SCALE MODELS IN HYDRAULIC ENGINEERING

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BY

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WITH DIAGRAMS AND PHOTOGRAPHS

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PREFACE

I THINK it will be commonly accepted that very great importance is now attached to the use of scale models as an aid to hydraulic engineering design. Consequently, no apology appears to be necessary for a book possessing the scope of the present volume. Indeed, I have heard many practising engineers, students, research workers and scientific people not directly associated with engineering, express the need for such a book. It is for my readers, of course, to decide whether my effort to satisfy this demand has been at all successful.

My object, however, has been to try to set down the principles which underlie the technique of many kinds of hydraulic model experiments and, as far as practicable, to provide a critical survey of the present situation, drawing attention to the limitations as well as to the advantages of the method. I have also tried to indicate the scope for further systematic study, especially in the direction of a more precise determination of the nature and the magnitude of " scale-effects ".

It is true that there exists an extensive literature on these subjects, but mainly in the form of papers scattered through the journals of the learned societies and the pages of the technical press. Clearly, without the widest possible reference to these publications and a very free use of the information contained in them, it would have been quite useless to attempt a book like this. I hope most sincerely that the detailed references which I have tried to make of these invaluable sources of data will be accepted as a measure of my indebtedness to the many authors, societies and publishers involved. This applies not only to the text but also to many of the diagrams, which necessarily have had to be either copied or adapted from other authors' publications. Similarly, some pages and illustrations have been extracted with little or no modification from my own published papers.

In particular, much of my material issues from the pages of *The Engineer* and *Engineering*, the *Proceedings* and *Journal* of the Institution of Civil Engineers, the *Proceedings* and *Transactions* of the American Society of Civil Engineers, and the *Bulletins* of the U.S. Waterways Experiment Station, Vicksburg, Mississippi. At the same time, I owe a great deal to the late Mr. John R. Freeman's *Hydraulic Laboratory Practice*, published by the American Society of Mechanical Engineers, while permission has been granted to me by the Controller of H.M. Stationery Office, London, to utilize portions of Professor Gibson's *Tidal Model of the Severn Estuary*, especially in my Chapters numbered V and VI. Whenever I have appealed to these or others—authorities, engineers, editors, societies, and publishers alike—for permission to make use of previously printed matter, I have received the utmost kindness and co-operation.

A large part of the book will be found to deal with river problems, to which my directly personal experience has been chiefly confined and in the investigation of which I have been fortunate to be, as it were, apprenticed to Professor A. H. Gibson, himself a disciple of Osborne Reynolds. If my book possesses any merit, it is entirely due to this connection and to the work of others whose publications I have so freely consulted.

I am deeply indebted to the printers and publishers for the care which they have expended, and to Mr. P. E. Brockbank, M.B.E., M.Sc., A.M.I.E.E., for his untiring zeal and unflagging cheerfulness in reading the proofs.

J. ALLEN

THE UNIVERSITY,
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CHAPTER I

FUNDAMENTAL CONSIDERATIONS IN MODEL TECHNIQUE

As our knowledge stands at the moment, the resistance of a body moving relative to a fluid can only be determined by pure calculation in the simplest cases of streamline flow. For example, analysis shows that the loss of head in streamline motion through a cylindrical tube of length l and bore d is

$$h=\frac{32\nu lv}{gd^2},$$

where v is the mean velocity (that is, the rate of discharge divided by the cross-sectional area), and ν is the kinematic viscosity of the fluid, measured in absolute units.* But even this statement is only applicable to the central portions of a long tube, because it neglects the loss of energy at the ends and it assumes the tube to be long enough to permit a parabolic distribution of velocity across a diameter to be attained as visualized in the ideal mathematical analysis. Corrections for these effects can only be accurately determined by experiment. Thus, if the tube connects two reservoirs containing water with a difference of level (or head) h, experiments indicate that

$$\frac{2gh}{v^2}=\alpha+\beta\frac{l}{d}\cdot\frac{1}{R},\dagger$$

where

$$\begin{cases}\alpha=2{\cdot}60-15{\cdot}5\dfrac{d}{l},\\ \beta=64{\cdot}00+1{\cdot}33\times10^6\left(\dfrac{d}{l}\right)^{3\cdot44},\\ R=\dfrac{vd}{\nu}.\end{cases}$$

* $\nu=\dfrac{\mu}{\rho}$, where μ is the viscosity and ρ is the density of the fluid. The shear stress tangential to a plane in the fluid is defined as $\mu\dfrac{dv}{dy}$, where $\dfrac{dv}{dy}$ is the velocity gradient normal to the plane. If this stress is measured in dynes per sq. cm., and dv in cm. per sec. and dy in cm., then μ becomes the coefficient of viscosity in absolute C.G.S. units, often called "poises". If ρ is in grm./cm.^{3}, ν becomes the kinematic viscosity in absolute C.G.S. units.

Similarly, if stress is defined in poundals/ft.^{2}; dv in ft./sec., and dy in feet, μ is the coefficient of viscosity in absolute ft.-lb.-sec. units, while measuring ρ in lb./ft.^{3}, results in ν being the coefficient of kinematic viscosity in absolute ft.-lb.-sec. units.

Expressed dimensionally, μ has the units $ML^{-1}T^{-1}$ and ν has those of L^2T^{-1} if force is measured in either poundals or dynes.

† "Some Experiments having Particular Reference to the Flow of Water along Short Capillary Tubes connecting Two Vessels with Free Surfaces", Allen, *Proc. Roy. Soc. Edin.*, vol. LVI, p. 26 ,March 1936.

If, then, complications of this kind are encountered in such relatively simple problems, how much more so may they be found in the type of phenomena with which the practising engineer is concerned. It is natural, therefore, that the method of model experiments should have commanded an increasing attention within recent years. Before the technique of such investigations can be appreciated, however, it is necessary to study the underlying principles of " dynamical similarity ".

If a model is to be similar, in a dynamical sense, to its prototype, two conditions are demanded:

(*a*) The paths described by corresponding particles shall be geometrically similar.

(*b*) Corresponding forces shall bear a constant ratio to one another. For example, if a force called into play by viscosity in the model is $\frac{1}{10}$ of the equivalent viscous force in the actual, so also a force due to surface tension should be $\frac{1}{10}$ of *its* counterpart. But the fundamental concept of a force is the product of mass and acceleration. Consequently whatever the origin of the forces considered—whether viscosity, surface tension, or otherwise—such forces should be proportional to mass × acceleration. Thus, let us consider a portion of fluid, of mass m, to be moving with velocity v in a circular path of radius r in the model, while the corresponding mass M in the actual system describes a path of radius R with velocity V. The corresponding radial forces are $\frac{mv^2}{r}$ and $\frac{MV^2}{R}$. But, in virtue of geometrical similarity, if l is any linear dimension in the model, and L the equivalent dimension in the actual, then $r/R = l/L$, while $m \propto \rho l^3$ and $M \propto PL^3$, where ρ, P represent densities.

It is clear, therefore, that the forces $\frac{mv^2}{r}$ and $\frac{MV^2}{R}$ are in the ratio of $\rho l^2 v^2$ to PL^2V^2. Moreover, a little reflection will show that precisely the same conclusion is dimensionally true of any product (mass × acceleration) ; since mass × acceleration * is proportional to

$$\text{mass} \times \text{velocity} \times \frac{\text{velocity}}{\text{distance}};$$

or to $\rho l^2 v^2$. The fundamental force-scale is, accordingly, represented by $\rho l^2 v^2 : PL^2V^2$, and this scale-ratio ought equally to apply to forces generated in virtue of viscosity or any other physical factor. The actual speeds at which such similarity exists between model and prototype are called " corresponding speeds ", and we shall presently find that in general

* Mass × acceleration $= m\frac{dv}{dt} = m\frac{dv}{dx}\cdot\frac{dx}{dt} = mv\frac{dv}{dx} \propto \rho l^3 v\frac{dv}{dx}$, or dimensionally $\propto \rho l^2 v^2$.

it is physically impracticable to ensure that forces caused by different effects are all in one ratio to those in the other system *at one and the same "corresponding speed"*.

An instrument which provides a powerful aid in our search for "corresponding speeds" is the principle of dimensional homogeneity, which requires all terms of a *true physical equation** to possess the same units in mass, length and time. $v = u + ft$ is a case in point, v, u and (ft) each having the dimensions LT^{-1}, or distance divided by time. We must realize, however, that $v = u + nft$ would be equally sound dimensionally if n were any number : i.e. given that the third term of the equation involved t we might infer that it was of the form $n\,ft$, where n must be found by other reasoning or by experiment.

As an example, let us consider the resistance to the flow of water through a cylindrical pipe. *Suppose it to be possible* to express this resistance in the form

$$R = Kl^{\alpha}\rho^{\beta}v^{\gamma}\mu^{x}d^{y}k^{z}, \qquad (1)$$

where R = resistance (in poundals or dynes),
K = a constant coefficient,
l = length of pipe (in feet or cm.),
ρ = density of fluid (lb. per ft.^{3} or grm. per cm.^{3}),
v = mean velocity (ft./sec. or cm./sec.),
μ = viscosity (in absolute units—lb.ft.$^{-1}$ sec^{-1} or grm.cm.$^{-1}$ sec^{-1}),
d = diameter of pipe (ft. or cm.),
k = height of projecting roughnesses from wall of pipe (ft. or cm.).

Using M, L, T to represent the fundamental units of mass, length and time, we have MLT^{-2} as the dimensions of R, while those of the right-hand side of eqn. (1) are

$$L^{\alpha}M^{\beta}L^{-3\beta}L^{\gamma}T^{-\gamma}M^{x}L^{-x}T^{-x}L^{y}L^{z}.$$

Equating the indices of like quantities, we obtain

$$\begin{array}{ll} \text{for } M, & 1 = \beta + x, \qquad (2) \\ \text{for } L, & 1 = \alpha - 3\beta + \gamma - x + y + z, \qquad (3) \\ \text{for } T, & -2 = -\gamma - x. \qquad (4) \end{array}$$

From (2), $$x = 1 - \beta. \qquad (5)$$

From (4), $$\gamma = 2 - x;$$
$$\therefore\ \gamma = 1 + \beta. \qquad (6)$$

Hence, from (3),
$$\begin{aligned} y &= 1 - \alpha + 3\beta - \gamma + x - z \\ &= 1 - \alpha + 3\beta - 1 - \beta + 1 - \beta - z \\ &= 1 - \alpha + \beta - z. \qquad (7) \end{aligned}$$

* For a more explicit definition, see "Dimensional Analysis", p. 41, Bridgeman (Yale Univ. Press or Humphrey Milford, 1943).

Substituting (5), (6), (7) in eqn. (1), we get

$$R = Kl^{\alpha}\rho^{\beta}v^{1+\beta}\mu^{1-\beta}d^{1-\alpha+\beta-z}k^{z},$$

or

$$R = K\left(\frac{l}{d}\right)^{\alpha}\left(\frac{k}{d}\right)^{z}\rho\, d^2v^2\left(\frac{vd\rho}{\mu}\right)^{\beta-1}. \qquad (8)$$

If the resistance is measured in terms of the head h of the fluid lost in flowing along the length l of pipe,

$$R = \frac{\pi d^2}{4}h\rho g. \qquad (8\text{A})$$

Consequently,

$$h = \frac{4K}{\pi}\cdot\frac{v^2}{g}\left(\frac{l}{d}\right)^{\alpha}\left(\frac{k}{d}\right)^{z}\left(\frac{vd\rho}{\mu}\right)^{\beta-1},$$

or

$$h = K_1\frac{v^2}{g}\left(\frac{l}{d}\right)^{\alpha}\left(\frac{k}{d}\right)^{z}\left(\frac{vd}{\nu}\right)^{\delta}. \qquad (9)$$

We are now in a position to design a set of experiments which might establish the full law of resistance. We will confine our attention for the time being to the case of " smooth " pipes. Keeping all other variables constant, we can change the gauge-length l over which the loss of head is measured. Such experiments will show h to be proportional to l; therefore $\alpha = 1$. Next, in a given pipe and with water at a constant temperature, let h be measured for different rates of discharge and consequently for different values of v. We shall find that up to a certain velocity, h is proportional to v; at higher velocities h is proportional to v^n, where n is nearer to 2 than 1. When h varies as the first power of v, we see from equation (9) that $2+\delta=1$, or $\delta=-1$; when h is proportional to v^n, $2+\delta=n$ or $\delta=n-2$. Again, if we experiment with different diameters of " smooth " pipe, we find that, for a given value of vd (the temperature and therefore the kinematic viscosity being kept constant), $\frac{hd}{v^2l}$ is constant; i.e. z in equation (9) is zero when the pipes are " smooth ".

As an alternative to keeping the temperature constant, it is more convenient actually to measure the temperature while making the tests and to employ the corresponding kinematic viscosity.* The only other measurements required (besides l and d) are h and the rate of discharge Q. From Q, we obtain $v\left(=\frac{4Q}{\pi d^2}\right)$ and eqn. (8A) enables us to calculate the total resistance R. Now suppose the quantity $\frac{Rd}{l\rho d^2v^2}$, or $\frac{R}{\rho l d v^2}$ to be plotted against $\frac{vd}{\nu}$. Experimental observations then show that, provided the pipes are smooth, the plotted points belong to one and the same law

* Values for air and for water will be found in Appendix A, p. 397.

whatever the pipe diameter and the fluid (water, air or oil, etc.). This method of plotting, in effect, was adopted by Stanton and Pannell,* who used $\frac{R_1}{\rho v^2}$ as ordinate plotted against $\log\left(\frac{vd}{\nu}\right)$. Here R_1 represents the resistance per unit area of the wall, i.e. $\frac{R}{\pi dl}$, and the log of $\frac{vd}{\nu}$ is used rather than $\frac{vd}{\nu}$ itself purely as a matter of convenience in that it serves to telescope the higher values of $\frac{vd}{\nu}$. In this way, they derived the classic

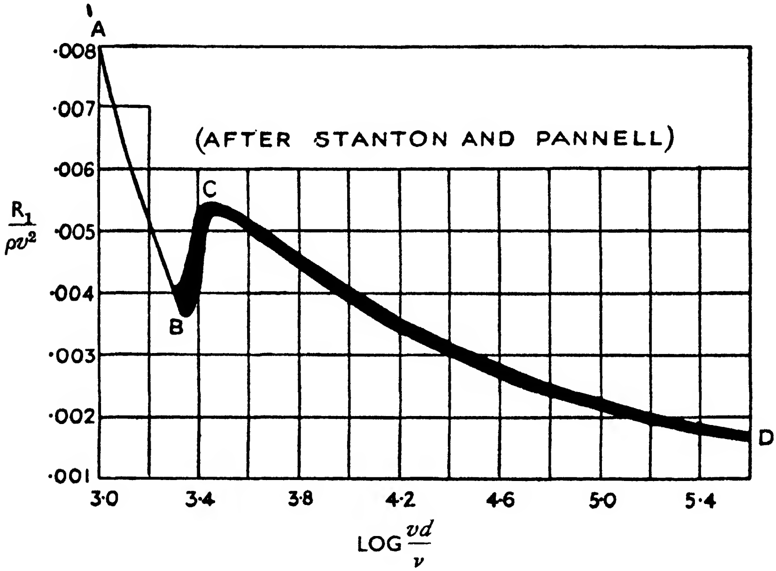


FIG. 1.

diagram shown in the sketch of Fig. 1, consideration of which indicates that over the region A–B,

$$\frac{R_1}{\rho v^2} \propto \frac{\nu}{vd},$$

or
$$R_1 \propto \frac{v\nu\rho}{d},$$

or
$$R \propto lv\nu\rho,$$

or
$$h \propto \frac{\nu lv}{gd^2} \qquad \text{(from (8A)).}$$

* *Phil. Trans. Roy. Soc.*, A, vol. 214, or *National Phys. Lab. Coll. Res.*, vol. XI, 1914, p.293.

Over this range (A–B) the flow is streamline and laminar, and the relationship just obtained is that $\left(\text{viz. } h = \frac{32\nu lv}{gd^2}\right)$ which also results from pure mathematical reasoning.

Over the range CD, however, $\frac{R_1}{\rho v^2}$ appears to be proportional to $\left(\frac{vd}{\nu}\right)$ raised to a power which itself apparently depends to some extent upon $\frac{vd}{\nu}$. If the line CD in Fig. 1 were horizontal, $\frac{R_1}{\rho v^2}$ would be constant and h would then be proportional to $\frac{lv^2}{dg}$, a law which would correspond with "fully turbulent flow".

To summarize the situation as far as we have gone, we have been led to the following principal conclusions :

(1) That the dimensionless quantity $\frac{vd}{\nu}$, now commonly called the Reynolds Number, is a criterion of the type of flow in cylindrical pipes. If this number is less than about 2,000,* the flow is laminar in character and (as would be anticipated from mathematical reasoning alone) h is then equal to $\frac{32\nu lv}{gd^2}$. For Reynolds numbers above about 3,000, h is proportional to $\frac{\nu^{2-n} lv^n}{gd^{3-n}}$, where n depends upon $\frac{vd}{\nu}$. An approximate equation, due to Davies and White,† is

$$h = \frac{0{\cdot}16 lv^2}{gd}\left(\frac{vd}{\nu}\right)^{-0{\cdot}25}, \quad \text{or} \quad \frac{0{\cdot}16 lv^{1{\cdot}75}\nu^{0{\cdot}25}}{gd^{1{\cdot}25}}.$$

This equation is said to agree with the observations of Stanton and Pannell within the limits of experimental accuracy over the range of $\frac{vd}{\nu}$ from 3,000 to 150,000.

(2) It is clear that in comparing the resistance of different pipes, we must regard the Reynolds number as the vital factor, for if in one case this number is low enough to favour streamline motion, the resistance is proportional to the first power of the velocity, while if in the other case $\frac{vd}{\nu}$ is high enough to favour turbulence, then the resistance varies more nearly as the square of v. This phenomenon is evidently one

* We are assuming that there is a sufficient length of pipe between its inlet and the gauge-length over which observations are made to allow initial disturbances to be damped out; i.e. for the motion to become laminar if it can.

† "A Review of Flow in Pipes and Channels" Davies and White, *Engineering*, July 19, 1929, p. 71.

which we must keep constantly in mind when dealing with any model experiments.

(3) If it is true that the index of v depends upon the Reynolds number, we are led to the view that equation (8) has been written in too simple a form, and indeed that it is more correctly

$$R = K\rho d^2 v^2 \times \text{some function of } \left\{\frac{l}{d}, \frac{k}{d}, \frac{vd}{\nu}\right\},$$

which a more rigorous mathematical analysis would have shown.

In engineering practice, it is usual to write the loss of head in pipes as

$$h = \frac{flv^2}{2gm},$$

where $\quad m$ = the hydraulic radius or hydraulic mean depth

$$= \frac{\pi d^2}{4} \Big/ \pi d$$

$$= \frac{d}{4}.$$

The coefficient f therefore becomes $\frac{ghd}{2lv^2}$, or $\frac{2}{\rho v^2} \cdot \frac{R}{\pi dl}$ (from eqn. 8A), which is the same as $\frac{2R_1}{\rho v^2}$, where R_1 is the resistance (in poundals per sq. ft. or dynes per sq. cm.) per unit area of the pipe walls.

Accordingly, for smooth pipes we may use $h = \frac{flv^2}{2gm}$, where

$$f = 16\left(\frac{vd}{\nu}\right)^{-1} \text{ for values of } \frac{vd}{\nu} \text{ up to about 2,000}$$

and $\quad f = 0{\cdot}08\left(\frac{vd}{\nu}\right)^{-0{\cdot}25}$ for values of $\frac{vd}{\nu}$ between 3,000 and 150,000.

Or again, if the Reynolds number exceeds about 4,000, we may use the more precise equation, due to Prandtl,*

$$\frac{1}{\sqrt{4f}} = 2{\cdot}0 \log_{10}(R_e\sqrt{4f}) - 0{\cdot}8, \quad \text{where } R_e = \frac{vd}{\nu}.$$

Between 2,000 and 3,000 lies a range of transition or instability for which it is difficult to give a precise law.

Before passing on to other considerations, it may be remarked that additional proof of the validity of the Reynolds number as a criterion of similarity in pipe flow was furnished by Stanton and Pannell,† who measured the ratio of the mean to the maximum velocity at a cross-section

* Prandtl, *Aerodynamic Theory*, vol. 3, p. 143.

† Stanton and Pannell, Nat. Phy. Lab., *Collected Researches*, vol. XI, p. 295, 1914; or *Phil. Trans. Roy. Soc.*, A, vol. 214.

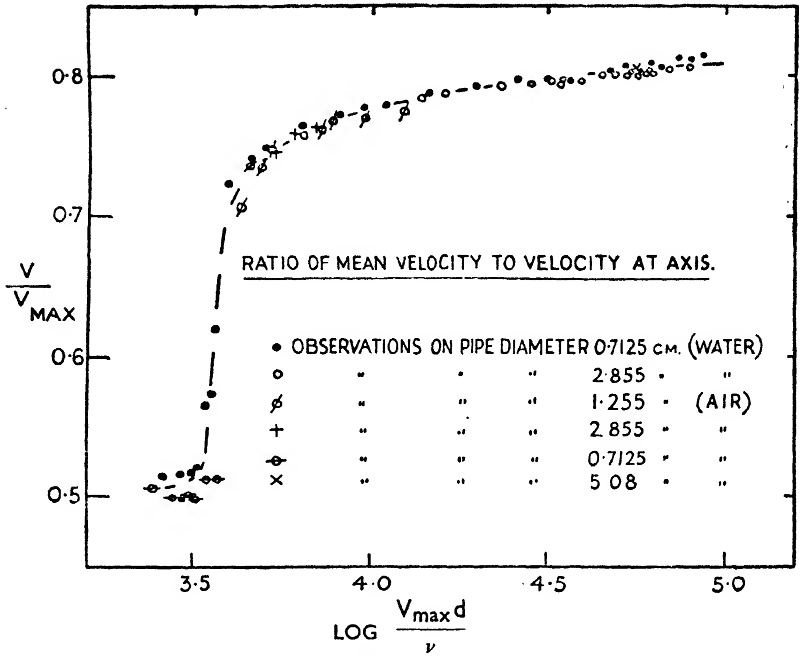


FIG. 2.

of a smooth pipe. Mathematical considerations show that with laminar motion this ratio should be 1 : 2. The results of Stanton and Pannell, using different fluids and pipe sizes, are shown in Fig. 2; in a previous Paper on the " Mechanical Viscosity of Fluids " (*Proc. Roy. Soc.*, A, vol. 85, p. 366), they had shown that the radial variation of the velocity of air in smooth brass pipes of 4·9 and 7·4 cm. diameter gave identical velocity-distribution curves only when $\frac{vd}{\nu}$ had the same value; they further found that the distribution of velocity was only independent of $\frac{vd}{\nu}$ when the surfaces of the pipes were so roughened that the resistance varied as the square of the velocity.

Pipes of Non-Circular Section. There is a good deal of experimental evidence to show that if the flow is turbulent, the value of f for various shapes of section is nearly the same as for a cylindrical pipe at the same value of $\frac{vm}{\nu}$, where m represents the hydraulic mean depth. The critical value of $\frac{vm}{\nu}$ above which flow initially turbulent will persist in being so, does appear, however, to vary with the shape. Thus, if we consider rectangular pipes, we realize that whereas all smooth cylindrical pipes are geometrically similar to one another, rectangular passages on the other hand must be considered in relationship to their ratio of width ($2a$) to depth ($2b$). Indeed, when such a passage is discharging an incompressible viscous fluid with *laminar* motion, mathematical analysis shows that the ratio of maximum to mean velocity and the position of the filament of mean velocity depend upon the ratio $a:b$,* while the rate of discharge is given by

$$Q = -\frac{4}{3}\frac{ab^3}{\mu}\cdot\frac{dp}{dz}\left\{1 - \frac{192}{\pi^5}\cdot\frac{b}{a}\left(\tanh\frac{\pi a}{2b} + \frac{1}{3^5}\tanh\frac{3\pi a}{2b} + \dots\right)\right\}\dagger$$

The Author believes ‡ that the critical $\frac{vm}{\nu}$ for turbulent flow is approximately inversely proportional to the theoretical ratio of maximum to mean velocity during laminar motion, thus:

TABLE I

$\frac{a}{b}$	*ratio* $v_{\max} : v_{\text{mean}}$	*Critical* $\frac{vm}{\nu}$
1·0	2·10	525
2·0	2·00	550
3·0	1·86	590
4·0	1·79	615
5·0	1·74	630
∞	1·50	730

[The critical $\frac{vd}{\nu}$ observed by Stanton and Pannell for cylindrical pipes is about 2,300, which is equivalent to a $\frac{vm}{\nu}$ of 575.]

It appears, therefore, that while for complete hydrodynamical similarity one rectangular passage must be geometrically similar to another, yet, provided the flow is turbulent, the Reynolds number based on the hydraulic

* Allen, " On the Ratio of the Maximum to the Mean Velocity, and the Position of the Filament of Mean Velocity, in the Laminar Motion of an Incompressible Viscous Fluid through a Pipe of Rectangular Cross-section ", *Phil. Mag.*, Ser. 7, vol. XVIII, p. 488, Sept. 1934.

† Cornish, " Flow in a Pipe of Rectangular Cross-section ", *Proc. Roy. Soc.*, A, CXX, pp. 691–700 (1928).

‡ Allen and Grunberg, " The Resistance to the Flow of Water along Smooth Rectangular Passages, and the Effect of a Slight Convergence or Divergence of the Boundaries ", *Phil. Mag.*, Ser. 7, vol. XXIII, p. 490, March 1937.

mean depth m is a sufficiently true criterion. In other words, the friction coefficient f for a given $\frac{vm}{\nu}$ depends upon $a:b$ if the flow is laminar, but is approximately independent of $a:b$ for turbulent flow. Again, the critical $\frac{vm}{\nu}$ is a function of the $a:b$ ratio.

Rough Cylindrical Pipes. When we wrote down equation (1) on page 13, we included a term k to represent the height of surface irregularities at the wall of the pipe, and by dimensional analysis we derived the expression

$$h \propto \frac{v^2}{g}\left(\frac{l}{d}\right)^{\alpha}\left(\frac{k}{d}\right)^{z}\left(\frac{vd}{\nu}\right)^{\delta},$$

or more generally,

$$h \propto \frac{v^2}{g} \times \text{a function of } \left\{\frac{l}{d}, \frac{k}{d}, \frac{vd}{\nu}\right\}.$$

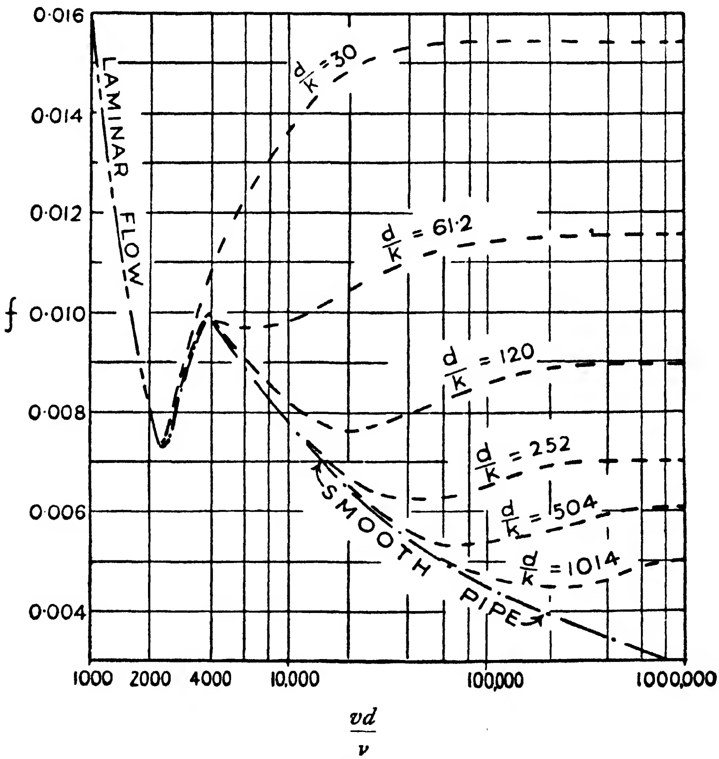


FIG. 3.

Accordingly, if h is equal to $\frac{2fl\,v^2}{gd}$, then

f becomes a function of $\left(\frac{k}{d}, \frac{vd}{\nu}\right)$.

The experimental investigation of the influence of the $\frac{k}{d}$ ratio offers a fertile field of research which has as yet been incompletely explored, but notable contributions have been made by workers such as Nikuradse* at Göttingen and Colebrook and White † at Imperial Co llege, London Nikuradse's results are shown in Fig. 3, from which it will be seen tha in the condition of laminar flow the roughness has no appreciable effect, and that round about the critical Reynolds number the curves for different $\left(\frac{d}{k}\right)$ ratios lie together within a narrow band. Thereafter, the curves depart from the smooth pipe law, and the earliest departure is shown by the roughest surface. Over a certain range, the value of f increases with $\frac{vd}{\nu}$: this is part of the so-called " transition region ", while there is a general tendency for f to become constant at sufficiently high Reynolds numbers.

An explanation of the development in the effect of roughness, largely due to Prandtl, is that first of all the surface excrescences lie inside a very thin viscous layer at the wall of the pipe even when the main motion in the pipe is turbulent; at higher values of $\frac{vd}{\nu}$, the roughnesses begin to project beyond the viscous layer, which has become thinner, and they start to shed eddies, for the maintenance of which additional energy is absorbed. Ultimately, all the excrescences stand outside the viscous layer and their own resistance (which actually depends upon their shape) predominates.‡

Prandtl § and von Kármán || find that Nikuradse's results may be made to lie on one curve, or within one narrow band, by plotting the quantity

* Nikuradse, J., *Forschungsh. Ver. dtsch. Ing.*, No. 361, 1933.

† Colebrook and White, "Experiments with Fluid Friction in Roughened Pipes", *Proc. Roy. Soc.*, A, CLXI, p. 367, 1937; also " The Reduction of Carrying Capacity of Pipes with Age ", *Journ. Inst. C.E.*, Nov. 1937, p. 99.

‡ For a discussion of the mechanism of turbulence, see " Turbulence ", the Twenty-fifth Wilbur Wright Memorial Lecture, by Theodor von Kármán, *Journ. Roy. Aeronautical Society*, Dec. 1937.

§ Prandtl; "Z. Ver. dtsch. Ing.", 77, No. 5, p. 105, 1933; "Aerodynamic Theory", vol. 3.

|| von Kármán, *Proc. Fourth Intern. Congr. Applied Mech.*, Cambridge, 1935: *Nach Ges. Wiss. Göttingen*, 1930.
Also see *Forschungsheft V.D.I.*, 361. 1933.

$\frac{1}{\sqrt{4f}} - 2 \log \frac{r}{k}$, r being the pipe radius, against a new Reynolds number, viz. $\frac{V_* k}{\nu}$, in which V_* represents $v\sqrt{\frac{f}{2}}$.*

When $\frac{V_* k}{\nu}$ exceeds 60, Nikuradse has found that the friction coefficient f is constant for a given pipe, i.e. the resistance is independent of viscosity and is, in fact, proportional to v^2. Under such conditions of fully turbulent flow,

$$\frac{1}{\sqrt{4f}} - 2 \log \frac{r}{k} = 1{\cdot}74,$$

which may be written as

$$\frac{f}{2} = \frac{\tau_0}{\rho v^2} = \frac{1}{8}\left(2 \log \frac{3{\cdot}7d}{k}\right)^{-2} = \frac{1}{8}\left(2 \log \frac{14{\cdot}8m}{k}\right)^{-2} \quad \ldots\ldots\ldots\ldots(10)$$

There are three considerations, however, which we have not yet mentioned: the possibility that the shape of the roughening irregularities, the density of their distribution, and the uniformity or otherwise of their individual grains may influence the resistance. Certainly one would expect this to be the case, especially when the excrescences extend for any appreciable distance outside the viscous boundary layer of fluid near the pipe wall. In other words, our original equation (1) ought to have included terms which are a measure of shape, uniformity of size and intensity of distribution.

Colebrook and White † have in fact attacked the problem of the intensity of distribution, and, while their results do not lead to any general solution, they are nevertheless of great interest. Working with a pipe 5·35 cm. diameter they tried first the effect of roughening its surface with sand grains 0·035 cm. diameter " in a very uniform manner just touching each other ", and found that the resistance was indistinguishable from

* If τ_0 represents the shear stress at the wall of the pipe, $2\tau_0 l\pi r = R = \pi r^2 h\rho g$

or
$$\tau_0 = \frac{rh\rho g}{2l} = \frac{mh\rho g}{l},$$

and if
$$h = \frac{flv^2}{2gm},$$

$$\tau_0 = \frac{flv^2}{2gm} \cdot \frac{m\rho g}{l} = \frac{fv^2\rho}{2},$$

or calling $V_* = \sqrt{\frac{\tau_0}{\rho}}$, we have $V_* = v\sqrt{\frac{f}{2}}$, which is thus seen to have the dimensions of a velocity, since f itself is dimensionless.

† *Proc. Roy. Soc.*, A, CLXI, p. 367.

that of a smooth pipe when $\frac{vd}{\nu} \approx 6{,}000$, but as the speed increased, " transition to rough law begins and is completed at $\frac{vd}{\nu} = 100{,}000$ ". The effect of superimposing grains of ten times this size on about 2 per cent. of the area was to increase the resistance by 12 per cent. at high speeds and by 20 per cent. at $\frac{vd}{\nu} = 10{,}000$. Doubling the number of large grains produced about twice these percentage differences, while this number of large grains alone gave resistance coefficients 7 per cent. lower at $\frac{vd}{\nu} = 100{,}000$ and 40 per cent. higher at 10,000, compared with the uniform roughening with the smaller grains.

The work of Colebrook and White * has been discussed at some length by the Author in a Paper entitled " Roughness Factors in Fluid Motion through Cylindrical Pipes and through Open Channels ", published in the *Journal* of the Institution of Civil Engineers, April 1943, p. 91. This analysis indicated that :

" (*a*) If a pipe is covered with excrescences of size k, evenly distributed over p per cent. of its area, its resistance follows the law of a smooth pipe until such a Reynolds number is reached as would cause departure from the smooth law if the pipe were *wholly* covered with grains k. Then follows a stage in which the law appropriate to the *completely* roughened surface is obeyed until the excess of f over that for a smooth surface is equal to the difference between the " smooth " f and that of a pipe uniformly roughened with grains of size $\frac{pk}{100}$ at that Reynolds number when " complete turbulence " would set in. The final stage is a line parallel to that which would apply if the surface were wholly roughened with grains $\frac{pk}{100}$.

" (*b*) In the second instance, if the given pipe is covered uniformly over x_1 per cent. of its area with fine grains k_1, and over x_2 per cent. with coarser grains k_2, the departure from the smooth law follows that appropriate to a surface completely covered with the coarser grains k_2 until the excess of the f so obtained over that of a smooth pipe is equal to the difference between the smooth f and that of the pipe uniformly roughened

* Colebrook and White, " Experiments with Fluid Friction in Roughened Pipes ", *Proc. Roy. Soc.*, A, CLXI, p. 367, 1937; " The Reduction of Carrying Capacity of Pipes with Age ", *Journ. Inst. C.E.*, Nov. 1937, p. 99; and Colebrook, " Turbulent Flow in Pipes, with particular reference to the Transition Region between the Smooth and Rough Pipe Laws ", *Journ. Inst. C.E.*, Feb. 1939, p. 133.

with grains of size $\frac{x_2 k_2}{100}$ when " complete turbulence " is attained. Then follows a stage in which the line of resistance lies parallel to that of a pipe wholly roughened with grains of $\frac{x_1 k_1}{100}$."

Straight Open Channels. The following are the principal features of flow through straight open channels.

(1) On the assumption that the velocity gradient at the surface, perpendicular to the surface, is zero, it is possible to calculate * the friction coefficient f for *true laminar flow*, f being the usual number in the equation

$$\frac{dr}{dl}=\frac{v}{g}\cdot\frac{dv}{dl}+\frac{fv^2}{2gm},\dagger$$

where r = the depth of the surface of the stream below some chosen datum,
l = the length,
v = the mean velocity at a given cross-section,
m = the hydraulic mean depth.

According to this theory, $f=K\left(\frac{\nu}{vm}\right)$, where K depends in complex fashion upon the dimensions of the channel section. Experiments on *rectangular* channels,‡ however, show close agreement with the calculated f only up to a value of about 300 for the Reynolds number $\frac{vm}{\nu}$. Then follows a range in which the resistance is higher than might be expected, and the Author attributes this phenomenon to the fact that the motion, while streamline, is not laminar, and the velocity gradient is no longer zero at the water surface. Superimposed upon the laminar flow, there are transverse currents which increase the loss of energy.

(2) The critical velocity for turbulence is given approximately by $\frac{vm}{\nu}=1{,}400$, a value about 1·9 times as great as that for rectangular *pipes* having a large aspect ratio of width to depth. This value of 1,400 § is

* Cornish, " Flow in a Pipe of Rectangular Cross-section ", *Proc. Roy. Soc.*, A, CXX, p. 691, 1928.

† In uniform flow, with a mean velocity constant from section to section along the length of the channel, this equation assumes the familiar form $v=C\sqrt{mi}$, where $C^2=2g/f$.

‡ Allen, " Streamline and Turbulent Flow in Open Channels ", *Phil. Mag.*, Ser. 7, vol. XVII, p. 1081, June 1934.

§ Thus, in a channel 100 ft. wide and 3 ft. deep, the critical velocity is only about 0·07 inch per second, when water at 15° C. is flowing.

independent of the roughness of the channel, at any rate for moderate roughnesses or certainly if $\frac{m}{k}$ is greater than about 25, k being as before the size of the excrescences.

(3) In *turbulent flow* through smooth rectangular open channels, the equation

$$f=0{\cdot}0234\left(\frac{vm}{\nu}\right)^{-0{\cdot}15}$$

may be taken to apply with considerable accuracy. There are indications, however, that the resistance of *semicircular* channels tends to follow more nearly the law for circular pipes, if $\frac{vm}{\nu}$ is sufficiently high. Thus, when $\frac{vm}{\nu}=100{,}000$, the value of f for smooth semicircular channels is approximately $\frac{f_p+2f_c}{3}$, where f_p satisfies Prandtl's equation for circular pipes when written in the form

$$\frac{1}{\sqrt{4f_p}}=2\log_{10}\left\{\left(\frac{4vm}{\nu}\right)\sqrt{4f_p}\right\}-0{\cdot}8,$$

or

$$\frac{1}{\sqrt{f_p}}=2{\cdot}012+4\log\left\{\frac{vm}{\nu}\sqrt{f_p}\right\},$$

and where

$$f_c=0{\cdot}0234\left(\frac{vm}{\nu}\right)^{-0{\cdot}15}.$$

When $\frac{vm}{\nu}=300{,}000$, f for semicircular channels is approximately $\frac{2f_p+f_c}{3}$ and when $\frac{vm}{\nu}=1{,}000{,}000$, it is about equal to f_p.

(4) If both the sides and bed of the *rectangular* channel are rough, f assumes a constant value * for a given m if the Reynolds number $\frac{vm}{\nu}$ is in general higher than 4,000. Moreover, a formula due to Bazin then applies with remarkable accuracy, viz.

$$C=\sqrt{\frac{2g}{f}}=\frac{157{\cdot}6}{1+\frac{N}{\sqrt{m}}}$$

* The loss of head is then proportional to the square of the mean velocity.

In this formula, m is to be measured in feet, g in ft./sec.^{2}, and N for typical surfaces is as follows :

TABLE II

Smooth cement or planed wood - - -	$N=0{\cdot}109$
Planks, bricks or cut stone - - - -	$N=0{\cdot}290$
Rubble masonry - - - - - - -	$N=0{\cdot}833$
Earth channels of very regular surface, or revetted with stone - - - - -	$N=1{\cdot}54$
Ordinary earth channels - - - -	$N=2{\cdot}35$
Exceptionally rough earth channels (bed covered with boulders) or weed-grown sides	$N=3{\cdot}17$

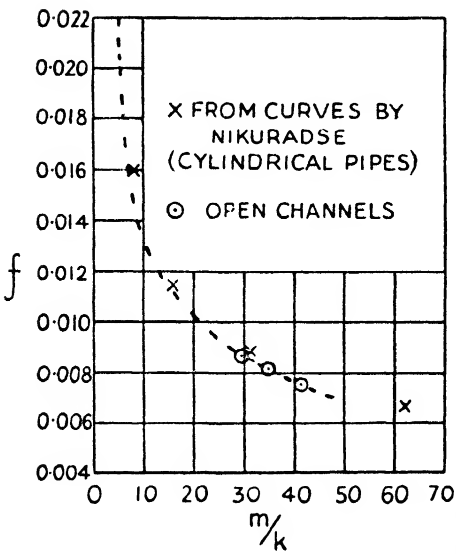


FIG. 4.

The Author * has tried the effect of roughening a channel with the application of a coat of paint intermixed with sand, and Fig. 4 † shows his results in comparison with Nikuradse's cylindrical pipes. From this it appears that the open-channel tests give the same values of f as the pipe tests if the roughness k/m is about 10 per cent. greater than that of the pipes. This is an amazingly close agreement, when it is realized that the shapes of the roughening grains, and the intensity of their distribution, could hardly be the same in the two nvestigations.

A detailed analysis has also been made of the Bazin and Manning formulae,‡ which are respectively

$$C=\frac{157{\cdot}6}{1+\dfrac{N}{\sqrt{m}}},$$

and

$$C=\frac{1{\cdot}486}{n}m^{\frac{2}{3}},$$

m being measured in feet.

* *Phil. Mag.*, June 1934.

† Allen, " The Resistance to Flow of Water along a Tortuous Stretch of River and in a Scale Model of the Same ", *Journ. Inst. C.E.*, Feb. 1939, pp. 115–132.

‡ Allen " Roughness Factors in Fluid Motion through Cylindrical Pipes and through Open Channels ", *Journ. Inst. C.E.*, April 1943, p. 91.

Now the work of Nikuradse on rough *cylindrical pipes* has led to the equation

$$\frac{1}{f}=16\left(\log\frac{3\cdot 7d}{k}\right)^2,$$

where f is the usual coefficient in $h=\dfrac{flv^2}{2gm}$ and d is the pipe diameter and k the height of the roughening excrescences.

Writing $d=4m$ and $f=2g/C^2$, this formula for rough pipes becomes

$$C=37\cdot 6+32\cdot 1\ \log_{10}(m/k),$$

the units of C being ft.$^{\frac{1}{2}}$ sec.$^{-1}$

The Author has shown that this rough *pipe* formula gives essentially the same relationship between C and m as applies over a wide range of conditions to *open channels.* Indeed, for a great variety of channels, including *all* earthen ones having any longitudinal slope likely to be found of importance in practice and having any hydraulic mean depth between 9 inches and 10 feet, the average of the two values of C obtained from the formulas of Bazin and Manning agrees with the relationship

$$C=37\cdot 6+32\cdot 1\ \log_{10}(m/k)$$

within ± 5 per cent. This margin of discrepancy is quite probably within the accuracy of many of the data upon which the channel formulas in current use are based.

(5) Considering next the case of open channels with rough bed and smooth sides, it would be anticipated that if the breadth of the stream is great compared with the depth, the resistance should approximate closely to that of an entirely rough section. Experiments indicate that if the breadth is half the depth, the friction coefficient for a wholly rough channel is about 16 per cent. greater than if only the bed is rough, while if the breadth is 1·5 times the depth, the corresponding percentage is about 9.

Similarity of Straight Open Channels. Considering uniform flow,* let the suffix 1 refer to the full-size channel and the suffix 2 to the model. Calling $2a$ the width and B the depth, while m is the hydraulic mean depth and i the slope, then if the motion is streamline, i_1 will $=$ i_2 provided $(2a:B)$ is the same in the actual and in the model, and provided

$$\left(\frac{vv}{B^2}\right)_1=\left(\frac{vv}{B^2}\right)_2,\ \text{or alternatively,}\ \left(\frac{vv}{a^2}\right)_1=\left(\frac{vv}{a^2}\right)_2.$$

With turbulent flow, however, $f=0\cdot 0234\left(\dfrac{vm}{\nu}\right)^{-0\cdot 15}$ in the case of smooth rectangular sections.

* In uniform flow, $\dfrac{dv}{dl}$ is zero and $\dfrac{dr}{dl}$ then $=i=\dfrac{fv^2}{2gm}$, or in the Chezy form, $v=C\sqrt{mi}$.

Then $$\frac{i_2}{i_1}=\left(\frac{v_2}{v_1}\right)^{1\cdot 85}\left(\frac{m_2}{m_1}\right)^{-1\cdot 15}\left(\frac{\nu_1}{\nu_2}\right)^{-0\cdot 15}$$

$$\left[\text{from } \frac{h}{l}=i=\frac{0\cdot 0234\left(\frac{vm}{\nu}\right)^{-0\cdot 15}v^2}{2gm}\right].$$

In the particular case of geometrically similar channels, having the same horizontal and vertical scale ratios $1:x$,

i_1 has to equal i_2,

and m_1 is equal to xm_2,

so that the corresponding velocity for operation of the model will be given by

$$\left(\frac{v_2}{v_1}\right)^{1\cdot 85}=x^{-1\cdot 15}\left(\frac{\nu_2}{\nu_1}\right)^{-0\cdot 15}.$$

If water at the same temperature is used in the two cases, so that $\nu_1=\nu_2$, this simplifies to

$$\left(\frac{v_2}{v_1}\right)=\left(\frac{1}{x}\right)^{0\cdot 62},$$

and a change of viscosity of 10 per cent.* would involve a discrepancy of only about 1·5 per cent. in the slope.

Supposing next that the sides and bed are rough and that f, for turbulent flow, is given by

$$C=\sqrt{\frac{2g}{f}}=\frac{157\cdot 6}{1+\frac{N}{\sqrt{m}}},$$

then if the horizontal scale of the model is $1:x$ and the vertical is $1:y$ v_2 will be chosen as $=\frac{v_1}{\sqrt{y}}$ and x will have to be such that

$$\frac{1+\frac{N_2}{\sqrt{m_2}}}{1+\frac{N_1}{\sqrt{m_1}}}=\sqrt{\frac{m_2 x}{m_1}},$$

in order that $\frac{h_2}{h_1}$ shall $=\frac{1}{y}$.

An example of model investigation in which this equality was found to hold is provided by the upper reaches of the Severn between Gloucester and Framilode,† a distance of rather over 12 miles. The effect of neap

* The kinematic viscosity of water falls by 10 per cent. if the temperature rises from 15° C. to 18·5° C.

† A. H. Gibson, *Construction and Operation of a Tidal Model of the Severn Estuary* p. 63, H.M. Stationery Office, 63-78-2, 1933.

tides is not felt above Framilode, and accordingly this part of the river should, at neap tides and at low water of other tides, be subject to the laws of uni-directional flow. The mean width of the river is about 265 ft., and when discharging 4,100 cusecs its mean m is 5·92 ft., its mean channel depth 6·2 ft. and its mean velocity 2·5 ft. per sec., the fall of water surface level between Gloucester and Framilode being 3·7 ft.—a gradient of 1 in 17,000. These data make Chezy's $C=134$ and Bazin's $N=0{\cdot}43$.

Now, in the first Severn model the horizontal scale was 1 : 8,500 and the vertical 1 : 100; i.e. $x=8{,}500$ and $y=100$. The scale of discharge was $1:xy^{\frac{3}{2}}$ and the scale of velocity $1:y^{\frac{1}{2}}$. At a flow equivalent to 4,100 cusecs, the value of m in the model was 0·0124 ft. and v was 0·25 ft. per sec., the fall of level being 0·037 ft. These values make $C=31{\cdot}9$, whereas if N had been 0·43, as in the river itself, C would have been 32·4.

Again, in the second Severn model, x was 8,500 and y was 200. On the basis of $1:xy^{\frac{3}{2}}$ as the scale ratio of discharge, the mean velocity corresponding to 4,100 cusecs was 0·17 ft. per sec. and m was 0·0104 ft., the fall between Gloucester and Framilode being 0·0185 ft. These values make $C=33{\cdot}5$, compared with 30·2 as obtained by substituting $N=0{\cdot}43$ in $C=\dfrac{157{\cdot}6}{1+\dfrac{N}{\sqrt{m}}}$.

Thus it seems that in this particular case the roughness of the model channels was only a little less than that of the river itself, i.e. N_2 was rather smaller than N_1, a conclusion which is reasonable in view of the facts that the bed was composed of sand not very different from that of the estuary and that the channel contains a number of bends.

If we consider the criterion

$$\frac{1+\dfrac{N_2}{\sqrt{m_2}}}{1+\dfrac{N_1}{\sqrt{m_1}}}=\sqrt{\frac{m_2 x}{m_1}},$$

in more general terms, for the case when $N_1=N_2=N$, say, we find that it is equivalent to writing

$$m_2^{\frac{1}{2}}=\frac{1+\sqrt{1+4\alpha N}}{2\alpha}.$$

where $$\alpha=\frac{N\sqrt{x}}{m_1}+\sqrt{\frac{x}{m_1}}=\left(\frac{N}{m_1}+\frac{1}{\sqrt{m_1}}\right)\sqrt{x}.$$

But in general this will approximate to

$$m_2^{\frac{1}{2}}=\frac{\sqrt{4\alpha N}}{2\alpha}=\sqrt{\frac{N}{\alpha}},$$

or
$$m_2=\frac{N}{\alpha}=\frac{N}{\left(\frac{N}{m_1}+\frac{1}{\sqrt{m_1}}\right)\sqrt{x}},$$

which means that in these circumstances ($N_2=N_1=N$), the value of m_2 does not decrease as quickly as does $1/x$. In other words, the *exaggeration* of the vertical scale relative to the horizontal should increase with increasing values of x.

Thus, if the prototype channel is rectangular in cross-section and has a breadth β times its depth, and if the vertical distortion of the model $=\frac{x}{y}=\gamma$,

then
$$\frac{m_2}{m_1}=\frac{\beta+2}{\left(\frac{\beta}{\gamma}+2\right)x},$$

from the definition $m=\frac{\text{area}}{\text{perimeter}}\cdot$ In that event, therefore, we should obtain from $m_2=N/\alpha$ the result

$$\frac{\beta+2}{\left(\frac{\beta}{\gamma}+2\right)x}m_1=\frac{N}{\left(\frac{N}{m_1}+\frac{1}{\sqrt{m_1}}\right)\sqrt{x}},$$

or
$$\frac{\beta}{\gamma}+2=\frac{(\beta+2)}{N\sqrt{x}}\left\{N+\sqrt{m_1}\right\},$$

in which β, $N(=N_1=N_2$ by hypothesis), and m_1 are fixed for a given prototype. This equation signifies that as x increases, the vertical distortion should also increase.

Again considering the example of *rough* channels, we see that if there is no distortion of scale and x therefore is equal to y,

$\frac{N_2}{N_1}$ would have to equal $\frac{1}{\sqrt{x}}$ in order that i_2 be equal to i_1 if $\frac{v_2}{v_1}=\frac{1}{\sqrt{x}}\cdot$

This condition $\left(\frac{N_2}{N_1}=\frac{1}{\sqrt{x}}\right)$ would be impossible to attain in some small-scale models, however smooth the model channel were made compared with the full-size channel. Instead, it would be necessary either to have

$$\frac{i_2}{i_1}=\left(\frac{1+\dfrac{N_2}{\sqrt{m_2}}}{1+\dfrac{N_1}{\sqrt{xm_2}}}\right)^2$$

or to ensure that $i_2=i_1$ by choosing a velocity scale from

$$\frac{v_2}{v_1}=\frac{1+\dfrac{N_1}{\sqrt{xm_2}}}{1+\dfrac{N_2}{\sqrt{m_2}}}\sqrt{\frac{1}{x}}\,.$$

Tortuous Channels. The resistance to flow along irregularly winding river-channels contains complications of such a nature that very little is known which is of universal application. For instance, the proportionate effect of skin-friction, in relation to the loss of energy caused by the bends and sudden changes of cross-section, clearly depends upon the particular river under consideration: the nature of the material of its bed and banks, and the abruptness or otherwise of its bends. When rivers of this kind are made the subject of model investigation, the question of "scale-effect" * assumes a peculiar significance. Considering it, for the time being, purely in relation to hydraulic resistance and not to questions of bed-movement or siltation, this "scale-effect" is essentially influenced by:

(1) the manner of the motion involved, that is, whether it is streamline or turbulent;

(2) the textural roughness of the model and its prototype;

(3) the effect of shape, both in plan and in cross-section.

It is commonly assumed that of these three influences, the most important and the most difficult to simulate in a model is the degree of roughness. The Author is of opinion, however, that in the wide class of rivers containing more or less abrupt changes of curvature, the resistance offered by these bends may be at least equal to that caused by textural roughness, and moreover that it is fortunately practicable to operate a model of such a river at such low values of Reynolds number as would hitherto have been considered certain to result in a marked "scale-effect" due to viscosity. These statements will now be discussed by reference to an investigation † of a portion of the River Mersey, shown in Fig. 5.

* "Scale-effect" is simply the conventional expression for possible discrepancies involved in extrapolating the results obtained on models to the full scale. It will be discussed in some detail with reference to particular types of investigation.

† Allen, "The Resistance to Flow of Water along a Tortuous Stretch of River and in a Scale Model of the Same". *Journ. Inst. C.E.*, Feb. 1939, pp. 115–132.

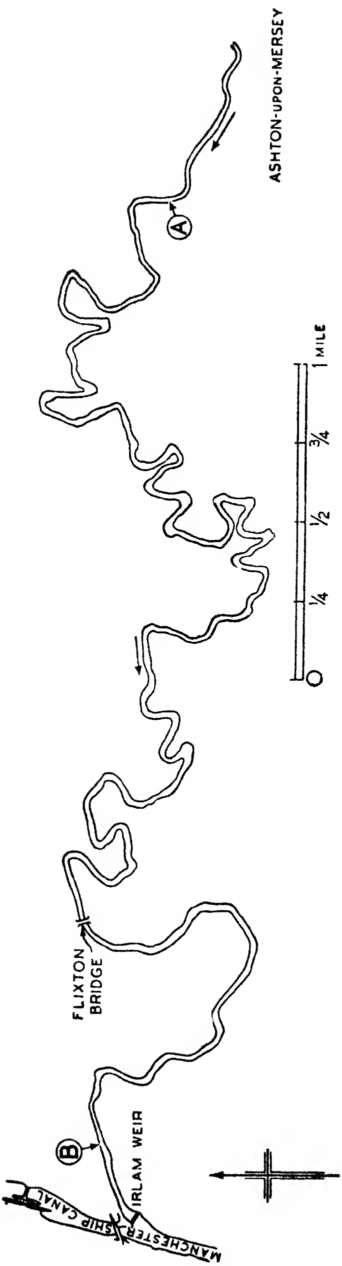


FIG. 5. River Mersey.

Measured along the centre-line of the river, this river is 8·30 miles long, or 2·25 times the shortest distance between the two ends. At the lower end, the river discharges over Irlam weir into the Manchester ship canal.

The model (illustrated in Plate I) was made to scales of 1 : 800 horizontally and 1 : 120 vertically. There was, therefore, a vertical distortion in the ratio of 6·67 to 1, and the scale of velocity was based on the square root of the vertical scale, i.e

$$\frac{v_2}{v_1}=\sqrt{\frac{1}{120}}=\frac{1}{10\cdot 96}.$$

The scale of rate of discharge was

$$1:(800\times 120\times\sqrt{120})$$

or

$$1:1\cdot 05\times 10^6.$$

The effect of Irlam Weir was reproduced by a block of teak constructed to a scale of 1 : 120 for its height and for its dimensions *parallel* to the direction of flow. Estimates of the rate of flow in the natural river were based upon the recorded head over Irlam Weir, which has been calibrated partly by means of current-meter observations and also by means of a 1 : 50 scale model of the weir and its approach channel.

The river model was made of an average mixture of 1 part by volume of Portland cement to 5 parts of fine sand (mean diameter 0·009 inch), and, as will be explained at a later stage, its bed was finally roughened by a sparse sprinkling of coarse sand. The model, thus moulded, was enclosed in a wooden casing whose upper edge was carefully levelled to act as a datum corresponding to +90·00 ft. O.D.

Water was fed to the upper end of the model through one of a selection of calibrated orifices screwed into the bottom of a cylindrical vessel open at the top and fitted with a gauge-glass for measurement of head. The stream thus provided entered the moulded part through screens of perforated zinc and, as a check, the discharge was always measured by direct collection at the downstream end of the model.

Analysis indicates that the *percentage* loss due to textural roughness in the actual river is of the order of 25 at dry weather flow (350 cusecs) and 45 to 50 for discharges ranging between 2,500 and 7,000 cusecs, and that the river is equivalent, as far as its textural roughness is concerned, to a channel of rectangular section roughened with projections about 6 inches high.

Immediately after the preliminary moulding of the model, water was supplied at rates corresponding to 2,550 and 7,000 cusecs in nature. The observed total loss of head, over the whole length of the model, was then seen to be true to nature within 6 and 12 per cent. respectively, the model giving too small a drop in both cases. As then tested, the sides and bed of the model were in a rough state, no attempt having been made to smooth out the chisel-marks and the general surface irregularities of the concrete.

Next, the artificial projections and indentations of the sides left by the preliminary moulding were carefully smoothed and a thin layer of neat cement was applied; altogether, this would result in making a very slight reduction in channel-width, probably by not more than 1 per cent., and of course the surface was rendered very considerably smoother. Yet the observed total drop differed from the first observations by less than ½ per cent.

Now it has often been suggested that the standard of finish imparted to the cement or concrete surface in a model must have an important bearing on the results obtained. The experiments just described are important, therefore, in proving that in a model of this character, the surface-finish has, within wide limits, no significant effect whatever on the total resistance of the channel.

It was clear at this stage, in fact, that minor adjustments of the configuration at the sharp bends would affect the resistance far more than any alteration in surface-roughness. This brings us to another point, namely that however detailed within practicable limits a survey of a river may be, it cannot completely describe the degree of severity of the changes of curvature. Ultimately, in fact, the criterion for the final moulding of the model at these particular sections must be the hydraulic results obtained.

This being the case, the shape of the bends was next adjusted in detail until a close overall hydraulic agreement between model and nature was achieved. Further modifications of the channel-section then made the water-levels at all gauge-stations still closer to those observed in nature for a number of stages between 2,550 and 7,000 cusecs. In order to produce similar agreement at a discharge of 350 cusecs, when the average depth of the model channel was only of the order of ½ inch, a handful of coarse sand (0·10 by 0·075 by 0·05 inch) was distributed over the bed, by trial and error, until the appropriate water-levels were obtained.

Having thus adjusted the channel to give the appropriate water-levels, we examined its dimensions by pressing plasticine templets to such a shape as to fit the channel-section at each of twenty-four points along its length. From these templets, the corresponding cross-sections were drawn so that, with the assistance of a planimeter and a map-distance-gauge, their areas and hydraulic mean depths could be measured.

It was thus found that, to scale, the mean area of cross-section agreed very closely with the natural river, for *a given depth of water*. On the other hand, the mean level of the deepest points in the model-channel was found to be the equivalent of 1·0 ft. higher than in nature.* For a given *mean water-surface level*, therefore, the cross-sectional area of the model-channel was less, proportionately, than in the actual river. The physical mechanism of this phenomenon is as follows:

The river-sections generally contain a more or less localized depression represented diagrammatically in Fig. 6 (*a*).

If the vertical scale of Fig. 6 (*a*) is exaggerated 7:1 times, the equivalent section becomes that shown in Fig. 6 (*b*), and as the reproduction of the thin depression in a small-scale model becomes either uncertain or impracticable, the section then degenerates into that represented by Fig. 6 (*c*). As, in this hypothetical instance, the area of Fig. 6 (*c*) is only 6·5 per cent. less than that of Fig. 6 (*b*), while the wetted perimeter is reduced by 28·5 per cent., it is necessary to compensate for the disproportionate increase of hydraulic mean depth by reducing the area, that is, by decreasing the width if the dimension 2 is left unaltered.

While maintaining the scale ratio $1:(800 \times 120 \times \sqrt{120})$ for rates of discharge in this model, it was necessary, therefore, to adjust the section of the model-channel until it was less than that appropriate to the scales of length and depth, the average reduction of area being about 12 per cent.

Another interesting point which emerged from this work will now be

* A depth of 1·0 ft. was represented in the model by 0·10 inch. The width of the water-surface, in the model, averaged about 2·18 cm. (0·86 inch) with the lowest discharges, and 4·55 cm. (1·79 inch) with the highest.

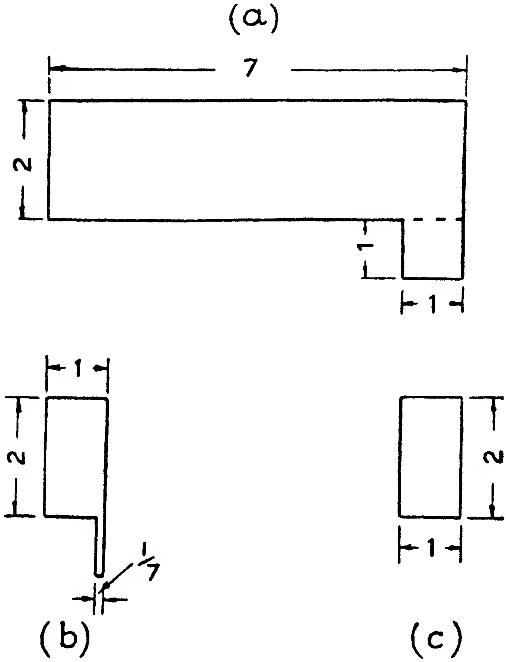


FIG. 6.

mentioned. Calling h the total observed drop of the water-surface between the upper and lower limits A and B, and making allowance for the change of kinetic head $\frac{V_B^2 - V_A^2}{2g}$, we will write

$$h' = h - \frac{V_B^2 - V_A^2}{2g}.$$

Now let $\frac{h'}{l} = i$ and $V = Km^{1/x}i^{\frac{1}{2}}$.

It was found that

$$\left.\begin{array}{l} x = 1{\cdot}63 \\ K = 28{\cdot}2 \end{array}\right\} \text{for } Q = 350 \text{ cusecs,}$$

$$\left.\begin{array}{l} x = 1{\cdot}77 \\ K = 46{\cdot}2 \end{array}\right\} \text{for } Q = 2{,}550 \text{ cusecs,}$$

$$\left.\begin{array}{l} x = 1{\cdot}76 \\ K = 49{\cdot}2 \end{array}\right\} \text{for } Q = 7{,}000 \text{ cusecs,}$$

in the actual river, and that the same values of x and K applied equally well to the model at corresponding discharges.

Moreover, experiments conducted in the model with hot and cold water (viscosity changing by from 60 to 100 per cent.) showed almost identical losses of head, despite the Reynolds numbers involved being as low as $vm/\nu = 36$.

The Separation of Viscosity and Inertia Effects.

If the hydrodynamical resistance of an object is dependent upon l, v, ρ, ν and g, where l represents some linear dimension of the object, v its velocity, ρ the density and ν the kinematic viscosity of the fluid, then the principle of dimensional homogeneity indicates that the resistance is proportional to $\rho l^2 v^2 \phi\left\{\left(\frac{vl}{\nu}\right), \left(\frac{gl}{v^2}\right)\right\}$, $\phi\left\{\left(\frac{vl}{\nu}\right), \left(\frac{gl}{v^2}\right)\right\}$ standing for "a function of the Reynolds number and of the quantity $\left(\frac{gl}{v^2}\right)$"; the latter is sometimes called the Froude number.

Considering two similar bodies, then, numbered (1) and (2),

$$R_1 = K\rho_1 l_1^2 v_1^2 \phi\left\{\left(\frac{v_1 l_1}{\nu_1}\right), \left(\frac{g l_1}{v_1^2}\right)\right\},$$

$$R_2 = K\rho_2 l_2^2 v_2^2 \phi\left\{\left(\frac{v_2 l_2}{\nu_2}\right), \left(\frac{g l_2}{v_2^2}\right)\right\},$$

and if $\frac{R_1}{R_2}$ is to be equal to $\frac{\rho_1 l_1^2 v_1^2}{\rho_2 l_2^2 v_2^2}$, as should be the case from the fundamental conception of force being proportional to mass $\times$ acceleration, it follows that

$$\phi\left\{\left(\frac{v_1 l_1}{\nu_1}\right), \left(\frac{g l_1}{v_1^2}\right)\right\} \text{ should } = \phi\left\{\left(\frac{v_2 l_2}{\nu_2}\right), \left(\frac{g l_2}{v_2^2}\right)\right\}.$$

Now if the "corresponding speeds" for dynamical similarity are chosen from

$$\frac{v_1 l_1}{\nu_1} = \frac{v_2 l_2}{\nu_2},$$

it is not generally practicable to have *simultaneously*

$$\frac{g l_1}{v_1^2} = \frac{g l_2}{v_2^2},$$

since this would involve choosing two fluids for which

$$\left(\frac{\nu_1}{\nu_2}\right)^2 = \left(\frac{l_1}{l_2}\right)^3.$$

For example, if the body designated (2) is a model of that called (1) to a scale of 1 : 10, then $\frac{l_1}{l_2} = 10$ and the model would require to be tested in a fluid whose kinematic viscosity is only $\frac{1}{\sqrt{1000}}$ of that in which its full-size prototype operates. The speed of working the model would be $\frac{1}{\sqrt{10}}$ of that at which its prototype works and complete dynamical similarity would be obtained *if the necessary fluid could be found.* Accordingly, if a certain force, x, were measured in such a model at a speed v_2, it could be assumed that the equivalent force experienced in the prototype at the corresponding speed $v_2\sqrt{10}$ would be

$$\left(\frac{\rho_1 l_1^2 v_1^2}{\rho_2 l_2^2 v_2^2}\right) x, \text{ or } 1{,}000 \frac{\rho_1}{\rho_2} x.$$

If, however, it is impracticable to use a fluid of the appropriate viscosity, some other device must be considered. In the first place, let us suppose that it is " gravity forces " which are believed to be predominant. The speed at which to operate the model may then be chosen from the relationship $\frac{v_2}{v_1} = \sqrt{\frac{l_2}{l_1}}$, but some doubt will remain as to whether, in virtue of the influence of viscosity, some " scale-effect " will invalidate the translation of the results by the force factor

$$\left(\frac{\rho_1 l_1^2 v_1^2}{\rho_2 l_2^2 v_2^2}\right)$$

to the full-size system. A direct test of this effect may be made by taking observations on models constructed to different scales and thus accumulating sufficient data to provide an empirical relationship between " scale-effect " and size of model. Alternatively, the model can be tried with fluids of different viscosities (e.g. water at different temperatures). Yet again, the magnitude of the " scale-effect " may be assessed by calculation, as in the case of *ship-models*, which will now be discussed.

The resistance experienced by a *ship's hull* moving through water is due to two causes:

(i) " skin-friction ";

(ii) surface-waves and eddies.

A technique due to Froude consists of estimating the first of these from experimental results obtained with flat planes towed endwise, and of determining the second by means of a scale-model (generally made of paraffin wax) towed through still water at a speed v_2 such that $v_2 = \frac{v_1}{\sqrt{S}}$, where $1 : S$ represents the scale of the model. The skin-frictional resistance is given by fAv^n, where A is the area of the wetted surface and f and n depend upon the nature of the surface and the velocity.* The method will be explained by using a numerical example:

"A model of a ship is made to a scale of 1 : 25. At a speed of 6 knots its skin-frictional resistance is *estimated* to be 1·95 lb., and its total resistance is *measured experimentally* as 3·80 lb. in fresh water. Compute the probable total resistance of the ship itself, at the corresponding speed, in sea water of specific gravity 1·028."

In this problem we have:

$$v_2 = 6 \text{ knots} = 10{\cdot}1 \text{ ft. per sec.}$$

From experimental data on skin-friction of surfaces towed through fresh water, let us suppose that $f_2 = 0{\cdot}0034$ and $n_2 = 1{\cdot}94$.

Then $1{\cdot}95 = 0{\cdot}0034\, A_2 (10{\cdot}1)^{1{\cdot}94}$;

$$\therefore\ \mathbf{A_2 = 6{\cdot}40 \text{ sq. ft.}}$$

We deduce, therefore, from the given information, that the wetted area of the model is 6·40 sq. ft. (In practice, A_2 would be known, f_2 and n_2 assumed, and the figure of 1·95 lb. calculated.)

Hence $$A_1 = 6{\cdot}40 \times 25^2 = \mathbf{4{,}000 \text{ sq. ft.}}$$

The corresponding speed of the ship

$$= v_1 = v_2\sqrt{S} = 10{\cdot}1 \times 5$$
$$= \mathbf{50{\cdot}5 \text{ ft. per second}}\ (=30 \text{ knots}).$$

$\therefore$ Skin-friction of ship $= f_1 A_1 (50{\cdot}5)^{n_1}$, and if it is believed that

$$f_1 = 0{\cdot}0025 \text{ and } n_1 = 1{\cdot}83,$$

this becomes

$$0{\cdot}0025 \times 4{,}000 \times 50{\cdot}5^{1{\cdot}83},$$

i.e. **13,100 lb.**

Now the wave-making and eddy resistance of the model

$$= 3{\cdot}80 - 1{\cdot}95 = \mathbf{1{\cdot}85 \text{ lb.}} \text{ (in fresh water)};$$

$\therefore$ the corresponding value for the ship itself, in sea water,

$$= 1{\cdot}85\left(\frac{\rho_1 l_1^2 v_1^2}{\rho_2 l_2^2 v_2^2}\right)$$
$$= 1{\cdot}85 \times 1{\cdot}028 \times 25^2 \times 25$$
$$= \mathbf{29{,}800 \text{ lb.}}$$

Hence the total estimated resistance of the ship

$$= 13{,}100 + 29{,}800$$
$$= \mathbf{42{,}900 \text{ lb.}}$$

* See Froude, *Brit. Assoc. Rpt.*, 1872; and A. H. Gibson, Chapter on "Hydrodynamical Resistance", in *The Mechanical Properties of Fluids*, Blackie, London, 1937.

If the ship had been a fully submerged *submarine* moving steadily at a depth such that surface disturbances were negligible, then the corresponding speed of the model should have been derived from

$$\frac{v_2 l_2}{\nu_2} = \frac{v_1 l_1}{\nu_1},$$

or

$$v_2 = \frac{\nu_2}{\nu_1} S v_1.$$

The same rule ought to apply to a model of an *airship* or *aeroplane* in steady motion, and a brief discussion of the difficulties involved in this kind of investigation may be found instructive, even though outside the immediate scope of this book. If, then, the model aircraft is tested in a stream of air at atmospheric pressure for which $\nu_2 = \nu_1$, it appears that in order to obey the rules of dynamical similarity, the air in the model must move with a velocity S times as high as that of the full-size 'plane. Thus, if the scale were 1 : 10 and if the actual machine had a speed of 300 miles per hour, the model would have to be tested in a wind of 3,000 m.p.h. and its resistance would be the same as that of its prototype. Clearly this would involve considerable power, and in addition the complication that such a speed would be roughly four times the velocity of sound in air: it is known that if the speed approaches that of sound, the compressibility of the air enters the problem as an added complication.

Now at a temperature of 15° C., the *kinematic* viscosity of air is nearly 13 times that of water, so that it would appear desirable to test the model in water instead of air. But even so, the speed required would be $\frac{300 \times 10}{13}$ or 230 m.p.h. Taking the density of water at normal temperature and pressure to be about 810 times that of air, the resistance of the model would be $\frac{810}{10^2} \cdot \left(\frac{230}{300}\right)^2$, or 4·76 times that of the machine itself. It is evident, therefore, that the power to drive the water past the model would be very great, and the model would have to be mounted on very robust supports.

Alternatively, the model might be tested in compressed air. Thus, if the air is compressed to a pressure of 25 atmospheres, its kinematic viscosity is reduced to about $\frac{1}{25}$ of its value at one atmosphere. The speed then required would be $\frac{300 \times 10}{25}$ or 120 m.p.h., and the resistance would be $25 \times \frac{1}{10^2} \times \left(\frac{120}{300}\right)^2$, or 0·04, of its prototype's. But even this solution presents great practical difficulties, because it involves placing the model in a shell of considerable diameter subjected to an internal pressure of some 370 lb. per sq. in.

The difficulties which we have noted in the course of these remarks are aggravated by the fact that the size and speed of aircraft have both increased during recent years. It would appear, therefore, that in general the investigation of such problems must be confined to one or more of the following methods:

(*a*) Model tests in air at atmospheric pressure, but at velocities lower than those demanded by strict adherence to dynamical similarity.

(*b*) Model tests in compressed air tunnels, again at speeds lower than those appropriate to the full Reynolds number.

(*c*) Tests of portions of the aircraft structure, either full size or to scale, in wind tunnels with velocities of the order of, say, 120 m.p.h. Thus, the National Advisory Council for Aeronautics in America has a tunnel of oval section 30 ft. × 60 ft. employing an air jet at atmospheric pressure and at 118 m.p.h., operated by an 8,000 h.p. motor, while the Royal Aircraft Establishment at Farnborough, England, has a 24 ft. tunnel utilizing speeds up to 115 m.p.h. and requiring 2,000 h.p., for model tests of complete machines and for model or full-size reproductions of parts of machines.*

In order to complete the information gained from such a model test, experience with full-sized machines in flight is required, and also with models built to different scales, but the value of the model test as a basic approach is not questioned, and it may be well for us to bear this fact in mind when we come to discuss certain types of hydraulic model investigations whose limitations have not been *scientifically* assessed and whose value has consequently not been properly appreciated by some critics.

A Water Tunnel for the Investigation of Flow Problems.

An ingenious apparatus has been described by A. Fage and J. H. Preston,† " especially suitable for the observation of flow near the surface of a long streamline body of revolution at Reynolds numbers up to $1{\cdot}3 \times 10^6$." Where such an investigation is to be made by a visual observation of solid particles moving in the fluid, it is found more convenient to experiment with water than with air, since for the same flow-pattern the

* " The 24-ft. Wind Tunnel at Farnborough ", *The Engineer*, vol. CLIX, 1935, pp 351, 380, 386, 468, 516, 519, 534.

Since the above was written, it has been announced that the British Government proposes to establish an aeronautical research station near Bedford, at which it is intended to have several wind tunnels of 40,000 h.p. and ultimately one of 100,000 h.p. (see *The Engineer*, March 30, 1945, p. 251).

There are at the moment a few wind tunnels in existence for small-scale model research on the resistance of objects at " supersonic " speeds.

† *Journ. Roy. Aeronautical Soc.*, vol. XLV, 1941, p. 124.

velocities involved in water are only about one-thirteenth of those in air. This follows from the argument that for the Reynolds number to be the same,

$$\left(\frac{vl}{\nu}\right)_{\text{air}} \text{ must} = \left(\frac{vl}{\nu}\right)_{\text{water}},$$

and consequently, if the linear dimension l is unaltered,

$$v_{\text{water}} = v_{\text{air}} \left\{\frac{\nu_{\text{water}}}{\nu_{\text{air}}}\right\}$$

$$= \text{about } \frac{v_{\text{air}}}{13}.$$

Moreover, it is found that the choice of particles suitable for observation is less restricted in water than in air, while for the same size and shape of the testing tunnel, the horse-power required to circulate the water is roughly one-third of that if air is used as the working fluid to give the same Reynolds number.

The water tunnel described by Messrs. Fage and Preston is shown diagrammatically in Fig. 7. The observation chamber itself was 32 inches

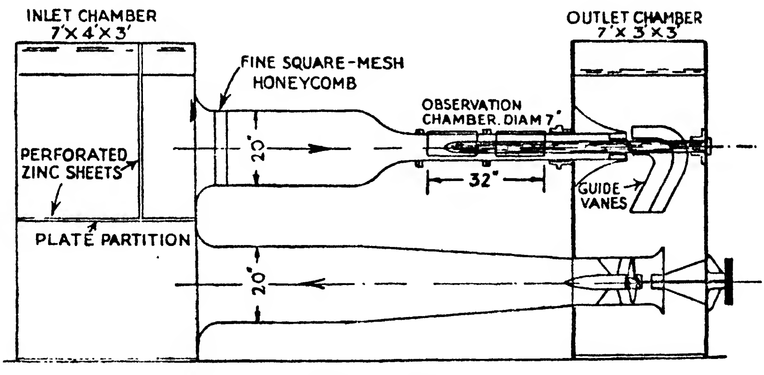


FIG. 7. Elevation of Water-Tunnel.
Fage and Preston, *Journ. Roy. Aero. Soc.*, Vol. XLV, 1941, p. 124.

long and 7 inches in diameter, made from steel castings and equipped with two semicircular perspex windows, each of which was enclosed in a water cell having a vertical glass wall and a free water surface: this device was used to eliminate astigmatic distortion. The water was circulated by means of a four-bladed screw, and stroboscope discs were mounted on the shaft outside the tunnel and viewed by a Neon lamp connected to the 50-cycle alternating current mains to measure the screw-speed and to provide a means of indicating steady operating conditions. Baffles, honeycomb and guide vanes were provided as indicated in the sketch of Fig. 7. Velocities up to 7 ft. per sec. could be generated in the observation chamber.

The general character of the flow could be observed by filaments of photographic white ink injected from a fine bore tube with a speed approximately equal to that of the surrounding water. These filaments were photographed against a dark background with the aid of a narrow beam of light at right angles to the axis of the camera. It was found that at speeds above about 1·6 ft. per sec. a filament remote from the wall waved "spasmodically with an amplitude increasing with the downstream distance, and with a frequency increasing with the velocity of the stream, but there is no tendency for the filament to break down or to lose its identity.* A filament band at the wall begins to wave at a lower stream speed (about 1·1 ft. per sec.) than that for a band at some distance from the wall, and, at high speeds, the tail of a band breaks down, diffuses rapidly and eventually disappears. The boundary layer at the tunnel wall is therefore turbulent at high speeds, and simultaneous observation of two filaments, one at the wall and the other well within the stream, suggests that the waviness of the filament in the stream is associated with the breakdown of flow in the boundary layer at the wall.

"It was observed that filament bands on the surface of a long streamline body of revolution were steady over the entire length of the body under observation (26 ins.) at all speeds up to the maximum speed of the tunnel (7 ft./sec.); and filament bands near and just outside the boundary layer were steady up to 1·6 ft./sec., and slightly wavy from this speed to the highest speed of the tunnel. The kinematic viscosity of the water at the time of observation was $11{\cdot}46 \times 10^{-6}$ ft.^{2}/sec., so that the flow in the boundary layer of the body was laminar up to a Reynolds number, U_0X/ν, about $1{\cdot}3 \times 10^6$, where U_0 is the stream velocity, and X is the axial distance from the nose. The general conclusion to be drawn from the observations of the behaviour of filament bands is that the stream in the observation chamber, although slightly disturbed at the higher speeds, is substantially free from turbulence at all speeds and throughout its entire length, except at the wall." This demonstrates the effectiveness of the inlet chamber, perforated sheets, honeycomb and convergent entrance nozzle in straightening out and stabilizing the flow.

More detailed observations were facilitated by two specially designed instruments, which may have opened up a new range of possibilities in

* The Author has observed a swaying or oscillation of a thread of dye in an open channel (see *Phil. Mag.*, Ser. 7, vol. XVII, June 1934, p. 1098), as also have Messrs. Piercy, Hooper and Winny (*Phil. Mag.*, XV, No. 99, March 1933, p. 673) in flow through pipes with cores.

the study of fluid motion and the transition from laminar to turbulent flow in the boundary layer.

The first has been called a fluid-motion microscope (originally " ultra-microscope "), and in principle it renders the movements of small illuminated particles visible and measurable through a microscope of medium magnification. " With the magnification used (about 40 : 1) the actual length of particle tracks in the field of view is about 0·12 in. In laminar flow, the tracks are parallel; in turbulent flow, they continually change direction and appear, owing to persistence of vision, to be crossing one another."

The second instrument, in which the fluid-motion microscope is equipped with an interrupter, enables the actual speed of a particle to be measured by the length of its track as recorded for a known time of exposure on a photographic plate. It can be used to determine the velocity distribution in a boundary layer " to a distance of about four-thousandths of an inch from the surface ".

For a full description of these important devices, the reader is advised to consult the Paper by Fage and Preston in the *Journal* of the Royal Aeronautical Society, vol. XLV, 1941, page 124; also Fage and Townend, *Proc. Roy. Soc.*, A, vol. 135, 1932, p. 656 ; and Fage, *Phil. Mag.*, vol. 21, 1936, p. 80.

The Sea-going Qualities of Vessels in a Rough Sea.

The first suggestion of the application of dynamical similarity to the study of the qualities of ships—in particular of lifeboats—appears to have been made by Osborne Reynolds.* Even if the size of the boat does not preclude full-scale experiments in heavy seas and strong winds, such work is dangerous, and it may be that the most hazardous combination of wind and wave is only rarely obtained. The idea is, therefore, to test the qualities of the vessel in a small scale-model under moderate wind and wave conditions produced either naturally or artificially. Supposing the model to be similar to the original in form and distribution of weight, it may be shown † that the angular roll will be the same if the height of the waves is proportional to the length l of the vessel, the wind velocity proportional to $\sqrt{l}$, and the frequency of the waves proportional to $\sqrt{l}$. Accordingly, the behaviour of a model 3 ft. long in waves 2 ft. high and a wind of 20 miles per hour might be expected to correspond with that of a boat 27 ft. long in waves 18 ft. high and a wind of 60 miles per hour.

* *Papers on Mechanical and Physical Subjects*, Reynolds, vol. II, p. 321.

† " The Principle of Dynamical Similarity, with Special Reference to Model Experiments ", A. H. Gibson, *Engineering*, vol. 117, 1924, p. 422.

A More General Consideration of Dimensional Analysis.*

On page 13, we wrote the equation

$$R = Kl^{\alpha}\rho^{\beta}v^{\gamma}\mu^{x}d^{y}k^{z} \qquad \text{.................................(1)}$$

as one which might represent the resistance to flow through a cylindrical pipe, and by expressing this equation in dimensional form we obtained the result:

$$R = K\rho d^2 v^2 \left(\frac{l}{d}\right)^{\alpha}\left(\frac{k}{d}\right)^{\gamma}\left(\frac{vd\rho}{\mu}\right)^{\beta-1},$$

a more general form of which is

$$R = K\rho d^2 v^2 \psi\left\{\frac{l}{d}, \frac{k}{d}, \frac{vd}{\nu}\right\}, \qquad \text{.............................(2)}$$

because really we are concerned with a function (ψ) which is the product of three infinite series in the dimensionless groups of terms $\frac{l}{d}$, $\frac{k}{d}$ and $\frac{vd}{\nu}$.

Now actually the equation (1) involves seven quantities R, l, ρ, v, μ, d, k, and the equation (2) has four dimensionless groups $\frac{R}{\rho d^2 v^2}$, $\frac{l}{d}$, $\frac{k}{d}$, $\frac{vd}{\nu}$. This number (four) is in fact equal to the number (seven) of the quantities originally stated in equation (1), less the number (three) of the primary quantities mass, length and time on which the dimensional analysis was based.

The same result as equation (2) could, then, have been derived as follows:

(*a*) By choosing three quantities (i.e. the same number as that of the primary quantities) from equation (1) such that together they involve all the primary quantities mass, length and time. For example, we might select R, l, ρ.

(*b*) Expecting finally to obtain $(7-3)$ or 4 dimensionless groups, we may next choose one other quantity from equation (1), say v.

(*c*) Calling a dimensionless group N, write

$$N_1 = R^{a_1} l^{b_1} \rho^{c_1} v.$$

* "Notes on the Method of Dimensions", Buckingham, *Phil. Mag.*, XLII, No. 251, Nov. 1921. *Dimensional Analysis*, Bridgman (Humphrey Milford), 1943. "The Principle of Dynamical Similarity, with Special Reference to Model Experiments", Gibson, *Engineering*, pp. 325, 357, 391, 422, March–April 1924. "Research in Mechanical Engineering by Small-Scale Apparatus", Johansen, *Proc. I.Mech.E.*, March 1929. "The Use of Models in Hydraulic Engineering", Gibson, *Trans. Inst. of Water Engineers*, London, vol. XXXIX, 1934, p. 172. "Aerodynamical Research and Hydraulic Practice, Fage, *Proc. I.Mech.E.*, April 1935. *Applied Fluid Mechanics*, O'Brien and Hickox, (McGraw-Hill) 1937.

Expressing this dimensionally,

$$O = M^{a_1}L^{a_1}T^{-2a_1}L^{b_1}M^{c_1}L^{-3c_1}LT^{-1},$$

or, for mass, $O = a_1 + c_1,$

for length, $O = a_1 + b_1 - 3c_1 + 1,$

for time, $O = -2a_1 - 1.$

From these simultaneous equations, we find that

$$a_1 = -\tfrac{1}{2},$$
$$c_1 = \tfrac{1}{2},$$
$$b_1 = 1;$$

$$\therefore\ N_1 = R^{-\frac{1}{2}}l\rho^{\frac{1}{2}}v = \frac{\rho^{\frac{1}{2}}lv}{R^{\frac{1}{2}}}.$$

(*d*) In similar fashion,

$$N_2 = R^{a_2}l^{b_2}\rho^{c_2}\mu,$$

which gives

$$N_2 = \frac{\mu}{R^{\frac{1}{2}}\rho^{\frac{1}{2}}}.$$

(*e*) $$N_3 = R^{a_3}l^{b_3}\rho^{c_3}d;$$

$$\therefore\ N_3 = \frac{d}{l}.$$

(*f*) $$N_4 = R^{a_4}l^{b_4}\rho^{c_4}k;$$

$$\therefore\ N_4 = \frac{k}{l}.$$

(*g*) Expressing N_1 in terms of N_2, N_3, N_4, we write

$$N_1 = \phi_1(N_2, N_3, N_4),$$

where ϕ_1 represents " some function of " ....
This gives

$$\frac{\rho^{\frac{1}{2}}lv}{R^{\frac{1}{2}}} = \phi_1\left(\frac{\mu}{R^{\frac{1}{2}}\rho^{\frac{1}{2}}}, \frac{d}{l}, \frac{k}{l}\right),$$

and it is equally true that

$$\frac{R}{\rho l^2 v^2} \text{ may be considered} = \phi_2\left(\frac{\mu}{R^{\frac{1}{2}}\rho^{\frac{1}{2}}}, \frac{d}{l}, \frac{k}{l}\right).$$

Again, since we are dealing with dimensionless quantities, this result is not invalidated if we replace one of the quantities by a function of itself, or if we multiply one quantity by a function of any other, so long as we do not change the number of dimensionless groups.

Thus we may multiply $\dfrac{\mu}{R^{\frac{1}{2}}\rho^{\frac{1}{2}}}$ by $\dfrac{R^{\frac{1}{2}}}{\rho^{\frac{1}{2}}lv} \cdot \dfrac{l}{d}$ and $\dfrac{k}{l}$ by $\dfrac{l}{d}$, which gives

$$\frac{R}{\rho l^2 v^2} = \phi\left(\frac{\mu}{\rho d v}, \frac{d}{l}, \frac{k}{d}\right),$$

or

$$\frac{R}{\rho l^2 v^2} = \phi\left(\frac{vd}{\nu}, \frac{k}{d}, \frac{l}{d}\right) \text{ as before.}$$

Other Examples of the Principles of Similarity.

(i) Discharge over Weirs or through Orifices under Gravity.

We might expect the rate of discharge Q (volume per unit time) to depend upon the head h, the density ρ, the size of orifice as defined by one typical dimension l in the case of geometrically similar weirs or orifices, the viscosity μ, the acceleration due to gravity g, and the surface tension of the fluid σ (force per unit length).*

If this is correct, then seven quantities Q, ρ, h, l, μ, g and σ are involved, and among them these contain the primary quantities of mass, length and time. We therefore expect to obtain (7 – 3) or 4 dimensionless groups.

Let

$$N_1 = Q^{a_1}\rho^{b_1}h^{c_1}l,$$

so that

$$M^\circ L^\circ T^\circ = L^{3a_1}T^{-a_1}M^{b_1}L^{-3b_1}L^{c_1}L$$

and

$$\therefore\ a_1 = 0;\quad b_1 = 0;\quad c_1 = -1;$$

$$\therefore\ N_1 = \frac{l}{h}.$$

$$N_2 = Q^{a_2}\rho^{b_2}h^{c_2}\mu;$$

$$\therefore\ M^\circ L^\circ T^\circ = L^{3a_2}T^{-a_2}M^{b_2}L^{-3b_2}L^{c_2}ML^{-1}T^{-1}$$

and

$$\therefore\ a_2 = -1;\quad b_2 = -1;\quad c_2 = 1;$$

$$\therefore\ N_2 = \frac{\mu h}{Q\rho}.$$

$$N_3 = Q^{a_3}\rho^{b_3}h^{c_3}g;$$

$$\therefore\ M^\circ L^\circ T^\circ = L^{3a_3}T^{-a_3}M^{b_3}L^{-3b_3}L^{c_3}LT^{-2}$$

and

$$\therefore\ a_3 = -2;\quad b_3 = 0;\quad c_3 = 5;$$

$$\therefore\ N_3 = \frac{gh^5}{Q^2}.$$

---

* Surface tension will influence the shape of the freely discharging jet from a weir or orifice; it will also cause the pressure inside the jet to be greater than atmospheric. Thus, at the vena contracta of a jet issuing from an orifice, where it is usual to assume atmospheric pressure, the pressure inside will exceed atmospheric by an amount $\frac{\sigma}{gr}$ lb. per sq. ft. if σ is in poundals per ft. and r is the radius of the jet in feet.

$$N_4 = Q^{a_4}\rho^{b_4}h^{c_4}\sigma;$$

$$\therefore\ M^{\circ}L^{\circ}T^{\circ} = L^{3a_4}T^{-a_4}M^{b_4}L^{-3b_4}L^{c_4}MLT^{-2}L^{-1};$$

and $$\therefore\ a_4 = -2;\quad b_4 = -1;\quad c_4 = 3;$$

$$\therefore\ N_4 = \frac{h^3\sigma}{Q^2\rho}.$$

Accordingly,

$$\psi\left(\frac{l}{h}, \frac{\mu h}{Q\rho}, \frac{gh^5}{Q^2}, \frac{h^3\sigma}{Q^2\rho}\right) = 0,$$

or $$\frac{Q^2}{gh^5} = \phi_1\left(\frac{l}{h}, \frac{\mu h}{Q\rho}, \frac{h^3\sigma}{Q^2\rho}\right),$$

or $$\frac{Q^2}{gh^5}\left(\frac{h}{l}\right)^4 = \phi_2\left(\frac{l}{h}, \frac{\mu h}{Q\rho}\cdot\frac{l}{h}, \frac{h^3\sigma}{Q^2\rho}\right),$$

or $$\frac{Q^2}{ghl^4} = \phi_2\left(\frac{l}{h}, \frac{\mu l}{Q\rho}, \frac{h^3\sigma}{Q^2\rho}\right),$$

and since a, the area of an orifice, is proportional to l^2, this is equivalent to stating that the coefficient of discharge of an orifice, as usually defined, is a function of

$$\frac{l}{h}, \frac{\mu l}{Q\rho}, \frac{h^3\sigma}{Q^2\rho}.$$

Alternatively, we may write

$$\frac{Q^2}{gh^5}\cdot\left(\frac{h}{l}\right)^2 = \phi\left(\frac{l}{h}, \frac{\mu l}{Q\rho}, \frac{h^3\sigma}{Q^2\rho}\right),$$

or $$\frac{Q^2}{gl^2h^3} = \phi\left(\frac{l}{h}, \frac{\mu l}{Q\rho}, \frac{h^3\sigma}{Q^2\rho}\right),$$

which is equivalent to stating that the coefficient of a weir, defined as $\frac{Q}{lh^{\frac{3}{2}}}$, where l is its breadth, is a function of

$$\frac{l}{h}, \frac{\mu l}{Q\rho}, \frac{h^3\sigma}{Q^2\rho}.$$

If, then, it is required to find the coefficient of discharge of a weir or orifice (1) by measurements on a model, (2), built to a scale of $1 : S$, it is strictly speaking necessary that

(*a*) the model shall be geometrically similar to its prototype;

(*b*) it shall be similarly situated with reference to the water surface and the boundaries of the channel or vessel in which it is contained;

$$(c)\ \frac{\nu_2}{\nu_1} = \frac{SQ_2}{Q_1};\qquad (d)\ \frac{\sigma_2}{\sigma_1}\cdot\frac{\rho_1}{\rho_2}\cdot\frac{Q_1^2}{Q_2^2} = S^3.$$

If the same fluid, at the same temperature, is used in the model as in the full-size weir or orifice, conditions (*c*) and (*d*) imply that

$$\frac{Q_2}{Q_1}=\frac{1}{S} \text{ and } \frac{Q_2}{Q_1}=\left(\frac{1}{S}\right)^{\frac{3}{2}}$$

respectively: these conditions cannot be obtained simultaneously. The indications are, however, that of the two effects, viscosity and surface-tension, the former is more important except when the head is relatively small. Thus, in experiments on flow over weirs, it appears that if h is less than about $\frac{1}{4}$ inch, surface-tension plays a proportionately greater part than would be the case in the full-size weir having a head of S times that in the model. At the other extreme, if the case under consideration is that of an orifice submerged on both sides, there will be *no* surface tension effect as far as the jet itself is concerned, and the capillary effect in the vessels containing the orifice will also in general be quite negligible, so that altogether the surface-tension of the fluid will then cease to enter the problem.

It is a remarkable fact that many of the most interesting experiments on the discharge of orifices and weirs are scientifically deficient in that the conditions of dynamical similarity do not appear to have been completely observed, and consequently it is impracticable to draw precise conclusions from them. Thus, in investigating the effect of the h/l ratio, the water temperature has not generally been recorded, so that the influence of viscosity and surface-tension is uncertain; in other cases where attention has been concentrated upon viscosity and surface-tension the h/l ratio appears to have been ignored. A discussion of the probable scale-effect in this type of problem is reserved for a later chapter.

(ii) Screw Propellers.

Here we may suppose the efficiency, η, of the screw to be influenced by the diameter l, the speed of advance v, the speed of rotation n revolutions per second, the density ρ and viscosity μ of the fluid, and (unless the propeller is deeply submerged) the acceleration g due to gravity.

Of the seven quantities here involved η itself is dimensionless (being a ratio of work done to energy applied). Consequently we may expect to find that η is a function of $(6-3)$ or three dimensionless quantities which may be obtained as follows:

$$N_1=\rho^{a_1}l^{b_1}v^{c_1}n;$$

$$\therefore\ M^\circ L^\circ T^\circ=M^{a_1}L^{-3a_1}L^{b_1}L^{c_1}T^{-c_1}T^{-1},$$

since n has the dimensions of $1/T$.

Hence, equating the indices of M, L and T in turn,

$$a_1 = 0, \quad b_1 = 1, \quad c_1 = -1;$$

$$\therefore \; N_1 = \frac{ln}{v}.$$

$$N_2 = \rho^{a_2} l^{b_2} v^{c_2} \mu;$$

$$\therefore \; M^\circ L^\circ T^\circ = M^{a_2} L^{-3a_2} L^{b_2} L^{c_2} T^{-c_2} M L^{-1} T^{-1};$$

$$\therefore \; a_2 = -1, \quad b_2 = -1, \quad c_2 = -1;$$

$$\therefore \; N_2 = \frac{\mu}{\rho l v} = \frac{\nu}{l v}.$$

$$N_3 = \rho^{a_3} l^{b_3} v^{c_3} g;$$

$$\therefore \; M^\circ L^\circ T^\circ = M^{a_3} L^{-3a_3} L^{b_3} L^{c_3} T^{-c_3} L T^{-2};$$

$$\therefore \; a_3 = 0, \quad b_3 = 1, \quad c_3 = -2;$$

$$\therefore \; N_3 = \frac{lg}{v^2}.$$

Hence we find that if our original assumption is correct as to the factors involved, the efficiency is a function of

$$\left(\frac{ln}{v}, \frac{\nu}{lv}, \frac{lg}{v^2}\right).$$

Now ln is proportional to the peripheral speed of the propeller blades. Consequently the first condition is that the efficiency of the full-size propeller may only be expected to be the same as that of the model if the ratio of speed of rotation to speed of advance is the same in the two cases. We are then left with two other conditions, namely that $\frac{\nu}{lv}$ and $\frac{lg}{v^2}$ shall have simultaneously the same respective values: a problem identical *in form* with that of ship models which we have already examined in pages 37–39. In the present case, however, velocities are usually so high that the motion of the fluid in the vicinity of the screw is intensely turbulent; viscosity then almost ceases to play any *direct* part,* and the speed of advance may be chosen from

$$\left(\frac{lg}{v^2}\right)_1 = \left(\frac{lg}{v^2}\right)_2,$$

where the suffix (1) refers to the actual and the suffix (2) to the model. If at the same time the propeller is submerged deeply enough to prevent

* By this is meant not that the fluid becomes inviscid, but that the motion in both model and prototype being essentially turbulent, there is no real necessity to make the Reynolds numbers identical for the sake of ensuring similarity in the flow-patterns.

the formation of surface waves, then gravity also disappears from the consideration, and in that case any speed of advance might be chosen. In any case, the thrust T_2 as measured in the model may be translated into the corresponding thrust for the full-size propeller by writing

$$\frac{T_1}{T_2}=\frac{\rho_1 l_1^2 v_1^2}{\rho_2 l_2^2 v_2^2}.$$

It is instructive, however, to pursue this problem further, and to try to take account of other factors which may contribute to the phenomenon. At relatively high speeds (say above about 600 ft. per second) the effect of the compressibility of the air may be appreciable in the case of *air screws* or *fans*. Also the distortion of the blades under stress ought really to be considered: this involves the density ρ' and the modulus of elasticity E of the material, while the compressibility of the air brings in its bulk modulus of compressibility K. The quantities involved are then increased in number to ten: T, l, v, ρ, ρ', μ, n, g, K and E. Of these, K and E are stresses having accordingly the dimensions of force per unit area or (mass × acceleration) per unit area, written symbolically

$$(MLT^{-2})L^{-2} \text{ or } ML^{-1}T^{-2}.$$

Proceeding as before and equating the indices of mass, length and time, in the expression $N_1=\rho^{a_1}l^{b_1}v^{c_1}T$; $N_2=\rho^{a_2}l^{b_2}v^{c_2}\rho'$, and so on, we obtain

$$T=\rho l^2 v^2 \phi\left\{\frac{\rho'}{\rho}, \frac{vl}{\nu}, \frac{nl}{v}, \frac{gl}{v^2}, \frac{K}{\rho v^2}, \frac{E}{\rho v^2}\right\}.$$

The fifth quantity contained within the brackets implies that if the model air screw is also tested in air, so that $\frac{K_2}{\rho_2}=\frac{K_1}{\rho_1}$, then $v_2=v_1$, or the velocity of translation of the model screw relative to the surrounding air must be the same as that of the full-size propeller. From the third quantity, $\frac{nl}{v}$, it next follows that the number of revolutions per minute of the model screw must be greater than that of the prototype in the ratio of $\frac{l_1}{l_2}$, while the first and last numbers inside the brackets imply that the model shall be made of the same material as the large screw (or, to be precise, of a material having the same density and elastic properties). The difficulty arises with the second and fourth terms which demand that

$$\frac{\nu}{vl_1}=\frac{\nu}{vl_2} \text{ and } \frac{gl_1}{v^2}=\frac{gl_2}{v^2},$$

since v_1 has already been made $=v_2=v$. Clearly these conditions cannot be satisfied and only experiment can show whether neglect of them introduces a serious error. The normal procedure is, of course, to test the model propeller, rotated by a small electric motor, in a wind-tunnel through which air is circulated at a measured speed. In the following table * are quoted the results obtained on an air-screw of diameter 8·91 feet, running at 540 r.p.m., compared with those measured by Eiffel on a $\frac{1}{3}$ scale model at 1,600 r.p.m. with the same translational speed:

TABLE III

Translational speed (m.p.h.)	*Measured thrust (lb.)*	*Thrust predicted from model* † *(lb.)*	*Measured efficiency (per cent.)*	*Efficiency predicted from model* † *(per cent.)*
0	200	200	0	0
11·0	186	182	32	34
22·0	156	148	58	58
32·5	109	105	67	67

It will be appreciated that in the case just mentioned, the rotational speed of the model (1,600 r.p.m.) was such as to give a screw tip-speed sensibly equal to that of the prototype which was three times as big in diameter and ran at 540 r.p.m.

It appears from experiment, however, that the effect of compressibility and viscosity may very often be neglected. For example, experiments have been made at the National Physical Laboratory ‡ on two air screws having a scale ratio of 7·5 : 1 and *run at very different tip-speeds*. Using the equation $T \propto \rho l^2 v^2$, a reasonable degree of accuracy was obtained. Thus, " the predicted thrust was only 4 per cent. less than the measured thrust and the efficiency 3 per cent. lower ".§

(iii) Hydraulic Pumps or Turbines.

If we assume that the efficiency of such machines is influenced by Q, the volume of water discharged per second, d the diameter of the impeller or runner, n the number of revolutions per second, the head h, and by the quantities ρ, μ and g, then writing:

* Extracted from Table I of *Research in Mechanical Engineering by Small-Scale Apparatus*, F. C. Johansen, *Proc. Inst. Mech.E.*, March 1929.

† Using $\left.\begin{array}{l}\dfrac{T_1}{\rho_1 v_1^2 l_1^2} = \dfrac{T_2}{\rho_2 v_2^2 l_2^2} \\ \text{and } \eta_1 = \eta_2\end{array}\right\}$ when $\rho_1 = \rho_2$ and $v_1 = v_2$.

‡ " Adv. Comm. for Aeronautics ", *Tech. Rep.*, 1911–12, p. 36; quoted by Gibson in his article, " The Principle of Dynamical Similarity, with Special Reference to Model Experiments ", *Engineering*, vol. 117, 1924, p. 359.

§ Gibson, *loc. cit.*

$$N_1 = \rho^{a_1} d^{b_1} n^{c_1} Q,$$
$$N_2 = \rho^{a_2} d^{b_2} n^{c_2} \mu,$$
$$N_3 = \rho^{a_3} d^{b_3} n^{c_3} g,$$
$$N_4 = \rho^{a_4} d^{b_4} n^{c_4} h,$$

we find that

$$N_1 = \frac{Q}{d^3 n},$$
$$N_2 = \frac{\mu}{\rho n d^2},$$
$$N_3 = \frac{g}{n^2 d} = \frac{g}{n^2 d} N_4 = \frac{gh}{n^2 d^2},$$
$$N_4 = \frac{h}{d}.$$

For strict hydrodynamic similarity, each of these four quantities should have the same value in the pump or turbine (1) as in its model (2).

The direct effect of viscosity, however, is small,* and it is usual to prescribe the speed of the model by the relationship

$$\left(\frac{h}{n^2 d^2}\right)_2 = \left(\frac{h}{n^2 d^2}\right)_1,$$

or

$$n_2 = n_1 \frac{d_1}{d_2} \sqrt{\frac{h_2}{h_1}} = S n_1 \sqrt{\frac{h_2}{h_1}},$$

the model being a geometrically similar representation of the machine to a scale of $1:S$.

At the same time, the discharges are related by

$$\left(\frac{Q}{d^3 n}\right)_2 = \left(\frac{Q}{d^3 n}\right)_1,$$

or

$$\frac{Q_2}{Q_1} = \left(\frac{d_2}{d_1}\right)^3 \frac{n_2}{n_1} = \left(\frac{d_2}{d_1}\right)^3 \frac{d_1}{d_2} \sqrt{\frac{h_2}{h_1}}$$
$$= \left(\frac{d_2}{d_1}\right)^2 \sqrt{\frac{h_2}{h_1}}$$
$$= \frac{1}{S^2} \sqrt{\frac{h_2}{h_1}}.$$

* Experiments by F. Reimerschmid (*Trans. Hydr. Inst. of Munich Tech. Univ.*, Bulletin 3, English translation published in 1935 by Am. Soc. Mech. Engs.) on a model Francis turbine with a runner diameter of 10·9 cm. showed the efficiency to be increased from about 78·6 per cent. to 81·7 per cent. as a result of reducing the kinematic viscosity of the water from 0·016 to 0·004 cm.^{2}/sec. by raising its temperature from 3° to 70° C. The rate of flow was correspondingly increased from about 13·40 to 13·68 litres per sec. In these tests the head was maintained constant at 1 metre and the guide vanes and the speed of the turbine controlled so as to operate the machine as close as possible to its maximum efficiency.

Moreover, for a given mechanical efficiency, the horse-power P absorbed by the pump or generated by the turbine is proportional to ρQh;

$$\therefore \frac{P_2}{P_1}=\frac{Q_2h_2}{Q_1h_1} \quad \text{if } \rho \text{ is the same in the model and full-size installations.}$$

$$\therefore \frac{P_2}{P_1}=\frac{1}{S^2}\left(\frac{h_2}{h_1}\right)^{\frac{3}{2}}.$$

The process, then, is to measure P_2, Q_2, h_2 and n_2 in the laboratory and to estimate P_1, Q_1 and n_1 for a head h_1 from the expressions:

$$\frac{P_1}{P_2}=S^2\left(\frac{h_1}{h_2}\right)^{\frac{3}{2}},$$

$$\frac{Q_1}{Q_2}=S^2\sqrt{\frac{h_1}{h_2}},$$

$$\frac{n_1}{n_2}=\frac{1}{S}\sqrt{\frac{h_1}{h_2}}.$$

It should be realized in this connection that mechanical friction has been assumed to have the same proportionate effect in the model as in the actual machine, that viscosity plays but a small part as regards scale-effect, and that over the range of conditions involved the head in the model installation may be chosen arbitrarily to suit the convenience of the experimenter—that is, we have not observed the rule that $\left(\frac{h}{d}\right)_2$ should $=\left(\frac{h}{d}\right)_1$. Owing to these simplifications, we may anticipate some scale-effect. To make allowance for this, it is common practice among turbine engineers * to employ a correcting factor, such as that due to Camerer:

$$\eta_T=1-(1-\eta_0)\left[\frac{1{\cdot}4\sqrt{d_0}+\sqrt{K_d}}{1{\cdot}4\sqrt{d_0}+1}\right],$$

or that of Moody:

$$\eta_T=1-(1-\eta_0)[K_d^{\frac{1}{4}}\,.\,K_h^{\frac{1}{10}}],$$

in which D = tip diameter of full-scale turbine (ft.),
d_0 = tip diameter of model turbine (ft.),
H = head of actual runner,
h_0 = head of model runner,
η_T = turbine efficiency of full-scale turbine,
η_0 = turbine efficiency of actual model turbine,

$$K_d=\frac{d_0}{D}; \quad K_h=\frac{h_0}{H}.$$

* "The Development of the Kaplan Turbine", J. R. Finniecome, *Engineering*, vol. 150, 1940, p. 381.

For example, if $\eta_0 = 0{\cdot}82$, $d_0 = 1$ ft., $D = 3{\cdot}00$ ft., $h_0 = 64$ ft., $H = 80$ ft., we find that η_T is 0·85(2) or 0·86(6) respectively when estimated on the basis of the Camerer or the Moody formula. If η_0 were 0·90 in this example, the corresponding values of η_T would be 0·91(8) and 0·92(7).

At a first glance the adjustments required by these formulae may appear to be remarkably small, and so indeed they are, having regard to all the considerations involved. But it has been pointed out * that in large hydraulic power plants one per cent. in efficiency may represent an operating cost of some thousands of pounds annually. With this in mind, turbine and pump engineers and researchers will no doubt continue their efforts towards an even more precise knowledge of the scale-effect inherent in such model experiments, and especially perhaps of the influence of the hydraulic roughness of the various surfaces used in the model and in the full-scale machine.

A very interesting and instructive article † concerning model turbine tests appeared in *Engineering*, vol. 122, 1926, pp. 374, 405 and 439. In it the author, Mr. Hadar Lind, remarks that " the results which are obtained in the laboratory and testing station are used as a basis for calculations and the design of turbine plants and for determining guarantees. Many years of experience have shown that the results obtained in actual plants as regards output, speed and efficiency, correspond very closely with those obtained from tests on the model turbine. It is, of course, presupposed that both turbines are similar throughout . . ." Mr. Lind also describes the arrangements made to study the phenomenon of " cavitation ", that is, the disturbance caused by low pressures and the consequent accumulation of vapour as a result of excessive suction height and centrifugal forces in the moving water. Not only are the output and efficiency of the model measured under various heads and suction heights but the flow is studied visually through a glass case. " To the end of the turbine shaft is coupled a contact which can be adjusted so that it closes a circuit once, twice or four times for each turbine revolution. The arrangement is connected up with a Neon lamp, which thus receives a current impulse each time the contact is closed. The periods of illumination of the lamp are so short that the blades of the runner appear to be stationary, and are quite sharp even at speeds of 2,000 r.p.m. or more. Because of the extremely small air bubbles which accompany the water, one can clearly see the water flow through the apparently motionless

* " Dimensional Analysis of Centrifugal Pumps ", *Engineering* editorial, Nov. 8, 1940, vol. 150, p. 371.

† " The Hydraulic Laboratories at Verkstaden, Kristinehamn, Sweden ", by Hadar Lind, *Engineering*, vol. 122, 1926, pp. 374, 405 and 439. When comparing the efficiency of the full-size machine with that of the model, Mr. Lind states that " the method of calculation indicated by Camerer " (see p. 53) " has proved very useful ".

runner and can further closely study the whole cavitation phenomenon. ... The researches which have already been made in the cavitation laboratory have also resulted in a degree of certainty previously unknown when fixing the suction height for Kaplan and propeller turbines, and have also extended the useful range of these turbines far above what one dared to hope a couple of years ago." Mr. Lind further describes a laboratory devoted to model tests on the impulse as distinct from the reaction type of turbine, in which investigations have been made of the most suitable type of bucket, the relation between the wheel diameter and the jet diameter, the best number of buckets, the shape of the needle and nozzle, the spacing of the nozzles when more than one jet impinges on a runner, the arrangement of the casing around the runner, and the form of the guides employed to prevent part of the water, after leaving the buckets, from being thrown back on to the wheel.

In addition to the features of turbine design already mentioned, other problems may arise which also may profitably become the subject of model investigation. These include the correct shape of the inlet structure and the tailrace, the transition of the suction-pipes or draught-tubes into the tailrace, the size and shape of surge-tanks,* and so on.

A notable study of inlet and tailrace problems was made, in connection with the Albbruck-Dogern Hydro-electric Power Station on the Rhine,† at the Federal Technical University at Zürich, in the River Structures laboratory at Karlsruhe, and at the Prussian Institute of Hydraulics and Boat Construction, Berlin. This comprehensive investigation included experiments on a model of scale 1 : 125, embracing the tailrace and a portion of the Rhine itself in order to determine the nature of possible siltation. "Coal briquette particles of 1 to 3 millimetres fineness" were adopted as mobile bed material, and at the conclusion of his paper Dr. Gruner remarks that each of the various experiments made previous to the construction of the plant "gave most satisfactory results, as their application in nature gave results which the laboratory investigations had foreseen. Thus this series of extensive experiments proved anew that investigations on hydraulic problems by scientifically-operated laboratories form an indispensable implement for the design of hydraulic projects".

Concerning tests on *centrifugal pumps*, it is of interest to observe that experiments with models using *air* may be convenient and may provide very valuable data for the designer of large *water* pumps. Such a model possesses advantages in the way of simplicity of construction—" light

* See page 57 for an account of model surge-tank experiments.

† "Research work in relation to the Albbruck-Dogern Hydro-electric Power Station on the Rhine", H. E. Gruner, *Trans. Inst. of Water Engineers*, vol. XXXIX (1934), p. 144. For a description of the Hydraulic Laboratory of the Federal Institute of Technology, Zürich, see *Engineering*, July 1, 1938, p. 3.

castings, wood, sheet-metal, even plasticine " *—and, correspondingly, adaptability to rapid modification for the purpose of studying the effects of changes in design.

If the effects of viscosity and compressibility are negligible, it follows that a pump which delivers Q gallons per minute of water against a head of h feet of water will at the same speed deliver Q gallons per minute of *air* against a head of h feet of *air*. Mr. R. W. Allen quotes results † comparing the characteristics of an air model with the final full-power trials of a pump with water: over a range of total head from 120 to 180 feet, the maximum discrepancy in rate of delivery appears to have been about 4 per cent., while the general agreement was within 1 or 2 per cent.

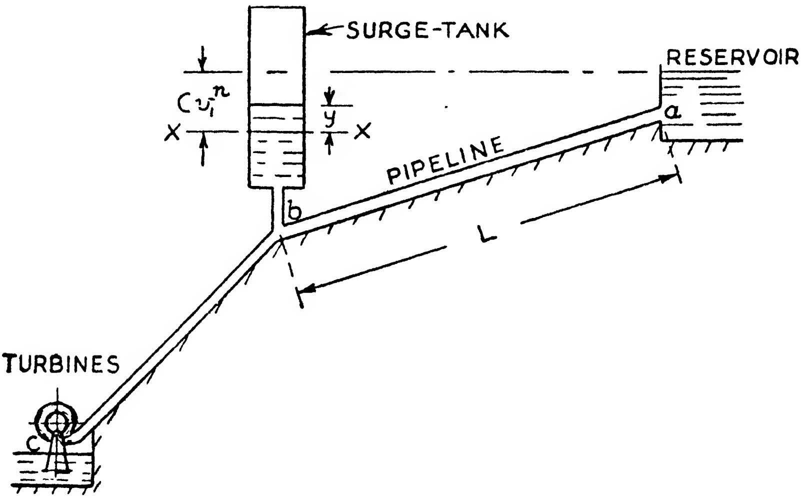


FIG. 8. Surge-Tank.

It should be realized, however, that a model discharging air instead of water cannot be used to study cavitation, nor in general can it provide a reliable measure of efficiency, if only because the power absorbed is so relatively small, and accordingly the losses in bearings may assume a disproportionate importance. Thus, the horse-power being proportional to ρQh and the density ρ of air being only $\frac{1}{800}$ (approximately) of the density of water, it follows that a pump performing a certain duty with air will absorb only about $\frac{1}{800}$ of the power required with water.

* *Some Experiences of the Use of Scale Models in General Engineering*, R. W. Allen Brit. Assoc., Section G (Cambridge), Aug. 19, 1938; also reproduced in *Engineering*' vol. 146, 1938, p. 243.

† *loc. cit.*

(iv) Models of Surge-Tank Installations.*

It is often desirable, and sometimes essential, to incorporate a "surge-tank" in a hydro-electric installation. The purpose of the tank is to avoid excessive surges of pressure when the hydraulic turbine gates are closed and to provide a sufficient supply of water at times of increasing load. (See Fig. 8.)

Let L represent the length of the pipe-line ab between the reservoir and the surge-tank;

- A the sectional area of the pipe-line;
- R the ratio of sectional area of the surge-tank to that of the pipe-line;
- V_1 the velocity in the pipe-line before a change of load;
- V_2 the velocity in the pipe-line when conditions have settled down after the change of load;
- V the instantaneous velocity in the pipe-line during the change;
- V' the velocity (simultaneous to V) in the pipe bc between the surge-tank and the turbines, during the change;
- y the height of the water surface in the surge-tank, above that level which is obtained under steady conditions with the initial velocity V_1.

Let us suppose that the loss of head in the main pipe-line (of length L) is CV_1^n at velocity V_1 and CV^n at velocity V. (This can only be an approximation, since C and n will depend, to some extent, upon the velocity.)

Now if the suffix m be used to define a quantity in the model, we must have:

(a) The scale of vertical surges the same as the scale of friction heads, i.e.

$$\frac{y}{y_m} = \frac{CV^n}{C_m V_m^n}. \qquad \text{(1)}$$

(b) At a certain instant, the head producing retardation in the pipe-line is $y - C(V_1^n - V^n)$, for if the conditions were steady at the instantaneous velocity V, the level in the surge-tank would be $CV_1^n - CV^n$ above the datum XX in Fig. 8.

Consequently,

$$y - C(V_1^n - V^n) = -\frac{L}{g}\frac{dV}{dt}. \qquad \text{(2)}$$

* "The Investigation of the Surge-Tank Problem by Model Experiments", A. H. Gibson, *Proc. Inst. C.E.*, vol. 219, Pt. 1, 1924–25, p. 161. "A Comparison of the Results of Observations on Surge Tank Installations and on their Scale Models; with an Investigation of the Dead-Beat Surge Tank, and of Surge Tanks of Non-Uniform Section", Gibson and Cowen, *Proc. Inst. C.E.*, vol. 235 (1934), p. 327.

But the rate of flow into the surge-tank, plus that in the pipe between the tank and the turbines, must equal that in the main pipe-line;

$$\therefore\ R\frac{dy}{dt}=V-R'V', \quad\quad (3)$$

if the sectional area of the pipe *bc* bears a ratio R': 1 to that of the pipe *ab*.

It is the complex mathematical nature of the equations derived above which makes it desirable to carry out a model investigation if such is practicable.

Now for equation (2) to be dimensionally homogeneous, y must have the dimensions of CV^n and of $\frac{LV}{gt}$;

$$\therefore\ \frac{y}{y_m}=\frac{LVt_m}{L_mV_mt}. \quad\quad (4)$$

Combining equations (1) and (4), we obtain

$$\frac{CV^n}{C_mV_m{}^n}=\frac{LVt_m}{L_mV_mt},$$

or

$$\frac{t}{t_m}=\frac{LC_mV_m{}^{n-1}}{L_mCV^{n-1}}, \quad\quad (5)$$

which fixes the time-scale t/t_m for any similar happenings in the model and its prototype.

Again, for equation (3) to be homogeneous, R must have the dimensions of $\frac{Vt}{y}$,

and

$$\therefore\ \frac{R}{R_m}=\frac{Vt}{y}\frac{y_m}{V_mt_m}. \quad\quad (6)$$

Equation (6) combined with (1) and (5) gives

$$\frac{R}{R_m}=\frac{L}{L_m}\left(\frac{C_m}{C}\right)^2\left(\frac{V_m}{V}\right)^{2n-2}, \quad\quad (7)$$

which determines the appropriate size of surge-tank for use in the model.

A numerical example will now be calculated: A pipe-line 500 ft. long has a diameter of 4 feet. It is equipped with a surge-tank of diameter 11·33 ft. The initial flow is at a speed of 4·77 ft. per sec., and this is suddenly reduced so as to settle down to 1·94 ft. per sec. The loss of head in the pipe is 0·61 ft. at 4·77 ft. per sec. In constructing a model of this installation, a pipe of diameter 3 inches is available, for which experiment shows a frictional loss of head $=0{\cdot}086V^{1{\cdot}82}$. Assuming the same index of velocity to apply to the full-size pipe, and adopting an initial velocity of 1 ft. per sec. for the model, determine the final velocity required in the model and the diameter of the model surge-tank. What will be the scales of time and of amplitude of surge?

Here we have:

$$\text{the final velocity in the model} = 1{\cdot}00 \times \frac{1{\cdot}94}{4{\cdot}77} = \mathbf{0{\cdot}406}\ \text{ft./sec.}$$

Also
$$\frac{y}{y_m} = \frac{CV^n}{C_m V_m{}^n} = \frac{0{\cdot}61}{0{\cdot}086 \times 1{\cdot}00^{1{\cdot}82}} = \mathbf{7{\cdot}09}.$$

$\therefore$ from (4),
$$7{\cdot}09 = \frac{500}{33{\cdot}3} \cdot \frac{4{\cdot}77}{1{\cdot}00} \cdot \frac{t_m}{t}, \text{ if } L_m = 33{\cdot}3 \text{ ft.,}$$

$$\frac{\mathbf{t}_m}{\mathbf{t}} = \frac{\mathbf{1}}{\mathbf{10{\cdot}1}}.$$

Again, from (6),
$$\frac{R}{R_m} = \frac{V y_m t}{V_m y t_m} = \frac{4{\cdot}77}{1{\cdot}00} \cdot \frac{1}{7{\cdot}09} \cdot \frac{10{\cdot}1}{1}$$
$$= 6{\cdot}80.$$

But
$$R = \left(\frac{11{\cdot}33}{4}\right)^2 = 8{\cdot}06.$$

$$\therefore\ R_m = 1{\cdot}18.$$

$$\therefore \text{ Area of model surge-tank} = 1{\cdot}18 \times \frac{\pi 3^2}{4} \text{ sq. in.}$$

$\therefore$ **Diam. of model surge-tank = 3·26 in.**

Hence, if the model were running initially at 1·00 ft. per second, it would be necessary to arrange to close it down to a final velocity of 0·406 ft. per second and to provide it with a surge-tank of 3·26 inches diameter. If, then, a surge of height x were observed to take place in the model during a time t_m, the anticipated corresponding movement in the prototype would be $7{\cdot}09x$ in a time $10{\cdot}1t_m$.

For a more detailed discussion and for evidence as to the close reliability of this type of model, the reader is referred to the publications already cited in the footnote to page 57, but at this stage it is well to note that this type of model uses different horizontal and vertical scales.

Regarding the principles from a fundamental basis again, and taking pipe friction as proportional to v^2 so that viscosity may (from that viewpoint) be neglected, we have the height y apparently depending upon L, t, ρ, v, g.

Calling $N_1 = L^{a_1} t^{b_1} \rho^{c_1} y$, we find dimensionally that

$$a_1 = -1,\ \ b_1 = 0,\ \ c_1 = 0, \text{ or } N_1 = \frac{y}{L}.$$

Similarly,
$$N_2 = L^{a_2} t^{b_2} \rho^{c_2} v = \frac{vt}{L},$$

and
$$N_3 = L^{a_3} t^{b_3} \rho^{c_3} g = \frac{gt^2}{L},$$

so that the time scale and velocity scale should be connected with the linear scale (now common to both y and L) by the expressions $t \propto \sqrt{L}$ and $v \propto \sqrt{L}$.

But the actual pipe-line is usually very long so that to reproduce its length in a laboratory would involve a comparatively small scale and, correspondingly, oscillations in the surge-tank so small as to be difficult to observe with accuracy; moreover, the time of valve closure or of surging in the model would be inconveniently short and the velocity in the model pipe-line so low as to make it probable that friction losses would not even approach the square law of resistance. In any event it is known that the friction loss in such pipes does *not* vary as the square of the velocity, but more nearly as $v^{1 \cdot 85}$, so that one of our original assumptions in the last paragraph was only an approximation.

Here, then, is an example of model investigation in which the vertical scale is deliberately exaggerated for both theoretical and practical reasons. The vertical scale is chosen as defined by the ratio of the friction heads in the model and full-scale pipe-lines, and not from the linear scale-ratio of the pipes. Compensation for this is then made by choosing the size of surge-tank and the time scale so as to be consistent with the equation (2) for the head producing retardation in the pipe-line and with equation (3) for the continuity of flow.

(v) A Problem involving the Transference of Heat.

In many problems concerning the reception of heat by one fluid from another or from a solid surface, in relative motion, the temperature-differences involved are such as to make radiation effects quite unimportant. It is often true, indeed, that the only physical factors likely to influence the phenomenon are:

H, the flow of heat per unit time;

T, the temperature difference;

k, the conductivity of the fluid;

σ, the specific heat of the fluid;

ρ, the density of the fluid;

μ, the viscosity of the fluid;

l, some representative linear dimension;

v, the velocity.

Now, heat may be regarded as a form of energy; temperature is regarded for dimensional purposes as a quantity, T, not yet reducible in terms of the fundamental units of mass M, length l and time t.

Accordingly, we may write, in dimensional form,

$$
\begin{aligned}
H &\propto \text{energy per unit time;}\\
&\propto M \times (\text{velocity})^2 \text{ per unit time;}\\
&\propto Ml^2t^{-3};\\
k &\propto \frac{H \times \text{length}}{\text{sectional area} \times \text{temp. difference}};\\
&\propto \frac{H}{lT};\\
&\propto Mlt^{-3}T^{-1};\\
\sigma &\propto \frac{\text{Heat per unit mass}}{\text{rise in temp.}};\\
&\propto \frac{Ht}{MT};\\
&\propto \frac{Ml^2t^{-2}}{MT};\\
&\propto l^2t^{-2}T^{-1}.
\end{aligned}
$$

In assuming H dependent upon l, ρ, v, μ, k, σ and T, eight quantities are introduced, involving in effect four primary quantities, mass, length, time and temperature. We may expect, therefore, that $(8-4)$, or four dimensionless groups, will result from the process of dimensional analysis.

Call

$$
\begin{aligned}
N_1 &= H^{a_1}l^{b_1}\rho^{c_1}T^{d_1}v,\\
N_2 &= H^{a_2}l^{b_2}\rho^{c_2}T^{d_2}\mu,\\
N_3 &= H^{a_3}l^{b_3}\rho^{c_3}T^{d_3}k,\\
N_4 &= H^{a_4}l^{b_4}\rho^{c_4}T^{d_4}\sigma.^*
\end{aligned}
$$

By equating the indices of mass, length, time and temperature, of the quantities on the right-hand side of these equations, to zero, it is found that

$$
\begin{aligned}
N_1 &= \frac{vl^{\frac{2}{3}}\rho^{\frac{1}{3}}}{H^{\frac{1}{3}}},\\
N_2 &= \frac{\mu}{H^{\frac{1}{3}}l^{\frac{1}{3}}\rho^{\frac{2}{3}}},\\
N_3 &= \frac{lTk}{H},\\
N_4 &= \frac{\sigma T\rho^{\frac{2}{3}}l^{\frac{4}{3}}}{H^{\frac{2}{3}}},
\end{aligned}
$$

* The reader may find it instructive to carry out the dimensional analysis by making

$$
\begin{aligned}
N_1 &= v^{a_1}l^{b_1}\rho^{c_1}T^{d_1}H,\\
N_2 &= v^{a_2}l^{b_2}\rho^{c_2}T^{d_2}\mu,
\end{aligned}
$$

etc.

whence

$$\frac{H}{\rho l^2 v^3} \propto \phi\left\{\frac{\mu^3}{Hl\rho^2}, \frac{lTk}{H}, \frac{T^3\sigma^3\rho^2 l^4}{H^2}\right\}$$

$$\propto \phi\left\{\frac{\mu}{vl\rho}, \frac{Tk}{\rho v^3 l}, \frac{T\sigma}{v^2}\right\}$$

$$\propto \phi\left\{\frac{\sigma T}{v^2}, \frac{k}{\sigma\mu}, \frac{\mu}{vl\rho}\right\}.$$

Experiment shows that if radiation be neglected, heat flow is directly proportional to temperature-difference; it follows that $\phi\left(\frac{\sigma T}{v^2}\right)$ must be proportional to $\frac{\sigma T}{v^2}$, and therefore

$$\frac{H}{\rho l^2 v^3} \propto \frac{\sigma T}{v^2}\phi\left\{\frac{k}{\sigma\mu}, \frac{\mu}{vl\rho}\right\},$$

or

$$H \propto \rho l^2 v \sigma T \phi\left\{\frac{k}{\sigma\mu}, \frac{\mu}{vl\rho}\right\}.$$

Again, if $H \propto v^a$, $\phi\left(\frac{\mu}{vl\rho}\right)$ must $\propto \left(\frac{\mu}{vl\rho}\right)^{1-a}$, and correspondingly

$$H \propto \rho^a l^{a+1} v^a \mu^{1-a} \sigma T \phi\left(\frac{k}{\sigma\mu}\right).$$

Now, if the same fluid be used in the model—and at the same temperature—as in the prototype, $\frac{k}{\sigma\mu}$ will have the same value in both, so that

$$H \propto l\left(\frac{vl\rho}{\mu}\right)^a \mu\sigma T. \quad \text{.............................(1)}$$

Assuming this equation to represent the law of heat transference sufficiently accurately, it is next desirable to try to establish a relationship between a and the index n of velocity in the corresponding law of fluid resistance. Suppose, then, that the force of resistance F to fluid motion is proportional to v^n. Dimensionally,

$$F \propto \text{mass} \times \text{acceleration},$$

$$\propto Mlt^{-2}.$$

But

$$\frac{H}{\sigma T} \propto \frac{Ml^2t^{-3}}{l^2t^{-2}T^{-1}T},$$

$$\propto Mt^{-1}.$$

Hence

$$\frac{F}{\frac{H}{\sigma T}} \propto lt^{-1} \propto v. \quad \text{.................................(2)}$$

Now, if it is true, as implied by equation (1), that

$$\frac{H}{\sigma T} \propto v^a,$$

it follows that

$$\frac{F}{\frac{H}{\sigma T}} \propto \frac{v^n}{v^a}.$$

But $\dfrac{F}{\frac{H}{\sigma T}} \propto v$, from equation (2).

Hence $$n - a = 1,$$
or $$a = n - 1.$$

In fully turbulent flow, $n = 2$; in streamline flow, $n = 1$. Consequently, reference to equation (1) suggests that if the flow is turbulent,

$$H \propto l^2 v \rho \sigma T, \quad \text{...(3)}$$

while if it is streamline,

$$H \propto l \mu \sigma T. \quad \text{...(4)}$$

Now let the suffix (1) refer to the prototype and the suffix (2) to the model, the scale of which is $1 : x$; then

$$\left.\begin{aligned} H_1 &\propto l_1^2 v_1 \rho_1 \sigma_1 T_1 \\ H_2 &\propto \frac{l_1^2}{x^2} v_2 \rho_2 \sigma_2 T_2 \end{aligned}\right\} \text{for turbulent flow.}$$

Assuming $\rho_1 = \rho_2$ and $\sigma_1 = \sigma_2$ and the velocity scale to be given by $v_2 = v_1/\sqrt{x}$, it follows that

$$\frac{H_1}{H_2} = \frac{T_1}{T_2} x^{\frac{5}{2}}. \quad \text{....................................(5)}$$

At this stage, it is advisable to recall that T_1 and T_2 represent temperature *differences*; H_1 and H_2 the quantities of heat transferred *in unit time* from the hot body to the colder fluid or from the hot fluid to the colder fluid. Let us suppose that $T_1 = T_2$, that is, the temperature difference in the prototype is the same as in the model. In the following time t_1 seconds, a mass M_1 in the prototype will have its temperature raised (under a temperature difference T_1) by $\dfrac{H_1 t_1}{M_1 \sigma}$. In the model, a mass M_2 of the same fluid will have its temperature raised by $\dfrac{H_2 t_2}{M_2 \sigma}$ in time t_2. If these temperature rises are to be the same,

$$\frac{H_1 t_1}{M_1} = \frac{H_2 t_2}{M_2},$$

or

$$\frac{H_1}{H_2} = \frac{M_1 t_2}{M_2 t_1}.$$

But $\qquad t_1 = t_2\sqrt{x} \quad \text{if} \quad v_1 = v_2\sqrt{x},$

and $\qquad M_1 = x^3 M_2.$

Hence
$$\frac{H_1}{H_2} = \frac{t_2 M_1}{t_1 M_2} = \frac{x^3}{\sqrt{x}} = x^{\frac{5}{2}}.$$

This agrees with equation (5) on our hypothesis that T_2 is made $= T_1$.

According to this reasoning, if the velocity scale is taken to be $1 : \sqrt{x}$ and the time scale to be $1 : \sqrt{x}$, the *same* temperatures will be obtained in the model of scale $1 : x$ as in the prototype, provided the flow in both cases is fully turbulent.

If, on the other hand, the flow in both prototype and model is streamline, the requirement becomes

$$\frac{t_2 M_1}{t_1 M_2} = \frac{l_1 \mu_1}{l_2 \mu_2} \quad \text{(from equation (4), taking } \sigma_1 = \sigma_2\text{)};$$

$$\therefore \frac{\rho_1 l_1{}^3 t_2}{\rho_2 l_2{}^3 t_1} = \frac{l_1 \mu_1}{l_2 \mu_2};$$

$$\therefore \frac{\rho_1 l_1}{\mu_1} \cdot \frac{l_1}{t_1} = \frac{\rho_2 l_2}{\mu_2} \cdot \frac{l_2}{t_2};$$

$$\therefore \frac{\rho_1 l_1 v_1}{\mu_1} = \frac{\rho_2 l_2 v_2}{\mu_2},$$

or for the same temperatures to be found in the model as in the prototype, the Reynolds numbers must be identical. Using the same fluid, this condition demands that

$$v_2 = \frac{v_1 l_1}{l_2} = v_1 x,$$

which means, for example, that if the scale of the model is $1 : 100$, the velocities in it must be 100 times as great as in the prototype—a requirement hardly likely to be practicable. Moreover, even if practicable, the velocities so increased would almost certainly lead to turbulence in the model and consequently to a breakdown in the whole theory of comparison with a prototype having streamline flow.

An opportunity to test the validity of the reasoning given above occurred recently in Professor Gibson's laboratory at Manchester on a model, with a scale of $1 : 150$, of a portion of a river from which water is taken for the condensers of a Power Station. After passing through the condensers, this water is discharged (at a higher temperature) back into the river; the problem was to arrange the position and direction of the outlet in such a way as not to cause a serious heating of the water passing into the intake.

A portion of this Power Station having been in operation for some time, and temperature observations having been made in the actual river, it was possible to reproduce the appropriate conditions and to measure the temperatures in the model at the corresponding points. A very close agreement was discovered. For example, in one series of observations the agreement was within 0·2° F. at four of five points of comparison; at the fifth, there was a discrepancy of 2·0° F., but this discrepancy could be entirely eliminated by moving the thermometer in the model a distance equivalent to 15 feet in nature.

In this model, the scale of velocity adopted was $1 : \sqrt{150}$ and the scale of discharges $1 : (150)^{\frac{5}{2}}$. Water was drawn out at the appropriate point through a pipe fitted with a valve which could be so adjusted as to regulate the flow to a value, as measured through a calibrated orifice, of $\dfrac{Q}{150^{\frac{5}{2}}}$, Q representing the known rate of flow through the prototype condensers. The water so extracted was allowed to go to waste. At the same time, a flow of $\dfrac{Q}{150^{\frac{5}{2}}}$ of heated water was supplied to the outlet pipe projecting into the model river, whose depth was adjusted to represent the known depth in nature and whose current velocity was made equal to that of the natural river divided by $\sqrt{150}$.

The fact that the *same* temperature differences from point to point in the model were registered as in the prototype confirmed the theory already discussed in relation to turbulent flow, and it would appear that this theory still applies even with very low speeds * of the main current of the river. Indeed, when a jet of heated water is projected into a stagnant mass of colder water, there is pronounced turbulence in the mixing zone, as shown by the observation of dye introduced with the water issuing in the jet.

For further information concerning dynamical similarity in relation to heat problems, the reader is advised to consult *The Mechanical Properties of Fluids*, Chapter IV (Blackie), and " Research in Mechanical Engineering by Small-Scale Apparatus ", by F. C. Johansen, *Proc. I.Mech.E.*, 1929.

In concluding this Chapter, it may be stated that the subject of dynamical similarity has been discussed at some length for two main purposes:

(*a*) To try to establish the fundamentals on which the theory and practice of model technique are based; and

(*b*) To indicate the difficulties involved and the present imperfections even in well-established applications such as those of aircraft, turbine and pump models, which nevertheless are acknowledged to provide invaluable data.

* There is scope for further research to attach a more definite value to this phrase " very low speeds ".

When we come to discuss other examples of hydraulic laboratory practice we shall encounter additional difficulties and complications, but the reader may be disposed to agree that such investigations may also be of the utmost value, even though sometimes involving a technique not yet so fully developed or understood.

CHAPTER II

MODEL EXPERIMENTS ON WEIRS, SPILLWAYS, AND SLUICE GATES

(a) Sharp-crested Rectangular Weirs.

If the head, h, and the length of the weir, b, are measured in feet, we may write

$$Q = C\sqrt{2g}\,bh^{\frac{3}{2}},$$

where Q is the rate of discharge in cusecs.

Suppose a model has been made of such a weir to a scale of $1 : S$. If the suffix (1) refers to the original weir and the suffix (2) to its model, then for geometrical similarity b_2 should $= \frac{b_1}{S}$ and $h_2 = \frac{h_1}{S}$. Provided there is no scale-effect due to any other cause, it is possible to calculate Q_1 from $Q_1 = \frac{b_1 h_1^{\frac{3}{2}}}{b_2 h_2^{\frac{3}{2}}} Q_2 = S^{\frac{5}{2}} Q_2$, but we might expect this simple relationship to be upset by the operation, in proportionately different degrees for the model and its prototype, of viscosity and surface tension. There is the complication, also, in the case of suppressed weirs discharging a jet of width equal to the channel itself, of the effect of aeration or ventilation of the "nappe", that is, the stream which pours over the weir crest. Should part of the air occupying the space between the underside of the nappe and the back of the weir be carried away in the stream, then the lower surface of the nappe will be depressed and the rate of discharge increased. It is evident, therefore, that conditions of ventilation should be similar—the most positive solution being that of ample provision of air vents in the side-walls of the channel; the discharge then being given, according to work by Rehbock at Karlsruhe (about 1912), by writing

$$C = \frac{2}{3}\left(0{\cdot}605 + \frac{1}{320h - 3} + \frac{0{\cdot}08h}{p}\right),$$

where p is the height of the weir crest above the bed of the approach channel.

To take an example, suppose $h_1 = 2$ ft., $\frac{h}{p} = \frac{1}{3}$ and $S = 25$. Then $C_1 = 0{\cdot}422$ and $C_2 = 0{\cdot}451$, or the scale-effect in the model amounts to 7·0 per cent. If, however, the scale of the model is $1 : 50$, C_2 becomes

0·490 and the scale-effect with a head corresponding to $h_1 = 2$ ft. is 16·2 per cent. In both cases, the model discharge is relatively too great.

Possible causes of this marked scale-effect are:

(i) Surface-tension,

(ii) Viscosity,

(iii) Velocity-distribution in the approach channel: this affects the kinetic head due to velocity of approach, and correspondingly it follows that if the velocities in the model and in the full-size channel are distributed differently, then the proportionate effects of kinetic head will also be different.

Surface tension is likely to influence the phenomenon in so far as it leads to an increased pressure inside the curved boundaries of the nappe. This excess of pressure over that of the surrounding atmosphere is inversely proportional to the radius of curvature of the boundary and directly proportional to the surface tension; since the radii of curvature of the model and the full-size nappes are different, there will be different surface tension effects. Rehbock * has, in fact, found empirically that his formula may be reduced to the rather simpler expression:

$$Q = \frac{2}{3}\left(0{\cdot}605 + 0{\cdot}08\,\frac{h}{p}\right)\sqrt{2g}\,b\,(h + 0{\cdot}00281)^{\frac{3}{2}} \text{ cusecs, if } h,\ b \text{ are in feet.}$$

As to the influence of viscosity on the distribution of velocity both in the nappe and the approaching stream, and consequently on the discharge, Rehbock has found † that the rate of discharge is not measurably changed by a difference of 12° C. in the water temperature, while Yoshinori Shimoyama ‡ apparently has discovered that viscosity has a completely negligible effect but that surface tension is very important at low heads.

Another question which arises is as to where the head should be measured: evidently it should not be observed on the rapidly falling profile of the stream, since then a slight displacement of the point of measurement would produce an appreciable error. On the other hand, the point should not be chosen so far back from the weir as to involve appreciable friction head in the approach channel. The British Standard Specification No. 599 (for Pump Tests), 1935, recommends a distance from the weir of about six times the maximum head: the reader will also find in the Specification recommendations as to the formulae to adopt when using sharp-edged weir plates as measuring devices—and again he

* *Hydraulic Laboratory Practice*, ed. Freeman, Am. Soc. Mech. Engs., 1929, p. 149.

† *Hydraulic Laboratory Practice*, ed. Freeman, Am. Soc. Mech. Engs., 1929, p. 151.

‡ Kyushu Imperial University, *Memoirs of College of Engineering*, 7, 185–207· abstract available in *Eng. Abstracts*, Inst. C.E., April 1935, p. 185.

will find details of the minimum area of ventilating passages desirable in the case of suppressed rectangular weirs.

(b) Other Shapes of Weir.

For river-gauging "in the field", sharp-edged weirs suffer from the disadvantage of being comparatively easily damaged; moreover, the ventilating holes placed between the weir and the overflowing sheet of water may become clogged with debris or ice. Again, many rivers have in the past been provided with weirs for quite other purposes such as the raising of the water level to meet the needs of navigation, industry or irrigation: if it is practicable to estimate the coefficient of discharge of such a weir by model experiments, the river engineer is then able to assess the flow of his river at various times by keeping a record of the head over the existing weir. In this way, invaluable quantitative information may be, and is being, collected for rivers whose discharge was hitherto known only very roughly, and having this more precise data in his possession the engineer is in a far stronger position when confronted with the design of any proposed regulating or control works.

Rehbock states (*Hydraulic Laboratory Practice*, ed. Freeman, Am. Soc. Mech. Engs., 1929, pp. 168 and 193) that comparative measurements have been made at Karlsruhe on models of weirs to different scales. The models were of brass or wood, in some cases roughened with sand to determine the influence of roughness. For weirs having semicircular crests, the scales varied between 1 : 100 and 1 : 5 (radius of crest 1·25 cm. to 25 cm.). For other types of round-topped weir, the range was smaller, while sharp-crested weir plates had heights which "varied nine-fold". The results showed that with rounded crests, no deviations from the law of similarity could be traced; for the same values of head : radius and head to height of weir, the coefficients were "identical". Nor could any effect be traced to roughness, length * of weir investigated or temperature of water (within the limits tried). On the other hand, in the sharp-crested weirs "considerable deviations from the law of similitude were noted for low heads".

According to Professor A. H. Gibson (in 1934),† "the general conclusion to be drawn from the available data is that so long as upstream conditions are maintained in the model similar to those in the original, the results of model tests on round-topped or ogee-section weirs can be relied upon as giving results for which only a very small scale correction

* In a model test of a weir placed across an irregular-shaped river, it is not safe, however, to investigate a so-called representative portion of the weir only, because in general there will be a complex variation of velocity and of water level across the section of the river approaching the weir.

† "The Use of Models in Hydraulic Engineering", A. H. Gibson, *Trans. Inst. Water Engs.*, XXXIX, 1934, p. 177.

needs to be made, so long as the smallest head in the model is not less than 0·25 inch. The experimental results suggest that writing $Q = Kbh^{\frac{3}{2}}$, the relationship

$$K_m = K\left(1 + \frac{0{\cdot}00056}{S}\right),$$

where S is the scale ratio ($\frac{1}{25}$, $\frac{1}{50}$, etc.), will give the large-scale weir coefficient K with an accuracy of the order of 1 per cent. ".

It will be seen that, according to this formula, the coefficient K in the model is likely to be about 0·6, 1·4 or 2·8 per cent. too high for the corresponding head in the original installation when the scale is 1 : 10, 1 : 25 or 1 : 50 respectively.

It is only reasonable to point out, however, that cases have been recorded in which the model discharge has apparently been relatively too *low*. For example, Mr. E. Soucek has reported measurements on a model with a scale of 1 : 12 (to which a more detailed reference will be made on p. 73), where the flow averaged about 5 per cent. less than the equivalent discharge of the prototype. Similarly, Mr. J. W. Johnson (see p. 75) has described another case in which an alteration of scale from 1 : 40 to 1 : 100 *decreased* the coefficient by some 3·5 to 5·0 per cent. On the other hand, Mr. G. H. Hickox (*Proc. Am. Soc. C.E.*, vol. 68, No. 8, Pt. 1, Oct. 1942, p. 1334) has discussed experiments made on a 1 : 72 model of a spillway which revealed coefficients averaging 3·9 per cent. *larger* than those indicated by observations on the full-size structure.

The difficulties which beset any attempt at a definite statement concerning the scale-effect are of two main categories: firstly, the doubtful accuracy of the gaugings on the actual structures, and secondly the fact that many of the model tests have been conducted only on so-called representative sections or restricted lengths of the spillways. Consequently, all that it appears justifiable to assert is that with any scale likely to be adopted in practice, a model of a river weir, even of complex shape, will yield coefficients which may be applied to the full-size weir with an error of only a few per cent.: in general, about ±5 per cent., *provided that the head in the model is not less than* 0·25 *inch, and that a sufficient length of the upstream portion of the river is reproduced to ensure that the flow conditions in the region of the weir are similar to those found in nature.*

As an example of this kind of investigation, a description will now be given of a model investigation made at Manchester University of the Irlam Weir, over which the River Mersey discharges into the Manchester Ship Canal (see Fig. 9 and Plate II). The model (scale 1 : 50) was originally built to embrace the River Mersey from the weir to a distance of some 88 ft. upstream of the site of the Recorder House, which, in the actual river, is equipped with instruments providing a continuous autographic record of heads. To facilitate the moulding of the shore and bed of the river, a number of cross-sections were cut out of thin cardboard: these sheets acted as templates which were mounted in their appropriate positions inside a waterproofed wooden casing. Cement mortar (about 1 part of Portland cement to 4 of sand) was cast between the templates. On trial, it was evident that at high discharges the readings might be affected by the unsettled conditions at the entrance to the model, which

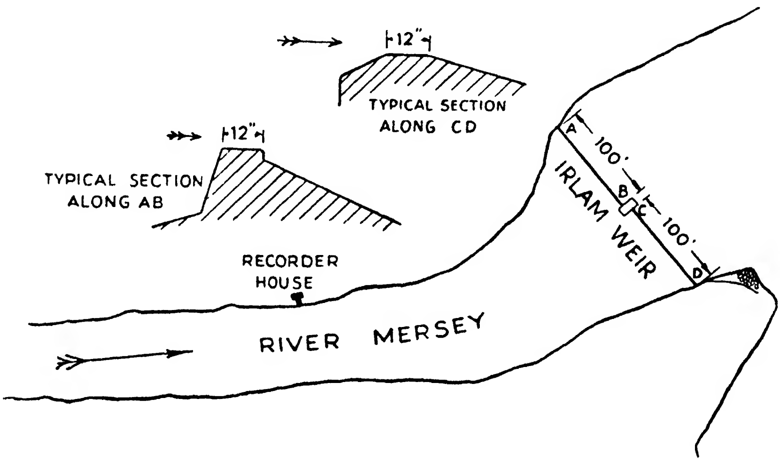


FIG. 9.

was supplied with water through a perforated pipe. Accordingly the tests were repeated after extending the approach length by about 5 ft. (equivalent to 250 ft. in the actual river). Results thus obtained after this modification showed no appreciable difference for discharges less than the equivalent of 5,000 cusecs, but with 9,000 cusecs the head as measured at the recorder was only 4·22 ft. before extending the inlet, as compared with 4·39 ft. afterwards.

Calling H the head, Q the rate of discharge, and A a sectional area of waterway, and letting the suffix 1 represent the actual weir and the suffix 2 the model, we have:

$$Q \propto \text{velocity} \times \text{area} \propto \sqrt{H} \,.\, A;$$

therefore
$$\frac{Q_1}{Q_2} = \frac{A_1}{A_2}\sqrt{\frac{H_1}{H_2}} = 50^{\frac{5}{2}},$$

since the scale was 1 : 50.

A flow of 1 cusec in the model would therefore represent 17,700 cusecs in nature.

The discharges in the model were measured at the lower rates by taking the time to fill a tank having a capacity of 22·1 cubic feet, and at the higher flows by means of a calibrated orifice in a diaphragm which had been fixed within the supply pipe.

Heads were measured by means of pointer gauges, fitted with verniers, reading directly to 0·01 inch, equivalent to 0·5 inch in nature. It was possible, however, to estimate within ±one-quarter of this magnitude, i.e. to within ±0·125 inch in nature.

As a matter of general interest, and also to obtain alternative standards of reference in the event of the mechanical failure of any one gauge in the actual river, the heads were measured not only at the site of the Recorder House, but also at a point in midstream and at the site of an existing post-gauge known as the " Manchester Ship Canal Gauge ". To represent the Recorder House, a basin about 1¼ inches square in plan was constructed of sheet copper; one face of this basin was flush with the coastline into which the basin was built, and this face contained a hole $\frac{1}{16}$ inch diameter, to allow water to pass from the model to the basin, where the level was read by a vernier pointer gauge.

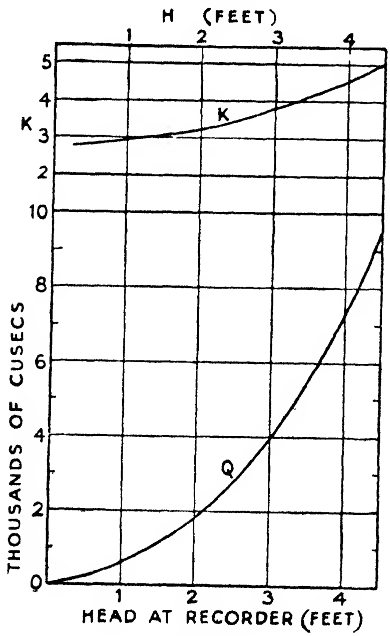


FIG. 10.

The core and crest of the weir itself were made of wood, soaked in paraffin wax to prevent distortion. Fig. 10 shows the results obtained at the recorder gauge, as translated into full-scale units. The diagram also shows coefficients of discharge (K in $Q = Kbh^{\frac{3}{2}}$, the breadth b being 200 ft. in this case) plotted against head. It was interesting to observe that the curve of discharge as originally plotted against head showed a discontinuity at a head of about 1·0 ft. (0·24 inch in the model). For heads lower than this critical value, the observed discharges were evidently too small, as judged from continuing the curve smoothly to the origin. Actually, however, estimates of discharge in this low-head region were available from current observations made in the river itself. It was also interesting to observe that the coefficients of discharge based on readings at the different gauges (Recorder House, midstream opposite the recorder and " Manchester Ship Canal Gauge ") were different from one another, especially at the higher heads. Accordingly it is not surprising that the values obtained in the model at the site of the recorder differed from those based on experiments with a " representative " length of the weir tested in a simple straight channel.

In another model tested at Manchester—that of Scotland Weir on the River Irk, Lancashire—a similar device to that already described for Irlam Weir was adopted for the Recorder House, except that the basin was fitted with a glass tube, projecting underwater some distance out into the stream. This tube represented to scale a 3-inch pipe with its end bent down to within about 16 inches of the river bed. Any localized velocity effect resulting from this proposed pipe was therefore taken into consideration: one object of this investigation was to find a suitable spot for measurement of head, not critically sensitive to the draw-down towards the weir, the idea then being to connect such a chosen spot with the recorder by means of the pipe as described.

If there is any possibility that the discharge may be affected by the *downstream* water rising above the weir-crest, this effect may easily be simulated in a model by the use of a regulating weir at the outlet. This regulating device may be so adjusted as to produce the correct tailwater evel, as shown by measurements in the actual river, in relation to the head. Actually, it is surprising how little is the flow influenced by appreciable drowning of a weir: in general, the discharge will be reduced by only 2 or 3 per cent. if the submergence does not exceed 0·2 of the head, and indeed the influence of submergence, which depends upon the shape of weir involved, may easily be less than this. For example, Mr. Edward Soucek has described * experiments on a 1 : 12 scale model of the spillway shown in Fig. 11, which demonstrated that for submergences lower than 45 per cent., the submergence had no measurable effect upon the general law of discharge. Mr. Soucek's investigation is also of great interest as providing a comparison between the model and the prototype dam, for which discharges were based upon current meter observations made in the tailwater 470 ft. downstream of the dam. The model was constructed of a galvanized iron sheet fastened to a welded steel frame, and was tested in a glass-sided flume 26 ft. long, 2·5 ft. deep and 2·56 ft. wide. Since the length of the actual structure was 273·5 ft., and the scale was 1 : 12,

* " Meter Measurements of Dam Discharge ", E. Soucek, *Proc. Am. Soc. C.E.*, vol. 68, No. 8, Part 1, Oct. 1942, p. 1352 (Contribution to a Symposium on Conformity between Model and Prototype).

the model in effect comprised only $\left(\frac{2{\cdot}56 \times 12}{273{\cdot}5}\right)$, or 1/11·2 of the overall length of the spillway. The rate of flow over the model spillway was measured by means of a calibrated sharp-crested, fully suppressed and aerated rectangular weir.

Mr. Soucek relates the rate of discharge to the "energy head", H_e, defined as the measured head above the crest of the dam plus $\frac{v^2}{2g}$, where v is the mean velocity in the approach channel. Calling Q the rate of

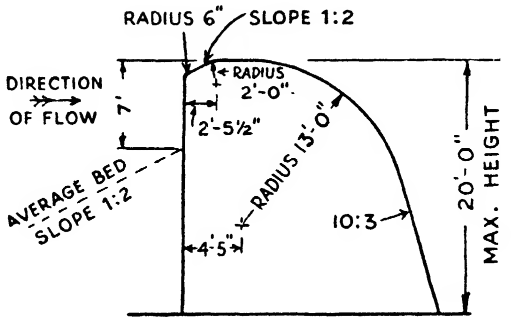


FIG. 11.

discharge and b the length of the weir, $Q = CbH_e^{\frac{3}{2}}$, and if there were no scale-effect,

$$\frac{Q_1}{Q_2} \text{ would } = \frac{b_1}{b_2}\left(\frac{H_{e_1}}{H_{e_2}}\right)^{\frac{3}{2}} = \frac{b_1}{b_2}(12)^{\frac{3}{2}},$$

the suffix 1 referring to the full-size and the suffix 2 to the model dam having a scale of 1 : 12. Since $b_1 = 273{\cdot}5$ ft. and $b_2 = 2{\cdot}56$ ft., the expression becomes

$$\frac{Q_1}{Q_2} = 4{,}430.$$

Values of Q_1 obtained from $Q_1 = 4{,}430$ times the measured model discharge were compared with the discharges estimated as applicable to the prototype at corresponding "energy heads", the estimate for the prototype being based, as previously mentioned, upon current meter observations. Over the whole range of 23 measurements, including 11 made while the dam was submerged, and covering prototype heads between 1·7 and 6·3 feet, the model discharge averaged about 5 per cent. less than the equivalent flow in the actual installation. The discrepancies varied between extreme limits of 15·0 per cent. less to 5·9 per cent. greater than the prototype discharge, but in only two cases did the model discharge appear to exceed the equivalent actual discharge. Taking into account the fact that the prototype discharges could only have been determined approximately by the method of current meter measurements adopted and the fact that the discharges varied between 1,900 and 16,000 cusecs, the results found from the model may be fairly described as satisfactory.

In discussing Mr. Soucek's work, Mr. J. W. Johnson has called attention * to some very interesting work † on models of the Upper Narrows Dam on the Yuba River. To take normal flows, a central section of this dam, 4 ft. lower than the remainder, was provided with a length of 189·4 ft. and an elevation of 527 ft., the elevation of low flow in the original stream being 291·2 ft. In order to study the weir-coefficient and the profile of the nappe in a typical stretch of this dam, three models were used, having scales of 1 : 25, 1 : 40 and 1 : 100. All the models were 6 inches wide, about 24 inches high (thus representing only a varying upper portion of the structure), and were tested in a flume 6 inches wide equipped with forebay and baffles. The flow was measured by means of an orifice meter in the pipe delivering water from a constant-head tank. " Because of the relatively strong surface tension forces at the heads below 0·03 ft., the nappe would cling to the face of the model—a condition which probably would not exist on the prototype at a corresponding head.... All runs were made with the nappe completely aerated by means of a 1-in. pipe through which air could flow to the underside of the nappe." Fig. 12 shows the cross-section of the dam.

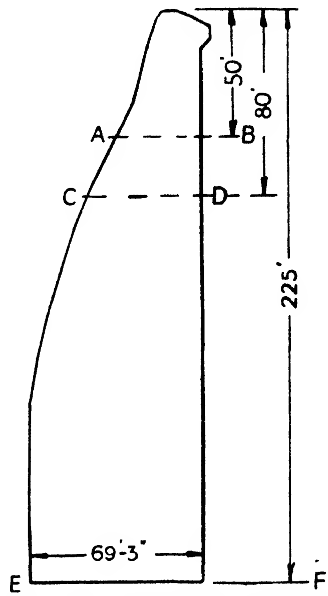


FIG. 12. Cross-Section of Upper Narrows Dam.

AB: *Limit of* 1 : 25 *Model.*
CD: *Limit of* 1 : 40 *Model.*
EF: *Limit of* 1 : 100 *Model.*

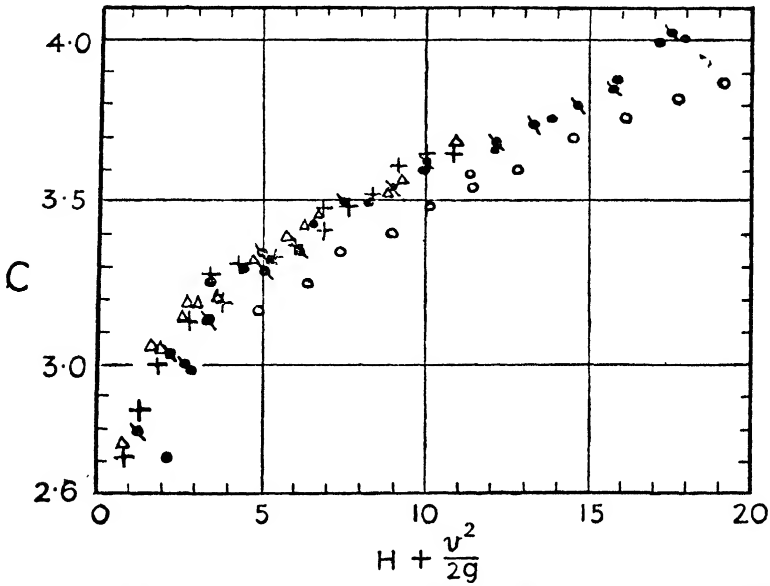
The models were constructed of smooth, varnished wood, but tests were also made on the 1 : 25 and 1 : 40 models with a roughened surface, using sand (passing a 100-mesh and being retained upon a 200-mesh sieve)

* *Proc. Am. Soc. C.E.*, vol. 69, No. 1, Jan. 1943, p. 171.

† " Models of Upper Narrows Dam Spillway ", W. M. Stubblefield and J. P. Henck, unpublished thesis presented to the University of California, December 1939, in partial fulfilment of the requirements for the degree of Bachelor of Science.

sprinkled over the newly varnished surface. Fig. 13 shows the measured coefficients C corresponding with the formula

$$Q = Cb\left(H + \frac{v^2}{2g}\right)^{\frac{3}{2}} = CbH_e^{\frac{3}{2}}.$$



Δ 1:25 Model   + 1:25 Model (Sanded)   ● 1:40 Model

● 1:40 „ (Sanded)   O 1 :100 „

FIG. 13.

The significant points of interest in these results appear to be:

(*a*) that the values for the 1 : 25 and 1 : 40 models, smooth *or* roughened, may be represented fairly reasonably by an average curve, and this despite the fact that the *heights* of the models were not to scale;

(*b*) the discharge coefficient of the 1 : 100 model was consistently less than that of the others—by some 3·5 to 5·0 per cent., proportionately;

(*c*) according to Mr. J. W. Johnson, the Upper Narrows model (1 : 25 and 1 : 40) results may be represented by the equation

$$Q_0 = 2{\cdot}79\left(H + \frac{v^2}{2g}\right)^{1{\cdot}62},$$

where Q_0 stands for the discharge in cusecs per foot of crest of the prototype dam under an " energy head " of $\left(H + \frac{v^2}{2g}\right)$ feet.

The index of 1·62 is of interest when compared with the values presented by Mr. W. F. Martin in a paper entitled " Discharge Measurements and Formulas for some Large Overflow Dams ", *Engineering News*, New York, vol. 64, No. 12, Sept. 29, 1910, p. 321. Mr. Martin has con-

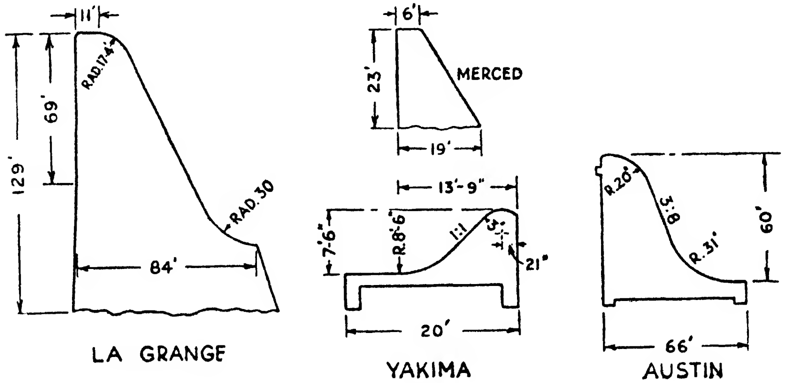


FIG. 14.

sidered the current-meter ratings for four structures (see Fig. 14). Of these, the La Grange Dam was curved in plan while the others were straight, and the formulae deduced by Mr. Martin are:

Merced Dam - - - -	$Q = 2{\cdot}491\ b\ H^{1{\cdot}77}$
La Grange Dam - - - -	$Q = 3{\cdot}088\ b\ H^{1{\cdot}50}$
Yakima Dam - - - -	$Q = 3{\cdot}110\ b\ H^{1{\cdot}65}$
Austin Dam - - - -	$Q = 3{\cdot}196\ b\ H^{1{\cdot}59}$

In these formulae, H does not include any allowance for velocity of approach which was apparently appreciable; it is probable that in the Upper Narrows example this effect was relatively much less important, but the index of 1·62 lies well within the range of Mr. Martin's values.

The method of model-experiments has also been applied to the determination of the length of spillway required for the discharge of flood waters from a reservoir. An interesting reference to this kind of work is contained in a statement by Mr. W. F. H. Creber: * " He (Mr. Creber) had proposed a certain arrangement for extending dams in accordance with the findings of the Floods Committee, and he had anticipated a rather stiff fight in Parliament with two of the Past-Presidents of the Institution of Water Engineers. Professor Gibson had said that this was a case in which mathematical demonstration was almost impossible, but they had made certain assumptions and had worked out figures which appeared to

* *Trans. Inst. of Water Engs.*, XXXIX, 1934, p. 196.

show that the weirs it was proposed to put in would do the work required. Then Professor Gibson had come to his aid and had made the model weirs, which had shown that the calculations were on the low side, and that the weirs would do what was required. The result was that instead of there being a fight in both Houses of Parliament with the two 'fierce' Past-Presidents of the Institution, it was found unnecessary to use the gloves at all, and they had all discussed the matter round a table and had agreed that the work was feasible. At a cost of less than £200, therefore, a saving of several thousands of pounds was effected."

Other examples of weir model experiments may be found in Freeman's *Hydraulic Laboratory Practice*, 1929, e.g. on p. 284 and p. 452.

(c) Sluice Gate Models.

If such a model is built to a scale of 1 : S, then, by reasoning similar to that already applied in the case of weirs, the discharge of the actual sluice-gate may be predicted by multiplying that measured in the model by the factor $S^{\frac{5}{2}}$. Concerning the magnitude of any possible scale-effect, Messrs.

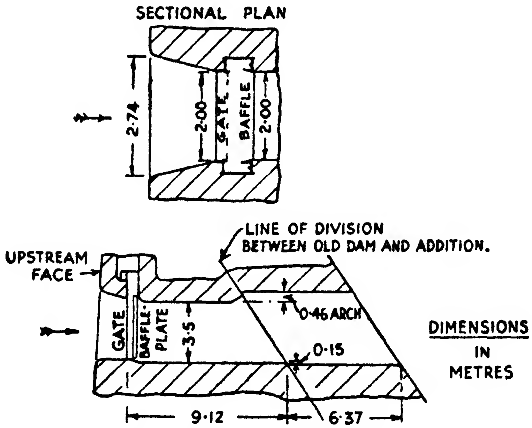


FIG. 15. Section on Vertical Centre Line, Assuan Dam.

Hurst and Watt * have made an extremely interesting comparison between models of different sizes and one of the Assuan Dam sluices (see Fig. 15). Direct measurements of the discharge of one of these sluices had previously been carried out in a measuring tank.† The scales of the models

* "The Similarity of Motion of Water through Sluices and through Scale Models: Experiments with Models of Sluices of the Assuan Dam", Hurst and Watt, *Min. Proc. I.C.E.*, vol. CCXVIII, 1925, p. 72.

† "The Measurement of the Discharge of the Nile through the Sluices of the Assuan Dam", Sir Murdoch MacDonald and H. G. Hurst, *Min. Proc. I.C.E.*, CCXII, 1921, p. 228.

were 1 : 33⅓, 1 : 50 and 1 : 66⅔, and the greatest discrepancy (−4·6 per cent.) between the model and the full-scale discharge occurred with the smallest model: the mean difference over the whole range of tests with the various models was only +0·4 per cent., and the maximum discrepancy with the biggest model was −3·7 per cent. In this summary, a + sign indicates that the model gave too small a discharge.

Experiments were also tried with the water in the reservoir not reaching the lintel of the fully-open sluice so that the sluice then discharged as an abnormally wide flat-crested weir. In this case, the models always showed relatively smaller discharges than the full-size structure: the results in general are not so consistent or so convincing as those obtained under gate (as distinct from weir) conditions, although the *mean* discrepancy for heads ranging between 1·0 and 3·5 metres is about 5 per cent. At a head of 0·5 metre (represented by 0·59, 0·394 and 0·295 inch respectively in the three models), all the models gave discharges at least 12 per cent. too small. As the depth of water at points remote from the inlet would be considerably less than the head, it is probable that surface tension was responsible for some of this discrepancy.

A further point of significance is that Messrs. Hurst and Watt report that in the case of the fully-open Assuan sluice operating with reservoir levels between 6·75 and 10·25 metres above sill level, *two* possible discharges, differing by some 7 per cent., existed for one and the same head. The cause of this was not established, but it was found that the mean model results lay between the two limits observed on the actual sluice. Actually, a similar instability has been observed by some observers on *model weirs*. Thus, at the Hydraulic Institute of Munich,* in experiments on round-topped weirs, divergences were observed, sometimes amounting to more than 6 per cent., in the discharges for a given head. When the upstream face of the weir was roughened with sand (average grain-size 0·12 in.), the divergences were much smaller than when a smooth face was used. With the smooth type, it was found that in general a reduction in the volume of flow led to a larger coefficient than when the reverse procedure was followed: once the flow had been set to a particular discharge, the head could apparently assume any value between certain limits (differing by about 6 per cent. for the most part), and thereafter the head remained sensibly constant at its selected value for a time of, say, thirty minutes. Moreover, the " instability " appeared to be more pronounced with low discharges. Evidently, there is scope for further investigation of this phenomenon, which is certainly not yet fully understood.

* " Hydraulic Laboratory Practice", ed. Freeman, *Am. Soc. Mech. Engs.*, 1929, pp. 446–451.

Reverting to a consideration of the Assuan sluice models, it is of interest to note that Messrs. Hurst and Watt examined the effect of roughening the surface of the 1 : 33⅓ scale-model: "its surface was varnished, and, while the varnish was sticky, sand was carefully and regularly blown over it." Two grades of sand were tried in separate experiments; their estimated mean diameters being about 0·25 and 0·50 millimetre. With a gate opening of 2 metres, these roughnesses caused a decrease of discharge of approximately 2·4 and 3·1 per cent., the effect being fairly regular for all heads. With a fully-open sluice, however, the effect increased with head, reaching values of 6·5 and 11·0 per cent. respectively when the reservoir level was some 11 metres above the sill of the sluice. On the general average, the finer sand produced a reduction of 5 per cent. in the discharge compared with the original smooth model, and Messrs. Hurst and Watt conclude that "for works of smooth-dressed stone, models of the scales dealt with in this paper should have surfaces of smooth metal, or painted or varnished wood. In accordance with the experiments carried out by Mr. Baker,* it is to be expected that finely-polished surfaces would not give results appreciably different from those obtained with the smooth surfaces of the present models. . . . Models can be used to determine the discharge of large sluices with an average accuracy as good as that obtainable by current-meter measurements. The scale of the model depends upon the product of the velocity on the actual sluice and its linear dimensions. The limit of smallness in the foregoing experiments occurs with weir conditions when the head above the sill is about 3 centimetres, the depth about 2 centimetres at the gate, and the velocity about 0·4 metre per second. Until further experiments on models of other structures define the limiting conditions more closely, it will be well to keep the product of velocity, in centimetres per second, and smallest dimensions of the orifice, in centimetres, above, say, 100, and in general not to use orifices of less than 3 centimetres in their smallest dimension."

On the whole, it is true to state that this kind of investigation has now been accorded general recognition as a very important and useful instrument for the guidance of engineers concerned with the design of barrages and similar regulating works. For example, the Egyptian Government recently expended the sum of approximately £E42,000 on the construction of a Hydraulic Research Laboratory.†

Dr. Hasan Zaky ‡ has described model experiments carried out (at the

* G. S. Baker, *Nat. Phys. Lab. Coll. Res.*, XIII (1916).

† "The Mohammad Aly Barrages, Egypt", A. G. Vaughan-Lee, *Journ. Inst. C.E.*, Feb. 1941, p. 237.

‡ "Model Experiments on the Gebel Aulia Dam", H. Zaky, *Journ. Inst. C.E.*, June 1941, p. 351; see also "Wave-Formation in Regulating Sluices", H. Zaky, *Journ. Inst. C.E.*, Dec. 1938, p. 276.

Delta Barrage Laboratory) with the object of studying designs for the Gebel Aulia dam. "These [experiments] proved", he writes, "to be highly satisfactory and of great practical value. In many aspects, the final design that was approved and executed was the outcome of these tests." Exploratory tests were first made on a $\frac{1}{50}$-scale model comprising two sluices: these led to alterations in the proposed number of sluices, the level of the sill and the length of apron. Experiments on two $\frac{1}{50}$ models then caused two important modifications connected with training-walls and fish-ladder. Finally, experiments were made to find coefficients of discharge under drowned conditions. For gate openings of less than 0·50 metre and for low upstream levels, a full-size gate was adopted; three models having scales of 1 : 25, 1 : 10 and 1 : 3 were constructed for tests on gate-openings greater than 1·0 metre and for various water-levels.

Writing $Q = CA\sqrt{H}$ cubic metres per second, where A represents the sluice area (sq. metres) and H the difference (in metres) between upstream and downstream levels, Dr. Zaky reports the following results:

TABLE IV

Model scale	*Opening : metres*			
	1·00	2·00	3·00	3·50
	Coeff. of discharge, $C =$			
1 : 25	3·07	3·14	3·32	3·62
1 : 10	3·02	3·15	3·55	3·81
1 : 3	3·05	3·18	3·50	3·69
Mean C:	3·04	3·16	3·47	3·70

It will be observed that in this table, the maximum difference between the 1 : 25 and 1 : 3 scale-model results is less than 6 per cent.

In some cases, it may be convenient, or desirable, to use an exaggerated vertical scale ratio in a model sluice gate—for example, if such a structure is only one part of a river problem which is under investigation. The question then arises as to how far the coefficient of discharge may be affected by the distortion of scale. Some light may be thrown on this by a study of the results obtained in experiments on vertical rectangular sharp-edged orifices. Comparing values quoted by Gibson * and Seely,† it is found that a sharp-edged orifice 12 inches wide and 2 inches deep

* Gibson, *Hydraulics and its Applications*, Constable, 3rd edn., pp. 140, 142.

† Seely, *The Flow of Water through Submerged Orifices*, University of Illinois Eng. Expt. Station Bulletin No. 105, May 1918, p. 35.

has, within 1 per cent., the same coefficient as one 6 inches wide and 2 inches deep under the same head of 12 inches. In this case, the vertical exaggeration of scale of the two orifices is 2 : 1. Again, within 1 per cent., the coefficient of a $\frac{1}{2}$-inch square orifice under a head of 1 foot is the same as that of an orifice 12 inches wide and 1·5 inches deep under a head of 3 feet. Here the exaggeration of vertical scale relative to horizontal for one orifice compared with the other is 8 : 1. Data of this kind together with the Author's general experience, lead to the conclusion that if the vertical scale is $1 : y$ and the horizontal $1 : x$ in the case of a model of a sluice or similar structure, then the discharge is related to that of the prototype in the ratio of $1 : xy^{\frac{3}{2}}$ with sufficient accuracy for many practical purposes, even when x is large compared with y, and that in the comparatively rare cases where the similarity is not sufficiently true the scale-effect may be corrected by a relatively small adjustment to the area of the model.

(d) Siphon Spillways.

Probably the most useful information yet obtained concerning the scale-effect in models of siphon spillways is contained in a paper by Gibson, Aspey and Tattersall,* who tried two models having throat depths of 1 inch and 4 inches respectively and found coefficients of discharge as given below:

TABLE V

Head (ft.)	*Coefficient of discharge*	
	Small model	*Large model*
0·1	0·664	—
0·2	0·692	0·772
0·5	0·725	0·792
1·0	0·745	0·800
1·5	0·751	0·804
2·0	—	0·805

At similar heads, therefore, there was a marked scale-effect: e.g. the larger model at a head of 2·0 ft. giving a coefficient 11 per cent. higher than that of the smaller model under a head of 0·5 ft. When, however, the coefficient was plotted against (vd), it was found that the results of the two models lay sensibly on one and the same curve, v being the

* "Experiments on Siphon Spillways", Gibson, Aspey and Tattersall, *Proc. I.C.E.*, vol. 231, 1930–1, Pt. 1, p. 203. See also "Model Experiments on Bellmouth and Siphon-Bellmouth Overflow Spillways", G. M. Binnie, *Journ. Inst. C.E.*, Nov. 1938, p. 65.

velocity (ft. per sec.) and d the depth (feet) at the throat. The parameter (vd) is in fact a Reynolds number if constant water temperature (and therefore viscosity) is assumed. The co-ordinates of this curve proved to be as follows:

TABLE VI

vd (*ft.*^{2}/*sec.*)	*Coefficient of discharge*
0·2	0·686
0·225	0·695
0·5	0·742
1·0	0·778
1·5	0·794
2·0	0·801
3·0	0·806
3·5	0·806

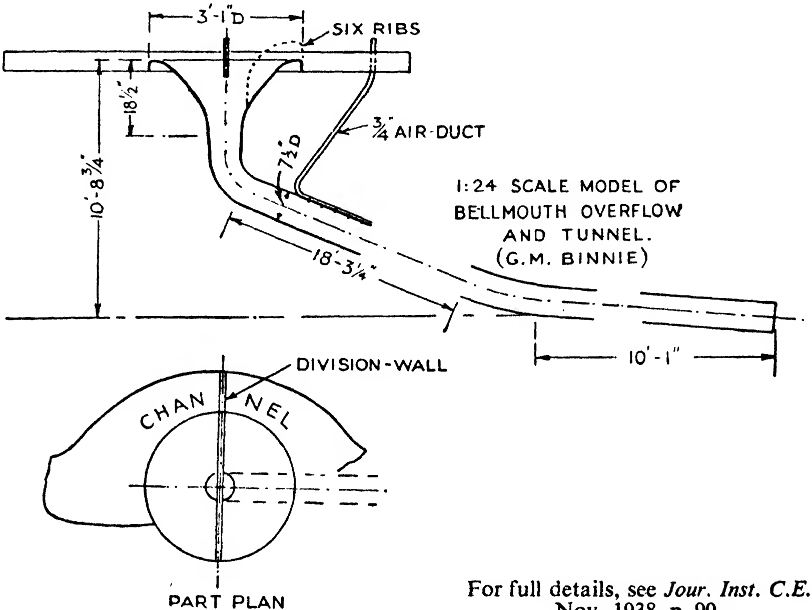
From these results, it is argued that if the model is made of such a size that vd exceeds 1·8 ft.^{2} per sec., "its lines of flow and coefficient of discharge may be expected to be sensibly the same as its prototype for all heads, and therefore for the all-important ratio of heads at which $h:H=d:D$."

To take an example, suppose a proposed siphon is to have a throat depth of 3 feet and is to work at a head of 15 feet, its coefficient of discharge being assumed to be of the order of 0·8. The velocity ($C\sqrt{2gh}$) would then be about 25 ft. per sec. If a model is made to a scale of 1 : x and is to have the same coefficient, v will $=25/\sqrt{x}$ and d will $=3/x$. Putting $\dfrac{25\times 3}{x^{\frac{3}{2}}}=1{\cdot}8$, x becomes 12, and it is desirable that the model shall not be made to a scale smaller than 1 : 12; with this scale, its coefficient may be expected to be only about 1 per cent. less than that of the full-size siphon, whereas if a scale of 1 : 48 be adopted, the product vd in the model will become approximately 0·225 and, according to the last table of data, its coefficient will need to be multiplied by $\dfrac{0{\cdot}806}{0{\cdot}695}$ or 1·16.

In this example, however, it has been *assumed* that the coefficient will be of the order of 0·8. For *general* purposes, it is preferable to deal in terms of $d\sqrt{h}$ rather than vd, since v is originally unknown. Messrs. Gibson, Aspey and Tattersall find that for siphons of the general design considered in their investigation, close similarity between model and actual may be expected if $d\sqrt{h}$ is not less than about 0·28 ft.$^{\frac{3}{2}}$.

(e) Bellmouth Overflow Spillways.

A number of tests have been made by different observers on the behaviour of bellmouthed overflows as a device for discharging surplus water from reservoirs.* A particular layout investigated by Mr. G. M. Binnie † is shown in Fig. 16. If the scale of the model is $1 : S$, the scale



For full details, see *Jour. Inst. C.E.*, Nov. 1938, p. 90.

Fig. 16.

of rate-of-discharge will be $1 : S^{\frac{5}{2}}$, since the flow is proportional to (area $\times$ velocity), and velocity itself is proportional to $\sqrt{\text{head}}$. Mr. Binnie experimented with four models, having scales of 1 : 19, 1 : 24, 1 : 29·4, 1 : 43·5. Of these, the first three gave similar results, whereas the last exhibited an obvious scale-effect, both in rate of discharge and in the vacua measured at various points in the tunnel. Calling H the height of the lip of the bellmouth above the centre of the outlet-end of the tunnel, Mr. Binnie concluded that, to avoid scale-effect, $d\sqrt{H}$ must be larger than 1·5 ft.$^{\frac{3}{2}}$, d being the pipe diameter. He considers, however, that this

* See, for example, " Bellmouthed Weirs and Tunnel Outlets for the Disposal of Flood-Water ", W. J. E. Binnie, *Trans. Inst. Water Engs.*, vol. 42, p. 103 (1937), for a description of model-experiments on which the design of several overflows was based. Also " Laboratory Experiments on Bellmouth Spillways ", A. M. Binnie and R. K. Wright, *Journ. Inst. C.E.*, Jan. 1941, p. 197.

† " Model-experiments on Bellmouth and Siphon-Bellmouth Overflow Spillways ", G. M. Binnie, *Journ. Inst. C.E.*, Nov. 1938, p. 65.

value (1·5) can be applied only to spillways having the same general proportions as that which he investigated, in which the length of tunnel was about three times the height H, since it is to be anticipated that as the length of the tunnel increases, the proportionate effect of viscosity will increase also. Consequently in cases where the tunnel or pipe is longer than $3H$, it is assumed that the critical value of $d\sqrt{H}$ will be higher than 1·5.

An important comparison of model and full-scale results in this type of problem has been provided by Mr. W. J. E. Binnie: * "A bellmouth overflow discharging into a tunnel was constructed to dispose of flood water at the Burnhope reservoir, the overflow being designed to deal with a maximum quantity of 2,670 cusecs, whilst the effective length of the crest was 154 feet. When the reservoir overflowed the water passed over a weir 200 feet in length and entered the channel leading to the bellmouth. The water flowing from the tunnel entered a basin, finally discharging into the river over another weir 110 feet in length. Self-recording instruments were established so that h was measured for both weirs and the bellmouth.

"Model-experiments were carried out to determine the coefficient C in the formula $Q = Clh^{\frac{3}{2}}$, where l denoted the length of the crest and h the depth of overflow. A flood occurred on 24th and 25th October, 1936, when the maximum value of h recorded at the bellmouth reached 1·7 ft. The crest of the 200-ft. weir was similar to one which had been tested in America,† and the crest of the 110-ft. weir was similar to one which had been tested in Egypt.‡ The coefficients determined by those experiments were therefore adopted, and the mean value of the results obtained for the readings of h on both weirs was assumed to be correct. The value of C for the bellmouth weir was calculated for increments of 3 inches between depths of 0·25 and 1·7 ft. on the crest, and was found to vary from 4 to 3·95, with the exception of one reading . . . further . . . owing to the turbulence which occurred in the basin into which the tunnel discharged, it was possible that coefficients derived from observations of h on the 200-ft. weir might be more accurate, in which case the value of C would vary from 3·99 at a depth of 3 inches to 4·16 at 1·7 ft.

"The tests carried out with the model to a scale-ratio of $\frac{1}{24}$ indicated a value of 3·84, confirming the impression which had been formed from experiments carried out elsewhere that the prototype would give rather better results than those indicated by model-tests."

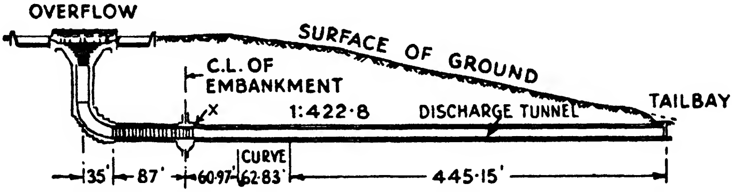
(The ratio of 3·99 to 3·84 is 1·04, and 4·16/3·84 is 1·08.)

* W. J. E. Binnie, in correspondence concerning a paper by G. M. Binnie: see *Journ. Inst. C.E.*, Oct. 1939, p. 277.

† *Water Supply and Irrigation*, U.S. Geol. Survey, Paper No. 200, p. 109.

‡ *Report on Submerged Weirs and Standing Wave Weirs*, Diagram VII, Type No. 9. Government Press, Cairo, 1923.

A particularly interesting design of overflow bellmouth is that adopted for the two overflows, situated one near each end of the earthwork embankment of the Ladybower Reservoir, River Derwent, Derbyshire.* A longitudinal section of the bellmouth and its tunnel is shown in Fig. 17A, while details of the bellmouth design itself appear in Fig. 17B, from which



× *Marks Transition from Circular to Horseshoe Shape Tunnel.*

FIG. 17A.

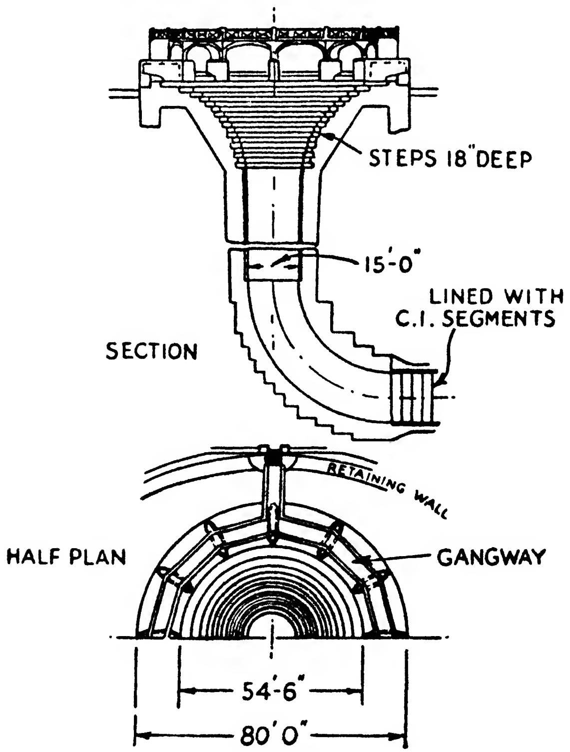


FIG. 17B.

* "The Ladybower Reservoir", illustrated article in *The Engineer*, vol. 168, pp. 440–442, Nov. 3, 1939.

it will be seen that the diameter at the top is 80 ft., diminishing to 15 ft. at the junction with the vertical shaft. The interior of the bellmouth possesses an unusual feature in that it is constructed in masonry steps and not as a smooth surface. This idea effects an economy in construction, but was not finally adopted until model tests by Professor A. H. Gibson at Manchester University showed that the steps would improve rather than detract from the efficiency of discharge. It will also be observed that the sill of the overflow is covered by a bridge, the twelve piers of which are utilized as radial cutwaters for the purpose of preventing any tendency to swirl as the water passes down the shaft. The bridge also provides access for dealing with obstructions such as debris or ice floes. Three photographs of Professor Gibson's model are reproduced in Plates III, IV and V.

(f) Sluice Gates of the " Tilting Flap " Design.

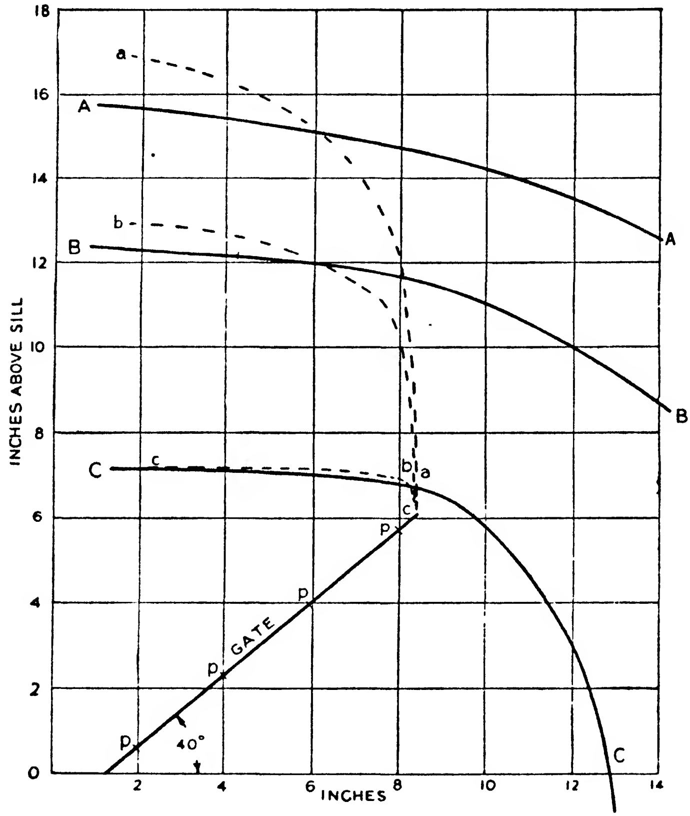
One type of sluice gate consists of a flap hinged at the upstream end and mechanically balanced to suit the variable head under which it operates. In making such a design it is necessary to be able to estimate not only the rate of discharge, but also the effective force on the flap. Laboratory experiments may help to give both these sorts of information.* The flap may be drilled and tapped for connections to pressure tubes for the purpose of examining the *effective* heads on the gate itself: in general it will be found that, owing to the velocity and acceleration of the stream, these heads are quite different from the " static head " measured vertically from the water surface to the gate. Fig. 18 shows typical results obtained with the layout of Fig. 19.

(f_1.) The reference made in the last paragraph to the measurement of effective pressures may perhaps be usefully supplemented by considering a piece of work reported by Messrs. J. C. Stevens and R. B. Cochrane.† In this case, a model was constructed to a scale of 1 : 5, in a flume 2 ft. wide, 10 ft. deep and 10 ft. long, to represent a 10-ft. length of the crest of a dam equipped with a vertical gate so arranged that water could flow between the bottom of the gate and the top of the dam. The primary object of the investigation was to determine the effect of possible slight irregularities in the construction of the surface of the prototype dam, but the interesting feature for our present purposes is that several pressure observations were made on both the model and the completed dam, the piezometer holes in the prototype being made in a steel plate 10 inches

* " Experiments on a Model Sluice Gate of the Tilting Flap Design ", Allen, Inst. C.E., *Sessional Notices*, May 1934.

†" Pressure Heads on a Western Dam ", J. C. Stevens and R. B. Cochrane, *Proc. Am. Soc. C.E.*, vol. 68, No. 8, Pt. 1, Oct. 1942, p. 1343, a contribution to a symposium on *Conformity between Model and Prototype.*

wide set flush with the concrete and bent to the shape of the crest. From these piezometer holes, pipes were led to a Bourdon gauge situated in an



p: PIEZOMETER HOLES IN GATE

A A: MEAN OF WATER SURFACE MEASUREMENTS ACROSS STREAM, HEAD OVER SILL 17·57"
B B: DITTO " " " 13·25"
C C: DITTO " " " 7·25"

a a: MEAN OF PIEZOMETER MEASUREMENTS ACROSS GATE, " " " 17·57"
b b: DITTO " " " 13·25"
c c: DITTO " " " 7·25"

FIG. 18.

inspection gallery. Two gate locations were tried: the normal one, 8 ft. downstream of the axis of the dam, and an emergency one (to be available in case of repairs) 9 ft. upstream of the axis.

With the gate in its downstream position, fair agreement was found to exist between the model pressure observations and those made on the

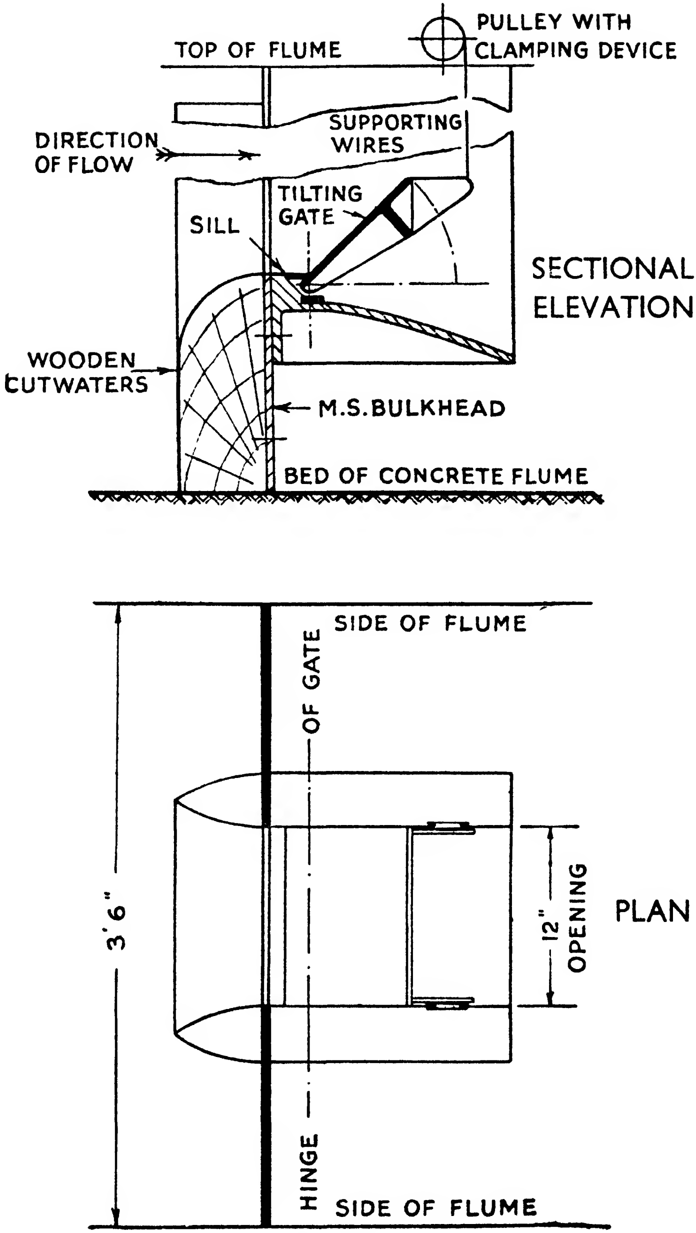

TOP OF FLUME
PULLEY WITH CLAMPING DEVICE
SUPPORTING WIRES
DIRECTION OF FLOW
TILTING GATE
SILL
SECTIONAL ELEVATION
WOODEN CUTWATERS
M.S. BULKHEAD
BED OF CONCRETE FLUME
SIDE OF FLUME
HINGE — OF GATE
3′ 6″
12″ OPENING
PLAN
SIDE OF FLUME

FIG. 19.

actual dam. This was also true of measurements conducted in the middle one of three bays of a 1 : 36 scale-model. Typical results are indicated in Fig. 20.

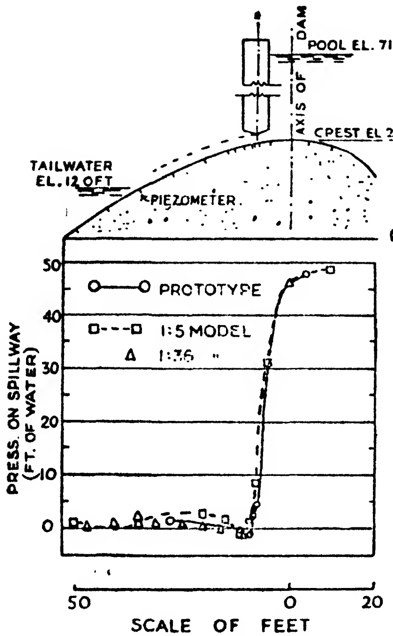


Fig. 20. Gate in Downstream Position (Opening 2 ft.).

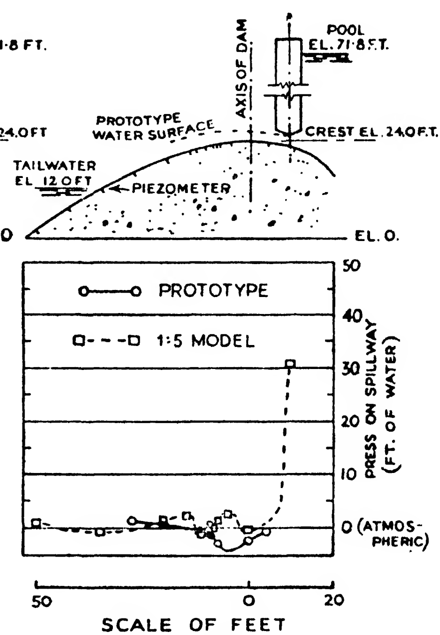


Fig. 21. Gate in Upstream Position (Opening 2 ft.).

With the gate in the upstream position, measurements were not made on the 1 : 36 scale, but those carried out on the 1 : 5 model revealed appreciable discrepancies compared with the prototype. For example, inspection of Fig. 21 will show that at one point the model indicated approximately 1·4 and the prototype −3·0 ft. of water. A more detailed comparison is afforded by the data given in Table VII: This table shows, for five different gate-openings ranging from 0·25 ft. to 3·00 ft., the discrepancy between the pressure-heads as registered at different points along the surface of the actual dam and those which would have been expected from the 1 : 5 scale model observations at the corresponding points.

TABLE VII

Difference between Pressure Heads (ft. of water) for the Gate in the Upstream Position

(A negative sign means that prototype heads were lower than those in the 1 : 5 model)

Gate opening (feet)	*Distance from axis of dam (feet)*					
	0·0	7·5	8·5	9·0	11·0	20·0
0·25	−0·3	−0·7	0·1	−0·4	0·2	−0·4
0·50	−1·1	−0·9	0·1	−0·4	0·0	−0·5
1·00	−1·5	−2·7	−0·3	−0·2	0·5	−0·4
2·00	−2·3	−4·4	−1·5	−1·3	0·0	−0·6
3·00	−3·3	−5·3	−2·4	−2·4	−0·9	−0·5

Messrs. Stevens and Cochrane state that they " can offer no valid explanation for the anomalous results between model and prototype with the gate in the upstream position. Under such high velocities—45 ft. per sec. for the prototype and 20 ft. per sec. for the model—the difficulty of obtaining accurate pressure heads through piezometers was fully recognized and every precaution was taken to eliminate such sources of error. The departures from conformity are not of this nature. The model consistently refused to show negative pressures where such pressures should have obtained just below the crest ".

In discussing this investigation, the late Mr. C. I. Grimm (*Proc. Am. Soc. C.E.*, vol. 69, No. 1, Jan. 1943, p. 170) has suggested that " the difference between the model and the prototype may be due to non-similitude of atmospheric pressure. . . . A model study of spillway crests under controlled atmospheric pressure would be timely and of decided value as a research project to supplement present knowledge ".

One or two other points come to mind. It would have been extremely interesting to have had results from the 1 : 36 model with the gate in the upstream position, since this model, although smaller, did include proportionately a bigger length of the structure; the results might also have indicated a general trend of the scale-effect. Again, it might have been illuminating to have tried the influence of slight changes in the upstream location of the gate; it is possible that the pressures would be very sensitive to this in view of the rising curve of the surface of the dam, and it is likely that the precise shape of this curve would materially affect the results. If indeed the observed anomalies were in the nature of a true " scale-effect ", it would be interesting to know whether this was brought about by a lack of similarity due to viscosity and surface tension in the degree of contraction of the stream issuing from the aperture between the gate and the dam.

Again considering the phenomenon of sub-atmospheric pressures, it may be worth while to quote the experience of Mr. R. J. Pafford, Jr., as described in the *Proc. Am. Soc. C.E.*, vol. 69, No. 4, Pt. 1, April 1943, p. 539. His experience related to a spillway stilling basin equipped with baffle piers for energy dissipation; some concern was felt about the possibility of cavitation troubles. Two hydraulic laboratories were concerned in the investigation. One attacked the problem by measuring and analyzing the pressures on baffle surfaces in a model under atmospheric pressure; the other used the method of visual observation of a model erected in a vacuum tank enabling pressures to be reduced in the correct scale ratio. Both laboratories first made tests on models of an *existing* stilling basin where pitting had occurred along the sides of the baffles during a heavy flood.

Mr. Pafford affirms that " the results of independently conducted tests at each laboratory were in reasonable agreement with conditions observed at the existing prototype and with conditions indicated at the other laboratory. . . . The original descriptions and photographs of conditions at the existing structure had indicated pitting only along certain areas on the sides of the baffle piers. During testing of the model of this structure in the vacuum tank, it was seen that conditions along the stilling basin floor in certain areas between the baffle piers were apparently nearly as severe as along the sides of the baffle piers where pitting had occurred, and there were also indications of incipience of cavitation at the extreme upstream edge of the tops of the baffles. Incidentally, indications of areas on the sides of the baffles in which cavitation might be suspected in the model were in excellent general agreement with pitted areas in the prototype. Some time after observing the foregoing phenomena at the laboratory, the writer [Mr. Pafford] visited the existing prototype for the specific purpose of further investigation of the extent of pitting in the stilling basin area. Visual observation revealed a minor degree of pitting at the leading edge of the tops of the baffle piers in the areas indicated by the model. The stilling basin floor was covered by a thin layer of mud and 4 or 5 ft. of murky water, making visual inspection there impracticable at the time. Careful probing, however, definitely established that there was considerable pitting of the apron floor in the very areas indicated by the model. As nearly as could be determined by ' feel ' through the sticks used for probing, the degree and depth of pitting were very similar to the pitting on the sides of the baffle piers where the depth ranged up to 2 or 3 in. below the original concrete surface".

(g) The Boulder Dam Experiments.

Before leaving the subject of model tests on spillways and the like, it appears appropriate to draw the attention of the reader to the very comprehensive and illuminating experiments made in connection with the Boulder Dam. The Bureau of Reclamation, Denver, Col., or Washington, D.C., U.S.A., has issued a long list of bulletins as Final Reports on the Boulder Canyon Project. Numbers 1 and 2 of Part VI, entitled *Model Studies of Spillways* and *Model Studies of Penstocks and Outlet Works*, may be of special interest. It was necessary to provide for a flow of 400,000 cusecs, for which two overflow channel-spillways were adopted, one on each side of the canyon, discharging through inclined shafts into tunnels which had previously been used for diversion. Models of three different scales were used, 1 : 20, 1 : 60 and 1 : 100, and it is stated that " the close agreement of results obtained in the similar models creates confidence that the results on the models will be duplicated in the prototype and that the Boulder Dam spillways will perform in a satisfactory manner ". A study of the Reports shows that important features embodied in the final design resulted from the model tests. In addition to the " usual " experiments on spillway forms, an investigation was made as to the possibility of the concrete lining of the tunnels being eroded by the high velocities (upwards of 180 ft. per sec.) which computation and model experiments showed might occur. This investigation consisted of subjecting concrete blocks to the action of a continuous stream of water issuing from a nozzle at velocities from 100 to 175 ft. per sec., and while the only tests of this series really applicable to working conditions were those carried out with small jet angles, it is remarkable that a block which suffered for five weeks a continuous jet of clear water *normal* to its surface at 175 ft. per sec. exhibited a pitting only to an average depth of ½ inch. Certain of the experiments on models of the junction of the 30-ft. diameter headers and the 13-ft. penstocks were made with transparent *pyralin* (celluloid) pipes to facilitate *visual* observation of the flow. A point which should be emphasized, however, is that quantitative results of junction-losses were appreciably lower than those measured by Professor Thoma, of Munich, on a geometrically similar model of about one-sixth the size of the American model. Thus, Professor Thoma's experiments on a brass pipe having a diameter of 1·69 in., connected with a 0·59 in. branch pipe, were repeated on a model having corresponding pipe-sizes of 10 in. and 3·49 in. respectively. The two models agreed in indicating that, for a given type of Tee, the junction-loss depends not on the absolute water-quantities, but upon the ratio of flows in the main and branch pipes, but the actual coefficients obtained from the models showed marked discrepancies, from which it was concluded that " the increased loss indicated by the smaller junction is primarily due to viscous effects and to the fact that the straight pipe friction is seldom to scale, although the junctions may be geometrically similar". A scale of 1 : 36 was adopted in the U.S. experiments on the 30-ft. header and 13-ft. penstock junction.

No such marked scale-effect is reported of the spillway experiments on models of different scales. "The only point", we are told, "in the spillway tests at which the viscosity forces were significant was in the model of the spillway tunnel at the Montrose laboratory. In this case, the model tunnel was constructed on a grade which compensated for the relatively greater viscosity effects in the model. . . . The agreement of results from models of different sizes tends to emphasize and encourage the use of comparatively small models."

This statement will serve to remind us of what is the most vital consideration in the practice of model-experimentation: the choice of scale and the practicability of relatively small scales. The cost of large models may not always be undertaken lightly; moreover, in many cases the difficulty of handling and of making measurements increases greatly as the size of model is increased, and a large model is not easily modified for the purpose of studying various proposed schemes.

If the scale-ratio is $1 : S$ and the corresponding scale of rate of discharge is $1 : S^{\frac{5}{2}}$, a structure discharging 200,000 cusecs, like each of the Boulder Dam spillways, requires a model flow of nearly 112 cusecs when $S = 20$. Even if the scale is reduced to 1 : 60, the discharge involved is over 7 cusecs —a considerable flow for even a well-equipped University laboratory in this country at the present time.

An attempt has been made in this chapter to indicate the kind of scale limitations which exist in order to avoid undue scale-effect, but the reader will appreciate that much is yet to be done before such limitations can be more precisely defined, and indeed it is desirable that every effort be made to determine the scale-effect of *small* models, so that such may possibly be used and their results adjusted by an accurately established correcting factor.

A further discussion of the question of choice of scale will be made in other connections at a later stage.

CHAPTER III

THE EFFECT OF ENGINEERING STRUCTURES ON THE RÉGIME OF WATERWAYS, WITH SPECIAL REFERENCE TO BRIDGE PIERS, SLUICE-DAMS AND SPILLWAYS

THIS subject, which is one of very great breadth and variety, is concerned with the prediction of the influence of proposed structures, such as bridges, spillways, barrages and training works, upon the behaviour of the river, canal or estuary in which they are to be constructed. It may be necessary to study the effect upon drainage, irrigation, fisheries, sewage disposal or navigation, and these in turn depend upon the magnitude and direction of currents, the rise and fall of the tides or the disposition of sand banks, of gravel or of mud, as well as upon the amount of material borne in suspension. No engineering problem could be more complex, and while it is true that engineers have by experience evolved certain general principles of design on which there is common agreement, nevertheless it is not uncommon (in questions of river control, for example) to find wide divergence of opinion among men of equal experience and reputation. To quote only a few examples, there are the cases of the Ribble estuary in Lancashire,* the Mersey,† the Seine,‡ and again the River Great Ouse, which discharges into the Wash and for which at least four schemes have been suggested by experienced authorities during the last fifteen years, not to mention the earlier history of that romantic river.§ Here, then, is a field in which the laboratory may play a part of the greatest possible value, provided that the results obtained from small-scale experiments are reliable. In many instances, reliability in only a qualitative sense may be of great practical use: if a model can truthfully predict that a certain channel will be deepened or shallowed as a result of some particular

* See *Tidal Rivers*, W. H. Wheeler, pp. 384–98 (Longmans, 1893); also *A History of the Ribble Navigation*, J. Barron (Corporation of Preston, 1938).

† p. 308 of "The River Seine", L. F. Vernon-Harcourt, *Proc. I.C.E.*, vol. LXXXIV 1886, p. 210.

‡ "The Principles of training Rivers through Tidal Estuaries, as illustrated by Investigations into the Methods of improving the Navigation Channels of the Estuary of the Seine", L. F. Vernon-Harcourt, *Proc. Roy. Soc.*, vol. XLV, 1888–89, p. 504; also "The River Seine", L. F. Vernon-Harcourt, *Proc. I.C.E.*, LXXXIV, 1886, p. 210.

§ *The Draining of the Fens*, H. C. Darby (Camb. Univ. Press, 1940).

works even though the magnitude of the effect remains uncertain, an advance will have been made which in the more intricate examples will represent an achievement of no mean order. An attempt will be made in the following pages to describe the position as it stands to-day in this kind of work, to emphasize the precautions which are essential to success, and to define the limitations as well as the advantages of the method.

Broadly speaking, there are two methods of attack:

(*a*) To explore in the model the water-levels and current velocities with and without the proposed works, and from this exploration to deduce the probable effect on sand formation, and so forth. This kind of experiment may be carried out in a model with a fixed bed.

(*b*) To observe the behaviour of a mobile bed of sand or other material in the model with and without the proposed works. At the same time, measurements of water-levels and velocities would also be made, as providing general data of importance in questions of drainage, navigation, and so on.

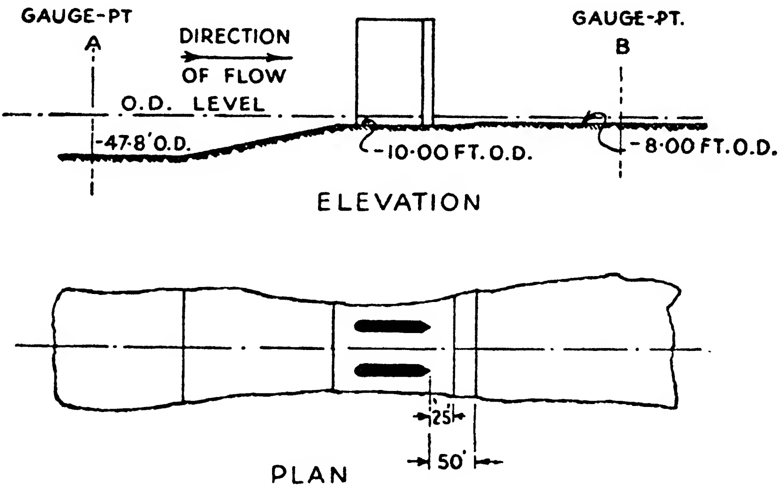


Fig. 22.

As an example of the first type, consider the arrangement of piers for a sluice dam,* as shown in Fig. 22, having an overall length of 6,825 feet and built on a rock plateau. The main purpose of the investigation was to determine the difference of water-level through the dam when the sluice gates were fully open, and the effect of changing the shape and pitch of the piers. Partly from considerations of size and water-supply, a scale of

* "The Design of Piers for a Bridge or Sluice Dam: an Investigation with the Aid of Model Experiments", Allen and Deshpande, *Inst. C.E. Selected Engineering Paper No.* 151, 1934.

1 : 221 was first adopted, and instead of the total length of 6,825 feet it was decided to include only a representative length of 210 feet.

Let Q_a denote the discharge, in cusecs, of the full-size dam;

Q_a' „ „ discharge, in cusecs, of a portion of length 210 ft.;

Q_m' „ „ discharge of the model dam, representing 210 ft. of the actual dam;

V_a „ „ velocity at a given point in the full-size project;

V_m „ „ velocity at the corresponding point in the model;

$1:H$ „ „ scale of the model;

then
$$\frac{V_m}{V_a}=\frac{1}{\sqrt{H}},$$

and
$$\frac{Q_m'}{Q_a'}=\frac{\text{mean } V_m \times \text{area of waterway in model}}{\text{mean } V_a \times \text{area of waterway in actual}}$$
$$=\frac{1}{\sqrt{H}}\cdot\frac{1}{H^2}=\frac{1}{H^{\frac{5}{2}}},$$

or, neglecting end-effects, which may be assumed small in the case of a long structure like this,

$$\frac{Q_m'}{Q_a}=\frac{1}{H^{\frac{5}{2}}}\cdot\frac{210}{6825}=4{\cdot}24\times10^{-8},\ \text{with } H=221.$$

In this type of investigation, the question arises as to where to measure the water-levels in order to obtain a true idea of the influence of the piers themselves. Preliminary tests indicated that recovery of head after flow through the dam was complete at a distance equivalent to 150 feet from the downstream end of the piers, and that the only measurable change in level between 150 and 220 feet was caused by friction. Accordingly, a standard position was adopted at a distance of 310 feet from the upstream end of the piers; the downstream gauge point varied in its distance from the downstream tip of the piers from 166 ft. with the longest piers to 219 ft. with the shortest.

In the model, the piers were constructed of pine-wood treated with linseed oil to prevent warping; the bed of the model was also of wood except for the highest portion, which consisted of a plate roughened with sand sprinkled over its varnished surface. Many designs of pier, such as those shown in Fig. 23, were tried, and typical results are given in Tables VIII and IX, from which it appears that, *from the viewpoint of reducing the loss of head*, the so-called " aerofoil section " is best, but of the other designs (which are easier to construct), that having a tip angle of about 17 degrees is the most efficient. There is little to be gained, and in certain circumstances a loss of efficiency may result, from the use of a tip angle of, say, 57 degrees rather than an absolutely blunt end. Experiment showed no significant changes as a result of filling in the slots and grooves of the piers or of roughening the piers with sand of mean diameter 0·009 inch sprinkled over a coat of varnish.

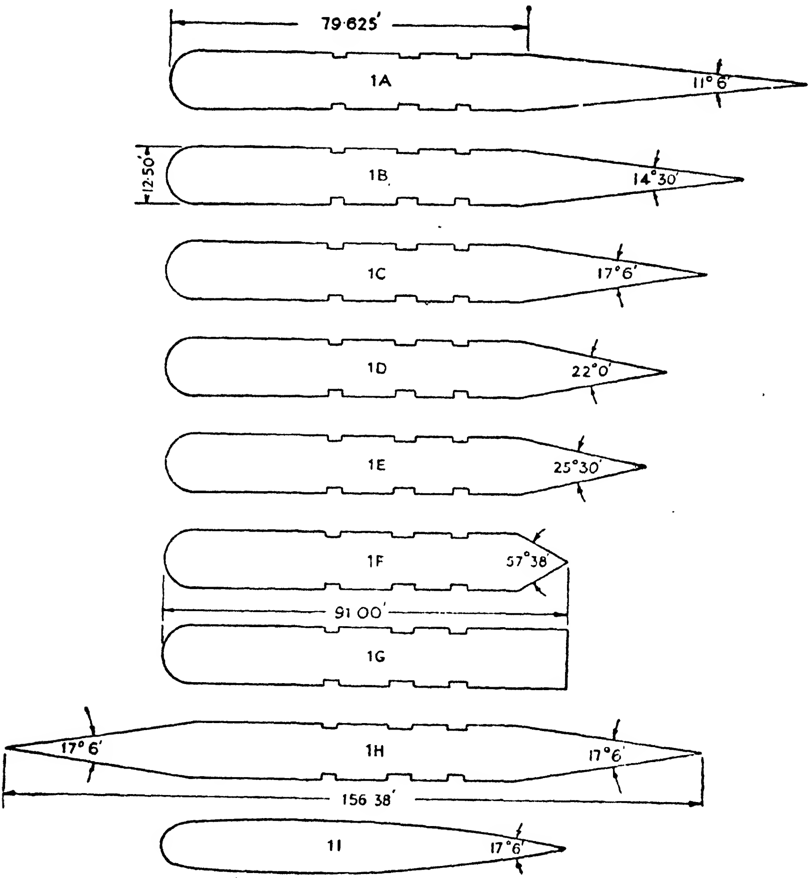


FIG. 23.

In the experiments so far quoted, the pitch of the piers was 52·5 feet and the width of each gap 40 feet; the overall length of the piers varied in the different designs. Experiments were made, however, with the same pitch (52·5 ft.) and gap (40 ft.), but with designs as described in Table X.

TABLE VIII

(Pitch of Piers 52·5 ft.)

Q_a (*millions of cusecs*)	*Water level at A (ft. above O.D.)*	*Drop in level in feet between A and B, for pier number*								
		1*A*	1*B*	1*C*	1*D*	1*E*	1*F*	1*G*	1*H*	1*I*
0·300	2·5	0·55	—	0·37	—	0·45	0·45	0·64	—	—
·729	7·5	1·16	1·01	0·84	0·88	0·92	1·29	1·38	0·92	0·64
·900	7·5	2·11	1·94	1·47	1·84	1·84	1·94	2·39	1·84	1·47
1·11	17·5	0·83	—	0·64	0·85	0·74	0·85	1·01	—	—
1·45	17·5	1·33	1·29	1·06	1·31	1·47	1·60	1·66	1·38	0·92
1·55	20·0	1·20	1·10	0·92	1·19	1·32	1·42	1·47	1·10	0·74
2·07	16·5	2·20	—	2·02	—	2·39	2·57	2·76	—	—

TABLE IX

Loss of head between *A* and *B* (in feet)

(Pitch of Piers 52·5 ft.)

Level at A: feet above O.D.	700,000 cusecs				1,000,000 cusecs			
	1*A*	1*C*	1*D*	1*F*	1*A*	1*C*	1*D*	1*F*
4·0	2·33	1·35	2·07	2·45	—	—	—	—
10·0	0·66	0·59	0·78	0·97	1·62	1·25	1·55	1·60
15·0	0·41	0·37	0·45	0·56	0·90	0·62	0·90	0·90
20·0	0·31	0·25	0·30	0·37	0·59	0·42	0·54	0·55
25·0	0·20	0·16	0·19	0·24	0·42	0·31	0·36	0·36
	1,250,000 cusecs				1,600,000 cusecs			
10·0	2·85	1·94	2·75	3·72	—	—	—	—
15·0	1·35	1·02	1·28	1·48	2·45	1·78	2·00	2·83
20·0	0·79	0·63	0·83	0·85	1·29	0·96	1·26	1·55
25·0	0·58	0·39	0·56	0·58	0·86	0·62	0·91	1·03

[NOTE: *A* is the upstream gauge-point;
B is the downstream gauge-point.]

TABLE X

Pier number	*Overall length (feet)*	*Shape of end*	
		Upstream	*Downstream*
2*A*1	91·0	Semicircular	Tapered, 11 degrees 6 minutes
2*C*1	,,	,,	,, 17 ,, 6 ,,
2*D*1	,,	,,	,, 22 ,, 0 ,,
2*F*1 (=1*F*)	,,	,,	,, 57 ,, 38 ,,
2*A*2	144·0	Semicircular	Tapered, 11 degrees 6 minutes
2*C*2	,,	,,	,, 17 ,, 6 ,,
2*D*2	,,	,,	,, 22 ,, 0 ,,
2*F*2	,,	,,	,, 57 ,, 38 ,,

These tests showed that the degree of obstruction produced by the piers is not only a function of the shape of their ends, but also, in very complex fashion, of their overall length; the effects are bound up with conditions of discharge and depth of water. Generally, however, a downstream tip-angle of about 17 degrees still gave the least loss of head. In practice, the best overall length may depend upon considerations other than hydraulic in nature, such as economy of construction, width of roadway (if any), space required by tackle for lifting the gates, and so on.

With a closer pitch, reducing the gap between the piers to 17·5 feet, the differences in efficiency of the various designs were less apparent, but the optimum angle of the downstream tip lay between 11 and 17 degrees.

The designer of a structure of this kind will also be interested in the effect of the *spacing of piers of a given design*. In this connection, the *structural* considerations of the layout are likely to be of great importance as influencing the stresses induced in any superstructure and in the sluice-gates when they are dropped in place, but the curves shown in Fig. 24 suggest that enlarging the width of waterway beyond a certain point may yield only a small return in enhanced hydraulic efficiency.

In the course of this investigation, a number of experiments was also carried out using a scale of 1 : 60·5; for this purpose, a concrete flume having a width of 42 inches was utilized, the flow being provided by a centrifugal pump circulating water through the flume and back to the forebay along a length of overhead pipe. Discharges ranged from 1·00 to 6·25 cusecs. The upstream gauge-point, *A*, was 380 feet (to scale) from the semicircular end of the piers, and the distance between the two gauges, points A and B, was equivalent to 925 feet. Between A and B the bed of the flume had a gradual rise of 3·93 feet (to scale), and calling the level of the bottom $-10{\cdot}0$ ft.O.D. at the site of the piers, that at A was

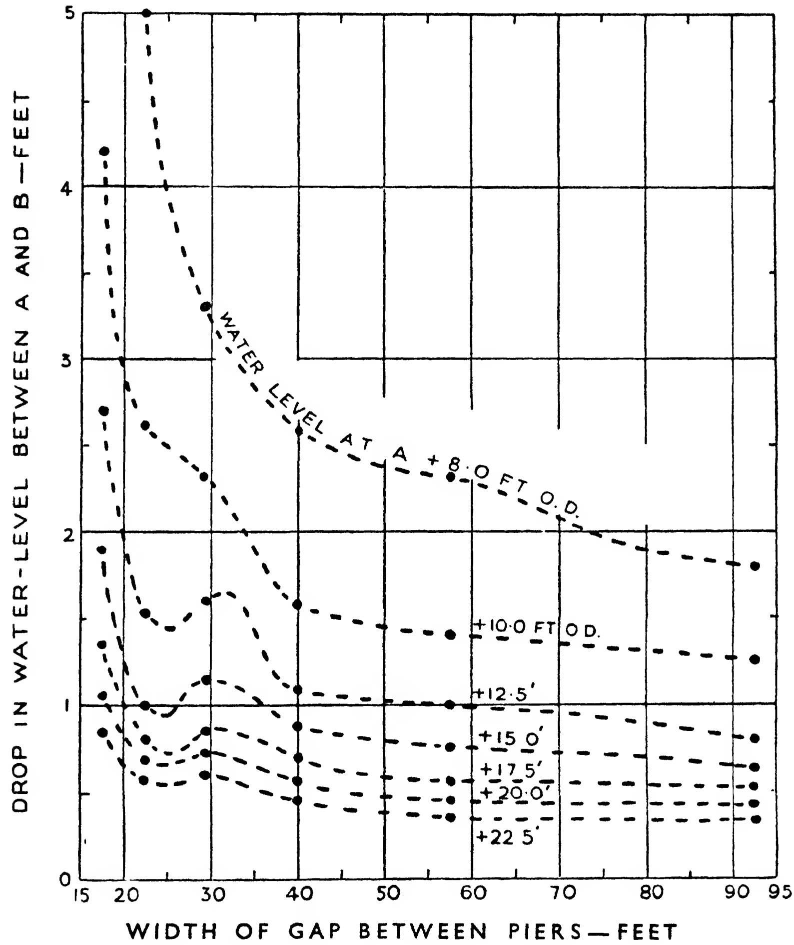


FIG. 24.

$-11{\cdot}06$, and that at B $-7{\cdot}13$ ft. O.D. The general arrangement was, therefore, not dissimilar from that in the right-hand portion of the smaller model (see Fig. 22), and the piers were pitched at the equivalent of 52·5 feet centres, leaving a gap between piers of 39·9 feet. Five designs were tried, as detailed in Table XI:

TABLE XI

Pier number	*Upstream end*	*Downstream end*	*Overall length (ft.)*
A	Semicircular	Tapered, 11 degrees 0 minutes	140·3
C	”	” 17 ” 6 ”	116·8
D	”	” 22 ” 0 ”	107·2
F	”	” 56 ” 48 ”	84·0
G	”	Blunt, 180 degrees	63·8

In these larger scale-model experiments, the difference in behaviour of the various designs of pier was not so marked, apparent discrepancies between the two models being attributed to the following causes:

(*a*) The dimensions of the piers not being quite comparable.

(*b*) The bed-levels not being identically equivalent. By actual measurement it was discovered that the removal of the raised ledge downstream of the piers in the smaller model reduced the loss of head between *A* and *B*, even without piers, by 4 or 5 per cent.

(*c*) The possibility of scale-effect, and in particular of an increased efficiency as the *absolute* dimensions increase and the size of the model approaches more closely that of its prototype. Detailed analysis of the results indicated actually that the scale-effect as such was relatively small.

In order to effect a direct comparison, coefficients of discharge were calculated on the following basis:

Without piers, there would be a loss of head between the upstream (*A*) and downstream (*B*) gauge-points of h such that $\frac{V^2_A}{2g}+h=\frac{V^2_B}{2g}+$(friction and other losses occurring even without piers).

With piers *in situ*, let the suffix P be applied. Then

$$\frac{V^2_{AP}}{2g}+h_P=\frac{V^2_{BP}}{2g}+\text{(friction and other losses occurring without piers)}$$
$$+\text{(losses due to piers)}.$$

For a given flow and upstream level, $V_{AP}=V_A$, and

$$\frac{V^2_{AP}}{2g}+h_P=\frac{V^2_{BP}}{2g}+\left(\frac{V^2_A}{2g}+h-\frac{V^2_B}{2g}\right)+\text{(losses due to piers)};$$

$$\therefore\ h_P=\frac{V^2_{BP}}{2g}+\left(h-\frac{V^2_B}{2g}\right)+\text{(losses due to piers)}.$$

As a criterion of performance, let the coefficient of discharge C be defined as

$$C = \frac{Q}{bd_{BP}\sqrt{2gH}},$$

where Q denotes the discharge of each opening (in cusecs),

b „ „ width of each opening (in feet),

d_{BP} „ „ depth at B as measured with piers *in situ*,

H „ „ effective head in feet, measured as the drop in level (with piers) between A and B, plus the velocity head $\frac{V^2_{AP}}{2g}$.

Allowance ought, however, to be made for such friction and eddy losses as would *in any event* occur without piers. Accordingly, experiments were made to obtain the drop in level between A and B without piers, thus rendering it possible to calculate the quantity

$$\frac{V^2_A}{2g} + h - \frac{V^2_B}{2g},$$

which for a given discharge and upstream level is the same as

$$\frac{V^2_{AP}}{2g} + h - \frac{V^2_B}{2g}.$$

This head was then subtracted from the effective head *with* piers, namely $\left(\frac{V^2_{AP}}{2g} + h_P\right)$, and the coefficient of discharge estimated as

$$C = \frac{Q}{bd_{BP}\sqrt{2g\left(h_P - h + \frac{V^2_B}{2g}\right)}}.$$

On this basis, the comparison between the two models is shown by the typical coefficients in Table XII.

Having regard to the points already discussed, the agreement between the two models is far from being unsatisfactory, especially when it is realized that the comparison is only practicable for relatively great depths when the losses are comparatively small and difficult to measure.

Regarding the comparative efficiency of the different designs, the average coefficients of discharge obtained in the larger model are presented in Table XIII.

TABLE XII

" Coefficients of Discharge ":

Pier number	Upstream level+15·0 ft. O.D.			Upstream level+22·5 ft. O.D.		
	Q_a $1{\cdot}00 \times 10^6$	Q_a $1{\cdot}25 \times 10^6$	Q_a $1{\cdot}60 \times 10^6$	Q_a $1{\cdot}00 \times 10^6$	Q_a $1{\cdot}25 \times 10^6$	Q_a $1{\cdot}60 \times 10^6$
1*A**	1·10	1·15	1·14	1·06	1·12	1·16
A†	1·22	1·24	1·23	1·23	1·25	1·26
1*C**	1·28	1·30	1·27	1·21	1·28	1·30
2*C*1*	1·23	1·18	—	1·21	1·17	—
C†	1·24	1·25	1·24	1·24	1·26	1·27
1*D**	1·10	1·18	1·23	1·09	1·12	1·14
2*D*1*	1·16	1·14	—	1·17	1·18	—
D†	1·20	1·21	1·22	1·22	1·24	1·25
1*F**	1·10	1·11	1·09	1·10	1·07	1·07
F†	1·16	1·18	1·19	1·17	1·17	1·18

* 1 : 221 model. † 1 : 60·5 model. Q_a discharge (cusecs) of prototype dam over all its length of 6,825 feet.

TABLE XIII

Level at A (ft. above O.D.)	*A* (11·0 *degrees*)	*C* (17·1 *degrees*)	*D* (22·0 *degrees*)	*F* (56·8 *degrees*)	*G* (180 *degrees*)
15·04	1·227	1·236	1·205	1·178	1·180
23·75	1·241	1·251	1·243	1·185	1·183
42·98	1·249	1·249	1·263	1·194	1·186

According to these results, the best angle tends to increase with increasing depth of water, an effect also noted in the smaller model.

This particular investigation has been discussed here at some length partly because of its intrinsic interest, but largely because it shows that in such a case a comparatively small model (which is cheaper to operate as well as to construct and easier to handle than a large one) is capable of giving the essential data concerning the comparative virtues of different designs: moreover, the absolute values may be quite close enough for practical application in a given case. To sum up, the smaller model showed (and the larger one confirmed) that there is a magnitude of the included angle at the downstream tip of the pier which gives a minimum loss of head. This magnitude is approximately 17 degrees, but depends on conditions of depth and discharge, tending to increase as the depth increases; to taper off the piers downstream to a tip-angle of, say, 50 or 60 degrees has no advantage over a blunt (180 degree) end.

But it should be emphasized that a sharp tip, while efficient from the aspect of loss of head, is worse than a blunt end from the point of view of erosion: visual observation of the smaller model showed this just as clearly as observation of the larger. Thus, with blunt noses, there was a region of back-flow towards the piers which would tend towards a deposit of material rather than a scour, whereas with a 17-degree tip a layer of small eddies was observed along the sides of the tip itself, but the major eddy-flow back towards the pier itself was nearly eliminated (see Fig. 25), the flow making ultimately downstream from the tip and therefore favouring erosion. A similar conclusion was apparently reached by C. Keutner,* who as a result of experiments with a mobile bed recommended a semicircular downstream end to minimize erosion. The investigation described above provides an example, therefore, of a model in which the observation of currents and eddies may lead to useful qualitative conclusions concerning erosion even though the model itself has an immobile bed.

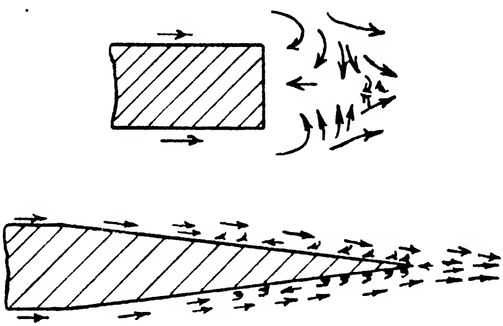


FIG. 25. Flow-Pattern Round Piers.

A more complicated problem arises when the bridge or sluice-dam or other obstruction is to be built in a *tidal* river. Until quite recently it was generally assumed that any obstruction would naturally have the effect of lowering high-water and raising low-water level at points above the site of the structure. In 1938, however, Professor A. H. Gibson published a paper † which inspired considerable controversy because it contained experimental evidence showing that such an assumption may in many instances be quite false; as to whether the levels will be raised or lowered depends upon the estuary as well as upon the size of the obstruction.

Thus, in a tidal model of the Humber, having a horizontal scale of

* *Die Bautechnik*, 10, pp. 161–70, abstracted in *Inst. C.E. Engineering Abstracts* No. 52, July 1932, p. 161. To cause little obstruction and little erosion of the bed, a sloping upstream edge, at 1 : 2·5, formed by two circular arcs intersecting at 135 deg., and a semicircular downstream end were apparently recommended.

† " An Experimental Investigation of the Effect of Bridge-Piers and other Obstructions on the Tidal Levels in an Estuary ", A. H. Gibson, *Journ. Inst. C.E.*, March 1938, p. 210.

1 : 7,040 and a vertical scale of 1 : 192, it was found that the effect of a proposed bridge spanning the waterway between Barton and Hessle, some six miles above Hull, was too small to measure accurately: it was certainly less than 0·1 foot. This bridge was to have sixteen piers producing an obstruction of about 4·5 per cent. of the high-water sectional area at its site. The same conclusion was found to apply when the piers were doubled in width, while the effect of completely blocking the main span of the bridge—924 feet in length (thus obstructing about 30 per cent. of the section)—was to lower high-water level of spring tides at Goole by approximately 0·1 foot and to raise low water by a similar amount: at points nearer the bridge, the effect became progressively smaller. On the other hand, experiments on a tiny model of the Severn estuary (horizontal scale 1 : 40,000 and vertical scale 1 : 366; overall length 6 feet) showed that obstructions up to 35 per cent. resulted in an elevation of high-water level everywhere upstream of the obstructions. These results might be open to question in view of the small size of the model were it not for the fact that measurements of tide levels and of the drift of floats, prior to the introduction of the obstructions, showed virtually perfect agreement with the values observed in Nature. Two photographs of this tiny model of the Severn estuary are reproduced in Plates VI and VII.

Again, experiments on a suggested embankment and bridge across the mouth of the Dee estuary between Point of Air and Hilbre Point were made in two models of very different sizes. In one, the horizontal and vertical scales were 1 : 7,040 and 1 : 192 respectively; in the other they were 1 : 40,000 and 1 : 400. The smaller model, incidentally, embraced only the Dee estuary itself, while the other included the Mersey and a considerable area of Liverpool Bay. With an obstruction amounting to 74 per cent. of the waterway, the mean high-water level at spring tides was lowered by the equivalent of 1·2 and 1·0 ft. respectively in the large and small models. Such examples as these, in addition to their immediate interest, promote confidence in the use of *small* models, at any rate where information concerning tide levels is all that is sought.

Other examples of the apparently anomalous effects of obstructions will now be cited. In the smaller model of the Dee referred to above, an obstruction of 35 per cent. blocking the Hilbre Swash on the northern side of the mouth *increased* the level of high water at Connah's Quay (some 15 miles upstream) by 8·0 inches; blocking the entrance to Mostyn Deep on the southern side had no appreciable effect, while the two obstructions together *lowered* the high-water level by 4·5 inches.

In a model of the River Parrett (Somerset) with scales of 1 : 3,000 and 1 : 260 horizontally and vertically, a *reduction* in the width of certain sluice-gate openings from 300 to 200 feet was found to produce a small increase in high water and a small reduction of low water in the upper reaches, during flood-river discharge.* Thus in this case, an increase in the degree of obstruction led to an overall increase in the amplitude of the tide in the upper reaches.

These effects are, of course, of an immediate nature; they may be changed as time goes on after the introduction of an obstruction by resulting modifications in the bed-formation of the estuary. If, however, a relatively small model shows *immediate* effects of a *harmful* kind, the particular scheme may be condemned on that evidence alone. Should the immediate effect not be so obviously deleterious, further experiment will be necessary, generally on a larger scale, with mobile bed, to test the long-term consequences. For example, in the Dee-with-Mersey model (horizontal scale 1 : 7,040; vertical 1 : 192), a 74 per cent. obstruction of the mouth of the Dee had the *immediate* effect of lowering the mean level of high water above the embankment and bridge by 1·2 ft., and of raising low water by 0·6 ft. But after forty years of tides, high water was found to be 0·4 ft. *higher* and low water 0·3 ft. higher than before the introduction of the obstacle. During these " forty years ", scour at the bridge had increased the area of waterway so that the obstruction amounted to about 66 per cent. of the original high-tide section; in addition, the general configuration of the estuary had been materially changed.

These complex phenomena are to be attributed to the fact that the tides present a problem in vibrations. The mass of water contained within an estuary would, if isolated from the sea and set in oscillation, possess a natural period of vibration. In its actual state the estuary has superimposed upon this natural period a forced vibration or vibrations due to the tides of the outer sea. If the tidal rise is small in comparison with the depth of water, mathematical reasoning † shows that in a channel with horizontal bed and straight parallel sides,

$$\eta_x = \frac{a \cos kx}{\cos kl} \cos(\sigma t + \epsilon),$$

where η_x = the tidal rise at a distance x from the *closed* end of the estuary,
a = the rise of tide at the mouth when $\cos(\sigma t + \epsilon) = 1$, that is, a denotes the height of the high-water surface *at the mouth*, above datum,
$k = \sigma/c$,
$\sigma = 2\pi/$tidal period,
$c = \sqrt{gh}$,
h = depth of water,
t = time,
ϵ = an arbitrary time,
l = length of estuary.

* Allen, " Schemes of Improvement for the River Parrett: an Investigation with the Aid of a Tidal Model ", *Journ. Inst. C.E.*, Dec. 1942, p. 85.

† *Hydrodynamics*, Lamb (Camb. Univ. Press), 4th ed., 1916, pp. 258 and 267.

Now it is possible that the introduction of an obstruction will change the *effective* length of the estuary, i.e. will alter the value of l. If that is so, the tidal rise within the estuary will also be altered. Indeed, Sir Horace Lamb has shown * that a cylindrical obstacle introduced near the middle of a long and narrow rectangular tank has "the effect of virtually increasing the length of the tank".

Another way of regarding the problem is to think of it in terms of an approximate mechanical analogy, say that of a vertical spring fixed at its upper end and carrying a weight W at its lower (free) end. Suppose a periodic force $Q \sin \alpha t$ to be applied vertically to W. This force has a period $\frac{2\pi}{\alpha}$. Suppose the motion of the system to be subjected to the resistance of the air or other medium surrounding it. For small velocities this resistance may be assumed proportional to the first power of the velocity.†

In that case, the amplitude of the vibrations becomes ‡

$$A = \frac{a}{\sqrt{\left(1 - \frac{T^2{}_N}{T^2{}_F}\right)^2 + \frac{T^2{}_N \gamma^2}{T^2{}_F}}},$$

where a = the deflexion which would be caused by the *maximum* disturbing force Q if applied statically;

T_N = the natural period of vibration of the system;

T_F = the period, $\frac{2\pi}{\alpha}$, of the disturbing force $Q \sin \alpha t$;

γ = a quantity proportional directly to the magnitude of the damping forces of resistance and to the natural period of vibration.

A consideration of this expression for A will reveal that various possibilities arise. For example, if T_F is very large, A approaches the value of a and the deflexion is sensibly the same as that which would result from a statical application of the extraneous force. On the other hand, if T_F is very small (the disturbing force having a high frequency), A will tend towards the value zero. Again, if T_F approaches T_N, the amplitude will tend to become very large, a state of resonance being created.

Again, if we regard the obstacle in an estuary as analogous to additional frictional resistance in the mechanical system, we see that due to the

* *Loc. cit.*, pp. 433–34.

† In an estuary, resistance is actually more nearly proportional to v^2, but the mechanical system is only quoted for purposes of approximate analogy.

‡ *Applied Elasticity*, Timoshenko and Lessels (Westinghouse Technical Night School Press, East Pittsburg, Pa., 1925), p. 330; or other text-books in Applied Mathematics or Mechanics.

quantity γ in the expression for A, the amplitude will be reduced. On the other hand, if this added resistance increases the period of natural vibration, the amplitude may be either increased or diminished according to the relative values of T_N, T_F and γ. In general, the natural period of an estuary is certainly shorter than the tidal period, and anything which lengthens the natural period will bring the two periods closer together: other things being equal, this in turn will increase the amplitude or tend to raise the high water and to lower the low-water levels.

Before leaving this topic, reference will be made to three recorded cases in actual engineering practice which support the ideas set out above:

According to W. H. Wheeler,* "At Shoreham, the widening and deepening of the harbour mouth resulted in depressing the rise of spring tides from 18 feet to 16 feet. At Newhaven also, the works carried out for improving the entrance to the harbour, although causing the tide to be earlier, have made the rise 2 feet less than formerly."

Again, Mr. Oscar Borer † has stated that "in the construction of a bridge across the Holland Deeps at Meerdijk, which was about 2 kilometres wide, it had been found possible to introduce an obstruction of at least 30 per cent. with no apparent effect on the river above".

An interesting example of the type of river model in which conclusions are based on current observations is provided by an investigation ‡ made at the U.S. Waterways Experiment Station (Vicksburg, Mississippi) of a portion of the Ohio River which includes two islands known as the Manchester Islands. (See Figs. 26*a* and 26*b*; also Plate VIII.) This model had a horizontal scale of 1 : 300 and a vertical scale of 1 : 80; it was built for the purpose of deciding which of the six areas lettered A, $B \dots F$ in Fig. 26*b* would be the most advantageous for the disposal of spoil obtained from dredging operations. "The problem was attacked from the standpoint of currents and velocities. At each of five stages —ranging from lowest (pool) to highest (41 ft. above pool)—the model was subjected to a thorough analysis of currents and velocities. Current directions were traced over the model through use of floats—submerged to 3 ft. for lowest stages; to 10 ft. for higher ones. Velocities were measured at 100-ft. intervals and at $\frac{6}{10}$ depth along the ranges indicated on the map of Fig. [26*b*]. The results of the current-velocity surveys were studied deductively, and estimates thereby developed as to the prob-

* *Tidal Rivers*, W. H. Wheeler (Longmans, Green & Co., 1893), p. 181.

† In discussing Professor Gibson's paper on "Bridge Piers", *Journ. Inst. C.E.*, March 1938, p. 241.

‡ *The U.S. Waterways Expt. Station Bulletin* (*Hydraulics*), vol .1, No. 2, Oct. 1, 1938, p. 7; and vol. 2, No. 1, Feb. 1, 1939, pp. 6–9.

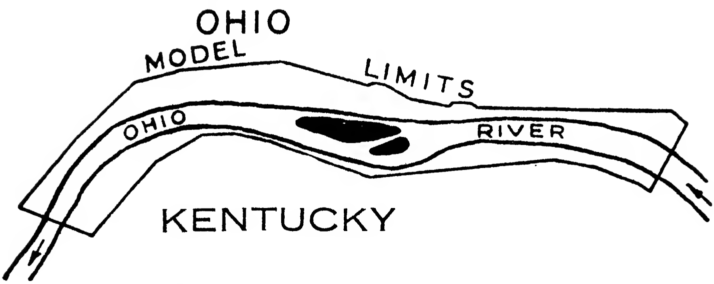


FIG. 26*a*. Area reproduced in Manchester Islands Model.

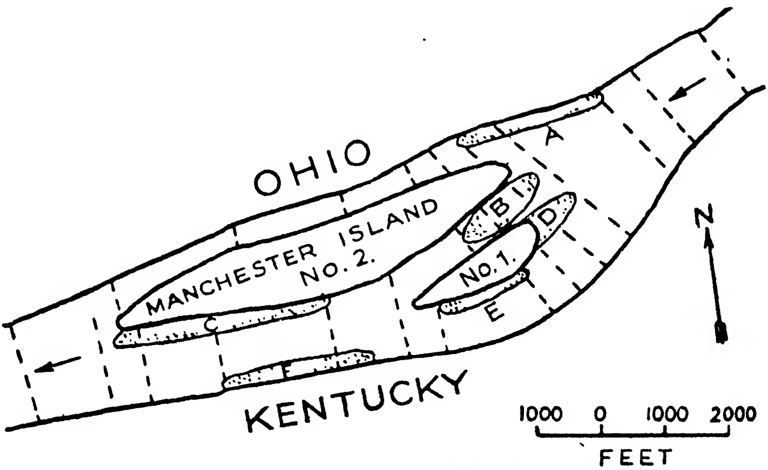


FIG. 26*b*. Broken lines indicate velocity ranges.

able effects of spoiling over the areas in question." It was in this way decided that areas *A*, *C* and *F* offered " the most advantageous locations for deposition of dredge spoil ".

Before carrying out the main investigation in this case, the performance of the model was first compared with that of the actual river: velocity measurements in the actual river were made at 100-ft. intervals and at 0·2 and 0·8 depths along the line *ab* (Fig. 26*c*). These observations were done at several stages of the river, as defined by water-levels read on appropriate gauges. With the model adjusted to these stages, velocities were measured in it along the corresponding line; at the same time soundings were taken to provide a direct check by comparison with the shape of section as obtained in nature. On this basis of calibration, it was found that when a given stage was reproduced, the discharge relation-

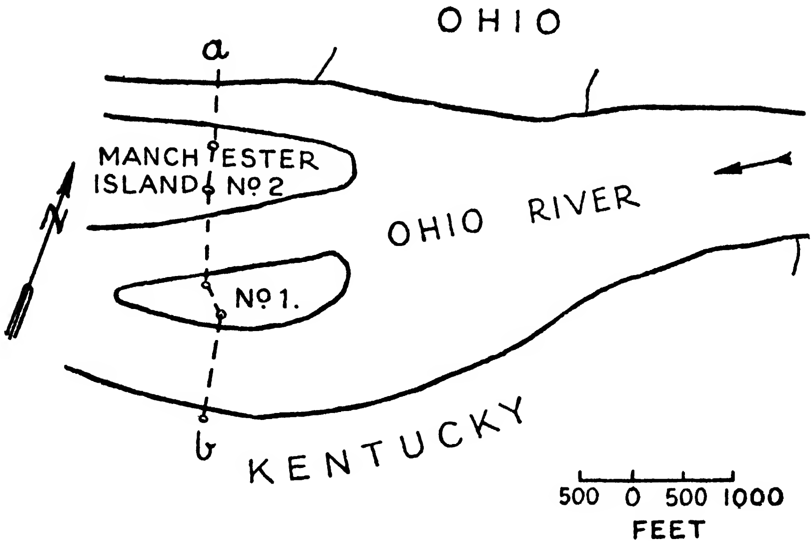


FIG. 26c.

ship was 1 : 160,000 (on the average). Since discharge is proportional to (area × velocity) and the scale of sectional areas was 1 : 300 × 80, it follows that the velocity scale derived by calibration was 1 : (160,000 ÷ 24,000) or 1 : 6·7 nearly, as compared with the nominal value of 1 : $\sqrt{80}$, or 1 : 9, say. The point apparently is, however, that on multiplying the model velocities by 6·7, it was considered that the *distribution* of velocity over the section was sufficiently closely simulated to warrant the continuance of the investigation along the lines described above, that is, to make a thorough exploration of currents in the model (a process which presumably would have entailed far more time, labour, and general difficulty if carried out " in the field "), and to deduce the required data from that exploration.

As a further example, we may usefully consider the experiments made at Dresden in 1905 * concerning the formation of mud deposits at the old entrance to the Kaiser Wilhelm Canal. At that time, no facilities existed in the Dresden laboratory for the production of ebb and flood tides; recourse was made therefore to two reproductions of the entrance works in one and the same flume (see Fig. 27), the orientation being such as to subject one to flood- and the other to ebb-tide currents. Float observations revealed eddy currents as indicated in Figs. 28*a* and 28*b*. In these experiments the floats were placed close up to the mole-heads; when placed only a short distance downstream of the mole-heads, they passed

* *Hydraulic Laboratory Practice*, ed. Freeman, Am. Soc. Mech. Eng., 1929, p. 99.

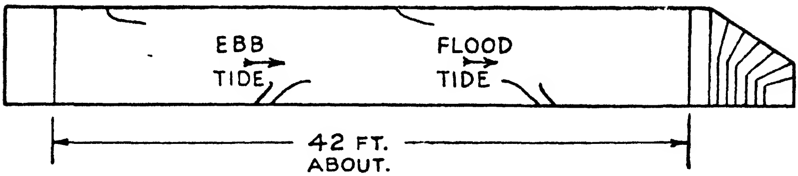


FIG. 27. Arrangements of Moles in Flume.

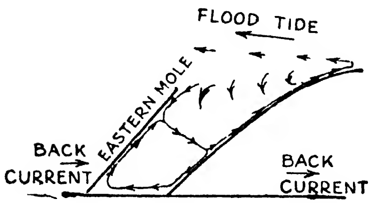


FIG. 28*a*.

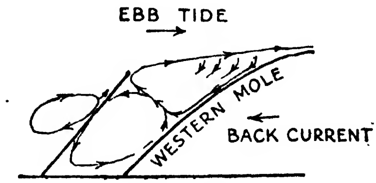


FIG. 28*b*.

the entrance and were carried away. This led the investigators to the conclusion that the mud shoals in the entrance were made up of those materials moving in close proximity to the mole-heads and along the bottom of the river bed. Consequently, two submerged training walls were constructed (see Fig. 29), which "decreased the mud deposits to a

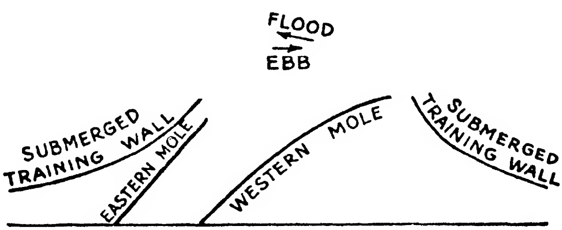


FIG. 29.

great extent ". Furthermore, a contemplated lengthening of the east mole was abandoned as a result of tests on the model.

Yet another example of this kind of study is provided by a 1 : 200 scale-model, with fixed bed, of the approach to a lock and dam on the Monongahela River.* The general features of this installation are shown diagrammatically in Fig. 30. Difficulties were experienced in navigating

* "Hydraulic Conditions at Approach to a Lock and Dam", W. J. Hopkins, *Proc. Am. Soc. C.E.*, vol. 68, No. 8, Pt. 1, Oct. 1942, p. 1369.

the approach to the lock as originally built, the main currents being found to accelerate towards the lock and to swing awkwardly across the entrance

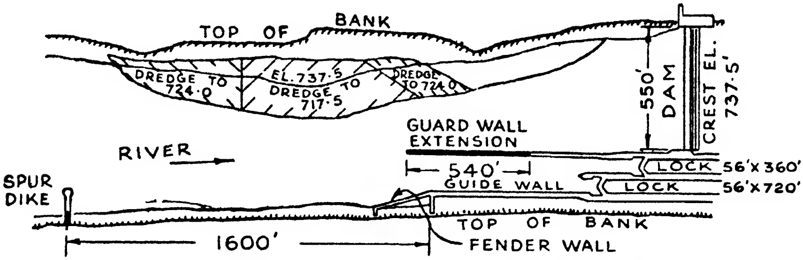


FIG. 30. Modified Design of Upper Lock Approach, Monongahela River (Sketch approx. to scale).

and over towards the spillway dam. The model embraced about 2·4 miles of the river, mostly on the side upstream of the lock in order to permit the full development of the flow pattern in the important portion. The vertical scale was the same as the horizontal (that is, 1 : 200), and the

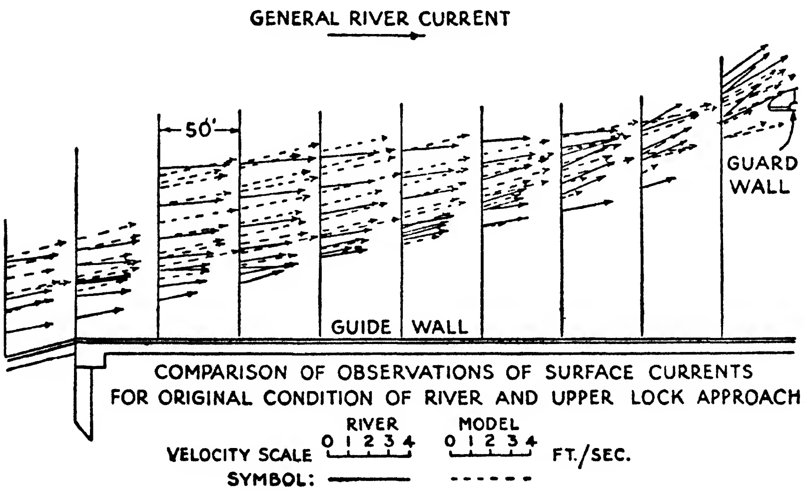
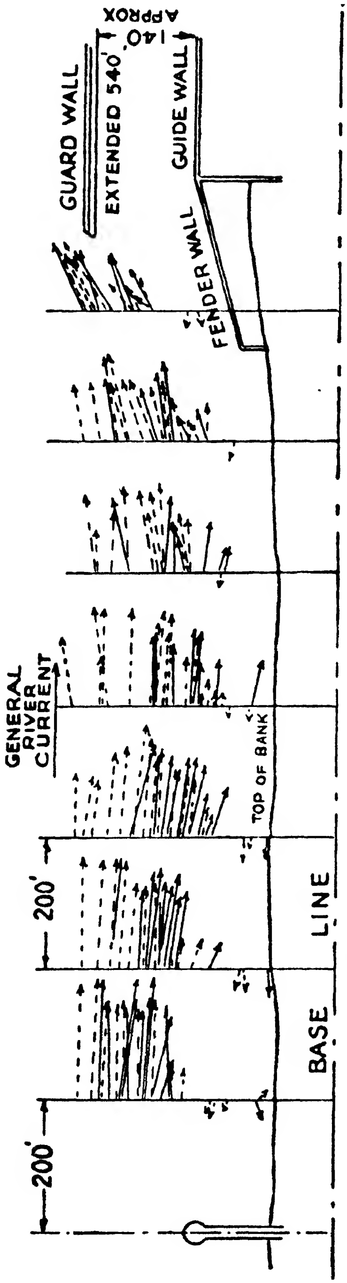


FIG. 31*a*.

method of attack was to observe velocities, by means of floats and confetti, with the layout of the structure as already built and afterwards with various individual and combined modifications. As a result, a scheme was devised, and carried out in the actual river, involving the extension



COMPARISON OF OBSERVATIONS OF SURFACE CURRENTS FOR ALTERED CONDITIONS OF RIVER AND UPPER LOCK APPROACH

(Based on Fig. 61, *Proc. Am. Soc. C.E.*, Oct. 1942, p. 1374, W. J. Hopkins)

	RIVER	MODEL	
VELOCITY SCALE	0 1 2	0 1 2	FT./SEC.
SYMBOL:	——	- - - -	

FIG. 31*b*.

of the guard wall by 540 ft., the construction of a spur dyke some 2,360 ft. upstream of the lock, and the dredging of a bar on the side of the river opposite the lock entrance. These arrangements proved to behave satisfactorily in practice, and a comparison of surface velocities as observed in the model and its prototype is afforded by Figs. 31*a* and 31*b*. In the case of the observations made prior to the modification of the structure, the head of 5·6 ft. on the dam was correspondingly reproduced in the model; in the other case the conditions were not quite comparable, the head then being 6·6 ft. in the prototype and 6·0 ft. in the model.

Models with a Mobile Bed.

We come next to a consideration of the other type of model—that in which the behaviour of a *mobile bed* is observed in an attempt to gain information about erosion and siltation—information which it is hoped will be at worst *qualitative* and at best approximately *quantitative* in character.

A comparatively simple example is furnished by experiments on the protection of dams, weirs and sluices against scour. The rate of discharge of water from structures of this kind is often such as would produce dangerous scour of the river-bed unless specially designed protective devices are provided in the design. There is no doubt that much reliable guidance has resulted from model-experiments in the field, an extremely interesting study being that made by Dr. Burns and Dr. White at the Imperial College of Science and Technology, London.* Their apparatus consisted of a glass-sided flume 20 feet long, 3 feet deep and 11½ inches wide, supplied with water from a constant-head overflow-tank, the control of discharge and downstream level being given by a gate-valve in the delivery pipe and an adjustable gate near the downstream end of the flume. The type of spillway under study is shown in Fig. 32: experiments were made on two models, one 21 inches high and the other 4·2 inches. The whole subject of the choice of bed materials for use in models is discussed in a separate chapter; for the moment it will suffice to state that Burns and White found that " standard Leighton Buzzard sand, 0·036 inch diameter, had fewest defects. It was particularly clean, and the water remained clear enough for the bed-movements to be readily visible. The individual grains were large enough to be seen individually, and yet were small enough to be picked up and carried downstream by the velocities attained in the 21-inch model. Certain other sands of smaller diameter satisfied these conditions equally well, but appreciable amounts were carried out of the flume, and the loss had to be made good before

* " The Protection of Dams, Weirs and Sluices against Scour ", R. V. Burns and C. M. White, *Journ. Inst. C.E.*, Nov. 1938, p. 23.

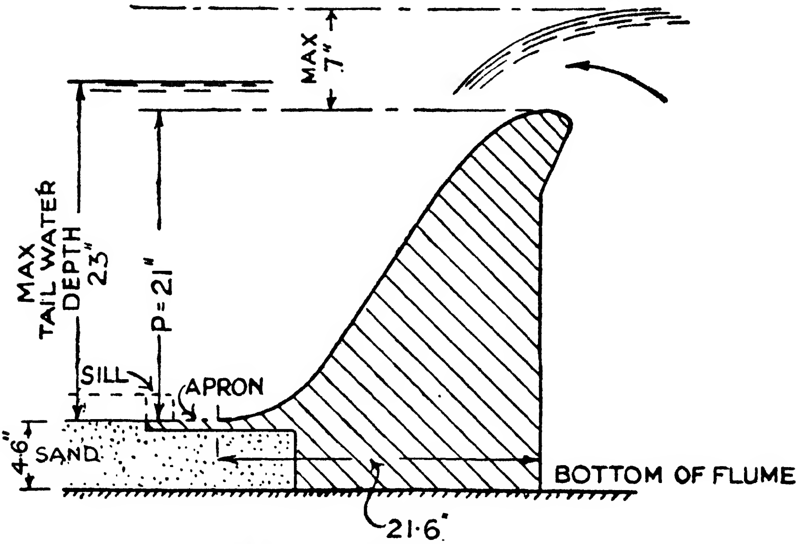


FIG. 32. Section of Model Weir. (Burns and White.)

each new experiment ". Remarkably close agreement was found in the final bed-formations obtained with the 21-inch model when operated with (*a*) Leighton Buzzard sand (0·036 inch) and (*b*) Aylesford sand (0·010 inch). This agreement is vividly demonstrated in Fig. 33.

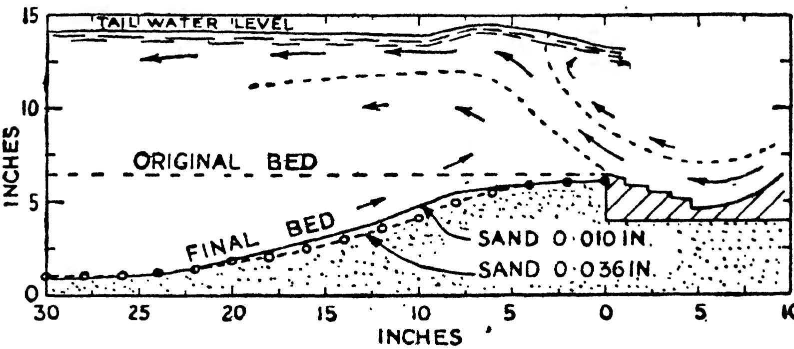


FIG. 33. Comparison between Two Bed-Materials.

Head over Weir-Crest 3·68 in.
Discharge 0·645 cusec. per ft. of Crest.
Duration of Test 6 hours.

Comparing the two models, it was found (when using the same sand, namely Aylesford 0·010 inch diameter) that while the beds scoured at different *rates* in the two models, the final configurations were remarkably similar (see Fig. 34). In the opinion of the experimenters, however, " the 4·2 inch model was almost too small for exploratory work. The stresses * on it were insufficient to move sands either smaller or larger than that used, and it was only the one size that would move freely ".

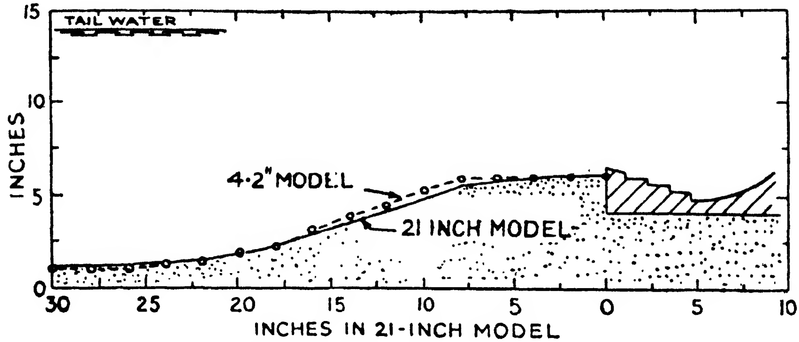


FIG. 34. Comparison between Two Models, One Five Times the Height of the Other. (Burns and White.)
Head over Weir-Crest 0·175P,
where P denotes Height of Crest above Apron.
Tailwater Depth above Sill = 0·36P.

Concerning the important practical question of the best duration of experiment, Dr. Burns and Dr. White found that it was advisable to adopt a compromise between (i) the desirability not to prolong the tests unnecessarily so that a large number of alternative spillway-toes could be examined, and (ii) the requirement of attaining a final bed-formation. As experiment showed that by the end of half an hour the bed-profile over a region of some 8 inches downstream of the toe had become stable, it was decided to adopt a standard run of half an hour on each design of toe.

For details of the large number of fascinating experiments made in the course of this investigation on many designs of protective toe, the reader is recommended to consult the original paper of Burns and White, but it may be stated briefly here that a simple sloping sill as illustrated in Fig. 35 behaved well over a wide range of flows and depths, although the absolute optimum value of θ was found to depend upon the tailwater-level. Similar experiments with an entirely different shape of weir have since been made by the Author.†

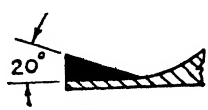


FIG. 35.

* The term " stresses " here signifies the hydraulic drag of water over the bed.

† Allen, in correspondence about the paper of Burns and White, *Journ. Inst. C.E.*, Oct. 1939, p. 251.

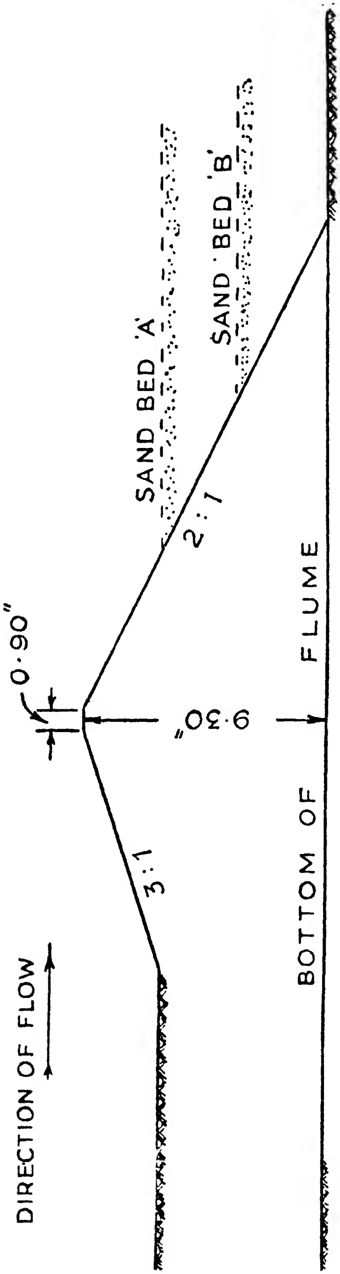


FIG. 36.

This weir is shown in Fig. 36; it was tested in a glass-sided flume 5·94 inches wide and 18 feet long. The bed of the channel upstream of the weir was fixed at a depth of 3·0 inches below the crest; experiments were performed with two initial levels *A* and *B* of the downstream bed, namely, 3·0 and 6·0 inches respectively below the weir-crest. For the mobile downstream bed, the material used was a Leighton Buzzard sand similar to that tried by Burns and White. A control-notch at the downstream end of the flume enabled tests to be made with various tailwater-levels, but in beginning any experiment water was first admitted to the *lower* part of the flume up to the crest of the weir in order to prevent the artificial scour otherwise produced by the first rush of water from above the weir. Runs of 30 minutes were made, during which the bed near the toe of the weir attained approximate stability. The result of a typical experiment *without* any protective device is shown in Fig. 37; the scoured material was deposited downstream where the bed, some 56 inches from the weir-crest, rose as much as 2 inches. A comparable performance with a protective apron and sill is demonstrated in Fig. 38, while the results of a test starting with a lower initial downstream bed are summarized in Fig. 39. The general conclusion reached was that an angle θ = about 27 degrees behaved well over the range of depths and discharges explored; it is possible that a different length of apron would have favoured an angle more like the

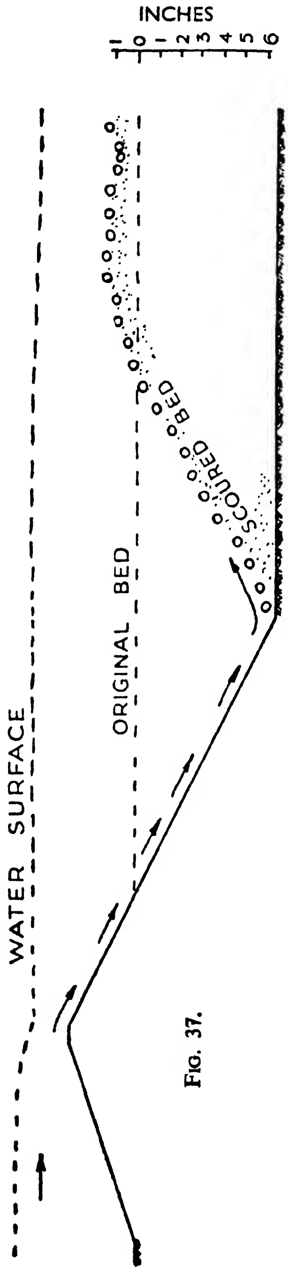

INCHES
-1 0 1 2 3 4 5 6
WATER SURFACE
ORIGINAL BED
SCOURED BED

Fig. 37.

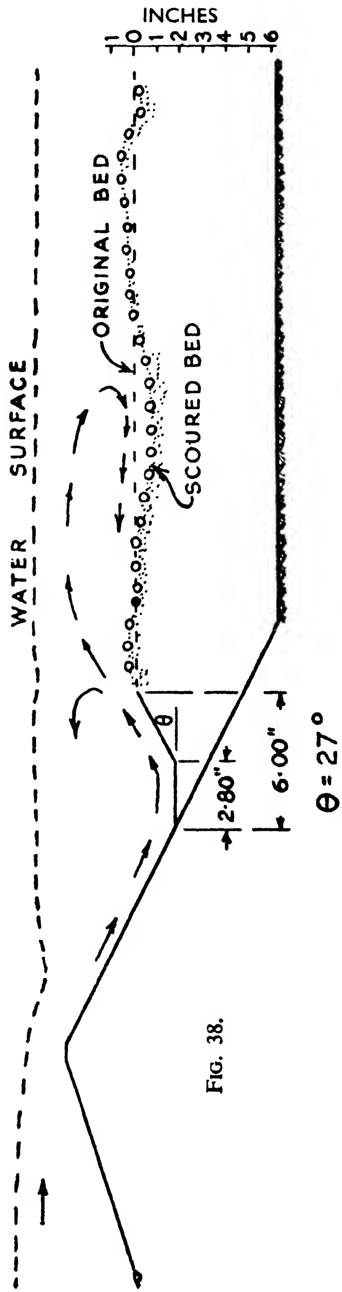

INCHES
-1 0 1 2 3 4 5 6
WATER SURFACE
ORIGINAL BED
SCOURED BED
θ
2·80"
6·00"
θ = 27°

Fig. 38.

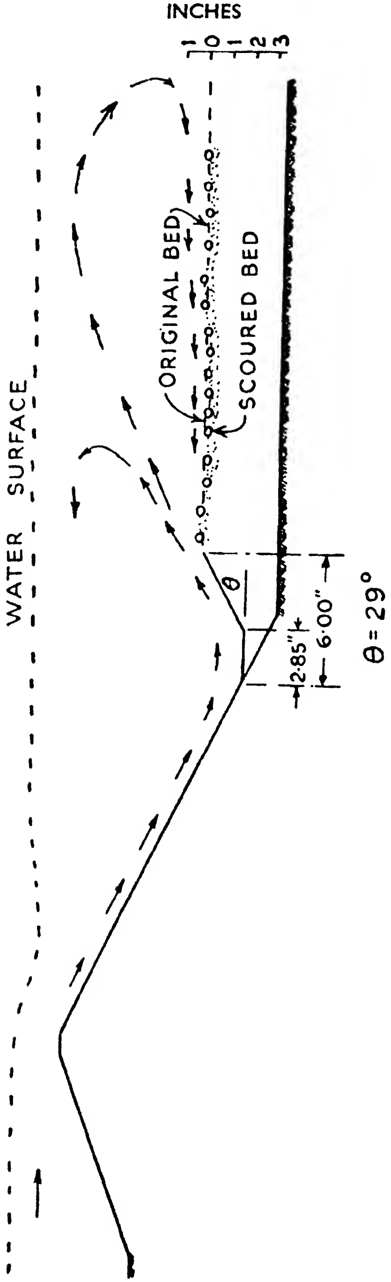

INCHES
1 0 1 2 3
WATER SURFACE
ORIGINAL BED
SCOURED BED
θ
6·00"
2·85"
θ = 29°

FIG. 39.

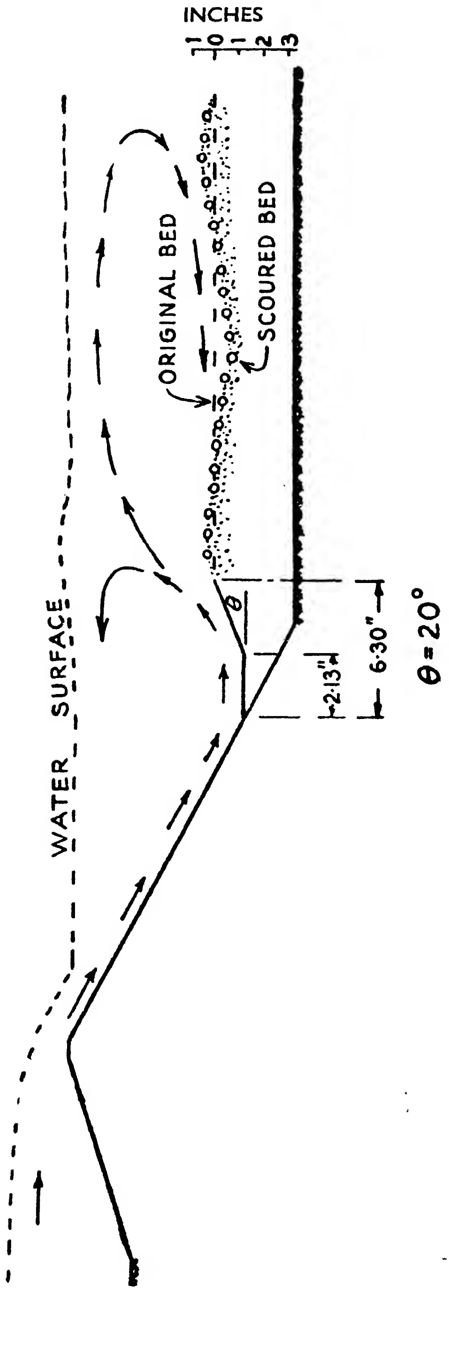

INCHES
1
0
1
2
3
ORIGINAL BED
SCOURED BED
WATER SURFACE
θ
2·13"
6·30"
θ = 20°

FIG. 40.

20 degrees found to give excellent results by Burns and White for their entirely different shape of weir. A pretty phenomenon was observed with the toe shown in Fig. 40: with a head of 2·39 inches above the weir crest and a downstream or tailwater level of 0·40 inch above the crest, material was for a time scoured from the bed and carried by the reverse eddy upstream to the sill, where it formed a bank which reached such a height that the régime of the flow was altered and a *lee eddy* produced, whereupon the bank moved downstream in the form of a travelling ripple, leaving the end of the sill exposed. After a short time, the original eddy-system re-appeared and again transported material back to the sill, when the whole process was repeated.

A considerable amount of valuable work of this kind has been done under the direction of Dr. Rehbock in the River Hydraulics Laboratory at Karlsruhe.* To quote but one of many examples, there was the investigation in 1922–24 of the prevention of scour below the flood discharge installation for the Friedland power plant on the Alle (East Prussia), for which experiments were made on two partial models (scales 1 : 10 and 1 : 40) and on two complete models (scales 1 : 40 and 1 : 200), the first three of which were provided with "a movable bed of varying size of grain" below the overfall. It is stated that good agreement of all phenomena was obtained on the different scales, that the tests led to the discovery of the "dentated sill" (see Fig. 41), and that the construc-

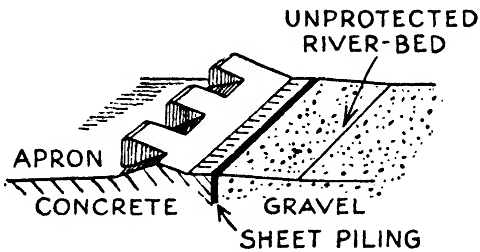


FIG. 41. Rehbock "Dentated" or Notched Sill.

tion of such a dentated sill in the actual plant led to "completely satisfactory behaviour in the great floods of the spring of 1924". In this type of protective device, the principle is to lift the stream of water away from the bed and, by means of the currents flowing *between* the teeth to prevent the lifted stream from dropping *suddenly* back to the bed.

Mr. W. E. Doran † has described a striking example of model investiga-

* See *Hydraulic Laboratory Practice*, ed. Freeman, Am. Soc. Mech. Engs., 1929, p. 111.

† In correspondence on the paper by Burns and White, *Journ. Inst. C.E.*, Oct. 1939, p. 258.

tion in connection with a sluice of 18 feet span, 6 feet high, on one of the Fenland rivers. This structure had been provided with a straight, flat apron 8 ft. 6 in. long on its discharge side; a scour-hole 9 feet deep had formed below it, so that the apron was severely undermined. A model was accordingly built, and its bed was moulded to the conditions obtained when the structure was built. After running under water-levels equivalent to those recorded in the history of the structure, the model developed scour in extremely close agreement with that experienced in nature (see Figs. 42*a* and *b*). Experiments then made with the object of discovering

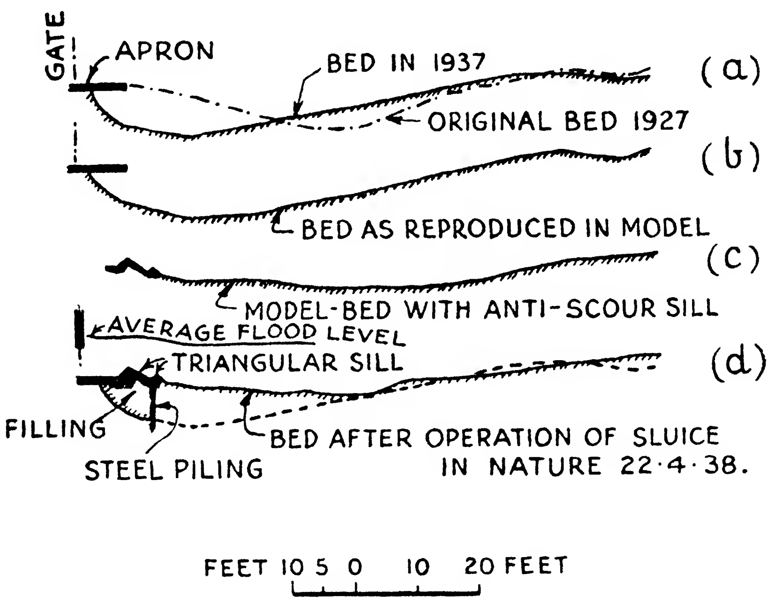


FIG. 42*a*, *b*, *c*, *d*.

remedial works led to the design of a reinforced concrete triangular sill, together with " streamline wing-walls " on either side. Construction according to these findings resulted in a complete vindication of the model, as demonstrated by Figs. 42*c* and *d*. Mr. Doran proceeds to remark that " towards the end of 1938 some workmen who wished to carry out some minor bank repairs opened the sluice, and left it open at levels at which it was not intended to be used and at which model-experiments had shown that scour would take place. That action resulted in considerable scour in quite a short time. . . . It was decided to wait until higher levels were available, and then to operate the gate to see whether or not the sill would pull back the bed-material. The gate was left full open during the floods which occurred at the end of January, and when surveyed on the 9th February, 1939, the scour-hole was found to be completely filled in and the bed-material piled up to the top of the primary sill. . . . That result showed that it was not necessary to have filled in the hole by dumping from barges, as the sill would have done that if left to itself."

Mention of the importance of *wing-walls* in the example just recounted will serve as a reminder that before coming to a *final* decision in such matters it will generally be necessary to experiment with a model which embraces the whole of the layout (not just a " representative " portion of it), and which includes a stretch of river both upstream and downstream of the proposed structure sufficient in length to ensure (as indicated by the observed behaviour of the model) that any important effects not confined to the immediate vicinity have been discovered. A feature especially worthy of note is the possibility of eddies being created which have a *vertical* axis, in addition to those with a horizontal one. These eddies or vortices rotating about a *vertical* axis are frequently caused by lack of symmetry in the constructional works; they may be responsible for scour of the same importance as, or even greater importance than, the other kind of vortex.

Any discussion of the model investigation of spillway-scour would be quite incomplete without reference to the admirable work of Butcher and Atkinson,* who have made a very comprehensive study of the structures shown in Figs. 43*a* and 43*b*, by means of scale models ranging from 1 : 100

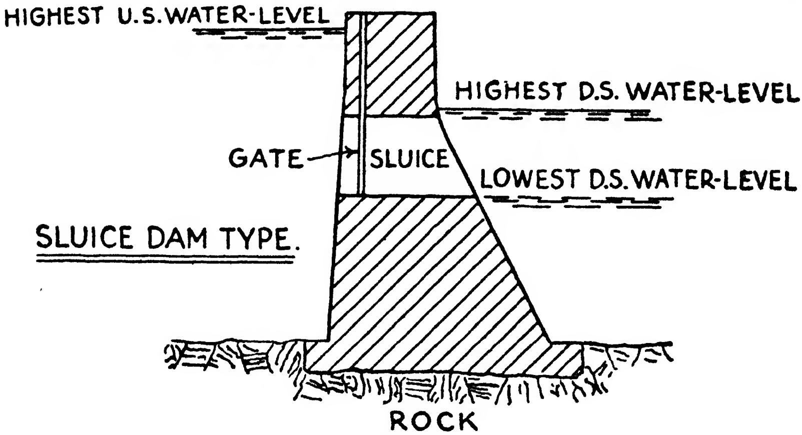


FIG. 43*a*.

* " The Causes and Prevention of Bed Erosion, with Special Reference to the Protection of Structures Controlling Rivers and Canals ", Butcher and Atkinson, *Proc. Inst. C.E.*, vol. 235, 1934, p. 175.

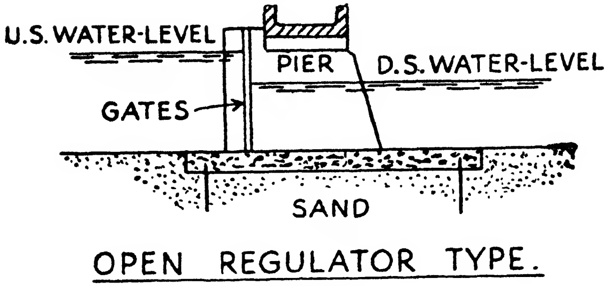


FIG. 43*b*.

to 16·7 : 100, comprising one, two, or more sluices and piers according to the scale. The order of similarity of flow observed in these models is typified by Fig. 44. Again, it is recorded that " a chance observation of

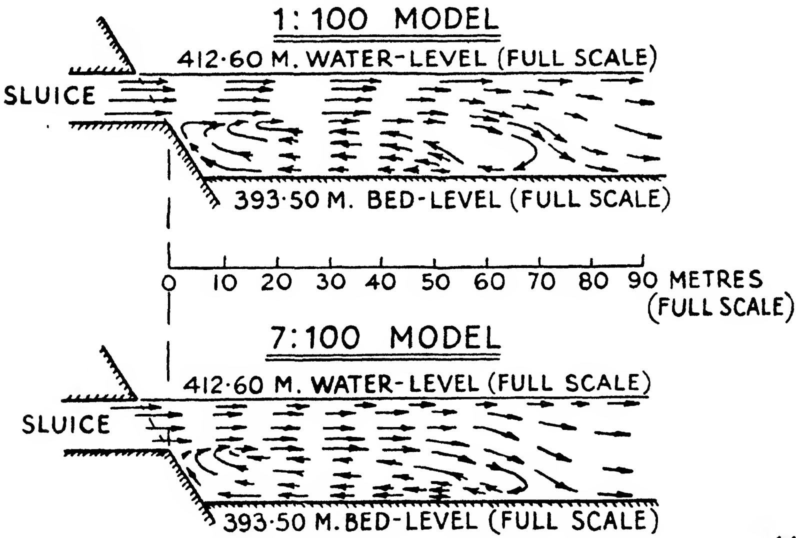


FIG. 44. Sennar Dam Models. Position of " Positive Vortex " as observed in two models.

the conditions of flow at the Delta barrage during flood established that there was an obvious reversal in the direction of the surface flow at some distance downstream ". By means of a current-direction indicator (Fig. 45) this point was located at 25·4 metres from the gates. " The levels and gate-positions were ascertained and reproduced on the 2 : 100 model and the currents were investigated. The results were, however, entirely different from those obtained on the barrage: so unlike were they that some error of observation was suspected, and re-examination of the model showed that the gates had been put in the wrong grooves. . . . The model gate-positions were corrected, and the flow immediately assumed the same form as that originally observed on the barrage, with a reversal of current at the surface 0·51 metre from the gates, corresponding to 25·5 metres on the full scale. Many other demonstrations of current similarity could be cited."

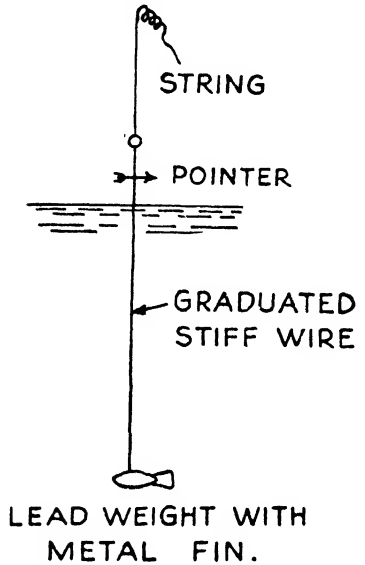


FIG. 45. Current Direction Indicator.

Very striking direct comparisons of erosive effect were also obtained, both between models of different scales and between model and prototype. Two of these are well worth quoting: the first is that of the Esna barrage, which in 1913 was subjected to severe scouring action just beyond its masonry floor, the resulting debris being piled in a bar 2 or 3 metres high and about 20 metres beyond the centre of the scour. The conditions of flow were reproduced in a 1:50 scale model which had a horizontal *sand* bed as a continuation of the masonry floor. It should be noted that the bed material in the prototype was composed of " thrown stone ". Despite the difference in materials, the resulting behaviour of the model appeared to be in close harmony with that observed in nature, as is indicated in Fig. 46. (No actual sections of the

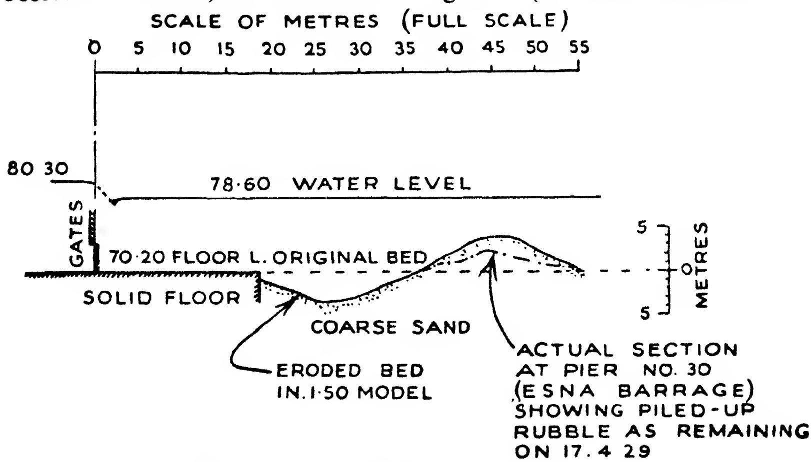
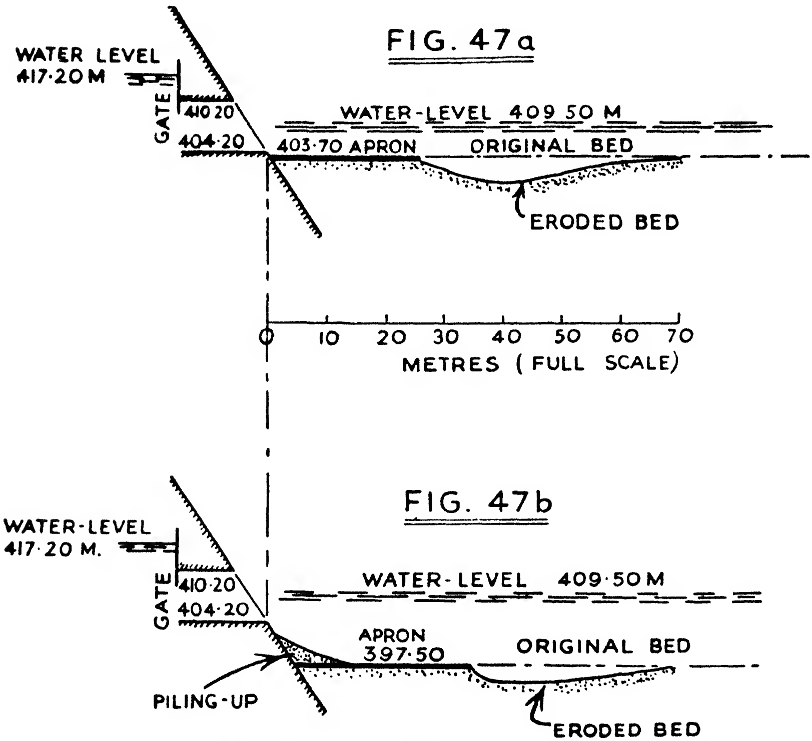


FIG. 46. Esna Barrage, Typical Erosion.

Esna 1913 scour were in existence; the comparison is therefore made with measurements of the bar as still remaining in 1929).

The second comparison is concerned with the Sennar Dam, in which two types of horizontal apron were built: one was 30 metres long and situated 6·70 metres below the sluice-floor level; the other was 25 metres long and 0·5 metre below the sluice floor. Experiments with a 1:100 model revealed different kinds of movement of the sand bed beyond the apron: with the lower apron, there was a certain amount of scour at the end of the masonry, and a bank of sand was deposited on the apron itself against the face of the dam. With the higher apron, however, severe scour took place beyond the apron, which itself was entirely clean of sand. These effects are shown in Figs. 47*a* and 47*b*.



Sennar Dam : Erosive tendency in 1 : 100 Model with high- and low-level aprons.

Some time later, the prototype at Sennar came into action under conditions approximately equivalent to those of the model tests: the results as actually surveyed were virtually identical with the model indications in both shape and magnitude: and this despite the different bed material.

Indeed, the material accumulated on the *lower* apron against the face of the dam contained " old rails, pipes, parts of decauville trucks, and other contractor's material which had been used during the construction of the aprons, showing conclusively that the accumulation was from material lying downstream of the dam and brought back by the positive vortex ". This " positive vortex " had previously been noted in the model experiments.

Messrs. Butcher and Atkinson also made a study of the question as to whether a so-called permanent floor or apron of a particular design would be adequate to resist the erosive forces brought into play. For example, would a bed of large blocks or of thrown stone be satisfactory without recourse to *continuous masonry*?

The fundamental argument advanced is that the weight in water, and therefore the inertia, of a cubical block of side L and specific gravity S, is equal to its effective density multiplied by its volume, and is consequently proportional to $(S-1)L^3$. If the force of water tending to shift the block is taken as proportional to the area of the face exposed to the current, that is, to L^2, it follows that the resistance of the block is measured by the quantity $(S-1)L$, which Messrs. Butcher and Atkinson call the " immovability ", denoted by I. The notation I_n is then used to signify a block for which $(S-1)L=n$, when L is measured in some *stated* units, say cm. Thus a block having $S=2{\cdot}0$ and $L=1$ may be described as I_1, as also may be one having $S=3{\cdot}0$ and $L=0{\cdot}5$. Different frictional values between the block and the floor are ignored in this theory, or at any rate considered to be of small account, on the argument that " in general the blocks are either completely at rest or are in a continuous state of agitation, due to the turbulence of the water, in neither of which states is true sliding friction a consideration ".

For experimental purposes, cubical blocks were made out of aluminium, drilled and loaded with cork or lead as demanded by various specific gravities ranging from 1·5 to 4·0. Each unit of this series of blocks $I_{0{\cdot}5}$, $I_{1{\cdot}0}$, etc., was lowered on to the apron and its behaviour observed. For example, a series of cubes of 7 cm. side was tested in a 7 : 100 scale model of a sluice-dam of the type shown in Fig. 43*a*. To avoid movement at any point, the full-scale value of I required was found to be 200 (or more); this appears to mean that a 7 cm. block of specific gravity 3·0 remained unmoved in the model, and the implication of the theory outlined above is therefore that a block 100 cm. in side and of specific gravity 3·0, or one of side 200 cm. and specific gravity 2·0, would be unmoved in the prototype under corresponding conditions of flow.* The experiments also

* In order to be justified in applying the model results to the prototype in this fashion, it is necessary to assume that the force exerted by the water on the cube is *not only* proportional to the area of the exposed face but also to the square of the velocity.

showed that, judged by the degree of movement of the cubes (described as "only just moves", "moves by sliding", and "moves freely"), the transporting force of the stream was greatest at a distance equivalent to about 35 metres from the sluice. When the experiments were repeated on a 1 : 100 scale, with corresponding scale blocks, the same conclusions were reached.

Further, Messrs. Butcher and Atkinson describe experiments (on a 1 : 50 model) to compare the size of blocks required to simulate certain effects in the Esna barrage as recorded during 1914. "Before that date, scour-holes immediately downstream of the floor had been filled with rectangular concrete blocks laid in courses but unfitted. During the following season, some, but not all, of these blocks were found to have been displaced for short distances, and in certain cases they had been piled one on the other, a state of affairs which showed that their "immovability" was nearly, but not quite, enough to resist the erosive forces at work. The most severe conditions of level and flow were duplicated on the 2 : 100 model, and the scour-hole which developed at the end of the floor was then tested with the standard blocks. Blocks $I_{0\cdot5}$ and $I_{1\cdot0}$ were quickly removed and thrown downstream, but $I_{1\cdot5}$ were practically stable." Apparently, the actual dimensions of the Esna blocks were $1{\cdot}5 \times 1{\cdot}0 \times 0{\cdot}70$ metre and the weight was 2·4 tons. The volume of each block was therefore 37·0 cubic feet, the density 145 lb. per cubic foot, and the specific gravity 2·33. A cube of the same volume would have a side of length 3·33 feet or 1·02 metre, and its "immovability factor" would be $1{\cdot}33 \times 1{\cdot}02$ or 1·36. On this argument, "the concordance between model results and the observed facts was therefore almost perfect".

Having satisfied themselves in this manner about the practicability of model tests on individual cubical blocks, Messrs. Butcher and Atkinson proceeded to predict, "allowing a reasonable factor of safety",* the size

Then, if the model scale of linear dimensions is $1 : x$ and if the velocity scale is $1 : \sqrt{x}$, the ratio of (the force tending to move the block) : (the weight of block) is represented by $[v^2L^2 : (S-1)L^3]$ or $[v^2 : (S-1)L]$, where v is the model velocity. In the prototype the corresponding ratio will be $[(v\sqrt{x})^2(Lx)^2 : (S-1)(Lx)^3]$ or $[v^2 : (S-1)L]$, which is the same as in the model.

It then follows that if a cube of side L and specific gravity S is just on the point of shifting in a model of linear scale $1 : x$ and velocity scale $1 : \sqrt{x}$, a cube of side Lx and specific gravity S will also be just on the point of moving in the full-size stream, provided that the bed-frictional coefficient is the same and that flow-pattern similarity exists.

The subject of transportation of blocks and other bed-materials will be discussed in greater detail in Chapter XI.

* Since the "immovability" of a cubical block is by the theory outlined in p. 128 proportional to the cube-root of the volume, it follows that to raise the factor of safety from 1 to 2 in the case of such a block *made of a given material* the volume must be increased eightfold. For example, a one-ton block would become eight tons in weight.

of unbonded blocks necessary for the apron of a certain dam. This they did by making such an apron, resting on fine gravel, in a model and painting straight lines across each course so as to facilitate the detection of any movement. "The model was then tested under the worst conditions likely to apply, and no displacement of the blocks took place except a slightly irregular settlement. The interstices also filled gradually with sand brought from downstream by a positive vortex. When the downstream level was lowered so that a negative or destructive vortex formed, the blocks were immediately displaced and hurled pell-mell downstream."

When the implications of this valuable investigation are considered, it is found that there is an apparent anomaly: in such models, a bed of fine sand is scoured in a fashion which agrees remarkably well with the scour observed in nature, even when the bed of the prototype consists of debris which bears no apparent scale relationship to the sand of the model. On the other hand, a definite scale relationship is essential when the movement of blocks is to be determined. Professor Gibson has suggested * that the explanation may be found "in the fact that whereas in the impact on a large body the forces involved depend only on the mean velocity of the stream, and vary as the square of that velocity, the forces acting to produce motion of the particles of a sand bed depend on the activity of the erosive layer immediately overlying the bed—the so-called boundary-layer—which is affected by factors other than the mean velocity of the stream". It is also possible that there exists a different law of behaviour for individual prismatic objects or for such objects laid in regular fashion compared either with the same blocks arranged in random manner or with a mass of pieces having irregular shapes. This aspect of the problem will be discussed in Chapter XI in the light of more recent experimental evidence.

To return to the subject of scour in the sand bed, however, it is worthy of note at this stage that work under the direction of Dr. Ing. Theodor Rehbock at Karlsruhe indicated † that the "stable final form of the model-bed was, within wide limits, much the same with different sizes of bed-material". Thus, in experiments on the scour produced by bridge-piers, very similar final results were obtained with three materials, one of 0·06 inch diameter and more, a second of 0·04 inch, and a third of 0·02 inch; the excavations were rather deeper in fact with a coarser than with a finer sand, but the times taken to attain the final excavation and

* A. H. Gibson, "Tidal and River Models", Vernon-Harcourt Lecture, 1935–36, *Journ. Inst. C.E.*, Supplement to No. 8 (1935–36), Oct. 1936, p. 699.

† Rehbock: in discussion of paper by Burns and White, *Journ. Inst. C.E.*, Oct. 1939, p. 260; also in discussion of paper by Butcher and Atkinson, *Proc. Inst. C.E.*, vol. 235 (1934), p. 256; also *Hydraulic Laboratory Practice*, ed. Freeman (Am. Soc. Mech. Eng., 1929), pp. 201–203, p. 228.

maximum depth of scour were appreciably longer with the coarser material. In order to facilitate the trial of various schemes, a material which reaches its ultimate state most rapidly is to be preferred; in Dr. Rehbock's laboratory it was discovered that the duration of tests on the Ryburg-Schworstadt weir (Upper Rhine) could be reduced from 24 hours to 40 minutes by the artifice of replacing fine gravel (0·35 inch diameter) by sand (0·03 inch diameter). Dr. Rehbock states, however, that the similarity of the scour-holes would be lost "if the mean diameter of the particles forming the model-bed were to become sensibly smaller than 0·5 millimetre" (0·02 inch).

The experiments on the scouring effect of bridge piers, to which reference has just been made, were conducted in a channel 1·64 ft. (19·7 inches) wide containing two piers 7·88 inches long and 1·57 inch wide, which were symmetrically disposed about the centre line of the channel. The rate of flow was 0·354 cusecs and the depth of water 0·164 ft. (1·97 inches), so that the mean velocity in the full width of the flume must have been 1·31 ft. per sec. over the initially level bed.

In the experiments of Burns and White comparing sands of diameter 0·010 and 0·036 inch (see Fig. 33), the *mean* tailwater velocity appears to have been about 1·0 ft. per sec. with the original undisturbed bed, while in their comparison between two models, one five times the height of the other (see Fig. 34), using a bed material of 0·012 inch sand, the initial *mean* tailwater velocities were of the order of 1·0 and 0·45 ft. per sec. respectively in the large and small models.

These data are presented as a guide to estimating the sort of conditions for which the bed materials described by Rehbock and by Burns and White are likely to be reliable in unidirectional-flow models of this general type.

Another important practical point is that the rate of scour is far more rapid during the earlier than during the later periods of the test. In fact, it will often be found that three-quarters or more of the ultimate scour takes place in the first third of the experiment: consequently, it may be possible frequently to discontinue an experiment after a comparatively short time owing to undesirable features of the particular design having by then made themselves quite apparent. It is also of interest to note that a completely *stable* final condition may not exist; instead there may be a fluctuation about some mean state.

In the foregoing discussion, an attempt has been made to provide a reasonably safe guide as to the choice of scale and bed material for models dealing with scour created by sluice-gates, spillways, barrages, bridge piers and the like. The mechanism of the phenomena involved is, however, far from being completely understood. For example, when it is stated that the bed formation is sensibly independent of the scale and of the material *within wide limits*, what precisely are these limits? Clearly there must be an upper limit of grain-size for use in a model of given scale, because one may readily visualize a material too coarse to move at all. But evidence is also available that if the material is too fine, then, owing to its adhesion or some other property, it behaves quite differently from coarser grains.

In an attempt to provide a practical solution to these problems, Professor E. Scimemi of the University of Padova has developed a technique * which consists of experimenting successively with gravel beds (specific gravity about 2·7) of different grain-sizes, starting, say, with a relatively coarse one and gradually working through a range of smaller diameters. The range of sizes explored by him has been from 30 millimetres down to 1 mm.; with the models as used he has found that below this smallest size there became apparent actions of agglomeration or of movement in mass which differed in character entirely from the general behaviour of the coarser materials. Professor Scimemi's method then consists of estimating the scour which will happen in the prototype from the maximum limit of scour indicated by any of the workable bed materials in the model. A few examples will serve to demonstrate the technique.

First, there is the case of the dam shown in Fig. 48, from which the

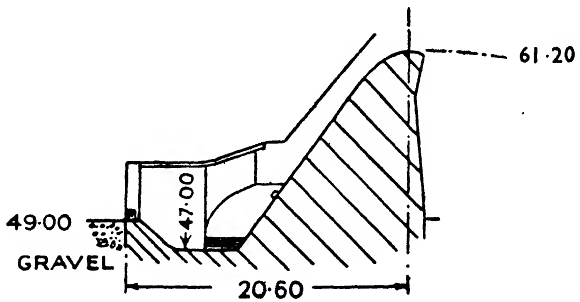


FIG. 48. Dimensions in Metres.

results indicated in Figs. 49*a* and 49*b* were obtained when using in a 1 : 20 model three bed materials consisting of gravel 10–20 mm., 6·5–10 mm. and 3·5–6·5 mm. respectively. By extrapolation of the curve shown in Fig. 49*b*, Professor Scimemi estimates that the maximum scour in the prototype will not exceed 2·20 metres; he proceeds to justify this method of interpretation by applying it to models of actual structures in which the scour has been determined.

* Scimemi, E., *Relation entre les affouillements obtenus sur l'original et sur modèles* (" The Relation between Scour in the Prototype and in the Model "), International Association for Hydraulic Structures Research, Liége, 1939 (published 1940), p. 91.

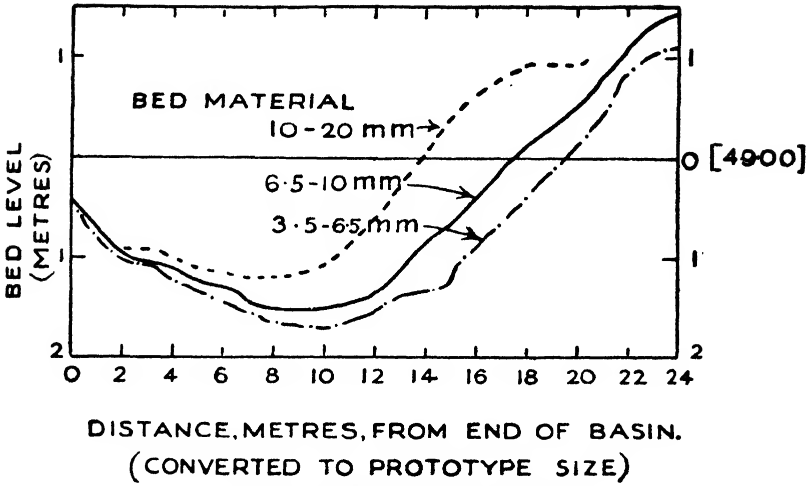


FIG. 49*a*. Results from 1 : 20 Model.

For example, he has constructed a 1 : 50 model to embrace three openings of the Conowingo Dam on the Susquehanna, U.S.A., for which the results of an American model investigation were already available and had

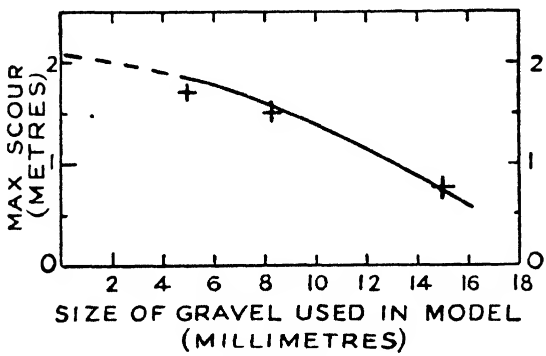


FIG. 49*b*.

been found to yield results resembling the occurrences in the actual structure.* The various bays of this spillway were 11·6 metres wide, the piers 2·14 metres thick and 17·70 metres long. The discharge through each opening was 452 cm. per sec. (16,000 cusecs), and the tailwater height 5·80 metres above the downstream apron. Scimemi's results are

* L. N. Reeve, "Erosion below Conowingo Dam proves Value of Model Test", *Eng. News Record*, 1932, I, p. 127.

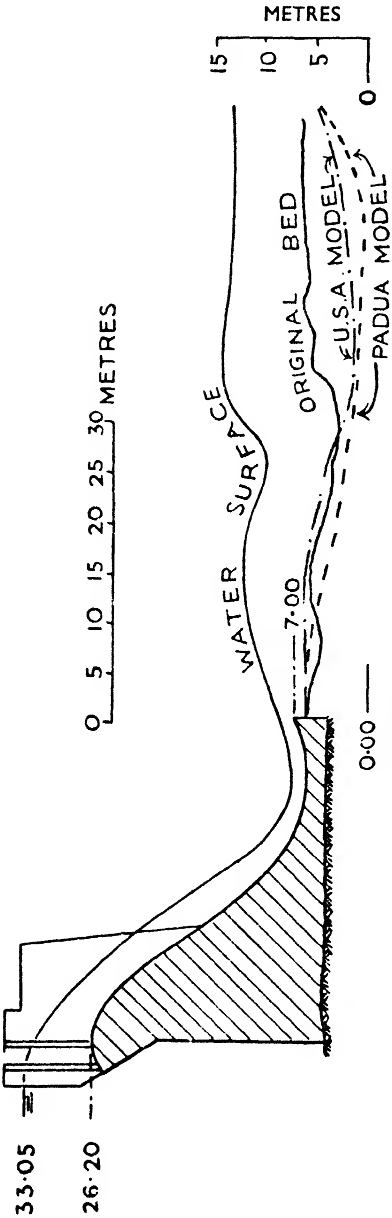


FIG. 50*a*. Conowingo Barrage.

summarized in Figs. 50*a* and 50*b*; his estimate of maximum depth of scour is 6·20 metres compared with about 6 metres obtained in the prototype. Fig. 50*b* at once raises an interesting speculation; when various experimenters have reported to the effect that they have observed no significant differences between the results obtained with different sizes of material, is it that, with the model-scales which they happen to have adopted, their range of materials has fallen along the relatively flat portion of a curve such as that shown in Fig. 50*b*?

An instructive demonstration of the influence of scale is provided by Scimemi's study of the Augst-Wyhlen Barrage on the Rhine. His results for two model-scales, 1 : 100 and 1 : 50, are shown in Fig. 51, from which he concludes that the 1 : 100 scale was too small for practical purposes but nevertheless showed almost exactly the same *maximum* scour as did the 1 : 50 or the prototype itself. The general tendency appears to be for the scour to become more and more nearly independent of the diameter of the bed material as the model size increases, or perhaps it is more correct to say as the *absolute* size of the structure and the *absolute* magnitude of the hydraulic forces in it increase. This argument is consistent with another extremely important feature of the investi-

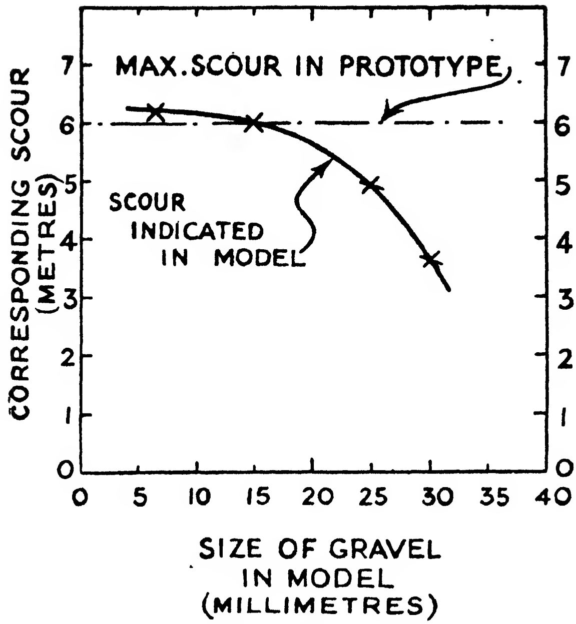


FIG. 50*b*.

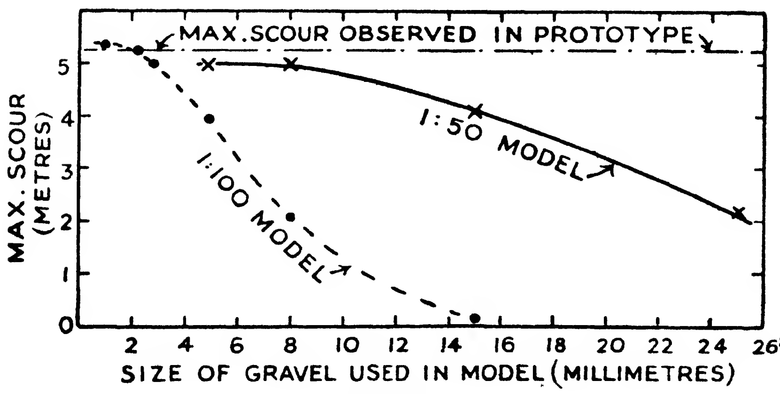


FIG. 51.

gations of Scimemi, Butcher and Atkinson, Rehbock and others, namely that the bed materials *in the prototype structures* which they have studied varied between very wide limits: from the large boulders, friable rock and heavy debris of the Sennar Dam to the sand (0–2 millimetres diameter) at the Barrage de la Grave on the Meuse.

So far, no specific mention has been made of the influence of density as distinct from grain-size of the bed material. An investigation has been made at Manchester by Mr. C. H. Hsia,* however, on the scour at the foot of the weir shown in Fig. 52. This weir was placed in a glass-sided

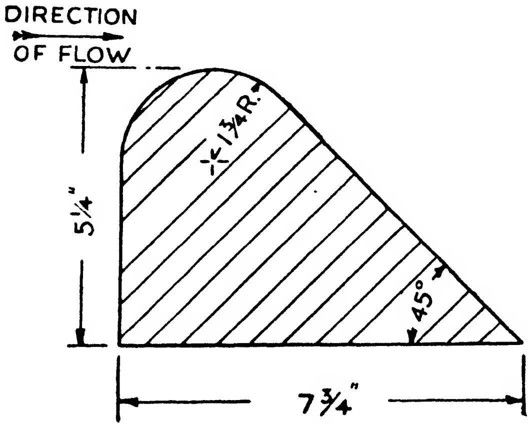


FIG. 52.

flume 8 inches wide; the head was fixed at 0·30 inch and the discharge was 0·00734 cusec. The bed material was initially levelled so that its surface was 1·6 inches below the weir-crest; this bed was retained at the downstream end of the flume by a piece of wood of the same depth as the bed. No other device was employed to control the height of the tailwater.

The materials tested under these conditions were as follows:

TABLE XIV

Sample	*Specific gravity, σ'*	*Mean dia., d, observed under microscope*	*Statical angle of repose under water*
Sand *A* - - -	2·62	0·0237"	30·4°
„ *B* - - -	2·63	·0157"	30·4°
„ *C* - - -	2·63	·0093"	30·1°
Powdered emery -	3·89	·0097"	36·9°
„ pumice -	1·96	·0162"	32·2°

* C. H. Hsia, M.Sc. thesis, Manchester University, April 1940.

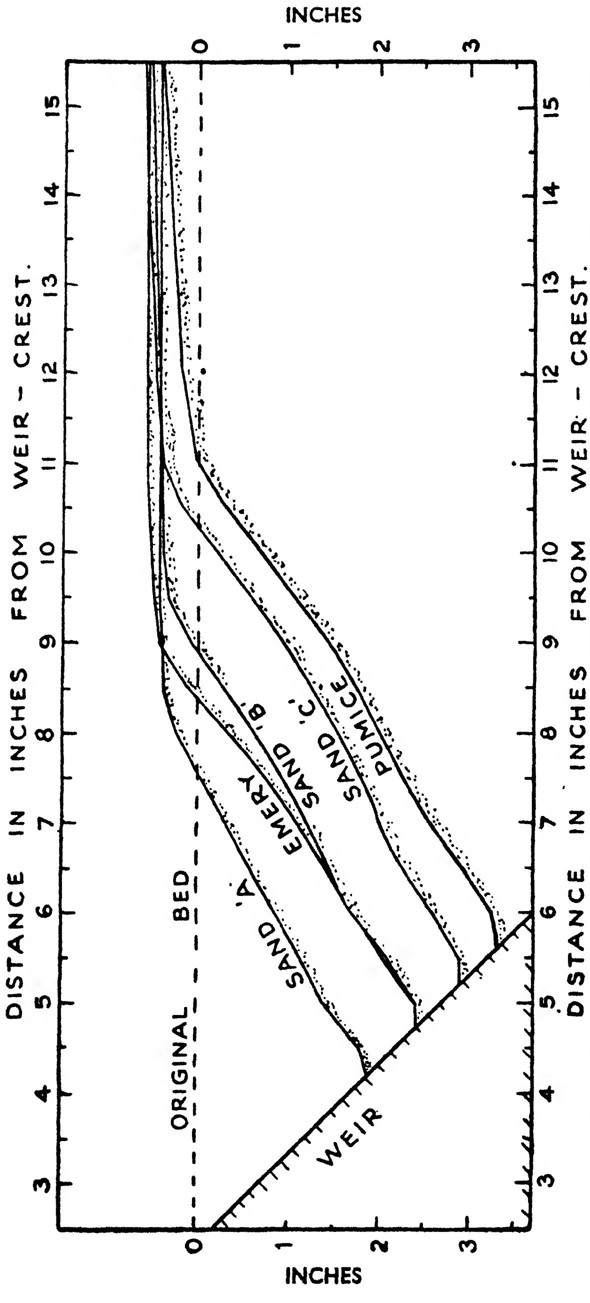


FIG. 53. Bed-Formations with various Materials. (Head of Water over Crest of Weir 0·30 Inch ; Discharge 0·00734 Cusecs).

The ultimate bed formations recorded by Mr. Hsia are shown in Fig. 53, from which it will be seen that the scour in the *immediate* vicinity of the weir differed appreciably in the five materials. Nevertheless, the final maximum depth of scour seems to bear a relationship to the quantity $(\sigma' - 1)^{0 \cdot 27} d^{0 \cdot 27}$ which, in the absence of any data concerning the shape and uniformity of the grains is an approximation to a criterion that will be mentioned in another connection in Chapter VIII. Thus, Mr. Hsia's results lead to the following:

TABLE XV

Material	$(\sigma' - 1)^{0 \cdot 27} d^{0 \cdot 27}$ [(*inches*)$^{0 \cdot 27}$]	*Max. depth of scour* (*inches*)
Sand *A*	0·42	1·9
Emery	·38	2·4
Sand *B*	·37	2·4
Pumice	·33	3·3
Sand *C*	·32	2·9

Incidentally, Mr. Hsia found that the scour induced in the pumice in approximately 5 minutes was of the same order as that obtained with sand *A* after 1,500 minutes; he further discovered that, with all the materials tried, about 70 per cent. of the final scour took place in the first 5 per cent. of the full time of the test.

We proceed in the next chapter to deal with more elaborate examples of non-tidal river models.

CHAPTER IV

MODELS OF NON-TIDAL RIVERS

IT may not be inappropriate to take as our first examples one or two of the model-investigations made at Dresden, where in 1898 Dr. Hubert Engels began operating the first permanent River Hydraulics Laboratory.

Experiments to a scale of 1 : 250 were made regarding the formation of deposits at the entrance to the Freudenauer Winter Harbour near Vienna, where 20,000 cubic yards of dredging was carried out annually.* In the beginning, float observations were made which, during a rise of water, gave the results shown in Fig. 54*a*. Next, experiments were performed

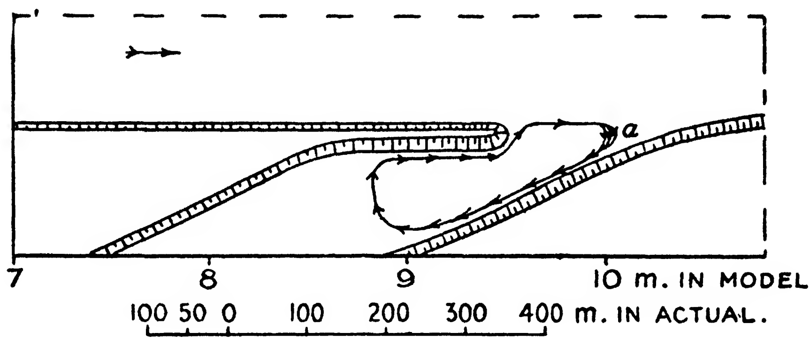


FIG. 54*a*. Current Phenomena at Rising Water in Model of Scale 1 : 250.
(*a*) Destructive cut where sand-bar might have been expected.
(From Fig. 89 of *Hydraulic Laboratory Practice*, ed. John R. Freeman.)

by introducing powdered soft coal, a photograph reproduced in Freeman's *Hydraulic Laboratory Practice* showing a very close similarity to the deposits found in the prototype (Fig. 54*b*); a remarkable feature was the separation of the two banks by a channel, the cause of which was attributed to the scouring action of the currents at *a* (Fig. 54*a*) as shown by the floats in the model. Incidentally, the larger deposit is described in the chart of the actual harbour as fine sand and mud, the smaller as gravel: yet the two banks in the model were reproduced by the one material (powdered coal). From the action of the model, it was concluded that the deposits were formed only during a rise of water in the

* *Hydraulic Laboratory Practice*, ed. Freeman, Am. Soc. Mech. Engs., 1929, p. 100.

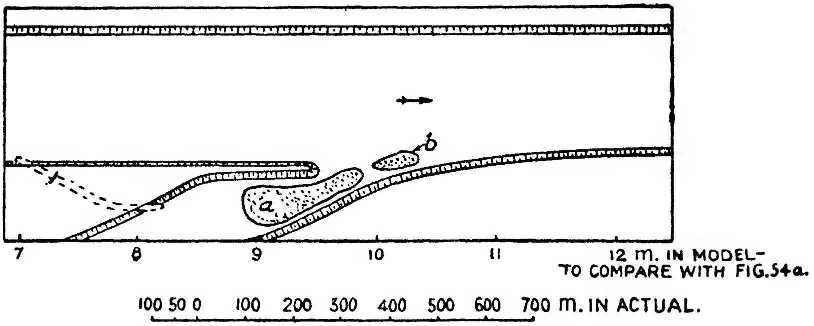


FIG. 54*b*. Entrance of Freudenhauer Winter Harbour.
Actual Formation of Deposits: (*a*) Fine Sand and Mud, (*b*) Gravel.
(From Fig. 88 of *Hydraulic Laboratory Practice*, ed. John R. Freeman.)

Danube, and accordingly a suggestion (apparently not acted upon because of its cost) was put forward for the construction of a scouring canal to create a counter-current in opposition to the sediment-carrying currents during a rise of the Danube.

Secondly, a reference may be made to Dr. Engels' laboratory work on the movement of material in river bends.* In this case, soft coal (1 to 2 mm. diameter) was introduced at various points in the model channels and showed results similar to those previously found through the agency of coloured sand, namely that " the moving sediment crosses from one bank to the other ".

The two examples just quoted may be said to represent a transition stage between the purely qualitative method of visual observation of currents and of sedimentary material introduced only locally in the model, and the more exacting method of experimenting on a completely mobile bed with a view to obtaining results of a more quantitative character.

Attention is invited next to a study of the Rhine made in Dr. Rehbock's River Hydraulics Laboratory at Karlsruhe in 1926–27.† In this investigation, a model was built in a reinforced concrete channel laid to a slope of 1:500, with a linear scale of 1:200, of a slightly curved reach of the Rhine some 3 miles in length and 900 ft. in width. This model would therefore be approximately 80 ft. long and 4 ft. 6 in. wide. It was operated under two rates of discharge, 0·312 and 0·063 cusecs respectively, which, according to a discharge scale of $200^{\frac{5}{2}}$, would represent some 177,000 and 35,000 cusecs in nature.

When a bed of river sand was used, intense ripple formation ensued

* *Loc. cit.*, p. 105.
† *Hydraulic Laboratory Practice*, p. 207, Expt. No. 8.

without the shifting bars characteristic of the river itself, and as a result of tests made in a smaller model having a horizontal scale of 1 : 1,000 and a vertical scale of 1 : 100 (thus containing a tenfold vertical distortion), the sand bed was replaced by " a lighter and somewhat coarser material, i.e. pumice sand ". This pumice, having a specific gravity of 1·7 and grains from 1 to 6 mm. (0·039 to 0·24 inch) in diameter, resulted in the production of slowly travelling bars instead of unnatural ripples, and details seem to have been reproduced which were in close agreement with features of the Rhine itself. In the course of the experiments, material washed out of the bed was recovered and re-introduced at the upstream end.

Two points are particularly noteworthy in this example: the use of powdered pumice as a bed material and the fact that preliminary experiments on a small (and distorted model) were of real assistance. The ease and speed of trying alternative materials would of course be greatly to the advantage of the smaller model.

Experiments have also been made at Karlsruhe concerning the influence of the height and length of a number of groins on the régime of the channel in a 5-kilometre stretch of the Rhine between Sondersheim and Mannheim.* The model scale was 1 : 200 and the bed material pumice (1 to 6 mm., specific gravity 1·7), and it is stated that " a surprisingly good agreement with the actual changes in the river bed of the Rhine has been obtained. The experiments made it possible to detect the unfavourable action caused by the lack of some groins on the left bank, resulting in strong attacks on the first ones just below. Through the subsequent construction of these groins, a much more favourable formation of the channel was produced, as further experiments have shown ". It is also remarked that " investigations with a distorted model could not be made on account of the wrong influence on the flow over the groins ". What this means is not precisely clear from the words used, but presumably † it refers to the well-known difficulty in all experiments with spur dykes or groynes, namely that such structures produce intense scour and steep slopes in the natural river; consequently, if the model has a vertical exaggeration or distortion of scale, the natural angle of repose of the bed material prohibits a quantitative reproduction of the scour. Nevertheless, other experiments made at Karlsruhe ‡ showed that with a distortion of 2·5 : 1 the *depth* of scour was correctly reproduced, although the area

* *Loc. cit.*, p. 219, Expt. No. 5.

† On reading over this passage again, it has occurred to me that more probably was meant the phenomenon which is discussed later (on p. 151) in connection with a model of the Ohio River. [J.A.]

‡ *Hydraulic Laboratory Practice*, p. 204, Expt. No. 4.

of scour was larger than in the undistorted model. In the United States Waterways Experiment Station it has been common practice to conduct model experiments on problems involving spur dykes with a distortion of the order of 8 : 1, and satisfactory results have been obtained. A further reference to these American investigations will be made later in this chapter; meanwhile it seems appropriate to call attention to a statement made by Mr. R. D. Gwyther.*

A number of model-tests were made, Mr. Gwyther states, in connection with a barrage on the river Tigris. The scales of the model were: horizontal, 1 : 250; vertical, 1 : 60. A vertical distortion of rather more than 4 : 1 was therefore present. We are told that the side slopes in the river-bed itself did not generally exceed 1 : 4 and that the sand used could stand under water at a slope of 1 : 1, so that the distortion as adopted was reasonable from this viewpoint, and moreover ensured that turbulent flow would exist when defining velocities as proportional to the square-root of the vertical scale (that is, a velocity of 1 ft. per second in nature was translated into $1/\sqrt{60}$ ft. per second in the model). This model was about 42 feet long and approximately one-half of its length was occupied by baffles and stilling-pools (presumably introduced to ensure a smooth entrance of water to the experimental part proper). The bed material consisted of river-sand, the banks of clay.

"A test was made", Mr. Gwyther reports, "of the model by moulding the river-bed to represent conditions previous to a flood of the river Tigris in December, 1936; the rise and fall of the river was then reproduced and the resulting changes in the river-bed measured. The agreement between the changes shown by the model and the recorded changes in the river-bed was remarkably close. During the experiment it was also confirmed that scour was caused by the increased velocity of flow which accompanied a rapid rise of the river, and it was estimated that that increase in velocity amounted to about 40 per cent. of the normal velocity of the river at the particular height. That figure was of special interest in cases when it was desired to calculate discharges on a rapidly-rising river."

It is presumed that in reproducing the rise and fall of the river, the time-scale as stated by Mr. Gwyther was observed. This time-scale is given as 1 : 32 and is evidently obtained by the argument that time is proportional to (distance divided by velocity), that is, the time-ratio in this model is $\frac{\sqrt{60}}{250}$ or 1/32·2.

* R. D. Gwyther, *Journ. Inst. C.E.*, March 1938, p. 237 (in discussing Professor Gibson's paper on "Bridge Piers and Other Obstructions").

An important concluding remark of Mr. Gwyther's is to the effect that this model was constructed at the site of the works and the results agreed very closely with actual occurrences in the river, even though the facilities and refinements of a laboratory were not available.

By this development in the discussion, we have reached a stage where it is evident that a non-tidal river model may reproduce natural conditions to the satisfaction of a practising engineer, even when the model is made with some distortion of vertical dimensions. But so far no positive guidance has emerged concerning the choice of scales or of bed-material.

In the first place, it is known that in almost all natural streams the motion of the water is turbulent. Thus, in Chapter I, p. 24, it has been shown that the critical Reynolds number $\frac{vm}{\nu}$, even in straight open channels with smooth and uniform rectangular section, is of the order of 1,400, and that consequently in a channel 100 ft. wide and 3 ft. deep the critical velocity is only about 0·07 inch per second at normal water temperatures. For similarity of the law of fluid resistance and to minimize scale-effects due to viscosity, it may therefore be taken as a safe rule that the Reynolds number in the model must not be less than 1,400, although in dealing with *tortuous* rivers it has been found (Chapter I, p. 31) that a model *may* be operated at much smaller values than 1,400 without appreciable error from this cause. A simplified form of this criterion for turbulence has been adopted by the United States Waterways Experiment Station, namely that the product of depth and velocity shall, for " safety ", not be less than 0·02 if the depth is measured in feet and the velocity in feet per second. They also recognize,* however, that in a model with bends and irregular section, a turbulent condition may be found at much lower velocities than those demanded by the criterion $vd \nless 0{\cdot}02$, which was based originally on experiments in a comparatively smooth flume.

But having by observance of these " safe " criteria ensured that the flow in the model shall be turbulent, it remains to be seen whether the change in water-level from one point to another will be reproduced to scale. The theoretical difficulties behind this have been outlined in Chapter I, pp. 28–31, and may with advantage be restated briefly:

Considering the case of a rough channel to which the Bazin expression may be applied, namely

$$C=\frac{157{\cdot}6}{1+\frac{N}{\sqrt{m}}}$$

in ft. sec. units, two cases arise according as to whether the vertical dimen-

* *Paper* 17 of the U.S. Waterways Expt. St., Vicksburg, Mississippi, 1935, pp. 53, 54

sions of the model are or are not distorted relative to the horizontal lengths.

Suppose the horizontal scale is $1:x$ and the vertical scale $1:y$, and let the suffix (1) refer to the prototype and the suffix (2) to the model.

Since, for turbulent flow in rough channels, a loss of head is proportional very nearly to v^2, it is natural to suppose the velocity scale to be given by

$$\frac{v_2}{v_1}=\frac{1}{\sqrt{y}},$$

and the corresponding scale of rates of discharge to be

$$\frac{q_2}{q_1}=\frac{1}{xy\sqrt{y}}=\frac{1}{xy^{\frac{3}{2}}}.$$

If this is so, and if the " frictional loss " of head in the model is to be $1/y$ of that in its prototype, then it is necessary that

$$\frac{1+\dfrac{N_2}{\sqrt{m_2}}}{1+\dfrac{N_1}{\sqrt{m_1}}}=\sqrt{\frac{m_2 x}{m_1}},$$

a condition which may often be satisfied if only by a trial-and-error process of adjusting the roughness of the model until it *is* satisfied.

But if there is *no* distortion of the vertical scale, m_2 will $=m_1/x$, and it follows that the requirement then becomes

$$\frac{N_2}{N_1}=\sqrt{\frac{1}{x}},$$

which is likely only to be practicable in relatively large models. Thus, if the prototype belongs to the class of very rough earthen channels, N_1 may $=3{\cdot}0$, say. Correspondingly, if the model has an undistorted scale of $1:200$ (that is, $x=200$), the roughness number N_2 for the model must be about $\frac{3{\cdot}0}{\sqrt{200}}$, or 0·21, which is a possible value for a channel of *fairly* smooth wood or cement. But if the scale is $1:2{,}000$, the model must have a surface defined by $N_2=0{\cdot}067$, a value only two-thirds of that appropriate to *smoothed* cement or *planed* wood. This difficulty might be overcome by deliberately setting the bed of the model to an exaggerated longitudinal slope (in other words, by tilting the model in the direction of its length), or by determining the velocity scale *experimentally*, that is, by measuring the discharge in the model which is found to produce the required water-surface slope corresponding to a certain discharge in nature. The additional complication arises, however, that the angle of tilt so produced, or the scale of discharge thus empirically determined, may not apply to other stages of flow.

As a demonstration of the advantage, from this point of view, of distorting the vertical dimensions of the model, the following examples will serve: Let $N_1 = 3{\cdot}0$ as before, and suppose the natural channel 400 ft. wide $\times$ 20 ft. deep. In the first place, let the model scales be 1 : 200 (horizontally) and 1 : 20 (vertically)—a distortion of 10 : 1. Neglecting any secondary effect of channel shape upon the Bazin formula,

$$C = \frac{157{\cdot}6}{1 + \dfrac{N}{\sqrt{m}}},$$

we should have

$$\frac{1 + \dfrac{N_2}{\sqrt{m_2}}}{1 + \dfrac{N_1}{\sqrt{m_1}}} = \sqrt{\frac{m_2 x}{m_1}},$$

and substituting $N_1 = 3{\cdot}0$, $m_1 = 18{\cdot}2$, $m_2 = 0{\cdot}5$, and $x = 200$, we find that

$$N_2 \text{ should} = 2{\cdot}1 \text{ instead of } 0{\cdot}21,$$

as found above for no distortion (that is, for $x = y = 200$). Again, if the model scales are 1 : 2,000 (horizontal) and 1 : 200 (vertical)—also giving a distortion of 10 : 1—, it is found that N_2 should $= 0{\cdot}67$ instead of $0{\cdot}067$.

In the second case, therefore, a decided practical benefit has resulted from the adoption of a vertical distortion in that $N_2 = 0{\cdot}67$ implies a reasonably possible roughness, whereas $N_2 = 0{\cdot}067$ means an unattainable *smoothness*. The advantage in the first case ($x = 200$; $y = 20$) may not be so obvious, and it appears that, as regards the practicability of producing a state of roughness which will give proportionate friction losses, there is much in favour of a vertical distortion of scale—but more especially in the smaller-scale models. Moreover, since it is true that, for a given horizontal scale, the velocity and the hydraulic mean depth and, therefore, the Reynolds' number, increase with increasing distortion, the practicability of obtaining turbulent flow in the model is enhanced by the adoption of a vertical exaggeration of scale. Here, again, this is likely to be more valuable in the case of the smaller models, which otherwise tend more towards operation in the non-turbulent condition.

So far, however, the problem of bed movement has not been specifically considered in this connection, but it will presently appear that from this viewpoint also some vertical distortion may be desirable, if not indeed essential. It is evident that the flow in the model must be at least sufficient to produce motion of the mobile bed.

Experiments on flat beds of granular material of a given density, shape, uniformity and size indicate that the *mean* current velocity in a rectangular channel, to generate motion, is proportional to $h^{0 \cdot 06}m^{0 \cdot 22}$, where h is the depth of water and m the hydraulic mean depth of the channel.* The indices of h and m are of such an order that, if the mean velocity is proportional to the square root of the vertical scale, an increase of distortion will lead to a proportionately higher increase of available current velocity than of velocity required to move the bed material. For example, in the simplest case of a stream sufficiently broad to permit the substitution of h for m, the effect of doubling the vertical distortion of scale would be to increase the mean velocity by some 40 per cent. while only raising the velocity needed for bed movement by about 22 per cent.

Furthermore, the advantage gained in this manner is once again more likely to be valued in the smaller type of model where velocities in general may tend to be too low for bed transportation.

As far as the argument has yet proceeded, it appears that, contrary to the usual ideas on this subject, there are definite points in favour of exaggerating the vertical scale relative to the horizontal. The counter-argument is, of course, that by doing so the limiting angle of repose of the bed material is given scope to reduce the possibility of simulating the slopes of banks and channels found in Nature: in fact, that ultimately the similarity will completely fail (in a quantitative sense) through inability of the material to stand at the gradients demanded by the distortion of scale. Clearly this difficulty will be greatest in those cases where the natural river itself contains steep-sided channels.

On the whole, then, it is true that *up to a point* a vertical distortion is desirable and is justified; the *necessity* for distortion is only to be avoided by the use of large-scale models which may on other grounds be uneconomical or impracticable. In *tidal* models, an exaggeration of vertical dimensions is usually indispensable if measurable tides are to be employed; and at this stage some remarks of Osborne Reynolds † may be considered appropriate:

" In one respect the great difference between the model and the estuary calls for remark: this is the much greater depth of the model as compared with its length and breadth. The vertical scale being 33 feet to an inch, and the horizontal scale 880 feet to an inch, so that the vertical heights are nearly twenty-seven times greater than the horizontal distances, such

* Allen, " An Investigation of the Stability of Bed Materials in a Stream of Water ", *Journ. Inst. C.E.*, March, 1942, p. 1.

† Reynolds, *On Certain Laws relating to the Régime of Rivers and Estuaries, and on the Possibility of Experiments on a Small Scale*, Rep. Brit. Assoc., 1887; or *Papers on Mechanical and Physical Subjects*, vol. II, p. 334.

a difference is necessary to get any results at all with such small-scale models; and it is only natural to suppose that it would materially affect the action. As a matter of fact, however, it does not seem to do so. And, further, it would seem that, notwithstanding the general resemblance on the *régime* of the beds of large and small streams running over sand, there is in these a similar difference in vertical scale, the smaller streams not only having a greater slope, but also having greater depth as compared with their breadth and steeper banks. . . . In the model it certainly seems that the general *régime* is determined by the momentum effects, and from the almost exact resemblance which this *régime* bears to that of the estuary, it would seem that, although the momentum effects may be diminished by the greater resistance on the bottom, they are still the prevailing influence in determining the configuration of the banks. Further investigation will doubtless explain this, and also determine the best proportional depths. From my present experience in constructing another model, I should adopt a somewhat greater exaggeration of the vertical scale."

Professor Gibson has developed one of Reynolds' arguments in the following way.*

" It is perhaps worth noticing in passing that what is in effect a distortion of scale is usual in nature, since small streams flowing through alluvial ground have much steeper side slopes and gradients than large rivers of similar *régime* in similar ground. In a very large river such as the Mississippi, the Ganges, or the Irrawaddy, the maximum depth will rarely exceed 1 : 50 of the maximum width, while in a small stream in similar ground this ratio will seldom be less than 1 : 5."

In the foregoing discussion, it has been assumed that the scale of velocities to be aimed at is proportional to the square root of the *vertical* scale of the model. On general grounds of dynamical similarity this is equivalent to ignoring the direct effects of viscosity, to using the so-called " Froude number ", $\frac{gl}{v^2}$, as the criterion of similarity and to interpreting l as a vertical dimension, h. Moreover, if the river embraces works such as weirs or sluice-gates the velocities appropriate to them must be taken as proportional to $\sqrt{\text{vertical heights}}$, while the loss of head—essentially an item measured on the vertical scale—due to sudden changes of cross-section or of shape may also be assumed to depend on v^2. Yet again, if the model is tidal, it will in general be found (as will be seen later) essential to adopt a velocity scale-ratio proportional to the square root of the vertical scale in order to ensure correct reproduction of the tides. In

* A. H. Gibson, " The Use of Models in Hydraulic Engineering ", *Trans. Inst Water Engs.*, vol. XXXIX, 1934, p. 172.

such a tidal model, in order to preserve a homogeneous scale of velocity, the non-tidal upper reaches would have to operate according to the rule $v \propto \sqrt{h}$ as well.

We have already seen that to satisfy this velocity requirement a trial-and-error method of adjusting the textural roughness of the model surfaces may be the only solution. The difficulty involved in this is likely to be least in the case of tortuous or irregular rivers where the losses of head due to bends and changes of section amount to a very appreciable proportion of the total loss (see Chapter I, pp. 28-29 and pp. 31-36). There are many devices available for adjusting surface-roughness, among them the use of stuccoed concrete, exposed sand or pebble aggregate, the ribbing of the cement or concrete surface by chisels or other tools, and the introduction of wire screens along the surfaces of the permanent banks.* As for the roughness of the mobile bed, this may be modified to some extent by the adoption of different grain-sizes, but the final choice of mobile material is far more likely to be dictated by considerations of scour.

Another approach to the interdependent problems of velocity-scale and roughness is through the application of the Manning formula for channel flow, which reads

$$v = \frac{1{\cdot}486}{n} m^{\frac{2}{3}} i^{\frac{1}{2}} \text{ ft. per sec.,}$$

where m is the hydraulic mean depth measured in feet and i is the slope. The quantity n has various values according to the type of channel concerned, typical ones being 0·009 for well-planed and perfectly continuous timber, 0·018 for earthen channels in very good order or heavily silted in the past, and 0·030 for channels in bad order.

As before, let the suffix (1) refer to the natural river and the suffix (2) to the model, and let A and Q represent cross-sectional area and rate of discharge respectively.

Then with this notation,

$$\frac{v_2}{v_1} = \left(\frac{n_1}{n_2}\right) \left(\frac{m_2}{m_1}\right)^{\frac{2}{3}} \left(\frac{i_2}{i_1}\right)^{\frac{1}{2}};$$

$$\frac{A_2}{A_1} = \frac{1}{xy}, \text{ the horizontal and vertical scales being } 1:x \text{ and } 1:y \text{ respectively,}$$

$$\frac{Q_2}{Q_1} = \frac{A_2 v_2}{A_1 v_1} = \frac{1}{xy} \left(\frac{n_1}{n_2}\right) \left(\frac{m_2}{m_1}\right)^{\frac{2}{3}} \left(\frac{i_2}{i_1}\right)^{\frac{1}{2}}.$$

* For photographs of model surfaces of different degrees of roughness, see *The U.S. Waterways Experiment Station Bulletin* (*Hydraulics*), vol. 2, No. 1, Feb. 1, 1939, p. 12. Remarks by H. D. Vogel (*Proc. Am. Soc. C.E.*, vol. 60, 1934, p. 1226) will also be of interest.

But $\qquad \frac{i_2}{i_1}=\frac{x}{y};$

$$\therefore \frac{Q_2}{Q_1}=\frac{1}{x^{\frac{1}{2}}y^{\frac{3}{2}}}\left(\frac{n_1}{n_2}\right)\left(\frac{m_2}{m_1}\right)^{\frac{2}{3}}.$$

With broad shallow streams and relatively undistorted models thereof, $\frac{m_2}{m_1}$ would approximate to $\frac{1}{y}$, but in general, owing to the complex nature of the quantity m in terms of the shape and dimensions of the channel, it is impossible to write $\left(\frac{m_2}{m_1}\right)$ in specific terms of x and y. For example, in the model of a non-tidal portion of the river Mersey to which reference has been made in pages 31–36, the scale-ratio of m_2: m_1 varied from 1: 200 at low stages to 1: 308 at high stages, compared with the vertical scale of 1: 120. Similarly, in the Severn tidal model having a horizonta scale of 1: 8,500 and a vertical scale of 1: 200, the value of m_2 in the upper reaches was 0·0104 foot at a rate of flow corresponding to 4,100 cusecs in the natural Severn, for which $m_1=5{\cdot}92$ ft. In this case, therefore, $\frac{m_1}{m_2}=\frac{5{\cdot}92}{0{\cdot}0104}=570$, compared with $y=200$.

However, having decided tentatively upon certain linear scale-ratios (that is, upon x and y), the investigators may draw typical cross-sections both of the river itself and of the proposed model. By measuring the area and wetted perimeter of these sections, the values of m_1 and m_2 may be computed and the expression

$$\frac{Q_2}{Q_1}=\frac{1}{x^{\frac{1}{2}}y^{\frac{3}{2}}}\left(\frac{n_1}{n_2}\right)\left(\frac{m_2}{m_1}\right)^{\frac{2}{3}}$$

will then serve to estimate the rate of discharge Q_2 required for the model, provided Q_1 and n_1 are known and n_2 can be assigned from previous experience of model experiments. Actually, in many cases, n_2 will approximate to n_1, more especially if the river under study is tortuous and irregular and if the sand used on the model bed is of the same order of size as that in the river itself.

If, then, n_2 is taken $=n_1$,

$$Q_2 \text{ becomes} = \frac{Q_1}{x^{\frac{1}{2}}y^{\frac{3}{2}}}\left(\frac{m_2}{m_1}\right)^{\frac{2}{3}},$$

and on this basis an estimate is possible, at any rate which will provide a guide as to whether the water-supply facilities available are adequate for a model of the contemplated scales.

This technique has been used by the United States Waterways Experiment Station,* and a numerical example † may serve to clarify the method. Here,

$$x = 720,$$
$$y = 72,$$
$$\frac{i_2}{i_1} = \frac{y}{x} = 10,$$
$$\frac{A_2}{A_1} = \frac{1}{xy} = \frac{1}{52000},$$
$$\frac{m_2}{m_1} = \frac{1}{80},$$
$$\left.\begin{aligned} \frac{v_2}{v_1} &= \left(\frac{m_2}{m_1}\right)^{\frac{2}{3}} \left(\frac{i_2}{i_1}\right)^{\frac{1}{2}} = \frac{1}{5{\cdot}87} \\ \frac{Q_2}{Q_1} &= \frac{v_2 A_2}{v_1 A_1} = \frac{1}{306000} \end{aligned}\right\} \text{ assuming } n_2 = n_1.$$

Since the estimated maximum natural flood was 600,000 cusecs in this case (Ohio River), it appeared that the laboratory supply would need to be of the order of 2 cusecs. Computation of the Reynolds' number $\left(\frac{v_2 m_2}{\nu}\right)$ was next made, and indicated that the flow would probably be turbulent in the model even at the lowest stages. Thus, the lowest anticipated value of $v_2 m_2$ was 0·032, corresponding with $v_2 = \frac{1{\cdot}86}{5{\cdot}87} = 0{\cdot}32$ ft. per sec. and $m_2 = 0{\cdot}10$ feet. The product $v_2 m_2$ was therefore, as already stated, expected to be 0·032, and the equivalent Reynolds' number at 15° C. 2,600. Furthermore, estimated velocities at a number of typical sections were in all cases appreciably lower than $\sqrt{gh}$, so that no trouble was anticipated from the phenomenon of " shooting flow ".

Trial tests in the model did, in fact, show that turbulent flow existed at all stages, and the average ratio of $Q_2 : Q_1$ necessary to give water profiles demanded by the chosen scales proved to be 1 : 311,000, which is extremely close to the value, 1 : 306,000, calculated above. It has been pointed out in a previous paragraph, however, that an anomaly arises out of the use of a velocity-scale derived in this way because the method leads to a velocity which is *not* proportional to $\sqrt{y}$, whereas the velocity-scale appropriate to any weirs, sluice-gates or similar structures incorporated in the experiments *should* be $1 : \sqrt{y}$. This discrepancy was actually found to

* H. D. Vogel, " Practical River Laboratory Hydraulics", *Proc. Am. Soc. C.E.*, vol. 59 (1933), p. 1413.

† *Model Study of Effects of Dikes on the River Bed at Walker's Bar, Ohio River*, Paper L of the U.S. Waterways Expt. Station, Vicksburg, Mississippi.

exist in this particular model of the Ohio River. The scales of this model—1 : 720 and 1 : 72—require a nominal discharge scale of

$$1 : (720 \times 72 \times \sqrt{72}),$$

that is 1 : 440,000, as compared with the adopted scale of 1 : 311,000.

Now part of the works consisted of dykes or weirs joining the mainland to the shores of certain islands. In order to maintain a proper relationship between the flow over these weirs and that taking place through other channels, it was necessary to give the weirs a length 440,000 ÷ 311,000, or 1·4 times their nominal scalar length, while at the same time maintaining them at their proper elevation.

This device (of increasing the length of the dyke) could not be applied to the *spur* dykes or groynes which were also included in the scheme, because if it had been the exposed toes of the groynes would have projected a disproportionate distance into the river. In so far as these groynes acted in the manner of submerged weirs, an estimate of their *effective* height was made as follows: Let b represent the length of a groyne, Q the discharge over it under a head H. Then

$$\frac{Q_1}{Q_2} = \frac{b_1}{b_2}\left(\frac{H_1}{H_2}\right)^{\frac{3}{2}}.$$

But $$\frac{Q_1}{Q_2} = 311000 \text{ and } \frac{b_1}{b_2} = 720.$$

Consequently, $$\frac{H_1}{H_2} = 57,$$

instead of the figure of 72, as would be appropriate to the vertical scale of the model. On these grounds, it was judged that the height of such groynes in the actual river ought to be made between two and three feet greater than the corresponding height of the groynes in the model. It is desirable to note, however, that this conclusion can only be approximately true, since it is based on an argument concerning the discharge of water over the groynes relative to that in the channel: the scouring effect will not necessarily obey the same argument. In another respect also this investigation serves to show how the results obtained from a model must be interpreted with discretion: it was discovered in a trial of a certain scheme that the most effective formation of a navigable channel was prevented by an eddy which caused a deposit of material. Two additional groynes served to break up the objectionable eddy, but the investigators were of opinion that the eddy itself might be exaggerated in the model, and they therefore recommended that the two additional groynes be regarded rather as providing a " margin of assurance ", to be adopted if the main works proved, after a period of observation in the natural river, to demand such an extra device.

The bed material adopted in this investigation was a river sand of mean diameter 0·0128 inch, a bed of silt-loam having been rejected owing to its compacting unevenly. The sand found in the prototype itself had a diameter of 0·0359 inch. A short portion of the model bed at the upstream end was moulded in concrete so as to preserve a permanent entrance of the proper shape; the quantity of water entering the model was set to a desired rate by " regulating the depth over a sharp-crested rectangular weir ". The resulting turbulence below the weir " was damped in a stilling box by a rack made of vertical wooden posts, and the current was given a proper entrance direction by two sets of movable vanes which were set by trial. The depth of water at the lower end of the model was controlled by raising or lowering an adjustable weir, or tailgate ".

In order to see whether the chosen bed material was suitable, the sand was placed in the model in a moist state and moulded to the configuration of a river survey. The model was then filled with water, precautions being taken to avoid the initial scour which would result from rapid filling. A run of five hours' duration was then made, during which " sufficient sand was added . . . to maintain at all times a thin layer on the section of the bed surfaced with concrete at the upstream end. Satisfactory evidence of sand movement was indicated by the formation of riffles [ripples] in the bed and by the formation of a deposit in the sand trap at the lower end of the model ".

Certain changes were observed in the bed while this test was being made, and accordingly a second run was carried out " to determine whether the changes would be cumulative or transitory in effect. Upon completion of the second run further changes were found to have taken place, but the general shape of the bed appeared to agree more closely with Nature than after completion of the first run. This led to the conclusion that a sufficiently similar movement of sand was taking place in the model to justify the initiation of actual experiments with the proposed contraction works ".

At a later stage, however, a further test was made, using a cycle of river discharges based upon five years' gauge readings in the river itself. The duration of each cycle in the model was 12 hours, and the test run lasted altogether 36 hours, thus comprising three cycles, each made up of a sequence of successively increasing flows up to a certain maximum followed by a number of successive reductions. Observations in this experiment led to the conclusion that " a substantial agreement was obtained between the configuration of the bed of the model and the bed of the river, and that the model indicated channel depths which are approximately three feet less than the depths obtained in the river ". The length of run was chosen as a result of experiments which indicated that it was amply sufficient to allow the bed of the model " to reach equilibrium ". No direct reference is made in the published paper to suggest that any virtue was attached to the *nominal* time-scale of the model, which, taking time as proportional to distance divided by velocity would be $1 : \frac{720}{5{\cdot}87}$, or 1 : 123.

Despite the various complications inherent in this model-investigation, the results have evidently been of considerable value, and a survey of the river has been made to discover the effect of the dykes as constructed. " Benefits to the channel were unquestioned although the survey was made before the end of the high-water season." *

It will be instructive, however, to examine what would have been the possibility, through the adoption of different scales, of avoiding some of the complications encountered in this investigation. The horizontal scale of 1 : 720 was " chosen primarily to keep the size of the model within limits imposed by the available laboratory floor space. This ratio appeared, in the light of previous experience, adequate for the production of a workable model ". If, then, this horizontal scale be accepted, the choice of vertical scale remains to be debated. We have seen that the one chosen (namely 1 : 72) resulted in a velocity scale-ratio of 1 : 5·87 (very nearly) instead of $1 : \sqrt{72}$, that is, 1 : 8·48. It follows, therefore, that the model surfaces were relatively too smooth, although having the same roughness value, n, on Manning's scale as the river itself. The question then arises as to whether the model roughness might by artificial means have been increased. Applying the equation

$$\frac{v_2}{v_1} = \left(\frac{n_1}{n_2}\right)\left(\frac{m_2}{m_1}\right)^{\frac{2}{3}}\left(\frac{i_2}{i_1}\right)^{\frac{1}{2}},$$

it is found that, for $\frac{v_2}{v_1}$ to equal $\frac{1}{\sqrt{72}}$, n_2 would have to be 1·44 times n_1, which means that the roughness of the model in this case would have needed to be increased by something approaching 50 per cent. It is most unlikely that such an increase could have been achieved by any artificial means, for the resistance of the river must have been caused, to some appreciable extent, by changes of sectional shape and area from place to place. Moreover, since the channel-width was relatively great compared with its depth, any change in the resistance of the sides would not be as effective as a change in the character of the bed. But to make any substantial alteration in the bed-roughness might easily have entailed the use of a bed material which would fail to behave reasonably from the point of view of sand-bank formation.

If, then, it be argued that the model roughness could not have been

* H. D. Vogel, " Hydraulic Laboratory Results and their Verification in Nature, " *Proc. Am. Soc. C.E.*, vol. 61 (1935), p. 70.

much changed from its actual magnitude (which proved to be such as to make $n_2 = n_1$), the remaining device would be that of choosing a different vertical scale, so as to provide such a value of m_2 as to *make* the velocity scale equal to $1:\sqrt{y}$, the vertical scale being $1:y$. Using the appropriate equality,

$$\frac{v_2}{v_1} = \frac{1}{\sqrt{y}} = \left(\frac{m_2}{m_1}\right)^{\frac{2}{3}} \left(\frac{i_2}{i_1}\right)^{\frac{1}{2}} = \left(\frac{m_2}{m_1}\right)^{\frac{2}{3}} \left(\frac{720}{y}\right)^{\frac{1}{2}},$$

we find that

$$\frac{m_2}{m_1} = \left(\frac{1}{720}\right)^{\frac{3}{4}} = \frac{1}{139}.$$

Since, with $y = 72$, $\frac{m_2}{m_1}$ was found to be 1/80, this result means that the revised vertical scale would need to be such as to make the mean hydraulic radius of the model $\frac{80}{139}$, or 0·575, of what it was with the scales actually adopted. In order to effect this change, the distortion of scale would have had to be reduced, and a *rough* idea of the kind of reduction required may be arrived at in the following way:

Suppose the full-size river channel to be replaced by a rectangular section of width β times its depth. Then

$$\frac{m_2}{m_1} = \frac{\beta + 2}{\left(\frac{\beta}{\gamma} + 2\right) x}, \text{ where } \gamma = \frac{x}{y}.$$

Now, with $x = 720$ and $y = 72$ (or $\gamma = 10$), we are told that $\frac{m_2}{m_1} = \frac{1}{80}$. It follows that the equivalent or effective value of β is 160. But it is now required that $\frac{m_2}{m_1} = \frac{1}{139}$.

Hence $$\frac{1}{139} = \frac{162}{720\left(\frac{160}{\gamma} + 2\right)}, \text{ or } \gamma = 5{\cdot}45,$$

and therefore $$y = \frac{720}{5{\cdot}45} = 132.$$

According to this approximate analysis, therefore, the conclusion is reached that had the vertical scale been about 1 : 130, instead of 1 : 72, the scale of velocities would have been approximately $1:\sqrt{130}$ while maintaining the correct water surface gradient. The Reynolds' number would, however, have been reduced thereby from 2,600 to $2{,}600 \times \frac{5{\cdot}87}{\sqrt{130}} \times \frac{80}{139}$, that is, to 770, and only the irregularities of the channel could then have prevented the motion from ceasing to be turbulent: moreover, it might have proved difficult to discover a suitably mobile bed material.

As a result of this discussion, it will be more than ever appreciated that the choice of scale, depending as it does on so many variables, requires a compromise, and correspondingly demands that the most careful attention be paid to the subject of scale-effect when the model results are assessed. On the whole, it seems doubtful after all whether a much more favourable choice of scales could have been made than that actually adopted for this study of the Ohio River dykes at Walker's Bar.

An interesting device, introduced in some laboratories with the particular object of facilitating bed movement, is the provision of *additional* longitudinal slope in the model.* The argument in this connection is that the vertical distortion of scale in itself may not provide a slope adequate for proper bed movement; accordingly the model is in effect tilted downstream without further alteration in the vertical scale. In practice, this result may be obtained by adjustment of the datum-level on the templates to which the bed and banks are moulded.

The method of deciding what additional slope is desirable rests upon the theory of "tractive force", which appears to have originated in a paper, "Le Rhône et les Rivières à Lit Affouillable", by M. P. du Boys, in *Annales des Ponts et Chaussées*, XVIII, 1879. Considering *uniform* flow in a channel of area A, length l and slope $i=\frac{h}{l}$, the resolved part of the weight of the water in the direction of motion is ρlAi lb., if ρ is the density (lb. per cu. ft.) and l and A are measured in ft. units. Since in uniform flow the velocity is constant, this resolved weight is expended in overcoming friction and not in producing acceleration. Hence the tractive force of resistance per unit area of the wetted walls and bed may be considered as equal to $\frac{\rho lAi}{Pl}$, where P is the perimeter. This quantity in turn reduces to ρmi lb. per square foot, m being the hydraulic mean depth in feet. If the channel is sufficiently wide to justify the approximation $m=$depth, D, the "tractive force per unit area" becomes ρDi lb. per sq. ft.

Again, in the usual notation $h=\frac{flv^2}{2gm}$ or $i=\frac{fv^2}{2gm}$. Hence an alternative expression for the tractive stress is $\frac{\rho fv^2}{2g}$ lb. per sq. ft.

On the assumption that the tractive stress is the direct mechanism whereby the bed material may be moved, a technique has been developed which appears to have been applied with considerable success in the

* H. D. Vogel, "Practical River Laboratory Hydraulics", *Proc. Am. Soc. C.E.*, vol. 59, 1933, p. 1413. Also *Paper* 11 of the U.S. Waterways Expt. Station, Vicksburg June 1933.

United States Waterways Experiment Station. In order to explain this method, we cannot do better than follow the steps of the argument in a numerical example as given by Lieut. H. D. Vogel:*

Considerations of space, water supply, turbulent flow and angle of repose have led to the adoption of certain linear scales for a particular model. For purposes of illustration, suppose these scales to be 1 : 500 (horizontally) and 1 : 125 (vertically). If " general movement " of the bed occurs at a certain stage in the river—say, when the water depth is z feet—it should also occur in the model at the corresponding stage of $\frac{z}{125}$ feet. The first step is to estimate the stage (z) at which such " general movement " does begin in Nature. To do this, a sample of the river sand is placed level in a flume in the laboratory. Let it be supposed that this sample comes from a portion of the river where the average slope is 0·0001. For convenience of experimentation, the flume is tilted to a slope of, say, $20 \times 0{\cdot}0001$, that is, 0·0020. Water is then supplied and so controlled by a weir at the discharge-end of the flume that the water surface is maintained parallel to the bed. In this way, the flow being gradually increased, a depth of water is reached with which " general movement " is observed in the flume. Suppose this depth to be 6 inches. The corresponding critical tractive stress is then $\rho(0{\cdot}5)(0{\cdot}002)$, and if this is also the critical tractive stress in Nature,

$$\rho z(0{\cdot}0001) = \rho(0{\cdot}5)(0{\cdot}002), \text{ or } z = 20 \times 0{\cdot}5 = 10 \text{ ft.}$$

This is taken to signify that " general movement " begins in Nature when the water depth is 10 ft. and the slope 0·0001. Consequently, it is required that " general movement " shall begin in the proposed model when the water depth is $\frac{10}{125}$, or 0·08 ft.

A sand is then selected for use in the model and is similarly tested in the flume; it is found that for its " general movement ", a tractive stress of, say, 0·008 lb. per sq. ft. is necessary. To create this with a water depth of 0·08 ft., it is estimated that the appropriate slope, i_2, is such as to make $\rho(0{\cdot}08)(i_2) = 0{\cdot}008$, and substituting $\rho = 62{\cdot}4$ lb. per cub. ft., i_2 becomes 1 : 624 or 0·0016, whereas the slope based upon the linear scales of 1 : 500 and 1 : 125 is only $0{\cdot}0001 \times \frac{500}{125}$, or 0·0004. The final procedure then is to mould the bed of the model in the selected sand so that the width and depth at any section are respectively determined by the chosen horizontal and vertical scales (1 : 500 and 1 : 125), but so that the longitudinal slope is equal to that in Nature multiplied by $\frac{500}{125} \times 4$, or 16. In effect, therefore, the model is given an additional " tilt " and the water

* H. D. Vogel, " Practical River Laboratory Hydraulics ", *Proc. Am. Soc. C.E.*, vol. 59, 1933, pp. 1413–39.

supply to it is increased in order to maintain the correct depths. For the hypothetical example reviewed above, the modified discharge would be tn the ratio of $\sqrt{4}:1$, or $2:1$, compared with the discharge to be provided if no additional tilt were employed. Correspondingly there will be, due to the artificial extra tilt, a still greater discrepancy in the actual velocity scale compared with the square root of the vertical scale.

One trouble which arises in connection with this technique is the difficulty of giving a precise definition to the term " general movement ". In an attempt to overcome this obstacle, a tentative proposal was advanced in 1935 by the U.S. Waterways Experiment Station * as follows: General movement is obtained when both of two conditions are attained, namely:

" 1. The material in transportation is reasonably similar in composition to the material composing the original bed.

" 2. The rate of movement is equal to or exceeds one pound (dry weight) per foot width of channel per hour."

This definition is intended to be confined to models and to sands of mean grain size less than about 0·6 mm. (0·0236 inch). The first of the two conditions is intended to ensure that the tractive stress adopted in the model shall exceed that necessary for ripple-production; otherwise it is contended that the *rate* of bed movement might be retarded by the ripples at values of the tractive stress greater than a lower adopted critical value. The second condition is imposed as the result of observations on a model of the Mississippi River which indicated that a minimum of about one lb. per foot width of channel per hour was desirable to give " satisfactory development of the model bed ".

The phenomenon of ripple-formation is one which has excited considerable interest in model-experiments. Reynolds himself was much intrigued by the appearance of sand ripples in his *tidal* models.† After remarking that such ripples formed a conspicuous feature of his models, he says (1889):

" The ripple marks on the strands are too well-known to need description, and there is nothing surprising that similar ripple marks should appear in the beds of the models. But although presenting a very similar appearance, and being about of the same size, the ripple marks seen in all the plans are essentially different in their origin and in the position they take in the régime of the sand in the models from that held by the

* Studies of River Bed Materials and their Movement, with Special Reference to the Lower Mississippi River, Paper 17, U.S. Waterways Expt. Station, Vicksburg, Jan. 1935, p. 33.

† *Reports of the Committee appointed to investigate the Action of Waves and Currents on the Beds and Foreshores of Estuaries by Means of Working Models*, Brit. Assoc., 1889–90–91; or *Papers on Mechanical and Physical Subjects*, Reynolds, vol. II, pp. 399, 489.

observed ripple marks on the shore sands. This last is caused by the alternating currents produced by the small swell running inshore, while that in the model is produced by the alternating action of the tide. There may seem nothing remarkable in this, considering that these currents in magnitude and direction are not dissimilar—but if the models are similar to the results obtained in estuaries, the converse should hold, and the estuaries should be similar to the models. In which case we are face to face with a very striking conclusion, that in the estuaries there should be —call it ripple mark or wave mark, produced by the action of the tide, similar to that on the models and on a scale proportional to the height of tide in the estuary. Thus some of the ripples in the models are from hollow to crest as much as one-fourth the mean rise of the tide, the distance between them being 12 times their height. This, in an estuary, would mean 7 or 8 feet high and 80 to 100 feet in distance.

" These ripples in the model are almost confined to the surface of the sand which is below low-water mark, though in places their somewhat eroded ends protrude up the slope of the low-water channels. The existence of these ripples very much enhances the effect of the water to shift the sand—this was noted in the experiments 2 and 3 on the bars, tank *A*; on the smooth walls of the sand the current, which would be about 6 inches a second, did not drift the sand at all, except close to the ridge, and then there was no apparent effect till after 1,700 tides, when ripples were just beginning, yet when the ripple once formed in another 1,200 tides the top of the bar had spread to 12 inches.

" The ripples also serve to show in which way any shift of the sand is taking place, as they have a steep side looking in the direction of motion, and when the slopes are equal it is an indication of equilibrium."

In a later Report (1891), Reynolds returns to the theme by remarking:

" The large tidal sand ripples below low water in the model estuaries, with the flood and ebb taking the same course, constitute a feature which it is impossible to overlook, yet the existence of corresponding ripples had been entirely overlooked in actual estuaries, until, when they were looked for, they were found to exist, having been first seen in the models. The reason that they were overlooked before is, no doubt, explained by the fact that the bottom is not visible below low water in actual estuaries; but this is not all. In the estuaries, these ripples, where found, have been confined to the bottom and sides of the narrow channels between high sand-banks, and they do not occur on the level sands below low water towards the mouths of estuaries to anything like the same extent as in the models. By rendering the estuary unsymmetrical and so causing the ebb and flood to take different courses, this effect, as explaining the greater prevalence of ripples with symmetrical estuaries, would be tested.

" These considerations led to the repetition of Experiment V in tank *F*, at first with a single groin extending from the right bank into the middle of the estuary at the mouth, and subsequently to the introduction of three more groins from alternate sides of the estuary to the middle, up the estuary, and then to the introduction of similar groins into tank *E*, during Experiment VII, with spring and neap tides.

" The result of these experiments is to show conclusively: . . . That in the models with boldly irregular boundaries the tidal ripples are much less frequent than in the symmetrical models, being confined to places where there are no cross-currents, as in actual estuaries."

A word or two may be added, by way of amplification, to Reynolds' explanation of the difficulty of observing the sand ridges in actual rivers. In the first place, it is not easy, from soundings taken during the course of a survey, to form any clear idea of the shape of such ridges as may exist. Experience with several models shows that for the purpose of drawing any detailed contoured plan of such formations a very large number of soundings is required; clearly the same must apply in surveying natural rivers. A further point is that a survey in Nature may take quite a long time to complete, and during the process the ripples or ridges of sand may move so that their existence or their situation at one particular moment is not registered in the measurements as taken. It is on record that in deep, swift water in the Mississippi sand-waves have been measured with a height of 20 feet, moving at 80 feet per day, and that the deeper and stronger the current, the larger and further apart are the waves.*

The Author has heard it said, moreover, that observers on the deck of salvage ships have noticed fluctuations in the gauges on the pipes connected with divers working below water as the divers walk along the bed, the fluctuations being consistent with the picture of ridge-configuration.

Some extremely interesting examples of tidal ridges on banks which are exposed at low water have been beautifully described by Dr. Vaughan Cornish.† On the Dun Sand (a little upstream of the English Stones in the Severn Estuary) he measured fifteen consecutive ridges and found their average length, from ridge to ridge, to be 37 ft. 8½ in., with an average

* Thomas and Watt, *The Improvement of Rivers* (John Wiley, New York; Chapman & Hall, London), 1913, vol. 1, p. 22.

† Vaughan Cornish, *Geogr. Journal*, vol. 18 (1901), pp. 170–201, illustrated by photographs. The Paper deals with tidal sand-ripples at Barmouth (North Wales), Grange (Lancashire), Findhorn (N.B.), Montrose (N.B.), Mundsley (Norfolk), the Goodwin Sands, Pegwell Bay (Kent), the Severn between Gloucester and the Tunnel, and Aberdovey (Wales).

Also Vaughan Cornish, *Tidal Sand Ripples above L.W. Mark*, Brit. Assoc. Rept., 1900, p. 733.

Also Vaughan Cornish, *Ocean Waves and Kindred Geophysical Phenomena* (with additional notes by Harold Jeffreys), Camb. Univ. Press, 1934, in which appears a chapter on " Waves in Sand and Snow formed and propelled by Wind and Current ".

height from trough to crest of 1 ft. 11·2 in. The maximum wave-length of this series was 54 ft. and the maximum height 2 ft. 9½ in. Similar features are exhibited in the saw-tooth profile of sand-banks such as the Whiting Shoal in the Wash (East Anglia), and indeed one is led to speculate as to whether sometimes even a system of complete sand-banks may not in reality be an extreme case of ridge-formation. This idea receives support from an examination of oblique aerial photographs of the estuary of the Cheshire Dee, on which is visible a large number of sand-ripples having a wave-length of approximately 40 to 70 yards: certain of the banks appear to be composed of great tidal ridges lying in folds across comparatively flat shoals. Some of those major ripples are clearly several feet high from trough to crest and ¼ mile or more apart.* One of these photographs is reproduced in Plate IX.

There is, however, no reason why such sand-ripples should be confined to *tidal* rivers, though the question arises as to how they come to be made at all. Suppose, then, that the bed of a stream is initially flat and made of a granular material. If the current velocity near the bed is sufficiently high, a lee eddy may be expected to form behind any protruding irregularity in the nominally flat surface of the bed, and the stream transporting grains over the irregularity may be expected, on mixing with this lee eddy, to deposit some of its grains and so to build up a still larger irregularity and a more pronounced eddy. The presence of some initial small excrescence to set this mechanism in motion is not unreasonable, because no sand bed encountered in Nature consists of perfectly uniform grains or is free from odd grains which project above the general surface of the bed. This apart, however, Dr. Harold Jeffreys † has shown that the turbulence of the stream itself can set the mechanism in motion. His explanation is worded so as to refer to aeolian sand-waves, but the argument is equally applicable to subaqueous:

"The beginnings of any sand-wave seem to depend on turbulence. In an actual wind the velocity is not uniform, nor is the pressure at a given height. The effects of this can be seen even on water when the velocity is insufficient to generate waves. The surface in anything but an absolute calm has a slightly mottled or granular appearance, presumably due to these slight variations of pressure. A sudden gust gives a 'cat's paw', a local region of intense mottling due to short-crested waves a few centimetres long; from this the regular train of waves emerges. Similar

* Allen, *Journ. Inst. C.E.*, Oct. 1939, p. 271, in correspondence concerning a Paper, "An Aerial Survey of the Estuary of the River Dee, employing a Simple Method of Rectifying Oblique Photographs", by J. L. Matheson, *Journ. Inst. C.E.*, Nov. 1938, p. 47.

† In the Additional Notes to Vaughan Cornish's *Ocean Waves*, Camb. Univ. Press, 1934.

gustiness when the wind blows over sand will lead to local scouring and therefore to the formation of irregularities of the surface. Then any irregularity produces a lee eddy, so that air driving sand over it mixes with air from the lee eddy, loses some of its velocity, and drops the sand. Originally, therefore, we should expect small disturbances to increase with time. . . .

" Many striking theoretical results about the formation and motion of sand-dunes, ripple-mark, sand-banks, and the meanders of streams have been given by Exner in the paper already mentioned.* Meandering is shown to depend on very much the same processes as the formation of ripple-mark. When a stream silts up at one side, the sand-bank forms a lee eddy, with a vertical axis this time, and deflects the stream to the opposite bank with extra velocity, so that there is rapid erosion. This in time leaves the bank further down projecting; hence there is a lee eddy behind the new projection and further silting there. Thus silting and erosion occur alternately on each side and a river, originally straight, becomes sinuous.

" It is sometimes said that meandering is a characteristic of slow streams. But I have never seen a lowland stream meander quite so much as the torrents that cross the peat on the slopes of Ingleborough. It seems that meandering cannot depend on slope or velocity alone, but must be influenced in some way by the nature of the bed."

Since the current velocity will be higher over the crest than above the trough, it would be expected, other things being equal, that erosion would be greatest at the crest of a ripple, and probably this effect acting opposite to that already mentioned determines the ultimate height attained under the action of a given stream. The normal sand-ripple advances in the same direction as the stream, but if the current velocity is sufficiently high the crests are smoothed out; experiments † have shown that with still faster currents, the ripples re-appear and may travel upstream against the current.

Cohesion of the particles composing a bed tends to prevent the formation of ripples. For example, cohesive mud is not ripple-marked, although it is frequently honeycombed by a beautifully elaborate network of drainage channels in the region between low- and high-water. (See Plate X.)

The subject of sand-ripples has been discussed above partly because of its intrinsic interest, but also because there is no doubt that such ripples have appeared in some models with an exaggerated size out of proportion

* Exner, *Ergebnisse der kosmischen Physik*, vol. 1, 1931, pp. 373–445.

† Experiments by Gilbert and Cornish, cited in *A Treatise on Sedimentation*, by W. H. Twenhofel and Others (Baillière, Tyndall, Cox, 1926), p. 466.

to the scales of the models. They have then overwhelmed the rest of the bed configuration so far as to prevent any similarity to the sand-banks of the natural river. Many experiments have been devoted towards trying to discover a bed material which will reduce the ripple-formation to reasonable proportions. Powdered lignite, pumice, emery, sawdust from pressure-creosoted timber, and finely-ground gilsonite * are among the substances tried, in addition to various sizes and mixtures of sand. The particular case of *tidal* models will be enlarged upon later; in non-tidal work, we have already quoted an example (page 141) of the satisfactory use of pumice, and we have noted the American practice of using an artificially increased slope: this enables a coarser sand to be moved, and has proved satisfactory from the point of view of ripples. Indeed, it is stated in the Vicksburg Paper No. 17 (p. 55), that:

" While the formation of riffles in the sands in use in models at the Station has been troublesome at times, it is not believed that they have been so severe as to vitiate the results of the studies. Nevertheless, the elimination of large riffles will make for easier operation of the models, and will to some extent reduce the element of judgment necessary in the interpretation of the results. The present studies have indicated that there is little riffle formation with sands of about 0·6 mm. mean grain size, and that the critical tractive force, using the ' model ' criterion for general movement, is a minimum at about this same grain size. Hence the first efforts will be to mix a sand of about this size, having a very small percentage of fine grains." [The " model " criterion here mentioned is that defined on page 157.]

A noteworthy attempt to deal quantitatively with the general problem of bed movement, including ripple-formation, has been made by Capt. Hans Kramer, whose original paper † and the discussion thereon make highly interesting reading. Kramer's experimental investigation was made in a flume 31¾ inches wide, 12 inches deep and nearly 46 ft. long. Approximately 2¾ inches of sand were used to form the bed, and the walls were also roughened with sand. During any experiment, the water surface was maintained parallel to the bed, so as to ensure uniform flow, by a tail-gate which was operated by the slope observer as he watched the manometer board. Slopes of 1: 400, 600, 800 and 1,000, and three different sands of specific gravity 2·70 were used:

* Gilsonite: a nearly pure, hard and brittle bitumen obtained from Colorado and Utah (*Mechanical Engineers' Handbook*, ed. Marks, 3rd ed. (1930), McGraw Hill, p. 757). Its specific gravity appears to be of the order of 1·30.

† H. Kramer, "Sand Mixtures and Sand Movement in Fluvial Models", *Proc. Am. Soc. C.E.*, vol. 60, p. 443, April 1934.

TABLE XVI

Sand	Size limits		Median size *		Uniformity modulus,† M
	mm.	*inches*	*mm.*	*inches*	
I	0·0 to 5·0	0·0 to 0·197	0·53	0·021	0·358
II	0·0 to 1·77	0·0 to 0·070	0·51	0·020	0·461
III	0·385 to 5·0	0·015 to 0·197	0·55	0·022	0·414

The analysis of the sands was carried out by sieving in a sifting machine, and material was added at the top of the channel in a moist condition on an adjustable distributing board, which also served as a float for stilling the inflow. The rate of feed of this sand was arranged to correspond with the rate of discharge of sand at the exit.

Kramer contends that, up to a point, decreasing stages of flow or reduced tractive forces at constant slope have a tendency to flatten out the ripples previously produced at higher stages, and he defines two limits of usefulness of sand for model purposes, namely:

the " lower limit " at which grains up to and including the largest are in motion;

the " upper limit " at which ripples of height greater than 8 per cent. of the depth of water are formed during a run of one hour, and such that subsequent lower stages fail to remove the ripples satisfactorily.

Kramer's experiments showed that with each of the three materials tried the lower critical tractive stress for that material was constant (that is, independent of the slope); the upper initial tractive force was *not* constant.

One interesting feature of these results, shown in Table XVII. on p. 164, is that Sand III, which was made by sieving out the finer constituents of Sand I, has a lower tractive stress than Sand I. The difference is small it is true—approximately 6 per cent.—but Kramer states that the effect was quite consistently displayed in his experiments, and he argues that the addition of finer material to a given sample has two conflicting effects: (*a*) it reduces the mean grain volume, and so tends to increase the mobility of the original sample; (*b*) it reduces the voidage, thereby increasing the

* 50 per cent. of the grains are larger and 50 per cent. smaller than the *median* grain diameter.

† The Uniformity Modulus is obtained by plotting a curve whose ordinates, y, represent the percentage (by weight) of the sample which is finer than a grain diameter, d, whose successive values are the abscissae. The space between this curve and the axis of y is then divided into two areas. One of these, A_A, lies between the horizontal lines $y=50$ per cent. and $y=100$ per cent., while the other area A_B lies between $y=0$ and $y=50$ per cent. The Uniformity Modulus, M, is then $=\frac{A_B}{A_A}$.

TABLE XVII

Sand	*Critical tractive stress*			
	Lower		*Upper*	
	grm. per sq. metre	*lb. per sq. ft.*	*grm. per sq. metre*	*lb. per sq. ft.*
I	52	0·0106	79, $i=0{\cdot}001$ 110, $i=0{\cdot}002$ 125, $i=0{\cdot}0025$	0·0161 0·0224 0·0255
II	39	0·0080	60, $i=0{\cdot}001$ 72, $i=0{\cdot}002$ 79, $i=0{\cdot}0025$	0·0122 0·0147 0·0161
III	49	0·0100	93, $i=0{\cdot}001$ 119, $i=0{\cdot}002$ 131, $i=0{\cdot}0025$	0·0190 0·0243 0·0268

(In the above table, i represents slope.)

TABLE XVIII

Showing the Range of Water-depths (inches) within the Zone of Usefulness appropriate to Kramer's Sands

Slope, i, 1 in:	1,000	800	600	400
Sand I	$\underbrace{2{\cdot}03 \text{ to } 3{\cdot}15}_{1{\cdot}12^*}$	$\underbrace{1{\cdot}62 \text{ to } 2{\cdot}76}_{1{\cdot}14}$	$\underbrace{1{\cdot}22 \text{ to } 2{\cdot}36}_{1{\cdot}14}$	$\underbrace{0{\cdot}81 \text{ to } 1{\cdot}97}_{1{\cdot}16}$
Sand II	$\underbrace{1{\cdot}53 \text{ to } 2{\cdot}36}_{0{\cdot}83}$	$\underbrace{1{\cdot}22 \text{ to } 1{\cdot}99}_{0{\cdot}77}$	$\underbrace{0{\cdot}91 \text{ to } 1{\cdot}62}_{0{\cdot}71}$	$\underbrace{0{\cdot}61 \text{ to } 1{\cdot}22}_{0{\cdot}61}$
Sand III	$\underbrace{1{\cdot}93 \text{ to } 3{\cdot}70}_{1{\cdot}77}$	$\underbrace{1{\cdot}54 \text{ to } 3{\cdot}15}_{1{\cdot}61}$	$\underbrace{1{\cdot}14 \text{ to } 2{\cdot}60}_{1{\cdot}46}$	$\underbrace{0{\cdot}77 \text{ to } 2{\cdot}06}_{1{\cdot}29}$

* This figure (1·12) is the difference of 2·03 and 3·15; it therefore represents the range of practicable depths in inches.

area of friction surfaces whose resistance the original grains must overcome in order to move. Another point would appear to be that the addition of finer grains may reduce the extent to which the coarser grains project above the general level of the bed: this effect would tend to prevent their motion by exposing them to small velocities and to decreased turbulence.

The essential virtue of Capt. Kramer's work, however, lies in the technique which it suggests for the design of models, that is, to determine by preliminary flume experiments the " lower " and " upper " critical tractive stresses of the material which it is proposed to adopt, and then so to choose the scales and slope of the model as to ensure that it will be operated within that " zone of usefulness " whose limits are described by the " lower " and " upper " values of the quantity ρDi, or more precisely ρmi.

For example, in the case of the sands tried by Kramer, the zone of usefulness, defined by the range of water-depths over which successful operation might be expected, worked out approximately as shown in Table XVIII on p. 164.

From this table, it may be concluded that:

(i) The magnitude of the range or zone of practicable water-depths satisfying Kramer's criteria of the " lower " and " upper " critical tractive stresses is not markedly affected by the slope over a fairly wide range of slopes and sand mixtures.

(ii) Sand III, obtained by sieving out the *finer* elements of Sand I, has a consistently higher range of usefulness than Sand I itself, and moreover the zone of Sand III embraces that of Sand I at both limits of depth.

(iii) Sand II, obtained by sieving out the *coarser* elements of Sand I, exhibits an appreciably restricted range compared with Sand I itself, although its lower limits are smaller. By a change of vertical scale in the model, this defect could be remedied if the slope were comparatively flat; not so, however, with steeper slopes. Thus, considering $i = 1:1{,}000$, the ratios of 2·03 to 1·53 and of 3·15 to 2·36 are each 1·33 (very nearly), but in the case of $i = 1:400$, the ratio of 0·81 to 0·61 is 1·33 while that of 1·97 to 1·22 is 1·61; consequently a change of vertical scale designed to accommodate Sand II equally as well as Sand I at small slopes would fail to do so at steep slopes.

On the balance, then, Kramer's results favour the use of comparatively coarse and well-graded sands, more especially from the standpoint of avoiding trouble through inordinate ripple-formation. His work provides one of many examples which prove that it is readily possible to adopt a bed material which is too *fine* for successful operation.

His quantitative classification of excessive ripples as those which exceed

8 per cent. of the depth of the water should, however, in the Author's opinion, be accepted with reserve. *At present* the only safe technique appears to be that of choosing a bed material guided by the work of the various investigators as summarized in these pages, and whenever possible to compare the actual behaviour of that material in the model with the phenomena observed in the particular natural river under comparable conditions. Having thus " calibrated " the model, the investigators may feel confident that the effect of proposed works afterwards inserted in the model will be demonstrated with considerable accuracy. As an alternative or additional measure the proposed works may be tried in models with different scales and any " scale-effects " judged from the comparative results so obtained.

Incidentally, Kramer has suggested *tentatively* that the " lower " critical tractive stress may be represented by the expression:

$$T_0 = \frac{100}{6} \cdot \frac{D(\rho_1 - \rho)}{M} \text{ grammes per sq. metre,}$$

where D is the mean, *not median*, grain diameter in millimetres,

$(\rho_1 - \rho)$ = the effective density in water (grammes per c.c.),

M = the uniformity modulus.

The *median* diameter and uniformity modulus have previously been defined (p. 163) in relation to a curve whose ordinates represent the percentage by weight of the sample finer than certain grain diameters against which they are plotted and which form the abscissae. The *mean* diameter D is simply the average abscissa of this curve, that is, the weighted average size of grain.

Later work at Vicksburg * suggests that a truer representation is

$$T_0 = 29\sqrt{\frac{D(\rho_1 - \rho)}{M}},$$

or if $(\rho_1 - \rho) = 1{\cdot}65$ grm. per c.c., which is a common value for sand,

$$T_0 = 37\sqrt{\frac{D}{M}} \text{ grm. per sq. metre.}$$

In " British units ", this equation may be written

$$T_0 = 0{\cdot}0038\sqrt{\frac{D(\rho_1 - \rho)}{M}} \text{ lb. per sq. ft.,}$$

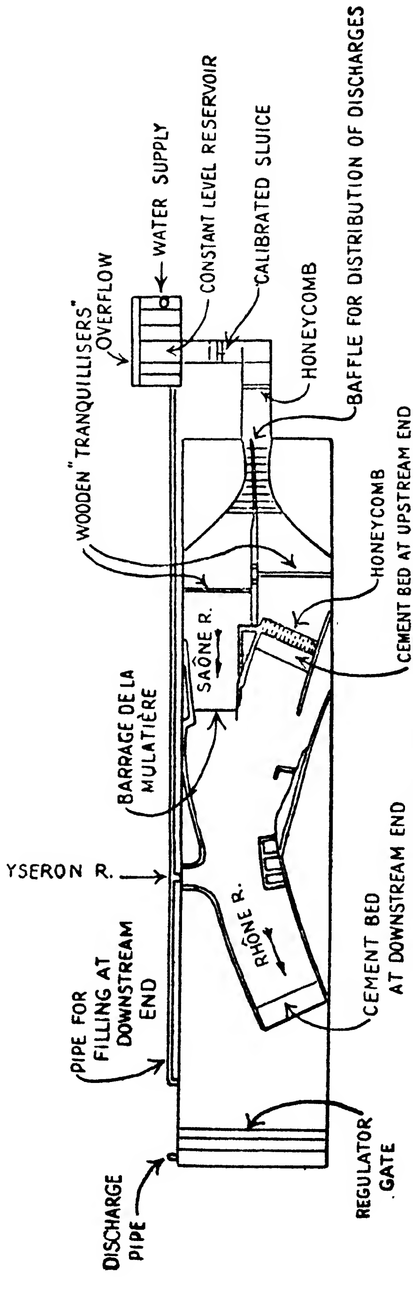
where D is in inches and $(\rho_1 - \rho)$, usually about 103, is in lb. per cubic foot.

Again it should be emphasized that the T_0 considered in these expressions is not the same as that suggested by the Vicksburg Laboratory as

* *U.S. Waterways Expt. Station Paper* 17 (Jan. 1935), p. 29.

a criterion for " model " operation, namely that " the material in transportation shall be reasonably similar in composition to the material composing the original bed " and " the rate of movement is equal to or exceeds one pound (dry weight) per foot width of channel per hour ". With many materials, this criterion yields appreciably higher values of the tractive stress than the T_0 of Kramer's " lower limit ".

To conclude this chapter, a description will be given of some particularly interesting river models. First let us consider a model of the River Rhône in the region of the Barrage de la Mulatière. This model was built in 1934 at the laboratory of L'École des Ingénieurs-Hydrauliciens, Grenoble, France; its object was to study the formation of a shoal which persisted in the vicinity of the mouth of the tributary River Yseron. The model has been described by M. L. Chadenson in the *Annales des Ponts et Chaussées,* 1935, pp. 988–1019 (" Essais sur Modéles Réduits pour des Rivières à Fond Mobile "). The general arrangement of the model is shown in Fig. 55; its mean width was about 6·6 ft. and its length approximately 26 ft. (excluding approach and exit portions). A horizontal scale of 1 : 250 and a vertical



(Model surrounded by rails to facilitate surveying)
Fig. 55.

scale of 1 : 50 were adopted, while the bed material used was a very clean quartzy sand of mean diameter 0·295 mm. (0·0116 inch). The choice of this sand appears to have been influenced by considerations of time, delivery and price: it is not claimed that it was necessarily the best that could have been found, and indeed M. Chadenson considered it to contain elements too fine for ideal performance. In accordance with the adopted distortion (5 : 1) of the vertical scale, the slope would have been 5 times that of the natural river, but as a result of experiments (which again appear to have been restricted by time considerations) the control weir at the downstream end was adjusted to provide a slope of about 6·5 times the natural water-slope.

A check on the performance of the model was made by comparing it with known events in Nature as influenced by high and low discharges and by the action of the tributary River Yseron.* This check having provided reasonably satisfying results, attention was then devoted to the study in the model of proposed remedial works; a scheme was devised and later constructed in the river itself. The testimony of soundings and of pilots' reports confirmed the model findings and gave confidence in the use of another model, this time of a stretch of the Rhône in the vicinity of the Edouard Herriot Dock at Lyons. This model, having a horizontal scale of 1 : 150 and a vertical scale of 1 : 50, was also constructed in the École des Ingénieurs-Hydrauliciens de Grenoble, and has been described in a paper by Monsieur L. Levin, entitled " Étude sur modèle réduit de l'entrée du port Édouard Herriot, à Lyon ", in *Le Génie Civil*, Tome CXIII, No. 14, Oct. 1, 1938, pp. 277–82. An article in English, based upon Monsieur Levin's paper, appeared in *The Engineer*, vol. CLXVII, p. 490, April 21, 1939. The outline of the river as included in the investigation of the Edouard Herriot Dock entrance is shown in Fig. 56. With its horizontal scale of 1 : 150, the moulded portion of the model river must therefore have been upwards of 70 feet in length, not reckoning the inlet and outlet devices. The flow along the reach concerned is made up of the discharges of the Saône and the Rhône, details of which appear in Fig. 57. As the river flows past the entrance to the dock, its current is split into two main components, one continuing along the principal direction of the river, and the other turning in towards the dock itself where it encounters still water and deposits the material which it has been carrying. A complicated system of eddies or swirls exists in the region between the calm water of the dock and the main current of the Rhône.

The substratum of the model bed consisted of cement; above it was

* M. L. Chadenson's paper (*Annales des Ponts et Chaussées*, 1935, pp. 988–1019) also gives his views upon the general theory of river models.

placed the mobile material—powdered pumice of specific gravity 1·57—to a thickness equivalent to some 25–40 feet. In the pumice adopted, about 60 per cent. of the grains ranged between 1·5 mm. (0·059 inch) and 2 mm. (0·079 inch) diameter, while 20 per cent. were between 0·2 mm. (0·008 inch) and 0·5 mm. (0·002 inch): very few grains occurred in the range between 0·5 mm. and 1·5 mm.

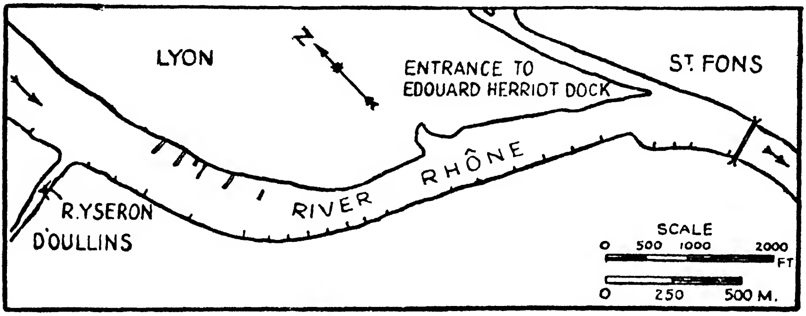


FIG. 56.

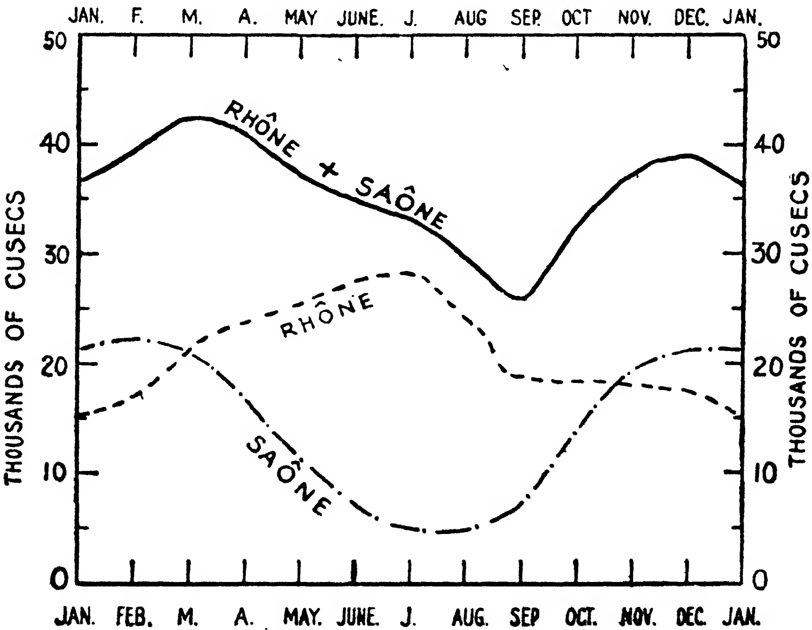


FIG. 57. Mean Monthly Discharges.

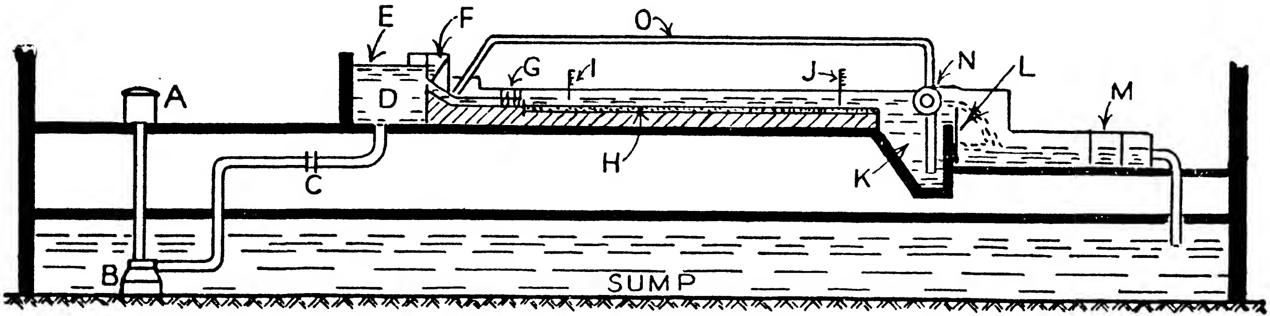


FIG. 58. Scale Model of River Rhône.
Diagrammatic Sketch of Apparatus (not to scale).

A. 30 KW Motor.
B. Centrifugal Pump, 380 litres/sec. capacity.
C. Diaphragm Meter.
D. Constant Level Feed Chamber.
E. Spillway.
F. Calibrated Inclined Gates giving discharge to represent Rhône and Saône.
G. Steadying Baffles.
H. Pumice bed.
I, J. Gauges for measurement of upstream and downstream levels.
K. Settling Chamber.
L. Spillway to give appropriate gradient, operated by hand or by remotely controlled motor.
M. Additional trap for lighter particles.
N. Sand Pump to return material from K via pipe O to upstream end.

The general outline of the apparatus for this investigation is shown in Fig. 58, from which it will be seen that a very neat and compact arrangement was adopted, the water being circulated through the model and the sump by means of a centrifugal pump having a capacity of approximately 13·5 cu. ft. per sec. (or 5,000 gallons per minute). Specially interesting features are the devices for feeding the pumice to the model bed: the pump *N* (with a delivery of about 0·14 cusec) returned the solid matter deposited in the settling tank *K*, according to the rate of settlement, through the pipe *O* to the inlet end of the model. Any pumice finding its way beyond the settling tank could not be dealt with by this means, but a trap *M* was inserted in the exit passage and when some 2 or 3 litres (0·07 or 0·106 cu. ft.) was collected in this trap it was transferred to the upstream end of the model. The vertical gate *L*, for control of the downstream level of the water in the model, could be actuated either by hand or by a remotely controlled motor.

Preliminary tests with the arrangement indicated in Fig. 59 showed

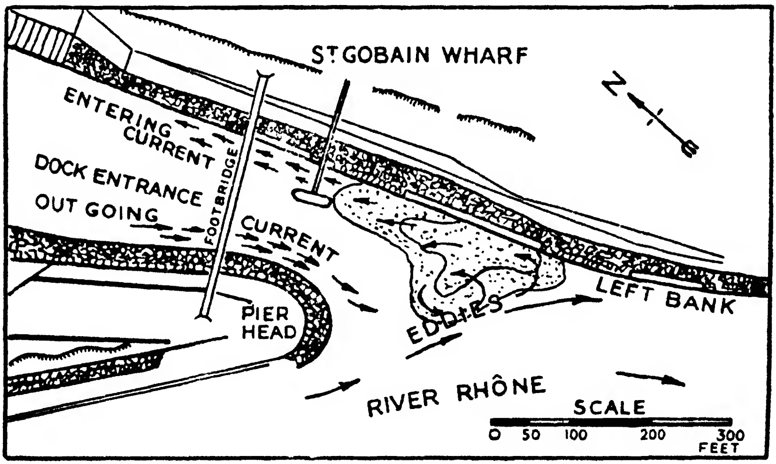


FIG. 59. Currents and Deposits at Entrance to Edouard Herriot Dock as observed in Model Experiments.

quite clearly the action of the current entering the dock entrance from the left bank of the river and producing a shoal. From this it was argued that means should be devised to prevent the bed current from entering the dock in that way. This conclusion was confirmed when a dam was constructed up to low-water level across a large part of the entrance: this effectively prevented the entry of bed currents and correspondingly of solid matter. Such a device was impracticable however from the point of view of navigation as well as of construction.

Later experiments showed the beneficial effects likely to result from an extension of the jetty in the form of an additional wall submerged at the higher river stages, and an exhaustive study of this groyne, as to its best length, width, height and shape led to the scheme shown in Fig. 60.

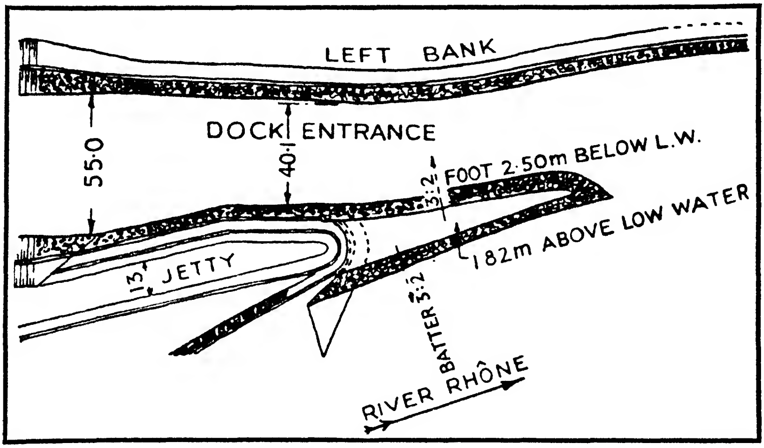


FIG. 60. Final Scheme for Edouard Herriot Dock Entrance. (Dimensions in Metres.)

In his concluding remarks, M. Levin states that experiments on a model of this type lead to correct results (" résultats exacts ") provided that

(*a*) there is a small distortion of scale (not exceeding 5: 1 [" Admettre une distortion faible (ne pas depasser 5) "];
(*b*) a suitable bed material is chosen;
(*c*) the hydraulic elements (slope, velocity, discharge) are determined and verified on the model in reproducing a state of the bed known to exist in Nature.

Concerning M. Levin's statement about distortion of scale, it should be noted that models have been successfully operated at Vicksburg, Missis-sipi, with distortions of 8:1 and 10:1, even though the effect of spur dikes or groynes has been involved. We shall also deal later with tidal models in which still bigger distortion has been employed.

The next investigation to be discussed is that made at the U.S. Water-ways Experiment Station concerning the improvement of the Mississippi at Brooks Point.* For several years, the crossing just below the Point had given trouble to navigation, and extensive dredging had been required

* *Model Studies for Channel Stabilization, Mississippi River*, Paper 15 of the U.S. Waterways Expt. Station, Vicksburg, Jan. 1934, pp. 15–33; also p. 62 of " Hydraulic Laboratory Results and their Verification in Nature ", H. D. Vogel, *Proc. Am. Soc. C.E.*, vol. 61, 1935, pp. 57–73.

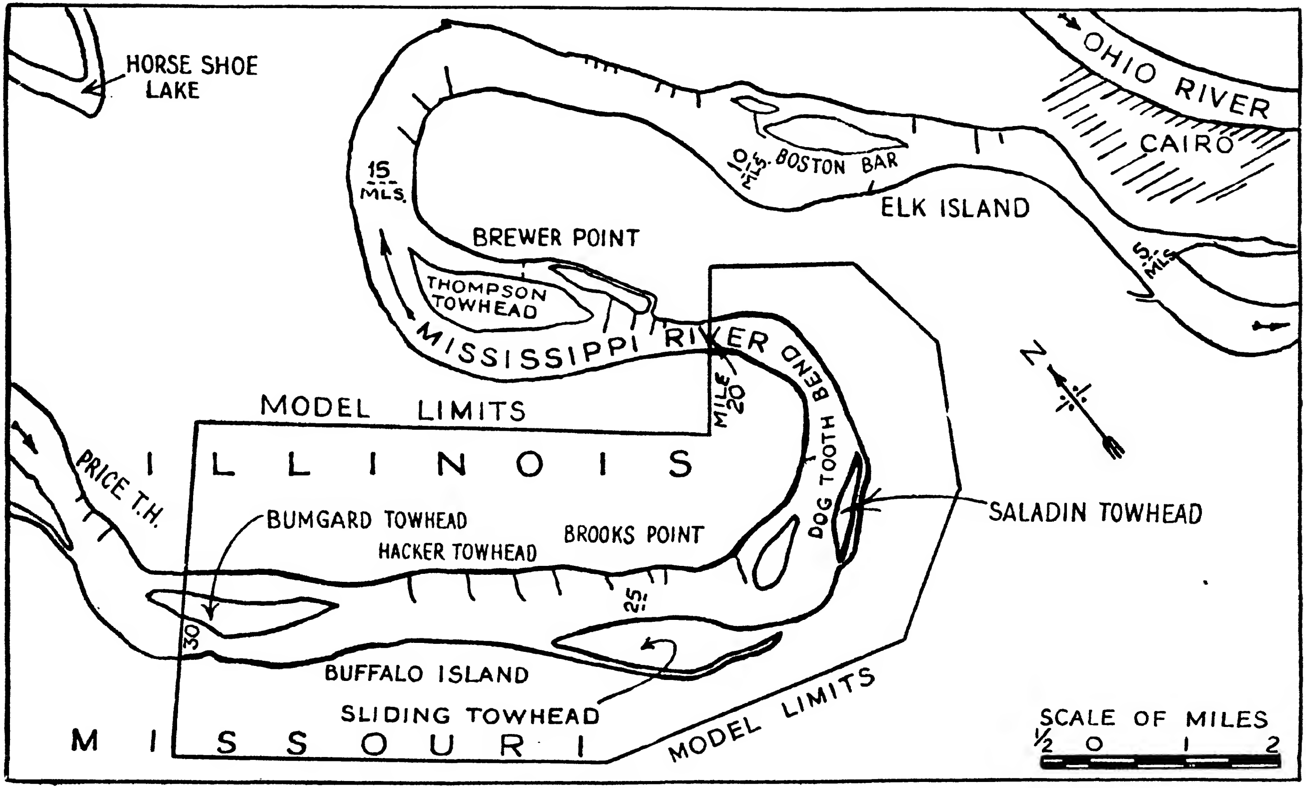


FIG. 61. Sketch-Map of Mississippi in vicinity of Brooks Point.
(Based on Plate 11 of Paper 15, U.S. Waterways Experimental Station, Vicksburg.)

to provide the necessary channel. Moreover, the current was attacking an existing revetment below Saladin Towhead (see Fig. 61), thus threatening to break through the neck less than 1·5 miles wide in the loop of the river and possibly to endanger the city of Cairo, Illinois.

A proposal was made to remove part of the existing dike system on the left bank and to construct fresh ones. One general survey only (September 1932) was available, and in the absence of other data the model was subjected to a " flat-bed verification test " in addition to a " stability test ".

The horizontal scale was 1 : 1,000 and the vertical scale 1 : 125; the moulded length of the channel being 10 miles, represented by some 53 feet in the model. A programme of river stages was devised from data covering about twelve years, and an operating cycle (apparently found by a process of trial) of sixteen hours was adopted, with stages varying by 5-foot intervals from low to high and back, the duration of the stages being in proportion to those revealed by hydrographs in Nature. Preliminary experiments were devoted to the adjustment of slopes, discharges and roughness, and with the bed moulded to 1932 conditions an 8-hour cycle was run during which no marked changes occurred; " this stability of stream bed provided evidence that the model was correctly designed and operated ", but a further test was carried out by moulding the bed flat, with the existing dikes in place, and subjecting the model to 7 cycles of operation, totalling some 100 hours, at the end of which it was found that the bed closely resembled that of the prototype survey of 1932 (compare Figs. 62*a* and 62*b*). Sand was introduced at the upstream end at the same rate as it was found to collect in the sand-trap at the downstream end.

Experiments were then devoted to studying the possibilities of improving the channel; these tests resulted in the selection of a scheme involving the building of five spur-dikes on the right bank just above the crossing and the removal of three dikes on the left bank. A survey made about two months after the completion of the new groynes but before the complete removal of the three old ones is shown in Fig. 63*a*, and appears to represent a transition stage towards the model indications shown in Fig. 63*b*.

Lieut. Vogel remarks that " while the experiments on the model of Brooks Point were in progress the Station was visited several times by the engineers in charge of the channel improvement works in the vicinity represented by the model. They were unanimous in voicing their opinion that the model simulated to a marked degree the conditions of Nature ". A general view of the Brooks Point model is reproduced (by kind permission of the U.S. Waterways Experiment Station) in Plate XI.

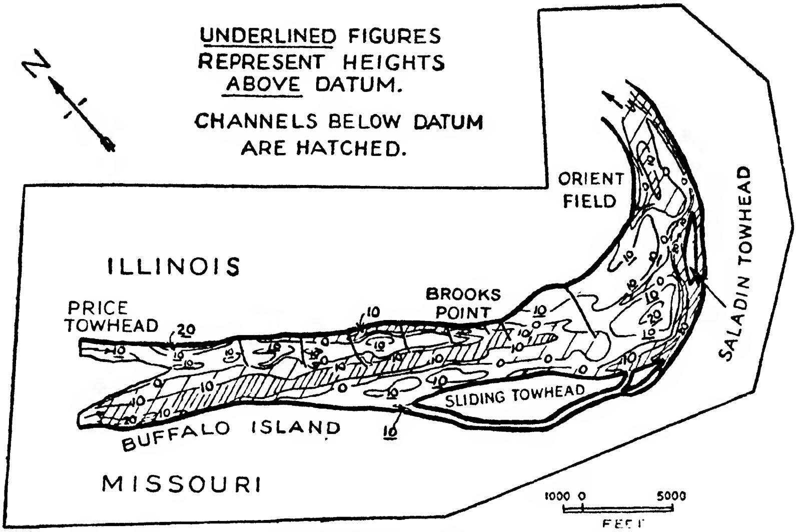


FIG. 62*a*. Principal Features of Actual River, Sept.–Oct., 1932.
(Based on Plate 13 of U.S. Waterways Experimental Station, Paper 15.)

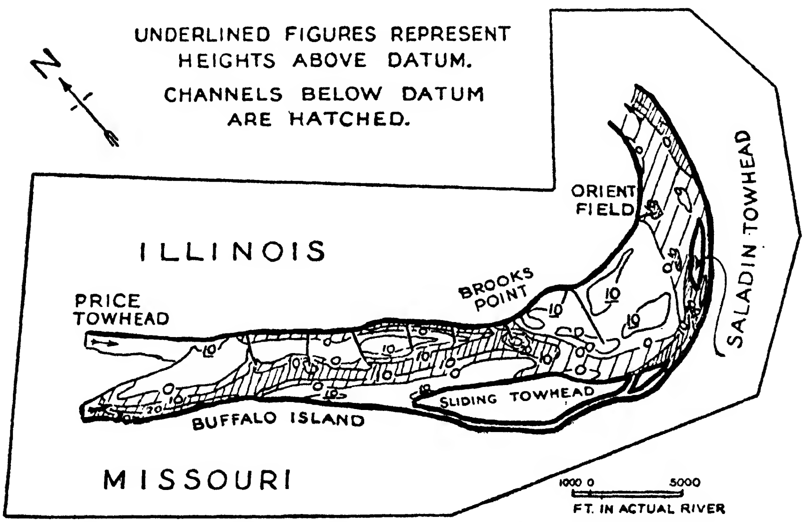


FIG. 62*b*. Principal Features of Model at end of test, starting with a flat bed. (Compare with Fig. 62*a*).
(Based on Plate 15 of U.S. Waterways Experimental Station, Paper 15.)

Another instructive study of the Mississippi is that made (also at Vicksburg) of the Point Pleasant reach.* In this case, three permeable pile-dikes of total length 8,232 lin. ft. had been completed during 1932, but

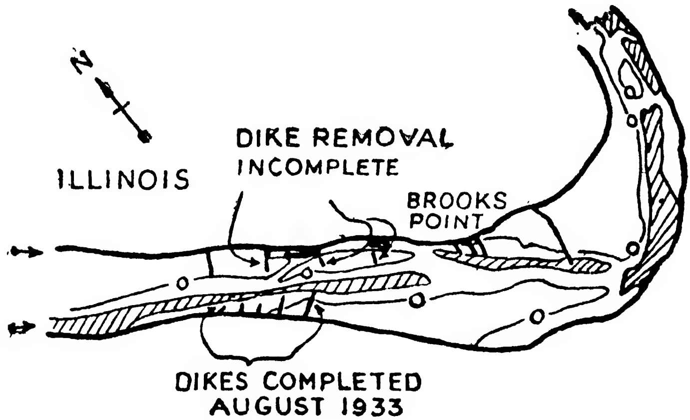


FIG. 63*a*. Principal Features at Brooks Point, October 1933. Hatched areas represent depths greater than 10 ft.

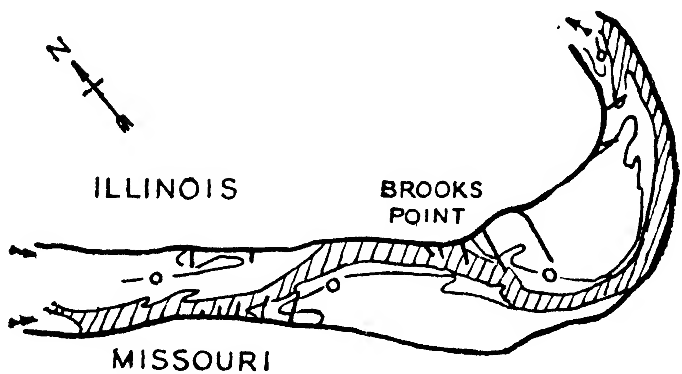


FIG. 63*b*. Indicated Solution by Model Experiments of Channel Improvement at Brooks Point, Ill.

(Figs. 63*a* and 63*b* based on *Proc. Am. Soc. C.E.*, vol. 61, 1935, pp. 63, 62.)

* Paper 15 of U.S. Waterways Expt. Station, Vicksburg, Jan. 1934, pp. 44–56; also p. 65 of "Hydraulic Laboratory Results and their Verification in Nature", H. D. Vogel, *Proc. Am. Soc. C.E.*, vol. 61, 1935, pp. 57–73.

observations made in August 1932 revealed no resulting improvement in the channel. Accordingly, it was suggested that the dikes had been incorrectly placed and should be removed. Model tests indicated, however, that the Point Pleasant groynes were properly placed and would in time produce navigable depths, provided that they were capable of with-

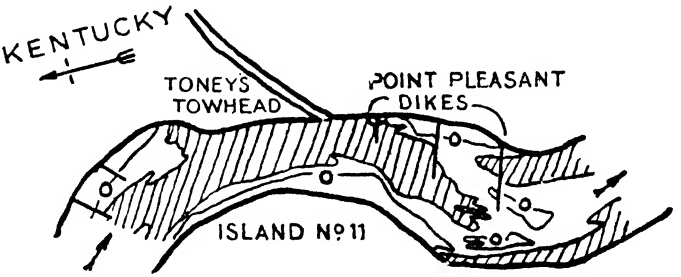


Fig. 64*a*. Condition at Pt. Pleasant, Ill., August, 1932.
Hatched areas represent depths greater than 9 ft. below mean low water.

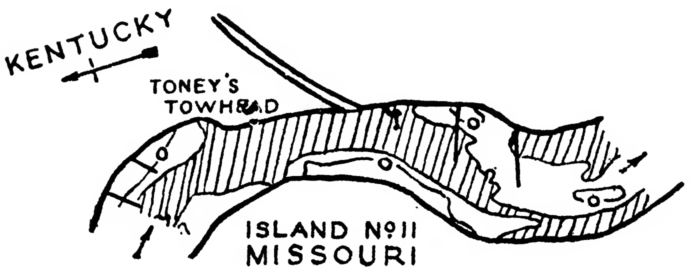


Fig. 64*b*. Laboratory Model Tests indicated eventual effectiveness of structures in about one year.
Hatched areas represent depths greater than 9 ft. below mean low water.
(Based on *Proc. Am. Soc. C.E.*, vol. 61, 1935, p. 67.)

standing the scouring effect of the current. Indeed, the indications were that in periods of low flow a navigable channel with controlling depths of approximately 10 ft. at mean low water would be obtained.

Fig. 64*a* shows the condition of the river itself in August 1932, while Fig. 64*b* represents the model findings as to the condition likely to ensue in about one year.

In July 1933, the Area Engineer did, in fact, find depths of 10 feet over almost all the critical region, and Lieut. Vogel states that " during the next month a continuous channel, 10 feet deep, would probably have developed, as scouring occurs on the crossings during low water " (see Fig. 65). This

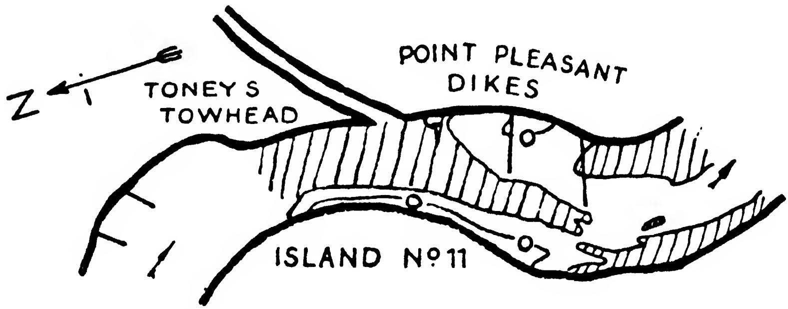


FIG. 65. Condition of Channel in Pt. Pleasant Reach in July 1933, eleven months after original survey.

Hatched areas represent depths greater than 9 ft. below mean low water.

(Based on *Proc. Am. Soc. C.E.*, vol. 61, 1935, p. 67.)

would have given a complete verification of the model predictions. In order that there should be no jeopardy to navigation, however, some dredging was done at that time (July). Thereafter, the Area Engineer reported under date of October 27, 1933, that the channel gave no trouble whatever. In fact, as of that date, it had scoured approximately 3 ft. deeper. This was the first time in years that the crossing was not dredged several times.

" Thus, there is—in the foregoing case—an instance in which, through the instrumentality of a model, the eventual effectiveness of existing structures was shown. Such results may be termed ' ratifying '. The aggregate cost of the Point Pleasant dikes, at an average cost per linear foot of between \$30 and \$35 was approximately \$265,000. Since it is frequently as expensive to remove dikes from a stream as to build them, it would seem that results which lead to such economies are of as great value as those which indicate new plans of attack."

The scales of the Point Pleasant model were 1 : 1,000 (horizontally) and 1 : 125 (vertically), and slope distortion was provided to ensure proper bed movement. The slope in the model was 0·0015 at mean low water, as compared with 0·0001 in Nature. The boundaries of the model were chosen so as to embrace the area which would be inundated by a 40-foot stage, and an operations programme was computed from five years' records. A cycle of eight hours was found to be satisfactory, during which

10-foot interval stages from 10 ft. to 40 ft. (rising) back to 10 ft. were simulated in proportion to the periods of the composite stage cycle as based on the five years of observations in Nature.

A general view of the Point Pleasant model is reproduced (by kind permission of the U.S. Waterways Experiment Station) in Plate XII.

CHAPTER V

MODELS OF TIDAL RIVERS

THE first serious attempt to construct a model of a tidal river along scientific lines was made by Professor Osborne Reynolds in 1885.* He seems to have started only with the idea of demonstrating the circulation of the water and the accompanying eddies in the estuary of the Mersey, and seeing whether or not his theories concerning this circulation and its secondary effect in producing channels were correct. His first model, embracing the region between the Liverpool narrows and a point some distance above Runcorn, had a horizontal scale of 1 : 31,800 and a vertical scale of 1 : 960, giving a distortion of approximately 33 : 1. The bottom was flat and the coastline represented by a vertical boundary. Sand was placed level in the model and tides were produced by means of a hinged tray or pan, which will be described in Chapter IX, at first operated by hand but later by a continuously-driven shaft, because the early trials gave promise of interesting results both from the standpoint of tidal phenomena and that of bed movement. Reynolds remarks that the working of the model by hand revealed that only one period—about 40 seconds—gave a correct imitation of the tidal phenomena in the actual Mersey; " a result that might have been foreseen from the theory of wave motions, since the scale of velocities varies as the square roots of the scales of wave heights, so that the velocities in the model which would correspond to the velocities in the channel would be as the square roots of the vertical scales—about 1/33—and the ratios of the periods would be the ratio of horizontal scales divided by this ratio of velocities ". This step in Reynolds' reasoning is of vital importance as being the first attempt, it is believed, to introduce the time-factor. This time-scale may be derived quite simply, then, by Reynolds' argument, as follows: the velocity of the propagation of the tidal wave is proportional to the square root of the depth, H, of water in which it is travelling—this is a mathematical and experimental fact. Let h represent the corresponding water-depth in the model, and let v, V represent velocities in the model and the prototype respectively. Then $\frac{v}{V}=\sqrt{\frac{h}{H}}$. But the time taken by such a

* *On Certain Laws relating to the Régime of Rivers and Estuaries and on the Possibility of Experiments on a Small Scale*, Reynolds, Brit. Assoc. Rpt., 1887; or *Papers on Mechanical and Physical Subjects*, vol. II, p. 326 (Camb. Univ. Press, 1901).

wave to cover a distance L (or l in the model) is proportional to L and to $\frac{1}{V}$.

Therefore corresponding times t, T in model and in Nature must be connected by the relationship

$$\frac{t}{T}=\frac{\frac{l}{v}}{\frac{L}{V}}=\frac{l}{L}\cdot\frac{V}{v}=\frac{l}{L}\sqrt{\frac{H}{h}},$$

which is equivalent to writing

$$\frac{t}{T}=\frac{1}{x}\sqrt{y},$$

where the horizontal scale is $1:x$ and the vertical scale is $1:y$.

Thus, if the tidal periods t_0 and T_0 are to be related by this same time-scale,

$$\frac{t_0}{T_0}=\frac{\sqrt{y}}{x}.$$

Suppose then that the mean tidal period in Nature is 12·4 hours (44,600 seconds) and that $x=31{,}800$ and $y=960$. In this case,

$$t_0=\frac{44600\times\sqrt{960}}{31800}=43\cdot5 \text{ seconds},$$

which is the appropriate period for the model. The period actually used by Reynolds was "42 seconds (about)".*

During the course of 2,000 tides (presumably spring tides) the sand was shaped by the action of the currents so as to agree closely with the configuration in Nature: † "the eastern bank, with the deep sloynes on the Cheshire side, the Devil's Bank and the Garston Channel, the Ellesmere Channel and the deep water in Dungeon Bay and at Dingle Point—all these were very marked in character and closely approximate in scale.

"And, what is as important, the causes of these as well as all minor features could be distinctly seen in the model."

A check test was then made, starting with a flat bed of sand, and again the general features of the estuary emerged. "So interesting were these

* There appears to be some discrepancy in Reynolds' *Scientific Papers* (*loc. cit.*), p. 332, as there the tidal period in Nature is taken as "11·25 hours or 40,700 seconds". On the other hand, the square root of 960 is taken as about 33 instead of 31, the net result being a figure of "42 seconds (about)" for the model. If *12·25* hours is meant, the period becomes 43 seconds, using $\sqrt{960}=31$.

† For comparative charts, see *Hydraulic Laboratory Practice*, ed. Freeman (Am. Soc. Mech. Engs., 1929), p. 46.

results that it was decided to try a larger scale. A model, having a horizontal scale of 6 inches to a mile" (1 : 10,560), " and a vertical scale of 33 feet to an inch " (1 : 396) " was therefore made, and the tide produced as before. The calculated period of this model is 80 seconds,* and experiment bears this out, any variation leading to some tidal phenomena, such as bonos or standing waves, which are not observed in the estuary ".

After a run of 6,000 tides, a survey of the model showed results presenting as great a resemblance to any one of the 1861, 1871 and 1881 charts of the Mersey as they did to each other.

Moreover, Reynolds points out that, having regard to the changes always going on, within certain limits, in the bed of both estuary and model and the resultant influence of these changes on currents and tides, he discerned agreement in current and tide phenomena. His views about the distortion or exaggeration of the vertical scale have already been quoted on page 146, but it is not inappropriate to repeat one important sentence: " From my present experience in constructing another model, I should adopt a somewhat greater exaggeration of the vertical scale."

Reynolds himself gives no indication of the size of sand which he employed in these models, but Professor A. H. Gibson states† that he " has been able to obtain a sample of Calais sand as used in the models of the Mersey estuary. This has a mean diameter ‡ of 0·00648 in. and a specific gravity of 2·54. Samples of sand from a series of points in the estuary of the Mersey itself have been supplied to the writer [Professor Gibson] by the courtesy of Mr. T. M. Newell, late Chief Engineer to the Mersey Docks and Harbour Board. The mean diameter ‡ of these is 0·00825 in., so that the ratio of diameters in this case is 1·27 : 1 ".

Following upon these classic experiments, Reynolds pursued further tests (on behalf of a Committee of the British Association) with the object of finding how far the bed configuration in such models is affected by the horizontal and vertical dimensions—and the relationship between these dimensions, as well as their connection with the tidal period. His findings on these problems are contained in *British Association Reports*, 1889, 1890 and 1891, and are conveniently reproduced in his *Papers on Mechanical and Physical Subjects*, vol. II, pp. 380–518 (Camb. Univ. Press, 1901). This part of his researches was made on sand contained in tanks of " geometrical shape ": rectangular, V-shaped, and V-shaped with

* $\frac{44600 \times \sqrt{396}}{10560} = 84{\cdot}2$ seconds; for 80 seconds it is necessary to assume a period of about 11·75 instead of 12·4 hours in Nature.

† *Construction and Operation of Severn Tidal Model*, A. H. Gibson, H.M. Stationery Office, 63-78-82, 1933, p. 23.

‡ As measured under a microscope.—J. A.

straight tidal rivers added at the upper ends. These were built to different scales and worked at correspondingly different tidal periods; in the later stages of the work, he also tried the effect of land water introduced at the top of the tidal through a calibrated orifice and—for the first time as far as the Author is aware—of a cycle of spring and neap tides generated mechanically by means of a system of gearing " designed by Mr. Greenshields ". In certain experiments, training walls and groynes were introduced.

Briefly, his more important conclusions, based on general observations and on detailed comparison of the behaviour of the different models, may be summarized as follows:

(*a*) When water flows through a channel at speeds higher than the critical, the eddies contained in it create an unevenness of the surface and a consequent distortion of reflections, thereby giving the appearance of a swirl to the surface. In cases where similarity failed, this swirl was absent at the beginning of the experiments.

(*b*) To ensure that the motion shall be turbulent and that symptoms of bed dissimilarity (such as the configuration being overwhelmed by excessive ripples) shall not appear, it is necessary that

$$h^3e \text{ shall not be less than } 0{\cdot}09.$$

Here h represents the spring tidal range, in feet, at the mouth of the model and e the vertical exaggeration of scale as compared with a 30-ft. natural tide. In other words, for comparative purposes, the exaggeration is to be determined, with the chosen tidal period of the model, by reference to a 30-ft. tide as standard. This will be made more clear by resort to symbols:

Let the period in the model be t seconds.
Let the period in the actual be T seconds.
Let the horizontal scale of the model be $1:x$.
Let the vertical scale of the model be $1:y$.
Let the spring tidal range at the mouth of the model be h feet.
Let the spring tidal range at the mouth of the actual be H feet.

Now if the range of tide in the actual estuary were 30 ft. instead of H, the vertical scale would not be $1:y$ but would be $1:y_1$, where

$$y_1 = \frac{30}{h}.$$

Also, for the given tidal period t the effective horizontal scale $1:x_1$ would be given by

$$\frac{t}{T} = \frac{\sqrt{y_1}}{x_1},$$

or
$$x_1 = \frac{T}{t}\sqrt{y_1}.$$

But
$$\frac{T}{t} = \frac{x}{\sqrt{y}};$$

$$\therefore\ x_1 = x\sqrt{\frac{y_1}{y}};$$

$$\therefore\ \frac{x_1}{y_1} = e = \frac{x}{\sqrt{y}}\,\frac{1}{\sqrt{y_1}} = \frac{x}{\sqrt{y}}\sqrt{\frac{h}{30}}.$$

Hence
$$h^3 e = h^3 x\sqrt{\frac{h}{30y}} = h^3\frac{x}{y}\sqrt{\frac{hy}{30}}.$$

But $hy = H$, by definition of the given vertical scale;

$$\therefore\ h^3 e = h^3\frac{x}{y}\sqrt{\frac{H}{30}}.$$

According to Reynolds, then, this last quantity, namely $h^3\frac{x}{y}\sqrt{\frac{H}{30}}$, should be not less than 0·09 if the tidal model is to work satisfactorily. Yet, if we consider Reynolds' own models of the Mersey, we find that they must have worked at much lower values of the criterion than 0·09. In those cases, H must have been of the order of 30 ft., so that in the first Mersey model ($x = 31{,}800$, $y = 960$, $h = \frac{30}{960}$, about), $h^3 e$ must have been approximately $(\frac{1}{32})^3(33)$ or 0·001. In the second Mersey model, $x = 10{,}560$, $y = 396$ and $h = \frac{30}{396}$, about, so that $h^3 e$ becomes $\left(\frac{1}{13\cdot 2}\right)^3(26\cdot 7)$ or 0·0125. Despite these low criteria, the models worked well, and it is therefore desirable to examine the validity of the criterion more closely.

In the first place, it was chiefly based upon experiments in two models of different sizes, over the greater area of the bed of which sand was initially laid. In the vast majority of the tests this sand started flat with a level equal to mean tide level at the mouth of the model, and in most of the experiments the model estuary was symmetrical in form. The magnitude assigned to h was that of the observed range of tide *at the conclusion of the experiment*—that is, after a state of near-stability had been created by the running of several thousand tides. It is doubtful whether the effect of asymmetry and of different initial bed levels was explored sufficiently to yield final conclusions; indeed it is true that to cover the whole range of possibilities exhaustively would have involved a very great extension of the already considerable programme of Reynolds' work.

But it is instructive to study his experiments on models rendered artificially asymmetrical by the introduction of " groynes " such as those indicated in Fig. 66. Supposing the tide generated at the mouth of the model to represent a 30-ft. natural tide, the vertical scale is calculable, and hence, from the formula for tidal period, so also is the horizontal

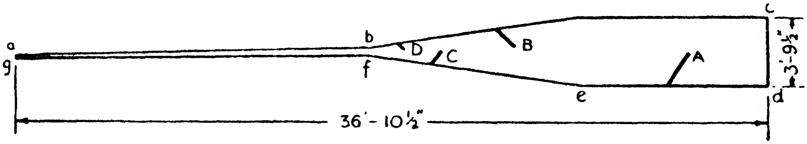


FIG. 66. *abcdefg*; Model Estuary (Osborne Reynolds); (*cd* is seaward end).
A, B, C, D: Groynes or spur dikes, introduced to render the model asymmetrical.

scale for the period as chosen. Such reasoning implies that the seaward " groyne " *A* in Fig. 66 represents a projecting irregularity some 11 miles in length, half a mile broad and 100 feet high up to high-water level; consequently the tidal currents are rendered very complex and a pronounced eddy is superimposed on the general hydraulic régime. This device, then, would certainly serve to imitate the non-symmetrical conditions of flow common in actual estuaries. It was tried in two models, designated *E* and *F*, *E* being about twice the length of *F* and having about a 50 per cent. longer tidal period. Model *E* was operated on a spring-neap cycle (29 tides from one spring to the next), while *F* worked on spring tides only. Correspondingly the rate of progression in *E* was slower than in *F*, but despite differences in initial conditions, similar results were obtained after 60,000 tides in *E* and 36,000 in *F*. Reynolds declares, in fact, his conviction that had it been practicable to prolong the test on model *E* sufficiently the final condition would have been " precisely similar " to that exhibited by *F*. Now the remarkable feature of these tests is that the h^3e criterion for *E* was of the order of 0·10, while that of *F* was only 0·03. One concludes therefore that in the case of estuaries rendered unsymmetrical, as in Nature, not only by irregularities of coastline near the mouth but also throughout their length, the condition that h^3e shall not be less than 0·09 is well on the safe side. This conclusion is substantiated by his earlier models of the Mersey and by his tests on longitudinal training walls in his " geometrical " estuaries *E* and *F* where *E* had a criterion of nearly 0·10 and *F* one of only 0·03.

The mathematical form of the criterion h^3e was justified by Reynolds as follows:

the velocity in uniform flow along an open channel of hydraulic mean depth m and slope i is given by $v = C\sqrt{mi}$, where C is a " constant ". If i is proportional to e, the exaggeration of scale, and if m is proportional to the tidal range h, this becomes $v \propto \sqrt{he}$. But the critical velocity for turbulence is inversely proportional to m, for a given water temperature (or viscosity), and since m has been assumed proportional to h, this means $v \propto \frac{1}{h}\cdot$ Consequently $\frac{1}{h} \propto \sqrt{he}$, or $h^3e = \text{constant}$. "The function $h^3e = \text{constant}$ is thus a criterion of the conditions under which similarity in the rate and manner of action of the water on the sand ceases." With some diffidence, however, it appears desirable to point out that in the original expression $v = C\sqrt{mi}$, C is not strictly constant but is itself a function of m, while again, due to the exaggeration of the vertical scale, m is not simply proportional to h or to any vertical dimension; rather is m a complex function of h and the horizontal scale-ratio combined. To sum up this part of the discussion, therefore, it seems reasonable to assert that if the Reynolds criterion $h^3e \nless 0{\cdot}09$ is satisfied—and a suitable bed material is chosen—there is every probability of success, however regular may be the shape of the estuary concerned; on the other hand, it is highly possible to succeed with much lower values than 0·09 in the average case of the irregular, unsymmetrical estuaries found in Nature. A different criterion will be advocated at a later stage.

(*c*) The next point to be discussed as emerging from Reynolds' investigation is his emphasis upon the importance of keeping the surface of the water free from scum. Unless this is done the surface velocities are very appreciably reduced, and correspondingly the distribution of velocity through the depth and width of the stream is considerably affected. In order to maintain a clean surface, Reynolds adopted an extremely simple and efficient device: near the seaward end of the model an overflow weir was installed. During those experiments in which land water was supplied at the top end of the model, this weir was adjusted so as to lick off the surface and preserve a constant volume of water in the model; in other words, its average rate of discharge was equal to the rate of input of land water. In other tests, when no land water was being supplied, an artificial flow was admitted at the seaward end so as to enable the overflow weir or "scummer" to be put into operation. This may appear to be a minor point, but in practice some form of scummer such as that shown in Fig. 67 is vital to the successful working of a tidal model; it is astonishing how tenacious may be the film of greasy dust deposited on the surface of the water from the atmosphere, how great is its effect on velocities, and yet how completely effective is the scummer in removing the offending film. Reynolds' models were also, incidentally, covered over with glass, partly to assist in keeping the model sand-banks clean, and partly as an aid in surveying the bed. The glass was marked out in reference squares by means of a network of black thread stretched against the lower surface of the glass. For the purpose of surveying, points on the contour of the boundary of sand and static water were referred to this system of squares,

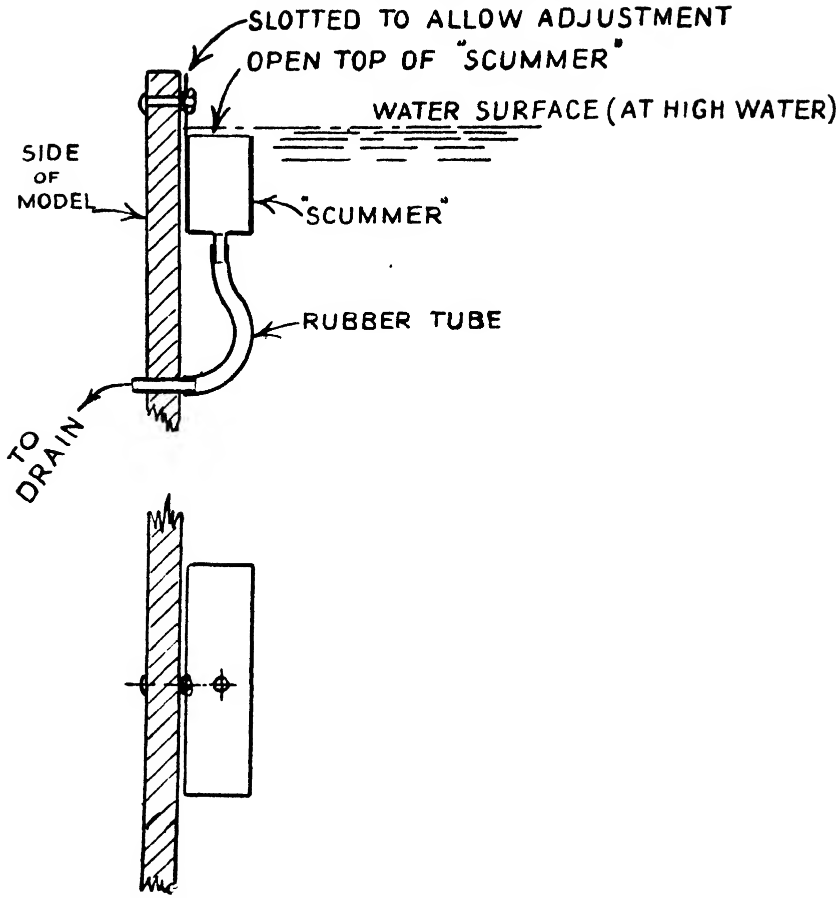


FIG. 67. *Above*—Sectional Elevation. *Below*—Part Plan.

the points being observed through an instrument, resting on the top of the glass, such as is sketched in Fig. 68.

(*d*) Reynolds appears to have been the first to suggest the use of such models in the study of river pollution. "The land water, one quart a minute, was brought from the town's mains in lead pipes. It is very soft, bright water, and was introduced at the top of the estuary. This went on for about three weeks. At the commencement the sand was all pure white, and remained so throughout the experiment except in the tidal river. At the top of the river a dark deposit, which washes backwards and forwards with the tide, began to show itself after commencing the experiment, gradually increasing in quantity and extending in distance. At the end of the experiment the sand was quite invisible from a black deposit at the head of the river and for 5 or 6 feet down; this, then, gradually shaded

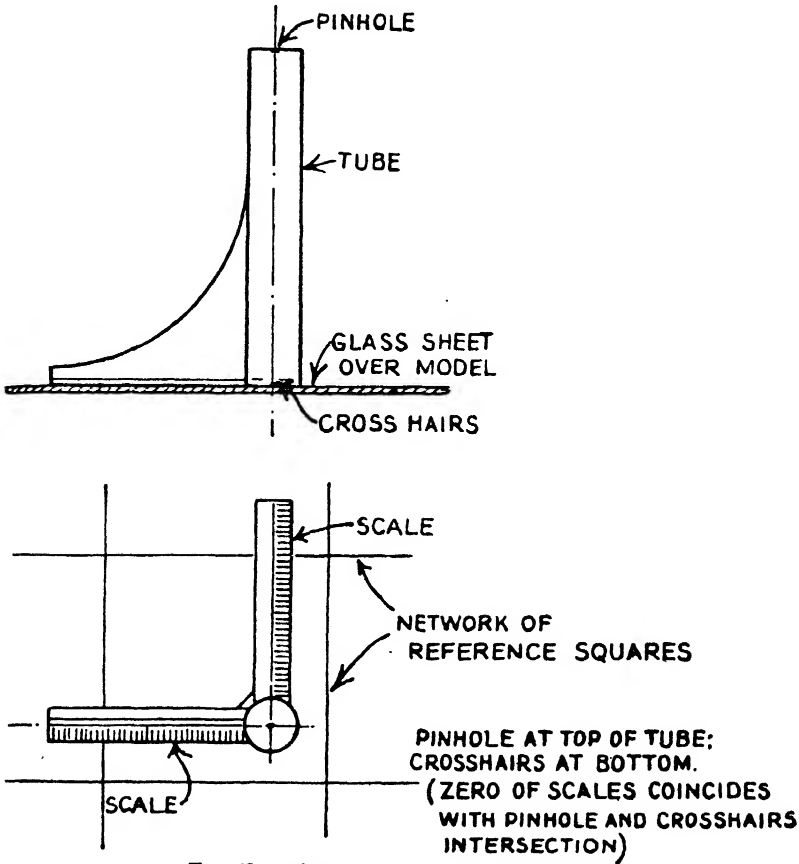


FIG. 68. *Above*—Elevation. *Below*—Plan.
Diagrammatic arrangement of Surveying Apparatus for Tidal Model (Osborne Reynolds).

off to a distance of 12 feet. Nor was it only a deposit, for the water was turbid at the top of the river and gradually purified downwards.

" On the other hand, in the precisely similar experiment, without land water the sand remained white and the water clear right up to the top of the river. This seems to suggest that these experiments might be useful to those interested in river pollution."

(*e*) Concerning the effects of spring-and-neap tides as compared with uniform spring tides, Reynolds found the state of " final equilibrium " of the bed to be identical, but attainment of this condition occurred in two-fifths the time when spring tides only were employed. The ratio of the spring and neap tidal ranges provided at the mouth of his models was 3 : 2. Further, he found that if constant or uniform tides of range equal to the mean of springs and neaps were employed, *essentially different* results ensued. In so far as these statements may be applied to the purely tidal portion of an estuary, they are probably true in a marked degree and frequently justify the convenient and relatively speedy device of experimenting with only spring tides in the preliminary stages of an investigation of proposed works. But later experience shows quite clearly that in the upper reaches of an estuary and more especially in regions above the point of uttermost penetration of neap tides, the results obtained with spring tides only are *not* the same as with any number of spring-neap cycles. Nor is it reasonable to suppose otherwise, since in the upper reaches (above the limit of neap tides) the régime must be predominantly influenced for long periods of time by the river water alone.

(*f*) Concerning the influence of land or river water, Reynolds laid great stress on the fact that even a comparatively small river flow is important, because it keeps the upper reaches open which otherwise might be completely blocked by sand driven upstream on the flood tide.

The next tidal model investigation to be undertaken in this country appears to have been by Mr. L. F. Vernon-Harcourt,* and was more particularly devoted to a study of methods of improving the navigation channels of the Seine estuary, although an attempt was made to draw conclusions of a more general nature. This investigation deserves a special place in the history of tidal models for several reasons: firstly, it reveals a distinguished consulting engineer of wide practical experience inspired by the experiments of Reynolds to operate a model in his own office; secondly, his painstaking trial of many kinds of bed material; thirdly, his careful analysis of the results obtained; and fourthly, his expressed conviction that " direct experiment for each estuary is undoubtedly preferable to abstract reasoning, where such experiment is possible, as it reproduces the special conditions of the estuary to be investigated ".

In his communication to the Royal Society, Mr. Vernon-Harcourt emphasizes the fact that whenever important training works through estuaries were proposed the opinions of engineers were of a very conflicting nature: in the case of the Seine fourteen schemes had been published

* L. F. Vernon-Harcourt, " The Principles of training Rivers through Tidal Estuaries, as illustrated by Investigation into the Methods of improving the Navigation Channels of the Estuary of the Seine ", *Proc. Roy. Soc.*, vol. XLV, 1888–89, p. 504.

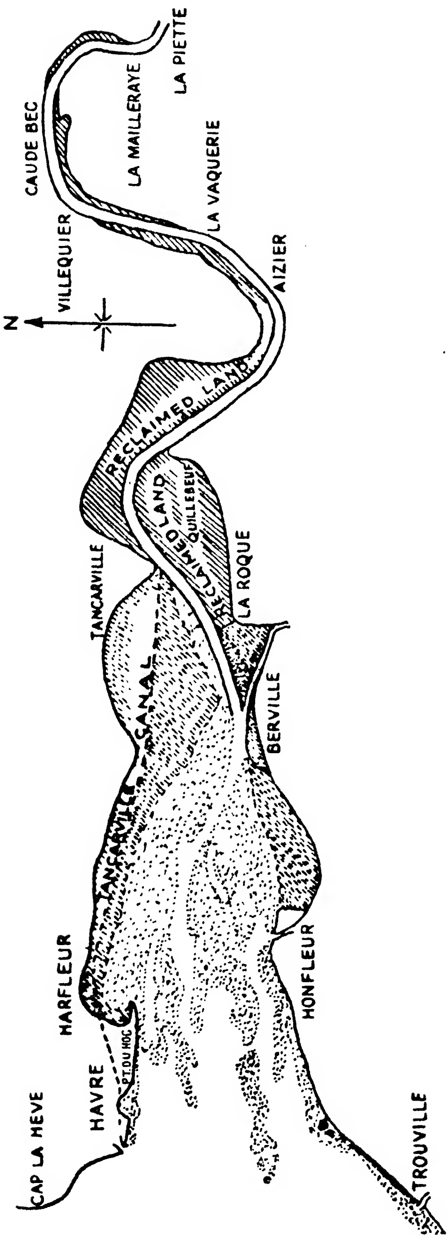


FIG. 69. 1880 Plan of Seine Estuary.
(From Plate 4, Fig. 1, *Min. Proc. I.C.E.*, vol. LXXXIV.)

in France, seven of them in five years, and "hardly any part of the estuary below Berville had not been traversed by some proposed trained channel, except the portion lying north of a line between Hoc and Tancarville points, which is too far removed from Honfleur to be admissible for any scheme".

Training works in the lower part of the tidal Seine had been started in 1848 and had reached Berville by 1870; they were stopped because of large unexpected accretions behind the walls and at the sides of the wide estuary below them and a consequent feeling of anxiety for the port of Havre. Any further works would need to train the shifting channel between Berville and the sea, and to improve access to Honfleur without prejudicing the approaches to Havre (see Fig. 69).

Vernon-Harcourt adopted the following scales for his model of the Seine: horizontal, 1 : 40,000; and vertical, 1 : 400. The model was nearly 9 feet long, and he made "the vertical scale one hundred times the horizontal, as the fall of the bed of the tidal Seine is very slight, and the rise of spring tides at the mouth, being 23 feet 7 inches, amounted to an elevation of the water in the model of only 0·71 inch ". The Reynolds criterion, $h^3 \frac{x}{y}\sqrt{\frac{H}{30}}$, must, incidentally, have been 0·018, and the tidal period appropriate to the chosen scales would be 22·5 seconds if the period in Nature were 12·4 hours: the period actually employed is stated to have been about 25 seconds, though there is no record of any mechanical means of ensuring this other than by raising and lowering the hinged tray used as a tide generator by means of the " screw of a letter press ". An important point, however, is that the tide-producing tray was arranged at such an angle that the tidal flow approached the estuary from a point a few degrees west of north-west: this representing a careful attempt to reproduce the correct entry conditions, as also did his device of introducing permanent solid mounds in place of sand over the hard portions of two banks near the mouth, an examination of the charts having revealed that these banks were persistent features of the estuary and contained outcrops of rock and gravel. Again, in places where a rocky bottom was known to exist near Havre and Villerville, the bed was moulded in cement; otherwise the cement substratum was kept well below the lowest known points of the channels and was covered with mobile material. Provision was made for fresh-water flow at the top of the model.

The first experiments at once revealed the presence of a bore and of a reverse current just before H.W. near Havre, as in Nature, and the creation of channels, from a bed originally moulded to *average* level, in agreement with the conditions known to exist before the beginning of the training walls. One notable feature was the instability of the main channel; another the formation of a small channel by Harfleur and Hoc Point as clearly shown in the chart of 1834. The channels were, however, deficient in depth, and the silver sand used was by degrees covered with a film or crust which reduced its mobility. A search was therefore made for a material which would be partly carried in suspension and which would facilitate experiments on training walls.

Powdered pumice was tried, but the kind used is described as having proved to be " too sticky "; flower of sulphur was " too greasy to be easily immersed in water ". Pounded coke was found to be too dirty and particles of it floated on the surface; violet-powder, fuller's earth and lupin seed were too pasty. Coffee grains were " too large " and too easily moved, while fine sawdust swelled and was transported so readily that no definite channels formed. Powdered Bath brick offered considerable hope of success, but gradually became too compact and resistant to motion, although Vernon-Harcourt remarks that " as it is probable that some sticky material is used in the manufacture of Bath bricks, it is quite possible that if I had succeeded in my endeavour to obtain the silt of the River Parret(t), from which the bricks are made, in its natural state, the material might have proved more subject to scouring influence ".

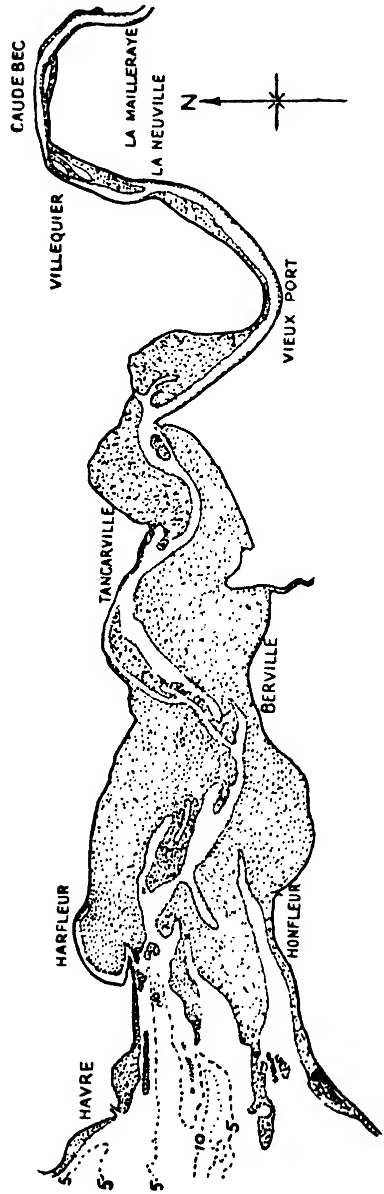


FIG. 70. 1834 Plan of Seine Estuary.
(From Plate 5, Fig. 1 of *Min. Proc. I.C.E.*, vol. LXXXIV.) Numbers 5 and 10 indicate depths in metres.

None of these substances proved to be as good as the silver sand already tried, but at last a fine sand was tried from " Chobham Common, belonging to the Bagshot beds, with a small admixture of peat. This sand, besides containing some very fine particles, was perfectly clean, so that water readily percolated through it; and it accordingly combined the advantages possessed by silver sand with a considerably greater fineness ". No indication is given by Vernon-Harcourt of the size of the Bagshot grains, but Professor Gibson states * that he has received a sample through the courtesy of Mr. Weale of the Mersey Docks and Harbour Board: it measured 0·00666 inch (under the microscope), as compared with the 0·00648 inch of Reynolds' Calais sand.

With this Chobham or Bagshot sand, the Seine model was found to give channels closely resembling those of the 1834 estuary chart (see Fig. 70), although the depths were smaller except at points of considerable scour. Vernon-Harcourt was of opinion that this deficiency in depths would have been largely elimin-

* A. H. Gibson, *Severn Model Reports*, H.M. Stationery Office, 63-78-2, 1933, p. 24.

ated by the use of a larger model, and especially if the bed had not been so nearly level as in the Seine. The existing training walls were then inserted in the form of strips of tin gradually introduced in sections. Accretion was found to take place behind these walls; the channel confined between them was scoured out and the foreshores at the back of the walls were raised up to H.W. level in places. Moreover, accretion extended beyond the walls on each side down to Hoc Point on the right bank, obliterating the inshore channel near Harfleur, and down to Honfleur on the left. Also, the main channel beyond the walls was comparatively shallow and unstable. All these effects were in striking agreement with known phenomena in the estuary (compare Figs. 69 and 70).

In view of these results, six schemes of walls were tried, and generalized conclusions drawn from the observations made. These may be found in the *Proceedings of the Royal Society*, volume XLV, already cited. Before leaving this discussion of Vernon-Harcourt's work, however, attention may well be drawn to his views concerning the effects of gales and waves. Having described the action of the waves as main agents in the erosion of cliffs and the stirring up of sand in shallow places and thus creating material which may affect accretion, he expresses the view that such circumstances cannot be reproduced in a model. Nevertheless, he was of the opinion that "the main forces acting in any tidal estuary are the tidal ebb and flow and the fresh-water discharge, which are constantly at work; and they regulate the size of the channels in an estuary, and for the most part their direction, as well as the limits of accretion. These are the forces which can be reproduced in miniature in a model, as proved by the close concordance in the channels obtained by experiment with the actual conditions of the Mersey, and with a previous state of the Seine estuary; and this similarity of results would not have occurred if the other influences noticed above were at all equally potent.

"Training walls mainly modify the direction and action of the tidal ebb and flow and fresh-water discharge; and, therefore, it is reasonable to suppose that the results in a model, due to these alterations, would correspond to their actual effects in an estuary, provided the important element of accretion could be also reproduced....

"... the further the training walls are extended, and the more an estuary is protected by works ... the more is the modifying influence of waves eliminated, and therefore the more are experiments in a model likely to correspond with the conditions of estuaries under similar conditions."

These questions of the influence of gales and of waves and coast erosion will be discussed in further detail on pages 237 and 254, but it does seem that, unless the estuary is in a highly exposed situation, the main argument of Vernon-Harcourt in favour of the importance of normal tidal action is essentially valid.

Other early tidal models worthy of note were constructed by Messrs. W. H. Wheeler and Herbert Wheeler (*Tidal Rivers*, Longmans, 1893, p. 332) and Mr. W. R. Kinipple (*Min. Proc. C.E.*, vol. C, 1890, p. 156). Messrs. Wheelers' model, which was 10 ft. 3 in. long and 6 ft. wide, was apparently designed to study the general phenomena of sand movement in hypothetical estuaries of geometrical form, and much the same effects were observed as in Reynolds' experiments. The cost of the model, including a wooden house with glass roof, was £50. Mr Kinipple's description is not easy to follow: " From the small models he had used, he was sure tolerably trustworthy results could be obtained, and especially by one 6 feet in diameter, having sides 3 or 4 inches high, a glass bottom, radial arms 3 feet across, driven by clock-work from a centre-pin, at a speed sufficient to move fine silver sand, and capable of being worked from the right or left, to represent ebb-tide and flood-tide; also a wave-maker, likewise driven by clock-work, and mounted in such a manner that any angle could be obtained; and if small models of piers, groynes, training-banks, mouths of rivers, narrows, estuaries, etc., were made of cast-tin and placed on the glass in proper relative positions to the currents and waves, and fine sand was dropped into the water, the gradual formation of banks, bars, the deepening at concave faces, and of narrows, arresting of the travel of sand by groynes, formation of spits, etc., could be pictorially displayed, and watched with interest, especially if observed in a dark room, with a light beneath the model."

We come next to consider a communication submitted to the Institution of Civil Engineers at its Engineering Conference held in 1921,* in which Mr. G. E. W. Cruttwell described a model which he made in 1903 while engaged in advising the Thames Conservancy on the improvement of the navigable channel across the Leigh Middle shoals in Sea Reach. This model, having horizontal and vertical scales of 1 : 10,560 and 1 : 384 respectively, embraced the river between Teddington and Shoeburyness. A tide-generating mechanism similar in essentials to Reynolds' (page 286) was used, and operated by a small overshot water-wheel supplied with water from a tap. The hinged tray or reservoir at the seaward end was ingeniously balanced by supporting it on floats immersed in water inside

* G. E. W. Cruttwell, *The Utility of Models for Estuarial Experiments*, Inst. C.E. Engineering Conference, 1921; also reported in *Engineering*, vol. 112, July 15, 1921, p. 130.

a fixed tank. As the reservoir rose and lost weight due to some of it contents flowing into the estuary on the flood tide, the floats also rose from the water in the fixed tank and compensated for the previous effect by their own loss of buoyancy. The tidal period was 83 seconds, corresponding to 12 hr. 25 min. in Nature; river-water was introduced at Teddington and at a point lower down to represent the combined inflow of the smaller tributaries below Teddington. Hydraulic effects were faithfully reproduced: the directions and relative velocities of the currents and the times of high and low water at various points were found to be in close agreement with the actual phenomena.

When the suggested dredged channels were scooped out of the sand which formed the bed, however, no movement or deposit of the shoals was discernible, although the sand used was " extremely fine ". Now the mean tidal range at the lower end of Sea Reach was taken as $14\frac{1}{2}$ feet, which would be represented by 0·45 inch, or 0·038 ft., in the model. The corresponding Reynolds' criterion must have been

$$h^3 \frac{x}{y} \sqrt{\frac{H}{30}} = 0{\cdot}038^3 \, \frac{10560}{384} \sqrt{\frac{14{\cdot}5}{30}}$$
$$= 0{\cdot}000054 \times 27{\cdot}5 \times \sqrt{\frac{1}{2{\cdot}07}} = 0{\cdot}00103,$$

which, of course, is very small compared with the " safe value " of 0·09, but is almost exactly the same as the value for Reynolds' first—and successful—Mersey model. The validity of the criterion is accordingly again brought under question. It is possible, of course, that Mr. Cruttwell's sand was unsuitable: either too coarse or, since it is described as " extremely fine ", so fine as to have been markedly cohesive. But a significant point is that the vertical scale in Reynolds' Mersey model having the same criterion (0·001) as Mr. Cruttwell's was 1 : 960, so that if the mean water-depths in the two estuaries were of the same order, those in the Mersey model would be only $\frac{384}{960}$ or 0·40 of the depths in the Thames model. Again, a velocity representing, say, V knots in the Mersey model would be $\sqrt{\frac{384}{960}}$ or 0·63 of that representing the same velocity of V knots in the Thames model. If, then, as the Author believes, the velocity required to move a given material is approximately proportional to the 0·28 power of the depth, it would only be necessary for velocities in the Mersey to be $\frac{0{\cdot}40^{0{\cdot}28}}{0{\cdot}63}$, or about 1·25 times as high as in the Thames, for the Reynolds' model just to exhibit bed movement while Mr. Cruttwell's just failed to do so. It is virtually certain that a vertical scale of, say, 1 : 160 instead of 1 : 384 would have remedied the defect of the Thames model.

Despite its inability to transport the sand, however, the Thames model did evidently provide data of real value. The value of the observations would, it is true, have been greatly enhanced by movement of the bed, " but even without any shifting of the materials ", Mr. Cruttwell says, " the author considers that valuable and trustworthy information can be obtained from the models as to the probable effect of dredging, training, groynes and other contemplated works ". This statement presumably refers to the valuable guidance which a careful analysis of tide and current observations in such a model may provide, and anyone experienced in the working of such models is likely to confirm his statement absolutely.

Indeed, Mr. Cruttwell was led by his model-tests to the conclusion that in order to maintain a deep-water mid-river channel, any training banks in Sea Reach would need to be constructed far from the shore and at a heavy cost " which could be more usefully expended in dredging a channel through the shoals ".

Finally, Mr. Cruttwell states that " the cost of a model and the apparatus for working it is considerable, amounting to about 300*l.* in the case of the Thames model. The greater part of the model and the whole of the working apparatus could be adapted for experiments with other estuaries at a trifling cost; so the author [Mr. Cruttwell] suggests that it would be most advantageous for the engineering profession if the National Physical Laboratory, or some similar institution, would instal the necessary apparatus, which could then be adapted to suit any particular case for a moderate fee to cover the necessary adaptations and investigations ".

While on the subject of the cost of such models, which in more recent and more elaborate investigations must have greatly exceeded the figure quoted by Mr. Cruttwell,* it is desirable to point out that much depends upon the magnitude of the survey operations which may have to be made in the real estuary in order to provide indispensable data for the model. These operations may involve, in addition to hydrographic surveys for cross-sections and longitudinal profiles, the procuring of sand, silt and water samples and the measurement of velocities by means of floats and current meter; also the measurement of tides at representative places, either on post-gauges observed by eye or on gauges equipped with automatic recording apparatus. The ordinary processes of hydrographic surveying may often be supplemented to great advantage by photographic aerial surveys, and in this connection the reader is recommended to consult a paper by Mr. J. L. Matheson entitled " An Aerial Survey of the Estuary of the River Dee, employing a Simple Method of rectifying Oblique Photo-

* Sir Leopold Savile, K.C.B., Journ. Inst. C.E., June 1939, p. 52, has quoted £10,000 as being approximately the expenditure on the model-experiments of the Rangoon tidal model.

graphs", in the *Journal* of the Institution of Civil Engineers, November 1938, p. 47. This method is not claimed to yield a degree of accuracy at all comparable with that of the true aerial survey methods, but it may often give information which is of the greatest value at a relatively low cost. Mr. Matheson describes, with the aid of photographs, an extremely simple apparatus which may be set up in any engineer's office whereby an oblique photograph produced by a camera held at almost any angle in an aeroplane may be rectified and converted into a plan. If such aerial photographs are taken on different occasions at, say, low water of ordinary spring tides, they enable maps to be drawn which show the significant changes in the shape of the channel. From this point of view the method has the advantage that each map so obtained represents an instantaneous picture of the estuary, so that the differences between the various maps are not masked by the time-effect inherent in a long hydrographic survey. Again, if a series of such photographs be taken at various known stages of one tide, the result is to obtain a "contoured" plan of the estuary between the limits of low and high water.

By this method a plan was obtained of the complicated low-water channel of the Dee estuary, which proved to be extremely useful in a model-investigation of means of improving that river.* The cost of the field-work, including the hire of an aeroplane and the processing of the photographs, is stated to have been £55 and, once the apparatus for rectifying the photographs had been devised and practice obtained in its use, two engineers were able to rectify a photograph in rather over a day. Concerning the accuracy possible, the reader is advised to consult Mr. Matheson's paper.

The next model to be considered is that of the Bombay Harbour. It was designed by Mr. John McClure, M.Inst.C.E., and described by him in a paper called "Bombay Harbour Survey and Tidal Model", which may be found in the *Minutes of Proceedings* of the Institution of Civil Engineers, vol. 232, 1932, p. 66. The horizontal and vertical scales were 1 : 7,296 and 1 : 96 respectively, and the tidal period, taken to be 46,560 seconds at Bombay, was correspondingly made equal to $\frac{46560 \times \sqrt{96}}{7296}$, that is, 62·54 seconds. The ordinary spring tidal range in the harbour being about 14 feet, the Reynolds' criterion $h^3 \frac{x}{y} \sqrt{\frac{H}{30}}$ would incidentally be $(\frac{14}{96})^3 \frac{7296}{96} \sqrt{\frac{14}{30}}$, or $(0{\cdot}146)^3 (76)(0{\cdot}684)$, that is, 0·162. The neap tidal range is about 6 feet.

* "Schemes of Improvement for the Cheshire Dee: an Investigation by means of Model Experiments", J. Allen, *Journ. I.C.E.*, June 1939, pp. 30–51.

Inspection of Fig. 71 will show that this model was quite a large one compared with those of Reynolds, Vernon-Harcourt or Cruttwell; moreover, its tide-making apparatus (the principle of which is outlined in Chapter IX) was extremely ingenious and original. The model was operated on a sequence of eight tides * varying from 15 feet to 4 ft. 6 in., an approximation to a full lunar cycle. Coastline, islands, rocks and other permanent features were moulded in concrete deposited between

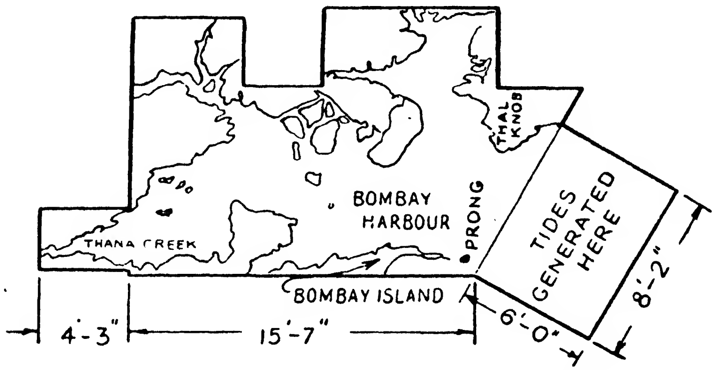


FIG. 71. Scope of Tidal Model of Bombay Harbour.

(From Plate 2, Fig. 6, of *Min. Proc. I.C.E.*, vol. 232.)

vertical templates made of zinc, except that the harbour-front, with its intricate details of docks, wharves and piers, was carved in teak. The whole model was contained within a table or box of seasoned teak-wood, dove-tailed at the corners, and having a floor of 2½-inch tongued-and-grooved planking—a very substantial construction—all joints being set in whitelead, and two coats of bituminous composition being applied over the inside surfaces, so providing a highly satisfactory watertight casing. Complete with all fittings, the cost was £1,050.

A bed material consisting of sand obtained by " upward-flow washing of the trap silt dredged from Bombay harbour " was employed; this sand could pass through a sieve having 10,000 holes per square inch. The restoration of bed-levels at the beginning of each test was facilitated by zinc templates suspended from the ceiling by means of pulleys, ropes and concrete counterbalance weights.

The directions of the currents in the model are stated to have been " exactly in accordance with the results of surveys made in the actual harbour ", and a maximum velocity of 4 ft. per sec. obtained over a certain course in the harbour was found to be represented by 0·42 ft. per second

* This apparently means 4 descending and 4 ascending tides.

over the corresponding course in the model. The ratio of 4 : 0·42 is 9·5(2) : 1, as compared with the nominal velocity scale of $\sqrt{96}$: 1, or 9·8 : 1; the discrepancy between these figures may easily be accounted for by errors or uncertainties in measurement.

On the other hand, model tide-curves reproduced in Mr. McClure's paper exhibit marked discontinuities, surges or jerks not so obvious in the natural tide-curves, although Mr. McClure has stated that " actual gauge disturbances of the rise and fall due to natural temporary arrestments of the flow " do occur in Nature and " were observable in the model tides, though considerably magnified in the case of the Thana record ".

The principal subject studied in the Bombay model was that of the most suitable lines for dredged channels. " Tests carried out over several thousand tides showed that a low travelling bar would be formed at the north end of the proposed dredging-area, which was judged to be inimical to the docks and main entrance-channel thereto, and influenced the trustees to set definite limits to the north boundary of the proposed new work."

Another important application of the model was to the study of sewage distribution. A scheme had been advanced for the discharge of crude sewage into the north end of the harbour, the promoters assuming that " the whole area of the harbour, about 120 square miles, would be effective in diluting the sewage ". However, " a model sewage-station, consisting of a tin can with a fine copper pipe and stop-cock, was fixed in the model at Pir Pau and discharged a fine stream of lime milk. The shortest travel of the tide during neaps is about 1¼ mile, and the travel increases gradually to about 8 miles at springs. The demonstration was begun in the model at low neaps, and rendered visible in a very convincing manner the cumulative results of the heavy sewage-discharge proposed, which was shown to confine itself to the commercial side of the harbour ". As a result of this demonstration, the trustees successfully opposed the scheme.

A study was also made of the disposition of silt dredgings. For this purpose a paste of finely-crushed bright vermilion Mangalore roofing-tiles was introduced at the site of the spoil ground in spoonfuls during a model run of some thousands of tides; when the model was drained and the sand allowed to dry, the intensity of distribution of the spoil was clearly shown by the graduated crimson colouring of the otherwise chocolate-coloured bed material.

Mention of this draining of the model previous to surveying of the bed makes it desirable to call attention to the fact that great care must be exercised in such a process. Otherwise artificial channels are apt to be cut through the lower banks by the water as the model is emptied; in some cases it is impossible to avoid this, however carefully the draining is carried out, and then it is essential to survey with water left in the model. Another point is that to expose the bed for any time to the air is to accelerate the troublesome tendency of the bed material to develop a skin which renders it immobile under the currents available.

The discussion which appears in the *Minutes of Proceedings* of the Institution of Civil Engineers, vol. 232, concerning Mr. McClure's paper, is almost as interesting and important as the paper itself. It makes quite clear the fact that at that time (1932) there were two sharply contrasting and conflicting schools of thought concerning the use of such models, one (represented notably by the late Sir Frederick Palmer, a Past-President) which confessed itself as never having been able "to understand the advantages of tidal models similar to that described in the Paper, nor able to follow the contention that results with models made to such a distorted scale could really serve as a guide under natural conditions"; the other school (equally notably represented by another Past-President, the late Sir John Purser Griffith) convinced of "the great value of models in determining the probable influence of structural works for the training of rivers and tidal estuaries", even to the extent that "no better or safer course could be followed for the solution of many of the difficult problems associated with the construction of river and marine works for the improvement of ports". An intermediate school of thought was also represented by Captain F. W. Mace, R.N.R., then Marine Surveyor and Water Bailiff of the Mersey Docks and Harbour Board, who "did not wish to pass any remarks on models generally, but his experience was that results from them must be accepted with considerable reserve".

It is the Author's belief that since 1932 there has been rather a change of attitude, and a more generally favourable disposition towards the acceptance of the model as a working instrument, at any rate for guidance, is now discernible in the profession as a whole. This is not to say that the adverse criticisms have been completely resolved or dissolved; indeed, it is hoped that a perusal of these pages will serve to indicate the limitations of the method and the fact that much fundamental work remains to be done. And clearly much will depend upon the accumulation of evidence as to the behaviour of structures actually made as compared with model forecasts. To claim at this stage a complete reliability would be as scientifically unsound as to reject the method because the models have distorted scales and use bed materials whose individual grain-sizes are out of all proportion to the linear scales. Already, a case for distorted scales has been argued on both theoretical and experimental grounds (as, for instance, in pages 141–147), and the question of grain-size has been discussed in pages 146–169; further attention will be paid to it as the book proceeds. But one point quite naturally emphasized by Sir Frederick Palmer—and often raised by engineers in general—was that if the sand used in the Bombay model passed through a sieve of 100 apertures per linear inch, and if the wires took up as much room as the apertures, it followed that there must have been about 200 grains to the linear inch, or on the horizontal scale of the model, 200 grains of sand in 600 feet. Each grain would therefore be about 3 feet long, to scale, and because of the vertical exaggeration of scale, would be about ½ inch thick. The "fine" sand thus relied upon to give indications of erosion was really equivalent to slabs 3 feet long and ½ inch thick. Three points appear to be overlooked in this argument: (*a*) the fact that the scouring effect of a shallow stream is greater than that of a deep one of the same *mean* velocity; (*b*) although the separate grains are large *to scale*, they are still only of "negligible" size in relation to a bank of which they form a part. A sand-bank 1 cubic inch in volume contains a few million grains at 200 per linear inch, even allowing for the voids; (*c*) there is evidence of experiments such as those of Rehbock at Karlsruhe (see page 130), which showed closely similar scour in models of bridge piers, even though the bed material was changed from 0·06 to 0·02 inch diameter. This and other experimental evidence already quoted (and to be supplemented in later pages), shows that there is a fundamental error in the idea of translating the size of the model grains into prototype boulders when the régime under consideration is that of banks formed in a mass of irregular shaped material.

In 1926, the construction was begun under the direction of Professor A. H. Gibson at Manchester University of a tidal model of the Severn estuary, embracing the area shown in Fig. 72. At the time, this model was the largest to be attempted in Britain, and as the opportunity was taken to carry out many experiments of a fundamental nature in addition to the tests for which it was specifically built, the publication in 1933 of a long report * concerning the whole investigation aroused widespread interest. So important has this work come to be regarded that a somewhat lengthy account of it in these pages may be considered appropriate, especially as the Severn model proved to be the forerunner of a series of such investigations in the Manchester laboratory, as well as of at least two other major studies conducted elsewhere in England.

The Severn model had a horizontal scale of 1 : 8,500 (or 7·46 inches to one statute mile); its overall length of some 50 feet covered a length

* *Tidal Model of the Severn Estuary: Two Reports by Professor A. H. Gibson*, H.M. Stationery Office, 63-78-2, 1933.

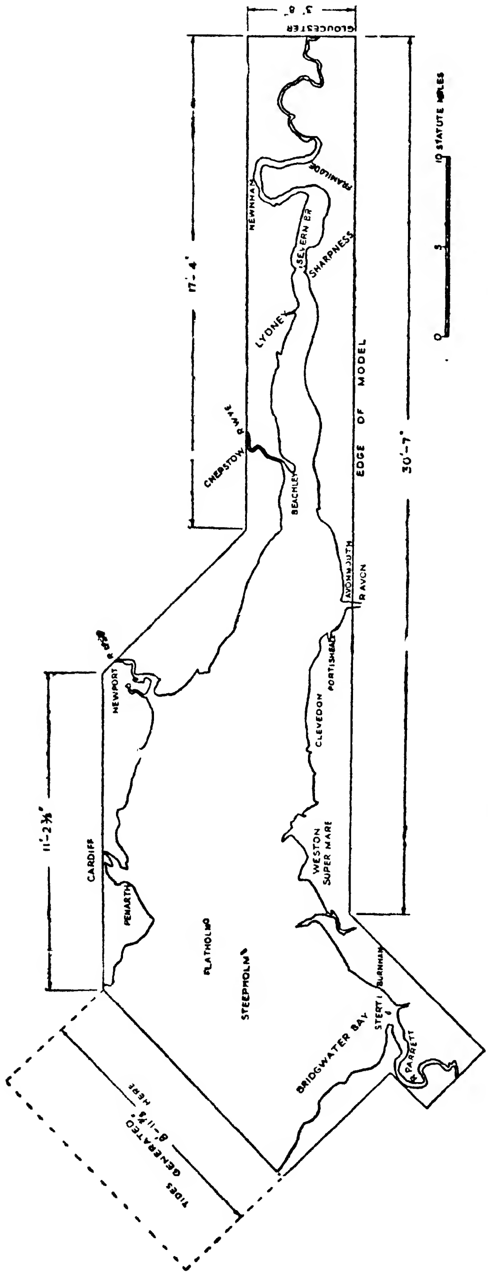


FIG. 72. Scope of Severn Tidal Models built with horizontal scale of 1 : 8500.

of channel of approximately 84 miles from Gloucester downwards. Its original vertical scale was 1 : 100, and this was later changed to 1 : 200, after which all the more important tests were repeated as well as fresh ones made. Corresponding to the vertical scales of 1 : 100 and 1 : 200, the tidal periods were 52·2* and 73·9 seconds respectively, these being calculated from the formula $t_0 = \frac{T_0\sqrt{y}}{x}$ with T_0 (the tidal period in Nature), assumed as being 12 hours 20 minutes.

The investigation was made on behalf of H.M. Government at the instigation of the Expert Co-ordinating Committee of the Severn Barrage Committee: its object was to determine the probable effects of a proposed tidal barrage upon the general régime of the estuary. The model findings also largely influenced the design of the barrage as finally

* Actually it was found that the mean running speed in the 1 : 100 model was 51·2 seconds, instead of 52·2, but the gears introduced in the driving mechanism when the vertical scale was modified ensured a period of 73·9 seconds.

advocated by the Committee. The purpose of the suggested barrage was to develop power from the tides. In this respect the Severn is a most favoured site,* because the tides in it are the second or third highest in the world, an ordinary spring tide at Avonmouth (port of Bristol) having a range of some 40 feet.

Briefly, the idea consisted of erecting a dam or barrage across the estuary and of installing within this structure a number of sluice-gates and of turbines. As the flood tide reached the dam on its seaward side, the gates would open to allow tide water to flow into the upper estuary or basin. At high water, these gates would be closed, thus impounding the water imprisoned above the barrage until such time as the fall of the ebb tide outside created sufficient difference of level between the basin and the outer sea to justify the operation of the turbines. The turbines would then be started up and would continue to work, discharging water from the tidal basin, until on the succeeding flood tide the head was reduced to the minimum economical value for operation.† From time to time, various sites have been suggested for such a barrage, but the most promising appeared to be either the Beachley-Aust Cliff narrows or the English Stones (see Fig. 73), and the latter was the site fixed upon as to be made the subject of detailed examination in the models. Compared with the Beachley site, it has the advantages of increasing the size of the estuary impounded, and therefore of increasing the amount of power available.‡ At the same time, the English Stones would provide a suitable foundation for a large portion of the structure, the Shoots would offer a natural tail-race for the discharge of the turbines, and the width of the estuary at this point would be adequate for a length of dam of comparatively simple shape sufficient to house the necessary sluice-gates and turbines. An objection to the English Stones site compared with that

* In such a scheme, the power available from a tidal basin of given area varies approximately as the square of the tidal range.

† It will be appreciated that with a scheme of this kind, the output of power is restricted to certain times and its magnitude varies in any one tide as well as from tide to tide. In order to correct for this irregular action, the Severn Barrage Committee in 1933 proposed the construction of a secondary power station fed by water from a storage reservoir above the Wye valley. The water in this storage reservoir was to be drawn from the Wye, using energy generated at the barrage itself during its peak periods. Thus the secondary station would act as a standby and supplementary source of power to render the output of the scheme as a whole more uniform and more controllable. This scheme for pumped storage is, however, *not* favoured by the panel of engineers appointed by the Minister of Fuel and Power in November 1943. Their findings have just been published (Feb. 1945); see *Report on the Severn Barrage Scheme*, H.M. Stationery Office, price 2s. 6d.

‡ The 1945 *Report on the Severn Barrage Scheme* (H.M. Stationery Office) still favours the English Stones site, although suggesting certain modifications in the layout of the barrage, notably the division of the turbines into two sections. It also recommends that the road and railway crossings be treated as schemes independent of the barrage itself.

near Beachley is that the River Wye becomes situated upstream of the barrage and might add to siltation problems: this was one of the questions with which the investigation was concerned.

From one point of view, the Severn is an ideal estuary for model treatment; the tides are so high and the currents so fast that the difficulties of

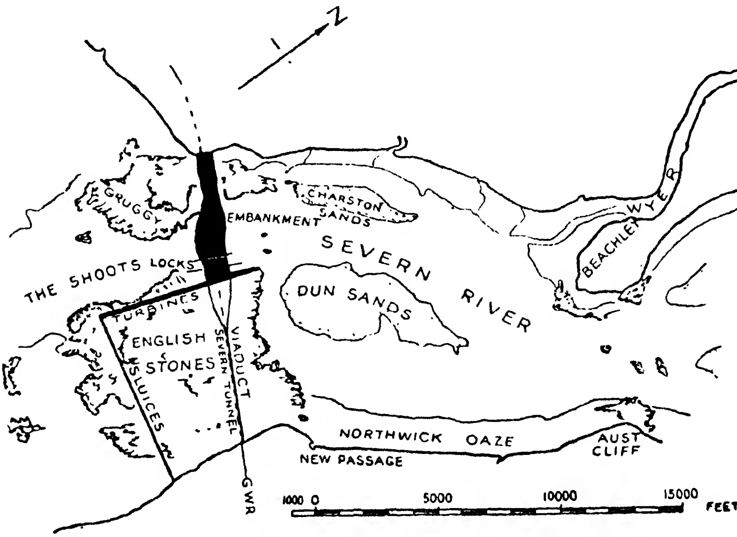


FIG. 73. Sketch Map showing Severn Barrage Works as proposed in 1933.

obtaining velocities in the model sufficient to transport bed material and of having tides large enough for accurate measurement are reduced to a minimum. Moreover, it is safe to assume (as experiment in fact confirmed) that the bed régime is overwhelmingly influenced by pure tidal action rather than by spasmodic gales or winds.

On the other hand, the Severn is well known as an exceedingly muddy estuary, and the reproduction of this phenomenon of water-borne silt had to be tackled as a major factor in the study of the barrage.

The investigation was favoured with a very comprehensive set of data concerning the estuary itself; several Admiralty charts were available, dating from the first-class survey of Captain Beechey (1849) to a great survey made specially in 1927 with the needs of the investigation in mind. Soundings were supplemented by tide and current observations, many of them carried out specifically for the work in hand, and samples of sand and silt were supplied for microscopic and other analysis. Captain Beechey's observations of the Severn bore (*Report of the Severn Survey of* 1849) were supplemented by Dr. Vaughan Cornish's (*Geographical Journal*, vol. 19, 1902, p. 52, and vol. 29, 1907, p. 28, and *Ocean Waves*, Camb.

Univ. Press, 1934), and by the staff of the Admiralty survey during 1926–27.

Inspection of Fig. 72 will show that the tidal portions of the rivers were not included in entirety. To compensate for this, an extra box was screwed on the outside of the wooden casing of the model for each of the important rivers. In the case of the tributary rivers, the Avon, Wye*, Usk and Parrett, this box communicated through a tube in the wooden side with the end of the river as embraced by the model. The box had a plan area representing the tidal portion of the river omitted from the moulded model itself. In making the final adjustment, coarse sand was placed in the box in the form of a sloping bank, and a stop-cock on the communicating tube was so adjusted that the tidal range in the box was one-half of that just inside the model. In the case of the Severn river itself, the box was given a plan area equal to the omitted tidal portion of the Severn and was in direct communication with the model through a slot cut in the wooden casing. This vertical slot extended from below bed level to above extreme high-water level, and its width was equal to that of the river at the moulded limit of the model. On the box side the vertical edges of the slot were rounded-off, and a sloping bank of coarse sand was placed inside the box to simulate the variation in area of the river between low and high water. Judging by observations of tide-levels and current velocities, these arrangements proved to be adequate.

The main islands—Flatholm, Steepholm, Denny and Stert—were made of wood well soaked in hot paraffin wax dusted over with sand. The coastline and those portions of the bed described as rock on Admiralty charts were moulded in Portland cement-and-sand mortar, and many of the smaller rocks such as the Firefly in King Road, together with the complicated surface of the English Stones, were made in playwax and plasticene. The use of plasticene for the English Stones was of particular advantage from the point of view of periodically erecting the model barrage and then restoring the conditions after withdrawing the barrage.

Upstream of the English Stones, a number of borings had been made by Captain Beechey in 1848–9, and where these indicated a substratum of rock, the model-bed was moulded in cement. Otherwise, the allowance made for the depth to bedrock was such as to permit any tendency to scour, and valuable guidance was provided by a study of those changes which had occurred since 1849 and of those which were known to be liable to happen at various times. For example, a comparison of the 1868, 1888 and 1923 editions of Admiralty Chart No. 2682 revealed dif-

* For the purpose of certain additional tests made subsequent to the main investigation, an extra box was fitted to the original casing of the model and the River Wye was moulded in this box up to the limit of tidal action.

ferences such as to prove that there is well over 50 feet of sand or other mobile material in the area of the Welsh Hook and Welsh grounds.

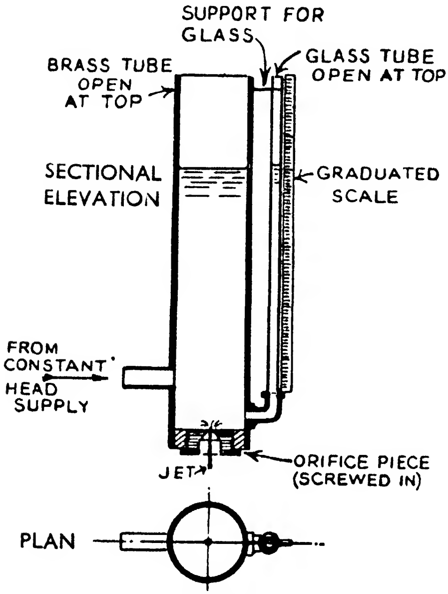


FIG. 74. Diagrammatic Sketch of River-Feed Apparatus.

Each of the five main rivers—the Severn, Wye, Avon, Usk and Parrett—was fed to the model from a small tank having a calibrated orifice, the head on which was read on a gauge-glass equipped with a scale (see Fig. 74). A constant-head delivery tank, fitted with a ball-valve, provided the main supply to the rivers. In determining the flow required to simulate the estimated discharges of the natural rivers, the reasoning followed was based purely upon the linear and velocity scales. Thus, if q represents the river-flow in the model and Q that in Nature, and if the horizontal scale is $1:x$ while the vertical is $1:y$, then

$$\frac{q}{Q}=\frac{\text{velocity in model}\times\text{area of model section}}{\text{velocity in nature}\times\text{area of actual section}}$$

$$=\sqrt{\frac{1}{y}}\cdot\frac{1}{xy}\quad\text{or}\quad q=\frac{Q}{xy^{\frac{3}{2}}}\text{ cusecs,}$$

if Q itself is in cusecs. With $x=8{,}500$ and $y=100$, this becomes

$$\frac{q}{Q}=\frac{1}{8\cdot5\times10^{6}},$$

while with $x=8{,}500$ and $y=200$,

$$\frac{q}{Q}=\frac{1}{24\cdot0\times10^{6}}.$$

Thus a mean discharge of 4,270 cusecs for the Severn became 0·000503 and 0·000178 cusecs in the two models respectively. These rates were, however, varied from time to time to simulate drought and spate conditions; including two or three floods in each year of tides, each lasting for the equivalent of about three days at a flow of five times the mean discharge.

Several minor tributaries, rivulets or "pills" also empty into the estuary, and observations in the preliminary model tests indicated that two of these, namely, Brim's Pill (above Severn Bridge) and Oldbury Pill (between Aust Cliff and Sharpness), might be of more than local importance. They were therefore incorporated.

Provision was made for the injection of silt (finely-divided mud) in suspension with the river water, and also direct into the estuary itself just below the English Stones. The effect of the salt water was also reproduced, as will be described later, probably for the first time in any model investigation.

Tidal heights and currents, both at springs and neaps, were measured and compared in great detail with the Admiralty observations, and it was found that once the shape and stroke of the plunger used for producing the tides had been so adjusted as to generate the correct tide-curve near the mouth, then the tidal phenomena throughout the model were faithfully reproduced. Table XIX shows the results obtained from the 1:100 vertical scale, compared with those observed in the estuary, which are given in brackets.

TABLE XIX

Place	*Mean spring tide*			*Mean neap tide*		
	H.W.	*L.W.*	*Range (feet)*	*H.W.*	*L.W.*	*Range (feet)*
Cardiff (Penarth)	20·0 (19·7)	−15·9 (−16·7)	35·9 (36·4)	10·3 (9·9)	−8·2 (−8·4)	18·5 (18·3)
Newport	21·0 (21·1)	−17·3 (−17·0)	38·3 (38·1)	10·4 (−)	−9·0 (−)	19·4 (−)
Avonmouth	21·9 (21·9)	−18·4 (−18·4)	40·3 (40·3)	11·3 (11·3)	−9·7 (−9·7)	21·0 (21·0)
Beachley	22·8 (22·2)	−19·9 (−19·2)	42·7 (41·4)	12·1 (11·4)	−10·5 (−10·8)	22·6 (22·2)
Sharpness	25·2 (24·5)	−2·8 (−3·3)	28·0 (27·8)	13·3 (12·7)	−4·0 (−4·5)	17·3 (17·2)
Framilode	26·9 (26·0)	+16·2 (16·0)	10·7 (10·0)	16·0 (15·8)	16·0 (15·8)	zero (zero)
Gloucester	25·8 (25·8)	+19·8 (20·0)	6·0 (5·8)	19·7 (19·5)	19·7 (19·5)	zero (zero)

(River discharge: " average ".)

For the purpose of this comparison, the tide-producing gear was adjusted to give the proper levels at Avonmouth. The corresponding observations on the 1:200 vertical scale are presented in the following Table:

TABLE XX

Place	Mean spring tide			Mean neap tide		
	H.W.	*L.W.*	*Range (feet)*	*H.W.*	*L.W.*	*Range (feet)*
Cardiff (Penarth)	19·7 (19·7)	−16·5 (−16·7)	36·2 (36·4)	10·1 (9·9)	−8·2 (−8·4)	18·3 (18·3)
Newport	20·8 (21·1)	−17·3 (−17·0)	38·1 (38·1)	11·1 (—)	−8·6 (—)	19·7 (—)
Avonmouth	21·9 (21·9)	−18·4 (−18·4)	40·3 (40·3)	11·3 (11·3)	−9·7 (−9·7)	21·0 (21·0)
Beachley	22·1 (22·2)	−19·2 (−19·2)	41·3 (41·4)	11·6 (11·4)	−10·6 (−10·8)	22·2 (22·2)
Sharpness	24·3 (24·5)	−2·8 (−3·3)	27·1 (27·8)	13·0 (12·7)	−3·8 (−4·5)	16·8 (17·2)
Framilode	25·9 (26·0)	15·8 (16·0)	10·1 (10·0)	15·6 (15·8)	15·6 (15·8)	zero (zero)
Gloucester	25·6 (25·8)	19·8 (20·0)	5·8 (5·8)	19·5 (19·5)	19·5 (19·5)	zero (zero)

Assuming the accuracy of O.D. records, it is estimated that the tide levels quoted above for the Severn itself are true within ±0·25 ft., and in view of this, the performance of the model was remarkably good. It was, incidentally, even better with the vertical scale of 1: 200 than with the 1: 100, the reasons being somewhat complex: firstly, the correction to the periodicity (see the footnote to page 202); secondly, the truer shape of tide curve generated in the 1: 200 model by a modification to the shape of the plunger; and thirdly, the more accurate reproduction of channel depths, especially in the vicinity of Beachley.

At a later stage, extended observations by the Admiralty revealed rather different levels of average spring, mean and neap tides at Avonmouth, and a further comparison was made on the 1: 200 vertical scale, which yielded the results given in Table XXI. Here again the estuary figures are quoted in brackets:

TABLE XXI

Place	Springs		Means		Neaps	
	H.W.	*L.W.*	*H.W.*	*L.W.*	*H.W.*	*L.W.*
Avonmouth	22·5 (22·5)	− 17·9 (− 17·9)	17·8 (17·8)	− 14·6 (− 14·6)	11·9 (11·9)	− 9·9 (− 9·9)
New Passage	23·2 (23·6)	− 18·4 (− 17·1)	18·3 (18·8)	− 15·0 (− 14·0)	12·7 (12·7)	− 9·8 (− 9·7)
Beachley	23·4 (23·4)	− 18·8 (− 17·6)	18·9 (18·5)	− 15·5 (− 14·8)	12·9 (12·3)	− 10·5 (− 10·5)
Shepperdine	23·7 (23·7)	− 12·6 (− 12·6)	19·0 (18·9)	− 11·6 (− 12·1)	13·1 (12·5)	− 11·3 (− 10·0)
Sharpness	25·2 (25·5)	− 3·4 (− 3·4)	19·8 (19·9)	− 3·5 (− 4·2)	13·4 (13·4)	− 4·9 (− 5·2)

As for the relative times of high water at various places, it may be stated that they agreed with the estuary within the limits of observational accuracy possible in either the model or the estuary. For example, the times of high water at Beachley and Gloucester were observed in the 1 : 200 model to be the equivalent * of 7 and 152 minutes after H.W. at Avonmouth. On a comparable tide in the Severn itself, the times are 6 and 156 minutes respectively.

Again, the ratio of the times of ebb and flow at various places was reasonably well reproduced, as will be seen from Table XXII:

TABLE XXII

Place	*Ratio of times of ebb and flow*			
	Spring		*Neap*	
	Estuary	*Model*	*Estuary*	*Model*
Penarth	1·11	1·08	0·95	0·96
Avonmouth	1·21	1·20	0·92	0·96
Beachley	1·47	1·48	1·03	1·02
Sharpness	3·52	3·63	2·69	2·70

* The time-scale being $1 : \frac{8500}{\sqrt{200}}$, or 1 : 600, an interval of 7 minutes would be represented by 0·70 second.

Similarly striking results were observed in the comparison of current velocities and float observations. During the Admiralty survey, 1926–27, a float had been dropped at the lower end of the Shoots at the beginning of a spring flood tide, and was followed upstream a distance of some 35 miles until it came to rest at slack water opposite Framilode, the time taken being 5 hours 52 minutes. This time corresponded to 24·8 seconds in the model with a vertical scale of 1: 100 and to 35·2 seconds in the 1:200 model.

When this experiment was tried in the model it was sometimes found that the float was caught in an eddy and swept out of the normal track, but when the float was found to make a continuous journey it reached points varying between ½ mile downstream and ½ mile upstream of Framilode,* the times of travel ranging between 23·5 and 25·5 seconds with the 1: 100 vertical scale and between 34·6 and 36·0 seconds with the 1:200 vertical scale. Thus, while in both cases the distance travelled was sensibly correct, the average speed was rather too quick (about 1·5 per cent.) with the 1: 100 scale: this discrepancy, so far as it is significant at all, might be due to the tide-producing machinery running rather too fast.

On another occasion, a float released at Framilode at the beginning of the ebb of a spring tide was found to take 3 hours 54 minutes to cover the 15 miles down to Severn Bridge. This time (3 hours 54 minutes) would correspond to 16·5 and 23·4 seconds on the 1: 100 and 1: 200 vertical scales respectively. In the model, the observed times with the 1: 100 scale ranged between 14·6 and 15·6 seconds, while with the 1: 200 scale they lay between 22·4 and 23·6 (mean time 23·2 seconds). The average time obtained in the 1: 100 case was 15·1 seconds, which was about 9 per cent. too short, an effect attributed to the smaller degree of accuracy with which the shallow channel, especially between Hock Crib and Severn Bridge, was reproduced.

The examples cited above represent only a small fraction of many such checks on the drift and speed of currents, all of which demonstrated the extraordinary pitch of accuracy exhibited in the hydraulic phenomena. River-water levels also were found to be well reproduced in both models. For instance, the lowest recorded level on the Severn below Llanthony Weir is apparently 16·05 ft. (in July, 1921), and according to data supplied by Professor S. M. Dixon, the smallest flow at Bewdley during that month

* I remember the first occasion on which this fascinating experiment was tried. An Officer of the Admiralty Hydrographic Department, on a visit to the laboratory, told us that the float observations referred to had been made in the Severn a few days previously. Professor Gibson replied that the next spring tide in the model was due in about five minutes and would provide an opportunity for the desired comparison. As it happened, the first float had an uninterrupted run, not being caught in any side-eddies, and came to rest within ½ inch of the correct place. [J. A.]

was 200 cusecs. On the assumption that the ratio of the total run-off for the region down to Gloucester was the same as under conditions of normal flow, Professor Gibson has estimated that the discharge at Gloucester must have been about 380 cusecs. In the models, the water-level obtained with this discharge proved to be 16·15 ft., as compared with the figure of 16·05 in the river itself. Another aspect of the river-level problem has already been discussed on pages 28 and 29.

In this investigation, no fewer than thirteen bed materials of different sizes and densities were tried, as listed in Table XXIII:

TABLE XXIII

Bed materials tried in the Severn Investigation

(1)	(2)	(3)	(4)	(5)	
Material	*Mean diam. (in.)*	*Specific gravity*	*Mean Ratio of longest to shortest diam.*	*Angle of repose in water (degrees)*	
				Statical	*Dis-turbed*
SANDS:					
1. " 120 mesh silica " -	0·00582	2·63	1·19	28·4	22·4
2. " Fine " Cobham -	·00657	2·63	1·34	30·2	24·3
3. " 80 mesh silica " -	·00700	2·62	1·23	29·9	23·5
4. Calais - - -	·00684	2·54	1·46	30·5	24·5
5. Portishead (sieved) -	·00814	2·60	1·23	30·2	23·9
6. " Coarse " Cobham -	·0100	2·63	1·23	30·5	24·5
7. " 24–40 mesh " Leighton Buzzard -	·0223	2·64	1·21	33·0	25·9
EMERY:					
" 120 mesh " emery -	0·00505	3·89	1·49	32·7	26·8
" 80 mesh " emery -	·00825	3·89	1·33	33·8	28·4
" 50 mesh " emery -	·0156	3·89	1·43	36·5	30·1
PUMICE:					
A - - - - -	0·0122	1·99	1·57	30·7	23·9
B - - - - -	·0153	1·95	1·54	31·3	24·9
C - - - - -	·0192	1·90	1·49	31·6	26·9

In explanation of this Table, it should be stated that the mean grain diameter was found by examining at least 50 grains under a microscope equipped with an eyepiece scale which had been calibrated with reference to a standard millimetre divided into 100 equal parts. Observations were made of the length and breadth of each individual grain: the ratio of these dimensions is what is given in column 4; it is a measure of the angularity of the grain shape. The average of the length and breadth gives the dimension quoted in column 2. The specific gravity was found by the usual specific gravity bottle, and the angle of repose was measured in a glass-sided vessel 6 in. long and 4 in. wide. A bank of sand was heaped under water against one wall of the vessel at an angle greater than its natural angle of repose. When allowed to settle, the bank assumed an angle which is quoted in column 5 (statical). Next, the vessel was gently tapped and this caused the sand to assume a somewhat smaller slope, called the "disturbed angle of repose". Professor Gibson's analysis of the results has led to the formula:

$$\tan \phi = Kd^{0 \cdot 125} \rho^{0 \cdot 19} r^{0 \cdot 25},$$

where ϕ = the angle of repose under water,
d = mean grain diameter in inches,
ρ = specific gravity of material minus 1,
r = mean ratio of longest to shortest diameters,
$K = 0 \cdot 92$ for the statical angle of repose,
$K = 0 \cdot 71$ for the disturbed angle of repose.

The symbol ρ in this formula is a measure of the effective density of the material *in water*; thus for sand number 1, $\rho = 1 \cdot 63$, while for pumice 4, $\rho = 0 \cdot 99$. Over the range of materials tried, the formula agrees with the observed $\tan \phi$ within extreme limits of $+4 \cdot 8$ and $-3 \cdot 0$ per cent., a + sign meaning that the observed value is higher than that given by the formula.

In order to compare the behaviour of the different bed materials, the portion of the estuary between Narlwood Rocks and Hempsted was chosen (see Fig. 75). Over this region, the channel and shoal formations are extremely complex. The length of channel involved is about 38 miles, over which the width of the channel varies between 150 and 2,000 yards. The procedure in the case of the first model (horizontal scale 1 : 8,500, vertical 1 : 100) was to mould the bed to the configuration of 1849 in order to find the volume of material involved. This volume was then remoulded as a flat bed at the level of Ordnance Datum and was found to extend between Guscar Rocks and Newnham. Under the action of the tides it was fascinating to watch the growth of the various channels and banks, such as the Lydney and Shepperdine Sands, progress being particularly rapid in the earlier stages. During the experiments the behaviour was kept under almost continuous observation and cross-sections were measured after 1, 2, 3, 5, 20 and 40 hours, this final time representing about 2,800 tides. Actually, with the sands, an approximation to the ultimate configuration was attained after 350 tides, while the coarser emeries reached a similar stage of development in about 700 tides.

In the case of the second model (horizontal scale 1 : 8,500, vertical 1 : 200) the procedure adopted for most of the materials was rather different: the bed was not initially flat, but was moulded approximately to the correct contours. Each test then consisted of 2,440 tides (50 hours), but the two coarsest emeries (see Table XXIII) were omitted, while the Calais sand, which had not previously been employed, was given a trial, as also were three grades of powdered pumice.

The main conclusions concerning materials other than pumice may be summarized as follows:

A. On the 1 : 100 *vertical scale.*

(i) Even after reaching its ultimate general form, the bed showed fluctuations, the height of the banks varying from time to time by 2 or 3 feet.

(ii) Over two portions, the formation was subject to great variations from time to time. Between the Narlwood Rocks and the opposite (western) shore, the bed was extremely unstable and the channel shifted periodically from one side to the other. Both the 1849 and 1927 Admiralty surveys, however, showed it to lie close under the Narlwood Rocks.

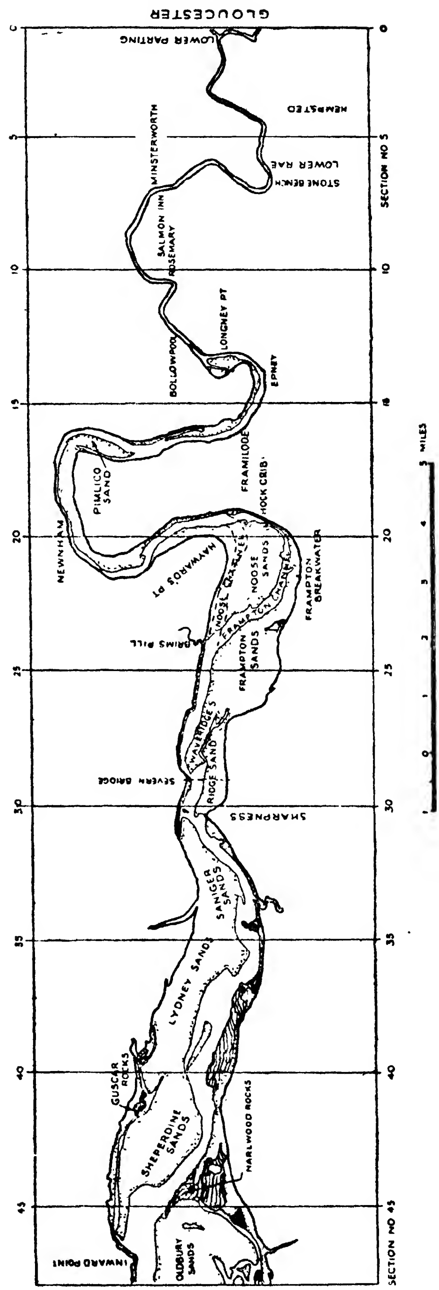


FIG. 75.

Between Tites Point and Hock Crib, where the Frampton, Noose and Waveridge sands are located, the two Cobham sands, the Portishead sand, the " 120-mesh " silica sand and the coarsest of the emeries gave a channel hugging the eastern shore as far as Tites Point and the Waveridge Sand disappeared. On the other hand, the two finer emeries, the " 80-mesh " silica sand and the Leighton Buzzard sand reproduced all the shoals and showed the low-water channel to cross over from Frampton Breakwater to Brimspill, thereafter following the western shore to Severn Bridge.

(iii) Over the remainder of the area, all materials gave the same general configuration, but produced very different maximum heights of bank, the finest sands yielding the biggest and the coarsest emery the smallest.

(iv) The best overall agreement with natural phenomena was exhibited by the " 80-mesh " silica sand (diameter 0·00700 inch), although the finest of the emeries behaved very nearly as well. The sand having a diameter of 0·00582 was too easily carried upstream on the flood tide, without any compensating scour downwards on the ebb, and that having a diameter of 0·00814 inch did not possess as high a tendency to stabilize itself in the form associated with the Admiralty surveys. Except at points of pronounced local scour—in the nature of potholes—the best materials resulted in fairly reasonable channel depths. Table XXIV provides the relevant data for the sand of diameter 0·00700 inch:

TABLE XXIV

Level of Bottom of Low Water Channel, ft. above O.D.

Section	*Model*	*Estuary*	
		1849	1927
4	11	13·5	—
8	5	4·5	—
12	6	6·5	—
16	9	9	10
19	2·5	5	6
20	3·5	−3	4
22	3	2	3
25	1	−3	−2
27	−3	−2	−3
34	−7	−8	−21
38	−10	−12	−16
41	−18	−14	−21
43	−17	−20	−26
45	−18·5	−18	−20

In general, however, the maximum heights of the banks proved to be lower than in Nature. Omitting sections 29 and 47 (where pronounced potholes occur beyond the capacity of the angle of repose in the model), and considering the region between Hock Crib and the lower end of Oldbury Sands, the mean difference between the maximum and minimum heights of cross-sections proved to be some 73 per cent. of the corresponding difference in the 1849 survey and 60 per cent. of that in the 1927.

B. On the 1 : 200 *vertical scale.*

(i) The general reproduction was better than on the 1 : 100 scale. In particular, the instability observed in the Narlwood region completely disappeared, the channel maintaining the more easterly route. Conditions in the vicinity of the Noose and Frampton sands were also improved, although the Calais sand, the finer Cobham, and the Portishead sand all showed a tendency to permit the channel to keep to the eastern shore as far as Tites Point.

(ii) The Leighton Buzzard sand (diameter 0·0223 inch) did not move freely in the model with 1 : 200 scale; transportation took place in the low-water channels, but the tops of the banks remained as originally moulded.

(iii) The following materials were all found to behave well:

(1) Sand of diameter 0·00700 inch,
(2) Sand of diameter 0·00582 inch,
(3) Sand of diameter 0·0100 inch,
(4) Emery of diameter 0·00505 inch,

although the finest of the sands listed here proved to be carried upstream rather too readily, this effect resulting in a shoaling of the narrow upper reaches. A detailed comparison of channel depths is afforded by inspection of Table XXV, while maximum heights of sections attained are recorded in Table XXVI (see p. 217).

From these observations, it appeared that with the 0·00700 inch sand the average difference between the maximum and minimum height at each section (excluding the abnormal sections 29 and 47) was 1·10 times the corresponding difference shown in the Beechey survey of 1849 and 0·90 of that in the 1927 survey, and it is noteworthy that at a later stage it was discovered that over the whole area between Gloucester and Barry Dock the relative heights at the various cross-sections averaged 0·90 of those in the latest Admiralty surveys.

Coming next to the tests on the powdered pumice, the procedure consisted of cleaning out the bed of the model above Narlwood Rocks down to bedrock, and of placing a bed of pumice, to the level of Ordnance Datum, over the 22-mile stretch between section 42 and Framilode.

TABLE XXV

Levels of Bottom of Low Water Channel (ft. above O.D.)

Section	Sand, 0·00582 inch	Sand, 0·00700 inch	Sand, 0·0100 inch	Emery, 0·00505 inch	Estuary	
					1849	1927
4	17	14	11	15	13·5	—
6	10	6	—	7	6	—
8	7	4·5	6	8·5	4·5	—
12	7	11	10	10·5	6·5	—
16	14	12	10	11·5	9	10
20	2	1	2	3	−3	4
22	2	2	5	5	2	3
25	1	−1	−2	1	−3	−3
27	−1	−3	−1	−3	−2	−3
28	−11	−8	−9	−4	−11	−6
29	−14	−14	−12	−14	−35	−35
34	−27	−18	−12	−15	−8	−21
38	−12	−12	−11	−17	−12	−16
41	−15	−14	−17	−14	−14	−21
43	−20	−17	−17	−20	−20	−26
45	−20	−19	−15	−18	−18	−20
46	−19	−19	−17	−23	−20	−23
47	−25	−24	−26	−29	−53	−50
48	−24	−28	−23	−25	−22	−26

Between sections 42 and 45 the pumice bed was tapered off at a moderate slope to join the lower channel, and the bed downstream of the Narlwood Rocks had previously been established in sand and mud in a condition approximating to the 1927 survey. The pumice developed, under tidal action, at great speed towards a configuration which showed a very remarkable general agreement with the estuary. Some material was transported upstream towards Gloucester; some drifted on the ebb past Beachley and into the wider estuary. After 4·5 years of tides (that is, 3,170 tides), conditions became fairly stable, and a measure had been obtained of the general mobility of the material and of its tendency to attain from an unnatural origin a state approximating to the estuary itself. Pumice was then added to build up the banks to the full heights shown in the 1927 survey, and the channels were adjusted, where required, to that condition also. A run of 6·6 years of tides was then made, during which frequent measurements were taken; during each "year", two floods were admitted to the Severn at a rate five times the mean discharge. Finally, an exactly similar test was conducted on the sand having a dia-

TABLE XXVI

Maximum Height of Section (ft. above O.D.)

Section	Sand, 0·00582 inch	Sand, 0·00700 inch	Sand, 0·0100 inch	Emery, 0·00505 inch	Estuary 1849	Estuary 1927
20	15	16	15	14	16	18
22	16	12·5	11	14	11	10
25	25	23	21	17	12	16
27	14	14	11	14	13	21
28	18	15	14	15	18	19
29	22	20	20	25	16	18
30	15	15	10	5	15	15
34	10	14	17	15·5	12	9
38	18	17	21	18·5	3	22
41	−4	−2	−3	−4	−2	0
43	−2	−6	−3	−7	−7	−5
45	−5	−6	−12	−14	−3	−14
46	−1	2	3	−5	0	2
47	4	0	5	2	−2	3
48	2	1	−2	0	5	5

meter of 0·00700 inch, which had proved to be the best of all the materials previously tried. Thus a direct comparison of this sand with the pumice powders was possible. The more significant findings may be summarized thus:

(i) The tendency towards ripple-formation was smaller with pumice than with sand (or with emery). It should be noted, however, that experience indicates that in any event the introduction of silt (finely-divided mud) in suspension reduces ripple-formation, whatever basic bed material is adopted. Consequently, in experiments made with the full provision of silt, any advantage which the pumice may possess compared with the sand, in respect of ripples, is not so important.

(ii) With all three grades of pumice, there was at one time or another a tendency towards unnatural instability in the region of the Frampton Sand. Thus at one period with pumice *B*, the channel made its way along the extreme eastern shore, completely eroding Frampton Sand, which then piled up towards the opposite shore. Later, however, Frampton Sand re-developed in its customary place and remained so for five years, in fact to the end of the trial. The Noose continually changed in height and extent, with all materials, but particularly so with pumice *B*, which at various times showed conditions in agreement with Beechey's 1849 survey, the 1927 survey and the Ordnance sheets.

With pumice *A* and *C*, the instability of Frampton Sand was more marked, and after a period of building up, the bank was scoured away, and the channel thereafter persisted in hugging the eastern shore.

One phenomenon was peculiar to pumice *B*—the bodily transportation of the Ridge Sand from its original (proper) place on the east side near Sharpness to the west coast. This condition lasted only for some few months of tides in the transition stage towards approximate stability but was never obtained with any other material.

(iii) Of the three kinds of pumice, grade *B* was very definitely the best and indeed approximated to the standard of the sand. Grade *C* behaved better than *A*, but it is significant that *B* is intermediate in size between the other two, and for that reason it appeared that the tests had definitely fixed, within very close limits, the best diameter of pumice to use.

(iv) As a *quantitative* measure of the results, various " gradients " have been calculated. These are:

(*a*) The " *lateral gradient* ", taken to be the average value of the difference in level between the highest and lowest points of sections 20 to 45, omitting the section 29 where scour did take place in the model but was inevitably (owing to the angle of repose) smaller than in Nature, so that its inclusion in the general comparison would lead to somewhat fictitious results.

(*b*) The " *lateral-longitudinal gradient* ", defined as the average of the maximum heights of sections 20 to 25, minus the average level of the main channel between 40 and 45.

(*c*) The " *longitudinal gradient* ", or the difference between the average level of the banks and channels from sections 4 to 14 and the average level of sections 35 to 45.

With these definitions, let each gradient in the estuary itself, averaged between 1849 and 1927, be called unity. Then the relative values obtained in the 1 : 200 model with different bed materials are listed in Table XXVII (p. 219).

The earlier tests on different sands and emeries had shown that, for a given material, the gradient increases as the grains diminish in size, and the results gained from the coarser grades, *B* and *C*, of pumice agree with this finding, whereas the finest grade, *A*, produced gradients definitely flatter than those of *B* and *C*. It would appear, therefore, that the manner in which the finest pumice grains are transported differs essentially from that of the coarser grains and of the sands and emeries within the range investigated. This was confirmed by general observation: pumice *A* was seen to be very easily carried into suspension during the flood-tide, rather than to be moved along the bed, but it was not so readily scoured

TABLE XXVII

Material	*Lateral gradient*	*Lateral-longitudinal gradient*	*Longitudinal gradient*	*Mean percentage error* *
Estuary, mean of 1849 and 1927 -	1·00	1·00	1·00	—
Pumice *A* - -	0·73	0·98	0·87	−14
„ *B* - -	0·95	1·06	1·05	+2
„ *C* - -	0·84	1·00	0·93	−8
Sand, diam. 0·00700 inch - - -	0·97	0·98	1·04	−0(·3)

by the ebb currents, and consequently the low-water channels were not so deep as with the coarser pumices or with the sands.

It will be seen from Table XXVII that the medium-sized pumice showed bigger lateral and lateral-longitudinal gradients than either of the other two. This was due largely to the fact that the channel between Frampton and Narlwood was scoured deeper with *B* than with *A* or *C*. Numerically, the channel with *B* was 33 per cent. deeper than with *A* and 13 per cent. deeper than with *C*. The banks, however, were 23 per cent. higher with *B* than *A* and 13 per cent. with *B* than *C*. Thus, the effect of the channel depths was predominant with the finest grade, *A*. This effect is analogous to one described by Dr. H. Chatley (*Proc. I.C.E.*, vol. CCXII, 1920–1, Part II, p. 411), namely that the velocity required to erode coarse silt may be higher than that for fine sand.

A detailed analysis of all the results thus obtained has indicated that the gradients attained by the various materials may be related to the size, density and shape of the particles by the expression

$$\text{gradient} \propto d^{-0\cdot287}\,\rho^{-0\cdot262}\,r^{-0\cdot15},$$

in which d represents the mean dimension of the average grain, as viewed under a microscope, ρ the *effective* density of the material in water, i.e. the true specific gravity minus the specific gravity (assumed unity) of water, and r the mean value of the ratio of the longest to the shortest diameter of an individual grain as seen under the microscope. Adopting the 0·0070 inch sand as a basis of comparison, the mean gradients observed with the other materials obey this formula within extreme limits of ± 3 per cent., excepting the finest pumice powder, whose gradients were some 10 per cent. lower than would be expected from the formula.

* A positive sign indicates that the material gives a gradient higher than that obtained in the estuary itself.

The general results also indicated that if a material gave the correct longitudinal gradient (compared with Nature), the lateral gradient was about 8 per cent. too small. Since the gradient is proportional to $d^{-0 \cdot 287}$ for a given value of ρ and r, this implies that the grain diameter of material which would provide the correct lateral gradient would be some 28 per cent. too small to give the proper longitudinal gradient. Experience shows, however, that the introduction of silt in suspension with the river water, and the additional injection of silt at some convenient spot in order to maintain a concentration of the same order as in the estuary, together with a means of reproducing the salinity of the water, closely rectifies this inconsistency, tending as it does to build up the highest portions of the banks and to increase the lateral gradient to a greater degree than the longitudinal.

Experiments with Water at Different Temperatures.

Assuming that the fluid force or drag on the grains comprising the bed is proportional to v^n, where v is the stream velocity, then the principle of dynamical similarity implies that the drag must also be proportional to ν^{2-n}, where ν is the kinematic viscosity of the water. If the value of n should happen to be 2, the direct effect of viscosity would vanish; that is to say, there would be no scale-effect from this cause. It is especially desirable to have some quantitative assurance on this point, because the temperature of the water in Nature must vary from time to time, and there was no attempt in the model to simulate these fluctuations, the water in the model being allowed, in fact, to undergo such temperature changes as happened to follow the conditions in the laboratory from time to time.

Accordingly, it was decided to make a direct test of temperature effects in the Severn model. For this purpose, the bed between Gloucester and Narlwood was moulded, in the sand of diameter 0·0070 inch, so as to represent the estuary conditions of 1927. The model was then operated on spring tides only for a period of 19 hours at " normal " or " room " temperature in order to allow the banks to become adjusted to their natural condition of approximate stability. A period of 27 hours of spring tides was next run, with the river water heated by passing it through a coil of copper pipe over a gas-burner, and with the temperature of the mass of water in the model itself also raised by means of a jet of hot water introduced into a deep rock channel, the Shoots, remote from the experimental portion. A survey was made on the completion of this run, and a similar trial was next made with the model water cooled by a supply of iced water injected at the Shoots and introduced in the Severn river supply.

The mean temperature over the experimental portion during the first test was 31·2° C. and in the second test 9·8° C., so that the kinematic viscosity in the second was about 66 per cent. higher than in the first trial.

The surveys showed conclusively that the effect of a change of viscosity even of this order was, in the Severn model, exceedingly small; in fact, the lateral and longitudinal gradients of the bed were approximately 0·9 and 0·8 per cent. respectively higher with the colder water than with the heated water. Accordingly, if the drag is proportional to ν^{2-n}, and if, as the results imply, an increase of 66 per cent. in ν increases the drag by about 1 per cent., it follows that $1{\cdot}66^{2-n} = 1{\cdot}01$ approximately, or $(2-n) = 0{\cdot}02$ or $n = 1{\cdot}98$. The motion in the model must, therefore, have been *essentially turbulent.* It is true that these tests were made with spring tides only, but general observation, together with surveys and tide observations made at different times, would tend to prove that the same conclusion sensibly applied to the model when operated on the full spring-neap cycle of tides.

As to the effect of temperature upon the tidal phenomena, it may be definitely stated that, for a given tide at the mouth, the tide levels obtained in the upper estuary, and the speed of the bore, were identical (as near as could be observed) at 9·8 and 31·2° C. Actually, a rise of temperature reduces both the viscosity and the surface tension of water, and the two effects tend rather to cancel one another in their influence on the speed of a bore wave in the model (see Chapter XII).

The Problem of Silt or Mud. Both from the point of view of the general régime of the estuary and the effect of the proposed barrage, it was of the utmost importance to attempt to reproduce the presence of fine silt in the Severn model. Since the technique involved in this is of considerable interest and importance in itself, a full discussion will be given in Chapter VI; meanwhile it will suffice to say that quantities of mud, dredged from the Severn itself near Portishead, were introduced both in the river waters and in the open estuary of the model, while at the same time a serious attempt was made to imitate the action of the salts of the sea in their important influence upon the tendency of such water-borne particles to be deposited.

Specific Results from the Severn Investigation.

For details of a comparison between the behaviour of the model and the estuary over a period representing 1849 to 1927, and for the conclusions reached as to the probable effect of the proposed barrage upon tides, currents, navigation, floods, sewage disposal * and so forth, the

* The effect on sewage disposal was examined in the model by introducing a burst of dye at the upper end and observing the time taken for this, with and without the barrage, to reach the open estuary.

reader is recommended to consult the following publications of H.M. Stationery Office (1933):

(*a*) *Report of the Severn Barrage Committee* (price 6d.), numbered 63–78. This provides a general summary of the findings concerning power output, price per unit of energy developed, and effect on navigational and other interests.*

(*b*) *Appendix to* 63–78: *Report of Expert Co-ordinating Sub-Committee*, numbered 63–78–1, price 15s. 0d., including plans and diagrams. This deals with the technical aspects of the choice of site for the barrage, its probable effect on tide levels and siltation, the civil, mechanical and electrical works required, and so forth.

(*c*) *Appendix to* 63–78: *Professor Gibson's Reports of the Model Investigations*, numbered 63–78–2, price £2 2s. 0d. (including diagrams).

A point worthy of some emphasis, perhaps, is that the *layout* of the barrage was considerably modified during the deliberations of the Expert Co-ordinating Sub-Committee very largely as a direct result of the work in the laboratory. Thus, the final design was simpler and neater and less costly than the preliminary: it is likely that an evolution of this kind in the design of engineering structures will frequently result from the study of model behaviour, which tends to clarify and crystallize ideas.

Another point is that in the model having a vertical scale of 1: 200, " it was decided to reinforce the lower portion of some of the steepest of the banks in order to prevent the initial settling of the material into the adjacent deep water channels. . . . This reinforcement is not carried up to the level of the top of the sand-banks, but finishes some 20 ft. below this level, so that the upper portions of the banks are perfectly free to move under the action of the currents. At all but a few points the reinforcement becomes completely covered by a layer of sand and, while preventing the excessive slumping of the banks noted in the previous experiments, does not affect the free movement of the surface layers ".†

Clearly an artificial device of this kind, designed to mitigate the influence of the angle of repose of the bed material, is one to be adopted with great caution and with due regard to any observed tendency for the currents to wish to erode the slopes so reinforced. It forms part of the

* There is now (Feb. 1945) a *Report on the Severn Barrage Scheme*, published by H.M. Stationery Office (price 2s. 6d.), which gives the findings of a panel of three engineers appointed by the Minister of Fuel and Power in Nov. 1943 " to review the conclusions of the Severn Barrage Committee in the light of later engineering experience and practice and of other developments, and to suggest what modifications, if any, should be made in the proposed scheme, in the programme for its execution and in the estimates of its costs ".

† Professor Gibson's *Reports*, p. 19.

whole question of choice of scales and vertical exaggeration as discussed elsewhere in this book.

Finally, concerning the Severn tidal models, it may be said that it would be difficult to imagine any models more fascinating to watch than they were; the great sweep of the flood tides covering a vast expanse of sand and mud a few seconds previously exposed at low-water; the hurried progress of the bore at springs along the upper reaches and, in the higher stretches, the astonishing changes in the detailed shape of the banks under the eye of the observer can hardly have been surpassed for interest and excitement in this field of work.

Plates XIII–XVIII illustrate some aspects of the Severn model investigation.

CHAPTER VI

SILT AND SALINITY

If sufficient data is available from observations made in Nature, it is practicable to manipulate the supply of water to the rivers in a model according to a programme which contains reasonably chosen periods of dry- and wet-weather discharge, and the percentage concentration of silt or mud mixed with the model water supply can also be regulated. In so far as this mud may be expected to deposit in regions of reduced velocity or at times of slack water, it may be expected that such silt will proceed to form shoals in qualitative agreement with Nature. A complication at once arises, however, when the vertical scale is different from the horizontal. Thus, if the horizontal scale is $1:x$ and the vertical scale $1:y$, the velocity scale for horizontal movement is, by dimensional reasoning, $1:\sqrt{y}$ and the time scale $1:x/\sqrt{y}$. But since all vertical depths are exaggerated in the ratio of $x:y$, it follows that a suspended particle should have a *vertical* motion $\frac{x}{y}\cdot\frac{1}{\sqrt{y}}$, or $\frac{x}{y^{\frac{3}{2}}}$ times as fast as a corresponding particle in Nature. For example, if (as in the second Severn model) the horizontal scale is $1:8,500$ and the vertical $1:200$, the silt in the model should have a vertical motion $\frac{8500}{200^{\frac{3}{2}}}$, or three times as fast as in the actual river. For the time being, let us concentrate attention upon the necessity for the silt to *fall* at this rate. When we do so, we discover yet another complication, namely that the salt of the sea itself may accelerate the rate of deposit, say n times, so that in effect what is required in the model is some means of ensuring that the silt falls at a rate $\frac{nx}{y^{\frac{3}{2}}}$ times as fast as the same silt would fall in fresh water.

One way of achieving this result would be to use heavier particles of silt. If $n=8$, say, and $\frac{x}{y^{\frac{3}{2}}}=3$, it would be necessary either to employ silt of the same size but of 24 times as great effective density in water, or to use silt of the same density but nearly 5 times as large as in Nature, or again a combination of increased size and density.* This, in practice, is

* This argument is based on the assumption that Stokes' law of the terminal velocity of a sphere in a viscous fluid is applicable (see *Scientific Papers*, Sir G. G. Stokes, vol. III, p. 60).

not so easy as it looks, especially as the particle sizes present in a sample of the natural silt vary over a wide range.

A more tractable method would appear to consist in utilizing the electrolytic properties of certain solutions such as potash alum or calcium chloride, so as to produce a natural coagulation of the particles and hence their more rapid deposit. The approach to this problem, as followed in the Severn investigation,* may now be recapitulated. The aim was to use a saline solution of water such that the *finest* silt in the model would fall $3n$ times as quickly as in fresh water.

That the addition of certain salts to muddy water may cause a very considerable acceleration in the rate of clearing has been known certainly for over a hundred years. For example, references have been made to it by Siddell (Mississippi Delta Enquiry, 1837), Skey (*Chemical News*, vol. 17 (1868), p. 160), Schloesing (*Comptes Rendus*, vol. 70, 1870, p. 1345), and Amor (*Min. Proc. I.C.E.*, vol. CXVIII, 1893–4, p. 107),† and it might be inferred that the appearance of bars at the mouths of some rivers is partly due to the precipitating effect of the sea salts as they mix with the silt-laden fresh water. There is no justification for treating the effect as necessarily of secondary importance; this is shown by a striking example quoted by C. J. v. Mierlo (*Assoc. Ing. Gand.*, 16, pp. 231–354), concerning the Old Fish Basin at Ostend into which a limited quantity of fresh water flows. The bed level maintained in this basin without dredging is some three feet below the equilibrium level of precisely similar basins in the vicinity *not* supplied with fresh water.

The electrochemical theory of the phenomenon has been well described by such writers as Burton (*Phil. Mag.*, 12, 1906, p. 472), Hatschek (*An Introduction to the Physics and Chemistry of Colloids*, p. 27 (1913)), and H. C. Jones (*The Nature of Solution*, 1917). If the particles present in a suspensoid are charged with the same kind of electricity, say positive, they will repel one another. On the other hand, the effect of surface tension will be to try to draw the suspended particles together so as to form the smallest volume for a given mass. If now a substance is added which in the presence of water dissociates into charged ions, it is possible for

* Appendix B to Professor Gibson's *Severn Model Reports*, p. 258 (H.M. Stationery Office, 1933).

† Mr. Amor translates from *Traité de Géologie*, M. Lapparent, 1882, première partie, livre deuxième, Section I, p. 169, as follows: "The accumulation of sediment (at the bottom of the sea) is helped by the property possessed by sea-water of retaining fine matter in suspension for a much shorter time than fresh water does. Thus, according to Mr. Sidell (quoted in Dana's *Manual of Geology*), sea-water clears itself in fifteen times less time than river-water does; notwithstanding that, on account of its greater density, it causes bodies immersed in it to lose one-fortieth more of their weight. By experiments Mr. Sidell proves that precipitation which required ten to fourteen days to be completed in fresh water, took place in fourteen to eighteen hours in saline solutions."

negative ions to be absorbed by the positively charged particles whose force of mutual repulsion may thus in effect be reduced and a state of affairs favourable to coagulation may be created. On the other hand, it may be possible to introduce a surplus quantity of salt so that the stability of the suspensoid is increased instead of being diminished.* We could visualize this as being equivalent to a change of sign opposite to the initial one being built up on the particles, and correspondingly a force of mutual repulsion being again brought into play.

The required saline solution for use in the Severn model experiments was determined as follows: A sample of mud from the Severn itself was allowed to dry slowly, it being feared that direct application of heat might alter its physical condition. It was then rolled into a powder which was passed through sieves of 0·097 mm. (0·0038 inch) nominal opening, the material emerging from the sieve in the form of a soft dust, which on shaking with water instantly formed a homogeneous cloudy suspension. Equal portions from the same heap of dry dust were weighed in a balance sensitive to 0·01 gramme and were gently dropped into glass vessels containing equal volumes of fresh water or of saline solution. By shaking the vessels a known number of times, a similar agitation was given to both mixtures; the times of agitation were noted and the turbid solutions left to clarify side by side. Observations were then made of (*a*) the depth of deposit on the bottom of the vessel, (*b*) the colour of the suspended cloud, (*c*) the ease with which print of different sizes could be read when placed behind the vessels, and (*d*) the time at which the bottom became visible through the depth. In general, it was found impossible to state that a suspension became clear at a certain instant, but it was practicable to fix the instant within a limiting period, and whenever opportunity served the test was repeated, observations then being taken at differently spaced intervals from those in the first experiment, so that another estimate was obtained of the limiting period at the beginning of which the suspensoid was definitely not clear and at the end of which it *was* clear. Conclusions were finally based on the average of the estimates of the time occupied in the ultimate clarification of the silt mixtures.

The concentrations of silt employed covered a range between 1 in 2·5 and 1 in 1,019 parts by weight. The volumes of solution varied from 50 to 1,000 c.c., and the sizes of beaker or glass container from 2·12 cm. diameter to 10·25 cm.

Now the action of precipitation may be considered to be comprised of two processes, (*a*) settlement of the heavier or larger particles, and (*b*) "clearing", or the deposit of the lighter or finer grains which remain

* Burton, *Phil Mag.*, 12, 1906, p. 472.

after the completion of the first process. Saline solutions may have little effect in accelerating " settlement " but, by coagulating the finer particles, may very greatly assist " clearing ". With a silt concentration as high as 1 in 2·5 by weight, however, the only advantage of sea-water of specific gravity 1·019 compared with fresh water was obtained immediately after agitation, when the fall of the well-defined border between the dense silt and the almost clear solution above it was some 1·5 times quicker in the sea than in the tap-water mixture. Afterwards, settlement was at the same rate in each (0·18 cm. per minute), and the test was unique among all those tried in that *clearing* caught up with *settlement* early on, and the ultimate time for complete clarification in the tap-water in this instance was only some 2 per cent. longer than in the sea-water. Silt concentrations of 1 in 5·1 and 1 in 10·2 proved to belong to an intermediate stage between the peculiar behaviour of the very closely packed particles and the behaviour of the degrees of concentration more normally experienced in natural rivers.

These effects recall a statement by Bancroft (*Applied Colloid Chemistry*, 1921, p. 193) concerning the settling of slimes or pulps: " If there is a large amount of water present and the pulps are consequently very dilute, each particle separates practically unaffected by the others. The upper part of the liquid remains cloudy owing to the presence of the very fine particles, there is no sharp line of demarcation in the liquid, and there is, or may be, a sharp line at the bottom separating the coarser particles which have settled and the supernatant liquid.* With increasing concentration, the coarser particles tend to interfere with the finer ones and to carry them down. This suspended pulp seems to settle as a whole, the upper portion of the liquid is practically clear with a sharp line separating it from the settling mud, and there is no visible dividing line at the bottom of the vessel between the slimes which have settled and those which are settling. According to Free † the change from one type to the other came at about 8–9 per cent. with the kaolin suspension that he took, the higher concentrations giving what he calls consolidation settling and the lower concentrations subsidence settling."

Returning now to a consideration of the Severn mud experiments for concentrations of the order of those likely to be found in rivers,

let R = the accelerating factor of the saline solution, that is, the ratio of the time of final clearing of the silty suspension in fresh water to that in saline solution.

Then the curve of Fig. 76 shows R plotted against the specific gravity of sea-water. This curve is based on silt concentrations ranging between

* Owens, *Geogr. Journ.*, 37, 1911, p. 68.
† Free, *Eng. Min. Journ.*, 109, 1916, p. 601.

1 in 50 and 1 in 1,000 by weight and the source of the fresh water used was Manchester tap-water, while the sea-water (diluted with tap-water in some cases to find the effect of reduced salinity) was obtained from Clevedon (Somerset), Southport (Lancashire), Ansdell (Lancashire) and Fleetwood (Lancashire). Fig. 76 shows an accelerating factor of about 7·5 in open sea-water and about 9·0 in sea-water diluted to a specific gravity of 1·004.

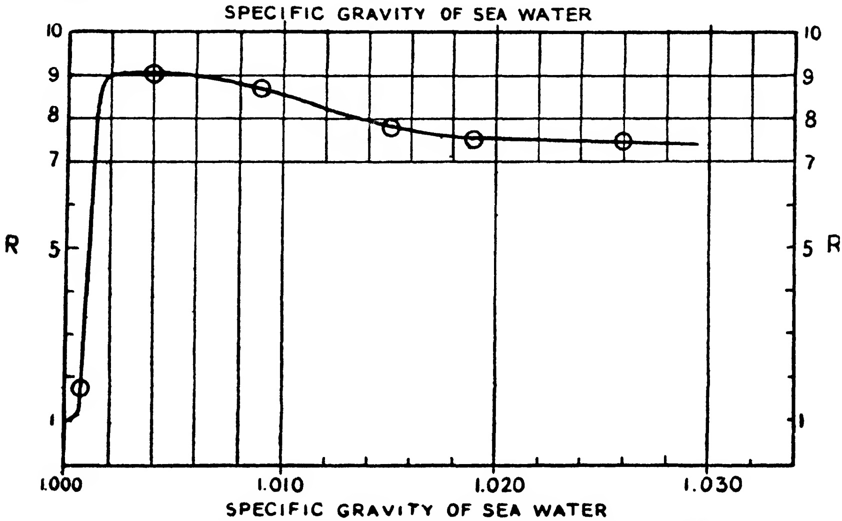


FIG. 76. Approximate Relationship between Accelerating Factor of Sea-Water and Spec. Grav. of Sea-Water.
(For concentrations of Portishead Mud between 1 : 50 and 1 : 1000 by weight.)

Comparing next the behaviour of other kinds of salt with the sea salts, it was found that approximately 4 times the weight of common salt would be required to produce clarification as rapidly as an average sea-salt solution. Calcium chloride, however, was 5 times as active (by weight) as common salt, while a solution of tap-water and potash alum containing 0·03 parts of alum by weight to 100 parts of water induced clarification approximately 2·7 times as quickly as sea-water of specific gravity 1·020. Throughout the whole of the tests, the temperature variation was between 11·4 and 18·3° C.; no significant effect of temperature was discernible within this range.

The conclusions deduced from this investigation may be stated to be in general agreement with those presented by Dr. H. Chatley in a very valuable paper, "The Constitution of Clay-Mud", *Inst. C.E. Selected Engineering Paper No.* 52, 1927. That there were discrepancies in detail is not surprising in view of the fact that Dr. Chatley's experiments were made with suspensions of Yangtze mud whose physical nature and history may yield different characteristics from Severn mud.

Interesting secondary phenomena noted during the Severn silt tests were that the colour of the silt deposited in fresh water was almost golden-brown, while in salt water it was " harder " or greyer in tone accompanied by a vaguer merging of the lowest stratum (resulting from the initial settlement) with the finer layers above; on two or three occasions, samples of turbid water left to clarify in an eastern window-sill were discovered in the morning to display a cloud of settling particles which drooped towards the window. This rather suggests some ionizing effect of sunlight, and it is noteworthy that Quincke * found that flocks of kaolin may sometimes settle on the light side, sometimes on the dark, depending on the history of the suspension and on whether the clearing was " spontaneous " or " artificial ".

Application of the Experimental Results to the Operation of the Severn Model.

The specific gravity of the open sea may be reasonably assumed to average about 1·028, while tests made on samples of the Severn indicate that off Clevedon at spring tides it is approximately 1·020 at one-third flood or one-third ebb. Other values are given in Table XXVIII:

TABLE XXVIII

Place	*At H.W.*	*At L.W.*
Off Avonmouth -	1·020	1·014
The Shoots - -	1·020	1·012
Off Beachley -	1·014	1·009

It thus appeared that a silt suspensoid in an alum solution of 0·03 per cent. by weight would clear 2·7 times as quickly as in the water found at H.W. between Avonmouth and the Shoots. The corresponding ratio for Avonmouth and the Shoots at L.W. and for Beachley at H.W. is 2·5: 1, while comparing with Beachley L.W. conditions it is 2·3: 1.

On the whole, these ratios seemed to offer a satisfactory solution to the problem, namely that in the model with vertical scale of 1: 200, silt particles should fall at a rate three times as great as in the estuary. More especially does it appear to be good enough when account is taken of the fact that the quantity of silt supplied to the rivers was purposely exaggerated in many of the model trials, and when it is realized that the

* Quincke, *Ueber die Klärung trüber Lösungen*; Drüde's *Annalen der Physik*, 4 Folge, VII, 1902, p. 86.

absence of wave-action—particularly at low tide—increases the tendency for silt once deposited to remain undisturbed.

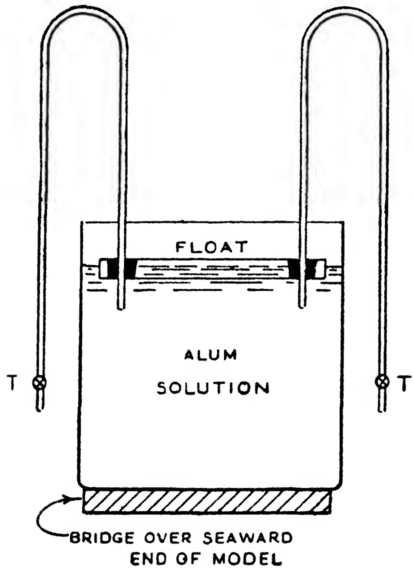


FIG. 77.

The alum was made up into a strong solution inside two tanks supported by a plank which spanned the seaward end of the model. Each tank was equipped with a float carrying two siphons (see Fig. 77), so that altogether four streams of drops of alum solution were supplied to the model estuary. Since the siphons fell bodily as the volume of solution in the tanks was being reduced, their rate of discharge was constant; it was so adjusted by the taps *T* as to maintain the required concentration (initially created by direct introduction of a quantity of alum to the model), the rate of inflow of the diluting fresh water supplied at the head of the rivers being known. Actually, samples were taken from the model and subjected to chemical analysis as a check on the maintenance of the required salinity. The weight of alum used per 24 hours of running was 12·0 ounces, which could conveniently be dissolved in the siphon tanks every day.

Silt Concentrations adopted.

On the average, the Severn river at Worcester contains only about 1 part of solid suspended matter in 27,700 parts of water,* and even at the limiting points of tidal action the concentration in this river (at Upton) and in the Wye (at Biggesweir) is no more than 1/30th or 1/50th of that in the estuary between Avonmouth and Cardiff. At Swansea and Ilfracombe, the water is virtually clear, so that there is a mass of suspended silt in the *middle* estuary which swings back and forth with the tides; whatever portion is lost by depositing within the middle region, or by drifting outside it, is made up by the contribution of the various inflowing rivers.†

The proportion of silt in suspension in the estuary is greatest at times

* *The Investigation of Rivers*, Final Report, *Roy. Geogr. Soc.*, 1916.

† Prof. Sollas, *Journ. Geol. Soc.*, 1883, p. 611.

between low-water and half-flood and least at high-water, except in the upper tidal reaches when the high-water concentrations tend to become the richest because of the rush of material carried up by the flood tide. In general there is *proportionately* more silt present in dry than in wet weather, the increase in the river water more than counterbalancing the increase in the silt-charge. Concentrations at neap tide are markedly less than at springs, especially in the top layers of the water.* Averaging all observations available, it appears that the mean concentration in the region of Avonmouth and the Shoots is about 1 : 500 by weight, while *at Framilode* (*the limit of Severn neap tides*) it is of the order of 1 : 10,000. In the preliminary tests, the river water supplied to the Severn, Avon, Usk, Wye and Parrett was arranged to include silt of an average concentration of 1 : 8,500, the idea being that such a supply would ensure that the silt effect would not be underestimated. Observation showed, however, that with such an arrangement, the mean concentration near Avonmouth did not exceed 1 : 5,000 (compared with 1 : 500 in Nature), and that even if the model were initially brought up to 1 : 500 by extra silt introduced near Avonmouth itself, the proportion fell to 1 : 5,000 in a period equivalent only to a few weeks.

This is an effect which deserves special consideration since it is likely to recur in some degree in all tidal models of silt-laden estuaries. Despite the undoubted discrepancy as noted, the soundings in the model after a long period of operation from an originally moulded state proved to be in reasonable agreement with Nature. This fact appears to present a paradox and can only be explained in the following general terms: suppose that in the present year the mass of solid matter maintained in suspension is M_0 and that a hundred years hence, say, the yearly average becomes M_1. During the period of 100 years, the rivers have supplied a mass m of fresh silt and a mass m' has been deposited on the bed of the estuary.

$$\text{Therefore } M_1 = M_0 + m - m'.$$

If then $M_1 = M_0$, that is, if the amount of suspended material remains (on the average) sensibly constant, the total amount deposited on the bed is simply that brought in from outside sources and is quite independent of M_0 or of M_1. Accordingly, although M_0 in the estuary itself is equivalent to 1 part in 500, say, while M_0 in the model is equivalent to but one-tenth of this, the total accretion in the model, being in effect practically equal to the silt introduced in the rivers over a long period, will be correct. Moreover, since the suspended silt has been caused to fall at about the correct rate, it is likely that the *disposition* of the deposits will be reasonably true as well.

* Professor Gibson's *Severn Model Reports*, 1933, p. 8.

As to why the concentration of silt present in the model estuary should reach a lower equilibrium value than in Nature, the disturbing influence would appear to be chiefly that of breaking waves or of wave-action in general, especially in comparatively shallow water. When the suspended silt falls, it forms a bed of a compact, sticky consistency extremely difficult to move by ordinary currents; in Nature, however, waves may stir some of this material again into suspension.

There must, accordingly, be an *initial* period of operation during which deposits in the model are relatively too thick, followed by a period during which (the total silt in suspension having attained an equilibrium or approximately equilibrium value) the deposits are reasonably correct.

Quite a different line of argument must apply, however, if tests are being conducted on the effects of a tidal barrage situated in a muddy estuary. For suppose the upper portion of such an estuary to be isolated from the outer sea by means of a system of gates. If these gates open during the flood tide and admit tidal water to the upper basin, this water will contain silt of a certain concentration which, through the agency of waves and currents in the outer estuary, is kept more or less constant from tide to tide. At high tide, these gates close and impound the turbid water until such time as other gates (or turbines) open during the ebb. In the course of the quiescent period in the upper basin, it is possible for some or all of the silt which entered on the flood tide to be deposited and, maybe, not subsequently removed, the upper basin being quite probably itself sheltered from pronounced wave-action or subjected to lower current velocities after the erection of the barrage than before. From this viewpoint, therefore, it is essential that the water entering the corresponding basin in the model through the sluice gates on the flood tide should contain the proper quantity of suspended mud, and in the Severn experiments this was ensured by introducing an additional silt supply, at a point a little seaward of the barrage, from an overhead tank equipped with a motor-driven stirrer.

The question arises, however, as to what would be the concentration of silt in the water flowing towards the barrage after the barrage had been constructed and put into operation. Would the concentration be the same as before the building of the barrage? It is only to be expected that the introduction of such a structure would materially alter the current velocities in its vicinity; measurements of such currents in the Severn model with and without the barrage served to show what these changes of velocity would be; they were found to depend upon the shape of barrage adopted and upon the times, relative to high water, at which the sluice and turbine gates were opened and closed. Now the proportion of silt present in the upper layers * of water in the region of Avonmouth and the Shoots is at present 2·15 times as high at springs as at neaps. The corresponding mean ratio of flood tide currents is 1·9 : 1, so that the silt concentration appears to be proportional † approximately to $v^{1 \cdot 2}$. Incidentally a similar sort of law has been observed by C. J. van Mierlo (*Assoc. Ingl. Gand.*, vol. 16, pp. 231–354) for the silt concentration at Ostend: the silt content of the upper layers at that place is halved by a reduction of current velocity from 3·25 to 2·0 ft. per second, giving a concentration proportional to $v^{1 \cdot 43}$; for the bottom layers it was $v^{1 \cdot 1}$.

If, then, the introduction of a certain design of barrage causes the flood currents near Avonmouth to be reduced in the ratio of 1 : 1·8 at springs and 1 : 1·74 at neaps, and if the silt proportion varies as $v^{1 \cdot 2}$, it may be inferred that the silt concentration will be lowered in the ratio of 1 : 2·02 at springs and 1 : 1·94 at neaps. Suppose next that the present spring and neap concentrations in Nature are 2·13 and 0·99 parts per 1,000 by weight respectively and that the estimated volume of water entering the sluice gates at springs is 1·67 as great as at neaps, then the anticipated mean silt concentration passing into the upper compartment after the introduction of the barrage becomes

$$\frac{\frac{2 \cdot 13}{2 \cdot 02} \times 1 \cdot 67 + \frac{0 \cdot 99}{1 \cdot 94}}{2 \cdot 67} \text{ per } 1{,}000,$$

or 0·85 per 1,000,

or 1 in 1,180 by weight,

whereas that obtained under present conditions (by averaging a number of results taken at various times) is 1 in 500.

The technique adopted in assessing the probable siltation above this particular barrage was, therefore, to inject a highly concentrated stream of silt between Avonmouth and the site of the barrage at such a rate that samples taken from the model on the flood tide in the vicinity of the proposed sluice gates averaged 1 : 500 by weight when no barrage was in place and 1 : 1,000 (that is, rather more than the anticipated

* The *upper* layers are considered because in this design of barrage it would be that part of the water which would enter the tidal basin through the sluice gates.

† It now occurs to the writer that the argument used in this connection most probably led to the adoption of a silt concentration for the barrage tests which erred on the safe side. The reason for this is that in stating the concentration to be proportional to $v^{1 \cdot 2}$, the whole of the reduction between springs and neaps is attributed to velocity, whereas the smaller depth at neaps would favour an *increase* of silt concentration. The full law of variation would, therefore, be concentration proportional to $v^n \times \text{depth}^m$ and n would be greater than 1·2. In support of this, it may be stated that there are several known instances in the river Parrett (Somerset) of higher silt concentrations at neaps than springs, the corresponding depths of water being sometimes twice as great on springs than on neaps.

value) with the barrage in operation. Starting with the same initial bed-condition, pairs of tests were thus made, without and with the barrage, to cover a period of twenty or more years of tides. The final conditions of the bed resulting from these experiments were compared to estimate the direct effect of the barrage itself. Thus, during twenty years of operation at 1 : 500 without the barrage, the siltation in the upper basin amounted to 40·0 million cubic yards, while during twenty years with the barrage (not the final design evolved) it was 51·6 million, the silt concentration for the barrage test being 1: 1,000. A deposit of 11·6 million cubic yards was therefore attributed to the barrage. When, for purposes of general information, the test with the barrage was repeated using a silt concentration of 1 : 500, the corresponding amount attributable to the barrage became 32·0 million cubic yards, suggesting that the total siltation in the upper basin varied approximately as the 1·46 power of the silt concentration used during the test on the barrage, and this " law " proved to be closely concordant with the results obtained when silt was supplied only to the rivers, and not also to the model estuary itself.

Quite apart, however, from the effect upon the problem of the barrage of the device of introducing additional silt, an important question concerns the influence of this *extra* silt upon the general behaviour of the model under present-day conditions (that is, without barrage). Certainly some local improvements were observed compared with the behaviour when silt was supplied only to the rivers, but on the whole the agreement with natural phenomena, as measured by soundings to the bed, was not so good. Especially is this latter statement true of the portion of the model seaward of Portishead (see Fig. 72, page 202), where, over certain regions, appreciable and unnatural shoaling took place. The indications are that some extra silt injected direct to the model estuary would have been advantageous, but not more than about one-half of what was actually injected. The system adopted was, therefore, to base conclusions concerning the probable effect of the barrage on the configuration *within the basin upstream of the structure* upon the results of the tests made *with* the additional silt supply, while the effect on the configuration of the *lower estuary* was, in general, based upon the experiments with silt supplied only to the rivers.

As to the effect of the alum as observed in the model itself, it may be stated that the general appearance of the bed, together with the detailed production of some of its features, was found to be visibly improved by the combined action of alum and silt compared with silt alone. For example, the mud flats forming the foreshore between Portishead and Avonmouth and between the Royal Edward Dock (Avonmouth) and

Severn Beach became well defined, whereas previously the silt which had entered with the Avon had been widely diffused.

It is probably true that the effectiveness of the salt depends upon the rate and thoroughness of its admixture with the suspensoid. Linder and Picton (*Journ. Chem. Soc.*, 87, 1905, p. 1906) observed that precipitation follows in the trail of the salt as it progresses through the mixture, while Baylis (*Engineering News Record*, vol. 92, 1924, p. 768) has described cases of poor coagulation due to slow initial mixture of alum with turbid water. Clearly, an effect of this kind must play its part in the formation of deposits in estuaries, but it will often be found that, as in the Severn, the admixture of sea- and river-water is very thorough, and the maximum coagulation for the given strength of salt can then occur.

The Density of the Silt.

In computations concerning dredging and siltation, the engineer is often interested in the density of silty deposits. If a sample of Severn silt be allowed to dry thoroughly and after reduction to a powder is tested by means of the specific gravity bottle, it is found to have a true specific gravity of 2·58,* the corresponding true density being 161 lb. per cubic ft. But the *effective* or *apparent* density of the same mud as deposited on the river bed may be far less than this, owing to the interstices between the particles. Thus, a sample of dredging taken at the delivery pipe while loading at the entrance to the Royal Edward Dock, Avonmouth, was found to measure, in its wet state, 0·0121 cubic foot and to weigh 1·016 lb. After drying in a warm atmosphere for a period of 15 days until successive weighing proved it to have lost its moisture, its weight was reduced to 0·426 lb. Its *effective* density, therefore, defined as the *dry* weight per cubic foot of *wet* deposit, was 35·2 lb. per cubic ft. Some surplus water may, however, have been present in a sample obtained in this fashion.

A number of samples were taken from various parts of the Severn model by sinking into the bed a thin-walled brass tube having an internal diameter of 0·687 inch. While still in the bed, the lower end of the tube was closed by means of a bent strip of zinc, and the sample was then raised. A depth-gauge served to measure its volume and the sample was weighed both wet, as dredged, and dry, after which it was passed through sieves to examine the relative silt (or mud) and sand contents. The depth to which the tube was sunk would correspond to some 15 feet in Nature.

The results derived from this analysis yielded no clear connection between the effective density and the nature of the sample. The average

* In examining a large number of samples from the Wash and the River Great Ouse (East Anglia), I have found a progressive decrease in the true specific gravity as the bed becomes more nearly pure silt or mud in the higher reaches of the river. Typical values are 2·64 at Seal Sand; 2·56 at Free Bridge, King's Lynn; 2·47 at Denver Sluice; and 2·35 in the Hundred Foot River, 1 mile below Earith Bridge. [J. A.]

value, however, was 87·9 lb. per cubic ft., while the lowest was 40·0, this relating to a sample from the Berrow Flats at a point where the slope of the bank was almost at the angle of instability. It is evident, then, that a great deal depends upon the location of the deposit, the strength and turbulence of the currents above it, the length of time during which the deposit has been consolidating, and upon whether it forms part of the face of a steep incline, when its effective density will tend to a minimum, or lies compacted at a depth below the surface, when the value will approach the maximum.

Silt and Salinity in Models other than the Severn

1. Model of Liverpool Bay and the Tidal Mersey.*

Horizontal scale 1 : 7,040, or 9 inches per statute mile; vertical scale 1 : 190; tidal period 87·1 seconds. With these scales, silt should fall $\frac{7040}{190^{\frac{3}{2}}}$, or 2·68 as quickly as in Nature. The specific gravity of the sea water at two representative places was found to be as follows:

TABLE XXIX

		At L.W.	*At H.W.*
In mid-river, off Prince's Landing Stage - - - - - -	Spring	1·019	1·023
	Neap	1·020	1·022
Off Formby Lightship - - -	Spring	1·023	1·025
	Neap	1·023	1·025

An investigation of the rate of coagulation and settlement of Mersey mud (as chosen for the model) in sea water of the mean salinity experienced near the Formby Lightship, and of the corresponding rates in alum solutions, led to the adoption of an alum concentration in the model of 0·01 per cent. by weight. Under conditions of normal river flow, this meant the use of 0·09 lb. of alum per 24 hours, introduced from siphons placed near the seaward limit of the model.

The main bed material of the model was a sand of mean diameter 0·0071 inch, or approximately 0·77 times that of the average sand in the Bay and in the approach channels to the port of Liverpool, but in addition the suspended mud was supplied to the rivers as they entered the model at such a rate as to maintain a mean silt concentration in mid-river off Prince's Landing Stage of about 1 : 11,000 by weight; samples from

* For further information concerning this model, see p. 246.

this locality in Nature had shown average values of approximately 1 : 6,000 and 1 : 17,000 at springs and neaps respectively, as compared with 1 : 12,000 and 1 : 24,000 at springs and neaps near the Formby Lightship.

In this model the supplies of silt-laden water for the rivers were fed through calibrated orifices situated in the bottom of vessels which were connected to a copper pipe. This pipe came out of and returned into a tank containing mud and water, which latter was supplied through a ball-valve. The amount of mud was replenished periodically by fresh charges —less than two ounces a day—and a motor-driven impeller kept it in suspension both in the tank and in the copper pipe through which it circulated and from which the appropriate river supplies were withdrawn.

Some allowance for wave-action was also made in this investigation by a battery of fans mounted over the mouth of the model and blowing from a direction between West and W.S.W. These fans were operated for a period equivalent to three months in each year of tides and produced surface waves corresponding in height to some 3 ft. in Nature. The chosen wind-direction was based upon five years' observations at Liverpool made at fixed hours—07.00, 13.00, 18.00 and 21.00 hours. These indicated that of winds having an intensity between 4 and 7 on the Beaufort scale * 64 per cent. were between S.W. and W.N.W., while all winds more intense than number 7 were between S.W. and W. Taking the Bay to be sheltered from the east and south and having regard to the fact that north winds are relatively rare, it was argued that the important winds from the point of view of the régime of the sand-banks are those lying between W. and W.S.W. The observations also showed that winds of intensity 4–12 blow for about 38 per cent. of the time, and that those of this intensity coming from between S.W. and W.N.W. are prevalent for some 25 per cent. of the time.

Observation of the model revealed that waves generated by the fans certainly had some effect, but that this was almost entirely confined to the highest portions of the sand-banks and to the more exposed foreshores. Ripples or ridges which had formed on these banks during a period of calm were largely smoothed out by the waves, and any silt or mud deposited on the higher spots tended to be licked off and carried elsewhere. But these phenomena could only be of a qualitative character.

In order to form some idea as to the likelihood or otherwise of heavy gales in the Bay lifting bed material from the back of training walls and dropping it in the main channels, a wooden board was mounted across the seaward end of the model. This board, which dipped into the water, was carried on a horizontal shaft to which a lever was attached. The

* No. 4 Beaufort scale represents a moderate breeze of about 15 m.p.h. velocity; No. 7 a high wind of about 35 m.p.h.

lever was worked by hand for an occasional period of four or five tides, and the resulting motion of the paddle produced waves equivalent to about 16 feet in height, causing silt to be brought into thorough suspension and certain stretches of foreshore and sand-bank to be eroded as much as 3 feet.

2. Model of the Rangoon River and Approach Channels.*

The scales of this model were 1 : 8,060 horizontally and 1 : 192 vertically, so that the required scale of *vertical* velocities was $\frac{8060}{192^{\frac{3}{2}}}$ or 3·04 : 1; this ratio of model " settlement-rate " to natural " settlement-rate " of silt was obtained by the introduction of potassium alum to the model sea at such a rate as to maintain a solution of 0·25 per cent. by weight, or roughly ten times the dose of alum adopted in the Severn or the Liverpool Bay models. On the other hand, the silt used in the Rangoon model was finely divided commercial clay, not natural marine mud.

The various tributary rivers received their water supply from a 250-gallon tank fed by the main through a ball-valve. Two separate pipes delivered water to each river; one was always running, while the second was arranged to increase the flow to the mean monsoon rate: this second pipe came into operation automatically during monsoon seasons through the action of a $\frac{1}{20}$ horse-power motor-driven timing gear. A hopper-and-belt feed carried the silt into the 250-gallon river supply tank, and the silt was kept in suspension inside this tank by means of a propeller. The speed of the belt-and-hopper apparatus was varied automatically between monsoon and dry seasons by means of a timing gear fixed to the main tide-generating machinery.

In addition to this supply of silt to the rivers themselves, a serious attempt was made to simulate the littoral drift of silt along the Gulf of Martaban. An analysis of silt samples taken in the Gulf itself on both flood and ebb tides showed that a larger mass of silt enters the Gulf on the flood tide, along the line chosen for the western limit of the model, than leaves it along that line during the ebb. Accordingly this excess amount of silt (or " tidal silt ", as it was called) was introduced in suspension with alum-charged water from a perforated trough spanning the mouth of the model. The perforations in the trough were so arranged as to discharge the greater part of the silt close inshore, and tests made during the operation of the model indicated that a good agreement was being obtained between the *distribution* of silt in the model and that in Nature. The combined silt-and-alum mixture for this injection was kept

* O. Elsden, " Investigation of the Outer Approach-Channels to the Port of Rangoon by means of a Tidal Model ", *Journ. Inst. C.E.*, June 1939, p. 3. This model is further discussed in Chapter VII.

stirred in an overhead tank. As might be expected, not all the silt fed to the river- and tidal-silt supply-tanks found its way into the model, but the residues in each tank were dried and weighed from time to time so that the silt feeds could be adjusted to ensure the proper net silt-discharge in the model.

South-westerly monsoon winds were reproduced by a battery of seven fans, which were brought into action automatically at the appropriate times through the seasonal control-gear. They made waves of about the correct height from trough to crest.

In order to determine the relative importance of tidal and river silt in the formation of the Outer bar, two differently coloured clays were at first adopted, but they became so intermingled as to defy visual separation of their respective contributions to any particular shoal. Two alternative methods were then developed by the staff of Sir Alexander Gibb and Partners (who designed, constructed and operated the model) in consultation with the Department of Chemistry at University College, London. These methods depended upon chemical detection. In the one, prussian-blue powder was added to one of the silts; in the other method, two different clays were used " which contained a different but constant proportion of titanium dioxide ".* Simple chemical tests served to show the proportions of the two silts present in a mixture extracted from some part of the model. It was thus established that " the major part of the Outer bar deposits had come from the China Bakir river or from further west, and that only a small proportion of the silt leaving the Rangoon river found its way on to the bar ".†

Other features of this fascinating model of the Rangoon river and its approaches, such as the treatment of the problem of coast erosion, are discussed in Chapter VII.

3. **Model of Bridgwater Bay and the River Parrett (Somerset).‡**

Horizontal scale, 1 : 3,000.

Vertical scale, 1 : 260.

Scale of horizontal velocities, $1 : \sqrt{260}$, or 1 : 16·1.

Scale of vertical velocities, $\dfrac{3000}{260^{3/2}}$, or 0·716 : 1.

It will be seen that the scales chosen for this model were such as to demand that particles should fall rather *more slowly* in the model than in Nature. It was still necessary, however, to reproduce the coagulating effect of salinity by utilizing a saline solution for the model Bridgwater

* Elsden on the Rangoon model, *Journ. Inst. C.E.*, June 1939, p. 22.

† Elsden, *loc. cit.*, p. 24.

‡ For further information about this model, see Chapter VII.

Bay of a strength sufficient to ensure that the silt as adopted (which was dredged from the Parrett) should fall 0·716 times as quickly as in the *sea water* of the Bay itself. Even after multiplying by this factor of 0·716, the rates of settlement are much greater than in *fresh* water. Preliminary tests made in the usual way in glass beakers indicated that the required alum-content was 0·01 per cent. by weight, and this was maintained, in the customary fashion, by siphons at the seaward end of the model. Many exploratory tests were carried out in this model *without* the introduction of silt and alum, conclusions being then based on the movement of the sand or pumice bed alone. This technique is discussed in Chapter VII. When suspended silt was introduced, however, account was taken of the fact that the major source of such silt deposits in the Parrett is the Severn estuary itself, into which the Parrett has its outfall. Accordingly the silt was introduced in suspension near Burnham,* and any artificial deposits created in the immediate vicinity of this point of injection were periodically removed. It may be appropriate to describe at this stage a new form of technique which was developed in this investigation with reference to accumulations of silt.

One of the projects under consideration was a tidal barrage or sluice-gate, which it was thought might be erected about two miles upstream of Bridgwater with the principal object of reducing the quantity of silt and sand at present transported to the upper reaches of the Parrett by the flood tide. At the selected site, the cross-section of the actual river is of trapezoidal shape, having a bottom width of some 40 feet and a top width (between flood banks) of about 135 feet. In the model, the barrage was represented by a gate operated electrically through a solenoid from contacts on the main tide-mechanism. The width of the gate was equivalent to 80 feet, and its sill was some 6 inches above the bed level of the river; its schedule of operation was as follows:

(i) During a period with flood river discharge, the gate was *lifted clear* of the water surface *throughout the tide.*

(ii) During a period of drought, the gate was *closed* throughout the tide, thus completely preventing any tidal water from passing above the site of the barrage.

(iii) During a period of "normal" river flow, the gate was closed just as the flood tide reached the level of the sill. On spring tides this was about 30 minutes † after the beginning of the flood tide at Bridgwater dock. The gate remained closed for approximately 2·5 hours,† at the end of which period it was found that the water-levels on the upstream

* For map, see Fig. 91, p. 269.

† These are equivalent data in Nature, not the *actual* times or dimensions in the model.

and downstream sides were equalized, the tide on the downstream side of the gate then standing at the same level as the river water impounded on the upstream side. At this instant of equalization of water-levels, the gate was opened so that its lower edge was 3 feet * above the sill. During the subsequent period of ebb tide—and indeed until the arrival of the next flood tide—water could escape from the upper region and was concentrated as a scouring agency upon the river bed below the barrage.

Two tests were run, one without and one with the barrage. In each case, silt was supplied in concentrated form between Burnham and Stert Point at such a rate as to maintain an average concentration of 1 : 125 by weight on the flood tide in the region of Combwich under conditions of " normal " river flow. This value (1 : 125) appears to be of the same order as that found in the estuary itself at the corresponding place, and so it was ensured that the correct amount of silt was being carried into the upper reaches of the tidal river, there to be available for the formation of deposits. At the start of *each* test, the *sand* bed of the tidal river was moulded to a level everywhere about 3 feet below the profile indicated in a longitudinal section of the actual river, dated December 1936. The model was then put into operation with silt and alum but no river discharge, until the deposit of silt brought the bed up to the standard 1936 level at the lowest points. At some places the bed was then too high, but these high spots were removed so as to give the standard levels. In this way, a layer of fresh, fine silt having a depth equivalent to 3 feet was produced before the start of each test proper. It should be mentioned also that during this work, the irregularities of the river banks themselves were simulated by a number of vertical wires $\frac{1}{32}$ inch in diameter, spaced out along the banks of the model in its upper reaches: their effect was to promote or increase eddy-formation. Each test proper lasted for 4·2 years of tides, and during this time the river discharge of the Parrett was varied according to the following sequence:

117	tides with no river flow.		114	tides with low river flow.	
26	,, ,,	flood river flow.	37	,, ,,	flood river flow.
673	,, ,,	" normal " river flow.	320	,, ,,	" normal " river flow.
36	,, ,,	flood river flow.	85	,, ,,	flood river flow.
150	,, ,,	low river flow (about one-third of " normal ").	582	,, ,,	" normal " river flow.
119	,, ,,	flood river flow.	112	,, ,,	low river flow.
534	,, ,,	" normal " river flow.	46	,, ,,	flood river flow.
			Total 2,951	tides.	

* These are equivalent data in Nature, not the *actual* times or dimensions in the model.

The flood river discharge was in general three times the "normal", with occasional bursts of *four* times. "Normal" discharge represented a flow of 1,100 cubic feet per sec. The programme of river flow detailed above had been evolved by a process of trial and error, so that the *mean* bed-level above Bridgwater at the end of the test of 4·2 years of tides *without* the barrage was within 6 inches of the level at the start, whilst between Bridgwater and Dunball it was sensibly identical at the beginning and end. (These phenomena are in close agreement with those found in the natural river over a complete cycle of conditions embracing "normal", flood and low river flows.) By comparing the surveys made during the pair of tests, the effect of the barrage was found to be as follows:

TABLE XXX

Sections	*After:*			
	117 *tides* *	143 *tides* †	2,153 *tides*	2,951 *tides*
62–59	−2·17	−0·81	−2·17	−2·19
57½–51½	+0·14	+0·73	−0·32	−0·34
50½–43½	−0·21	−0·04	−0·11	+0·19
Dunball-Combwich			−0·40	−0·60
Combwich-Stert Pt.				−0·70

* With *no* river flow. † Last 26 tides with flood river.

The positions of the places and sections are shown in Fig. 78. The

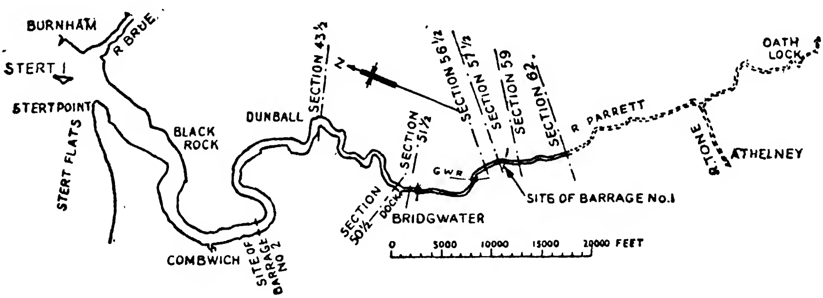


FIG. 78.

figures quoted in the Table are measured in feet; a plus sign indicates a higher bed-level with the barrage than with the open river.

These and other tests on this barrage led to the conclusion that beneficial results could definitely be obtained from such a structure in this particular river, that tidal silt could be excluded from the stretches of the Parrett and the Tone lying above the site of the sluice-gate and that excessive siltation would not be caused below it. Observation showed, however, that any lowering of the levels of high water in times of heavy river-discharge would not amount to more than a few inches, and accordingly conditions at high water of spring tides combined with such river flows would not be materially relieved except in so far that the barrage would certainly tend to *conserve* any enlargement or improvement of the channel in the upper reaches as might be effected by artificial widening, dredging or other contemplated works. In that sense, the barrage might confer important benefits; and of course the gates themselves would provide a means, during low river flows, of maintaining any required depth of water in the upper reaches.

It is, however, desirable to examine critically at this stage the question as to how far it is likely that the model indications concerning the effect of this barrage upon the disposition of silt would be borne out in practice. In so far as experience suggests that silt once deposited upon the bed of a narrow model-channel is less easily eroded than in Nature, one would expect a still bigger scour in the actual river. But the additional complication arises that the sides of the model river in its upper reaches are, even with the relatively low distortion of scale (11·5:1) adopted in this instance, almost vertical, whereas in Nature they are sloping. Except in times of flood river, therefore, some of the silt deposited in Nature would lie above water level during the more active periods of scour, whereas in the model practically all the silt surfaces, at any rate above Dunball, are subject to scour.

Two possibilities therefore arise: the current in the actual river, being concentrated on only a portion of the silt deposits, might scour the *bed* to a greater depth than that indicated by the model; on the other hand, the silt deposited on the *sides* of the natural river *might* accumulate to a somewhat greater extent with than without the barrage, an effect which could not be accurately revealed in the model. Even so, it is likely that the material thus (possibly) accumulated would be scoured away by the first river-floods.

Taking into account all the considerations enumerated above and the fact that the barrage was tested in the model both with and without silt and under different programmes of river flow, it was believed that if such a structure were erected, its effect upon the régime of the river would be very much as the model experiments indicated.

Two effects of some importance have not so far been mentioned: over a distance of roughly half-a-mile downstream of the barrage, there was a marked siltation during periods of drought, but the shoal thus formed was in fact very largely and rapidly scoured away by the next flood river in the model. For example, after the first 117 tides—all run with the supply of water to the Parrett river completely stopped and the sluice-gate closed throughout the period—the mean bed-level over this region was 3·6 feet higher than after 117 tides, accompanied by complete drought, without the barrage. But at the end of the succeeding 26 tides with the river in spate, this comparative siltation due to the barrage was reduced to 0·8 foot.

The second point is that considerable scour took place on the *immediate* downstream side of the gate during flood periods. This scour had the effect of exposing the substratum of heavier bed-material beneath the surface layer of silt and of eroding some of the material thus exposed and depositing it downstream in the form of a bar. When, however, the bed just below the gate was protected against local erosion by means of plasticene, to imitate the use of a protective apron, this objectionable phenomenon disappeared.

It may also be worth while to note that an alternative barrage scheme was studied in this model. Indeed, the proposed method of operating this alternative barrage is of considerable interest in itself. The suggested site for it was a point some 5,000 feet above Combwich (see Fig. 78), where the river is 600 feet wide between the inside edges of the saltings. The idea was that the central 300 feet of this gap should be occupied by sluice-gates, leaving a wing wall on either side with its crest-level at +11·0 feet O.D., or roughly 18 inches and 9 ft. 6 in. below the high-water level of ordinary neap and spring tides respectively in this vicinity. The level of the sluice-sill was to be −1·5 ft. O.D., and the proposed scheme of operation (due to the late Mr. C. G. Du Cane, O.B.E., B.A., M.Inst.C.E.) was as follows:

(i) When the river was in flood, the gates were to be lifted clear of the water throughout the tide, but the obstruction of the wing walls would remain.

(ii) Under conditions of low river flow, or of " normal " flow, the gates were to be lifted (so that their lower edge was above high-water level) when the flood tide began to overtop the wing walls. The gates were then to remain open until on the ebb the tide fell to the level of the wing walls. While the gates were closed, water gathering in the river behind them could be discharged over the *top* of the gates and wing walls.

In this way, it was hoped to prevent the passage of silt up the river during the earlier stages of the rising tide (except when the river was in spate), and in fact observations showed that floats dropped just above the barrage at the moment of the beginning of a spring flood tide in the model were carried upstream to a point only about 1 mile above Dunball with " normal " or no river flow, and only to a point some 4,200 yards above Dunball when the river was in flood and the gates consequently open throughout the tide. Accordingly, in times of low river discharge, when the possibility of siltation in the upper reaches is greatest, it was evident that fresh silt brought to the site of the barrage by the flood tide. could not be transported on that same tide as far upstream as Bridgwater, whereas under present-day conditions it can travel many miles *above* Bridgwater.

Experiments in the model, lasting 2,951 tides, showed that this barrage should give lower bed-levels in the river upstream of its site than under present-day conditions, the effect being most marked at the more remote regions, while there was a little siltation between the barrage and Combwich dying out further seawards. The model indications in fact favoured this barrage in comparison with the one already discussed, but the advantage was judged to be outweighed by other considerations of cost, constructional complications and the necessity to provide locking accommodation for shipping.

CHAPTER VII

TIDAL MODELS OF THE MERSEY, RANGOON RIVER, DEE AND PARRETT

THIS chapter will be devoted to a discussion of four tidal model investigations carried out in England during the past fifteen years: our object is not so much to describe the apparatus and to catalogue all the experiments made, but rather to draw attention to features of special interest as contributing to the general technique of such investigations.

1. Model of the tidal Mersey and Liverpool Bay.

Horizontal scale, 1: 7,040 (or 9 inches to one statute mile).
Vertical scale, 1: 190.
Vertical exaggeration, 37: 1.
Scale of horizontal velocities, 1: 13·8.
Scale of vertical velocities, 2·68: 1.
Scale of river discharges, 1: $18{\cdot}4 \times 10^6$.
Tidal period 87·1 seconds.

The object of the investigation was to study schemes of training walls with a view to improving the approach channels for the Port of Liverpool and to reducing the amount of dredging required in the maintenance of the approaches. The scope of the model is shown in Fig. 79*a*; as a precaution, the estuary of the Dee was included as possibly having some influence upon the conditions in Liverpool Bay. The seaward limit of the model lay a little outside the position of the Bar Lightship, or some 4 miles north-west of the end of the Queen's Channel. Some concern was felt about the choice of the northerly boundary, since this had to be represented by an artificial wall and not by a shoreline. The line adopted, however, was based upon float observations in the natural Bay, which indicate that the general tendency is for the flood-tide currents north of this boundary to make northwards towards the Ribble, and for those south of the boundary to proceed into the Mersey.

The estuary of the Mersey was moulded up to a point about one mile below Warrington, a small compartment being added to represent the area of the remaining portion of the river up to the limit of spring tides. For space considerations, the canalized portion of the Dee between Connah's Quay and Chester was replaced by a zigzag channel or labyrinth of appropriate length and width.

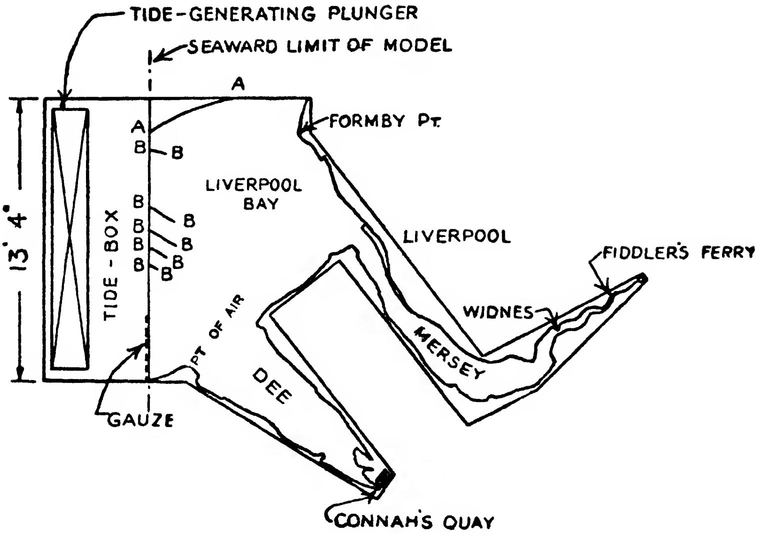


FIG. 79*a*.

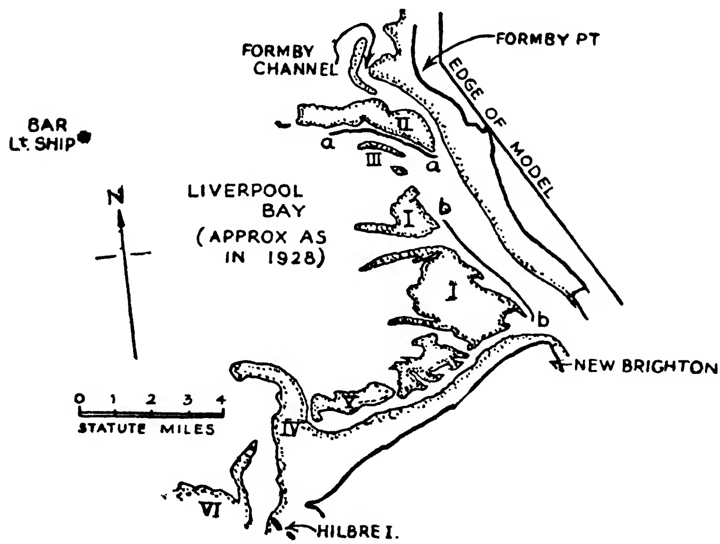


FIG. 79*b*. *aa*: *bb*: Training Walls (since greatly extended).

I: Great Burbo Bank.	II: Taylor's Bank.	III: Askew Spit.
IV: East Hoyle Bank.	V: Spencer's Spit.	VI: West Hoyle Bank.

Main approach to Liverpool lies between Banks II and III.

The tides were produced by the vertical motion of a balanced steel plunger of 21 square feet mean sectional area in plan, the stroke of which was varied from springs to neaps by epicyclic gearing; the main crank-pin of the driving mechanism was constrained by a slotted bar pivoted at one end, this device ensuring that the flood tide should occupy a shorter time than the ebb.

One difficulty immediately encountered was that, owing to conditions outside the scope of the model, the flood-tide currents set in various directions from the chosen westerly boundary. The broken lines in Fig. 80 show the track of floats released at the beginning of the flood on natural

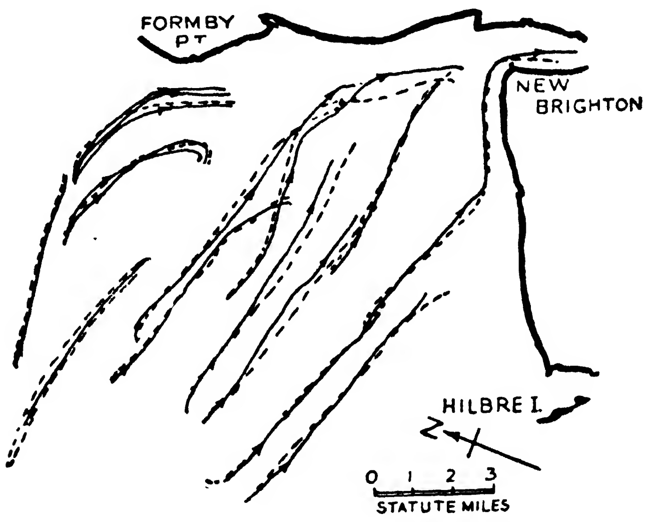


FIG. 80. Drift of Floats on Flood-Tide (Spring).
Full lines: Model. Broken Lines: Liverpool Bay itself.

tides of different ranges; the full lines show the comparable *model* observations, and it is fair to regard the agreement as satisfactory in view of possible wind effects or slight differences in the shape of the bed in the actual Bay. This agreement was only attained, however, after much experimenting with baffles. Thus, the curved boundary designated *AA* in Fig. 79*a* was found to be necessary in order to effect the proper run of those floats which sweep into the northern end of the Formby channel. Five short baffles, extending up to H.W. level and having lengths from 5 to 16½ inches, were also required to direct the floats moving round the East Hoyle Bank towards Spencer's Spit (see baffles called *BB* in Fig. 79*a*). Finally, in order to obtain the correct velocity of the flood current (about 2½ knots) past the Point of Air, it was necessary to insert a sheet of copper gauze as indicated in Fig. 79*a*; the local reduction of velocity produced by this gauze corresponds to that caused in Nature by the Chester Flats, which lie outside the scope of the model.

Exhaustive measurements were made to test the accuracy with which tides and current velocities were reproduced throughout the model. The system was to modify the stroke and shape of the plunger until a close agreement was obtained between the tide curves at the mouths of the Dee and Mersey with corresponding natural curves; the tides and currents then experienced at other points were examined in detail, and virtually perfect reproduction was found to exist. For example, Fig. 81 shows

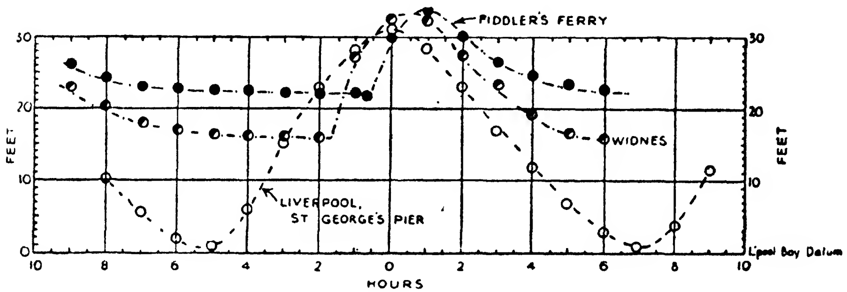


FIG. 81. Curves represent tides as observed in Model. Circles represent tides as observed in Nature.

spring tides at three places in the Mersey: Liverpool (St. George's Pier), Widnes and Fiddler's Ferry. The only apparent discrepancy of any significance was found at Connah's Quay, in the Dee, where a surge was observed at high water of spring tides. This phenomenon was also discovered in a model afterwards made of the Dee estuary alone and is discussed in that connection later in this chapter (page 262).

The bed material adopted for the model was a sand of mean diameter 0·0071 inch, very nearly three-quarters of the average size of sand found in those parts of the estuary and Bay with which the investigation was chiefly concerned.

The subjects of silt, wind and waves have already been discussed in Chapter VI, and it will suffice here to say that in a preliminary test the bed of the model was moulded approximately to the contours of 1900. Following this, the model was operated for a period equivalent to 4·25 years, and it was found that there was marked erosion along the concave face of Taylor's Bank together with accretion of Askew Spit: these effects are in general agreement with the occurrences in Nature during the years after 1900. In later work on proposed schemes of training walls, the experiments began with the bed moulded as nearly as practicable to the shape of the 1929 survey. It is noteworthy, however, that due to the vertical exaggeration of scale combined with the angle of repose of the sand, certain features could not be moulded in detail. The method employed, therefore, was to mould the sand under water and, after running a few tides to permit initial settlement, to survey the configuration actually obtained: this was taken as the basis of reference and was used as the condition to which future " 1929 " mouldings were made. For example, in the region of Taylor's revetment the moulded depth represented some 45 feet, as compared with a maximum of about 62 feet in Nature, while in the Burbo Cut the model depth was 18 feet compared with a maximum of 31 feet. Actually, however, the figures of 45 and 18 feet did allow some small further deepening under the action of the currents, and the likelihood of pronounced scour would be indicated by such a deepening.

One of the questions under examination was that of the best *height* of training walls, as well as the best position, and the study of this question of *height* proved to be one of the most interesting and valuable parts of the whole investigation. Other things being equal, the best system of walls is likely to be that which gives the greatest ratio of ebb to flood velocities, and especially if the velocities are specified as those occurring *late* in the ebb and *early* in the flood, since at those times of low depths of water the bed movement is probably at its maximum. Accordingly, a systematic exploration of current velocities was made with various heights of training wall in any particular scheme. The effect was also tried of varying the level of the top of the wall along its length.

These measurements indicated that the general effect of raising the level of the walls was to increase the flood velocities proportionately more than the ebb. Indeed, in some instances, ebb velocities were *reduced* by raising the level of the more seaward portions of the walls relative to the level of the rest of the walls.

Altogether, in fact, it became evident that, quite apart from any question of cost or ease of construction, the general level of the walls should be kept as low as possible, consistent only with two factors, namely that they should provide adequate guidance to the currents along the channel and that they should protect the channel against cross-currents.

Such " hydraulic " effects as these are very easily and reliably studied in a model by the observation of floats, aluminium dust, or bursts of dye, and it was remarkable to find how effective could be the guidance given to the stream by comparatively low walls.

Plates XIX and XX represent general views of the Mersey tidal model.

2. The Rangoon Model.

Horizontal scale, 1: 8,060 (or 9 inches to 1 sea mile).
Vertical scale, 1: 192.
Vertical exaggeration, 42: 1.

Scale of horizontal velocities, 1 : 13·9.
Scale of vertical velocities, 3·04 : 1.
Scale of river discharges, $1 : 21{\cdot}5 \times 10^6$.

As certain portions of the approach channels to the Port of Rangoon appeared to be deteriorating, the Port Commissioners in 1929 invited Sir Alexander Gibb and Partners " to state their views regarding a programme for future study and observation of the approach channels across the outer bar. Field surveys and investigations were started, and in 1931 it was decided to complete these by means of tidal-model experiments ".*

The model was constructed and operated in a basement of University College, London, two rooms each about 40 feet by 60 feet being occupied one for the model and the other for a workshop, storeroom and office. The moulding of the coastline and bed was made in a wooden tank, but instead of the " Manchester system " of waterproofing the wood by means of calico hot-ironed into marine-glue or bitumen, the Rangoon model tank was " lined with asphalt, laid on felt and expanded metal ".

Owing to the phenomenon of " diurnal variation of tide ", two plungers were used in this model, one running at half the speed of the other. The design of this kind of device is discussed in Chapter IX. Typical tide curves are shown in Fig. 82, and while it cannot be claimed that there is close agreement between the model and the natural phenomena *throughout* any individual tide, nevertheless the *average* shape is reasonably well reproduced, and this is especially true of the later ebb and earlier flood which happen to be the most vital in their effect on the movement of the bed.

In order to provide automatic control of the seasonal variations in the discharges of silt and fresh water, and in the mean sea-level, a timing gear was included in the main driving mechanism. It consisted of a reduction gear rotating a shaft once for every 704 tides or " model year ". The shaft carried two cams, each closing an electric switch for half a revolution of the shaft, that is, for six " model months ". The cams were so arranged that when one switch was open the other was closed. Thus the action of one cam was to complete an electric circuit and to put into operation a number of small motors which brought the model under monsoon conditions for " 6 months ". When, at the end of this period, the circuit was broken, the second cam closed the " dry weather circuit ", bringing all operating conditions to their dry-weather state. The motors themselves manipulated valves in the river-supply pipes, changed the speed of the hopper-and-belt feed of silt to the river supply tank, and raised or lowered a weir plate which discharged water to waste at a rate corresponding

* Elsden, " Investigation of the Outer Approach-Channels to the Port of Rangoon by means of a Tidal Model ", *Journ. Inst. C.E.*, June 1939, p. 3.

with the dry-weather or monsoon inflow of the rivers, thus maintaining appropriate mean sea-levels in the model. On the assumption that over

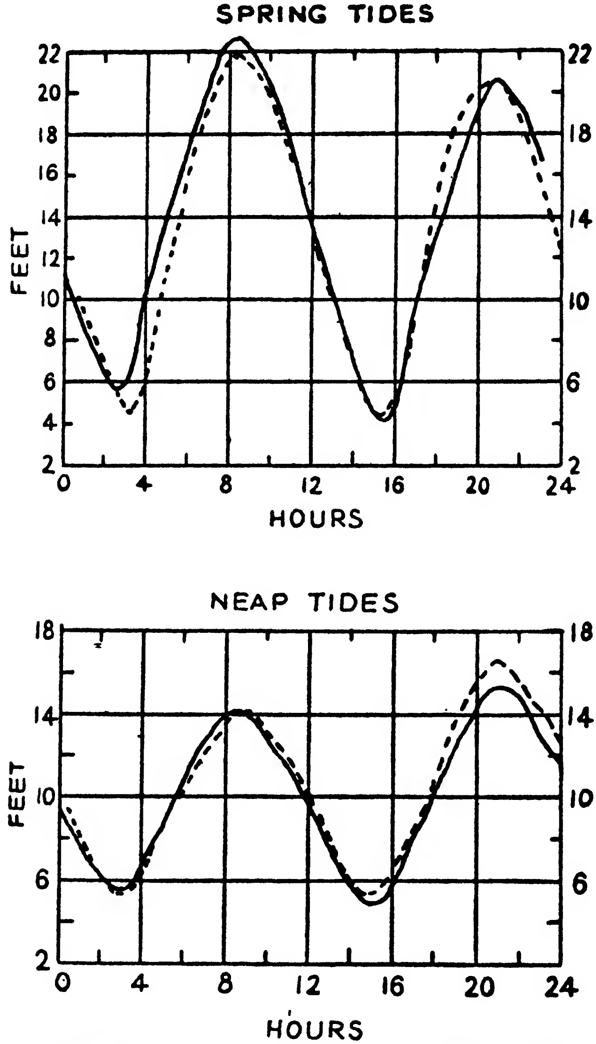


FIG. 82. Broken lines: Tides in Model. Full lines: Tides in Nature. (at Elephant Point, dry season)

(Based on Figs. 8, Plate 1, p. 68, *Journ. Inst. C.E.*, June, 1939.)

a long period of time, the final effect on banks and channels may be attributed to *average* conditions, the model-rivers were arranged to give *average* monsoon discharges alternating with *average* dry-weather flows, and the tidal cycle similarly took no account of abnormal or spasmodic variations. The general lay-out of the model is depicted in Fig. 83, and

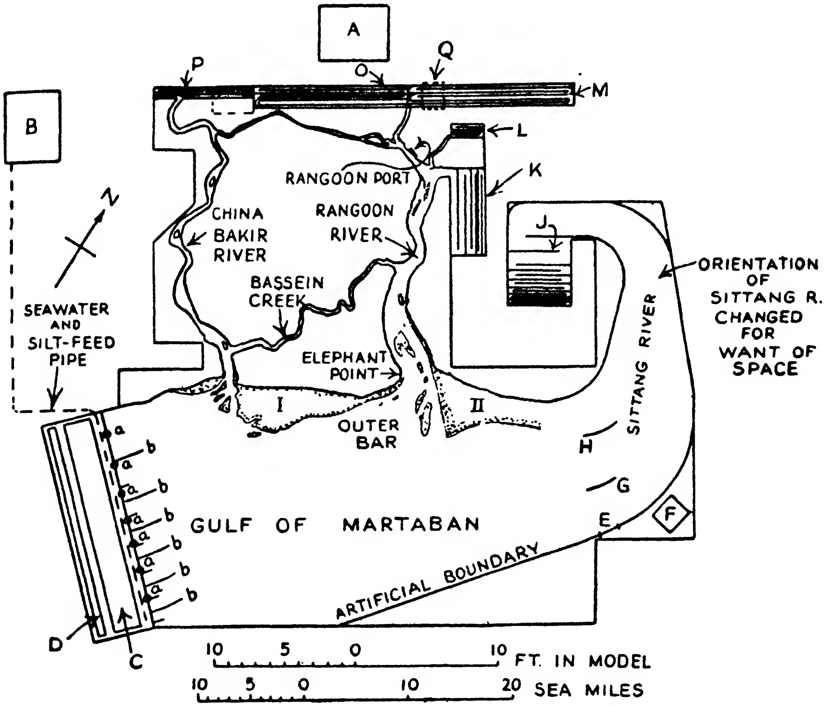


FIG. 83. Layout of Rangoon Tidal Model.
(Based on Plate 1, Fig. 3, of *Journ. Inst. C.E.*, June, 1939, p. 68.)

I. China Bakir Flats. II. Eastern Grove Flats.
A. Silt Tank for River Supply. B. Alum and Silt Tank for Sea Supply.
C. Main Tide-Producing Plunger. D. Diurnal Tide-Producing Plunger.
E. Weir for maintaining mean tide level. F. Weir-control motor.
G, H. Baffles to correct centrifugal effects. J. Labyrinth for Sittang R.
a. Fans for monsoon winds. *b*. Adjustable baffles controlling tidal stream.
K. Labyrinth for Pegu R. L. Labyrinth for Pazundaung Creek.
M. Labyrinth for Hlaing and Rangoon R. O. Labyrinth for Panhlaing R.
P. Labyrinth for China Bakir R. Q. River control motor.

the technique developed in relation to silt and monsoon winds is described in Chapter VI.

It will be seen that the Gulf of Martaban and the Sittang River had to be given artificial boundaries, and the portion of the tank containing the model version of the Gulf was made rectangular; the internal boundary was put in after asphalting the whole tank, so that the alignment of the arbitrary boundary could, if necessary, be changed. The criterion adopted for this was the requirement that the tidal range should increase eastwards. At the same time, the flow supplied to the Sittang River was only one-half of the estimated discharge of that river, as it was considered that only half of the Sittang can flow into the area embraced by the model. For space considerations also, the upper portions of the tidal rivers were represented by labyrinths which approximately reproduced the convergence due to the banks of the rivers; the beds of these labyrinths were moulded in sand to the correct longitudinal slopes. The more intricate parts of the Rangoon River boundaries were moulded in paraffin wax instead of concrete or cement mortar.

The choice of sand for the mobile bed was based upon the Severn model investigation, which had indicated that the best size was about three-quarters of the size of the actual sand found on the river bed. Accordingly, samples of Rangoon sand were examined under the microscope, and commercial sands were adopted of about three-quarters the size thus found. Two such commercial sands were chosen: one a silver sand from Cornwall which was employed in the more important parts of the model, and the other a very much cheaper ferruginous sand from the Sheffield district. Each had sensibly the same specific gravity, 2·65, as the Rangoon sand. It should be noted, however, that while certain preliminary or trial runs were started with the bed moulded in sand, which was believed to represent closely the conditions of the sea bed in the year 1875, a different procedure was followed in "predictive runs" on the model. The Port Commissioners' charts indicated that large portions of the modelled area had become covered, by 1932, with silty material some feet thick, and accordingly the model was for later experiments moulded with a bed of that nature.

The trial, or "calibration", tests were started from the 1875 régime and continued for the number of tides required to reach the year 1932; surveys were also made of the "1897" and "1910" conditions. In the first pair of trial runs, an attempt was made to represent certain *erodible* stretches of *river bank* or *coastline* in a material which would erode at the rate known to have taken place in the actual river. Although this work was supplemented by separate experiments in a flume which was given a rocking motion so that the water in it surged backwards and forwards with a period equal to that of the tidal model, the effort had finally to be abandoned. The materials tried included sand-clay mixtures and cement-sand compositions of proportions down to 1 part of cement in 300 parts of sand. This piece of research in itself is of the greatest importance: it was found that in no case did any material, among the many which were examined, simultaneously satisfy the three requirements of eroding at the correct rate, maintaining the proper side-slope and discharging appropriate amounts of material into the model. The procedure adopted, therefore, was to mould the eroding coastline in a heavy puddle-clay which did not itself erode at all. This clay bank was cut back at frequent and regular intervals to agree with the recession shown on charts of the river. At the same time, the proper amounts of sand and silt were added daily to the length of river containing the eroding banks. Measurements made in the actual river had indicated that there was an approximate ratio of 19 : 1 for the fine to the coarse constituents of the eroding alluvial banks; consequently the mixture added to the model was made up of 95 per cent. of silt and 5 per cent. of sand. Other experiments were made to determine the weight of this mixture per unit of volume after settlement and consolidation under water; these experiments formed the basis for deciding the weight of mixture to be added daily to the model in order to make up for the volume of coastline artificially cut away.

As a result of four trial runs, it was considered that the model behaved satisfactorily, and it was established that the greater part of the Outer bar deposits came from the China Bakir river or from further west, only a small proportion of the silt leaving the Rangoon river finding its way on to the bar. This point is amplified in Chapter VI.

On the whole, the principal changes known to have occurred in Nature were also found in the model, but the formation of ripples led to complicated model-contours which were not discernible in the actual charts. In order to facilitate the process of comparison, therefore, a rectangle 5 miles by 2 miles, embracing the region of the Outer bar, was plotted on all the charts, both of the actual and model river, and the *average* sounding within this area was calculated. Any diminution in the average figure thus derived was taken as a direct measurement of siltation. Mr. Elsden gives the following information (see Table XXXI, p. 256).

It will be seen that the percentage values are based on the siltation in Nature between 1875 and 1932 as 100 per cent., and having regard to all the difficulties present in this particularly complicated problem the model results are extremely gratifying. While the accretion is rather on the low side, it will be observed that it is consistently maintained over the whole period and that the model confirms the finding in Nature of very nearly the same siltation in the 22 years following 1910 as in the 35 years before 1910. This is a point of great significance.

When we consider the next stage of the Rangoon investigation, namely that of predictive runs, starting with the bed moulded to the conditions of the year 1932, a further complication becomes apparent in connection with the phenomenon of erosion of the west bank of the Rangoon River

TABLE XXXI

Comparison of Actual and Model Siltation on Outer Bar

	Decrease of average sounding on bar since 1875		
	1875	1910	1932
Admiralty and Port Commission charts	0	38 inches =51 per cent.	74 inches =100 per cent.
Model surveys in trial runs	0	32 inches =44 per cent.	63 inches =85 per cent.

above Elephant Point. During the trial runs, the technique of cutting away this bank and of introducing the corresponding amount of silt and sand for dispersion about the river was both practicable and justified, because the *rate* of such erosion was known from the recorded history of the river. Would this *rate* of erosion be maintained in the future? The answer to this question would depend upon the possibly varying nature of the strata within the erodible banks. Accordingly, two predictive tests were made, each covering the period from 1932 to 1982. In the first, the erosion of the river coastline was continued at a rate equal to about *one-tenth* of the actual 1875–1932 rate, while in the second, the rate was kept the *same* as in the 1875–1932 period. The line of argument followed in making this decision was that the truth would lie between these two extremes and that a definite indication would be provided of the extent to which the régime as a whole is influenced by the widening of the river. As an additional check, the second predictive run was prolonged for a period of 15 years beyond 1982 with a doubled rate of introduction of silt. This, it was considered, might approximately reproduce the effect of 30 years of extra tides; the method of doubling the silt quantity was adopted as preferable to that of using spring tides only for the reason that the relatively quiescent periods at neap tides may have an important influence.

It is not our object here to repeat the detailed conclusions reached from these experiments, except in a particular case which represents one of the most striking features of any model investigation so far undertaken: one conclusion arrived at was that

"The deterioration of the approaches is unlikely to continue much beyond the present stage, and further improvement will probably occur."

Now, in the discussion at the Institution of Civil Engineers on Mr. Elsden's paper, Mr. J. Guthrie Brown * affirmed that

* *Journ. Inst. C.E.*, June 1939, p. 55.

"The confidence felt in the accuracy of the model in foretelling the future enabled Sir Alexander Gibb to indicate, as a result of its operations, that the conditions at the bar, which had suffered as the result of deltaic degeneration of the river mouth, with consistent loss of navigation depth, for the greater part of a century, would appear to have reached their climax. The Commissioners, who had contemplated as an essential requirement an extensive and expensive system of river-training, were advised that any such expenditure would be worse than useless, that a policy of masterly inactivity was all that was necessary, and that the conditions would become no worse and would in all probability improve. Such advice would have been inconceivable without the guidance of the model, and it was gladly accepted by the Commissioners. Whilst that was most gratifying to those concerned with the model, what was even more encouraging was that the conditions had improved as forecast by the model.

" The first reasonably detailed chart available of the river mouth was dated 1860, when there was a ruling depth over the bar of 24 feet at low water spring tides. This depth continually decreased until in 1931 the depth available was only 12 feet; the model experiments were then commenced. Fig. [84] showed the depth of water over the bar during

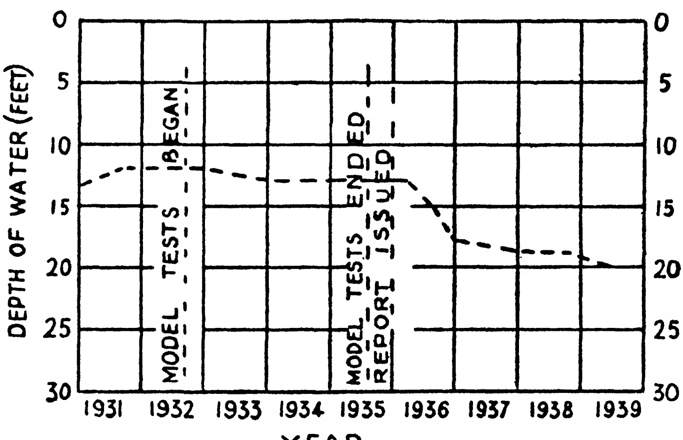


FIG. 84. Rangoon Outer Bar: Ruling Depths at Low Water of Spring Tides.
(Based on *Journ. Inst. C.E.*, June, 1939, p. 56.)

the last 9 years. There had been a period of 3 years, during which the model was operated, when the 12-foot ruling depth was still about the same. Up to that time there had been no justification for hoping for an improvement. The model results, however, indicated that the conditions would, in fact, improve, and in due course they had started to improve; the results had proved the accuracy of the forecast, because up to date there was a ruling depth of 20 feet over the bar, there having been a gradual and continuous improvement."

Despite the conclusion discussed above, the opportunity was not lost of examining methods whereby the limiting depth might be made greater than that resulting from the action of natural forces alone. These methods comprised training walls and dredging, together with the most advantageous site for dumping the spoil. They were studied partly by means of observations of current velocities and partly by direct measurement of bed movements, accretion and scour. In dredging, "the bed material was stirred up by a rotating templet and carried away by the flood tide".

An investigation was also made with the object of discovering how far a model might be used to study the behaviour of dredged channels. This was done by comparing two channels, one dredged parallel to the direction of flow and the other normal to the main tidal current. Mr. Elsden remarks that "The channel whose alignment lay parallel to the direction of flow was found to be the more permanent, but the difference between the two channels was very much smaller in the model than would probably be the case in actual fact, since the siltation of the model cross-channel was considerably affected by the ripple formation. This was taken into account when considering the possibilities of dredging from the full-size point of view".

Here, then, is an example of how engineers engaged in this field of laboratory practice themselves fully realize the necessity for interpreting the results of the model experiments with caution and discretion; another example of this attitude is provided by the discussion, in Chapter VI, of the investigation of a proposed barrage on the River Parrett.

Plates XXI and XXII, representing views of the Rangoon tidal model, are reproduced through the kindness of Sir Alexander Gibb and Partners and the Commissioners of the Port of Rangoon.

3. Model Investigation of the "Cheshire" Dee.*

This investigation† was directed towards discovering means of improving the drainage and the navigable qualities of the River Dee, which suffers from two principal troubles. There is a continual tendency for bed

* The word "Cheshire" is placed in inverted commas here in deference to a fascinating description of this great river by Captain G. A. Wright, M.C., Chief Engineer to the Dee Catchment Board, who was the instigator of the investigation. Captain Wright has remarked that only 5 miles of its beautiful course lie in Cheshire, the remainder being in Wales or forming the boundary between Wales and England.

† "Schemes of Improvement for the Cheshire Dee: an Investigation by Means of Model-Experiments", Allen, *Journ. Inst. C.E.*, June 1939, p. 30.

material to be transported upstream on the flood tide, which at Connah's Quay occupies about 2 hours compared with the 10 hours or so of ebb. It is also true that the channel seaward of Connah's Quay is tortuous, shallow and subject to considerable fluctuations of position and configuration. The main features of the estuary are shown in Fig. 85. At

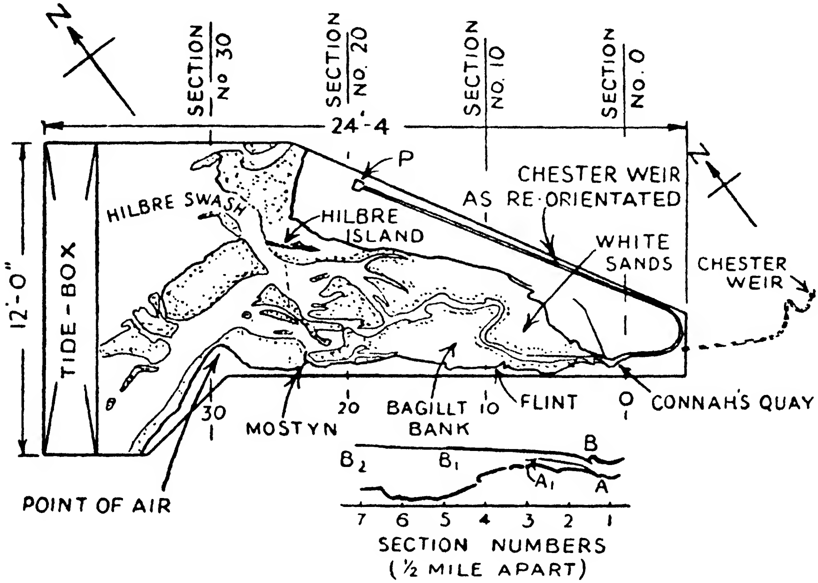


FIG. 85. *Above*—Lay-out of major Dee Model (river supply enters at P). *Below*—Enlarged Detail of Training Walls.

the time of the investigation there were in existence two training walls. One of these, AA_1, referred to as the South Wall, is covered at high water, its crest-level being about 21 feet above Liverpool Bay datum. The other, BB_1B_2, called the North Wall, is above high water of spring tides over the portion BB_1, and has a crest-level over B_1B_2 which averages 22·0 ft. L.B.D. (Liverpool Bay Datum is 14·67 ft. below O.D., and an ordinary spring tide at Connah's Quay may rise to about 29·5 ft. above L.B.D.).

It has long been thought that these walls might be better extended; another suggestion has been to erect a barrage or tidal sluice-gate near Connah's Quay, so regulated as to prevent the upstream travel of material and to promote scour downstream. The points investigated were therefore:

(1) Are the existing walls beneficial?
(2) Could they be improved by altering their crest-levels?
(3) Would an extension be desirable, and, if so, along what line?

(4) Is a barrage, equipped with locks and sluices, likely to effect an improvement?

For this purpose, two models were constructed. The larger one had scales of 1 : 5,000 horizontal and 1 : 200 vertical. Corresponding to these scales, its tidal period was 126·4 seconds, representing 12·4 hours in Nature so that one year of tides occupied about 24 hours. Its scope is shown in Fig. 85, from which it will be seen that, for space considerations, the canalized portion of the Dee up to Chester Weir was bent round and embraced within the general waterproofed wooden casing of the model. Despite this, the tide levels at Chester Weir relative to those at Connah's Quay, and the phenomenon of the bore, were accurately reproduced. River water was admitted at the upper end of the canalized river through calibrated orifices, the mean flow being made equivalent to 1,070 cusecs and the maximum flow 7,500 cusecs. Tides were generated by the displacement of a steel plunger driven through epicyclic gearing for the lunar cycle, and, as in the combined Mersey and Dee model already described, the last wheel of the epicyclic train carried a pin which rotated in a brass block which was able to slide along a slotted bar pivoted at one end; this served to make the time of rise of the plunger longer than the fall, and hence the ebb tide to last longer than the flood, as was demanded by the Liverpool Bay tide curves.

The bed material used to form the sand-banks in the model was a sand of specific gravity 2·63, a mean grain size as seen under the microscope of 0·0071 inch, and a mean ratio of longest to shortest diameters of its individual grains of 1·53. Certain tests were, however, repeated with a coarser sand of diameter 0·0092 inch and mean ratio 1·58, and also with powdered pumice of specific gravity 2·00, mean size 0·0115 inch and mean ratio 1·58.

The other tidal model used in this investigation was made with a horizontal scale of 1 : 40,000 and a vertical scale of 1 : 400; its tidal period was 22·3 seconds, and although it was only about 4 feet in length it proved to be of considerable assistance. Tidal heights were found to be reproduced with a high degree of accuracy, the appropriate tide-curve at the mouth of the model being generated, as in the larger model, by a motor-driven steel plunger. Preliminary experiments made in the smaller model served as a useful guide in discriminating between schemes of training walls which offered no hope of success and those which appeared to be promising; the latter were then tried in the larger model.

It is not inappropriate to emphasize the value of a "miniature model" of this type, which possesses great virtues in mechanical simplicity and cost as well as being so easily and quickly adjustable to different conditions. For example, the bed of such a model can be rapidly moulded, and the *general* effect of training walls in the form of thin metal plates may be discerned in a short time. Moreover, since tidal phenomena are so well reproduced, such a small model may well serve as a means of assisting the design of the tide-producing mechanism for a more elaborate model in which modifications by trial and error are much more difficult and expensive.

No attempt was made in the investigation to compare the behaviour of the major model with the natural estuary of the Dee over a period of time (as was done in the Severn and Rangoon investigations). But the practice was followed of running pairs of comparative tests in the model; that is, to test a proposed scheme of works by operating with and without that scheme from a standard initial condition of the bed. Tide levels and current velocities were also frequently measured as well as surveys of the bed, it being argued that any alteration which prolongs the flood- and shortens the ebb-tide must tend to improve the condition of the river between Connah's Quay and Chester. Since, however, most of the movement of the bed takes place during the first half of the flood and the second half of the ebb, observations of these times were regarded as important, as well as those of the overall times of flood and ebb. It should also be noted that the use of two models built to different scales provided a means of detecting discrepancies due to scale-effect; no material discrepancy was actually revealed. Unless otherwise stated, the following discussion refers to the work on the larger model.

The tide-curves shown in Fig. 86 will serve to illustrate one or two important points. In the first place, the agreement between the rates of rise and fall at Hilbre Island (mouth of the estuary) is probably as close as could be achieved by the use of one generating plunger alone. It is not so easy, however, to pass judgment on the curves shown for Connah's Quay, although clearly the *general* effects are faithfully reproduced. The difficulty arises when a *detailed* analysis is attempted; the rate of rise of the natural tide at Connah's Quay is known to vary considerably from time to time. Thus, Mr. A. Caradoc Williams, Assoc.M.Inst.C.E., who has very great local knowledge, has stated (Liverpool Engineering Society, November 1929) that " the flow at Connah's Quay lasts for a period of from 2 hours 15 minutes down to 1 hour 50 minutes. . . . During neap-tides there may be no rise at all at Chester, and in one case recently there was only a three-foot rise at Connah's Quay." When it is realized that the neap-range at Connah's Quay and at Chester Weir is frequently 7·5 and 2·5 feet respectively, the difficulty of comparative analysis will be appreciated. The model in fact showed that for a given tide at Hilbre, the level of high water at Connah's Quay could be raised by as much as 17 inches simply by lowering the Bagillt bank. There was also often a surge of tide at high water; that is, the tide rose to a certain level, began to fall, and after a short interval rose again before proceeding with the process of ebbing; this effect was found to depend upon the bed configuration. In extreme cases, the fall from the first high water amounted to the equivalent of some 12 inches on spring tides, followed by a rise of several inches to the second high water. It is not generally believed that

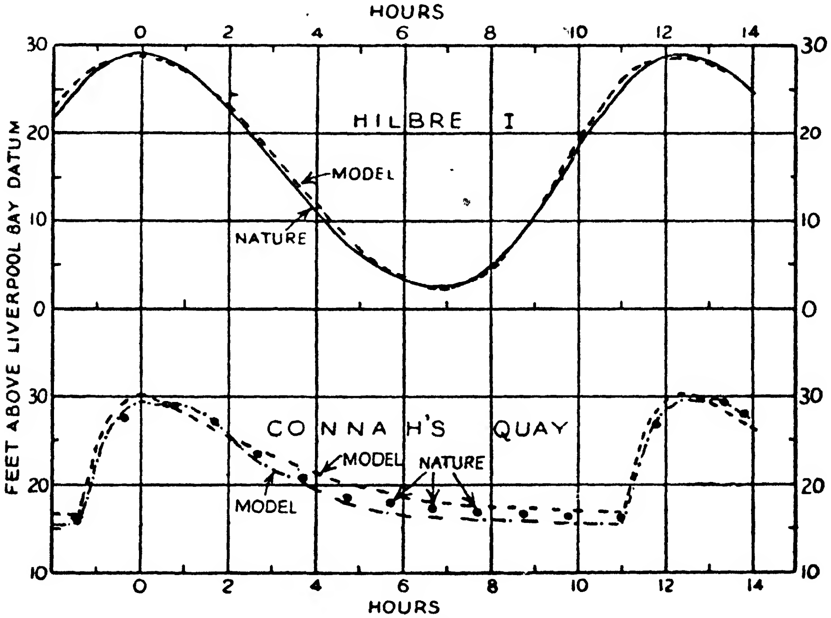


FIG. 86.

(Based on *Journ. Inst. C.E.*, June, 1939, p. 35.)

this effect occurs in Nature, although there exist tide-curves for this part of the Dee estuary which do, at any rate, exhibit jerks and pauses of a complex character.

It is interesting to note that the level of low water in the model at Connah's Quay on neap tides was below that of spring tide low water (for a given river-discharge and configuration of the bed). This is believed to be in agreement with the actual river, and indeed is an effect not at all uncommon in the upper reaches of tidal rivers: the greater volume of water passing upstream on the spring tide resulting really in an increased river-discharge for the ebb. In Fig. 86, two model tide-curves are shown for Connah's Quay, taken at different times and with different low-water levels; an estuary-curve is also provided for comparison.

For full information concerning the experiments made with various schemes of training walls, attention is invited to the Paper already cited,* but some features of special interest may be recapitulated here. Firstly, it was demonstrated that the existing training walls *are* of value; when they were taken out of the model, it was found that high-water levels at Connah's Quay and Chester Weir rose 6 and 5 inches respectively, this effect being accompanied by a reduction of 37 per cent. in the time taken by the first half of the flood tide at Connah's Quay. Consequently, the existing walls may be considered to act beneficially upon drainage and upon the transport of bed material upstream.

In order to effect any further improvement, in particular to the channels seaward of Connah's Quay, it was found necessary to extend the existing walls for a considerable distance, the most promising scheme† being that indicated in Fig. 87. It should be borne in mind that this scheme was not only tried by inserting the walls in entirety for a start, but also by

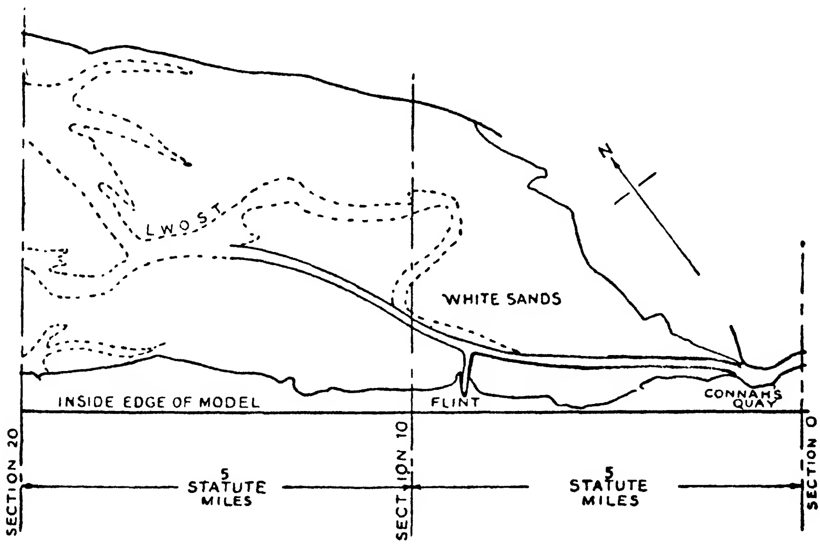


FIG. 87. Training walls, shown in thick lines, to be above high water of spring tides.

(Based on *Journ Inst. C.E.*, June, 1939, p. 48.)

* "Schemes of Improvement for the Cheshire Dee", *Journ. Inst. C.E.*, June 1939, p. 30.

† In other experiments made after the publication of the Paper in the June 1939 *Journal of the Institution of Civil Engineers*, it was found that this scheme might be still further improved as regards its effects on current velocities. The general line of the walls, however, remained as shown in Fig. 87.

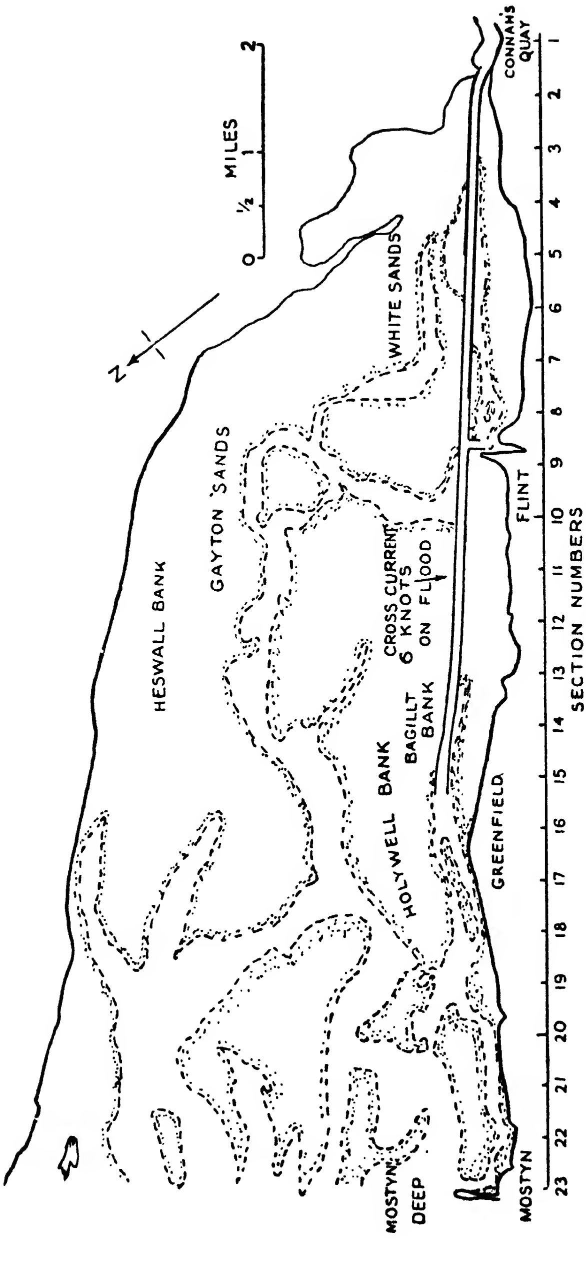
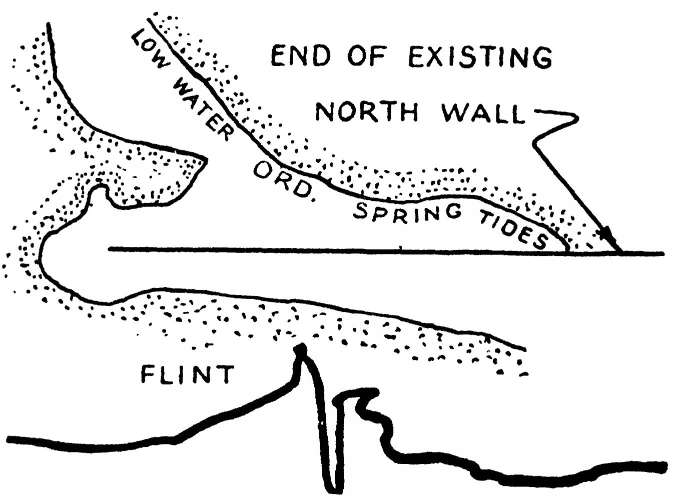


FIG. 88. Low Water configuration in Model after prolonged test with training walls through Bagillt Bank.
(Walls in thick lines are above H.W.S.T.)
(For further details, see *Journ. Inst. C.E.*, June, 1939, p. 37, and p. 68, Plate 2, Fig. 4.)

constructing them in stages to a definite plan spread over some five years of tides and accompanied by progressive dredging, the spoil being deposited over specified areas. Incidentally, in experiments of this sort, where the training walls are made of thin metal plates, it is probable that the local scour at the exposed end of the wall is exaggerated, due to the sharp edge: this effect may be counteracted by protecting the bed at the toe of the wall with a small flat plate resting on the bed and thus simulating the action of a mattress, fascine or layer of dumped stone.

A good deal of time was devoted to studying the possibility of training a straight channel through the Bagillt bank, as indicated in Fig. 88. In this connection, dredging, training walls, and a combination of the two methods were considered, but it was found that the amount of dredging required to establish and maintain a navigable channel along this line was very large, that the channel suffered from the essential instability of a straight course, that the ebb current seaward of the toe of the walls was likely to erode any foreshore not heavily protected, and that in any event the approach to the mouth of such a channel from the Hilbre side of the estuary would be difficult. One of the troubles experienced is illustrated in Fig. 89, from which it will be seen that while the advancing training



10 9 8 7

Section Numbers in Model (spaced at distances equivalent to ½ mile apart).

FIG. 89. (For further details see *Journ. Inst. C.E.*, June, 1939, p. 37.)

wall is scouring a hole in the Bagillt bank lying ahead of it yet the channel persists in doubling back *behind* the wall in order to rejoin the existing channel. This was still found to be so when the bed material was replaced by a coarser sand or by pumice powder.

In order to determine what sort of configuration of sand-banks would be produced by the action of the tides if the bed of the estuary were initially level, four experiments, one with sand and three with pumice as a bed material, were tried in the *smaller* model. For two of these tests, one with sand and one with pumice, all training walls were removed; in the other tests, a plate representing the existing North Wall was introduced. The results are shown in Figs. 90*a–d*; they demonstrate the

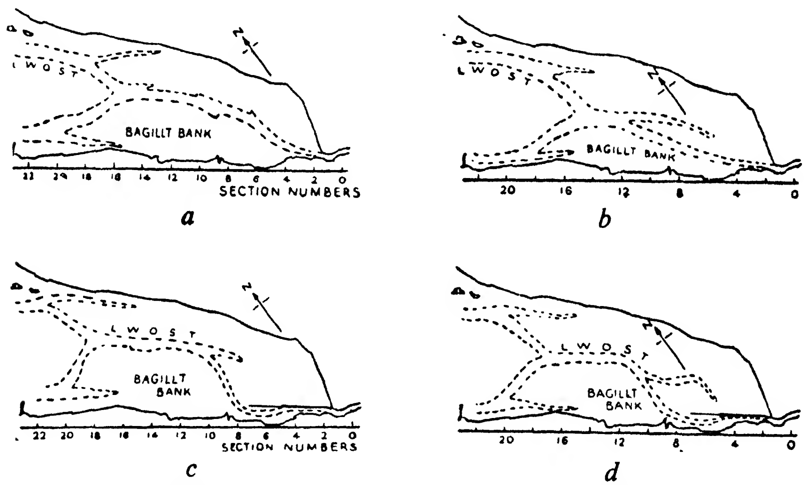


FIG. 90. Results of Tests in smaller Dee Tidal Model starting with *Bed Moulded level.*

(Based on *Journ. Inst. C.E.*, June, 1939, pp. 40 and 41.)

a. After 4,300 Tides without training walls. Bed Material, Sand (mean diam. 0·0071 inch, spec. grav. 2·64).

b. After 1,000 Tides without training walls. Bed Material, Pumice (mean diam. 0·0115 inch, spec. grav. 2·00).

c. After 1,200 Tides with northern training wall. Bed Material, Pumice.

d. As 90*c*, but after 4,000 Tides.

tendency of the channel to make towards the northern or Wirral side of the estuary: nor did the introduction of the training wall effectively reduce this tendency beyond the toe of the wall. In every case the Bagillt bank, produced partly by direct transportation of material on the flood tide and partly by an eddy contained in the flood tide and curving round towards the Welsh shore, developed as a prominent feature of the régime.

In this investigation, a barrage built across the channel at the end of the existing South Wall gave disappointing results. It consisted of a gate raised by an electrical solenoid operated by a timing device attached to the main tide-gear of the larger model. During the rising tide, the gate was shut so that no tide water could carry sand into the upper reaches of the river. During the latter part of the ebb tide the gate was opened to discharge a high velocity jet of river water which had accumulated above the barrage during the period of closure. Such a scheme offered the doubly attractive prospect of preventing the drift of sand up the river and of encouraging scour in the lower channels on the ebb. Experiment indicated, however, that the scoured material would be redeposited in the channel upstream of Flint, and in fact that there would be a general deterioration between Flint and the barrage. Even an initially dredged channel could not be maintained, for during periods of little river flow, when the volume of water available for discharge through the sluices would be small, fresh material would be piled up near the barrage on every rising tide.

Plates XXIII, XXIV and XXV are photographs of the Dee tidal model.

4. Model of Bridgwater Bay and the River Parrett (Somerset).

Horizontal scale, 1 : 3,000.
Vertical scale, 1 : 260.
Vertical exaggeration, 11·5 : 1.
Scale of horizontal velocities, 1 : 16·1.
Scale of vertical velocities, 1 : 1·40.
Scale of river discharges, 1 : $12{\cdot}6 \times 10^6$.

This investigation * was of special interest in that it involved experiments with practically every known method of river improvement: groynes, longitudinal training walls, cut-off channels to replace existing bends in the tidal river, and barrages. Of all these methods, only the barrage appeared on test to offer a practical solution. This conclusion is in striking contrast to the investigation of the Dee estuary with which we have just dealt, and this fact emphasizes the danger of generalization in the treatment of river problems.

The Parrett is navigable to Bridgwater bridge, about 13 miles above Burnham pier (see Fig. 91), and vessels up to some 800 tons use it as far as Bridgwater, while those up to about 1,500 tons can proceed to Dunball. The Parrett is tidal to Oath Lock and its tributary, the Tone, to Ham Mills, these points being 9·14 and 12·9 miles respectively above Bridgwater dock. The range of spring tides at Burnham-on-Sea is approximately 37 feet; at

* "Schemes of Improvement for the River Parrett: An Investigation with the Aid of a Tidal Model", Allen, *Journ. Inst. C.E.*, Dec. 1942, p. 85.

Bridgwater it is of the order of 14 feet, but this figure depends essentially upon the volume of fluvial water passing down the river.

Above Bridgwater, a large area has been subject to flooding, and it has been surmised that any enlargement or improvement of the channels above Bridgwater, which might be designed to assist drainage, would probably become useless in the absence of any means of reducing the present tendency of the flood tide to carry silt into the upper reaches. But in designing any works with the object of reducing this siltation, the aim must be kept in mind of avoiding interference with navigation.

At Burnham, a spring tide rises in about 4¾ hours, yet at Bridgwater it may take only 1¾ hours. A tidal bore is often observed, and it is remarked on Admiralty Chart No. 1,152 (March 1931) that " first of flood at spring tides rushes up the river Parrett with great force, and in the vicinity of Stag Island takes the form of a bore, which attains a height of over 2 feet at Bridgwater. At neap tides the bore is imperceptible ". The site of Stag Island (no longer exposed at all stages of the tide) is shown in Fig. 91.

In the year 1938, the late Mr. C. G. Du Cane, O.B.E., was invited by the Somerset Rivers Catchment Board and the Corporation of Bridgwater to advise concerning the improvement of the river, for which a scheme had been tentatively drafted, and after studying the data then available, Mr. Du Cane decided that a tidal model might be of value. Professor A. H. Gibson was accordingly requested to construct and operate such a model.

The scope of this model is shown in Fig. 91; in order to ensure the accurate reproduction of tidal phenomena and to be reasonably satisfied that the hydraulic conditions at the seaward limit would not be sensibly altered by any of the proposed works, it was deemed essential to include a large area of Bridgwater Bay. The seaward limit chosen was some 6·5 miles west of Burnham along a line approximately normal to the set of the currents. The upper limit of the moulded model was about 3 miles above Bridgwater, and the remaining tidal portion was represented by a labyrinth into the top of which river water could be admitted through a calibrated orifice at a rate equivalent to 1,100 cusecs for " normal " and 3,300 cusecs for flood discharge. In certain tests, short bursts of 4,400 cusecs and an elaborate programme of dry weather and spate conditions were employed (see Chapter VI). The sand used for the mobile bed was similar to that found best in the Severn investigation, but some check runs were made with the upper portions of the tidal river moulded in powdered pumice of specific gravity 2·00 and mean grain-diameter 0·0115 inch (that is, 60 per cent. greater than the sand, the specific gravitv of which was 2·63).

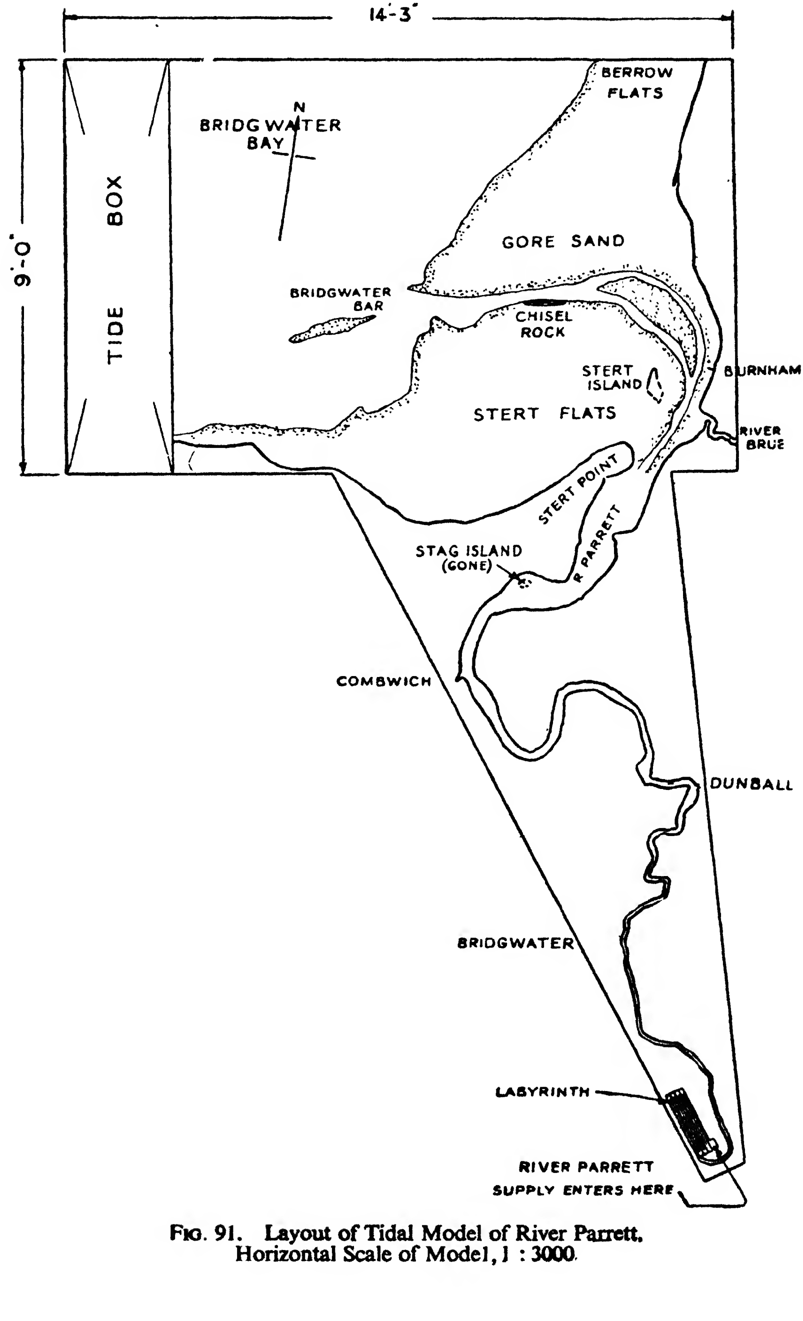


FIG. 91. Layout of Tidal Model of River Parrett. Horizontal Scale of Model, 1 : 3000.

The tide-producing mechanism was very like that of the Severn experiments, being made up of a counterbalanced plunger driven by a constant-speed motor with nearly simple harmonic motion. Provision was made for a cycle of spring-neap tides, but some exploratory trials of proposed schemes were made with the simpler and more convenient artifice of spring tides only, during which tide and current effects and bed movements were all observed and compared with corresponding measurements in the model before the introduction of the proposed schemes.

According to the chosen horizontal and vertical scales the tidal period of the model should have been 240 seconds, to represent 12·4 hours, but experiment showed that closer agreement with tide-curves from the estuary and with the phenomenon of the bore was obtained with a shorter period; and finally, as a result of experiment, a period of 210·5 seconds was adopted. This period is 0·876 of that (namely 240 seconds) appropriate to the scales according to the usual expression

$$\frac{t}{T}=\frac{l}{L}\sqrt{\frac{H}{h}}=\frac{y^{\frac{1}{2}}}{x},$$

the horizontal scale being $1:x$ and the vertical scale $1:y$. This example is then of peculiar interest, since it is the only one of the Manchester models (the Severn, the Humber, the Mersey, the Dee and the Parrett) in which such a departure from the use of the nominal time-scale was found to be necessary. Similarly, the period employed for the Great Ouse and the Rangoon models was also consistent with the linear scales. It is not easy to account for the one exception of the Parrett, although in one respect that model differed essentially from the others, namely in the ratio of the horizontal and vertical scales. Thus, the vertical exaggeration was 3,000: 260, or 11·5:1, a value much smaller than in the earlier models. Indeed, the smallest exaggeration used in the others was 25:1 (in the Dee). It may be, therefore, that the relatively low exaggeration led to hydraulic resistances which were proportionately too low, and accordingly it was necessary to accelerate the currents by way of compensation. This situation rather reminds us of what happened in the case of the model of the *non-tidal* Mersey, which has been discussed in pages 31 to 36. In that example, where the horizontal scale was 1:800 and the vertical scale 1:120, giving an exaggeration or vertical distortion of 6·67:1, the channel section was reduced in area by some 12 per cent., and the bed roughened with coarse sand, in order to obtain the proper resistance at all stages of the river, whose rate of discharge was still based upon the scale-ratio $1:800\times 120\times\sqrt{120}$. In effect, therefore, the mean velocities of flow were accelerated by some 12 per cent. by means of an adjustment to the sectional area of the channel, whereas in the Parrett tidal model the current velocities accompanying the tide were increased by practically the same percentage by shortening the tidal period. The suggestion that the necessity to use a shorter period might have been due to otherwise deficient resistance is supported by the effect on the bore. A high resistance tends to favour the creation of a bore, even though the ultimate decay of the bore is caused by the further resistance which the wave encounters in its propagation along the river.

There is, however, one other factor which may have contributed to the difficulty experienced in this model, namely that the precise levels of the vast expanse of the flats which dry out at low water along the shores of Bridgwater Bay have an important bearing upon tidal phenomena. If, then, the levels of those flats were not sufficiently well defined by the data available on the Charts, it is possible that the contours to which they were actually moulded at the start of every major test in the model were just sufficiently inaccurate to make an alteration in the tidal period necessary.

In connection with all that has been written above, the question will naturally be asked: why was such a relatively small vertical distortion (11·5:1) adopted, since in earlier investigations in the same laboratory at Manchester University, the lowest distortion had been 25:1? The answer lies in one of the types of schemes which had to be investigated in the Parrett problem, namely a complicated system of groynes or spur dikes which commonly produce scour-holes with steep sides. In such a case, therefore, it is desirable to reduce the vertical exaggeration with a view to improving the chances of faithful reproduction in the model, where the angle of repose of the sand and silt has to be contended with.

As an illustration of the way in which a model of this kind may often yield incontestible results of the greatest value in a very short time, we may with advantage refer to the experiments on the Parrett model concerning the effect of a proposed barrier bank between Stert Point and Stert Island. If, as has been suggested, a large quantity of silt is stirred up by the flood tide as it advances over the Stert flats and is carried through the gap between Stert Point and Stert Island and thence into the tidal river, it might be profitable to close the gap either partially or completely by means of a barrier bank. This would then offer a chance of decreasing the amount of tidal silt available for accretion in the upper estuary.

Observations of currents in the model with floats and dye revealed, however, that on a spring tide, the tide coming up the main channel past Burnham reached Stert Island and Stert Point rather earlier than that coming in directly across the Stert flats. Consequently, there was a short period of *outflow* between the Point and the Island until, some 2½ hours before high water, inflow began and floats dropped at that instant behaved in various ways according to the precise position and instant of releasing them. Most were caught in an eddy round the Point and got no further upstream; none succeeded in reaching more than ½ mile above Combwich. Nor did any inflow take place through the gap between the Point and the Island from the region seaward of a line about 1,200 yards west of the Point and the Island.

Again, the maximum velocity of inflow appeared on measurement to be the equivalent of approximately 1 knot, whilst the outflow through the gap attained 2·7 knots, and floats released at the high-water turn of spring tides travelled seaward of the Chisel rocks before coming to rest.

Calculations based on these observations led to the conclusion that not more than about 3 per cent. of the water entering the estuary on a spring tide does in fact come over the Stert flats and through the passage between Stert Point and Stert Island. Moreover, this water enters during the later stages of the flood tidal rise, and only a small fraction of it travels more than one mile upstream, while none of it gets beyond approximately half a mile above Combwich. It appeared, therefore, that the inflow between the Point and the Island can contribute only a very small quantity to the silt deposits in the upper reaches. If it be argued that the same conclusion might have been reached from float observations in the estuary itself without recourse to a model, the answer is that such observations in Nature are not so easily undertaken, having regard to weather and other practical difficulties, and that it is impossible to get the same *general* picture as in a model. It is sometimes stated, with justification, that a model may be too small to indicate local or detailed effects, but it is equally true that in visual observations of an actual river where the field of vision is necessarily restricted, such localized phenomena may assume an importance out of all proportion to the complete régime. There is the further point, too, that by dropping a small crystal of permanganate of potash on the bed of a model, the deep-water currents may be seen, thus supplementing the study of the surface movement by means of dust or floats.

Reverting to the subject of the proposed barrier bank, it should be remarked that the proposal was to construct the embankment, if at all, not up to high-water level, because if it were built so high it would cut off one means of escape for the ebb tide, and also because a comparatively low wall might be effective in trapping the major portion of the silt carried on the flood tide. Accordingly, experiments were made in the model with an embankment having an average top-level about 10 feet below high water of ordinary spring tides, the mean height of the wall above the existing bed being 3 feet and the maximum height 13·5 feet. Inflow then began only some 15 minutes later than it did without the embankment, and occasional floats reached Combwich, that is, a point only half a mile short of the spot reached with the open gap.

Every consideration, therefore, supported the conclusion that such a barrier bank would have an inappreciable influence in any scheme of improvement designed to reduce siltation in the upper estuary, and it was pointless to examine its effects on bed movement or with silty water.

As an example of tests made on the movement of the bed without the added complication of silt and alum, we may consider the investigation of the proposed groynes between Combwich and Stert Point. As a preliminary, the bed of the model was moulded in sand to conform with Admiralty Chart No. 1,152 of the 1922–8 survey by H.M.S. *Flinders* (with later corrections), over the area seaward of Stert Point, and with plans and sections dated 1938 supplied by Mr. E. L. Kelting, Chief Engineer of the Somerset Rivers Catchment Board, for the tidal river. The model was then operated for 2,000 tides in order to enable the bed material to settle, and the condition at the end of this run was surveyed and adopted as a standard initial condition with which to begin comparative tests without and with proposed works.

In the first trial of the groynes, the model was run for four years of tides, all springs, after inserting the groynes to their full designed dimensions, and an exactly similar test was made, from the same starting condition, without the groynes. Surveys and tide and current observations made at the completion of this pair of experiments showed that the groynes produced very undesirable results, including a general increase of flood velocities *in the river* accompanied by a decrease of ebb velocities, a threatened erosion of the foreshore near Burnham where there was a local increase of the ebb currents, and the creation of a tortuous channel beset with a complex system of eddies and intense local velocities. Moreover, with the river Parrett in spate, the level of low water at Bridgwater was 1·3 foot higher with the groynes than without them, whilst high-water level was raised by 0·9 foot.

Since the groynes had been put into the model at full height (up to about half-tide level) instead of in stages, and since the experiment had been made with spring tides only, it was felt that the scheme might not have been given a fair trial, and accordingly a further test was made in which the full cycle of spring and neap tides was employed and the groynes were built up gradually of metal plates, over a period of five years of tides. A protective mattress in the form of a thin plate was also maintained, resting on the sand around the sides and toe of each groyne during its construction. The test was continued for four years after the completion of the period of building.

This experiment showed some differences in detail compared with the other, the magnitude of the influence on tide and current phenomena being smaller, but the general results were very similar. The low-water configuration as finally surveyed is shown in Fig. 92.

Various modifications to the layout of the scheme did not produce any general benefit, and when a regular channel was formed by dredging and depositing the spoil landwards of the channel, so as to give conditions which it was hoped the groynes themselves would have created, the channel so formed noticeably deteriorated in a period of one year of tides.

In view of all these results, it was considered unnecessary to carry out any experiments on the groynes with the addition of silt to the model. When dealing with the proposal for a tidal barrage, however, the use of silt proved to be essential, and that part of the work is discussed in Chapter VI, while the reader may wish to consult the Paper in the December 1942 *Journal of the Institution of Civil Engineers* (p. 85) for information concerning the investigation of training walls. A description will also be found there of experiments on the elimination of sharp bends in the river by the introduction of artificial cuts. The influence of these upon tide and current phenomena (and hence upon the bed régime as a whole) is complicated by the fact that such cuts reduce the resistance to the flood as well as the ebb and affect the natural period of vibration of the water in the estuary.

A photograph of the Parrett tidal model is reproduced in Plate XXVI.

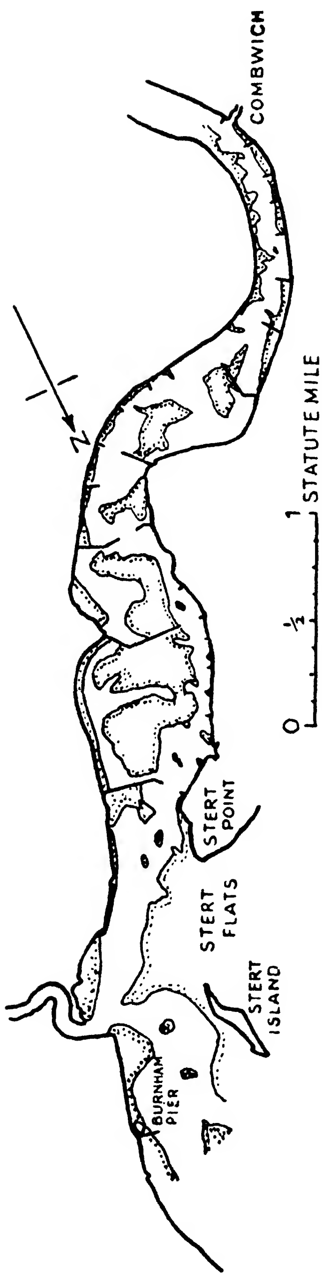


FIG. 92. Contour of L.W.O.S.T. as obtained in Parrett Tidal Model 4 " years " after completion of groynes.
(Based on *Journ. Inst. C.E.*, Dec. 1942, p. 98.)

CHAPTER VIII

GENERAL QUESTIONS CONCERNING THE CHOICE OF SCALES FOR A TIDAL MODEL

SUPPOSE the horizontal scale to be $1:x$ and the vertical scale $1:y$.

It is hoped that useful guidance as to a reasonable choice of values for x and y may be provided by a perusal of Chapters V, VI and VII, but this subject is of such great practical importance as to call for some further discussion.

In the first place, there will be a general tendency to adopt as small a value of x as will enable the model to be accommodated in the space available, keeping in mind also the available water supply, and, if the estuary carries appreciable quantities of suspended silt, the amounts of such silt which will need to be provided during each day's operation. Thus, if Q represents the total maximum rate of discharge of the rivers into the estuary, it has to be remembered that a maximum discharge equal to $\dfrac{Q}{xy^{3/2}}$ will need to be supplied to the model, while if Q_1 cubic feet per second is the *mean* discharge of the natural rivers, provision will have to be made for feeding a quantity of water equal to $\dfrac{24 \times 3600 \times Q_1}{xy^{3/2}}$ cubic feet to the model every 24 hours. Moreover, if these rivers contain an average concentration of $1:C$ by weight of suspended silt, then the daily supply of silt in the river waters will amount to $\dfrac{24 \times 3600 \times Q_1 \times 62{\cdot}4}{Cxy^{3/2}}$ pounds. In a large model, this weight of material may prove to be quite formidable, especially if Q_1 itself is large or if, as in the Severn, Rangoon and Parrett investigations, allowance has to be made for *tidal* silt as distinct from purely river-borne. These considerations, in addition to those of cost, simplicity of construction both of the model and the mechanical equipment, and ease of handling, suggest that it is not always advisable to make the model as large as may be physically practicable, and it is believed that the many examples discussed in this book may offer a reliable guidance in the final choice to be made, while, at the risk of mere repetition, attention is again drawn to the value of quite small models, if only as a preliminary to more ambitious ventures.

The choice of the horizontal scale is primarily dependent, however, upon the *scope* of the model, the first requirement being that the seaward limit shall be sufficiently remote from the site of the works concerned to make it tolerably certain that the conditions at the entrance to the model will be sensibly unaffected by the proposed works. It is also necessary that the upper reaches of the tidal rivers shall be included at least to the limit of spring tidal action, although some portion may quite safely be reproduced by a labyrinth, a tidal compartment, or by bending the river round into a more compact space. There is also the alternative of terminating the moulded part of the model well below the limit of tidal action and ensuring that the tide phenomena are reproduced by inserting an automatically controlled sluice, weir, or valve at the chosen upper boundary of the model. An American device of this kind is described in Chapter IX. In many cases, however, this method could not be applied, since one of the objects of the experiments may be to examine the effect of proposed works upon the tides in the upper estuary; in such cases, those tides cannot be standardized, but must be allowed *to adjust themselves* in conformity only with a given (or sensibly unaltered) tide at the *seaward* end of the model. It is a curious fact that the most difficult of all kinds of model to design is that which is required for the study of comparatively small or localized effects. For example, suppose the problem to be that of a sand or mud deposit at the entrance to a dock. In the nature of the case, a small value of x will have to be adopted (the horizontal scale being $1:x$), since relatively small details have to be reproduced. But the volume of water passing the entrance to the dock on its way towards the top of the river every flood-tide is itself considerable, and this fact, coupled with the small value of x, leads to considerable practical difficulties. One cannot be more precise than to state that, at the judgment of the designer, it may be necessary in such cases to sacrifice some of the actual width of the river and to reproduce only a fraction of it, extending from the bank in which the dock is situated to some distance out which is considered adequate for the influence it may bear upon the detailed problem under examination.

Having then decided upon a suitable horizontal scale, we have next to consider the *vertical* scale. What are the primary points to be kept in mind? Firstly, the motion of the water must be turbulent so that the possibility of scale-effect " due to viscosity " is reduced to a minimum; secondly, the velocities obtained must be sufficiently strong to move a bed made of, say, sand having a diameter approximately three-quarters of that of the natural sand. At the same time, however, the value of y must not be too *small*, because then the vertical distortion of scale ($x:y$) will impose a severe strain upon the angle of repose of the sand and ultimately it will become impossible to reproduce the lateral gradients of the channels or the steep sides of scour-holes. Nor should the value of

y be so *large* as to give trouble in the measurement of the resulting small tidal ranges in the model, although if this occurs it is very likely that the chosen vertical scale will be unsound from the primary points of view of turbulence and bed movement.

We have seen that a vertical distortion $(x{:}y)$ as great as 100:1 has been used where hydraulic effects and *qualitative* bed-movements only were desired; at the other end of the scale, a distortion as low as 11·5:1 has been adopted in a tidal model where the effect of groynes or spur dikes has been of vital importance. In general terms, a vertical distortion of about 40:1 appears promising for the production of approximately quantitative results, except in the case of groynes or similar structures. when a lower value of, say, 10:1 is desirable. Higher values than 40:1 will tend to be preferable when the current velocities and tidal ranges in the actual estuary are themselves small, while lower values may be chosen with advantage when the natural tidal phenomena are well pronounced and the estuary contains steep-sided channels.

In connection with this subject of the relationship between x and y, a theory advanced by Mr. Gerald Lacey, M.Inst.C.E., is of considerable interest.* It implies that for a given horizontal scale $1:x$, and for a bed material of the same order of size as that in the prototype, the vertical scale $1{:}y$ should be designed from the formula $y=x^{\frac{2}{3}}$; correspondingly, the vertical distortion would become $\frac{x}{y}=\frac{x}{x^{\frac{2}{3}}}=x^{\frac{1}{3}}$. In a letter to *Water and Water Engineering*, May 1937, p. 276, Mr. Lacey stated that: "The equations which the writer has derived were deduced as a by-product of a research into régime flow in channels large and small with sandy beds, They were subsequently applied by the writer to model work." He also pointed out that his theory concerning the vertical distortion of scale was in reasonable agreement with the scales chosen by Osborne Reynolds for his models of the Mersey estuary. Thus, for $x=31{,}800$, Reynolds chose $y=960$, or $\frac{x}{y}=33{\cdot}2$, compared with $(31{,}800)^{\frac{1}{3}}=31{\cdot}6$. Again, in a second model of the Mersey, having $x=10{,}560$, Reynolds made $y=396$ and $\frac{x}{y}$ therefore equal to 26·7, which is in fair agreement with Mr. Lacey's $x^{\frac{1}{3}}=10{,}560^{\frac{1}{3}}=21{\cdot}9$. It may be significant, however, that the discrepancy in the second case is larger than in the first, and it is only fair to recall that Reynolds himself wrote: "From my present experience"—that is, with *both* models—"in constructing another model, I should adopt a

* "Stable Channels in Alluvium", Lacey, *Min. Proc. Inst. C.E.*, vol. 229, pp. 259–384, and especially see p. 380 (1930); "Uniform Flow in Alluvial Rivers and Canals", Lacey, *Min. Proc. Inst. C.E.*, vol. 237, 1933–34, Pt. 1, p. 421.

somewhat greater exaggeration of the vertical scale." Attention may also be called to the scales adopted in certain other investigations:

TABLE XXXII

Model	*Laboratory*	*x adopted for horizontal scale*	*Vertical exaggeration actually adopted*	*Vertical exaggeration required according to Mr. Lacey's theory*
Seine -	Vernon-Harcourt's	40,000	100	34·2
Severn -	Professor Gibson's, Manchester University	(*a*) 8,500 (*b*) 8,500	85·0 42·5	20·4 20·4
Mersey -	Manchester University	7,040	37·0	19·2
Humber	Manchester University	7,040	36·6	19·2
Dee -	Manchester University	(*a*) 5,000 (*b*) 40,000	25·0 100·0	17·1 34·2
Parrett -	Manchester University	3,000	11·5	14·4
Rangoon	Sir Alexander Gibb & Partners	8,060	42·0	20·0
Great Ouse and Wash	Great Ouse Catchment Board	2,500	41·7	13·6

This detailed comparison suggests, therefore, that specific experimental research would be needed to enable one to pass a final judgment upon Mr. Lacey's theory.

It may be useful to recall at this point that the vertical scale originally adopted in the Severn model was 1 : 100 and that it was afterwards changed to 1 : 200. In general, however, the indications of the probable effect of the proposed barrage were similar in the two models. A striking comparison is given, for example, in pages 122 and 135 of Professor Gibson's *Severn Model Reports* (H.M. Stationery Office, 1933).

This suggests that in some tidal model investigations it may be definitely advantageous to start with a rather large vertical exaggeration of scale, since the measurement of tides is thereby facilitated and, due to the relatively short tidal period accompanying a large vertical exaggeration, results may be quickly obtained regarding the effect of schemes of improvement such as training walls. These results may not be conclusive, but they will very often be sufficiently qualitative to warrant a decision as to the *comparative* virtues of the various schemes. The most promising scheme or schemes may then be subjected to more detailed examination on a reduced vertical exaggeration: it will generally be found perfectly practicable to reconstruct a model to a *lower* vertical exaggeration, whereas the reverse process is so full of mechanical and structural difficulties as to render it almost as costly as building an entirely new model.

As an indication of whether the motion of the water in the model will in fact be turbulent, it is suggested that the following procedure should be worth while. The maximum current velocity, v knots, during the flood tide near the mouth of the estuary will usually occur at about half-tide when the *mean* depth of water in the approach channels is, say, H feet. In the model, the corresponding velocity is $\frac{1{\cdot}69v}{\sqrt{y}}$ feet per second and the depth $\frac{H}{y}$ feet. Knowing v and having provisionally chosen a value for y, then $\frac{1{\cdot}69v}{\sqrt{y}} \cdot \frac{H}{y}$ should not be less than 0·0169, or, in "round numbers", y should not be greater than $20(vH)^{\frac{2}{3}}$. This suggested criterion is based upon the Author's experimental determination of the critical Reynolds number $\frac{vm}{\nu} = 1{,}400$ for straight rectangular open channels (see page 24) and upon the value of ν appropriate to water at ordinary temperatures of about 15° C. It is, of course, true that the channels in an estuary are far from being straight or rectangular and that the hydraulic mean depth m in the model may be very different from the average depth $\frac{H}{y}$, but experience lends support to the view that $y \not> 20(vH)^{\frac{2}{3}}$ offers a reasonably safe check upon the chosen vertical scale from the aspect of turbulence, especially if v and H relate to neap tides. The reason for this is that the effect of the channels being irregular in both shape and cross-section compensates for the use of an average depth instead of a hydraulic mean depth. Experience also tends to indicate that if the criterion $y \not> 20(vH)^{\frac{2}{3}}$ is satisfied when applied to the deeper channels at the *mouth* of the model estuary, then the turbulence generated in that region does not have time to decay completely during the slack-water period at high tide; it is, moreover, transmitted into the model estuary, where sudden changes of width and depth together with pronounced features of the coastline also have a chance of contributing eddy-formation. It may, however, be desirable to assist turbulence in the uppermost narrow reaches of the tidal river by lining the banks with wires, as already described on page 241 in connection with the model of the River Parrett. The resistance of small-scale tortuous channels has also been discussed in some detail in Chapter I, with particular reference to a model of a non-tidal stretch of the Mersey.

It still remains to make an estimate of whether the chosen vertical scale will enable the bed material of the model to be shifted under the action of the available currents. The capacity of a stream to disturb the bed over which it flows depends upon many factors, such as the size, shape, density and compactness of the bed material, the depth and speed of the stream and the degree of turbulence in it. Researches are still in progress in many places with a view to filling the gaps in our present knowledge of this important and complicated subject. Meanwhile the Author proposes his own line of attack.* An analysis of experiments made in straight-sided channels of rectangular cross-section indicates that the mean velocity of the stream, $\bar{v}$ feet per second, required to move grains forming an initially flat bed is given by

$$\bar{v}=K\left(\frac{L+B}{2l}\right)^{0\cdot44}(\sigma'-\sigma)^{0\cdot22}l^{0\cdot22}M^{-0\cdot22}h^{0\cdot06}m^{0\cdot22}, \quad \ldots\ldots\ldots\ldots(1)$$

where

$K=1\cdot10$ for movement of the first few particles,

$K=1\cdot59$ for movement of many particles,

$\frac{L+B}{2}=$ mean dimension of the grains as viewed under a microscope (inches),

$l=$ length of side of a cube having the same weight as the average grain (inches),

$\sigma'=$ specific gravity of material,

$\sigma=$ specific gravity of water (say unity),

$M=$ uniformity modulus of material (see page 163),

$h=$ depth of water (inches),

$m=$ hydraulic mean depth of stream (inches).

Similarly, for the first disturbance of the particles forming a *mound* of material,

$$\bar{v}=1\cdot30(\sigma'-\sigma)^{0\cdot27}l^{0\cdot27}M^{-0\cdot27}h^{0\cdot21}m^{0\cdot02} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

* "An Investigation of the Stability of Bed Materials in a Stream of Water", Allen, *Journ. Inst. C.E.*, March 1942, p. 1. See also Chapter XI.

whilst in order to flatten out the crest of the mound,

$$\bar{v} = 1{\cdot}37\left(\frac{L+B}{2l}\right)^{0\cdot54}(\sigma' - \sigma)^{0\cdot27}l^{0\cdot27}M^{-0\cdot27}h^{0\cdot21}m^{0\cdot02}. \qquad \text{(3)}$$

In equations (2) and (3), $\bar{v}$, h and m refer to the stream passing over the crest of the mound.

Now, if we consider the Severn investigation as an actual example for the application of these results to tidal model work, we find that $\frac{L+B}{2}$ is 0·0089 inch in the estuary itself and 0·0070 inch in the model (for the " best " sand tried). For the estuary sand, $\sigma' = 2{\cdot}60$, $\frac{L+B}{2l} = 1{\cdot}27$ and $M = 0{\cdot}60$ approximately, while the corresponding values for the model sand are $\sigma' = 2{\cdot}62$, $\frac{L+B}{2l} = 1{\cdot}25$ and $M = 0{\cdot}75$. According to equation (1), the velocities (feet per second) required for the first few grains to move on an initially flat level bed in a straight channel of rectangular cross-section would be

$\bar{v} = 0{\cdot}51h^{0\cdot06}m^{0\cdot22}$ for the estuary sand

and $\bar{v} = 0{\cdot}46h^{0\cdot06}m^{0\cdot22}$ for the model sand,

h and m being measured in inches.

Considering these expressions in conjunction with the tide and current data, it is found that on a spring tide, the velocities off Avonmouth are sufficient for movement of the sand-bed during about three-quarters of the flood and three-quarters of the ebb. In the model,* they are sufficient during about 20 per cent. of the flood and 30 per cent. of the ebb; but such movement would be enough to create small ripples or surface irregularities, and so to reduce the velocities required for further movement, at least to the order of those associated with a mound. In fact, using the formula (2) for the initial disturbance of a mound, it appears that, in the model, velocities would be sufficiently high during some 40 per cent. of the flood and 40 per cent. of the ebb.

A similar treatment of the portion of the estuary near Sharpness, on the basis of the level-bed formula, indicates that in the estuary itself the velocities are sufficient during approximately $\frac{3}{4}$ of the flood and $\frac{3}{4}$ of the ebb, whilst in the model the corresponding proportions are 45 per cent. and 30 per cent. But if the " mound formula " be applied, the percentage times in the model are at once increased to 55 on the flood and 40 on the ebb: in practice, these times might be more nearly equalized by the appreciable bed-gradient in this part of the model coupled with the vertical exaggeration of scale.

* With vertical scale of 1 : 200.

All these conclusions are based upon the formulae derived from bed movement in straight, parallel-sided channels. It is clear that in a river or estuary, with its irregularities of coastline and of sectional area, the velocities required for movement of a sand-bed are very likely to be appreciably reduced owing to the pronounced eddy-formation created by the sudden changes of section.

Again, in the Severn investigation, it was discovered that (for materials of a given ratio of L, the longest dimension, to B, the shortest dimension, of the individual grains seen under the microscope), the lateral and longitudinal gradients assumed by the bed were proportional to $d^{-0\cdot287}\rho_1{}^{-0\cdot262}$, where $d=\frac{L+B}{2}$ and ρ_1 denotes the *effective* weight in water of unit volume of the material. Now ρ_1 is itself proportional to $(\sigma'-\sigma)$, and it is a remarkable fact that the numerical values of the indices of d and ρ_1 are almost the same as those attached to l and $(\sigma'-\sigma)$ in equations (2) and (3). This definitely tends to support the idea that $(\sigma'-\sigma)^{0\cdot27}l^{0\cdot27}M^{-0\cdot27}$ may be a criterion of the *comparative* behaviour of different materials in a tidal model. Let this criterion, $(\sigma'-\sigma)^{0\cdot27}l^{0\cdot27}M^{-0\cdot27}$, be called C; then the following Table XXXIII gives values of C for a number of the materials tried in the Severn experiments:

TABLE XXXIII

Material	$\frac{L+B}{2}$ (*inches*)	l (*inches*)	σ'	C
120-mesh silica sand -	0·00582	0·00485	2·63	0·291
80- „ „ „ -	0·00700	0·00560	2·62	0·304
50–80 Cobham sand -	0·0100	0·00788	2·64	0·330
120-mesh emery - -	0·00505	0·00336	3·89	0·303
80- „ „ - -	0·00825	0·00606	3·89	0·354
Pumice *A* - - -	0·0122	0·00754	1·99	0·289
„ *B* - - -	0·0153	0·00957	1·95	0·305
„ *C* - - -	0·0192	0·0123	1·90	0·311
Severn estuary sand -	0·00893	0·00702	2·60	0·340

Now the material which gave the best overall reproduction was the 80-mesh silica sand ($C=0\cdot304$). But the 120-mesh emery ($C=0\cdot303$) and pumice *B* ($C=0\cdot305$) behaved in very similar fashion. The 120-mesh silica sand ($C=0\cdot291$) was also good, but showed a tendency to be carried too easily upstream, whilst pumice *A* ($C=0\cdot289$) suffered also from this defect. It would seem, therefore, that C furnishes a remarkably sensitive criterion of the behaviour of materials having very different sizes and densities.

Considering formula (2) again, it will be observed that a reasonably close approximation is

$$\bar{v} = 1{\cdot}30(\sigma' - \sigma)^{0\cdot 27} l^{0\cdot 27} M^{-0\cdot 27} h^{0\cdot 23},$$

which becomes

$$\bar{v} = 1{\cdot}60 l^{0\cdot 27} h^{0\cdot 23} \quad \text{.................................(4)}$$

if $\sigma' - \sigma = 1{\cdot}64$, as for the average sand, and if $M = 0{\cdot}75$, which is about the usual value for the sand employed in a tidal model.

In expression (4), l represents the length of a cube having the same weight as the average sand grain. If L stands for the length and B for the breadth of the grain seen in plan-view under the microscope, then it may be taken that, for the type of sand usually associated with tidal model experiments, l = very nearly $0{\cdot}80 \dfrac{L+B}{2}$. Consequently, formula (4) may be changed to

$$\bar{v} = 1{\cdot}50 \left(\frac{L+B}{2}\right)^{0\cdot 27} h^{0\cdot 23}. \quad \text{..............................(5)}$$

Having checked the vertical scale $1:y$ from the aspect of turbulence by seeing whether y is not greater than $20(vH)^{\frac{2}{3}}$, where v is the maximum flood current velocity (knots) and H the corresponding mean depth of water (feet) in the natural estuary (as defined on page 279), we suggest, therefore, that the question of bed movement be resolved as follows:

(*a*) Assume a mean grain size $\left(\dfrac{L+B}{2}\right)$ inches for use in the model, say about $\frac{3}{4}$ of the value of $\dfrac{L+B}{2}$ for the prototype sand.

(*b*) Calculate the magnitude of h (in inches) as equal to the mean-tide depth of water in the channel at a few important places in the model. (This value of h may be taken to be $\dfrac{12}{y}$ times the corresponding channel depth in feet derived from the charts of the natural estuary).

(*c*) Work out the value of $1{\cdot}50 \left(\dfrac{L+B}{2}\right)^{0\cdot 27} h^{0\cdot 23}$. If this proves to be not greater than the anticipated maximum flood or the anticipated maximum ebb velocity in the model, on spring tides, at the places under consideration, then there is a very good chance indeed that the proposed vertical scale will be satisfactory from the aspect of bed movement. (The "anticipated maximum flood or the anticipated maximum ebb velocity in the model" may be estimated from the prototype velocities divided by $\sqrt{y}$.)

Thus, if $\frac{L+B}{2} = 0{\cdot}007$ inch, $\sigma' - \sigma = 1{\cdot}64$, $M = 0{\cdot}75$, and $h = 6$ inches, the model velocity would have to be not less than 0·59 ft. per second. With a vertical scale of 1 : 200, this velocity would represent nearly 5 knots, while the mean-tide channel depth of 6 inches would correspond with 100 feet in Nature. Again, using the same bed material, the maximum flood or ebb velocity in the model channel when $h = 1$ inch would need to be at least 0·39 ft. per second, representing 3·26 knots in the prototype having a corresponding channel depth of 200 inches, or $16\frac{2}{3}$ feet.

If, for any reason such as cost, it is proposed to use a different material from that so far visualized (namely, a sand three-quarters of the size of the estuary sand), it may be assumed that such a material will behave very much the same, provided that it has sensibly the same value of $(\sigma' - \sigma)^{0{\cdot}27} l^{0{\cdot}27} M^{-0{\cdot}27}$.

Incidentally, the value of l may be computed, with sufficient accuracy for materials of mean diameter $\frac{L+B}{2}$ less than 0·015 inch, from the formula $l = \left(\frac{L+B}{2}\right) \cdot \frac{B}{L}$. This is convenient because L (the length) and B (the breadth) of the grains may be readily estimated by microscope observations, and hence the mean value of $\frac{L+B}{2}$ and the mean ratio of $B : L$ for individual grains are easily obtained.

The one other factor which may influence the choice of the vertical scale is the motion of water-borne silt. We have seen that this silt should fall $\frac{x}{y^{\frac{3}{2}}}$ times as fast in the model as in the prototype, and the smaller this value can be kept, consistent with the other considerations, the better, since the difficulty of selecting a suitable silt and coagulant is likely thus to be minimized.

CHAPTER IX

TIDE-GENERATING MECHANISM

THE tide-making machinery adopted by Reynolds is shown diagrammatically in Fig. 93, from which it will be seen that water was displaced by means of a tray hinged to the seaward end of the model. The upward motion of the tray thus produced a flood tide, while the downward movement allowed water to leave the model on the ebb. A flexible rubber joint was used to give a watertight joint between the tray and the model itself, while a system of counterbalance weights reduced the load on the driving motor to a minimum. Reynolds remarks * that "owing to the careful balance of the tanks and the use of spur instead of worm gearing, the work required is not more than 0·008 horse-power, so that five-sixths of the pressure is spent in overcoming the fluid resistance, which increasing as the square of the speed, affords a very important means of regulating the speed, which, indeed, is thus rendered very regular". The driving motor itself was a double-acting oscillating water-engine having a $\frac{3}{4}''$ piston and 4″ stroke, while the available water pressure was 50 lb. per sq. inch. Reynolds also conducted some of his experiments with tides varying harmonically between spring and neap; for this purpose the stroke of the hinged tray was varied automatically through a set of gearing. He further included provision for the production of waves with a period 1/200th of the tidal period.

A similar device was adopted by Mr. L. F. Vernon-Harcourt in his model of the Seine estuary,† except that the hinged tray was raised and lowered by means of the screw of a letter-press, *apparently* worked by hand.

The Reynolds tilting tray was also employed by Messrs. W. H. Wheeler and Herbert Wheeler, motive power being supplied by a "small percussion motor" using water from "the town mains under a pressure of 25 lb. per square inch. The motor required about 60 gallons of water an hour, and ran at 350 revolutions a minute. The generating machinery was arranged to give both spring and neap tides in the proportion of 6 to 4" (W. H. Wheeler, *Tidal Rivers* (Longmans, 1893), p. 332).

Mr. W. R. Kinipple (*Min. Proc. I.C.E.*, vol. C, 1890, p. 156) refers to small models which he had driven by clockwork, but the description is not very clear. He did, however, make provision for "a wave-maker,

* *Papers on Mechanical and Physical Subjects*, vol. II, p. 384.

† *Proc. Roy. Soc.*, vol. XLV, 1889, p. 504.

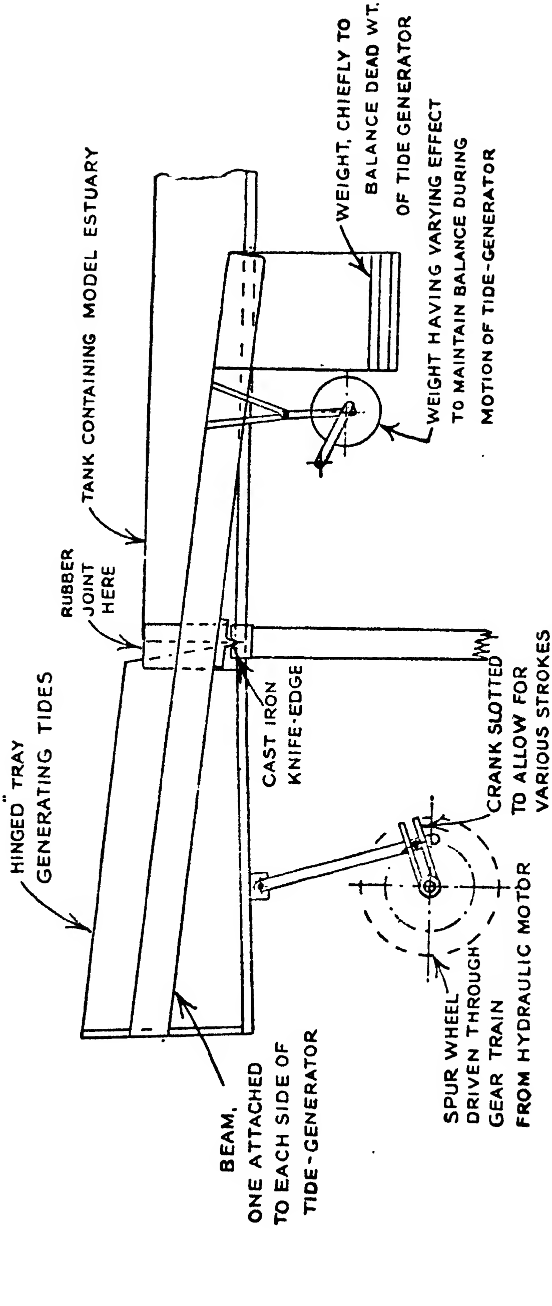


FIG. 93. Diagrammatic Arrangement of Reynolds' Tide-producing Mechanism.

likewise driven by clockwork, and mounted in such a manner that any angle could be obtained ". (See also page 194.)

While the hinged tray may be quite suitable for comparatively small models, it is likely to give mechanical trouble if used on a larger scale; nor is it so easy to modify or adjust as is the plunger type of tide generator which will now be described. In this method, the plunger (see Fig. 94) is suspended from a lever which is driven by a constant speed electric motor through reduction gear and an epicyclic train of wheels, the function of which is to give the lunar variation from spring to neap tides. The plunger is connected to a balancing system involving a cam from which is suspended a balance weight. Thus, at low water of spring tides the balance weight has its greatest effective radius, corresponding with the minimum buoyancy of the plunger. Conversely, at high water of spring tides, when the plunger is almost completely immersed, the cam is in a position giving the least leverage to the balance weight.

The plunger itself may be constructed of mild steel plates stiffened with angle-iron and with cross-battens or mid-feathers. It is loaded with sand or gravel ballast so chosen that it does not float when at its lowest position and so that the effective dead-weight may be supported by a cam and balance weight of convenient shape and size. It is readily possible to design such a plunger with a varying width and of such dimensions that, in conjunction with the vertical motion provided, it will displace water at the rate demanded by the estimated plan-area of the estuary and the tidal rise and fall in it. Owing to the highly complicated nature of the tide phenomena, however, the design so obtained is likely only to give an approximation to the required tides; a wise precaution therefore is to make the steel portion of the plunger only about 9/10ths of the designed size, and to make up the remainder with wooden side-pieces bolted to the main steel structure. These side-pieces may be easily modified, by experiment, to give the accuracy required.

To illustrate the method of design, the steps involved may be summarized thus:

(1) From the charts, plans and sections of the estuary, estimate the maximum total volume of water displaced on a flood tide. Let this volume be V cubic feet. If the horizontal scale of the model is $1:x$ and the vertical scale $1:y$, then the corresponding volume of water to be displaced in the model is $v=\dfrac{V}{x^2y}\cdot$ For a first approximation, add, say, 5 per cent. for the volume which will have to be displaced in the model tide-box, i.e. in that portion, outside the moulded seaward limit of the model, in which the plunger itself works.

Calling l the length, b the mean breadth and s the vertical travel or stroke of the plunger itself (all in feet), then as a first approximation we have

$$slb = 1{\cdot}05v.$$

Choosing a suitable value for l in relation to the mouth of the model

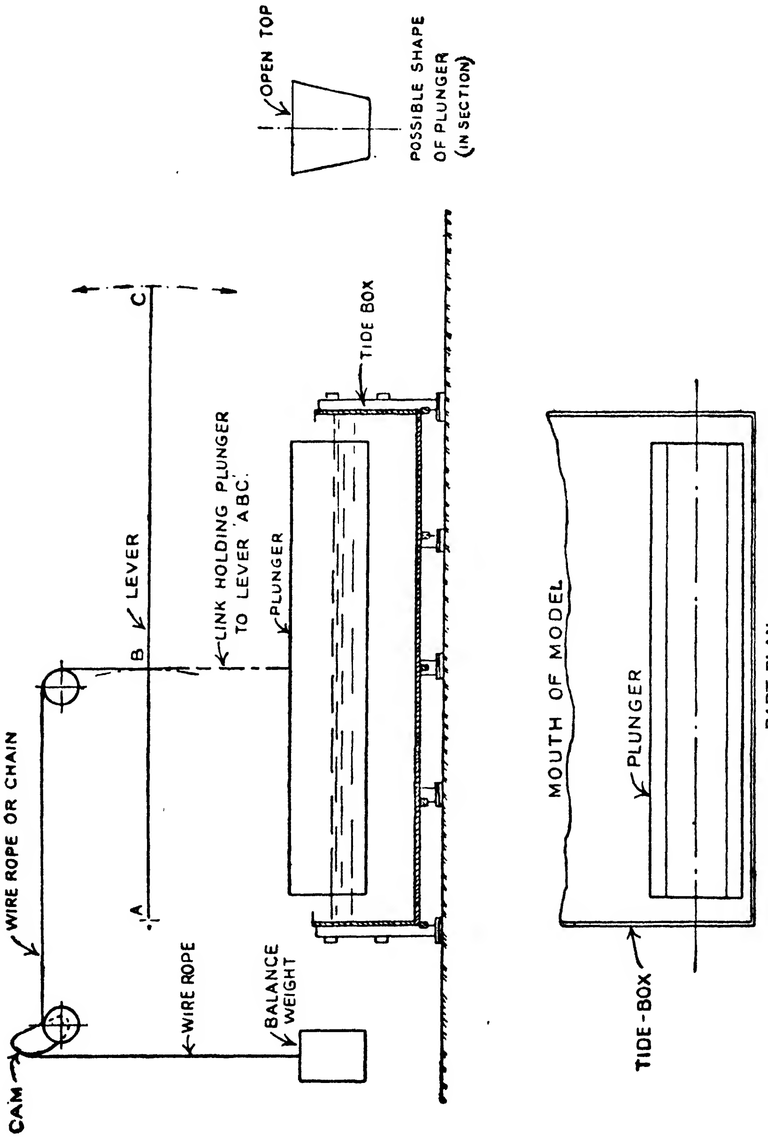


FIG. 94. (Lever "ABC" is hinged at "A" and cscillated by mechanism operating on end "C".)

and assigning some convenient value to the stroke s, a preliminary estimate of b is obtained.

(2) If the tide at the mouth takes the same time to rise as to fall, then a "donkey-pump" mechanism, as illustrated in Fig. 95, may be adopted.

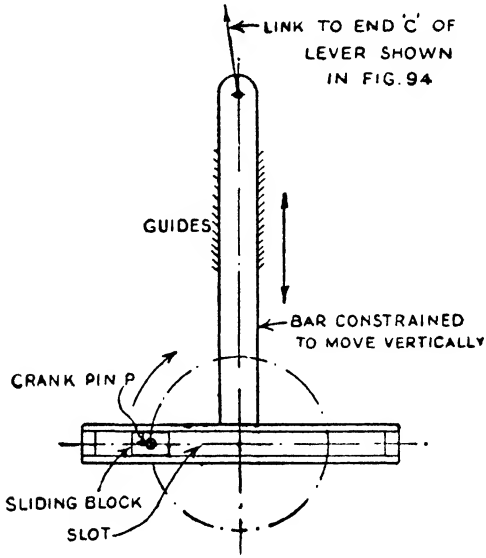


FIG. 95.

But if, as is common, the tide takes longer to ebb than to flow, then a "slotted lever" may be used as shown in Fig. 96, the ratio of times of ebb and flood being $\alpha : \beta$. This ratio will vary, with such a simple mechanism, from spring to neap as the crank-pin circle diminishes, but in practice it will very often be found that just such a variation is needed.

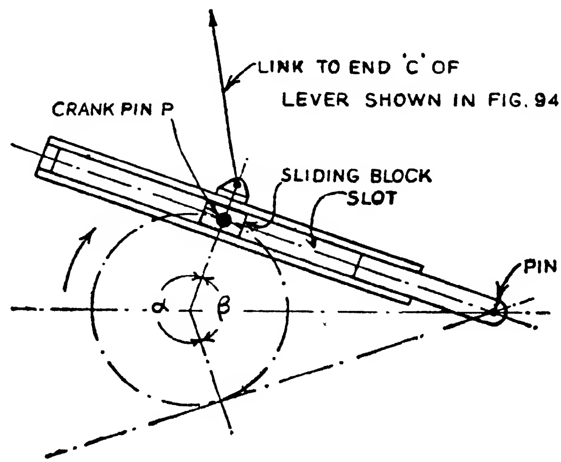


FIG. 96.

(3) Having obtained the preliminary estimate of the lengths of links and levers required, we are in a position to calculate or to obtain graphically if preferred a curve having the vertical movement of the plunger plotted as ordinate against time as abscissa. Let this curve be designated graph *A*, and let us call graph *B* one which shows the relationship between the volume of water to be displaced in the moulded part of the model and the time reckoned from the beginning of the flood at the mouth. The total volume displaced is v cubic feet; suppose that in the last " half-hour " of the flood it is v_1 (obtained from graph *B*) while the plunger descends a distance s_1 (obtained from graph *A*) in the corresponding time.

Then, approximately, the top breadth, b_1, of the plunger, will have to be $\dfrac{v_1 sb}{vs_1}$, if the length l is constant. Allowing a clearance all round the plunger of, say, 1 inch in the smallest to 6 inches in the largest models, we now have the dimensions of the tide-box—say, a length L and breadth B in the plan view.

(4) Next suppose the bottom of the plunger to be just touching the water surface in the tide-box at low water (at the end of the design, the depth of the plunger would be increased so as to make it be immersed, say, 1 to 2 inches at low water). We wish to find the mean width b' of plunger required for the first " half-hour " of the flood. This may be obtained from

$$lb'(s'+h') = \left.\begin{array}{l}\text{vol. displaced in model during first} \\ \quad\text{" half-hour,"}\end{array}\right\} + LBh',$$

where
$$\left.\begin{array}{l} s' = \text{the actual vertical descent of the plunger} \\ \qquad\text{(from graph } A) \\ h' = \text{the desired rise of the tide at the mouth}\end{array}\right\}\begin{array}{c}\text{in the} \\ \text{" half-hour ".}\end{array}$$

The volume displaced in the model is to be found from graph *B*.

Similarly, the mean width b'' of the plunger as immersed at the end of the second half-hour may be found from

$$lb''(s''+h'') = (\text{volume of water displaced in model during the first hour}) + LBh'',$$

where s'' = the vertical descent of the plunger during the first " *hour* ",
h'' = the required rise of the tide at the mouth during the first " *hour* ".

Proceeding step by step in this way, the cross-section of the plunger is obtained, and usually a simple geometrical shape such as  will provide a sufficiently close approximation, the final modification being done after experiment. In addition to the possibility then of altering the shape of the plunger, adjustments are available in the throw of the crank-pin on the main driving mechanism and in the point of attachment of the connecting rod to the slotted lever.

Plungers of this kind have been used in a wide variety of sizes, the smallest being some 12 inches long × 4 inches wide with a stroke of $2\frac{3}{4}$ inches, and the largest 31′ 6″ long × 5′ $10\frac{1}{2}$″ wide at top, spring-tide stroke 23·5 inches and loaded weight 12·5 tons. This big plunger was designed for a model * of the Wash and the River Great Ouse. Its great length and weight presented peculiar difficulties, and the methods of suspending and balancing it are shown in Fig. 97, from which it will be seen that two cams were employed, one connected to each end of the plunger. The trouble with this device is that the plunger becomes unstable.

* This model, having a horizontal scale of 1:2,500 and a vertical scale of 1:60, was designed primarily to investigate problems concerned with the drainage of the Fens. It was constructed in Cambridge, in a laboratory specially erected for the purpose, by the River Great Ouse Catchment Board. See *Tidal Model of the River Great Ouse and the Wash*, D. B. O'Shea, Joint Meeting of the Eastern and South Midland Districts of the Inst. of Mun. and Cty. Engs., Cambridge, 16th May, 1936.

A very interesting account of the history of drainage in the Fenlands has been written by Dr. H. C. Darby, *The Draining of the Fens* (Camb. Univ. Press, 1940); and a delightfully written and beautifully illustrated book of impressions of the Fen Counties is Miss Iris Wedgwood's *Fenland Rivers*, with drawings by Henry Rushbury, R.A., Rich and Cowan, 1936).

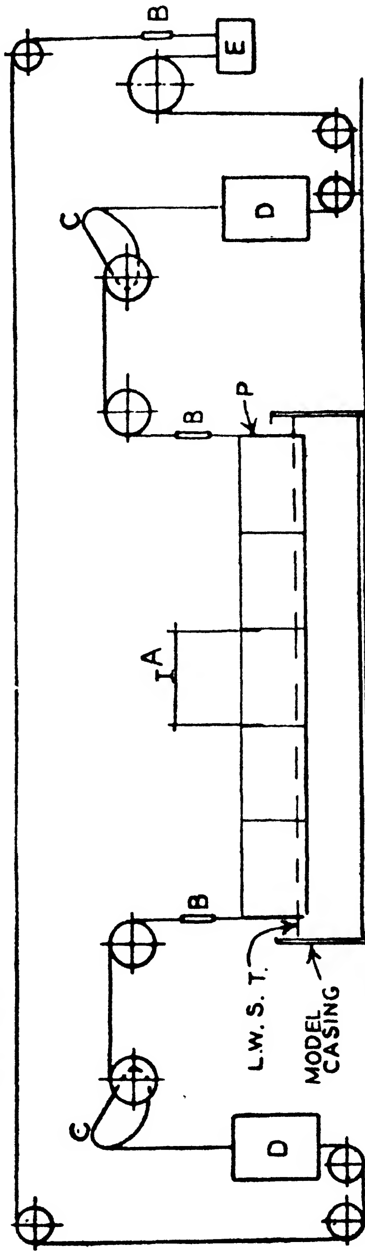


Fig. 97. Diagrammatic Sketch indicating a method of stabilising Plunger when two Cams are used.

P. Plunger suspended from oscillating arm "A". B. Adjustable turnbuckles for regulating length of steel ropes.

C. Cams carrying Balance-weights "D", which are connected to subsidiary weight "E".

Thus, if one end moves upwards relative to the other, its cam assumes a leverage which is more than sufficient to maintain balance. Consequently the tilt of the plunger, instead of staying constant or being corrected, is increased. In order to overcome this tendency, the balance weights were reduced in size and the weight so removed was connected to *both* of the balancing systems through additional ropes. This arrangement gave virtually complete balance throughout the stroke, but of course a given proportion of out-of-balance in such a heavy plunger means a greater *absolute* unbalanced force than in the lighter plungers. Friction in the more complicated balancing system is also proportionately higher. These effects make it essential to have a much more robust slotted lever for the main driving mechanism in order to withstand bending stresses.

The epicyclic train of wheels as used on many tidal models for the purpose of the spring-neap cycle of tides is shown in some detail in Fig. 98

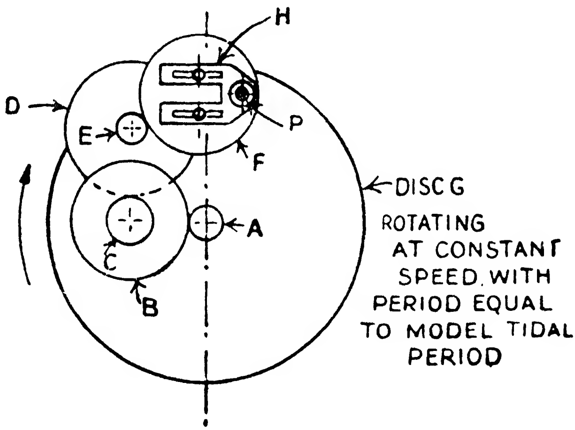


FIG. 98.

A is stationary (17 teeth). B has 53 teeth. C has 20 teeth.
D has 58 teeth. E has 18 teeth. F has 56 teeth.
Fork H is clamped to F. Crankpin P is fixed to H.
P is shown also in Figs. 95 and 96.

and Plates XXVII and XXVIII. The crank-pin is part of a cast-iron fork *H* bolted to the last wheel *F* of the train. This wheel, while revolving about the centre of the whole mechanism, makes one twenty-eighth of a revolution about its own axis. Accordingly, if the gearing is put in motion with the crank-pin originally at its greatest radius, it will be at its minimum radius of action 14 revolutions of the main mechanism later, i.e. after 14 tides. Thereafter the effective radius of the crank-pin increases until, 28 tides after starting, it is back in its original spring-tide position, having meanwhile imparted varying strokes to the plunger corresponding to the full lunar cycle.

If it is desired to alter the range of the spring tide, this may be done quite readily by adjusting the position of the cast-iron fork *H* on the wheel *F*. Again, if it is required to dispense with the full cycle of tides for any particular experiment, it is necessary only to remove one or both of the wheels *B–C*, *D–E*, and then to clamp the wheel *F* in any desired position by means of studs to the cast-iron disc *G*. For example, the mechanism may be arranged to give spring tides only.

An entirely different type of tide-producing machinery has been put into operation at the United States Waterways Experiment Station, Vicksburg. This highly ingenious device was adopted, for instance, on the model of Mare Island Strait,* where the tidal range is of the order of 6 ft. and is accompanied by a marked diurnal inequality. In this case, the *mean* daily tidal cycle was reproduced. At each end of the model a tailway was constructed (see Fig. 99), and a constant supply of water was introduced to each tailway. The tailways were provided with gates consisting of a lower fixed plate and an upper movable plate raised or lowered by a screw and bevel drive from a ¼-h.p. reversible motor. Part of the water supplied was discharged via the gates to a sump, whence it was re-circulated; the remainder flowed through the model.

Each gate was controlled by a cam and float, the tide-cam *a* being rotated on a horizontal shaft by a 110-volt, 60-cycle, synchronous motor *b* through reduction gears *c*, giving a cam-speed of 1 revolution per 16 min. 45 sec., the equivalent of one lunar day corresponding to the horizontal scale of the model (1 : 800) and the vertical scale (1 : 80).

The cams were shaped to simulate the *mean* tidal curve of the particular place at which they were stationed; the cam-motion was transferred through a roller rod to contact points *d*, which were insulated from one another and separately connected to one side of an 8-volt circuit leading to a control board. A vertical rod was attached to a float *e* situated in a stilling chamber which communicated with the model at each cam-station. At the top of the rod was a brass disc *f* connected with the other side of the 8-volt circuit. The revolution of the cam caused the contact points *d* to rise and fall, current flowing in one direction or another according as to whether the circuit was closed through the upper or lower connection. The circuit, being thus closed through the battery and interrupter *g*, then energized the primary coil of one or other of two mercury-type 8–220 volt relays and closed the 220-volt circuit to operate the gate motor in the appropriate direction. If the water in the model rose above the correct level then the disc *f* came into contact with the upper contact point, and

* For a fully illustrated description, see *Engineering*, Oct. 1, 1937, p. 361.

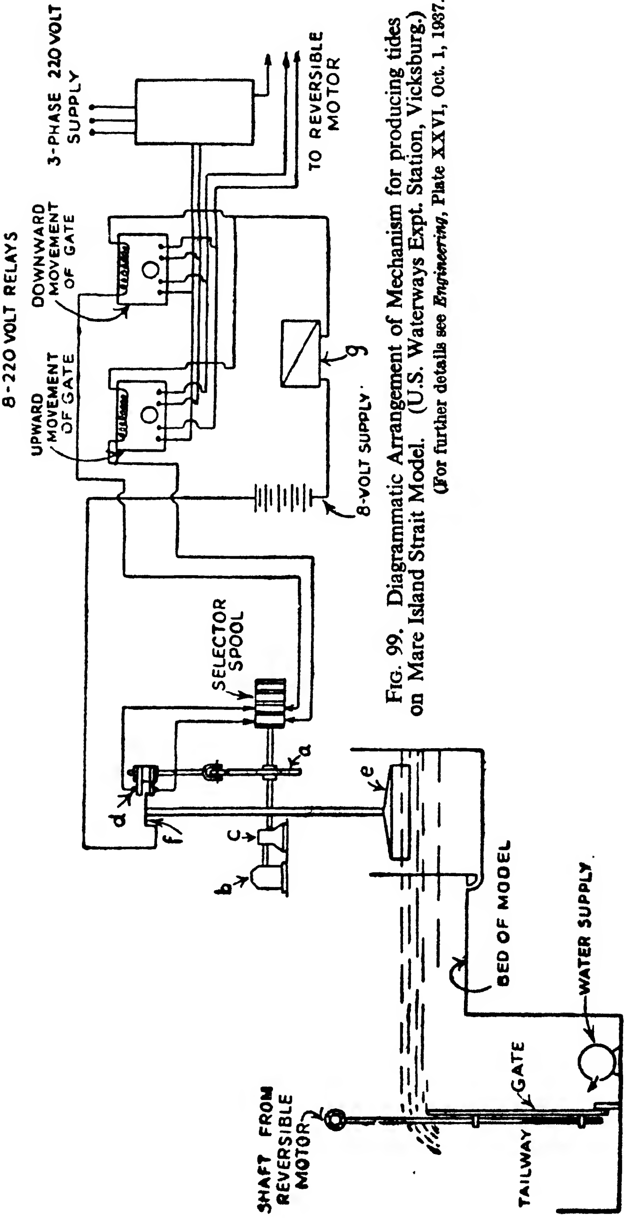


FIG. 99. Diagrammatic Arrangement of Mechanism for producing tides on Mare Island Strait Model. (U.S. Waterways Expt. Station, Vicksburg.)
(For further details see *Engineering*, Plate XXVI, Oct. 1, 1937.)

the 8-volt circuit was closed in such a way as to cause the motor to lower the gate. Similarly, when a portion of cam corresponding to a rise of tide came into action, the lower contact point touched the disc and the relay controlling the *upward* motion of the gate was then effective.

A very ingenious tide-producing mechanism was designed by Mr. McClure for the Bombay tidal model.* In principle, it consists of an airtight inverted dish made of steel plates called a " displacer-tank " as illustrated in the *diagrammatic* sketch of Fig. 100. An air-vent allows air to escape

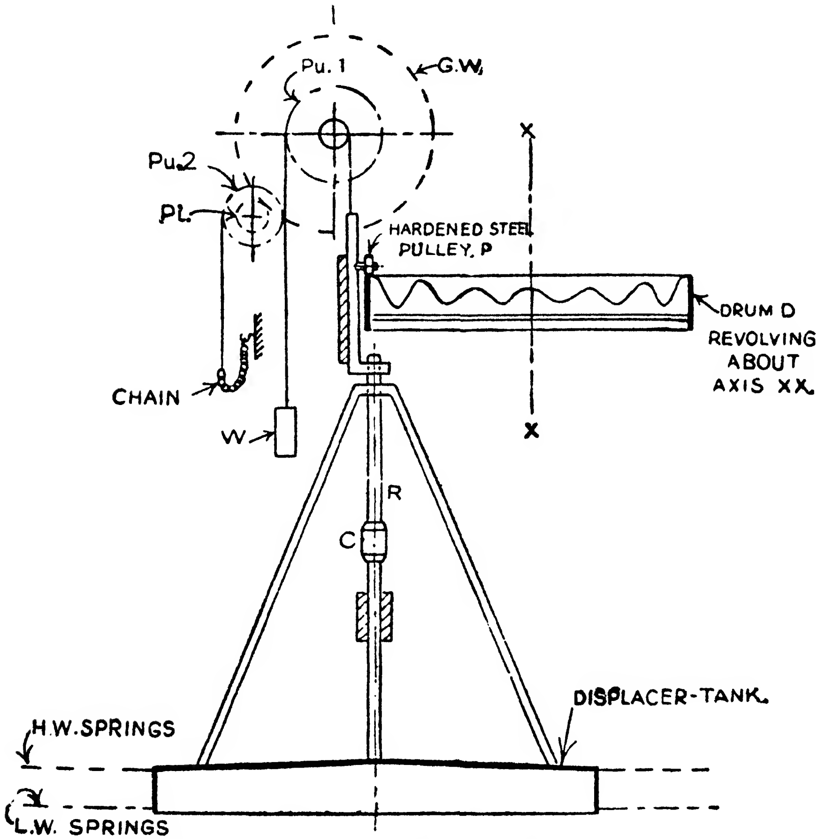


FIG. 100. Showing Principle of Tide-producing Mechanism of Bombay Tidal Model (Mr. J. McClure).
(For details, see Plate 2, Fig. 7, *Min. Proc. Inst. C.E.*, vol. 232, 1932.)

from this tank when the model is filled with water; the vent is then closed, so that as the tank rises vertically it lifts the water held in its interior; in

* " Bombay Harbour Survey and Tidal Model ", J. McClure, *Min. Proc. I.C.E.*, vol. 232, p. 66, 1932.

consequence, the water-level in the model falls and so experiences the action of an ebb tide, to be followed by the flood tide when the tank is lowered. The displacer-tank is suspended from a length of rod, *R*, suitably guided and containing a screwed coupling *C* to allow of adjustment. On a side-bracket attached near the top of the rod *R* is a hardened steel pulley *P* which engages with a drum *D*. This drum is cut to the profile appropriate to the tide cycle and is made to rotate at the proper speed about a vertical axis *XX*, thus communicating various strokes or vertical movements, in successive tides, to the displacer-tank. The fabrication of the tide-template drum *D* is also of interest: the shape of eight tides varying from a spring of 15 feet to a neap of 4 ft. 6 in. and back to spring was scribed on a flat mild-steel plate 12 ft. long and 8 in. deep and ½ in. thick. By drilling, cutting and trimming to this scribed-line the required profile was obtained, and the flat plate was bent to a circle in a cold-rolling mill.

In order to balance the weight of the displacer-tank and its supporting rod, weights *W* are suspended from pulleys *Pu*1, and on the same horizontal shaft is a gear-wheel *GW* in mesh with a pinion *Pi* mounted on a second horizontal shaft to which are fastened two pulleys such as *Pu*2. To one of these is attached a length of chain so arranged as to provide a varying weight to counterbalance the effective weight of the displacer-tank which alters during its stroke. The other pulley carries small weights used in the final adjustment.

While this arrangement presents novel and interesting mechanical features, it must in the Author's view be realized that it is not so flexible or so easily capable of adjustment as is the mechanism consisting of a plunger driven through epicyclic gearing. Moreover, in order to do the same as the epicyclic train, it would be necessary to have 28 tide-curves cut on the revolving drum *D*, that is, 14 tides from spring to neap followed by 14 from neap to spring.

Tides produced by more than one Plunger.

In the nature of the case, the shape of tide-curve which may be generated by a single plunger is restricted, and while there will be many models for which this simple arrangement will be quite adequate, there remain examples for which more than one plunger will have to be adopted. This may best be illustrated by consideration of an estuary subjected to appreciable " diurnal inequality " in its tide phenomena. In the simplest theory of the tides, the sun and moon are assumed to be situated in the plane of the earth's equator, whereas at any particular time the planes of their orbits may be appreciably inclined to the plane of the equator. This " declination " gives rise to the effect known as " diurnal inequality ", so that the level of high water at a certain place this morning may be sensibly different from the level at the same place to-night.* To quote an example at random, the following figures are extracted from the tidal predictions for Shatt-al-Arab bar, Persian Gulf (*Admiralty Tide Tables*, 1930):

TABLE XXXIV

Date	*Low water height—feet*	*High water height—feet*
Jan. 1	−0·3 5·7	7·8 9·8
Jan. 2	−0·3 5·7	7·9 9·7

Now an effect of this *kind* may be obtained by compounding two sine curves, one having twice the period of the other. This is demonstrated in Fig. 101, where the curve $y = a \sin \theta$ is combined with $y = -b \cos (\theta/2)$

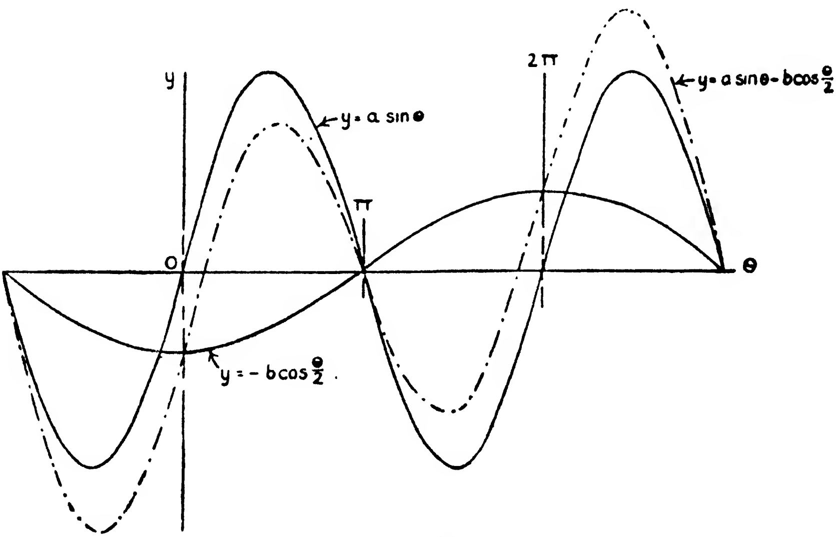


FIG. 101.

to give the resultant $y = a \sin \theta - b \cos (\theta/2)$. The crests and hollows (or turning points) of the resultant curve are given by

$$\frac{dy}{d\theta} = a \cos \theta + \frac{b}{2} \sin \frac{\theta}{2} = 0,$$

* Engineers will find an extremely readable and lucid outline of tide theory in vol. 1 of *Plane and Geodetic Surveying*, by the late Professor David Clark (Constable).

or
$$a\cos\theta = -\frac{b}{2}\sin\frac{\theta}{2},$$

or
$$a\{1-2\sin^2\theta/2\} = -\frac{b}{2}\sin\frac{\theta}{2},$$

or
$$\sin^2\frac{\theta}{2} - \frac{b}{4a}\sin\frac{\theta}{2} - \tfrac{1}{2} = 0,$$

i.e.
$$\sin\frac{\theta}{2} = \frac{\dfrac{b}{4a} \pm \sqrt{\dfrac{b^2}{16a^2}+2}}{2}.$$

For example, *if* $b = \frac{4}{10}a$, the successive maxima and minima will occur when

$$\sin(\theta/2) = 0{\cdot}759 \text{ or } -0{\cdot}659,$$

i.e. when $\theta = 98{\cdot}8°, 261{\cdot}2°, 442{\cdot}4°, 637{\cdot}6°, 818{\cdot}8° \ldots .$

If the period of the larger vibration (amplitude a) is 12·4 hours and that of the smaller is 24·8 hours, then 360° represents 12·4 hours, so that the turning points occur at times 3·4, 9·0, 15·2, 22·0, 28·2 ... hours. Also, by substituting $\theta = 98{\cdot}8°, 261{\cdot}2° \ldots$ in the equation $y = a\sin\theta - b\cos(\theta/2)$, the corresponding levels of high and low water become

$$0{\cdot}728a,\quad -0{\cdot}728a,\quad 1{\cdot}292a,\quad -1{\cdot}292a,\quad 0{\cdot}728a \ldots .$$

With this combination of simple harmonic curves, then, the second high water would rise 0·564a higher than the first or third, and the second low water would be 0·564a lower than the first or third low water. Thus, if a were 5 feet and b were 2 feet, the successive high and low waters would be 3·64, −3·64, 6·46, −6·46, 3·64 ... feet.

This suggests, therefore, that there is wide scope for the representation of diurnal inequality by two tide-generating plungers, the smaller one running at half the speed of the other. Such an arrangement has in fact been tried experimentally, as in the model of the Rangoon river.* It should be realized, however, that without resort to exceedingly elaborate machinery, the two plungers alone can only produce an *average* lunar cycle of tides. In Nature, the diurnal inequality may vary appreciably between the equinoxes and the solstices, but for practical purposes it will generally suffice to adopt the following procedure:

(1) Assuming the general form of the tide-curves to be as shown in Fig. 102, extract from the tide-records of the place concerned (a) the spring tide values of LWa, HWa, LWb and HWb, and (b) the neap tide values for each of the twelve months of a year. Averaging these, we obtain two

* "Investigation of the Outer Approach-Channels to the Port of Rangoon by means of a Tidal Model", O. Elsden, *Journ. Inst. C.E.*, June 1939, p. 3.

dimensioned diagrams, such as Fig. 102, one describing the *average* spring phenomena and the other the *average* neap.

(2) A process of trial and error may then be employed to decide what combination of two curves would give the required spring tides and what combination of two curves would give the required neap. Correspondingly, a preliminary design of the two plungers may be made.

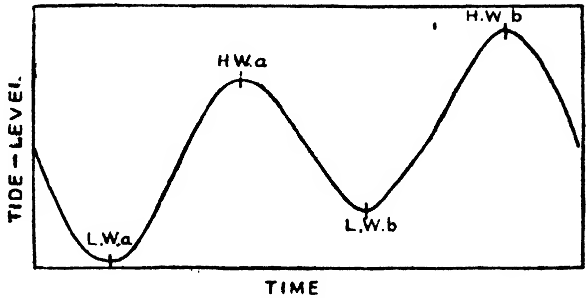


FIG. 102.

(3) The design so reached may be checked and, if necessary, modified by a full consideration of the volumetric displacements taking place in the model with the two plungers in action simultaneously.

Let V_1 = the volume of the semidiurnal plunger immersed at time T'.
$_1V_1$ = the volume of the semidiurnal plunger immersed at time $_1T'$.
V_2 = the volume of the diurnal plunger immersed at time T'.
$_1V_2$ = the volume of the diurnal plunger immersed at time $_1T'$.
V = the volume of water entering the moulded part of the model in the time between T' and $_1T'$.
A = the complete plan area of the tide-box, i.e. of the compartment (seaward of the moulded portion of the model) in which the plungers work.
h = the rise of tide-level at the seaward end of the model during the interval T' to $_1T'$. The required value of h is known from the tide-curves obtained during step (1).

Then

$$V = ({}_1V_1 + {}_1V_2) - (V_1 + V_2) - Ah,$$

and V so calculated should agree with the volume of water estimated, from the contoured charts of the estuary together with the assumed tide-curve, to be required to enter the model in that interval of time $_1T' - T'$.

As for the imitation of the lunar cycle between the average spring and the average neap tide, this may again be realized by altering the strokes of the plungers through epicyclic gearing, although it is possible that in some cases a sufficiently close approximation may be given by driving only the semi-diurnal plunger through epicyclic gearing, the diurnal plunger being provided with a constant stroke.

Each plunger may be counterbalanced by a pulley and cam system, different balance weights being used for the two plungers, but possibly the same size and shape of cam being found practicable for both if the loaded weights of the plungers are appropriately chosen.

A subsidiary electrical device of some interest, adaptable to tidal models for the automatic control of sluice-gates, river-supply systems, or other ancillary mechanisms, is briefly described in Professor Gibson's *Severn Model Reports* (H.M. Stationery Office, 1933), p. 257. It consists of one or more brass strips fastened by countersunk set-screws to a part of the rim of the disc G shown in Fig. 98. As the strip moves round with the disc G, it raises a lever to the end of which is attached a contact-maker. The act of raising this contact-maker from the bottom of a brass cup containing oil, serves to break the electrical circuit through a solenoid which operates the sluice-gate or other mechanism. It is a simple matter to alter the duration of the circuit by changing the length of the brass strip mounted on the disc G.

To give *approximate indications* of the effects of proposed structures in a reach of tidal water, the apparatus shown in Fig. 103 is possible. Here,

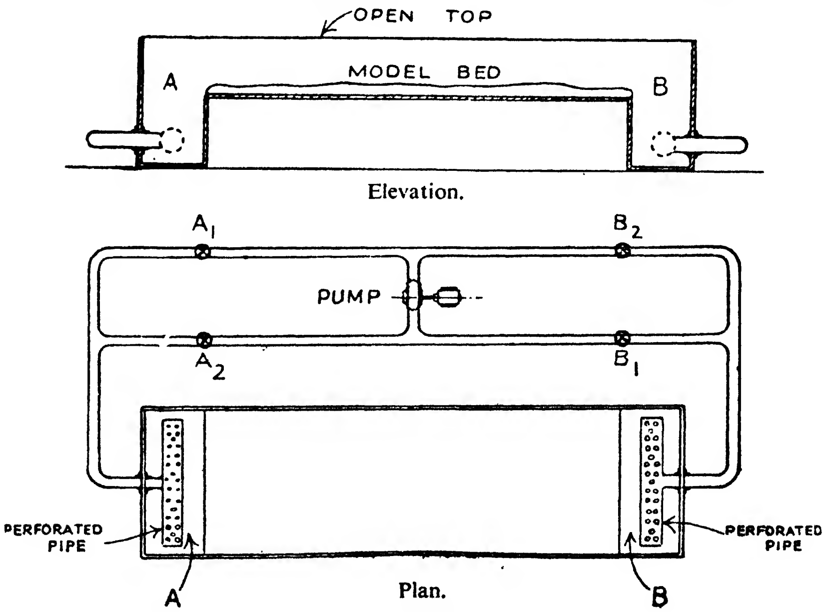


Fig. 103.

situated at each end of the model, is a sump containing a perforated pipe. Water may be pumped from one end A to B or from B to A alternately, according as to whether the valves A_1, B_1 or A_2, B_2 respectively are open, the other two valves being closed. The simplest procedure, and one which might be adequate for certain purposes, is to determine from tide and current observations in the natural tideway the average of the current velocities on both flood and ebb greater than, *say*, 1½ knots, the time during which such currents persist, and the mean depth of water in this period when the currents exceed 1½ knots. Assuming then that the active or important movement of the bed takes place while the currents are greater than 1½ knots, the model may be run with the suction and delivery valves so adjusted as to give the equivalent currents, time of flood, and mean water depth. By closing these valves and opening the other pair, the "mean effective" ebb current may next be generated, and so on.

CHAPTER X

NOTES ON SOME INVESTIGATIONS MADE IN HYDRAULICS LABORATORIES IN HOLLAND, INDIA, U.S.S.R., ITALY, CHINA AND CEYLON

In the course of this book, reference has been made to several laboratory investigations of river problems, but the examples quoted cannot be claimed to do other than touch the fringe of the work which has been undertaken in various countries. Some further reference at this stage may serve to provide a rather more complete picture.

Holland.

Professor J. T. Thijsse * of Waterloopkundig Laboratorium, Delft, has stated that in 1914 there were ten hydraulics laboratories in Germany and not one in Holland. Data for the Juliana canal and the sluices at Ymuiden were obtained from Berlin and for the Zuider Zee works from Karlsruhe. Later, the Dutch Government Department for Hydraulic Engineering, in collaboration with the Technical Academy at Delft, constructed a special laboratory for scale model investigations of harbours, locks, sluices and other devices. A great deal of work has been done at this place, some of the models costing apparently a thousand pounds or more.

It may be of interest to give the scales adopted in a few of the Delft investigations: †

(a) Zeebrugge Harbour.

Horizontal scale 1 : 400; vertical scale 1 : 60.

The principal object was to discover a means of preventing the creation of a great eddy on the rising tide. The flood tide attains a current velocity of about 2·33 knots and the ebb 1·55 knots. The pattern of the currents found in the model agreed closely with that in the harbour itself.

(b) The Port of Abidjan (Ivory Coast).

Horizontal scale 1 : 270; vertical scale 1 : 120.

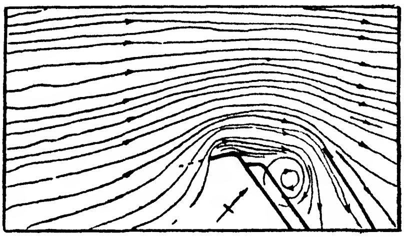
The object was to deflect sand from the harbour into a very deep hole in the sea bed. The average range of tide is 0·80 metres (2·6 feet).

* *De Ing.* (*Bouw. -en Waterbouwkunde*), vol. 49, pp. 175–184; abstracted in *Inst. Civ. Engs. Engineering Abstracts*, Jan. 1935, p. 151.

† J. B. Schijf, XVIth International Congress of Navigation, Brussels, 1935.

(c) The Mouth of the Rotterdamsche Waterweg (Hook of Holland).
Horizontal scale 1 : 1,000; vertical scale 1 : 150.

This model was used for a preliminary study of the problem associated with the flood tide which produced an eddy, accompanied by a shoal, along the southern mole. Experiment indicated that some improvement could be obtained by the construction of simple additional works. Fig. 104*a* shows the directions of the currents in the model before the insertion



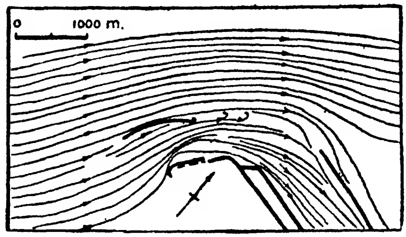


FIG. 104*a*. FIG. 104*b*.
Flow-lines observed in model of Hook of Holland (Delft Laboratory).

of the extra walls, while Fig. 104*b* indicates their directions after the introduction of a rounded nose and a simple guide wall. The sketches reproduced in Figs. 105*a* and 105*b* show the corresponding bed contours.

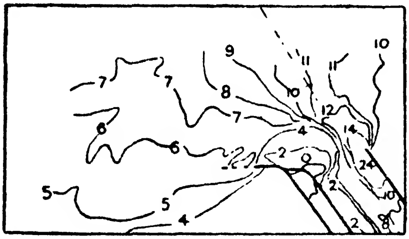


FIG. 105*a*. FIG. 105*b*.
Depths in Model after the Flood Tide.

(d) The Port of Wilhelminahaven at Vlaardingen.
Horizontal scale 1 : 200; vertical scale 1 : 75.

In this model, muddy water was introduced to give a measure of siltation, and it was found that considerable improvement could be expected if the groynes shown in either Fig. 106*b* or 106*c* were constructed. These devices served in the model to deflect silt-laden water from the mouth of the harbour, but on general grounds, including navigational difficulties, they were considered otherwise objectionable. An alternative, but much less effective, device proved to be that of shortening, and raising above high tide level, the western jetty or groyne as indicated in Fig. 106*d*. This

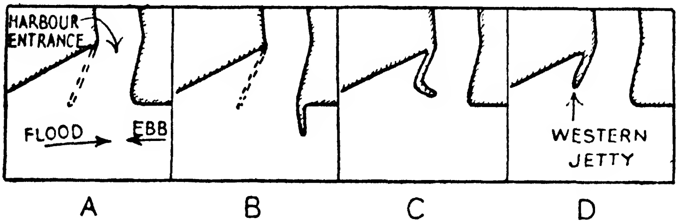


FIG. 106.

arrangement reduced the rate of siltation to some extent and was not considered objectionable on navigational or other grounds.

(e) The Heyplaat Dokhaven.

Horizontal scale 1: 125; vertical scale also 1: 125.

In this case, very muddy water was known to enter the harbour through its wide entrance on the Waterweg and to deposit in the shipping channels. The introduction of groynes in the model, reducing the width of the entrance by more than one-third, was found to reduce the siltation, but these groynes or jetties (see Fig. 107) were considered to be inadvisable from consideration of the manœuvring of large vessels. The introduction of model groynes within the harbour area itself, with a view to deepening the channels and confining siltation to other areas, was unsuccessful.

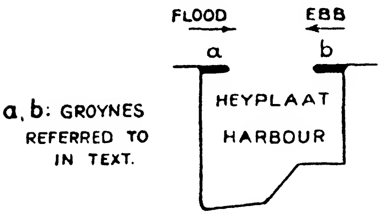


FIG. 107.

(f) Breakwaters recently completed at the Port of Leith (Scotland).

In his Inaugural Address as Chairman of the Edinburgh and District Association of the Institution of Civil Engineers, 11th October, 1944, Mr. J. Dalgleish Easton, M.Inst.C.E., has given a fascinating account of experiments made in the Netherlands Government Laboratory at Delft as a preliminary to the design of two new breakwaters at Leith. The Author is greatly indebted to Mr. Easton for permission to quote the following particulars:

" The purpose of the breakwaters was to form a new entrance to the Port, farther out into the estuary, to take the place of the old entrance which was built in 1852, and at the same time to enclose a considerable area of the sea so as to provide room for future extensions to the Port of Leith.

"The breakwaters are constructed on the same principle as the great dyke formed some years ago by the Netherlands Government across the Zuyder Zee, and I think they are the first breakwaters of this type to be constructed in this country. . . .

"The experiments were divided into two main series:

"1. To find a satisfactory cross-section for the breakwater so that it would withstand the heaviest storm possible at Leith.

"2. To find the most suitable positions of the seaward ends of the two breakwaters to form a new entrance to the Port and in such a position as to reduce as much as possible the disturbance of the water in the harbour during storms.

"Each breakwater consists of a mound of pumped sand encased within a thick layer of boulder clay, the toes of the slopes being formed by two parallel mounds of tipped boulder clay, the tops of which are a little above low-water level. . . . The whole structure * is formed with long flat 4 to 1 and 3 to 1 slopes, with three berms where the planes of the slopes would intersect. To protect the boulder clay from the action of the waves, the whole surface of the slopes and berms above low-water level is covered by a layer of heavy whinstone blocks laid on a foundation of rubble, while the tipped boulder clay mounds at the toes are protected by a tipped rubble mound with stones varying in weight up to 2½ tons.

"The principle of the design is to provide long flat slopes and berms in such a manner that they are sufficient to allow the waves to run up the slopes and so dissipate their energy, and the object of the tests† is to find the slopes required and the position and number of berms to do this. The berms hold the waves and thus cause them to form a water cushion.

"The maximum width of the base of the East Breakwater is 360 feet, with a maximum height of 43 feet above the bed of the sea.

"The East Breakwater is about ½ mile long and the West Breakwater is ¾ mile long. The area enclosed is about 250 acres."

For the experiments designed to discover the best profile for the breakwaters, a *covered* flume 13·2 ft. wide and 88 ft. long was used. The depth of water could be varied, waves produced by a machine at one end of the flume, and a current of air blown over the water surface. This wind was made by fans in a tunnel underneath the flume and in communication with it at each end. The scale ratio adopted for the model tests was 1 : 25, and correspondingly the velocity scale would be $1 : \sqrt{25}$ or 1 : 5.

The width of the flume (13·2 ft.) enabled two models with different profiles to be tested simultaneously. They were placed next to one an-

* The general idea of the construction is shown in Fig. 108.

† i.e. the first series of tests.

other at one end of the flume and were constructed of wood. Each model thus occupied a 6·6 ft. width of the flume, but it was found that only about 13¾ inches of the width of each model needed to be completed with the rubble layer, tipped stone or stone pitching. The rubble was represented by fine stone chips, the pitching by small marble blocks having the same specific gravity as whinstone, and the tipped stone by weighted pieces of whinstone. To facilitate observations, the sides of the flume in the region of the models were made of glass.

"The range of ordinary spring tides at Leith is approximately 16 feet, but it was not considered advisable to carry out the tests with a rising and falling water level if accurate observations were to be made.

"It was therefore decided to divide the tide range of 16 feet into 10 sections and to carry out a test for each of these with the corresponding water level, beginning at low water and proceeding to high water and then back to low water in steps of 1·6 feet, two sets of measurements being taken for each water level. For the purpose of verification, however, measurements were several times carried out with a constantly increasing and then decreasing depth of water and it was found that the results were not less favourable.

"The wave dimensions were also increased by steps for the same reason as the water depths, and for each profile the test was always commenced with a wave-height well below that which was likely to damage the breakwater and stepped up by stages to the maximum wave. The period for the first stage was 5½ seconds and that of the last 8½ seconds."

Mr. Easton describes the results obtained with several different profiles from which the final design was developed; this was subjected to an additional test on a larger scale for the tipped-stone toes of the breakwater and the results formerly obtained were confirmed, although it was decided as a result to provide heavier stone for this purpose and to place 2½-ton stones along the crest of the lowest berm because of the great force exerted by the backwash of the waves. It was further decided to grout the whole of the pitching on the sea-face up to high-water level with cement mortar.

Mr. Easton remarks, however, that the resistance of stone pitching is "greatly influenced by the degree of tightness between the various stones. In actual practice in a well-pitched slope the stones will be very tightly packed, but after some time they may be loosened on account of settlement or as a result of wave action.

"It was found impossible to imitate satisfactorily the stone pitching in the models and it was therefore decided to carry out all the experiments with very loose pitching with the stones hardly supporting each other. By this means the model breakwaters were tested under the least favourable conditions, as each stone was separately attacked by movement of the water, and it was therefore assumed that if the model had sufficient resistance an actual breakwater would be safe under all possible circumstances . . . a slight damage in the model caused by the largest wave was considered allowable, as the pitching stones in the actual breakwater are much more compact than in the model.

" With the tipped stones at the toes of the breakwater the difficulty experienced with the pitching did not arise because their actual condition could be exactly reproduced in the model. . . .

" The visual observations of the behaviour of the pitching and toes of the various profiles could not, in the scientific sense, be considered ' accurate measurement ', as they are to a considerable extent dependent on the

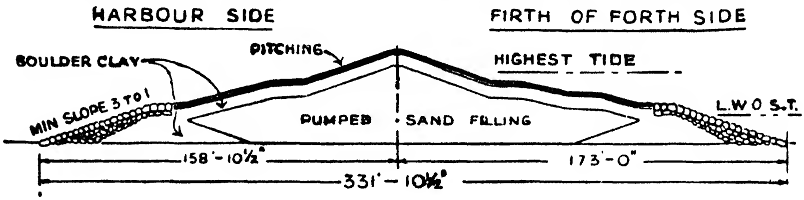


FIG. 108.
(For details see Inaugural Address to *Inst. C.E.*, Edinburgh and District Association, 11th Oct., 1944; J. Dalgleish Easton, M.Inst.C.E.).

person who makes the observations. To get over this difficulty, arrangements were made for all the tests to be carried out by the same person, so that the manner of appreciation was the same for all the experiments. For the same reason the placing of the pitching stones on the model was always done by the same workman."

Before starting the second series of experiments (designed to find the most suitable positions of the seaward ends of the two breakwaters), it was agreed that:

" (*a*) the new harbour should not be less tranquil than the existing harbour;
(*b*) the greatest height of waves alongside quays should not exceed 2 feet;
(*c*) any alteration to the positions of the two pier heads should always be such that a ship could enter and leave in an approximately straight line; and
(*d*) the bottom width of the entrance channel should be 400 feet."

The model of the harbour was constructed, to a scale of 1: 180, inside a brickwork tank about 45 feet by 28 feet, built on the concrete floor of the laboratory. A fixed bed of rubble and sand covered with cement mortar was adopted for the bed of the sea; the harbour portion was moulded in sand to the profile of a number of copper templates. This sand enabled modifications of the lay-out to be made with the minimum of difficulty. The modelling of the coastline, piers and breakwaters was executed in cement mortar, while the new quays of the harbour were made of wood to facilitate placing and removal.

Two wave-making machines were used to generate waves from different directions. Each machine consisted of "a motor coupled to a flywheel giving an oscillating rotation to an axle by means of an eccentric, and the period and amplitude of the rotation could be regulated. The wave-boards were hinged to the foundation plates with their upper ends attached to the axle by means of push rods, which caused the boards to swing. . . . The push rods were fitted with ball and socket joints at both ends so that the direction of the boards and, consequently, the waves could be varied. In order to prevent reflection of the waves against the south-west boundary of the tank, a slope of sand was laid down on which the waves could break and thus be absorbed."

A point worthy of special notice is that at the two places where the wave-making boards were situated, the depth of water in Nature is about 40 feet. To the scale of 1 : 180, this would be represented by 2·67 inches, a depth too small for the production of regular waves. The depth in these regions was therefore arbitrarily increased to about 7 inches. The maximum wave outside the harbour—13½ feet—was represented by a height of 0·9 inch in the model.

Eleven arrangements for the harbour entrance were tested in this way, and one of these has been actually constructed "to the satisfaction of the Pilots and Shipmasters using the Port".

India.

Large models have been constructed at the Khadakvasla Hydrodynamic Research Station near Poona, and Mr. Gerald Lacey's suggestion for relating the horizontal scale, $1 : x$, to the vertical scale, $1 : y$, by the equation $y = x^{\frac{2}{3}}$ appears to have been adopted for river models at this laboratory. One specially interesting example of the investigations made at Poona under the direction of Mr. C. C. Inglis (now Sir Claude Inglis) is concerned with the protection of the Hardinge Bridge over the River Ganges, for details and criticism of which the reader is referred to "The Principles of River-Training for Railway Bridges, and their Application to the Case of the Hardinge Bridge over the Lower Ganges at Sara", Sir Robert Richard Gales, *Journal Inst. C.E.*, Dec. 1938, pp. 136–200, with discussion on pp. 201–224; and in *Journal Inst. C.E.*, Oct. 1939, pp. 279–332; and to "River Training in Connection with Bridges", *Engineering* editorial, vol. CXLVI, p. 649, Dec. 2, 1938.

The Annual Reports of the Central Irrigation and Hydrodynamic Research Station, Khadakvasla (Indian Waterways Experiment Station, Poona), from 1939 onwards contain a mass of invaluable information which, unfortunately, it is quite impossible to summarize adequately in this book. This Hydraulics Station began to operate, we believe, about twenty-five years ago, and has actively pursued a large number of investigations, including some which are of a very fundamental character. Thus, the *Research Publication No.* 8, prepared by Sir Claude Inglis and his Staff, contains a map of India indicating "24 places visited by the Director during the year 1943 in an advisory capacity, 4 places where advice was given without inspection or experiment, and 18 places in connection with which model experiments were in progress, after inspection".

Among the many investigations cited in this publication are:

- the control of floods entering the port of Vizagapatam;
- the control of erosion due to littoral drift at Vizagapatam;
- a scheme to augment Bombay's water supply;
- the cause of the change of course of the Indus near Chasma, and how to control it;
- siphon spillways at Mysore;
- the diversion of the River Gandak, which had outflanked the railway bridge at Bagaha, U.P.;
- the exclusion of sand from canals taking off the Son River;
- the prevention of dangerous bank erosion on the Hooghly (Port of Calcutta) (2 tidal models);
- the form-drag of cylinders in connection with bridges;
- whether material available is suitable for constructing a dam in the Solani Khadir;
- the design of the Lower Sind Barrage;
- the effect of the Right Bank Approach Channel on silting of the Indus at Sukkur Barrage.

In addition, fundamental research was in progress concerning problems such as the transportation of pebbles in flumes, the formation and behaviour of bed ripples; the grading of sand and silt by washing (elutriation); the reduction of waves caused by swell in a harbour; empirical run-off formulae in connection with rainfall.

Extremely important hydraulic model work has also been carried out at Lahore and other Indian research establishments. In common with the U.S. Experiment Station at Vicksburg, Mississippi, many of the Indian river-models are constructed in the open air.

U.S.S.R.

A great deal of work of very considerable scientific and practical importance has been done in the Soviet Union. In 1933, the Scientific Research Institute of Hydrotechnics (People's Commissariat of Heavy Industries, Central Department of Power Production and Distribution) published from Leningrad a fascinating volume entitled *Description of Laboratories of the State Scientific Research Institute of Hydrotechnics and the Leningrad High School on Hydrotechnics*, edited by J. B. Egiazaroff, Professor of Hydroelectric Power. Various sections of this well-illustrated book have synopses in English, and the titles of the illustrations are also provided with English translations. The construction of the Leningrad Hydrotechnical Laboratory No. 1 was completed early in 1930; it occupied a building 70 metres (230 feet) long and from 11 to 14 metres (36 to 46 feet) wide. This is only one of the many laboratories described in the volume, and altogether it is evident that a vast field of research—both pure and applied—was being actively explored. For example, there are details of work on soil mechanics; sedimentation; percolation; transport o bed materials; stability of dams; the hydromechanics of fluids of very different viscosities; backwater effects; siltation in rivers and the most suitable lay-out of control works and hydro-electric plant; siphon spillways; surge-tanks; wave propagation. Photographs are printed showing experiments on the drift of ice-blocks in the region of river-works, the ice in the model being in some cases natural ice and in others being simulated by blocks of wood. In connection with Hydrotechnical Laboratory No. 1, it is stated (p. 46) that:

" During two years of its existence, the staff of the Laboratory has made 47 investigations of various kinds for 12 different hydroelectric plants.

" As all forces of the Laboratory were concentrated on current needs of projects under construction, always urgent and indispensable, the scientific research work, not immediately connected with current construction, could not be developed to any considerable extent.

" Notwitstanding this circumstance, a series of scientific works may be noted, for which the results of practical experiments have served as a starting point and a base."

Italy.

Reference has been made in Chapter III to the work of Scimemi at Padua on the scour at the toe of a spillway. An account of a novel tidal model investigation carried out in the Laboratorio di Idraulica del R. Istituto Superiore di Ingegneria di Padova has been presented by E. Scimemi and E. Indri in a Paper entitled " Esperienze sulla formazione della barra nel Porto Lagunare di Lido (Venezia) ", *Atti del Reale Istituto Veneto di Scienzi, Lettere ed Arti*, 1934–35, vol. XCIV, part II, p. 993. In this model, built to a horizontal scale of 1 : 2,000 and a vertical scale of 1 : 200, the bed material was taken from the foreshore of the Lido itself; the hydraulic effects were produced by a motor-driven wave-maker, together with a tidal current generated by admitting water through a hand-operated valve and discharging the water over one or other of two weirs according to flood or ebb conditions. The model was calibrated, to determine the most suitable time- and velocity-scales, by moulding its bed originally to the natural configuration of the year 1872 and operating it for two sequences of 50 tides each, at the end of which periods the model was surveyed and compared with the 1882 and 1892 prototype surveys. With experimentally chosen velocity- and time-scales, the agreement was sufficiently close to justify a study of the formation of the bar by observing the effects apparent in the model. As the Author understands it, this method differs essentially from the normal approach to the problem of similitude in tidal models. The normal approach consists in fixing the time- and velocity-scales in relation to the linear scales of the model by the principle of dynamical similarity and adhering to these time- and velocity-scales, except in so far as small modifications may be suggested by a comparison of tide and current phenomena with the corresponding phenomena in Nature. With the time-scale so fixed in accordance with hydraulic effects, a number of tides, n, in the model is taken to be equivalent to the same number, n, of tides in Nature and therefore to $\frac{n}{705}$ years, taking 705 tides per annum. In calibrating the model with reference to bed changes, the procedure then follows of comparing the behaviour of the model during its n tides of operation with that in the prototype during the period $\frac{n}{705}$ years, and if necessary the type of bed material, the kind and concentration of suspended silt, the quantity of any coagulating salt and the schedule of fresh water river-discharges are modified to give the required degree of accuracy. This process would appear to have been reversed in the model at Padua. Knowing from the experience of earlier experimenters that the size of the bed material to be used in the model is at any rate *of the same order* as that in Nature, it was decided to adopt the identical sand from the area concerned and to operate with such velocity- and time-scales for waves and tide as gave reasonable agreement between the changes observed in the model during 50 tides and those in Nature during ten years. The scales adopted were as follows:

Horizontal - - -	1 : 2,000
Vertical - - -	1 : 200
Time (for tide) - -	1 : 18
Time (for waves) - -	1 : 12
Velocity - - -	1 : 16
Discharge - - -	1 : 4,000,000

According to the principle of dynamical similarity, the time-scale for tidal effects should have been $1:\left(\frac{2000}{\sqrt{200}}\right)$ or 1 : 141·4, while the scales of velocity and discharge would be 1 : 14·14 and 1 : 5,656,000. The time-scales actually adopted correspond more closely with what would be the theoretical value if there were no vertical distortion of scale. Thus, if the horizontal scale were 1 : 200, the time-scale would be 1 : 14·14 and the velocity scale also 1 : 14·14.

While this investigation is certainly of interest, it does not appear as though the method of attack—in particular the arbitrary choice of a certain number of tides to represent a certain period of years—is likely to lend itself to general application, and especially not to those cases in which surface waves are of minor importance compared with normal tidal action. The tidal range at the place under consideration (Venice) is small: the *Encycl. Brit.* (14th Ed.), vol. 23, p. 60, states it to be about 2 ft. on the average, but under certain conditions of wind to become 5 ft. and over. To a scale of 1 : 200, 2 feet would be represented by 0·12 inch and 5 feet by 0·30 inch.

China.

It is well known that the rivers of China are associated with problems on a truly enormous scale. It has been stated that, "taking the world as a whole, roughly one person in every ten lives in the Yang-tse basin. For centuries the Yang-tse, which is well over 3,000 miles long, has been the aorta of trade for an area of some 750,000 square miles ".* Vessels drawing up to 15 ft. of water can pass all the year round from the sea to Hankow—700 miles—but navigation is hindered by a bar across the south channel of the estuary and by several shoals between Hankow and Woosung, which become awkward during the periods of winter low-water level. One reach, near Matang, was being studied by the Central Hydraulic Research Institute at Nanking in a provisional laboratory in 1937. The horizontal scale of the model used for this purpose was 1 : 1,250 and the vertical scale 1 : 90. In this particular region, the silt content in suspension in the Yang-tse was considered unimportant and accordingly clear water was adopted for the model. The bed material was granulated coal from 0·5 to 2·0 mm. in size (0·02 to 0·08 inch). The volume of water supplied to the model river at maximum stage was 80 litres per second (2·83 cubic feet per second). In the spring of 1937, a scheme was being proposed, based on the model investigation, for the widening of one of the channels concerned.†

* R. B. Whittington, *Engineering in China*, the Institution of Civil Engineers, North-Western Association, Manchester, April, 26, 1944.

† Whittington and Sanger, *Models in Engineering*, Shanghai Association of the Inst C.E. and Eng. Soc. of China, Nov. 20, 1939.

A permanent hydraulics laboratory had also been designed and construction on it started in August 1937.

Tremendous problems are inherent in the control of the Yellow River (the Hwang-Ho) as well. In this case the proportion of solid matter in suspension is said to attain phenomenal values; in some of the tributaries as much as 50 per cent. by weight. To quote Mr. R. B. Whittington,* "the loess is an extremely fine brownish-yellow earth, which tends to fracture vertically, so that in these regions the river now flows through deep gorges, of which the faces may be several hundred feet high. These cliffs are easily undermined and break off in large vertical slices to be borne away by current and deposited either in the plain or in its submarine extension. . . .

"For more than a hundred generations the aim has been to keep the river bounded within dykes; but the history of China is grimly punctuated with records of catastrophic breaches in the defences. The last great catastrophe took place in 1852, when the river changed its outlet from south to north of the Shantung peninsula. It seems impossible to exaggerate the effects of major or even minor disasters in terms of human lives lost. While the difficulties of control are immense, the committee of experts† (of which the late Mr. A. T. Coode was the British member) which visited China in 1935 considered that they were not altogether insuperable. Among their suggestions was the erection of local dykes, especially to surround the inhabited centres, converting these into islands at flood times, and so saving the populations from drowning. To prevent starvation, the introduction of quick-growing crops was suggested. These might be harvested before the flood season.

"Other suggestions were: the construction of weirs in the upper reaches and tributaries, to hold back some of the flood waters and reduce erosion; and the extension and organization of the old 'ditch' system, by which one-ninth of every cultivated field was traditionally devoted to storage of the rainfall.

"Confidence in irrigation schemes may have been weakened by the irrigation works completed by the Suiyuan Provincial Government in 1932, at an estimated cost of Ch. $860,000. The scheme is reported to have been a 'a total failure almost unparalleled in the history of irrigation.'"

If problems of this magnitude are tackled in models, the models themselves tend to become small artificial rivers and to be more favourably undertaken in the open, with some light protection against the weather

* *Engineering n China*, loc. cit.

† *Report of the Comm. of Experts on Hydraulic and Road Questions in China*, League of Nations, No. C91M34, Geneva, 1936.

where necessary, rather than being completely housed within a building. One difficulty lies in the almost inevitable division of the river into component reaches for individual study: precautions have then to be taken to cover the possibility of improvement works in one stretch causing undersirable effects in another. This may be far from easy. One method would consist of ensuring that any changes in velocity or depth at the end *B* of a stretch *AB* are reproduced at the entrance *B* to the model of a stretch *BC*, but even so it remains possible that schemes tried out in the model *BC* may produce secondary repercussions in *AB*. Here, the choice of the subdivisions of the river into separate parts for analysis will call for both skill and experience. Another method would consist of a detailed examination of the effects of proposed works in component models, followed by a final test of more general effects in a smaller scale model of the various reaches combined.

Ceylon.

During the last few years, a Hydraulics Laboratory has been constructed at Colombo under the direction of Dr. R. V. Burns, whose work with Dr. C. M. White in London on the scour at the toe of a spillway has already been mentioned in Chapter III. This Research Laboratory of the Irrigation Department of Ceylon is equipped for model tests, and a tidal model investigation has already been completed leading to proposed Flood Protection Schemes for the Nilwala-ganga.* This model has a horizontal scale of 1 : 2,400 and a vertical scale of 1 : 60; the velocity scale was therefore $1:\sqrt{60}$, or 1 : 7·7, and the tidal period 143 seconds, the time-scale being $1:\dfrac{2400}{\sqrt{60}}$, or 1 : 313. The tides were generated by a motor-driven plunger in the same way as in the Severn investigation; the problem under consideration may be regarded as one of flooding and drainage combined. Some flooding is necessary for the rice fields, but the crops are killed if water stays on them too long, as they also are by tidal water running up-river when the river itself is nearly dry. From the viewpoint of a model investigation, a troublesome feature is the small tidal range of 18 inches, and yet this tide travels upstream some 4 to 6 miles. The value of the Reynolds' criterion $h^3\dfrac{x}{y}\sqrt{\dfrac{H}{30}}$ with the scales as chosen must have been 0·00014, which provides further evidence in support of the use of low values of this criterion in tidal models of natural, irregularly-shaped estuaries.

Dr. Burns states that: " The bottom of the river channel was cut 1 inch

* *Final Report on the Nilwala-ganga Flood Protection Schemes*, by R. V. Burns, Ceylon Government Press, Colombo, 1940.

deeper (equivalent to 5 ft. in Nature) than the depths as determined in the field, and this portion was later moulded with clean sharp sand (passing 40-mesh and held on 80-mesh sieves).... In checking the performance of the model, it was first tested at several points to see that it reproduced the conditions as they exist in Nature, especially as to flood flows. This was a long and tedious process, as several errors were detected; in fact the model was remoulded several times before it was found to perform correctly. Later, the most obvious solutions were tried out and their effects studied. Each proposal in turn was tested, estimated, and analyzed until finally a combination of different proposals was evolved, and this is now put forward as being the most desirable solution.... Apart from the question of gauge readings, manometer readings, velocity measurements, and movements of bed materials, records of the important tests were made by means of a 16 mm. Ciné Kodak motion picture camera."

CHAPTER XI

MODEL EXPERIMENTS ON THE STABILITY OF BLOCKS BROKEN STONE AND SAND

THE discussion which forms the body of this Chapter is based upon a Paper published in the *Journal of the Institution of Civil Engineers*, March 1942.* The need has long been felt for quantitative information as to the least size and weight of material which may safely be employed in the construction of embankments, causeways and similar structures across a river, estuary or tidal sound. The experiments to which reference is now made had as their object, therefore, the determination of the current-velocities causing movement of regular-shaped blocks, either built systematically—without cement or other binding agency—in successive rows so as to form an embankment, or alternatively laid in random fashion to produce a mound. The investigation was also directed towards finding the velocities if instead of using, say, cubical concrete blocks, the work is carried out in flexible bolsters containing stone chippings. Finally, experiments were made on flat beds and mounds of chippings.

Considering first the theoretical aspect of the stability of a single cube on the bed of a stream, we may attribute the movement of the block to three causes:

(i) the impact of the stream on the upstream face;

(ii) the reduction of pressure on the top and lee faces;

(iii) the drag of the current, in particular along the top and sides of the block.

Each of these effects may be considered to be approximately proportional to the square of the current velocity, and the first of them may be approached in terms of the principle of momentum.

If tbe cube rests on the horizontal bed of a stream, its resistance to overturning is measured by a moment about the downstream lower edge of magnitude

$$\frac{(\rho' - \rho)\, l^4}{2},$$

where ρ' denotes the density of the cube, ρ the density of water, and l the length of the side of the cube.

* " An Investigation of the Stability of Bed Materials in a Stream of Water ", Allen, *Journ. Inst. C.E.*, March 1942, p. 1.

Again, its resistance to sliding is a force of magnitude

$$\mu(\rho' - \rho) l^3,$$

where μ is the coefficient of friction between the cube and the bed.

Now let v denote the velocity of the current at a height y above the bed, and let it be assumed that the stream is broad in comparison with l, so that v may be taken to be constant across the *width* of the cube.

Then the normal impact of the current against the upstream face of the block amounts to a horizontal force of

$$\frac{K\rho l}{g}\int_0^l v^2 dy.$$

Here K is a coefficient intended to take account of the fact that not all the forward momentum of the impinging stream is destroyed, since the filaments near the edges of the face exposed to the current retain an appreciable forward momentum. The value of K appears to be about 0·70.*

The overturning moment of the impinging stream is

$$\frac{K\rho l}{g}\int_0^l v^2 y\, dy.$$

Hence, neglecting any other forces which may tend to upset the block, it will overturn if

$$K\int_0^l v^2 y\, dy > \frac{g(\rho' - \rho) l^3}{2\rho},$$

or will slide if

$$K\int_0^l v^2\, dy > \frac{\mu g(\rho' - \rho) l^2}{\rho}.$$

These results, while representing only part of the story, serve to demonstrate quite clearly that the stability of the block is essentially dependent upon the way in which the velocity is distributed through the depth of the stream, and it is instructive to examine certain possibilities.

Let v_s denote the surface velocity of the stream, $v_{\max}$ the maximum velocity and $\bar{v}$ the mean velocity.

In the first place, suppose the velocity to be quite uniform, so that $v_s = v_{\max} = \bar{v}$. The condition for overturning then becomes

$$v^2_s > \frac{g(\rho' - \rho)}{K\rho} l,$$

and that for sliding becomes

$$v^2_s > \frac{\mu g(\rho' - \rho)}{K\rho} l.$$

* *Hydraulics and its Applications*, A. H. Gibson, 4th ed., p. 389, Constable, 1930.

In this case, therefore, the velocity at which the cube will be disturbed is independent of the depth of water, is proportional to the square root of the dimension of the block, and is in general smaller for sliding than for overturning if the cube rests on a plane bed.

Secondly, suppose the velocity to vary linearly from zero at the bed to v_s at the water surface, the total depth being D. Then

$$\frac{v}{y}=\frac{v_s}{D},$$

$$\bar{v}=\frac{v_s}{2},$$

$$v_{\max}=v_s,$$

and the cube will overturn if

$$v_s^2>\frac{2g}{K}\left(\frac{\rho'-\rho}{\rho}\right)\left(\frac{D}{l}\right)^2 l,$$

and will slide if

$$v_s^2>\frac{3\mu g}{K}\left(\frac{\rho'-\rho}{\rho}\right)\left(\frac{D}{l}\right)^2 l.$$

In this case, therefore, the velocity to produce movement is directly proportional to (D/l) and to $\sqrt{l}$, whilst the cube will overturn rather than slide if $\mu>2/3$.

Finally, suppose the velocity-distribution to follow a parabolic law from zero at the bed to a maximum at the water surface; such a distribution might be approximately attained in a channel having a rough bed of uneven contour.

In this case,

$$\frac{v}{v_s}=\sqrt{\frac{y}{D}},$$

$$v_{\max}=v_s,$$

$$\bar{v}=\tfrac{2}{3}v_s,$$

and for overturning,

$$v_s^2>\frac{3g}{2K}\left(\frac{\rho'-\rho}{\rho}\right)\left(\frac{D}{l}\right)l,$$

whilst for sliding,

$$v_s^2>\frac{2\mu g}{K}\left(\frac{\rho'-\rho}{\rho}\right)\left(\frac{D}{l}\right)l.$$

With this assumed parabolic velocity-distribution, therefore, the block will overturn rather than slide if $\mu>\frac{3}{4}$; in a stream of given depth, the velocity required to shift the cube is independent of the size of the cube and is proportional to the square root of the depth of stream. If, then, the whole of the mechanical system of forces tending to disturb equilibrium could be expressed in this general manner, namely $v^2 \propto \left(\frac{\rho' - \rho}{\rho}\right)\left(\frac{D}{l}\right) l$, it would follow that, given such a parabolic distribution of velocity, the *same size and weight of block* would be moved in a model stream as in its full-scale prototype, provided that the velocity-scale were chosen to be proportional to the square root of the depth.

Another way of regarding the results is that all of them are of the form

$$v^2 \propto \left(\frac{\rho' - \rho}{\rho}\right)\left(\frac{D}{l}\right)^n l.$$

With a uniform velocity, $n = 0$; with a linear distribution, $n = 2$; and with a parabolic, $n = 1$. Suppose, then, that the suffix (1) refers to a full-size cube in an actual stream, and the suffix (2) to a geometrically similar cube in a model stream having a depth $\frac{1}{S}$ times that in the prototype. The scale of the model is thus $1 : S$ and v_2 may be chosen $= v_1\sqrt{\frac{1}{S}}$. In that case, since $\left(\frac{D}{l}\right)_2$ is made $= \left(\frac{D}{l}\right)_1$, it follows that the model stream of velocity $v_2 = v_1\sqrt{\frac{1}{S}}$ will be just sufficient to disturb the model cube $\left(\text{having } l_2 = \frac{l_1}{S}\right)$ if the prototype stream of velocity v_1 is just strong enough to disturb the full-size cube of side l_1 and of material having the same density as the model block. This assumes that the distribution of velocity is similar and the value of K identical in the two streams.

As to the velocity-distribution actually obtained in open channels, it may be said that the general tendency is for the filament of maximum velocity to approach more closely to the surface as the stream becomes shallower and its bed rougher; Fig. 109 shows a *mean* or *typical* curve based upon experiments in a concrete channel.

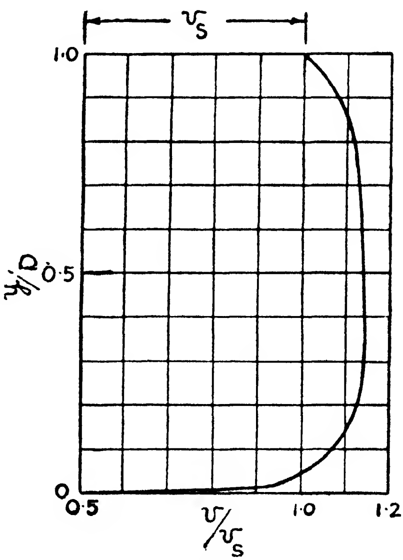


FIG. 109

This curve has been arithmetically integrated to give the summations

$$\frac{K\rho l}{g}\Sigma_0^l v^2 y\,\delta y \quad \text{and} \quad \frac{K\rho l}{g}\Sigma_0^l v^2\,\delta y,$$

assuming $K = 0{\cdot}70$.

Writing the condition for overturning

$$\frac{0{\cdot}70\rho l}{g}\Sigma_0^l v^2 y\,\delta y = \frac{(\rho' - \rho) l^4}{2}$$

in the form

$$v_s = \alpha\sqrt{\left(\frac{\rho' - \rho}{\rho}\right) l};$$

and the condition for sliding

$$\frac{0{\cdot}70\rho l}{g}\Sigma_0^l v^2\,.\,\delta y = \mu(\rho' - \rho) l^3$$

in the form

$$v_s = \beta\sqrt{\mu\left(\frac{\rho' - \rho}{\rho}\right) l},$$

it was thus found that α and β have the following values:

TABLE XXXV

$\frac{D}{l}$	$\frac{h}{l}$	α	β
1	0	6·06	6·15
2	1	6·09	6·21
5	4	6·30	6·55
10	9	6·65	6·96
15	14	6·96	7·30

In this table, h represents the depth of stream measured above the top of the cube, so that $D = h + l$. It may be stated at once that observed values of v_s in the concrete flumes on which the velocity curve of Fig. 109 was based gave values of α within 6 per cent. of those shown in Table XXXV, the experimental cubes ranging between 1 inch and 6 inches in side.

The concrete flumes employed in the experiments were respectively 12 and 21 inches wide, and water was circulated through them by a centrifugal pump, the rate of flow being measured on a calibrated venturi meter, while localized velocities were obtained by means of a pitometer.

Preliminary observations revealed that the way in which a cubical block first moved depended upon just where the block was placed on the bed of the flume and upon which side was in contact with the bed. Sometimes it would slide, remain stationary for some time, slide again for some distance, overturn and roll. On other occasions its first movement would be by overturning. Friction tests were made by dragging cement blocks along the bed with a cord and spring balance; these indicated an average coefficient of friction, μ, of 0·62.

Having regard to these phenomena, it was decided to standardize the conditions by fixing a sheet-iron plate 0·094 inch thick across the width of the flume. The various objects used were then placed with one edge abutting this plate, and the flow of water was increased in gradual and regular steps until movement was observed. The pitometer and depth readings were taken immediately before this movement occurred. Calculations indicate that the velocities necessary to overturn the blocks are 10 per cent. higher when the sheet-iron plate is used than if no such device were adopted. This figure of 10 per cent. refers to a block 1 inch deep; the corresponding percentages for blocks 2 inches and 6 inches deep are 5 and 1·5 respectively. Observed velocities were accordingly reduced by these adjusting percentages.

For the purpose of checking observations, many of the tests were repeated several times: these justify the statement that the velocities interpreted as being those for initial movement are consistent within about ± 10 per cent.; often they lie within a very much narrower band. Nor is any greater accuracy likely to be attained in such tests, considering the difference which might be anticipated from slight irregularities in the shape of the object tried, and consequently its varying behaviour according to which face receives the impact of the current. Moreover, there is the well-known difficulty in this kind of experiment of estimating the precise moment of initial movement.

The object of using flumes of different widths was firstly to determine any scale effect due to width, and secondly to enable the higher velocities as demanded by the larger objects to be obtained with the pump available.

Tests on Single Blocks placed in the Centre of the Flume.

For this series, blocks were cast in cement mortar; two batches were made from different mixes, and after soaking in water the densities were found to be 150·6 and 130·4 lb. per cubic foot respectively. The corresponding values of $\frac{\rho' - \rho}{\rho}$ were therefore 1·41 and 1·09. Blocks of plasticene were also used, for which $\frac{\rho' - \rho}{\rho}$ was found to be 0·733.

As a result of these experiments, it appeared that

$$\bar{v} = \frac{L}{l}\left[10{\cdot}0 - \frac{5300}{\left(32 + \frac{h}{l}\right)^2}\right]\sqrt{\frac{\rho' - \rho}{\rho} \cdot l}, \quad \text{.................(I)}$$

where $\bar{v}$ = mean velocity in feet per second of stream for which the block overturns;
= rate of discharge divided by (the area of cross-section of the channel minus the area of section of the block);
L = length of block measured parallel to the direction of the current (feet);
l = depth of block (feet);
h = depth of water above top of block (feet).
ρ' = density of block, lb. per cu. ft.;
ρ = density of water, lb. per cu. ft.

This formula certainly applies for any width of block (normal to the current) up to twice the depth l, and for $L:l$ up to 2:1.

Taking as an example a concrete cube 3 ft. × 3 ft. × 3 ft. of density 150 lb. per cubic foot, $\frac{\rho'-\rho}{\rho}=1\cdot40$ and $\frac{L}{l}=1$. The following table shows the values of $\bar{v}$ to produce overturning:

TABLE XXXVI

Total depth, D feet.	*Depth over cube, h feet*	*$\bar{v}$ ft. per sec.*
6	3	10·5
9	6	11·1
12	9	11·6
18	15	12·5
33	30	14·3

If the block were a prism 3 feet by 3 feet in section and 6 feet long, with its long side normal to the current, the velocities would be practically the same as those in Table XXXVI. With the 6-foot side parallel to the current, the velocities would be twice as great, and this block would have twice the weight of the original 3-foot cube, that is, its volume would be 54 cubic feet and its weight 3·62 tons. The same weight of material in cubical form would give a cube of side 3·77 feet, for moving which the velocities would be as follows:

TABLE XXXVII

D feet	*h feet*	*$\bar{v}$ ft. per sec.*
6	2·23	11·5
9	5·23	12·1
12	8·23	12·6
18	14·23	13·4
33	29·23	15·3

A further point of interest which emerged from the observations was that the *maximum* velocity found at the instant of movement anywhere between the top of a cube and the water surface was very nearly equal to $\sqrt{\frac{2g(\rho'-\rho)l}{\rho}}$ feet per second, measuring l in feet.

Tests on more than One Cube. These were intended to cover the possibility of constructing, in stages, an embankment or mound of loose cubical blocks. The following arrangements of blocks, stretching across the full width of the flume, were investigated:

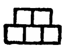
Case (*a*)  a single row;

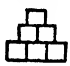
„ (*b*)  two rows;

„ (*c*)  three rows;

„ (*d*)  four rows;

„ (*e*)  one on two;

„ (*f*)  two on three;

„ (*g*)  one on two on three;

„ (*h*)  three on four;

„ (*i*) A pell-mell wall of cubes, dropped at random, to form an embankment of crest-height approximately the same as in case (*g*).

Any small gap left between the end of the wall and the side of the flume was filled with plasticene or other material pressed lightly into position.

In analyzing the results, allowance for the plate 0·094 inch thick was made in case (*a*), but not otherwise, as it was seen that with more than one row it frequently happened that the first disturbance took place by one or more blocks being lifted from the upstream rows right over the remainder of the blocks. In cases (*e*) and (*g*), it was generally observed that the top row first shifted into a slightly arched line, the central part being further downstream than the ends, before one of its component cubes overturned.

The mean velocities $\bar{v}$ of the stream passing over the crest of these obstructions, at which the first failure occurred, is shown in the following table as a proportion of the corresponding $\bar{v}$ for a single cube:

TABLE XXXVIII

$\bar{v}$ relative to $\bar{v}$ for a Single Cube, as given by formula (I), p. 321

$\frac{h}{l}$	Case:									
	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	
									*	†
1	0·58	0·94	1·25	1·28	0·60	0·91	0·41	1·00	0·47	1·06
2	0·70	0·93	1·21	1·30	0·68	0·95	0·52	1·05	0·55	1·10
5	0·83	1·05	1·30	1·35	0·83	1·08	0·77	1·22	0·77	1·23
10	0·87	1·10	1·20	1·25	0·91	1·06	0·91	1·20	0·88	1·25

* Only one or two cubes moved.

† Many cubes moved; mound flattened to about two-thirds of its original height.

Several significant conclusions emerge from these results; for example:

(i) The stability of a single row of cubes in relatively shallow water is far less than that of one of its component parts. This is attributed to the more pronounced lee suction.

(ii) The stability of one tier of successive rows of cubes (cases *a, b, c, d*) approaches a limiting value. Thus, two rows have the same order of stability as a single isolated cube; three rows will withstand about 25 per cent. higher velocities than two rows, but four rows only involve a further 5 per cent. increase.

(iii) Except with a relatively long flat bed of a single thickness of cubes, the effect of increasing the h/l ratio from, say, 1 to 10, is far more pronounced than in the case of an isolated cube: but in all cases the indications are that the effect of water-depth becomes comparatively small for values of h/l exceeding 10.

To provide a direct comparison with Table XXXVI (which refers to an solated 3-foot cube weighing 1·81 tons), the following table has been prepared to show the velocities required for initial disturbance of a mound formed by dumping the same size and weight of blocks in random manner:

TABLE XXXIX

h, depth over crest of mound, feet -	3	6	15	30
$\bar{v}$, ft. per sec. - - - - - -	4·9	6·1	9·6	12·6

Tests on Flexible Bolsters. A possible way of constructing an underwater embankment is by the use of broken stone enclosed in wire cages.

To study the stability of this type of construction, experiments were made with bolsters consisting of thin cotton bags filled with granite chippings. The true specific gravity of these chippings was 2·64; their average weight per piece was 0·00316 lb. and their average dimensions:

maximum length of piece - -	0·80 inch.
maximum breadth of piece - -	0·49 „
maximum thickness of piece - -	0·29 „

The bolsters made in this way had a mean outside diameter of 2·1 inches and a length of 3·54 inches; the *dry* weight of each bolster was 0·571 lb., equivalent to a density of 80·3 lb. per cubic ft. Adopting this figure of ρ', $\dfrac{\rho' - \rho}{\rho}$ becomes 0·288. The *dry* weight of the bolster is sensibly the same as that of a 2-inch cement cube having a *wet* density of 130·4 lb. per cubic foot.

Let us first consider the results obtained with *one* such bolster placed in the centre of the flume with its length *normal* to the current. Taking l to be 2·1 inches, it was found that the mean velocity $\bar{v}$ at the site of the bolster, to cause it to overturn, was always very nearly two-thirds of the $\bar{v}$ withstood by the 2-inch cube of the same dry weight for the same value of h/l.

But when the bolster was lying wet on the bottom of the flume, it was found to have a depth of 1·8 inch. Its mean width would consequently be of the order of $\dfrac{(2 \cdot 1)^2}{1 \cdot 8}$, or 2·45 inches, while its length was still 3·54 inches. The velocities required to overturn it were practically identical with those for a solid rectangular prism 3·54 inches long × 2·45 inches wide × 1·8 inch deep, situated with its longest dimension normal to the stream, the density ρ' of the prism being 80·3 lb. per cu. ft., that is, the same as the *dry* density of the bolster.

Dealing next with the tests made on various arrangements of such bolsters, the same nomenclature will be adopted as in the case of the cubes:

Case (*a*)  single row;

„ (*b*)  two rows;

„ (*c*)  three rows;

„ (*d*)  four rows;

„ (*e*)  one on two;

Case (*f*)  two on three;

„ (*g*)  one on two on three;

„ (*h*)  three on four;

„ (*i*) A pell-mell wall of bolsters, dropped at random to form an embankment of height approximately the same as in case (*g*).

The total depth of the embankment in (*e*), (*f*) and (*h*) was approximately 3·2 inches; in cases (*g*) and (*i*) it was 4·5 inches. In the experiments, the bolsters were placed with their long dimension *normal* to the current, except in the case of the pell-mell mound, when they were dropped from just clear of the water surface and allowed to settle in random fashion.

Table XL below shows the ratios of the mean velocities sustained by these arrangements at the first disturbance to the corresponding $\bar{v}$ with 2-inch cubes of wet density 130·4 lb. per cubic foot:

TABLE XL

$\frac{h}{l}$	*a*	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i* *
1	0·94	0·73	0·82	0·76	0·91	0·78	1·38	0·71	1·03
2	0·87	0·78	0·80	0·73	0·93	0·76	1·17	0·70	1·02
5	0·85	0·81	0·74	0·69	0·96	0·73	0·94	0·71	0·99
10	0·80	0·74	0·73	0·73	0·92	0·67	0·82	0·69	0·98

* Initial movement of one or two pieces only.

It appears that there is a decided tendency for a mound constructed in regular fashion (case *g*) to be more stable than a random formation (case *i*) in small depths, but less stable in greater depths of water. The opposite is the case when cubical blocks are used, and pitot measurements, in fact, revealed that the velocity-distribution through the depth of water is different in the two examples of bolsters and cubes.

Now the weight of a 3-foot concrete cube of density 150 lb. per cubic foot is 4,050 lb. A bolster having a density of 80·3 lb. per cubic foot and a length: diameter ratio of 3·54:2·1 would have dimensions of 5·66 feet long by 3·37 feet mean diameter for a weight of 4,050 lb.

The following table shows the mean velocities at which this cube and bolster of the same weight are expected, from the model experiments, to move:

TABLE XLI

Total depth, D, feet:	6	9	12	18	33
$\bar{v}$, ft. per second:					
Cube - - - -	10·5	11·1	11·6	12·5	14·3
Bolster - - - -	6·2	6·4	6·8	7·4	8·7

When a comparison is made for the case of a random mound, however, the bolsters do not come out so badly:

TABLE XLII

Depth, h, over crest of mound, feet	*$\bar{v}$ ft. per sec.*	
	3-foot cubes	*Bolsters*
3	4·9	4·6
6	6·1	5·6
15	9·6	8·1
30	12·6	10·9

Tests on Less Flexible Bolsters. *A priori*, one would think that the flexibility of the bolsters would have some effect upon their resistance to motion, since it will affect their shape when they are resting on the bed of the channel or on one another. To form some idea of the importance of this effect, bolsters were made of cotton bags containing stone chippings *and sand.* The total weight of 1·25 lb. per bolster included 0·18 lb. of sand having a mean diameter of the order of 0·01 inch; the chippings were similar to those already described in connection with the lighter bolsters. The new bolsters, which were 5·2 inches long, had an average diameter of 2·1 inches and a dry density of 119·8 lb. per cu. ft. They were tested in water of between 10 and 11 inches total depth.

The following Table XLIII shows the velocities required to move these heavier bolsters, relative to those for the lighter bolsters in the same depth of water (see Table XLIII, p. 328).

Now taking $\sqrt{\frac{\rho' - \rho}{\rho}}$ as a measure of $\bar{v}$, we should expect the velocities necessary to move the heavier bolsters to be some 1·79 times those for the lighter—that is, if the cross-section and depth of water had the same values in the two cases. On this basis, therefore, it would appear that the heavier ones were relatively less stable. Again, it has been stated on page 325 that the solitary lighter bolster had the same stability as a rect-

TABLE XLIII

Arrangement	$\bar{v}$ *for heavy bolster relative to* $\bar{v}$ *for light bolster*
Solitary	1·43
a	1·53
b	1·20
c	1·09
d	1·11
e	1·43
f	1·22
g	1·37
h	1·23
i (initial movement)	1·47

angular block 1·8 inch deep and 2·45 inches wide (parallel to the current) and of the same density (80·3 lb. per cubic foot) as the bolster. Suppose now that this block has a density $\rho' = 119{\cdot}8$ lb. per cubic foot (that is, the same as the dry density of the heavier bolster): in 10·5 inches of water, such a block would overturn at $\bar{v} = 3{\cdot}1$ feet per second, whereas the heavier bolster was found to roll over at only 2·4 feet per second. On this basis as well, therefore, the heavier bolster is relatively less stable than the lighter. This is due to the reduction in flexibility, which results in the heavier bolster retaining a much more cylindrical shape, so that in fact it moved at almost exactly the same velocity as a block having $\rho' = 119{\cdot}8$ lb. per cubic foot and a *square* section of 2·1 inches by 2·1 inches.

Furthermore, it is reasonable to suppose that the improved interlocking of the more flexible bolsters would increase their stability if they were used for the construction of an embankment. Accordingly, the investigation was extended to determine the velocities necessary to disturb flat beds and mounds made of *stone chippings and sand alone*, not in bolsters.

Five kinds of chippings were tried; a summary of their physical properties is presented in Table XLIV, p. 329.

A curious feature of the characteristics tabulated is that in all cases the average thickness is almost exactly equal to the length, *l*, of a cube having the same volume as the average piece of aggregate.

All the materials had a high degree of uniformity of size. For example, ten successive groups of five pieces of grade *A* weighed 0·47, 0·62, 0·62, 0·41, 0·72, 0·47, 0·56, 0·59, 0·66 and 0·50 lb. respectively, giving an average of 0·112 lb. per piece as compared with 0·104 lb. for 300 pieces. Two successive samples of grade *E*, each containing 120 pieces, weighed 19·2 and 19·4 grammes respectively.

TABLE XLIV

Material	*A*	*B*	*C*	*D*	*E*
Average weight per piece: lb. - -	0·104	0·0651	0·0216	0·00316	0·000356
Average of:					
Maximum length L (inches) - -	1·93	1·64	1·27	0·80	0·29
Maximum breadth B (inches) - -	1·45	1·25	0·91	0·49	0·22
Maximum thickness (inches) - -	1·05	0·88	0·62	0·29	0·15
Ratios of:					
Length/thickness -	1·84	1·86	2·05	2·76	1·93
Length/breadth -	1·33	1·31	1·40	1·63	1·32
Breadth/thickness -	1·38	1·42	1·47	1·69	1·46
$\frac{L+B}{2l}$ - - - -	1·64	1·64	1·79	2·02	1·59
Length, l, of a cube of the same weight as the average piece (inches) - - -	1·03	0·88	0·61	0·32	0·16
Specific gravity, σ' -	2·64	2·64	2·64	2·64	2·56

In the tests on flat beds, a layer of material was spread across the width of the flume; the uniform thickness of this layer ranged between 1 and 2 inches in the various experiments, while the length of the bed parallel to the axis of flow was between 4 and 12 inches. At its ends, the layer was tapered to the bottom of the flume at an angle of about 30 degrees. Experiment showed that the *mean* velocities $\bar{v}$ for movement with a bed 4 inches long were sensibly the same as with one 12 inches long.

A difficulty experienced by all investigators in this field is to define when motion of the bed material actually begins, or to describe the manner of motion once it has been initiated. In the investigation now under review, the procedure adopted was as follows: The velocity was increased gradually until one or two particles were seen to move; the velocity and depth of water were then measured. In general, it was found that an appreciable increase in this velocity caused no further movement, but presently several particles shifted. Velocity and depth measurements were made at this stage, and the flow was again increased until it became enough to cause general movement—not the simultaneous motion of all the surface particles, but a condition, usually accompanied by bursts of activity at different points, such that it was evident that the whole surface would in time be scoured away.

In analyzing the data, an average was taken of the first and second stages: this average was called the velocity at which the *first few particles move.* An average was also calculated for the second and third stages, and was called the velocity at which *many particles move.*

The results are shown graphically in Figs. 110*a* and 110*b*, where $\bar{v}$ is the mean velocity of the stream flowing over the flat bed of material, h the

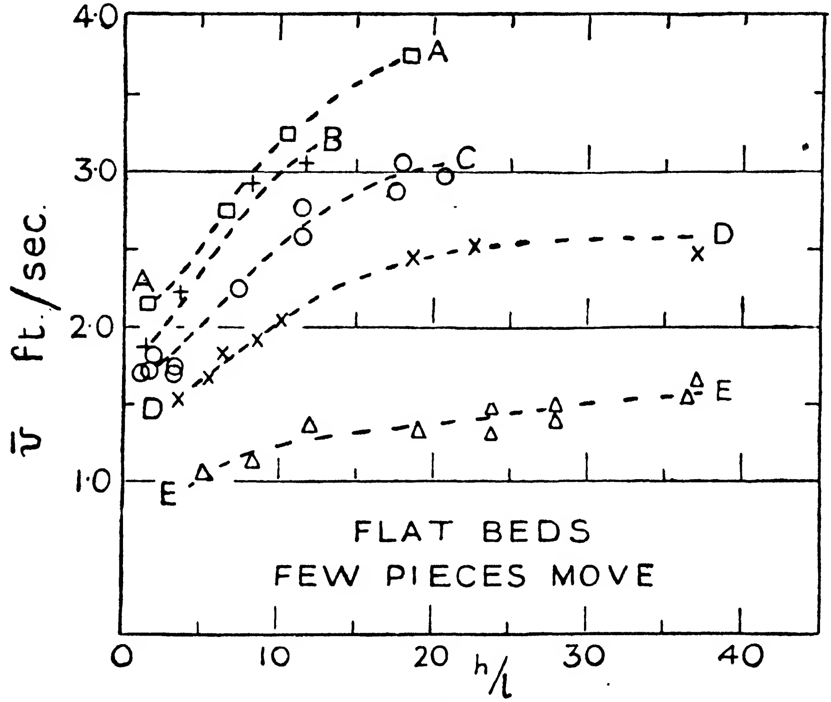


FIG. 110*a*.

depth of this stream above the aggregate and l the equivalent cube length of the material. These diagrams indicate that for a given value of h/l, $\bar{v}$ is nearly proportional to $\sqrt{(\sigma' - \sigma)l}$, or taking the specific gravity of the water at the time to be sensibly unity, nearly proportional to $\sqrt{(\sigma' - 1)l}$, except in the case of material D when the velocity necessary for movement is about 16 per cent. higher than might be inferred from the results with grade A on the basis of $\bar{v} \propto \sqrt{(\sigma' - 1)l}$. The reason for this exception appeared to lie in the fact that the pieces of grade D were definitely dissimilar in shape from those of the rest of the materials (see Table XLIV). An attempt was made, therefore, to introduce an allowance for this factor, together with the influence of the ratio of hydraulic mean depth m

to the simple depth h, and the uniformity modulus M of the material. The ratio of m to h might have a bearing upon the velocity-distribution in the stream, and the uniformity modulus M presumably affects the

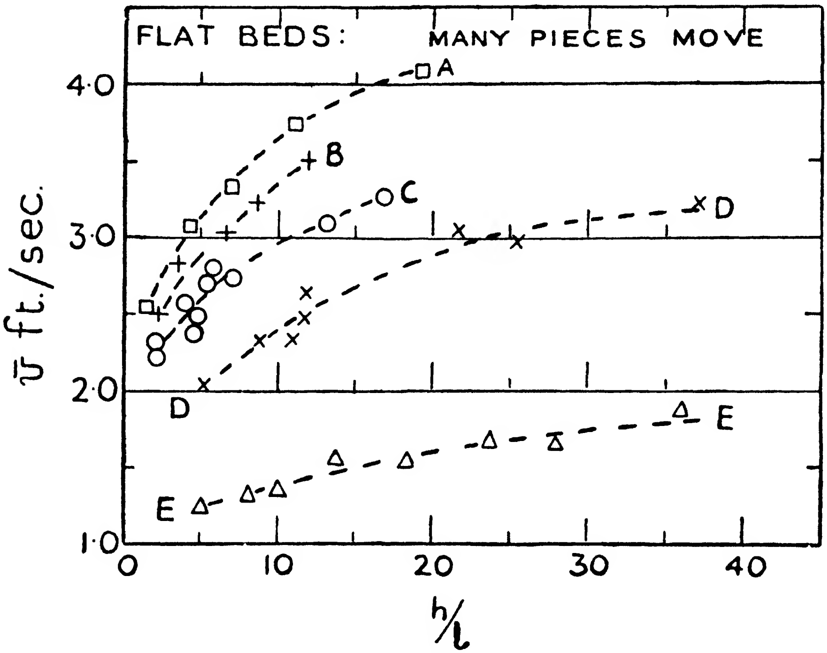


FIG. 110*b*.

interlocking of the grains. M is defined by plotting as ordinate the percentage, by weight, of particles finer than a certain grain-size against that grain-size as abscissa: dividing the area which lies between the vertical axis of ordinates and the resulting curve into two parts, one above and one below the 50 per cent. line, M becomes the ratio of the area below to that above the 50 per cent. dividing line; see Fig. 111. For materials *A–D*, M is of the order of 0·90, whilst for material *E* it is approximately 0·80.

In order to estimate the effect of the several variables, the results of two other investigations were also considered,

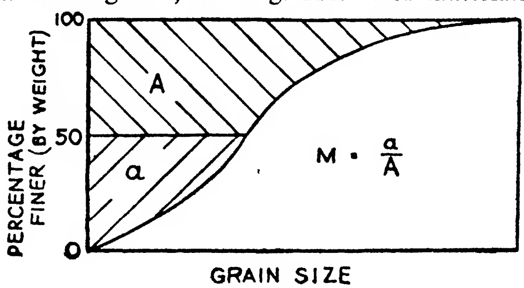


FIG. 111.

namely, " A Study of the Problem of Transportation of Bed Materials in River and Estuary Flow and of the Investigation of River Flow Problems by Means of Models ", described in a Ph.D. thesis to the Victoria University of Manchester by Dr. T. L. Chou in June 1936, and " Studies of River Bed Materials and their Movement, with Special Reference to the Lower Mississippi River " (Mississippi River Commission Print, January 1935). These investigations materially increased the field of conditions available for analysis, so that the final conclusions were based upon materials ranging in size from equivalent cube-lengths of 0·00384 inch to 1·03 inch and specific gravities of 2·016 to 3·89. The channel used by Dr. Chou was 6 inches wide, while the American flume (at the U.S. Waterways Experiment Station, Vicksburg, Mississippi) was 27·8 inches wide. In the tests on aggregates *A–E*, the ratio of $h : m$ varied between 1·03 and 4·29; in the appropriate Vicksburg experiments it attained a maximum value of 1·23 and in Dr. Chou's work it ranged from 1·49 to 4·99.

The formula finally evolved was

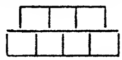
$$\bar{v} = K\left(\frac{L+B}{2l}\right)^{0\cdot44} (\sigma' - \sigma)^{0\cdot22} l^{0\cdot22} M^{-0\cdot22} h^{0\cdot06} m^{0\cdot22}, \quad \ldots\ldots\ldots\ldots\text{(II)}$$

where $\bar{v}$ is in feet per second and l, L, B, h and m are all in inches;

$K = 1\cdot10$ for movement of the first few particles,

$K = 1\cdot59$ for movement of many particles.

The term $\frac{L+B}{2l}$ arises from analogy to the case of a rectangular block of length L and thickness l, for which $\bar{v}$ is proportional to L/l, L being parallel to the current, whereas in the case of a random bed of aggregate, some particles may be arranged with either L or B parallel to the direction of motion.

It is interesting to compare the velocities required to disturb a flat bed of 3-foot cubes ($\sigma' = 2\cdot40$, say) weighing 4,050 lb., with irregular pieces of the same l and weight. The cubes will be assumed to have been laid in orderly, successive rows, three on four thus: . The irregular pieces of stone will be assumed to have $l = 36$ inches, $\frac{L+B}{2l} = 1\cdot50$, $M = 0\cdot90$, $K = 1\cdot10$, and the stream will be supposed to be comparatively broad so that m may be treated as sensibly $= h$. Table XLV shows the comparable figures (see p. 333).

This comparison suggests that if the stream is relatively deep there is little to be gained by the use of regularly-shaped pieces placed systemati-

TABLE XLV

Depth over bed, h feet	$\bar{v}$ for initial movement; ft. per sec.	
	3-*foot cubes*	*Aggregate*
3	10·8	8·8
6	12·2	10·6
15	14·9	13·7
30	16·9	16·6

cally in rows and layers, rather than a chance arrangement of irregularly-shaped pieces having the same weight and volume.

Tests on Mounds made of Broken Stone. This part of the investigation dealt with the materials called A, B, C and D. In the case of aggregate A, the mound was made 4 inches high and 9 inches wide at the base; it stretched across the full width of the channel. The mounds made of the smaller aggregates were of geometrically similar cross-section in proportion to their equivalent cube-length l. It was found impracticable to try material E because of the small dimensions which would be needed to make the mound geometrically similar to the others.

Observations were taken just after the movement of one or of a very few pieces from the crest of the mound: usually the first piece to move was one situated near the centre of the channel and orientated with its longest dimension normal to the current. After the necessary observations had been made at this stage, the flow was increased and, in general, it was seen that particles began to move up the side facing the current. Further observations were taken when the crest of the mound had been so far eroded as to reduce its height to between two-thirds and three-quarters of its initial value.

Analysis of the results showed that the effect of the ratio of the actual depth to the hydraulic mean depth is not so marked as in the case of the flat beds, and this fact is attributed to the more even distribution of velocity across the width of the stream as it flows over the crest of the mound.

The method of plotting the results is shown in Figs. 112*a* and 112*b*; it was evolved by a process of trial-and-error guided by a rational consideration of what quantities could affect the phenomena under consideration and by analogy to the fundamental example of the stability of a cube. Both ordinate and abscissa represent *dimensionless* quantities; or to be more precise, the abscissae are dimensionless as they stand, and the ordinates would also be if multiplied by $\frac{1}{\sqrt{g}}$, which was regarded as a constant.

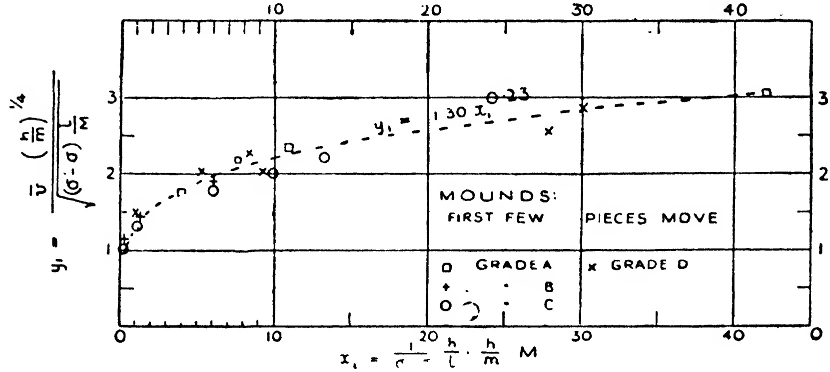


FIG. 112*a*.

The equations of the curves found to fit the plotted points reasonably closely proved to be

$y_1 = 1\cdot30x_1^{0\cdot23}$ for movement of the first few pieces,

$y_2 = 1\cdot37x_2^{0\cdot23}$ for "flattening of the crest" to between 2/3 and 3/4 of its initial height.

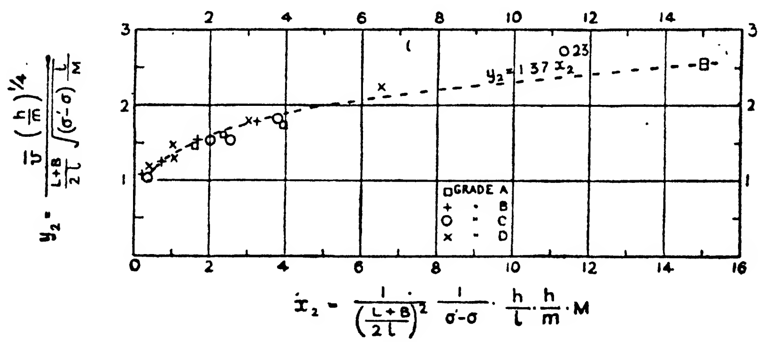


FIG. 112*b*.

These equations imply that:

(*a*) for initial disturbance of the mound,

$$\bar{v} = 1\cdot30\,(\sigma' - \sigma)^{0\cdot27} l^{0\cdot27} M^{-0\cdot27} h^{0\cdot21} m^{0\cdot02}; \quad \ldots\ldots\ldots\ldots\ldots\text{(III)}$$

(*b*) for "flattening the crest of the mound",

$$\bar{v} = 1\cdot37\left(\frac{L+B}{2l}\right)^{0\cdot54} (\sigma' - \sigma)^{0\cdot27} l^{0\cdot27} M^{-0\cdot27} h^{0\cdot21} m^{0\cdot02}. \quad \ldots\ldots\ldots\text{(IV)}$$

The fact that no $\left(\frac{L+B}{2l}\right)$ term appears in the first of these equations is presumably due to the particles being originally more or less loosely or delicately poised on the top instead of being interlocked as in the body of the mound. Even so, the effect of particle-shape is not entirely absent, since the equivalent cube-length l itself depends upon the shape. The mean velocity $\bar{v}$ of the stream flowing over the mound is given by the quoted formulae in feet per second if L, B, l, h and m are specified in inches. It will be seen that if the channel is fairly broad compared with its depth, $\bar{v}$ is proportional to $h^{0 \cdot 23}$.

Let us now compare the stability of mounds made of cubes and broken stone respectively. Suppose the cubes to be 3 ft. × 3 ft. × 3 ft., each weighing 4,050 lb., so that $\sigma' = 2{\cdot}40$. If aggregate of the same weight be used instead, having $l = 36$ inches, $\sigma' = 2{\cdot}40$, $M = 0{\cdot}90$, and if this aggregate is similar in shape to those materials covered in this investigation, then the following Table XLVI provides the desired comparison:

TABLE XLVI

Depth over crest of mound laid in random fashion (feet).	*$\bar{v}$ for initial movement (ft. per sec.)*	
	3-foot cubes	*Aggregate, $l = 3$ feet, $M = 0{\cdot}90$*
3	4·9	8·8
6	6·1	10·3
15	9·6	12·7
30	12·6	14·9

In this case, therefore, as distinct from that of the flat beds represented by Table XLV, there is a very great advantage in using pieces of irregular shape when the stream is comparatively shallow.

It is also interesting to make a comparison with the results previously given for a mound of bolsters, also laid in random fashion. Table XLI was prepared for bolsters 5·66 feet long and 3·37 feet in diameter weighing 4,050 lb. The more flexible (and more stable) bolsters used in the laboratory experiments measured 3·54 inches by 2·1 inches, that is, they represented, in their linear dimensions, a model of the full-size bolsters to a scale of 1 : 19·2. These laboratory bolsters were filled with aggregate D, for which $l = 0{\cdot}32$ inch, so that the size of aggregate for the full-size bolster weighing 4,050 lb. would be $l = 0{\cdot}32 \times 19{\cdot}2$, or 6·15 inches. Again, for aggregate D, $\sigma' = 2{\cdot}64$ and $M = 0{\cdot}90$, while each piece having $l = 6{\cdot}15$ inches would weigh 22·1 lb.

The comparison would therefore appear to be as follows:

TABLE XLVII

Depth over crest of pell-mell mound (feet)	$\bar{v}$, *ft. per sec., for initial movement of mound of:*		
	3-*foot cubes* (4,050 *lb.*)	*Bolsters* (4,050 *lb.*)	*Aggregate* $l = 6{\cdot}15$ *inches*; 22·1 *lb.*
3	4·9	4·6	5·7
6	6·1	5·6	6·7
15	9·6	8·1	8·2
30	12·6	10·9	9·7

The relatively small chippings are thus seen to be remarkably stable compared with the heavier objects and are indeed preferable for use in the shallower streams. Their success in practice, however, would depend upon the possibility of building such a mound during quiescent periods.

This particular investigation has been discussed in some detail here for the reason that, in addition to any intrinsic value which the results themselves may possess, the technique of the experimentation and the treatment of the observations may be found of interest.

It is important to appreciate that formulae II, III and IV apply only to materials within the range of shapes covered by the experiments, but that is likely to embrace such aggregates as would be employed in practice; to take an extreme illustration, the formulae II, III and IV *cannot* be applied to cubical blocks by writing $L = B = l$ and $M = 1$. Calculations concerning cubical blocks must, in fact, be based on the experiments devoted specifically to them.

The principal results of the investigation may now be summarized by stating that:

(*a*) A close estimate of the mean current-velocity required to overturn a rectangular prism may be obtained by using equation I.

(*b*) An *isolated* bolster is appreciably less stable than an *isolated* cubical block of the same weight, but the difference in the case of a random mound is far less marked.

(*c*) Flexibility is an important factor in the resistance of the bolsters.

(*d*) Higher velocities are resisted by embankments built of irregularly-shaped pieces of stone than by cubical pieces of the same weight.

(*e*) In comparatively shallow streams, a mound made of stone chippings is more stable, once it has been constructed, than if the chippings were enclosed in cages to form bolsters of very considerable weight.

In the investigation considered in this chapter, no account is taken of wave-action. For descriptions of the experimental technique when waves are involved, see Chapters X and XII.

CHAPTER XII

WAVES

1. Model Studies of Beach Formation under Wave Action.

THE U.S. Waterways Experiment Station in 1939 investigated the possibility of using a small-scale reproduction of wave action to determine the stable slopes of beaches or dam faces.* Their tests were conducted in a basin 60 feet long × 20 feet wide × 4 feet deep, and waves were generated by a horizontal 6-inch cylinder mounted eccentrically and driven from a 2 H.P. variable speed motor by a chain. The desired characteristics of wave were obtained by adjusting the eccentricity and speed of rotation of the cylinder.

At the start of each experiment, the beach was moulded at a 1 : 5 slope from the bottom of the tank to well above the upper limit of wave action. After being subjected to wave action for a period of three hours, the beach was found to have attained approximate stability, and four longitudinal profiles were then measured at chosen places across the 20-ft. width of flume.

Using the same " fine sand ", three experiments were made; in each of these a ratio of length to depth of wave of 20 : 1 was closely adhered to, as this ratio appeared by trial operation to be " best adapted to flume conditions ". In the first experiment, the depth of water was 6 inches, in the second 12 inches, and in the third 18 inches, while the heights of wave employed were respectively 0·67, 1·38 and 2·04 inches. The wave-heights were, therefore, nearly proportional to the depths of water, as also were the distances—12, 24 and 36 ft.—from the toe of the moulded slope to the rotor. At the same time, the revolutions per minute of the wave-generator were arranged to be inversely proportional to the square root of the depth of water; they were actually 125, 88 and 72 r.p.m., so that the wave periods themselves (0·480, 0·680 and 0·833 second respectively) were directly proportional to the square roots of the linear scales of the models.

Despite the use of one and the same sand in all three models, the mean bed slopes attained, namely 1 : 7·87, 1 : 8·89 and 1 : 7·94, were in fairly close agreement, and this evidence was taken as indicative that " a small-scale study of wave action on various materials could be used in studying full-scale wave action on *identical* materials ". Tests on powdered coal (specific gravity 1·35) were, however, considered less reliable, this material

* *U.S. Waterways Expt. Station Bulletin (Hydraulics)*, vol. 3, No. 1, Feb. 1, 1940, p. 17.

being thrown into semi-suspension by the waves and thus failing to present "a well-compacted beach upon which a wave could advance and break".

A good deal of very careful and interesting work on beach-formation has also been carried out by Major (now Brigadier) R. A. Bagnold. Unfortunately, this study was curtailed by the outbreak of the war in 1939, but the results obtained up to that time have been reported to the Institution of Civil Engineers.* Brigadier Bagnold's experiments, made in Dr. C. M. White's Laboratory at the Imperial College of Science and Technology, London, were conducted in a flume only 21 inches wide, but closer attention was paid to the detailed shape of the beach-profiles. The waves were generated by a paddle instead of an eccentric roller; the wave-generator is, in fact, described in an earlier Paper by Brig. Bagnold, namely "Interim Report on Wave-Pressure Research", *Journn. Inst. C.E.*, June 1939, p. 202. The mobile materials adopted were rounded beach pebbles ranging in diameter from 0·5 to 0·9 cm., with a mean diameter of 0·7 cm., and sands ranging from 0·05 to 0·3 cm.

Bagnold points out that the velocity c of advance of a surface-water wave is connected with the wave-length λ, and the depth H_0 of the water by the equation

$$c^2 = \frac{g\lambda}{2\pi} \tanh \frac{2\pi H_0}{\lambda}.$$

Now as $\frac{2\pi H_0}{\lambda}$ increases, the value of $\tanh \frac{2\pi H_0}{\lambda}$ approaches nearer and nearer to unity; indeed, if $H_0 = \frac{\lambda}{2}$, so that $\frac{2\pi H_0}{\lambda} = \pi$, the hyperbolic tangent = unity with an error of less than one per cent. It follows that should H_0 exceed, say, one-half of the wave-length,

$$c^2 = \frac{g\lambda}{2\pi} \text{ very nearly,}$$

or the wave-velocity is independent of the depth, as may be established directly by a consideration of "deep-water" waves.†

On the other hand, as H_0 diminishes relative to λ, c approaches the value $\sqrt{gH_0}$; that is, for a "long wave", the wave-velocity is independent of the wave-length. Thus, if $H_0 = 0{\cdot}1\lambda$, $c = 0{\cdot}94\sqrt{gH_0}$, and if $H_0 = 0{\cdot}05\lambda$, $c = 0{\cdot}99\sqrt{gH_0}$.

Reverting to the consideration of waves approaching a beach, Bagnold states that "the actual wave-length decreases as the wave continues to advance, because the foremost waves are travelling more slowly in the

* "Beach Formation by Waves; some Model-Experiments in a Wave Tank", Bagnold, *Journ. Inst. C.E.*, Nov. 1940, p. 27.

† "*A Treatise on Hydromechanics*, Part II (Hydrodynamics)", Ramsey, p. 270 (Bell, 1942); *Hydraulics and its Applications*, Gibson, p. 409 (Constable, 1930).

shallower water and tend to be overtaken by those behind, which are still in deeper water. When the wave-length is about 7 times the amplitude * the wave becomes unstable and breaks. If, however, the sea bottom is bordered by a comparatively steep shingle beach, the wave's velocity cannot slow down sufficiently fast to keep pace with the rapidly decreasing depth; the equation, $c=\sqrt{gH_0}$, no longer holds good, and the fall in wave-velocity between the beginning of the beach and its extreme crest may be negligible. The wave suffers one of two possible fates. (1) If its wave-length/amplitude ratio was previously approaching 7 the wave is broken by the sudden distortion caused by the slope of the beach.... (2) If the wave-length/amplitude ratio was previously low the wave passes through the stage of maximum instability, when its forward face is raised to the steepest angle, and does not break.... The main difference between the two cases is that in (1) the foremost water of the surge up the beach has come from the overturned crest, whereas in (2) it has come from the foot of the wave face. These two cases are but extremes of a complete range of possibilities corresponding to a total break, various degrees of partial break, and no break at all.

"The experiments showed that on a shingle beach it makes no appreciable difference to the ultimate profile whether the wave breaks or not."

Fig. 113 shows the characteristic beach profile as described by Bagnold. This profile contains an "upper beach" *BC* and a "lower beach" *CD*

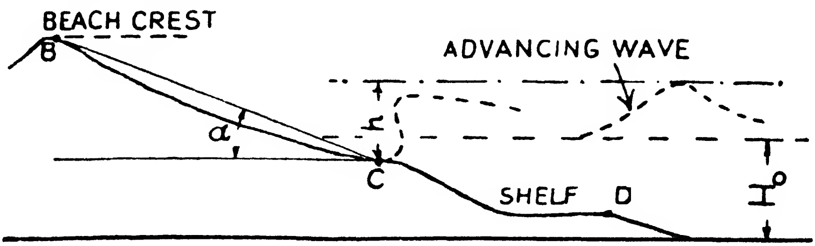


FIG. 113.

(Based on Fig. 2c, p. 30, *Journ. Inst. C.E.*, Nov. 1940.) (R. A. Bagnold.)

separated by the convex "step" *C*. The bottom part of the "lower beach" he calls the "shelf"; α the "beach angle". He considers that the profiles of the upper and lower beaches depend on different sets of factors, and therefore obey different model-rules. The character of the

* Bagnold defines the wave-amplitude as the difference of level between the crest and the trough of the wave "out at sea", but having regard to the limited length of his experimental tank and to the difficulty of defining the "mean level", he modifies the definition of "wave amplitude", for the purpose of his Paper concerning model experiments on beach formation, to "the difference between the crest level 'out at sea' and the level of the lowest exposed beach-line immediately in front of the advancing wave"

lower beach " seems to depend only on the value of the ratio R of wave amplitude to particle size, that of the upper beach . . . on the absolute value of each of these quantities independently, the height of the upper beach being proportional to the wave amplitude h, and its angle α depending on the porosity of the material, which is itself a function of the grain diameter ".

It would appear that the restricted width of Bagnold's experimental tank led to complications. Thus, he remarks that observations of natural beaches formed on the seashore from large and highly porous shingle show that if the waves advance perpendicular to the shore-line they " retain this same direction throughout the rise and fall of the surge ". To this he attributes his observed fact that " experimental shingle beaches thrown up within the narrow confines of the wave tank appear . . . to imitate very closely the profiles of real shingle beaches in Nature ". Indeed, in discussing the selective grading of pieces of different sizes over a pebble beach, he affirms that " on the whole, it may be said that the model-beach of shingle imitates, in a remarkable way, all the features of its full-scale counterpart ".

He found, however, that the correspondence between artificial and natural beaches of *sand* was not so close, there being a marked tendency towards the production of a steeper beach-angle than is commonly experienced in Nature. This feature he discusses in terms of the lateral water-movement over sand beaches, and instances a case of sand beach formation near Mersa Matruh on the Egyptian coast west of Alexandria, where the upper beach takes the shape of a regular succession of bays at the promontories of which the beach angle approaches 14 degrees, although the angle found in the middle of the bays may be as low as 3 degrees. " The two side streams from the promontories on either side meet in the centre and together form a return surge down the hollow of considerably greater intensity than the previous upward surge there. It should be noted also that owing to the introduction of a sideways flow there is no longer any dead period (except on the front of each promontory) when the water and the sand grains it carries come momentarily to rest. The grains are kept in continual movement and therefore do not get a chance to settle on the bed.

" There can be little doubt that both of these effects, the incipient beginnings of which were previously noted in the wave-tank experiments . . . are responsible for the fact that the angle of a natural sand beach is less than the experimental 14-degree friction angle, α_0."

2. Model Studies of Waves in Harbours.

It is frequently necessary to choose a location for breakwaters or other works with the object of protecting a harbour against waves. As a guide in solving this difficult problem, a model of the fixed bed type is often of very considerable value. Such a model may be moulded to the appropriate contours in concrete or cement mortar, with features such as jetties, wharves, breakwaters and groynes constructed in waterproofed wood, concrete or metal.

In such problems, waves of various kinds may be involved, for example (i) deep-water waves of oscillation, (ii) shallow-water waves of oscillation, (iii) long waves whose wave-length is great compared with the depth of water into which they travel. Now the velocity, c, of the wave is given by the expression

$$c^2 = \frac{g\lambda}{2\pi} \text{ .... for type (i),}$$

λ being the wave-length.

In this case, therefore, c^2 is proportional to a horizontal length dimension. The assumption made here is that the particles of water near the surface describe circular orbits in a vertical plane.* In shallower water, however, the orbit becomes an ellipse of horizontal axis a and vertical axis b, and the wave-velocity c is then defined by

$$c^2 = \frac{g\lambda b}{2\pi a} \text{ .... (type ii),}$$

which depends upon both horizontal and vertical dimensions.

As the water gets still shallower, however,

$$c^2 \text{ approximately} = gH_0 \text{ .... (type iii),}$$

where H_0 is the depth.

For this type of wave, then, the velocity depends upon a vertical dimension only.

Two extremes are, therefore, possible; namely, that in which the velocity is influenced only by horizontal dimensions, and that in which vertical dimensions are all-important, while between the two extremes comes the case where both vertical and horizontal distances affect the wave-velocity.

In order to cater for all these possibilities in one and the same model, it is necessary to make the horizontal scale the same as the vertical scale, but it will be seen later that in the particular case of a tidal bore extremely good results may be obtained in a distorted model.

Other influences at work are those of viscosity, bed-roughness and surface-tension, but in all save the smallest models these are of very definitely secondary importance. In short waves, the curvature of the

* *Hydraulics and its Applications*, Gibson, 4th ed., p. 407 (Constable, 1930).

surface may become relatively great, and surface-tension may play an important part by increasing the pressure below the surface of the wave-crest and decreasing it below the trough, as may be visualized through the mechanical analogy of an elastic skin. But this effect is only likely to become serious if the wave-length is less than one or two inches. " No wave or ripple can be produced by a moving object whose velocity is less than this critical value of 9·2 inches per second, and a breeze of velocity less than this has no power to ruffle the surface of a still pool " (Gibson).

Before embarking upon model tests of this nature, it is necessary to decide upon the height, period and direction of approach of the waves likely to be encountered in the area surrounding the works under consideration. The most satisfactory procedure, of course, consists in observing these phenomena in Nature over a long period of time; otherwise the waves selected for reproduction in the model must be decided upon by the engineer's judgment and experience, making sure that a comprehensive range of waves—both large and small—is covered in the experiments. Judgment is also required to ensure that a sufficiently large area is included within the boundary of the model, outside the harbour or other works under test, for the maintenance of the proper form of waves entering the works.

The U.S. Waterways Experiment Station at Vicksburg, Mississippi, has undertaken several wave-action studies of a fascinating character.* The scales adopted were either 1 : 50 or 1 : 100, and the wave machine consisted of a triangular-shaped plunger to which a vertical motion was given, through gearing, by a variable-speed motor. By adjusting the stroke of the plunger, the required size of wave could be generated; the period of the waves could be arranged by controlling the speed of the plunger. The machine was mounted on wheels so that it could be moved to produce waves from different directions.

With a scale of $1 : x$, the velocities in the model are $1/\sqrt{x}$ of those in the prototype, while the time-scale also is $1 : \sqrt{x}$. Taking energy as proportional to (mass times the square of velocity), the wave-energy in the model is $\rho_2/\rho_1 x^4$ of that in Nature, ρ_1 being the density of the natural water and ρ_2 that of the water used in the model.

Fig. 114 illustrates the general layout of a model of the harbour of Port Washington, on the western shore of Lake Michigan.† Existing works were found inadequate to prevent damage to wharves and ships during

* *U.S. Waterways Experiment Station, Hydraulics Bulletin*, vol. 4, No. 1, May 15, 1941.

† *U.S. Waterways Experiment Station, Hydraulics Bulletin*, vol. 4, No. 1, May 15, 1941; p. 8.

even moderate onshore storms. "A new breakwater, the north breakwater, had been constructed but was not wholly successful, as the entrance width of 725 feet remaining between the southerly end of the breakwater and the outer end of the privately constructed south breakwater admitted excessive wave action in the harbour during easterly and southeasterly storms. During the winter of 1934–35 several severe storms inflicted damage estimated at $80,000 to docks and small craft."

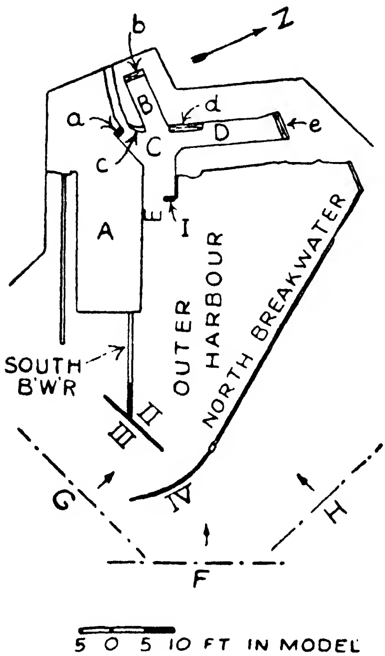


FIG. 114.

1 : 50 Model of Port Washington.

(Based on U.S. Waterways Expt. Station Bulletin No. 10, Fig. 2.)

Scheme I. 85 ft. Stub Pier.
" II. S. Breakwater Extension.
" III. Detached Breakwater.
" IV. N. Breakwater Extension.
" V. Rock-filled Cribs *a, b, c, d,* and Riprap *e.*

A. Coal Wharf.
B. West Slip.
C. Basin.
D. North Slip.
E. Inner Channel.
F. Wave-machine Direction No. 1.
G. " " " " 2.
H. " " " " 3.

The model scale was 1 : 50; three wave-machine pits were provided to enable waves from three directions to be created by the portable machine having a plunger 50 feet long. To cover the range of storm waves in the lake, waves of height equivalent to 7, 10, 15 and 20 feet were tried in each direction.

The model experiments indicated that waves piled up against the face of the Milwaukee Electric Railway and Light Company coal wharf, thus giving increased height to the waves entering the inner basin. There was also reflection of these waves from the vertical bulkhead sides of the slips, the resulting waves being often as high as those out in the lake.

Experiment then revealed that breakwater extensions, including those originally suggested and others developed during the investigation, would not materially reduce the height of waves in the inner basin due to storm disturbances moving approximately along the axis of the channel. Plate XXIX shows the conditions in the region of the curved north breakwater extension.

The problem was then attacked from the standpoint of reducing the wave reflection " by absorbing the energy of the primary waves in the inner basin ". With this end in view, two types of work were tried: riprap absorbers (Plate XXX) at a 1 : 2 slope, and rock-filled cribs (Plate XXXI). Riprap arranged in three places was found to reduce the wave heights in the inner basin to about one-half of those experienced with no improvements or with the extended breakwaters. The crib absorbers reduced the wave heights by three feet. Finally, it was recommended that a combination of sloped riprap and rock-filled cribs be constructed. The Vicksburg Report goes on to state that:

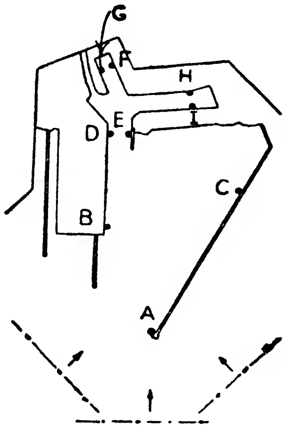
" Because of insistent demands from local fishing interests for protection from easterly to southerly storms, the south breakwater extension was built. This produces benefits from southerly storms, but a series of wave-height observations at various points in the harbour, before and after construction of the extension, confirms the results of the model tests, that the structure would be of little benefit for storms entering the harbour about along its axis. Although the south breakwater extension has aided in reducing the wave heights in the harbour to some extent, there is still too much disturbance for safe mooring of small boats in the slips or outer basin or for the unloading of large coal boats during major storms running directly into the harbour. Wave absorbers have not been constructed to date. A direct comparison of model and field data shows a marked similarity of wave heights. All this sums up to prove that the model has correctly evaluated the effect to be obtained from the extension of the south breakwater."

Fig. 115 demonstrates the comparison between the observed wave heights in the model and the prototype. When the difficulties of measurement and of ensuring precisely similar initial conditions are kept in mind, the agreement appears to be remarkably close.

Plates XXXII and XXXIII, reproduced from the *U.S. Waterways Experiment Station Hydraulics Bulletin*, vol. 4, No. 1; May 15, 1941; pp. 16–17, illustrate wave conditions as seen in a 1 : 100 scale model of San Juan Harbour on the north coast of Puerto Rico. These plates demonstrate quite clearly that the 24-foot wave breaks much farther seaward than the 19-ft. wave; correspondingly, the 19-ft. ocean waves lead to more severe conditions in the harbour than the 24-ft. waves, which lose energy at an earlier stage in their approach.

To quote the *U.S. Waterways Experimental Station* again:

" In the simpler cases of, say, a dish-shaped harbour with a uniform bottom, a reasonably accurate estimate of wave action within the harbour might be possible. However, in the majority of cases, the most difficult ones, reduction of energy entering the harbour by use of contraction breakwaters at the entrance will not provide the desired protection. In the three cases discussed in this bulletin this is true. Hence, protection within the harbour is necessary. The complexity of the problem is



STAFF GAUGE	WAVE HEIGHT PROTOTYPE	MODEL
A	10·0 FT.	9·0 FT.
B	2·0	3·0
C	6·0	5·0
D	4·5	4·0
E	4·5	3·0
F	4·0	4·0
G	4·0	4·0
H	3·5	3·0
I	3·0	2·0

FIG. 115. Prototype and Model Wave Heights. (Port Washington.)
(Based on U.S. Waterways Expt. Station Hydraulics Bulletin No. 10, Fig. 8.)

illustrated by the fact that local protection at one location in a harbour may be secured by a breakwater which will aggravate conditions in other parts of the harbour. . . . Such results are not predictable by any known method of rational analysis. The obvious conclusion is that hydraulic model treatment is the only feasible design procedure."

3. Wave-Impact Pressures on Engineering Structures.

It is unfortunately not possible at the moment to present any very definite information, quantitatively, concerning the pressures exerted on sea-walls and similar structures or in models of them. Especially is it impracticable to state any precise rules of connection between model and prototype measurements. The reasons for this state of affairs may be summarized as follows:

(i) So much depends upon the character of the wave itself; slight differences may give very different effects, so that there may be only a comparatively slender chance of high shock pressures being observed.

(ii) Until fairly recently, the experimental instrument for recording such pressures had not been developed, the difficulty being the short duration of the maximum shock. The piezo-electric unit has, however, opened up new possibilities.

(iii) The phenomenon is affected by the amount of air present in the breaking wave; this again depends upon the chemical nature of the water, and is likely to change appreciably from one place to another.

Professor A. H. Gibson in 1912 (*Min. Proc. I.C.E.*, vol. CLXXXVII, p. 274) carried the ideas then held appreciably further by suggesting that " while, on the assumption of simple hydrostatic transmission of pressure, the effective internal pressure due to wave-impact cannot exceed that exerted by wave-impact on the sea-face of a breakwater, the pressures produced, if the energy of the wave is devoted to compression of air in the open joints, may amount to approximately twice this magnitude. If, however, conditions are favourable to the production of water-hammer, considerably greater internal pressures, up to some fifteen times the face-pressure with very high velocities of impact, are to be regarded as possible."

Coming to much later investigations, it is known that the U.S. Waterways Experiment Station in 1941 were in process of conducting experiments in a tank 5 ft. high, 117 ft. long and 18 ft. wide, with a view to measuring the pressure distribution of waves striking model breakwaters.* The tank was equipped with a plate-glass window near the breakwater and waves of height up to 12 inches were produced, while pressures were indicated by electric dynamometers. Unfortunately, the writer is not aware of the results derived from this promising investigation.

In this country, the most comprehensive and authoritative study yet made is that reviewed in the Interim Report on Wave-Pressure Research by Brigadier R. A. Bagnold, F.R.S. (*Journ. Inst. C.E.*, June 1939, p. 202). This work was started, as a result of the International Navigation Congress at Brussels (1935), by the Research Committee of the Institution of Civil Engineers and was carried out under the supervision of Assistant Professor C. M. White at the Imperial College of Science and Technology, London. Model experiments were made in a flume or wave-tank 21 inches wide, and close contact was maintained with the French Department of Ponts et Chaussées, under whose direction full-scale measurements were being carried out.

Bagnold points out that:

" In deep water the advancing wave does not break, but contact with the wall deflects the water upwards as a *clapoti*. The pressures are therefore small, but they last for periods of many seconds while the water is rising and falling. The mathematical theory of the *clapoti* has been worked out by Sainflon (" Essai sur les dignes maritimes verticales ", *Ann. Ponts et Chaussées*, vol. 98 (II), 1928, p. 5), and the results have been satisfactorily

* *Loc. cit.*, p. 21.

verified, both by French investigators at Dieppe and by Italian investigators at Genoa. In localities where the water-level is constant—for example, in the Mediterranean—a sufficient depth of water is maintained at the wall to ensure that no impact other than that of the *clapoti* can ever occur. The maximum pressures to be dealt with are therefore known as soon as reliable data have been collected regarding the maximum waves likely to be encountered." (This presumably refers to the pressures on the wall face, but Gibson's analysis already quoted shows that quite different effects may be communicated through joints to the interior on account of air compression, or even direct impact of water on water in a confined space.)

" Where the tide varies, however, the depth of water at the wall may be such that at low tide an oncoming wave may strike the wall at the moment of breaking, so that the advancing water face is nearly parallel with that of the wall. In this case, although the ultimate fate of the wave is the same as before—namely, it forms an upward jet which gives rise to a long-period pressure of low intensity—there also occurs, superimposed on the other, a shock pressure of great intensity but of very short duration.

" Such shock pressures have been recorded at Dieppe up to a maximum value of 100 lb. per sq. inch. They occurred fitfully, however, and not more than 2 per cent. of the waves which struck the measuring apparatus gave any pressure at all. Not only has no useful correlation been obtained between the pressure maxima and the characteristics of the waves producing them, but no physical explanation has been forthcoming regarding how these pressures can arise. The problem is made more difficult by the entire absence of any theory or mathematical background for the mechanism of the breaking wave.

" It was considered that the most fruitful method of studying such a problem was likely to be the use of a model wave-tank in which the characteristics of the wave causing the pressures could be maintained under close control."

The pressure element used by Bagnold is shown in Fig. 116. It consists essentially of a stainless steel capsule mounted in a brass plug. The capsule contains a pair of quartz crystals, and the electric charge generated by the mechanical compression of these crystals is carried " through a screened cable of low capacity and high insulation-resistance " to a two-valve amplifier, whence the impulses pass through " a second direct-current paraphase amplifier " to a 12-cm. cathode-ray tube oscillograph provided with a convenient time-base. The instrument is calibrated by connecting a gauge and pump to a pneumatic cap at the front of the pressure-unit. A spot deflection of 1 cm. was arranged to represent 10 lb. per sq. in. of pressure.

In the experiments, a single wave was made to travel from the paddle to the wall containing the pressure measuring device; the wave was broken as the result of the bed of the channel being set to a slope upwards

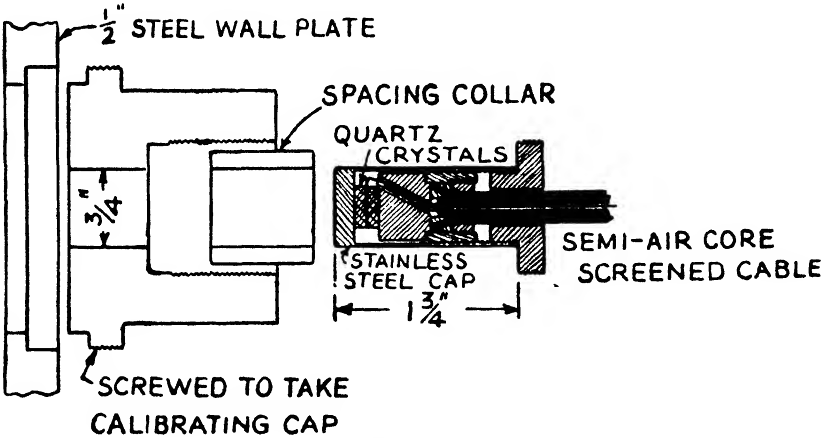


FIG. 116.

(Based on Fig. 5, p. 205, *Journ. Inst. C.E.*, June 1939.) (R. A. Bagnold.)

towards the wall. Being reflected from the wall, the wave ran back to the paddle which had been stationary meanwhile, but which was so arranged as to make another stroke at the appropriate time and to drive the same wave forward again. The total amplitude of the wave employed was 10 inches, the velocity of its summit 6·8 ft. per second, and the velocity of the vertical wave-front just before impact with the wall was 8 ft. per sec. Fig. 117 illustrates the general state of affairs.

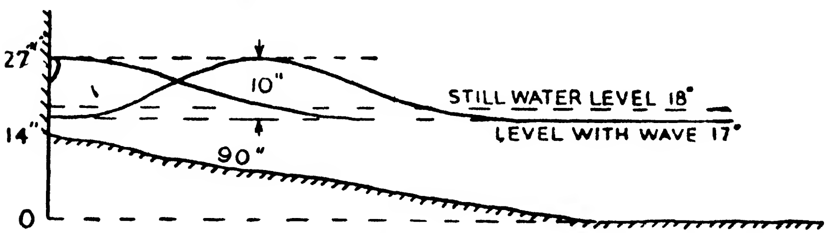


FIG. 117.

(Based on Fig. 13, p. 210, *Journ. Inst. C.E.*, June 1939.) (R. A. Bagnold.)

In order to emphasize the fundamental difficulties encountered in work of this kind, it should be stated that although a breaking wave of constant amplitude could be repeated indefinitely, yet a mains-voltage fluctuation of about 3 per cent. was sufficient, when the wave machine was driven by a direct-current motor, to make it impracticable to control the position and timing of the break in such a way as to ensure a succession of impacts all giving measurable shock pressures. As in the French full-scale observations, " the shock pressures occurred fitfully. In 90 per cent. of the impacts there were no shock pressures at all; the majority of the shock pressures when they did occur did not exceed 10 lb. per square inch, but very occasionally maxima as great as 35 lb. per sq. in. were observed ".

After installing a synchronous motor, however, and spending much time in adjustments to the wave-producing mechanism, it was found that shock pressures could be registered at every impact, yet a change of only 0·3 per cent. in the paddle-timing could effect the complete disappearance of the pressures. Moreover, even with the " ideal " adjustments and nominally constant conditions, the variation in the values of maximum pressures observed during a sequence of wave impacts was as great as in the earlier work. It appeared that increasing the probability of shock pressures being created had led only " to increasing the probability that still higher pressures would occasionally be observed. The highest pressure recorded was 80 lb. per sq. inch. . . ."

In order to appreciate the reason for this essential instability or the erratic effects resulting from apparently constant conditions, it is desirable to follow Bagnold's argument concerning the characteristics of wave which lead to shock. Fig. 118 shows a " delayed break " of wave in which the water-line marked *E* has started to rise from its lowest position previous to the wave-tip making contact with the wall at *A*. The water-line *E* is rising very quickly, and should the break happen only a fraction of a second too late, *E* will have reached the elevation of *A before* contact is made there by the wave-crest. In that event, no air is enclosed and neither noise nor shock takes place; instead there is a steady run of the water-line *E* up the face of the wall.

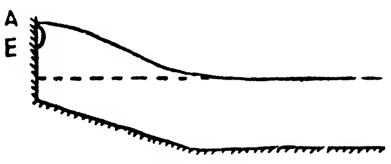


FIG. 118.
(Based on Fig. 12, p. 210, *Journ. Inst. C.E.*, June 1939.) (R. A. Bagnold.)

Shock pressures happen only when an air cushion is enclosed between the wall and the approaching wave-front; the magnitude of the pressure depends essentially upon the size and shape of the cushion. Accordingly, vastly different maximum pressures may be expected from waves which *appear* superficially to be identical, since comparatively small irregularities in the profile of the wave-front may have an important effect. Indeed, the wave-front will contain irregularities sufficient, when the air-cushion is thin enough to favour high shock pressures, to divide the air pocket into a number of cells: only if these cells are identical and suffer identically the same rates of compression wave after wave will successive observations be perfectly consistent.

The argument is, then, that " the shock pressure exerted by a breaking wave is due to the violent simultaneous retardation of a certain limited mass of water which is brought to rest by the action of a thin cushion of air, which in the process becomes compressed by the advancing wave-front ". Further, Bagnold suggests that " the volume of water concerned is approximately equal to that of a horizontal column whose cross-section parallel to the wall is that of the frontal aspect of the air cushion, and whose length K is half the vertical width of the cushion. . . . The maximum pressure is given approximately by

$$(p_{\max} - p_0) = 2{\cdot}7\rho U^2 \frac{K}{D}."$$

In this formula, U is the initial velocity of approach of the wave and D is the mean thickness of the air cushion; ρ is the density of the water. The formula is derived by considering the water as a free piston compressing the air adiabatically.

Since, according to the work under consideration, K is half the vertical width of the air cushion, the formula implies that the greatest shock pressure will occur when compression takes place simultaneously over a large area and when the cushion-thickness D is small. For this condition to obtain, whatever the original form of the advancing wave, its front must become approximately plane and parallel to the wall at its near approach. Bagnold believes the maximum value of K to be of the order of 0·2 times the wave-amplitude $2h$; on that supposition the maximum pressure-rise becomes

$$(p_{\max} - p_0) = 0{\cdot}54\rho U^2 \frac{2h}{D},$$

from which he reasons that "It seems probable that in the very disturbed water inevitably found near sea walls during storms, enough air is entrained or held on the surface as foam to provide in itself a lower limit to the possible values of D, and that a limit is thereby set to the maximum shock pressure which can be exerted. This, however, could only be established by full-scale measurements ". Possibly in this case the crux of the whole matter lies in the last sentence. Thus, reverting to the equation

$$(p_{\max} - p_0) = 2{\cdot}7\rho U^2 \frac{K}{D},$$

and supposing K to be really proportional to the wave-amplitude, and U to be approximately proportional to the square root of the water-depth, one would expect the pressures measured in a model of scale $1:x$ to be

$1/x$ of the prototype pressures, provided the wave-shapes are similar and that D adjusts itself according to the linear scale. Now the highest peak pressure detected in the model was 80 lb. per sq. inch. Taking $x = 12$, this would be equivalent to 960 lb. per sq. inch in the prototype, the wave-amplitude then being 10 feet as compared with 10 inches in the model, and the velocity of the wave-front just before impact being $8\sqrt{12}$ or 27·7 ft. per sec. The highest peak pressure recorded at Dieppe was, however, only 100 lb. per sq. inch. Clearly some of this discrepancy may be attributed to the relatively small number of waves of the " given " shape and timing observed in the difficult natural conditions, but Bagnold attributes the main cause to the " very different physical properties of the surfaces of sea water and fresh water ", in particular the enhanced formation and maintenance of air-water emulsions in sea water as compared with fresh water, and the froth and foam associated with the violently agitated water near a sea wall in heavy weather.

On the other hand, he finds that the area of pressure-time diagrams tends to approach a definite value, since a pressure which rises to a higher peak persists for a shorter time. The impulse per unit area of wall is given by

$$I = \int p\,dt = 2\rho\, UK,$$

and taking K to have a maximum possible value of $0{\cdot}2 \times$ the wave-amplitude $2h$, this becomes

$$I = 0{\cdot}8\rho\, Uh.$$

He affirms that " when maximum impulses are compared, full-scale measurements at Dieppe are consistent with those made on the model ".

It will be appreciated from what has been written on this subject that the study of wave-impact pressures presents peculiar difficulties, both mathematically and experimentally; the "statistical" nature of the problem is one feature, and the reproduction of natural conditions in the model is far from easy. Great strides have been made, however, in the development of methods of measuring and recording shock pressures of shor duration, and it will be agreed that the model investigations so far made have considerably clarified and extended our knowledge of the mechanism of the phenomena involved. It is greatly to be hoped that the programme for further research advocated at the end of the " Interim Report on Wave-Pressure Research " (*Journ. Inst. C.E.*, June 1939) may be carried out in the near future.

Even if the problem of reproducing quantitatively all the factors which ought to be included in such models should prove to be insoluble, there is no doubt that further light will be thrown on the nature of the forces brought into play; this itself will guide the observer in the field of full-scale research by indicating the effects to which attention should specially be devoted. For further information about wave action on vertical breakwaters, the reader may wish to consult papers by J. Larras (*Sc. et Ind.* (*Travaux*), vol. 19, p. 357), G. Ferro (*Ann. Lav. Pubb.*, vol. 74, p. 764 and p. 935), and A. Stuky and D. Bonnard (*Sci. et Ind.* (*Travaux*), vol. 21, p. 13).

A description of model experiments relating to the design of two breakwaters for the Harbour of Leith (Scotland) has already been given in Chapter X.

4. The Effect of Waves on the Discharge over Weirs.

This has been investigated theoretically and experimentally by Professor A. H. Gibson (*The Effect of Surface Waves on the Discharge over Weirs*, Selected Engineering Paper No. 99, Inst. C.E., 1930). The type of wave considered is one of oscillation, in which the particles of water concerned in the wave motion are not themselves carried in the direction of propagation, but oscillate about a mean position in a vertical plane parallel to the direction of propagation of the wave profile. The path described by a particle in such motion in deep water is sensibly circular, and the wave-form assumes the shape of an inferior trochoid or prolate cycloid, that is, the path described by a point situated between the centre and the circumference of a circle which rolls without sliding along a horizontal plane.

The equation to the surface profile of a trochoidal wave is

$$\begin{cases} y = a(1 - \cos\theta) \\ x = R\theta + a\sin\theta, \end{cases}$$

where a is the radius of the orbital circle of the surface particles and is consequently one-half of the wave height; R is the radius of the generating circle of the trochoid. The wave-length is $2\pi R$; y denotes the height of the water surface measured above the trough of the wave, x is the abscissa of the point measured horizontally from the trough, and θ is zero at the trough and π at the succeeding crest. When $a = R$, the crest of the wave assumes a sharp point or cusp.

Gibson shows that the ultimate influence of the wave on the discharge over a weir is very much the same as that of a wave of the same profile but containing no *internal* motions; in other words, the effect on the *mean* discharge during a period occupied by the passage of one or more complete waves is sensibly the same as would result from a very slow wave whose rate of rise and fall is such that the flow is able to adjust itself at any instant in proportion to $H^{3/2}$, where H is the head at that instant measured in the usual way.

He then proceeds to demonstrate that the effect of a trochoidal wave system depends on both the height and the length of the waves, while for purely sinusoidal waves the effect depends only upon the wave height (as also is the tendency for trochoidal waves when the ratio $R:a$ is large).

These ideas were examined experimentally in a concrete flume 3 ft. 6 in. wide and 36 feet long. The head over the weir-crest was measured in a stilling-basin communicating with the flume through a $\frac{1}{4}$-inch pipe opening flush with the bottom of the channel at the centre of its width and 8 feet upstream of the weir-crest.

The required surface waves were generated by a motor-driven reciprocating paddle, the stroke, depth of immersion and period of which could be varied within wide limits so that the experiments covered waves ranging from 4 to 56 inches in length, 0·5 to 7·5 inches in height from trough to crest and frequencies between 75 and 260 per minute.

It was found that with a rectangular weir not extending over the full width of the flume but having end-contractions, or with a triangular weir, the waves were reflected from the exposed weir-plate, and in order to ensure regularity of wave-form it was necessary in those cases to make the distance between the paddle and the weir equal to a whole number of wave-lengths.

At first, the lengths of wave—ranging from 4 to 20 inches—were measured by the simultaneous observation of the position of two successive crests on a horizontal scale adjusted so as to touch the crests, while the heights were given by a gauge whose pointer was arranged alternately to touch the crest and the trough; these heights were measured 6 feet from the weir. Later, however, the wave-form was recorded by an instantaneous photograph of a 6-ft. length extending from 3 to 9 ft. upstream of the weir. These photographs were taken against the background of a white vertical plate, ruled in black 2-inch squares, fixed to the side of the flume. By this method, mean heights could be measured to within about 0·1 inch and mean lengths to within 0·2 inch. The number of waves passing the weir in unit time was also observed, and by multiplying this frequency by the wave-length, the wave velocity relative to the fixed bed of the channel was obtained. Subtracting the velocity of the stream then gave the velocity of propagation of the waves in *still* water. The corresponding theoretical velocities would be

$$V=\sqrt{gl/2\pi} \text{ for a sinusoidal wave,}$$

and $$V=\sqrt{\{(gl/2\pi)[1+(4\pi^2a^2/l^2)]\}} \text{ for a trochoidal wave.}$$

Some typical values are quoted in Table XLVIII:

TABLE XLVIII

Wave-length, l inches	*Frequency per min.*	*l/a*	*V ft./sec.*		
			Observed	*Calculated*	
				Sinusoidal	*Trochoidal*
4	260	16	1·34	1·31	1·41
9	183	14	2·05	1·97	2·16
15	130	25	2·55	2·54	2·61
52	75	16	4·99	4·70	5·06

The observed values thus come intermediate between the two calculated values.

In determining the effect of the waves upon the discharge, the technique adopted was to regulate the supply-pump so as to produce a discharge corresponding to some head H. The wave mechanism was then started and adjusted to give a wave of a certain height and length, when the new head H' was observed in the stilling-basin. For rectangular weirs, the discharge was taken to be proportional to $(\text{Head})^{3/2}$, and for triangular weirs to $(\text{Head})^{5/2}$. Accordingly the *percentage* increase in discharge due to the waves was assessed as $\frac{3}{2}\left\{\frac{H'-H}{H}\times 100\right\}$ in the case of rectangular weirs and $\frac{5}{2}\left\{\frac{H'-H}{H}-100\right\}$ in the case of triangular weirs.

As a result of this theoretical and experimental investigation, Professor Gibson recommends the following correction-factors:

I. Expressing the discharge as $(1+k)$ times the discharge under the same recorded head with undisturbed water in the approach channel,

$k = 0{\cdot}19(a/H)^2$ for sinusoidal waves and a thin-crested rectangular weir;

$k = 0{\cdot}94(a/H)^2$ for sinusoidal waves and a 90-degree V-notch.

II. With waves of trochoidal form, the effect is smaller and k assumes the values quoted in Table XLIX (see p. 356).

In Table XLIX (p. 356), *a* represents one-half of the wave height from trough to crest, and it is recommended that the wave height be measured at a point distant about 12 times the mean head upstream of the weir-crest and, for a weir having end-contractions or for a V-notch, at a point about midway between the side and centre of the approach channel.

TABLE XLIX

Values of k

$\dfrac{\text{Wave-length}}{\text{Wave height}}$	Type of weir	
	Rectangular	*90-deg. Vee*
π (sharp-crested)	0·07 $(a/H)^2$	0·50 $(a/H)^2$
4	0·12 ,,	0·66 ,,
5	0·14 ,,	0·73 ,,
6	0·155 ,,	0·79 ,,
8	0·165 ,,	0·84 ,,
10	0·175 ,,	0·88 ,,
15	0·182 ,,	0·92 ,,

(For longer waves, the correction for sinusoidal waves is sufficiently close.)

A subtle point is noted by Gibson in connection with the observed effect when using a *broad-crested* weir. Here the effect was much smaller than that discovered for a sharp-crested rectangular weir having no end-contractions. Moreover, the effect was more irregular, and even in some cases apparently negative. This Professor Gibson attributes to the fact that the thickness of the stream on the top of a broad-crested weir adjusts itself until the discharge is a maximum; any change in the thickness reduces the discharge. The thickness of the stream may be shown thus to adjust itself to a value approximately $\frac{2}{3}$ of the effective head.* "If then a cyclical fluctuation of surface-level is superposed on a weir of this type, such an adjustment is no longer possible, so that the effect of surface waves in increasing the discharge will be less than in the case of a thin weir and may conceivably become negative, as apparently occurred in certain of the experiments of this series."

5. " Waves of Transmission " and Tidal Bores.

In a wave of transmission or translation, as distinct from one of oscillation, not only does the wave-form advance, but in addition there is a translation or displacement of a volume of water and particles of water are carried forward to new positions. The ideal example consists of a solitary wave, such as might be produced by the sudden opening of a sluice-gate admitting water to a canal, or by the rapid immersion of a solid body at one end of a trough. More complicated examples are provided by the onset of a sudden burst of rainfall communicated to the upper reaches of a waterway or by the phenomenon of the tidal bore (in

* *Hydraulics and its Applications*, A. H. Gibson, 4th ed., p. 175 (Constable, 1930).

some places called the " eagre "), as found in many estuaries, such as the Severn, Trent, Dee, Parrett (England), Seine and Garonne (France), Amazon (South America), Hooghly (India), Tsien-Tang (China), and many others.

A fascinating description and discussion of some of these phenomena appears in Dr. Vaughan Cornish's *Ocean Waves* (with additional notes by Dr. Harold Jeffreys, F.R.S.), while the Severn bore in particular has been described by Captain Beechey (*Report of the Severn Survey of* 1849), Dr. Vaughan Cornish (*Geographical Journal*, vol. 19, 1902, p. 52, and vol. 29, 1907, p. 28) and by the staff of the Admiralty survey of 1926–7. A long and painstaking series of observations of the Trent bore is presented by Messrs. Champion and Corkan in the *Proceedings of the Royal Society of London*, Ser. A, March 1936; and Commander Moore, R.N., has described the awe-inspiring phenomena on the Tsien-Tang, with a solitary wave 8–11 feet high, in his *Report on the Bore of the Tsien-Tang Kiang* (Admiralty Publication, 1888).

Photographs of the Severn bore appear in Plates XXXIV and XXXV.

A classical investigation of waves propagated along straight channels of simple geometrical shape will be found in Scott Russell's *Report on Waves*, British Association, 1844, p. 311, but what we are more especially concerned with here is the practicability of reproducing such phenomena in small-scale models, and in this connection the pioneering work is due to Professor A. H. Gibson with his very detailed study of the height and speed of progress of the bore in the models of the Severn (*Construction and Operation of a Tidal Model of the Severn Estuary*, H.M. Stationery Office, 1933, Section IX and Appendix D). It should be realized at once that no special device is used for the production of the bore in the model beyond the mechanical generation of the proper rate of rise and fall of the tide at the *mouth* of the model, the shape and configuration of the channels doing the rest, although in the case of a more recent model of the River Parrett, the tidal period had to be shortened somewhat, compared with that computed from the horizontal and vertical scales, in order to give a closer agreement with tidal phenomena, including the bore (see Chapter VII).

Discussing the effects as observed in the Severn model having a horizontal scale of 1 : 8,500 and a vertical scale of 1 : 100, Professor Gibson states:

" In the model the height of the bore is observed against a vertical gauge graduated to represent feet, and the time required to travel between the given points is measured by a chronometer registering to 0·01 second. In every case the heights of bore and the times are observed for three or four tides on each side of spring tide. The observed results are then plotted

. . . and the height and time of the bore corresponding to spring tide is taken from the smooth curve representing the results.

" Observations on the model show that the first indications of the bore are noticeable below Sharpness. It first becomes pronounced about midway between Severn Bridge and Hayward's Point and gradually increases in height to become a maximum about Longney Point and Epney, afterwards diminishing in height towards Gloucester.

" There is no bore on a neap tide. In the model with a normal spring tidal range the first measurable bore at Longney Point during a lunar cycle occurs about the eighth tide before spring tide. The height increases at each successive tide until with normal river flow, and with a 41·5 ft. tidal range at Avonmouth, it has a height of about 3·1 ft. at Longney Point. With a 45 ft. range at Avonmouth the height under similar conditions is approximately 3·8 ft.

" The height of bore, however, with a given tidal range depends appreciably on the level and configuration of the bed of the channel above Beachley, and more especially on the levels and configuration of the Noose and Frampton Sands. Thus, with a 45 ft. tide at Avonmouth the height at Longney Point during the range of the experiments has been as high as 4·6 ft. and as low as 3·4 ft. This change has been found to correspond to a change in the levels and configuration of the Noose and Frampton Sands.*

" The first observations on the model were carried out with a view of determining how nearly the height and speed of the bore compare with the values obtained in the Severn itself. With the river flow adjusted to its mean value and with a tidal range of 42·5 ft. at Avonmouth, the mean height of the bore at Longney Point as observed in a series of tests was 3·6 ft., and the times of its passage between various points were as follows:

Time for bore to travel from	*Secs.*	*Corresponding time corrected for scale ratio (mins.)* †	*Time observed in estuary (mins.)*
Hayward's Point to Framilode -	4·68	66·4	69
Hayward's Point to Stonebench Inn - - - - - -	7·70	109·0	110
Hayward's Point to Lower Parting - - - - -	9·13	128·5	128

† The time scale multiplier is $\frac{L}{l}\sqrt{\frac{h}{H}}=\frac{8500}{\sqrt{100}}=850.$

* Compare with Dr. Vaughan Cornish, *Ocean Waves*, p. 118: " It is interesting to note that the flood tide operates upon sand-banks so as to avoid the formation of a

" The time of passage between Hayward's Point and Lower Parting is affected somewhat by a variation in the mean tide level in the model. The maximum time taken during any observation over the whole of this series of tests was 9·25 seconds and the minimum 8·70 seconds. These times would correspond to 131 minutes and 123 minutes in the estuary. . . . These observations were carried out during a period corresponding to the years 1905–1910 in the estuary. In the period corresponding to the end of 1927, further observations were made to check Admiralty observations made in this period.

" In the model, under conditions similar to those during the Admiralty observations of 14th September, 1927, with the same tidal range of 45·2 ft. at Avonmouth, the height of bore was 3·5 ft. as compared with 3·3 ft. in the estuary itself and the time between Hayward's Point and Lower Parting was 9·18 seconds—equivalent to 130 minutes in the estuary—as compared with the observed time of 131 minutes. . . ."

From these quotations, it is evident that quite a remarkably high degree of accuracy is practicable in this kind of work. This is important as indicating that reliable quantitative measurements may be made in such models to determine the effect of any proposed works upon the formation of a bore; it is also significant as being one further confirmation of the faithful reproduction of the "hydraulic" phenomena associated with tides.

It might be feared, however, that a change of temperature would have an appreciable influence upon the formation of the bore in the model. A rise of temperature reduces both the viscosity and the surface tension of water, and the reduction of viscosity would tend to increase the height of the wave in the upper reaches of the model, since the forces of damping would be reduced. Such an increase in the height of the bore would be accompanied by a higher speed of propagation, but on the other hand the reduction of surface tension associated with the rise in temperature would tend slightly to decrease the velocity of travel. In the Severn model, measurements made with water temperatures of 9·8°C. and 32·2°C. respectively showed no measurable difference in either height or speed of the bore, although the change in temperature would involve a reduction of some 72 and 4 per cent. in kinematic viscosity and surface tension respectively. Professor Gibson's final conclusion is that " the speed of the bore depends on the depth of water, the height of the bore, and

bore. It is after a wet season, when the ebb channel between Hock Cliff and Severn Bridge is deeply incised so that the tide comes up this way, that, according to local testimony, there is a large bore in the river between Newnham and Gloucester. Below Severn Bridge the flood tide has sufficient preponderance to keep an alternative channel open in all seasons; and here (again according to local testimony) no bore is formed, or at most one which has only a short run."

the velocity of the river current, as indicated by the theoretical formula

$$V_p = \sqrt{\frac{2g(h+k)^2}{2h+k}} - v \text{ ft. per sec.,}$$

and if h is taken as the depth of the deep-water channel of the river, the actual velocity is approximately 95 per cent. of that given by the formula". In this formula (derived in Gibson's *Hydraulics and its Applications*, 4th ed., p. 405) V_p denotes the velocity of propagation of the wave-crest, relative to the fixed boundaries of the stream, v is the velocity of the stream itself, h the depth of the stream before the arrival of the bore and k the height of the wave-crest above the water surface of the stream. In writing the minus sign in $V_p = \sqrt{\frac{2g(h+k)^2}{2h+k}} - v$, it is assumed that the stream is flowing contrary to the direction of motion of the wave. If we call V_p' the velocity of propagation of the wave-crest *relative to the stream*,

$$V_p' = \sqrt{\frac{2g(h+k)^2}{2h+k}} \ . \qquad \text{......(1)}$$

An alternative formula, derived by different reasoning in Lamb's *Hydrodynamics*, 4th edn., p. 272 (1916), is

$$V_p' = \sqrt{\frac{g(h+k)(2h+k)}{2h}} \ . \qquad \text{......(2)}$$

Formulae (1) and (2) differ from one another by less than 1 per cent. for values of k/h up to 0·25; if $k=h$, formula (1) gives a velocity 6 per cent. less than formula (2). Again, if k is very small compared with h, both formulae are sensibly reduced to

$$V_p' = \sqrt{gh}.$$

The Author has made a study of the propagation of such waves in various small channels.* This investigation included experiments on two river models, one of which (the Mersey) was non-tidal but extremely tortuous (for further details, see Chapter I), and the other (the Dee) was tidal. Work was also done on small, straight channels of rectangular, trapezoidal and triangular cross-section. Certain of the observations were made with wave heights as great as 0·5h. Let us first consider the work on the non-tidal Mersey model.

Wave velocities in this case were measured over a length AB of 1510 cm., the upstream gauge-point (A) being some 105 cm. below the upper limit of the model, and the downstream gauge-point (B) some 55 cm. above

* Allen, "Experiments on Water Waves of Translation in Small Channels", *Phil. Mag.*, Ser. 7, vol. XXV, p. 754, May 1938.

Irlam Weir. The width of the water surface averaged 2·18 cm. with the lowest river discharges and 4·55 cm. with the highest. The experimental procedure was as follows:

(1) A steady stream of water was supplied at the upstream limit of the model. The corresponding rate of flow, q, was determined by timing the collection of a measured volume at the downstream end of the model.

(2) A pointer-gauge mounted over the station B was adjusted until the pointer stood 0·01 inch above the water surface.

(3) A quantity, x, of water was then poured into the model at its upstream end. This quantity produced a wave of translation superimposed upon the general flow of the stream.

(4) A stop-watch graduated in tenths of a second was started as the wave passed station A, and was stopped when the wave arrived at B. The pointer-gauge at B greatly assisted the latter observation, since the instant of the wave's rising into contact with the pointer was very sharply defined; in some cases it was even possible to detect the arrival of the wave by visual observation of the reduction of the 0·01 inch gap between the pointer and the water surface.

The quantity x of water introduced at the top of the model to create a wave was frequently made 2,000 c.c.; sometimes it was increased to 8,000 c.c. The time of passage of the wave between A and B was sensibly unaltered by this change in x, although with 8,000 c.c. it was easier to follow the progress of the wave in the lower stretches of the model. The reason for the agreement in times observed with these very different volumes of added water is evidently that, while the wave height *near the point of its formation* was appreciable, yet the *average* height in either case was small compared with the depth of water in which the wave travelled. For the purpose of analyzing the results in this case, therefore, the wave height k was treated as negligible compared with the depth h. Dividing the whole length l between stations A and B into elements δl over which the *deep water* channel depth h was approximately uniform,

$$V_p' = \frac{l}{\Sigma\left(\frac{\delta l}{\sqrt{gh}}\right)},$$

and
$$V_p = V_p' + v,$$

where v was calculated from the measured areas of cross-sections and the known discharge q, account being taken of the spacing of the sections along the river.

The results showed that the measured V_p was always less than the calculated V_p, but that for practical purposes it was sufficiently close to calculate the velocity as though the wave were propagated through *still* water of depth equal to that measured along the deep water channel. The following Table L provides the essential comparison on this basis:

TABLE L

q (c.c./sec.) - -	11·1	25·9	59·7	75·0	107*	126	161*	214*
Mean h (cm.) of deep water channel - -	1·24	1·73	2·54	2·78	3·18	3·56	3·92	4·62
v (cm./sec.) - -	5·21	8·32	11·7	13·0	13·5	15·5	15·0	15·8
Ratio of measured V_p to V_p' calculated as $\dfrac{l}{\Sigma\left(\dfrac{\delta l}{\sqrt{gh}}\right)}$	1·10	1·08	0·98	1·03	0·97	0·98	0·92	0·95

The observations marked * were made after roughening the bed of the model with coarse sand; a value of q equal to 214 c.c. per second represents 8,000 cubic feet per sec. in Nature.

As a result of further investigation, it appeared that the disappearance of the effect of the speed of the stream in which the wave is travelling was due to two causes. In the first place, the wave velocity itself is less than that corresponding to the depth of water in the *deep* water channel. On the other hand, only a portion of the velocity of the stream is superimposed upon the wave, because of transverse currents at the bends. As a net result, therefore, the measured V_p agrees reasonably closely with the calculated V_p'.

The scales of this model were 1 : 800 horizontally and 1 : 120 vertically, and translating the model results into data for the prototype by the discharge scale $1 : 800 \times 120^{\frac{3}{2}}$ and the velocity scale $1 : \sqrt{120}$, it appeared that if the river Mersey itself, when discharging some 400 cusecs, were to receive flood-water from the reaches upstream of Ashton-upon-Mersey, a time of approximately 50 minutes would elapse before the effect was felt in the vicinity of Irlam Weir, where the river flows into the Manchester Ship Canal. The distance between the two places, measured along the tortuous channel, is some 8·3 miles as compared with 3·7 miles *direct* distance.

Experiments were also made, during this investigation, on the velocity of propagation of a solitary wave in straight channels of triangular and of trapezoidal cross-section. These tests showed that the *effective* depth h_m in the case of a triangular section is very closely equal to the average depth

—a conclusion previously reached by Scott Russell. With a trapezoidal section, however, the effective depth h_m is approximately

$$\frac{bh+\frac{1}{2}Sh^2}{b+Sh},$$

where b is the bottom width, h the water depth measured at the centre, and S is the side-slope (horizontal/vertical ratio).

Finally, observations were made of the tidal bore in a model of the Dee estuary (scales 1:5,000 horizontal and 1:200 vertical). In this model a bore was often observed to form on the flood-tide some distance below Connah's Quay and to proceed upstream to Chester Weir. The height of the sudden rise due to this wave, and the speed of its translation, varied considerably according to the configuration of the sand-banks, the flow of water down the river, and the presence in the model of various proposed schemes of training walls. When the bed and the training walls were adjusted to represent present-day conditions, the time elapsing between the first sharp rise of the flood-tide at Connah's Quay and at a point near Chester Weir was 9·8 seconds. The distance between the points was 247 cm., so that the speed of propagation averaged 25·2 cm. per sec. The velocity scale of the model being $1:\sqrt{200}$, this represents 355 cm. per sec., or 7·94 miles per hour in Nature, a figure which is closely in agreement with that of 8 m.p.h. supplied by eye-witnesses of the natural phenomenon. It was further observed in the model that the surface current followed the bore at a speed of about 0·75 of that of the wave-crest—this again being in close agreement with Nature. Yet the average width of this channel in the model was only about 2·5 cm.

The opportunity was taken, with the Dee model, to measure the height and speed of the bore under different conditions, and to compare the results with Professor Gibson's formula:

$$V_p=\sqrt{\frac{2g(h+k)^2}{2h+k}}-v.$$

The method of measuring the height of the wave was:

(*a*) to adjust a pointer-gauge so as to touch the water just before the arrival of the bore, and

(*b*) to re-adjust the pointer, by trial and error, so that the first sharp rise of the flood, due to the bore, just touched the pointer.

The difference between readings (*a*) and (*b*) then gave the height of the bore.

In applying the formula, v was taken to be $\frac{2}{3}$ of the maximum surface velocity of the stream, previous to the arrival of the bore, as indicated by a shallow float. The value of v adopted would not differ greatly, therefore, from the mean velocity over the cross-section, at any rate in its proportional influence on the final calculated V_p. The depth substituted for h was the mean depth of the water in the deep water channel (measured to the lowest point of the channel at any particular place) just before the arrival of the bore. Observations made under various conditions revealed velocities in agreement with the formula within ± 3 per cent. when the channel was straight or gently curved and free from sand-bank obstructions; even when the bed of the channel was beset with sand-banks, the observed speed was only 10 or 15 per cent. less than that given by the formula. In certain of the Dee experiments, h was as low as 0·36 cm. and k as high as one-half of h.

Very little of a quantitative mathematical nature appears to be known concerning the *origin* of the bore in a tidal river, but some light results from the analogy of the production of a solitary wave by the sudden immersion of a body in water. If, at some point in an estuary, the tide rises so quickly that, owing to the constriction in width or depth at that point it is impossible for the tidal flow to pass gradually upstream, a wave will result and will persist until either its energy is dissipated in friction or its height is reduced on its passing into a wider section. It would be possible artificially to create the bore by preventing the ingress of flood-tide water by means of a gate which, after a time, is quickly opened. Looked at in this way, resistance favours the creation of the bore, but also resistance promotes its ultimate decay as the wave advances upstream.

The idea, commonly held, that the bore is a very rare phenomenon associated only with certain few rivers such as the Severn, is quite erroneous. To some degree it appears, with high tides, on many tidal rivers. The Author was walking along the southern shore of the River Wyre (Lancashire) on 9th August, 1944, and when at a point some 1·1 miles above Shard Bridge, or about 5·4 miles (direct distance) above Wyre Dock, he was fortunate enough to witness the absolutely sudden initiation of the flood-tide. This occurred at 3.28 p.m. D.B.S.T., the predicted time of high water at Fleetwood being 4.47 p.m. and the predicted rise there 26 ft. 2 in. As he saw it, the flood-tide began in the form of a definite bore. Although the wave was only about 3 inches high, the eye could readily follow its travel upstream with a sharp line of demarcation extending from bank to bank of the river. It was followed by an impressively swift current.

6. The Standing Wave (or Hydraulic Jump).

If, due to obstructions or other causes, the surface of the water flowing along an open channel is not parallel to the bed, even though the width of the channel is constant, it is clear that the velocity varies from section to section; the motion is non-uniform, and allowance must be made in any analysis for changes in the kinetic energy, whereas Chézy's equation $v = C\sqrt{mi}$ assumes that the work done by gravity goes simply to overcome friction.

In a broad channel, the hydraulic mean depth approximates to the actual depth h. For non-uniform flow h varies from point to point along the channel; let H be the depth in the same channel for *uniform* flow at the same rate of discharge. It may be shown * that the variation of h along the channel then depends upon $\frac{h}{H}$ and upon $\frac{iC^2}{g}\left(\text{or } \frac{2i}{f}\right)$, i being the longitudinal slope of the bed of the channel.

To take an example, consider the stream issuing from beneath a sluice-gate. If the slope of the channel is sufficiently small or if its bed is sufficiently rough, $\frac{2i}{f}$ will be less than unity. It is also quite possible that the depth h of the stream issuing from the gap below the gate is such that $h^3 < \frac{2i}{f} H^3$. In these circumstances, the value of h will increase downstream until a steep-fronted standing wave is formed as shown in Fig. 119. On

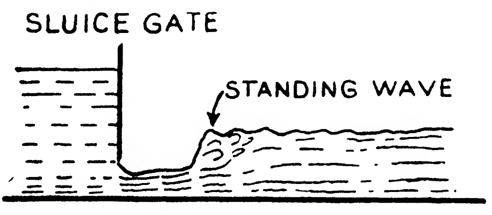


FIG. 119.

the other hand, no such wave would be created, even if the depth of the stream passing out of the gap were less than H, should the channel be sufficiently smooth or its bed-slope steep enough to make $\frac{2i}{f} > 1$. In that case, the depth downstream of the sluice-gate rises *gradually* to the magnitude H appropriate to uniform flow (unless there is some other obstruction further downstream).

Again, a standing wave may be formed by an underwater obstruction if the channel is smooth or steep enough to make $\frac{2i}{f} > 1$, and if, due to the

* *Hydraulics and its Applications*, Gibson, 4th ed., p. 309 (Constable).

obstruction, h can become equal to $H\sqrt[3]{\frac{2i}{f}}$. This case is shown in Fig. 120.

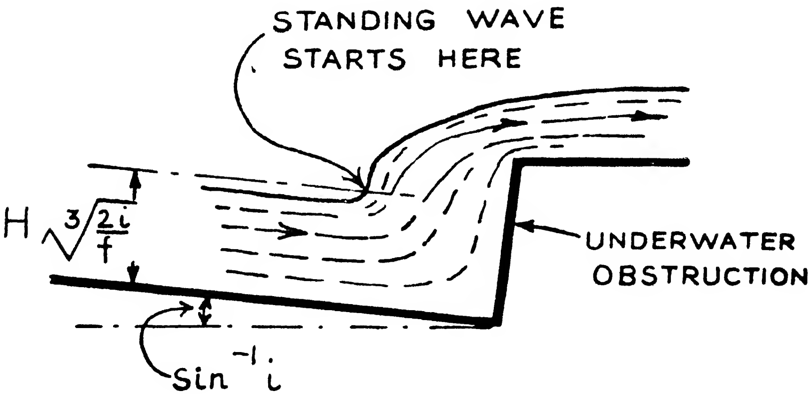


FIG. 120.

Now experiments * confirm the validity of a mathematical expression,

$$(h_1 - h_2)\left(\frac{h_1 + h_2}{2} - \frac{h_1}{h_2} \cdot \frac{v_1^2}{g}\right) + \left(h_2 - \frac{d}{2}\right) d = 0,$$

where h_1 and h_2 have the meaning indicated in Fig. 121, v_1 is the mean velocity before the jump, and d is the fall in the bed of the stream between the points at which h_1 and h_2 are measured.

FIG. 121.

This equation is derived by considerations of momentum. If the bed of the channel is level, $d = 0$ and the expression simplifies to

$$h_2 = -\frac{h_1}{2} + \sqrt{\frac{h_1^2}{4} + \frac{2h_1 v_1^2}{g}},$$

or x = the height of the standing wave $= h_2 - h_1$

$$= \sqrt{\frac{h_1^2}{4} + \frac{2h_1 v_1^2}{g}} - \frac{3}{2} h_1 \quad \text{.............................(I)}$$

To be more precise, account should be taken of the fact that, owing to the variation of velocity across the section, the true kinetic head is greater

* "The Formation of Standing Waves in an Open Channel", A. H. Gibson, *Proc. Inst. C.E.*, vol. CXCVII, 1914, p. 233.

than the apparent kinetic head defined simply as $\frac{(\text{mean velocity})^2}{2g}$, so that formula (I) should read

$$x = \sqrt{\frac{h_1^2}{4} + \frac{2h_1(1+\alpha)v_1^2}{g}} - \frac{3}{2}h_1 .$$

Since, however, it is known that α rarely exceeds 0·05, and is generally about 0·03, in artificial channels, and since $\frac{2h\alpha v_1^2}{g}$ appears only as an additive term under a square-root sign, this is a refinement without significance in practical applications.

The experimental technique appropriate to this kind of investigation may be illustrated by reference to Professor Gibson's experiments ("The Formation of Standing Waves in an Open Channel", *Proc. Inst. C.E.*, vol. CXCVII, 1914, p. 233). These were made in a concrete flume 3 feet wide and 45 feet long with a slope of only 0·1 inch over its whole length. Water was supplied through an 8-inch pipe to a forebay, 6 feet wide, equipped with baffles, and the rate of discharge was measured in an 8,000 gallon tank fitted with a float and a rod calibrated in terms of the volume of water collected. A vertical sluice-gate made of $\frac{1}{4}$-inch steel plate was inserted, as indicated in Fig. 122, and the depth h_1 of the stream issuing

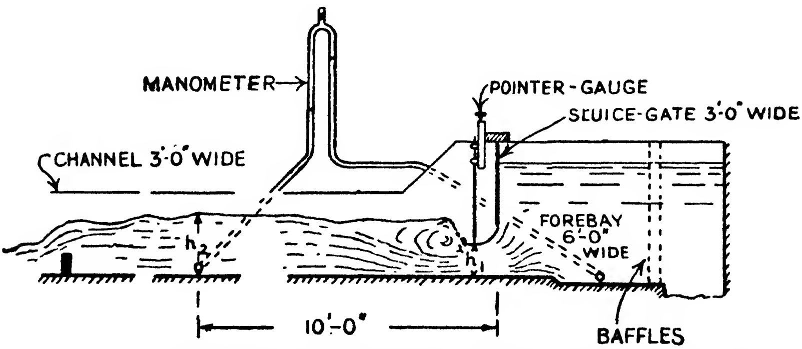


FIG. 122. Diagrammatic arrangement of Professor Gibson's apparatus for Standing Wave Experiments.

(*Proc. Inst. C.E.*, vol. CXCVII, 1914, p. 233.)

from beneath this gate was determined by a pointer gauge, while h_2, the depth at the wave crest was obtained from observations on an inverted U-tube manometer whose two limbs were connected with the forebay and a point 10 feet below the gate respectively. "This method has the advantage that any oscillations due to subsidiary waves superposed on the main standing wave are damped out. In no case did the oscillation of the top of the measuring column amount to so much as 0·01 foot, and in the majority of cases it was much less than this. Owing to the gradual readjustment of velocities in the stream below the jump, and to a certain conversion of kinetic energy into pressure energy in the process, the mean surface level rises gradually below the jump for some little distance downstream. A careful examination of the surface profile showed, however, that this action appeared to be complete at the point (10 feet downstream) at which the depth h_2 was measured.

" In carrying out an experiment the discharge from the flume is baffled until the front of the standing wave . . . is brought as nearly as possible up to the point-gauge. The difference of level between the water in the forebay and at the standing wave is then read off on the U-tube gauge and the depth in the forebay is measured. From this difference of level the depth h_2 at the standing wave is computed. This value requires slight correction for the friction loss in the forebay, and in the 10-foot length of channel below the sluice. Knowing the discharge and depth below the sluice, and the value of C for the channel, this loss can readily be computed. The correction is in every case small, seldom amounting to as much as 0·02 foot, and has in each case been added to the observed value of h_2. . . ."

An interesting and important application of standing-wave experiments to a practical problem is exemplified in the " back-water suppressor " as applied to a hydro-electric installation. Generally speaking, whenever a hydro-electric plant utilizes river-flow, a flood in the river is accompanied by a bigger rise of tail-water than of head-water level. This phenomenon is responsible for a reduction of working head, and might even thereby cause a fall in output, despite the increased volume of water at the disposal of the plant. The obvious recourse to a larger number of turbines than that necessary for low river discharges, so as to have some extra machines available for use in times of heavy discharge, may not be economical, and another method has been developed—the " back-water suppressor ".

In this device, the surplus flood-water passes over the top of a spillway, at the toe of which it possesses considerable velocity and impinges upon the much slower stream emerging from the turbine draught-tubes. The two streams together move on into the tail-race, where a standing-wave is formed which causes the working head on the turbines to be apparently increased by the amount h (indicated in Figs. 123 *a* and *b*), compared with what it would be in the absence of the " back-water suppressor ".

Mr. J. A. Sirnit has described * model tests of this device, using scales of 1 : 24 and 1 : 10; the results from the two different models were found to be in close agreement. Professor A. H, Gibson also has made a

* " Preventing Loss of Power due to High Backwater ", J. A. Sirnit, *Engineering News Record*, vol. 88, No. 23, June 8, 1922, p. 964.

model investigation, with an analytical discussion,* in which he studied the effect of altering the shape of the toe of the spillway, of introducing a step in the bed of the tail-race (as shown in Fig. 123 *b*), and of the length of tail-race necessary for full development of the standing wave.

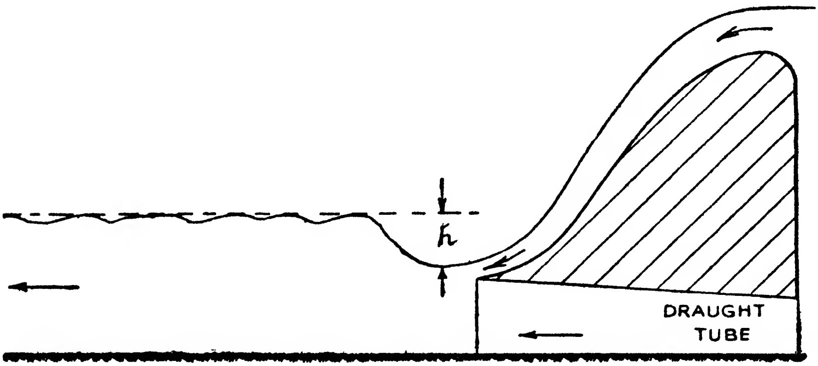


FIG. 123*a*.

Gibson's first experiments were on a 1 : 24 model of a projected installation, in which the width of the draught-tube, spillway and tail-race was to be 25 feet, the depth of the draught-tube at its outlet 4·8 feet, and the turbine discharge 500 cusecs at full load. The depth of water at the end

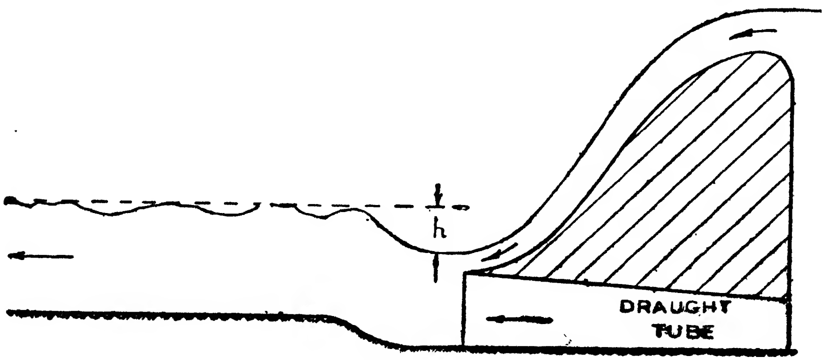


FIG. 123*b*.

of the tail-race was 5·0 feet during dry weather and 9·0 feet under flood conditions, when a surplus of 500 cusecs could be discharged over the top of the spillway. Under dry-weather conditions the working head was 13 feet. The rates of discharge in the model were made equal to those in the prototype divided by $24^{5/2}$, so that 1,000 cusecs must have been

* "An Investigation of the Back-water Suppressor as applied to a Hydro-electric Installation", Gibson, *Selected Eng. Paper No.* 92, Inst. C.E., 1930.

represented by 0·355 cubic ft. per second in the model, while the tail-race width of 25 feet must have been represented by 12·5 inches.

The models were made of smoothly finished, shellac-varnished wood, and water levels were measured at various points by means of open glass gauges connected by rubber and copper tubing to small orifices finishing flush with the surface of the bottom of the tail-race along its centre-line. An additional direct measurement was also made of the depth of the surface of the trough of the standing wave below the top of the side-walls. An adjustable sluice-gate fitted at the downstream end of the model enabled the depth of water in the tail-race to be regulated according to the prescribed conditions or to simulate any other conditions for the purpose of a more general study. The method of measuring discharges is of some interest. Preliminary tests were carried out in order to establish the relationship between the head of water over the spillway and the rate of flow over its crest. These experiments were made *without* discharge through the draught-tube. When experimenting with the full device in operation, the combined flow from the spillway and draught-tube was measured by collection in a tank of known capacity, and the turbine discharge (that is, the rate of discharge through the draught-tube) was computed by deducting the quantity (known from the preliminary tests) passing over the top of the spillway.

The main results of Professor Gibson's experiments may be summarized as follows:

(*a*) The gain of head due to the suppressor depends upon the ratio of the volume of spillway water to that of the turbine discharge, and increases with that ratio.

(*b*) The best form of suppressor is that in which the bottom of the tail-race is raised above the invert level of the draught tube. With such a design, he was unable to calculate the magnitude of the effect, and came to the conclusion that model experiments were therefore essential, although in the case of the less efficient arrangement where the tail-race is at the same level as the draught tube, he was able to suggest a theory, involving the mutual impact of two streams, which gave results in excellent agreement with the experiments.

(*c*) The *real* gain of head is in general less than that based simply upon the difference in level between the trough of the standing wave and the level at the end of the tail race. This is because the pressure at the outlet of the draught tube is " greater than that corresponding to the level of the water surface in its vicinity, the excess pressure being due to centrifugal action accompanying changes in the direction of flow in this region."

(*d*) In order to take full advantage of the fact that the height of the standing wave continues to increase for some distance beyond the toe of

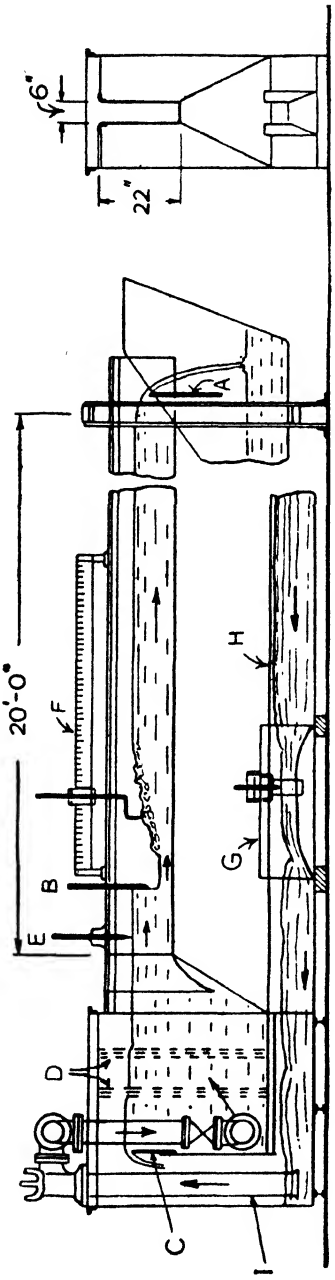


FIG. 124.

(Based on Fig. 4, p. 639, *Trans. Am. Soc. C.E.*, vol. CI, 1936. Bakhmeteff and Matzke.)

A. Adjustable Weir-Plate. B. Adjustable Sluice. C. Adjustable Overflow.
D. Baffles. E. Head-Gauge. F. "Profilometer".
G. Critical Depth Meter. H. Return Flume. I. Vertical Pump.

the spillway, the length of the tail-race should be some twelve times the depth of the draught tube.

This last point raises the general question as to how far a model may be relied upon to reproduce the longitudinal, in addition to the vertical, dimensions of the standing wave. Not only is this important in the example we have just considered; it may be a vital consideration in the design of protective aprons at the toe of a spillway or dam or at the exit of sluice-gates. In these cases, it may be essential to provide a sufficient length of concrete or other "permanent" apron on which the wave formation shall be complete.

Messrs. Bakhmeteff and Matzke * have devoted experiments especially to the investigation of the longitudinal dimensions. The lay-out of their apparatus is illustrated in Fig. 124; the channel was 6 inches wide, 22 inches deep, and 20 feet long, and was made of welded steel. A 5-inch vertical pump supplied water to the inlet basin provided with baffles and an adjustable

* "The Hydraulic Jump in Terms of Dynamic Similarity", Bakhmeteff and Matzke, *Trans. Am. Soc. C.E.*, vol. 101, 1936, p. 630.

overflow. Owing to space and foundation considerations, the sump was arranged as shown, beneath the supply basin, and in communication with the return channel. The rate of flow was measured in this return channel by means of a " critical depth meter " (a form of Venturi flume), calibrated by volumetric measurement and by comparison with a sharp-crested weir. The investigators were satisfied that the discharge was known to within 1 per cent.

Standing waves were produced by an adjustable sluice and the down-stream level regulated by a weir-plate; the bottom of the flume was horizontal. In measuring the profile of the wave, the pointer gauge was so adjusted that over a period of time the water surface oscillated around its point as an average position.

Fig. 125 is reproduced from the published Paper. It shows values of $\frac{y}{h}$ plotted against $\frac{x}{h}$, x and y being co-ordinates of points on the surface

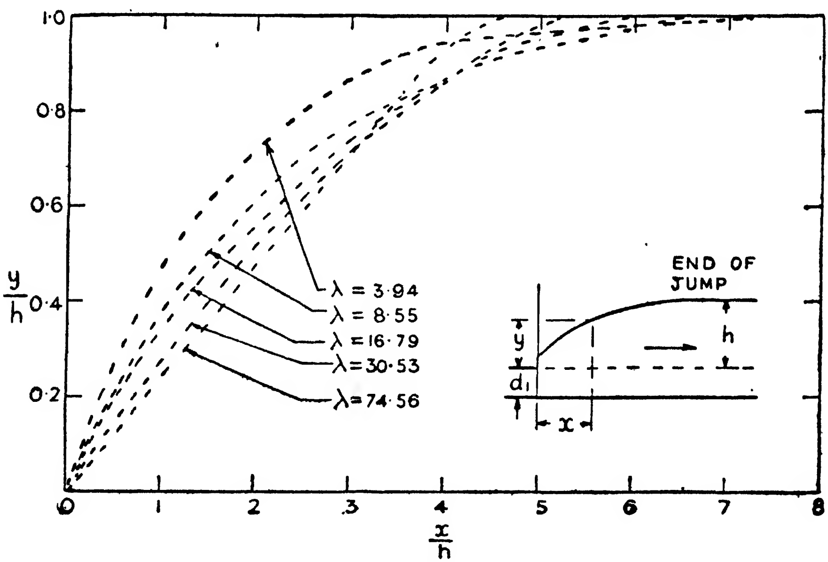


FIG. 125.

(Based on Fig. 8, p. 644, *Trans. Am. Soc. C.E.*, vol. CI, 1936. Bakhmeteff and Matzke.)
Plotted points omitted for clarity in this reproduction.

profile and h the height of the jump. The implication is that the shape of the wave is dependent upon the value of the dimensionless quantity λ defined as $\frac{q^2}{gd_1^3}$, q being the discharge (cubic ft. per sec.) per unit width. Further, this would imply that the profile in the model will be similar to that in the prototype if the value of $\frac{q^2}{gd_1{}^3}$ is identical in the two, and this would mean that

$$\frac{Q_2}{Q_1} \text{ must } = \left(\frac{1}{S}\right)^{\frac{5}{2}},$$

where Q_2 is the total rate of discharge in the model, Q_1 that in the prototype, and the model scale is $1:S$.

But this is precisely the relationship of discharges demanded by the principle of dynamical similarity on the assumption that the standing wave is predominantly governed by gravity rather than by viscosity; it is moreover the relationship found experimentally to give the correct *height* of jump. It would appear from this investigation, therefore, that if the rate of flow in the model is made equal to $\frac{Q_1}{S^{\frac{5}{2}}}$, both horizontal and vertical dimensions of the standing wave may be reproduced with considerable success.

There may be cases, however, where the horizontal scale of the model is not the same as the vertical. A tidal model is a case in point and has a horizontal scale of, say, $1:S_x$ and a vertical scale of $1:S_y$. In that event, rates of discharge are related to those in the estuary by the ratio $1:S_x{}^2S_y{}^{\frac{1}{2}}$, and if a sluice-gate or other obstruction leads to the formation of a standing wave, the height of the jump may be translated into the height expected in the actual estuary by multiplying by S_y. The length of the jump to be expected in the prototype will also be approximately obtained from that observed in the model by multiplying by S_y, *not* by S_x.

A further application of the standing-wave phenomenon is found in the Venturi or Parshall flume *, in which an open channel is provided with a constricted region, either by side pieces as in Fig. 126*a* or by a hump as

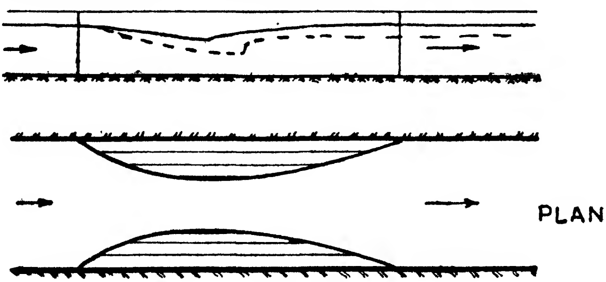


FIG. 126*a*.

* "Notes on Standing Wave Flumes and Flume Meter Falls", C. C. Inglis, Govt. of Bombay Public Works Dept., Tech. Paper No. 15 (Bombay, 1928); "The Development of the Venturi Flume", Jameson, *Water and Water Engineering*, March 1930; "Non-Uniform Flow of Water", Engel, *The Engineer*, 21st and 28th April and 5th May, 1933; "The Venturi Flume", Engel, *The Engineer*, 3rd and 10th August, 1934.

in Fig. 126*b*, or by a combination of the two. This device is coming into increasing use for the measurement of discharges through open channels, as it possesses certain definite advantages compared with a weir.

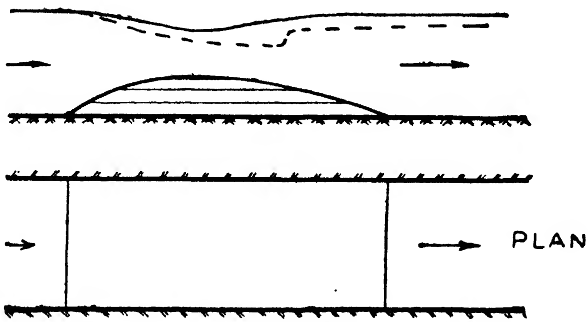


FIG. 126*b*.

For example, it does not prevent the passage of solid matter; it does not require " vent-holes ", and again the loss of head which it produces is relatively low. Moreover, under favourable conditions only one measurement—the depth of water at the upstream gauge-point—is required. This stage is generally described as " free discharge ", to distinguish it from the condition when a small change in the level downstream of the constriction affects the upstream level, this being known as " drowned " discharge. Dr. Engel has shown (*The Engineer*, 1933) that the presence of a standing wave does not in itself ensure completely free discharge. He calls attention to five distinct types of flow (the type of flume with which he deals had side constrictions only, as shown in Fig. 126*a*):

(i) Laminar, in which the water surface in the throat is quite smooth, and " eddies form only in the boundary layer in the divergent outlet ".

(ii) Intermediate between laminar and turbulent flow; the water surface is disturbed by ripples.

(iii) The turbulent region, in which waves are observed at the throat and downstream.

(iv) Intermediate stage between turbulent and rapid flow; this is accompanied by a " V-crested wave, a wave which shows indications of the beginning of side rollers, a type of surface roll which covers a part of the wave only. The V-crested wave is followed by further undulations downstream ".

(v) Rapid flow, in which the water surface is " comparatively smooth in the convergent entrance and in the throat. A hydraulic jump (standing wave) appears in the divergent outlet or in the following part of the channel; no further waves follow the jump ".

The general equation given by Engel for the Venturi flume is:

$$Q = C\frac{b_2 d_2}{\sqrt{1-\left(\frac{b_2 d_2}{b_1 d_1}\right)^2}}\sqrt{2g(d_1-d_2)},$$

where Q is the rate of discharge,
b_2 the breadth at the throat,
d_2 the depth of water in the throat,
d_1 the depth of water in the channel upstream of the throat,
b_1 the breadth of the upstream channel,
and C is the coefficient of discharge.

In type (i) of flow, he finds C to depend upon the Reynolds number which for his purpose he defines as

$$\frac{\text{velocity} \times \text{“ mean hydraulic radius ”}}{\text{kinematic viscosity}},$$

the so-called " mean hydraulic radius " being $\frac{2bd}{2d+b}$, or twice what is usually termed the " hydraulic mean depth ".

At the other extreme—type (v)—he finds C to be simply a function of what he calls the Boussinesq number, defined as

$$\frac{v}{\sqrt{gr}} = \frac{v}{\sqrt{g\frac{2bd}{2d+b}}}.$$

He also finds that if this quantity, as calculated from the velocity and dimensions at the narrowest part of the flume where the water attains its least depth, is equal to unity, the flow changes in character from type (iv) to type (v), being definitely " rapid " for values of the Boussinesq number greater than unity. (The Boussinesq number introduces the effect of width in relation to depth.)

At the " limit of free discharge ", corresponding with the highest downstream level that can be used for a given rate of flow without influencing the upstream depth, a large number of Dr. Engel's tests indicated that the Froude number $\frac{v}{\sqrt{gd}}$ and the Boussinesq number as calculated for the foot of the jump had numerically equal values, meaning that d then $= \frac{2bd}{2d+b}$, or the depth is one-half of the breadth.

For the condition of " free discharge ", the formula for the rate of flow may be simplified to read

$$Q = 3{\cdot}09 K b_2 d_1^{\frac{3}{2}},$$

K being the appropriate coefficient (of the order of 0·97), b_2 the throat breadth, and d_1 the upstream depth. Alternatively, the effect of the velocity of approach may be introduced by writing

$$Q = 3{\cdot}09 K_1 b_2 \left(d_1 + \frac{v_1^2}{2g}\right)^{\frac{3}{2}},$$

v_1 being the mean velocity at the upstream gauge-point.

For details as to satisfactory proportions of the flume and the conditions under which the various formulae apply, the reader is advised to consult Dr. Engel's papers in *The Engineer.* The one entitled " The Venturi Flume " and dated 3rd and 10th August, 1934, also gives calculations to demonstrate the design of free discharging Venturi flumes. Our object in referring to this work in these pages is not so much to detail the principles of the device itself as to call attention to Dr. Engel's use of the various dimensionless quantities, and especially his novel introduction of the " Boussinesq number ".

CHAPTER XIII

CONSTRUCTION OF RIVER MODELS; DEVICES FOR MEASURING DEPTHS AND VELOCITIES

1. Construction of River Models.

Passing reference has already been made to various details in the construction of hydraulic models, and some of the photographic illustrations give, it is hoped, a clear picture of the general appearance of a variety of such models when ready for testing. Furthermore, the photographs reproduced in Plate XIII and Plates XXII *a*, *b* and *c* are intended to depic a river model under construction.

In principle, it will be understood that the river model itself is moulded inside a water-tight casing, and this casing has a carefully levelled upper edge which acts as a datum for levels or depths. Alternatively, there may be a rail, supported on adjustable screws, which serves the same purpose. In either case, this " datum-edge or surface " is first set up with the aid of a spirit-level or a surveyor's level, and is afterwards checked by taking measurements down to the surface of still water in the model. For the purpose of subsequent observations, allowance may be made, if necessary, for any slight discrepancies detected by this method of checking.

The water-tight casing itself may be made of wood, supported on a number of frames or trestles with suitable cross-bearers and legs sufficiently strong to ensure a rigid structure. For levelling adjustments, it is an advantage to arrange each leg on pairs of wedges (" Fox wedges "). One method of waterproofing consists of coating the inside of the casing with a layer (say $\frac{1}{16}$ to $\frac{1}{8}$ inch thick) of bitumen or marine-glue, over which is spread a sheet of calico, running flat along the bottom of the casing and being continued vertically up the sides. Hot irons are then pressed over the whole surface of this calico so that the bitumen or marine-glue thoroughly impregnates the fabric. Finally, two or three coats of bitumastic paint or varnish are applied.

Asphalt provides another means of ensuring water-tightness (see Section 2 of Chapter VII), while, instead of a timber casing, it may be desirable or economical in some cases to adopt brickwork lined with cement-mortar or concrete. Another alternative is steel plates, welded, riveted or bolted together. Pre-cast concrete slabs may also be used.

It will be appreciated that a saving of filling-material may be effected by varying the depth of the box from one region to another, according to the slope of the river and the soundings in its deep-water channels. Indeed, the deeper portions, in some cases, are best accommodated in a space excavated *below* floor-level.

Again, instead of supporting the box on legs *above* floor-level, it may sometimes be an advantage to use an existing concrete floor as the base-slab.

A portion of the tidal model of the Great Ouse and the Wash was actually built upon the laboratory floor, and to facilitate observation a concrete-lined trench was constructed alongside, so that the observers could stand in this trench without suffering the cramped position otherwise imposed.

When this box or casing has been completed and tested for water-tightness, the next stage consists of placing inside it a number of vertical sheets, or templates, which are cut to the shape of cross-sections of the banks and the immobile bed of the river. These templates may be made of cardboard, about $\frac{1}{16}$ inch thick, which have been thoroughly soaked in hot paraffin wax after the profiles have been cut, or some of sheet-metal such as copper (a more expensive method).

The cutting of these templates will be understood by reference to Fig. 127*a*, from which it will be seen that two lines are marked on the sheet:

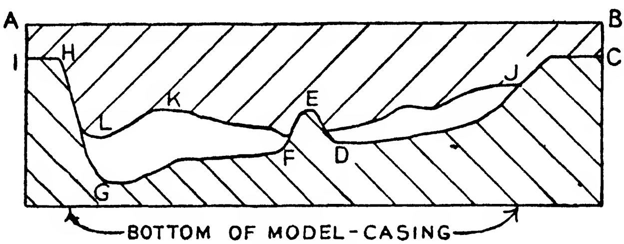


FIG. 127*a*.

a lower one (*CDEFGHI*) representing the permanent section and an upper one (*ABCJEKLHI*), so drawn that the space between upper and lower lines represents the *mobile* material (sand or mud, etc.).

By cutting along the lines, two templates are obtained: one which is placed permanently inside the model casing, and another which is retained as an aid in placing the sand or other movable material.

If, by borings or trial-pits, the level of true bed-rock or other hard surface has been ascertained, the lower templates may be made to suit. This, however, is not generally the case, and it is accordingly necessary to use judgment in allowing a thickness of mobile material sufficient to provide for any likely scour. The changes of depth which are known to have occurred in the history of the river itself may serve as a guide in this process; otherwise an arbitrary allowance equivalent to, say, 10 feet of sand may be adopted.

The space between the lower templates as fixed in the model-casing is filled in partly with clinker, broken stone, bricks, gravel or other convenient substance, and the rest (about ½ inch in thickness) with cement-mortar. It is an advantage, in order to simplify later modifications and the carving of details which come between templates, if this mortar is made comparatively weak, for example, 1 part of Portland cement to 6 parts of sand (diameter about 0·01 inch).

After the cement-mortar moulding has been completed in this way, a systematic check is made by detailed reference to the charts, and any details in the form of piers, breakwaters, islands, are introduced. These may be made of metal, wood or cement as seems most convenient, but if wood is adopted it is necessary generally to safeguard against warping by varnishing the wood or soaking it in paraffin wax.

Next comes the moulding of the mobile bed. This is accelerated by using the *upper* templates. The sand or other material is well moistened before being placed in position. The upper templates are then removed.

While it is evident that the upper templates very greatly assist this initial moulding, yet it is essential for accuracy to check over this work, after water has been admitted to the model, by the method of "straight-edge and sounding stick", which is described in the present chapter in connection with surveying.

In comparatively wide parts of a model, a considerable economy of time may be effected, provided that the bed is free from rock-formation, by making the *lower* templates to cover only a limited portion of the width, confined to the coastline. This system is illustrated in Fig. 127*b*.

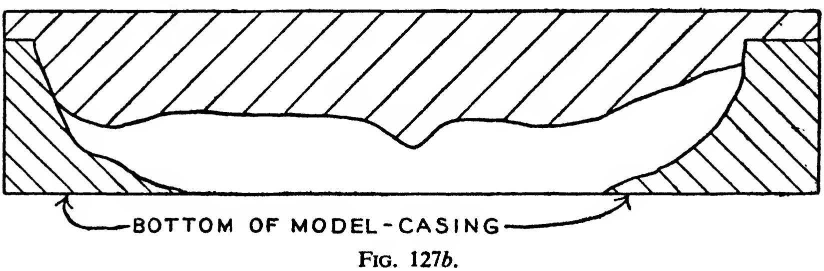


FIG. 127*b*.

It will be appreciated that, in order to facilitate the work of the observers, a wide model has to be spanned either by a plank resting on portable trestles placed outside the model-casing, or by a more elaborate structure in the form of a light bridge which may be carried on runners capable of rolling along side-rails.

2. Methods of Surveying River Models.

A straightforward, if laborious, method of surveying consists of having carefully levelled surfaces round the model on which a straight-edge may be moved to any desired section. The straight-edge is graduated, say in inches and fractions thereof, and a " sounding stick " is held against it so that the pointed end of the stick just touches the bed * of the model river (Fig. 128). This sounding stick is graduated in feet or fathoms (to

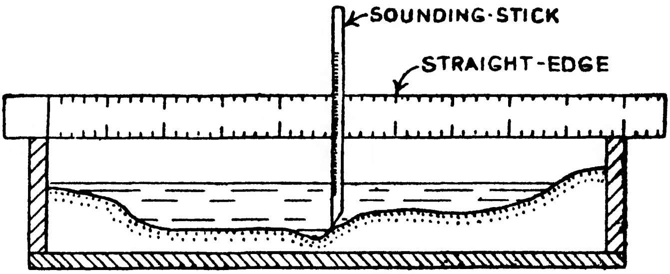


FIG. 128.

the vertical scale of the model) with reference to some chosen datum, and the observer calls to his assistant the distance as measured on the straight-edge, followed by the depth as given by the sounding stick. The assistant records these data in a book (say a land-surveyor's field-book), and sometimes it is an advantage to have a second assistant simultaneously plotting the results on a plan. On the plan thus plotted, either during or after the survey, contours are interpolated. With practice this method may become quite rapid (for example, the whole of the Severn tidal model (horizontal scale 1 : 8,500) was on occasion surveyed in 1½ working days), and it possesses some advantages compared with more elaborate systems. Thus, it is quite flexible in that the observer may survey any sections demanded by the shape of the channels at a particular time; as he proceeds he is able to make quite sure that all the relevant data is being obtained; a permanent record is obtained by the entries in the field-book in a form which is extremely convenient for analysis. Again, the map produced from the information is in a form exactly similar to a hydrographic chart of the actual river, while any cross-sections may also be drawn. The method may, of course, be usefully supplemented on occasion by a photograph or series of photographs showing special features.

An alternative system consists of setting the water in the model to a series of particular levels in turn and surveying, either by the straight-edge and sounding-stick method or by photography, the contour of con-

* The points chosen for sounding are preferably such as to define contours, changes of slope and maximum channel depths.

tact between the water and the model bed. It still remains, however, to obtain the depths of the channels, and this fact, combined with the additional time involved in setting the water levels, makes the system in no way superior to that already described. A variation of this system consists of marking the outline of contours by means of a number of strings which are afterwards photographed.

Yet again, there is a method of mounting across the model a pantographic recording mechanism, which may be moved on wheels until it lies above a certain line of section. A sounding rod is then traversed over the bed and the profile of the section simultaneously plotted, through the pantographic mechanism, on a sheet of drawing paper attached to a vertical board which forms part of the apparatus. It still remains, however, to convert the cross-sections so drawn into plan-form for mapping purposes.

3. Methods of Measuring Current Velocities in Models.

The method chosen for velocity observations depends essentially on the magnitude of the current, the dimensions of the model channel and the type of model, i.e. whether tidal or non-tidal. For non-tidal work, the pitot tube or pitometer is often ideal: it may be supported on a bracket which slides along a movable beam spanning the model, so that the pressure holes of the pitometer may be set at any place or at any depth below the surface of the stream. A simple form of pitometer is shown in Fig. 129. This type of pitometer is very easily made, as will be appreciated from the sketch, by brazing together two bent tubes each containing an orifice. The tubes are arranged so that one of the two orifices faces downstream if the other is looking upstream, and while it is true that such a meter does not possess the merits of the more elaborate types (e.g. close constancy of its coefficient over a wide range of velocities and high accuracy of locating the direction of the current under measurement), nevertheless it may be quite adequate in many cases, provided of course that its coefficient of performance is first determined by calibration

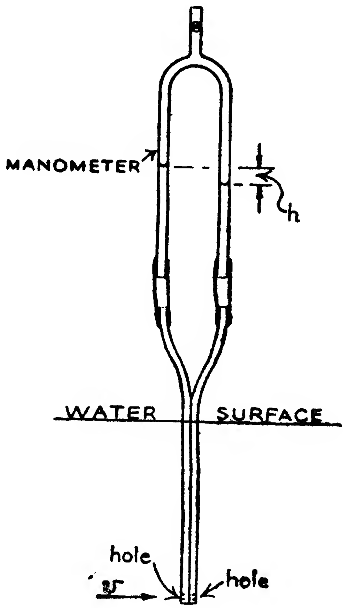


FIG. 129.

under conditions similar to those obtained when the instrument is in use.

One method of calibration consists of comparing the pitometer reading with velocities measured by submerged floats or coloured matter in a glass-sided flume. Another way is to traverse a cross-section of a pipe or channel, that is, to take readings with the tube at a number of points throughout the width and depth of the stream, whose mean velocity $\bar{v}$ is determined by measuring the dimensions of the cross-section and by collecting the discharge in a vessel in a measured time.

Calling h the differential head (measured in feet of water) corresponding with a velocity v feet per second at a particular point as indicated by the pitometer, then

$$v = C\sqrt{2gh},$$

where C is the coefficient of the pitometer.

The value of C depends to some extent on the velocity v itself; also upon the absolute size and shape of the pitot tube, and upon whether the meter is calibrated by holding it fixed in a moving stream or by moving the pitot tube through still water. (See an article by Prof. A. H. Gibson in *The Mechanical Properties of Fluids* (Blackie), p. 176.)

A difficulty is likely to arise, however, in the measurement of the small velocity head corresponding to comparatively low velocities. Three methods are available for meeting this difficulty:

(*a*) To fix the inverted U-tube manometer with its limbs inclined at an angle α to the horizontal. The differential head is then magnified in the ratio cosec α : 1, compared with its value if the U-tube were vertical.

(*b*) To use a liquid rather lighter than water as the fluid above the water in the manometer. Calling ρ the specific gravity of the liquid relative to water at the same temperature, the magnification becomes

$$\left(\frac{1}{1-\rho}\operatorname{cosec}\alpha\right) : 1,$$

where α is again the inclination of the gauge. Thus, if α is 90° (gauge vertical) and toluene is used, the magnification obtained is as follows:

TABLE XLVIII

Temp. °*F.*	50	59	68
Temp. °*C.*	10	15	20
$\frac{1}{1-\rho}$	8·00	7·75	7·52

(*c*) The pitometer may be connected to a Chattock Tilting Manometer, with which small pressure-differences may be detected. This instrument has been frequently described, e.g. in:

"Experiments with a Tilting Manometer for Measurement of Small Pressure Differences", J. R. Pannell, *Engineering*, Sept. 12, 1913, p. 343;

Hydraulics, p. 12, F. C. Lea (Arnold);

An Introduction to Fluid Mechanics, p. 222, Alex. H. Jameson (Longmans);

Hydraulic Measurements, p. 25, H. Addison (Chapman & Hall);

Hydraulics and the Mechanics of Fluids, p. 432, E. H. Lewitt (Pitman).

The U.S. Waterways Experiment Station makes use of the Bentzel Velocity Tube (see Fig. 130), an instrument constructed to measure a range of velocities between 0·10 and 4 or 5 feet per second. To quote from page 14 of Paper 17 from this Station, entitled *Studies of River Bed Materials and their Movement, with Special Reference to the Lower Mississippi River* (January 1935):

"It has been found to be ideally suited for velocity measurements at the Station, where it has entirely superseded the use of the Pitot tube and other current-measuring devices. The principle on which the meter works is as follows:

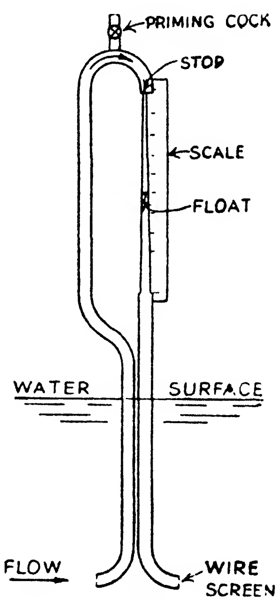


FIG. 130. Velocity tube invented by Carl E. Bentzel, Research Assistant, U.S. Waterways Expt. Station.

(Based on Plate 10, p. 15, Paper 17 of U.S. Waterways Expt. Station, Vicksburg, Mississippi, Jan. 1935.)

"The water flowing into the upstream leg of the tube causes a velocity head to be created; the velocity head on the downstream leg, on the other hand, is negative. This difference in head causes the circulation through the tube of a small quantity of water, the amount depending upon the velocity of the water flowing through the flume. In the downstream leg of the tube, which has an even taper inside, is a small float, made of a piece of capillary glass tubing, closed at both ends, and so constructed that it has a very slight buoyancy.

"When there is no flow through the tube, this float rises until it rests against a wire stop in the top of the tapered tube. When water is flowing through the tube, however, the impact of the flowing water causes the float to be pushed down the tapered tube. At some point within the length of the tapered section, the unit impact force of the water, reduced by the enlarged section, exactly balances the buoyancy force of the float, which then comes to rest. It has been found that this instrument can be calibrated very closely by towing it through still water, and that for every velocity of flow, within the range of the instrument, there is a corresponding position of the float within the tapered section, which will not vary in successive trials by more than 1 or 2 per cent. By the use of floats of various specific gravities, and tapered tubes of varying inside dimensions, almost any velocity can be measured with the tube. Necessarily the effect of temperature on the viscosity and the density of the water must be considered, although a variation in temperature of only a few degrees has only a small effect."

The Bentzel Tube was invented by Mr. Carl E. Bentzel, Research Assistant, U.S. Waterways Experiment Station.

A method commonly used for velocity measurement in either tidal or non-tidal work is that of timing some kind of float over a measured distance marked by fine wires. If the float consists of a patch of aluminium powder, true surface velocities will be measured; sub-surface estimates may be made by using a small quantity of coloured matter such as permanganate of potash or aniline black. Alternatively cork or wooden floats, weighted with a blob of wax or plasticene or with a drawing pin, may be arranged so as to have a depth of submergence equivalent, say, to that of the floats used in the actual river. In any case, the precautions to take are (i) the release of the float some little distance from the leading wire marking the measured distance, so that the inertia and initial disturbance have time to be overcome; (ii) the repetition of the observations, so as to obtain a sufficient number of consistent values to give a reasonably reliable average.

At the laboratory of l'École des Ingénieurs-Hydrauliciens de Grenoble,* the examination of currents at various depths has been made by means of a solution of benzene and trichlorethylene, having the same density as that of water, on contact with which it forms coloured droplets.

A method used extensively on the Rangoon Tidal Model has been described by Mr. Oscar Elsden (" Investigation of the Outer Approach-Channels to the Port of Rangoon by Means of a Tidal Model ", *Journ. Inst. C.E.*, June 1939). He states: " This method of examination was

* " Experiments with a Scale Model of the Edouard Herriot Dock at Lyon ", *The Engineer*, vol. CLXVII, April 21, 1939, p. 490—based on an article by L. Levin in *Le Génie Civil*, 1st Oct., 1938.

made possible by the development by Mr. T. L. Norfolk, M.Inst.C.E., of a new type of current-meter which was used by him in the model of the Mersey estuary. The device consists of a small bead suspended in the water by a fine hair. Any motion of the water will deflect the bead from its vertical position; the amount of the horizontal deflection bears a definite relation to the velocity of the stream, and is determined by calibration, whilst the direction of the deflection is of course the same as the direction of flow. The deflection is measured by a vertical sighting tube, which is continually moved by hand so that the bead remains in the line of sight. As the tube carries an autographic recording pencil operated 15 times per tide by a special switch on the main engine, the pencil draws a polar graph of successive positions of the bead."

The current velocity observations so far discussed are those at localized points or over comparatively short stretches of channel. It is also frequently valuable to observe the drift of the currents over a long distance. This may be done by plotting the course of aluminium dust, floats or coloured matter; for this purpose it is useful to divide the top of the model into a number of squares by means of horizontal string or wire and to number these squares for identification purposes. As the drifting material crosses the line of one of the strings or wires, the observer, who is stationed on a movable plank or bridge just above the model, may mark the point of crossing with a wire clip or other device.

An elaboration of this technique was made use of on the Rangoon Model, and Mr. Elsden writes:* . . . " a sheet of plate glass was obtained, 6 feet by 3 feet in size, and was mounted in a strong wooden frame, which could be supported over any area by a movable wooden gantry. By this means the glass sheet was suspended horizontally over the model, as close as possible to high-water level. The glass was divided into ¼-mile squares by a network of threads on the underside and was placed over any area where current readings were desired. If a float were released so as to pass beneath the plate glass, its successive positions could be recorded thereon at 1-second intervals by means of paint-spots: these formed a continuous track across the glass, and were subsequently replotted on squared paper to form a permanent record. The timing was done by a metronome arranged to beat at 1-second intervals. This scheme is due to Mr. Ralph Freeman, M.Inst.C.E., and worked perfectly."

Again, with the aid of photography, it is possible to use some kind of float as a measuring device for either localized velocities or drift observations. For this purpose, the float may consist of a lighted candle or alternatively a small patch of aluminium powder may be adopted. If the camera is provided with a timing mechanism which opens and closes its

* *Loc. cit.*, p. 22.

shutter at known intervals, a series of spots or lines will be photographed from which both direction and speed may be ascertained. In the absence of this apparatus, the camera shutter may sometimes be opened at suitable intervals by hand and the corresponding times registered by placing a "seconds-hand" watch in the field of view.

4. Measurement of Water Levels.

The simplest way of measuring water levels is by estimating the reading, on a vertical graduated gauge, corresponding to the surface of the surrounding water (see Fig. 131). The gauge itself may be a strip of thin copper, painted white and marked in Indian ink with scale divisions equivalent to feet or inches in the full-size prototype. Care and practice are needed in estimating the gauge-reading corresponding with the lowest point of the meniscus, that is, the surface of the surrounding water.

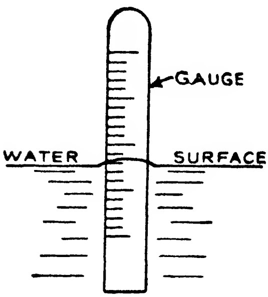


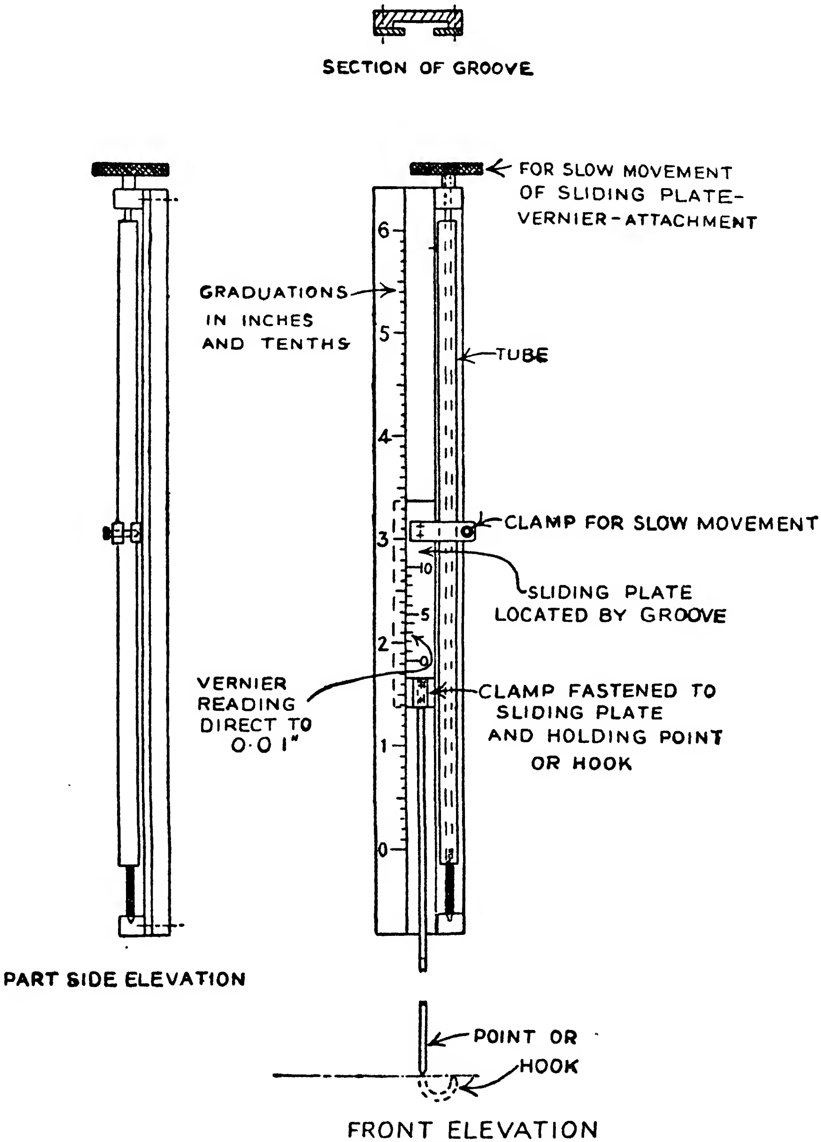
FIG. 131.

If the model is tidal, a tide curve connecting water level with time may be obtained by having two observers, one of whom, *A*, keeps his eye fixed on the gauge and calls out readings to the other, *B*, as observed at 5-second intervals. Observer *B*, who records the readings called out to him by *A*, has a stop-watch so that he can announce the times thus: "3, 4, 5" (seconds) . . . "3, 4, 5" . . . , *A* taking his reading on the word "five", and *B* recording it during seconds 1 and 2. Alternatively, a metronome may be used to beat the required time-intervals.

Another and more accurate device is to read the levels by means of a pointer or hook-gauge connected either to a vertical scale equipped with a vernier reading directly to 0·01 inch, say, or to a micrometer-head reading directly to 0·001 inch. These arrangements are sketched in Figs. 132*a* and *b*. In tidal work, the pointer type is used for rising levels and the hook for falling. A tide curve may be obtained by setting the pointer- or hook-gauge to any vernier or micrometer reading and observing with a stop-watch the time which elapses between the beginning of the flood-tide or any other convenient instant and the water surface touching the pointer or hook. Though more accurate, this method takes a considerably longer time.

The water gauge is arranged preferably to give a direct measurement in the channel itself, but it is sometimes more convenient to take readings on the water surface in a stilling-basin which is in adequate communication with the channel.

Again, it is convenient in some models to have a pressure hole or a series of pressure holes bored in the bottom or sides of the channel and connected with rubber to glass gauge tubes or, if pressure differences only



G. 132*a*

are required, a pair of pressure holes may be connected to the two sides of a differential manometer.* It is important to have the pressure holes *flush* with the bottom or side of the channel. Referring particularly to

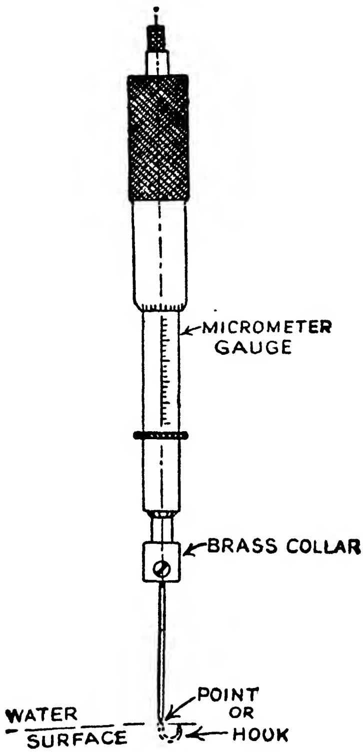


FIG. 132*b*.

the arrangement shown in Fig. 133, it is to be observed that the water surface in the gauge tube will tend to be higher (due to capillary action) than in the channel itself. This discrepancy may be of the order of a tenth of an inch with a glass tube ½ inch in diameter, but its value for a given tube is likely to vary from time to time with the state of cleanliness. For clean tubes the effect is *inversely* proportional to their diameter. Another point is that owing to dynamic effects, a pressure reading of this kind may not be a direct measure of the water-surface level (see pages 88 and 370 for striking examples of this phenomenon).

The zero levels of the various water gauges in the model may be

* For details concerning many possible types of manometer, the reader may wish to consult *Hydraulics and its Applications*, Gibson (Constable), and the publications cited on page 383 of the present chapter.

established relative to one another by observing the gauge readings when water is standing still in the model. Such observations are called " static readings ".

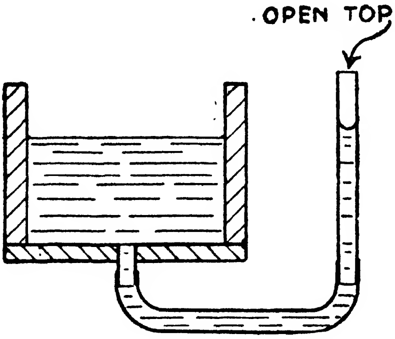


FIG. 133.

Yet another very simple device—capable of an accuracy quite good enough in many cases—consists of attaching a brass rod, say 2 inches long and ⅛-inch diameter, to the end of a steel rule graduated in hundredths of an inch. The end of the rod remote from the rule is ground to a sharp point. Fig. 134 illustrates this simple idea, which gives a measurement

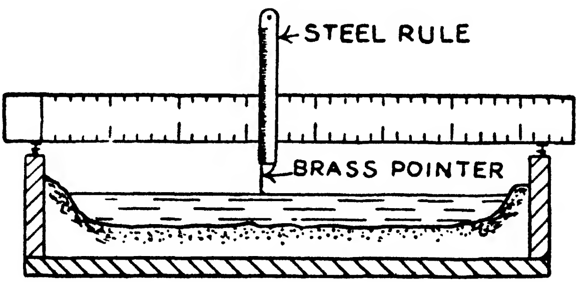


FIG. 134.

of the depth of the water surface below a horizontal line marked on a straight-edge. The straight-edge rests on guides erected along the edges of the model, and the beauty of the method lies in the speed with which the observation may be made and the point of observation transferred to another position in the model. As a check on the level of the guides supporting the straight-edge, readings are taken, at the various positions, to a static water surface.

5. Water Supply; Silt Supply; Mechanical Generation of Tides and Waves.

These subjects have already been discussed in this book, especially in Chapters IV, V, VI, VII, IX, X and XII.

CHAPTER XIV

MODEL EXPERIMENTS CONCERNING FILTERS, EARTH DAMS (PERMEABILITY), AND STRESSES IN DAMS

1. A brief reference will now be made to the use of models in the study of the flow of fluids through porous media. Examples of such problems which come to mind are the seepage of water through and under dams and the flow systems of water or oil wells. In his classical experiments on the flow through sand filters, Darcy * discovered that the rate of flow was directly proportional to the head between the inlet and outlet faces of the bed and inversely proportional to the thickness of the bed. This law of linear proportionality between resistance and velocity is known to be valid only over a limited range, in fact, provided the Reynolds number $\frac{vd}{\nu}$ does not exceed a certain value. Here v is the rate of flow divided by the cross-sectional area of the bed, ν is the kinematic viscosity of the fluid and d the effective diameter of the grains which make up the filter-medium. According to Muskat, † a safe lower limit for this critical Reynolds number is 1 (unity), " with d chosen as any reasonable average diameter of the sand grains ". The same authority states ‡ that " in the great majority of flow systems of physical interest the flow will be strictly governed by Darcy's law, except possibly in very localized parts of the porous medium of very limited dimensions ".§

Laboratory experiments have frequently been of value in demonstrating the streamlines in such examples as beneath a dam constructed with or without sheet piling at the heel or toe. Similarly models of permeable earth dykes have been made in tanks having glass sides through which the paths of flow could be observed when rendered visible by the injection

* Darcy, H., *Les fontaines publiques de la ville de Dijon*, 1856.

† *The Flow of Homogeneous Fluids through Porous Media*, Muskat (McGraw-Hill, 1937), p. 67.

‡ *Loc. cit.*, p. 68.

§ Concerning laboratory experiments on the flow both of liquids and gases through beds of granular material, the reader is advised to consult the very interesting and comprehensive Papers by Dr. H. E. Rose, published in the *Proceedings* of the Institution of Mechanical Engineers, 1945. The titles of these Papers are: " An Investigation into the Laws of Flow of Fluids through Beds of Granular Materials "; " The Isothermal Flow of Gases through Beds of Granular Materials "; " On the Resistance Coefficient-Reynolds Number Relationship for Fluid Flow through a Bed of Granular Material ". Among the subjects discussed by Dr. Rose are the relationships between the resistance and the Reynolds number, the voidage of the bed, the size of the container and the depth of the bed.

of dyes at various points along the face of the dam. One method is to use a solution of potassium chromate as the seepage fluid and to introduce silver nitrate solution at points in the surface of the sand or othe permeable material.

Use has also been made of the analogies existing between the quantities of hydrodynamics and those of electrostatics and electro-conduction. Thus pressure is analogous to electrostatic potential or to voltage; the ratio of permeability to viscosity is analogous to dielectric constant or to specific conductivity; surfaces of equal pressure to equipotential surfaces; streamlines to tubes or lines of force or flow, while " Darcy's law " of the proportionality of resistance to velocity is represented by Maxwell's law of dielectric displacement in electrostatics and by Ohm's law in current electricity. For a discussion of the implications involved and their application to electrical models of fluid flow, the reader is recommended to consult the beautiful book by Dr. Muskat, already cited, viz. *The Flow of Homogeneous Fluids through Porous Media* (McGraw-Hill, 1937); one example only will be given here to whet the reader's appetite:

Fig. 135 represents the circuit used for studying the seepage of water through a dam with vertical faces. *AEDF* is a *high-resistance* sheet which

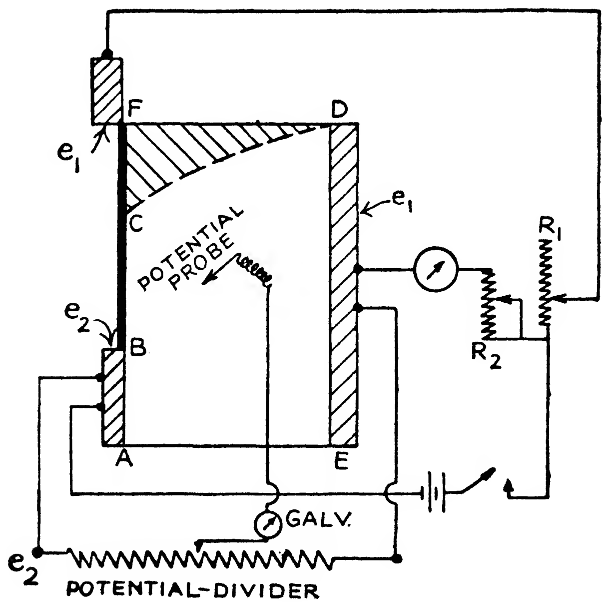


FIG. 135.

(Based on Fig. 107, entitled "The Electrical Circuit for Model Expts. with Gravity-Flow Systems (after Wyckoff and Reed, *Physics*)", of "*The Flow of Homogeneous Fluids Through Porous Media*" (Muskat).) (*By courtesy of McGraw-Hill Publishing Co., Ltd.*)

serves for the cross-section of the permeable dam. It is constructed of Bristol board or similar material uniformly sprayed with 12 to 20 coats of graphite colloid such as aquadag. ED is a highly conducting electrode (simulating the inflow face); it is maintained at a potential e_1; AB is the outflow surface maintained at e_2. It is supposed that in the actual structure water stands to the top at the back DE of the dam, and only up to the level of B at the front. BF is a resistance strip, its terminals being at potentials e_1 and e_2. The resistance of BF is much smaller than that of the sheet $AEDF$, and consequently the current in BF greatly exceeds that in $AEDF$. Accordingly, the potential along BF remains closely linear. The portion DFC is cut away along the line DC, which is found by trial and error to be such that the potential of any point in DC is directly proportional to the distance of that point above AE. The boundary CD is then established as the electrical equivalent of the free water surface of percolation through the body of the dam: once CD has been found, the potential distribution within $ABCDE$ may be mapped out, and the curves which intersect the potential contours thus obtained become streamlines (see Fig. 136). Moreover, if the current I through the meter with the potential-divider circuit open is measured for a potential-drop $e_1 - e_2$, the flux or rate of flow in the corresponding dam may be estimated from the formula

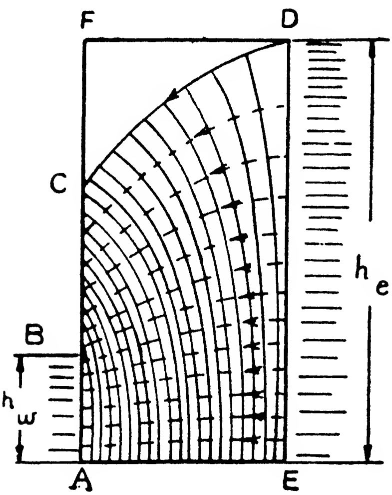


FIG. 136.

(Based on Dr. Muskat's Fig. 108, entitled "Potential and Streamline Distributions in a Dam with Vertical Faces, as obtained by Electrical-Model Experiments (after Wyckoff and Reed, *Physics*).)"
(*By courtesy of McGraw-Hill Publishing Co., Ltd.*)

$$Q = \frac{K\gamma g(h_e - h_w)}{\mu} \cdot \frac{\sigma I}{e_1 - e_2},$$

where Q = flux,

K = permeability of material of dam,

h_e = depth of water on upstream face,

h_w = depth of water on downstream face,

σ = specific resistance of sheet $AEDF$ used in electrical model,

μ = viscosity of water,

γ = density of water.

This ingenious method is, of course, applicable to the more complicated shapes of sections as commonly built (see Fig. 137).

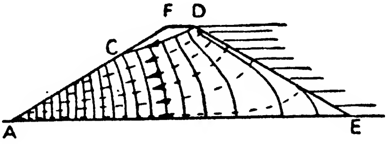


FIG. 137.

(Based on Dr. Muskat's Fig. 110, entitled "Potential and Streamline Distributions in a Dam whose Faces have an Inclination of 30 deg., as obtained by Electrical-Model Experiments (after Wyckoff and Reed, *Physics*).)" (*By courtesy of McGraw-Hill Publishing Co., Ltd.*)

If the dam contains an impermeable central core, a corresponding hole is cut in the conducting sheet; the effect of variations in permeability is investigated by varying the number of coats of graphite which are sprayed over the sheet.

2. Stresses in Dams.

Suppose the stress at any point to be S, while l represents any representative linear dimension of the dam. Let the material of the dam have a density w, while the fluid pressing against it has a density ρ. Suppose also that an external load W is applied and that the material of the dam has a Young's modulus of direct elasticity E and a Poisson's ratio σ. We may then assume that, for a given shape of dam, S depends upon l, w, ρ, E, σ, g and W. Here we have introduced 8 quantities, involving altogether the three primary ones of mass, length and time. We may anticipate, therefore, 5 dimensionless groups in our dimensional analysis.

Thus, calling

$$N_1 = S^{a_1} l^{b_1} w^{c_1} \rho,$$
$$N_2 = S^{a_2} l^{b_2} w^{c_2} E,$$
$$N_3 = S^{a_3} l^{b_3} w^{c_3} \sigma,$$
$$N_4 = S^{a_4} l^{b_4} w^{c_4} g,$$
$$N_5 = S^{a_5} l^{b_5} w^{c_5} W,$$

and carrying out the usual process of analysis, we find

$$N_1 = \frac{\rho}{w},$$

$$N_2 = \frac{E}{S},$$

$N_3 = \sigma$ (itself a ratio and therefore dimensionless),

$$N_4 = \frac{lgw}{E},$$

$$N_5 = \frac{W}{Sl^2}.$$

Hence
$$\frac{E}{S} \propto \phi\left\{\frac{\rho}{w}, \sigma, \frac{lgw}{E}, \frac{W}{Sl^2}\right\},$$
$$\propto \phi\left\{\frac{\rho}{w}, \sigma, \frac{lgw}{E}, \frac{W}{Sl^2}\cdot\frac{S}{E}\right\},$$
$$\propto \phi\left\{\frac{\rho}{w}, \sigma, \frac{lgw}{E}, \frac{W}{l^2E}\right\},$$
or stress
$$\propto E\psi\left\{\frac{\rho}{w}, \sigma, \frac{lgw}{E}, \frac{W}{l^2E}\right\}.$$

Now a change of length (or a deflection) is proportional to $\dfrac{\text{stress}\times l}{E}$.

Hence
$$\frac{\text{change of length, or deflection,}}{l} \propto \psi\left\{\frac{\rho}{w}, \sigma, \frac{lgw}{E}, \frac{W}{l^2E}\right\}.$$

If a scale model be constructed, of scale $1: x$, it appears from the foregoing argument that in order that its several displacements or deflections shall equal those at corresponding points in the prototype divided by x (or, in other words, if deflection is to be proportional to l), then the quantities

$$\frac{\rho}{w}, \ \sigma, \ \frac{lgw}{E} \text{ and } \frac{W}{l^2E},$$

must be the same in the model as in the prototype. Using the suffix (1) to identify the full-size dam and (2) the model,

$$\frac{\rho_2}{w_2}=\frac{\rho_1}{w_1}; \quad \text{......(1)}$$

$$\sigma_2=\sigma_1; \quad \text{......(2)}$$

$$\frac{l_2gw_2}{E_2}=\frac{l_1gw_1}{E_1}, \text{ or } \frac{w_2}{E_2}=\frac{xw_1}{E_1}; \quad \text{......(3)}$$

$$\frac{W_2}{l_2{}^2E_2}=\frac{W_1}{l_1{}^2E_1}, \text{ or } \frac{W_2}{W_1}=\frac{E_2}{x^2E_1}. \quad \text{......(4)}$$

Combining (1) and (3),

$$\frac{\rho_2 l_2}{E_2}=\frac{\rho_1 l_1}{E_1},$$

or

$$\frac{\rho_2}{\rho_1}=\frac{w_2}{w_1}=\frac{l_1}{l_2}\cdot\frac{E_2}{E_1}=\frac{xE_2}{E_1}.$$

The model ought, therefore, to be made of such material that its physical properties satisfy the conditions

$$\begin{cases} \sigma_2=\sigma_1, \\ \dfrac{w_2}{w_1}=\dfrac{xE_2}{E_1}. \end{cases}$$

Moreover, any external loading should be applied according to the relationship,

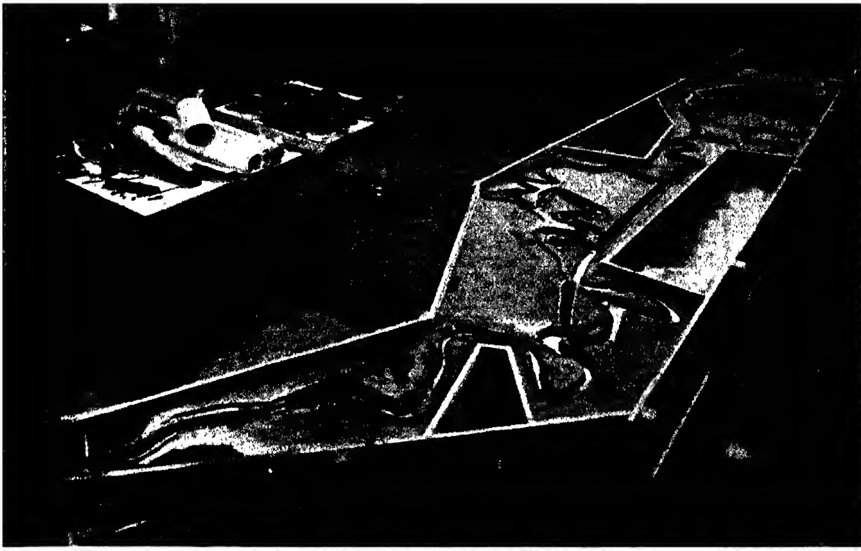
$$\frac{W_2}{W_1}=\frac{E_2}{x^2E_1},$$

and still further, if the actual dam is subjected to the pressure of a depth H of water, the model must be subjected to a depth $\frac{H}{x}$ of a fluid whose density equals that of water multiplied by $\frac{w_2}{w_1}$ or by $\frac{xE_2}{E_1}$. Stresses in the model may then be determined, say by strain measurements combined with a knowledge of the elastic modulus E_2, and such stresses multiplied by $\frac{E_1}{E_2}$ to give the corresponding stresses in the full-size dam.

An experimental investigation along these lines was actually conducted in connection with the design of the Boulder Dam, the loading fluid in the model being mercury. (See the U.S. Govt. publication on *Dams and Control Works*, 1938, or *Models in Engineering*, by R. B. Whittington and F. J. Sanger, Shanghai Association of the Institution of Civil Engineers and Engineering Society of China, Nov. 20, 1939.)

It is, of course, tacitly assumed that the stresses do not exceed the limit of proportionality of stress to strain in either the prototype or its model.

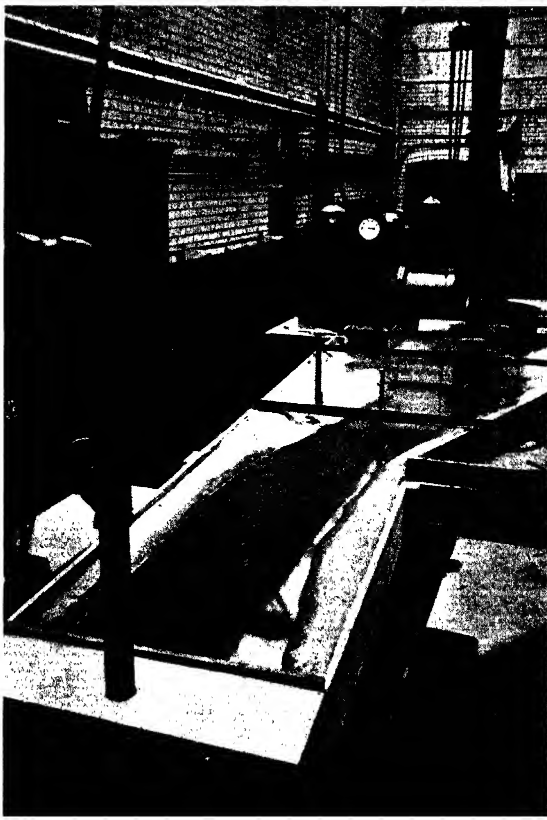
PLATES



I. PROFESSOR GIBSON'S MODEL OF A NON-TIDAL REACH OF THE RIVER MERSEY

At the left-hand end, water is being supplied at Ashton-upon-Mersey

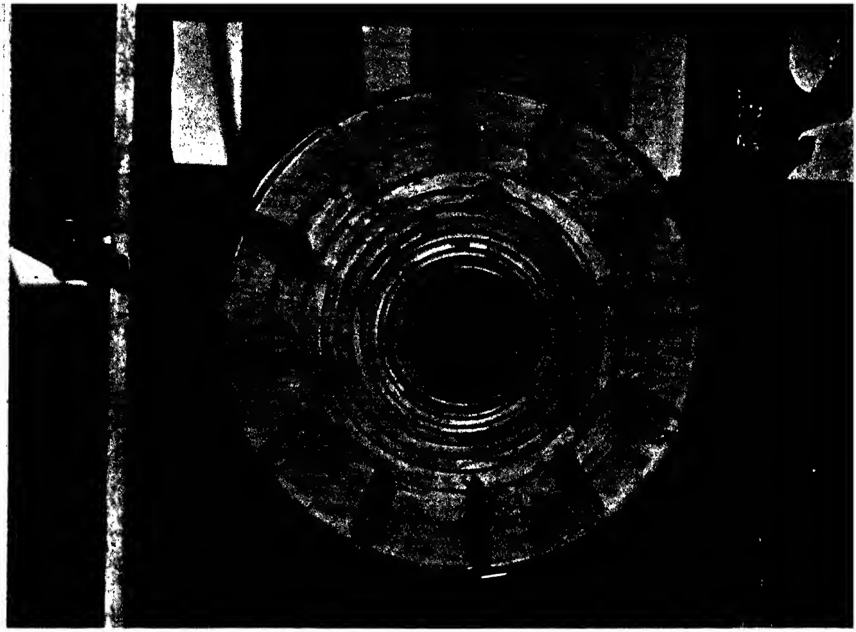
SCALES: Horizontal 1:800
Vertical 1:120



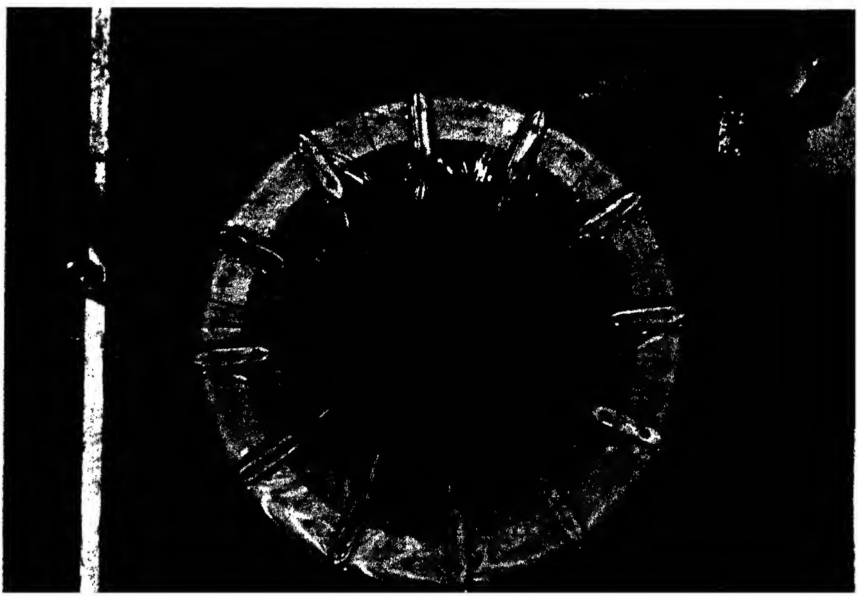
II. MODEL OF IRLAM WEIR AND A PORTION OF THE RIVER MERSEY

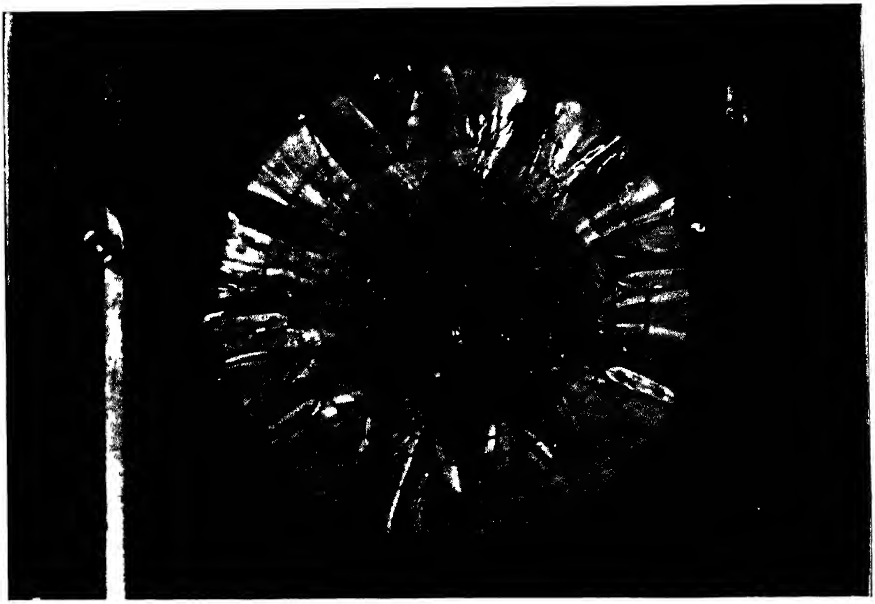
Erected over the concrete flume in the Whitworth Engineering Laboratories at Manchester University

SCALE: 1:50



II. & IV. PROFESSOR GIBSON'S MODEL OF OVERFLOW BELLMOUTH FOR LADYBOWER RESERVOIR, DERBYSHIRE

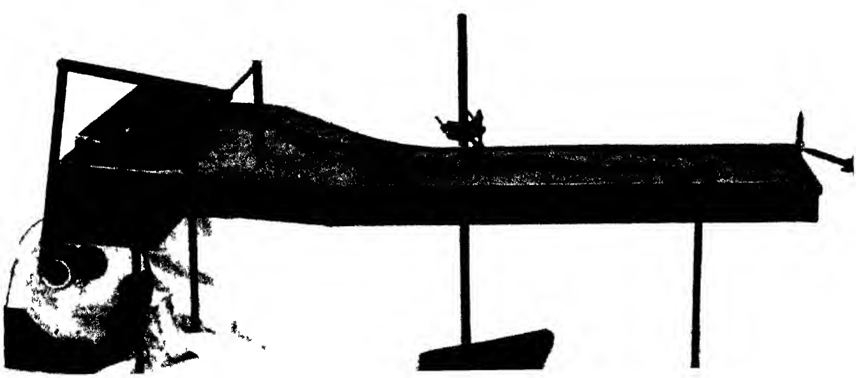


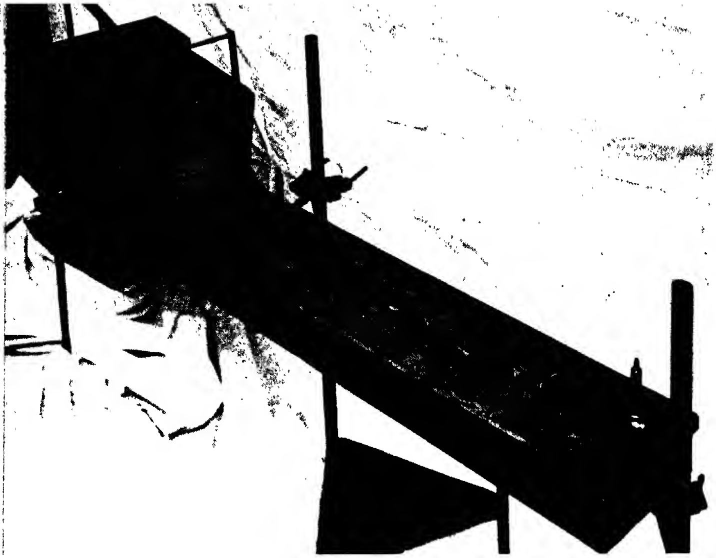


V. PROFESSOR GIBSON'S MODEL OF OVERFLOW BELLMOUTH FOR LADYBOWER RESERVOIR, DERBYSHIRE

VI. PROFESSOR GIBSON'S SMALL MODEL OF THE SEVERN ESTUARY

SCALES Horizontal 1 40 000
Vertical 1 366



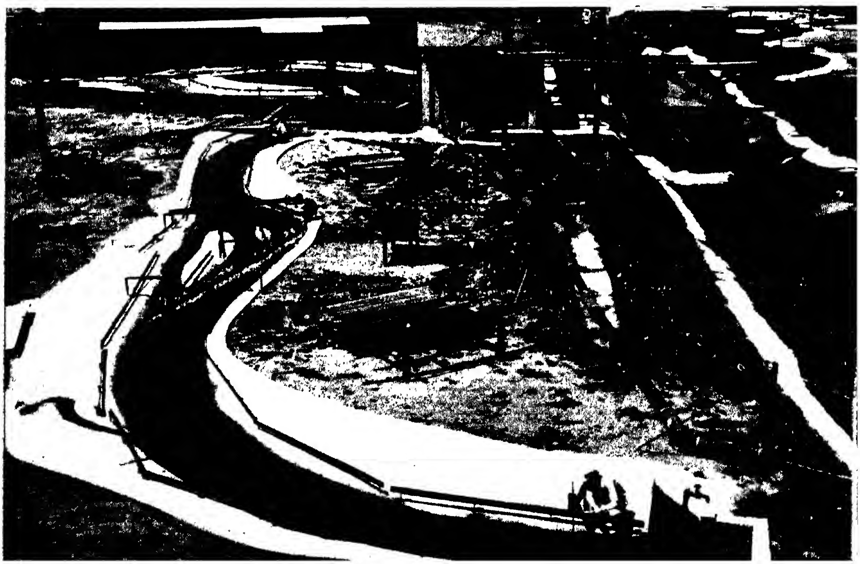


VII. PROFESSOR GIBSON'S SMALL MODEL OF THE SEVERN ESTUARY

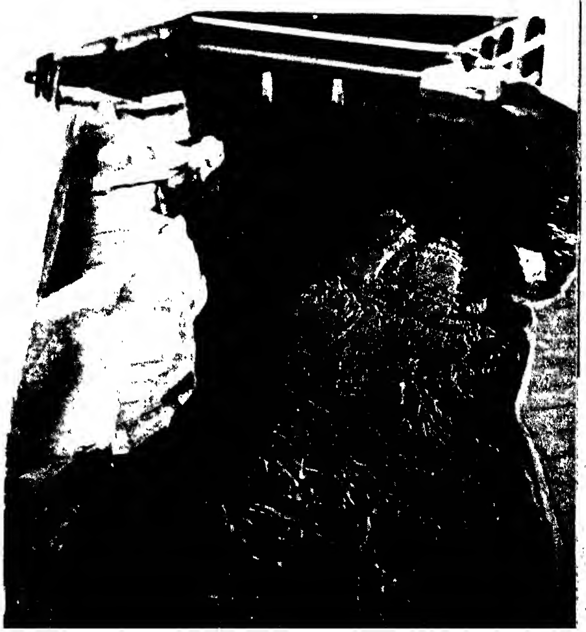
SCALES: Horizontal 1 40,000
Vertical 1:366

VIII. MODEL OF MANCHESTER ISLANDS REACH OF OHIO RIVER

SCALES: Horizontal 1:300
Vertical 1:80

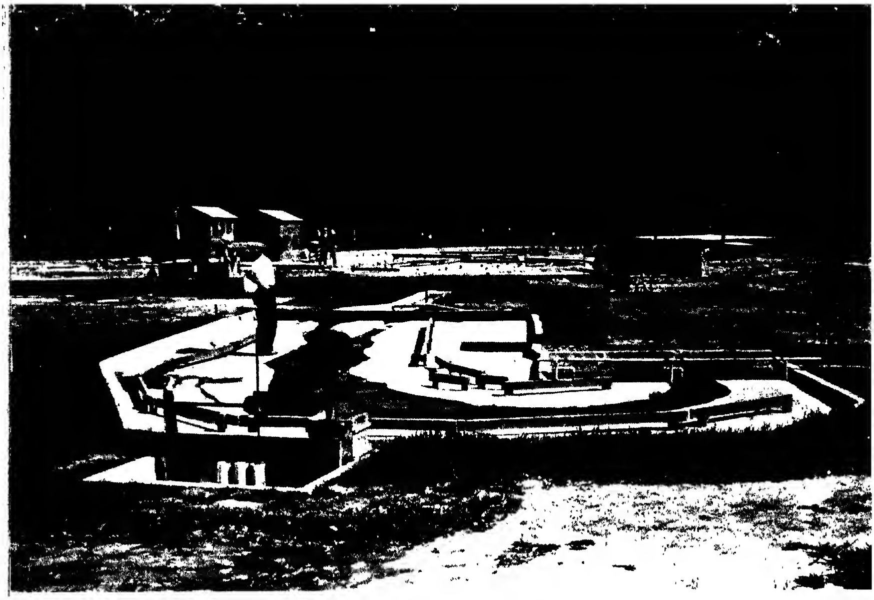


IX. AERIAL PHOTOGRAPH OF THE DEE ESTUARY



X. MODEL OF THE SEVERN ESTUARY

SCALES: Horizontal 1:8,500
Vertical 1:200

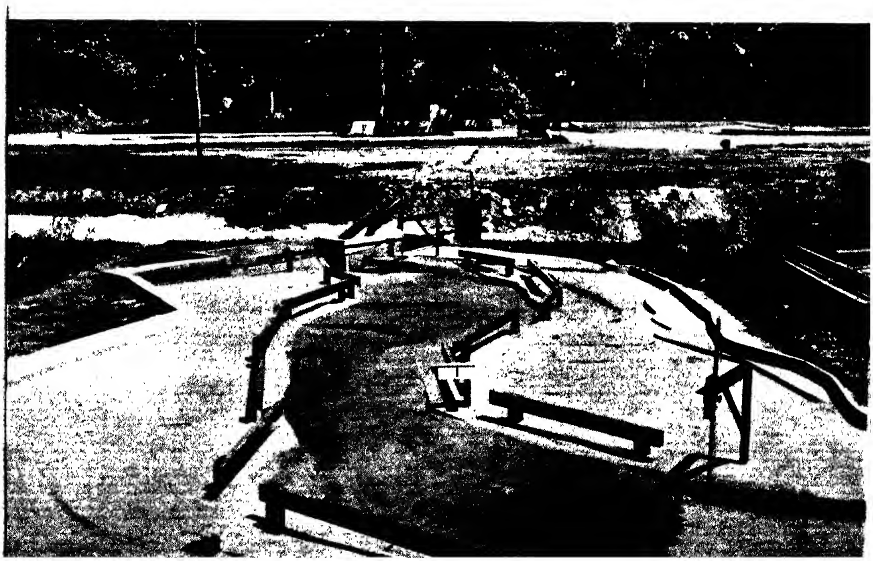


XI. GENERAL VIEW OF BROOKS POINT MODEL

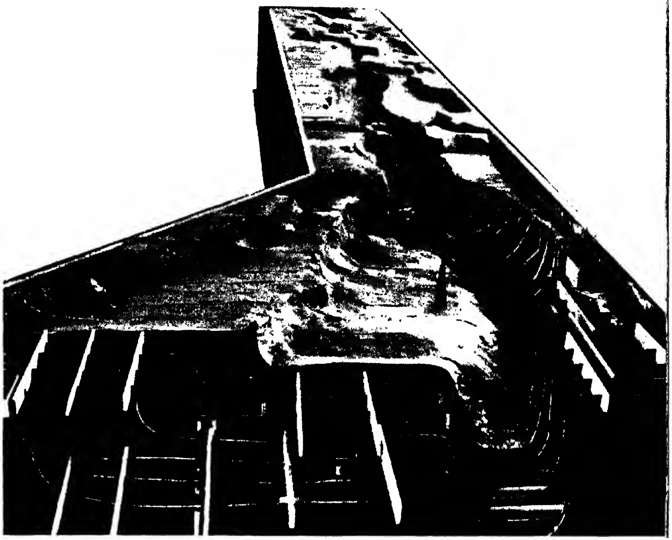
SCALES: Horizontal 1:1000
Vertical 1:125

XII. GENERAL VIEW OF POINT PLEASANT MODEL, WITH POINT MOULDED TO CONFIGURATION SHOWN BY SURVEY OF AUGUST, 1932

SCALES: Horizontal 1:1000
Vertical 1:125 [See Text]



SCALES: Horizontal 1:8500
Vertical 1:100



XIII. CARDBOARD TEMPLATES IN POSITION FOR CEMENT-MORTAR MOULDING OF FIRST SEVERN MODEL

XIV. THE SEVERN TIDAL MODEL SEEN FROM THE SEAWARD END

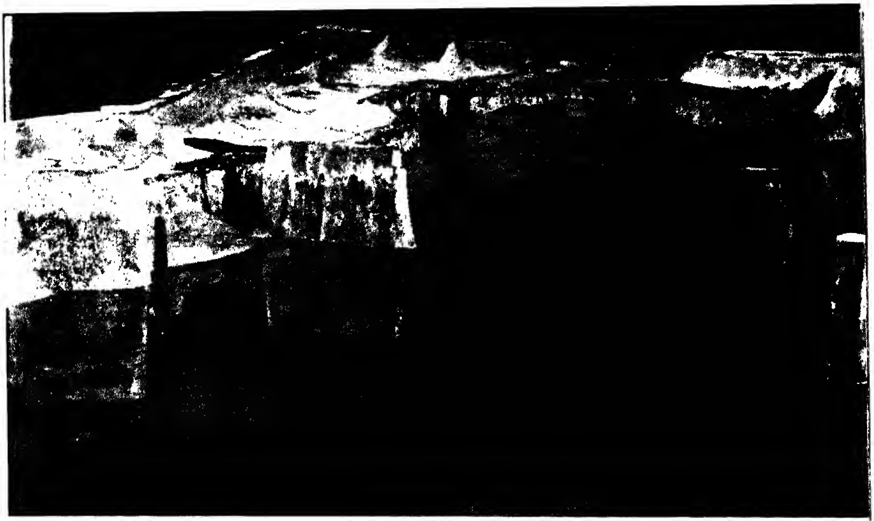
The model barrage is seen in this view, with its two solenoids for operating the gates

SCALES Horizontal 1:8500
Vertical 1:200

XV. THE SEVERN TIDAL MODEL SEEN FROM THE UPSTREAM END

On the right-hand side is a projecting wooden box incorporating a portion of the River Wye not included in the model as originally built

SCALES: Horizontal 1:8500
Vertical 1:200



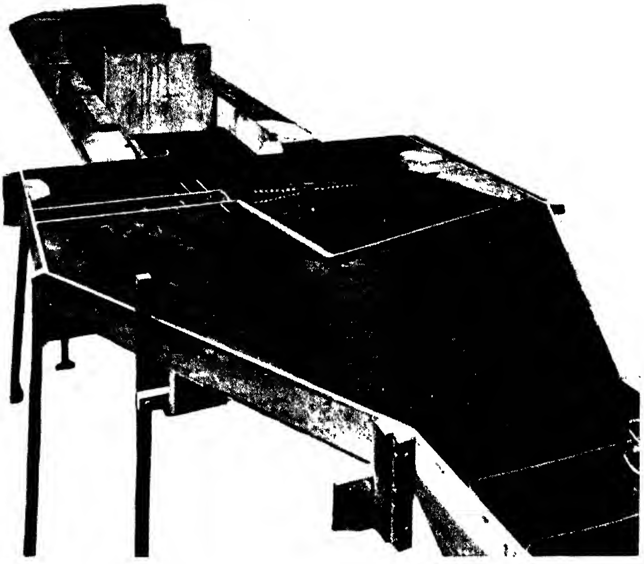
XVI. THE SEVERN TIDAL MODEL

A corner showing, in the middle background, the estuary of the Parrett; on the right-hand side, the two islands Flatholm and Steepholm

SCALES Horizontal 1 8500
Vertical 1 100

XVII. THE ENGLISH STONES AND THE DEEP CHANNEL KNOWN AS 'THE SHOOTS'

SCALES: Horizontal 1 8500
Vertical 1. 100



XVIII. VIEW OF A DETAIL MODEL OF THE SEVERN BARRAGE AS PROPOSED IN 1933

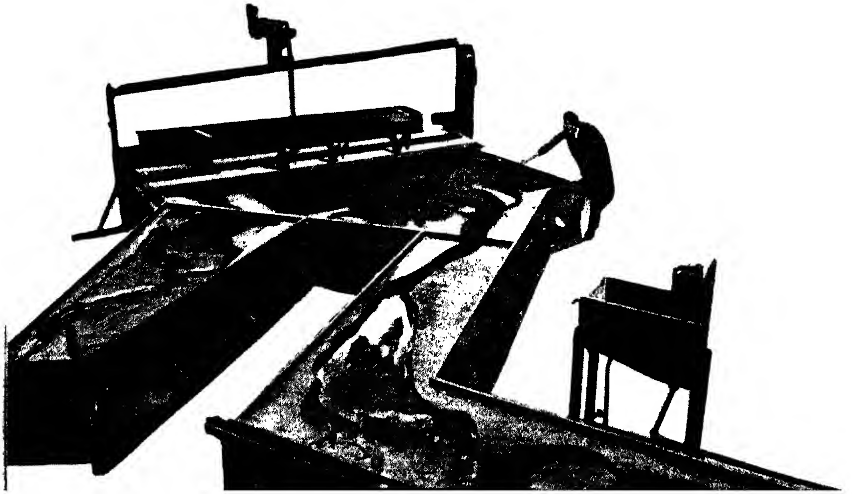
This model was used in an experimental investigation to determine the optimum amount of rock-excavation on the English Stones from the point of view of head-losses

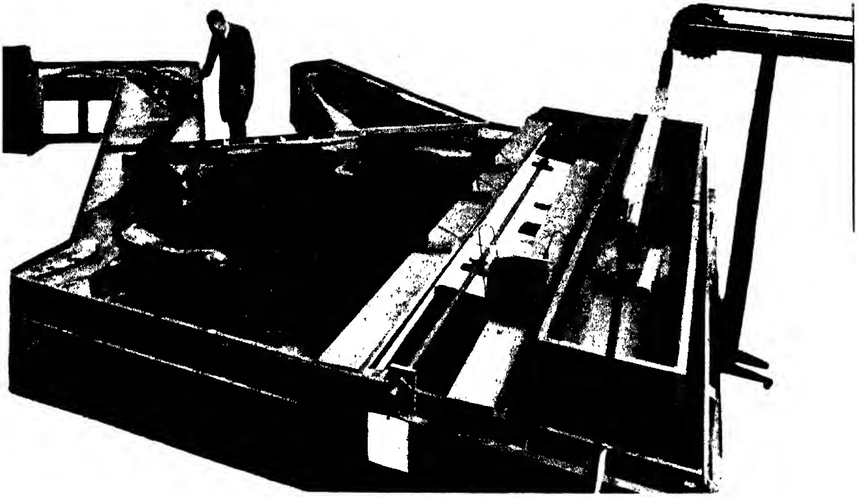
SCALES. Horizontal 1 1500
Vertical 1 300

XIX TIDAL MODEL OF LIVERPOOL BAY AND THE MERSEY AND DEE ESTUARIES

View looking seawards, with the Mersey on the right and the Dee on the left. Behind the Mersey is the silt-box containing a motor-driven agitator which pumps silt-laden water through a copper pipe. From this pipe, supplies are tapped-off for the rivers

SCALES. Horizontal 1 7040
Vertical 1.190

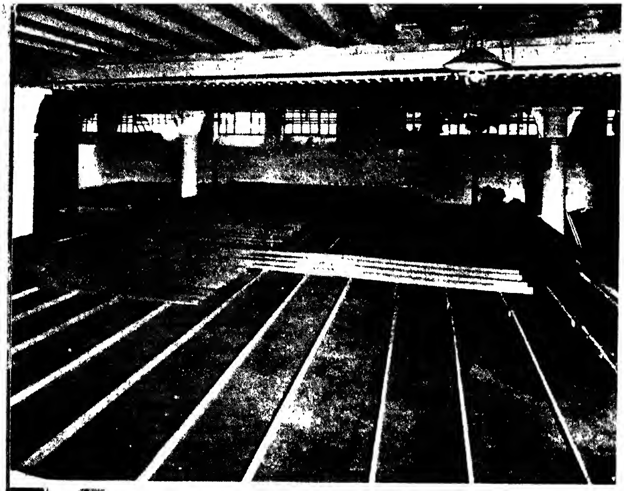




XX. TIDAL MODEL OF LIVERPOOL BAY AND THE MERSEY AND DEE ESTUARIES

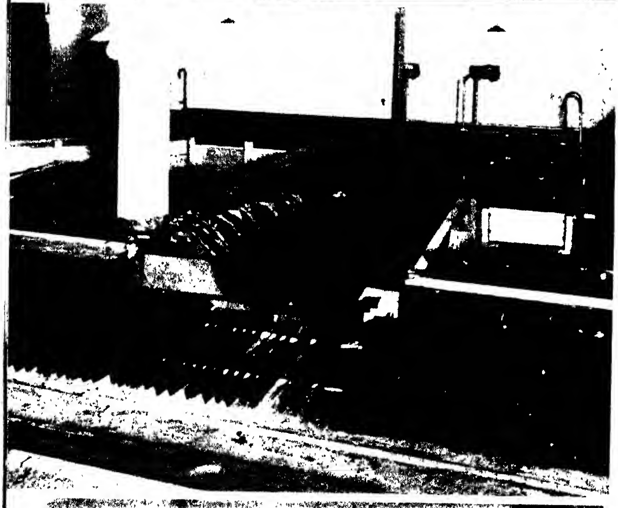
Note.—The tide-producing plunger, loaded with pig-iron and sand; the three fans for reproducing the prevailing wind; the two alum-solution tanks mounted on a plank which spans the tide-box near the plunger; the network of strings used for identifying points in Liverpool Bay

XXI. ROCKING TANK MECHANISM



XXII (a)

METHOD OF LAYING BOTTOM SECTIONS BEFORE CONCRETING



XXII (b)

SHOWING CONCRETING AND RANGOON RIVER

XXII (c)

CONCRETE LAYER FINISHED

XXIII. TIDAL MODEL OF THE DEE ESTUARY

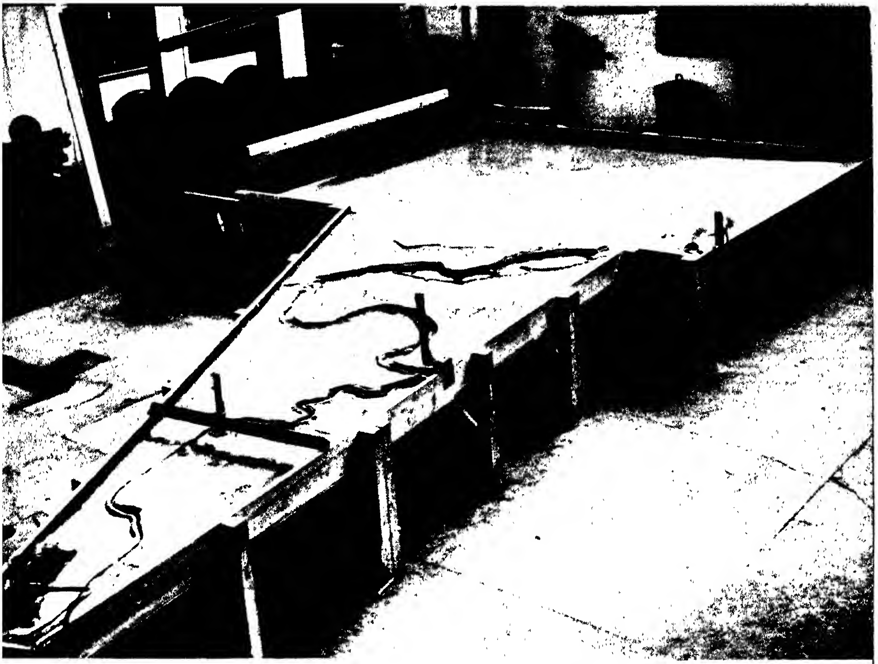
The farther observer on the right-hand side is standing opposite to Flint

SCALES: Horizontal 1:5000
Vertical 1:200

XXIV. SANDBANKS IN THE DEE TIDAL MODEL

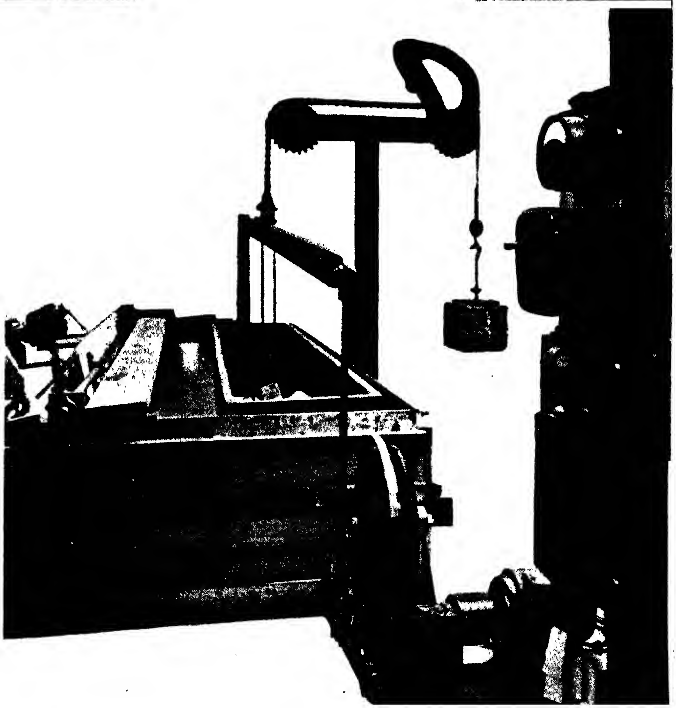
Almost half-way up the right-hand side is the projecting point of the coastline at Flint: above that is the northern training wall

XXV. A SCHEME OF TRAINING WALLS PARTLY CONSTRUCTED IN THE DEE TIDAL MODEL



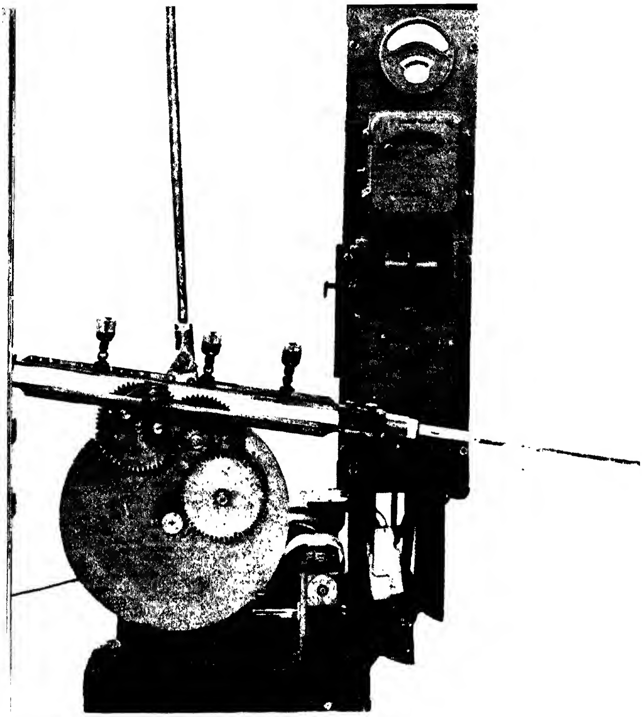
XXVI. A GENERAL VIEW OF THE PARRETT TIDAL MODEL, LOOKING SEAWARDS

SCALES: Horizontal 1:3000
Vertical 1:260



XXVII. TIDE-PRODUCING MECHANISM OF MERSEY TIDAL MODEL

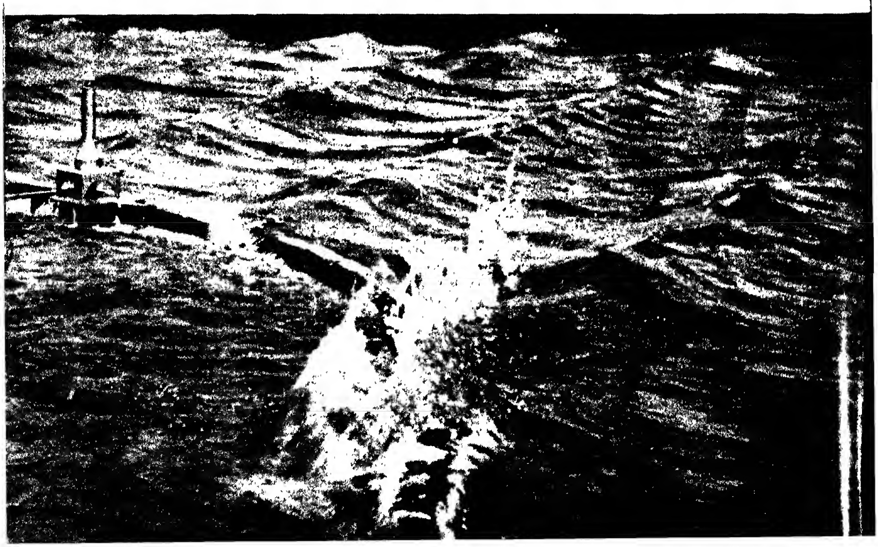
Note the cam and counterbalance weight

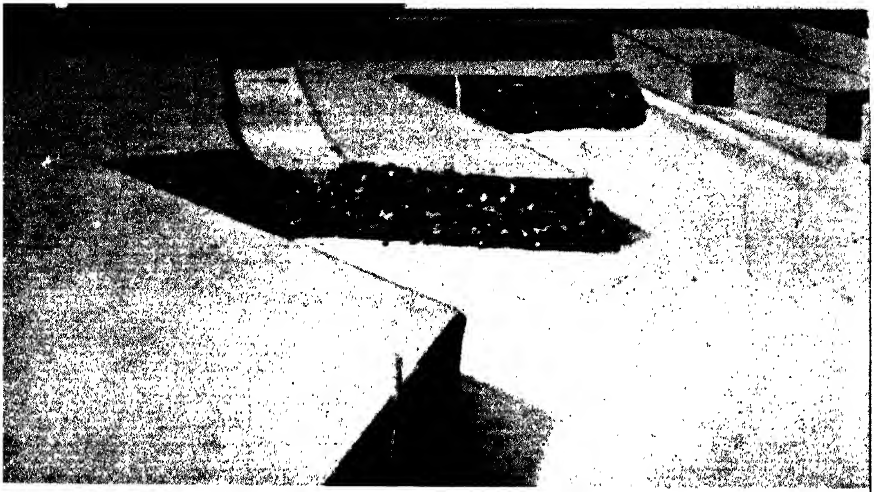


XXVIII. DRIVING MECHANISM FOR MERSEY TIDAL MODEL

Note the epicyclic train of gearwheels for reproducing the lunar cycle of tides

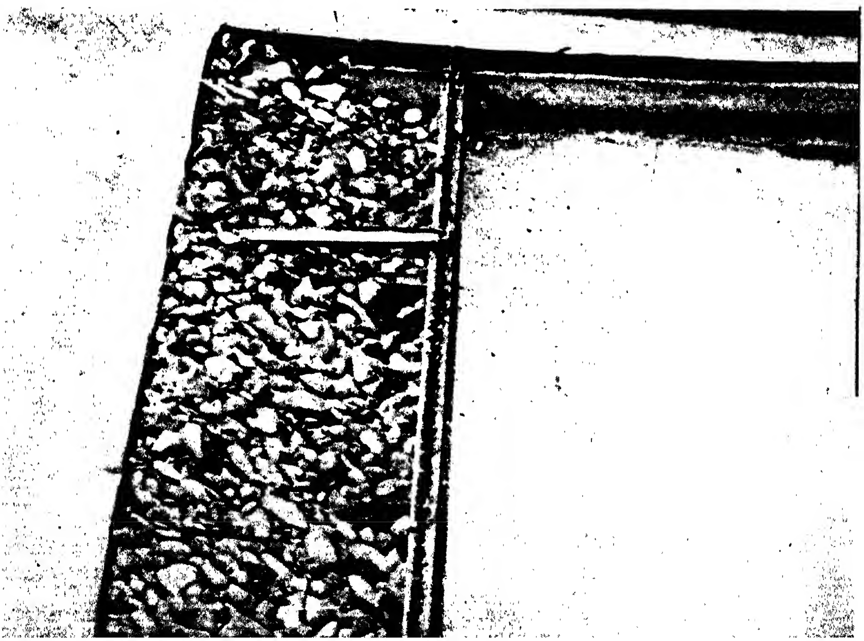
XXIX. CONDITIONS AT CURVED NORTH BREAKWATER EXTENSION





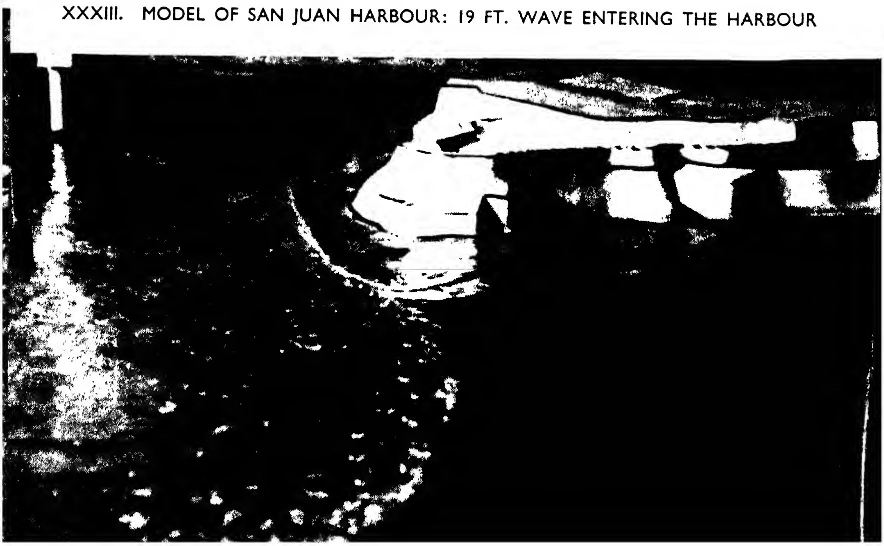
XXX. RIPRAP ABSORBERS

XXXI. ROCK-FILLED CRIB ABSORBER



XXXII. MODEL OF SAN JUAN HARBOUR: 24-FT. WAVE ENTERING THE HARBOUR
MODEL SCALE 1:100

XXXIII. MODEL OF SAN JUAN HARBOUR: 19 FT. WAVE ENTERING THE HARBOUR



XXXIV.
THE SEVERN BORE

XXXV.
THE SEVERN BORE

APPENDIX A

Values of μ and ν for Water and Air

Temp. °C.	Water		Air *	
	μ gm./cm./sec.	ν cm.²/sec.	$10^4\mu$ gm./cm./sec.	ν cm.²/sec.
0	0·0179	0·0179	1·71	0·132
5	·0152	·0152	1·73	0·136
10	·0131	·0131	1·76	0·141
15	·0114	·0114	1·78	0·145
20	·0101	·0101	1·81	0·150
25	·00894	·00897	1·83	0·155
30	·00801	·00804	1·86	0·159
35	·00723	·00727	1·88	0·164
40	·00656	·00661	1·90	0·169
50	·00549	·00556	1·95	0·179
60	·00469	·00477	2·00	0·188
80	·00357	·00367	2·09	0·209
100	·00284	·00296	2·18	0·230

Note that the figures quoted in relation to the viscosity of air are the values of $10^4\mu$. Thus, the coefficient of viscosity of dry air at atmospheric pressure and at 0° C. is $1{\cdot}71 \times 10^{-4}$ gm./cm./sec.

APPENDIX B

A few additional References

" Laboratory Investigation of Flume Traction and Transportation", Y. L. Chang ; *Trans. Am. Soc. C.E.*, vol. 104 (1939), p. 1246.

" The Optimum Size of Models for Studying Flow through Nests of Tubes ", R. Pendennis Wallis ; *Engineering* (London), vol. 147, p. 487, April 28, 1939.

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" River Meandering and the Earth's Rotation ", M. S. Quraishy ; *Curr. Sci.*, Oct. 1943, 12, 278.

" Experimental Fluid Dynamics Applied to Engineering Practice"— Twelfth Andrew Laing Lecture, delivered by Dr. G. A. Hankins, North

(* Dry air at atmospheric pressure.)

East Coast Inst. of Engs. and Shipbuilders, 29th Oct., 1943 (*Trans. N.E. Coast Inst. Engs. and Shipbuilders*, vol. 60, p. 23).

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" British Standard Code for Flow Measurement ", B.S.S. 1042; British Standards Institution, 1943.

" Hydraulic Model of the Mississippi ", *Engineering News-Record* (New York), May 31, 1945, p. 72 (vol. 134, p. 766)—gives particulars of a model now under construction, with a horizontal scale of 1 : 2000 and a vertical scale of 1 : 100, of the entire Mississippi River basin and its tributaries, at an estimated cost of $3,000,000.

" Training Works in the Rupnarain River in Bengal ", Sir Claude Inglis ; *Inst. C.E. Maritime and Waterways Eng. Div.*, Feb. 5, 1946.

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" The Effect of Obstructions in Tidal Estuaries ", A. M. Binnie ; *Engineering* (London), March 15, 1946, p. 241.

APPENDIX C

A Further List of References added November 1950

1. *Concerning Rivers or Tidal Waters*

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In September 1949, the International Association for Hydraulic Structures Research became the International Association for Hydraulic Research.

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I. *Names of Engineers and others to whose work reference is made*

[1] Excluding Appendix C.

II

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III

General Subjects