QUALITY CONTROL
and
STATISTICAL
METHODS

by

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The combination of phenomena is beyond the grasp of the human intellect. But the impulse to seek causes is innate in the soul of man. And the human intellect, with no inkling of the immense variety and complexity of circumstances conditioning a phenomenon, any one of which may be separately conceived of as the cause of it, snatches at the first and most easily understood approximation, and says here is the cause.

Count Leo Tolstoy, "War and Peace"
Part XIII, I, p. 928
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Random House, Inc., New York, N.Y.

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PREFACE

The purpose of this book is to present as clearly as possible, as briefly as practicable, and yet adequately, the most generally useful modern techniques of quality control and statistical methods as applied to industrial problems of product quality. It is intended for those who are new to the field (or relatively so) and who want to improve their effectiveness in appraising and controlling (or helping to bring about control of) quality.

There is an old saying that "there is more than one way to skin a cat." So also there are many ways of appraising and controlling quality; however, the day is rapidly passing when an industrial organization beset with problems of product quality (and yours is probably no exception) can remain progressive and competitive without extensive use of statistical methods of quality control. The term "industrial" is used here in its broadest sense. It includes such diverse things as the electrical industry, growing and packaging of food, steel products, errors in filling mail orders, clothing, lumber, controlling overtime, optical instruments, biologicals, textiles, and polls of public opinion. In short, it involves any field where the quality of product must be appraised and controlled.

For the convenience of the reader, all sections within chapters have been numbered by the decimal system; for example, Section 16.3 is the third section of chapter 16. All figures and tables have been given numbers corresponding to the section in which they are located. Where there is more than one figure or table in a section, they are identified by small letters; thus, Fig. 6.9c is the third figure in Section 6.9.

An effort has been made to avoid using the same symbol for more than one meaning. Where this was not practicable because of widespread usage, the meaning should be clear from the context. In general, the symbols are those most commonly used. The most notable exception is that the capital letter N is used throughout for sample size and the small letter n is reserved for degrees of freedom. The Greek small letter sigma (σ) is used to indicate standard deviation (in conformance with common custom) and is supplemented by the use of S and s when it is desired to distinguish between cases where there is a correction made for sample size and those not so corrected.

Sooner or later some of your friends or co-workers will approach you with questions about odds in various card and gambling games. Instead of spending your own time figuring out the answers it is suggested that you obtain a copy of Oswald Jacoby's book, "How to Figure the Odds."* It will provide answers for most of their questions.

* Doubleday & Company, Inc., Garden City, N. Y., 1947.

While it would be impossible to list here the names of all who have had a part in the development of the material and ideas presented in this book, the author is particularly indebted to Walter A. Shewhart, Edwin G. Olds, Holbrook Working, W. Edwards Deming, and Harold F. Dodge. Their genius and leadership in developing the field has been a constant source of inspiration and guidance.

I am indebted to Professor Ronald A. Fisher and to Messrs. Oliver & Boyd Ltd., Edinburgh, for permission to reprint a portion of Table IV (Table 17.3) from their book "Statistical Methods for Research Workers."

Erie, Pa. EDWARD M. SCHROCK
August, 1949

INTRODUCTION

There are three questions and one objection that often occur to the person just being introduced to quality control and statistical methods. We shall deal briefly with them here.

Question 1: Why are statistical methods of quality control so important?

You will agree that truth is one of the most important things in the world. Knowing the facts of a case is prerequisite to dealing effectively with it. The most effective way that has so far been developed for getting at the facts about quality characteristics is to use statistical methods. You will probably hear it said that these methods are tools for dealing with quality problems. Actually they are much more than that. As Dr. Walter A. Shewhart puts it, they provide us with a way of looking at the Universe. They pervade all phases of existence. They lead us to a better understanding of universal truth.

Inherent variability is present in all things. It is this that prevents us from ever achieving an "exact" measurement of anything. We have nothing "exact" to measure with! As Dr. W. Edwards Deming puts it, the science of exactness becomes the science of dealing most effectively with inexactness. Statistical methods offer us the most effective means of dealing with inexactness.

So important are statistical methods that some day they will be just as integral a part of the engineer's training as mathematics. The universally useful concepts may even be taught in our secondary schools.

Question 2: How can I sell the desirability of modern statistical quality control to management?

How does one go about selling anything? Either the customer must want what is being offered for sale or the desire for it must be created. The customer must be brought to realize his need for it. If he does not really need it you won't fool him for long. Remember what Lincoln said about fooling some of the people. Far too large a segment of industry has already adopted modern statistical quality control for there to be any question about the existence of a real need for it. But suppose your boss is different; suppose he believes that statistics are useful only to insurance companies and government bureaus. Then tackle one of your most difficult quality problems. Use statistical methods to their fullest advantage. Then when you have brought about better quality at the same cost, the same quality at lower cost, or better quality at lower cost, lay your story before him. If he is not pleased with that sort of result, you have our sympathy. If he is a reasonable man (there will probably always be a few people in

this world who are not), you won't need to sell him; the facts will sell themselves.

Question 3: Where does quality control fit into my organization?

There are many ramifications to this question. Unfortunately there is no simple answer. Let us consider a few of the ramifications and some general principles to guide us.

(a) To whom should quality control report? The answer to this question will depend upon how important you consider the quality of your products to be. The more important it is, the higher the level at which it should report. Above all, it should report to a level high enough to insure that its hands will not be tied in an effort to do effective work. It must be able to bring about changes in specifications where needed, to insist upon changes in design where they are faulty, and to bring about changes in factory practice where they are defective. It must be able to attack any source of poor quality and bring about improved conditions.

(b) How many people should there be in the quality control group? This will depend largely upon the overall size of your organization and the type of product you make. One of your best guides will be to find out how big a group other companies have who are using statistical quality control and who have an overall size and type of product comparable to your own.

(c) What sort of people should be in the quality control group? In addition to the need for administrative and organization abilities, there should be at least one competent quality control engineer. Unfortunately there is a dearth of these at present. In due time colleges and universities should help relieve this situation. There should also be one or more engineers familiar with the technical aspects of the product and the manufacturing processes. In addition there will be a need for one or more clerks to record data and perform routine computations. All these workers should have a natural aptitude for dealing with numerical values.

(d) What are the general fields of activity of a quality control group? First, the quality control group must make certain that there is adequate incoming acceptance inspection of raw materials. This will also involve visits (in cooperation with the purchasing department) to the vendors to be sure quality needs are understood and that the vendor is equipped to make and test the material before he sends it to you. Secondly, process controls should be instituted in your own organization, in order that further processing and assembly needs will be met and to insure ultimate quality of the product. This will involve the need for changing specifications where faulty. Thirdly, there should be an audit of quality of finished product, so that corrective steps may be taken where quality is defective. This may also involve audits of sub-assemblies.

(e) How closely should quality control be integrated with the inspection

function? Since the general purpose of inspection is to determine the nature of the quality in any given lot of material and to pass upon its acceptability, there should be a high degree of integration between quality control and inspection. This integration should be as complete as possible.

Objection: Quality control is fine, but my product is different. It is not easily subject to control. It is too complicated. It is a not infrequent source of amazement to see the number of people who seek examples of modern statistical quality control and then who say, "That's fine, but my field is different!" The difference is almost always superficial. There is no field of human material experience that does not function in a statistical manner. As long as you use materials, men, methods, and machines you have problems of quality variation. As long as these exist, you have need of modern methods of statistical quality control.

Consider what scrap and rework cost you on an annual basis. Include in this what defective product costs you in terms of customer good-will and reputation. Certainly you can afford to spend some portion of this amount to bring about a greater reduction in losses and an improved competitive position.

CHAPTER 1

WHAT QUALITY CONTROL AND STATISTICAL METHODS HAVE TO OFFER

1.1: The quality/cost ratio. Throughout the ages the business man has sought to outdo his competitors. He has found that one of the most successful ways of doing this is to increase the value of the ratio

$$\frac{\text{Quality}}{\text{Cost}}$$

This he does by offering better quality at the same cost, by offering the same quality at a lower cost, or by both raising the quality and lowering the cost.

Although most people realize that there is such a thing as quality variation, few appreciate the nature of such variations or how to evaluate them. It will be our purpose here to provide a basis for a better understanding of the variations that exist in quality characteristics, to show how statistical methods can help achieve control of quality variations, and to describe some of the statistical methods that can be used to evaluate quality variations.

1.2: How quality control and statistical methods can help raise the value of the quality/cost ratio. The advantages of quality control and statistical methods inhere in the fact that directly or indirectly they help to improve quality or to lower costs, or both. With respect to quality they help achieve a better quality level and better uniformity of quality. With respect to costs they help achieve better utilization of raw materials, more efficient utilization of equipment, and less scrap and rework. In addition, they help achieve better inspection, improved producer-consumer relations and better specifications. It is difficult to place these advantages in discrete categories; hence the above classification is primarily one of convenience. Let us consider these items individually.

1.3: Better quality level. At one time or another virtually everyone has computed an average, the common method being to add the separate items and divide by the number of items. The layman is generally inclined to accept this figure as constituting all he needs to know about the quality level. While in some cases this may be so, there are times when it will be very inadequate.

Specifications of measurable quality characteristics generally state one

of three things: (1) an aim value with plus and minus tolerances, (2) a maximum value only, (3) a minimum value only. If a tolerance band is specified, the aim value is usually in the center of the band, *e.g.*, $1.055'' \pm 0.001''$. Sometimes we encounter lop-sided tolerances, *e.g.*, $1.052'' \begin{smallmatrix} +0.001'' \\ -0.002'' \end{smallmatrix}$. These immediately raise the question whether it is possible to aim away from the center of the band and still not exceed either tolerance limit. Unless the band is wider than that resulting from chance fluctuations or the distribution of the quality characteristic markedly lop-sided (see section 4.4), such specifications are unrealistic.

If the aim is at the center of the tolerance, we are faced with the question of whether we must hold closely to the aim value. If it is possible to let the quality level fluctuate to some extent, how far can such fluctuations go before the danger of producing defective items arises? A statistical analysis will provide us with the basis for answering this question.

The mere fact that the average of recently produced items lies at the center of the tolerance band (or at some other desired point) is not enough in itself. Only if virtually all items fall within the band and *if we can reasonably expect them to do so in the immediate future* is the situation a healthy one. Only if the process is controlled, *i.e.*, if such fluctuations as occur are due to chance factors and not to identifiable and economically removable causes, and the level is satisfactory, can we be satisfied with the process.

If only one tolerance limit is stated, *e.g.*, 65,000 psi minimum or 0.050 per cent sulfur maximum, the problem becomes one of how closely the quality level may be permitted to approach the limit. To keep the quality level a great distance away is usually costly. To let it approach too close means the production of defective items. Statistical methods provide a means of arriving at the best answer.

1.4: Better uniformity of quality. As recently as 160 years ago it was believed that many items could be made exactly alike if sufficient care was taken in the manufacturing process. Not until many years later was it realized that exact duplication of anything is impossible, and that a certain amount of fluctuation in any quality characteristic is inherent. As long as this variation is such that no assignable causes can be economically indentified, the operation is said to be statistically controlled.

In general, manufacturers would probably like to make statistically controlled products, but it is safe to say that most of them today are undoubtedly issuing relatively uncontrolled products. In a few rare cases, the product may be found to be more uniform than is needed to function satisfactorily. Where such uniformity is costly to obtain (and it usually is), it is economically wasteful.

1.5: Better utilization of raw materials. Where more than one

source of raw materials is available, statistical methods provide the most effective means of determining the relative merits of each source. Where the costs associated with each source are essentially the same, the source or sources that will provide suitable quality levels and suitable uniformity may be identified. Where costs differ appreciably, quality and costs may be balanced to obtain the most economical results.

1.6: More efficient utilization of equipment. Several machines may presumably be alike, yet vary considerably in the quality of the product they turn out. The prompt identification of machines needing repair or adjustment may be an important step in keeping a mass production system flowing smoothly. A control chart on each machine will promptly identify a machine needing attention.

Records of the inherent precision of each of several machines available to do a certain type of job will permit the most efficient selection of the right machine—a machine capable of meeting the precision required without waste of precision ability. A statistical analysis offers the best evaluation of a machine's capabilities.

1.7: Less scrap and rework. The proper way to make most products is to make them right the first time. Unless the tolerances are very wide, this will be impossible in a process that is not operating in a statistically controlled manner. As control is gradually achieved through the identification and elimination or control of assignable causes, defective work will decrease. To realize their full potentialities, modern mass production industries must be operated in a statistically controlled manner. It is the only way to avoid a constantly recurring scrap and rework problem.

1.8: Better inspection. The primary purpose of inspection is to determine the existing state of quality with a view to acceptance or rejection. Inspection procedures may be placed in three general classifications: more than 100 per cent inspection, 100 per cent inspection, and sampling inspection.

The fact that some critical quality characteristics are inspected two or more times on each item is an obvious admission that so called "100 per cent inspection" is not perfect. In general, good 100 per cent inspection may be expected to eliminate only 85 to 95 per cent of the defective items.

If the test method is destructive or is costly to perform, sampling inspection is used. The sampling plans that most readily suggest themselves to the layman (*e.g.*, inspect ten items and allow no defectives, or inspect 10 per cent of the product and allow some stated small number of defectives) rarely accomplish their intended purpose. Sampling theory has so many ramifications that the advice of a statistician familiar with the nature of sampling variations should always be sought before putting a sampling plan into effect.

In connection with the general subject of inspection, it should be remem-

bered that quality cannot be inspected into a product; it must result from the manufacturing process.

1.9: Improved producer-consumer relations. When a manufacturer plans to use semi-finished or finished parts supplied by a vendor as a part of his raw materials, he generally assumes that the material supplied will be essentially constant in its important quality characteristics. When it is not, the manufacturing schedule may be seriously disrupted. Lack of an adequate appraisal of quality may make claim adjustments difficult.

If the vendor brings his process under statistical quality control, and can produce the evidence to show that his product is satisfactory, he is less likely to get claims and is in a position to refute unjust claims. Similarly, the consumer who can produce statistical evidence establishing lack of control is more likely to get proper attention from the vendor. When both vendor and consumer have an adequate appraisal of product quality, relationships are almost certain to improve. Statistical methods offer the best way of getting such appraisals.

The consuming public also expects uniformity in the products it buys. Only as such uniformity is found can the producer establish a reputation for dependability.

1.10: Better specifications. The general subject of specifications involves so many ramifications that we shall only attempt to touch lightly on some of the statistical implications. In writing specifications, it is now generally recognized that tolerance limits must be placed upon measurable quality characteristics. These tolerance limits must be reconciled with actual needs and ability to produce. There is no use in specifying a closer tolerance than is needed or than can be produced. All too little attention has been given in the past to what could be produced. Statistical methods offer the best way of appraising the potentialities of a process so that in turn specifications may be written more soundly.

CHAPTER II

CASE HISTORIES

2.1: General. Prior to World War II there was comparatively little material published that dealt with case histories of industrial statistical quality control. The stimulus of the recent war effort has resulted in a veritable flood of articles in many technical and trade magazines. The

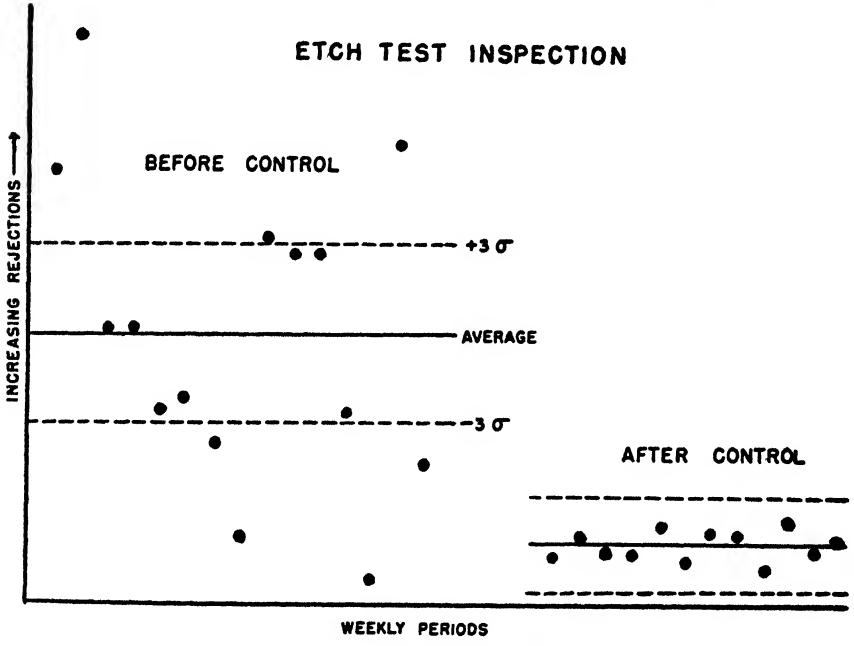

Fig. 2.2. Improvement in a quality level.

purpose of this chapter is simply to illustrate with a few such case histories how statistical quality control methods can be effective in dealing with problems of quality. The details of applying the techniques will be discussed in subsequent chapters.

2.2: Improving a quality level. This case involved a certain grade of steel in which internal soundness was very important. Internal soundness is evaluated by taking small cross-sections of the steel in billet form (about two inches square) and etching them in acid. Unduly porous material is quickly revealed by this test. A control chart of the process (Fig. 2.2) revealed a serious lack of control, many of the plotted points falling outside

the dotted control chart limits. A fundamental change was then made in the manner in which the molten steel was deoxidized in the ingot mold. It is clearly evident from the right side of Fig. 2.2 that the control was greatly improved (results now falling between the dotted lines) and the portion of defective material greatly reduced.

2.3: Improvement of uniformity. Fig. 2.3a shows a machining operation that was very unsatisfactory. Over 25 per cent of the pieces inspected

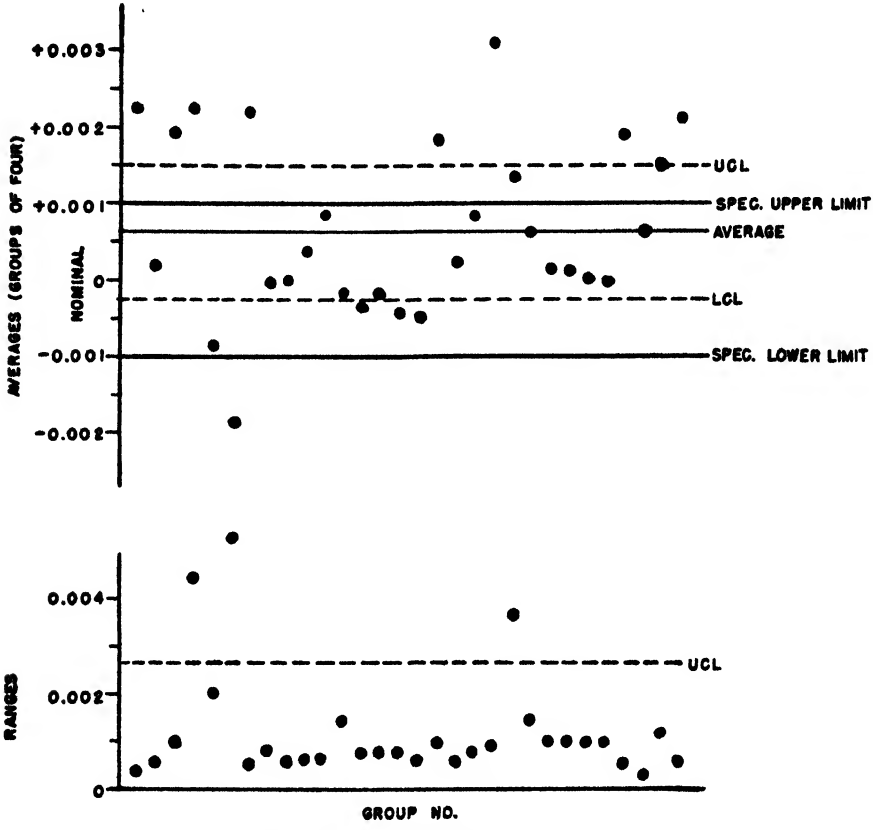

Fig. 2.3a. A process having too much variability and lack of control.

were outside specifications, the degree of control was very poor (many points outside control chart limits), and inherent variability much too great.

After the machining operation had been studied, a change in tooling was recommended. The results obtained are shown in Fig. 2.3b. The degree of control is greatly improved (fewer points outside control chart limits), although there is still room for improvement, and the inherent variability is much smaller. As long as the process level stays close to the midpoint of

the specification band and the process is controlled, defectives should be very few. The change also resulted in greatly increased tool life.

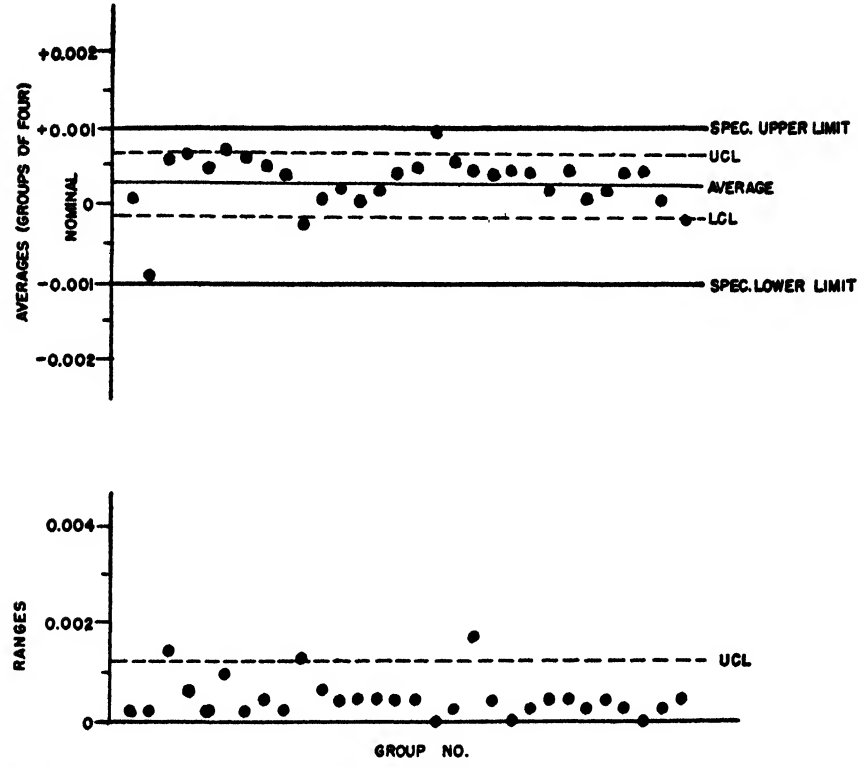

Fig. 2.3b. Process of Fig. 2.3a improved.

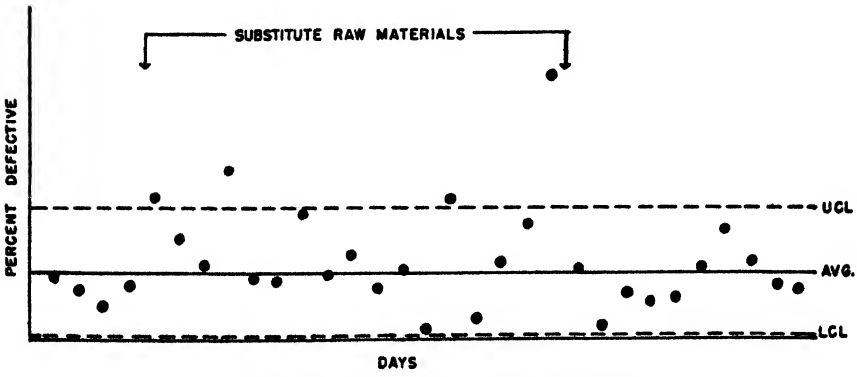

Fig. 2.4. Effect of inferior raw materials.

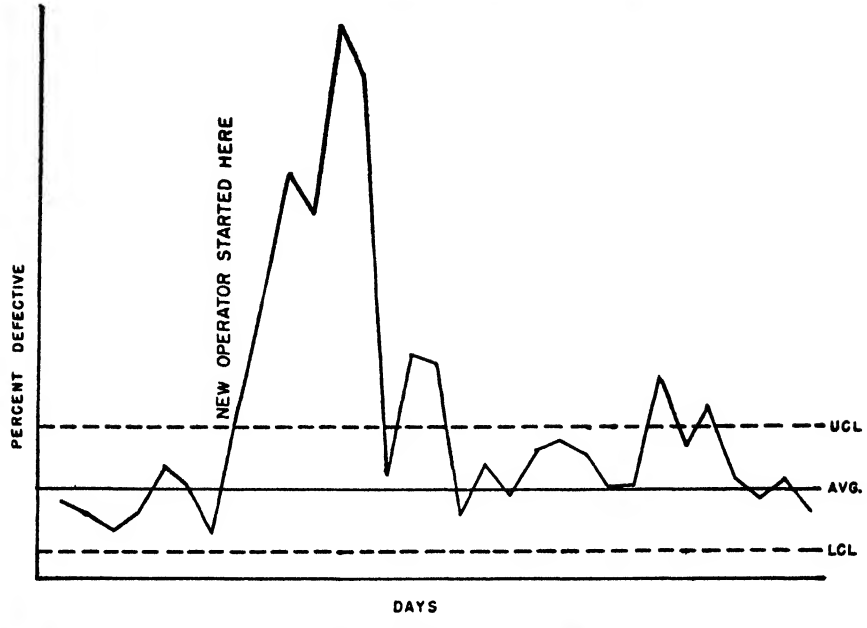

Fig. 2.5. Effect of inexperienced operator.

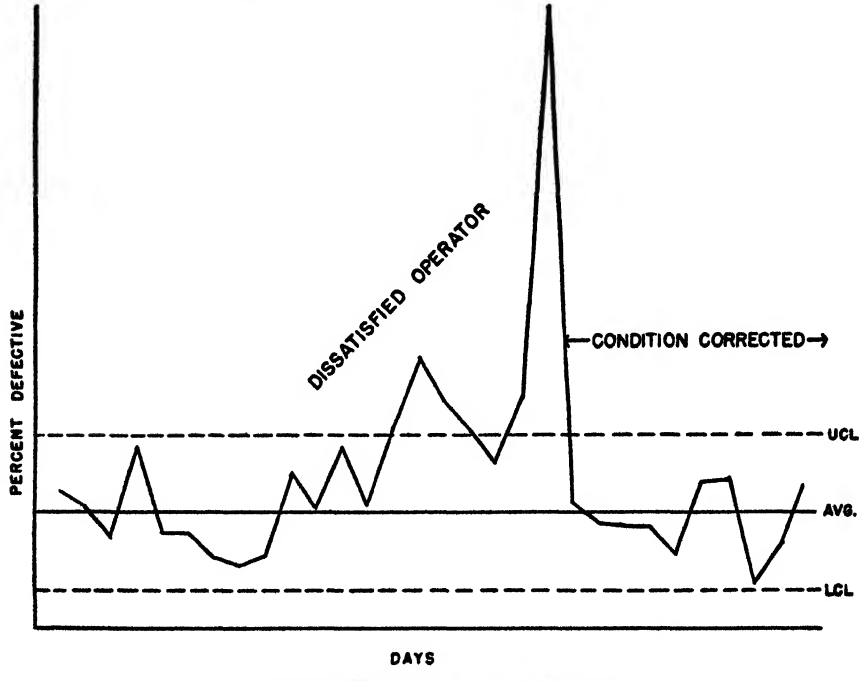

Fig. 2.6. Effect of operator's attitude.

2.4: Effect of inferior raw material. For a period of 17 days it became necessary to use materials of an inferior nature on an assembly operation in order to keep production going. Fig. 2.4 shows the results of this condition. The portion of defective product immediately increased and a number of points went out of control (above dotted upper control limit). As soon as the regular supply of raw materials became available the portion of defective material decreased and control was regained.

2.5: Effect of operator inexperience. Fig. 2.5 shows what happened to quality as a result of having to place a new operator on an assembly line involving a skilled operation before the operator was adequately trained. Rejections were abnormally high for several days, and it was several weeks before the operator settled down to a satisfactory performance. As a result of this study, a special training program was developed for new operators before being placed on this job.

2.6: Effect of operator's attitude. In any large organization it is almost inevitable that someone will become dissatisfied with his or her job. Will such dissatisfaction have an adverse effect upon the quality of the work done? Fig. 2.6 says yes. Not until the rejections reached their peak did the operator voice her complaints, but one can see in the rising trend of the curve for almost two weeks prior to that time the growing dissatisfaction of the operator. Her complaints were adjusted immediately and the very next day control was re-established.

CHAPTER III

THE MEANING OF NUMBERS

3.1: Our early contacts with numbers. One of the first things a child is taught is how to count. At first, to count to ten is considered a fine accomplishment, but it pales into insignificance as the series is lengthened to twenty, fifty, or a hundred. As the mind matures, we learn the definitions of a thousand, million, billion, trillion, and so on. Unfortunately, we tend to learn the names of numbers before the mind has matured enough for us to fully grasp the concepts involved. Thus we arrive at the mental maturity needed to understand the meaning of numbers having already acquired a longstanding familiarity with their names. We succumb to the human weakness of believing that because we know the name of a thing we fully understand it. The development of our concepts of numbers is therefore largely neglected. The ultimate result is generally the unwitting acquisition of a host of misconceptions.

3.2: Figures never lie. There is an old saying to the effect that figures never lie, but that liars can figure. Any number merely represents a result arrived at under certain conditions by some person or persons. Obviously, if the wrong conclusion is reached as a result of the figure obtained, it is because the conditions of the experiment were not all they should have been, or the results were incorrectly observed or recorded, or were not properly interpreted. Numbers cannot lie. Only human beings can lie, or err in judgment. Because of our errors in judgment we sometimes misinterpret results by reading in meanings that are not there or overlooking meanings that are there.

3.3: Precision and accuracy.[1] There are probably few words as loosely used by scientists as *precision* and *accuracy*. Let us first consider the definitions of these words. If you examine a dictionary you will probably find that precision involves exactness while accuracy involves freedom from error. Each will also probably be defined in terms of the other. It is not unusual to find them used interchangeably in scientific writings.

Perfect precision is a concept which human experience may approach, but never reach. Sufficiently sensitive measuring equipment always reveals that no matter how hard we try to make two or more items alike with respect to any measurable quality characteristic, there are variations in the product.

[1] For a thoroughgoing discussion of these terms beyond the scope of this book see W. A. Shewhart, "Statistical Method from the Viewpoint of Quality Control," Chapter IV, the Graduate School, the Department of Agriculture, Washington, D. C., 1939.

The smaller these variations, the more *precise* we say the process is. Thus, any number obtained as the result of a specific process is merely one of many such numbers that may be obtained. It is necessary that many such numbers be obtained before the degree of precision present can be evaluated.

The purpose in making repetitive measurements on the same item (presumably in the same manner) may be to make sure that there is no error in the reading, recording or results. Thus, *accuracy* may be the goal. Incorrectly recorded results are generally worse than no results at all. They are very likely to lead to wrong conclusions.

A recent scientific report referred to the accuracy of the test involved as being from plus and minus ten to plus and minus 35. We are not informed as to whether the author means that errors of this magnitude might occur due to improper use or reading of instruments, or whether this variation is to be expected as a result of the lack of precision in the measuring method. Nor are we told what percentage of the test results obtained fell within these limits. Obviously, statements of this type can have only a very vague and ill-defined meaning for those who read them. Before such statements can be meaningful, it is necessary that we know what percentage of the results fall within the stated limits, and most important of all, that we know whether the data were obtained under what is known as statistically controlled conditions (see section 6.2).

Another example of the type of situation here involved concerns three physicians who were separately questioned as to the significance of basal metabolism measurements. One observed that the results are not significant unless they exceed plus or minus ten. Another stated that results beyond plus or minus nine are significant, while the third claimed that the results are not significant unless they exceed plus or minus 15. In all probability each physician specified a different level as being significant because of his own evaluation of what constitutes significant effects (or the opinion of the particular author he read). The important consideration clearly is the percentage of individuals outside any given limits that reveal undesirable conditions with respect to health—conditions resulting from the causative factor involved.

3.4: Samples. Industrial progress has brought about an increasing demand for highly precise manufacture. Tolerances that are considered commonplace today would have been termed fantastic a few decades ago. In some cases it is considered necessary to measure each item manufactured in order to be sure that the dimension produced is satisfactory. This procedure is commonly referred to as 100 per cent inspection, as compared to sampling inspection in which only a fraction of the items produced are inspected.

It is generally assumed that 100 per cent inspection will give perfect

protection against the acceptance of any rejectable material. While in some few cases this goal may be very nearly reached, generally the results are a long way from the perfect inspection implied by the figure "100 per cent." Human fatigue will eventually result in inspection errors, so that in critical cases it may be necessary to conduct what is called 200 or 300 per cent inspection, that is, every article is examined two or three times.

Many measurements do not lend themselves to a simple inspection procedure. Suppose the dimension required of a shaft is a diameter of 0.6845 plus or minus 0.0002 inch. When can we say that any given shaft has met this requirement? Any circle has an infinite number of diameters, and any shaft contains an infinite number of circles. Even procedures which appear to be relatively simple may yield different results when followed by different individuals. Thus, in the final analysis, all measurements are merely samples of what might be obtained under certain stated conditions.

A certain manufacturer has the ironclad policy that every melt of steel must have a complete physical test. What is actually meant is that a tension-test specimen must be taken from every melt. It is a known fact that two different tension-test specimens taken from adjacent portions of metal will almost invariably give measurably different test results. Any adequate appraisal of a melt of steel would therefore involve taking a great many tests. On the other hand, as long as successive melts do not differ significantly from one another (*i.e.*, the process is operating in a controlled manner), the testing of every melt may well be a waste of time and materials.

If the manufacturer makes any rejectable material, some will be accepted unless what is made is so undesirable that none of the product is accepted. The inspection and test procedures prescribed to eliminate the rejectable material will inevitably result in some of the acceptable material being rejected and some of the rejectable material being accepted. The best guard against this is good 100 per cent inspection, but even here human error and such factors as fatigue, lack of interest, and distraction will result in the acceptance of rejectable material and the rejection of acceptable material. There is the further consideration that 100 per cent inspection is sometimes a physical impossibility. For example, the determination of the tensile strength of the metal in each of a number of metal parts establishes only the result obtained on the portions of the metal tested. Other portions of metal from the same articles would have yielded different test results. Thus, in any product involving the possibility of more than one test determination, it can never be said to be truly 100 per cent inspected unless all such points are tested. Obviously when there is an infinite number of places in the item from which the test specimens may be taken, 100 per cent inspection is impossible. Under such conditions we are forced to deal with the distribu-

tion of the quality characteristic involved if we wish to evaluate adequately the material submitted for inspection.

It is important that samples taken be as representative as possible of the material from which they come. Samples should generally not be taken from a larger volume of material than can reasonably be expected to be essentially uniform. Where there are important variations in the material, the sample should properly represent the various strata. The importance of this has been clearly demonstrated by national polls of public opinion. A widely read newspaper columnist recently observed that the possibility of one person's answer representing thousands of persons is sheer bunk. This might be true if no consideration were given to the problem of who should be questioned. If the individuals selected for questioning adequately represent the important differences that exist among human beings, the result obtained from a poll will have a high degree of reliability.

3.5: Points of reference. All things in this world are relative. This is especially evident in the realm of physical measurements. Our units of length, weight, and volume are all derived from certain standards which are kept in the International Bureau of Weights and Measurements near Paris. We then prepare what we believe to be satisfactory copies of these standards and use the copies in our everyday life. The copies cannot be perfectly identical with the originals, but since the differences are exceedingly small we do not worry about them.

Unfortunately, numbers are sometimes used without adequate points of reference. Following one of our national holidays it was reported that the state of New York led the nation with 33 accidental deaths. No reference was made to the fact that New York state has a greater population than any other state in the Union or to what the accidental death rate was per capita. It may have been that New York's record on this occasion was really much better than many of the other states on a per capita basis.

A newspaper columnist recently observed that average taxation in the United States is $357 per person as compared to $291 in Britain. No reference was made to the average income in the United States as compared to Great Britain, or to what a dollar will buy in each of the countries.

During a safety campaign in a large city, one of the city newspapers published a picture of a traffic intersection and stated that a large percentage of the traffic accidents which occurred at the corner were due to youths in a certain age bracket. No reference was made to what percentage of the drivers going through this intersection were youths in the age bracket involved. The inference that youth is an important cause of accidents at that intersection may be entirely unfounded. This possibility is increased by the fact that the intersection involved bordered on the campus of a large university and that another large school is only a few blocks away.

Before numbers can have adequate meaning there must be satisfactory points of reference. In developing these, all factors must be considered that have an important bearing on the numbers to be evaluated.

3.6: Gathering of data. The first consideration involved in gathering data is the purpose for which it is to be gathered. Unless this can be clearly stated there will be little point in going any further. Once a purpose has been defined, it must be possible to carry out the testing procedure in an operationally verifiable manner; otherwise, the figures derived will be absolutely useless, except insofar as they may have kept someone out of mischief while engaged in gathering them.

Suppose the purpose is to determine how good a rubber ball is. Unless we define what we mean by "good," any figures obtained will be useless. The ball could be examined to determine its weight, balance, diameter, and density, but these things would not necessarily tell us what we really want to know about the ball. If the requirement of the rubber ball is that it bounce as high as possible, we can devise a test procedure that will evaluate the ball in an operationally verifiable manner.

It must be borne in mind that nothing in this world can ever completely represent anything else. Every caution must therefore be exercised to make the numerical results obtained as representative as possible of the material from which they come. If samples are to be taken from a box or tray they should be taken from various locations and not all from one corner. Consideration should be given to whether the contents of the box are the work of one machine and one operator. All factors that might affect the measurements obtained should be taken into account.

There are five common sources of errors in figures:

1. failure to measure the thing we want to measure;
2. errors due to the measuring equipment;
3. errors of operators due to carelessness, fatigue or inadequate instruction;
4. transcription errors;
5. errors involved in the use of computing equipment.

Consideration should be given to each of these sources of error before establishing a testing program.

Walter Yust, Editor-in-Chief of the "Encyclopaedia Britannica," has observed that the hardest thing to get in this world is a fact. An example he cites was his effort to obtain information on the length of the Ozark River. Five different authorities gave five different answers.

3.7: Attributes and variables. An attribute measurement is one in which a quality characteristic is present or absent, or falls into a limited number of discrete categories. The "go no-go" gage provides a measurement by attributes. The item examined is considered satisfactory or unsatisfac-

tory. A box may contain a number of balls of different colors. An examination of the box would reveal how many of each color are present.

Measurements by variables involve the use of a scale which can theoretically be infinitely subdivided. The length, weight, or density of an object are examples of measurements by variables.

Measurements by attributes are generally less expensive to make than measurements by variables, but have the shortcoming that many more measurements must be taken to obtain the same amount of information that can be had by a small number of measurements by variables. This is because there is no way of telling to what extent an item fails to meet the desired quality level when it is rejected on an attribute basis. If the item is acceptable there is no way of telling whether it just barely meets the required quality level or is well above it. Generally, from 300 to 500 items must be inspected by attributes to derive as much information as can be had from about 50 to 100 items inspected by variables. In addition, the smaller the portion of defective items when measured by attributes, the more items must be inspected to determine the prevailing process quality level. A process average of 40 per cent defective would normally be expected to produce in the neighborhood of 40 defective items in each 100. Five hundred items would be expected to contain about 200 defective items. A process average of 0.5 per cent defective would be expected to contain defectives at the rate of one in about every 200 items examined, so that 500 items would not be expected to contain more than about 2 or 3 defectives. Unless the defectives are being produced at very consistent intervals (which is not likely), 500 items might contain no defectives or as many as 7, just due to chance fluctuations.

3.8: Variability. Efforts to make two or more items identical have always revealed that when sufficiently precise measuring equipment is available the items vary from one another. Things which are said to be "alike" are generally items in which the existing variability is unimportant from a practical point of view. A certain species of bird is said always to select blades of grass of the same length for its nest. It is doubtful that any of our feathered friends are able to make highly precise measurements. What is really meant is that the difference in length among the blades of grass selected are very small in comparison to the differences among all the blades of grass in the vicinity of the ones selected. We generally refer to things as being "alike" when the differences among them are so small that we don't care about them.

The remark, "You can tell he's an engineer, his work is so precise" reflects the layman's impression of the engineering profession. While it is true that the engineer often works with a degree of precision quite foreign to the layman, the layman often overestimates it. An illustration of this

occurred in an article written by a newspaper columnist in which interstate trucking rules were being discussed. The writer stated that by comparison the railroads operate on a much sounder basis. He contended that everything on rails is standardized, and that engineers know to the ounce (this word is an exact quote) what every bridge and viaduct will carry. As a matter of fact the engineer cannot predict loads much more closely than several hundred pounds for small structures and thousands of pounds for large ones. This is one of the reasons why a factor of safety is used in such construction.

There is the further consideration that material things change with time. With respect to material things, we have a very human tendency to believe in fixed values in a world in which nothing is fixed.

CHAPTER IV

SUMMARIZATION OF DATA

4.1: General. The inspection of a large number of items, or the taking of many observations on the same item, may result in the accumulation of so many numbers that the mind has considerable difficulty in determining their significance. We therefore endeavor to condense into one number or a few numbers what we consider to be the important aspects of the original numbers. Very commonly this consists of computing the mean of the numbers, that is, the numbers are added together and the sum divided by the number of different items.

The layman sometimes endeavors to evaluate the scatter in observed data by scanning to observe the range which includes most of the numbers. Occasionally the maximum and minimum may be noted to determine the full width of the range. Most people appreciate the fact that there may be an occasional "wild" number which gives an unduly large range. Hence, more importance is generally attached to that range which includes about 95 per cent of the observations. We shall now consider in more detail these and other methods of summarizing data.

4.2: Central trends. Probably the most commonly used measure of central trend is the *arithmetic mean*, which consists of the sum of the numbers divided by the number of items. The term *average* is often used interchangeably with *arithmetic mean*. A differentiation should be made since *average* is a generalized term and includes a number of different ways of measuring central trends, whereas *arithmetic mean* has a specific meaning as just indicated.

The *median* is that number which is exceeded by as many numbers as fall below it. It is most conveniently determined by first arranging the numbers in sequence of magnitude. When there is an odd number of items it will always be one of the observed numbers. When there is an even number of items it is considered to be midway between the two most central numbers. Thus the median of the numbers 2, 7, and 12 is 7. For the numbers 1, 4, 5, and 9 it is 4.5. The chief advantages of the median are that it is generally comparatively easy to determine and it is unaffected by extreme items.

The *mode* is that value which occurs most frequently in a series of numbers. Where no two numbers are the same it may be determined by arranging the numbers into class intervals. The mode then is that class interval in which the greatest number of observations fall.

The computation of the median is somewhat more involved in the case

of class intervals. Two procedures may be used: the midpoint of the class interval in which the mid-most value occurs may be used, or it may be assumed that the items in the class interval distribute themselves uniformly throughout the interval. The latter procedure gives a better approximation of the true median, but will generally not justify the trouble necessary for its computation.

4.3: Measures of dispersion. Measures of dispersion are designed to evaluate the extent to which observations scatter around the mean. The most commonly used measures of dispersion are standard deviation and range. The latter is becoming increasingly popular for use in connection with control charts because of its ease of calculation.

Standard deviation may be defined as the root mean square of the differences between the observed values and the mean. It is commonly indicated by the small Greek letter sigma (σ). It is obtained from ungrouped data by determining the difference between each observation and the mean, squaring each of these differences, summing them, dividing by the number of observations, and extracting the square root according to the formula

$$\sigma = \sqrt{\frac{\Sigma (X - \bar{X})^2}{N}}$$

where the capital Greek letter sigma (Σ) means "sum of", X is an observation, $\bar{X}$ is the mean, and N is the number of observations. In making computations, the following equivalent formula will be more convenient:

$$\sigma = \sqrt{\frac{\Sigma X^2}{N} - \bar{X}^2}.$$

The symbols capital letter S and small letter s are also used to designate standard deviation, the former being used when a correction is made for the effect of sample size, and the latter when no correction is made for sample size (see section 14.5 for the effect of sample size). Following is an example of the computation of the standard deviation.

X	X^2
4	16
7	49
5	25
5	25
6	36
5	25
9	81
7	49
48	306

$$\bar{X} = \frac{\Sigma X}{N} = \frac{48}{8} = 6.0 \qquad \bar{X}^2 = 36.0$$

$$\frac{\Sigma X^2}{N} = \frac{306}{8} = 38.25$$

$$\frac{\Sigma X^2}{N} - \bar{X}^2 = 38.25 - 36.0 = 2.25$$

$$\sigma = \sqrt{2.25} = 1.5$$

Where there are sufficient data it may be more convenient to use class intervals. The formula for class intervals is

$$\sigma = i\sqrt{\frac{\Sigma fd'^2}{N} - c^2}$$

where i is the size of the class interval, f is the frequency in the class interval, d' is the number of intervals away from the guessed mean interval, N is the number of items, and $c = \frac{\Sigma fd'}{N}$. It is not necessary that the true mean lie in the guessed mean interval as the correction c compensates for any error in guessing; however, the selection of the proper interval will reduce the size of the numbers in the computations. Following is an example of the computation of standard deviation for grouped data.

Class Interval	f	d'	fd'	fd'^2
52 0–55.9	3	5	15	75
48.0–51.9	9	4	36	144
44 0–47.9	13	3	39	117
40.0–43.9	27	2	54	108
36 0–39.9	41	1	41	41
32.0–35.9	60	0	0	0
28.0–31.9	57	−1	−57	57
24 0–27.9	37	−2	−74	148
20.0–23.9	24	−3	−72	216
16.0–19.9	15	−4	−60	240
12 0–15.9	9	−5	−45	225
8.0–11.9	5	−6	−30	180
$N = 300$			−153	1551

$$c = \frac{\Sigma fd'}{N} = \frac{-153}{300} = -0.51$$

$$c^2 = 0.2601$$

$$\sigma = i\sqrt{\frac{\Sigma fd'^2}{N} - c^2}$$

$$= 4.0\sqrt{\frac{1551}{300} - 0.2601}$$

$$= 4.0\sqrt{4.9099}$$

$$= 4.0(2.2158)$$

$$\sigma = 8.86$$

An excellent approximation of the true mean of grouped data may be obtained by adding to the guessed mean (midpoint of the guessed mean interval) the correction ic. The guessed mean above was 34.0. The correction would be 4.0 (−0.51) or −2.04. The mean would then be 34.0 − 2.04 = 31.96.

Most data obtained from natural sources tend to give a distribution which has come to be known as the normal or Gaussian Curve (Fig. 4.3). Early investigations of distributions of such quantities as lengths of blades of grass, or height and weight of people revealed a bell-shaped type of curve. Because this curve was found to be so common in nature, it came to be called the *normal* curve. It was thought that any marked departure from this curve was non-normal, and due to some abnormal condition. Subsequent investigations have revealed many other types of distributions which are just as normal for the types of cause systems that produce them as the

normal curve is for the type of cause system that produces it. This means that care must be exercised in assuming that a particular type of distribution exists in a given situation.

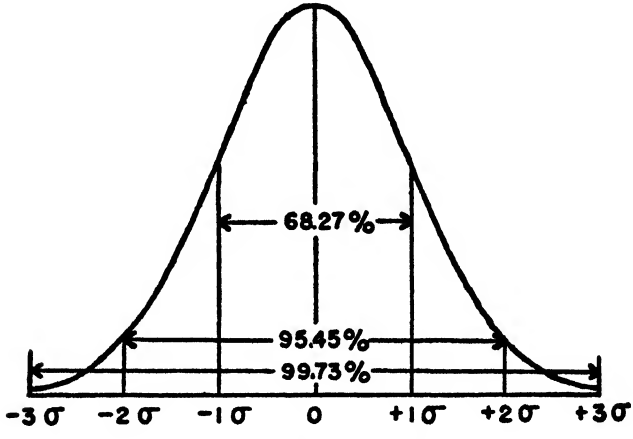

Fig. 4.3. Normal curve.

TABLE 4.3. AREAS OF THE NORMAL CURVE IN ONE DIRECTION FROM THE MEAN

σ	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.0	.00000	.03983	.07926	.11791	.15542	.19146	.22575	.25804	.28814	.31594
1.0	.34134	.36433	.38493	.40320	.41924	.43319	.44520	.45543	.46407	.47128
2.0	.47725	.48214	.48610	.48928	49180	.49379	.49534	.49653	.49744	.49813
3.0	.49865	.49903	.49931	.49952	49966	.49977	.49984	.49989	.49993	.49995

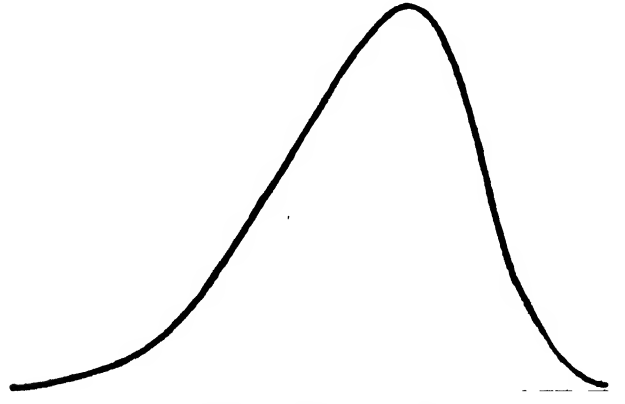

Fig. 4.4a. Negatively skewed curve.

If a normal distribution is drawn and three standard deviations marked off along the base line in either direction from the mean, it will be found

that very little of the area under the curve remains in the tails. If the area under the curve is considered unity, that portion remaining in one of the tails is 0.00135. Thus, three standard deviations plus and minus from the mean include 99.73 per cent of all the area under the curve. Fig. 4.3 shows the portion of the area under several sections of the curve. Table 4.3 gives

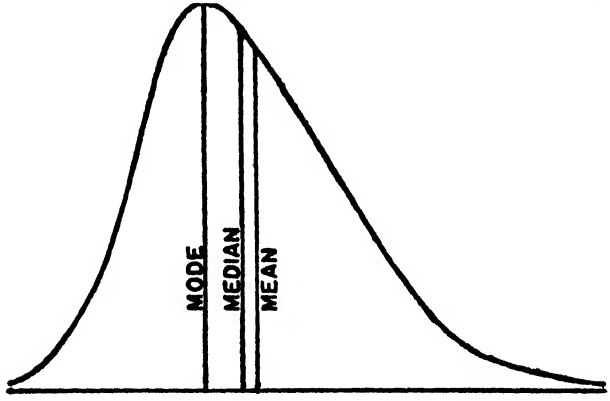

Fig. 4.4b. Mode, median, and mean of a positively skewed curve.

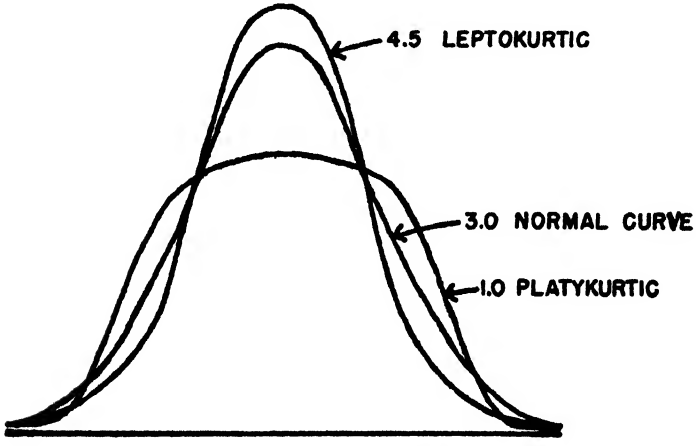

Fig. 4.5a. Kurtosis.

areas of the normal curve enclosed by the base line, the ordinate at the mean, the curve, and the ordinate at various sigmas from the mean.

4.4: Skewness. It sometimes happens that the cause system involved will produce a lop-sided distribution. This is illustrated in Fig. 4.4a. It is obvious that the areas under such a curve for any given number of standard deviations from the mean will differ for the two sides of the curve and also from the normal curve. Where the amount of skewness is small it may

usually be disregarded without any serious consequences; however, one cannot afford to ignore large amounts of skewness. Fortunately, these are exceptional. The normal curve is said to have zero skewness. Skewness values extend from zero both positively and negatively.

In a normal distribution, the mean, median, and mode coincide. On a skewed distribution these three points are separated, as shown in Fig. 4.4b.

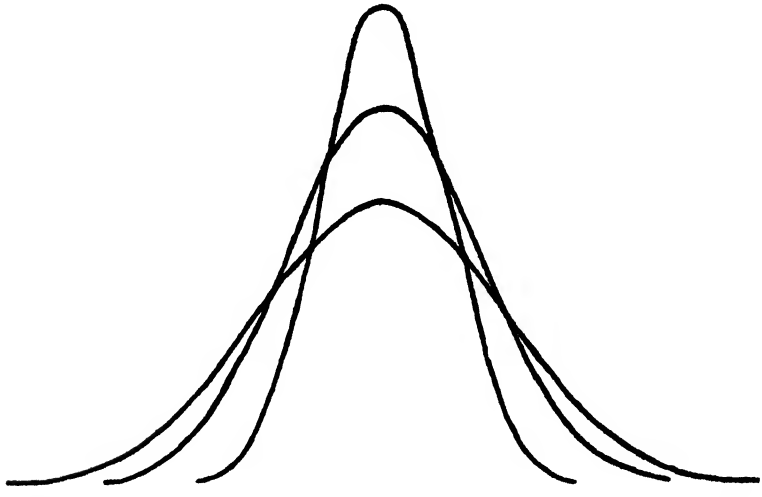

Fig. 4.5b. Three normal curves differing only in the amount of scatter.

4.5: Kurtosis. Kurtosis is a measure of the degree of peakedness of a curve. A curve that is more peaked than the normal curve is said to be *leptokurtic*, the normal curve is said to be *mesokurtic*, and a curve less peaked than the normal curve is said to be *platykurtic*. These types are illustrated in Fig. 4.5a. The normal curve has a kurtosis value of 3.0. As in the case of skewness, kurtosis is seldom an important consideration.

It should be noted that there is a basic difference between a platykurtic curve and one that appears flattened merely because the scale of the base line has been spread out. This difference will be noted by comparing Figs. 4.5a and 4.5b. Note that the tails of the three curves in Fig. 4.5a all come very close to the base line at about the same point, whereas the tails of the curves in Fig. 4.5b are spread apart.

CHAPTER V

PICTORIAL PRESENTATION OF DATA

5.1: General. There are many different ways of presenting data pictorially. We shall mention only a few of the most widely used and generally convenient methods for quality control work.

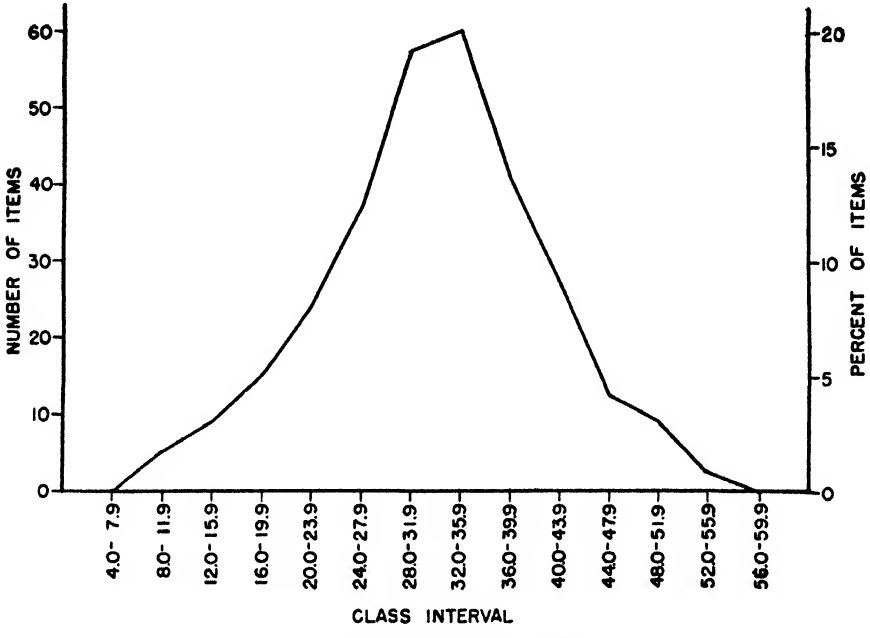

Fig. 5.2. Frequency polygon.

5.2: Frequency polygon. The frequency polygon provides an effective way of presenting a "picture" of a frequency distribution. Fig. 5.2 shows the class interval data of Section 4.3 in a frequency polygon. If it is desired to show the frequency of occurrence in terms of both number of items and percentage of items, it is suggested that the two vertical scales be placed on opposite sides of the chart. This obviates the visual confusion that occurs when both scales are placed on the same side of the chart.

Essentially, the chart consists of a series of points, the height of which above the base line indicates the frequency of occurrence of items for each of the steps or class intervals of measurement. These successive points are connected by straight lines for the sole purpose of aiding the eye in getting the pattern made by the points. No attempt should be made to read fre-

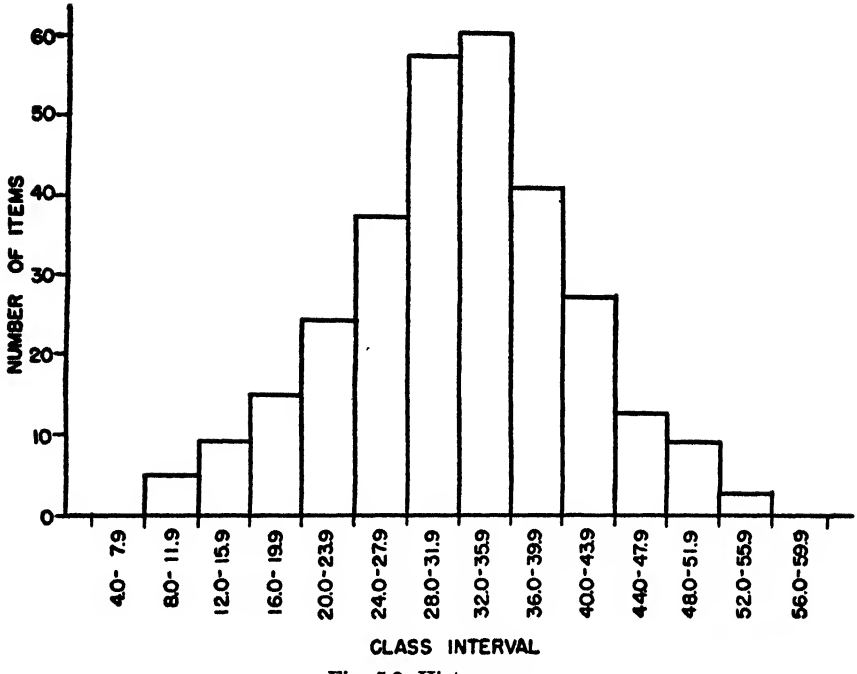

Fig. 5.3. Histogram.

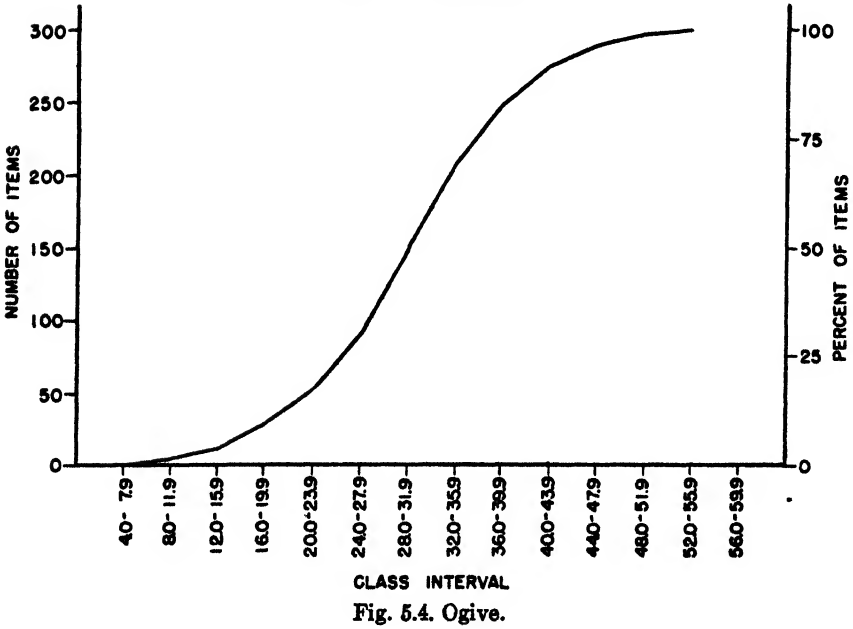

Fig. 5.4. Ogive.

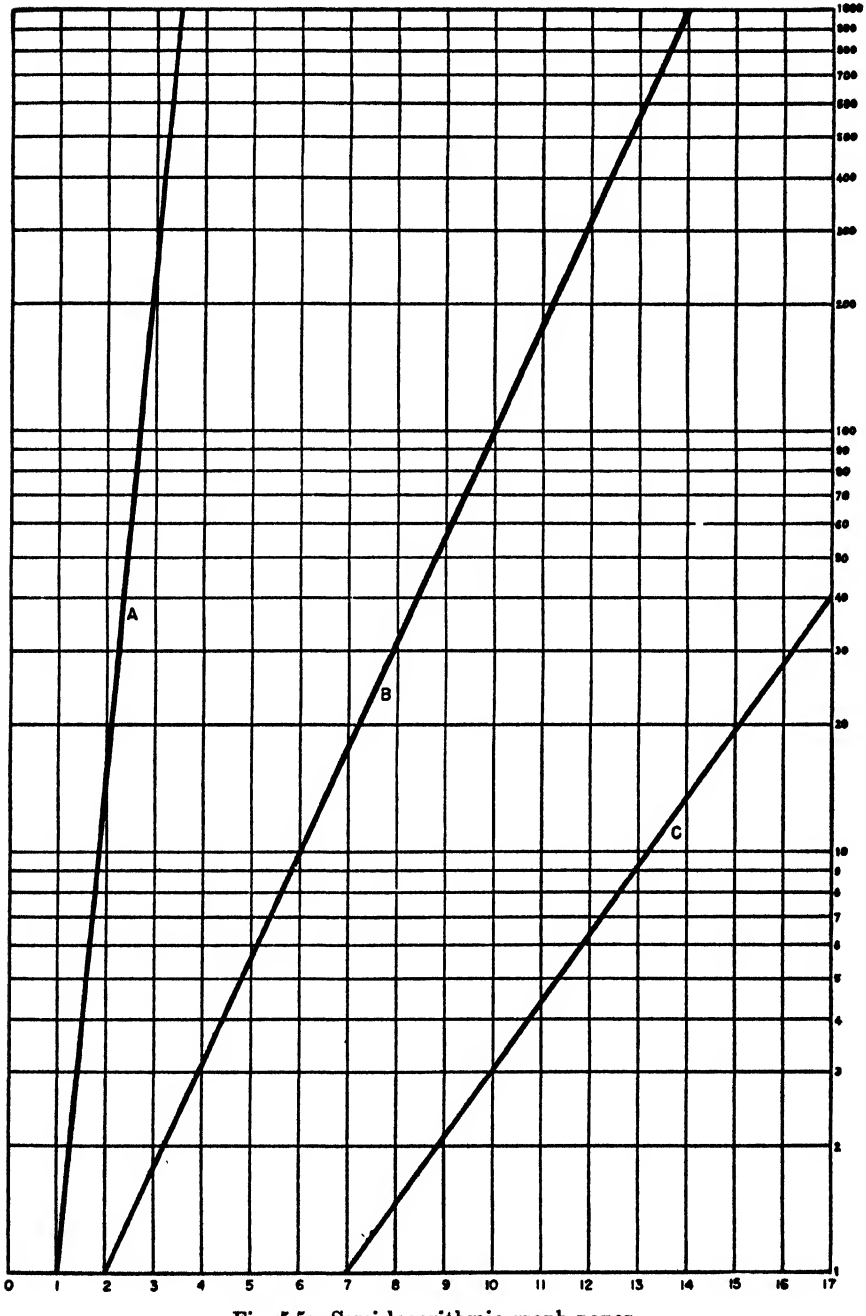

Fig. 5.5a. Semi-logarithmic graph paper.

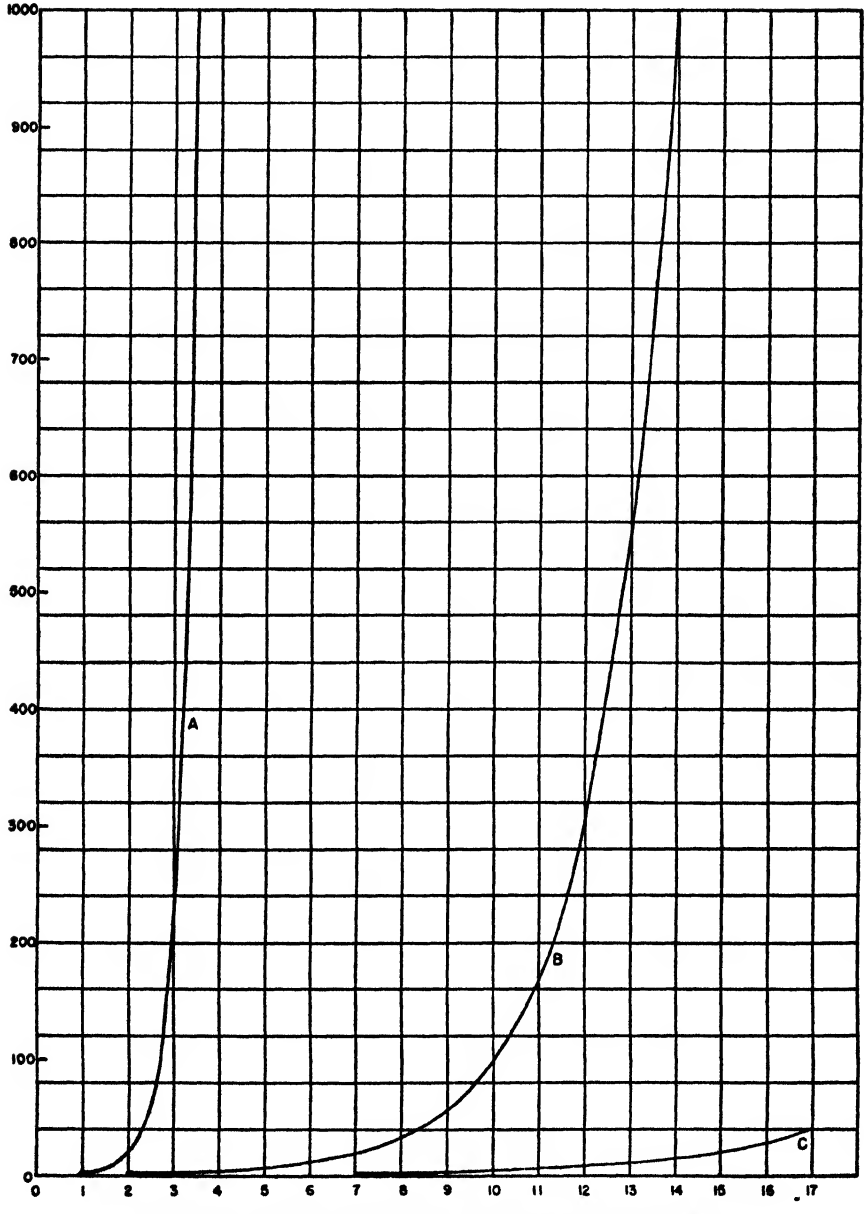

Fig. 5.5b. Curves of Fig. 5.5a on equal spaced graph paper.

quencies at locations along these lines at other than the plotted points. The plotting of class intervals opposite tick marks along the base line as shown in Fig. 5.2 will help discourage such attempts.

5.3: Histogram. The histogram consists of a series of columns, the areas of which represent the number of items in each interval. Fig. 5.3 presents the class interval data of Section 4.3 in the form of a histogram. If desired, the sides of the rectangles may be omitted between the inside of the horizontal steps and the base line.

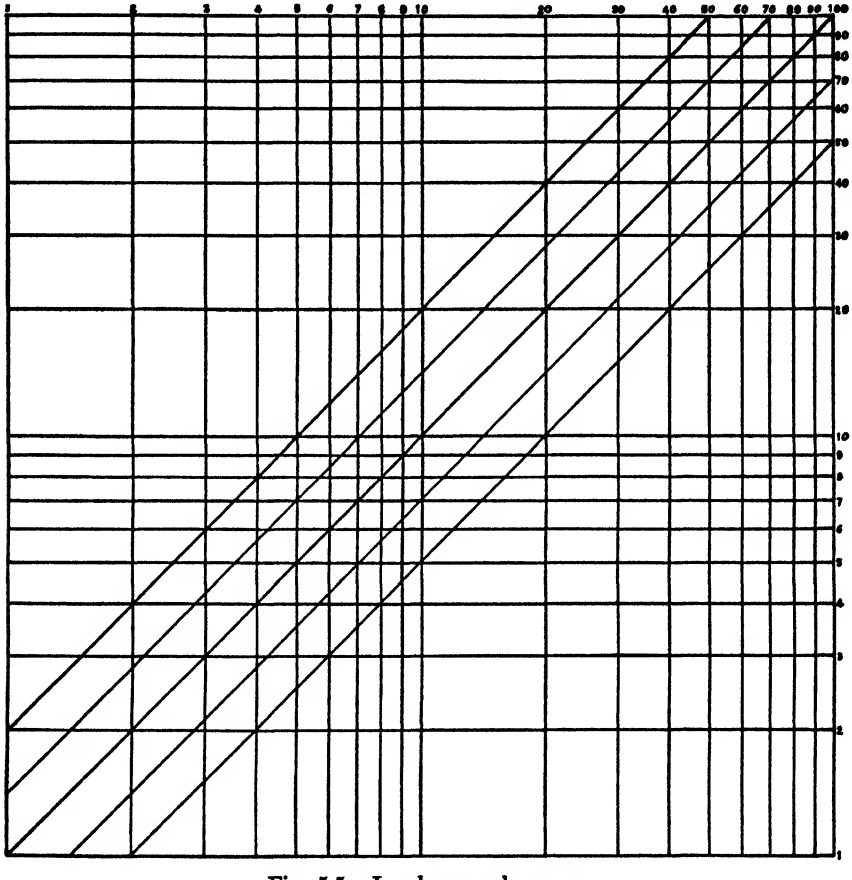

Fig. 5.5c. Log-log graph paper.

Whether a frequency polygon or histogram should be used is largely a matter of preference; however, the frequency polygon can generally be used to better advantage if two or more distributions are to be plotted on the same diagram. The frequency polygon implies that all the values in an interval are concentrated at the midpoint, while the histogram implies that they are spread uniformly over the interval. In practice, both implications are usually incorrect.

5.4: Ogive. The ogive is a cumulative frequency curve. Fig. 5.4 shows the class interval data of Section 4.3 in an ogive. The chief advantage of the ogive is the ease with which cumulative percentages can be read. It must be remembered that the plotted points represent the cumulative percentages to the upper limits of the class intervals. Another advantage is the relative ease with which a curve can be smoothed through the plotted points.

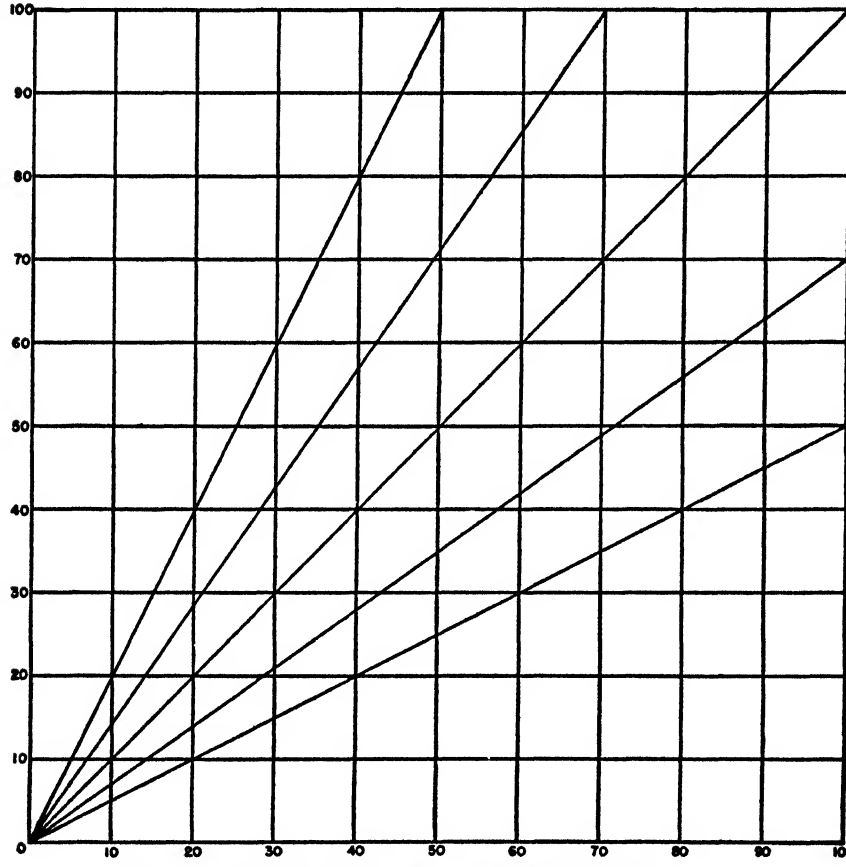

Fig. 5.5d. Curves of Fig. 5.5c on equal spaced graph paper.

5.5: Graph paper. Of the many varieties of graph paper now commercially available, three types are especially useful to the statistical worker in industry. One of these is semi-logarithmic paper. On this paper, one of the scales consists of equal size subdivisions. The other scale is logarithmic. Fig. 5.5a shows this type of graph paper. The advantage of such paper is shown by the three straight lines which are also plotted in Fig. 5.5b on

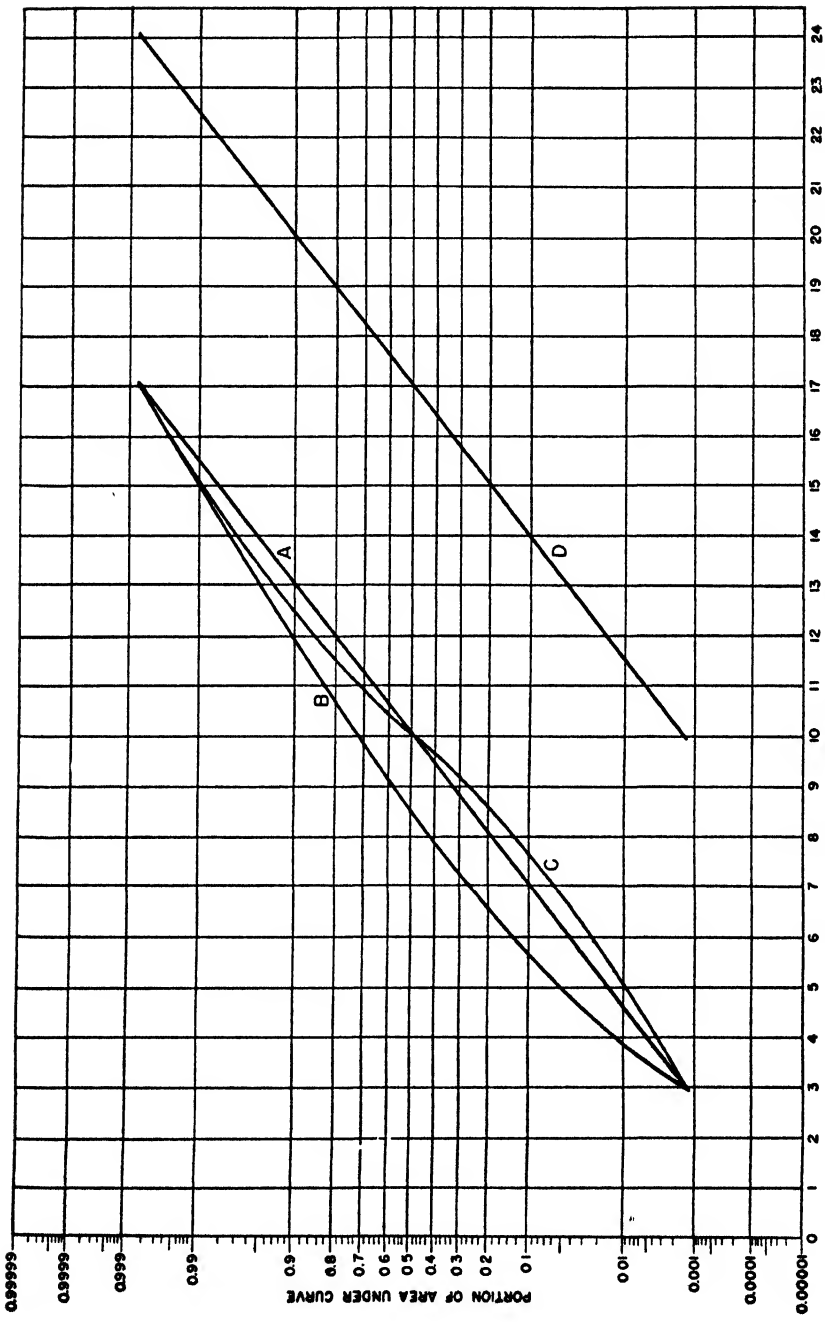

Fig. 5.5e. Probability graph paper.

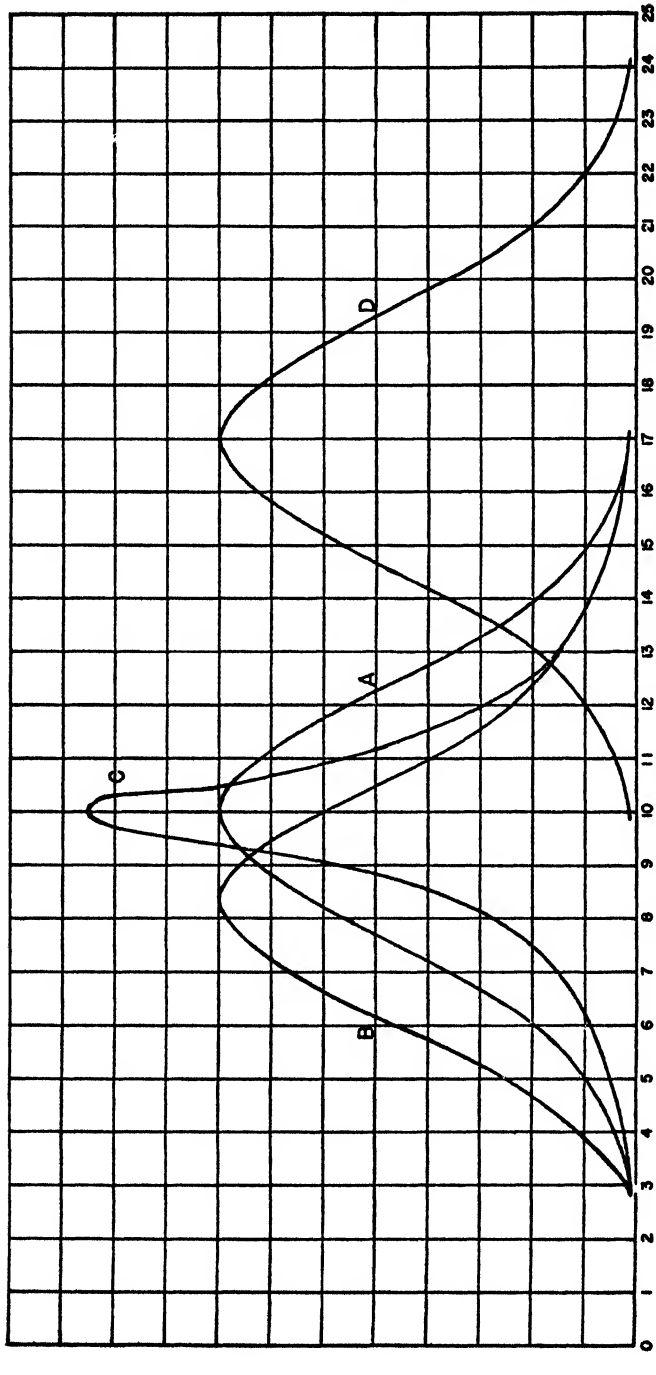

Fig. 5.5f. Curves of Fig. 5.5e on equal spaced graph paper.

graph paper having equal size subdivisions in both directions. In addition to permitting the plotting of logarithmic relationships as straight lines, semi-log paper also permits greater precision when reading smaller values on the log scale.

Another type of graph paper is log-log paper. This type has logarithmic scales in both directions. It is illustrated in Fig. 5.5c. The lines drawn on it are also shown in Fig. 5.5d on graph paper having equal subdivisions on both scales. In addition to permitting greater precision when reading smaller values, log-log paper eliminates the convergence of lines (straight or curved) having a common origin on equal interval paper.

The other type, probability paper, provides a way of quickly checking the normality of a distribution. Line A on Fig. 5.5e represents a normal curve as also shown on Fig. 5.5f. A positively skewed distribution (line B) curves above line A in Fig. 5.5e, while a negatively skewed distribution would fall below line A in a similar manner. A leptokurtic distribution forms a broad S as in line C. A platykurtic distribution would form a broad reverse S in a similar way. To plot a distribution, all that is necessary is to compute the cumulative percentages of the items for the various steps across the base line. Results from small samples should be considered tentative unless extremely non-normal and consistent in a number of samples. A fairly precise determination of the shape of a distribution would require about 3000 to 5000 items obtained under statistically controlled conditions.

Another advantage of probability paper is that two or more distributions may be drawn on the same diagram without any overlapping of the curves as illustrated by curves A and D in Figs. 5.5e and 5.5f.

CHAPTER VI

THE QUALITY CONTROL CHART FOR VARIABLES

6.1: The quality control chart for variables. Fig. 6.1 shows a quality control chart for a process that is operating in a statistically controlled manner. The data for this chart were obtained by drawing numbered chips from a bowl. Since the cause system (bowl and operation of drawing chips sight unseen) remained constant throughout the operation, it would be expected to show control as defined in the next section. When one is first confronted with such a chart, many questions flood upon the mind. It will be our purpose in this chapter to deal with the more important and outstanding ones.

6.2: The state of statistical control. A process is said to be *in a state of statistical control* when the variations in successive observations of the quality characteristics involved are due solely to a constant system of chance causes. It will be noted that there is no suggestion here that anything happens just "by accident" or without any cause whatsoever. It is definitely stated that causes of quality variation are at work, but identifies them as "chance causes;" that is, they are permitted to vary as they may without any effort being made to control or eliminate them, or even to identify them. Individually, their effect on quality variation is negligible; collectively they make up what is known as *inherent variability*. This is the variation in the production process with which we must be satisfied, or else change the process fundamentally to bring about a new constant system of chance causes. It is seldom possible to identify, and it is never economical to control or eliminate chance causes.

Assignable causes of quality variation are those that may be more readily identified and that may be controlled or eliminated economically. They can usually be identified with comparatively little study or investigation. Sometimes they may be difficult to identify; however, it will be wise to search for an assignable cause each time the presence of one (or more) is indicated. We shall see farther on how the control chart indicates the presence of such a cause.

6.3: Gathering data for a control chart. No control chart can be any better than the data on which it is based. We should first assure our selves that we are measuring the thing we want to measure, that the measuring equipment is operating properly, that the operators are properly instructed and free from bias, that transcription errors are avoided by adequate checking, and that any computing equipment used is operating

properly and is properly operated. The next problem is the number of observations needed. There is no hard-and-fast rule, but in general at least 100 would be desirable, and 40 would constitute a bare minimum.

Closely allied with the question of how many observations to take is the matter of group size. This will have a bearing on the exact number of observations used to start a chart. By group size is meant the number of successive observations that are considered together for control chart purposes. For example, the first four items produced might constitute group one, the next four group two, the next four group three, etc. In the next section we shall discuss the reason for the grouping data.

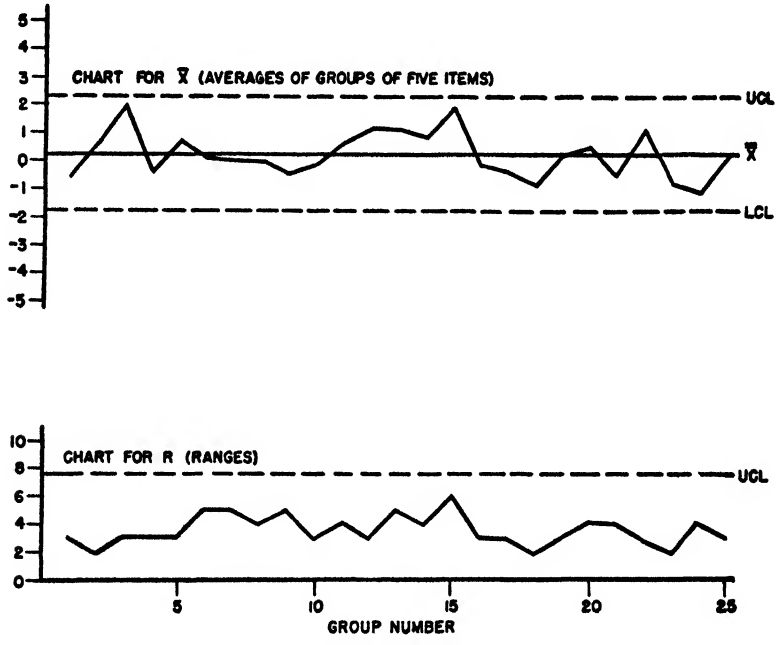

Fig. 6.1. The quality control chart for variables.

6.4: The reason for grouping data. Since the primary purpose of a control chart is to reveal the presence of assignable causes of variation so that we may act promptly to control or remove them, it is desirable that the control chart limits be based on only the variability that is inherent in the process. If we lump all the data together and compute the standard deviation of the individual measurements, we shall include in that determination all the assignable causes as well as chance causes of variation. As a result, our estimate of variability will be too large if the process is not actually in control. If the process is in control (few processes are when first investigated) it will not make much difference. Since we cannot know the degree of

control until after the chart is constructed, we must always avoid lumping all the data and computing the standard deviation of the individual items on this basis.

The data below shows the estimated population standard deviation of a controlled and an uncontrolled process as computed on the basis of the lumped individual items and as grouped into successive five's in the usual control chart procedure (this procedure will be discussed later). Each series consisted of 125 successive observations.

	Estimate of population standard deviation	
	Lumped Individuals	Normal Control Chart Group Method
Controlled process	1.58	1.56
Uncontrolled process	2 88	1 53

It will be noted that when there is lack of control, the individual values increase in their scatter to such an extent that they give an abnormally high estimate of population variability. Actually in the above illustration, three different levels of quality were being produced during collection of the 125 uncontrolled process observations. The inherent variability at each of the three levels, however, was the same.

Even though a process is not controlled, items produced in a short span of time will be likely to come from the same constant system of chance causes. Assignable causes entering or leaving a process seldom affect more than one or two groups with respect to the range of the groups. Since most of the group ranges are unaffected, the average group range will very nearly reflect only inherent process variability, whether or not the process level (averages of groups) is actually controlled. Of course, if the group ranges are seriously out of control, no estimate of the inherent capabilities of the process can be made until the ranges are brought into control. Fortunately, ranges (which reflect process variability) seldom get seriously out of control as compared to group averages (which reflect process level).

6.5: How to determine the group size. The group size may be anything from two up. If it is at all possible, it is generally desirable to avoid group sizes of two or three. If it is necessary to take data by two's and three's, serious consideration should be given to grouping two consecutive such groups to form a group of four or six. There are two reasons for this. One is to avoid too frequent computation of group averages and ranges, and the other is the fact that if the parent distribution is markedly non-normal, the distribution of group averages of two's or three's will also be definitely non-normal, whereas for group sizes of four or more the distribution of group averages is essentially normal, virtually without regard to the shape of the parent distribution. Thus for group sizes of four or more, the three sigma

limits for averages are valid even though the parent population is definitely non-normal.

One advantage in using a group size of four instead of some larger number is that points are plotted more frequently and indications of lack of control thereby caught more quickly. This is partly offset by the fact that smaller

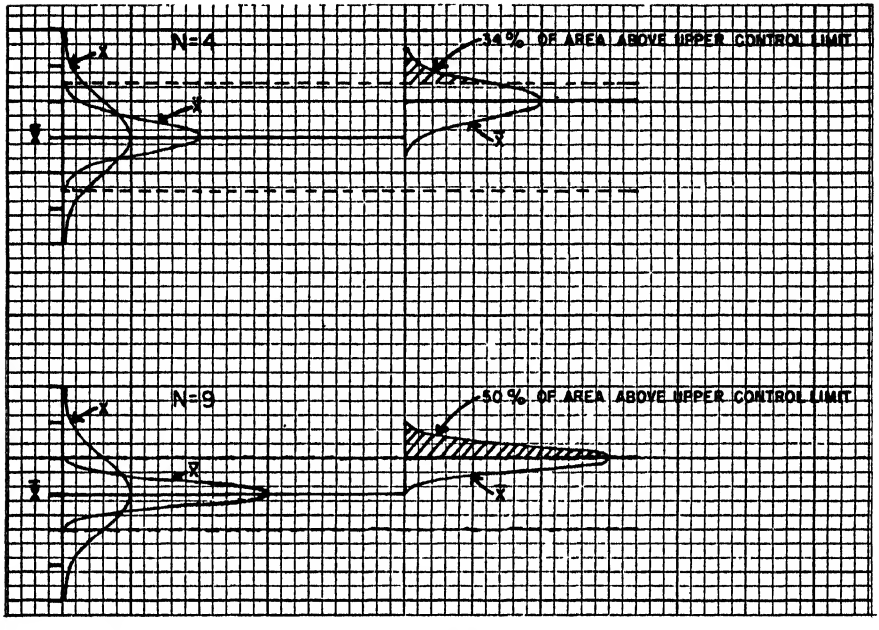

Fig. 6.5. Effect of sample size on variability of sample average.

Let us assume that the process level and variability of individuals (X) are established as shown on the left. In both of the above cases a subsequent shift in process level of one standard deviation of individuals is shown. When the sample size is 4 it will be noted that the probability of a sample average ($\bar{X}$) now falling above the upper control limit (based upon the sample averages of 4 as originally distributed) is 0.34. When the sample size is 9, this probability is increase to 0.50.

group sizes are less sensitive to small shifts in process levels than are larger groups. This follows from the formula

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{N}}$$

in which $\sigma_{\bar{X}}$ is the standard deviation of group averages, σ is the standard deviation of the parent population, and N is the sample size (group size). In other words, the variability of group averages varies inversely as the square root of the sample size. Fig. 6.5 illustrates this relationship.

Another advantage is the fact that a small group size will be more likely

to contain only inherent variability than a larger group. Still another is that where the individual items are approximately normally distributed and it is desired to check them, the control limits for individuals can be very easily computed, since they are exactly twice as far from the central line as the control limits for averages of fours.

Sample groups of five have the advantage that the average of the five numbers observed can be very easily obtained. All that is necessary is to add the five numbers, multiply by two and shift the decimal point one place to the left as follows:

73	
69	
47	
55	
61	
305	Sum of the five observations
305	Adding the same figure is equivalent to multiplying by two
610	
61.0	Average obtained by shifting decimal point one place to left

One company uses this procedure to advantage on a tape-printing adding machine. On each run of data there are first printed as non-add numbers the identifying order, part, or chart number, and date. Then the five observations are recorded and subtotaled. The subtotal is then recorded again and a grand total recorded. Now all that is necessary is to shift the decimal point one place to the left and the average is obtained. Next, the highest value in the group is recorded and then the lowest value is subtracted from it, giving the range of the group. The tape then forms a permanent record for future reference.

While these conveniences are desirable, these group sizes should be used only when such a group size forms a logical basis for grouping, or time sequence is the only basis on which grouping can logically be made. Insofar as possible, groups should be formed on some logical basis. If six items are tested per shift, four samples taken from one machine, then four from another, etc., seven tests made per batch, or eight items examined from every heat, these group sizes should be used. When all the samples in a group come from some natural manufacturing unit they are more likely to represent inherent variability only, and thus provide desirable control limits. When a lack of control occurs, it will then be possible to relate that fact to a logical unit of material and more effectively determine the nature of the factors responsible for the lack of control. A large part of the success that any quality control engineer will have depends upon how effectively he plans the grouping for his control charts.

In general, it is recommended that group sizes be kept as small as practicable except that groups of two or three should be avoided. Group sizes that are larger than 15 when formed on a logical basis of grouping should be broken into two or more groups.

6.6: Determination of control limits. Control chart limits may be derived from either standard deviations or ranges. The range of a group is determined by subtracting the lowest value from the highest value in the group. Since the relationship between ranges and standard deviations is known, it is a simple matter to convert from one to the other by the use of factors which vary in accordance with the group size. Fig. 6.6 shows control limits derived both ways from the same set of data. It will be noted that there is virtually no difference between the two sets of limits, and the general pattern of the points for standard deviation is the same as the pattern of points for ranges. In view of the large amount of labor involved in computing the standard deviations, it is generally preferable to use ranges. Table 6.6 gives the factors for determining the three standard deviation limits for groups of various sizes.

After recording the individual values for the group, the average and range of the group is determined. The average of all the group averages ($\bar{\bar{X}}$) and the average of all the ranges ($\bar{R}$) is determined and the control chart limits determined as shown in Fig. 6.6 using the proper factors from Table 6.6.

It is recommended that at least 25 groups of data be accumulated before starting a chart. Fewer than this may be used if necessary, but less than ten should not be used except under the most unusual circumstances.

6.7: Significance levels. The quality control chart consists essentially of a solid line plotted on a scale to represent the average level of the quality characteristic involved. Around this central line are placed dotted parallel lines to indicate that band within which virtually all the observed results should fall as long as the process is statistically controlled. Control charts for variables are usually supplemented with a control chart for some measure of variability, such as range.

As has previously been indicated, three standard deviations plus and minus from the mean includes 99.73 per cent of all the items in a normal distribution. Because of this fact the most commonly used practice for establishing quality control limits is to place them at three standard deviations above and below the mean. When limits are placed close to the mean, observations will fall outside them more frequently just due to chance.

Two kinds of errors may be made in taking action on the results of a quality control chart. Action may be taken on the basis of a point falling outside control limits when there is no lack of control (the point outside limits being a chance occurrence), or action may not be taken when a lack of control actually exists, the point falling inside control limits because the

CONTROL LIMITS FOR AVERAGES

$\bar{\bar{X}} \pm A_2\bar{R}$	$\bar{\bar{X}} \pm A_1\bar{\sigma}$
$0.21 \pm 0.577(3.6)$	$0.21 \pm 1.596(1.31)$
0.21 ± 2.08	0.21 ± 2.09
UCL = 2.29	UCL = 2.30
LCL = −1.87	LCL = −1.88

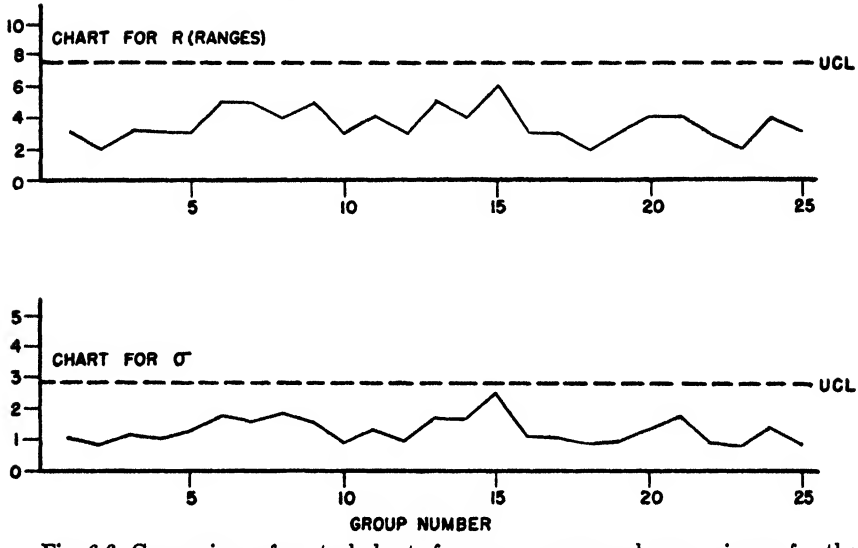

Fig. 6.6. Comparison of control charts for group ranges and group sigmas for the same date. These charts are based upon the same data used in preparing Fig. 6.1. The original data were obtained by drawing at random from a bowl containing chips making up a normal distribution. The data were tabulated in the following manner:

Group No.	1	2	3	4	5	$\bar{X}$	R	σ
1	0	−1	−2	−1	1	−0.6	3	1.02
2	0	2	1	0	0	0.6	2	0.80
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
25	2	−1	0	0	0	0.2	3	0.98
Averages						$0.21(\bar{\bar{X}})$	$3.6(\bar{R})$	$1.33(\bar{\sigma})$

The upper part of the figure shows the computation of limits for the control chart for average shown in Fig. 6.1. Since both methods give essentially the same results, factors (A_1) for computing limits from sigmas are not given in Table 6.6. In comparing the patterns of the range and sigma charts it will be noted that both tend to rise and fall in the same manner. The patterns are not identical since the range uses only the two extreme values in each group while the sigma uses all values in each group.

shift in level is slight. We must therefore balance the error of looking for trouble when none exists against the cost of failing to look for trouble when it actually exists. Where there is no unusual economic aspect to the situation, the three standard deviation limits have been found most useful in practice. At times two sigma limits are used as warning limits. The action limits in connection with some statistical tests are stated in terms of proba-

TABLE 6.6. FACTORS FOR COMPUTING THREE SIGMA CONTROL LIMITS

No. of observations in sample	Chart for Averages Factors for Control Limits	Chart for Ranges			Factors for Individual Items
		Factor for Central Line	Factors for Control Limits		
N	A_2	d_2	D_3	D_4	I_2
2	1.880	1.128	0	3.268	2.659
3	1 023	1 693	0	2 574	1.772
4	.729	2 059	0	2 282	1.457
5	.577	2.326	0	2 114	1.289
6	.483	2.534	0	2 004	1.183
7	.419	2 704	.076	1 924	1.109
8	.373	2 847	.136	1 864	1.053
9	.337	2 970	.184	1.816	1.010
10	.308	3 078	.223	1 777	.974
11	.285	3.173	.256	1 744	.945
12	.266	3 258	.284	1.717	.920
13	.249	3.336	.308	1.692	.899
14	.235	3 407	.329	1 671	.880
15	.223	3 472	.348	1 652	.864

Formulas

Chart for	Central Line	Control Limits
Averages	$\bar{\bar{X}}$	$\bar{\bar{X}} \pm A_2\bar{R}$
Ranges	$\bar{R}$	$D_3\bar{R}$ and $D_4\bar{R}$
Individuals	$\bar{\bar{X}}$	$\bar{\bar{X}} \pm I_2\bar{R}$

$$\frac{\bar{R}}{d_2} = \sigma \text{ of the population}$$

Columns A_2, d_2, D_3, and D_4 of this table are taken from Table I, Supplement B, of the "A. S. T. M. Manual on Presentation of Data" by permission of the American Society for Testing Materials, Philadelphia, Pa.

bilities. The values 0.01, 0.05, and 0.1 are often used. Commonly used sigma and probability limits are given in Table 6.7.

It must be borne in mind that in dealing with quality control charts (and other statistical tests) no exact probability values can be attached to the control limits because the observed average and observed inherent variability are only estimations of the true values for the process. The control

level and limits almost invariably are slightly misplaced; however, this is normally of no consequence and need not be a matter of concern. Usually the three sigma limits will be entirely satisfactory, and it is recommended that the beginner confine himself to the use of such limits.

6.8: Most processes are out of control when first investigated. When plotting a control chart for the first time, the beginner is often dismayed to find the process out of control. He is likely to say, "There's no use doing anything more with this process as it is hopelessly out of control." As a matter of fact, few if any processes are ever hopelessly out of control. Almost invariably steps can be taken to bring them under control. Further, the existence of causes of lack of control is exactly what we are using the control chart to reveal. We should be pleased that we have found indications of lack of control as we can now have confidence that our search for trouble sources will probably be rewarded. In the next chapter some of the

TABLE 6.7. SIGNIFICANCE LEVELS

Sigmas from mean toward one tail of normal curve	Probability		
	Portion of area still remaining under one tail	Odds for an event in one tail	Odds for an event in either tail
1.28	0 1	1 in 10	1 in 5
1.50	0 06681	1 in 15	1 in 7
1.64	0.05	1 in 20	1 in 10
2.00	0.02275	1 in 44	1 in 22
2.33	0.01	1 in 100	1 in 50
3.00	0.00135	1 in 741	1 in 370
3.09	0.001	1 in 1000	1 in 500

ways of using the control chart to help locate assignable causes will be mentioned.

6.9: The effect of mixing cause systems. Figs. 6.9a, 6.9b, and 6.9c show three different parent populations especially prepared for demonstration purposes. If the three distributions are added, the result obtained is shown in Fig. 6.9d.

Physically similar chips were prepared with numbers on them corresponding to the first three distributions (Figs. 6.9a, 6.9b, and 6.9c) and mixed together in one bowl. Six chips were drawn from the bowl. The numbers on the chips were recorded and restored to the bowl, mixed up and the operation repeated until 30 groups were obtained. The results are shown in Fig. 6.9e. It is obvious that a state of good control appears to be in operation. The chips were then separated into the three distributions. Six chips were drawn at a time from bowl A until ten groups were obtained

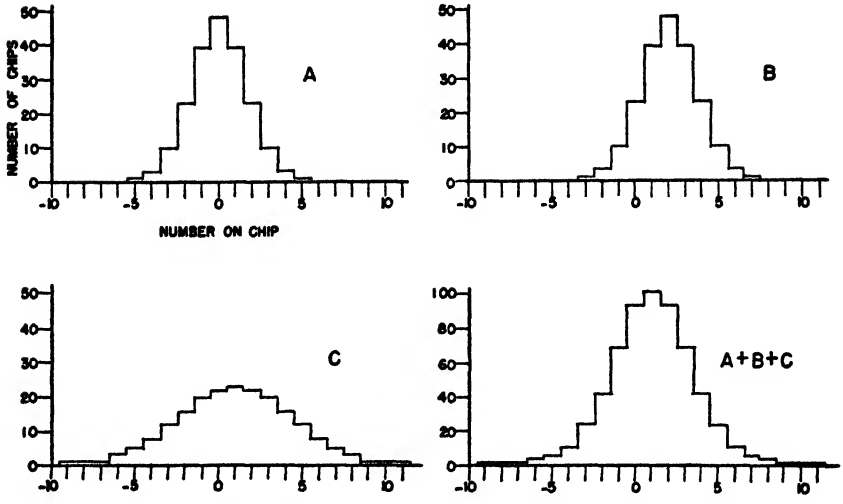

Fig. 6.9a. (Upper left) Distribution with mean of zero and sigma of 1.715.
Fig. 6.9b. (Upper right) Distribution with mean of two and sigma of 1.715.
Fig. 6.9c. (Lower left) Distribution with mean of one and sigma of 3.470.
Fig. 6.9d. (Lower right) Distribution with mean of one and sigma of 2.578.

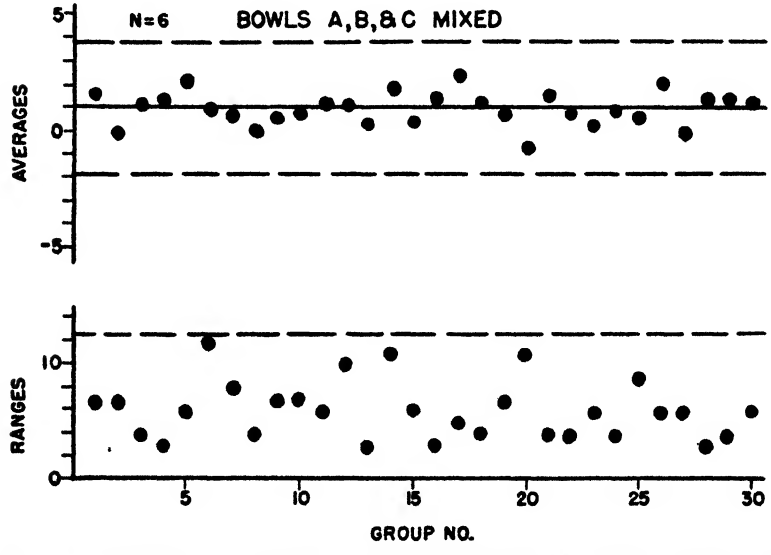

Fig. 6.9e. Appearance of control resulting from random sampling of mixed populations.

(the chips were replaced and mixed after each group was drawn.) The same was done for bowls B and C. (The small number of ten groups was used for convenience in preparation of the illustration.) Fig. 6.9f shows the results. It is obvious that some of the points on each chart would be out of control if judged by the control limits of one of the other charts. A control chart does not reveal assignable causes if the samples are taken at random with respect to the cause. A new basis for grouping will sometimes reveal causes of trouble otherwise hidden.

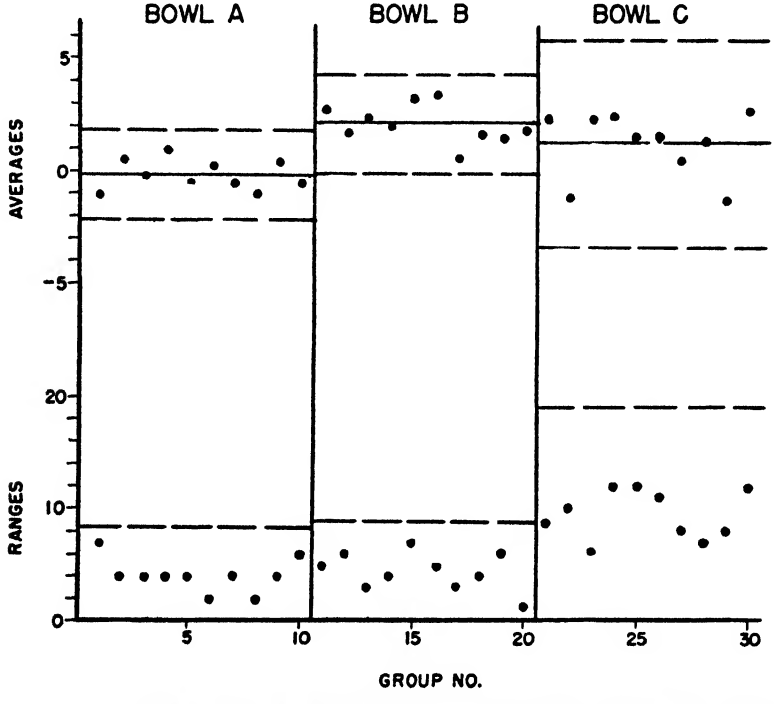

Fig. 6.9f. Drawings of samples of six from the populations that were mixed together in Fig. 6.9e.

6.10: Some practical aspects of the quality control chart. Unfortunately some people have been misled into believing that mass production is necessary before the quality control chart can be used. As a matter of fact the only restriction that must be made with respect to any application of the quality control chart is that the process involved must be one in which an effort is made to produce many items or to make many measurements on the same item, all of which under ideal conditions would be expected to be exactly alike with respect to some measurable quality characteristic. Thus the quality control chart can be applied to one item.

Lack of control may be due to three different situations. The most com-

mon of these is the one in which the inherent variability remains essentially constant, but the quality level shifts from time to time. This is illustrated in Fig. 6.10a. The level of quality may remain essentially constant while the variability changes from time to time, as shown in Fig. 6.10b. The last condition is one in which both quality level and inherent variability shift at the same time (or during the same short period), as in Fig. 6.10c. The idea of a controlled situation is one in which both the quality level and inherent variability are essentially constant (Fig. 6.10d).

It is sometimes said that there must be a distribution (constant system of chance causes) before a useful prediction can be made; however, there is a distribution cause system that gives rise to every observation. Hence, before a useful prediction can be made, there must be a reasonably constant cause system for an adequate length of time—long enough for a reasonably satisfactory determination of the nature of the distribution of the results obtained.

Steel items which require heat treatment often become mixed in the manufacturing process so that a heat-treatment furnace charge often contains items from several different melts. If a sampling program is planned, the question immediately arises as to whether the samples should be grouped according to heat-treatment lots or according to the melts from which the pieces come. Fig. 6.10e illustrates the arrangement of eight groups of four test results, each according to heat-treatment lots and according to melts. In the case of the heat-treatment groups, each group contains test results from two, three, or four melts. In the grouping of melts, each group contains test results from one melt only. It will be noted that all groups in the heat-treatment arrangement fall inside the control limits, while the arrangement by melts shows three of the eight averages outside the control limits. There is clearly some cause (or causes) operating in the melting process to cause significant differences among the melts. When the test results are mixed without regard to the melt, these indications of lack of control are lost.

When several different plants or production lines are making the same type of production, differences in quality levels and inherent variability will be the normal experience, even though all are operating in a statistically controlled manner within themselves. This is illustrated in Fig. 6.10f. It will be noted that each plant has a different quality level and a different inherent variability. If the quality level is desired at the highest possible point, Plant A would be the desired source of supply. If the least possible variation in the products is the important consideration, Plant D would be the one to select. If a combination of A's level and D's variability is needed, it will probably be much simpler to get Plant D to shift its level to A's rather, than to try to get A to decrease its variability to match D's. Shifting levels is generally a much simpler job than reducing inherent variability.

If assignable causes of variability are arranged in order according to the

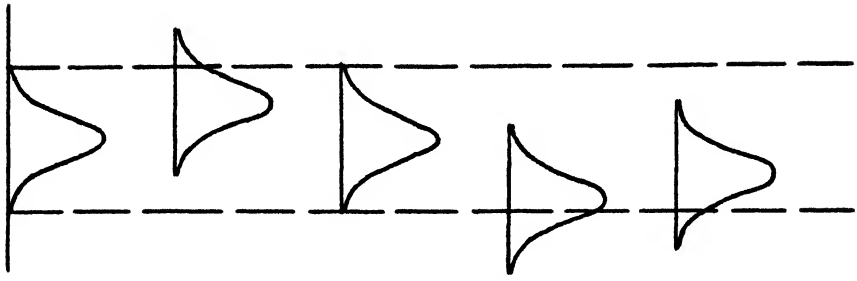

Fig. 6.10a. Lack of control due to a shifting quality level.

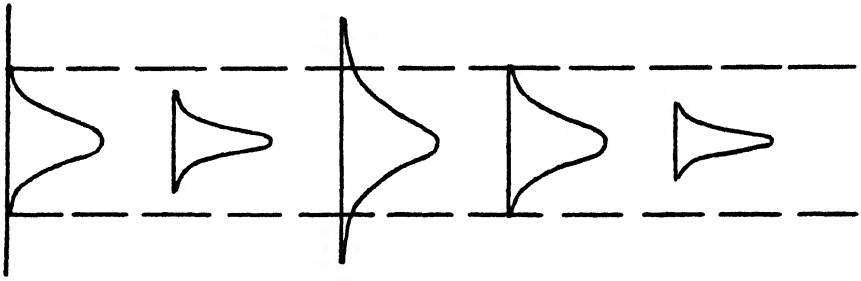

Fig. 6.10b. Lack of control due to changes in inherent variability.

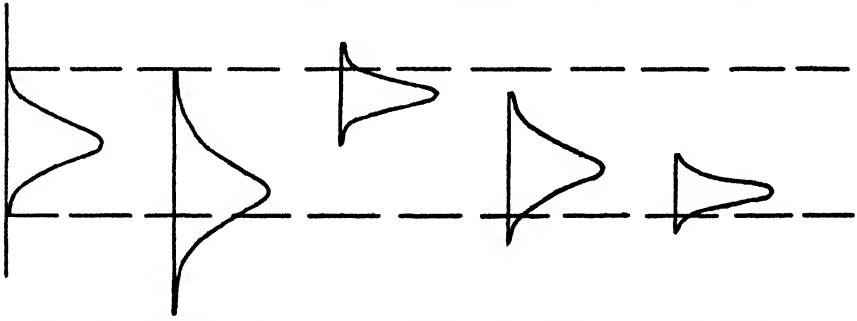

Fig. 6.10c. Lack of control due to changes in both quality level and inherent variability.

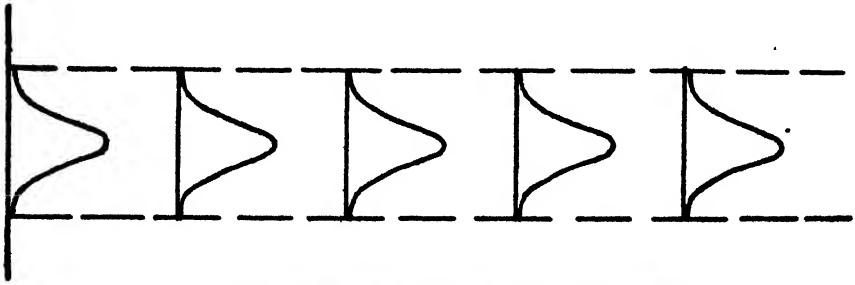

Fig. 6.10d. A statistically controlled process.

magnitude of their effect upon a specific quality characteristic, the first few will generally be found to far outrank the remaining causes with respect to their importance. There are seldom more than about seven or eight causative factors of outstanding importance with respect to any given quality characteristic.

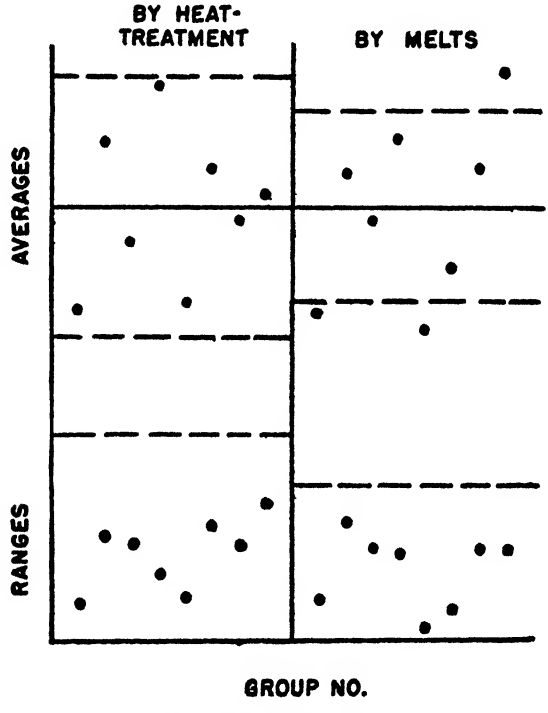

Fig. 6.10e. Effect of selecting proper basis for grouping of test results. Plotted points are groups of four steel physical property test results. Each group on the left came from the same heat-treatment batch, but contains tests from two or more different melts. Each group on the right came from the same melt, but contains tests from two or more heat-treatment batches. The same test results are involved in both charts.

6.11: Drawing the quality control chart. While rough charts may be drawn in any convenient manner, it is highly desirable that charts that will be examined by other people, especially supervisors and management, be drawn with considerable care. In particular the following points should be considered:

(a) The size of the chart should be such as to be convenient for filing. There seems to be a rather pronounced tendency to make them too large. A control chart does not have to be big to make a big impression. Of course, blown-up charts can be very useful for large gatherings.

(b) The chart should be as neat as possible. Whether we like it or not some people still judge books by their cover and people by their clothes. Since a chart must at times do a selling job (especially in the beginning), it should be as attractive as possible. There should be a good balance of lettering sizes, scale used and plotting of points.

(c) The chart should provide for the addition of a reasonable number of future points as data are accumulated. To accomplish this the points should not be placed too far apart along the horizontal scale.

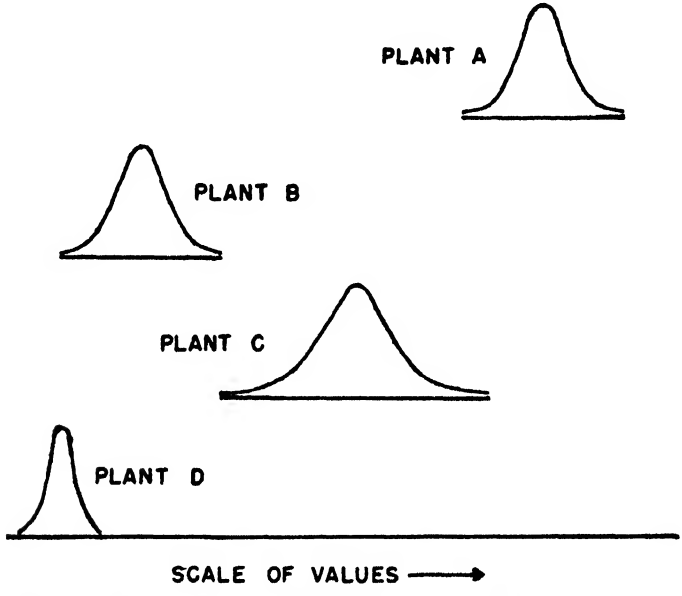

Fig. 6.10f. Normally expected differences among plants all making the same product.

(d) The extent to which the chart is labeled should be determined by the background and experience of those for whom the chart is intended. There is a tendency to underlabel charts so that insufficient information is provided for an accurate interpretation of the chart, and at times they are overlabeled, with the result that those who examine the chart become confused.

(e) At times it is desirable to revise control chart limits when additional data have been accumulated. Such revised limits should not be drawn over points previously covered by the original limits. The effect of double sets of limits is only one of confusion.

(f) Plotted points should be quite legible. Open circles or large dots are generally used for this purpose. Successive points are sometimes connected with straight lines. Remember, the purpose of a series of plotted points is

to give a picture of the extent of the scatter or trends and not to provide a means of reading exact values from a scale.

(g) In conformance with general practice it is recommended that solid lines be used for averages and dotted lines for control limits. There is no specific reason for this arrangement other than common practice.

(h) In selecting a style of graph paper, choose one with the minimum number of background lines necessary to plot and read the chart. Too many fine subdivisions confuse the eye and thus detract from the story the chart has to tell. Anywhere from five to ten divisions per inch should fill nearly all needs.

CHAPTER VII

USING THE QUALITY CONTROL CHART TO IDENTIFY ASSIGNABLE CAUSES

7.1: General. In its conventional use in process control, the quality control chart is considered to be a highly effective means for detecting the existence of assignable causes of variation in product quality. It is a device that is often described by saying that it tells when to act, when to do something to the process to bring it back in line. The point the so-called "practical man" often makes is that while that is fine as far as it goes, what he wants to know is the identity of the assignable cause that needs to be controlled or eliminated. While engineering knowledge and special investigation are generally necessary for this final step, judicious use of the quality control chart can often be of considerable help in locating the assignable cause.

7.2: Two fundamental aspects of the quality control chart. Before developing the specific subject with which we are here concerned, let us first briefly consider two fundamental aspects of the quality control chart as used in process control. First, most industrial processes involve a complex mixture of chance and assignable causes, so complex that a highly precise determination of the distribution pattern of the chance causes is economically impossible or operationally impractical. For this reason it will generally be wise to avoid the use of measurement control charts which plot the results of single observations, although it is realized that in special cases such charts may be very useful. Control charts of groups of four or more are desirable, since they obviate some of the difficulties involved in definitely non-normal distributions, even though they are less precise in pinning down the exact time when lack of control becomes apparent. The estimate of inherent variability should, of course, be based on small groups of observations, to eliminate as far as possible the effect of assignable causes on the estimate.

Secondly, the quality control chart does not necessarily tell us the exact time when a shift in process level or variability occurred. A small shift in level or variability may continue for some time before being detected, since such shifts only slightly increase the probability that a point will fall outside control limits. Fortunately, the smaller the shift the less important it is likely to be. In addition, unless all items are tested and results plotted, shifts may originate between samples. It must be remembered that a point outside limits is a much better basis for assuming that an assignable cause has

entered the process than a point inside limits is for assuming that no assignable causes have developed.

In spite of the foregoing limitations, the quality control chart continues to be a highly effective means of detecting the development of assignable causes. Let us now consider how the chart can be used to even greater advantage to unearth specific assignable causes.

7.3: Tracking down assignable causes. The beginner is cautioned to avoid making too many control charts at first in view of the dissipation of effort and effectiveness. One or two charts on the most troublesome quality characteristics is a general recommendation. When some familiarity with the use and operation of the control chart has been gained, an attempt to use it to help identify assignable causes is in order. Regardless of how complex a process may be, it can be broken down into four general sources of product quality variation: raw materials, machines, men, and methods. If the assignable cause or causes can be pinned down to one of these areas, considerable progress has been made toward final identification. If two or more sources of a raw material are used, separate charts may be kept on each source. Similarly, separate charts may be kept on different machines (all presumably operating alike), different operators or turns, or on different methods of processing. In addition, the length of time required for an obvious shift in quality level may be a clue to the nature of the assignable cause involved.

7.4: A case of charts based on machines (furnaces). In the following illustration (and in all others in this chapter), the original data have been greatly modified so that only the general nature of the situation is reflected.

Four glass melting furnaces were involved in the problem of cordiness (defects in glass). Control charts were kept on samples of the product from each of the furnaces. Samples were taken at approximately the same time from each furnace. The results are shown in Fig. 7.4.

It is immediately apparent that the shifts in quality level tend to be of the same nature in all the furnaces. This indicates that the assignable cause is some factor common to all the furnaces. Factors such as differences in the individual furnaces and furnace crews are immediately eliminated. Attention is narrowed to factors common to the four furnaces. In this case the raw materials are all fed into the furnace from the same bin; hence it would appear that the quality variations in the final product are primarily due to variations in the raw material. In any event, a careful study of the quality of the raw materials would be desirable. It offers much promise for fruitful investigation.

In this case we started with separate charts according to machines (fur-

naces) and concluded that the assignable cause was associated with the raw materials.

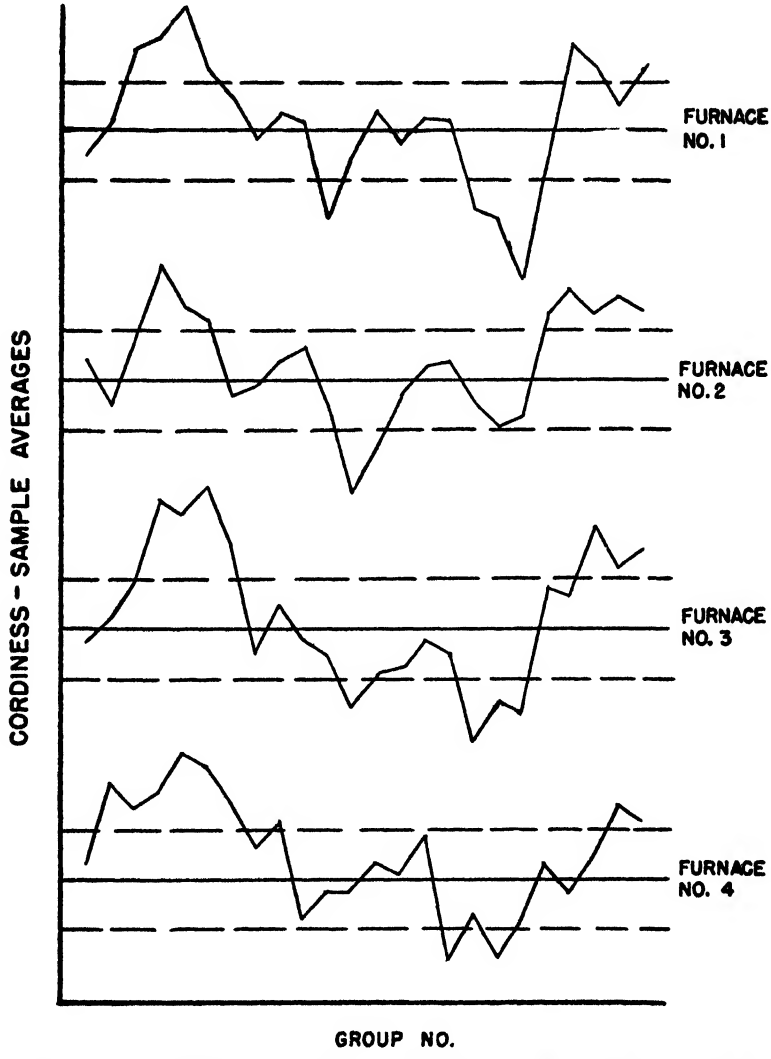

Fig. 7.4. Four glass melting furnaces showing the same quality trends.

7.5: A case of charts based on men (crews). In this illustration, it was suspected that either the crew or the shift was responsible for the trouble that was occurring. Too many defectives were passing through visual inspection. The crews were rotated each week and charts prepared as indicated in Fig. 7.5. It was immediately apparent that the source of the

trouble was the day shift and that the crews were all doing about equally good jobs. Special lighting had been provided for the two night shifts, but in the daytime this was supplemented with considerable natural light. It was suspected that the daylight was interfering with the effectiveness of the inspection operation. The windows were painted black so that only the special artificial light was used for the inspection operation. The trouble

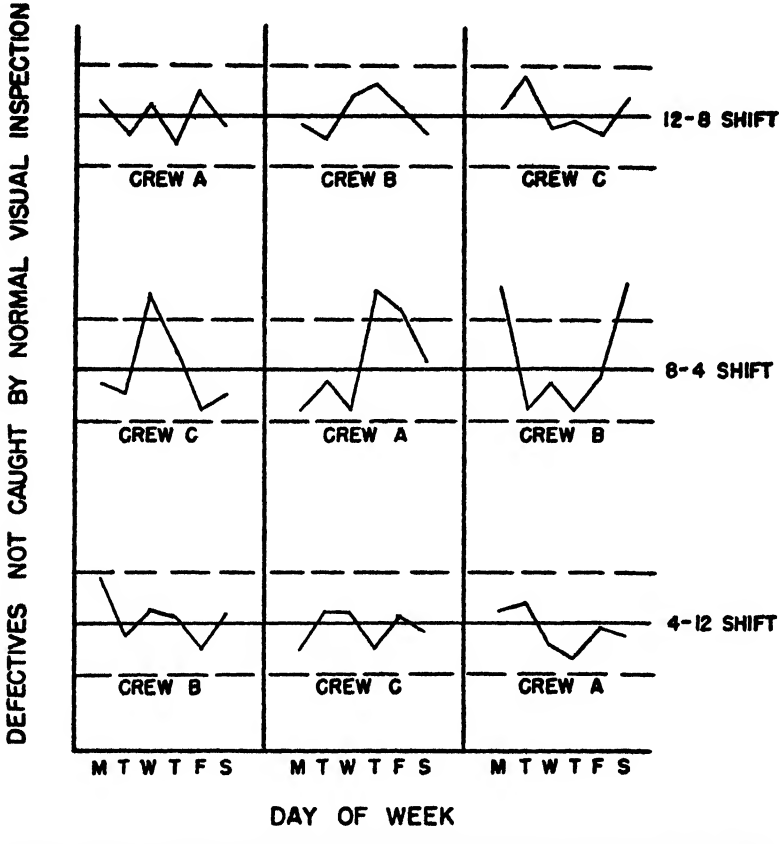

Fig. 7.5. Control chart arrangement revealing daylight inspection as an assignable cause.

that had been occurring on the day shift immediately cleared up. In this case we started with separate charts according to men (crews) and concluded that the assignable cause was associated with the method of inspection.

7.6: A case not requiring a chart. In another illustration, tool marks were found which were cause for rejection. The analysis in this case was so simple and the result so obvious that control charts were not needed;

however, the method of attack was in line with the sort of thing we are considering here and hence is appropriate.

The parts were made on a drum type machine having four cutters and six positions. The part had four trunnions. The trunnions on each of 125 pieces were identified and processed. Thirty-three of the 500 trunnions

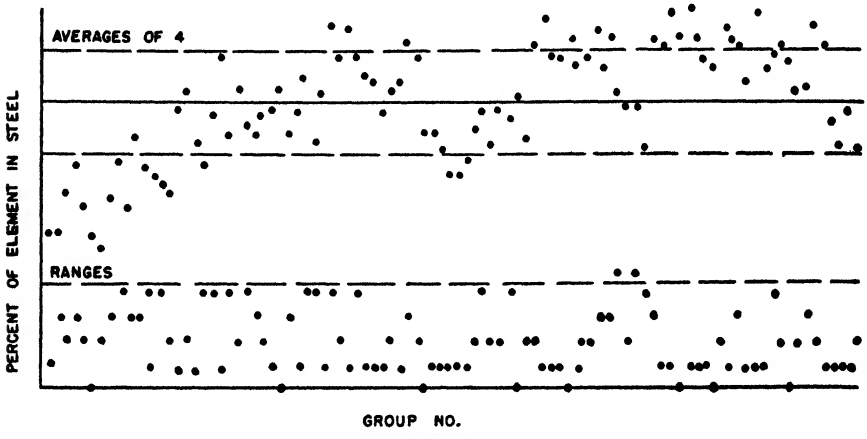

Fig. 7.7. Control chart showing gradual shifting of level with time.

were found to be defective. The following table indicated the location of the trouble:

Drum Position	Cutter No. 1	2	3	4	Totals
A	0	0	3	1	4
B	1	0	2	0	3
C	1	0	4	0	5
D	0	2	5	1	8
E	0	2	4	0	6
F	0	0	5	2	7
Totals	2	4	23	4	33

It was found that the piece was not being properly centered at Cutter No. 3. In this instance, judicious selection of data and a special inspection trial run solved the problem. The importance of proper identification of parts and steps in the process is clearly indicated.

7.7: A case of a chart based on time. In this illustration the time element was of considerable importance. It involved the chemical analysis of Bessemer steel for one of the elements in it. An analysis was made on a sample of steel from each blow and the results grouped by fours. The chart shown in Fig. 7.7 covered a period of about two months.

While the ranges show nice control, it is obvious that the process level is

gradually shifting up and down, about one to three weeks being required for an up or down swing. Since there are many potential assignable causes that vary from a few seconds to many days in the time they require to produce a detectable quality variation in the product, the information Fig. 7.7 gives is particularly helpful. All factors that would cause an up or down shift in quality level in a few hours or less can be eliminated from investigation and effort concentrated on the longer-interval factors.

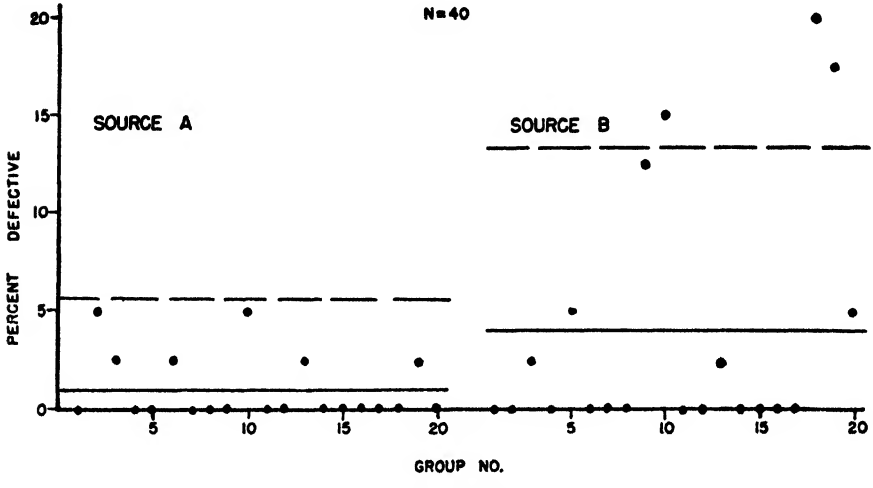

Fig. 7.9. Comparison of two suppliers of the same product. Since the lower control limit theoretically would fall below zero (actually impossible), no lower limit is shown.

7.8 A case of a chart based on method. Figs. 2.3a and 2.3b illustrate how control charts may be used effectively to reveal the results obtained with two different methods of processing material.

7.9: A case of charts based on materials. Fig. 7.9 illustrates how two sources of raw materials may be compared using quality control charts. It will be noted that source A has a much lower percentage defective and good control, whereas source B not only supplies poorer material, but also goes out of control occasionally.

CHAPTER VIII

APPLICATION AND INTERPRETATION OF THE QUALITY CONTROL CHART

8.1: General. Any new application of the quality control chart is almost certain to raise a number of questions concerning its applicability and interpretation. It is our purpose in this chapter to consider some of the most common misconceptions that may arise.

8.2: There are too many variables in the manufacturing process. Any process designed to produce successive items, all of which are intended to be essentially alike with respect to certain measurable quality characteristics, will involve a number of variables. As a normal minimum there will be at least the four major variables discussed in the last chapter —raw materials, methods, men, and machines. Each of these major variables may in turn be broken down into as many subdivisions as the product involves.

For a comparatively simple process, the problem of determining the importance of each variable is not too difficult. As the process grows more complex, the need for statistical methods to evaluate the significance of results increases. In any case, statistical methods are invaluable in distinguishing among borderline cases where the significance of obtained differences is not obvious.

In most manufacturing operations, the objective is to make many pieces at a satisfactory quality level, all the pieces being essentially alike. This is the very thing the quality control chart is designed to measure. Therefore, the number of variables in the manufacturing process is of no concern with respect to the suitability of the quality control chart for this purpose.

8.3: There are too many changes made in the process from time to time.

Change in material things inheres in the fact that the minute particles that make up all matter are in constant motion and are subject to constantly shifting intra- and inter-atomic forces. The changes that occur in the material world vary all the way from spectacular ones that occur in a fraction of a second to the virtually imperceptible ones that require many years for completion. The important thing is that all material things change with time.

Efforts to establish a manufacturing process that will produce an indefinite succession of identical items are futile. The possibility of two or more things exactly alike exists only in the mind as a concept and has no verification in human experience. Even if the portions of raw materials

were identical, we would be defeated by the fact that manufacturing equipment is subject to constant change. Friction, temperature fluctuations, and other constantly shifting chemical and physical forces continually alter the machinery used. The net result is that no two items ever come from identically the same process.

With sufficiently homogeneous raw materials and proper engineering design, it is usually possible to produce successive items that are *nearly enough* alike to function satisfactorily for the purpose intended. When such items are produced under the condition of statistical quality control, no further reduction in the amount of variability inherent in the process can be achieved, unless fundamental changes are made in the nature of the raw materials or in the process. When operations are proceeding in this manner, the changes occurring in the process (whether readily detectable or not) are unimportant. When changes occur that significantly shift the quality level or variability of the product, this fact will promptly reveal itself on the control chart.

8.4: The sample is not sufficiently representative of the lot.

It is agreed that sometimes the sample does not properly represent part or all of the lot from which it comes; however, it does represent the production process and enables us to determine whether the process is in control. When it is not in control, the likelihood of substandard material being made is greatly increased. At such times it is highly desirable to increase the amount of testing done.

8.5: The test method is not precise enough.

Methods of testing vary considerably in their precision. Most test methods used do enable us to distinguish various quality levels. In general, we may state that any test method that is considered good enough to form a basis for acceptance or rejection of the material tested is suitable for application of the quality control chart.

8.6: The method has never before been used for this application. How do we know it will work?

The point may be raised that the quality control chart technique has been in use only since 1924 and that this is the first time it has been applied in this manner. After all, how are we sure its application here is sound? The answer to this question lies in the function of the quality control chart. That function is to measure the degree and kind of control existing in a repetitive process designed to produce successive items all essentially alike. Therefore, the mere fact that a specific application has not been made before need not be a matter of concern. The 25 years the method has been in use have been far more than enough to establish its validity and reliability: indeed, they were established virtually from the beginning.

8.7: With time, my ranges will decrease and I will eventually be forced out of control.

When production is begun on a new product, it is generally expected that some production difficulties will occur that will be cleared up as time goes on. It is also expected that some way may be found to improve product uniformity. When a number of manufacturers are making the same sort of new product, it would be most surprising if some of them did not succeed in making this improvement.

As ways are found to improve the product, the production job may become more difficult; however, when a new level of quality or reduced variability has been established, the probability that a point will fall outside the new control limits just by chance is the same as it was before. There never will come a time when the manufacturer will of necessity fall out of control just because he is making an improved product.

It is natural for the manufacturer to desire tolerance limits as wide as possible. This tends to make his job easier. The way to better quality, however, requires something more than mere maintenance of existing standards or close adherence to minimum standards of acceptance. Efforts to obtain wider control limits through the deliberate introduction of greater variability into the product are to be strictly avoided. The lack of randomness almost certain to be associated with such efforts will quickly reveal itself as a lack of control. In any event, the task will undoubtedly prove to be more laborious and costly than any so-called advantage obtained would justify.

To illustrate the relationship between changes in the cause system and quality control chart shifts in quality level and variability, three charts have been constructed using random numbers. A series of random numbers consists of digits so selected that each time one is chosen, one digit is just as likely to occur as any other digit (insofar as it is humanly possible to create this condition). The following is a series of 50 such numbers:

0 9 1 1 5 1 8 6 3 5 1 2 2 5 3 7 5 9 1 3 6 7 2 0 2
1 1 3 3 3 6 1 6 2 6 0 0 6 2 1 1 9 4 6 9 3 8 0 8 9

Each five successive numbers were added. The maximum possible number that could be thus obtained would be 45 (five 9's) and the minimum would be 0 (five 0's). These newly obtained numbers tend to approximate closely the normal distribution. One hundred numbers were accumulated in this manner. They were then grouped by successive fours and the average and range of each group of four determined. The expected grand average would be 22.5. Control chart limits were then calculated for the 25 groups. The results are shown as Chart A of Fig. 8.7.

The entire process was then repeated after all 0's, 1's, 8's, and 9's had been removed from the original list of random numbers. The maximum

number now obtainable would be 35 (five 7's) and the minimum would be 10 (five 2's). The expected grand average would still be 22.5; but the average range would be expected to be considerably smaller, thus bringing the control chart limits on the chart for averages much closer together. The results are shown as Chart B of Fig. 8.7.

The entire process was again repeated with only the 0's, 1's, 2's, and 3's removed from the original list of random numbers. The maximum number now obtainable would be 45 (five 9's) and the minimum would be 20 (five 4's). The expected grand average would now be 32.5, and the average range would be expected to be virtually the same as that for Chart B. The results are shown as Chart C of Fig. 8.7.

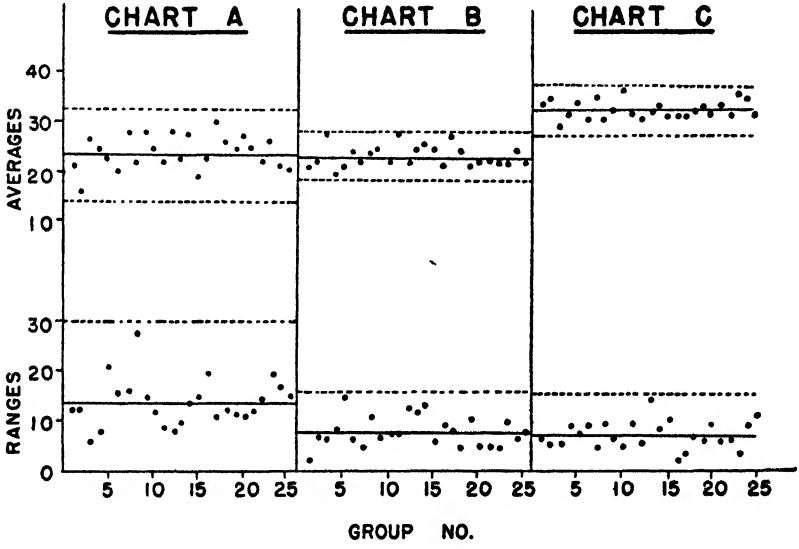

Fig. 8.7. Control charts reflecting changes in the population.

	Chart A	Chart B	Chart C
Calculated results:			
Grand average	22.83	22 69	32.29
Upper control limit	32.31	27 76	37.13
Lower control limit	13 35	17.62	27.45
Average range	13 00	6.96	6.64
Upper control limit	29.67	15.88	15.15
Lower control limit	0	0	0

The important thing to note in connection with these charts is that when significant shifts occurred in either quality level (as reflected by the grand average) or inherent variability (as reflected by the average range), it was due to a change in the cause system producing the results. When

no change was made in the cause system, there was no significant shift in grand average or range. No matter how long a process continues, significant shifts are very unlikely to occur unless there is a significant change in the raw materials or in the manufacturing process. If, after a process has continued for some time, a significant shift in quality level or inherent variability has occurred, the cause of the shift will sometimes be apparent. If it is not, the manufacturer should search diligently for the cause until it is clear that further search would probably not be profitable.

8.8: It is possible to go out of control limits and get back in without doing anything to the process.

Quality level shifts that are of small magnitude or of short duration can readily occur without immediately revealing themselves. Shifts may be gradual or sudden, uniformly in one direction or erratic, controlled or uncontrolled. Any combination of these conditions may occur with any degree of intensity. Shifts that are of slight magnitude but long duration may result in only an occasional point falling out of control limits. Thus, if the manufacturer makes no deliberate change in the process during this time, it appears to him that the process has gone out of control and back in without any effort on his part. The truth is that the process has probably not been in control at the previous level or with the same magnitude of inherent variability as previously at any time during the period involved.

The greater the shift that occurs, the greater will be the likelihood of a point falling outside control limits, and the more serious the situation will be. Any point that falls outside control limits should be regarded as a danger signal, and every effort made (within economically practicable limits) to determine the reason for the indication of lack of control. Only very rarely do indications of lack of control occur without an assignable cause.

8.9: The control limits have shifted and now a point is outside control limits, whereas formerly it would have been within limits. Why is this point not considered a satisfactory condition?

Each plotted point on the control chart tells a most eloquent story in relation to its own control limits. Thus two points widely separated on the chart may have the same numerical value, but different meanings. Suppose the first of these points falls within control limits. We can assume from this fact only that the product is continuing to be made in a controlled manner. Before the second point with which we are concerned occurs, many other points are accumulated which reveal that control has been achieved at a higher level. When the second point occurs it may now fall below the new lower control limit. It signifies that control is probably not being maintained at the new level. Any indication of lack of control must always be regarded as a danger signal regardless of when or where the indication oc-

curs. In addition to our interest in the point's numerical value, we are also interested in its position relative to the control limits which apply to it.

8.10: Why is going above the upper limit for averages undesirable when working to a specification minimum? Is this not a great deal better than can be expected?

If the point that goes above the upper limit is preceded by a gradual upward trend of the group averages in what appears to be a controlled manner, there is little likelihood of defective material being produced. If, however, a point suddenly jumps out of the upper limit without any forewarning, it probably represents a condition of lack of control. When the manufacturing process is out of control, almost anything can happen, including a sudden drop in quality. Frequently, such high points are followed almost immediately by test failures. While a high level of quality is desirable, it is also very important that control be maintained. Without control we cannot expect to have a uniform product.

8.11: If points for averages are permitted to go above their upper control limit to encourage shifts to higher levels, why are not points for ranges permitted to go above their upper limit?

The function of the averages is to reveal the level on which the process is operating. The function of the ranges is to reveal the amount of variability inherent in the process. The ranges do this independently of the process quality level and reveal nothing about its quantitative value. The average range, however, does enable us to establish the limits within which virtually all the group averages should fall. These limits are then placed above and below the process average quality level. Any given group average then indicates both a quality level and whether there is any indication of lack of control. Any given group range reveals both variability and any indication of lack of control. Thus, group averages that go above the upper control limit may possibly be the result of a gradually rising quality level, but a range that goes above its upper control limit can indicate only a lack of control.

8.12: Is not an uncontrolled product at a higher level more desirable than a controlled product at a lower level?

While the average quality level of the process is important, the degree of control in effect at that level is also important—in fact vitally important. Without the existence of controlled operations, we can never be certain within what limits future observations will be likely to lie, within what limits untested material already made lies (when the control chart is derived from samples), or whether the existing level of average quality will be maintained.

In general, Source B of Fig. 8.12 would be definitely preferred to Source

A, in spite of the fact that the average quality level shown for B is lower than for A. Since A shows several indications of being out of control, we cannot be sure that the average quality level shown is the true quality level for all material made during the period involved. Nor can we be sure as to the amount of untested material that would also fall outside the control limits, or what portion of it, if tested, would fall below the specification minimum, or what the future level or variability will be. On the other hand, in the case of Source B, we have a high degree of assurance that the quality average and variability of the untested material are essentially the same as for the tested material; that extremely little, if any, of the untested material would fall below the specification minimum; and that the

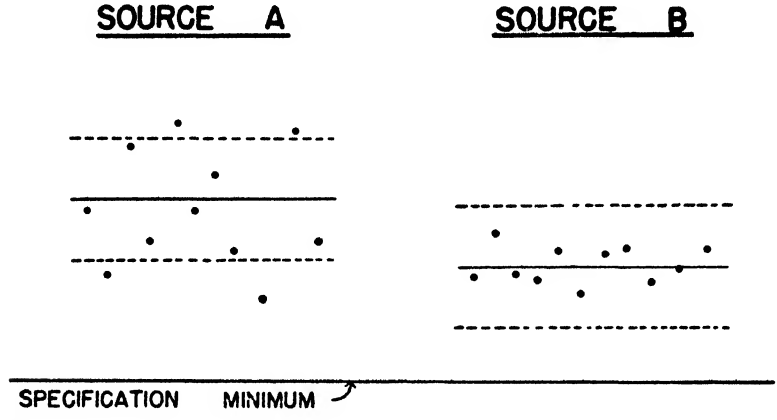

Fig. 8.12. Contrast of level and control. Source A has a higher level for the items tested, but there is no telling what the rest of the lot is like and there is no basis for predicting the future. Source B would normally be preferred.

existing level and amount of variability probably will be maintained in the immediate future.

8.13: What good is a control chart on a certain property if it is a different property in which we are interested?

If the two properties are not related, such a chart will not be helpful. In some cases two different properties are closely related although it is feasible to measure only one of them. Wherever the two properties are related, such a chart can be very useful.

8.14: If the level of averages is high enough (when working to a minimum only), is it not possible for a point to fall at least a little below the lower control limit without danger of failures?

In this situation there may be no immediate danger of failures on the quality characteristic charted. If there are a number of related quality

characteristics of importance, trouble may be encountered with one or more of them. The safe thing is never to ignore indications of lack of control.

8.15: If one is interested in controlling a certain quality characteristic, should control charts be started immediately on all factors known or believed to affect the quality involved?

Yes, if there are only a few factors involved, say two to four. No, if there are many. In the latter case it will be better to start with a control chart only for the quality characteristic it is desired to control. As indications of lack of control occur and causes are identified, control charts on causative factors may be added.

8.16: Summary of principal points.

1. Wherever the aim is to make many items essentially alike, the quality control chart is applicable regardless of the number of variables in the manufacturing process.

2. It is applicable regardless of the number of changes made in the manufacturing process from time to time.

3. It is applicable regardless of whether the sample is adequately representative of any so-called "lot."

4. Any test method with sufficient precision to form the basis of acceptance or rejection of the material tested will be suitable for application of the quality control chart. In general, the amount of possible error in any individual test result should be small in comparison to the scale over which test results will normally vary.

5. The quality control chart is applicable to any process designed to make many items all essentially alike, regardless of whether any applications specifically comparable to the one in question have ever been made before.

6. Inherent variability in a type of product does not change significantly with the passing of time unless fundamental changes occur in the production process.

7. Any point that falls outside control chart limits indicates (with a high degree of probability) that a significant shift in quality level has occurred. Such indications may result from either a slight or large shift.

8. The significance of any point on a control chart (with respect to the degree of control it indicates), depends upon its position in relation to the control limits that apply to it.

9. A point falling above the upper control limit for averages must be regarded as a danger signal even if the only requirement is a specification minimum. It indicates an unstable process that may suddenly drop in quality level.

10. A point for ranges that falls above its upper limit reflects significantly increased variability in the product, a condition generally regarded as undesirable.

11. A controlled product at a somewhat lower quality level (provided minimum requirements are adequately met) is generally preferable to an uncontrolled product at a higher quality level.

12. A quality control chart based upon a certain quality characteristic can be helpful in gaining control of related properties.

13. No matter how high the average quality level may be, any point falling outside the control limits should be regarded as a danger signal.

14. In establishing a quality control program, control charts should first be constructed for the quality characteristic desired. Charts for causative factors may be added later as appears feasible.

CHAPTER IX

THE QUALITY CONTROL CHART FOR ATTRIBUTES

9.1: Attributes measurements. Sometimes it is neither economical nor otherwise practical to obtain measurements of quality characteristics. The mere presence of an undesirable condition, regardless of the degree to which it is present, may cause the items to be classed as defective. For example, radiators that leak are considered unsatisfactory, whether they leak slowly or rapidly. Thus, with respect to water-tightness, there are only two classes of radiators—those which leak and those that do not.

Some small piece parts are made in such large quantities that dimensional measurements by variables on each part would be uneconomical. Such parts are often inspected with go no-go gages. If measured with proper gages, assurance is provided that the part is either within the tolerance specified or not.

The greatest weakness of attributes measurements is that if a part is good, it is not known how good, and if bad, it is not known how bad. Thus it takes several times as many measurements by attributes to provide a comparable appraisal of a quality level as measurements by variables. Balanced against this is the fact that several times as many measurements by attributes can generally be made for the same cost as measurements by variables. The best inspection method for any given situation must be decided on the basis of the practical aspects of the case.

9.2: Defect and defective. The term "defect" refers to failure to meet a specified quality standard. If the quality standard is that a radiator be water-tight, then a leak is a defect. Two leaks in a radiator would be two defects in the one item.

The term "defective" designates an item that contains one or more defects. A radiator that contains any leaks is defective regardless of the number of leaks present.

In this chapter we shall concern ourselves only with defectives. In the next chapter we shall consider defects.

9.3: The control chart for fraction defective. Fig. 9.3 shows a control chart for fraction defective. The data were obtained by drawing samples of beads from a box containing ten per cent red beads (representing defective items) and 90 per cent white beads (representing effective items).

Many of the comments made in Chapter VI in connection with control

charts for variables are also applicable here. This especially applies to Sections 6.2, the last paragraph of 6.6, 6.7, 6.8, 6.9, and 6.11.

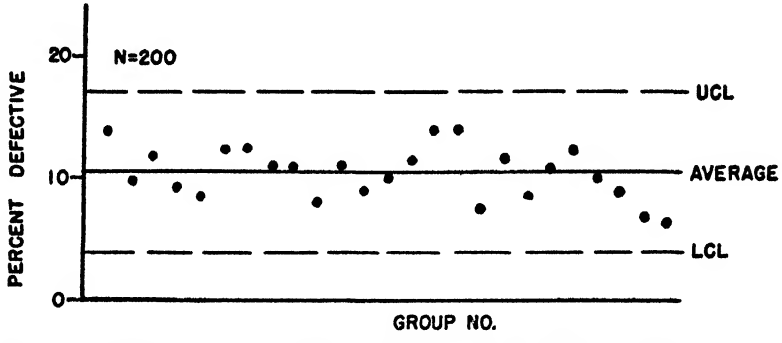

Fig. 9.3. Control chart for fraction defective. The fraction defective is shown here as percent defective (fraction defective times 100). The data were as follows:

Group No.	No Def.	Frac. Def.	Group No.	No. Def.	Frac Def.
1	28	0.140	16	28	0.140
2	20	0.100	17	15	0.075
3	24	0.120	18	23	0.115
4	19	0.095	19	17	0.085
5	17	0.085	20	22	0.110
6	25	0.125	21	25	0.125
7	25	0.125	22	20	0.100
8	22	0.110	23	18	0.090
9	22	0.110	24	14	0.070
10	16	0.080	25	13	0.065
11	22	0.110			
12	18	0.090			
13	20	0.100			
14	23	0.115			
15	28	0.140	Total.....	524	0.1048 (avg.)

$$\frac{524}{5000} = 0.1048 = \bar{p} = 10.48\%$$

$$\text{UCL} = 16.99\%$$

$$\text{LCL} = 3.97\%$$

9.4: Determination of control limits. Control chart limits are obtained by the formula

$$\bar{p} \pm 3\sqrt{\frac{\bar{p}(1-\bar{p})}{N}}$$

where $\bar{p}$ is the average fraction defective and N is the number of items per

sample. It will be noted that there is no chart comparable to the chart for range in this case. This follows from the fact that regardless of the nature of the production process, the inherent variations in attribute sampling are completely determined by the fraction defective and the sample size. Thus for a given average fraction defective and a given sample size the control chart limits are always the same.

9.5: The problem of variable sample size. It sometimes happens that successive samples are of different size. This need not cause any inconvenience as long as the largest sample size does not exceed the smallest sample size by more than 20 per cent of the smallest sample size. Control limits may be based on the average sample size as in Fig. 9.5a. There are two indications of lack of control in this illustration. At sample 9 the product appears to be significantly better than can be accounted for by chance, and at samples 21 and 22 (two points but only one event) it is significantly worse.

As the sample size variations exceed the 20 per cent limitation, the effect upon the location of control chart limits becomes increasingly important. Separate control chart limits must then be calculated for any unusually large or unusually small samples. This is illustrated in Fig. 9.5b. It will be noted that the central line remains fixed while the control limits come closer together for larger samples and are farther apart for smaller samples. Note also that some of the points for the small sample size would be out of control if judged by the limits for the larger sample sizes. Actually, a state of control was maintained during the whole period.

Effectiveness of workers may change if the rate of production changes greatly. While it will not necessarily do so, it should be watched as a possible assignable cause of lack of control.

9.6: Control chart for number of defectives. The number of defectives (pN, where p is the fraction defective in the sample and N is the sample size) may be used directly for a control chart. This is illustrated in Fig. 9.6 which uses the data of Fig. 9.3. The formula

$$\bar{p}N \pm 3\sqrt{\bar{p}N(1 - \bar{p})}$$

where $\bar{p}N$ is the average number of defectives per sample and $\bar{p}$ is the average fraction defective, gives the three sigma control limits.

It will be noted that if the sample size changes, the expected number of defectives per sample will also change. This means that a different central line as well as different control limits must be used on the chart for each different sample size. For this reason the pN chart is generally used only in connection with samples of constant size.

When $\bar{p}$ is less than 0.05, the value of $1-\bar{p}$ approaches sufficiently close to unity that it may be dropped from the above formula. It then becomes

$$\bar{p}N \pm 3\sqrt{\bar{p}N}$$

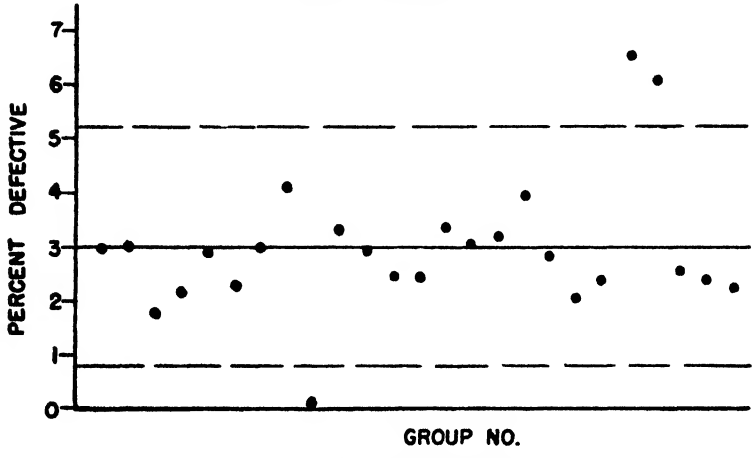

Fig. 9.5a. Control chart with variable sample size using constant control limits. The data for this chart were as follows:

Group No.	Sample Size	No. Def.	% Def.	Group No.	Sample Size	No. Def	% Def.
1	523	16	3.06	16	553	18	3.25
2	584	18	3.08	17	575	23	4.00
3	502	9	1.79	18	591	17	2.88
4	596	13	2.18	19	536	11	2.05
5	575	17	2.96	20	572	14	2.45
6	513	12	2.34	21	502	33	6.57
7	566	17	3.00	22	511	31	6.07
8	531	22	4.14	23	533	14	2.63
9	540	1	0.19	24	536	16	2.99
10	591	20	3.38	25	526	12	2.33
11	515	15	2.91				
12	518	13	2.51				
13	563	14	2.49				
14	551	19	3.45				
15	522	16	3.07	Total....	13625	411	3.02 (avg.)

$$\frac{411}{13625} = 3.02 = \bar{p}$$

$\bar{N} = 545$ UCL = 5.22%
LCL = 0.82%

It will be noted that the largest sample size (596) exceeds the smallest sample size (502) by only 19 per cent, hence constant control limits may be used.

9.7: Quality control tables for attribute inspection. Such considerations as judicious selection and number of quality characteristics on which to run quality control charts, the selection of group size and the way of

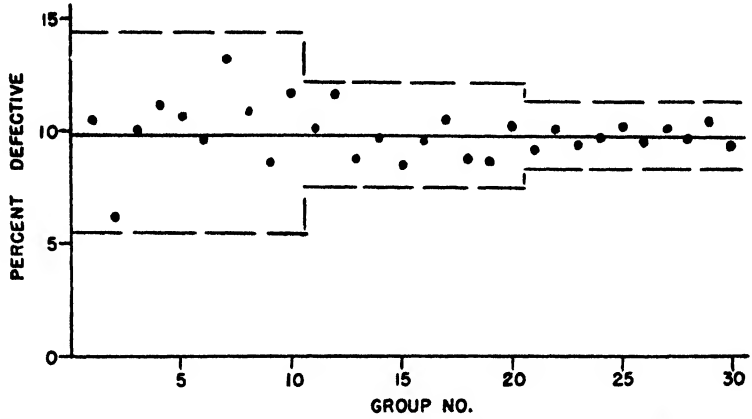

Fig. 9.5b. Adjustment of control limits for variation in sample size. A total of 56000 items were inspected giving the following results:

Groups	Sample size	UCL	LCL		
1-10	400	14.40%	5.44%	Total inspected:	56000
11-20	1600	12.16%	7.68%	Total defective:	5557
21-30	3600	11.41%	8.43%	Per cent defective:	9.92%

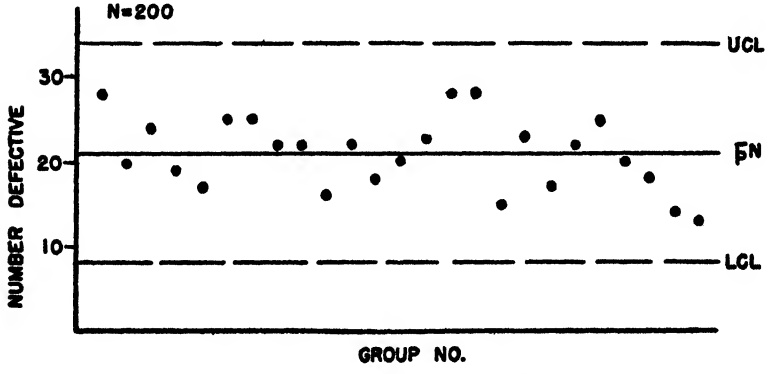

Fig. 9.6. Control chart for number of defectives. Determination of limits:

$$\bar{p}N = \frac{524}{25} = 20.96$$

$$20.96 \pm 3\sqrt{20.96(1-0.1048)}$$

$$20.96 \pm 13.00 = 33.96 \text{ and } 7.96$$

obtaining samples, and the manner of presenting results will govern to a large extent the success of a quality control program. Of even greater importance is the matter of setting up the system so that action can be taken quickly when indications of lack of control occur. In order to assist in this connection, three items are offered.

Item 1. Table of width of three sigma band in per cent. Table 9.7a is intended for use in expediting the plotting of the control limits on fraction defective charts. The arguments provide for process levels of per cent defective in convenient intervals from 0.1 to 99.9 per cent defective, and for sample sizes from 10 to 10,000 in convenient steps.

To illustrate the use of the table, suppose we are dealing with a process that has established a process level of 1.95 per cent defective and a sample size of 832. We select the nearest column and row (2.0 per cent and 800) and read the entry 1.48 per cent. This value (which is the width of the three sigma band) is then added to and subtracted from the mean value of 1.95 per cent, giving an upper control limit of 3.43 per cent and a lower control limit of 0.47 per cent. These values are entirely satisfactory approximations of the more precisely (and more laboriously) calculated values of 3.39 and 0.51 per cent.

When observed values fall exactly midway between arguments, the column or row that should be chosen will depend on whether one wishes to run a slightly greater or slightly lesser risk of looking for trouble when none exists. The narrower limits give slightly greater risks, the wider limits slightly smaller risks. In any event, the difference in risks will be very small and usually of no practical consequence.

The following tabulation for five days shows how conveniently the table may be used. Three quality characteristics are measured on each day's sample.

Date	No. Insp.	Quality Characteristic A				Quality Characteristic B				Quality Characteristic C			
		No. Def.	% Def.	UCL*	LCL*	No. Def.	% Def.	UCL	LCL	No. Def.	% Def.	UCL	LCL
May level used for June.			10.31				0.96				0.69		
June 2.....	1070	172	16.07†	13.02	7.60	14	1.31	1.86	0.06	6	0.56	1.33	0.05
June 3	931	138	14.82†	13.31	7.31	3	0.32	1.96	0	5	0.54	1.40	0
June 4.....	396	36	9.09	14.81	5.81	2	0.51	2.45	0	2	0.51	1.76	0
June 5.....	1004	119	11.85	13.16	7.46	5	0.50	1.90	0.02	8	0.80	1.36	0.02
June 6.....	964	120	12.45	13.16	7.46	6	0.62	1.90	0.02	3	0.31	1.36	0.02

* UCL = Upper control limit, LCL = Lower control limit.

† Out of control limits.

TABLE 9.7a

TABLE OF WIDTH OF THREE SIGMA BAND IN PER CENT

Add and subtract entry to process level to get control limits. Use nearest arguments.

	Process Level in Percent Defective											
	0.1	0.25	0.5	1.0	1.5	2	3	4	5	6	7	8
N	99.9	99.75	99.5	99.0	98.5	98	97	96	95	94	93	92
10	3.00	4.74	6.69	9.44	11.53	13.28	16.18	18.59	20.68	22.53	24.21	25.74
20	2.12	3.35	4.73	6.67	8.15	9.39	11.44	13.15	14.62	15.93	17.12	18.20
30	1.73	2.74	3.86	5.45	6.66	7.67	9.34	10.73	11.94	13.01	13.98	14.86
40	1.50	2.37	3.35	4.72	5.77	6.64	8.09	9.30	10.34	11.26	12.10	12.87
50	1.34	2.12	2.99	4.22	5.16	5.94	7.24	8.31	9.25	10.08	10.82	11.51
60	1.22	1.93	2.73	3.85	4.71	5.42	6.61	7.59	8.44	9.20	9.88	10.51
70	1.13	1.79	2.53	3.57	4.36	5.02	6.12	7.03	7.81	8.52	9.15	9.73
80	1.06	1.67	2.37	3.34	4.08	4.70	5.72	6.57	7.31	7.97	8.56	9.10
90	1.00	1.58	2.23	3.15	3.84	4.43	5.39	6.20	6.89	7.51	8.07	8.58
100	0.95	1.50	2.12	2.98	3.65	4.20	5.12	5.88	6.54	7.12	7.65	8.14
200	0.67	1.06	1.50	2.11	2.58	2.97	3.62	4.16	4.62	5.04	5.41	5.76
300	0.55	0.86	1.22	1.72	2.11	2.42	2.95	3.39	3.77	4.11	4.42	4.70
400	0.47	0.75	1.06	1.49	1.82	2.10	2.56	2.94	3.27	3.56	3.83	4.07
500	0.42	0.67	0.95	1.33	1.63	1.88	2.29	2.63	2.92	3.19	3.42	3.64
600	0.39	0.61	0.86	1.22	1.49	1.71	2.09	2.40	2.67	2.91	3.12	3.32
700	0.36	0.57	0.80	1.13	1.38	1.59	1.93	2.22	2.47	2.69	2.89	3.08
800	0.34	0.53	0.75	1.06	1.29	1.48	1.81	2.08	2.31	2.52	2.71	2.88
900	0.32	0.50	0.71	1.00	1.22	1.40	1.71	1.96	2.18	2.37	2.55	2.71
1000	0.30	0.47	0.67	0.94	1.15	1.33	1.62	1.86	2.07	2.25	2.42	2.57
1100	0.29	0.45	0.64	0.90	1.10	1.27	1.54	1.77	1.97	2.15	2.31	2.45
1200	0.27	0.43	0.61	0.86	1.05	1.21	1.48	1.70	1.89	2.06	2.21	2.35
1300	0.26	0.42	0.59	0.83	1.01	1.16	1.42	1.63	1.81	1.98	2.12	2.26
1400	0.25	0.40	0.57	0.80	0.97	1.12	1.37	1.57	1.75	1.90	2.05	2.18
1500	0.24	0.39	0.55	0.77	0.94	1.08	1.32	1.52	1.69	1.84	1.98	2.10
1600	0.24	0.37	0.53	0.75	0.91	1.05	1.28	1.47	1.63	1.78	1.91	2.03
1700	0.23	0.36	0.51	0.72	0.88	1.02	1.24	1.43	1.59	1.73	1.86	1.97
1800	0.22	0.35	0.50	0.70	0.86	0.99	1.21	1.39	1.54	1.68	1.80	1.92
1900	0.22	0.34	0.49	0.68	0.84	0.96	1.17	1.35	1.50	1.63	1.76	1.87
2000	0.21	0.34	0.47	0.67	0.82	0.94	1.14	1.31	1.46	1.59	1.71	1.82
2100	0.21	0.33	0.46	0.65	0.80	0.92	1.12	1.28	1.43	1.55	1.67	1.78
2200	0.20	0.32	0.45	0.64	0.78	0.90	1.09	1.25	1.39	1.52	1.63	1.74
2300	0.20	0.31	0.44	0.62	0.76	0.88	1.07	1.23	1.36	1.49	1.60	1.70
2400	0.19	0.31	0.43	0.61	0.74	0.86	1.04	1.20	1.33	1.45	1.56	1.66
2500	0.19	0.30	0.42	0.60	0.73	0.84	1.02	1.18	1.31	1.42	1.53	1.63
2600	0.19	0.29	0.42	0.59	0.72	0.82	1.00	1.15	1.28	1.40	1.50	1.60
2700	0.18	0.29	0.41	0.57	0.70	0.81	0.98	1.13	1.26	1.37	1.47	1.57
2800	0.18	0.28	0.40	0.56	0.69	0.79	0.97	1.11	1.24	1.35	1.45	1.54
2900	0.18	0.28	0.39	0.55	0.68	0.78	0.95	1.09	1.21	1.32	1.42	1.51
3000	0.17	0.27	0.39	0.54	0.67	0.77	0.93	1.07	1.19	1.30	1.40	1.49
3100	0.17	0.27	0.38	0.54	0.65	0.75	0.92	1.06	1.17	1.28	1.37	1.46
3200	0.17	0.26	0.37	0.53	0.64	0.74	0.90	1.04	1.16	1.26	1.35	1.44
3300	0.17	0.26	0.37	0.52	0.63	0.73	0.89	1.02	1.14	1.24	1.33	1.42
3400	0.16	0.26	0.36	0.51	0.63	0.72	0.88	1.01	1.12	1.22	1.31	1.40
3500	0.16	0.25	0.36	0.50	0.62	0.71	0.87	0.99	1.11	1.20	1.29	1.38
3600	0.16	0.25	0.35	0.50	0.61	0.70	0.85	0.98	1.09	1.19	1.28	1.36

TABLE 9.7a continued

	Process Level in Percent Defective											
	0.1	0.25	0.5	1.0	1.5	2	3	4	5	6	7	8
N	99.9	99.75	99.5	99.0	98.5	98	97	96	95	94	93	92
3700	0.16	0.25	0.35	0.49	0.60	0.69	0.84	0.97	1.07	1.17	1.26	1.34
3800	0.15	0.24	0.34	0.48	0.59	0.68	0.83	0.95	1.06	1.16	1.24	1.32
3900	0.15	0.24	0.34	0.48	0.58	0.67	0.82	0.94	1.05	1.14	1.23	1.30
4000	0.15	0.24	0.33	0.47	0.58	0.66	0.81	0.93	1.03	1.13	1.21	1.29
4200	0.15	0.23	0.33	0.46	0.56	0.65	0.79	0.91	1.01	1.10	1.18	1.26
4400	0.14	0.23	0.32	0.45	0.55	0.63	0.77	0.89	0.99	1.07	1.15	1.23
4600	0.14	0.22	0.31	0.44	0.54	0.62	0.75	0.87	0.96	1.05	1.13	1.20
4800	0.14	0.22	0.31	0.43	0.53	0.61	0.74	0.85	0.94	1.03	1.10	1.17
5000	0.13	0.21	0.30	0.42	0.52	0.59	0.72	0.83	0.92	1.01	1.08	1.15
5500	0.13	0.20	0.29	0.40	0.49	0.57	0.69	0.79	0.88	0.96	1.03	1.10
6000	0.12	0.19	0.27	0.39	0.47	0.54	0.66	0.76	0.84	0.92	0.99	1.05
6500	0.12	0.19	0.26	0.37	0.45	0.52	0.63	0.73	0.81	0.88	0.95	1.01
7000	0.11	0.18	0.25	0.36	0.44	0.50	0.61	0.70	0.78	0.85	0.91	0.97
7500	0.11	0.17	0.24	0.34	0.42	0.48	0.59	0.68	0.76	0.82	0.88	0.94
8000	0.11	0.17	0.24	0.33	0.41	0.47	0.57	0.66	0.73	0.80	0.86	0.91
8500	0.10	0.16	0.23	0.32	0.40	0.46	0.56	0.64	0.71	0.77	0.83	0.88
9000	0.10	0.16	0.22	0.31	0.38	0.44	0.54	0.62	0.69	0.75	0.81	0.86
9500	0.10	0.15	0.22	0.31	0.37	0.43	0.53	0.60	0.67	0.73	0.79	0.84
10000	0.09	0.15	0.21	0.30	0.36	0.42	0.51	0.59	0.65	0.71	0.77	0.81

	Process Level in Percent Defective												
	9	10	12	14	16	18	20	25	30	35	40	45	
N	91	90	88	86	84	82	80	75	70	65	60	55	50
10	27.15	28.46	30.83	32.92	34.78	36.45	37.95	41.08	43.47	45.25	46.48	47.20	47.43
20	19.20	20.12	21.80	23.28	24.59	25.77	26.83	29.05	30.74	32.00	32.86	33.37	33.54
30	15.67	16.43	17.80	19.01	20.08	21.04	21.91	23.72	25.10	26.12	26.83	27.25	27.39
40	13.57	14.23	15.41	16.46	17.39	18.22	18.97	20.54	21.74	22.62	23.24	23.60	23.72
50	12.14	12.73	13.79	14.72	15.55	16.30	16.97	18.37	19.44	20.24	20.78	21.11	21.21
60	11.08	11.62	12.59	13.44	14.20	14.88	15.49	16.77	17.75	18.47	18.97	19.27	19.36
70	10.26	10.76	11.65	12.44	13.15	13.78	14.34	15.53	16.43	17.10	17.57	17.84	17.93
80	9.60	10.06	10.90	11.64	12.30	12.89	13.42	14.52	15.37	16.00	16.43	16.69	16.77
90	9.05	9.49	10.28	10.97	11.59	12.15	12.65	13.69	14.49	15.08	15.49	15.73	15.81
100	8.59	9.00	9.75	10.41	11.00	11.53	12.00	12.99	13.75	14.31	14.70	14.92	15.00
200	6.07	6.36	6.89	7.36	7.78	8.15	8.49	9.19	9.72	10.12	10.39	10.55	10.61
300	4.96	5.20	5.63	6.01	6.35	6.65	6.93	7.50	7.94	8.26	8.49	8.62	8.66
400	4.29	4.50	4.87	5.20	5.50	5.76	6.00	6.50	6.87	7.15	7.35	7.46	7.50
500	3.84	4.02	4.36	4.66	4.92	5.15	5.37	5.81	6.15	6.40	6.57	6.67	6.71
600	3.50	3.67	3.98	4.25	4.49	4.71	4.90	5.30	5.61	5.84	6.00	6.09	6.12
700	3.24	3.40	3.68	3.93	4.16	4.36	4.54	4.91	5.20	5.41	5.55	5.64	5.67
800	3.04	3.18	3.45	3.68	3.89	4.07	4.24	4.59	4.86	5.06	5.20	5.28	5.30
900	2.86	3.00	3.25	3.47	3.67	3.84	4.00	4.33	4.58	4.77	4.90	4.97	5.00
1000	2.72	2.85	3.08	3.29	3.48	3.64	3.79	4.11	4.35	4.52	4.65	4.72	4.74
1100	2.59	2.71	2.94	3.14	3.32	3.48	3.62	3.92	4.15	4.31	4.43	4.50	4.52
1200	2.48	2.60	2.81	3.00	3.18	3.33	3.46	3.75	3.97	4.13	4.24	4.31	4.33
1300	2.38	2.50	2.70	2.89	3.05	3.20	3.33	3.60	3.81	3.97	4.08	4.14	4.16
1400	2.29	2.41	2.61	2.78	2.94	3.08	3.21	3.47	3.67	3.82	3.93	3.99	4.01
1500	2.22	2.32	2.52	2.69	2.84	2.98	3.10	3.35	3.55	3.69	3.79	3.85	3.87

TABLE 9.7a continued

	Process Level in Percent Defective												
	9	10	12	14	16	18	20	25	30	35	40	45	
n	91	90	88	86	84	82	80	75	70	65	60	55	50
1600	2.15	2.25	2.44	2.60	2.75	2.88	3.00	3.25	3.44	3.58	3.67	3.73	3.75
1700	2.08	2.18	2.36	2.52	2.67	2.80	2.91	3.15	3.33	3.47	3.56	3.62	3.64
1800	2.02	2.12	2.30	2.45	2.59	2.72	2.83	3.06	3.24	3.37	3.46	3.52	3.54
1900	1.97	2.06	2.24	2.39	2.52	2.64	2.75	2.98	3.15	3.28	3.37	3.42	3.44
2000	1.92	2.01	2.18	2.33	2.46	2.58	2.68	2.90	3.07	3.20	3.29	3.34	3.35
2100	1.87	1.96	2.13	2.27	2.40	2.52	2.62	2.83	3.00	3.12	3.21	3.26	3.27
2200	1.83	1.92	2.08	2.22	2.34	2.46	2.56	2.77	2.93	3.05	3.13	3.18	3.20
2300	1.79	1.88	2.03	2.17	2.29	2.40	2.50	2.71	2.87	2.98	3.06	3.11	3.13
2400	1.75	1.84	1.99	2.12	2.24	2.35	2.45	2.65	2.81	2.92	3.00	3.05	3.06
2500	1.72	1.80	1.95	2.08	2.20	2.31	2.40	2.60	2.75	2.86	2.94	2.98	3.00
2600	1.68	1.77	1.91	2.04	2.16	2.26	2.35	2.55	2.70	2.81	2.88	2.93	2.94
2700	1.65	1.73	1.88	2.00	2.12	2.22	2.31	2.50	2.65	2.75	2.83	2.87	2.89
2800	1.62	1.70	1.84	1.97	2.08	2.18	2.27	2.45	2.60	2.70	2.78	2.82	2.83
2900	1.59	1.67	1.81	1.93	2.04	2.14	2.23	2.41	2.55	2.66	2.73	2.77	2.79
3000	1.57	1.64	1.78	1.90	2.01	2.10	2.19	2.37	2.51	2.61	2.68	2.72	2.74
3100	1.54	1.62	1.75	1.87	1.98	2.07	2.16	2.33	2.47	2.57	2.64	2.68	2.69
3200	1.52	1.59	1.72	1.84	1.94	2.04	2.12	2.30	2.43	2.53	2.60	2.64	2.65
3300	1.49	1.57	1.70	1.81	1.91	2.01	2.09	2.26	2.39	2.49	2.56	2.60	2.61
3400	1.47	1.54	1.67	1.79	1.89	1.98	2.06	2.23	2.36	2.45	2.52	2.56	2.57
3500	1.45	1.52	1.65	1.76	1.86	1.95	2.03	2.20	2.32	2.42	2.48	2.52	2.54
3600	1.43	1.50	1.62	1.74	1.83	1.92	2.00	2.17	2.29	2.38	2.45	2.49	2.50
3700	1.41	1.48	1.60	1.71	1.81	1.89	1.97	2.14	2.26	2.35	2.42	2.45	2.47
3800	1.39	1.46	1.58	1.69	1.78	1.87	1.95	2.11	2.23	2.32	2.38	2.42	2.43
3900	1.37	1.44	1.56	1.67	1.76	1.85	1.92	2.08	2.20	2.29	2.35	2.39	2.40
4000	1.36	1.42	1.54	1.65	1.74	1.82	1.90	2.05	2.17	2.26	2.32	2.36	2.37
4200	1.32	1.39	1.50	1.61	1.70	1.78	1.85	2.00	2.12	2.21	2.27	2.30	2.31
4400	1.29	1.36	1.47	1.57	1.66	1.74	1.81	1.96	2.07	2.16	2.22	2.25	2.26
4600	1.27	1.33	1.44	1.53	1.62	1.70	1.77	1.92	2.03	2.11	2.17	2.20	2.21
4800	1.24	1.30	1.41	1.50	1.59	1.66	1.73	1.88	1.98	2.07	2.12	2.15	2.17
5000	1.21	1.27	1.38	1.47	1.56	1.63	1.70	1.84	1.94	2.02	2.08	2.11	2.12
5500	1.16	1.21	1.31	1.40	1.48	1.55	1.62	1.75	1.85	1.93	1.98	2.01	2.02
6000	1.11	1.16	1.26	1.34	1.42	1.49	1.55	1.68	1.77	1.85	1.90	1.93	1.94
6500	1.06	1.12	1.21	1.29	1.36	1.43	1.49	1.61	1.71	1.77	1.82	1.85	1.86
7000	1.03	1.08	1.17	1.24	1.31	1.38	1.43	1.55	1.64	1.71	1.76	1.78	1.79
7500	0.99	1.04	1.13	1.20	1.27	1.33	1.39	1.50	1.59	1.65	1.70	1.72	1.73
8000	0.96	1.01	1.09	1.16	1.23	1.29	1.34	1.45	1.54	1.60	1.64	1.67	1.68
8500	0.93	0.98	1.06	1.13	1.19	1.25	1.30	1.41	1.49	1.55	1.59	1.62	1.63
9000	0.90	0.95	1.03	1.10	1.16	1.21	1.26	1.37	1.45	1.51	1.55	1.57	1.58
9500	0.88	0.92	1.00	1.07	1.13	1.18	1.23	1.33	1.41	1.47	1.51	1.53	1.54
10000	0.86	0.90	0.97	1.04	1.10	1.15	1.20	1.30	1.37	1.43	1.47	1.49	1.50

Although the relationships are curvilinear, straight line interpolation between rows or columns may be used for all practical purposes.

Item 2. Quality control tables for number of defectives. Foremen, inspectors, and others who cannot conveniently take the time to plot a number of quality control charts, but who still want a quick check on whether several quality characteristics are in control will find these tables convenient.

The tables have been prepared in two sections. Section One is intended for use in connection with an established process level per cent defective where varying sample sizes are involved. The proper table may be selected by dividing the total number of defective items by the total number inspected during at least the last 25 days (lots, shifts, etc.) and multiplying by 100 to convert to per cent. Using the table for the nearest per cent defective, check each day (lot, shift, etc.) to determine those that exceed the number in the column headed "Rejects Must Not Exceed." Read the number inspected to the nearest 100. Although the relationships are curvilinear, straight line interpolation may be used for all practical purposes (similar interpolations may be made between tables if desired). Eliminating all figures for those days with excessive rejects, recompute the process level. Select the proper table for use in judging the next 25 days' production (one month, or other practical interval).

Section Two is intended for use in situations where several different quality characteristics, each with a different quality level, are checked on samples of the same size.

Excessive defectives indicate the presence of assignable causes. Such events call for prompt investigation to identify and control or eliminate the assignable causes. Fewer defects than expected should also be investigated to determine what occurred to make the product significantly better. If such causes can be brought into constant operation the product may be permanently improved.

These tables are based upon the three standard deviation control limits. The per cent defective control chart limits are also listed in Section One for the convenience of those who wish to plot control charts.

Those who want to obtain limits for sample sizes other than those tabulated above or for quality levels in excess of 35 per cent defective may use Figs. 9.7a and 9.7b. Lower numbers of defectives can be read exactly. Higher numbers may involve a small error. These errors will generally be less than one per cent of the correct value for the number of defectives. Readings will be improved if several are made at regular intervals of sample size and corrected to give "smooth" successive differences.

The smallest number of defectives shown on Fig. 9.7b is one. Under some conditions of sample size and process level, zero defectives would be sig-

(Text continued on page 122)

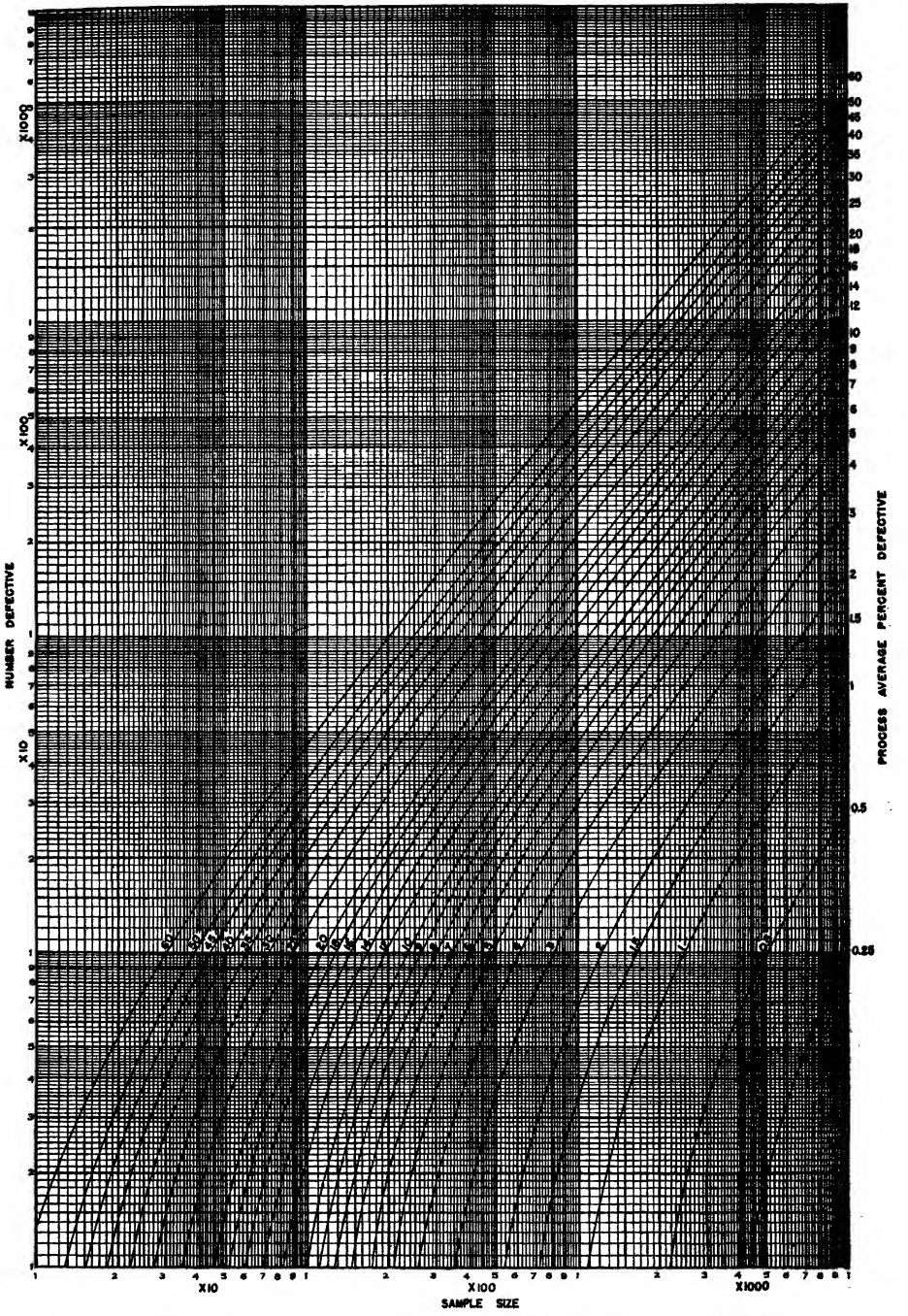

Fig. 9.7b. Curves for lower control chart limit for number defective.

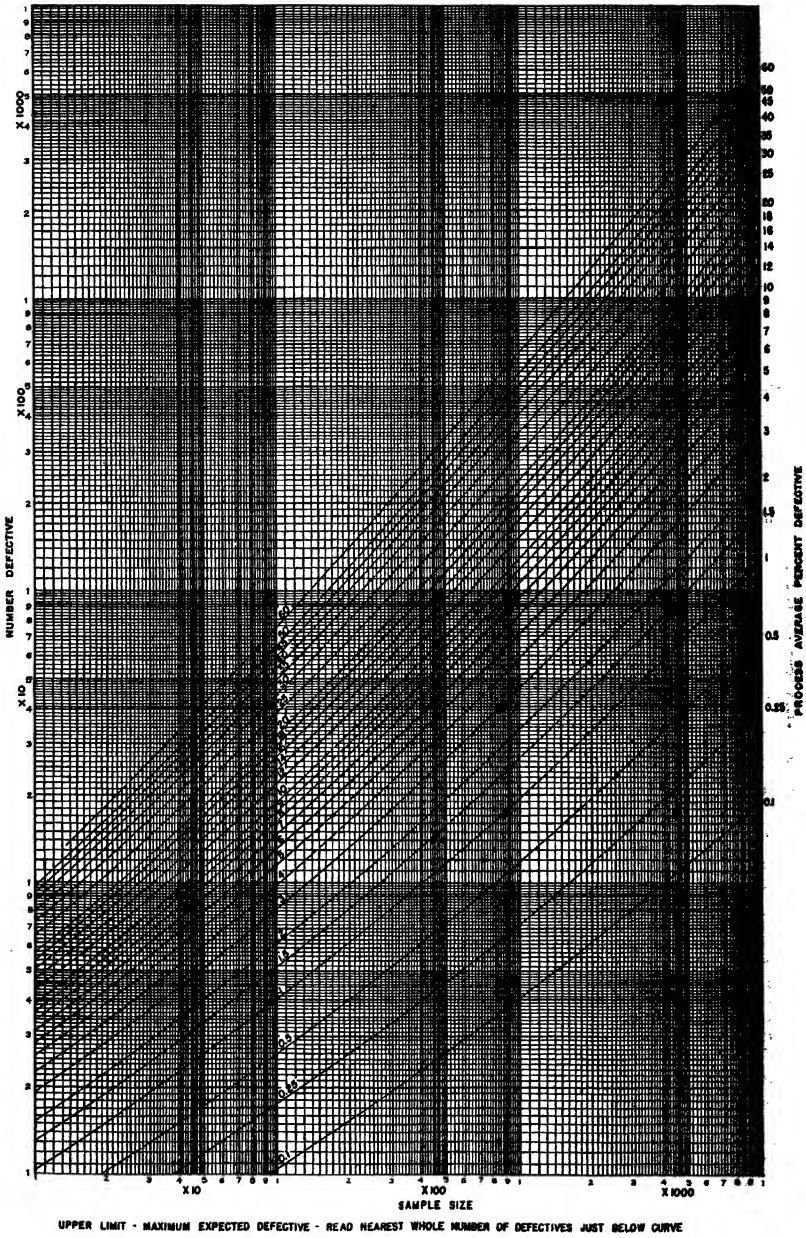

Fig. 9.7a. Curves for upper control chart limit for number defective.

TABLE 9.7b

QUALITY CONTROL TABLES FOR NUMBER OF DEFECTIVES
SECTION ONE
PROCESS LEVEL 0.1% DEFECTIVE

Number Inspected	UCL	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected	LCL
100	1.05	1	0.1	-	0
200	0.77	1	0.2	-	0
300	0.65	1	0.3	-	0
400	0.57	2	0.4	-	0
500	0.52	2	0.5	-	0
600	0.49	2	0.6	-	0
700	0.46	3	0.7	-	0
800	0.44	3	0.8	-	0
900	0.42	3	0.9	-	0
1000	0.40	3	1.0	-	0
1100	0.39	4	1.1	-	0
1200	0.37	4	1.2	-	0
1300	0.36	4	1.3	-	0
1400	0.35	4	1.4	-	0
1500	0.34	5	1.5	-	0
1600	0.34	5	1.6	-	0
1700	0.33	5	1.7	-	0
1800	0.32	5	1.8	-	0
1900	0.32	6	1.9	-	0
2000	0.31	6	2.0	-	0
2100	0.31	6	2.1	-	0
2200	0.30	6	2.2	-	0
2300	0.30	6	2.3	-	0
2400	0.29	7	2.4	-	0
2500	0.29	7	2.5	-	0
2600	0.29	7	2.6	-	0
2700	0.28	7	2.7	-	0
2800	0.28	7	2.8	-	0
2900	0.28	8	2.9	-	0
3000	0.27	8	3.0	-	0
3100	0.27	8	3.1	-	0
3200	0.27	8	3.2	-	0
3300	0.27	8	3.3	-	0
3400	0.26	8	3.4	-	0
3500	0.26	9	3.5	-	0
3600	0.26	9	3.6	-	0
3700	0.26	9	3.7	-	0
3800	0.25	9	3.8	-	0
3900	0.25	9	3.9	-	0
4000	0.25	9	4.0	-	0
4200	0.25	10	4.2	-	0
4400	0.24	10	4.4	-	0
4600	0.24	11	4.6	-	0
4800	0.24	11	4.8	-	0
5000	0.23	11	5.0	-	0
5500	0.23	12	5.5	-	0
6000	0.22	13	6.0	-	0
6500	0.22	14	6.5	-	0
7000	0.21	14	7.0	-	0
7500	0.21	15	7.5	-	0
8000	0.21	16	8.0	-	0
8500	0.20	17	8.5	-	0
9000	0.20	17	9.0	1	0
9500	0.20	18	9.5	1	0
10000	0.19	19	10.0	1	0.01

TABLE 9.7b continued

PROCESS LEVEL 0.25% DEFECTIVE

Number Inspected	UCL	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected	LCL
100	1.75	1	0.25	-	0
200	1.31	2	0.50	-	0
300	1.11	3	0.75	-	0
400	1.00	3	1.00	-	0
500	0.92	4	1.25	-	0
600	0.86	5	1.50	-	0
700	0.82	5	1.75	-	0
800	0.78	6	2.00	-	0
900	0.75	6	2.25	-	0
1000	0.72	7	2.50	-	0
1100	0.70	7	2.75	-	0
1200	0.68	8	3.00	-	0
1300	0.67	8	3.25	-	0
1400	0.65	9	3.50	-	0
1500	0.64	9	3.75	-	0
1600	0.62	9	4.00	-	0
1700	0.61	10	4.25	-	0
1800	0.60	10	4.50	-	0
1900	0.59	11	4.75	-	0
2000	0.59	11	5.00	-	0
2100	0.58	12	5.25	-	0
2200	0.57	12	5.50	-	0
2300	0.56	12	5.75	-	0
2400	0.56	13	6.00	-	0
2500	0.55	13	6.25	-	0
2600	0.54	14	6.50	-	0
2700	0.54	14	6.75	-	0
2800	0.53	14	7.00	-	0
2900	0.53	15	7.25	-	0
3000	0.52	15	7.50	-	0
3100	0.52	16	7.75	-	0
3200	0.51	16	8.00	-	0
3300	0.51	16	8.25	-	0
3400	0.51	17	8.50	-	0
3500	0.50	17	8.75	-	0
3600	0.50	17	9.00	1	0
3700	0.50	18	9.25	1	0
3800	0.49	18	9.50	1	0.01
3900	0.49	19	9.75	1	0.01
4000	0.49	19	10.00	1	0.01
4200	0.48	20	10.50	1	0.02
4400	0.48	20	11.00	2	0.02
4600	0.47	21	11.50	2	0.03
4800	0.47	22	12.00	2	0.03
5000	0.46	23	12.50	2	0.04
5500	0.45	24	13.75	3	0.05
6000	0.44	26	15.00	4	0.06
6500	0.44	28	16.25	5	0.06
7000	0.43	30	17.50	5	0.07
7500	0.42	31	18.75	6	0.08
8000	0.42	33	20.00	7	0.08
8500	0.41	35	21.25	8	0.09
9000	0.41	36	22.50	9	0.09
9500	0.40	38	23.75	10	0.10
10000	0.40	39	25.00	11	0.10

TABLE 9.7b continued

PROCESS LEVEL 0.5% DEFECTIVE

Number Inspected	UCL	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected	LCL
100	2.62	2	0.5	-	0
200	2.00	3	1.0	-	0
300	1.72	5	1.5	-	0
400	1.56	6	2.0	-	0
500	1.45	7	2.5	-	0
600	1.36	8	3.0	-	0
700	1.30	9	3.5	-	0
800	1.25	9	4.0	-	0
900	1.21	10	4.5	-	0
1000	1.20	11	5.0	-	0
1100	1.14	12	5.5	-	0
1200	1.11	13	6.0	-	0
1300	1.09	14	6.5	-	0
1400	1.07	14	7.0	-	0
1500	1.05	15	7.5	-	0
1600	1.03	16	8.0	-	0
1700	1.01	17	8.5	-	0
1800	1.00	17	9.0	1	0
1900	0.99	18	9.5	1	0.01
2000	0.97	19	10.0	1	0.03
2100	0.96	20	10.5	1	0.04
2200	0.95	20	11.0	2	0.05
2300	0.94	21	11.5	2	0.06
2400	0.93	22	12.0	2	0.07
2500	0.92	23	12.5	2	0.08
2600	0.92	23	13.0	3	0.08
2700	0.91	24	13.5	3	0.09
2800	0.90	25	14.0	3	0.10
2900	0.89	25	14.5	4	0.11
3000	0.89	26	15.0	4	0.11
3100	0.88	27	15.5	4	0.12
3200	0.87	27	16.0	5	0.13
3300	0.87	28	16.5	5	0.13
3400	0.86	29	17.0	5	0.14
3500	0.86	30	17.5	5	0.14
3600	0.85	30	18.0	6	0.15
3700	0.85	31	18.5	6	0.15
3800	0.84	32	19.0	6	0.16
3900	0.84	32	19.5	7	0.16
4000	0.83	33	20.0	7	0.17
4200	0.83	34	21.0	8	0.17
4400	0.82	36	22.0	8	0.18
4600	0.81	37	23.0	9	0.19
4800	0.81	38	24.0	10	0.19
5000	0.80	39	25.0	11	0.20
5500	0.79	43	27.5	12	0.21
6000	0.77	46	30.0	14	0.23
6500	0.76	49	32.5	16	0.24
7000	0.75	52	35.0	18	0.25
7500	0.74	55	37.5	20	0.26
8000	0.74	58	40.0	22	0.26
8500	0.73	62	42.5	23	0.27
9000	0.72	65	45.0	25	0.28
9500	0.72	68	47.5	27	0.28
10000	0.71	71	50.0	29	0.29

TABLE 9.7b continued

PROCESS LEVEL 1% DEFECTIVE

Number Inspected	UCL	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected	LCL
100	3.98	3	1	-	0
200	3.11	6	2	-	0
300	2.72	8	3	-	0
400	2.49	9	4	-	0
500	2.33	11	5	-	0
600	2.22	13	6	-	0
700	2.13	14	7	-	0
800	2.06	16	8	-	0
900	2.00	17	9	1	0
1000	1.94	19	10	1	0.06
1100	1.90	20	11	2	0.10
1200	1.86	22	12	2	0.14
1300	1.83	23	13	3	0.17
1400	1.80	25	14	3	0.20
1500	1.77	26	15	4	0.23
1600	1.75	27	16	5	0.25
1700	1.72	29	17	5	0.28
1800	1.70	30	18	6	0.30
1900	1.68	32	19	6	0.32
2000	1.67	33	20	7	0.33
2100	1.65	34	21	7	0.35
2200	1.64	36	22	8	0.36
2300	1.62	37	23	9	0.37
2400	1.61	38	24	10	0.39
2500	1.60	39	25	11	0.40
2600	1.59	41	26	11	0.41
2700	1.57	42	27	12	0.43
2800	1.56	43	28	13	0.44
2900	1.55	45	29	13	0.45
3000	1.54	46	30	14	0.46
3100	1.54	47	31	15	0.46
3200	1.53	48	32	16	0.47
3300	1.52	50	33	16	0.48
3400	1.51	51	34	17	0.49
3500	1.50	52	35	18	0.50
3600	1.50	53	36	19	0.50
3700	1.49	55	37	19	0.51
3800	1.48	56	38	20	0.52
3900	1.48	57	39	21	0.52
4000	1.47	58	40	22	0.53
4200	1.46	61	42	23	0.54
4400	1.45	63	44	25	0.55
4600	1.44	66	46	26	0.56
4800	1.43	68	48	28	0.57
5000	1.42	71	50	29	0.58
5500	1.40	77	55	33	0.60
6000	1.39	83	60	37	0.61
6500	1.37	89	65	41	0.63
7000	1.36	94	70	46	0.64
7500	1.34	100	75	50	0.66
8000	1.33	106	80	54	0.67
8500	1.32	112	85	58	0.68
9000	1.31	118	90	62	0.69
9500	1.31	124	95	66	0.69
10000	1.30	129	100	71	0.70

TABLE 9.7b continued

PROCESS LEVEL 1.5% DEFECTIVE

Number Inspected	UCL	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected	LCL
100	5.15	5	1.5	-	0
200	4.08	8	3.0	-	0
300	3.61	10	4.5	-	0
400	3.32	13	6.0	-	0
500	3.13	15	7.5	-	0
600	2.99	17	9.0	1	0.01
700	2.88	20	10.5	1	0.12
800	2.79	22	12.0	2	0.21
900	2.72	24	13.5	3	0.28
1000	2.65	26	15.0	4	0.35
1100	2.60	28	16.5	5	0.40
1200	2.55	30	18.0	6	0.45
1300	2.51	32	19.5	7	0.49
1400	2.47	34	21.0	8	0.53
1500	2.44	36	22.5	9	0.56
1600	2.41	38	24.0	10	0.59
1700	2.38	40	25.5	11	0.62
1800	2.36	42	27.0	12	0.64
1900	2.34	44	28.5	13	0.66
2000	2.32	46	30.0	14	0.68
2100	2.30	48	31.5	15	0.70
2200	2.28	50	33.0	16	0.72
2300	2.26	51	34.5	18	0.74
2400	2.24	53	36.0	19	0.76
2500	2.23	55	37.5	20	0.77
2600	2.22	57	39.0	21	0.78
2700	2.20	59	40.5	22	0.80
2800	2.19	61	42.0	23	0.81
2900	2.18	63	43.5	24	0.82
3000	2.17	64	45.0	26	0.83
3100	2.15	66	46.5	27	0.85
3200	2.14	68	48.0	28	0.86
3300	2.13	70	49.5	29	0.87
3400	2.13	72	51.0	30	0.87
3500	2.12	74	52.5	31	0.88
3600	2.11	75	54.0	33	0.89
3700	2.10	77	55.5	34	0.90
3800	2.09	79	57.0	35	0.91
3900	2.08	81	58.5	36	0.92
4000	2.08	83	60.0	37	0.92
4200	2.06	86	63.0	40	0.94
4400	2.05	90	66.0	42	0.95
4600	2.04	93	69.0	45	0.96
4800	2.03	97	72.0	47	0.97
5000	2.02	100	75.0	50	0.98
5500	1.99	109	82.5	56	1.01
6000	1.97	118	90.0	62	1.03
6500	1.95	126	97.5	69	1.05
7000	1.94	135	105.0	75	1.06
7500	1.92	144	112.5	81	1.08
8000	1.91	152	120.0	88	1.09
8500	1.90	161	127.5	94	1.10
9000	1.88	169	135.0	101	1.12
9500	1.87	178	142.5	107	1.13
10000	1.86	186	150.0	114	1.14

TABLE 9.7b continued

PROCESS LEVEL 2% DEFECTIVE

Number Inspected	UCL	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected	LCL
100	6.20	6	2	-	0
200	4.97	9	4	-	0
300	4.42	13	6	-	0
400	4.10	16	8	-	0
500	3.88	19	10	1	0.12
600	3.71	22	12	2	0.29
700	3.59	25	14	3	0.41
800	3.48	27	16	5	0.52
900	3.40	30	18	6	0.60
1000	3.33	33	20	7	0.67
1100	3.27	35	22	9	0.73
1200	3.21	38	24	10	0.79
1300	3.16	41	26	11	0.84
1400	3.12	43	28	13	0.88
1500	3.08	46	30	14	0.92
1600	3.05	48	32	16	0.95
1700	3.02	51	34	17	0.98
1800	2.99	53	36	19	1.01
1900	2.96	56	38	20	1.04
2000	2.94	58	40	22	1.06
2100	2.92	61	42	23	1.08
2200	2.90	63	44	25	1.10
2300	2.88	66	46	26	1.12
2400	2.86	68	48	28	1.14
2500	2.84	70	50	30	1.16
2600	2.82	73	52	31	1.18
2700	2.81	75	54	33	1.19
2800	2.79	78	56	34	1.21
2900	2.78	80	58	36	1.22
3000	2.77	82	60	38	1.23
3100	2.75	85	62	39	1.25
3200	2.74	87	64	41	1.26
3300	2.73	90	66	42	1.27
3400	2.72	92	68	44	1.28
3500	2.71	94	70	46	1.29
3600	2.70	97	72	47	1.30
3700	2.69	99	74	49	1.31
3800	2.68	101	76	51	1.32
3900	2.67	104	78	52	1.33
4000	2.66	106	80	54	1.34
4200	2.65	111	84	57	1.35
4400	2.63	115	88	61	1.37
4600	2.62	120	92	64	1.38
4800	2.61	125	96	67	1.39
5000	2.59	129	100	71	1.41
5500	2.57	141	110	79	1.43
6000	2.54	152	120	88	1.46
6500	2.52	163	130	97	1.48
7000	2.50	175	140	105	1.50
7500	2.48	186	150	114	1.52
8000	2.47	197	160	123	1.53
8500	2.46	208	170	132	1.54
9000	2.44	219	180	141	1.56
9500	2.43	230	190	150	1.57
10000	2.42	241	200	159	1.58

TABLE 9.7b continued

PROCESS LEVEL 5% DEFECTIVE

Number Inspected	UCL	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected	LCL
100	8.12	8	5	-	0
200	6.62	13	6	-	0
300	5.95	17	9	1	0.05
400	5.56	22	12	2	0.44
500	5.29	26	15	4	0.71
600	5.09	30	18	6	0.91
700	4.93	34	21	8	1.07
800	4.81	38	24	10	1.19
900	4.71	42	27	12	1.29
1000	4.62	46	30	14	1.38
1100	4.54	49	33	17	1.46
1200	4.48	53	36	19	1.52
1300	4.42	57	39	21	1.58
1400	4.37	61	42	23	1.63
1500	4.32	64	45	26	1.68
1600	4.28	68	48	28	1.72
1700	4.24	72	51	30	1.76
1800	4.21	75	54	33	1.79
1900	4.17	79	57	35	1.83
2000	4.14	82	60	38	1.86
2100	4.12	86	63	40	1.88
2200	4.09	89	66	43	1.91
2300	4.07	93	69	45	1.93
2400	4.04	97	72	47	1.96
2500	4.02	100	75	50	1.98
2600	4.00	104	78	52	2.00
2700	3.98	107	81	55	2.02
2800	3.97	111	84	57	2.03
2900	3.95	114	87	60	2.05
3000	3.93	118	90	62	2.07
3100	3.92	121	93	65	2.08
3200	3.90	124	96	68	2.10
3300	3.89	128	99	70	2.11
3400	3.88	131	102	73	2.12
3500	3.87	135	105	75	2.13
3600	3.85	138	108	78	2.15
3700	3.84	142	111	80	2.16
3800	3.83	145	114	83	2.17
3900	3.82	148	117	86	2.18
4000	3.81	152	120	88	2.19
4200	3.79	159	126	93	2.21
4400	3.77	165	132	99	2.23
4600	3.75	172	138	104	2.25
4800	3.74	179	144	109	2.26
5000	3.72	186	150	114	2.28
5500	3.69	202	165	128	2.31
6000	3.66	219	180	141	2.34
6500	3.63	236	195	154	2.37
7000	3.61	252	210	168	2.39
7500	3.59	269	225	181	2.41
8000	3.57	285	240	195	2.43
8500	3.56	302	255	208	2.44
9000	3.54	318	270	222	2.46
9500	3.53	334	285	236	2.47
10000	3.51	351	300	249	2.49

TABLE 9.7b continued

PROCESS LEVEL 4% DEFECTIVE

Number Inspected	UOL	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected	LOL
100	9.88	9	4	-	0
200	8.16	16	8	-	0
300	7.39	22	12	2	0.61
400	6.94	27	16	5	1.06
500	6.63	33	20	7	1.37
600	6.40	38	24	10	1.60
700	6.22	43	28	13	1.78
800	6.08	48	32	16	1.92
900	5.96	53	36	19	2.04
1000	5.86	58	40	22	2.14
1100	5.77	63	44	25	2.23
1200	5.70	68	48	28	2.30
1300	5.63	73	52	31	2.37
1400	5.57	77	56	35	2.43
1500	5.52	82	60	38	2.48
1600	5.47	87	64	41	2.53
1700	5.43	92	68	44	2.57
1800	5.39	96	72	48	2.61
1900	5.35	101	76	51	2.65
2000	5.31	106	80	54	2.69
2100	5.28	110	84	58	2.72
2200	5.25	115	88	61	2.75
2300	5.23	120	92	64	2.77
2400	5.20	124	96	68	2.80
2500	5.18	129	100	71	2.82
2600	5.15	133	104	75	2.85
2700	5.13	138	108	78	2.87
2800	5.11	143	112	81	2.89
2900	5.09	147	116	85	2.91
3000	5.07	152	120	88	2.93
3100	5.06	156	124	92	2.94
3200	5.04	161	128	95	2.96
3300	5.02	165	132	99	2.98
3400	5.01	170	136	102	2.99
3500	4.99	174	140	106	3.01
3600	4.98	179	144	109	3.02
3700	4.97	183	148	113	3.03
3800	4.95	188	152	116	3.05
3900	4.94	192	156	120	3.06
4000	4.93	197	160	123	3.07
4200	4.91	206	168	130	3.09
4400	4.89	214	176	138	3.11
4600	4.87	223	184	145	3.13
4800	4.85	232	192	152	3.15
5000	4.83	241	200	159	3.17
5500	4.79	263	220	177	3.21
6000	4.76	285	240	195	3.24
6500	4.73	307	260	213	3.27
7000	4.70	329	280	231	3.30
7500	4.68	350	300	250	3.32
8000	4.66	372	320	268	3.34
8500	4.64	394	340	286	3.36
9000	4.62	415	360	305	3.38
9500	4.60	437	380	323	3.40
10000	4.59	458	400	342	3.41

TABLE 9.7b continued

PROCESS LEVEL 5% DEFECTIVE

Number Inspected	UCL	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected	LCL
100	11.54	11	5	-	0
200	9.62	19	10	1	0.38
300	8.77	26	15	4	1.23
400	8.27	33	20	7	1.73
500	7.92	39	25	11	2.08
600	7.67	46	30	14	2.33
700	7.47	52	35	18	2.53
800	7.31	58	40	22	2.69
900	7.18	64	45	26	2.82
1000	7.07	70	50	30	2.93
1100	6.97	76	55	34	3.03
1200	6.89	82	60	38	3.11
1300	6.81	88	65	42	3.19
1400	6.75	94	70	46	3.25
1500	6.69	100	75	50	3.31
1600	6.63	106	80	54	3.37
1700	6.59	111	85	59	3.41
1800	6.54	117	90	63	3.46
1900	6.50	123	95	67	3.50
2000	6.46	129	100	71	3.54
2100	6.43	134	105	76	3.57
2200	6.39	140	110	80	3.61
2300	6.36	146	115	84	3.64
2400	6.33	152	120	88	3.67
2500	6.31	157	125	93	3.69
2600	6.28	163	130	97	3.72
2700	6.26	168	135	102	3.74
2800	6.24	174	140	106	3.76
2900	6.21	180	145	110	3.79
3000	6.19	185	150	115	3.81
3100	6.17	191	155	119	3.83
3200	6.16	196	160	124	3.84
3300	6.14	202	165	128	3.86
3400	6.12	208	170	132	3.88
3500	6.11	213	175	137	3.89
3600	6.09	219	180	141	3.91
3700	6.07	224	185	146	3.93
3800	6.06	230	190	150	3.94
3900	6.05	235	195	155	3.95
4000	6.03	241	200	159	3.97
4200	6.01	252	210	168	3.99
4400	5.99	263	220	177	4.01
4600	5.96	274	230	186	4.04
4800	5.94	285	240	195	4.06
5000	5.92	296	250	204	4.08
5500	5.88	323	275	227	4.12
6000	5.84	350	300	250	4.16
6500	5.81	377	325	273	4.19
7000	5.78	404	350	296	4.22
7500	5.75	431	375	319	4.25
8000	5.73	458	400	342	4.27
8500	5.71	485	425	365	4.29
9000	5.69	512	450	388	4.31
9500	5.67	538	475	412	4.33
10000	5.65	565	500	435	4.35

TABLE 9.7b continued

PROCESS LEVEL 6% DEFECTIVE

Number Inspected	UCL	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected	LCL
100	13.12	13	6	-	0
200	11.04	22	12	2	0.96
300	10.11	30	18	6	1.89
400	9.56	38	24	10	2.44
500	9.19	45	30	15	2.81
600	8.91	53	36	19	3.09
700	8.69	60	42	24	3.31
800	8.52	68	48	28	3.48
900	8.37	75	54	33	3.63
1000	8.25	82	60	38	3.75
1100	8.15	89	66	43	3.85
1200	8.06	96	72	48	3.94
1300	7.98	103	78	53	4.02
1400	7.90	110	84	58	4.10
1500	7.84	117	90	63	4.16
1600	7.78	124	96	68	4.22
1700	7.73	131	102	73	4.27
1800	7.70	138	108	78	4.30
1900	7.63	145	114	83	4.37
2000	7.59	151	120	89	4.41
2100	7.55	158	126	94	4.45
2200	7.52	165	132	99	4.48
2300	7.49	172	138	104	4.51
2400	7.45	178	144	110	4.55
2500	7.42	185	150	115	4.58
2600	7.40	192	156	120	4.60
2700	7.37	199	162	125	4.63
2800	7.35	205	168	131	4.65
2900	7.32	212	174	136	4.68
3000	7.30	219	180	141	4.70
3100	7.28	225	186	147	4.72
3200	7.26	232	192	152	4.74
3300	7.24	238	198	158	4.76
3400	7.22	245	204	163	4.78
3500	7.20	252	210	168	4.80
3600	7.19	258	216	174	4.81
3700	7.17	265	222	179	4.83
3800	7.16	271	228	185	4.84
3900	7.14	278	234	190	4.86
4000	7.13	285	240	195	4.87
4200	7.10	298	252	206	4.90
4400	7.07	311	264	217	4.93
4600	7.05	324	276	228	4.95
4800	7.03	337	288	239	4.97
5000	7.01	350	300	250	4.99
5500	6.96	382	330	278	5.04
6000	6.92	415	360	305	5.08
6500	6.88	447	390	333	5.12
7000	6.85	479	420	361	5.15
7500	6.82	511	450	389	5.18
8000	6.80	543	480	417	5.20
8500	6.77	575	510	445	5.23
9000	6.75	607	540	473	5.25
9500	6.73	639	570	501	5.27
10000	6.71	671	600	529	5.29

TABLE 9.7b continued

PROCESS LEVEL 7% DEFECTIVE

Number Inspected	UCL	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected	LCL
100	14.65	14	7	-	0
200	12.41	24	14	4	1.59
300	11.42	34	21	8	2.58
400	10.83	43	28	13	3.17
500	10.42	52	35	18	3.58
600	10.12	60	42	24	3.88
700	9.89	69	49	29	4.11
800	9.71	77	56	35	4.29
900	9.55	85	63	41	4.45
1000	9.42	94	70	46	4.58
1100	9.31	102	77	52	4.69
1200	9.21	110	84	58	4.79
1300	9.12	118	91	64	4.88
1400	9.05	126	98	70	4.95
1500	8.98	134	105	76	5.02
1600	8.91	142	112	82	5.09
1700	8.86	150	119	88	5.14
1800	8.80	158	126	94	5.20
1900	8.76	166	133	100	5.24
2000	8.71	174	140	106	5.29
2100	8.67	182	147	112	5.33
2200	8.63	189	154	119	5.37
2300	8.60	197	161	125	5.40
2400	8.56	205	168	131	5.44
2500	8.53	213	175	137	5.47
2600	8.50	221	182	143	5.50
2700	8.47	228	189	150	5.53
2800	8.45	236	196	156	5.55
2900	8.42	244	203	162	5.58
3000	8.40	251	210	169	5.60
3100	8.37	259	217	175	5.63
3200	8.35	267	224	181	5.65
3300	8.33	274	231	188	5.67
3400	8.31	282	238	194	5.69
3500	8.29	290	245	200	5.71
3600	8.28	297	252	207	5.72
3700	8.26	305	259	213	5.74
3800	8.24	313	266	219	5.76
3900	8.23	320	273	226	5.77
4000	8.21	328	280	232	5.79
4200	8.18	343	294	245	5.82
4400	8.15	358	308	258	5.85
4600	8.13	373	322	271	5.87
4800	8.10	389	336	283	5.90
5000	8.08	404	350	296	5.92
5500	8.03	441	385	329	5.97
6000	7.99	479	420	361	6.01
6500	7.95	516	455	394	6.05
7000	7.91	554	490	426	6.09
7500	7.88	591	525	459	6.12
8000	7.86	628	560	492	6.14
8500	7.83	665	595	525	6.17
9000	7.81	702	630	558	6.19
9500	7.79	739	665	591	6.21
10000	7.77	776	700	624	6.23

TABLE 9.7b continued

PROCESS LEVEL 8% DEFECTIVE

Number Inspected	UCL	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected	LCL
100	16.14	16	8	-	0
200	13.76	27	16	5	2.24
300	12.70	38	24	10	3.30
400	12.07	48	32	16	3.93
500	11.64	58	40	22	4.36
600	11.32	67	48	29	4.68
700	11.08	77	56	35	4.92
800	10.88	87	64	41	5.12
900	10.71	96	72	48	5.29
1000	10.57	105	80	55	5.43
1100	10.45	114	88	62	5.55
1200	10.35	124	96	68	5.65
1300	10.25	133	104	75	5.75
1400	10.22	142	112	82	5.78
1500	10.10	151	120	89	5.90
1600	9.13	160	128	96	5.97
1700	9.97	169	136	103	6.03
1800	9.92	178	144	110	6.08
1900	9.87	187	152	117	6.13
2000	9.82	196	160	124	6.18
2100	9.78	205	168	131	6.22
2200	9.74	214	176	138	6.26
2300	9.70	223	184	145	6.30
2400	9.66	231	192	153	6.34
2500	9.63	240	200	160	6.37
2600	9.60	249	208	167	6.40
2700	9.57	258	216	174	6.43
2800	9.54	267	224	181	6.46
2900	9.51	275	232	189	6.49
3000	9.49	284	240	196	6.51
3100	9.46	293	248	203	6.54
3200	9.44	302	256	210	6.56
3300	9.42	310	264	218	6.58
3400	9.40	319	272	225	6.60
3500	9.38	328	280	232	6.62
3600	9.36	336	288	240	6.64
3700	9.34	345	296	247	6.66
3800	9.32	354	304	254	6.68
3900	9.30	362	312	262	6.70
4000	9.29	371	320	269	6.71
4200	9.26	388	336	284	6.74
4400	9.23	405	352	299	6.77
4600	9.20	423	368	313	6.80
4800	9.17	440	384	328	6.83
5000	9.15	457	400	343	6.85
5500	9.10	500	440	380	6.90
6000	9.05	543	480	417	6.95
6500	9.01	585	520	455	6.99
7000	8.97	628	560	492	7.03
7500	8.94	670	600	530	7.06
8000	8.91	712	640	568	7.09
8500	8.88	755	680	605	7.12
9000	8.86	797	720	643	7.14
9500	8.84	839	760	681	7.16
10000	8.81	881	800	719	7.19

TABLE 9.7b continued

PROCESS LEVEL 9% DEFECTIVE

Number Inspected	UCL	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected	LCL
100	17.59	17	9	1	0.41
200	15.07	30	18	6	2.93
300	13.96	41	27	13	4.04
400	13.29	53	36	19	4.71
500	12.84	64	45	26	5.16
600	12.50	75	54	33	5.50
700	12.24	85	63	41	5.76
800	12.04	96	72	48	5.96
900	11.86	106	81	56	6.14
1000	11.71	117	90	63	6.29
1100	11.59	127	99	71	6.41
1200	11.48	137	108	79	6.52
1300	11.38	147	117	87	6.62
1400	11.29	158	126	94	6.71
1500	11.22	168	135	102	6.78
1600	11.15	178	144	110	6.85
1700	11.08	188	153	118	6.92
1800	11.02	198	162	126	6.98
1900	10.97	208	171	134	7.03
2000	10.92	218	180	142	7.08
2100	10.87	228	189	150	7.13
2200	10.83	238	198	158	7.17
2300	10.79	248	207	166	7.21
2400	10.75	258	216	174	7.25
2500	10.72	267	225	183	7.28
2600	10.68	277	234	191	7.32
2700	10.65	287	243	199	7.35
2800	10.62	297	252	207	7.38
2900	10.59	307	261	215	7.41
3000	10.57	317	270	223	7.43
3100	10.54	326	279	232	7.46
3200	10.52	336	288	240	7.48
3300	10.49	346	297	248	7.51
3400	10.47	356	306	256	7.53
3500	10.45	365	315	265	7.55
3600	10.43	375	324	273	7.57
3700	10.41	385	333	281	7.59
3800	10.39	394	342	290	7.61
3900	10.37	404	351	298	7.63
4000	10.36	414	360	306	7.64
4200	10.32	433	378	323	7.68
4400	10.29	452	396	340	7.71
4600	10.27	472	414	356	7.73
4800	10.24	491	432	373	7.76
5000	10.21	510	450	390	7.79
5500	10.16	558	495	432	7.84
6000	10.11	606	540	474	7.89
6500	10.06	654	585	516	7.94
7000	10.03	701	630	559	7.97
7500	9.99	749	675	601	8.01
8000	9.96	796	720	644	8.04
8500	9.93	844	765	686	8.07
9000	9.90	891	810	729	8.10
9500	9.88	938	855	772	8.12
10000	9.86	985	900	815	8.14

TABLE 9.7b continued

PROCESS LEVEL 10% DEFECTIVE

Number Inspected	UCL	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected	LCL
100	19.00	18	10	2	1.00
200	16.36	32	20	8	3.64
300	15.20	45	30	15	4.80
400	14.50	57	40	23	5.50
500	14.02	70	50	30	5.98
600	13.67	82	60	38	6.33
700	13.40	93	70	47	6.60
800	13.18	105	80	55	6.82
900	13.00	116	90	64	7.00
1000	12.85	128	100	72	7.15
1100	12.71	139	110	81	7.29
1200	12.60	151	120	89	7.40
1300	12.50	162	130	98	7.50
1400	12.41	173	140	107	7.59
1500	12.32	184	150	116	7.68
1600	12.25	195	160	125	7.75
1700	12.18	207	170	133	7.82
1800	12.12	218	180	142	7.88
1900	12.07	229	190	151	7.93
2000	12.01	240	200	160	7.99
2100	11.96	251	210	169	8.04
2200	11.92	262	220	178	8.08
2300	11.88	273	230	187	8.12
2400	11.84	284	240	196	8.16
2500	11.80	294	250	206	8.20
2600	11.77	305	260	215	8.23
2700	11.73	316	270	224	8.27
2800	11.70	327	280	233	8.30
2900	11.67	338	290	242	8.33
3000	11.64	349	300	251	8.36
3100	11.62	360	310	260	8.38
3200	11.59	370	320	270	8.41
3300	11.57	381	330	279	8.43
3400	11.54	392	340	288	8.46
3500	11.52	403	350	297	8.48
3600	11.50	413	360	307	8.50
3700	11.48	424	370	316	8.52
3800	11.45	435	380	325	8.55
3900	11.44	446	390	334	8.56
4000	11.42	456	400	344	8.58
4200	11.39	478	420	362	8.61
4400	11.36	499	440	381	8.64
4600	11.33	521	460	399	8.67
4800	11.30	542	480	418	8.70
5000	11.27	563	500	437	8.73
5500	11.21	616	550	484	8.79
6000	11.16	669	600	531	8.84
6500	11.12	722	650	578	8.88
7000	11.08	775	700	625	8.92
7500	11.04	827	750	673	8.96
8000	11.01	880	800	720	8.99
8500	10.98	932	850	768	9.02
9000	10.95	985	900	815	9.05
9500	10.92	1037	950	863	9.08
10000	10.90	1090	1000	910	9.10

TABLE 9.7b continued

PROCESS LEVEL 12% DEFECTIVE

Number Inspected	UCL	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected	LCL
100	21.75	21	12	3	2.25
200	18.89	37	24	11	5.11
300	17.63	52	36	20	6.37
400	16.87	67	48	29	7.13
500	16.36	81	60	39	7.64
600	15.98	95	72	49	8.02
700	15.68	109	84	59	8.32
800	15.45	123	96	69	8.55
900	15.25	137	108	79	8.75
1000	15.08	151	120	89	8.92
1100	14.94	164	132	100	9.06
1200	14.81	177	144	111	9.19
1300	14.70	191	156	121	9.30
1400	14.61	204	168	132	9.39
1500	14.52	217	180	143	9.48
1600	14.44	230	192	154	9.56
1700	14.36	244	204	164	9.64
1800	14.30	257	216	175	9.70
1900	14.23	270	228	186	9.77
2000	14.18	283	240	197	9.82
2100	14.13	296	252	208	9.87
2200	14.08	309	264	219	9.92
2300	14.03	322	276	230	9.97
2400	13.99	335	288	241	10.01
2500	13.95	348	300	252	10.05
2600	13.91	361	312	263	10.09
2700	13.88	374	324	274	10.12
2800	13.84	387	336	285	10.16
2900	13.81	400	348	296	10.19
3000	13.78	413	360	307	10.22
3100	13.75	426	372	318	10.25
3200	13.72	439	384	329	10.28
3300	13.70	452	396	340	10.30
3400	13.67	464	408	352	10.33
3500	13.65	477	420	363	10.35
3600	13.62	490	432	374	10.38
3700	13.60	503	444	385	10.40
3800	13.58	516	456	396	10.42
3900	13.56	528	468	408	10.44
4000	13.54	541	480	419	10.46
4200	13.50	567	504	441	10.50
4400	13.47	592	528	464	10.53
4600	13.44	618	552	486	10.56
4800	13.41	643	576	509	10.59
5000	13.38	668	600	532	10.62
5500	13.31	732	660	588	10.69
6000	13.26	795	720	645	10.74
6500	13.21	858	780	702	10.79
7000	13.17	921	840	759	10.83
7500	13.13	964	900	816	10.87
8000	13.09	1047	960	873	10.91
8500	13.06	1109	1020	931	10.94
9000	13.03	1172	1080	988	10.97
9500	13.00	1235	1140	1045	11.00
10000	12.97	1297	1200	1103	11.03

TABLE 9.7b continued

PROCESS LEVEL 14% DEFECTIVE

Number Inspected	UCL	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected	LCL
100	24.41	24	14	4	3.59
200	21.36	42	28	14	6.64
300	20.01	60	42	24	7.99
400	19.20	76	56	36	8.80
500	18.66	93	70	47	9.34
600	18.25	109	84	59	9.75
700	17.93	125	98	71	10.07
800	17.68	141	112	83	10.32
900	17.47	157	126	95	10.53
1000	17.29	172	140	108	10.71
1100	17.14	188	154	120	10.86
1200	17.00	204	168	132	11.00
1300	16.89	219	182	145	11.11
1400	16.78	234	196	158	11.22
1500	16.69	250	210	170	11.31
1600	16.60	265	224	183	11.40
1700	16.52	280	238	196	11.48
1800	16.45	296	252	208	11.55
1900	16.39	311	266	221	11.61
2000	16.33	326	280	234	11.67
2100	16.27	341	294	247	11.73
2200	16.22	356	308	260	11.78
2300	16.17	371	322	273	11.83
2400	16.12	386	336	286	11.88
2500	16.08	402	350	298	11.92
2600	16.04	417	364	311	11.96
2700	16.00	432	378	324	12.00
2800	15.97	447	392	337	12.03
2900	15.93	462	406	350	12.07
3000	15.90	477	420	363	12.10
3100	15.87	491	434	377	12.13
3200	15.84	506	448	390	12.16
3300	15.81	521	462	403	12.19
3400	15.79	536	476	416	12.21
3500	15.76	551	490	429	12.24
3600	15.73	566	504	442	12.27
3700	15.71	581	518	455	12.29
3800	15.69	596	532	468	12.31
3900	15.67	611	546	481	12.33
4000	15.65	625	560	495	12.35
4200	15.61	655	588	521	12.39
4400	15.57	685	616	547	12.43
4600	15.53	714	644	574	12.47
4800	15.50	744	672	600	12.50
5000	15.47	773	700	627	12.53
5500	15.40	847	770	693	12.60
6000	15.34	920	840	760	12.66
6500	15.29	993	910	827	12.71
7000	15.24	1067	980	893	12.76
7500	15.20	1140	1050	960	12.80
8000	15.16	1213	1120	1027	12.84
8500	15.13	1285	1190	1095	12.87
9000	15.10	1358	1260	1162	12.90
9500	15.07	1431	1330	1229	12.93
10000	15.04	1504	1400	1296	12.96

TABLE 9.7b continued

PROCESS LEVEL 16% DEFECTIVE

Number Inspected	UCL	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected	LCL
100	27.00	26	16	6	5.00
200	23.78	47	32	17	8.22
300	22.35	67	48	29	9.65
400	21.50	85	64	43	10.50
500	20.92	104	80	56	11.08
600	20.49	122	96	70	11.51
700	20.16	141	112	83	11.84
800	19.89	159	128	97	12.11
900	19.67	176	144	112	12.33
1000	19.48	194	160	126	12.52
1100	19.32	212	176	140	12.68
1200	19.18	230	192	154	12.82
1300	19.05	247	208	169	12.95
1400	18.94	265	224	183	13.06
1500	18.84	282	240	198	13.16
1600	18.75	299	256	213	13.25
1700	18.67	317	272	227	13.33
1800	18.59	334	288	242	13.41
1900	18.52	351	304	257	13.48
2000	18.46	369	320	271	13.54
2100	18.40	386	336	286	13.60
2200	18.34	403	352	301	13.66
2300	18.29	420	368	316	13.71
2400	18.24	437	384	331	13.76
2500	18.20	454	400	346	13.80
2600	18.16	472	416	360	13.84
2700	18.12	489	432	375	13.88
2800	18.08	506	448	390	13.92
2900	18.04	523	464	405	13.96
3000	18.01	540	480	420	13.99
3100	17.98	557	496	435	14.02
3200	17.94	574	512	450	14.06
3300	17.91	591	528	465	14.09
3400	17.89	608	544	480	14.11
3500	17.86	625	560	495	14.14
3600	17.83	641	576	511	14.17
3700	17.81	658	592	526	14.19
3800	17.78	675	608	541	14.22
3900	17.76	692	624	556	14.24
4000	17.74	709	640	571	14.26
4200	17.70	743	672	601	14.30
4400	17.66	776	704	632	14.34
4600	17.62	810	736	662	14.38
4800	17.59	844	768	692	14.41
5000	17.56	877	800	723	14.44
5500	17.48	961	880	799	14.52
6000	17.42	1045	960	875	14.58
6500	17.36	1128	1040	952	14.64
7000	17.31	1212	1120	1028	14.69
7500	17.27	1295	1200	1105	14.73
8000	17.23	1378	1280	1182	14.77
8500	17.19	1461	1360	1259	14.81
9000	17.16	1544	1440	1336	14.84
9500	17.13	1627	1520	1413	14.87
10000	17.10	1709	1600	1491	14.90

TABLE 9.7b continued

PROCESS LEVEL 18% DEFECTIVE

Number Inspected	UCL	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected	LCL
100	29.53	29	18	7	6.47
200	26.15	52	36	20	9.85
300	24.65	73	54	35	11.35
400	23.76	95	72	49	12.24
500	23.15	115	90	65	12.85
600	22.71	136	108	80	13.29
700	22.36	156	126	96	13.64
800	22.07	176	144	112	13.93
900	21.84	196	162	128	14.16
1000	21.64	216	180	144	14.36
1100	21.48	236	198	160	14.52
1200	21.33	255	216	177	14.67
1300	21.20	275	234	193	14.80
1400	21.08	295	252	209	14.92
1500	20.98	314	270	226	15.02
1600	20.88	334	288	242	15.12
1700	20.80	353	306	259	15.20
1800	20.72	372	324	276	15.28
1900	20.64	392	342	292	15.36
2000	20.58	411	360	309	15.42
2100	20.52	430	378	326	15.48
2200	20.46	450	396	342	15.54
2300	20.40	469	414	359	15.60
2400	20.35	488	432	376	15.65
2500	20.31	507	450	393	15.69
2600	20.26	526	468	410	15.74
2700	20.22	545	486	427	15.78
2800	20.18	564	504	444	15.82
2900	20.14	584	522	460	15.86
3000	20.10	603	540	477	15.90
3100	20.07	622	558	494	15.93
3200	20.04	641	576	511	15.96
3300	20.01	660	594	528	15.99
3400	19.98	679	612	545	16.02
3500	19.95	698	630	562	16.05
3600	19.92	717	648	579	16.08
3700	19.89	736	666	596	16.11
3800	19.87	755	684	613	16.13
3900	19.85	773	702	631	16.15
4000	19.82	792	720	648	16.18
4200	19.78	830	756	682	16.22
4400	19.74	868	792	716	16.26
4600	19.70	906	828	750	16.30
4800	19.66	943	864	785	16.34
5000	19.63	981	900	819	16.37
5500	19.55	1075	990	905	16.45
6000	19.49	1169	1080	991	16.51
6500	19.43	1262	1170	1078	16.57
7000	19.38	1356	1260	1164	16.62
7500	19.33	1449	1350	1251	16.67
8000	19.29	1543	1440	1337	16.71
8500	19.25	1636	1530	1424	16.75
9000	19.21	1729	1620	1511	16.79
9500	19.18	1822	1710	1598	16.82
10000	19.15	1915	1800	1685	16.85

TABLE 9.7b continued

PROCESS LEVEL 20% DEFECTIVE

Number Inspected	UCL	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected	LCL
100	32.00	31	20	9	8.00
200	28.49	56	40	24	11.51
300	26.93	80	60	40	13.07
400	26.00	104	80	56	14.00
500	25.37	126	100	74	14.63
600	24.90	149	120	91	15.10
700	24.54	171	140	109	15.46
800	24.24	193	160	127	15.76
900	24.00	215	180	145	16.00
1000	23.79	237	200	163	16.21
1100	23.62	259	220	181	16.38
1200	23.46	281	240	199	16.54
1300	23.33	303	260	217	16.67
1400	23.21	324	280	236	16.79
1500	23.10	346	300	254	16.90
1600	23.00	367	320	273	17.00
1700	22.91	389	340	291	17.09
1800	22.83	410	360	310	17.17
1900	22.75	432	380	328	17.25
2000	22.68	453	400	347	17.32
2100	22.62	474	420	366	17.38
2200	22.56	496	440	384	17.44
2300	22.50	517	460	403	17.50
2400	22.45	538	480	422	17.55
2500	22.40	559	500	441	17.60
2600	22.35	581	520	459	17.65
2700	22.31	602	540	478	17.69
2800	22.27	623	560	497	17.73
2900	22.23	644	580	516	17.77
3000	22.19	665	600	535	17.81
3100	22.16	686	620	554	17.84
3200	22.12	707	640	573	17.88
3300	22.09	728	660	592	17.91
3400	22.06	749	680	611	17.94
3500	22.03	770	700	630	17.97
3600	22.00	791	720	649	18.00
3700	21.97	812	740	668	18.03
3800	21.95	833	760	687	18.05
3900	21.92	854	780	706	18.08
4000	21.90	875	800	725	18.10
4200	21.85	917	840	763	18.15
4400	21.81	959	880	801	18.19
4600	21.77	1001	920	839	18.23
4800	21.73	1043	960	877	18.27
5000	21.70	1084	1000	916	18.30
5500	21.62	1188	1100	1012	18.38
6000	21.55	1292	1200	1108	18.45
6500	21.49	1396	1300	1204	18.51
7000	21.43	1500	1400	1300	18.57
7500	21.39	1603	1500	1397	18.61
8000	21.34	1707	1600	1493	18.66
8500	21.30	1810	1700	1590	18.70
9000	21.26	1913	1800	1687	18.74
9500	21.23	2016	1900	1784	18.77
10000	21.20	2119	2000	1881	18.80

TABLE 9.7b continued

PROCESS LEVEL 25% DEFECTIVE

Number Inspected	UCL	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected	LCL
100	37.99	37	25	13	12.01
200	34.19	68	50	32	15.81
300	32.50	97	75	53	17.50
400	31.50	125	100	75	18.50
500	30.81	154	125	96	19.19
600	30.30	181	150	119	19.70
700	29.91	209	175	141	20.09
800	29.59	236	200	164	20.41
900	29.33	263	225	187	20.67
1000	29.11	291	250	209	20.89
1100	28.92	318	275	232	21.08
1200	28.75	344	300	256	21.25
1300	28.60	371	325	279	21.40
1400	28.47	398	350	302	21.53
1500	28.35	425	375	325	21.65
1600	28.25	451	400	349	21.75
1700	28.15	478	425	372	21.85
1800	28.06	505	450	395	21.94
1900	27.98	531	475	419	22.02
2000	27.90	558	500	442	22.10
2100	27.83	584	525	466	22.17
2200	27.77	610	550	490	22.23
2300	27.71	637	575	513	22.29
2400	27.65	663	600	537	22.35
2500	27.60	689	625	561	22.40
2600	27.55	716	650	584	22.45
2700	27.50	742	675	608	22.50
2800	27.45	768	700	632	22.55
2900	27.41	794	725	656	22.59
3000	27.37	821	750	679	22.63
3100	27.33	847	775	703	22.67
3200	27.30	873	800	727	22.70
3300	27.26	899	825	751	22.74
3400	27.23	925	850	775	22.77
3500	27.20	951	875	799	22.80
3600	27.17	977	900	823	22.83
3700	27.14	1004	925	846	22.86
3800	27.11	1030	950	870	22.89
3900	27.08	1056	975	894	22.92
4000	27.05	1082	1000	918	22.95
4200	27.00	1134	1050	966	23.00
4400	26.96	1186	1100	1014	23.04
4600	26.92	1238	1150	1062	23.08
4800	26.88	1290	1200	1110	23.12
5000	26.84	1341	1250	1159	23.16
5500	26.75	1471	1375	1279	23.25
6000	26.68	1600	1500	1400	23.32
6500	26.61	1729	1625	1521	23.39
7000	26.55	1858	1750	1642	23.45
7500	26.50	1987	1875	1763	23.50
8000	26.45	2116	2000	1884	23.55
8500	26.41	2244	2125	2006	23.59
9000	26.37	2373	2250	2127	23.63
9500	26.33	2501	2375	2249	23.67
10000	26.30	2629	2500	2371	23.70

TABLE 9.7b continued

PROCESS LEVEL 30% DEFECTIVE

Number Inspected	UCL	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected	LCL
100	43.75	43	30	17	16.25
200	39.72	79	60	41	20.28
300	37.94	113	90	67	22.06
400	36.87	147	120	93	23.13
500	36.15	180	150	120	23.85
600	35.61	213	180	147	24.31
700	35.20	246	210	174	24.80
800	34.86	278	240	202	25.14
900	34.58	311	270	229	25.42
1000	34.35	343	300	257	25.65
1100	34.15	375	330	285	25.85
1200	33.97	407	360	313	26.03
1300	33.81	439	390	341	26.19
1400	33.67	471	420	369	26.33
1500	33.55	503	450	397	26.45
1600	33.44	534	480	426	26.56
1700	33.33	566	510	454	26.67
1800	33.24	598	540	482	26.76
1900	33.15	629	570	511	26.85
2000	33.07	661	600	539	26.93
2100	33.00	692	630	568	27.00
2200	32.93	724	660	596	27.07
2300	32.87	755	690	625	27.13
2400	32.81	787	720	653	27.19
2500	32.75	818	750	682	27.25
2600	32.70	850	780	710	27.30
2700	32.65	881	810	739	27.35
2800	32.60	912	840	768	27.40
2900	32.55	944	870	796	27.45
3000	32.51	975	900	825	27.49
3100	32.47	1006	930	854	27.53
3200	32.43	1037	960	883	27.57
3300	32.39	1068	990	912	27.61
3400	32.36	1100	1020	940	27.64
3500	32.32	1131	1050	969	27.68
3600	32.29	1162	1080	998	27.71
3700	32.26	1193	1110	1027	27.74
3800	32.23	1224	1140	1056	27.77
3900	32.20	1255	1170	1085	27.80
4000	32.17	1286	1200	1114	27.83
4200	32.12	1349	1260	1171	27.88
4400	32.07	1411	1320	1229	27.93
4600	32.03	1473	1380	1287	27.97
4800	31.98	1535	1440	1345	28.02
5000	31.94	1597	1500	1403	28.06
5500	31.85	1751	1650	1549	28.15
6000	31.77	1906	1800	1694	28.23
6500	31.71	2060	1950	1840	28.29
7000	31.64	2215	2100	1985	28.36
7500	31.59	2369	2250	2131	28.41
8000	31.54	2522	2400	2278	28.46
8500	31.49	2676	2550	2424	28.51
9000	31.45	2830	2700	2570	28.55
9500	31.41	2983	2850	2717	28.59
10000	31.37	3137	3000	2863	28.63

TABLE 9.7b continued

PROCESS LEVEL 35% DEFECTIVE

Number Inspected	UCL	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected	LCL
100	49.31	49	35	21	20.69
200	45.12	90	70	50	24.88
300	43.26	129	105	81	26.74
400	42.15	168	140	112	27.85
500	41.40	206	175	144	28.60
600	40.84	245	210	175	29.16
700	40.41	282	245	208	29.59
800	40.06	320	280	240	29.94
900	39.77	357	315	273	30.23
1000	39.52	395	350	305	30.48
1100	39.31	432	385	338	30.69
1200	39.13	469	420	371	30.87
1300	38.97	506	455	404	31.03
1400	38.82	543	490	437	31.18
1500	38.69	580	525	470	31.31
1600	38.58	617	560	503	31.42
1700	38.47	654	595	536	31.53
1800	38.37	690	630	570	31.63
1900	38.28	727	665	603	31.72
2000	38.20	763	700	637	31.80
2100	38.12	800	735	670	31.88
2200	38.05	837	770	703	31.95
2300	37.98	873	805	737	32.02
2400	37.92	910	840	770	32.08
2500	37.86	946	875	804	32.14
2600	37.81	982	910	838	32.19
2700	37.75	1019	945	871	32.25
2800	37.70	1055	980	905	32.30
2900	37.66	1092	1015	938	32.34
3000	37.61	1128	1050	972	32.39
3100	37.57	1164	1085	1006	32.43
3200	37.53	1200	1120	1040	32.47
3300	37.49	1237	1155	1073	32.51
3400	37.45	1273	1190	1107	32.55
3500	37.42	1309	1225	1141	32.58
3600	37.38	1345	1260	1175	32.62
3700	37.35	1382	1295	1208	32.65
3800	37.32	1418	1330	1242	32.68
3900	37.29	1454	1365	1276	32.71
4000	37.26	1490	1400	1310	32.74
4200	37.21	1562	1470	1378	32.79
4400	37.16	1634	1540	1446	32.84
4600	37.11	1707	1610	1513	32.89
4800	37.07	1779	1680	1581	32.93
5000	37.02	1851	1750	1649	32.98
5500	36.93	2031	1925	1819	33.07
6000	36.85	2210	2100	1990	33.15
6500	36.77	2390	2275	2160	33.23
7000	36.71	2569	2450	2331	33.29
7500	36.65	2748	2625	2502	33.35
8000	36.60	2927	2800	2673	33.40
8500	36.55	3106	2975	2844	33.45
9000	36.51	3285	3150	3015	33.49
9500	36.47	3464	3325	3186	33.53
10000	36.43	3643	3500	3357	33.57

TABLE 9.7b continued

SECTION TWO

NUMBER INSPECTED 100

Process Level Percent Defective	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected
0.1	1	0.1	-
0.25	1	0.25	-
0.5	2	0.5	-
1.0	3	1	-
1.5	5	1.5	-
2	6	2	-
3	8	3	-
4	9	4	-
5	11	5	-
6	13	6	-
7	14	7	-
8	16	8	-
9	17	9	1
10	18	10	2
12	21	12	3
14	24	14	4
16	26	16	6
18	29	18	7
20	31	20	9
25	37	25	13
30	43	30	17
35	49	35	21

NUMBER INSPECTED 200

0.1	1	0.2	-
0.25	2	0.5	-
0.5	3	1	-
1.0	6	2	-
1.5	8	3	-
2	9	4	-
3	13	6	-
4	16	8	-
5	19	10	1
6	22	12	2
7	24	14	4
8	27	16	5
9	30	18	6
10	32	20	8
12	37	24	11
14	42	28	14
16	47	32	17
18	52	36	20
20	56	40	24
25	68	50	32
30	79	60	41
35	90	70	50

TABLE 9.7b continued

NUMBER INSPECTED 300

Process Level Percent Defective	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected
0.1	1	0.3	-
0.25	3	0.75	-
0.5	5	1.5	-
1.0	8	3	-
1.5	10	4.5	-
2	13	6	-
3	17	9	1
4	22	12	2
5	26	15	4
6	30	18	6
7	34	21	8
8	38	24	10
9	41	27	13
10	45	30	15
12	52	36	20
14	60	42	24
16	67	48	29
18	73	54	35
20	80	60	40
25	97	75	53
30	113	90	67
35	129	105	81

NUMBER INSPECTED 400

0.1	2	0.4	-
0.25	3	1	-
0.5	6	2	-
1.0	9	4	-
1.5	13	6	-
2	16	8	-
3	22	12	2
4	27	16	5
5	33	20	7
6	38	24	10
7	43	28	13
8	48	32	16
9	53	36	19
10	57	40	23
12	67	48	29
14	76	56	36
16	85	64	43
18	95	72	49
20	104	80	56
25	125	100	75
30	147	120	93
35	168	140	112

TABLE 9.7b continued

NUMBER INSPECTED 500

Process Level Percent Defective	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected
0.1	2	0.5	-
0.25	4	1.25	-
0.5	7	2.5	-
1.0	11	5	-
1.5	15	7.5	-
2	19	10	1
3	26	15	4
4	33	20	7
5	39	25	11
6	45	30	15
7	52	35	18
8	58	40	22
9	64	45	26
10	70	50	30
12	81	60	39
14	93	70	47
16	104	80	56
18	115	90	65
20	126	100	74
25	154	125	96
30	180	150	120
35	206	175	144

NUMBER INSPECTED 600

0.1	2	0.6	-
0.25	5	1.5	-
0.5	8	3	-
1.0	13	6	-
1.5	17	9	1
2	22	12	2
3	30	18	6
4	38	24	10
5	46	30	14
6	53	36	19
7	60	42	24
8	67	48	29
9	75	54	33
10	82	60	38
12	95	72	49
14	109	84	59
16	122	96	70
18	136	108	80
20	149	120	91
25	181	150	119
30	213	180	147
35	245	210	175

TABLE 9.7b continued

NUMBER INSPECTED 700

Process Level Percent Defective	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected
0.1	3	0.7	-
0.25	5	1.75	-
0.5	9	3.5	-
1.0	14	7	-
1.5	20	10.5	1
2	25	14	3
3	34	21	8
4	43	28	13
5	52	35	18
6	60	42	24
7	69	49	29
8	77	56	35
9	85	63	41
10	93	70	47
12	109	84	59
14	125	98	71
16	141	112	83
18	156	126	96
20	171	140	109
25	209	175	141
30	246	210	174
35	282	245	208

NUMBER INSPECTED 800

0.1	3	0.8	-
0.25	6	2	-
0.5	9	4	-
1.0	16	8	-
1.5	22	12	2
2	27	16	5
3	38	24	10
4	48	32	16
5	58	40	22
6	68	48	28
7	77	56	35
8	87	64	41
9	96	72	48
10	105	80	55
12	123	96	69
14	141	112	83
16	159	128	97
18	176	144	112
20	193	160	127
25	236	200	164
30	278	240	202
35	320	280	240

TABLE 9.7b continued

NUMBER INSPECTED 900

Process Level Percent Defective	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected
0.1	3	0.9	-
0.25	6	2.25	-
0.5	10	4.5	-
1.0	17	9	1
1.5	24	13.5	3
2	30	18	6
3	42	27	12
4	53	36	19
5	64	45	26
6	75	54	33
7	85	63	41
8	96	72	48
9	106	81	56
10	116	90	64
12	137	108	79
14	157	126	95
16	176	144	112
18	196	162	128
20	215	180	145
25	263	225	187
30	311	270	229
35	357	315	273

NUMBER INSPECTED 1000

0.1	3	1	-
0.25	7	2.5	-
0.5	11	5	-
1.0	19	10	1
1.5	26	15	4
2	33	20	7
3	46	30	14
4	58	40	22
5	70	50	30
6	82	60	38
7	94	70	46
8	105	80	55
9	117	90	63
10	128	100	72
12	151	120	89
14	172	140	108
16	194	160	126
18	216	180	144
20	237	200	163
25	291	250	209
30	343	300	257
35	395	350	305

TABLE 9.7b continued

NUMBER INSPECTED 1100

Process Level Percent Defective	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected
0.1	4	1.1	-
0.25	7	2.75	-
0.5	12	5.5	-
1.0	20	11	2
1.5	28	16.5	5
2	35	22	9
3	49	33	17
4	63	44	25
5	76	55	34
6	89	66	43
7	102	77	52
8	114	88	62
9	127	99	71
10	139	110	81
12	164	132	100
14	188	154	120
16	212	176	140
18	236	198	160
20	259	220	181
25	318	275	232
30	375	330	285
35	432	385	338

NUMBER INSPECTED 1200

0.1	4	1.2	-
0.25	8	3	-
0.5	13	6	-
1.0	22	12	2
1.5	30	18	6
2	38	24	10
3	53	36	19
4	68	48	28
5	82	60	38
6	96	72	48
7	110	84	58
8	124	96	68
9	137	108	79
10	151	120	89
12	177	144	111
14	204	168	132
16	230	192	154
18	255	216	177
20	281	240	199
25	344	300	256
30	407	360	313
35	469	420	371

TABLE 9.7b continued

NUMBER INSPECTED 1300

Process Level Percent Defective	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected
0.1	4	1.3	-
0.25	8	3.25	-
0.5	14	6.5	-
1.0	23	13	3
1.5	32	19.5	7
2	41	26	11
3	57	39	21
4	73	52	31
5	88	65	42
6	103	78	53
7	118	91	64
8	133	104	75
9	147	117	87
10	162	130	98
12	191	156	121
14	219	182	145
16	247	208	169
18	275	234	193
20	303	260	217
25	371	325	279
30	439	390	341
35	506	455	404

NUMBER INSPECTED 1400

0.1	4	1.4	-
0.25	9	3.5	-
0.5	14	7	-
1.0	25	14	3
1.5	34	21	8
2	43	28	13
3	61	42	23
4	77	56	35
5	94	70	46
6	110	84	58
7	126	98	70
8	142	112	82
9	158	126	94
10	173	140	107
12	204	168	132
14	234	196	158
16	265	224	183
18	295	252	209
20	324	280	236
25	398	350	302
30	471	420	369
35	543	490	437

TABLE 9.7b continued

NUMBER INSPECTED 1500

Process Level Percent Defective	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected
0.1	5	1.5	-
0.25	9	3.75	-
0.5	15	7.5	-
1.0	26	15	4
1.5	36	22.5	9
2	46	30	14
3	64	45	26
4	82	60	38
5	100	75	50
6	117	90	63
7	134	105	76
8	151	120	89
9	168	135	102
10	184	150	116
12	217	180	143
14	250	210	170
16	282	240	198
18	314	270	226
20	346	300	254
25	425	375	325
30	503	450	397
35	580	525	470

NUMBER INSPECTED 1600

0.1	5	1.6	-
0.25	9	4	-
0.5	16	8	-
1.0	27	16	5
1.5	38	24	10
2	48	32	16
3	68	48	28
4	87	64	41
5	106	80	54
6	124	96	68
7	142	112	82
8	160	128	96
9	178	144	110
10	195	160	125
12	230	192	154
14	265	224	183
16	299	256	213
18	334	288	242
20	367	320	273
25	451	400	349
30	534	480	426
35	617	560	503

TABLE 9.7b continued

NUMBER INSPECTED 1700

Process Level Percent Defective	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected
0.1	5	1.7	-
0.25	10	4.25	-
0.5	17	8.5	-
1.0	29	17	5
1.5	40	25.5	11
2	51	34	17
3	72	51	30
4	92	68	44
5	111	85	59
6	131	102	73
7	150	119	88
8	169	136	103
9	188	153	118
10	207	170	133
12	244	204	164
14	280	238	196
16	317	272	227
18	353	306	259
20	389	340	291
25	478	425	372
30	566	510	454
35	654	595	536

NUMBER INSPECTED 1800

0.1	5	1.8	-
0.25	10	4.5	-
0.5	17	9	1
1.0	30	18	6
1.5	42	27	12
2	53	36	19
3	75	54	33
4	96	72	48
5	117	90	63
6	138	108	78
7	158	126	94
8	178	144	110
9	198	162	126
10	218	180	142
12	257	216	175
14	296	252	208
16	334	288	242
18	372	324	276
20	410	360	310
25	505	450	395
30	598	540	482
35	690	630	

TABLE 9.7b continued

NUMBER INSPECTED 1900

Process Level Percent Defective	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected
0.1	6	1.9	-
0.25	11	4.75	-
0.5	18	9.5	1
1.0	32	19	6
1.5	44	28.5	13
2	56	38	20
3	79	57	35
4	101	76	51
5	123	95	67
6	145	114	83
7	166	133	100
8	187	152	117
9	208	171	134
10	229	190	151
12	270	228	186
14	311	266	221
16	351	304	257
18	392	342	292
20	432	380	328
25	531	475	419
30	629	570	511
35	727	665	603

NUMBER INSPECTED 2000

0.1	6	2	-
0.25	11	5	-
0.5	19	10	1
1.0	33	20	7
1.5	46	30	14
2	58	40	22
3	82	60	38
4	106	80	54
5	129	100	71
6	151	120	89
7	174	140	106
8	196	160	124
9	218	180	142
10	240	200	160
12	283	240	197
14	326	280	234
16	369	320	271
18	411	360	309
20	453	400	347
25	558	500	442
30	661	600	539
35	763	700	637

TABLE 9.7b continued

NUMBER INSPECTED 2100

Process Level Percent Defective	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected
0.1	6	2.1	-
0.25	12	5.25	-
0.5	20	10.5	1
1.0	34	21	7
1.5	48	31.5	15
2	61	42	23
3	86	63	40
4	110	84	58
5	134	105	76
6	158	126	94
7	182	147	112
8	205	168	131
9	228	189	150
10	251	210	169
12	296	252	208
14	341	294	247
16	386	336	286
18	430	378	326
20	474	420	366
25	584	525	466
30	692	630	568
35	800	735	670

NUMBER INSPECTED 2200

0.1	6	2.2	-
0.25	12	5.5	-
0.5	20	11	2
1.0	36	22	8
1.5	50	33	16
2	63	44	25
3	89	66	43
4	115	88	61
5	140	110	80
6	165	132	99
7	189	154	119
8	214	176	138
9	238	198	158
10	262	220	178
12	309	264	219
14	356	308	260
16	403	352	301
18	450	396	342
20	496	440	384
25	610	550	490
30	724	660	596
35	837		

TABLE 9.7b continued

NUMBER INSPECTED 2300

Process Level Percent Defective	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected
0.1	6	2.3	-
0.25	12	5.75	-
0.5	21	11.5	2
1.0	37	23	9
1.5	51	34.5	18
2	66	46	26
3	93	69	45
4	120	92	64
5	146	115	84
6	172	138	104
7	197	161	125
8	223	184	145
9	248	207	166
10	275	230	187
12	322	276	230
14	371	322	273
16	420	368	316
18	469	414	359
20	517	460	403
25	637	575	513
30	755	690	625
35	873	805	737

NUMBER INSPECTED 2400

0.1	7	2.4	-
0.25	13	6	-
0.5	22	12	2
1.0	38	24	10
1.5	53	36	19
2	68	48	28
3	97	72	47
4	124	96	68
5	152	120	88
6	178	144	110
7	205	168	131
8	231	192	153
9	258	216	174
10	284	240	196
12	335	288	241
14	386	336	286
16	437	384	331
18	488	432	376
20	538	480	422
25	663	600	537
30	787	720	653
35	910	840	770

TABLE 9.7b continued

NUMBER INSPECTED 2500

Process Level Percent Defective	Rejects Must Not Exceed	Average Number Inspected	Fewest Rejects Expected
0.1	7	2.5	-
0.25	13	6.25	-
0.5	23	12.5	2
1.0	39	25	11
1.5	55	37.5	20
2	70	50	30
3	100	75	50
4	129	100	71
5	157	125	93
6	185	150	115
7	213	175	137
8	240	200	160
9	267	225	183
10	294	250	206
12	348	300	252
14	402	350	298
16	454	400	346
18	507	450	393
20	559	500	441
25	689	625	561
30	818	750	682
35	946	875	804

NUMBER INSPECTED 2600

0.1	7	2.6	-
0.25	14	6.5	-
0.5	23	13	3
1.0	41	26	11
1.5	57	39	21
2	73	52	31
3	104	78	52
4	133	104	75
5	163	130	97
6	192	156	120
7	221	182	143
8	249	208	167
9	277	234	191
10	305	260	215
12	361	312	263
14	417	364	311
16	472	416	360
18	526	468	410
20	581	520	459
25	716	650	584
30	850	780	710
35	982	910	838

TABLE 9.7b continued

NUMBER INSPECTED 2700

Process Level Percent Defective	Rejects Must Not Exceed	Average Number Inspected	Fewest Rejects Expected
0.1	7	2.7	-
0.25	14	6.75	-
0.5	24	13.5	3
1.0	42	27	12
1.5	59	40.5	22
2	75	54	33
3	107	81	55
4	138	108	78
5	168	135	102
6	199	162	125
7	228	189	150
8	258	216	174
9	287	243	199
10	316	270	224
12	374	324	274
14	432	378	324
16	489	432	375
18	545	486	427
20	602	540	478
25	742	675	608
30	881	810	739
35	1019	945	871

NUMBER INSPECTED 2800

0.1	7	2.8	-
0.25	14	7	-
0.5	25	14	3
1.0	43	28	13
1.5	61	42	23
2	78	56	34
3	111	84	57
4	143	112	81
5	174	140	106
6	205	168	131
7	236	196	156
8	267	224	181
9	297	252	207
10	327	280	233
12	387	336	285
14	447	392	337
16	506	448	390
18	564	504	444
20	623	560	497
25	768	700	632
30	912	840	768
35	1055	980	905

TABLE 9.7b continued

NUMBER INSPECTED 2900

Process Level Percent Defective	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected
0.1	8	2.9	-
0.25	15	7.25	-
0.5	25	14.5	4
1.0	45	29	13
1.5	63	43.5	24
2	80	58	36
3	114	87	60
4	147	116	85
5	180	145	110
6	212	174	136
7	244	203	162
8	275	232	189
9	307	261	215
10	338	290	242
12	400	348	296
14	462	406	350
16	523	464	405
18	584	522	460
20	644	580	516
25	794	725	656
30	944	870	796
35	1092	1015	938

NUMBER INSPECTED 3000

0.1	8	3	-
0.25	15	7.5	-
0.5	26	15	4
1.0	46	30	14
1.5	64	45	26
2	82	60	38
3	118	90	62
4	152	120	88
5	185	150	115
6	219	180	141
7	251	210	169
8	284	240	196
9	317	270	223
10	349	300	251
12	413	360	307
14	477	420	363
16	540	480	420
18	603	540	477
20	665	600	535
25	821	750	679
30	975	900	825
35	1128	1050	

TABLE 9.7b continued

NUMBER INSPECTED 3100

Process Level Percent Defective	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected
0.1	8	3.1	-
0.25	16	7.75	-
0.5	27	15.5	4
1.0	47	31	15
1.5	66	46.5	27
2	85	62	39
3	121	93	65
4	156	124	92
5	191	155	119
6	225	186	147
7	259	217	175
8	293	248	203
9	326	279	232
10	360	310	260
12	426	372	318
14	491	434	377
16	557	496	435
18	622	558	494
20	686	620	554
25	847	775	703
30	1006	930	854
35	1164	1085	1006

NUMBER INSPECTED 3200

0.1	8	3.2	-
0.25	16	8	-
0.5	27	16	5
1.0	48	32	16
1.5	68	48	28
2	87	64	41
3	124	96	68
4	161	128	95
5	196	160	124
6	232	192	152
7	267	224	181
8	302	256	210
9	336	288	240
10	370	320	270
12	439	384	329
14	506	448	390
16	574	512	450
18	641	576	511
20	707	640	573
25	873	800	727
30	1037	960	883
35	1200	1120	1040

TABLE 9.7b continued

NUMBER INSPECTED 3300

Process Level Percent Defective	Rejects Must Not Exceed	Average Number Inspected	Fewest Rejects Expected
0.1	8	3.3	-
0.25	16	8.25	-
0.5	28	16.5	5
1.0	50	33	16
1.5	70	49.5	29
2	90	66	42
3	128	99	70
4	165	132	99
5	202	165	128
6	238	198	158
7	274	231	188
8	310	264	218
9	346	297	248
10	381	330	279
12	452	396	340
14	521	462	403
16	591	528	465
18	660	594	528
20	728	660	592
25	899	825	751
30	1068	990	912
35	1237	1155	1073

NUMBER INSPECTED 3400

0.1	8	3.4	-
0.25	17	8.5	-
0.5	29	17	5
1.0	51	34	17
1.5	72	51	30
2	92	68	44
3	131	102	73
4	170	136	102
5	208	170	132
6	245	204	163
7	282	238	194
8	319	272	225
9	356	306	256
10	392	340	288
12	464	408	352
14	536	476	416
16	608	544	480
18	679	612	545
20	749	680	611
25	925	850	775
30	1068	990	912
35	1273	1190	1107

TABLE 9.7b continued

NUMBER INSPECTED 3500

Process Level Percent Defective	Rejects Must Not Exceed	Average Number Inspected	Fewest Rejects Expected
0.1	9	3.5	-
0.25	17	8.75	-
0.5	30	17.5	5
1.0	52	35	18
1.5	74	52.5	31
2	94	70	46
3	135	105	75
4	174	140	106
5	213	175	137
6	252	210	168
7	290	245	200
8	328	280	232
9	365	315	265
10	403	350	297
12	477	420	363
14	551	490	429
16	625	560	495
18	698	630	562
20	770	700	630
25	951	875	799
30	1131	1050	969
35	1309	1225	1141

NUMBER INSPECTED 3600

0.1	9	3.6	-
0.25	17	9	1
0.5	30	18	6
1.0	53	36	19
1.5	75	54	33
2	97	72	47
3	138	108	78
4	179	144	109
5	219	180	141
6	258	216	174
7	297	252	207
8	336	288	240
9	375	324	273
10	413	360	307
12	490	432	374
14	566	504	442
16	641	576	511
18	717	648	579
20	791	720	649
25	977	900	823
30	1162	1080	998
35	1345	1260	1175

TABLE 9.7b continued

NUMBER INSPECTED 3700

Process Level Percent Defective	Rejects Must Not Exceed	Average Number Inspected	Fewest Rejects Expected
0.1	9	3.7	-
0.25	18	9.25	1
0.5	31	18.5	6
1.0	55	37	19
1.5	77	55.5	34
2	99	74	49
3	142	111	80
4	183	148	113
5	224	185	146
6	265	222	179
7	305	259	213
8	345	296	247
9	385	333	281
10	424	370	316
12	503	444	385
14	581	518	455
16	658	592	526
18	736	666	596
20	812	740	668
25	1004	925	846
30	1193	1110	1027
35	1382	1295	1208

NUMBER INSPECTED 3800

0.1	9	3.8	-
0.25	18	9.5	1
0.5	32	19	6
1.0	56	38	20
1.5	79	57	35
2	101	76	51
3	145	114	83
4	188	152	116
5	230	190	150
6	271	228	185
7	313	266	219
8	354	304	254
9	394	342	290
10	435	380	325
12	516	456	396
14	596	532	468
16	675	608	541
18	755	684	613
20	833	760	687
25	1030	950	870
30	1224	1140	1056
35	1418	1330	1242

TABLE 9.7b continued

NUMBER INSPECTED 3900

Process Level Percent Defective	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected
0.1	9	3.9	-
0.25	19	9.75	1
0.5	32	19.5	7
1.0	57	39	21
1.5	81	58.5	36
2	104	78	52
3	148	117	86
4	192	156	120
5	235	195	155
6	278	234	190
7	320	273	226
8	362	312	262
9	404	351	298
10	446	390	334
12	528	468	408
14	611	546	481
16	692	624	556
18	773	702	631
20	854	780	706
25	1056	975	894
30	1255	1170	1085
35	1454	1365	1276

NUMBER INSPECTED 4000

0.1	9	4	
0.25	19	10	1
0.5	33	20	7
1.0	58	40	22
1.5	83	60	37
2	106	80	54
3	152	120	88
4	197	160	123
5	241	200	159
6	285	240	195
7	328	280	232
8	371	320	269
9	414	360	306
10	456	400	344
12	541	480	419
14	625	560	495
16	709	640	571
18	792	720	648
20	875	800	725
25	1082	1000	918
30	1286	1200	1114
35	1490	1400	1310

TABLE 9.7b continued

NUMBER INSPECTED 4200

Process Level Percent Defective	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected
0.1	10	4.2	-
0.25	20	10.5	1
0.5	34	21	8
1.0	61	42	23
1.5	86	63	40
2	111	84	57
3	159	126	93
4	206	168	130
5	252	210	168
6	298	252	206
7	343	294	245
8	388	336	284
9	433	378	323
10	478	420	362
12	567	504	441
14	655	588	521
16	743	672	601
18	830	756	682
20	917	840	763
25	1134	1050	966
30	1349	1260	1171
35	1562	1470	1378

NUMBER INSPECTED 4400

0.1	10	4.4	-
0.25	20	11	2
0.5	36	22	8
1.0	63	44	25
1.5	90	66	42
2	115	88	61
3	165	132	99
4	214	176	138
5	263	220	177
6	311	264	217
7	358	308	258
8	405	352	299
9	452	396	340
10	499	440	381
12	592	528	464
14	685	616	547
16	776	704	632
18	868	792	716
20	959	880	801
25	1186	1100	1014
30	1411	1320	1229
35	1634	1540	1446

TABLE 9.7b continued

NUMBER INSPECTED 4600

Process Level Percent Defective	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected
0.1	11	4.6	-
0.25	21	11.5	2
0.5	37	23	9
1.0	66	46	26
1.5	93	69	45
2	120	92	64
3	172	138	104
4	223	184	145
5	274	230	186
6	324	276	228
7	373	322	271
8	423	368	313
9	472	414	356
10	521	460	399
12	618	552	486
14	714	644	574
16	810	736	662
18	906	828	750
20	1001	920	839
25	1238	1150	1062
30	1473	1380	1287
35	1707	1610	1515

NUMBER INSPECTED 4800

0.1	11	4.8	-
0.25	22	12	2
0.5	38	24	10
1.0	68	48	28
1.5	97	72	47
2	125	96	67
3	179	144	109
4	232	192	152
5	285	240	195
6	337	288	239
7	389	336	283
8	440	384	328
9	491	432	373
10	542	480	418
12	643	576	509
14	744	672	600
16	844	768	692
18	943	864	785
20	1043	960	877
25	1290	1200	1110
30	1535	1440	1345
35	1779	1680	

TABLE 9.7b continued

NUMBER INSPECTED 5000

Process Level Percent Defective	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected
0.1	11	5.0	-
0.25	23	12.5	2
0.5	39	25	11
1.0	71	50	29
1.5	100	75	50
2	129	100	71
3	186	150	114
4	241	200	159
5	296	250	204
6	350	300	250
7	404	350	296
8	457	400	343
9	510	450	390
10	563	500	437
12	668	600	532
14	773	700	627
16	877	800	723
18	981	900	819
20	1084	1000	916
25	1341	1250	1159
30	1597	1500	1403
35	1851	1750	1649

NUMBER INSPECTED 5500

0.1	12	5.5	-
0.25	24	13.75	3
0.5	43	27.5	12
1.0	77	55	33
1.5	109	82.5	56
2	141	110	79
3	202	165	128
4	263	220	177
5	323	275	227
6	382	330	278
7	441	385	329
8	500	440	380
9	558	495	432
10	616	550	484
12	732	660	588
14	847	770	693
16	961	880	799
18	1075	990	905
20	1188	1100	1012
25	1471	1375	1279
30	1751	1650	1549
35	2031	1925	1819

TABLE 9.7b continued

NUMBER INSPECTED 6000

Process Level Percent Defective	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected
0.1	13	6	-
0.25	26	15	4
0.5	46	30	14
1.0	83	60	37
1.5	118	90	62
2	152	120	88
3	219	180	141
4	285	240	195
5	350	300	250
6	415	360	305
7	479	420	361
8	543	480	417
9	606	540	474
10	669	600	531
12	795	720	645
14	920	840	760
16	1045	960	875
18	1169	1080	991
20	1292	1200	1108
25	1600	1500	1400
30	1906	1800	1694
35	2210	2100	1990

NUMBER INSPECTED 6500

0.1	14	6.5	-
0.25	28	16.25	5
0.5	49	32.5	16
1.0	89	65	41
1.5	126	97.5	69
2	163	130	97
3	236	195	154
4	307	260	213
5	377	325	273
6	447	390	333
7	516	455	394
8	585	520	455
9	654	585	516
10	722	650	578
12	858	780	702
14	993	910	827
16	1128	1040	952
18	1262	1170	1078
20	1396	1300	1204
25	1729	1625	1521
30	2060	1950	1840
35	2390	2275	2160

TABLE 9.7b continued

NUMBER INSPECTED 7000

Process Level Percent Defective	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected
0.1	14	7	-
0.25	30	17.5	5
0.5	52	35	18
1.0	94	70	46
1.5	135	105	75
2	175	140	105
3	252	210	168
4	329	280	231
5	404	350	296
6	479	420	361
7	554	490	426
8	628	560	492
9	701	630	559
10	775	700	625
12	921	840	759
14	1067	980	893
16	1212	1120	1028
18	1356	1260	1164
20	1500	1400	1300
25	1858	1750	1642
30	2215	2100	1985
35	2569	2450	2331

NUMBER INSPECTED 7500

0.1	15	7.5	-
0.25	31	18.75	6
0.5	55	37.5	20
1.0	100	75	50
1.5	144	112.5	81
2	186	150	114
3	269	225	181
4	350	300	250
5	431	375	319
6	511	450	389
7	591	525	459
8	670	600	530
9	749	675	601
10	827	750	673
12	984	900	816
14	1140	1050	960
16	1295	1200	1105
18	1449	1350	1251
20	1603	1500	1397
25	1987	1875	1763
30	2369	2250	2131
35	2748	2625	2502

TABLE 9.7b continued

NUMBER INSPECTED 8000

Process Level Percent Defective	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected
0.1	16	8	-
0.25	33	20	7
0.5	58	40	22
1.0	106	80	54
1.5	152	120	88
2	197	160	123
3	285	240	195
4	372	320	268
5	458	400	342
6	543	480	417
7	628	560	492
8	712	640	568
9	796	720	644
10	880	800	720
12	1047	960	873
14	1213	1120	1027
16	1378	1280	1182
18	1543	1440	1337
20	1707	1600	1493
25	2116	2000	1884
30	2522	2400	2278
35	2927	2800	2673

NUMBER INSPECTED 8500

0.1	17	8.5	-
0.25	35	21.25	8
0.5	62	42.5	23
1.0	112	85	58
1.5	161	127.5	94
2	208	170	132
3	302	255	208
4	394	340	286
5	485	425	365
6	575	510	445
7	665	595	525
8	755	680	605
9	844	765	686
10	932	850	768
12	1109	1020	931
14	1285	1190	1095
16	1461	1360	1259
18	1636	1530	1424
20	1810	1700	1590
25	2244	2125	2006
30	2676	2550	2424
35	3106	2975	2844

TABLE 9.7b continued

NUMBER INSPECTED 9000

Process Level Percent Defective	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected
0.1	17	9	1
0.25	36	22.5	9
0.5	65	45	25
1.0	118	90	62
1.5	169	135	101
2	219	180	141
3	318	270	222
4	415	360	305
5	512	450	388
6	607	540	473
7	702	630	558
8	797	720	643
9	891	810	729
10	985	900	815
12	1172	1080	988
14	1358	1260	1162
16	1544	1440	1336
18	1729	1620	1511
20	1913	1800	1687
25	2373	2250	2127
30	2830	2700	2570
35	3285	3150	3015

NUMBER INSPECTED 9500

0.1	18	9.5	1
0.25	38	23.75	10
0.5	68	47.5	27
1.0	124	95	66
1.5	178	142.5	107
2	230	190	150
3	334	285	236
4	437	380	323
5	538	475	412
6	639	570	501
7	739	665	591
8	839	760	681
9	938	855	772
10	1037	950	863
12	1235	1140	1045
14	1431	1330	1229
16	1627	1520	1413
18	1822	1710	1598
20	2016	1900	1784
25	2501	2375	2249
30	2983	2850	2717
35	3464	3325	3186

TABLE 9.7b continued

NUMBER INSPECTED 10000

Process Level Percent Defective	Rejects Must Not Exceed	Average Number Expected	Fewest Rejects Expected
0.1	19	10	1
0.25	39	25	11
0.5	71	50	29
1.0	129	100	71
1.5	186	150	114
2	241	200	159
3	351	300	249
4	458	400	342
5	565	500	435
6	671	600	529
7	776	700	624
8	881	800	719
9	985	900	815
10	1090	1000	910
12	1297	1200	1103
14	1504	1400	1296
16	1709	1600	1491
18	1915	1800	1685
20	2119	2000	1881
25	2629	2500	2371
30	3137	3000	2863
35	3643	3500	3357

nificant. The following table gives the minimum sample size necessary for zero defectives to be significant at the various process levels:

Process Average Percent Defective	Minimum Sample Size for Zero Defectives to be Significant
0.1	8991
0.25	3591
0.5	1791
1.0	892
1.5	591
2	441
3	291
4	217
5	172
6	141
7	120
8	104

Process Average Percent Defective	Minimum Sample Size for Zero Defectives to be Significant
9	92
10	81
12	67
14	56
16	48
18	41
20	36
25	27
30	21
35	17
40	14
45	11
50	9
60	6

Figs. 9.7a and 9.7b may also be used to determine the limits within which a process level probably lies when all that is known is the number of defectives in the sample. For example, suppose there are ten defectives in a sample of 200 items. The sample is five per cent defective. Lacking any other information, this is the best guess as to where the true process level lies. To determine how much higher than this it may be, we consult Fig. 9.7b and locate the intersection of ten defectives and a sample size of 200. The process level curve at this point is approximately 12 per cent. From Fig. 9.7a we read a process level curve of two per cent. We can then say that the process level is probably not higher than 12 per cent nor lower than two per cent. We cannot attach specific probabilities to these estimates as there is no known way to predict process levels exactly from sample values; however we can be highly confident that the true process level will not be beyond the limits as determined above.

Item 3. An inspection reporting procedure using the Quality Control Tables (*Table 9.7b*). Where there are many quality characteristics to be kept under control and the plotting of control charts is not practicable, a daily inspection report that shows which items are out of control can be helpful. Such a report form is shown in Fig. 9.7c.

The column headed "Major Defects Found" permits the listing of reasons for rejection such as oversize, undersize, burrs, etc. The column headed "% Rejs." may be omitted if desired as an indication of lack of control can be determined from the columns headed "Total Insp." and "Total Rejs." The Quality Control Tables for Number of Defectives are

DAILY PARTS REPORT

Date.

Part Name	Major Defects Found	Total Insp.	Total Rejs.	% Rejs.	Out of Limits
Part A.....		3443	1248	36.2	
Part B.....		1712	315	18.4	
Part C.....		2632	14	0.5	
Part D.....		1049	29	2.8	
Part E...		2216	65	2.9	
Part F.....		1571	46	2.9	
Part G.....		4877	10	0.2	
Part H.....		3157	39	1.2	
Part I.....		6782	58	0.9	
Part J.....		3537	53	1.5	
Part K.....		3905	102	2.6	
Part L.....		3578	31	0.9	
Part M.....		2215	2	0.1	
Part N.....		2292	50	2.2	
Part O.....		725	11	1.5	
Part P.....		4805	32	0.6	
Part Q.....		2163	41	1.9	

Fig. 9.7c.

used as explained in Item 2 above and an X placed in the column "Out of Limits" when rejects are excessive. This will immediately focus attention

on those items where the prospects for improvement of quality are most promising. These parts should be promptly investigated.

Some so-called "practical men" say that they know when they are in trouble. What they want to know is what is causing the trouble. Unfortunately, they actually know they are in trouble only when the trouble is exceedingly serious. Often when the lack of control is only slight, they will not be aware of it, although it would virtually always be economical for them to do something about the matter. As an illustration of the inability of the unaided mind to recognize all indications of lack of control, it is suggested that you make a list of those parts in Fig. 9.7c which you believe are out of control on the high side. To aid you, Fig. 9.7d gives the process quality levels as established during the previous month. Do not refer to the tables in Item 2 (Table 9.7b) above in making up your list. Make your selections before reading further.

Part Name	% Defective	Part Name	% Defective
Part A	4	Part J	2
Part B	4	Part K	3
Part C	1.5	Part L	1
Part D	1	Part M	3
Part E	2	Part N	0.5
Part F	2	Part O	5
Part G	0.25	Part P	1
Part H	0.5	Part Q	1.5
Part I	0.5		

Fig. 9.7d.

If your selection has been correct, your list will include Parts A, B, D, E, H, I, and N. Referring to the tables in Item 2 (Table 9.7b) above we find the maximum allowable defects for the respective parts is 170, 92, 57, 19, 63, 48, 22, 27, 56, 94, 148, 53, 89, 21, 19, 68, and 50. Some of the decisions should have been fairly easy, but unless you used statistical methods to arrive at your answers (which you were not supposed to do) you probably made some mistakes.

CHAPTER X

THE QUALITY CONTROL CHART FOR DEFECTS PER UNIT

10.1. The nature of the inspection situation. In the last chapter we distinguished between the terms "defective" and "defect," and discussed defectives. In this chapter we shall consider defects.

It will be recalled that one defective may contain more than one defect. Fig. 10.1 illustrates this situation. Of the five items inspected, three are defective. They contain a total of ten defects or an average of two defects per item. The inspection may be for such things as surface defects per unit area, the number of flaws per unit area of cloth, or insulation defects per unit length of wire. The sample may consist of five square yards, 1000 feet of length, a sample of five test specimens, or some other specific quantity. The defects to be counted may be seams in steel, slivers, scratches, pinholes, surface irregularities, etc.

Fig. 10.1. Comparison between defects and defectives.

It is obvious that to rate articles as merely satisfactory or defective under this sort of inspection would be inadequate. It is desirable in this situation to take into account how bad each defective item is. In order to do this, we conceive of the item as being capable of containing defects in many places (actually this must not be less than ten). Usually it will be an indefinitely large number, the only limiting factor being the size of the defect in relation to the size of the sample piece. Each such place will be big enough to contain only one defect and so will be OK or defective. If p represents the portion of the places defective on one item or unit (this must not exceed 0.10) and N the number of places, then pN, or as it is more commonly designated, c, will represent the number of defects on the unit.

10.2. Determining the control chart limits. It will be recalled that the control chart limits for $\bar{p}N$ were determined from the formula $\bar{p}N \pm 3\sqrt{\bar{p}N\,(1 - \bar{p})}$. In dealing with defects per unit, $\bar{p}$ must not exceed 0.10 and it will usually be a great deal smaller. We may then drop the quantity $1 - \bar{p}$ from the formula (since it will be very close to unity) and substitute $\bar{c}$ for $\bar{p}N$ giving

$$\bar{c} \pm 3\sqrt{\bar{c}}$$

If it can be conveniently arranged, it may be desirable to use a unit that gives a $\bar{c}$ of 15 or larger in order to have a lower limit on the control chart. Without this limit, there can be no indications of significantly better than average quality. Note that there is no lower limit using the above formula until $\bar{c}$ exceeds nine ($9 \pm 3\sqrt{9}$, UCL = 18, LCL = 0).

It is not necessary to compute separate control limits for each type of defect unless so desired. In cases where there are many closely related types of defects this might be impractical.

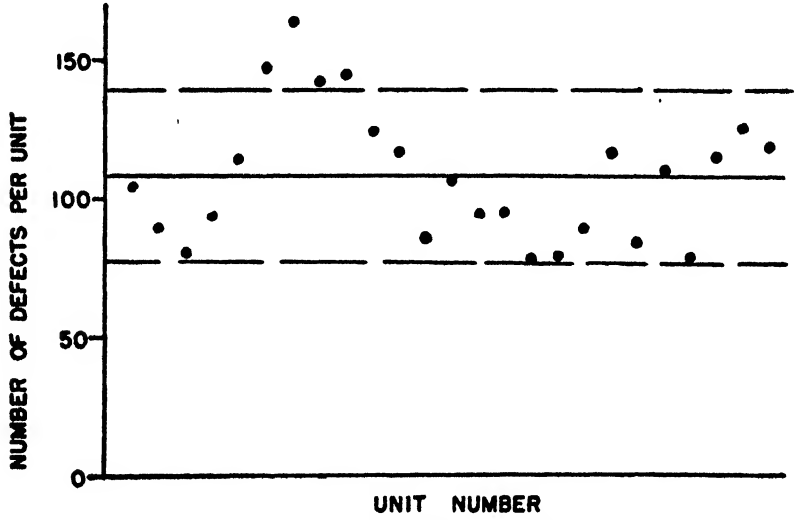

Fig. 10.2. Control chart for number of defects per unit. The lack of control beginning with the sixth unit resulted from putting a new operator on the job. There were 2715 defects in the 25 units inspected giving a $\bar{c}$ of 108.6. Upper and lower control limits are located at 139.8 and 77.4 respectively.

Any change in unit size will result in changing $\bar{c}$ as well as the control limits. For this reason the sample size should be kept constant if possible. Where not possible, a new $\bar{c}$ must be computed and plotted in addition to new control limits.

If several units are grouped together, either of two procedures may be used: (1) add all the defects together for all the units in the group and treat the group as one unit, or (2) determine the average number of defects per unit within each group of units and use the formula

$$\bar{\bar{c}} \pm \sqrt{\frac{\bar{\bar{c}}}{N}}$$

where N is the number of units per group of units and $\bar{\bar{c}}$ is the average of all the group averages. Fig. 10.2 shows a control chart for defects per unit.

CHAPTER XI

MODIFIED CONTROL CHART LIMITS

11.1. Full statistical control not always required. There is a common saying that it is possible to get too much of a good thing. This is just another way of expressing the law of diminishing returns. It is certainly uneconomical to give the consumer a degree of product quality uniformity that he does not need or want when increased uniformity raises production costs.

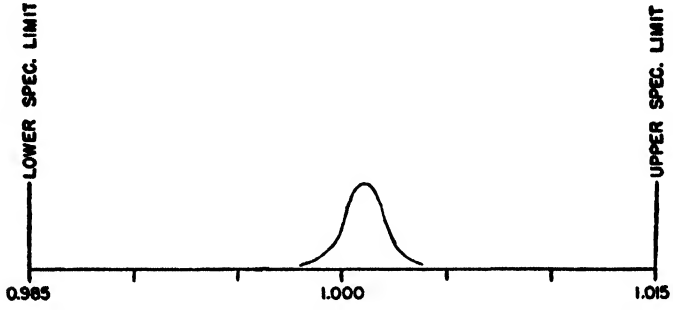

Fig. 11.1. A process that is much more uniform than needed to meet the specifications.

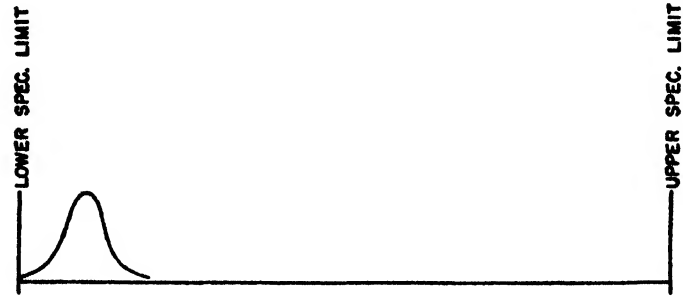

Fig. 11.2a. Process must not be permitted to shift any lower than shown.

Suppose the customer has specified a dimension of 1.000″ plus or minus 0.015″and an investigation of the production process shows that a standard deviation of individual values of 0.001″ and a mean of 1.001″ is being produced. This situation is shown in Fig. 11.1. While an essentially normal distribution is assumed in this discussion, the principles involved would apply equally well to a markedly nonnormal distribution if proper allowance for the nonnormality is made.

First consideration should be given to the use of less uniform material (provided that it is cheaper) or to the use of less expensive equipment or

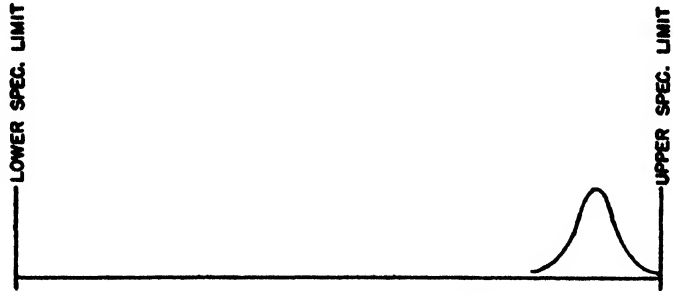

Fig. 11.2b. Process must not be permitted to shift any higher than shown.

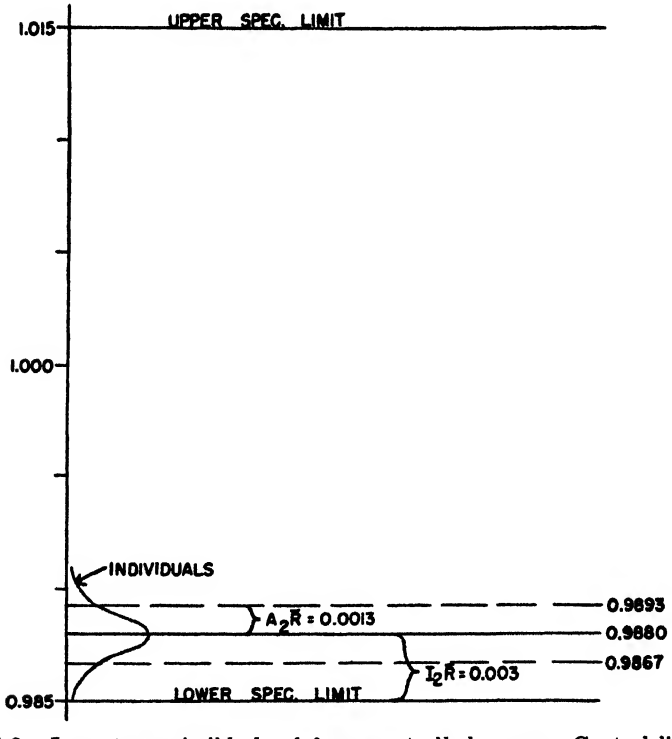

Fig. 11.3a. Lowest permissible level for a controlled process. Control limits are for averages of samples of five items each.

processing. If there is no practicable solution of this sort possible, we then consider what degree of control should be required. It is obvious that we do not need to require maintenance of the existing quality level as shown

in Fig. 11.1. Considerable shifting in the quality level may occur without the danger of parts being produced outside the specification limits.

11.2. Limits of quality level shifts. The problem with which we are now confronted is how closely the quality level may be permitted to approach either specification limit. Figs. 11.2a and 11.2b indicate the permissible extremes if defectives are to be kept to a minimum. At these extremes it is obvious that a state of statistical control must be maintained.

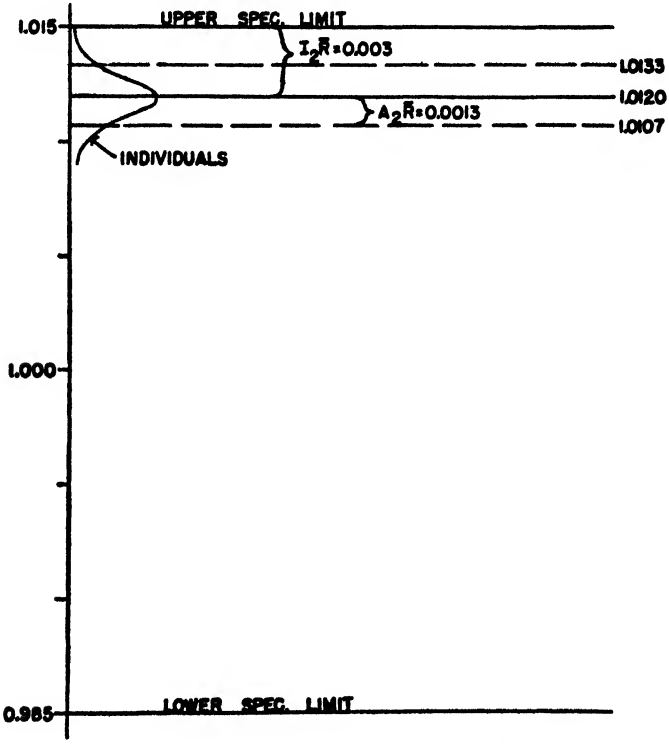

Fig. 11.3b. Highest permissible level for a controlled process. Control limits are for averages of samples of five items each.

A marked shift of level in the wrong direction would quickly result in defective material.

11.3. Adaptation to the quality control chart. Thus far we have been discussing the distribution of individual values. Quality control charts for variables usually are based on sample averages and ranges and the standard deviation of individuals is not computed. The question then arises as to how far in from the specification limits the process average must be in terms of individual values. This may be determined by using Table 6.6.

The factor (I_2) in the Table when multiplied by the average range ($\bar{R}$) will give three standard deviations of individual values. This product will be the distance in from specification limits that the process level must be to avoid any appreciable quantity of defective material.

In the illustration above, the observed standard deviation of individual values was found to be 0.001. This corresponds to an expected average range of 0.0023 for samples of five ($\bar{R} = d_2\sigma$). We will assume this to be

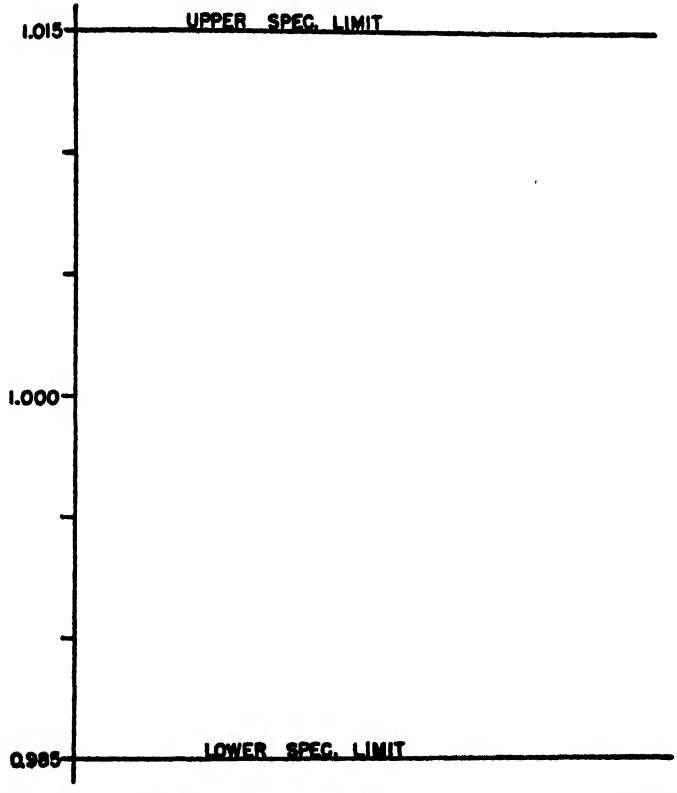

Fig. 11.4a. Average of a sample of five items from an uncontrolled process. Average is 1.002.

the observed value of $\bar{R}$ and the only estimate of variability available. From Table 6.6 we have 1.289 for I_2 which when multiplied by 0.0023 gives 0.003 (which we already know to be correct). Figs. 11.3a and 11.3b illustrate this situation (the scale is now vertical as compared to the horizontal scale of Figs. 11.2a and 11.2b and control chart limits have been added).

Sample averages are permitted to vary between 0.9867 and 1.0133, a total spread of 0.0266 as compared to the spread of 0.0026 at a fixed process

level; however, there is a limitation that must be observed. At the extreme process levels shown in Figs. 11.3a and 11.3b, control of both process level and inherent variability must be maintained. As soon as lack of control develops, there is the immediate prospect of substantial quantities of defective material. The process level and variability must be stable or additional limitations imposed as described in the next section.

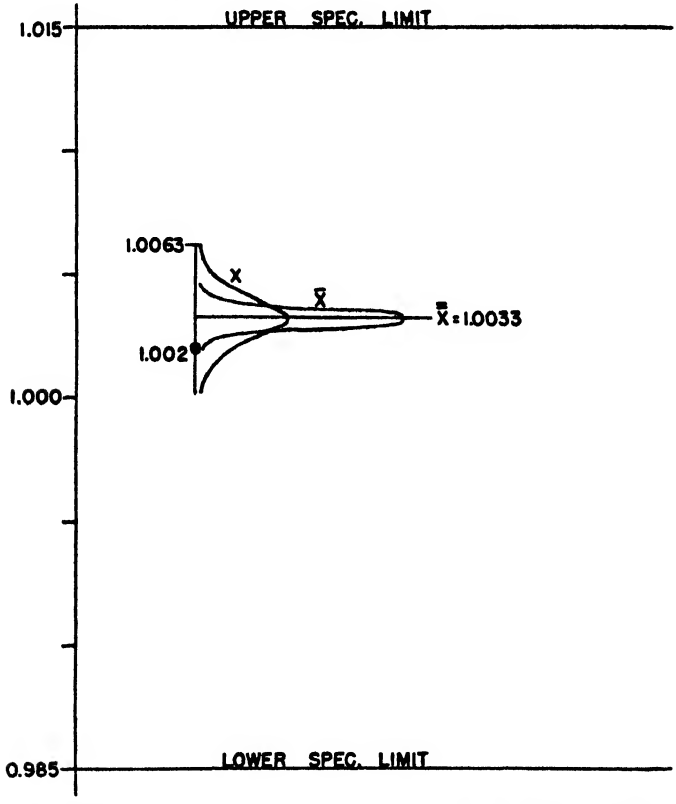

Fig. 11.4b. Upper confidence limit for process level ($\overline{\overline{X}}$) based on a sample average from an uncontrolled process. Curves are shown for distribution of individuals (X) and sample averages ($\overline{X}$) if process level is actually at 1.0033.

11.4. Modified limits for unstable processes. Some processes are normally very difficult to bring under statistical quality control. In such cases it will be neither necessary nor economical to do so provided two conditions are met:

1. The inherent variability must remain essentially the same regardless of the process level (*i.e.*, the ranges must show statistical control even though the averages do not).

2. The distance between the specification limits must be materially greater than six times the standard deviation of individuals (inherent variability of the process).

Condition 2 is nicely met by the illustration in Figs. 11.3a and 11.3b. In this case the distance between the specification limits is 30 times the standard deviation of the process inherent variability. Let us assume for

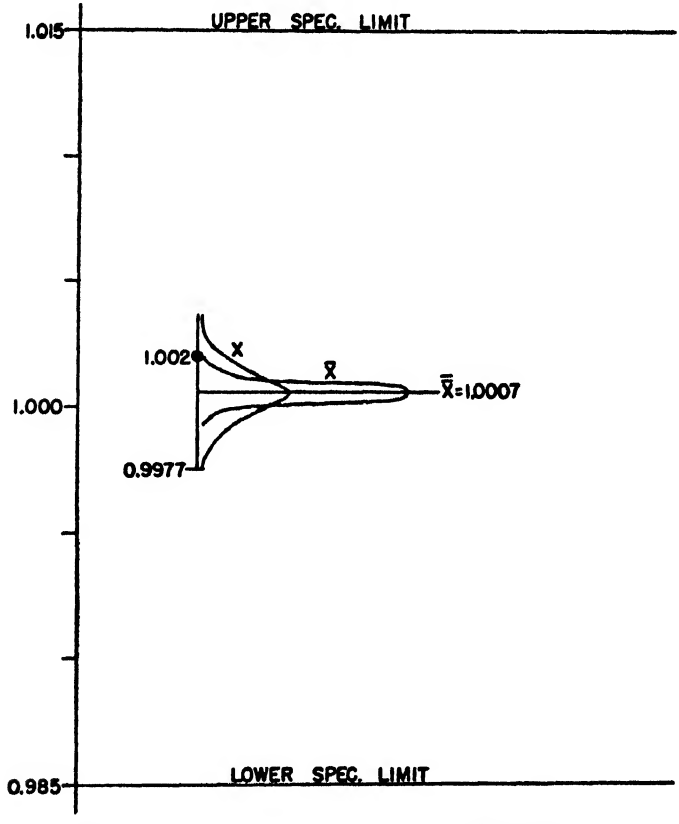

Fig. 11.4c. Lower confidence limit for process level ($\bar{X}$) based on a sample average from an uncontrolled process. Curves are shown for distribution of individuals (X) and sample averages ($\bar{X}$) if process level is actually at 1.0007.

purposes of illustration that the ranges show statistical control. If the process level tends to shift often and unexpectedly, all we can say about the process level at any given time is that it is probably somewhere near the most recently observed measurements. In Fig. 11.4a the plotted point represents the average of the most recently observed group of five items. Since the prior group averages are of little or no help in deciding where the

process level is located at the present time, we must rely on this one average only. Fig. 11.4b represents one possible extreme. If the observed average is three sigmas below the present process level (it is very unlikely that it will ever be more than this), then the highest individual value likely to occur if many measurements were made at this instant would be at three sigmas of averages (this brings us up to the process level) plus three sigmas

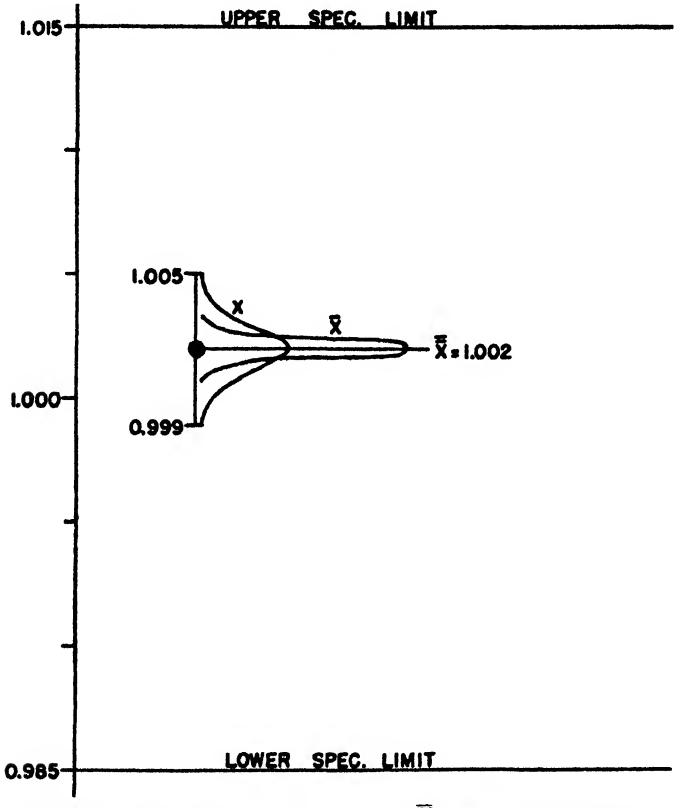

Fig. 11.4d. Most likely value of process level ($\bar{\bar{X}}$) based on a sample average from an uncontrolled process. Curves are shown for distribution of individuals (X) and sample averages ($\bar{X}$) if process level is actually at 1.002.

of individuals above the observed average. This value would be 1.002 + 0.0013 + 0.003 = 1.0063, which is well inside the specification limit of 1.015.

Similarly, Fig. 11.4c represents the opposite extreme. The lowest individual value likely to occur if many measurements were made at this point would be at 1.002 − 0.0013 − 0.003 = 0.9977, which is well inside the specification limit of 0.985.

Fig. 11.4d represents the most likely location of the process distribution

since the average of the observed values is always the best estimate of the process mean.

If virtually no values outside the specification limits can be tolerated, the modified control limits will be placed at

$$\text{Upper specification limit} - 3\sigma_{\bar{x}} - 3\sigma_{\bar{x}}, \text{ and}$$
$$\text{Lower specification limit} + 3\sigma_{\bar{x}} + 3\sigma_{\bar{x}}.$$

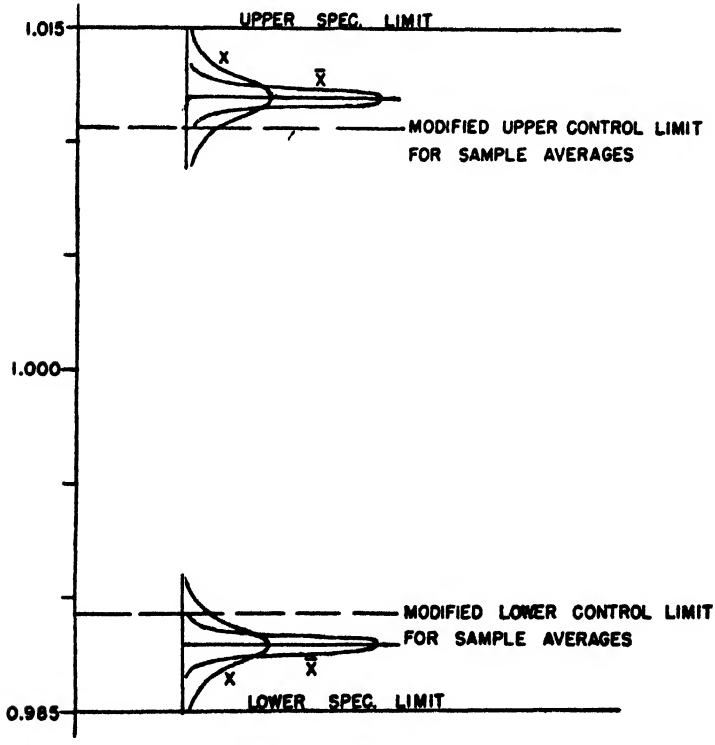

Fig. 11.4e. Modified control limits for an unstable process. The distribution curves are the worst situations expected to prevail when an average of a group of five falls on the modified limit.

This is illustrated in Fig. 11.4e. For the case we have been discussing these limits will be at $1.015 - 0.003 - 0.0013 = 1.0107$, and $0.985 + 0.003 + 0.0013 = 0.9893$. The difference between these two limits is 0.0214, or more than 16 times the standard deviation of group averages.

If some small risk of getting a defective item can be tolerated, the modified limits may be permitted to approach the specification limits more closely. Unfortunately, there is no way of predicting for an unstable process how far above or below the sample average the process average will likely

be at any given time. It is suggested that the modified limits may be set as shown below with a risk (usually) of getting a defective item no greater than about one in 100 to 200 items:

$$\text{Upper specification limit} - 1.2\sigma_x - 3\sigma_z, \text{ and}$$
$$\text{Lower specification limit} + 1.2\sigma_x + 3\sigma_z.$$

The factor by which σ_x is to be multiplied generally need not exceed 3 and should seldom be less than 1.2. Experience with a specific application may indicate the desirability of some intermediate value.

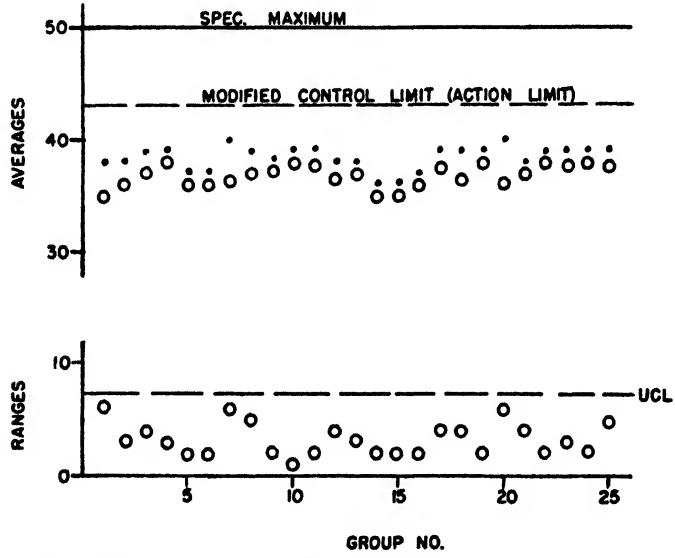

Fig. 11.4f. Modified control limit for a satisfactory process. This chart is adapted from a process that is often very unstable. In this instance it would even show control with ordinary control chart limits. There is no specification minimum in this case.

It must be remembered that in setting modified limits of the type here described, the ranges must show good statistical control. Whenever a range goes out of control, there is the strong possibility of defective material occurring.

Fig. 11.4f shows a modified control limit chart for a process in which only a maximum is specified. The circles represent averages and ranges for groups of four. The dots are the highest individual values in each group of four. The modified control limit (perhaps better called by the more general term of action limit since control in the usual sense is not required) is located by coming down three sigmas of individuals plus three sigmas of averages from the specification maximum. The process is satisfactory since

none of the group averages exceed the action limit and the ranges show good control.

Fig. 11.4g shows a similar chart for an unsatisfactory process. Many of the group averages exceed the action limit and two of the ranges are exces-

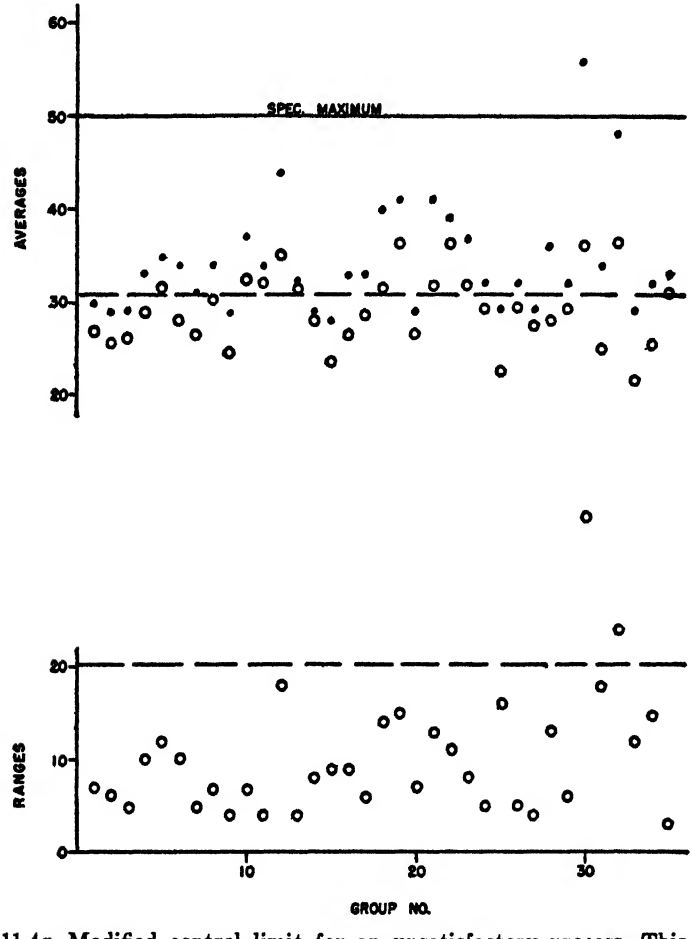

Fig. 11.4g. Modified control limit for an unsatisfactory process. This chart is adapted from the same process as that involved in Fig. 11.4f but in a different plant.

sive. The defective item produced in group 30 occurred when both the range and group average were beyond the limits. For such a process, if the specification limit cannot be raised, either the product must be inspected 100 per cent and bad items sorted from the good, or something done to improve the process (unless it is possible to tolerate the defectives produced).

CHAPTER XII

USE OF CONTROL CHART WHEN KNOWN TREND EXISTS

12.1. Nature of trend. The type of trend with which we are here concerned is a gradual shift in quality level which results from an assignable cause that it is impossible or impractical to control. Probably the best known such cause is tool wear and its effect upon dimensions. Other causes include such things as gradual erosion of furnace linings (*e.g.*, bessemer converter and open hearth for making steel), wear of machine parts (usually insignificant in a properly designed and properly lubricated machine), and changes in ambient air temperature or humidity.

12.2. Special procedure necessary for trends. It is obvious that conventional control chart limits will be unsatisfactory when dealing with trends, since values will eventually go out of control due only to the trend. It is also apparent that specification limits must be wide enough to allow a marked shift in process level if defective items are to be avoided.

12.3. Determination of nature of trend. Data for several runs should first be examined to determine whether the trend is straight line or curvilinear, and the approximate rate at which the level shifts. This process can be expedited by increasing the amount of inspection data obtained during this period.

One method of locating the trend line is to average 50 to 100 measurements involving a very short interval of time. This may be repeated at several places along the trend line. These points should be sufficient to locate the trend line.

Another method is to average at least the first 50 successive observations both horizontally and vertically. This gives one point on the trend line. This procedure must be repeated for at least the last 50 successive observations for a straight line trend. If the scatter of the items indicates a curvilinear trend, additional points must be obtained or the first method used.

12.4. Determination of control limits. With either of the methods described in Section 12.3 control limits may be obtained by grouping the successive measurements (preferably a small group size such as four or five to avoid appreciable trend rise within the group) to get an observed average range. The average range is then multiplied by the appropriate A_2 factor from Table 6.6. The resulting value is added to and subtracted from the

value of the trend line at at least two widely separated places for straight line relationships and at several places for curvilinear trends.

12.5. Starting and stopping the process. If the assignable cause involved can be adjusted (such as tool setting), the process should be started with a quality level as close as possible to a point three sigmas of individuals from the specification limit. This is illustrated in Fig. 12.5. The quality level produced by the process at the beginning may be obtained by measur-

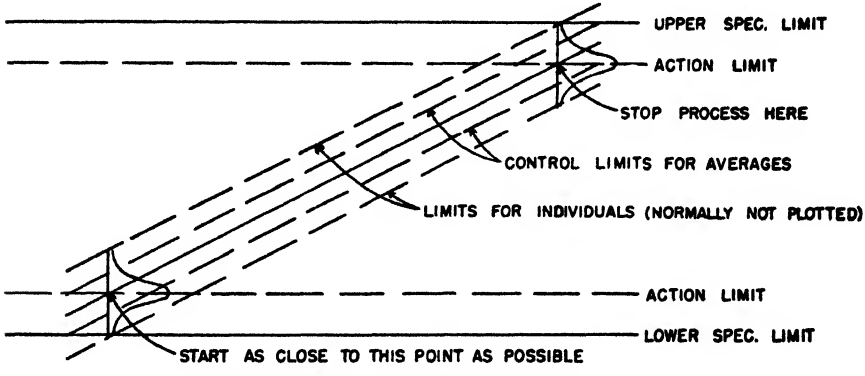

Fig. 12.5. Control chart for a known trend.

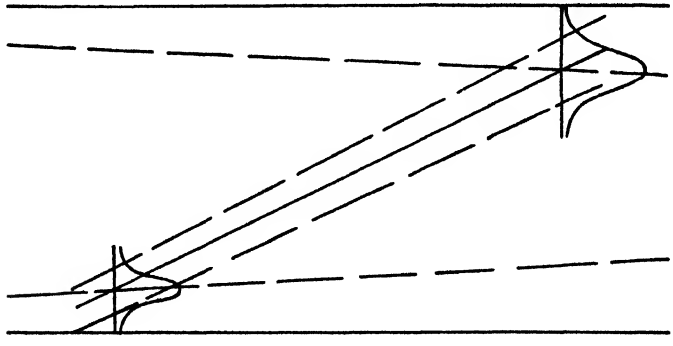

Fig. 12.6. Adjustment of a trend chart for increasing dispersion. Compare with Fig. 12.5.

ing 50 to 100 items. Whatever machine adjustment is necessary may then be made upon the basis of the average of these measurements. *A machine adjustment should never be attempted on the basis of one or two items.* Such adjustments are futile. They lead only to defective items and abnormally high down time. They seldom if ever result in the correct adjustment being made.

The process level may be permitted to rise until the trend line reaches a point three sigmas of individuals (use I_2 factors from Table 6.6) from the

specification limit as shown in Fig. 12.5. If some small portion of defectives can be tolerated, the process may be permitted to continue beyond this point; however, the longer it continues the greater the portion of defective material will become. Even if no defective material can be tolerated, it may be economical to let the process continue a short while and 100 per cent inspect the last material made.

12.6. Effect of gradual increase in dispersion. A control chart for ranges should be kept in connection with the chart for averages. If it is clear that the average range increases as time goes on, the inner limit lines must come closer together as shown in Fig. 12.6. It is desirable to have at least ten ranges to average both horizontally and vertically to determine the average range at any given point.

CHAPTER XIII

QUALITY CONTROL BY LIMIT GAGING*

13.1. Conditions favoring the use of limit gaging. Most industrial plants producing large quantities of small piece parts have been accustomed for some time to inspection by the attribute method (go no-go gaging is a very common form). It may be impracticable in many cases to convert to a system of variables inspection. There may be a large investment in existing gaging equipment and necessary standards for checking it, the training of existing personnel to use variables measurements and apply variables quality control charts may be hampered by lack of necessary skills and the cost of hiring new personnel may be prohibitive, or the components may be so small that it is difficult to handle them on a variables measurement basis.

13.2. Inadequacy of patrol inspection. While patrol inspection generally provides for the taking of any sample size at any time interval (even though some sort of rough schedule may be followed), its greatest weakness lies in the fact that routine samples are seldom large enough to catch any but the most drastic quality level shifts and the system may degenerate to the point where the time intervals are much too large and the standards are much too lax. For example, one inspector took a handful (indefinite quantity) whenever it suited him and passed the sample if it was not over 50 per cent defective. The material then continued to flow into production until another sample was taken.

13.3. Limit gaging only a substitute method. It should be borne in mind that the limit gage method is not suggested for wholesale replacement of quality control by measurements where such a system is functioning satisfactorily or where it is feasible to install such a system. Measurements provide the most generally satisfactory means of appraising quality levels and process control.

13.4. Permissible variation in process level. Many repetitive operations tend to retain essentially the same variability over long periods of time even though the process level may shift due to such things as tool wear and resetting of tools. The limits within which the process level may be permitted to vary for the purpose of limit gaging are illustrated in Fig. 13.4. The distance of four standard deviations from the specification limit

* This chapter is based (with permission) upon an article of the same title appearing in Vol. 3, No. 23, October 1944, of the Production and Engineering Bulletin issued by the Ministry of Labour and National Service and the Ministry of Production (London).

to the mean instead of three (minimum generally allowed for a controlled process in connection with the control chart for variables), is to provide an added margin of safety since we are dealing with attributes inspection instead of variables.

13.5. Setting of gage limits. The weakness in conventional attribute inspection is that when a process is producing satisfactory product we expect very few defective items and a very marked shift in quality level must occur before we become aware of the situation. For this reason we use a special limit gage with closer tolerances than those of the specification. The slope of the normal curve is steepest at plus and minus one standard deviation (the inflection points on the curve). Thus with gage limits at

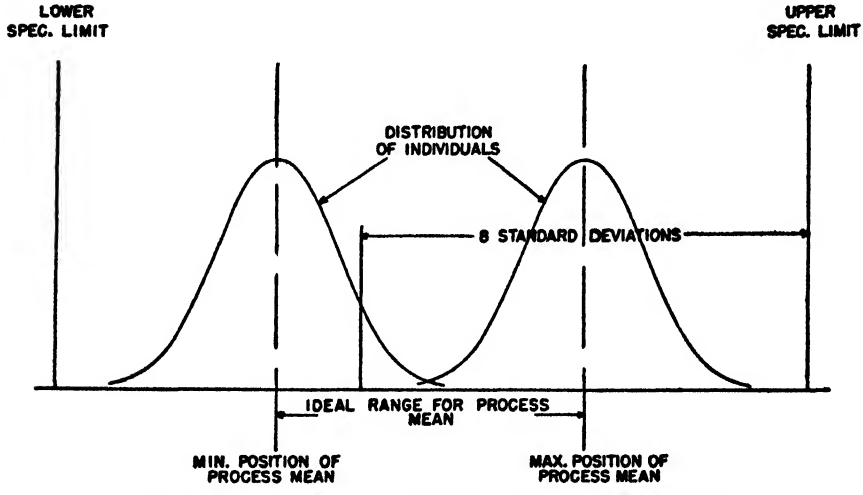

Fig. 13.4. Ideal range for process mean with limit gaging.

this point, a given shift in quality level will have the maximum effect upon the number of items falling beyond the gage limits.

Gage limits for a process just able to meet the specification are illustrated in Fig. 13.5a. Fig. 13.5b illustrates the gage limits for a process able to keep well within the specification limits.

The most sensitive position for the limit gage limits varies somewhat with the sample size, but for small samples it is very close to the mean plus and minus one standard deviation. The setting of these limits does not mean that the specification tolerance is reduced, but rather that a certain portion of items will be expected to fall outside the limit gage limits when the process is operating under control. This will greatly increase the sensitivity of the attribute inspection to shifts in quality level. Further, a larger sample may be taken than could be handled under variables measurements thus increasing the power of the inspection.

The first step in determining the actual limits is to obtain an estimate of the standard deviation of the process. At least 50 to 100 items should be measured on a variable scale. The process should be kept under conditions

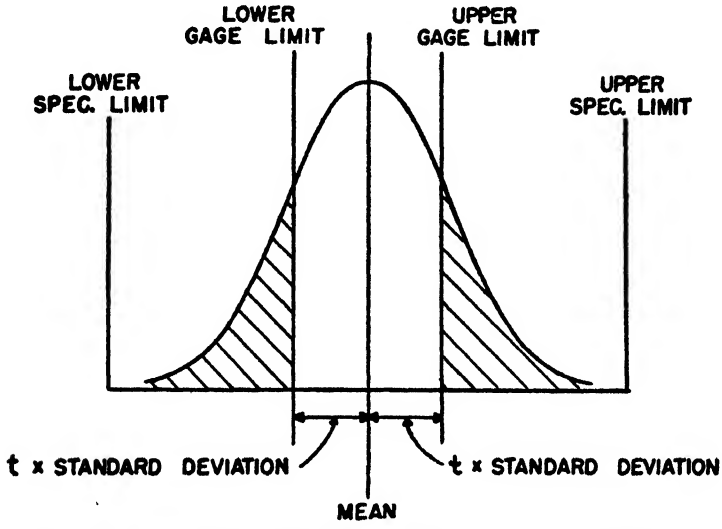

Fig. 13.5a. Gage limits for a process just able to meet the specification.

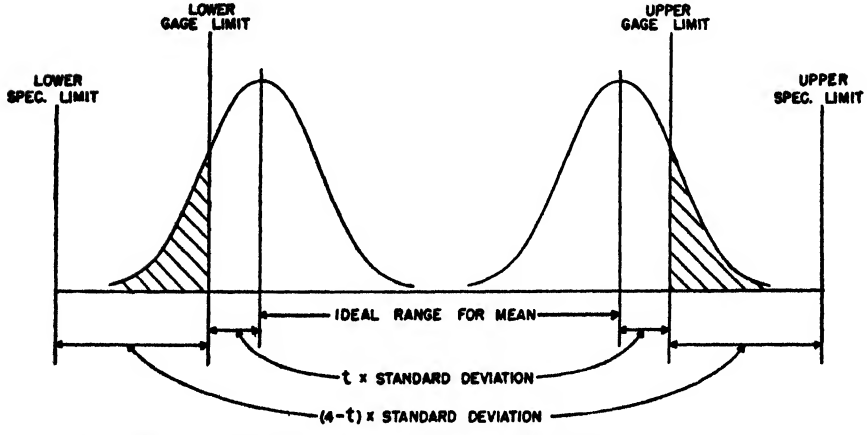

Fig. 13.5b. Gage limits for a process able to keep well within the specification limits.

as constant as possible during the collection of this data. To assure the presence of control, the data should be checked by plotting a control chart.

If the standard deviation is $\frac{1}{6}$ to $\frac{1}{8}$ of the specification tolerance, the process is just satisfactory. Set the gage limits at the middle of the tolerance range plus and minus t sigma (see Table 13.5). If sigma is over $\frac{1}{8}$ the toler-

ance band, the same procedure may be used, but some defectives (out of specification limits) must be expected as a matter of course. This follows whatever the inspection method.

TABLE 13.5

Sample Size	Factor t	Maximum Allowable Outside Gage Limits	
		Action Limit	Warning Limit
25	1 08	8	5
20	1.06	7	4
15	1 00	6	4
10	1.14	4	2
5	0 90	3	2

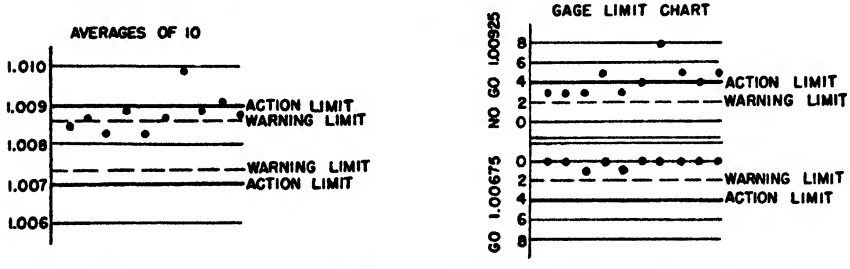

Fig. 13.6a. Comparison of actual measurements and limit gage chart for process just able to meet the specification.

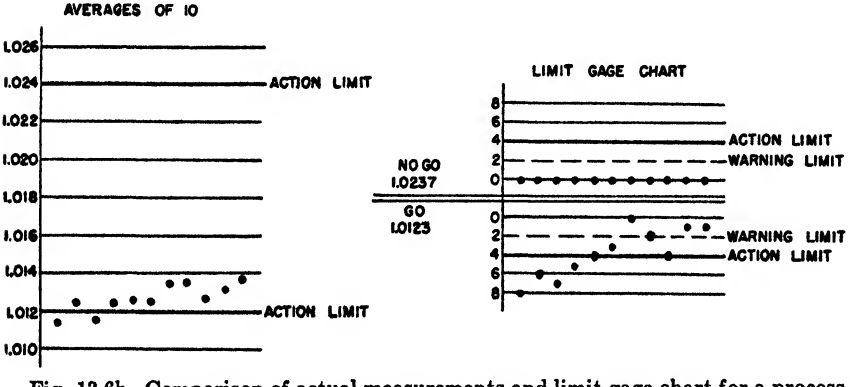

Fig. 13.6b. Comparison of actual measurements and limit gage chart for a process able to keep well within the specification limits.

If the standard deviation is less than $\frac{1}{8}$ of the tolerance band, come in from the specification limits $(4 - t)$ standard deviations. This will take advantage of the extra margin of variation the process allows.

13.6. Examples. In the first example, the specification called for a

dimension of 1.008″ plus or minus 0.0035″. Ten samples of ten items each were taken and the standard deviation found to be 0.0011″. This was just a little less than $\frac{1}{6}$ of the tolerance band (0.0070″). The gage limits were therefore set at 1.008 plus and minus (1.14 × 0.011), or at 1.00925 and 1.00675. A comparison between the actual measurements and the limit gage chart is shown in Fig. 13.6a.

In the second example, the specification called for a dimension of 1.018″ plus or minus 0.008″. Eleven samples of ten each gave a standard deviation of 0.0008″. This was 1/20 of the tolerance band. The gage limits were therefore set at 1.010 + [(4 − 1.14)0.0008] = 1.0123 and 1.026 − [(4 − 1.14)0.0008] = 1.0237. Fig. 13.6b compares the results for the actual measurements and gage inspection for this example.

13.7. Cautions. Several practical points should be kept in mind when using limit gages:

(a) If several machines are being used simultaneously to make the same item, use the standard deviation from the least consistent machine. If there is too much difference among the machines, their output cannot be lumped together. Practical judgement must be used in making this decision.

(b) Check the standard deviation occasionally. Any material change in this value will affect the validity of the limits.

(c) Guard against marked skewness. Moderate amounts of skewness will not detract from the value of the procedure.

(d) Check the gages frequently for wear. Since many items will just pass or fail to pass the gage, the wear will be greater than on ordinary gages.

(e) Plot the results on charts placed on or near the machines. The charts will tell the story of quality trends much more effectively than the numbers by themselves.

(f) Be sure to explain to the workmen that the specification tolerances are not reduced by this method.

CHAPTER XIV

RELATIONSHIP BETWEEN CONTROL LIMITS AND SPECIFICATION LIMITS

14.1. Origin of specification limits. In the final analysis all specifications have their origin in some human need or want. Sometimes these needs or wants are not clearly understood. Sometimes it is difficult to state in concrete terms how they may be satisfied or how the quality characteristics are to be measured. It is small wonder that some specifications are ambiguous or unrealistic.

More specifically, specification limits are based upon engineering considerations that must be met in order that the product will perform as intended. Unless specification limits must be met in order to fill a human need or want, or to contribute to that end, there is no justification for them. Specification limits that require unnecessary precision of manufacture or insufficient precision cause enormous losses annually in scrap, rework, and excessive costs of manufacture. Note that what we have said so far is independent of the ability to make that which is specified.

14.2. Control chart limits. Control chart limits are determined by the process involved and reflect what the process is capable of making. If the process is uncontrolled, they do not represent the full potentialities of the process and may not provide an adequate basis for deciding whether the process can meet the specifications; however, if at least the chart for ranges shows control, it can be determined whether the process is capable (when control of the averages is achieved) of working to specification limits.

14.3. When is a process under a satisfactory degree of control? Theoretically, when dealing with a normal distribution only one group average in 370 will exceed the three sigma control limits on the average just due to chance. In practice we have only an estimation of the true process level and inherent variability. These estimates are subject to chance variation which while small in magnitude (usually) nevertheless tend to increase the probability of going out of limits just due to chance. In general a process may be considered under satisfactory control if no groups out of 25 are out of limits, not over one group out of 35 groups, and not over two groups out of 100.

14.4. Relating the control chart limits to the specification. It must be remembered that specifications normally relate to individual test results. Control charts for variables are usually based upon grouped results. Hence control chart limits for averages must not be directly related to the

specification limits. The three sigma limits for individual values may be obtained by multiplying the width of the three sigma band for averages by the square root of the group size, by multiplying $\bar{R}$ by I_2 (see Table 6.6), or by dividing $3\bar{R}$ by d_2 (see Table 6.6). The resulting value is then added to the average of the process and subtracted from it. The process may now be judged in relation to the specification in accordance with Fig. 14.4.

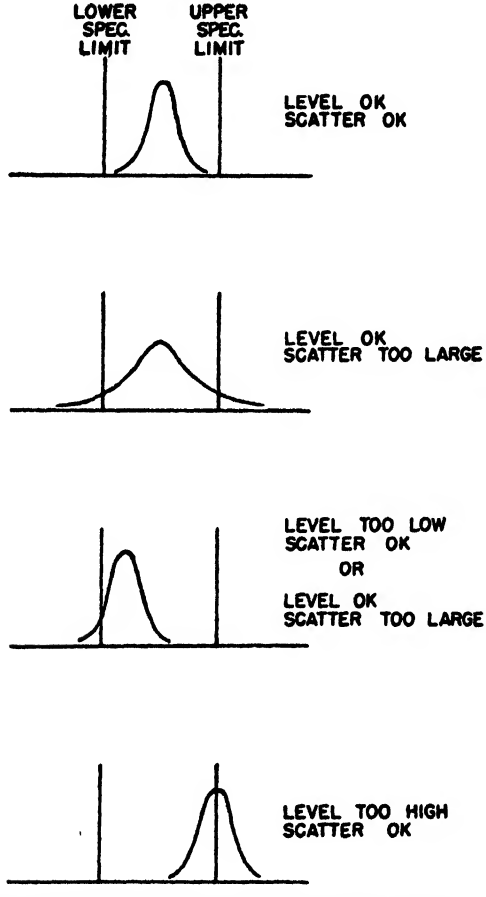

Fig. 14.4. Relating the process to specification limits.

In connection with many physical properties, such as tension tests where only a minimum is specified, it will be found to be common industrial practice to work to such a quality level that the lower control limit for individuals falls below the specification minimum. Rarely, however, will it fall more than $1\frac{1}{2}$ sigmas (of individuals) below. When it has reached this point, approximately 7 per cent of the items in a normal distribution will be ex-

pected to fall below the specification minimum but not below the lower control limit for individuals.

The reason why industrial firms work to such quality levels is that it is very common to permit retests. Retests are permitted because testing equipment sometimes lacks the degree of precision we would like it to have and because test specimens sometimes contain flaws which are not characteristic of the product involved. Let us suppose that a certain controlled product is being manufactured at a level such that 0.07 (7 per cent) of the product can be expected to fall below the specification minimum. When an item from such a controlled process falls below the specification minimum and a retest is taken without retreating the product, the probability that the retest will fail is only 0.07. Thus, if only one retest is permitted, the expectancy is that only 0.0049 (0.07×0.07) of the product will be rejected. If a second retest is permitted, only 0.000343 (0.07×0.0049) of the product will be rejected or a little more than three hundreths of one per cent. Therefore, it is normally not economical for the producer to work to a higher quality level since occasional retests are often less costly than raising the general quality level of the product.

Where both a maximum and minimum are specified, it is generally desirable that the control limits for individuals come somewhat closer together than the specification limits. To be considered a satisfactory situation, the width of the control limit band for individuals should not exceed three-fourths of the width of the specification band.

14.5. Estimating the population standard deviation from sample standard deviations. If the sample is fairly large, say 50 or more, it will generally give a satisfactory estimate of the population standard deviation when computed from the formula

$$s = \sqrt{\frac{\Sigma(X - \bar{X})^2}{N}}$$

However, sample standard deviations are biased and tend to give a smaller value than the population value. This bias is small when the sample size is large, but becomes increasingly important as the sample size becomes smaller. Even with samples as small as 25 it may generally be disregarded, but below this figure it becomes increasingly more important.

For a single sample, the bias may be compensated for by using the formula

$$S = \sqrt{\frac{\Sigma(X - \bar{X})^2}{N - 1}}$$

or the simpler form for computational purposes

$$S = \sqrt{\frac{N\Sigma X^2 - (\Sigma X)^2}{N(N-1)}}.$$

S^2 is an unbiased estimate of σ^2 (population variance). Several values of S^2 obtained by sampling from a population may be averaged together to obtain an improved estimate of population variance. The square root of this avarage will give an unbiased estimate of the population standard deviation. If several values of S are to be averaged together, they must first be divided by c_3 (see Table 14.5) since $E(S) = c_3\sigma$, where E is the symbol for expected value.

If the formula for s is used to obtain the sample standard deviations, then they must first be divided by c_2, since $E(s) = c_2\sigma$. Values for c_2 and c_3 are given in Table 14.5.

TABLE 14.5. VALUES OF c_2 AND c_3 WHERE $c_3 = \sqrt{\frac{N}{N-1}}\, c_2$

Sample Size N	c_2	c_3	Sample Size N	c_2	c_3
2	0 56419	0 79788	16	0.95225	0.98348
3	0 72360	0 88623	17	0 95511	0.98451
4	0 79788	0.92132	18	0 95765	0.98541
5	0 84075	0 93999	19	0 95991	0 98621
			20	0 96194	0 98693
6	0 86863	0 95153	21	0.96378	0 98758
7	0 88820	0 95937	22	0 96545	0 98817
8	0 90270	0 96503	23	0 96697	0 98870
9	0.91387	0 96931	24	0 96837	0 98919
10	0.92275	0 97266	25	0 96965	0 98964
11	0 92996	0.97535	50	0 9849	0 9949
12	0.93594	0 97756	75	0 9900	0 9966
13	0 94098	0 97941	100	0 9925	0 9975
14	0 94529	0 98097			
15	0 94901	0 98232			

14.6. What to do when the process is unsatisfactory. If the process is inherently incapable of meeting the specification tolerances, there are three alternatives:

(a) Change the specifications

(b) Change the process fundamentally

(c) Use 100 per cent inspection to sort out the defective items

It is suggested that an effort be made to obtain a change in the specifications before attempting the other solutions. The reason for this is that so few specifications in the past have been developed upon sound consideration of the statistical aspects involved and the real needs of the processing.

If the specifications have been soundly established so that the stated

requirements must be met in order to obtain a satisfactory product or to continue further processing operations, then the process must be altered fundamentally if possible. This may mean obtaining a source of more uniform raw materials, a better type of raw material, improved machinery, the addition of new steps in the processing, or better trained or better supervised personnel.

If it is not economical or otherwise practicable to take either of the above steps, then 100 per cent inspection must be instituted in order to sort good items from bad. In any event, the economic aspects of these solutions should be carefully considered before action is taken.

Process changes should always involve a trial run to determine whether the new process will be satisfactory. Several changes may be needed before a suitable solution is found.

14.7. Adoption of standard values. Standard values are indicative of what a process can do under controlled conditions. They should never be adopted until the process has shown control. Once adopted, they provide a goal for the future based upon the analysis of past data. Standard values should never be adopted arbitrarily without reference to the process. The mistake is sometimes made of using such arbitrarily adopted values to plot control chart limits. The weakness of such a procedure lies in the fact that while points outside such limits will indicate that the process is not operating to the standards, the limits plotted do not permit a determination of whether the process is operating in a controlled manner at its natural level. In other words, such a control chart can not be used to evaluate control, but merely divergence (whether controlled or not) from some arbitrary standard. On the other hand, a control chart based on the process itself can both evaluate control, and be related to the specification limits.

CHAPTER XV

USE OF THE VARIABLES CONTROL CHART AS A BASIS FOR REDUCING VOLUME OF INSPECTION

15.1. Establishment of initial eligibility for reduced testing. The usual procedure will be to provide that normal testing shall be that testing required by the specification involved. The next step should be the determination of what shall constitute reduced testing. As a matter of economics, it generally will not be feasible to establish a program unless the testing can be reduced at least one-half. The maximum reduction that can reasonably be allowed should be provided for. This might be $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{10}$ or less of normal testing.

A. *Initial eligibility should require that a minimum number of successive groups be accumulated with none falling outside control limits.* Whenever possible, at least 25 successive groups of data should be plotted on the chart. Where immediately past data is available, there should be no difficulty with this requirement. Where the process is new and the data is accumulated rapidly, there should be no difficulty. If the data are accumulated slowly, it may be necessary to start the control chart with less than 25 groups. The minimum number of groups that should normally be used is ten (as many more than ten as practicable should be used). Where more than 25 groups are used, it may be desirable to permit one or two groups to be outside control limits as discussed in Section 14.3. It should not be expected that all groups will fall inside control limits at all times since even under theoretically ideal conditions an average of one in every 370 groups can be expected to exceed control limits just due to chance. When we add to this fact the consideration that our observed average and inherent variability are only estimations, we can expect chance groups outside control limits somewhat more frequently than one in 370. On the other hand, one should be cautious to avoid being much more liberal than indicated above. Where less than 25 groups are used for a basis of initial eligibility, none of the points should be permitted to fall outside control limits.

B. *The average quality level must be far enough away (in terms of the variability inherent in the process) from the specification requirements.* This means that control limits for individuals should be computed in order that their relationship to the specification limits may be determined. If no items can be tolerated beyond the specifications, the control limits for individuals

must not exceed the specification limits. If some small percentage of items beyond the specification limits can be tolerated, then the control limits for individuals may exceed the specification limits to the appropriate extent. This can be determined by consulting Table 4.3 and locating the portion of the area of the curve that will be permitted beyond the specification limits. This in turn can be translated into the number of standard deviations involved. Thus, if only a specification minimum is required and approximately 7 per cent of the individual items can be tolerated below the specification minimum, we find by consulting Table 4.3 that the area beyond $1\frac{1}{2}$ sigmas from the mean includes 6.681 per cent of the area under the curve. Thus, when using averages of groups of four, we would permit the lower control limit for averages to fall as low as the specification minimum (the control limit for averages of four is at $1\frac{1}{2}$ sigmas of individuals from the mean; see Section 6.5).

Another device that can be used is to state the specification limits in terms of the group size. For a group size of four this would mean that the specification limits are set half as far from the midpoint of the specification range. For the group size of five they would be set 1/2.236 as far from the midpoint of the specification range. In general, the fraction involved can be determined by setting 1 over the square root of the group size.

C. *No actual rejections should be permitted in the data used for initial eligibility.* In view of the fact that occasional items beyond the specification limits may be permitted and that retests are generally allowed, no actual rejections should be permitted. A rejection is considered to occur when the original test and all retests permitted by the specification have failed. A failure is a test result which does not meet specification requirements, whereas a rejection involves scrapping or a final refusal to accept material offered for inspection.

15.2. Conditions requiring the re-establishment of eligibility. There are two conditions under which eligibility should be re-established that are normally not serious enough to necessitate the meeting of initial eligibility requirements.

D. *The occurrence of an indication of lack of control.* If no groups are permitted outside the control limits, then the occurrence of any group outside limits should necessitate the re-establishment of eligibility. If one group in any 35 or 50 or some other number is permitted outside limits, then when the second group in such a run falls outside control limits, re-establishment of eligibility should be required.

E. *The making of a possibly harmful change in raw materials or the manufacturing process.* Changes which involve past experience that has clearly indicated that no significant shift in level or variability is to be expected

should be permitted without requiring a return to normal testing. Changes which may have a harmful effect with respect to the quality level or variability are usually not made deliberately, but may be the result of exhausting the desired source of raw materials or of mill breakdowns requiring the substitution of temporary equipment. In such cases, it is better to increase the testing for the time being, to be on the safe side.

15.3. Re-establishment of eligibility. In view of the fact that the process has already established its ability to meet the requirements of initial eligibility, it may reasonably be assumed that the condition requiring re-eligibility is temporary. Hence, it will not be necessary to require as many groups to fall within control chart limits as initially required. It will generally be satisfactory to re-establish eligibility on the basis of half as many groups as initially required. Where necessary or feasible, this might be reduced slightly.

F. *Minimum number of successive groups with none outside control limits.* In this case no groups should ever be permitted to fall outside control limits.

G. *No actual rejections in data required for re-eligibility.* The occurrence of a rejection is normally sufficient grounds to require the obtaining of initial eligibility before reduced testing can again be put into effect; however, a rejection may be tolerated once initial eligibility has been obtained provided the control limits for individuals are permitted to exceed the specification limits. In such a case this requirement might even be waived in connection with re-eligibility.

H. *The average quality level must continue to be far enough away (in terms of the variability inherent in the process) from the specification limits.* Normal procedure will involve recomputing the control limits periodically on the basis of the new data as accumulated. Thus, the control limits computed on the basis of the first 25 groups may be projected ahead to cover the next 25 groups. Control limits for the next 25 groups may be based upon the last 25 or last 50 groups, but in any event it is desirable to avoid using more than the last 50 groups as a basis for projecting control limits. This is necessary in order to avoid the influence of "ancient history", and is an acknowledgment of the fact that slight and gradual shifts may occur in something resembling a controlled manner over a long period of time. When such a recomputation becomes necessary, it may result in shifting somewhat the plotted average quality level or inherent variability.

I. *There must be no possibly harmful change in raw materials or the manufacturing process during the data involved.* See comments above under heading E.

J. *The last eight (or more) groups must not be all on or above, or all on or below the central line for averages.* A run of eight or more consecutive groups,

all on one side of the mean may be considered an indication of a significant shift in quality level. When such an event occurs, the requirements of initial eligibility should be imposed.

15.4. Conditions under which obtaining initial eligibility again should be required.

K. *If at any time a recomputed control level falls too low or too high, initial eligibility requirements should be met.* See heading H above.

L. *If the last eight (or more) groups are all on or above, or all on or below the central line for averages, the requirements of initial eligibility should again be met.* See heading J above.

M. *If the requirements of section 15.3 above are not met at a time when re-eligibility is required, the requirements of initial eligibility should again be met.* At any time when re-eligibility or initial eligibility is required, the immediate past data should be tested to determine whether the requirements of initial eligibility are met by such data. In gaining initial eligibility it should not be necessary to wait for the accumulation of completely new data. This follows from the fact that trends to new levels, or control at new levels may have occurred some time prior to the condition requiring re-eligibility or initial eligibility.

CHAPTER XVI

THE BINOMIAL AND POISSON DISTRIBUTIONS

16.1. The binomial distribution. Some situations cannot be satisfactorily described by a continuous curve such as the normal curve. Consider, for example, the matter of tossing a coin. Barring the possibility of standing on edge (or of disappearing into thin air), the result of a toss can only be a head or a tail (also barred are two-headed or two-tailed coins). If two coins are tossed simultaneously, the only possible results are both heads, one head and one tail, or both tails. Things of this sort can best be described by the binomial distribution, which is given by the expression $(\bar{q} + \bar{p})^N$, where $\bar{p}$ (population or process fraction defective) is less than 1, $\bar{q}$ is $1 - \bar{p}$ and N (sample size) is any positive integer.

Suppose we toss five coins simultaneously (or one coin five times); what is the probability that all five coins will come down heads, that there will be four heads and one tail, etc.? If q represents heads (H) and p tails (T), the expression now becomes $(H + T)^5$, or $H^5 + 5H^4T + \frac{5.4}{1.2}H^3T^2 + \frac{5.4.3}{1.2.3}H^2T^3 + \frac{5\ 4.3.2}{1\ 2\ 3.4}HT^4 + T^5$. In a normal coin, the likelihood of a head or a tail will be the same, for all practical purposes. Thus the possibility of either one of the events is one out of two, or 0.5 (unity, or 1.0 represents all possible events). Substituting 0.5 for both H and T we get $0.03125 + 0.15625 + 0.3125 + 0.3125 + 0.15625 + 0.03125$. Thus we expect to get all five heads 3.125 per cent of the time on the average, four heads and one tail 15.625 per cent of the time, etc.

In Fig. 16.1 we see the lack of symmetry characteristic of the binomial except when p is 0.5. It will be noted that the distribution for $p = 0.1$ becomes more nearly symmetrical as N increases. For large values of N, it appears to be almost symmetrical. When p is exactly 0.5 it is always symmetrical regardless of the value of N, but does not begin to resemble the normal distribution closely until N is about 10. As N increases, the binomial approaches the normal distribution more nearly, but never reaches it, since the limiting form of the binomial as N is increased while $\bar{p}N$ remains constant is the Poisson distribution, which will be discussed in Section 16.4.

16.2. Requirements for strict applicability of the binomial distribution. The first requirement for strict applicability of a binomial distribu-

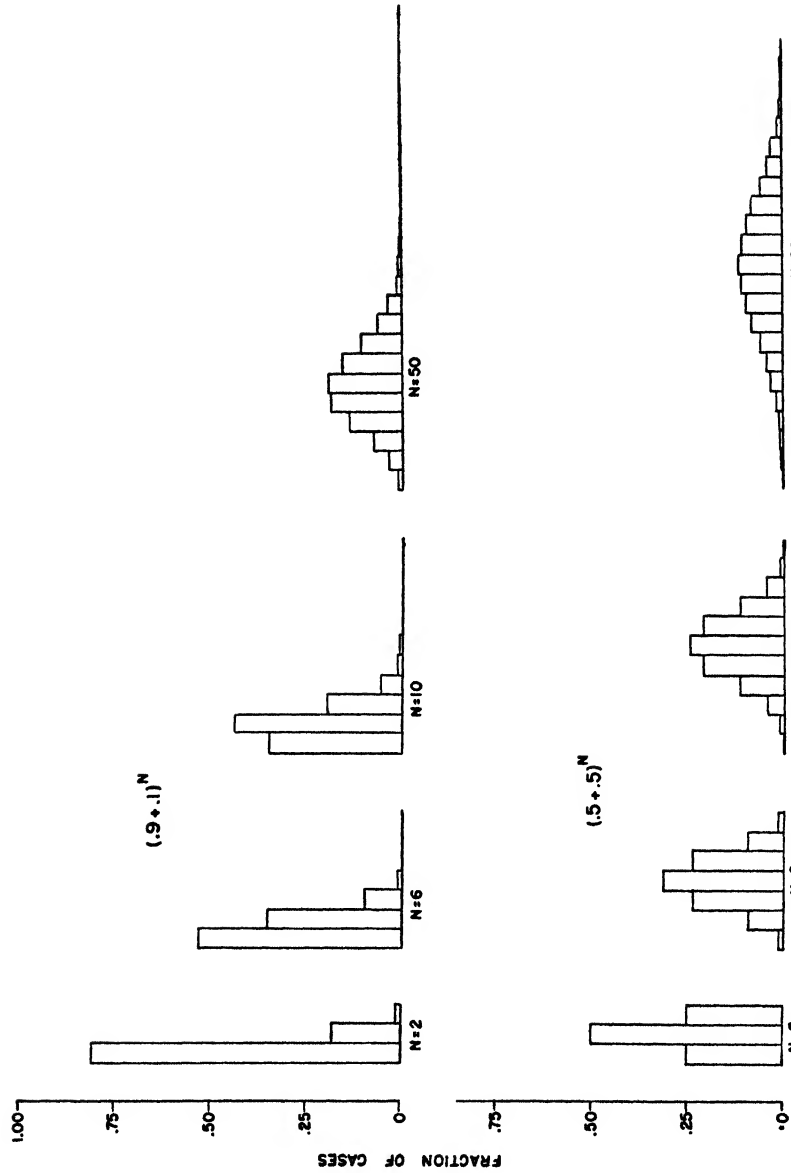

Fig. 16.1. Two binomial distributions for several values of N.

tion is that the value of $\bar{p}$ must be constant. Any shift in the value of $\bar{p}$ will change the probabilities with which the various events will occur.

The second requirement is that the sample size must be constant, or adjustment made for any variations. As in the case of $\bar{p}$, any change in sample size changes the probabilities involved.

16.3. Use of the binomial distribution in connection with control charts. For control chart purposes, the use of the binomial is very much simplified by the formula for the standard deviation of $\bar{p}N$,

$$\sigma_{\bar{p}N} = \sqrt{\bar{p}N(1-\bar{p})}.$$

For most binomial distributions, about 0.001 to 0.005 is beyond $\bar{p}N + 3\sigma_{\bar{p}N}$. This is reasonably comparable to the area of the normal curve beyond three standard deviations. While the lower control limit tends to be somewhat lower than is necessary (except when $\bar{p}N$ is about 50 or more), this is generally unimportant.

16.4. The Poisson distribution.* The Poisson distribution is the limiting form of the binomial, as N is increased while $\bar{p}N$ remains constant. The expression for the Poisson distribution is

$$e^{-\bar{c}} + \bar{c}e^{-\bar{c}} + \frac{\bar{c}^2}{2!}e^{-\bar{c}} + \frac{\bar{c}^3}{3!}e^{-\bar{c}} + \cdots + \frac{\bar{c}^N}{N!}e^{-\bar{c}} = 1$$

where $e = 2.71828 +$ (the base of natural or Naperian logarithms) and $\bar{c} = \bar{p}N$. The mean is $\bar{c}$ and $\sigma_{\bar{c}} = \sqrt{\bar{c}}$. The distribution is highly skewed when $\bar{p}N$ is small, and becomes more nearly symmetrical as $\bar{p}N$ increases, approaching the normal distribution. If $\bar{p}$ is near 0.5, it is nearly normal even when $\bar{p}N$ is small.

The Poisson distribution gives an acceptable approximation to specific binomial distributions when $\bar{p}$ is equal to or less than 0.1 and N is equal to or greater than 10.

* For practical aids to applying the Poisson distribution see "A Guide to Utilization of the Binomial and Poisson Distributions," by Holbrook Working, Stanford University Press, Stanford University, California, and "Poisson's Exponential Binomial Limit," by E. C. Molina, D. Van Nostrand, which provides convenient tables of probabilities. In Molina's tables, the symbol a is equivalent to $\bar{p}N$.

CHAPTER XVII

SIGNIFICANCE OF OBTAINED DIFFERENCES OF SAMPLE MEANS AND STANDARD DEVIATIONS

17.1. The significance of a difference. Although large or unusual differences in observed data can generally be recognized by the layman, many real differences cannot be properly evaluated without the aid of statistical methods. These differences may involve enormous sums of money in mass production situations where a slight modification of production practice may result in a real but not readily detected improvement in quality of product. There is a natural human tendency to consider a difference significant only when it obviously involves practical convenience or economic advantage. What the layman considers significant may or may not be statistically significant.

There is the classic example of the mill superintendent who each morning examined the record of scrap losses for the previous day. Each time the losses were lower than those of the previous day he was very much pleased, and each time they exceeded those of the previous day he was very much disturbed, and would call in his assistant to demand the reason for the increase. Somehow the assistant always managed to supply a "reason"; however, as the statistician well knows, most of the data fluctuations would probably be due to chance causes and the mill superintendent was obviously unduly concerned about matters over which no control could reasonably be exercised.

One of the general principles that should be borne in mind is that large differences may be statistically significant even though the volume of data is small, and that small differences often may be statistically significant when the volume of data is large. A true increase of one per cent in yield may mean a savings of thousands of dollars annually where the volume of the output is large. Small differences in comparative test results should not be disregarded just because they are small. Similarly, a large difference should not be thrown out just because there is only one item of each kind in the test. Under such circumstances it will normally be highly desirable to obtain some additional data in order to determine the true significance of the obtained difference.

All too many technical reports are written with such statements as,

"there is some indication that this procedure improves the results obtained"; "it does not appear to benefit the other class of items in a similar manner"; "none of the special materials appears to have a definite effect on the characteristics of either type of item." Such generalizations usually reflect either a lack of sufficient data to establish the significance of the results, or an attempt to judge the results without making use of a suitable statistical analysis. In view of the large expense often involved in experimentation, it is highly desirable that the maximum amount of information be obtained from the observed results. Statistical methods provide us with the means of accomplishing this purpose. Before we consider these methods it will be desirable to discuss what is meant by the phrase "degrees of freedom".

17.2. Degrees of freedom. Let us suppose that we have two samples of four items each and have computed the mean of each sample. We now wish to know whether the two means are significantly different. Our comparison of the two means must be based upon the number of degrees of freedom present, that is, the extent to which the numbers being compared are free to vary. When we compute the mean of a sample of four items and say that this is one of the values to be compared, it actually represents only three items that are able to vary independently, for given any three numbers and an average that includes a fourth number, the fourth number is constrained to have only one value. If the first three numbers are 9, 5, and 6, and the average of all four numbers is 7, the fourth number must of necessity be 8—that is, it has no freedom to vary. Thus, having computed the sample mean, it now has degrees of freedom equal to the sample size minus one. Similarly, each additional statistic, such as standard deviation or skewness computed for a sample, removes an additional degree of freedom from the sample. If we computed as many statistics from a sample as there were items in the sample, we would have fully constrained the sample so that it would have no degrees of freedom in it. When comparing differences we must base the comparison upon the number of items that are free to take on any value whatsoever. In the above case where we wished to compare the means of two samples of four items each, there would be a total of six degrees of freedom, three from each sample.

17.3. Difference between two sample means (variables).

Case I involves the same number of observations in each sample and a total of over 30 degrees of freedom. The first step is to compute the mean of each sample. The smaller mean is subtracted from the larger mean to obtain the magnitude of the difference. The second step is to obtain the square root of the sum of the variances of the sample means. This will give the standard deviation of the differences in sample means. The result of the

second step is then divided into the result of the first step. The formula for these steps is

$$\frac{|\bar{X}_1 - \bar{X}_2|}{\sqrt{S^2_{\bar{x}_1} + S^2_{\bar{x}_2}}}$$

where $S^2_{\bar{x}_1} = \frac{S_1^2}{N_1}, S^2_{\bar{x}_2} = \frac{S_2^2}{N_2}$, and $S_{(\bar{x}_1 - \bar{x}_2)} = \sqrt{S^2_{\bar{x}_1} + S^2_{\bar{x}_2}}$

(the subscripts refer to the two samples). If the value of this formula equals or exceeds 3.00, the difference in the sample means may be assumed to be significant. There is very little likelihood that both sample means could have come from the same population. If it is equal to or over 2.50 but is less than 3.00, the difference may be real. The collection of additional data will be desirable if it is at all practicable. If it is equal to or over 2.00 but is less than 2.50, it is doubtful that the difference is significant although it is possible. The collection of additional data will be desirable if it can be obtained easily or if it is imperative to show up a real difference that may be of small magnitude. If it is less than 2.00, the difference may be assumed to be a chance difference. Bear in mind that this does not preclude the possibility that there is a real difference of relatively small magnitude. It means only that for the sample sizes used the samples might well have come from the same population, although there is no assurance whatsoever that they did. It is assumed in this procedure that the items in the two samples are uncorrelated and that they are obtained under controlled conditions. Results of the test will not be valid if these conditions are not met.

Case II involves a different number of observations in each sample and over 30 degrees of freedom. The procedure is the same as in Case I except for the denominator of the test formula, which is changed as follows to provide the proper weighting for the sample size differences:

$$S_{(\bar{x}_1 - \bar{x}_2)} = \sqrt{\frac{(N_1 + N_2)(\Sigma d_1^2 + \Sigma d_2^2)}{N_1 N_2[(N_1 - 1) + (N_2 - 1)]}}$$

where d is the difference between an observation X and the mean $\bar{X}$. We now apply the test as before:

$$\frac{|\bar{X}_1 - \bar{X}_2|}{S_{(\bar{x}_1 - \bar{x}_2)}}$$

If grouped data are used, the value of Σd^2 may be obtained by using the formula

$$\Sigma d^2 = i^2\left[\Sigma f d'^2 - \frac{(\Sigma f d')^2}{N}\right]$$

where i is the size of the group interval and d' on the right side of the equation is in terms of group intervals.

Case III involves the same number of observations in each sample and 30 degrees of freedom or less. While the sampling distribution of $\frac{|\bar{X} - \bar{X}_2|}{S_{(\bar{x}_1 - \bar{x}_2)}}$ is essentially normal when N is large, the nonnormality begins to be important when the total degrees of freedom is as small as 30 and increases in importance as they become fewer. The nonnormality takes the form of a more widely dispersed distribution than the normal curve and is known as

TABLE 17.3. VALUES OF t*

Degrees of freedom	Level of significance		Degrees of freedom	Level of significance	
	0.05	0.01		0.05	0.01
1	12 706	63.657	16	2 120	2.921
2	4.303	9.925	17	2 110	2 898
3	3 182	5.841	18	2 101	2.878
4	2 776	4 604	19	2 093	2.861
5	2 571	4.032	20	2 086	2.845
6	2 447	3 707	21	2 080	2 831
7	2 365	3 499	22	2 074	2 819
8	2 306	3 355	23	2 069	2.807
9	2 262	3 250	24	2 064	2 797
10	2 228	3 169	25	2 060	2 787
11	2.201	3 106	26	2.056	2 779
12	2 179	3.055	27	2 052	2.771
13	2 160	3 012	28	2 048	2.763
14	2.145	2 977	29	2 045	2 756
15	2.131	2 947	30	2 042	2 750
			∞	1.960	2 576

* Table 17.3 is abridged from Table IV of R. A. Fisher: "Statistical Methods for Research Workers," Oliver & Boyd Ltd., Edinburgh, by permission of the author and publishers.

the t distribution. This increased dispersion is the result of the fact that the population standard deviations are unknown and estimates (which of course are subject to variability) are used as derived from the samples.

As in Case I, we compute the value of the test formula, but say that it is equal to t:

$$t = \frac{|\bar{X}_1 - \bar{X}_2|}{S_{(\bar{x}_1 - \bar{x}_2)}}$$

The value of t that must be equalled or exceeded for two different levels of significance is given in table 17.3.

Case IV involves a different number of observations in each sample and 30 degrees of freedom or less. The procedure here is similar to Case II except that the formula is set equal to t:

$$t = \frac{|\bar{X}_1 - \bar{X}_2|}{\sqrt{\frac{(N_1 + N_2)(\Sigma d_1^2 + \Sigma d_2^2)}{N_1 N_2[(N_1 - 1) + (N_2 - 1)]}}}$$

Values of t that must be equalled or exceeded before the difference may be considered significant are given in Table 17.3.

17.4. Difference between two proportions (attributes). To test the significance of the difference between two proportions (fractions defective), the absolute value of the difference is divided by the standard error of the difference. The standard error of the difference is given by the formula

$$\sigma_{(p_1 - p_2)} = \sqrt{\frac{p_1 q_1}{N_1} + \frac{p_2 q_2}{N_2}}$$

The subscripts identify the two groups of data. If the value of the formula

$$\frac{|p_1 - p_2|}{\sigma_{(p_1 - p_2)}}$$

equals or exceeds 3.00, the difference may be assumed to be significant; if 2.50 or over but less than 3.00, the difference probably is significant. The collection of additional data will be desirable if at all practicable. Values of 2.00 or over but less than 2.50 will justify additional investigation if it is desirable to show up a real difference that may be of small magnitude. Values of less than 2.00 may be assumed to indicate a chance difference.

Except when $p = 0.5$, the distribution of p is skewed; however, the amount of the skewness will generally be unimportant as long as the value of pN is 5.0 or over.

Example. Crews A and B were both making the same product. At the end of two weeks Crew A had produced 5000 items of which 24 were defective (0.0048). Crew B produced 5500 items of which 44 were defective (0.0080). Did Crew A do a significantly better job than Crew B? Substituting in the formulas, we obtain

$$\sigma_{(p_A - p_B)} = \sqrt{\frac{0.0048(0.9952)}{5000} + \frac{0.0080(0.9920)}{5500}} = 0.00155$$

$$\frac{|p_A - p_B|}{\sigma_{(p_A - p_B)}} = \frac{0.0032}{0.00155} = 2.06$$

The value of 2.06 is so low that we may conclude that Crew A probably did not do a better job than Crew B. If the figures of 5000 and 5500 represent the total output, then we know that Crew A actually did produce

a smaller proportion of defectives, but we cannot be very sure that in a repeat trial Crew A would again do better than Crew B, or in other words, that there is a real difference in ability between the crews. If the figures of 5000 and 5500 were only samples of the output, say one-twentieth of the total output, then we could only say that if there is a real difference in the total fraction defective produced by each of the two crews, it is probably a small difference—one that we could not be sure of without considerably more sampling. Of course, if we are willing to accept a low level of significance we may say that Crew A did do a better job than Crew B.

17.5. Difference between two sample standard deviations. The standard error of the standard deviation σ_σ for a normal population is

$$\sigma_\sigma = \frac{\sigma_P}{\sqrt{2N}} = 0.7071068\sigma_{\bar{x}}$$

where σ_P, is the population standard deviation. From this basic fact we may test the difference between two sample standard deviations.

Case I. Each of the samples consists of more than 31 items. The standard error of the difference between two sample standard deviations is

$$\sigma_{(\sigma_1-\sigma_2)} = \sqrt{\frac{\sigma_1^2}{2N_1} + \frac{\sigma_2^2}{2N_2}}$$

An observed difference may be tested by the following formula:

$$\frac{|S_1 - S_2|}{S_{(S_1-S_2)}}$$

S values (see Section 14.5) will be used in practice since population values are seldom if ever available. The results may then be evaluated as described in Case I of Section 17.3 above.

Example. Group A consists of 50 observations having $S_1 = 18$. Group B consists of 45 observations having $S_2 = 11$.

$$\frac{18 - 11}{\sqrt{\frac{324}{100} + \frac{121}{90}}} = \frac{7}{\sqrt{4.58}} = \frac{7}{2.14} = 3.27$$

Since the result obtained exceeds 3.00, there can be little doubt that Group A has significantly more variability than Group B. It is assumed in this test that the two sets of observations are uncorrelated and that the parent populations are approximately normal.

Case II. Either or both samples consist of 31 items or less. Since the sampling distribution of σ becomes markedly nonnormal for small samples, a different procedure must be used to evaluate sample differences when N is small. We compute

$$F = \frac{S_1^2}{S_2^2}$$

or we may compute $\frac{S_1}{S_2}$ and square the resulting figure. The larger value is always used in the numerator. We then enter the table of F (Table 17.5) using the appropriate arguments (use nearest argument given when $n > 6$, and ∞ when $n > 30$). If the entry is exceeded for the probability level selected, the difference between the two samples may be said to be significant.

TABLE 17.5*. $F = S_1^2/S_2^2$

n^2	Probability levels			n^2	Probability levels		
	0.05	0.01	0.001		0.05	0.01	0.001
	$n_1 = 1$				$n_1 = 6$		
1	161.45	4,052.2	405,280.0	1	233.99	5,859.0	585,940.0
2	18 51	98.50	998.5	2	19 33	99 33	999.3
3	10.13	34.12	167.5	3	8 94	27.91	132.81
4	7.71	21.20	74.14	4	6.16	15 21	50.53
5	6.61	16.26	47 04	5	4 95	10 67	28.84
6	5.99	13.74	35 51	6	4.28	8.47	20.03
8	5.32	11.26	25.42	8	3.58	6 37	12.86
12	4.75	9.33	18 64	12	3 00	4 82	8.38
24	4.26	7.82	14 03	24	2 51	3 67	5.55
∞	3.84	6.63	10 83	∞	2 10	2 80	3.74
	$n_1 = 2$				$n_1 = 8$		
1	199.50	4,999 5	500,000 0	1	238.88	5,981 6	598,140 0
2	19.00	99.00	999 0	2	19.37	99 37	999.4
3	9.55	30.82	148.5	3	8 85	27.49	130.6
4	6.94	18.00	61.25	4	6.04	14 80	49.00
5	5.79	13.27	36 61	5	4 82	10 29	27.64
6	5.14	10 92	27.00	6	4 15	8.10	19.03
8	4.46	8 65	18.49	8	3.44	6.03	12.04
12	3 89	6.93	12 97	12	2.85	4.50	7.71
24	3 40	5.61	9.34	24	2.36	3.36	4.99
∞	3.00	4 61	6.91	∞	1 94	2 51	3.27
	$n_1 = 3$				$n_1 = 12$		
1	215.71	5,403.3	540,380.0	1	243.91	6,106 3	€10,670.0
2	19.16	99.17	999.2	2	19.41	99.42	999.4
3	9 28	29 46	141 1	3	8.74	27.05	128.3
4	6.59	16 69	56 18	4	5 91	14.37	47.41
5	5 41	12.06	33 20	5	4.68	9.89	26.42
6	4.76	9 78	23 70	6	4.00	7.72	17.99
8	4.07	7.59	15.83	8	3.28	5.67	11.19
12	3.49	5.95	10.80	12	2 69	4.16	7.00
24	3.01	4.72	7.55	24	2.18	3.03	4.39
∞	2.60	3.78	5.42	∞	1.75	2.18	2.74

TABLE 17.5.—*Continued*

n^2	Probability levels			n^2	Probability levels		
	0.05	0.01	0.001		0.05	0.01	0.001
	$n_1 = 4$				$n_1 = 24$		
1	224 58	5,624.6	562,500 0	1	249 05	6,234 6	623,500.0
2	19.25	99 25	999.2	2	19 45	99 46	999 5
3	9 12	28 71	137 1	3	8 64	26 60	125 9
4	6.39	15 98	53 44	4	5 77	13 93	45 77
5	5 19	11 39	31 09	5	4 53	9 47	25 14
6	4 53	9 15	21 90	6	3 84	7 31	16 89
8	3 84	7 01	14 39	8	3 12	5 28	10.30
12	3 26	5 41	9 63	12	2 51	3 78	6 25
24	2 78	4 22	6 59	24	1 98	2 66	3.74
∞	2 37	3 32	4 62	∞	1 52	1 79	2 13
	$n_1 = 5$				$n_1 = \infty$		
1	230 16	5,763 7	576,400 0	1	254 32	6,366.0	636,620 0
2	19 30	99 30	999 3	2	19 50	99 50	999 5
3	9 01	28 24	134 6	3	8 53	26.12	123 5
4	6 26	15.52	51 71	4	5 63	13 46	44 05
5	5 05	10 97	29 75	5	4 36	9 02	23 78
6	4 39	8 75	20 81	6	3 67	6 88	15.75
8	3 69	6 63	13 49	8	2 93	4 86	9.34
12	3 11	5 06	8 89	12	2 30	3 36	5.42
24	2 62	3 90	5 98	24	1 73	2 21	2 97
∞	2 21	3 02	4 10	∞	1 00	1 00	1 00

* Table 17.5 is abridged from Table I of Frederick E. Croxton and Dudley J. Cowden: "Two Extensions of the F Table," Prentice-Hall, Inc., New York, by permission of the authors and publisher.

Example. Sample A consists of 6 items having $S_1^2 = 54.8$. Sample B consists of 26 items having $S_2^2 = 12.6$. Is the standard deviation of sample A significantly greater than the standard deviation of sample B? Computing F we obtain $\frac{54.8}{12.6} = 4.35$. Entering the F table we use n_1 (degrees of freedom) $= 5$ (associated with the larger square) and $n_2 = 25$ (enter the table at $n_2 = 24$, since this is the nearest value given). The F values at the 0.01 and 0.001 levels are 3.90 and 5.98 respectively. We conclude that the samples are significantly different at the 0.01 level, but not at the 0.001 level. The action we then take will be based upon the risk we are willing to run of being wrong. If we wish a high level of assurance that the difference is significant and it is practicable to obtain them, we will get more samples.

CHAPTER XVIII

ACCEPTANCE SAMPLING

18.1. Lot-by-lot inspection by attributes. One of the basic problems that faces almost everyone concerned with evaluating product quality is how to decide on the basis of a sample whether a lot should be considered as being of acceptable quality. By a "lot" we generally mean a quantity of material that can be conveniently handled as a segment of production. It may consist of a certain number of items, a case, a day's production, a carload, or such similar quantity. These lots might be described as "convenience lots." The items in such a lot are not necessarily all essentially alike.

A "statistical lot" is one in which the variations in quality from item to item are chance variations. Thus in such a lot, all of the items are essentially alike. There may be any number of items from one up in such a lot. Lot-by-lot sampling plans are customarily based upon convenience lots, that is, they are designed to deal with such lots. Unless otherwise specified, our references to lots will mean "convenience lots".

We have already seen that 100 per cent inspection does not guarantee a perfect product. Also, 100 per cent inspection often is not practicable because of time or cost. Good sampling inspection, where the items inspected are examined carefully may even provide better assurance of product quality than 100 per cent inspection with its elements of fatigue, carelessness, etc.

When asked what sort of quality level he wants in the product, the designer or customer frequently replies that he expects all items to meet the specification; however, unless the specification is very liberal with respect to what the process can make, he is very unlikely to get "perfect product". If a very large quantity of any item is made, some defective items are almost certain to be made. Further, if any defective items are made, sooner or later some of the defective items will be accepted. It is generally agreed that good 100 per cent inspection will remove only about 85 to 95 per cent of the defective material, while very good 200 or 300 per cent inspection may remove over 99 per cent of the defective items, particularly if one or more of the inspections is automatic and mechanical. Even then we cannot hope for 100 per cent.

Since 100 per cent inspection is generally not justifiable economically, the problem becomes one of deciding upon the amount of sampling that will be necessary to provide a reasonable degree of assurance as to the quality of the material being inspected. Unfortunately, ordinary judgment is not to be

trusted as a basis for a plan of accepting material on samples. Such plans as taking a sample consisting of 10 per cent of the lot and not allowing over 2 per cent of the sample to be defective, or taking a sample of 10 items and allowing no defectives almost invariably fail miserably to achieve the purpose for which they are intended. These two plans will be discussed more fully in Section 18.14.

A fundamental principle to remember is that in the long run, what is accepted from a controlled process will be essentially the same as what is offered for inspection. Even from an uncontrolled process this will still be generally true. The only possible improvement comes from the defective items that are caught in the samples and removed or replaced with good items. The uninspected portions of the lots accepted will be the same in the long run as the material offered for inspection.

18.2. Acceptable quality level (AQL). Assuming perfect 100 per cent inspection (which of course is not humanly attainable), the only way to discriminate perfectly between acceptable and unacceptable lots would be to inspect them 100 per cent. Slightly less than 100 per cent inspection would be possible if some defectives are allowed. Thus in a lot of 1000 items, if 4 per cent of defectives are allowed, it would be necessary only to inspect the lot until 960 good items (or 41 bad items) were found; however, for good lots this is so close to 100 per cent inspection that we immediately discard it as impracticable.

It is obvious that if we go to a sample size that is only a small part of the lot, there will be some risk of making a mistake in judging an individual lot. It will not be possible, for example, to accept all lots 1 per cent defective or less and reject all lots over 1 per cent. Fortunately, such a separation is not realistic from the viewpoint of practical operations. A process that can be operated economically with material 1 per cent defective probably will not suddenly become uneconomical at a level of 1.1 per cent defective, though it may become uneconomical if the material is as much as 4 per cent defective.

The first step in deciding upon a satisfactory sampling plan will be to decide upon a quality level that is acceptable. This level (AQL) is commonly designated by the symbol p_1. It is the fraction defective that can be tolerated without serious effect upon further processing operations or customer reaction. Nearly all such lots should be accepted (only perfect inspection would assure acceptance of ALL of them). The AQL must be determined by practical considerations.

18.3. Lot tolerance (p_t). The only real protection against unsatisfactory material is to use a sampling plan that will reject most of the lots offered that would seriously interfere with further processing operations or cause too much unfavorable customer reaction. Lots of this quality level are com-

monly designated by the symbol p_t or p_2. The lot tolerance must also be determined by practical considerations.

18.4. Producer's risk (α). The producer's risk is the risk the producer runs of having lots of quality p_1 rejected. It is desired to keep this risk as low as possible. The most practicable level is generally considered to be about 0.05 (5 per cent). This means that in the long run about one lot in 20 will be rejected, provided the lots are coming from a process controlled at quality level p_1. To reduce this risk much further would require an increase in sampling that would generally be uneconomical. Of course, the producer may decrease his risk by producing material at a better quality level than p_1. The extent to which this will be practicable will be governed by the economical considerations involved.

18.5. Consumer's risk (β). The consumer's risk is the risk the consumer runs of accepting lots of quality p_2. This risk is generally set at about 0.10. This means that in the long run, only about one lot in ten of quality p_2 will be accepted. This will generally be ample protection, as few producers can afford to have nine out of ten lots rejected. As the quality becomes worse, the portion of rejected lots increases.

18.6. The determination of sample size and acceptance number. As a working basis for the determination of sample size and acceptance number (allowable number of defectives in the sample, indicated by the symbol c) for lot quality protection, we may start with the proposal that we shall consider the sampling plan satisfactory if it rejects nine out of ten lots containing a stated per cent defective. Suppose we set p_2 at 4 per cent (0.04) defective and start with a sample size of 50 and an acceptance number (c) of 1. The probability of accepting a lot 4 per cent defective under this plan can be determined by consulting Fig. 18.6. The base line is pN on a logarithmic scale. N is the sample size and p is the process level proportion defective expressed as a decimal. In the present case $p = 0.04$ and $N = 50$, so $pN = 2.0$. We then follow the vertical line for $pN = 2.0$ until it intersects the curved line for $c = 1$. From this point we follow across to the scale on the left margin where we read the probability, 0.42, that there will be one or no defectives in the sample. This is the probability of acceptance of such a lot. The balance to make up 1.00 is 0.58 which is the probability of rejection. Since we want a probability of rejection of 0.90, this plan is unsatisfactory.

It is immediately obvious that we can increase the probability of rejection (and correspondingly decrease the probability of acceptance) by lowering the acceptance number c to 0. This gives a probability of rejection of 0.86. Since this is still not high enough we must increase N. This time let us start on the probability scale at the left at a probability of acceptance of 0.10 (probability of rejection of 0.90) and read across horizontally until we reach

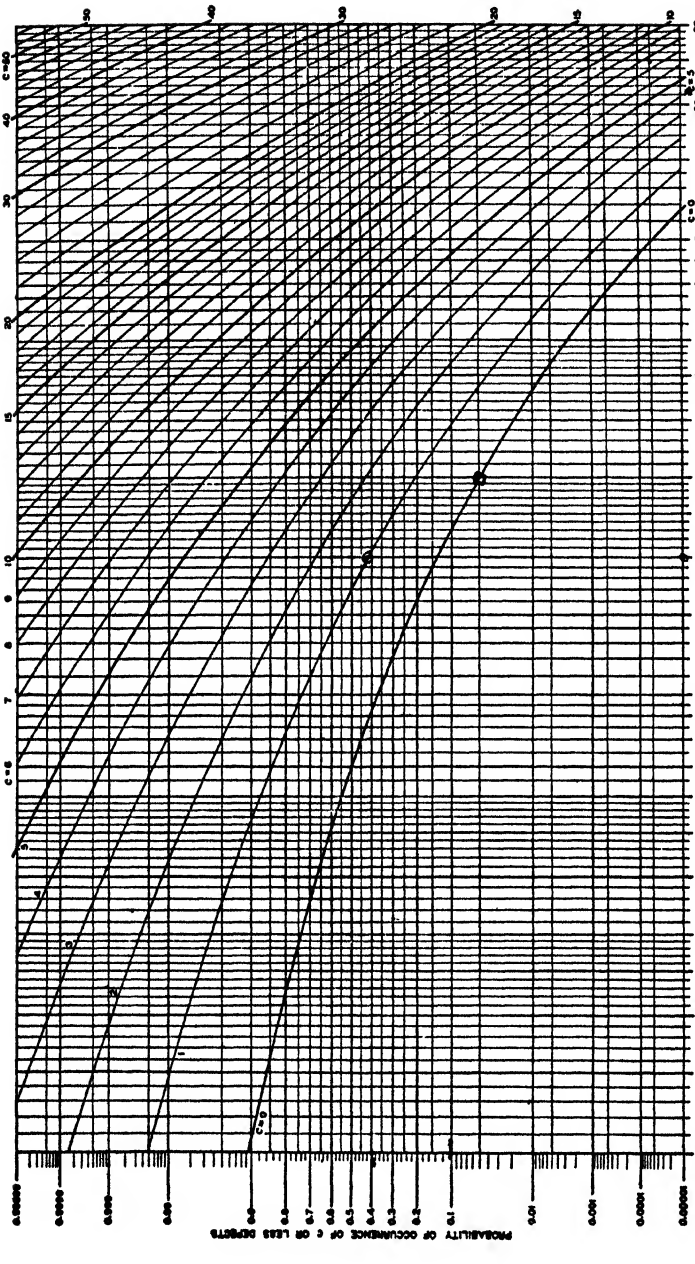

Fig. 18.6. Cumulative probability curves—Poisson exponential. For determining probability of occurrence of c or less defects in a sample of N pieces selected from an infinite universe in which the fraction defective is p (a modification of chart by Miss F. Thorndike, *Bell System Technical Journal, October, 1926*). Reproduced from H. F. Dodge and H. G. Romig, Single and Double Sampling Inspection Tables, *Bell System Technical Journal*, vol. 20, no. 1, pp. 1–61, January, 1941, by permission of the *Bell System Technical Journal*.

the intersection at $c = 0$. Then reading down from this point we find the value of pN is 2.3. Since p is set at 0.04, N must be $\frac{2.3}{0.04} = 57.5$. It is impossible to sample half an item, and since either 57 or 58 would be inconvenient to remember, it is suggested that we set the sample size at 60. This gives us a probability of rejection of 0.91 (which would be satisfactory as far as we have gone). These results are tabulated in the first four lines of Table 18.6a. Subsequent lines in the table show the effect of the plan $N = 60$, $c = 0$ upon the probability of acceptance as the quality offered (p)

TABLE 18.6a. SAMPLING PLAN FOR A GIVEN CONSUMER'S RISK

N	p	c	pN	Prob. of acceptance	Prob. of rejection
50	0 04	1	2.0	0 42	0.58
50	0 04	0	2 0	0.14	0 86
57.5	0 04	0	2 3	0.10	0.90
60	0.04	0	2 4	0 09	0 91
60	0.01	0	0 6	0.55	0 45
60	0.02	0	1.2	0.30	0.70
60	0.03	0	1.8	0.16	0.84
60	0.04	0	2.4	0.09	0.91
60	0 05	0	3.0	0 05	0.95
60	0 06	0	3 6	0.03	0.97
60	0.01	1	0.6	0 87	0.13
100	0.01	1	1.0	0.74	0.26
100	0 02	1	2 0	0 38	0 62
100	0 03	1	3 0	0.21	0.79
100	0.04	1	4 0	0 09	0.91
100	0.05	1	5.0	0.04	0.96
100	0.06	1	6.0	0 02	0.98

varies, the plan $N = 60$, $c = 1$ at a quality level of 0.01 and the plan $N = 100$, $c = 1$ for various quality levels. Note that at a quality level of $p = 0.01$, and with $N = 60$, lowering c from 1 to 0 increases the probability of rejection. Also, allowing an acceptance number of 1 with a sample size of 100 gives the same protection against material that is 0.04 defective as an acceptance number of 0 does for a sample size 60, but allows greater acceptance of material that is only 0.01 defective.

Thus far we have arrived at a sampling plan that will provide a satisfactory consumer's risk at lot tolerance (p_2) quality, but have not considered the producer's risk. If we set the AQL at 0.01 and the producer's risk at 0.05,

neither the plan of $N = 60$, $c = 0$ nor $N = 100$, $c = 1$ will be satisfactory, as the respective probabilities of rejection are 0.45 and 0.26.

To arrive at a suitable producer's risk while retaining the desired consumer's risk, we construct Table 18.6b, using a constant $p_2 = 0.04$, $\beta = 0.10$ and increasing c by 1 for each line. Each line is completed by entering Fig. 18.6 from the left margin at 0.10, following across to the curve for the appropriate c and then reading down to get the value of pN. This value is divided by p (0.04) to give the value of N, which is then entered in the right side of the table under the producer's risk. Using the same c and $p_1 = 0.01$, pN is computed and the value of α determined from Fig. 18.6. We find that the sampling plan $N = 200$ and $c = 4$ gives the desired producer's and consumer's risks.*

18.7. The operating characteristic curve.† An operating characteristic curve portrays graphically the way a sampling plan operates as the

TABLE 18.6b. SAMPLING PLAN FOR GIVEN CONSUMER'S AND PRODUCER'S RISK

Consumer's Risk					Producer's Risk				
N	p	c	pN	β	N	p	c	pN	α
57 5	0 04	0	2 3	0 10	57 5	0 01	0	0 575	0.44
97 5	0 04	1	3 9	0 10	97 5	0 01	1	0 975	0 26
132 5	0 04	2	5 3	0 10	132 5	0 01	2	1 325	0.15
167.5	0.04	3	6 7	0 10	167 5	0 01	3	1 675	0 09
200	0 04	4	8 0	0 10	200	0 01	4	2 0	0.05

incoming quality level varies. From it may be read the probability of acceptance of lots according to the fraction defective in the material submitted for sampling inspection. Since material containing no defectives will always be accepted regardless of the value of c, all operating characteristic (OC) curves start with a probability of acceptance of 1.00. As long as the lots are required to contain at least one good item, lots 100 per cent defective will always be rejected and the OC curves will all terminate with a probability of acceptance of zero. The two most important points on an OC curve are, of course, the probabilities of acceptance at p_1 and p_2. Table 18.7 gives the probability of acceptance for the two plans $N = 200$, $c = 4$, and $N = 60$,

* For a graphical method that may be used in connection with single sampling plans, see: A. R. Burgess, "A Graphical Method of Determining a Single Sampling Plan," *Industrial Quality Control*, **4**: No. 6, 25–27 (May, 1948).

† So named by Col. H. H. Zornig (Research Associate in Metallurgy at the Hanford Engineering Works of the General Electric Research Laboratory) just prior to World War II, at which time he was in charge of the Ballistic Research Laboratory at Aberdeen Proving Ground, Maryland.

$c = 0$, at six different quality levels. These values are shown graphically in Fig. 18.7 (to plot the actual curves, more points than this would have to be calculated). The absence of an inflection point for the plan allowing no

TABLE 18.7. A COMPARISON OF TWO SAMPLING PLANS

N	p	c	pN	Prob. of Acceptance	N	p	c	pN	Prob. of Acceptance
200	0 01	4	2 0	0 95	60	0 01	0	0 6	0.55
200	0.02	4	4 0	0 63	60	0 02	0	1.2	0 30
200	0 03	4	6 0	0 29	60	0 03	0	1 8	0 16
200	0 04	4	8 0	0 10	60	0 04	0	2 4	0.09
200	0 05	4	10 0	0 03	60	0 05	0	3 0	0.05
200	0 06	4	12 0	0 008	60	0 06	0	3 6	0 03

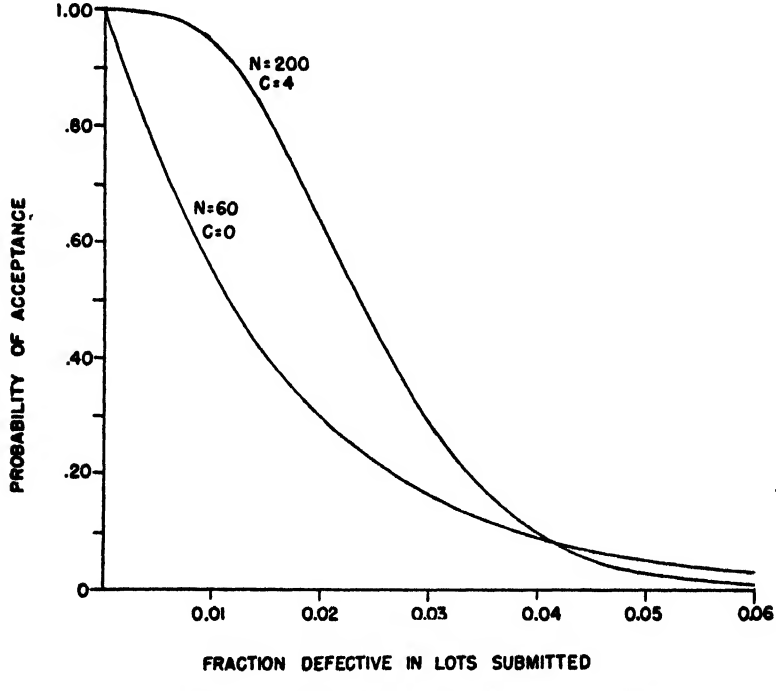

Fig. 18.7. Operating characteristic curves for two sampling plans.

defectives is characteristic of plans where $c = 0$. Since such plans provide a satisfactory producer's risk only when the fraction defective is extremely low, they are seldom used.

The type of sampling plan we have just described is generally referred to

as a single sampling plan, since it involves taking only one sample and deciding to accept or reject the lot on the basis of this sample.

18.8. Double sampling plans. Double sampling plans provide for the taking of a second sample when the results of a first sample are marginal, as will often be the case when lots are of borderline quality. Such plans are commonly based upon four requirements:*

N_1 = first sample size
c_1 = acceptance number for first sample
N_2 = second sample size
c_2 = acceptance number for $N_1 + N_2$

If c_1 is not exceeded in the first sample, the lot is accepted. If c_2 is exceeded in the first sample, the lot is rejected. If, in N_1, c_1 is exceeded, but c_2 is not, a second sample, N_2, is taken and the lot accepted if c_2 is not exceeded in the total sample and rejected if c_2 is exceeded.

Since a second sample is most frequently required when dealing with lots of marginal quality, it is possible to make N_1 smaller than the sample size for a single sampling plan having essentially the same OC curve. This means that good quality material will be accepted most of the time on the basis of a smaller sample than required by a comparable (with respect to the OC curve) single sampling plan. Also, bad lots will generally be rejected on the basis of the first sample. The general reduction in the amount of average inspection afforded by double sampling plans is one of their strong advantages (see Fig. 19.3).

A second advantage of double sampling plans is their strong psychological advantage in giving lots of marginal quality a "second chance." This feature has a strong appeal to practical minded production men.

The procedure for computing the OC curve of a double sampling plan may be illustrated with the following plan:

$N_1 = 100$ $\quad c_1 = 1$
$N_2 = 200$ $\quad c_2 = 3$

---

* Paul Peach in "An Introduction to Industrial Statistics and Quality Control," p. 103, Edwards & Broughton Co., Raleigh, N. C., 1947, adds a fifth requirement, k_1, which he designates "retest number." If not exceeded when c_1 is exceeded from N_1, a second sample is taken. If k_1 is exceeded, the lot is rejected without taking a second sample. This is a refinement which considerably complicates application of the plan and will generally be undesirable unless carefully trained and especially competent inspectors are available. When used it will reduce the average amount of inspection somewhat when dealing with lots of marginal quality.

In order to compute the probability of acceptance at a given level of incoming quality, say 0.01 fraction defective, we may set up the following table:

Possibilities	1st Sample	2nd Sample	Prob. of Acceptance
a	0 or 1	—	0 74
b	2	0 or 1	0 07*
c	3	0	0.01†
Total....			0 82

* This is the product of the probability of finding exactly two defectives in a sample of 100 (0.18), and the probability of finding 0 or 1 defective in a sample of 200 (0.41). To find the probability of exactly two defectives from Fig. 18.6, find the probability of two or less (curve for $c = 2$) and subtract from this the probability of one or less (curve for $c = 1$). The result is the probability of finding exactly two defectives.

† This is the product of the probability of finding exactly three defectives in a sample of 100 (0.06), and the probability of finding no defectives in a sample of 200 (0.14).

18.9. Sharpening the OC curve for double sampling plans when dealing with small sample sizes. Let us assume that practical limitations prevent using larger sample sizes than $N_1 = 5$ and $N_2 = 10$. If we set $p_1 = 0.10$, $p_2 = 0.50$, $\alpha = 0.03$ and $\beta = 0.10$, we cannot find suitable acceptance numbers. $c_1 = 1$ and $c_2 = 3$ give the desired producer's risk, but give a consumer's risk that is too high (0.30). $c'_1 = 0$ and $c'_2 = 2$ give the desired consumer's risk, but give a producer's risk that is too great (0.14). The OC curves for these plans are shown in Fig. 18.9.

We now need a way of providing that, when incoming lots are of approximately p_1 quality, we shall use the acceptance numbers $c_1 = 1$ and $c_2 = 3$, and that when incoming lots are of approximately p_2 quality that we shall use the acceptance numbers $c'_1 = 0$, $c'_2 = 2$. This may be accomplished by examining the number of defective items in the first samples (N_1) from the last six lots (30 items), each time a lot is inspected (strictly speaking, each series of six lots should be treated independently, but this refinement is not essential and disregarding it will not have any important consequence). If there are over seven defectives in these 30 items, use the c' acceptance numbers. This plan remains in effect until there are not over three defectives in the last six lots (N_1). Then use the c acceptance numbers and proceed as above.

In order to compute the OC curve for the combined effect of using both c and c' values, we let

A = probability of getting three or fewer defectives in 30 items
B = probability of getting four, five, six, or seven defectives in 30 items
C = probability of getting eight or more defectives in 30 items
L_i = probability that the ith lot is tested with the more liberal values c
S_i = probability that the ith lot is tested with the stricter values c'

Then

$$L_i = \frac{A}{1-B} = \frac{A}{A+C} \quad \text{and} \quad S_i = \frac{C}{1-B} = \frac{C}{A+C}$$

The value of B may be disregarded by using $A + C$ as the denominator in each case. We now set up a table to compute the probability of acceptance at various levels of incoming quality (p). The probabilities in Table 18.9 were read from the curves of Fig. 18.6.

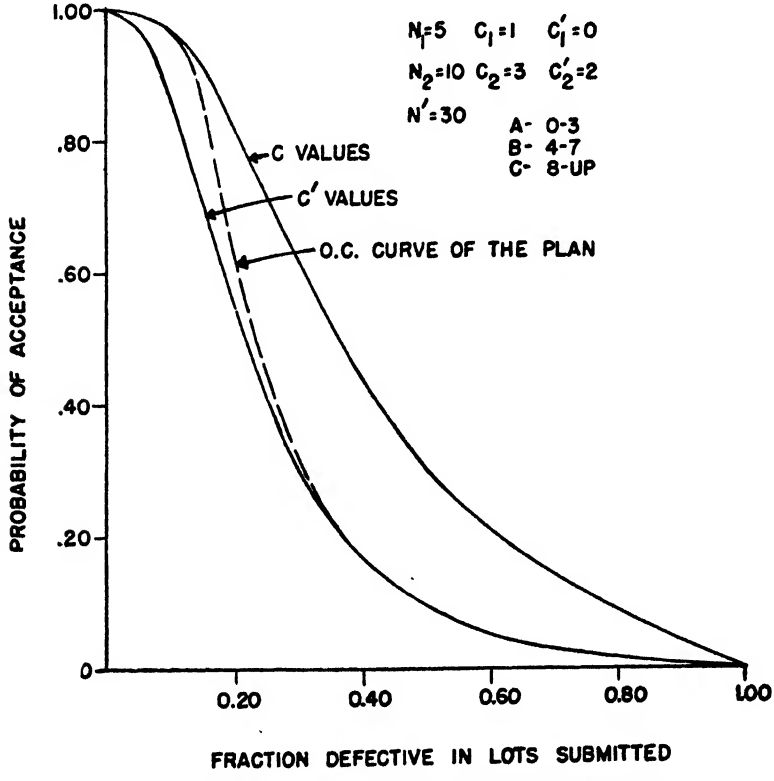

Fig. 18.9. Operating characteristic curve for a special plan.

The total probability of acceptance is plotted in Fig. 18.9 (dotted line) as the OC curve of the plan. The desired producer's risk of 0.03 is obtained and a consumer's risk of 0.095 (slightly better than the desired 0.10) is obtained.

While this illustration has been developed for a double sampling plan, the same sort of procedure may be used with single sampling plans.

18.10. Relationship between sample size and lot size. It will be noted that the discussion of the probability of acceptance of a lot has been

concerned only with the process level of fraction defective, the sample size, and the acceptance numbers. This is because of the fact that as long as the sample is a small portion of the lot (10 per cent of the lot size, or less), the size of the lot has no bearing on the probability of acceptance. A sample can only tell so much about the lot from which it comes, regardless of whether that lot consists of 500 items, 2000 items, or any other number. This is illustrated in Fig. 18.10. It will be noted that the width of the sampling fluctuations is related to the sample size and not to the lot size.

TABLE 18.9. DEVELOPMENT OF THE OC CURVE FOR A SPECIAL DOUBLE SAMPLING PLAN

p	N	pN	A	C	L_i	S_i	Frac. of lots acc. on basis of		Tot. prob. of acc.‡
							c values*	c' values†	
0.05	30	1.5	0 93	0 0002	0 9998	0 0002	0 9978	0 0002	0.998
0 08	30	2 4	0 78	0 003	0 996	0 004	0 981	0 004	0.985
0 10	30	3 0	0 65	0 012	0 982	0 018	0 95	0 015	0 97
0 12	30	3 6	0 52	0 03	0 945	0.055	0.90	0 04	0 94
0 15	30	4.5	0 34	0 09	0 79	0 21	0 72	0 15	0.87
0.20	30	6.0	0 15	0.26	0.37	0 63	0.30	0 34	0 64
0 25	30	7 5	0 06	0.48	0.11	0.89	0 08	0 36	0 44
0 30	30	9 0	0 02	0 68	0.03	0 97	0 02	0 29	0.31
0 40	30	12 0	0 002	0 91	0 002	0 998	0 001	0.17	0.171
0 50	30	15.0	0 0002	0 98	0 0002	0 9998	—	0 095	0.095
0 60	30	18 0	—	0 997	—	0 9999+	—	0 053	0.053
0 80	30	24 0	—	0 99995	—	0 9999+	—	0 018	0 018

* The entries in this column are obtained by multiplying the L_i column by the probability of acceptance when using the c values. *E.g.*, if $p = 0.05$, the probability of acceptance using the c values is 0.998 (determined from Fig. 18.6 as discussed in Section 18.8), and 0.9998 times 0.998 is 0.9978.

† The entries in this column are obtained by multiplying the S_i column by the probability of acceptance when using the c' values. *E.g.*, if $p = 0.20$, the probability of acceptance using the c' values is 0.547 (determined from Fig. 18.6 as discussed in Section 18.8), and 0.63 times 0.547 is 0.34.

‡ This column is the sum of the two previous columns.

There is a natural tendency to feel that the sample size should be larger for larger lots. This is true only where p_2 and β are not specified and where it is desired to hold the fraction defective as low as possible. Under such conditions we shall want to increase the sample size as the lot size increases because there is more at stake economically in a larger lot and our judgment of the lot should be more precise, and there is more chance that the lot may be nonhomogeneous, *i.e.*, it may come from a process exhibiting lack of control during the manufacture of one lot.

18.11. Average outgoing quality limit (AOQL). The average outgoing

quality limit is the worst possible quality that will be accepted in the long run if all defectives found in samples are replaced with good items and all

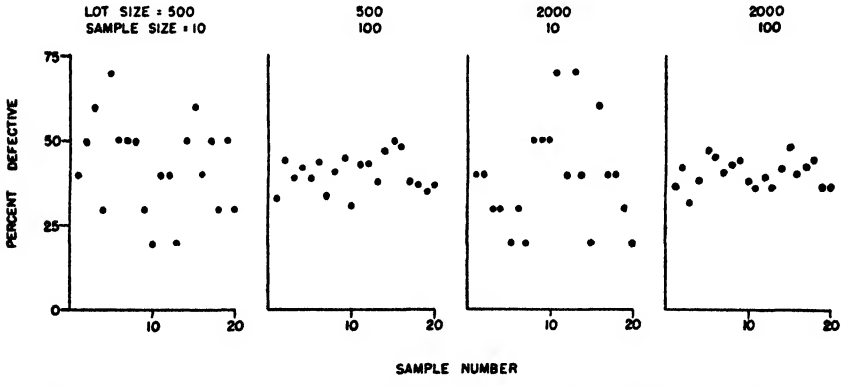

Fig. 18.10. Influence of lot size and sample size on sample variability. Note that sample variability is related to sample size, but independent of lot size. Drawings were made from lots 40% defective.

TABLE 18.11a. DEVELOPMENT OF THE AOQ CURVE FOR A GIVEN SAMPLING PLAN

$N = 100 \quad c = 2 \quad$ Lot size = 1000

p	Avg. no. def. in 100 lots	Average defectives removed from 100 lots							Def's. in acc. lots	AOQ
		Avg. def's. in samples			Avg. def's. in lots 100% insp.			Total def's. removed		
		Per lot	No. lots	Total	Per lot	No. lots	Total			
0 005	500	0 5	98 5	49.25	5	1 5	7.5	56 75	443.25	0.0044325
0 01	1000	1	92	92	10	8	80	172	828	0 00828
0.015	1500	1.5	81	121.5	15	19	285	406.5	1093 5	0 010935
0 02	2000	2	68	136	20	32	640	776	1224	0 01224
0 022	2200	2 2	63	138 6	22	37	814	952 6	1247 4	0.012474
0.0225	2250	2 25	62	139.5	22.5	38	855	994 5	1255 5	0.012555
0.023	2300	2.3	60	138	23	40	920	1058	1242	0.01242
0 025	2500	2.5	55	137 5	25	45	1125	1262 5	1237 5	0 012375
0.03	3000	3	43	129	30	57	1710	1839	1161	0.01161
0.04	4000	4	24	96	40	76	3040	3136	864	0 00864
0.05	5000	5	13	65	50	87	4350	4415	585	0 00585
0.06	6000	6	6	36	60	94	5640	5676	324	0 00324
0.07	7000	7	3	21	70	97	6790	6811	189	0.00189

rejected lots are inspected 100 per cent and then accepted after all defectives have been replaced with good items.

Let us suppose that samples of 100 are taken from lots of 1000 and that $c = 2$. Table 18.11a shows the average outgoing quality after all defectives

found have been removed. The results are shown graphically in Fig. 18.11. It is obvious that all incoming lots containing no defectives will be accepted

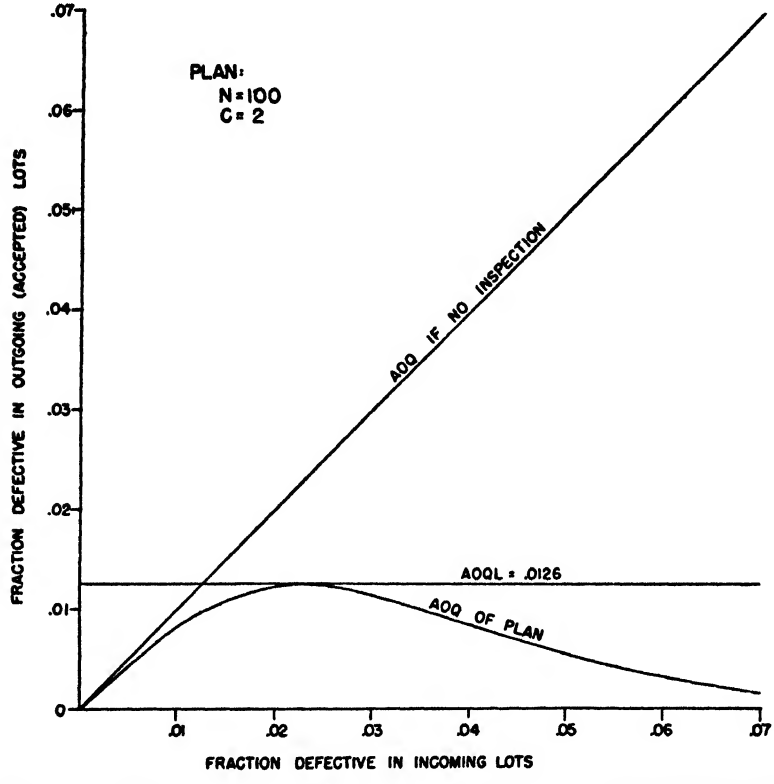

Fig. 18.11. AOQ curve for N = 100, c = 2. All defectives in samples and all defectives in lots inspected 100% must be replaced with good items.

TABLE 18.11b. SIMPLIFIED DEVELOPMENT OF THE AOQ CURVE FOR A GIVEN SAMPLING PLAN

Process fraction defective	Probability of acceptance	AOQ
0.01	0.92	0.0092
0.015	0.81	0 01215
0.02	0.68	0.0136
0.0225	0.62	0.01395
0 025	0.55	0.01375
0.03	0.43	0.0129
0.04	0.24	0.0096
0.05	0.13	0.0065
0.06	0.06	0.0036
0.07	0.03	0.0021

on a sampling basis, and there will be no defectives in the accepted material. Also, lots containing only defectives will all be rejected, inspected 100 per cent and all the material finally accepted will be free of defectives (100 per cent replacement of the defectives with all good items). Between these points, as the fraction defective in incoming lots becomes greater, the fraction defective will rise to a maximum and then fall off to zero when all lots are detailed.

It must be remembered that the AOQL is an average figure. Over a short period of time the AOQL may be exceeded. For the sampling plan of Fig. 18.11 the lot tolerance (with consumer's risk at 0.10) is 5.3 per cent defective although the AOQL is 1.26 per cent defective.

The procedure of computing the average outgoing quality (AOQ) can be greatly simplified by ignoring the matter of lot size and defectives found in the samples of accepted lots. The results will be generally satisfactory approximations. The process fraction defective is multiplied by the probability of acceptance to obtain the AOQ (since the lots 100 per cent inspected are entirely cleared of defectives and only the lots accepted on the sampling basis contain defectives). The data of Table 18.11a when treated in this manner give the results shown in Table 18.11b. This procedure gives an AOQL of 1.395 per cent, which is a little higher than that previously obtained (1.26 per cent).

18.12. Standardized sampling tables. While it is possible to construct a sampling procedure to suit your particular needs, it is recommended that, wherever possible, one of the previously published standardized tables be used. There are two advantages in doing this: first, considerable time and effort will probably be saved, and secondly, you will have the advantage of using a procedure previously tried and found satisfactory.

Among the more widely known standard tables are the single and double sampling inspection tables of Dodge and Romig*, and the Army Services Forces Standard Inspection Procedures†. The Dodge-Romig tables provide for either lot tolerance protection at a fixed consumer's risk of 0.10 or average outgoing quality limit protection. Sample sizes and acceptance numbers are provided for convenient intervals of lot size and process average. The producer's risk is not specified, but is kept satisfactorily low by providing different sample sizes and acceptance numbers when the process level shifts materially.

* H. F. Dodge and H. G. Romig, "Sampling Inspection Tables—Single and Double Sampling," John Wiley & Sons, Inc., New York, 1944.

† See G. Rupert Gause, "Quality through Inspection—Standardized Control Methods Insure Acceptable Ordnance Material," *Army Ordnance, 25:* No. 139, 117–120 (July–August, 1943). Also see E. L. Grant, "Statistical Quality Control," McGraw-Hill Book Co., Inc., New York, 1946, pp. 389–412.

The problem of how to adjust the relationship of N_1 to N_2 has been dealt with in the Dodge-Romig tables by using the combination that would minimize total inspection including detailed lots.

The Army Service Forces tables provide sample sizes and acceptance numbers for convenient intervals of Acceptable Quality Level (AQL) and lot size (designated sublot size in the tables). The AQL to be used is selected on the basis of the fraction defective that can be tolerated in the material accepted. This will involve customer reaction to unsatisfactory material, the extent to which defectives will be weeded out in further operations, and the extent to which assembly operations will be delayed by the presence of defectives. The judgment and experience of those most familiar with the process involved must be drawn upon to set up the most reasonable AQL.

Major defects are classified as those likely to result in the failure of the article to function as intended. *Minor* defects are those likely to cause the article to function with slightly reduced effectiveness or to interfere with subsequent assembly or repair. Here again, judgment and experience are necessary to classify specific defects.

The right side of Army Service Forces (ASF) Table III* (Table 18.12a) provides AOQL values through which the table may be entered when the producer is known to be supplying material worse than the AQL specified. ASF Table IV (Table 18.12b) provides for reduced inspection (one-fifth normal amount)when the stated conditions have been met. These conditions assure a process level that is probably better than the AQL. Through the use of c_c acceptance numbers applied to each five successive lots, the consumer's risk is kept essentially the same as in Table III. Since the ASF double sampling tables have been reproduced elsewhere (see footnote†) only the single sampling tables are given here. The double sampling tables have the psychological advantage of giving doubtful lots a second chance, and provide a smaller average sampling load than single sampling tables. Single sampling tables have the advantages of being easier to administer, and not requiring second samples, which can be very troublesome when dealing with a large variety of lots under crowded conditions. Other ASF tables that were developed during World War II are not discussed here because of their more specialized nature and restricted applicability.

The ASF double sampling tables provide for $N_2 = 2N_1$. This arrangement has a very definite psychological advantage as it is natural to want to increase the sample size materially when dealing with lots of marginal quality.

The producer's risk in the ASF tables is approximately 5 per cent. The

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TABLE 18.12a. ARMY SERVICE FORCES TABLE III—NORMAL SINGLE SAMPLING
TABLE III. SINGLE SAMPLING LOT BY LOT ACCEPTANCE INSPECTION

Sub-Lot Size		500 to 799	800 to 1,299	1,300 to 3,199	3,200 to 7,999	8,000 to 21,999	22,000 to 109,999	110,000 to 549,999	550,000 and Over	Approximate AOQL (Per cent Defective)
Sample Size		75	115	150	225	300	450	750	1,500	
Acceptable Quality Level (Per cent Defective)		Acceptance Number	Acceptance Number	Acceptance Number	Acceptance Number	Acceptance Number	Acceptance Number	Acceptance Number	Acceptance Number	
Major	Minor	c	c	c	c	c	c	c	c	
005- 010		*	*	*	*	*	*	*	*	.06
.011-.020	.005- 010	*	*	*	→	→	→	→	1	.10
.021-.030	.011- 020	*	*	→	→	→	→	1	2	.15
.031-.060	.021-.030	*	→	→	→	→	1	2	3	.25
.061- 10	.031- 060	→	→	→	→	1	2	3	5	35
.11-.15	.061- 10	→	→	→	1	2	3	4	6	.50
.16- 25	11-.15	→	→	1	2	3	4	5	8	.80
26- 50	.16-.25	1	2	3	4	5	6	8	13	1 0
.51-1 0	26- 50	2	3	4	5	7	9	13	23	2.0
1.1-2 0	.51-1.0	3	4	5	8	10	14	23	←	3.0
2.1-3.0	1 1-2 0	4	6	8	11	14	20	←	←	4.0
3 1-4 0	2 1-3.0	5	8	10	14	18	26	←	←	5.0
4.1-5 0	3 1-4 0	6	9	12	17	22	←	←	←	6.0
	4.1-5.0	8	11	15	21	26	←	←	←	8.0

* Table not applicable in this region. Use Table VI.

→ Use sample size in first column to right in which acceptance number is shown for Acceptable Quality Level involved, except as indicated under Note 5 below.

← Use sample size in first column to left in which acceptance number is shown for Acceptable Quality Level involved.

CONDITIONS UNDER WHICH SINGLE SAMPLING INSPECTION (TABLE III) MAY BE USED:

1. Selection or inspection of a second sample in accordance with Table I is impracticable, unusually difficult, or excessive in cost.

PROCEDURE:

2. For Major Defects:
   a. Select sample of size indicated in Table III for sub-lot size involved.
   b. Determine in sample the number of articles, d, which contain Major defects.
      (1) If d does not exceed the c indicated for the sample size and Acceptable Quality Level, Major, involved: Pass sub-lot for Majors.
      (2) If d exceeds c: Return sub-lot to contractor.
3. For Minor Defects:
   Carry out above procedure with "Minor" substituted everywhere for "Major," using same sample wherever feasible.

4. Return to contractor all defective articles observed in any of the above inspections.

NOTES:

5. Above Procedure is satisfactory when process average is equal to or better than Acceptable Quality Level. If process average is poorer than this, use acceptance number that corresponds with the AOQL value which is equal to or next better than the Acceptable Quality Level (however, if a right-hand arrow (→) or an asterisk (*) appears instead of such acceptance number, or if the Acceptable Quality Level is better than 06 per cent, use the Procedure of Table VI).
6. AOQL, the Average Outgoing Quality Limit, is the poorest average quality accepted if all sub-lots returned to contractor are inspected 100 per cent and accepted after removal of all defective articles.

TABLE 18.12b. ARMY SERVICE FORCES TABLE IV—REDUCED SINGLE SAMPLING TABLE IV. REDUCED SINGLE SAMPLING LOT-BY-LOT ACCEPTANCE INSPECTION

Sub-Lot Size		500 to 799		800 to 1,299		1,300 to 3,199		3,200 to 7,999		8,000 to 21,999		22,000 to 109,999		110,000 to 549,999		550,000 and Over	
Sample Size		15		23		30		45		60		90		150		300	
Acceptable Quality Level (Per cent Defective)		Acceptance Numbers		Acceptance Numbers		Acceptance Numbers		Acceptance Numbers		Acceptance Numbers		Acceptance Numbers		Acceptance Numbers		Acceptance Numbers	
Major	Minor	c	c_c	c	c_c	c	c_c	c	c_c	c	c_c	c	c_c	c	c_c	c	c_c
.005- 010		*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
.011-.020	.005-.010	*	*	*	*	*	*	→	→	→	→	→	→	→	→	1	1
.021-.030	.011-.020	*	*	*	*	→	→	→	→	→	→	→	→	1	1	1	2
.031- 060	021-.030	*	*	→	→	→	→	→	→	→	→	1	1	1	2	1	3
.061- 10	.031-.060	→	→	→	→	→	→	→	→	1	1	1	2	1	3	2	5
.11-.15	061-.10	→	→	→	→	→	→	1	1	1	2	1	3	2	4	2	6
.16-.25	.11- 15	→	→	→	→	1	1	1	2	1	3	2	4	2	5	3	8
.26-.50	.16-.25	1	1	1	2	1	3	2	4	2	5	2	6	3	8	5	13
.51-1 0	26-.50	1	2	1	3	2	4	2	5	3	7	3	9	5	13	8	23
1.1-2.0	.51-1 0	2	3	2	4	3	5	3	8	4	10	5	14	7	23	←	←
2.1-3.0	1 1-2 0	2	4	3	6	3	8	4	11	5	14	7	20	←	←	←	←
3.1-4 0	2.1-3 0	3	5	3	8	4	10	5	14	6	18	8	26	←	←	←	←
4 1-5.0	3.1-4.0	3	6	4	9	5	12	6	17	7	22	←	←	←	←	←	←
	4.1-5 0	4	8	5	11	6	15	7	21	9	26	←	←	←	←	←	←

* Table not applicable in this region.

→ Use sample size in first columns to right in which acceptance numbers are shown for Acceptable Quality Level involved.

← Use sample size in first columns to left in which acceptance numbers are shown for Acceptance Quality Level involved.

CONDITIONS UNDER WHICH REDUCED INSPECTION (TABLE IV) MAY BE STARTED FOR ANY PARTICULAR CLASS OF DEFECTS:

1. Production has been continuous for the immediately preceding two weeks, and for not less than 20 sub-lots; and
2. The last 20 sub-lots have been inspected under Table III, and none have been returned to the contractor because of failure to meet the requirements of Table III for the class of defects involved; and
3. The per cent defective, computed from the combined samples from these 20 sub-lots and all other sub-lots inspected during the last two week period, is less than the lower boundary of the acceptable quality range being used (left-hand columns of Table III).

PROCEDURE UNDER REDUCED INSPECTION (TABLE IV):

4. Select sample, pass sub-lot, return sub-lot to contractor, etc., as in Procedure for Table III, but using sample size and acceptance number, c, of Table IV.
5. In addition to action on individual sub-lots under paragraph 4, the first five of such sub-lots inspected must be considered as a group, the second five as a group, etc. As the 2d, 3d, etc., sub-lot in each group is inspected, cumulate the number of defectives (articles which contain defects of the class involved) in samples from that group.
6. Return to contractor all defective articles observed in any of the above inspections.

CONDITIONS UNDER WHICH NORMAL SINGLE SAMPLING INSPECTION (TABLE III) MUST BE RESUMED FOR THE CLASS OF DEFECTS INVOLVED:

7. Each succeeding sub-lot shall be inspected under Table III,
   a. If a sub-lot is returned to contractor under paragraph 4; or
   b. If at any time during the inspection of successive sub-lots in a group of five sub-lots, the cumulated number of defectives in the samples so far inspected exceeds the c_t indicated in Table IV.
8. Inspection under Table III shall then continue until the conditions of paragraphs 1, 2, and 3 are again met.

values of p_2 and β are not stated; however, for any given value of p_2, the consumer's risk decreases as the sample size becomes larger. This is a very reasonable condition as there is more at stake in larger lots. Further, adequate protection can generally be obtained by entering the tables through the AOQL column when poor quality material is being received consistently.

While either the Dodge-Romig tables or ASF tables may be used in connection with either in-process or incoming sampling inspection, the Dodge-Romig tables are particularly recommended for use in connection with in-process inspection, and the ASF tables in connection with acceptance of incoming inspection.

18.13. The Dodge plan for sampling inspection of continuous production. Dodge has developed a sampling inspection plan for continuous production that provides AOQL protection*. It is especially suited

* H. F. Dodge, "A Sampling Plan for Continuous Production," *The Annals of Mathematical Statistics*, **14**, No. 3, 264–279 (Sept., 1943). Fig. 18.13a is reproduced from this paper with the permission of the author.

to conveyor line or assembly line operations where there is a continuous flow of items. Dodge describes the procedure as follows:

(a) At the outset, inspect 100 per cent of the units consecutively as produced and continue such inspection until i units in succession are found clear of defects.

(b) When i units in succession are found clear of defects, discontinue 100 per cent inspection, and inspect only a fraction f of the units, selecting individual sample units one at a time from the flow of product, in such a manner as to assure an unbiased sample.

(c) If a sample unit is found defective, revert immediately to a 100 per cent inspection of succeeding units and continue until again i units in succession are found clear of defects, as in paragraph (a).

(d) Correct or replace with good units, all defective units found.

Fig. 18.13a provides a basis for selecting a combination of f and i for any given AOQL. It will be noted that many combinations of f and i are possible for any given AOQL. The choice of a given combination will depend upon such factors as the value of p_t (see Fig. 18.13a) that can be tolerated, the portion of the production the inspector can handle (if this is less than 100 per cent of the production, additional inspection personnel will be required when on the 100 per cent basis), and the extent to which fluctuations in the inspection load can be dealt with when large shifts occur in the process quality level. (see Fig. 18.13b).

A number of things are desirable to know about a given plan. These are designated by the following symbols:

h = average number of units inspected in a "failure sequence" (a sequence terminating in a defective and consisting of i or less units)
G = average number of failure sequences that will be encountered before finding i units clear of defectives
u = average number of units inspected on a 100 per cent inspection basis following the finding of a defective
H = average number of sample units inspected in a period of sampling inspection
v = average number of units that will be passed in a period of sampling inspection
F = fraction of the total product units inspected in the long run
p_A = average outgoing quality for a given p
p_I = incoming fraction defective for the AOQL

The formulas for computing these values are as follows:

$$h = \frac{1}{p(1 - q^i)}[1 - q^i(1 + pi)]$$

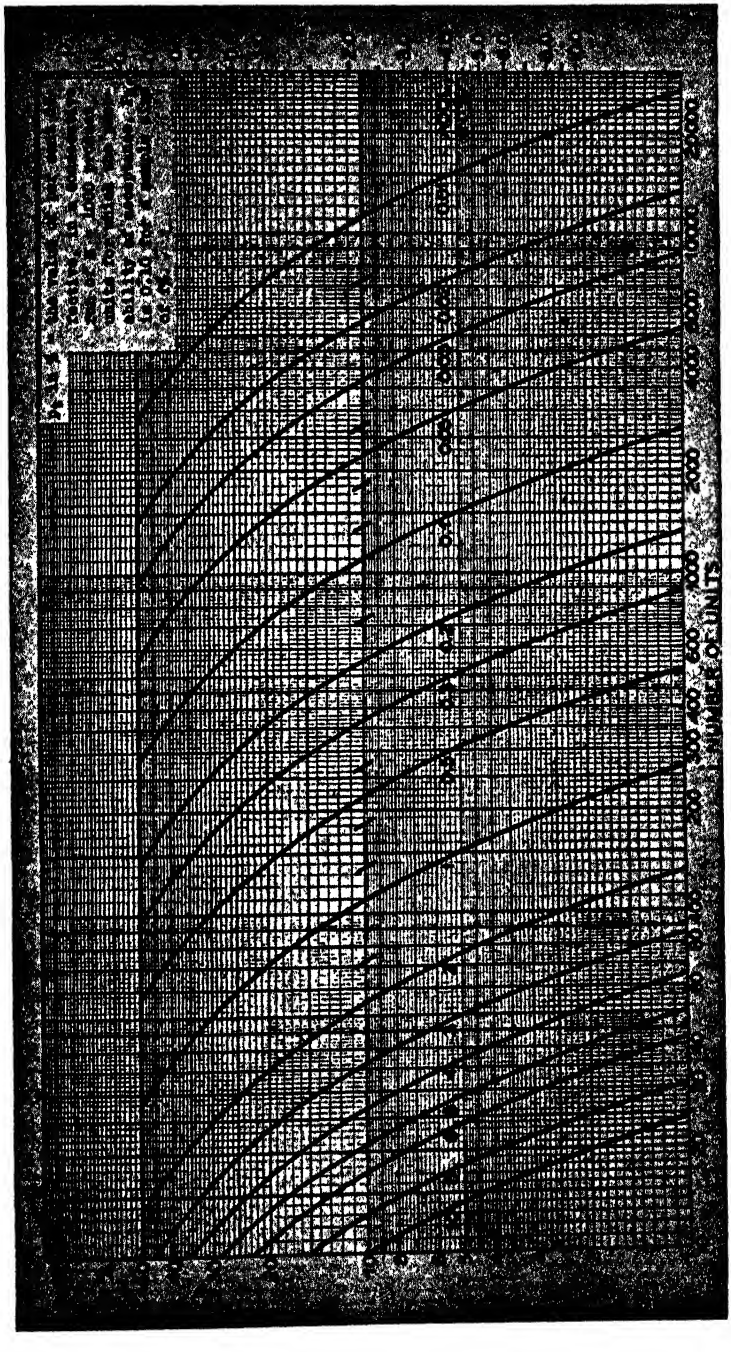


Fig. 18.13a. Curves for determining values of f and i for a given value of AOQL.

where $q = 1 - p$; q^i may be computed with the aid of logarithms to the base 10

$$G = \frac{1 - q^i}{q^i}$$

$$u = Gh + i = \frac{1 - q^i}{pq^i}$$

$$H = \frac{p}{(1 - q)^2} = \frac{1}{p}$$

$$v = \frac{H}{f} = \frac{1}{fp}$$

$$F = \frac{u + fv}{u + v}$$

$$p_A = p(1 - F)$$

$$p_I = \frac{1 + ip_L}{i + 1}$$

where p_L is the AOQL expressed as a fraction.

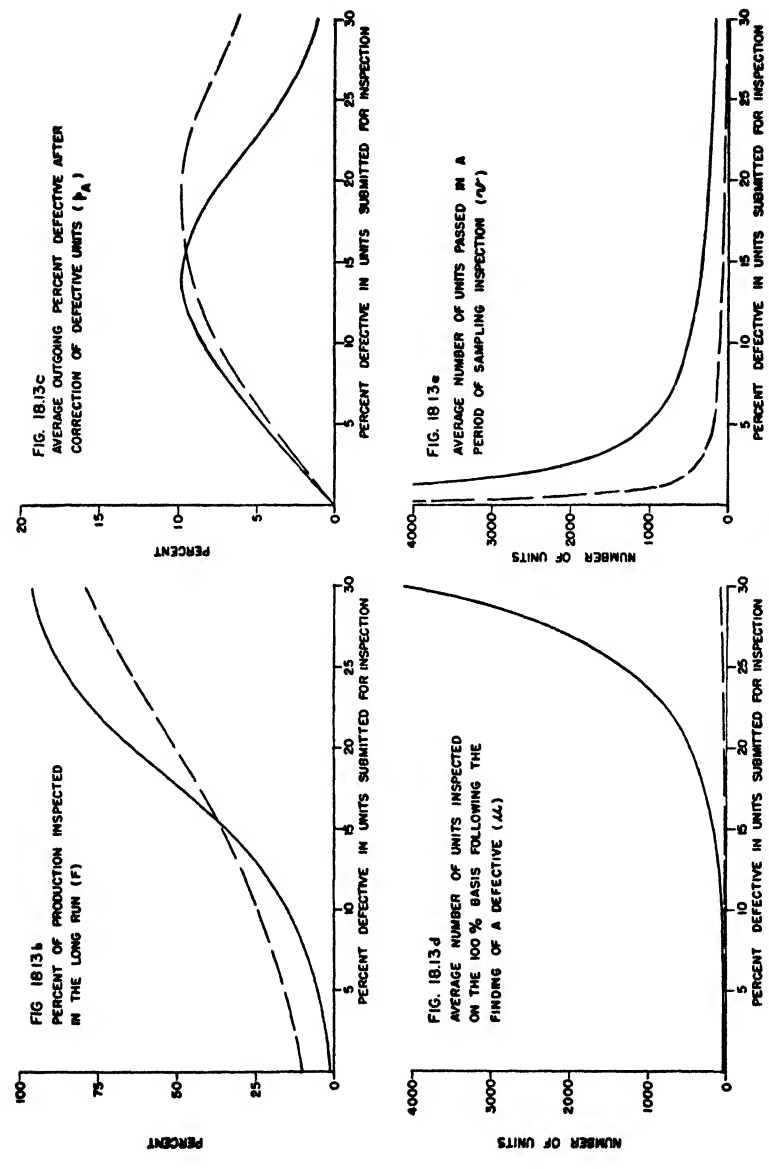
In the following table, these values are given for several values of p for the plan AOQL = 10%, $i = 20, f = 0.02$:

p	h	G	u	H	v	F	p_A
0 015	9.95	0 35	23.5	66 7	3333	0 027	0.015
0.05	8 83	1.79	35.8	20.0	1000	0.054	0.047
0.10	7.23	7.22	72.2	10 0	500	0.144	0.086
0 15	5.86	24 8	165 3	6.7	333	0 345	0.098
0.20	4.77	85 7	428.9	5 0	250	0 639	0.072
0.25	3.94	314.4	1258.6	4 0	200	0 866	0.034
0.30	3.32	1252.1	4177.1	3.3	167	0.962	0.011

$$p_I = 0.143$$

Figs. 18.13b, c, d, and e show the curves for F, p_A, u, and v for two plans, both having AOQL's of approximately 10 per cent. It will be noted that the one with a higher value for f gives a flatter curve for F (Fig. 18.13b), *i.e.*, a less severe change in the inspection load as p varies; also, the AOQL (Fig. 18.13c) is reached at a higher value of p, so that as long as p is less than 0.155, the AOQ will be slightly better. With the higher value of f there is not the likelihood that u will rise to impractical proportions should the quality level deteriorate markedly (Fig. 18.13d). On the other hand, the value of v will be smaller (Fig. 18.13e).

18.14. Defects of two common plans. In Section 18.1 we referred to two plans widely used in the past (and probably still used in some places) that generally fail to accomplish their intended purpose. One of these plans



Figs. 18.13b, c, d, and e. Aspects of Dodge continuous sampling.

was to inspect a sample of ten items and allow no defectives (the implication being that by allowing no defectives, a high standard of quality is being maintained).

Using Fig. 18.6 we compute the probability of acceptance of lots for several different values of incoming quality. This results in the following values:

Plan: $N = 10$ $c = 0$

p	pN	prob. of acceptance
0.01	0.1	0.91
0.02	0.2	0.82
0.05	0.5	0.61
0.07	0.7	0.50
0.10	1.0	0.37
0.14	1.4	0.25
0.23	2.3	0.10

If 2 per cent defective is good enough, the plan is hardly satisfactory, since it will accept only 82 per cent of such lots. On the other hand, material that is 7 per cent defective will be accepted 50 per cent of the time, 10 per cent defective material will be accepted 37 per cent of the time, and not until the material falls to 23 per cent defective do we reach the generally desired consumer's risk of 10 per cent.

The other plan was of the type that provides for taking a sample equal to 10 per cent of the lot and allowing not over 2 per cent of the sample to be defective. This plan implies that material 2 per cent defective or less is acceptable. The following table shows the probability of acceptance for two different lot sizes and four different quality levels:

p	lot size	sample size	c	prob. of acceptance
0.01	100	10	0*	0.91
0.01	20000	2000	40	0.99997
0.02	100	10	0*	0.82
0.02	20000	2000	40	0.54
0.03	100	10	0*	0.74
0.03	20000	2000	40	0.004
0.07	100	10	0*	0.50
0.07	20000	2000	40	nil

* Actually 0.2, but zero is the nearest whole number.

We immediately note that lots of different sample size under this plan do not have the same probability of acceptance even though the per cent defective in the lots is the same. Lots of 100 as much as 7 per cent defective

are accepted half the time, while lots of 20,000 are virtually never accepted. Lots that are 2 per cent defective are accepted only 82 per cent of the time, and lots of 20,000 only a little over half the time. While lots 1 per cent defective are accepted virtually all the time if consisting of 20,000 items, lots of 100 are accepted only a little over 9 out of 10 times.

One is forced to conclude that a "homemade" sampling plan should never be accepted as satisfactory until it has been adequately investigated.

CHAPTER XIX
SEQUENTIAL ANALYSIS

19.1. Multiple sampling. In Section 18.8 it was pointed out that one of the advantages of double sampling over single sampling is the fact that except for lots of marginal quality, the average amount of sampling required is less for the same protection. It would then be natural to suggest triple or quadruple sampling as a way to reduce the required amount of sampling still further. Unfortunately, such plans become very complex both to construct and to administer, and the small gain in reduction of sampling is insufficient to warrant them unless one goes all the way to sequential sampling.

19.2. Sequential analysis. The ultimate in multiple sampling is sequential analysis. It provides for examining sample items one at a time (or in small groups where desirable) and making a decision after each item inspected to accept the lot, reject the lot, or continue sampling. (See Fig. 19.2). This procedure provides a minimum amount of sampling. Although more complex than either single or double sampling, the savings in inspection required will often justify its use, especially where the testing is destructive or the samples costly to obtain.

Our consideration here of sequential analysis will be limited to the situation in which attribute inspection is used and the object is to accept or reject the lot. Applications of sequential analysis have been developed for other situations as follows*: "When the result of a single observation is a classification as good or bad and when the result of the test is a decision between two methods or products. When the quality being tested is measured and when the question is whether a standard is exceeded. When the quality being tested is measured and when the question is whether a lot differs from a standard. Variability of quality about the average."

19.3. Comparison of amount of sampling required by single and double sampling, and sequential analysis. Fig. 19.3 is based on three plans having essentially the same operating characteristic curves. The sequential analysis plan shows a marked advantage over single sampling regardless of quality of lots submitted. It shows very little advantage over double sampling until the per cent defective in lots submitted exceeds about 1 per cent. Above about 3 per cent defective, it shows a marked advantage over double sampling.

* See Statistical Research Group, Columbia University, "Sequential Analysis of Statistical Data: Applications," Columbia University Press, New York, 1945.

The curves of Fig. 19.3 apply strictly only to the plans shown. Other plans will produce different curves, but in general, the same basic relationships will hold.

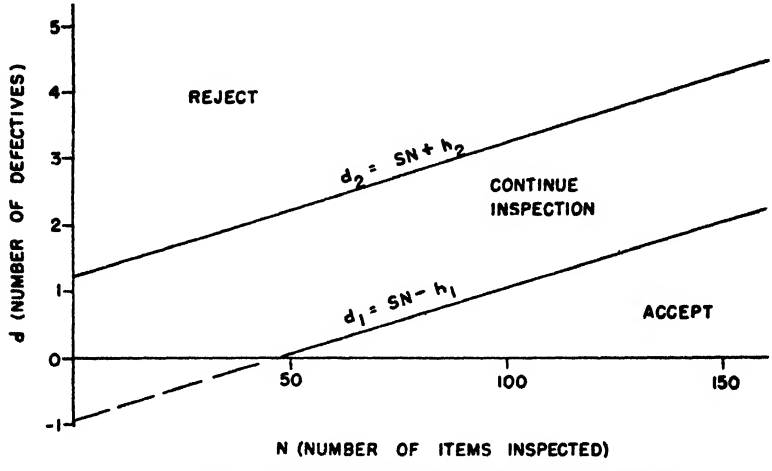


Fig. 19.2. A sequential analysis plan for attribute inspection.

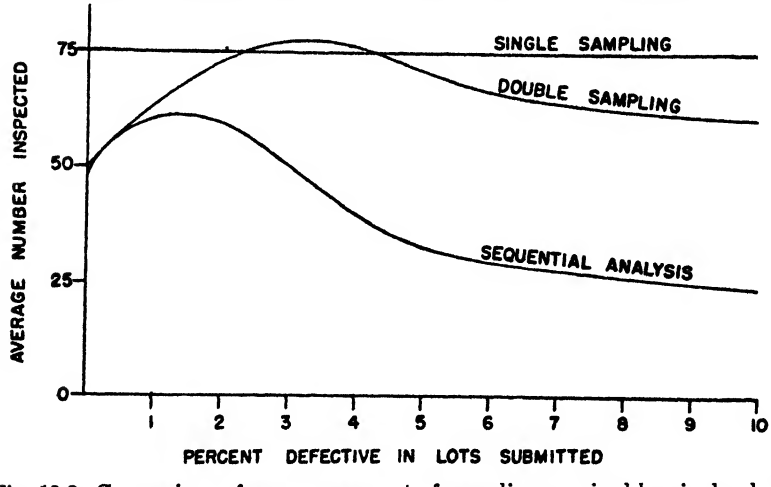


Fig. 19.3. Comparison of average amount of sampling required by single, double, and sequential plans having essentially the same operating characteristic curves. The single plan: N = 75, c = 1. The double plan: $N_1 = 50$, $N_2 = 100$, $c_1 = 0$, $c_2 = 2$. The sequential plan: $p_1 = 0.005$, $p_2 = 0.05$, $\alpha = 0.05$, $\beta = 0.10$ (see Section 19.4).

19.4. Computation of a sequential analysis plan. The first step in setting up a sequential analysis plan is to specify p_1 (good quality), p_2 (bad quality), α (producer's risk), and β (consumer's risk). For illustrative purposes we will use $p_1 = 0.005$, $p_2 = 0.05$, $\alpha = 0.05$, and $\beta = 0.10$. These

values must be based upon the practical and economic aspects of the product involved (see Sections 18.2, 18.3, 18.4, and 18.5).

The next step is the determination of the four values g_1, g_2, a, and b. These are obtained as follows:

$$g_1 = \log p_2 - \log p_1 = (8.69897\text{-}10) - (7.69897\text{-}10) = 1.00000$$
$$g_2 = \log (1 - p_1) - \log (1 - p_2) = (9.99782\text{-}10) - (9.97772\text{-}10) = 0.02010$$
$$a = \log (1 - \beta) - \log \alpha = (9.95424\text{-}10) - (8.69897\text{-}10) = 1.25527$$
$$b = \log (1 - \alpha) - \log \beta = (9.97772\text{-}10) - (9.00000\text{-}10) = 0.97772$$

From these values, the terms h_1, h_2, and s are computed for use in the equations that determine the sloping lines, as shown in Fig. 19.2.

$$h_1 = \frac{b}{g_1 + g_2} = \frac{0.97772}{1.02010} = 0.958455$$

$$h_2 = \frac{a}{g_1 + g_2} = \frac{1.25527}{1.02010} = 1.230536$$

$$s = \frac{g_2}{g_1 + g_2} = \frac{0.02010}{1.02010} = 0.019704$$

The acceptance and rejection lines are then determined by the following equations:

$$d_2 \text{ (rejection line)} = sN + h_2 = 0.019704N + 1.230536$$
$$d_1 \text{ (acceptance line)} = sN - h_1 = 0.019704N - 0.958455$$

In preparing a sampling table, d_2 is taken to be the next higher whole number to the computed value, and d_1 is taken to be the next smaller. If d_2 is equalled or exceeded after N items have been inspected, the decision is made to reject the lot. If d_1 is equalled or a value less than d_1 is obtained after N items have been inspected, a decision is made to accept the lot. If a value between d_1 and d_2 is obtained, inspection is continued.

If it is not convenient to take samples one by one, they may be taken in groups of say 10, 20, 50, 100 or any other convenient number at a time and the decision based upon the total inspected. For the above plan we might use a table as follows:

N	d_1	d_2
20	—	2
40	—	3
60	0	3
80	0	3
100	1	4
120	1	4
140	1	4
160	2	5

Note that acceptance at $N = 20$ or 40 is not permissible (even though there have been no defects thus far) since d_1 does not take on a positive value until $N = 49$.

While the maximum amount of sampling (on the average) required to make a decision will generally occur when the quality of lots submitted lies between p_1 and p_2, the average amount of sampling required at those two quality levels will generally suffice to appraise any given plan. These values may be determined as follows:

$$\bar{N}_{p_1} = \frac{(1-\alpha)h_1 - \alpha h_2}{s - p_1} = \frac{(0.95)0.958455 - (0.05)1.230536}{0.019704 - 0.005} = 58$$

$$\bar{N}_{p_2} = \frac{(1-\beta)h_2 - \beta h_1}{p_2 - s} = \frac{(0.9)1.230536 - (0.1)0.958455}{0.05 - 0.019704} = 33$$

It will be noted that the group sampling table above was terminated at $N = 160$. Theoretically it could be carried to an infinite sample size. Of course, for a lot of borderline quality, the sampling could go on for an unduly long time without a decision being reached. In practice, the sample size required to reach a decision will seldom exceed two or three times N_{p_1} or N_{p_2}. For practical reasons it is generally desirable to stop at such a point and make an arbitrary decision if one has not already been reached. The lot may be rejected if has not been accepted so far. This practice would have the effect of slightly increasing the producer's risk. The lot may be accepted if it has not been rejected so far. This would be accomplished above by changing the value for d_1 at $N = 160$ to 4. This practice would have the effect of slightly increasing the consumer's risk. It is the procedure most generally favored. Some other solution may be used, such as deciding to accept according to whether the number of defectives is closer to d_1 or d_2, or the lines for d_1 and d_2 may be tapered together in any way desired.

It may be desirable to quickly determine the smallest sample numbers at which acceptance (m_0) and rejection (m_1) are possible. These values are determined as follows:

$$m_0 \text{ is given by the next larger integer to } \frac{h_1}{s}$$

$$= \frac{0.958455}{0.019704} = 49$$

$$m_1 \text{ is given by the next larger integer to } \frac{h_2}{1-s}$$

$$= \frac{1.230536}{0.980296} = 2$$

Thus, acceptance first becomes possible under the above plan when 49 items have been inspected and all are good. Rejection first becomes possible when two items have been inspected and both are defective.

19.5. Aids to the use of sequential analysis. Much of the labor required in working out a sequential analysis plan has been eliminated by graphical and tabular aids prepared by the Statistical Research Group (see footnote, Section 19.2). In addition, the Navy Department has prepared a set of standard sampling inspection tables that provide single, double, and sequential sampling plans in a wide variety of combinations of p_1, p_2, α, and β values*. A unique feature of these tables is that the operating characteristic curve associated with each table is given.

* General Specifications for Inspection of Material, Appendix X, "Standard Sampling Inspection Tables for Inspection by Attributes," Issued by the Navy Department, April 1, 1946, Government Printing Office, Washington 25, D. C., or see "Sampling Inspection," by the Statistical Research Group, Columbia University, McGraw-Hill Book Company, New York, 1948.

CHAPTER XX

LEAST SQUARES AND CORRELATION

20.1. Scatter diagrams. One of the simplest and most effective ways to present a trend or relationship graphically is by the use of a scatter diagram. Fig. 20.1 is a scatter diagram based on the following data:

X Brinell Hardness Number	Y Tensile Strength (psi)
235	117,300
249	115,300
243	120,100
267	133,900
250	121,700
247	119,300
238	111,900
247	113,500
242	114,100
251	116,200

These data consist of ten pairs of readings taken on ten different samples of a certain kind of steel. For each sample a Brinell hardness number (BHN) and the tensile strength (TS) were determined. We would like to know if there is any trend in the tensile strength values as the BHN increases, since the tensile strength test is destructive and it would be desirable to substitute the nondestructive BHN test if it will show the same trend of results. An examination of Fig. 20.1 immediately indicates that TS values tend to increase as the BHN increases.

Later in this chapter we shall discuss the matter of the closeness of the relationship (correlation) of the two methods of testing. For the present we shall confine ourselves to the problem of predicting tensile strength from the BHN test.

20.2. Method of least squares. While we could draw a line by just looking at the plotted points of Fig. 20.1 that would seem to show the trend in Y values as X increases, such a line would probably be too inaccurate for our purposes. The method of least square offers a much more satisfactory method for locating the trend line properly. The method provides for the determination of a straight line that meets two requirements: first, the algebraic sum of the vertical deviations from the trend line equals zero (which would be true of any straight line through the point $\bar{X}$, $\bar{Y}$ other than a vertical line); and secondly, the sum of the squares of the vertical deviations must be less than the sum of the squares from any other straight line (this limits the line to one position).

In plotting two variables, it is customary to plot the independent variable (cause, or thing from which the prediction is being made) along the X axis (horizontal). The dependent variable (effect, or thing being predicted) is plotted along the Y axis (vertical).

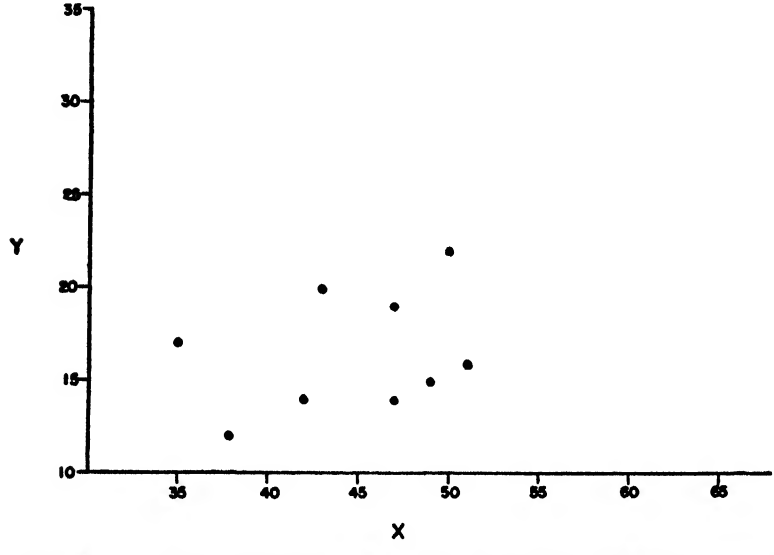


Fig. 20.1. A scatter diagram. X = Brinell hardness number and Y = Tensile strength in pounds per square inch. See section 20.2 for the abbreviated method of scaling the plotted values.

For the purpose of illustration, we may shorten the values tabulated in Section 20.1 as follows:

X = BHN	Y = TS
35	17
49	15
43	20
67	34
50	22
47	19
38	12
47	14
42	14
51	16

The dropping of 200 from each of the X values and 100,000 from each of the Y values in no wise affects the results. The hundreds in the case of the Y values are approximations and so are rounded off to the nearest 1,000.

The first step is to obtain the following sums: ΣX, ΣY, ΣXY, and ΣX^2.

These values are 469, 183, 8979, and 22691 respectively (using the shortened values). They are then substituted in the following simultaneous equations in which N is the number of pairs of observations:

$$\text{I. } \Sigma Y = aN + b\Sigma X$$
$$\text{II. } \Sigma XY = a\Sigma X + b\Sigma X^2$$

$$\text{I. } 183 = 10a + 469b$$
$$\text{II. } 8979 = 469a + 22691b$$

Solving, we get

$$\text{II. } 8979 = 469a + 22691b$$
$$\text{III. } 8582.7 = 469a + 21996.1b \text{ (I above times 46.9)}$$

Subtracting III from II we get

$$694.9b = 396.3$$
$$b = 0.571$$

Substituting $b = 0.571$ in I, we get

$$183 = 10a + (0.571)469$$
$$10a = -85$$
$$a = -8.5$$

Computed values of $Y(Y_c)$ may then be obtained from given values of X by using the estimating equation

$$Y_c = a + bX$$

In this case, $Y_c = -8.5 + 0.571X$. It must be remembered that this equation provides us with a basis for predicting the most likely value of Y for a given value of X, and not the reverse. To estimate the most likely value of X for a given value of Y, the values for X and Y may be interchanged and the above process repeated*. Also, care should be taken in extrapolating the relationship beyond the range of values in the original data. The trend may change for more extreme values.

From the estimating equation we find that when $X = 35.9$, $Y = 12$ and when $X = 67.4$, $Y = 30$. Fig. 20.2 shows a line plotted through these two points for the data of Fig. 20.1.

The use of simultaneous equations can be avoided by using the following formulas for a and b:

$$a = \frac{\Sigma X^2 \Sigma Y - \Sigma X \Sigma XY}{N\Sigma X^2 - (\Sigma X)^2}$$

$$b = \frac{N\Sigma XY - \Sigma X \Sigma Y}{N\Sigma X^2 - (\Sigma X)^2}$$

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* For a discussion of which regression line should be used, see Charles P. Winsor, "Which Regression?," *Biometrics Bulletin*, **2**, No. 6, 101–109 (Dec., 1946).

Only one of these values need be computed since

$$a = \bar{Y} - b\bar{X}$$

$$b = \frac{\bar{Y} - a}{\bar{X}}$$

and $\bar{X}$ and $\bar{Y}$ are readily available.

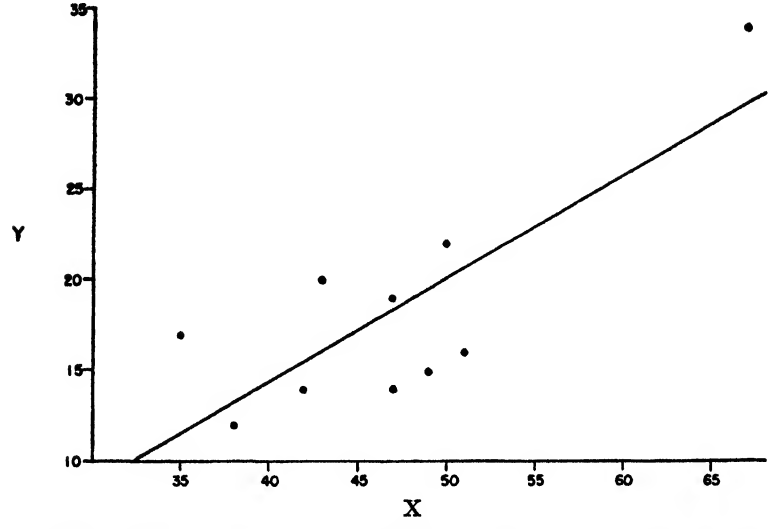

Fig. 20.2. Regression line for estimating most likely value of Y for a given value of X. Actually observed points will scatter around this line. This is the trend line or line of best fit obtained by the method of least squares.

20.3. The standard error of the estimate. Since our estimated value of Y, Y_c is only an estimate, we would like to know to what extent actual values may be expected to scatter around the estimate. This is given by

$$\sigma_{Y_e} = \sqrt{\frac{\Sigma Y^2 - (a\Sigma Y + b\Sigma XY)}{N}}$$

For the above data, $\sigma_{Y_e} = 3.68 (\sigma^2_{Y_e} = 13.55)$. Thus for a computed value of Y, observed values may be expected to lie between $Y_c \pm 3(3.68) = Y_c \pm 11.04$.

Since variance (standard deviation squared) is an additive quantity, the total variance of the Y values about the mean of Y is equal to the sum of the variance of the computed values of Y about the mean of Y, plus the variance of the observed values about the trend line (also referred to as a regression line). This is expressed in the following formula:

$$\sigma^2_Y = \sigma^2_{Y_e} + \sigma^2_{Y_e}$$

20.4. Correlation. Correlation provides a method for evaluating the extent to which variations in one variable are associated with variations in another variable. It does not determine whether there is a cause and effect relationship, but if this does exist, it permits an appraisal of the closeness of the relationship.

True correlation is generally considered as being confined to situations in which variation in one variable (dependent variable) is caused by a variation in the other variable (independent variable). For example, scientific experimentation has established the relationship distance equals velocity multiplied by time. If velocity is kept constant, then distance (dependent variable) increases with time (independent variable). The relationship is supposed to be, and is generally accepted, as an exact one. Recent theory suggests that the passage of time is slowed by increasing the velocity, that is, insofar as the object that is moving is concerned, but that the magnitude of the effect is inconsequential until one talks in terms of the speed of light; however, we cannot measure either time or distance with perfect precision. Experimental attempts to confirm the "law" will show some deviations from expected values if the measuring equipment is sufficiently precise. These would probably be so small that we would be inclined to discard them as unimportant. We would say that time and distance are almost perfectly correlated.

Two variables may seem to be related due to a common cause which affects each variable in the same or opposite ways. For example, an increase in the carbon content of steel causes (through changes in metallographic structure) the steel to be harder and to have a higher tensile strength. This correlation under some circumstances is good enough to permit the hardness test (nondestructive) to be substituted satisfactorily for the tensile strength test (destructive).

The two variables may be interacting. For example, higher prices may stimulate production, but excessive production may cause prices to fall.

Chance conditions may at times result in fairly high correlation values. For example, in a group of people it may be found that there is a very close correlation between the number of coins in their pockets and the heights of the individuals. Yet it would be difficult to explain why this should be so. Further, a different group of people might show widely different results.

20.5. Scale of correlation values. When variations in the dependent variable are due only to the independent variable, the variables are said to show perfect correlation (with sufficiently sensitive measuring equipment applied to the proper things we may approach but will never reach perfect correlation). Perfect correlation is expressed by the figure 1.00. If both variables increase simultaneously, the correlation is said to be posi-

tive. If one variable increases as the other decreases, the correlation is said to be negative. If there is no relationship at all between the two variables, the correlation is said to be neutral or zero. While correlation values may be computed to any number of decimals desired, it is customary to use only two decimal places.

Since traits of the mind are very difficult to measure, it is customary in educational and psychological problems to regard correlation values as unimportant unless they exceed plus or minus about 0.85. Industrial quality characteristics on the other hand are generally susceptible to much more precise measurement. Considerably lower correlation values may there-

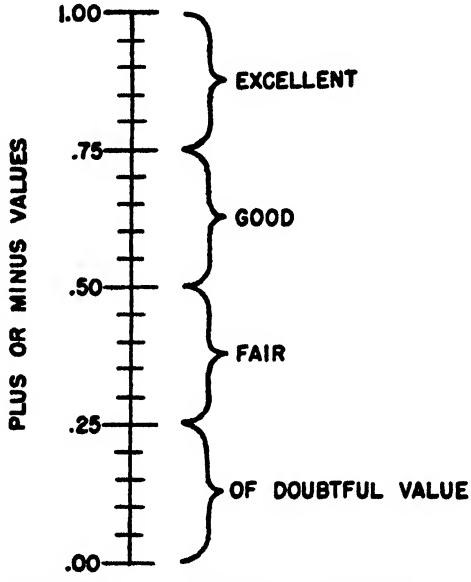

Fig. 20.5. Suggested scale of correlation values.

fore be considered important. The scale of Fig. 20.5 is suggested for general use. Of course such a scale would not apply in situations where prior knowledge indicates we should expect nearly perfect correlation, as in pairing repeat measurements on items highly uniform within themselves and when tested with equipment inherently capable of highly precise measurements. Care should be taken to avoid undue emphasis upon any correlation value, no matter how high, unless the sample size is large enough that the influence of chance effects may be disregarded (see Section 20.7).

It is not necessary that there be a true cause and effect relationship for a correlation to be useful. Two variables related to a common cause may be correlated with each other; however, in describing such a correlation,

all the relevant factors should be mentioned. In the example given above concerning the hardness and tensile strength of steel, the grade of steel (*i.e.*, its chemical composition and deoxidation practice) should be stated, the heat treatment (if any), and the extent to which the common cause(s) (such as carbon content or heat treatment) is varied. Without such information, attempts to repeat the experiment may result in widely different results.

20.6. Computation of correlation coefficient (r). The computation of the coefficient of correlation will be illustrated with the data of Section 20.2. Any one of three methods may be used.

Method a. If the regression equation has already been solved as in Section 20.2, the following equation may be used conveniently*:

$$r = \sqrt{\frac{(a\Sigma Y + b\Sigma XY) - \bar{Y}\Sigma Y}{\Sigma Y^2 - \bar{Y}\Sigma Y}}$$

$$= \sqrt{\frac{(-1555.5 + 5127.009) - 3348.9}{3707 - 3348.9}}$$

$$= \sqrt{\frac{222.609}{358.1}} \qquad = \sqrt{.6216}$$

$$= +0.79$$

The sign of r is the same as that of the b coefficient. In this case b was $+0.571$, so r also is positive.

Method b. If the standard error of the estimate has been calculated as in Section 20.3, the following equation may be used:

$$r^2 = 1 - \frac{\sigma_{Y_s}^2}{\sigma_Y^2}$$

From Section 20.3 we have $\sigma_{Y_s}^2 = 13.55$. $\sigma_Y^2 = \dfrac{\Sigma Y^2}{N} - \left(\dfrac{\Sigma Y}{N}\right)^2 = 370.70 - 334.89 = 35.81$. Hence,

$$r^2 = 1 - \frac{13.55}{35.81} = 1 - 0.3784 = 0.6216$$

$$r = +0.79$$

As before, r takes the sign of the b coefficient.

* All values needed except ΣY^2 were previously calculated. If a fully automatic calculating machine is used, the values of ΣX, ΣY, ΣX^2, ΣY^2, and $2\Sigma XY$ may all be obtained quickly and simultaneously in one operation.

Method c. If the regression equation is not of interest and the b coefficient not obtained, the following equation may be used:

$$r = \frac{N\Sigma XY - (\Sigma X)(\Sigma Y)}{\sqrt{[N\Sigma X^2 - (\Sigma X)^2][N\Sigma Y^2 - (\Sigma Y)^2]}}$$

$$= \frac{89790 - 85827}{\sqrt{(226910 - 219961)(37070 - 33489)}}$$

$$= \frac{3963}{\sqrt{24884369}}$$

$$= \frac{3963}{4988}$$

$$= +0.79$$

In this equation the correct sign for r is obtained automatically.

20.7. Chance limits of r. The chance distribution of r is symmetrical only when the population value of r is zero. It becomes more and more skewed as the population value approaches ± 1.00. It is also definitely non-normal when N is very small. An effort should be made to obtain an N of 30 (pairs of values) or more when computing a correlation coefficient.

To determine whether an observed r exceeds chance limits, assume that the two variables are unrelated. The standard deviation of r (when the population value is zero) is given by

$$\sigma_{(r=0)} = \frac{1}{\sqrt{N-1}}$$

If the observed r equals or exceeds $3\sigma_{(r=0)}$, assume the correlation coefficient to be significant; if it falls between 2.50 and 2.99, gather further data if at all feasible; if it falls between 2.00 and 2.49 sigmas, gather additional data if readily obtainable; if below 2.00 sigmas, assume it to be a chance result. When the observed r exceeds three sigmas we reject the hypothesis that the variables are uncorrelated, or in other words we say that the variables are correlated. Chance limits for a few selected sample sizes are given in Table 20.7.

A close approach to the true population value of the correlation coefficient requires either a large sample size or several repeated trials with smaller samples. In one investigation, only 12 pairs of observations were available in a period of a month. The study was continued for 14 months, each month a correlation coefficient being obtained. The successive monthly values were as follows: +0.73, +0.74, +0.10, +0.41, +0.66, +0.45,

+0.50, +0.42, +0.34, +0.68, +0.43, +0.37, +0.32, and +0.35. The average of these 14 values is +0.46. When it is noted that all 14 values are positive, there can be little doubt left that the two variables are correlated and that the true correlation is approximately +0.46.

20.8. Portion of observed variance explained by correlation. Going back to the formulas for r given in Section 20.6, we note that one of them is given in terms of r^2 as follows:

$$r^2 = 1 - \frac{\sigma_{Y_e}^2}{\sigma_Y^2}$$

Since σ_Y^2 represents the total variance of the Y values around $\bar{Y}$, and $\sigma_{Y_e}^2$, the variance of the Y values around the trend line (regression of Y on X) due to causes unexplained by the correlation, $\frac{\sigma_{Y_e}^2}{\sigma_Y^2}$ is the portion of the variance in Y values that is unexplained. Since the balance of the variance is the portion explained by the correlation (Section 20.3),

$$1 - \frac{\sigma_{Y_e}^2}{\sigma_Y^2}$$

is the portion of the variance of Y explained by the relationship, which, of course, is the same as r^2. Hence,

r^2 = portion of variance of Y explained by the correlation.

Thus, if $r = 0.50$, 0.25 (25%) of the variance in Y is due to the correlation of X and Y. The other three-fourths is due to some other variable or variables.

TABLE 20.7. CHANCE LIMITS OF r WHEN POPULATION VALUE IS ZERO

Entries are plus or minus values

N	2.0σ	2.5σ	3.0σ
30	0.37	0 46	0.56
50	0 29	0.36	0 43
100	0.20	0.25	0.30
200	0.14	0 18	0.21

20.9. Correlation of grouped data. When there are a large number of observations, it is generally more convenient to handle the data in terms of group intervals. To avoid loss of precision it is generally desirable to divide the range covered by each variable into at least 12 equal intervals. The following table uses fewer intervals for the purpose of convenience in

illustrating the procedure (the data used is an extension of the data presented in Section 20.2):

Class Lts. BHN / TS	Mid Value	17–24	25–32	33–40	41–48	49–56	57–64	65–72	73–80	fY	$d'Y$	$fYd'Y$	$\frac{fY}{(d'Y)^2}$
		20.5	28.5	36.5	44.5	52.5	60.5	68.5	76.5				
33–36	34.5						+8 1 8	+12 3 36	+16 1 16	5	4	20	80
29–32	30.5			−3 1 −3	0 1 0		+6 3 18		+12 1 12	6	3	18	54
25–28	26.5				0 3 0	+2 2 4	+4 2 8	+6 1 6		8	2	16	32
21–24	22.5			−1 1 −1		+1 2 2	+2 2 4	+3 3 9		8	1	8	8
17–20	18.5			0 3 0	0 3 0	0 2 0		0 1 0		9	0	0	0
13–16	14.5		+2 1 2	+1 8 8	0 6 0	−1 2 −2	−2 1 −2			18	−1	−18	18
9–12	10.5			+2 8 16	0 3 0	−2 1 −2				12	−2	−24	48
5– 8	6.5	+9 1 9	+6 1 6	+3 1 3	0 1 0					4	−3	−12	36
X		1	2	22	17	9	9	8	2	N=70		*	†
$d'X$		−3	−2	−1	0	1	2	3	4				
$fXd'X$		−3	−4	−22	0	9	18	24	8	‡		$\Sigma fd'Xd'Y = 157$	
$fX(d'X)^2$ ...		9	8	22	0	9	36	72	32	§			

* $\Sigma fYd'Y = 8$.
† $\Sigma fY(d'Y)^2 = 276$.
‡ $\Sigma fXd'X = 30$.
§ $\Sigma fX(d'X)^2 = 188$.

The first step is to tally the number of observations that fall in each block. This is indicated by the middle number in each block. The next step is to select a row in which it is believed the mean of Y falls and a column in which it is believed the mean of X falls. These need not be the correct columns as the formulas compensate for any error, but if the correct selections are made the computations will be easier. The next step is to indicate the moment of each block in terms of the number of intervals it is located away from the guessed mean intervals. Thus the block in which a Y of 26 and an X of 66 will fall has a moment of $+6$. It is two intervals above the mean Y row and three intervals higher than the guessed mean of X column. It will be noted that all blocks in the upper right and lower left quadrants will have positive moments, all blocks in the upper left and the lower right quadrants will have negative moments, and all blocks in the guessed mean intervals will have zero moment. The moments are indicated by the top figures in each block.

The next step is to multiply the block moment by the frequency of items in the block which gives the bottom figure in each block. The column headed fY is the sum of the items in each of the Y rows. The $d'Y$ column is the distance of the Y interval from the guessed mean interval in terms of number of intervals. The column headed $fYd'Y$ is the product of the previous two columns. The column headed $fY(d'Y)^2$ is the product of the two previous columns. These last two columns are summed (the $fYd'Y$ column being summed algebraically). A similar procedure is followed for the X values. The value in the lower right-hand corner of the table ($\Sigma fd'Xd'Y$) is obtained by summing algebraically the bottom figures in all the blocks.

The correlation coefficient is found by solving the following formula:

$$r = \frac{N\Sigma fd'Xd'Y - (\Sigma fXd'X)(\Sigma fYd'Y)}{\sqrt{[N\Sigma fX(d'X)^2 - (\Sigma fXd'X)^2][N\Sigma fy(d'Y)^2 - (\Sigma fYd'Y)^2]}}$$

$$= \frac{10990 - 240}{\sqrt{(13160 - 900)(19320 - 64)}} = \frac{10750}{\sqrt{12260(19256)}}$$

$$= \frac{10750}{\sqrt{236078560}} = \frac{10750}{15365} = +0.70$$

This illustration is based on only 70 items for the purpose of convenience in illustrating the procedure. This method is most useful when there are many more items to be correlated.

In order to determine the correct mean of the Y values, compute

$$\bar{Y} = \bar{Y}_d + \left(\frac{\Sigma fYd'Y}{N}\right)i$$

in which $\bar{Y}_d$ is the midpoint of the interval arbitrarily selected and i is the size of the group interval.

$$\bar{Y} = 18.5 + \left(\frac{8}{70}\right)4 = 18.5 + (0.114)4 = 19.0$$

$\bar{X}$ may be similarly determined by substituting X for Y throughout this equation.

$$\bar{X} = 44.5 + \left(\frac{30}{70}\right)8 = 44.5 + (0.429)8 = 47.9$$

The standard deviation of all Y's about their mean may be determined by computing

$$\sigma_Y = i\sqrt{\frac{\Sigma fY(d'Y)^2}{N} - \left(\frac{\Sigma fYd'Y}{N}\right)^2}$$

$$= 4\sqrt{\frac{276}{70} - \left(\frac{8}{70}\right)^2} = 7.92$$

The standard deviation of all X values may be determined in a similar manner. This value is found to be 12.72.

The estimating equation for Y is

$$Y_c = r\frac{\sigma_Y}{\sigma_X}(X - \bar{X}) + \bar{Y}$$

$$= 0.70\frac{7.92}{12.72}(X - 47.9) + 19.0$$

$$= 0.436(X - 47.9) + 19.0$$

$$= 0.436X - 20.9 + 19.0$$

$$= 0.436X - 1.9$$

The standard error of the estimate of Y_c is as follows:

$$\sigma_{Y_c} = \sigma_Y\sqrt{1 - r^2}$$

$$= 7.92\sqrt{1 - 0.49}$$

$$= 7.92(0.715) = 5.66$$

20.10. Spearman's rank correlation coefficient. Spearman's rank correlation coefficient (indicated by the small Greek letter rho, ρ) is useful where the number of items is small and only the general nature of the relationship is of interest. The equation for ρ is

$$\rho = 1 - \frac{6\Sigma D^2}{N(N^2 - 1)}$$

in which D is the difference in rank for the two variables. In the following illustration it will be noted that tied values are given the same rank.

Original Data		Rank		$D = (X-Y)$	D^2
X	Y	X	Y		
35	17	1	6	−5	25
38	12	2	1	1	1
42	14	3	2.5	.5	.25
43	20	4	8	−4	16
47	14	5.5	2.5	3	9
47	19	5.5	7	−1.5	2.25
49	15	7	4	3	9
50	22	8	9	−1	1
51	16	9	5	4	16
67	34	10	10	0	0
					79.5

$$\rho = 1 - \frac{6(79.5)}{10(99)} = 1 - 0.48 = 0.52$$

This procedure is simple to apply, but gives less precise results than the method discussed in the preceding sections.

GLOSSARY

A_2: a factor varying with sample size, which when multiplied by the average range gives three standard deviations of group averages for a normal population.

AOQ: average outgoing quality; for a given sampling plan and incoming fraction defective it is the average quality that will be accepted in the long run if all defectives found in the samples and in lots 100 per cent inspected are replaced with good items.

AOQL: average outgoing quality limit; for a given sampling plan it is the worst possible quality that will be accepted in the long run if all-defectives found in samples are replaced with good items and all rejected lots are inspected 100 per cent and accepted after all defectives have been replaced with good items.

AQL: acceptable quality level, the fraction defective that can be tolerated without serious effect upon further processing operations or customer reaction. It is commonly designated by the symbol p_1.

α: producer's risk (Greek small letter alpha).

Assignable cause: causes of quality variation that are identifiable and that may be eliminated or controlled economically.

Attribute measurements: one in which a quality characteristic is present or absent, or falls into a limited number of discreet categories.

β: consumer's risk (Greek small letter beta).

c: a correction used in computing a mean from a guessed mean interval.

c: defects per unit.

c: allowable number of defectives in the sample.

$\bar{c}$: average number of defects per unit

$\bar{\bar{c}}$: average of a series of $\bar{c}$'s.

Chance causes: causes of quality variation that can seldom be identified and that are never economical to control or eliminate. Individually their effect is negligible; collectively they make up the inherent variability in a constant cause system.

Consumer's risk: the risk the consumer runs of accepting lots of quality p_2.

Convenience lot: a number of items that can be conveniently handled for packing and shipping.

d: difference between an observation X and the mean $\bar{X}$.

d': number of class intervals away from a guessed mean interval.

d_2: a factor varying with sample size, which when multiplied by the population standard deviation of individuals gives the expected average group range.

D_3: a factor varying with sample size, which when multiplied by the average range gives the lower three sigma control limit for group ranges.

D_4: a factor varying with sample size, which when multiplied by the average range gives the upper three sigma control limit for group ranges.

Defect: a failure to meet a specified quality standard.

Defective: an item that contains one or more defects.

f: frequency of items in a class interval.

Group size: the number of successive observations that are considered together for control chart purposes.

i: size of a class interval.

I_2: a factor varying with sample size, which when multiplied by the average range gives three standard deviations of individuals for a normal population.

Kurtosis: a measure of the degree of peakedness of a curve.

LCL: lower control limit.

Lot tolerance: lots of a quality that would seriously interfere with further processing operations or cause too much unfavorable customer reaction. Lot tolerance is commonly designated by the symbol p_t or p_2.

Mean: the sum of the numbers divided by the number of items.

Median: that number which is exceeded by as many numbers as fall below it in any given group of numbers.

Mode: that value which occurs most frequently in a series of numbers.

N: the number of observations or items.

$\bar{N}$: average number of observations or items.

OC Curve: operating characteristic curve; a curve which portrays graphically the probability of acceptance of a lot according to the fraction defective in the lot, when sampled by a given sampling plan.

p: fraction defective.

$\bar{p}$: average fraction defective.

p_1: the acceptable quality level.

p_2: lot tolerance.

$| \ |$: magnitude of the difference between the numbers inside the parallel lines disregarding the sign of the difference.

pN: number of defective items in a group of N items.

$\bar{p}N$: average number of defective items in a series of groups of N items each.

p_t: lot tolerance.

Producer's risk: the risk the producer runs of having lots of quality p_1 rejected.

r: coefficient of correlation.

R: range; the difference between the largest and smallest number in a group of numbers.

$\bar{R}$: average range.

Random numbers: a series of random numbers consists of digits so selected that each time one is chosen, one digit is just as apt to occur as any other digit.

Range: see R above.

ρ: (rho) Spearman's rank correlation coefficient.

s: standard deviation computed without making a correction for sample size.

S: standard deviation computed with a correction for sample size.

Σ: sum of (Greek capital letter sigma).

σ: standard deviation, also called sigma (Greek small letter sigma).

Skewness: lop-sided or asymmetrical curve.

Standard deviation: the root mean square of the sum of the differences between the observed values and the mean.

Statistical lot: one in which the variations from item to item are chance variations.

t: used to designate a given number of standard deviations, as in $t\sigma$.

UCL: upper control limit.

Variables measurement: measurement on a scale which can theoretically be infinitely subdivided.

X: an observation on a quality characteristic expressed in a numerical value.

$\bar{X}$: the mean of a group of X's.

$\bar{\bar{X}}$: the mean of a group of $\bar{X}$'s; the overall mean.

Y_c: a computed value of Y based upon its correlation with X.

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