[2] Friis, H. T., and E. Bruce, "A Radio Field Strength Measuring System," *Proc. I.R.E.*, Vol. 14, No. 4, pp. 507–519, 1926; also in Bell Telephone Laboratories, Reprint B-209.

readings being taken of meter A_1 in its plate circuit for this purpose. The attenuator is calibrated in ratios of its impressed e.m.f. to its output e.m.f. The output of the attenuator excites the grid of the first tube in the 300-kc. amplifier; this latter excites the low-frequency detector and actuates its plate-circuit meter A_2. When the loop is receiving, the signal wave and beating oscillator on, the output voltage of the first detector is

$$E_1 = kE_xE_B,$$

according to the theory of the heterodyne detector.[1] E_1 is the difference- or beat-frequency, component of the alternating plate-circuit e.m.f. The attenuator is adjusted until a convenient

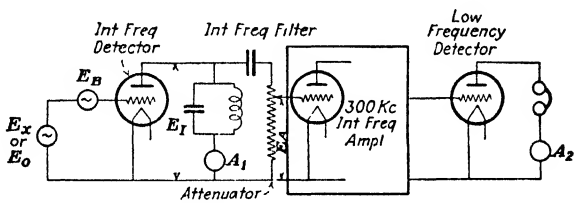


FIG. 5.—Fundamental circuit of field-strength measuring system. (*After Friis and Bruce.*)

deflection appears on meter A_2 and if the attenuation ratio is α_1 for this deflection,

$$E_A = \frac{kE_xE_B}{\alpha_1}. \tag{9}$$

The next step consists of substituting the e.m.f. E_0 of the local comparison oscillator for E_x at the same frequency but much larger in value; *i.e.*, $E_0 >> E_x$. It must be sufficiently large to give a reading on meter A_1, so that from the calibration of the first detector the value of E_0 in volts is obtained. During the measurement E_B is turned off. Next, E_B is again turned on, and the attenuator varied until the same reading of A_2 is obtained as for the first adjustment. If this second adjustment of the attenuator is α_2,

$$E_A = \frac{kE_0E_B}{\alpha_2}. \tag{9a}$$

[1] *Loc. cit.*, footnote, p. 191.

From Eqs. (9) and (9a), and dividing by the loop step up ratio β,

$$E_x = E_0\frac{\alpha_1}{\alpha_2\beta}. \tag{10}$$

From the value of E_x and the equation for the induced voltage in terms of dimensions and frequency (Eq. 6), the field strength in microvolts per meter can be determined. While the fundamental principle is simple and direct, the practical application is more difficult, due to the high frequency involved. The realization of an accurate workable equipment is attained after several important modifications which are brought out in detail by the authors Friis and Bruce, and the student should read the entire

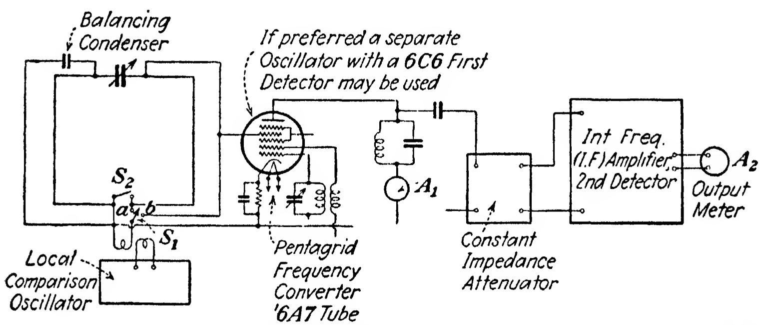


FIG. 6.—Schematic diagram of field-strength measuring equipment for high radio frequencies.

paper on this radio field-strength measuring system before attempting the assembly of such a measuring equipment. Englund and Friis[1] have outlined the problem of measuring field strength at various ranges of frequencies. In their paper they also give the fundamental principle regarding the introduction of local comparison voltage into the loop antenna by mutual induction instead of with the 1-ohm resistor. At frequencies above 1,500 kc., it is necessary to introduce this e.m.f. by mutual induction, for reasons previously mentioned.

A modification of the final form of the plate-modulated type of Friis and Bruce apparatus is illustrated diagrammatically in Fig. 6. Here the input-loop circuit and the introduction of the local voltage inductively into the center of the loop are due to the

[1] ENGLUND, C. R., and H. T. FRIIS, "Methods of Measuring Radio Field Strengths," *Trans. A.I.E.E.*, Vol. 46, pp. 492–497, 1927.

two authors just mentioned. Such a measuring equipment may be built up in the laboratory; and if so done, it is best to build it as a unit, preferably on rack-and-panel mounting rather than trying to use separately contained oscillator, superheterodyne receiver, etc. The loop, consisting of only two or four turns, wires separated one inch or more, is mounted directly at the top of the panel-mounted assembly, as shown in illustrations in Friis and Bruce's paper.

Referring to Fig. 6, a balanced loop is used, placing the tuning condenser at the top for symmetry. The beating oscillator is introduced as indicated and is not affected by tuning the loop. The operation is similar to that previously described except that the resonance stepup of the loop is found by throwing s_1 to b and then closing s_2, placing the local comparison voltage directly on the first-detector grid and noting the value of this voltage from the deflection of calibrated meter A_2. Then s_1 is thrown to a, s_2 is opened, and the deflection of A_2 again noted. The increase is due to *half* the resonance step up of the tuned loop circuit, and twice the ratio of the two voltages on the grid is equal to the resonance step ratio and is used in Eq. (10).

3. Field-strength Measurement by Other Methods. *Open-antenna Method.*—This method makes use of the mutual inductance to introduce the calibrating e.m.f. and uses the first detector, also, as a vacuum-tube voltmeter. The principle of this method is given by Friis and Bruce,[1] in their paper which also dealt with coil-antenna measuring sets and was previously quoted. Their method of using the first detector as a tube voltmeter with which to measure the strong locally generated comparison voltage is discussed in Art. 2. Figure 7 shows a similar circuit and apparatus schematically, to illustrate the method. The method may be briefly described as follows: With switch S to the left, and G open, the signal is tuned in with the aid of condenser C, and the beating oscillator adjusted for a desired beat note. It should be pointed out that the input of the beating oscillator is in the plate circuit of the first detector and that the balancing condenser BC neutralizes the first tube so that it will not feed back into the antenna circuit. (A screen-grid tube may be used also.) The attenuator is adjusted in order to obtain a suitable deflection on the output meter. The local comparison

[1] *Loc. cit.*, footnote, p. 196.

oscillator is then turned on, adjusted to the same frequency as the signal e.m.f. by zero beats; the beating oscillator is then shut off; and the locally generated comparison voltage increased until a reading is obtained on meter *A*, this being the indicator for the first detector tube when used as a calibrated voltmeter. It will be noted that the local voltage is introduced conveniently by means of a mutual inductance *M*. When the local comparison voltage is obtained, the beating oscillator is again started, and the attenuator adjusted to obtain the same output-meter deflection, as in the case of reception of the signal wave. It is not necessary to shut off the incoming waves during the use of the comparison oscillator, since the latter is much more powerful

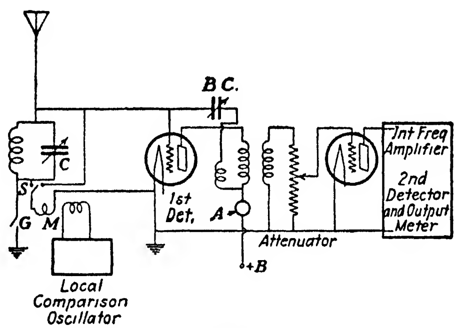


FIG. 7.—Schematic circuit of field-strength measuring system at high frequencies, using open antenna. (*After Friis and Bruce.*)

than the effect of the signal wave. Finally, switch *S* is thrown to the right, and *G* is closed; the local comparison voltage is now placed directly on the first detector grid, and the attenuator again adjusted for the same output-meter deflection. This latter step gives a correction for any resonance step up or trap effect and makes it possible to compare experimentally the local comparison voltage with antenna-induced voltage acting on the tuning circuit and detector.

The formula for the field strength in microvolts per meter will now be derived. When the signal wave is tuned in, and a reading obtained as explained above, the intermediate amplifier amplifies the difference frequency of two beating radio frequencies. The introduction of the beating frequency (by the beating oscillator) into the plate circuit of the first detector results in the production of beats because the plate-current characteristic is nonlinear, and the fact that the amplitude of the difference-frequency component is directly proportional to the product of the two component-frequency amplitudes may be proved in a way similar to that referred to in Art. 2. Then, for the case of the tuned-in signal wave of the field to be measured, the output-meter reading is

$$A_0 = \frac{kE_x'E_B}{\alpha_1}, \tag{11}$$

where E_x' is the voltage impressed upon the detector by the antenna. It is not the total induced voltage in the antenna because of resonance effects, etc. E_B is the voltage of the beating oscillator, and α_1 is the attenuation ratio required for a readable deflection A_0. When the local comparison voltage is substituted for E_x', it may be called E_0; but, as in the case for the antenna-induced voltage, it is not the voltage impressed on the first detector tube. This latter portion is E_0'. The relations between the actual voltages impressed in series with the antenna tuned circuit and directly on the tube input may be expressed thus:

$$E_0' = BE_0, \text{ and } E_x' = BE_x,$$

where B is a constant which takes care of resonant step-up effects, etc. From the foregoing relations,

$$\frac{E_0'}{E_x'} = \frac{E_0}{E_x}. \tag{12}$$

Coming back to the reading obtained when the comparison voltage is placed in series with the antenna and ground circuit, E_0' is impressed on the first tube, and the attenuation is α_2, or

$$A_0 = \frac{kE_0'E_B}{\alpha_2}. \tag{13}$$

Lastly, when the antenna tuned circuit is shorted by switches S and G and E_0 thrown directly on the first tube,

$$A_0 = \frac{kE_0E_B}{\alpha_3}. \tag{14}$$

It is convenient to take a reading of E_0' by means of the meter in the plate circuit of the first tube, used as a calibrated voltmeter.

Then E_0' is known, and solving Eqs. (11), (12), (13), and (14) for E_x,

$$E_x = E_0'\frac{\alpha_1\alpha_3}{\alpha_2^2} \text{ volts;} \tag{15}$$

E_x is the total voltage induced in the antenna by the field to be measured. The field strength is then

$$\mathcal{E} = \frac{\alpha_1\alpha_3 E_x \cdot 10^6}{\alpha_2^2 \cdot h_r} \text{ microvolts per meter,} \tag{16}$$

where h_r is the effective height of the measuring antenna in meters. The attenuation ratios are the ratios of the voltage input to the potentiometer (attenuator) of Fig. 7 to its voltage output.

For any antenna or set of antennas used, the effective heights could well be determined experimentally by creating a known field, a few wave lengths away from the standard transmitting coil antenna described in Chap. III. It would seem as if this method would be satisfactory for frequencies as high as for the coil-antenna equipment previously described. The first detector could be calibrated at these high frequencies. A known current passing through a condenser of known capacitance on small-diameter high-resistance wire gives voltage for calibration; or a calibrated mutual inductance may be used to provide a known voltage ωMI to the tube, where M is the mutual inductance, and I is the current in its primary coil. It is often said that most field-strength measuring equipment has a probable accuracy of not more than 20 per cent; but if the comparative values may be relied upon to an accuracy of 5 per cent, the value of such measurements is unquestioned. If the experimenter will establish a calculated field strength near a standard transmitting-coil antenna, as has been suggested several times previously, and later measure it with a carefully made measuring equipment, using a loop or open receiving antenna of such a type that its effective height may be calculated, and finds that the comparative measurements check within 10 to 20 per cent, he can be satisfied that the measuring set is fairly reliable.

Calibrating Receiving System with Aid of Transmitting-coil Antenna.—A receiving antenna may be connected to a radio-receiving set having a carrier-wave indicator, such as a second detector calibrated as a vacuum-tube voltmeter or an R-meter or other means. At a convenient distance a transmitting-coil antenna or loop is set up and excited by a suitable oscillator. From the current in the loop and its dimensions the field strength at the receiving antenna a known distance from the loop may be calculated. In Eq. (9), which appears in Art. 6, Chap. III, the current is

$$I_r = \frac{\mathcal{E} h_r}{R},$$

where $\mathcal{E}$ is the electric field in volts per meter when I_r is in amperes.

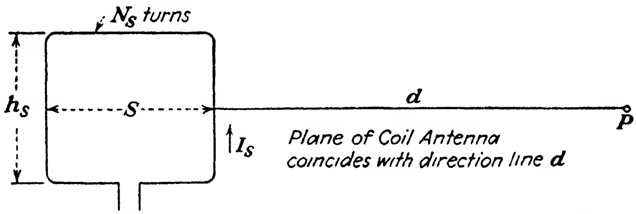


FIG. 8.—Transmitting-coil antenna or loop and dimensions.

Then, substituting this relation in the current equation referred to, it is evident that

$$\mathcal{E} = \frac{376 N_s h_s I_s}{\lambda d} \sin \frac{\pi_s}{\lambda}. \tag{17}$$

The loop dimensions are shown in Fig. 8. λ, the wave length, and the other dimensions are in meters. In the figure the open (or loop) receiving antenna is at point P, and by means of Eq. (17) the field strength at P may be calculated. The special oscillators and transmitting loops which produce the known field intensities are called *field generators* by Schelling, Burrows, and Ferrell.[1] These authors recommended that for ultra-short wave lengths the field generator be mounted several meters length above the earth and about one-half-wave length from the receiving antenna, the latter being a half-wave type at the same height above ground when calibrating on a 3- or 4-meter wave length. This was found to prevent errors due to proximity of the earth at such short wave lengths. For this case they also give the formula for field

[1] *Proc. I.R.E.*, Vol. 21, No. 3, pp. 430–431, March, 1933.

strength radiated from the loop as

$$\mathcal{E} = \frac{120\pi^2 NAI}{\lambda^2 d}\left(1 - j\frac{\lambda}{2\pi d}\right), \tag{18}$$

where $\mathcal{E}$ = field strength in volts per meter.
N = number of turns in loop.
A = area of loop, square centimeters.
I = current in loop in amperes.
d = distance between loop and receiving antenna in meters.
λ = wave length in meters.

Field-intensity Meters and Their Use.—Very useful devices for indicating comparative field intensities in the neighborhood of transmitting antennas have come into wide use and will be briefly described. The simplest form consists of a coil and shunt variable condenser having a grid leak-grid condenser-type vacuum-detector input connected in shunt just as in the ordinary tuned-detector input circuit. In this device a small milliameter is placed in the detector-plate circuit, and a short vertical *pickup* rod has its lower end connected to the grid end of the tuned circuit. When an electric field is present, it generates an e.m.f. in the rod which excites the tuned circuit into resonance, causing the plate milliammeter reading to decrease, the amount of decrease depending on the field strength. The indication is not linear.

A more sensitive and improved type with a linear response is shown in Fig. 9. The construction of this device is described in detail in the "Radio Amateur's Handbook."[1] This device is a type of tube that will function as a diode detector and direct-current amplifier. The normal plate current is about 1.4 milliamp.; and when the pickup rod intercepts a wave, the plate current drops, the decrease being a linear function of existing field strength. A value of the latter less than 1 millivolt per meter will cause a noticeable decrease in plate current. This and similar instruments may be made in portable form and used to explore field patterns in the neighborhood of a transmitting antenna. They are also useful in determining when an antenna such as the Hertz type is in resonance, as the excitation frequency is varied.

[1] See p. 340, 1937 ed., published by American Radio Relay League.

4. Logarithmic Decrement of Damped Waves. *Resistance-variation Method.*—The subject of decrements of circuits, damped currents and damped waves was formerly of much more importance than now and was treated in an exhaustive manner in many older treatises on radio communication. At the present time electrodeless discharge tubes, diathermy apparatus, etc., produce damped waves, and occasionally the knowledge of the decrement is of prime importance. It is well known that the logarithmic decrement, defined as the logarithm of the ratio

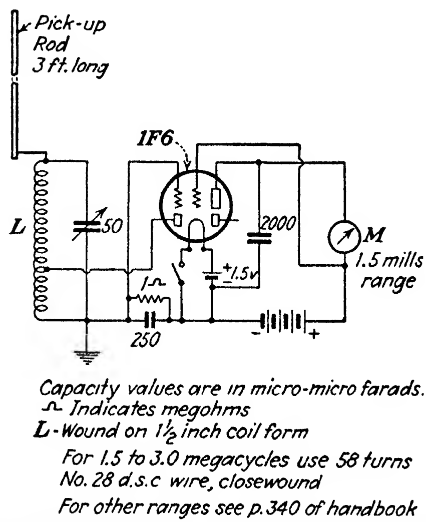


FIG. 9.—Field-strength meter. (*After Radio Amateurs Handbook.*)

of two succeeding maxima of damped alternating-current due to free oscillations in a circuit containing R, L, and C, is expressed by the relation

$$\delta = \frac{R}{2fL}.$$

This is also called the decrement of the oscillating circuit R, L, and C; and its determination necessitates only a measurement of the resistance, inductance, and frequency. The expression has other forms:

$$\delta = \pi R\omega C = \pi R\sqrt{\frac{C}{L}}.$$

The theory of the resistance-variation method of measuring the logarithmic decrement of a damped wave is based upon the expression for the effective value of a damped current as given by V. Bjerknes.[1] Referring to Fig. 10, it is assumed that circuit 1 carries damped currents produced by buzzer excitation (it may be a buzzer-excited wave meter). The damped currents excite circuit 2 by weak magnetic coupling; and the effective damped current squared in circuit 2, the test circuit, is, by Bjerknes' formula,

$$I_2^2 = \frac{NE_2^2(\alpha_1 + \alpha_2)}{16L_2^2\alpha_1\alpha_2} \cdot \frac{1}{(\omega_1 - \omega_2)^2 + (\alpha_1 + \alpha_2)^2},$$

where N is the number of damped-wave trains per second, and E_2 is the maximum e.m.f. induced in circuit 2 by the field due to the current in circuit 1. The damping α_1 of the oscillating current in circuit 1 is due to the damping factor of circuit 1 for pure impulse excitation; therefore, E_2 is damped to the same degree.

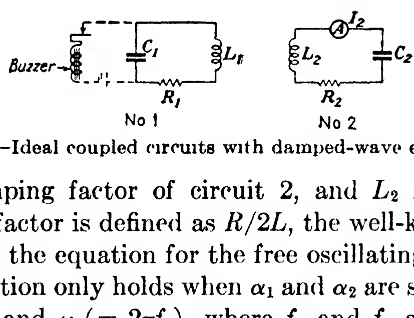


FIG. 10.—Ideal coupled circuits with damped-wave excitation.

α_2 is the damping factor of circuit 2, and L_2 its inductance. The damping factor is defined as $R/2L$, the well-known factor in the solution of the equation for the free oscillating circuit. The foregoing equation only holds when α_1 and α_2 are small compared to $\omega_1(= 2\pi f_1)$ and $\omega_2(= 2\pi f_2)$, where f_1 and f_2 are the natural frequencies of circuits 1 and 2. If both circuits are tuned to the same frequency, $\omega_1 = \omega_2$ and the foregoing equation becomes

$$I_2^2 = \frac{NE_2^2}{16L_2^2\alpha_1\alpha_2(\alpha_1 + \alpha_2)}. \tag{19}$$

If the resistance of circuit 2 is changed to R_2', the damping factor of the circuit becomes $\alpha_2' = \dfrac{R_2'}{2L_2}$ and the equation becomes

$$(I_2')^2 = \frac{NE_2^2}{16L_2^2\alpha_1\alpha_2'(\alpha_1 + \alpha_2')}.$$

[1] *Ann. der Physik*, Vol. 44, pp. 74–92, 1891; also, Vol. 55, p. 121, 1895. Note that Bjerknes' equation is given in many later books and papers; *e.g.*, see footnotes Art. 9, Chap. II.

Combining the two preceding equations, there results the expression

$$\frac{I_2^2}{(I_2')^2} = \frac{\alpha_2'(\alpha_1 + \alpha_2')}{\alpha_2(\alpha_1 + \alpha_2)}.$$

Solving for α_1,

$$\alpha_1 = \frac{(\alpha_2' I_2')^2 - (\alpha_2 I_2)^2}{\alpha_2 I_2^2 - \alpha_2'(I_2')^2}. \tag{20}$$

If the resistance of circuit 2 is doubled, $R_2' = 2R_2$, and $\alpha_2' = 2\,\alpha_2$, and Eq. (20) becomes

$$\alpha_1 = \alpha_2 \frac{[4(I_2')^2 - (I_2)^2]}{[I_2^2 - 2(I_2')^2]}. \tag{21}$$

Now, the frequency of each circuit was made equal, $\omega_1 = \omega_2$, and hence the ratio of the decrements of the two circuits is equal to the ratio of their damping factors, or

$$\frac{\delta_1}{\delta_2} = \frac{\alpha_1}{\alpha_2}.$$

Using this relation, Eq (21) becomes

$$\delta_1 = \delta_2 \frac{[4(I_2')^2 - (I_2)^2]}{[I_2^2 - 2(I_2')^2]}, \tag{22}$$

from which the decrement of circuit 1, and therefore the decrement of the damped e.m.f. impressed on circuit 2, may be determined, provided the decrement δ_2 of circuit 2 is known. This latter value may be determined by measuring the high-frequency resistance, capacitance, and natural period of circuit 2, by methods described in Chap. I.

Practical Procedure for Damped Waves.—While the foregoing theory deals with two lumped constant circuits with weak magnetic coupling, it also applies to the measurement of the damping and decrement of an electromagnetic wave in space, the damping of the e.m.f. induced in the receiving antenna being identical with that of the wave and the transmitting-antenna current. Figure 11 shows a simple antenna excited into damped oscillation by being weakly coupled to a condenser discharge circuit containing a quenched spark gap which produces a very highly damped discharge current, allowing the antenna to oscillate freely between discharges of the gap.[1] Any of the several known methods of producing damped waves may be used. The test circuit 2 is merely the frequency-meter circuit shown, whose resistance may be increased. Readings of the induced current in this circuit are taken for known values of R_2 and R_2', as explained in the theory, and the logarithmic decrement of the e.m.f. induced in the test circuit and hence of the emitted wave is calculated. For convenience, Eq. (22) is reproduced below in a more convenient way:

$$\delta = \delta_t \frac{[4I_b^2 - I_a^2]}{[I_a^2 - 2I_b^2]}, \tag{23}$$

where δ is the logarithmic decrement of the emitted wave, δ_t that of the test or frequency-meter circuit, I_a the current in amperes

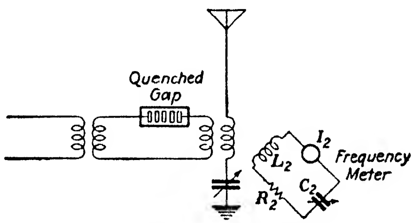


FIG. 11.—Antenna excited by damped e.m.f., and coupled frequency-meter circuit (*cf.* Fig. 10).

before the resistance of the circuit was increased, and I_b the current when the resistance was doubled. This measurement may be made at a distant point with a receiving antenna and set, but the over-all decrement of the latter must be known. Measurements of decrements of waves at distant points of reception are not usually made.

Reactance-variation Method.—This method is much like that described for measuring resistance by reactance variation, as described in Chap. I, Art. 13, except that the damped wave is used to excite the variable-tuned test circuit which may be a frequency meter with current-squared resonance indicator. The following derivation is given in outline.[2] Rewriting Bjerknes'

[1] LAUER and BROWN, "Radio Engineering Principles," 2d ed., p. 51; it is also discussed in many other treatises.

[2] MORECROFT, J. H., "Principles of Radio Communication, 2d ed., p. 313 also see ZENNECK, J., "Wireless Telegraphy" (transl. A. E. Seelig), p. 114

equation, as given preceding Eq. (19),

$$I_2^2 = \frac{NE^2}{16L_2^2}\frac{(\alpha_1 + \alpha_2)}{\alpha_1\alpha_2}\frac{1}{(\omega_1 - \omega_2)^2 + (\alpha_1 + \alpha_2)^2}. \tag{24}$$

The capacitance of the test circuit (Fig. 11) is varied for resonance, and $\omega_1 = \omega_2$. The equation for current squared is then

$$I_r^2 = \frac{NE^2}{16L_2^2}\frac{1}{\alpha_1\alpha_2(\alpha_1 + \alpha_2)}. \tag{25}$$

Dividing Eq. (24) by Eq. (25) and transposing,

$$\frac{I_r^2 - I_2^2}{I_2^2} = \frac{(\omega_1 - \omega_2)^2}{(\alpha_1 + \alpha_2)^2},$$

or

$$\alpha_1 + \alpha_2 = (\omega_1 - \omega_2)\sqrt{\frac{I_2^2}{I_r^2 - I_2^2}}.$$

Now, it must be assumed that for the cases of Eqs. (24) and (25) the natural periods of circuits 1 and 2 are nearly the same in order that these equations may hold. For this state of affairs the damping factors on the left side of the foregoing equation may be expressed in terms of the natural frequency of circuit 2 and the logarithmic decrements, or

$$\alpha_1 = f_1\delta_1 = f_2\delta_1,$$
$$\alpha_2 = f_2\delta_2.$$

The equation may then be written

$$f_2(\delta_1 + \delta_2) = 2\pi(f_1 - f_2)\sqrt{\frac{I_2^2}{I_r^2 - I_2^2}},$$

or

$$\delta_1 + \delta_2 = 2\pi\left(\frac{f_1 - f_2}{f_2}\right)\sqrt{\frac{I_2^2}{I_r^2 - I_2^2}}. \tag{26}$$

For the case of resonance [Eq. (25)] the natural frequency of circuit 2 was tuned to that of circuit 1, so that this value of indicated frequency of circuit 2 may be designated f_r, and

$$f_r = f_1.$$

Also, it will be realized that the current-squared indication I_2^2 was obtained for the case of the natural frequency of circuit 2 being at value f_2 slightly off resonance. These points are often not clear to the student who uses the formulas given for decrement measurements. It was therefore thought best to use the subscripts in the order of the numbered coupled circuits, as shown in Fig. 10 and followed in Eqs. (19)–(26). Equation (26) is usually given in other treatises and handbooks as

$$\delta_1 + \delta_2 = 2\pi\frac{(f_r - f)}{f_r}\sqrt{\frac{I^2}{I_r^2 - I^2}}, \tag{27}$$

where the meanings of f and I are evident. It is convenient to set the scale of the frequency meter so that $I^2 = I_r^2/2$ and the radical term in Eq. (27) becomes unity. Since f and f_r are very near together, f_r being at the peak of the resonance curve of current versus scale, setting of the frequency meter is difficult to determine accurately, and greater precision is obtained by finding the frequency-meter setting f' on the other side of the resonance-curve peak corresponding to I^2 (obtained for f). This is shown by the resonance curve of Fig. 12. It may be shown that

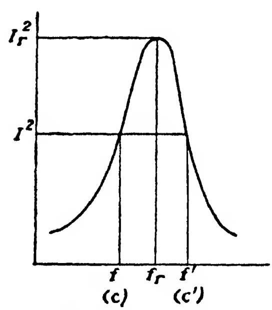


FIG. 12.—Resonance curve illustrating principle of decrement measurement by Bjerknes' method.

$$\delta_1 + \delta_2 = 2\pi\left(\frac{f' - f_r}{f_r}\right), \tag{28}$$

when

$$\frac{I_r^2}{2} = I^2.$$

Combining this with Eq. (27), there results

$$\delta_1 + \delta_2 = \pi\left(\frac{f' - f}{f_r}\right), \tag{29}$$

where δ_1 is the logarithmic decrement of the damped incident wave or exciting primary current, and δ_2 that of the test or frequency-meter circuit. The latter decrement may be determined by repeating the measurement when the test circuit is excited by undamped currents or waves from a vacuum tube or other generator.

It is pointed out in *U. S. Bureau of Standards, Circular* 74 (page 186, revised edition) that Bjerknes' classical proof shows that the sum of the decrements of the emitted wave or primary current and the secondary circuit is given by the same expressions as those which give the decrement of a circuit when measured by the reactance-variation method using undamped currents or waves from any radio-frequency generator or vacuum-tube oscillator. Hence, the formulas may be derived in a very similar way to those developed in Chap. I, Art. 3. If the frequency-meter condenser is calibrated in capacitance values, a very convenient form of equation results from this method of derivation:

$$\delta_1 + \delta_2 = \pi \frac{C' - C}{C + C'} \sqrt{\frac{I^2}{I_r^2 - I^2}}. \tag{30}$$

where C and C' are shown in Fig. 12. The preceding equations have given rise to the development of special apparatus, known as decremeters, for the measurement of logarithmic decrement, and these were formerly in common use. Representative types are the Marconi Company and Kolster decremeters.[1]

5. Direction-finding Measurement with Coil Antenna. *Method and Procedure.*—The apparent horizontal direction of propagation of an incoming electromagnetic wave is now almost universally measured with the coil or loop antenna. The word *apparent* is used because of the fact that the direction of propagation changes during the progress of the wave, owing to the presence of various bodies or mediums. The measured direction, then, may be very different from the true direction of the source of the radiated wave. Fundamental theory shows that the maximum e.m.f. induced in a coil antenna by an incident wave occurs when the plane of the coil is in the direction of propagation of the wave at the point where the said coil is located and that the induced e.m.f. is zero when the plane of the coil and the direction

[1] For discussion of the principle of the Marconi decremeters, see J. Zenneck "Wireless Telegraphy" (transl. A. E. Seelig), pp. 126–127. The principle of the Kolster decremeter is described in "Radio Instruments and Measurements," *U. S. Bur. Standards, Rev. Circ.* 74, pp. 196–199.

of the wave are at right angles to each other. When the wave is vertically polarized so that the electrostatic field is vertical at the surface of the earth and the electromagnetic lines are horizontal and the wave propagation is in a horizontal direction, the coil antenna is placed with its plane vertical and rotated about a vertical axis through the plane of the coil and through its geographical center. The induced e.m.f. at the terminals of the coil varies with the direction of the incident wave along a cosine curve and is graphically represented in Fig. 13. This is a plan view of the vertical coil rotated about its center O, and the circles forming the cosine curves are the loci of relative values of induced e.m.f. at the terminals of the coil for varying horizontal directions of the incident wave of constant field intensity. A slight angular movement of the coil from the position in the

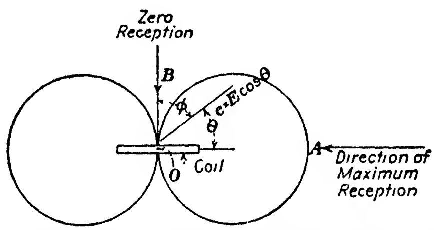


FIG. 13.—Curve of intensity of reception of waves with direction-finding loop.

figure gives very little relative change in response in the receptor apparatus connected to the coil. (The coil is being used as a receiving-loop antenna.) If the coil is set at 90 deg. to this position, or if the wave is incident at B instead of at A, it may be easily seen and may be proved that a very slight angular change of the coil position will result in a relatively large change in terminal e.m.f.

The common practice consists of setting the coil for a small e.m.f. so that a deflection may be read on the output meter of the detector-amplifier system connected to the coil; then the coil is rotated past the point of zero reception until the same deflection is obtained. The line BO, the direction of the incident wave, is then the mean of these two positions. If the coil is slightly unsymmetrical, the tendency toward error of measurement is corrected by swinging the coil through 180 deg. and repeating the observations. The mean of the two sets of observations is then taken.

Degree of Precision Obtainable.—The degree of precision of direction finding just described is dependent upon the degree of noticeable change in the reading of the output meter as the loop is turned. The degree of accuracy obtained when the method of measurement is carried out as described, using the mean of the deflections on each side of the zero setting, also depends upon the value of angular position of the coil to obtain a *readable deflection* of the output meter. This error will be analyzed.

Let ϕ be the angle between the normal to the plane of the coil and the direction of the incident wave (see Fig. 13). Assuming that a *square-law* detector is used in the coil-antenna receiving apparatus, the deflection of the output meter is proportional to the square of the terminal e.m.f. of the direction-finder coil or loop, as already pointed out. Then

$$I = \mathcal{E}^2 \sin^2 \phi.$$

When ϕ is increased a small amount $\Delta\phi$ to obtain a just noticeable change in the value of I, the new value of I will be I_1, and

$$I_1 = \mathcal{E}^2 \sin^2 (\phi + \Delta\phi).$$

Dividing the preceding expressions into each other and transposing,

$$\sin^2 (\phi + \Delta\phi) = \frac{\sin^2 \phi}{R},$$

where $R = I/I_1$. Using the relation $\sin^2 x = \frac{1}{2} - \frac{1}{2} \cos 2x$ and solving for $\Delta\phi$,

$$\Delta\phi = \frac{1}{2}\left[\cos^{-1} \frac{(R - 1 + \cos 2\phi)}{R}\right] - \phi. \qquad (31)$$

In this equation, ϕ is the angle used for obtaining a reading I. For the setting on one side of the zero, the position may be actually nearly equal to $\phi + \Delta\phi$ without noticing any change in I, and for the other side it may also be nearly $\phi - \Delta\phi$; it is then apparent that the mean of these positions would be off the true position for zero coil e.m.f. by the angle $\Delta\phi$, and this would be the probable error in the measured direction. Equation (31) shows that for a certain value of R the value of $\Delta\phi$ will depend in a complex way upon ϕ. Assuming that a change of 2 per cent in I is noticed on the output indicator, the values of $\Delta\phi$ calculated for various values of ϕ are shown as follows:

ϕ, degrees	$\Delta\phi$
2	1 5 min.
10	10 min.
45	34 min.
75	2 deg. 17 min.

These calculations show that if a readable deflection is obtained when the loop is displaced 10 deg. from the zero position, the direction of the incident wave should be determined to less than 0.2 deg. when taking the mean of the angular position for the same output-meter reading, the positions being not greater than 10 deg. on each side of the zero. Now, the ability to attain these results depends upon the degree of amplification of the receiving apparatus. If the receiving amplifier is less effective so that it would require 45 deg. off the zero position for a readable deflection, the error in this method of measurement is 34 min., or about 0.5 deg. The whole range of probable direction is 180 deg., so that the percentage of errors is quite small.

It will be appreciated that the foregoing method measures only the position of the direction line along which the wave travels. To obtain the absolute direction, it is necessary to use special apparatus for the purpose.[1] This comprises a simple earthed vertical antenna rod placed at the center of the loop and inductively coupled to the loop-output e.m.f. The theory and manipulation in using this device are given in the reference cited and will not be repeated here.

In commercial radio direction finding, the precision of measurement is often not so high as that indicated in the analytical study given above. This is due to several factors, especially the presence of a sky wave at the point of measurement. Shielded and balanced loops have been developed to improve measurement precision,[2] and special open-antenna combinations may be used, but these are not so convenient as the loop.

[1] For example, see Glasgow, R. S., "Principles of Radio Engineering," p. 440; also *Proc. I.R.E.*, vol. 17, p. 425, March, 1929. The same principle for precision holds for this special apparatus as for the system described above.

[2] See *U.S. Bur. Standards, Sci. Paper* 245.

6. Miscellaneous Wave Measurements. *Vertical and Horizontal Polarization.*—The coil or loop antenna may be used for determining whether or not the wave is vertically polarized, *i.e.*, whether or not the electric field is vertical and the magnetic field is horizontal. As mentioned in the preceding section, the coil must have its plane vertical for maximum reception of such waves. If the coil is so placed and has its plane in the direction of the incident waves, the terminal e.m.f. of the loop will be a *maximum;* but if the coil is now rotated about an axis through its own plane and parallel to the direction of the incident waves until the coil plane is horizontal, the loop terminal e.m.f. will be reduced to zero. This fact is illustrated in Fig. 14. If the e.m.f., and hence the connected receiver output, is maximum when the plane of the loop is at an angle of 30 deg. with the ver-

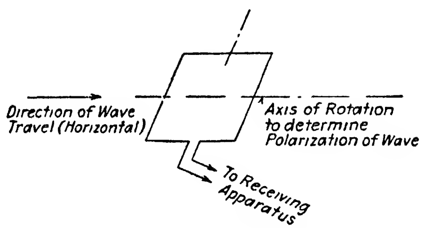


FIG. 14.—Use of receiving coil or loop to determine the orientation of an electromagnetic wave.

tical, the fields are displaced from their normal vertical and horizontal positions by the same angle of shift. It is convenient to measure the polarization of very short waves with a Hertz receiving antenna arranged so that it may be rotated about an axis through its center and perpendicular to its length. The arrangement is shown in Fig. 15. Bamboo poles are fixed to a hub on a shaft so that they may be rotated in the manner indicated. The wires of the Hertz receiving antenna are tied to the poles with cord, and the antenna system may be thus orientated so that the antenna is rotatable in a plane perpendicular to the direction line of the wave. If the wave is vertically polarized, for maximum reception the receiving-antenna wire should also be vertical. If the antenna is horizontal, the reception will be a minimum but will not be zero, owing to the total earthed-antenna effect of the entire receiving system. If the direction line of the wave travel is perpendicular to the shaft supporting the antenna of Fig. 15, the reception will be a minimum when the length of the antenna is again horizontal, *i.e.*, in line with the wave travel. If the wave front is tilted, the antenna will not be horizontal for minimum receiver response. The entire apparatus of Fig. 15 should be mounted on a rotating table so that the rotating arm may be turned for use on waves coming from any horizontal direction. As in the case of the loop receiving antenna used for direction measurements, it will be found that the most sensitive adjustment is for minimum response rather than for maximum.[1] In using this apparatus it

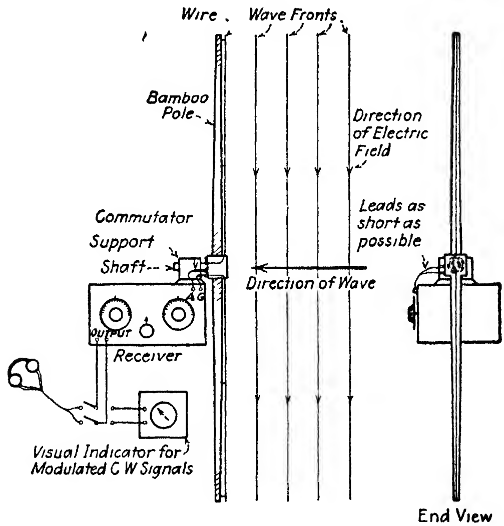


FIG. 15.—Apparatus for determination of orientation of short waves.

must be remembered that reliable indications can be obtained only if the field is fairly complete in polarization in one direction. If the wave is heterogenously polarized, no definite orientation can be detected. If the incident wave is known to be a downcoming sky wave, *i.e.*, a reflection or refraction from the ionosphere, the polarity of the electric or magnetic field, as indicated by the apparatus, will be that for the *resultant* of the incident wave and the component reflected from the earth. Hence the vertical angle of the received sky wave cannot be found in this

[1] Excellent apparatus incorporating the half-wave rod and loop and instructions for their use is given by Smith-Rose and Barfield, *Experimental Wireless and Wireless Engineer*, Vol. 2, pp. 739–740, September, 1925.

manner, except as will be pointed out later in the discussion of such measurements.

Radiation Power.—It was pointed out in the Introduction to this chapter that the figure of merit of transmission is measured by the *meter-amperes* attained by the transmitting or the radiator device. The power radiated from a transmitting antenna may be calculated from elaborate measurements of field intensity in all directions from the radiator and not more than two wave lengths or so away from it. This, however, is impractical, and it is common practice to determine the radiated power from the simple expression

$$P_R = I^2 R_z,$$

where I is the current at the base of the earthed transmitting antenna, and R_z its radiation resistance. Several methods of measuring this radiation resistance are described in Chap. III. A convenient method of determining the power radiated from an earthed transmitting antenna is in use by the Department of Commerce. The radiated power is given as

$$P_R = 1.58 \times 10^3 \left(\frac{Ih}{\lambda}\right)^2 \text{ watts,} \tag{32}$$

where h is the effective height of the earthed transmitting antenna in meters, λ the wave length in the same units, and I the current at the base of the transmitting antenna in amperes. This equation is recognized as a form of the expression I^2R_z, where R_z is in terms of the effective height squared and wave length squared. The effective height may be determined by the methods described in Chap. III. The method used by the Department of Commerce in its work consists of calculating the effective height from the field strength measured near the transmitting antenna. The measurements are made with a standard calibrated measuring equipment employing the coil-antenna receiver and described earlier in this chapter. The effective height is then calculated by a formula that is merely a convenient form of one of the standard transmission formulas. It is given as

$$h = \frac{\varepsilon \lambda D}{3.77 \cdot 10^5 \cdot I} \text{ meters,} \tag{33}$$

where $\mathcal{E}$ is the field strength in millivolts per meter, λ the wave length in meters, I the transmitting-antenna current in amperes, and D the distance from the transmitting antenna to the point where the field intensity is measured in meters. In order to check the reliability of the measurements of $\mathcal{E}$, measurements are made with increasing values of D, and a curve plotted between the values of D and $\mathcal{E}$. If the curve is a smooth hyperbolic form with no irregularities, it is evident that the measurements at the various points are not influenced by reflections or absorptions of near-by objects such as trees and buildings.

Measurement of Absorption and Damping Coefficients.—The current in a receiving antenna at great distances from the transmitter is dependent not only upon the dimensions of the transmitting antenna and its current, and the distance, but also upon a coefficient known as the *absorption factor;* Austin's formula for the absorption over sea water[1] is given as

$$F_1 = \epsilon^{-0.000047\frac{d}{\sqrt{\lambda}}}, \tag{34}$$

where d and λ are the distance and wave length in meters. If the transmitting-antenna current is damped, another factor, F_2, given by Austin, must be introduced, where δ' is the logarithmic decrement of the damped wave; L and R are the inductance and resistance of the receiving antenna in microhenrys and ohms, respectively; and λ is the wave length in meters, thus:

$$F_2 = \sqrt{\frac{1}{1 + \frac{600L\delta'}{R\lambda}}}. \tag{35}$$

The true value of the field intensity at a distance from the transmitting antenna is the value calculated from the transmission formula times the factor or factors concerned, as given by Eqs. (34) and (35). The absorption factor may be measured by simply measuring the field strength at various distances from the transmitting antenna. The difference in the measured values is, of course, due to absorption. The factor F_2 due to damping of the wave may experimentally be determined by measuring the

[1] DELLINGER, J. H., "Radio Transmission," *Trans. A.I.E.E.*, Vol. 38, Part 2, p. 1375, 1919.

field strength for damped and for undamped current in the transmitting antenna or with two known values of damping.

Measurement of the Angle of Incidence of Received Sky Waves.—The determination of the angle at which a received sky wave strikes the earth is very useful in radio-transmission engineering and in investigations of short-wave propagation and ionosphere refracting phenomena. One method, usable for ordinary broadcast frequencies, say 1,000 to 3,000 kc., will be briefly described; and the other method, suitable for 5,000 kc. and higher frequencies, will be mentioned but not fully described,

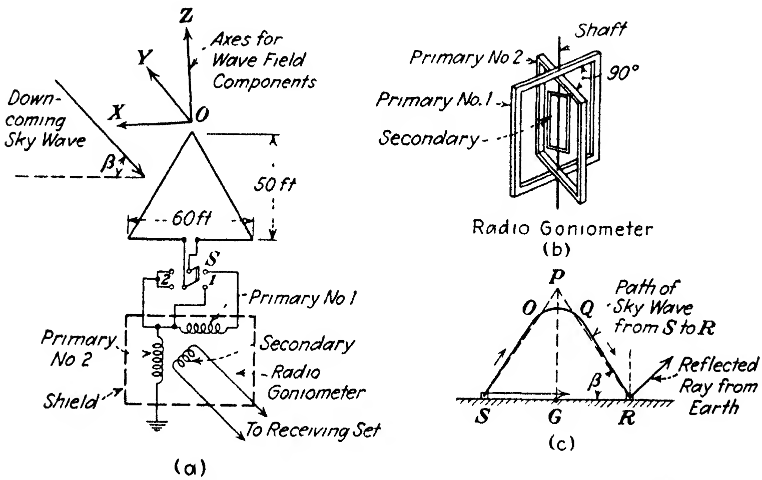


FIG. 16.—Apparatus for determining angle of incidence of received sky wave. (*After Smith-Rose and Barfield.*)

owing to the complex apparatus and technique. The methods will be designated *a*, *b*, etc.

a. The first method is due to Smith-Rose and Barfield.[1] They used a large triangular loop, as shown in Fig. 16, and a device known as a *radio goniometer*. This consists of two primary coils wound on wood frames 6 to 10 in. square and one placed within the other, their planes being at right angles. The secondary is placed within these two primaries and rotatable about the shaft, as indicated at (*b*) in the figure. It is not advisable to go into the theory of this to any great extent, but a

[1] *Experimental Wireless and Wireless Engineer*, Vol. 4, March, 1927, see especially pp. 137–138; also *Proc. Roy. Soc. London*, Vol. 110A, p. 580, 1926.

description of the procedure will aid in making the principle of the method more clearly understood. The procedure will be given in two steps.

1. The antenna system (*a*) is designed to function alternately as a loop and as an earthed receiving antenna. A near-by transmitter or a standard field generator (previously discussed) is set up. S is closed on position 1, and the triangular loop is rotated until its plane is parallel with the wave-travel direction so that maximum receiver output is obtained. The output from the goniometer secondary will also vary with its angular position with primary coil 1. It may at first be set 45 deg. with the latter. The received voltage and receiver output will be due to a certain magnetic field strength H', this field being vertically polarized; *i.e.*, the lines of magnetic induction are in horizontal planes, and the electrostatic field is vertical. Switch S is then closed on position 2, converting the loop into a plain vertical earthed antenna. The receiver pickup is now from primary 2 and will be stronger for the open antenna, of course. It may be pointed out here that the *same magnetic field strength* actuates both antennas. The apparatus is now to be adjusted until the same response is obtained with switch S in either position. This may be done by decreasing the turns in primary 2 and by rotating the goniometer secondary coil until it is more nearly at right angles with primary 2. S is shifted back and forth while the secondary coil is slowly rotated as above indicated until the receiving-set output voltage is the same for either position of the switch.

2. The test equipment is now adjusted so that if an incident wave is vertical, no response will be obtained from the vertical antenna; and if it is horizontal, the response from both antennas is the same when the secondary coil is left unmoved. Now, when a downcoming sky wave makes some angle with the horizontal, the secondary coil must be rotated from its original position to obtain equal response with both positions of switch S, because the vertical component of the electric field to which the vertical antenna responds is lessened. Let the angle that the secondary must be moved through, as explained, be θ. The voltage from the loop is

$$e_L = H_v N_L K_r \sin \theta,$$

Where H_y is the horizontal component of the magnetic field, N_L is the loop-induction factor, and K_r is the reflection characteristic of the earth. The total response will be affected by the reflected ray from the earth. The voltage from the vertical antenna is

$$e_v = \mathcal{E}_z h_v K_r \cos \theta;$$

$\mathcal{E}_z$ = vertical component of the electric field. Since $e_L = e_v$,

$$\tan \theta = \frac{\mathcal{E}_z}{Hy},$$

since H_L and N_L are in effect on the receiver made equal by step 1 of the procedure. From the theory of electromagnetic-wave reflections it may be shown that

$$\cos \beta = \frac{\mathcal{E}_z}{Hy} = \tan \theta,$$

where β will be the angle that the downcoming wave makes with the earth.

b. If it is desired to obtain the angle of the downcoming wave by means of a half-wave rotating rod or a tilting loop, either one may be set for minimum response, but it will be necessary to know the conductivity of the earth near the surface to correct for the reflected component of the resultant field before the inclination angle of loop or rod can be used to determine the angle β of the downcoming wave.[1] This may be done by conductivity measurements. When the direction of the downcoming wave is found, the effective height *GP* of the ionosphere may be found by triangulation, as shown in (*c*) (Fig. 16).

c. The pulse method, for finding the downcoming direction of the sky wave, has been employed successfully at lower wave lengths (5,000 kc.) by Friis, Feldman, and Sharpless.[2] The different reception characteristics at various angles above the horizontal for half-wave and full-wave antennas are taken advantage of in determining the vertical angles of a downcoming sky wave. This sky wave may be in the form of a single pulse from the transmitting station. The pulses received on the two receiv-

[1] See SMITH-ROSE and BARFIELD, *loc. cit.*

[2] *Proc. I.R.E.*, Vol. 22, No. 1, p. 47, 1934.

ing antennas will be different in amplitude in accordance with the vertical angle of the received sky-wave component. If the ground wave reaches the receiving antenna, a second pulse will be observed a very small interval later. These can be best observed by oscillograph-photographic methods. Space does not permit an adequate description of this excellent method, but interested students will find adequate description of the principle involved in the paper referred to.

d. Friis used a cathode-ray oscilloscope in a very interesting way for measuring both the horizontal- and vertical-direction

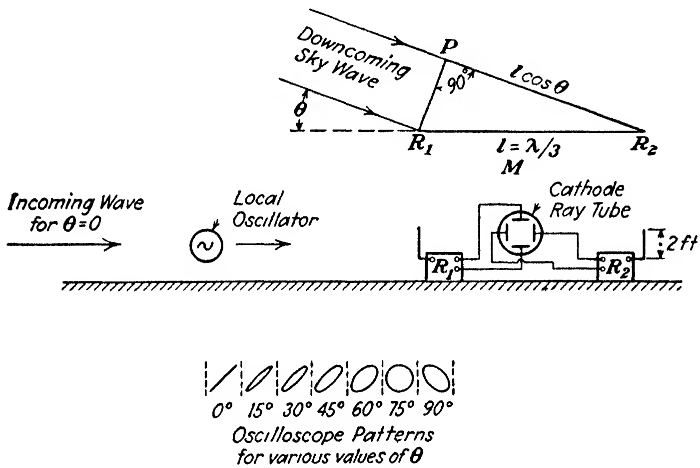


FIG. 17.—Test arrangement and diagrams illustrating Friis' method of vertical angle of incidence measurement

angles of reception of 16-meter waves.[1] For the measurement of the angle of incidence of downcoming sky waves the arrangement of apparatus was as indicated in Fig. 17. Two identical superheterodyne receiving sets (double-detection type) are placed a distance apart equal to one-third of the wave length being received ($\lambda/3$) and with the two short vertical-rod antennas connected to the receivers also $\lambda/3$ meters apart and in line with the great-circle direction to the transmitting station. A local oscillator is also placed at a convenient distance from the receivers, as shown, and also in the same direction line but between the receivers and distant transmitter. The local oscillator is tuned so that it beats on the incoming wave with a convenient

[1] *Proc. I.R.E.*, Vol. 16, No. 5, p. 658, May, 1928.

audio-frequency beat note, say 500 cycles, in each receiver. The audio-amplifier outputs of the receivers are connected to the oscilloscope deflector plates as shown and will cause a pattern to be traced on the screen of the cathode-ray tube which depends on the amplitude and phase of the two 500-cycle output voltages. If the wave from the distant station is horizontal, as indicated at the left, the phase of the difference frequency at receiver R_1 is the same as the phase of the difference frequency at R_2. This is due to the fact that both the local oscillator wave and the distant incoming wave arrive at each receiving antenna in the same phase. Two voltages in phase with each other and impressed upon right-angle pairs of deflection plates of the oscilloscope will produce a straight line, as shown in Chap. II, Art. 7. When the incoming wave comes down from a high angle, say the vertical, the foregoing is not the case. The incoming wave arrives at the two receiving antennas simultaneously, but the wave from the local oscillator, of course, does not; hence the output voltages will differ in phase. The arrival of the incoming wave from an angle θ, above the horizontal, will cause the two 500-cycle output voltages to be out of phase, and the pattern on the oscilloscope screen will change to an ellipse. When the two voltages are in quadrature but equal, the pattern is a circle. The phase difference of the waves at the two antennas, and hence the phase difference between the audio-output voltages for an angle θ of the downcoming wave, will be derived. For the effect at a distance d of a sinusoidal source Maxwell gave the equation

$$\mathcal{E}_d = \mathcal{E}_0 \sin \omega\left(t - \frac{d}{c}\right),$$

where $\omega = 2\pi f$, t is the time, and c is the velocity of propagation of the effect. At the position of receiver R_1 the phase of the wave from the local oscillator and the phase of the received sky wave at angle θ will be a certain value, and at point P (Diagram M, Fig. 17) the phase of the sky wave is also the same as at R_1. In the foregoing equation $\omega d/c$ is the phase of the wave in radians. It may be assumed that the phases of the fields at R_1 due to the local oscillator wave and the downcoming sky wave are zero. Then the phase of the field at R_2 of the local oscillator wave is $\omega l/c$, and the phase at R_2 of the downcoming sky wave is

$\omega l \cos \theta / c$. Hence, the phase difference of the 500-cycle output voltages from the receivers is

$$\phi = \frac{\omega l}{c} - \frac{\omega l \cos \theta}{c}.$$

Now,

$$c = \lambda f,$$

and

$$\omega = 2\pi f;$$

hence,

$$\phi = \frac{2\pi}{3}(1 - \cos \theta) \text{ radians,}$$

when $l = \lambda/3$, as was used in the test setup. Using this formula for ϕ, the proportions of the elliptical patterns on the oscilloscope screen can be calculated for any assumed value of θ, and then by observing the type of pattern for a skywave received the value of θ may be found. The effects of the reflected waves from earth at R_1 and R_2 are alike and hence produce no change in the phase difference between the two output voltages. Hence this method is independent of the reflection effects from the earth. In Fig. 17 are shown patterns calculated for various assumed values of θ; these are taken from the paper referred to.

Ionosphere Height, Selective Fading, and Short-wave Transmission Measurements.—These subjects would, if adequately treated, comprise a sizable volume. They require advanced technique and experience of a trained investigator. It is permissible, in the author's opinion, to omit such involved measurements in a course for undergraduates in electrical engineering or physics. The fundamental principles underlying these measurements and the measurement technique are fully described in papers on these subjects. The advanced student may readily proceed with the aid of the bibliography noted below.[1]

[1] KIRBY, BERKNER, and STUART, "Studies of the Ionosphere and Their Application to Radio Transmission," *Proc. I.R.E.*, Vol. 22, p. 481, April, 1934. Also, for comprehensive bibliography, *Elec. Eng.*, Vol. 54, No. 8, p. 844, August, 1935.

CHAPTER V

MEASUREMENT OF ELECTRON-TUBE COEFFICIENTS AND AMPLIFIER PERFORMANCE

1. Introduction. *Audio- and Radio-frequency Measurements.*—The general field of audio-frequency electrical measurements is not within the scope of this book; however, in order to make the material fairly comprehensive, it will be necessary to treat briefly a few audio-frequency methods of measuring electron-tube output characteristics. The *tube-output* coefficients measured by audio-frequency methods are fairly applicable to tubes used at radio frequencies, and it is in fact often possible to use radio frequencies in output-coefficient measurement.

Fundamental Concepts and Terminology.—The reader is probably familiar with the concepts of three fundamental tube coefficients, *viz.*, amplification factor, mutual conductance on transconductance, and alternating-current plate resistance. Other coefficients, such as detection coefficient and undistorted power output, are derived from the former and the degree of symmetry or asymmetry of the plate- and grid-current characteristics. These quantities are all, or in part, derived in several recent treatises on radio communication and electron tubes.

2. Static and Dynamic Characteristics. *Method of Obtaining Static Characteristics.*—The characteristics are of such form that it is necessary to provide a continuously variable plate-supply voltage and grid-bias voltage. It is quite necessary to have available a multirange milliammeter to read plate currents, even though the tube tested be a fairly high-power type with normal average plate currents of 100 to 250 milliamp. For low plate potentials and for high negative grid-bias potentials, the readings of plate currents at and below 1 milliamp. will be sought, so that a single range meter will prove inadequate. A type of milliammeter having a range of 1 amp., 100, 10, 1, and 0.1 milliamp. will be found convenient, especially if provided with range-changing knob or switch. A setup requiring only one such meter for measuring both plate and grid currents is shown in Fig. 1. The flat-type ammeter jacks J_1 and J_2 are provided so that the multirange milliammeter may be inserted for either plate- or grid-current measurement. The type of plug and jack used must be one that will allow reversing of the ammeter connections by turning the plug. It will be noted that the plate- and grid-circuit return connections to the filamentary cathode are made at the contact of a potentiometer P_3. Connecting the plate return to the positive-filament terminal and the grid return to the negative-filament terminal introduces potentials due to the gradients, complex in form, along the filament wire. If the grid is connected to the plate, it will be found that a small current will flow in the plate circuit from

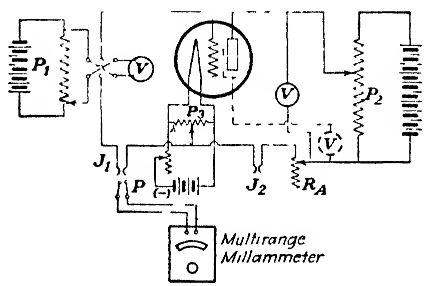


FIG. 1.—Circuit arrangement for obtaining static characteristics of thermionic tubes.

plate to filament, even though the plate return is connected to the negative filament. This current may be of the order of 10 to 40 μa for small amplifier tubes. The connection to a middle point of a tapped resistor or potentiometer, as shown by P_3, is probably a good compromise among several difficulties. With this connection it may be safely assumed that the net average grid and plate potentials are those given by the voltmeters shown. If the tube is to be used with the grid return connected to the negative end of the filament, the potentiometer contact should be moved to the left terminal, as shown dotted in Fig. 1, to establish test conditions similar to conditions of later use.

One other uncertainty introduced, when working with small tubes, is the effect of the high resistance of some low-range milliammeters. In some cases their resistances are upward of 2,500 ohms, and errors due to the IR drop through such meters are serious, unless a correction is made for the plate voltage. Fortunately, this is easily done if the resistance of the plate-current meter is known. For high-resistance tubes such as the screen-grid type, a high-sensitivity D'Arsonval galvanometer with standard shunts or calibrated with a reliable milliammeter and improvised shunt is quite desirable (screen-grid bias circuit shown dotted). If a milliammeter with plug is used, it is convenient to provide resistance R_A which is cut into the plate circuit to substitute for the resistance of the removed milliammeter. For the case of electron tubes containing an indirectly heated cathode, the plate-circuit return is of course brought directly to the cathode terminal, and potentiometer P_3 is not needed.

Methods of Obtaining Dynamic Characteristics.—The conventional dynamic characteristics of electron tubes are obtained by partially graphical methods from *families* of static characteristic curves. The grid-voltage plate-current dynamic characteristic is obtained from the corresponding family of static curves, and the plate-voltage plate-current dynamic characteristic is obtained from its corresponding family of static curves. The families of static characteristics are easily determined in the manner previously explained, changing the fixed plate voltage or fixed grid bias for each new static-curve determination of the family. Since these dynamic characteristics are used in determining undistorted power output, the exact procedure with the given family of static characteristics will be given in Art. 8 of this chapter on the determination of undistorted power output of a class A power-amplifier tube.

Formerly much experimenting was done on the technique of obtaining dynamic characteristics by a momentary recording or indicating process, an oscillograph or oscilloscope being used for the purpose. The usefulness of this procedure is limited to certain cases. With a reactive impedance in the plate circuit of the tube, the dynamic characteristic (plate-current grid-voltage) assumes an elliptical form, which is not so easily derived from static characteristics as for a resistance plate load.[1] The dynamic characteristic may be shown on the cathode-ray oscilloscope in a manner shown in Fig. 1*a*. If the grid-voltage and plate-current variations were in phase, the trace on the cathode-ray tube screen would be a straight diagonal line; but when they

[1] See footnote 1, p. 256.

are out of phase, the trace becomes an ellipse (see Chap. II, Art. 7).

Characteristics in the Positive-grid Region.—It is not always feasible to attempt to obtain static characteristics in the region of positive-grid potential, owing to excessive heating of the plate of the tube being tested. Some idea of these may be obtained with the aid of the previously described procedure using the circuit of Fig. 1*a*, biasing the tube beyond cutoff, and impressing enough sweep voltage on the grid to drive it positive. It may be necessary to introduce a one-stage resistance-coupled vacuum-tube amplifier between the sweep-oscillator output and the grid of the tube under test. This will reverse the polarity of the sweep voltage, if necessary, to make it increase in a positive direction with time.

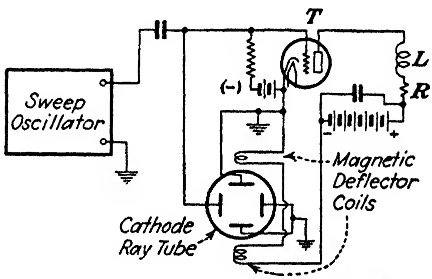


FIG. 1*a*.—Test circuit for obtaining dynamic characteristic on a cathode-ray-tube screen.

An ingenious method of obtaining positive grid characteristics is described by Kozanowski and Mouromtseff.[1] In this method a condenser is charged with a sufficiently high potential and, in turn, discharged through a resistance connected between the grid and filament. The grid goes positive for a short interval, and the potential and current variations are recorded by a magnetic oscillograph. The test circuit is shown in Fig. 1*b*. The *synchronous switch* is opened mechanically at the instant of opening the oscillograph light shutter. For a complete description and discussion of this method the reader may obtain the paper referred to.

3. Measurement of Amplification Factor. *Static Characteristic Method.*—The amplification factor is defined as the ratio of the effectiveness of the increments of grid-potential change on

[1] *Proc. I.R.E.*, Vol. 21, No. 8, p. 1082, 1933.

the resulting plate-current changes to the effectiveness of the increments of plate-potential changes necessary for the same plate-current changes. In brief, it is the ratio of effectiveness of the grid to the plate in controlling the plate current. The mathematical concept of amplification factor need not be fully

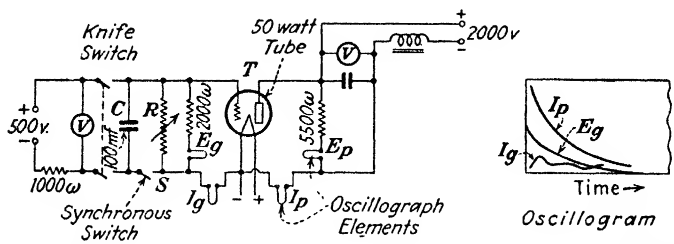


FIG. 1b.—Test circuit for obtaining tube characteristics in the positive-grid-potential region. (*After Kozanowski and Mourontseff.*)

treated here but may be found in full in the literature.[1] The commonly known tube-voltage amplification factor is given by most writers as

$$\mu = \frac{\partial E_P}{\partial E_g}(i_p = \text{constant}). \tag{1}$$

Figure 2 (*a*) shows a series of curves plotted between grid potential

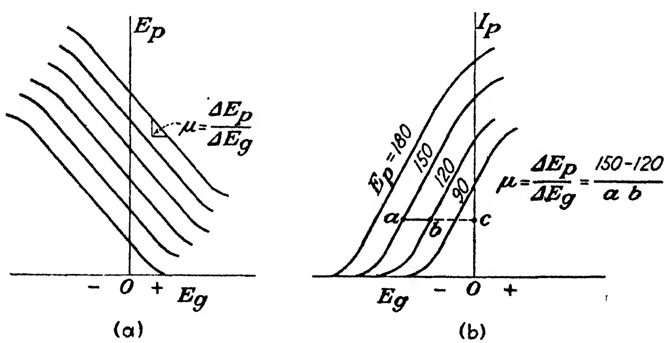


FIG. 2.—Use of static characteristic family for finding amplification factor "μ."

and plate potential for constant plate currents. The method of obtaining these curves consists of starting with as high a positive-plate potential and negative-grid potential as is desired, to obtain

[1] VAN DER BIJL, H. J., "The Thermionic Vacuum Tube," pp. 155–156; CHAFFEE, E. LEON, *Proc. I.R.E.*, Vol. 17, No. 9, p. 1633, 1929; also textbook, "Vacuum Tubes."

a certain plate current. The plate potential is then lowered a certain amount, and the grid potential is changed (lowered, if formerly negative) until the plate current has regained its former value. The required values of E_P and E_g are then plotted. This procedure is continued until the increases of grid potential become too great for practical purposes, and the curve changes slope, as shown in the figure. The slope of this curve $\Delta E_p/\Delta E_g$ is constant through a considerable range of potentials and is therefore equal to

$$\frac{\partial E_P}{\partial E_g} = \mu$$

for the limits indicated. It is noted that most of the curves are parallel, indicating, of course, that the amplification factor is constant. The foregoing expression for amplification factor also means that it can be obtained from the grid-voltage plate-current family, as indicated in (*b*), Fig. 2. Here points *a* and *b* are for the same plate current, and the corresponding plate voltages are shown, so that for the portions selected the amplification factor is

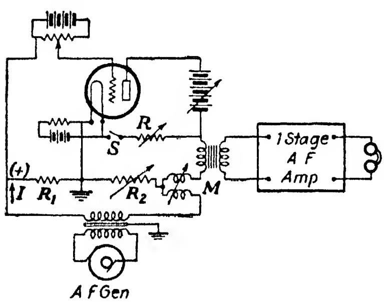


FIG. 3.—Circuit for measuring amplification factor and alternating-current plate resistance. (*After J. M. Miller*)

$$\mu = \frac{\Delta E_P}{\Delta E_g} = \frac{150 - 120}{ab}. \tag{2}$$

It may also be obtained from proper increments obtained from the plate-voltage plate-circuit family in an obvious manner.

The Alternating-current Method.—This method was given by J. M. Miller[1] and is widely used. The apparatus and circuit arrangements are as shown in Fig. 3. The audio-frequency generator produces a current I in the resistors R_1 and R_2 and hence small e.m.fs. in the plate and grid circuits of the tube. At the instant when the current I has the direction shown, the resulting potential drop through R_1 places a positive potential on the grid but a negative potential on the plate, due to the drop in R_2. If the drop through R_2 is made larger than that through

[1] *Proc. I.R.E.*, Vol. 6, No. 3, pp. 141–148, 1918.

R_1 by a certain optimum amount, there will be no change in plate current. The increments of potential thus introduced into the grid and plate circuits in opposite phase are IR_1 and IR_2 when their values are small compared to the plate-supply battery voltage. When no sound is heard in the telephone receiver, connected through an amplifier into the plate circuit as shown, it indicates that the effects of the incremental alternating grid and plate potentials are equal and opposite. When a null balance is attained by adjusting R_2 to a certain value, there is no alternating-current plate current; hence,

$$\mu \Delta E_g - IR_2 = 0;$$

but $\mu \Delta E_g = \mu IR_1$, the effective alternating voltage in the grid circuit. Then

$$\mu IR_1 - IR_2 = 0;$$

$$\mu = \frac{R_2}{R_1}; \tag{3}$$

μ is the amplification factor of the tube.

As was mentioned above, it is necessary to keep the introduced alternating-current potentials small compared to the steady potentials, and of course they will be quite small when a fairly sensitive telephone receiver with a one-stage amplifier is used to indicate a null balance. The use of an inductive impedance in the plate circuit, such as a telephone receiver or repeating transformer, results in phase relations of the incremental alternating-current potentials acting in the plate circuit, not quite in opposite phase, so that the balance is actually for minimum sound; silence cannot be attained. This difficulty is obviated by placing a resistance in the plate circuit in place of the transformer and coupling it with condensers to a two-stage amplifier. Even with resistances there still usually exists a small phase difference between $\mu \Delta E_g$ and IR_2 due to the interelectrode capacitances of the tube to be tested. This may be eliminated when tube resistances are low compared to tube-element reactances by using the mutual inductance M, connected as shown in Fig. 3, to add a small component in quadrature with IR_2 to make the balance complete. The quadrature component ωMI is usually small compared to IR_2 and may be neglected in the calculation of amplification factor. The resistance R_1 is often made a constant value of 10 ohms, and R_2 then may be calibrated directly in values of amplification factor. The use of the one-stage amplifier improves the precision of balance.

4. Measurement of Alternating-current Plate Resistance. *Miller's Null-balance Method.*—This method was also described in the paper referred to in Art. 3. The apparatus and circuit arrangements are shown in Fig. 3. The arrangement is used as for the measurement of amplification factor, except that switch S is closed, introducing resistance R at the place shown. The general plate-circuit theorem[1] shows that when a small alternating-current e.m.f. e_g is introduced into the grid circuit, the resulting alternating component of the plate current is

$$i_P = \frac{\mu e_g}{r_P + R}.$$

When switch S is closed, plate current flows through R as given by the foregoing equation, and the drop through R is

$$i_P R = \left(\frac{\mu I R_1}{r_P + R}\right) R, \tag{4}$$

where $e_g = IR_1$. The $i_P R$ drop is in phase with the plate current, which in turn is in phase with the voltage IR_1 but in opposite phase with IR_2. Therefore R_2 may be adjusted until it is equal to $i_P R$, when no sound will be heard in the receivers. Thus,

$$i_P R = IR_2$$

by adjustment, and R_2 is not used in the plate circuit [Eq. (4)]. Substituting the above equality in Eq. (4),

$$IR_2 = \left(\frac{\mu I R_1}{r_P + R}\right) R,$$

from which

$$r_P = R\left(\mu \frac{R_1}{R_2} - 1\right). \tag{5}$$

Switch S is left open to measure the amplification factor μ, after

[1] Given in J. M. Miller's paper; also see J. R. Carson, *Proc. I.R.E.*, Vol. 7, p. 187, 1919.

which it is closed, and the plate resistance obtained. It is necessary to use a fairly high resistance for R; a 0- to 10,000-ohm decade box is handy with an extra 10,000-ohm unit to add in series. R_1/R_2 may be made unity for measurements of plate resistance of the common detector and voltage-amplifier tubes.

Wheatstone-bridge Method.—Stuart Ballantine described a method[1] of measuring the alternating-current plate resistance of a tube in which the plate circuit may be placed in one arm of a simple alternating-current resistance bridge, as shown in Fig. 4. The plate current flows through the shunt resistance around the output transformer. The condenser C_P is placed in parallel with the plate resistance and battery to by-pass any alternating plate

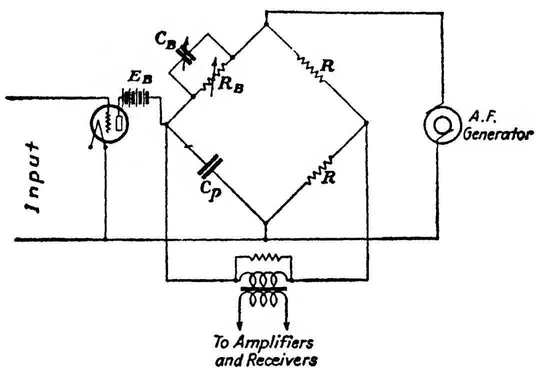


FIG. 4.—Circuit for measurement of alternating-current plate resistance with bridge. (*After Ballantine.*)

current if the tube is functioning as a detector or amplifier at the time of measurement. The ratio arms R are made equal, so that for balance $R_B = R_P$. The balance is, of course, imperfect unless a variable condenser C_B is used, as shown, to compensate for the phase-shift effect of C_P and the plate-to-filament capacitance. The advantage of this method lies in its use at the same time that a tube is functioning as a detector amplifier. It will be found that the plate resistance decreases with increasing input voltages to the tube under test, as well as supply. If the tube under test is functioning as an audio amplifier or detector of modulated radio-frequency current, C_P should be increased in value until the tone in the receiver, due to the alternating plate current, is sufficiently low so as not seriously to interfere with

[1] *Proc. I.R.E.*, Vol. 17, No. 7, p. 1164, 1929.

the bridge-balancing operation. This is greatly facilitated by having a different frequency supplied by the generator from that which is used for modulating the tube under test. If the tube is working on unmodulated radio frequencies, this difficulty does not present itself.

The alternating-current plate resistance is really the reciprocal of the slope of the plate-voltage plate-current characteristic and so may be readily found using any curve of this family.

5. Measurement of Transconductance. *Principle of Direct Measurement.*—The conductance of the plate circuit is defined as the slope of the plate-current plate-voltage curve, or

$$g_P = \frac{\partial I_P}{\partial E_P},$$

and of course may be readily found if such a static curve or family of curves is at hand. The average slope of the plate-current grid-voltage curve is shown in general vacuum-tube theory to be μ times as great as the slope of the above-mentioned curve, or

$$\frac{\partial I_P}{\partial E_g} = \mu \frac{\partial I_P}{\partial E_P}. \tag{6}$$

The average alternating-current plate resistance is the reciprocal of the average conductance, or

$$R_P = \frac{1}{g_P}.$$

The term $\partial I_P/\partial E_g$ may be defined as an equivalent conductance involving the rate of change of plate current with grid voltage and is given the name *transconductance*, symbol g_m. Substituting the relations just explained in Eq. (6),

$$g_m = \mu g_P = \frac{\mu}{R_P}. \tag{7}$$

The transconductance or mutual conductance may be calculated from measured values of μ and R_P[1]; but it has been found that

[1] This has been more fully treated in J. H. Morecroft, "Principles of Radio Communication," 2d ed., p. 518. A more comprehensive theory is given in recent treatises on electron tubes, or electronics.

this output coefficient is of particular value in comparing the relative figures of merit of tubes of the same type and rating; hence, it is desirable to measure it by some quick and positive method.

The transconductance is measured by a null method, somewhat similar to that for plate-resistance measurement but using a different circuit arrangement and e.m.f. balance. The apparatus

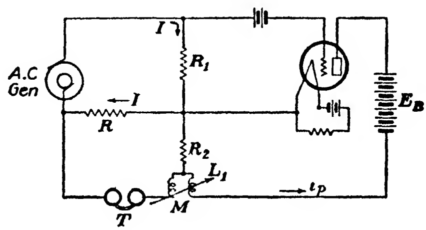


FIG. 5.—Circuit for measuring transconductance.

and circuit arrangement is shown in Fig. 5. Using an expression similar to that explained in Art. 4 of this chapter,

$$i_P = \frac{\mu e_g}{R_P + R_2}$$

and

$$e_g = IR_1;$$

hence,

$$i_P R_2 = \left(\frac{\mu I R_1}{R_P + R_2}\right) R_2 = IR$$

for a null balance in the telephone receivers. This equation holds when the primary inductance L_1 of the mutual inductance M (Fig. 5) is small compared to $R_P + R_2$. In this particular setup, R_2 is purposely small compared to R_P, so that it may be neglected in the denominator of the preceding equation. The solution of the equation for the value of μ/R_P is then given as

$$\frac{\mu}{R_P} = \frac{R}{R_1 R_2} = g_m.$$

The foregoing expression gives g_m in mhos; but if R_1 and R_2 are made equal to about 100 ohms each,

$$g_m = 100R \text{ micromhos},$$

where R is measured in ohms. The small mutual inductance M, already mentioned, is often helpful in obtaining a sufficiently complete balance for rapid measurement work. It has a similar function to that in Fig. 3. Usually, however, it need not be incorporated in the measuring setup. Its use is based upon the principle that in some cases the alternating plate current is not exactly in phase with the applied alternating grid potential, and therefore the potential drops IR and $i_P R_2$ are not exactly in opposite phase, and the balance will be imperfect. The function of the mutual inductance in improving the balance consists of introducing a small e.m.f. in quadrature with the $i_P R_2$ drop, making the total e.m.f.

$$[(i_P R_2)^2 + (\omega M i_P)^2]^{1/2},$$

exactly in opposite phase to IR in the telephone-receiver circuit. Obviously, the phase shift through the tube is arc tan $\omega M/R$.

6. Multielement-tube Coefficient Measurement. *General Principles and Procedure.*—For the case of multi-electrode electron tubes the same principles as for triodes are applicable. The slope of a plate-current plate-voltage curve determines the alternating-current plate conductance ($1/R_P$), and amplification factors and various conductances may be measured from certain families of static characteristics. However, in many cases the extreme values of the coefficients for these tubes make their determination from curve slopes and spaces between curves uncertain; the possibility of large error is great.

Alternating-current methods using suitable null-balance circuits must be relied upon. If one of the methods previously described for triodes is used, it must be with considerable caution because of the fact that the capacity reactance between the tube elements is *not* high compared to the grid or plate resistances. This often makes a null balance indefinite and the result uncertain if it is attained. This procedure may be tried using as low-frequency alternating test voltage (see Figs. 3, 4, 5) as is practical in order to increase the capacitance reactances compared to the resistances that they shunt. Preliminary measurements should be made on tubes whose output coefficients are known in order to check the accuracy of the method.

W. N. Tuttle has analyzed the effects of interelement reactances and has shown how these effects may be compensated for and accurate indications obtained from the ratios at null balance.[1] In Fig. 6*A* are shown his diagrams of tube-element resistances and reactances and the introduction of voltages whose effects are balanced out in the telephone receiver. The circuits

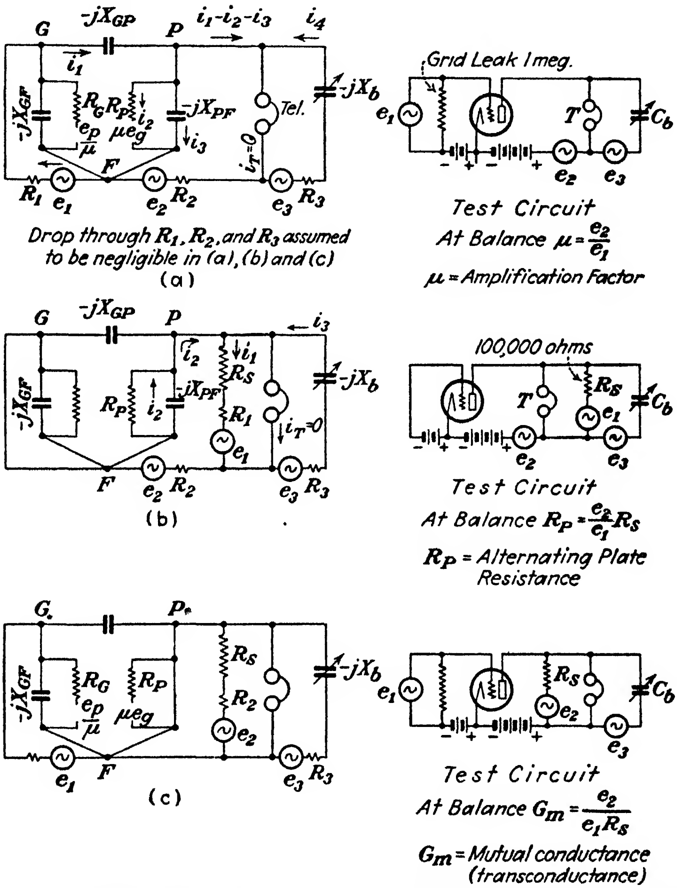


FIG. 6*A*.—Fundamental circuits of Tuttle bridge for vacuum-tube testing.

are shown for the three principle tube coefficients. In diagram (*a*) of Fig. 6*A* is shown the arrangement for amplification-constant measurement by introduction of e_1 and e_2 as shown; this is, of course, similar in principle to introducing voltage drops across resistances at similar points in Fig. 3. R_1, R_2, and R_3 are the

[1] *Proc. I.R.E.*, Vol. 21, No. 6, p. 844, 1933.

internal resistances of the sources of e_1, e_2, and e_3. If these internal resistances of the sources of e_1, e_2, and e_3 must be taken into account, the proofs of the working equations are tedious. These resistances, however, may be easily made quite low; in the actual testing equipment the sources are the output terminals of attenuators having resistances across the output terminals of only a few ohms. Hence voltage drops across such low resistances may be entirely neglected compared with those across the comparatively high interelement resistances and reactances. Considering the current directions indicated in (*a*), Fig. 6*A*, (in which the grid-biasing and plate-voltage batteries are not shown), voltage-drop relations may be written for the assumed condition of balance, *i.e.*, zero alternating current in the telephone receiver. Since the receiver is connected in the circuit, it would also be true that the voltage across it is *also zero* if there were no current flowing. These assumptions are made in the equations. The currents are assumed to be, in general, complex, but the test voltages are assumed to be in phase with each other, or they may be put in opposite phase with reversing switches. Voltage drops give the equations

$$e_1 - i_1(-jX_{GP}) - i_2R_P - \mu e_1 = 0, \tag{8}$$

$$-e_2 + \mu e_1 + i_2R_P = 0, \tag{9}$$

where μe_1 is the alternating voltage developed in the plate circuit by e_1 applied between grid and filament ($e_1 = e_g$).

Also,

$$i_3(-jX_{PF}) - e_2 = 0, \tag{10}$$

$$e_3 - i_4(-jX_b) = 0. \tag{11}$$

Since the telephone-receiver current is zero at balance,

$$i_1 - i_2 - i_3 + i_4 = 0. \tag{12}$$

Equations (9), (10), (11), and (12) may now be substituted in (8) to eliminate all current terms, giving

$$e_1 = \left[\frac{(e_2 - e_1)}{R_P} + \frac{e_3}{-jX_b} - \frac{e_2}{-jX_{PF}}\right](-jX_{GP}) - \left(\frac{e_2 - \mu e_1}{R_P}\right)R_P - \mu e_1 = 0. \tag{13}$$

This is an equation containing real and imaginary terms; and if the entire quantity equals zero, the sum of the real terms equals zero, and the sum of the imaginary terms is zero also. Equating the imaginary terms to zero,

$$\left(\frac{e_2 - \mu e_1}{R_P}\right)(-jX_{GP}) = 0,$$

from which

$$\mu = \frac{e_2}{e_1} \tag{14}$$

for no current through the telephone receiver. Equation (14) is the simple balance relation when e_1 and e_2 are properly adjusted, from which the amplification constant is found. This relation is true for the circuit when the quadrature-balancing voltage e_3 and the balancing reactance $(-jX_b)$ are adjusted to the proper values. The result is a complete balance. Using the real components in Eq. (13), it is easily shown that

$$C_{GP} = C_b\left(\frac{e_3 - e_2}{e_2 - e_1(1 - 2\mu)}\right) \tag{15}$$

at balance. From Eq. (15) the effective grid-plate capacitance may be found.

The test circuit for measuring the alternating plate resistance is shown in Diagram (*b*), Fig. 6*A*. In this measurement a rather high standard resistance R_s must be placed in the plate-filament circuit as shown, and e_1 in series with it. The grid-filament connection is through the biasing battery. Again, balance is attained when e_1, e_2, e_3, and C_b are properly adjusted to give zero current in the telephone receivers. Voltage-drop equations are

$$e_2 - i_2\left(\frac{-jX_{PF}R_P}{R_P - jX_{PF}}\right) - i_1R_s - e_1 = 0, \tag{16}$$

$$e_3 - i_3(-jX_b) = 0, \tag{17}$$

$$i_1R_s + e_1 = 0. \tag{18}$$

Also,

$$i_2 - i_1 + i_3 = 0. \tag{19}$$

Using Eqs. (17), (18), and (19) to eliminate the current terms from Eq. (16), there results

$$e_2\left(\frac{R_P - jX_{PF}}{-jX_{PF}R_P}\right) - \frac{e_3}{jX_b} - \frac{e_1}{R_s} = 0,$$

or

$$\frac{je_2}{X_{PF}} + \frac{e_2}{R_P} + \frac{je_3}{X_b} - \frac{e_1}{R_s} = 0. \tag{20}$$

Equating real terms to zero gives the working equation

$$R_P = \frac{e_2}{e_1}R_s. \tag{21}$$

Here, again, e_1, e_2, e_3, and C_b are properly adjusted for complete balance. From the imaginary terms in Eq. (20) it is easily shown that

$$C_{PF} = C_b\frac{-e_3}{e_2}. \tag{22}$$

The minus sign merely means that e_3 must be reversed to provide the required quadrature-balancing component. A reversing switch is provided for this purpose.

The measurement of transconductance or mutual conductance is very easily made, by using the circuit of Diagram (*c*), Fig. 6*A*. Here e_2 is placed where e_1 was for plate-resistance measurement, and e_1 is placed in the grid circuit, as shown. Since e_1 is placed as in Diagram (*a*), the voltage drop and current relations are stated in a manner similar to the equations for Diagram (*a*). The balance conditions give the working equation

$$G_m = \frac{e_2}{e_1}\frac{1}{R_s} \tag{23}$$

for mutual conductance. The student may prove this for himself after a study of the foregoing proofs of Eqs. (14) and (21).

In Fig. 6*B* is shown the apparatus described by Tuttle for supplying the testing voltages. The transformers must be similar and carefully designed to provide testing voltages that are exactly in phase. A_d is called a *decimal-reading attenuator*, having a constant-output resistance R_0 for all settings. The *L pad* R_4R_5 presents this same resistance to T_1—T_1 and T_2

being alike. The secondary winding providing e_3 draws practically no current. The step attenuators R_1 and R_2 are identical; each has a low-output resistance and a constant-input resistance.

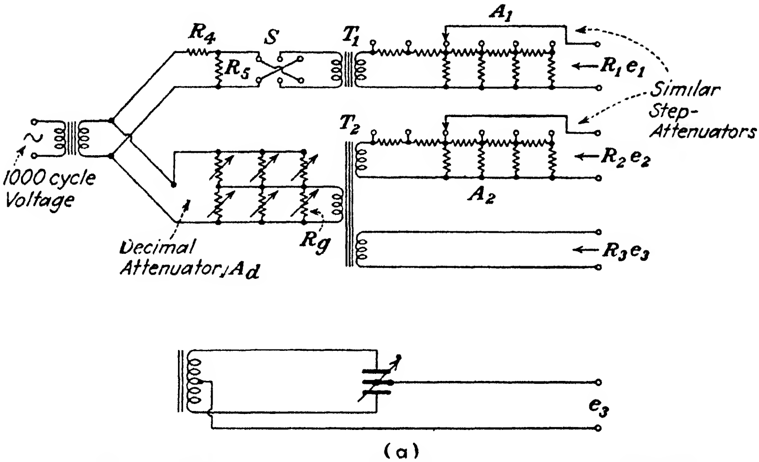


FIG. 6B.—Circuit diagram of Tuttle bridge for vacuum-tube testing.

They may be designed similar to that described in Art. 6, Chap. VI. Further considerations in the proper design of this apparatus may be found in Tuttle's paper, previously referred to.

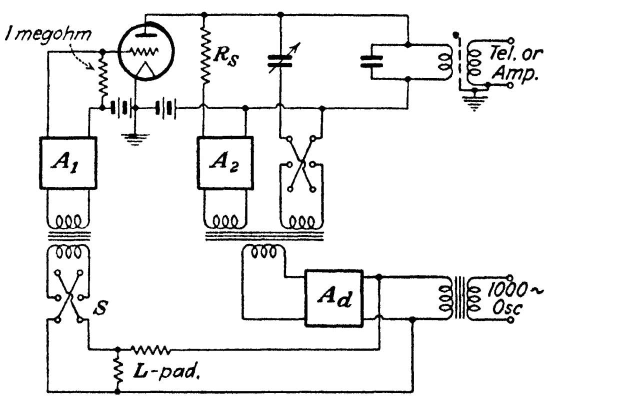


FIG. 6C.—Complete circuit arrangement for mutual conductance (transconductance); see also Figs. 6A, 6B. (*After Tuttle.*)

This author also suggests that a *split-stator* variable condenser be used instead of C_b, Fig. 6A, providing a quadrature balancing voltage, as shown at (a), Fig. 6B. Figure 6C shows the complete test arrangement for measuring mutual conductance. An output transformer having an electrostatic shield between windings is provided to supply the telephone receiver or an amplifier; the primary of this is shunted by a suitable capacitance which improves selectivity against extraneous sounds. It is possible to omit the decimal attenuator A_d supplying e_2 and e_3, connecting an ordinary voltage divider across the output of attenuator A_1 supplying voltages e_1. This provides values of e_1 lower than e_2, and continuously variable for similar positions of the dial switches of A_1 and A_2. For measuring the other tube coefficients, A_1 and A_2 outputs (e_1 and e_2) are switched into the proper parts of the test circuit by a six-pole three-position switch.[1] An accuracy of 1 to 2 per cent is claimed for this measuring apparatus. Its range is given as from 0.02 to 20,000 micromhos for mutual conductance (transconductance) measurement and from 0.001 to 10,000 for amplification measurement. This apparatus can be prepared in the laboratory if precautions mentioned in the paper referred to are taken.

Tube-emission Checker.—The plate current of an electron tube having a certain grid bias (or zero bias) at a certain plate voltage is a fair index of the condition of any one particular type. From known values for tubes in good condition, a tube being tested would be discarded as defective if its plate current were more than a predetermined percentage less than that of the average good tube. The low-priced tube testers are of this type. The limitations of such a test are obvious, as no information as to the actual values of output coefficients can be obtained.

Grid-shift-type Tube Testers.—In this device the change of plate current caused by a shift in the grid-bias voltage is used to test the condition of the tube. Diagram (*a*), Fig. 7, shows a possible test circuit. *T* is the tube under test. Part of the cathode-biasing resistor R_C is varied until the milliammeter reads a low value of plate current. Then an additional fixed portion is shorted by closing key *K*, causing a small increase in plate current. For good tubes a certain increase is expected, and as the tube deteriorates the increase becomes less. With these testers the tubes may be rated as *good*, *fair*, and *poor*. This tester may easily be built in the laboratory or service shop.

[1] A testing equipment is manufactured by the General Radio Company, Cambridge, Mass. It is known as a *vacuum-tube bridge*.

In commercial forms the rectifier filter is usually omitted, and the self-rectification of the tube under test causes pulsating currents to flow, producing the bias potential and giving readings of average value of the plate-current pulsations. Also, the meter *M* may be used as a voltmeter to adjust alternating-current line voltage by switching provision, and a copper oxide rectifier unit to make it usable on alternating-current supply.

Mutual-conductance Tester.—This device gives more accurate comparison than the other just described. It is illustrated in

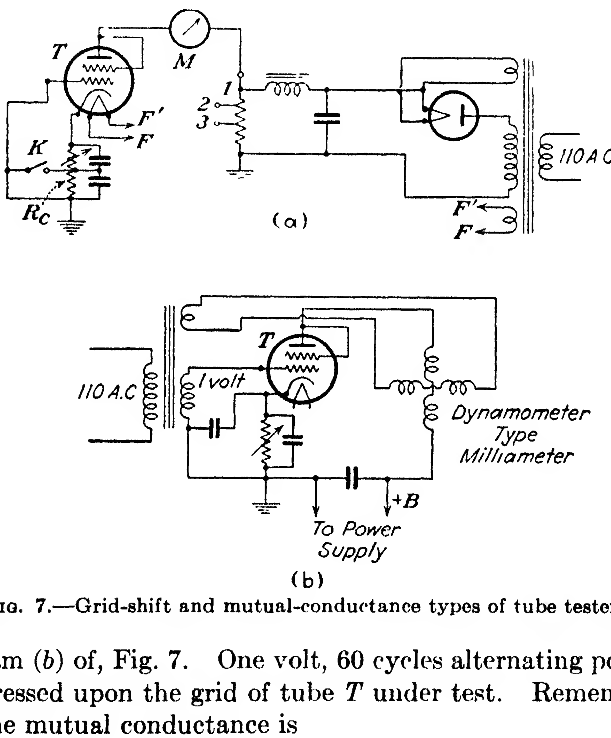


FIG. 7.—Grid-shift and mutual-conductance types of tube testers.

Diagram (*b*) of, Fig. 7. One volt, 60 cycles alternating potential is impressed upon the grid of tube *T* under test. Remembering that the mutual conductance is

$$G_m = \frac{\Delta I_P}{\Delta Eg}.$$

If 1 volt is impressed on the grid-filament circuit, the alternating component of plate current will be the mutual conductance. If an alternating-current milliammeter is used, G_m in micromhos is equal to one thousand times the plate current in milliamperes.

Since the plate current is pulsating, its alternating component is a distorted wave and unsymmetrical. Hence the second, third, fourth, etc., harmonics are strong, and the alternating-current milliammeter should give the effective value of the actual wave. The best instrument for this purpose is the dynamometer type, which is the type shown in the figure. To make it read on low currents and make it less affected by external fields, one coil of the dynamometer is excited separately, as shown. Comparison of tubes by mutual-conductance measurements is considered to be the most accurate and reliable of any test.

7. Measurement of Detection Coefficient. *General Principles of Measurement.*—The figure of merit of a detector tube is the value of its ability to deliver a certain average change in its plate current when a given radio-frequency e.m.f. is impressed on its input terminals. When the plate circuit contains an impedance, the change in plate current produces a certain voltage drop across the latter, thus giving a certain output voltage. Literature on the subject of detectors contains many theoretical treatises on the detecting ability of a tube, and nearly all of them define and use a term called *detection coefficient.*[1] Without attempting to give any analysis, it may be briefly stated that it has been shown by both theoretical and experimental investigators, such as those referred to in the footnote, that the degree of change in the average value of plate current varies with the square of the input e.m.f. to the detector device. This holds for any detector device, whether it is a simple crystal detector, two-element thermionic vacuum-tube rectifier, or the later three-electrode thermionic vacuum-tube detector with or without the grid condenser and leak. These devices all have the well-known square-law characteristic within a certain range of impressed-voltage variation. The special case of the linear diode will be treated later.[2] For a modulated wave impressed on a detector device the effective value of the signal-frequency component

[1] An exhaustive treatment with important bibliography is given in a paper entitled "A Theoretical and Experimental Investigation of Detection for Small Signals," *Proc. I.R.E.*, Vol. 15, No. 2, by E. L. Chaffee and G. H. Browning. An excellent treatment of detection is given in R. S. Glasgow, "Principles of Radio Engineering."

[2] *Diode efficiency* is usually measured instead of *detection coefficient* for diodes, see p. 366.

of detector current is

$$I_m = \frac{1}{2}\frac{\partial^2 i}{\partial e^2}\frac{K\mathcal{E}^2}{\sqrt{2}}. \tag{24}$$

In this expression, it must be remembered that $\mathcal{E}$ is the amplitude of the radio-frequency carrier voltage before modulation is effected. To obtain the detection coefficient for the detector device when used as a detector of a modulated impressed e.m.f., it is desirable to measure the signal-frequency component I_m of the detector current. This may be done with the aid of a suitable amplifier and output voltmeter and a low-pass filter effectively to cut off the distortion component of the detector current. The definition of the current-detection coefficient may or may not include the coefficients of $\mathcal{E}^2$ in Eq. (24). If it is defined as the asymmetry of the characteristic or the second-order differential parameter $\partial^2 i/2\partial e^2$, the detection coefficient is

$$\eta = \frac{\sqrt{2}I_m}{K\mathcal{E}^2} = \frac{I_m}{KE^2}, \tag{25}$$

where E is the effective value of the radio-frequency carrier voltage. The output voltage E_0, obtained when I_m flows through an impedance in the detector circuit, may be used in Eq. (25) to obtain the voltage-detection coefficient

$$\eta = \frac{E_0}{KE^2}, \tag{25a}$$

where E_0 is the effective value of the signal-frequency component of the output voltage. The measurement of output voltage must be made directly, because introducing the impedance into the circuit modifies the derived value of I_m, as shown by several writers.[1]

The foregoing outline of the problem of detection-coefficient measurement applies specifically to a simple two-terminal rectifier device such as a crystal detector or a two-element vacuum-tube detector or rectifier. In the case of the more complex three-

[1] For instance, see p. 118 of E. L. Chaffee's paper referred to in footnote 1, p. 242.

electrode vacuum-tube detector, it has been shown that the detection coefficient is obtained by measuring the same quantities as given by Eqs. (25) and (25*a*). However, the detection coefficient is dependent upon the differential parameters of both plate and grid circuits of the tube, and Morecroft,[1] gives for the case of an undamped e.m.f., impressed upon a vacuum-tube detector with grid condenser and leak, the change in plate current as

$$\Delta I_P = \frac{E^2}{4}\frac{d^2I_g/dE_g^2}{dI_g/dE_g}\frac{dI_p}{dE_g}, \tag{26}$$

where the subscripts g and p apply to the grid and plate circuits, respectively. As is well known, ΔI_p in Eq. (26) is not an increase but a decrease in the average steady plate current. The methods

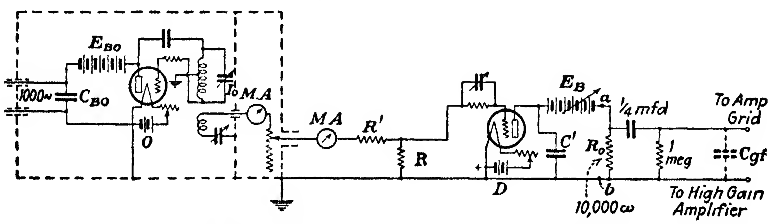


FIG. 8.—Circuit arrangement for measuring detection coefficient, direct method.

of measuring the current and voltage-detection coefficients of vacuum-tube detectors will be now described.

Direct Method.—This is called a direct method because it involves the direct experimental determination of the input e.m.f. and output current or voltage. The apparatus and circuit arrangement are as shown in Fig. 8. A 1,000-cycle modulating potential is superimposed on the plate supply of the shielded oscillator *O*. A modulated radio-frequency current of value indicated by *MA* passes through known resistances *R'* and *R*, so that a known modulated radio-frequency e.m.f. is impressed on the input circuit of the detector tube *D* under test. The grid condenser and leak resistance are shown variable so as to obtain optimum response of the detector. The audio-frequency output current is determined by measuring the voltage drop across the resistance R_0. There will be a certain amount of

[1] "Principles of Radio Communication," 2d ed., p. 537. Reprinted by permission of John Wiley & Sons, Inc.

error in the measurement of the output voltage E_0 due to the impedances connected across R_0, as shown in the diagram. The error can be calculated and corrected. This output voltage is very small, and a high-gain amplifier with output indicator must be used when measuring the detection coefficient for small input e.m.fs., such as 0.05 volt. It was pointed out in the theory that it is desirable to base the detection coefficient on the value

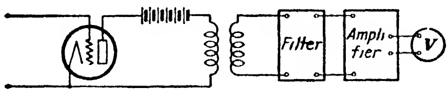


FIG. 9.—Output transformer and filter used in measurement of detection coefficient.

of the signal or modulating-frequency component of the output voltage or current and exclude the distortion components, which are frequencies twice the modulating frequency and higher. To do this, a low-pass filter may be placed between the output of the detector tube under test and the amplifier. If the filter input is connected in place of the 1-megohm resistance, its characteristic impedance must be high. It is generally preferable to use a 1 : 1 ratio output transformer in place of the resistance R_0, as shown in Fig. 9 especially when the tube is to be used with an impedance in its plate circuit. No difficulties due to attenuation effects occur, because the calibration of the amplifier and output indicator is obtained by connecting the output points a, b, Fig. 9A to a known 1,000-cycle calibrating e.m.f. This may be done using the calibrating potentiometer shown in Fig. 9A. R_2 of this device must be very low compared to the impedance of the terminals ab (looking into the output circuit). The limit of R_2 may be calculated if the approximate impedance of terminals ab is known. Another and more certain method of calibrating when this impedance is not known is to use the step attenuator described in Chap. VI, Art. 6. A standard T-type attenuation-resistance network may also be used if the current of ab can be measured; but if it is of the order of 100 μa or less, this is impractical.

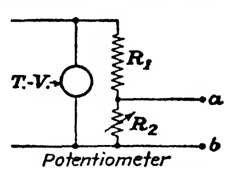


FIG. 9A.—Voltage divider or potentiometer used in calibrating measuring apparatus of Fig. 9.

The formulas given for detection coefficient include the value of the degree of modulation. It is not advisable to measure directly the degree of modulation, because of the small output of the circuit driver. It is most easily predetermined by a test on the circuit-driver characteristic which is described later in connection with Chaffee and Browning's null-balance method of measuring detection coefficient. Where the effective value of the unmodulated radio-frequency impressed voltage E, the fractional modulation K, and the modulation-frequency component E_0 (of the output voltage) have been measured, the voltage-detection coefficient may be calculated, using Eq. (25*a*).

It will be realized that this measurement of the signal component of output current or voltage and the measurement of the percentage of modulation entail complications in the measuring process. For comparison tests of vacuum tubes as detectors, it is convenient and sufficiently accurate to provide a completely modulated radio-frequency e.m.f. to the detector, by supplying a 500- or 1,000-cycle supply voltage to the circuit driver. The effective values of the complex wave-form input and output voltages are then measured, and the detection coefficient calculated from the simple expression

$$\eta_t = \frac{E_0}{E^2}, \tag{27}$$

where η_t is a total approximate value. For this type of measurement, it is desirable to use the known resistance R_0 in the plate circuit of the detector, from which the audio component of detector-plate current, and hence the current-detector coefficient, may be determined from Eq. (25), letting $K = 1$.

Chaffee and Browning Null Method.—This method is described by Chaffee and Browning on pages 144 to 147 of their paper already referred to here.

The apparatus and circuit arrangement reproduced from their paper in slightly modified form are shown in Fig. 10. The input voltage before modulation occurs is measured by the drop I_0R_0 through R_0 which may be a four-terminal attenuation resistance or even a low-value *single* resistance. The output voltage is obtained from a known mutual inductance M_b, having a known primary inductance L_b. C_b is a radio-frequency by-pass condenser. The voltage induced in the secondary is balanced in magnitude and phase against the drop across R and the secondary of another mutual inductance M. The effective alternating audio-input current of the detector is ΔI_p, and the voltage across the secondary of M_b is

$$E_{0b} = \Delta I_p \omega M_b \text{ at balance,}$$

when no current flows in the secondary. The resistance R' is made high compared to L, so that I_m is nearly E_m/R'. The

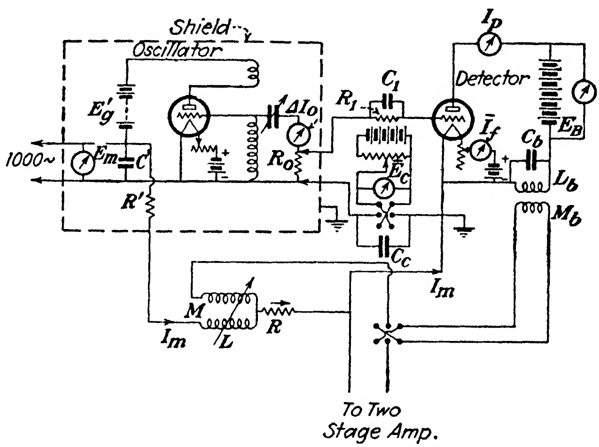


FIG. 10.—Apparatus and circuit arrangement for measuring detection coefficient by null balance. (*After Chaffee and Browning.*)

voltage opposed to E_{0b} is made up of the drop of I_m through R and the quadrature secondary e.m.f. $\omega M I_m$ of the secondary of M.

$$E_{0M} = I_m\sqrt{R^2 + (\omega M)^2},$$

and

$$E_m = I_m R'.$$

From these relations,

$$E_{0M} = E_m \frac{\sqrt{R^2 + (\omega M)^2}}{R'}.$$

At balance,

$$E_{0M} = E_{0b};$$

and, substituting values of these expressions, a formula for ΔI_p is obtained:

$$\Delta I_p = \frac{E_m\sqrt{R^2 + (\omega M)^2}}{R'\omega M_b}. \tag{28}$$

The authors give a form of current-detection coefficient which they term (Det. $I)_l$ and give this quantity equal to

$$(\text{Det. } I)_l = \frac{\Delta I_p}{\sqrt{2mR_0^2(\sqrt{2}I_0)^2}},$$

where m is the degree of modulation. Substituting ΔI_p from Eq. (28) in this expression, they give

$$(\text{Det. } I)_l = \frac{Em\sqrt{R^2 + \omega M)^2}}{\sqrt{2mR'R_0^2\omega M_b}}. \tag{29}$$

Often $\omega = 2\pi \times 1{,}000$, that is, the modulating potential has a 1,000-cycle frequency. The authors referred to have employed a symbol $\Delta^2 I_p$ (instead of ΔI_p used in this text) to represent the effective audio-frequency component of output current of the detector. They also used ΔI_0 instead of the I_0 term in Eq. (29).

In order to measure the degree of modulation, the authors of this method plot a curve showing the relation between output current of the oscillator and battery voltage, as shown in Fig. 11. This curve is linear between points P and Q. The battery potential is then set at N on the curve, and the crest value of the modulating potential set at a value equal to or less than $PN(NQ)$. If it is equal to PN, the modulation is 100 per cent; and if it is one-half the value of PN, the degree of modulation is 0.5. If the curve is not quite a straight line, the percentage of modulation must be calculated from the current corresponding to N and that corresponding to N minus the crest modulating voltage, or N plus the crest value. Let the crest modulating voltage be

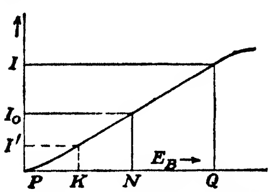


FIG. 11.—Operating characteristic of oscillator for determining percentage modulation. (*After Chaffee and Browning.*)

$$N - K.$$

The degree of modulation is then

$$m = \frac{I_0 - I'}{I_0},$$

where the values are shown in Fig. 11. The authors also give the values that they have used for the various circuit constants of the measuring setup.[1] The value of L_b and M_b is given as approximately 0.65 henry; R', 60,000 ohms; M, 10,000 μh maximum.

A feature of this method lies in the use of a telephone receiver and two-stage amplifier to detect the condition of balance. By this aural method, the fundamental tones can be balanced against each other, even though a double-frequency distortion component is present in the output voltage of the detector. By making the degree of modulation 0.5 or less, the ratio of the amplitude of the fundamental modulating-frequency component of detector-output current to the double-frequency term will be 8 : 1, and for such a low degree of modulation there is very little interference from the double-frequency component when balancing. Of course, it is realized that these satisfactory conditions assume that the modulation potential is sinusoidal or nearly so. Some method described in Chap. VII should be employed to check the wave form of this voltage.

8. Measurement of Undistorted Power Output. *Discussion of Methods.*—The measurement of power output of the thermionic vacuum tube used as a class A amplifier may be accomplished by the use of the output coefficients, input e.m.f., and known output impedance of the vacuum-tube circuit, but the wave form of the reproduced e.m.f. and current in the output circuit may be badly distorted from the original input; and such a measurement would then be no criterion of the ability of the tube to deliver power with undistorted output current and voltage wave form. Methods of experimentally determining the so-called power output required with elaborate apparatus or considerable calculation might be mentioned, but it is sufficient to outline briefly one such method. This method consists of observing the wave forms of the input voltage to the tube and of the current in the output circuit as the amplitude of the input voltage is increased. When the device for observing wave form shows that the output current begins to be noticeably distorted, the point of maximum undis-

[1] The modern modulated signal generator (Fig. 6, p. 349) may well replace the oscillator of Fig. 10.

torted power output has been attained, and with the output circuit of a known resistance the power output may be determined by measuring the alternating current flowing in this resistance. One of the cathode-ray oscilloscopes, described in Chap. VII, may be conveniently used for this purpose. Of course, the limit of undistorted power output may be passed before a change in wave form can be detected; probably a harmonic having 5 per cent of the amplitude of the fundamental would be present without a perceptible change in the wave shape, especially if the original input wave is a sinusoid. If greater accuracy is desired, it is necessary to employ a qualitative method of harmonic analysis, such as is described in Chap. VII. It would be exceedingly laborious to repeat the qualitative analysis for varying input voltages, so that the oscilloscope would be convenient to show an upper limit of voltage at which a qualitative harmonic analysis could be quickly made. When an amplifier tube is overexcited, a second harmonic appears in the output wave form to a greater degree than any higher harmonic, so that a very brief modification of the heterodyne method of harmonic analysis could be used to detect the presence of a second harmonic, this modified method being permissible only for sinusoidal input voltage.

The second method, which involves calculation of the power output from the static characteristic of the tube, is much used in practice and has been the subject of considerable theoretical as well as experimental investigation.

Method by Calculation from Static Characteristics.—The method about to be described was given by E. W. Kellogg,[1] in which he shows how to derive the dynamic characteristic from the family of plate-current grid-voltage static characteristics, determining the operating limits on the dynamic characteristic for negligible distortion and finally calculating the power output to a resistance load whose value was previously assumed. Later, Hanna, Sutherlin' and Upp review this method[2] in their paper on the development of new amplifier tubes and give some inter-

[1] "Design of Non-distorting Power Amplifiers," *Trans. A.I.E.E.*, Vol. 44, pp. 302–315.

[2] *Proc. I.R.E.*, Vol. 16, No. 4, pp. 462–475. For W. J. Brown's proof, see *Proc. Physical Soc. London*, Vol. 36, p. 221. Another proof may be found in R. S. Glasgow, "Principles of Radio Engineering," pp. 179–181.

esting data concerning the exact ratio of load resistance to alternating-current plate resistance for maximum power output in working at plate voltages greater than the rated value. Figure 12 shows an audio-frequency power-amplifier circuit with load-circuit resistance R_L. The power derived from the plate-battery supply is used in heating the plates of the power-amplifier tube and in supplying the power dissipated in R_l, the choke L being assumed to have no resistance. It is also assumed that the power consumed in the grid circuit is negligible. W. J. Brown,[1] to whom the authors of the first-mentioned paper refer, has shown that from elementary theoretical considerations the load resistance must be twice alternating-current plate resistance for

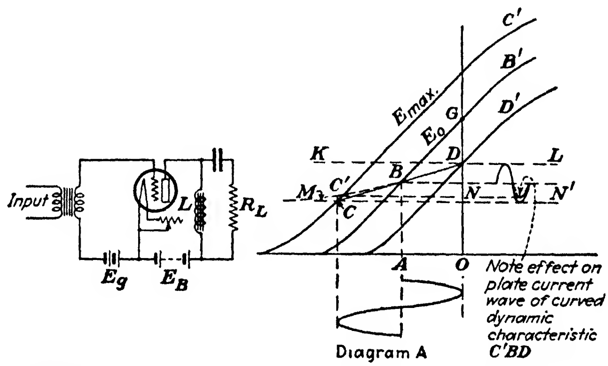


FIG. 12.—Diagram of test circuit and fundamental-dynamic-characteristic curve. (*After W. J. Brown.*)

maximum undistorted power output. In giving this proof, assumptions must be made that the output circuit must be purely resistive and that the wave is sinusoidal. Strictly speaking, this is applicable only to a sustained pure note, but the formula applies equally well for comparison purposes to any wave form. In the operation of the amplifier of Fig. 12, the plate-current variation causes the plate potential to vary, with the result that the dynamic grid-voltage plate-current characteristic does not follow a single static characteristic for one plate potential; but owing to the drop in the load resistance, the plate voltage is continually changing from one static characteristic to another. Therefore, in Fig. 12, the dynamic characteristic is actually the straight line *CBD*. The power output is undistorted as long

[1] See preceding footnote.

as the alternating plate-current wave form is very nearly the same as that of the input voltage. For such a result *CBD* is a straight line. This is true whether the load is a resistance directly in the plate circuit or connected to the secondary of a transformer whose primary is in the plate circuit or connected as shown in Fig. 12. The determination of this characteristic is given in papers referred to.[1] The procedure will not be given here, as this method of determining output is not now used so much as formerly.

The plate family of characteristics provide a very convenient and widely used means of output and distortion limits of class A

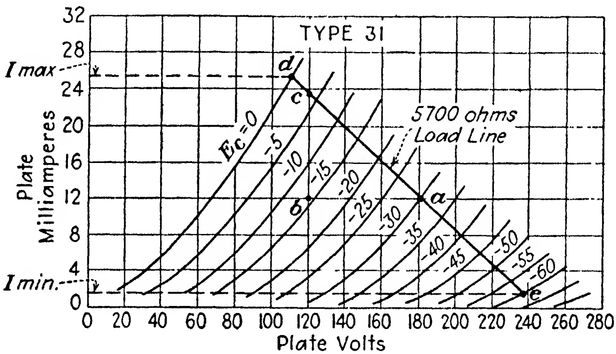


FIG. 13.—Plate-characteristic family for determination of power output and distortion.

power amplifiers.[2] In Fig. 13 is shown the characteristic plate family for a type '31 power amplifier triode. These static characteristics are obtained in the usual way. The rating given for this type of tube specifies the maximum operating *plate-supply* potential as 180 volts at a grid bias of -30 volts. This corresponds to point *a*, Fig. 13. As an example it will be assumed that a load resistance of 5,700 ohms is used, and it is required to determine the power output obtainable when the grid-excitation voltage has a maximum value of 30 volts. The class A amplifier is always thus operated, and it is customary to say that the *grid swing* is 30 volts for this case. Then a straight line, known as the

[1] See paper by Hanna, Sutherlin, and Upp, *loc. cit.* The procedure is also described in H. A. Brown, "Radio Frequency Electrical Measurements," 1st ed., pp. 252–254.

[2] In Kellogg's paper (*loc. cit.*), credit for the origin of this method is given to J. C. Warner.

load line, is drawn through point a at a cotangent slope equal to $ab/bc = 5{,}700$; ab and bc are, of course, read on the corresponding coordinate scales. The straight line will intersect the curve for zero grid potential at d, and the curve for -60 volts grid bias at e. At d the plate current ordinate is $I_{max} = 25$ milliamp., and at e it is $I_{min} = 1.4$ milliamp. Thus 30 volts grid swing will cause the total plate-current change of $25 - 1.4 = 23.6$ milliamp. The voltage is seen to vary from 109 volts for 24 mils to 238 volts for 1.4 mils, a double amplitude of 129 volts. The alternating component of plate voltage is present across the load resistance, and it is shown in the papers referred to that the average power output P_0 is

$$P_0 = \frac{(E_{max} - E_{min})(I_{max} - I_{min})}{8}. \tag{30}$$

For the example considered,

$$P_0 = \frac{(25 - 1.4)(238 - 109)}{8} = 380 \text{ milliwatts.}$$

This is very near the rated value of 375 for maximum undistorted power output.

It is important to determine the amount of distortion of the excitation-voltage wave shape, as it will appear in the wave of alternating current in the load resistance; it is by taking account of this effect that the so-called *maximum undistorted power* output is determined. It will be noticed when Fig. 13 is examined that the grid swing from -30 volts to zero gives a positive plate-current change of 13.0 milliamp., but the swing from -30 to -60 gives 10.6 mils, a smaller change. This means that the dynamic characteristic in Fig. 12 departs from a straight line to the left of B and assumes the dotted-line form BC', and the effect on the plate current is also shown. The main result is the production of a second-harmonic component of alternating plate current (and load current). The higher order harmonics are usually not sufficiently prominent as to need consideration for the case of a single-tube class A amplifier when the amplitude of the second harmonic is less than 6 to 8 per cent of the fundamental. It is shown in Kellogg's paper and others[1] that the percentage

[1] A good discussion and proof of this is found in R. S. Glasgow, "Principles of Radio Engineering," p. 181.

second-harmonic distortion is

$$\% \text{ 2d har.} = \frac{\frac{1}{2}(I_{max} + I_{min}) - I_b}{(I_{max} - I_{min})} \times 100 \tag{31}$$

$$= \frac{\frac{1}{2}(25 + 1.4) - 12.1}{25 - 1.4} = 4.7 \text{ per cent}$$

for the example given. The maximum undistorted output, distortion not greater than 5.0 per cent, say, may be found by varying the slope of load line *ead* in Fig. 13 and finding the values of output and percentage second harmonic for each slope (load resistance is the cotangent of the slope), from which the maximum value will be found. It happens that, for the example given, the 380 milli-

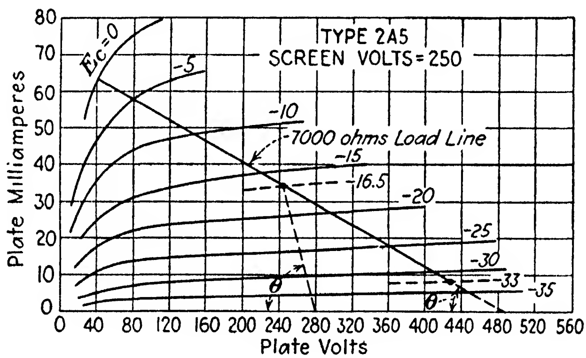


FIG. 14.—Plate-characteristic family and load line for maximum undistorted output for pentode-type power-amplifier tube.

watts is the maximum undistorted output and is, of course, for 5,700 ohms load resistance. Figure 14 shows how similar determinations are determined for a type 2A5 pentode tube. Owing to the presence of a screen and a suppressor grid in this type of tube, the load resistance for maximum undistorted power output is 7,000 ohms and is much lower than the alternating-current plate resistance of 100,000 ohms, as seen from the slope of the plate-family curves.

The foregoing measurements were made on amplifier tubes in which it was assumed that the plate-supply voltage E_B is present between plate and filament and that there was no resistance drop in the choke coil in the plate circuit of the test circuit of Fig. 12. The same would be true if an output transformer were used which had a primary winding of zero direct-current resistance connected in series with the plate circuit, and the load resistance, as usual, connected to the secondary winding. The load resistance reflected into the plate circuit would include effective connected load-circuit resistance, secondary-winding effective resistance, and core-loss resistance. Now, the choke and the primary winding will have some direct-current resistance, and the presence of this will require a higher plate-supply voltage than that present at the plate for normal grid bias and no grid excitation. Assuming that this direct-current resistance is 1,050 ohms, a line may be drawn in Fig. 14 from the point where the plate-current curve for the operating grid bias (−16.5 volts) intersects the load line, to the abscissa at a slope θ whose cotangent is 1,050, giving 280 volts supply voltage for 250 volts on the plate.[1] If the load resistance is directly in the plate circuit, the plate-supply voltage is obtained by extending the load line to the abscissa, giving 490 volts in Fig. 14. There are some conditions where the *effective* alternating-current load resistance is less than the direct-current resistance in the plate circuit, such as in resistance-coupled amplifiers, in which case a line is drawn at a slope that is less than that of the load line, indicating a higher required supply voltage than that found by extending the load line to the abscissae line.

If it is desired to obtain increased undistorted power output, higher plate-operating voltage and higher negative grid-bias potential are tried out, using the same family of characteristics; and trials will show that the load resistance must be greater than twice the alternating-current plate resistance for maximum power output for the case of triodes. If the load impedance is not a pure resistance, the dynamic characteristic is not a straight line, and for a high reactive load impedance it has the form of an ellipse. It is pointed out by Kellogg (footnote 1, page 250) that if such a load circuit is used, and its impedance is higher than twice the plate resistance, no distortion will be encountered if the same plate and grid voltages are maintained. It is also pointed out that if the amplifier tubes work into the primary of an output transformer whose secondary has largely resistance load, and whose leakage reactance is fairly small due to good design, the input impedance will be mostly noninductive and may be made equal to twice or greater than the alternating-current plate

[1] The data for the curves of Figs. 13 and 14 was obtained from the RCA Cunningham Radiotron Manual, Series *RC*-12.

resistance for maximum undistorted output of the amplifier tube.[1]

9. Direct Measurement of Audio-frequency Power-amplifier Output. *Method for Class A Amplifier.*—The power output of a class A amplifier under varying conditions may be directly measured, and the various electrode potentials and plate-load resistance may be varied to obtain optimum operating conditions. Figure 15, Diagram (*a*), shows a flexible test circuit for this purpose arranged to test a pentode-type power tube. The alternating-current power output is, of course, the I^2R value in the load resistance. Distortion can be measured by one of the several

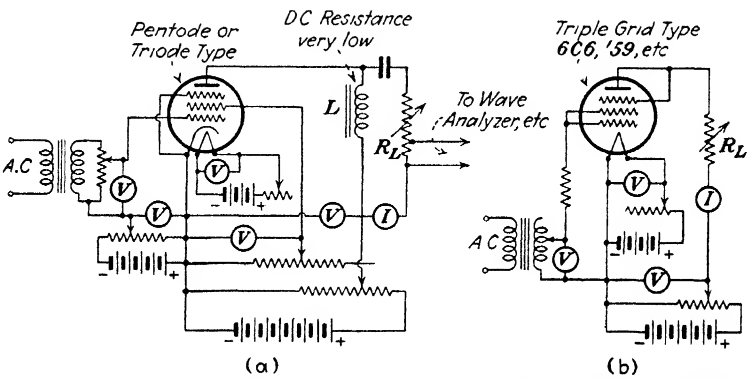


FIG. 15.—Test circuits for direct measurement of power output of classes A and B amplifiers.

methods discussed in Chap. VII. Analysis of the input and output waves is, of course, the most desirable. If the input is made sinusoidal with the aid of selective circuits and low-pass filters, the amplifier distortion is easily measured. The determination of the percentage total harmonic residue with the aid of the Belfils' bridge described in Chap. VII is often convenient with the aid of a sensitive amplifier-electron tube voltmeter, which is connected to the output terminals of the well-designed Belfils' bridge. The cathode-ray oscilloscope connected across the load resistance, or a portion of it, is a convenient way to observe when distortion of a pure wave form of input voltage becomes noticeable.

[1] The theoretical elliptical dynamic characteristic is derived and discussed in a paper entitled "On the Theory of Power Amplification," *Proc. I.R.E.*, Vol. 16, No. 2, pp. 193–207, by Manfred von Ardenne.

Method for Class B Amplifier.—Remembering that the plate current is zero for no grid excitation of a class B amplifier tube, the load resistance may be placed in the plate circuit for power-output determination, as shown in Diagram (*b*), Fig. 15. The plate current is in half-wave pulses due to bias at cutoff, and the milliammeter I reads the average value of the current pulses. By integrating over a cycle of current and voltage, the average power output can be found. A practical approximate formula is recommended[1] and gives the output as

$$\text{Watts output} = \frac{(\text{average } I_{d-c} \text{ in amperes})^2 \times R_L}{0.405}, \qquad (32)$$

where R_L is the load resistance.

It is often important to determine when the grid excitation becomes so great that the positive-peak grid voltage exceeds the minimum plate voltage and robs the plate circuit of a portion of the maximum value of the plate-current pulses. The cathode-ray oscilloscope will indicate when the plate-current pulses show a decided flattening of its peaks due to this condition. It is, however, best detected by connecting the oscilloscope input to the output-load resistance of a complete push-pull (or rather *push-push*) class B amplifier having a sinusoidal wave of input voltage.

10. Measurement of Power-amplifier Operating Current and Potential Curves. *Oscillographic Method.*—The power output and the efficiency of a radio-frequency class B or C power amplifier is often worked out with the aid of plate-grid-current waves and the wave of plate voltage. The actual losses on the plate and grid can be calculated by taking product values of $e_p i$ and finding average ordinates from which average power values are found. The different families of static characteristics are also often used in this connection.[2] Oscillographs may be used to give actual pictures of the positive pulse of plate current and the corresponding dip in the plate voltage to its minimum value and the grid voltage and its positive-region current pulse. The cathode-ray oscilloscope is convenient for this purpose, and its use is illustrated in the test circuit of Fig. 16. One cathode-ray tube, deflector

[1] RCA Cunningham Radiotron Manual, Series RC-12, p. 140.

[2] PRINCE, D. C., "Vacuum Tubes as Power Oscillators," *Proc. I.R.E.*, Vol. 11, pp. 275, 425, 527; see also GLASGOW, R. S., "Principles of Radio Engineering," p. 273; TERMAN, F. E., "Radio Engineering," pp. 119–225.

plate of a pair, is of course grounded, while the other is connected first to the grid, giving its potential wave, and then to the plate terminal, giving the potential wave of the plate. The connection may be alternated rapidly with the aid of a two-section rotating commutator, making both potential waves appear simultaneously upon the screen.[1] The plate and grid currents, in cases of small amplifier tubes, is rather small, and deflections due to these may be made of sufficient magnitude by passing these currents through small deflecting coils so placed that they will cause the beam to be deflected at right angles to the sweep-voltage deflection. For larger power tubes, the plate- and grid-current pulses are of

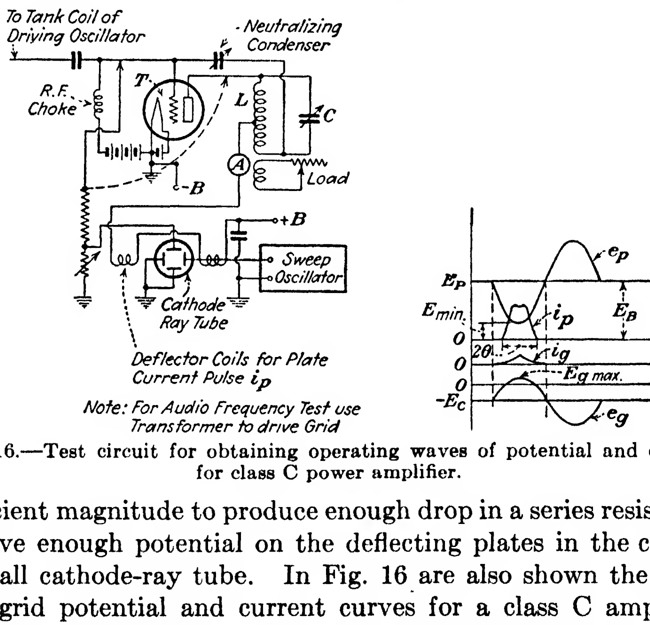


FIG. 16.—Test circuit for obtaining operating waves of potential and current for class C power amplifier.

sufficient magnitude to produce enough drop in a series resistance to give enough potential on the deflecting plates in the case of a small cathode-ray tube. In Fig. 16 are also shown the plate and grid potential and current curves for a class C amplifier. Testing the amplifier at low radio frequencies gives a good indication of the relative shape and phase of these curves; but if it is desired to operate the power-amplifier tube at frequencies above 300 or 400 kc., a special radio-frequency sweep oscillator is needed for the time-axis deflection of the cathode-ray beam.[2]

[1] An electron-tube device for doing this has been developed at Purdue University. It is manufactured by Allan B. Dumont, Upper Montclair, N. J.

[2] A sweep-voltage supply which functions up to 12 Mc. is described by Goldsmith and Richards, see p. 329 of this book.

The amplitudes of the deflections are obtained by calibrating the cathode-ray tube with the plate-supply voltage. By comparison with the plate-voltage wave the value of the double angle of plate-current flow 2θ (see wave diagrams, Fig. 16) may be measured. The waves of Fig. 16 may be traced on translucent coordinate paper placed on the screen end of the cathode-ray tube, or they may be photographed.

Use of Peak Voltmeter for Plate- and Grid-voltage Measurement.—If a magnetic oscillograph is used to obtain an observed or photographic record of the plate-current pulses, it must be with caution. When a steep wave-front current returning to zero value passes through the oscillograph element, the latter will pass through zero and show a noticeable reversed deflection due to its inertia. This is known as *overshooting*. A special high-frequency element must be used in the oscillograph to prevent this. These elements have the bridges supporting the suspension loop closer to the mirror (cemented on the loop wires) than does the standard element. The special element is, of course, considerably less sensitive than the standard. If the standard element is used, the frequency of the grid-driving voltage must be made quite low. It must also be remembered that it requires 100 milliamp. to produce a ½- to 1-in. deflection with the standard element. The measurement of the maximum grid potential and of minimum plate voltage is not very precise when carried out as explained in the preceding paragraph. They may be measured by using a specially designed peak-trough voltmeter using a kenotron tube instead of the triode ordinarily used in the peak voltmeter. The reason for this is the adaptation of the kenotron to high voltages and its resistance made very high by varying its filament current. For lower voltage amplifier tubes, the triode-type peak voltmeter may be used. Figure 17 shows how arrangements may be made to use a single-tube voltmeter to measure maximum grid voltage and minimum plate voltage of a neutralized radio-frequency power-amplifier triode. At the same time, the oscilloscope is used to obtain the plate- and grid-current pulses, as previously described. When the porcelain-base double-pole double-throw switch is closed in position 1, the kenotron plate is connected to the grid of the power-amplifier tube under test; and if the voltage V across potentiometer P is not quite so great as the positive maximum of the grid potential, pulses of current will flow through the kenotron, causing milliammeter A to have deflection. P is varied until A reads zero, at which V indicates the maximum positive grid potential. When the switch is in position 2, the kenotron filament is connected to the power-amplifier tube plate, and no current will flow in A unless voltage V is higher than the minimum value of the pulsating plate voltage of the power amplifier, and the kenotron functions as a *trough voltmeter*.

11. Plate-efficiency Direct Measurement in Power Amplifiers. *Class B of Linear-amplifier Efficiency.*—It is sometimes desirable to determine whether the actual plate efficiency of such a radio-frequency power amplifier approximates the theoretical value of 78.5 per cent ($\pi/4 \times 100$). The plate-circuit input is, of course, $E_B I_B$, the plate-voltage and plate-current supply values being

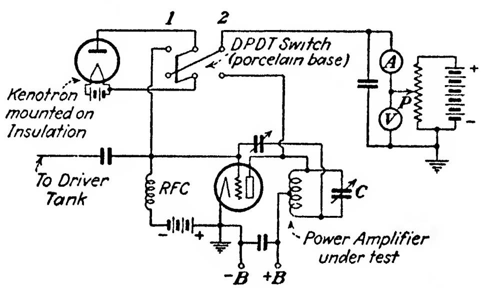


FIG. 17.—Peak-trough voltmeter arranged to measure maximum positive grid and minimum plate potential of a power-amplifier tube in operation.

read on direct-current meters which read average values of unidirectional pulsating quantities. The plate-supply voltage E_B will, of course, be continuous, but the current I_B will be pulsating; it may be proved that the average power is equal to the product of these quantities. Figure 18 shows the test circuit. The load may be a resistance added in series in the tank circuit, but a more flexible arrangement is inductively to couple the load as shown. Variation of the coupling and of the resistance may be made to obtain desired plate-current and tube load. A direct-current milliammeter in the grid lead will show when the grid receives adequate excitation. This should not exceed 25 per cent of the rated plate current. Optional *link coupling* to obtain variable excitation is indicated. When all meter readings have been obtained, the resistance of the tank circuit is measured, with the load still coupled, using preferably the resistance-variation method, as explained in Chap. I. To facilitate this, the tank-tuning condenser C should be a high-grade calibrated type, and a coupled-current indicator is more convenient than the tank-circuit ammeter for this latter measurement. An inductively coupled load may be conveniently measured with an incandescent lamp enclosed in a light-tight box, and in this box is also placed a photronic cell. This latter is connected to a small microammeter, and a deflection is obtained which is proportional to the light from the lamp bulb and hence to the radio-frequency current squared. Calibration is made with direct current. This device is shown in Fig. 28, Chap. VI.

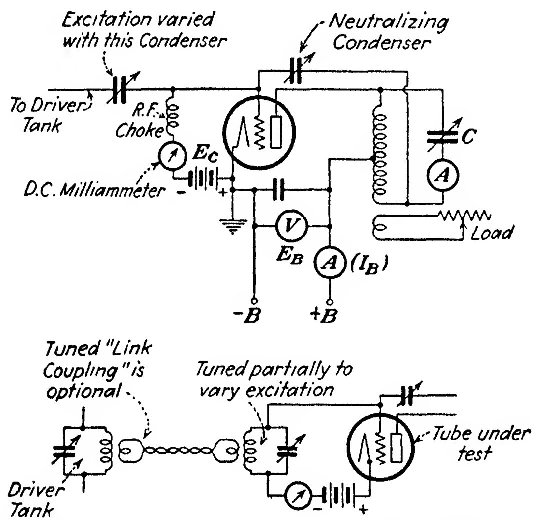


FIG. 18.—Test circuit for measuring output and plate efficiency of a class B amplifier.

12. Oscillator and Amplifier Efficiency by Other Methods. *Method by* I^2R.—This method of measuring efficiency consists of measuring the resistance of the load circuit by a suitable means discussed in Chap. I, followed by a measurement of the alternating current in the oscillating circuit and the measurement of the plate-supply voltage and average value of the plate-supply current. The procedure is similar to that explained in the preceding article. Thus the power output and the power input to the tube and associated circuits is determined with an accuracy that is sufficient, although it may not be better than 5 per cent. It is the practice of some to add noninductive resistance to the oscillating circuit until the oscillator current decreases 50 per cent, whence the resistance of the oscillating circuit is equal to that of the added resistance. This assumes that the excitation of the oscillating circuit remains constant, but usually the decrease of oscillating circuit is accompanied by a decrease in grid excitation and hence a decrease in the effective voltage μE_g, which excites the oscillating circuit, so that it is better to make a measurement of oscillator-circuit resistance by a suitable resistance-measurement method for the frequency of the generated current.

Calorimeter Method.—Everitt and Byrne in this paper, referred to in Chap. III, Art. 2, mentioned a method of measuring the power dissipated in an oscillating vacuum tube by immersing the tube in an oil calorimeter. The power dissipated is calculated from the rising temperature of the oil and its specific heat during a specified time, and the power input to the vacuum-tube oscillator complete is measured by means of instruments, perferably a wattmeter. In cases where the vacuum-tube oscillator is exciting coupled-output circuits such as antennas (and these circuits having a nonquasi-stationary current distribution), it is not easy to measure the effective radio-frequency resistance, and this method offers a very reliable means of determining the power output and efficiency of oscillating vacuum tubes under almost any conditions of high-frequency and complex dissipative circuits. The plate loss of a high-power water-cooled amplifier tube is easily found by measuring the temperature rise of the cooling water and the amount flowing in a given time.

Efficiency of Vacuum-tube Generators with Surface Pyrometer.—A means of determining the power dissipation in an oscillating or power-amplifying vacuum tube has been given by A. Crossley and R. M. Page.[1] They evolved the method after experimenting with an optical pyrometer for measurement of the temperature of the tube electrodes and found that a device known as a *surface pyrometer* could be placed in contact with the glass envelope of the vacuum tube under test, obtaining a temperature measurement which was a function of the mean electrode temperature and hence depended upon the mean watt dissipation within the tube. This surface pyrometer is an especially constructed

[1] "A New Method for Determining the Efficiency of Vacuum-tube Circuits," *Proc. I.R.E.*, Vol. 16, No. 10, pp. 1375–1383.

device whose thermal element is in the form of a flat strip and is supported with the aid of a heavier metal strip to an adjustable framework. It is manufactured by the Cambridge Scientific Instrument Company for use in obtaining the temperature of flat or cylindrical surfaces. It was found that the surface pyrometer could be attached to the tube envelope at a point opposite the flat or cylindrical surface of the anode and that temperature readings could be taken for various impressed-plate supply voltages with the tube in the nonoscillating state, thus giving surface temperatures which are a function of the power dissipated on the plate of the tube. Next, the tube was placed in circuit with a suitably tuned output circuit and feedback arrangement, so that it functioned as an oscillator or generator of radio-frequency power. After sufficient time had elapsed to assure stable conditions, the temperature was again read with the aid of the pyrometer, and from the previous calibrated data the power dissipated on the electrode, which was the power lost, was obtained. This value subtracted from the watts input to the oscillator from the plate-supply battery gave the power output. It was assumed that the power dissipation on the grids contributes to the mean anode temperature, and hence the power dissipated is due both to grid-circuit loss and to plate loss. The form of the pyrometer and method of attaching it to the tubes is best understood by the illustrations given in the paper referred to.

CHAPTER VI

ELECTROMOTIVE FORCE, CURRENT, POWER

1. Introduction. *Requirements in Measurement of E.M.F. Radio Frequencies.*—The measurement of e.m.f. and of current at radio frequencies presents limitations and difficulties that are not encountered at power frequencies. Fortunately, the degree of precision of such measurements has been permissibly less at radio frequencies, owing in part to the fact that the circuit constants become more uncertain in nature and value as the frequency increases. However, progress is being made in the reliability and precision of design and measurement of circuit constant standards and in measurement apparatus, so that a higher degree of precision is becoming more and more common.

The production and measurement of very small values of current and voltage are of paramount importance; microvoltages are used quite extensively, but the problem of the measurement of such quantities has not yet been completely solved. The measurement of currents of the order of a few microamperes at radio frequencies presents another important problem, and such measurements can now be made with a fair degree of precision, provided the frequency is below ultra-high limits; beyond these limits, extreme skin-effect phenomenon and appreciable shunt admittance make the measurements uncertain. Measurements of such small currents and of small voltages, as well as measurements of large currents and ordinary voltages, will be described in due course.

2. Measurement of E.M.F. with Electron-tube Voltmeter. *Fundamental Principles; Theory.*—The electron-tube voltmeter, commonly called the vacuum-tube voltmeter, has come into wide use for the measurement of voltages ranging from a fraction of 1 volt to over 100 volts. This method of measurement has the advantage over all other methods in that very little or no current is taken by the measurement device and the fact that the frequency limits for fair degrees of accuracy are considerably higher.[1] The fundamental principle incorporated in a tube voltmeter is most easily explained by discussing the behavior of the apparatus and circuit arrangement of Fig. 1. With no voltage impressed on the grid of the vacuum tube, the grid bias is placed at a value that allows a small plate current to flow as indicated in the diagram at the right of the figure. When the high-frequency voltage is impressed at the terminals shown, the mean value of plate current increases according to the type and curvature of the tube characteristic, as illustrated in the diagram. This phenomenon is also present in the vacuum-tube detector without

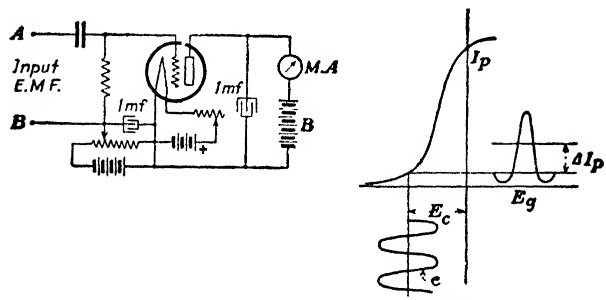


FIG. 1.—Circuit diagram and characteristic of thermionic-tube voltmeter.

grid condenser or leak. The anode current may be expressed as a function of the impressed grid voltage by the following equation:

$$i = a_0 + a_1 e + a_2 e^2 + a_3 e^3 \cdot \cdot \cdot + a_n e^n,$$

where a_0 is the steady anode or plate current when $e = 0$, and

$$a_1 = \frac{\partial i}{\partial e}, \qquad a_2 = \frac{\partial^2 i}{\partial e^2}, \text{ etc.}$$

If an alternating e.m.f. containing a fundamental and second harmonic of the form

$$e = \mathcal{E}_1 \sin \omega t + \mathcal{E}_2 \sin (2\omega t + \theta)$$

is impressed on the input terminals A and B (Fig. 1), the expression for the anode current is obtained by substituting this value in

[1] The electrostatic voltmeter has been used somewhat, but it has an input capacitance of 100 mmf. at full scale and will not give deflections below 10 to 20 volts.

the equation of anode current, and

$$i = a_0 + a_2\left(\frac{\mathcal{E}_1^2}{2} + \frac{\mathcal{E}_2^2}{2}\right) + \left(\text{terms at frequencies } \frac{\omega}{2\pi}, \frac{\omega}{\pi}, \text{ etc,}\right)$$

results, assuming that the characteristic of the tube can be fully expressed by the first three terms of the series. The equation for i contains the constant term a_0, a series of terms that show the existence of components of plate current at frequencies $\omega/2\pi$ and multiples of this frequency, and a term

$$\begin{aligned}\Delta I_p &= a_2\left(\frac{\mathcal{E}_1^2}{2} + \frac{\mathcal{E}_2^2}{2}\right)\\ &= a_2(E_1^2 + E_2^2)\\ &= a_2E^2,\end{aligned}$$

which shows that the change in the average value of anode

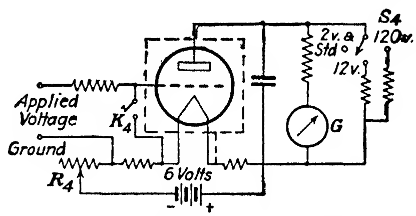


FIG. 2.—Circuit of Moullin vacuum-tube voltmeter.

current is proportional to the square of the effective value of the total e.m.f. impressed upon the tube. Thus it is seen that the only obtainable indication on the direct-current milliammeter in the plate circuit of the tube is due to the rectification characteristic.

If the impressed e.m.f. is greatly increased, the equation for plate current will include third- and fourth-power terms of the series. The usual practice is to operate the tube as a voltmeter within the region where the change in average plate current varies as the square of the input voltage or the plate current is expressed by the first three terms of the series given above. This is very important from the standpoint of wave-form error, as will be discussed later.

Types, Features.—A large number of thermionic-voltmeter designs have been given in the past, and some of them have novel features or are applicable to certain requirements. A very brief description of a few of these will be given. One of the early forms which is now on the market is shown in Fig. 2 and was developed by E. B. Moullin.[1] The novel feature of this instrument is the complete absence of an anode battery, the plate voltage being obtained by connection of the plate-circuit return to the positive filament. By using shunt resistances across the galvanometer and series resistor the instrument is given three ranges. The volt ranges are indicated at the shunt-switch positions. Moullin and Turner describe a form of tube voltmeter adaptable to the measurement of smaller voltages by the simple process of balancing out a portion of the steady plate current from the plate-current indicator device. This form of tube voltmeter is shown in Fig. 3.[2] Many tube-voltmeter circuits of this type have been used, and with care in manipulation a sensitive microammeter may be used in the circuit, giving the instrument a full-scale deflection for voltages below 0.2 volt.

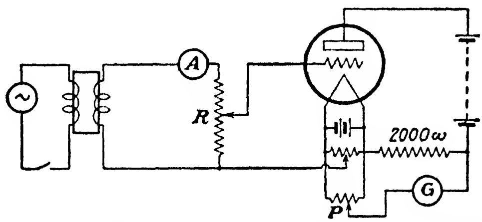


FIG. 3.—Vacuum-tube (thermionic) voltmeter with compensation for normal plate current. (*After Moullin and Turner.*)

A compensated-tube electron-tube voltmeter is described by H. M. Turner,[3] in which changes in the filament temperature are automatically rendered innocuous and have no effect upon the calibration curve. Figure 4, taken from Turner's paper, shows the circuit arrangement of this tube voltmeter and the calibration curves for compensated and noncompensated filament-current variation. Jansky and Feldman have described a two-range tube voltmeter,[4] which was carefully designed for efficiency and convenience. The circuit arrangement and apparatus are shown in Fig. 5, reproduced from their paper. H. R. Lubcke has pub-

[1] This instrument is manufactured by the Cambridge Instrument Company. It is described by Moullin in his book "Radio Frequency Measurements," 1931 ed., pp. 140–142.

[2] See *Jour. I.E.E.*, Vol. 60, p. 708, 1922.

[3] *Proc. I.R.E.*, Vol. 16, No. 6, pp. 799–801, 1928.

[4] *Trans. A.I.E.E.*, Vol. 47, p. 126, 1928.

lished a paper giving design rules for the best circuit combinations possible for a required range of the vacuum-tube voltmeter.[1]

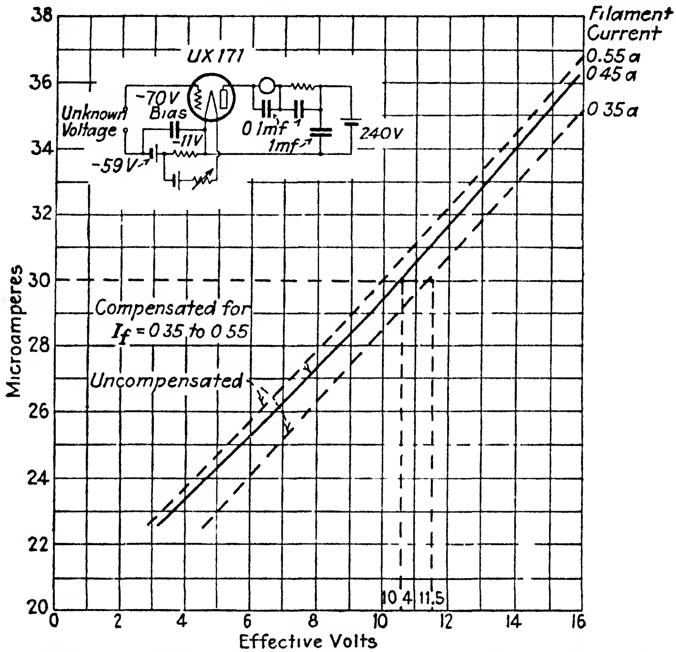


FIG. 4.—Special compensated vacuum-tube voltmeter and calibration characteristic. (*After H. M. Turner.*)

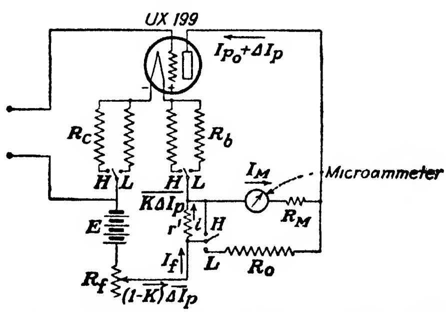


FIG. 5.—Flexible vacuum-tube voltmeter circuit. (*After Jansky and Feldman.*)

In addition to these are several types incorporating a balanced Wheatstone bridge with which the normal plate current is elimi-

[1] "Vacuum Tube Voltmeter Design," *Proc. I.R.E.*, Vol. 17, No. 5, pp. 864–872.

nated from the indicating microammeter.[1] They may be easily set up in the laboratory but are no longer commercially manufactured. The screen-grid tube may also be used in a tube-voltmeter circuit and supplied with the well-known grid condenser and grid leak, as in the grid-leak detector. The plate current, of course, decreases in this type when the e.m.f. to be measured is applied to the input terminals. If a compensation circuit, as previously described, (Fig. 3) is used, changes of plate current may be read on a microammeter for very small input voltages. It is necessary to calibrate such a voltmeter at the frequency of measurement.

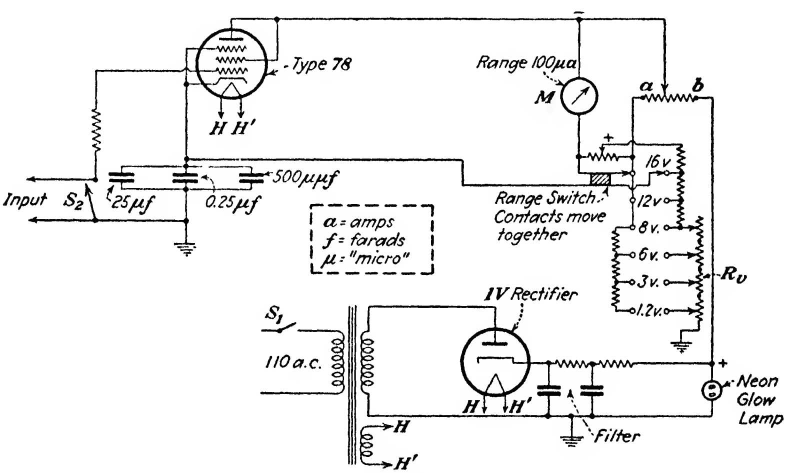


FIG. 6.—Modern multirange vacuum-tube voltmeter with calibration adjustments. (*Weston model* 669.)

A modern medium-range type tube voltmeter, accurate up to 20 Mc., is shown in Fig. 6. The double-range switch provides convenient change from 1.2- to 16-volt ranges. The variable resistors R_v are used to calibrate for the lower ranges and to compensate for other variables. No readjustment of these is necessary when changing tubes if the latter have fairly uniform characteristics. The three fixed condensers by-pass alternating-current plate current components around the biasing resistors. The high capacitance by-passes very low frequencies successfully,

[1] See *Trans. A.I.E.E.*, Vol. 46, p. 541, 1927. Two of these are described in H. A. Brown, "Radio Frequency Electrical Measurements," 1st ed., p. 269.

but low-capacitance condensers by-pass the higher radio frequencies more successfully because at these frequencies high-capacitance paper condensers function more as inductances than as capacitances. Plate-current compensation in the indicating microammeter is also provided.[1]

The new acorn-type electron tubes, designed for high frequencies, are well utilized in the tube-voltmeter circuit of Fig. 7.

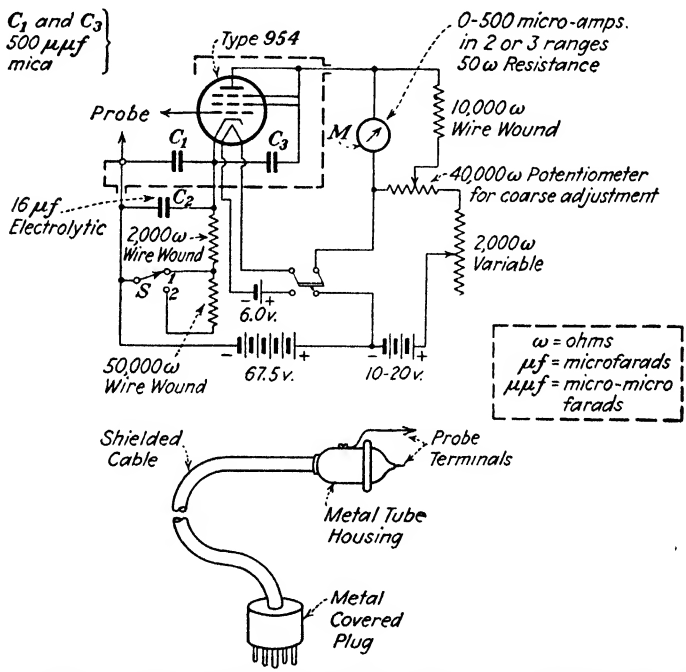


FIG. 7.—Vacuum-tube voltmeter for radio frequencies up to 50 Mc. in two ranges.

Here the tube is used as a triode and has an input capacitance of 1.4 mmf. Self-bias for low and high ranges is provided by resistors, as shown. With switch *S* on 1 and 2, calibration curves shown in Fig. 8 are obtained. This instrument is made to read accurately at frequencies up to 50 Mc. and higher, by a so-called *probe construction*, shown in Fig. 7, wherein the acorn tube is placed in a metal housing at the end of a shielded cable. The probe is the control-grid terminal. The leads carry direct

[1] Information concerning this instrument was kindly furnished by J. H. Miller of the Weston Electrical Instrument Corporation.

current, and by-pass condensers C_1 and C_3 are in the tube housing.[1]

Another recent commercial development makes use of an acorn tube as a diode rectifier, this tube being supported in a shielding case at the end of a flexible shaft. Two probe terminals attach to the plate and cathode of the tube.[2] The circuit is

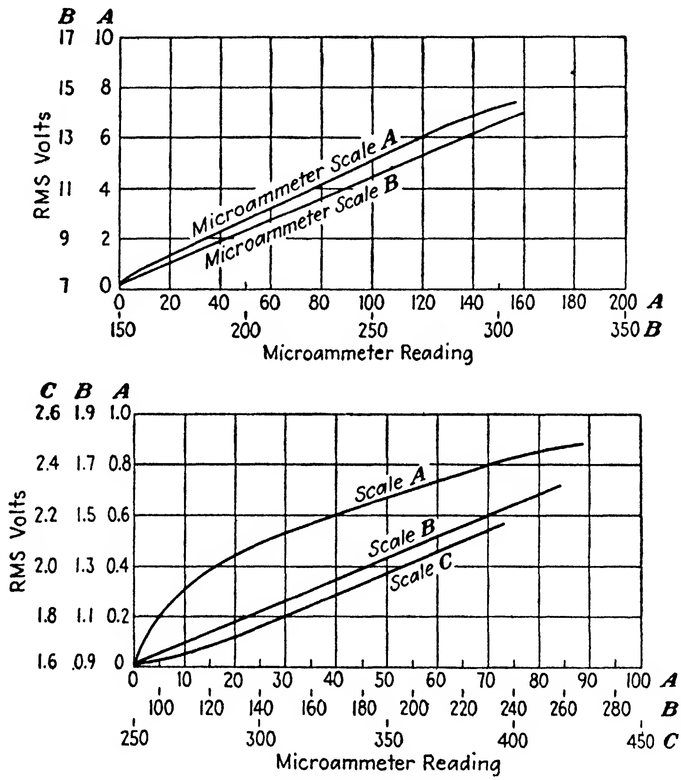


FIG. 8.—Sample calibrations for vacuum-tube voltmeter of Fig. 7.

shown in Fig. 8*a*. The condenser C_1 is charged up to nearly the peak value of the input voltage through the diode rectifier, and the degenerative direct-current amplifier using the type 75

[1] The complete description of the design of this instrument is given in R.C.A. Manufacturing Company, Inc., Application Note 47. Figures 7 and 8 are also taken from this paper.

[2] For description see *General Radio Experimenter*, Vol. 11, No. 12, May, 1937. The instrument is manufactured by the General Radio Company.

triode amplifies the potential existing across C_1, the value of this voltage showing on the calibrated meter M possessing uniform scales in volts. The reference should be consulted for detailed discussion. Its readings are within 3 per cent of correct values at 100 Mc. and less than 1 per cent below 50 Mc. This is essentially a peak voltmeter but of course may be used to measure effective values if the wave form is sinusoidal, as it usually is in tuned radio-frequency circuits.

Accuracy, Wave-form Error.—The two most troublesome sources of error in the measurement of voltage with a tube voltmeter are the frequency error and that due to turnover.

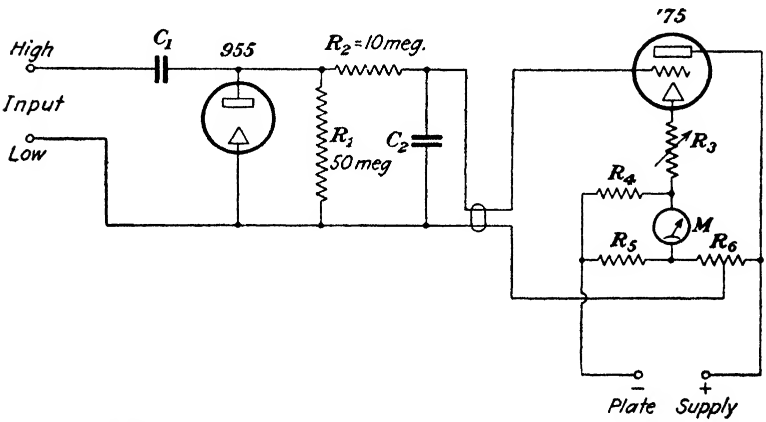


FIG. 8a.—High-frequency vacuum-tube voltmeter using acorn tube as diode rectifier. (*After General Radio Company.*)

These two errors can be minimized by suitable precaution, but there are limits due to these effects beyond which the measurement is impracticable. The frequency error is due to the fact that the input capacitance offers an input shunt admittance which increases with the frequency, and the tube takes more and more current as the frequency is increased. A vacuum-tube voltmeter may well indicate the correct voltage of a source to which it is connected with full accuracy, even though it draws current from the source. If the leads to the tube have very low resistance and inductive reactance, there will be no error caused by capacitive current as far as the tube voltmeter is concerned when the wave form is sinusoidal. If the voltage to be measured is that of a high-impedance generator, the current taken by the voltmeter may of course cause the generator terminal voltage to be quite different while being measured. Leads to a tube voltmeter may have inductive reactance, causing a rise in voltage between the terminals of the source and the tube electrodes. If the lead inductance and tube input capacitance form a resonant circuit at the measuring frequency, very great error will result. The latter must be less than 10 per cent of the former for negligible effect. A frequency error will also be present in the plate-current indicating meter unless the radio-frequency components are adequately by-passed around the meter by a large condenser, the 1-mf. size being commonly used for this purpose. A 1-mf. condenser may even be inductive at very high frequencies. While it is the usual practice to calibrate the tube voltmeter at audio or power frequencies, it is better to calibrate at radio frequencies when the tube is to be so used. Calibration at a frequency of 1,000 kc. is practicable with the aid of a measured mutual inductance, the current in the primary of the mutual inductance being measured with a reliable thermocouple current meter, so that the voltage ωMI_1, induced in the secondary, is connected to the input terminals. Calibration of vacuum-tube voltmeters at very high radio frequencies (up to 100 Mc.) may now be readily accomplished by passing current at the calibrating frequency through a very fine short high-resistance wire to provide the known calibrating voltage, the current being accurately measured with a suitable high radio-frequency thermocouple.[1]

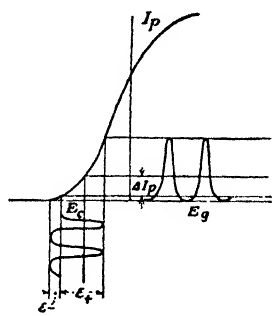


FIG. 9.—Tube characteristic and unsymmetrical impressed e.m.f. illustrating turnover.

The so-called *turnover* is that due to the presence of even harmonics in the wave form of the e.m.f. to be measured, resulting in an unsymmetrical wave. Referring to Fig. 9, a highly unsymmetrical e.m.f. wave is impressed upon the grid of the tube voltmeter having its grid bias at E_c. It will be appreciated that the value of ΔI_p obtained is due to the square of the large amplitude of the assumed positive half wave, or E_+^2. For this connection,

$$\Delta I_{p+} = a_2 \frac{\mathcal{E}_+^2}{2}.$$

[1] See Fig. 15*A*, p. 286.

If the connections to the source of this e.m.f. are reversed, the average change in plate current is due to E_-^2, and

$$\Delta I_{p-} = a_2 \frac{\mathcal{E}_-^2}{2}.$$

A simple graphical analysis will show that for some unsymmetrical wave forms the average change in the direct-current plate current is different when the conditions have been reversed. This phenomenon is known as the turnover, and the measurement of effective e.m.f. will be in error known as a wave-form error. It is claimed by some investigators that the error is practically eliminated if the readings of plate current for the two connections are averaged and if the voltage is obtained from the calibration curve for the average value of the plate-current readings.

A very interesting analysis of the error due to the foregoing cause has been given by M. Reed,[1] in which he defines turnover as

$$\text{Turnover} = \frac{\Delta I_{p-}}{\Delta I_{p+}} = 1 - \frac{I_0}{\Delta I_{p+}},$$

where I_0 will be defined presently. This author has shown that when the tube characteristic is a cubic equation, the readings obtained for reversed connections of the input voltage will not be the same as for the original connections (under certain conditions). Denoting the difference in the two readings by I_0, he showed that $I_0 = 0$ when the impressed-voltage wave contains a series of harmonics and that certain of these harmonics are double the frequency of some lower ones, *i.e.*, a second and a fourth or a third and a sixth, and so on. Then, too, these harmonics must have a certain phase with each other in order to give a value of I_0. Reed showed that there may be certain phase relations between a pair of such harmonics that result in $I_0 = 0$, even though the characteristic of the tube is a cubic or higher degree equation. Hence the presence of an unsymmetrical wave in the impressed voltage on a tube-voltmeter with a characteristic which is of a higher degree than a quadratic does not

[1] "The Property of 'Turn-over,'" *Experimental Wireless and Wireless Engineer*, Vol. 6, pp. 310–315; by permission of the publishers of *Experimental Wireless and Wireless Engineer.*

always mean that there will be a wave-form error. Whatever the number and phase of the harmonics, $I_0 = 0$ when the tube characteristic is expressible by a quadratic equation. This latter fact is also brought out in the paper but was previously shown by Jansky and Feldman in their paper referred to.[1] They assumed a quadratic characteristic and an impressed-voltage wave possessing n harmonics expressed in the following manner:

$$e_B = \mathcal{E}_{B_1} \sin \omega t + \mathcal{E}_{B_2} \sin (2\omega t + \theta_2) + \mathcal{E}_{B_3} \sin (3\omega t + \theta_3) + \cdots \mathcal{E}_{B_n} \sin (n\omega t + \theta_n).$$

The effective value is

$$E_B = \sqrt{E^2{}_{B_1} + E^2{}_{B_2} + E^2{}_{B_3} + \cdots E^2{}_{B_n}}.$$

This is made equal to a simple sinusoid E_A, expressible as

$$e_A = \mathcal{E}_A \sin \omega t.$$

The two voltages $E_A = E_B$ are then impressed on the input terminals of a tube voltmeter, and it was shown by substituting in the expanded expression that

$$\Delta I_P = \Delta E_g \frac{dI_P}{dE_g} + \Delta E_g{}^2 \frac{d^2 I_p}{dE_g{}^2}$$

and that the average change in plate current is

$$\overline{\Delta I_{P_A}} = \frac{1}{2} \frac{d^2 I_p}{dE_g{}^2} \left(\frac{\mathcal{E}_A{}^2}{2} \right) = \frac{1}{2} \frac{d^2 I_p}{dE_g{}^2} (E_A{}^2) \tag{1}$$

for the simple sinusoid e_A.

$$\overline{\Delta I_{P_B}} = \frac{1}{2} \frac{d^2 I_p}{dE_g{}^2} \left(\frac{\mathcal{E}_{B_1}{}^2}{2} + \frac{\mathcal{E}_{B_2}{}^2}{2} + \cdots + \frac{\mathcal{E}_{B_n}{}^2}{2} \right)$$
$$= \frac{1}{2} \frac{d^2 I_p}{dE_g{}^2} (E_{B_1}{}^2 + E_{B_2}{}^2 + \cdots E_{B_n}{}^2) \tag{2}$$

for the complex unsymmetrical wave. But by assumption the effective values are equal, or, from Eqs. (1) and (2),

$$\overline{\Delta I_{P_A}} = \overline{\Delta I_{P_B}},$$

and hence the fact that the voltage E_B contains n harmonics

[1] *Loc. cit.*

does not affect the expected value of the average change in plate current of the tube voltmeter.[1]

To insure against a wave-form error for unsymmetrical waves, certain precautions are important. Reed shows curves of the variation of turnover (previously defined) with the amount

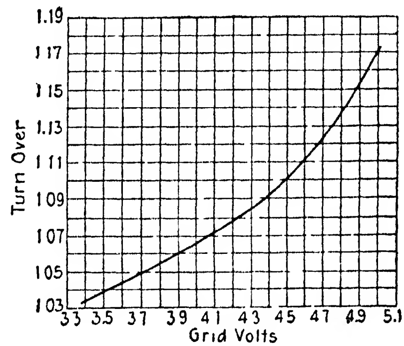


FIG. 10.—Variation of turnover with grid bias. (*After Reed.*)

of negative grid-bias voltage such as is shown in Fig. 10. It is noted that to reduce the value of turnover, and hence the wave-form error, it is necessary to reduce the negative grid bias. This, as pointed out by the above-mentioned writers, brings the tube characteristic within the quadratic range. Jansky and Feldman recommend that a curve be plotted as shown in Fig. 10*a*, and the voltmeter operated in the range where this square-root curve is a straight line. Of course, for measuring larger voltages in this range some sort of attenuator must be used. The practice often followed in the laboratory has been to note the reading of the plate-current meter for the two positions of a reversing switch between the unknown voltage and the tube input. If the readings are different, there is a value of turnover, and a wave-form error will result. Caution must be used in doing this, because errors may result due to the possible existence of

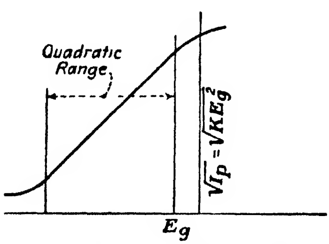


FIG. 10*a*.—Special graph of vacuum-tube characteristic to show quadratic range.

[1] It will be noted that the phase of the harmonics has no effect on the effective value.

different capacitances between the terminals of the tube-voltmeter and ground. The grid bias is then lowered until this difference I_0 becomes negligibly small. If such a point cannot be found, the input voltage is lowered in amplitude, and the test repeated. The tube voltmeter is then calibrated for this value of grid bias. This method has the advantage over the others in that it insures placing the grid bias and input amplitude within the quadratic range on the dynamic characteristic. This insures proper adjustment of grid bias which should lie at the mid-point of the quadratic range shown in Fig. 10*a*. This results in a pure *square-law* tube-voltmeter.

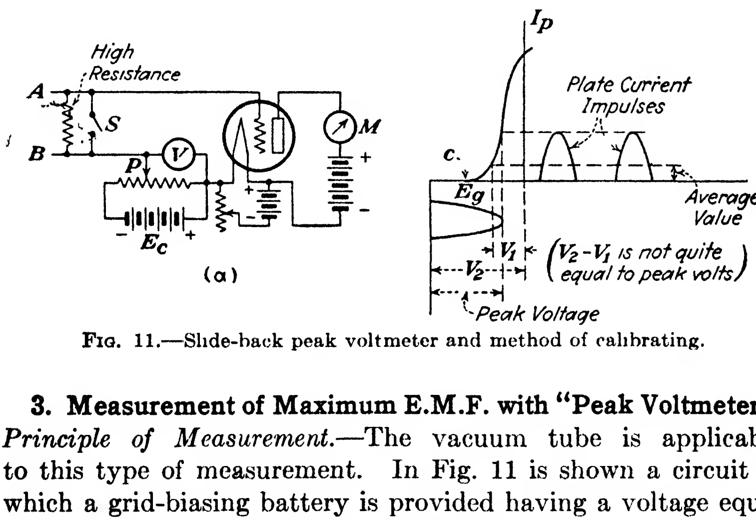


FIG. 11.—Slide-back peak voltmeter and method of calibrating.

3. Measurement of Maximum E.M.F. with "Peak Voltmeter." *Principle of Measurement.*—The vacuum tube is applicable to this type of measurement. In Fig. 11 is shown a circuit in which a grid-biasing battery is provided having a voltage equal to or greater than that of the maximum value to be measured. With switch *S* closed, the sliding contact of *P* is varied until the low-range plate milliammeter *M* shows a small current flowing, and this well down on the steep portion of the $I_p - E_g$ characteristic, as shown at the right of the figure. *S* is then opened, a calibrating voltage of known peak value applied at *AB* is added in series, potentiometer *P* readjusted until the plate current is the same as previously, and the reading of the grid-bias voltmeter *V* noted. The known applied voltage is varied, the corresponding grid-bias voltages for the restoration of initial plate current recorded, and a calibration curve plotted between peak volts and readings of grid-bias voltage *V*. This is a fairly high-accuracy measuring procedure, but it is obvious that it requires that the electron-tube potentials be held very constant.

If the tube characteristic of Fig. 11 has considerable slope at C, the cutoff point, the change of grid bias necessary to reduce the plate current to zero when an unknown is applied is a fairly accurate measure of the peak value of the latter voltage. This method of measuring is widely used where high accuracy is not essential. This type is often known as the *slide-back* peak voltmeter. A useful modification, incorporating a similar principle, is merely a two-element rectifier tube and variable voltage

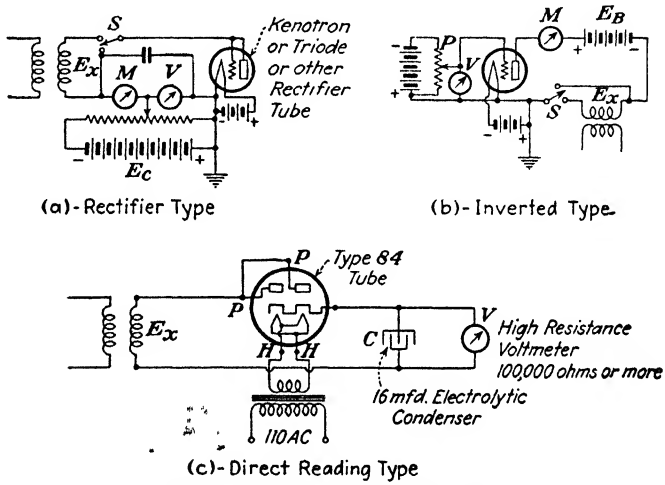


FIG. 12.—Types of peak voltmeters.

in series, as shown at (*a*), Fig. 12. Before the unknown voltage is applied, a small current may flow through the low-range milliammeter of microammeter M due to some of the emitted electrons from the filament passing through the circuit by their momentum. Usually, connecting the battery to the negative filament terminal of the rectifier tube introduces sufficient negative potential to reduce this residual current to zero. If not, the potentiometer can be given an initial setting so as to give a small reading to V. Then when the unknown voltage is applied, the change in the reading of V required to reduce the current in M to zero is very nearly equal to the peak, or maximum, value of the unknown voltage.

Other Types of Peak Voltmeters.—A somewhat different type, described by Hund,[1] is shown at *b*, Fig. 12. Before the unknown voltage is applied, potentiometer P is varied for zero plate current, giving a grid-bias voltage reading V_1. The unknown voltage is applied, and P readjusted for a new grid-bias value V_2, at which the plate current is again zero. It is shown by the author referred to that the peak value of the unknown voltage is then

$$E_x = E_B\left(\frac{E_2}{E_1}\right) - 1, \tag{3}$$

where E_B is the plate-supply voltage. The instrument may be calibrated if preferred. The advantage claimed for this is that the grid-biasing battery, being μ times as effective as for the slide-back type (μ is the tube-amplification factor), need be only about E_x/μ volts, or $1/\mu$ as high as for the slide-back and rectifier types. Another type, recently described by W. R. Jones,[2] is shown at (*c*) Fig. 12, wherein an electrolytic condenser, charged to the peak-voltage potential, discharges through a high-resistance voltmeter. At 60 cycles and higher frequencies, the charging impulses are so frequent that the condenser potential is not appreciably lowered by the small discharge current through the very high-resistance voltmeter. The drop through the rectifier tube is negligible.

Peak, or slide-back, voltmeters are coming more and more into use, not only in the laboratory but also in radio-service work and among amateurs. Several new and useful forms have been described. In Fig. 12*A* is shown a simple peak tube-voltmeter which makes use of a type 6E5 *magic-eye* tube to indicate when the direct-current voltmeter[3] V reads the peak, or maximum, value of the voltage (radio or audio frequency) applied at AB. This occurs when the shadow in the magic eye is brought to a narrow line. R_5 is used for preliminary adjustment for line shadow with AB short-circuited. With the voltmeter tube mounted in a *gooseneck* at the end of a shielded flexible cable, good accuracy is obtained for the higher frequencies, but considerable error enters for low voltages. To avoid this, the device

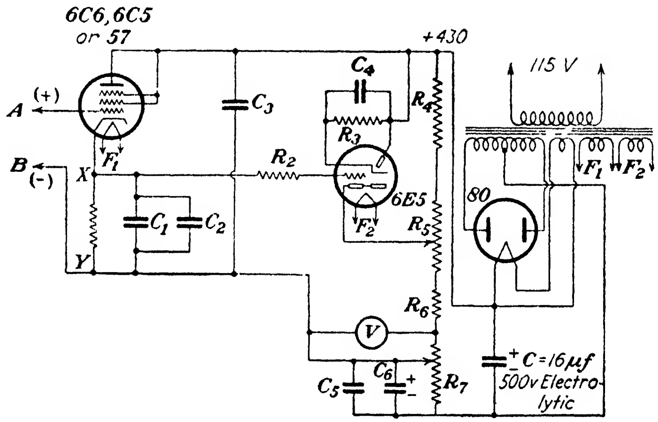
[1] "High Frequency Measurements," p. 143.

[2] *Sylvania News*, Vol. 6, No. 4.

[3] WALLER, L. C., *Q.S.T.*, Vol. 20, No. 10, p. 35, October, 1936.

should be calibrated with known 60-cycle voltages. Another interesting peak voltmeter makes use of a direct-current amplifier instead of a magic eye and may also be built in the laboratory.[1] The tube is mounted at the end of a gooseneck adapter, and an accuracy of within 2 per cent is claimed at 15 Mc.

4. Measurements of Dip, or Trough, E.M.F., Average Value, Etc. *Measurement of Dip, or Trough, E.M.F.*—It is often impor-



R_1 - 2 megohms, ½ watt R_3 - 1 megohm, ½ watt
R_2 - 100,000 ohms, 1 watt R_4 - 10,000 ohms, 10 watts
R_5 - 1000 ohm wire-wound linear potentiometer
R_6 - 500 ohms, 2 watts
R_7 - 10,000-ohm, 10-watt wire-wound linear potentiometer
C_1 - 4 μf, 400 v, paper (low leakage)
C_2, C_3 - 0.01 μf., mica, 300 v.
C_4 - 0.1 μf., paper, 300 v, C_5 - 0.01 μf., mica
C_6 - 8 μf., 350 v., electrolytic
V - 10, 250, 750 volts range - 1000 ohms per volt

FIG. 12A.—Slide-back-type vacuum-tube voltmeter using "magic eye" indicator.

tant to measure the *dip* in a voltage, or rather the minimum value to which a pulsating voltage drops. This is important in the case of power-amplifier and oscillator tubes in which the plate voltage rises and dips above and below the average or mean value and does this at a radio-frequency rate in radio-frequency amplifiers. Figure 13, (*a*) shows means for measuring this minimum value, sometimes referred to as the ***trough voltage***. Preliminary adjustments must be made (as in the description of peak-voltage measurements) to reduce the residual plate current

[1] *Radio News*, Vol. 19, No. 1, p. 14, July, 1937.

to zero. Then the pulsating e.m.f., shown at (*b*) in the figure, is applied at *AB*, the potentiometer *P* readjusted to reduce the plate current to zero, and the reading of *V* will be the minimum or trough voltage. In this circuit, plate current from battery E_B will flow if the minimum voltage is below the value obtained from the battery and potentiometer *P*. For all higher values of the applied voltage to *AB* no current will flow. Condenser C_b by-passes the alternating-current components of current impulses around the meters.

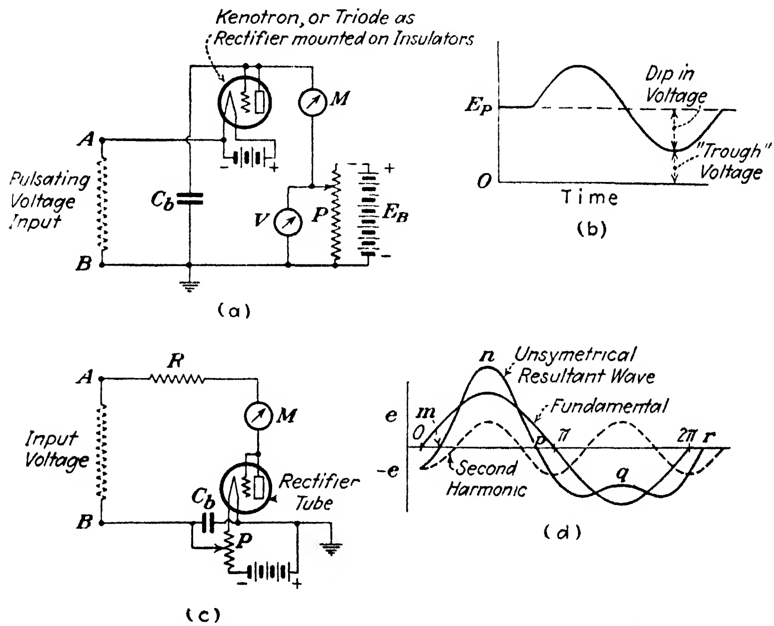


FIG. 13.—Circuits for measuring trough voltage, and average voltage.

Measurement of Average Voltage.—A direct-current milliammeter *M* and linear rectifier, such as shown at (*c*), Fig. 13, may be used to measure average values, after previous calibration with sine waves whose average values may be easily calculated from the measured effective values. The presence of resistance *R* in the rectifier circuit makes the tube characteristic more nearly linear, and sufficient voltage must be available to insure linearity.

When the rectifier characteristic is linear, the turnover is zero when measuring an unsymmetrical wave consisting of a fundamental and harmonics. The phases of the harmonics are immaterial. A simple case to illustrate this is shown at (*d*), Fig. 13, where a wave is made up of a fundamental and a second-harmonic component, and the phase of the latter is such that the maximum values of the resultant wave above and below the axis are quite different. But analysis will show that for any phase of the harmonic the area added to the positive half-wave fundamental equals the area subtracted from it, and the same is true for the negative half-wave of the fundamental components. The reading of the d'Arsonval-type direct-current milliammeter in the linear rectifier circuit will be proportional to the average value of the half wave of current passing through the rectifier and hence to the half wave of voltage applied to the rectifier. Then the meter deflection is

$$D = kE_{av},$$

where E_{av} is the average value over a half-wave cycle of the fundamental frequency. For a wave

$$e = E_1 \sin \omega t + E_2 \sin (2\omega t + \theta),$$

$$E_{av} = \frac{1}{\pi}\int_0^\pi e\, d(\omega t) = \frac{2}{\pi}E_1. \quad (4)$$

The deflection, and hence the average value, does not change by addition of a second, or higher, even harmonic, nor does it change with the phase θ of the harmonic. For reversed connections of the input voltage (Fig. 13(*c*) the same half-wave area [$n = q$ in (*d*), Fig. 13] is applied to the tube at (*c*)), giving the same reading regardless of the phase of the harmonic. The same result will be obtained for an odd harmonic or for any number of harmonics superimposed on the fundamental.

Copper oxide rectifiers have a linear characteristic and are admirably adapted to the measurement of the average value of a half wave at audio and low radio frequencies. The high capacitance of the copper oxide film may short-circuit a high radio-frequency voltage. Figure 14 shows applications of copper oxide rectifier instruments. When this device is used to measure *effective values*, it should be done with caution because errors caused by wave forms other than sinusoidal on which it is assumed to be calibrated may be serious. Wolf shows that a 50 per cent second harmonic may cause a 10 per cent error in effective value, and a 50 per cent third will cause as much as a 16 per cent error as read by such an instrument.[1] Modern diode tubes such as the 6H6 are well adapted to use as rectifiers for output meters and with 10,000 ohms in series give a nearly linear characteristic. Such a device is shown in Fig. 14,[2] and is

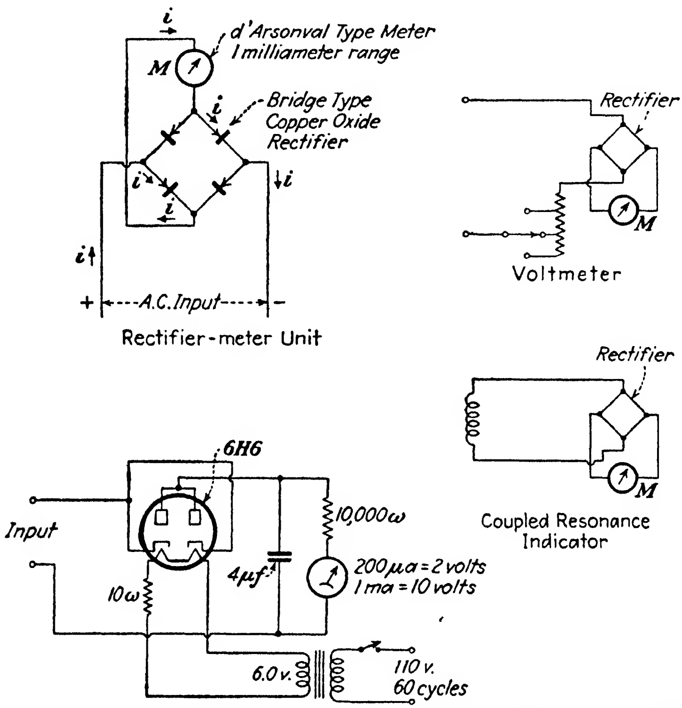


FIG. 14.—Principle of application of copper oxide rectifiers and diodes to indicating meters.

similar to the arrangement of Fig. 12, but with different recommended constants.

5. Direct Measurement of E.M.F. at Radio Frequencies. *Measurement of Maximum Value.*—In checking the accuracy of electron-tube voltmeters at radio frequencies, it is necessary

[1] *Proc. I.R.E.*, Vol. 19, No. 4, p. 653, 1931. Rectifier and rectifier instruments for general voltage and current measurement are discussed by Sahagen, *Proc. I.R.E.*, Vol. 19, No. 2, p. 233, 1931; and by F. E. Terman, *Proc. I.R.E.*, Vol. 23, No. 3, p. 234, 1935.

[2] *Radio News*, Vol. 19, No. 1, p. 35, July 1937.

to have some means of measuring the effective value of the voltmeter-calibrating voltages at radio frequencies. The calibrating voltage may easily be made sinusoidal by simple resonant circuits as filters. The maximum value of the sine wave may be found by charging a condenser with this voltage with the aid of a series-connected electron-tube rectifier and then discharging the condenser through a ballistic galvanometer. The condenser charges up to the maximum value of applied voltage, and this is

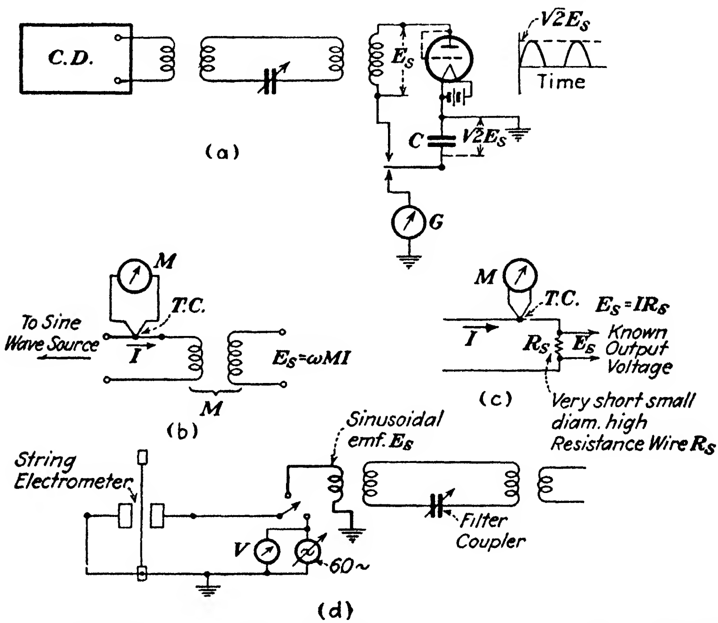


FIG. 15.—Methods of absolute measurement of e.m.f. at radio frequencies.

found by subsequently charging the condenser with a known direct potential from a battery and varying this until the same galvanometer deflection is obtained. This standardizing circuit is shown at (*a*), Fig. 15.

Measurement of Effective Value.—A sine wave of current of known effective value may be caused to flow through the primary of a known mutual inductance as shown at (*b*), Fig. 15. The secondary induced e.m.f. is $E_s = \omega MI$ and is effective volts if I is effective amperes. I is measured by a reliable thermocouple, as indicated. For high radio frequencies, this becomes uncertain owing to self- and mutual capacitances of the coils of M, and the potential drop along a concentric tube is more reliable. This latter can be built sufficiently large, and known large currents may be passed through it to give potential drops of several volts. The device is described on p. 351. A very small diameter and very short length of high-resistance wire will have an inductive reactance that is negligible compared to its resistance. Hence if a known effective-value ultra-high frequency current is passed through this resistance wire, a known effective-value voltage of the same frequency is obtained. Such a standardizing circuit is shown at (*c*), Fig. 15. The effective resistance R_s of the wire at radio frequency may be calculated or found from suitable correction tables (see footnote, p. 308; see also Fig. 15*A*). Vacuum thermocouples which are quite accurate at 100 Mc. are readily available for measuring the current I (see Art. 8).

The string electrometer of Stebbins or Lutz[1] may be used for measuring effective values. These are said to have electrostatic capacitances as low as 4 mmf. For use on alternating voltage they are connected *idiostatically; i.e.*, one of the fixed knife-edge electrodes is connected to the string, as shown in the test circuit at (*d*), Fig. 15. This decreases its sensitivity greatly, so that several volts are required to produce noticeable deflection of the string. A precision telescope with an ocular micrometer is used to measure the deflection of the string. As indicated, the electrometer is calibrated with known 60-cycle variable voltage.

The electrostatic voltmeter is built on the principle of the electrometer and is calibrated in effective values. Its indication is independent of frequency and of wave form, and, often, it is a convenient instrument for measuring radio-frequency volts. It has, however, 100 mmf. or more capacitance, hence should be used with caution. This means that at radio frequencies the measured voltage wave should be sinusoidal. The instrument would have different input impedances to waves containing harmonics. The frequency at which the instrument is used must be much lower than the resonant frequency of the leads with the input capacitance. On account of the weak electrostatic attraction, the lowest readable deflection is obtained at about 20 volts. With the aid of a condenser potentiometer, voltages exceeding

[1] Lutz, C. W., *Physik. Ztsch.*, Vol. 13, p. 924, 1912. The Stebbins instrument is similar.

the range of the instrument may be measured. The piston attenuator[1] of the capacitance type is also adaptable for producing small known voltages. A known voltage for use as a standard by which to calibrate tube voltmeters, etc., has recently been described,[2] which produces the known voltage by passing a known current through a very short thin high-resistance wire. It is the same principle as that of Fig. 15, Diagram (c). The current is measured by a separate heater-type thermocouple. The circuit arrangement is shown in Fig. 15*A*, and the dimensions and wire material are also shown in the figure. The inductance of the resistor (7 ohms) is about 1.6 cm. ($1.6 \times 10^{-9}h$), giving an inductive reactance of 1 ohm at 100 Mc. Since this is added in quadrature to the resistance, the impedance is increased only 1 per cent. At this frequency, the skin effect is negligible on

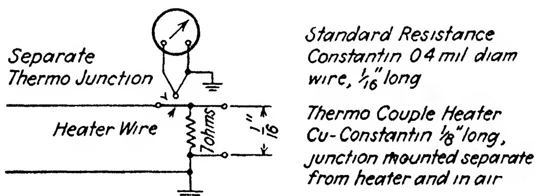


Fig. 15*A*.—Arrangement of thermocouple and resistance for voltage standard for frequencies up to 100 Mc.

wire 0.4 mil in diameter. If 0.25-mil wire were used, the inductive error would be 1 per cent at 250 Mc.

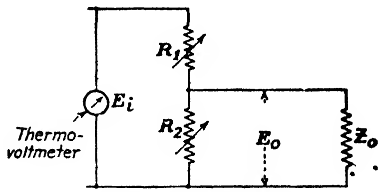
6. Measurement of Small E.M.F. *Potentiometer-voltage Divider.*—The direct measurement of very small voltages requires the use of an amplifier of suitable characteristics and some such device as a vacuum-tube voltmeter or output meter. The production of a known small e.m.f. for calibration of the indicating device presents the most serious aspect of this problem. The potentiometer-voltage divider, or *voltage divider*, illustrated in Fig. 16 presents a convenient means of providing the required known small e.m.f. from a larger easily measurable supply voltage. The output voltage is

[1] See paper by Harnett and Case, *Proc. I.R.E.*, Vol. 23, No. 6, 1935; the capacitance type is described in Art. 6.

[2] Arguimbau, L. B., "A High Frequency Voltage Standard," *General Radio Experimenter*, Vol. 12, No. 1, June, 1937.

$$E_0 = E_i\frac{R_2}{R_1 + R_2}, \tag{5}$$

where E_i is the measured supply voltage. The current taken from the output terminals must be zero, or very small; or the relation above will not hold. It is not necessary to have the impedance of the circuit fed by the potentiometer infinite in value for a required degree of precision in the determination of E_0. This output voltage may be calculated for any input voltage E_i, potentiometer R_1 and R_2, and value of vector impedance Z_0. Also, it is possible to calculate the error for various values of Z_0; but a much simpler analysis of the accuracy will be given, assuming that Z_0 is noninductive, giving it the term R_0. By applying Ohm's law, it is easily shown that



Rayleigh Potentiometer

FIG. 16—Circuit of simple voltage-divider potentiometer.

$$E_0 = E_i\frac{1}{(g_2 + g_0)R_1 + 1},$$

where $g_2 = 1/R_2$, and $g_0 = 1/R_0$. It is evident from this equation that if g_0 is negligibly small compared with g_2, there will be no appreciable error in the value of E_0. When Z_0 is infinite, Eq. (5) holds true; but when Z_0 equals R_0, a finite value, the output voltage is reduced to a value KE_0, where K represents the multiplying factor to obtain the changed value of output voltage. Then

$$KE_0 = E_i\frac{1}{(g_2 + g_0)R_1 + 1}. \tag{6}$$

From Eqs. (5) and (6) it is easy to solve for K. From the difference between the values E_0 and KE_0 the percentage of error resulting from using the simple potentiometer reading of Eq. (5) can be obtained. The difference in the result is, of course, $1 - K$, and

$$K = \frac{R_1R_0 + R_2R_0}{R_1R_0 + R_1R_2 + R_2R_0}. \tag{7}$$

The potentiometer is often used to obtain very small known voltages, and under this condition the expression for E_0 is simply

$$E_0 = E_i\frac{R_2}{R_1}. \tag{8}$$

When the output circuit possesses resistance R_0, the output voltage is

$$KE_0 = E_i\frac{R_2R_0}{R_2 + R_0} \times \frac{1}{R_1}. \tag{9}$$

From these expressions the change is

$$1 - K = \frac{R_2}{R_2 + R_0}. \tag{10}$$

Of course it is quite convenient to calculate the correct value KE_0 when R_0 is known, but it is often only approximately known, and for rapid measurements it is more convenient to use the simpler Eq. (8), and Eq. (10) then gives a good idea of the resulting percentage of error. This simple form of voltage divider has many useful applications.

Attenuation Network.—When the impedance of the circuit into which the desired reduced voltage is fed is low, so that it draws considerable current, it is often convenient to divide the supply voltage by means of a T-type connection of known resistance, commonly called an attenuation network. For certain conditions of circuit constants and required degree of voltage attenuation,

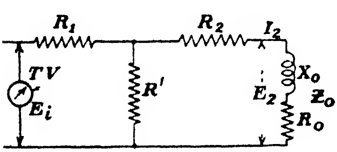


FIG. 17.—T-type attenuation network for obtaining reduced voltages.

where the supply voltage E_i may be measured directly with a thermionic voltmeter or other suitable means, the network connections are as shown in Fig. 17. It is necessary to know the vector impedance Z_0 of the circuit into which the attenuator feeds. One method of calculating E_2 is carried out by assuming the output voltage to be the reference vector and solving the circuit just as in a T-type transmission line. It is then easy to show that

$$E_i = E_2\sqrt{\left(1 + \frac{R_1}{R'} + \frac{R_0A}{Z_0^2}\right)^2 + \left(\frac{X_0A}{Z_0}\right)^2}, \tag{11}$$

where

$$A = R_1 + R_2 + \frac{R_1 R_2}{R'},$$

from which E_2 is derived when E_i and the circuit constants are known. A convenient arrangement for some measurement work consists in letting

$$R_1 = R_2 = R' = R_0 = Z_0,$$

from which the expression

$$E_i = 5E_2 \tag{12}$$

is obtained. This makes a desirable means of attenuating a voltage to one-fifth of its original value, provided the output circuit has no other impedance in parallel with R_0 (except that of the grid-circuit filament of an amplifier tube or other very high impedance).

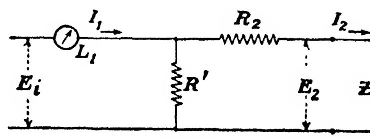


FIG. 18.- Attenuation network using current measurement.

In cases where E_i, the input voltage, is small, it is sometimes convenient to measure the input current I_1 and use the attenuator connected as shown in Fig. 18. Using I_2 as the reference vector for solving the circuit, it is readily shown that

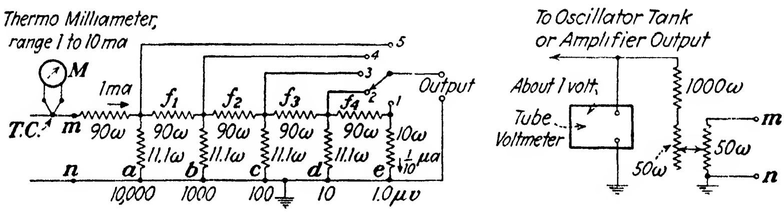
$$I_1 = I_2\sqrt{\left(1 + \frac{R_2}{R'} + \frac{R_0}{R'}\right)^2 + \left(\frac{X_0}{R'}\right)^2} = BI_2. \tag{13}$$

From this expression the value of E_2 may be obtained in terms of Eq. (13) and Z_0:

$$E_2 = I_2 Z_0 = \frac{I_1 Z_0}{B}. \tag{14}$$

These attenuation networks are not used in the same manner as in their application to communication circuits where the input and output impedances have equal values. When such values are known, the network can be designed to attenuate in predetermined proportions. Different types of attenuation network are treated in books on telephone lines and communication-circuit problems.

The Step Attenuator.—Step attenuators are designed to attenuate a known voltage in known decimal proportions. Figure 19 shows the circuit and constants of such a device. Part (*a*) shows the circuit constants. When the input current at *mn* is 1 milliamp., as measured by the thermocouple and milliammeter *T.C.* and *M*, the output voltage at switch point 1 is 1 microvolt. At the other switch points, it increases in multiples of 10 as the indicated switch-point numbers increase. A simple way to design such an attenuator is to start at the right with shunt section *e*. Letting the resistance of *e* be 10 ohms, a current of $\frac{1}{10}$ μa flowing through will produce a drop of 1 microvolt across *e*. This current flowing through a resistance of 90 ohms will give a drop of 9 microvolts, and this added to the 1 microvolt across *e* gives 10 microvolts across *d* if the resistance



(a) (b)

FIG 19.—Design and operation of resistance-step attenuator for microvoltage output.

of series section f_4 is 90 ohms. The drop across f_3 should be 90 microvolts in order than it may add to the 10 microvolts across *d* to give 100 across *c*. This means that the resistance of *d* should be 11.1 ohms to give a total of 10 μa in f_3. The same scheme carried from right to left through the attenuator network results in all series elements except *e* being 11.1 ohms. The resistance at the output terminals is 9.1 ohms at all switch points except No. 5, for which it is 10 ohms. For 1 microvolt output the input current is seen to be 1 milliamp. Now, the output voltage at switch point 1 may be increased uniformly by increasing the input current in like manner. Then for uniform variation of output voltage using current variation and different switch points is from 1 microvolt to 0.1 volt for a current range of from 1 to 10 milliamp. If another shunt section of 11.1 ohms is placed across input terminals *mn*, the current range will be from 10 to 100 milliamp. It may be found that two thermocouples will be needed to cover this range unless a millivoltmeter M having 1 or 2 and 5 millivolts scale range is available. A vacuum-tube voltmeter and a constant-impedance input attenuator of the L type may be substituted for the thermocouple, as shown in Diagram (*b*) of Fig. 19. This is done in some modern signal generators. The input voltage is adjusted to a constant value such as 1 volt, and the input L attenuator is calibrated using thermocouples in series at m. This has the advantage that the load resistance presented to the supply oscillator or power ampli-

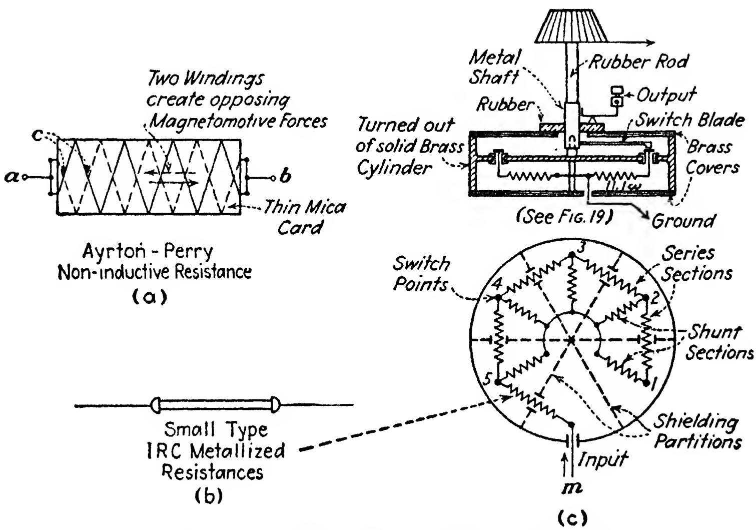


FIG. 20.—Details of construction of resistance-step attenuator.

fier is constant. The disadvantage lies in the fact that the variable resistances in the constant-impedance L attenuator are of quite critical design for high radio frequencies.

If a radio-frequency step-resistance attenuator is constructed in the laboratory, much care and precaution are needed to make it function with less than 10 or 20 per cent error. Nonreactive resistances at frequencies up to 25 Mc. present a serious problem. Constant-temperature coefficient resistance wire wound on thin mica cards, the *Ayrton-Perry* style of winding being used to reduce self-capacitance and inductance, is used quite widely. This type of winding is shown at (*a*), Fig. 20. Two windings in the opposite sense create opposing magnetomotive forces. The two terminals *a* and *b* place the resistances in parallel. The self-capacitance is reduced by having the insulated wires touch each other with a minimum area of contact *c* and by having the terminals at opposite ends of the card. The inductance of this winding can be calculated using special formulas, but it will be found experimentally that unless the wires are very small and quite close together, considerable reactance will result at high radio frequencies. The small 0.1-watt-type metallized resistor shown at (*b*) Fig. 20, has satisfactory characteristics at high radio frequencies and is also used quite extensively in attenuators. Detail sketch (*c*), Fig. 20 shows how either of these units may be mounted in a shielding compartment which is divided into cells, as shown, as it has been found necessary to shield each section from the higher potentials and magnetic fields of preceding sec-

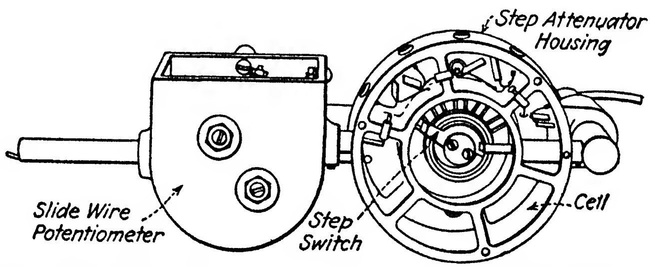


FIG. 20*A*.—Assembled step attenuator and slide-wire potentiometer. (*After General Radio Company.*)

tions. The complete attenuator with associated input-meter radio-frequency power source and means of modulating the output is called a *signal generator;* these generators are further described in Chap. VIII, Art. 3. A modern commercial step attenuator is shown in Fig. 20*A*.

Other Types of Attenuators.—Many other interesting and useful types have been developed and deserve consideration, but it is felt that space in this book can be spared for only a brief mention of a few such devices. Harnett and Case have developed capacitance and mutual-inductance attenuators of the *piston type.*[1] Two examples are shown in Fig. 21. The capacitance type shown in Diagram (*a*) is simple and functions at all frequencies. Its output voltage, however, depends upon the capacitance of the output circuit, as indicated at (*b*), and it constitutes a capacitance attenuator. *A* is, of course, a constant

[1] *Proc. I.R.E.*, Vol. 23, No. 6, p. 578, June, 1935.

in the formula for C/C_0 and is given in decibels per inner radius a of the copper tube. The mutual-inductance type shown at (c), Fig. 21, is more easily constructed than is the capacitance type. The flux penetrates the inner wall of the copper cylinder, and the effective radius is $a + p$, but p is negligible at high radio

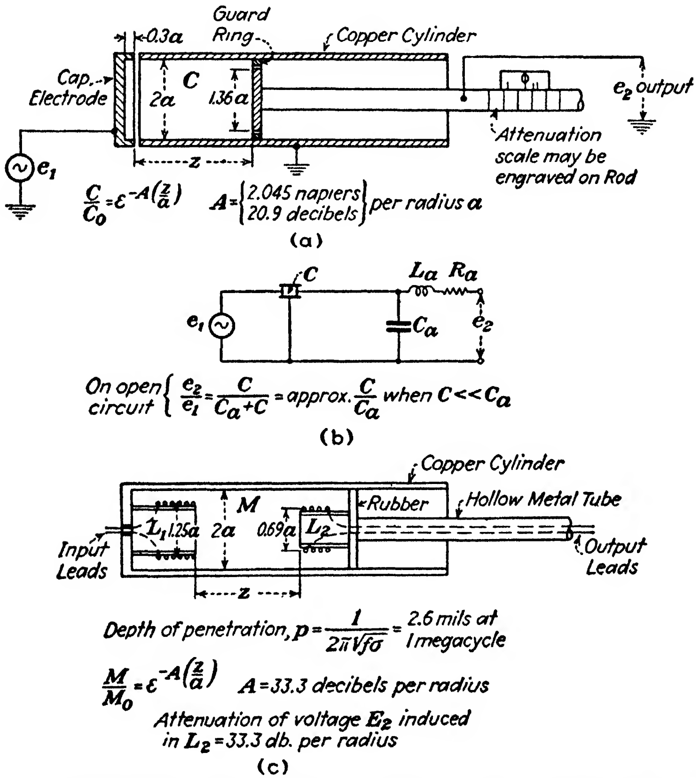


FIG. 21.—Capacitance and inductance types of piston attenuators. (*After Harnett and Case.*)

frequencies and not really needed at 1,500 kc. A vacuum-tube voltmeter may be connected to L_2, and the latter moved toward L_1 until the voltage induced in L_2 is 1 volt. Then for any position Z the attenuation is $33.3\frac{Z}{a}$ db below 1 volt, or

$$20 \log_{10} \frac{1}{e_2} = 33.3\frac{Z}{a},$$

from which e_2 may be found if desired. The reader should read the paper referred to before attempting to construct and use these attenuators. They have proved very valuable in factory-production testing of radio-receiving sets. The mutual-inductance type shown at (*c*), Fig. 21, is probably the most practical and easily constructed in which L_1 may be an oscillator-tank coil. It may be made a plug-in type so that the oscillator-attenuator will have a wide frequency range. The dimension $2a$ should not be greater than 2 or 3 in., and the length can be calculated for a required maximum attenuation. In using this type, it must be remembered that resonance rise or step up of the output voltage is a possibility if L_2 is connected to a capacitive load.

Concentric metallic-tube impedance-drop attenuators are simple to construct, and although the range of output voltage is rather limited, it is easily adapted to radio receiver test-voltage requirements. The impedance of the output terminals is very low. One such device is shown and described fully in Art. 3, Chap. VIII. Another advantage lies in the ease with which the output voltage may be calculated with a very simple formula and the easily performed experimental check, wherein several amperes radio-frequency current is passed through the attenuator and the output voltage is measured with a vacuum-tube voltmeter.

Checking Accuracy of Step Attenuators.—This presents a serious problem because it involves a form of measurement of extremely low voltages of medium and high radio frequencies. A *ratio checking* procedure is probably the most practical solution. The maximum voltage output may be 1 volt, and this is easily measured on a vacuum-tube voltmeter.

For checking, a good-grade amateur-communication-type radio receiver is desirable if it possesses a radio-frequency or intermediate-frequency sensitivity control and the well-known carrier intensity, or *R*, meter. If it does not possess this latter feature, it should have the usual beat oscillator for code reception and a rectifier-type voltmeter connected across the voice coil of the dynamic speaker. Either of these provisions will give indications of the input voltage to the receiving set. Some form of *field generator*, such as described in Art. 3 (Chap. VI), must also be provided. Any convenient type of *shielded oscillator* and small transmitting loop containing a current-indicating device may be used. The procedure in making a check will be given in four steps:

1. The receiver-input terminals are connected to the highest voltage-output switch point of the attenuator to be checked, which has had this output voltage previously measured in some manner mentioned above. The sensitivity control and receiver tuning are adjusted so that a maximum meter-scale reading is obtained. The receiver-input terminals are then transferred by means of a change-over switch to a convenient open or loop receiving antenna, and the field-generator frequency is, of course, being adjusted to resonance with the receiver tuning.

2. If the output voltage on the next switch point of the attenuator being tested is supposed to be one-tenth of that above, the current in the field-generator transmitting loop is reduced to one-tenth of its former value, and this will reduce the voltage at the input terminals of the receiver also to one-tenth of its former value. The deflection of the output meter may be reduced to such a low value that it cannot be accurately read; and if this is the case, the sensitivity control may be changed to increase the sensitivity so as to obtain an easily read output-meter deflection.

3. The receiver-input terminals are then transferred to the output terminals of the attenuator with the switch on the next switch point, as designated above. The attenuator-output voltage should now be one-tenth of its former value, and the output meter of the receiving set should show the same deflection as for the 10 per cent transmitter-loop current. If this voltage is in error, the percentage difference can be determined by changing the transmitter-loop current sufficiently to give a receiver-output reading equal to the attenuator output.

4. The degree of attenuation from the second to the third switch point of the attenuator may be determined by a procedure exactly similar to the foregoing. This means that the sensitivity of the receiver will have to be increased so as to obtain full receiver-output reading from the attenuator third switch point, and so the field generator must be placed at a greater distance, or its transmitting loop must be newly orientated if necessary. In this way, the attenuator output may be checked for each step, even down to the 1- or 10-microvolt output.

Other means[1] or modifications will suggest themselves to the student. For instance, the mutual-inductance-type piston attenuator with an oscillator may be used instead of the field generator. The current in the primary is then changed from full value to one-tenth of that value, as for the field generator. It is often advisable to use two thermoammeters of widely different ranges to obtain accurately read full and one-tenth values of primary current. The secondary coil is shifted as necessary for checking different ranges. If this testing attenuator is accurately made, the change from full to one-tenth value receiver-input voltage may be accomplished by withdrawing the piston with its secondary coil by an amount indicated by the formula given (see Fig. 21).

An ingenious method of calibrating a signal generator or its attenuator was recently described by Arguimbau.[2] Outlining the method very briefly, it is well known that if in the case of heterodyne detection one of the voltages impressed upon the detector is large, say 1 volt, and the other impressed voltage is small, the component of detector current at the heterodyne frequency, or *beat frequency*, will vary in direct proportion to the value of the lower impressed voltage over a wide range. This voltage may vary from 0.1 microvolt to 50 millivolts, and the difference-frequency component of plate current will vary in direct proportion. Voltage from a radio-frequency voltage standard is first superimposed on that from a heterodyning oscillator, the latter having the 1-volt output. These voltages are fed into a detector, and the difference-, or beat-frequency (15 kc.) output is measured by suitable means. Then the output voltage from the signal generator or attenuator under test is substituted for that from the radio-frequency voltage standard, and the detector output again obtained. The ratio of these outputs is the ratio of the standard unknown radio-frequency voltages. The actual test apparatus is quite elaborate, requiring auxilliary attenuators, audio oscillator, etc., and space cannot be given here to its complete description.

[1] TERMAN, F. E., "Measurements in Radio Engineering," p. 232, describes a method in which the second detector of the radio receiver is calibrated as a tube voltmeter.

[2] *General Radio Experimenter*, Vol. 12, Nos. 3 and 4, August and September, 1937

7. Measurement of Voltage Amplification.—The voltage amplification of one or more stages in a radio-frequency amplifier is usually measured by means of the vacuum-tube voltmeter. A stage usually includes the amplifier tube with the plate circuit connected to the terminals of the coupling-transformer primary winding. The output voltage is that obtained across the secondary of the coupling transformer when its primary is connected in series with the plate circuit of the associated amplifier tube. Figure 22 shows a stage from its input terminals *AB* to the point where the output voltage is measured, *CD*.[1] The voltage amplification is, of course, the ratio of output voltage E_{CD} to input

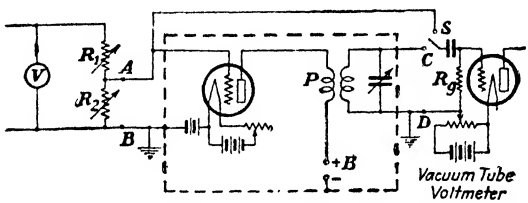


FIG. 22.—Circuit for measurement of radio-frequency amplification.

voltage E_{AB}. Voltage amplification is often indicated in the expression for power amplification in decibels, thus:

$$A = 20 \log \frac{E_2}{E_1},$$

if the input and output conductances are the same. The significance of this unit and the formula for its determination are discussed in Art. 10 of this chapter.

The measuring circuit may be conveniently connected, and switches provided so that the input terminals to a voltmeter may be switched from a calibrating source to the terminals *CD* for measurement of the output voltage of the amplifier stage. The calibrating source may also be used simultaneously to excite the amplifier; these arrangements are shown in Fig. 22. In case it is desired to measure the alternating-current output voltage across the plate coil *P* of the radio-frequency stage of Fig. 22, it is necessary to place a condenser in series with the grid of the tube voltmeter to prevent the drop in *P* due to the steady

[1] This method and those that follow assume radio frequencies and that the voltage dividers are for radio-frequency use. At frequencies below 50 kc., standard attenuation networks calibrated in decibels are desirable.

amplifier-tube plate current from altering the adjustment of the tube voltmeter. In case a series condenser is used, the grid must be biased through a high resistance connected as indicated in the figure. Precaution as to the proper shielding of the input and output coupling transformer coils from each other are important, and neutralization must be used in case of a triode-amplifier tube if it is to be used in a neutralized amplifier.

It is possible to measure the voltage amplification of an amplifier of preferably several stages by the null-balance method. The apparatus and circuit arrangements for this method are very much the same as for the method of detection coefficient, as given by E. L. Chaffee and described in Chap. V, Art. 7 (Fig. 10).

Attenuation Method.—The voltage amplification per stage may be measured by a method that does not require the absolute measurement of input and output voltages. The method is due to Sylvan Harris and is fully described by him.[1] His summary of the advantages of the method will be briefly restated. The measurements are independent of the values of input and output voltages, and no connections need be made to the stage under measurement except a switch for cutting it in or out of the amplifier circuit. No special apparatus is required other than that provided in most physics or radio laboratories, and the measurements are made on a stage while actually in a radio receiver under operating conditions. The method is explained, in the paper referred to, somewhat as follows: Let K be the voltage gain per stage of a shielded radio-frequency amplifier of n stages, and let K_d be the detector constant, and K_a be the voltage gain in the audio-frequency amplifier. If e is the input e.m.f. to the first stage, the output voltage of a square-law detector will be

$$E_{0d} = [eK^n]^2 K_d, \tag{15}$$

and the audio-amplifier output voltage will be

$$E_{0a} = K_d K_a [eK^n]^2.$$

This output voltage excites a tube voltmeter through a voltage divider, such as is shown in Fig. 23. If one stage which provides a voltage gain of K is switched out of the amplifier, the output

[1] "Measurements of Radio Frequency Amplification," *Proc. I.R.E.*, Vol. 15, No. 7, pp. 641–648, 1928.

voltage will be changed to

$$E'_{0a} = K_d K_a [eK^{n-1}]^2. \tag{15a}$$

The voltage amplification K will be obtained by dividing Eq. (15) by Eq. (15a) and solving:

$$K = \sqrt{\frac{E_{0a}}{E'_{0a}}}.$$

Referring to Fig. 23, the value of R_2 of the voltage divider will be inversely proportional to $\sqrt{E_{0a}}$ for a constant reading of the milliameter MA, so that

$$K = \sqrt{\frac{R_2'}{R_2}}, \tag{16}$$

where R_2' corresponds to E'_{0a}. This holds true provided the detector tube is not overloaded. The voltage divider is adjusted

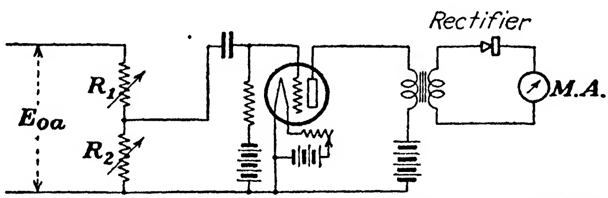


FIG. 23.—Voltage divider and vacuum-tube voltmeter for measuring circuit of Fig. 22.

so as to obtain a *datum* deflection when the stage to be measured is switched out of the amplifier.[1] This gives the setting R_2'. The stage is then switched in, and the voltage divider set at R_2 to obtain the same deflection as before. Of course it is not necessary that all stages have the same gain K. The amplifier may have three stages of gain K_1, K_2, K_3 and the gain of stage 2 measured by switching it out of the circuit; and a deduction similar to the foregoing shows that

$$K_2 = \sqrt{\frac{R_2'}{R_2}}. \tag{16a}$$

In the derivation for this case, the voltage impressed on the detector input is $eK_1K_2K_3$ for three stages.

[1] A copper oxide rectifier may be substituted for the crystal rectifier in Fig. 23 (see Art. 3 of this chapter).

It is also possible to measure the voltage amplification, or gain, of an isolated stage by this method. The stage to be measured is connected so as to excite another radio-frequency amplifier of a suitable number of stages, and readings of R_2 are taken for the same deflection of the output meter when the first stage is in and out. The author of the paper on this method also explains how the input voltage e may be measured, using another vacuum-tube voltmeter and amplifier to substitute in place of the amplifier-detector apparatus. From previous calibration of this second tube voltmeter, the input voltage is easily determined. It is also possible to determine when the detector in the *amplifier*

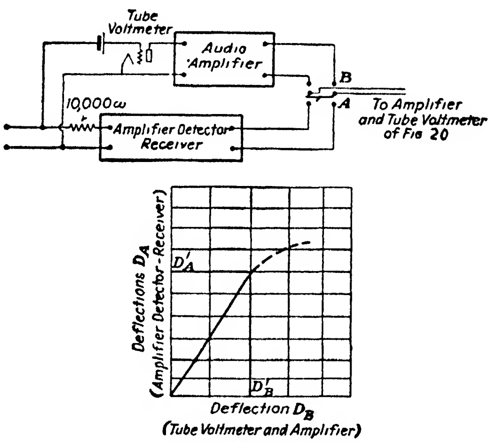


FIG. 24.—Circuit arrangement used in measurement of radio-frequency amplification. (*After Harris.*)

detector becomes overloaded by comparing deflections obtained with it and with the second amplifier and tube voltmeter. This tube voltmeter is one having a square-law characteristic; and if deflections are obtained for increasing input voltages and plotted in a curve, the relation will be linear up to the point where the detector begins to overload. The arrangement for this test is shown in Fig. 24, together with a curve showing the results. This plot may also be logarithmic, as the change from linearity will be more marked. For input voltages above those for which the deflections D_A' and D_B' were obtained, the detector becomes overloaded and departs from the square law. The so-called *power detectors* in use at the present time are overloaded in this sense. Of course this test assumes that the characteristic of the tube voltmeter is a quadratic through the whole range of test voltages. Its characteristic would have to be examined by some such means as that suggested in Art. 2, preceding. All of the tests described must be made with much precaution against stray fields, and the shielding is quite elaborate. The radio-frequency supply voltage must be modulated to a known extent. The reader is referred to Harris' paper for shielding and other details.

8. Measurement of Current at Radio Frequencies. *Special Requirements.*—Current-measuring instruments used for commercial and lower audio frequencies mostly incorporate stationary and movable inductive coils, but at radio frequencies it is impractical to attempt to measure current with such devices. Many measuring devices, adapted to alternating currents regardless of frequencies, have been evolved in the past 40 years. One of the earliest devices was due to Heinrich Hertz and is described in his "Miscellaneous Papers." This device consisted of a heating wire stretched between two supports; close to and parallel to this heating device was a second fine wire which would expand when current flowed through the heating wire, and the expansion allowed the fine wire to stretch and move the pointer of a needle to which it was attached.

Later a device called the *hot-wire air-thermometer heater* was developed, which consisted of a heater wire in a closed glass chamber and a small U-tube manometer containing some colored liquid within the glass chamber. When current flowed through the heater wire, causing it to warm the air within the closed chamber, the increased pressure therein would be shown by the difference in levels of the liquid in the U-tube manometer. This device was constructed in a form that was quite sensitive and was used as a resonance indicator for the Doenitz wave meter. J. Zenneck[1] describes this instrument.

Another early device, known as Tissot's bolometer, was formerly used for measuring small currents at radio frequencies. It consisted essentially of a Wheatstone bridge, and in one arm of the bridge was placed a fine high-resistance wire through which passed the high-frequency current to be measured. The bridge was first balanced for zero deflection of the galvanometer, and after passage of the high-frequency current for a sufficient time

[1] "Wireless Telegraphy," pp. 72–73.

to attain considerable temperature of the heating wire it was rebalanced, thus measuring the change in resistance, or the hot and cold resistances of the heating wire. From the known coefficient of the resistance increase with current, the effective value of the radio-frequency current could be determined. One form of this measuring device is shown in Fig. 25; the heating wire in this case is arranged in the diamond form, and the high-frequency current has no tendency to be shunted around through the other arms of the bridge.[1]

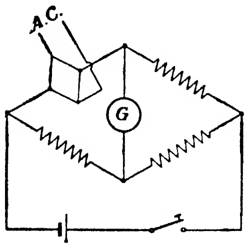


FIG. 25.—One form of Tissot's bolometer for measuring high-frequency current.

The hot-wire ammeter was later developed in a practical form and until recent years was widely used commercially.

Current Thermocouples.—The thermocouple already mentioned in this chapter is now in wide use for the measurement of radio-frequency currents ranging from 1 milliamp. to the large tank-circuit currents in modern high-frequency transmitters. In its practical form for current measurement, it is called a thermoammeter and comprises the thermocouple and millivoltmeter in the same container case. Thermocouples for the measurement of very small currents recently have a high-resistance heater wire; *e.g.*, the thermocouple whose normal heater current is 1 milliamp. has a heater resistance of about 1,000 ohms. They are designed so as to produce some 5 or 6 millivolts potential at the terminals of the junction with the normal heater current. When the heater current is of the order of 100 milliamp., the heater resistance is of the order of 5 ohms or less. These heater-resistance values are of greatest importance in the use of thermocouples for current measurement because the introduction of the heater wire into a radio-frequency circuit may alter the circuit resistance to such an extent that desired conditions cannot be maintained. A very low alternating-current rating necessitates using a high-resistance heater wire to develop enough heat to generate a given open-circuit junction potential, hence the direct-current volts developed in the junction per watt is higher for the

[1] For fuller description of the bolometer, see "Radio Instruments and Measurements," *U. S. Bur. Standards, Circ.* 74, pp. 161–163; also J. Zenneck (*loc. cit.*), pp. 72–73.

low-rating thermocouples than for the high-rating ones. A few values will be given for couples whose junction potential is 6 millivolts.

Alternating-current rating, amperes	Heater resistance	Direct-current volts per watt
0.010	32.60	2.27
0.025	9.50	1.18
1.000	0.27	0.03

Methods of absolute or comparative current measurements which do not introduce appreciable circuit series resistance have

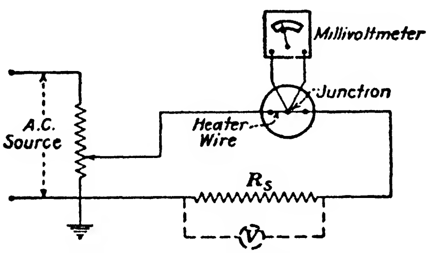


FIG. 26.—Circuit arrangement for calibration of thermocouple.

been evolved and will be discussed later. The *Vacuo-thermo-junctions* are widely used for small-current measurement in the radio-frequency laboratory, and in most cases consist of a very fine heating wire welded to platinum supports with a copper constantan junction lightly soldered to the middle of the heating wire. Such a device is shown schematically in Fig. 26. There is a novel form of thermo junction made for the measurement of currents from 0.1 to 1 amp., which has a short standard heater wire fastened by the two supports and 10 thermo junctions connected in series and placed side by side close to the heating wire but not touching it. This is called an independent junction and has several advantages. There is a tendency in the case of the thermocouple soldered junction for a very high-frequency current to be shunted from the heater wire through the junction and millivoltmeter to ground, but this is eliminated in the later type of junction. The Peltier effect is also absent. Figure

27 shows several typical thermo junctions. The type shown at (*a*) in the figure is readily understood by the study of the circuit. The alternating current flowing through the two wires composed of copper and constantan raises the temperature of the junction of the two dissimilar materials, causing a continuous potential to be generated and a small direct current of something less than 100μa to flow through the d'Arsonval-type meter, as indicated. This type is no longer in general use

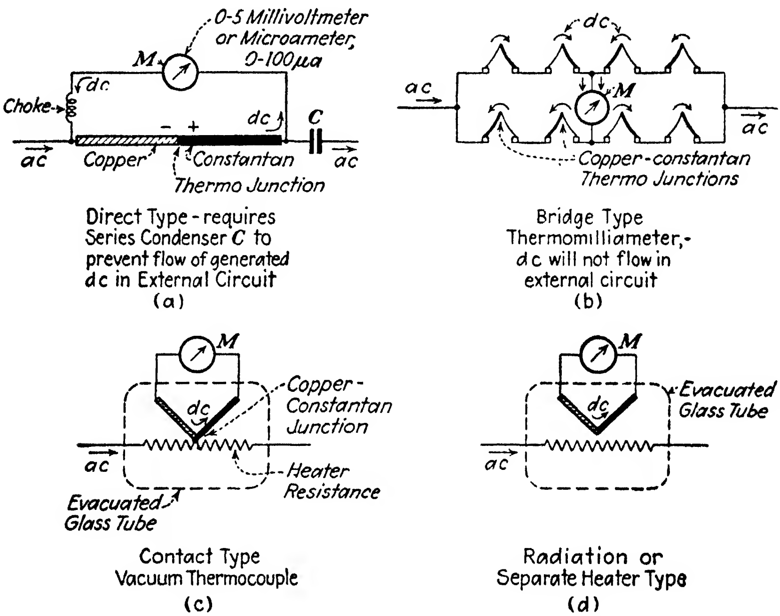


FIG. 27.—Principal types of audio- and radio-frequency thermocouple meters.

because of the fact that the source of alternating current will partially shunt the meter *M* unless a series condenser *C* is introduced as shown. This disadvantage is overcome in the bridge-type thermocouple combination and direct-current meter shown at (*b*) in Fig. 27. A bridge circuit is formed by the eight thermocouples arranged as shown to form a bridge with the meter connected across one of the diagonals. All direct-current generated potentials are directed through the meter, but there is no tendency for the direct current to flow in the external circuit. In addition to this, the use of eight thermocouples instead of a possible four in such a bridge circuit increases the sensitivity. This type of thermoammeter is in common use for ranges from $\frac{1}{10}$ amp. to several amperes. The thermocouples in such an instrument are usually enclosed in a thick-walled bakelite case so as to prevent excessive heat loss and resulting errors.

For small currents below 1 or 200 milliamp., it is customary to mount a single thermocouple of the contact type or of the radiation type in an evacuated glass envelope to prevent as much loss of heat radiation as possible. These types are shown at (*c*) and (*d*), Fig. 27. In the latter (radiation type), shunting of radio-frequency current to ground through *M* is prevented.

J. H. Miller[1] describes a new type of vacuum thermocouple which is useful at 100 Mc. These instruments have a 3-mil diameter heater wire about $\frac{1}{32}$ in. long. Welding a small thermo junction to the center of the heater wire has been successfully accomplished. Another development incorporated a platinum-alloy tube having a wall only 1 mil thick, and it was found that the correction factor was only 0.95 at 80 Mc. In all of the thermocouples described above, great care must be used in reducing the undesirable effects of radio frequencies. The heater wires must have sufficient resistance and small diameter so as to not give an appreciable increase in effective resistance due to the well-known skin effect. It is often desirable not to ground the circuit at the point of contact in the heater wire, so that the radiation or separate-heater type has been used in rather high radio frequencies.[2]

Calibration of Thermocouples.—Thermocouples may be calibrated in three principal ways:

a. Direct current is passed through the junction and through a standard milliammeter, ammeter, or calibrated shunt and galvanometer. To compensate for the Peltier effect, readings of the millivoltmeter connected to the junction are taken for both directions of the flow of the direct calibrating current through the heater. The average of these two deflections is then taken as that obtained for any alternating current through the heater, equal in value to the calibrating current. This method, while not highly recommended, is very convenient when alternating-current precision apparatus is not available.

[1] *Proc. I.R.E.*, Vol. 24, No. 12, p. 1567, 1936.

[2] This type is manufactured by the General Radio Company.

b. For calibration of the larger thermo junctions whose range is 100 milliamp. or above, very high-precision standard commercial-frequency alternating-current milliammeters are available. The thermocouple may then be calibrated by passing 60-cycle current through the heater circuit and through the standard low-frequency milliammeter and calibrating the millivoltmeter connected to the junction from the readings of the standard milliammeter. Some vacuum junctions are constructed so as to have a very small capacitance between terminals, so that a 60-cycle calibration will hold when the device is used at very high radio frequencies. However, this calibration probably is unreliable at frequencies higher than 3 Mc.

c. Precision calibration of thermo junctions for the measurement of very small high-frequency currents presents a difficult problem. Standard low-frequency-current measuring devices are not available, so it becomes necessary to establish temporary standard circuit conditions. An ideal method consists in determining the voltage drop across a standard resistor with a precision voltmeter which requires no current, but the realization of this is not easy. A thermocouple of the vacuum type connected in series with a precision standard resistor and to a low-potential source of alternating current is shown in Fig. 26. If the voltage drop across the standard resistor R_s is accurately known, the current flowing in the circuit is determined. If a quadrant electrometer, which has been previously calibrated for deflection with known impressed voltage, is connected across R_s, it will draw extremely small current owing to its capacitance, which is as low as 12 cm. It requires an alternating e.m.f. of several volts to produce a readable deflection on the scale of a quadrant electrometer; hence R_s must be high enough so that the current flowing will give a drop of several volts, and this means that R_s must be several thousand ohms. A more common way is to calibrate a vibration galvanometer connected in series with a high resistance and use this previously calibrated voltmeter device to measure the drop across R_s. The known voltage for the calibration of the vibration galvanometer may be obtained by passing a known alternating current through a precision standard low resistance. In conducting this calibration, it is necessary carefully to observe precautions as to constancy of standard resistances and series resistors and the frequency of the alternating-current source. The vibration galvanometer must be tuned to a certain frequency, after which the frequency must be held as nearly constant as possible. The most ideal calibration of thermocouples would be that which could be done at radio frequencies, and this is possible if the laboratory technician has satisfied himself that his measurement of the calibrating high-frequency current by means of a bolometer, standard resistance or capacitance and electrometer, or other means is reliable to the required accuracy of calibration.

Ambient temperature effects sometimes must be taken into consideration in the calibration. Often, when the ambient temperature changes sufficiently to cause a noticeable difference in the deflection of the meter connected to the thermo junction, the trouble can be eliminated by placing the thermocouple in a vacuum thermos bottle provided with a perforated cork. The thermos bottle should also be packed with cotton so as to support the vacuum thermo junction and to keep out a large volume of air which may otherwise circulate while it is being heated, thus causing a slowly varying temperature within the thermos jar. Sometimes it is feasible to provide two thermocouples and pass the alternating current through the heater of one of them. Both junctions are then connected in series in opposition and to the galvanometer or millivoltmeter. Therefore any tendency for the deflection to change due to the ambient temperature near the one junction whose heater carries the alternating current is opposed by the same temperature effect in the other junction whose heater carries no current. Such provision is sometimes made on thermocouples used for the precise measurement of temperature.

The calibration of thermocouples at high and ultra-high radio frequencies is accomplished in an effective way described by Miller.[1] The test circuit for this method is shown in Fig. 28. The radio-frequency current passes through a small tungsten-filament lamp, the filament being short and straight and the current sufficient to bring the filament up to the incandescence. The deflection of the microammeter connected to the photronic cell is, of course, dependent upon the effective value of the radio-frequency current. This current is passed through the thermocouple being tested, and the deflection of the millivoltmeter *M*

[1] *Loc. cit.*

may be designated as I_1. The lamp circuit is then transferred to a 60-cycle source, and the latter is varied until a value I_2 is obtained at which the photronic-cell meter shows the same deflection. When this condition obtains, the effective value of the radio-frequency current would be equal to the easily measured calibrating 60-cycle current, provided the resistance of the lamp filament was the same for the direct current for equal light given off. The resistance of the tungsten filament is then measured with a direct current which will give the same light. The effective resistance R_{ef} at the testing frequency is then found from the Smithsonian tables[1] for the given tungsten-filament diameter. The ratio $m = R_{ef}/R_{dc}$ is the radio-frequency

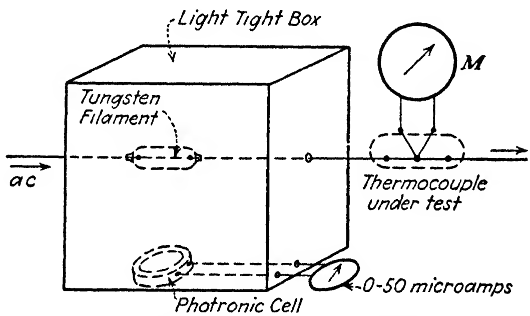


FIG. 28.—Photoelectric measurement of ultra-high-frequency current, and calibration of thermocouples.

increase factor. For equal light values and deflections of the photronic cell meter,

$$I_1^2 R_{ef} = I_2^2 R_{dc},$$

from which

$$I_1 = I_2 \frac{1}{\sqrt{m}}. \tag{17}$$

In this way, a high-frequency accurate calibration curve or a correction curve for the 60-cycle calibration may be obtained.

An interesting chart is shown in Miller's paper giving correction factors needed for various high radio frequencies for different ranges of certain models of thermoammeters. Below the 500-millamp. range, the correction factor is from 0.99+ to 0.9;

[1] Smithsonian Physical Tables, 8th ed., Tables 532–534, pp. 449–451.

i.e., the error is less than 10 per cent at 150 Mc. For meters of 3 and 8 amp., the correction becomes large, the meters reading as much as 100 per cent high owing to skin effect in the heater wire at frequencies above 50 Mc. Below 5 Mc., the meter indications are within 5 per cent of the correct value for the high-range instruments. H. M. Turner and P. C. Michel recently described an electrodynamic method of measuring currents at frequencies from 1 to 100 Mc.[1] The current passes through a single-turn circular primary coil of about 4 cm. diameter, mounted in a vertical plane. A single-turn 2.4-cm.-diameter secondary coil of No. 26 wire is suspended by a quartz fiber in a vertical plane near the plane of the vertical primary; the secondary is enclosed in a thin-walled unevacuated glass bulb. The center of the secondary turn lies on the axis of the primary coil. The plane of the secondary, when at rest, is normally displaced about 12 deg. from the position of zero coupling with the primary (planes of the two coils perpendicular for zero coupling). The primary current causes induced currents in the secondary, and the interaction causes the secondary coil to turn toward the position of zero coupling. But the inertia of the latter causes it to swing beyond this position, resulting in reversed torque on the secondary, and a new swing toward the zero coupling position. Thus a damped oscillation of the secondary is set up, until the coil finally comes to rest in the position of zero coupling.

The frequency of this damped mechanical torsional oscillation is proportional to the primary current,

$$I = CF, \text{ amperes,}$$

where C is a constant, which may be determined with the aid of a calibrated radio-frequency ammeter at any frequency between 1 and 5 Mc.; and F is the damped oscillation frequency, which may be observed by counting the time for a number of complete oscillations. The method is remarkably accurate at 50 to 100 Mc. Its limitations are discussed in the paper referred to.

High-current Measurement Using Shunts.—For the measurement of large radio-frequency currents an ordinary meter shunt would be accurate at only the frequency at which calibration was made. The error due to increasing frequency above this value is termed the *frequency effect*. For radio-frequency work, shunts

[1] *Proc. I. R. E.*, Vol. 25, No. 11, p. 1367; November, 1937.

have been made so that the frequency effect is reduced to a minimum. The inductive drop across the shunt is, of course, responsible for the frequency error in such devices; and even if the inductance is eliminated by noninductive arrangement, the increase in resistance at radio frequencies results in errors. Shunts have been constructed of many thin-wire or flat-metal strips and arranged in parallel to form a cylinder. One of these strips is made the expanding wire of a hot-wire ammeter or consists of the leads to the heater circuit of the associated thermocouple. Such a device is shown in Fig. 29. All of these devices are subject to objections.[1]

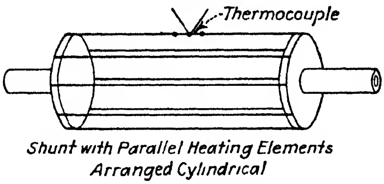


FIG. 29.—Type of high-frequency shunt for use with thermocouple.

Another method of measuring large radio-frequency currents is by the use of the condenser shunt. In its simple form, it consists of two condensers and the thermoammeter connected as shown in Fig. 30(*a*). The line current flows mainly through

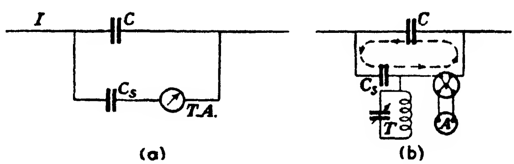


FIG. 30.—Arrangement of shunting condenser and thermoammeter for measuring high-frequency current. (*After Nyman.*)

condenser C, and only a small portion thus passes through the comparatively low capacitance C_s and thermoammeter in series. The impedance of the heater circuit of the thermocouple of TA, being of the order of 10 to 40 ohms, is very low compared to the capacitive reactance of C_s. Hence, the ratio of the currents in C_s and C can be taken in direct proportion to their capacitance values. It is then quite simple to design the condenser shunt to suit the line current and type of thermoammeter to be used. Alexander Nyman describes improved forms of the condenser shunting the measurement of large currents.[2] He points out

[1] See "Radio Instruments and Measurements," *U. S. Bur. Standards*, *Circ.* 74, pp. 144–150.

[2] *Proc. I.R.E.*, Vol. 16, No. 2, pp. 208–217. The simple form of condenser shunt is also described in "Radio Instruments and Measurements," *U. S. Bur. Standards*, *Circ.* 74, p. 151.

that the circuit comprising C, C_s, and the thermoammeter may form a resonant closed circuit at a high frequency, most probably a harmonic of the current being measured, thus causing error in the current reading. To prevent this possibility, he provides another tuned circuit electrically in contact with this circuit, which will prevent circulating currents by the absorption of energy. The arrangement of this condenser shunt is shown

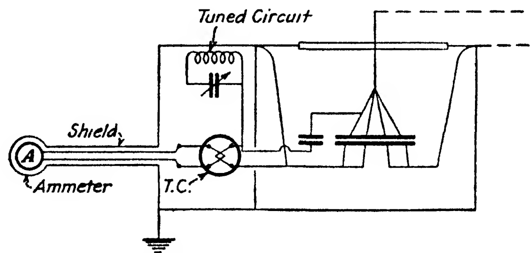


FIG. 31.—Practical form of apparatus illustrated in Fig. 30.

in Fig. 30 (*b*). The closed circuit T is tuned to the natural frequency of the condenser shunt circuit, and at all other frequencies its effect is negligible. At very high frequencies, the inductive drop in the leads becomes an important factor, and so do the mutual effects; so for the practical operation of this current-measuring device, it is necessary to shield the various parts carefully and to make the thermocouple leads very short and close together. For a full description of these construction details, the reader is referred to Nyman's paper, in which is also shown the arrangement of this device for practical operation, as reproduced in Fig. 31.

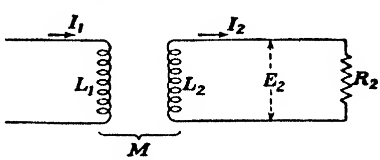


FIG. 32.—Mutual inductance for current measurement.

Radio-frequency Current Transformer.—A calibrated mutual inductance, used as an air-cored transformer, provides a simple and reliable means of measuring radio-frequency currents of several amperes or more, depending upon the frequency. Figure 32 shows a schematic diagram of such a transformer. The fundamental principle of this transformer is based upon the value of the voltage E_2, induced in the secondary, where E_2 equals $\omega M I_1$. This is equal to the impedance drop in the secondary circuit comprising the secondary coil L_2, thermocouple ammeter, and the entire resistance R_2 of the secondary circuit. This impedance drop is equal to

$$I_2\sqrt{R_2^2 + (\omega L_2)^2} = \omega M I_1.$$

The current ratio N is

$$\frac{I_1}{I_2} = \frac{\sqrt{R_2^2 + (\omega L_2)^2}}{\omega M}.$$

The low-loss secondary coil would have an impedance that is mostly inductive; then

$$\frac{I_1}{I_2} = \frac{L_2}{M}. \tag{18}$$

For the lower radio frequencies, this simple relation is entirely satisfactory, and no frequency correction is necessary if the windings of the transformer have a low coefficient of coupling and are carefully designed so as to reduce self-capacitance. At high radio frequencies, a correction may be needed because of the capacitance effect between the coils. A method of correcting for the frequency error due to this capacitance effect is given by Möller and Schrader.[1] Let the true ratio between the primary and secondary currents be N. The correction for the current ratio N is

$$N_1 = \frac{N}{1 + \omega^2 K}, \tag{19}$$

where N_1 is the apparent ratio at the frequency $\omega/2\pi$, and K is a constant depending on the frequency and capacitance between coils. K can be determined experimentally by obtaining apparent ratios N_1 and N_2 for ω_1 and ω_2. Then

$$K = \frac{N_1 - N_2}{\omega_2 N_2 - \omega_1 N_1}. \tag{19a}$$

From these equations it is easy to design the current transformer, but it is best to check the calibration for the frequency range in which it is to be used. An actual calibration curve may be made for lowered values of I_1 where an ammeter for its measurement

[1] *Jahrbuch der drahtlosen Telegraphie,* Vol. 22, p. 64, 1923.

is available, provided I_2 is still measurable with a low-range thermomilliammeter in the secondary.

9. Measurement of Very Small Currents. *Low-range Thermocouple Instruments.*—The measurement of the very small current by means of thermocouples and galvanometers is fraught with the difficulty of obtaining sufficient heat to warm the thermo junction sufficiently to produce a small e.m.f. which will deflect a sensitive galvanometer. This is readily accomplished in the thermogalvanometer invented by W. Duddell.[1] This remarkable galvanometer, while evolved many years ago, is one of the most sensitive devices at hand for the measurement of small radio-frequency currents. The essential principle is illustrated in Fig. 33, which shows the circuit arrangement and the position of the permanent magnet field. A sensitive thermocouple *Bi*, *Sb* is suspended above and close to a heater wire made in the form of a plate coil or grid of zigzag wire arrangement. The e.m.f. developed by the junction causes a minute current to flow in the short-circuited turn to which the thermo junction is connected as shown. The short-circuited turn, together with a small mirror, is in turn suspended by a fine quartz thread so that the very small magnetic effect of the current flowing from the permanent magnetic field will give the coil and mirror an angular deflection. The thermo junction, of course, rotates with the coil to which it is fastened. In spite of the fact that the thermo junction is placed at a finite distance from the source of the heating wire, deflections are obtained for surprisingly small currents. The current- and voltage-sensitivity data, as given by the Cambridge Scientific Instrument Company, are shown in the table on p. 314.

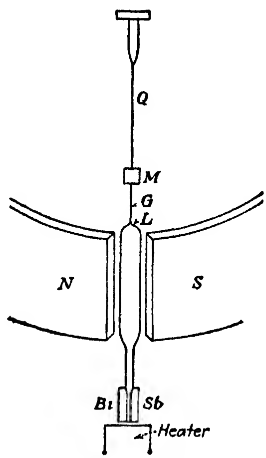


FIG. 33.—Element of the Duddell thermogalvanometer.

The table shows the instrument to be capable of measuring radio-frequency currents of a very few microamperes. Using the

[1] *Proc. Physical Soc. London,* Vol. 19, p. 233; also *Phil. Mag.,* Vol. 8, p. 91, 1904. The instrument in its most recent form is discussed and illustrated in the catalogues of the Cambridge Scientific Instrument Company.

1,000-ohm heater, 1 μa would give a noticeable deflection at the scale distance of 1 meter. It is to be noted that if this device is used in series with a known resistance or high-reactance condenser, it can be used to advantage in the measurement of e.m.f. at radio frequencies. It will be noticed that the instrument can be introduced directly into the tuned circuit of a radio-frequency amplifier so that it will measure the current,

Scale Distance, 1 Meter

Approximate resistance of heater, ohms	Current to give 10 mm. deflection, microamperes	E.m.f. for 10 mm. deflection, millivolts
1,000	20	20.0
100	60	6.0
10	200	300.0
4	300	1.2
1	600	0.6

even if the tube is of only moderate capacity. If it is placed in the load resistance connected across the plate-circuit choke of an amplifier, when the latter is being tested for output, as directed in Chap. V, Art. 9, it will measure the alternating current in this output circuit. Suppose that an amplifier tube has a signal voltage of 0.1 volt impressed upon its input terminals; the alternating-current component of plate current is obtained by dividing the effective voltage acting in the plate circuit by the total resistance. If the amplification factor is 5, and the alternating-current plate resistance and load-circuit resistance are each 10,000 ohms, the alternating-current plate current will be

$$I_p = \frac{\mu E_g}{R_p + R} = \frac{5 \cdot 0.1}{20{,}000} = 25\ \mu\text{a}.$$

Hence, for measuring the alternating component of plate current of this small amplifier tube, the 1,000-ohm heater is required.

It is also possible to measure quite small currents by the use of a low-range thermocouple of the vacuum type, using a sensitive D'Arsonval galvanometer to indicate the junction current.

For this purpose, vacuo junctions whose normal heater current is of the order of 1 or 2 milliamp. are used. Such thermocouples have a heater resistance of about 1,000 ohms, but the open-circuit voltage at the junction is only a very few millivolts. The resistance of the junction is also low, sometimes not more than 10 or 15 ohms. It therefore requires a galvanometer that has a high current sensitivity and the lowest possible resistance, to give the maximum deflection when connected to the junction with normal current flowing in the heater. Low-range microammeters have resistances from 150 to upwards of 2,000 ohms, while some D'Arsonval galvanometers have a resistance of 15 or 20 ohms and will give full-scale deflection for a current of 500 or 600 μa; *e.g.*, a 2-milliamp. thermocouple has a junction resistance of 12 ohms and develops 7 millivolts. If this is connected to a 15-ohm galvanometer, the total current flowing is $\frac{7}{15 + 12}$, or 260 μa, a little less than average full-scale deflection. On the principle that the junction e.m.fs., and hence the deflections, are proportional to the square of the heater current, it is readily shown that for a high-frequency heater current of 200 μa, the deflection obtained with such a galvanometer will be about 2.6 mm., or scale divisions. Current sensitivity such as this is all or more than is usually required for radio-frequency currents.[1]

It is also possible to measure very small radio-frequency currents with the aid of a vacuum-tube voltmeter and high-value standard resistance. Shunting the standard resistance with the input circuit of the tube voltmeter places limitations upon the usefulness of this device at high frequencies, but there are limitations within which this method of current measurement may be very useful. J. M. Eglin has recently described a direct-current amplifier for measuring small current,[2] in which a modification of a U. S. patent issued to P. I. Wold is used. Figure 34 is a schematic diagram showing the connections and arrangement of this amplifier. The amplifier tube T_1 and the balancing tube T_2 are connected in the arms of a Wheatstone bridge, and a galvanometer or sensitive microammeter M may be used.

[1] Galvanometers having a low resistance and high sensitivity, as given above, are manufactured by the Leeds & Northrup Company and the Cambridge Scientific Instrument Company.

[2] *Jour. Optical Soc. Am.*, Vol. 18, pp. 393–402, 1929.

Adjustments of the variable resistances are carefully made for zero deflection without any input e.m.f. across R_{G1}. When a direct or alternating current is passed through this resistance, the average value or steady component of plate current in T_1 changes; this, of course, is due to a change in plate-current resistance of T_1, and an unbalance of the bridge occurs. The device may be calibrated for deflections of M. A remarkably high current-measurement sensitivity is given for this device. For instance, when the input resistance R_{G1} is 1 megohm, the lowest measurable current is 10^{-10} amp. For the proper design and adjustment of this amplifier, the reader should consult the paper referred to above. The electrometer tube of Nelson[1] has possibilities for measuring currents in the region of micromicroamperes. For this purpose a sensitive grid-leak grid-condenser type of vacuum-tube detector is used to produce an average potential to apply to the electrometer tube input-electrode. It is claimed that direct current of the order of 10^{-15} amp. may be measured.

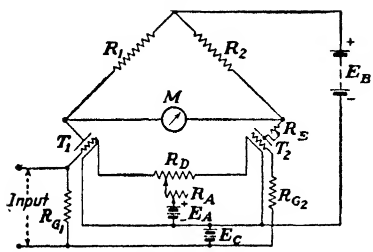


FIG. 34.—Amplifier for very small currents. (*After Wold-Eglin.*)

10. Measurement of Power. *Radio-frequency Power-measurement Considerations.*—The dynamometer-type electrical measuring instrument gives a satisfactory means of measuring power at commercial or audio frequencies, but the inability to use this device at radio frequencies makes power measurement a more difficult problem. If the effective radio-frequency resistance of an absorbing circuit is known, it is very easy to obtain the power absorbed, by measuring the effective value of the radio-frequency current flowing in the circuit with the simple I^2R formula; or the effective value of the radio-frequency voltage may be measured, and the E^2/R formula used. If the radio-frequency current and e.m.f. impressed upon an absorbing circuit are measured, the phase relation between these two quantities must be determined by a special radio-frequency potentiometer designed for this purpose, such as is described in the preceding section. Of course it is assumed that the wave shapes are nearly sinusoidal. In many well-equipped laboratories, the calorimeter

[1] *Rev. Sci. Instruments*, Vol. 1, p. 281, 1930.

method is used. If the absorbing circuit can be placed in a calorimeter, this method often provides the most desirable method of measuring power, especially at very high radio frequencies. This method has already been mentioned in Chaps. I and V. Most of the quantitative work done at radio frequencies has not required a direct measurement of power, but comparative power indications have often been made by comparing values of voltage squared or current squared.

Output Power Ratios.—Expressions for power ratios have been much used in the past in communication engineering at telephone frequencies. Mathematical expressions for these ratios are usually employed in the design and testing of telephone-communication circuits at audio frequencies, but they are just as applicable to radio-frequency circuits. Owing to the logarithmic characteristics of the sensation of hearing as a function of sound power, there have come into use for power ratios formulas that are logarithmic in character. These formulas are also convenient for use in connection with the transmission formulas for telephone lines and cables. Another advantage is that when combining two power devices, the decibels expressing the power ratio of the two add to give the total power ratio.

Telephone-transmission engineers are interested in the power ratio of the output of a line to its input; this means the ratio of the power P_2 absorbed in the circuit of the load to the power P_1 delivered to the line. The definition follows:

$$N = 10 \log_{10} \frac{P_2}{P_1}, \tag{20}$$

N being in units termed *decibels.*[1] This figure of merit may be applied to any power-*absorbing* or -*generating* device. In dealing with an amplifier, radio receiving set, microphone, antenna, er any other device that delivers energy, interest lies in how much it will deliver to a load of impedance Z ohms. To express this in decibels, a base level of 6 milliwatts, and more recently 1 milliwatt, has been selected as the standard; it is often called the zero level. In Eq. (20), then, $P_1 = 0.006$ watt, but it is not always necessary or intended that this value of P_1 be used.

[1] MARTIN, W. H., "The Decibel," *Bell System Tech. Jour.*, Vol. 8, p. 1, January, 1929.

For an output P_2 absorbed in Z, the load

$$P_2 = E_2^2 g,$$

where conductance $g = R/Z^2$, and R is the resistance component of Z. For the base level or any value P_1 lower than P_2,

$$P_1 = E_1^2 g.$$

From these two relations,

$$N = 20 \log_{10} \frac{E_2}{E_1} \text{ db.} \qquad (20a)$$

This means that the vacuum-tube amplifier or other power-converting device will of course deliver a higher output voltage E_2 to the load in order to produce the higher value P_2 than for the assumed base level of P_1 at voltage E_1.

Power-amplifier Ratings.—At the present time, power-amplifier ratings are given in decibels, but the base level of 6 milliwatts is not always implied. If it is implied, the actual power output of the amplifier can be calculated from Eq. (20). If the base level is not specified, as in the case of low-power output amplifiers, the decibel rating means power *gain* available by using the amplifier. Taking an example of each, a certain amplifier has an output, 6 watts, and will have an output of 30 decibels (db) above the 6-milliwatt base level, but this does not indicate what voltage gain is realized in the amplifier itself. Again, the same amplifier with the 6 watts output may be rated as having a *gain* of 90 db. This, of course, does not mean that the output is 90 db above the 6 milliwatt but that the base level according to Eq. (20) may be calculated. The 90-db gain means that the voltage amplification of the amplifier may be found by Eq. (20*a*); or if the output voltage E_2 is found from the power output in the specified load resistance suited to the amplifier, the required input voltage E_1 needed to drive the amplifier is found from the decibels-gain rating and Eq. (20*a*),[1] the necessity being avoided of actually measuring power in watts.

Decibel Measurement.—The measurement of decibels attenuation through an absorbing circuit, or the decibels gain through an amplifier, can, as has been pointed out, be accomplished by

[1] For further discussion of this subject, see R. S. Glasgow, "Principles of Radio Engineering," p. 171

measuring the voltage ratios when the device is feeding a final load circuit from an input circuit. There is also a method that makes use of a standard attenuator for attenuating the input to the amplifier under test, so that the same reading is obtained on a detector that is obtained for the input e.m.f. For the measurement of attenuation through an absorbing circuit, the standard

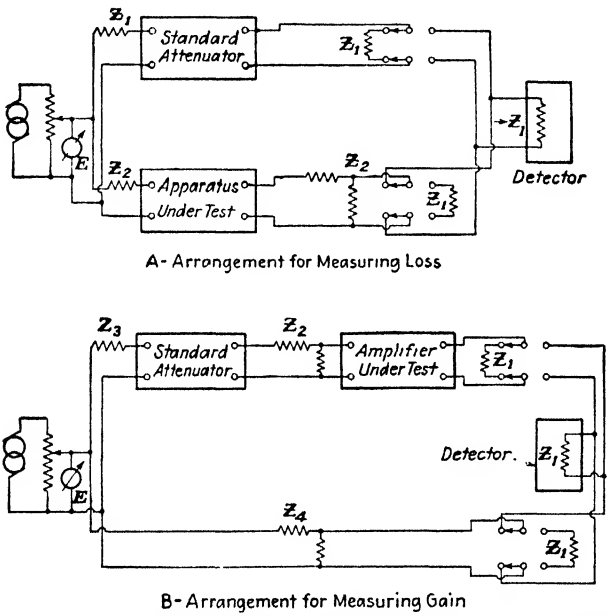


FIG. 35.—Circuits for measuring gain or loss in decibels. (*After Shackleton and Ferguson.*)

attenuator is adjusted to give the same output voltage as the absorbing circuit; both should feed into the same value of load impedance for this comparison. A description of these measurements is given by Shackleton and Ferguson.[1] Figure 35, taken from their paper, shows the arrangement of the apparatus and circuits for these measurements. They give the upper limit of frequency as 150 kc. for accurate attenuation measurements. The manipulation of the apparatus will be understood from a study of the diagrams. The extra impedances provided at the terminals of the double-pole double-throw switches keep the total load on the supply generator constant. Z_1 is the output-

[1] "High Frequency Measurement," *Trans. A.I.E.E.*, Vol. 46, p. 526, 1927.

circuit impedance and is equal to the load impedance. For a complete discussion of the steps in adjusting the various matching impedances, the reader should consult the Shackleton and Ferguson paper.

A calibrated standard attenuator is made up of variable-resistance units connected in the well-known H-type or T-type arrangement. Figure 36 shows a typical variable H-type attenuator. These devices are designed to work from a circuit of impedance Z called a *source* to another like impedance or load called a *sink*. This value is the so-called characteristic imped-

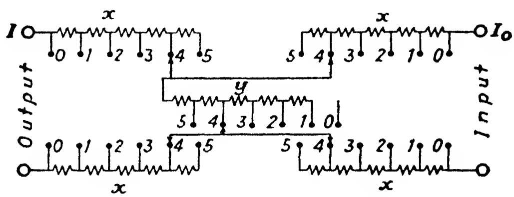


FIG. 36.—Type H calibrated attenuation circuit. (*After Lamson.*)

ance. The design of the resistances x and y for predetermined values of decibels attenuation through the device is given by H. W. Lamson, from whose paper Fig. 36 is taken.[1] He gives the values of x and y as

$$x = \frac{Z}{2}\left(\frac{K - 1}{K + 1}\right),$$

$$y = 2Z\left(\frac{K}{K^2 - 1}\right),$$

where $K = I_0/I > 1$, where x and y are as shown in Fig. 36. Now, it was shown previously that the decibels-attenuation ratio may be expressed as

$$N = 20 \log_{10} \frac{I_0}{I}.$$

From this it is easily shown that

$$K = 10^{\frac{N}{20}} = \text{antilog} \frac{N}{20}.$$

Therefore, if a certain attenuation N is desired, K is determined

[1] "Design and Use of Attenuation Networks," *General Radio Experimenter*, Vol. 2, No. 4, p. 2, 1927.

from which x and y may be calculated for any value of characteristic impedance Z by the foregoing relations.[1]

Power Measurement with Radio-frequency Wattmeter.—Turner and McNamara[2] have described the vacuum-tube wattmeter shown at (*a*), Fig. 37. The tubes T_1 and T_2 function in the square-law region, and their characteristics must be identical. The input voltage to T_1 is $[E + R(i - ji')]$, and, assuming that the shunt resistance across the line takes negligible current,

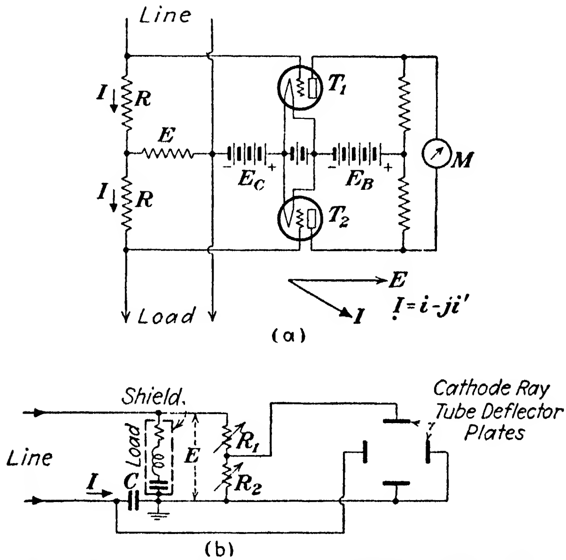


FIG. 37.—Vacuum-tube and cathode-ray wattmeters.

the input voltage to T_2 is $[E - R(i - ji)]$. Assuming the second-power law, it can be easily proved that when the current flows in the load, direct plate current changes so that the change flows in plate milliammeter M. The change in plate current in T_1 is proportional to $(E + iR)^2 + (Ri')^2$, and for T_2 it is $(E - iR)^2 + (Ri')^2$. Then it is easily shown that the difference flowing in M is proportional to Ei, the average power consumed in the load. In using this method, calibration of the wattmeter is made on a load of resistance R_s in which the power is measured by using an ammeter in the load and finding the resulting I^2R_s of the standard

[1] A comprehensive paper on attenuation-network design is given by P. K. McElroy, *Proc. I.R.E.*, Vol. 23, No. 3, p. 213, February, 1935.

[2] *Proc. I.R.E.*, Vol. 18, No. 10, p. 1743, October, 1930.

load. The balance condition may be checked by opening the load circuit at which the reading of M should be zero.

A development of the cathode-ray tube wattmeter which functions at frequencies as high as 40 Mc. is described by A. H. Taylor.[1] A diagram of the measuring circuit is shown at (*b*), Fig. 37. Deflection of the cathode-ray beam in the vertical direction is due to a portion of the voltage E across the load, and deflection in the horizontal is due to the drop resulting from flow of load current I through C. The result will be a closed figure whose area is proportional to the power in the load. The area of this closed loop is

$$A = \int x dy,$$

where the deflections $x = K_1 e$ and $y = K_2 i = K_2 \frac{q}{C}$, e and i being instantaneous values of E and I.

Now,

$$\frac{dy}{dt} = \frac{K_2}{C} i, \text{ or } dy = \frac{K_2}{C} i dt.$$

Then

$$dA = (K_1 e) \frac{K_2}{C} i dt,$$

or

$$A = \frac{K_1 K_2}{C} \int_0^T e i dt \tag{21}$$

But power, where A is the area under the cathode-ray trace.

$$\begin{aligned} P &= \frac{1}{T} \int_0^T e i dt = f \int_0^T e i dt \\ &= KfCA, \end{aligned} \tag{22}$$

where f is the frequency, $K = 1/K_1 K_2$, and C and A are as given above. This interesting measurement may be easily performed in the instructional laboratory. K_1, K_2, and A may be determined quantitatively using the same scale values, A being easily found by placing millimeter coordinate paper and counting squares. Also, a standard load may be used for calibration power

[1] *Proc. I.R.E.*, Vol. 24, No. 10, p. 1342, 1936. In this paper, a comprehensive discussion of the applications of this instrument is given.

in this, and the area of the trace measured, this being called A_s. Then, for the unknown load of area A_x,

$$P_{ai} = (I^2R_s)\frac{A_x}{A_s}. \qquad (23)$$

The condenser C is dispensed with in a more flexible and elaborate apparatus, described in the paper referred to, using two pairs of push-pull amplifier tubes, with tuned inputs and current transformers, but unfortunately space cannot be given here to its description and explanation.

Other Power-measuring Devices.—Space does not permit the inclusion of other circuits and means of radio-frequency power measurement. The light-tight box and photronic cell described in the preceding chapter (Art. 8, Fig. 28) is useful in finding the power in a resistance load coupled to an amplifier or oscillator-tank circuit.[1] The method is readily understood after the material referred to has been read.

A new vacuum-tube wattmeter development using pentagrid tubes of the 2A7 or 6A7 type is described by Pierce.[2] The quadrant electrometer, described by Bradford,[3] is also a new and interesting radio-frequency power-measurement development. In one form, a drop through a resistance due to the flow of load current is impressed between the two sets of quadrants, and a portion of the load potential is applied between one set of quadrants and the suspended vane, or *needle*. The deflection can then be made proportional to the power in the load. Errors due to capacitances in the electrometer, etc., must be corrected for.

[1] This was described in *Q.S.T.*, Vol. 16, p. 38, June, 1932, for measuring antenna current.

[2] *Proc. I.R.E.*, Vol. 24, No. 4, p. 577, 1936.

[3] *Proc. I.R.E.*, Vol. 23, No. 8, p. 958, 1935.

CHAPTER VII

MEASUREMENT OF WAVE FORM

1. Introduction. *Distortion of Radio Frequency, E.M.F., and Current.*—The wave form of high-frequency alternating e.m.f. or current may be distorted from the ideal sinusoidal shape as for the case of power frequencies and often with the added complexity of unsymmetrical wave shapes due to even harmonics. As many as 300 harmonics have been detected in the wave form of the e.m.f. developed in a piezoelectric oscillator, but it is a general rule that the amplitude of the harmonics decreases as the order becomes higher. In the case of an overmodulated amplifier tube or an overexcited power-oscillator tube, the second harmonic may be more than 50 per cent of the fundamental in amplitude. The determination of wave form at radio frequencies or the measurement of the number and amplitude of the various harmonics present in a wave constitutes a laboratory problem of considerable importance.

Evolution of Wave-form Measurement.—From the early method applicable to alternators, in which point-by-point methods and search coils were used, followed the development of the magnetic oscillograph. The graphic records obtained by these methods were analyzed by laborious integration processes to obtain the amplitude and phase of the harmonics. Later, analyzer machines were developed to facilitate the analysis of the oscillograph record. All such methods were slow of procedure and required voluminous calculations.

One of the outstanding departures from the foregoing methods was developed by R. L. Wegel and C. R. Moore,[1] for use at telephone frequencies. Since in telephone engineering it is necessary to know only the amplitude but not the phase of the harmonics, they found it possible to develop a machine that automatically obtained the amplitude of all the important harmonics present and photographically represented a plot of the

[1] *Trans. A.I.E.E.*, Vol. 43, pp. 457–465.

harmonic amplitudes on sensitized graph paper. The entire procedure required only five minutes or so. Since this development, new methods have evolved which require less expensive apparatus, nearly as little time, and are applicable to radio frequencies as well. The discussions of two such quantitative methods will be given, but, first, a few simple and approximate methods will be briefly reviewed.

2. Observed Wave Forms with Cathode-ray Oscillograph. *The Simplest Methods.*—First of all, the cathode-ray oscillograph can be used for examining the unknown wave for the presence of harmonics. One method consists of applying to two deflector plates of the cathode-ray tube the alternating e.m.f. to be tested, and if the amplitude of the e.m.f. thus applied is of the

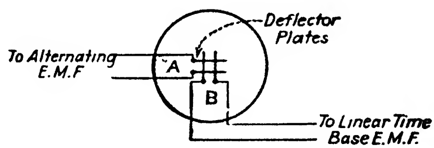


FIG. 1.—Connections to cathode-ray tube for observing presence of harmonics in a wave.

order of 12 volts, the cathode-ray beam will trace a straight line on the fluorescent screen. If the e.m.f. contains harmonics, bright spots and abrupt changes in the width of the straight line traced will be observed. However, this is not an infallible test; for instance, a wave may be slightly peaked, owing to a third harmonic, and the effect is not easy to discern. Another method consists of applying to one pair of deflector plates the e.m.f. to be measured and supplying a separately generated sine wave at the same frequency to the other pair of deflector plates placed at right angles to the first pair. The arrangement of the deflector plates is as indicated in Fig. 1. This second e.m.f. is derived from the drop across a variable resistance and inductance in series so that its phase is easily changed. When it is 90 deg. out of phase with the e.m.f. to be examined, an ellipse or circle is obtained when the latter e.m.f. contains no harmonics. If harmonics are present, only distorted closed figures are obtained, and for small high-order harmonics the figure is a serrated ellipse or circle.[1]

[1] For the fundamental theory of the elliptical trace obtained, see Chap. II, Art. 7. See also Fig. 16, Chap. II.

Wave-shape Observation with Unidirectional Time-base Deflection.—As is well known, the cathode-ray oscillograph has been successfully used to photograph high-frequency electrical disturbances in a very short time, of the order of a few microseconds, high accelerating potentials and the photographic plates being used in the evacuating chamber in front of the incident beam. For recurring waves, it is much more convenient to observe the trace made by the cathode beam on the sensitized screen, when the trace can be made to represent the wave form in the usual rectangular coordinates. It is possible that if the second pair of

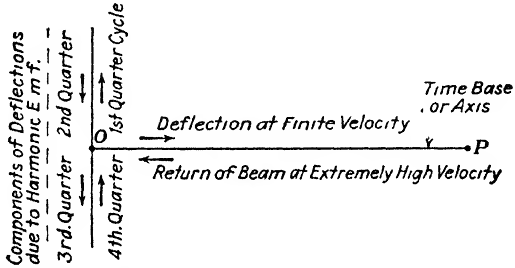


FIG. 2.—Deflection paths of cathode-ray beam in the production of waves.

deflector plates, at right angles to those to which the unknown wave shape e.m.f. is connected, is excited by an alternating e.m.f. at a frequency several times greater than that of the unknown e.m.f., complex Lissajous figures will be obtained. If the beam is made to move along a time base at a finite speed until it has traveled completely across the screen and then returned at extremely high speed back to its starting point, then, if this travel is repeated at the same rate, it would trace the wave form of the unknown e.m.f. so rapidly that an apparently steady trace of the wave, due to the persistence of vision, would appear on the screen. The principle is illustrated by Figs. 1 and 2. The deflector plates *A* produce a to-and-fro deflection due to the alternating e.m.f. to be studied. The deflector plates *B* are excited by an e.m.f. which increases more or less linearly with time, deflecting the beam at a certain velocity from *O* to *P*, Fig. 2, and this is followed by a sudden collapse of the deflecting e.m.f., allowing the beam to return to its starting point *O* almost instantaneously. While it is moving from *O* to *P*, it may be assumed that the spot is also deflected from zero to maximum amplitude in the four quarter-cycle positions of the unknown e.m.f. wave, resulting in the trace of a wave, shown in Fig. 3. The figure shows the particular case where the time required for the movement from O to P (Fig. 2) is just equal to the time for one complete cycle of the e.m.f. applied to plates A. If the speed of the spot from O to P is only one-half of the assumed preceding value, two complete cycles will appear in the trace made on the sensitized screen. If this trace is sufficiently steady, it may be photographed with a camera, an exposure of 45 sec. to 1 min. being used for a lens speed of f.6.3. Often a piece of tissue paper can be placed over

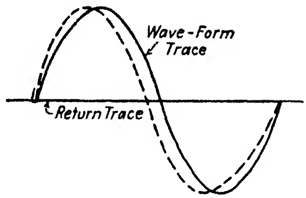


FIG. 3.—Trace of a wave resulting from use of linear time-base voltage for the horizontal deflection of cathode beam (see Figs. 1 and 2).

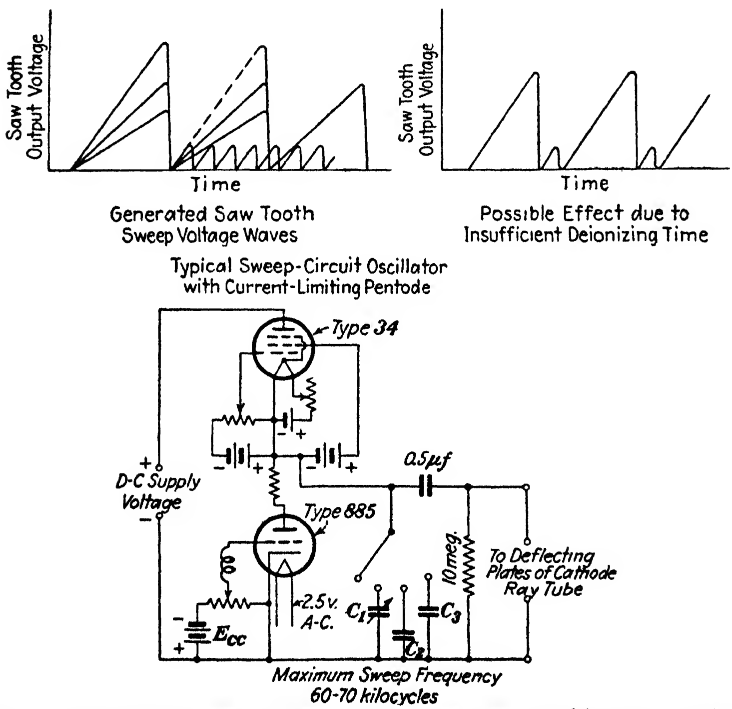


FIG. 4.—Conventional battery-biased sweep-voltage supply, ½ cycle to 70 kc., and typical generated *saw-tooth* wave forms.

the crowned end of the cathode-ray tube, and the trace will shine through the paper so that it may be retraced with a pencil.

Sweep, or Saw-tooth, Oscillators.—The required linear time-base deflection with quick return, as just described, is obtained with a modern sweep oscillator.[1] Figure 4 shows such an oscillator having a maximum sweep frequency of 60 to 70 kc., with the condenser switch on position *O*. The use of insulated biasing batteries, shown in Fig. 4, results in low inherent capacitance between portions of the circuit and hence highest possible sweep frequencies. The type 34 tube provides linear charging of the

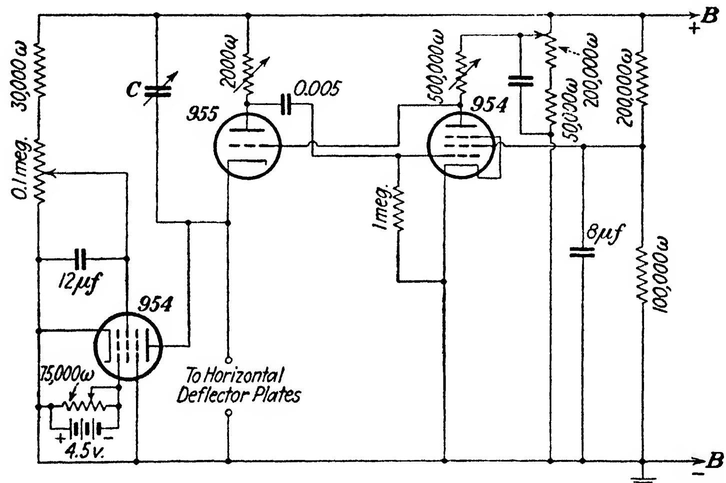


FIG. 4A.—Multivibrator-type sweep-voltage generator, 50 to 500 kc., using acorn tubes with 200-volt power supply.

condenser capacitance in circuit due to its constant plate-current characteristic. The type 885 is a gaseous-discharge tube which breaks down, giving the high-speed return of the spot to point *O*, Fig. 2. The maximum frequency is limited by the deionizing time in the gas-discharge tube. A useful booklet of instruction is published by the R.C.A. Manufacturing Company, in which complete details for an alternating-current-operated sweep-voltage supply are given. Commercial oscilloscopes incorporate sweep circuits at the present time having sweep frequencies up to 80 kc., but most of them have about a 16-kc. limit, and a new commercial form will appear in the near future, whose upper

[1] An early form of cathode-ray oscilloscope with a saw-tooth neon-tube oscillator is described by Bedell and Reich, *Trans. A.I.E.E.*, Vol. 46, p. 546, 1927.

limit is 300 kc. In Fig. 4*A* is shown a multivibrator-type sweep-voltage generator, in which the fundamental frequency may be easily increased to 500 kc. when *acorn*-type tubes are used.[1]

A radio-frequency sweep-voltage supply having maximum frequencies of some 12 Mc. is described by Goldsmith and Richards.[2] It is shown schematically in Fig. 5. The capacitance C, merely that of the plate to filament of the type 59 tube, is charged by the constant charging current through the plate-filament circuits of four type 58 tubes in parallel, giving a linear sweep. The type 59 tube is connected as a class B triode, and

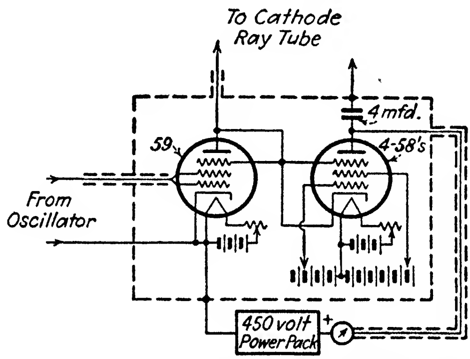


FIG. 5.—Radio-frequency sweep-voltage generator, maximum frequency 12 Mc. (*After Goldsmith and Richards.*)

a small negative bias beyond the cutoff is provided. A radio-frequency voltage from a radio-frequency oscillator-tank coil is supplied in series with the 59 tube grid bias; and when the grid is driven positive by the oscillator-output voltage, the condenser (plate-filament cap. of 59 tube) is discharged in an extremely short interval. The bias is adjusted so that only the peaks of the oscillator-output voltage are effective in swinging the 59 tube grid positive. For further particulars regarding successful performance, the reader should consult the paper referred to above.

3. Approximate or Qualitative Harmonic Detection.—The foregoing methods for visual observation of wave form are simple in principle but require elaborate and expensive apparatus. For qualitative detection of the presence of harmonics of a wave,

[1] Information and the circuit diagram of this apparatus were kindly furnished by H. D. Cooper of the Clough-Brengle Company.

[2] *Proc. I.R.E.*, Vol. 23, No. 6, p. 653, June, 1935; a good bibliography is given in footnotes, p. 653.

selective tuned circuits with resonance indicators may often be used. One useful application of this principle is to test for the presence of harmonics of the output of the oscillator, or of transmitting current, by coupling a frequency meter provided with a sensitive resonance indicator to the output current in question and varying the tuning through the entire ranges corresponding to the second and higher harmonics of the fundamental of the transmitter current. If this resonance indicator consists of a vacuum-tube detector, regeneration may be provided to make the resonance in the frequency-meter circuit very much sharper to enable it to detect one harmonic at a time. This method is of doubtful practical value for low-amplitude harmonics; probably it could not be used satisfactorily when the amplitude of the latter is less than 10 per cent of the fundamental, unless the comparatively powerful fundamental were suppressed by some suitable filter. A heterodyne oscillator may be used to detect the presence of harmonics, provided the heterodyning e.m.f. is sinusoidal in form. Quantitative methods, making use of this principle, will be described presently.

4. Quantitative Harmonic Analysis by Heterodyne Methods. *Tube-voltmeter Heterodyne Measurement.*—This method is adapted to the measurement of the amplitudes of harmonics of a complex wave but cannot be used to determine their phase. It is described by C. G. Suits,[1] has the feature of extreme simplicity of apparatus required, and will give the amplitude of a harmonic as low as 10 per cent of the fundamental, to an accuracy of about 1 per cent. A heterodyning oscillator adds its e.m.f. to that of the e.m.f. whose harmonics are to be determined, at the input terminals of a sensitive high-grade tube voltmeter. The connections of the measuring circuit are as shown in Fig. 6. A heterodyne oscillator is adjusted to beat on the fundamental or any harmonic component of the unknown wave form, with a beat frequency of about 1 cycle per second or less. The amplitude of plate-current variation is observed by the swing of the needle of meter M, the steady plate current being balanced out by the compensating shunt. Theory shows that if two harmonic e.m.fs. in series are impressed on a detector or amplifier tube having square-law rectification characteristics, the frequencies beating on each other produce sum-frequency and difference-frequency components of current in the

[1] *Proc. I.R.E.*, Vol. 18, No. 1, pp. 178–192, 1930.

output circuit. Using expressions for the complex and heterodyning e.m.fs. as follows:

$$e_h = \mathcal{E}_h \cos \phi_h \tag{1}$$

for the heterodyning voltage; and

$$e = \mathcal{E}_1 \cos \phi_1 + \mathcal{E}_2 \cos \phi_2 + \cdots \mathcal{E}_n \cos \phi_n \tag{2}$$

for the complex e.m.f. to be analyzed, where $\phi_2 = 2\phi_1$, etc. The author of the foregoing paper uses the expressions in forms similar to those above and substitutes them in the power series

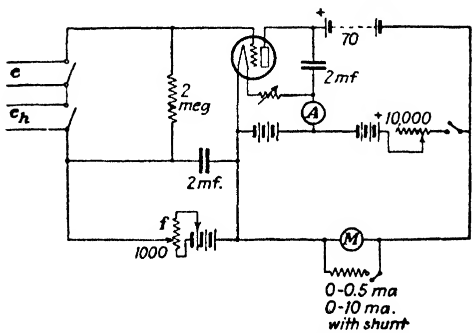


FIG. 6.—Circuit for wave-form analysis by heterodyne phenomena. (*After Suits.*)

for a typical rectifier, such as the biased-grid vacuum tube. This series is of the form

$$i_p + I_p = a_0 + a_1e_0 + a_2e_0^2 + a_3e_0^3, \tag{3}$$

where $e_0 = e + e_h$, and where higher powers than the second are neglected in the power series, because the square-law or quadratic characteristic of the plate-current variation is assumed. Equations (1) and (2) are substituted in Eq. (3), and the terms summed up. This mathematical work is carried out by the author referred to above, and he gives the expression

$$\delta i_p = \frac{a_2}{2}\sum_{0}^{n} \mathcal{E}_n^2 + a_2 \sum_{1}^{n} \mathcal{E}_h\mathcal{E}_n \cos (\phi_h - \phi_n). \tag{4}$$

A part of the change in plate current has a summed-up direct-current term and a term representing the summation of the difference frequencies of a harmonic n and the heterodyne frequency h. However, only one of the harmonics will beat on the heterodyning frequencies at the period below that of needle M at a time; hence the needle will swing with an amplitude and frequency of

$$\delta i_p = a_2 \mathcal{E}_h \mathcal{E}_n \cos (\phi_h - \phi_n), \tag{5}$$

where N is the frequency of the harmonic to which H was adjusted. To beat at a frequency of 1 cycle per second or less, the beat frequency of the next higher or lower harmonic with that of $\mathcal{E}_h$ would be

$$BF_{n+1} = (2n) - h \text{ or } h - \frac{n}{2}.$$

But let $n = 10^6$ cycles per second, and let $h = 10^6 + 1$ cycles per second. Then,

$$\begin{aligned} BF_{n+1} &= 2 \times 10^6 - (10^6 + 1) \text{ or } 10^6 + 1 - \frac{10^6}{2} \\ &= (10^6 - 1) \text{ cycles, or } (\tfrac{1}{2} \times 10^6 + 1) \text{ cycles,} \end{aligned}$$

which are, of course, radio frequencies.

As a more simple example, let

$$e_h = \mathcal{E}_h \sin (\omega_h t + \theta_t)$$

and

$$e = \mathcal{E}_1 \sin \omega_1 t + \mathcal{E}_2 \sin (2\omega t + \theta_2);$$

substituting in the quadratic portion of Eq. (3),

$$i_p + I_p = a_0 + a_1[\mathcal{E}_h \sin (\omega_h t + \theta_t) + \mathcal{E}_1 \sin \omega_1 t + \mathcal{E}_2 \sin (2\omega t + \theta_2)] + a_2[\mathcal{E}_h \sin (\omega_h t + \theta_t) + \mathcal{E}_1 \sin \omega_1 t + \mathcal{E}_2 \sin (2\omega t + \theta_2)]^2.$$

All terms of the equation will consist of high-frequency terms, zero-frequency terms, and one term of the form

$$2\mathcal{E}_2 \mathcal{E}_h \cos (2\omega t + \theta_2) \cos (\omega_h t + \theta_h).$$

This term may be expanded to show that it contains two components,[1] one of which is a difference-frequency term

$$\delta i_{p(-)} = a_2 \mathcal{E}_2 \mathcal{E}_h \cos [(2\omega_1 - \omega_h)t + \theta_2 - \theta_h].$$

For example, if

$$\frac{\omega_1}{2\pi} = 10^4 \text{ and } \frac{\omega_h}{2\pi} = 10^4 + 1,$$

$$\delta i_{p(-)} = a_2 \mathcal{E}_2 \mathcal{E}_h \cos 2\pi t; \tag{6}$$

[1] For a similar expansion, see J. H. Morecroft, "Principles of Radio Communication," 2d ed., p. 773.

and the maximum amplitude of this expression is

$$\delta\mathcal{I}_{p(-)} = a_2\mathcal{E}_2\mathcal{E}_h,$$

or

$$\mathcal{E}_2 = \frac{\partial\mathcal{I}_{p(-)}}{a_2\mathcal{E}_h}; \tag{7}$$

the total swing of needle from positive to negative is twice this, or

$$\mathcal{E}_2 = \frac{\delta I_{pt}}{2a_2\mathcal{E}_h}, \tag{8}$$

where δI_{pt} is the total swing of the needle, and maximum values are shown in Eq. (7) by the capital script letters $\mathcal{E}_2$ and $\mathcal{E}_h$. To determine the value of the denominator, a known sinusoidal e.m.f. E is impressed, and the reading is, according to the vacuum-tube-voltmeter theory, $\Delta I_p = a_2E$, from which a_2 is found.

Even though the value of a_2, the second-order differential parameter of the tube, is not determined by a separate test, the comparative amplitudes of a fundamental and harmonics may be found by observing the comparative swings of the needle as the heterodyning voltage is tuned successively to the various harmonics of the unknown e.m.f. Each harmonic given must be used to adjust the heterodyning frequency to about the same value, which, of course, is easily estimated by the speed of the swing of the needle of the meter M. It is very important to shield the sources of the unknown and heterodyning e.m.fs. so that the tube voltmeter will not be excited by a modulated wave. Such modulation would give rise to other low-frequency components, resulting in serious error.

Methods of Selective Amplifier and Balanced Modulator.—A somewhat different method of employing the heterodyne principle is described by A. A. Landeen,[1] wherein ingenious use is made of a selective amplifier and balanced modulator. These two devices are shown in Figs. 7 and 8. If the input circuit to the amplifier of Fig. 7 is connected to the tuned circuit L, C, R, as shown, and if this circuit is tuned to resonance with the fundamental of a complex wave, the higher harmonics will be more effectively suppressed if the amplifier input is connected across

[1] *Bell System Tech. Jour.*, Vol. 6, No. 2, pp. 230–247.

the condenser C, as shown. Then the components of e.m.f. impressed on the amplifier due to the harmonics become lower and lower compared to that due to the fundamental, as the order of the harmonics becomes higher. To illustrate, it may be assumed that a constant voltage E is impressed on circuit (A) shown in Fig. 7. R_s is a constant resistance, and R is the resist-

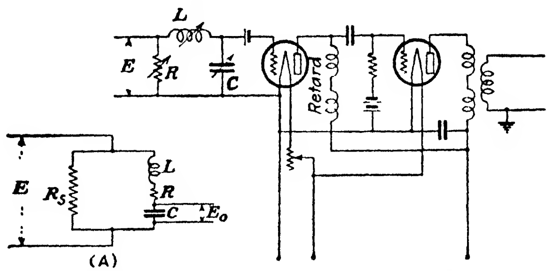


FIG. 7.—Elementary-circuit selective amplifier for wave analysis. (*After Landeen.*)

ance of the inductance coil L. It is readily shown by the circuit equations that, at any frequency,

$$\left(\frac{E}{E_0}\right)^2 = (\omega CR)^2 - \left[\left(\frac{f}{f_r}\right)^2 - 1\right]^2,$$

where E_0 is the voltage across C, and $f_r(= 1/2\pi\sqrt{LC})$ is the frequency of resonance of the branch LRC. It will be noted that when $f = f_r$,

$$E_0 = \frac{E}{\omega CR} = \frac{\omega LE}{R} = QE,$$

and may be many times E, if R is sufficiently small. As f becomes higher and higher above the value of f_r, the ratio E_0/E becomes smaller and smaller. Another analysis will show that the drop across L becomes smaller and smaller as the frequency decreases below the value to which the L, C, R circuit is tuned. In general, the impedance of the source is quite high, and E will not remain constant, but the same general comparisons hold true. Now, the variable-tune circuit of Fig. 7 may be tuned to any harmonic of the voltage E, so that this harmonic is stepped up in voltage owing to the resonance effect and passed on through the amplifier to another similar selective amplifier stage. Thus a particular harmonic desired is amplified with the effective exclusion of the other harmonics and is finally impressed on the balanced modulator of Fig. 8. The input from the heterodyning oscillator is connected to the common grid lead as shown, and the heterodyning frequency is set to about 800 cycles off the frequency f, coming through the input transformer T. The theory of this modulator shows that there will be components in its output of frequencies $(f - f_h)$ and $(f + f_h)$ but none at the frequencies f or f_h.[1] These sum and difference frequencies are similar to the well-known side-band frequencies of modulated radio communication. The low-frequency selective circuit, shown coupled to the output of the modulator in Fig. 8 is designed to effectively pass a difference

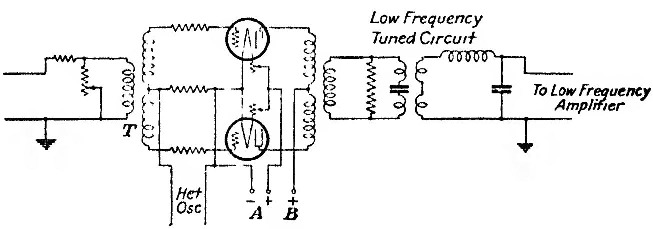


FIG. 8.—Balanced modulator used in wave analyzer. (*After Landeen.*)

frequency of 800 cycles or so, which finally excites a low-frequency amplifier, giving a reading on an output meter. The reading will, of course, be proportional to the amplification of the particular harmonic to which the selective tuned circuits of Fig. 7 are adjusted. The balanced modulator is particularly desirable because the distorting frequency components due to beating of upper and lower side bands on each other are practically eliminated. Only a very general description of this apparatus and method has been given. The measurement of wave form by the method outlined would require quite elaborate apparatus, which latter could be perfected only after considerable study and experimental tests. The complete details of the measurement apparatus are not within the scope of this text, so the reader is advised to study the paper by J. W. Horton, on the method and apparatus described above, before undertaking the actual work.[2]

[1] The fundamental theory of the balanced modulator is outlined in R. S. Glasgow, "Principles of Radio Engineering," p. 331.

[2] "Empirical Analysis of Complex Electric Waves," *Trans. A.I.E.E.*, Vol. 46, pp. 535–541. 1927.

Piezoelectric quartz crystals have extremely narrow pass-band characteristics when used as wave filters. These may be used to great advantage in the output circuits of the harmonic analyzer of the balanced modulator type in place of the low-frequency tuned circuit of Fig. 8. A well-known commercial-type harmonic analyzer utilizing quartz resonators is shown in Fig. 8*A*.[1] By comparing Figs. 8 and 8*A*, it will be noted that they are analyzers similar in principle but that that of Fig. 8*A* makes use of modern

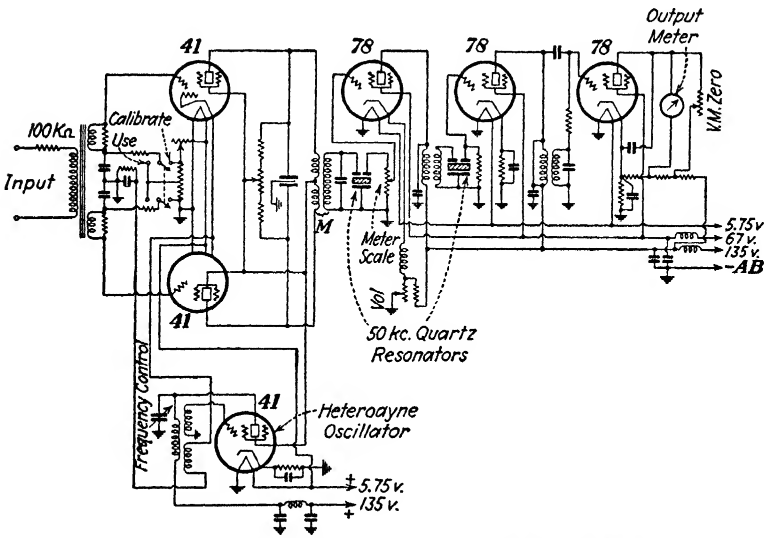


Fig. 8*A*.—Balanced modulator-type harmonic analyzer utilizing piezoelectric quartz filters. (*General Radio Company.*)

tubes and has convenient adjustments for calibration, and 50-kc. quartz resonators, giving a two-stage highly selective filter.

5. Purity of Wave Form with Belfils' Bridge. *Description and Method.*—The purity of a wave may be measured by means of the well-known resonance bridge in the manner described by G. Belfils.[2] The resonance bridge shown in Fig. 9 is connected in series with a source that is the unknown e.m.f. wave. If it is desired to test the e.m.f. wave itself, a resistance load should be provided in series with the leads to the bridge terminals *A* and

[1] This type of wave analyzer is manufactured by the General Radio Company, Cambridge, Mass.

[2] "Measuring the Residue of Voltage Waves by a Filter Method," *Rev. gén. élec.*, Vol. 19, No. 14, pp. 523–529, 1926.

B, so that the wave is not further distorted by the possible effect of the inductance element on higher harmonics. If the bridge is placed in series with a line leading from a generator to any type of load circuit, the bridge is then adapted to test the current wave. In this case, of course, a shunt resistance should be connected across terminals A and B; returning to the manner in which the bridge itself functions, it may be assumed that a tube voltmeter, which takes no current, is connected to the diagonal CD. This voltmeter V will measure the so-called *residue* of the harmonic components when the fundamental of the unknown wave is balanced out by the tuned-circuit branch LC. If the fundamental is effectively removed in this manner, it is readily shown by fundamental vacuum-tube-voltmeter theory that

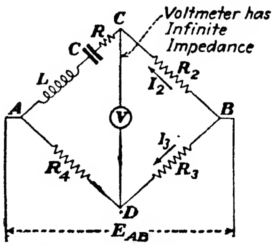


FIG. 9.—Resonance bridge adapted to measure harmonic residue of a wave. (*After Belfils.*)

$$\Delta I_p = K(E_2^2 + E_3^2 + E_4^2) \cdot \cdot \cdot , = KE^2,$$

where E is the effective value of the total residue, and E_2, E_3, etc., are the effective values of the second-, third-, etc., harmonic components. The constant K is, of course, eliminated by calibration. If such a reading is to be obtained from this bridge when tuned to the fundamental, it must be assumed that the impedance of branch AC is practically infinite at all the harmonic frequencies and that the impedance of V is infinite. If R_4 is small compared to R_3, and $R_2 = R_3$, then $R_4 = R$, and V gives a reading due to the total harmonic residue of voltage E_{AB}.

Degree of Precision Obtainable.—In order to realize the limitations of this bridge for measuring the harmonic residue, let an e.m.f. containing a fundamental and second harmonic be tested; and let the fundamental be tuned out at $\omega_r = 2\pi f_r$. Let it be assumed that the total harmonic residue of E is present across AB and that only a portion of this value is present across the diagonals CD due to impedance drops. An equation may be derived to give the ratio of the harmonic-residue portion of E_{AB} to E_{CD}, the total harmonic voltage actually measured by V, but the expression is cumbersome to handle. It is much simpler to substitute trial values in the simpler complex-circuit relations and then obtain E_{CD} directly for the ratio. If E'_{AB} represents the harmonic-residue portion of E_{AB} in the circuit of Fig. 9,

$$I_3R_3 = \frac{E'_{AB}R_3}{R_3 + R_4},$$

and

$$\underset{\cdot}{I}_2R_2 = E'_{AB}\frac{R_2}{R_2 + R + jX},$$

where $X = \left(\omega L - \frac{1}{\omega C}\right)$, and $\underset{\cdot}{I}_2$ is the complex form of I_2. From these equations, it follows that

$$\underset{\cdot}{E}_{CD} = E'_{AB}\left[\left(\frac{R_3}{R_3 + R_4} - \frac{R_2}{(R_2 + R)^2 + (X)^2}\right) + j\frac{X}{(R_2 + R)^2 + X^2)}\right]$$

Now, taking the hypothetical wave of fundamental frequency $\omega_r/2\pi$, let $\omega_r = 10^5$. This wave contains only a second harmonic, and for it

$$\omega = 2 \times 10^5.$$

C may be 0.001 mf., and L equals 100 mil-henry. The accuracy of the residue measurement may be illustrated by assuming that the harmonic component E'_{AB} at its frequency of twice the value for which the bridge is tuned is impressed upon the terminals A and B. The tube voltmeter V will read E_{CD}, and the ratio of E'_{AB} to this voltage may be found from the equation above. It is assumed, as a simple example, that R is 10 ohms (which is really low for inductance L) and $R_2 = R_3 = 100$ ohms. Substituting in the foregoing equation and finding the real value of E_{CD}, by combining the real and imaginary terms, it is found that

$$\frac{\underset{\cdot}{E}_{CD}}{E'_{AB}} = 0.91.$$

This means that E_{CD} reads about 9 per cent below E'_{AB}; hence the measured second-harmonic component will be in error about 9 per cent. If, however, R_2 and R_3 are increased to 1,000 ohms, it is found that $E_{CD}/E_{AB} = 0.986$, and the precision of measurement of the second harmonic is much improved. If the wave contains higher order harmonics, the precision of measurement is much higher than for this lowest possible harmonic of the fundamental. The vector diagram of Fig. 10 shows the e.m.f. relation for these latter assumptions. The vector diagram is drawn for the case where the reactance of the L, C branch is 15,000 ohms for the

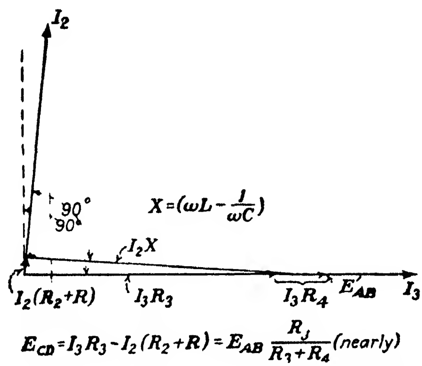


FIG. 10.—Vector diagram for resonance (Belfils) bridge.

frequency of the second harmonic, and it is noted that I_2 leads E'_{AB} by nearly 90 deg., and that E_{CD} is nearly equal to E'_{AB} when R_4 and R are made very small. Calculations made for frequencies of the order of 1,000 cycles show that this ideal arrangement is very difficult to attain and that large errors are hard to avoid in the measurement of the lower order of harmonic residue. It thus seems that the Belfils bridge is well adapted to testing radio-frequency wave-form purity, provided the bridge-resistance arms are carefully proportioned

CHAPTER VIII

MODULATION, RECEIVER, AND PIEZOELECTRIC CRYSTAL MEASUREMENTS

1. Introduction. *Useful Measurements on Generator and Receiver Apparatus.*—The radio engineer faces the necessity of making radio-frequency measurements to determine the gross efficacy, power efficiency, and frequency characteristics and other important qualities in the design, development, calibration, and testing of modern transmitting and receiving equipment. Many methods of making measurements are now officially or unofficially recognized as standard, and manufacturers have for several years produced convenient forms of measuring apparatus for the commercial market. These special complete measuring apparatus are a valuable asset to the experimental- as well as the commercial-testing radio laboratory. The importance of considerable precision in radio-frequency electrical measurements has emphasized the need of reasonably priced but reliable standards, and these have also been made available. With such standards at hand, apparatus for even the complex measurements may be assembled and calibrated or may even be produced in the moderately equipped shop.

The output and efficiency of power oscillators and amplifiers is an important measurement problem. These devices are rated by their power output in watts, and their efficiencies are actually measured by determining quantitatively the ratio of the power output to the power input. It is interesting to know, at this point, that it has been proposed to rate radio-transmitting equipment on the bases of the mean electromagnetic field strength produced by such equipment under certain standardized conditions.[1] Several methods of measuring power output and efficiency of oscillator and amplifier tube have been discussed

[1] See Edwards, S. W., and J. E. Brown, *Proc. I.R.E.*, Vol. 16, No. 9, pp. 1173–1193, 1928.

previously in this text.[1] Thereforê, the problem of measurement of power input and power output of tube transmitters, which is essentially the same as that for vacuum tubes, will not be treated in detail in this chapter. Precise and convenient means of measuring the degree of percentage of modulation of the carrier wave in radio-telephone transmission is of prime importance, since the transmission system's merit depends upon the degree of modulation as well as upon the total power radiated by the transmitter. In the reception field, the new developments make it necessary to measure quantitatively sharpness of resonance or selectivity, efficacy of receiver response or sensitivity, and over-all frequency responses, in addition to other items. The methods of measuring the just-mentioned quantities are discussed at some length in the present chapter. Some of the additional items or quantities not treated in this chapter include tuning-range test, undistorted power output by direct measurement, volume-control tests, and quantitative tests on the amount of hum produced by the power-supply voltage.

Piezoelectric Measurements.—Because of the recent advent of apparatus utilizing piezoelectric phenomena, the field of quantitative measurements is still new and rather small. However, there have been evolved some piezoelectric measurements such as determination of damping of crystals, determination of the motional admittance, and, most important of all, the frequency-temperature characteristic of the crystal. A few piezoelectric measurements will be described in due course.

2. Measurement of Percentage Amplitude Modulation. *Statement of Measurement Problem.*—Briefly stated, various results obtained from the technical process of modulating the amplitude of a carrier-frequency current or e.m.f. are shown in Fig. 1 for a single sinusoidal modulating e.m.f.[2] Curve I gives the result when the modulated-power amplifier is functioning properly. *P*, the *positive peak* of modulation, is equal to the negative peak; *n*, the envelope of the radio-frequency carrier, is sinusoidal; and the

[1] For measurements of power output and power input of vacuum tubes, see Chap. V, Arts. 8 to 10; for power-gain or loss measurements, see Chap. VI, Art. 10; and for the principles of the differential thermometer calorimeter, see Chap. I, Art. 15.

[2] These curves are taken from a paper by W. N. Tuttle, discussing the subject more fully; *General Radio Experimenter*, Vol. 5, No. 10, p. 2, March, 1931.

modulation is termed *symmetrical.* The unmodulated carrier amplitude C is equal to the average value A of the modulated wave, as shown. In curve II, where the power amplifier may be of insufficient capacity, the positive peaks are less than the negative; and in curve III, certain conditions obtain to reduce the negative peaks in amplitude compared to the positive. Again, the modulating voltage may have an unsymmetrical wave shape,

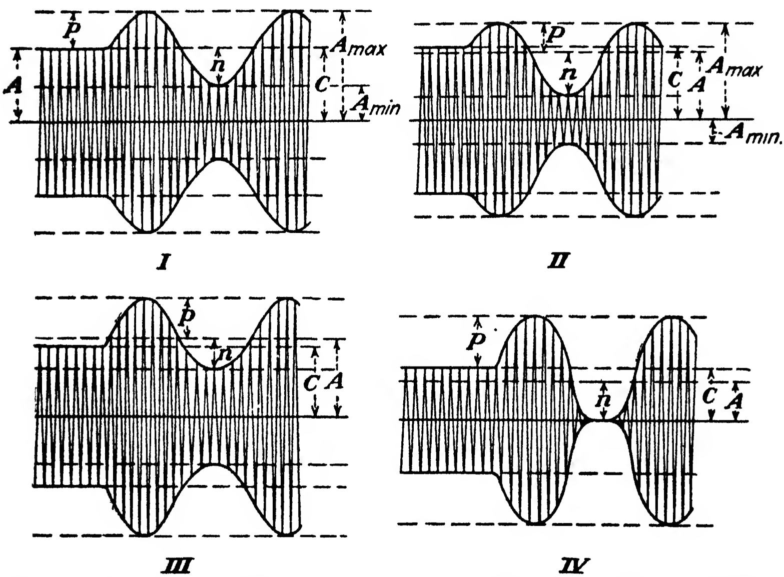


FIG. 1.—Types of modulated waves illustrating peak and trough modulation, symmetrical modulation, and overmodulation.

and envelope II or III may be the correct shape. In curve IV, overmodulation occurs on the negative peak; while for the positive peaks, insufficient power capacity results in suppression of increasing amplitudes below their intended values. In curves II, III, and IV, the average amplitude during modulation is not equal to the unmodulated amplitude.

The foregoing brings out the fact that to measure actual percentage modulation of the carrier amplitude requires separate measurements for positive and negative peaks.

The percentage positive and percentage negative peaks are, respectively, defined as p/A and n/A. These are known as percentage positive- and negative-peak modulations. The best single value to express the average percentage modulation M is given as the average of these values:[1]

$$M = \frac{1}{2}\left(\frac{p}{A} + \frac{n}{A}\right). \tag{1}$$

Now, in Fig. 1,

$$A_{max} = A + p,$$
$$A_{min} = A - n.$$

Substituting,

$$M = \frac{1}{2}\left(\frac{A_{max} - A_{min}}{A}\right). \tag{2}$$

But

$$A = \frac{A_{max} + A_{min}}{2}.$$

Hence,

$$M = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}. \tag{3}$$

These are convenient formulas for use in making measurements,[2] of which a few will be described.

Percentage Modulation Measurement with Cathode-ray Oscilloscope—Three Methods.—I. Figure 2 shows how this device may be used with its associated sweep-voltage supply (see Figs. 4 and 5, Chap. VII) for measuring percentage modulation on positive peaks, negative peaks, or the average single value. A radio-frequency voltage is picked up from a power-amplifier tank or antenna circuit, and the trace on the screen will be a solid variable-width band of light, because the radio frequency is so high that individual cycles cannot be seen for a sweep frequency in the audio range. The envelope of this band of light should, of course, be an exact reproduction of the modulating e.m.f. wave shape. The case for sinusoidal modulation is shown in Fig. 2. The measurements to be made on the screen are indicated by Fig. 1 and Eqs. (1), (2), or (3). It is convenient to measure the double-amplitude values D_{max} and D_{min}. There is considerable halation

[1] "I.R.E. Yearbook," p. 55, 1929.

[2] Equation (3) is the same as that given from a somewhat different viewpoint for percentage modulation by Brown and Keener, *Univ. of Ill. Eng. Exp. Sta. Bull.* 148, 1925.

around the edges of the light band on the cathode-ray tube screen, and the existence of harmonics of the radio-frequency carrier has the effect of making the edges of the light band indistinct, so that the modulation measurement is not closer than probably 5 per cent.

II. Increased precision of measurement is obtained by passing the radio-frequency input through a linear rectifier and filter.

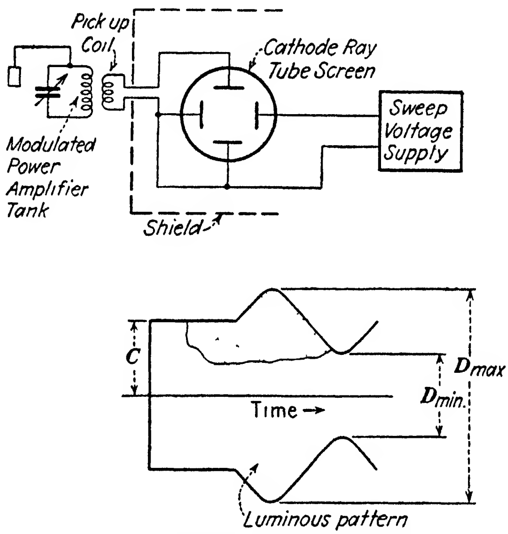


FIG. 2.—Test circuit for measuring modulation factor (or per cent modulation) with cathode-ray oscilloscope.

This results in a single line for no modulation and a single-line wave when modulation occurs and when the sweep frequency is an integral submultiple of or equal to the modulation frequency. The test circuit and oscilloscope trace are shown at (*a*) and (*b*), Fig. 3. A much smaller electron-beam intensity is required in the cathode-ray tube in this method than in I, so that the trace is more sharply defined, and closer measurements of deflections may be made with a scale or dividers. If the sweep circuit is omitted, and deflector plate 3 of the cathode-ray tube is grounded, a single spot displaced from the zero position on the screen will appear. When modulation is obtained, a straight-line trace will appear, as indicated at (*c*), Fig. 3. For 100 per cent modulation, this line will touch the position of zero input voltage to the plates.

It can be readily seen that the three types of modulation may be measured by this method.

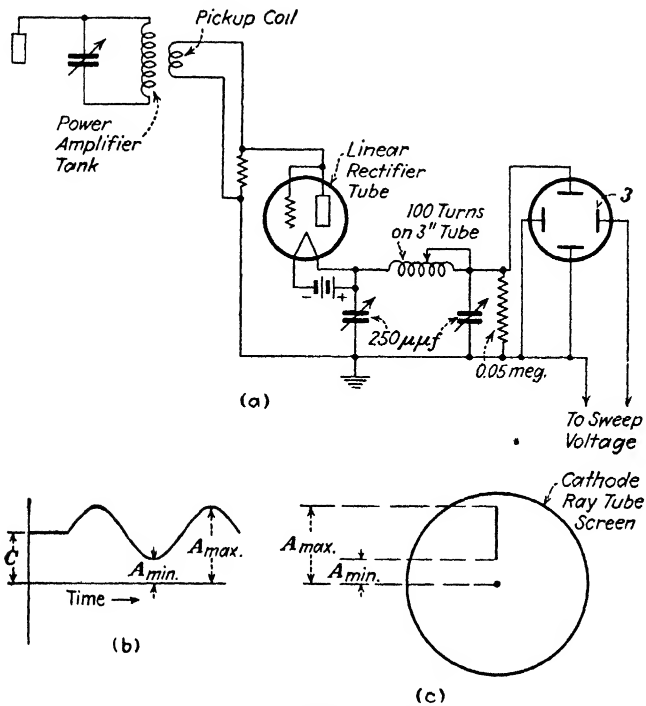


FIG. 3.—Modulation measurement using cathode-ray oscilloscope, linear rectifier, and filter.

III. A very simple scheme which is widely used is illustrated in Fig. 4, where the trace will be a trapezoid for moderate

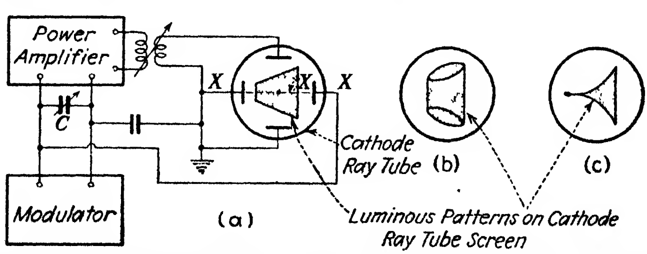


FIG. 4.—Test circuit for trapezoidal method of modulation-factor measurement.

degrees of modulation and a triangle for 100 per cent modulation. As shown, the modulating voltage deflects the beam along the

X-axis, and of course the carrier amplitude is simultaneously varied, resulting in the pattern shown. As in the two previous methods, the three types of modulation may be measured. If the trace has edges that appear elliptical, as indicated at (*b*), Fig. 4, the modulating voltage suffers a phase shift in the power amplifier, and the effect is corrected by adjusting the capacitance of condenser *C* connected across the modulation-voltage supply. At (*c*) is shown the effect of overmodulation, if the modulation is not linear; *i.e.*, the positive- and negative-modulation peaks are unequal, and the sloping edges of the trapezoid trace will not be straight. In this method of modulation, halation and radio-

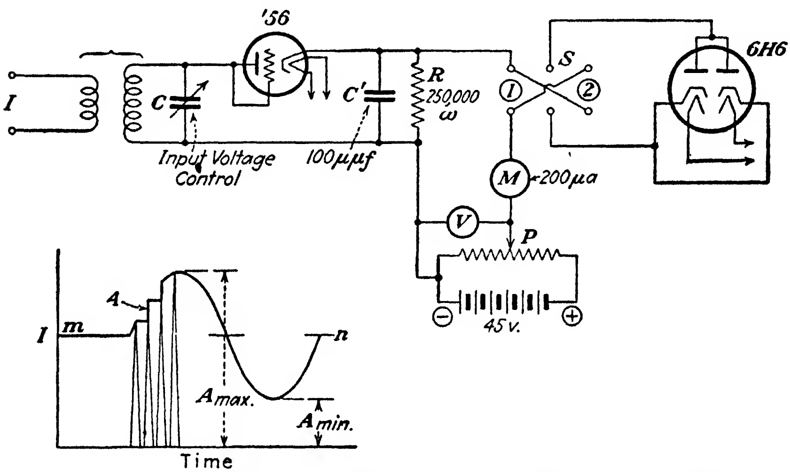


FIG. 5.—Test circuit for measuring positive- and negative-peak modulation.

frequency harmonics limit the precision of measurement; a 5 per cent error, at least, is probable.

Measurement with Linear Rectifier and Peak-trough Voltmeter.— The method of Van der Pol and Posthumus[1] yields measurements with fair precision. In the test circuit of Fig. 5, modulated voltage is impressed at *I*; and with the aid of *C*, the input voltage to the type 56 (diode) is sufficient to make it function as a linear rectifier. By means of the radio-frequency filter, the current that flows through the load resistance *R* will be the smooth half-envelope wave *mn* of the rectified wave diagram. It is necessary that *R* be high compared to the plate resistance of the '56 and that *C'* be low in capacitance so that the modulation envelope will

[1] *Experimental Wireless and Wireless Engineer*, Vol. 4, p. 140, 1927.

be faithfully reproduced. In all diode detectors, R is about 250,000 ohms, and the by-pass condenser capacitance is small, so that the latter may discharge at a rate greater than the maximum slope of the modulating envelope; but it should have sufficient capacitance to smooth out the radio-frequency pulsations of potential across R.[1] C' is usually given as 100 mmf. for receiver circuits using a diode detector.

When switch S is on position ①, the modulation envelope e.m.f. is impressed on the plate of the peak-voltmeter tube which is also a diode (the double diode 6H6 may be used as well as a triode), and current will flow, giving a reading on M unless the opposing voltage from potentiometer P equals the positive-peak value A_{max}; V reads this latter value. Switch S is then changed to position ②, the trough-voltmeter connection is made, and current will not flow unless the voltage of P exceeds A_{min} in value, and this is equal to the reading of V when M indicates slight current flow. The average percentage modulation is then given by Eq. (3). In a commercial development of this apparatus,[2] the potentiometer P is calibrated to read directly in percentage modulation, and voltmeter V is not used. This requires some special considerations which are discussed in the paper referred to.

Other Methods and Types of Modulation Measurements.—Other methods of amplitude-modulation measurements include observation of percentage increase in radio-frequency current when modulation sets in, specially calibrated ammeters which measure the alternating component of modulation plate current, special use of thermocouples, continuously reading peak-voltmeter-linear-rectifier combinations, etc.

Frequency modulation and phase modulation may also be measured, modulating the carrier wave with a very low or commercial frequency.[3] These modulation measurements have special applications but will not be treated in this book.

3. Radio-receiver Sensitivity Measurement. *Definition of Sensitivity.*—Former definitions of this quantity have been used and later revised.[4] At the present time, sensitivity is

[1] Note exaggerated zigzag line A, Fig. 5. This is discussed in the 1937 edition, RCA "Receiving Tube Manual," pp. 26, 27.

[2] *General Radio Experimenter*, Vol. 5, No. 10, March, 1931.

[3] HUND, AUGUST, "High Frequency Measurements," pp. 373–375, 378.

[4] Some of these are discussed in H. A. Brown, "Radio Frequency Electrical Measurements," 1st ed., pp. 333–335.

understood to be the 30 per cent modulated microvolts input to a receiver, through the standard dummy antenna, to give 50-milliwatts output in the load circuit connected to the power amplifier tube—this usually being the dynamic speaker. Sometimes the microvolts input for 1-watt output with 30 per cent modulation of the carrier signal is specified. The more sensitive the receiver the lower the input for standard output. Hence, it has recently become customary to rate the sensitivity in decibels below 1-volt input for standard output. The sensitivity is then

$$S_v = 20 \log_{10} \frac{10^6}{E_{i-m}}, \tag{4}$$

where E_{i-m} is the input in microvolts for 50-milliwatts output (or for 1-watt output). The greater the value of S_v the more sensitive the receiving set.

Apparatus Needed.—The known input voltage is a laboratory and sometimes a commercial problem. It is most desirable to provide a modern *signal generator*, comprising a well-shielded oscillator with modulation of 30 to 50 per cent at 400 cycles, a step attenuator, and convenient controls and indicators. Such a signal generator is shown in Fig. 6. If it is necessary to construct suitable equipment in the laboratory, the step attenuator may be readily constructed and checked by procedure explained in Art. 6, Chap. VI. A simpler device is to use a stable shielded oscillator and the easily constructed mutual-inductance-type piston attenuator (Fig. 21, Chap. VI).

In the construction of a signal generator, shielding must be carefully done, and the well-known principle of grounding to a common point must be adhered to. Heavy-gage metal must be used for the shield cans or boxes, and the covers must fit tightly. When the oscillator of Fig. 6 is modulated by varying the plate-supply voltage, the oscillator frequency varies somewhat; this is called *frequency modulation.* In well-designed commercial forms, it is claimed that the variation is less than 100 cycles, except for extreme cases. In Fig. 6, the frequency-variation effect is reduced by adjustment of the stabilizing resistance R in the oscillator plate-feed connection. In a recent commercial development of the signal generator, a stabilized oscillator drives an aperiodic

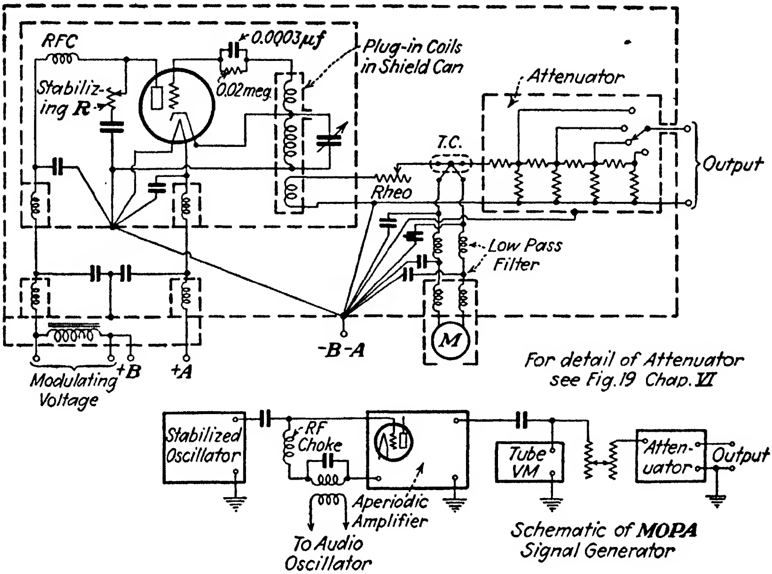


FIG. 6.—Laboratory signal generator with resistance step attenuator.

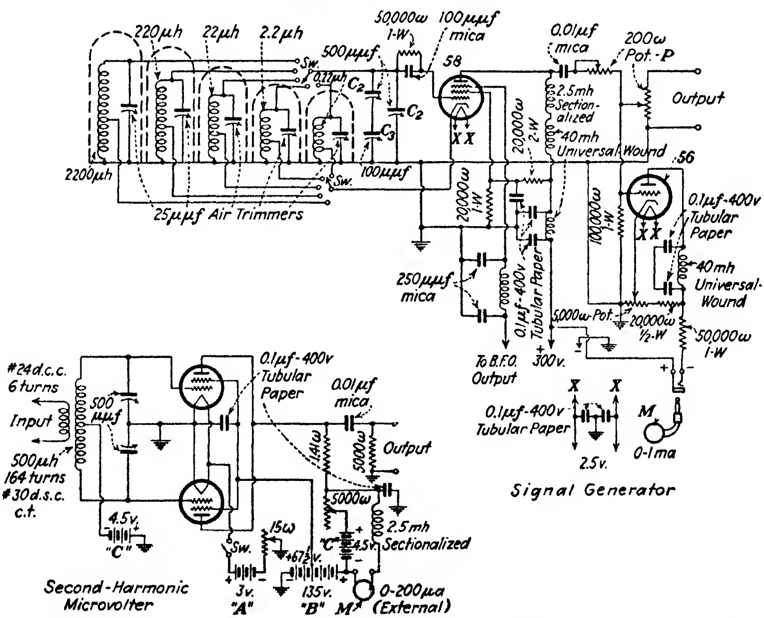


FIG. 6A.—Laboratory-constructed signal generator, and second-harmonic microvolter. Method of shielding is similar to that of Fig. 6.

amplifier, and grid modulation is applied to the latter. This eliminates frequency modulation and requires much less modulator power than does plate modulation of the signal generator of Fig. 6. A schematic diagram of this new master-oscillator power-amplifier-type (*MOPA*) signal generator is also shown in Fig. 6.

An interesting and useful signal generator which can be used for receiving-set testing is shown in Fig. 6*A* and can be readily constructed in the laboratory.[1] The oscillator-coil units are in shield cans and incorporate individual trimmer condensers. A special arrangement of variable tuning condensers facilitates shifting the frequency an equal amount in obtaining selectivity curves. To accomplish this, the two condensers C_2 are ganged on the same shaft. The 200-ohm output potentiometers *P* are noninductive. The type-56 tube functions as a vacuum-tube voltmeter. This signal generator may be suppressor grid modulated by connecting the audio voltage at *B.F.O.* Known output microvoltages are produced by a separate device known as a *second-harmonic microvolter*, also shown in the figure. The input of this device is connected to the output of the signal generator, and the tuned circuit of the microvolter is tuned to resonance with the second harmonic of the signal generator. The direct current in the plate circuit of the push-pull amplifier will be proportional to the microvolts alternating-current output; and when the plate resistor in series with microammeter *M* is 1.41 ohms, the output voltage is in microvolts. For further details of construction, the paper referred to should be consulted.

A newly developed commercial signal generator has a maximum output of 2.5 volts and a frequency range of 50 kc. to 25 Mc. In addition, the same manufacturer has developed a high-frequency type, range 20 to 100 Mc., with 50,000 microvolts output at 100 Mc. Space does not permit showing circuit diagrams of these instruments.[2]

An interesting and simple device for producing known output voltages to test receivers is shown in Fig. 7. A known inductance is made up of two concentric brass tubes, as described by A. W.

[1] DeSoto, C. B., *Q.S.T.*, Vol. 20, No. 10, p. 41, October, 1936.

[2] Manufactured by the Ferris Instrument Company. The circuit diagrams of both instruments may be found in catalogue pamphlets issued by the manufacturer.

Hull[1]; and the voltage drop along the length of inner tube can be calculated from the dimensions and the current flowing. Figure 7 shows a longitudinal section through the device together with connections. The concentric brass tubes A and B are soldered to a brass end plate E, and the inductance of either tube from the

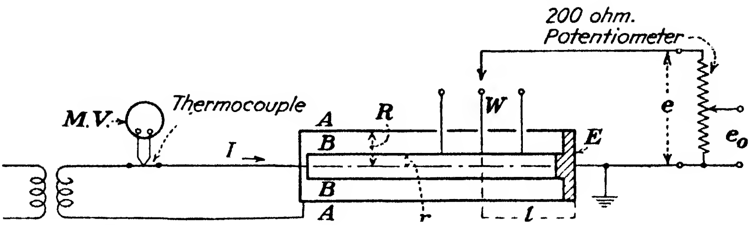


FIG. 7.—Special calibrated concentric tube for producing known low potentials.

open end to the end plate, for any length of the inner tube, may be calculated from the well-known formula

$$L' = 2 \log_\epsilon \frac{R}{r},$$

which becomes

$$L = \frac{0.009}{10^6} \log_{10} \frac{R}{r},$$

where L is in henrys per centimeter length of tube B. The e.m.f. drop is then equal to the inductive reactance along the tube length used l times the current flowing, or

$$e = 2\pi f l I (0.009) \log_{10} \frac{R}{r} \text{ microvolts,} \tag{5}$$

where f is the frequency, I is in amperes, and l is in centimeters. Lead wires are soldered to the inner tube and lead out through the outer tube, thus making it convenient to use different lengths along the inner tube.

The voltage drop at any of the output leads can be calculated with Eq. (5), and an experimental check may be made by passing 3 or 4 amp. along the inner rod at the desired test radio frequency and measuring the voltage drop e with a vacuum-tube voltmeter. This has been found to check closely with Eq. (5) calculations.

[1] *Phy. Rev.*, Vol. 25, ser. 2, pp. 645–670, 1925; also see *Q.S.T.*, Vol. 14, No. 9, pp. 78–82, 1930.

At *e* there is shown connected a 100- or 200-ohm potentiometer of a graphite-composition type,[1] and the output voltage for testing sensitivity is derived from the strip-and-roller contact. A scale may be provided for the rotating contact, and the output voltage actually measured with the tube voltmeter for several amperes in the concentric tube. Low output-voltage calibrations must be obtained by applying the voltage of a dry cell to the potentiometer and measuring output current with a microameter and 20,000-ohm resistance in the output circuit.

In testing of receiving sets using the coil or loop antenna, the microvolts per meter, and hence the terminal voltage of the

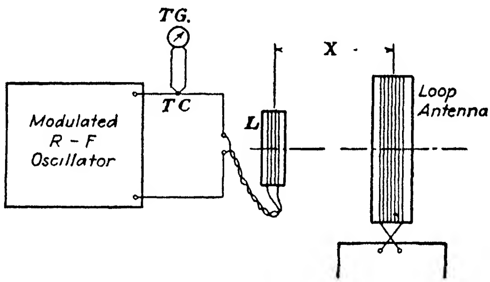


FIG. 8.—Standard transmitter-loop and receiver-loop antenna for sensitivity measurements. (*After I. R. E. Specifications.*)

loop, may be nicely provided by placing the loop antenna in a known electromagnetic field which is provided by a modulated coil-antenna transmitter placed at a proper distance. The arrangements for this may be provided as explained for antenna and field-strength measurements, Chaps. III and IV; Fig. 8, taken from the Institute of Radio Engineers recommendations,[2] shows in a simple diagrammatic form the relative positions of transmitter and receiver loops for inductive excitation. The axes of the transmitting-loop antennas are in line, as shown, which means that the receiving-loop antenna is affected by the induction field, from the transmitting loop. The convenient formula given for the microvolts per meter, due to the transmitter, is

$$\mathcal{E} = \frac{18{,}850 N_1 A^2 I \cos B}{(A^2 + X^2)^{3/2}}, \tag{6}$$

[1] Such a device suitable for radio frequencies is manufactured by Centralab, Milwaukee, Wis. A radio-frequency wire-wound potentiometer is manufactured by General Radio Company for use in their signal generator.

[2] *Proc. I.R.E.*, Vol. 18, No. 8, p. 1282, 1930.

where N_1 is the number of turns in the coupling coil L.
A is the radius of the coupling coil in centimeters.
I is the ammeter reading in microamperes.
X is the distance in centimeters between the center of the coupling coil and the center of the loop antenna.
B is the angle, if any, between the axis of the loop antenna and the line between the coil centers.

The voltage induced in the receiving-loop antenna is

$$E = \mathcal{E}Q,$$

where Q, the effective height of the receiving-loop antenna, is

$$Q = 2N_2h \sin\left(\frac{fs}{300{,}000}\right), \tag{7}$$

where N_2 is the number of turns in the loop antenna.
h is the height of the loop antenna in meters.
s is the length of the loop in meters.
f is the frequency in kilocycles per second.

These convenient formulas are derived on the fundamental principles outlined in Chap. IV. The voltage induced in the loop antenna is conveniently varied by adjusting the distance X or transmitting-antenna current I. The distance X must be large compared to the dimensions of the loop, but, of course, it is small as compared to a full wave length, in order to obtain a sufficiently high induction field. The latter equation for effective height applies only to rectangular loops. Another method which is often convenient is the introduction of the voltage e, in series at the center of the receiving-loop antenna. If this is done, it corresponds to the voltage induced in the loop inductance by the electromagnetic field, and it is well to remember that the voltage across the loop terminals is the voltage e times the step-up ratio, as explained in Chap. IV, Art. 2, if the receiver-loop circuit is tuned.

In measuring the wattage output in sensitivity tests, it is best to connect a resistance load in the power-tube plate circuit with the aid of a 1:1 ratio transformer of fairly low direct-current resistance. The high load resistance must not be connected directly into the power-tube plate circuit because of the resulting high drop in supply voltage available at the plate. A high-resistance rectifier-type voltmeter may be connected across a known portion of the load resistance, from the readings and resistance proportions of which the power output may be calculated. Power-output meters carrying built-in resistance loads are a convenience. It is sometimes permissible to assume that the dynamic speaker-transformer primary presents the proper load resistance and very little reactance to the power tube when the secondary is connected to the speaker voice coil. Then a

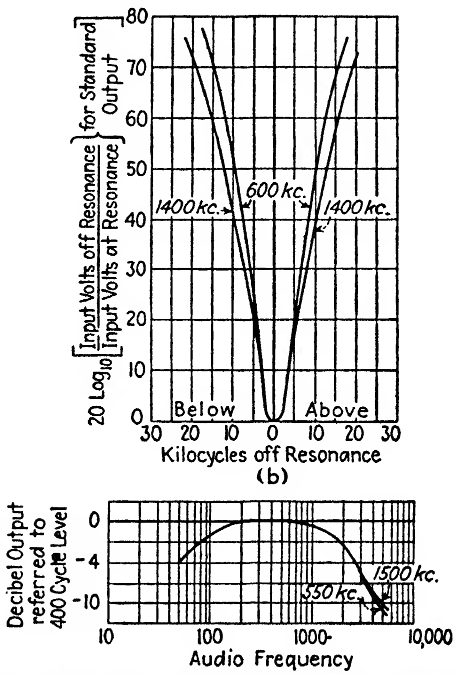


FIG. 9.—Typical selectivity and fidelity curves for radio receiving set.

suitable high-resistance rectifier-type voltmeter is connected across the primary winding. If this voltmeter has an input transformer associated with it, there will be no residual reading due to plate-current resistance drop in the dynamic speaker-transformer primary winding. In service testing, the output voltmeter is often connected to the voice-coil terminals.

4. Selectivity Measurement. *Definition of Selectivity of Radio Receiver.*—This was formerly determined by measuring the Q of the tuned-receiver circuit by the reactance-variation method but is no longer in use. At the present time, selectivity is defined as the ratio of the input voltage at a certain frequency off resonance to give the same output as that obtained at resonance. This has different values for different values of detuning, so it may be taken at 5 or at 10 kc. off resonance. If expressed in decibels, it is 20 $\log_{10}$ times the preceding ratio. Selectivity curves of the *crevice* type are usually obtained for receivers. One such is shown at (*b*), Fig. 9

Measurement Procedure.—The foregoing definition indicates the manner in which data should be taken for receivers having no volume control. After obtaining the output reading with the receiver tuned to the signal-generator 30 per cent, 400-cycle, modulated carrier, this carrier is to be tuned slightly off resonance with the receiver tune. One method of doing this is to provide a laboratory circuit driver or oscillator and, with the 400-cycle modulation turned off, tune this oscillator to zero beat with the signal generator. Then the signal-generator frequency is changed until a 2,000-cycle beat note is obtained. This may be measured by an audio-frequency bridge or by comparison with a 2,000-cycle tuning fork. The signal-generator frequency is now 2 kc. off resonance, its modulation is again turned on, and the modulated output voltage is increased until the receiver output is the same as formerly. The procedure of shifting the signal-generator frequency another 2 kc. is now repeated. If the increments for taking readings are 5 or 10 kc., the frequencies of the laboratory oscillator (circuit driver) may be measured by an absorption-type wave meter.

When the radio receiver being tested has an automatic volume control, complications set in, because when the signal generator is tuned off resonance, the receiver gain automatically increases. The two-signal generator method is then needed for selectivity measurement. Connections are made as in Fig. 10. Because two signal generators are connected in parallel, the dummy antennas have double resistance and reactive values. The primary purpose of a second signal generator is to hold the receiver gain constant while the first signal generator is detuned from resonance, as previously explained. Briefly, the procedure is to turn on signal generator 1, apply the 30-per cent 400-cycle modulation, and adjust the receiver input to obtain 50-milliwatts output with full receiver gain. Then, with modulation switch *S* (Fig. 10) open, signal generator 2 is turned on and adjusted to a frequency, say 5 kc., off resonance with the receiver tune. With S at 2, 30-per cent, 400-cycle modulation is now applied to signal generator 2, which is now 5 kc. off resonance with the receiver, and the output voltage of No. 2 is raised until the 50-milliwatt output of the receiver is restored. While this is being done, of course, the full unmodulated carrier voltage from No. 1 is maintained on the receiver to hold constant automatic volume-

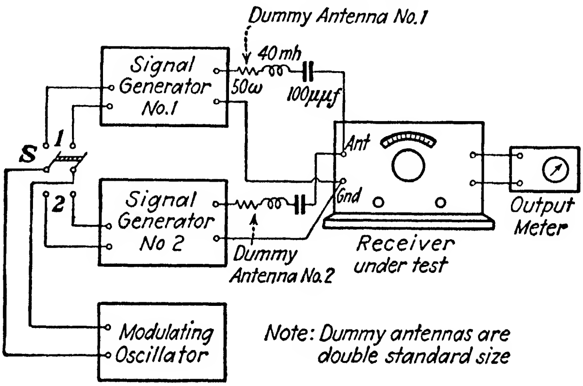


FIG. 10.—Arrangement of apparatus for measuring selectivity of receiving set incorporating automatic volume control.

control setting. It has become customary to rate receiver selectivity in *decibels down* (db) for a certain value or values off carrier frequency of resonance. Then decibels down off resonance

$$= 20 \log_{10} \frac{\text{input voltage off resonance}}{\text{input voltage at resonance}} \text{ for standard output}$$

Selectivity is often given simply as the ratio itself without applying the logarithm.

The advantage of the decibel-down rating is the use of a uniform scale of ordinates on which values to be plotted do not extend into high numbers. When the ratios themselves are plotted, a logarithmic scale of ordinates must be used for convenience, because the ratios often exceed 10,000. This may be seen in Fig. 9.

5. Measurement of Fidelity of Receiving Sets. *Definition.*—The output voltage e_m at a modulation frequency f_m necessary to produce the same power output as that obtained by an output voltage e_s at a modulation frequency of 400 cycles furnishes a comparative means of determining fidelity of sound reproduction in a radio receiver. The fidelity expression is

$$F = 20 \log_{10} \frac{e_m}{e_s}$$
$$= 20 \log_{10} \left. \frac{\text{output voltage at modulation frequency } f_m}{\text{output voltage at 400 cycles}} \right\} \text{for constant input.}$$

Data for the fidelity curve may be plotted from calculations from the foregoing expression, using measured values of e_m at different modulation frequencies f_m of the input-carrier voltage applied to the receiver through the standard dummy antenna. A typical fidelity curve is shown at the bottom of Fig. 9.

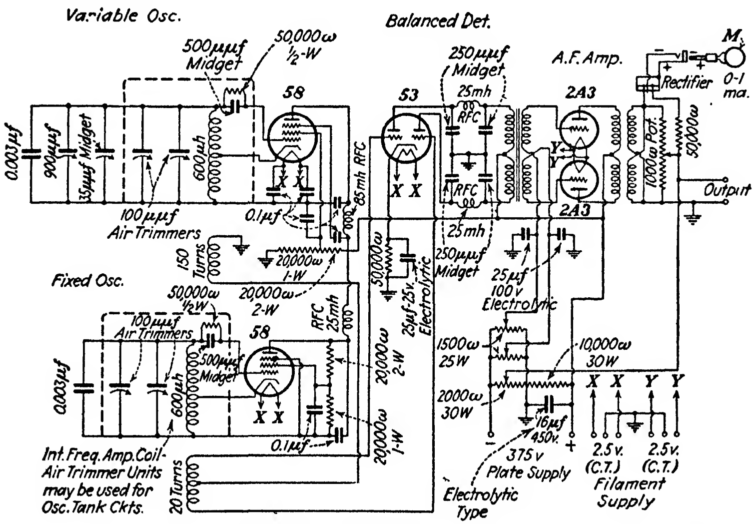


FIG. 11.—Beat-frequency audio-oscillator for fidelity tests on radio receivers.

Test Procedure.—A modulation-voltage supply whose frequency may be varied from 40 or 50 cycles to about 10,000 cycles is needed to modulate the signal-generator output through the required modulation-frequency range. A beat-frequency oscillator is convenient for this purpose, since its frequency is varied with a single variable condenser with calibrated dial, and it has practically constant output voltage through this frequency range. Many commercial forms are on the market. However, it may be constructed in the laboratory if economy must be considered. A circuit diagram of such a device is shown in Fig. 11. This beat-frequency oscillator has the unique feature in that one of the component frequencies is applied to both grids of the type 53 balanced detector in the same phase, and the other frequency is applied to the grids in opposite phase, making the detector function as a balanced modulator. This results in its output being confined to sum and difference frequencies.[1] The only filter needed is in the plate circuit of the detector. The oscillators are the self-compensating pentode type (not electron-coupled), and the pickup coils coupled to their tank circuits do not pick up any appreciable harmonics. Complete instructions for constructing this device are given in DeSoto's paper, referred to. Considerable descriptive material and discussion of practical value will be found in the literature on the subject of beat-frequency oscillators.[2]

The signal generator, which is arranged to be 30-per cent modulated by this variable-frequency modulation-voltage supply, is connected to the receiver input through the standard dummy antenna (200 mmf., 25 ohms, 20 μh in series). For short waves, a 400-ohm resistance is used as a dummy antenna. The output voltage is set at a convenient value with manual gain control well advanced, while the input voltage is modulated at 400 cycles. The frequency of modulation of the signal generator is then varied, and readings of output voltage are obtained. A complete test involves the loud-speaker device; and for such a test, a high-quality microphone, such as the moving-coil type, is placed at a distance from and in front of the loud-speaker opening. This pickup microphone excites a high-grade, high-gain, resistance-coupled amplifier vacuum-tube voltmeter combination. Sometimes the pickup microphone is rotated through a certain radius arc over, say, 90 deg., and an average reading obtained for this angle to eliminate nodal or interference effects in the laboratory at various sound frequencies. If it is desired only to obtain a fidelity test on the receiver without the loud-speaker, a resistance load should be substituted, using a high-grade transformer to couple the load to the power-tube stage-plate circuit. Fidelity tests are often made by connecting a suitable

[1] This instrument is described by C. B. DeSoto, *Q.S.T.*, Vol. 20, No. 4, p. 41, April, 1936. For principle of the balanced modulator, see R. S. Glasgow, "Principles of Radio Engineering," p. 331.

[2] See Hund, August, "High Frequency Measurements," particularly pp. 185, 186. An excellent modern design is found also in *Experimental Wireless and Wireless Engineer*, Vol. 11, p. 234, May, 1934.

output voltmeter having a high resistance to the terminals of the dynamic-speaker voice coil. A rectifier-type meter or a modern multirange tube voltmeter may be used for this purpose (see Art. 2, Chap. VI, especially Fig. 6). Basing fidelity on measurement of voltage applied to the voice coil assumes that the impedance of the voice coil is constant and yields a measurement of electrical fidelity that is only approximate. Motional impedance variation and sound characteristics of loud-speakers are special problems in sound engineering.

6. Miscellaneous Radio-receiver Measurements. *Intermediate-frequency Amplifier-resonance Curve.*—The fidelity and selectivity are highly dependent on a nearly rectangular resonance curve with the desired band spread, this particularly true in the intermediate-frequency amplifier stage or stages. Hence, a commonly made test is the actual instantaneous visual reproduction of the resonance or tuning curve in this part of the radio receiver. This is accomplished with the aid of a cathode-ray oscilloscope and a voltage input to the intermediate-frequency amplifier, this input voltage being varied rapidly through the range

$$f_i \pm f'_{max},$$

where f_i is the rated intermediate frequency, and f'_{max} is the maximum modulating frequency that should be passed on to the second detector and audio system.

In one type of testing apparatus, this is accomplished by having in the tank circuit of the oscillator furnishing the intermediate-frequency voltage a small variable trimmer condenser whose rotary plates are rapidly rotated by electric motor. A more modern testing equipment incorporates an electronic frequency modulator, of which one available commercial form will be briefly described.[1]

In Fig. 12 is shown a schematic diagram of the frequency modulator, or *wobbulator*. An ordinary test-signal generator furnishes an unmodulated input voltage to the input terminals of the balanced modulator, incorporating the two type 76 tubes, this input voltage being applied through a special iron-core trans-

[1] This interesting device is manufactured by the Clough-Brengle Company, Chicago, Ill. The circuit diagram of Fig. 12 was kindly furnished by the manufacturers.

former in opposite phase to the grids of the tubes. Another voltage generated by the type 6C5 tube oscillator and attenuated by the attenuator is applied to the grids of the balanced-modulator tubes in the same phase. As is well known in balanced-modulator theory, the output voltage of the balanced modulator in the jack terminals marked "output" will contain sum and difference frequencies of the input-signal generator and 6C5 oscillator. It is desired that one of the component output frequencies, say at difference frequencies, be adjustable to the intermediate

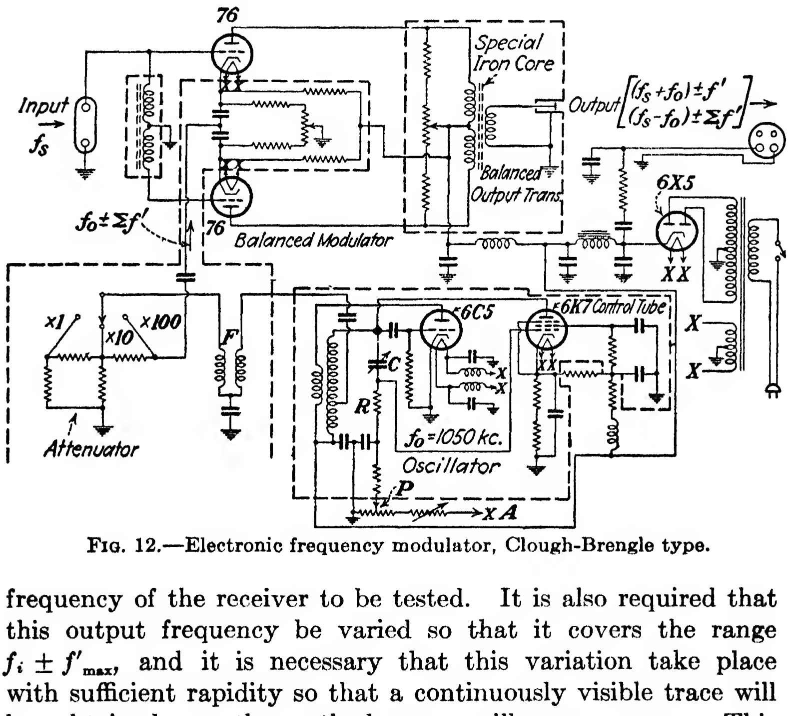


FIG. 12.—Electronic frequency modulator, Clough-Brengle type.

frequency of the receiver to be tested. It is also required that this output frequency be varied so that it covers the range $f_i \pm f'_{max}$, and it is necessary that this variation take place with sufficient rapidity so that a continuously visible trace will be obtained on the cathode-ray oscilloscope screen. This frequency modulation is accomplished sixty times per second, and this result is brought about with the aid of the type 6K7 control tube. The control grid of this tube is connected across a small resister *R* which is in series with the tank condenser *C* of the oscillator tube (type 6C5). This causes the plate of the control tube to draw alternating-current plate current which is in phase with the voltage drop across *R* and hence in phase with the current in *C*, because *R* is small compared to the reactance of *C*.

The result is that the alternating-current plate current of the control tube is a portion of the tank-circuit capacity current. If the bias of the control grid of the 6K7 is changed, the latter will of course draw a new value of alternating-current plate current, which is equivalent to changing the value of C and hence will change the frequency of the oscillator. The control-tube grid bias is therefore varied at a 60-cycle rate by having the control-grid lead connected to earth through the potentiometer P at the bottom of the figure. Sixty-cycle alternating current from the power transformer passes through this potentiometer from the point A to ground. The degree of change in oscillator frequency is, of course, increased by increasing the 60-cycle control-grid bias variation of the 6K7 control tube. The normal frequency of the oscillator is about 1,050 kc., and this 1,050 plus and minus the frequency variation passes through the coupled filter system F to the attenuator input. The output from the output transformer of the balance modulator will therefore be

$$(f_s + f_0) \pm \Sigma f' \text{ and } (f_s - f_0) \pm \Sigma f'.$$

If f_s = 1,515 kc.,

$$f_i = 1{,}515 - 1{,}050 = 465 \text{ kc.}$$

The range of $\pm\Sigma f'$ is shown in the resonance-curve trace Fig. 12A. The balanced modulator together with the coupled filter is very effective in eliminating undesirable responses and overloading of the receiver under test. Space cannot be given here to a discussion of these effects and their elimination.

If the output jack of the frequency modulator is connected to the grid of the first detector of a superheterodyne receiver, the output terminals of the last stage of its intermediate-frequency amplifier are connected to two deflector plates of the cathode-ray oscilloscope, and a 60-cycle sweep voltage is applied to the other two deflector plates, a pattern will appear upon the oscilloscope screen, the outline of the pattern being the actual resonance curve of the intermediate-frequency amplifier. It is, of course, assumed that the signal generator is first tuned so that its frequency minus the 6C5 tube oscillator frequency equals the intermediate carrier frequency. With the added potentiometer P this carrier frequency can be varied so that the maximum deviation of intermediate-frequency is as much as 30 kc. If a linear rectifier and filter are connected between the intermediate-frequency amplifier output and oscilloscope deflector plates, the pattern on the screen is a single-line trace having the shape of the intermediate-amplifier resonance curve. The second detector of the receiver can often be made use of to obtain the required rectification. A schematic arrangement of the test circuit is shown in

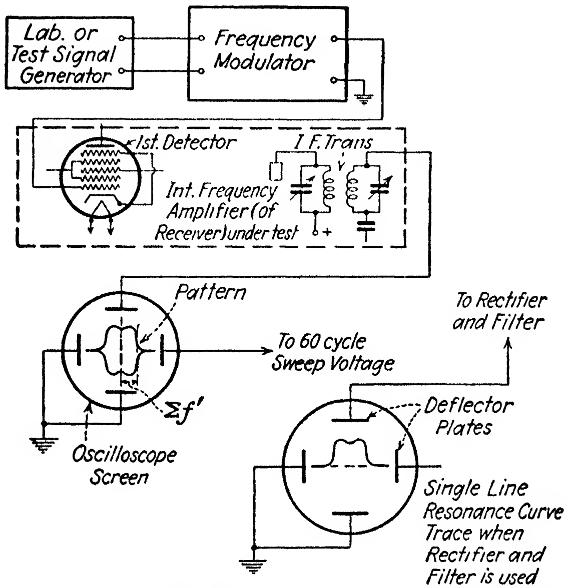


FIG. 12A.—Test-circuit arrangements for obtaining intermediate-frequency-amplifier resonance curves.

Fig. 12A, and the patterns that may be obtained on the oscilloscope screen are also shown.

Additional Tests.—Several types of tests and measurements, in addition to those above described, have been used and informally standardized. These include cross-talk measurement, overload test, automatic-volume-control (a.v.c.) test, harmonic-distortion test, maximum-undistorted-output test,[1] and conversion gain measurement in the first detector or mixer.[2]

It is not within the scope of this book to go into a detailed instructional treatment of these various tests. The overload

[1] PENDER, McILWAIN, "Electrical Engineers' Handbook," 3d ed., pp. 7–152.

[2] Conversion gain is defined as the ratio of intermediate-frequency voltage output to radio-frequency voltage input. See R.C.A. Application Note No. 87, February, 1937.

test is, however, of more than passing interest, because it shows the point at which distortion sets in from overloading as the microvolt input increases. The method of test procedure is to supply a 30-per cent modulated 400-cycle input voltage to the receiver and to vary the latter from 120 to 0 db below 1 volt or from 1 microvolt to 1 volt, obtaining readings of power output. A curve, plotted between power output as the ordinate and microvolt input as the abscissa, has somewhat the appearance of a magnetization curve. At the point where this curve suffers a sharp decrease in slope (approaches saturation), receiver overloading sets in. Above this point, there is very little increase in output, with further increase in input voltage.

Cross-talk measurements were formerly made as a routine test on older models of radio receivers which do not incorporate the modern multi-mu tubes. For this measurement, two signal generators are needed, and an attempt is made to obtain modulated output from the first signal generator which is not modulated by the second signal generator, the latter providing a modulated input.

An unwanted-response test is interesting and is very easily made. The receiver is tuned to the carrier wave of one signal generator, after which the frequency of a second signal generator connected to the radio receiver through a dummy antenna is varied through a wide range. The unwanted response due to input from the variable-signal generator reaching the grid of the first detector is then noted for this position of the tuning dial on its calibrated scale.[1]

7. Determination of Power-detection Characteristic. *Definition of Power Detection.*—By power detection is meant the operation of the detector tube on input voltages which will cause the device to function on the straight portion of its dynamic characteristic. It has long been realized that the response of modulated waves would be considerably improved in fidelity of tone if a detector possessing linear characteristics were used, but such a device has not been made commercially available until quite recently.[2] Most receiving sets are now designed for *power*

[1] *Proc. I.R.E.*, Vol. 23, p. 1164, October, 1935.

[2] For discussion of linear rectifier and also the necessary input voltage to obtain linear operation, see Stuart Ballantine, "Detection at High Signal Voltages," *Proc. I.R.E.*, Vol. 17, pp. 1163–1177.

detection, so it is sometimes desirable to test them for this characteristic of operation. If the dynamic characteristic of the detector tube is as represented in Fig. 13, a radio-frequency e.m.f. modulated to about 30 per cent may be impressed on its input, with the result that the output ΔI_p in the detector-tube plate circuit will give an undistorted reproduction of the modulating envelope. Without going into the details of a quantitative analysis, it will be realized from a study of Fig. 13 that this assumption is warranted. Considering the figure, it should be

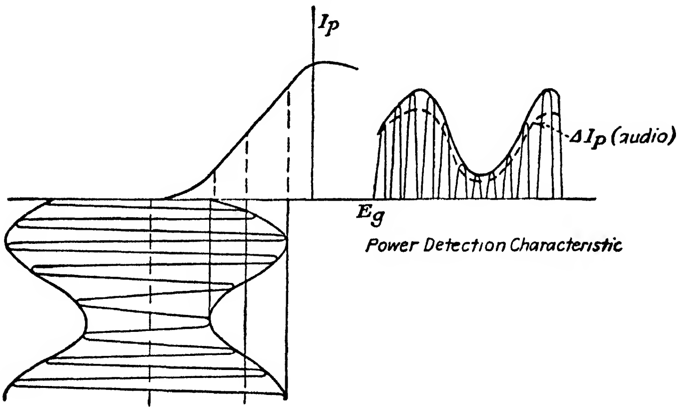


FIG. 13.—Power-detection characteristic showing limits of allowable modulation.

noted that if the modulation is 100 per cent, or if the maximum amplitude of the input e.m.f. is small compared to that shown, the usual distortion will occur due to operation of the detector on the curved portion of the characteristic. With automatic volume control provided, modulation into the curved portion is usually prevented.

Method of Measurement of Characteristic.—A very complete paper on the features, analysis, and measurement of power detection is published by Stuart Ballantine,[1] in which he describes a method of measuring the input to the detector necessary to keep its performance within the limits of no distortion. Briefly, it consists of impressing upon the detector input a modulated radio-frequency e.m.f. and measuring the distortion ratio on the output voltage as the input voltage is varied. An average of some 30 or 40 per cent modulation may be selected for the

[1] See footnote preceding.

measurement. The input voltage may be measured by some suitable means described in Chap. VI. The distortion of the audio-frequency output of the detector from the wave shape of the modulating potential may be obtained by means of harmonic analysis or by a visual approximate comparison. The measurement of wave form is dealt with in Chap. VII, and some of the methods there described may be used. Ballantine describes the measurement of a factor that he has termed the *distortion ratio*, with the aid of Belfils' bridge. He defines the distortion ratio in the following way: The effective value of the audio-frequency component of the detector output voltage is

$$E = \sqrt{E_1^2 + E_2^2 + E_3^2 + \cdots},$$

where E_1, E_2, E_3, etc., are the effective values of the fundamental and harmonic components, respectively. If the fundamental is removed from the output voltage by the use of Befils' bridge, the voltmeter across the bridge diagonals will give a reading that is the effective value of the residue of harmonics of this fundamental and is

$$E_h = \sqrt{E_2^2 + E_3^2 + \cdots}.$$

The distortion ratio, then, is the ratio

$$\sqrt{\frac{E_2^2 + E_3^2 + \cdots}{E_1^2}} = \sqrt{\frac{\Sigma E_n^2}{E_1^2}}. \tag{8}$$

Usually, E_1^2 is much larger than ΣE_n^2, especially at the point where distortion is just noticeable, so that E equals E_1, nearly. If not, E_1 may be determined by solving from the expressions given above. When the input voltage is of such a value that the distortion ratio is zero or nearly so, the detector tube is working on the straight portion of its dynamic characteristic. Then the determination of the output voltage of the radio-frequency amplifier of the receiving set should be made experimentally in order to determine whether or not the detector tube is receiving sufficient input e.m.f. for power detection. This test may be carried out at different modulating frequencies covering the useful modulating range, and a complete performance characteristic of the detector may be thus obtained. Among the useful curves of power-detection performance of a tube are a curve of the audio-output fundamental versus the radio-frequency carrier voltage (to show the point of overloading of the detector) and a curve of distortion ratio plotted against radio-frequency carrier voltage for different degrees of modulation. Diodes function as high signal voltage detectors in modern receivers. Diode efficiency is defined as the ratio of output voltage developed across the diode load resistor to the envelope portion of the modulated input voltage to the diode.[1] These may be measured with peak and r.m.s. tube-voltmeters (Chap. VI, Arts. 2, 3).

8. Piezoelectric Measurements. *Measurement of Frequency and Frequency Variation of Piezoelectric Crystals.*—Among the most important measurements that should be made of piezoelectric crystals is that of accurately measuring the fundamental frequency of the piezoelectric-driven oscillator. Since quartz crystals maintain extremely constant frequency, it is necessary to use a precision method of frequency measurement. Some crystals have three fundamental frequencies which are sometimes very close together, and the lowest one, being the most prominent or the one to which the crystal will resonate most vigorously, is present when the piezoelectric crystal is being used to drive an oscillator. The fundamental frequencies sometimes change when the crystal is disturbed mechanically. Such a crystal is regarded as defective and should be discarded if a test shows that it will change its frequency when turned from its nominal position with plates horizontal to another position. In the succeeding discussions, it will be assumed that the reader is familiar with the cutting, mounting, and other features of piezoelectric technique; for the benefit of those who are not familiar with this particular subject, it is discussed at some length in the literature.[2] One of the precision methods of Chap. II must be used to accurately measure a fundamental frequency at which a quartz crystal is driving an oscillator, but for quick, fairly accurate measurements the heterodyne-frequency measurement is warranted.

The variation of frequency is of prime interest and importance. Some special designs of piezoelectric crystal shapes, such as the ring shapes, have zero-temperature coefficient; most plain-cut crystals have either a positive or a negative coefficient of frequency change with temperature. An excellent paper on the subject of the features of the differently cut quartz-crystal

[1] See TERMAN, F. E., "Radio Engineering," 2d ed., pp. 425-426; also Fig. 5, p. 346, and footnote 1, p. 347.

[2] See HUND, AUGUST, *Proc. I.R.E.*, Vol. 14, No. 4, p. 447.

blanks, and on the temperature-coefficient feature on each, is given by Lack.[1] Two types of frequency variation of a piezoelectric oscillator with temperature are considered. One is the variation for a temperature change of 0.1 to 0.2°C. This small variation will be discussed first, since quartz crystals for frequency control of oscillators are usually placed in a temperature-controlled space wherein a thermostat is used to operate on a temperature change of 0.1 to 0.2°C. The usual perpendicular- or *X*-cut crystals have a negative temperature coefficient of 10 to 25 parts per million per degree centigrade; therefore, it will be expected that for a change of 0.1°C., the frequency would change up to 2.5 cycles per second in a crystal oscillator working at 1,000 kc., or the change would be as much as 25 cycles for 1° change in temperature. The new *A*-cut and *AT*-cut crystals have a temperature coefficient of less than three parts per million per degree centigrade. If the crystal temperature can be accurately changed by such a small amount, the frequency change could be obtained by counting visual beats between the crystal oscillator and a heterodyning circuit driver, as suggested by Hund.[2] However, it is more practical to make the measurement of frequency change when the temperature is changed 5 or even 10°C. and to assume that the frequency change is proportional for smaller increments of temperature variation (this is not always true for the special-cut crystals described by Lack in the paper referred to in a previous footnote). When a certain increment of temperature change changes the frequency 250 cycles or so, it is best to measure the frequency change instead of the absolute value of frequency. This may be done in several ways based more or less upon the setup shown in Fig. 14. At the initial temperature, a constant-frequency circuit of preferably the *high-C* type[3] is adjusted to beat on the piezo-oscillator frequency with a beat note of about 500 cycles. The frequency stability of the oscillator *C.D.* must be frequently checked against secondary standard. Methods of doing this are described in Chap. II. A 500-cycle tuning fork is used to set the beat note at very nearly this value by audible comparison, by obtaining the condition of zero beating of the two 500-cycle tones. Next, a beat-frequency oscillator is adjusted to 500 cycles output by the

[1] *Proc. I.R.E.*, Vol. 17, No. 7, pp. 1122–1141, 1929.

[2] *Loc. cit.*

[3] The electron-coupled oscillator of Fig. 3, Chap. II, may be used.

same comparison method. The beat-frequency oscillator is shielded from the other oscillator. The beats are compared by listening simultaneously to the two telephone receivers, one connected to the beat-frequency oscillator, and the other connected in the plate circuit of the piezo oscillator. The beat-frequency oscillator should have a calibration curve for its frequency-varying condenser dial, and the above-described first adjustment to a tuning fork checks a point on the calibration curve. The other calibration points are comparatively fairly

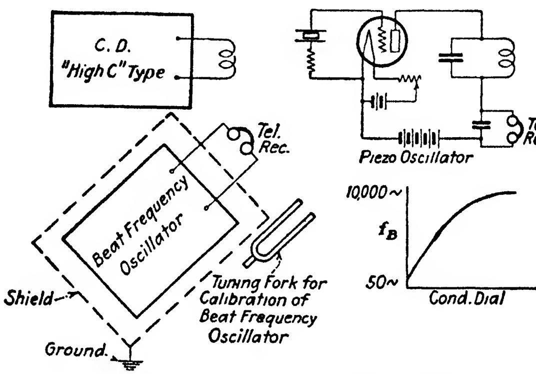


Fig. 14.—Apparatus and circuits for measuring changes in piezo-oscillator frequency.

correct. When these are permanently adjusted, the temperature of the piezo-oscillator crystal is changed 5°C. or so, and its change in frequency will result in a change in the pitch of the beat note heard in the telephone receiver in the piezo-oscillator plate circuit. The beat-frequency oscillator is then readjusted to give zero audio beat with the new beat note, and the change in the frequency is determined from the difference in the new setting of the beat-frequency oscillator and 500 cycles. If the frequency of the piezo oscillator is f_1 at 25°C., and the frequency of the fork is 500 cycles, the frequency of the heterodyning circuit driver may be $f_1 - 500$. When the temperature of the crystal is increased to 30°C., the frequency of the piezo oscillator is f_2, and the beat frequency is then $f_2 - (f_1 - 500)$. Now, let

$$f_2 - (f_1 - 500) = f',$$

where f' is the new reading on the beat-frequency oscillator calibration curve when the latter has been readjusted for the new zero-beat tone. Hence the change in the frequency of the piezo oscillator is

$$f_2 - f_1 = f' - 500.$$

If the temperature coefficient is negative, this expression gives a negative result, as expected. The calibration curve of the beat-frequency-curve oscillator is shown in Fig. 14. This oscillator

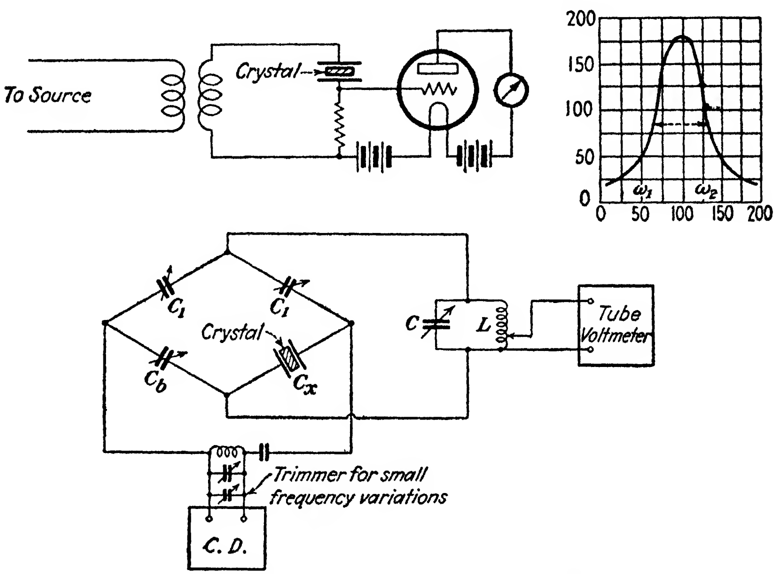


FIG. 15.—Circuits for measuring the damping of a piezoelectric crystal.

must be shielded so that one of its component frequencies does beat on the other oscillators. It is appreciated that if the calibration curve of the beat-frequency oscillator is within 1 per cent, this represents also the accuracy of the measurement of the change of the piezo-oscillator frequency.

Measurement of Damping of a Piezoelectric Crystal; Resonance Curves.—In a paper on the resonance characteristics and nature of piezoelectric crystals, Meissner describes a method of determining the damping of the crystal by obtaining a resonance curve.[1] He suggests two possible setups for obtaining the resonance-curve data. One is shown in Fig. 15. If the vacuum

[1] *Proc. I.R.E.*, Vol. 15, No. 4, pp. 281–296; see especially p. 283.

tube is used, it is necessary to eliminate the steady plate current with a compensating shunt such as is used for vacuum-tube voltmeters, etc. The deflections are plotted for various values of frequency f, or for values of

$$\omega = 2\pi f,$$

as the frequency is varied through the value for resonance with the crystal; and the resonance curve, as shown in Fig. 15 is obtained. The peak occurs where the exciting frequency is equal or nearly equal to one of the fundamental frequencies of the crystal with which the crystal resonates most vigorously. The capacitance bridge circuit shown in Fig. 15 is also convenient for obtaining resonance curves for a crystal. The capacitances are adjusted for balance of the bridge at a frequency removed from the crystal-resonance value, and at such frequencies the crystal acts practically as a capacitance. As resonance within the crystal is approached, the crystal-reactance change and loss-resistance increase destroy the bridge balance to a greater and greater extent, and the unbalanced e.m.f. component across the bridge diagonals (and hence that applied to the tube voltmeter) vary accordingly. The tuned circuit LC is sometimes used to increase the sensitivity, and it is kept at resonance for all applied test frequencies so as to present a high noninductive impedance to the crystal bridge. The resonance of the piezo crystal is very sharp, and the frequencies must be measured by precision methods to obtain values on the steeper portions of the resonance curve. It is convenient simply to measure the change in frequency of the variable-frequency circuit driver used to obtain the resonance curve. This is done as described just previously, and the measurement setup of Fig. 14 is used, except that the variable-frequency circuit driver mentioned is used in place of the piezo oscillator of Fig. 14.

The *damping* of the piezoelectric crystal may be taken as the logarithmic decrement, as obtained by the resonance curve; but it is customary to use the constant term of the exponent of the damping term

$$\epsilon^{-\frac{Rt}{2L}}$$

in the expression for a *damped-free oscillation*. The term $R/2L$ is generally called the *damping*.[1] If the resonance curve of

[1] MORECROFT, J. H., "Principles of Radio Communication," 2d ed., pp. 247–249.

Fig. 15 is obtained by reading the meter deflections, these vary with the current squared; hence the ordinates of the resonance curve shown are proportional to the current squared flowing through the piezo crystal. A term known as *sharpness of resonance* is generally used as a figure of merit of a tuned circuit containing R, L, and C in series with the e.m.f. induced magnetically in L, the inductance. It may be shown that defining this sharpness of resonance, Q, as $\omega L/R$ results in an expression

$$\frac{\omega_r L}{R} = \frac{\omega_r}{(\omega_2 - \omega_1)}\sqrt{\frac{(I_r)^2 - (I_1)^2}{I_1^2}}, \qquad (9)$$

where I_r is the current at resonance, and I_1 and I_2 are currents at radian frequencies ω_1 and ω_2, respectively.[1] Then if two values of (ω) are selected, where $I_1^2 = I_r^2/2$, and the radical term of Eq. (9) is unity, and the expression

$$\frac{L}{R} = \frac{\omega_1 + \omega_2}{(\omega_2^2 - \omega_1^2)} = \frac{1}{\omega_2 - \omega_1}, \text{ as in Eq. (9).}$$

From this, the damping Δ is

$$\Delta = \frac{R}{2L} = \frac{\omega_2 - \omega_1}{2}.$$

Hence, for the damping measurement it is necessary only to observe I_r^2 and then obtain ω_1 and ω_2 as indicated above. The damping may be increased by exerting pressure on the metal plates between which the piezo crystal is placed. In case there is an air gap between the crystal and the metal plate opposite to any one of its faces, damping increases with the length of the air gap and for certain critical air-gap lengths increases enormously. D. W. Dye[2] reported this effect, and he also showed that when the damping is increased enormously for an air gap of 0.72 mm. for a certain crystal, the same increase is again obtained, at 1.44 mm. air-gap length. This, he pointed out, occurs every time that the air-gap length is an integral multiple of one-half wave length of the supersonic air wave corresponding to the frequency of the piezoelectric resonator.

[1] This is proved in several texts; see H. A. Brown, "Radio Frequency Electrical Measurements," 1st ed., p. 330.

[2] *Proc. Phys. Soc. London*, Vol. 38, pp. 399–458, 1926; see p. 427 for discussion of this effect.

Damping of a piezo quartz crystal may also be measured by observing the rate of decay of oscillations of the crystal when the driving e.m.f. is removed, as was done by Van Dyke.[1] The logarithmic decrement for damped or decaying oscillations is

$$\delta = \log_e \frac{A_1}{A_2} = \frac{1}{n} \log_e \frac{A_1}{A_n}, \tag{9a}$$

where A_1, A_2, and A_n are successive positive amplitudes, as indicated by the order of the subscripts. In the work referred to, photographs were made of the amplitudes at rapid intervals with the aid of a cathode-ray oscillograph in which the sensitive film is placed in the evacuated chamber so as to obtain photographic records during very short intervals. Hund[2] suggests obtaining the comparative deflections by magnetically coupling a ballistic galvanometer and rectifier and suitable key in series to the oscillating crystal and observing the deflections at successive known time intervals of closing the galvanometer key after the crystal driving e.m.f. has been removed. More detailed instructions for carrying out this measurement are found in the book referred to.

Piezo quartz crystals have a very high Q factor[3] and hence very low damping and logarithmic decrement. The latter may vary from 10^{-4} to 0.54×10^{-5}, this latter value being found for a specially prepared 67.5-kc. quartz crystal by Van Dyke. This means that the decaying oscillations may last from ½ to as much as 10 sec. In an interval t_n from amplitude A_1 to A_n, the number of oscillations n may be easily found from the frequency of oscillation. Hence, measurements of comparative amplitudes ½ sec. or more apart give a ratio A_1/A_n which is large and hence can be obtained with fair precision in a carefully controlled procedure.

Measurement of Motional Admittances.—The motional admittance of a piezoelectric crystal is analogous to that of a telephone-receiver diaphragm and is defined as

$$Y_m = \frac{(2b_e)^2}{Z_m}, \tag{10}$$

[1] *Proc. I.R.E.*, Vol. 23, No. 4, 1935; see pp. 391, 392, especially.

[2] "High Frequency Measurements," p. 431.

[3] Direct measurement of crystal Q is described in *Bell System Tech. Jour.*, Vol. 13, p. 431, July, 1934. Also *Elec. Eng.*, Vol. 57, No. 2, p. 105, February, 1938.

where $2b_\epsilon$ is the stress per unit potential difference between the faces, and Z_m is the mechanical impedance, as pointed out by Y. Watanabe in his paper on the determination of the motional-admittance circle diagram and of the equivalent electrical network of the piezo crystal in its metal mounting plates.[1] His methods of measuring these quantities will be briefly explained with equations and diagrams taken from his paper. For the

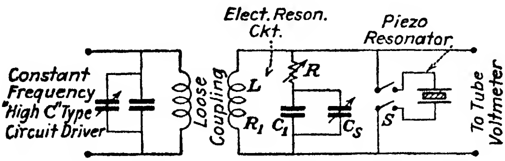


FIG. 16.—Circuit for measuring motional admittance of a piezo crystal. (*After Watanabe.*)

first method, the apparatus and circuit arrangement are as in Fig. 16. The piezo crystal acts as a resonator, and its electrical admittance is greatest when the frequency of the circuit driver is the same as a natural frequency of the crystal. The circuit driver, which must be one of very constant frequency, is set in operation at a desired frequency setting. Switch S is closed, and the precision-standard variable condenser C_S is varied until the tube voltmeter shows maximum deflection, indicating that the circuit is in resonance with the source. C_1 is a known variable or fixed capacitance of suitable size, and R is standard variable resistance. After this first adjustment, S is opened, and C_S is varied by an amount ΔC_S to regain resonance as indicated by the tube voltmeter, and the variable resistance R is adjusted so as to obtain the same deflection as formerly. In the paper referred to, Watanabe points out that the equivalent conductance of the piezoelectric resonator, for the frequency at which the test was made, is derived by equating expressions for total impedance of the parallel combinations of resonator and network R, C_S, C_1. From the expressions of this impedance, when switch S is closed and open, it may be shown that the equivalent conductance of the piezo resonator is

$$\begin{aligned} g_m &= R\omega^2(C_1 + C_S + \Delta C_S)^2 \\ &= R\omega^2(C_1 + C_S)^2 \text{ (nearly),} \end{aligned}$$

where $\omega = 2\pi f$, and f is the testing frequency. ΔC_S is usually

[1] *Proc. I.R.E.*, Vol. 18, No. 4, pp. 695–717, 1930.

small compared to $C_1 + C_S$. The equivalent electrical network of the piezo-crystal resonator, with its air gap, is shown in Fig. 17, Diagram A, where C_a is the capacitance of the air gap, and C_d is the geometrical capacitance of the space occupied by the crystal. C_m, L_m, and R_m constitute the effective equivalent impedance of the piezo crystal itself. Referring to the above-described experimental procedure, it will be realized that ΔC_S replaces the total equivalent capacitance of the resonator circuit complete. This may be expressed as

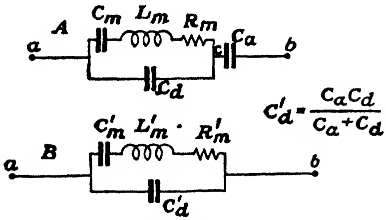


FIG. 17.—Equivalent electrical networks of a piezo crystal.

$$\Delta C_S = \frac{(C_d + C_m)C_a}{C_a + C_d + C_m}.$$

Values of g_m may be obtained by this method, as ω is varied, and for the value ω_0, which is that for resonance with the piezo crystal,

$$R_m = \frac{1}{g_{m0}},$$

$$L_m = \frac{R_m}{2\Delta} = \frac{R_m}{\omega_2 - \omega_1},$$

since Δ is the damping $R_m/2L$, as explained previously.

$$C_m = \frac{1}{\omega_0^2 L_m}. \qquad (11)$$

$1/R_m = g_{m0}$ indicates that the piezo-resonator equivalent conductance is at the frequency of resonance $f_0 = \omega_0/2\pi$.

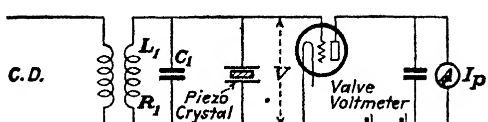


FIG. 18.—Circuit for obtaining the inverted resonance curve of a piezo crystal.

Figure 18 shows the arrangement and connections for apparatus used in obtaining an inverted resonance curve, or crevice, for the piezo-crystal resonator. L_1 and R_1 of the input coil are previously measured, and C_1 is a fixed capacitance, large compared to C_m of the crystal. The frequency of the circuit driver at

C.D. is varied, and readings of the tube voltmeter give data with which to plot the inverted resonance curve *A* of Fig. 19. In this case, the minimum deflection is obtained when the frequency $f = f_0$, that for resonance with the crystal, because the admittance of the latter is very much increased at resonance, as shown in the previous measurement described. The increase of admittance near resonance means that more current is taken through this branch of the circuit, resulting in a lowered terminal voltage across L_1 (Fig. 18). In the paper cited above, the author points out that if *V* is the voltage across the crystal, as measured by the tube voltmeter, then, under certain assumptions, an expression may be written of the form

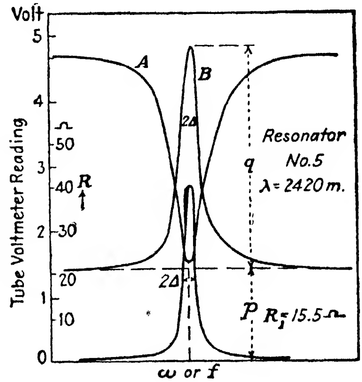


FIG. 19.—Resonance curves and inverted resonance curve (or "crevice") of piezo crystal.

$$\frac{1}{V} = j\frac{\omega_0 C_1 R_1}{E} + j\frac{1}{\omega_0 C_1 E}\left[\frac{1}{R_m + j\left(\omega L_m - \dfrac{1}{\omega C_m}\right)}\right],$$

where *E* is the voltage induced in coil *L*. If the reciprocal of curve *A* of Fig. 19 is plotted, curve *B* will result, and the equation for this latter curve is given by the foregoing expression. It contains a constant term and terms that depend on ω; the constant term of the equation is denoted by ordinate *p* of the line asymptotic to curve *B*; and from the preceding equation,

$$p = \frac{\omega_0 C_1 R_1}{E};$$

the second term represents the ordinates above the height *p*, and at the point of resonance its maximum ordinate occurs, denoted by *q*; from the preceding equation,

$$q = \frac{1}{\omega_0 C_1 R_m E},$$

since

$$\omega_0 L_m = \frac{1}{\omega_0 C_m}$$

at resonance. Also, it is readily understood that the maximum ordinate of curve B from the zero line of ordinates is $1/V_0$, where V_0 is measured by the tube voltmeter; for $\omega_0 = 2\pi f_0$. From the expression given above for q, it is readily shown that

$$R_m = \frac{p}{\omega_0^2 C_1^2 R_1 q},$$

where p and q are known from the relations just given, and all other quantities are also known. Again, as in the previous method, L_m and C_m are found by substituting in Eq. (11) given in the description of the previous method. The damping is readily obtained from resonance curve B (Fig. 19).

Other Piezoelectric Measurements.—The so-called characteristic curves for a piezoelectric resonator are of much interest in connection with a theoretical and experimental investigation of piezo oscillators. The significance and method of obtaining these curves were first described by W. G. Cady in his pioneer work on the subject.[1] He describes the use of a circuit shown in Fig. 20, taken from his paper.

It is felt that it is not within the scope of this book to go into the detailed description of the use of Cady's circuit for obtaining data for studying crystal characteristics or into the analysis of the impedance circles that may be obtained. Dye also used the circuit of Fig. 20 for his researches on the validity of the impedance circle[2] and for his measurements of resonant-frequency behavior. He also measured a quantity termed σ_1:

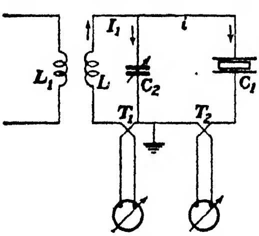


FIG. 20.—Circuit for obtaining characteristic curves of piezo crystal. (*After Cady.*)

$$\sigma_1 = \frac{I_1^2}{I_0^2},$$

where I_1 is the current shown in Fig. 20 when the piezo crystal is in resonance with the supply frequency, I_0 is the current when the crystal is removed, and C_2 is changed to regain resonance. This manifestation is very similar to that described previously for the measurement of motional conductance g_m.

Cady describes in his paper, previously referred to, a method of measuring the stabilizing effect of a piezo quartz crystal.

[1] *Proc. I.R.E.*, Vol. 10, No. 2, pp. 83–114, 1922.

[2] See footnote 3, p. 371.

The crystal in its mounting is placed in parallel with the variable condenser c of a vacuum-tube oscillator. To increase the oscillator frequency when the crystal is not present, the capacitance of C is decreased, and the frequency change is along a curve of the form

$$f^2 = \frac{K}{C}.$$

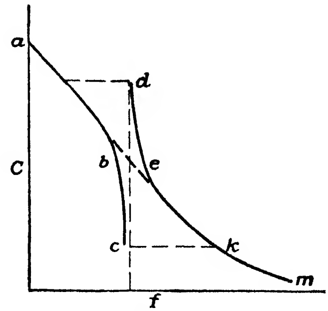


FIG. 21.

But when the piezo crystal is connected as described, the results are as shown in Fig. 21, when the frequency approaches the value for crystal resonance. As C is decreased, the curve departs from the usual form and follows bc. At c, the frequency jumps suddenly over to K and then follows km. If the beginning at value m or k, C is increased, the frequency of the oscillator follows kcd, after which it jumps suddenly to the left, as shown. This is exactly analogous to the well-known drag-loop effect between two closely coupled circuits. It will be noted that the stabilizing effect is in the near neighborhood of the quartz-crystal resonant frequency. A complete bibliography of piezoelectric investigations is given by Cady in the *Proceedings of the Institute of Radio Engineers*, Vol. 16, No. 4, pages 521 to 535, 1928.

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