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KINEMATICS OF MACHINES

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BY THE LATE

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FOURTH EDITION

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PREFACE TO THE FOURTH EDITION

The original object in writing this book, namely, to provide in concise form the necessary material for an elementary course, has been kept in view in the present edition. Hence the length of the book has been only slightly increased.

Some changes have been introduced to bring the material into better accord with present practice; others, for the sake of lucidity. A few sections have been altered in such a way as to place less dependence on the students' prior knowledge.

The present edition will be found to contain a considerable addition to the number of problems. The chapter on gears has been revised to conform with recent standards. Brief descriptions of some mechanisms of kinematic interest and present practical importance have been added.

The author wishes to express his appreciation of many useful suggestions made by Professors L. J. Bradford, M. S. Gjesdahl, and C. G. Vandegrift of the Pennsylvania State College.

G. L. Guillet

May 29, 1940

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KINEMATICS OF MACHINES

CHAPTER I

GENERAL CONSIDERATIONS

1. **Kinematics of Machines** is that portion of the study of machines which deals with the motions of the parts of which they are composed.

In general, the design of a machine must be carried out in four stages: first, the purpose for which the machine is to be used must be considered and the necessary motions must be studied; second, some device must be selected which will produce the required motions; third, the forces acting in the members must be calculated; and finally, a choice of materials must be made and the parts properly proportioned to withstand the forces determined above. The final form is necessarily influenced by many factors whose consideration is outside the province of this book, in which, as far as possible, we shall confine our attention to the study of motions in machinery. The four stages just mentioned, comprising the subject of Machine Design, are, however, interdependent to such an extent that an entirely separate consideration of any one of them would be useless from the standpoint of practical design. Thus, theoretical forms derived from kinematic considerations alone must nearly always be modified on application to real machines, for the reason that account must be taken of other factors, such as strength, wearing qualities, ease of production, etc.

2. **A Machine** is a combination of parts of resistant materials having definite motions and capable of transmitting or transforming energy. A machine must always be supplied with energy from an external source. Its usefulness consists in its ability to alter

the energy supplied so as to render it available for the accomplishment of a desired service.

A steam engine transforms the pressure energy of the steam into mechanical work which is delivered to the crank shaft. This machine, therefore, transforms one kind of energy into another.

A transmission gear may be used to connect two shafts which are required to rotate at different rates. Its function will then be to alter the mechanical work supplied to it, as regards the speed of rotation and twisting moment, the result being to put the energy into a form more suitable for some specific purpose.

Resistant Materials are those that do not easily become distorted or change their physical form when forces are applied.

3. **A Mechanism** is a combination of pieces of resistant materials whose parts have constrained relative motions. A machine is composed of one or more mechanisms. When we speak of a mechanism, we think of a device that will produce certain mechanical movements and we set aside the question as to whether it is capable of doing useful work. A mechanism may or may not be able to transmit an appreciable amount of energy; a machine must do so. The latter is, therefore, a practical development of the former.

A working model of any machine, the works of a watch, and the moving parts of an engineering instrument are all termed mechanisms because in these cases the energy transmitted is very small, just enough to overcome friction, and the motions produced are the important consideration.

A **Structure** is a combination of pieces of resistant material capable of carrying loads or transmitting forces, but having no relative motion of its parts.

A machine frame may be built of several metal parts rigidly fastened together so as to allow no movement to take place between different sections. It is, therefore, a structure. Other examples include bridges, buildings, etc.

4. **A Link** is a part of a machine or mechanism connecting other parts which have motion relative to it. A link may serve as a support, guide other links, transmit motion, or function in all three ways.

Thus, in a steam engine, the connecting rod, crank, crosshead, and frame are links, since each functions in at least one of these ways. A single link may be composed of several pieces of material, provided that the several portions are so fastened together as to move as a unit; the connecting rod consists of the rod, brasses, shims, adjusting wedge, bolts, etc., all of which are rigidly connected and make up one link.

In connection with general mechanical work, the term " link " is applied to a slotted bar, as shown in Figs. 1–1 and 1–2, or to a section of an ordinary chain. In Kinematics it is used in a more general way, as noted above.

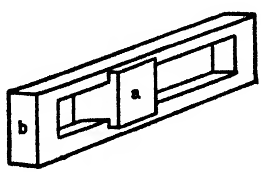


FIG. 1–1
Slider and Straight Guide.

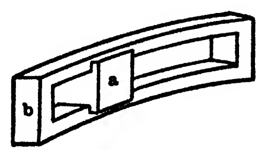


FIG. 1–2
Slider and Curved Guide.

Rigid Links, such as the steam-engine parts just mentioned, are capable of transmitting either a thrust or a pull. To this class belong most of the metal parts of machines. There are, however, many examples of **Flexible** Links, which in general are so constituted as to offer resistance in one manner only. Thus, **Tension Links,** such as ropes, belts, and chains, will transmit a pull but not a thrust, while **Compression** or **Pressure** Links, such as the water in a hydraulic accumulator and pump system, or the oil in a hydraulic braking system on a car, are capable of carrying only a thrust.

When a number of links are connected to one another in such a way as to allow motion to take place in the combination, it is called a **Kinematic Chain.** A kinematic chain is not necessarily a mechanism; it becomes one when so constructed as to allow constrained relative motion among its parts.

5. Pairs of Elements. — Links may make contact with one another in various ways. Contact may take place over a surface, along a line, or at a point. Those portions of two links making contact are known as a pair of elements. **Sliding Pairs** are those in which there is relative motion of the points of contact. (See *a* and *b* in Figs. 1–1 and 1–2.)

When two bodies are so connected that one is constrained to rotate about a fixed axis passing through the other, the contact surfaces are known as a **Turning Pair.** The pin joints of Figs. 1–3 and 1–4 belong to this class. Turning pairs in practical machines generally have sliding contact and it would therefore seem that they might be classified as sliding pairs. Theoretically, however, a turning pair requires line contact only (i.e., along the axis of rotation), so that no sliding action is kinematically necessary.

Rolling Pairs are those in which there is no relative motion of the points of contact. Ball bearings and roller bearings contain examples of this kind of pairing. In Fig. 1–11 is shown a section of a tapered roller bearing in which rolling pairs are formed by the contact points of rollers and races.

Lower Pairing is obtained when two links are in contact over finite surfaces, the two elements of the pair being geometrically

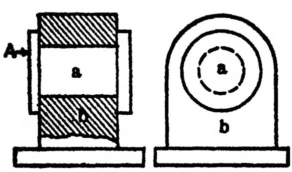


FIG. 1–3
Pin Joint.

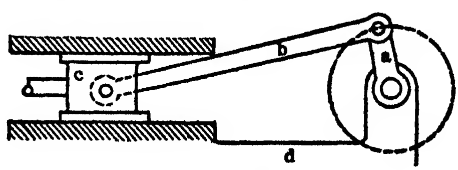


FIG. 1–4
Direct Acting Engine Mechanism.

similar. A revolving shaft fitted with plain bearings, a threaded bolt and nut, an engine crosshead and its guides — these will all serve as illustrations of lower pairing. (See Figs. 1–3 and 1–4.) This is the most common form of pairing in machinery, because it has the practical advantage of large wearing surfaces.

Higher Pairing exists when two links make contact along a line or at a point. Here the contact surfaces are not similar in form. Higher pairing is found in gears, the teeth making contact at points or along lines; also in ball and roller bearings, the rollers and balls making line and point contact, respectively, with the races.

Higher pairs with rolling contact have small friction losses, and the wear on the contacts is not severe.

It should be noted that in practical machines, since the materials employed possess a little elasticity, some distortion of the contact surfaces takes place when pressure is applied. Thus, when theoretically line or point contact should occur, actually we always have some surface contact. This condition, combined

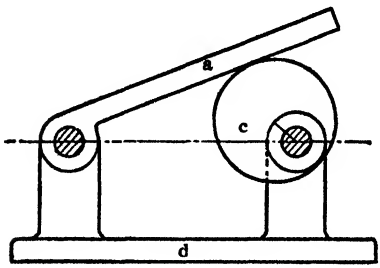


FIG. 1–5*a*
Cam Mechanism with Higher Pairing.

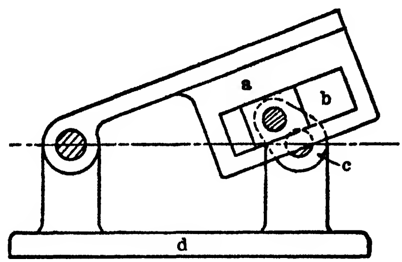


FIG. 1–5*b*
Mechanism with Lower Pairing derived from that of Fig. 1–5*a*.

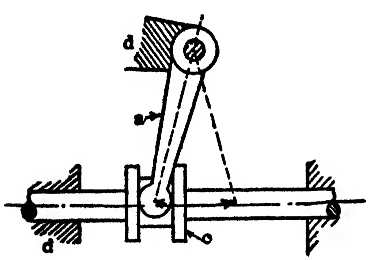


FIG. 1–6*a*
Valve Motion of Steam Pump with Higher Pairing.

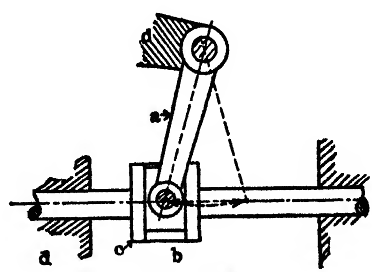


FIG. 1–6*b*
Valve Motion of Steam Pump with Lower Pairing only.

with the use of lubricants, makes it possible to use sliding pairs with line or point contact in cases where the sliding velocity is low.

Alteration from higher to lower pairing can be accomplished without any change in the motion of the original links. Figure

1–5*a* shows a circular cam driving an oscillating follower with line contact. Figure 1–5*b* is an equivalent mechanism with lower pairing. The motions of links denoted by the same letters are identical in the two mechanisms. Figure 1–6*a* shows the mechanism used to drive the valves of a Duplex Steam Pump, higher pairing again being in evidence between one pair of links. Figure 1–6*b* shows this mechanism changed to one with lower pairing only.

In both of the cases just mentioned, lower pairing has been brought about by the addition of one link. This is the method which must usually be adopted.

6. Link Types. — Links forming parts of machines move in such a variety of ways that no general classification as regards motion is possible. The following are three types in common use:

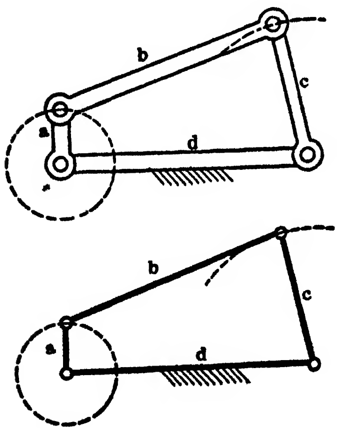


FIG. 1–7
Quadric Crank Mechanism.

A **Crank** is a link in the form of a rod or bar, which executes complete rotations about a fixed center. (See *a*, Fig. 1–7.)

A **Lever** is a link in the form of a rod or bar which oscillates through an angle, reversing its sense of rotation at certain intervals. (See *c*, Fig. 1–7.)

A **Slider** is a link in the form of a rod, block, or slotted bar, which slides over the surface of a second link. It may move in a straight line, as does the crosshead of the steam engine of Fig. 1–4, or it may move in a curve, as does the block in Fig. 1–2.

7. Motion can be defined as a change of position. Motion is always a relative term; that is, we cannot conceive of any motion of a body except by reference to another body. It is usual to regard the earth as a fixed body, and we therefore speak of the **Absolute Motion** of a body when we mean its motion relative to the earth. We apply the term **Relative Motion** to the movement of one body relative to another moving body, the earth again being considered as stationary. Strictly speaking, we have relative motion in both cases, and the above designations are only a conventional means of distinguishing them.

Constrained Motion. — A body is said to have constrained motion when it is so guided by contact with other bodies, or by external forces, that any point on it is obliged to move in a definite path. Any link in a mechanism has constrained motion.

Partial Constraint exists when the movement of a body is only restrained in certain directions, or so as to take place within certain boundaries. For example, considering the kinematic chain shown in Fig. 1–8, consisting of five links with turning pairs, it is evident that the links *a* and *b* are only partially constrained, since a point, such as *A*, may move anywhere on a surface whose boundaries are fixed by the lengths of the links. If, however, we add to the chain another link, such as *f*, with pin joints, then constraint becomes complete, since any point on any link we may select will have a definite path of motion. The chain is now a mechanism.

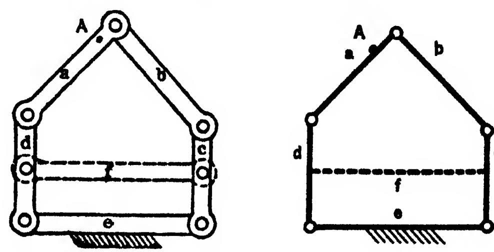


Fig. 1–8
Compound Mechanism.

The following common classes of kinematic chains have complete constraint, provided that the pairs are so formed as to maintain contact:

(1) Chains containing four links, each link bearing elements of two lower pairs. Figures 1–4, 1–5*b*, 1–6*b*, and 1–7 illustrate this class.

(2) Chains with three links, each link containing elements of two pairs, higher pairing being used between two links and lower pairing for the other two connections. Figures 1–5*a* and 1–6*a* show two of these mechanisms.

These chains are known as **Simple Mechanisms.**

There are many mechanisms known as **Compound Mechanisms,** which have more than four links and in which certain of the links contain elements of more than two pairs. Figure 1–8, when link *f* has been added, illustrates a mechanism of this kind. A compound mechanism is frequently a combination of two or more simple mechanisms.

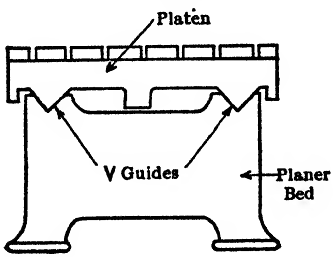


FIG. 1–9
Method of Constraining Motion of Planer Platen.

In many mechanisms constraint is not effected entirely by the form of the links, the action of gravity, spring pressure, centrifugal force, etc., being utilized. For example, the platen of a planer is held in contact and alignment with the bed by gravity, which prevents the V-shaped guides on its lower side from coming out of engagement with the corresponding grooves on the top of the bed. (See Fig. 1–9.) Also, in many cam mechanisms the follower is kept in contact with the cam by means of a spring.

8. Inversion of a Mechanism. — In any mechanism we have one link which is "fixed"; i.e., at rest relatively to the earth, or to the body on which it is mounted. Exactly the same system of links may often be rendered suitable for a different purpose if the link originally fixed is allowed to move, while some other link is held stationary. Thus, in Fig. 1–4, the ordinary engine mechanism, *d*, is the fixed link. By fixing the crank *a* and allowing *d* to move, we obtain a device which is used as a quick-return motion in certain machine tools. The latter mechanism is called an **Inversion** of the former one.

It is important to note that the inversion of a mechanism does not in any way alter the relative motion of the links which compose it. In Fig. 1–4, for example, no matter which link is fixed, the motion of *a* relative to *d* is that of rotation, and *c* always slides in a straight line on *d*.

9. Classification of Motions. — Most of the motions which occur in mechanisms fall into one of the following classes:

(*a*) **Plane Motion** of a body is obtained when all points in it move in parallel or coincident planes.

When studying plane motion we disregard the thickness of the links perpendicular to the plane of motion and speak of the center of rotation, instant center, etc., instead of the axis of rotation, instant axis, etc., which are the correct terms for the real bodies. This somewhat simplifies the treatment and does not detract from the value of the information obtained, since all points in any one perpendicular to the plane of motion move in an identical manner. Plane motion is common to all mechanisms shown in Figs. 1–1 to 1–9. Our diagrams may be simplified by drawing lines to represent links, thus obtaining what are called "skeleton diagrams." In Fig. 1–8 is shown the projection of a mechanism, also a skeleton diagram.

Rectilinear Motion is a form of plane motion in which all points in the body considered move in parallel straight lines. (See *c*, Fig. 1–4.)

(*b*) **Helical Motion** is executed when a body rotates about an axis and at the same time moves parallel with the same axis, the two motions bearing a fixed ratio to each other. The motion of a nut on a threaded bolt is a very common example. Any point on a body with this form of motion describes a curve called a **helix**. Figure 1–10 shows the projection of a helical curve on a plane parallel with the axis of rotation.

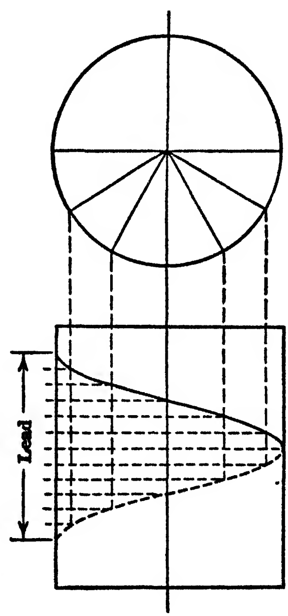


FIG. 1–10
Projection of a Helix.

The contact surfaces in helical motion are called a **Screw Pair**.

(*c*) **Spherical Motion.** — A body is said to have spherical motion when moving in such a way that any point in it remains at a constant distance from a fixed point. Any point on the body, therefore, moves on the surface of a sphere. Figure 1–11 shows a cross-section of a tapered roller bearing. Generally the inner race X revolves with the shaft Y, and the outer race Z is stationary. Any point, such as A, on one of the rollers moves on the surface of a sphere ALM, whose center is at the point O; hence the rollers have spherical motion. Other examples are found in certain ball bearings, and in bevel gears, universal joints, etc.

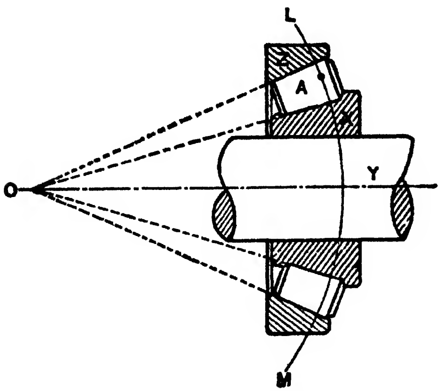


Fig. 1–11
An Example of Spherical Motion.

10. Determination of the Motion of a Body. — The motion of a body is studied by consideration of the motion of certain points on it. The number of points to be considered depends on whether the body can move in any manner, or whether its motion is limited to some special type, as, for example, plane motion, helical motion, etc.

In general, to determine the motion of a body completely, we must know the motion of three non-collinear points on it. This can easily be demonstrated as follows: If we take any body and fix three points on it, it is evident that no motion is possible unless these points lie in a straight line. Likewise, if we move each of the three points along a definite path in space, any other point on the body will also follow a definite path, and constraint is complete.

When a body has **Plane Motion**, by the same reasoning it will be seen that it is only necessary to control the motion of two points in order to secure complete constraint.

When the motion is **Rectilinear**, the motion of one point determines that of any other in the body, since all points on it have exactly the same motion.

QUESTIONS — CHAPTER I

1. Define the terms "machine" and "mechanism." Give one example of each.

2. Distinguish between a structure and a mechanism.

3. What is a link? What is meant by (*a*) a rigid link, (*b*) a flexible link, (*c*) a pressure link, (*d*) a tension link?

4. Define (*a*) pair of elements, (*b*) sliding pair, (*c*) turning pair, (*d*) rolling pair.

5. Explain why lower pairing is generally more desirable than higher pairing with sliding contact.

6. Distinguish between higher and lower pairing and give an example of each.

7. Give an example of how a linkage with higher pairing can be changed to one with lower pairing, and point out the nature of the alteration required.

8. What is a crank, a lever, a slider? Sketch mechanisms containing examples of each.

9. Explain what is meant by partial and complete constraint. To what extent is unconstrained motion possible?

10. Name three classes of kinematic chains in which the motion of the links is completely constrained.

11. What is (*a*) a simple chain, (*b*) a compound chain?

12. What is done when a mechanism is inverted? How does the act of inversion affect (*a*) the relative motion, (*b*) the absolute motion of the parts?

13. Define (*a*) motion, (*b*) plane motion, (*c*) rectilinear motion, (*d*) helical motion, (*e*) spherical motion. Give an example of each of the four varieties of motion just mentioned.

14. In order to determine completely the motion of a body, how many points on it must be considered, (*a*) in general, (*b*) when it has plane motion, (*c*) when it has rectilinear motion?

15. Mention two methods employed for securing closure of a mechanism by external forces, and illustrate each one.

CHAPTER II

DISPLACEMENT, VELOCITY, AND ACCELERATION

1. Definitions. — The **Displacement** of a body is its change of position with reference to a fixed point. Both direction and distance are necessarily stated in order to define completely the displacement of a point or body.

Velocity is the rate of change of position or displacement of a body. A body may change its position by translation through space or by angular movement. Thus it may have **linear** or **angular** velocity.

Linear Velocity is the rate of linear displacement of a point or body along its path of motion. It includes two factors, namely, speed and direction of motion. Linear velocity is often measured in feet per second, or in miles per hour, though other units are found more suitable in special cases. A linear velocity can always be represented graphically by a line, the direction showing the direction of movement and the length representing the magnitude of the velocity.

Angular Velocity is the rate of change of angular position of a body or line. In order to state it completely, the sense of rotation should be given. Angular velocity is commonly measured by the angle turned through per unit of time. The angle may be expressed in radians, degrees, revolutions, etc. Thus, "radians per minute," "degrees per second," and "revolutions per minute" are often used as units.

2. Relation between Angular and Linear Velocity. — Let B, Fig. 2–1, represent a point on a body which is rotating about O. O is therefore a fixed point, and B is moving, at the instant considered, in a direction perpendicular to the line OB, its linear velocity being V feet per second as indicated on the diagram.

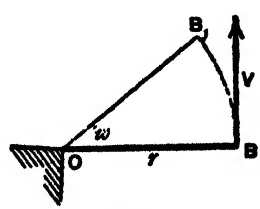


Fig. 2–1

Assuming that B continues to move at the same speed, after an interval of one second it will have reached the position B_1, the arc BB_1 having a length equal to V, by definition. The angle ω turned through by line OB during the same interval is measured in radians by

$$\frac{\text{Arc } BB_1}{OB} = \frac{V}{r}.$$

Therefore,

$$\omega = \frac{V}{r} \qquad \text{or} \qquad V = \omega r, \tag{2-1}$$

where ω = angular velocity of the body in radians per unit of time. We may, therefore, state the following law:

The linear velocity of a point on moving a body is equal to the angular velocity of the body multiplied by the distance of the point from the center of rotation.

3. Acceleration is the rate of change of velocity with respect to time. Since velocity may be either angular or linear, we likewise have both angular and linear acceleration.

Linear Acceleration is the rate of change of linear velocity. If a moving body or point has a linear velocity at a certain instant of 10 ft. per sec., and a second later the velocity has become 18 ft. per sec., we have a change of linear velocity of 8 ft. per sec. in the one-second interval. Thus, the acceleration for the interval considered is 8 ft. per sec. per sec. Note that, as calculated, this value is only an **average** acceleration for the given one-second period and does not tell us how the acceleration may have varied during the interval. More information would be necessary in order to get the acceleration existing at a particular instant. Linear acceleration, like linear velocity, can always be represented graphically by a line which shows the magnitude and direction.

Angular Acceleration is the rate of change of the angular velocity. When the angular velocity is measured in radians per second, our corresponding unit of angular acceleration will be " radians per second per second "; or, if in revolutions per minute, it may be either " revolutions per minute per minute " or " revolutions per minute per second."

For linear motion, the relationships between displacement (s), velocity (v), and acceleration (a) may be expressed mathematically as follows:

$$v = \frac{ds}{dt}, \quad a = \frac{dv}{dt}. \tag{2-2}$$

Here v is the instantaneous velocity or velocity at a certain instant and ds/dt expresses the rate of change of the displacement. Similarly, a is the instantaneous acceleration and dv/dt the corresponding rate of change of velocity.

When a particle or body starts from rest and is **uniformly accelerated** to velocity v in time t,

$$\begin{aligned} s &= \text{Average velocity} \times \text{Time} \\ &= \frac{v + 0}{2} \times t = \frac{vt}{2} \end{aligned} \tag{2-3}$$

and

$$v = at. \tag{2-4}$$

Therefore

$$s = \tfrac{1}{2} at^2. \tag{2-5}$$

By elimination of t in equations (2-4) and (2-3) we have

$$v^2 = 2\,as. \tag{2-6}$$

When dealing with **angular motion**, if

θ = angular displacement,
ω = angular velocity,
α = angular acceleration,

then, by definition,

$$\omega = \frac{d\theta}{dt}, \quad \alpha = \frac{d\omega}{dt}.$$

4. Normal and Tangential Acceleration. — The velocity of a moving point may change in two ways: (a) its linear speed along its path may increase or decrease; or (b) the direction of its motion may change.

(*a*) The rate of change of speed in the direction of motion is the **Tangential Acceleration,** since this involves an acceleration acting along the path of motion.

(*b*) The change in the direction of the motion is due to the **Normal** or **Centripetal Acceleration,** which acts in a direction normal to the direction of the path of motion. We may therefore define these terms as follows:

The **Tangential Acceleration** is the linear acceleration in the direction of motion at the instant considered, and is measured by the rate of change of velocity along its path.

The **Normal** or **Centripetal Acceleration** is that acceleration which causes the direction of motion of a body to change. It acts along a line perpendicular to the path of motion and toward the center of curvature of this path. For example, suppose a point *A* (Fig. 2–2) is traveling along the curved path *XY*. *O* is the center of curvature of *XY* at the point *A*. Assume that the linear velocity of *A* is increasing. Then *A* at the instant considered has a tangential acceleration represented by the tangent *AB* to the path of motion. The value of this acceleration is dV/dt, where *V* and *t* represent velocity and time, respectively. *A* also has a normal acceleration acting along *AO* and represented by a line *AC*. Calculation of the value of the normal acceleration will be considered in the next article. The resultant acceleration of *A* is evidently represented by the diagonal *AD* of the parallelogram *ABDC*.

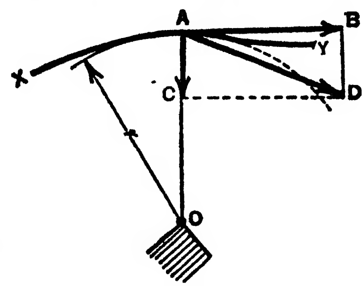


FIG. 2–2

5. Value of the Normal Acceleration. — This acceleration may be expressed in terms of the velocity of the point and the radius to the center of curvature of its path. In Fig. 2–3 a point moves along a curved path *XY*. Let *A* represent the position it occupies at a certain instant, its velocity then being equal to *V*. After a short interval of time Δt, the point has moved to position A_1 and its velocity is now $V + \Delta V$. Let lines *AB* and A_1B_1 represent V and $V + \Delta V$, respectively. These lines are tangents to the curve XY at A and A_1. The center of curvature of the small portion AA_1 of XY is O, and the radius of curvature $r = OA = OA_1$.

The change in velocity is found graphically as follows: Draw a velocity triangle abc in which ab is equal and parallel to AB, and ac is equal and parallel to A_1B_1, these two sides thus representing the initial and final velocities for the small time interval

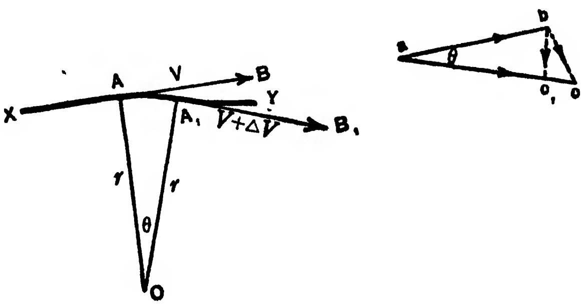


FIG. 2-3

Δt. The third side of the triangle, bc, represents the **change** in velocity. Find a point c_1 on ac such that $ac_1 = ab$, representing V.

Assuming that θ has an infinitesimal value, bc_1 becomes sensibly parallel to both AO and $A'O$ and represents the change in normal velocity, while c_1c measures the change in tangential velocity. The angle bac $(= \theta) = \omega \cdot dt$, where ω is the angular velocity about the center of curvature. The normal acceleration is calculated from bc_1. From the figure,

$$bc_1 = ab \cdot \theta = ab \cdot \omega \cdot dt = V \cdot \omega \cdot dt.$$

Now, the normal acceleration $= \dfrac{\text{change in normal velocity}}{dt}$

$$= \frac{bc_1}{dt} = \frac{V \cdot \omega \cdot dt}{dt} = V\omega = \frac{V^2}{r} \quad \text{or} \quad \omega^2 r. \qquad (2\text{-}7)$$

For a point revolving about a fixed center, the normal acceleration has the same value in terms of the velocity and radius, because only instantaneous conditions were considered in deriving the above formula.

6. Relation between Tangential Acceleration and Angular Acceleration. — These two quantities bear the same relationship to each other as do linear velocity and angular velocity. For, since

$$v = \omega r,$$

by differentiating both sides of the equation and dividing by dt,

$$\frac{dv}{dt} = \frac{d\omega}{dt} \cdot r,$$

or

$$a = \alpha r. \tag{2-8}$$

Thus

The tangential acceleration of a point on a moving body is equal to the angular acceleration of the body multiplied by the distance from the point to the center of rotation.

Example. — The flywheel for a metal shearing machine is 4 ft. in diameter and rotates at a normal speed of 180 R.P.M. During the shearing period, which lasts 2 sec., the flywheel speed is reduced to a final value of 150 R.P.M. Assuming constant angular deceleration, determine the normal and tangential acceleration of a point on the rim at the instant when the speed is 160 R.P.M.

Solution. — The angular velocity in radians per second corresponding to 160 R.P.M. equals

$$\frac{160 \times 2\pi}{60} = 16.74.$$

The normal acceleration of a point on the rim equals

$$\omega^2 r = (16.74)^2 \times 2 = 562 \text{ ft. per sec. per sec.}$$

Since the wheel speed changes from 180 to 150 R.P.M. in 2 sec., with constant acceleration, the value of this acceleration is

$$(150 - 180) \div 2 = -15 \text{ R.P.M. per sec.},$$

which equals

$$-\frac{15 \times 2\pi}{60} \quad \text{or} \quad -1.57 \text{ radians per sec. per sec.}$$

Therefore, the tangential acceleration of a point on the rim is

$$a_T = -1.57 \times 2 = -3.14 \text{ ft. per sec. per sec.}$$

The negative sign indicates a deceleration.

7. Simple Harmonic Motion. — When a point A (Fig. 2–4) moves in a circle with uniform velocity about a fixed point O, B, the projection of A on any diameter such as XY, moves with simple harmonic motion.

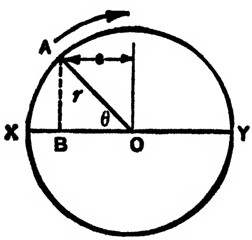


FIG. 2–4

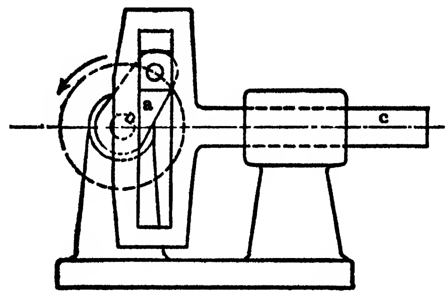


FIG. 2–5
Scotch Yoke.

The **Period** is the time required for a complete revolution of the generating line OA (Fig. 2–4).

The **Amplitude** is the distance between the points of reversal of motion, or twice the length of the generating radius.

The **Phase** is the angular position of the generating line at any instant with respect to a reference line.

The mechanism shown in Fig. 2–5, commonly called a "Scotch Yoke," gives simple harmonic motion to the link c when the crank a rotates with uniform velocity.

8. Displacement, Velocity, and Acceleration for S.H.M. — Suppose OA (Fig. 2–4) starts from an initial position OX and revolves clockwise. Let ω equal its constant angular velocity. Then, after a period of time t, the angle turned through will be $\omega \cdot t = \theta$, and the displacement of B from its mean position at O will be

$$s = OB = OA \cos\theta = r \cos \omega t. \qquad (2\text{–}9)$$

The velocity of B equals

$$v = \frac{ds}{dt} = \frac{d(r\cos\omega t)}{dt} = -\frac{r(\sin\omega t)\,d(\omega t)}{dt} = -r\omega\sin\omega t. \quad (2\text{–}10)$$

The acceleration of B equals

$$a = \frac{dv}{dt} = \frac{d(-r\omega\sin\omega t)}{dt} = -\frac{r\omega(\cos\omega t)\,d(\omega t)}{dt} = -\omega^2 r\cos\omega t. \quad (2\text{–}11)$$

Simple Harmonic Motion can therefore be defined as a motion in which the acceleration is directly proportional to the displacement, and acts in a direction toward the point of zero displacement.

Linear Displacement Curve for S.H.M. — In order to construct this curve, we take a base line representing 360° motion of the generating line (Fig. 2–6), and divide it into any number of equal spaces representing equal angles. Draw a circle whose radius is equal to that of the generating line, with center on the base line

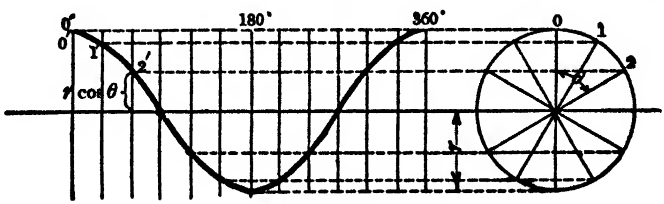


FIG. 2–6

Linear Curve of Displacement for Body with S.H.M.

produced. Divide this circle into angles of the same value as the base-line intervals. The zero position is at 90° to the base line, as indicated on the diagram. Project horizontally from points 0, 1, 2, etc., to points 0′, 1′, 2′, etc. The latter are points on the linear curve of displacement on a base representing the crank angle, for it is evident that the ordinate for any angle θ is equal to $r\cos\theta$.

Linear Velocity Curve for S.H.M. — (See Fig. 2–7.) The base line, as before, is taken to represent 360° motion of the generating line and is divided into equal spaces of a convenient width. A circle is drawn with center on the base line produced, its radius representing the linear velocity at the end of the generating line. This circle is divided into equal angles of the same value as the base-line divisions, starting with zero position along the base line as indicated. Taking points 0, 1, 2, etc., project horizontally to points 0′, 1′, 2′, etc., which are points on the required curve, for at any angle θ the ordinate is equal to $-\omega r \sin \theta$ which is the value of the velocity, by equation (2–10).

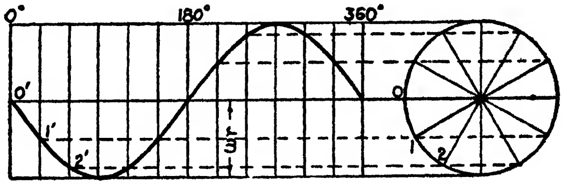


FIG. 2–7
Linear Curve of Velocity for Body with S.H.M.

Linear Acceleration Curve for S.H.M. — A base line is taken to represent a complete revolution of the crank (Fig. 2–8). A circle is drawn with center on this base line produced, with radius representing $\omega^2 r$. The base and circle are divided into equal angles of the same value, the zero position in the circle being at 90° to the base line, as for the displacement curve. Project horizontally from points 0, 1, 2, etc., to points 0′, 1′, 2′, etc., which are points on the required curve. For any angle θ the ordinate to the curve thus obtained is evidently equal to $-\omega^2 \cdot r \cos \theta$, the value of the acceleration of a point moving with S.H.M., by equation (2–11). It will be observed that the graphical constructions for displacement and acceleration curves are identical, except for positive and negative signs.

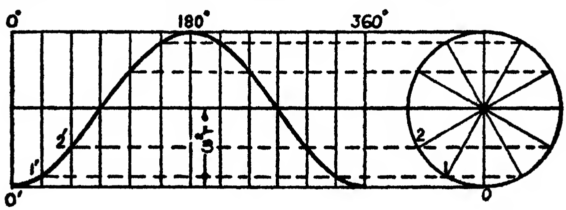


FIG. 2–8
Linear Curve of Acceleration for Body with S.H.M.

9. Polar Curves for S.H.M. — The construction for plotting a polar curve for the displacement of a point having S.H.M. is shown in Fig. 2–9. Let XY be the path of motion, its length representing the amplitude. Draw a circle with center O and XY as a diameter. Draw circles with OX and OY as diameters. Then these circles are polar displacement curves on an angle base as required. This may be proved as follows: Take a radius OR in any position, making an angle θ with XY. This intercepts one of the small circles at S. Join XS. Then the angle $XSO = 90°$, since it is an angle subtended by a semicircle. Therefore,

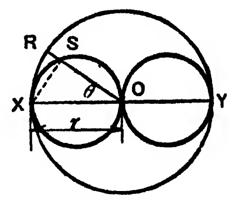


FIG. 2–9
S.H.M. — Polar Curve of Displacement.

$$OS = OX \cdot \cos\theta = r\cos\theta.$$

Consequently, the intercept OS is the displacement of the point, by equation (2–9).

The **Polar Velocity** curve is found as follows: Take a line OX representing the linear velocity of the outer end of the generating line (Fig. 2–10). Describe a circle with OX as radius. Draw RS perpendicular to XY. Construct two circles with RO and OS as diameters. Then these circles are the required velocity curves.

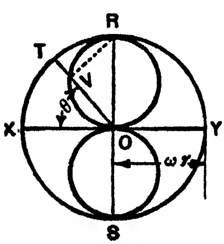


FIG. 2–10
S.H.M. — Polar Curve of Velocity.

Proof: Take any line OT making an angle θ with the line XY, intersecting the small circle at V. Join VR. Then angle $OVR = 90°$, being the angle in a semicircle. From the figure,

$$OV = OR \cdot \cos(90 - \theta) = \omega r \sin\theta.$$

Therefore, the distance OV represents the velocity of the point with S.H.M. by equation (2–10), which proves the construction to be correct.

The **Polar Acceleration** curve of Fig. 2–11 is constructed in the same manner as the **Displacement** curve, except that the radius OX now represents $\omega^2 r$. The ordinate OS at an angle θ can be shown to have the value $\omega^2 r \cos\theta$, which proves the construction.

10. Composition of Simple Harmonic Motions of Equal Periods. — Machines sometimes contain two or more parts which are driven with simple harmonic motions from the same rotating member; thus they have equal periods, though they may differ in phase and amplitude. This condition is approximately attained in certain kinds of valve gears on steam engines. When designing, it is necessary to study the relative motion of such parts.

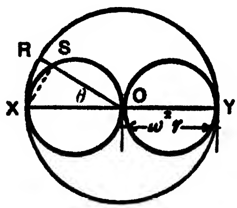


FIG. 2–11
S.H.M. — Polar Curve of Acceleration.

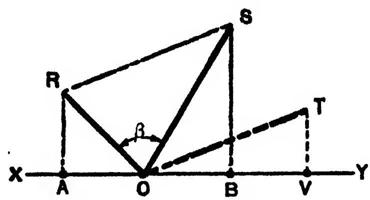


FIG. 2–12
Composition of Two Simple Harmonics.

Suppose that A and B (Fig. 2–12) represent the instantaneous positions of two points moving with S.H.M. along a line XY. The generating radii OR and OS may be unequal and have a phase difference of β. The relative displacement or distance between the points is AB at the instant considered. From O draw OT equal and parallel to RS. Then OV, the projection of OT on XY, is obviously equal to AB. It is therefore evident that OT will act as a generating radius producing S.H.M. of the point V whose displacement from mid-position is the relative displacement of A and B.

Thus, two points moving with S.H.M. of equal periods along the same straight line have a relative motion which is also simple harmonic, the phase and amplitude of this motion being determinable by the construction just outlined.

11. Linear Velocity-time and Acceleration-time Curves. — Frequently it is desirable to analyze the motion of a body, where data are available as to its position at certain instants, but no data are directly obtainable as to its velocity and acceleration. Information regarding the latter quantities can be obtained if a linear displacement-time curve is first plotted and a graphical method is then used to derive velocity-time and acceleration-time curves from the displacement-time curve.

In Fig. 2–13 is shown a curve obtained by plotting the displacement of a moving body from a fixed reference point, on a base representing time. Thus at any point on the curve, such as A, the displacement and time are, respectively, s and t. At point B, after a short interval of time Δt has elapsed, the displacement is $s + \Delta s$ at time $t + \Delta t$. Now the average velocity during the short interval is v where $v = \Delta s/\Delta t = \tan\theta$. θ is the angle of slope of the line BAD. In the limit where Δt is an infinitesimal, the distance from A to B also becomes infinitesimal, and the line BAD is then a tangent to the curve. Velocity v becomes the **instantaneous velocity,** that is, the velocity existing during the infinitesimal period dt. It follows, therefore, that the instantaneous velocity of a body is measured by the tangent of the angle of slope of the displacement-time curve at the point considered.

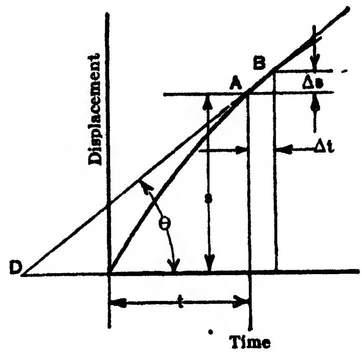


FIG. 2–13

Since acceleration bears the same relation to velocity as velocity bears to displacement, it can be shown in the same manner that the **instantaneous acceleration** of the body is equal to the tangent of the angle of slope of the velocity-time curve at the point under consideration.

12. Graphical construction of velocity and acceleration curves from a known displacement curve may be carried out as follows: Select any convenient number of points on the displacement curve (Fig. 2–14), as A_1, A_2, A_3. Construct the triangles $A_1B_1C_1$, $A_2B_2C_2$, $A_3B_3C_3$ in which the "AB" lines are tangents to the curve and the horizontal "BC" lines are all of equal length.

The velocities at A_1, A_2, A_3 are proportional to the quantities

$\frac{A_1C_1}{B_1C_1}, \frac{A_2C_2}{B_2C_2}, \frac{A_3C_3}{B_3C_3}$, since these are the tangents of the slope angles. By our construction, $B_1C_1 = B_2C_2 = B_3C_3$, and therefore it is evident that the velocities are represented graphically by the lengths A_1C_1, A_2C_2, A_3C_3. These lengths are plotted as ordinates on a time base, giving the **Velocity-time Diagram** of Fig. 2–14.

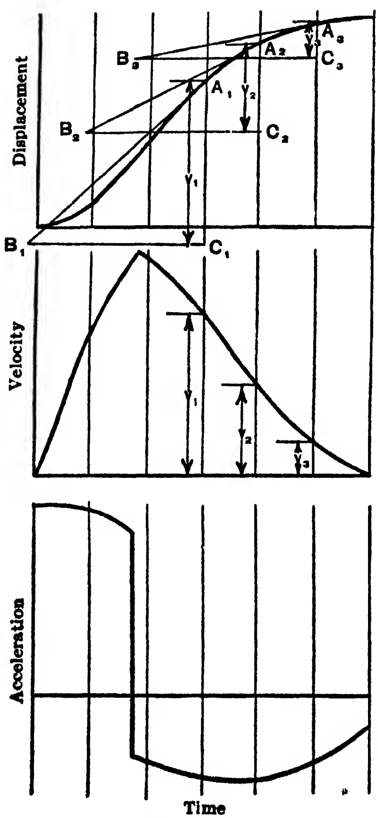


FIG. 2–14

Since acceleration bears the same relationship to velocity as velocity bears to displacement, a repetition of the construction just outlined, if applied to the velocity curve, will enable us to draw an **Acceleration-time Diagram,** as shown in Fig. 2–14.

It is somewhat difficult to draw accurate tangents to a curve of irregular form; hence the alternative construction which follows, though only an approximate one, is often held to produce better results.

We shall assume that the following data are available in regard to the motion of a car in which distances are measured from a fixed reference point at stated time intervals:

Time (Seconds)	*Distance* (Feet)	*Time* (Seconds)	*Distance* (Feet)
0	0	25	412
5	11	30	472
10	57	35	501
15	138	40	511
20	298		

We shall plot our displacement-time diagram, using the following scales:

Distance	1 in. = 160 ft.
Time	1 in. = 10 sec.

For convenience in projection of points, we shall locate our curves as shown in the diagram, Fig. 2–15, the base line always representing time, and the same instant in time being represented by points on the three bases which are in the same vertical line. The construction for one point on the velocity curve corresponding to the 15–20-sec. interval is as follows. Find the points a and b where the curve crosses the 15-sec. and 20-sec. lines, respectively. Draw ac horizontally, thus obtaining a distance bc ($= x$). Transfer x to the velocity-time diagram, plotting it opposite the middle of the 15–20-sec. strip, namely, at $17\frac{1}{2}$ sec. Distance x can be stepped off 2, 3, or 4 times on this line if desirable, in order to increase the height of the velocity curve. In the diagram as drawn, it is stepped off twice.

Then b' is one point on the velocity-time curve. The proof of this is as follows.

Since ad represents the displacement of the body at the end of 15 sec., and be the displacement at the end of 20 sec., $(be - ad)$ or x represents the displacement during the 5-sec. interval.

Similar lengths, y, z, will represent in the same way the displacement during the 20–25-sec. and the 25–30-sec. interval, respectively. Now the average velocities during equal time intervals are proportional to the displacements which are obtained during the corresponding time intervals. Hence x, y, z are proportional to these velocities. If we make the assumption that the average velocity for each interval is equal to the actual velocity at the middle of the interval, an assumption which is reasonably correct if the time intervals are short, we may proceed to obtain a velocity curve by plotting x, y, z (or multiples of these distances if more convenient) opposite the middle of each interval. It will be noted, however, that the error due to the approximation decreases as the time intervals are made shorter; hence, for accuracy, we must divide up our time base into narrow strips.

Since acceleration bears the same relationship to velocity as velocity does to displacement, it follows that the acceleration-time curve is obtained from the velocity-time curve in exactly the same manner as that in which the velocity-time has been derived from the displacement-time. This curve is shown in the figure, the ordinates being doubled as before, in order to magnify variations in the shape of the curve.

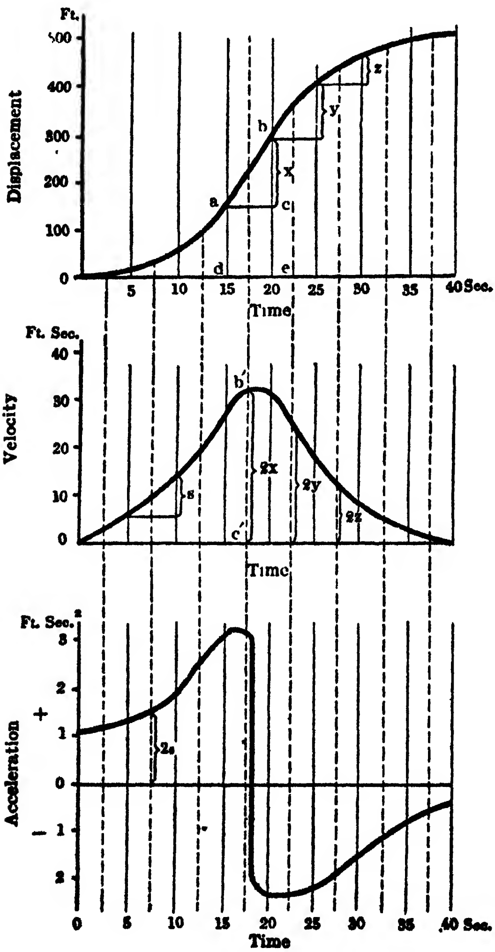


Fig. 2–15

A Scale for a diagram is a statement of the numerical value of the quantity represented by unit distance on the diagram. Thus the velocity scale in Fig. 2–15 is a statement of the number of feet per second represented by an ordinate one inch long on the velocity-time diagram. To find the numerical value of this scale, let us assume that we have an ordinate of this length at some point on the curve. Suppose, for example, that $b'c'$ equals 1 in. That is, referring to the velocity curve,

$$b'c' = 2\,x = 1 \text{ in.},$$

and, returning to the displacement-time diagram,

$$bc = x = \tfrac{1}{2} \text{ in.}$$

But our displacement scale is 1 in. = 160 ft.

Therefore bc represents 80 ft.

Also, bc is the change in displacement in 5 sec. The average velocity during this 5-sec. interval is therefore $80 \div 5 = 16$ ft. per sec. Consequently the 1-in. ordinate on the velocity diagram represents 16 ft. per sec., or in other words the velocity scale is

$$1 \text{ in.} = 16 \text{ ft. per sec.}$$

A length of 1 in. was assumed for $b'c'$ simply for ease in calculation. Any value will give the same scale.

Similarly, if we assume that any ordinate on the acceleration diagram, say that marked "$2s$," equals 1 in., then the corresponding ordinate s on the velocity diagram will have a length of $\tfrac{1}{2}$ in., since the ordinates were doubled. This represents (by the velocity scale just calculated) $16 \div 2 = 8$ ft. per sec., which is the change in velocity during the 5-sec. period. This is equivalent to an average acceleration for the period of $8 \div 5 = 1.6$ ft. per sec. per sec. Hence the acceleration scale is

$$1 \text{ in.} = 1.6 \text{ ft. per sec. per sec.}$$

13. Relative Velocities of Three Bodies. — The symbol V_{ab} means the relative linear velocity of point or body a with respect to point or body b. Considering three bodies, we have three relative velocities as follows: V_{ab}, V_{bc}, V_{ac}. There are also velocities V_{ba}, V_{cb}, and V_{ca}, but these are not independent quantities, since

$$V_{ab} = -V_{ba};\ V_{bc} = -V_{cb};\ V_{ac} = -V_{ca}.$$

The truth of these statements is self-evident if we consider the following example. A passenger on an east-bound train sees any landmark receding to the west at the speed of the train. Thus two vectors, representing velocity of passenger relative to earth and of earth relative to passenger, are of equal length but have arrows representing directions pointing, respectively, east and west, or one velocity is −1 times the other. Again, if the passenger walks forward through the train it is obvious that his velocity relative to the ground is the **arithmetic** sum of his velocity relative to the train plus the velocity of the train relative to the ground. If, however, he walks in another direction, say diagonally across the car, then his velocity to the ground is found by taking the **vectorial** summation of his velocity relative to the train (V_{ab}) and the velocity of the train relative to the ground (V_{bc}) or

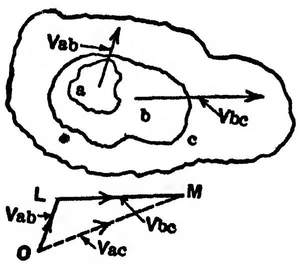


Fig. 2–16

$$V_{ab} \nrightarrow V_{bc} = V_{ac}.$$

The sign +→ is used to indicate the vectorial rather than the arithmetic sum. In Fig. 2–16 is shown the graphical method of summing the vectors V_{ab} and V_{bc}, the **resultant being equal to the closing line V_{ac} of the triangle.** The direction of the arrows in this figure should be carefully observed since here errors often occur. Where we have a vectorial sum, as for V_{ab} and V_{bc}, the arrows point around the figure in the same sense. As a guide to correct vectorial summation let us write our vectorial equation as follows:

$$V_{a/b} \nrightarrow V_{b/c} = V_{a/c}.$$

If we treat the subscripts a/b, b/c as quantities to be multiplied, the result is $(a/b) \times (b/c) = a/c$. On the other hand, the sum $V_{a/b}$ +→ $V_{c/b}$ would equal V_{ac/b^2} by the same method, and this quantity has no significance as a velocity vector. This would show that the left-hand side of the equation is wrongly written.

When two bodies have known velocities with respect to a third, the **Velocity Ratio** is the quotient obtained by dividing one velocity by the other. Both velocities may be linear, or both may be angular. If both are angular the velocity ratio should bear a negative sign when the two bodies turn in opposite senses.

Example. — Find the velocity of the crank pin A (Fig. 2–17) on a locomotive driver when the locomotive is moving at the rate of 36.1 miles per hour.

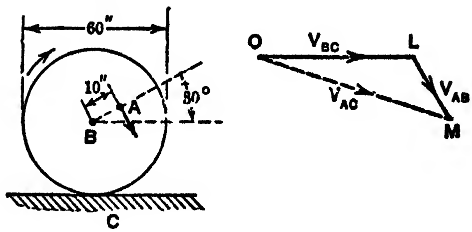


FIG. 2–17

Solution. — The velocity of wheel center B relative to the track C is $\dfrac{36.1 \times 22}{15} = 53$ ft. per sec.

The angular velocity of the wheel is $\dfrac{53 \times 12}{30} = 21.2$ radians per second.

The linear velocity of A relative to B ($v = \omega r$) is

$$21.2 \times 10 = 212 \text{ in. per sec.}$$
$$= 17.66 \text{ ft. per sec.}$$

Starting from a point O, draw vectors OL and LM representing V_{BC} and V_{AB} respectively. The closing line OM of the triangle OLM is the required velocity V_{AC}. By this method OM is found to represent a velocity of 63.7 ft. per sec. at an angle of 14° with the horizontal.

QUESTIONS — CHAPTER II

1. Define (*a*) displacement, (*b*) velocity, (*c*) linear velocity, (*d*) angular velocity. Name a common unit for each.

2. Define (*a*) acceleration, (*b*) linear acceleration, (*c*) angular acceleration. Name a common unit for each.

3. What is meant by (*a*) tangential acceleration, (*b*) normal acceleration, of a moving point? What is the normal acceleration of a point moving in a curved path of radius r feet at a speed of v feet per second?

4. A train moves at the rate of 45 miles per hour. What is its velocity in feet per second? *Ans.* 66 ft. per sec.

5. The wheel of an automobile is turning at the rate of 350 R.P.M. What is the angular velocity of the wheel in radians per second? What is the speed of the car in miles per hour? The tire diameter is 30 in.

6. A piece of cast iron 6 in. in diameter is to be turned in a lathe with a cutting speed of 90 ft. per min. At what speed must the work be rotated?
Ans. 57.3 R.P.M.

7. In a shaper the cutting tool has a stroke of 15 in. The cutting stroke takes twice the time required for the return stroke. If it is desired to obtain an average speed during the cutting stroke of 25 ft per min., what must be the R.P.M. of the driving crank? *Ans.* 13.34 R.P.M.

8. An automobile is accelerated uniformly from 10 to 30 miles per hour in 8 sec. Find the value of the acceleration in foot-second units. If the wheels are 28 in. in diameter, find their angular acceleration in radian-second units.

9. A flywheel rotates at 250 R.P.M., its radius being 6 ft. Find (*a*) its angular velocity in radian measure, (*b*) the linear velocity of a point 3 ft. from the axis, (*c*) the normal acceleration of a point on the rim.

10. A flywheel 4 ft. in diameter is speeded up from 120 to 380 R.P.M. with constant angular acceleration in 2 sec. Find the total acceleration of a point on the circumference of the wheel when the speed reaches 180 R.P.M.
Ans. 711 ft. per sec. per sec.

11. A point on a body moves along a curved path. At a certain instant the point has a velocity of 27 ft. per sec., a tangential acceleration of 40 ft. per sec. per sec., and a normal acceleration of 54 ft. per sec. per sec. Find (*a*) the radius of curvature of the path, (*b*) the total acceleration of the point, (*c*) the angular velocity and the angular acceleration of the body.

12. The speed of a flywheel rim is 50 ft. per sec. and the R.P.M. are 200. Find (*a*) the radius of the wheel, (*b*) the normal acceleration of a point on the rim.

13. A reciprocating steam engine has a stroke of 24 in. and rotates at 225 R.P.M. Find (*a*) the linear velocity of the crank pin, (*b*) the normal acceleration of the crank pin. *Ans.* (*a*) 23.5 ft. per sec. (*b*) 554 ft. per sec. per sec.

14. A revolving body has an angular acceleration of 30 radians per sec. per sec. A point on it is distant 12 in. from the axis of revolution. When the speed of the body reaches 200 R.P.M., find the normal and tangential acceleration of the point, and show how the total acceleration can be obtained graphically. *Ans.* 30 ft. per sec. per sec. 438 ft. per sec. per sec.

15. What is meant by "simple harmonic motion"? Explain the meaning of the terms "period," "phase," and "amplitude" as applied to this form of motion.

16. Show how to draw a linear curve of (*a*) velocity, (*b*) acceleration, for a point having S.H.M., given the period and amplitude.

17. Show how to draw a polar curve of (*a*) displacement, (*b*) velocity, for a point having S.H.M., given the period and amplitude. Prove your constructions.

18. A point has S.H.M. of period 2 sec., amplitude 6 in. Find (*a*) its maximum velocity, (*b*) its maximum acceleration.

19. A point has simple harmonic motion of period 2 sec., amplitude 6 in. Find (*a*) its displacement, (*b*) its velocity, when the revolving line is at 30° with the line of motion. *Ans.* 2.6 in., 0.392 ft. per sec.

20. In the mechanism shown in Fig. 20P, the crank *a*, 12 in. long, rotates at 100 R.P.M. Find (*a*) the velocity of *b*, and (*b*) the acceleration of *b*, when the crank *a* is at 60° with the line of stroke.

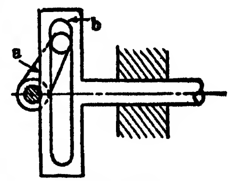


FIG. 20P

21. The slotted link in a Scotch yoke is found to attain a maximum velocity of 6 ft. per sec., the crank length being 4 in. At what average speed does the slotted link move if the crank rotates uniformly? What is the maximum acceleration?

Ans. 3.86 ft. per sec.; 108 ft. per sec. per sec.

22. A single-cylinder steam engine has a slide valve driven by an eccentric, the eccentric radius being advanced 120° ahead of the engine crank. The total movement of the valve is 4 in., and it can be considered as simple harmonic. Find the displacement, velocity, and acceleration of the valve at the instant when the engine crank is on the dead center. The engine turns at 300 R.P.M.

23. Given a distance-time curve, show how to obtain graphically a velocity-time and acceleration-time curve.

24. If, in Question 23, the distance scale is 1 in. = s ft., the time base is divided into intervals representing n sec., and the ordinates are doubled in transferring, show that the velocity scale is

$$1 \text{ in.} = \frac{s}{2n} \text{ ft. per sec.}$$

25. If, in Question 23, the velocity scale is 1 in. = v ft. per sec., the time base is divided into intervals representing n sec., and ordinates are doubled in transferring to the acceleration diagram, show that the acceleration scale is

$$1 \text{ in.} = \frac{v}{2n} \text{ ft. per sec. per sec.}$$

26. A polar curve for the velocity of a point with S.H.M. is drawn with a maximum ordinate of 2 in. (*a*) If the length of the generating radius is 6 in. and its angular velocity 150 R.P.M., what is the velocity scale for the polar curve? (*b*) If a polar acceleration curve for the same point has a maximum ordinate of 2 in. what is the acceleration scale?

27. How many different relative motions are there when three bodies are considered? when four bodies are considered? Show graphically how to find one of the relative velocities when the others are known.

28. A bee enters the open window of an automobile and leaves at the corresponding point on the opposite window, 4½ ft. away, just ½ sec. later. If the automobile is traveling at the rate of 12 ft. per sec., find the speed and direction of flight of the bee. *Ans.* 15.0 ft. per sec., 30° 52′ to auto axis.

29 A locomotive has driving wheels of 62 in. diameter. The tender wheels are 36 in. diameter. Find the angular velocity of each and the velocity ratio when the locomotive is traveling at the rate of 40 miles per hour.

Ans. 22.7 rad. per sec., 39.1 rad. per sec. Ratio 1 : 1.72.

30. A locomotive, running at 60 miles per hour has a stroke of 32 in. Find the velocity of the crank pin relative to the ground when the crank is in the position shown in Fig. 30P.

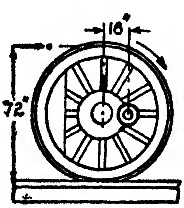


Fig. 30P

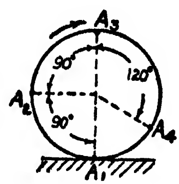


Fig. 31P

31. A locomotive is traveling at the rate of 45 miles per hour. The driving wheels are 6 ft. in diameter. Find the speed of a point on the rim of the driver when the point occupies the positions A_1, A_2, A_3, A_4 in Fig. 31P.

32. A street car with wheels 30 in. in diameter is traveling at the rate of 35 miles per hour. (*a*) What is the angular velocity of the wheels? (*b*) What is the linear velocity of the highest point on the tread of a wheel, relative to the track? *Ans.* (*a*) 41 rad. per sec. (*b*) 102.5 ft. per sec.

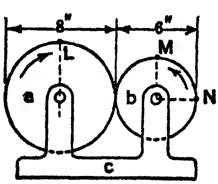


Fig. 33P

33. The pair of friction wheels *a* and *b* shown in Fig. 33P are respectively 8 and 6 in. in diameter. They roll together without slipping, *a* turning at 210 R.P.M. Show how to find graphically the relative velocities of point *L* to point *M*, of point *L* to point *N*, and of point *M* to point *N*.

34. An airplane has a landing speed of 50 miles per hour. Find the velocity of the end of the propeller blade relative to the ground, at the instant when the blade is in a horizontal position when landing. The propeller is 7 ft. in diameter and is turning at 550 R.P.M.

35. Two points lying on a revolving disc in the same radial straight line have a relative velocity of 60 ft. per sec. If the disc turns at 420 R.P.M., what is the distance between the points?

36. The hour hand of a clock is 3 in. long and the minute hand 5 in. long. What is the relative velocity, in inches per minute, of points at the ends of the hands at nine o'clock?

37. A pilot flies a straight course from city A due north to city B, 400 miles distant. His plane has an air speed of 180 miles per hour. A cross wind blows due east at 60 miles per hour. In what direction must the plane be headed, and how long will the trip take? *Ans.* N 19° 30′ W, 2 hr. 21 min.

38. A boatman heads his boat across a river 1 mile in width. He can propel the boat at the rate of 4 miles per hour. The current in the river flows at 5 miles per hour. How long does it take him to reach the opposite bank? What is his velocity relative to the shore in direction and magnitude? How far downstream does he land?

39. An airplane takes off from city A to fly to city B, due west and distant 600 miles. The wind is estimated to be blowing at 50 miles per hour in a direction E 30° S and the plane's course is set on this basis. If the wind actually blows at 30 miles per hour in the direction given, how far from the starting point will the plane be at the time it should have landed at its destination? Obtain by vector diagram the distance of the plane from its objective at that time. The plane has a cruising speed of 200 miles per hour.

CHAPTER III

INSTANT CENTERS

1. General. — Parts of machines with plane motion may be divided into three groups: (*a*) those with angular rotation about a fixed axis; (*b*) those with angular movement but not about a fixed axis; and (*c*) those with linear but not angular motion. All these motions may be studied by the use of **Instant Centers.**

For kinematic purposes, as pointed out in Chapter I, we shall disregard the thickness of the bodies perpendicular to the plane of motion, and deal with the projections of the bodies on this plane.

Let *c*, Fig. 3–1, represent a body having plane motion relative to a second body *d*. A point *A* on *c* lies in the position A_1 at a certain instant, and point *B* is at B_1 at the same instant. An instant later *A* has moved to A_2 and *B* to B_2. Since *A* and *B* are points on the same body, A_1B_1 and A_2B_2 are of equal length. If we bisect A_1A_2 and B_1B_2 at right angles by *KL* and *MN*, respectively, and find the intersection of these two lines at *O*, then it is evident that the movement of *A* from A_1 to A_2 and of *B* from B_1 to B_2 could be accomplished by rotation of the body *c* about *O*. If distances A_1A_2 and B_1B_2 are infinitely small, then *O* becomes the **Instant Center** for the relative motion, and is called O_{cd}, meaning "the instant center for the motion of *c* with respect to *d*." O_{cd} can be regarded as the position of a pair of superimposed points, one on each body, the two points having for the instant no motion with respect to one another. The instant center may therefore be defined in either of the following ways:

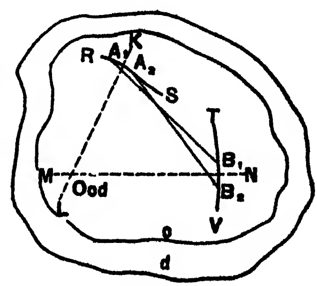


Fig. 3–1

(*a*) When two bodies have plane relative motion, the instant center is a point on one body about which the other rotates at the instant considered.

(*b*) When two bodies have plane relative motion, the instant center is the point at which the bodies are relatively at rest at the instant considered.

2. Locating Instant Centers. — Instant centers are extremely useful in finding the velocities of links in mechanisms. Their use sometimes enables us to substitute for a given mechanism another which produces equal motions and which is mechanically more serviceable. The methods of locating the instant centers are therefore of great importance. When a body has constrained motion, any point on it traces out a **point path** in space. When the motion is plane, the point path is a plane figure of some sort. Very frequently we can easily find the point paths corresponding to the motions of two points on a body. When this is the case the instant center is found as follows: In Fig. 3–2 body c has plane motion relative to d. Curves RS and TV are respectively the paths traced out on d by two points A and B on c. The instantaneous motions of the two points must be along tangents v and v_1 to the paths of motion, and the instant center must be so placed as to give motions in these directions. To cause A to move in direction v, the body must be swung about a center somewhere on the line KL perpendicular to v. Likewise, to cause B to move along v_1, the center must be somewhere along MN. The intersection of these lines at O_{cd} is the only point which will satisfy both requirements, and this point is therefore the instant center.

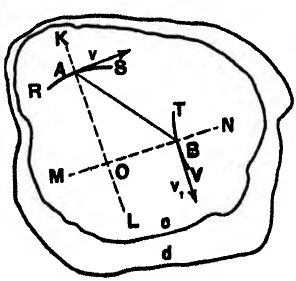


FIG. 3–2

3. Special Cases. — (*a*) When two links in a mechanism are connected by a pin joint, as for example c and d, Fig. 3–3, it is evident that the pivot point is the instant center for all possible positions of the two bodies, and is therefore a permanent center as well as an instant center.

(*b*) When a body has rectilinear motion with respect to a second body, as in Fig. 3–4 where block *c* slides between the flat guides *d*, the instant center is at infinity. This must be the case, since if we

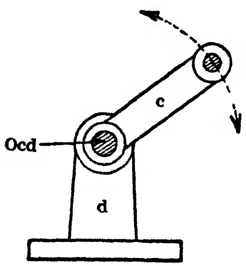


FIG 3–3

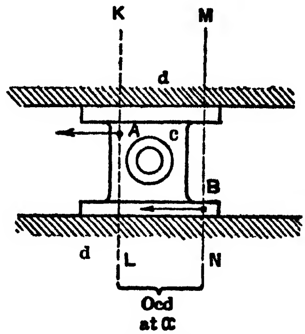


FIG. 3–4

take any two points, such as *A* and *B*, on *c* and draw *KL* and *MN* perpendicular to the directions of motion, these lines are parallel and meet at infinity.

(*c*) Where two bodies slide on one another, maintaining contact at all times, as *c* and *d*, Fig. 3–5, the instant center must lie along the perpendicular to the common tangent. This follows, since the relative motion of contact point O_1 on *c* to contact point O_2 on *d* is along the common tangent *XY*; otherwise the two surfaces would either separate or cut into one another. Relative motion along the common tangent can be effected only by swinging about a center somewhere along the perpendicular *KL*; hence the instant center is on this line.

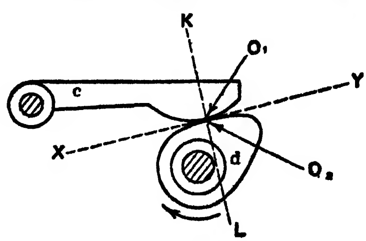


FIG. 3–5

(*d*) Where one body rolls on the surface of a second, as in Fig. 3–6, where *c* rolls on *d*, the instant center is then the point of contact, since at this point the bodies have no relative motion.

4. Approximate Construction for Locating Instant Centers. — When the instant center must be obtained from point paths that are not either circles or straight lines, the following approximate method can often be profitably substituted for that described in Art. 2. It avoids the difficulty of drawing normals to the curves. In Fig. 3–7, *RS* and *TV* are respectively the paths traced out by two points *A* and *B* on a body *c* having plane motion with respect to another body *d*. A_1 and B_1 are positions reached by *A* and *B* at a certain instant. After a short time interval, *A* moves to A_2

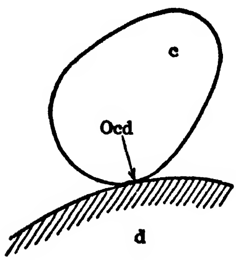


Fig. 3–6

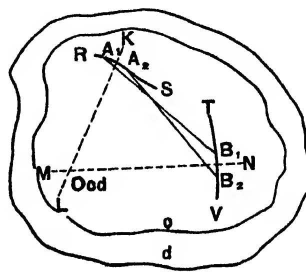


Fig. 3–7

and *B* to B_2. Evidently, A_1B_1 equals A_2B_2, since *A* and *B* are points on the same body a fixed distance apart. To find the approximate position of the instant center corresponding to the short time interval while this movement takes place, we bisect A_1A_2 at 90° by *KL*, and B_1B_2 at 90° by *MN*. The two perpendiculars meet at O_{cd}, the required point. This construction is exact only when A_1A_2 and B_1B_2 are circular arcs with O_{cd} as a common center, or when A_1A_2 and B_1B_2 are infinitesimals. The error in other cases is not great if A_1A_2 and B_1B_2 are small.

5. Kennedy's Theorem. — This theorem states that the instant centers for any three bodies having plane motion lie along the same straight line. It is proved as follows:

Let *a*, *b*, *c* (see Fig. 3–8) be any three bodies having plane motion with respect to one another. Let O_{ab}, O_{bc}, O_{ac} be the three instant centers.

O_{ab} is a point on either *a* or *b*, because it is an instantaneous pivot about which one body swings with reference to the other.

First consider O_{ab} as a point on a. Then it is moving relatively to c about the instant center O_{ac}, and its direction of motion is perpendicular to the line O_{ac}—O_{ab}. Next consider O_{ab} as a point on b. It is now moving relatively to c about the instant center O_{bc}, and its direction of motion is perpendicular to the line O_{bc}—O_{ab}. But the point O_{ab} cannot have two different motions relative to c at the same instant. Therefore, the perpendiculars to the lines O_{ac}—O_{ab} and O_{bc}—O_{ab} must coincide. Consequently, O_{ac}—O_{ab}—O_{bc} is a straight line.

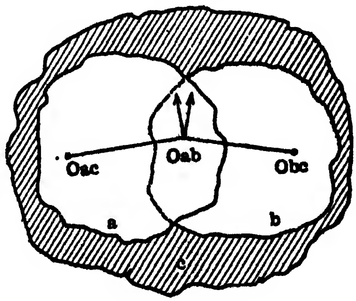


FIG. 3-8

Kennedy's Theorem is very useful in locating instant centers in mechanisms in cases where two instant centers for three links are known and the third has to be found. Examples given later in the chapter illustrate its application for this purpose.

6. Number of Instant Centers. — In any mechanism having plane motion, there is one instant center for each pair of links. The number of instant centers is therefore equal to the number of pairs of links. With n links, the number of instant centers is equal to the number of combinations of n objects taken two at a time, namely, $\frac{n(n-1)}{2}$.

7. Quadric-crank Mechanism. — (Fig. 3-9.) This consists of four links connected by turning pairs at K, L, M, N. The number of instant centers is, by Art. 6, $\frac{4 \times 3}{2} = 6$. Four of these centers are found at the pivot points, by Art. 3(a). These are O_{ab}, O_{bc}, O_{cd}, and O_{da}. The remaining two, namely O_{ac} and O_{db}, may be located (a) by the point-

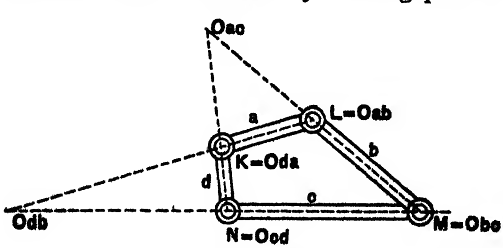


FIG. 3-9

path method of Art. 2, or (*b*) by the use of Kennedy's Theorem. Both methods will now be described.

(*a*) In applying the point-path method to find the instant center of two bodies, it is convenient to consider one of the bodies as being fixed and then to note the direction of motion of two points on the other. Thus, to find O_{ac}, we regard *c* as a fixed link and observe the direction of movement of points *K* and *L*. Point *K* is moving about a center at *N* in a direction normal to *KN*. Likewise, *L* is moving about *M* in a direction normal to *LM*. The intersection of *KN* and *LM* is, therefore, O_{ac}, by Art. 2.

In a similar way, by regarding *b* as fixed, we can find O_{db} at the intersection of *KL* and *MN*.

(*b*) In using Kennedy's Theorem to find O_{ac}, we select two groups of bodies, each group consisting of the two bodies *a*, *c*, plus a third. The instant centers for each group must lie in one straight line, by the theorem. Taking *a*, *c*, *b* as one group, O_{ac} must lie on a straight line with O_{ab}—O_{bc}. Taking *a*, *c*, *d* as the other group, O_{ac} must lie along O_{da}—O_{cd}. Therefore, O_{ac} is at the intersection of the lines O_{ab}—O_{bc} and O_{da}—O_{cd}. The instant center O_{db} is found in the same manner.

8. Instant Centers for the Slider-crank Mechanism. — It is important that the student be able to recognize the Slider-crank Mechanism in any of its many forms since it is applied to a large variety of practical uses. It may be described as a four-link chain in which one pair of links have rectilinear motion with respect to each other, while the relative motion of any other pair of adjacent links is that of rotation about a permanent center. The mechanism, therefore, contains three turning pairs and one sliding pair.

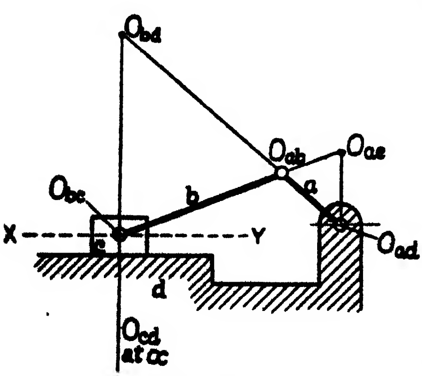


Fig. 3-10

Figures 3–10, 3–11, and 3–12 illustrate three forms of the Slider-crank Mechanism, corresponding links bearing the same letter. There are six instant centers, three of them, O_{ad}, O_{ab}, O_{bc}, being located at the pivot points. One, O_{cd}, is at infinity, because the relative motion of c and d is rectilinear. The two remaining centers, O_{ac} and O_{bd}, may be found as follows: For O_{bd}, d is regarded as fixed; then the point O_{bc} moves along XY and the point O_{ab} moves perpendicularly to the center line of the link a. As these are two points on link b, the instant center O_{bd} is at the intersection of the perpendiculars

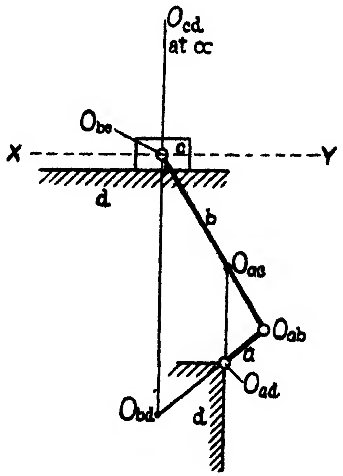


FIG. 3–11

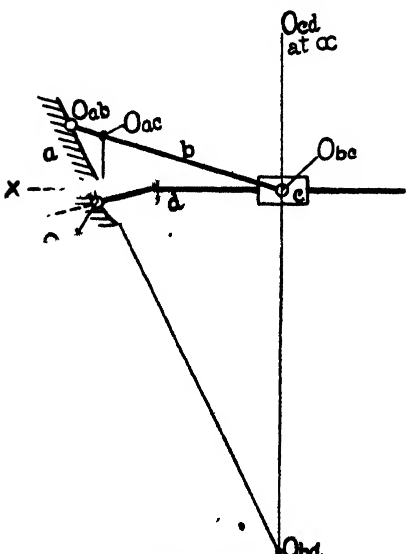


FIG. 3–12

to the directions of motion of these points. (See figures.) Similarly, for O_{ac}, if c is fixed, O_{ad} is constrained to move parallel to XY, and point O_{ab} perpendicularly to the center line of b. The normals to these directions of motion meet at O_{ac}. It is possible to state a general rule, for finding the two centers for opposite links (O_{ac} and O_{bd}), which will apply to any form of the chain. Draw perpendiculars to the line of slide (XY) at the pivot points of the two links containing elements of the sliding pair. The two intersections of these lines with the center lines of the other links (which have pivot elements only) will be the required centers.

9. Circle Diagram. — A diagram of the form shown in Fig. 3–14 is useful when finding instant centers since it gives a visual indication of the order in which the centers can be located by means of Kennedy's Theorem and also, at any stage in the process, it shows what centers remain to be found. The circle diagram will be used for finding the centers in the six-link mechanism of

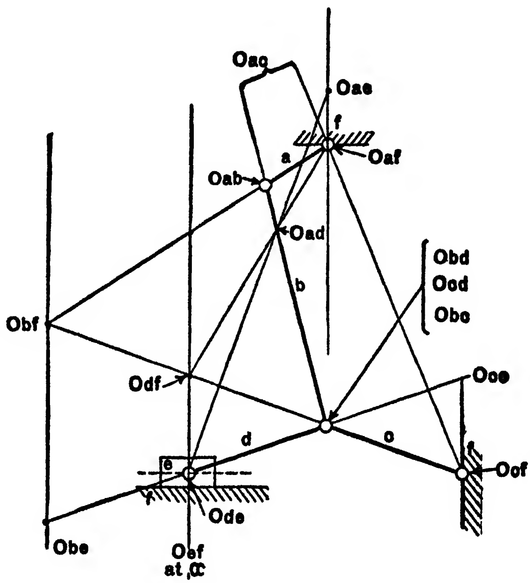


Fig. 3–13
Crusher Mechanism.

Fig. 3–13. We note (by Art. 6) that the number of instant centers is fifteen. The following procedure is used to locate them:

(*a*) Draw a circle, as in Fig. 3–14, and mark points, *a*, *b*, *c*, *d*, *e*, *f*, around the circumference, representing the six links in the mechanism. As the instant centers are located, draw lines connecting the points with corresponding letters on this diagram. Thus *ab* is drawn when the instant center O_{ab} is found. The figure will have lines connecting all pairs of points when all instant centers have been determined; five lines will radiate from each point on the circumference. Numbers on the lines, indicating the sequence in which they are drawn, facilitate checking.

(*b*) Pivot points are instant centers for the links connected at these points. Therefore, locate O_{af}, O_{ab}, O_{bc}, O_{cd}, O_{de}, O_{bd}, O_{cf}, and draw lines 1–7 in figure.

(*c*) When the relative motion is rectilinear, the instant centers are at infinity. Thus O_{ef} is at infinity. Draw line 8 on the diagram.

(*d*) Links *cdef* form a slider-crank mechanism. Therefore, O_{df} and O_{ce} are located as in Art. 8 and lines 9 and 10 are drawn.

(*e*) Kennedy's Theorem will be used to locate the rest of the instant centers, though in this particular case two more could be

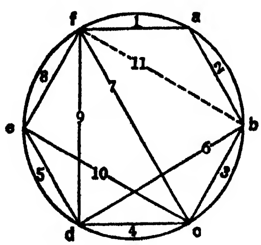


Fig. 3–14

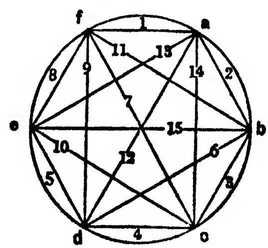


Fig. 3–15

found by considering the **Quadric-crank Mechanism** *abcf*. From inspection of our diagram, now in the condition shown in Fig. 3–14, we note that by joining *fb* we complete two triangles *abf* and *dbf*. Since this is the case, we locate the instant center O_{bf} at the intersection of O_{ab}—O_{af} and O_{df}—O_{bd}. Had we drawn *ea* instead, only one triangle, namely *eaf*, would have been formed; hence the center O_{ae} could not be found by the theorem at this stage, though it can be after O_{ad} (line 12) is placed. The line *ea* is therefore taken as number 13. The procedure is the same for the remaining points, which by inspection are seen to be O_{ac} and O_{be}.

If each line is first drawn as a broken line, while the center is being located, and made solid as soon as the center is found, errors are less likely to occur. Figure 3–13 shows the location of all the instant centers, and Fig. 3–15 the completed diagram.

10. Centrodes. — The locus of the instant center of a moving body is known as a centrode. Referring to Fig. 3-16, A and B, two points on a moving body c, trace out point paths RS and TV on a fixed body d. A_1, A_2, A_3 are instantaneous positions of point A, and B_1, B_2, B_3 are corresponding positions of point B at the same instants in time. Erecting perpendiculars to the point paths at A_1 and B_1, we find the instant center O_1 at the intersection. O_2 and O_3 are found in similar fashion by intersections of perpendiculars at A_2, B_2, and A_3, B_3, respectively. O_1, O_2, O_3 are points on the centrode for the relative motion as drawn on the fixed body d.

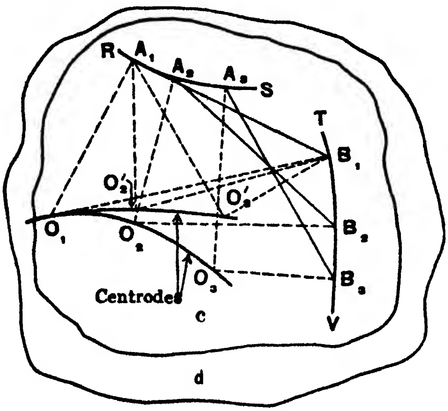


Fig. 3-16

Since the instant center is a point on either body, we can find a second centrode traced out by the instant centers on the surface of c. That is, we now consider O as a point attached to and moving with c. To construct this centrode we must choose a "reference" position of the points A and B, say at A_1B_1, and refer the instant centers O_2 and O_3 back to it. O_1 then becomes a common point on both centrodes, and the points O_2' and O_3' are found by sliding back the triangles $A_2B_2O_2$ and $A_3B_3O_3$ so that the "AB" sides coincide with A_1B_1. Thus, triangle $A_1B_1O_2'$ is made equal to $A_2B_2O_2$, and triangle $A_1B_1O_3'$ equals $A_3B_3O_3$. A curve drawn through $O_1O_2'O_3'$ is the centrode attached to c.

An alternative method of drawing the second centrode attached to the moving link is to invert the mechanism, making this link the fixed one. By this method, c in the figure would become the fixed link and d would be allowed to move. This construction becomes difficult when the point paths have an irregular shape. A case suitable for this method will be considered in the next article.

11. Properties of Centrodes. — When one body has constrained motion relative to another, it will be observed that, for any relative position of the two bodies, the two centrodes are in contact at a point, this point being the instant center for that position. As relative motion continues, it follows that the two centrodes will roll on one another. Consequently, it is possible to substitute for a given mechanism an equivalent mechanism containing two rolling surfaces which will produce the same motion of a selected link as it had in the original. An example of this will now be shown.

In Fig. 3–17 is illustrated one form of the **Double Slider-crank Mechanism,** consisting of a bar link *a* pivoted to blocks *b* and *c*, the latter sliding in guides on a frame *d*. Points *A* and *B* on *a* have straight-line motion along *XY* and *XZ*, respectively. In

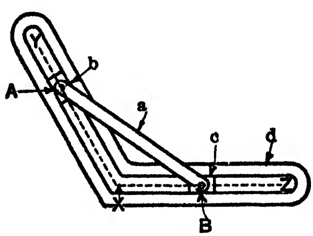


FIG. 3–17

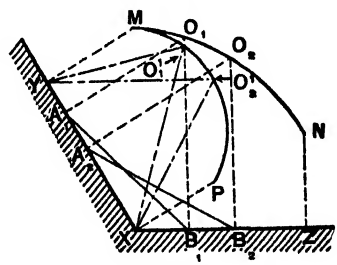


FIG. 3–18

the initial position A_1, B_1 (Fig. 3–18), the instant center is at O_1, the intersection of perpendiculars to XY and XZ at A_1 and B_1, respectively. The centrode attached to the frame *d* is drawn through points O_1, O_2, etc., the latter being found in the same way as O_1. This is the curve MO_1O_2N.

Two methods may be used to find the centrode attached to *a*.

(*a*) In Fig. 3–18 is shown the same construction as was used in Art. 10. For the reference position, AB is taken as coincident with XY. Triangle XYO_1' is made equal to $A_1B_1O_1$; XYO_2' equals $A_2B_2O_2$, etc. The curve $MO_1'O_2'P$ is the centrode attached to *a*.

(*b*) In Fig. 3–19 link *a* is held fixed and *d* is moved around it. As the frame, represented by the lines XY and XZ, is moved around the body a, represented by the line AB, XY must always pass through A and XZ must pass through B. The angle YXZ being constant, the apex X will trace out a circular arc AX_2X_1B. When X is in position X_1, the instant center is at O_1', a point found by the intersection of perpendiculars to X_1Y_1 and X_1Z_1 through A and B, respectively. Similarly, O_2' corresponds to position X_2. The centrode attached to a is the curve $MO_1'O_2'P$.

We can now construct a mechanism as shown in Fig. 3–20 in which we have replaced the sliding blocks b, c and their guides

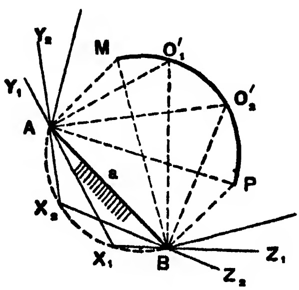


FIG. 3–19

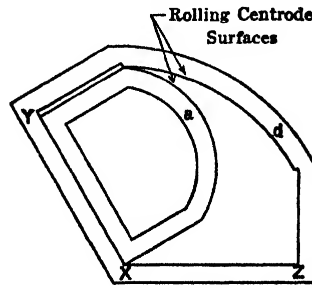


FIG. 3–20

(Fig. 3–17) by two surfaces whose profiles are the two centrodes, one surface being attached to a and the other to d. The motion of a relative to d in Fig. 3–20, obtained by rolling together the centrode surfaces, is the same as for the corresponding links of Fig. 3–17.

Figure 3–20 is an example of the substitution of higher pairing with rolling contact for lower pairing with sliding contact. This principle has been employed in the valve gears of certain gas engines of German manufacture.

QUESTIONS — CHAPTER III

1. Define "instant axis." What convention do we use, for simplicity, when referring to the instant axis of a body having plane motion?

2. Show how to find the instant center for the motion of a body when the directions of motion of two points on it are known.

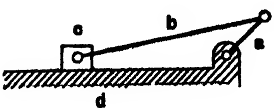
3. Show how to find one position of the instant center of a moving body when the paths of two points on it are known.

4. Prove that when three bodies have plane relative motion the instant centers must lie in one straight line.

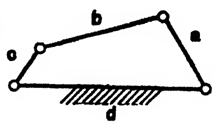
5. Prove that the number of instant centers for n bodies equals

$$\frac{n(n-1)}{2}.$$

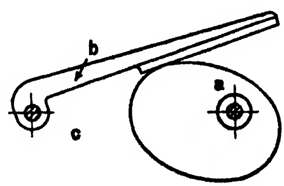
6. Locate all the instant centers in the mechanisms shown in Figs. 6A to 6H. In each case explain the methods employed.



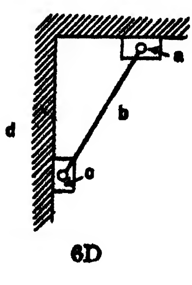
6A



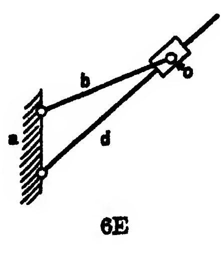
6B



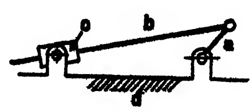
6C



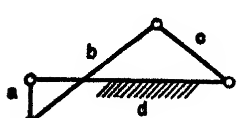
6D



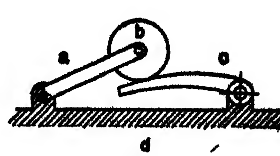
6E



6F

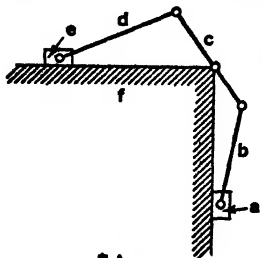


6G

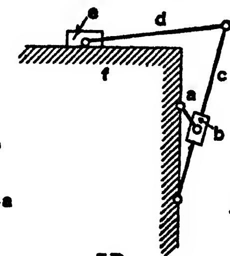


6H

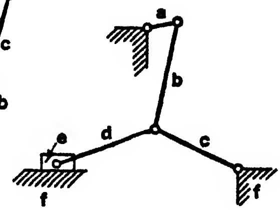
7. In the compound mechanisms shown in Figs. 7A to 7G (*a*) determine the number of instant centers, (*b*) locate all the instant centers.



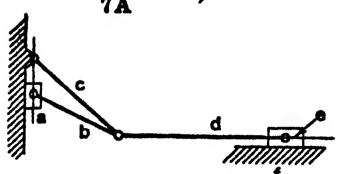
7A



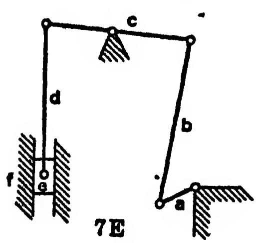
7B



7C



7D



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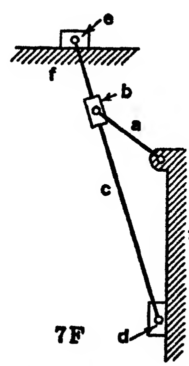

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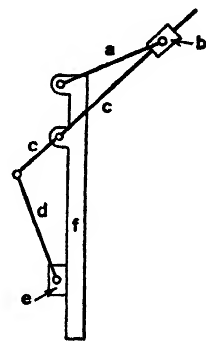
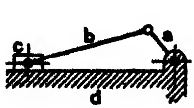
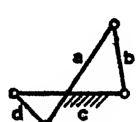


7G

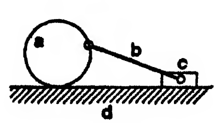
8. Show how to plot the centrodes for the motion of link *b* relative to *d* in the mechanisms shown in Figs. 8A to 8D. (Find at least two points on each centrode.)



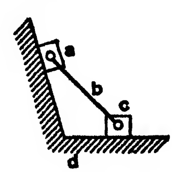
8A



8B



8C

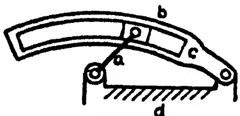


8D

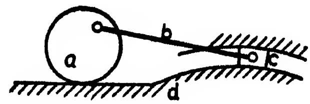
9. What is the valuable property of centrodes as regards their application to the design of mechanisms?

10. Illustrate the manner in which a mechanism can be altered so as to use rolling centrodes instead of turning and sliding pairs.

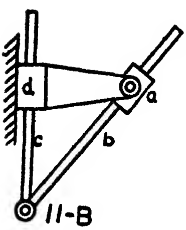
11. Locate all the instant centers in the mechanisms shown in Figs. A to E In each case explain the methods employed.



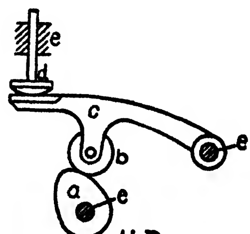
11-A



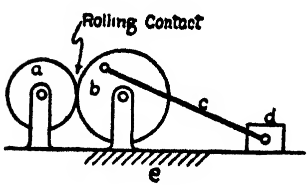
11-C



11-B



11-D



11-E

CHAPTER IV

VELOCITY AND ACCELERATION IN PLANE MOTION

1. Velocities from Instant Centers. Points on One Link.— When a body revolves about a fixed center, as c (Fig. 4–1), which is pin-connected to the fixed frame d, any point on the moving body has a velocity which varies directly as its distance from the center of rotation. The velocity of a point P on c is $V_P = \omega_{cd} \cdot r_P$. Similarly, any other point Q on the same body c, has a velocity $V_Q = \omega_{cd} \cdot r_Q$.

Dividing the two equations,

$$\frac{V_Q}{V_P} = \frac{r_Q}{r_P} \qquad \text{or} \qquad V_Q = V_P \times \frac{r_Q}{r_P}. \tag{4–1}$$

As P and Q are any two points on the body c, it follows that if the velocity of one point on a moving body and the location of the pivot point are known, the velocity of any other point can be found. A simple graphical construction is often found convenient. A vector V_P (Fig. 4–1) is drawn perpendicular to r_P, representing the known velocity of P. With O as center and radius OP, a circle is drawn intersecting OQ, produced if necessary, at S. Since S and P are the same distance from the center of rotation, their velocities are equal in magnitude but differ in direction. The vector ST is drawn perpendicular to OS to represent V_P. Line QW, drawn parallel to ST, forms a triangle OQW, similar to OST. Therefore,

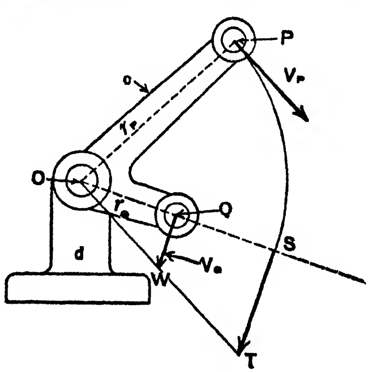


Fig. 4–1

$$\frac{QW}{ST} = \frac{OQ}{OS} = \frac{r_Q}{r_P} = \frac{V_Q}{V_P} \text{ from equation (4–1).}$$

Hence, if ST represents V_P, then QW represents V_Q to the same scale. As instantaneous conditions only have been considered, this graphical construction applies whether the point about which the body rotates is an instant center or a permanent center.

Sometimes the instant center is inaccessible and the above graphical construction becomes impossible. In this case the following alternative is useful:

Let P and Q (Fig. 4–2) represent two points on a body c, in motion with reference to a second body d. The known velocity of P is represented in magnitude and direction by the vector V_P. The instant center is at some inaccessible point O_{cd} which is located at the intersection of perpendiculars to V_P and V_Q.

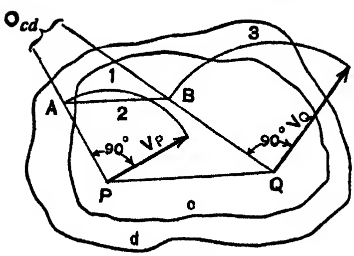


FIG. 4–2

With P as center, arc 1 is swung through 90°, locating the point A on the line $P—O_{cd}$, the normal to the direction of motion of P. From A the line 2 is drawn parallel to PQ meeting $Q—O_{cd}$ at B. With center Q and radius QB, arc 3 is described, subtending 90° at Q. Then it can be shown that vector V_Q represents the velocity of Q. The proof follows.

Since AB is parallel to PQ, triangles $O_{cd}AB$ and $O_{cd}PQ$ are similar. As a result,

$$\frac{O_{cd}—Q}{O_{cd}—P} = \frac{BQ}{AP}.$$

But, by equation (4–1),

$$\frac{V_Q}{V_P} = \frac{O_{cd}—Q}{O_{cd}—P}.$$

Hence,

$$\frac{V_Q}{V_P} = \frac{BQ}{AP}.$$

If V_P is represented in magnitude by AP, then BQ represents V_Q to the same velocity scale.

2. Velocities from Instant Centers. Points on Different Links. — Very frequently it is necessary to find the velocity of a point on a certain link of a mechanism from the known velocity of another point located on a different link. Several methods are usually available, each having advantages for particular cases. A discussion of these methods follows.

(*a*) **Direct Method.** — In applying this method, we **first find the instant center for two links, one containing the point of known velocity and the other containing the point whose velocity is to be determined.** Regarding this instant center as a point where the links in question have a common velocity, we can work through it from one link to the other. For example, in Fig. 4–3 is shown a quadric-crank mechanism in which link *d* is fixed. We shall assume that the velocity of a point *P* on link *a* is known and that we require the velocity of a point *Q* on *c*. Our first step is to find the instant center O_{ac}.

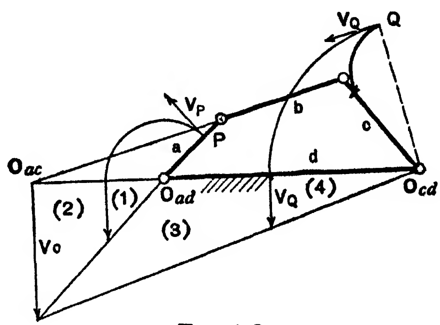


FIG. 4–3

Considering the two points P and O_{ac}: these are points on link *a*, and since the velocity of *P* is known, the velocity of O_{ac} (V_0) can be found graphically by the method of Art. 1. Thus link *a* pivots about O_{ad} and the triangle (1) is drawn, one side representing V_P. Triangle (2) similar to (1) will have a side representing the velocity of O_{ac} ($= V_0$) as indicated. As a point in link *c*, O_{ac} has the same velocity V_0; hence we now know the velocity of one point on *c* and can find the velocity of any other point, such as *Q*. Since link *c* pivots about O_{cd}, we construct the triangle (3) and then draw the similar triangle (4). The latter has a side representing V_Q. This length is set off perpendicular to $Q—O_{cd}$ and represents the velocity of *Q* in magnitude and direction.

The construction can be applied to any form of mechanism, provided that the instant center for the two links on which the points are located is accessible. When it is not accessible (as for example when it is located at infinity), some other method must be used.

(*b*) **Connecting-link Method.** — This is a step-by-step method whereby we start with the link on which is located the point of known velocity, and work through its instant center with respect to a connecting link, then along the connecting link to its instant center with respect to the next link. Continuing in this manner, we finally reach the link containing the point whose velocity is required. In general, it is necessary to begin by **locating all instant centers with respect to the fixed link and the instant centers of each link with respect to its adjacent link.**

For comparison with the direct method, we shall use the same mechanism (Fig. 4–4) as was shown in Fig. 4–3, namely, a quadric-crank mechanism, in which the velocity of P on link a is known and the velocity of Q on c is required. In this example links a and c are connected by link b, and we work through the latter from a to c.

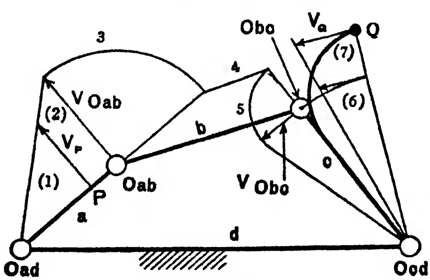


FIG. 4–4

We first locate O_{ad}, O_{ab}, O_{bc}, O_{cd}, as shown in the figure. The vector V_P, at 90° to $P—O_{ad}$, represents the known velocity of P. Considering the two points P and O_{ab} as points on a, we make the construction (by Art. 1) shown by the similar triangles (1) and (2) to find the velocity vector $V_{O_{ab}}$ for the point O_{ab}.

Next, taking the points O_{ab} and O_{bc} (two points on b), we construct arc 3, draw line 4 parallel to b, and swing arc 5 through 90° (by Art. 1), obtaining vector $V_{O_{bc}}$ as the velocity of O_{bc}. O_{bc} and Q are both points on c which pivots about O_{cd}. Drawing the similar triangles (6) and (7), we find vector V_Q representing the required velocity.

In this example, pin joints connect the three links a, b, and c. The connection between these links may, however, take any form and the method can be applied to any mechanism, provided that the needed instant centers are accessible.

The direct method may require the use of instant centers, the location of which requires much labor. For this reason the connecting-link method is sometimes preferable.

3. Linear Velocities by Resolution. — Knowing the magnitude and direction of motion of one point on a moving body, and the direction of motion of a second point on the same body, the magnitude of the velocity of the latter point can be found by resolution. This method depends on the fact that the distance between the two points is constant, if the body is rigid.

Let P and Q (Fig. 4–5) be two points on a body c in motion with respect to body d. The velocity of P is indicated in magnitude and direction by the vector V_P at the instant considered. The point Q has motion in the direction QA at the same instant. The distance PQ is constant, and so the components of the velocities in a direction parallel to PQ must be the same; otherwise the distance between them would increase or decrease. By drawing triangle (1) we find the vector V_0, the component parallel to PQ. Triangle (2) can now be drawn, since the vector V_0', representing the component of Q's velocity parallel to PQ, is equal to V_0; also one side is perpendicular to PQ and the third lies along QA. The side last mentioned is V_Q and represents the velocity of Q.

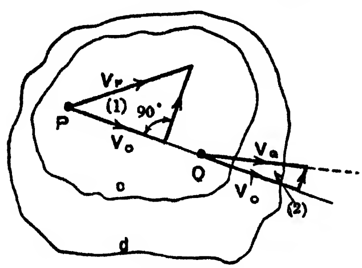


FIG. 4–5

By working from point to point through connecting links, the method of resolution can be used in many cases to find the velocity of any point on a mechanism when the velocity of one point, not necessarily on the same link, is known.

Example 1. — In Fig. 4–6 is illustrated a compound mechanism often used in shapers as a means of driving the ram which carries the cutting tool. The slope of link d has been exaggerated in order to illustrate the construction more clearly.

We shall suppose that the velocity of the point P on the driving crank *a* is known and that the velocity of the point *Q* on the ram *e* is required.

Point *P* on link *a* and a coincident point *P′* on link *c* must have the same velocity normal to the line of slide of *b* on *c*. If this were not the case, *P* would move off the line R—O_{cf}; this is an impossibility, owing to the constraining effect of the sliding pair. If V_P is resolved into two components parallel and normal to R—O_{cf} by drawing triangle (1), then the normal component represents $V_{P'}$, the velocity of point *P′* on *c*.

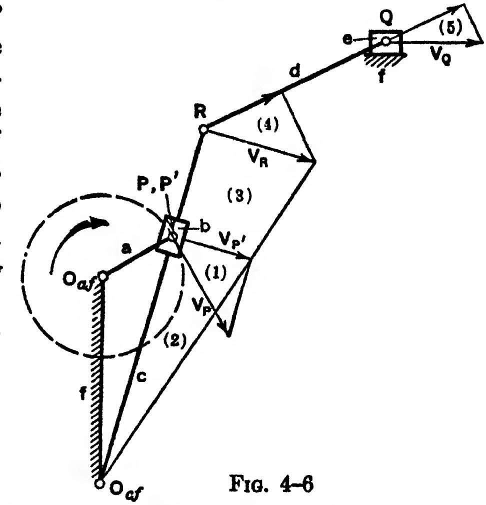


Fig. 4–6

P′ and *R* are two points on the link *c* pivoting about O_{cf}. By use of the graphical construction given in Art. 1 and shown by triangles (2) and (3), we find vector V_R representing the velocity of *R*. The resolution method cannot be used here, because $V_{P'}$ has a zero component along R—O_{cf}.

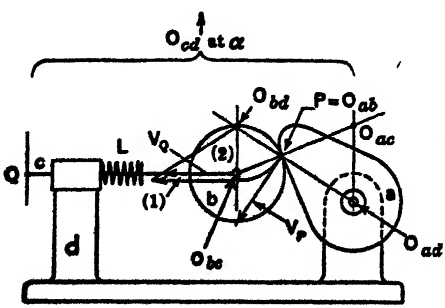


Fig. 4–7

Finally, *R* and *Q* are points on the link *d* and therefore have equal velocity components along *d*. The resolution method, requiring the construction of triangles (4) and (5), fixes the length of the vector V_Q which represents the velocity of *Q*.

Example 2. — In Fig. 4–7 is illustrated a cam mechanism with rolling contact between links *a* and *b*. Constraint is effected by the spring *L* which keeps these two links in contact, though instead this could be provided for by causing b to move in a groove cut on the face of a. Kinematically, the mechanism is the same in either case.

We shall assume that the velocity of any point Q on c is required for the position of the mechanism shown in the figure, when the angular velocity of a relative to the fixed frame d is known.

Links a and b make contact at a point P which is evidently the instant center O_{ab}, because at this point the two links have no relative motion.

In order to use the **Direct Instant-center Method** of Art. 2(a) we require the location of O_{ac}. Centers O_{bc}, O_{ad} are at pivot points; O_{cd} is at infinity. O_{ac} is found by applying Kennedy's Theorem after locating the other centers.

The linear velocities of Q and O_{ac} are the same, since c has rectilinear motion. Also, O_{ac} is a point on a. Therefore,

$$\text{Velocity of } Q = \text{Velocity of } O_{ac} = \omega_{ad} \times (O_{ac}\text{—}O_{ad}).$$

ω_{ad} being known, the velocity of Q can be calculated, and the only graphical construction needed is that involved in the location of O_{ac}.

The **Connecting-link Method** of Art. 2(b) is carried out graphically as illustrated in Fig. 4–7. Center O_{bd} is first located by use of Kennedy's Theorem. Point $P(O_{ab})$ has a linear velocity equal to $\omega_{ad}(O_{ab}\text{—}O_{ad})$, which we represent by the vector V_P perpendicular to the line $O_{ab}\text{—}O_{ad}$. Now, considering points O_{bc} and P as two points on b, which pivots about O_{bd}, the graphical construction shown by triangles (1) and (2) provides a means of finding the velocity of the former point. This point (O_{bc}) has the same velocity as Q, both being represented by the vector V_Q in the figure.

Example 3. — Figure 4–8 shows a mechanism used on the pressure riveter. Link a is the air-cylinder piston which drives the riveting die e through intermediate links b, c, d. We make the assumption that the velocity of a point R on e is required when the velocity of P on a is known.

The Connecting-link Method will be found most suitable; the direct method would require the use of O_{ae}, which is not easily located. The instant centers of b and d with respect to the fixed link f are found by noting the directions of motion of two points on each of these links. Their positions are shown in Fig. 4–8.

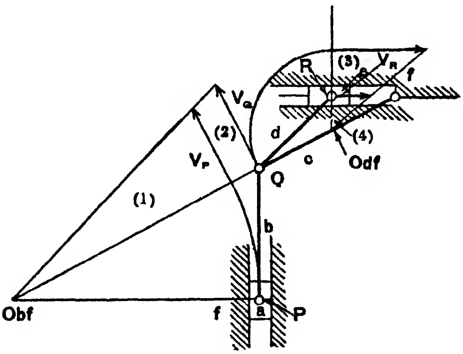


Fig. 4–8

Taking a vector V_P to represent the linear velocity of P and using the two points P and Q on b, we construct similar triangles (1) and (2) with common apex at O_{bf}. Thus the vector V_Q is found.

Points Q and R are points on links c. Similar triangles (3) and (4) with common apex at O_{df} are therefore drawn, determining the vector V_R, which represents the required velocity of R.

4. Angular Velocities. — When two bodies are in motion it can be shown that their instantaneous angular velocities with respect to a third body are inversely as the distances from their instant center to the instant centers about which they are pivoting on the third body. Thus, in Fig. 4–9, a and b are two bodies in motion with respect to c. The three instant centers O_{ac}, O_{ab}, O_{bc} are assumed to be located as shown in the figure, lying in one straight line in accordance with Kennedy's Theorem.

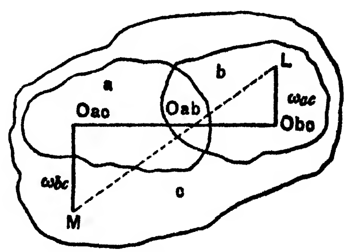


Fig. 4–9

O_{ab} is a point common to a and b. As a point in a, its instantaneous linear velocity is $\omega_{ac}(O_{ab}—O_{ac})$. As a point in b, O_{ab} is moving with a linear velocity $\omega_{bc}(O_{ab}—O_{bc})$. Therefore,

$$\omega_{ac}(O_{ab}-O_{ac}) = \omega_{bc}(O_{ab}-O_{bc}),$$

or

$$\frac{\omega_{ac}}{\omega_{bc}} = \frac{O_{ab}-O_{bc}}{O_{ab}-O_{ac}}. \qquad (4\text{–}2)$$

When one of these angular velocities is known the other can be determined graphically. The construction is indicated in Fig. 4–9. Suppose ω_{ac} is known and ω_{bc} is to be determined. Draw O_{bc}—L perpendicular to O_{bc}—O_{ac} of a length representing ω_{ac}. Join L—O_{ab} and produce this line to meet O_{ac}—M, parallel to (O_{bc}—L). From similar triangles,

$$\frac{O_{ac}-M}{O_{bc}-L} = \frac{O_{ab}-O_{ac}}{O_{ab}-O_{bc}} = \frac{\omega_{bc}}{\omega_{ac}}.$$

Hence, O_{ac}—M represents ω_{bc} to the same scale as O_{bc}—L represents ω_{ac}.

When O_{ab} falls between O_{ac} and O_{bc}, the bodies a and b turn in opposite senses; but when O_{ab} is on O_{ac}—O_{bc} produced, the bodies a and b turn in the same sense.

Example. — Figures 4–10 and 4–11 show the same slider-crank mechanism in two positions. In each case, assuming that the angular velocity of the crank a (ω_{ab}) is known, show how to find graphically the angular velocity of link d (ω_{db}).

Construction. — Find the three instant centers for links a, d, and b (the fixed link). These centers lie on one straight line by

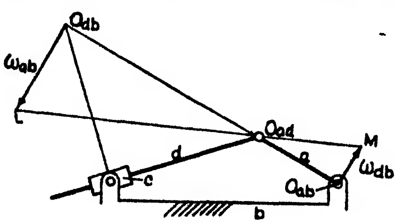


Fig. 4–10

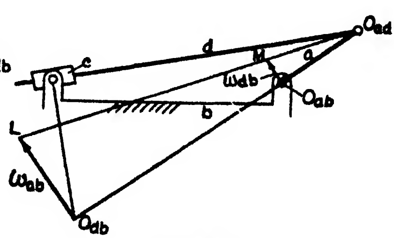


Fig. 4–11

Kennedy's Theorem. Draw the triangle L—O_{db}—O_{ad} in which O_{db}—L, perpendicular to O_{db}—O_{ad}, represents the known angular velocity ω_{ab}. Draw O_{ab}—M, parallel to O_{db}—L, meeting L—O_{ad}, produced if necessary, at M. Then O_{ab}—M represents the desired angular velocity ω_{db}.

5. The Image Method. — A graphical method of determining velocities and accelerations of points in mechanisms which has proved to have wide application and very considerable practical importance will now be considered. This is generally known as the "Image Method." It is given by Professor Burmester in his "Kinematik." The construction of the acceleration diagram often requires the prior determination of certain velocities, hence the latter problem will be taken up first.

6. The Velocity Image. — If there are two points A and B on a body in plane motion, then the absolute velocity of B is equal to the vectorial sum of (1) the relative velocity of B with respect to A and (2) the absolute velocity of A. The Velocity-Image method is based on this statement.

Suppose in Fig. 4–12 A and B represent two points on the same body. Assume that the absolute velocity V_A of A and the

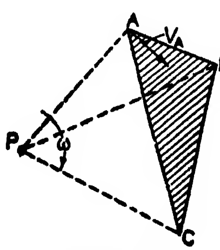


FIG 4–12

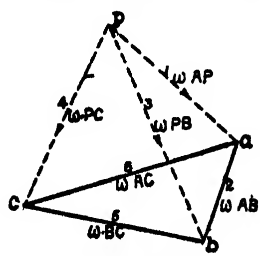


FIG. 4–13

angular velocity ω of the body are known. The instant center P of the body with respect to a fixed plane can be easily located since it lies on line PA perpendicular to the direction of motion of point A, and the length PA is equal to $V_A \div \omega$. If we now require the velocity of B, we find its direction at 90° to PB and its magnitude is $\omega \cdot PB$.

Instead of finding V_B through use of the instant center, we may draw a Velocity-Image diagram as shown in Fig. 4–13. From a point or "pole" p, the line pa is drawn at 90° to PA, representing V_A (equal to $\omega \cdot PA$) to any convenient velocity scale. The relative velocity of B to A (V_{BA}) is equal to $\omega \cdot AB$ in a direction at 90° to AB. This must be true since AB is a fixed length and ω

the angular velocity of the body. Therefore to find the vectorial sum of V_A and V_{BA}, from point a in Fig. 4–13, lay off a distance ab at 90° to AB, representing $\omega \cdot AB$ to the selected velocity scale. Join pb. Then pb will represent V_B to the same scale as chosen above, since by construction the velocity triangle pab is similar to the body triangle PAB, and hence if $pa = \omega \cdot AP$ then $pb = \omega \cdot PB = V_B$.

If we take a third point C on the same body (Fig. 4–12) then since the relative velocity of C to B must be in a direction at right angles to CB and the relative velocity of C to A lies at right angles to CA, point c on the velocity diagram is found by drawing bc at 90° to BC and ac at 90° to AC. The intersection c is joined to point p. It is evident from the similarity of the Figs. 4–12 and 4–13 that $pc = \omega \cdot PC = V_C$.

It will be noted that the velocity triangle abc has sides ab, ac, bc respectively perpendicular to the corresponding sides AB, AC, BC of the body triangle. The velocity triangle is in fact the body triangle drawn to another scale and turned through 90° in the sense of rotation of the body. Hence the name "Velocity Image."

Example. — In the Quadric-crank Mechanism of Fig. 4–14, link AB rotates with a constant angular velocity ω. It is required

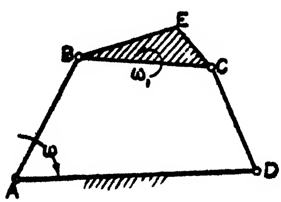


Fig. 4–14

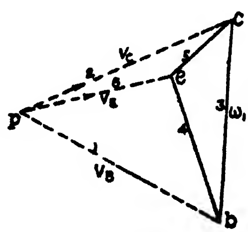


Fig. 4–15

to find the absolute velocities of points B, C, and E on the adjacent link.

Considering the link BC, the velocity of B can be calculated: it is equal to $\omega \cdot AB$ in a direction at 90° to AB. Also, we know that point C has absolute motion in a direction at 90° to CD, and that the relative motion of C with respect to B is at 90° to BC. Hence from pole p, Fig. 4–15, draw $pb = V_B$, to a convenient scale, perpendicular to AB. Next find point c at the intersection of lines pc and bc respectively perpendicular to CD and BC. Then pc is the absolute velocity of C.

To find V_E, on bc as a base construct the triangle bce similar to BCE. It is possible to construct two such triangles, but bce correctly drawn should occupy a position at 90° to BCE in the sense of rotation of BC. Inspection of the mechanism shows that when AB turns clockwise, BC turns counter-clockwise, and therefore e is located as shown, to the left of bc. If points p and e be joined, pe is equal to the absolute velocity of E. If desired, the angular velocity ω_1 of BC can now be calculated, since $bc = \omega_1 \cdot BC$ or $\omega_1 = bc \div BC$, where bc is measured in velocity units and BC is full size. The numbers on the lines of the velocity diagram indicate the order in which the lines are drawn.

7. The Acceleration-Image Method. — This is based on a principle concerning accelerations similar to that underlying the Velocity-Image construction, which may be stated as follows:

For two points on a body in plane motion, the absolute acceleration of the second is equal to the vectorial sum of the acceleration of the second relative to the first and the absolute acceleration of the first.

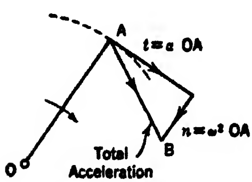


FIG. 4–16

As pointed out in Chapter II a point A (Fig. 4–16) on a body moving about an instant center O is subject to a tangential acceleration t acting in the direction of motion, and a normal acceleration n acting toward the center of rotation, where

$$t = \alpha \cdot OA$$

and

$$n = \omega^2 \cdot OA$$

ω and α being respectively the angular velocity and the angular acceleration of the body. Distance AB representing the total acceleration is equal to the vectorial sum of t and n.

The assumption that O is the instant center with respect to motion over a fixed surface means that O is a fixed point, and hence AB is the absolute acceleration of A. If, on the contrary, A and

O are both in motion then AB is the relative acceleration of A with respect to O.

In Fig. 4–17 A, B, C are three points on the same body which has a known angular velocity ω and angular acceleration α. Assuming that the velocity V_A of one point A is also known, we may

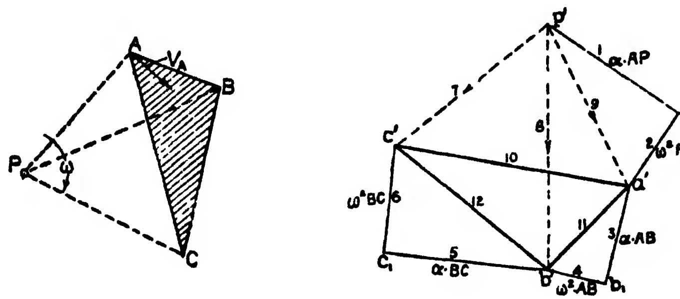


FIG. 4–17

proceed to find the absolute accelerations of all three points. The construction is made as follows:

Locate the instant center P by drawing AP in a direction perpendicular to the direction of motion of A, of length equal to $V_A \div \omega$. Calculate the value of $\alpha \cdot AP$, the tangential acceleration of A, and draw from a pole p' a line $p'a_1$ at 90° to AP representing $\alpha \cdot AP$ to a convenient acceleration scale.

Calculate the value of $\omega^2 \cdot AP$, the normal acceleration of A, and from a_1 draw a line a_1a', parallel to AP, to represent this acceleration. Join $p'a'$. Then $p'a'$ represents the absolute acceleration of A. The relative acceleration of B to A is next determined. This requires the calculation of $\alpha \cdot AB$ and $\omega^2 \cdot AB$. Starting from a' lines $a'b_1$ and b_1b' are drawn respectively perpendicular and parallel to BA. Point b' is joined to p' and the line $p'b'$ represents the absolute acceleration of B.

Similarly by calculation of $\alpha \cdot BC$ and $\omega^2 \cdot BC$ and by drawing lines $b'c_1$ and c_1c' to represent these quantities c' is located. Joining p' to c' we obtain a length representing the absolute acceleration of C.

From the geometry of the figure it can easily be shown that the triangle $a'b'c'$ is the body figure ABC altered in scale and rotated through an angle $\left[90 + \tan^{-1}\frac{\omega^2}{\alpha}\right]^\circ$ in the same sense as the angular velocity. Hence the name "Acceleration Image."

In constructing the acceleration diagram, care should be exercised to insure that the normal acceleration vector for each point is drawn in a direction **toward** and not away from the other point to which the acceleration is referred. The tangential acceleration line must be drawn in the direction opposite to that of the velocity of the point if the angular acceleration is negative in character, that is, when the angular velocity is decreasing. Thus, in Fig. 4–17, if α were negative or opposite in sense to ω, then $p'a_1$ must be drawn in the reverse direction to that shown in the figure.

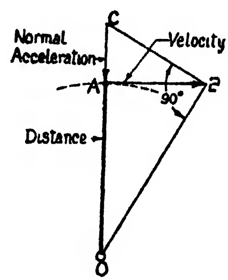


FIG. 4–18

8. Graphical Calculation of the Normal Acceleration. — When the velocity of a point relative to a second point on the same body is known, also its distance from the other point, then the relative normal acceleration may be found graphically. In Fig. 4–18, let AO represent the distance (S) between points A and O to a scale of $1'' = k$ feet. Also let AB at 90° to AO represent the velocity (V_{AO}), to a scale $1'' = m$ feet per second. Thus $S = k \cdot AO$ feet and $V_{AO} = m \cdot AB$ feet per second, where AO and AB are in inch units.

Now the relative normal acceleration of A to O is

$$\frac{(V_{AO})^2}{S} = \frac{(m \cdot AB)^2}{k \cdot AO} = \frac{m^2}{k} \cdot \frac{(AB)^2}{AO}.$$

In Fig. 4–18 draw BC perpendicular to BO, meeting OA produced at C. From the similarity of triangles CAB and BAO

$$\frac{CA}{AB} = \frac{AB}{AO} \quad \text{or} \quad CA = \frac{(AB)^2}{AO}.$$

The normal acceleration of A is therefore equal to

$$\frac{m^2}{k} \cdot CA.$$

In other words, CA represents the acceleration to a scale of 1″ $= n$ feet per second per second where $n = \frac{m^2}{k}$ or $m = \sqrt{k \cdot n}$.

The example which follows will indicate the method of using this graphical construction in drawing an acceleration diagram.

Example 1. — In the Slider-crank Mechanism of Fig. 4–19, the angular velocity of the crank AB is constant and equal to ω. Find the accelerations of three points B, C, and D on the connecting rod.

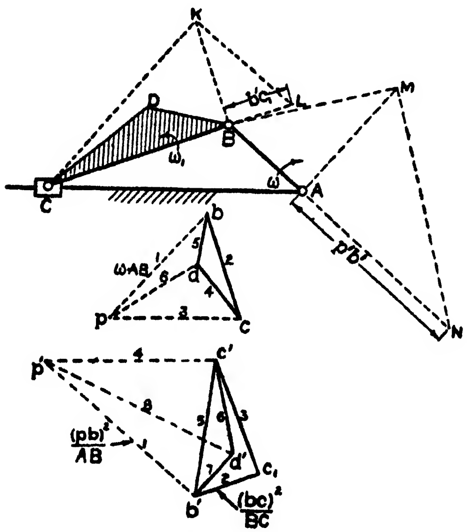


FIG. 4–19

The velocity diagram will be considered first, since it is needed in constructing the acceleration diagram.

To make it possible to find the normal accelerations by the graphical method just outlined, the scale for the velocity curve must be 1″ $= \sqrt{k \cdot n}$ velocity units. Generally it is best to fix upon suitable values for the displacement and acceleration scales and then calculate the velocity scale.

Point B is moving in a direction at 90° to AB with a velocity $\omega \cdot AB$ which must be calculated. C has absolute velocity along CA, and velocity relative to B in a direction normal to BC. With these data we draw the triangle pbc. The velocity image of DBC is the similar triangle dbc, advanced 90° in the sense of rotation of BC. Inspection of the figure will show that BC is turning counter-clockwise when AB turns clockwise. The velocity figure is completed by drawing the straight line pd.

Pole p' is the starting point for the acceleration diagram. Point B has no tangential acceleration, its total acceleration being equal to $\omega^2 \cdot AB$ acting toward A. The value of this quantity is found graphically by drawing AM normal to AB and equal to pb and then completing the right angled triangle BMN from which the length AN, equal to $(pb)^2 \div AB$ or $\omega^2 \cdot AB$, is determined. On the acceleration diagram we draw $p'b'$ equal to AN in a direction parallel to BA.

The normal acceleration of C relative to B is next found by means of the right angled triangle CKL in which $BK = bc$. Distance BL is equal to $(bc)^2 \div BC$. Hence $b'c_1$ equal to BL is drawn on the acceleration diagram.

Point C has straight line motion along CA and therefore in regard to the fixed link its only acceleration is a tangential one along CA. Point C has, however, a tangential acceleration relative to B acting normal to BC. Hence from p' draw a line parallel to CA and from c_1 a line perpendicular to BC. The intersection locates the point c'. The diagram is completed by drawing the triangle $b'c'd'$ similar to BCD and joining $p'd'$. For the position of the mechanism in Fig. 4–19, the angular acceleration of the connecting rod is such as to reduce its angular velocity, hence α has a negative sign. By reference to the expression for the angular relationship given in Art. 7 it follows that $b'c'd'$ occupies an angular position corresponding to the rotation of body BCD through an angle somewhat greater than 180°, in a sense the same as the angular velocity of that body.

The absolute accelerations of points B, C, D are represented by the dotted lines $p'b'$, $p'c'$, $p'd'$.

Example 2. — Fig. 4–20 shows a cam mechanism with a pivoted roller follower. The cam profile is in the form of a circular arc with center at B. It is required to find the angular velocity and angular acceleration of the follower, assuming a constant angular velocity of the cam.

The distance BC from center of curvature of the cam profile to center of the roller is constant. Hence the equivalent mechanism is a Quadric-crank Mechanism with links AB, BC, CD and DA (fixed).

The velocity triangle pbc has sides respectively perpendicular to AB, BC and CD. Length pb is equal to $\omega \cdot AB$ where ω is the angular velocity of the cam. Length pc represents the velocity of C. The angular velocity of the follower is equal to $pc \div CD$ when pc is expressed in velocity units and CD is the real length of the arm.

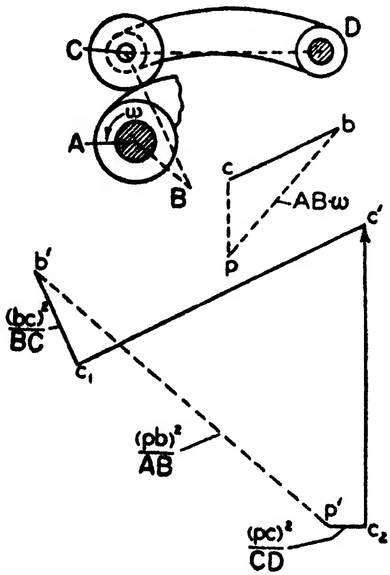


Fig. 4–20

For the acceleration diagram, the lines are drawn in the order shown by figures on the diagram starting from pole p'.

The three normal accelerations $p'b'$, $b'c_1$ and $p'c_2$ may be either calculated or found graphically. In the latter case the scales of displacement, velocity and acceleration must bear to one another the relationship stated in Art. 8. Line $p'b'$ is drawn parallel to BA, $b'c_1$ parallel to CB and $p'c_2$ parallel to CD. Point c' is found by drawing line c_1c' at 90° to CB and c_2c' at 90° to CD. The required angular acceleration of the follower is equal to the tangential acceleration of C, namely c_2c', divided by the length CD.

The Acceleration Image method can be applied to all cam mechanisms which have Quadric-crank or Slider-crank mechanisms as their kinematic equivalent. Thus it can be used for circular arc cams with both pivoted and reciprocating followers.

Example 3. — Fig. 4–21 represents the linkage for two cylinders of a nine cylinder radial engine employing articulated connecting rods. AB is the master connecting rod and ACD one of the articulated rods, the point C representing a pin joint where CD is attached to the master rod. The crank is assumed to rotate clockwise with uniform angular velocity, and the Image Diagrams will be used to find the velocities and accelerations of the pistons at B and D.

Velocity Diagram. — Line pa has a length equal to $\omega \cdot OA$ to the scale selected. Its direction is perpendicular to OA. Vector pb, parallel to OB, and ab perpendicular to AB complete the triangle pba. Triangle abc, similar to ABC, turned 90° in the sense of rotation of AB, is next constructed. Line cd at 90° to CD and pd parallel to DO intersect at d. Then the velocities of the pistons at B and D are equal respectively to pb and pd to scale.

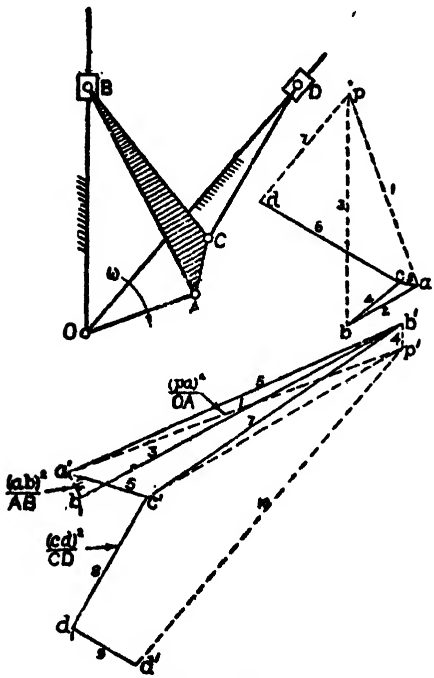


FIG. 4–21

Acceleration Diagram. — Line $p'a'$ parallel to AO is equal to the normal acceleration of A about O, of value $(pa)^2 \div OA$. Its length may either be calculated or obtained graphically as described in Art. 8. In the latter case the scale relationship must be as specified in that article. Line $a'b_1$ of length $(ab)^2 \div AB$, calculated or graphically determined, is next drawn. From b_1 draw b_1b' at 90° to AB and from p', $p'b'$ parallel to OB. The intersection is b'. Join $a'b'$ and using this line as a base, construct a triangle $a'b'c'$, the acceleration image of ABC. Calculate or find graphically the relative normal acceleration of D to C, equal to $(cd)^2 \div CD_1$ and draw $c'd_1$. The tangential acceleration of D to C acts at 90° to CD, and D has absolute total acceleration along DO. Hence point d' is found by the intersection of a line from d_1 perpendicular to CD and a line from p' parallel to DO.

The accelerations of pistons at B and D are respectively equal to $p'b'$ and $p'd'$ to scale.

QUESTIONS — CHAPTER IV

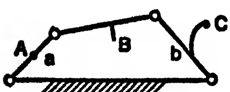
1. State the relationship between the linear velocities of two points on a moving link whose instant center is known.

2. Two points are located on different links of the same mechanism, the instant center of the links being known. When we know the velocity of one point, what property of the instant center is used in finding the velocity of the second point?

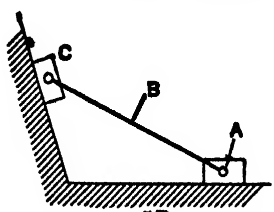
3. Show how to find by the resolution method the velocity of a point *B* which moves in a known direction, it being assumed that the velocity of a second point *A* on the same body is known both in magnitude and direction.

4. When two bodies are so connected as to have relative rectilinear motion, what components of their linear velocities with respect to a third body have equal value?

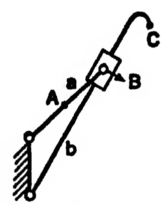
5. In Figs. 5A to 5G, assume a known velocity for the point *A* and show how to find graphically the velocities of points *B*, *C*. Mark the positions of all instant centers used in the constructions you employ.



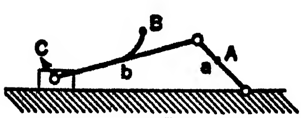
5A



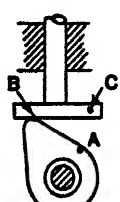
5B



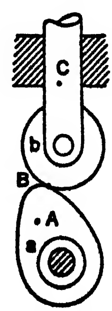
5C



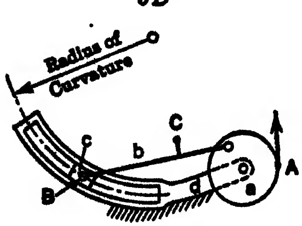
5D



5E



5F

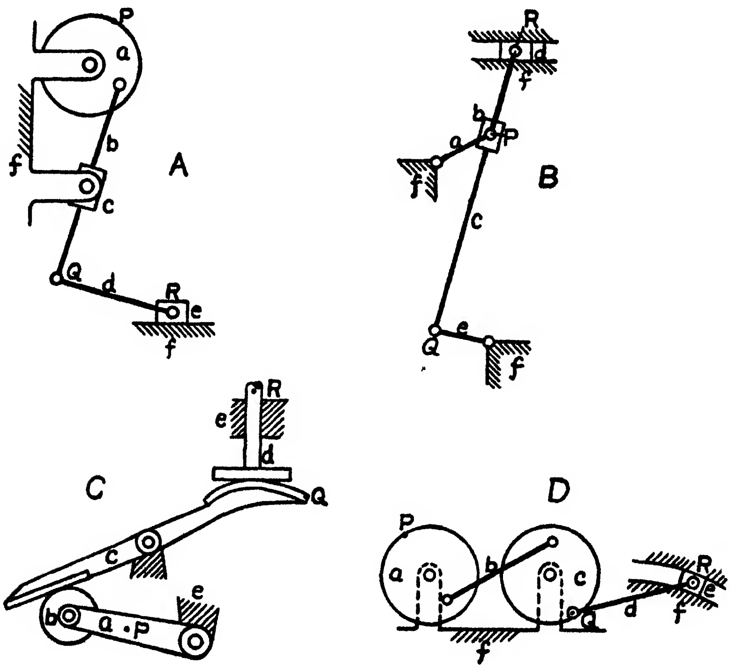


5G

6. In Figs. 5A, 5C, 5D, 5F and 5G assume a known angular velocity for link *a* and find the angular velocity of link *b*.

7. (*a*) In Figs. A to D, assume a known linear velocity for point P, and show how to find graphically the velocities of points *Q* and *R*.

(*b*) In Figs. A to D, assume a known angular velocity for link *a*, and show how to obtain graphically the angular velocities of links *b* and *c*.



8. In Fig. 5A, assume that link *a* rotates at a known angular velocity in a clockwise sense. By use of the Image method, show how to find graphically (1) the linear velocity and (2) the linear acceleration of the point *B*.

9. Using Fig. 5D, find by the Image method the linear velocity and linear acceleration of the point *B*, assuming that the link *a* has a known angular velocity in a counterclockwise sense.

CHAPTER V

SLIDER-CRANK MECHANISMS

1. General. — The uses of the **slider-crank mechanism** in its various forms are so many and important that it merits careful consideration. It can be described as a simple four-link mechanism with plane relative motion among its parts, three pairs of constraining surfaces being pin joints and the fourth a slider and guide allowing relative rectilinear movement of a pair of adjacent links.

Figures 5–1, 5–2, and 5–3 show a process of development of the slider-crank mechanism from a quadric-crank mechanism. Figure 5–1 illustrates a quadric-crank mechanism; Fig. 5–2 shows a device derived from it by altering the form of the constraining surfaces.

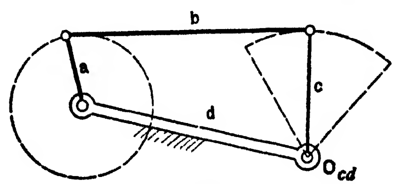


FIG. 5–1

The pin joint between links *c* and *d* in Fig. 5–1 has been altered to a block and circular slotted guide in Fig. 5–2. If, however, the mean radius of the slot in *d* is made equal to the length of *c* in the former mechanism, the movements of corresponding links in both are identical. The material pin at O_{cd}, about which *c* moves with respect to *d* in the quadric mechanism, is replaced by an imaginary pivot O_{cd} in the latter.

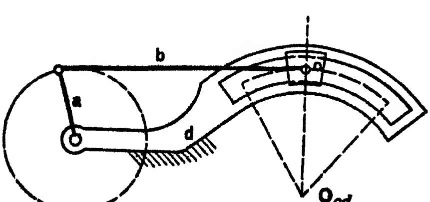


FIG. 5–2

If the linkage is further altered by giving the slot in *d* an infinite radius, so that O_{cd} moves out to infinity, it becomes a common form of the slider-crank mechanism as illustrated in Fig. 5–3.

The slider crank having four links, any one of which may be fixed, four inversions are possible. These inversions will be taken up in detail in the paragraphs which follow.

2. First Inversion. Sliding Block Linkage. — In this mechanism, shown in Fig. 5–3, link *d* becomes the stationary member. As applied to reciprocating engines, *d* is the frame, *a* the crank, and *b* the connecting rod. Link *c* is the piston in some engines having no crosshead; in others it consists of the crosshead, piston rod, and piston, since these parts move as a single rigid piece of material.

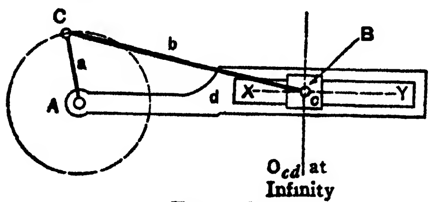


Fig. 5–3

The mechanism is said to be " offset " when (as in Fig. 5–3) the straight line *XY*, which is the path of motion of the point *B*, does not pass through point *A*.

The crank, in practical engines employing this mechanism, generally rotates with an angular velocity which is approximately constant. For purposes of design it is necessary to analyze the velocity and acceleration of the piston. The analysis is commonly made under the assumption that the crank velocity is exactly constant, the error involved being of small proportions.

3. Piston Velocity. Graphical Method. — The Direct Instant Center method as described in Art. 2, Chapter IV, may be used to find the piston velocity when the crank-pin velocity is known. However, the alternative method shown in Fig. 5–4 is shorter and generally more convenient. The construction in this figure is as follows:

The center line of the connecting rod *b* is produced to meet at *D* a line *AD* drawn in a direction perpendicular to the line of stroke. It can be shown that the distance *AD* represents the piston velocity to the same scale as the crank line *AC* represents the crank-pin velocity. This statement is proved as follows:

Produce *AC* to meet at *E* a line *BE* drawn perpendicular to the path of *B*. Then *E* is O_{bd}, and hence

$$\frac{\text{Linear velocity of } B}{\text{Linear velocity of } C} = \frac{EB}{EC}.$$

Also $\dfrac{EB}{EC} = \dfrac{AD}{AC}$ (from similarity of triangles BEC and CDA).

Therefore,

$$\frac{V_B}{V_C} = \frac{AD}{AC} \quad \text{or} \quad V_B = \frac{V_C}{AC} \times AD. \tag{5-1}$$

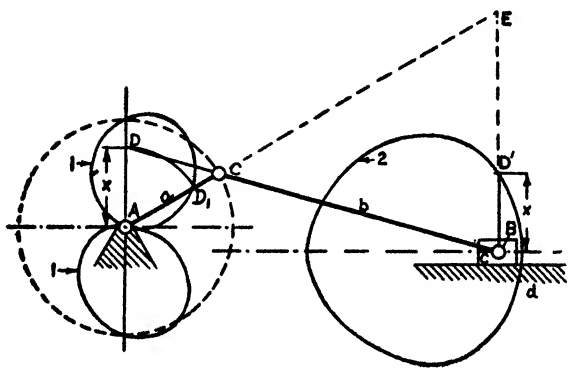


FIG. 5-4

Now V_C is the crank-pin velocity, which is constant when the crank rotates at a uniform speed. AC, also, is a fixed length.

Consequently, we may write

$$\text{Piston velocity} = V_B = \text{a constant} \times AD.$$

When AD has a length of 1 in., $V_B = \dfrac{V_C}{AC} \times 1$; that is, 1 in. represents $\dfrac{V_C}{AC}$ units of velocity. As an easy way of remembering the scale we may note that **the piston velocity is represented by the length *AD* to the same scale as the crank length *AC* on our drawing represents the crank-pin velocity.**

A Polar Curve of piston velocity on a crank-angle base is shown at (1) in Fig. 5-4. Point D_1 on this curve is found by making the length AD_1 equal to AD.

A Velocity-displacement Curve is also drawn at (2) in Fig. 5-4. Point D' on this curve corresponds to the initial position of the mechanism and is found by erecting an ordinate BD' equal to AD.

A Velocity-time Curve (Fig. 5–5) is constructed by plotting the same velocity ordinates on a base on which equal crank angles are represented by equal spaces; time and angular crank displacement are proportional to each other and, the crank velocity being constant, the same base will serve for both. Thus, distance x in Fig. 5–5 is made equal to the length similarly marked in Fig. 5–4.

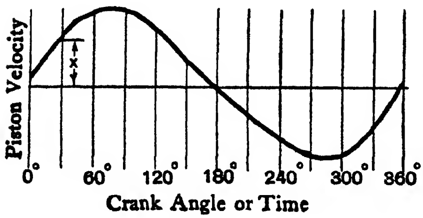


FIG. 5–5

4. Characteristics of Piston Motion. — Figure 5–6 shows the **Velocity-displacement Curve** for the piston motion in an engine with no offset. It will be observed that the maximum velocity is attained somewhat before mid-stroke, when the piston is moving away from the out dead center, the curve being unsymmetrical about a vertical line at half stroke but symmetrical about the horizontal axis. When offset exists, as in Fig. 5–4, it is unsymmetrical in both ways. Point F (Fig. 5–6), the projection of the crank-pin center on the line of stroke, has simple harmonic motion when the crank rotates with uniform velocity.

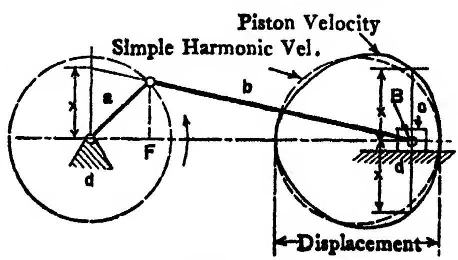


FIG. 5–6

The curve (a circle) drawn in broken lines represents the velocity of F (see Art. 9, Chapter II). This curve differs somewhat from the piston-velocity curve. If the connecting rod always made a constant angle with the line of stroke, its projection BF on that line would have a constant length. That is, points B and F would have equal velocity at all times; the piston would move with simple harmonic motion. Since this is not the case, B cannot move with simple harmonic motion. The larger the angular movement of the connecting rod, the greater the variation from simple harmonic motion becomes. By increasing the length of the connecting rod in proportion to the crank length we decrease the angular displacement of the former, and the piston motion tends to approach simple harmonic. If the connecting rod were of infinite length this condition would be exactly attained.

This distortion of the piston motion from simple harmonic has been aptly termed the "connecting-rod effect." The design of valve gears and the balancing of engines would be much simplified if it did not exist. By reference to Fig. 5–6 it will be seen that it tends to increase the piston velocity during periods before and after the crank passes the out dead center and has the opposite effect during the other portions of the stroke. Maximum piston velocity is attained somewhat before half stroke.

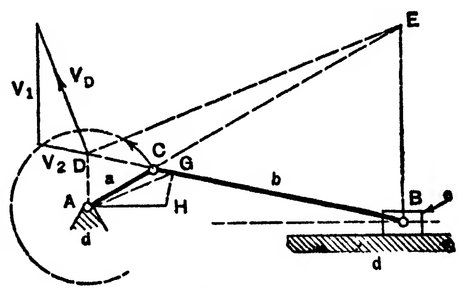


FIG. 5–7

5. Piston Acceleration. Graphical Method. — A line whose length represents the piston acceleration may be obtained as shown in Fig. 5–7. Points D and E are, respectively, the instant centers O_{ac} and O_{bd} and these are first located. From A, the crank pivot, a line AG is drawn parallel to DE. At G a line GH, perpendicular to the connecting rod, is erected. This meets AH parallel to the line of stroke of the piston at H. It can be shown that the distance AH represents the piston acceleration to the same scale as the distance AC represents the normal acceleration of the crank-pin center. That is,

$$\frac{\text{Piston acceleration}}{\text{Crank-pin acceleration}} = \frac{AH}{AC},$$

or

$$\text{Piston acceleration} = \frac{V_C^2}{r} \times \frac{AH}{AC},$$

r being the crank length. If our diagram is drawn full size, $r = AC$ and the piston acceleration is equal to

$$\frac{V_C^2}{(AC)^2} \times AH = \omega^2 \times AH,$$

where ω = angular velocity of the crank. Proof of these statements will be found in the next article.

6. Proof for Graphical Piston-acceleration Construction.[1] — Point D (Fig. 5–7), for any position of the mechanism, lies along the straight line BC. Considering C and D as two points on link b, which pivots for the instant about E,

$$\frac{V_D}{V_C} = \frac{ED}{EC}$$

and, since triangles EDC and AGC are similar,

$$\frac{ED}{EC} = \frac{AG}{AC}.$$

Therefore,

$$V_D = V_C \times \frac{AG}{AC}. \qquad (5\text{–}2)$$

It was shown in Art. 3 that the piston velocity is equal to $V_C \times \frac{AD}{AC}$. The piston acceleration, or rate of change of piston velocity, is equal to the rate of change of the quantity $V_C \times \frac{AD}{AC}$. But because V_C and AC are constant, we may write

$$\text{Piston acceleration} = \frac{V_C}{AC} \times (\text{rate of change of } AD).$$

The distance AD changes only through motion of the point D in a direction perpendicular to the line of stroke, i.e., along AD. The rate of change of length AD is consequently equal to the rate of

[1] The following method of proof is taken from "Kinematics of Machines" by R. J. Durley.

movement or velocity of D in this direction, provided that our figure is drawn full size. Taking a vector V_D at 90° to ED, representing the velocity of D as a point on link b, and drawing the velocity triangle as indicated in the figure, in which component V_1 is perpendicular to the line of stroke and V_2 lies along BC, the former (V_1) is the rate of change of length AD. We can therefore write

$$\text{Piston acceleration} = \frac{V_C}{AC} \times V_1. \qquad (5\text{–}3)$$

In the triangle of velocities just mentioned, the three sides are respectively perpendicular to the three sides of the triangle AHG, whence

$$\frac{V_1}{V_D} = \frac{AH}{AG} \qquad \text{or} \qquad V_1 = V_D \times \frac{AH}{AG}. \qquad (5\text{–}4)$$

Substituting in equation (5–4) the value of V_D from equation (5–2),

$$V_1 = V_C \times \frac{AG}{AC} \times \frac{AH}{AG} = V_C \times \frac{AH}{AC}.$$

Taking equation (5–3) and writing in this value of V_1,

$$\text{Piston acceleration} = \frac{V_C}{AC} \times V_C \times \frac{AH}{AC}$$

$$= \frac{V_C^2}{(AC)^2} \times AH = \omega^2 \times AH, \qquad (5\text{–}5)$$

where ω = angular velocity of the crank.

These results are obtained on the assumption that the diagram is full size. This being the case, the acceleration scale is 1 in. = ω^2 in. per sec. per sec., where AH is measured in inches and ω is in radians per second. When the diagram is drawn to a distance scale 1 in. = n in. the acceleration scale must be n times as large, i.e., 1 in. = $\omega^2 \cdot n$ in. per sec. per sec., or 1 in. = $\dfrac{\omega^2 n}{12}$ ft. per sec. per sec.

7. Klein's Construction. — Instant centers required for the construction given in Art. 5 are sometimes inaccessible, and some other method must then be used.

From the geometry of Fig. 5–7 it will be seen that the point G on the connecting rod is so located that

$$CG : CD = CA : CE = CD : CB$$

or
$$(CD)^2 = CG \cdot CB. \tag{5-6}$$

Any construction for finding G that will give this relationship is satisfactory. The well-known **Klein's Construction,** shown in Fig. 5–8, is one of the best for this purpose.

In Fig. 5–8 point D is found, as in Fig. 5–7, by producing BC to meet AD perpendicular to the line of stroke of the piston. A

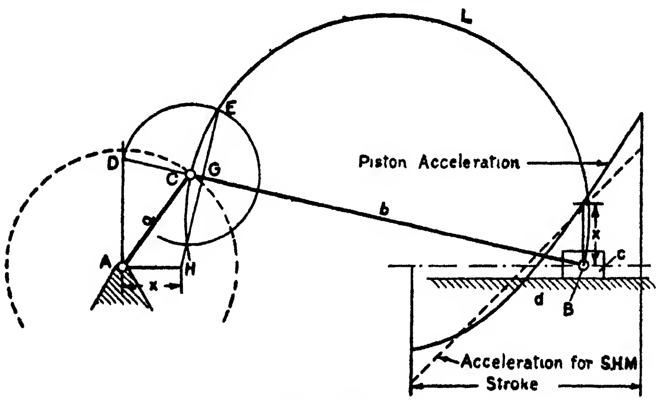


Fig. 5–8

semicircle CLB is then drawn, with CB as a diameter. This is intersected at E by an arc drawn with C as center and CD as radius. From E a line EGH perpendicular to BC is drawn, meeting at H a line AH parallel to the line of stroke. Then G and H are the same points as found by the construction of Fig. 5–7. This must be the case, since when CE and EB are joined two similar triangles CEG and CEB are formed and

$$CG : CE = CE : CB.$$

But

$$CE = CD.$$

Therefore,

$$(CD)^2 = CG \cdot CB.$$

Hence the Piston Acceleration is equal to $\omega^2 \times AH$ as in Fig. 5–7.

8. Piston Velocity and Acceleration. Analytical Method.— Although the graphical method of analysis is usually to be preferred, there are cases where an analytical method is necessary. The case of a mechanism with no offset will be considered. In Fig. 5–9, let r be the crank length and nr be the connecting-rod length, n being the ratio of connecting-rod length to crank length. Suppose the crank to be at any angle θ with the line of stroke, and ϕ the corresponding inclination of the connecting rod. x is the distance from the center of the crosshead pin to the center of the crank shaft. At mid-stroke, evidently, $x = nr$. At any crank angle θ the piston displacement from mid-position $= x - nr$.

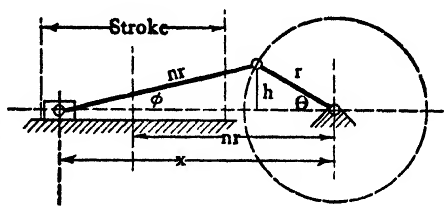


FIG. 5–9

From Fig. 5–9,

$$x = r\cos\theta + nr\cos\phi$$

and the piston displacement $= x - nr = r\cos\theta + nr\cos\phi - nr$

$$= r(\cos\theta + n\cos\phi - n). \qquad (5\text{–}7)$$

Also,

$$\sin\theta = \frac{h}{r} \qquad \text{and} \qquad \sin\phi = \frac{h}{nr}.$$

By division,

$$\sin\phi = \frac{\sin\theta}{n}.$$

Now,

$$\cos\phi = \sqrt{1 - \sin^2\phi} = \sqrt{1 - \frac{\sin^2\theta}{n^2}} = \frac{1}{n}\sqrt{n^2 - \sin^2\theta}.$$

Substituting this value of $\cos\phi$ in (5–7),

$$\text{Piston displacement} = r(\cos\theta + \sqrt{n^2 - \sin^2\theta} - n). \tag{5–8}$$

Thus we have the piston displacement in terms of the crank angle.

If the piston moved with simple harmonic motion its displacement at crank angle θ would be $r\cos\theta$. The "connecting-rod effect," due to the obliquity of this member with the line of stroke, is represented by the term $r(\sqrt{n^2 - \sin^2\theta} - n)$.

The **Piston Velocity** is equal to $\frac{ds}{dt}$ where s is the piston displacement. Substituting the value of s from equation (5–8),

$$\text{Piston velocity} = \frac{d}{dt}\left[r(\cos\theta + \sqrt{n^2 - \sin^2\theta} - n)\right]$$

$$= r\left[-\sin\theta\cdot\frac{d\theta}{dt} + \tfrac{1}{2}(n^2 - \sin^2\theta)^{-\frac{1}{2}} \times \frac{d(n^2 - \sin^2\theta)}{dt}\right]$$

$$= -r\frac{d\theta}{dt}\left[\sin\theta + \frac{2\sin\theta\cdot\cos\theta}{2\sqrt{n^2 - \sin^2\theta}}\right]$$

$$= -r\cdot\omega\left[\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right], \tag{5–9}$$

since $\frac{d\theta}{dt} = \omega =$ (angular velocity of the crank).

An approximate form of this equation is obtained by neglecting $\sin^2\theta$ in the denominator. The error involved is not very large, the value of n in engine design being seldom less than 4, and $\sin^2\theta$ being equal to 1 as a maximum. Equation (5–9) then reduces to the following form:

$$\text{Piston velocity} = -r\omega\left(\sin\theta + \frac{\sin 2\theta}{2n}\right). \tag{5–10}$$

The minus sign has no special significance and may be omitted.

The **Piston Acceleration** is equal to $\frac{dv}{dt}$. Taking the exact velocity equation (5–9), differentiating and dividing by dt, it can be shown that

$$\text{Piston acceleration} = r\omega^2\left[\cos\theta + \frac{\cos^4\theta + \cos 2\,\theta(n^2 - 1)}{(n^2 - \sin^2\theta)^{3/2}}\right]. \quad (5\text{–}11)$$

Treating the approximate equation (5–10) in the same manner,

$$\text{Piston acceleration} = \frac{d}{dt}\left[r\omega\left(\sin\theta + \frac{\sin 2\,\theta}{2\,n}\right)\right]$$

$$= r\omega^2\left(\cos\theta + \frac{\cos 2\,\theta}{n}\right). \quad (5\text{–}12)$$

Equation (5–12) is so much simpler than (5–11) that it is generally used where extreme accuracy is not required.[2]

When n is equal to 4 the approximate equation shows a maximum error of about 0.6 per cent of the greatest acceleration.

9. Quick-return Motion. — The Sliding-block Mechanism can be used as a quick-return motion when offset as shown in Fig. 5–10. That is, the piston link c executes its strokes to right and left in unequal periods of time. The mechanism is shown by broken lines in the two positions where the piston link has reached the end of its travel to right and left, respectively. At these positions the crank and connecting-rod links lie along the same straight line. With the crank turning clockwise, the piston stroke to the left takes place while the crank rotates through the angle θ_1 and the return stroke requires a crank movement of θ_2. Assuming a constant crank velocity, the time ratio of the two strokes is equal to θ_1/θ_2. This ratio is unity when the offset is zero and increases with the offset.

10. Second Inversion. Swinging-block Linkage. — In this mechanism, shown in Fig. 5–11, the link b, corresponding to the connecting rod in the direct-acting engine mechanism, is the fixed link. Figure 5–12 illustrates the application to an oscillating

[2] Tables and curves for displacement, velocity and acceleration, derived by use of the exact formulae, will be found in an article by J. L. Bogert in *Marine Engineering*, December, 1920.

steam engine, link *c* taking the form of a cylinder pivoted so as to oscillate about trunnions at *B*. Link *d* becomes the piston and piston rod.

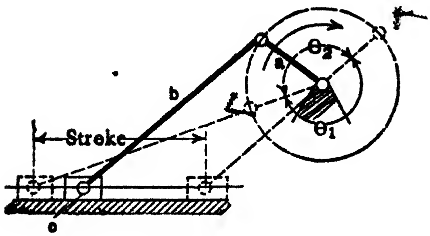


Fig. 5–10

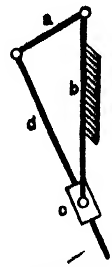


Fig. 5–11

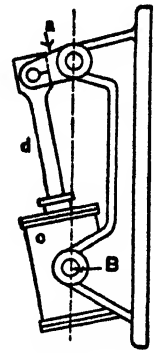


Fig. 5–12

Figure 5–13 shows the **Crank-shaper Quick-return Motion,** another application of the second inversion. Link *a* is the driving crank, attached to which is the block *d*. The latter slides between

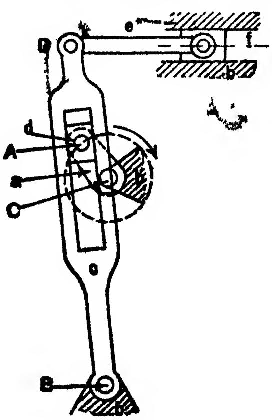


Fig. 5–13

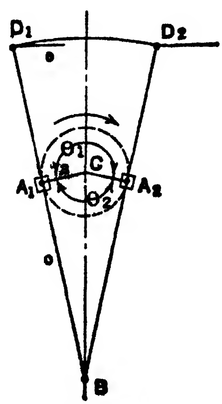


Fig. 5–14

guides formed on lever *c*, driving ram *f* through rod *e*. As applied to the shaper, ram *f* carries the cutting tool. This has a reciprocating motion, the return stroke being performed at a higher speed than the cutting stroke. Supposing, for example, that the crank turns clockwise; the lever c will reach its extreme position to the left when the crank is at A_1C (Fig. 5–14) perpendicular to BA_1D_1. Likewise, c will reach the other extreme position when the crank is in position A_2C. The crank meanwhile turns through an angle θ_1. The return stroke takes place during a crank movement θ_2. Consequently, assuming constant crank velocity, the ratio of times of cutting to return strokes is equal to θ_1/θ_2. This ratio can be given any required value by proper choice of ratio $\dfrac{\text{Length } AC}{\text{Length } BC}$.

11. Third Inversion. Turning-block Linkage. — This is illustrated in Fig. 5–15, link a, corresponding to the crank in the direct-acting engine mechanism, being the fixed link. Figure 5–16 shows the **Whitworth Quick-return Motion,** a well-known application often employed in machine tools and in other cases where it is desired to produce a reciprocating motion with a rapid return stroke.

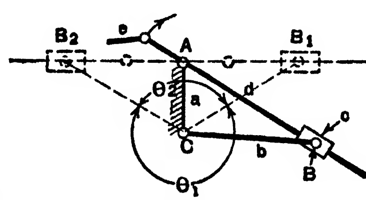


FIG. 5–15

In Fig. 5–16, b is the driving crank rotating with constant velocity and driving the slotted link d by means of block c. Link d revolves with variable velocity, and a connecting link e may be attached to drive a reciprocating member if required.

Referring to Fig. 5–15, link d turns clockwise from a horizontal position AB_1 through 180° to position AB_2, while the driving crank is turning through angle θ_1. It executes the next half revolution while the driving crank moves through an angle θ_2. The time ratio is therefore θ_1/θ_2.

Reduction of the length of link a without altering that of b will cause the ratio θ_1/θ_2 to decrease, its value becoming unity in the limit when a has zero length.

12. Fourth Inversion. Fixed-block Linkage. — The remaining inversion of the Slider-crank Chain is obtained by fixing the block c (Fig. 5–17). The most common application of this linkage is found in hand-operated water pumps. It is also used in certain direct-driven reciprocating steam pumps. In the hand-pump application, c becomes the pump barrel and d the pump rod, to

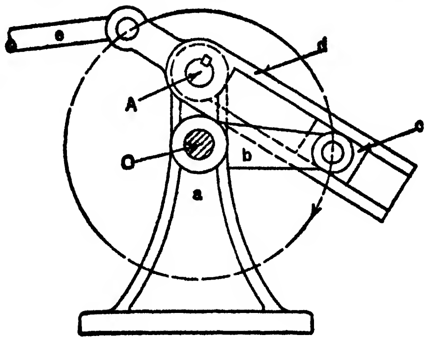


FIG. 5-16

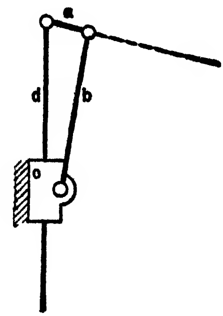


FIG. 5-17

the lower end of which the plunger is attached. The dotted extension of a forms the pump handle.

QUESTIONS — CHAPTER V

1. Draw skeleton diagrams of the four inversions of the slider-crank mechanism. Name a practical application of each.

2. Show how to draw a polar curve of piston velocity on a crank-angle base for the sliding-block mechanism.

3. Show how to find graphically the velocity of the piston in the direct-acting steam-engine mechanism. Prove your construction to be correct.

4. Show how to draw velocity and acceleration curves for the crosshead of a direct-acting engine on a base representing the crosshead position. How is the scale for the diagram obtained?

5. Sketch and explain Klein's construction for finding the acceleration of the piston in the direct-acting steam-engine mechanism. If the diagram is drawn to a scale of 1 in. = n in., what is the acceleration scale when the ordinates are measured in inches and ω is in radians per second?

6. Prove that the velocity of the piston in the direct-acting steam engine is not simple harmonic. How can the motion be made to approach simple harmonic?

7. A steam engine has a stroke of 12 in. and the connecting rod is 24 in. long. Show how to find graphically the piston velocity and acceleration when the crank makes an angle of (a) 45° and (b) 90° and (c) 120° with the line of stroke. Also determine the velocity and acceleration scales in foot-second units if the drawing of the mechanism is one-fourth full size and the engine rotates at 240 R.P.M. *Ans.* 1 in. = 8.38 ft. per sec., 1 in. = 210.5 ft. per sec. per sec.

8. A steam engine, 8-in. bore and 10-in. stroke, runs at 300 R.P.M. Assuming a constant angular velocity of the crank, calculate the piston velocity and acceleration corresponding to the following crank angles: 0°, 45°, 90°, 135°, 180°. The connecting rod is 25 in. long. *Ans.* At 45°, Vel. = 10.56 ft. per sec. Acc. = 290.5 ft. per sec. per sec.

9. If the average piston speed of an internal-combustion engine, $3\frac{1}{2}$-in. bore and 5-in. stroke, with a connecting rod 11 in. long, is 3000 ft. per min., at what speed does the engine run? Find, also, the maximum piston velocity and maximum piston acceleration. At what angles are these maximum values obtained?

10. In a four-cylinder gasoline engine with a stroke of $4\frac{1}{2}$ in. and a connecting rod 9 in. long, two cranks are on the top dead center when the other two are on the bottom dead center. Compare the piston accelerations for this position of the cranks if the R.P.M. is 3600. What is the average piston velocity?

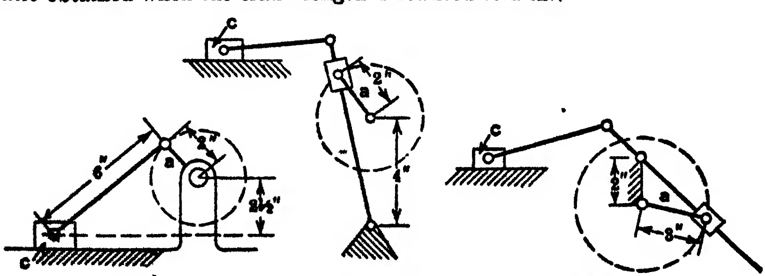
11. Compare the calculated piston accelerations of problem 10 with those obtained under the assumption that the pistons have S.H.M. Express the differences as a percentage.

12. Sketch the crank-shaper quick-return motion. How is the time ratio of advance to return obtained graphically? How is the stroke of the shaper altered? Does this affect the time ratio of the strokes?

13. Sketch the Whitworth quick-return motion. Show how to find the time ratio of advance and return strokes. What alteration would be required in order to increase this ratio?

14. Sketch an inversion of the slider-crank mechanism used in hand pumps, and name and indicate the essential parts in this application.

15. A crank-shaper quick-return motion has a driving crank 4 in. long. Find the distance between the pivot points of driving crank and swinging link if the time ratio of advance to return stroke is 3 : 1. What is the new time ratio obtained when the crank length is reduced to 2 in.?



16A 16B 16C

16. Find graphically the time ratio of advance to return for the driven block *c* in the mechanisms shown in Figs. A to C, assuming that the driving crank *a* rotates with constant angular velocity.

CHAPTER VI

CAM MECHANISMS

1. Cam mechanisms are extensively used in machinery because of the ease with which the design can be carried out to produce any desired motion. The motions needed in machine parts are often of such a nature that it would be difficult to obtain them by any other mechanism of equal simplicity and practicability. Thus cam mechanisms are commonly used for operating valves in automobile, stationary, and marine internal-combustion engines, in printing machinery, shoe-making machinery, automatic screw machines, stamp mills, clocks, locks, etc. It is difficult to find a machine of the type which we term "automatic" which does not employ one or more cam mechanisms.

All cam mechanisms are composed of at least three links: (*a*) the **Cam,** which has a contact surface either curved or straight; (*b*) the **Follower,** whose motion is produced by contact with the cam surface; and (*c*) the **Frame,** which supports and guides the follower and cam.

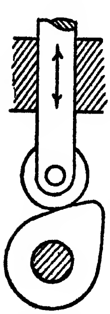


Fig. 6–1

2. Types: Disc or Plate Cam. — Here the cam takes the form of a revolving or oscillating disc, as in Fig. 6–1, the circumference of the disc forming the profile with which the follower makes contact. The follower moves in a plane perpendicular to the axis of rotation of the cam.

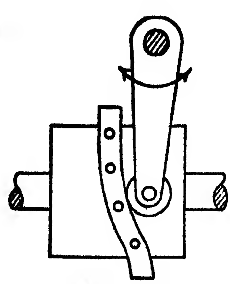


Fig. 6–2

Cylinder Cam. — (See Fig. 6–2.) In this mechanism the profile which acts on the follower is formed on the surface of the cylinder, so as to move the follower in a plane parallel to the axis about which the cam rotates or oscillates.

Translation Cam. — Here the cam profile is cut on one side of a block of metal or other material and the cam has reciprocating motion along a flat surface. (See Fig. 6–3.)

All cams may be regarded as wedges having surfaces of uniform or variable slope, more frequently the latter. When the wedge is

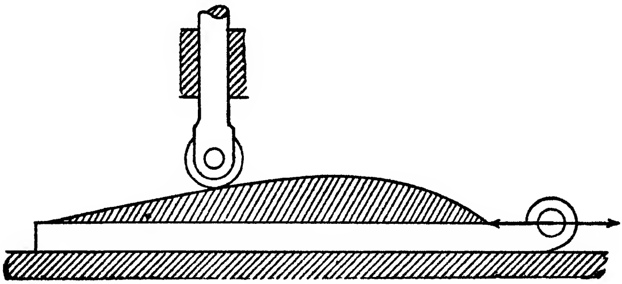


Fig. 6–3

caused to slide back and forth on a flat surface we call it a **Translation Cam** (Fig. 6–3). When the wedge is wrapped around the circumference of a circular disc (Fig. 6–4) it becomes a **Disc Cam.**

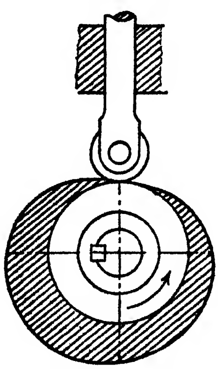


Fig. 6–4

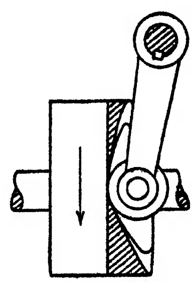


Fig. 6–5

When the wedge is bent to form a ring and applied to the flat end of a cylinder (Fig. 6–5) a **Cylinder Cam** results.

3. Constraint of Follower. — In the mechanisms shown in Figs. 6–3, 6–4, and 6–5 it will be noted that the form of the cam is such that it does not completely constrain the motion of the follower, as no means of maintaining contact between cam and follower is indicated. Continuous contact is usually effected by utilizing either the forces of gravity or spring pressure.

4. A Positive-motion Mechanism (Fig. 6–6) is one in which the follower is compelled to move in a definite path by constraining surfaces and without the application of external forces. Failure to do so can be due only to breakage of some part. Methods of accomplishing this result will be taken up in detail in a later article.

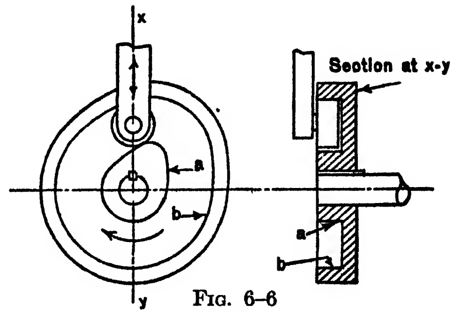


FIG. 6–6

DISPLACEMENT DIAGRAMS

5. Profile Design. — The design of a cam profile is governed by the requirements in regard to the motion of the follower. These requirements depend on the function which the mechanism performs in the machine to which it is applied. The cycle of events for the follower, determined by such considerations, may call for certain "rest" periods during which no follower motion occurs, and certain periods of motion of a specified nature. It is generally found convenient to start on the cam-design problem by first making a graphical representation of the follower movement, which we call a **Displacement Diagram.** This is a linear curve in which abscissæ represent cam displacements and ordinates represent follower displacements. Since both members may have either linear or angular motion, these displacements may be either linear or angular, depending on the particular form of mechanism under consideration. Linear follower displacement is often referred to as the "lift," even though the movement may not be in a vertical direction.

Followers in practical applications frequently move exactly or approximately in accord with one of the following conditions:

(*a*) Motion at **Constant Velocity;**
(*b*) Motion with **Constant Acceleration** or **Deceleration;**
(*c*) **Simple Harmonic Motion.**

The **Displacement Diagrams** corresponding to these three cases, together with certain modifications, will next be considered.

The cam shaft, where the cam has angular motion, is assumed to rotate at a **constant** speed. The discussion which follows is based on this assumption. Here the **Displacement Curve** is one in which the base represents **Time** as well as **Cam Displacement,** the two quantities being proportional to each other.

6. Constant Velocity. — In Fig. 6–7 is shown a displacement diagram for a cam mechanism in which the follower rises 2 in. during 90° motion of the cam at constant velocity, rests for 90°, falls 2 in. with constant velocity during 90°, and rests for the balance of the cycle.

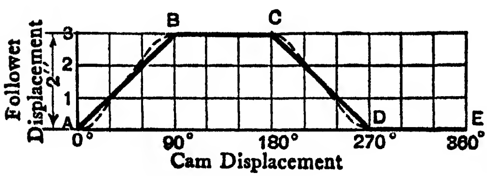


FIG. 6–7

When a body moves with constant velocity, its displacement is directly proportional to the elapsed time. Assuming a constant cam velocity, the follower displacement is consequently proportional to the cam displacement. The curve *AB* for the first 90° must therefore be a straight line. During the second 90° period a horizontal straight line *BC* represents the rest period. The drop period during the next 90° of cam motion is indicated by another straight line *CD*, since here again we have constant velocity. *DE* is drawn horizontally for the final period.

For a practical application the diagram would probably be modified to the form shown by the broken lines, unless the cam turns very slowly. This is done to avoid sudden changes of motion when the lift begins and ends, and substitutes a gradual change of velocity which eliminates shock and noise. Further reference to this matter will be made later.

7. Constant Acceleration. — For any moving body with constant acceleration, $s = \frac{1}{2}at^2$, where s is the displacement, a the acceleration, and t the time interval. The distance moved is therefore proportional to the square of the time. Taking time intervals of 1, 2, 3, 4, etc., time units, the displacements of the body at the ends of these intervals will be relatively proportional to the quantities 1^2, 2^2, 3^2, etc., or 1, 4, 9, etc. This principle is applied in the case of the displacement diagram shown in Fig. 6–8. Here the requirement is that the follower shall move a distance AC during a cam displacement AB. The construction is as follows:

AB is divided into any number of equal spaces; in the figure these are four in number. Each of these spaces represents an equal time interval under the assumption that the cam has uniform velocity. The follower displacements up to the ends of these intervals are proportional to the numbers 1, 4, 9, 16. But AC is the displacement at the end of the fourth interval. Therefore, we divide AC into sixteen equal parts and project from the first, fourth, ninth, and sixteenth division, as shown in the figure, thus locating points on the required curve.

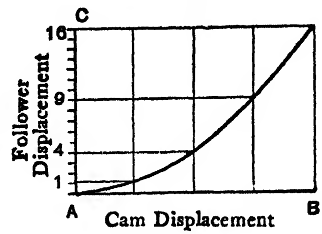


Fig. 6–8

8. Constantly Accelerated and Decelerated Motion. — Acceleration lasting to the end of the follower travel would result in maximum velocity being attained just before the follower comes to rest, and this would cause a shock unless the cam speed were very slow. Consequently, the acceleration period should last only part of the lift interval and be succeeded by a "deceleration," which will bring the follower to rest gradually. Giving these quantities constant values will often result in smooth cam action. The constant acceleration may or may not be equal to the constant deceleration; a cam profile can be designed to give any desired ratio of acceleration to deceleration. The displacement diagram for such a case will next be considered.

Let a_1 be the constant acceleration during the first part of the follower motion, s_1 and t_1 being the corresponding displacement and time. Let a_2 be the deceleration during the latter part of the motion, s_2 and t_2 being the displacement and time for the same interval. The ratio $\frac{a_1}{a_2}$ is the given acceleration-deceleration ratio. Now $S = s_1 + s_2$, where S is the total follower movement.

If v = the velocity at the end of the acceleration period,

$$v^2 = 2\,a_1 s_1 = 2\,a_2 s_2 \qquad \text{or} \qquad \frac{a_1}{a_2} = \frac{s_2}{s_1}.$$

Also,

$$v = a_1 t_1 = a_2 t_2 \qquad \text{or} \qquad \frac{a_1}{a_2} = \frac{t_2}{t_1}.$$

That is, the displacements and time intervals are to each other inversely as the acceleration-deceleration ratio.

Example. — Draw the displacement diagram for a cam mechanism in which the follower moves 2 in. during 180° of cam displacement, acceleration and deceleration being constant and having the ratio of 3 to 1.

From the above discussion it will be evident that the displacements and times corresponding to the two intervals are in the ratio of 1 to 3. For the acceleration period the displacement is therefore one fourth of the total displacement and the period lasts for one fourth of the total time, ending at 45° cam displacement (Fig. 6–9). This fixes the position of the point B on the 45° line, the ordinate being ½ in. The construction for other points on the **Acceleration Curve** is the same as that used in Fig. 6–8. Points on the **Deceleration Curve** BC are found in the same way by working from C toward the left.

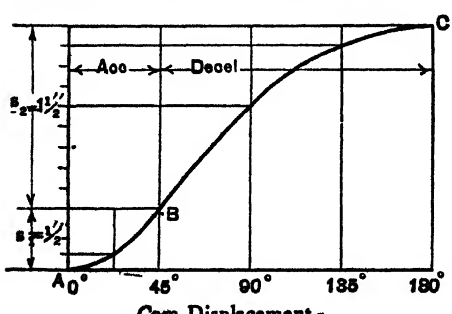


FIG. 6–9

9. Practical Modification of the Constant-velocity Diagram. — As noted in Art. 6, the displacement diagram for the constant-velocity cam is modified somewhat from the theoretical form in practical applications, for the purpose of avoiding sudden changes of velocity at the beginning and end of the lift period.

This modification can best be made by using a short period of constant acceleration at the beginning of lift, lasting until a suitable velocity has been attained. The follower then moves with constant velocity until near the end of the lift period, when a constant deceleration is applied, and the follower is brought to rest without shock.

The construction of the lift diagram for such a case will now be considered.

Suppose it is specified that the follower is to lift during 150° of cam motion, the displacements being those due to constant acceleration for 30°, constant velocity for 90°, and constant deceleration for the remaining 30°.

When a body is accelerated uniformly from rest to a velocity v in t units of time, it is evident that the average velocity for the period is $v/2$ and the distance moved is $vt/2$. If, instead, the body has a constant velocity v, it will move the same distance $vt/2$ in time $t/2$. Consequently, the follower in question will move the same distance during the first 30°, where it has constant acceleration, as it does in subsequent 15° intervals with constant velocity. The total lift can therefore be regarded as composed of eight equal increments, the first being executed in the first 30° period, the next six in the succeeding six 15° intervals, and the last in the final 30° period. We therefore divide the total lift (Fig. 6–10) into eight equal parts, obtaining the points 1, 2, 3, etc., and project from 1 to 1′, 2 to 2′, etc. Connecting 1′, 2′, 3′ by a smooth curve completes the diagram. Intermediate points on the acceleration and deceleration curves may be found as in Fig. 5–8.

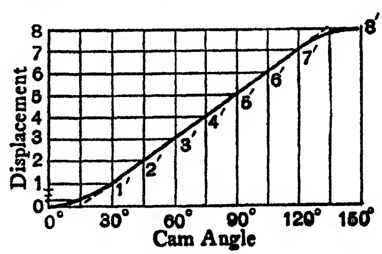


Fig. 6–10

10. Simple Harmonic Motion. — The construction of the displacement diagram for harmonic motion of the follower is the same as that described in Art. 8, Chapter II, for drawing a linear-displacement curve of a point with harmonic motion. Figure 6–11 illustrates a case in which the follower lifts 2 in. during 180° of cam motion, then rests for 90°, falls to initial position in 90°, and rests for the balance of the cycle.

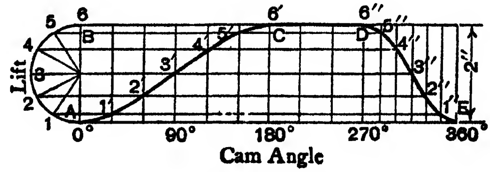


Fig. 6–11

A semicircle is drawn as shown, the lift being used as a diameter. The cam angle for the lift period, 180°, is divided into any convenient number of equal parts; in the figure each of these represents 30°. The semicircle is divided into the same number of equal arcs, and thus points 1, 2, 3, 4, etc., are found. Horizontal projection then locates points 1′, 2′, 3′, etc., on the required curve. For the "drop" period, projection from the same points, 1, 2, 3, may be made if the cam angle corresponding to this period is divided into the same number of parts as the semicircle.

CAM PROFILE CONSTRUCTION

11. General Methods. — So far we have only discussed the method of drawing displacement diagrams for required follower motions. The next step to be considered is that of finding the cam profiles necessary to produce these movements. The construction is altered in detail with different types of followers, but we may map out a general method which is applicable to all cases irrespective of the form of the displacement curve or the variety of follower in use. It can be applied to disc cams, cylinder cams, and translation cams, and comprises the following steps:

(*a*) The cam is considered as being the fixed link in the mechanism instead of the frame which carries the cam shaft and follower guides. That is, we deal with an inversion of the actual mechanism. As noted in Art. 3, Chapter I, the relative motion of any pair of links remains unaltered when the mechanism is inverted. Therefore, the cam and follower will have the same relative motion whether the frame or the cam is considered as the fixed member.

(*b*) The portion of the follower that acts on the cam is then drawn in various positions which it will occupy at different instants during its cycle of motion relative to the stationary cam. The contact surface of the follower may consist of the surface of a roller, a knife edge, a flat, convex, or concave sliding face, etc. In Fig. 6–13, in broken lines, is shown the position of the follower corresponding to angular displacements of 30°, 60°, 90°, etc., from an arbitrary zero radius. The choice of the angular intervals depends on the number of points which it is desirable to locate on the cam profile.

(*c*) The cam profile is found by drawing a smooth curve tangent to the follower contact surface in all of its various positions.

The contact surface of the follower is located as required in (*b*) by first finding the position of some selected point on the follower. The point chosen, which we shall call the "**reference point,**" should be one that is easily located from data furnished by the displacement curve and also one from which the working face of the follower can be conveniently drawn. For example, where a roller is used, the roller center is the best point for the purpose; where the follower is flat-faced, the point where the follower axis intersects the contact face is satisfactory.

It will be noted that the constructions which are described in the following articles differ from one another only because of variations in the form of follower employed and differences in the way in which it is constrained to move with reference to the frame and cam.

DISC CAMS

12. Knife-edge Follower. — In this mechanism, the follower has contact with the cam along a line represented by the point *A* in Fig. 6–13, in all positions. This style of follower is suitable only for very light service because the edge cannot be effectively lubricated, pressure is concentrated on this portion and wear is likely to be excessive.

Assuming that data are given whereby the displacement diagram (Fig. 6–12) can be plotted by methods already outlined, we shall proceed to discuss the method of drawing the cam profile.

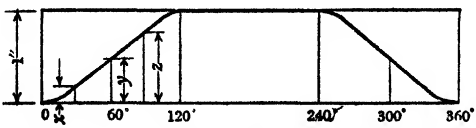


Fig. 6–12

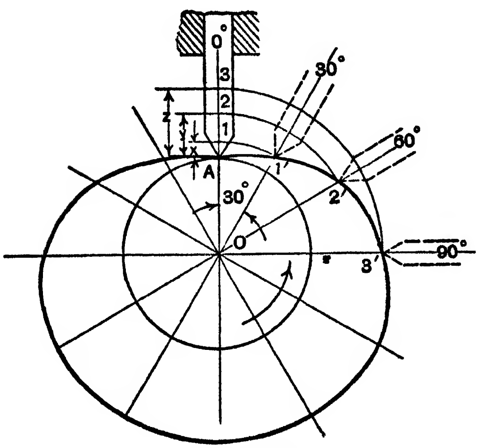


Fig. 6–13

The diameter of the base circle is taken as 2 in. and the lift 1 in. Distances x, y, z, etc., in Fig. 6–12, represent the follower displacements after 30°, 60°, 90°, etc., of cam movement. Any other convenient angles may, of course, be used. The base circle is first drawn (Fig. 6–13) and a zero radius is chosen as a reference line representing the initial position of the follower axis. In the initial position, shown in solid lines, the knife edge of the follower touches the base circle.

In accordance with the general plan outlined in Art. 11, we consider the cam as the fixed link and move the follower around it. Point A is the most convenient "reference" point, and its successive positions are first located. To find the position of A after 30° of cam movement, we set off the distance x from A outwardly along the path of motion of this point; thus point 1 is determined. Next, with O as center and O–1 as radius, we swing an arc 1–1′ in a sense opposite to that of the cam movement, and subtending an angle of 30° at the point O. Then 1′ will be the new position of A corresponding to 30° angular motion. Using y, z, etc., as displacements, we find points 2′, 3′, etc., in the same way. As the cam always touches the follower at A we complete the construction by drawing a smooth curve through A, 1′, 2′, 3′, etc.

The follower edge does not always move in a straight path passing through the cam axis; Fig. 6–14 shows the case where the follower is offset, that is, A moves along a line passing to one side of the cam center. The description of the construction required to obtain the cam profile in Fig. 6–13 will apply without change to Fig. 6–14.

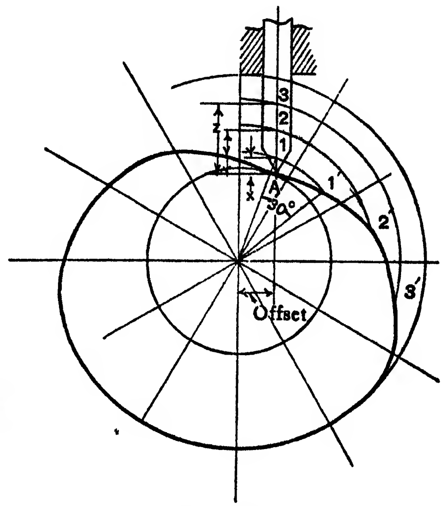


Fig. 6–14

13. Roller Follower. — The follower is usually either guided so as to move with rectilinear motion, or it is pivoted so as to swing about a fixed point. The general method outlined in Art. 11 applies in either case. The center of the roller is used as a reference point whose path is first determined, and from which the follower contact surface, namely the circumference of the roller, is located in various positions.

(*a*) **Roller Follower with Rectilinear Motion.** — The displacement diagram, Fig. 6–15, is assumed to specify the motion requirements. The base circle (Fig. 6–16) is first drawn and the roller located in its initial position touching this circle. The path of the roller center, A–A', is drawn. A zero radius, for convenience, parallel to A–A', is next located, and angular intervals of 30° are laid off from it about O. Keeping the cam stationary, we then find the position of the roller center A, after 30° displacement of the follower. The displacement diagram indicates a displacement x at 30°; this distance is set off along A–A', giving point 1. With center O and radius O–1, an arc 1–1′ is described in a sense opposite to that of the cam movement, and of such length as to subtend an angle of 30° at O. Point 1′ can most easily be located by making chord 1–1′ equal to chord *L–M* or 1–*L* equal to 1′–*M*.

Points 2′, 3′, 4′, etc., are found in a similar way. Using these points as centers and the roller radius, corresponding positions of the follower contact surface are drawn. The required cam pro-

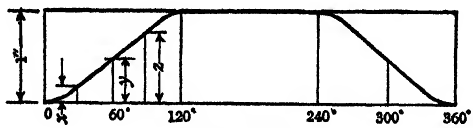


Fig. 6–15

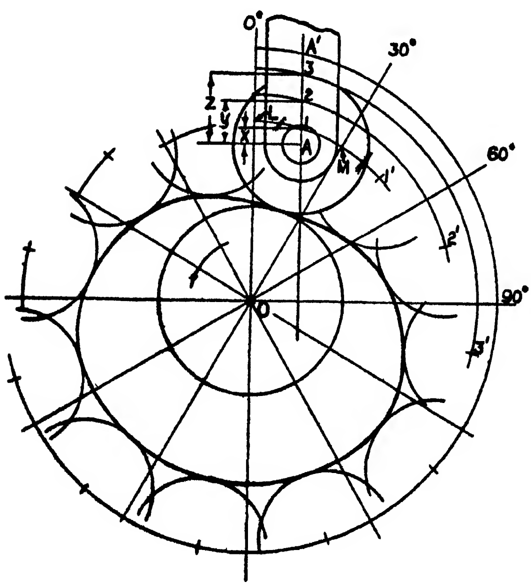


Fig. 6–16

file is evidently a curve drawn tangent to each of these circles. This curve is made as smooth as possible.

In Fig. 6–16, the line *AA′* does not pass through the cam axis, hence the follower is said to be "offset." An offset is sometimes provided to reduce side thrust during the lift period.

Figure 6–17 illustrates the cam obtained when the follower has no offset, that is, when A–A' passes through O. Points 1′, 2′, 3′ then fall, respectively, on the 30°, 60°, 90° radii.

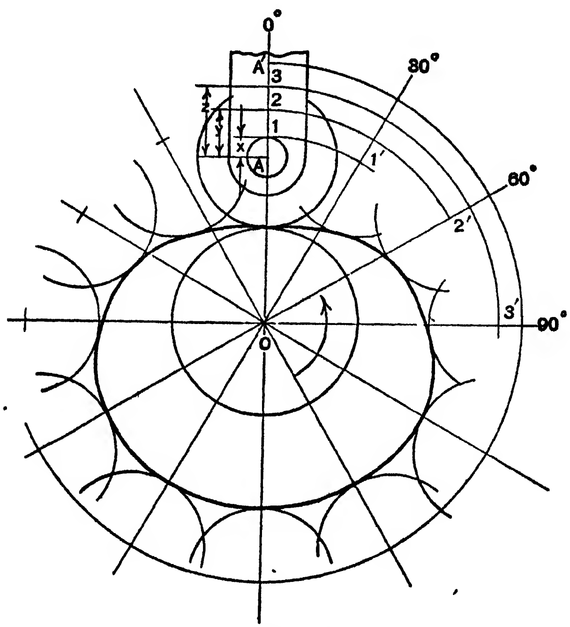


Fig. 6–17

(*b*) **Pivoted Roller Follower.** — Here the angular motion of the follower is assumed to be specified, the total displacement being $\theta°$. A displacement diagram drawn for the angular motion of the follower will also serve as a linear-displacement diagram for the motion of the roller center A, since the two quantities are directly proportional to each other ($v = \omega r$). This consideration is the basis for the construction which follows. It is assumed that the base circle, roller diameter, length of follower, and position of pivot are known. In Fig. 6–19 the mechanism is first drawn with the roller touching the base circle. An arc A–A', with center

B and radius BA, of such a length as to subtend an angle of $\theta°$ at B, is the path of motion of the roller center.

The displacement diagram, Fig. 6–18, is next drawn, the rectified length of A–A' being used to represent the angle θ. The

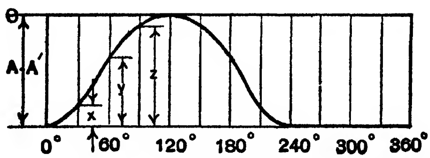


FIG. 6–18

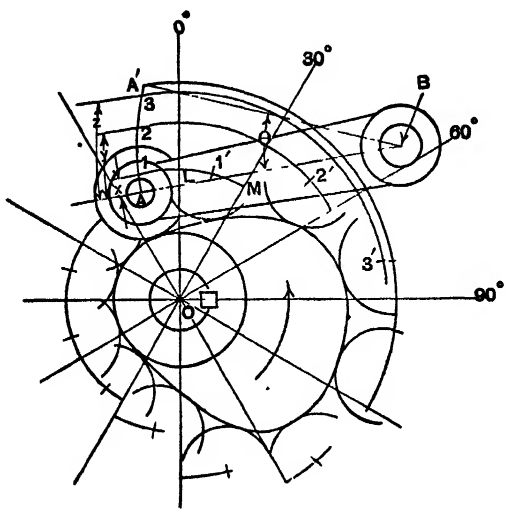


FIG. 6–19

method of doing this is exactly the same whether the follower displacements are linear or angular. (See Art. 5.) From this point on, the construction is identical with that used for Fig. 6–16. Distance x, representing the displacement at 30°, is set off along the arc A–A', giving point 1. With O as center and radius O–1 an arc is constructed, and a chord 1–1′ is laid off on it, of length equal to chord L–M (or 1–L = 1′–M). Points 2′, 3′, etc., are found in the same way. Roller circles are drawn with 1′, 2′, 3′

as centers, and finally the cam profile is formed so as to touch all these circles.

14. Follower with Convex Sliding Face. — (See Fig. 6–20.) Such followers usually have contact surfaces formed by circular arcs. If we take a roller follower and fasten the roller so that it cannot rotate on the follower, the motion of the latter remains unchanged. A sliding follower is therefore equivalent kinematically to a roller follower having a contact face of the same shape, as far as the follower motion is concerned. The reference point used (with a roller follower) in constructing the cam profile is the roller center. If the center of curvature (A, Fig. 6–20) for the sliding follower is used as the reference point, the construction as outlined in Art. 13 may be applied without change.

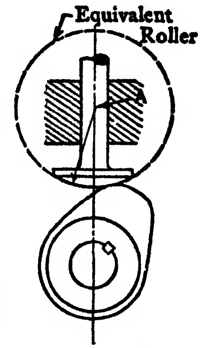


FIG. 6–20

15. Flat-faced Follower. — Here two cases will be considered: (*a*) the case where the follower has rectilinear motion, and (*b*) the case where the follower has angular movement about a pivot.

(*a*) **Flat-faced Follower with Rectilinear Motion.** — Figure 6–22 illustrates this case. Assuming that the displacement diagram has been obtained and is of the form shown in Fig. 6–21, we proceed as follows:

Draw the base circle for the cam and divide it into convenient angular divisions. Draw the follower in its initial position BC, tangent to the base circle. Point A where the cam touches the base circle in this position is chosen as the reference point. Set off distances x, y, z, etc., obtained from the displacement diagram, along the path of motion of A, obtaining points 1, 2, 3, etc. With O as center and O–1 as radius, swing an arc 1–1′. Point 1′ is the position of A after 30° displacement. Points 2′, 3′, etc., are found in the same way from displacements y, z, etc. Through 1′ draw a line perpendicular to the radius O–1′; this represents the follower face for 30° displacement. Draw similar lines through 2′, 3′, etc., each perpendicular to the corresponding radius. The cam profile is found by drawing a curve to touch each of these lines.

It will be noted that the intersections of these lines form triangles, shown by the cross-hatched surfaces in the figure. The drawing of the cam profile will be facilitated if it is remembered that the required curve touches the bases of each of the triangles at about their mid-point.

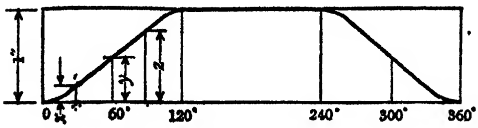


Fig. 6–21

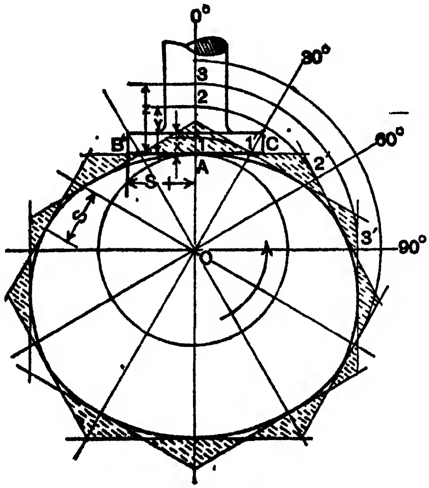


Fig. 6–22

The construction just given may in some cases result in the condition shown in Fig. 6–23 where it is impossible to draw a curve to touch all lines, such as 1–1′, 2–2′, 3–3′, etc. The cause is too rapid acceleration or deceleration of the follower, and the remedy is to increase the base-circle diameter. When the base circle is enlarged a certain amount, three of the lines will meet at a point; then the profile will have a sharp corner which is liable to wear away rapidly. Further increase in the base-circle diameter will cause this corner to disappear.

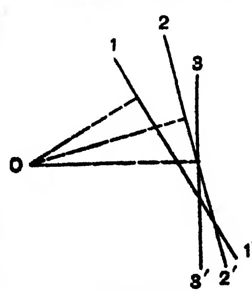


Fig. 6–23

The necessary length of follower face *BC* in Fig. 6–22 can readily be determined by inspection of the figure. The face is usually a circular disc, free to rotate about the follower axis. The point of contact is only on the axis in "rest" positions and moves out toward *B* or *C* as the follower velocity increases. The distances AB and AC must be great enough so that the contact point never passes B or C. By inspection of the diagram, the length of the longest tangent S can be found; AB and AC should be at least equal to S, and preferably slightly greater.

By offsetting the follower slightly, as shown in Fig. 6–24, a slow rotation of this member is induced. This tends to cause even wear on the contact face.

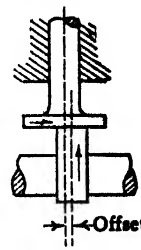


Fig. 6–24

(b) **Pivoted Flat-faced Follower.** — Figure 6–25 illustrates this mechanism, the follower turning about a fixed pivot at B. To construct the cam profile, any point, such as C, on the follower face is selected as a reference point. Arc C–C', with center B, is the path of motion

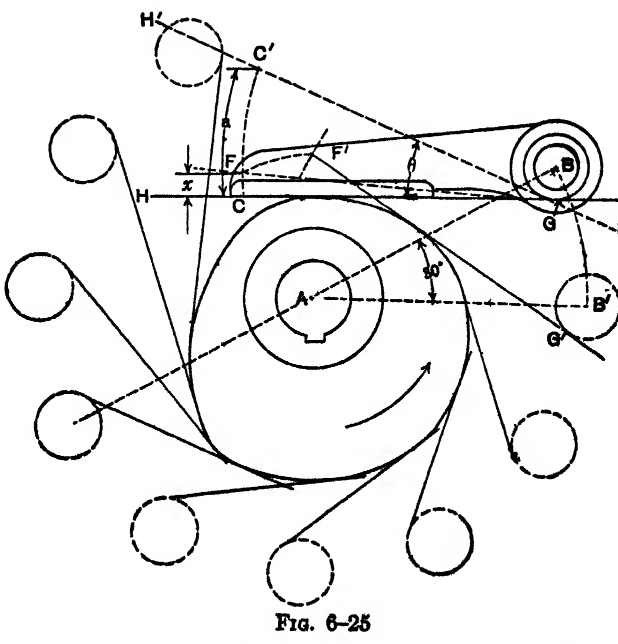


Fig. 6–25

of C, assuming that the follower is to have a total displacement of $\theta°$. The displacement diagram, Fig. 6–26, is drawn in the usual manner, the rectified length of C–C' (a) being used to represent follower displacement. The form of the curve depends on the motion specifications. The construction for a point on the cam profile at 30° cam displacement is indicated in the figure. Distance x represents the angular displacement of the follower at this instant; this distance is set off along the arc C–C', thus giving

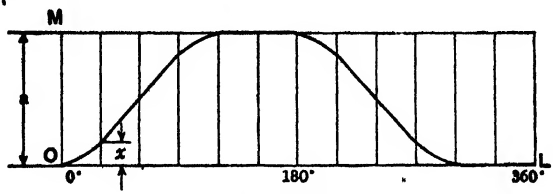


FIG. 6–26

point F. The follower is next rotated 30° in a sense opposite to that of the cam movement, which causes F to move to F' and B to B'. F' is easily located, since the angle $BAB' = 30°$ and $BF = B'F'$. By drawing, with B' as center, a circle of radius BG, the tangent $F'G'$ will represent the new position of the follower face. Repetition of this construction for other cam angles gives the series of lines shown in the figure, to which the cam profile must be tangent.

16. Primary and Secondary Follower. — The mechanism of Fig. 6–28 has a pivoted follower, on the back of which a second follower with rectilinear motion makes contact. We shall refer to these, respectively, as the "primary" and "secondary" followers. The advantages of such an arrangement are, (a) that the secondary follower is relieved of most of the side thrust, and (b) a large movement is obtainable with a small cam. Furthermore, the axis of the secondary follower may be offset a considerable amount from the cam axis.

It will be assumed that the motion of the secondary follower is definitely specified, so that a displacement diagram (as in Fig. 6–27) can be drawn; also, that sufficient data are given to enable the mechanism to be drawn in the position shown by the full lines of Fig. 6–28 with the roller in contact with the base circle.

Roller center E evidently moves in an arc EF with center at the pivot point D. The straight line AY is the path of point A on the secondary follower; this point we shall use as our reference point in tracing the follower motion. The construction for the roller position corresponding to 60° displacement from initial position is shown in the figure. The displacement diagram shows a follower displacement of x at this instant. This distance is set off along AY, thus giving the point A'. The follower end in its new position is drawn as shown, in broken lines, A' being used to locate the arc forming the contact surface. We next draw $B'C'$ touching this arc, representing the new position of the top face of the primary follower. $B'C'$ crosses the arc EF at J'. If $J'E'$, equal to JE, is set off along EF, evidently E' is the new location of the roller center. Our next step is to rotate the primary follower 60° in a sense opposite to that of the motion of the cam. Point E' moves in an arc with O as center to the point M, the arc $E'M$ subtending an angle of 60° at O. If OH, an arbitrary zero radius, is drawn, and angles of 30°, 60°, 90°, etc., laid off from it, then point M is easily obtained by making chord

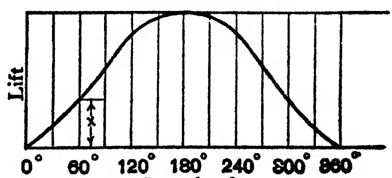


Fig. 6-27

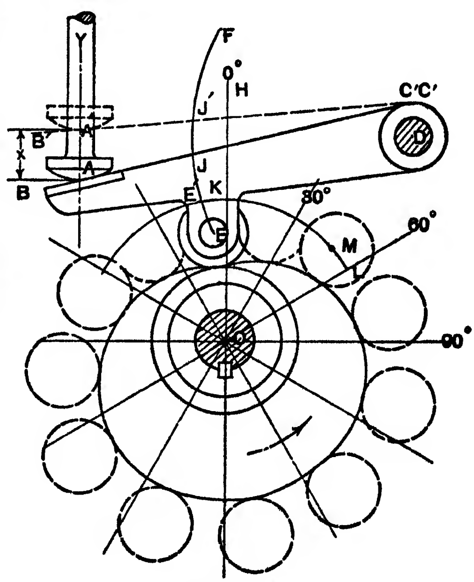


Fig. 6-28

LM equal to *KE'*. The cam profile is tangent to the roller circle with *M* as center.

17. Positive-motion Cam Mechanisms. — This variety was mentioned in Art. 4 as being one in which provision is made for controlling the motion of the follower in both directions by the use of two contact surfaces. For disc cams this is accomplished in the following ways.

(*a*) By the use of a grooved disc and roller follower, as in Fig. 6–6.

(*b*) By providing two contact surfaces on the follower, located on opposite sides of the cam axis, both bearing on the same cam. (See Fig. 6–29.)

(*c*) By using two contact surfaces on the follower as in Type 2, but causing each to bear on a separate cam. (See Fig. 6–31.) A brief discussion of each type follows.

Type (*a*). — In Fig. 6–6, *a* and *b* are the two contact surfaces. The inner one, *a*, is constructed just as though the cam were of the ordinary non-positive kind. Then circles of diameter equal to that of the roller are drawn at convenient angular intervals, touching the surface *a*. A curve drawn to touch each of these circles on the outside will outline the surface *b*. A certain amount of clearance is necessary, the groove being made slightly wider than the roller.

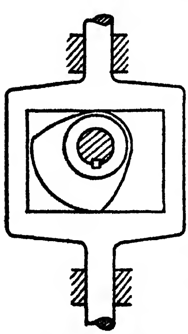


Fig. 6–29

It will be observed that when the roller rolls against *a* and slides on *b* it turns counter-clockwise, whereas when rolling against *b* it turns clockwise. Each of these conditions exists at least once during a revolution, hence the roller must reverse its angular movement at least twice per revolution. This is bound to cause slippage, which may produce excessive wear at some points on the contact surfaces.

Type (*b*). — When a flat-faced follower with reciprocating motion is used and the mechanism has the form shown in Fig. 6–29, the mechanism can be designed to give any required motion for 180°, subject only to limitations which apply to any cam with a flat-faced follower. The motion during the other half of the revolution, however, will be the same as that obtained during the first half, since it is caused by contact between the same cam profile and a follower face of the same shape, the only difference being that the direction is reversed. This type is not suitable, therefore, for an application in which the follower motion must be different on the lift and return strokes.

Where the follower has angular motion about a pivot, the motion can be specified for 180° plus the angular displacement of the follower.

In Fig. 6–30 is shown an assumed lift diagram for 180° of motion and the details of construction for a cam mechanism as in Fig. 6–29. The half cam surface ABC is found by the construction

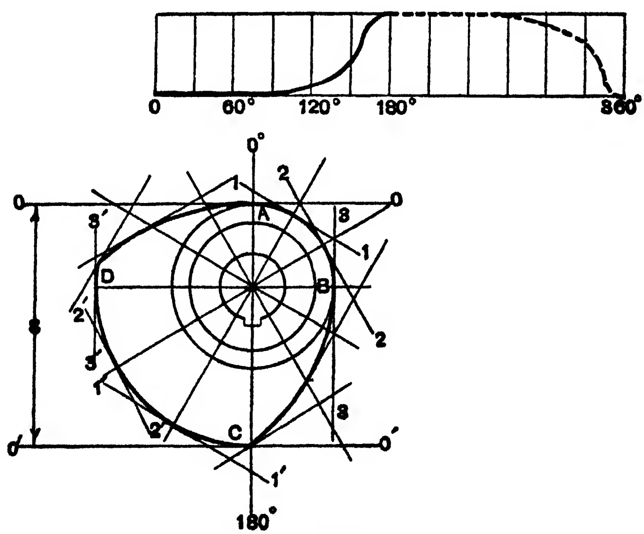


Fig. 6–30

of Art. 15. The lines 0–0, 1–1, 2–2, etc., show the positions occupied by the upper face at the ends of equal angular intervals of 30°. The necessary distance S from the upper to the lower face of the follower is evidently equal to

Base-circle Diameter + Lift.

To find the profile of the portion CDA of the cam, draw lines

0′–0′, 1′–1′, 2′–2′, etc., respectively parallel to and at distance S from 0–0, 1–1, 2–2, etc. Curve CDA is then drawn tangent to these lines. Clearance is provided by making the distance between the follower faces somewhat larger than S.

The preceding construction produces a figure such that the distance between any pair of parallel tangents is constant.

Type (*c*). — This consists of two disc cams, mounted on the same shaft, acting on a follower with two contact faces or rollers placed on opposite sides of the cam shaft, each bearing on one of the cams. (See Fig. 6–31.) In designing the cam profiles, the first or "**motion**" cam is drawn in the same manner as a non-positive cam bearing on one of the rollers or faces. The second or "**return**" cam is then drawn so that its profile will maintain contact with the other roller or face of the follower. It is usually

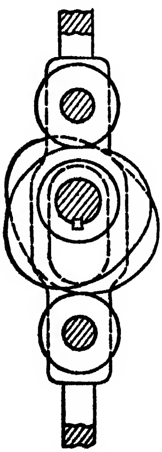


Fig. 6–31

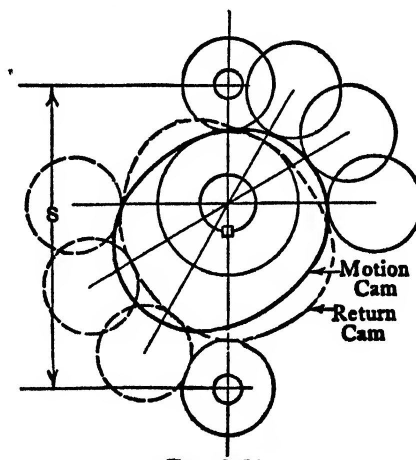


Fig. 6–32

most convenient to make both the base circles of the same size. In this case the distance S, in Fig. 6–32, from center to center of rollers is equal to 2 × Roller Radius + Base-circle Diameter + Lift.

Figure 6–32 shows the construction. The roller circles drawn in broken lines, to which the profile of the return cam is tangent, are located diametrically opposite the circles in solid lines, tangent to the motion cam, the distances between the centers of each pair on the same diameter being equal to S.

While more complicated and expensive to make, this type imposes no limitations on the motion as does Type *b* with single disc. Neither does it have the undesirable reversal of roller rotation which occurs in Type *a*.

CYLINDER CAMS

18. Types. — These may have a follower guided so as to move in a straight line along an element of the cylinder (Fig. 6–33) or a follower pivoted so as to move about an axis perpendicular to the cam axis (Fig. 6–35). The roller, if cylindrical, cannot have pure rolling contact because of the difference in the surface speed at top and bottom of the groove. Consequently it is sometimes made in the form of the frustum of a cone (Fig. 6–34) with the apex on the axis of revolution of the cam. While this promotes pure rolling action, it also introduces an undesirable thrust, tending to move the roller away from the cam.

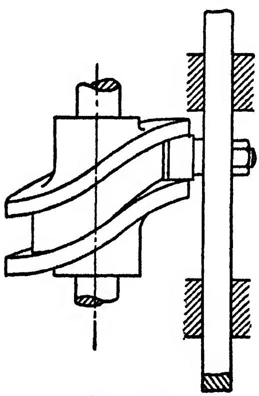


Fig. 6–33

19. Cylinder Cam with Rectilinear Motion of Follower. — Figure 6–36 shows the construction of a cylinder cam which meets the following specifications:

Cylinder Diameter	6 in.
Roller Diameter	$1\frac{1}{2}$ in.
Groove Depth	$\frac{11}{16}$ in.

The roller is cylindrical. The follower moves in a straight line along a cylinder element, rising from initial position with constant acceleration for 45°, with constant velocity for 60°, with constant deceleration for 45° to top position. It then rests for 30° and returns with the same motion as on the lift. The total lift is 4 in.

A cam angle-displacement diagram is first drawn (Fig. 6–36), the base being taken of such length that it represents the cylinder circumference (6π in.) to a convenient scale. Taking centers at a number of points along the curve, and with roller radius, draw circles as indicated in the figure. Draw curves tangent to these circles above and below; they will be the developed outline of the groove on the cylinder surface. If a templet of this form is made and wrapped around the cylinder, the required profile can be marked on the latter. The development of the root cylinder, of length $4\frac{3}{8}\pi$ in., is also shown in the figure. For strictly accurate results this is needed for drawing the elevation of the root lines. The shorter method adopted in Fig. 6–36, using only the outer cylinder development, is approximately correct for the root lines.

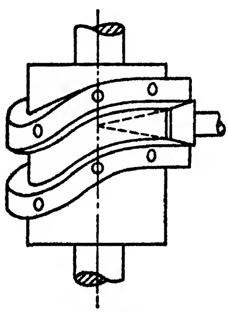


FIG. 6–34

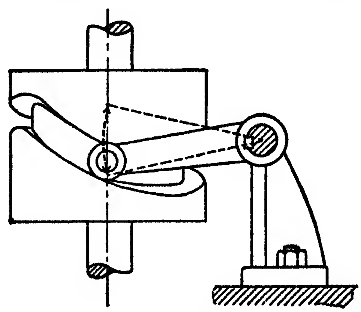


FIG. 6–35

20. Pivoted-follower Cylinder Cam. — Figure 6–37 shows the construction of a cam of the type illustrated in Fig. 6–35. The arm, from fixed point to roller center, is $4\frac{1}{2}$ in. long. The motion specifications must in this case call for certain angular displacements of the follower at given cam angles. For comparison, the total angular displacement of the follower has been taken as θ, corresponding to a vertical displacement of the roller center of 4 in., the same as in the mechanism of Fig. 6–36. Also, the specifications for angular motion have been taken the same as those for linear displacement in the preceding problem. The displacement curve will therefore have the same form.

Construction: Draw plan and elevation of the cam cylinder. Locate the pivot point c so that the roller center will move as nearly as possible along an element of the cylinder. This means that at mid-stroke the arm must be at right angles to the element

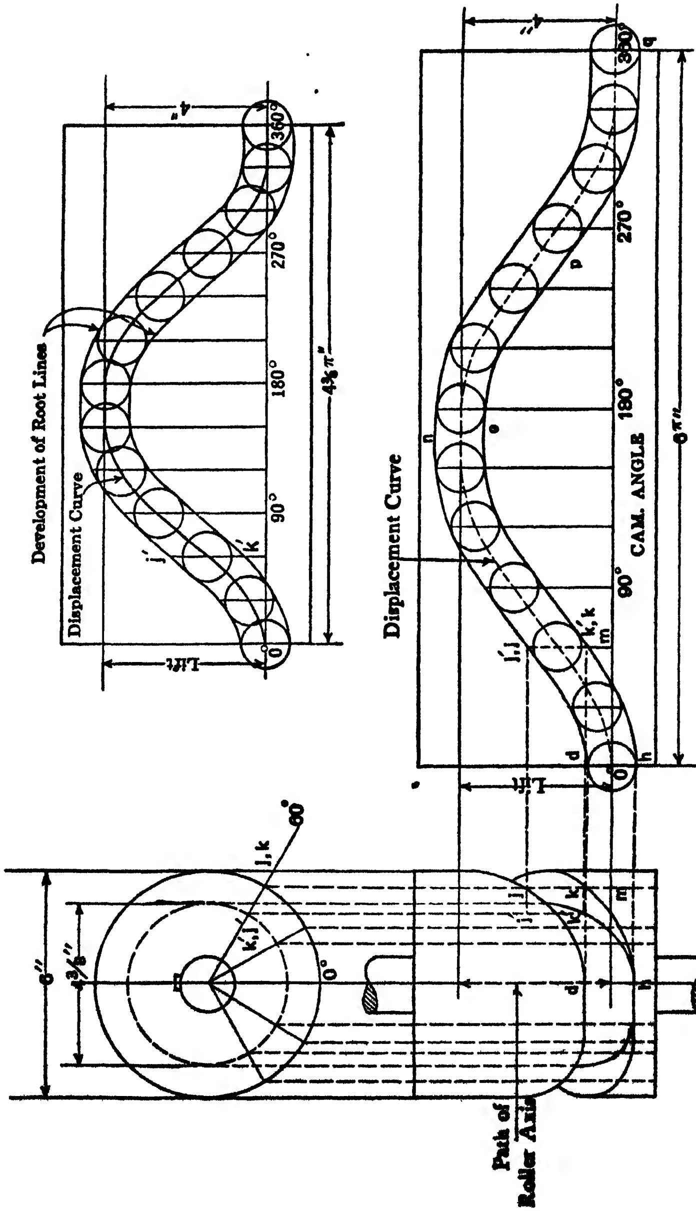

Development of Root Lines
Displacement Curve
360°
270°
180°
90°
0
Lift
4"
4⅜π"
Displacement Curve
CAM. ANGLE
6π"
6"
4⅜"
Path of Roller Axis

Fig. 6-36

pq, also that the arc *ab*, which is the path of the roller center, must be so placed as to extend equal distances on both sides of *pq*.

The rectified length of arc *ab* (*s*) is next obtained, and a displacement diagram is drawn, *s* being used as the total displacement. Draw a rectangle of length 6π in. to represent the development of the cylinder surface. Draw, on the development, arcs, as *dfm*, spaced at equal distances, and of radii equal to the arm length *ac*. On the diagram the arcs are spaced so as to represent 30° intervals. From the displacement diagram at a 60° cam angle, it will be seen that the follower displacement is an amount x. Transfer x to the development, marking off the arc length (not a chord length) $de = x$. With *e* as center and radius of the roller, construct a circle. Repeat this operation for each of the other angular positions. Draw a curve above and below, tangent to all the roller circles. The resulting diagram is a development of the groove on the cylinder surface.

A development of the groove form having been obtained, the elevation of the cam, shown at the lower left of Fig. 6–37, is next drawn by ordinary projection methods, using the development in combination with the end view of the cam cylinder shown at the upper left of the figure. Projection lines are shown in the figure for points on the 60° element. It should be noted that projection is made from the points where the groove curves cross the cylinder element *kl* and not from points on the arc *dfm*.

The roller is assumed to be conical, the diameter at the large end being 1¼ in. By drawing a diagram of the roller as shown at the lower left of the figure, we may obtain the diameter at the small end. With this diameter, and with centers at *e* and similar points, draw circles on the cam development Curves tangent to these circles, shown in broken lines in the figure, though not true developments of the root lines, may be used with fair accuracy to draw the elevation of such lines. A more exact method would necessitate drawing an accurate development of the root cylinder, on a base line of length equal to the root cylinder circumference, similar to that shown in Fig. 6–36. Generally this refinement is unnecessary, and we may draw our elevation as shown in the drawing. Projection lines are drawn for the 60° element in this figure.

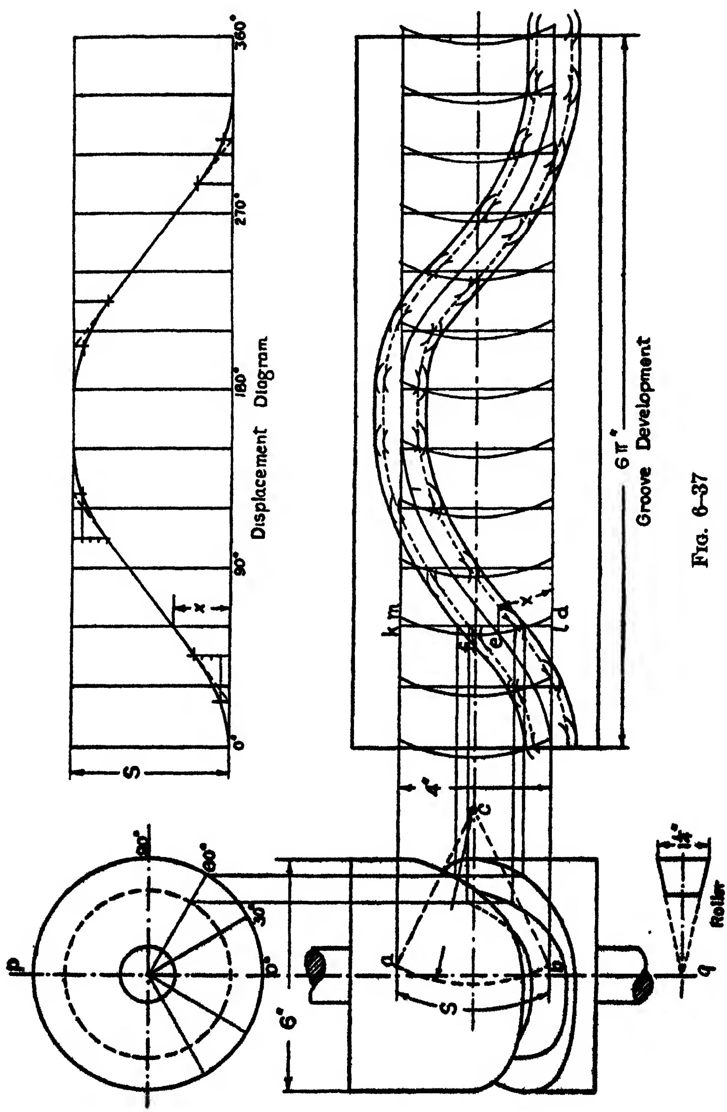

Displacement Diagram
0°
90°
180°
270°
360°
S
x
Groove Development
6π"
4"
x
k m
l d
e
c
a
b
p
q
S
0°
30°
60°
90°
6"
Roller

FIG. 6–37

CIRCULAR-ARC CAMS

21. General. — Many cams have profiles formed by circular arcs. There are three reasons for using such outlines in preference to other curves: (1) the drawing office specifications are more easily made for the use of the shop; (2) the process of manufacture is cheaper; (3) the completed cam can be checked more easily and with greater accuracy. Valve cams used in automobile and other internal-combustion engines, as well as many others, are usually of this class.

By proper choice of the radii and centers of the arcs, theoretical requirements as to follower movements can be approximated very closely. The process of design can be carried out by first drawing a displacement diagram of the desired motion to a large scale and from it laying out the cam. Then, by trial, arcs and radii are chosen which will approximate the true form. Finally, the resulting cam is checked by working back to a displacement diagram which is compared with the original one. If the revision of the cam is found to have altered the displacement curve to an unsatisfactory form, a further revision may be necessary.

For high-speed cams it is necessary to draw an acceleration curve for the follower, since the spring pressure needed in the non-positive type is dependent to a large extent on the weight of the follower and attached parts, and on the acceleration. Starting with the displacement diagram and treating it as a displacement-time curve, by the method of Art 13, Chapter II, we may construct a velocity-time and an acceleration-time curve, the latter giving the desired information for calculation of the spring. The Velocity Image method of Chapter IV can also be used. See Arts. 5 to 8 and Example 2 in this chapter. Other methods of arriving at the velocity and acceleration of the follower are described in the articles which follow. These the author has found to give accurate results with little labor.

22. Circular-arc Cams with Flat-faced Follower. — In Fig. 6-38 is shown a simple form of cam of this type, consisting of a circular disc rotating about a point O other than its geometric center A. The distance r from the center A to the face of the follower is evidently constant for any position of the cam. Therefore, the vertical motions of point A and the follower are the same. The vertical motion of A is simple harmonic when the cam rotates at constant angular velocity, since it is then moving in a circle about O at constant speed. It will be observed that B, the point of contact, is at the foot of a perpendicular from A on the cam face. If a perpendicular OC is drawn from O to AB, the following relationships hold, by Art. 8, Chapter II.

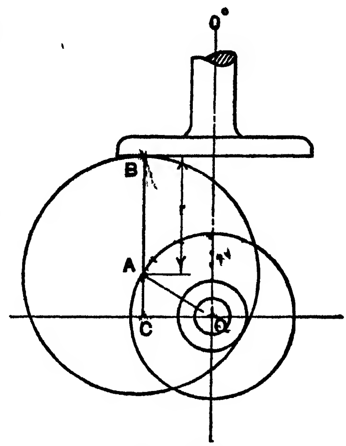


Fig. 6–38

Follower Velocity = Vertical Velocity of $A = \omega \cdot OC$,

Follower Acceleration = Vertical Acceleration of $A = \omega^2 \cdot AC$, where ω = angular velocity of the cam.

Plotting the lengths OC and AC on a base representing cam angles will therefore determine points on the velocity and acceleration curves for the follower motion, as shown in Fig. 6–39. These are evidently sine and cosine curves.

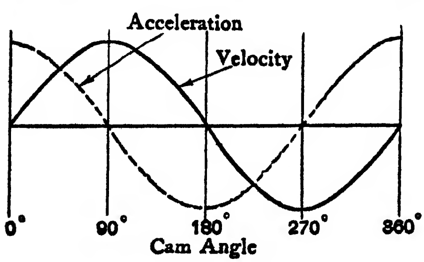


Fig. 6–39

The same construction may be used for a cam whose profile is made up of several circular curves; the follower motion will then be composed of corresponding sections of harmonics varying in amplitude and phase in accordance with the locations of the center of curvature.

Figure 6–40 illustrates the application to a portion of a cam outline composed of a circular arc KL with center at A', and LM with center at A. Here the cam is kept stationary and the follower is moved around it, as in previous cam constructions.

When the angular displacement is θ from a reference position, the follower face lies along XY and touches the cam at the point B at the foot of the perpendicular AB. OC is drawn from O perpendicular to AB. From the above discussion it is evident that OC $(= v)$ and AC $(= a)$ represent graphically the velocity and acceleration of the follower at cam displacement θ. These distances are therefore transferred to the velocity and acceleration diagrams of Figs. 6–41 and 6–42 and plotted as ordinates at angle θ. Other points on the curves are obtained in a similar manner. The follower begins contact with the arc LM at cam angle α and terminates contact at angle β when the cam is assumed to have counter-clockwise rotation. In this event the acceleration represented by AC is positive and has consequently been plotted above the base line. If C should fall on AB produced, the sign would be the opposite.

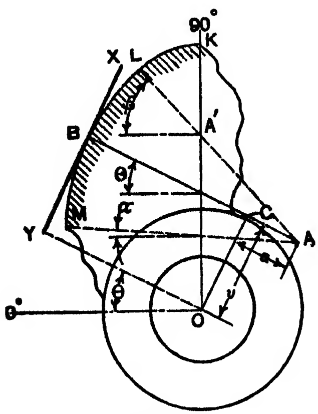


Fig. 6–40

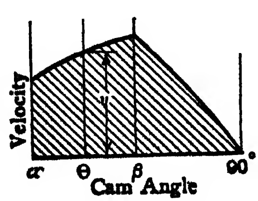


Fig. 6–41

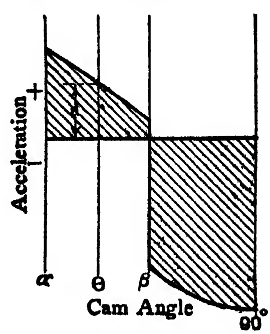


Fig. 6–42

Assuming that the cam turns at 900 R.P.M. and that the cam is drawn twice full size, the velocity and acceleration scales can be found as follows:

The angular velocity of the cam $(\omega) = 2\pi \times \frac{900}{60} = 94.2$ radians per second.

The follower velocity

$$= \omega \times CO \text{ where } CO \text{ is actual size}$$

$$= \omega \times \frac{CO}{2} \text{ where } CO \text{ is twice full size}$$

$$= \frac{94.2}{2} \times CO \text{ in. per sec. where } CO \text{ is in inches}$$

$$= 3.92 \times CO \text{ ft. per sec.}$$

For the velocity curve, therefore, 1 in. = 3.92 ft. per sec. Similarly, for the acceleration curve, 1 in. $= \frac{\omega^2}{2 \times 12} = 369$ ft. per sec. per sec.

23. Circular-arc Cams with Roller Follower. — As pointed out by Goodman in his book "Mechanics Applied to Engineering," the motion of the follower in this mechanism is the same as the motion of the piston in a direct-acting engine of proper proportions. The cam mechanism is shown in solid lines in Fig. 6–43; the equivalent mechanism is indicated in broken lines in the same figure. The crank length R is equal to the distance from the center of curvature of the cam surface to the center of rotation of the cam. The connecting rod has a length equal to the sum of the radius of the cam surface plus the roller radius. Keeping this analogy in mind, it is evident that the follower velocity and acceleration may be determined by the same formulæ or by the same graphical construction which we apply in finding the piston velocity and acceleration in the direct-acting engine.

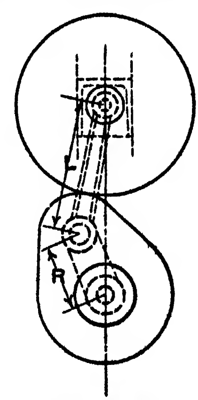


Fig. 6–43

Figures 6–44*a*, 45*a*, 46*a*, and 47*a* show graphical velocity and acceleration constructions for different cases as follows:

Figures 44*a* and 46*a* deal with the toe of the cam. In the former, L, the connecting-rod length, is greater than R, the crank length; while in the latter L is less than R.

Figures 45*a* and 47*a* deal with "**flank**" arcs. In the former L is greater than R; in the latter L is less than R.

The method of obtaining a length representing the follower velocity is the same in all the above cases and is that described in Art. 3, Chapter V. A line OC is drawn perpendicular to the "line of stroke" OE to meet the "connecting rod" EA, produced if necessary, at C. The distance OC $(= v)$ represents the follower velocity to a scale of 1 in. $= \frac{\omega}{12}$ ft. per sec., where ω is in radians per second and the diagram is actual size. The corresponding velocity curves shown in Figs. 6–44*b*, 45*b*, 46*b*, and 47*b* are found by plotting OC and lengths obtained in similar fashion for other cam angles on a base representing the cam angles. In Fig. 6–44*a*, OC represents the velocity at a cam displacement of 75° from the arbitrary zero position.

Klein's Construction (Art. 7, Chapter V) is next applied to find the acceleration in the cases shown in Figs. 6–44*a* and 6–45*a*, where R is not greater than L. With A as a center and radius equal to AC, an arc is drawn to meet a semicircle which has AE as its diameter. From the intersection at B, BF is next drawn perpendicular to EA, meeting the line OE at D. In Art. 7, Chapter V, it was shown that

$$\text{Follower Acceleration} = \omega^2 \times OD.$$

OD, therefore, represents this acceleration to a scale of 1 in. $= \frac{\omega^2}{12}$ ft. per sec. per sec. when the diagram is full size. Points on the acceleration curves of Figs. 6–44*b* and 6–45*b* are therefore found by plotting distances such as OD, as ordinates at the corresponding angles..

Where R is greater than L, as in Figs. 6–46*a* and 6–47*a*, Klein's Construction becomes impossible since no intersection of the arcs takes place. This construction is, however, just a means of locating the point F on the line EA such that $AF \cdot AE = (AC)^2$.* In Figs. 6–46*a* and 6–47*a* this point is found as follows:

At E erect a perpendicular to AE. This intersects at B an arc drawn with A as center and radius AC. At B draw BF per-

* See Chap. V, Art. 7.

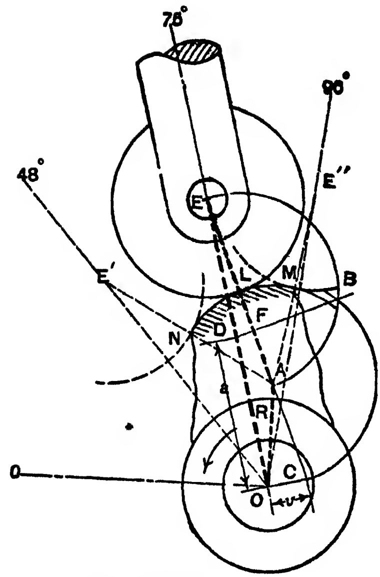


FIG. 6-44*a*

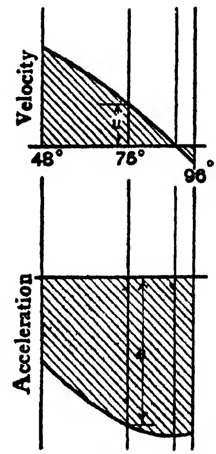


FIG. 6-44*b*

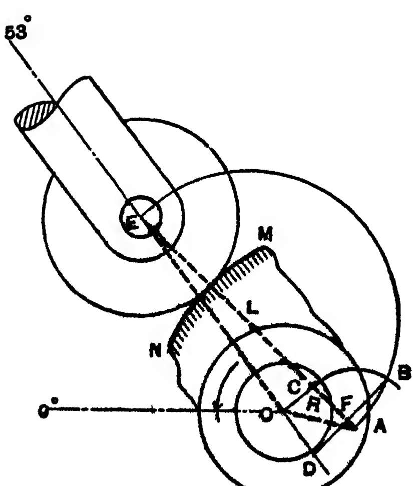


FIG. 6-45*a*

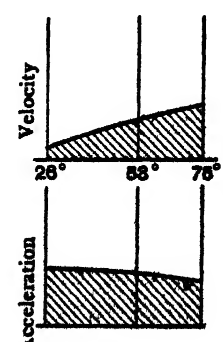


FIG. 6-45*b*

pendicular to AB, meeting AE produced at F. Evidently, from the two similar triangles AEB and ABF,

$$EA : AB = AB : AF$$

or

$$AF \cdot AE = (AB)^2 = (AC)^2.$$

Therefore F is the required point, and if FD is drawn perpendicular to AE, then

$$\text{Follower Acceleration} = \omega^2 \times OD.$$

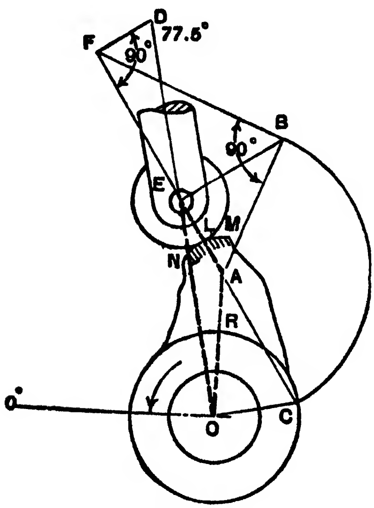


FIG. 6–46a

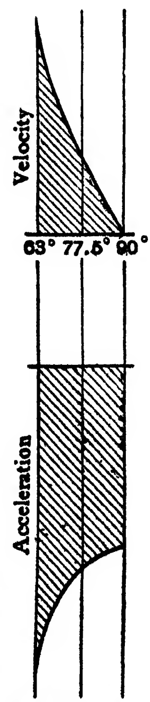


FIG. 6–46b

OD is therefore plotted as an ordinate to obtain a point on the acceleration curve. (See Figs. 6–46b and 6–47b.)

The ratio of L to R governs the shape of the acceleration curve. By choosing a proper value of $\frac{L}{R}$ the acceleration can be maintained practically constant for a limited cam displacement. Thus, a ratio of $\frac{L}{R} = 3.5$, if used in the mechanisms of 6–44a and 6–45a, would give an acceleration of approximately constant value for cam displacements of 45° on either side of the dead-center position of the equivalent mechanism. Similarly, if the ratio $\frac{L}{R} = .8$ were used in Figs. 6–46*a* and 6–47*a*, the follower acceleration would

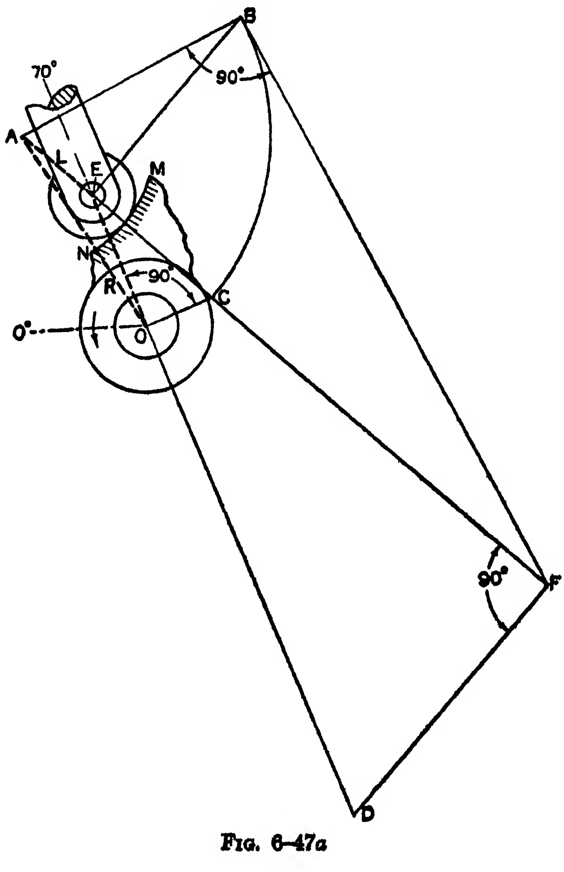


FIG. 6–47*a*

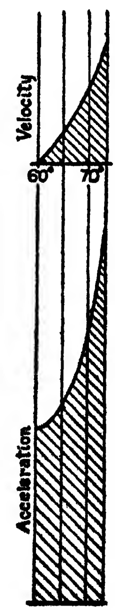


FIG. 6–47*b*

be about constant for 25° of cam displacement from the dead-center position.

The flat-surfaced or "tangent" cam has not yet been considered. Figure 6–48 illustrates this case. The velocity is found by drawing EC perpendicular to the cam face, to meet OC perpendicular to OE. The velocity is $\omega \times OC$ and this distance is used to determine a point on the velocity curve, Fig. 6–49. As regards the acceleration, neither of the constructions used for the circular-arc profiles can be applied. If the triangle OFC be drawn, in which OF is parallel to the cam face and FC parallel to OE, then

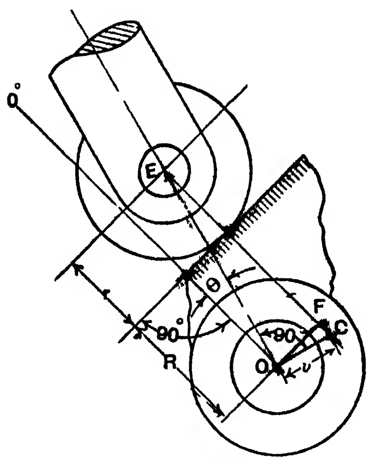


FIG. 6–48

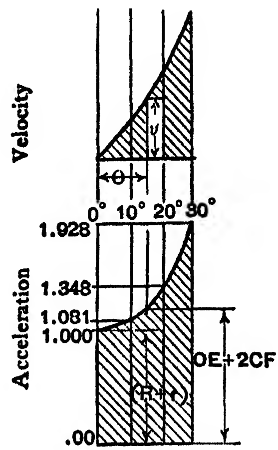


FIG. 6–49

it can be shown that the acceleration is equal to $\omega^2(OE + 2\,CF)$.

The proof for these statements is as follows:

The follower displacement from zero velocity position at cam displacement θ, from the figure, is equal to

$$OE - (R + r) = \frac{R + r}{\cos\theta} - (R + r).$$

$$\text{The velocity } (v) = \frac{ds}{dt} = \frac{d}{dt}\left[\frac{R + r}{\cos\theta}\right] = \frac{\omega(R + r)}{\cos\theta} \times \tan\theta$$

$$= \omega \times OE \times \tan\theta = \omega \times OC.$$

$$\text{The acceleration } (a) = \frac{dv}{dt} = \frac{d}{dt}\left[\frac{\omega(R + r)}{\cos\theta} \times \tan\theta\right]$$

$$= \omega^2(R + r)\left(\frac{1 + 2\tan^2\theta}{\cos\theta}\right).$$

This may be written in the form

$$a = \frac{\omega^2(R + r)}{\cos \theta} + \frac{2\,\omega^2(R + r)\tan^2 \theta}{\cos \theta};$$

and, since

$$OE = \frac{R + r}{\cos \theta},$$

$$OC = OE \tan \theta \quad \text{and} \quad CF = OC \times \tan \theta = OE \times \tan^2 \theta,$$

we may write

$$a = \omega^2(OE + 2\,CF).$$

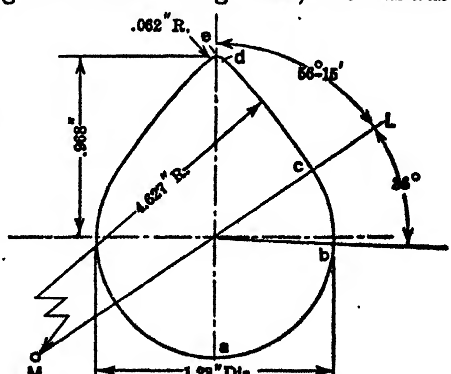
The distance $(OE + 2\,CF)$ is plotted as an ordinate on the acceleration diagram of Fig. 6–49. As before, the scale is 1 in. $= \frac{\omega^2}{12}$ ft. per sec. per sec. for a diagram drawn full size.

It should be observed that the circular-arc cam constructions given in Arts. 22 and 23 only apply where the follower has rectilinear motion, though similar graphical constructions are available for pivoted followers.[1]

24. Automobile Engine Valve Cams. — The Intake Valve Cam used in the Auburn car engine is shown in Fig. 6–50; the Exhaust Valve Cam of the same motor is illustrated on page 264.

These cams are of the circular arc variety and are designed to operate with a flat-faced follower. The half-cam profile is composed of four sections each fulfilling a definite function in the movement of the valve lifter, as follows:

(*a*) The valve is closed



(Lycoming Motor Co.)
FIG. 6–50

[1] For additional information, see paper by H. Schreck on "Kinematics of Cams" in the *Transactions* of the A.S.M.E., 1926; also article by G. L. Guillet on "Graphical Analysis of Circular Arc Cams" in the *American Machinist*, Oct. 21 and 28, 1926, and June 16, 1927.

when the follower makes contact with the base circle portion *a–b*.

(*b*) The cam profile is a regular curve for portion *b–c*, subtending an angle of 36°, during which the radial distances to the profile increase uniformly at the rate of 0.00033 in. per degree. The total lift of the follower during this period is thus equal to 0.012 in. for the whole interval. This slow lift at nearly constant velocity closes up the clearance between the follower and the valve stem without noise.

(*c*) The portion of the cam profile *c–d*, which is a circular arc with center along *L–M* and radius as indicated, produces a high acceleration of follower and valve, resulting in rapid lift.

(*d*) The "toe" portion of the cam *d–e*, a circular arc of small radius, produces a comparatively low deceleration of the follower at the end of which the valve velocity is decreased to zero at the top of its travel.

The cam is symmetrical about *e–a*. The acceleration during *c–d* is usually from three to four times the deceleration during *d–e*. The valve spring keeps the follower in contact with the cam during the latter period, and by designing for a low deceleration it is possible to avoid the use of a heavy spring.

QUESTIONS — CHAPTER VI

1. Plot displacement diagrams for the follower motions as specified in A to E. Show in each case sufficient construction lines, points, and notation to indicate the methods employed.

A. A follower lifts with constant velocity to highest position during 120° of cam displacement, rests for 60°, and falls with simple harmonic motion during 45° to initial position, where it remains for the balance of the revolution.

B. A follower lifts with constant and equal acceleration and deceleration to the top of its travel during 180° of cam movement, rests for 30°, returns to its initial position with constant velocity during 120°, and rests for the balance of the revolution.

C. The follower motion is the same as for 1B, except that during the lift period the acceleration is twice as great as the deceleration.

D. A pivoted follower moves through a total angle of 20°. Its outward motion is accomplished with constant acceleration during a cam displacement of 45°, then at constant velocity for 90°, and finally with constant deceleration for 45° to the end of its travel.

E. A pivoted follower, starting from the extreme outward position, moves in with constant and equal acceleration and deceleration to the other end of its travel, the total displacement being 25° during a cam movement of 90°. It then rests for 45° and returns to its initial position during 60° with motion of the same character as on the other stroke. A rest period makes up the balance of the revolution.

2. Why is it impracticable to use an unmodified constant-velocity cam at high speeds? How should it be modified in order to obtain best results?

3. A follower lifts ½ in. during a cam displacement of 90° at constant velocity, the cam rotating at a constant speed of 120 R.P.M. (*a*) Find the velocity of the follower. (*b*) If the follower lifts with constant and equal acceleration and deceleration, find the value of the acceleration and the maximum velocity attained. *Ans.* (*a*) 4 in. per sec. (*b*) 128 in. per sec. per sec., 8 in. per sec.

4. A follower lifts ¾ in. during a half-revolution of the cam, the latter rotating at a constant speed of 480 R.P.M. The constant acceleration for the first part of the lift period is three times as great as the constant deceleration during the latter part of this period. Find the value of the acceleration and the cam displacement during which it takes place.

5. Calculate the maximum velocity and acceleration of a follower which moves through a distance of 1 in. with simple harmonic motion during 120° of cam displacement, the cam rotating at 200 R.P.M. *Ans.* 15.7 in. per sec. 492 in. per sec. per sec.

6. In each of the following cases, A to I, assume a displacement diagram to be given and show how to plot the cam profile which will give the required motion to the follower. Show sufficient construction lines and notation to indicate the method employed in each case.

A. Disc cam rotating clockwise, with knife-edge follower, the edge moving in a straight line passing through the cam axis.

B. Disc cam rotating clockwise, with knife-edge follower moving in a straight line passing to the left of the cam axis.

C. Disc cam rotating counter-clockwise with roller follower having rectilinear motion, the roller center moving in a straight line which intersects the cam axis.

D. Disc cam rotating counter-clockwise with roller follower having rectilinear motion, the roller center moving along a straight line passing somewhat to the right of the cam axis.

E. Disc cam rotating clockwise with roller follower pivoted to the left and somewhat above the cam axis.

F. Disc cam turning clockwise with follower having convex sliding face. Follower moves without angular displacement, its axis passing through the cam axis.

G. Disc cam, turning counter-clockwise, having flat-faced follower with rectilinear motion. How is the necessary breadth of the follower face determined in this case?

H. Disc cam, turning counter-clockwise, with flat-faced follower pivoted to the right and somewhat above the cam axis.

I. Disc cam with roller-type, pivoted primary follower and convex-faced secondary follower having rectilinear motion.

7. Sketch and explain the action in the three types of positive-motion cam mechanisms.

8. (a) Where a positive-motion cam mechanism has a single cam acting on two faces of a yoke follower, what limitation is imposed on the motion of the follower? (b) What is the main objection to the use of positive-motion cam mechanisms with a single roller acting on two cam faces?

9. Assuming that the form of the displacement curve is known, show how to find the cam profiles in each of the following positive-motion cam mechanisms.

A. Single-disc cam turning clockwise with yoke-type follower having two parallel, flat contact faces. Follower has rectilinear motion.
B. Single-disc cam turning clockwise with yoke-type follower carrying two rollers. Roller centers move in a straight line passing through the cam axis.
C. Single-disc cam turning clockwise with yoke-type follower carrying two rollers. Follower pivots about a point on the right side of the cam axis.
D. Slotted disc cam with two contact faces acting on a roller-type follower. The follower reciprocates along a straight line passing through the cam axis.
E. Mechanism with motion and return cams each bearing on one of two parallel flat faces of yoke follower with rectilinear motion.

10. State one advantage and one disadvantage of using a conical roller in a cylinder-cam mechanism as compared with the use of a cylindrical roller.

11. Show how to draw a development of the profile of a cylinder cam when the form of the displacement curve is known. Show the construction for obtaining the elevation of the cam. Assume that the follower has rectilinear motion along an element of the cam cylinder and that the roller is cylindrical.

12. State three practical advantages in using circular-arc profiles instead of curves of other kinds.

13. Show how to find graphically the velocity and acceleration of the follower under conditions stated in A to E. State the velocity and acceleration scales in each case.

A. Follower has a flat face, moves with rectilinear motion, and makes contact with a circular-arc profile of a disc cam, the cam dimensions and speed being known.
B. Roller-type follower with rectilinear motion and no offset is in contact with a circular-arc cam toe. The latter has a profile such that L is greater than R in the equivalent direct-acting engine mechanism.
C. Roller-type follower with rectilinear motion and no offset is in contact with a circular-arc cam toe. The latter has a profile such that L is less than R in the equivalent direct-acting engine mechanism.
D. Same as Question B except that the follower is in contact with the cam flank.
E. Same as Question C except that the follower is in contact with the cam flank.
F. Roller-type follower with rectilinear motion and no offset is in contact with a flat surface on the cam.

14. A cam for a four-cycle internal-combustion engine exhaust valve is designed to have a period of valve opening which lasts for 110° of crank-shaft rotation. As is usual, the cam rotates at one-half crank-shaft speed. If the cam is designed for constant acceleration and deceleration with an acceleration-deceleration ratio of 3 : 1, find (*a*) the maximum velocity of the valve during the opening period and (*b*) the maximum acceleration of the valve. The engine speed is 3600 R.P.M., and the valve lift $\frac{1}{4}$ in.

15. How would you alter a cam mechanism in order to overcome the following difficulties? (*a*) Too much side thrust on the follower. (*b*) Inability to secure sufficiently rapid lift with a flat-faced follower. How would you avoid (*c*) the necessity of using a spring, (*d*) difficulties in specifying the form of cam profile?

CHAPTER VII

ROLLING CONTACT

1. Conditions for Rolling Contact. — When two bodies in contact move with respect to each other in such a way that there is no relative motion at the point of contact, the bodies are said to have **Pure Rolling Contact.** It follows that the points in contact have, for the instant, the same velocity relative to a third body. Moreover, by Art. 1, Chapter III, the instant center of the two bodies is located at the contact point.

When two bodies with pure rolling contact turn about **instant or permanent centers** on a third body, the point of contact must always lie on the straight line joining these centers. This may be proved by reference to Fig. 7–1, in which a and b have rolling contact and turn respectively about centers O_{ac} and O_{bc}. P is the point of contact at the instant and, since at this point no relative motion exists, P is the instant center O_{ab}. By Kennedy's Theorem (Art. 5, Chapter III), O_{ac}, O_{bc}, and O_{ab} lie in one straight line.

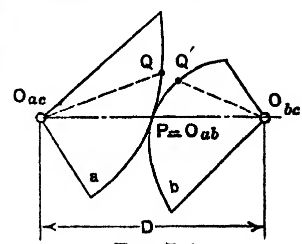


FIG. 7–1

It has just been shown that the point of contact of a pair of rolling bodies is on the line joining their instantaneous centers or pivots. We shall consider the case where two bodies with pure rolling contact turn about **fixed pivots**; in Fig. 7–1, O_{ac} and O_{bc} now become permanent centers as well as instant centers. If we select any point Q on the profile of body a, measure the length of the profile from P to Q, and lay off an equal length PQ' along the profile of b, then evidently when the bodies rotate, at some time Q and Q' will coincide; otherwise slippage will have taken place. Since Q and Q' meet on the line O_{ac}—O_{bc},

$$(O_{ac}—Q) + (Q'—O_{bc}) = (O_{ac}—P) + (P—O_{bc}) = D, \qquad (7–1)$$

where D is the distance between the permanent centers. Thus, bodies of any form may have pure rolling contact; but they will have a fixed distance between their centers of rotation, and will therefore be capable of turning about permanent centers on a third body, only when the condition stated by equation (7–1) holds true. This condition is that the **sum of the radiants** to any pair of points which pure rolling will bring in contact **must be constant.**

2. Profile Construction. — The following approximate construction may be used to find the profile of a body which is required to have pure rolling contact with a second body of known form, both bodies being assumed to rotate about permanent centers.

The construction is based on the properties of rolling bodies discussed in Art. 1, Chapter VII. Let a (Fig. 7–2) be the body of known form rotating about O, and suppose it is required to find the profile of a second body rotating about O' which will roll with the first. By joining O and O' we find P, which must be the contact point in the position of the mechanism shown in the figure.

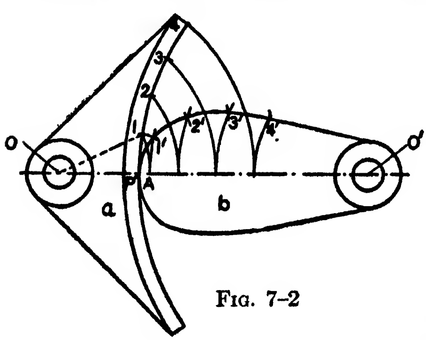


Fig. 7–2

A convenient number of points 1, 2, 3, etc., are selected on the profile of a. To find a point on b which will come in contact with 1 as the bodies roll together, we use O as center, radius equal to $O1$, and draw an arc intersecting OO' at A. We next use P as center and radius $P1$, constructing a second arc. Finally, with O' as center and radius $O'A$, we strike an arc intersecting the latter at $1'$. Now the sum of the distances $O1$ and $O'1'$ equals OO', by construction. Since the distances $P1$ and $P1'$ are equal, if the profiles of the bodies between these points are sensibly similar, the lengths of these profiles will also be approximately equal, and hence 1 and $1'$ will coincide during rotation. Point $1'$ may therefore be taken as a point on the profile of b. Point $2'$ is found in similar manner by making the distance $1'2'$ equal to 12, and $O'2'$ equal to OO' minus $O2$. Finally a smooth curve is drawn through P, $1'$, $2'$, etc. This construction becomes exact when the points 1, 2, 3, are an infinitesimal distance apart.

3. Angular Velocity Ratio. — In Fig. 7–3 two bodies in rolling contact make contact for the instant at a point P. If V_P is the linear velocity of the common point and we consider P as a point on a, $V_P = \omega_{ac} \times (OP)$.

Regarding P as a point on b,

$$V_P = \omega_{bc} \times (O'P).$$

Thus,

$$\omega_{ac} \times (OP) = \omega_{bc} \times (O'P)$$

or

$$\frac{\omega_{bc}}{\omega_{ac}} = \frac{OP}{O'P}. \qquad (7\text{–}2)$$

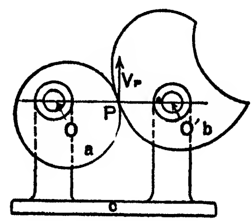


Fig. 7–3

In the case considered it is obvious that the bodies a and b rotate in opposite senses. If the two rotations are considered as positive and negative, respectively, the velocity ratio should bear a negative sign.

Instantaneous conditions only were considered; consequently, points O and O' in Fig. 7–3 **need only be instant pivots** and not necessarily fixed pivots.

The above equation, stated in words, means that **the velocity ratio of a pair of bodies in rolling contact is inversely proportional to the distance from their point of contact to their respective pivots.** When the point of contact falls between their pivots, they must rotate in opposite senses. When it falls to one side of both pivots, the reverse is true.

For constant velocity ratio, OP and $O'P$ must have a constant ratio, which is true only when P occupies a fixed position on the line of centers. A pair of circles are the only curves that fulfill this condition; consequently, bodies that roll together with constant velocity ratio must have circular sections perpendicular to their axes of revolution, and such bodies will have velocities inversely proportional to their radii.

4. Velocity Ratio of Rolling Cones. — Figure 7–4 shows two cones used to connect shafts meeting at an angle θ. The distances

BC and CD are the radii of the circular bases. At C the cones have a common velocity V_C. Then,

$$V_C = \omega_{ac} \times BC = \omega_{bc} \times CD,$$

or

$$\frac{\omega_{bc}}{\omega_{ac}} = \frac{BC}{CD}.$$

The velocities are therefore inversely proportional to the radii or diameters of the bases.

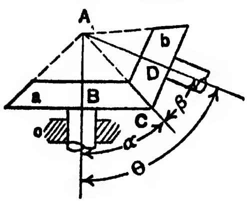


FIG. 7-4

Let α and β be the angles of the cones. From the figure,

$$\sin \alpha = \frac{BC}{AC} \qquad (1)$$

$$\sin \beta = \frac{CD}{AC}. \qquad (2)$$

Dividing (1) by (2),

$$\frac{\sin \alpha}{\sin \beta} = \frac{BC}{CD} = \frac{\omega_{bc}}{\omega_{ac}}.$$

But

$$\alpha = \theta - \beta.$$

Therefore,

$$\frac{\sin (\theta - \beta)}{\sin \beta} = \frac{\omega_{bc}}{\omega_{ac}},$$

or

$$\frac{\sin \theta \cdot \cos \beta - \cos \theta \cdot \sin \beta}{\sin \beta} = \frac{\omega_{bc}}{\omega_{ac}}.$$

Dividing numerator and denominator by $\cos \beta$,

$$\frac{\sin \theta}{\tan \beta} - \cos \theta = \frac{\omega_{bc}}{\omega_{ac}},$$

or

$$\tan \beta = \frac{\sin \theta}{\frac{\omega_{bc}}{\omega_{ac}} + \cos \theta}. \qquad (7\text{-}3)$$

In the same way it can be proved that

$$\tan \alpha = \frac{\sin \theta}{\frac{\omega_{ac}}{\omega_{bc}} + \cos \theta}. \qquad (7\text{–}4)$$

For rolling cones with internal contact (see Fig. 7–5) it may be proved in a similar manner that:

$$\tan \beta = \frac{\sin \theta}{\frac{\omega_{bc}}{\omega_{ac}} - \cos \theta} \qquad (7\text{–}5)$$

$$\tan \alpha = \frac{-\sin \theta}{\frac{\omega_{ac}}{\omega_{bc}} - \cos \theta}. \qquad (7\text{–}6)$$

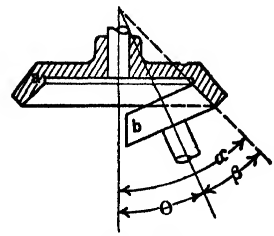


FIG. 7–5

These formulæ enable us to calculate the cone angles when the angle between shafts and the velocity ratio are known.

5. Rolling Cones. Graphical Method.—As an alternative to the calculation of the angles of rolling cones, a graphical solution can easily be made.

OA and *OB* (Fig. 7–6), represent the axes of intersecting shafts to be connected by rolling cones with a speed ratio of 5 to 2. These may have either external or internal contact; the former will be considered first.

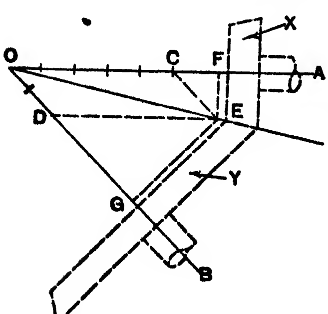


FIG. 7–6

A distance of five units is laid off along *OA*, thus locating the point *C*. Similarly, point *D* on *OB* is found by making *OD* equal two units. From *C*, *CE* is drawn parallel to *OB*, and from *D*, *DE* parallel to *AO*. The intersection *E* is a point on the contact line of cones which give the desired speed ratio. Join *OE* and drop perpendiculars *EF* and *EG* to the axes *AO* and *OB*. From the geometry of the figure, it can be shown that

$$EF : EG = OD : OC = 2 : 5.$$

Furthermore, a pair of perpendiculars drawn from any point on OE on the two axes will have the same ratio of lengths. At x and y (Fig. 7–6), are shown two cone frustums drawn with a pair of these perpendiculars as base radii. The speed ratio is

$$\omega_y : \omega_x = EF : EG = 2 : 5.$$

The cone angles to give the speed ratio specified are AOE and EOB.

Figure 7–7 illustrates the case where cones with internal contact are desired with the same speed ratio as before. The construction differs from that of Fig. 7–6 only to the extent that OD is laid off along BO produced. It should be noted that the use of internal instead of external contact reverses the sense of rotation of the driven member.

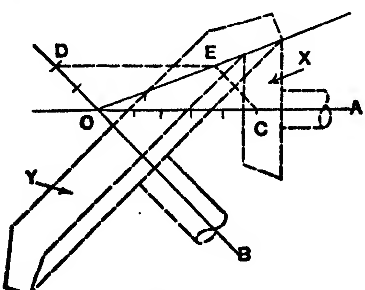


Fig. 7–7

6. Rolling Ellipses. — Two equal ellipses, initially placed as in Fig. 7–8, with all foci lying along the same straight line, and each turning about one of its foci, can be shown to have pure rolling contact. If O and O' denote the foci that are the centers of rotation, the distance between these points is obviously equal to the

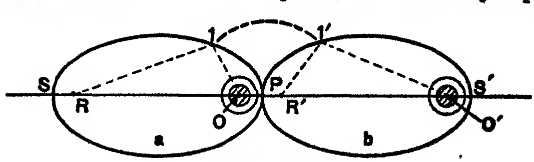


Fig. 7–8

major axis of either ellipse. It must be shown that the sum of the radii to any pair of points that the rolling of the curves will bring into contact is constant. (See Art. 1, Chapter VII.)

Using the initial point of contact P as center and any radius, we strike an arc cutting the curves at 1 and 1′. Since chords $P1$ and $P1'$ are equal, from the symmetry of the figure it will be evident that the elliptical arcs $P1$ and $P1'$ are of the same length. Therefore, pure rolling will bring 1 and 1′ together. We must show that $O1 + O'1'$ equals OO'. If R, R' are the other two foci, from the properties of the ellipse we know that $O1 + 1R$ equals the major axis. But, since the ellipses are equal in all respects, $1R = O'1'$. Therefore,

$$O1 + O'1' = O1 + 1R = \text{Major Axis} = OO'.$$

Thus the requirement of equation (7–1) is complied with.

The angular velocity ratio $\omega_b \div \omega_a$ of the ellipses in the relative position shown in Fig. 7–8 is equal to $OP \div O'P$, by Art. 3, Chapter VII. When ellipse a has rotated 180°, S and S' will come in contact, and at that instant the velocity ratio will be $OS \div O'S' = O'P \div OP$. These are, respectively, minimum and maximum values of the velocity ratio, the latter being the reciprocal of the former. During each half-revolution of the ellipses, the ratio changes from one value to the other.

7. Ellipses for Desired Velocity Ratios. — It is possible to construct ellipses which will give any desired variation in velocity ratio. Figure 7–9 illustrates the construction for a case where the driven ellipse is to have, as a maximum, three times the angular velocity of the driver. It follows that the minimum velocity of the driven member is one-third that of the driver. The distance OO', between the centers of rotation, is assumed to be known.

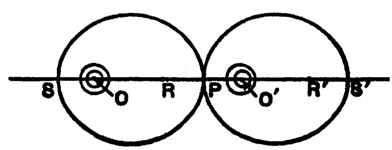


FIG. 7–9

In Fig. 7–9, OO' is first divided into two parts OP and PO', such that $OP \div O'P = 3 \div 1$. If OS, equal to $O'P$, is laid off along OO' produced, then PS will be the major axis of one ellipse. PS' equal to PS is the major axis of the second ellipse. The foci R and R' are located by making RP and $R'S'$ equal to PO'. Knowing the foci and major axes, we may draw the ellipses by any of the usual methods.

8. Friction Drives may be defined as being those in which power is transmitted by the rolling contact of driving and driven members, friction at the contact surfaces being depended upon to avoid appreciable slippage. They are practical applications of mechanisms having rolling contact. **Cylindrical wheels** with internal or external contact are commonly employed to connect parallel shafts. Wheels taking the form of **frustums of cones** are used for connecting intersecting shafts. These may have external contact (as in Fig. 7–4) or internal contact (as in Fig. 7–5). The frustums must be those of cones having a common apex in order that pure rolling conditions may be approached.

Practically, a certain amount of slippage is bound to take place in a friction drive when power is being transmitted. This type of drive is most serviceable for light duty. A heavy contact pressure is necessary when transmitting a large amount of power; this tends to cause friction losses and wear on the bearings of the wheel axles as well as on the contact surfaces.

For the purpose of increasing the power which may be transmitted by friction wheels for a given contact pressure, the wheels are sometimes provided with V-shaped circumferential grooves. It can easily be shown, however, that such construction renders pure rolling contact impossible, and therefore tends toward increased wear and friction losses.

9. Brush Wheel and Plate. — A friction drive of the form shown in Fig. 7–10 is sometimes used where it is desired to obtain a speed ratio which can be varied at will. The **Brush Wheel** *a* is usually the driver, making frictional contact with the driven plate *b*. Wheel *a* is mounted so that it may be shifted in an axial direction, thus moving in a line parallel to the surface of the plate. The speed ratio of driven and driving wheels depends on its position. Reversal of the sense of rotation is effected by moving *a* to the opposite side of the disc axis. A depression at the center of the disc causes the two members to break contact when *a* is in mid-position, giving a "neutral" point in which no drive is obtained.

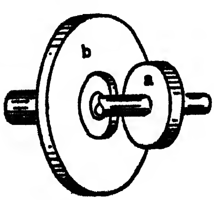


Fig. 7–10

Pure rolling takes place only when a has no sensible thickness and consequently makes point contact. Such a condition in the practical machine could be approached if the power transmitted were very small. A wide brush wheel with line contact will have pure rolling contact at or near its center point, with increasing sliding velocity as the edges are approached. This tends to cause rapid wear at the sides.

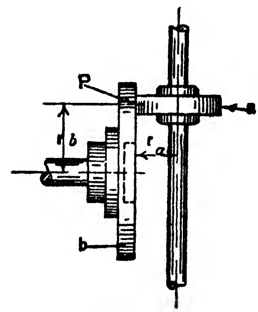


FIG. 7-11

If P (Fig. 7-11) is the point at which pure rolling takes place, then V_P is the same when calculated from the angular velocity of either body. Therefore,

$$V_P = \omega_a \times r_a = \omega_b \times r_b$$

or

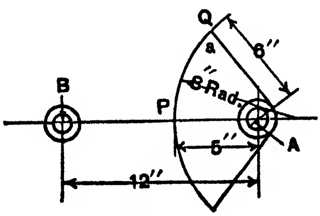
$$\frac{\omega_a}{\omega_b} = \frac{r_b}{r_a}.$$

When ω_a is constant, ω_b will have its maximum value when r_b is least and its minimum value when r_b is largest. These radii may be selected to give the desired range of speeds. P, the point of pure rolling, is generally assumed to be located at the middle of the face of the wheel a.

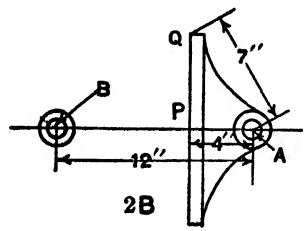
QUESTIONS — CHAPTER VII

1. (*a*) What is meant by pure rolling contact? (*b*) When any two bodies have pure rolling contact, what can be said regarding the location of the contact point? (*c*) When two bodies turn about fixed pivots and have pure rolling contact with one another, what condition must be satisfied in regard to their profiles? Prove this statement.

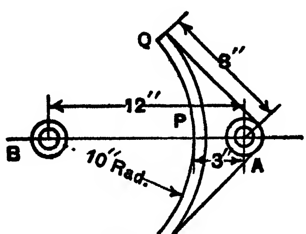
2. In Figs. 2A to 2C, show how to find the profile of a body rotating about fixed pivot *B* which will have pure rolling contact with the body turning about a fixed pivot at *A*. Calculate the angular velocity ratios when the contact point is at *P* and at *Q*.



2 A



2B



2C

3. Prove that a pair of equal ellipses rotating about fixed centers at their foci may have pure rolling contact.

4. Two parallel shafts are at a distance of 15 in. center to center. They are to be connected by rolling ellipses so as to obtain a maximum velocity ratio of 4 : 1. (*a*) Find the lengths of their major axes and the distances between their foci. (*b*) What is the minimum velocity ratio of the shafts?

5. Rolling cones with external contact are required to connect two shafts at 60°, the velocity ratio to be 3 : 2. Show how to find graphically the cone angles.

6. Rolling cones with internal contact are required to connect two shafts at 30°, the velocity ratio to be 3 : 1. Show how to find graphically the cone angles.

7. Two shafts intersecting at an angle of 60° are connected by means of rolling cones with external contact. One cone has a center angle of 20° and rotates at 300 R.P.M. Find the center angle of the other cone and its R.P.M.

Ans. 40°; 160 R.P.M.

8. A rolling cone with a center angle of 15° turning at 240 R.P.M. makes external contact with a second cone which turns at 360 R.P.M. Find the angle between the shafts and the apex angle of the second cone.

9. Rolling cones with internal contact are to be used to secure a speed ratio of 4 : 1. The smaller cone has a center angle of 10°. What is the center angle of the larger cone, and what is the angle between the shafts?

10. A pair of equal rolling ellipses have a distance between their foci of 4 in. The major axes are 7 in. long. Find the maximum and minimum speed ratios. *Ans.* 1 : 3.67, 3.67 : 1.

11. A large bucket elevator has a head sprocket with six teeth, the arrangement requiring a speed variation of the sprocket from a minimum of 10 R.P.M. to a maximum of 11½ R.P.M. in order to secure a uniform bucket speed. This variation in the sprocket speed is obtained by means of a pair of non-circular gears with elliptical pitch surfaces. If the center-to-center distance of the gears is 30 in., find the length of the major axes and the distance between the foci.

12. What is a friction drive? Sketch a form of friction drive which permits alteration of the velocity ratio when in service. How can it be arranged to reverse the direction of motion of the driven member? Why is pure rolling impossible in this device?

13. A brush-wheel-and-plate friction drive are required to connect two shafts, the driver turning at 300 R.P.M., and the driven plate to turn at a maximum speed of 100 R.P.M. and a minimum of 25 R.P.M. The driving wheel is 5 in. in diameter and 1 in. wide. Find the maximum and minimum diameter required for the plate.

14. What maximum and minimum speeds are obtainable at the driven shaft in a brush-wheel-and-disc friction drive in which the wheel, 8 in. in diameter and 1½ in. wide, rotates at 400 R.P.M. and in which the disc has a maximum diameter on the contact surface of 26 in., the minimum diameter being 14 in.? Allow 3 per cent for slippage.

CHAPTER VIII

TOOTHED GEARING

1. Toothed Gears are commonly employed for transmitting power from one revolving shaft to another. In comparison with other forms of drives, they are especially adapted for cases where a constant velocity ratio is required, or where driving and driven members must have definite phase relationships.

Since the interlocking action of the teeth makes the drive positive, and friction is not depended upon to avoid slippage, the pressure required to keep the gears in contact when power is being transmitted is much less than in an equivalent friction drive. This results in lower bearing pressures, less wear on the bearing surfaces, and greater efficiency.

Pitch Surfaces. — The pitch surface of a gear wheel may be defined as an imaginary surface on which the tooth construction is based. For any gear the form and dimensions of the pitch surface must be known before the teeth can be properly designed. Figure 8–1 shows the cylindrical pitch surface for a spur gear.

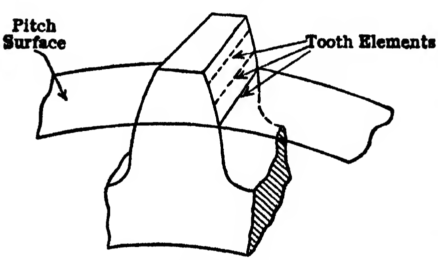


FIG. 8–1

Tooth Element. — A gear tooth may be regarded as a surface swept up by a line moving through space. The line is not always straight or even of constant shape. This profile generator in any one of its consecutive positions is known as a tooth element. Tooth elements always connect corresponding points on tooth sections taken perpendicular to the pitch surfaces. In Fig. 8–1 are shown tooth elements for a spur gear, which are straight lines parallel to one another.

Pitch Surfaces of mating gears may have either (*a*) **Pure Rolling Contact** or (*b*) **Sliding Contact.** This matter is of importance, since it determines whether or not the teeth have relative sliding along the tooth elements. When this kind of sliding action occurs it places certain limitations on the form of the elements.

Gear wheels are illustrations of **Higher Pairing,** because line or point contact only is obtained.

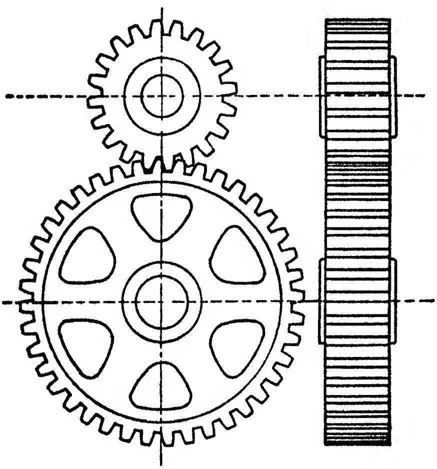


Fig. 8–2
Plain Spur Gears.

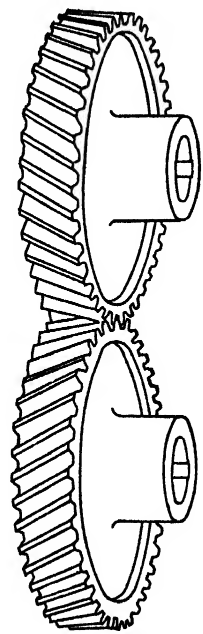


Fig. 8–3
Twisted-tooth Spur Gears.

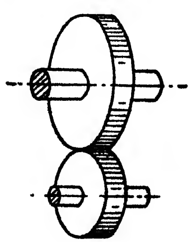


Fig. 8–4
Spur Gear Pitch Surfaces.

When the pitch surfaces of gear wheels have line contact, the gear teeth likewise have line contact. When the pitch surfaces have point contact so also have the gear teeth, the only common exception being found in some worm gears.

2. Gear Classification. — Gears may be classified according to the relative position of the axes of revolution. The axes may be (*a*) parallel, (*b*) intersecting, (*c*) neither parallel nor intersecting. We shall first make a brief survey of the common forms, and later discuss each in more detail.

(*a*) **Gears for Connecting Parallel Shafts.** — Here we may employ the **Common Spur Gears** as shown in Fig. 8–2 or the **Twisted-tooth Spur Gears** of Fig. 8–3. In both, the pitch surfaces are cylindrical with pure rolling contact as illustrated in

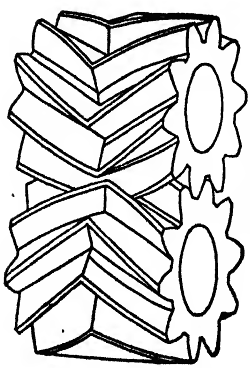


FIG. 8–5
Herringbone Gears.

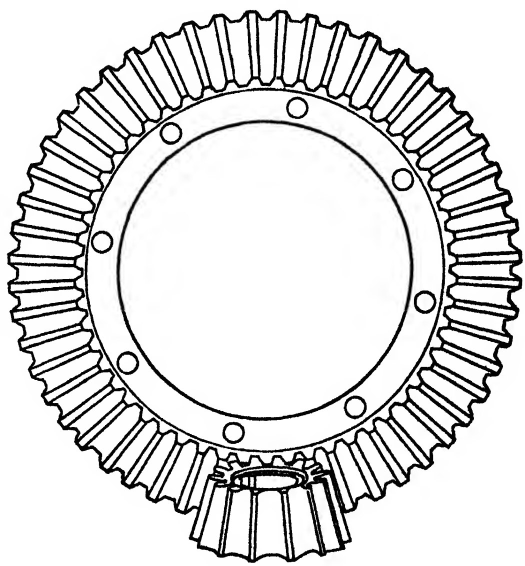


FIG. 8–6
Plain Bevel Gears.

Fig. 8–4. Tooth elements in the former are straight lines parallel with the axis of the gear, while in the latter these elements are helices. The twisted-tooth gear generally operates more quietly than the other type, the difference in this respect being particularly noticeable at high speed. The main disadvantage of the twisted-tooth type lies in the end thrust produced when the gear is transmitting power.

In the **Herringbone Gear** of Fig. 8–5 the end thrust set up by one side is balanced by an equal and opposite thrust due to the action on the other side. This gear can be regarded as composed of two twisted gears of similar dimensions, one having a right-handed and the other a left-handed helix.

(*b*) **Gears for Intersecting Shafts.** — In this case the **Plain Bevel Gear,** as in Fig. 8–6, or the **Spiral Bevel** shown in Fig. 8–7,

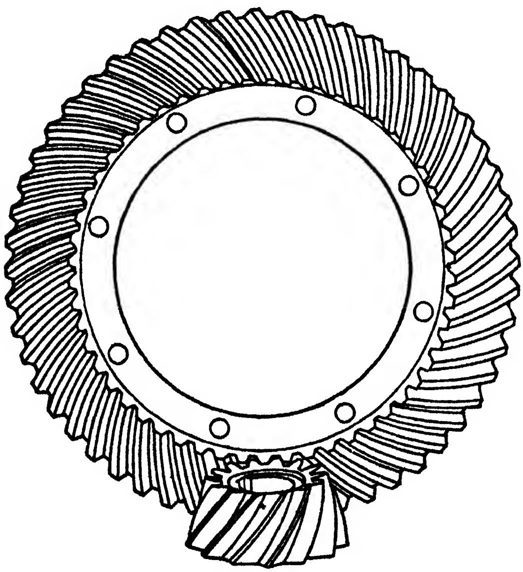


FIG. 8–7
Spiral Bevel Gears.

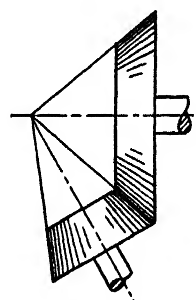


FIG. 8–8
Bevel Gear Pitch Surfaces.

is employed. In both cases the pitch surfaces are cones having a common apex as shown in Fig. 8–8. In the **Plain Bevel** the tooth elements are straight lines, while in the **Spiral Bevel** they are conical helices.

The **Spiral-bevel Gear** has a decided advantage over the **Plain-bevel Gear** as regards quietness of operation.

(*c*) **Gears for Connecting Shafts neither Intersecting nor Parallel.** Here **Helical Gears** or **Skew Bevels** are suitable.

Helical 'Gears (Fig. 8–9) are used to connect parallel as well as non-parallel shafts. In the former case they are sometimes called twisted-tooth spur gears. Their pitch surfaces (Fig. 8–10) are cylindrical and the tooth elements are helices. Where the shafts are non-parallel,* the pitch surfaces touch at a point and have sliding contact; here the teeth also make point contact and slide along the elements.

The **Worm Gear** shown in Fig. 8–11 is a special form of helical gear, the two members being known as the **Worm** and the **Worm Wheel.** The **Worm,** as compared with that which we commonly call a helical gear, has a small helix angle in proportion to its face width, with the result that each tooth extends a long distance around the circumference. It is customary to speak of worm teeth as " threads " on account of the resemblance which the worm bears to a threaded bolt. Hence we refer to a " single-threaded worm," a " double-threaded worm," etc., depending upon the number of teeth formed on the cylindrical surface. The **Worm Wheel** generally has the tooth surface concaved as shown in Fig. 8–11, for the purpose of obtaining line instead of point contact of the teeth.

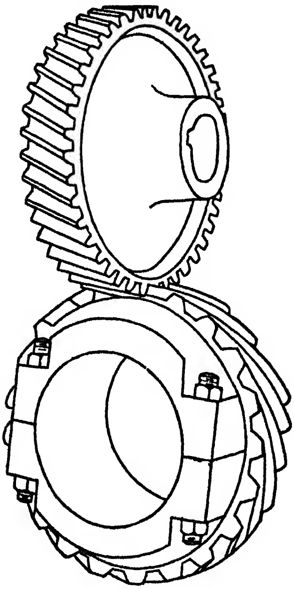


Fig. 8–9
Helical Gears.

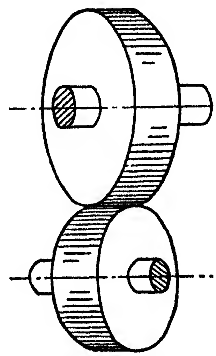


Fig. 8–10
Pitch Cylinders for Helical Gears.

Skew Bevels somewhat resemble other Bevel Gears in general appearance. Their pitch surfaces are not cones, however, but hyperboloids of revolution, i.e., figures swept out by the revolution

* In this case they are often mis-called "Spiral Gears."

of a generating line about the non-parallel axes of the gears. Contact between the two pitch surfaces thus formed takes place

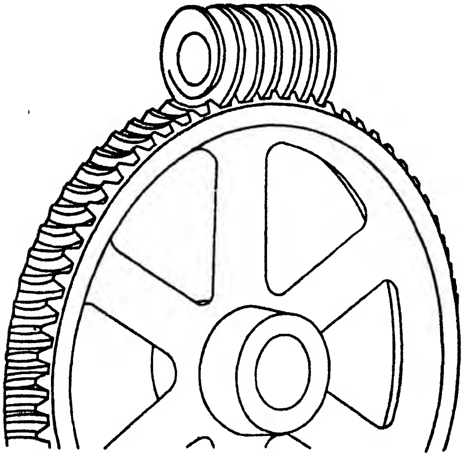


FIG. 8–11
Worm Gear.

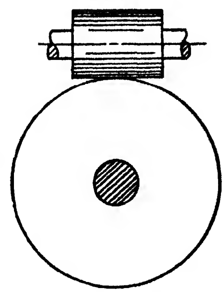


FIG. 8–12
Worm Gear Pitch Surfaces.

along this line in its position common to the two rotations. Figure 8–13 shows two such hyperboloids in contact along the line A—B. The portions of the pitch surfaces used for **Skew Bevels** are frustums of the figures remote from the smallest sections as indicated by X and Y in the figure.

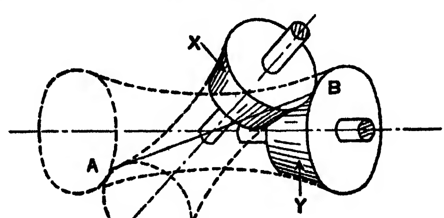


FIG. 8–13
Skew Gear Pitch Surfaces.

Evidently, pure rolling between the pitch surfaces is not possible, and sliding takes place along the elements.

Such gears are difficult to produce and are seldom used. Standard gears of this form are not obtainable.

3. Velocity Ratio. — A general rule for pairs of toothed gears is that the **angular velocity ratio is inversely proportional to the**

numbers of teeth. This law applies to all common classes of gears, such as spur, bevel, and helical gears.

When two gears are in motion it is evident that equal numbers of teeth on each gear pass any fixed point in a definite time interval, since the teeth on one gear mesh in consecutive order with the tooth spaces on the other gear. A gear having N_a teeth makes one turn while N_a teeth pass the fixed point. A meshing gear with N_b teeth will therefore make $\frac{N_a}{N_b}$ turns during the same interval.

If ω_a and ω_b are, respectively, the angular velocities of the two gears, then

$$\frac{\omega_a}{\omega_b} = \frac{1}{N_a \div N_b} = \frac{N_b}{N_a}, \tag{8-1}$$

which proves the above rule.

SPUR GEARS

4. Gear Terms. — Figure 8–14 illustrates many of the definitions which follow.

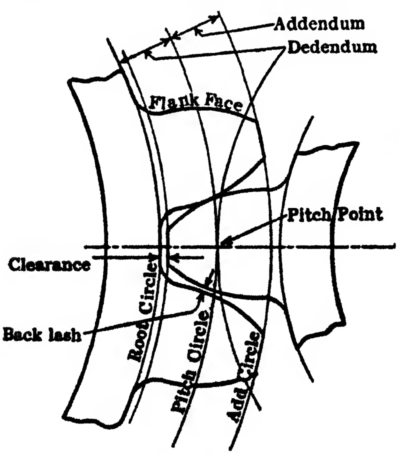


Fig. 8–14
Spur Gear Notation.

Pitch Diameter. — The diameter of the cylinder which is the pitch surface of a spur gear is known as the pitch diameter. Since the pitch cylinders of two spur gears roll together, the angular-velocity ratio is the inverse ratio of the pitch diameters, by Art. 3, Chapter VII. A pair of mating spur gears have numbers of teeth proportional to their pitch circumferences, because both must have the same spacing of the teeth in order to obtain pure rolling of the pitch circles. Thus, for two such gears, *a* and *b*,

$$\frac{\omega_a}{\omega_b} = \frac{D_b}{D_a} = \frac{N_b}{N_a}, \tag{8-2}$$

where N, D, and ω represent, respectively, number of teeth, pitch diameter, and angular velocity.

Pitch Point. — That point on the line joining the centers of two gears at which the pitch circles touch is called the pitch point.

Addendum. — The distance from the pitch circle to the outer end of the tooth, measured radially, is known as the addendum.

Clearance. — The clearance is the amount by which the points of the teeth on one gear clear the roots of the teeth on the mating gear. This is measured along the line of centers.

Dedendum.[1] — The radial distance from the pitch circle to the root circle is called the dedendum.

The **Whole Depth** is the sum of the addendum and dedendum.

The **Working Depth** is the whole depth minus the clearance.

The **Face** of the tooth is that portion of the profile between the pitch circle and outer end of the tooth.

The **Flank** of the tooth is that portion of the profile between the pitch circle and the root circle.

Backlash. — The minimum distance between the non-driving side of a tooth and the adjacent side of the mating tooth is called the backlash.

Gear and Pinion. — When two gears mesh with each other, the larger is commonly referred to as " the gear " and the smaller as " the pinion."

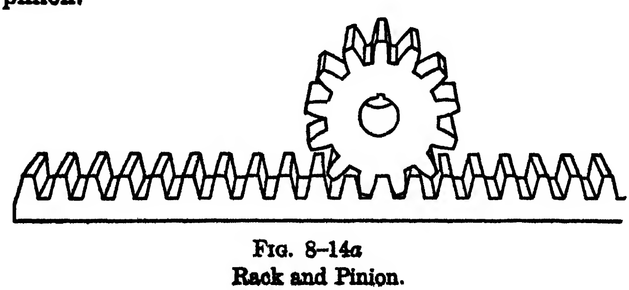


Fig. 8–14*a*
Rack and Pinion.

Rack. — When teeth are cut along the side of a straight bar it is known as a rack. Figure 8–14*a* shows a pinion and rack. The pitch surface of the latter is a plane.

[1] As defined by the A.G.M.A. and A.S.M.E. code. By other authorities the dedendum does not include the clearance.

Angle and Arc of Action. — The angle turned through by the driver for the period during which one of its teeth remains in contact with a mating tooth on the driven wheel is known as the **Angle of Action** of the driver. The angle turned through by the driven wheel in the same period is the **Angle of Action** of the driven wheel. The corresponding arcs on the pitch circle are called the **Arcs of Action.**

Evidently, the arc of action must be greater than the circular pitch; otherwise contact between one pair of teeth would cease before the next pair made contact. In general, the longer the teeth the greater the arc of action. This consideration has been an important factor in fixing the length of standard teeth.

The **Angle of Approach** is the angle turned through by the gear from the instant a pair of teeth make contact to the instant at which they are in contact at the pitch point.

The **Angle of Recess** is the angle turned through by the gear from the instant a pair of teeth are in contact at the pitch point to the instant when contact between the same teeth ceases.

The **Angle of Action** is equal to the **sum** of the angles of approach and recess.

The **Width of Face** of a gear is measured on the pitch surface in a plane containing the axis of revolution. **Face of Tooth** should not be confused with **Face of Gear** for the two are entirely different.

5. Pitch. — The pitch of a gear is a measure of the size of the teeth; all tooth dimensions in standard systems are based on the pitch. Gears that are intended to run with each other must have the same pitch, as well as tooth profiles of proper form. Two common methods of stating gear pitches are as follows:

The **Circular Pitch** is the distance between corresponding points on adjacent teeth, this distance being measured along the circumference of the pitch circle. When p denotes the circular pitch,

$$p = \frac{\pi D}{N}. \qquad (8\text{-}3)$$

The **Diametral Pitch**[2] is the result obtained by dividing the number of teeth by the pitch diameter. Stated otherwise, it is the number of teeth per inch of pitch diameter. Where P is this pitch,

$$P = \frac{N}{D}. \tag{8-4}$$

It should be observed that the **Circular Pitch** is a linear dimension, ordinarily expressed as so many inches. The **Diametral Pitch,** on the other hand, is just a ratio.

Relation between Circular and Diametral Pitch. — Multiplying equation (8–3) by equation (8–4), we have

$$pP = \pi. \tag{8-5}$$

The **Circular-pitch** method of specifying tooth sizes is the older one, but the **Diametral-pitch** method has advantages which have resulted in very general use, especially for small teeth. One advantage is shown as follows: A gear with 19 teeth of 2 D.P. has a pitch diameter of $19 \div 2 = 9\frac{1}{2}$ in., by equation (8–4). A gear with 19 teeth of 2-in. circular pitch has a pitch diameter of $\frac{19 \times 2}{\pi} = 12.095+$ in., by equation (8–3). The calculation is easier in the former case, and the result is always a rational number.

Gear teeth of 1 D.P. or smaller are commonly on the diametral-pitch system; those of 3-in. circular pitch or larger are on the circular-pitch system.

6. Gears are generally of circular section and give a constant angular velocity ratio of the connected shafts, though non-circular gears are employed where a variable-speed ratio is desired. Whether circular or non-circular, **the teeth should be so shaped as to produce pure rolling contact of the pitch surfaces.** The pitch surfaces, therefore, comply with the laws governing bodies having pure rolling contact as discussed in Chapter VII. Thus

[2] The term "module" is sometimes used to define the pitch. The module is the pitch diameter divided by the number of teeth, or the reciprocal of the diametral pitch.

the point of contact of two pitch surfaces, known as the **Pitch Point,** is the common instant center for the two gears.

If gear teeth are so designed as to give pure rolling of the pitch surfaces then the following law of gear teeth will result:

The common normal to the tooth surfaces at the point of contact must always pass through the pitch point.

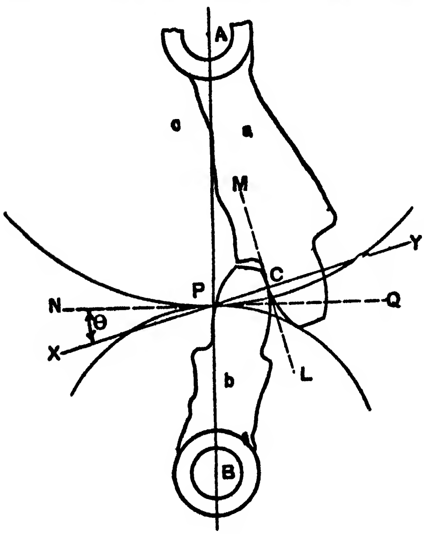


FIG. 8–15

This law is proved as follows: Figure 8–15 shows two gears making contact at C. The pitch point is at P. It is required to show that the common normal to the tooth profiles at C will pass through P. Proof is made by two statements:

(*a*) The mating gears are two bodies in sliding contact at C; hence the relative motion at the contact point is along the common tangent LM, otherwise the teeth would tend to either overlap or break contact. Therefore the instant center O_{ab} must lie along the common normal XY.

(*b*) The pitch surfaces have pure rolling contact at P. Hence P is the instant center O_{ab}.

Therefore the common normal at C passes through P.

For rolling bodies, as shown in Chapter VII, Art. 3,

$$\frac{\omega_{ac}}{\omega_{bc}} = \frac{BP}{AP}.$$

If the pitch surfaces are of circular section, P will occupy a fixed position on the line joining A and B. Hence BP/AP will be of constant value, and the angular velocity ratio of the gears will be constant.

If the pitch surfaces are non-circular, the value of the varying speed ratio can be calculated at any instant by means of the above equation, if the position of P is known.

7. Sliding Action of Teeth. — When a pair of teeth touch at the pitch point, they have, for the instant, pure rolling contact, since the point of contact is then the instant center for the gears. It follows that **in any other position they must slide** on one another, for then they meet at a point other than the instant center. The velocity of slide is directly proportional to the distance from the instant center to the point of contact at the instant considered. Maximum sliding velocity occurs when the teeth are just beginning or ending contact, the contact point being then most remote from the pitch point.

The magnitude of the sliding velocity at any instant can be determined graphically:

In Fig. 8–16 is illustrated a pair of conjugate teeth in contact at point C. The common normal at C is the line XY passing

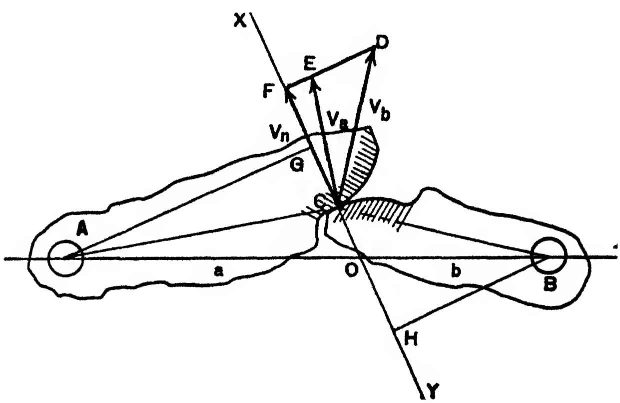


FIG. 8–16

through the pitch point O. The linear velocity of C, considered as a point on gear a, is represented by the vector CE perpendicular to the radius CA. The velocity of C, considered as a point on gear b, is represented by the vector CD, perpendicular to CB. Vectors CE and CD may be resolved into components parallel and perpendicular to the normal XY. The components of each parallel to XY must be the same normal velocity V_n, as the teeth maintain contact and yet do not cut into one another. The algebraic difference of the velocity components perpendicular to XY, namely, $DE = FD - FE$, represents the rate of slide of one surface on the other, or the sliding velocity. Inspection of the figure will show that this velocity decreases as the contact point moves toward point O and increases when it moves in the reverse direction.

8. Tooth Profiles. — In general, it is possible to select teeth of any shape for a spur gear, and then proceed to form teeth on a second gear which will be conjugate to the first, satisfying the law of constant velocity ratio. The process whereby the teeth are formed on the second gear might be carried out as follows: Suppose the blank to be made of a plastic material. The gear and blank are mounted on shafts and run together so that the pitch surfaces have the same linear velocity. Teeth are thus "rolled in" the surface of the soft blank. These teeth will have the correct outline. "Generating" processes of gear cutting, often used in commercial production, are carried out along similar lines. For example, in the Fellows gear shaper, the cutting tool takes the form of a gear wheel and is reciprocated across the blank. Between strokes the cutter and blank are both rotated slightly, the relative motion being equivalent to the rolling of the pitch surfaces. The resulting teeth will have the same form as obtained by the "rolling-in" process described above, without requiring the use of a plastic blank.

If a tooth shape is selected at random, and a conjugate tooth of theoretically correct shape is formed, it does not necessarily follow that the use of such teeth would be practical. Strength and wearing qualities must also be taken into account. In engineering practice we find that only two curves are in common use for profiles, namely, the **Cycloid** and the **Involute.**

CYCLOIDAL TEETH

9. The Cycloid, as used in circular gears, is a curve described by a point on a circle which rolls internally or externally on another circle. The rolling circle is known as the describing circle, and in

forming a gear-tooth outline it is rolled internally and externally on the pitch circle. Internal rolling forms the flank of the tooth; external rolling forms the face. In Fig. 8–17, *a* and *b* are the pitch circles of the gears. Circle *c* is rolled internally on *a*, the point *O* on *c* describing the curve *OB*, which forms the flank of the tooth on the upper wheel. Circle *c* is then rolled externally on *b*, point *O* on *c* now tracing out the curve *OC* which is the face of the tooth on the lower gear. Likewise, curves *OD* and *OE* are obtained by rolling circle *d* internally on *b* and externally on *a*. These curves form, respectively, the flank of the lower and the face of the upper gears. The two describing circles, *c* and *d*, need not be the same size, but the same circle must be used for the face of one gear and the flank of the other which works with it. In practice, to secure interchangeable gears, the same radius of describing circles is used throughout a series.

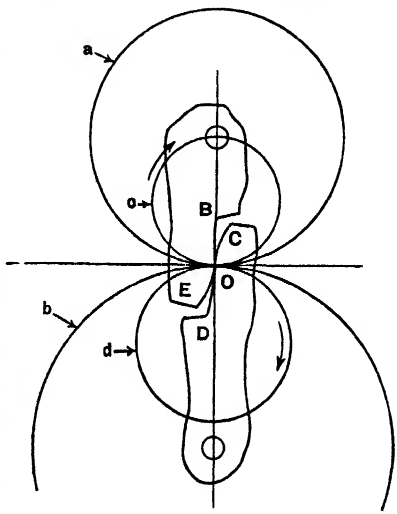


FIG. 8–17

When the diameter of the describing circle is one-half the pitch diameter of the gear, the flank of the tooth becomes a radial straight line (see *OB*, Fig. 8–17) and the tooth is somewhat narrow at the root. If the describing circle is made larger, the tooth becomes still narrower at the root and lacks strength; also, if the describing circle is made large enough, it may be impossible to cut the gear teeth by use of a milling cutter, because the space between the teeth widens out from the pitch circle toward the root. In the Brown and Sharpe standard system, the diameter of the describing circle is made equal to one-half the pitch diameter of a gear with fifteen teeth. This renders it possible to cut a twelve-tooth pinion with a milling cutter, this pinion being the smallest generally used.

10. Cycloid Construction. — In Fig. 8–18, LM is a portion of a pitch circle, and a describing circle of radius r is rolled on it internally, as required for the flank portion of a tooth. Point O on the describing circle traces out the cycloid which we wish to construct when this circle is rolled in a clockwise sense. The construction is indicated in the figure for points P, Q, R on the required curve.

When the describing circle has rolled so as to make any point such as 1 the contact point, the center of the describing circle will lie at C_1, on the line $1A$, and its circumference is shown at m. The new position of O at P is found by making the rectified length of the arc $1P$ equal to the rectified length of the arc $O1$. These arcs must be equal on account of the pure rolling action between describing and pitch circles. For contact at points 2, 3, etc., corresponding points Q, R, etc., are found on the cycloid by the same method, the curve being continued until it can be rounded into the root circle by a fillet arc.

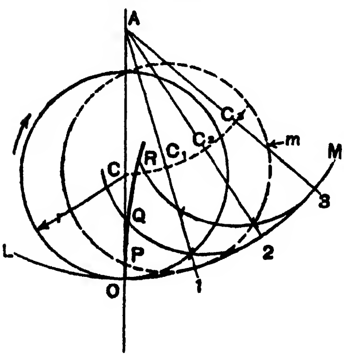


Fig. 8–18

The construction for finding the curve when the describing circle rolls externally is shown in Fig. 8–19. The method and description just given will apply without any alteration. The resulting curve can be used as the profile of a tooth face and is extended to its intersection with the addendum circle.

11. Proof of the Essential Property. — In order to show that cycloids may be used for gear teeth, it is necessary to prove that the normal at the point of contact of a pair of these curves, generated by the same describing circle, will always pass through the pitch point, in accordance with the law of Art. 6, Chapter VIII.

In Fig. 8–20, a and b are the pitch circles for a pair of spur gears, and O is the pitch point.

Cycloid *PM*, attached to *b*, is traced out by a point on describing circle *c* when it rolls externally on *b*.

Cycloid *PL*, attached to *a*, is described by a point on *c* as the latter circle rolls internally on *a*.

The two cycloids are in contact at *P* for the position of the gears shown in the figure. Point *O*, where *a*, *b*, and *c* are in con-

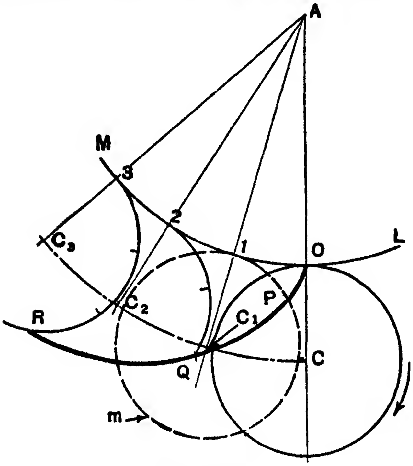


FIG. 8-19

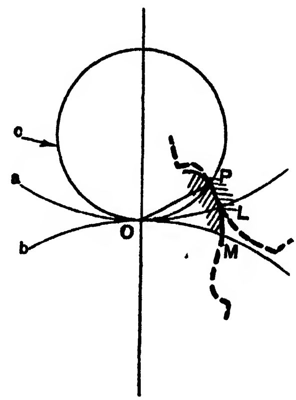


FIG 8-20

tact, is the common instant center for relative motions of these bodies, since at this point they have no relative motion. Therefore, *P*, as a point on *c*, has motion relative to both *a* and *b* in a direction at right angles to *OP*. Consequently, *OP* must be the common normal to the curves *PL* and *PM*.

These statements are true for any position of the gears, and *O* is a fixed point; therefore, the cycloids have the required property for gear-tooth profiles, as expressed by the law of Art. 6.

12. Path of Contact. — In Fig. 8-21 a pair of mating tooth profiles of the cycloidal type are shown in three positions, namely, a_1b_1, when just making contact; a_2b_2, when in contact at the pitch point; and a_3b_3, when about to break contact. The path of the point of contact must be the curve *COD*, which is composed of portions of the circumference of the two generating circles with centers at *L* and *M*.

Point C where contact begins is located at the intersection of the upper describing circle with the addendum circle of the gear,

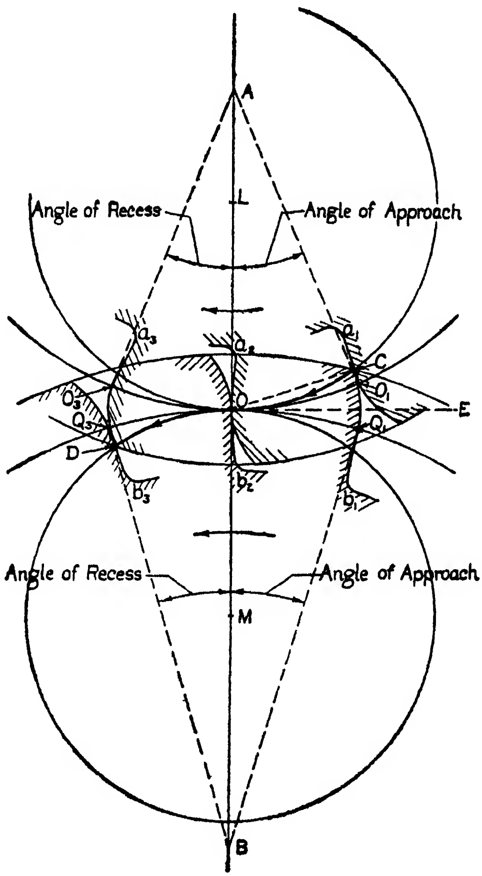


Fig. 8–21

while D is found at the intersection of the lower describing circle with the addendum circle of the pinion. Lengthening the teeth evidently moves the points C and D further apart and increases the period of contact. By joining CO and drawing OE perpendicular to the line of centers AB, we find the pressure angle COE for that position of the gears in which C is the contact point. This angle diminishes to zero as the point of contact approaches O and thereafter increases to another maximum at D. The cycloidal form of tooth is therefore characterized by a variable pressure angle, which is zero when the teeth make contact at the pitch point.

13. Angles of Approach and Recess. — Referring once more to Fig. 8–21, since a_1, a_2, and a_3 represent three positions of the same tooth on the pinion, O_1, O, and O_3 are three positions of one point on the tooth, and the angles O_1AO and OAO_3 show the corresponding angular movements of the pinion. These angles are, respectively, the **approach** and **recess** angles for the pinion. From points Q_1OQ_3 the angle Q_1BO, of approach, and OBQ_3, of recess, for the gear are found.

The **arcs of action** are the same for both gear and pinion and are equal to either O_1O_3 or Q_1Q_3.

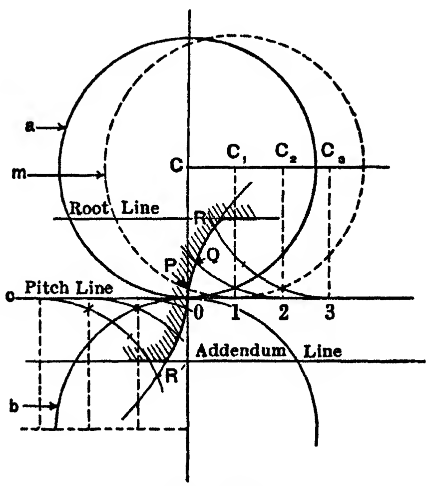


FIG. 8–22

Cycloidal Rack. — A rack may be regarded as a portion of a gear wheel with an infinitely large pitch diameter, the pitch circumference being a straight line. The generating circles for rack teeth, as a and b, Fig. 8–22, are rolled along a pitch line c in order to form the addendum and dedendum portions of the teeth.

To draw the cycloid, select any number of points 1, 2, 3, etc., along the pitch line. The rolling circle a, initially in contact at O, will reach the new position m when 1 is the contact point. Its center is then at C_1, where $1C_1$ is perpendicular to the pitch line. By laying off an arc $1P$ whose rectified length is equal to

$O1$, we find the point P which lies on the required curve. Points Q, R, etc., are found in a similar manner.

The curves are continued until the addendum and root lines are reached. The re-entrant angle where the tooth flank joins the root line is rounded off by a circular fillet to increase the strength of the tooth.

INVOLUTE TEETH

14. Involutes. — In general, if we take a curve of any form and roll on it a straight line, a point in this line will describe a path known as an involute of the curve. For nearly all gears with involute teeth, the involute is formed by rolling the straight line on a circle. The only exceptions to this are gears of non-circular section. When we speak of involutes in connection with gears, without further definition, the involute of a circle is meant. This circle is commonly called the **Base Circle** of the Involute.

15. Mechanical Development of Involute Curves. — A device shown in Fig. 8–23 illustrates a method of development of conjugate involute curves. A cord CD, $C'D'$ is wrapped around two circular discs a and b. Disc a has a transparent disc m attached to its face, and b has a similar disc n fastened to it. If a is rotated, the cord, acting as a belt, will drive b in the opposite sense. A point P on the cord will then trace out on the disc m an involute KL, and on the disc n an involute MN.

By keeping a and b in fixed position and then cutting the cord at P, the same curves may be traced out by the two ends when the loose portions are wound on and unwound from the discs to which they are attached.

Considering the above methods for obtaining the curves, two facts are evident: (a) that the point of contact always lies along the line of the cord, namely along CD; (b) that since the tangent portion of the cord is always swinging with reference to a disc about its point of tangency with the disc in question, the relative motion is always perpendicular to the tangent line. CD is therefore the common normal to the two curves at all times and it crosses the line of centers at a fixed point O. Moreover, the pressure angle is constant and equal to *DOQ* where *OQ* is a normal to the line of centers of the discs.

To summarize, it has been shown that involute profiles:

(*a*) Have the essential property for correct gear-tooth forms.
(*b*) Have a straight-line path of contact.
(*c*) Have a constant pressure angle.

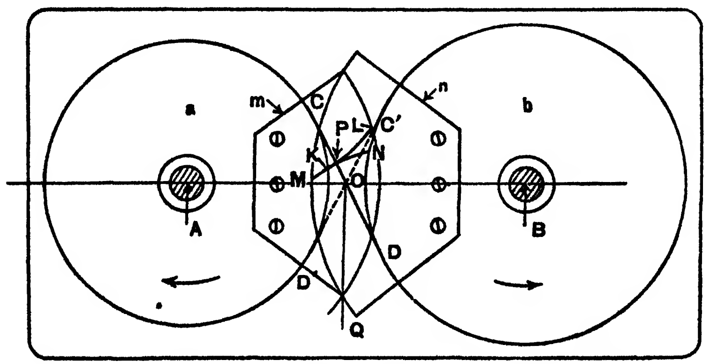


FIG. 8–23

Point O (Fig. 8–23) where the line CD intersects the line of centers is the instant center of wheels a and b. Hence the angular velocity ratio $\frac{\omega_a}{\omega_b}$ is equal to $\frac{BO}{AO}$, and O is therefore the pitch point, by Art. 6, Chapter VIII.

When the pitch point O and the pressure angle are known, the base circles may be found, since they are tangent to the line CD, whose position is fixed by these data.

16. Graphical Construction. — As already pointed out, the involute can be thought of as the path traced out by the end of a string which is unwound from a cylinder, the motion being, of course, in a plane perpendicular to the cylinder axis. The length of the unwound or tangent portion of the string is, in any position, equal to the length of the arc on the base circle from which it was unwrapped. Putting it another way, the tangent to the base circle from any point on an involute is equal in length to the arc from the point of tangency to the starting point of the curve. We make use of this property in constructing the curve. In Fig. 8–24 we shall suppose that a is the base circle and that we wish to draw an involute to pass through a given point A.

From A draw AB, tangent to circle a. Obtain an arc BC equal in length to AB. This can be done approximately in the drafting room by dividing AB into any number of equal parts, four being used in the diagram. Taking one of these equal lengths on our dividers, we step off four divisions, starting from B around the arc, thus locating C. The point C is the inner end of the involute curve. To obtain a point between A and C, take point D on the arc at the second division, draw DF tangent at D and step off on it two lengths with the same divider setting as before, obtaining E; a third point on the involute. Other points are found in a similar manner. A smooth curve is finally drawn through the points and this forms the required involute. The curve is continued outward until the addendum circle is reached.

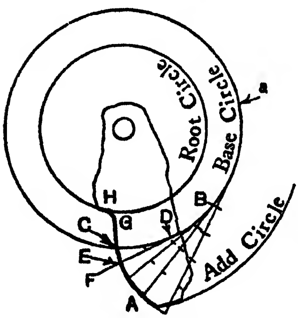


FIG. 8–24

In completing the tooth flank, the curve must be extended inside the base circle to allow space for the mating tooth, plus a clearance. As the involute cannot penetrate the base circle, a different form is necessary for this portion. Generally we use a radial straight line CG, terminating in a small circular arc GH at the junction with the root circle. Contact does not take place on CG: its outline is determined by considerations of strength and ease of production. The purpose of the arc GH is to strengthen the tooth at the root, and it should be of as large radius as possible without causing interference with the teeth on the mating gear.

17. Other Properties of the Involute. — Involute curves have an outstanding advantage over cycloidal or other curves which might be employed for working profiles in circular gears, namely, that the center-to-center distance can be changed without destroying conjugate tooth action or changing the angular velocity

ratio. That is, a pair of these curves will comply with the gear-tooth law no matter what the center-to-center distance may be. Proof of this statement can be seen by reference to Fig. 8–25. When the gear centers A and B are moved farther apart, the common tangent CD to the base circles will incline at a greater angle with the vertical, hence increasing the pressure angle. As before, the common tangent will cross the line of centers at the same point because we are still dealing with contact of the same involutes. The pitch radii AP and BP become larger, but the ratio of BP to AP remains unchanged and hence the speed ratio is unaltered. For by reference to the figure

$$AP = AC \div \cos\alpha \quad \text{and} \quad BP = BD \div \cos\alpha.$$

Now
$$\frac{\omega_a}{\omega_b} = \frac{BP}{AP} = \frac{BD}{\cos\alpha} \div \frac{AC}{\cos\alpha} = \frac{BD}{AC}.$$

But BD/AC is independent of the center-to-center distance. The speed ratio is in consequence dependent only on the relative diameters of the base circles and does not change when the center-to-center distance is varied.

A second deduction that can be made from Fig. 8–25 is that an involute profile has neither pitch circle nor pressure angle peculiar to itself, but obtains both of these by virtue of its location in regard to a second involute. Thus a gear which meshes with two others may have two pitch circles of different diameters, each corresponding to one contact. Improved tooth forms are sometimes made possible by taking advantage of this property.

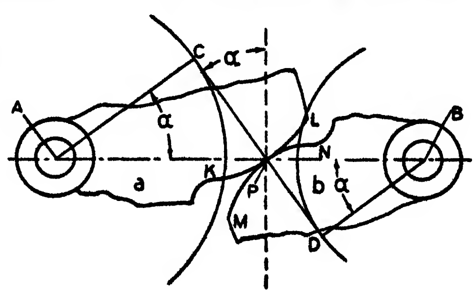


Fig. 8-25

A third deduction easily made from the geometry of Fig. 8–25 is that the diameter of the base circle is equal to the diameter of the pitch circle multiplied by the cosine of the pressure angle.

A pressure angle of $14\frac{1}{2}°$ was early adopted for involute gear teeth. The sine of this angle is approximately .25, which simplifies the work of laying out the teeth. The same pressure angle is still extensively used since it usually results in satisfactory tooth forms. Larger angles, up to 23°, are not uncommon, particularly where small tooth numbers are required.

Practical advantages resulting from the use of involute profiles may be summarized as follows:

(*a*) Involute gears may be mounted with small initial inaccuracies in the center-to-center distance, or this distance may change as the result of bearing wear, tooth contact still complying with the fundamental gear-tooth law.

(*b*) Involute gears may be used for applications such as driving rolls in steel mills, where the center-to-center distance constantly varies.

(*c*) The working surface of the involute rack is of the simplest possible form, a plane. This reduces to a minimum the difficulty of producing accurate conjugate teeth — a manufacturing advantage.

(*d*) Where the teeth are cut by formed milling cutters, the number of cutters needed in covering the range from the smallest pinion to rack is less than would be necessary for cycloidal profiles. This is due to the slow change in tooth curvature as the tooth numbers are increased.

The main disadvantage of involute teeth lies in the fact that interference is obtained with pinions having small tooth numbers. No interference difficulty is encountered in cycloidal teeth.

Involute teeth never have concave faces or flanks except in internal gears. On the other hand, cycloidal gears with more than fifteen teeth have concave flanks. Hence in the cycloidal form a concave flank normally mates with a convex face and surface contact is more nearly approximated than when using involute profiles. From this standpoint the cycloidal system would appear to be somewhat better adapted to carrying heavy loads.

In present engineering practice, the involute form of tooth has so completely displaced its rival, much used in earlier years, as to render the cycloidal form practically obsolete. Later, reference will be made to a **composite** form of tooth which uses a profile composed of both involute and cycloidal curves.

18. Tooth Action. — In Fig. 8–26 is shown a pair of involute gear profiles in three positions. We shall assume that the gears rotate as indicated by the arrows on the diagram. At a_1, b_1 the teeth are just beginning to make contact; at a_2, b_2 the teeth are in contact at the pitch point; and at a_3, b_3 contact is just about to cease.

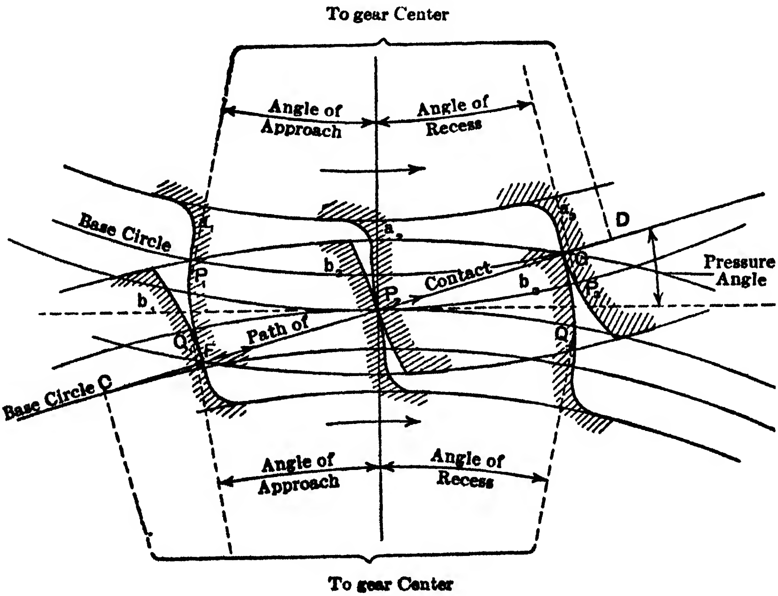


Fig. 8–26

The path of contact must lie at all times on the straight line CD, this line being the common tangent to the base circles. Contact begins at F and ends at G. P_1, P_2, P_3 on the pitch circle of the upper gear are corresponding positions of one point on this gear, while Q_1, P_2, Q_3 are similarly the three positions occupied by one point on the lower gear. Since the pitch circles have pure rolling contact, arcs $P_1P_2P_3$ and $Q_1P_2Q_3$ are of equal length. Also, by definition, P_1P_2 is the arc of approach and P_2P_3 the arc of recess for the upper gear, while arcs Q_1P_2 and P_2Q_3 have the same values for the lower gear. The angles of approach and recess for the former are found by joining P_1 and P_3 to the center of the upper gear as indicated on the figure. A similar construction locates the angles of approach and recess for the lower gear. The pressure angle is noted in the figure.

It will be observed that points F and G, where contact begins and ends, are found at the intersections of the addendum circles with the pressure line CD. F and G are located between points C and D in this figure, but with other tooth proportions and numbers of teeth on the gears it may happen that either F or G or both F and G fall on line CD produced. This leads to **Interference.**

19. Interference. — Gear teeth are said to interfere when they tend to overlap or cut into the mating teeth. Under certain conditions interference will take place when the teeth have true involute profiles. Such a situation is illustrated in Fig. 8–27. Here point G, the intersection of the addendum circle of the lower gear with the contact line CD, falls on CD produced. This condition is always accompanied by interference, as may be seen from the following discussion.

Considering the two gears in Fig. 8–27 to turn in the sense indicated by the arrows, and observing a pair of teeth, a, b initially in contact at F, we note that the contact point traces out the line FD as the gears revolve. Contact does not cease at D, but thereafter it must take place between the involute portion LM of the tooth on the lower gear and the flank QR, inside the base circle of the upper gear. This flank is not of involute form, since the involute cannot be extended inside the base circle. If QR is a radial flat surface, overlapping or " interference " will take place. Modification of the tooth forms is then necessary. The following methods are possible:

(*a*) The height of the teeth may be reduced.

(*b*) The pressure angle may be increased.

(*c*) The radial flank of the pinion may be cut back. This is technically known as " undercutting " when any material is removed inside a radial line from the junction point of the involute and its base circle. (See Fig. 8–30.)

(*d*) The face of the gear tooth may be relieved.

Methods (*a*) and (*b*) are employed in conjunction with each other in what are known as stub-tooth gears, to which reference will be made later.

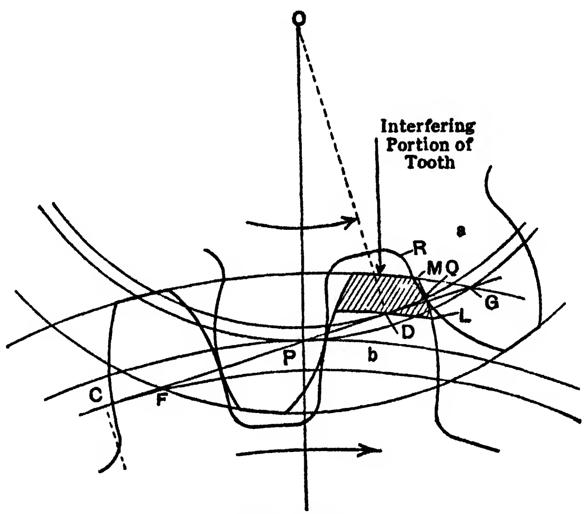


Fig. 8–27

Method (*c*) is often undesirable for two reasons: first that it weakens the pinion tooth, and second that it may result in a short arc of action, since excessive undercutting may remove the inner portion of the involute as well as the straight part of the tooth below the base circle.

Method (*d*) is satisfactory where manufacturing methods permit of its use.

In the "composite" type of tooth, interference is avoided by use of cycloidal curves for those portions of the profile which would interfere if made of involute form.

20. Maximum Radius of Addendum Circle without Interference. — It is evident that the interference encountered in the gears of Fig. 8–27 could be eliminated if the teeth on the lower gear were shortened by cutting off the portion which is shown cross-hatched. This tooth would then have its **maximum addendum without interference.** In Fig. 8–28, gears with centers at A and B have pressure angle α and base circle radii AC and BD. The maximum addenda radii that can be used for the gears without introducing interference are respectively AD and CB, the circles then passing through the interference points C and D.

Since AC and BD are perpendicular to CD, we may complete a rectangle $ACDF$. From the geometry of the figure

$$CD = AF = AB \sin \alpha.$$
$$AD = \sqrt{(AC)^2 + (CD)^2} = \sqrt{(AC)^2 + (AB)^2 \times \sin^2 \alpha}.$$
$$BC = \sqrt{(BD)^2 + (CD)^2} = \sqrt{(BD)^2 + (AB)^2 \times \sin^2 \alpha}.$$

Hence the maximum addendum radius without interference is equal to $\sqrt{(\text{Base circle radius})^2 + (C\text{.-to-}C\text{. distance})^2 \times \sin^2 \alpha}$.

Example. — Find the maximum addendum radius of two equal gears having 23 teeth of 1 diametral pitch, the pressure angle being 14½°.

Solution. — Pitch radius of gear = 11.5 in.
Base circle radius = 11.5 × cos 14½°.
= 11.5 × .968 = 11.13.
Center-to-center distance = 23 in.

Hence

Maximum addendum radius =
$$\sqrt{(11.13)^2 + (23)^2 \times (.2504)^2} = 12.5 \text{ in.}$$

Therefore the maximum addendum = 12.5 − 11.5 = 1.0 in. If the gears are made with full-depth proportions, the addendum is $1/P$ = 1 in. This indicates that two equal gears having 23 teeth of standard 14½° involute form will not interfere, but are just at the limit of addendum length. Gears with fewer teeth will show interference.

21. Correct Modification for Interference. — Where the modification for interference consists in cutting back the proper part of the tooth face on the rack or larger gear, the resulting surface should be of a form conjugate to the flat radial surface inside the base circle of the pinion. A straight line is a cycloid generated by a rolling circle half the diameter of the directing circle. Hence the conjugate curve on the larger gear should be a cycloid generated by a rolling circle half the diameter of the pinion pitch circle when rolled externally on the pitch circle of the former Thus in Fig. 8–27 the profile *LM* should be a cycloid formed by rolling a circle of diameter *OP* externally on the pitch circle of gear *b*. This curve is conjugate to the straight line *QR* on pinion *a*.

22. Involute Rack Teeth. — Since a rack can be regarded as a portion of a gear wheel with infinite pitch radius, the base circle

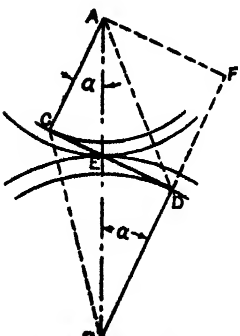


Fig. 8–28

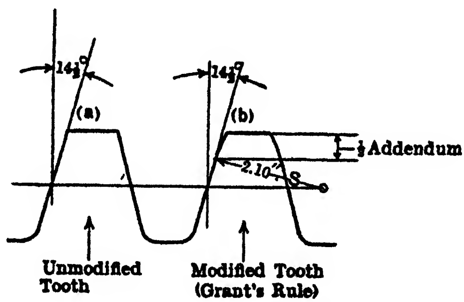


Fig. 8–29
Involute Rack Teeth.

for the rack involute is also of infinite radius, and the involute itself is a straight line. Rack teeth on the involute system therefore have straight working surfaces, except where modification becomes necessary to avoid interference. Figure 8–29 (*a*) illustrates a rack tooth of this kind. The angle between the side of a tooth and the perpendicular to the pitch line is equal to the pressure angle or angle of obliquity. The amount of modification on the face of the tooth which is really required to eliminate interference depends on the size of pinion with which the rack is to mesh. For the 14½° system George B. Grant recommends rounding off the upper half of the face by an arc of radius equal to 2.10 in. ÷ diametral pitch, or 0.67 in. × circular pitch. See

Fig. 8–29(*b*). This gives a satisfactory approximate form for cast teeth.

23. A simple device which will draw a gear tooth of proper form to work with a rack tooth of any selected shape is shown in Fig. 8–30. This also serves to illustrate the principle of operation of spur-gear-tooth generators which use a rack-tooth cutter. A frame *A* has a slot on its upper side in which the rack *B* slides.

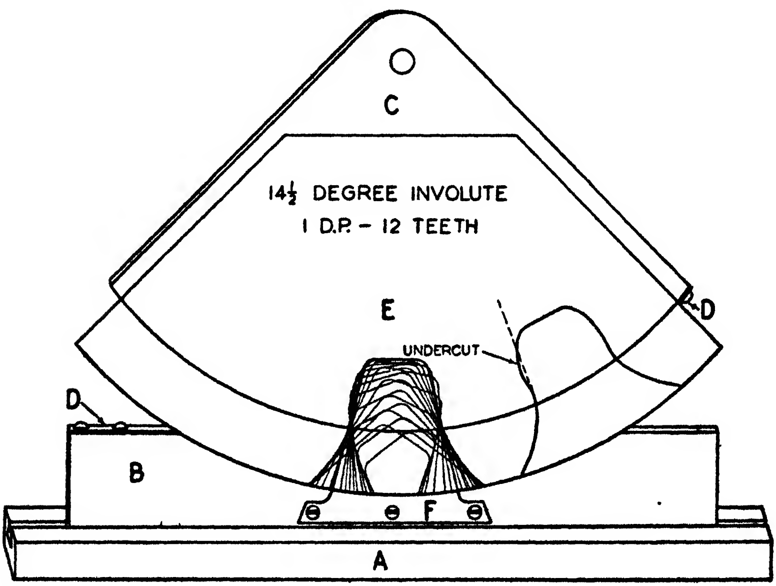


Fig. 8–30

The frame also has a vertical bar in the rear, not visible, which carries a pivot about which the pitch-surface sector *C* swings. Parts *B* and *C* are connected by means of a flexible steel strip *D* so as to cause *B* to have rolling contact with *C*. The tooth is generated on a paper sector *E* which is pinned to the face of *C*. The rack tooth *F* is preferably made of celluloid or other transparent material. The tooth form is determined by moving the rack along by small displacements and in each position drawing the outline of the rack tooth on the paper sector. The tooth profile is the tangent curve or envelope of the rack-tooth outlines.

The tooth form shown in the figure is that for a twelve-tooth pinion of $14\frac{1}{2}°$ full-depth involute type. The standard rack tooth has been lengthened by an amount $0.157 \div P$, in order to provide clearance at the root of the pinion tooth. Considerable undercutting is noticeable, part of the involute between base and pitch circle being removed as well as the material lower on the flank. This gives a short arc of action which is characteristic of the $14\frac{1}{2}°$ involute full-depth tooth when the tooth numbers are small.

24. Tooth-form Requirements. — Aside from the fact that the working faces of gear teeth must comply with fundamental law of Art. 6, there are several other requirements which have influenced the choice of standard gear-tooth forms and proportions.

(*a*) The teeth must be capable of accurate production at low cost.

(*b*) The tooth form should have good wearing qualities. Low rubbing speeds and close approach to surface contact are both favorable. The latter condition is secured when mating teeth both have large radii of curvature. Tooth pressures are distributed over a wider strip of surface when large radii are used. The result is a lower intensity of pressure and less wear.

(*c*) The tooth form must result in good "beam strength." In service, forces act on the tooth side tending to bend it like a beam. Beam strength is greatest in a short tooth with a wide section across the root.

(*d*) The arc of contact must be at least equal to the circular pitch; otherwise there would not be continuous contact between gears. An arc of action greater than 1.4 times the circular pitch is generally held to be good design. Below this limit, noisy action of the gears is likely unless they are very accurately cut.

(*e*) Interchangeability of a series of gears of the same pitch is generally desirable, although it is unnecessary with many gears which are of the "special purpose" type.

25. Standard Tooth Forms. — In view of the requirements for gear-tooth forms, four types of teeth have been standardized by the American Gear Manufacturers Association. These are known as:

(*a*) The 14½° Composite System.
(*b*) The 14½° Full-depth Involute System.
(*c*) The 20° Full-depth Involute System.
(*d*) The 20° Stub Involute System.

Tooth proportions are given in Table 1.

TABLE 1

PROPORTIONS OF STANDARD TEETH

	14½° Composite	14½° Full-depth Involute	20° Full-depth Involute	20° Stub Involute
Addendum	$\frac{1}{P}$	$\frac{1}{P}$	$\frac{1}{P}$	$\frac{0.8}{P}$
Minimum dedendum	$\frac{1.157}{P}$	$\frac{1.157}{P}$	$\frac{1.157}{P}$	$\frac{1}{P}$
Whole depth = addendum and dedendum	$\frac{2.157}{P}$	$\frac{2.157}{P}$	$\frac{2.157}{P}$	$\frac{1.8}{P}$
Clearance	$\frac{0.157}{P}$	$\frac{0.157}{P}$	$\frac{0.157}{P}$	$\frac{0.2}{P}$

P = diametral pitch.

It will be noted that the composite, the 14½° full-depth, and the 20° full-depth systems all have teeth of the same proportions. These proportions are exactly the same as those used in the earlier "Brown and Sharpe Standard" system.

Each of the four standard systems has gear teeth which are conjugate to a "basic rack." The basic rack is therefore the standard or reference form in cutting teeth on gears of any size.

The 14½° Composite System. — The basic rack (approximate form) is shown in Fig. 8–31. It will be observed that the rack-tooth sides are composed of circular arcs at the top and bottom of the tooth connected by a straight portion. There is also a fillet arc where the tooth side joins the root circle. The straight portion, comprising about the middle third, is the involute section. The circular arcs forming the outer portion of the face and inner portion of the flank are selected to approximate cycloidal curves.

This system was early developed from the cycloidal tooth forms which were at one time in almost universal use. While possessing some of the advantages of the straight involute system, it avoids the interference and undercutting of small pinions, which are characteristic of that system. It is the common form of tooth cut

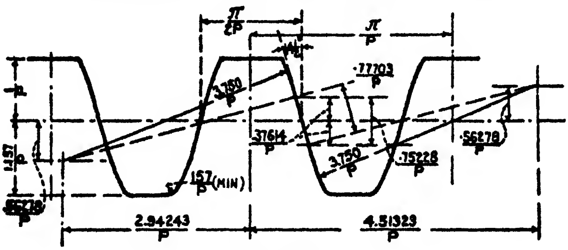


Fig. 8-31

in the small shop where the formed milling-cutter method is employed. It is suitable for gears with twelve or more teeth, and this range of gear sizes can be produced by means of a set of fifteen standard cutters. (See Fig. 8-32.) Each cutter can be used over a range of tooth numbers as shown in Table 2. Each is correct for the smallest number in the range and somewhat in error for the higher numbers. However, errors introduced in this way can at least be partially corrected by slightly varying the depth of cut in the blank. Standard tables, approved by the Gear Manufacturers Association, are available[3] which show the depth of cut required for best results with various sizes of gears.

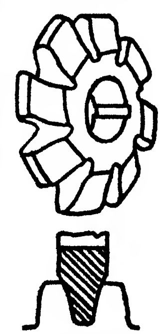


Fig. 8-32

While the basic rack is not so simple in form as that for pure involute teeth, the composite system gives a sufficiently large arc of contact and a tooth form otherwise quite satisfactory for the range of tooth numbers mentioned above. As it is difficult to attain high standards of accuracy in teeth cut by formed cutters, this type is most satisfactory when loads and speeds are moderate.

14½° Full-depth Involute. — This system has a basic rack with the same tooth addendum and dedendum as the composite system. The rack tooth is straight-sided as shown in Fig. 8-33

[3] See "Manual of Gear Design," Vol. 2, by Buckingham.

TABLE 2

COMPOSITE TOOTH MILLING CUTTERS

STANDARD 14½° SYSTEM

Cutter Number	Tooth Numbers	Cutter Number	Tooth Numbers
1	135 to rack	4½	23 to 25
1½	80 to 134	5	21 to 22
2	55 to 79	5½	19 to 20
2½	42 to 54	6	17 to 18
3	35 to 41	6½	15 to 16
3½	30 to 34	7	14
4	26 to 29	7½	13
		8	12

except for a fillet arc at the root to strengthen the tooth. This system is used for generated teeth.

Interference occurs when the tooth number of equal pinions is less than twenty-three or when a rack engages a gear with less than

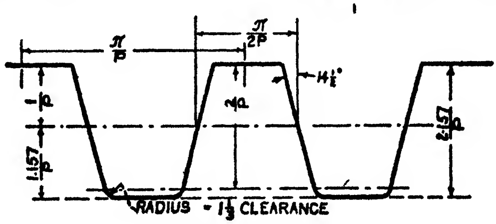


FIG. 8-33

thirty-two teeth. Undercutting is therefore necessary with small tooth numbers, and as a result the arc of action then becomes unsatisfactory. For example, with two twelve-toothed pinions the arc of contact is 0.034 times the circular pitch, an unusable value. The desired arc of action, namely, 1.4 times the circular pitch, is obtainable with gears having twenty, twenty-one, or twenty-two teeth, the exact value depending on the tooth number of the mating gear. This type of tooth is very satisfactory, however, when tooth numbers are large.

20° Full-depth Involute System. — The basic rack (Fig. 8-34) is the same as that for the 14½° system except for the pressure angle. The use of a larger pressure angle leads to better tooth action when the tooth numbers are small. For example, an arc of action equal to 1.4 times the circular pitch is obtainable with equal gears of fourteen teeth. In this respect, this type of tooth gives

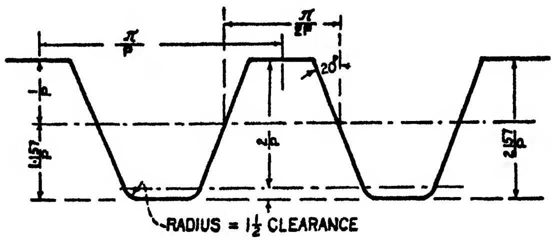


Fig. 8-34

the best results of any of the four standard types with low tooth numbers. It is used for generated teeth.

20° Stub Involute System. — The basic rack (Fig. 8-35) has a tooth about 18 per cent shorter than that of the full-depth systems. Interference difficulties are much reduced as compared with the other standard involute teeth. Thus a stub tooth rack will mesh

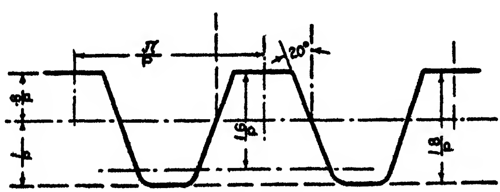


Fig. 8-35

with a seventeen-tooth pinion without interference. A pair of twelve-tooth pinions gives an arc of action equal to 1.19 times the circular pitch. This is a usable value. However, the arc of action does not increase rapidly with the increase in tooth numbers, a 27-30 combination having a value only 1.35 times the circular pitch. On this account accuracy of cutting is especially important if noisy action is to be avoided.

Figure 8-36 shows a graphical comparison of 20° stub and 14½°

full-depth teeth of the same pitch for gears of equal size. The short tooth with wide root, characteristic of the former, gives high beam strength and accounts for its suitability for use where subject to heavy shocks.

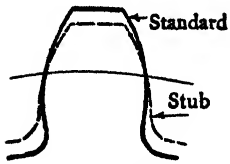


FIG. 8-36
Comparison of Stub Tooth and Standard 14½° Tooth.

The 20° stub tooth was designed primarily for use in automobile transmissions, and it is also employed for heavy mill gearing. The teeth are cut by generating processes.

26. Approximate Gear Tooth Profiles. — When gear teeth are cast instead of being cut on the gear blank, circular-arc profiles are often substituted for the theoretically correct involute or cycloidal curves. This lowers the cost of both drawings and patterns, because the tooth forms can be specified more easily by the drafting room and laid out more rapidly by the pattern maker. The errors introduced by this approximation are generally small if the centers and radii are properly chosen. In any event, cast teeth are likely to be distorted on cooling, the surfaces are somewhat rough, and extreme accuracy in making the patterns is not justified.

To save time, gear teeth are often drawn by the circular-arc method even though the teeth are to be cut to true involute form.

Many methods for the laying out of circular-arc teeth have been used. Among the better known are the **Grant Odontograph** and the **Brown and Sharpe** methods.

27. Approximate Involute Teeth: Grant Odontograph. — The **Grant Method** of constructing approximate tooth outlines is one of the best known and most useful. Table 3 contains the information for teeth of 15° pressure angle and standard proportions, the true involute being approximately by two circular arcs. The radius of the arc for the face of a required tooth is found by using the figures in the "Face Radius" column corresponding to the number of teeth on the gear. These figures are either divided by the diametral pitch or multiplied by the circular pitch, and the result is the required radius. The radius of the flank arc is found from the data in the "Flank Radius" column by the same method. The centers from which the arcs are drawn are located on the base circle of the true involute. Inside the base circle the tooth profile

TABLE 3

GRANT INVOLUTE ODONTOGRAPH

No. of Teeth	Divide by Diam. Pitch		Multiply by Circular Pitch	
	Face Radius, in.	Flank Radius, in.	Face Radius, in.	Flank Radius, in.
10	2.28	0.69	0.73	0.22
11	2.40	0.83	0.76	0.27
12	2.51	0.96	0.80	0.31
13	2.62	1.09	0.83	0.34
14	2.72	1.22	0.87	0.39
15	2.82	1.34	0.90	0.43
16	2.92	1.46	0.93	0.47
17	3.02	1.58	0.96	0.50
18	3.12	1.69	0.99	0.54
19	3.22	1.79	1.03	0.57
20	3.32	1.89	1.06	0.60
21	3.41	1.98	1.09	0.63
22	3.49	2.06	1.11	0.66
23	3.57	2.15	1.13	0.69
24	3.64	2.24	1.16	0.71
25	3.71	2.33	1.18	0.74
26	3.78	2.42	1.20	0.77
27	3.85	2.50	1.23	0.80
28	3.92	2.59	1.25	0.82
29	3.99	2.67	1.27	0.85
30	4.06	2.76	1.29	0.88
31	4.13	2.85	1.31	0.91
32	4.20	2.93	1.34	0.93
33	4.27	3.01	1.36	0.96
34	4.33	3.09	1.38	0.99
35	4.39	3.16	1.39	1.01
36	4.45	3.23	1.41	1.03
37–40	4.20		1.34	
41–45	4.63		1.48	
46–51	5.06		1.61	
52–60	5.74		1.83	
61–70	6.52		2.07	
71–90	7.72		2.46	
91–120	9.78		3.11	
121–180	13.38		4.26	
181–360	21.62		6.88	

is extended along a radial line and rounded into the root circle by a fillet arc. Face and flank arcs coincide with the true involute at three points; hence this method is sometimes referred to as a **Three-point Odontograph.**

Figure 8–37 illustrates the application to a gear having fourteen teeth of 2 diametral pitch. In the table, opposite " 14 teeth," the face-radius column shows the value 2.72, and the flank-radius column 1.22. Dividing these numbers by the diametral pitch, we obtain 1.36 in. and 0.61 in. The former radius is used to draw an arc *OB* extending from the pitch to the addendum circle. The latter is the radius of the arc *OC* extending from the pitch circle inward to the base circle. A radial straight line *CD* and a fillet arc *DE* complete the tooth profile.

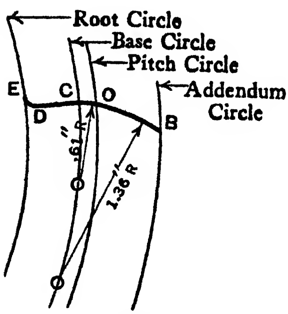


FIG. 8–37

28. Other Tooth Forms. — A system of involute teeth known as the **Long-and-Short-Addendum** system has of late found many industrial applications. In this system the pinion tooth, generally of standard length, is constructed so that it projects further outside the pitch circle than the standard tooth, resulting in a long addendum and a short dedendum. The conjugate tooth on the gear must of necessity have a short addendum and a long dedendum. Reference to Fig. 8–38 indicates the resulting form of the teeth in comparison with the standard involute teeth. The pinion tooth becomes wider and stronger at the root section where the greatest bending moment will act. The gear tooth is but slightly stronger than the standard tooth, but this is of no importance since both forms are stronger than the pinion tooth. Furthermore, interference is eliminated over the ordinary range of tooth numbers. The usual pressure angle is about 20°. The gears made by the General Electric Co. for railway and industrial haulage, and the Westinghouse-Nuttall gears, are of this type.

In the Maag system of gear cutting, a straight-sided rack is used in a special machine to produce a true involute tooth by a generation method, a process which lends itself to accurate production. The depth of the teeth and the pressure angle are not the same for all tooth numbers. Teeth of standard depth and low pressure angle are used for gears with large tooth numbers, while short

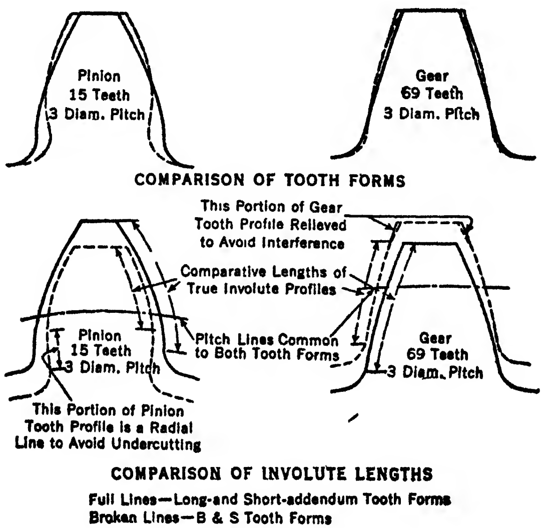


FIG. 8–38
(General Electric Co.)

teeth and large pressure angle (up to 23°) are used for small tooth numbers. Between the two extremes the tooth heights and pressure angles are graded, the object being to obtain the most suitable combination for each particular case. By this method it is possible to avoid interference and have strong teeth even when the tooth number is very small. This is accomplished, however, by the sacrifice of interchangeability. Figure 8–39 shows standard and Maag gears with eight and sixteen teeth. The standard $14\frac{1}{2}°$ involute teeth at the left of the figure are evidently of much weaker form than the shorter Maag teeth which have a larger pressure angle. The Maag system also incorporates the long-and-short-addendum principle as indicated by examination of the figure.

29. The Variable Center Distance System takes advantage of the fact that the involute curve has no inherent pitch circle. The objective is to obtain better tooth action where tooth numbers are small. It incorporates the following features:

(1) The same standard rack cutters are employed as for the standard $14\frac{1}{2}°$ involute system.

(2) Pinions with small tooth numbers have tooth heights less than standard, but center-to-center distances are larger.

(3) Each size of gear has a fixed root-circle diameter: it is independent of mating-gear diameter.

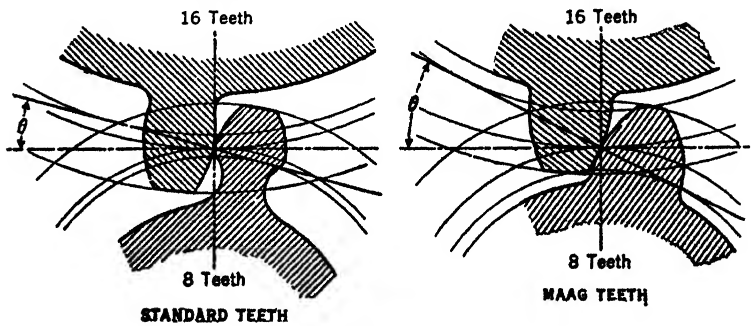


Fig. 8-39
(Niles-Bement-Pond Co.)

(4) Tooth heights depend on the combination, becoming smaller as tooth numbers are decreased.

The net result is to increase beam strength and the arc of action in small pinions. Tooth heights and center-to-center distances become standard for the larger gears.

Complete details will be found in the "Manual of Gear Design" by Earl Buckingham. Comparative data for a 12–14 tooth combination of 1 D.P. is as follows:

	Tooth Height, in.	Center-to-center Distance, in.	$\frac{\text{Arc of Action}}{\text{Circular Pitch}}$
Standard $14\frac{1}{2}°$	2.157	13	0.166
Variable center distance ...	1.8351	13.8905	1.090

30. Problem. — We shall take the following specifications for a gear and pinion, and proceed to draw the teeth.

	Gear	*Pinion*
Number of teeth	24	12
Diametral pitch	2	2
Bore	2 in.	1¼ in.
Diameter of hub	3¾ in.	2¾ in.
Full depth, 14½° involute teeth.		

From the above data we make the following calculations:

	Inches
Pitch diameter of gear	12
Pitch diameter of pinion	6
Addendum	$= 1/P = 0.5$
Diameter of addendum circle of gear	$= 12 + 2 \times 0.5 = 13$
Clearance	$= 0.1571 \div 2 = 0.0785$
Diameter of root circle of gear	$= 12 - 2\ (0.5 + 0.0785) = 10.843$
Distance between centers	$(12 + 6) \div 2 = 9$
Diameter of addendum circle of pinion	$= 6 + 2 \times 0.5 = 7$
Diameter of root circle of pinion	$= 6 - 2\ (0.5 + 0.0785) = 4.843$

Construction. — Locate two centers A and B, 9 in. apart. (See Fig. 8–40.)

Draw pitch circles touching at pitch point O.

Draw addendum and dedendum circles.

Draw OP perpendicular to AB.

Lay off 14½° angles from OP, thus obtaining QX and YZ.

With centers A and B, draw circles tangent to QX and YZ. These are the base circles for the involutes.

Starting from the pitch point O, divide the pitch circumference of the gear into 24 equal parts. This can be done by calculating the angles subtended at the center by the pitch arcs and then laying off the arcs from the angles. In the case of the gear, the pitch arc subtends an angle of $\frac{360°}{24} = 15°$. Points 1, 2, 3, etc., are thus found. The pitch circumference of the pinion is similarly divided into 12 equal parts, and we thus obtain points 1′, 2′, 3′, etc.

We now have the base circles for the involutes and one point on each involute where it crosses the pitch circle. Selecting any one of the points on the gear pitch circle, we next draw an involute or approximate involute curve by one of the methods already outlined. We duplicate this curve through each of the other points 1, 2, 3, etc., taking care that each of these curves has the same angular relationship to a radius through its starting point.

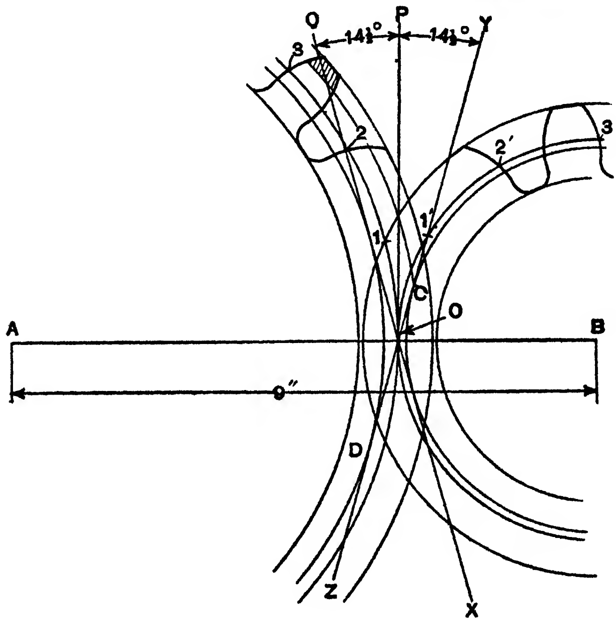


FIG. 8-40

If the gear is to have cut teeth, no backlash need be provided for, and we bisect the distances 0–1, 1–2, 2–3, etc. Through the points of bisection we draw the involute curve already obtained but in the reversed position, so as to form the other sides of the teeth. The same construction is employed for the pinion.

The tooth flanks may be completed by a radial line and fillet arc.

Modification of the tooth points is necessary to avoid interference. To determine the interfering portion of the teeth, we first locate point C where line YZ is tangent to the base circle.

With A as center and radius AC, describe an arc. The cross-hatched portion of the gear tooth (Fig. 8–40) lying outside this arc must be modified. The teeth on the pinion do not interfere, since the interference point D falls outside the addendum circle of the pinion.

31. Internal or Annular Gears. — One of these gears is illustrated in Fig. 8–41, the teeth being cut internally on a hollow cylinder or ring. Except for the clearances, an internal gear and an external gear of the same pitch, pitch diameter, and tooth proportions have the same profile, the tooth spaces on one corresponding exactly to the teeth on the other

Internal gears are required when driving and driven members must rotate in the same sense. The drive is more compact than when external gears with the same velocity ratio are used

(*Stephens-Adamson Mfg Co*)

FIG 8–41
Internal Gear

32. Twisted-tooth Spur Gears. — (Fig. 8–3.) A plain spur gear may in theory be converted into a twisted-tooth spur gear by first cutting it into an infinite number of thin sections, by passing through it planes perpendicular to the axis of revolution, and then rotating these sections so that each is somewhat in advance of the adjacent one. By making the increments of angular advance of equal value, elements of the combined teeth become true helices. This is usually done for ease in manufacture, though a helical form is not essential; since no sliding takes place along the tooth elements these may have any desired shape, provided that the mating gear teeth are formed with corresponding curves of the elements.

When made with true helical tooth elements, the twisted-tooth spur gear and the helical gear of the same diameter, pitch, and helix angle used for connecting non-parallel shafts do not differ in form. Both are commonly called helical gears The same method of designing the teeth applies to both. This method is discussed later.

The superiority of the twisted-tooth spur gear as regards quiet action has already been pointed out. Noise in gear operation is due to the impact of the teeth as they come into contact. In the plain spur gear, contact takes place along the whole length of a tooth at the same instant, whereas in the twisted-tooth gear contact begins at one end of the tooth and progresses to the other during an appreciable time interval. The latter action produces little noise.

The **Angle of Cut** is the angle between the helical pitch element and the parallel to the axis of revolution. The angle of cut is usually made so large that there is always one pair of teeth in contact at the pitch point. This means that the helix must advance at least one pitch in the width of the gear. Too large an angle causes excessive end thrust. In practice the angle ranges from 15° to 30°.

HELICAL GEARS

33. Tooth Action. Sliding Velocity. — Gears of this kind are used to connect non-intersecting shafts at any angle (see Fig. 8–9). The cylindrical pitch surfaces of the helical gears touch along a line only when the axes of the cylinders are parallel, in which case they are often called "twisted-tooth spur gears." Otherwise the pitch surfaces touch at a point, and consequently the teeth also have point contact. Two cylinders cannot have pure rolling contact unless their axes are parallel. Thus, in helical gears connecting non-parallel shafts, the teeth slide on one another along the tooth elements. The sliding action is provided for by making the teeth of uniform cross-section and the tooth elements true helices.

In Fig. 8–42, a and b represent the pitch cylinders of a pair of helical gears connecting two shafts whose axes are AB and CD, at an angle θ. The pitch cylinders make contact at a point P, and the line LM is the common tangent to the helical tooth elements through P.

Point P, regarded as a point on a, has a velocity represented by a vector PQ ($= V_a$) perpendicular to AB. Point P considered as a point on b has a velocity represented by a vector PR ($= V_b$) at 90° to CD. Vectors PQ and PR must have a common component PS ($= V_n$) in a direction normal to the common tangent ML. If this were not true the teeth would either come out of contact or interfere. The algebraic difference of the components of PQ and PR in a direction parallel to ML, namely, QS plus SR, represents the sliding velocity at the pitch surface along the tooth elements.

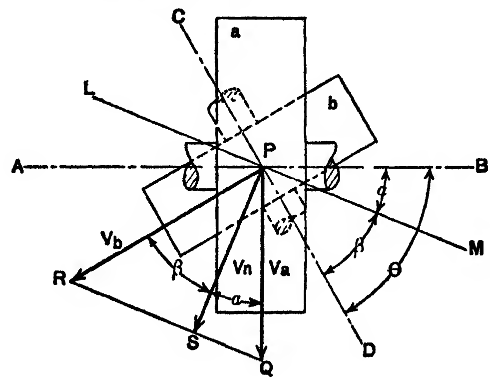


FIG. 8–42

34. Angular Velocity Ratio. — The **Angle of Cut** or **Helix Angle** of the teeth is the angle between a tangent to the helix at a point on the pitch surface and a line parallel to the gear axis. In Fig. 8–42, α is the angle of cut for the gear a, and β for the gear b. Evidently, from the figure,

$$\alpha + \beta = \theta.$$

Also, by construction,

$$\text{the angle } SPQ = \alpha, \qquad \text{the angle } SPR = \beta,$$

and therefore

$$V_a \cdot \cos\alpha = SP = V_b \cdot \cos\beta \qquad \text{or} \qquad \frac{V_a}{V_b} = \frac{\cos\beta}{\cos\alpha}.$$

Now,

$$\omega_a = V_a \div r_a \qquad \text{and} \qquad \omega_b = V_b \div r_b.$$

Therefore, the angular velocity ratio is

$$\frac{\omega_a}{\omega_b} = \frac{V_a \cdot r_b}{V_b \cdot r_a} = \frac{r_b \cdot \cos\beta}{r_a \cdot \cos\alpha}. \tag{8-6}$$

It will be noted that the velocity ratio depends not only on the pitch diameters but also on the angles of cut of the teeth. With unchanged diameters a different speed ratio can be obtained from a pair of these gears by altering the angles of cut.

35. Graphical Construction for Helical Gears. — Figure 8–43 illustrates a graphical method of finding the angles of cut α and β for a pair of helical gears when the angle between shafts θ, the diameters of the gears d_a and d_b, and the velocities ω_a and ω_b are known.

Construction:

Draw two lines OX and OY at an angle θ.

Set off distances from O representing d_a, d_b, ω_a, ω_b, to any convenient scales, as in the figure.

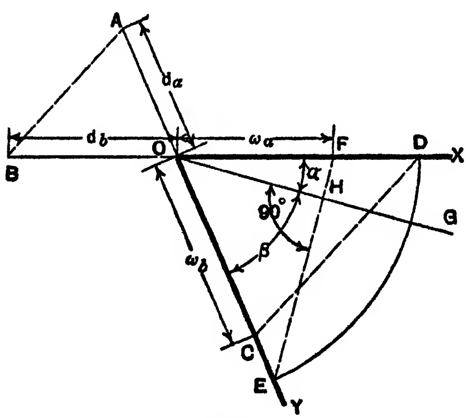


FIG. 8–43

Join AB and draw CD parallel to it.

On OY find a length OE equal to OD. Join EF.

Draw OG perpendicular to EF.

The required angles of cut are the angles α and β as indicated in the figure.

Proof:

From the geometry of the figure,

$$\frac{d_a}{d_b} = \frac{OC}{OD} = \frac{\omega_b}{OE} = \frac{\omega_b}{OH \div \cos\beta} = \frac{\omega_b \cos\beta}{OF \cos\alpha} = \frac{\omega_b \cos\beta}{\omega_a \cos\alpha}.$$

Therefore,

$$\frac{\omega_a}{\omega_b} = \frac{d_b \cos\beta}{d_a \cos\alpha}.$$

Hence α and β are the required angles of cut, from equation (8–6).

When the numbers of teeth (N_a and N_b) are known, instead of the velocity ratio, since $N_a/N_b = \omega_b/\omega_a$, we may alter the diagram by taking OF to represent N_b and OC to represent N_a.

36. Normal Pitch. — A helix, such as AB, Fig. 8-44, drawn on the pitch surface, normal to the tooth elements, is known as the normal helix. The circular pitches of a mating pair of helical gears must be equal when these pitches are measured along the normal helix. This pitch we call the **Normal Circular Pitch.** The **Normal Diametral Pitch** is found by dividing the normal circular pitch into π, diametral and circular pitch therefore bearing the same relationship to one another as in common spur gears.

The normal pitch is the pitch of the cutter that must be used to produce the teeth. Thus a helical gear of 10 normal diametral pitch and a plain spur gear of 10 diametral pitch are both produced by the use of 10 D.P. cutters. It will be shown later that the same shape of cutter cannot be used in both cases if the two gears referred to have equal numbers of teeth. This is due to the fact that the profiles on the normal plane are different.

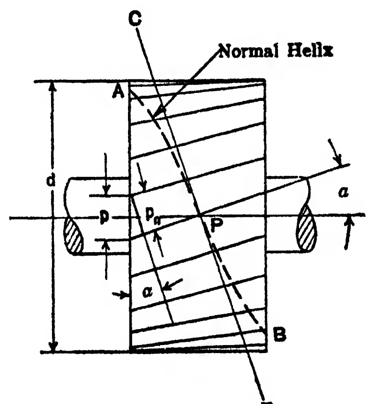


FIG. 8–44

We shall next proceed to express the relationship between normal pitch, pitch diameter, and numbers of teeth for the helical gear. In Fig. 8–44, let

p_n = normal circular pitch,
p = circular pitch measured in the plane of revolution (known as the circumferential pitch),
P_n = normal diametral pitch ($= \pi \div p_n$),
N = number of teeth,
d = pitch diameter,
α = angle of cut.

From the figure,

$$p = \frac{p_n}{\cos \alpha} \tag{8-7}$$

The pitch circumference $= \pi d = pN = \dfrac{p_n N}{\cos \alpha} = \dfrac{\pi N}{P_n \cos \alpha}$,

or

$$d = \frac{N}{P_n \cos \alpha}. \tag{8-8}$$

37. Velocity Ratio in Terms of the Number of Teeth. — Equation (8–6) expresses the angular velocity ratio of helical gears in terms of the pitch diameters and angles of cut. For two gears a and b,

$$\frac{\omega_a}{\omega_b} = \frac{d_b \cos \beta}{d_a \cos \alpha},$$

where α and β are the angles of cut.

By equation (8–8),

$$d_b = \frac{N_b}{P_n \cdot \cos \beta} \quad \text{and} \quad d_a = \frac{N_a}{P_n \cdot \cos \alpha}.$$

Substituting these values of d_b and d_a in the above,

$$\frac{\omega_a}{\omega_b} = \frac{N_b}{N_a}.$$

In helical as in spur gears, therefore, the angular velocities are inversely proportional to the numbers of teeth.

38. Tooth Forms. — In forming the teeth on a helical gear the cutter moves along the tooth elements, and sections at right angles to the path of movement will have the same profile as the cutter unless the teeth are generated. A study of such sections is therefore of importance.

In Fig. 8–44 is illustrated a pitch cylinder for a helical gear, the curve AB being the normal helix. A plane CE, tangent to AB at a point P, will intersect a tooth in a direction normal to its elements. The tooth action may be regarded as that due to rolling two pitch surfaces in a plane CE plus a sliding action parallel to the tooth elements. The latter motion has no influence on the normal tooth section. This section can therefore be made the same as required for a spur gear of pitch equal to the normal pitch of the helical gear and of pitch radius equal to the radius of curvature of the normal helix.

This radius is equal to the radius of curvature at P of the ellipse formed by the intersection of plane CE with the pitch cylinder. This ellipse has a minor axis of length d and major axis $d \div \cos\alpha$, where α is the angle of cut. The radius of curvature of any ellipse at the end of its minor axis is equal to

$$\frac{(\frac{1}{2}\text{ major axis})^2}{\frac{1}{2}\text{ minor axis}},$$

or, in the case considered, it is

$$\frac{(\frac{1}{2}d \div \cos\alpha)^2}{\frac{1}{2}d} = \frac{d}{2\cos^2\alpha}. \qquad (8\text{–}9)$$

This is the pitch radius of a spur gear having teeth of the same form. If N = number of teeth on the helical gear, $\pi d/N$ = circular pitch in a plane perpendicular to the gear axis. The normal circular pitch is therefore $\pi d \cos\alpha/N$, by equation (8–7). The number of teeth on the equivalent spur gear having a pitch radius equal to the radius of curvature of the normal helix is the pitch circumference divided by the circular pitch, or

$$2\pi\left(\frac{d}{2\cos^2\alpha}\right) \div \frac{\pi d\cos\alpha}{N} = \frac{N}{\cos^3\alpha}. \qquad (8\text{–}10)$$

The conclusion is that **a helical gear of N teeth requires the same shape of teeth as a spur gear of $\left(\frac{N}{\cos^3\alpha}\right)$ teeth.**

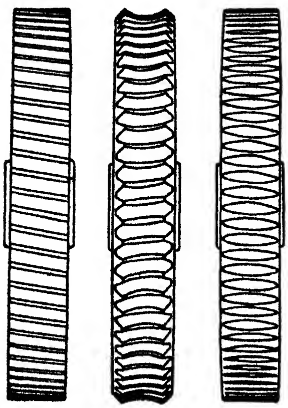
When using milling cutters for tooth production, the space will not have the same cross-section as the cutter. For this reason the cutter used should be one which is suitable for a spur gear having

$$\left(\frac{N}{\cos^3\alpha} + P_n D_c \tan^2\alpha\right) \text{ teeth.}$$

D_c is the diameter of the cutter at the middle of the profile.

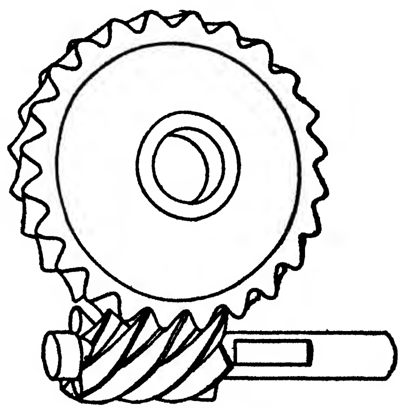
WORM GEARING

39. Properties and Uses. — The properties of the worm gear are such as to render it suitable for many uses. Though common for connecting non-intersecting shafts at 90°, it is sometimes used for other shaft angles. When properly constructed it will operate with little noise and show good efficiency. For large speed reductions it is more compact than other types of gear drive. Under certain conditions the drive is "irreversible"; that is, the combination of worm and wheel can be driven only from the worm end. For this reason it has been employed in hoists, steering gears for automobiles, etc., where reversibility is not desired.



(*a*) (*b*) (*c*)

Fig. 8–45
Types of Worm Wheels.



(Foote Bros. Gear Co.)

Fig. 8–46
Multiple-threaded Worm and Wheel.

A simple form of worm wheel, illustrated at *a*, Fig. 8–40, is just an ordinary helical gear. The pitch surfaces are cylindrical, and the teeth have point contact only.

Line contact, with a consequent improvement in wearing qualities and load-carrying capacity, is secured in the gear shown at *b*, Fig. 8–45. Here the worm-wheel surface is concaved so as to conform with the worm outline, and the active section is not confined to the central plane. This is the style generally used.

At *c*, Fig. 8–45, is shown a third form of worm wheel. This presents a fairly smooth outer surface, an advantage when the wheel is sometimes turned by hand.

Figure 8–46 shows a variety of worm gear with multiple-threaded worm, used for rear-axle drives in automobiles.

In a special type of worm developed in England, known as the **Hindley Worm,** the axial section of the pitch surface is curved to fit the pitch arc of the worm wheel. A large number of teeth are thus brought into a simultaneous contact. Proper action requires that accurate endwise adjustment of the worm be maintained.

40. Worm-gear Terms. — Figure 8–47 illustrates the way in which worm-gear dimensions are indicated.

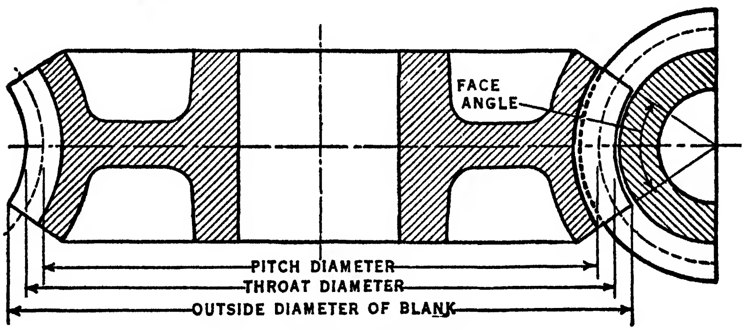


Fig. 8–47
Worm-gear Dimensions.

The **Pitch** of the **Worm** is the distance from a point on one thread or tooth to the corresponding point on the next tooth, measured in a direction parallel to the axis of revolution. Some authorities call this the **Axial Pitch.** The **Pitch** of the **Worm Wheel** is the distance from a point on one tooth to a similar point on the next tooth, measured on the pitch surface in the plane of revolution.

The **Lead** is the amount the worm helix advances along the axis per turn. When a worm is " single threaded," the pitch and lead are equal to one another; when " double threaded," the lead is twice the pitch. In general, where the worm has n threads, the lead is n times the pitch.

The **Face Angle** is measured as indicated in Fig. 8–47. This usually has a value between 60° and 90°.

41. Velocity Ratio. — When the worm has a **single** thread, the worm wheel is advanced one tooth per turn of the worm; hence if the worm wheel has N teeth the worm must make N revolutions per turn of the worm wheel. The **Velocity Ratio** of wheel to worm is therefore equal to $1/N$. When the worm is double threaded the worm wheel is advanced two teeth per turn of the worm, and the velocity ratio becomes $2/N$. The general expression is n/N, where n equals the number of threads on the worm. Worm gears, therefore, comply with the ordinary law for tooth gears.

The velocity ratio of a worm gear can be found directly from the lead and worm-wheel pitch diameter. Per turn of the worm, a

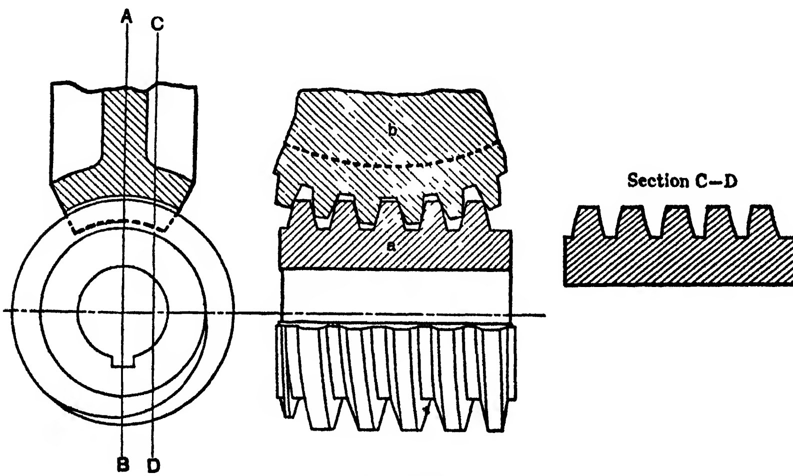


Fig. 8-48

point on the pitch circumference of the wheel is advanced a distance equal to the lead. The angular movement of the worm wheel, in revolutions, for one turn of the worm, is therefore equal to

$$\frac{\text{Lead of worm thread}}{\text{Worm-wheel circumference}}. \qquad (8\text{-}11)$$

This quantity is the **angular velocity ratio** of wheel to worm.

42. Tooth Action. — Considering the worm and wheel shown in section in Fig. 8-48, it will be observed that the wheel can be rotated by imparting either of the following motions to the worm:

(*a*) sliding it along its axis without rotation,
(*b*) rotating it about its axis without endwise motion.

The action of the teeth on one another is the same in both (*a*) and (*b*), except that in (*b*) the teeth have relative sliding along the tooth elements. Both motions require teeth of the same shape. We may therefore regard the worm as a rack and the worm wheel as a special form of gear meshing with it.

The conditions for constant velocity ratio are satisfied when section *a*, Fig. 8–48, found by passing a plane *A–B* through the worm axis, has the form of a spur rack, and section *b* of the worm wheel in the same plane has the form of a conjugate spur-gear tooth.

In the figure, an involute rack section with straight sides has been employed for *a*. To correspond, section *b* is that of an involute spur gear of the same pitch and diameter as the worm wheel. The cross-sections on any other parallel plane, such as *C–D*, must be conjugate tooth forms. The worm section on plane *C–D* is different from *a*, the former showing teeth with curved sides and unsymmetrical form. The conjugate tooth section of the wheel must vary correspondingly. Generally, the method of producing worm gears makes it unnecessary to determine any of the worm-wheel sections, since these are generated by means of a cutter shaped like the worm. One of these is shown in Fig. 8–49. This cutter is first turned to the shape of the worm selected; it is then slotted or "gashed" to form cutting edges, and finally hardened and ground for clearance back of the cutting edges. The cutter and worm-wheel blank are then mounted on arbors and driven so that their relative motion is the same as that of the finished worm and wheel. Generated teeth of the correct shape are thus cut on the blank. The only tooth section that must be specified for the whole process is that for the worm in a plane containing its axis.

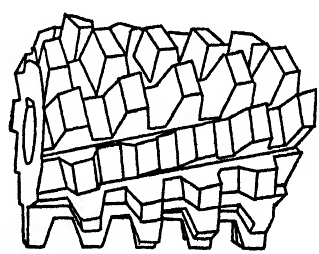


Fig. 8–49
Worm-wheel Hob.

BEVEL GEARS

43. General. — Bevel gears may be used to connect intersecting shafts making any angle with one another. The pitch cones of conjugate bevel gears of the ordinary type have a common apex; this results in pure rolling contact of the pitch cones along their elements. The use of pitch cones not having a common apex would increase the difficulties of design and production and the tooth forms would be less serviceable. Referring to Fig. 8–50, which shows a section through a pair of bevel gears, it will be noted that all tooth elements are straight and radiate from a common apex. This results in similar tooth forms at all points along the cone elements.

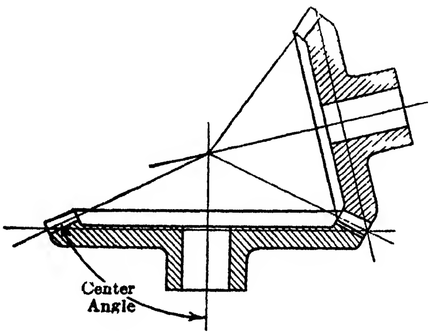


Fig. 8–50
External Bevel Gears.

Bevel gears may be classified according to the size of the pitch

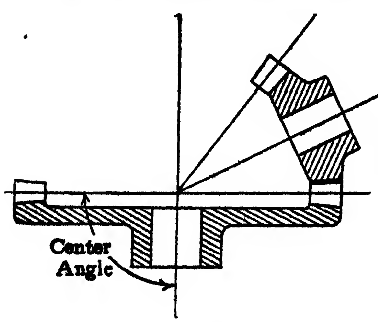


Fig. 8–51
Crown Gear and Pinion.

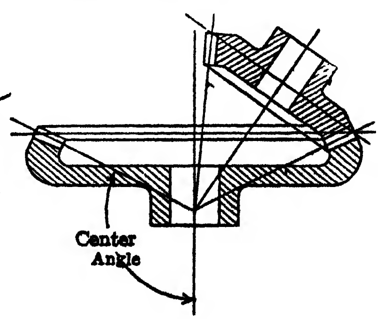


Fig. 8–52
Internal Bevel Gear and Pinion.

cone angle. Those having a center angle less than 90° are known as **External Bevels** (Fig. 8–50). In a **Crown Gear** (Fig. 8–51) the center angle is 90° and the cone becomes a plane surface. **Internal Bevels** (Fig. 8–52) have a center angle greater than 90°, the cone

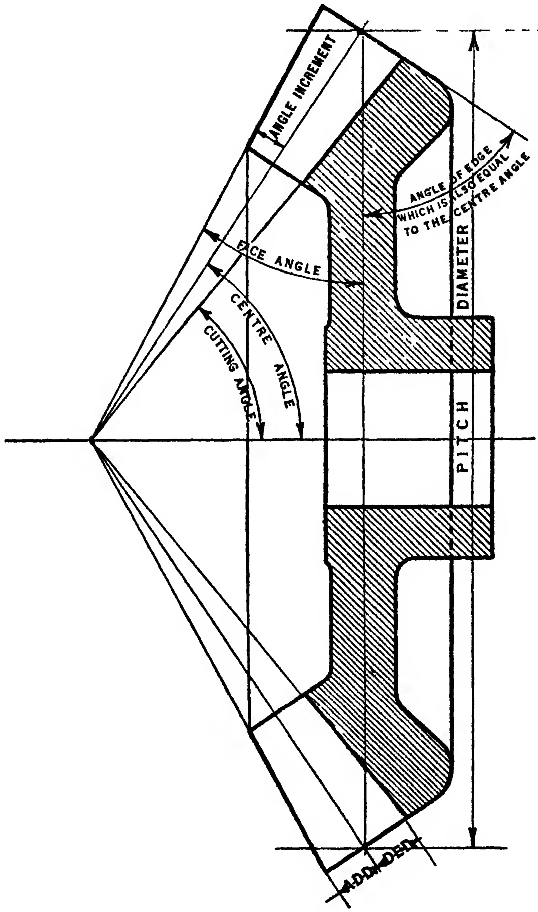


Fig. 8–53
Bevel-gear Notation.

being inverted. Equal bevel gears connecting shafts at 90° are known as **Miter Gears.**

44. Bevel-gear Terms. — Figure 8–53 indicates the meanings of the terms **Cutting Angle, Center Angle, Edge Angle,**

Angle Increment, Pitch Diameter, etc., as applied to bevel gears.

Pitch. — The **Circular Pitch** of the teeth in a bevel gear diminishes as we pass along the surface toward the apex of the pitch cone. Strictly speaking, a statement regarding the pitch of a gear of this kind should be supplemented by information as to where it is measured. In accordance with common usage, however, the circular pitch, unless otherwise specified, means the pitch measured at the outer ends of the teeth. The same statement applies to the pitch diameter, diametral pitch, addendum, dedendum, etc.

45. The Velocity Ratio of bevel gears follows the usual law for toothed gearing, the angular velocity ratio of a pair being inversely proportional to the numbers of teeth. For a given pitch, the numbers of teeth vary directly as the pitch diameters, that is, as the diameters of the bases of the pitch cones. Diameters are dependent on the cone angles. When the angle between the gear axes and the velocity ratio are known, the proper pitch cone angles may be determined by the method of Art. 4, Chapter VII, which applies to rolling pitch cones.

In view of the fact that different speed ratios require different cone angles, bevel gears must be designed in pairs for a particular ratio, and interchangeability does not exist to the same extent as in spur gears.

46. Tooth Action. — Bevel gears have **relative Spherical Motion,** since all points remain at fixed distances from the common apex of the pitch cones. The teeth should in consequence be laid out on spherical surfaces for the same reason that spur-gear teeth are laid out on plane surfaces. It is convenient to consider a bevel gear to be composed of a large number of thin laminae, each having the form of a portion of a thin, hollow sphere as shown in Fig. 8–54. Each of these laminae makes contact with a corresponding lamina on the mating gear, the tooth form of one member being conjugate to that of the other.

Figure 8–55 shows the formation of a pair of bevels from a sphere which is intersected by two planes EC and CD, touching one another at C. The pair of circular intersections, represented by lines EC and CD, may be used as the bases of pitch cones OEC and OCD for the bevel gears. These gears will revolve about axes OB and OA, touching one another along cone elements The surfaces of sections EMC and CND may be regarded as the exteriors of a pair of mating spherical laminae referred to above. Instead of attempting to find the proper forms of the teeth on these spherical surfaces, which is somewhat difficult, we resort to a simplified method known as **Tredgold's Approximation.** By this method we find two cones EBC and ACD, tangent to the sphere along the circumferences of the circles represented by lines CD and CE. These are known as the **Normal Cones,** because each has elements which are normal to intersecting elements on the corresponding pitch cones. We observe that these cones very nearly coincide with the spherical surface near their points of tangency around the circles CD and CE. Teeth laid out on the conical surfaces, using CD and CE as pitch circles, will closely resemble in their action the teeth of ordinary spur gears. Hence the profiles on the normal cones may be made the same as those of spur gears of the same circular pitch, little error being introduced in so doing.

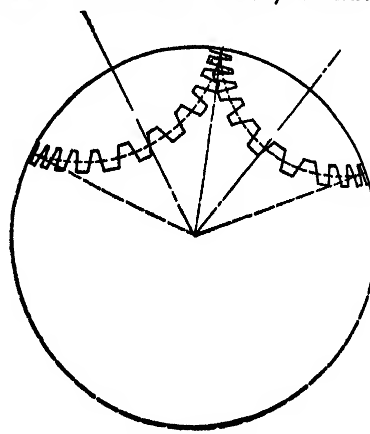


Fig. 8–54

To draw the tooth sections on the normal cones, we first develop the cone surfaces and thus reduce the problem to one of drawing spur-gear teeth of known pitch. We use the outer circular edges of the developed surfaces as pitch circles. In Fig. 8–55, $FGHI$ is the development of normal cone EBC, and $JKML$ that of cone ACD. Assuming the gear whose pitch cone is OCD to have N teeth, the circular pitch is $\pi(CD)/N$. Around the arc KML, whose length is $\pi(CD)$, we shall have N teeth. The complete circle of which KML is a part has a circumference equal to

$2\,\pi(JK) = 2\,\pi(AC)$, and it will contain

$$N_1 = N \times \frac{2\,\pi(AC)}{\pi(DC)} \text{ teeth}$$

$$= N \times \frac{\text{Slant height of normal cone}}{\text{Pitch radius of bevel gear}}. \qquad (8\text{–}12)$$

The tooth forms we require are therefore those of a spur gear with N' teeth and of the same circular pitch as the bevel gear teeth.

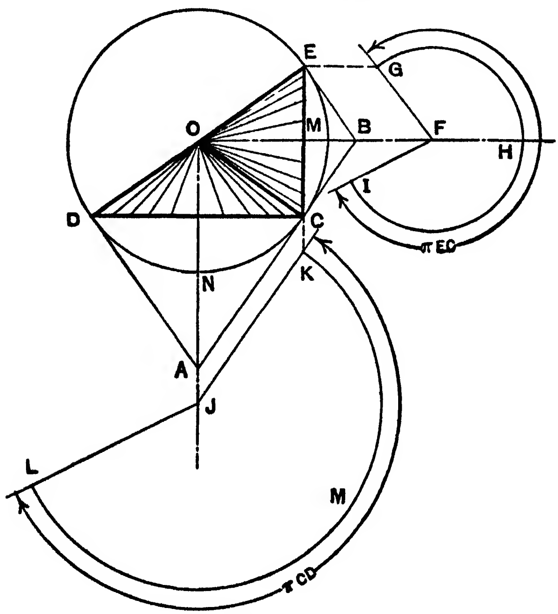


Fig. 8–55

Example. — In Fig. 8–56 is shown the tooth development by Tredgold's method for a standard involute gear of twenty-four teeth, 5 diametral pitch, meshing with an eighteen-tooth pinion, for connecting two shafts at 90°. The construction is as follows:

The axes OA and OB are drawn making an angle AOB equal to 90°.

The pitch diameter of the gear = 24 ÷ 5 = 4.8 in.

The pitch diameter of the pinion = 18 ÷ 5 = 3.6 in.

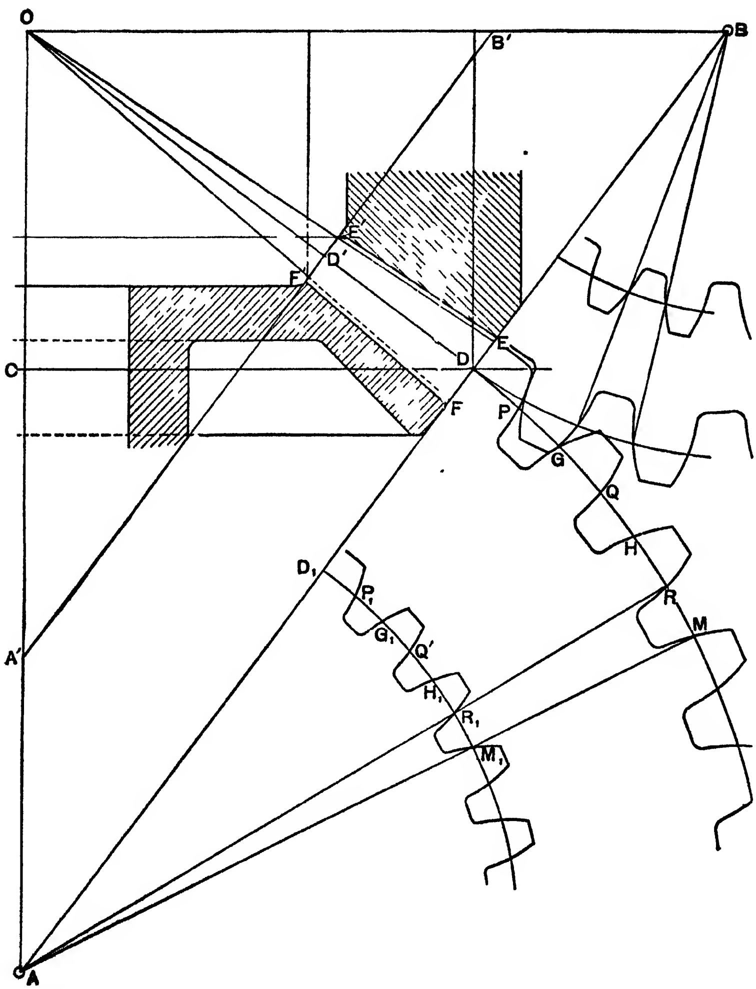


Fig. 8–56
Development of Bevel Gear Teeth on the Normal Cones.

Draw $OC = (3.6 \div 2)$ in., and set off $CD = (4.8 \div 2)$ in. parallel to OB. D is a point on the pitch cones. Connect OD,

$$\text{The addendum of the teeth} = \tfrac{1}{5} = 0.2 \text{ in.}$$
$$\text{The clearance} = 0.1571 \div 5 = 0.0314 \text{ in.}$$

Draw DE at 90° to $OD = 0.2$ in.

Draw DF at 90° to $OD = 0.2$ in. $+ 0.0314$ in. $= 0.2314$ in.

Join OE and OF. These are the addendum and root lines of the gear tooth.

Produce DFE to A and B.

Then A is the apex of the normal cone for the gear.

With center A and radius AD, draw arc DM.

With center A and radii AF and AE, draw arcs defining points and roots of teeth on the normal cone.

The circular pitch of the teeth $= \pi/5 = 0.628$ in.

Lay off distances DG, GH, HM on the pitch arc, equal to 0.628 in.

Draw involute tooth profiles through G, H, M, etc., as for a spur gear of pitch radius AD and 5 D.P.

Bisect DG, GH, HM, obtaining points P, Q, R. Draw involute profiles through these points as before, to form the other sides of the teeth.

To draw the tooth development on the normal cone at the inside of the gear, use A as center once more, $A'D'$ as radius, and draw the arc $D_1G_1M_1$.

Join MA and RA. These lines intersect the arc at M_1 and R_1. Space off distances around the arc equal to M_1R_1, obtaining points P_1, G_1, Q_1, etc.

Draw circles with A as center and radii $A'E'$ and $A'F'$. These define the addenda and roots of the teeth.

Construct involute tooth profiles passing through D_1, P_1, G_1, Q_1, etc., as for a spur gear of pitch radius $A'D'$ and circular pitch D_1G_1.

We have now obtained the shape of the teeth as they appear on the development of the normal cones at the outside and inside of the gear. It will be noted that the tooth forms are similar figures at the two sections.

The construction for the pinion teeth is carried out in the same manner.

Interference of teeth can be investigated from the normal cone developments, as for the spur gear, and modifications of the tooth faces may be made if necessary.

47. Hypoid Gears. — These gears, illustrated in Fig. 8–57, possess certain of the characteristics of both the hyperboloidal and the spiral bevel gears. Like the former, they connect non-intersecting shafts. As in the latter, the tooth elements are of spiral shape so as to give the progressive contact which leads to quiet operation.

Instead of using the hyperboloidal pitch surfaces of the true hyperboloidal gear as shown in Fig. 8–13, cones which approximate these surfaces are substituted. This decreases the difficulties of production. Inspection of Fig. 8–57 will indicate that the teeth slide on one another along the tooth elements. This dictates the use of curves of a particular form for these elements; in this respect also, the hypoid gear differs from the hyperboloidal, which has sensibly straight tooth elements.

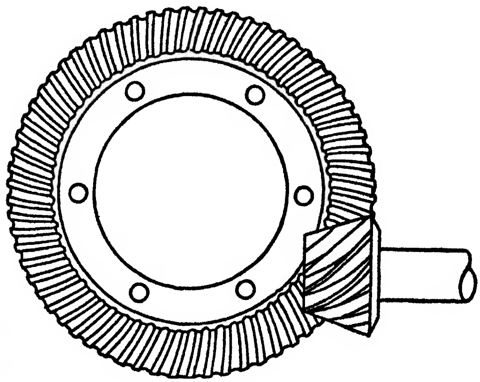


Fig. 8–57
Hypoid Gear.

The hypoid gear, developed by the Gleason Gear Company, has found an important use in the automobile drive, though the fact that the shafts are offset and may be extended past one another has led to other industrial applications. The advantages for automotive use are as follows:

(*a*) The pinion axis can be placed below the gear axis, rendering possible a lower body design for the car.

(*b*) The circumferential pitch of the teeth on the pinion is greater than that of the teeth on the gear, owing to the difference in the helix angles which are used for the two members. This results in a pinion which is larger and stronger than that in a spiral bevel drive with the same speed ratio and the same size of gear.

(*c*) Surface contact is more nearly attained, thus permitting higher tooth pressures to be carried.[4]

A considerable amount of sliding along the tooth elements is present in hypoid gearing. On this account the rubbing velocity of the teeth is higher than in other bevel gears, and hence inaccuracy in mounting or lack of rigidity in mounting is more likely to result in failure of the teeth by scoring. To offset this tendency, special lubricants are generally recommended for hypoids. Nevertheless, the fact that this form of gearing has been adopted for truck drives by some of the larger manufacturers indicates that it is dependable and durable under heavy service.

QUESTIONS — CHAPTER VIII

1. What are the advantages of a gear drive as compared with a friction-wheel drive?

2. Explain the meaning of the following terms as applied to gear wheels: (*a*) pitch circle, (*b*) circular pitch, (*c*) diametral pitch, (*d*) pitch point.

3. Derive an expression for the relationship between the circular and diametral pitch of a spur gear.

4. Define the following terms as applied to gears: (*a*) addendum, (*b*) dedendum, (*c*) face of tooth, (*d*) flank of tooth, (*e*) face of gear, (*f*) backlash.

5. What are conjugate gears? What is the fundamental law governing the shape of conjugate teeth?

6. What are (*a*) the clearance of gear teeth, (*b*) the arc of action, (*c*) the angle of approach, (*d*) the angle of recess? How is the arc of action affected by lengthening the teeth?

7. What are the forms of the pitch surfaces in the following varieties of gears: (*a*) helical, (*b*) plain spur, (*c*) bevel, (*d*) worm?

8. What three common kinds of gears are used for connecting parallel shafts? Sketch.

9. When shafts intersect, what two kinds of gears are used to connect them? Sketch.

10. What two kinds of gears are used to connect shafts that are neither parallel nor intersecting?

11. Sketch (*a*) a plain spur gear, (*b*) a helical spur gear, (*c*) a bevel gear, (*d*) a spiral bevel gear.

12. Sketch (*a*) a herringbone gear, (*b*) a worm gear, (*c*) a skew gear.

[4] For additional information see paper by A. L. Stewart and E. Wildhaber in the S.A.E. Journal, June, 1926.

13. What are the geometrical forms of pitch surfaces for (*a*) a spur gear, (*b*) a herringbone gear, (*c*) a plain bevel gear, (*d*) a spiral bevel gear, (*e*) a worm gear?

14. Why is it necessary that the teeth on mating gears be so formed that a constant velocity ratio is obtained? What must be true of the tooth profiles to produce this result?

15. Prove that in conjugate gears the normal to the surfaces at the point of contact must pass through the pitch point.

16. Why are cycloids and involutes used for tooth profiles in gear wheels? Can other curves be used?

17. What is the "describing" circle in a cycloidal gear? What is a cycloid, and how are the flanks and faces of a cycloidal gear formed?

18. In standard cycloidal gears, what is the size of the describing circle? Why is this size used instead of a larger one?

19. What is an involute? Show how to draw an involute when one point on it and the diameter of the base circle are given.

20. What are the proportions of the standard full-depth involute teeth?

21. What is the path of contact of a pair of involute gears? What is meant by interference of gear teeth? Show by sketch how a pair of gear teeth may be examined for interference.

22. What four modifications may be made in involute gear teeth of standard proportions to avoid interference? What method is commonly employed?

23. Under what conditions is interference obtained in two equal gears with standard $14\frac{1}{2}°$ involute teeth? With a rack and pinion?

24. Why is it impossible to modify an involute rack for interference so that it will have a profile of theoretically correct form to work with more than one size of pinion?

25. By the Grant Odontograph method, what form is given to the working profile of the involute tooth?

26. What two considerations fix the size of the fillet arc at the root of a spur-gear tooth?

27. What is a stub tooth? What are its advantages and disadvantages as compared with standard teeth, and for what kind of service is it used?

28. What is the pressure angle of a pair of involute gears? Does it change during the period of contact of a pair of mating teeth? What values are used (*a*) for standard involute gears, (*b*) for stub-tooth gears? What is the effect on the shape of tooth when the pressure angle is increased? On the bearing pressure?

29. What are the practical advantages of involute as compared with cycloidal gears? How are the pressure angle and backlash in involute gears affected by using a greater center-to-center distance than standard?

30. A gear with standard $14\frac{1}{2}°$ involute teeth has a pitch diameter of 6 in. and a diametral pitch of 4. Calculate the following: (*a*) number of teeth, (*b*) addendum, (*c*) working depth, (*d*) clearance, (*e*) root diameter, (*f*) outside diameter, (*g*) base-circle diameter.

31. Same as Problem 30 but teeth are of standard A.G.M.A. stub-tooth form.

32. Same as Problem 30 except that gear diameter is 4 in. and diametral pitch is 5.

33. A gear has 20 teeth of 2-in. circular pitch, the teeth being of standard $14\frac{1}{2}°$ involute form. Calculate (*a*) the pitch diameter, (*b*) the addendum, (*c*) the clearance, (*d*) the working depth, (*e*) the root diameter, (*f*) the outside diameter, (*g*) the base-circle diameter.

34. Two shafts 10 in. apart are to be connected by spur gears with external teeth, one shaft running at 400 R.P.M., and the other at 600 R.P.M. Find the pitch diameters of the gears and the number of teeth if the diametral pitch is 4. What is the value of the circular pitch?

Ans. 12 in., 8 in., 48, 32, 0.7854 in.

35. *A* and *B* are two mating spur gears. *A* has 30 teeth of 3 diametral pitch and *B* has a pitch diameter of 15 in. The teeth are of standard $14\frac{1}{2}°$ involute form. Find: (*a*) number of teeth on *B*; (*b*) center-to-center distance of shafts; (*c*) circular pitch; (*d*) addendum; (*e*) dedendum; (*f*) root diameter of *A*; (*g*) outside diameter of *A*; (*h*) diameter of base circle of *A*; (*i*) speed ratio of *A* to *B*.

36. Spur gear *A* has 20 teeth of 2-in. circular pitch and meshes with gear *B* having 28 teeth. Find the center-to-center distance of the shafts and the speed ratio. *Ans.* 15.27 in., 1.4.

37. Shaft *A* carries a spur gear with internal teeth. Its pitch diameter is 20 in. and the diametral pitch is 3. Shaft *B* carries a mating gear with 12 teeth. Find the center-to-center distance and the speed ratio of the shafts.

38. A pair of gears having teeth of 20° stub involute form, make internal contact. The shafts are 10 in. from center to center, and the angular velocity ratio is 3 : 1. The diametral pitch is 4. Find: (*a*) pitch diameters of the gears; (*b*) number of teeth on each gear; (*c*) addendum for each gear; (*d*) dedendum for each gear; (*e*) diameter of base circle for the pinion.

39. What is an annular gear? What relationship exists between the teeth of a spur gear and those of an annular gear of an equal pitch and diameter?

40. A pair of spur gears *a* and *b* have internal contact on *b*. The number of teeth on *a* is 20, and the angular velocity ratio of *a* to *b* is 5 : 2. The center-to-center distance is $7\frac{1}{2}$ in. Teeth are $14\frac{1}{2}°$ involute form. Find: (*a*) number of teeth on *b*; (*b*) pitch diameter of *a* and *b*; (*c*) diametral pitch; (*d*) outside diameter of *a*; (*e*) height of teeth.

41. How do the tooth actions of pairs of helical gears differ from one another: (*a*) when the shafts are parallel, (*b*) when the shafts are not parallel?

42. Show that in a pair of helical gears *a* and *b*

$$\frac{\omega_a}{\omega_b} = \frac{d_b \cos \beta}{d_a \cos \alpha},$$

where ω_a and ω_b are the angular velocities, d_a and d_b the pitch diameters, and α and β the angles of cut.

44. A helical gear of 6-in. pitch diameter has an angle of cut of 25° and there are 24 teeth. Find the values of the circular and diametral pitches measured both circumferentially and normal to the teeth.

Ans. $P = 4$, $p = 0.7854$ in., $P' = 4.42$, $p' = 0.712$ in.

45. A helical gear of 10-in. pitch diameter has a normal diametral pitch of 4. The angle of cut is 20°. Find the normal circular pitch, also the diametral and circular pitches measured circumferentially.

46. If it be desired to cut a helical gear with 18 teeth of 6 diametral pitch and angle of cut of 15°, how would a spur-gear cutter be selected for the work?

47. A helical gear of 10-in. pitch diameter has an angle of cut of 30°, and there are 30 teeth. Find values of the circular and diametral pitches measured both circumferentially and normal to the teeth. Also give specifications for spur-gear cutter suitable for the helical gear.

48. A helical gear with 26 teeth, pitch diameter 6 in. has a normal diametral pitch of 5. Find the angle of cut.

49. A helical gear of 8-in. pitch diameter has 47 teeth, the angle of cut being 21°. Find the normal circular pitch.

50. A pair of helical gears are used to connect two shafts at 60°. One gear has a pitch diameter of 8 in. and an angle of cut of 35°. Its angular velocity is 1.1 times that of the other. Find for the second gear (*a*) angle of cut, (*b*) pitch diameter. *Ans.* (*a*) 25°, (*b*) 7.94 in.

51. A pair of helical gears connect two shafts at an angle of 45°. The pitch diameters are, respectively, 8 in. and 12 in., and the speed ratio 4 : 3. Find graphically the angles of cut of the teeth. *Ans.* 14°, 31°.

52. Two helical gears having 21 teeth and 63 teeth, respectively, are used to connect a pair of shafts at an angle of 60°. The pitch diameters of the gears are 7 in. and 14 in. Find graphically the angles of cut of the teeth.

53. A pair of helical gears *a* and *b* connect shafts located at 90°. Gear *a* has an angle of cut of 30° and a pitch diameter of 10 in. Gear *a* rotates at 1200 R.P.M. and *b* at 800 R.P.M. Determine the pitch diameter of *b*.

54. Helical gears of 6-in. and 8-in. pitch diameters, respectively, have a speed ratio of 3 : 2. The shafts meet at an angle of 60°. Show how to find graphically the angles of cut of the teeth.

55. Two helical gears *a* and *b* are mounted on shafts intersecting at an angle of 75°. Gear *a* has a pitch diameter of 8 in., and *b* of 12 in. Gear *a* has an angle of cut of 30°. Find the speed ratio of the gears.

56. Show by sketch the meaning of the following terms: pitch, lead, and helix angle of a worm. How would you calculate the helix angle when the pitch diameter and lead are known?

57. Indicate by sketch the general form of three types of worm wheels. Which is most commonly used, and why?

58. What is the nature of an axial section of a worm? Of a worm wheel?

59. A quadruple-threaded worm has two threads per inch. When it is mated with a certain worm wheel the angular velocity ratio is 20 : 1. Determine the pitch diameter and number of teeth on the worm wheel.

60. A worm gear consists of a worm with triple thread driving a worm wheel with 48 teeth, to which is attached a 24-in. pulley. The worm is driven at 900 R.P.M. Find the linear speed at the pulley face.

Ans. 353 ft. per min.

61. A worm has a lead of $\frac{1}{4}$ in. and meshes with a worm wheel of 8-in. pitch diameter. Determine the number of turns required on the worm shaft to revolve the worm wheel one turn.

62. A worm has a pitch diameter of 2 in. and is triple threaded. It meshes with a worm wheel of 12-in. pitch diameter and the speed ratio is 30 : 1. What is the axial pitch of the worm?

63. A double threaded worm has an axial pitch of $\frac{1}{2}$ in. and meshes with a worm wheel with a pitch diameter of 6 in. Find the speed ratio.

64. In a worm-gear drive the worm wheel is 12.73-in. pitch diameter and is driven by a worm which is double threaded. The axial pitch of the worm is $\frac{1}{5}$ in. If the worm rotates at 800 R.P.M. what is the speed of the worm wheel?

65. Explain the meaning of the following terms as applied to a bevel gear: (*a*) pitch diameter, (*b*) face of gear, (*c*) addendum, (*d*) dedendum.

66. Explain the meaning of the following terms as applied to bevel gears: (*a*) cutting angle, (*b*) center angle, (*c*) edge angle, (*d*) face angle.

67. Explain why the diametral pitch is different at the inside and outside of the teeth in a bevel gear.

68. What do we mean by the "normal cone"? For what is it used?

69. What is Tredgold's Approximation as applied to bevel gears?

70. A pair of bevel gears connecting two shafts at 90° have a speed ratio of 3 to 2. Find the center angle for both. If the larger wheel has 30 teeth of 4 diametral pitch, find the pitch diameter and the number of teeth on the smaller gear. *Ans.* 33°.40′, 56°.20′, 5 in., 20.

71. A bevel gear of 8-in. diameter, 6 diametral pitch, has a pitch cone angle of 60°. Find the number of teeth on the spur gear whose tooth form is the same as that of the bevel gear on the normal cone development. *Ans.* 96.

72. A bevel gear, with a center angle of 30°, has 32 teeth of 4 diametral pitch. What is the number of teeth on a spur gear with teeth of the same form?

73. A bevel gear with 50 teeth and 5 diametral pitch has a center angle of 60°. Determine the number of teeth on the corresponding spur gear.

74. A pair of bevel gears used to connect shafts at 90° have a speed ratio of 4.5 to 1. The smaller gear has a 2-in. pitch diameter, and the teeth have a diametral pitch of 5. The teeth are of standard stub involute form. Find the pitch diameter and number of teeth on the larger gear, also the center angles of both gears. What are the values of the addendum and whole depth of the teeth?

75. A pair of bevel gears having shafts intersecting at 75° give a velocity ratio of 2 : 1. The larger gear has a diameter of 14 in. The diametral pitch of the teeth is 5. Find the center angles. Determine the number of teeth on the spur gear whose tooth form is the same as that of the smaller gear.

76. A pair of bevel gears connect two shafts at 60° and have a speed ratio of 3 : 5. If the larger gear has 45 teeth of 5 diametral pitch, find the pitch diameter and number of teeth on the smaller gear. Also, find the center angles.

CHAPTER IX

GEAR TRAINS

1. Train Value. — A mechanism which transmits motion from a driving to a driven shaft by use of two or more gear wheels is called a **Gear Train.** Problems involving the calculation of the velocity ratios of such trains will be considered in this chapter.

The **Train Value** we shall define as the ratio

$$\frac{\text{Angular Velocity of the Last Wheel (Driven)}}{\text{Angular Velocity of the First Wheel (Driver)}}.$$

These velocities are measured in the ordinary gear train with reference to the fixed frame which supports the gear shafts.

A positive sign for the train value indicates that the first and last wheels turn in the same sense, while a negative sign is used to indicate rotation in the opposite sense.

In Art. 3, Chapter VIII, it was shown that the same general law for the velocity ratio applies to any pair of toothed gears, whether spur, helical, or bevel, etc. This law states that the velocity ratio of a pair of gears is the inverse ratio of the numbers of teeth. Hence the method of finding the train value in terms of the numbers of teeth is the same for all gear trains, no matter what variety or varieties of gears they contain.

2. A Simple Gear Train is one in which no two wheels are rigidly fastened to the same shaft so as to rotate at the same velocity. Figure 9–1 shows a train of this kind. Here motion is transmitted from a to d through intermediate wheels b and c. By definition, the train value is $\frac{\omega_{de}}{\omega_{ae}}$. The pitch circles of the gears roll together without slipping; therefore the pitch-line velocity is the same for all. It follows that wheel d, through contact with c, will turn at the same rate as though it meshed with a. The sizes of intermediate wheels b and c, or the numbers of teeth they contain, evidently have no effect on the train value. For this reason, b and c are usually termed **idlers**. This is somewhat of a misnomer, since these wheels transmit power as well as a and d.

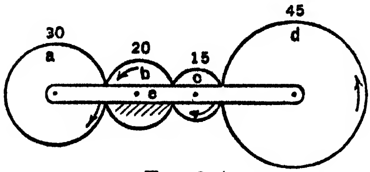


FIG. 9–1

If wheel c were removed from the train and d then driven from b, d would still have the same speed but in the reverse sense. Thus the **number of idlers** controls the sign of the train value.

In view of the foregoing it is evident that the train value for a simple gear train is equal to the inverse ratio of the numbers of teeth on the first and last wheels. For the train of Fig. 9–1,

$$\frac{\omega_{de}}{\omega_{ae}} = -\frac{N_a}{N_d}, \tag{9-1}$$

where N_a and N_d are the numbers of teeth.

By inspection, wheels a and d are observed to rotate in opposite senses, which accounts for the negative sign. Substituting the numbers of teeth indicated in the figure,

$$\frac{\omega_{de}}{\omega_{ae}} = -\frac{30}{45} = -\tfrac{2}{3}.$$

Figure 9–2 shows a simple gear train containing an annular or

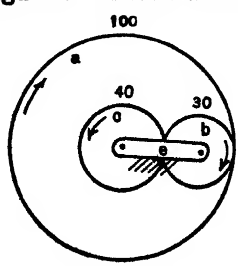


FIG. 9–2

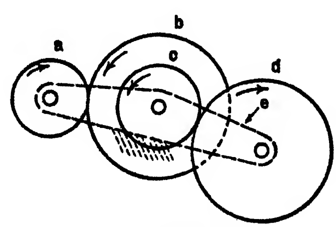


FIG. 9–3

internal gear, driving an idler b which in turn drives wheel c. The train ratio is

$$\frac{\omega_{ce}}{\omega_{ae}} = -\frac{100}{40} = -2\tfrac{1}{2}.$$

We use the minus sign, since the driving wheel turns clockwise when the driven wheel turns anti-clockwise.

3. A Compound Gear Train is one in which at least one pair of wheels are rigidly attached to the same shaft so that both revolve at the same angular speed. One of these trains is shown in Fig. 9–3. In this train the drive is through a, b, c, d, in order, and wheels b and c are keyed to the same shaft. To find the train ratio we may proceed as follows:

Considering wheels a and b,

$$\frac{\omega_{be}}{\omega_{ae}} = \frac{N_a}{N_b}. \tag{1}$$

Also, considering wheels c and d,

$$\frac{\omega_{de}}{\omega_{ce}} = \frac{N_c}{N_d}. \tag{2}$$

Multiplying (1) and (2),

$$\frac{\omega_{be}}{\omega_{ae}} \times \frac{\omega_{de}}{\omega_{ce}} = \frac{N_a \times N_c}{N_b \times N_d}.$$

But,

$$\omega_b = \omega_c,$$

since these wheels are keyed to the same shaft. Therefore,

$$\frac{\omega_{de}}{\omega_{ae}} = \frac{N_a \times N_c}{N_b \times N_d}. \tag{9–2}$$

Calling the first wheel in each pair of meshing gears *a driver* and the second *a driven* wheel, we may write

$$\text{Train Value} = \pm \frac{\text{Product of Nos. of Teeth on Drivers}}{\text{Product of Nos. of Teeth on Driven}}.$$

The sign, as before, depends on whether rotation at the driven end of the train is the same or opposite to that at the driving end. Compound trains are often used where the speed reduction is large. In such cases a simple train with the same speed ratio might require the use of one very large gear.

4. Reverted Gear Trains. — A gear train is said to be reverted when the first and last gears turn about the same axis. The gear trains in an automobile transmission which are in use on "low," "intermediate" or "reverse" are of this type. The first and last wheels are co-axial, so that they can be coupled together when the car is in "high." The back gears of the lathe form part of a reverted train. In Fig. 9–4 is shown a reverted train of four spur gears, b and c being keyed to the same shaft. The distance from center to center of the shafts is

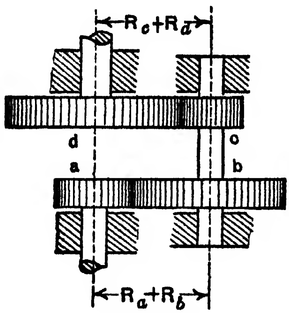


FIG. 9–4

$$R_a + R_b = R_c + R_d.$$

If all wheels have teeth of the same pitch, the numbers of teeth are proportional to the pitch radii. Hence, if

$$R_a = C \times N_a,$$

then

$$R_b = C \times N_b, \qquad R_c = C \times N_c, \qquad R_d = C \times N_d,$$

where C is a constant. Substituting these values in the above equation, we have

$$N_a + N_b = N_c + N_d. \tag{9–3}$$

Example. — A reverted gear train of four gears, arranged as shown in Fig. 9–4, has a train value of $\frac{1}{6}$. Wheel a has 20 teeth, wheel b 40 teeth. Find the number of teeth for c and d, assuming that the pitch of the teeth is the same for all wheels.

The train value is

$$\frac{\omega_{ds}}{\omega_{as}} = \frac{1}{6} = \frac{N_a \times N_c}{N_b \times N_d} = \frac{20 \times N_c}{40 \times N_d}.$$

Therefore,

$$\frac{N_c}{N_d} = \frac{1}{3} \qquad \text{or} \qquad N_c = \frac{N_d}{3}. \qquad (1)$$

Also, from equation (9–3),

$$N_a + N_b = N_c + N_d$$
$$20 + 40 = N_c + N_d. \qquad (2)$$

Equations (1) and (2) may be solved for N_c and N_d. We thus find $N_c = 15$, $N_d = 45$.

5. Sliding-gear Automobile Transmission. — Figure 9–5 illustrates a common form of automobile transmission which provides

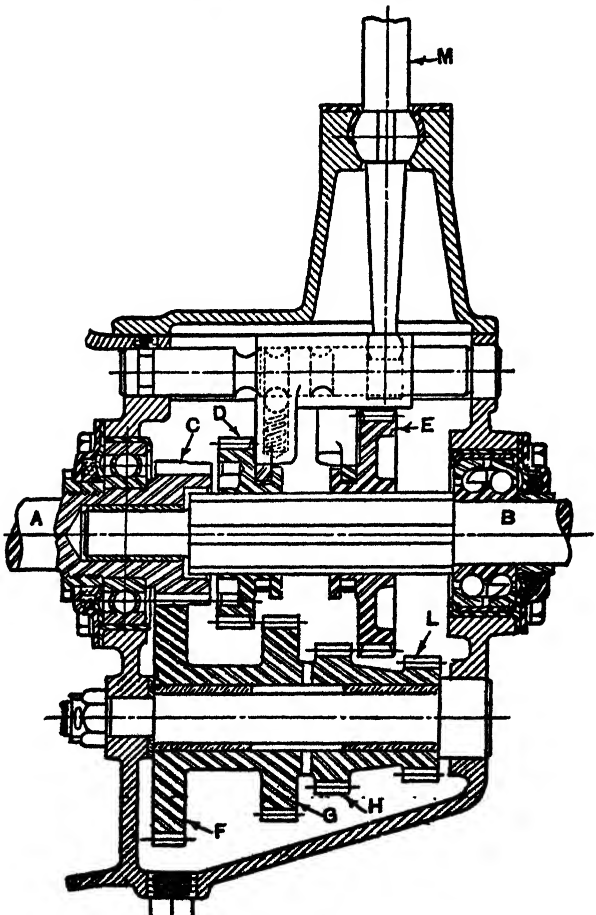


FIG. 9–5
Sliding-gear Automobile Transmission.

three speeds forward, neutral, and reverse. The more important members consist of driving shaft *A* and co-axial driven shaft *B*, sliding gears D and E which turn with B, and gears F, G, H, L, rigidly connected together and rotating on a lay shaft. The illustration shows the transmission in neutral position. Gears C and F are always in mesh, so that unit F, G, H, I is always in motion.

The gear system is controlled by lever M which slides gears D or E to right or left as desired. The transmission operates as follows:

(*a*) *Third Speed* (*direct drive*). — Gear D is moved to the left, the internal teeth on D engaging with C. Shafts A and B now rotate at the same speed.

(*b*) *Second Speed.* — Gear D is shifted to the right, engaging gear G. The reverted gear train C, F, G, D causes B to rotate in the same sense as A but at a reduced speed.

(*c*) *First Speed.* — Gear E is moved to the left, engaging H. The gear train C, F, H, E drives B in the same sense as A with a speed reduction of larger value than in the second speed position on account of the decrease in the ratio of tooth numbers, $N_H : N_E$ as compared with $N_G : N_D$.

(*d*) *Reverse.* — Gear E is moved to the right, engaging an idler located behind the plane of section and meshing with L. This idler is not shown in the figure. Motion is now transmitted through C, F, L to the idler and through it to E. The addition of the idler causes B to rotate in a sense opposite to that of A.

In the Buick transmission shown in Fig. 9–5*a*, helical gears are employed to insure quiet operation. To avoid clashing of gears during the engagement of high and intermediate speeds, the transmission is provided with a synchronizing device which insures that shafts to be connected are rotating at the same speed. This device consists of two cone clutches, one for use when changing into high and the other when changing into intermediate speeds. One of these clutches is engaged by the primary movement of the control lever, the further motion of the lever afterward connecting a positive clutch which takes the form of a toothed gear mating with an internal gear.

The speed ratios obtainable and the gears used for each are as follows:

High gear: direct connection of driving shaft A with driven shaft B is made by locking the positive clutch L.

Intermediate gear: the drive is through gears C, D, E, and F. Gear F is obliged to rotate with driven shaft B only when positive clutch M, M_1 is engaged; otherwise this gear turns freely on a bronze bush mounted on shaft B.

Low gear: here the drive is through C, D, G, and H. H is a sliding gear mounted on splines to rotate with B.

Reverse: in this case the gear train consists of C, D, I, thence to an idler mounted on a shaft behind the plane of section and then to H.

The figure shows the transmission in neutral. Movement of shifter shaft N to the right engages low speed by bringing sliding gear H in mesh with G. Movement of shifter shaft N to the left engages the reverse speed by bringing H in contact with the idler mentioned above.

When the second shifter shaft O is moved to the right, clutch L, L_1 is engaged and shafts A and B rotate together. Movement of O to the left engages clutch M, M_1 which connects gear F to shaft B and provides the intermediate speed.

The synchronizing portion of the mechanism consists essentially of positive clutch members L and M mounted on splines so as to rotate with B, cam sleeve P, and shifter sleeve Q. Clutch member L, M has arms which pass through slots in the cam sleeve and rigidly connect the former member to the shifter sleeve on the outside. The cam sleeve carries at its ends the cone clutch members R and S. The clutch member, cam sleeve, and shifter sleeve rotate as a unit with shaft B.

Assuming that the car is moving and that it is desired to engage the intermediate speed, the shifter shaft O is moved to the left, carrying with it cam sleeve P and clutch member L, M. The cone clutch S first engages and thus brings gear F to the same speed as shaft B. Resistance to further axial motion of the cam sleeve causes balls T to spring out of a circumferential groove in the cam sleeve and allows the clutch member to continue motion to the left until positive clutch M and M_1 is engaged. The function of the slots in the cam sleeve, and the "cam" which projects through

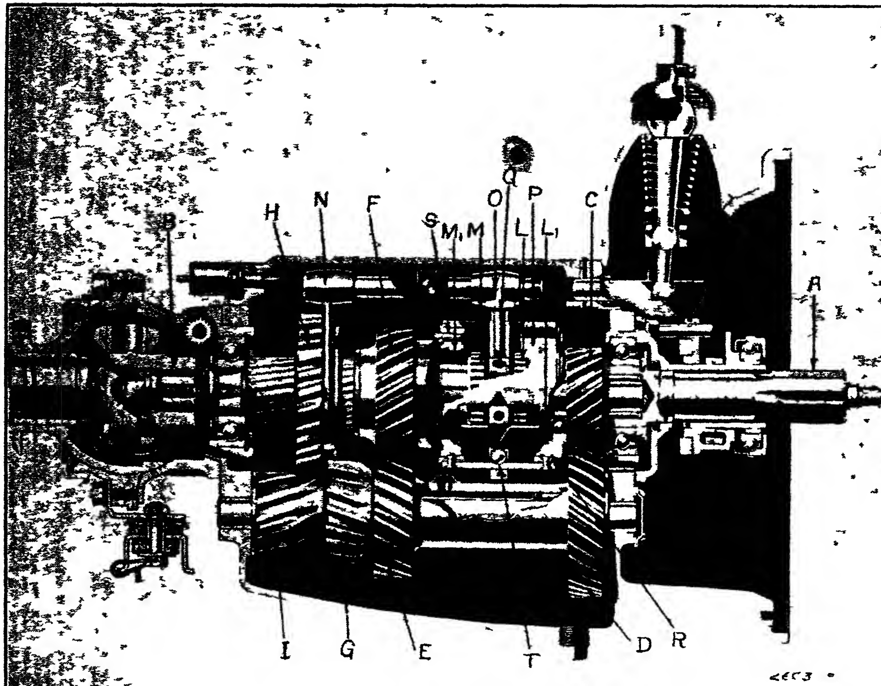
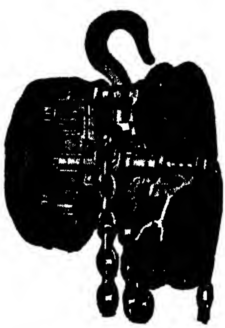


Fig. 9–5a. Buick "Synchromesh" Transmission.

them, is to cause a slight angular movement of the two clutch members M and M_1 during engagement. This avoids jamming of the teeth.

A similar action takes place when " high " is engaged. In this case, cone clutch R is used to synchronize the speed of shaft A with that of B before positive clutch L, L_1 is engaged.

6. Spur-geared Hoisting Block. — Figure 9–6 shows an application of a compound reverted gear train to a hoisting block. The device is operated by a hand chain on the right, which runs on a sprocket of comparatively large diameter. This sprocket is connected to a shaft which transmits motion to the gear train on the left. This train is an epicyclic with fixed internal gear, having as a driven member a cage supporting the small revolving pinions. This cage is keyed to a sleeve which also carries the hoisting chain sprocket at the middle of the block. An automatic clutch in the right-hand side of the case holds the load until the hand chain is pulled in the lowering direction.



(Yale and Towne Mfg. Co.)
FIG. 9–6
Spur-geared Hoisting Block.

By proper choice of the sprocket diameters and gear sizes the device is designed for any desired speed ratio of hoisting chain to hand chain. Neglecting friction losses, the ratio of the load-chain pull to the hand-chain pull is the reciprocal of their speed ratio, since the work done by both is the same.

7. Epicyclic or Planetary Gear Trains. — In the ordinary gear trains already discussed, the wheels revolve about fixed axes, the frame supporting the wheels being the fixed link in the mechanism. In an epicyclic gear train, on the other hand, the axes of certain of the wheels are in motion and one of the gears becomes the fixed link. An ordinary gear train may be converted into an epicyclic train by fixing one of the wheels and causing the frame

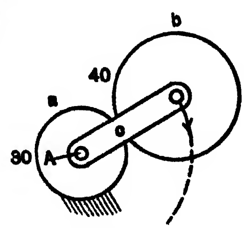


FIG. 9–7

carrying the wheel axles to revolve. The epicyclic train of Fig. 9–7 has a stationary wheel *a*, and frame *c* revolves about the pin at *A* with the result that *b* rolls around on *a*.

What we often want to know about an epicyclic is the ratio of the angular velocity of the driven wheel to the angular velocity of the frame carrying the wheel axles. In Fig. 9–7 this is $\frac{\omega_{ba}}{\omega_{ca}}$, both velocities being measured with respect to the fixed wheel. This quantity we may call the **Epicyclic Value** and we will consider two methods of calculating it.

8. First Method. — The evaluation of the Epicyclic Value may be made by applying two fundamental principles concerning the motion of any three bodies.

(1) If we have three moving bodies, the angular velocity of the third relative to the first is equal to the angular velocity of the second relative to the first plus the angular velocity of the third relative to the second. (See Art. 13, Chapter II.) Thus if *a, c, b*, in Fig. 9–7, are the three bodies, then

$$\omega_{ba} = \omega_{ca} + \omega_{bc}. \qquad (A)$$

(2) If we have two bodies, the angular velocity of the first relative to the second is equal numerically to the angular velocity of the second relative to the first, but of opposite sign. Hence,

$$\omega_{ca} = -\omega_{ac}. \qquad (B)$$

Referring to Fig. 9–7, the epicyclic value equals

$$\frac{\omega_{ba}}{\omega_{ca}} = \frac{\omega_{ca} + \omega_{bc}}{\omega_{ca}}, \text{ by Equation } (A),$$

$$= 1 + \frac{\omega_{bc}}{\omega_{ca}} = 1 - \frac{\omega_{bc}}{\omega_{ac}}, \text{ by Equation } (B).$$

But $\frac{\omega_{bc}}{\omega_{ac}}$ is the train value when the frame *c* is the fixed link. Calling this train value *T*,

$$\textbf{Epicyclic Value} = 1 - T. \qquad (9\text{–}4)$$

This formula can be applied to any epicyclic gear train. Care must be exercised in two directions: (*a*) in obtaining the proper sign for T and (b) in calculating its value. The **denominator** in the fraction expressing the train value is the **angular velocity of the wheel which becomes the fixed link in the epicyclic.**

Example 1. — Suppose c, Fig. 9–7, makes one turn clockwise; find the number of turns made by b. Wheel a has 30 teeth and wheel b 40 teeth.

The train value $(T) = \dfrac{\omega_{bc}}{\omega_{ac}} = -\dfrac{30}{40}\cdot$ The minus sign is required, since a and b rotate in opposite senses.

By equation (9–4), the epicyclic value $\dfrac{\omega_{ba}}{\omega_{ca}} = 1 - T = 1 - (-\tfrac{3}{4})$

$$= +1\tfrac{3}{4}.$$

While c makes $+1$ turn, b makes $+1\tfrac{3}{4}$ turns.

Example 2. — An epicyclic gear (Fig. 9–8) has a stationary wheel a. Arm e turns at 50 R.P.M. clockwise. Find the speed and direction of rotation of d.

Wheels b and c are evidently idlers.

The train value $\dfrac{\omega_{de}}{\omega_{ae}} = +\dfrac{130}{40}\cdot$

The epicyclic value $\dfrac{\omega_{da}}{\omega_{ea}} = 1 - T = 1 - \dfrac{130}{40} = -2\tfrac{1}{4}.$

If e makes 50 revolutions clockwise, d makes $50 \times (-2\tfrac{1}{4}) = -112.5$ R.P.M.

The minus sign indicates anti-clockwise rotation.

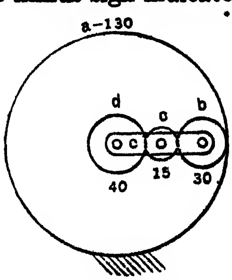


FIG. 9–8

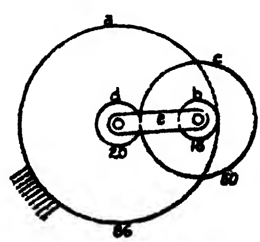


FIG. 9–9

Example 3. — Figure 9–9 represents an epicyclic gear train in which a is fixed. Find the number of turns made by d while e makes one turn clockwise.

The train value (T) equals

$$\frac{\omega_{de}}{\omega_{ae}} = -\frac{86 \times 50}{16 \times 20} = -13.4.$$

Using the epicyclic formula,

$$\frac{\omega_{da}}{\omega_{ea}} = 1 - T = 1 + 13.4 = 14.4.$$

Hence, if e makes one turn clockwise, d makes 14.4 turns clockwise.

Example 4. — Figure 9–10 shows a reverted epicyclic train arranged to give a large speed reduction from the driving shaft A to the driven shaft C. Shaft A drives an arm e which supports B to which are keyed gear wheels b and c. Wheel a is fixed. Rotation of A therefore causes b to roll around on a, c meanwhile driving d, which is keyed to driven shaft C. We shall suppose that the numbers of teeth on the wheels are as follows: a — 60 teeth, b — 61 teeth, c — 60 teeth, d — 61 teeth. Shaft A turns at 100 R.P.M. Find the speed of C.

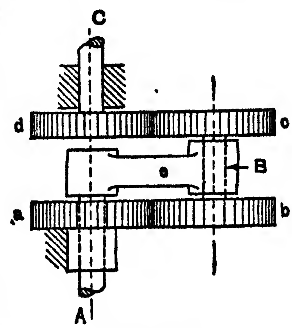


FIG. 9–10

The train value (T),

$$\frac{\omega_{de}}{\omega_{ae}} = \frac{N_a \times N_c}{N_b \times N_d} = +\frac{60 \times 60}{61 \times 61} = +\frac{3600}{3721}.$$

For the epicyclic,

$$\frac{\omega_{da}}{\omega_{ea}} = 1 - \frac{3600}{3721} = \frac{121}{3721}.$$

If A makes 100 R.P.M., C makes $100 \times \frac{121}{3721} = 3.25$ R.P.M.

This case illustrates the method by which a large speed reduction can be obtained by means of an epicyclic, using wheels which are all about the same size. The gear train can thus be made in a compact form.

9. Second Method. — As an alternative to the method already explained, the following procedure is useful in calculating the velocity ratio of epicyclic trains. It consists of two steps:

(1) The epicyclic train is converted into an ordinary train by locking the epicyclic arm on which certain gears are mounted and at the same time releasing the fixed gear. The gear formerly fixed is now rotated one turn clockwise and the number of turns made by other members is recorded.

(2) The gears are locked so that they can have no relative motion and the whole mechanism is rotated one turn counter-clockwise. As a result each member of the train will make one turn counter-clockwise.

The initial and final positions of the "fixed" gear are the same, hence the angular displacement of the other gears is the same as though the train had remained an epicyclic. From these angular displacements the velocity ratio of the train can be calculated.

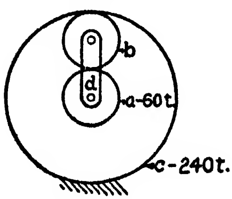


FIG. 9–11

Applying the above method to the epicyclic train of Fig. 9–11.

(1) arm d is locked and gear c rotated one turn in a positive direction and then (2) the gears are locked and the whole mechanism rotated one turn in a negative sense. The results may be tabulated as follows:

Step	Turns			
	a	b	c	d
1	−4	–	+1	0
2	−1	−1	−1	−1
Total	−5	–	0	−1

Thus, while d makes −1 turn, a makes −5 turns and the ratio $\frac{\omega_a}{\omega_d}$ equals +5.

In Fig. 9–12 is illustrated a compound epicyclic in which the epicyclic arm f carrying gears b and c is neither the driver nor driven link. Driver a engages gear b which in turn meshes with stationary annular wheel e. Gears b and c are keyed to a shaft supported by arm f, which is free to turn on shaft A. Gear c engages annular gear d which is keyed to driven shaft B.

The first step in finding the train ratio is to lock the arm f and rotate e one turn in a positive sense. The second step is to lock

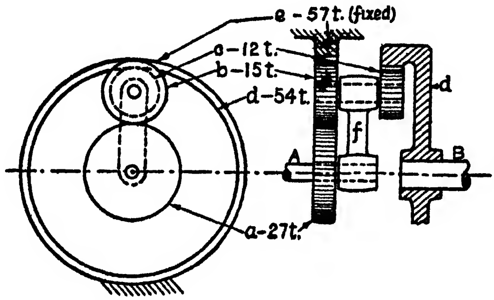


FIG. 9–12

the gears and rotate the whole mechanism one turn in a negative sense, so as to return e to its initial position. The tabulation is as follows:

Step	Turns					
	a	b	c	d	e	f
1	$-\frac{57}{27}$	$+\frac{57}{15}$	$+\frac{57}{15}$	$+\frac{57}{15}\times\frac{12}{54}$	+1	0
2	−1	−1	−1	−1	−1	−1
Total	$-\frac{28}{9}$	–	–	$-\frac{7}{45}$	0	−1

Thus while a makes $-\frac{28}{9}$ turns, d makes $-\frac{7}{45}$ turns.

The speed ratio $\frac{\omega_d}{\omega_a} = (-\frac{7}{45}) \div (-\frac{28}{9}) = +\frac{1}{20}$. Hence while a makes twenty turns d makes one turn, in the same sense.

Obviously this method can be applied to solve any of the problems on epicyclic trains in this chapter.

10. Speed Reducers. — Where a reduction in the rate of revolution must be made between the prime mover and the driven machine, as for example where an electric motor is used to drive a slow-speed machine, a speed reducer of the geared type shown in Fig. 9–13 is often employed as a substitute for belts, chains, or exposed gears.

The casing gives rigid support for the gears, protects them from dirt, and permits them to be operated in an oil bath. The drive

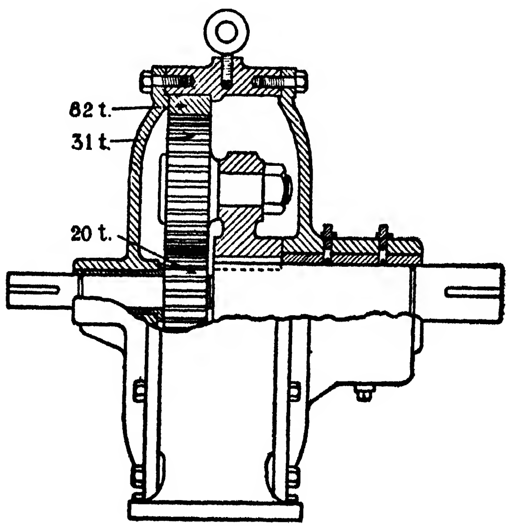


FIG. 9–13
Speed Reducer with Epicyclic Gear Train, Speed Ratio 1 : 5.1.

is compact and efficient and requires little attention. The gear train may be composed of spur, helical, or worm gears arranged in several different ways, the design depending on the service and the ratio of reduction required. The speed reducer illustrated contains an epicyclic train, an annular gear being the fixed member.

QUESTIONS — CHAPTER IX

1. Define the following terms as applied to gear trains: (*a*) train value, (*b*) simple gear train, (*c*) compound gear train, (*d*) reverted gear train.

2. A gear train consists of an internal or annular spur gear *a* and a gear *b* meshing with it. The two wheels revolve about fixed centers 6 in. apart. Wheel *a* turns at 30 R.P.M. and *b* at 120 R.P.M. Find (*a*) the pitch diameters of both gears, (*b*) the number of teeth on each if the diametral pitch is 4.

Ans. (*a*) 16 in., 4 in. (*b*) 64, 16.

3. Find the train value in the following train of gears:

15 teeth,
30 teeth,
20 teeth,
45 teeth.

4. Find the train value in the following train:

30 teeth,
75 teeth — 10-in. pitch diameter,
12-in. pitch diameter — 50 teeth,
60 teeth. *Ans.* $\frac{5}{18}$.

5. Find the train value in the following gear train:

10-tooth spur gear,
30-tooth spur gear — 40-tooth spur gear,
120-tooth spur gear — single-threaded worm,
40-tooth worm wheel.

Ans. $\frac{1}{360}$.

6. A compound reverted spur-gear train with four wheels, *a*, *b*, *c*, *d*, has a train value of 1 : 4½, all gear teeth having the same pitch. *a* has 40 teeth, *b* has 60 teeth. Find the number of teeth in *c* and *d*. If the diametral pitch is 3, find the distance between shaft centers.

Ans. $N_c = 25$, $N_d = 75$. 16.667 in.

7. The back gear train of a lathe is made up as follows: Wheel *a* on the cone pulley with 16 teeth drives *b* with 72 teeth. Wheel *c* is rigidly connected to *b* and drives *d*, connected to the live spindle of the machine. The train value is 1 : 13½. Find the numbers of teeth on *c* and *d*.

8. When an auto is in intermediate gear, the gear train in use is composed of the following gears, the first one being directly connected to the engine:

24 teeth,
41 teeth — 34 teeth,
31 teeth — 11-tooth bevel pinion,
56-tooth bevel gear.

(*a*) Find the ratio of the rear-axle speed to the engine speed.

(*b*) If the tires are 30 in. in diameter, find the engine speed corresponding to a car speed of 20 miles per hour. *Ans.* (*a*) 1 : 7.93. (*b*) 1775 R.P.M.

9. In the automobile transmission shown in Fig. 9–5, calculate the four possible speed ratios of shaft *B* to shaft *A* when the numbers of teeth on the gears are as follows:

$$C - 22, F - 39, G - 32, D - 29, E - 38, H - 23, L - 18.$$

10. What is an epicyclic gear train? How may an ordinary gear train be converted into an epicyclic?

11. An epicyclic train consists of a simple train of three wheels *a*, *b*, and *c*, carried on a frame *e*. Wheel *a* is fixed and *e* turns clockwise at 75 R.P.M. Wheel *a* has 60 teeth, *b* — 20 teeth, *c* — 30 teeth. Find the speed and sense of the motion of *c*. *Ans.* 75 R.P.M. counter-clockwise.

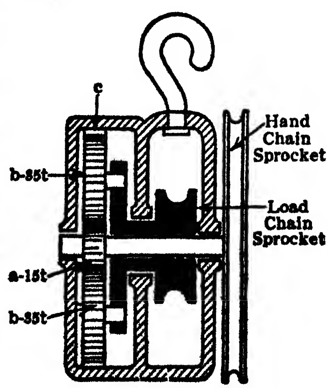
12. An epicyclic gear consists of a simple gear train of two wheels *a* and *b*, carried on a frame *e*. Wheel *a* is fixed, *b* turns counter-clockwise at 100 R.P.M. Wheel *a* has 40 teeth, *b* — 50 teeth. Find the speed and direction of motion of *e*. *Ans.* 55.56 R.P.M. counter-clockwise.

13. In a reverted epicyclic train the gears in order are *a*, *b*, *c*, *d*. Arm *e* pivots about the center of *a* and carries the wheels *b* and *c*, keyed together at its outer end. Wheel *a* has 30 teeth, *b* — 50, *c* — 20, *d* — 60. The arm *e* is driven at 150 R.P.M. Find the speed of *d* when *a* is fixed.

Ans. 120 R.P.M.

14. An epicyclic train is composed of a fixed annular wheel *a*, with 150 teeth. Meshing with *a* is a wheel *b* which drives wheel *d* through an idler *c*, *d* being concentric with *a*. Wheels *b* and *c* are carried on an arm which revolves clockwise at 100 R.P.M. Wheel *b* has 25 teeth, *c* — 30 teeth, *d* — 40 teeth. Find the speed and sense of rotation of *d*.

15. An epicyclic train contains a fixed annular wheel *a*, with 200 teeth. Meshing with *a* is wheel *b* with 90 teeth, which drives *c* with 20 teeth. Wheel *c* is concentric with *a*. Wheel *b* is carried on an arm *e* which revolves about the axis of *a*. Wheel *c* is the driver, and it rotates at 100 R.P.M. clockwise. Find the speed of arm *e*.



16. The epicyclic train used in a hoisting block is shown diagrammatically in the figure. The hand chain sprocket is 12 in. in diameter and the load chain sprocket is 6 in. in diameter. Gear *a* is keyed to the hand-chain sprocket shaft. Gears *b*, *b* are pivoted on arms attached to the load chain sprocket. Annular gear *c* is stationary. The numbers of teeth are as indicated in the figure. Find (*a*) ratio of the chain speeds, (*b*) the load which can be lifted by a pull of 100 lb. on the hand chain (friction neglected). *Ans.* (*a*) 1 : 13⅓. (*b*) 1333 lb.

17. Make the calculation for the speed ratio of the epicyclic train shown in Fig. 9–13.

18. Find the speed ratio obtained by use of the gear train shown in Fig. P-18.

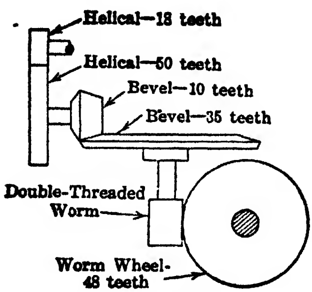


Fig. P-18

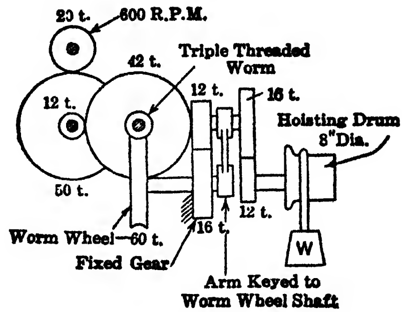


Fig. P-19

19. Determine the rate at which the weight *W* is raised by use of the gear train shown in Fig. P-19.

20. In the epicyclic gear train shown in Fig. P-20, shaft *A* is rotated at 350 R.P.M. in a clockwise sense. Gear *b* has 20 teeth, *c* has 28 teeth, and *d*, 35 teeth. Gears *b* and *c* are attached to arm *e*. Find the number of teeth on the fixed gear *a* and the speed and sense of rotation of shaft *B*.

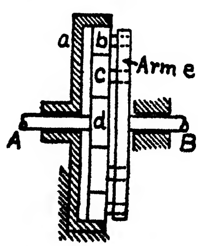


Fig. P-20

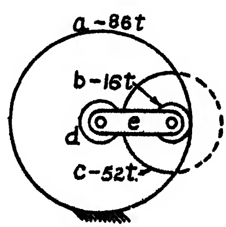


Fig. P-21

21. In the epicyclic train shown in Fig. P-21, gears *b* and *c* are keyed to a shaft carried on revolving arm *e*. Arm *e* turns about the axis of gears *a* and *d*. (*a*) If all the gear teeth have the same pitch, find the number of teeth on *d*. (*b*) If *d* rotates at 600 R.P.M. clockwise, find the angular velocity and sense of rotation of arm *e*. *Ans.* 18 *t*, 36.3 R.P.M. clockwise.

22. In Fig. P-22, *a* is a fixed internal gear. Shafts *A* and *B* are coaxial. Arm *f* is keyed to *A* and carries at its outer end gears *b*, *c*, *d*. Gear *b* has 40 teeth; *c*, 18; *d*, 20; *e*, 50. (*a*) If all gears have the same pitch, find the number of teeth on *a*. (*b*) If shaft *A* rotates at 200 R.P.M. clockwise, find the speed and sense of rotation of shaft *B*.

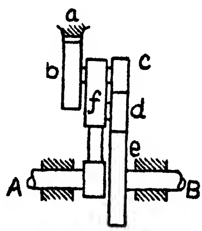


FIG. P-22

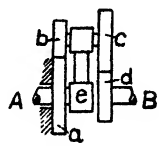


FIG. P-23

23. In the epicyclic shown in Fig. P-23 all gears have the same pitch. Gear *a* has 80 teeth, and *b*, 40 teeth. Shaft *A* with arm *e* rotates at 100 R.P.M. clockwise, and shaft *B* rotates at 180 R.P.M. counter-clockwise. Determine the number of teeth on gears *c* and *d*.

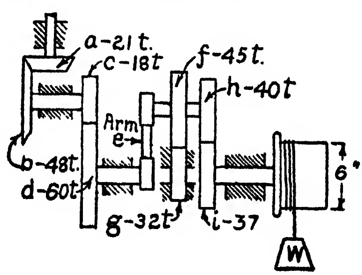


FIG. P-24

24. Gear *a*, Fig. P-24, is rotated at a speed of 200 R.P.M. Find the speed at which weight *W* is raised.

CHAPTER X

FLEXIBLE CONNECTORS

1. Belts, Ropes, and **Chains** are the important members of the class of links which we term non-rigid or flexible, since their form changes while in motion. They are adapted for transmitting a pull, but are incapable of carrying a thrust.

Ropes and belts do not give a positive drive, because their ability to transmit power depends on friction between the band and pulleys. Thus they must be given an initial tension, which causes a pressure on the bearings much higher than in equivalent chain or gear drives.

BELT DRIVES

2. Belts are the most suitable for cases where high speeds are necessary. They run with greatest economy at 3000 to 5000 feet per minute, hence they are often used in connection with high-speed machinery. Their operation is affected by exposure to the weather and by the presence of water, oil, grease, etc., which alter the frictional forces at the contact surfaces.

The drive not being positive, a definite phase relationship cannot be maintained between driver and driven unit. This precludes the use of belts for many purposes, as, for example, for the operation of cam shafts and timing devices in internal-combustion motors, for driving the lead screw in lathes when thread cutting, and for connecting moving sections of many automatic machines.

3. Speed Ratio in Belt Drives. — We shall assume that no slippage takes place and that the material is inextensible. Neither of these assumptions is correct in practice and therefore our calculated theoretical speed must be modified to take care of these factors.

In Fig. 10–1 we have a belt drive in which r_a, r_b are the radii of the driving and driven pulleys, respectively, and t is the belt thickness. Evidently, when the belt is bent round a pulley the outside fibers will stretch and the inside fibers contract. There will be a neutral surface about the middle of the belt at which neither expansion nor contraction will take place. The speed ratio of the drive will, therefore, be equal to that of a pair of pulleys of radius $r_a + t/2$ and $r_b + t/2$ connected by a belt of infinitesimal thickness.

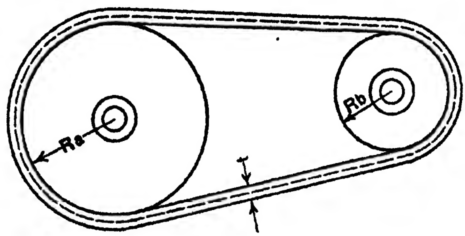


FIG. 10–1
Open Belt Drive.

Let V = belt speed = linear velocity at the neutral surface. Then,

$$\omega_a = \frac{V}{r_a + \frac{t}{2}}, \qquad \omega_b = \frac{V}{r_b + \frac{t}{2}}.$$

Therefore,

$$\frac{\omega_b}{\omega_a} = \frac{r_a + \frac{t}{2}}{r_b + \frac{t}{2}}. \tag{10–1}$$

Usually t is small in comparison with r_a and r_b. Therefore we may write as a close approximation,

$$\frac{\omega_b}{\omega_a} = \frac{r_a}{r_b}. \tag{10–2}$$

When the speed of a driven shaft is calculated from that of the driver by one of the above equations, the result is known as the **theoretical speed.** The **actual speed** of the driven shaft will always be somewhat less, the difference being due to "slip," which in practice amounts to 2 to 4 per cent of the theoretical speed. Thus

$$\text{Actual R.P.M.} = \text{Theoretical R.P.M.} \left(1 - \frac{\text{Per cent slip}}{100}\right).$$

A belt of given size will transmit maximum power if run at a linear speed of 4000 to 5000 ft. per min. It is therefore desirable to design the drive so as to keep within this range, unless the R.P.M. of the driver is low, in which event a lower belt speed may be necessary in order to avoid the use of large and expensive pulleys.

Exact speeds may be unobtainable if stock pulleys are to be used because such pulleys are made only in certain sizes, usually of even-inch diameters. A close approximation is secured by proper selection.

Example. — A line shaft is to be driven at 500 R.P.M. from an electric motor turning at 1750 R.P.M. The belt speed should be about 4000 ft. per min. Provide for a slip of 4 per cent. Find suitable diameters of stock pulleys, which are obtainable in even-inch diameters.

Solution.

$$\text{Belt speed} = \pi D_a N_a,$$

where D_a = driver diameter, and N_a = R.P.M.

$$4000 \times 12 = \pi D_a \times 1750,$$

or

$$D_a = \frac{4000 \times 12}{\pi \times 1750} = 8.75 \text{ in.}$$

Also,

$$\text{Theoretical R.P.M.} = \frac{\text{Actual R.P.M.}}{1 - \text{Slip}} = \frac{500}{1 - 0.04} = 521.$$

Hence, by equation (10–2),

$$\frac{D_a}{D_b} = \frac{521}{1750}, \quad \text{or} \quad D_b = D_a \times 3.36.$$

If	D_a =	7	8	9	10 in.
then	D_b =	23.5	26.8	30.2	33.6 in.
		(23)	(27)	(30)	(34)

Final choice might be influenced by relative cost of pulleys, but, considering desired belt and driven pulley speeds, the values $D_a = 8$ in. and $D_b = 27$ in. should be selected as giving closest approximation to requirements.

4. Crossed and Open Belt Drives. — The drive of Fig. 10–1, in which both pulleys turn in the same sense, is known as an **open belt drive.** In Fig. 10–2 is shown the **crossed belt drive**; here the pulleys turn in opposite senses.

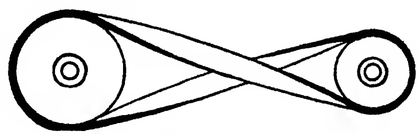


Fig. 10–2
Crossed Belt Drive.

Both forms give satisfactory service, though the crossed drive tends to wear out sooner on account of the rubbing action where the belt crosses itself. This is more pronounced when the belt is wide and the drive a short one. The larger arcs of contact between belt and pulleys in the crossed drive are advantageous.

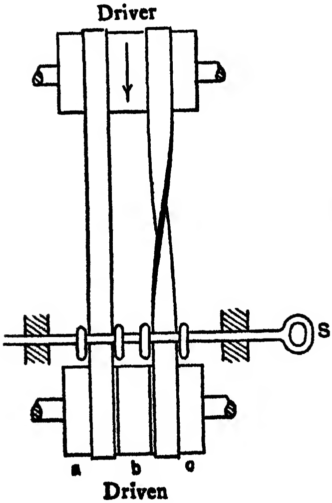


Fig. 10–3

A combination consisting of one open and one crossed belt in conjunction with loose pulleys (see Fig. 10–3) is used as a means of driving a machine in either direction, or of stopping its motion. The belts are moved sidewise by means of a belt shifter, S. The belts run on loose pulleys a and c when the shifter is in its mid-position; then the driven shaft is stationary. Moving the shifter to the right throws the open belt on the fast pulley b, whereas moving it to the left brings the crossed belt on the pulley b. The motion of the driven shaft may, therefore, be clockwise or counter-clockwise, depending on the position of the shifter. The shifter should act on the slack side of the belt, near the driven pulley. This drive is quite common in connection with machine tools.

5. Crowned and Flat Pulleys. — A Crowned Pulley has a larger diameter at the middle of the face than at the ends, whereas a **Flat Pulley** has the same diameter throughout. When flat pulleys are used, a very slight misalignment of the shafts or a small defect in the form of the belt will cause the belt to run off the pulleys. This can be avoided by means of a belt guide which has arms to hold the belt in the required position, by flanging the pulleys, or by using crowned pulleys. Guides and flanges cause wear on the edges of the belt; hence crowned pulleys are preferable.

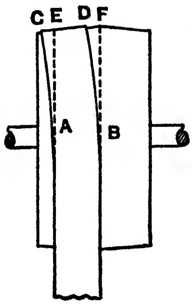


Fig. 10–4

The reason why the crown pulley causes the belt to run centrally may be seen by reference to Fig. 10–4. Here the belt is shown in a position at one side of the pulley. The belt tends to stretch more on the side *BD* than on the side *AC*; it therefore assumes a curved form. As the pulley rotates, section *AB* will move along the paths indicated by the broken lines *AE* and *BF*. The belt therefore runs toward the central plane of the pulley and takes up a position where its center line coincides with the ridge of the crowned surface.

The taper on a crown pulley varies from about $\frac{3}{4}$ in. (on the diameter) per foot width of face on narrow pulleys to $\frac{1}{4}$ in. per foot of width on wide ones. The taper may be **uniform** as shown in

Fig. 10–5

Fig. 10–6

the rim section of Fig. 10–5, or the profile may be a circular arc as in Fig. 10–6.

The face of the pulley is usually a little wider than the belt in order to prevent overhang if the belt is not running exactly central.

6. Non-parallel Shaft Drives. Quarter-twist Drives. — Belts are most commonly employed for connecting parallel shafts, but they may be used to connect non-parallel shafts as well. In any belt drive the following conditions must be observed in regard to the location of the pulleys in order to keep the belt from running off.

The belt must leave each pulley in the central plane of the pulley face toward which it moves.

A drive connecting two shafts at 90° is known as a **Quarter-twist** belt drive. One form is shown in Fig. 10–7. The pulleys are located to conform to the law just stated. Note the plan view of the drive, in which the pulleys intersect at *A*, the mid-point of both faces.

Evidently the belt can be run only in the direction indicated by the arrows. The point *X*, where the belt leaves the upper pulley; is in the plane *XY* passing through the middle of the lower pulley, also, point *R*, where the belt leaves the lower pulley, is in the plane *RS* passing through the middle of the upper pulley. This form of drive can also be used for connecting shafts at any angle between 0° and 180°. When the angle is 0° we have an ordinary open belt drive; at the other extreme it becomes a crossed belt drive.

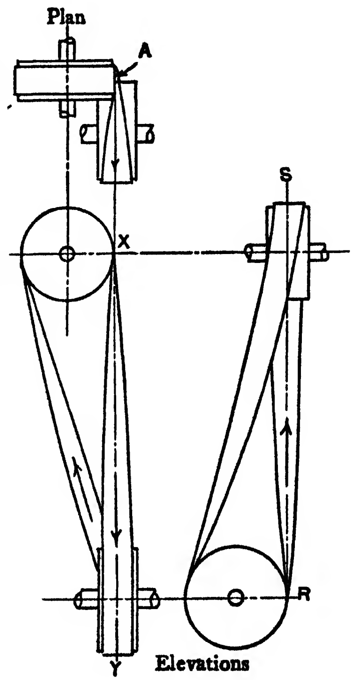


Fig. 10–7
Quarter-twist Belt Drive.

An alternative variety of quarter-twist drive is shown in Fig. 10–8. This has a guide pulley, or **Mule Pulley**, on the slack side of the drive. The mule pulley incidentally increases the arc of contact of the driving pulley; its main object is to bring the belt leaving this pulley into the plane passing through the middle of the driven pulley face. In this drive, a uniform tension is obtained across the belt section on the tight side, since the center of the belt is always in the plane passing through the middle of the driven pulley. This is not true of the drive of Fig. 10–7. By tilting the forward end of the mule-pulley shaft upward to the proper angle, the tension can be made uniform across the section on the slack side also. This drive is suitable for rotation in one direction only, because reversal would put the guide pulley on the tight side of the belt.

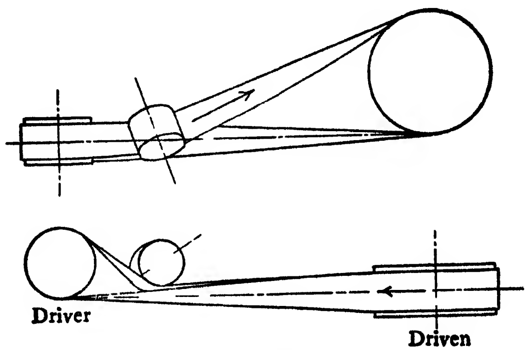


FIG. 10–8

Quarter-twist belts should be avoided wherever possible, as the wear on such belts is excessive. They give fairly good service with narrow belts on long drives.

7. Calculation of Belt Length. — Knowing the center-to-center distance L and the pulley radii R_a and R_b, we can calculate the

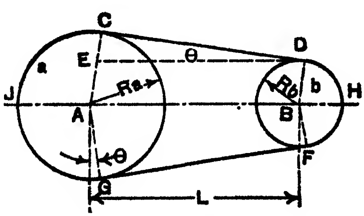


FIG. 10–9

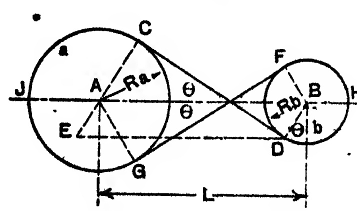


FIG. 10–10

theoretical belt length. (See Figs. 10–9 and 10–10.) In both figures we draw DE parallel with AB, meeting CA at E. The total length of belt = $FG + DC +$ arc $FHD +$ arc CJG.

Now,

$$FG + DC = 2\,(DC) = 2\sqrt{(DE)^2 - (CE)^2} \quad (\text{Angle } DCE = 90°.)$$
$$= 2\sqrt{L^2 - (R_a \pm R_b)^2}. \qquad (10\text{–}3)$$

The positive sign is used for crossed belts, and the negative for open belts.

Let θ be the angle made by the straight portion of the belt with the line of centers.

(*a*) **Open Belts.** — From Fig. 10–9, where the angles are expressed in radians,

$$\text{arc } CJG = (\pi + 2\,\theta)R_a \qquad \text{and} \qquad \text{arc } FHD = (\pi - 2\,\theta)R_b.$$

The total length of the open belt

$$FG + DC + \text{arc } CJG + \text{arc } FHD$$

is therefore equal to

$$2\sqrt{L^2 - (R_a - R_b)^2} + \pi(R_a + R_b) + 2\,\theta(R_a - R_b)$$
$$= 2\sqrt{L^2 - (R_a - R_b)^2} + \pi(R_a + R_b)$$
$$+ 2\,(R_a - R_b)\sin^{-1}\left(\frac{R_a - R_b}{L}\right). \qquad (10\text{–}4)$$

This formula is somewhat unwieldy, and for ordinary use a simpler approximate form, known as **Rankine's Equation,** is satisfactory. This is derived from equation (10–4) as follows:

(1) The first term can be put in the form $2\,L(1 - x)^{\frac{1}{2}}$ where $x = \left(\frac{R_a - R_b}{L}\right)^2$. This expression is expanded, and terms containing x^2 and higher powers of x are neglected. This is permissible since x is a small fraction.

(2) The angle θ is small, and little error results in writing $\theta = (R_a - R_b)/L$. Making these approximations and substituting pulley diameters for radii, we have

$$\text{Belt length} = 2\,L + \frac{\pi}{2}(D_a + D_b) + \frac{(D_a - D_b)^2}{4\,L}, \qquad (10\text{–}5)$$

where D_a and D_b are the pulley diameters.

According to Carl G. Barth (*American Machinist,* March 12, 1903) this formula has a maximum error of about 2 per cent. It is exact when the pulleys are the same size, and the error increases as θ increases.

(*b*) **Crossed Belts.** — It can be shown by Fig. 10–10 that the exact length of these belts is given by the expression

$$2\sqrt{L^2 - (R_a + R_b)^2} + \pi(R_a + R_b) + 2\,\theta(R_a + R_b)$$
$$= 2\sqrt{L^2 - (R_a + R_b)^2} + (R_a + R_b)\left[\pi + 2\sin^{-1}\left(\frac{R_a + R_b}{L}\right)\right]. \quad (10\text{–}6)$$

The value of this expression is constant as long as $(R_a + R_b)$ and L are constant. We reach the important conclusion that the length of a crossed belt is constant when the center-to-center distance and sum of the radii of the pulleys are constant.

8. Stepped-cone Pulleys. — A pair of pulleys, as shown in Fig. 10–11, is frequently used where a variable speed is required on the driven shaft, a different speed ratio being obtained with each pair of steps. These pulleys must be so designed that the same length of belt is required for any pair of steps.

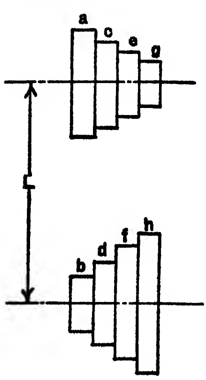


Fig. 10–11 Cone Pulleys.

When using a crossed belt this condition is satisfied by having pulley radii such that

$$(R_a + R_b) = (R_c + R_d) = (R_e + R_f), \text{ etc.}$$

(See Fig. 10–11.)

For an open belt the determination of the pulley sizes is more difficult, and to avoid tedious calculation several graphical methods have been devised. One of the most satisfactory of these is that due to Burmester, which is illustrated in Fig. 10–12.

9. Burmester's Construction. — Cone pulleys for open belts must be designed to suit a definite center-to-center distance of shafts, whereas for crossed belts the same pulleys can be used for any center-to-center distance, since then the only requirement is that the sum of the radii or diameters of each pair of steps shall be the same.

The speed of the driving shaft, the desired speeds of the driven shaft, and the center-to-center distance usually form part of the specifications of the drive. Knowing these quantities, we can find the radii of one pair of pulleys by consideration of the desired belt speed or the allowable diameter of the largest step. We shall suppose, therefore, that we are designing a pair of pulleys similar to that shown in Fig. 10–11 for an open belt, the distance L from center to center of shafts and the radii R_a and R_b for one pair of steps being known. The quantities to be determined are the radii of the other steps. The Burmester method of finding these graphically is as shown in Fig. 10–12. The construction of this diagram is as follows:

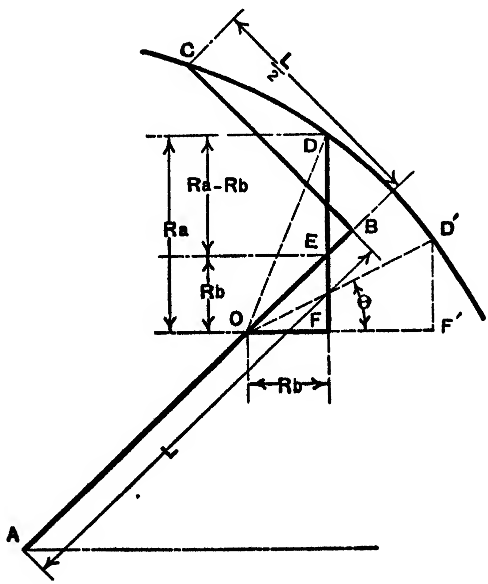


FIG. 10–12
Burmester Construction.

Draw a line AB of length equal to L, making an angle of 45° with the horizontal. From B draw BC, equal to $\frac{1}{2}L$, perpendicular to AB. With center A and radius AC, draw an arc CDD'.

Calculate the length $R_a - R_b$, and by trial find the position of a point D on the arc CDD' such that the vertical distance DE to AB is equal to $R_a - R_b$.

Produce DE to F, making EF equal to R_b.

From F draw a horizontal line meeting AB at O.

Then $FO = R_b$ (since the angle $EOF = 45°$). Also

$$DF = R_a.$$

It will be noted that

$$\tan DOF = \frac{R_a}{R_b} = \frac{\omega_b}{\omega_a}.$$

If we then take any point, such as D', on the arc and draw from it a vertical line to meet OF produced at F', from the essential property of the diagram, the distances $D'F'$ and OF' are the radii of a pair of steps which may be used for the cone pulley, giving a speed ratio equal to $OF'/D'F'$.

Thus, to obtain the radii R_c and R_d, for example, make θ an angle whose tangent is ω_d/ω_c. The distances $D'F'$ and OF' will then be equal to R_c and R_d, respectively. Other radii are found in the same manner.

Though not exact, this construction will generally give results sufficiently accurate for practical purposes.

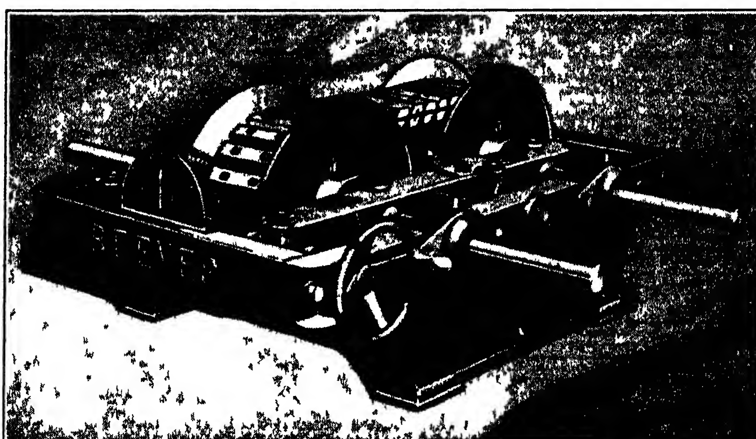


Fig. 10–13
Variable-speed Belt Drive.

10. Reeves Drive. — (See Fig. 10–13.) This device makes it possible to vary at will the speed ratio of the driving and driven shafts. These shafts are parallel, and each carries a pair of cone-shaped discs mounted with the apices facing one another. Each pair forms a pulley over which runs a V-shaped belt. The four discs are keyed to the shafts on which they are mounted but are free to slide axially. They are held in position by two levers interconnected in such a way that when one pair of discs is brought closer together the other pair is moved farther apart. This movement increases the diameter of the circle at which the belt runs on one pair, and decreases the diameter on the other pair, hence causing a change in the speed ratio of the shafts. The position of the levers and discs is controlled by a hand wheel mounted on a threaded shaft.

The Reeves Drive has been used in connection with paper-making and textile machinery, conveyors, etc., where variable speeds are required.

ROPE DRIVES

11. Ropes are run on grooved pulleys. The groove acts as a guide for the ropes, and the wedging action in the groove makes it possible to transmit power with less initial tension than would be required with flat pulleys. Drives with several turns of rope running side by side on the same pulleys are quite common. In the English system each loop is a separate piece of rope; in the American system the rope is continuous. The American system possesses the advantages of more uniform distribution of tension over the different turns and permits of the use of one tightener for the drive. On the other hand, in the English system, breakage of one turn does not put the whole drive out of commission, but much splicing of ends is necessary and no convenient way of properly taking up the stretch is possible.

Rope drives are practically always used in preference to other mechanical drives for long-distance transmission of power, on the ground of both cost and suitability. However, the general adoption of electric power, accompanied by the practice of driving each machine or group of machines by a separate motor, has rendered the rope drive almost obsolete.

The calculation of the speed ratio for a rope drive differs in no way from that of a belt drive. Kinematically, the two drives are identical.

V-BELTS

12. Belts of trapezoidal section, known as V-belts, have become a popular form of mechanical drive during the past few years, and for some purposes they have entirely replaced the older type of flat belt.

The V-belt is almost always of the endless type constructed of canvas, cotton cords, and rubber, the whole being molded and vulcanized together. Figure 10–14 shows a typical section. The sides of the belt are slightly concave and the included angle is usually 42°. The belts run in grooved pulleys, the angle of the groove being about 36° in the smaller-diameter pulleys and 38° to 40° in larger pulleys. Pulley-groove angles are made less than belt angles, which become smaller when the belt is bent around the pulley.

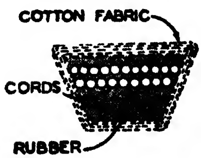
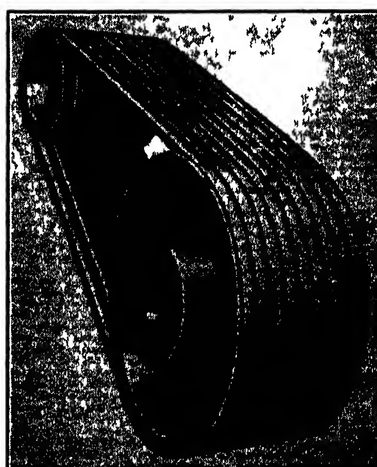


FIG 10–14

V-belts possess two valuable characteristics. First, they may be operated very satisfactorily with a short center-to-center distance. This distance need not be greater than the diameter of the larger pulley. Second, they require little adjustment to compensate for wear or stretch, since, owing to the wedging action in the groove, this style of belt will transmit a considerable amount of power without excessive slip even when the initial tension is practically nil.



(Dodge Mfg. Co.)

FIG. 10–15. V-Belt Drive.

It is usual for a V-belt drive to consist of several belts run in parallel grooves (see Fig. 10–15) where the power to be transmitted exceeds the capacity of a single belt. The belts should then be "matched," particularly as to length; otherwise the load will not be equally distributed among the belts and uneven wear will result. For similar reasons it has not been found satisfactory to attempt to run old and new belts in parallel.

CHAIN DRIVES

13. Chains are made of a series of jointed metal links in a variety of forms. They may be classified in accordance with their uses as (*a*) hoisting and hauling chains, (*b*) elevator and conveyor chains, and (*c*) power-transmission chains.

The **Coil Chain** (Fig. 10–16) is the usual form of hoisting chain. It is made of iron of circular section bent to the proper form and welded at the joint. The **Sprocket Wheels** for this chain are shown in Figs. 10–17 and 10–18. The plain **Grooved Sheave** is suitable only as a guide and not as a means of transmitting energy to or from the chain. The **Pocket Sheave** (Fig. 10–18) has a central groove and depressions which conform to the profile of the links, a liberal clearance being allowed at the end of these depressions to provide for stretching of the chain or irregularities in the pitch.

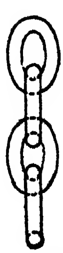


Fig. 10–16

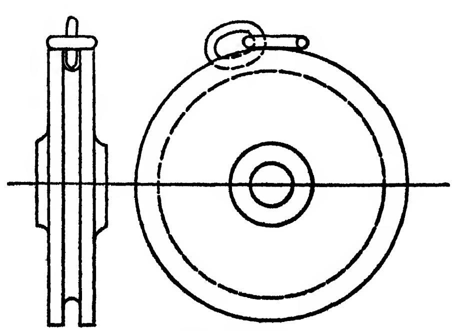


Fig. 10–17

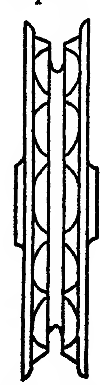


Fig. 10–18

Conveyor Chains. — The **Ewart Chain** (Fig. 10–19), which has easily detachable links, and the **Pintle Chain** (Fig. 10–20) are two of a large class of chains employed in mills, mines, and factories for elevators and conveyors handling a variety of materials. For such purposes, buckets, flights, etc., are connected to the chains by various kinds of attachment links. These chains are generally of malleable iron and are run on cast sprockets. This construction is suitable for rough service and slow speeds. These chains are operated at about 100 ft. per min.

Power-transmission Chains. — These are generally of stronger materials and more accurately made than the classes of chains de-

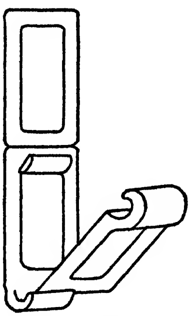


FIG. 10–19
Ewart Chain.

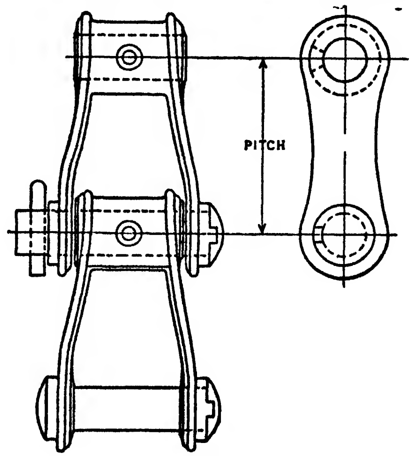


FIG. 10–20
Pintle Chain.

scribed above. Wearing surfaces are of steel, hardened and ground, and the chains are run on sprockets with cut teeth. Consequently, they are more costly but may be operated at higher speeds.

The **Roller Chain,** illustrated in Fig. 10–21, is used mainly as a power-transmission chain. The construction at the joints is such as to obtain as large bearing surfaces as possible. The pin connecting two sections is either riveted to the outer pair of links or fastened in such a way that it cannot turn relatively to them. The bushing through which this pin passes is riveted to the inner pair of links. When the chain bends in passing on or off a sprocket, the pin and bushing

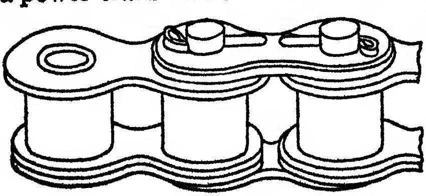


FIG. 10–21
Roller Chain.

form a turning pair and their contact surfaces slide on one another. Here the area is large, extending nearly the full width of the chain. The area of contact of pin and links is small, and if they had relative motion rapid wear would result. Outside the bushing is a hardened-steel roller which makes contact with the sprocket teeth.

Silent Chains (Fig. 10–26) are used entirely for power transmission. These will be considered later.

14. Sprocket Profiles. — Two practical difficulties are encountered in designing sprockets for chain drives. The first is that the chain pitch varies somewhat because of inaccuracies in manufacture. This applies more particularly to the rougher forms of chain, though it is true to a minor extent in the high-grade power-transmission chains. The second is that wear at the joints causes the chain to elongate, an old chain having a greater pitch than it had when new. Wear does not increase the sprocket pitch; consequently, if the pitches of chain and sprocket were originally the same, after service they will differ somewhat. For both the reasons just given, the design must be such as to permit of satisfactory operation when the pitches are not equal.

Let us first consider the form of sprocket tooth for roller chains which we might use if no pitch difference existed. The profile of such a tooth is shown by the solid lines in Fig. 10–22. The portion AB of the space between teeth is a circular arc of radius equal to that of the pin or roller which fits into it. The portion BC, extending to the addendum of the tooth, is a circular arc with center at the axis D of the adjacent roller. This form causes the roller to maintain contact with the tooth when leaving the sprocket. Such a tooth would be satisfactory only so long as the chain pitch remained equal to that of the sprocket. Elongation of the chain would cause the roller to strike hard on the tooth side.

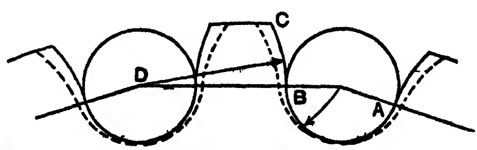


Fig. 10–22

Modification of the theoretical tooth form to allow for a difference in pitch of chain and sprocket is made in two ways. One of these, now used only on the slow-speed (conveyor) type of chain, is illustrated by the broken lines of Fig. 10–22. The tooth is made narrower in order to obtain "pitch-line" clearance; also the addendum is rounded off to reduce the friction as the rollers move in and out of contact. Figures 10–23 and 10–24 illustrate in an exaggerated form the effect of these modifications on the chain action. Figure 10–23 shows the engagement of a new chain and sprocket, the chain pitch being somewhat shorter than that of the sprocket. It is evident that roller *A*, just about to leave the driving sprocket, is carrying all the load. Roller *B*, which

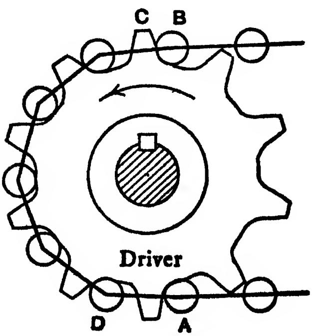


Fig. 10–23

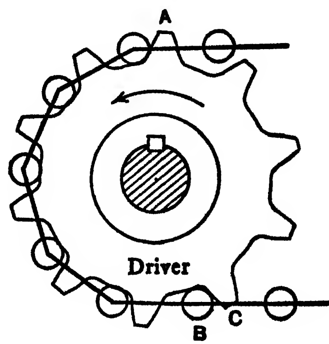


Fig. 10–24

has just come into engagement, barely clears the back of the tooth *C*. When *A* goes out of engagement it rolls up the side of the tooth and allows the chain to slip back slightly on the sprocket so that roller *D* will take the load. At a certain point in the life of the chain, when its pitch has increased to that of the sprocket, all the rollers will bear equally against the mating teeth if the wear is uniform. Later, when it has worn so that its pitch is greater than that of the sprocket, conditions will be as illustrated in Fig. 10–24. Here the roller *A*, which is the last to engage the sprocket, is carrying all the load which it picked up as it rolled down the side of the tooth. During this period the whole chain would slide ahead somewhat on the sprocket. Therefore, whether the pitch of the chain is larger or smaller than that of the sprocket, the load is carried by one tooth at a time. The chain slippage leads to noise and vibration at high chain speeds.

The above method of shaping the sprocket teeth to allow for chain elongation was formerly used for power-chain sprockets. It has been superseded by other forms which have been found superior for high-speed chains. The latter were developed by Hans Renold in England and the Diamond Chain Company in the United States. The sprocket tooth adopted as standard for roller chains by the American Society of Mechanical Engineers and other engineering bodies is essentially the same in its action, though the tooth shape is not the same as that of either the Renold or the Diamond sprocket.[1]

15. Standard A.S.M.E. Roller-chain Sprocket. — The standard tooth is illustrated in Fig. 10–25. The tooth profile is composed of three circular arcs *AB*, *BC*, *DE*, and a straight line *CD*.

Arc *AB* has a radius equal to that of the roller plus a small clearance. This arc terminates on a line *BO*, making an angle of $\left(35 + \frac{60}{T}\right)^\circ$ with *XY*. *T* equals the number of teeth on the sprocket. Line *BO* is produced to *P*, making $OP = 0.8 \times$ roller diameter. Line *PC* is next drawn by making the angle $BPC = \left(18 - \frac{56}{T}\right)^\circ$. Arc *BC* is drawn with *P* as center. Line *CM* is drawn perpendicular to *PC*. *OS*, the

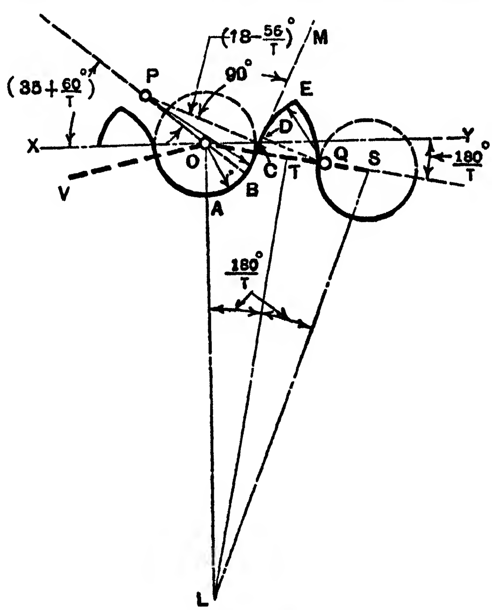


Fig. 10–25
Standard Tooth for Roller-chain Sprocket.

[1] For details of these teeth, see "Mechanics of Machinery," by R. C. Heck.

center line of the link, makes an angle of $(180/T)^\circ$ with XY. Point Q is located on this line by making OQ equal to $1.24 \times$ roller diameter. Taking Q as center, arc DE is drawn tangent to line CM.[3]

No pitch-line clearance is provided, because it is not required. Extension of the chain due to wear is taken care of automatically. When its pitch becomes larger it rides nearer the tooth points and thus travels in a pitch polygon with larger sides. This result is obtained by the use of a suitable pressure angle BOS. It will be observed that this is the angle between the normal at the point of contact and the pitch line of the chain link.

Considering the roller with center at O (Fig. 10–25), the pressure between the tooth and roller acts along OB and hence the pull on the chain along OS tends to move the roller outward until it is stopped by a pull along OV. The pull along OV in turn moves the next roller outward, and so the action continues around the sprocket until each tooth carries a share of the load. The load is not equally divided among the teeth, since the first tooth carries about half of the total; neither do the chain pins move in a perfect circle. Nevertheless, the chain runs more smoothly than with the older type of tooth having pitch-line clearance.

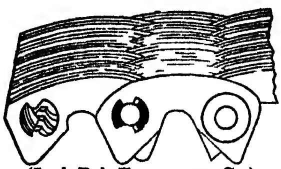
Roller chains are manufactured in double, triple, and quadruple widths to meet large power requirements.

16. Silent Chains. — This type, illustrated in Fig. 10–26, is used for power transmission. Its construction is such that it can very easily be made in any width to suit the load to be carried. Consequently it is suitable for transmission of large amounts of power. The characteristic feature is the hooked form of link, made of stampings from sheet steel.

This chain was invented by Renold in England. The Morse Silent Chain and that made by the Link-Belt Engineering Company are two well-known varieties manufactured in the United States. Both of these have constructions at the joint differing somewhat from the original Renold chain, the object being to improve the wearing qualities.

[3] For further details see "Machine Design," by Norman, Ault, and Zarobsky; see also American Gear Manufacturers Association data sheets.

The sprocket teeth are straight on the sides and make surface contact with the straight portions AB and CD (Fig. 10–27) of the links. The angle between AB and CD is from 55° to 60°. When the chain pitch increases through wear the chain rides nearer the points of the teeth, the action being the same as that of the roller chain with the standard sprocket. The comparative absence of noise can be attributed to three factors: (*a*) automatic adjustment of position on the sprockets to compensate for wear, (*b*) large



(Link-Belt Engineering Co.)

FIG. 10–26
Silent Chain.

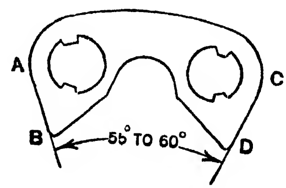


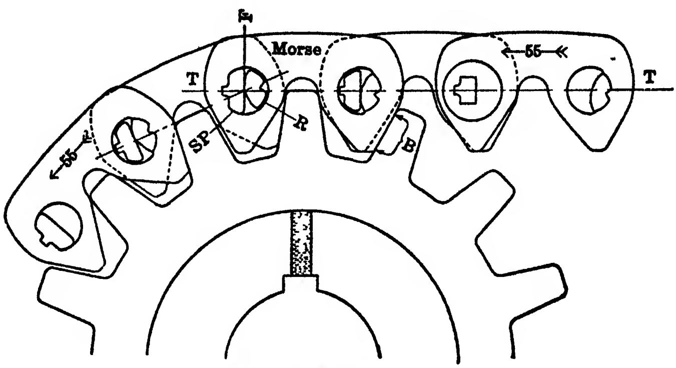
FIG. 10–27
Link for Silent Chain

surface contact with sprockets, and (*c*) sliding action during contact, due to the oblique direction of relative motion of the surfaces at the instant preceding contact.

Figures 10–26 and 10–27 show a joint construction consisting of a round pin fitted into a bushing made in two segments each of which is keyed to one row of links, with the result that sliding motion, due to bending of the chain, takes place between the pin and segments. This construction has two advantages over a plain pin-and-hole method of connecting the links: first, it practically doubles the bearing surface; and second, it improves the lubrication on account of the long unbroken bearing surface on each segment.

In the form of chain, extensively used, illustrated in Fig. 10–28, the pin itself is made in two segments each keyed to a row of links. The segments are so shaped as to approximate rolling contact with each other, the sliding action being very slight. This tends to eliminate friction losses and results in an improvement in durability and efficiency in the transmission of power.

17. Speed Variation in Chain Drives. — When a chain passes around a sprocket, it takes the form of a polygon whose sides have a length equal to the chain pitch. Suppose that the driving sprocket is rotating at uniform speed; the chain will then have a linear velocity which varies somewhat on account of the polygonal form in which it moves. This variation is small in sprockets with many teeth, and becomes of importance only in chain-driven



(Morse Chain Co.)

FIG. 10–28

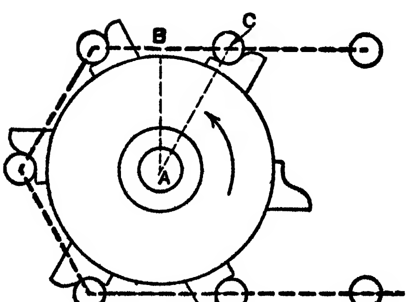


FIG. 10–29

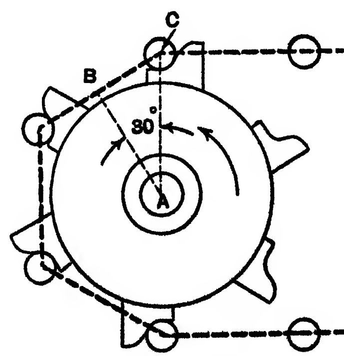


FIG. 10–30

elevators using six- or eight-tooth sprockets in combination with a long-pitch chain.

Figure 10–29 shows a six-tooth sprocket in a position where a roller at C has just come in contact with a sprocket tooth. The chain velocity is evidently equal to $\omega \times AB$, where ω is the angular velocity of the sprocket. After the sprocket has revolved 30° the condition is that of Fig. 10–30, and the chain velocity is $\omega \times AC$. The former expression evidently fixes the value of the minimum chain velocity; the latter gives the maximum chain velocity.

The ratio

$$\frac{\text{Maximum chain speed}}{\text{Minimum chain speed}} = \frac{AC}{AB} = \frac{1}{\cos 30°} = 1.155$$

for the six-tooth sprocket.

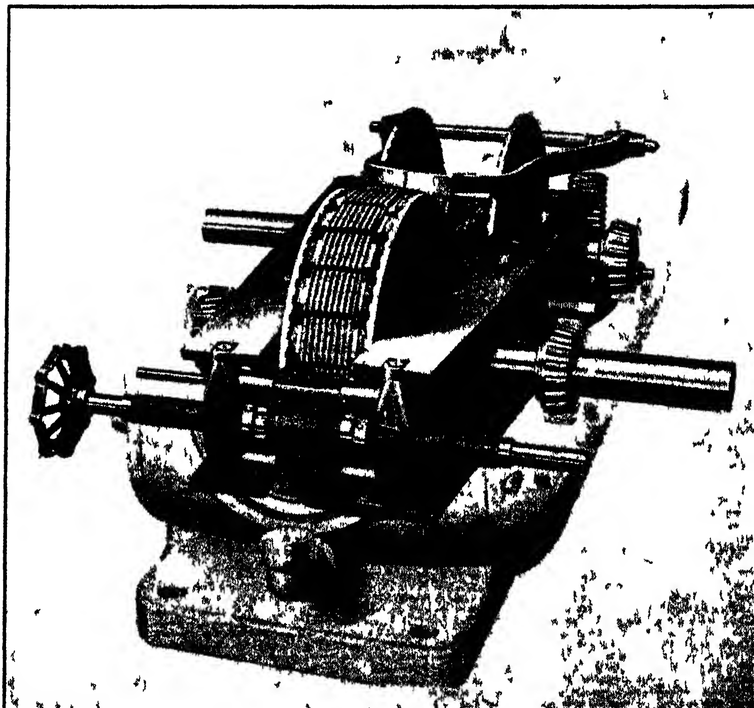
With an eight-tooth sprocket this ratio is 1.082; with a ten-tooth, 1.051; and diminishing values are obtained as the number of teeth is further increased.

The variation in chain speed may in a measure account for noise produced in some chain drives. In bucket elevators it has on occasion led to the installation of compensating devices in the drive, which cause the driving-sprocket speed to vary in order to obtain a constant speed of the chain. In this case, however, the object is not to reduce noise but to avoid irregular motion of the elevator. Such devices are never required when the sprocket has more than eight teeth.

When transmitting power from one shaft to another, it is obvious that if the driving sprocket is the same size as the driven sprocket and both have at any instant the same angular position of the teeth with reference to the tight side of the chain, then the speed variation due to the action of the chain on the driving sprocket will be neutralized by an opposite action taking place at the driven sprocket. Hence, under these conditions, the driving and driven shafts will have a constant velocity ratio.

Where the sprockets are not of the same size a partial compensation can still be made by so adjusting the center distance that the teeth are in the same phase. With respect to noise, rapidity of wear, and efficiency, a noticeable improvement can be made by such an adjustment.

18. Positive Infinitely Variable Transmission. — An interesting form of chain drive which has an unusual combination of properties, namely, positive drive and an infinite number of speed ratios, is shown in Fig. 10–31. The device consists essentially of two sprocket wheels and a chain of special type. The sprockets are composed of two sliding discs with fluted, conical working surfaces. The chain contains pockets filled with flat steel plates which are free to move in a direction at right angles to the length of the chain.



(Link-Belt Co.)

FIG. 10–31. Positive Variable-Speed Transmission.

When the chain engages the fluted sides of the sprocket, the plates are pushed over by the ridges on one side of the sprocket and enter the hollows on the opposite side of the sprocket. In effect the sprockets continuously form mating teeth on the chain as it makes contact, and thus a positive drive is secured. The sprocket sides are adjusted by a hand wheel and lever mounting in such a way that one pair move toward each other as the others move apart, the result being to change the pitch diameters at which the chain operates, and hence alter the speed ratio.

In the usual design, the speed of the driven shaft at maximum setting of the transmission is from two to six times its minimum speed.

QUESTIONS — CHAPTER X

1. Under what conditions are belts a suitable form of drive? In what cases is it impossible to use a belt drive?

2. What is the relationship between the speed ratio and the pulley diameters in a belt drive? How can the exact formula be modified when the belt thickness is small compared with the pulley radius?

3. Find the pulley diameters which must be used on motors turning at the following revolutions per minute if the belt speeds are to be 3200 ft. per min.: (*a*) 3600 R.P.M.; (*b*) 1750 R.P.M.; (*c*) 900 R.P.M.; (*d*) 600 R.P.M.

4. Split steel pulleys are available in stock sizes varying by inch increments. What diameters should be selected for a grinder turning at 4000 R.P.M. if the motor runs at 1750 R.P.M. and 4 per cent slip is estimated? The belt speed is to be approximately 3300 ft. per min.

5. A multiple V-belt drive using *C* section belt is employed to connect a Diesel engine running at 1600 R.P.M. to a line shaft running at 600 R.P.M. The manufacturer recommends a belt speed not higher than 5000 ft. per min. and pulley diameters not less than 9 in. Select stock pulleys (no fractional inch sizes), allowing for 3 per cent slip.

6. A belt drive is to connect a motor running at 900 R.P.M. to a line shaft which is to turn approximately at 500 R.P.M. The belt speed is not to exceed 3850 ft. per min. Standard pulleys are obtainable in even-inch diameters. Select pulley diameters (*a*) neglecting belt thickness and slip, (*b*) allowing for belt thickness of $\frac{3}{8}$ in., (*c*) allowing also for slippage of 5 per cent.

Ans. (*a*) 15 in., 27 in. (*b*) 16 in., 29 in. (*c*) 15 in., 26 in.

7. A machine which should run at 200 R.P.M. is to be belt-driven through one countershaft running about 600 R.P.M. by a motor running at 1800 R.P.M. The belt speed is not to be higher than 4000 ft. per min. The largest pulley in the drive must not be over 36 in. in diameter. Select suitable pulley of stock sizes (even-inch diameters) for the drive, allowing for a total slip of 8 per cent.

8. What are "crossed" and "open" belt drives? Show by sketch how a pair of belts can be used in conjunction with tight and loose pulleys to produce a reversible drive. Name a common use for an arrangement of this sort.

9. What three methods are used to prevent a belt from running off the pulleys? Explain why crowning the pulley tends to cause the belt to run centrally. What two forms are used for the pulley profile?

10. Show two methods of connecting shafts at 90° by means of a belt drive. Where a mule pulley is employed, on which side of the belt should it act?

11. What is the law, regarding the location of the pulleys in any belt drive, which must be observed in order to keep the belt from running off the pulleys?

12. Calculate the belt length in an open belt drive connecting two shafts 10 ft. apart, the pulleys being respectively 12 in. and 24 in. in diameter.

Ans. 24 ft. $8\frac{3}{4}$ in.

13. A V-belt drive has pulleys respectively 6 in. and 18 in. in diameter. The center-to-center distance should not be less than the diameter of the larger pulley. Find a satisfactory belt length for shortest drive. Stock belts are available in lengths varying by 2-in. increments.

14. Explain the Burmester construction for obtaining the diameters of the steps in a drive using an open belt on stepped-cone pulleys. Are the results exact?

15. Two shafts are to be connected by stepped-cone pulleys with three pairs of steps, for a crossed belt. The driving shaft turns at 400 R.P.M. The driven shaft is to turn at 200, 400, and 600 R.P.M. The largest step on the driving pulley is 18 in. in diameter. Find the diameters of the other steps.

Ans. Driving — 18, 15, 10 in. Driven — 12, 15, 20 in.

16. Two shafts are connected by stepped-cone pulleys with four pairs of steps for a crossed belt. The first pair of steps are, respectively, for the driver 24 in. and for the driven 12 in. The speed ratios of the driven to the driver for the other steps are, respectively, 3 : 2, 1 : 1, 1 : 3. Find the diameters of all steps.

17. A stepped-cone pulley for a lathe is to give spindle speeds of 150, 280, 540, and 1000 R.P.M. when driving from a line shaft rotating at 400 R.P.M. A crossed belt is used. The largest step on the driver is 11 in. in diameter. Find the diameters of all other steps on the pulleys to the nearest even $\frac{1}{16}$ in.

18. Take the data of Problem 17 with the exception that the cone pulleys are to be used for an open belt, the center-to-center distance being 34 in. Find the pulley diameters by the Burmester method.

19. Name two reasons for using grooved pulleys in rope drives. What are the American and English systems of rope driving? Point out the advantages and disadvantages of each.

20. Under what conditions are V-belt drives to be preferred to other forms of mechanical drives?

21. Sketch four common types of chains. State the kind of service for which each is best adapted.

22. What methods are employed in the design of sprocket teeth to take care of variation in chain pitch, in the case of conveyor and similar slow-speed chains?

23. What two forms of chains are especially adapted for power transmission? What features of construction make them suitable for operation at high speeds?

24. What form of tooth would be suitable for a roller chain if the pitch were perfectly uniform and the elongation due to wear could be neglected?

25. In designing sprocket teeth for slow-speed conveyor chains, by what means can we provide for a small difference of pitch between chain and sprocket?

26. How is the elongation of the chain due to wear taken care of in the standard type of sprocket for roller chains? What form of tooth is used in this sprocket?

27. Sketch the link used in the silent chain. What part of the link bears against the sprocket, and what is the shape of the contact surface?

28. When a silent chain bends, what surfaces form the turning pair? Why has the original form of silent chain been altered, as regards the construction at the joints?

29. To what factors can the quiet operation of the silent chain be attributed?

30. Explain why a variation in the speed ratio is obtained in a chain drive during each revolution when the sprockets are of unequal size? How is this variation affected by the number of teeth on the sprockets? When the sprockets are of equal size, how must they be arranged in order to obtain a constant speed ratio?

31. A silent chain with $\frac{5}{8}$-in. pitch is used to connect an engine turning at 1500 R.P.M. to a machine rotating at approximately 350 R.P.M. The chain speed is to be not more than 1800. The sprockets should not have less than 18 teeth nor more than 105 teeth. (*a*) Find suitable tooth numbers for both sprockets. (*b*) What length of chain is required for a center-to-center distance of about 28 in.?

32. A chain of $\frac{1}{2}$-in. pitch is driven from an 18-tooth sprocket turning at 2000 R.P.M. Calculate the maximum and minimum linear velocity of the chain.

33. In a chain drive the smaller sprocket has 12 teeth. By proper arrangement of the phase position of the larger sprocket it is possible to secure a 50 per cent compensation of the speed variation due to the smaller sprocket. Will the drive be satisfactory if the speed variation must be kept below 1 per cent?

CHAPTER XI

MISCELLANEOUS MECHANISMS

1. Ratchets. — A ratchet mechanism in its most common form consists of a device whereby two members capable of rotation are connected so that one will rotate the other in a certain sense, but not in the opposite sense.

Its use, however, is not limited to members having rotational motion. Often the driver has a reciprocating straight-line motion and compels linear motion of the driven member in one direction only.

The ratchet wheel *b* of Fig. 11–1 is turned in a clockwise sense by the action of pawl *c* when driver *a* turns clockwise. When *a* turns counter-clockwise, *b* remains stationary. A second pawl, *d*, pivoted on the frame of the mechanism, is sometimes used to prevent the ratchet wheel from running backward.

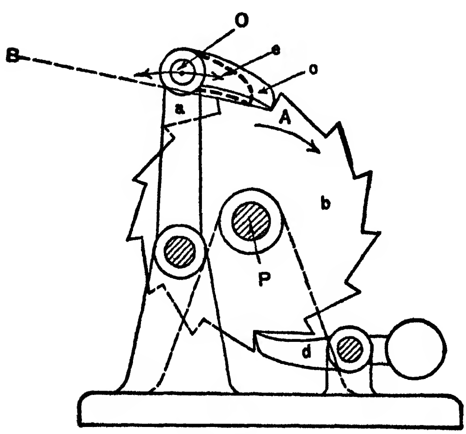


Fig. 11–1
Toothed Ratchet.

Two points should be noted in regard to the teeth on the ratchet wheel.

(*a*) The normal *AB* to the face of the tooth at the point of contact should pass between centers *O* and *P*. Otherwise the pawl tends to slip out of contact when the driving force is applied.

(*b*) A certain amount of " lost motion " may occur if the point of the pawl is not against a tooth face at the instant when the driver begins to move in the driving direction. This lost motion may vary from zero to the tooth pitch as a maximum, depending on conditions. A small pitch, therefore, insures a small amount of lost motion. By the addition of another pawl, *e*, of different length (see Fig. 11–1), the possible lost motion may be reduced to one-half its former value.

Figure 11–2 shows the mechanism of a lifting jack in which the driven link is given rectilinear motion when the handle is oscillated.

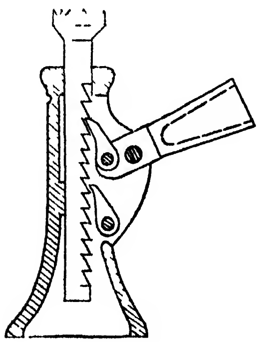


FIG. 11–2
Lifting Jack.

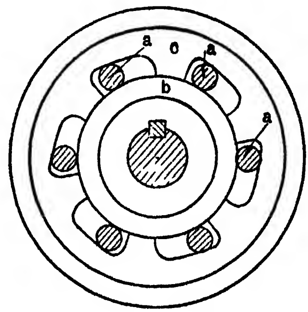


FIG. 11–3
Silent Ratchet.

Figure 11–3 shows a form of **silent ratchet.** Rollers *a, a* are placed in slots which taper. Rotation of the driver *b* in a clockwise sense causes the rollers to move toward the narrow ends of the slots where they wedge tightly and thus lock together driver *b* and driven member *c*. This mechanism can be arranged to allow little lost motion, and the noisy action of the pawl-and-tooth type is avoided. Positive action is lacking, since friction alone is responsible for the transmission of motion to the driven member.

Figure 11–4 illustrates another form of silent ratchet with friction pawls whose action is assisted by the use of springs.

2. Oldham's Coupling. — (Fig. 11–5.) This is used to connect shafts which are parallel, but not co-axial. It is a four-link mechanism, derived from the slider crank by the substitution of a sliding pair for one of the turning pairs. Link *c* has the form of a disc with a bar or key on each face, the keys being located at 90°

with one another. These form sliding pairs with slots cut in the faces of discs b and d.

The connections between b, c, and d are evidently such that all three members must turn through equal angles during the same

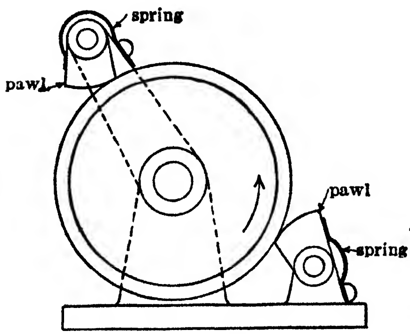
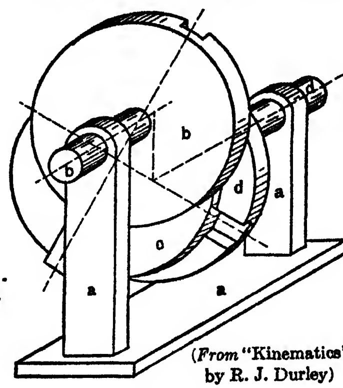


FIG. 11–4
Silent Ratchet.



(*From* "Kinematics" by R. J. Durley)

FIG. 11–5
Oldham's Coupling.

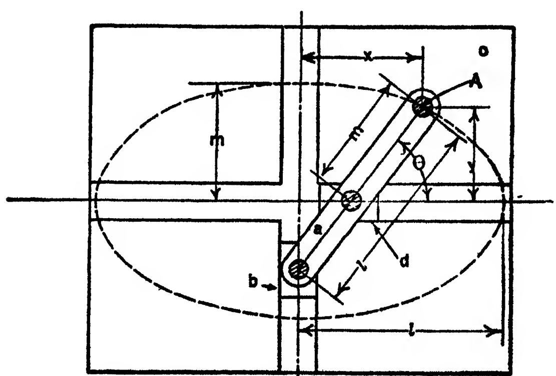


FIG. 11–6
Elliptical Trammels.

time interval. Hence the velocity ratio of b to d is constant and equal to unity.

3. Elliptical Trammels. — (Fig. 11–6.) This device is serviceable in drawing ellipses. In a form somewhat different from that shown in the figure, it has been employed as an **Elliptical Chuck** for machining parts of elliptical section. Like the **Oldham Coupling,** it is a four-link mechanism containing two sliding and two turning pairs. The fixed link c has in this case elements of two sliding pairs. Any point A on arm a can be shown to trace out an elliptical path on c. This is proved as follows:

Let x and y represent the coördinates of point A for any position of the mechanism. From the figure,

$$y = m \sin \theta,$$
$$x = l \cos \theta.$$

Hence,

$$\frac{x^2}{l^2} + \frac{y^2}{m^2} = \sin^2 \theta + \cos^2 \theta = 1.$$

This is the equation for an ellipse with major axis equal to $2\,l$ and minor axis equal to $2\,m$.

4. Straight-line Motions. — Where a link is required to have rectilinear motion, constraint is generally effected by the use of a "slide and guide" with plane surfaces in contact. Rectilinear motion can be obtained, however, by several mechanisms containing turning pairs only. These are generally termed "straight-line motions." Such motions can be divided into two classes: (*a*) approximate straight-line motions, and (*b*) accurate straight-line motions.

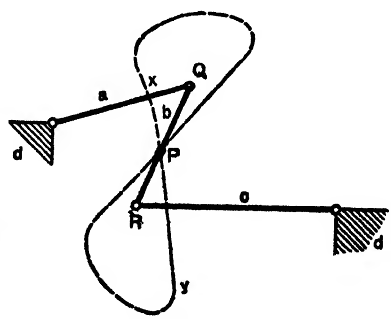


FIG. 11–7
Watt Straight-line Motion.

The **Watt** straight-line motion is of historical interest only, being little used nowadays. Watt employed it in his engines as a substitute for the present-day crosshead and guide, for the reason that in his time it was difficult to form accurate plane surfaces in metal. The device consists of a quadric-crank mechanism, a, b, c, d, as in Fig. 11–7. A point P on b traces out the dotted path shown in the figure. This will be observed to have a portion xy which is **approximately straight,** provided that point P is so selected that

$$PQ : PR = c : a.$$

Crosby Indicator Motion. — A mechanism shown in Fig. 11–8, composed of links connected by turning pairs, is used to connect the indicator piston rod with the recording pencil on the **Crosby Engine Indicator.** The pin A, connected to the end of the piston rod, is constrained to move in a vertical straight line. The pencil point at P traces out the indicator card on the drum b. In order that the instrument may present an accurate record of the pressure changes in the engine cylinder, it is necessary that the linkage connecting the piston rod and recording point fulfill the following requirements:

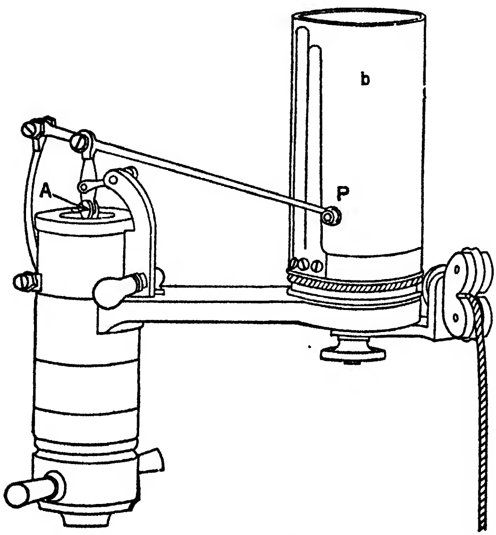


FIG. 11–8
Crosby Indicator.

(*a*) It should cause the point P to move in a straight line parallel to the direction of motion of the indicator piston.

(*b*) It should magnify the piston motion in order to draw an indicator card of reasonable size, the ratio of magnification being constant for all positions of the mechanism.

Neither of these objects is accomplished with mathematical accuracy by the Crosby motion, but the errors involved are so small as to be negligible for practical uses.

The **Peaucellier straight-line motion** (Fig. 11–9) belongs to the class of accurate straight-line motions, since it can be shown that point C moves in a straight-line path CD, perpendicular to the center line of the fixed link f. The lengths of the links must have the following relative values:

$$a = b = c = d,$$
$$e = f,$$
$$g = h.$$

This mechanism has too many joints to be of great practical importance.

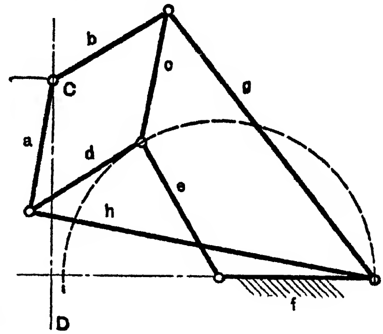


FIG. 11–9
Peaucellier Straight-line Motion.

5. The Pantagraph is generally used as a means of reproducing drawings or maps to a smaller or larger scale. It has also been employed as a reducing motion in connection with engine indicators. Two forms are shown in Figs. 11–10 and 11–11. In both figures the length OA equals BC; also OC equals AB. The figure $OABC$ is therefore a parallellogram, O being the fixed point or pole.

If we place the mechanism in any position, select any point P on BC, and then find a point Q on AB such that O, Q, P fall in the same straight line, it can be proved that points P and Q will trace out similar figures. In the practical form of the pantagraph, P forms the tracer point, which is run around a map or figure, and a copying point at Q will reproduce the diagram to a smaller scale.

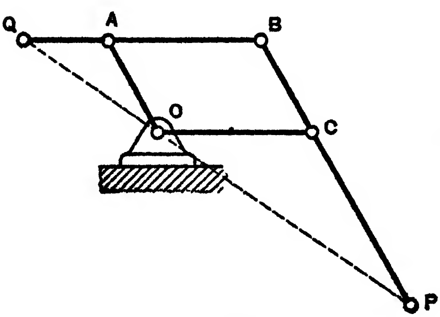


FIG. 11–10
Pantagraph.

To prove the essential property of the pantagraph, it is necessary to show (1) that Q remains on the straight line OP for any position of the mechanism, and (2) that $OQ : OP$ is constant. The figures traced out by P and Q will then be similar, the linear dimensions being proportional to lengths OP and OQ.

In the initial position of the mechanism, in which Q lies on the straight line OP, we have similar triangles QBP and OCP and

$$QB : BP = OC : CP.$$

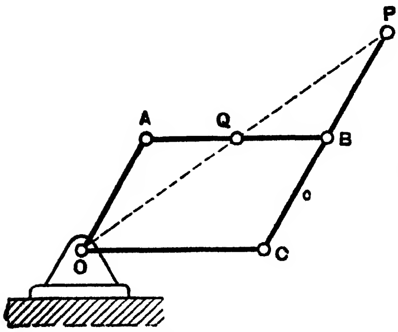


FIG. 11–11
Pantagraph.

This holds true for any position, since QB, BP, OC, CP are fixed lengths. Also QB and OC are always parallel. Therefore, triangles QBP and OCP are always similar. It follows that Q always lies on the straight line OP and that

$$OQ : OP = BC : CP$$
$$= \text{a constant.}$$

6. Hooke's Joint.—This mechanism, shown in Fig. 11–12, is often called a **Universal Coupling.** It is used to connect two shafts which intersect, but which are not necessarily in the same straight line. A device of this kind is essential for connecting a driving and driven shaft where the angle between them changes in service. Such a condition is encountered in the transmission of power to the rear axle of a motor car. Here the drive shaft connecting the engine to the rear axle does not make a fixed angle with the axis of rotation of the engine crank shaft, because of the action of the car springs.

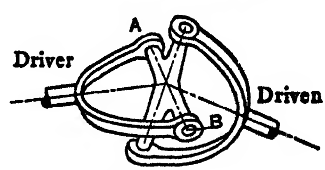


FIG. 11–12
Universal Joint.

It can be shown that a single coupling does not transmit motion with a constant angular velocity ratio, except when the shafts are in line with each other.

The velocity ratio at any instant is equal to

$$\frac{\omega_2}{\omega_1} = \frac{\cos\theta}{1 - \sin^2\theta\sin^2(\alpha + 90°)},$$

where θ = angle between the shafts,
α = angular displacement of the driving shaft from the position where the pins on the drive shaft yoke lie in the plane of the two shafts,
ω_1 = angular velocity of the driving shaft,
ω_2 = angular velocity of driven shaft.

The angular velocity ratio ($\omega_2 \div \omega_1$) varies from maximum to minimum value during an angular displacement of 90°. The maximum and minimum values of this ratio are shown by the curves of Fig. 11–13. It will be observed that the speed variation increases rapidly with the shaft angle.

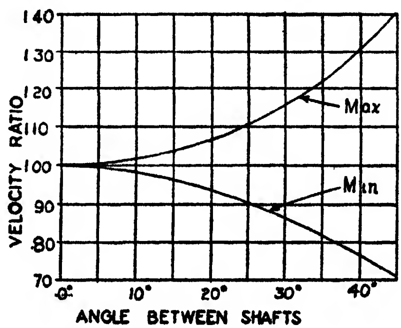


Fig. 11–13

Constant-speed Universal Joints. — In order to avoid trouble due to the speed variation of the Hooke's Joint, other forms of coupling having a constant angular velocity ratio have been developed. One of the best of these is the Bendix-Weiss "rolling ball" universal joint shown in Fig. 11–14. This is composed of two yokes on which curved grooves are cut, forming races on which roll four large balls. A fifth and smaller "guide" ball, centrally located, serves as a locking device to keep the assembly in position.


(Bendix Corp.)

Fig. 11–14. Constant-Speed Universal Joint.

The geometric requirement to secure a constant angular velocity ratio is that the centers of all four balls shall lie in a plane which bisects the angle between the shafts. This requirement is met by proper design of the grooves.

A common use for this joint is in the "front-drive" automobile. Here the power must be transmitted to the front wheels, which must at the same time be capable of swinging through a large angle in order to steer the car. In this application of the universal joint, a non-constant angular velocity ratio would be very unsatisfactory.

Constant Velocity Ratio by Use of Two Universals. — Two shafts lying in any relative position may be connected by a pair of universal joints and an intermediate shaft so that they will have the **same angular velocity** at any instant. To accomplish this result, the connecting shaft is located so as to make equal angles with the main shafts, and the driving pins on the yokes attached to the connecting shaft are placed at such an angle that each lies in the plane of the adjacent shafts at the same instant. Under these conditions, a reduction or increase of the speed of the intermediate shaft, as compared with that of one of the main shafts, caused by the interposed coupling, will be exactly neutralized by an equal but opposite change of speed of the other main shaft as compared with that of the intermediate shaft, due to the second coupling. The net result is that both main shafts will have the same speed at any instant. The compensating action of the couplings is due to their symmetrical arrangement with regard to the two planes containing the adjacent shafts.

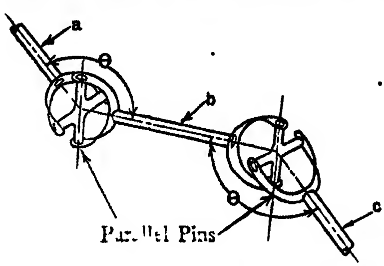


Fig. 11–15
Compensated Drive using Two Universal Joints.

A drive for connecting parallel shafts, compensated in the manner just described, is shown in Fig. 11–15. Since the planes containing the axes of adjacent shafts are here coincident, the driving pins on the connecting shaft yokes are parallel. The compensating effect of the two couplings under these particular circumstances will be clear from inspection of the figure. If the connecting shaft is cut by a transverse plane and one-half of the arrangement is rotated about the axis of this shaft, the main shafts may be brought to any desired non-parallel position. When the ends of the connecting shaft are rejoined, compensation is evidently still maintained, and the relative position of the couplings is that already specified as being necessary.

7. Geneva Stop. — In certain automatic machinery the **Geneva Stop** is used when it is desired to obtain alternate periods of rest and angular motion for a driven member, when the driver rotates continuously in the same sense.

We find examples of it in watches, motion-picture machines, can-making machinery, and indexing devices employed in the machine shop.

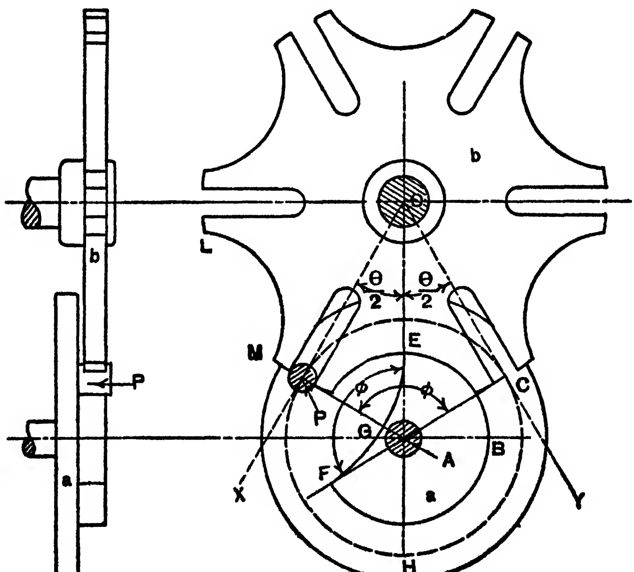


Fig. 11–16
Geneva Stop.

It is suitable for movements of the driven link not exceeding 90° for each revolution of the driver. In Fig. 11–16 the driver *a* consists of a circular disc to which is attached a driving pin *P* and a second disc *EBFG*. The pin *P* engages for a certain portion of each revolution with radial slots in the driven member *b* which it carries forward through an angle θ. When pin P passes out of a slot the surface EBF engages one of the geometrically similar surfaces LM on the driven member and holds it stationary until P enters the next slot.

In order that the driven member may be started and stopped without shock, it is necessary that the mechanism be designed in such a way that in the position illustrated the lines OX, OY are tangent to the circle PCH through the axis of the driving pin.

Thus, in laying out the mechanism, an angle $\theta/2$ is measured on each side of the line OH, where θ is the desired angular movement of the driven shaft.

Any circle, as PCH, tangent to OX and OY, may be used as the path of the driving-pin axis. The radius of this circle is the crank length for the driving pin, and its center locates the axis of the driving disc. The arc EGF is chosen so that the surface will clear the points of the driven disc. The angle EAF (ϕ) must be made equal to the angle PAC in order that the driven member may be alternately locked in position and released at the proper instants.

QUESTIONS — CHAPTER XI

1. For what purpose is a ratchet mechanism employed? Sketch one form (*a*) of ratchet with toothed wheel, (*b*) of silent ratchet.

2. Show by sketch the working parts of the ordinary lifting jack.

3. What two methods may be used to reduce the lost motion in a ratchet mechanism?

4. In a ratchet wheel, how must the teeth be shaped so that the pawl will not tend to slip out of contact?

5. Sketch the Oldham coupling. For what purpose is this device used?

6. Sketch the elliptical trammel and prove its essential property.

7. Sketch one form of the pantagraph mechanism. Prove that the copying and tra er points describe similar figures.

8. What two requirements must the link motion of an engine indicator fulfill? Sketch the linkage of the Crosby indicator.

9. Show by sketch the arrangement, and note the relative lengths of the links, in a Peaucellier straight-line motion.

10. Under what conditions is a Hooke's Joint a suitable method of connecting two shafts? Show by sketch how you would arrange two such joints to maintain a constant velocity ratio.

11. Sketch and explain the action of the Geneva stop.

12. In laying out the Geneva stop, the circle, which is the path of the crank-pin center, is tangent to the center lines of adjacent slots on the driven disc for one position of the mechanism. Why is this necessary?

DRAFTING-ROOM PROBLEMS[1]

The drawings are to be made on a 12 × 18 in. sheet with a border $1\frac{1}{2}$ in. wide on the left-hand side and $\frac{1}{2}$ in. wide on the other three sides. A space 2 × 6 in. is provided for the title in the lower right-hand corner.

In the illustrations, the dimensions shown in circles, thus ($2\frac{1}{2}''$), are distances measured from the border line. **These dimensions are always to be taken full size,** regardless of what scale is specified for the drawing.

[1] The author wishes to acknowledge the valuable assistance of Professor J. I. Clower in the original compilation of many of these problems.

PROBLEM 1

CROSBY INDICATOR STRAIGHT-LINE MOTION

The diagram below shows the straight-line motion mechanism used in the Crosby indicator for magnifying the motion of the indicator piston.

Plot the mechanism *four times full size* in the position shown. Then move the piston C upward in steps of $\frac{1}{16}$ in. each for a total distance of $\frac{3}{8}$ in. and plot the path of the pointer P.

Determine and note on your drawing the **ratio of magnification** of the motion of point P to C.

DATA:

$A—B = 1\frac{3}{4}$ in.	$C—E = \frac{7}{32}$ in.
$B—D = \frac{5}{8}$ in.	$E—F = \frac{29}{64}$ in.
$B—P = 3\frac{3}{4}$ in.	$D—E = \frac{17}{32}$ in.

Lines $A—B$ and $D—C$ are parallel and vertical in the position shown.

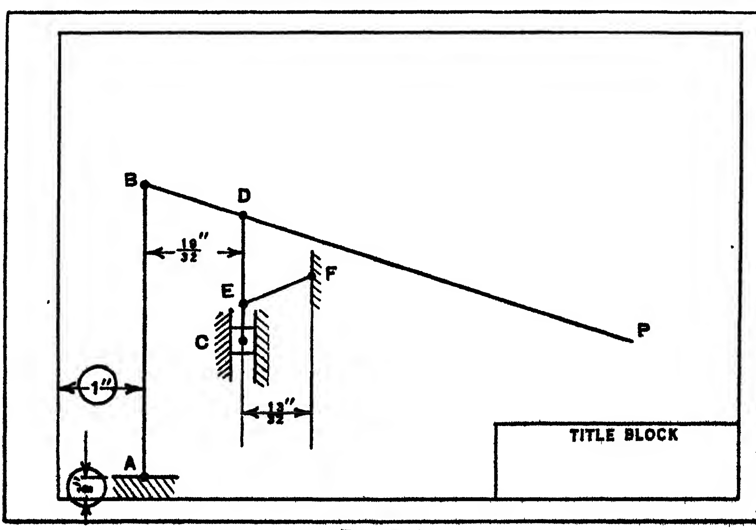


PLATE 1

PROBLEM 2

TABOR INDICATOR MECHANISM

1. Plot the linkage, *four times full size,* with *BP* horizontal as shown. Move the pencil point *P* upward, ¼ in. at a time, in a vertical line for a total travel of 2 in., starting 1 in. below the initial position. Locate the corresponding positions of points *C*, *D*, and *E*.

2. Determine and note on your drawing the **ratio of magnification** of the motion of point *P* to *D*.

3. Determine and indicate on the drawing the **radius and center of the circular arc** which most closely approximates the path of point *E*. Draw the **outline of the slot** which is required to guide the roller.

DATA:

A—B = 1¼ in.	*C—E* = ⅝ in.
B—C = ⅝ in.	*C—D* = 1 in.
B—P = 3⅛ in.	Diameter of roller = 3/16 in.

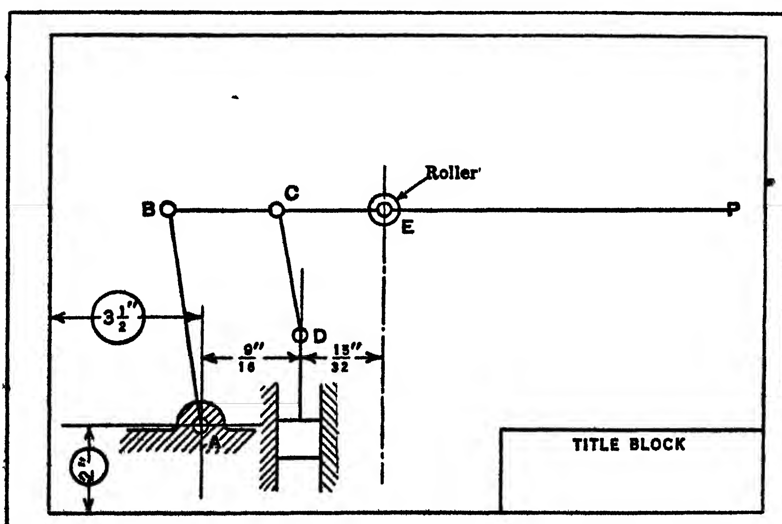


PLATE 2

PROBLEM 3

DISPLACEMENT, VELOCITY, AND ACCELERATION CURVES

1. Draw the Scotch Yoke shown in Plate 3, to a scale of 3 in. = 1 ft.

2. Calculate the normal and tangential acceleration of the crank pin at an instant when the crank velocity is 120 R.P.M. and the crank acceleration is 180 radians per sec. per sec. Find graphically the total acceleration of the crank pin, using a scale of 1 in. = 50 ft. per sec. per sec.

3. Assuming that the crank rotates at a uniform rate of 300 R.P.M., plot polar and linear curves for the displacement, velocity, and acceleration of the slotted link. For each curve make the maximum ordinate $1\frac{1}{4}$ in. long. Determine and record the scale for each curve.

4. With the same assumption as in **3**, calculate by formulae the displacement, velocity, and acceleration of the slotted link at crank angles of 0°, 30°, 60°, 90°, 120°, 150°, and 180°. Tabulate these values.

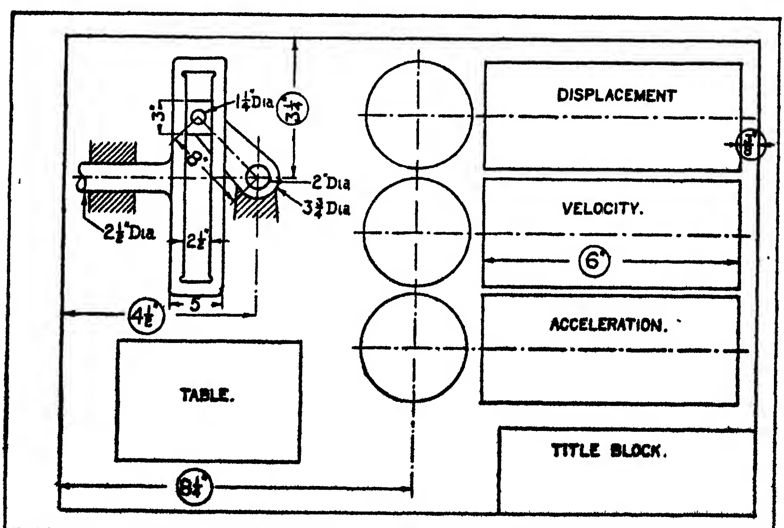


PLATE 3

PROBLEM 4

DISPLACEMENT, VELOCITY, AND ACCELERATION CURVES

Refer to Plate 3 and data of Problem 3 except that the crank length for the Scotch Yoke Mechanism is to be made 7 in. long.

PROBLEM 5

DISPLACEMENT, VELOCITY, AND ACCELERATION CURVES

1. Draw the Scotch Yoke Mechanism as shown in Plate 3, to a scale of 3 in. = 1 ft.

2. Calculate the normal and tangential acceleration of the crank pin at an instant when the crank is at an angle of 45° with the horizontal, the crank velocity 100 R.P.M., and the angular acceleration of the crank 420 R.P.M. per sec. Find graphically the total acceleration of the crank pin, using a scale of 1 in. = 40 ft. per sec. per sec.

3. Assuming that the crank rotates at a uniform rate of 500 R.P.M., plot polar and linear curves for the displacement, velocity, and acceleration of the slotted link. For all curves, make the maximum ordinate 1½ in. long. Determine and record the scale for each curve.

4. With the same assumptions as in 3, calculate by formulae the displacement, velocity, and acceleration of the slotted link at crank angles of 0°, 30°, 60°, 90°, 120°, 150°, and 180°. Tabulate these values.

PROBLEM 6

DISPLACEMENT, VELOCITY, AND ACCELERATION-TIME CURVES

The following data relate to a moving body:

Time	Distance	Time	Distance
0 sec.	0 ft.	6 sec.	40.00 ft.
1	13.75	7	42.20
2	22.20	8	43.80
3	28.50	9	44.80
4	33.50	10	45.15
5	37.10		

PLOT: (*a*) Using the above data, a **Distance-time Curve** on a base line located as shown on the sketch below. Scales: Distance — 1 in. = 6 ft. Time — 1 in. = 1 sec.

(*b*) A **Velocity-time Curve.** — Obtain points for each 1-sec. interval. Double the ordinates.

(*c*) An **Acceleration-time Curve.** — Obtain points at intervals of 1 sec. Double the ordinates. Use *A* — *B* as base line for this curve. Calculate velocity and acceleration scales, expressing them in foot-second units. Construct graphical scales.

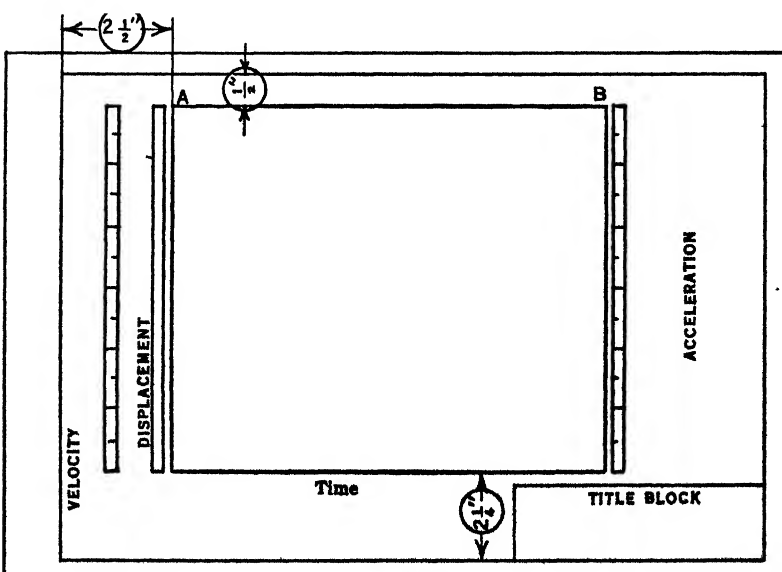


PLATE 4

PROBLEM 7

DISPLACEMENT, VELOCITY, AND ACCELERATION-TIME CURVES

Refer to Plate 4.

The following data relate to a moving body:

Time	Distance	Time	Distance	Time	Distance
0 sec.	0 ft.	3½ sec.	9.8 ft.	7 sec.	36 ft.
½	0.2	4	12.8	7½	39.67
1	0.8	4½	16.2	8	42.67
1½	1.8	5	20	8½	45
2	3.2	5½	24	9	46.67
2½	5.0	6	28	9½	47.67
3	7.2	6½	32	10	48

PLOT: (*a*) From above data, a **Distance-time Curve** on a base located as shown in Plate 4. Scales: Distance — 1 in. = 6 ft. Time — 1 in. = 1 sec.

(*b*) A **Velocity-time Curve.** — Obtain points for each ½-sec. interval. Multiply ordinates by 4.

(*c*) An **Acceleration-time Curve.** — Obtain points at intervals of ½ sec. Use as base a horizontal line 4 in. below *A—B*. Multiply the ordinates by 6.

Calculate velocity and acceleration scales, expressing them in foot-second units. Construct graphical scales.

PROBLEM 8
INSTANT CENTERS

1. Draw the four mechanisms illustrated in Plate 5 to full scale.
2. Locate all the instant centers.
3. In Figs. *A* and *B*, assume that point *P* has a velocity of 12 ft. per sec. Represent this velocity by a vector 1½ in. long and find graphically the instantaneous velocities of points *Q* and *R*.
4. In Figs. *C* and *D*, assume that the link *a* is rotating at a rate of 75 R.P.M. and find graphically the instantaneous angular velocity of links *b* and *c* in both mechanisms, in R.P.M. Represent the angular velocity of link *a* by a vector 1½ in. long. Indicate the sense of rotation of *b* and *c*.

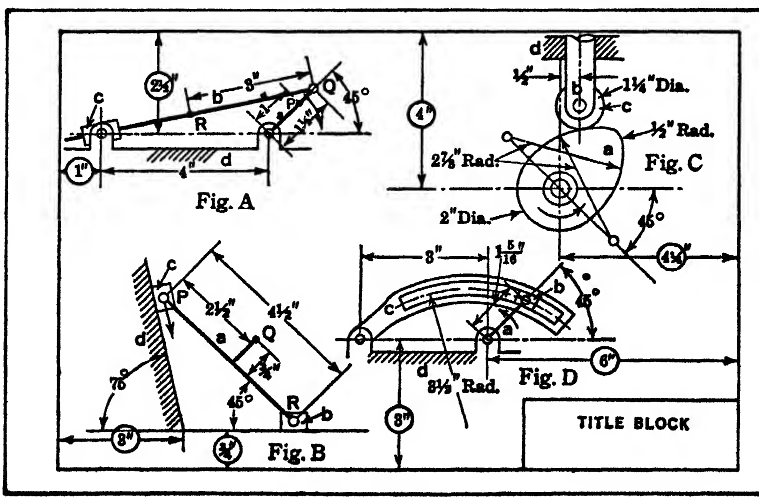


PLATE 5

PROBLEM 8A
INSTANT CENTERS

1. Draw the four mechanisms illustrated in Plate 5*A* to full scale.
2. Locate all the instant centers.
3. In Figs. *A*, *B*, *C*, and *D*, assume that point *L* has a velocity of 15 ft. per sec. Represent this velocity by a vector 1½ in. long, and find graphically the instantaneous velocities of points *M* and *N*.
4. In Figs. *A*, *B*, and *C*, assume that the link *a* is rotating at the rate of 75 R.P.M., and find graphically the instantaneous angular velocities of links *b* and *c* in all mechanisms. Represent the angular velocity of link *a* by a vector 1½ in. long. Indicate the sense of rotation of *b* and *c*.

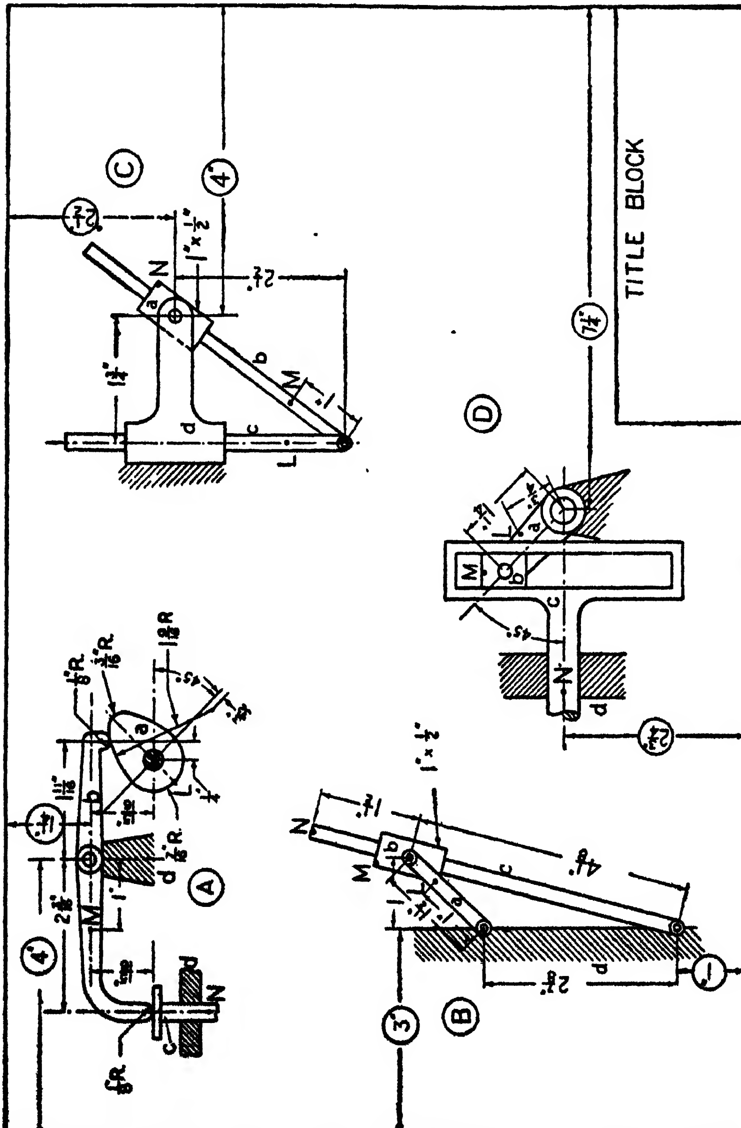


PLATE 5A

PROBLEM 9

INSTANT CENTERS, LINEAR AND ANGULAR VELOCITIES

Refer to Plate 6.

The skeleton drawing represents a six-link quick-return mechanism. Lay out the mechanism according to the dimensions shown on the drawing.

1. Locate all the **instant centers.**
2. Determine graphically the **linear velocities** of points *P*, *Q*, *R*, and *S*, for the position of the mechanism shown in the figure when the driving link *a* makes 15 R.P.M. in a counter-clockwise direction. Tabulate the numerical values on the drawing. Velocity Scale: 1 in. = 20 ft. per min.
3. Representing the angular velocity of *a* by a line 1 in. long, determine graphically the corresponding **angular velocities** of links *b*, *c*, and *d*. Tabulate the numerical values on the drawing.

Scale: 4 in. = 1 ft.

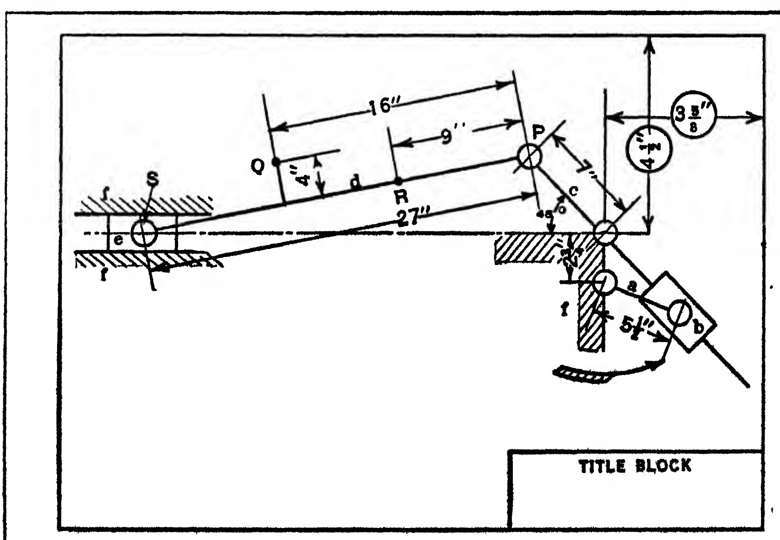


PLATE 6

PROBLEM 10

DIAGRAMS FOR SLIDER-CRANK MECHANISM

Illustrated in Plate 7 is a skeleton diagram of the **Slider-crank Mechanism** used to transform rectilinear motion into rotary motion. The crosshead *c* has a stroke of 24 in. The connecting rod *b* is 5 times as long as the crank *a*. The crank turns at a constant speed of 160 R.P.M. in a clockwise direction.

Construct the following diagrams for a complete revolution of the crank *a*; obtain points at 15° intervals.

1. **Polar Velocity Diagram** showing crosshead velocities on corresponding crank positions.
2. **Velocity-displacement Diagram** showing crosshead velocities on positions of the crosshead pin.
3. **Velocity-time Curve,** on a base line 9 in. long, located as shown, representing the time of one revolution of the crank *a*.
4. **Acceleration-displacement Diagram,** using same base line as **2.**
5. **Acceleration-time Diagram,** base line same as **3.**

Label all diagrams, using notation above. Calculate and construct graphical scales for velocity and acceleration. Show the computations on the drawing.

Scale: 2 in. = 1 ft.

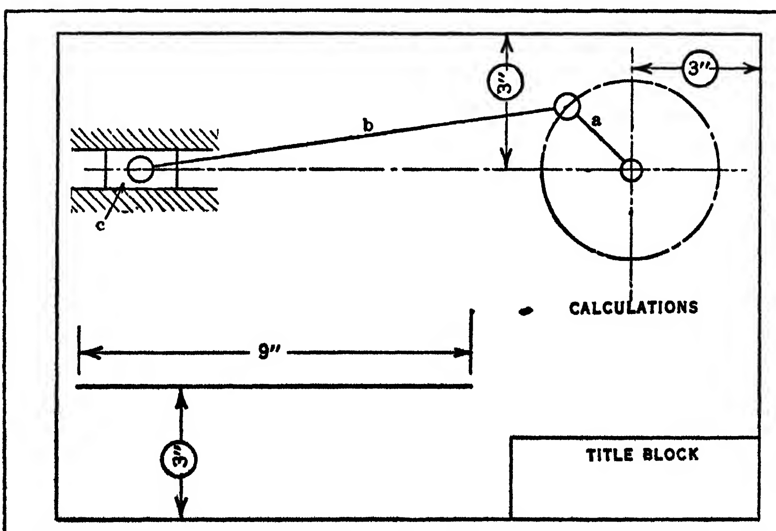


PLATE 7

PROBLEM 11

PISTON VELOCITY AND ACCELERATION

A six-cylinder gasoline engine has a bore of $3\frac{1}{4}$ in. and a stroke of $4\frac{1}{4}$ in. It develops maximum power at a speed of 3600 R.P.M. The connecting rod is 8 in. long.

Locate the crank-shaft center as shown in Plate 7 and construct the following diagrams, obtaining points at intervals of 15°, and assuming the engine to be turning at the speed for maximum power.

1. Polar Velocity Diagram, showing piston velocities on corresponding crank positions.

2. Velocity-displacement Diagram, showing piston velocities on a base representing wrist-pin positions.

3. Velocity-time Diagram, on a base 9 in. long, located as in Plate 7, representing the time of one revolution of the crank.

4. Acceleration-displacement Diagram, using the same base as in **2.**

4. Acceleration-time Diagram, on the same base as for **3.** Calculate and construct graphical scales for velocity and acceleration. Assuming that the reciprocating parts weigh 1.25 lb. per cylinder, construct for the acceleration curve a graphical scale of the accelerating (or "shaking") force.

Show all computations on the drawing.

Scale: Full size.

PROBLEM 12

WHITWORTH QUICK-RETURN MOTION MECHANISM

Make a skeleton drawing of the mechanism according to the dimensions, Plate 8, using the following information: Length of connecting rod = 32 in.; length of *A—C* = $11\frac{1}{2}$ in.; *B—D* = $7\frac{3}{4}$ in.; *A—B* = $7\frac{1}{2}$ in.; and *B—C* = 5 in.

1. Find the stroke of the ram.

2. Construct a full-stroke **velocity diagram** for the ram, taking as a base the line of travel of the point at which the connecting rod is attached to the ram. For the construction of this diagram, let the velocity of point *C* be represented by a velocity vector $1\frac{1}{8}$ in. long.

3. Calculate the **velocity scale** when crank *A—C* turns 80 R.P.M. Construct a graphical scale as indicated on the sketch. Show calculations on drawing.

4. Locate all the **instant centers** for the position shown.

5. Determine the **time ratio** of the cutting stroke to the return stroke. Note this ratio on your drawing.

Scale: 3 in. = 1 ft.

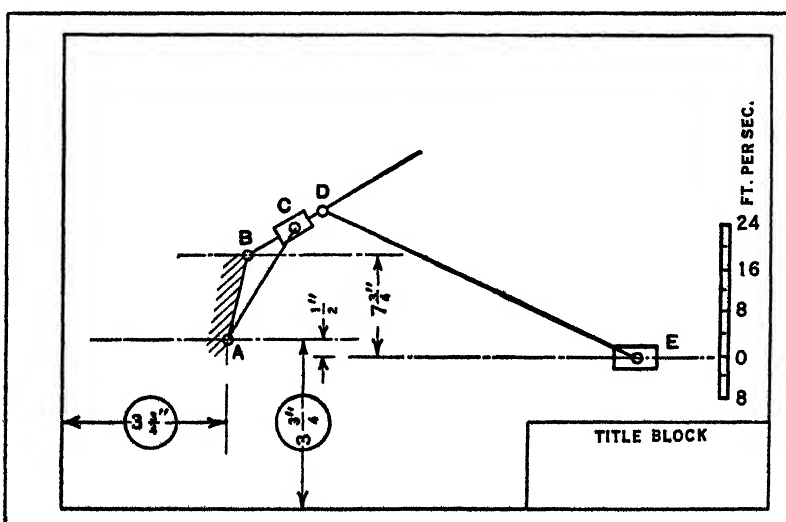


PLATE 8

PROBLEM 13

CRANK-DRIVEN QUICK-RETURN MOTION

Draw the quick-return motion shown in Plate 9. The driving crank *a* has a length of $3\frac{3}{4}$ in. and rotates clockwise at 240 R.P.M. The driven slider *e* is to have a stroke of 16 in., and the time ratio of advance to return is to be 2 : 1.

1. Draw the mechanism as shown, with the block *e* at the left end of its stroke. Find the length of lever *c* necessary to give the proper stroke and time ratio.

2. Draw a skeleton diagram of the mechanism as indicated by the dotted lines in the figure, with crank *a* at 45° with the vertical position.

3. Locate all instant centers for the mechanism in the position specified in paragraph 2.

4. By use of instant center O_{ae}, determine the instantaneous velocity of *e*. Represent the velocity of pin *M* on the driving crank by a line 2 in. long.

5. By use of instant centers O_{ac} and O_{cf}, determine graphically the velocity of *e* and see that it checks with the value obtained in paragraph 4.

Scale: $\frac{1}{2}$ in. = 1 in.

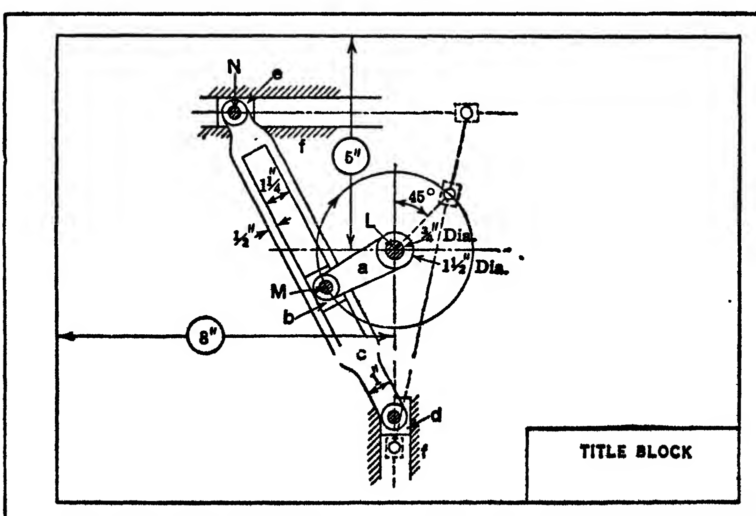


PLATE 9

PROBLEM 14

VELOCITY CURVE FOR QUICK-RETURN MOTION

1. Refer to Plate 9 and proceed as directed in paragraph 1 of Problem 13.

2. Divide the stroke of point N into four equal parts.

3. Determine the velocity of N in the three intermediate positions thus obtained, for both advance and return strokes.

4. Plot a velocity curve for sliding block e.

The following method may be employed for determination of the velocity of e:

Represent the constant velocity of crank pin M by a line $1\frac{1}{2}$ in. long. Find O_{cf}. The absolute velocity of M on a is known, and the coincident point M on c has a movement relative to the former point in a direction along the line of slide of b on c. Furthermore, M on c has an absolute velocity about the center O_{cf} which can therefore be found graphically. From this point proceed to N.

5. Draw a graphical scale of velocity for N.

PROBLEM 15

DISC CAMS WITH ROLLER AND FLAT-FACED FOLLOWERS

See Plate 10.

Design two disc cams, each imparting the same motion to its follower. Locate follower axes as shown, or offset them $\frac{1}{2}''$ to the left as instructed. One follower is flat-faced; the other is provided with a roller. The outward motion takes place during a cam displacement of 135°. The follower then rests for 45° and returns during the next half-revolution. For the outward motion the velocity is constant except for periods at the beginning and end, each occupying one-twelfth of a revolution, during which acceleration and deceleration are uniform. The return motion is to be simple harmonic. The cams rotate clockwise.

Determine points on the profiles at intervals of 15°.

DATA:

Diameter of cam shafts	$1\frac{3}{16}$ in.	Diameter of base circles	$2\frac{3}{4}$ in.
Diameter of cam hubs	$2\frac{3}{8}$ in.	Diameter of roller	$1\frac{1}{2}$ in.
Keyways	$\frac{1}{4} \times \frac{1}{8}$ in.	Follower Displacement	$1\frac{3}{4}$ in.

Scale: Full size.

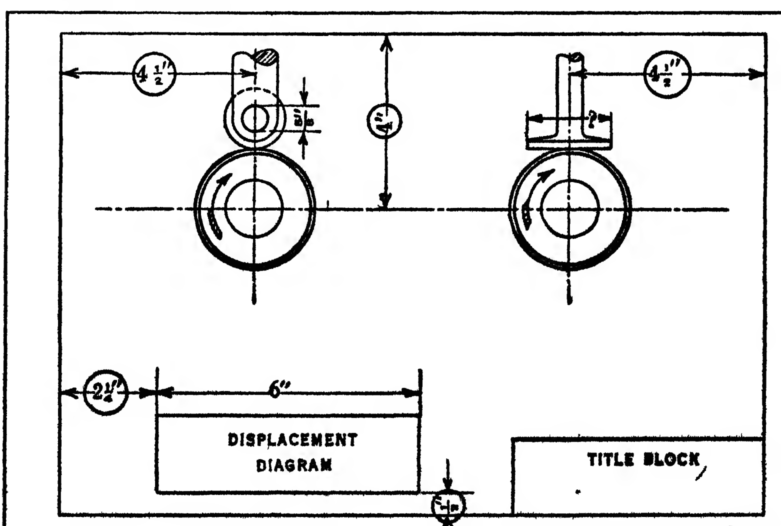


PLATE 10

PROBLEM 16

DISC CAM WITH PIVOTED ROLLER FOLLOWER

A pivoted roller follower as illustrated in Plate 11 turns through an angle of 30°. The outward motion is accomplished with constant velocity during 180° of cam displacement, except for 30° periods at the beginning and end of the movement where the acceleration and deceleration are constant. Two rest periods of equal length are provided in the extreme positions. During the return motion which requires a cam displacement of 120°, the follower is uniformly accelerated and decelerated, the acceleration-deceleration ratio being 1 : 2. The cam turns clockwise. Obtain points on the cam profile at 15° intervals. Determine the form of the follower arm necessary to clear the cam.

DATA:

Cam shaft diameter	$1\frac{3}{16}$ in.	Follower shaft diameter	$1\frac{1}{4}$ in.
Cam hub diameter	$2\frac{3}{8}$ in.	Follower hub diameter	$2\frac{1}{2}$ in.
Keyways	$\frac{1}{4} \times \frac{1}{8}$ in.	Roller pin diameter	$\frac{5}{8}$ in.
Roller diameter	$1\frac{5}{8}$ in.		

Scale: Full size.

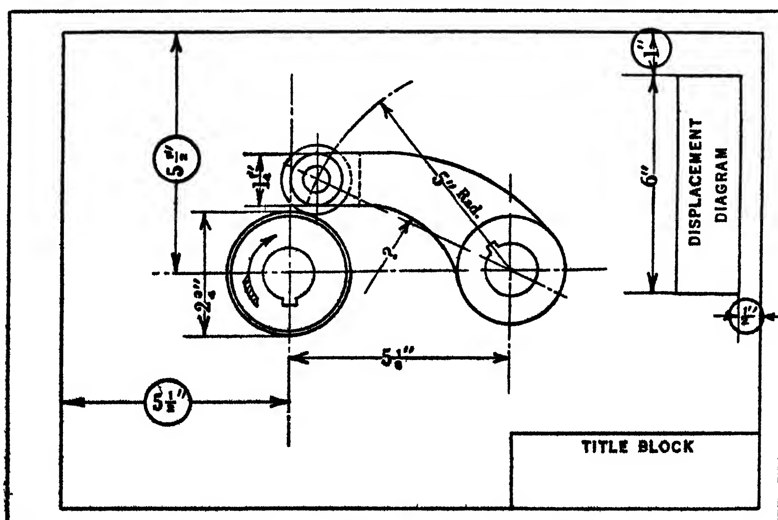


PLATE 11

PROBLEM 17

DISC CAM WITH PIVOTED FLAT-FACED FOLLOWER

See Plate 12.

The follower has an angular displacement of 30°. It moves outward during 12 of the 24 equal time periods required for one revolution of the cam and returns to the initial position during the following 8 time periods. Both motions are simple harmonic. The cam rotates clockwise. The follower face when produced is tangent to the hub.

Obtain points on the cam profile at 15° intervals and determine the necessary length of the contact surface of the follower.

DATA:

Cam shaft diameter	$1\frac{3}{16}$ in.	Base circle diameter	$2\frac{3}{4}$ in.
Cam hub diameter	$2\frac{3}{8}$ in.	Follower shaft diameter	$1\frac{3}{16}$ in.
Keyways	$\frac{1}{4} \times \frac{1}{8}$ in.	Follower hub diameter	$2\frac{3}{8}$ in.

Scale: Full size.

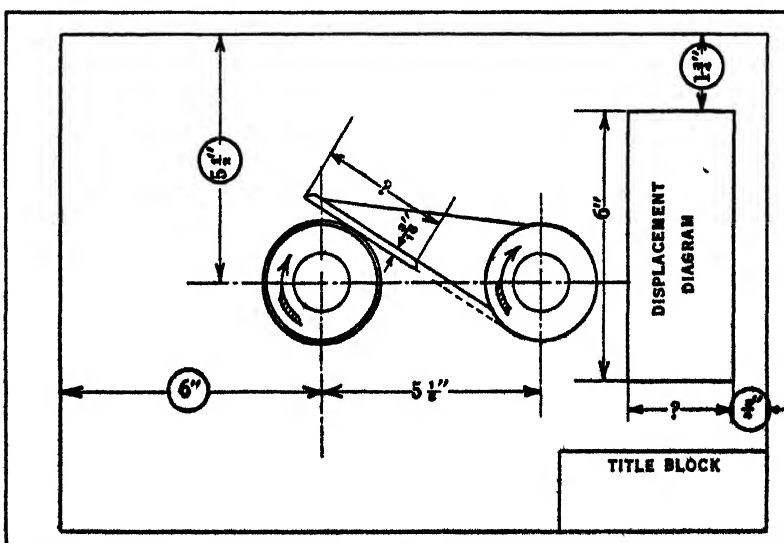


PLATE 12

PROBLEM 18

DISC CAM WITH PRIMARY AND SECONDARY FOLLOWER

In the cam mechanism shown in Plate 13 the secondary follower has a movement of $2\frac{1}{4}$ in. Starting from the lowest position, this follower rises vertically with simple harmonic motion during 12 of the 24 equal time periods required for one revolution of the cam. It then rests for 6 time periods and falls with uniformly accelerated and decelerated motion during the remaining 6 time periods. The cam rotates clockwise. Find points on the cam profile at intervals of 15°.

DATA:

Cam shaft diameter	$1\frac{1}{4}$ in.	Base circle diameter	3 in.
Cam hub diameter	$2\frac{1}{2}$ in.	Roller diameter	$1\frac{1}{2}$ in.

Scale: Full size.

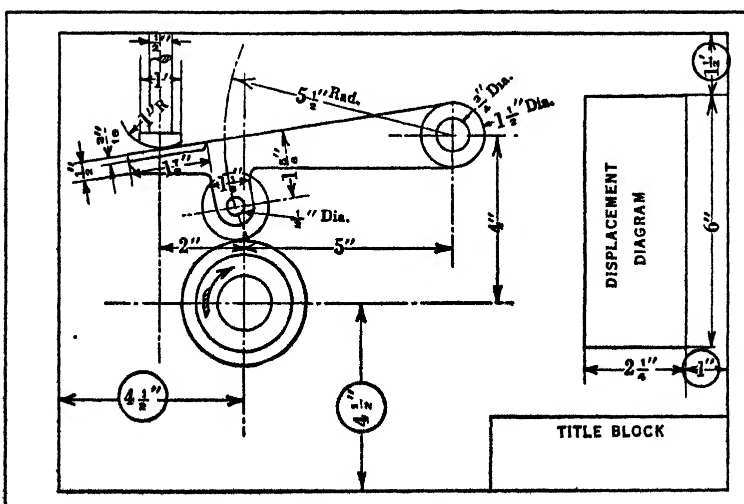


PLATE 13

PROBLEM 19

POSITIVE-RETURN CAM MECHANISM

See Plate 14.

Design a cam mechanism with two disc cams located on the same shafts and arranged for positive return of the follower. Starting from its lowest position, the follower rises with simple harmonic motion during a cam displacement of 180°. It then returns with uniform acceleration and deceleration to the initial position while the cam rotates 120°, and rests during the final 60°.

DATA:

Diameter of cam shaft	$1\frac{1}{8}$ in.	Diameter of rollers	$1\frac{1}{4}$ in.
Diameter of cam hub	$2\frac{1}{4}$ in.	Diameter of roller pins	$\frac{1}{2}$ in.
Diameter of base circles	3 in.	Keyway	$\frac{1}{4} \times \frac{1}{8}$ in.
		Lift	2 in.

The cams rotate counter-clockwise.

Obtain points on the profiles at intervals of 15°.

Scale: Full size.

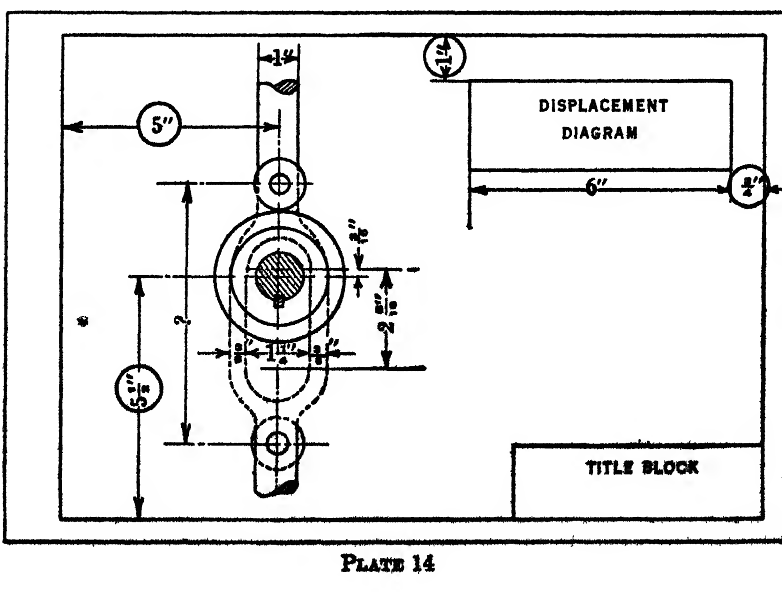


PLATE 14

PROBLEM 20

DOUBLE-DISC POSITIVE-RETURN CAM MECHANISM

This mechanism is illustrated in Plate 15.

1. Draw the first or **motion** cam and the follower by use of the given data.

2. Determine the profile of the second or **return** cam, obtaining points at 15° intervals.

3. On a base line 6 in. long, representing a displacement of 90° of the cam, plot the displacement diagram for the follower. Obtain points at 15° intervals and double the ordinates.

4. On the same base, plot a velocity curve for the follower. Divide the base line into 7½° intervals and obtain twelve points. Quadruple the ordinates.

5. Construct displacement and velocity scales.

DATA:

Diameter of rollers	2 in.	Diameter of roller pins	¾ in.
Diameter of cam shaft	1¼ in.	Keyway	¼ × ⅛ in.
Diameter of cam hub	2½ in.	*A—B* is a straight line.	
		R.P.M. of cam	180

Scale: Full size.

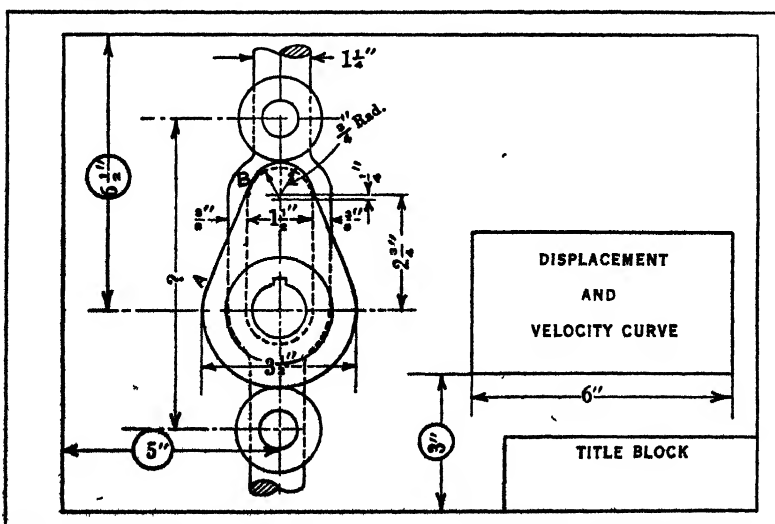


PLATE 15

PROBLEM 21

CYLINDER CAM WITH RECIPROCATING FOLLOWER

See Plate 16.

Design a cylinder cam imparting motion to its roller follower which moves in a straight line parallel to the cam axis:

The upward motion is accomplished while the cam rotates through 180° with constant velocity except during the first and last 30° where it has uniformly accelerated and decelerated motion. The follower rests at the top position while the cam rotates through 45°. The return motion, taking place in 135°, is uniformly accelerated and decelerated during equal time periods.

DATA:

Follower displacement	$2\frac{1}{4}$ in.	Length of cam hub	$4\frac{1}{4}$ in.
Diameter of cam hub	$1\frac{7}{8}$ in.	Keyway	$\frac{1}{4} \times \frac{1}{8}$ in.
Bore of hub	$\frac{15}{16}$ in.	Depth of groove	$\frac{3}{8}$ in.
Outside diameter of cam	$3\frac{3}{16}$ in.	Diameter of roller	$\frac{1}{2}$ in.

Obtain points at intervals of 15° and draw (*a*) a displacement diagram and development of the cam surface, (*b*) a plan view and elevation of the cam. Completed cam to be of form shown in Fig. 6-33, allowing $\frac{5}{16}$-in. thickness of metal for walls of groove.

Scale: Full size.

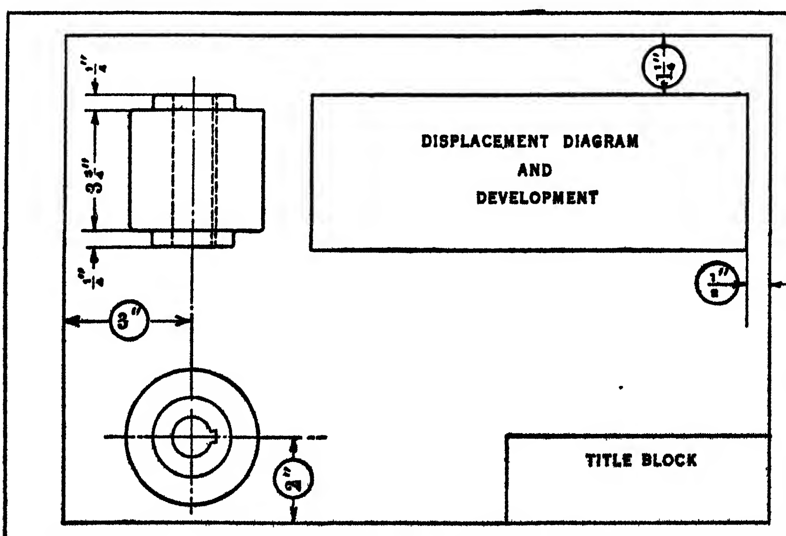


PLATE 16

PROBLEM 22

CYLINDER-CAM MECHANISM

See Plate 17.

A cylinder-cam mechanism is required to move a pivoted follower through an angle of 40° (20° on each side of the horizontal position). The upward motion of the follower is accomplished during one-half revolution of the cam. This motion takes place with constant velocity except during the first and last 45° of cam displacement when the acceleration and deceleration are constant and of equal value. The follower then rests for a cam displacement of 30°. The return motion is accomplished with constant acceleration and deceleration during equal periods while the cam rotates the remaining 150°. The cam rotates clockwise. The follower is provided with a conical roller.

Draw (*a*) a displacement diagram, (*b*) a development of the cylinder, (*c*) plan and elevation of the cam.

DATA:

Diameter of cam shaft	$1\frac{3}{16}$ in.	Diameter of cam hub	$2\frac{1}{4}$ in.
Maximum roller diam.	$\frac{3}{8}$ in.	Keyway	$\frac{3}{8} \times \frac{3}{16}$ in.
Depth of groove	$\frac{3}{8}$ in.	Length of cam hub	$4\frac{1}{8}$ in.
Radius of follower arm	$3\frac{1}{2}$ in.	Diameter of cam	$3\frac{3}{16}$ in.

Scale: Full size.

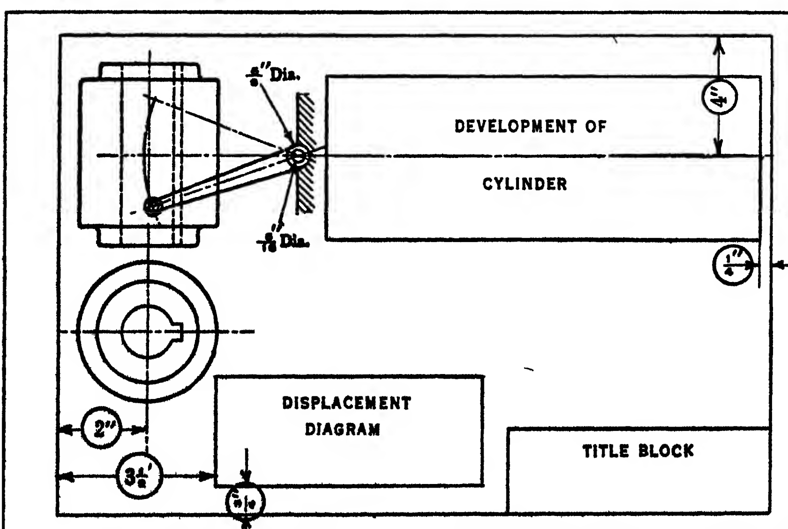


PLATE 17

PROBLEM 23

AUTOMOBILE ENGINE VALVE CAM

An automobile engine has an exhaust-valve cam as shown in Plate 18, designed for use with a flat-faced follower.

1. Plot the half cam *three times full size.*

2. On a base line 9 in. long, representing 90° motion of the cam, located as shown, plot a **displacement diagram.** Obtain points at 5° intervals. Quadruple the ordinates in plotting this curve.

3. On the same base, plot a **velocity curve,** using the method of Art. 22, Chapter VI, and doubling the ordinates.

4. On a base line 6 in. above the lower border line, plot an **acceleration curve.** Use the method mentioned above. Halve the ordinates.

5. Calculate the **displacement, velocity,** and **acceleration scales,** expressing them as 1 in. = —— in. (displacement); 1 in. = —— ft. per sec. (velocity); 1 in. = —— ft. per sec. per sec. (acceleration). Assume a motor speed of 3000 R.P.M.

6. Construct **graphical scales** and determine the maximum values of the acceleration, both positive and negative, and the maximum velocity. Indicate these values on the drawing.

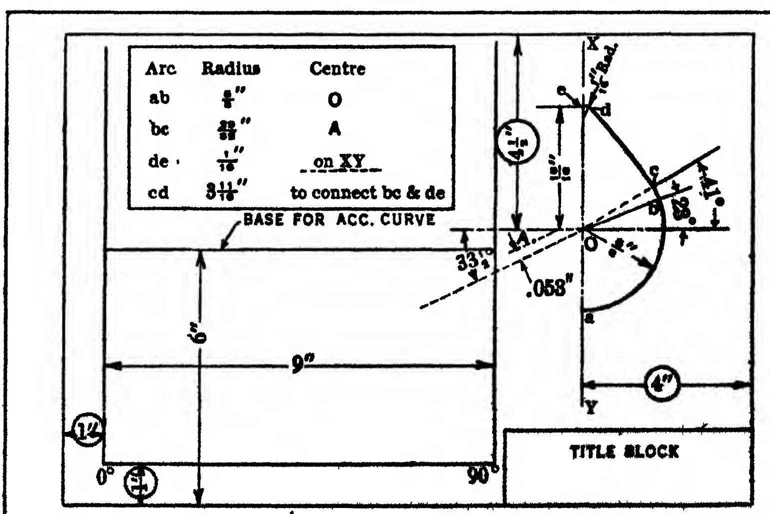


PLATE 18

PROBLEM 24

AUTOMOBILE ENGINE VALVE CAM

Substitute the Engine Exhaust Valve cam shown in Plate 19 for the cam shown in Plate 18. Proceed to plot displacement, velocity, and acceleration curves, etc., for the follower motion as directed in Problem 23. The follower is of the flat-faced type.

Find also the required diameter of the follower face.

The cam profile is composed of circular arcs as shown in the figure. Radii for arcs *ab*, *bc*, *de* are given. Center of arc *cd* is to be located so that this arc will be tangent to the adjacent arcs *bc* and *de*.

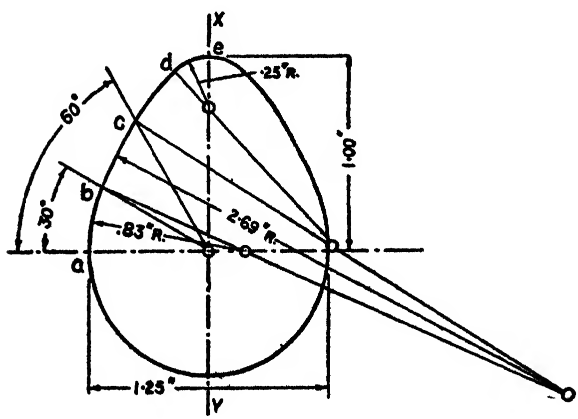


PLATE 19

PROBLEM 25

INVOLUTE GEARS

See Plate 20.

Draw a pair of involute gears and a rack having teeth of 2 diametral pitch, zero backlash, and standard $14\frac{1}{2}°$ involute proportions to meet the following requirements:

Gear. — Pitch diam. 12 in. Keyway $\frac{1}{2} \times \frac{1}{4}$ in. Bore of hub $2\frac{3}{16}$ in.
Pinion. — Pitch diam. 6 in. Keyway $\frac{3}{8} \times \frac{3}{16}$ in. Bore of hub $1\frac{1}{16}$ in.
Rack. — 20 teeth.

1. Use the Grant Odontograph method for drawing the tooth profiles.
2. Construct the exact involute for one tooth of each gear as a check on the accuracy of the Odontograph method.
3. Examine the tooth profiles for interference and, by cross-hatching, indicate the interfering portions.
4. Indicate the centers and radii used in laying out the teeth.
5. Determine the angle of action, angle of approach, and angle of recess for each gear, giving numerical values on drawing.
6. Indicate the path of the point of contact.

Scale: Full size.

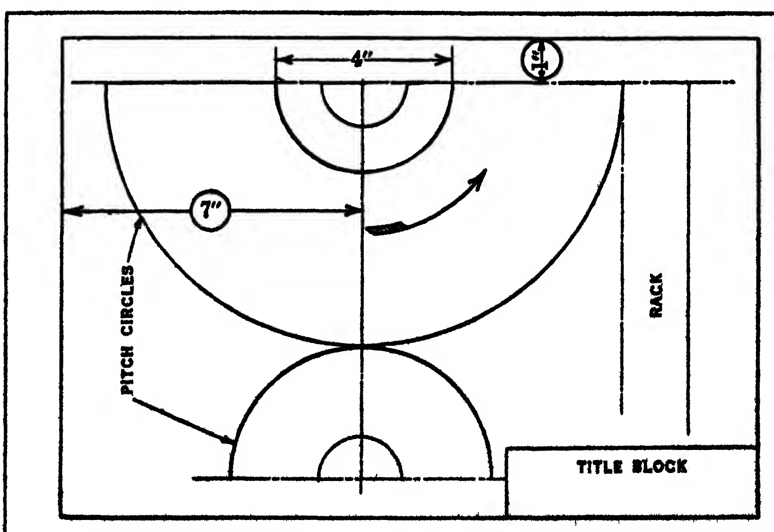


PLATE 20

PROBLEM 26

INVOLUTE GEARS

Same data as for Problem 25 except that teeth are of standard A.G.M.A. stub-tooth form. Draw one true involute tooth profile for each gear and reproduce as required for other teeth by means of a template. Obtain other data as directed in Problem 25, items 3, 4, 5, and 6.

PROBLEM 27

INVOLUTE GEARS

See Plate 20.

Draw a pair of involute gears and a rack having teeth of $2\frac{1}{2}$ diametral pitch, zero backlash, and standard $14\frac{1}{2}°$ involute proportions to meet the following requirements:

Gear. — 28 teeth. Keyway $\frac{1}{2} \times \frac{1}{4}$ in. Bore of hub $2\frac{3}{16}$ in.

Pinion. — 16 teeth. Keyway $\frac{3}{8} \times \frac{3}{16}$ in. Bore of hub $1\frac{1}{8}$ in.

Proceed as indicated in items 1–6 of Problem 25.

PROBLEM 28

CYCLOIDAL GEARS

Draw a gear, pinion, and rack to meet the following requirements:

Gear. — 24 teeth. Diameter of hub 4 in. Bore of hub $2\frac{3}{16}$ in.
Pinion. — 12 teeth. Diameter of hub $3\frac{7}{16}$ in. Bore of hub $1\frac{15}{16}$ in.
Rack. — 18 teeth. Keyseats $\frac{1}{2}$ in. by $\frac{1}{4}$ in.

The diametral pitch is 2 and the tooth proportions are standard full depth. Allow no backlash. The width of face is three times the circular pitch.

For comparison, draw the **exact** profile of one tooth on each wheel.

Indicate the centers and radii used in constructing the tooth profiles, also the path of the point of contact, the angle of approach and recess, and the maximum pressure angle.

Place on the drawing the following dimensions: width of face, pitch diameters, outside diameters, bore and outside diameters of hub, width and depth of keyseats, length of rack, and height of teeth.

Scale: Full size.

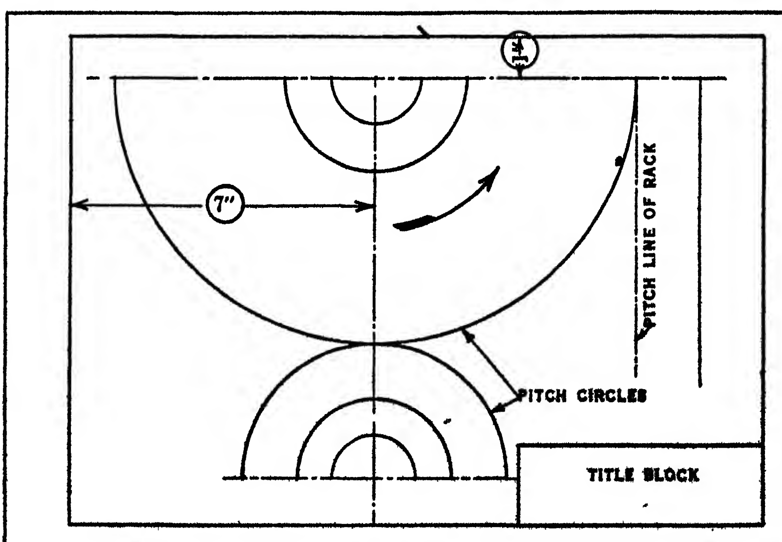


PLATE 21

PROBLEM 29

INVOLUTE BEVEL GEARS

Make a drawing in section of a pair of involute bevel gears as illustrated below. The diametral pitch is 2. The teeth have standard $14\frac{1}{2}°$ involute proportions with no backlash.

Show the tooth forms on the developments of the normal cones at the large and small ends of the teeth by use of the Grant Odontograph method.

Dimension the drawing and tabulate the following information: pitch diameter, edge angle, number of teeth, diameters of blanks, angle between shafts, diametral pitch, and face angle.

Determine the interfering portion of the tooth and indicate it by cross-hatching.

DATA:

Angular velocity ratio	10 : 7	Bore of pinion hub	$1\frac{3}{8}$ in.
Pitch diameter of gear	10 in.	Keyways	$\frac{3}{8} \times \frac{3}{16}$ in.
Bore of gear hub	$1\frac{1}{2}$ in.		

Scale: Full size.

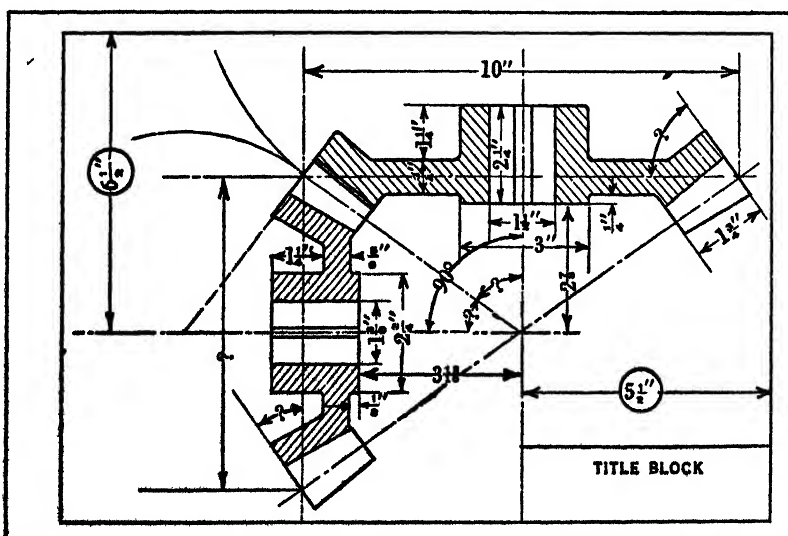


PLATE 22

PROBLEM 30

INVOLUTE BEVEL GEARS

Same as Problem 29, except use a speed ratio of 10 : 5.

PROBLEM 31

INVOLUTE BEVEL GEARS

Same as Problem 29, except teeth are of standard A.G.M.A. stub-tooth form.

PROBLEM 32

CROWN GEAR AND PINION

Make a drawing of a crown gear and pinion in section, as illustrated in Plate 23. The involute teeth have standard $14\frac{1}{2}°$ involute proportions, no backlash being provided. The diametral pitch is 2.

Show the tooth forms on the developments of normal cones at both ends of the teeth, constructing the profiles by the Grant Odontograph method.

Tabulate pitch diameters, numbers of teeth, diametral pitch, circular pitch, angle between shafts, face angles, edge angles, and diameters of blanks.

DATA:

Angular velocity ratio 6 : 10	Bore of pinion hub $1\frac{3}{16}$ in.
Pitch diameter of gear 10 in.	Keyways $\frac{3}{8} \times \frac{3}{16}$ in.
Bore of gear hub $1\frac{7}{16}$ in.	

Scale: Full size.

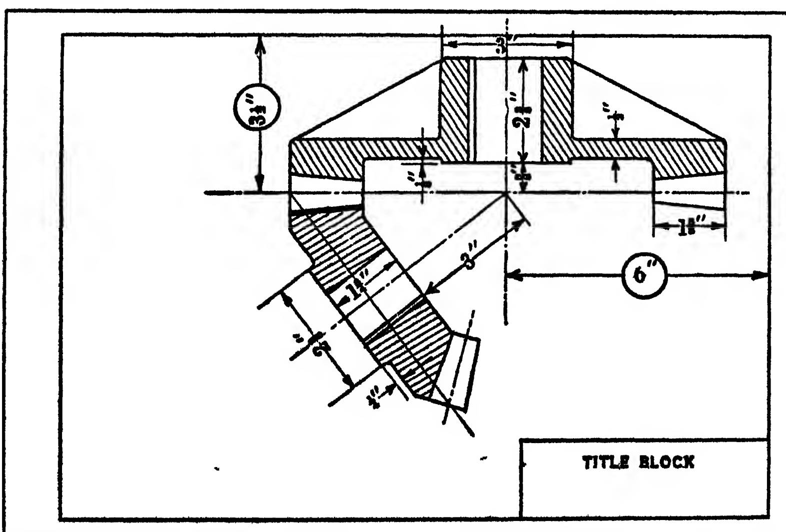


PLATE 23

PROBLEM 33

CROWN GEAR AND PINION

Same as Problem 32, except that angular velocity ratio is 7 : 10.

PROBLEM 34

WORM AND WORM WHEEL

Make a drawing of a worm and worm wheel with involute teeth, for a velocity ratio of 26 to 1.

DATA:

Worm — Steel, case-hardened, $14\frac{1}{2}°$ involute proportions, single right-hand thread, outside diameter $2\frac{3}{4}$ in., bore $\frac{15}{16}$ in., keyway $\frac{1}{4} \times \frac{1}{8}$ in.

Worm Wheel — Bronze, face angle 60°, bore $1\frac{7}{16}$ in., circular pitch 0.8125 in., keyway $\frac{3}{8} \times \frac{3}{16}$ in.

Dimension drawing fully, using decimals for the distance between centers, the throat diameter, the throat radius of the wheel, and the root diameter of the worm wheel.

Scale: Full size.

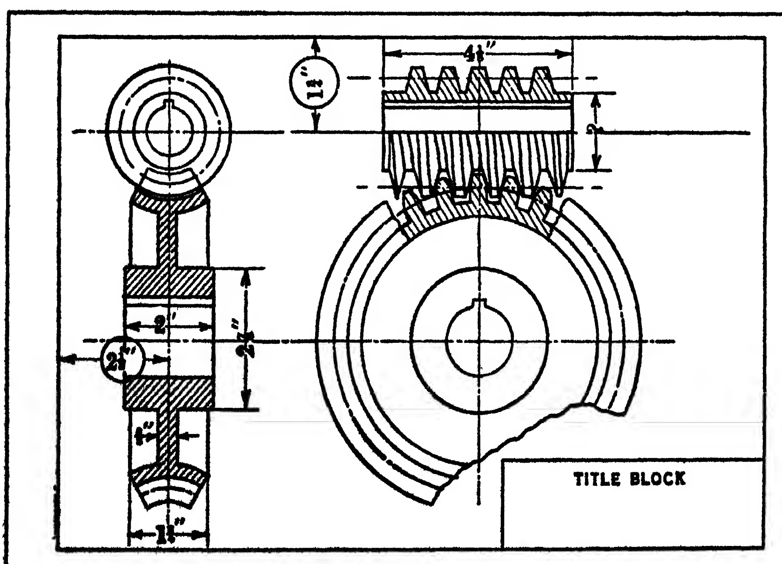


PLATE 24

PROBLEM 35

CONE PULLEYS

By the Burmester method, design a pair of cone pulleys, each with four steps, to connect, by means of an open belt, parallel shafts 30 in. center to center. The driver turns at 255 R.P.M., and the speed ratios of driver to driven are 3 : 1, 2 : 1, 1 : 1, and 1 : 2.5. The maximum belt speed is 1200 ft. per min. Each step is $2\frac{1}{4}$ in. wide with $\frac{1}{16}$-in. crown. The rim thickness is $\frac{1}{2}$ in. and the shafts are $1\frac{15}{16}$-in. diameter with keyways $\frac{3}{8} \times \frac{3}{16}$ in. The cones are attached to the hubs by four ribs each $\frac{1}{2}$ in. thick. The hubs are 5 in. long and $3\frac{3}{4}$ in. outside diameter, located centrally along the length of the cone pulley.

The pulley diameters found by the Burmester construction are to be rounded off to the nearest even $\frac{1}{8}$ in.

Calculate the length of belt for **each pair** of steps by using formula (10-5) in the text. Tabulate these lengths on the drawing.

Scale: 3 in. = 1 ft.

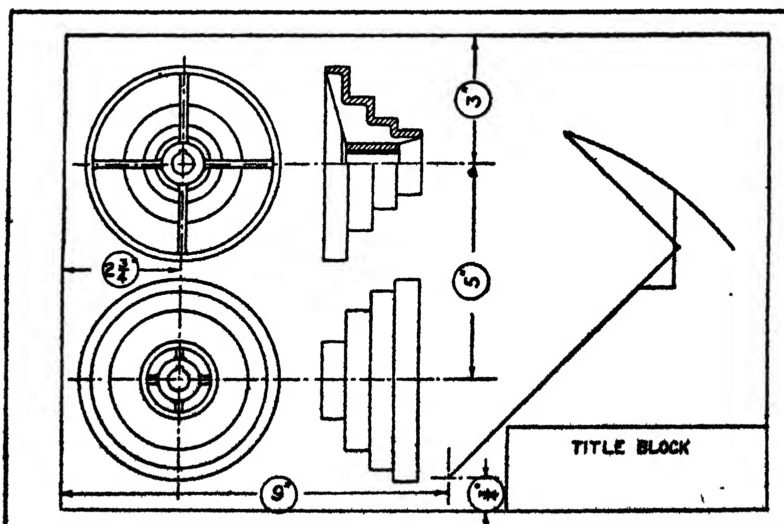


PLATE 26

PROBLEM 36

CONE PULLEYS

Refer to Plate 26 and follow the instructions given in Problem 35, but substitute for the data given in this problem the following:

The driver turns at 360 R.P.M. and the speed ratios of driver to driven shaft are 1 : 3, 1 : 1.5, 1 : 1, 2 : 1. The maximum belt speeds is 1700 ft. per min.

All other data as in Problem 35.

PROBLEM 37

GENEVA STOP MOTION

Design a Geneva Motion to give the driven member one-sixth of a revolution per revolution of the driving member, as shown in Plate 27.

The driving pin at *C* is 1 in. in diameter and the distance *BC* is 4 in. The driving shaft at *B* is 1½ in. in diameter, and the driven shaft at *A* is 1¾ in. in diameter. The disc carrying the driving pin is ¾ in. thick and has a diameter of 9½ in. The slotted member is ¾ in. thick.

Assume that the driving shaft rotates at a constant speed of 200 R.P.M. and plot an Angular Velocity-time Curve of the driven member in radians per second. Obtain the required angular velocities graphically for each 15° of motion of the driver using a vector 4 in. long to represent the angular velocity of this member.

Construct a graphical velocity scale on the velocity diagram, recording all calculations.

Scale: 6 in. = 1 ft.

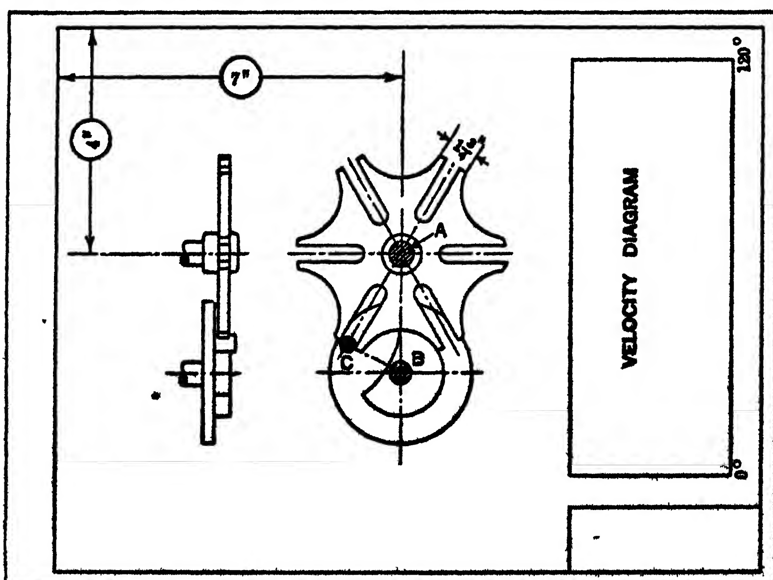


PLATE 27

PROBLEM 38

THE PANTAGRAPH

1. Draw the two forms of the pantagraph shown in Plate 28.

2. In the upper figure, move the tracing point *P* to other positions *Q*, *R*, *S*, *T*, *U*, located at uniform intervals around the circumference of a circle 1½ in. in diameter. Find the corresponding positions of the copying point *V*.

3. In the lower figure, move the tracing point *A* to points *B*, *C*, *D*, *E*, *F*, located on a triangle of the form shown in the plate. Find the corresponding positions of the copying point *G*.

4. For both pantagraphs, find the ratio of magnification.

Scale: Full size.

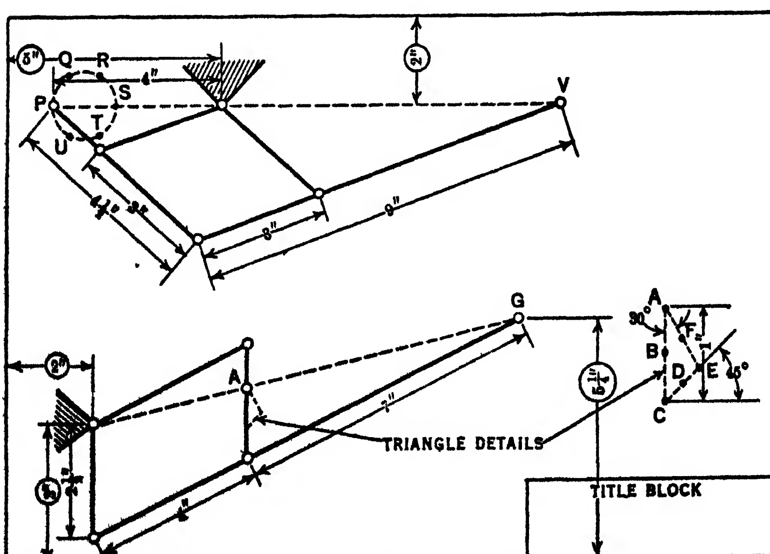


PLATE 28

PROBLEM 39

POSITIVE-MOTION CAM MECHANISM

A yoke-type positive-motion cam mechanism of the form shown in Plate 29 has a pivoted follower which swings through a total angle of 30°. The angular motion of the follower during its clockwise displacement is composed of constant acceleration and deceleration, the acceleration-deceleration ratio being 5 : 2, the complete movement taking place in 210° of clockwise cam displacement.

The mechanism has the following dimensions:

Diameter of cam shaft	1 in.
Cam hub diameter	2 in.
Follower shaft diameter	¾ in.
Follower hub diameter	1½ in.
Base circle diameter	3 in.

1. Find the width of the follower yoke.
2. Plot the displacement diagram from 0° to 210°.
3. Plot the cam profile.
4. Find the length of the arms on yoke and length of contact surface.
5. Plot the remainder of the displacement curve.

Scale: Full size.

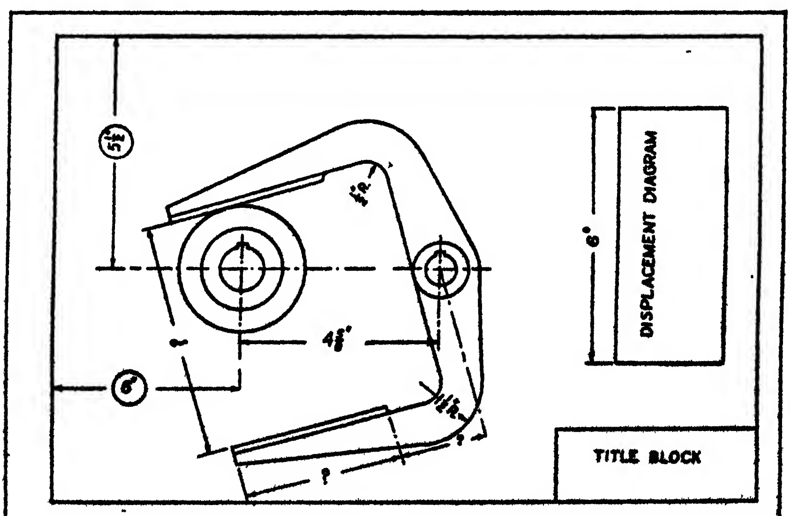


PLATE 29

PROBLEM 40

DRAG LINK COUPLING

A drag-link coupling shown on Plate 30 is used to produce variable speed on a driven shaft. The driving shaft turns at a uniform speed of 20 R.P.M. in a counter-clockwise sense. Crank *a* is keyed to the driving shaft and crank *c* to the driven shaft, the two being connected by drag link *b*.

1. Using the angular-velocity theorem of Article 4, Chapter IV, plot a curve showing the angular velocity of the driven shaft on a time base. Use an angular velocity scale of 1 in. = 8 R.P.M.

2. Find the driving crank angles at which the driven shaft has maximum and minimum speeds of rotation, also the angles at which its speed is equal to that of the driving shaft.

Scale: 6 in. = 1 ft.

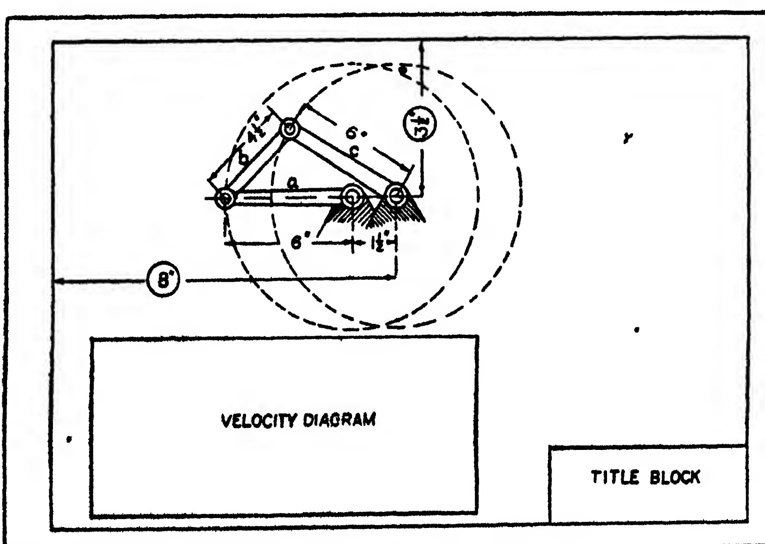


PLATE 30

INDEX

A

B

C