Electricity and Magnetism Classical and Modern '

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PREFACE

The incessant demand of our numerous students for a more or less comprehensive introductory account, both theoretical and experimental, of the main topics of the subject of electricity and magnetism has led us to write this text. In determining the scope and the mode of treatment of this book the needs and the mathematical equipment of students going in for B. Sc. (Pass) and B. Sc. (Hons.) degrees have been taken into account.

The book has been divided into two main parts : (1) Classical and (2) Modern. In the classical part which has been spread over the first twenty chapters, the fundamental principles of subjects such as electro and magnetostatics, magnetism, direct and alternating currents, electrical machinery, thermoelectricity, units and standards and Maxwell's theory of electromagnetic field have been dealt with. There are indeed elaborate treatises on these subjects, but being more mathematical they are unsuited to a student of physics with limited mathematical attainments. Emphasis is, therefore, laid on explanation of physical concepts with the aid of experiments and just the necessary amount of mathematical theory.

In the modern part which has been spread over the remaining eleven chapters we treat outlines of important recent developments such as the electron, electronics, the positive rays, atomic structure, x-rays, photoelectricity, natural and artificial radioactivity, the electron waves, cosmic rays, terrestrial magnetism and atmospheric electricity.

On these subjects also there are several excellent books available. They are, however, suitable more to the specialists than to the type of student for whom the present book is written. And, therefore, outlines of important recent developments in these various subjects, constituting what is called 'Modern Physics' are presented in a clear and simple manner. In this part as well, attention has been paid to the experimental as well as theoretical aspects of modern physics. It is hoped that these outlines will serve as a useful and stimulating introduction to more advanced studies.

An intimate grasp of a physical principle is obtained by solving numerical problems pertaining to the principle. Hence, some carefully selected problems with their solutions have been appended to many articles in the body of the text. It is hoped that with the help

of these examples a students will be able to solve any numerical problem given to him. As the text is not intended to serve as a reference book no bibliography is given.

This book makes no pretensions to completeness; but a glance at the table of contents will reveal that no important topic, perhaps with the exception of Electrical Images, has been left out. We propose to remove this deficiency in the next edition.

In preparing this book many standard works and original writings by eminent men of science have been freely consulted; our thanks are due to all these authors. We are indebted to all those who have given us help and encouragement when the text was being written. We are particularly grateful to, among others, Professor W.L. Bragg, Professor Blackett, Professor R. A. Millikan, Professor Anderson, Professor H. E. White, Professor O. Klemperer, Messers D. Van Nostrand Company, Messers Bell and Sons, the editor, American Journal of Physics and the Secretary, Physical Society, London, for permission to reproduce certain figures.

We will feel grateful to those of our readers who will be good enough to communicate to us helpful criticisms and suggestions directed towards further improvement of this introductory text.

Allahabad
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G. B. DEODHAR
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CONTENTS

CHAPTER XII

CHAPTER XIII

CHAPTER XIV

CHAPTER XX

CHAPTER XXI

CHAPTER XXII

CHAPTER XXIII

CHAPTER XXIV

Page

CHAPTER XXIX

De Broglie relation. The experiment of Davisson and Germer. G. P. Thomson's experiment. The dualistic nature of matter. The Schrödinger wave equation.

CHAPTER XXX

Discovery of cosmic rays. Variation of cosmic ray intensity with altitude and latitude. Interaction of cosmic ray with matter. The positron. The heavy electron or mesotron.

CHAPTER XXXI

Magnetic elements of earth's field. Experimental determination of declination, H and dip. Magnetic maps. Variations of terrestrial field. Theories of terrestrial magnetism. Atmospheric and terrestrial electricity. Electricity of thunderstorms.

PART I

CLASSICAL

ELECTROSTATICS

CHAPTER I

FUNDAMENTAL ELECTROSTATIC PHENOMENA

1. Positive and negative electrification. As early as about 25 centuries ago the Greek philosophers found that a piece of amber rubbed with silk acquired the property of attracting light, tiny bodies such as pith balls, bits of paper, etc. Later on Gilbert, a physician of Queen Elizabeth, showed that similar property is acquired by many other substances. This state of bodies is called electrification and the agency which causes the attraction and allied phenomena is called *electricity*, apparently from the Greek word for amber viz. "$\eta\lambda\varepsilon\kappa\tau\rho\rho\nu$". Electrostatics is the name given to that branch of physics which deals with forces acting between charged bodies. Here electricity is not in motion.

In contrast with this, that branch of physics which deals with the phenomena associated with electricity in motion or electric currents is called electrodynamics.

There are two kinds of electricity which we must suppose to exist—positive and negative. This can be easily shown as follows :—Rub a glass rod with silk and touch the knob of a gold leaf electroscope or a suspended gilded pith ball. The gilded pith ball becomes electrified. Proceed in the same way and electrify another gilded pith ball with the glass rod. The two pith balls are found to repel. If the experiment is now repeated by touching the pith balls with silk after rubbing it against a glass rod the pith balls again repel each other. If, however, we have two pith balls, one electrified by the glass rod and the other by the silk, they are found to attract each other. Thus we are led to conclude that there are two kinds of electricity and that two bodies similarly charged repel each other while two bodies oppositely charged attract each other. In quite an arbitrary manner we

assume that glass when rubbed with silk becomes positively charged and the silk negatively charged.

2. Triboelectric series. From observations on different substances it has been possible to prepare a list of substances such that a substance acquires positive charge when it is rubbed with a substance lower down in the list. The other substance of course acquires a negative charge. Such a list is called a *triboelectric series*.

It may be noted, however, that the position of a substance depends, to some extent, on factors like temperature, humidity, etc. The following is a triboelectric series taken from Smithsonian tables : fur, glass, mica, silk, cotton, amber, resins, metals, sulphur, platinum, celluloid.

3. Conductors and insulators. All substances are broadly divided into these two main classes. Those substances which on touching the electrified body remove its charge are called conductors. Those which do not remove, are called insulators. It must be noted, however, that there is no perfect conductor or no perfect insulator. The difference between the two lies only in degree. Metals in general are good conductors, solutions of acids and salts are next best. Oils, wax, quartz are good insulators and so are gases which are not ionised.

4. Electrification by induction. In explaining the positive and negative charges we charged the pith balls by bringing them into contact with glass or silk. The balls are charged by conduction. A body can be electrified by simply bringing the charged body close to the uncharged one. This is what is called *induction* process. Simple experiments tell us that the end of the body close to the inducing body receives a charge opposite in sign to that of the inducing body ; while the remote end receives a charge of the same sign (Fig $1 \cdot 1$, a). If the body on which charges are induced is divided into two and the two parts are widely separated while still under the influence of the inducing body we obtain two separate charges of opposite sign (Fig. $1 \cdot 1$, b). If

the two bodies are kept in contact and one is touched or earthed momentarily (Fig. 1·1, *c*) the negative charge flows to earth ; on
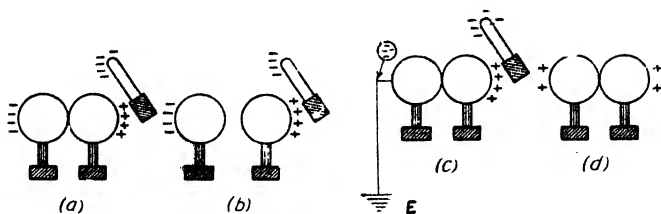
Fig. 1·1 Electrification by induction.

now removing the inducing body the remaining charge spreads over the two bodies which are thus charged positively by induction, (Fig. 1·1, *d*).

5. Theories of electrification. It will be interesting to give here a brief review of the old and new theories regarding production of electric charges.

About 200 years ago Benjamin Franklin proposed a *one-fluid theory* to explain electrical phenomena. According to this theory a certain indestructible weightless electric fluid can exist in all substances in different degrees. When a substance is uncharged there is some normal content of this electrical fluid. When a body is charged by rubbing there is a transfer of this fluid from one body to the other. The body whose normal content is increased by this transfer gets positively charged, while the other body, due to decrease in its normal content of the electrical fluid, gets negatively charged. In this way production of equal and opposite charges by friction is explained. The particles of matter were supposed to repel each other and attract the electric fluid. In the neutral or uncharged state the attraction and repulsion balanced each other and so no electrical effect is shown by uncharged bodies. It should be noted here that the universal law of gravitational attraction remains unaccountable on this view of electrification.

Later a *two-fluid theory* was introduced to explain electrical phenomena. According to this theory an electrified body

contains positive and negative electrical fluids in equal quantities. A charged body contains either an excess of positive fluid or an excess of negative fluid according as it is positively or negatively charged. Similar fluids repel each other while dissimilar fluids attract each other.

There is still another theory supported by Faraday, Maxwell and others. This is the *dielectric theory*. According to this theory electrical phenomena are mere manifestations of a certain kind of strain in the ether surrounding the charged body. To discharge a body means to remove this strain. Due to difficulties in the matter of proving the existence of ether this theory falls to the ground.

New concepts regarding the structure of matter gave rise to the *modern theory of electrification*. According to these concepts an atom consists of central positively charged nucleus round which electrons revolve in close orbits like the planets around the sun. The electron is the smallest particle having a negative charge of magnitude 4.80×10^{-10} e.s.u. Its mass is 9.0×10^{-28} gm. The nucleus consists of protons and neutrons. The proton has the same charge as that on the electron but opposite in sign. It has a mass of 1.66×10^{-24} gm. which is nearly the same as that of the hydrogen atom. It is thus about 1856 times heavier than the electron. The neutron is a neutral counterpart of proton but is slightly heavier than the latter. The nucleus is confined to a small sphere of diameter of the order of 10^{-13} cm. The electronic orbits have the dimensions of the order of 10^{-8} cm. The outer electrons are distributed in different electronic shells according to the Pauli principle (Ch. XXIV). The electrons in the deepest shell are most tightly bound to the nucleus. The strength of binding decreases as we go outwards. The electrons in the outermost shell are very loosely bound and are called the valence electrons. In a neutral atom the total number of electrons equals the total number of protons in the nucleus. For example a neutral atom of sodium has 11 electrons and 11 protons. Since the mass of the sodium atom is 23 atomic units, the number of neutrons in the nucleus is 12. The neutron contributes only to the mass of the

atom. When the sodium atom loses one electron it is left with 10 electrons and 11 protons. Thus there is a net unit positive charge on the atom giving us a sodium positive ion (Na⁺).

Substances, with which we normally deal, are aggregates of atoms and molecules. When glass is rubbed with silk some of the electrons of the surface atoms of glass being more loosely bound get detached and are transferred to the surface atoms of silk. The silk thus has an excess of electrons and is negatively charged; while the glass is deficient in the same number of electrons and is thus positively charged.

6. Coulomb's law. In 1785 Coulomb experimentally studied the force between two small charged balls. The apparatus used by him is called a torsion balance. The principle of this apparatus is roughly indicated in fig. 1·2. An insulating horizontal arm AB with a small metal ball B at one end is suspended at its mid point M by a fine torsion suspension S. The upper end of S is fixed to a graduated drum D. The ball B, is charged. A second ball C is mounted on an insulating handle I. When C is charged and brought near B the latter is repelled or attracted according as the electrification of B and C is of the same or opposite sign. In any case the suspension S is twisted. The amount of twist is measured by

Fig. 1·2 Coulomb's torsion balance.

turning the graduated drum D in the opposite sense so that B is brought back to its original position. The measured twist is proportional to the force of repulsion or attraction between B and C. The distance between B and C could be accurately measured. By varying this distance Coulomb found that the *force between two*

charged bodies is inversely proportional to the square of the distance.
Further, by keeping the distance fixed and varying the charges it
was found that the force is proportional to the product of the two
charges q', q.

$$F = \pm cqq'/r^2,$$

where c is a constant. For vacuum $c = 1$. Positive value indicates
repulsion and negative value attraction.

7. Experimental proof of the inverse square law.

There are several sources of uncertainty in Coulomb's experi-
ment. They are briefly as follows : (1)
the charges are not situated at points
and are not evenly distributed over
the spheres due to mutual influence,
(2) the effects of induced charges upon
the case and other parts, (3) the
charges gradually leak away due to
imperfect insulators supporting the
charged balls and (4) the amount of
torsion and the distance between the
balls cannot be measured accurately.
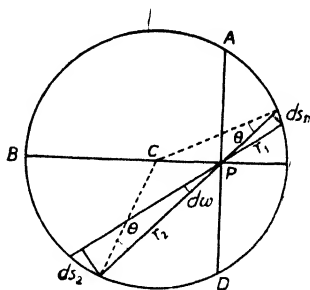
Fig. 1·3

To avoid these sources of error indirect proof of the inverse
square law was given by Cavendish and later by Faraday and
Maxwell. The principle of the method is as follows :—ABD is a
conducting hollow sphere with centre at C (Fig. 1·3). Let it be
charged. Consider a point P within the sphere. Draw through P
as vertex a double cone with a small solid angle d, to cut the sphere
in small areas ds_1 and ds_2. If r_1 and r_2 be the distances of P from
ds_1 and ds_2 respectively the area of the right circular cone at ds_1 is
$r^2_1 d\omega$ making an angle θ with ds_1. Thus $ds_1 = r_1^2 d\omega/\cos\theta$. Similarly,
$ds_2 = r_2^2 d\omega/\cos\theta$. If the sphere is symmetrically situated with
respect to the neighbouring conductors it will have a uniform
charge of surface density ρ. Thus the charge on ds_2 is $r_1^2 d\omega\rho/\cos\theta$
and that on ds_2, $r_2^2 d\omega\rho/\cos\theta$. Suppose that at P there is placed a
unit charge. Also suppose that the force varies inversely as the
nth power of the distance. Thus the force at P due to charge on

ds_1 is $r_1^2 d\omega\rho/r_1^n \cos\theta$ and that due to charge on ds_2 is $r_2^2 d\omega\rho/r_2^n \cos\theta$. These are oppositely directed and so will cancel each other if $n=2$. The whole spherical surface can be divided into similar pairs. Thus the resulting force at P is zero. If $n>2$ and the surface positively charged it can be easily seen that the resultant force is directed towards the centre of the sphere and that if $n<2$ it is directed away from the centre.

Cavendish and later Maxwell constructed two conducting con-centric spheres B and A separated from each other by insulators I (Fig. 1·4). The outer sphere had a very small opening H through which B could be connected to A by a suitably arrang-ed piece of wire W. At first A is positively charged and connected to *B*. Connection is then removed leaving B again insulated. As shown above B is charged positively if $n>2$, nega-tively if $n<2$ and uncharged if $n=2$. Cavendish using a pithball electroscope
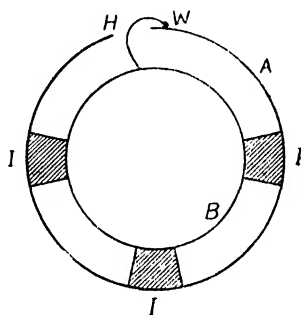
Fig. 1·4
Experiment of Cavendish

to detect charge on B concluded from the experiment that n lies within one per cent of the value of 2. Maxwell repeating Caven-dish's experiment in 1870 with an electrometer concluded that n cannot differ from 2 by more than $1/21600$.

Recently in 1936 Plimpton and Lawton carried out a very accurate experiment to prove the inverse square law. They placed a sensitive resonance electrometer within the spheres to measure any variable potential difference between them. A harmonically alternating high potential greater than 3000 volts was applied to the outer sphere and although the sensitivity of the electrometer was of the order of 10^{-3} volt no effect upon the instrument was detected. If the exponent in the law of force were not exactly 2 but $2\pm m$ then from the experiments it was concluded that $m<2\times10^{-9}$. For full details the student should read the original paper (Phys. Rev. Vol. 50, p. 1066, 1936).

8. Unit of charge. It is thus established beyond doubt that Coulomb's law is true. For vacuum we write it as $F = qq'/r^2$. It gives us immediately the definition of unit of charge. Put $F = r = 1$; then assuming the charges to be equal we have $q = q' = 1$. Thus the C. G. S. unit of electrical charge is that charge which repels a like equal charge placed at a distance of 1 cm. in vacuum with a force of 1 dyne. This unit and all the c. g. s. units based on Coulomb's law are known as electrostatic units (shortened as e. s. u.)

ELECTROSTATIC FIELD IN FREE SPACE

1. Electric Field. A region in which electric forces are acting is called an electric field.   To explore this field we have to move a small positively charged body from point to point and find the force exerted on it at each point.   If e is the charge on the small body and E is the force acting on unit charge at any point in the field, then the force on the charge e is given by

$$F = e\,E.$$

E is called the electric intensity, the electric vector or the electric field.   In exploring the field as described above one should be careful to see that the charges producing the field are fixed in their positions so as not to disturb the electric field due to repulsive or attractive action of the test charge e.   It is also necessary to see that when the force is measured the test charge e should remain fixed in position relative to the observer.   In general E varies in magnitude and direction at every point of the field although there may be points at which $E = 0$.

The electric intensity at a point at distance r from a charge q is given by $E = q/r^2$ by Coulomb's law.   If the field be due to a number of point charges the intensity at a point is found by combining vectorially the intensities due to the various charges. Except in some special cases this is a troublesome process.   The idea' of potential is introduced to simplify the methods for calculating the resultant intensity.

2.   Electric potential.   The concept of potential is of very great importance in the theory of electrostatic fields.   It asserts that for every electrostatic field there is a single valued potential function such that the electric intensity at a point is equal to its space rate of decrease or its negative gradient.   If V is the poten-

tial function at a point P (x, y, z)

$$E = -\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)V = -\text{grad. } V, \quad \cdot \quad \cdot \quad \cdot \quad (2\cdot1)$$

where the components of E along the three rectangular coordinate axes are

$$E_x = -\frac{\partial V}{\partial x}, \; E_y = -\frac{\partial V}{\partial y}, \; E_z = -\frac{\partial V}{\partial z}.$$

If a positive charge be placed in an electrostatic field it will be pushed from a region of higher to a region of lower potential, while opposite will be the case with a negative charge.

The potential V discussed above can be readily interpreted in terms of work. Let a unit charge be moved in an electrostatic field from a point P to a point Q, (fig. 2·1), without out disturbing the field, along any arbitrary path s. The work done by the electric intensity is
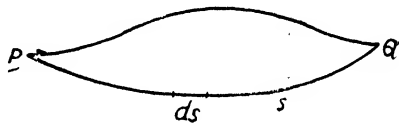
Fig. 2·1

$$W = \int_P^Q E_s \, ds,$$

where Fs is the electric intensity along ds. But by the concept of the potential $Es = -\frac{\partial V}{\partial s}$. Therefore the work done is

$$W = -\int_P^Q \frac{\partial V}{\partial s} \, ds = \left(V_P - V_Q\right) \quad \cdot \quad \cdot \quad \cdot \quad (2\cdot2)$$

That is, the work done by the field in this process is the excess of potential at P over that at Q. It is independent of the path by which the charge is taken from P to Q.

Potential is thus measurable only by its differences. If the field is limited we may suppose that for distant points $V_Q = 0$. The potential at a point P is then equal to the work done by the field in taking a unit positive charge from P to infinity. It can also be defined as the amount of work required to bring a unit positive charge from infinity to that point. It must be assumed that during this process the charges producing the field retain

their positions uninfluenced by the encroachment of the unit test charge. Physically speaking, potential is identical with the work per unit charge or potential energy per unit charge placed at the point in question. As potential is work or energy it is a scalar quantity. The work is taken positive when it is done upon the charge by an external agency and negative when it is done by the field.

A conductor has a constant value of potential. This follows from the fact that in electrostatic equilibrium the electric intensity in a conductor is zero.

3. Potential due to a point charge. In fig. 2·2, P is the point at which potential is to be found out due to charge q situated at O. The electric intensity at a point Q on a path PS is

$$E = q/l^2$$

in the direction along OQ. The work done in moving a unit positive charge from R to
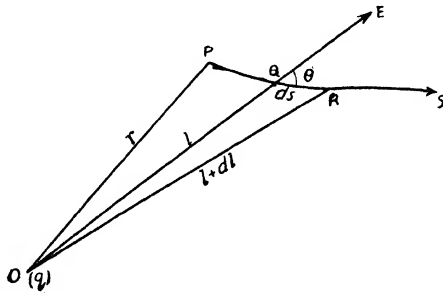
Fig. 2·2.

Q through a distance ds against the action of the electric field is

$$dV = \frac{q}{l^2} \cos \theta \, ds = -\frac{q}{l^2} \, dl \qquad \because \; ds \cos \theta = -dl$$

If the unit positive charge is moved against the field from ∞ to the point P the total work done is given by

$$V = -\int_{\infty}^{r} \frac{q}{l^2} dl = q/r \qquad \cdot \quad \cdot \quad \cdot \quad (2·3)$$

If the field is due to a number of point charges q_1, q_2, q_3 etc., situated at distances r_1, r_2, r_3, etc., from the point at which the potential is to be found, then as the potential is a scalar quantity it is given by the sum

$$V = \frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \cdots$$

$$= \sum_i \frac{q_i}{r_i} \qquad \qquad \cdots \quad (2\cdot4)$$

4. Potential of a charged sphere. Let S be an isolated metal sphere of radius R with charge $+q$ (fig. 2·3) The problem is to calculate the potential at a point P at a distance r from the centre of the sphere. The surface density of charge on the sphere is $\sigma = q/4\pi R^2$.
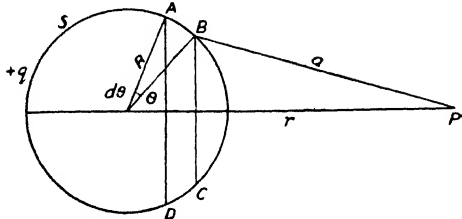
Fig. 2·3 Potential due to a charged sphere.

Consider an annular strip ABCD of width $AB = ds$. We take the following two cases.

CASE (i) When the point P is outside.

Potential at P due to the annular strip is

$$\delta V_P = \text{area of strip} \times \sigma/a$$

$$= 2\pi R \sin\theta \, R \, d\theta \cdot \sigma/a \qquad \cdots \quad (2\cdot5)$$

Now, $a^2 = R^2 + r^2 - 2Rr\cos\theta$ $\qquad \cdots \quad (2\cdot6)$

Hence the potential due to the whole sphere is

$$V_P = \int_0^\pi \frac{2\pi R^2\sigma \sin\theta \, d\theta}{\sqrt{(R^2+r^2-2Rr\cos\theta)}}$$

$$= 2\pi\sigma \int_{r-R}^{R+r} \frac{R^2 a \, da}{R r a} \qquad \text{by differentiating (2.6)}$$

$$= \frac{2\pi\sigma R}{r}\left[a \right]_{r-R}^{r+R}$$

$$= \frac{4\pi R^2\sigma}{r}$$

$$= \frac{q}{r} \qquad \qquad \cdots \quad (2\cdot7)$$

$$-\frac{q}{r_1{}^2}\cos\theta_1\,ds_1 + \frac{q}{r_2{}^2}\cos\theta_2\,ds_2 - \frac{q}{r_3{}^2}\cos\theta_3\,ds_3 + \frac{q}{r_4{}^2}\cos\theta_4\,ds_4,$$

or

$$-q\,d\omega + q\,d\omega - q\,d\omega + q\,d\omega = 0. \qquad \ldots \quad (2{\cdot}11)$$

The positive and negative signs have been taken according as the angle θ is actute or obtuse or according as the electric intensity E is directed outwards or inwards (the normals are always drawn outwards and taken positive by convention). Hence by drawing cones in this way it can be seen that the total normal induction over the whole surface will be zero.

We will now consider some of the applications of Gauss's theorem.

8. Electric intensity near a charged sphere. In fig. 2·6, S is a uniformly charged sphere of surface density σ and radius a. P is a point, distant r from the centre of S, at which the electric intensity is to be determined. Imagine a concentric sphere of radius r passing through the point P. By symmetry the electric intensity is E at all points of the surface of the sphere and is everywhere normal.
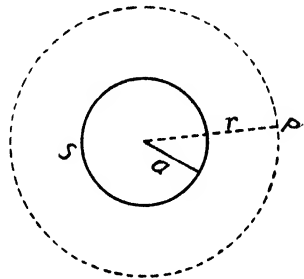
Fig. 2·6

The total normal induction is $4\pi r^2 E$ and by Gauss's theorem, we have,

$$4\pi r^2 E = 4\pi \int \sigma ds = 4\pi q,$$

where q is the total charge on the sphere.

Hence
$$E = \frac{q}{r^2}. \qquad \ldots \quad (2{\cdot}12)$$

Eq. (2·12) shows that at any external point the electric intensity is the same as if the total charge were concentrated at the centre of the spherical surface. This equation also gives us the intensity, if instead of the charge distribution being on the surface it is in the body of the sphere. If the volume distribution is ρ and it is uniform then for q we must use $\frac{4}{3}\pi a^3\rho$.

9. Electric intensity at any point inside a sphere having a uniform volume distribution of electric charge ρ. In fig. 2·7 P is a point within a sphere of radius a having uniform volume distribution of electric charge ρ.
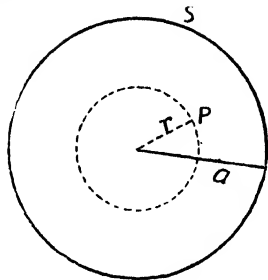
Divide the sphere into thin concentric shells. The effect of each shell which does not enclose P is as though the charge were concentrated at the centre, eq. (2·12).

Fig. 2·7

Combining the effect of all the shells except those which enclose P we have

$$E = \tfrac{4}{3}\pi\, r^3\, \rho/r^2 = \tfrac{4}{3}\pi r \rho. \qquad \ldots \quad (2\cdot13)$$

The shells which enclose P have no field at that point. Eq. (2·13) shows that the field varies directly as the radius and so is maximum at the surface of the sphere.

10. Uniformly charged cylinder. C is the uniformly charged cylinder of infinite length (fig. 2·8) having a charge σ per unit length. By symmetry the intensity of the field is normal to the axis and is the same for points situated at the same distance from the axis.
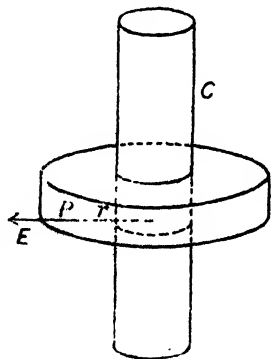
Let P be a point, distant r from the axis at which E is to be determined. Draw a coaxial cylinder through P of unit length. The intensity E is parallel to the end faces, and hence the contribution to the T. N. I. is due to the curved surface only. The area of the curved surface is $2\pi r$. Hence the T. N. I. is equal to $2\pi r$E and so by Gauss's theorem

Fig. 2·8

$$2\pi r\mathrm{E} = 4\pi\sigma,$$

or

$$\mathrm{E} = 2\sigma/r. \qquad \ldots \quad (2\cdot14)$$

It should be noted that eq. (2·14) is independent of the radius of the charged cylinder and so holds good for a linear charge. It also shows that E is greatest at the surface.

11. Uniformly charged plane. Let X Y be an infinite plane with uniform charge distribution of surface density σ, fig. (2·9). By symmetry the electric intensity is normal to the plane everywhere. Construct a prism cutting the plane, with equal surfaces AB and CD parallel to the plane and equal sides AC and BD perpendicular to it. As the electric intensity is parallel to the sides AC and BD the T. N. I. over all the sides is $2Es$ where s is the area AB or CD. The charge enclosed

Fig 2·9

by the prism is $s\sigma$. Therefore, by Gauss's theorem,

$$2\ Es = 4\pi s\sigma,$$

or
$$E = 2\pi\sigma. \qquad \cdots \quad (2·15)$$

Eq. (2·15) shows that the field is uniform on each side of the plane and its magnitude is independent of the distance from the plane. It should be clearly understood that the charge is not situated upon a conductor but is merely a sheet of charge.

12. Two uniformly charged infinite parallel planes. Consider two uniformly and oppositely charged infinite planes AB and CD parallel to each other and separated by a distance d, and having a surface density of charge σ, (fig. 2·10). At a point P the field due to AB is $2\pi\sigma$ to the right. This is also the field in magnitude and direction due to CD. Therefore the resultant field is

$$E = 4\pi\sigma, \qquad (2·16)$$

and is uniform between the planes.

At an outside point P′ the electric intensity due to AB $2\pi\sigma$ to the right and that due to CD is $2\pi\sigma$ to the left. Therefore the resultant intensity is zero. Thus at all points situated outside the two planes the field vanishes.
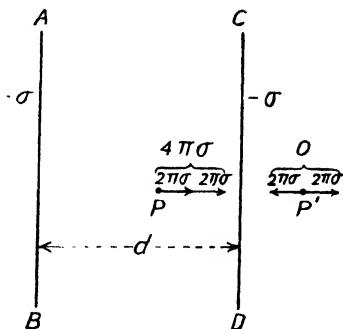
13. Field intensity just outside a conductor. Coulomb's law. In fig. 2·11 AB, CD are the two surfaces of a conductor with surface charge density $+\sigma$.

Fig. 2·10

Since in the steady condition no charge moves in the interior of the conductor, AB, CD are equipotential surfaces. The lines of force are, thus, everywhere normal to AB and CD.

Construct a prism LMNQ with equal sides LM and NQ each of area s parallel to the conducting surface. The face LM of this prism lies inside the conductor. The sides LQ and MN are parallel to the normal. The electric intensity being parallel to LQ and

Fig. 2·11

MN the normal induction over these sides is zero. The normal induction over LM also vanishes since there is no field inside the conductor. Thus the T.N.I. is equal to that over the side NQ only, i.e. Es. The charge enclosed by the prism is $s\sigma$. Therefore, by Gauss's theorem

$$\text{T. N. I} = \text{E}\,s = 4\pi\sigma s,$$

or $$\text{E} = 4\pi\sigma. \qquad \ldots \quad (2\cdot17)$$

Eq. (2·17) is called Coulomb's law.
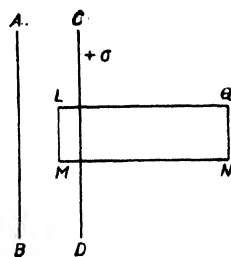
14. Region inside a conducting surface. Let S be a charged, conducting, closed surface (fig. 2·12). This is an equipotential surface. It can be shown by Gauss's theorem that the space within S is at uniform potential when there is no electric charge enclosed by the surface.

If the space within S is not at the same potential as S, a
second equipotential surface S' can
be drawn just inside S. Let V be
the potential of S and V' that of
S'. Then $(V-V')$ is the potential
difference between S and S'. If
this is positive the field is directed
from S to S'; if negative, it will be
directed from S' to S.   By Gauss's
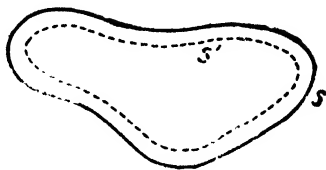
Fig. 2·12

theorem T.N.I.$=0$, since there is no charge enclosed. Since E
has the same sign all over the surface, $E=0$ at all points of the
surface. Thus $V=V'$. Similarly, dealing with the whole of the
enclosed space it can be seen that it is at uniform potential and
that there is no field inside it. Absence of field means absence of
lines of force. As lines of force originate from positive charges or
end on negative charges this absence means absence of charges
inside a conductor. Thus we see that whole of the charge is
located on the surface of the conductor.

15. Hollow conductor. Let S and S' be the outer and
inner surfaces respectively of a hollow conductor, (fig. 2·12).
Imagine a closed surface to be drawn between S and S' surround-
ing S'. The intensity over this surface is everywhere zero since
it lies in the interior of the conductor Therefore by Gauss's
theorem total charge within it is zero.   Thus if in the cavity
within S there is a charge q there must be an equal and opposite
charge on S'.   Hence in a hollow closed conductor if there is any
charge there must be equal and opposite charge on the inner
surface of the conductor.

If the empty space inside a hollow conductor does not con-
tain any charge there is no electric intensity anywhere in the
empty space.

16. Theorems regarding lines of force.

Theorem I. The lines of force are continuous in a region
which does not contain any charge. Consider a closed surface
AB bounded by lines of force and cross sections S_1 and S_2. The
flux through the sides is zero since there is no component of electric

intensity perpendicular to the sides. Therefore the total flux through the surface is the flux through S_1 and S_2. By Gauss's theorem, since there is no charge enclosed, flux through S_1 = flux through S_2 or by eq. (2·9) the number of tubes of force coming in through S_1 = the number of tubes of force passing out through S_2. Thus lines of force are continuous in the region occupied by the section of the tube under consideration and hence for any space where there is no charge.

Theorem II. The number of tubes of force diverging from a charge $+q$ or ending on a charge $-q$ is $4\pi q$. This immediately follows from Gauss's law and eq.(2·9). The former gives us the flux as $4\pi q$ and the latter the number of tubes of force N. Thus with each tube a charge $1/4\pi$ is associated.

This property of a hollow conductor underlies the practice of using electric screens to shield delicate electrical apparatus from the influence of stray electric fields. The intensity due to outside charges is reduced to zero inside a space surrounded by a closed conductor and so the electrical apparatus is uninfluenced by any outside charges.

17. Mechanical force on the surface of a charged conductor. We have seen that electric charges can move freely in a conductor, yet they cannot escape from the surface. This means that in the field set up by a charged conductor the latter exerts a force on each element of charge on its surface so as not to allow it to move normally. Since action and reaction are equal and opposite the conductor is thus subject to mechanical normal forces equivalent to the electrical forces on the surface charges.

Consider a surface S with uniform electrification of surface density $+\sigma$, (fig. 2·13.) The charge on the element AB of area ds is σds. We will proceed to find the resultant force on this charge.

The required force on the element ds is obviously due to charges other than that on ds.
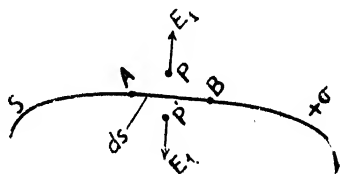
Fig. 2·13

Let P be a point very close to the surface element ds. The intensity at P due to the charged conducting surface S is $4\pi\sigma$ by Coulomb's law. This is made up of two parts : (1) E_1, the intensity due to the element ds and (2) E_2, the intensity due to the remaining surface. That is,

$$E_1 + E_2 = 4\pi\sigma.$$

Take a point P′ within the conductor on the other side of ds and infinitely close to P. At P′ the value of E_2 is the same but E_1 is equal and opposite to that at P. Hence the total force at P′ is $E_2 - E_1$ which is zero since P′ is situated inside the conductor. That is, $E_2 = E_1$ and hence from eq. (2·18) $E_2 = 2\pi\sigma$.

Therefore, the mechanical force F per unit area on the charged conductor is given by

$$Fds = 2\pi\sigma \cdot \sigma \, ds,$$

i.e., by

$$F = 2\pi\sigma^2,$$

$$= E^2/8\pi. \qquad \qquad . \quad . \quad . \quad (2·19)$$

where $E = 4\pi\sigma$ is the resultant electric intensity.

EXAMPLE. Two grams of gold are beaten into a thin leaf 10^{-4} sq. cm. in area. A small piece of this is put upon the surface of a conductor. Find the surface density of charge required by the conductor to just lift up the gold foil piece.

The weight of the gold foil per sq. cm. is $mg = 2 \times 980/10^{-4} = 0·1958$ dyne. The mechanical force is $2\pi\sigma^2$, where σ is the required surface density of charge. Therefore, $2\pi\sigma^2 = 0·1958$ or $\sigma = 0·1765$ unit.

18. Energy of the electrostatic field. In the previous article we have seen that the force per unit area of a conductor is $E^2/8\pi$. Now if the surface is displaced normally through a distance dl, the work done per unit area is $E^2/8\pi \times dl$. The

volume of the space covered is dl. Hence the energy per unit volume of the electric field is

$$W = E^2/8\pi. \qquad \qquad . \quad . \quad . \quad (2\cdot20)$$

19. Electrified soap bubble. The outward mechanical force can be demonstrated by electrifying a soap bubble. On increasing the charge on a soap bubble it is found that the outward mechanical force causes an increase in its size. We will establish a relation between the charge on a closed soap bubble and its radii before and after electrification.

Let r be the radius of a closed soap bubble S, (fig. $2\cdot14$). Imagine a small circular sur-
face element AB with centre C
cut off from the surface of the
soap bubble by a right circular
cone AOB of semi-angle ϕ.
Area of the circular element AB
is $\pi r^2 \phi^2$. The forces acting on
the element AB are
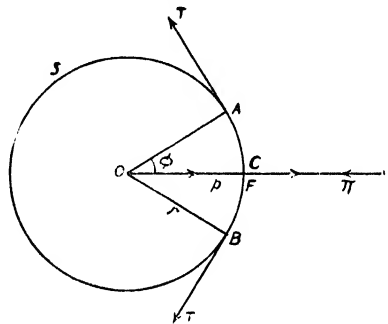
(a) The atmospheric pressure
$\pi . \pi r^2 \phi^2$ acting normally in-
wards

Fig. $2\cdot14$

(b) The internal pressure (p), varying inversely as the volume equal to $\dfrac{\lambda}{r^3} \pi r^2 \phi^2$ acting normally outwards, where λ is a constant.

(c) The mechanical force (F) due to electrification equal to $2\pi\sigma^2 \times \pi r^2 \phi^2$ acting normally outwards.

(d) The surface tension acting in the surface of the bubble across the boundary of the element. If T denotes the surface tension per unit length, the tension across the length dl of the circular element is T dl acting in the direction ϕ with the tangent plane at C. This has two components Tdl cos ϕ in the tangent plane and T dl sin ϕ along CO. Combining the forces along the circumference of the circular element AB we find that the components in the tangent plane annul each other while

those along CO combine into a resultant $2\pi r\phi T \times \sin\phi$, *i.e.*, $2\pi r T \phi^2$ for a small value of ϕ.

The element AB will be in equilibrium when

$$\pi\cdot\pi r^2\phi - \frac{\lambda}{r^3}\pi r^2\phi^2 - 2\pi^2\sigma^2 r^2\phi^2 + 2\pi r \ T \ \phi^2 = 0,$$

or

$$\pi - \frac{\lambda}{r^3} - 2\pi\sigma^2 + \frac{2T}{r} = 0. \qquad \ldots \quad (2\cdot21)$$

If q be the charge put on the soap bubble and if r be its radius when so charged, the above equation can be written as

$$\pi - \frac{\lambda}{r^4} - q^2/8\pi r^4 + 2T/r = 0. \qquad \ldots \quad (2\cdot22)$$

If r_0 be the radius of the soap bubble when it is uncharged $(\sigma=0)$ the condition of equilibrium is

$$\pi - \lambda/r_0^3 + 2T/r = 0. \qquad \ldots \quad (2\cdot23)$$

Taking T to be constant and eliminating it from $(2\cdot22)$ and $(2\cdot23)$ we have

$$\pi(r - r_0) - \lambda(1/r^2 - 1/r_0^2) = q^2/8\pi r^4 \qquad \ldots \quad (2\cdot24)$$

If the bubble is blown on a tube open to the atmosphere eq. $(2\cdot21)$ reduces to

$$\pi\sigma^2 = T/r. \qquad \ldots \quad (2\cdot25)$$

Taking the bubble roughly to be a uniformly charged sphere **eq.** $(2\cdot25)$ becomes

$$q^2 = 16\pi r^3 T. \qquad \ldots \quad (2\cdot26)$$

Or in terms of potential V of the bubble eq. $(2\cdot26)$ reduces to $\qquad V^2 = 16\pi r T. \qquad \ldots \quad (2\cdot27)$

Eq. $(2\cdot27)$ gives the radius of the bubble if the potential V and the surface tension T are known. For small values of T the electrification produces large change in the radius.

CHAPTER III
CONDENSERS

1. Capacity of a conductor. The capacity of a conductor is defined as the ratio of its charge to its potential, the conductor being completely isolated or in the neighbourhood of conductors at zero potential. Since the potential of a charged sphere is given by $V = q/a$ (eq. 2·7), its capacity according to the above definition is equal to its radius.

2. Spherical condenser. A condenser is a combination of two conductors insulated from each other and generally, though not necessarily, so arranged that a surface of one is parallel to a surface of the other.

Consider a conducting sphere of radius a surrounded by a concentric conducting spherical shell of internal radius b and external radius b' (fig. 3·1). Let a charge $+ q$ be placed upon the sphere a. This will induce a charge $- q$ on the inner surface of the shell. Thus if $+ q'$ be the total charge on the shell there will be a charge $q + q'$ on the outer surface of the shell. But we know that in the space between a and b the electric field is due to the inner sphere a only, the charge on the outer sphere producing no field in the space enclosed. The electric intensity at distance r from the centre of the inner sphere is q/r^2. The potential of the shell is uniform and equal to, say, V_b. If V_a be the potential of the inner sphere,
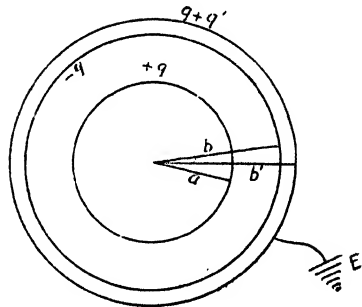
Fig. 3·1

$$V_a - V_b = \int_b^a q/r^2 \, dr = \frac{q}{a} - \frac{q}{b} \ . \qquad \cdots \quad (3\cdot1)$$

Eq. (3·1) shows that this difference of potential is independent of the thickness of the shell and the charge on its outer surface. If the shell is earthed we have from eq. (3·1),

$$V_a = q\,(b - a)/ab,\ \text{since}\ V_b = 0.$$

Hence the capacity of a spherical condenser is given by

$$C = \frac{q}{V_a} = \frac{ab}{(b - a)}. \qquad \qquad \cdots \quad (3\cdot2)$$

Eq. (3·2) shows that the capacity of a sphere is increased by surrounding it with a concentric spherical conductor kept at constant potential, say that of the earth. Also the capacity increases as the distance between the two spheres diminishes.

Let us now suppose that the inner sphere is earthed and the outer sphere insulated as shown in (fig. 3·2). If $+q'$ be the charge on the outer surface of the shell and $+q$ on the inner, the charge on the sphere is $-q$. The total charge on the shell is $(q + q')$. Let V_b denote the potential of the shell. The electric intensity outside the shell is q'/r^2 and in the space between the two spheres is $-q/r^2$. Hence,
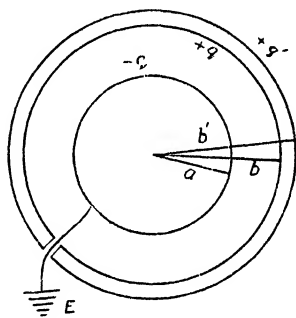
Fig. 3·2

$$V_b - 0 = \int_{b'}^{\infty} q'/r^2\,dr = q'/b',$$

$$\text{and}\ \ 0 - V_b = -\int_{a}^{b} \frac{q}{r^2}\,dr = \frac{q}{b} - \frac{q}{a}.$$

From these two equations we have

$$q + q' = V_b\left(\frac{ab}{b - a} + b'\right),$$

and
$$C = (q + q')/V_b = \frac{ab}{(b - a)} + b'. \qquad (3\cdot3)$$

3. Cylindrical condenser. It consists of two concentric cylinders insulated from each other, (fig. 3·3). Let the radius of the

inner cylinder be a and that of the outer b. If $+q$ be the charge per unit length on the inner cylinder there will be a charge $-q$ on the inner surface of the outer cylinder, and so if $+q'$ be the total charge on the outer cylinder, the charge on the outer surface is $(q+q')$. This charge produces no field intensity in the space enclosed. The electric intensity due to a is $2q/qr$ at a distance r from the axis of the inner cylinder. Therefore the potential difference between the inner and outer cylinders is
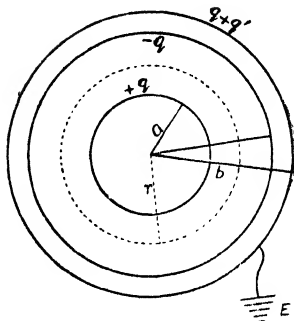
Fig. 3·3

$$V_a - V_b = \int_a^b \frac{2q}{r}\ dr = 2q \log_e \left(\frac{b}{a} \right). \qquad (3·4)$$

If the outer cylinder is earthed *i.e.* $V_b = 0$, the capacity per unit length of the cylinder is given by

$$C = q/V_a = 1/2 \log_e \left(\frac{b}{a} \right). \qquad (3·5)$$

4. A parallel plate condenser. A and B (fig. 3·4) are two parallel conducting plates at potentials V_A and V_B respectively. Let t be the distance between the two plates and σ the surface charge density on A and $-\sigma$ that on B. By eq. (2·16) the electric intensity between the plates is $4\pi\sigma$ normal to them

Fig. 3·4

and outside it is zero. The work done in taking a unit positive charge from A to B is $4\pi\sigma t$ which is, by definition, the potential difference $(V_A - V_B)$ between the plates.
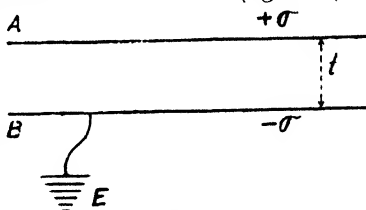
Hence the capacity C per unit area

$$= \sigma/(V_A - V_B),$$
$$= 1/4\pi t. \qquad \ldots \quad (3·6)$$

If the area of the plate is A the capacity of this condenser becomes $A/4\pi t$. The plates A and B may be both insulated or

one of them, say, B may be earthed as shown in the figure. V_B is then zero. By eq. (2·19) the mechanical force of attraction between the plates is $2\pi\sigma^2 A = \dfrac{A}{8\pi t^2}(V_A - V_B)^2 = \dfrac{A}{8\pi t^2}V^2{}_A$, if $V_B = 0$.

5. The guard ring. In deriving eq. (3·6) we supposed that the electric field in-tensity between the plates is uniform, which is, however, not the case at the edges. This introduces a certain error in calcula-

Fig. 3·5

ting the capacity. Lord Kelvin showed that if the central portion of one of the plates, say A, is separated from the rest of the plate by a narrow gap G this edge effect is eliminated. The outer portion R of the plate is called a *guard ring*.
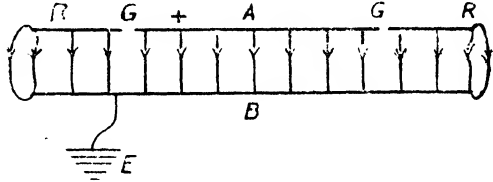
As shown in fig. (3·5) the lines of force between the guarded plate A and the earthed plate B are perpendicular to the surfaces right up to the edge of the central plate A, the region of non-uniformity of the lines of force being shifted to the outer edges of R. The so-called edge effects have thus been eliminated for the plate A. It must be borne in mind that some lines of force fall upon a portion B which is in front of the gap. If we assume that half of them start from A and half from R the effective area of the condenser plate becomes A', where A'=A+half the area of the gap between A and R approximately, as shown by Kelvin. The capacity of the condenser thus becomes $C = A'/4\pi t$.

6. Useful condensers. A common form of condenser is the Leyden jar (fig. 3·6). This is a glass jar coated inside and outside for about 2/3 of its depth with tin foil. The rest of the glass surface is coated with shellac varnish for insulation purposes. The contact
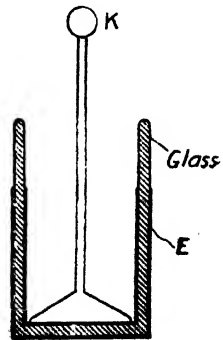
Fig. 3·6

with the inner coating is made with a metallic knob K. The outer coating is earthed. If A be the area of the inner coating and t the thickness of the glass the capacity is roughly $KA/4\pi t$, where $K(=6)$ is the *dielectric constant* of glass. The influence of the dielectric will be discussed in the next chapter.

Many standard condensers are made of layers of tin foil separated by mica sheets. Mica is obtainable in thin sheets and it is an extremely good insulator. We can thus have large capacity without undue increase in the bulk.

7. Variable condensers. There are various forms of variable conden-

Fig. 3·7

sers. A convenient form due to Lord Kelvin is represented diagrammatically in fig. (3·7). It is a sliding cylindrical condenser. A and C are two cylinders of the same diameter placed coaxially with a slight gap G between them. B is another coaxial cylinder of smaller diameter with an index I which gives its position upon the graduations marked on C. A is insulated and B and C earthed. By sliding B inside A the capacity of the condenser can be varied by known amounts. If l is the distance through which B slides inside A the change in capacity of A is $l/2 \log_e \frac{b}{a}$, where a and b are the radii of the cylinders A and B respectively. Movements of conductors in the neighbourhood of such a condenser causes variations in its capacity. To eliminate this influence A is surrounded by an earthed metallic sheet. This condenser is used where known alterations of capacity are to be made.
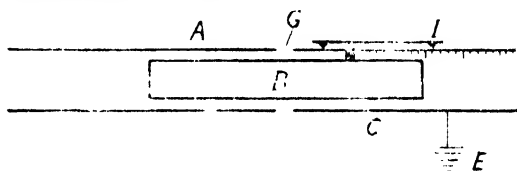
Another useful type of variable condenser is made out of a number of semi-circular conducting plates. A number of these are fixed one above the other parallel to each other separated by a certain fixed distance. Between and parallel to them there are a

number of similar plates mounted upon a rotatable axle. By rotating this axle the moving plates may be made to lie entirely within or entirely outside the fixed plates. The capacity can thus be varied from a maximum to a minimum value. This arrangement is nothing but a multiple parallel-plate condenser. The relative position of the plates indicating the covered area is determined on a circular scale by means of an index attached to the rotating axle. The scale must be calibrated by experimental comparison with standard condensers of known capacity. A condenser of this type is commonly used in radio sets for tuning.

8. Condensers in series. Condensers are said to be arranged in series when the plate of lower potential of one is connected to the plate of the higher potential of the next as shown
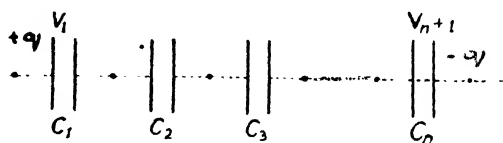
Fig. 3·8

in fig. (3·8). Let $C_1, C_2, C_3, \ldots$ etc. be the capacities of the various condensers. If $+q$ be the charge on the first plate of condenser C_1, the second plate of the last condenser will have a charge $-q$. If V_1 be the potential of the first plate of C_1 then V_{n+1} will be the potential of the second plate of C_n. Now,

$$V_1 - V_2 = q/C_1, \quad V_2 - V_3 = q/C_2, \quad \ldots \quad V_n - V_{n+1} = q/C_n$$

By adding these we have

$$V_1 - V_{n+1} = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots + \frac{1}{C_n}\right)$$

Hence C, the resultant capacity of the condensers in series, is given by

$$(V_1 - V_{n+1})/q = 1/C = \sum_{k=1}^{k=n} 1/C_k \qquad \ldots \quad (3\cdot7)$$

If $\qquad C_1 = C_2 = C_3 = \ldots = C_n = C'$

Then $\qquad C = C'/n. \qquad \ldots \quad (3\cdot8)$

3 E

i.e., the resultant capacity of a system of n condensers of equal capacity, in series, is equal to $1/n$ *of the value of the capacity of a single condenser.*

9. Condensers in parallel. Condensers are said to be arranged in *parallel* when all the plates of higher potential are connected to one point and all those of lower potential to another point as shown in fig. 3·9. Let V_1 be the higher potential and V_2 the lower.
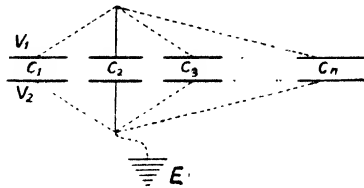
Fig. 3·9

The charges on the condensers are given by

$$q_1 = C_1 (V_1 - V_2), \ q_2 = C_2 (V_1 - V_2), \dots q_n = C_n (V_1 - V_2)$$
$$\therefore q + q_2 + q_3 + \dots + q_n = (C_1 + C_2 + C_3 + \dots$$
$$+ C_n)(V_1 - V_2) \quad \dots \quad (3·9)$$

If C be the resultant capacity the total charge q on the condensers is given by

$$q = C (V_1 - V_2) \qquad \dots \quad (3·10)$$

From eqs. (3·9) and (3·10) we get

$$C = \sum_{k=1}^{k=n} C_k \qquad \dots \quad (3·11)$$

Obviously if the individual capacities are each equal to C' eq. (3·11) reduces to $C = nC'$, *i.e., the resultant capacity of a system of n condensers of equal capacity, in parallel, is equal to n times the capacity of a single condenser.*

10. Energy of a charged condenser. The process of charging a conductor involves a certain amount of expenditure of energy. The work done in charging, say, by friction or by electrophorous or by Wimshurst's machine, is stored up as potential energy of the charged body. This is used up in driving the current when the conductor is discharged.

The energy of the charged conductor is measured by work done in putting a charge q upon it. Consider an isolated conductor possessing capacity C. In the process of charging let V' be the

potential when the charge on the conductor is q'. We have $q'=CV'$. Work done in increasing the charge dq' is $V'dq'$, since the addition of a small charge dq does not appreciably alter the potential V'. Thus the increase of potential energy of the conductor for a small increment dq' in its charge is

$$dU = V'dq' = \frac{1}{C} q'dq'.$$

Therefore, the total potential energy of the conductor when a charge q is placed upon the conductor is

$$U = \int dU = \frac{1}{C} \int_0^q q'dq' = \frac{1}{2} \frac{q^2}{C} = \tfrac{1}{2}CV^2 = \tfrac{1}{2}qV, \quad (3\cdot12)$$

where V is the final potential of the conductor due to the endowment of the final charge q.

11. Loss of energy on sharing charge. When a charge passes from a place of higher potential to a place of lower potential electrical energy is lost. This can be shown as follows :

Consider two conductors of capacity C_1 and C_2 having V_1 and V_2 as their potentials, $V_1 > V_2$. When the two are connected a charge passes from C_1 to C_2 and finally the potential of each becomes V.

If q be the charge, we have

$$q = (V_1 - V)\ C_1 = (V - V_2)\ C_2,$$

or
$$V = (C_1 V_1 + C_2 V_2)/(C_1 + C_2).$$

Energy before connecting the two together is
$$E_0 = \tfrac{1}{2}\ C_1\ V_1{}^2 + \tfrac{1}{2}\ C_2\ V_2{}^2.$$

Energy after connecting them together is
$$E = \tfrac{1}{2}\ (C_1 + C_2)(C_1 V_1 + C_2 V_2)^2/(C_1 + C_2)^2,$$
$$= \tfrac{1}{2}(C_1 V_1 + C_2 V_2)^2/(C_1 + C_2).$$

Hence the change in energy is given by

$$E_0 - E = \tfrac{1}{2}\ C_1\ C_2\ (V_1 - V_2)^2\ /(C_1 + C_2). \qquad . \quad . \quad . \quad (3\cdot13)$$

As the quantity on the right hand side of eq. $(3\cdot13)$ is positive, $E_0 > E$, *i.e.*, there is a loss of energy. This loss appears as heat in the connecting wire or as a spark between the two conductors.

CHAPTER IV

DIELECTRICS

1. The inverse square law of force. In the foregoing chapters we considered electric fields, capacity, etc., due to charged conductors in air. It has long been known that the medium in which the charges are situated have a profound influence upon the absolute values of the electrical force. Thus Cavendish and later Faraday discovered that when instead of air some other non-conducting substance is put between the plates of a parallel-plate condenser, its capacity is always increased in a constant ratio, K, the value of which depends upon the nature of the substance introduced between the plates. K is called the *specific inductive capacity* or *dielectric constant* of the substance which has received the name *dielectric*. The value of K for air at atmospheric pressure is about 1·00059 ; for vacuum it is taken to be unity. Thus the expressions derived in the foregoing chapters, although rigorously true for vacuum, may be taken to be true for air. We shall derive below an expression for inverse square law of force taking into account the constant K.

Consider two parallel-plate condensers A and B, each having the distance t between their plates. Suppose that in A the medium between the plates is air and that the medium in B is a dielectric of specific inductive capacity K. The upper plates of A and B have potentials V_1 and V_2 and the lower plates are earthed. If σ is the charge per unit area for each and C and C′ the capacities per unit area we have by definitions of K and capacity

$$C' = K \times C, \qquad \qquad \cdots \quad (4\cdot1)$$

and
$$C = \sigma/V_1, \ \ C' = \sigma/V_2. \qquad \cdots \quad (4\cdot2)$$

From eqs. (4·1) and (4·2) we have

$$V_2 = V_1/K. \qquad \qquad \cdots \quad (4\cdot3)$$

Eq. (4·3) shows that *for the same charges* the drop of potential in the dielectric is $1/K$ times that in air.

Now if E is the electric intensity between the plates of the condenser A and E′ that between the plates of the condenser B we have, by the definition of potential,

$$Et = V_1 \text{ and } E't = V_2,$$

or from eq. (4·3)

$$E' = E/K. \qquad \qquad . \quad . \quad . \quad (4·4)$$

Eq. (4·4) shows that the electric intensity in the condenser filled with a substance of specific inductive capacity K is 1/K times that in the condenser in air. Generalising we say that the force of repulsion between two point charges q and q' placed distant r apart in a medium of uniform specific inductive capacity K is given by

$$F = qq'/Kr^2. \qquad \qquad . \quad . \quad . \quad (4·5)$$

We have already seen that in air

$$F = qq' /r^2 ,$$

the force in air is K times the force in a dielectric.

The capacity of a cylindrical condenser with a dielectric K is $K/2 \log_e b/a$ and that of a parallel-plate condenser is $K/4\pi t$.

2. Parallel-plate condenser with a dielectric slab. Let t be the distance between the plates and A their area. A dielectric slab of area A, thickness d and dielectric constant K, is introduced between the plates partly projecting so that B is its uncovered area, (fig. 4·1). Let a be the distance of one surface of the slab from plate I. The lines of force are perpendicular to the plates as shown by the arrows.
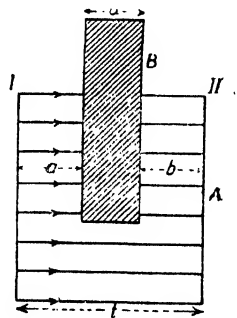
Fig. 4·1

The electric intensity E in air is $4\pi\sigma$ and the electric intensity E′ in the slab is $4\pi\sigma/K$, where σ is the charge per unit area.

Now,

$$V_1 - V_2 = 4\pi\sigma \times a + \frac{4\pi\sigma}{K} \times d + 4\pi\sigma \times [t - (a+d)]$$

$$= 4\pi\sigma\{t - d(1 - 1/K)\}$$

The condenser may be supposed to be made up of two condensers C_1 and C_2 joined in parallel, the former of area

(A—B) with a dielectric between the plates and the latter a parallel-plate air condenser of area B.   Thus the total capacity C is given by

$$C = C_1 + C_2$$

$$= \frac{(A-B)}{4\pi \left\{ t - \left(1 - \frac{1}{K}\right) \right\}} + \frac{B}{4\pi t} \quad . \quad . \quad . \quad (4\cdot6)$$

Eq. (4·6) reduces to $C = A/4\pi t$ for $A = B$ and to $C = A/4\pi\{t - d(1 - 1/K)\}$ for $B = 0$. Thus as B increases C diminishes. The energy of the condenser is $\frac{1}{2} Q^2/C$, where Q is the charge. If Q is kept constant it is seen that the energy increases as the slab is withdrawn.   There must be a mechanical force tending to resist this withdrawal.   The dielectric slab will thus be drawn in between the plates of the condenser.   It is acted upon by forces tending to move it from weaker to stronger parts of the electric field.

3. Maxwell's displacement theory.   Maxwell attempted to picture electrostatic phenomena by his conception of " electric displacement."   According to Maxwell electric intensity acting in all media including ether produces motion of electricity in that medium.   In a conductor electricity continues to move so long as the electric intensity exists.   In an insulator or free ether there is motion of electricity ; but this motion is limited to a small displacement of electricity exactly proportional to the electric intensity and in the same direction in isotropic media.   The displacement is thus a vector.   Its magnitude in any direction is equal to the total quantity of electricity per unit area kept perpendicular to that direction during the time taken by the electric intensity to rise from zero in the beginning to E in the end.

Like Faraday, Maxwell imagined the existence of lines of force and tubes of induction in an electric field.   The energy **of** this field is due to tension along the tubes and pressure sideways. Consider a small element of a tube of induction enclosed between two equipotential surfaces V_1, V_2, and the curved sides of the

tube, (fig. 4·2). Let S be the cross-section of the tube and dl the normal distance between the two equipotential surfaces. The volume enclosed is thus $S dl$. E is the electric intensity finally reached. A certain fraction of this, say p, will give us the electric intensity pE at any time. If α is the constant of proportionality between the electric displacement and E, then at the same moment, the electric displacement per unit area is $\alpha\, p$ E. If $(p+\delta p)$E be the electric intensity at any subsequent

Fig. 4·2

time, the electric displacement will be $\alpha\,(p+\delta p)$ E or the increase in the displacement is $\alpha E\,\delta p$ per unit area. This means that a quantity $\alpha E\delta p S$ enters at one end and passes out through the other end of the volume element. Work done in transferring this electricity through a potential difference $V_1 - V_2$ is $\alpha\,E\,\delta p\,S\,(V_1 - V_2)$. But $(V_1 - V_2) = pE dl$. Thus the total work W done in displacing the electricity in the volume element dl is given by
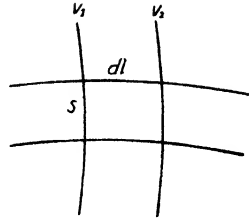
$$W = \int_0^1 \alpha E^2 p \delta p S dl$$

$$= \tfrac{1}{2}\,\alpha\,E^2\,S dl$$

According to Maxwell this energy is stored up in the volume $S dl$ and is thus equal to $\dfrac{E^2}{8\pi} \times S dl$ (see eq. 2·20). Thus putting $\tfrac{1}{2}\alpha E^2\,S dl = \dfrac{E^2}{8\pi}\,S dl$ we have $\alpha = \dfrac{1}{4\pi}$ and so the displacement D is given by

$$D = E/4\pi. \qquad \qquad . \quad . \quad . \quad (4\cdot7)$$

If the volume element $S dl$ is taken in a dielectric of specific inductive capacity K the energy can be shown to be equal to $KE^2/8\pi$.

Hence for a dielectric

$$D = KE/4\pi. \qquad \qquad . \quad . \quad . \quad (4\cdot8)$$

The displacement and intensity in an electric field are related to each other like strain and stress in elastic bodies. A tube of induction starts from a positive charge and ends upon a negative

charge. Thus a positive charge may be looked upon as a displacement at one end of the tube and a negative charge as a displacement in the opposite direction at the other end of the tube. Throughout the tube the displacement is continuous but its value depends upon the area of the cross-section of the tube.

In article 7 we introduce the displacement vector as a vector sum of the electric intensity and polarisation in a dielectric medium.

4.   Electric doublet. Polarisation of dielectric.   Two equal and opposite charges separated by a very small distance constitute an *electric doublet.* An electric doublet can, therefore, be compared to a very small magnet the north pole of which corresponds to the positive charge of the doublet and the south pole to its negative charge. Like the magnetic moment of a magnet the electric doublet has an electric moment given by *edl*, where *e* is the charge and *dl* the distance of separation. The axis of the electric moment is the line joining the negative charge to the positive charge. Just as a magnet when placed in a magnetic field tends to orient itself parallel to the field so also does an electric doublet when placed in an electric field.

A dielectric is made up of atoms, which, according to modern ideas, consist of a positively charged massive nucleus surrounded by electrons. In non-polar dielectrics the centres of gravity of the positive and negative charges coincide and the atom as a whole is electrically neutral. When, however, the

Fig. 4·3

dielectric is subjected to electric intensity the positive and negative charges of each atom are relatively displaced forming an electric doublet which produces an electric field outside (fig. 4·3). The dielectric is then said to be *polarised.*
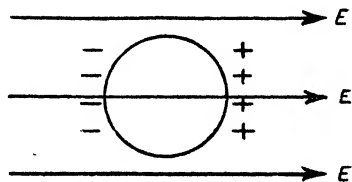
In an isotropic dielectric the direction of polarisation is the same as that of the external electric field and is indicated by the line joining the negative to the positive charge. This kind of dielectric polarisation is independent of temperature since it does not involve

molecular orientation. The susceptibility of the dielectric is described as " dia-electric."

In polar dielectrics the atoms and molecules individually possess intrinsic dipole moments and we get a case of para-electric susceptibility. In the absence of an electric field the random orientations of the dipole moments of the molecules make the resultant moment zero. An external electric field produces a sort of orderly arrangement turning each molecular dipole moment more in its own direction. There is thus a resultant dipole moment in the direction of the field the magnitude of which increases with that of the field. The alignment of the dipoles along the field is opposed by their thermal motion, thus, bringing a temperature factor in the expression for para-electric susceptibility.

5. Potential and field due to an electric doublet. Let $-qq$ be an electric doublet of moment ql, which is designated by p, (fig. 4·4).

The electric moment is a vector, the direction of which is taken positive along a line from $-q$ to $+q$. The potential at P due to the two
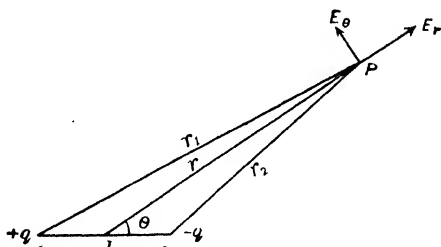
Fig. 4·4

charges is $V = \dfrac{q}{r_1} - \dfrac{q}{r_2} = \dfrac{q}{r - \dfrac{l}{2}\cos\theta} - \dfrac{q}{r + \dfrac{l}{2}\cos\theta}$

$$= ql\cos\theta / \left(r^2 - \dfrac{l^2}{4}\cos^2\theta\right)$$

$$= ql\cos\theta / r^2, \qquad \text{neglecting } l^2,$$

$$= p\cos\theta / r^2. \qquad \qquad . \quad . \quad . \quad (4\cdot9)$$

The intensities along and perpendicular to the radius vector in the directions of increasing r and θ are respectively

$$\left.\begin{array}{l} E_r = -\dfrac{\partial V}{\partial r} = 2p\cos\theta / r^3 \\[2mm] E_\theta = -\dfrac{\partial V}{r\partial\theta} = p\sin\theta / r^3 \end{array}\right\} . \qquad . \quad . \quad . \quad (4\cdot10)$$

The resultant field due to the dipole is given by

$$E = \sqrt{E^2_r + E^2_\theta} = \frac{p}{r^3} \sqrt{3 \cos^2\theta + 1} \,. \qquad \ldots \quad (4\cdot11)$$

6. Volume density of electric charge in a polarised dielectric.

Consider a small volume $\delta V = \triangle x . \triangle y . \triangle z$ in a dielectric, (fig. 4·5). This will contain a large number of atoms in chaotic condition. The mean density of charge in this volume
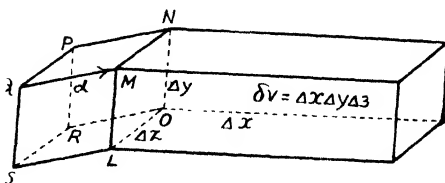
Fig. 4·5

is zero when there is no polarisation. When there is electric intensity imposed on the medium the bound electrons are displaced in each atom. The resulting charge density in this volume will be obtained by finding out the excess of the charge entering the volume over that leaving it. To simplify calculations suppose that the negative charge is fixed and the positive is displaced. Let d be the displacement in the direction QM. If ρ_1 is the positive charge density in the volume MNOLQPRS the total charge in this volume is $\rho_1 \, dx \triangle y \triangle z$, where dx is the x-component of the displacement. This is the charge entering the volume $\triangle x \triangle y \triangle z$ when the medium is polarised. The charge leaving this volume is

$$\left\{ \rho_1 dx + \frac{\partial}{\partial x}(\rho_1 \, dx) \triangle x \right\} \triangle y \triangle z, \text{ where } \rho_1 \text{ and } dx \text{ may change}$$

at the second face. Hence the excess of charge entering is

$$- \frac{\partial}{\partial x}(\rho_1 \, dx) \triangle x \triangle y \triangle z.$$

Now $\rho_1 d$ is the electric moment per unit volume or polarisation P. Thus $(\rho_1 \, dx)$ will be the x-component P_x of P. The excess of charge entering the volume $\triangle x \triangle y \triangle z$ over that leaving it along the Y and Z axes can be found similarly. Adding these we get the total excess per unit volume as

$$\rho_P = - \left(\frac{\partial P_x}{\partial x} + \frac{\partial P_v}{\partial y} + \frac{\partial P_z}{\partial z} \right). \qquad \ldots \quad (4\cdot12)$$

If P is constant in the medium the right hand side of eq. (4·12) vanishes and we have no volume density of charge in the volume $\triangle x \triangle y \triangle z$. The only charge is that on the surface.

The surface charge can be easily calculated out by considering the portion of the medium in the form of a prism of length l and cross-section S. If $+\sigma_P$ and $-\sigma_P$ be the charge density on the two ends, the electric moment is σ_P Sl and so the polarisation P by definition is given by

$$\sigma_P = P \qquad \qquad . \quad . \quad . \quad (4·13)$$

The charge passing into the volume dV can also be expressed as a surface integral. In fig. 4·6 S is the surface enclosing the volume dV of the dielectric. Consider the surface element $AB = dS$. The displacement is making an angle α with the outward drawn normal n. The volume of the prism ABCD is
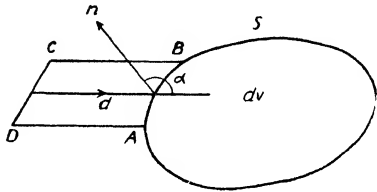
Fig. 4·6

$- d \cos \alpha \, dS$. If ρ_1 is the charge density in this, the total charge in this volume is $- \rho_1 \, d \cos \alpha \, dS$. It is this charge which passes through dS when the medium is polarised. Therefore, the total charge entering dV when the medium is polarised becomes

$$q_P = - \int_S \rho_1 \, d \cos \alpha \, dS = - \int_S P \cos \alpha \, dS. \qquad . \quad . \quad . \quad (4·14)$$

Eq. (4·14) can be used to obtain a general expression for the charge density on the surface of a dielectric. Let LMPQ be a flat prism (fig. 4·7) enclosing an area dS of the dielectric K· The height LP of this prism is small compared to the width PQ of the base. Apply eq. (4·14) to the surface of the prism. The value of the integral over LM is zero, since P=0 outside the dielectric. The integral over
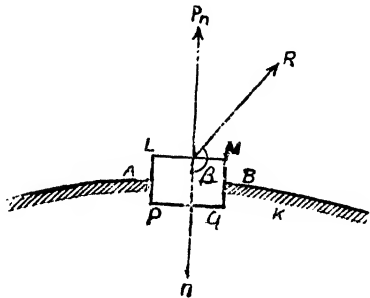
Fig. 4·7

sides at right angles to the surface is negligible since their area is small. Thus the required integral is that over the base dS, *viz.*, P cos β dS.

If σ_{p} be the charge per unit area on the surface of the dielectric the charge entering dS is σ_{p} dS and hence by eq. (4·14)

$$\sigma_{\mathrm{p}} dS = \mathrm{P} \cos \beta \, dS = \mathrm{P}_n \, dS,$$

or $$\sigma_{\mathrm{P}} = \mathrm{P}_n \qquad \qquad \ldots \quad (4 \cdot 15)$$

Eq. (4·15) gives the charge per unit area of the surface of the dielectric due to polarisation.

If the polarisation is not uniform we get in addition to the surface charge a volume distribution of charge given by eq. (4·12). Thus the electric field intensity near a dielectric is the resultant of the imposed field due to outside charges and the fields due to the surface and volume charge distributions determined by eqs. (4·15) and (4·12).

7. Gauss's law for dielectrics. Displacement vector.

S is any closed surface enclosing free charge Σq in a dielectric. Let E denote the electric intensity and θ the angle it makes with the outward drawn normal at the element dS. Due to polarisation effects there is a charge $q_{\mathrm{p}} = - \int \mathrm{P} \cos \alpha \, dS$ in the volume contained by the surface. Hence by Gauss's law

$$\int \mathrm{E} \cos \theta \, dS = 4\pi(q_{\mathrm{p}} + \Sigma q) = 4\pi(- \int \mathrm{P} \cos \alpha \, dS + \Sigma q),$$

or $$\int (\mathrm{E} \cos \theta + 4\pi \, \mathrm{P} \cos \alpha) dS = 4\pi \Sigma q. \qquad \ldots \quad (4 \cdot 16)$$

The left hand side of eq. (4.16) can be replaced by $\int \mathrm{D} \cos \phi \, dS$, where D is called the *electric displacement,* a vector defined by the vector sum of E and 4π P, i.e., by

$$\mathrm{D} = \mathrm{E} + 4\pi \, \mathrm{P}. \qquad \qquad \ldots \quad (4 \cdot 17)$$

Gauss's law then takes the form

$$\int \mathrm{D} \cos \phi \, dS = 4\pi \Sigma q, \qquad \qquad \ldots \quad (4 \cdot 18)$$

where the displacement D makes an angle ϕ with the outward drawn normal to the surface. Eq. (4·18) states that *the surface integral of the normal component of the displacement is equal to 4π times the free charge contained within a closed surface in a dielectric.*

8. Isotropic dielectric. In such a medium P is in the same direction as E and is also proportional to it, i.e. $P = \varepsilon E$, where ε is called the *electric susceptibility.* It is practically constant for steady electric field. From eq. (4·17) we have

$$D = E + 4\pi\varepsilon E = (1 + 4\pi\varepsilon) E = KE. \qquad . \quad . \quad (4\cdot19)$$

$K = (1 + 4\pi\varepsilon)$ is called the specific inductive capacity or *dielectric constant* of the medium.

In an isotropic dielectric Gauss's law takes the form

$$\int KE \cos\phi\, dS = 4\pi \Sigma q. \qquad . \quad . \quad (4\cdot20)$$

K is constant for a homogeneous dielectric. If there is no free charge in the interior of a dielectric eq. (4·20) states that the flux of E over any closed surface situated wholly in the medium is zero. Hence there is no polarisation in such a medium. All charges reside on the surface.

9. Lines and tubes of electric displacement. These are lines so drawn that everywhere the direction of the line is the same as the direction of D. Bundles of a number of lines of displacement are called tubes of displacement. The number of tubes of displacement per unit area is equal to D, the displacement. In analogy with the properties of lines of force discussed on p. 22 we state the properties of the lines of displacement as follows: (1) In regions where there are no free charges lines of dispalcement are continuous, (2) 4π tubes of displacement originate on each unit positive charge and end on each unit negative free charge, and (3) lines of displacement are continuous in passing across a boundary separating two media provided no free charges reside on the surface of separation. In contrast to this the lines of force are not continuous across the boundary.

10. Boundary conditions. (1) Consider two dielectrics K_1 and K_2 in contact, (fig. 4·8 *a*). Let E_1 and E_2 be the tangential components of the electric intensity in the two media. Draw two equipotential surfaces V_1 and V_2 separated by an infinitesimal distance l. Then V_1 and V_2 are
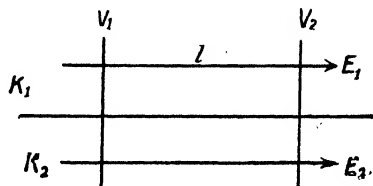
Fig. 4·8 (*a*)

parallel and perpendicular to the surface. By the definition of potential

$$V_1 - V_2 = E_1 l = E_2 l,$$

or $$E_1 = E_2, \qquad \dots (4\cdot21)$$

i.e., the tangential components of the electric intensity in the two media are the same.

(2) Take a small closed surface with ends parallel to the surface of separation and sides perpendicular to it, (fig. 4·8 *b*). Let D_1 and D_2 be the normal components of the displacement in the two media. Since there is no free charge inside the small surface under consideration we have by Gauss' law
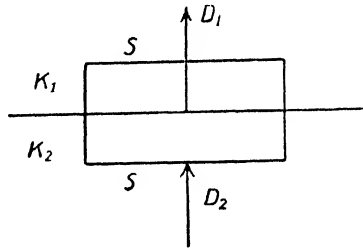
Fig. 4·8 (*b*)

$$D_1 S - D_2 S = 0,$$

or $$D_1 = D_2 \qquad \dots (4\cdot22)$$

i.e. the normal components of the electric displacement in the two media are equal.

11. Law of refraction. The foregoing boundary conditions enable us to find the change in the direction of the lines of displacement across the boundary. If θ_1 and θ_2 be the angles which the lines make with the normal on the two sides we have by the boundary condition (4·21)
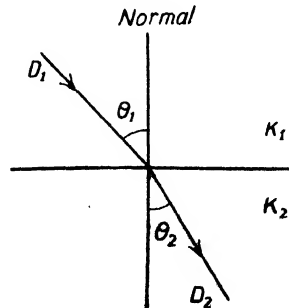
Fig. 4·9

$$E_1 \sin \theta_1 = E_2 \sin \theta_2, \qquad \dots (4\cdot23)$$

and by the boundary condition (4·22)

$$K_1 E_1 \cos \theta_1 = K_2 E_2 \cos \theta_2. \qquad \dots (4\cdot24)$$

Dividing eq. (4.23) by eq. (4.24) we have

$$\tan \theta_1 / \tan \theta_2 = K_1 / K_2. \qquad \dots (4\cdot25)$$

We see from eq. (4·25) that if $K_1 = 1$ (empty space) $\tan \theta_2 > \tan \theta_1$ or $\theta_2 > \theta_1$, i.e., the lines of displacement are bent away from the normal to the boundary surface.

42. Electrostatic field in a dielectric sphere. Clausius Mossotti's law. The field at a point in a dielectric is due to surface and volume distribution of electric charges inside the dielectric and the polarization of atoms. If the potential function is definitely determined we can obtain the intensity of the field at a point. This is not, however, usually possible. We deal below with the special case of an uncharged dielectric sphere placed in a uniform field.

Let an uncharged isotropic sphere of radius a be placed in a uniform horizontal electric field (fig. 4·10.). Under the action of this field each atom of the dielectric will be polarised and the dielectric sphere will possess a dipole moment $M = \frac{4}{3}\pi a^3 pn$, where n is the number of atoms per unit volume of the dielectric and p the dipole moment per atom.
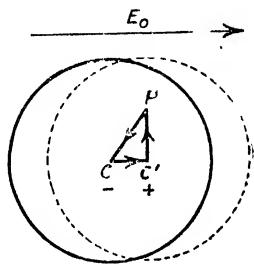
Fig. 4·10

The field at a point outside the sphere is the resultant of E_0 and the field due to the dipole moment M at the centre of the sphere, the dielectric being removed. The field at an inside point P (fig. 4·10) due to M may be pictured as that due to two spheres, one possessing a uniform positive electrification of volume density $+ \rho$ and the other a uniform negative electrification of volume density $- \rho$, whose centres originally coincide (resultant dipole moment zero) but are separated under the action of the uniform electric field E_0 (resultant dipole moment M), the line of separation being parallel to E_0. The electric intensity at P due to the sphere of positive electrification with centre C'

is $\frac{\rho}{K} \cdot \frac{4}{3}\pi C'P^3/C'P^2 = \frac{\rho}{K} \cdot \frac{4}{3}\pi C'P$ and that due to the negative

electrification with centre C is $\frac{\rho}{K} \cdot \frac{4}{3}\pi CP^3/CP^2 = \frac{\rho}{K} \cdot \frac{4}{3}\pi CP$ along the directions shown by arrows in the figure. C'P and CP thus represent these fields in magnitude and direction on a certain scale. From the triangle of forces C'PC we get CC' as the resultant in magnitude and direction. As this is independent of the position of P, we see that inside the sphere the field is everywhere the same and parallel to CC', i.e., to E_0, the outside field.

If a is the radius of the sphere and θ the angle made by a radius vector with the direction of E_0, we have, for points just outside and inside the sphere, the tangential components of the field given respectively by $E_0 \sin\theta - M\sin\theta/a^3$ and $E_i \sin\theta$, eq. (4·10). Similarly the normal components of the field intensity just outside and inside the sphere are respectively $E_0\cos\theta + \frac{2M}{a^3}\cos\theta$ and $E_i\cos\theta$.

By boundary conditions (4·21) and (4·22) we have

$$E_0 \sin\theta - \frac{M}{a^3}\sin\theta = E_i \sin\theta \qquad \ldots \quad (4\cdot26)$$

and $\quad \frac{1}{4\pi}\left(E_0\cos\theta + \frac{2M}{a^3}\cos\theta\right) = \frac{K}{4\pi}E_i\cos\theta, \qquad \ldots \quad (4\cdot27)$

where K is the dielectric constant of the sphere. From (4·26) and (4·27) we have

$$E_i = \frac{3}{(K+2)}E_0, \qquad \ldots \quad (4\cdot28)$$

and $\qquad M = \frac{(K-1)}{(K+2)}a^3 E_0. \qquad \ldots \quad (4\cdot29)$

Putting $M = \frac{4}{3}\pi a^3 P$, where P is the dipole moment per unit volume in (4·29) we have

$$P = \frac{3}{4\pi} \cdot \frac{(K-1)}{(K+2)}E_0. \qquad \ldots \quad (4\cdot30)$$

If α denotes the ratio of polarization per atom to the imposed field we have $P = \alpha E_0 n = \alpha E_0 N\rho/m$, where m is the molecular weight, ρ the density, n the number of atoms per unit volume and N Loschmidt number, $6\cdot06 \times 10^{23}$. Thus from (4·30) we have

$$\frac{\alpha N \rho}{m} = \frac{3}{4\pi} \cdot \frac{K-1}{K+2}$$

or $$\frac{K-1}{K+2} = \beta \rho, \text{ where } \beta \text{ is a constant.} \quad . \quad . \quad (4 \cdot 31)$$

Eq. $(4 \cdot 31)$ is Clausius-Mossotti's law which is found to be true indicating the validity of the assumptions made in its derivation. From the electromagnetic theory of light, $\mu^2 = K$, μ being the refractive index of the material and K its dielectric constant. Hence in optics Clausius-Mossotti's law takes the form

$$\frac{\mu^2 - 1}{\mu^2 + 2} = \beta \rho,$$

which has also been tested by experiments.

For infinitely large values of K eq. $(4 \cdot 28)$ gives $E_i = 0$ which in electrostatics means a conducting sphere. Putting $K = \infty$ in eq. $(4 \cdot 30)$, P the polarization per unit volume of an insulated conducting sphere is given by

$$P = \frac{3}{4\pi} E_0 , \qquad \qquad . \quad . \quad . \quad (4 \cdot 32)$$

a result also obtainable directly.

The total dipole moment of such a sphere is thus given by

$$M = \frac{4\pi a^3}{3} \times \frac{3}{4\pi} E_0 = a^3 E_0 , \qquad . \quad . \quad . \quad (4 \cdot 33)$$

and the polarizability by

$$\alpha = \frac{M}{F_0} = a^3. \qquad \qquad . \quad . \quad . \quad (4 \cdot 34)$$

CHAPTER V

ELECTROSTATIC INSTRUMENTS

1. Absolute electrometer. The principle of this instrument is illustrated diagrammatically in fig. 5·1. It is essentially a parallel-plate guard ring condenser. A is the guarded plate which is attracted by the lower plate B. G is the guard ring. V_1 and V_2 are the potentials of the plates.
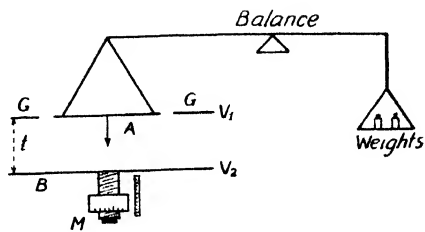
Fig. 5·1

The plate A is attached to one end of the arm of a balance and normally lies in the plane of G. A is maintained at a constant potential and B is connected in turn to the bodies the difference of potential of which is to be measured. If A′ be the effective area of the plate A (practically equal to the actual area of the plate + half the gap), the total force on the plate A is

$$F = 2\pi\sigma^2 A'.$$

Now, $$(V_1 - V_2)/t = E = 4\pi\sigma,$$

where t is the distance between the plates.

Therefore $$F = 2\pi\left(\frac{V_1 - V_2}{4\pi t}\right)^2 A'.$$

Hence, $$(V_1 - V_2) = t\sqrt{\frac{8\pi F}{A'}}. \qquad \ldots \quad (5·1)$$

Use. (1) All the plates are properly earthed and the instrument so adjusted that the plate A lies exactly in the plane of G.

(2) A and G are now disconnected. They are charged to a constant potential V_1 by a small electric machine. The constancy of V_1 is suitably checked and controlled.

(3) The plate B is now connected to the body at potential V_2. As a result the plate A will be pulled down. It is then brought back to its former position by putting suitable weights $W = mg$ in the balance pan.

By eq. (5·1) we have

$$V_1 - V_2 = l \sqrt{\frac{8\pi mg}{A'}}. \qquad \cdot \quad \cdot \quad \cdot \quad (5·2)$$

Similarly, if $m'g$ is the balancing weight when the plate B is connected to the second body at potential V_3, we have

$$V_1 - V_3 = l \sqrt{\frac{8\pi m'g}{A'}}. \qquad \cdot \quad \cdot \quad \cdot \quad (5·3)$$

From eqs. (5·2) and (5·3) we have

$$V_2 - V_3 = l \left\{ \sqrt{\frac{8\pi m'g}{A'}} - \sqrt{\frac{8\pi mg}{A'}} \right\}. \quad \cdot \quad \cdot \quad (5·4)$$

As it is difficult to know exactly the distance l the instrument is modified a little. The lower plate B is fixed to a micrometer screw M, accurately calibrated. When the plate B is connected to the body at potential V_2 the counterpoise weight is mg as before and we have

$$V_1 - V_2 = t \sqrt{\frac{8\pi mg}{A'}}. \qquad \cdot \quad \cdot \quad \cdot \quad (5·5)$$

Let the plate B be now connected to the body at potential V_3 and the micrometer screw is worked till the balance is obtained, the counter-poise weight mg being all along in the pan. If the distance between the plates is now t', we have

$$V_1 - V_3 = t' \sqrt{\frac{8\pi mg}{A'}}. \qquad \cdot \quad \cdot \quad \cdot \quad (5·6)$$

From eqs. (5·5) and (5·6) we have

$$V_2 - V_3 = (t' - t) \sqrt{\frac{8\pi mg}{A'}}. \qquad \cdot \quad \cdot \quad \cdot \quad (5·7)$$

The difference $t' - t$ can be measured accurately on the micrometer screw although not the actual values t and t'.

Thus the potential difference between the two bodies is obtained in terms of measurable fundamental quantities. The instrument does not require any calibration with a standard and hence it is called an absolute electrometer.

2. Dolezalek quadrant electrometer. This is a very
useful electrostatic instrument of great
sensitivity and accuracy. It has been
adapted to a variety of uses. Figs. 5·2 (a)
and (b) show a typical arrangement of
the main parts of this instrument. Four
similar hollow metal qudrants are sup-
ported on small insulating pillars of amber
or quartz. The opposite quadrants AA
and BB are connected by fine metallic
wires (not shown). A very light

Fig. 5·2 (a)

aluminium or metal-coated paper vane C, called the needle,
is suspended within these quadrants by a fine torsion fibre carrying
a light mirror M which together with a lamp and scale gives the
deflexion of the needle when the fibre is twisted. The whole is
enclosed in a metal case which acts as an electric screen. The
needle is charged to a certain constant potential. When the
quadrants A and B are at the same
potential the needle hangs symmetrically
between them. On creating a difference
of potential between A and B the electric
field produces a couple which causes the
needle to rotate until the twist in the
fibre brings it to rest. The amount of
deflection θ is proportional to the difference
of potential of the quadrants. The type
of quadrant electrometer briefly out-
lined here is that designed by F. Dolezalek.
instrument is shown in fig. 5·3.
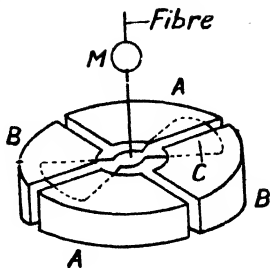
Fig. 5·2 (b)

A full view of this
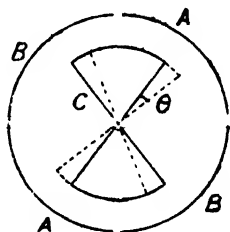
Let V_a, V_b, V_c be the potentials of the quadrants and the
needle respectively. The quadrants A and the needle form one
system of condensers of capacity C_a while the quadrants B and
the needle form another system of condensers of capacity C_b.

For any. deflexion of the needle, the potential energy of the
system is given by

$$W = \tfrac{1}{2} C_a(V_a - V_c)^2 + \tfrac{1}{2} C_b (V_b - V_c)^2 + W_0, \qquad (5\cdot8)$$

where W_0 is the extra term arising on account of the capacity of the quadrants with respect to earth, (assuming that the shielding is perfect).
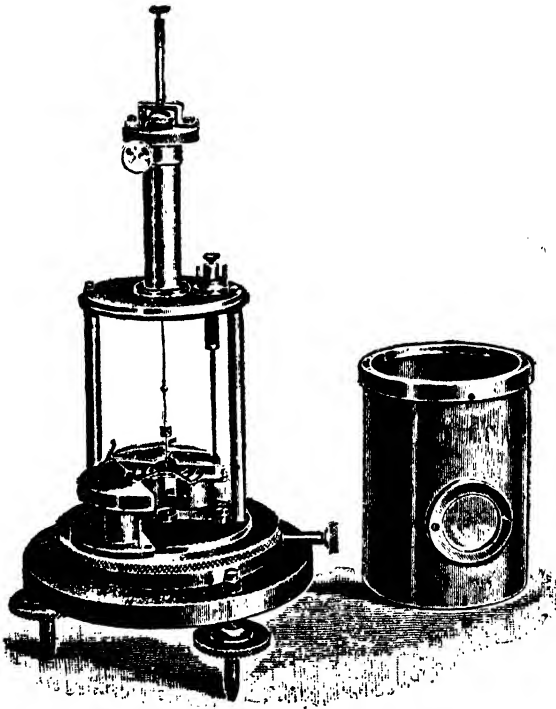
Fig. 5·3

The deflecting couple is given by

$$c = \frac{\partial W}{\partial \theta} = \tfrac{1}{2} \frac{\partial C_a}{\partial \theta} (V_a - V_c)^2 + \tfrac{1}{2} \frac{\partial C_b}{\partial \theta} (V_b - V_c)^2. \qquad . \quad . \quad (5\cdot9)$$

Now,

$$\frac{\partial C_a}{\partial \theta} = -\frac{\partial C_b}{\partial \theta}, \qquad . \quad . \quad . \quad (5\cdot10)$$

since the increase in capacity of one pair of quadrants is equal

to the decrease in the capacity of the other pair, as the needle moves from one pair into the other.   Also

$$\frac{\partial C_a}{\partial \theta} = \alpha, \text{ a constant,} \qquad . \quad . \quad (5 \cdot 11)$$

because the increase or decrease of capacity per unit deflexion is a function of the geometry of the quadrants and the needle alone. Substituting eqs. $(5 \cdot 10)$ and $(5 \cdot 11)$ in eq. $(5 \cdot 9)$ we have

$$c = \tfrac{1}{2} \alpha \{(V_c - V_a)^2 - (V_c - V_b)^2\}. \qquad . \quad . \quad (5 \cdot 12)$$

In the equilibrium position the deflecting couple will be equal to the torsional couple, i.e.,

$$c = \tau \, \theta, \qquad \qquad . \quad . \quad (5 \cdot 13)$$

where τ is the torsional couple per unit twist. From eqs. $(5 \cdot 12)$ and $(5 \cdot 13)$ we have

$$\theta = \tfrac{1}{2} \frac{\alpha}{\tau} \left\{ (V_c - V_a)^2 - (V_c - V_b)^2 \right\},$$

$$= K \left(V_a - V_b\right) \left(V_c - \frac{V_a + V_b}{2}\right), \qquad . \quad . \quad (5 \cdot 14)$$

where $K = \tfrac{1}{2} \alpha / \tau$, a constant.   The shape of the needle is of no consequence so long as the change in area within each pair of quadrants is proportional to the deflection.   For this purpose the outer edge of the needle is circular and the radial edges lie sufficiently well within quadrants.

This instrument can be used in two different ways: (1) In the *heterostatic* use $V_a \neq V_b \neq V_c$ and $V_c \gg V_a$ and V_b. Eq. $(5 \cdot 14)$ reduces to

$$\theta = K_h \left(V_a - V_b\right), \qquad \qquad . \quad . \quad (5 \cdot 15)$$

where K_h is another constant.

(2) In the *idiostatic* use the needle is connected to one pair of quadrants, say B. Eq. $(5 \cdot 14)$ reduces to

$$\theta = K_i \left(V_a - V_b\right)^2, \qquad \qquad . \quad . \quad (5 \cdot 16)$$

where K_i is another constant.

Since θ depends upon the square of $(V_a - V_b)$ the electrometer can measure alternating voltage when used idiostatically.

3. Compton electrometer. The Dolezalek instrument described above has been modified by various workers. In 1919 A. H. Compton and K. T. Compton introduced an electrometer of the quadrants type in which one quadrant can be raised or lowered with respect to the other three. Further dissymmetry is introduced by tilting the needle initially. Due to this dissymmetry the voltage sensitivity of the instrument is made as high as 5000 divs/volt (for full details see Phys. Rev. 14, 85, 1919).

4. Lindemann electrometer. This is another very convenient form of quadrant electrometer shown diagrammatically in fig. 5·4(a). A, B and A', B' are the quadrants. NN is a small needle suspended at the centre of a silvered quartz fibre FF. At the end of the needle there is a small rod. The movements of the electrometer needle are observed with the aid of a microscope fitted with about $\frac{1}{2}''$ objective and an eyepiece with a scale giving a magnification of about 8 times. The needle may also be projected on a screen for more convenient observation. A scheme of connections for a fair degree of accuracy is shown in figure 5·4(b). R, R are safety resistances, each of at least 250,000
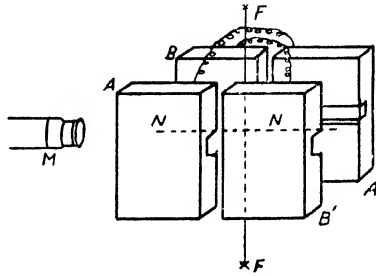
Fig. 5·4 (a)
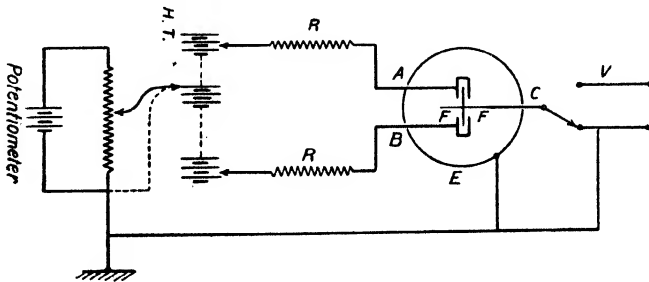
Fig. 5·4 (b)

ohms.   A H. T. battery with 3 volt tappings is used for giving potentials to the quadrants.

To start with, the needle and the quadrants are all connected to the earthed case.   The needle is then at its mechanical zero. By adjusting the body of the electrometer the needle is brought to any required position in the eyepiece field.   With the needle still earthed connect the quadrant plates to two points on the H. T. battery so that the potential applied is a little less than the maximum stated in the certificate sent with the instrument. The potentiometer is then adjusted so that the needle returns to its zero position.   The needle potential is then raised to 1 volt and by reading the resulting deflexion on the scale of the microscope the sensitivity of the electrometer is known. To measure any unknown potential V, it is connected to the terminal C of the needle..   The deflexion of the needle is noted on the microscope scale and from the value of the sensitivity determined previously the potential is readily known.

(For fuller details see the original paper by Lindemann and Keeley. Phil. Mag. Vol. 47, 1924).

5.   Practice with the Dolezalek electrometer.   We give below some details of the method of using this instrument in a laboratory.   For all the experiments with the electrometer certain preliminary adjustments must be made.   They are as follows :

Adjustments.   (1) Set up the electrometer and adjust the lamp and scale so that the position of the needle can be read upon the scale.   Note the reading when the quadrants and the needle are earthed.

(2) Charge the needle to a potential of say 100 volts, the quadrants remaining earthed.   Care should be taken in selecting the voltage on the needle so as not to make it unstable.   Note the reading.   If this differs from the previous reading it means that the needle is not placed symmetrically within the quadrants. Adjust the levelling screws of the instrument so that the needle

is brought back to its original position when the quadrants and the needle were earthed. Check the adjustment (1) again.

(3) Adjust the scale normal so that the deflections on the two sides of the zero for equal and opposite charges are equal.

(4) Test for any leakage of charge put upon the quadrants and improve the insulation of the support if any leakage is found.

Capacity of quadrants. For many experiments it is necessary to know the capacity of the electrometer. For this connections are made as shown in fig. 5·5.

The electrometer is connected to a two-way key S_1 and a one-
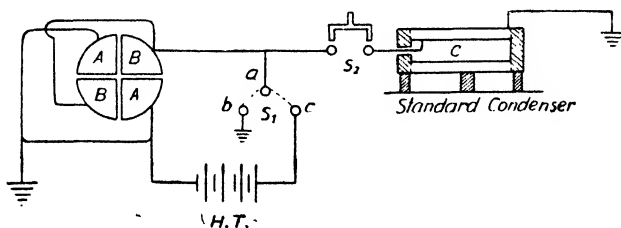
Fig. 5·5

way key S_2. First the quadrants B are connected to earth by connecting a to b. Note the reading on the scale. Disconnect a from b and connect it to c. The quadrants B receive a certain potential V corresponding to the voltage of the battery H. T. Note the deflection θ_1. Disconnect the quadrants B from the battery and connect them through the key S_2 to the inner cylinder of the cylindrical condenser of capacity C. Note the deflection θ_2. The quadrants have shared their charge with the condenser. Let c be the capacity of the electrometer. Assuming the deflection to be proportional to the potential the charge on the electrometer is $kc\theta_1$, where k is a constant. When the electrometer is connected to the condenser the total capacity becomes $(c+C)$. Hence the charge on the electrometer and the condenser is $k(c+C)\theta_2$. Since the charges are the same we have

$$c = C\,\theta_2/(\theta_1 - \theta_2).$$

Thus the capacity of the quadrant electrometer is obtained from the value of the capacity of the standard condenser and the de-

flections θ_1 and θ_2. If a is the external diameter of the inner cylinder, b the internal diameter of the outer cylinder and l the length, the capacity $C = l/2 \log_e \dfrac{b}{a}$. C is thus obtained in absolute measure.

6. Measurement of small potential differences. In eq. (5·15) if the constant K_h is known we can find out any unknown difference of potential from the observed deflection θ. To determine K_h the quadrants A are earthed. The quadrants B are then raised to a few potentials and the corresponding deflections are noted. A graph is then plotted with the deflection θ as the ordinate and the corresponding potential V as the abscissa. The slope of the straight line thus obatined gives K_h. The quadrants B are then earthed to ensure that no charge is left on them. The position of the needle is noted. The quadrants are then disconnected and connected to the unknown potential to be measured. Noting the deflection again eq. (5·15) gives us the potential since K_h is known.

7. Measurement of ionization current. Small electric currents produced in gases by ionizing agencies like X-rays, γ-rays, α-rays, etc., can be measured accurately with a quadrant electrometer. The experimental arrangement is shown in fig. 5·6. The ionization chamber consists of two metal plates P_1, P_2
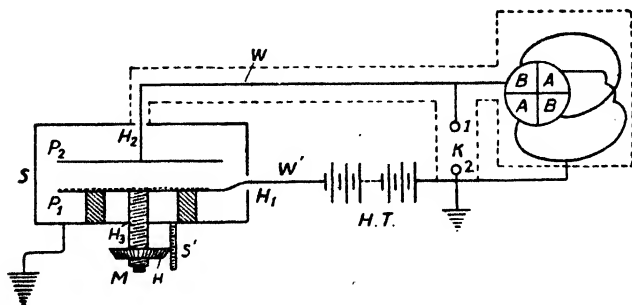
Fig. 5·6

of suitable size mounted horizontally inside a metal box which acts as an electric screen. P_1 is fixed to the end of a micrometer screw M capable of vertical movements which can be accurately measured with the help of the vertical scale S' and the graduated micrometer head H. The plate P_1 is isolated from the case by a suitable insulating ring H_3. Similarly, the fixed plate P_2 is isolated from the case by the insulating ring H_2. The wire W connects P_2 to the electrometer quadrants B and to the terminal 1 of the earthing key K. The terminal 2 of K and the quadrants A of the electrometer are earthed. The wire W, the key and the electrometer are all suitably screened as shown by the dotted line. The wire W' passing through insulating ring H_1 connects the plate P_1 to one pole of a high tension battery H. T. the other pole of which is connected to earth. The radioactive material such as uranium oxide used as the ionizing agent is put upon the plate P_1.

The electrometer is first adjusted as in experiments 1 and 2 and the capacity of the quadrants B together with that of the connecting wires and the plate P_2 is determined (see p. 54-55).

The rate of leakage of the charge on the quadrants is then measured. P_1 is first kept clean and a suitable small potential is put upon P_2 and B. The resulting deflexion would maintain itself for any length of time when B is kept isolated from earth and the charging potential. However, due to faulty insulation, inspite of all precautions, the deflexion slowly diminishes with time. If S_E be the electrometer sensitivity (number of scale divisions per unit potential in volts determined previously) and C the capacity of the system in e.s.u., the charge q on the quadrants B and the plate P_2, at any time corresponding to a deflexion ϕ, is given by $q = \dfrac{C}{S_E \times 300} \phi$. Noting ϕ at various times a graph is plotted connecting ϕ as ordinate and t as abscissa. The graph represents the leakage curve of the electrometer. The leakage current at any time is given by $\dfrac{dq}{dt} = \dfrac{C}{300 S_E} \cdot \dfrac{d\phi}{dt}$ in electrostatic units,

which must be taken into account in any subsequent measurement of current by the electrometer.

Now place a thin layer of uranium oxide on the plate P_1, separate P_1 from P_2 by, say, a centimetre, adjust V to a potential of few volts and connect the key K. On opening K the electrometer needle begins to move as P_2 catches ionic charges of one sign. Note the deflexion θ at various times and plot a graph with θ as ordinate and t as abscissa. Select the portion of the curve
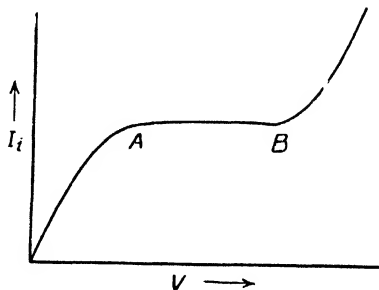
Fig. 5·7

where the motion of the needle is uniform. If $\dfrac{d\theta}{dt}$ is the rate of increase of θ in this part of the curve, the ionization current going through the electrometer is (allowing for the leakage)

$$I_i = \frac{1}{300 \times S_E} \cdot \frac{d}{dt}(\theta - \phi) \text{ e.s.u} = \frac{C}{300 \times S_E \times 3 \times 10^9} \frac{d}{dt}(\theta - \phi) \text{ amp.}$$

$$. \quad . \quad . \quad (5·17)$$

Repeat the observations for increasing values of V and plot the ionization current I_i as the ordinate and V as the abscissa. A curve similar to that shown in fig. 5·7 will be obtained. The current assumes a saturation value between A and B and then rapidly increases with further increase of V.

Next repeat the observations for I_i with varying distance between P_1 and P_2 and fixed value of V. It will be seen that I_i increases as the distance between the plates increases.

EXAMPLE. An idea of the magnitude of ionization current measured by electrometer may be obtained by the following. In a certain ionization experiment the following observations were obtained : capacity of the quadrant-plate system, 80 cms ; voltage sensitivity 500 mms ; leakage drift 5 mms per second and the total drift due to ionization and leakage 50 mms per second. Calculate the value of the ionization current.

Allowing for the leakage the rate of drift is 45 mms per second,

we have $\dfrac{1}{S_E} \times \dfrac{d(\theta - \phi)}{dt} = \dfrac{1}{500} \times \dfrac{45}{1} = \dfrac{9}{100}.$

Therefore,

$$I_i = \frac{80 \times 9}{300 \times 100 \times 3 \times 10^9} = 8 \times 10^{-12} \text{ amp.}$$

Very much lower currents may be measured by electrometers if the capacity of the system is considerably reduced. Electroscopes possess a very low capacity and so are capable of measuring currents of the order of 10^{-15} amp.

8. Measurement of dielectric constant. Boltzmann's method. The dielectric constant of a substance may be determined with a quadrant electrometer in the following manner. The capacity of a parallel-plate air condenser of area A is $A/4\pi t$, where t is the distance between the plates.

The capacity of the same condenser when a plane-parallel slab of dielectric constant K, area A and thickness d is introduced between the plates is $A/4\pi \left[t - d\left(1 - \dfrac{1}{K} \right) \right]$. Knowing the difference between the two capacities the dielectric constant is determined.

The connections are made as shown in fig. 5·5 . The condenser used being a guard ring condenser of which the guarded plate is connected to switch S_2 and the lower plate connected to earth. The lower plate is fitted with a micrometer screw by working which the plate can be raised or lowered and the amount of movement accurately measured. To start with, the electrometer is adjusted as usual. The key S_2 is lowered and a is connected to c in the two-way switch S_1. The condenser and the quadrants thus become charged to a potential V corresponding to the voltage V of the battery. The defletion θ_1 of the electrometer needle is noted. The connection a-c is now broken and the the dielectric slab introduced between the condenser plates. As the capacity is increased and the charge has not altered we get a lowering of the original potential V to V′ reducing the deflexion

from θ_1 to θ_2. Read the micrometer screw of the lower con-
denser plate and move the plate so that the deflection increases
from θ_2 to θ_1. Take the reading of the micrometer screw again.
The difference between the two readings m gives us d $(1 - 1/\text{K})$, i.e.,

$$\text{K} = d/(d - m).$$

The attracted disc electrometer may be similarly used to deter-
mine K.

By suitable modifications of the apparatus it has been found
possible to determine the dielectric constant of liquids and gases.
A suitable container may be filled with a liquid or a gas and in-
serted between the plates. Proceeding as before the dielectric
constant for the container and the liquid or gas is determined
and subtracting from this the dielectric constant for the container
we obtain that for the liquid or the gas. In the case of gases
the gas between the plates of a guard ring condenser is subjected
to increased pressure. It is found that the dielectric constant
of a gas is proportional to its pressure. Thus taking $\text{K} = 1$ for
vacuum we have K at a pressure p given by

$$\text{K} = 1 + ap/76,$$

where a is a constant.

It is also found that temperature has got an influence upon
the dielectric constant. For some substances there is a decrease
and for others an increase in K. To have some idea of the change
involved we give below the values for carbon bisulphide and
glass. For carbon bisulphide K decreases by $0\cdot004\%$ per degree
rise in temperature. For glass K increases at $30°\text{C}$ by $0\cdot2\%$
per degree rise in temperature.

It should be noted that these values for the dielectric constant
are for steady electric fields. For varying fields it is a function
of the frequency of the electrical oscillations. The reason is that
the electric strain created in the dielectric by the field takes a
finite time to change when changes in the field take place.

9. Electrostatic generators. Machines which develop
voltages especially high on the basis of *electrostatic induction* are
called electrostatic generators. Later on we shall deal with the

other type of generators the working of which depends upon the phenomeno n of electromagnetic induction.

The student is assumed to be already familiar with the working of Wimshurt and Voss machines developed long ago for producing high voltages by making use of the phenomenon of electrostatic induction. These machines suffer from certain disadvantages, the chief among which are their low power output and bad regulation. The v oltage decreases rapidly when the load increases.

Van de Graaff generator. This type of generator is superior to the Wimshurt and Voss machines as it gives large power output. Its principle is shown diagrammatically in fig. 5·8. A large hollow metal sphere S is supported on an insulating column I. A belt B of some insulating material such as rubber or silk travels at a very rapid rate in the column between two pulleys P_1 and P_2, the former motor driven and put outside and the latter an idling pulley fixed inside the sphere. T_1, T_2, T_3 and T_4 are the electrodes. E is a small battery used for giving a small positive charge to the electrode T_1. The belt as it moves up is charged positively when it comes close to the pointed end of T_1 by what is called *corona* discharge. When the belt reaches the electrode T_3 it transfers its positive charge to this electrode again by *corona* discharge. The electrode T_4 then becomes negatively charged by induction. The down-going belt then gets negative charge from this
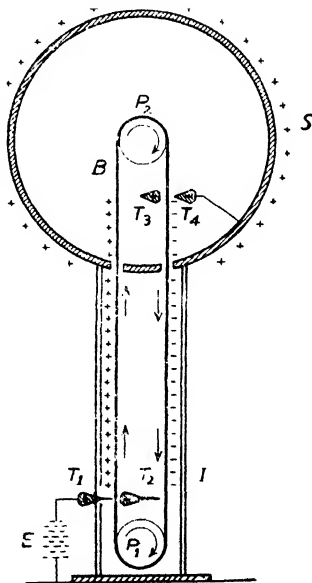
Fig. 5·8 Van de Graaff generator

electrode. As the belt reaches the electrode T_2 the latter gets some negative charge which by induction enhances the positive charge received by the up-going belt. The rest of the negative charge is lost to the ground through pulley P_1. The rate of this transference of charge goes on increasing and the sphere soon gets a high potential with respect to the ground. The mutual repulsion of the positive charges makes them confined to the outer parts of the sphere. The electrode T_4 is thus left free to receive more charge. The sphere, owing to the surface smoothness and large radius of curvature, loses very little charge by corona and accumulates very large charge and potential. Discharge in air is reduced by enclosing the whole in a tank containing air at high pressure.

In recent times this type of generator has been considerably improved to give very high voltages. In an experimental model the diameter of the sphere which is made of an aluminium alloy is raised to 15 feet. It is expected to yield a voltage of the order of ten million volts and an output of the order of 20 kilowatts. Van de Graaff type of generator has been found very useful in the study of atomic nuclei.

10. Electrostatic voltmeters. Kelvin's electrostatic voltmeter. Several electrostatic voltmeters have been designed on the principle of the quadrant electrometer which is indeed an electrostatic voltmeter for measuring small potential differences. The fundamental principle underlying all electrostatic voltmeters is the attraction of electrified bodies ; we give below a brief description of Kelvin's instrument. The essential parts inside an earthed metallic case are shown in fig. 5·9. SS is a fixed metallic sector. The movable metallic conductor $N_1 N_2$ is sup-
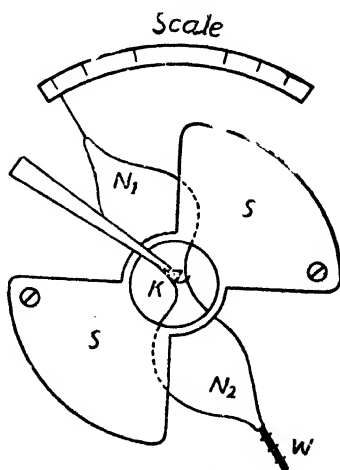
Fig. 5·9

ported on a knife-edge K in the centre. At one end of $N_1 N_2$ there is a long light pointer moving over a circular scale. The other end of $N_1 N_2$ carries a small weight W. When a difference of potential is established between S and $N_1 N_2$, N_1 moves over the scale under the action of an electric couple the magnitude of which depends upon the potential difference. This electric couple is balanced against gravity. The scale of the instrument is calibrated with known potential differences and so it is self-recording. Its range is varied by altering the weight W and under suitable conditions it can read up to several thousand volts.

Attracted disc type. Electrostatic voltmeters based on the principle of attracted disc electrometer have been constructed in a variety of forms for various ranges. When a difference of potential is established between the fixed plate and the movable plate the latter is attracted. The movements of this disc are transmitted to a suitably arranged mechanical pointer which moves over a scale. When calibrated this instrument gives voltages directly.

Sphere gap electrostatic voltmeter. This is the simplest and most widely used voltmeter suitable for moderate and high voltages of the order of 100 kilovolts. It consists principally of two similar metal spheres S (fig. 5·10) of the same diameter mounted
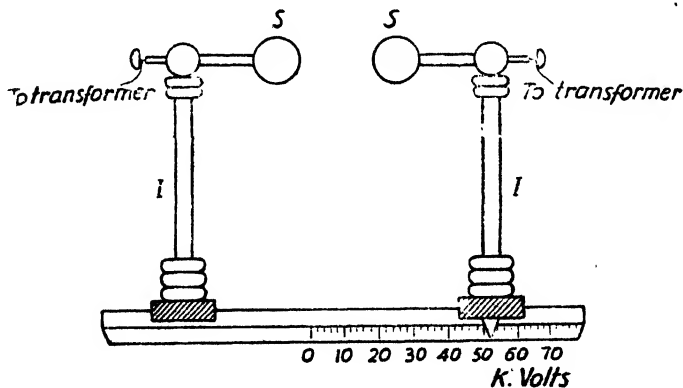
Fig. 5·10. Sphere gap voltmeter

5 E

on insulated pillars I on a rigid frame-work. One of them is fixed while the other is movable. The latter has an index attached to it which moves over a linear scale. The standard sizes of the spheres vary from 5 mms. to 2000 mms. in diameter. The voltage to be measured is applied to the spheres and the distance between them is adjusted so that a spark just jumps across the air gap. The distance between the spheres gives a measure of the voltage. This instrument is first calibrated with known voltages to make it direct reading. Certain precautions are necessary for accurate determinations. They are : (a) the width of the spark i.e., the distance between the spheres should not exceed the radius of either sphere, otherwise the field between the spheres will not remain uniform, (b) the spheres should be cleanly polished for consistent sparking, (c) a resistance of the order of 1000 ohms per kilovolt measured should be inserted in series with the gap. This, among other things, limits the current and saves the spherical surfaces from pitting and (d) neighbouring objects should be sufficiently away from the gap to minimise the effects due to induced charges.

MAGNETOSTATICS

1. Introduction. The science of magnetism had its origin in the observation of the ancients that a certain mineral ore found in Asia Minor had the peculiar property of attracting small pieces of iron. It was called lodestone or leading stone.

If a bar of iron was rubbed with a lodestone and then plunged into iron filings, the latter were found to adhere most strongly at the ends of the bar. Those regions where the magnetism was strongest were called the poles of the magnet and the line joining them as the axis of the magnet. If such a magnetised bar was suspended freely it always pointed in a definite direction ; the end which pointed towards the geographical north was called the *north pole* and that which pointed towards the south was called the *south pole*. Like poles repelled and unlike poles attracted each other with a force which is inversely proportional to the square of the distance. All this was established experimentally much before Gilbert, the founder of modern magnetism. It was also discovered that it was impossible to isolate a magnetic pole like an electric charge.

Though the two sciences of electricity and magnetism developed independently of each other, we know now that they should be regarded as different aspects of electric charge at rest or in motion. We shall see later that a moving electric charge constitutes an electric current which is always accompained by a magnetic field.

2. Molecular theory of magnetism. If we take a magnet and go on subdividing it we shall see that it is impossible to isolate a north pole from the south pole. Thus according to

this theory all magnetic substances are made up of a large number
of elementary small magnets.
In an unmagnetized material
the molecular magnets are in a
chaotic state, (fig. 6·1 *a*). Hence
an ordinary piece of iron will
exhibit no magnetism. The pro-
cess of magnetization consists in

 (a)

 (b)

Fig. 6·1

arranging the molecular magnets in an orderly fashion, (fig. 6·1
b). The regular arrangement of the elementary magnets is
opposed partly by friction and partly by their mutual magnetic
interactions. When all the magnets have been arranged the
substance exhibits magnetic saturation, i.e., the magnetization
will not increase further.
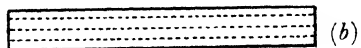
On the present view each atom of a magnetic substance is
an elementary magnet. The magnetism is due to the motion
of electrons in the atom. As we know that the electrons are not
only going round the nucleus but they are also spinning about
their own axis. The orbital motion of the electrons gives rise to
a feeble magnetism (called paramagnetism), whereas the stronger
magnetism of iron is due to the spin motion of the electrons (called
ferromagnetism).

3. Magnetic law of force. The force between two poles
of strength m_1 and m_2 in vacuum at a distance r apart is given by

$$F = \frac{m_1 m_2}{r^2}. \qquad \qquad . \quad . \quad . \quad (6·1)$$

In eq. (6·1) F is equal to one dyne if m_1, m_2 and r are each equal
to unity. A unit magnetic pole is that pole which exerts a force
of one dyne on an equal pole placed at a distance of one cm., both
poles being situated in vacuum. (6·1) is analogous to Coulomb's
law of force in electrostatics.

The intensity of a magnetic field at any point is given by the
force which would be experienced by a unit pole placed at that
point. The unit of magnetic field is *gauss*.

The force on a pole of strength m at a point where the intensity of the magnetic field is H is given by

$$F = mH. \qquad \qquad \ldots \quad (6\cdot2)$$

4. Magnetic lines of force.

Analogous to the electric line of force a magnetic line of force is a line the direction of which at any point gives the direction of the magnetic intensity at that point. All lines of force are regarded as starting from a N-pole and ending on a S-pole. No two lines of force can cross each

Fig. 6·2 (a)

other. Lines of force due to a bar magnet are shown in fig. 6·2 (a), while those due to two bar magnets placed parallel to each other are shown in figs. 6·2 (b) and 6·2 (c).
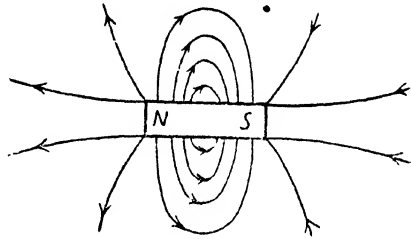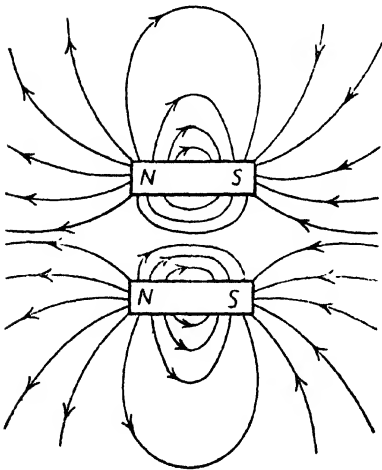
Fig. 6·2 (b)
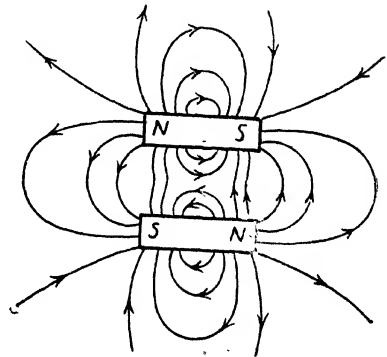
Fig. 6·2 (c)

5. Magnetic potential. The magnetic potential at a

point in a magnetic field is
defined as the work done
in bringing a unit pole from
infinity to that point. The
unit of magnetic potential
is ergs per unit pole.

The field H at point
P_1, (fig. 6·3), due to a pole of
strength m at O, is
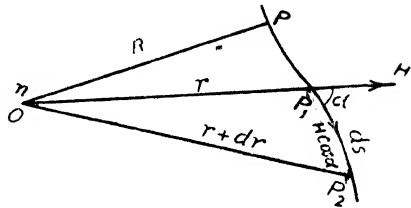
Fig. 6·3

$$H = \frac{m}{r^2} \text{ , from eq. } (6·1).$$

The work done in carrying a unit pole from P_2 to P_1 is

$$dV = \frac{m}{r^2} \cos \alpha \, dS,$$

$$= -\frac{m}{r^2} dr, \quad \text{since } dS \cos \alpha = -dr.$$

The potential at P is $V_P = -\int_{r=\infty}^{r=R} \frac{m}{r^2} dr = \frac{m}{R}.$ \hfill (6·3)

From the above deduction it is clear that the potential at any
point is a function of the coordinates of the point and is independ-
ent of the path along which the unit pole is brought from infinity
to that point. The components of the magnetic field at the point
P $(x, y, z,)$ are given by

$$Hx = -\frac{\partial V}{\partial x}, \quad Hy = -\frac{\partial V}{\partial y}, \quad Hz = -\frac{\partial V}{\partial z}. \tag{6·4}$$

6. Magnetic moment. The magnetic moment of a
magent of length $2l$ and pole strength m is defined as the product
of the pole strength and the distance between the poles, i.e.,

$$M = 2ml. \qquad \qquad . \quad . \quad . \quad (6·5)$$

If a freely suspended magnet is turned through an angle θ from the magnetic meridian (fig. 6·4), it will have a tendency to come back to its original position. The couple required to keep it in the displaced position is given by
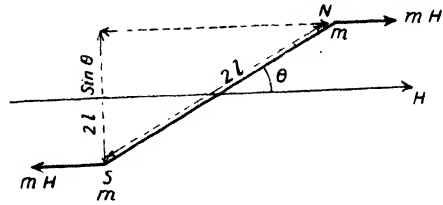
Fig. 6·4

$$C = 2Hml \sin \theta. \qquad \ldots \quad (6·6)$$

If $\theta = 90°$ and $H = 1$ gauss

$$C = 2ml = M.$$

Therefore a magnetic moment can also be defined as the couple required to keep the magnet at right angles to the field of unit strength, H being the only force acting.

7. Intensity of magnetization. It is defined as the magnetic moment per unit volume, i.e.,

$$I = \frac{M}{V}, \qquad \ldots \quad (6·7)$$

where V is the volume of a magnetic material and M its magnetic moment.

Let us take the case of a rectangular magnet of length $2l$ and cross-section α. Then,

$$I = \frac{M}{V} = \frac{2ml}{2l\alpha} = \frac{m}{\alpha}. \qquad \ldots \quad (6·8)$$

Thus the intensity of magnetization can also be defined as the pole strength per unit area.

8. Magnetic susceptibility. If a magnetic material under the influence of a magnetising force H acquires an intensity of magnetization I, the magnetic susceptibility κ is defined as

$$\kappa = \frac{I}{H}. \qquad \ldots \quad (6·9)$$

9. Gauss' law. Consider a pole m enclosed by a closed surface S of any shape, (fig 6·5). Let ds be an elementary area of the surface. The normal induction over the surface ds is H cos α ds. According to this law the total normal induction N over any closed surface enclosing a pole m is equal to $4\pi m$.

Fig. 6·5

Now,
$$dN = H \cos \alpha \, ds,$$
$$= \frac{m}{r^2} \cos \alpha \, ds = m d\Omega,$$
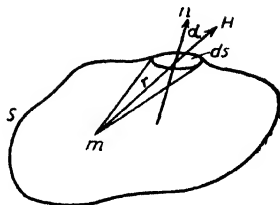
where $d\Omega$ is the solid angle subtended by ds at m.

Hence
$$N = \int m d\Omega = 4\pi m. \qquad \qquad . \quad . \quad (6\cdot10)$$

If the pole is outside the surface it can be shown that
$$N = 0.$$

10. Analogy between electrostatic and magnetostatic phenomena. The elctrostatic and the magnetostatic phenomena show a great similarity in two important respects, viz. (1) like poles repel and unlike poles attract each other, and (2) the force between two poles is given by the inverse square law.

There are, however, certain fundamental differences between the two phenomena : (1) It is impossible to isolate either a north pole or a south pole ; the two poles always go together. On the other hand in the electrical case the positive and negative charges can be isolated. (2) There is no such substance known in which the magnetic poles move under the influence of an impressed magnetic field, i.e., there are no magnetic conductors. On the other hand we have electric conductors in which the electric charges move under the influence of the impressed electric field.

11. Force due to a magnetic dipole. Two poles $-m$

and $+m$ very close together constitute a magnetic dipole, (fig. 6·6).
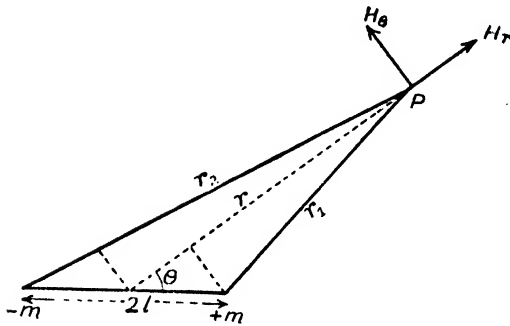
Fig. 6·6

Potential at P due to the dipole having length $2l$ and magnetic moment M is

$$V_P = + \frac{m}{r_1} - \frac{m}{r_2},$$

$$= \frac{m}{r - l \cos \theta} + \frac{m}{r + l \cos \theta},$$

$$= \frac{2ml \cos \theta}{r^2 - l^2 \cos^2 \theta} = \frac{M \cos \theta}{r^2 - l^2 \cos^2 \theta}. \qquad (6·11)$$

The field along the radius vector at P is

$$H_r = - \frac{\partial V}{\partial r} = - \frac{\partial}{\partial r} \frac{M \cos \theta}{r^2 - l^2 \cos^2 \theta},$$

$$= \frac{2M \cos \theta}{(r^2 - l^2 \cos^2 \theta)^2}. \qquad . \quad . \quad . \quad (6·11a)$$

The field prependicular to the radius vector is

$$H_\theta = - \frac{1}{r} \frac{\partial V}{\partial \theta} = - \frac{1}{r} \frac{\partial}{\partial \theta} \frac{M \cos\theta}{(r^2 - l^2 \cos^2\theta)},$$

$$= \frac{M \sin \theta \ (r^2 + l^2 \cos^2 \theta)}{r(r^2 - l^2 \cos^2 \theta)^2}. \qquad (6·11b)$$

If $r >> 2l$ $V_p = \dfrac{M \cos \theta}{r^2}$,   . . . (6·12)

then $H_r = \dfrac{2M \cos \theta}{r^3}$,   . . . (6·13)

and $H_\theta = \dfrac{M \sin \theta}{r^3}$.   . . . (6·14)

For $\theta = 0$ $H_r = \dfrac{2M}{r^3}$,   . . . (6·15)

and for $\theta = \pi/2$ $H_{l2} = \dfrac{M}{r^3}$.   . . . (6·16)

12. Potential energy of a magnet in a magnetic field. The couple C acting on a magnet placed in an external field H is

$$C = - MH \sin \theta \text{ from eq.}(6\cdot6).$$

The minus sign indicates that the couple is tending to decrease θ.

The potential energy of the magnet in the displaced position is

$$U = - \int - MH \sin \theta \, d\theta = - MH \cos \theta, \quad . . . \quad (6\cdot17)$$

since $\text{couple} = - \dfrac{\partial V}{\partial \theta}$.

13. Vibration magnetometer. It consists of a light brass stirrup S suspended by means of a torsionless fibre F from the torsion head T, (fig. 6·7 a). The stirrup S can carry a small magnet and a brass rod and is also provided with a light aluminium pointer P which moves over a circular scale at the base of a small rectangular box B enclosing the suspended system. The box is mounted on a base provided with three levelling screws. It is provided with two collinear graduated arms on which slide two cradles C_1 and C_2 which carry the auxiliary magnet to deflect the suspended needle. The centre of the circular scale coincides with the zero of the graduated arms. In a more accurate instrument the suspension is provided with a small mirror and the

deflections are measured by a lamp and scale arrangement.
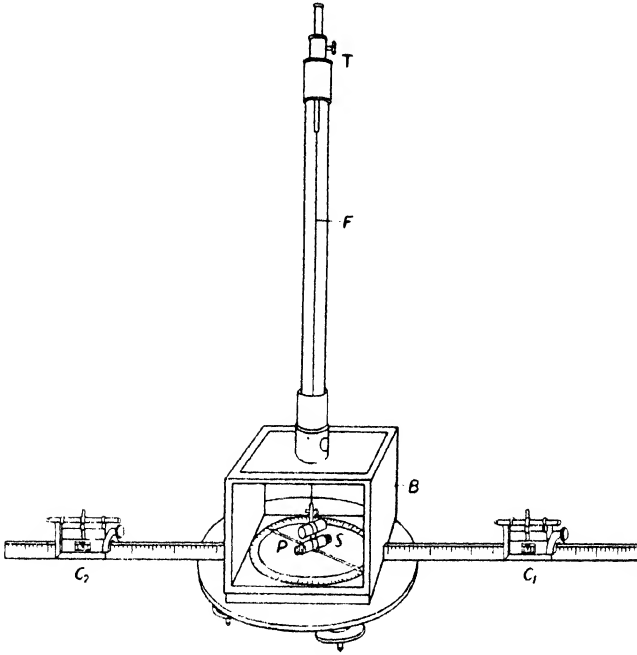
Fig. 6·7(a). Vibration magnetometer

Determination of the Earth's horizontal field. To determine H we use the simple vibration magnetometer des-
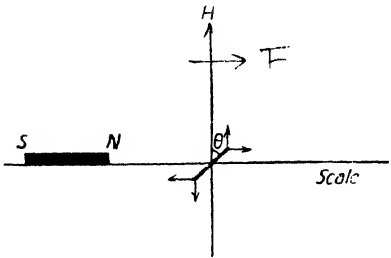
Fig. 6·7 (b)
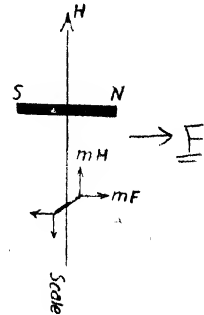
Fig. 6·7 (c)

cribed above.   It can be used either in gauss A position or in gauss B position (figs. 6·7 *b* and 6·7 *c*).

In the former the arms of the magnetometer are set perpendicular to the magnetic meridian and the deflecting magnet is placed along the arms ; whereas in the latter the arms are set along the magnetic meridian and the deflecting magnet is placed perpendicular to the arms.   In both the cases the magnet points east-west.

Adjustments :

(1) Determine the magnetic meridian by means of an ordinary magnetic compass and set the two arms in a direction perpendicular to it.

(2) Remove the torsion of the fibre.

(3) Adjust the levelling screws so that the suspension passes through the centre of the circular scale.

A small magnet of magnetic moment M is placed at a distance from the oscillating magnet of moment M_1.   We suppose that the oscillating magnet is so small that in the region occupied by it there is no variation of magnetic field due to M.   Under the influence of M, M_1 will be deflected through an angle θ with respect to the magnetic meridian.   The couple on M_1 due to the earth's field H is $HM_1 \sin \theta$ and that due to F (the field of M) is $FM_1 \cos \theta$.   In the equilibrium position the two couples must be equal, i.e.,

$$HM_1 \sin \theta = FM_1 \cos \theta. \qquad . \quad . \quad . \quad (6·18)$$

From eq. (6·15) $$F = \frac{2M}{d^3},$$

provided the magnet is very small in comparison to *d*.

From the above two eqs.   we have,

$$\frac{M}{H} = \frac{d^3}{2} \tan \theta. \qquad . \quad . \quad . \quad (6·19)$$

To eliminate M we perform a vibration experiment. M is suspended and allowed to oscillate in the earth's field.   Its equation of motion is

$$I \frac{d^2\theta}{dt^2} = - MH \sin \theta,$$

$$= - MH\,\theta, \text{ if } \theta \text{ is small,}$$

where I is the moment of inertia of M about the vertical axis.

The period T is

$$T = 2\pi \sqrt{\frac{I}{MH}}. \qquad \ldots \quad (6.20)$$

To eliminate I a small brass rod is placed along with the magnet in the stirrup and the period is determined.

Now, $\qquad T_1 = 2\pi \sqrt{\frac{I}{MH}}$, with magnet alone,

and $\qquad T_2 = 2\pi \sqrt{\frac{I + I_1}{MH}}$, with magnet + the brass

bar of moment of inertia I_1.

From the above equations, we have

$$\frac{1}{4\pi^2}\left(T_2{}^2 - T_1{}^2\right)MH = I_1, \qquad \ldots \quad (6.21)$$

Eliminating M from eqs. (6.19) and (6.21) we have

$$H = 2\pi \sqrt{\frac{2I_1}{d^3 \tan\theta \,(T_2{}^2 - T_1{}^2)}}. \qquad \ldots \quad (6.22)$$

In deducing the above expression we have assumed that the distance d is very large in comparison to the length of M. In practice if d is very large the deflection is too small for accurate measurement. Hence we need a more accurate formula for F, in which the square of the length of M is retained. From eq. 6.11(a)

$$F = \frac{2Md}{(d^2 - l^2)^2} = \frac{2M}{d^3}\left(1 + \frac{2l^2}{d^2} + \ldots\right), \qquad \ldots \quad (6.23)$$

since $\theta = 0$.

Eq. (6.19) now takes the form

$$\frac{M}{H}\left(1 + \frac{2l^2}{d^2}\right) = \frac{d^3}{2}\tan\theta. \qquad \ldots \quad (6.24)$$

If θ_1 and θ_2 are the deflections at distances d_1 and d_2 respectively we have

$$\left.\begin{array}{l} \dfrac{M}{H}\left(1 + \dfrac{2l^2}{d_1{}^2}\right) = \dfrac{d_1{}^3}{2}\tan\theta \\[2ex] \dfrac{M}{H}\left(1 + \dfrac{2l^2}{d_2{}^2}\right) = \dfrac{d_2{}^3}{2}\tan\theta \end{array}\right\}. \qquad \ldots \quad (6.25)$$

and

Eliminating l^2 from the above two equations we have

$$\frac{M}{H} = \frac{d_1^5 \tan \theta_1 - d_2^5 \tan \theta_2}{2(d_1^2 - d_2^2)} . \qquad . \quad . \quad . \quad (6{\cdot}26)$$

If we use eq. (6·26) to determine H knowing M from eq. (6·21) we need not find the length of the magnet, which involves uncertainty.

METHOD. Place M in the sliding cradle at a distance d and read the two ends of the pointer. Turn M end for end and read the two ends of the pointer again. Transfer M to the other arm the distance remaining the same and repeat the above procedure. The angle θ is given by the mean of the eight readings. This lengthy procedure is adopted to avoid errors due to lack of any mechanical or magnetic symmetry. To determine the period a large number of oscillations say 100, must be taken.

EXAMPLE.—A bar magnet is brought up in the end-on position to a point due east of a vibrating magnetometer which is thereby deflected through an angle θ. Show that the time of oscillation of the magnetometer is changed in the ratio $(\cos\theta)^{\frac{1}{2}}$: 1.

Before the magnet is brought the field is H and

$$T = 2\pi \sqrt{\frac{I}{MH}}.$$

When the magnet is brought the resultant field is

$$\sqrt{H^2 + H^2 \tan^2\theta} = H \sec\theta,$$

and

$$T_1 = 2\pi \sqrt{\frac{I}{MH \sec\theta}}.$$

Hence

$$T_1 : T = (\cos\theta)^{1/2} : 1.$$

Proof of the inverse square law by vibration magnetometer. When the auxiliary magnet is in gauss A position the field F_1 at O is

$$F_1 = \frac{2M}{d^3} \text{ approx.}$$

$$= H \tan\theta_1, \qquad . \quad . \quad . \quad (6{\cdot}27)$$

and when it is in gauss B position the field is

$$F_2 = \frac{M}{d^3} = H \tan\theta_2. \qquad . \quad . \quad . \quad (6{\cdot}28)$$

From eqs. (6·27) and (6·28)

$$\tan \theta_1 = 2 \tan \theta_2. \qquad \ldots \quad (6·29)$$

By varying d we obtain a large set of θ_1 and θ_2 . And if the inverse square law holds, $\dfrac{\tan \theta_1}{\tan \theta_2}$ will always be equal to 2.

If we do not neglect the length of the magnet then in gauss B position

$$H \tan \theta = \frac{M}{(d^2 + l^2)^{3/2}},$$

or

$$(d^2 + l^2) = \left(\frac{M}{H} \cot \theta \right)^{\frac{2}{3}}. \qquad \ldots \quad (6·30)$$

If $(\cot \theta)^{2/3}$ is plotted against d^2, we get a straight line if the inverse square law holds.

14. Uniformly magnetized sphere. A magnetic dipole may be thought as formed by pulling apart a short distance δx two initially coincident equal and opposite poles. Let us take a uniformly magnetized sphere of radius a, (fig. 8·6). We may regard its magnetic condition to have been produced by pulling apart a short distance $OO' = \delta x$ two initially coincident spheres each of magnetic density $+ \rho$ and $- \rho$ respectively. The intensity of magnetization I is equal to $\rho \delta x$.
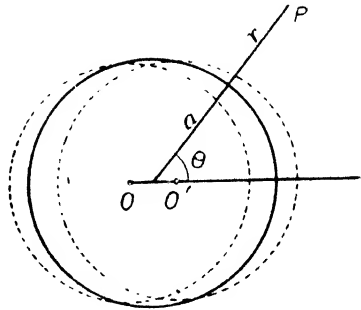
Fig. 8·6

Our uniformly magnetized sphere is, therefore, equivalent to a dipole OO' whose moment is given by

$$M = \rho \delta x V = \rho \delta x \tfrac{4}{3} \pi a^3, \qquad \ldots \quad (6·31)$$

where V is the volume of the sphere.

Hence the potential at the point P (r, θ) is

$$V_P = \rho \delta x \tfrac{4}{3} \pi a^3 \frac{\cos \theta}{r^2},$$

$$= \tfrac{4}{3} \pi a^3 . \frac{I \cos \theta}{r^2}. \qquad \ldots \quad (6·32)$$

Thus the potential at any external point is the same as that given by a magnet of moment $\frac{4}{3}\pi a^3$ I placed at the centre of the sphere.

EXAMPLE.—Treating the earth as uniformly magnetized and supposing the horizontal force to be 0·18 gauss in the magnetic latitude 52°, find the intensity of magnetization and the dip in this latitude.

Potential at any external point is

$$V = \frac{M \cos\theta}{r^2},$$

where $M = \frac{4}{3}\pi a^3 I.$

Now $(H_r)_p = -\left(\frac{\partial V}{\partial r}\right)_{r=a} = \frac{2M\cos\theta}{a^3}.$

and $(H_\theta)_p = -\left(\frac{1}{r}\cdot\frac{\partial V}{\partial\theta}\right)_{r=a} = \frac{M\sin\theta}{a^3}.$

But H_θ is given to be 0·18, therefore,

$$0{\cdot}18 = \frac{4}{3}\pi a^3 I \frac{1}{a^3} \sin(\pi/2 + 52°),$$

Hence $I = \dfrac{0{\cdot}18 \times 3}{4\pi\cos 52°},$

$$= 0{\cdot}0698 \quad \text{pole strength/area.}$$

The angle of dip is given by

$$\theta = \tan^{-1}(2\tan 52),$$
$$= 68°,39'.$$

15. Mutual potential energy of two small coplanar magnets. Consider two small magnets of magnetic moments M_1 and M_2. Let θ_1 and θ_2 be the angles which M_1 and M_2 make respectively with the line joining their centres, (fig 6·9).

F-6·9

Fig. 6·9

The potential energy of M_1 in the field due to M_2 is

$$W = -M_1 H_\theta \times \text{cosine of the angle}$$

which the field at M_1 makes with the axis of M_1.
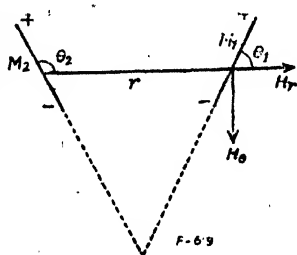
$\qquad\qquad$ . . . (6·33)

Now $H_r = \dfrac{2\,M_2 \cos\theta_2}{r^3}$ and $H_\theta = \dfrac{M_2 \sin\theta_2}{r^3}$.

Their components along the axis of M_1 are respectively $\dfrac{2\,M_2 \cos\theta_2}{r^3} \cos\theta_1$ and $-\dfrac{M_2 \sin\theta_2 \sin\theta_1}{r^3}$.

Hence $W = -\dfrac{M_1\,M_2}{r^3}(2\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2)$ . . . (6·34)

CASE (1) If the two magnets are equal and parallel to each other, i.e., $M_1 = M_2 = M$ and $\theta_1 = \theta_2 = 90^\circ$

$$W = \frac{2\,M^2}{r^3}.$$ . . . (6·35)

CASE (2) If the two magnets are equal and lie in the same straight line with their north and south poles pointing towards each other, i.e., $\theta_r = \theta_2 = 0$

$$W = -\frac{2\,M^2}{r^3}.$$ . . . (6·36)

The potential energy is minimum in case (2) and hence that would be a stable position.

16. Magnetic shell. A thin sheet of magnetic material magnetized normally at every point of the surface is called a magnetic shell. The strength ϕ of a shell at any point is given by the product of the intensity of magnetization I and the thickness t at that point. For a uniform shell,

$$\phi = It = \frac{M}{V}t = \frac{M}{S},$$ . . . (6·37)

where V is the volume of the shell, and S its area.

Therefore, the strength of a shell can also be defined as the magnetic moment per unit area of the shell.

A uniform shell can be regarded as made up of small magnets all directed along normals to the sheet, with their positive poles on one surface and the negative poles on the other.

6 E

17. Potential due to a magnetic shell of strength φ.

Consider an elementary area *ds* of the shell (fig. 6·10). This is equivalent to a small magnet of moment ϕds.

The potential at P due to this elementary area is

$$\delta V_P = \frac{\phi ds \cos\theta}{r^2}, \quad . \quad . \quad (6\cdot38)$$
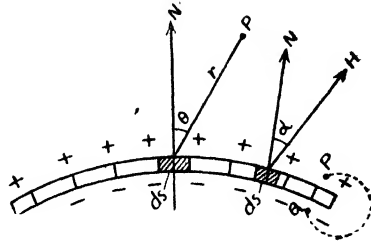
Fig. 6·10

where θ is the angle which the positive direction of the normal (the positive direction is taken when the normal is drawn from $-ve$ to $+ve$ side of the shell) makes with the radius vector; or

$$\delta V_P = \phi d\Omega,$$

where $d\Omega$ is the solid angle subtended at P by *ds*.

The potential due to the whole shell is, therefore,

$$V_P = \int \phi d\Omega = \phi\Omega. \quad . \quad . \quad . \quad (6\cdot39)$$

Thus the potential at any point due to a magnetic shell of strength ϕ is ϕ times the solid angle subtended at that point by the shell. The potential is independent of the form of the shell but depends only on its boundary. For a closed spherical shell the potential at any outside point is zero and at any inside point is $-4\pi\phi$.

The work done in carrying a unit $+ve$ pole from the $+ve$ to the $-ve$ side of the shell along a path which goes round the edge is

$$W = V_P - V_Q, \quad . \quad . \quad (6\cdot40)$$

where P and Q are two points very near the shell and on the $+ve$ and $-ve$ side of the shell respectively. By eq. (6·39) we have

$$W = \phi\Omega - \{-(4\pi - \Omega)\phi\}$$
$$= 4\pi\phi, \quad . \quad . \quad (6\cdot41)$$

where Ω is the solid angle subtended by the shell at P.

18. Potential energy of a magnetic shell in a magnetic field H.

Consider an elementary area *ds* of the shell (fig. 6·10). Let the field H make an angle α with the normal to *ds*.

The potential energy of the element ds of the shell is

$$dU = -\phi ds\, H \cos \alpha.$$

and for the whole shell

$$U = -\phi \int_S H \cos \alpha\, ds$$

$$= -\phi N, \qquad \qquad . \quad . \quad . \quad (6\cdot42)$$

where $N = \int_S H \cos \alpha\, ds$ is the magnetic flux through the shell.

EXAMPLE.—A uniform magnetic shell of strength ϕ is bounded by a circle of radius a. Find the mechanical force on a small magnet of moment M placed along the axis at a distance z and also find its period of oscillation, I being the moment of inertia of the magnet.

The potential at P, (fig. 6·11), is given by $V_P = \phi\Omega$, where Ω is the solid angle subtended by the shell at P.

But $\quad \Omega = 2\pi(1 - \cos\theta)$

$$= 2\pi\left(1 - \frac{z}{\sqrt{a^2 + z^2}}\right).$$

Hence the magnetic force H at P is

$$H = -\frac{\partial V}{\partial z} = \frac{2\pi\phi a^2}{(a^2 + z^2)^{3/2}}$$

directed along the axis.

Fig. 6·11

The period T of the magnet is

$$T = 2\pi\sqrt{\frac{I}{MH}} = 2\pi\sqrt{\frac{I(a^2 + z^2)^{3/2}}{2\pi\phi a^2}}.$$

The mechanical force at the two ends of the magnet is

$$-mH \quad \text{and} \quad m\left(H + \frac{\partial H}{\partial z}\,\delta z\right).$$

Therefore, the resultant force is

$$m\delta z\,\frac{\partial H}{\partial z} = M\frac{\partial H}{\partial z} = -\frac{6\pi\phi Ma^2 z}{(a^2 + z^2)^{5/2}}.$$
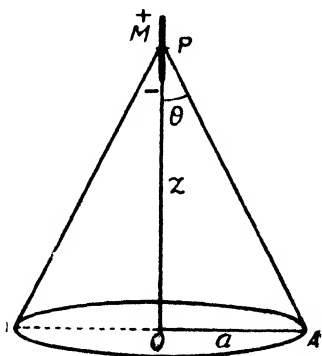
CHAPTER VII
PRIMARY AND SECONDARY CELLS

1. Electromotive force. In electrostatics we were concerned with the phenomena of electrostatic field due to charges not in motion. Consider the case of two bodies A and B with charges $+q$ and $-q$ respectively. Let V_1 be the potential of A and V_2 that of B. The difference of potential $V_1 - V_2$ is the work done in taking unit positive charge from V_2 to V_1 against the repulsion due to $+q$ and the simultaneous attraction due to $-q$. When the two bodies are connected by a conducting wire there is a flow of electrons from B to A till equality of potentials is obtained. In other words there is a current of electricity from A to B in analogy with the current of water flowing through a pipe connecting two reservoirs of water filled to different levels. The speed with which the equalisation of water levels is effected depends upon the resistance of the pipe, the speed being small when this resistance is high and vice versa. In the same way the speed with which the voltages are equalised will depend upon the conducting power of the connecting wire.

In order to maintain a steady current of water there must be some agency to maintain the level difference constant in the two reservoirs. In the same way to maintain a steady flow of electric current in the wire connecting the two charged bodies we must connect them to some agency which will maintain their difference of potential constant. In other words we require some kind of electric pump for removing electrons from A and transferring them to B so as to maintain their positive and negative charges. Various kinds of such electron pumps exist. They are listed below : (1) Electrostatic generators, (2) Electro-chemical cells, (3) Electromagnetic generators, (4) Thermocouples and (5) Photoelectric cells.

We will take the help of device no. 2, i.e., an electrochemical cell, to maintain the difference of potential between A and B.

Suppose that the wire connecting A and B is removed. The electrochemical action in the cell transfers electrons from A to B and thus the former becomes positively charged and the latter negatively. Thus the potential of A is raised with respect to that of B. This tends to drive the surplus electrons from B to A through the cell. The electrochemical action in the cell, however, brings about a condition of equilibrium so that the difference of potential between A and B is constant. The electrochemical energy spent in this is known as the electromotive force (e.m.f.) of the cell. This is exactly of the same nature as the potential difference (p.d.) between A and B and is expressed in volts.

When A and B are connected by a conducting wire the surplus electrons from the negative terminal B flow through the wire to the positive terminal A. This loss of electrons by B is just compensated by the e.m.f. of the cell transferring electrons from A to B through the cell so as to keep the p.d. between A and B constant. In general the p.d. between A and B on open circuit is greater than that when the circuit is closed by connecting A to B by a wire. This is obviously due to part of the e.m.f. of the cell being used up in driving the current through the internal resistance of the cell ; the balance appears as p.d. between A and B. The e.m.f. of a cell is equal to the p.d. between A and B on open circuit. In a closed circuit the e.m.f. is the rate of work to maintain unit current in the circuit ; if i is the current and e the e.m.f. the total work done in maintaining the current in the circuit for time t is given by

$$W = eit. \qquad . \quad . \quad . \quad (7\cdot1)$$

In a given circuit the e.m.f. acts in one direction. If a number of elecromotive forces exist in a circuit then the resultant e.m.f. is given by the algebraic sum of the individual e.m.fs'. Although as we pointed out above, the nature of e.m.f. is the same as that of p.d. there is a distinction between the two. In a given circuit e.m.f. has always the same direction whereas the direction of p.d. depends upon the direction of the current.

2. Primary cells. A primary cell is a device for transforming chemical energy into electrical energy. The simplest

form of such a cell is obtained by immersing two dissimilar con-
ducting plates in an electrolyte kept in a vessel so that the plates
partly project from the electrolyte. The plates are found to be
at different potentials so that the cell has an e.m.f.

The value of the e.m.f. depends upon the material of the
plates. High e.m.f. corresponds to a large separation of elements
constituting the plates in what is called the electrochemical series
given in Table 6—I below:

<div align="center">TABLE 6—I</div>

Electropositive

Caesium	Nickel	Rhodium
Rubidium	Thallium	Platinum
Potassium	Indium	Osmium
Sodium	Lead	Silicon
Lithium	Cadmium	Carbon
Barium	Tin	Boron
Strontium	Bismuth	Nitrogen
Calcium	Copper	Arsenic
Magnesium	Hydrogen	Selenium
Aluminium	Mercury	Phosphorus
Chromium	Silver	Sulphur
Manganese	Antimony	Iodine
Zinc	Tellurium	Bromine
Gallium	Pallad um	Chlorine
Iron	Gold	Oxygen
Cobalt	Iridium	Fluorine

<div align="right">Electronegative</div>

In this table each element is electropositive to all those that
follow. The exact order may vary slightly according to con-
ditions.

Thus zinc and copper or zinc and platinum or zinc and carbon
are suitable substances for forming the plates. When the terminals
of the plates called poles are connected externally current flows
in the outside circuit from the electronegative (say carbon) to
electropositive (say zinc) plate.

Within the electrolyte it flows from electropositive to electronegative plate. It is the electropositive plate that by passing into solution in the electrolyte gives most of the energy by combining with the negative ions in the electrolyte. A simple voltaic cell is shown in fig. 7·1. Here the electrolyte is

dilute $H_2 SO_4$ and the plates are of platinum and zinc. When the two poles of the cell are joined together by a wire electric current flows in the external circuit from Pt to Zn; but in the electrolyte it flows from Zn to Pt as shown by arrows. When the current passes in the electrolyte the positive hydrogen ions are deposited upon the electronegative Pt plate while the acid radical (SO_4) is deposited in equivalent amount on the electropositive plate of Zn.

Fig. 7·1

At this plate $ZnSO_4$ is formed which goes into the electrolyte.
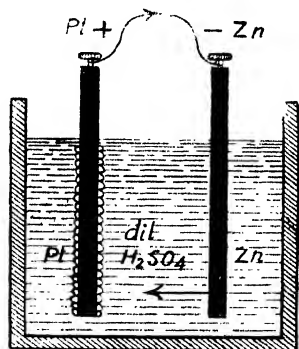
Polarisation. Hydrogen does not combine with Pt nor does it freely come out but tiny hydrogen bubbles keep hanging on the plate. These virtually transform the Pt plate into a hydrogen plate. Now H_2 is above Pt in the electrochemical series and thus is more positive. Therefore the e.m.f. of the cell which is now virtually $H_2 - Zn$ cell is reduced. This reduction of e.m.f. is called polarisation. Polarisation is chiefly noticeable at the electronegative plate. Polarisation of a simple cell is a serious drawback and so such a cell is of little practical utility.

Depolarisation. Polarisation of a cell may be reduced by substituting a harmless ion in place of the harmful hydrogen ion. It may be reduced preferably by oxidising the hydrogen. The former method is adopted in the Daniell cell shown diagrammatically in fig 7·2. Zinc and sulphuric acid are contained in a porous pot placed in a jar in which there

is copper sulphate solution with a cylindrical copper electrode. When the external circuit is closed current begins to flow in the circuit. Within the electrolyte (SO_4) ions go to Zn producing $ZnSO_4$. In the $CuSO_4$ part of the cell Cu^+ ions go to the copper plate which is more electronegative than zinc. The hydrogen gas reaching the copper sulphate forms H_2SO_4. Thus in this cell copper instead of hydrogen is deposited on the copper plate. Polarisation is thus eliminated. This is the most perfect method of depola-

Fig. 7·2

risation but it is not always practicable since it means the use of an electrolyte containing the positive ion of the metal of the electronegative plate.
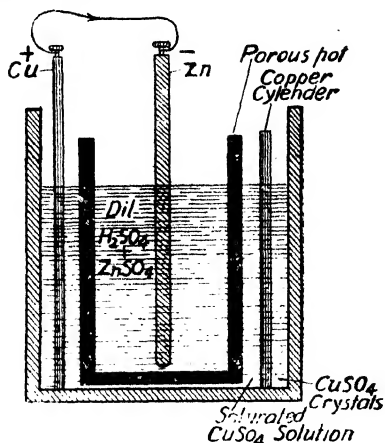
For removing polarisation by the method of oxidation various oxidising agents—solids, liquids and gases—have been used ; such as, MnO_2 in Leclanche' cell, nitric acid in Grove cell and atmospheric oxygen in Fery's cell.

Local action. Besides polarisation there are two more parasitic actions found in voltaic cells, to a greater or smaller extent, lowering the efficiency of the cell. The first one is what is called *local action*. Consider a cell with copper and zinc electrodes and dilute sulphuric acid as the electrolyte. Commercial zinc contains impurities like lead, iron, etc. Lead is electronegative to zinc and so is iron. They are in contact and both are surrounded by the electrolyte. Thus local short-circuited cells are formed giving rise to local currents producing zinc sulphate. And this is happening even when the Zn and Cu electrodes are not externally connected by a metal wire. Thus the chemical energy instead of being converted into useful electric currents is wasted away in the heat produced by local currents.

This defect may, however, be removed by using chemically pure zinc or amalgamating the zinc plate with a small quantity of mercury.

In a two-fluid cell like the Daniell cell there is another action prejudicial to the proper functioning of the cell. The copper sulphate in the Daniell cell diffuses through the porous pot and on coming into contact with zinc deposits copper either as the black CuO or as metal on the zinc electrode with the evolution of hydrogen. The partially coated zinc plate behaves more like a copper electrode and thus the e.m.f. of the cell is lowered. By reducing the porosity of the porous pot the diffusion may be made less but this increases the internal resistance of the cell.

3. Standard cells. Weston cell. A standerd cell is of great importance in electric measurements where a standard e.m.f. is needed. There are many patterns of standerd cell. We deal here with a cell due to E. Weston of the United States. This cell was recommended as a standard of e.m.f. by the International Conference on units and standards which met in London in 1908.

In this cell the positive electrode is mercury and the negative electrode is a cadmium amalgam containing 12·5% of cadmium. These are enclosed in an H-shaped glass container. Connection is made to the two electrodes by platinum wires to the glass walls, (fig. 7·3).

A layer of paste of a mixture of mercurous suphate, powdered crystals of cadmium suphate and a saturated aqueous solution of cadmium sulphate is put on the top of
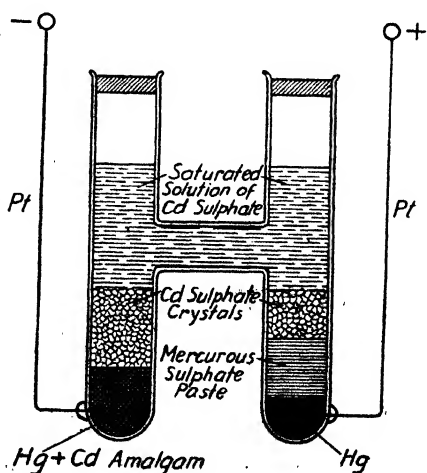
Fig. 7·3

mercury. A layer of cadmium sulphate crystals is then put into each limb. Finally the cell is filled with a saturated solution of cadmium sulphate to a convenient level. The tops of the two limbs are then sealed off.

Great care must be taken to obtain the purest samples of the materials used in this cell. This cell must be used with great care so that an extremely small current is drawn from it for a very short time.

The e.m.f. of the cell is taken to be $1 \cdot 0183$ volts at 20^0 C. The variation of e.m.f. with temperature over the range 0^0 to 40^0 C is given by the formula

$$E_t = E_{20} - 0 \cdot 0000406(t-20) - 0 \cdot 00000095(t-20)^2$$
$$+ 0 \cdot 00000001(t-20)^3 \text{ volt,}$$

recommended by the International Conference of 1908 mentioned above. Thus the temperature coefficient is very small and may usually be neglected.

Clark cell. In this form of standard cell zinc sulphate is substituted for cadimum sulphate in the Weston cell. The e.m.f. of this cell is $1 \cdot 433$ at 15° C. The temperature coefficient as shown by the formula

$$E_t = E_{15} - 0 \cdot 00119 (t-15) - 0 \cdot 000007 (t-15)^2,$$

is very much higher than that of the Weston cell. This is a serious disadavantage with the Clark cell.

We give below in Table 6-II a brief summary of some of the important practical primary cells.

4. Secondary cells. Lead accumulators. In addition to the primary cells discussed above there are secondary cells called storage cells or accumulators. These are reversible in the sense that they are charged by sending a current through them in such a direction that the normal chemical action producing the e.m.f. of the cell is reversed.

The commonest is the lead accumulator shown diagrammatically in fig. 7·4. It consists of a number of plates kept dipped in dilute sulphuric acid of sp. gr. 1·25. The plates are generally

made in the form of grids or grooved sheets those intended for positive plates being covered with red lead (Pb_3O_4) made into a paste with H_2SO_4. Those intended for negative plates are covered with litharge or PbO paste. To charge the cell current

is passed through the sulphuric acid from the red lead plate to the litharge plate. The red lead becomes oxidised to PbO_2 (chocolate colour) whilst the litharge is reduced to spongy metallic lead and becomes slaty grey. The cell is then charged. When used as a current
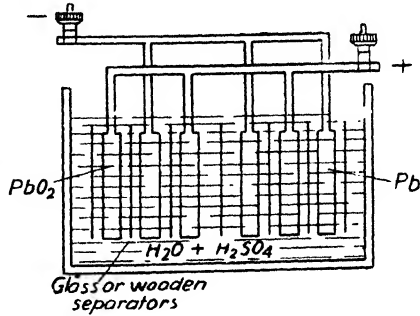
Fig. 7·4

generator PbO_2 and Pb are changed into $PbSO_4$. The current on discharge flows from the PbO_2 plates to the Pb plates through the outer circuits. The acid density falls as water is formed in the discharge process. The cell should be recharged when the density falls to 1·1 by sending current through the cell from the positive to the negative terminal. When fully charged the e.m.f. is about 2·05 volts; when fully discharged it is about 1·8 volts. The internal resistance is low, being of the order of 0·01 ohm and there is no appreciable polarisation. The capacity, i.e., the number of ampere hours it gives on discharge, of lead accumulators, varies from 10 to 1000 ampere hours. Care should be taken to add distilled water from time to time to compensate for the loss by evaporation so that the acid level is maintained about half an inch above the top of the plates. To prevent spoiling of accumulators they should be regularly charged before the voltage has fallen to 1·8 volts. Extra care should be taken not to short-circuit the cell; otherwise due to low internal resistance huge currents will be generated. The internal heat, being proportional to square of the current, will assume large value sufficient to cause buckling of the plates and increased chemical action. Their life is thus considerably shortened. To

TABLE 6—II. Practical primary cells (not for continuous use)

Name of the cell	Electrodes +	Electrodes −	Electrolyte	Depolariser	e. m. f. in volts
Bichromate	Carbon	Amalgamated Zinc	Dil. H_2SO_4	Potassium bichromate or chromic acid dissolved in the electrolyte; or sometimes zinc electrode is placed in a porous pot in which potash bichromate is placed. The porous pot is surrounded by dilute sulphuric acid.	1·9 to 2·0
Lalande	Copper frame with cupric oxide blocks	Zinc	Caustic soda in water 1 : 3	Cupric Oxide	1·0 to 1·1
Leclanché	Carbon plate surrounded by MnO_2	Zinc rod amalgamated.	Ammonium chloride sol. 2 to 4 oz. in one pint.	...	1·5 falling to 1·0 if not used intermittently.

					E.M.F.
Daniell	Cylindrical copper sheet surrounded by saturated solution of copper sulphate.	Zinc rod in Zn SO_4 and dil. H_2SO_4 put in porous pot.	Dilute H_2SO_4 mixed with Zn SO_4.	$Cu\,SO_4$.	1·07 to 1·14 according to strength of solutions.
Grove	Platinum sheet in a flat narrow porous pot containing nitric acid.	Zinc heavy casting round both sides of porous pot. $H_2\,SO_4$.	Dilute $H_2\,SO_4$.	Strong nitric acid.	1·9 to 2·0
Bunsen	Carbon rod or plate in nitric acid placed in ro. nd porous pot.	Zinc cylindrical sheet surrounding porous pot dipped in dil. H_2SO_4.	$H_2\,SO_4$.	Strong nitric acid.	1·9 to 2·0
Dry cell	Carbon surrounded by depolariser.	Zinc plate used as container.	A paste of ammonium chloride, zinc flour, zinc chloride and a little water.	A paste of $Mn\,O_2$ (10 gms), carbon (10 gms), ammonium chloride (2 gms), zinc chloride (1 gm) and a little water.	…
Weston	Mercury	Cadmium amalgam containing 12·5% of cadmium.	…	…	1·0183

avoid fragmentation of the plates the charging should not be done at a very high rate.

The chemical changes are represented below:

Charging:

(−) *ve* plate.—$Pb\,SO_4 + H_2 = Pb + H_2SO_4$

(+) *ve* plate.—$Pb\,SO_4 + SO_4 + 2\,H_2O = Pb\,O_2 + 2\,H_2\,SO_4$

Discharge:

(−) *ve* plate.—$Pb + SO_4 = Pb\,SO_4$

(+) *ve* plate.—$PbO_2 + H_2 + H_2\,SO_4 = PbO + H_2O + H_2SO_4$
$$= Pb\,SO_4 + 2\,H_2O$$
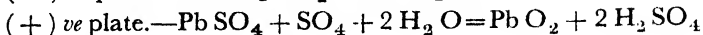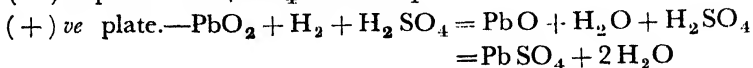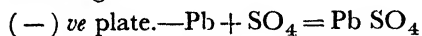
It is clear from these equations that during discharge water is formed and so the sp. gr. of the acid falls down. During charging process reverse is the case. The sp. gr. is 1·15 when the cell is fully discharged and becomes 1·21 when fully charged. The state of charge should be judged by the specific gravity and so a hydrometer should be kept ready at hand.

The lead peroxide plate is 'formed' in two distinct ways:

(1) By electrolysis, using lead plate as anode. This is a very lengthy process and is very costly. (2) The easily obtainable red lead is compressed into lead grids. When immersed in dilute sulphuric acid PbO_2 is formed immediately according to the chemical reaction

$$Pb_3\,O_4 + 2\,H_2\,SO_4 = Pb\,O_2 + 2\,Pb\,SO_4 + 2\,H_2O.$$
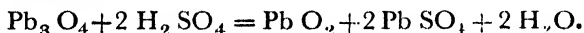
The lead sulphate is converted to $Pb\,O_2$ by charging over a moderate time.

Lead accumulators are very heavy . A lighter cell is described below.

The Edison or iron-nickel cell. The charged cell consists of $Ni\,(OH)_4$ as the positive plate, iron as the negative plate and potassium hydrate with a small quantity of lithium hydrate disolved in water as the electrolyte. To increase the capacity a number of plates are used as in lead accumulators. On discharge the (OH) ion of the electrolyte goes to the negative iron plate thus oxidising it. The K ion goes to $Ni\,(OH)_4$ and reduces it to $Ni\,(OH)_2$. On charging, the reverse action takes place. The process may be represented by the following chemical equation:

$$Ni\ (OH)_4 + KOH + Fe \rightleftarrows Ni\ (OH)_2 + KOH + Fe(OH)_2$$
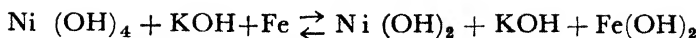
One important feature of this process is that the electrolyte as a whole remains unchanged; it acts merely as a carrier of (OH) ion from one plate to the other. Due to this no change in the sp. gr. of KOH takes place. The maximum value of e.m.f. of an Edison cell on full charge is about 1·4 volts. As in the case of lead accumulators distilled water is added from time to time to make good the loss by evaporation. All the metal parts of an Edison cell are made of nickel-plated sheet steel. Edison cell is very rugged and stands rough usage and idling much better than the lead accumulator. Because of these properties it is used on a wide scale for driving electric vehicles.

We summarise below some of the most important characteristics of a good cell

(1) The e.m.f. should be high and steady.

(2) The internal resistance should be low.

(3) The cell should possess long life and should be capable of giving large constant current.

(4) On open circuit it should be totally free from any chemical changes.

(5) The electrodes and electrolytes should consist of cheap and durable materials.

(6) It should be easy to handle and should not emit any corrosive vapours.

Lead accumulators satisfy almost all the requirements except for its rather large weight.

5. Gibbs-Helmholtz formula for e.m.f. of a cell. Various theories have been proposed to explain the origin of the e.m.f. of a cell. We will consider here briefly the thermo-chemical and thermal effects in a cell as giving rise to the e.m.f.

We have already seen that when the circuit of a cell is completed an electric current flows, the electrical energy, being given by the product of the e.m.f. and the charge circulated. This current gives rise to chemical changes which are accompanied by evolution of a net amount of heat. Assume that chemical changes take place only when the external circuit is completed. From the

principle of the conservation of energy the chemical energy must appear as electrical energy wholly and if the former is known, the latter and from that the e.m.f. may be found out.

As an illustration we will calculate the e.m.f. of a Daniell cell. Zinc is dissolved in sulphuric acid and copper is deposited on the copper electrode.   According to the thermo-chemical experiments about 106×10^3 calories of heat are given out when 65 gms. of zinc are dissolved in dilute sulphuric acid and about 56×10^3 calories when 63·5 gms of copper are dissolved.   In the actual cell reaction copper is evolved and so 56×10^3 calories are absorbed.   The net amount of heat generated, during the time 65 gms. of zinc are dissolved, will be $(106-56) \times 10^3$, i.e., 50×10^3 calories.   If nothing of this is wasted by local action or in any other way the whole of it must appear as electrical energy.   In symbols if e be the e.m.f., q the charge circulated, H_C the heat of chemical reaction and J the mechanical equivalent of heat, we have, electrical energy $= eq =$ chemical energy $= H_C J$ and therefore,

$$e = J\, H_C/q. \qquad \qquad . \quad . \quad . \quad (7\cdot2)$$

The quantity q can be found from the electro-chemical equivalent of copper or of zinc.   Thus $q = 63\cdot5/0\cdot0003286$ coulombs $= 63\cdot5/0\cdot003286$ c.g.s. units.   Inserting the values in eq. $(7\cdot2)$ we have

$$e = \frac{50 \times 10^3 \times 4\cdot2 \times 10^7 \times 0\cdot003286}{63\cdot5},$$

$$= 1\cdot086 \times 10^8 \text{ c.g.s. units,}$$
$$= 1\cdot086 \text{ volts.} \qquad \qquad . \quad . \quad . \quad (7\cdot3)$$

Such calculations were first made by Kelvin.

The formulae obtained for mono-valent and di-valent metals are

$$e = H_C/23 \times 10^3 \qquad \text{volts (monovalent metal)} \quad . \quad . \quad . \quad (7\cdot4)$$
and $$e = H_C/46 \times 10^3 \qquad \text{volts (divalent metal)}, \qquad . \quad . \quad . \quad (7\cdot5)$$
where H_C is the heat of formation in calories of a grammolecule of the compound.

Helmholtz modified Kelvin's formula to allow for any possible contribution to the total e.m.f. by thermo-electric effects. If H_T denotes the amount of heat due to the thermal effect and H_C that due to chemical reactions we have $qe = H_C + H_T$.   For finding out H_T we proceed as follows :

Consider a perfectly reversible cell. A cell in which chemical reactions are completely reversed by sending a charging current through it is called a reversible cell. A Daniell cell approximates to this.

. Take the reversible cell round a complete cycle of operations represented by $a\,b\,c\,d$ in fig 7·5.
Let e be the e.m.f. of the cell at the absolute temperature T, its state being represented by the point a in the indicator diagram. Raise its temperature by a small amount by supplying heat adiabatically so that it arrives
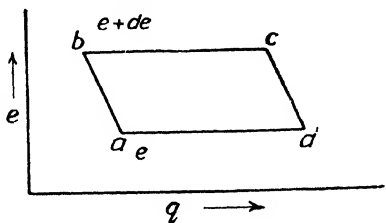
Fig. 7·5

at the point b where the e.m.f. is $e+de$. At this temperature allow it to generate q coulombs of charge giving electrical energy equal to $q(e+de)$. The cell now arrives at the point c. Extract an amount of heat from the cell so that it falls back to temperature T adiabatically. Its state is now represented by d. Lastly pass q coulombs of charge through the cell in opposition to the e.m.f. The electrical energy supplied is qe. The cell now arrives at its original condition a. The heat supplied during the rise of temperature from T to $(T + dT)$ is regained during the fall of temperature from $(T + dT)$ to T. The chemical reactions have also been reversed.

There is, however, a small irreversible portion of energy which is the heat generated when current i flows through the internal resistance r of the cell. This amount is i^2r which may be made small by diminishing i to an infinitesimal amount.

The net result of the above operations is a gain in electrical energy $q(e+de) - qe = qde$. This has to come from the heat of chemical reactions since during the cycle they are reversed. It must be due to the part H_T in the energy equation $qe = H_C + H_T$. Now according to thermodynamics when a certain amount of heat H flows from a body A at temperature $(T + dT)$ to a body

7 E

B at temperature T a certain fraction of H given by $H \dfrac{dT}{T}$ is transformed into work. Therefore, we have

$$qde = H_T \, dT/T,$$

or $\qquad H_T = qT \dfrac{de}{dT}.$ $\qquad$ . . . (7·6)

Substituting this value of H_T in the energy equation $qe = H_C + H_T$

we have $\qquad\qquad e = H_C + T \cdot \dfrac{de}{dT},$ $\qquad$ . . . . (7·7)

or numerically for divalent metals

$$e = \frac{H_C}{46 \times 10^3} + T \frac{de}{dT} \text{ volts.} \qquad \text{. . . . (7·8)}$$

Eq. (7·7) is known as Gibbs-Helmholtz formula. This important formula is an improvement upon Kelvin's formula embodied in eqs. (7·4) and (7·5).

Some interesting conclusions can be drawn from the Gibbs-Helmholtz formula :—

(1) The e.m.f. of a cell is equal to the energy of the chemical reaction in the cell only when $\dfrac{de}{dT} = 0$.

(2) If $\dfrac{de}{dT}$ is positive, i.e., if the e.m.f. increases with rise in temperature $e > \dfrac{H_C}{46 \times 10^3}$ and so the additional heat required is drawn from the surroundings. The cell is thus cooled down when it generates current.

(3) If $\dfrac{de}{dT}$ is negative the cell is warmed up when it generates current.

(4) In the case of Daniell cell $\dfrac{de}{dT}$ is very small and so the heat of chemical reaction H_C gives the e.m.f. as calculated before.

(5) The heat of formation of most of the compounds that can be utilised in cells is equivalent to about 2 volts or less. Thus it is impossible to obtain a high e.m.f. such as 5 or 6 volts from a single voltaic cell.

CHAPTER VIII

LAWS OF STEADY CURRENTS

1. Ohm's law. Every substance offers resistance to the passage of electric current through it. If the material is taken in the form of a wire it is found that its resistance to electric current is inversely proportional to the area of cross section A of the wire and directly proportional to its length l, i.e., the resistance $R = \sigma l / A$; σ is called the specific resistance of the material of the wire. It varies with the substance. If $l = 1$ and $A = 1$, $\sigma = R =$ the resistance of unit cube of the substance. In practice it is found that σ varies with temperature.

According to Ohm's law the current I between two points of a circuit with a potential difference E between the two points, is

given by $$I = \frac{E}{R},\qquad\qquad \cdots \quad (8\cdot1)$$

where R is the resistance, provided the circuit is made out of a conductor at constant temperature and provided there is no active source of e.m.f. between the two points.

2. Unit of resistance. From Ohm's law we get the unit of resistance, Ohm. Putting $E = I = 1$, we get $R = 1$. Ohm is the resistance that will develop a potential difference of 1 volt between its terminals when it is made to carry a current of 1 amp.

The international Ohm is the resistance of a column of mercury 106·3 cms long, of constant cross-sectional area, weighing 14·4521 grams and held at 0°c. As it is easy to reproduce this standard Ohm it is commonly employed as a primary standard in making precise measurements.

3. Joule's Law. If r is the resistance between two points of a conductor carrying current i, e the p.d. between them, and t seconds the time for which the current flows, the work done is given by

$$W = e\,i\,t = E\,I\,t \times 10^7 \text{ ergs}, \qquad \cdots \quad (8\cdot2)$$

where E is expressed in Volts and I in amperes.

If this work appears as heat H we have

$$W = J\,H = EIt \times 10^7,$$

where J is the mechanical equivalent of heat. Substituting $J = 4 \cdot 2 \times 10^7$ ergs/cal, we have

$$H = E\ I\ t/4 \cdot 2$$
$$= I^2\ R\ t/4 \cdot 2, \qquad (8 \cdot 3)$$

by Ohm's law. Eq. $(8 \cdot 3)$ is known as the Joule's law.

The practical unit of electrical energy is one *Joule*. Thus, 1 Joule = 1 volt × 1 coulomb = 1 volt × 1 amp × 1 sec = 10^7 ergs. The rate of working or *electrical power* has also got a unit. This is watt. It is one Joule per second. In symbols,

Power P = E I.

1 watt = 1 volt × 1 amp. or 1 Joule per second or $0 \cdot 7372$ foot pound per sec.

1 horse power = 746 watts = $0 \cdot 746$ kilowatt.

Board of Trade unit of energy. Although the Joule is 10^7 ergs it is found to be only too small for commercial purposes. The unit adopted by the Board of Trade for this purpose is the *Kilowatt-hour* or 1000 watt-hours. The Board of Trade unit of energy is the amount of work done in a circuit when the power exerted in watts multiplied by the time in hours during which it is exerted equals 1000. Thus 1 B. O. T. unit = 1000 watt-hours = 36×10^5 Joules = 36×10^{12} ergs = $1\frac{1}{3}$ horse power hour roughly.

4. Combination of resistances. Resistances in series.
In fig. $8 \cdot 1$ if r_1, r_2, etc., are the n resistances
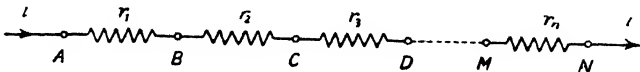
Fig. $8 \cdot 1$

joined in series, V_A, V_B, etc, are the potentials at the various extremities of the resistances and i the common current flowing in all of them we have

$$V_A - V_B = ir_1, \quad V_B - V_C = ir_2, \text{ etc.}$$

By addition we have

$$V_A - V_N = i \sum_{k=1}^{k=n} r_k \qquad \dots \quad (8 \cdot 4)$$

But $\qquad V_A - V_N = i\,R, \qquad \dots \quad (8 \cdot 5)$

where R is is the combined resistance.

Hence from eqs. $(8 \cdot 4)$ and $(8 \cdot 5)$ we have

$$R = \sum_{k=1}^{k=n} r_k \qquad \dots \quad (8 \cdot 6)$$

Resistances in parallel. n resistances, r_1, r_2, r_3, etc., are joined in parallel as shown in fig. $8 \cdot 2$. If i is the total current between A and B, i_1, i_2, i_3 etc.,the parts of i in the various resistances and R the total resistance, we have
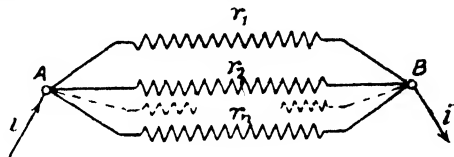
Fig. 8·2

$$iR = V_A - V_B = i_1 r_1 = i_2 r_2 = \dots = i_n\,r_n, \qquad \dots \quad (8 \cdot 7)$$

and $\qquad i = i_1 + i_2 + i_3 + \dots + i_n. \qquad \dots \quad (8 \cdot 8)$

From eqs. $(8 \cdot 7)$ and $(8 \cdot 8)$ we have

$$\frac{1}{R} = \sum_{k=1}^{k=n} \frac{1}{r_k} . \qquad \dots \quad (8 \cdot 9)$$

5. Combination of cells. Cells can be combined either in series or in parallel, the choice depending upon the use to which they are to be put. Economy requires that very little electrical energy should be wasted as heat internally. Since the amount of heat is proportional to $i^2 r$, the internal resistance must be made small. Hence the cells are connected in parallel.

For very quick action in a circuit containing inductance the time constant L/R (Ch. XII, Art. 11) must be made small and hence R must be large. Thus the cells must be grouped in series.

We shall now deduce a condition to obtain maximum current in a circuit of external resistance R. Consider n rows of cells joined in parallel, each row containing m cells in series. If e is the e.m.f. of each cell and r its internal resistance, we have total number of of cells $= N = mn$, total e.m.f. $E = me$, and by Ohm's law the total current

$$I = me / \frac{mr}{n} + R,$$

From these relations we get

$$I = Nne / Nr + Rn^2. \qquad . \quad . \quad . \quad (8 \cdot 10)$$

To get the maximum value of I equate to zero the first differential coefficient of I with respect to n. Thus 1 is maximum when

$R = mr/n =$ total internal resistance of n rows of cells. $\qquad (8.11)$

Eq. ($8 \cdot 11$) states that I is maximum when the external resistance is equal to the combined internal resistance of the n rows of cells.

6. Kirchhoff's Laws. In the previous articles we considered the application of Ohm's law to simple circuits. In actual practice many complicated net works of conductors are to be dealt with. Some of the groups of conductors even include an active e.m.f. The solution of all such net works readily comes out by the application of two general laws enunciated by Kirchhoff about 1842. They run as follows :—

(1) The algebraic sum of the currents at any junction in a circuit is zero, that is, $\Sigma i = 0$

(2) In any closed circuit the algebraic sum of the products of the current and resistance of each part of the circuit is equal to the algebraic sum of the electromotive forces in that circuit, that is, $\Sigma ir = \Sigma e$.

In applying these laws care should be taken in giving proper signs to the various currents. Generally the currents flowing towards a point are taken to be positive while those flowing away from it are taken to be negative. In applying the second law positive sign is assigned to each active e.m.f. if the source of the e.m.f. is traversed from its negative to its positive terminal, otherwise a negative sign is used. The other ir values should be given a negative or a positive sign according as the direction of current flow is

current entering at A and i' that received at G, the current $i-i'$ leaks to the ground at E'. Let r be resistance of the leak, x the resistance between A and L and R the total resistance of the telegraph line AG. Applying Kirchhoff's laws to the circuits $E_1ALE'E_1$ and $LGE_2E'L$ we have

$$ix + r(i-i') = e, \qquad \qquad \text{. . . } (8{\cdot}16)$$

$$i'(R-x) - r(i-i') = 0, \qquad \qquad \text{. . . } (8{\cdot}17)$$

Eliminating i from eqs. $(8{\cdot}16)$ and $(8{\cdot}17)$ we have

$$\{R(x+r) - x^2\}\, i' = er,$$

or

$$i' = er/\{R(x+r) - x^2\}. \qquad \qquad \text{. . . } (8{\cdot}18)$$

The current i' received at G will thus be least if $R(x+r)-x^2$ is greatest i.e., if $x = R/2$,

That is, the fault should be situated in the middle of the wire.

8. Electrolysis. When an electric current passes through some substances, chemical action takes place at the electrodes. This chemical action is known as electrolysis and the substances in which it takes place are called *electrolytes*. Majority of the electrolytes are liquids or aqueous solutions of acids, salts or bases. It has been found that the products obtained by electrolysis depend upon the nature of the solvent, the solute, the electrodes as well as upon the area of the electrodes immersed and the current passing through. To illustrate the influence of the nature of electrodes consider the case of aqueous solution of copper sulphate. With copper electrodes copper is deposited at the cathode and taken off from the anode; with platinum electrodes copper is deposited at the cathode while oxygen is liberated at the anode.

9. Faraday's Laws. As a result of his work on electrolysis Faraday enunciated two important laws which are now known as Faraday's laws of electrolysis.

(1) *The mass of any substance liberated is proportional to the product of current and the time for which it passes, that is, to the total quantity of electricity which has passed.*

(2) *The masses of different substances liberated by the passage of the same quantity of electricity are proportional to their chemical equivalents* (atomic weight divided by valency)

Faraday's laws may be expressed symbolically thus :

$$m \propto It \text{ (First law)},$$
$$m \propto \eta \text{ (Second law)},$$
$$m = k\eta It = zIt, \qquad . \quad . \quad . \quad (8\cdot19)$$

where m is the mass liberated, I the current, t the time, η the chemical equivalent (atomic weight÷valency), k a constant and z the electrochemical equivalent (e.c.e.). From eq. ($8\cdot19$) we see that the e.c.e. of a substance is its amount deposited by one coulomb ; also $z=k\eta$ and k=e.c.e. of hydrogen, taking chemical equivalent as 1 in place of $1\cdot008$.

Thus the e.c.e. of a substance=e.c.e. of hydrogen multiplied by the chemical equivalent of the substance. It, therefore, follows that if the e.c.e. of some standard substance (silver) is accurately known that of any other substance can be easily calculated. By the definition of ampere on the international agreement the e.c.e. of silver is $0\cdot0011183$ gram of silver per coulomb. Thus for hydrogen the e.c.e. is $0\cdot0011183 \times 1\cdot008/108 = 0\cdot00001045$ gm./coulomb and that for copper (valency 2) is $0\cdot0011183 \times 31\cdot8/108 = 0\cdot000329$ gm./coulomb.

10. Nature of electrolysis. Ions. Faraday. Electrolysis starts as soon as current is switched on to the electrodes. In this respect an electrolyte behaves exactly like a solid conductor. In solid conductors the flow of current is due to the motion of free electrons which already exist in the conductor. According to Arrhenius the current in an electrolyte is due to the positive ions (*cations*) moving towards the cathode and to the negative ions (*anions*) moving towards the anode. These ions are formed by dissociation of the solute molecule immediately the solute is dissolved. Thus when copper sulphate is dissolved in water the solution contains positive copper ions and negative (SO_4) ions. Electrolysis consists of migration of these ions to the electrodes. The migration velocity is very small but is measurable directly.

In a dilute solution the dissociation is complete while in a concentrated solution the dissociation is partial due to greater recombination of the positive and negative ions. Conduction in a solution is similar to that in a gas which will be treated in Ch. XXI.

The charges on ions of one kind are all the same. All univalent ions carry the same charge; all divalent ions double the charge; all trivalent ions three times this charge and so on.

The charge on a univalent ion can be easily calculated. By Faraday's laws a certain fixed quantity of electricity is required to liberate one gram equivalent of any substance. This is called *Faraday* and is denoted by letter F. Its value is 96,540 coulombs. Now one gram equivalent of any univalent ion contains $N = 60.6 \times 10^{22}$ ions. Therefore, charge carried by one univalent ion is $9.654 \times 3 \times 10^{10} / 60.6 \times 10^{22} = 4.78 \times 10^{-10}$ e.s. units. This is the same as the charge on the electron determined directly.

According to the electron theory a positive ion is due to loss of electrons and the negative ion is due to gain of electrons. Thus when copper sulphate is dissolved in water dissociation takes place transferring two electrons from copper to (SO_4)

$$Cu\ SO_4 \rightleftarrows Cu^{++} + (SO_4)^{=}$$
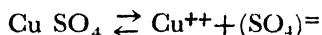
On reaching the electrodes under the action of the electric field these ions give up their charges to the electrodes.

We will now examine Ohm's law as applied to electrolytes. A voltage applied to the electrodes of an electrolytic cell is used up in two distinct processes: (1) passage of current through the solution (2) maintaining the chemical change, if any. If there is no chemical change then current is proportional to the applied p.d. or in other words Ohm's law is obeyed. Illustration is copper voltameter using small voltages. In a water voltameter, however, decomposition of water into hydrogen and oxygen takes place as well. If the variation of current with the applied voltage is studied and the observations plotted for this voltameter a curve similar to that shown in fig. 8·6 is obtained. The curve

shows that up to about 1·7 volts the
current is very small. Beyond this
critical voltage the current increases
rapidly with voltage according to
linear law indicating that the cell
obeys Ohm's law for voltages much
exceeding 1·7 volts.
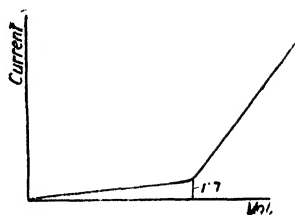
Fig. 8·6

Applications of electrolysis. Electrolysis has found a
variety of applications in industry. A short illustrative list is
given below.

(1) Electroplating. Cheap metals like brass, copper, etc., are
given a coating of precious metals like gold and silver by elec-
trodeposition. The article to be coated is made the cathode
while a pure sample of the metal to be deposited is the anode.
As the electrolyte, double cyanides of gold or silver, $KAu(CN)_2$
or $KAg(CN)_2$ are used. If the article is an insulator it is first
coated with graphite to make it conducting. To obtain success
in electroplating a good choice of the electrolyte, suitable cur-
rent and cleanliness of the article are essential. For uniform
deposition the article should be slowly rotated.

(2) Production of metals. Aluminium is now obtained mainly
by this process. The oxide Al_2O_3 is dissolved in fused cryolite
Na_3AlF_6 as solvent. Pure aluminium forms at the cathode.

(3) Copper refining. Crude copper obtained by smelting
is used as anode and a pure copper rod as the cathode in a copper
voltameter. Arsenic and other impurities in the crude copper
fall to the bottom.

(4) Alloys. Alloys like Cu-Sn, Cu-Zn can be formed directly
by electrolysis using certain concentrations of mixed solutions
of metals.

(5) Electrotyping. Metal types can be built up by using
plaster or wax moulds coated with graphite as the cathode.
Suitable solutions are used.

MAGNETIC FIELD OF STEADY CURRENTS

1. Ampere's law. The phenomenon that an electric current is accompanied by a magnetic field was first discovered by Oersted in 1819. After an extensive study of this phenomenon Ampere found that there is a force of attraction between uni-directional straight parallel currents and a repulsion between currents flowing in opposite directions. He also found that a current flowing in a helix behaves like a bar magnet. For the direction of the field Ampere gave the following rule : If an observer imagines himself to be swimming along the current with his face downwards so that the current enters at his feet, the north pole of a compass needle will be deflected to his left. There are other equivalent rules which are considered to be much simpler than the
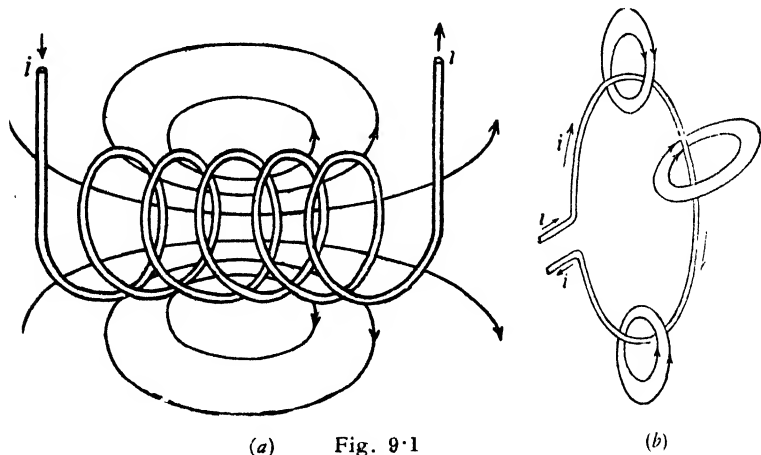
(a)   Fig. 9·1   (b)

original Ampere's rule. The *right-hand screw* or the *corkscrew* rule states that if we imagine the electric current to flow in the direction of rotation of a right-handed screw then the direction of advance of the screw point gives the direction of the

magnetic field. The righthand rule states that if a current carrying conductor is held in the right hand with the extended thumb pointing in the direction of the current then the fingers will encircle the wire in the direction of the magnetic field. The magnetic lines of force accompanying a current are represented qualitatively in figs. 9·1 (a) to 9·1 (d) in a few cases.
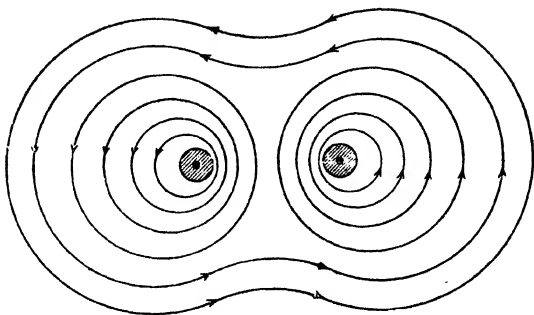
Fig. 9·1 (c)
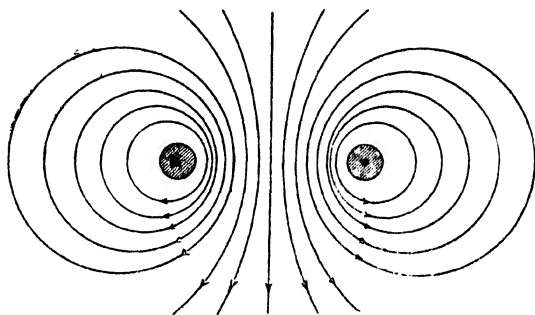
Fig. 9·1 (d)

The magnitude of the magnetic field produced by an element dl of a conductor (fig. 9·2) carrying a current i at a point P distant r from the element in a direction θ is given by
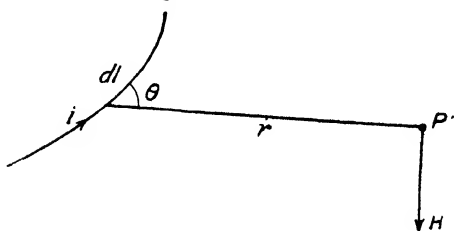
Fig. 9·2

$$d\,H = idl \sin \theta / r^2, \qquad \qquad \cdot \quad \cdot \quad \cdot \quad (9·1)$$

where i is to be expressed in electromagnetic units. The direction of the field is at right angles to the plane containing the element and the line joining it to the point. This is Ampere's law. Originally it was given by Ampere for conduction currents. According

to modern ideas a current of electricity is only a drift of electrons with a certain velocity. Hence the law is equally applicable to moving electric charges.

2. Effect of magnetic field upon current. Eq. (9.1) gives the field intensity at the point P. If a point pole of magnetic strength m is situated at P the force on it by the current will be

$$dF = md\text{H} = midl \sin \theta / r^2. \qquad . \quad . \quad . \quad (9\cdot 2)$$

According to the principle of action and reaction the force exerted by m on the current is equal and opposite to this. Now the magnetic field due to m at the element is

$$\text{H} = -m/r^2.$$

Substituting this value in eq. (9·2) the force dF on the current element dl is given by

$$dF = i\text{H} \, dl \sin \theta. \qquad . \quad . \quad . \quad (9\cdot 3)$$

Since eq. (9·3) is independent of m it gives the force on a current element dl in any external magnetic field H. To obtain the effect of the medium we must multiply H by μ, the magnetic permeability, to give us the induction B. Eq. (9·3), then, becomes

$$dF = i\mu\text{H}dl \sin \theta,$$
$$= i\text{B}_n \, dl, \qquad . \quad . \quad . \quad (9\cdot 4)$$

where B_n is the flux normal to the element. Eq. (9·4) gives another definition of absolute ampere, *abampere*. The abampere is a current which when flowing in 1 cm. of a straight conductor placed perpendicular to the lines of induction of a magnetic field having flux density of 1 gauss will experience a side thrust of 1 dyne.

The direction of the force experienced by a current-carrying conductor in a magnetic field can be found by finding the direction of magnetic field by the current according to Ampere's rule. It may, however, be found directly by the application of *Fleming's left hand rule* which runs as follows: Hold the thumb, the forefinger and the middle finger of the left hand mutually perpendicular to one another as shown in fig. 9·3; then if the forefinger points along the

Fig. **9·3**

3 E
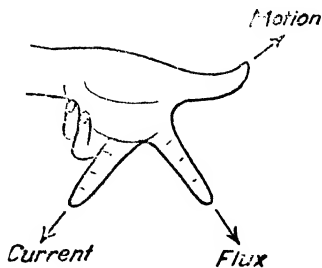
flux or field direction, the middle finger points along the current then the thumb will show the direction of the force.

3.   Field due to a circular coil at an axial point.   Let r be the radius of the coil, i the current and dl a peripheral element subtending an angle $d\phi$ at the centre O of the coil, (fig. 9·4.). P is an axial point situated at distances x and l from O and B respectively. The field due to the current element dl is, according to eq. (9·1),

Fig. 9·4

$$dH = ir\,d\phi/l^2 \text{, since } \theta = \pi/2 \text{ and } dl = rd\phi.$$

Its direction is PH perpendicular to the current direction and to l. It will have two components $dH \cos \alpha$ and $dH \sin \alpha$ respectively parallel and perpendicular to the axis OP. The diametrically opposite current element A'B' will produce a field at P which will also have two components one along and the other perpendicular to OP, but downwards. Thus the normal components annul each other while the axial components add up. Dividing the whole coil in this manner into diametrically opposite current elements we see that the resultant field is along the axis OP the magnitude of which is given by

$$H = \int_0^{2\pi} ir\,d\phi \cos \alpha/l^2,$$

$$= 2\pi i r^2/(r^2+x^2)^{3/2}. \qquad \cdots \quad (9·5)$$

If the coil consists of n similar turns, eq. (9·5) becomes

$$H = 2\pi n i r^2/(r^2+x^2)^{3/2}. \qquad \cdots \quad (9·6)$$

If the point lies at the centre, $x = 0$ and we have

$$H = 2\pi n i/r. \qquad \cdots \quad (9·7)$$

4.   Field at any point in the plane of a circular coil.   In fig. 9·5 P is a point at a distance $PO = \beta a$ from the centre O of the coil of radius a, β being a proper fraction.
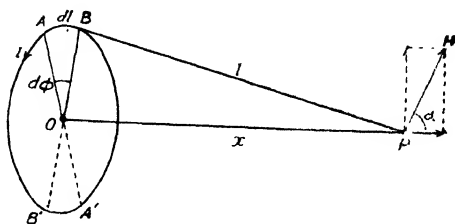
The field at P due to a current element $AB = dl$ is given by

$$dH = idl \sin\theta / r^2, \qquad \ldots \quad (9.8)$$

where r is the radius vector PA. Now,

$$dl = ad\phi,$$

$$\sin\theta = (r^2 + a^2 - \beta^2 a^2)/2ra,$$

and $\quad r^2 = a^2 + \beta^2 a^2 - 2\beta a^2 \cos\phi.$ Substituting these values in eq. (9.8) we have, after some reductions,

$$dH = i(1 - \beta\cos\phi)\,d\phi/a(1+\beta^2 - 2\beta\cos\phi)^{3/2}. \quad . \quad . \quad (9.9)$$

Hence,

$$H = \frac{2i}{a}\int_0^\pi \frac{(1 - \beta\cos\phi)\,d\phi}{(1 + \beta^2 - 2\beta\cos\phi)^{3/2}}.$$

$$\ldots \quad (9.10)$$

Fig. 9·5

The right hand side of eq. (9.10) is an elliptic integral, the values of which can be obtained from mathematical tables. For $\beta = 0$ (P coinciding with O) eq. (9.10) reduces to eq. (9.7). For $\beta = 1$, $H = \infty$, that is, for an ideal wire of zero cross-section the field at the wire is infinite. If the field H_P is evaluated for other values of β and compared with H_0 the field at the centre, it is found that near the centre the field is fairly constant up to about $\beta = 0.2$. Beyond this central region the field increases rapidly as we approach the wire.

5. A pair of circular coaxial coils.

Consider two coaxial coils A and B of turns m and n and radii a and b respectively, placed with their planes parallel to
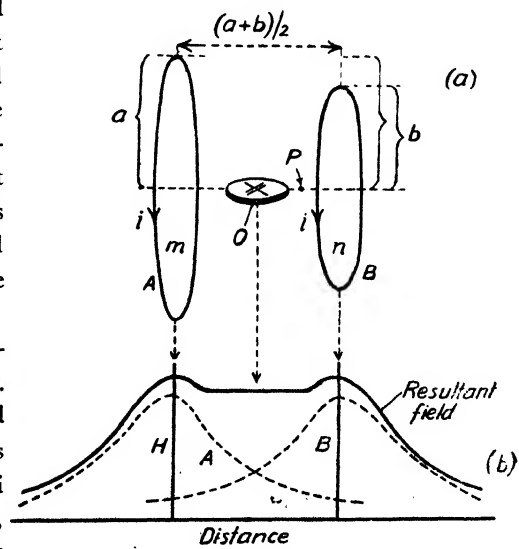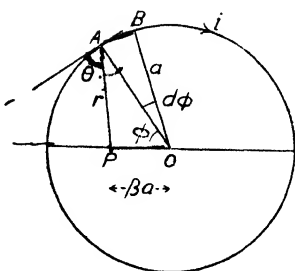
Fig. 9·6

each other separated by a distance $(a+b)/2$ (fig. 9·6a.). Take a point O at a distance $a/2$ from A and $b/2$ from B as origin. The field at an axial point P at a distance x from O is, by eq. (9·5), given by

$$H = \frac{2\pi nib^2}{\left\{ b^2 + \left(\frac{b}{2} - x\right)^2 \right\}^{3/2}} + \frac{2\pi mia^2}{\left\{ a^2 + \left(\frac{a}{2} + x\right)^2 \right\}^{\frac{5}{2}}}$$

$$= \frac{4\pi ni}{\sqrt{5}\, b} \left(1 - \frac{4}{5}\frac{x}{b} + \frac{4}{5}\frac{x^2}{b^2}\right)^{-\frac{3}{2}} + \frac{4\pi mi}{\sqrt{5}\, a} \left(1 + \frac{4}{5}\frac{x}{a} + \frac{4}{5}\frac{x^2}{a^2}\right)^{-\frac{3}{2}}$$

$$= \frac{4\pi i}{\sqrt{5}} \left\{ \frac{n}{b}\left(1 + \frac{6}{5}\frac{x}{b} - \frac{32}{25}\frac{x^3}{b^3}\right) + \frac{m}{a}\left(1 - \frac{6}{5}\frac{x}{a} + \frac{32}{25}\frac{x^3}{a^3}\right) \right\}, \quad (9·11)$$

neglecting terms higher than x^3.

Now,

$$\frac{dH}{dx} = \frac{4\pi i}{\sqrt{5}} \left\{ \frac{n}{b}\left(\frac{6}{5b} - \frac{32}{25}\cdot\frac{3}{b^3} x^2\right) + \frac{m}{a}\left(-\frac{6}{5a} + \frac{32}{25}\cdot\frac{3}{a^3} x^2 \right) \right\},$$

$$\frac{d^2H}{dx^2} = \frac{4\pi i}{\sqrt{5}} \left\{ -\left(\frac{n}{b}\cdot\frac{32}{25}\cdot\frac{3}{b^3}\cdot 2x\right) + \left(\frac{m}{a}\cdot\frac{32}{25}\cdot\frac{3}{a^3}\cdot 2x\right) \right\},$$

and $$\frac{d^3H}{dx^3} = \frac{4\pi i}{\sqrt{5}} \left\{ -\left(\frac{n}{b}\cdot\frac{32}{25}\cdot\frac{3}{b^3}\cdot 2 \right) + \left(\frac{m}{a}\cdot\frac{32}{25}\cdot\frac{3}{a^3}\cdot 2 \right) \right\}.$$

We see that

$$\frac{dH}{dx} = \frac{d^2H}{dx^2} = 0 \text{ for } x = 0,$$

$$\text{if} \qquad \frac{m}{a^2} = \frac{n}{b^2}, \qquad \qquad \cdot \quad \cdot \quad \cdot \quad (A)$$

and further $$\frac{d^3H}{dx^3} = 0 ,$$

$$\text{if} \qquad \frac{m}{a^4} = \frac{n}{b^4}. \qquad \qquad \cdot \quad \cdot \quad \cdot \quad (B)$$

Conditions (A) and (B) are simultaneously satisfied when $m=n$ and $a=b$. Hence the field at the origin will be very uniform when the two coils have the same number of turns and have equal radii as can be seen from fig. 9·6 (b). This result is utilised in the construction of Helmholtz Galvanometer.

6. Field on the axis of a solenoid.   In fig. 9·7 AB represents a solenoid of length l and internal radius r having uniformly distributed n turns of wire carrying a current i. Take an elementary section dx of the solenoid distant x from the point P which is

at a distance D from the centre O. The element will contain $\frac{n}{l}dx$

turns each car-
rying a cur-
rent i. For in-
finitesimal va-
lues of dx the
field at the
point P, by
eq. (9·6), is
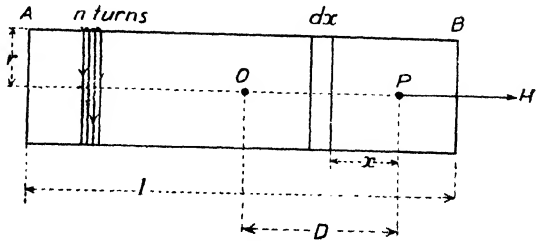
Fig. 9·7

$$dH = 2\pi \left(\frac{n}{l}\right) dx \; i \; r^2 / (r^2 + x^2)^{3/2}$$

Hence,

$$H = \frac{2\pi n i r^2}{l} \int\limits_{-\left(\frac{l}{2} - D\right)}^{\left(\frac{l}{2} + D\right)} (r^2 + x^2)^{-3/2} \, dx$$

$$= \frac{2\pi n i}{l} \left\{ \frac{\frac{l}{2} + D}{\sqrt{r^2 + \left(\frac{l}{2} + D\right)^2}} + \frac{\left(\frac{l}{2} - D\right)}{\sqrt{r^2 + \left(\frac{l}{2} - D\right)^2}} \right\} \quad . \quad (9·12)$$

At the centre of the solenoid (D = 0) the field is given by

$$H_0 = 2\pi n i / \sqrt{r^2 + l^2/4.} \qquad . \quad . \quad . \quad (9·13)$$

If $l >> r$ this reduces to

$$H_0 = 4\pi n i / l. \qquad . \quad . \quad . \quad (9·14)$$

The product $\frac{n}{l} i$ is called *ampere-turns* per unit length. Thus eq.
(9·14) can be put in the form

$$H_0 = 4\pi \times \text{ampere turns per unit length.} \qquad . \quad . \quad (9·15)$$

From eq. (9·12), putting $D = \frac{l}{2}$, we get the field intensity at the
end of the solenoid,

$$H_e = 2\pi n i / l. \qquad . \quad . \quad . \quad (9·16)$$

Eq. (9·12) can be used to show the variation of H along the axis. For a long solenoid of small radius it can be shown that the field is fairly uniform along the axis. It falls abruptly at the ends. The conditions are roughly represented in fig. 9·1 (*a*).

7. Field due to a straight current. In fig. 9·8 AB is a conductor carrying a current *i* along the *y*-axis. P is a point on the *x*-axis situated at a distance R from the origin. The field at P is perpendicular to the plane containing the radius vector *r* and the current. Its magnitude to a current element *dl* is, by Ampere's law, given by

$$dH = idl \sin \theta / / r^2.$$

Fig. 9·8

Let the angles between OX and the lines joining the extremities A and B of the conductor to P be α_1 and α_2 as shown in the figure. Now $dl = rd\alpha / \sin\theta$ and $\frac{1}{r} = \frac{\cos\alpha}{R}$. Substituting these values in the above equation and integrating between the limits, α_1 and α_2 we have

$$H = \frac{i}{R} \int_{\alpha_1}^{\alpha_2} \cos\alpha \, d\alpha = \frac{i}{R} (\sin \alpha_2 - \sin \alpha_1). \qquad . \quad . \quad (9·17)$$

If the current conductor is infinite in length $\sin \alpha_2 = +1$ and $\sin \alpha_1 = -1$. Eq. (9·17) then reduces to

$$H = 2i/R. \qquad . \quad . \quad . \quad (9·18)$$

Eq. (9·18) is known as the *law of Biot and Savart*. The lines of force are concentric circles with their common centre lying on the linear conductor. The sense in which they are described is given by the cork-screw rule. For two parallel conductors the lines of force are shown in figs. 9·1(*c*) and 9·1(*d*). For a single conductor they will be perfectly concentric.

8. Equivalent magnetic shell. In previous articles we calculated the magnetic field due to currents in some simple circuits directly using Ampere's law. In 1823 Ampere introduced the important concept of equivalent magnetic shell which is
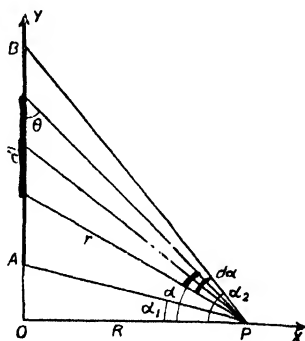
found of great use in solving many complicated problems about magnetic interactions between current-carrying circuits. By a series of experiments Ampere showed that the magnetic effect at distant points produced by a conductor carrying a current was the same as that produced by assuming the area enclosed by the conductor as a magnetic sheet or a magnetic shell whose polar faces, north and south, are bounded by the current. Such a magnetic sheet is called *equivalent magnetic shell* (for a magnetic shell see art. 16, chapter VI). A solenoidal current is equivalent to a bar magnet with ends coinciding with the faces of the solenoid. A circular wire carrying current is equivalent to a circular magnetic shell one face having N polarity and the other S polarity. The side whose polarity is N depends upon the direction of the current which can be found by Fleming's left hand rule.

9. Strength of an equivalent magnetic shell. Consider a very small circular coil of radius r and carrying a current i. The strength of the magnetic field at any point P on the axis at a distance R, large compared to r from the centre is, by eq. (9·5),

$$H = 2\pi r^2 i/R^3,$$

$$= 2 \times \text{area of the coil} \times \text{current}/R^3. \quad . \quad (9·19)$$

But from eq. (6·15) the field at any point on the axis of a magnetic dipole of moment M is

$$H = 2M/r^3.$$

. . . (9·20)

Comparing eqs. (9·19) and (9·20) we see that the current carrying coil behaves like a magnetic shell of moment M given by the product of the area of the coil and the current, the axis of the magnet being perpendicular to the plane of the circuit. The current in the circuit is equal to the magnetic moment per unit area of the shell, i.e., the strength ϕ of the shell. This result is quite
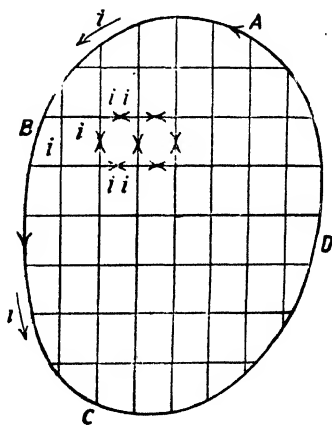
Fig. 9·9

general and will hold good for a closed circuit of any form.

Now consider a circuit ABCD of finite size. Let us divide it into a large number of small meshes each carrying the same current i round it in the same direction, fig. 9·9. It can be seen that each side of a mesh is common to two adjacent meshes and that the currents flowing in it are in opposite directions. The sum of the currents in the meshes is thus zero except for the current at the boundary ABCD. Now each mesh may be replaced by an equivalent magnetic shell of strength equal to the current in it. Thus the whole circuit ABCD is equivalent to a magnetic shell of strength i.

10. Application of the concept of an equivalent magnetic shell. The potential at any point due to a magnetic shell of strength ϕ is given by the product of the strength of the shell and the solid angle subtended by the shell at the point. Since any circuit carrying a current i is equivalent to a magnetic shell of strength i. The potential due to the circuit at any point is $i \times$ solid angle subtended by the circuit at the point. We shall give a simple application of this important concept.

Consider a circular coil of radius a and carrying a current i, (fig. 9·10). The potential at an axial point P is

$$V_P = i\Omega,$$

Fig. 9·10

$$= i\,2\pi\,(1 - \cos\theta) = i\,2\pi\left(1 - \frac{x}{\sqrt{a^2 + x^2}}\right).$$

By symmetry the resultant field along the axis is

$$H = -\frac{\partial V_P}{\partial x} = \frac{2\pi a^2 i}{(x^2 + a^2)^{3/2}}\ , \qquad \cdot\ \cdot\ \cdot\quad (9{\cdot}21)$$

which is identical with eq. (9·5).
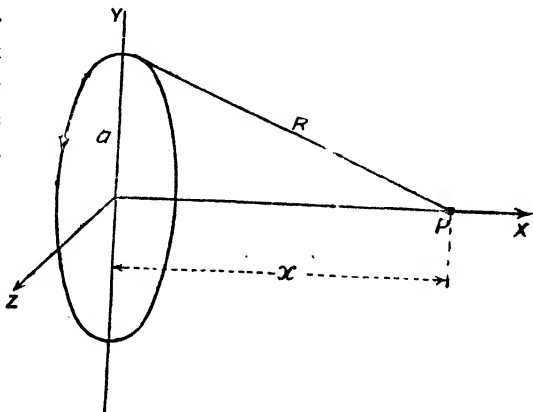
11. Potential in a magnetic field due to a current.
We have shown in art. 16 that the work done in taking
positive pole round a magnetic shell is $4\pi\phi$. If the magnetic
field is due to a current i we have simply to put i in place of ϕ
in accordance with the concept of equivalent magnetic shell. Thus
the work done in taking unit positive pole round a current i is $4\pi i$.
If the closed path is described again there is an additional work
$4\pi i$. By repeating this process the work is increased in steps
of $4\pi i$. The potential at a point in the magnetic field surrounding
a current is thus not a single-valued function. It has a number
of values differing by multiples of $4\pi i$.

12. Ampere's circuital theorem. Consider a small ele-
ment dl of a path $AB = l$ in
a magnetic field, (fig. 9·11).
Let H be the intensity of the
magnetic field produced by
a current i at any point of
the path and θ its incli-
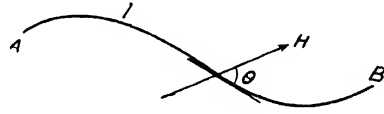
Fig. 9·11

nation with the tangent to the path at that point. The com-
ponent of the force along the path is H cos θ. The work done in
carrying unit pole through an infinitesimal distance dl along
this path is

$$dW = H \cos \theta \, dl,$$

and the total work done from A to B is given by

$$W = \int_A^B H \cos \theta \, dl. \qquad . \quad . \quad . \quad (9·22)$$

Eq. (9·22) is the *line integral* of the magnetic field round the
current ; and if the path is closed we have

$$W = \oint H \cos \theta \, dl = 4\pi i, \Big\} \quad . \quad . \quad . \quad (9·23)$$

or   curl H $= 4\pi i$.

The line integral along a closed path is also called *magnetomotive
force*, (m.m.f.), and is expressed in *gilberts*.

Eq. (9·23) is known as Ampere's circuital theorem. If the
closed path is not linked by any current,

curl H $= 0$. . . . (9·24)

13. Applications of Ampere's circuital theorem.
Straight linear current. By symmetry there is a constant field

H at a distance r from the linear current and at right angles to it. A line of force is thus a circle with its centre upon the current. The line integral is $2\pi r$H.   By eq. (9·23) we have

$$2\pi r H = 4\pi i,$$
$$\text{or} \qquad H = 2i/r. \qquad \qquad . \quad . \quad . \quad (9·23)$$

Current in a solid cylinder.   Let R be the radius of a solid cylinder carrying a uniform current i, (fig. 9·12). By eq. (9·23) the field intensity outside the cylinder is $2i/r$. Inside the cylinder at a point P, distant r_1 from the axis, the line integral is

$$\oint H dl = 2\pi r_1 H = 4\pi \times \text{current sur-}$$

rounded,

$$= 4\pi \times \frac{i}{\pi R^2} \times \pi r_1^2,$$
$$= 4\pi i \, r_1^2/R^2,$$

Fig. 9·12

Hence, $H = 2i r_1/R^2$. $\qquad . \quad . \quad . \quad (9·24)$

From eq. (9·24) we see that H is maximum at the surface of the cylinder and zero at the centre and at infinite distance.

Endless solenoid.   This is represented in fig. 9·13.   In this case the north face of the equivalent shell of each turn comes into contact with the south face of the equivalent shell of the next turn.   Dealing in this manner with the total number of turns n, each carrying current i, we get the equivalent magnetic shell in the form of a solid closed ring without any free polarity.   There is thus no magnetic field outside such a ring. Evidently the magnetic field H inside the ring
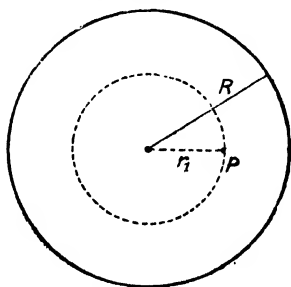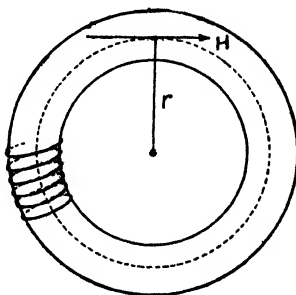
Fig. 9·13

at a distance r from the axis is tangential to the circumference of the circle of radius r.   The path is $2\pi r$ and it is linked n times with the current i.   Thus Ampere's circuital theorem gives

$$2\pi r H = 4\pi n i,$$
$$\text{or } H = 2ni/r. \qquad \qquad . \quad . \quad . \quad (9 \cdot 25)$$

If the radius of the solenoid is small in comparison to the radius of curvature of its axis about which the current goes, then eq. (9·25) indicates that H is uniform in the ring solenoid. Putting $2\pi r = l$, the total stretched length of the solenoid, eq. (9·25) reduces to $H = 4\pi ni/l$ which is the same as eq. (9·14) for a long solenoid.

14. Mechanical force on a current-carrying circuit placed in a magnetic field. Motion of translation. A current-carrying coil will experience a mechanical force when placed in a magnetic field, since it is equivalent to a magnetic shell.

Let i be the current flowing in a circuit S placed in a magnetic field H, and F the mechanical force per unit length of the circuit exerted by H, (fig. 9·14). Suppose that F moves S through dx into the new position S'. Let the current element ds make an angle θ with H.

The potential energy of the equivalent magnetic shell S of strength i is, by eq. (6·42),

$$W = -i\,N, \qquad (9 \cdot 26)$$

where N is the normal induction through the circuit. The mechanical force is given by
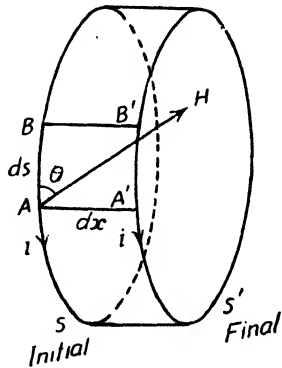
Fig. 9·14

$$F = -\frac{\partial W}{\partial x} = i\,\frac{\partial N}{\partial x}. \qquad . \quad . \quad . \quad (9 \cdot 27)$$

The work done by F in moving S through dx is Fdx. The change in potential energy is $dW = -i\partial N$.

We shall now calculate ∂N, the change in flux. Consider the closed region of space, bounded by the circuit in its two positions S and S' and the side curved surface formed by parallelograms like ABA'B'. If N and N' are the normal inductions for positions S and S' and N'' the normal induction over the curved

surface, then $N'-N+N''$ is the total normal induction over the space considered. As there is no magnetic charge placed in this region, we have by Gauss's theorem $N'-N+N''=0$ or $\partial N = N-N'=N''$. Now $N''=$ area $ABA'B' \times \mu\,H\sin\theta$ (where $\mu=$ magnetic permeability of the medium)$=ds\,dx\sin\phi$. $\mu\,H\sin\theta=B\,ds\,dx\sin\phi\sin\theta$, where ϕ is the angle between ds and dx.

Thus by eq. (9·27) we have

$$F=iB\sin\phi\sin\theta. \qquad . \quad . \quad . \quad (9\cdot28)$$

From eqs. (9·26) and (9·27) we see that the dirction is such that there is increase of induction and so diminution of its potential energy. The minimum potential energy corresponds to maximum value of F for a given direction of H. It is this condition which the circuit attains in the equilibrium position.

From (9·28) putting $\phi=\pi/2$ we have

$$\left.\begin{array}{l} F=iB\sin\theta, \text{ in a magnetic medium} \\ \text{and} \qquad F=iH\sin\theta, \text{ in air} \end{array}\right\} . \qquad (9\cdot29)$$

The direction of F is perpendicular to the direction of the current. Also $F=0$ in the direction of H since $\dfrac{\partial N}{\partial x}=0$. Hence the resultant force F must be perpendicular to H. Thus F is perpendicular to the plane of i and H or B. The direction of F can be easily remembered by Fleming's left-hand rule.

EXAMPLE. Calculate the mechanical force which will be exerted on the inner layer per inch length of a long solenoid with 40 turns per cm. and carrying a current of 10,000 amps.

Field along the axis of the solenoid is

$$H=\frac{4\pi m}{10}=\frac{4\pi \cdot 40 \cdot 10^4}{10},$$

$$=500,000 \text{ gauss}.$$

The mechanical force is

$$F=\frac{500,000\times2\cdot54\times10,000}{10}=12\cdot7\times10^8 \text{ dynes}.$$

$$=1\cdot28 \text{ tons weight}.$$

Rotation of plane current circuit in a uniform magnetic field. A coil S of area A carrying a current i is placed

in a uniform magnetic field H, (fig. 9·15). If θ is the angle between the plane of the coil and the direction of H, the potential energy is given by

$$W = -iN,$$

$$= -iAH \sin \theta. \qquad (9\cdot30)$$

There is no transverse motion of S since in this case $\delta N = 0$. The coil will, however, rotate if free to do so. The turning couple is given by

$$\tau = -\frac{\partial W}{\partial \theta},$$

$$= iAH \cos \theta, \qquad (9\cdot31)$$

acting in the direction of increasing θ. In the equilibrium position $\tau = 0$,

Fig. 9·15

and hence $\theta = \pi/2$, i.e., the coil sets itself perpendicular to the magnetic field, thus embracing maximum amount of flux of H (or of induction μH if the medium is magnetic).
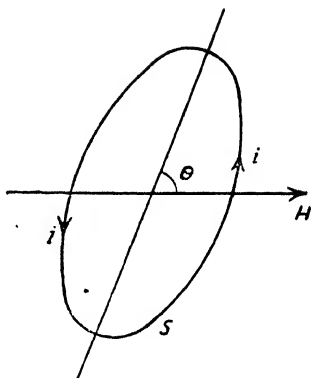
15. Mechanical force between parallel straight currents. AB and A'B' are two thin straight conductors carrying currents i and i' and separated by distance a, (fig. 9·16).

The intensity of the field at a point P on the wire A'B' due to the current in AB is $H = 2i/a$, directed at right angles to the plane containing AB and A'B'. By Fleming's left-hand rule, when i' is flowing in the direction as shown in the figure the mechanical force on A'B' is directed towards
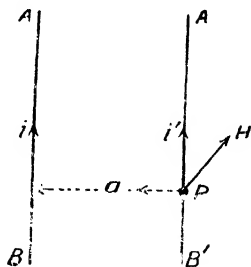
Fig. 9·16

AB along a and is of mangitude $2ii'/a$ per unit length, i.e., two currents flowing in the same direction attract each other.

If the direction of i' is opposite to that of i the magnitude of the force remains the same but its direction is now away from AB, i.e., there is a force of repulsion between two oppositely directed parallel currents. The lines of force in these two cases are shown in figs. 9·1(c) and 9·1(d.)

CHAPTER X

DIRECT CURRENT MEASURING INSTRUMENTS

1. Galvanometers. Galvanometers are the most important instruments used in the detection and measurement of small currents. They are based upon the fundamental principle of magnetic effect of electric current discussed in Chapter IX. They are divided into two main types named after their originators. One is called the Thomson type and the other the D' Arsonval type. In the Thomson type, the magnetic field of the coil carrying the current to be measured or detected acts upon a small suspended magnet ; in the D' Arsonval type, the current is passed through a light coil suspended between the poles of a magnet (generally permanent). We present below a brief account of the principle and construction of some of the most important of these instruments.

2. The tangent galvanometer. It consists principally
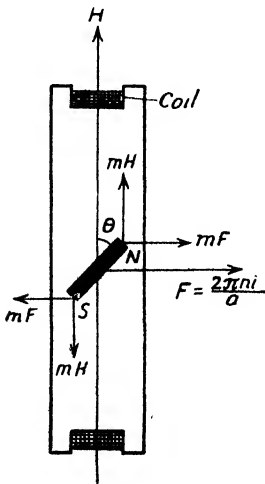
(a)   Fig. 10·1   (b)
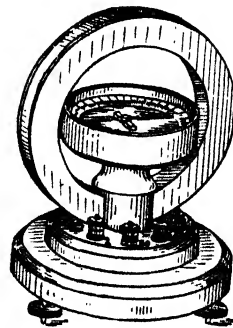
of a small magnet supported on a pivot or suitably suspended by a fibre so as to rotate freely. There is a light pointer fixed at right angles to this with the help of which the deflections of the needle can be measured on a circular scale with its centre coinciding with the pivot . The small magnet is situated at the centre of a rigidly supported vertical coil of n turns of wire. The radius of the coil must be large compared with the dimensions of the suspended magnet so that the latter may be supposed to lie wholly in the magnetic field at the centre of the coil . The principle of this galvanometer is demonstrated in fig. $10 \cdot 1$ (a), while the general appearance is shown in fig $10 \cdot 1$ (b).

The plane of this coil is first put in magnetic meridian when no current passes through it. When a current i passes through it, a magnetic field $F = \dfrac{2\pi ni}{a}$ perpendicular to the plane of the coil is produced, a being the mean radius of the turns. H is the horizontal component of the magnetic field due to the earth. Under these two perpendicular fields the small magnet NS will be deflected through an angle θ given by

$$\tan \theta = F/H = 2\pi ni/a H. \qquad . \quad . \quad . \quad (10 \cdot 1)$$

If the current is expressed in amperes I, eq. $(10 \cdot 1)$ takes the form

$$I = \left(\frac{10\,H\,a}{2\pi n}\right) \tan \theta = K \tan \theta. \qquad . \quad . \quad (10 \cdot 2)$$

The expression within brackets is constant for a given galvanometer and is called its *reduction factor*. If we know this constant and measure θ, I is determined. The reduction factor can be determined by the use of electrolytic action of current as will be described in the succeeding chapter. The principal drawbacks of this instrument are : (1) the plane of the coil must be accurately adjusted in magnetic meridian and must remain so during the experiment, (2) the deflexion must not be very large otherwise measurement will be inaccurate due to too rapid a variation of tan θ, (3) it is rather bulky and (4) the needle is not situated in a perfectly uniform magnetic field of the coil.

3. Helmholtz galvanometer. It consists of two similar parallel coils mounted rigidly upon a small rotating table. The distance between the two coils is equal to the radius a of either. The suspended magnet is arranged to lie with its centre at a distance $a/2$ from each coil. The advantage of this arrangement is that, as discussed in Art. 5, Ch. IX, the field at the middle is very uniform. Puttting $m = n$, $a = b$ and $x = 0$ in eq. (9·11), the field is given by

$$F = \frac{4\pi n\, a^2 i}{\left(\frac{5}{4}a^2\right)^{\frac{3}{2}}} = \frac{32}{5^{3/2}} \cdot \frac{\pi\, ni}{a} . \qquad \ldots \quad (10·3)$$

Arranging the coil in the magnetic meridian as in the case of the single coil instrument, the expression for the current I in amperes is

$$1 = \frac{10}{32} \frac{125\, a\, H}{\pi n} \tan \theta . \qquad \ldots \quad (10·4)$$

The principle of this galvanometer has been already illustrated in fig. 9·6 and its general appearance is shown in fig. 10·2 which repre- sents an apparatus made by Messrs Nalder Brothers.

4. Sensibility of tangent galvanometer. From eq. (10·2) we see that a given current will produce a large deflection if the reduction factor is small. This means that for large sensitiveness the radius of the coil must be made small and n the number of turns must be large. Evidently there is a limit to increased sensitiveness by this method.
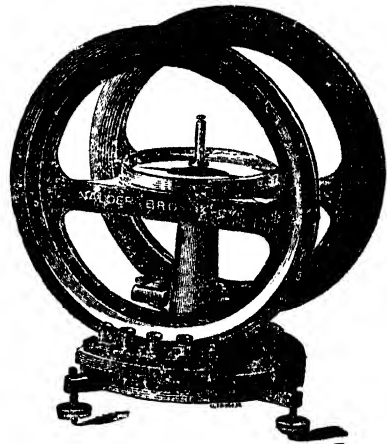
Fig. 10·2

The sensibility of a tangent galvanometer is measured by the ratio of the increment in deflection to the increment in current per unit current or by $\delta\theta / \frac{\delta i}{i}$, where $\delta\theta$ and δi are small quantities. From eq. (10·2) we have $\delta i = K \sec^2 \theta\, \delta\theta$. Hence $\delta\theta / \frac{\delta i}{i} = \frac{1}{2}\sin 2\theta$. This is maximum when $2\theta = \pi/2$, i.e., $\theta = \pi/4$. This means that the given

galvanometer is most sensitive when the current through it is equal to K, the reduction factor.

In order to increase the range of usefulness of a suspended magnet galvanometer a device is adopted to control the sensitiveness. This device is nothing but a controlling bar magnet which can be made to slide up and down along the upper portion of a rod which supports the suspension of the magnetic needle at its lower end. The S pole of this control magnet points towards the north and the N pole towards the south. Thus the restoring torque due to the magnetic field H of the earth is increased or diminished by lowering or raising the control magnet. The sensitiveness can thus be diminished or increased within wide limits.

5. Sine galvanometer. This is similar to a tangent galvanometer, but in this the current-carrying coil instead of being fixed, is rotatable about a vertical axis. As a current passes the plane of the coil is rotated until the coil and the needle lie in the same plane. The direction of the needle in equilibrium gives the direction of the resultant of the two forces H and $F = 2\pi ni/a$. Hence as shown in fig. 10·3,

$$H \sin \theta = 2\pi nI/10a, \qquad (10·5)$$

where I is in amperes.
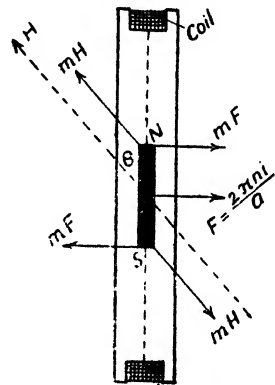
Fig. 10·3

6. Astatic galvanometer. This is another device for reducing the restoring torque on the suspended needle to its minimum value. It was first used by Lord Kelvin in his instrument called Thomson galvanometer. In this instrument Thomson used the so-called astatic system of light, tiny magnets. The construction of the suspended system is shown diagrammatically in fig. 10·4.

A very light and thin glass tube *ab* is suspended by a fine fibre *ca* of either quartz or phosphor bronze from the rigid support C at the top of the galvanometer. This tube carries two

9 ε

systems I and II of very thin and light magnets. The two are quite similar and parallel to each other. They are so arranged that their mangetic moments are equal and opposite. Such a system is astatic. The earth's magnetic field does not exert any restoring torque on this. The control torque is exerted by the torsion in the suspension *ca*. By using a thin suspension this restoring torque can be made very small and thus the sensitivity can be made high. There are two fixed small circular coils A and B surrounding the two systems I and II so that the centres of the coils coincide with the centres of the systems I and II. These two coils are connected in series, the ends being connected to terminals T_1 and T_2. Current passes through A and B in opposite senses. Because of this the torques on the components I and II of the astatic system are in the same direction and so the deflection of the system is the same as in the case of a single coil having a number of turns equal to the sum of those on the two coils.

Fig. 10·4
Astatic galvanometer

By making the system very light, say a few milligrams in weight, and by using a very fine quartz suspension it is possible to measure currents of the order of 10^{-1} ampere with such a galvanometer. The deflections are measured with a lamp and scale; for this purpose a small light mirror M is rigidly attached to the suspension. The whole apparatus is protected from the influence of external magnetic fields by surrounding it with one or more soft iron shells. This magnetic shielding becomes of especial importance when dealing with measurements of very small currents. The magnetic shields may be concentric spherical or cylindrical shells.

The astatic galvanometer is not an absolute instrument. It

must be calibrated with the help of an absolute instrument like a tangent galvanometer.

7. Broca galvanometer. This is a modern type of suspended magnet galvanometer embodying all the principles of an astatic galvanometer. The moving system consists of an astatic pair the components of which have got as nearly equal moments as possible. The sensitivity of the instrument can be varied by adjusting a small control magnet at the base of the instrument.

8. D'Arsonval galvanometer. This is the name given to the suspended coil galvanometer after the inventor. It has largely superseded the suspended magnet instrument for usual laboratory and other work where extremely high sensitivity is not essentially required. It has the following special advantages: (1) The deflections are not affected by external magnetic fields, (2) the instrument may be placed facing in any direction and (3) the suspended system may be made aperiodic, thus enabling the worker to avoid loss of time in waiting for it to come to rest. One of its disadvantages is that its sensitiveness cannot be varied at will.

(a)   (b)
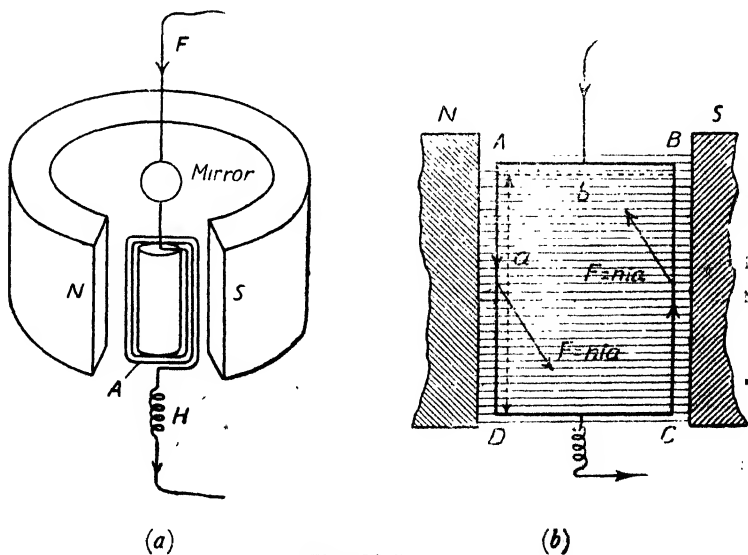
Fig. 10·5

A galvanometer may be designed for measurement of currents. It is then called a current galvanometer. It may be designed for measurements of small quantities of electricity. It is then called a ballistic galvanometer. We will deal here with the first type, a consideration of the second being postponed to a subsequent chapter.

In the moving-coil galvanometer the current-carrying coil is either rectangular or circular and is wound on a light frame. It is suspended between the pole pieces of a strong permanent horse-shoe magnet. The suspension is a phosphor-bronze ribbon. The arrangement and function of the essential parts are shown diagrammatically in figs. 10·5 (a) and (b).

The current enters through the phosphor-bronze strip F from above, and, after passing round the various turns in the coil A, comes out through a second phosphor-bronze strip in the form of a helix H.

Referring to fig. 10·5(b) we see that when a current flows through the coil it will be deflected through an angle θ with respect to the field H. The potential energy of the coil in the displaced position is, by eq. (9·30),

$$W = -inAH \sin \theta,$$

where A is the area of the coil and n the number of turns. Hence the deflecting couple is

$$C = -\frac{dW}{d\theta} = inAH \cos \theta. \qquad . \quad . \quad (10·6)$$

In the equilibrium position the deflecting couple C will be equal to the restoring couple $\tau\theta$, τ being the couple per unit twist of the suspension. That is,

$$\tau\theta = inAH \cos \theta,$$

or

$$i = \frac{\tau}{nAH} \cdot \frac{\theta}{\cos \theta}. \qquad . \quad . \quad . \quad (10·7)$$

Eq. (10·7) is very inconvenient to use in practice because of a scale varying as $\theta/\cos\theta$. This difficulty is avoided by a special device shown in figs. 10·6. The suspended coil, if rectangular, is made to surround a fixed cylinder of soft iron and if circular, it is made to surround a fixed soft iron ball. The pole pieces

of the magnet are also made circular. In this manner the field
threading the suspended coil is
made   strong   as well as radial
instead of uniform.   Because of
this   the plane of the coil in
every   position   coincides   with
the   field   direction,   i.e., $\theta = 0$
always. Eq. (10.7) then be-
comes
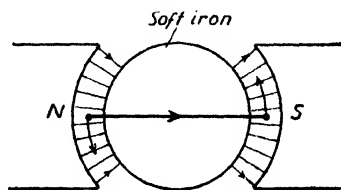
Fig. 10·6

$$i = \frac{\tau}{n\mathrm{AH}} \theta = \mathrm{K}\theta. \qquad . \quad . \quad . \quad (10\cdot8)$$

In eq. (10·8) τ, n, A and H are constant for a given galvanometer
and so the current has linear relation with θ.   The constant
K can be determined with the help of a tangent galvanometer
or with standard resistances and a standard e.m.f.   A well-made
D' Arsonval galvanometer can measure currents of the order of
of 10^{-9} amp.

9.   Sensitivity of galvanometers. The galvanometer sen-
sitivity is defined in various ways as follows :

(1) *Microampere sensitivity*. Deflection in mms. on a scale
placed normally at one metre distance from the galvanometer
mirror when a current of one microampere (10^{-6}) is passed through
the galvanometer. (2) *Microvolt sensitivity*.   The deflection in mms.
on a scale placed normally at 1 metre from the galvanometer
mirror when a p.d. of 1 microvolt is placed across the galvano-
meter terminals.   (3) *Megohm sensitivity*.   The number of meg-
ohms put in the galvanometer circuit so that an impressed p.d.
of 1 volt across the galvanometer terminals produces a deflec-
tion of one,mm. on a scale placed normally at 1 metre distance
from the   galvanometer   mirror.   (4) *Figure of merit*. When a
galvanometer is to be standardised and used for current measure-
ment this definition is found to be of practical utility.   Figure
of merit of a galvanometer is the current in amperes required to
produce a deflection of 1 mm. on a scale placed normally at 1 metre
from the mirror. (5) It is also sometimes defined as the number

of scale divisions deflection produced by one microampere when the scale is placed normally at a distance of 1000 divisions from the mirror, reduced to corresponding value for the same rate of expenditure of energy when the resistance of the galvanometer is taken as one ohm and the period of vibration as one second. It can be shown that when so reduced the sensibility is given by $\theta/T^2 \sqrt{R}$, where θ is the deflection for 10^{-6} amp., R the resistance and T the vibration period of the galvanometer. A method of determining the figure of merit of a galvanometer will be described in the next chapter.

10.   Ammeters.   Direct current ammeters are instruments used for reading directly the current flowing in a circuit in which they are introduced in series. Evidently their resistance must be so small that when introduced into the circuit they do not appreciably alter the current to be measured.

The most common type of ammeter for direct current measurement is a moving coil galvanometer of robust design. The mirror and phosphor-bronze suspension are replaced by a light mechanical pointer moving over a circular scale and a pivot. A hair spring exerts the required restoring couple. The resistance of the moving coil is reduced by using only a few turns of a conducting wire or by using a suitable low resistance called a shunt across it.

There are other types of ammeters called soft iron and hot wire ammeters. Since their deflection depends upon the square of the current they can be used for direct as well as for alternating currents. They will be described along with other A.C. instruments in Chapter XVI.

11.   Voltmeters.   Instruments used to measure potential differences are called voltmeters. There is a large variety of these instruments each having its own scope of application. Disregarding the particular principles in each we may broadly divide voltmeters into two types: (a) electromagnetic, and (b) electrostatic. The electrostatic type (already described in Ch. V) is

specially useful for measurement of comparatively large potential differences, 1000 volts and upwards. For ordinary voltages the electromagnetic type is a very suitable and accurate instrument. For direct current circuits the electromagnetic type of voltmeter is nothing but an ammeter with a high resistance in series.

In order to measure a potential difference the voltmeter must be connected in parallel to the circuit. Unless the resistance of the voltmeter is high it will draw a considerable portion of current from the main circuit and will thus produce a large disturbance in the current distribution. The voltmeter resistance must, therefore, be so great that it draws a minimum current from the main circuit. The ideal case is when no current is drawn at all by the voltmeter. This ideal is however not fulfilled by the electromagnetic type of voltmeter but it is completely realised by the electrostatic type of voltmeter. A moving coil galvanometer can be converted into a voltmeter by including a suitable high resistance in series with the coil. The same galvanometer can be converted into an ammeter by using a suitable shunt across its coil. The internal connections in the two cases are shown in fig. 10·7 which also gives the method of introducing them into the main circuit.
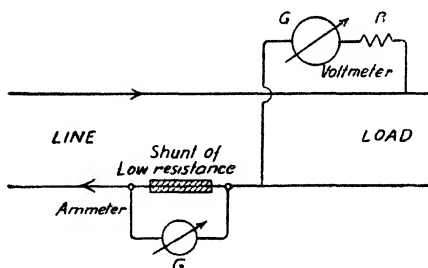
Fig. 10·7

12. Ammeters as voltmeters and voltmeters as ammeters. From the foregoing description of the construction of voltmeters and ammeters it follows that a given voltmeter can be converted into a required ammeter and vice versa. We shall illustrate this with the help of a few examples.

EXAMPLES.—(1) An ammeter of 25 amperes range is to be constructed out of a milli-voltmeter of 50 milli-volts range. Find the resistance of the necessary shunt.

Let S ohms be the resistance of the shunt. Then p.d. between its terminals for 25 amperes current is $25 \times S$ volts. This must be equal to 50×10^{-3} volt necessary to have a full scale deflection. Hence $S = \dfrac{50 \times 10^{-3}}{25} = 0.002$ ohm.

(2) A 50 milli-volt voltmeter has a resistance of 10 ohms. Find the value of a series resistance to convert it into 100 volt voltmeter.

Let R be the required resistance when 100 volts are put across the terminals A and B, (fig. 10.8.) The drop across M. V. must be 50×10^{-3} volt. The current for full scale deflection is

Fig. 10.8

$50 \times 10^{-3}/10$, *i.e.*, 50×10^{-4} amp. by Ohm's law. Hence the p.d. between A and B is

$$(10 + R) \times 50 \times 10^{-4} = 100 \text{ volts,}$$

or $\qquad\qquad\qquad R = 19,990 \ \Omega.$

(3) An ammeter of 17Ω resistance is graduated to read milli-amperes directly. Find a series resistance necessary to convert it into a voltmeter reading volts directly.

Let R be the series resistance. Suppose that x volts are imposed on the free terminals of the combination. The current passing through the instrument is, by Ohm's law, $\dfrac{x}{17 + R} \times 10^{3}$ amperes. As the modified instrument is reading volts directly this produces deflection x which on the original scale means x milli-amperes. Thus $\dfrac{x \times 1000}{17 + R} = x$ or $R = 983\Omega.$

(4) A galvanometer of resistance 150Ω has in series with it a resistance of 1000Ω. A shunt of 0.015Ω is placed across the free ends. If a p.d. of 1 volt produces deflection of 600 scale divisions calculate the change in the series resistance necessary to produce a deflection of 100 scale divisions when
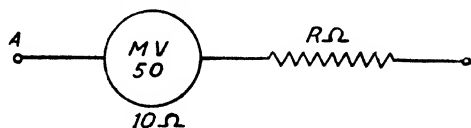
a current of 50 amps. is passed through the shunt. Assume that the scale is a proportional one.

Since the current through the galvanometer corresponding to 600 divisions is $1/(150+1000)$ amp., the current required to produce 100 divisions' deflection is $\dfrac{100}{600} \times \dfrac{1}{1150}$ amp. If r is the resistance in series corresponding to this deflection we have p.d. across the two terminals $= \dfrac{100}{600 \times 1150} \times (1150+r)$ volts. This must be equal to the p.d. 50×0.015 volt across the shunt, i.e.,

$$\frac{100}{600 \times 1150} \times (1150+r) = 0.75,$$

or
$$r = 4025\,\Omega.$$

This is the resistance which must be added.

13. Kelvin current balance. This is another instrument
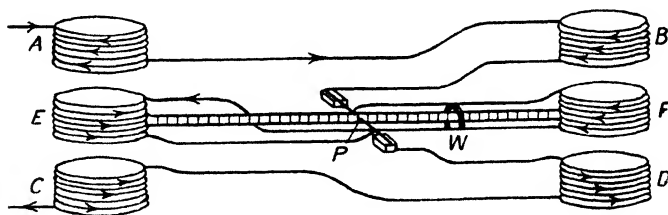
Fig. 10·9 Kelvin current balance.

for measuring currents. It is designed to operate not on the interaction of current and magnetic field as in the moving coil type of ammeters but on the magnetic interaction between current-carrying coils. The principle of its construction is demonstrated in fig. 10·9.

It consists principally of four fixed plane coils ABCD arranged at the corners of a rectangle with their faces horizontal. Two similar coils E and F are attached to a graduated balance arm on which there slides a counterpoise weight W. P is the pivot about which the arm is capable of swinging. The coils are connected in series. The current to be measured passes through these in such a direction that due to attractions and repulsions between currents the coil E is pushed downwards while at the

same time the coil F is pushed upwards. The arm is so adjusted that when no current passes through the coils the counterpoise W stands at zero mark, the arm being horizontal.   When current passes equilibrium is disturbed as explained above. The balance is restored by sliding W along the graduated arm. The torque due to the current is proportional to the product of the currents, i.e. i^2.   The restoring couple is equal to the product of the weight W and its displacement d from the zero position.

Hence $Ki^2 = Wd,$ . . .   (10·9)

where K is a constant of the instrument depending upon its construction.

Or $i = K' \sqrt{d},$ . . .   (10·10)

where K' is another constant.

Determining K' with the help of known currents the arm is calibrated to read currents directly. As d depends upon the square of the current the instrument can be used for both direct and alternating currents.

14. Power meters. When a current I flows in a circuit across which there is a potential difference E the electrical energy consumed in the circuit is EI watts per second (i.e. Power).   Instruments measuring power directly are called power meters. We deal here with two principal types of wattmeters: (1) those giving the rate of consumption of energy at any time, and (2) those recording the consumption of energy in a given time.   Instruments belonging to the latter class are used in electric installations and are called service meters.

Kelvin watt balance. The outward appearance of this instrument is the same as that of Kelvin current balance.   In
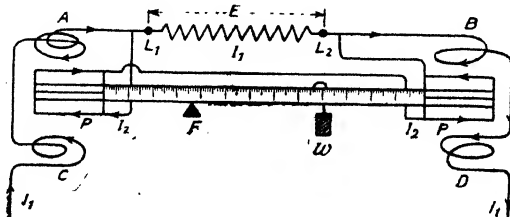
Fig. 10·10 Kelvin watt balance.

the current balance all the coils must be of low resistance so as not to cause any appreciable disturbance in the current distribution of the circuit to be measured. In Kelvin watt balance the two moving coils PP, (fig. 10·10), are of high resistance and the other four coils ABCD are of low resistance. The main current I_1 is passed through the four low-resistance fixed coils. The movable high resistance coils are put across the circuit in which power absorption is to be measured. If E is the potential drop across the circuit and I_2 the resulting current in the high resistance coils we have the energy consumed per second given by

$$W = EI_1 = (I_2R) I_1 = KI_1I_2,$$

$$= K_1 \times \text{force on the moving coils,}$$

$$= K_2d,$$

where R is the resistance of the moving coils, K, K_1, K_2 are constants and d is the displacement of the counterpoise weight from the zero position. After calibration the instrument can read directly the power absorbed at any time.

Electrodynamometer type. The principle underlying this type of meter is also the force of attraction and repulsion between parallel currents. Two coils F and M are mounted vertically, fig. 10·11. F is a low resistance fixed coil consisting of a few turns of thick wire and is put in series with the main circuit carrying current I_1. M is a high resistance suspended coil of many turns of fine wire, connected across the circuit in which power consumption is to be measured. It will carry current I_2 proportional to the voltage E across the circuit. T is a graduated torsion head which measures the twist

Fig. 10·11 Electrodynamometer.

on the coil M when current passes through it. To start with, the coils are put perpendicular to each other. Under the action of the current they try to set themselves parallel to each other.

The torsion head T is turned through a certain angle θ to maintain their perpendicular positions. The power absorbed is given by

$$W = EI_1 = KI_1I_2 = K'\theta,$$

where K and K′ are constants. The constant K′ is determined from known values of W and θ. The instrument thus gives W directly in terms of θ.

15. Energy meters. Motor form of meter. For commercial purposes it is necessary to know the amount of electrical energy consumed in a certain time instead of its rate at any time.

Various commercial meters have been developed for this purpose. Let us consider a wattmeter of the electrodynamometer type described in the previous article. In order that it may serve as an energy meter it must be so adapted that the moving coil goes on rotating continuously with time. This is not possible unless the current in it is reversed periodically. This is achieved in the motor form of energy meter. The fixed coil is the field coil and the moving coil, which is a composite coil consisting of a number of turns, forms the armature of the motor. The coils forming the armature are so arranged that as the motor rotates the position of any one coil
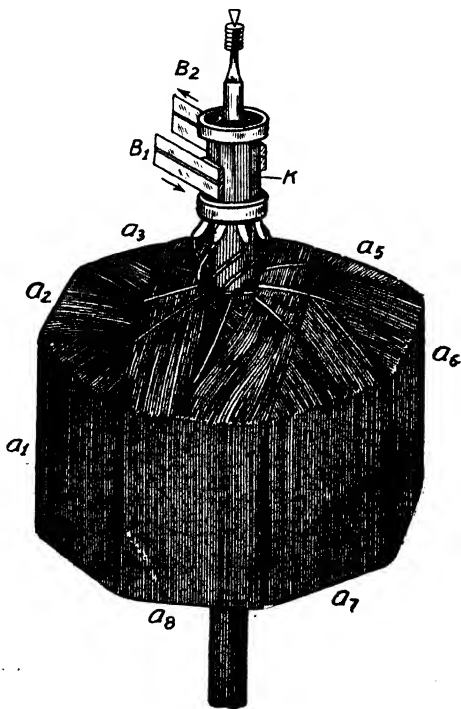
Fig. 10·12 (a)

ature of the motor. The coils forming the armature are so arranged that as the motor rotates the position of any one coil

in the magnetic field is taken up by the next coil; and if the number of coils on the armature is sufficiently large the torque exerted by the field coil on the armature remains constant inspite of its motion relative to the field coil.

As an illustration we take the armature in the Thomson energy meter. There are 8 coils $a_1, a_2, \ldots a_8$, wound on a light frame, (fig. 10·12 a). The end of each coil is connected to the beginning of the next and also to one strip of the eight-part commutator K by one of the eight wires $w_1, w_2, \ldots w_8$. A stationary resistance is put in series with the armature which is connected as a shunt to the circuit in which the consumption of energy is to be measured. The shunt current enters and leaves the armature by way of two stationary brushes B_1 and B_2.

The field coils FF, (fig. 10·12 b), are placed in series with the supply circuit in which the energy consumption is to be measured. The main current passing through these coils produces a stationary magnetic field very nearly at right angles to that produced by the armature current. Interaction between these two fields results in continuous rotation of the armature. This motion is transmitted to a counting mechanism through a suitable worm wheel. Since no iron is used in either the field coil or the
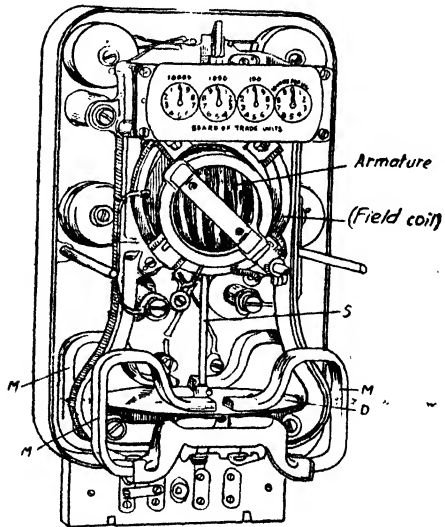
Fig. 10·12 (b) Energymeter.

armature the magnetic fields are proportional to the currents and hence the driving torque is proportional to IE , the power in watts, where I is the current and E the voltage on the circuit in which the energy is being consumed.

D is a horizontal aluminium disc fixed rigidly to the armature spindle S. There are four permanent horse-shoe magnets M with their N and S poles so arranged that the lines of force are vertical and perpendicular to the plane of the disc. The rotation of the disc with the armature generates in it by induction eddy currents of strength proportional to the angular velocity ω of the disc. The attracting action exerted by the magnets on these currents retards the motion of the armature spindle. The resisting torque is proportional to the strength of the eddy currents and hence to ω the angular velocity of the disc. When steady conditions are reached the driving and the resisting torques on the armature will balance, i.e.,

$$IE = K\omega, \qquad \qquad \ldots \quad (10\cdot11)$$

where K is a constant. It should be noted here that in stating this condition it has been assumed that the friction at the bearings, brushes and other mechanical parts is negligible. From eq. $(10\cdot11)$ we have, for a finite time t seconds,

$$IEt = K\omega t = K\theta, \qquad \qquad \ldots \quad (10\cdot12)$$

provided IE=constant. Eq. $(10\cdot12)$ shows that the angle turned through by the armature multiplied by a constant gives us the amount of energy consumed in watts which can be expressed in B.O.T. units. The meter is calibrated with the help of known powers to make it direct reading.

16. Quantity or ampere-hour meters. Instruments primarily designed to measure quantity of electricity, $(I \times t)$, passing through a circuit are called quantity meters. They are usually calibrated to give directly the amount of energy $(E \times I \times t)$ consumed in the circuit for a constant supply voltage E.

The simplest type of quantity meter is the electrolytic one. Most ampere-hour meters are, however, of the motor form which we illustrate by Chamberlain and Hookham meter. The principle of this meter is illustrated diagrammatically in fig. $10\cdot13$ (*b*) and the general assembly in fig. $10\cdot13$ (*a*). A light white metal disc D supported by the spindle S is placed in a flat cylindrical ebonite box BB filled with mercury and is free to rotate about the vertical

axis.   On the opposite faces of the box there are conical poles PP
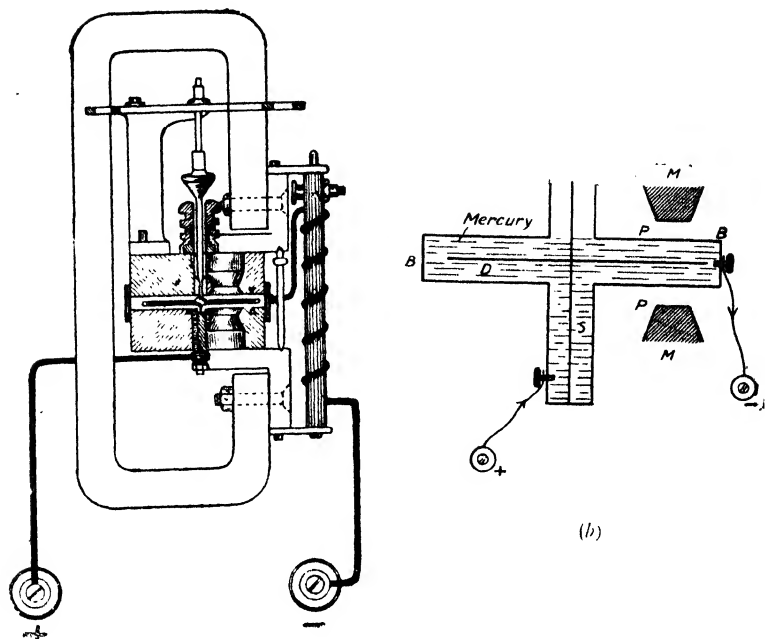
(a)

Fig. 10·13.   Chamberlain and Hookham Meter.

of a large permanent magnet M.   The load current enters the
mercury near the spindle and leaves it at a point of the rim of the
box so that the flow of current is radial and lies between the poles
of the magnet.   The reaction between the magnetic field and the
current makes the disc and mercury revolve under a driving torque
proportional to the current.   The magnetic field develops by in-
duction eddy currents in the revolving disc and mercury of strength
proportional to the velocity of revolution ω.   The reaction be-
tween these and the magnetic field gives rise to a retarding torque
on the disc proportional to the strength of the eddy currents and so
proportional to ω.   When the revolving system attains a constant
speed of rotation the driving and retarding torques are equal, i.e.

$$K_1 I = K_2 \omega,$$

or $I = K\omega$,   . . . (10·13)

where K_1, K_2 and K are constants.

From eq. (10·13)

$$I \times \text{time} = K\omega \times \text{time},$$

or quantity of electricity consumed $= K' \times$ number of revolutions of the disc in time t.   . . . (10·14)

The rotation of the disc is transmitted by a system of gears and wheels to a counting mechanism which is calibrated for a given supply voltage to read directly the energy consumed in B.O.T. units.

Motor type meters are not exactly correct under all circumstances and a certain percentage of error is tolerated.

EXAMPLE.

For a meter 4800 revolutions of the disc correspond to a reading of 1 B.O.T. unit. On testing it was found that the disc made 60 revolutions in 46″, when a current of 5 amperes at 200 volts was passed through it. What is the error of the meter ?

(I) Calculated energy $= \dfrac{5 \times 200 \times 46}{60 \times 60 \times 1000} = 23/1800$ B.O.T.

(II) Meter reading $= \dfrac{60 \times 1}{4800}$ $= 1/80$ B.O.T.

Diff. $(I - II) = 1/3600$, or the meter reads $2·17\%$ too low.

Evidently if a quantity meter is adjusted for a voltage V then when used on a line of supply of V′ volts the readings must be multiplied by $\dfrac{V'}{V}$ in order to get the readings in true B.O.T. units.

DIRECT CURRENT MEASUREMENTS

1. Wheatstone's bridge. One of the most convenient and accurate methods of determining an unknown resistance is the special arrangement of resistances called the Wheatstone's bridge or net, (fig. 8·4), the principle of which was discussed in Art. 7, Ch. VIII. The resistances P and Q form the ratio arms, R is a variable resistance and S the resistance to be determined. The condition of balance (zero deflexion of the galvanometer) is

$$P/Q = R/S,$$

from which S is readily calculated when P/Q and R are known.

The Wheatstone's bridge is obtained in various forms of which two most commonly available ones are the post office box and the slide wire or metre bridge.

2. Post office box. This is a compact form of Wheatstone's bridge. There is quite a large variety of post office boxes differing more or less in details · and outside form but essentially all of them are the same. A common

Fig. 11·1 (a)

form of this box is represented diagrammatically in fig. 11·1 (a) while the formation of corresponding Wheatstone's net is represented in fig. 11·1 (b). The ratio arms AB, BC have each three coils of resistance 10, 100, 1000 ohms 'respectively, placed symmetrically.

10 E

The variable arm AD consists of a number of suitable resis-
tance coils arranged in a
series; the smallest of these
is 1 ohm and the largest 5000
ohms. The total resistance of
all the coils in AD is 11110
Ohms so that the resis-
tance of AD can be varied
from this maximum value
down to 1 ohm. All the
resistances are wound non-
inductively and are made of
wire (generally manganin)
having low temperature
coefficient of resistance. All
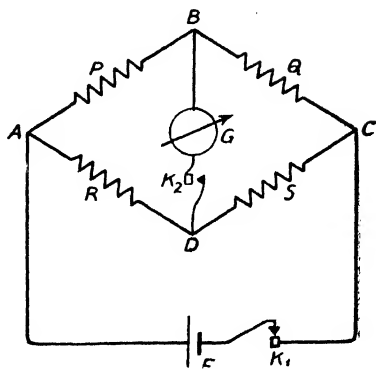
Fig. 11·1(b)

the coils are rated to carry a certain maximum current which should
on no account be exceeded, otherwise the coils will either be burnt
off or their values permanently altered so as to render the
measurements unreliable. The unknown resistance S, the cell E
and the galvanometer G are connected as shown in the figure;
the cell and the galvanometer may, however, exchange their
places, if so desired, to obtain better sensitivity of balance. K_1
and K_2 are two tapping keys introduced in the battery and galvano-
meter circuits. Care should be taken to depress the battery key
first in order to obtain steady current distribution in the bridge.

To measure S, first put P = 10, 100 or 1000 = Q so that P/Q = 1.
Press the battery key first, then the galvanometer key for a short
time and note the direction of deflection for R = 0. Next repeat
with R = ∞ and note the direction of deflection again. If the
connections are all right the directions of the two deflections will be
opposite. Now re-insert the infinity plug and increase R in steps
gradually till two values are obtained for which the galvanometer
deflexions are in opposite directions. The desired resistance lies
between these two limits, say 23 and 24 ohms.

The ratio P/Q is now made 10 and the above process is re-
peated to obtain two limits, say 237 and 238 ohms. The desired
resistance lies between 23·7 and 23·8 ohms. To measure S up to

second decimal place alter P/Q to 100 and redetermine the limits as before. Suppose they are 2375 and 2376. The resistance S then lies between 23·75 and 23·76 ohms. To obtain a closer value of S proceed as follows : suppose that for R=2375 ohms the deflection to the right is 10 divisions and that for R=2376 ohms it is 15 divisions to the left. Thus, for zero deflection

$$R = 2375 + \frac{10}{15 + 10} = 2375\cdot4 \text{ ohms and so } S = 23\cdot754 \text{ ohms.}$$

If S is greater than the total resistance of all the coils in the arm AD it is impossible to obtain any balance unless P/Q is made $\frac{1}{10}$ or $\frac{1}{100}$. Similarly if S is less than 1 ohm no balance is obtained unless P/Q is made 10 or 100. The highest resistance which a post office box is capable of measuring is the maximum available resistance in the arm R multiplied by the highest available ratio P/Q. Similarly the lowest resistance which can be measured is the lowest available resistance in the arm R multiplied by the lowest ratio P/Q. The post office box can thus be used to measure a wide range of resistances. For very accurate work the experimenter should always reverse the battery terminals and obtain the balance again; in this manner errors due to thermal effects and contact potential differences are eliminated.

3. Metre Bridge. This is a much simpler form of Wheat-
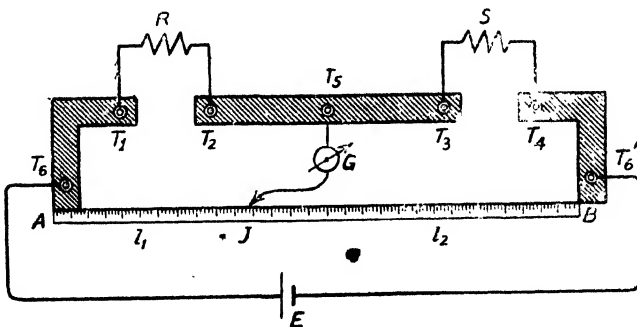
Fig. 11·2

stone's bridge and is shown diagrammatically in fig. 11·2. A

uniform eureka wire AB, 100 cms in length having a total resis-tance of about an ohm, is stretched along a metre scale fixed on a rectangular wooden board. The ends of the wire are soldered to stout copper strips (represented by shaded parts) of negligible resistance. The gaps T_1T_2, T_3T_4 in the strip receive the standard resistance R and the unknown resistance S. The galvanometer is connected between the terminal T_5 and a jockey J sliding over the wire. The cell E is connected between T_6 and T_6'. The balance is obtained by sliding J along the wire. If ρ is the resistance per unit length of the wire and J the position of the jockey corresponding to the balance point, we have

$$P/Q = R/S,$$
$$\text{or } S = R. \, Q/P = l_2/l_1 \, .R.$$

This form of Wheatstone's bridge offers continuous varia-tion of l_2/l_1. Since the error in the ratio l_2/l_1 increases as $(l_2 - l_1)$ increases, l_2 must be nearly equal to l_1 and hence R should be chosen to be nearly equal to S. For accurate measurements the following precautions must also be taken : (1) end corrections at the two junctions A and B must be determined and used in cal-culations and (2) the readings on the wire must be corrected for non-uniformity of cross-section along its length.

4. Kelvin double bridge. A detailed examination of eq. (8·15) shows that if one pair of resistances r_3, r_4 becomes small the current through the galvanometer in a Wheatstone's bridge becomes small. This renders the bridge insensitive for measuring low resistances. Moreover, since the resistance of connecting wires and the resistance at the contacts become comparable to the low resistance to be measured, a large error results in its de-termination.

The errors are avoided by using an arrangement of resistances called Kelvin double bridge shown diagrammatically in fig. 11·3. A galvanometer is connected between M_1 and M_2 the mid-points of two resistances ab and cd, the ends of which are connected to the standard variable low resistance S and the unknown low resistance X. The resistances r_1, r_2, r_3, r_4 are taken to be different to allow for any inaccuracy in bisecting exactly ab and cd at

M_1 and M_2 respectively.

After arranging the resistances in this manner a large current is passed through the bridge and the resistances S and X are adjusted by sliding the contacts until the galvanometer shows no deflection.
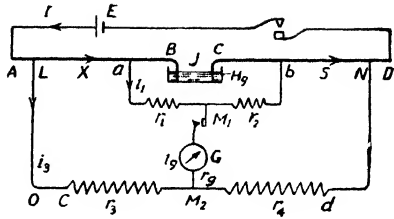
Fig. 11·3 Kelvin double bridge

Let i_1, i_3 and I be the currents in r_1, r_3 and the battery respectively. By applying Kirchhoff's laws we have

$$\left.\begin{array}{l}
\text{current in L}a = \text{I}-i_3 \\
\quad\text{,,}\quad\text{,,}\quad ab = \text{I}-i_3-i_1 \\
\quad\text{,,}\quad\text{,,}\quad r_2 = i_1-i_g \\
\quad\text{,,}\quad\text{,,}\quad r_4 = i_3+i_g \\
\quad\text{,,}\quad\text{,,}\quad b\text{N} = \text{I}-i_3-i_g
\end{array}\right\} \qquad . \ . \ . \quad (11·1)$$

and for the circuit LONEL, $i_3r_3+(i_3+i_g)r_4+\text{I}r_b=e$

for circuit LaM$_2$OL, $(\text{I}-i_3)\ \text{X}+r_1i_1+i_gr_g-r_3i_3=0$

$$\left.\begin{array}{l}
\text{,,}\quad\text{,,}\quad aba, (\text{I}-i_3-i_1)\text{R}-(i_1-i_g)r_2-r_1\,i_1=0 \\
\qquad\qquad \text{where R is the resistance of } ab. \\
\text{,,}\quad\text{,,}\quad \text{M}_1b\,\text{NM}_2\text{M}_1, (i_1-i_g)r_2+(\text{I}-i_1-i_g)\text{S}- \\
\qquad\qquad\qquad\qquad (i_g+i_3)r_4i_gr_g=0
\end{array}\right\} \quad (11·2)$$

From Eqs. (11·1) and (11·2) we can obtain the value of i_g. The numerator of the expression for i_g is

$(r_1+r_2)\ (r_4\ \text{X}-r_3\ \text{S})+\text{R}\ (r_1\text{X}-r_3\text{S})+\text{R}(r_1r_4-r_2r_3)$,

which vanishes when

$$r_4\text{X}-r_3\ \text{S}=r_1r_4-r_2r_3=0,$$

i.e., when $\qquad \dfrac{\text{X}}{\text{S}}=r_3/r_4=r_1/r_2.$ $\qquad$ . . . $\quad$ (11·3)

Eq. (11·3) is the condition of the balance from which X is known if r_1, r_2, r_3, r_4 and S are given.

5. An Ohm-meter. Megger.

Any unknown resistance can be measured by applying Ohm's law ; but this requires

the use of an ammeter and a voltmeter simultaneously. An ohm-meter originally devised by Perry combines the functions of the ammeter and the voltmeter into one single instrument.

The principle of an ohmmeter is similar to that of Siemen's electrodynamometer used for measurement of power. A plane coil M of low resistance and another coil S of high resistance are fixed at right angles to each other, (fig. 11·4). At their common centre a small magnetic needle is
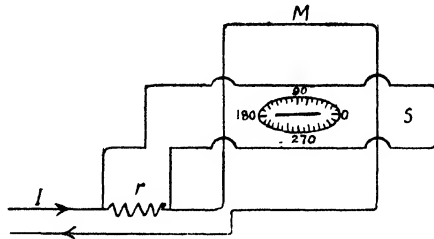
Fig. 11·4 Ohmmeter

arranged free to rotate over a circular scale. The resistance r to be measured is put in series with M and the coil S is put across r. If I is the current flowing through both r and M, and V the voltage across r, the current in the coil S is $V/R = Ir/R$, where R is the resistance of the coil S. The currents in the coils M and S produce magnetic fields perpendicular to each other and if no other field is allowed to affect the needle the latter will set itself in a direction θ with the coil S so that $\tan \theta \propto r$. . The scale of the ohm-meter is calibrated with known values of r, so that any unknown resistance can be read out directly.

A commercial form of ohm-meter constructed by Evershed is much used in testing leakages in electric instal-lations. Fig. 11·5 shows the scheme of connect-ions in Evershed's ohm-meter. For simplicity the magnet and pointer with the scale are not shown in this figure.
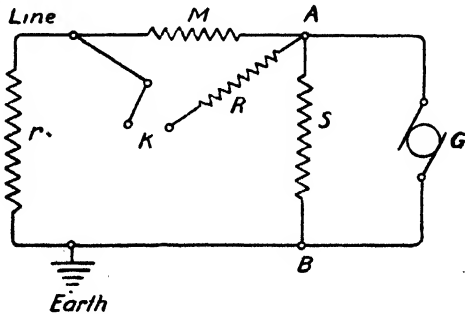
A portable generator G is used to produce the current necessary for

Fig. 11·5

working this instrument. If V is the potential difference across **AB,** I the current through M and r, and r_1 the resistance of the coil M we have $r+r_1=\text{V}/\text{I}$. Since r_1 is constant for a given instrument, the position taken up by the needle for $r=0$ is marked zero and the scale is graduated to read off r directly.

The range of the instrument can be increased by shunting the coil M with a resistance R brought into circuit by closing the key K. The shunt causes diminution of current in the main circuit and a higher reading on the instrument is obtained.

To eliminate errors due to extraneous magnetic fields Evershed introduced a new form of instrument called the "Megger", the principle of which is illustrated in fig. 11·6. C_1 and C_2 are two coils at right angles to each other and are mounted on a common spindle in a uniform magnetic field H
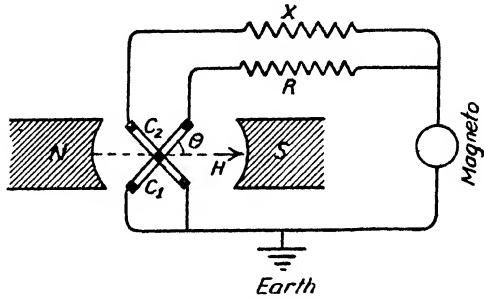
Fig. 11·6 Megger

of a permanent fixed magnet NS. Two ends of the coils are connected to one terminal of the magneto through the resistances **X** and **R** and the other two ends together with the other terminal of the magneto are earthed. The same magnet is used for the magneto also, thus combining in a single instrument the functions of an ohm-meter and a generator.

The potential energy of the two coils is given by

$$\text{W} = -i_1 \text{A} n_1 \text{H} \sin \theta - i_2 \text{A} n_2 \text{H} \sin (\theta + \pi/2),$$

where i_1 and i_2 are the currents in C_1 and C_2, n_1 and n_2 the number of turns in them, A the area of each coil, and θ the deflection of the coil C_1. The couple is given by

$$\text{C} = -\frac{d\text{W}}{d\theta} = i_1 \text{A} n_1 \text{H} \cos \theta + i_2 \text{A} n_2 \text{H} \cos (\theta + \pi/2),$$

$$= \frac{\text{E}}{\text{R}} \text{A} n_1 \text{H} \cos \theta - \frac{\text{E}}{\text{X}} \text{A} n_2 \text{H} \sin \theta,$$

since the e.m.f. for both the coils is the same.

In the position of minimum potential energy $C=0$, i.e.,

$$n_1 \cos \theta / R = n_2 \sin \theta / X,$$

or $\qquad \tan \theta = X/R. \ \ n_1/n_2 = \text{constant} . X. \qquad . \quad . \quad . \quad (11\cdot4)$

In deducing eq. $(11\cdot4)$ it has been assumed that the pivots are frictionless and that there is no restoring couple.

Eq. $(11\cdot4)$ gives the value of the unknown resistance X in terms of θ. The instrument is calibrated to read X directly.

6. Resistance of a galvanometer. Thomson's method.

The connections are made as shown in fig. $11\cdot7$. P and Q are the ratio arms of a post office box. The galvanometer is put in the arm conjugate to P. A short-circuiting key K_1 is placed between B and D. In the arm conjugate to this is placed the cell E through a rheostat W and a key K_2. Press the key K_2 and note the deflection θ of the galvanometer. Keeping K_2 pressed depress K_1. In general a change

Fig. 11·7

in θ will result. Adjust the resistance R so that on releasing or depressing the key K_1 no change in the original deflection is obtained. The resistance R will then be the resistance of the galvanometer.
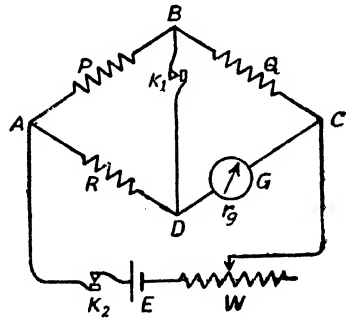
The principle underlying this method is exactly the same as that for determining any unknown resistance by the application of Wheatstone's bridge. When the relation $P/Q = R/r_g$ holds and so the bridge is balanced the conjugate arms BD and AC are independent of each other. That is why when the key K_1 is depressed and released so as to change the resistance of the arm BD from zero to infinity no change in the current distribution in the net-work and hence no change in the deflection of the galvanometer takes place.

7. Resistance of a cell. Mance's method.

A method similar to the foregoing method of Kelvin was introduced by Mance for determining the resistance of a cell. The connections are shown in fig. 11·8. The battery E is now put in the unknown resistance arm CD. A galvanometer is connected between B and D through a rheostat W and a key K_1. The conjugate arm AC consists simply of a short circuiting key K_2.
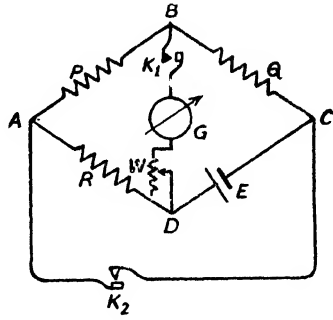
Fig. 11·8

On depressing key K_1 the galvanometer will show some steady deflection θ. Now on depressing K_2, the key K_1 being still closed, some change will be observed. Adjust R so that on releasing or depressing K_2 no change in θ takes place. One of the serious defects of this method is that some current is flowing through the cell all the time the experiment is carried on. The more accurate method of potentiometer will be described later.

8. Carey Foster's bridge. Comparison of two nearly equal resistances.

The diagram of this bridge is shown in fig. 11·9. It is principally a modified metre bridge with two
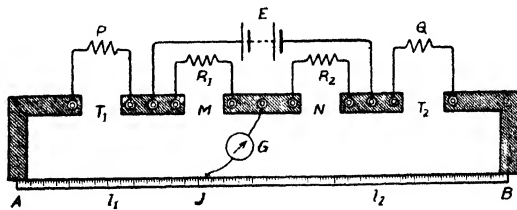
Fig. 11·9

additional gaps M and N to receive two ratio coil resistances R_1 and R_2. The gaps T_1 and T_2 receive two nearly equal resistances P and Q which are to be compared. AB is the slide wire stretched along a metre scale. Let α and β be the effective resistances of the connecting leads, metal strips, soldering, etc. on the sides A and B respectively. Let ρ be the resistance per

unit length of the slide wire. By sliding J let the balance be obtained at J dividing the wire into two portions l_1 and l_2. Then we have

$$\frac{R_1}{R_2} = \frac{P + \alpha + \rho l_1}{Q + \beta + \rho l_2} . \qquad \cdots \quad (11 \cdot 5)$$

Now if P and Q exchange their places, so that l_1' and l_2' are the new lengths for the balance, we have

$$\frac{R_1}{R_2} = \frac{Q + \alpha + \rho l_1'}{P + \beta + \rho l_2'} . \qquad \cdots \quad (11 \cdot 6)$$

From eqs. (11·5) and (11·6), by adding unity to each, we get

$$\frac{P + Q + \alpha + \beta + \rho (l_1 + l_2)}{Q + \beta + \rho l_2} = \frac{P + Q + \alpha + \beta + \rho (l_1' + l_2')}{P + \beta + \rho l_2'} . \qquad (11 \cdot 7)$$

Since $(l_1 + l_2) = (l_1' + l_2')$, the numerators of eq. (11·7) are equal and hence the denominators are also equal. Thus,

$$P + \beta + \rho l_2' = Q + \beta + \rho l_2$$

or $\qquad\qquad P - Q = \rho \ (l_2 - l'_2) = \rho(l_1' - l_1) \quad \cdots \quad (11 \cdot 8)$

From eq. (11·8) we see that the difference between two nearly equal resistances is equal to the resistance of the wire of length equal to the distance between the balance points before and after exchange. It should be noted that the result does not depend upon the individual values of R_1 and R_2 but only upon their constancy. The ratio coils should, therefore, be placed in a constant temperature bath during an experiment.

For accurate work ρ must be determined very carefully at various points over the entire length of the wire to take account of its non-uniformity. This is done by taking two standard resistances P and Q, one of which is shunted by a standard resistance box. By changing the value of the shunt, any value of $P - Q$ can be obtained. Proceeding as above, we get the value of $P - Q$ in terms of portions of the wire at various points along the wire. A calibration curve may then be drawn and used in all determinations with the bridge.

There are various convenient commercial forms of Carey Foster's bridge differing in constructional details only.

9. Temperature coefficient of platinum. It is determined by using a platinum thermometer which consists of a spiral of pure platinum wire wound on a mica frame and inserted in a silica or porcelain tube. The leads which are connected to the two terminals of the platinum spiral have got some resistance. A loop of wire, exactly identical to these leads in all respects, is enclosed in the tube and brought to two binding terminals. These are called the compensating leads. The experiment starts with the determination of the resistance of the platinum spiral at various temperatures in order to arrive at the law of its variation with temperature. Carey Foster's bridge may be used for this purpose. The platinum thermometer tube is put either in a bath the temperature of which can be controlled for which either a thermostat or some constant temperature baths such as melting wax or boiling water. The platinum leads are connected to the gap T_2 of the bridge while the compensating leads, through a series resistance go, to the gap T_1. By following the procedure described in Art. 8 the resistance of the thermometer is measured at various temperatures and the values are plotted against temperature. It is found that the law of variation of resistance with temperature t on the gas scale is given by

$$R_t = R_0 (1 + \alpha t + \beta t^2). \qquad . \quad . \quad . \quad (11 \cdot 9)$$

Now the temperature on platinum resistance scale is given by

$$T_{pt} = 100 \ (R_t - R_0) / (R_{100} - R_0) . \quad . \quad . \quad . \quad (11 \cdot 10)$$

Substituting the value of R_t from eq. $(11 \cdot 9)$ in eq. $(11 \cdot 10)$ we get

$$T_{pt} = t(\alpha + \beta t) \big/ (\alpha + 100 \beta),$$

$$\text{or} \quad (t - T_{pt}) = \frac{100 \beta}{(\alpha + 100 \beta)} \ t \left(1 - \frac{t}{100} \right),$$

$$= \frac{\triangle t}{100} \left(\frac{t}{100} - 1 \right), \qquad . \quad . \quad . \quad (11 \cdot 11)$$

where $\triangle = - 10^4 \beta / (\alpha + 100 \beta)$.

We may note here that if $\beta = 0,$ i.e., if the law of variation of resistance of platinum were a linear one, $\triangle = 0$; $t - T_{pt}$ would have then become zero or the platinum scale and the gas scale would have been identical.

But $\triangle \neq 0$; for platinum it is of the order of $1\cdot57$. It is determined from eq. $(11\cdot10)$ by finding T_{pt} at the melting point of sulphur $(440^{\circ}C)$ and substituting it in eq. $(11\cdot11)$. At higher temperatures $\triangle$ is determined by using melting points of gold and other metals as fixed points.

10. Callendar and Griffith's bridge. Although Carey

Foster's bridge is easily adapted to the measurement of resistance of a platinum thermometer, a bridge generally used for this purpose is the Callendar and Griffith's bridge. This is another modification of Wheatstone's bridge, the scheme of connections of which is shown diagrammatically in fig. $11\cdot10$. P and Q form the

Fig. $11\cdot10$

ratio arms adjusted to equality.   There is a gap T in the arm A_2B, to which are connected the leads pp of the platinum thermometer. The compensating leads cc are connected to the gap C in the arm A_1A.   AB is the bridge wire stretched along a parallel wire $A'B'$ of the same material.   A crosspiece J, also made of the same material, makes contact between AB and the galvanometer.   A set of resistance coils R having values one, two, four, etc. times the resistance of 20 cms. length of AB are placed in the arm A_1A of the bridge.   AB has usually a resistance of $1/200$ ohm per cm. and hence the various coils have resistances $0\cdot1$, $0\cdot2$, $0\cdot4$,...,$12\cdot8$ ohms.   The balance is obtained by sliding the cross-piece J.

Let R_t be the resistance of the thermometer at any temperature t, ρ the resistance per centimetre of AB, r_1 the resistance of each pair of the leads, l the distance of the cross-arm J from the centre O of the wire AB and R the resistance used for balancing. We have, since $P=Q$ by adjustment,

$$r_1 + R + (P \cdot O\,B + \rho l) = r_1 + Rt + (P \cdot OB - \rho l)$$

or
$$R_t = R + 2\,\rho l. \qquad . \quad . \quad . \quad (11 \cdot 12)$$

The above equation gives the resistance of the thermometer at any temperature. If the variation of resistance is measured over a certain range of temperatures, temperature changes can be read in terms of the displacement of the balance point from the mid point O of the slide wire. For example, if $R_{1000} - R_0 = 5$ ohms and $\rho = 0 \cdot 005$ ohm, l must change by $0 \cdot 05 / 2 \times 0 \cdot 005 = 5$ cms to keep up the balance when the temperature changes by $0 \cdot 1° C$. This means that a shift of one mm. of the balance point corresponds to a change of $1 / 50° C$.

11. Wire potentiometer. Comparison of E. M. Fs.

This is a unique electrical instrument possessing a very wide field of usefulness. Although primarily meant for comparing voltages it can efficiently compare currents and resistances also. There are various types of potentiometers differing from each other in
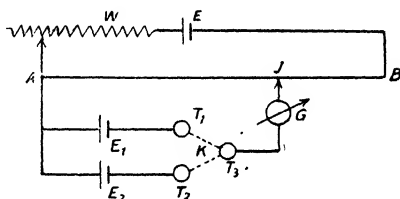
Fig. 11·11

constructional details. The fundamental principle of all of them is, however, the same and is illustrated by the wire potentiometer, (fig 11·11). A long manganin or platinoid wire AB of uniform cross-section is stretched along a scale. A battery E of large constant e.m.f. maintains a constant current through AB. The rheostat W is used to control the current in AB. If the polarity of E is as shown in the figure the potential of A will be higher than that of B. Along the length of the wire, since it is of uniform cross-section, there will be a constant drop of potential per unit length of the wire. The positive terminals of the batteries E_1 and E_2 whose E. M. Fs are to be compared are connected to the positive terminal A of the main cell E. whereas negative terminals are connected to the terminals T_1 and T_2 of a two-way key K, the third terminal T_3 of which is connected through a sensitive galvanometer G to a jockey J sliding along AB. The jockey is so

adjusted that the potential drop due to the battery E exactly equals the E. M. Fs E_1 and E_2 of the batteries E_1 and E_2 in turn. When this is so the galvanometer will show no deflexion. If l_1 and l_2 be the distances of J from A when E_1 and E_2 are balanced and σ the potential drop per unit length of AB, we have

$$E_1 = \sigma l_1, \qquad \qquad \text{. . . (11.13)}$$
$$E_2 = \sigma l_2. \qquad \qquad \text{. . . (11.14)}$$

Dividing the former by the latter we have

$$E_1/E_2 = l_1/l_2 \qquad \qquad \text{. . . (11.15)}$$

Eq. (11.15) gives the e.m.f. of a cell E_1 in terms of that of the cell E_2 and distances l_1 and l_2 on the wire.

The instrument can be made direct reading with the help of a standard cell (e.g. Weston cell) of known e.m.f. The method of calibration will be clear from the following example. Let $E_2 = 1.018$ volts. Suppose that AB contains 200 equal parts each part being divided into 10 equal parts. Put the standard cell in the potentiometer circuit and adjust the jockey J so as to read 101.8. Adjust the rheostat W so that the galvanometer shows no deflection. Then, we have $101.8 \times \sigma = 1.018$ volts or the fall of potential per large scale division $= \sigma = \dfrac{1.018}{101.8} = 0.01$ volt. Now switch the key K on to the cell of which the e.m.f. is to be measured. Without changing W adjust the jockey to obtain a balance. If the scale reading now is 145.6, the unknown e.m.f. is 1.456 volt. It should be noted that the accuracy of measurement depends upon the constancy of the potential gradient in AB.

It is clear, from the foregoing description of the use of the potentiometer arrangement for measuring e.m.f.'s, that no current is drawn from the source, and hence, this instrument is capable of giving the most accurate measurements of voltages in electrical circuits.

12. Measurement of large and small voltages. It should be noted that the voltage to be measured must never exceed that of the supply battery E otherwise no balance point can be obtained anywhere on the wire. This difficulty cannot

be overcome by increasing the voltage of the supply battery since this will cause undue heating of the potentiometer wire and the rheostat W. The standard voltage drop used is of the order of 2 volts per 100 cms. of the wire. When small voltages are to be measured this will be a serious source of error.

Low voltages of the order of few micro or milli-volts can be measured by including a suitable auxiliary resistance in series with the potentiometer wire. The auxiliary resistance is so adjusted that when a 2 volt supply cell is connected across the wire and the auxiliary resistance, the potential drop, across the wire is several milli or micro-volts.

Large voltages can be measured by using what is called a volt-box. This is a device for accurately dividing the given voltage into definite fractions of the total, small enough to fall within the range of the given potentiometer. Its principle is illustrated in fig 11·12. AB is high resistance r. Tappings are made accurately at
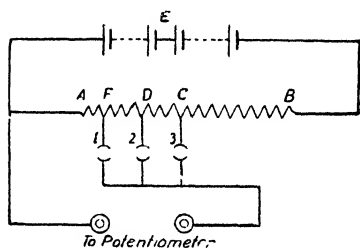
Fig. 11·12

points C, D, F so that the resistance between A and C is $r/2$, that between A and D $r/5$ and that between A and F $r/10$. Evidently the potential drops on these portions, when a certain current is passing through the whole resistance r, will be respectively $\frac{1}{2}$, $\frac{1}{5}$th and $\frac{1}{10}$th of that between A and B. The battery E whose voltage is to be measured is connected to A and B. By putting plugs in gaps 1, 2, 3, we offer to the potentiometer for measurement, voltages which are successively $\frac{1}{10}$th, $\frac{1}{5}$th and $\frac{1}{2}$ the voltage E. The potentiometer when first calibrated with a standard cell will thus give these fractions of the desired voltage. Multiplying the potentiometer readings by the reciprocals of these fractions we get the voltage E. Evidently tappings corresponding to smaller and smaller fractions must be provided in the volt-box if it is to be used for measurements of higher and higher

voltages. Thus to measure 100 volts the volt-box must have a tapping across which there is at most a potential of 2 volts which is supposed to be the range of the potentiometer.

In fig. 11·13 we give another arrangement for such fractional division of the voltage to be measured. The figure is self-explanatory. The potentiometer reading must be multiplied by 10,100 and 200 according as the volt-
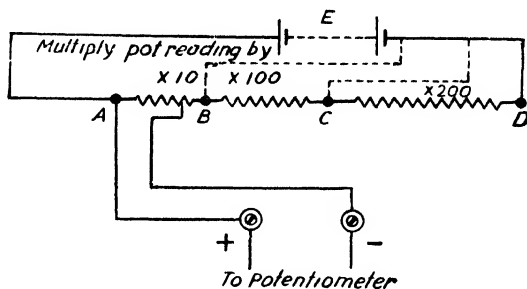
Fig.11·13

age E to be measured is impressed on AB, AC or AD. It is thus seen that with the help of a volt-box the range of a potentiometer can be increased very considerably. This type of volt-box is generally used with Leeds Northrup Student Potentiometer which is a commercial modification of the simple wire potentiometer discussed here. We give below the essentials of the construction of two commercial potentiometers.

13. Crompton potentiometer. This is a compact and accurate instrument enabling rapid and reliable measurements to be made· Its principle is illustrated in fig. 11·14. AB is a straight uniform manganin wire say 100 cms. in
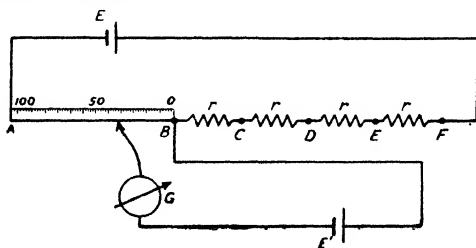
Fig. 11·14

length. BC, CD, DE, EF are resistances joined in series with the wire, each having a resistance equal to the resistance of 100 cms. of the wire. Suppose the balance is obtained at a point corresponding to 30 cms. of the wire when the other contact is at B.

The p.d. will be proportional to 30. It will, however, be proportional to 130, 230, 330 and 430 when the other contact is respectively at C, D, E, and F. The sensitiveness is thus in reased without affecting the range. The scheme of connections in this instrument is shown in fig. 11·15.
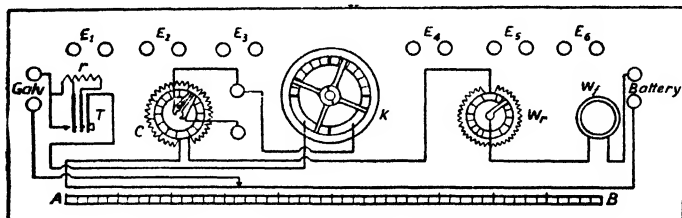
Fig. 11·15 Crompton potentiometer

AB is the balance wire. At C there are 14 equal coils each of which is equal in resistance to AB. They are so arranged that the potential terminal from the source whose voltage is to be measured may be connected to A or to the junction of any two of the 14 coils by means of a rotating arm. The sources whose e.m.f's are to be compared are connected at E_1, E_2, E_3, E , etc. They can be successively brought into the circuit by rotating the switch K. W_r and W_f are two adjustable rheostats for rough and fine adjustment of the main current. T is the tap key which brings the galvanometer into circuit through a resistance r which may be reduced to zero for final adjustment of the balance.

The potentiometer is first calibrated with the help of a standard cell, say Weston cell (1·0183 volts), one terminal of which is connected to A and the other through the rotating arm to the 10th coil so that 10 coils are switched in. The contact on the wire is set at 18·3. The main current is then varied by W_r and W_f till exact balance is attained. Each division of the potentiometer wire thus measures 0·001 volt or 1 millivolt. The voltage of any source can then be read immediately by connecting it to one of the pairs of terminals E_1, E_2, etc., and balancing it on the wire. If each large division of the scale is divided into 10 equal parts and

11 E

the balance is read up to half of the smallest division it is easy to measure voltages up to 1/20 millivolt.

By measuring the potential drop across any known standard resistance the current passing through it can be calculated. The potentiometer thus serves the purpose of an accurate ammeter satisfying rigidly the condition that an ideal ammeter should cause no disturbance in the current distribution in a circuit into which it is introduced.

14. Student potentiometer. This potentiometer is made
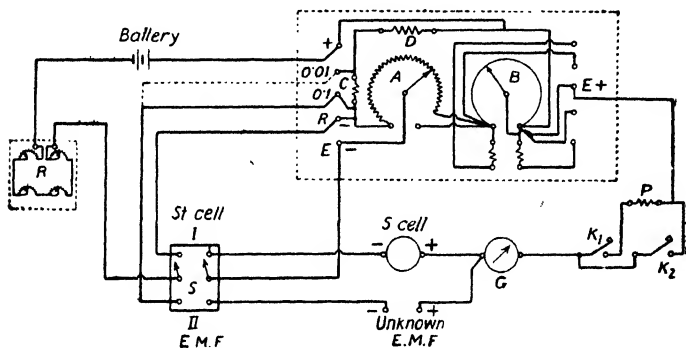
Fig. 11·16—Student potentiometer

by Leeds and Northrup Company of America. Although not a precision instrument of the K type it is excellent training for later work with this or other precision instrument of similar design.

The circuit in student potentiometer for voltage measurement is shown in fig. 11·16. The potentiometer mainly consists of only the calibrated resistances A and B, all accessory apparatus being external. A is made up of fifteen 10-ohm coils and B is a single 10 ohm circular slide wire as shown in the figure . Both are adjustable by dial switches. The resistances A and B are adjusted by the maker to a limit of error within 0·004%. The slide wire is divided into one hundred uniform divisions accurate to within 0·5 division.

The only sliding contacts within the potentiometer are at A and B ; all other contacts are soldered or brazed. The contacts

A and B being in the galvanometer circuit and not in the cali-
brated resistance circuit, any variation in the contact resistances
at A and B causes no error in the measurement.

The potentiometer is first standardized with a standard cad-
mium cell. For this purpose the double-pole double-throw switch
S is closed to the standard cell (position I). The contacts A and B
are then set to correspond to the certified cell voltage. The regulat-
ing rheostat R is then adjusted until the galvanometer G shows
no deflection when the key K is closed. The tapping key K serves
the purpose of obtaining a preliminary rough balance by including
a resistance P of about 10000 ohms in the galvanometer circuit.
The regulating rheostat is a four-dial resistance box containing a total
resistance of 999·9 ohms adjustable in steps of 0·1 ohm.

To find the e.m.f. of a given cell or p.d. between any two
points in a circuit, the switch S is to be thrown to the side marked
e.m.f. (position II). On first tapping the key K_1 a preliminary
rough balance is obtained by adjusting the sliding contacts A and
B. The final balance is then obtained by closing the key K_2. The
e.m.f. or p.d. is then read directly, part upon the dial A and the
remaining part upon the slide wire B.

There are two ranges in the potentiometer, 0 to 16 milli-
volts and 0 to 1·6 volts. The desired range is obtained by con-
necting to the proper binding terminal. The range is changed
by two shunt coils C and D. The resistances of these are so pro-
portioned that when connection is made to the binding terminal
0·1 the current in A and B is 100 times that through C and D. When
the connection is made to the binding terminal 0·01 the coil C
is in series with A and B and the current through this circuit drops
down to 1/100th that through D. The total current of course
remains the same.

15. Internal resistance of a cell. On joining the two
poles of a cell by thick copper rods of negligible resistance a finite
current circulates from one pole to the other through the electro-
lyte. In passing through the electrolyte the current has to over-
come a resistance r which is called the *internal resistance* of the cell.

The internal resistance of a cell can be very easily and accurately measured by a potentiometer. The connections are shown in fig. 11·17. E′ is the main cell and E is the cell of which the internal resistance is to be measured. An external resistance R is connected across this cell through a single-way key K. With the key
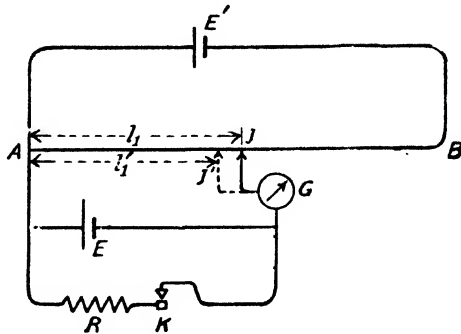
Fig. 11·17

K open, let the balance point be obtained at J distant l_1 from A. Now close K and redetermine the balance. The new balance point will be shifted to J′ towards A since the voltage of the cell drops down to $ER/(R+r)$. We have

$$E = K l_1, \qquad \cdots \quad (11·16)$$

and

$$R/(R+r)\, E = K l_1', \qquad \cdots \quad (11·17)$$

where K is a constant. From eqs. (11·16) and (11·17) we have

$$r(\text{the internal resistance}) = \frac{(l_1 - l_1')}{l_1'} R. \qquad \cdots \quad (11·18)$$

16. Comparison of two resistances. The connections are shown in fig. 11·18. R_1 and R_2 are the resistances which are to be compared. Pass a steady current I through them from a constant voltage supply B through a rheostat W. Balance in succession the potential drops across R_1 and R_2. If l_1 and l_2 are the corresponding lengths, then
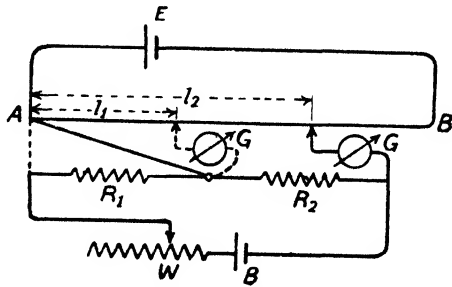
Fig. 11·18

$$IR_1/IR_2 = l_1/l_2 \text{ or } R_1/R_2 = l_1/l_2.$$

17. Determination of low resistance.   The connections
are shown in fig. 11·19.
A high resistance R, of
the order of say 10
ohms, is put in series
with the low resistance
r to be measured.   If
J′ and J are the
balance points corres-
ponding to the poten-
tial   drops   across

Fig. 11·19

$(R + r)$ and R respectively, we have
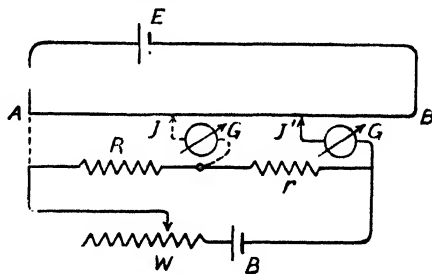
$$(R + r)/R = l_1/l_2,$$
or $$\qquad r = (l_1 - l_2)/l_2 \cdot R. \qquad (11·19)$$

**18. Measurement of current by potentiometer. Cali-
bration of ammeter**.   The
connections   are   shown   in
fig 11·20.   R is a standard
resistance inserted in the cir-
cuit in which the current
I to be measured is flowing.
The potential drop, I R = V,
across R, is measured directly
as usual by a potentiometer,
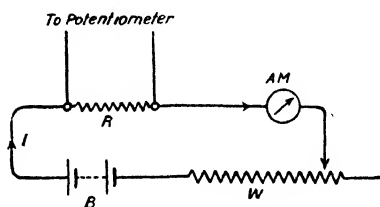
Fig. 11·20

if it is calibrated before by a standard cell.   The current in the
circuit is $I = V/R$.   The supply battery B is arranged to give a
constant voltage of say 10 or 12 volts.   Increase the current in the
circuit gradually by adjusting the rheostat W and measure the
current that produces about one-tenth of the full scale deflection.
Measure the fall of potential across R by the potentiometer.
Measure the drop of potential across R corresoponding to 8 or 10
uniformly distributed readings on the ammeter scale.   The standard
cell balance must be tested at each determination and any defect

in the balance must be corrected by the fine variation rheostat in the main supply circuit of the potentiometer. Note the zero reading of the ammeter before and after starting its calibration. Plot the difference between the ammeter readings and the true amperes as obtained by the potentiometer against the former. Join the successive points by straight lines. From this graph correction to any reading of the ammeter is at once obtained.

19. To calibrate a voltmeter. The scheme of connections is shown in fig. 11·21. PQ is a high resistance carrying a steady current from the accumulator supply battery B. O is a sliding contact for varying the voltage put across
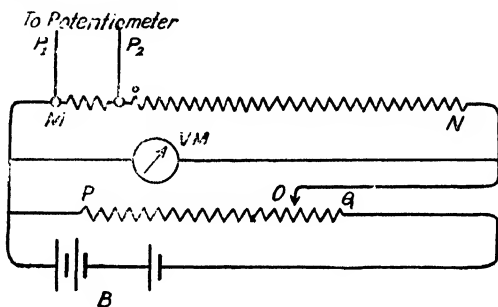
Fig. 11·21

the voltmeter VM and the voltbox MN. The terminals P_1 and P_2 from the volt box are connected to the potentiometer. The voltage of B should be high enough to produce full scale deflection of the voltmeter. First calibrate the potentiometer with the standard cell. Set the sliding contact for say 8 to 10 uniformly distributed readings of the voltmeter and note the corresponding readings of the potentiometer. Care should be taken to choose a suitable ratio on the volt box.

The correction curve for the voltmeter is then plotted in the manner described in the previous experiment.

20. Reduction factor. The scheme of connections for determining the reduction factor of a Helmholtz galvanometer is shown in fig. 11·22.

Adjustments. The adjustments are made in the following order.

(1)   The   coils must be in the vertical plane. Set up a plumb
line close   to   the
face of a   coil, and
adjust the levelling
screws   till   the face
of the coil becomes
parallel   to   the
plumb line.
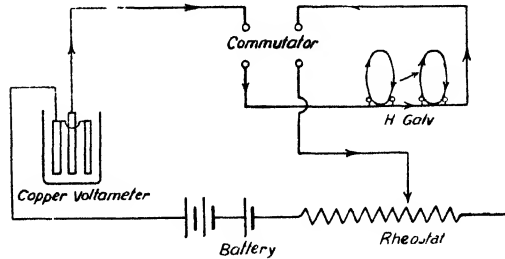
(2)   The   centre
of the needle must
be   at   the   middle
point of the line joining the centres of the two coils.   Raise
and lower the little box containing the magnetic needle so that its
centre is brought as accurately as possible on the line joining the
centres of the two coils.   It is assumed that it lies mid-way between
the two coils.   A little departure from this position is of no par-
ticular consequence since the field is uniform over a finite region
surrounding this point.

(3) The coils should lie in the magnetic meridian.   This
is an important adjustment.   Pass a suitable current through
the galvanometer and note the deflection. Rotate the coil so that
on reversing the current by the commutator key K the deflect-
ions on the two sides are the same.   To avoid   errors   due to
eccentricity of the scale the positions of both the ends of the pointer
 must be read.   The galvanometer coils should then be clamped
in position.   The proof of the above adjustment is as follows :

NS and N′ S′ (fig. 11·23) are the original and the deflected
positions of the needle.   The coil rotates about the vertical
axis through O.   Let θ and θ' be the deflections of the needle
before and after reversing the current and α the angle be-
tween the plane of the coil and the magnetic meridian.   The
forces acting on the needle are shown in the figure.   Taking
moments about O we have

$mHl \sin \theta = mFl \cos (\theta + \alpha)$ and $mHl \sin \theta' = mFl \cos (\theta' - \alpha)$,
where l is the half-length of the needle.

Thus, if $\theta = \theta'$, $\alpha = 0$, i.e., when the deflections of the needle before and after reversing the current in the coil are the same the plane of the coil lies in the magnetic meridian.

Having adjusted the galvanometer in this manner a test copper plate is put in the copper voltameter and the current is adjusted so that the deflexion is 45°. Replace the test plate by a clean, washed and accurately weighed copper plate. Pass the current for about 20′ in one direction and for another 20′ in the reverse direction.
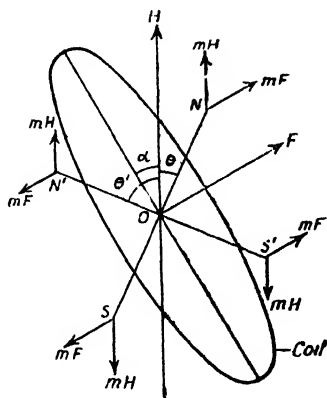
Fig. 11·23

If m is the mass of copper deposited, t the time for which the current is passed, z the electrochemical equivalent of copper (0·0003294), we have

$$I = m/Zt = K \tan 45°,$$

or
$$K = m/Zt \text{ ampere.}$$

21. Figure of merit or current sensitivity of a reflecting galvanometer.

The figure of merit or the current sensitivity of a galvanometer is the current in amperes required to produce a deflection of one mm. on a scale placed normally at a distance of one metre from the mirror of the galvanometer. When once this is known for the given galvanometer it can be used for measuring currents very accurately.
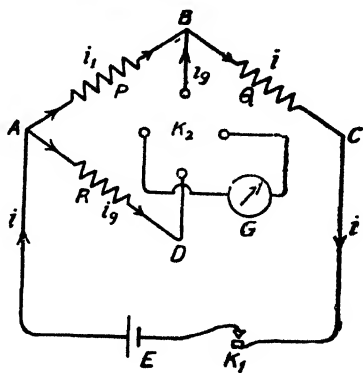
Fig. 11·24

current in the solenoid is 1 amp. ; find the potential difference in volts between the centre and the circumference of the disc.

The field H inside the solenoid is

$$H = 4\pi n i/10 = 4.\pi \times 50 \times \tfrac{1}{10} \text{ gauss}$$

where n is the number of turns per cm. and i the current in amps. The flux through the disc = area of disc $\times 4\pi \times 5$,

$$= \pi \times 8 \times 8 \times 4\pi \times 5.$$

Hence the e.m.f. generated is

$$= \pi \times 8 \times 8 \times 4\pi \times 5 \times 600/60,$$
$$= 1\cdot 26 \times 10^5 \text{ e. m. u.}$$
$$= 1\cdot 26 \times 10^5/10^8 \text{ volt.}$$
$$= 1\cdot 26 \times 10^{-3} \text{ volt.}$$

2. Elementary proof of $e = -\dfrac{dN}{dt}$. Consider a circuit as shown in fig. 12·2. The wire AB can slide over two parallel rails at a distance l apart. Let I steady current a m.u. flow in the circuit. A magnetic field of strength H is applied perpendicular to the plane of the diagram and directed into it.
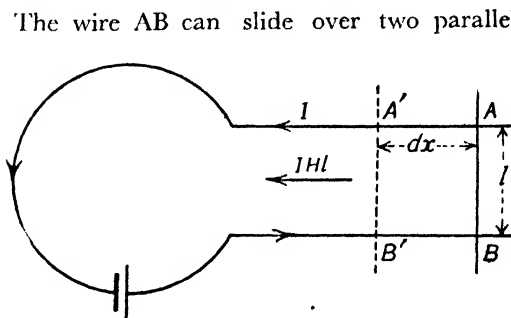
Fig. 12·2

The mechanical force acting on AB is IHl, whose direction is shown by the arrow (use Fleming's left-hand rule). Let the wire move a distance dx in time dt. The work done by the mechanical force is IH$l\ dx$, i.e.IdN, since ldx is the area swept in time dt.

Let e (e. m. u.) be the induced e. m. f., then the work done by the induced e.m.f. is $eIdt$ ergs. Hence

$$IdN = -eIdt,$$
$$e = -\frac{dN}{dt}.$$

3. Fleming's right-hand rule. The direction of the in-
duced e.m.f. can be found out by applying
Fleming's right-hand rule. Stretch the
thumb, middle finger and forefinger of
the right hand mutually at right angles to
each other. If the thumb indicates the
direction of motion of the conductor, the
forefinger the direction of the field, then the
middle finger denoted the direction of
the induced current, (fig. 12·3).
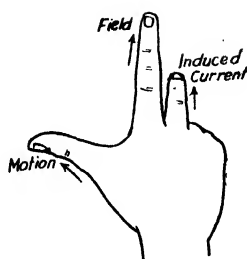
Fig 12·3 Fleming's right
hand rule

It can be seen that by reversing the direction of motion of the
conductor or that of the field the direction of the induced e.m.f.
is also reversed. If, however, the direction of both the field and
the motion is reversed, the induced e.m.f. remains unchanged
in direction.

**4. Mechanism of electromagnetic induction on the
electron theory**. Consider a metallic block AB placed in a
magnetic field H, directed away from the reader and perpendicular
to the plane of the paper. Suppose the block moves in the direct-
ion as shown in fig. 12·4.
When the conductor moves
in the magnetic field the
electrons and the positive-
ions are acted upon by a
force at right angles to the
field and to the direction of
motion. Under the action of
this impressed force the elec-
trons being mobile move
whereas the heavy positive

Fig. 12·4

ions remain stationary at the lattice points. The movement of the elec-
trons causes their deficit on one side and an increase on the other side
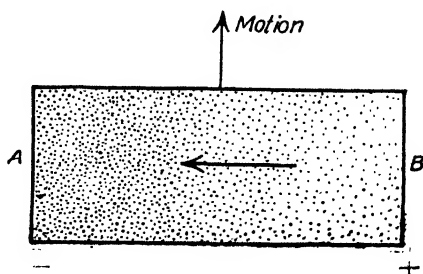
7. Self-inductance of a long solenoid. Consider a solenoid

AB of length l having radius a and n turns per cm. fig. 12·6. The magnetic field H at a point P on the axis of the solenoid is given by
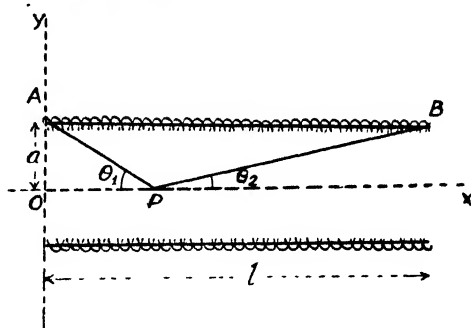
Fig. 12·6

$$H = 2\pi n i \,(\cos\theta_1 + \cos\theta_2) \text{ from eq. } (9\cdot12), \quad . \quad . \quad . \quad (12\cdot4)$$

$$= 2\pi n i \left(\frac{x}{\sqrt{x^2+a^2}} + \frac{l-x}{\sqrt{(l-x)^2+a^2}} \right),$$

where x is the distance of P from the origin O.

The magnetic flux through ndx turns in length dx is given by
$$dN = \pi a^2 \mu . H n dx,$$
where μ is the permeability of the material over which the solenoid is wound.

The total flux is, therefore,

$$N = 2\pi\mu n^2 a^2 i \left[\int_0^l \frac{x\,dx}{\sqrt{x^2+a^2}} + \int_0^l \frac{l-x}{\sqrt{(l-x)^2+a^2}} dx. \right],$$

$$= 4\pi^2 \mu n^2 a^2 i \left[\sqrt{l^2+a^2} - a \right]. \qquad . \quad . \quad . \quad (12\cdot5)$$

From the definition of self-inductance we have

$$L = 4\pi^2 \mu n^2 a^2 \left[\sqrt{l^2+a^2} - a \right]. \qquad . \quad . \quad . \quad (12\cdot6)$$

In the case of a long solenoid a can be neglected in comparison to l; therefore, eq. (12·6) reduces to

$$L = 4\pi^2 \mu n^2 a^2 l. \qquad . \quad . \quad . \quad (12\cdot7)$$

8. Mutual inductance of two solenoids.

Let the primary be a long solenoid of n_1 turns per unit length and of cross section A, in which a current i flows. Let the secondary be a small solenoid wound at the centre of the primary. The secondary has n_2 turns in all.

12 E

The field H along the axis of the primary is
$$H = 4\pi n_1 i.$$

The flux N through each turn of the secondary is
$$N = \mu HA = 4\pi n_1 \mu A i. \qquad \ldots \quad (12\cdot8)$$

Since there are n_2 turns in the secondary the total flux through it is $n_2 4\pi n_1 A\mu i$.

Hence , the mutual inductance is
$$M = 4\pi n_1 n_2 A\mu. \qquad \ldots \quad (12\cdot9)$$

9. Mutual inductance of two coils. Let i be the current flowing in the same direction in two coils of radii r_1 and r_2 and separated by a distance x. The strength of the magnetic field H due to the coil r_1 at the centre of r_2 is given by
$$H = \frac{2\pi i r_1^2}{(r_1^2 + x^2)^{3/2}}. \qquad \ldots \quad (12\cdot10)$$

If the coil r_2 is much smaller than the coil r_1, H can be taken to be uniform over the entire cross-section of r_2. Then the flux through r_2 is
$$N = \frac{2\pi i r_1^2}{(r_1^2 + x^2)^{3/2}} \pi r_2^2 \sin\theta, \qquad \ldots \quad (12\cdot11)$$

where θ is the angle which the plane of the coil r_2 makes with H. From the definition of mutual inductance, M will be given by
$$M = \frac{2\pi^2 r_1^2 r_2^2}{(r_1^2 + x^2)^{3/2}} \sin\theta. \qquad \ldots \quad (12\cdot12)$$

For $\theta = 0$, the two coils being perpendicular,
$$M = 0. \checkmark$$

For $\theta = \pi/2$ the two coils being parallel,
$$M = \frac{2\pi^2 r_1^2 r_2^2}{(r_1^2 + x^2)^{3/2}}, \checkmark \qquad \ldots \quad (12\cdot13)$$

i.e., there is a maximum coupling when the coils are parallel.

For two concentric coils eq. (12·13) reduces to
$$M = \frac{2\pi^2 r_2^2}{r_1}. \qquad \ldots \quad (12\cdot14)$$

10. The earth inductor. It consists of a coil of wire so mounted that it can be rotated in any orientation in the earth's mag-

netic field (fig. 12·7). The two free ends of the coil are connected to two terminals T_1 and T_2. They are sometimes also joined to a commutator. Let the area of the coil be A and n the number of turns in it At any instant t, let the plane of the coil make an angle θ with the earth's horizontal field H. The flux N through the coil in this position is
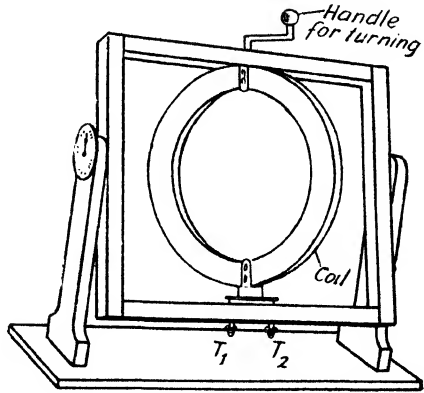
Fig. 12·7 The earth inductor

$$N = AH_t \sin \theta. \qquad \ldots \quad (12·15)$$

The induced e.m.f. in the coil is

$$e = -\frac{dN}{dt} = -AHn \frac{d}{dt}(\sin \theta).$$

If R denotes the resistance of the coil and the ballistic galvanometer connected with it in series, then the current i through the circuit will be given by

$$\left. \begin{array}{l} i = \dfrac{e}{R} = -\dfrac{AHn}{R}\dfrac{d}{dt}(\sin \theta), \\[2mm] \text{or} \quad \displaystyle\int i\,dt = -\dfrac{AHn}{R}\displaystyle\int d(\sin \theta), \end{array} \right\} \qquad (12·16)$$

if the inductor coil is placed initially at right angles to the magnetic meridian, i.e., at $t=0$, $\theta=\pi/2$ and during the time t it has been rotated through 180°, i.e., the plane of the coil is again at right angles to the magnetic meridian, i.e., at $t=t$, $\theta=-\pi/2$, we have

$$\int_0^t i\,dt = -\frac{AHn}{R}\int_{\pi/2}^{-\pi/2} d(\sin \theta). \qquad \ldots \quad (12·17)$$

The left-hand side of the above equation represents the total charge Q that has flown through the galvanometer in time t.
Therefore,

$$Q = \frac{2AHn}{R}, \qquad \ldots \quad (12·18)$$

or $\dfrac{2\mathrm{AH}n}{\mathrm{R}} = Q_s \theta \left(1 + \dfrac{\lambda}{2}\right)$, where θ is the throw of the galvano-meter and Q_s its quantity sensitivity.

Hence $\qquad \mathrm{H} = \dfrac{\mathrm{R}}{2\mathrm{A}n} \, \mathrm{C}_s \dfrac{\mathrm{T}}{2\pi} \, \theta \left(1 + \dfrac{\lambda}{2}\right), \quad \cdot \quad \cdot \quad \cdot \quad (12\cdot19)$

where λ is the logarithmic decrement, T the period of the galvanometer and C_s its current sensitivity (see Ch. XIII). Eq. $(12\cdot19)$ gives the intensity of earth's horizontal field H.

The vertical intensity of the earth's field can be known by rotating the coil with its axis horizontal and in the magnetic meridian. If in this position ϕ is the throw of the galvanometer the vertical intensity V is given by

$$\mathrm{V} = \dfrac{\mathrm{R}}{2\mathrm{A}n} \, \mathrm{C}_s \, \dfrac{\mathrm{T}}{2\pi} \, \phi \left(1 + \dfrac{\lambda}{2}\right). \qquad \cdot \quad \cdot \quad \cdot \quad (12\cdot20)$$

Hence $\qquad \tan(\mathrm{dip}) = \dfrac{\mathrm{V}}{\mathrm{H}} = \dfrac{\phi}{\theta}. \qquad \cdot \quad \cdot \quad \cdot \quad (12\cdot21)$

11. Growth and decay of currents. Consider a circuit containing a self-inductance L, a resistance R and a battery E (fig. 12·8 a). Just when the circuit is completed the current in it is zero. Eventually the current attains a
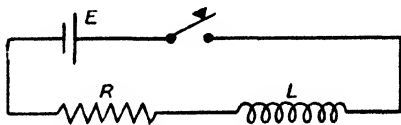
Fig. 12·8 (a)

final steady value I_0 given by Ohm's law $\mathrm{I}_0 = \mathrm{E}/\mathrm{R}$. Our problem is to investigate the behaviour of the current at any time t after the completion of the circuit and before it attains the steady value I_0. As the current I rises from zero to some finite value in time t the back e.m.f. developed in the circuit is $\mathrm{L}\dfrac{d\mathrm{I}}{dt}$. Therefore the effective e.m.f. which drives the current in the circuit is given by

$$\mathrm{RI} = \mathrm{E} - \mathrm{L}\,\dfrac{d\mathrm{I}}{dt}. \qquad \cdot \quad \cdot \quad \cdot \quad (12\cdot22)$$

Rearranging, we have

$$\dfrac{\mathrm{L}d\mathrm{I}}{\mathrm{E} - \mathrm{I}\mathrm{R}} = dt.$$

Integrating,

$$-\frac{L}{R}\log_e (E-IR) = t + \text{a const.} \quad . \quad . \quad . \quad (12.23)$$

When $t=0$, $I=0$, and so the constant is equal to $-\dfrac{L}{R}\log E$. We have then,

$$\log_e \frac{E-IR}{E} = -\frac{R}{L}t,$$

or

$$I = \frac{E}{R}\Big(1 - e^{-Rt/L}\Big).$$

If we put $\dfrac{E}{R} = I_0$, the final steady value of the current, then

$$I = I_0\Big(1 - e^{-Rt/L}\Big). \quad . \quad . \quad . \quad (12.24)$$

Eq. (12.24) shows that when $t \to \infty$, $I \to I_0$, i.e., the current never attains the value I_0 but only approaches it asymptotically.

From eq. (12.24) by differentiating we have

$$\frac{dI}{dt} = I_0 \,\frac{R}{L}\cdot e^{-Rt/L},$$

which shows that the rate of increase of current does not depend upon the separate values of R and L but on the ratio R/L. The ratio L/R is called the *time constant* of the circuit and is generally denoted by λ.

Therefore, $$I = I_0\Big(1 - e^{-t/\lambda}\Big). \quad . \quad . \quad . \quad (12.25)$$

We now see that for circuits whose λ is small the current takes much smaller time to approach the final steady value I_0 than for those which have large λ. Thus for circuits with very small inductance L the current takes only a very small fraction of a second to approach the value I_0. In fig. 12·8(b) the growth of the current for two differen values of λ is illustrated.

Decay of currents·
If after the current
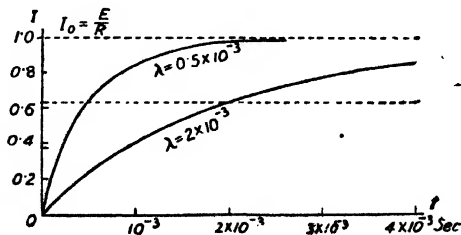
Fig. 12·8 (b) Growth of current.

has attained the final steady value the source of e.m.f. in the circuit is disconnected, eq. (12·22) reduces to

$$L\frac{dI}{dt} = -IR, \qquad \ldots \quad (12·26)$$

or

$$\frac{dI}{I} = -\frac{R}{L}\,dt.$$

Integrating, we have

$$\log_e I = -\frac{R}{L}t + a \text{ const.}$$

When $t=0$, $I=I_0$; therefore, the value of the constant is $\log_e I_0$. We then have

$$I = I_0 e^{-\frac{R}{L}t},$$

or

$$I = I_0 e^{-t/\lambda}. \qquad (12·27)$$

This equation also shows that the decay of current will be slow for large values of λ. The decay curves are given in fig. 12·8 (c). As the time taken by the current to reach its final steady value is ordinarily of the order of 10^{-3} seconds, the growth cannot be
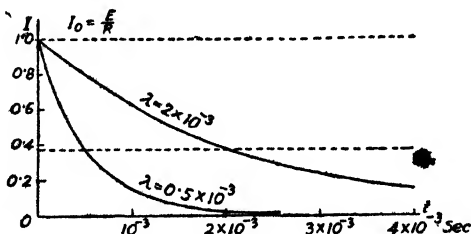
Fig. 12·8 (c) Decay of current

experimentally demonstrated by an ordinary galvanometer. We need for this purpose a special galvanometer such as vibration galvanometer which has very light moving parts and therefore a high frequency of vibration. It is worth noting here that inductance plays a part in electrical circuits similar to that played by mass or inertia in mechanics.

12. Charge and discharge of a condenser. Consider a

circuit containing a capacity C, a resistance R and a source of e.m.f. E (fig. 12 9). The poten-tial difference across the plates of the condenser is Q/C when
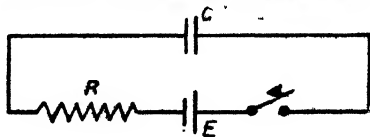
Fig. 12·9

Q is the charge on the plates at any instant t, and that across the resistance is RI. Hence

$$E = \frac{Q}{C} + RI. \qquad \cdots \quad (12 \cdot 28)$$

Eq. $(12 \cdot 28)$ has the same form as eq. $(12 \cdot 22)$. Hence its solution is

$$Q = EC \; (1 - e^{-t/CR}),$$
$$= Q_0 \; (1 - e^{-t/CR}), \qquad \cdots \quad (12 \cdot 29)$$

where Q_0 is the final value of the charge, the initial condition being $t = 0$, $Q = 0$. CR is called the time constant and is denoted by λ and therefore,

$$Q = Q_0 \; (1 - e^{-t/\lambda}). \qquad \cdots \quad (12 \cdot 30)$$

From the above equation we see that as $t \to \infty$, $Q \to Q_0$, *i.e.*, the charge never attains the final steady value Q_0 but approaches it asymptotically. The rate of increase of charge depends upon the product of C and R. The current at any instant in the circuit is given by

$$I = \frac{dQ}{dt} = \frac{Q_0}{CR} \left(1 - e^{-t/\lambda} \right). \qquad \cdots \quad (12 \cdot 31)$$

Discharge.

If now the e.m.f. E is disconnected eq. $(12 \cdot 28)$ becomes

$$\frac{Q}{C} + R\frac{dQ}{dt} = 0. \qquad \cdots \quad (12 \cdot 32)$$

Integrating, we have $Q = e^{-t/CR} + \text{const.}$

When $t = 0$, $Q = Q_0$; and the above equation becomes

$$Q = Q_0 \, e^{-t/CR}. \qquad \cdots \quad (12 \cdot 33)$$

The current during discharge is

$$I = \frac{dQ}{dt} = -\frac{Q_0}{CR} e^{-t/CR}. \qquad \cdots \quad (12 \cdot 34)$$

The curves for charge and discharge of a condenser will be similar to those shown for the growth and decay of currents.

13. Circuit containing inductance, capacity and resistance.

Consider a circuit containing a capacity C, a self-inductance L and a resistance R (fig. 12·10). The equation for the e.m.f. in the circuit is
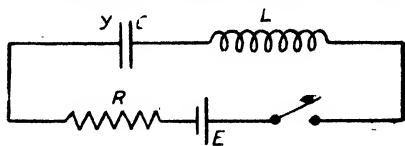
Fig 12·10

$$L\frac{dI}{dt} + RI + \frac{q}{C} = E. \qquad . \quad . \quad (12\cdot35)$$

Let us introduce a new independent variable Q such that

$$Q = q - C E. \qquad . \quad . \quad . \quad (12\cdot36)$$

Eq. (12·35), then, transforms into

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = 0, \qquad . \quad . \quad . \quad (12\cdot37)$$

since E is constant and does not vary with time.

Eq. (12·37) has a solution of the form

$$Q = e^{\lambda t}. \qquad . \quad . \quad . \quad (12\cdot38)$$

Substituting (12·38) in (12·37) we get

$$L\lambda^2 + R\lambda + \frac{1}{C} = 0. \qquad . \quad . \quad . \quad (12\cdot39)$$

The above quadratic equation has two roots of λ which are given by

$$\lambda = -\frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}. \qquad . \quad . \quad . \quad (12\cdot40)$$

Three different cases arise:

Case (1), $\dfrac{R^2}{4L^2} > \dfrac{1}{LC}.$

The solution of eq. (12·37) is

$$Q = e^{-\alpha t}(Ae^{\beta t} + Be^{-\beta t}), \qquad . \quad . \quad . \quad (12\cdot41)$$

where $\alpha = +\dfrac{R}{2L}$ and $\beta = \sqrt{\dfrac{R^2}{4L^2} - \dfrac{1}{LC}}$,

or $q = CE + e^{-\alpha t}(Ae^{\beta t} + Be^{-\beta t}). \qquad . \quad . \quad . \quad (12\cdot42)$

The constants A and B in the above equation are determined by the conditions:

$$q = 0 \quad , \quad t = 0 , \qquad (12\cdot43, a)$$

and $i = \dfrac{dq}{dt} = 0 \quad , \quad t = 0 . \qquad (12\cdot43, b)$

Hence the solution is

$$q = CE \left[1 - \frac{1}{2\beta} e^{-\alpha t} \left\{ (\alpha+\beta)e^{\beta t} - (\alpha-\beta)e^{-\beta t} \right\} \right]. \quad (12.44)$$

In this case the charge gradually acquires its final value q_0.

Case (2), $\dfrac{R^2}{4L^2} = \dfrac{1}{LC}$.

In this case the two roots of the equation are equal. Hence the solution is

$$Q = e^{-\alpha t}(A + B). \quad \ldots \quad (12.45)$$

Applying the initial conditions (12.43 a and b) we have

$$Q = 0,$$

i.e., $$q = CE. \quad \ldots \quad (12.46)$$

Case (3), $\dfrac{R^2}{4L^2} < \dfrac{1}{LC}$.

This is an important case which we shall discuss in detail.

Here $\lambda = -\alpha \pm i\beta$, where $\beta = \sqrt{\dfrac{1}{LC} - \dfrac{R^2}{4L^2}}$.

The solution is, therefore,

$$Q = e^{-\alpha t}. (Ae^{i\beta t} + Be^{-i\beta t}), \quad \ldots \quad (12.47)$$

which can be put in the form

$$Q = e^{-\alpha t}. \gamma \sin (\beta t + \epsilon), \text{ where } \gamma \text{ and } \epsilon \text{ are two new constants.}$$

From eq. (12.36), $q = CE + e^{-\alpha t}. \gamma \sin (\beta t + \epsilon). \quad \ldots \quad (12.48)$

Applying the conditions (12.43) we have

$$\left. \begin{array}{l} 0 = CE + \gamma \sin \epsilon, \\ 0 = \beta \cos \epsilon - \alpha \sin \epsilon, \end{array} \right\}. \quad \ldots \quad (12.49)$$

from which we have

$$\tan \epsilon = \frac{\beta}{\alpha} \text{ and } \gamma = -\frac{CE}{\sin \epsilon} = -CE \frac{\sqrt{\alpha^2 + \beta^2}}{\beta},$$

Hence $$q = CE \left\{ 1 - \frac{\sqrt{\beta^2 + \alpha^2}}{\beta} e^{-\alpha t} \sin (\beta t + \epsilon) \right\} \quad (12.50)$$

From this equation we see that the charge is oscillatory in character. As time goes on the amplitude of the charge goes on diminishing because of the exponential factor $e^{-\alpha t} = e^{-R/2Lt}$., which is known as the *damping factor*. Finally the charge attains the constant value CE.

The oscillatory character of the charge is shown in fig. 12·11 from which we see that at the point B of the curve the charge on the condenser is more than the final steady value CE. Hence, a condenser when connected to an electrical supply will attain a charge nearly
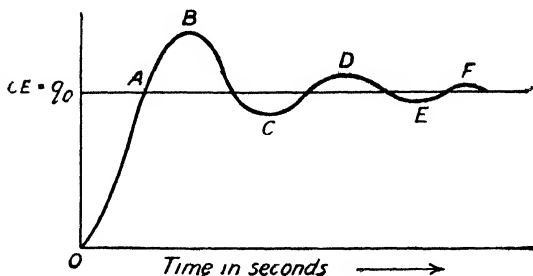
Fig. 12·11

twice the charge CE if the inductance in the circuit is large and the resistance small (since $e^{-R/2Lt} \sim 1$ for large L and small R). This will result in the break down of the dielectric because the potential on the condenser is now nearly double the value it can safely stand. To avoid the puncturing of the condenser a sufficient resistance is initially connected in series to damp the oscillations, and afterwards the resistance is removed.

14. Oscillatory discharge of the condenser. When the condenser has attained the final steady charge $q_0 = CE$ the source of e.m.f. is cut off. The equation for the e.m.f. in the circuit now becomes

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0. \quad . \quad . \quad . \quad (12\cdot51)$$

The method of solution for eq. (12·51) is the same as that used for eq. (12·37). We shall here take up case (3) which is of practical importance.

The solution of eq. (12·51) is

$$q = e^{-\alpha t}(Ae^{i\beta t} + Be^{-i\beta t}),$$
$$= \gamma e^{-\alpha t} \cdot \sin(\beta t + \varepsilon), \quad . \quad . \quad . \quad (12\cdot52)$$

where $\alpha = \dfrac{R}{2L}, \beta = \sqrt{\dfrac{1}{LC} - \dfrac{R^2}{4L^2}}$ and γ and ε constants.

The initial conditions are

$$q = q_0$$
$$i = \left(\frac{dq}{dt}\right) = 0 \quad \text{at } t = 0. \Big\} \quad . \quad . \quad . \quad (12 \cdot 53)$$

These two conditions give

$$\gamma \sin \varepsilon = q_0, \Big\} \quad . \quad . \quad . \quad (12 \cdot 54)$$
$$\text{and} \quad \beta \cos \varepsilon = \alpha \sin \varepsilon, \Big\}$$

from which we have

$$\tan \varepsilon = \frac{\beta}{\alpha} \text{ and } \gamma = q_0 \frac{\sqrt{\beta^2 + \alpha^2}}{\beta}.$$

The solution is, therefore,

$$q = \frac{\sqrt{\beta^2 + \alpha^2}}{\beta} \cdot q_0 \, e^{-\alpha t} \sin (\beta t + \varepsilon). \quad . \quad . \quad . \quad (12 \cdot 55)$$

The charge q as we see from eq. $(12 \cdot 55)$ is oscillatory in character. The frequency of oscillation is given by

$$\nu = \frac{\beta}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}, \quad . \quad . \quad . \quad (12 \cdot 56)$$

and is called the *natural frequency* of the circuit.

The period T is, therefore, given by $2\pi \Big/ \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}.$ \quad (15·57)

If $\dfrac{R^2}{4L^2} << \dfrac{1}{LC}$, the natural frequency is

$$\nu = \frac{1}{2\pi \sqrt{LC}}. \quad . \quad . \quad . \quad (12 \cdot 58)$$

The current in the circuit is

$$i = \frac{dq}{dt} = -\frac{\beta^2 + \alpha^2}{\beta} \cdot q_0 \, e^{-\alpha t} \sin \beta t$$
$$= i_0 \, e^{-\alpha t} \sin \beta t, \quad . \quad . \quad . \quad (12 \cdot 59)$$

where $i_0 = -\dfrac{\alpha^2 + \beta^2}{\beta} q_0$.

From eq. (12·59),

$$\frac{di}{dt} = i_0 e^{-\lambda t} \left\{ -\lambda \sin \beta t + \beta \cos \beta t \right\}.$$

For maxima $\frac{di}{dt} = 0$. Hence

$$\tan \beta t = \beta/\lambda,$$

i.e., $$t_m = \frac{1}{\beta} \tan^{-1} \beta/\lambda, \qquad \cdots \quad (12·60)$$

giving the time for i maximum.

EXAMPLE.—A condenser of capacity $0·25\mu$F is charged up to 100 volts and then discharged through a circuit of inductance 1/10 henry and resistance 100 ohms. Find the frequency of the electromagnetic waves generated and the value of the maximum current in the circuit.

The frequency of the oscillations is given by

$$\nu = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}},$$

$$= \frac{1}{2\pi} \sqrt{\frac{10}{0·25 \times 10^{-6}} - \frac{(100)^3}{4}} = 10^3 \text{ approx.}$$

The maximum value of the current is

$$i_0 = \frac{\beta^2 + \lambda^2}{\beta} q_0,$$

$$= \frac{\frac{1}{LC}}{\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}} CE = \frac{4 \times 10^7}{2 \times 10^3 \sqrt{10}} \times 0·25 \times 10^{-6} \times 100,$$

$$= \frac{1}{2\sqrt{10}} \text{amp. approx.}$$

The wave-length of the e.m. waves is,

$$\lambda = \frac{c}{\nu} = \frac{3 \times 10^{10}}{10^3} \text{ cms.}$$

$$= 300 \text{ kilometres.}$$

15. Induction coil. An induction coil is essentially an open core transformer with an automatic circuit interrupter. The

primary P has a few turns of thick wire, while the secondary S
has a large number
of turns of thin wire
(fig. 12·12 a). Both
are wound on the
same core D built up
of a number of soft
iron wires to minimise
eddy losses. The pri-
mary circuit is com-
pleted through a me-
chanical interrupter
which consists of a
soft iron armature A,
a spring B and a

Fig. 12·12 (a) Induction coil

thumb screw T. A condenser C is connected in parallel with the
primary circuit.

When a current flows in the primary the iron core gets mag-
netized and attracts the armature A. As the armature bends to
the left the primary circuit breaks; the core loses its magnetism
and A is thrown back to the right completing again the primary
circuit. The proper adjustment of the thumb screw makes the
armature vibrate back and forth, thus periodically making
and breaking the circuit. Both at the make and at the break
an induced e.m.f. is generated in the secondary because the flux
through it is changing. But the e.m.f. generated at the break
is greater than that generated at the make since the rate of decay
of current is greater than the rate of its growth. This can be
immediately seen without entering into a detailed calculation.
From Art. 11 the time constant of the primary circuit is L/R, the
magnitude of which is great when the circuit is closed because
R is small but its value is enormously reduced when the circuit
is broken because R is very large, almost infinity at the break.
Consequently, enough voltage is developed to cause a spark
in the secondary only during the break; the voltage during the
make may be insufficient to cause a spark. The secondary current

is, therefore, unidirectional, (fig. 12·12 *b*). During the make
the primary circuit is com-
pleted through the thumb
screw T and the armature
A. When the circuit breaks
the primary current is
forced to follow the path *via*
the condenser. Thus damped

Fig. 12·12 (*b*)

electrical oscillations are set up in the primary circuit, which last
until the current has decayed to the point where the coil has lost
enough of its magnetization. The armature swings to the right
and completes the primary circuit.

16. A simplified theory of the induction coil.

After the
break the equation for the e.m.f. in the primary circuit is

$$L_p \frac{di_p}{dt} + M \frac{di_s}{dt} + \frac{q_p}{C_p} = 0, \qquad \ldots \quad (12·61)$$

neglecting the resistance of the primary. The equation for the
secondary is

$$L_s \frac{di_s}{dt} + M \frac{di_p}{dt} + R_s\, i_s = 0. \qquad \ldots \quad (12·62)$$

Differentiating eq. (12·61) twice we have

$$L_p \frac{d^3 i_p}{dt^3} + M \frac{d^3 i_s}{dt^3} + \frac{1}{C_p} \cdot \frac{di_p}{dt} = 0. \qquad \ldots \quad (12·63)$$

Differentiating eq. (12·62) twice, we have

$$M \frac{d^3 i_p}{dt^3} = -L_s \frac{d^3 i_s}{dt^3} - R_s \cdot \frac{d^2 i_s}{dt^2}. \qquad \ldots \quad (12·64)$$

From eq. (12·63) and (12·64) by eliminating i_p, we have

$$(L_p L_s - M^2) \frac{d^3 i_s}{dt^3} + R_s L_p \frac{d^2 i_s}{dt^2} + \frac{L_s}{C_p} \cdot \frac{di_s}{dt} + \frac{R_s}{C_p} i_s = 0. \qquad (12·65)$$

Eq. (12·64) has a solution of the form $e^{i\gamma t}$. Substituting it in eq.
(12·64) we have

$$a\gamma^3 + b\gamma^2 + c\gamma + d = 0, \qquad \ldots \quad (12·66)$$

where $a = L_p L_s - M^2$, $b = R_s L_p$, $c = \dfrac{L_s}{C_p}$, and $d = \dfrac{R_s}{C_p}$.

The cubic eq. (12·65) has one real negative root and the other two roots are complex conjugate quantities where real part is negative. Let the roots be $-\gamma, -\alpha + i\beta$ and $-\alpha - i\beta$. The solution is

$$i_s = Ae^{-\gamma t} + Be^{-\alpha t} \sin(\beta t - \varepsilon). \quad . \quad . \quad . \quad (12\cdot 67)$$

From eq. (12·67) we see that i_s is the sum of two terms; one an exponentially decreasing term and the other a damped harmonic oscillation. The dotted lines in fig. 12·12 (c) represent these two components and the solid line gives eir sum
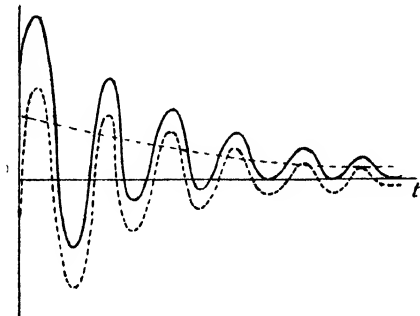
A little insight into eq. (12·67) gives the utility of the condenser in the primary circuit. In the absence of the condenser

Fig. 12·12 (c)

the current in the secondary would have been given by the exponential curve only, but in the presence of the condenser there is an oscillatory part of it as well as the exponential part, the resultant being the solid curve. The maximum of the solid curve is nearly twice the maximum of the exponential curve during the first half of the oscillation. Thus the presence of the condenser in the primary causes double the charge to pass through the secondary. Also the back e.m.f. in the primary due to its own self-inductance would cause a spark to pass between the platinum points, which would rapidly burn them away, but with the condenser there, the self-inductive e.m.f. charges the condenser rapidly, and then the charge surges back again, helping to destroy the self-inductive e.m.f. very quickly.

BALLISTIC GALVANOMETER AND ITS APPLICATIONS

1. Description and theory of ballistic galvanometer.
It consists of either a rectangular or a circular coil of wire, containing many turns and suspended between the pole pieces of permanent strong horse-shoe magnet by means of a phosphor-bronze wire. The suspension carries a small concave mirror. The upper end of the suspension is fixed to a torsion head and the lower end is helical in form. This whole system is enclosed in a case which is provided with levelling screws at its base. The coil surrounds a soft-iron core, the purpose of which is to concentrate the lines of induction in its interior and make the field radial (see figs. 10·5 *a* and 10·6).

A coil carrying a current *i* behaves like a magnetic shell of strength *i*. The potential energy of such a shell in a magnetic field of strength H is

$$W = - niAH \sin \theta, \qquad . \quad . \quad . \quad (13 \cdot 1)$$

where *n* is the number of turns, A the area of the coil and *θ* the angle between the field H and the plane of the coil. The couple C acting on the coil is

$$C = - \frac{dW}{d\theta} = niAH \cos \theta. \qquad . \quad . \quad . \quad (13 \cdot 2)$$

The negative sign is placed to indicate that the couple tends to increase *θ*. This deflecting couple is opposed by the couple due to torsion in the fibre and by the couple due to mechanical and electromagnetic damping. Assuming that there is no electromagnetic damping (i.e., the coil is on an open circuit), the equation of motion of the coil is

$$I \frac{d^2\theta}{dt^2} = niAH \cos\theta - \varkappa\theta - \beta \frac{d\theta}{dt}. \qquad . \quad . \quad . \quad (13 \cdot 3)$$

The second term on the right-hand side of eq. (13·3) gives the couple due to torsion in the fibre since it is proportional to *θ*; and

the third term represents the couple due to air damping since it is proportional to the angular velocity $\frac{d\theta}{dt}$ of the coil.

Since the field is radial, eq. (13·3) becomes

$$\text{I}\,\frac{d^2\theta}{dt^2} = ni\text{AH} - \varkappa\theta - \beta\,\frac{d\theta}{dt}. \qquad \ldots \quad (13\cdot4)$$

Integrating eq. (13·4) between the limits 0 and τ we have,

$$\text{I}\,\frac{d\theta}{dt} = n\text{AH}\int_0^\tau i\,dt - \varkappa\int_0^\tau \theta\,dt - \beta\int_0^\tau \frac{d\theta}{dt}\,dt. \quad \ldots \quad (13\cdot5)$$

The first integral on the right-hand side gives the total charge Q that has flown through the galvanometer during the small interval τ. The remaining two integrals are each equal to zero since we assume that the angle through which the coil has moved in time τ is inappreciable. Therefore eq. (13·5) becomes

$$\text{I}\omega = n\text{AHQ}, \qquad \ldots \quad (13\cdot6)$$

where ω is the angular velocity with which the coil begins to oscillate after the transient current has stopped.

The subsequent motion of the coil is governed by the equation

$$\text{I}\,\frac{d^2\theta}{dt^2} = -\varkappa\theta - \beta\,\frac{d\theta}{dt}, \qquad \ldots \quad (13\cdot7)$$

since the current has now stopped flowing. Putting eq. (13·7) in the standard form

$$\frac{d^2\theta}{dt^2} + 2b\,\frac{d\theta}{dt} + k^2\theta = 0, \qquad \ldots \quad (13\cdot8)$$

where

$$2b = \frac{\beta}{\text{I}}, \text{ and } \frac{\varkappa}{\text{I}} = k^2. \qquad \ldots \quad (13\cdot9)$$

The solution of eq. (13·8) is

$$\theta = \theta_0\,e^{-bt}\sin\left(\omega\,t + \varepsilon\right), \qquad \ldots \quad (13\cdot10)$$

where $\omega_0 = \sqrt{k^2 - b^2}$ and θ_0 and ε are constants.

The constants of motion can be determined by the initial conditions

$$\left.\begin{array}{ll} \theta = 0, & t = 0 \\[2mm] \left(\dfrac{d\theta}{dt}\right)_{t=0} = \dfrac{n\text{AH}}{\text{I}}\,\text{Q}. \end{array}\right\} \qquad \ldots \quad (13\cdot11)$$

and

13 E

Hence the solution is

$$\theta = \theta_0 \, e^{-bt} \sin \omega_0 t, \qquad \cdot \quad \cdot \quad \cdot \quad (13 \cdot 12)$$

where $\theta_0 = n\mathrm{AH} \dfrac{\mathrm{T}}{2\pi} \mathrm{Q}$ and T is the period $\left(\mathrm{T} = \dfrac{2\pi}{\omega_0} \right).$

Eq. (13·12) represents a simple harmonic vibration whose amplitude, however, diminishes exponentially with time. The damped vibration is shown in fig. 13·1. 12·12(c)

The frequency of the damped vibration is

$$\nu = \frac{1}{2\pi} \sqrt{k^2 - b^2} \, ,$$

and that of the undamped vibration

$$\nu_0 = \frac{1}{2\pi} \sqrt{k^2} = \frac{1}{\mathrm{T}_0} . \qquad \cdot \quad \cdot \quad \cdot \quad (13 \cdot 13)$$

The amplitudes of successive throws are

$$\theta_1 = \theta_0 e^{-b\mathrm{T}/4}, \ \theta_2 = \theta_0 e^{-3b\mathrm{T}/4}, \ \theta_3 = \theta_0 e^{-5b\mathrm{T}/4}.$$

The ratio of any two successive throws is $e^{\,b\mathrm{T}/2}$ and is constant. The natural logarithm of this constant ratio is called logarithmic decrement. Denoting it by λ we have,

$$10 \ \lambda = \log_e \frac{\theta_1}{\theta_{11}} . \qquad \cdot \quad \cdot \quad \cdot \quad (13 \cdot 14)$$

The maximum throw θ_0, when there is no damping, is given by

$$\theta_0 = \theta_1 e^{b\mathrm{T}/4} = \theta_1 e^{\lambda/2} = \theta_1 \, (1 + \lambda/2) \text{ approx.}$$

For no damping the equation of motion of the coil is

$$\mathrm{I} \frac{d^2\theta}{dt^2} = - \varkappa\theta. \qquad \cdot \quad \cdot \quad \cdot \quad (13 \cdot 15)$$

Integrating we have,

$$\mathrm{I}\dot{\theta}^2 = - \varkappa\theta^2 + \text{const.}$$

The constant of integration is found by using the initial conditions given by (13·11).

Hence

$$\mathrm{I}\dot{\theta}^2 = \frac{(n\mathrm{AH})^2}{\mathrm{I}} \mathrm{Q}^2 - \varkappa\theta^2. \qquad \cdot \quad \cdot \quad \cdot \quad (13 \cdot 16)$$

If θ_1 is the maximum value of the throw then $\dot{\theta}_1 = 0$ and from eq. (13·16) we have

$$\mathrm{Q} = \frac{\sqrt{\mathrm{I}\varkappa}}{n\mathrm{AH}} \theta_1. \qquad \cdot \quad \cdot \quad \cdot \quad (13 \cdot 17)$$

The period T_0 for the undamped oscillation is $T_0 = 2\pi\sqrt{I/\alpha}$. Eq. (13·17) now becomes

$$Q = \frac{T_0}{2\pi} \cdot \frac{\alpha}{n\mathrm{AH}}\theta_1.$$

The quantity sensitivity Q_s of the galvanometer is defined as

$$Q_s = \frac{Q}{\theta_1} = \frac{T_0}{2\pi} \cdot \frac{\alpha}{n\mathrm{AH}}. \qquad \text{. . .} \quad (13·18)$$

When a constant current i flows in the coil and produces a steady deflection ϕ, in the equilibrium position the restoring couple must be equal to the deflecting couple, i.e.,

$$n\mathrm{AH}i = \alpha\,\phi.$$

The current sensitivity C_s of the instrument is defined by

$$C_s = \frac{i}{\phi} = \frac{\alpha}{n\mathrm{AH}}. \qquad \text{. . .} \quad (13·19)$$

Hence from eqs. (13·18) and (13·19) we have

$$Q_s = C_s \frac{T_0}{2\pi}. \qquad \text{. . .} \quad (13·20)$$

This important relation will be often used.

Galvanometer damping.   In the discussion of the theory of the ballistic galvanometer we have neglected the electromagnetic damping.   If the external circuit connecting the coil is closed, there is an induced e.m.f. in the circuit as the coil oscillates in the magnetic field of the permanent magnet.   The induced e.m.f. will be given by

$$e = -\frac{d\mathrm{N}}{dt} = -\frac{d}{dt}\left(n\mathrm{AH}\sin\theta \right) = -n\mathrm{AH}\cos\theta\,\frac{d\theta}{dt}. \qquad (13·21)$$

If R is the total resistance of the circuit including that of the coil, the induced current i is

$$i = \frac{e}{\mathrm{R}} = \frac{-n\mathrm{AH}}{\mathrm{R}}\cos\theta\,\frac{d\theta}{dt}. \qquad \text{. . .} \quad (13·22)$$

The couple C due to electromagnetic damping is

$$\mathrm{C} = i\,\frac{d\mathrm{N}}{d\theta} = \frac{(n\mathrm{AH})^2}{\mathrm{R}}\cos^2\theta\,\frac{d\theta}{dt}. \qquad \text{. . .} \quad (13·23)$$

The complete equation of motion of the coil is

$$\mathrm{I}\,\frac{d^2\theta}{dt^2} = -\alpha\theta - \beta\,\frac{d\theta}{dt} - \frac{(n\mathrm{AH})^2}{\mathrm{R}}\cos^2\theta\,\frac{d\theta}{dt}. \qquad (13·24)$$

As the field is radial, $\cos^2\theta = 1$; eq. (13·24) becomes

$$\mathrm{I}\,\frac{d^2\theta}{dt^2} + \frac{d\theta}{dt}\left[\beta + \frac{(n\mathrm{AH})^2}{\mathrm{R}}\right] + \alpha\theta = 0. \quad . \quad . \quad (13\cdot25)$$

Writing eq. (13·25) in the standard form we have

$$\ddot{\theta} + 2b\dot{\theta} + k^2\theta = 0, \qquad . \quad . \quad . \quad (13\cdot26)$$

where $\qquad \dfrac{\beta + \dfrac{(n\mathrm{AH})^2}{\mathrm{R}}}{\mathrm{I}} = 2b$ and $\dfrac{\alpha}{\mathrm{I}} = k^2.$

There are three different solutions of eq. (13·26) according as $k^2 - b^2$ is $+ve$, zero, or $-ve$. (as seen in Art 13 Ch. XII).

Case (1). If $k^2 > b^2$ the galvanometer is said to be under-damped and the solution is

$$\theta = \theta_0\,e^{-bt}\sin\,(\omega t + \varepsilon). \qquad . \quad . \quad . \quad (13\cdot27)$$

The logarithmic decrement is

$$\lambda = \tfrac{1}{2}b\mathrm{T} = \frac{\mathrm{T}}{4\mathrm{I}}\left[\beta + \frac{(n\mathrm{AH})^2}{\mathrm{R}}\right], \quad . \quad . \quad . \quad (13\cdot28)$$

where T is now the period on closed circuit.

The free period is $\mathrm{T}_0 = \dfrac{2\pi}{k}$; the relation between the two periods is

$$\mathrm{T}^2 = \mathrm{T}_0^{\,2}\left(1 + \frac{\lambda^2}{\pi^2}\right). \qquad . \quad . \quad . \quad (13\cdot29)$$

It should be clearly understood that T and λ are the period and the logarithmic decrement respectively on closed circuit.

Case (2). Critical damping. The galvanometer is said to be critically damped, when the coil does not oscillate and returns back quickly to its zero position.

For critical damping $\quad \omega = 0.$

Hence $\qquad\qquad\qquad \lambda = \mathrm{T} = \infty.$

The condition for critical damping is

$$k = b,$$

or $\qquad\qquad \beta + \dfrac{(n\mathrm{AH})^2}{\mathrm{R}}\bigg/2\mathrm{I} = \sqrt{\dfrac{\alpha}{\mathrm{I}}}. \qquad . \quad . \quad . \quad (13\cdot30)$

in it. The resultant e.m.f. e in the coil is the difference between the e.m.f. $\dfrac{dN}{dt}$, due to the search coil cutting the flux and $nAH\,\dfrac{d\theta}{dt}$.

Therefore

$$e = \frac{dN}{dt} - nAH\,\frac{d\theta}{dt}\,. \qquad \cdot \quad \cdot \quad \cdot \quad (13\cdot55)$$

From eqs. $(13\cdot54)$ and $(13\cdot55)$ we have,

$$I\,\frac{d\omega}{dt} = \frac{nAH}{r}\left(\frac{dN}{dt} - nAH\,\frac{d\theta}{dt}\right). \qquad \cdot \quad \cdot \quad \cdot \quad (13\cdot56)$$

Integrating the above equation we have

$$I\,\left|\int_{0}^{\tau}\frac{d\omega}{dt}\,dt = \frac{nAH}{r}\,\right\}\,\left|\int_{0}^{\tau}\frac{dN}{dt}\,dt - AHn\int_{0}^{\tau}\frac{d\theta}{dt}\,dt\,\right\|\,.$$

The expression on the left-hand side is zero at both the limits since the coil is at rest both initially and finally. Hence

$$N = nAH\theta, \qquad \cdot \quad \cdot \quad \cdot \quad (13\cdot57)$$

where θ is the difference between the angles at the two instants 0 and τ. Thus the total change of the flux N linked with the search coil is independent of the time during which the change has taken place but depends only on the difference between the initial and the final positions of the pointer.

CHAPTER XIV

MAGNETIC PROPERTIES OF MATERIALS

1. Magnetic induction. Consider a ferromagnetic body placed in a uniform magnetic field H. In this field the body gets magnetized. Let I be the intensity of magnetisation. We shall assume for simplification that I and H have the same direction. The problem is to find the magnetic field inside the material of the body. Imagine that a small cavity is scooped out inside the body and a unit positive pole placed in it, (fig 14·1). According to the definition of magnetic force, the field inside the cavity is the mechanical force on a unit positive pole.

Let the cavity be cylindrical in shape, whose cross-section is α and length l. We suppose that the radius of the cross-section is
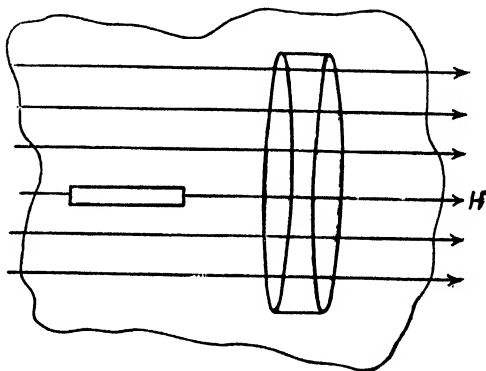
Fig 14·1

very small in comparison to the length of the cavity. Let the axis of the cylindrical cavity be parallel to the intensity of magnetisation. The act of creating a cylindrical cavity produces a free polarity of strength $I\alpha$ and $-I\alpha$ at its two ends (since I is the pole strength per unit area). The force on a unit positive pole at the centre of the cavity would be given by $\dfrac{2I\alpha}{l^2}$ in the direction of I. Hence, the total field inside the body is $B = H + \dfrac{2I\alpha}{l^2}$. But by hypothesis the radius of the cross-section is small in comparison to $2l$, and hence the magnetic field inside the needle-shaped cavity is simply H and is independent of I.

that when one of the ends A is withdrawn a little, the search coil which is attached to a spring can fly out. The whole flux in the bar is cut by the search coil, which can be measured by a ballistic galvanometer. The magnetic field can be known from the relation $H = \dfrac{4\pi n i}{10}$, where i is the magnetizing current in amps and n the mean number of turns per unit length of the bar between the inside surfaces of the yoke. The actual number of turns per cm. of M_1 and M_2 are not to be used because of the small gap in which the search coil lies. For accurate results the value of H must be corrected for (1) the leakage of magnetic flux at the slots A and B and at the cut of the bar and (2) the finite permeability of the yoke.

7. Magnetic circuit and Magneto-motive force.   Consider

a ferromagnetic closed circuit of irregular cross-section as shown in fig. 14·6.   The magnetic flux N $i.e.$, the number of tubes of induction is constant throughout the body of the specimen.
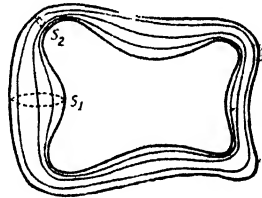
Now $B_1 S_1 = B_2 S_2 = \ldots = N$ (from the definition of B), where B_1 and B_2 are the values of magnetic induction at the cross-

Fig. 14·6

sections S_1 and S_2 respectively.   The work done in carrying a unit positive pole round the tube of induction in the closed circuit is given by $W = \int H dl,$ . . . (14·14)
where H is the magnetic field at any point along the closed path. W is called the *magnetomotive force*, m.m.f. The unit of m.m.f. is the *Gilbert*.   If the work required to carry a unit magnetic pole round a closed magnetic circuit is one erg, the *m.m.f.* is one Gilbert.   From eq. (14·14) we have

$$\text{m.m.f.} = N. \int \frac{dl}{\mu s}, \quad \text{since } B = \mu H \text{ and } Bs = N.$$

$$\text{or} \quad N = \frac{\text{m.m.f.}}{\int \dfrac{dl}{\mu s}}. \qquad . . . (14·15)$$

In an electrical circuit we have an analogous equation

$$\text{current} = \frac{\text{e.m.f.}}{\text{resistance}}. \quad \text{(Ohm's law)} \qquad (14\cdot16)$$

If we compare the equations (14·15) with (14·16) the quantity $\int \frac{dl}{\mu s}$ is analogous to the resistance and we call it as the magnetic resistance or *reluctance* of the circuit. Like electrical resistance the magnetic resistance too depends on the length, area of cross-section and nature of the material. If a magnetic circuit consists of several different parts we have for the whole circuit

$$\int \frac{dl}{\mu s} = \frac{l_1}{\mu_1 s_1} + \frac{l_2}{\mu_2 s_2} + \cdots \qquad \cdots \quad (14\cdot17)$$

The above law is for magnetic resistances in series. The law for magnetic resistances in parallel is similar to the law for electrical resistances in parallel. In fact there is one to one correspondence. It must be remembered, however, that the resemblance exists only in the form of two expressions, since there is no such thing as a magnetic current. The flux N, though analogous to current, is only a statical condition.

8. Anchor ring. Consider an anchor ring magnet of radius r with a small air gap of width d (fig. 14·7 a). Let S be the cross-section of the ring and μ the permeability of the material of the ring. The *m.m.f.* is $4\pi \times$ current linked with the circuit, or
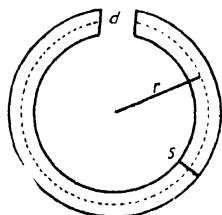
Fig. 14·7 (a) Anchor ring

$$\text{m.m.f.} = 4\pi.(2\pi r n i), \qquad \cdots \quad (14\cdot18)$$

where n is the number of turns per unit length, and i the current in *e.m.u.* From eq. (14·17) we have for the ring

$$\oint \frac{dl}{\mu s} = \frac{d}{\mu s_{air}} + \frac{2\pi r - d}{\mu s}, \qquad (14\cdot19)$$

where μ_{air} is the permeability of air.
From eqs. (14·15), (14·18) and (14·19)

we have

$$N = \frac{4\pi . (2\pi rni) \mu s}{2\pi + (\mu - 1)d}, \qquad . \quad . \quad . \quad (14\cdot20)$$

or

$$B = \frac{N}{S} = \frac{4\pi (2\pi rni)\mu}{2\pi r + (\mu - 1)d} \qquad . \quad . \quad . \quad (14\cdot21)$$

The field H within the gap is given by

$$H = \frac{B}{\mu_{air}} = B, \text{ since } \mu_{air} = 1.$$

The reluctance of a more familiar electromagnet as shown in fig. 14·7(b) is given by

$$\oint \frac{dl}{\mu s} = \frac{l}{s_1} + \frac{2l_1}{\mu_1 s_1} + \frac{2l_2}{\mu_2 s_2} + \frac{l_3}{\mu_3 s_3} \quad . \quad . \quad (14\cdot22)$$
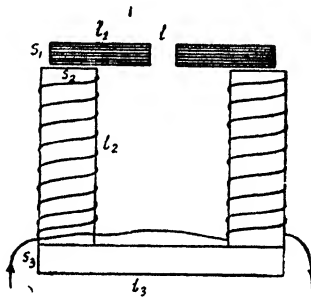
Fig 14·7 (b)

EXAMPLE. A ring electromagnet has a mean radius of 25 cms. and is to be excited by a current of 10 amps. to produce a magnetic field of 2500 gauss in a gap 0·5 cm. wide in the ring. If μ for the ring is 800, what must be the number of turns of wire on the ring ?

From eq. (14·22) we have

$$H = \frac{4\pi . (2\pi rni)\mu}{2\pi r + (\mu - 1)d},$$

i.e.,

$$2500 = \frac{(4 . \pi . 2\pi . 2 . n . \frac{10}{10}) \, 800}{2 . \pi . 25 + (800 - 1) \, 0.5} .$$

Hence the total number of turns $= 2.\pi.25.n$.

$$= 132 \text{ turns.}$$

9. Para, dia and ferromagnetism. So far as the behaviour of substances in a magnetic field was concerned Faraday divided them into three categories. A paramagnetic substance has a

tendency to move when suspended in a non-uniform magnetic field from weaker to stronger parts of the field and is characterized by a positive susceptibility.   On the other hand a diamagnetic substance has a tendency to move from stronger to weaker parts of the field and is characterized by a negative susceptibility. Substances like iron, cobalt and nickel, for which the susceptibility is much larger than unity, are called ferromagnetic. The last three substances show a ferromagnetic behaviour only up to a certain temperature beyond which they are paramagnetic in character.   Diamagnetism is a universal property of all substances.   It is present in para and ferromagnetic substances but is completely masked by their strong para and ferromagnetic behaviour Bismuth is a typical diamagnetic substance. The classic experiments of P. Curie on the susceptibility measurements established the following two very important laws :

(1) The diamagnetic susceptibility is independent of temperature.

(2) The paramagnetic susceptibility varies inversely as the absolute temperature.

10. Langevin's theory of Diamagnetism. On the modern theory of the structure of matter every atom consists of a central nucleus round which the electrons revolve in closed orbits which are, in general, elliptic.   In Langevin's time no such theory of matter existed, but it was known that all substances consist of electrons which revolve round a centre of force.   Consider a circular orbit of radius "a" round which an electron revolves with an angular velocity ω_0.   An electron revolving with an angular velocity ω_0 constitutes a current of magnitude $\frac{e\omega_0}{2\pi}$, since $\frac{\omega_0}{2\pi}$ is the period and e is the electronic charge in e.s.u.   The closed orbit will behave like a magnetic shell of moment given by

$$M = iA = \frac{e\omega_0}{2\pi c}.\pi a^2,$$

$$= \tfrac{1}{2}\frac{e\omega_0 a^2}{c}. \qquad \bullet \quad \bullet \quad \bullet \quad (14 \cdot 23)$$

In each atom there will be several such orbits. The total magnetic moment of the atom will be given by the vector sum of the moments of each orbit. If the resultant magnetic moment is not zero the atomic magnet will be acted upon by a torque when placed in a magnetic field and the substance will behave as paramagnetic.

Let H be the magnetic field applied perpendicular to the plane of the orbit fig. 14·8. The direction of the magnetic moment will depend upon the direction of rotation of the electron. The force f acting on the electron mov-
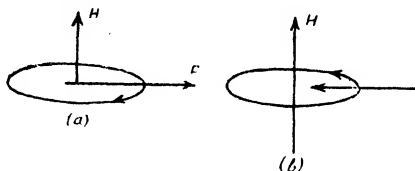
Fig. 14·8

ing in a magnetic field H is given by

$$f = \frac{Hea\omega}{c}, \qquad \qquad \ldots \quad (14·24)$$

where ω is the new angular velocity.
In case (a) the new equation of motion is

$$m\omega^2 a = F - \frac{Hea\omega}{c}, \qquad \qquad \ldots \quad (14·25)$$

$$= m\omega_0^2 a - \frac{Hea\omega}{c},$$

where F is the centrifugal force in the absence of the magnetic field[1], and m is the mass of the electron. From eq. (14·25) we get

$$\omega^2 - \omega_0^2 = - \frac{He\omega}{mc}$$

and so $\qquad \qquad \triangle \omega = - \frac{He}{2mc}$ approx. $\qquad \ldots \quad (14·26)$

Hence the change in the magnetic moment of the atom is

$$\triangle M = \tfrac{1}{2} \frac{ae^2}{c} \triangle \omega$$

$$= - \frac{e^2 H a^2}{4mc^2}. \qquad \qquad \ldots \quad (14·27)$$

From eq. (14·27) we see that the effect of the magnetic field is to decrease the magnetic moment by $\triangle M$ in case (a). It can easily be seen that in case (b) there will be an increase in the magnetic moment.

In an atom there will be several such orbits with a large number of electrons, and hence eqn. (14·27) is to be replaced by

$$\triangle M = -\frac{e^2 H}{4mc^2} \sum_i \overline{a_i^2}, \qquad \cdots \quad (14·28)$$

where the summation is taken over all the electrons of an atom and $\overline{a_i^2}$ denotes the average value of the (radius)^2 since the orbits are not all circular.

In deducing eq. (14·27) we have taken the plane of the orbit to be perpendicular to the magnetic field ; actually the orbits are oriented in all possible directions. If r_j^2 denotes the mean square value of the radius of the electron orbit projected on a plane perpendicular to the field, eq. (14·28) becomes

$$\triangle M = -\frac{e^2 H}{4mc^2} \sum_{j=1}^{j=n} \overline{r_j^2}, \qquad \cdots \quad (14·29)$$

where n is the number of electrons in the atom.

Now $\overline{r_j^2} = \overline{x_j^2} + \overline{y_j^2}$, where x_j^2 and $\overline{y_j^2}$ are the mean square co-ordinates of the jth electron. For a spherically symmetrical atom

$$\overline{x_j^2} = \overline{y_j^2} = \overline{z_j^2} = \tfrac{1}{3} \overline{r_j^2}$$

Hence $$\triangle M = -\frac{e^2 H}{6mc^2} \sum_{j=1}^{j=n} \overline{r_j^2}. \qquad \cdots \quad (14·30)$$

The diamagnetic susceptibility per atom will be

$$\chi_A = \frac{\triangle M}{H} = -\frac{e^2}{6mc^2} \sum_1^n \overline{r_j^2}.$$

As there are N atoms in a gm. atom, the susceptibility for a gm. atom will be given by

$$\chi_M = -\frac{Ne^2}{6mc^2} \cdot \sum_1^n \overline{r_j^2} \qquad \cdots \quad (14·31)$$

After substituting the numerical values we obtain

$$\chi_M = -2.83 \times 10^{10} \sum_1^n \bar{r_j^2}.$$

The essential condition for an atom to be diamagnetic is that the resultant magnetic moment of the atom must be equal to zero. The spectroscopist would say that for an atom to be diamagnetic the resultant angular and the spin moment of the atom must be equal to zero *i.e.* the atom must be in the state $^1S_0$. This condition is satisfied by the inert gases. From eqn. (14·31) we see that the diamagnetic susceptibility is independent of temperature.

11. Paramagnetism and Langevin's theory.

According to Weber's idea of magnetism every magnetic material consists of small molecular magnets arranged in a chaotic manner. In a magnetic field these magnets tend to align themselves parallel to the field. This tendency of the magnets is opposed by their thermal motion. Langevin gave a mathematical form to Weber's ideas.

Let μ be the magnetic moment of the molecular magnet inclined at an angle θ to the direction of magnetic field H. The couple acting on the magnet is

$$C = \mu H \sin \theta . \qquad \qquad \text{. . . (14·32)}$$

The potential energy W of the molecular magnet is

$$W = \int_0^\theta \mu H \sin \theta\, d\theta = \mu H (1 - \cos \theta). \quad \text{. . . (14·33)}$$

The number dn of molecular magnets in a solid angle $d\omega$ whose axes lie between θ and $\theta + d\theta$ is given by

$$dn = A e^{-\mu H(1-\cos\theta)/kT}\, 2\pi \sin \theta\, d\theta \quad \text{. . . . (14·34)}$$

(from Maxwell's law)

The number n of molecules per unit volume is

$$n = 2\pi A \int_0^\pi e^{-\dfrac{\mu H}{kT}(1-\cos\theta)} \sin \theta\, d\theta,$$

$$= B \int_0^\pi e^{\alpha \cos \theta} \sin\theta d\theta = B\,(e^{\alpha} - e^{-\alpha}), \qquad (14 \cdot 35)$$

where $B = 2\pi A e^{-\alpha}$

and $\alpha = \mu H / kT.$

The average magnetic moment μ along the field for a molecular magnet is

$$\bar{\mu} = \frac{\displaystyle\int_0^\pi \mu \cos \theta \, dn}{\displaystyle\int_0^\pi dn}$$

$$= \frac{B \displaystyle\int_0^\pi e^{\alpha \cos \theta} \sin \theta \; \mu \cos \theta \; d\theta}{B \displaystyle\int_0^\pi e^{\alpha \cos \theta} \sin \theta \; d\theta}$$

$$= \mu \left(\coth \alpha - \frac{1}{\alpha} \right)$$

$$= \frac{\mu \alpha}{3} \text{ approx.}$$

$$= \frac{\mu^2 H}{3kT}, \qquad\qquad \cdots \quad (14 \cdot 36)$$

The paramagnetic susceptibility per gm. atom is

$$\chi = \frac{N\mu^2}{3kT} = \frac{N^2\mu^2}{3RT}, \qquad \bullet$$

$$= \frac{\sigma_0^2}{3RT} = \frac{C_M}{T}. \qquad \cdots \quad (14 \cdot 37)$$

R is gas constant, N the Avogadro number and σ_0 is the gm. molecular magnetic moment when the magnets are aligned parallel to the field. This is the well-known *Curie Law*. C_M is called the Curie constant.

In deducing eq. (14·37) we have neglected the mutual interaction between the atomic magnets. Langevin's theory is therefore essentially a theory of a paramagnetic gas. Since the Curie Law

has been found to hold góod widely, it may be concluded that the mutual magnetic influence of the molecules is frequently small.

The phenomena of para and dia-magnetism can be easily visualised in terms of the metion of the electrons in close orbits in atoms and molecules. For a diamagnetic substance the total magnetic moment of the atom is zero, but when the substance is placed in a magnetic field there results a modification, as shown before, of electronic currents giving rise to a feeble magnetic moment of the atom; and hence as a first approximation the susceptibility will be independent of temperature and physical state. On the other hand for a paramagnetic substance as the atoms or molecules possess an intrinsic magnetic moment, there will be initial diamagnetic effect when the substance is placed in a magnetic field; but a paramagnetic effect will eventually be produced by the alignment of the molecular magnets parallel to the field. This tendency of the magnets will be opposed by their thermal motion. Hence the paramagnetic susceptibility will be a function of temperature.

At absolute zero, when there is no thermal agitation, all the molecular magnets will be quite free to turn along the field direction, and the smallest applied field should produce complete saturation in the case of a paramagnetic gas. This, however, will not be the case for two reasons; firstly the paramagnetic gas will solidify at that temperature and the mutual interaction will have to be taken into account, and secondly the Maxwell-Boltzmann's law will not hold good.

12. Ferromagnetism. Iron and other ferromagnetics are qualitatively distinguished magnetically from other substances by the fact that they may acquire a relatively high magnetization in a weak field. In a field of 10 gauss, for example, the specific intensity of magnetization of antimony, a diamagnetic substance, is -0.8×10^{-6} and of cobalt sulphate a paramagnetic, about 0.6×10^{-3}, while that of a typical specimen of soft iron is about 200. In general for ferromagnetics there is not a linear relation between magnetization and field strength, so that magnetic characteristics

of a ferromagnetic cannot be specified so simply as those of para-
and dia-magnetics.   The magnetization however is not a unique
function of field strength, but also depends upon the field strength
to which the specimen has been previously subjected.   If the fields
varied cyclically, the well-known hysteresis curve is traced out.
The most important fact about ferromagnetics is that they may
exist in permanently magnetized state when there is no external
field.   Above a certain critical temperature all the ferromagnetics
become paramagnetic, and when it is stated that a substance is
ferromagnetic it is to be understood that this refers to a range of
temperature below the critical temperature.

According to the well known Curie Law the paramagnetic
susceptibility of a paramagnetic gas   varies inversely as the ab-
solute temperature.   Extensive experimental determinations of
paramagnetic susceptibility at low temperature shows that the
Curie Law breaks down.   To explain this anomaly   in Curie
Law, Weiss modified the expression for the effective magnetic field
acting within the specimen.   He assumed the effective field to
consist of two parts: (1) the incident field H and (2) a field pro-
portional   to the intensity of magnetization.   If we denote the
constant of proportionality to be N, we get

$$H_e = H + NI, \qquad \ldots \quad (14\cdot38)$$

where H_e is the effective field.

The magnetic behaviour of a substance composed of mole-
cules of moment μ is given as before, by

$$\frac{\mu}{\mu} = \coth \alpha - \frac{1}{\alpha}, \qquad \ldots \quad (14\cdot39)$$

and

$$\alpha = \frac{\mu H_e}{kT}. \qquad \ldots \quad (14\cdot40)$$

If ρ is the density, M the molecular weight, σ the gram mole-
cular moment and σ_0 the saturation value, then

$$I = \frac{\rho\sigma}{M}, \qquad \ldots \quad (14\cdot41)$$

and

$$\frac{\sigma}{\sigma_0} = \coth \alpha - \frac{1}{\alpha}. \qquad \ldots \quad (14\cdot42)$$

From eqs. (14·38) and (14·40) we have

$$\alpha = \frac{\mu H}{kT} + \frac{\mu NI}{kT},$$

or

$$\alpha = \frac{\sigma_0 H}{RT} + \frac{\sigma_0 NI}{RT}, \qquad \ldots \quad (14·43)$$

Using (14·41) we have

$$\alpha = \frac{\sigma_0 H}{RT} + \frac{\sigma_0^2 N \rho}{MRT} \cdot \frac{\sigma}{\sigma_0}$$

or

$$\frac{\sigma}{\sigma_0} = \frac{MRT}{N \rho \sigma_0^2} \alpha - \frac{M}{N \rho \sigma_0} H. \qquad \ldots \quad (14·44)$$

When x is small $\coth x - \frac{1}{x} = \frac{\alpha}{3} = \frac{\sigma}{\sigma_0}$, therefore

$$\frac{\sigma}{\sigma_0} \left\{ \frac{3MRT}{N \rho \sigma_0^2} - 1 \right\} = \frac{M}{N \rho \sigma_0} H. \qquad \ldots \quad (14·45)$$

Hence

$$\chi_M = \frac{\sigma}{H} \cdot$$

$$= \frac{\sigma_0^2 / 3R}{T - \left(\frac{N \rho \sigma_0^2}{3MR} \right)} \cdot \qquad \ldots \quad (14·46)$$

This may be written as

$$\chi_M = \frac{C_M}{T - \theta}, \qquad \ldots \quad (14·47)$$

where

$$\left. \begin{array}{l} C_M = \dfrac{\sigma_0^2}{3R} \\[2mm] \theta = \dfrac{N \rho \sigma_0^2}{3MR} = \dfrac{N \rho C_M}{M} \cdot \end{array} \right\} \qquad \ldots \quad (14·48)$$

and

It has been found that the modified Curie-Weiss law agrees with the experimental results to a greater degree.

From eq. (14·44) we see that when the external field H is zero there is still some magnetization present given by

$$\frac{\sigma}{\sigma_0} = \frac{MRT}{N \rho \sigma_0^2} \alpha = \frac{T}{\theta} \cdot \frac{\alpha}{3} \qquad \ldots \quad (14·49)$$

Thus we have two simultaneous equations for $\frac{\sigma}{\sigma_0}$, namely (14·44) and (14·49), which can be solved by the graphical method. The two

graphs are shown in fig. 14·9. It is clear from the figure that the
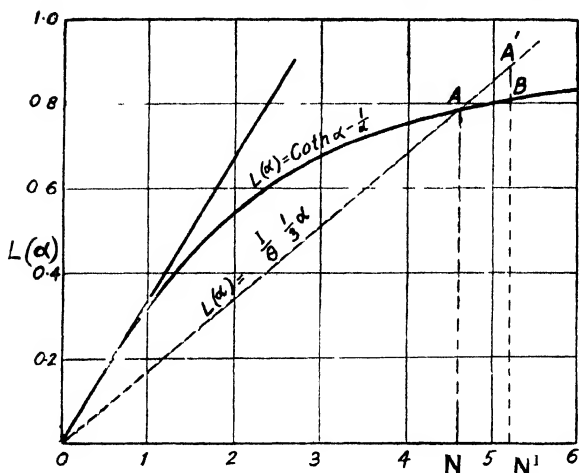
Fig. 14·9

two equations are satisfied simultaneously at two points O and A.
The state at A is the equilibrium state while that at O is unstable
because at O if we increase the value of $\frac{\sigma}{\sigma_0}$ a little, the value will
go on increasing till state A is reached. On the other hand at A
if we increase the value of $\frac{\sigma}{\sigma_0}$ to N'A', the increase in α being
NN', the new field will tend to reduce the magnetization to the
smaller equilibrium value N'B. It is also clear from the figure
that the spontaneous magnetization as assumed by Weiss shall
only occur if the straight line $\frac{\sigma}{\sigma_0} = \frac{T}{\theta} \frac{\alpha}{3}$ makes smaller angle
with the α-axis than the tangent to the Langevin curve. This
will happen only if

$$\frac{\sigma}{\sigma_0} < \frac{\alpha}{3},$$

or $$T < \theta, \qquad \qquad . \quad . \quad . \quad (14·50)$$

from eq. (14·49).

Thus a paramagnetic substance below its Curie temperature θ will become ferromagnetic, meaning thereby that there will be spontaneous magnetization present.

Below the Curie temperature the thermal agitation is not sufficient to prevent the molecules from orienting each other by their mutual interaction, so that a spontaneous magnetization results. But actually a piece of iron below the Curie temperature (758°C) appears to be non-magnetic. Perhaps the reason is that the axes of the spontaneously magnetized regions (called domains) have random directions. Only under the influence of an external magnetic field do their axes become more or less parallel (apparent magnetization).

The Curie-Weiss law gives a nice explanation of ferromagnetism, but the existence of a molecular field proportional to the intensity of magnetization is still a postulate which needs explanation. We can assume the molecular field to be the Lorentz-Lorenz polarisation field in which case the molecular field, as we have seen in Art. 12, Ch. IV, $NI = \frac{4}{3}\pi I$, $i.e.$, the value of the constant N is equal to $\frac{4}{3}\pi$. But experimentally it has been observed that this value of N is in many cases thousands of times too small to account for the observed results.

We shall now briefly summarize the basic ideas of the modern theory of ferromagnetism which is due to Weiss. In iron and other ferromagnetic substances there exist small spontaneously magnetized regions called the *domains*. Each domain contains a large number of atoms (between 10^9 and 10^{15}), each of which is a permanent magnet like a paramagnetic atom, but the magnetism of the individual ions is due to electron spin. The ferromagnetic behaviour of iron is not due to the properties of single ions. All the ions in a domain have their magnetic axes pointing in some direction; the alignment being brought about by a permanent intramolecular field which extends over the whole domain, but its origin is not well-known. This intramolecular field is the chief characteristic of ferromagnetism. At temperatures below the Curie point the thermal

15 E

agitation of the ions is not violent enough to destroy the intramolecular field; and hence below the Curie point each domain is practically saturated.   Above the Curie point it is the domain structure which is destroyed and not the magnetism of the individual ions and the substance becomes paramagnetic.   The disarrangement of the ions in the domain is reversible, *i.e.*, on cooling the substance the intramolecular field returns and ferromagnetism reappears.

The direction of magnetization of each domain is indeterminate, and if the directions are distributed at random, the substance as a whole will appear unmagnetized.   The large sized domains cannot turn round as such except in very high fields.   There is a direction of easy magnetization associated with each domain.

When a ferromagnetic substance is placed in an increasing external magnetic field, the successive stages of magnetization are represented by fig. 14·10, in which the arrows represent the directions of magnetization of various domains.

unmagnetized   saturation
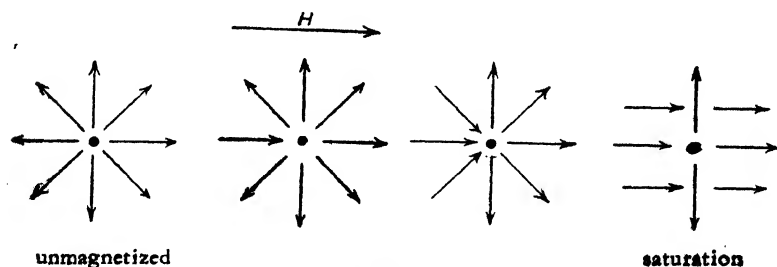
Fig. 14·10

Let an increasing field H be applied in the forward direction and let the coercive force for a domain be H_c. At first the external field will have no effect.   But as soon as H becomes equal to H_c, only that domain changes sign which is oriented with its direction of magnetization towards the backward direction.   As H increases further, more and more domains reverse their directions of magnetization; a domain oriented at an angle θ to the backward direction will reverse its direction of magnetization when $H \cos \theta$

becomes equal to H_c. This corresponds to the well-known steep irreversible part of the hysteresis curve. Finally there will be a reversible increase in magnetization due to the direction of magnetization of each domain tending towards the external field.

Barkhausen, in 1919, demonstrated experimentally that such sudden changes of direction of magnetization do take place, thereby proving the existence of domains. The experimental arrangement is shown in fig. 14·11.
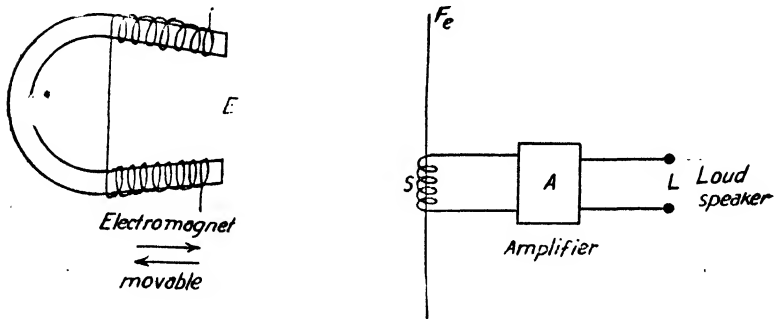

Fig. 14·11

A thin iron wire Fe is surrounded by a coil of wire S, the two ends of which are connected through an amplifier A to a loud-speaker L. An electromagnet E is brought slowly and uniformly towards the wire. As soon as the field due to the electromagnet exceeds the coercive field for a certain direction, the domains suddenly change their direction. This causes corresponding increases of magnetic flux through the coil, and consequently currents of short duration are induced in it. After proper amplification their induced current pulses are heard as clicks in the loud-speaker (*Barkhausen effect*).

13. Measurement of Susceptibility. Curie's balance. The method consists in measuring the pondermotive force when the specimen is placed in an non-homogeneous magnetic field. The potential energy of a specimen of volume v in a field H is

$$W = \int (\mu - \mu_0)\, \frac{H^2 dv}{8\pi}, \qquad \cdot \quad \cdot \quad \cdot \quad (14·51)$$

where μ is the permeability of the specimen and μ_0 that of the surrounding medium,

or $\qquad\qquad W = \frac{1}{2}(k-k_0) \int H^2 dv,$

since $\qquad\qquad \mu = 1 + 4\pi k.$

The force F is given by

$$F = \frac{\partial W}{\partial s} = \frac{1}{2}(k-k_0) \int \frac{\partial H^2}{\partial s} \, dv. \qquad . \quad . \quad . \quad (14\cdot52)$$

A non-homogeneous field with an axis of symmetry is produced by inclining two pole pieces of an electromagnet to each other. The force along the axis of x is

$$F_x = kvHy \frac{\partial Hy}{\partial x},$$

where we have taken $k_0 = 0$,

or $\qquad\qquad F = m\kappa Hy \frac{\partial Hy}{\partial x}, \qquad\qquad . \quad . \quad . \quad (14\cdot53)$

where m is the mass of the specimen, and κ the mass susceptibility.

A sketch of Curie balance is shown in fig 14·12. The specimen whose susceptibility is to be determined is placed in a glass tube of rather complicated form. It is hung from the arm of a balance in the region of the field where $Hy \dfrac{\partial Hy}{\partial x}$ is maximum. The other arm carries a platform Q for placing small weights and a copper plate P which acts as a damper. The plate P carries a long pointer which moves over a calibrated scale S. The displacement of the

Fig. 14·12

pointer gives the force acting on the specimen. The field Hy is determined by the usual search coil ballistic galvanometer method. $\dfrac{\partial Hy}{\partial x}$ is measured directly by moving the search coil along the x-axis with its plane parallel to the y-axis. But this procedure would rather be insensitive, since we shall be measuring a small change
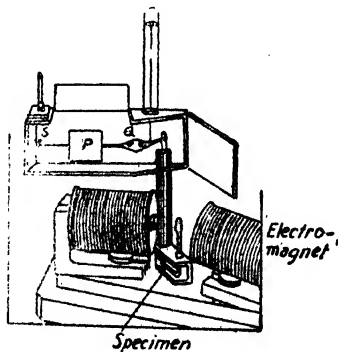
in a large quantity . The method adopted, therefore, is to measure $\frac{\partial Hx}{\partial y}$ by moving the search coil along the y-axis with its plane parallel to the y-axis, since $\frac{\partial Hy}{\partial x} = \frac{\partial Hx}{\partial y}$. Curves are then drawn for Hy, $\frac{\partial Hy}{\partial x}$ and $Hy \frac{\partial Hy}{\partial x}$ and the specimen is placed in the region where $Hy \frac{\partial Hy}{\partial x}$ is maximum. A serious source of error in this method lies in setting the specimen always in the same position. Curie's measurements were in general correct to 1 or 2 per cent. For measurements at different temperatures the specimen is enclosed in a special electric oven.

Gouy method. In this method the specimen is made in the form of a uniform cylinder and is suspended with one end in a homogeneous field ; the other end is in the region where the field is small. The axis of the cylinder is at right angles to the magnetic field, fig. 14·13. The force acting on the specimen is, as before,
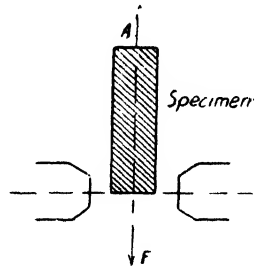

Fig. 14·13. Gouy method

$$F = \frac{k-k_0}{2} \int \frac{\partial H^2}{\partial x} \, dv = \frac{k-k_0}{2} \int A \, \frac{\partial H^2}{\partial x} \, dx,$$
$$= \tfrac{1}{2} (k-k_0) A (H^2 - H'^2), \quad . \quad . \quad . \quad (14·54)$$

where k is the volume susceptibility of the specimen, k_0 that of the surrounding medium, H the field between the poles, H' the field at the other end of the specimen and A the cross-section of the cylinder. If the length of the cylinder be such that H' is small, H'² may be neglected; then

$$F = \tfrac{1}{2}(k-k_0) AH^2 \quad . \quad . \quad . \quad (14·55)$$

The force F may be determined by putting weights in the other arm of a sensitive balance. This method is very simple. Its only drawback is that its sensitivity is not very high.

Quincke's method for liquids. The principle of this method is the same as that of Gouy. A U-tube of the form shown in fig. 14·14 is employed. The narrow limb of the tube

passes between the poles of a
strong electromagnet, the meniscus
being in the region where the
field is uniform. At the other end
of the liquid surface the field is
negligible. Under the influence
of the field the meniscus will rise
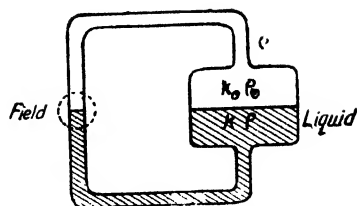or fall by an amount h such that

Fig. 14·14. Quincke Method

$$h \left(1 + \frac{s}{S} \right) (\rho - \rho_0)g = \tfrac{1}{2} (k - k_0)H^2, \qquad (14{\cdot}55)$$

where ρ and ρ_0 are the densities of the liquid and the gas respectively and s the cross-section of the narrow end, while S that of the wide end. The left-hand side of eq. (14·55) gives the resultant downward pressure. If S is much greater than s, then

$$k - k_0 = \frac{2hg\rho}{H^2}\left(1 - \frac{\rho_0}{\rho} \right),$$

or $$X = \frac{2hg}{H^2} + \frac{\rho_0}{\rho}X, \text{ approx.} \quad . \quad . \quad . \quad (14{\cdot}56)$$

If the gas is at atmospheric pressure, $1 - \dfrac{\rho_0}{\rho} = 1$.

For diamagnetic liquids a magnetic field of the order of 10^4 gauss is needed for appreciable displacement of the meniscus; whereas for paramagnetic salts a field of the order of 10^3 gauss will suffice.

14. Production of high magnetic fields. In modern nuclear and atomic research we need intense magnetic fields of the order of 10^5 gauss or even more. Such intense magnetic fields have been produced by Kapitza. We shall briefly outline how high magnetic fields can be generated with the help of electromagnets.

For a ring electromagnet, consisting of a cone of uniform cross-section S, length L, and permeability μ, having M windings carrying a current i amperes and having an interpole gap d, the field H is given approximately by

$$H = \frac{4\pi Mi}{10\left(\dfrac{L}{\mu} + d\right)} , \qquad . \quad . \quad . \quad (14{\cdot}57)$$

from eq. (14·21).

For an electromagnet with tapering pole-pieces of cross-section s, the field is

$$H = \frac{4\pi M i}{10} \Big/ \left(\frac{L}{S_\mu} s + d\right), \quad . \quad . \quad . \quad (14{\cdot}58)$$

assuming that there is no magnetic leakage.

From eq. (14·58) we see that the field strength can be increased by increasing the ampere turns, by diminishing the inter-space d and by increasing the permeability and the cross-section S. Therefore, for the generation of high magnetic fields we need very massive electromagnets.

Powerful electromagnets have been devised by Weiss. In such magnets the coils are mounted on coaxial cones which are supported as a massive yoke. Since very large currents are necessary, tubular windings are used, through which a stream of water flows. The conical pole pieces are made of ferrocobalt, a special alloy of high permeability. The adjustment of height and orientation of the whole magnet is controlled by mechanical devices. The following data will give an idea of the magnitudes involved: A Weiss magnet had 100,000 ampere-turns in the excitation coils and gave a field of 50,000 gauss between the poles when the diameter of the butt ends of the concial pole tips was 3·6 mms. and the distance between the butt ends was 1·1 mm. The direct-current voltage was about 90 volts and the current about 113 amperes.

15. Intense magnetic fields. Kapitza's method. For generating very intense magnetic fields greater than 50,000 gauss, air-cored coils are used, since no useful purpose is served by using an iron core as the external field far exceeds that due to the magnetization of iron itself (see eq. 14·38. NI is negligible compared to H). The basic idea of the method is to produce a field in the coil for a very short time only, say about 1/100 second, so that a high current can flow in the coil without fusing it. Suppose, for example, that a long solenoid is wound with 40 turns per cm. length and it is required to obtain a magnetic field of intensity 10^6 gauss at

the central part within the air core of the solenoid.   The current
necessary to produce the field will be

$$i = \frac{10^6}{4\pi \cdot 40} = 19{,}900 \text{ amperes.}$$

Such a huge current can be allowed to flow only for a very small
fraction of a second if the solenoid winding is not to be burnt
out.   Since the current used is transitory and the power used
is enormous, we should have a suitable appliance which is capable
of storing energy over a relatively long period and liberating it
in a short one.

In his earlier experiments Kapitza used special accumulators
with very low internal resistance, but these were afterwards re-
placed by a large A. C. generator, giving powers up to 50,000
kilowatts in the coil for 1/100 second.   The general principle is
to make use of a half-cycle of current, the circuit being made
when the generator e.m.f. is zero and current maximum, and broken
when the current is zero.   The time of make and break is con-
trolled by a falling plate (for details see Kapitza's paper in P.R.S.).
From a 2000 k. w. generator a current of 72,000 amps at 2,250
volts has been obtained on short circuiting.

CHAPTER XV

ALTERNATING CURRENTS

1. A coil rotating in a magnetic field. If a coil rotates in a magnetic field H, the flux N through the coil at any instant t when its plane makes an angle θ with the direction of H is given by

$$N = nAH \sin \theta, \qquad \cdots \quad (15 \cdot 1)$$

where A is the area of the coil, and n the number of turns in it. We see from eqn. (15·1) that the flux N changes from zero to nAH as the coil rotates from $\theta = 0$ to $\theta = \frac{1}{2}\pi$. It cuts the maximum flux when its plane is perpendicular to the magnetic field. The induced e.m.f. in the coil is

$$e = -\frac{dN}{dt} = -nAH \cos \theta \, \frac{d\theta}{dt},$$

$$= -nAH\omega \cos \theta, \qquad \cdots \quad (15 \cdot 2)$$

where ω is the uniform angular velocity of the coil. From eqn. (15·2) we see that the e.m.f. in the coil is maximum when $\theta = 0$,
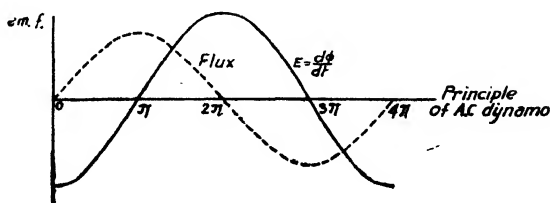

Fig. 15·1 (a). Coil rotating in a magnetic field.

i.e., when the flux in the coil is zero, and it is minimum when $\theta = \frac{1}{2}\pi$, *i.e.*, when the flux is maximum. The two are said to be opposite in phase. The behaviour of flux and the induced e.m.f. is shown in fig. 15·1(a).

If the coil is connected to an external circuit through two slip rings, fig. 15·1 (b), the current in the circuit will be in one direction for one half of the revolution and in the opposite direction for the other half. In one complete revolution each slip ring changes its polarity twice. Such an arrangement therefore acts as a source of alternating current. This is the simple principle on which an alternating current dynamo is based.
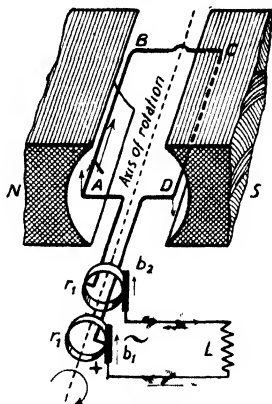

Fig. 15·1 (b).
Simple A. C. Dynamo.

2.   Alternating current circuit.

Consider a circuit containing a capacity c, an inductance L and a resistance R connected to an alternating e.m.f. $E_0 \sin \omega t$.   The equation for the e.m.f. in the circuit is

$$L \frac{di}{dt} + Ri + \frac{q}{c} = E_0 \sin \omega t,$$

or

$$L \frac{d^2i}{dt^2} + R \frac{di}{dt} + \frac{i}{c} = E_0 \omega \cos \omega t. \quad . \quad . \quad . \quad (15·3)$$

The solution of eq. (15·3) is of the form

$$i = i_0 \sin (\omega t - \phi). \quad . \quad . \quad (15·4)$$

From eqns. (15·3) and (15·4) we have

$$\{-L\omega^2 \cos \phi + R\omega \sin \phi + \frac{1}{C} \cos \phi\} i_0 \sin \omega t$$

$$+\{(L\omega^2 \sin \phi + R\omega \cos \phi - \frac{1}{C} \sin \phi) + i_0 - E_0\omega\} \cos \omega t = 0 \quad (15·5)$$

This is an identity which must hold good for all values of time.

Putting $\omega t = \pi/2$ in eq. (15·5) and $\omega t = 0$,

$$\tan \phi = \frac{L\omega - 1/C\omega}{R}, \quad . \quad . \quad . \quad (15·6a)$$

$$i_0 = \frac{E_0}{R \cos \phi + (L\omega - 1/C\omega) \sin \phi}, \quad . \quad . \quad . \quad (15·6 b)$$

Hence the solution is

$$i = \frac{E_0}{R \cos \phi + (L\omega - 1/C\omega)\sin \phi} \sin (\omega t - \phi).$$

or
$$i = \frac{E_0 \sin(\omega t - \phi)}{\sqrt{\{R^2 + (L\omega - 1/C\omega)\}^2}}, \quad \cdots \quad (15 \cdot 7)$$

From eq. (15·7) the maximum value i_0 of the current is given by

$$i_0 = \frac{E_0}{\sqrt{\{R^2 + (L\omega - 1/C\omega)\}}}. \quad \cdots \quad (15 \cdot 8)$$

The relation, therefore, between the maximum current and the maximum e.m.f. in the case of an alternating current is similar to the Ohm's law in the direct current circuit with the difference that we have an extra term $\left(L\omega - \dfrac{1}{C\omega}\right)^2$ under the radical sign.

3. Impedance, reactance and resonance. The quantity $\sqrt{R^2 + \left(L\omega - \dfrac{1}{C\omega}\right)^2}$ is called the *impedance* of the circuit. At very high frequencies the dominating term under the radical is $(L\omega)^2$. The quantity $L\omega$ is called the *reactance* of the circuit. The impedance is minimum and the current is maximum for the frequency

$$\gamma_0 = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}, \quad \cdots \quad (15 \cdot 9)$$

provided we neglect the resistance.

γ_0 is also the natural frequency of the circuit, and is called the *resonance* frequency. At resonance frequency $\phi = 0$ (since $L\omega = 1/C\omega$), i.e., the current and the e.m.f. are in the same phase. For frequencies less than the resonance frequency (since $L\omega < 1/C\omega$), ϕ is negative and hence the current leads the e.m.f., whereas for frequencies greater than the resonance frequency (since now $L\omega > 1/C\omega$), ϕ is positive and the current lags behind the e.m.f.

4. Virtual current and virtual e.m.f. The virtual or the effective current is defined as that current which will generate the same amount of heat in a resistance R as an alternating current actually flowing in it. The heat energy generated per second in R by the effective current i_e is Ri_e^2, and that generated by the alternating current i is

$$R\frac{\omega}{2\pi}\int_0^{2\pi/\omega} i^2 \, dt$$

(the average over the complete period).

By definition,

$$\mathrm{R}i_e^2 = \mathrm{R}\frac{\omega}{2\pi}\int_0^{2\pi/\omega} i^2 dt = \mathrm{R}i_0^2\frac{\omega}{2\pi}\int_0^{2\pi/\omega} \sin^2(\omega t - \phi)\, dt,$$

using eq. (15·4),

$$= \frac{\mathrm{R}i_0^2}{2}, \text{ since the value of the integral is } \tfrac{1}{2}\cdot\frac{2\pi}{\omega}$$

or
$$i_e = \frac{i_0}{\sqrt{2}}. \qquad \cdots \quad (15\cdot10)$$

i_e, the virtual current is $\dfrac{1}{\sqrt{2}}$ times the maximum-current i_0. As i_e is the square root of the mean value of the square of i, it is also defined as the *root mean square* value of the current, or, in short form, R.M.S. There is a similar definition for the e.m.f. By an exactly similar calculation the effective or the virtual e.m.f. E_e is given by

$$\mathrm{E}_e = \frac{\mathrm{E}_0}{\sqrt{2}}. \qquad \cdots \quad (15\cdot11)$$

It is the virtual current that is measured by an A.C. instrument such as a hot-wire ammeter. The maximum current in the circuit would be $\sqrt{2}$ times the value indicated by the instrument.

5. The power factor. At any instant t the power P in the circuit is $\mathrm{P} = \mathrm{E}i$

or
$$\mathrm{P} = \mathrm{E}_0 i_0 \sin \omega t \sin(\omega t - \phi). \qquad \cdots \quad (15\cdot12)$$

Since the e.m.f. and the current are not in phase, the power will sometimes be positive and sometimes negative during the course of one oscillation. The mean power averaged over one complete period is

$$\overline{\mathrm{P}} = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} \mathrm{E}i\, dt = \frac{\mathrm{E}_0\,\omega\,i_0}{2\pi}\int_0^{2\pi/\omega} \sin \omega t \sin(\omega t - \phi)\, dt$$

$$= \frac{\mathrm{E}_0 i_0\omega}{2\pi}\left[\cos\phi\int_0^{2\pi/\omega}\sin^2\omega t\, dt - \sin\phi\int_0^{2\pi/\omega}\sin\omega t\,\cos\omega t\, dt\right]$$

$$= \frac{E_0 i_0}{2} \cos \phi, \quad \text{since the second integral is zero.}$$

$$= \frac{E_0}{\sqrt{2}} \cdot \frac{i_0}{\sqrt{2}} \cos \phi = E_e i_e \cos \phi. \quad . \quad . \quad (15 \cdot 13)$$

Thus the power delivered in a circuit is the product of the virtual volt, virtual current and a factor $\cos \phi$, called the *power factor*. Power is maximum at only resonance frequency since $\phi = 0$. If the inductance of a circuit is so large that $\phi \to \frac{1}{2}\pi$ the current in the circuit is entirely watt-less ; the resistance being negligible.

6. Vector diagrams. The alternating e.m.f. $E = E_0 \sin \omega$ and the alter-
nating current
$I = I_0 \sin \omega t - \phi)$
may be re-
presented by
the projections
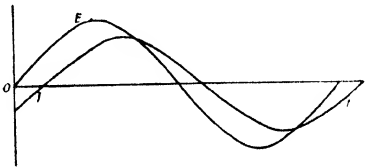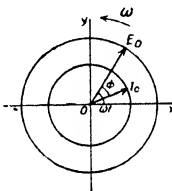on the y axis
of the vectors

Fig. 15·2 (a)

E_0 and I_0 respectively, which rotate with a constant angular velo-
city ω in counter-clockwise direction, the latter lagging behind the former by an angle ϕ, fig. 15·2 (a).

The trigonometric expression for the current I in a circuit containing an inductance L, a resistance R and an impressed e.m.f. $E = E_0 \sin \omega t$ is

$$I = \frac{E_0}{\sqrt{(R^2 + L^2 \omega^2)}} \sin (\omega t - \phi), \text{ where } \tan \phi = \frac{L\omega}{R}.$$

The maximum value of the current is

$$I_0 = \frac{E_0}{\sqrt{(R^2 + L^2 \omega^2)}},$$

or

$$E_0 = \sqrt{R^2 I_0^2 + L^2 \omega^2 I_0^2}.$$

E_0 is, therefore, given by the diagonal of the rectangle with sides RI_0 and $L\omega I_0$, as shown in fig. 15·2 (b).

If this figure is rotated about the origin O, with angular velocity ω in the counter clock-wise direction, the projections of the vectors E_0, RI_0 and $L\omega I_0$, on the y-axis give the instantaneous values of the impressed e.m.f. and the e.m.f. across the resistance and the inductance respectively.
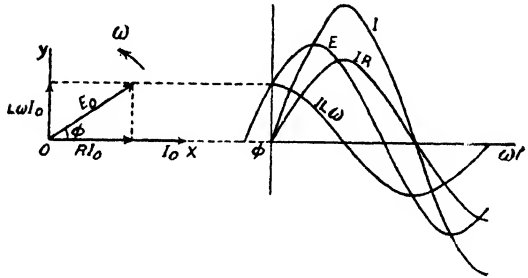

Fig. 15·2 (b)

For a circuit containing capacity, inductance and resisatance

$$I_0 = \frac{E_0}{\sqrt{\left\{ R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2 \right\}}} \quad \text{and} \quad \phi = \tan^{-1} \frac{L\omega - \frac{1}{C\omega}}{R}.$$

The vector diagram is shown in fig 15·2(c), when $L\omega = \frac{1}{C\omega}$, $\phi = 0$. E_0 and I_0 are then in phase. When $\frac{1}{C\omega} > L\omega$, ϕ is negative and the current leads the e.m.f.
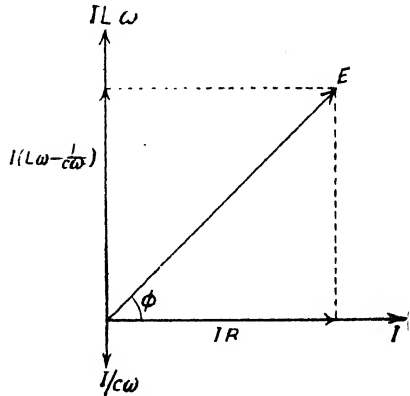

Fig. 15·2 (c)

7. Electromagnetic repulsion.

We shall investigate the phenomenon of repulsion that occurs between a conductor and an alternating current carrying circuit. The conductor is in the form of a copper ring A placed above an electromagnet M fed by an alternating e.m.f. (fig. 15·3.) Let the current in M be given by,

$$I = I_0 \sin \omega t. \qquad \cdots \quad (15\cdot14)$$

The magnetic flux N through A is

$$N = MI_0 \sin \omega t, \qquad \cdots \quad (15\cdot15)$$

where M is the mutual inductance of the two circuits. The induced e.m.f. in the ring A is

$$E = -\frac{dN}{dt} = -MI_0 \omega \cos \omega t.$$

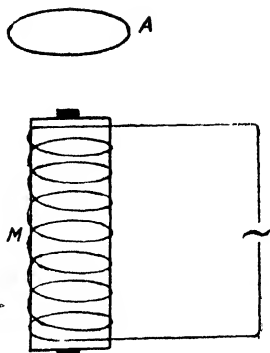If L denotes the self-inductance of the ring, and R its resistance, the equation for the e.m.f. in A is

Fig 15·3

$$L \frac{dI_1}{dt} + RI_1 = -MI_0 \omega \cos \omega t. \qquad \cdots \quad (15\cdot16)$$

Hence the current in the ring is

$$I_1 = \frac{-M\omega I_0}{\sqrt{(R^2 + L^2\omega^2)}} \cos (\omega t - \phi), \qquad \cdots \quad (15\cdot17)$$

where $\tan \phi = \dfrac{L\omega}{R}$.

The potential energy of the ring is given by

$$W = -NI_1 = +MI_0 \sin \omega t \frac{M\omega I_0}{\sqrt{(R^2 + L^2\omega^2)}} \cos (\omega t - \phi). \quad (15\cdot18)$$

The force F acting on the ring along the axis of z is

$$F = -\frac{\partial W}{\partial z} = + \frac{2I_0^2 \omega M}{\sqrt{(R^2 + L^2\omega^2)}} \cdot \frac{\partial M}{\partial z} \sin \omega t \cos (\omega t - \phi). \quad (15\cdot19)$$

A negative sign has been placed before $\dfrac{\partial M}{}$ to indicate that M decreases as z increases.

The average value of F over one complete period is

$$F = + \frac{2MI_0{}^2\omega.}{\sqrt{(R^2+L^2\omega^2)}} \frac{\partial M}{\partial z} \cdot \left[\frac{\omega}{2\pi} \int_0^{2\pi/\omega} \sin \omega t \cos \omega t \cos \phi \, dt \right.$$

$$\left. + \frac{\omega}{2\pi} \int_0^{2\pi/\omega} \sin^2 \omega t . \sin \phi \, dt \right]$$

$$= + \frac{MI_0{}^2\omega}{\sqrt{R^2+L^2\omega^2}} \cdot \frac{\partial M}{\partial z} \sin \phi, \quad . \quad . \quad (15\cdot20)$$

(The first integral is zero and the second one is $\frac{1}{2} \sin \phi$). The positive sign indicates that the force is of repulsion. If R is small and ω large, F may be large enough to balance a heavy ring against gravity.

EXAMPLE. What would be the height at which a ring of radius 12 cm. mass. 2 gms. and self-inductance 10^{-7} henry and resistance 10^{-4} ohms would be supported if held freely on an electromagnet which has a core of permeability 1000 and cross section 10 cm^2 wound with 10 turns per cm ? The 60 cycle alternating current passing through it has an amplitude of 10 amp. Assume that the magnet is so long that the effect of the distant pole can be neglected, and the intensity of magnetization is uniform and the pole can be treated as if concentrated on the axis.

The magnetic field B inside the solenoid is $= 4\pi\mu n i = 4\pi 1000.10.\frac{10}{10}$ gauss. Now $B = H + 4\pi I$, where I is the intensity of magnetisation.

Hence $\qquad I = \dfrac{B}{4\pi}$, neglecting H which is small.

$\qquad \qquad \therefore = 10^4$.   C.G.S. units.

The pole strength m of the magnet is by definition

$$m = \frac{I}{\text{area of the cross-section}} = \frac{10^4}{10} = 10^3.$$

The magnetic flux in the ring due to the pole at a distance x is

$$N = \pi . (12)^2. \frac{10^3}{x^2} ,$$

assuming the field at the ring to be uniform.

The mutual inductance M is given by

$$Mi = N,$$

i.e. $\qquad \therefore M = \pi . (12)^2 \cdot \dfrac{10^3}{x^2} \dfrac{10}{10}$ absolute units.

and
$$\frac{\partial M}{\partial x} = \frac{\pi \cdot (12)^2 \cdot 10^3 \cdot 2}{x^3}.$$

Also $\tan \phi = \dfrac{L\omega}{R} = \dfrac{10^{-7} \cdot 2\pi \cdot 60}{10^{-4}} = 0.377$ approx.

Now $\overline{F} = \dfrac{I_o{}^2 \omega}{\sqrt{R^2 + L^2\omega^2}} \cdot M \cdot \dfrac{\partial M}{\partial x} \cdot \sin \phi.$

$$\therefore \quad mg = \overline{F}.$$

So $2 \times 980 = \dfrac{\pi^2 (12)^4 (10)^6}{x^5} \dfrac{2 \cdot 2 \cdot \pi \cdot 60}{\sqrt{\{10^{-3} \cdot 10^{18} + 10^{-14} \cdot 10^{18}(2\pi \cdot 60)^2}} \cdot 38,$
which gives.

$$x = 12.5 \cdot \text{cms. nearly.}$$

8. Couple on a coil rotating in a magnetic field. Consider a coil of plane area A rotating with a uniform angular velocity ω in a magnetic field H. If at any instant t the plane of the coil makes an angle θ with the direction of H, the flux N through the coil is

$$N = AH \sin \omega t. \qquad \ldots \quad (15.21)$$

The induced e.m.f. in the coil is

$$e = -\frac{dN}{dt} = -AH\omega \cdot \cos \omega t. \qquad \ldots \quad (15.22)$$

If L denotes the self inductance and R the resistance of the coil, the equation for the e.m.f. in the coil is

$$L \frac{di}{dt} + Ri = -AH\omega \cdot \cos \omega t. \qquad \ldots \quad (15.23)$$

The current in the coil is, therefore,

$$i = \frac{-AH\omega}{\sqrt{(R^2 + L^2\omega^2)}} \cos(\omega t - \phi), \text{ where } \tan \phi \quad \frac{L\omega}{R}. \qquad (15.24)$$

The coil behaves like a magnetic shell of strength i whose potential energy in the field H is given by

$$W = -iAH \sin \omega t \qquad \ldots \quad (15.25)$$

Therefore the couple acting on the coil tending to increase θ is

$$c = -\frac{dW}{d(\omega t)} = iAH \cos \omega t \qquad \ldots \quad (15.26)$$

Substituting the value of i from eqn. (15.24) in eqn. (15.26), we have

$$c = -\frac{A^2H^2\omega}{\sqrt{(R^2 + L^2\omega^2)}} \cos \omega t \cdot \cos(\omega t - \phi), \qquad (15.27)$$

the minus sign indicating that the couple is opposing the rotation,

16 E

or $\qquad c = \frac{1}{2} \cdot \dfrac{A^2 H^2 \omega}{\sqrt{R^2 + L^2 \omega^2}} \left\{ \cos (2\omega t - \phi) + \cos \phi \right\} \cdot$ $\qquad$ (15·28)

The mean value of the couple is

$$\bar{c} = \frac{1}{2} \cdot \frac{A^2 H^2 \omega}{\sqrt{R^2 + L^2 \omega^2}} \cdot \left[\frac{\omega}{2\pi} \int_0^{2\pi/\omega} \cos (2\omega t - \phi) \, dt + \frac{\omega}{2\pi} \int_0^{2\pi/\omega} \cos \phi \, dt \right]$$

$$= \frac{1}{2} \cdot \frac{A^2 H^2 \omega}{\sqrt{R^2 + L^2 \omega^2}} \cdot \cos \phi$$

$$= \frac{A^2 H^2 R \omega}{2(R^2 + L^2 \omega^2)}. \qquad \ldots \quad (15 \cdot 29)$$

If instead of the coil rotating in the magnetic field, the field had been rotating with the angular velocity ω, the couple acting on the coil in the direction of the rotation of the field would have been given by eqn. (15·29). And if the coil was free to rotate it would begin to rotate in the same direction as the field. In fact $\bar{c}$ is the relative couple. The expression indicates that the couple vanishes both for $\omega = 0$ and $\omega = \infty$. The couple, therefore, would be maximum for some intermediate value of ω. The value of ω for which $\bar{c}$ would be maximum is given by

$$\frac{d \bar{c}}{d\omega} = 0 = \frac{A^2 H \cdot R}{2} \cdot \frac{L^2 \omega^2 + R^2 - 2L^2 \omega^2}{(R^2 + L^2 \omega^2)^2} \cdot$$

giving $\qquad\qquad \omega = \dfrac{R}{L} \cdot$

Hence $\qquad\qquad (c)_{max} = \dfrac{A \cdot H^2}{4L} \cdot \qquad \ldots \quad (15 \cdot 30)$

The value of ϕ for the maximum couple is $\pi/4$.

Fig. 15·4 gives a typical curve between the couple and the angular velocity. The couple at first increases with ω, reaches the maximum value and finally drops down to zero.
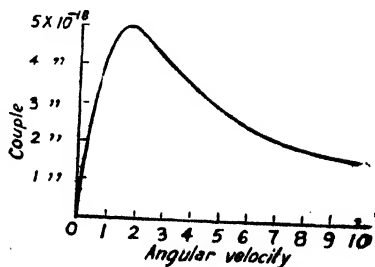
A most important use to which this problem of a coil rotating in a magnetic field has been put is in the construction of a dynamo

Fig. 15·4

9. Choke coil. In a direct current circuit the current is reduced by introducing an extra resistance in the circuit, the voltage remaining constant. This extra resistance involves a waste of energy equal to that which is lost in heating it, whereas in an alternating current circuit we have an economical device for reducing the current without involving any waste of energy. The device is to place in the ciruit a coil of large inductance L and small resistance R. The current in the circuit is

$$i = \frac{E_0}{\sqrt{\{R^2 + (L\omega - 1/C\omega)\}^2}} \cdot \sin(\omega t - \phi),$$

$$\text{where } \tan\phi = \frac{L\omega - \dfrac{1}{C\omega}}{R}.$$

If L is large and R small $\phi \to \pi/2$ and $\cos\phi \to 0$. Thus the current is reduced, and the power wasted is negligible. (since $P \to 0$)

Such a coil of large inductance and small resistance, which has the property of reducing the current without any waste of electrical energy, is called a choke. The following example will illustrate the use of a choke.

An electric lamp, which runs at 100 volts D. C. and 10 amps. current is connected to 200 volt A.C. mains. Calculate the inductance of the choke. The A.C. supply has a frequency of 50 cycles per second.

Resistance of this lamp is $\dfrac{100}{10} = 10$ ohms.

The current i_0 is given by

$$i_0 = \frac{E_0}{\sqrt{(L^2\omega^2 + R^2)}},$$

i.e. $$10 = \frac{200}{\sqrt{\{L^2(2\pi.50)^2 + 10^2\}}}.$$

Hence $$L = \frac{\sqrt{300}}{2.\pi.50} = 0.06 \text{ henries nearly.}$$

N.B.—If the lamp is to be used in a D. C. supply, the resistance required is $\dfrac{200-100}{10} = 10$ ohms.

In this case there will be a waste of electrical energy equal to $10.(10)^2$ watts.

ALTERNATING CURRENT MEASURING INSTRUMENTS

1. Principle of A. C. measuring instruments. A direct current reading voltmeter or an ammeter cannot be used on an alternating current supply; the reason is that the a.c. currents and voltages are varying so rapidly that only their mean values are indicated. The mean value, as can be seen, comes out to be equal to zero. The mean value of $E_0 \sin \omega t$ in one complete cycle is

$$\overline{E_0 \sin \omega t} = \frac{\int_0^{2\pi/\omega} E_0 \sin \omega t \, dt}{\int_0^{2\pi/\omega} \omega t \, dt} = 0. \qquad \ldots \quad (16{\cdot}1)$$

During first half of the cycle the mean value is positive and during the remaining half it is negative; the resultant in one complete cycle is zero. Therefore an alternating current instrument should be such that the deflections due to either current or e.m.f. should depend upon the square of the current and the square of the e.m.f. respectively, because in that case the mean value is not zero. The mean value of $E_0^2 \sin^2 \omega t$ is given by

$$\overline{E_0^2 \sin^2 \omega t} = \frac{E_0^2 \int_0^{2\pi/\omega} \sin^2 \omega t \, dt}{\int_0^{2\pi/\omega} \omega t \, dt} = \frac{E_0^2}{2}. \qquad \ldots \quad (16{\cdot}2)$$

The hot wire and the soft iron ammeters are instruments of this type.

2. Hot wire Ammeter. The action of the instrument depends upon the thermal expansion of a wire when a current i flows in it. In fig. 16·1 the current enters at A and leaves at B.

The wire W is kept taut by means of a spring S, one end of which is fixed at D and the other end is connected to another wire, which, after passing over the pulley P is joined to the main wire W. The pulley P is connected to a pointer which moves on a calibrated scale. On account of the heating by the current, the wire W becomes slack. The slack is taken up by the spring S. The movement of S rotates the pulley which in its turn moves the pointer on the scale. The electrical energy causing expansion is Ri^2 and if θ is the deflection of the pointer, then

Fig. 16·1. Hot wire ammeter

$$Ri^2 = k\theta, \qquad \cdot \ \ \cdot \ \ \cdot \quad (16\cdot3)$$

where k is some constant.

The scale is calibrated with the help of some standard instrument. The disadvantages of the instrument are :

(1) The scale, is not linear since θ is proportional to i^2.

(2) The mechanical construction is a source of uncertainty. A great advantage of the instrument is that it can be used both for the measurement of alternating and direct currents.

3. Soft iron ammeter. It consists of two soft iron rods P and Q lying inside. and para llel to the axis of a solenoid wound on a brass cylinder. One of the rods is fixed to the brass frame and the other one
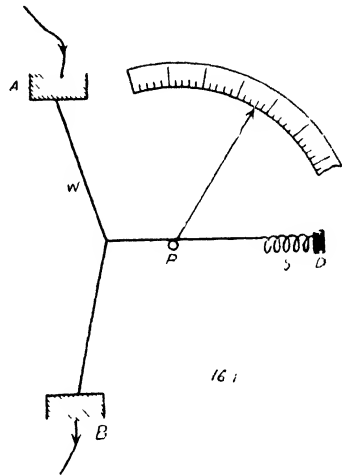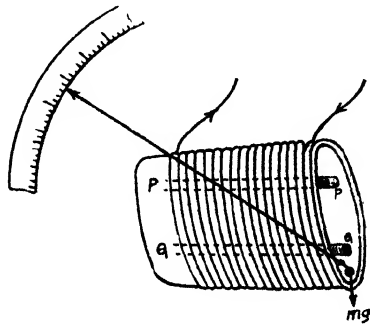

Fig. 16·2.

is pivoted on the axis of the solenoid. The movable rod Q is connected to a pointer moving on a calibrated scale. The movement of the rod is limited by a counterweight *mg*. The rods are magnetised when the current pasess through the solenoid with the two ends P and Q having the same polarity. The force of repulsion between the two ends is proportional to their pole strength which in turn is proportional to the current flowing in the solenoid, and hence the force is proportional to the square of the current.

The hot wire and soft iron ammeters can be used as voltmeters if a high resistance is introduced in series with the hot wire and the solenoid respectively.

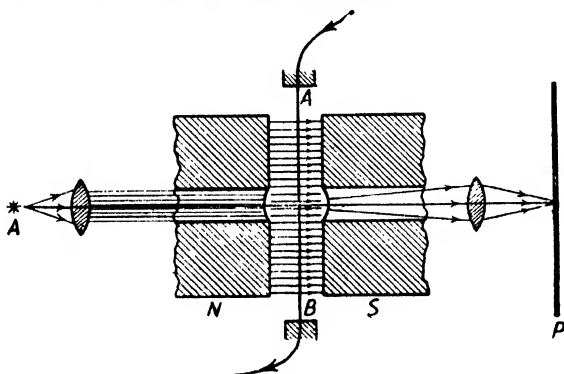**4. Einthoven string Galvanometer.** This instrument is

Fig. 16·3 (*a*). Einthoven string galvanometer

specially suited for the detection of very feeble alternating currents of high frequency and of short duration. It consists of a straight wire AB, usually of silvered quartz held in tension between the pole pieces of a powerful electromagnet NS, fig 16·3 (*a*). The pole pieces are drilled so that a beam of light from the source L throws a shadow of the wire on a photographic film P travelling parallel to the wire. When a discharge passes through the wire, a force perpendicular both to the wire

Fig. 16·3 (*b*)

and the magnetic field acts on it tending to deflect it towards the reader. The photographic record will be a succession of peaks on a straight line base, as shown in fig. 16·3(*b*). The height of a peak gives the magnitude of the corresponding discharge.

This galvanometer can also be used to measure small direct currents. When used for direct current measurement the deflections of the fibre are measured by a micrometer eye-piece sliding in the slot of the magnet.

5. Duddell Oscillograph. The main purpose of this instrument is to determine the wave form of an alternating e.m.f. or current. It is essentially a dead beat$_t$ galvanometer with a very high frequency of vibration. It consists of a fine strip of phosphor-bronze which passes over a pulley P and is placed between the pole pieces of a permanent magnet. The two free ends of the strip are screwed at terminals T, T. A small mirror is attached to the strip, which is kept in considerable tension by pulling the pulley upwards by means of a spring. When the current flows, each vertical portion of the

Fig. 16·4 Duddel oscillograph

strip is acted upon by an equal and parallel but oppositely directed force, thus · producing a couple. The rotation of the mirror is indicated on a screen by a beam of light reflected from it.
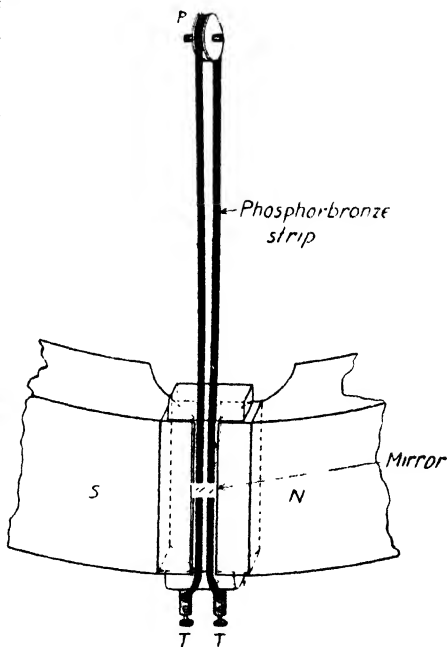
In order to determine the actual wave form, the spot of light after reflection from M is thrown or another mirror which rotates

about an axis perpendicular to that of the mirror M. The motion of the auxiliary mirror is controlled by a synchronous motor driven by the alternating current whose wave form is to be examined. We have thus superimposed two perpendicular vibrations of the same frequency giving us the required wave form.

If a permanent record of the wave form is desired, the auxiliary mirror is replaced by a photographic film mounted on a rotating drum driven by the synchronous motor.

6. Vibration galvanometer. The vibration galvanometer is mainly used for the detection of minute alternating currents of low frequencies. This instrument is similar to D' Arsonval galvanometer. It consists of a small coil carrying a very small mirror and suspended between the poles of a permanent magnet. It, however, differs in the method of suspension from the ordinary galvanometer. The coil is held in position by a taut phosphor-bronze ribbon, the effective length of which can be varied by means of a movable bridge carried on the upper screw. The tension in the ribbon can be varied by means of the lower screw and the spring. The length and tension in the ribbon are so varied that the period of mechanical oscillations of moving system is equal

Fig. 16·5. Vibration galvanometer

to the period of the alternating current in the coil. This is known as the tuning of the system. From the theory of forced oscillations we know that if the moving system is in resonance with the impressed force, the amplitude of the vibration is maximum. Thus the instrument will show practically no response to alternating currents to which it is not tuned. On the other hand the response will be great for currents for which it is tuned. The image
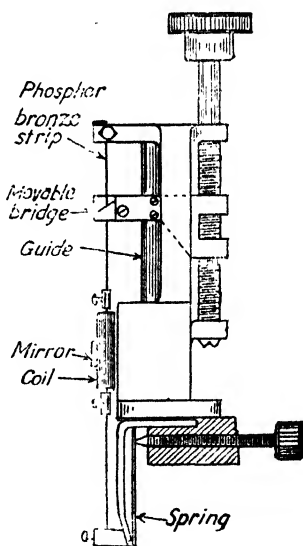
of an incandescent filament lamp is reflected by the mirror on a scale. When the system vibrates, the image is drawn out into a band whose length indicates the amplitude, while a slight movement of the coil is detected by a diminution in the sharpness of the line. The current sensitivity of the instrument is of the order of 10^{-6} amperes.

7. Wattmeter. It is an instrument for measuring power.

The power consumed in an A.C. circuit is given by the product of effective voltage and effective current and power factor. On account of the presence of the power factor the ammeter and the voltmeter method of measuring
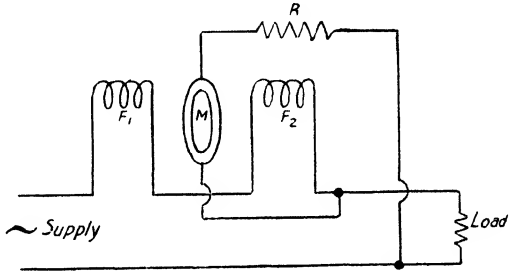

Fig. 16·6. Wattmeter

power cannot be used. It consists of a ·movable coil M, wound with fine wire and connected with the high resistance R in parallel with the load. The two fixed coils F_1 and F_2 are wound with a few turns of thick wire and are connected in series with the load. The current in the fixed coils is the current flowing in the load, whereas the current in the movable coil M is proportional to the voltage across the load. Therefore the couple on M at any instant is proportional to the product of the voltage and the current at that instant. The moving coil deflexion will be proportional to the average torque which is given by

$$\overline{C} = K\, E_e.\; I_e \cos \phi.$$

Hence $\theta = K_1\, E_e.\; I_e \cos \phi,$ . . . (16·4)

where K_1 is a constant.

CHAPTER XVII

ALTERNATING AND DIRECT CURRENT MACHINERY

1. Introduction. Faraday's discovery of electromagnetic induction opened up a vast field of electrical engineering with its profound influence on human civilization. Electrical machinery is broadly classified as (1) Generators and (2) Motors. Generators produce electrical energy by rotation of conductors in a magnetic field, while motors yield mechanical energy by the action of a magnetic field upon electric currents in conductors. Further, the machines are called alternating current when periodically reversing voltages and currents are involved and are called direct current when the voltages and currents do not change sign. Electrical engineering is a vast technical subject in which fundamental physical principles of electromagnetic induction are applied. We shall consider briefly in this chapter these applications to some simple types of alternating and direct current machinery, not from the point of view of the practical engineer, but from that of a student of general electricity.

2. Alternators. The essential principle underlying the construction of an alternator will be clear from fig. 17·1 (a). ABCD is a conducting loop arranged to rotate in a magnetic field NS about an axis (see dotted line) perpendicular to the field. The two ends of the loop are connected to two smooth metallic slip rings r_1, r_2. Connection to the external circuit is made by carbon brushes $b_1 \, b_2$ pressing lightly against r_1, r_2. The magnetic field N S is produced by passing a current through field coils wound on pole pieces N and S. We
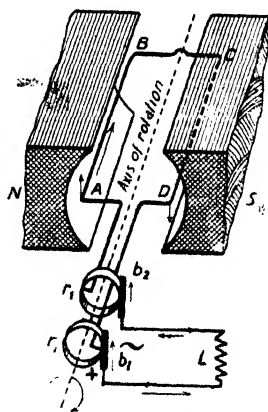

Fig 17·1 (a)

saw in Ch. XV Art 1, that when the conducting loop is made to rotate with a uniform angular velocity ω, an e.m.f. is set up in the
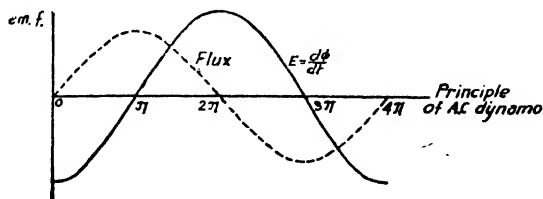

Fig. 17·1 (b)

loop due to increasing and decreasing flux through the loop given by $E = \omega\phi_m \sin \omega t$, where ϕ_m is the maximum flux through the coil, fig. 17·1 (b). When the circuit is closed through L, a current $I = Io \sin \omega t$ flows through it. Thus we get an alternating e.m.f. and current from this device, which is therefore called an alternator. The direction of e.m.f. can be easily determined by applying Fleming's right hand rule. The frequency of alternation for the two pole alternator of fig. 17·1 (b) is equal to the number of revolutions of the armature per second. If the alternator has more than one pair of poles, as is actually the case, the frequency of alternation is equal to the product of the revolutions of the armature per second and the number of pole pairs. For example, for a 50 cycle alternating current and ten poles the number of revolutions of the armature per second is 10.

The simple one loop alternator described above is not adequate for producing e.m.f.'s of practical utility. Actually an alternator armature consists of several groups of loops arranged in series. The distance between two groups is equal to that between two poles. The armature is so wound that the e.m.f.'s induced in all coils act in the same direction. Further, the density of the magnetic lines of force is increased by winding the armature coils on an iron cylinder and by suitably shaping the magnetic pole face so as to reduce the reluctance of the magnetic circuit to a minimum. In this way, using moderate speed of rotation, high e.m.f.'s are generated. If n is the number of coils each of one turn, the e.m.f. is now given by $E = 2\pi n \nu \phi_m \sin 2\pi\nu t$ with a corresponding expression for the current. When the armature rotates in a magnetic field, eddy currents are

generated in the iron core which cause heating. To prevent these eddy currents armature cores are laminated, that is, they are built up of, large number of thin iron discs separated from each other by very thin paper or varnish.

There are three principal classes of alternators : (1) Those in which the armature rotates (i.e., it is the rotor) and the field is at rest (i.e., it is the stator), (2) those in which the field is the rotor and the armature is the stator and (3) inductor type machines.

The principle of the machine of class (1) has already been described. In machines belonging to class (2) the rotating magnet coils are wound on poles projecting from a hub and are supplied direct current by slip rings; the stator armature is wound in slots on the inner side of a cylinder enclosing the rotor. The alternating current is generated in this and is simply taken off from its two extremities. This is the most common form of alternator and is very convenient for generating high voltage alternating current, since at the slip rings and brushes there is only low voltage d.c. supply for actuating the field magnets, and it is easy to insulate the stator winding. The inner arrangement of a machine of this type is diagrammatically represented in fig. 17·2.

Fig 17·2

In the inductor type of machines the armature and the field magnet windings are wound on projections inside the stator. The rotor simply consists of a drum with iron or steel projections.
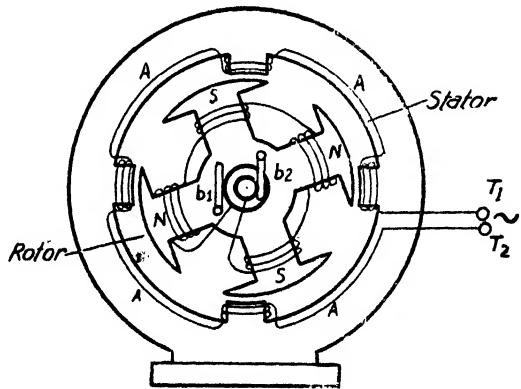
Polyphase Generators. The machine illustrated in fig 17·2 is a single phase generator. It produces between its terminals only one e.m.f. of the form $E = E_0 \sin \omega t$. A polyphase generator has a number of independent coils wound with a uniform spacing around the armature frame. The number of phases of the generated voltage is equal to the number of these coils. The most common form of polyphase generator is a three phase generator shown diagrammatically in fig 17·3. The e.m.f's generated are given by
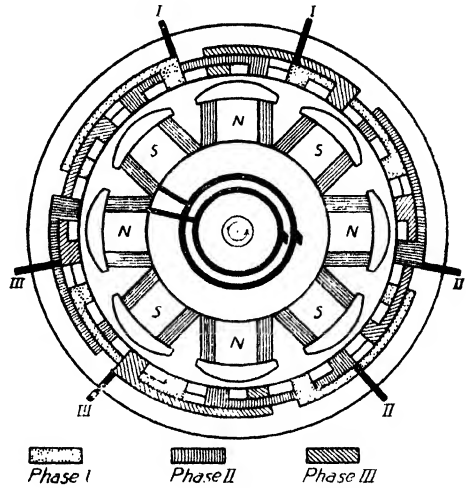

Fig. 17·3

$E_1 = E_0 \sin \omega t, \; E_2 = E_0 \sin \left(\omega t + \dfrac{2\pi}{3} \right)$ and $E_3 = E_0 \sin \left(\omega t + \dfrac{4\pi}{3} \right)$.
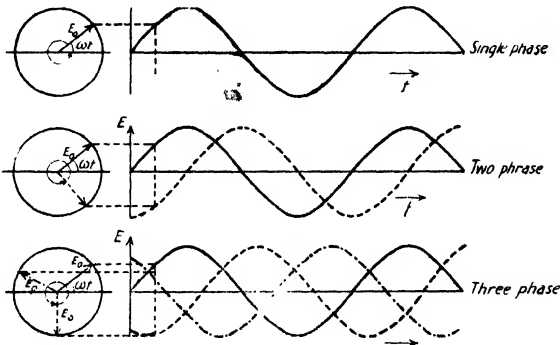

Fig. 17·4

These are shown in fig. 17·4. For transmission of power from a three phase machine four wires are generally required, one end of each phase being connected to a neutral.

3. A. C. Motors. Synchronous Motors. An A. C. generator can be reversed in function to serve as a motor.   Such a machine is called a synchronous motor.   Its speed of rotation is such that if used as a generator it would produce an e.m.f. of the same frequency as the voltage actually applied to its two terminals.   It requires, of course a constant d.c. current to excite its field.   The speed of the synchronous motor is given by the frequency of the applied voltage divided by the number of pole pairs and is thus constant. It is, therefore, used whenever a constant speed is the essential requirement.   With the help of an auxiliary winding the motor is first started and speeded up to the requisite speed which is then maintained by the applied a.c. voltage of suitable frequency.

Induction motors. In this type of a.c. motors the stator is similar to that in a synchronous motor; but the rotor consists of a core with some closed windings on it.   The rotor is energized by induction and not by any auxiliary current.   This is a great practical advantage.

A polyphase supply produces a rotating magnetic field in the region occupied by the rotor.   The reaction between the eddy current generated in the rotor and the rotating magnetic field gives rise to a torque resulting in the rotation of the motor.

4. Direct-current generators or dynamos.   These are machines which produce e.m.f's, and currents which do not change sign.   The principle of a single coil dynamo of this type is represented in fig. 17·5.   The slip rings of the alternator in fig. 17·1 (a) are replaced by a split ring constituting what is called the commutator, the two halves C_1, C_2 of which are connected to the two ends of the coil and rotate with it.   Current to the external circuit L is taken by means of two fixed carbon brushes b_1 , b_2 rubbing against the split ring.   It is clear from the figure that every time the direction of
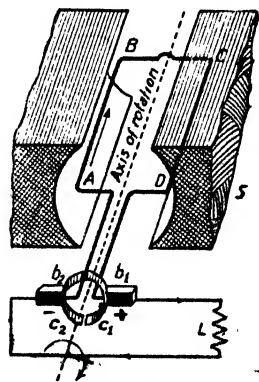

Fig. 17·5

the induced e.m.f. changes, the connections to the ends of the coil
are reversed.   The current sent through the external circuit is thus
unidirectional but sinusoidal and of varying value, even dropping
to zero twice during one rotation, fig. 17·6(a). Such wide variations
are considerably reduced and the current is made more constant,
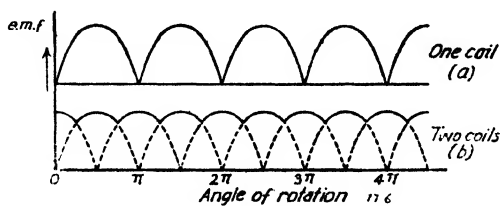as shown in fig. 17·6 (b), by using two coils mutually at right

Fig. 17·6

angles to each other and connecting their ends to the commutator
ring which is now divided into four sections, fig. 17·7. By
increasing the number of coils and corresponding sections of the
commutator the current can be further made steady.
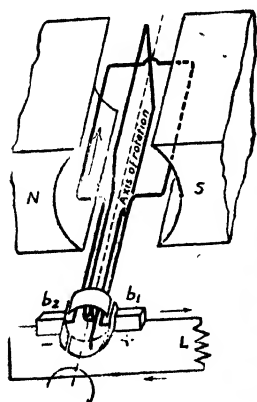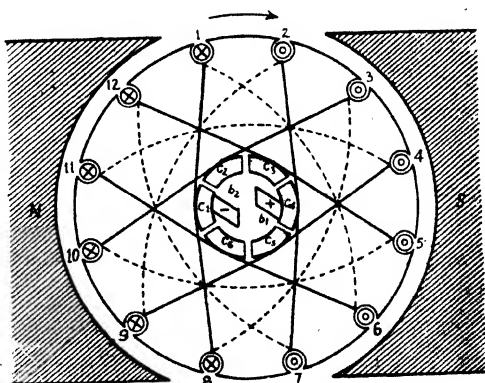

Fig. 17·7

Fig. 17·8

In modern generators the coils are not separate but are all in
series arrranged around the surface of a rotating drum like a yarn
on a ball. The method of winding will be clear from fig. 17·8,
which shows diagrammatically a six-coil armature . The conduc-

tors 1, 2, 3,...12 are arranged along the surface of a drum. The solid lines represent connections across the other end. Starting
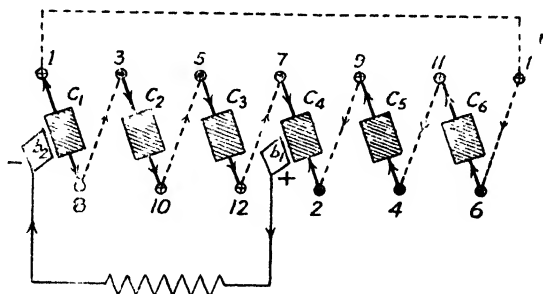

Fig. 17·9

from the conductor 1 we go to conductor 8; down 8 to 3; up 3 to 10 and on so via remaining conductors back to 1. The commutator rotating with the armature is divided into six equal sections C_1, C_2, etc. These are connected to the midpoints of the cross connections (1,8), (3,10), etc. Fig. 17·9 represents more simply the continuous winding described above. As can be easily seen from this figure, the induced e.m.f. between the brushes is equal to the sum of the e.m.f.'s induced in the coils 8, 3, 10, 5, 12 and 7 or in 2, 9, 4,11, 6 and 1. Thus voltages higher than those with the separate coils of the simple generators described previously are obtained.

The induced armature current produces a magnetic field perpendicular to the exciting field. The direction of the resultant field is thus changed and the brushes must be brought in alignment with this. This difficulty is avoided by using subsidiary electromagnets in series with the armature winding which neutralize the field due to the armature current.

E. M. F. of a dynamo. If ϕ is the flux per pole, a conductor of the armature cuts 2ϕ lines of force as it passes from a north pole to a succeeding south pole. Assuming that there are p pairs of poles round the armature which makes N revolutions per second, the rate of change of flux per conductor becomes $2pN\phi$ and so the induced e.m.f. is $2pN\phi \times 10^{-8}$ volts. If there are n conductors, the e.m.f. of the dynamo is given by $E = 2pNn\phi \times 10^{-8}$ volts.

When the dynamo is running on open circuit, the mechanical power supplied to the shaft has to overcome only the friction at the bearings, air resistance, etc.   When, however, the electrical circuit is closed, a current flows through the armature conductors which, due to their rotation in the magnetic field, experience an electro-magnetic torque according to Fleming's left-hand rule. The mechanical power supplied has to overcome this.   If T is this torque in foot pounds, N the rotations per second, E the e.m.f. generated and I the current in the armature circuit, we have

$$\frac{2\pi NT}{550} \ (i.e. \ \text{mechanical power})$$

$$= EI/746 \ (i.e. \ \text{electrical power})$$

Hence,
$$T = \frac{550 \cdot EI}{2\pi N \times 746},$$

$$= \frac{550 \times 2p Nn\phi \times 10^{-8} \times I}{2\pi N \times 746}.$$

Thus T varies directly as the flux and the armature current and is independent of the armature speed.

5. Classes of Dynamos.   There are two principal classes of dynamos.   Those in which the magnetic field is due to a perma-nent magnet and those in which it is due to an electromagnet. As strong fields by permanent magnets cannot be produced, only small dynamos with permanent magnet can be constructed.   The dynamo is then called a *magneto*. In all large machines the magnetic field in which the armature rotates is produced by an electromagnet. They are usually self-excited *i.e.*, the current necessary for the electromagnet is produced by the machine itself.   This is possi-ble due to the retentivity of iron and steel on which the coil of the electromagnet is wound.

Any small residual magnetism generates an e.m.f. in the armature when it is set rotating; and if the field coils are properly connected so that the residual field is increased, the e.m.f. will steadily mount up.

17 E

Types of self-excited dynamos.   (1) Series-wound.   In
this dynamo the field and armature
coils are connected in series so that
current through them is the same
as that in the external circuit.   The
windings consist of thick wire suffi-
cient to stand large currents.   Thus
the e.m.f. of such a dynamo varies
with the current drawn and so is
useless for most of the purposes
where a constant supply of e.m.f.
is essential.   This machine is repre-
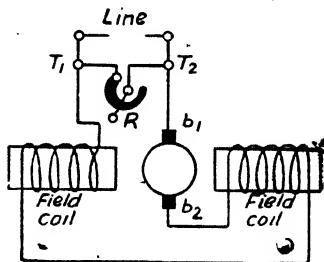sented schematically in fig. 17·10.

Fig. 17·10

The external characteristics of a series wound dynamo are
shown in fig. 17·13, curves 1, 2,
and 3.   Curve 1 shows the induced
e.m.f. E, for various armature and
field currents.   Curve 2 gives the
ohmic loss IR due to the combi-
ned resistance R of the armature
and field coils for a current I.
There is also some amount of
loss V_B due to the resistance of
brushes which is not shown in
the figure.   The out-put voltage

Fig. 17·11

is thus given by curve 3 which is obtained by subtract-
ing curve 2 from curve 1 (assuming $V_B=0$).   If there is
appreciable loss at the brushes, the out-put voltage is given
by $V=E-IR-V_B$.   From curve 3 we see that the out-put
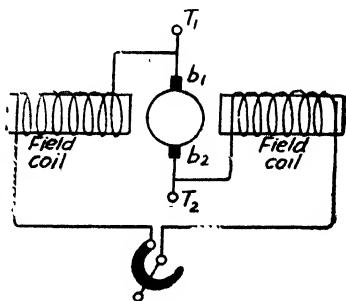voltage goes through a maximum value as the load current
is increased.

(2) Shunt wound.   In this the field magnet windings are
connected as a shunt to the external circuit so that a part of the main
current serves as the magnetizing current.   A shunt-wound dynamo

is shown schematically in fig. 17·11 and its external charac-
teristics in fig. 17·13 curve (a)
which shows how the terminal out-
put voltage varies with the load, i.e.,
with the current drawn from the
machine into the external circui$_t$
for some particular speed of rotation.
The terminal voltage steadily falls
down with the increase in load and

Fig. 17.12

for very large loads it drops to zero. The field current $I_f = V/R_f$
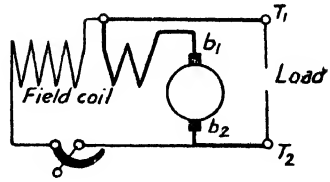then drops to zero and the dynamo ceases functioning.

(3) Compound wound. This is a combination of the
series and shunt dynamos designed to obtain advantages of both.
The connections are shown schematically in fig. 17·12 and the

external characteristic in fig.
17·13, curve (b). It is seen that
the terminal voltage is almost
independent of load. The
series wound has got rising
characteristic and the shunt
wound falling characteristic
and thus a suitable combi-
nation can be found to obtain
a level characteristic.
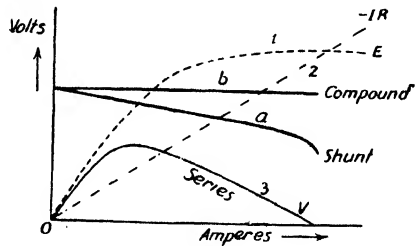
Fig. 17·13

6. Efficiency of dynamos. A portion of electrical power
generated for a given supply of mechanical power is always lost
due to (1) Ohmic resistance of the armature and field windings
and the brushes (Copper losses), (2) hysteresis in the armature
and pole pieces and eddy currents (Iron losses) and (3) friction
at the bearings and air resistance (Mechanical losses). As a result,
the available electrical energy is always less than the total elec-
trical power generated. The ratio of total electrical power to
mechanical power supplied is called *mechanical efficiency*. The
ratio of the available electrical power to the total electrical power
is called *electrical efficiency*. The ratio of available electrical

energy to the mechanical power supplied is called *commercial efficiency*. It easily follows from these definitions that commercial efficiency is equal to the product of mechanical and electrical efficiencies. In a good dynamo the commercial efficiency is as high as 95%.

EXAMPLE. What is the horsepower required to drive a 200 kilowatt dynamo when giving its full-rated load if the machine has got full load efficiency of 90%.

Now, commercial efficiency $= \dfrac{\text{power developed in watts}}{\text{Horsepower} \times 746}$,

since 1 horsepower $= 746$ watts. Therefore, from the data we have

$$\frac{90}{100} = \frac{200 \times 1000}{\text{H.P.} \times 746},$$

or   H. P. $= 297\cdot8$ approximately.

7. D. C. motors. Principle of a d. c. motor—Like a. c. motors we have d. c. motors which are the reverse of dynamos. Electrical energy supplied is converted into mechanical energy. Any d. c. dynamo can work as a d. c. motor but the actual design of a motor has got some special features.

The simplest type of a d. c. motor consists of a single coil placed between the poles of an electromagnet as shown in fig. 17·14. With b_1 as the positive terminal of the input voltage the coil rotates in the clockwise direction. This can be easily seen by applying Fleming's left hand rule to the conductor. The steady torque on the armature can be found out as follows :

If n is the number of turns of the coil, H the field, A the area of each coil and θ the angle made by the normal to the plane of the coil with the field we have

Fig. 17·14
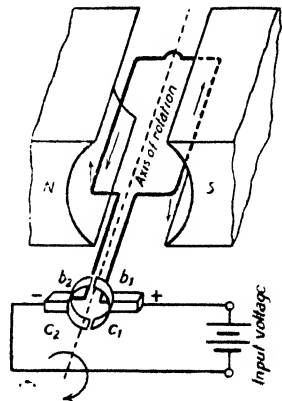
the flux $N = nAH \cos \theta$,

and the torque

$$\tau' = i \ \frac{d\mathrm{N}}{d\theta} = - ni\mathrm{AH} \sin \theta.$$

The steady torque is

$$\tau = - ni\mathrm{AH}, \qquad \cdots \quad (17\cdot1)$$

the minus sign indicating that the steady torque is in a direction opposite to that in which θ is measured, i.e., in the clock-wise direction.

As the coil rotates in the magnetic field an induced e.m.f. $E_B = -\dfrac{d\mathrm{N}}{dt} = n\mathrm{AH} \sin \theta \ \dfrac{d\theta}{dt}$ is produced. The steady value of this for $\theta = \frac{1}{2}\pi$ (when the plane of the coil and the position of the split rings C_1, C_2 are as shown in the figure) is given by

$$E_B = n\mathrm{AH}\omega, \qquad \cdots \quad (17\cdot2)$$

where $\omega = \dfrac{d\theta}{dt}$.

As the coil rotates in the same direction as the torque, ω is negative and therefore E_B negative. This induced e.m.f. E_B is called the *back electromotive force* of the motor opposing the impressed voltage V. If R is the resistance of the circuit and i the current

$$V - E_B = Ri, \qquad \cdots \quad (17\cdot3)$$

by Ohm's law. The armature resistance R is kept low to avoid copper losses, so that if the full running voltage V is applied to the motor when the armature is at rest, the resultant armature current will be so large that the armature windings may be damaged. For this reason a starting
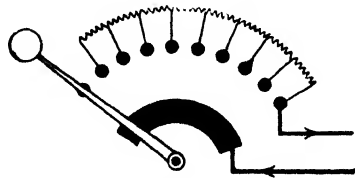
Fig. 17·15

resistance as shown in fig. 17·15 is used in the armature coils which is gradually cut down as the motor speeds up. From eqs. (17·1) and (17·2) we have

$$\text{Torque } \tau = i \times E_B/2\pi r, \qquad \cdots \quad (17\cdot4)$$

where r is the number of revolutions per second.

Eq. (17·4) shows that the torque is directly proportional to the product of the armature current and the back e.m.f. and inversely proportional to the number of revolutions of the motor.

Efficiency of a motor. This is defined as in the case of dynamos by the ratio of mechanical power output to the electrical power input. Thus if V is the applied voltage, i_a the current in the armature, R_a the armature resistance and E_B the back e.m.f. we have by eq. (17·3)

$$V i_a = R i_a{}^2 + E_B i_a.$$

In this $R i_a{}^2$ represents copper losses and $V i_a$ is the power supplied. Thus the balance $E_B i_a$ represents the electrical equivalent of mechanical power output.

Thus η the efficiency of a motor is given by

$$\eta = E_B i_a / V i_a = E_B / V. \quad . \quad . \quad (17·5)$$

Eq. (17·5) shows that η increases as E_B approaches V, i.e., for small values of the armature current. Thus the efficiency of a motor is very large when it is running light and taking very little power from the mains and falls off as the load increases.

Motors in practice. Shunt and series-wound motors. The simple type of machine described above is not useful in practice as it will rotate with jerks. For smooth running a drum armature like that in dynamos must be used. Like dynamos there are three principal types of motors : Series wound, Shunt wound and Compound wound. In a series wound motor the same current flows through the armature and field coils, fig. 17·16. From eq. (17·4) we see that a larger torque means a larger current and a smaller number of revolutions since E_B may be considered to be constant. When the motor is just started without any load the torque is small and therefore r large. It may be so large that the commutato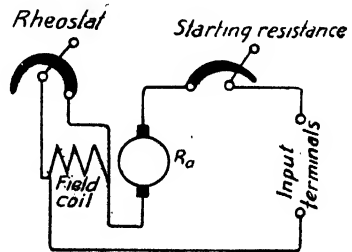r and armature may be damaged. In order to avoid such dangerous initial speeds a series motor should never be started without any load. On the other hand, a series motor develops a very large starting torque by keeping the speed small

Fig. 17·16

and adjusts its speed to changes in load. For this reason it is of great use in traction work in which power is required.

In a shunt motor the field and the armature coils are in parallel, fig. 17·17. Now since the field H is proportional to i the current in the field coil, eq. (17·1) shows that if i_f diminishes τ diminishes ; and from eq. (17·4.) we see that if τ diminishes r increases. Thus diminution of field current increases the speed, and vice-versa, other factors being constant. Also, since $\tau \propto i_a \times i_f$, [see eq. (17·1)]

Fig. 17·17

we see that if i_f is constant $\tau \propto i_a$ and therefore by eq. (17·4) r is constant since E_B is almost constant. Thus a shunt motor may be taken almost as a constant speed machine for varying values of load. It is used where constant speed is required when load is varying, for example, with a lathe.

The characteristics of a shunt and series motor are shown in fig. 17·18. The starting torque of a shunt motor is not so great as that of a series motor ; the motor should therefore be allowed to pick up speed before load is put on, otherwise the large armature current during the speeding up of the motor would give rise to undue heating effects.

Fig. 17·18

EXAMPLES :—

(1) A direct current shunt motor having 90% efficiency and 10 H. P. is connected to 240 volts mains. If the field current is 1 amp. and armature resistance 0·2, find the power input and the back e.m.f. and also the torque exerted if the speed is 600 r.p.m.
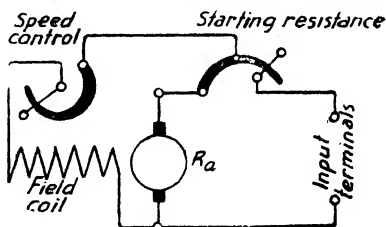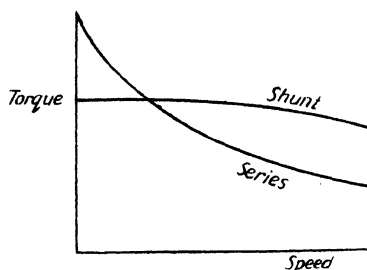
(i) $$\text{Efficiency} = \frac{\text{Power output}}{\text{Power input}} = \frac{90}{100}$$

$$\text{Power output} = 10 \text{ H. P.}$$
$$= 7460 \text{ watts}$$

$\therefore$ Power input $= \dfrac{100}{90} \times 7460$ watts.

$$= 8\cdot3 \text{ k. watts}$$

$\therefore$ Total current in the motor $= 8\cdot3$ k. watt / 240 **volt** $= 34\cdot6$ amps. and $i_a = 34\cdot6 - 1 = 33\cdot6$ amps.

Also if E_B is the back e.m.f., V the applied voltage, i_a the armature current and r_a the armature resistance we have
$$V - E_B = i_a \times r_a.$$

Putting V $= 240$ volts, $r_a = 0\cdot2 \, \Omega$ and $i_a = 33\cdot6$ amps. we have

$$E_B = 240 - 33\cdot6 \times 0\cdot2$$
$$= 240 - 6\cdot72 = 233\cdot28 \text{ or } 233\cdot3 \text{ volts nearly.}$$

Thus power input $= 8\cdot3$ k. watts,
and back e.m.f. $= 233\cdot3$ volts

(ii) $$\text{Torque} = \frac{\text{armature current} \times \text{back e.m.f.}}{2\pi \times \text{revolutions}}$$

$$= \frac{33\cdot6 \times 233\cdot3}{2\pi \times 10} \times 10^{-1} \times 10^8 \text{ dynes. cm.}$$

Dividing by $g = 980$, we have
$$\text{torque} = 12\cdot7 \text{ k. gram.metre}$$

(2) In the motor of the preceding example what will be the starting resistance to be put in series to limit the starting current to its full load value ?

If R is the starting resistance we have
applied voltage $240 =$ full load armature current $\times$
(armature resistance $+ R$),

$\therefore$ putting the full load current $= 33\cdot5$ amps. and armature resistance $= 0\cdot2 \, \Omega$, we have $R = 6\cdot93 \, \Omega$.

(3) A d.c. motor whose armature resistance is $0\cdot2$ ohm **takes** an armature current of 10 amperes when working at 1500 r.p.m. from 200 volts mains.   Find (a) efficiency of the machine, (b)

available horse power when the speed of the machine drops to 1200 r.p.m., (c) efficiency of the machine for the speed in (b).

(a) $\text{Efficiency} = \dfrac{\text{Power obtained}}{\text{Power supplied}}.$

Now   Power supplied = Voltage × armature current

$= 200 \times 10 = 2000$ watts.

Copper losses in armature = (armature current)^2 ×

(armature resistance)

$= 10^2 \times 0.2$

$= 20$ watts,

∴ available power = $2000 - 20 = 1980$ watts

and so $\text{efficiency} = \dfrac{1980}{2000} \times 100$

$= 99\%.$

(b) When speed is 1500 r. p.m.

back e.m.f. $(E_B) = n\text{AH} \times 2\pi \times r$, eq. (17·2)

$= n\text{AH} \times 2 \times 1500.$

When speed is 1300 r. p.m.

back e.m.f. $E_B = n\text{AH} \times 2\pi \times 1300$

$= \dfrac{E_B \times 1300}{1500}.$

Now $E_B =$ applied voltage − armature

current × arm. resistance

$= 200 - 10 \times 0.2$

$= 198$ volts,

∴ $E'_B = \dfrac{198 \times 1300}{1500}$

$= 171.6$ volts.

For this back e.m.f.

$\text{armature current} = \dfrac{\text{applied voltage} - E'_B}{\text{armature resistance}}$

$= \dfrac{200 - 171.6}{0.2}$

$= 142$ amps.

Power supplied $= 142 \times 200$

$= 28400$ watts.

$$\text{Copper \quad losses} = 142 \times 142 \times 0\cdot2$$
$$= 4032\cdot8 \text{ watts.}$$
$$\text{Power available} = 28400 - 4032\cdot8$$
$$= 24367\cdot2 \text{ watts}$$
$$= \frac{24367\cdot2}{746}$$
$$= 32\cdot6 \text{ H.P.}$$

(c) $$\text{Efficiency} = \frac{\text{Power available}}{\text{Power supplied}}$$
$$= \frac{24367\cdot2}{28400} \times 100$$
$$= 85\cdot9 \ \%$$

(3) A shunt wound dynamo having armature resistance $0\cdot2$ ohm and field coil resistance 40 ohms supplies a current of 20 amps. at 240 volts through an external circuit. Find (a) lost volts (b) e.m.f. of the dynamo.

(a) If E is the e.m.f. of the dynamo and V the terminal voltage we have lost volts

$$\text{E} - \text{V} = \text{armature resistance} \times \text{armature currents}$$
$$= 0\cdot2 \times (\text{field current} + \text{supply current})$$
$$= 0\cdot2 \times \left(\frac{240}{40} + 20\right) = 5\cdot2 \text{ volts.}$$

(b) $\text{E} = 5\cdot2 + \text{V} = 5\cdot2 + 240 = 245\cdot2$ volts.

(4) What is the horsepower required to drive a 200 k. watt dynamo when developing its full rated load, if the full load commercial efficiency is 90% (1 H. P. = 746 watts).

$$\text{Commercial efficiency} = \frac{\text{full wattage developed in external circuit}}{\text{horsepower necessary} \times 746}$$

$$\therefore \quad \frac{90}{100} = \frac{200 \times 1000}{\text{H.P.} \times 746}$$

$$\therefore \quad \text{H.P.} = \frac{2}{9 \times 746} \times 10^6$$

$$= 297\cdot8.$$

8.   D. C. motors and alternating current.   If the positive and negative terminals of a d.c. motor are interchanged it goes on rotating in the same direction as the directions of current

in the field and armature are both reversed. Thus a d.c. motor may be run by alternating current. The cores must, however, be laminated to take account of the alternations of the magnetization of the iron. Also due to the large self-inductance of the windings sparking occurs at the commutator and also a comparatively higher voltage is needed to obtain a given current when alternating current is used. To reduce this effect to minimum the air gap is made as small as possible, a compensating winding is used and frequency of alternating current is reduced. As such a motor is easily controllable it is used in electric railways, a frequency of only about 16 cycles per second being employed.

Another way to drive a d.c. motor with alternating current is to pass the a.c. through the field and compensating windings only, keeping the brushes short circuited. The reaction of the magnetic field with the induced current in the armature gives rise to the necessary driving torque. Because of this action a motor of this type is called repulsion motor. The same result can be obtained by skewing the short circuited brushes and passing the alternating current through the field windings alone.

9. Motor generators. Motor alternators. Rotary converters. We give here just a very brief account of these important devices of great practical utility.

Motor generators. The electric supply is sometimes of unsuitable voltage or may be d.c. when a.c. is required. In such cases the main supply is utilized to run a d.c. motor and the mechanical power so derived is used to run a d.c. generator or an alternator, as required for the particular conditions. Such an arrangement is called a motor generator or a motor-alternator according as the generated voltage is d.c. or a.c.

The motor and the generator may be two distinct machines with shafts coupled together or they may be built in one casting with separate pole pieces, armatures and commutators. In smaller machines they are in one casting with one set of pole pieces and one armature having two separate windings with its commutator

at either end. The motor used is shunt wound so that due to its practically constant speed the voltage of the generator is not dependent upon the vagaries of the supply. Most important uses of motor-generators are the low power supply for bells, telephones etc., for charging secondary batteries and for supplying steady currents at lower voltages to search-lights.

The motor-alternator. This consists of a motor and an alternator on one shaft. There is a starter for the motor, a field regulator for controlling the speed and another regulator for controlling the field current of the alternator. By these controls the speed of the alternator and hence its frequency can be controlled ; the regulator in the field of the alternator controls the flux and hence the a.c. voltage quite independently of frequency.

Rotary converter. This is a machine with one set of field magnets, usually shunt wound and excited with d.c. There is only one armature with its windings uniformly distributed.   This is connected to a commutator at one end and has tappings to two slip rings at the other end.   In a two pole machine these tappings are taken 180° apart while in a four pole machine they are taken 90° apart.   The following are the uses of this machine :—(1) A.C. generator. Supply d.c. at the commutator end and draw a.c. from the slip rings. (2) D.C. generators. Supply a.c. to the slip rings and draw d.c. from the commutator and (3) use it as a both d.c. and a.c. generator by revolving the armature by an engine.

10. Transformers. A transformer is an important and a convenient device for converting large alternating current at low voltage to small alternating current at high voltage and vice-versa. The former is called a step up and the latter a step down transformer.   The action is truly electromagnetic but there are no moving parts in a transformer as in the case of dynamos and generators.   Both these types are of great practical

importance especially in the systems of transmission of electrical power as will be seen in a succeeding article.   As a transformer is a static machine, it requires very little attention and its maintenance is low.   Its efficiency is high but the cost per k.w. is low when compared to other apparatus.   For very high voltages the problem of insulation is not so serious because the transformer windings can be immersed in oil.   All these factors go to make a transformer so important in modern industry.   It is responsible for the extensive use of alternating current.

Principle of a transformer.   A transformer consists essentially of two coils, a primary P and a secondary S, each wound separately on the two legs of a laminated iron core as shown in fig. 17·19. Actually the primary and secondary are
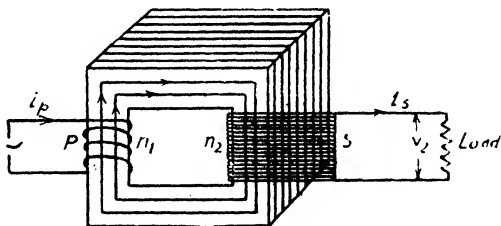

Fig. 17·19

wound on the same leg to reduce the magnetic leakage. Electrical energy is supplied to the primary, whence it is transferred to the secondary by electromagnetic induction.   For a step up transformer the number of turns in the secondary is much greater than that in the primary.   Actually the ratio of the secondary voltage to the primary voltage is equal to the ratio of the number of turns in the respective coils.   An ideal transformer is one in which the energy output in the secondary is equal to the energy input in the primary.   Actually there are many losses such as hysteresis and eddy current losses and the leakage of the magnetic flux.   The aim of the designer is to minimize these losses. The hysteresis loss is minimized by a suitable choice of the core material and the eddy losses are reduced by making the core laminated.   To reduce the leakage of magnetic flux both primary and secondary are wound on the same leg.

Consider the primary and the secondary as two separate circuits with self and mutual inductance, fig. 17·20. We shall first deduce a few relations which will be used later on. For a current i_p in the primary, the induced e. m. f. in P and S are given by
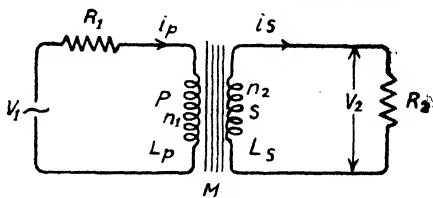

Fig. 17·20

$$e_p = - L_p \frac{di_p}{dt}, \text{ and } e_s = - M \frac{di_p}{dt}.$$

Hence
$$\frac{e_s}{e_p} = \frac{M}{L_p}. \qquad \ldots \quad (17·6)$$

Again for a current i_s in the secondary we have,

$$e_p = - M \frac{di_s}{dt} \text{ and } e_s = - L_s \frac{di_s}{dt}.$$

Hence
$$\frac{e_s}{e_p} = \frac{L_s}{M}. \qquad \ldots \quad (17·7)$$

From eqs. (17·6) and (17·7) we have
$$M^2 = L_p L_s. \qquad \ldots \quad (17·8)$$

In the deduction of eq. (17·8) we have assumed that there is no leakage of magnetic flux. Actually a fraction k is used in generating the induced e.m.f. and the remainder leaks out. If we take this factor into account, eqs. (17·6) and (17·7) will become respectively

$$M = k L_p \frac{e_s}{e_p}, \text{ and } M = k L_s \frac{e_p}{e_s}.$$

giving
$$\frac{M^2}{L_p L_s} = k^2, \qquad \ldots \quad (17·9)$$

k is known as the *coupling constant*, which is equal to unity when there is no magnetic leakage. Now, if n_1 and n_2 are the number of turns in the primary and secondary respectively we have

$$L_p \propto n_1^2, \qquad \ldots \quad (17·10)$$

and
$$L_s \propto n_2^2 \qquad \ldots \quad (17·11)$$

From (17·10) and (17·11) we have

$$\frac{L_p}{L_s} = \left(\frac{n_1}{n_2}\right)^2 = a^2, \qquad \cdots \quad (17\cdot12)$$

where a is called the *transformation ratio*.

If N denotes the flux per turn, the e.m.f's in the primary and the secondary are respectively

$$e_p = -n_1 \frac{dN}{dt} \quad \text{and} \quad e_s = -n_2 \frac{dN}{dt},$$

or

$$\frac{e_s}{e_p} = \frac{n_2}{n_1}, \qquad \cdots \quad (17\cdot13)$$

i.e., the ratio of the secondary to the primary voltage is equal to the ratio of the number of turns in the secondary to that of the primary.

The equations for the e.m.f's in the primary and secondary are respectively

and

$$\left. \begin{array}{l} L_p \dfrac{di_p}{dt} + M\dfrac{di_s}{dt} + R_1 i_p = V_1, \\[2mm] L_s \dfrac{di_s}{dt} + M \dfrac{di_p}{dt} + R_2 i_s = 0, \end{array} \right\} \quad \cdots \quad (17\cdot14)$$

neglecting the capacity of the two coils. Since the current is of the form $e^{j\omega t}$, $(j = \sqrt{-1})$, (17·14) takes the form

$$(jL_p\omega + R_1)\, i_p + jM\omega i_s = V_1, \qquad \cdots \quad (17\cdot15)$$

and

$$(jL_s\omega + R_2)\, i_s + jM\omega i_p = 0. \qquad \cdots \quad (17\cdot16)$$

Writing eqs. (17·15) and (17·16) in another form

$$V_1 = i_p Z_p + i_s Z_m, \qquad \cdots \quad (17\cdot17)$$

$$0 = i_s Z_s + i_p Z_m, \qquad \cdots \quad (17\cdot18)$$

where the impedances are

$$Z_p = jL_p\omega + R_1,$$
$$Z_s = jL_s\omega + R_2,$$

and

$$Z_m = j\omega M.$$

From eqs. (17·17) and (17·18) we have

$$i_p = \frac{V_1}{Z_p - \dfrac{Z_m^2}{Z_s}}, \qquad \cdots \quad (17\cdot19)$$

and
$$i_s = -\frac{Z_m}{Z_s} i_p. \qquad \qquad \cdot \; \cdot \; \cdot \; (17\cdot20)$$

The apparent impedance of the primary is

$$Z'_p = Z_p - \frac{Z_m^2}{Z_s}. \qquad \qquad \cdot \; \cdot \; \cdot \; (17\cdot21)$$

Thus the presence of the secondary adds a term $-Z_m^2/Z_s$ to the primary impedance.

Now, $-\dfrac{Z_m^2}{Z_s} = \dfrac{M^2\omega^2}{R_2 + jL_s\omega} = \dfrac{M^2\omega^2 R_2}{R_2^2 + L_s^2\omega^2} - j\dfrac{M^2\omega^2 L_s\omega}{R_2^2 + L_s^2\omega^2}.$

The apparent impedance Z'_p takes the form

$$Z'_p = R_1 + \frac{M^2\omega^2 R_2}{R_2^2 + L_s^2\omega^2} + j\left[L_p\omega - \frac{M^2\omega^2 L_s\omega}{R_2^2 + L_s^2\omega^2}\right]. \qquad (17\cdot22)$$

Thus the circuit equivalent to the above two circuits is shown in fig. 17·21.

The extra resistance term is

$$\frac{M^2\omega^2 R_2}{R_2^2 + L_s^2\omega^2}.$$

Since $L_s\omega$ is much greater than R_2, this reduces to

$$\frac{M^2 R_2}{L_s^2} = a^2 R_2.$$
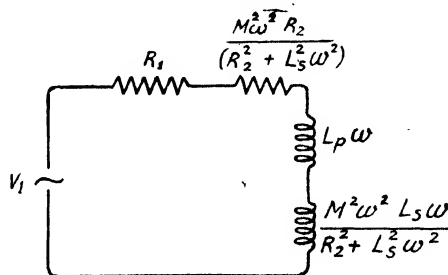

Fig. 17·21

The effective resistance in the primary circuit is, therefore,

$$R_1 + a^2 R_2. \qquad \qquad \cdot \; \cdot \; \cdot \; (17\cdot23)$$

The extra term in the reactance is $\dfrac{M^2\omega^2 L_s\omega}{R_2^2 + L_s^2\omega^2}$. Since $L_s\omega$ is much greater than R_2, using eq. (17·9) it reduces to $k^2 L_p\omega$, and so the apparent reactance becomes

$$(L_p\omega - k^2 L_p\omega) = L_p\omega(1 - k^2) = 2L_p\omega(1 - k),$$

since k is nearly equal to 1. $\qquad \qquad \cdot \; \cdot \; \cdot \; (17\cdot24)$

11. Rectification of alternating currents. Alternating voltage is rectified, that is, made unidirectional by several devices

such as a rotating sector and kenotrons (see Ch. XXII). It has
been found in recent years that
mercury arc rectifier has got
special advantages. It consists of
an evacuated vessel containing a
pool of mercury at the bottom
which serves as the cathode C
and a metal plate at the top
which serves as the anode A (fig.
17·22). Connections to the supply
transformer are made as shown
in the figure. D.C. is drawn
out from the terminals L_1 L_2. A
plunger is lowered by an automatic

Fig. 17·22

arrangement until it touches the mercury and then withdrawn. An
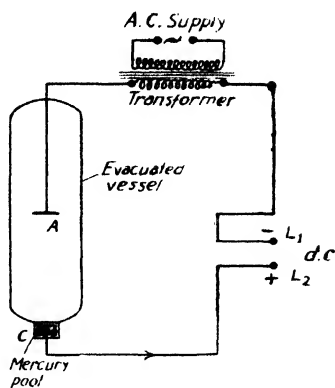arc starts which is then transferred to the anode. Current passes
only in one direction from the cathode into mercury; in the other
direction the resistance is very high. In actual practice for
·moother D.C. voltages a number of anodes are used, the A.C. on
secondary is made polyphase and a reactance coil is used in th·
D.C. leads of the rectifier.

12. Transmission of electrical power. In the foregoing
articles we have dealt with the essential principles of converting
mechanical energy into electrical energy and vice-versa and also
those of converting high A.C. voltages to low A.C. voltages and
vice-versa by transformers. Electrical engineering has to deal
in detail with the production and reconversion of electrical energy
as well as with its distribution. The production of electrical power
at one place is done by hydraulic turbines and by steam and
internal combustion engines. The transmission of this power to
distant places plays a very important role in the economic and
social life of a country and has duely received close attention
of the electrical engineer. In this article we will only give a very
brief introductory account of some important aspects of power
transmission, keeping closely in view the requirements of a ·student
of general electricity only.

18 E

Electrical energy can be transmitted either by continuous currents (D.C.) or by alternating currents (A.C.), each system having its own special advantages and disadvantages. The consumer may employ either D.C. or A.C. motors as appliance for doing useful mechanical work. The former have their own advantages over the latter but are of greater complexity since they require a commutator.

It is, however, in he distribution of power that the peculiarities of the two systems of power distribution are brought into strong contrast. Generally the consumers are situated at a very great distance from the generating centres. This necessarily involves miles of copper wire carried on good insulators. If P is the power to be transmitted we have $P = Vi$, where V is the voltage and i the current. In the transmission line there is loss of energy i^2R, where R is the line resistance. The fractional loss of power is thus $i^2 R/Vi = PR/V^2$. For large values of V it is small. Thus for a given transmission line it is more economical to transmit power by putting high voltage upon the line. At the receiving station this must be reduced to suitable working voltages to be supplied to the consumers because it is not convenient to transform electrical energy into mechanical energy at high voltages ; also high voltages are dangerous to the consumers. In the case of continuous currents conversion of voltages is done by a suitable rotation machinery. This is rather inconvenient and inefficient. In the case of alternating currents, however, conversion from high to low voltages and vice-versa is done more efficiently and economically by transformers ; and undoubtedly from this point of view transmission of electrical power by alternating currents is preferable and is actually in great vogue. At the generating station a comparatively low voltage can be stepped up by a transformer to a suitable high line voltage. At the distributing centre this may be stepped down to suitable low working voltages. Nor the alternating character of the voltage is a serious drawback because with suitable rectifiers the voltages, if so desired, may be made unidirectional. Due to this flexibility of operation the alternating current system of transmission is dis-

tinctly superior to the direct current system. In the former, however, the fractional loss is greater than in case of the latter. In the A.C. case power supplied to the line is $V_e i_e \cos \alpha$, V_e and i_e being the effective values of the voltages and current and $\cos \alpha$ the power factor. The joule loss is $i_e^2 R$. The fractional loss is thus

$$i^2_e R / V_e i_e \cos \alpha = i_e^2 R / V_e \cos \alpha.$$

As $\cos \alpha$ is always less than unity this is always greater than that in the direct current system where the power factor does not appear. Inspite of this drawback the A.C. system is widely used for transmission over long distance due to its peculiar flexibility.

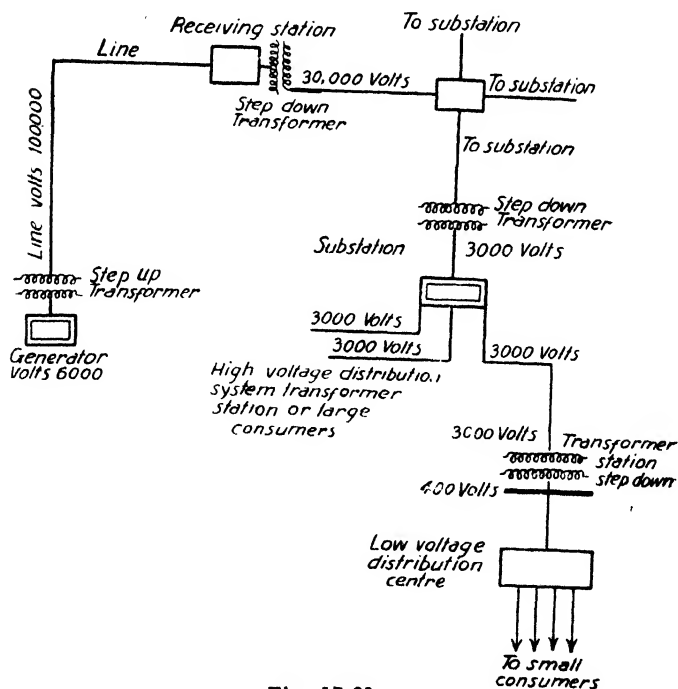It has been pointed out above that it is more economical to

Fig. 17·23

transmit power at high voltages and small currents. But high voltages increase insulating difficulties and are a menace to the safety of human lives. Because of these factors there is always

a higher limit set to the line voltage.   Generally for land stations it lies between 20 and 200 kilovolts although recently much higher voltages have been used.   The principal features of the method of transmitting electrical energy by alternating currents to consumers is indicated diagrammatically in fig. 17·23.   We thus realise the great importance of transformers in the transmission of energy Sometimes they are constructed on a very large scale as the relative losses are then reduced.   These large transformers are oil immersed for insulation and are cooled by special devices.

CHAPTER XVIII

THERMOELECTRICITY

1. Seebeck and Peltier effects. In 1826 Seebeck discovered that if junctions of two dissimilar metals are maintained at different temperatures an electric current flows round the circuit from the hot junction to the cold. Seebeck arranged a large number of metals in a series such that when any two form a circuit the current flows across the hot junction from the metal occurring earlier to that occurring later in the list. He gives the following series : Bi — Ni — Co — Pd — Pt — U — Cu — Mn — Ti — Hg — Pb — Sn — Cr — Mo — Rh — Ir — Au — Ag — Zn — W — Cd — Fe — As — Sb — Te. Thus if we construct a circuit out of Bi and Sb the flow of current is as shown in fig. 18·1.
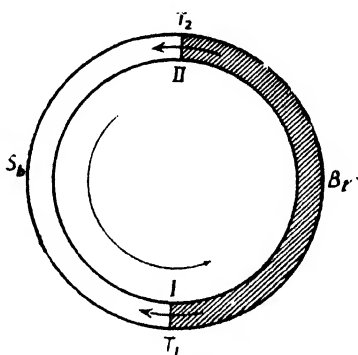
Fig. 18·1

In 1834 Peltier discovered the complementary phenomenon that when a current is passed across the junction heating or cooling results depending upon the direction of the current flow. For example a Bi—Sb junction is heated or cooled according as the current flows from Sb to Bi or vice-versa. This thermal effect of current should be clearly distinguished from the Joule effect. In the latter the amount of heat produced is given

Fig 18·2

by $(\text{current})^2 \times (\text{resistance})$ and so does not depend upon the direction of the current; also the process is irreversible. On the other
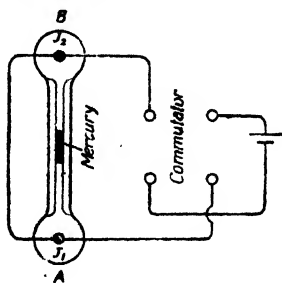
hand the Peltier effect depends upon the direction and is reversible. Peltier effect may be demonstrated with the apparatus represented in fig. 18·2. A and B are two bulbs filled with air. They enclose the junctions of two metals and are connected by a fine capillary containing a small drop of mercury. On passing current in one direction the junction A is heated up resulting in the expansion of air which pushes the droplet to the right. When the current is reversed the droplet moves in the other direction indicating cooling of A and heating of B. Seebeck and Peltier phenomena are both explained by assuming an e.m.f. to exist at the junction of two metals its direction being from a metal occurring earlier to that occurring later in the thermoelectric series. Thus if π_2 is the e.m.f or the Peltier coefficient across the junction II at temperature T_2 and π_1 that at the junction I at temperature T_1 then their directions being opposite the resulting e.m.f. in the circuit is $(\pi_2 - \pi_1)$. When the current flows round the circuit, at the junction II its direction is the same as π_2 and hence the junction cools down. At junction I the directions of the two are opposite and it heats up. Thus a total e.m.f. will exist so long as $T_2 \neq T_1$ and unless this difference in temperature is maintained by some external agency the thermoelectric current will soon bring the temperatures of the junctions I and II to equality.

2. Laws of addition of thermoelectric e.m.f's. Experiments have established two laws regarding the addition of thermal e.m.f's. These laws are of great importance in practice since in measuring the thermoelectric effect it is always necessary to introduce other metals in the form of wires, binding terminals, etc., between the original thermo-junctions and the measuring instrument such as a galvanometer. The laws are stated as follows :

(1) **The law of intermediate metals.** The total e.m.f. between two metals A and C is equal to the sum of the e.m.f's of the couples AB, BC, of metals A, B, C over the same temperature range. Any number of intermediate metals may be added provided the temperature is the same.

(2) **The law of intermediate temperatures.** The e.m.f. for a couple with junctions at temperatures T_1 and T_3 is equal to the sum of the e.m.f.'s of two couples of the same pair of metals, one couple having junctions at T_1, T_2 and the other at T_2, T_3 or symbolically $E_{T_1 T_3} = E_{T_1 T_2} + E_{T_2 T_3}$.

(3) **Thomson or Kelvin effect.** From the previous account of the Peltier effect it is clear that a thermo-couple behaves like a reversible heat engine absorbing heat at a higher temperature and delivering a part of it at a lower temperature. If T_2 be the higher and T_1 the lower temperature in absolute measure, we have, by thermodynamics,

$$\frac{\text{Heat absorbed at } T_2}{\text{Heat delivered at } T_1} = \frac{T_2}{T_1}$$

or if charge q is circulated round the circuit and π_2 and π_1 the Peltier coefficients, heat absorbed at T_2 is $\pi_2 q$ and that given up at T_1 is $\pi_1 q$ and therefore

$$\frac{\pi_2 q}{\pi_1 q} = \frac{T_2}{T_1},$$

or

$$\frac{\pi_2 - \pi_1}{\pi_1} = \frac{T_2 - T_1}{T_1}.$$

But $\pi_2 - \pi_1$ = resultant electromotive force e in the circuit and if T_1 is kept fixed and therefore π_1 we have

$$e = \frac{\pi_1}{T_1} \left(T_2 - T_1 \right). \qquad (18 \cdot 1)$$

Eq. (18·1) indicates that if one of the two junctions of a thermo-couple, say, Fe—Cu is kept at a constant temperature by immersing it in an ice bath and the other junction is heated the thermo-electric e.m.f. should vary linearly with T_2. This expectation is, however, not borne out by experiment. It is observed that the thermo-electric curve rises to a maximum value of e, then falls down to zero and is actually reversed.

This discrepancy led Lord Kelvin to conclude that there exists in addition to the Peltier effect a thermo-electric effect due to the temperature gradient along each of the metals forming the thermocouple. If A and B are the two metals, for each of these there is this additional e.m.f. Let σ_A and σ_B be the e.m.f's per unit

temperature difference along A and B. σ is assumed to be positive when the e.m.f. is directed from the lower to the higher temperature. The total e.m.f in the circuit is then

$$e = \pi_2 - \pi_1 - \int_{T_1}^{T_2} (\sigma_A - \sigma_B) dT. \qquad . \quad . \quad . \quad (18\cdot2)$$

The quantity σ is called the Kelvin or *Thomson coefficient* for the metal. It is the work done in taking unit charge through unit temperature interval. The existence of the coefficient involves absorption of heat when current flows in the conductor in the direction of this electromotive force ; in the opposite direction heat is evolved. σ is also called the *specific heat of electricity*.

Thomson effect may be demonstrated with an apparatus indicated in fig. 18·3 An iron rod is led into the shape shown in the figure. the two free ends being surrounded by mercury baths. P and Q are two resistance coils wound on the straight arms of the iron rod. They are put in the gaps of a metre bridge and are balanced. The loop is then heated red hot by the Bunsen
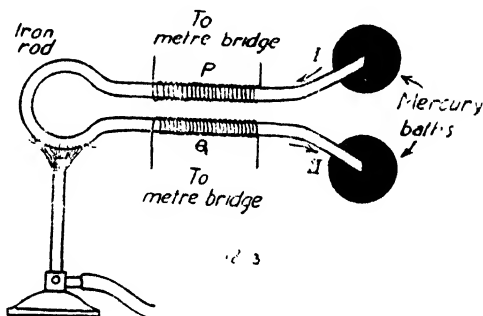

Fig 18·3

burner to produce a steep temperature gradient along the bar. P and Q are now rebalanced. On passing a current of about 10 amps. through the iron bar it will be found that the balance will be disturbed in a manner indicating that heat is given out when the current flows up the temperature gradient and that it is absorbed when the current flows down in the reverse direction.

4. Measurement of Thermoelectric e.m.f. The e.m.f. in a thermo-couple may be determined by a calibrated galvanometer of known resistance. It is more convenient, however, to use a microvoltmeter if the e.m.f. is varying rapidly due to rapid

changes in the temperature of the hot junction. If the changes are slow the e.m.f. is best measured by a potentiometer. The connections are indicated in fig. 18·4 A potentiometer wire AB having a resistance of about an ohm is connected in series with two variable resistances R_1 and R_2 each of the order of several thousand ohms.
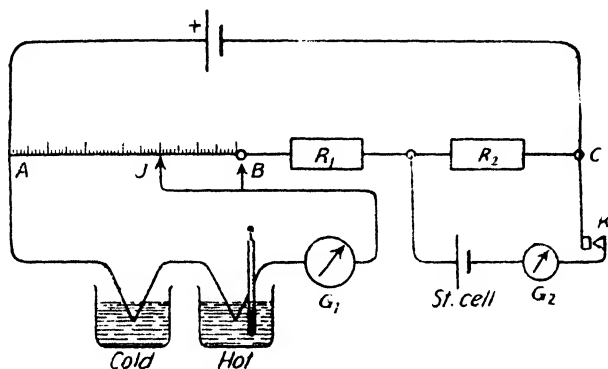

Fig 18·4

The resistance R_2 is varied, R_1 being also varied so as to keep $R_1 + R_2 =$ constant, till the standard cell is balanced. Then if I is the current flowing in AC, ρ the resistance per unit length of AB and l the balancing length AJ we have

$$E_{\text{th. couple}} = l \, \rho \, I$$

and

$$E_{\text{standard}} = R_2 \, I$$

From these we have

$$E_{\text{th. couple}} = E_{\text{standard}} \times \frac{l \, \rho}{R_2}$$

5. Thermoelectric curves. If the e.m.f's of a thermo-couple are plotted as ordinates and the corresponding temperatures of the hot junction as abscissae, the cold junction being maintained at a constant temperature, we get at thermo-electric curve shown in fig. 18·5 which brings out the fundamental characteristics of

thermo-electric curves for all metals. It is seen that a thermoelectric curve is parabolic in nature having a peak at N and that the descending part crosses the axis of temperatures at I. The meaning of this is that as the temperature of the hot junction of a thermocouple is gradually increased, the e.m.f. gradually in-

Fig 18·5

creases to the maximum value e_m at the temperature T_m called the *neutral temperature* dropping down to zero at temperature T_i called the *inversion temperature* beyond which the sign of the e.m.f. is reversed.
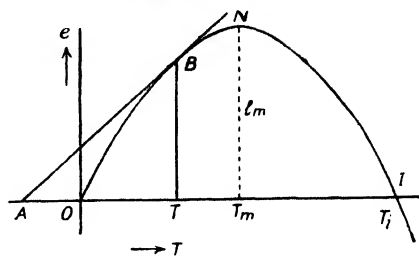
Thermoelectric Power. The total e.m.f. in a thermoelectric circuit is given by, eq. (18·2).

$$e = \pi_2 - \pi_1 - \int_{T_1}^{T_2} (\sigma_A - \sigma_B) \, dT$$

$$= \int_{T_1}^{T_2} \frac{d\pi}{dT} \, dT - \int_{T_1}^{T_2} (\sigma_A - \sigma_B) \, dT$$

$$= \Big|_{T_1}^{T_2} \left\{ \frac{d\pi}{dT} - (\sigma_A - \sigma_B) \right\} dT$$

Differentiating both sides with respect to T we have

$$\frac{de}{dT} = \frac{d\pi}{dT} - (\sigma_A - \sigma_B) \qquad \ldots \quad (18·3)$$

The quantity $\frac{de}{dT}$ is called the *thermo-electric power* P for the two metals forming a thermocouple. It is determined from the thermoelectric curve by calculating the slope of the curve at any point. For example, the value of P at B (fig. 18·5) is given by BT/AT.

If P is plotted against T we obtain what is called thermoelectric power curve which is a straight line. The use of this in describing the thermo-electric behaviour of metals will be discussed in a subsequent article.

6. Second law of thermodynamics and the thermoelectric e.m.f. A thermo-couple may be taken as a reversible heat engine like a voltaic cell since the Peltier and Thomson e.m.f's are reversible. Consider a thermo couple Sb—Bi (fig. 18·6) working between the absolute temperature limits T_2 and T_1, $T_2 > T_1$.
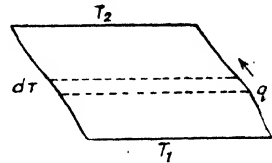

Fig. 18·6

Let a charge q be taken round the circuit in an anti-clockwise direction. Heat will be absorbed when the direction of the current is the same as the Peltier or Thomson e.m.f. and evolved when it is opposite. According to thermodynamics,

$$\Sigma \frac{Q}{T} = 0, \qquad \ldots \quad (18\cdot4)$$

where Q is the amount of heat absorbed or evolved. Suppose that the complete cycle is subdivided into a large number of elementary cycles working between temperature intervals dT (see the law of intermediate temperatures). For an elementary cycle we have

$$\frac{Q}{T} = q \left(\frac{\pi_{T+dT}}{T+dT} - \frac{\pi_T}{T} - \frac{\sigma_A \, dT}{T} + \frac{\sigma_B \, dT}{T} \right).$$

For the complete cycle we have

$$\Sigma \frac{Q}{T} = q \left[\int_{T_1}^{T_2} \frac{d}{dT} \left(\frac{\pi}{T} \right) dT - \int_{T_1}^{T_2} \sigma_A \frac{dT}{T} + \int_{T_1}^{T_2} \sigma_B \frac{dT}{T} \right]$$
$$= 0, \qquad \text{by eq. (18·4)}$$

Hence, $$\int_{T_1}^{T_2} \frac{d}{dT} \left(\frac{\pi}{T} \right) dT - \int_{T_1}^{T_2} \frac{\sigma_A - \sigma_B}{T} \, dT = 0$$

Differentiating with respect to T we have

or $$\frac{d}{dT} \left(\frac{\pi}{T} \right) - \frac{\sigma_A - \sigma_B}{T} = 0$$

$$\left(\sigma_A - \sigma_B \right) = T \frac{d}{dT} \left(\frac{\pi}{T} \right). \qquad \ldots \quad (18\cdot5)$$

By eq. (18·3) we have

$$\frac{de}{dT} = \frac{d\pi}{dT} - (\sigma_A - \sigma_B).$$

Substituting the value of $(\sigma_A - \sigma_B)$ in this from eq. (18·5) we have

$$\pi = T \frac{de}{dT}. \qquad \qquad . \quad . \quad . \quad (18·6)$$

Also from eqs. (18·5) and (18·6) we have

$$(\sigma_A - \sigma_B) = T \frac{d^2e}{dT^2} = T \frac{dP}{dT}. \qquad . \quad . \quad . \quad (18·7)$$

Eqs. (18·6) and (18·7) are very useful in determining the Peltier and Thomson coefficients from the slopes of the thermo-electric and thermo-electric power curves.

7. Thermo-electric diagrams. We shall now discuss the thermo-electric behaviour of metals with one common metal viz., lead are first plotted. Lead is used as the common metal because it approaches very closely an ideal metal for which $\sigma = 0$. Due to the parabolic nature of the thermo-electric curves their interpretation becomes inconvenient ; and so from these, new curves called thermo-electric power curves are plotted which being straight lines can be readily interpreted.

From the thermo-electric power diagrams e, π and σ can be calculated as areas as shown below :

Let I and II be the power diagrams for two metals B and C with respect to a metal A, Fig. 18·7. The following properties are shown by each curve.
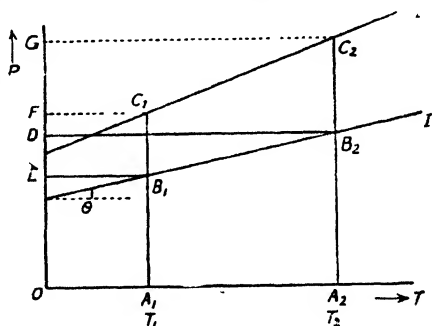

Fig. 18·7

Curve I. The total e.m.f. in the thermo-couple is

$$e = \int_{T_1}^{T_2} \frac{de}{dT} d'T$$

$$= \int_{T_1}^{T_2} P dT$$

$$= \text{Area } A_1 B_1 B_2 A_2. \qquad . \quad . \quad . \quad (18·8)$$

By eq. (18·6) the Peltier e.m.f. is given by $\pi = T\dfrac{de}{dT} = TP =$ area of rectangle enclosed between the coordinate axes and the perpendiculars drawn from a point on the thermo-electric diagram on the coordinate axes.   Thus

$$\pi_{T_2} = \text{Area } B_2 \text{ DOA}_2$$
$$\text{and} \quad \pi_{T_1} = \text{Area } B_1 \text{EOA}_1$$
$$\therefore \quad \pi_{T_2} - \pi_{T_1} = \text{Area } B_2 \text{DEB}_1 A_1 \quad . \qquad . \quad . \quad . \quad (18·9)$$

Also by eq. (18·2) we have the Thomson e.m.f. given by

$$\int_{T_1}^{T_2} (\sigma_B - \sigma_A)\ dT = (\pi_{T_2} - \pi_{T_1}) - e$$

$$= \text{Area } B_2 \text{ DEB}_1, \text{ by eqs. (18·8} \text{ and (18·9)}$$
$$. \quad . \quad . \quad (18·10)$$

Curve II. Similar quantities are obtained for this curve.   If a thermocouple is now formed with metals B and C then by the law of intermediate metals we have

$$\left.\begin{array}{l} e = \text{Area } C_1\ B_1\ B_2\ C_2 \\ \pi_{T_2} = \text{Area } C_1 \text{ GD } B_2 \\ \pi_{T_1} = \text{Area } C_1 \text{ FE } B_1 \end{array}\right\} \qquad . \quad . \quad . \quad (18·11)$$

Eq. (18·10) will then give the value of $(\sigma_B - \sigma_C)$.

Again by eq. (18·7) we have

$$(\sigma_B - \sigma_A) = T\frac{dP}{T},$$

and therefore, $\sigma_B = T\dfrac{dP}{dT} = T\tan\theta,$ $\qquad$ . . . $(18·12)$

since $\sigma_A = 0$ for the standard metal, where θ is the angle which the power curve makes with the axis of T. Eq. (18·12) shows that σ_B agrees in sign with $\dfrac{dP}{dT}$.

It is worth noting here that as some metals are positive and others negative with respect to the standard metal the corresponding slopes of the power lines will be in the reverse directions.

There are certain special cases of interest worth considering. For platinum and platinum-iridium alloy the power lines have the

same slope. Thus writing the equations for the power lines of platinum and platinum-iridium as $P_p = a\,T + b$ and $P_{pi} = a\,T + c$, we have the thermoelectric e.m.f. for a couple composed of these metals given by

$$e = \int_{T_1}^{T_2} (P_p - P_{pi})\ d\mathrm{T} = \int_{T_1}^{T_2} (b - c)\ d\mathrm{T} = (b - c)\,(T_2 - T_1).$$

We thus see that $P_t - P_t\,I_r$ couple will show linear variation of the thermo-electric e.m.f. with temperature and so can be usefully employed to measure temperatures.

Again certain metals like iron and nickel form exceptions to the general parabolic law of thermo-electric curves. They exhibit several points of inflexion at high temperatures. The power lines for these metals are not straight but form loops intersecting the power line say for silver at two points J and K as shown in fig. 18·8. The temperatures corresponding to points J and K are 310°C and 620°C. Since

Fig. 18·8

the Peltier e.m.f.'s at these temperatures are zero the e.m.f. acting round the couple is due to Thomson effect only. The area enclosed between the line and the loop represents this e.m.f. Here we have an interesting example of a couple working on the Thomson coefficients only of the two metals.
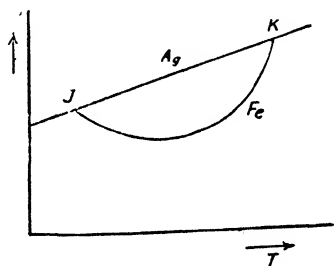
8. Applications of Thermoelectric effects. Since the thermo-electric e.m.f. depends upon the difference in temperatures of the two junctions, a thermo-couple along with the instrument used for measuring e.m.f.'s serves as a sensitive thermometer. The metals to be used depend upon the range of temperature which is to be measured. For low temperatures below 300°C pure iron and pure nickel may be used. For a temperature range 300°C to 1400°C the most suitable elements are pure platinum and an alloy of pure platinum with 10% of rhodium, the junction to be heated

being enclosed in a quartz tube. For temperatures up to 1700°C molybdenum-tungsten thermo-couple in an atmosphere of nitrogen or hydrogen is used.   Thermo-electric thermometers must be calibrated with the help of known standard temperatures.

9.   Conduction Electrons and Thermoelectric effects. According to electron theory every metal contains a conduction electron 'gas'.   It is the drift of these conduction electrons which constitutes an electric current in the metal.   The conception of electron gas enables us to form a qualitative picture of the thermo-electric effects. The number of electrons per unit volume or electron density varies from metal to metal at a given temperature. If we set up two dissimilar metals so as to form a thermo-couple with two junctions at the same temperature the interface at each junction will behave like a semi-permeable membrane through which the electron gas will diffuse from one metal into the other until an equilibrium condition is reached.   The   loss of electrons by one metal and the gain by the other involved in the above process sets up a difference of potential between the two sides of the interface.   The resulting electric field opposes further transfer of electrons when equilibrium is attained.   This difference of potential is termed the Peltier coefficient.   As the electron energy is larger at higher temperatures this coefficient increases with temperature.   Hence no permanent current in a thermo-couple is obtained unless a temperature difference between the two junctions is maintained by supplying heat.

The Thomson effect can be similarly explained by a possible temperature gradient in each metal.   Such a temperature gradient gives rise to a pressure gradient of the electron 'gas' which causes a transfer of electrons from one part of the metal to the other until the opposition of the resulting electric fields produces final equilibrium   conditions.

10.   Contact Potential Difference and Work function. In the previous article we considered the passage of electrons across a metal-metal surface.   The restraining force exerted upon the electrons gives rise to what is known as Peltier e.m.f.   In the case

of a metal-vacuum surface an electron is unable to leave the surface under normal conditions due to the existence of a potential barrier. A definite amount of energy per unit charge, called the *work function*, must be spent on the electron before it is able to leave the surface. The value of work function varies from metal to metal and for each metal changes slightly with temperature.

Consider a circuit formed of two metals A and B as shown in fig. 18·9. Let A′ and B′ be the free ends. The circuit is supposed to be situated in vacuum and at a uniform temperature. Let V_{AB} be a possible difference of potential between two points situated just outside the surfaces A′ and B′. At the interface AB there is as observed in the previous article a Peltier e.m.f. say π_{AB}. Also let ϕ_A and ϕ_B be the
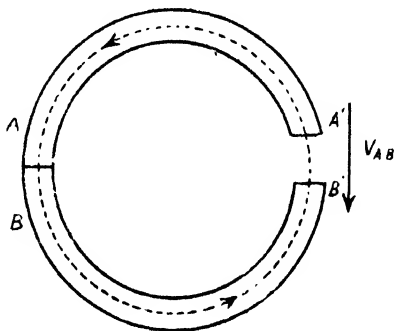

Fig. 18·9

work functions for the metals A and B. Consider that a unit charge is taken round the complete circuit as shown by the arrow. The total work done is equal to $\phi_B + V_{AB} - \phi_A - \pi_{AB}$. This must be zero since the forces are electrostatic in nature. Thus

$$V_{AB} = \pi_{AB} + \phi_A - \phi_B.$$

V_{AB} is known as *contact potential difference*. Now π_{AB} is found to be a very small quantity in comparison to ϕ. Hence neglecting this in the above equation we obtain

$$V_{AB} = \phi_A - \phi_B. \qquad \cdot \quad \cdot \quad \cdot \quad (18 \cdot 13)$$

Along with work function the contact potential difference is found to be of very great importance in the emission of electricity from hot bodies (thermionics) and also in the emission of electricity by light (photo-electricity).

CHAPTER XIX
UNITS, DIMENSIONS AND STANDARDS

1. Systems of Units. The quantitative formulation of experimental results in any field of scientific enquiry necessitates the adoption of a system of units. Thus in the study of electricity we must express electrical charge in terms of a unit charge and likewise a current in terms of a unit current. Similarly in the study of magnetism we must express, for example, magnetic pole strength in terms of a unit magnetic pole.

As pointed out earlier there are two distinct systems of units used in measuring electrical quantities : (1) the electrostatic (e.s.u.) and (2) the electromagnetic (e.m.u.). In the electrostatic system the fundamental unit is determined by the force of repulsion between two similar charges of electricity while in the electromagnetic system it is determined by the force of repulsion between two similar magnetic poles. In both of these systems all the quantities are expressed in terms of the fundamental units of length, mass and time. Hence the electromagnetic and electrostatic systems of units are termed " absolute ". For scientific purposes we normally empoly the c.g.s. system in which the units of length mass and time are respectively the centimetre, the gram and the second. We define here for convenience these three fundamental units. The gram expressed by the symbol M is 1/1000th part of the satndard kilogram kept at Sèveres. The centimetre expressed by the symbol L is 1/100 th part of the metre which is the distance between two marks on a standard bar kept at Sèveres ; this distance is very nearly equal to one-tenmillionth part of the earth's quadrant. The unit of time represented by the symbol T is $1/(60 \times 60 \times 24) = 1/86,400$ of the duration of the mean solar day the best known invariant measure of time.

The ratio between corresponding units of these two systems of electrical units is found to be some power of the velocity of light, 3×10^{10} cms/sec.

19 ε

In actual practice it is found that the quantities defined in the absolute electromagnetic and electrostatic systems of units are not of convenient magnitudes. Hence a third system of units has come in vogue. This is called *the pracictal system*. In this system all the units are decimal multiples of the corresponding electromagnetic units. It is the custom of international scientific conferences to honour scientists by giving their names to the relevant units. e.g. Ampere and Coulomb.

It is found in building the systems of units that we also require unit of some physical property of the medium. In the electrostatic system we have to introduce K the dielectric constant and in the electromagnetic system we have to bring in μ the magnetic permeability. They are expressed in terms of these constants for free space taken as unit. It should be noted that no name is given to these units.

2. Dimensions. As pointed out in the last article the units of all physical quantities are expressible in terms of these fundamental units of length, mass and time. The powers of the symbols L, M and T necessary to obtain any unit are called the dimensions of the unit. Thus Force = mass $\times$ acceleration = $[M]^1 \times [L]^1 \times [T]^{-2}$ and so the dimensions of unit force are 1 of M, 1 of L and -2 of T. Unit of force (dyne) is that force which produces unit acceleration in unit mass. The dimensions of an electrical quantity are similarly derived from the definition of that quantity. Thus in the electrostatic system the force between two charges q_1 and q_2 separated by a distance r situated in a medium of dielectric constant K, say, expressed in unknown units say [K], or Force = $q_1 q_2 / K r^2$

Writing this in dimensional form we have,

$$[M]^1 \times [L]^1 \times [T]^{-2} = [\text{Charge}]^2 \times [K]^{-1} \times [L]^{-2}$$

or $$[\text{Charge}]^2 = [K]^1 \times [M]^1 \times [L]^3 \times [T]^{-2}$$

and $\therefore$ $$[\text{Charge}] = [K]^{\frac{1}{2}} \times [M]^{\frac{1}{2}} \times [L]^{\frac{3}{2}} \times [T]^{-1}$$

In the same way dimensions of other physical quantities can be found out. We give in the following articles the definitions and dimensions of fundamental electrical quantities in the various systems.

3. Fundamental Electrical Units. Magnetic Units.

(1) Magnetic pole strength. The unit magnetic pole is a pole of such strength that it repels a like pole placed at a distance of one centimetre in vacuo with a force of one dyne. $[\mu_0]^{\frac{1}{2}}[M]^{\frac{1}{2}}[L]^{\frac{3}{2}}[T]^{-1}$, where $[\mu_0]$ is the dimension of the magnetic permeability (of value 1 for vacuum).

(2) Magnetic moment is the pole strength × length. $[\mu_0]^{\frac{1}{2}}[M]^{\frac{1}{2}}[L]^{\frac{5}{2}}[T]^{-1}$,

(3) Magnetic intensity s the force per unit pole. $[\mu_0]^{-\frac{1}{2}}[M]^{\frac{1}{2}}[L]^{-\frac{1}{2}}[T]^{-1}$

Electrostatic System. (1) *Charge.* A unit charge is that charge which reples a like charge kept at a distance of one centimetre in vacuo with a force o ne dyne. $[K_0]^{\frac{1}{2}}[M]^{\frac{1}{2}}[L]^{\frac{3}{2}}[T]^{-1}$, where $[K_0]^1$ is the dimension of the quantity K_0 called the dielectric constant (of value 1 for vacuum).

(2) *Current.* The unit current flows in a conductor when a unit charge flows across any plane in the conductor in one second. $[K_0]^{\frac{1}{2}}[M]^{\frac{1}{2}}[L]^{\frac{3}{2}}[T]^{-2}$

(3) *Potential difference.* The unit potential difference exists between two points when one erg of work is required to carry unit electrostatic charge from one point to the other. $[K_0]^{-\frac{1}{2}}[M]^{\frac{1}{2}}[L]^{\frac{1}{2}}[T]^{-1}$

(4) *Resistance.* A conductor is said to possess one electrostatic unit of resistance if, when carrying unit current, the p. d. across its terminals is one electrostatic unit. $[K_0]^{-1}[L]^{-1}[T]$

(5) *Capacity.* A condenser is said to possess unit capacity when one unit of p. d. across its terminals gives one unit charge to it. $[K_0][L]$

(6) *Inductance.* A coil is said to possess one unit of inductance if unit rate of charge of inducing current produces one electrostatic unit of electromotive force. $[K_0]^{-1}[L]^{-1}[T]^2$

Electromagnetic System. (1) *Current.* The electrmagnetic unit of current is that current which when flowing through an arc of one centimetre of a circle of radius equal to one centimetre produces unit magntic field at the centre. $[\mu_0]^{-\frac{1}{2}}[M]^{\frac{1}{2}}[L]^{\frac{1}{2}}[T]^{-1}$

(2) *Charge.* Unit charge is that charge which passes per second across any plane of a conductor in which electromagnetic unit current is flowing. $[\mu_0]^{-\frac{1}{2}}[M]^{\frac{1}{2}}[L]^{\frac{1}{2}}$

(3) *Potential difference.* Two points are said to possess unit P.D. when one erg of work is requred to carry unit electromagnetic charge from one point to the other. $[\mu_0]^{\frac{1}{2}}[M]^{\frac{1}{2}}[L]^{\frac{3}{2}}[T]^{-2}$

(4) *Resistance.* A conductor is said to possess one e.m. unit of resistance if when carrying one e.m. unit of current the P. D. across its terminals is one e.m. unit. $[\mu_0][L][T]^{-1}$

(5) *Capacity.* A condenser is said to possess one e.m. unit of capacity if one e.m. unit of P. D. across its terminals gives to it one e.m. unit of charge. $[\mu_0]^{-1}[L]^{-1}[T]^2$

(6) *Inductance.* A coil is said to possess one e.m. unit of inductance if unit rate of change of e.m. current produces one e.m. unit of e.m.f. $[\mu_0][L]$

Practical units. (1) Current. The unit is Ampere. It is one-tenth of an e.m. unit of current.

(2) *Charge.* The unit is *Coulomb*. It is that quantity of charge which passess per second across any plane of a conductor in which a current of one Ampere is flowing.

(3) *Potential difference.* The unit is *volt.* The P. D. between two points is one *volt* when one joule of work is required to carry one coulomb from one point to the other.

(4) *Resistance.* The unit is Ohm. It is the resistance offered by a conductor if the difference of potential across its terminals is one *volt* when a current of one Ampere flows through it.

(5) *Capacity.* The unit is Farad. It is the capacity of a condenser receiving one coulomb for one Volt across its terminals.

(6) *Inductance.* The unit is Henry. A coil possesses inductance equal to one Henry when a rate of change of one Ampere per second induces an e.m.f. of one volt.

(7) *Magnetic flux.* The unit is Maxwell. The total flux in a magnetic circuit is one Maxwell when it possesses one line of magnetic induction.

(8) *Magnetic induction.* The induction in a magnetic circuit is one *Gauss* when the flux density is one Maxwell per square centimetre.

(9) *Magnetomotive force.* The unit is *Gilbert.* If one erg of work is required to carry unit magnetic pole round a magnetic circuit the magnetomotive force is said to be one Gilbert.

(10) *Field strength.* If the magnetomotive force is one Gilbert per centimetre the magnetic field possesses unit strength.

(11) *Reluctance.* A magnetic circuit is said to possess a reluctance of one Oersted if a magnetomotive force of one Gilbert produces a flux of one Maxwell.

4. Relation between the systems of electrical units. The relations between the practical and absolute electromagnetic

units of various electrical quantities have already been stated in the previous article. Relation between the electromagnetic and electrostatic units can be easily deduced.

The dimensions of any physical quantity must be the same no matter how it is measured. Let us take for example the electric current. We have

$$[i] = K^{\frac{1}{2}} M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-2} \text{ in } e.s.u.$$

$$[i] = \mu^{-\frac{1}{2}} M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1} \text{ in } e.m.u.$$

Hence equating the two quantities we have

$$\sqrt{\frac{1}{\mu K}} = L^1 T^{-1} = \text{Velocity}$$

From the electromagnetic theory of light

$$\sqrt{\frac{1}{\mu K}} = c \text{ (the velocity of light.)}$$

Also,

$$\frac{e.m. \text{ unit of current}}{e.s. \text{ unit of current}} = \frac{M^{\frac{1}{2}} \mu^{-\frac{1}{2}} L^{\frac{1}{2}} T^{-1}}{K^{\frac{1}{2}} M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-2}} = \sqrt{\frac{1}{K\mu}} \Big/ L^1 T^{-1}$$

$$= 3 \times 10^{10} \text{ by experiment.}$$

Thus $\sqrt{\dfrac{1}{K\mu}}$
$$= 3 \times 10^{10} L^1 T^{-1}$$
$$= 3 \times 10^{10} \text{ cms/sec}$$
$$= \text{ velocity of light in free space } (c).$$

If any other electric or magnetic quantity is considered then from dimensions we find that the ratio of the e.m.u. to the e.s.u. is always c^{-2}, c^{-1}, c or c^2.

As pointed out earlier the absolute e.m. or e.s. units are of inconvenient size and so a system of practical units has been obtained. In this system the practical units are related to the absolute electromagnetic units by powers of 10. Such units are called derived practical units. Knowing the ratio of e.m.u. to the e.s.u. we can find out the relation between practical units and the e.s. units.

TABLE XIX—I

Quantity	Symbol	e.s.u. / e.m.u.	Practical unit.		
			Name	e.m.u.	e.s.u.
Electrical					
Charge	e	$1/c$	Coulomb	$=10^{-1}$	$=3 \times 10^9$
Current	i	$1/c$	ampere	$=10^{-1}$	$=3 \times 10^9$
Resistance	r	c^2	Ohm	$=10^9$	$=\frac{1}{9} \times 10^{-}$
e. m. f.	E	c	Volt	$=10^8$	$=1/300$
Electric field	F	c	Volt/cm	—	—
Capacity	C	$1/c^2$	micro-farad	$=10^{15}$	$=9 \times 10^5$
Self-inductance	L	c^2	Henry	$=10^9$	$=\frac{1}{9} \times 10^{-}$
Mutual-inductance	M	c^2	Henry	$=10^9$	$=\frac{1}{9} \times 10^{-}$
Energy	W	1	Joule	1 erg $= 1/10^7$ joule	
Power	P	1	Watt	1 erg per sec $= 1/10^7$	
Magnetic					
Pole strength	m	$1/c$	1 e.m.u.	1	3×10^{10}
Field strength	H	$1/c$	Gauss	1	3×10^{10}
Induction	B	$1/c$	Gauss	1	$\frac{1}{3} \times 10^{-10}$
Intensity of magnetization	I	c	—	—	—
Permeability	μ	c^2	—	—	—
Magnetic reluctance	S	c^2	Oersted	1	$1/9 \times 10^{20}$
Magnetomotive force	G	c	Gilbert	1	$1/3 \times 10^{10}$

We present in Table IX-I the commonly required units
in the three different systems.

5. International practical units. In the foregoing article
we mentioned that derived practical units related to the
absolute e.m. units in convenient powers of 10 have been
adopted. All practical units so defined are not, however, easily
reproducible with any great degree of accuracy. Hence the
International conference held in London in 1908 has adopted
certain reproducible standards which are close to the
true or derived practical units. These units are called " in-
ternational ".

The " international " ampere liberates 0·0011183 *gram of silver per
second in a silver voltameter under specified conditions.*

The " international " volt is 1/1·0183 *the e.m.f. of a standard Weston
cell at* 20°C.

*The " international " ohm is the resistance of a column of mercury of
one square millimeter cross section and* 106·3 *cms. length and of mass* 14·4521
grams at 0°C.

It must be noted that these " *international* " units do not per-
fectly agree with the true practical units derived from the absolute
e.m. units. The relation between the true and international units
is given below.   ·

1 " international " ampere=0·99997   abs.   Ampere.

1 " international "   Ohm=1·00052   ,,   Ohm.

1 " international "   Volt=1·00049   ,,   Volt.

This difference must be taken into account in very accurate
scientific work although ordinarily the error is negligible.

**6. Experimental determination of the relation bet-
wtea E. M. and E. S. units.** There are various methods
of determining this relation. We give below that due to
Maxwell.

The capacity of a parallel plate condenser is first calculated in e.s. units. Its value is then measured in e.m. units by an arrangement indicated in fig. 19·1. This is nothing but Wheatstone bridge arrangement. The capacity C is put into the unknown resistance arm C D in the manner shown. When the commutator contact T touches C_1 the condenser gets charged up to a potential difference between C and D; when it comes into contact with C the condenser is discharged. If the frequency of this charge and discharge is n per second, the rate of flow of the current in the arm C D is given by

$$i = nq$$
$$= nCV$$
$$= V/1/nC,$$
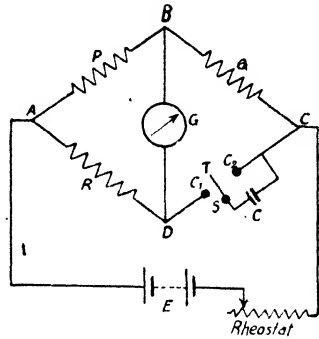
Fig. 19·1.

where q is the condenser charge and V the p.d. between C and D. This shows that a condenser of capacity C charging and discharging n times per second is equivalent to a resistance $1/nC$. If the bridge is balanced we have

$$\frac{P}{Q} = \frac{R}{1/nC}$$

and

$$C = \frac{P}{Q} \cdot \frac{1}{nR}.$$

Knowing P, Q, R in e.m. units and n we get C in e.m. units. As the capacity of this condenser is already calculated in e.s. units we get the relation between the two systems of units.

J. J. Thomson and Searle used a cylindrical condenser with guard rings of cylindrical form. Their mean value of this ratio is $2·9955 \times 10^{10}$. Rosa and Dorsey using this method found this value to be $2·9982 \times 10^{10}$.

Since the ratio of the e.m.u. to the e.s.u. of any electric or magnetic quantity is some power of c, the velocity of light in vacuum, we get probably the most reliable value of this ratio between the two systems of units by the direct determinatoin of the velocity of light.

7. Absolute Determinations. The Ampere. The absolute determination of ampere is really the absolute determination of the electrochemical equivalent of silver. A steady current measured in e.m. units by a suitable instrument is passed through a standard silver voltameter put in series. The following is a brief outline of the principle of the method used by Lord Rayleigh and Mrs. Sidgwick in 1884 (for a fuller account see Lord Rayleigh's papers, vol. II, p. 285).

A steady current is passed through two or three silver voltameters and through a system of coils arranged in series, fig. 19·2. Two fixed horizontal Helmholtz coils c_1 and c_2 are arranged parallel to each other and separated by a distance equal to their radius which was about 25 cms. A small coil c of radius of about 10 cms hanging from the arm of a sensitive balance is arranged parallel to c_1 and c_2 midway. The number of turns in c_1 and c_2 was 450 and that in c 242.
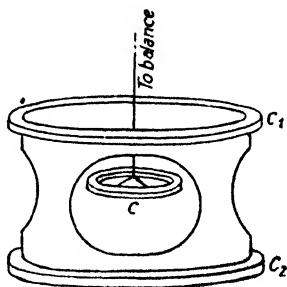
Fig. 19·2.

The same current passes through all the coils and the direction of flow is so arranged that the fields due to the larger coils c_1 and c_2 pull the small coil c either up or down. The force upon the coil c is found by taking the difference in the weighings when the current in c_1 and c_2 is reversed.

We can easily find an expression for the force in dynes exerted on c by the current in c_1 and c_2. Let n be the number of turns in each of the coils c_1 and c_2, a the common radius and i the current flowing.

The field at an axial point due to one of the coils is given by

$$F = 2\pi na^2 i/(a^2 + x^2)^{\frac{3}{2}}$$

The coil c is arranged at a distance $x = a/2$. Thus the gradient of the field at this distance is

$$-\left(\frac{dF}{dx}\right)_{x=a/2} = 2\pi na^2 i \times \tfrac{3}{2}\,(a^2+x^2)^{-5/2} \times 2x$$

$$= \frac{96}{25\sqrt{5}} \cdot \frac{\pi ni}{a^2}$$

and it can be seen that it is maximum. For the coils c_1 and c_2 together the gradient becomes $\dfrac{192}{25\sqrt{5}} \cdot \pi ni/a^2$.

If n' be the number of turns in c, A its area and i the current flowing through it its equivalent magnetic shell has a magnetic moment equal to iAn'.

Hence the force on c in dynes is

$$iAn'.\ \frac{dF}{dx} = \frac{192}{25\sqrt{5}} \cdot \frac{\pi ni}{a^2} \cdot \quad iAn' = \alpha i^2.$$

The constant α can be determined in fundamental units by direct measurement. The force is measured directly in dynes and so i^2 and hence i is determined in absolute units.

The best results are obtained when a solution of pure silver nitrate in water is used in the silver voltameter. The strength of the solution is 15 to 30% and a current of 1 ampere is passed for an hour.

As a mean result it was found that 0·0111794 gram of silver was deposited in a second by one c. g. s. unit of current.

In 1912 Rosa, Dorsey and Miller in America made some refinements in Rayleigh's apparatus. All the three coils were water cooled and g was measured very accurately. Another refined arrangement similar to that in Kelvin's ampere balance was used at the National Physical laboratory in England. The accuracy obtained in the measurement of current was 1 in 3×10^5.

Determination of Ohm. **(1) Weber's method of rotating-coil.** The principle of this method is indicated in fig. 19·3. A plane circular coil C rotates with a uniform angular velocity ω about a vertical axis through O in the anticlockwise direction. If n is the number of turns in C, a its radius and H the horizontal component of the earth's field the magnetic flux passing through
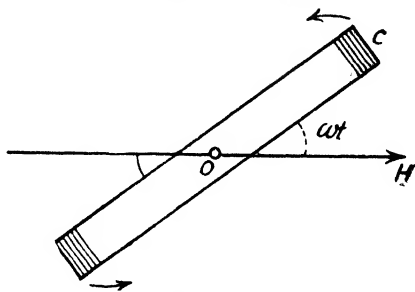

Fig. 19·3

the coil at any instant is $\pi a^2 n$ H $\sin \omega t$. The induced e.m.f. in the coil at any instant is $-\dfrac{d}{dt}(\pi a^2 n$ H $\sin \omega t) = -\pi a^2 n$ H $\omega \cos \omega t$. The corresponding induced current is $\dfrac{-\pi a^2 n\,\mathrm{H}\omega}{\sqrt{\mathrm{L}^2\omega^2 + \mathrm{R}^2}} \cos (\omega t - \alpha)$, where L is the self inductance and R the resistance of the coil and α the phase angle by which the current lags behind the e.m.f.

This current will have a momentary magnetic field at its centre perpendicular to the plane of the coil of magnitude $-\dfrac{2\pi n}{a} \times \dfrac{\pi a^2 n\mathrm{H}\omega}{\sqrt{\mathrm{L}^2\omega^2 + \mathrm{R}^2}} \cos (\omega t - \alpha)$. The mean value of this at right angles to the magnetic meridian is $-\dfrac{\pi^2 a\,n^2\,\mathrm{H}\omega}{\sqrt{\mathrm{L}^2\,\omega^2 + \mathrm{R}^2}} \cos \alpha$

Similarly the average magnetic field in the meridian is $-\dfrac{\pi^2 a\,n^2\,\mathrm{H}\omega}{\sqrt{\mathrm{L}^2\,\omega^2 + \mathrm{R}^2}} \sin \alpha$ and the total field in the meridian becomes $\mathrm{H} - \dfrac{\pi^2 a\,n^2\,\mathrm{H}\omega}{\sqrt{\mathrm{L}^2\,\omega^2 + \mathrm{R}^2}} \sin \alpha$. At the centre of the coil there is a suspended needle which will be deflected due to these two components. If θ is the deflexion we have

$$\tan \theta = \dfrac{\pi^2 a\,n^2\,\mathrm{H}\omega}{\sqrt{\mathrm{L}^2\omega^2 + \mathrm{R}^2}} \cos \alpha \;\Big/\; \mathrm{H} - \dfrac{\pi^2 a\,n^2\,\mathrm{H}\omega}{\sqrt{\mathrm{L}^2\,\omega^2 + \mathrm{R}^2}} \sin \alpha$$

$$= \pi^2 a\,n^2\omega \cos \alpha \;\Big/\; \sqrt{\mathrm{L}^2\omega^2 + \mathrm{R}^2} - \pi^2 a\,n^2\,\omega \sin \alpha$$

For small values of L in comparison to R, sin $\alpha = 0$ and cos $\alpha = 1$ $\left(\because \tan \alpha = \dfrac{L\omega}{R} \right)$ and the above expression reduces to

$$R = \pi^2 \, a \, n^2 \omega \, \cot \theta$$

The values of the quantities on the right hand side being obtained in absolute measure we get absolute value of R. It has the dimensions of velocity. In arriving at the final result small corrections due to the torsion in the suspending fibre and the induced effect in the coil due to the needle must be determined and allowed for. Using this method Lord Rayleigh found that 1 B.A. unit $= 0.98651$ earth quadrant /sec.

(2) **Method of Lorenz.** A circular brass disc D of diameter 31.072 cms. is set to rotate with constant speed about an axis passing through its centre and perpendicular to its plane, fig. 19·4. The axis of rotation is made to
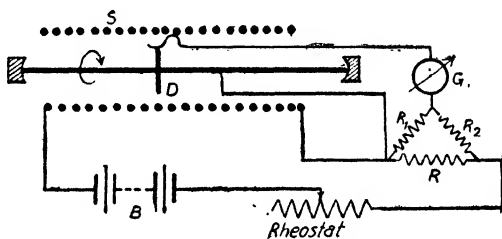
Fig. 19·4

coincide with the axis of a long solenoid S through which a constant current is passed from the accumulator battery B. R is the resistance whose value in absolute measure is to be found. Its value in international ohms is already determined by direct comparison with the mercury standard. R_1 and R_2 are two resistances accurately determined in terms of R and are adjusted to have a ratio R_2/R_1 of the order of 1000. The arrangement of the resistances and the galvanometer is as shown in the figure.

If H is the magnetic field due to the solenoid for a current i, a the radius of D and n the number of revolutions per second any radius of the disc cuts $\pi a^2 Hn$ lines of force per second. Thus the induced e.m.f. in the disc acting from the centre towards the peri-

phery is πa^2 Hn. This is also equal to Mni where M is the mutual inductance between the solenoid and the disc. A certain deflexion of the galvanometer will be obtained which is balanced by the fall of potential across R$_1$ due to the current i through the solenoid.

Now the current passing through $R_1 = \dfrac{Ri}{R_1 + R_2 + R}$

and so the fall of P. D. across it is $\dfrac{Ri}{R_1 + R_2 + R} \times R_1$. Therefore. for balance

$$\frac{Ri}{R_1 + R_2 + R} \times R_1 = Mni$$

from this we get

$$R = Mn \frac{R_1 + R_2 + R}{R_1}$$

which is thus known in absolute measure.

The principal sources of error are the thermo-e.m.f. due to rubbing of the brushes and the induced e.m.f. due to cutting of earth's field by the disc. These are balanced by the drop of potential across a low resistance when no current is passing through the solenoid. The speed of revolution was carefully determined by stroboscopic method with the help of a calibrated tuning fork standard. The mean of the results gave 1 B.A. unit $= 0.98677 \times 10^9$ c.g.s. units.

E.M.F. in International volts. The e.m.f. of a standard Weston cell in terms of International volts is determined by using a potentiometer to compare this e.m.f. with the drop of potential across a known resistance in international ohms through which a steady current known in international amperes passes. The current is also measured by a silver voltameter in series. The value found for Weston cell is 1.0188 volts at $20°$ C.

CHAPTER XX

1. Introduction. Electromagnetic Radiation. We saw in art. 14, Chapter XII, that under certain conditions if a charged condenser of capacity C is discharged through a resistance R and inductance L, the discharge is oscillatory and the frequency of oscillation is given by

$$f = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}. \qquad . \quad . \quad . \quad (20 \cdot 1)$$

Let us now examine how the oscillatory discharge of a condenser will affect conductors placed in its neighbourhood. Since an electric current is always associated with a magnetic field, the oscillatory current during the discharge of the condenser is accompanied by an oscillating magnetic field, which in its turn induces an oscillating e.m.f. in the neighbouring conductor. This induced e.m.f. causes a varying current in the conductor giving rise to an alternating magnetic field which again induces an alternating e.m.f. in a second conductor near it, and so on.

Maxwell suggested that a similar process takes place throughout space whenever an electric discharge occurs i.e. varying electric and magnetic fields are propogated in space. He showed from his electromagnetic equations, as we shall presently see, that varying electric and magnetic fields are propogated through space as a wave motion with the velocity of light i.e. light is an electromagnetic radiation. Before we deduce Maxwell's electromagnetic equations we shall express Ampere's theorem and Faraday's law in a more general form suitable for application to an infinite conductor.

2. Ampere's theorem. According to this theorem the work done in carrying a unit positive pole around the path em bracing the current I is $4\pi I$.

or
$$\oint H ds = 4\pi I, \qquad (20\cdot2)$$

where H denotes the component of the magnetic intensity parallel to each element of length ds

Let us draw the co-ordinate axes at any point of the conductor. The current density i at any point will in general have three components i_x, i_y and i_z. Let us write down the Ampere's theorem for each component separately. Consider a small rectangle ABCD of sides δ_y, δ_z at right angles to the direction of flow of i_x. The line integral along ABCD is
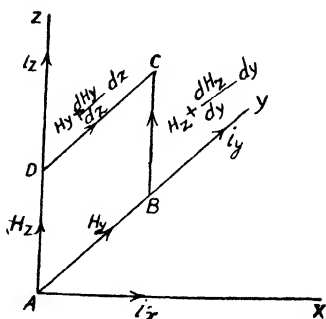
Fig. 20·1

$$H_y \delta y + \left(H_z + \frac{dH_z}{dy}\delta_y \right) \delta z - \left(H_y + \frac{dH_y}{dz}\delta z \right)\delta_y - H_z\,\delta z$$
$$= \left(\frac{dH_z}{dy} - \frac{dH_y}{dz} \right) \delta y\,\delta z.$$

Hence.

$$\left(\frac{dH_z}{dy} - \frac{dH_y}{dz} \right) \delta y\,\delta z = 4\pi i_x\,\delta y\,\delta z$$

or
$$\left(\frac{dH_z}{dy} - \frac{dH_y}{dz} \right) = 4\pi i_x \qquad . \quad . \quad . \quad (20\cdot3)$$

Similarly for the other two components we have,

$$\left(\frac{dH_x}{dz} - \frac{dH_z}{dx} \right) = 4\pi i_y \qquad . \quad . \quad . \quad (20\cdot4)$$

$$\left(\frac{dHy}{dz} - \frac{dHz}{dy} \right) = 4\pi iz \qquad . \quad . \quad . \quad (20\cdot5)$$

The current density is in electromagnetic units.

3. Faraday Neumann Law.

According to this law an induced $e.\,m.\,f.$ is generated in a circuit when it is placed in a changing magnetic field ; and the magnitude of the $e.\,m.\,f.$ is given by

$$e = -\frac{dN}{dt}. \qquad . \quad . \quad . \quad (20\cdot6)$$

where N is the total flux linked with the circuit. Since the electric intensity $E = -\dfrac{\partial \phi}{\partial s}$. (the negative potential gradient), the net *e. m. f.* in any circuit is given by

$$\phi = -\oint E \, ds. \qquad \cdot \quad \cdot \quad \cdot \quad (20 \cdot 7)$$

From eqns. $(20 \cdot 6)$ and $(20 \cdot 7)$ we have

$$c \int E \, ds = -\frac{dN}{dt}, \qquad \cdot \quad \cdot \quad \cdot \quad (20 \cdot 8)$$

i.e. the rate of change of flux is equal to the line integral of the electric force taken round the circuit. c is the velocity of light introduced to convert E into *e. m. u.*

Let us now express eq. $(20 \cdot 8)$ in Cartesian coördinates. Consider a rectangle $\delta y \, \delta z$ perpendicular to the component of the magnetic field. The flux threading this is

$$N_z = H_z \, \delta y \, \delta z$$

and the line integral of E round the rectangle is, as before,

$$\left(\frac{dE_z}{dy} - \frac{dE_y}{dz} \right) \delta y \, \delta z.$$

Hence,

$$\frac{dE_z}{dy} - \frac{dE_y}{dz} = -\frac{1}{c} \frac{dH_x}{dt}. \qquad \cdot \quad \cdot \quad \cdot \quad (20 \cdot 9)$$

Similarly for the y and z components we have

$$\frac{dE_x}{dz} - \frac{dE_z}{dx} = -\frac{1}{c} \frac{dH_y}{dt}. \qquad \cdot \quad \cdot \quad \cdot \quad (20 \cdot 10)$$

$$\text{and } \frac{dE_y}{dx} - \frac{dE_x}{dy} = -\frac{1}{c} \frac{dH_z}{dt} \qquad \cdot \quad \cdot \quad \cdot \quad (20 \cdot 11)$$

4. Equation of continuity.

In addition to the above
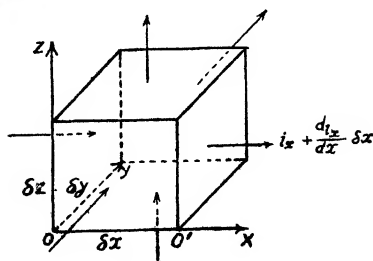
Fig 20·2

fundamental equations three more relations are to be satisfied.

20 E

These are the equations of continuity for the current density i, electric intensity E and magnetic intensity H.

Consider a parallelopiped δx, δy, δz, in an infinite conductor. The current entering the $\delta y \delta z$ face at O is $i_x \delta y \delta z$ while that leaving at O' in the same direction is

$$\left(i_x + \frac{di_x}{dx} \delta_x \right) \delta y \, \delta z$$

The net current leaving the parallelopiped in the direction of the axis of x is

$$\frac{di_x}{d_x} \delta y \, \delta z$$

Similarly for the other two directions we have

$$\frac{di_y}{dy} \delta_x \delta z \text{ and } \frac{di_z}{dz} \delta y \delta_x$$

Hence the total current leaving the parallelopiped is

$$\frac{di_x}{dx} + \frac{di_y}{dy} + \frac{di_z}{dz}.$$

If there is no accumulation of charge within the parallelopiped we have

$$\frac{di_x}{dx} + \frac{di_y}{dy} + \frac{di_z}{dz} = 0. \qquad . \quad . \quad . \quad (20\cdot12)$$

Similarly we have

$$\frac{dH_x}{dx} + \frac{dH_y}{dy} + \frac{dH_z}{dz} = 0, \qquad . \quad . \quad . \quad (20\cdot13)$$

which expresses the fact that the flux entering the parallelo iped is equal to that leaving it

and $\qquad\qquad \dfrac{dE_x}{dx} + \dfrac{dE_y}{dy} + \dfrac{dE_z}{dz} = 0 . \quad . \quad . \quad (20\cdot14)$

Eq (20·14) expresses the famous Gauss's theorem when the charge enclosed is zero.

5. Maxwell's hypothesis and electromagnetic equations in free space. We come across a fundamental difficulty when we try to apply the Ampere's theorem, eq. (20·2), to a circuit containing a condenser. We know that in a circuit containing a condenser and under the influence of a varying

electric force the current flows only in the connecting wires
and there is no flow of charge in between the condenser plates.
Hence it is not possible to draw the equivalent magnetic shell,
since the circuit is not continuous, which will enable us to deduce
the work law.

If q denotes the charge on the plates of a parallel plate con-
denser at any instant t, the current in the wire is

$$I = \frac{dq}{dt}$$

$$= C \frac{dV}{dt}, \qquad \qquad \text{. . .} \quad (20\cdot15)$$

where C is the capacity of the condenser and V the voltage across
the plates. Since $C = \frac{A}{4\pi d}$ and $E = \frac{V}{d}$ we have for the current

$$I = \frac{A}{4\pi c} \frac{dE}{dt}, \qquad \qquad \text{. . .} \quad (20\cdot16)$$

where c is the velocity of light since we are measuring the
current in e.m.u. It was suggested by Maxwell that the quantity
$\frac{1}{4\pi c} \frac{dE}{dt}$ should be regarded as a current, which he has called a
" *displacement current* ", bridging the gulf between the condenser
plates. It is the displacement current which flows between the
plates of the condenser whereas the conduction current flows
in the wire; hence the circuit is continuous. The displacement
current is characterized by a changing electric field. The total
current in the circuit is then

$$C = i + \frac{1}{4\pi c} \frac{dE}{dt}. \qquad \qquad \text{. . .} \quad (20\cdot17)$$

And since the displacement current has as much reality as
the conduction current the foregoing electromagnetic equations
will be modified as follows.

$$\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right) = 4\pi \left(i_x + \frac{1}{4\pi c} \cdot \frac{\partial E_x}{\partial t} \right) \qquad \text{. . .} \quad (20\cdot18)$$

$$\left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \right) = 4\pi \left(i_y + \frac{1}{4\pi c} \cdot \frac{\partial E_y}{\partial t} \right) \qquad \text{. . .} \quad (20\cdot19)$$

$$\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) = 4\pi \left(i_z + \frac{1}{4\pi c} \cdot \frac{\partial E_z}{\partial t} \right) \qquad \text{. . .} \quad (20\cdot20)$$

$$\frac{\partial C_x}{\partial x} + \frac{\partial C_y}{\partial y} + \frac{\partial C_z}{\partial z} = 0 \qquad \qquad \text{. . . (20.21)}$$

By using vector notation the above equations can be written in a more simplified form

$$\text{Curl } H = \nabla \times H = 4\pi i + \frac{1}{c}\frac{\partial E}{\partial t} \qquad \text{. . . (20.22)}$$

$$\text{Div. } C = \nabla . C = \nabla . \left(i + \frac{1}{4\pi c} . \frac{\partial E}{\partial t}\right) = 0 \quad \text{. . . (20.23)}$$

Eqs. (20.9) (20.10) (20.11) and (20.14) become

$$\nabla \times E = -\frac{1}{c}\frac{\partial H}{\partial t} \qquad \qquad \text{. . . (20.24)}$$

$$\nabla . E = 0 \qquad \qquad \text{. . . (20.25)}$$

For free space the conduction current is negligible and the above Maxwell's equations reduce to

$$\nabla \times H = \frac{1}{c}\frac{\partial E}{\partial t} \qquad \qquad \text{. . . (20.26)}$$

$$\nabla . E = 0 \qquad \qquad \text{. . . (20.27)}$$

$$\nabla \times E = -\frac{1}{c}\frac{\partial H}{\partial t}. \qquad \qquad \text{. . . (20.28)}$$

$$\nabla . H = 0 \qquad \qquad \text{. . . (20.29)}$$

Eq. (20.26) is actually made up of three equations,

$$\left.\begin{array}{l} \dfrac{\partial H_z}{\partial y} - \dfrac{\partial H_y}{\partial z} = \dfrac{1}{c}\dfrac{\partial E_x}{\partial t} \\[2mm] \dfrac{\partial H_x}{\partial z} - \dfrac{\partial H_z}{\partial x} = \dfrac{1}{c}\dfrac{\partial E_y}{\partial t} \\[2mm] \dfrac{\partial H_y}{\partial x} - \dfrac{\partial H_x}{\partial y} = \dfrac{1}{c}\dfrac{\partial E_z}{\partial t} \end{array}\right\} \qquad \begin{array}{l} \text{. . . (20.30 } a) \\[2mm] \text{. . . (20.30 } b) \\[2mm] \text{. . . (20.30 } c) \end{array}$$

and so also is eq. (20.28)

$$\left.\begin{array}{l} \dfrac{\partial E_z}{\partial y} - \dfrac{\partial E_y}{\partial z} = -\dfrac{1}{c}\dfrac{\partial H_x}{\partial t} \\[2mm] \dfrac{\partial E_x}{\partial z} - \dfrac{\partial E_z}{\partial x} = -\dfrac{1}{c}\dfrac{\partial H_y}{\partial t} \\[2mm] \dfrac{\partial E_y}{\partial x} - \dfrac{\partial E_x}{\partial y} = -\dfrac{1}{c}\dfrac{\partial H_z}{\partial t} \end{array}\right\} \qquad \begin{array}{l} \text{. . . (20.31 } a) \\[2mm] \text{. . . (20.31 } b) \\[2mm] \text{. . . (20.31 } c) \end{array}$$

Differentiating eq. (20.30 a) with respect to t we have

$$\frac{1}{c}\frac{\partial^2 E_x}{\partial t^2} = \frac{\partial}{\partial y}.\frac{\partial H_z}{\partial t} - \frac{\partial}{\partial z}.\frac{\partial H_y}{\partial t}. \qquad \text{. . . (20.32)}$$

Substituting for $\dfrac{\partial H_z}{\partial t}$ and $\dfrac{\partial H_y}{\partial t}$ from (20·31 b) and (20·31 c) in (20·32),
we have

$$\frac{1}{c^2}\frac{\partial^2 E_a}{\partial t^2} = -\frac{\partial}{\partial y}\left(\frac{\partial E_y}{\partial x}-\frac{\partial E_x}{\partial y}\right)+\frac{\partial}{\partial z}\left(\frac{\partial E_x}{\partial z}-\frac{\partial E_z}{\partial x}\right)$$

$$\cdots \quad (20·33)$$

Similarly, we have

$$\frac{1}{c^2}\frac{\partial^2 E_y}{\partial t^2} = -\frac{\partial}{\partial z}\left(\frac{\partial E_z}{\partial y}-\frac{\partial E_y}{\partial z}\right)+\frac{\partial}{\partial x}\left(\frac{\partial E_y}{\partial x}-\frac{\partial E_x}{\partial y}\right)$$

$$\cdots \quad (20·34)$$

$$\text{and } \frac{1}{c^2}\frac{\partial^2 E_z}{\partial t^4} = -\frac{\partial}{\partial x}\left(\frac{\partial E_x}{\partial z}-\frac{\partial E_z}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\partial E_z}{\partial y}-\frac{\partial E_y}{\partial z}\right)$$

$$\cdots \quad (20·35)$$

Adding eqs. (20·33), (20·34) and (20·35) and using eq. (20·27) we have

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}(E_x+E_y+E_z) = \frac{\partial^2}{\partial x^2}(E_x+E_y+E_z)+\frac{\partial^2}{\partial y^2}(E_x+E_y+E_z)$$

$$+\frac{\partial^2}{\partial z^2}(E_x+E_y+E_z)$$

or

$$\frac{\partial^2 E}{\partial x^2}+\frac{\partial^2 E}{\partial y^2}+\frac{\partial^2 E}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}=0 \quad \cdots \quad (20·36)$$

and similarly,

$$\frac{\partial^2 H}{\partial x^2}+\frac{\partial^2 H}{\partial y^2}+\frac{\partial^2 H}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2 H}{\partial t^2}=0 \quad \cdots \quad (20·37)$$

Eqs. (20·36) and (20·37) are Maxwell's fundamental equations for electromagnetic waves.

The above two equations have a form similar to that of the more familiar wave equations, where the velocity of propagation is c. Thus the electric and magnetic intensities are propagated in space with the velocity of light. It can be shown further (left as an exercise for the reader) that E and H are perpendicular to each other in the wave front. These facts led Maxwell to postulate that light waves are electromagnetic waves consisting of a complementary condition of electric and magnetic intensity travelling in space with the velocity c. This hypothesis of Maxwell was experimentally verified by Hertz.

6. Hertz Experiments. The oscillators used by Hertz to
produce electro-
magnetic waves
were of various
types. One of them
is represented in
Fig 20·3. P_1, P_2
are two rectangular
brass plates each
about 40 sq. cm·
in area with a thick
wire 30 cms. in
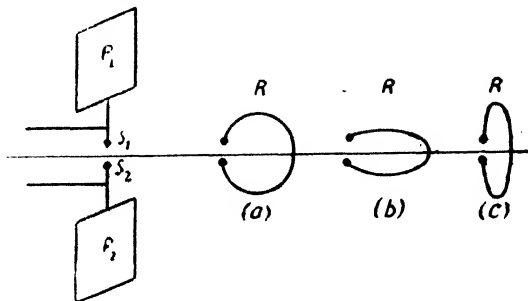
Fig. 20·3

length attached to each plate. At the end of the wire a spherical
brass knob was fixed. The brass knobs S_1, S_2 were connected
to the secondary terminals of an induction coil. The balls S_1, S_2
were gilt and highly polished. The space between them in
the spark gap was 2 or 3 cms. The discharge across the spark
gap was oscillatory, the period being about $1·4 \times 10^{-8}$ sec.

To detect the waves he used a resonator R, fig. 20·3 (a), (b)
and (c), consisting of a piece of wire in the form of a broken
circle terminating in suitable brass knobs separated by an air gap.
The diameter of this circle resonator was about 70 cms. The
length of the sparks gap could be adjusted by a micrometer screw
in an ebonite frame. The action on the resonator is as follows.

If the diameter of the resonator is properly chosen its natural
period of oscillation coincides with that of the oscillator. Owing
to the action of the oscillator the magnetic induction through the
resonator changes giving rise to an induced periodic e.m.f. Owing
to resonance effect the amplitude of the oscillating difference of
potential increases to such an extent that sparks pass between
the knobs. The length of the air gap between the knobs is taken
as the measure of the strength of induction through the resonator
and consequently that of the component of the magnetic intensity
perpendicular to its plane.

It was found that the sparks between the knobs of the resonator
were maximum in length when the oscillator and resonator were

parallel to each other fig. 20.3 (*a*) and (*c*) but vanish altogether
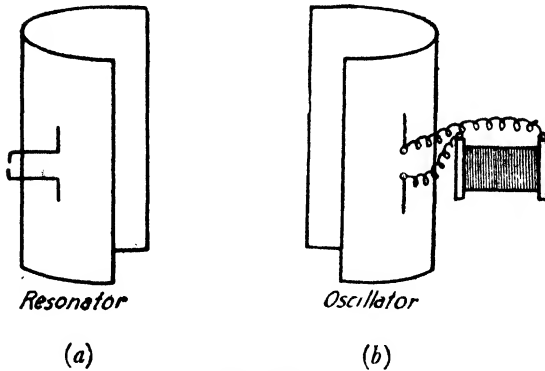
Fig. 20·4

when they were perpendicular to each other fig 20·3 (*b*). This experiment showed that the electromagnetic beam originating from the oscillator is plane polarized.

Hertz demonstrated the existence of stationary electromagnetic waves by allowing them to fall upon a large plane sheet of zinc. The radiation reflected from this sheet and the oncoming incident one together produced stationary waves, since it was found that by moving the resonator out from the sheet maxima at points corresponding to $\frac{\lambda}{4}$, $\frac{3\lambda}{4}$ etc., and minima at points corresponding to $\frac{\lambda}{2}$, λ etc., from the sheet could be detected. The wave-length being thus determined it was easy to calculate the velocity of propagation from the known value of the frequency of oscillation. In this way the velocity of about 3×10^{10} centimetres per second was obtained.

The refraction of electromagnetic waves was also demonstrated by Hertz. The oscillator was a parabolic zinc reflector with two brass cylinders mounted in its focal line to form a spark gap, fig. 20.4. (*b*). The resonator consisted of a similar parabolic reflector. Two pieces of thick wire were placed in its focal line and the spark gap was brought out at the back of the mirror for

convenience of observation, fig. 20.4 (*a*). The oscillator and the reflector were first arranged so that the path of the beam was determined. On inserting a large prism of 30° made of pitch between the oscillator and the resonator it was found that the resonator ceased sparking. The sparking could, however, be restored by moving the resonator sideways. It was thus clear that the prism deviated the beam. The angle of minimum deviation was about 20 degrees giving 1.69 as the refractive index of the prism for the electromagnetic waves.

PART II

MODERN

CHAPTER XXI

CONDUCTION OF ELECTRICITY THROUGH GASES

1. Electrical discharge through gases and the discovery of the electron. The phenomenon of electrical discharge through gases can rightly claim to have laid the first foundation of modern physics. It has led to the discovery of the fundamental particle, the *electron*. It was first in the year 1853 that an electric spark was sent by Maison, an obscure French Physicist, through a rearefied gas, and to his great surprise he discovered that the spark took the form of a bright glow. The interest in the phenomenon was stimulated several years later by Geisler, a German glass blower, who began manufacturing gaseous discharge tubes. It was Sir William Crookes, an English Physicist, who investigated the phenomenon in greater detail.

A glass tube 100 cms. in length and about 3 cms. in diameter has at its two ends two aluminium electrodes which are connected
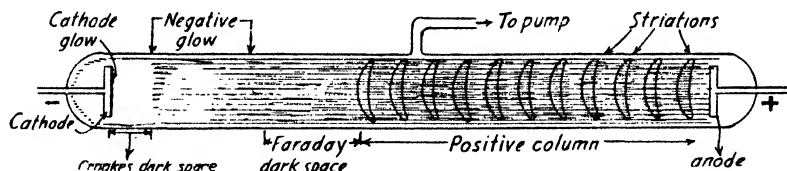
Fig. 21·1.

to the secondary of an induction coil giving a voltage of say 40,000 volts, fig. 21·1. There is an opening in the side of the tube which is connected to a mercury diffusion pump through a liquid air trap, not shown in the figure. The various stages through which the discharge passes as the pressure in the tube falls are described below in order :

(1) At first the discharge refuses to pass but when the pressure in the tube has fallen to 10 mms. of mercury it takes the form of blue streamers.

(2) At a pressure of about 5 mms. the discharge is pink and fills the whole tube.

(3) As the pressure drops to about 2 mms. a dark region called the *Faraday dark space* appears which divides the whole discharge into two parts, a long section which is pinkish in colour is called the *positive coloumn* and a short bluish section near the cathode which is called the *negative glow*.

(4) At a still lower pressure the negative glow recedes from the cathode producing a dark space called the *Crookes dark space* between it and the cathode. At this stage the positive column breaks up into equally spaced layers called the *striations*.

(5) As the pressure further lowers down the striations and the negative glow grow faint and finally at a pressure of 0·01 mm. they disappear completely and the Crookes dark space fills the whole tube.

2. Cathode rays. At this stage the glass of the tube begins to glow with a faint greenish light. This green fluorescence is due to rays coming from the cathode. These are the cathode rays which were first called by Crookes as the fourth state of matter, but as we know now they consist of tiny particles called the electrons. These rays possess the following important properties :

Fig. 21·2.

(*a*) They travel in straight lines. This property can be demonstrated by putting an object in their path and observing its shadow, fig. 21·2.

(*b*) They produce a bluish fluorescence when they fall on a zinc sulphide screen. This property is often utilized for their detection.
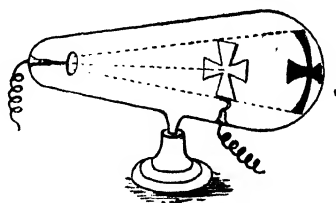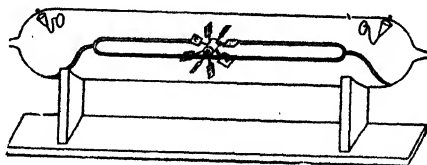
Fig. 21·3.

(c) That the cathode rays possess momentum and energy was demonstrated by Crookes first by putting a small mica vane on two rails inside the discharge tube, fig. 21·3. When the cathode rays fall on the vane they impart momentum to it and it begins to move. This simple experiment of Crookes proved conclusively that the rays possess mass, velocity and kinetic energy i.e. they are corpuscular in character.

(d) That the cathode rays are negatively charged particles was first shown by Jean Perrin, a distinguished French Physicist, fig. 21·4.

The cathode rays after passing through the perforated anode are deflected by a magnet into a Faraday cylinder connected
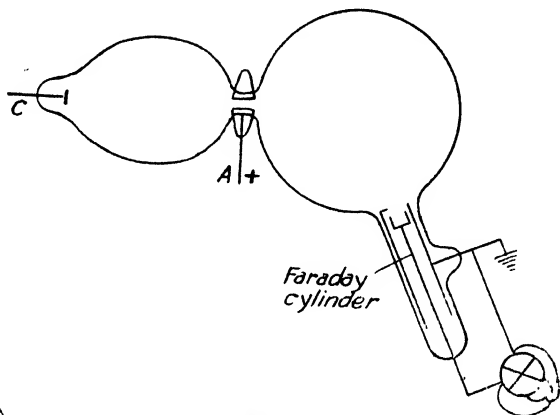
Fig. 21·4

to a quadrant electrometer which indicated a negative charge on the Faraday cylinder.

(e) That the cathode rays can pass through thin sheets of matter was first demonstrated by Lenard. He allowed the rays to pass through a thin aluminium window and detected the phosphorescence produced by them on the other side.

3. e/m of cathode rays by J. J. Thomson. If cathode rays are particles of negative electricity it should be possible to determine their charge and mass. Their specific charge was first determined by Sir J. J. Thomson. A beam of electrons from the cathode C passes through two pin hole diaphragms, D_1 and D_2, which limit the beam into a fine pencil passing between two metal plates PP and a magnetic field and finally falling on the

fluorescent screen F. The plates PP are charged to a high
potential, fig. 21·5.
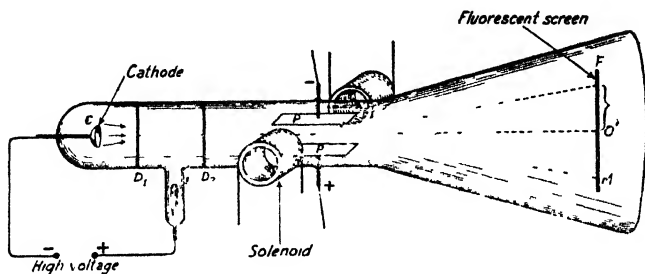
Fig 21 5

Thomson's method for determining e/m for electron.

Let the electrons move along the axis of x and let the electric
field be applied along the axis of z and the magnetic field along
the axis of y. In the absence of both the electric and the mag-
netic fields the electron beam strikes at O'. An electric potential
is applied between the plates with the lower plate as positive.
The electron during its journey between the plates is acted upon
by a downward force given by

$$F_E = Ee \qquad \qquad . \quad . \quad . \quad (21\cdot1)$$

where e is the charge on the electron in e.m.u. and E the electric
intensity in e.m.u. between the plates. The path of the electron
in the electric field is a parabola. As soon as the electron comes
out of the electric field it flies off along the tangent to the path
and strikes the fluorescent screen at M. A charge e moving with
velocity v constitutes a current of magnitude ev. If the electron
is acted upon by a magnetic field of strength H applied perpen-
dicular to its direction of motion, the magnetic force perpendicu-
lar both to the magnetic field and the direction of motion is

$$F_H = Hev \qquad \qquad . \quad . \quad . \quad (21\cdot2)$$

Under the influence of the magnetic field alone the electron will
therefore describe a circular path whose radius of curvature is
given by

$$Hev = \frac{mv^2}{\rho} \qquad \qquad . \quad . \quad . \quad (21\cdot3)$$

Since the centrifugal force $\dfrac{mv^2}{\rho}$ must be equal to the centripetal

force $\dfrac{Hev}{c}$, the electron will be deflected along the axis of z

either below or above O′ depending upon the direction of H.

If now the electric and magnetic field intensities are so adjusted that when the electron is moving simultaneously under the influence of both the fields it remains undeviated, the two forces must be equal and opposite.   We therefore have

$$eE = Hev, \qquad \cdots \quad (21\cdot4)$$

whence.
$$v = \dfrac{E}{H}$$

Thus the velocity of the electron can be known by knowing the magnitudes of E and H.

Having known the velocity of the electron Thomson measured the deflection of the electron under the influence of the magnetic field alone.   From eqs. $(21\cdot3)$ and $(21\cdot4)$ we have

$$\dfrac{e}{m} = \dfrac{E}{\rho H^2}. \qquad \cdots \quad (21\cdot5)$$

Now ρ can be calculated by knowing the deflection D and the dimensions of the apparatus since

$$\tan \theta = \dfrac{l}{\rho} = \dfrac{D}{\sqrt{L^2 + D^2}}.$$

Hence,
$$\rho = l\sqrt{1 + L^2/D^2} \qquad \cdots \quad (21\cdot6)$$

where l is the length of the plates P and L the the distance of the fluorescent screen from P.

If v is the potential difference in volts between the plates

$$E = \dfrac{V\cdot10^8}{d} \; e.m.u, \qquad \cdots \quad (21\cdot7)$$

where d is the distance between the plates. Using $(21\cdot6)$ and $(21\cdot7)$

$$\dfrac{e}{m} = \dfrac{V\cdot10^8}{H^2 d} \cdot \left(\dfrac{1+L^2/D^2}{l}\right)^{-1/2} \dfrac{e.m.u.}{gm} \quad \cdots \quad (21\cdot8)$$

All the quantities on the right hand side of eq. $(21\cdot8)$ are known and hence the value of $\dfrac{e}{m}$ can be determined.   Thomson's value

was $7 \cdot 7 \times 10^6$ *e.m.u./gm.* The present day accepted value is $1 \cdot 76 \cdot 10^7$ *e.m.u./gm.*

Example. Calculate the energy of an electron in electron volts which in a magnetic field of 10^3 gauss has a radius of curvature of 10 cms.

From eq. (21·3) $\text{H}ev = \dfrac{mv^2}{\rho}$,

whence $\qquad\qquad v = \dfrac{\text{H}e\rho}{m}.$

The kinetic energy of the electron is $\frac{1}{2} mv^2$. An electron volt is defined as the kinetic energy which the electron would gain in falling through a potential difference of 1 volt. If it falls through a potential difference of V volts, the kinetic energy would be eV. 10^8 Hence $\qquad e$V. $10.^8 = \frac{1}{2}mv^2$.

From the above two equations we have

$$\text{V} = \tfrac{1}{2}\, 10^{-8}\, \text{H}^2\rho^2 \left(\frac{e}{m}\right) = \tfrac{1}{2}.\, 10^{-8}.10^6.10^2.1\cdot76.10^7$$
$$= 8\cdot8.\, 10^6.\, e\text{V}.$$

N.B. The energy of an electron is generally expressed in electron volts but it can be expressed in ergs too. The conversion can be made by means of an equation which is very useful.

$$\text{E} = \tfrac{1}{2}mv^2 = e_m.\, 10^8 = \frac{e_s \text{V}}{300}.$$

e_m denotes the charge in e.m.u. and e_s denotes the in charge e.s.u.

$$1 e.\text{V} = \frac{e_s}{300}\, ergs = \frac{4\cdot8 \cdot 10^{-10}}{300} = 1\cdot6 \times 10^{-12}\, ergs.$$

4. Charge on the Electron. The earliest determinations of the electronic charge were made at the Cavendish laboratory by a band of distinguished physicists such as J. J. Thomson, C. T. R. Wilson and H. A. Wilson. Their experiments are now of only historical interest. It was R.A. Millikan, an American Physicist, who first determined the charge on the electron with great precision. We give below his method.

Millikan's oil drop method. Milikan's apparatus consists essentially of a parallel plate condenser connected to a high

voltage battery, through the switch S, fig. 21·6. The upper plate of the condenser has in its middle a fine pin hole P through which tiny oil drops find their way from the cloud on the top formed
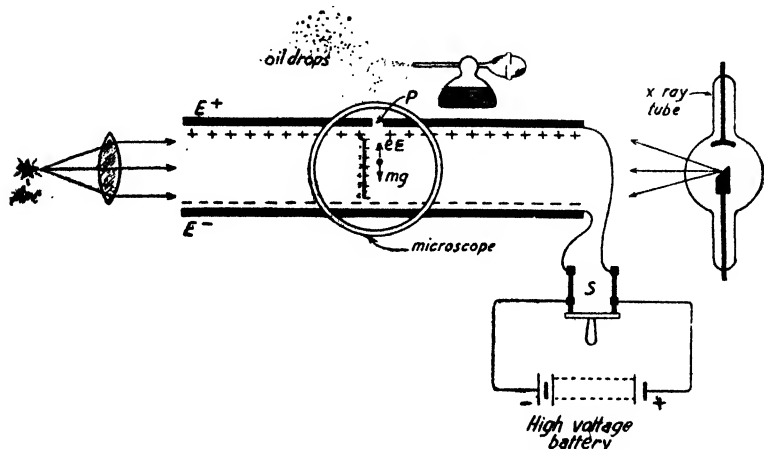
Fig. 21·6. Millikan's oil drop method for determining electronic charge.

by an oil spray. The oil drop is illuminated by an arc light and is observed by means of a low power microscope provided with horizontal parallel cross-wires and a vertical scale. The circle in the figure gives the field of view of the microscope.

During the process of spraying the oil droplets are charged with electricity by friction. At first both the condenser plates are earthed i.e. they are uncharged. The oil droplet is acted upon by a vertical downward force due to gravity and as it is moving in a viscous medium (viscosity of air) it is at the same time acted upon by an upward resisting force of the air. The two forces then balance and the droplet attains the terminal velocity before it has reached the upper cross-wire. Its velocity is determined by using a stop watch and measuring the time it takes to fall through the distance between the two cross-wires. The upward resisting force on the droplet is from Stokes law

$$F = 6\pi a\eta v, \qquad \qquad \cdots \quad (21·9)$$

where a is the radius of the droplet, η, the viscosity of the air

and v the velocity of the droplet.   When the particle has attained the terminal velocity v_1, F must be equal to mg, the downward force due to gravity.   Hence,

$$6\,\pi a\eta v_1 = mg \qquad \ldots \quad (21 \cdot 10)$$

If d is the distance between the cross-wires and t the time of fall, the velocity v_1 is given by

$$v_1 = \frac{d}{t_1} \qquad \ldots \quad (21 \cdot 11)$$

When the droplet nears the bottom plate the switch S is closed charging the upper plate as positive.   The charged droplet (if negative) will now begin to move upward with the velocity v_2. Equating the downward force to the upward force we have

$$6\,\pi a\,\eta\,v_2 + mg = E\,e_n, \qquad \ldots \quad (21 \cdot 12)$$

where e_n is the charge on the droplet and E the electric intensity between the plates.   The velocity v_2 is given by

$$v_2 = \frac{d}{t_2} \qquad \ldots \quad (21 \cdot 13)$$

The velocity v_2 can be suitably chosen by varying the field.   Care is taken to observe only that droplet amongst many which is moving straight with a slow velocity.

As the droplet nears the top plate the switch S is opened and the plates are earthed.   The droplet again begins to fall under gravity and the velocity of free fall is determined as before.   As soon as it nears the bottom plate the switch S is closed and the droplet rises once more.   In this way the same droplet can be made to move up and down a large number of times.   Each time both during fall and rise the velocity is measured.

Taking the oil droplet as a small sphere of radius a and density ρ, its mass is

$$m = \tfrac{4}{3}\pi a^3 \rho \qquad \ldots \quad (21 \cdot 14)$$

From eqns $(21 \cdot 9)$ and $(21 \cdot 14)$ we have.

$$6\pi a\,\eta v_1 = \tfrac{4}{3}\pi a^3 g(\rho - \sigma), \qquad \ldots \quad (21 \cdot 15)$$

where σ is the density of air.   Neglecting σ in (eq. $21 \cdot 15$) we have for the radius

$$a = \left(\frac{9\eta \, v_1}{2g\rho} \right)^{\frac{1}{2}} \qquad \qquad \text{. . . (21.16)}$$

From eqs. $(21 \cdot 10)$ and $(21 \cdot 12)$ we have

$$\frac{v_1 + v_2}{v_1} = \frac{E \, e_n}{mg}$$

and so using eqs. $(21 \cdot 14)$ and $(21 \cdot 16)$,

$$e_n = \frac{4\pi}{3} \left(\frac{9\eta}{2} \right)^{3/2} \left(\frac{v_1}{g\rho} \right)^{1/2} \frac{(v_1 + v_2)}{E} \, v_1^{\frac{1}{2}}. \quad \text{. . . (21·17)}$$

It was observed by Millikan that by exposing the space between the plates to x-rays the charge on the oil droplet could be increased or decreased almost at will. If e_n' is the charge on the droplet corresponding to the velocity v'_2 then from eq. $(21 \cdot 17)$

$$e_n' = A \, (v_1 + v'_2) \, v_1^{\frac{1}{2}}, \qquad \text{. . . (21·18)}$$

where $A = \frac{4\pi}{3} \left(\frac{9\eta}{2} \right)^{\frac{3}{2}} \left(\frac{v_1}{g\rho} \right)^{\frac{1}{2}} \frac{1}{E}$.

The velocity v_1 of free fall remains unchanged whatever the charge on the droplet may be because the total mass of the electrons on the droplet is very small compared to the mass of the droplet. From eqs. $(12 \cdot 17)$ and $(12 \cdot 18)$

$$e_n' - e_n = A v_1^{1/2} (v'_2 - v_2)$$

or $\qquad pe = A \, v_1^{1/2} \left(\frac{d}{t'_2} - \frac{d}{t_2} \right), \qquad \text{. . . (21·19)}$

where p is an integer and e the elementary electronic charge. Millikan actually found from a large number of experiments that the charge on a droplet is never less than a certain minimum and is always some integral multiple of this value. Millikan's value for the elementary charge is $(4 \cdot 774 \pm 0 \cdot 005) \times 10^{-10}$ e.s.u. While the latest and the most accurate value is $e = 4 \cdot 802 \times 10^{-10}$ *e.s.u.*

(5) Mass of the electron. Thomson's value of $\frac{e}{m}$ is $1 \cdot 76.10^7$.*e.m.u./gm*

or $\quad \frac{e}{m} = 1 \cdot 76.10^7 \cdot 3 \cdot 10^{10}$. *e.s.u./gm*. Charge on the electron is

$$e = 4 \cdot 80 \times 10^{-10} \, e.s.u.$$

Hence the mass of the electron is

$$m = \frac{4 \cdot 8.10^{-10}}{1 \cdot 76.3.10^{17}} gms = 9 \cdot 11 \times 10^{-28} gm.$$

EXAMPLE.   Calculate the velocity of an electron when it falls through a potential difference of a thousand volts.

$$\frac{e.V}{300} = \tfrac{1}{2} mv^2$$

$$\frac{4 \cdot 8.10^{-10} . 10^3}{300} = \tfrac{1}{2}(9.10^{-28})v^2$$

Hence $v = \sqrt{\dfrac{10^3.4 \cdot 8.2.10^{-10}}{300.9.\ 10^{-28}}} = 1.9.10^9 \, cms/sec.$

CHAPTER XXII
ELECTRONICS

1. Introduction. When a metal is heated it gives off electrons, higher the temperature more copious is the emission. This process is called *Thermionic Emission*. The mechanism of electron emission from heated metals is very simple. Inside the metals there are electrons which move freely from one atom of the metal to the other. In fact the electrons are as mobile as the molecules of a gas and we can consider them as forming an electron gas. But the electron gas inside the metal differs from the ordinary classical gas with which we are familiar in all very important respects that the constituent electrons do not obey like the gas molecules the usual Maxwell's law of distribution of velocities but another very important and fundamental law known as Fermi-Dirac law, the discussion of which cannot be undertaken here. Ordinarily these free electrons cannot escape from the surface of the metal since they have to overcome a potential barrier at the metal surface. But as the temperature rises the kinetic energy of the electrons rises, and those whose kinetic energy is greater than the surface restraint succeed in escaping. The minimum amount of work needed to overcome this surface restraint is known as the *thermionic work function* of the metal. For tungsten it has a value of 4·52 volts. The magnitude of the thermionic current is given by the well known Richardson's equation,

$$i = AT^2 e^{-\phi/kT}, \qquad \qquad \cdots \quad (22 \cdot 1)$$

where i is the saturation current in amps/sq. cm. of the filament surface. ϕ is the work function in ergs, T the absolute temperature of the filament and A a constant. The phenomenon of thermionic emission is the basis of the whole science of electronics.

An ordinary vacuum tube consists of an evacuated glass bulb in which is sealed a filament, which emits electrons when heated ;

a plate which is kept at a positive potential to attract the electrons and subsidiary electrodes between the plate and the filament to control the flow of electrons. Such an arrangement is called a thermionic tube. Tubes are classified as diodes, triodes, tetrodes, pentodes, etc. depending upon the number of electrodes present.

2. The Diode. The diode is the simplest vacuum tube which consists of a filament and a plate. The positive potential on the plate attracts the electrons from the heated filament, giving rise to a thermionic current the magnitude of which depends upon the number of electrons reaching the plate per second. The direction of the current within the tube is from the plate to the cathode, since by a convention which is purely arbitrary the direction of the current flow is opposite to that of the flow of the electrons. If the potential on the plate is not high enough to remove all the electrons as fast as they are emitted, near the surface of the filament a swarm of electrons collects which exerts a repulsive force on the electrons which have just left the filament. This cloud is called the *space-charge*. Hence the field near the filament is the resultant of two opposite forces, one the potential of the plate and the other the field of the space charge. Let us now investigate some of the characteristics of a diode.

Characteristics of a diode. The characteristics of the diode are studied with the arrangement shown in fig. 22·1 (a). H. T. is a high tension battery for varying the plate voltage and L. T. a low tension battery for heating the filament. A is a milliammeter to read the current in the plate and the filament circuits. The experimental results are shown graphically in fig. 22·1 (b). For a high negative plate voltage there is no plate current since all the electrons are repelled back. When the plate voltage is positive
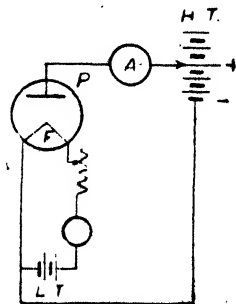
Fig. 22·1 (a)

and small, there is a current, though small, in the plate circuit
since some of the very fast
electrons are able to overcome
the resultant repulsive force.
As the voltage on the plate
increases the current increases
till it reaches a saturation value
corresponding to part BA of
the curve. Beyond the point
B there is no increase in the
plate current with the in-
crease in the plate voltage
which means that   the plate
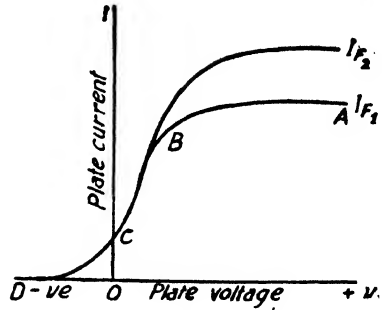
Fig. 22·1 (b)

voltage is high enough to attract all the electrons as fast as
they are emitted. If the filament current increases from I_{F_2}
to I_{F_1} the saturation current increases since there are more
electrons available; but at the same time a higher plate
voltage is needed to reach the saturation point. This arises
because of the space charge effect.   At values of plate
current less than saturation where the current is limited by
space charge, it can be shown theoretically for a special
case of plane parallel electrodes that the plate current is
given by

$$i_p = K V_P^{3/2}, \qquad \qquad . \quad . \quad . \quad (22·2)$$

where V_P is the plate voltage and K some constant.

Diode as a rectifier. We have seen that in a diode the current
will flow when the plate is positive with respect to the filament.
If an alternating e.m.f. is now applied between the filament and the
plate, the current will flow only during the positive half of the cycle
and no current would flow during the negative half of the cycle
i.e we have produced half-wave rectification by using a diode.
Diodes are used as rectifiers in modern radio receivers.   Gas-filled
diodes are used for rectifying heavy currents, for such purposes as
charging accumulators from A. C. mains.

3. The triode. Lee de Forest by introducing a third elec-
trode, what is called a *grid*, between the
filament and the plate provided an impor-
tant means for controlling the flow of
electrons in the tube. We shall presently
see that the grid enables the three ele-
ctrode tube, or triode to act as an ampli-
fier and oscillator. By making the po-
tential of the grid positive the space charge
can be neutralized thus increasing the plate

Fig. 22·2

current. On the other hand if the grid is negative the space charge
effect is augmented, decreasing the plate current. Since the grid is
situated nearer to the filament than the plate a small change in
the grid potential can produce a very much larger change in the
plate potential. If we consider the filament and the grid as a
condenser system with C_{gf} as its capacity, and the plate and the
filament as the other condenser system with C_{pf} as its capacity,
then the field is proportional

$$C_{pf} V_P + C_{gf} V_g, \qquad . \quad . \quad . \quad (22\cdot3)$$

where V_P and V_g are the potentials on the plate and the grid respec-
tively. And since the current is proportional to the 3/2 power of
the field just outside the filament, we have

$$i \propto (C_{pf}.V_P + C_{gf}.V_g)^{3/2},$$

$$\text{or} \qquad i = \alpha (V_P + \mu V_g)^{3/2}, \qquad . \quad . \quad . \quad (22\cdot4)$$

where α is a constant and $\mu = \dfrac{C_{gf}}{C_{pf}}$.

From eq. (22·4) we see that the current depends both on the plate
voltage as well as on the grid voltage. A potential on the grid is
μ times as effective as is a potential on the anode. The quantity
μ is known as the *voltage amplification factor* of the valve and is its
structural constant. μ in modern valves can be as high as 50.
The current leaving the filament is determined chiefly by the grid
voltage but most of the current goes to the plate i.e the grid does
work for the plate.
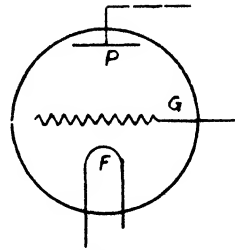
Triode characteristics. Since the plate current depends both on the grid and plate voltages, we have

$$I_p = f(V_g, V_p), \qquad \ldots \quad (22 \cdot 5)$$

where the filament current is kept constant at rated value. The variation of I_p with V_g is studied with an arrangement shown in fig. 22·3 (a).
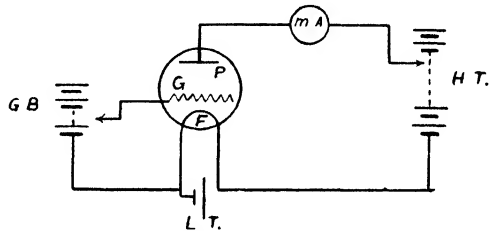
Fig. 22.3(a)

H.T. is a high tension battery for applying suitable voltages to the plate. G.B, or what is called a grid bias, is a battery
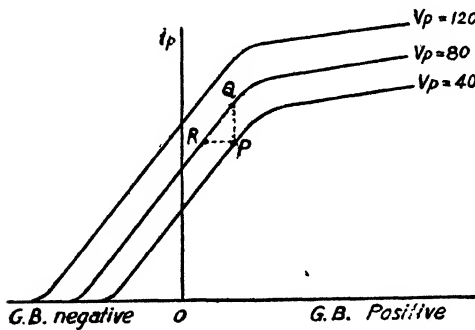
Fig. 22·3 (b)

with tappings every 1 volt and M. A. is a milliampere having suitable ranges to read the plate current. Fig. 22·3 (b) shows a family of characteristic curves illustrating the variation of plate current with grid voltage for various values of the plate voltage for a fixed filament current.

The *amplification factor* μ of a triode valve is defined as the ratio of the change in plate voltage to a change in grid voltage, the plate current remaining constant,

$$\mu = \frac{\partial V_p}{\partial V_g} \quad , \text{ for } dI_p = 0 \, .$$

In fig. 22·5, μ will be given by $\dfrac{40}{RP}$ since the increase PQ in the plate current could be achieved either by keeping the grid bias fixed at OR and increasing the plate voltage by 40 or by keeping the plate voltage fixed and increasing the grid bias by an amount RP.

The *mutual conductance* of the valve is defined by

$$g_m = \frac{\partial I_p}{\partial V_g} \quad , \qquad dV_p = 0.$$

i.e. it is given by the slope at any point on the curves of fig. 22·5. It is the ratio of the increase of the plate current and the corresponding grid voltage.

The *anode slope conductance* of the valve is defined as the ratio of the change in plate current to the change in the plate voltage *i.e.*

$$g_p = \frac{\partial I_p}{\partial V_p}, \qquad dV_g = 0.$$

The reciprocal of g_p is called the *anode slope resistance* and is generally denoted by r_p *i.e.*

$$r_p = \frac{1}{g_p} = \frac{\partial V_p}{\partial I_p} \, .$$

r_p is also called the *impedance* of the valve.

Now $\mu = \dfrac{\partial V_p}{\partial I_p} \cdot \dfrac{\partial I_p}{\partial V_g} = r_p \cdot g_m.$

i.e. amplification factor=impedance $\times$ gradient of the characteristic.

It is easy to see from the shape of the characteristic curves that the above three quantities are approximately constant over the straight portion of the curves, but they change considerably near the bends.

Triode as an amplifier. The amplifying action of a triode depends on the fact, as we have seen before, that a small change of potential applied to the grid is μ times as effective in influencing the plate current, as is the same change of potential applied to the plate. By employing a number of triodes in series any desired voltage amplification can be achieved, the amplified output voltage of one valve serving as the input voltage of the next.

The basic circuit of a triode as an amplifier is shown in fig. 22·4(a) The signal voltage to be amplified is applied between the grid and the filament. The grid bias G. B. is so adjusted that the instantaneous grid potential never becomes positive with respect to the filament as indicated
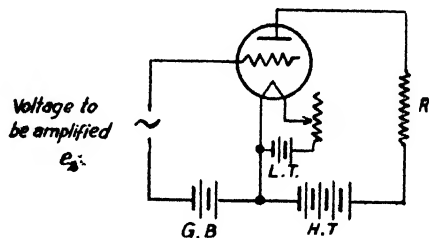
Fig. 22·4 (a)

in fig. 22·4 (b). If the grid is positive a current would flow in the grid circuit ; this would mean drawing of energy from the supply e_s which is rather undesirable. Instead of having a resistance R in the plate circuit we can have in general a load of impedance Z.
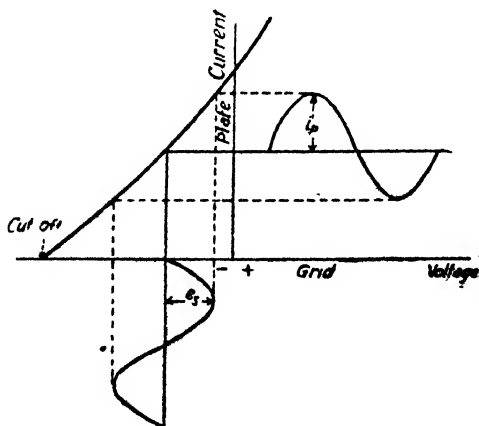
Let us now investigate by how much the signal

Fig. 22·4 (b)

voltage e_s is amplified. A voltage change e_s in the grid circuit is equivalent to a voltage change μe_s in the plate circuit. If

R_a denotes the internal resistance of the valve, the change I_p in the plate current is given by

$$I_p = \frac{\mu \, e_s}{R + Ra}, \qquad \qquad \cdots \quad (22 \cdot 6)$$

since $R + Ra$ is the total resistance.

The change of potential drop across R is

$$\delta V = R I_p = -\frac{R \, \mu^e s}{R + Ra} \; .$$

Hence the voltage amplification is

$$A_v = \frac{\delta V}{e_s} = \frac{R \, \mu}{R + Ra}. \qquad \cdots \quad (22 \cdot 7)$$

As R approaches infinity, A_v approaches μ as a limiting value. For $R = Ra$ it is equal to $\mu/2$. But a large R would mean a large potential drop across the load resistance R so that the voltage on the plate may be only a small fraction of the H.T. Hence, in order to get a high amplification in this case it is necessary to use a valve of high amplification factor or else use several stages of amplification.

If instead of having a pure resistance load R we have an impedance load Z, the alternating component of the plate current is

$$I_p = \frac{\mu \, e_s}{Ra + Z} = \frac{\mu \, e_s}{Ra + R + j\omega L}, \qquad \cdots \quad (22 \cdot 8)$$

where L is the inductance of the circuit.

$$\delta V = I_p Z = \frac{\mu \, e_s \sqrt{R^2 + L^2 \omega^2}}{\sqrt{(R_a + R)^2 + L^2 \omega^2}}$$

Hence
$$A_v = \frac{I_p Z}{e_s} = \frac{\mu \sqrt{R^2 + L^2 \omega^2}}{\sqrt{(Ra + R)^2 + E^2 \omega^2}} \qquad \cdots \quad (22 \cdot 9)$$

If the resistance R of the load is negligible eq. (22·9) becomes

$$A_v = \frac{\mu \omega L}{\sqrt{R^2 a + L^2 \omega^2}} \; . \qquad \cdots \quad (22 \cdot 10)$$

Since the above expression for A_v depends on the frequency ω of the incoming signal it might appear at first sight that there would not be a uniform amplification for all frequencies which would rather be an objectional feature of such an amplifier for it would lead to frequency distortion. This is, however, not the case as A_v is practically constant and equal to μ for values of Lω large

compared to R_a. And hence, if we so choose L that $L\omega$ is large compared to R_a for the lowest frequency with which we are concerned, the amplification will be practically constant for all the higher frequencies. Impedance-coupled amplifiers have one advantage over resistance-coupling in that the resistance of the coil to the direct current is relatively small, so that the average plate voltage impressed on the tube is almost equal to H. T. This allows lower values of plate supply voltage to be used. The resistance-coupled amplifier is capable of delivering constant amplification over a wider frequency range than the other type, particularly in the lower frequencies.

5. Triode as a radio frequency amplifier. Radio frequency amplifiers may be divided into tuned or untuned amplifiers: the former amplify only a narrow band of frequencies, all frequencies lying outside the selected band are discriminated against;
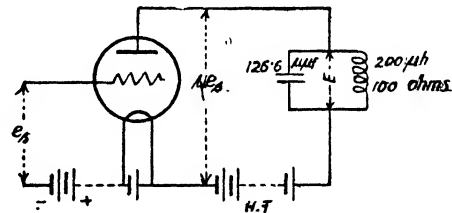
Fig. 22·5 (a)

the latter give a fairly uniform response over a wide range of frequencies. The impedance-coupled and other types of amplifiers discussed in the previous article can be used as untuned amplifiers. Tuned radio frequency amplifiers are used in modern receiving sets in order to secure a high degree of selectivity. We shall discuss here only tuned radio frequency amplifiers.

Tuned amplifiers. Fig. 22·5 (a) shows the effect of the internal resistance of the valve on the voltage across a parallel resonant circuit. If the internal resistance of the tube were zero, the output voltage is uniform over the entire frequency range and is equal to $\mu\, e_s$. As the resistance increases the curves begin to develop a peak symmetrical about the resonance frequency. For higher resistances the peak gets sharper i.e for frequencies both above and below the resonance frequency the output voltage falls abruptly, and that is what we need. Why it so

happens is easy to understand. At resonance, the impedance of the parallel resonant circuit falls down abruptly and the current which is inversely proportional to inpedance increases correspndingly.
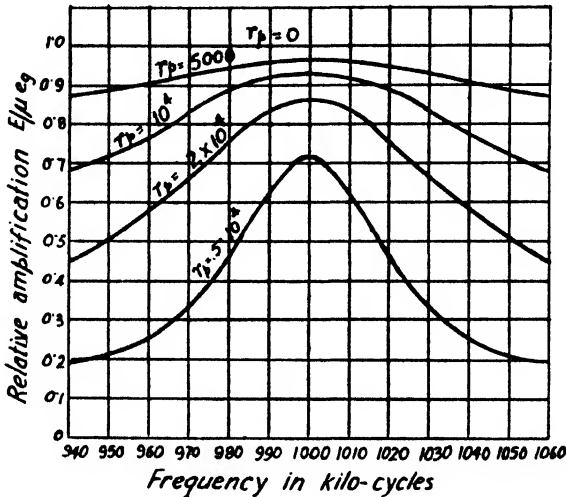
Fig. 22.5 (b)

The maximum amplification per stage both in the resistance-coupled as well as impedance-coupled amplifier is limited to the amplification factor μ of the triode. A higher amplification can be

Fig. 22·5 (c)

achieved by using a step-up transformer as shown in fig. 22·5 (c)
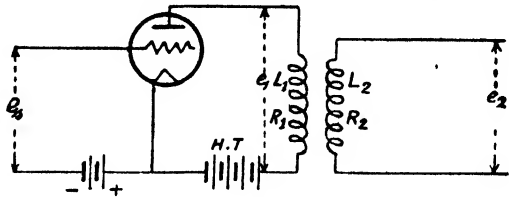
Assuming the secondary of the transformer to be on open circuit, the secondary voltage e_2 will be given by a times the primary voltage e_1 where a is the transformation ratio i.e.,

$$e_2 = a \frac{\mu e_s \sqrt{R_1{}^2 + L_1{}^2\omega^2}}{\sqrt{(R_a + R_1)^2 + L_1{}^2\omega^2}} \qquad . \qquad (22 \cdot 11)$$

If R_1 is very small compared to ωL_1 the voltage amplification A_v is

$$A_v = \frac{\mu a \omega L_1}{\sqrt{Ra^2 + L_1{}^2\omega^2}} \ . \qquad \cdot \quad \cdot \quad \cdot \quad (22 \cdot 12)$$

At the frequencies such that ωL_1 is very large compared to the valve resistance R_a, A_v becomes

$$A_v = \mu a. \qquad \cdot \quad \cdot \quad \cdot \quad (22 \cdot 13)$$

In an ideal transformer

$$A_v = \mu \frac{N_r}{N_p}, \qquad \cdot \quad \cdot \quad \cdot \quad (22 \cdot 14)$$

where N_s and N_p are the number of turns in the secondary and primary respectively. The above approximations are valid for frequencies below about 500 cycles. Transformer-coupled amplifiers are most commonly used in modern radio receivers, since they are capable of producing higher amplification per stage than the impedance-coupled type.

5. Triode as an oscillator. We know that a circuit containing an inductance L, a capacity C and a resistance R begins to oscillate, under certain conditions with frequency $\dfrac{1}{\sqrt{LC}}$. In a short time the condenser is discharged and the oscillations stop. Such oscillations which gradually decrease in amplitude and finally die out are called damped oscillations. These are not of much practical importance. In wireless telephony we need undamped oscillations which do not weaken and die out. With the help of a vacuum tube we can start and maintain oscillations.
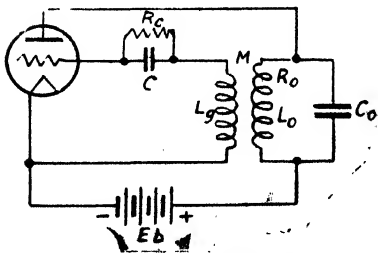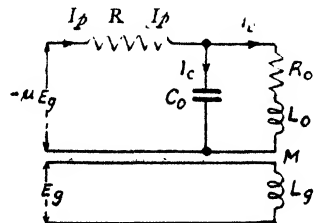
Fig. 22·6 (a)   Fig 22·6 (b)

One form of a circuit in which the triode acts as an oscillator is shown in fig. 22.6 (a)

The grid circuit contains a by-pass condenser **C** and a coil L_g which is inductively coupled to the coil L_O in the plate circuit. If I_L denotes the oscillatory current flowing in L_O, the voltage induced in the grid is $-j\omega MI_L$, depending upon the sign of the mutual inductance M, which as we shall see has to be negative in order that the circuit may be self-exciting. The coupling between L_g and L_O can be varied if desired.

The grid of the oscillator is biased negatively to a point considerably beyond cut-off. The required grid bias is obtained by means of a grid leak R_c. The grid current consists of a series of rectified impulses which flow through R_c. The condenser C acts as a radio frequency by-pass to enable the full value of excitation voltage induced in L_g to be effective.

If the filament current is already flowing the circuit will begin to oscillate as soon as the plate voltage is switched on. Initially the plate current I_p is zero and as it grows in L_O it induces a voltage in L_g equal to $M \dfrac{dI_p}{dt}$. The induced voltage in the grid would increase the plate current I_p if M is negative, which follows from elementary considerations. As the plate current approaches the saturation value $\dfrac{dI_p}{dt}$ decreases and finally drops down to zero. The increase in the plate current was caused by the voltage induced in the grid circuit, and when this ceases, the plate current falls. A decreasing plate current induces a voltage in the opposite direction in the grid circuit, which further decreases the current until cut-off is reached, and whole cycle starts over again.

Let us consider the equivalent circuit of fig. 22·6 (*b*). Applying Kirchhoff's laws to the equivalent circuit and assuming steady state conditions, we obtain the following equations.

$$I_p = I_L + I_c \qquad \qquad \cdots \quad (22 \cdot 15)$$

$$-\mu E_g = I_p R_p + I_L (R_0 + j\omega L_0) \qquad \cdots \quad (22 \cdot 16)$$

$$\frac{I_c}{j\omega C_0} = I_L (R_0 + j\omega L_0) \qquad \cdots \quad (22 \cdot 17)$$

$$E_g = -j\omega MI_L \qquad \qquad \cdots \quad (22 \cdot 18)$$

From (22·16) using (22·15) and (22·18) we have

$$i_{\mu\omega} MI_L = (I_L + I_c) R_{\not p} + I_L (R_0 + j\omega L_0) \quad . \quad . \quad . \quad (22·19)$$

Substituting the value of I_c from (22·17) in (22·19) we have

$$j_{\mu\omega} MI_L = j\omega C_0 I_L (R_0 + j\omega L_0) R_{\not p} + I_L (R_0 + R_{\not p} + j\omega L_0) \quad (22·20)$$

or $I_I . [(R_0 + R_{\not p} - \omega^2 L_0 C_0 R_{\not p}) + j\omega (L_0 + C_0 R_0 R_{\not p} - \mu M)] = 0 \quad (22·21)$

If the left hand side of (22·21) is to be zero each term in the square brackets must be separately equal to zero. From the second term we get

$$M = \frac{L_0 + C_0 R_0 R_{\not p}}{\mu} \quad . \quad . \quad . \quad (22·22)$$

which gives the minimum value of M for the circuit to oscillate.

From the first term we have

$$\omega = \sqrt{\frac{R_0 + R_{\not p}}{L_0 C_0 R_{\not p}}} = \omega_0 \sqrt{1 + \frac{R_0}{R_{\not p}}}, \quad . \quad . \quad . \quad (22·23)$$

where $\omega_0 = \dfrac{1}{\sqrt{L_0 C_0}}$.

From eq. (22·23) we see that ω depends upon the resistance of the load and the resistance $R_{\not p}$ of the tube. Hence any changes in them produces a slight change in the frequency of oscillation. In the above amplified treatment we have assumed that no current flows in the grid and the various currents are sinusoidal.

7. Modulation. Modulation is defined as the process where-
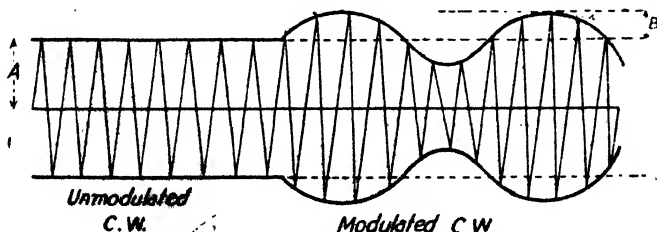
Fig 22·7

by the audiofrequency oscillations produced by the microphone circuit (arising out of the speech of the broadcaster) are super-

posed on the radio frequency continuous wave or C. W. oscillation called the *carrier wave*; the resultant wave form is known as the *modulated wave*. It should be clearly understood that a modulated wave is entirely a radio frequency oscillation which prior to detection exhibits no physical audiofrequency properties. Fig. 22·7 shows a carrier wave of unmodulated amplitude A modulated by a pure tone. The amplitude of the modulated wave varies between $A+B$ and $A-B$. B is called the *depth of modulation*.

Let us now consider a carrier wave (unmodulated) of amplitude A and frequency p. Let the frequency of modulation be ω and its amplitude K. The high frequency current i is given by

$$i = (A + K\sin\omega t)\ \sin pt$$

i e $\quad i = A\ (1 + m\sin\omega t)\ \sin pt,\qquad\qquad$ . . . (22·24)

where $m = \dfrac{K}{A}$

or $\quad i = A\ \sin p t - \tfrac{1}{2}\ Am\ \cos\ (p+\omega)\ t + \tfrac{1}{2}\ Am\ \cos\ (p-\omega)t.\quad$ (22·25)
From eq. (22·25) we see that the high frequency wave contains three waves ; the original carrier wave of frequency p and two others, $(p+\omega)$ lying above and $(p-\omega)$ lying below the carrier frequency. These sum and difference terms are called *side bands*. The electrical oscillations imposed on the carrier wave would be of a very complex nature which, however, can be resolved into a number of pure tones similar to that represented by $K\sin\omega t$. Hence the side band would be a continuous spectrum.

Side bands. We have seen that a wave radiated from a transmitting station can be analysed into a continuous wave oscillation having the carrier wave frequency and a number of superposed oscillations whose frequencies are greater and less than that of the carrier by amounts equal to the frequencies of the original
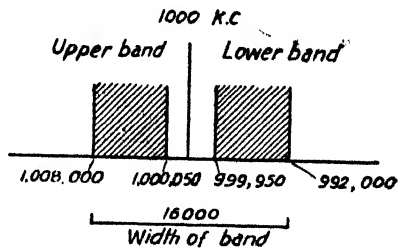
Fig. 22·8

pure tones in the speech or music. The band of such frequencies

above the carrier frequency is known as the *upper sideband*, and that below the carrier frequency as the *lower sideband*, for example, if the carrier frequency is 1000 kc/sec. and the limiting frequencies of the speech or music are 50 and 8000 cycles per second the frequency is as shown in fig 22·8.

The receiving circuit must therefore be adjusted so that all frequencies in the band are equally well received. It will, therefore, be clear that in order to prevent interference in reception the difference between the carrier frequencies of any two radio transmitters must be more than 8000 cycles/sec. (which is nearly the upper limit for music). The prewar international agreement was 900 cycles/sec. for adjacent broadcasting stations. These stations which are adjacent in frequency are widely spaced geographically.

Grid modulation. The modulation of a carrier wave can be secured in a variety of ways, and the simplest is shown in fig. 22·9. In this type of arrangement the secondary S of a transformer is put in the grid circuit and the primary contains the microphone (speech input). When the broad-caster speaks before the microphone the current in the secondary varies according to the words spoken ; this causes variations in the grid potential which give rise to variations in the plate current and consequently to the oscillations in the $L_0 C_0$ circuit.
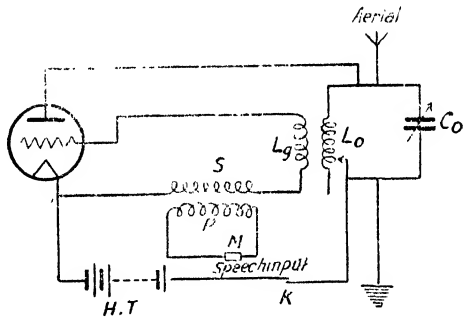
Fig. 22·9 Grid circuit modulation.

8. Detection. Detection is the name given to the process whereby the signal is extracted from the impressed modulated wave. The necessity of extracting the signal from the modulated wave

22 E

arises from the fact the human ear as well as the telephone are unable to respond to radio frequencies.

To produce this result the symmetrical wave form shown in fig. 22·10 (*a*) has to be rendered asymmetrical, as shown in fig. 22·10 (*b,c*) by which we mean that either the upper half or the lower half of the wave form is either reduced or entirely cut off. When this is done the average value of the out-put current varies
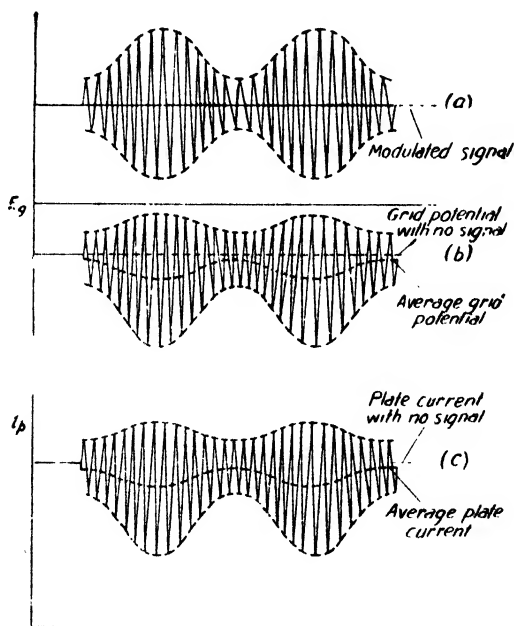
Fig. 22·10

in accordance with the modulated envelope of the impressed wave. The basic function of a detector is, therefore, to produce an asymmetrical variation of the impressed signal. Before detection the wave is entirely at radiofrequency, but after detection an audiofrequency component is present and may be separated by means of suitable circuits.

Triode as a rectifier. By making use of the non-linear portion of the characteristic curve of the triode, rectification can be achieved. Such a curved characteristic can be represented by a power series,

$$i = \alpha_0 + \alpha v + \beta v^2 + \quad , \qquad \bullet \ \bullet \ \bullet \quad (22 \cdot 26)$$

where i is the current through the rectifier when a potential v is applied to it.

When the amplitude of the impressed wave is small, the working portion of the characteristic may be approximately expressed by the first three terms of the power series,

$$i = \alpha_0 + \alpha v + \beta v^2.$$

Suppose that a modulated wave given by

$$v = E_0(1 + m \sin \omega t) \sin pt.$$

is impressed on the detector. The current through the detector is given by

$$i = \alpha_0 + \alpha E_0(1 + m \sin \omega t) \sin pt + \beta \ E_0^2(1 + m \sin \omega t)^2 \sin^2 pt. \quad (22 \cdot 27)$$

This current consists of two parts : A high frequency component

$$\alpha \ E_0(1 + m \sin pt - \frac{\beta \ E_0^2}{2} \ (1 + m \sin \omega t)^2 \cos 2 pt$$

which varies so rapidly that it cannot affect the telephones and a low frequency component

$$\tfrac{1}{2}\beta E_0^2 \ (1 + \frac{m^2}{2} + 2m \sin \omega t - \tfrac{1}{2}m^2 \ \cos 2\omega t) \qquad (22 \cdot 28)$$

which will be able to affect the telephone, producing a note of the same frequency ω as that originally impressed on the carrier wave. There is, however, in addition a term $\tfrac{1}{2} m^2 \cos 2\omega t$ which will produce an octave of the original note. The distortion term of frequency 2ω can be kept small by using a small value of m. For this reason it is usual in broadcasting to employ rather a small percentage modulation to diminish distortion. We also notice from $(22 \cdot 28)$ that the low frequency component varies as the square of the incoming signal strength, and so the receiver favours strong signals in comparison with weak ones. This is known as *square law detection.* The square law detectors are unsatisfactory for the reception of broadcast entertainment if m is more than 50%. We have seen above that a triode can be used as a rectifier by making use of the non-linear portion of the I_p-Vg

characteristic. This is shown graphically in fig. 22·11. The
variation in the average value of the plate current is shown by
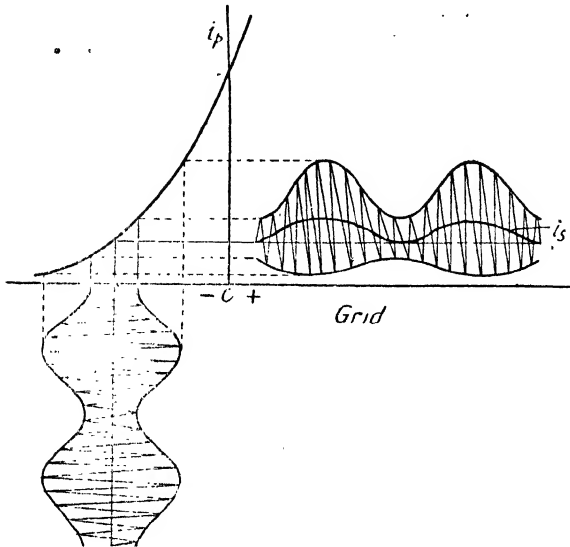
Fig. 22·11

the curve i_s. The variations in i_s follow the modulated envelope
of the impressed wave.

9. Diode as a rectifier. We have seen in the previous
article, using an anode bend
detector, that the rectified current
is proportional to the square of
the amplitude of the incoming
signal voltage. An essential con-
dition for distortionless rectifica-
tion is that the rectified current
should be proportional to the
first power of the signal voltage.
Hence, for satisfactory detection a
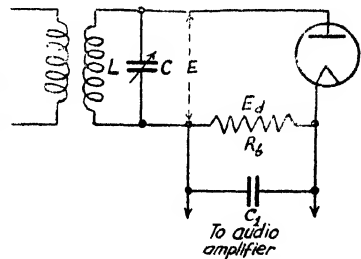
Fig. 22·12

linear detector is necessary. We have in a diode such a linear
detector. Diodes as detectors are commonly used in modern
receiving sets. A typical diode-detector circuit is shown in fig.22·12.

The tuned LC circuit is joined in series with the diode and the load resistance R_b, the value of which varies ordinarily from 5 to 1 megohm. The condenser C_1 whose value is nearly 150 $\mu\mu f$ acts as a radio-frequency by-pass. The performance of the diode as a detector depends on the fact that the potential across the load resistance is strictly proportional to the amplitude of the signal voltage over a very wide range. The audio-frequency variations of potential across the load may be passed on to suitable amplifying stages.

10. Heterodyne detection. When two notes of nearly the same frequency are sounded together we get the familiar phenomenon of beats. Let us now suppose that two high frequency e.m.f.s, $E_1 \sin p_1 t$ and $E_2 \sin p_2 t$ whose frequencies p_1 and p_2 are nearly equal, are superimposed on a rectifier whose characteristic is represented by

$$i = \alpha_0 + \alpha v + \beta v^2. \qquad . \qquad . \qquad (22\cdot29)$$

Substituting $v = E_1 \sin p_1 t + E_2 \sin p_2 t$ in $(22\cdot29)$ we have

$$i = \alpha_0 + \alpha(E_1 \sin p_1 t + E_2 \sin p_2 t) + \beta(E_1 \sin p_1 t + E_2 \sin p_2 t)^2$$

$$= \alpha_0 + \alpha(E_1 \sin p_1 t + E_2 \sin p_2 t) + \tfrac{1}{2}\beta E_1^2(1 - \cos 2 p_1 t)$$

$$+ \tfrac{1}{2}\beta E_2^2(1 - \cos 2 p_2 t) + \beta E_1 E_2 \{\cos(p_1 - p_2)t - \cos(p_1 + p_2)t\} \quad (22\cdot30)$$

The average value of the rectified current i will vary at the beat frequency rate. The process of impressing two voltages of nearly the same frequency upon a detector and extracting the beat frequency is called *heterodyne detection*. In super-heterodyne system which is most commonly used in radio reception, the incoming modulated signals are super-imposed on the oscillations of a local controllable oscillator having a frequency higher or lower than the received carrier by a desired amount. The amplifier following the detector can then be initially tuned to a fixed value of beat frequency. All incoming signals are converted to the same beat frequency by changing the frequency of the local oscillator i.e. the carrier frequency of the signal is changed to suit the amplifier, rather than adjusting the amplifier to suit the signal and this is a great advantage.

11. Crystal detector. A contact between two dissimilar pieces of certain crystals, such as galena, pyrites, carborundum, etc. or between a crystal and a fine metal wire has an important property of rectification. A crystal rectifier is rather the simplest detector.

12. Three-valve receiver set. In a modern three-valve receiver the first stage consists in amplifying the weak signals
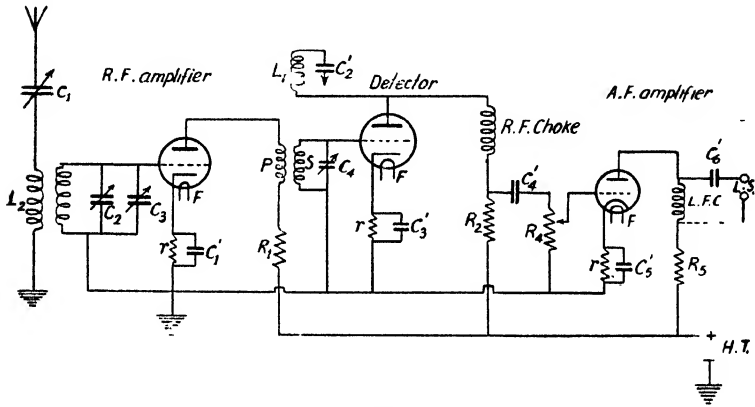
Fig. 22.13 (a)

arriving at the aerial. This is done by a radio-frequency amplifier. Corresponding but magnified changes in the potential across a
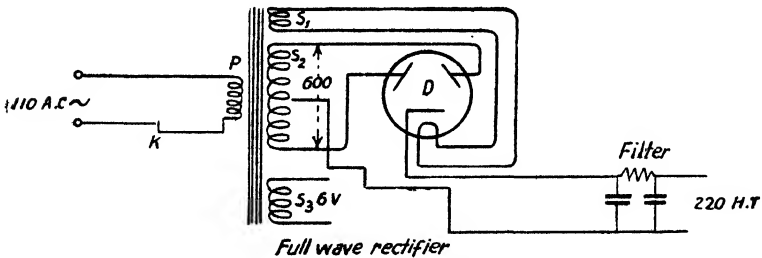
Fig. 22·13 (b)

resistance are produced in the plate circuit of this valve. The second stage consists in rectifying these amplified potential differences. This is achieved by a detector which makes the

variations unidirectional. The third stage consists in amplify-
ing the low frequency unidirectional
variations. This is done by a low
frequency amplifier. In the last stage
the low frequency amplified signals
are passed on to a loudspeaker which pro-
duces a faithful replica of the transmitted
signals. A very efficient and simple three-
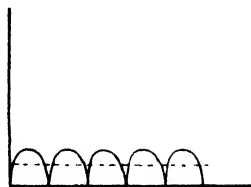
Fig. 22·13 (c)

valve receiver is shown in fig. 22·13 (a). The aerial is connected via
the variable condenser C_1 to a plug in coil L_2, which is tuned
to the incoming signals by the variable condensers C_2 and C_3; the
former is of relatively large capacity for the general coverage of
wave bands, and the latter of smaller capacity for band-spread
purposes. The resistance r in rC_1' circuit is for giving proper
negative bias to the grid and the condenser C_1' acts as a radio-
frequency by-pass. The amplified R. F. signals are passed on to
the grid circuit of the detector through a tuned transformer PS.
The resistance R_1 in series with P gives proper voltage to the
plate of the R. F. amplifier. Radio-frequency amplification does
not materially assist the loudness of the stronger signals, but by
increasing the strength of weak signals before detection, enables
them to be effectively rectified. It thus increases the actual
sensitivity of the receiver to which it is fitted. The reaction coil
L_1 is coupled to L_2 and the condenser C_2' acts as a stopper for the
anode voltage. The necessity for coupling L_1 and L_2 arises on
account of the following reason: The oscillatory potential differ-
ences set up in the aerial are applied between the grid and the
filament of the R.F. amplifiers. Now some of this received
energy is wasted in the resistance of the grid circuit and thus the
actual available energy is reduced. The purpose of the reaction
is to feed back some energy in proper phase to the grid circuit, to
reinforce the original signals. But there is a limit to which the
signal strength could be increased. Reaction increases not only
the amplification and sensitivity of the detector but increases
selectivity as well since the reaction is equivalent to the reduction
of the effective resistance of the grid circuit. A radio frequency

choke R.F.C. prevents excessive R.F. potential from reaching the A.F. amplifier. The coupling between the detector and A.F. amplifier is by means of a resistance-capacity coupling. The coupling resistance is R_2 and the condenser C_1'. The potentiometer R_1 across the grid circuit of the A.F. amplifier acts as a volume control. The anode circuit of the A.F. amplifier contains a resistance R_5 which limits the anode current and also the audible output to a comfortable level. The potentials set up across the low frequency choke L.F.C. are transferred to a telephone through the blocking condenser C_6', which prevents the H.T. from reacting the phone.

The receiver is fitted with a simple A.C. mains power supply unit shown in fig. 22·13 (b). The primary of the iron-core transformer P is fed by 110 A.C. (or 220 A.C.) The secondary consists of three parts S_1, S_2 and S_3. The voltage developed in S_1 and S_3 is 6 volts A.C. which is utilised for heating the filament of the double diode D and the filaments of the valves of the receiver respectively. The voltage developed in S_2 is say 600 volts. A.C. The two ends of the secondary S_2 go to the two plates of the diode. The tapping in the middle of S_2 acts as the negative of the H.T. Such an arrangement acts as a full-wave rectifier, fig. 22·13 (c).

13. Superheterodyne receivers. A simple three-valve receiver discussed in the previous article falls short of certain requirements such as selectivity and high sensitivity to modulated signals. The demand of present times has resulted in the increase of a large number of powerful transmitting stations thereby making the broad-casting conditions more and more difficult to cope with. Selectivity is therefore the chief quality to a modern receiver. The supereterodyne receiver has been specially designed to cambat the intolerable nuisance of overlapping—the creeping-in of unwanted stations. The selectivity obtainable from a single resonant circuit is hardly adequate.

The superheterodyne receiver differs from the receiver of the previous article in that the signal or carrier is amplified

at the original signal frequency and is then changed to another fixed intermediate frequency by imposing on the carrier a suitable frequency generated by a local oscillator. The reason for this arrangement is that by changing the frequency it is possible to choose an audio-frequency amplifier such that the amplification and selectivity are just what are required; also as this frequency is fixed by the design no further tuning of the amplifier is necessary.

14. Propagation of radio signals. Ionosphere. How do the radio signals from a transmitter reach radio receivers far and wide over the globe ? The obvious answer to this important question is that the signals or the electromagnetic waves follow the surface of the earth. In the early days of radio development it was supposed that the ground ray was responsible for radio transmission. As the distance from the transmitter increases, the ground ray grows weaker and weaker, the attennuation depending on the nature of the surface of the earth and the frequency in use. The ground range over sea water is much larger than that over very dry soil. The lower the frequency employed, the greater will be the ground ray range. For example the ground ray range for 16 kc./sec. is some thousands of miles; whereas for 60,000 kc./sec. it is only a few miles. But we know that stations like B.B.C. using frequencies of the order 15,000 kc./sec. are heard at great distances of thousands of miles. Therefore, the signals (high frequency) which reach such great distances cannot travel by ground. A hypothesis was put forward by Heaviside and Kenelly that the earth is surrounded by a conducting atmospheric shell which acts as a reflector for the radio waves. Later experiments have proved the existence of such a conducting layer now known as Heaviside layer.

Heaviside and Appleton layers. The densely ionised layer detected at a variable height between about 60 and 80 miles above the earth's surface is called the Heaviside layer or the E region.

Appleton in 1925 proved the existence of another more strongly ionized layer at a height greater than the Heaviside layer. The Appleton or F layer extends between 140 and 300 miles over the earth's surface. The ionization in these layers is mainly due to the presence of free electrons, produced by the ultraviolet radiation of the sun or the gases comprising the earth's upper atmosphere. During the day the upper atmosphere is strongly ionized down to a height of 50 miles. At night, when the sun's light is removed, the ionization in the lower regions falls because of the recombination of free electrons with ionized atoms and molecules, but in the upper regions the ionization is sensibly constant since the rate of recombination is negligible, owing to low pressure.

In addition to the E and F layers, another layer called the D layer which is weakly ionized has also been distinguished. These three layers, neglecting the fine structure, govern most of the observed radio phenomena. The free electrons in a layer not only deviate the waves, but also absorb energy from them. The amount of absorption is greater in regions of high pressure than in those where the pressure is low. In the day the ionization extends to regions where the pressure is high and hence the atmospheric ray is strongly absorbed ; while at night the ionization is conjoined to regions where the pressure is low and hence the signals at night are deviated without being appreciably weakened.

Layer reflection and refraction. The ray from the transmitter T travels onwards and upwards in a straight line with
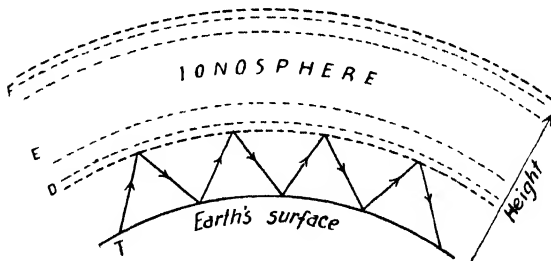
Fig. 22·14.

the velocity of light. When it enters the ionosphere, with its layers of conducting air, its behaviour alters. The behaviour is

similar to that of a light ray when it enters from air into a medium of variable refractive index. The refracting properties of the ionosphere for the radio waves are due to the presence of free electrons in it. The electron concentration is not constant throughout the medium but increases with height up to the point of maximum concentration. Suppose for simplicity we divide the whole region into a series of thin layers each having a constant electron concentration and each having greater electronic density than that next below it. The direction of the ray would be so altered by successive refractions that it would strike each succeeding layer more and more obliquely. Eventually it would strike a layer at an angle just greater than the critical angle where the ray would be totally reflected. The down-coming ray will be able to actuate a radio receiver at the point on the earth where it returns. Further more, on reaching the earth's surface it is reflected like a light ray from a mirror, and is sent off upwards again at the same angle at which it started. On reaching the ionosphere it is again refracted, suffers total reflection and comes back to the earth at a point twice as far from the transmitter as that at which it first came down. This process is repeated till the whole energy dies down. The radio wave, therefore, travels to a great distance on the earth in a series of hops, fig. 22·14.

Skip distance. Obliquely incident rays can be returned from the ionosphere with a much lower ionization in the layer than is required to return a vertically incident ray. Or with a given ionization density in the layer the greater the angle of incidence the higher the frequency which will be returned. Thus the critical frequency for vertical incidence is related to the highest frequency returned at oblique incidence, the latter being the *maximum usable frequency* (M. U. F.) for the distance considered.

For frequencies below the critical frequency all the upgoing rays would be reflected back to earth, no matter at what angle they struck the reflecting layers. None of these waves would penetrate the layer at such a low frequency. But for long distance transmission such low frequencies are of no use. We have to use the

highest frequency which it is possible to use. This is the M. U. F. for

the distance over which
we are transmitting, and
it is always above the
critical frequency. If we
use the M. U. F. then
all the higher angle rays
will penetrate the layer.
We have the situation
shown in fig 22·15. It
is necessary to go a
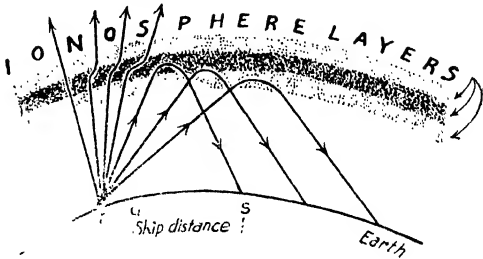
Fig. 22·15

distance T S from a transmitter T before we can find a ray which
makes a sufficiently flat angle to be bent down. No ray is received
within the distance TS. The ground ray is only up to a short
distance from the transmitter. Since the attenuation of short
waves is very rapid the ground ray effectively disappears past a
point G. Thus no signal is received within the distance GS;
this distance is called the *skip distance*. The skip distance depends
entirely on the ionization of the layer and on the frequency
used.

15. Cathode ray oscillograph.   It consists of an evacuated

long glass flask
F at the narrow
end o which
there is a system
of e l e c t r o d e s
while its broad
end is coated
with a fluores-
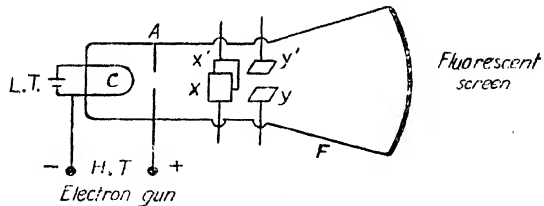
Fig. 22·16.   Cathode ray oscillograph

cent substance.   Electrons from the heated cathode C are
accelerated by a high potential say of the order of 5,000 volts
towards a perforated anode A which at the same time renders
the electron beam into a fine pencil.   The electron beam then
passes between two pairs of electrodes XX' and YY' which
are kept mutually at right angles to each other.   The beam then
finally falls on the fluorescent screen.

If an alternating potential difference is applied to XX′ the spot is drawn out into a horizantal line, the length of which is proportional to the peak value of the applied voltage. When an alternating p.d. is applied to each pair of plates, the spot describes what are known as Lissajous figures, since movement of the spot is the resultant of two simple harmonic motions at right angles to each other. When the two potentials have the same frequency the figure is an ellipse, the eccentricity of which depends upon their peak values.

A cathode ray oscillograph has been put to a variety of uses, such as tracing the hysteresis curve, knowing the wave form of an A.C. potential, measurement of voltages, etc. This instrument has now become as common as a galvanometer.

16. Television. Principle of television. Television means the reproduction of a scene with all its details and movements at a distant receiver, employing a radio or cable communication channel to transmit the necessary signals. The principle is given below briefly. The optical image of the scene to be televised is divided into a large number of small squares called elements; the changing values of light and shade pertaining to each element are translated into variations of potentials with the help of a photo-electric cell, which are then used to modulate the carrier. The brighter the element, the greater is the carrier amplitude. At the receiving station the entire process is reversed.

Scanning. The image of the television scene is divided into a large number of small squares called *elements*. This process of dividing the scene is known as *scanning*. If the elements are small enough each can be regarded as uniformly bright, and can thus be transmitted electrically as a single pulse. The number of elements determines the definition of the transmission. Modern television operates on the principle of plain
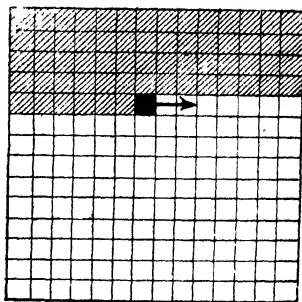
Fig. 22·17

sequential scanning which is illustrated diagrammatically in fig 22·17.

Here the elements are considered to be arranged in uniform rows across the image field, and are scanned along each row in turn with a uniform speed. The rate of scanning is determined by two factors ; first, the scanning must be sufficiently rapid to appear continuous to the eye and second, the whole field must be scanned a suffcient number of times each second to give the impression of continuous movement in the final image. The number of complete scannings needed to avoid flicker is nearly 25 where the image is rather dim, but at least 50 in the case of a bright image. The number of complete scannings per second is called the *frame frequency*.

Frequencies used in television transmission. Let us estimate the highest frequency which need be transmitted by a television system. If there are N scanning lines across a square field, there will be N^2 elements, and if the frame frequency be F, then the number of elements scanned per second will be N^2F. A detailed calculation however shows that the maximum modulation frequency is actually $N^2F/_2$. If $F = 25$ and $N^2 = 10,000$, frame is 125,000 cycles/second. For high definition television frame frequency is of the order of 4 M. C. Such a high frequency cannot possibly be used to modulate a transmitter working on moderately short-wave regions, where the carrier frequency is of the same order. Hence arises the necessity of using carrier frequencies around 40 M. C.

Simple transmission system. (Rotating Disc Scanning). Fig 22·18 (*a*) illustrates the essentials of a low definition television transmitter. The scene S to be transmitted is illuminated by powerful lamps. The lamp L forms a real image of the object on the scanning disc, as shown separately in fig. 22·18 (*b*). The scanning disc is made of aluminium pierced with a number of holes equal to the number of scanning lines to be used. The holes lie on a spiral track about the axis of rotation and are equally spaced along it. The action of the scanning disc is as

follows. Suppose the outermost scanning aperture is at the extreme left of the image. As the disc rotates this aperture passes across the image and allows light to pass to the photo-electric cell. The light that passes has swept only the first scanning line starting from left and ending in the right. When
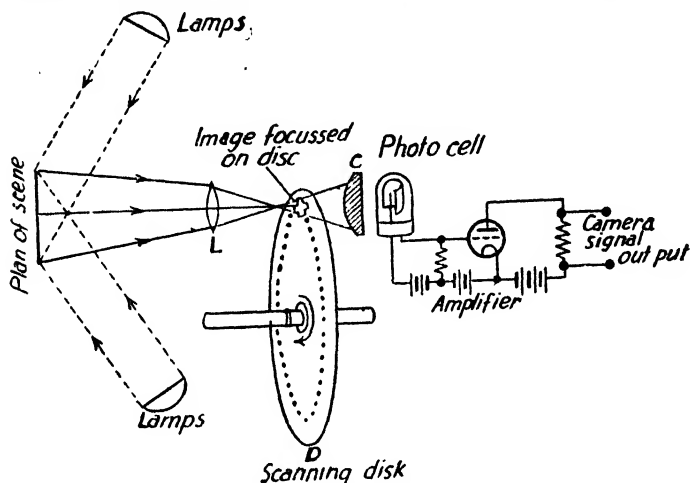
Fig. 22·18 (a)

the disk has rotated far enough to bring the outermost aperture to the extreme right of the image, the next aperture has reached the left hand edge of the second scanning line, and as the first aperture passes out of the frame the second aperture passes into it. This process repeats itself until the whole image is scanned. Since the whole image is scanned once for each rotation of the disk, the latter must rotate at a rate equal to that of the frame frequency. It is important that each hole should accurately fill the width of each scanning strip, so that the lines of demarcation will not be much noticeable between them.

Throughout the scanning process the light reaching the photo-tube at any instant comes only from one element of the image. As the illumination changes from element to element there is a corresponding change in the photo-electric current. The varying current, passing through a resistance, gives rise to a varying potential of the same form which is then amplified.

This simple system suffers from several disadvantages. Firstly, the equipment involves moving parts giving rise to mechanical difficulties. Secondly, the wheel has to be very big in size to minimise the curvature of the scanning lines and to make the speed

the same for all the scanning apertures. However, the major difficulty is of inadequate illumination, arising from loss of light.

Biard, in order to overcome the illumination difficulty, introduced the flying spot system in which the light from a concentrated source such as a carbon arc is focussed on the scene through the scanning apertures in the disk. The intense spot of light is caused to

Fig. 22·18 (b)

move over the scene in a series of adjacent lines as the scanning disk rotates. By using a battery of photo-electric cells a current may be developed which is several hundred times greater than that available from the previous method. The flying spot method has also its own limitations which we shall not discuss.
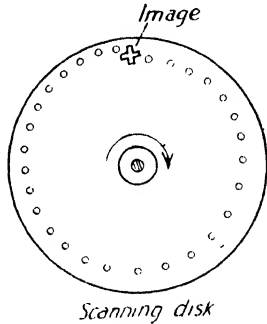
The iconoscope. Modern high definition television uses a cathode ray beam as a scanner. The principle underlying the
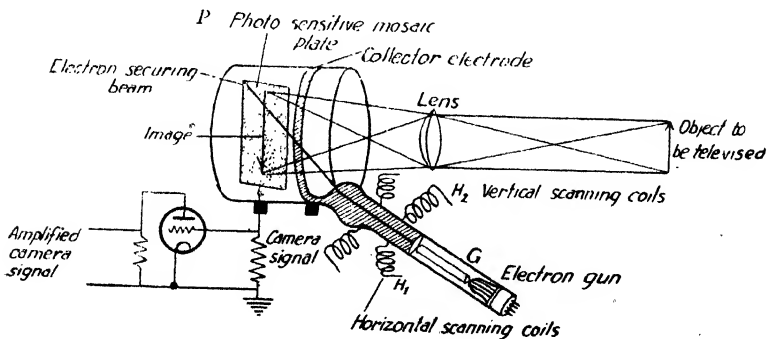
Fig. 22·19

iconoscope is illustrated in fig. 22·19. The essential part of the iconoscope tube is a flat plate P of mica on which are deposited

several million separately insulated globules of silver, which are coated with cesium oxide to make them photosensitive. The reverse side of the mosaic plate is coated with a thin *signal coat* of colloidal graphite which serves as the electrode through which the signal is transferred, during scanning, to the external circuit. In fact each globule is a miniature photo-tube cathode. The scene to be transmitted is focussed, through a lens on this plate. Under the influence of illumination each globule loses electrons, thereby the capacitance becomes charged in proportion to the illumination falling on it. As a result, the plate assumes an electrical charge deficiency the distribution of which is the same as that of the light in the optical image. In this way, the fundamental action of photoelectrical translation is performed and the optical image is translated into an electrical image.

Scanning is accomplished by a narrow beam of electrons emanating from the electron gun G. The beam can be moved both horizontally and vertically with the help of the two solenoids H_1 and H_2 situated outside the side arm of the iconoscope. As the beam passes over the globules it restores their previously lost charge, thereby discharging the capacitance. The discharge acting through the capacitance to the signal coating, appears as a current impulse in the signal circuit connected to the signal coating. Therefore, as the scanning beam moves across each line, the current impulses generated correspond in magnitude with the magnitude of the illumination present on the globules scanned.

In transmitting television programme, two separate transmitters are employed; one for sound transmission and the other for the picture transmission. The electrical signal from the camera tube, with synchronizing impulses superimposed, is impressed on the carrier wave. The combined signal is known as the *video signal.*

Television reception. At the receiver the electrical image of the scene has to be faithfully translated into the optical image. For a faithful reproduction each element of the reproduced image at the receiver has to be in perfect phase relationship with the corresponding element of the electrical image

23 E

at the transmitter. The picture carrier frequency is separated from the sound-carrier frequency by a super-heterodyne conversion process. The sound carrier is demodulated and then applied to the loud speaker after proper amplification. The picture carrier is similarly demodulated to obtain the picture frequencies, which after proper amplification are used to control the image-producing tube.

The cathode ray image-producing tube. It is a funnel-shaped glass tube at the narrow end of which is mounted an electron gun, fig. 22·20, which produces a fine pencil of electrons. The pencil is directed towards the wide end of the tube.
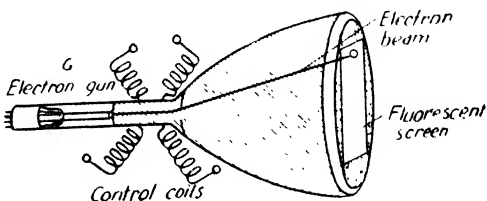
Fig. 22·20

The inner surface of the wide end is coated with a luminescent material which produces light under the impact of electrons. The electron beam in the picture tube is caused to move in the same manner as, synchronously with, the beam of electrons in the camera tube at the transmitter. The movement of the beam is controlled by two pairs of control coils, one of which causes the beam to move horizantally, whereas the other causes it to more vertically. The currents in the coils are controlled by synchronizing signals, as a result of which the electron beam woves across the luminescent screen in a series of horizontal lines. The reproduction of the picture is accomplished by changing the brilliance of the spot of light as it moves across the lines of the scanning pattern. This change of brilliance is obtained by changing the potential applied to a control electrode in the electron gun. This control potential is derived from the picture signal itself. Consequently, as the beam moves across each line in the pattern, the brilliance of the spot is controlled in direct accordance with the corresponding changes in image plate potential in the transmitting camera tube. Thus we obtain a faithful reproduction of the image. The process repeats itself at the rate of 30 pictures per second.

17. Electron microscope. Introduction. Since the invention of the microscope, about 300 years ago, constant attempts have been made to increase its resolving power in order to observe smaller and smaller objects. The practical limit of magnification with the visual microscope is about 2000×, whereas with ultraviolet microscope, magnifications of 6000× have been obtained. Magnifications of as much as 100,000× have been obtained with the electron microscope, thereby making visible the form and outline of bacteria, viri, colloids and other very small particles which up to now could be detected only by the effects they produce. The electron microscope has proved to be of immense benefit to the medical and biological sciences, and has opened up a new field of research. It is the practical embodiment of pure and perhaps highly abstract research of L. de Broglie.

The R. P. of a microscope is given by

$$d = \frac{\lambda}{N \sin \alpha}, \qquad \qquad \text{. . . (22·31)}$$

where d is the smallest distance between two parts of an object, λ the wave-length of the light with which the object is illuminated and ($N \sin \alpha$) is the numerical aperture of the objective ; N being the refractive index of the object space and α the semivertical angle of the cone of rays entering the objective. As the wavelength decreases the R.P. increases. Thus using ultraviolet light we get greater R.P than with the ordinary visible light. It was shown by L de Broglie that an electron with energy V electron-volts has a wave-length associated with it, given by

$$\lambda = \frac{12·2}{\sqrt{V}} \text{ Å.} \qquad \qquad \text{. . . (22·32)}$$

Electrons of energy 10^4 ev will have, for instance, a wave-length of 0·122 Å, a wave-length 50,000 smaller than the wave-length of the visible light. Hence if we use a beam of fast electrons instead of ordinary light we shall obtain much higher resolving power. It was shown by H. Busch that the action of short axially symmetrical magnetic fields on electron rays is similar to the action of a glass lens on light rays. This discovery opened up a new science of electron optics.

Principle of electron microscope. The principle of the instrument is illustrated in fig. 22·21 which also shows the corresponding optical analogue. The source of electron radiation, corresponding to the light source in the visual microscope, consists

ELECTRON MICROSCOPE   LIGHT PHOTO-MICROSCOPE
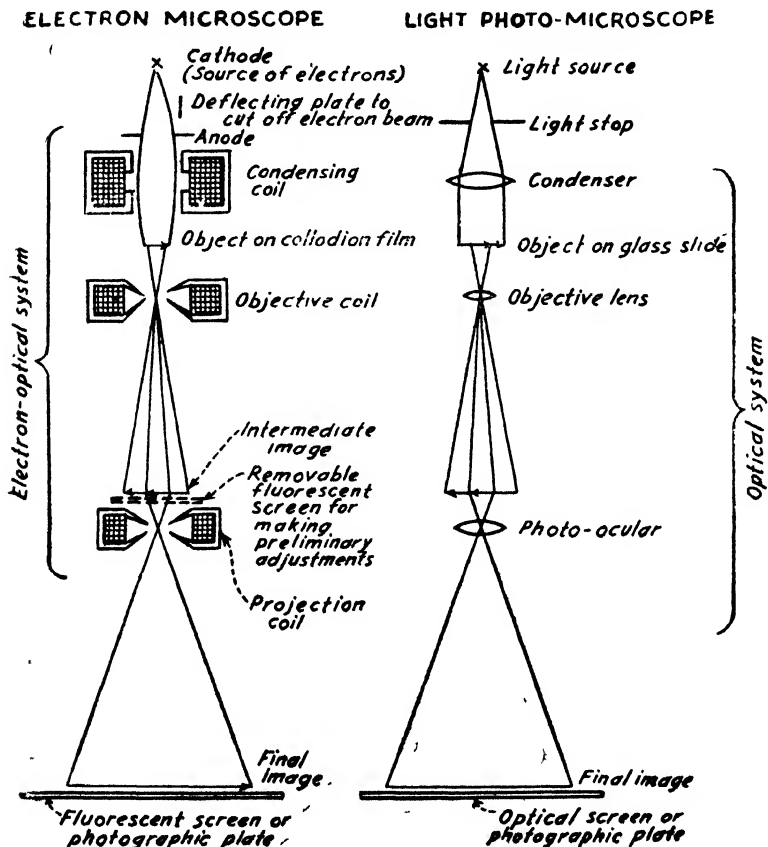
Fig. 22·21

of a hot cathode. The emitted electrons are accelerated to a high voltage of 10^5 volts. Such high voltages are necessary, as can be seen from eq. (22·32), to get electrons of very short wavelength, thereby increasing the R.P. and the magnification of the microscope. The electron beam after passing through an aperture

in the anode is condensed on the object by a condensing coil in a manner similar to the collimation of the light rays in the optical system. The object is held in a special locking and adjusting chamber. As in the optical microscope the objective coil forms an intermediate electron-optical image on a fluorescent screen. In the Siemens instrument the magnification in the intermediate stage is about 80 ×. By means of an object-shifting device, that part of the image which is to be further magnified is brought over an opening in the centre of the intermediate screen. The electron rays for this part of the image are then condensed by a projection coil in such a way that the intermediate image is further magnified as much as 350 ×. The resulant magnification of the final image is therefore 30,000 ×. The final image can either be observed on the fluorescent screen or can be photographed. A high speed diffusion pump maintains a high degree of vacuum inside the microscope. Object and photographic plates are introduced from outside by means of air-locks, so that only a few minutes after their introduction the vacuum is again good enough to obtain an exposure. The electron beam only strikes the relatively tender object during the short time (order of a second) of exposure ; the rest of the time the beam is deflected by a magnetic field set up in the deflection chamber.

The most important parts of the microscope are the electron lenses, since the refraction of the electron beam in traversing the electric or the magnetic fields provides the basis for electron microscopy. The focal length of the electron lenses is not fixed but
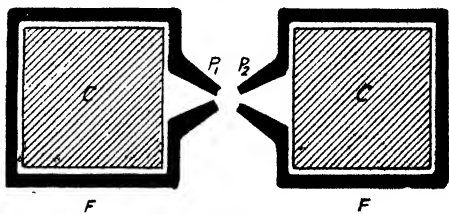
Fig. 22·22 (a)

may be adjusted by varying the electric or magnetic fields of the electron lens. In electron microscopes of high magnification, electron lenses of very short focal length have to be employed. Lenses of short focal length are obtained by large magnetic fields i.e. by increasing the current through the coils, or by using coils

of many turns of wire.　The shape of the ferro-magnetic circuit, and the use of materials of high permeability, with proper distribution of air gaps, is an important consideration in designing an electron lens.　Such lenses consist, as shown in fig.　$22\cdot22$ (a), of a coil C with many (say 2,000) ampere-turns, the coil being surrounded by a shield F of iron ending in two pole pieces P_1, P_2 which leave a small gap.　To maintain the focal lengths of the electron optical system sufficiently constant during the time of exposure, the current through the various coils must be maintained constant to a high degree of precision, for current variation would produce various types of aberrations and distortions.

Of the numerous applications of the electron microscope the

Bacillus tertius amylobacter flagellates.
[From Reports on Progress in Physics Vol. VII]
Fig. 22.23

most important are undoubtedly those in the fields of medicine and microbiology.　The beautiful photographs of bacteria, fig. $22\cdot23$, reveal new details which could never be seen by the optical microscope.

The least resolved distance using mean white light ($\lambda = 5\cdot5 \times 10^{-5}$ cm.) is 2×10^{-5} cm. ; while with 60 K.V. electrons ($\lambda = 4\cdot8 \times 10^{-10}$ cms) it is 3 to 5×10^{-7} cm.

CHAPTER XXIII
POSITIVE RAY ANALYSIS AND ISOTOPES

1. The discovery of positive rays. In 1886 Goldstein discovered canal rays what are now known as positive rays. He used a special discharge tube the cathode of which was perforated and the discharge tube extended beyond the cathode. He observed a beam of particles emerging out of the hole in the cathode travelling in straight lines and in the direction opposite to that of the cathode rays. To these new rays he gave the name canal rays, Kanal being the German word for channel. Goldstein's experiment is illustrated in fig. 23·1. By deflecting these canal rays in
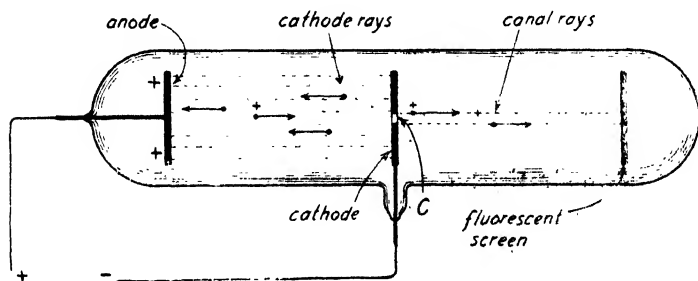
Fig. 23.1

a magnetic field W. Wien showed that they carry a positive charge. We know, now, that these rays are positively charged atoms of the residual gas in the discharge tube. Whence do these positively charged atoms come? The answer is quite simple. The electrons shooting out of the cathode are attracted towards the anode and during their journey from the cathode to the anode they occasionally collide with the gaseous atoms knocking out electrons from them. These positively charged atoms are now attracted by the cathode. Out of the many only those which are travelling along the hole are able to pass through it, thus constituting the canal rays.

2. Thomson's mass spectrograph. The nature of these positive rays was first studied by Sir J. J. Thomson by deflecting
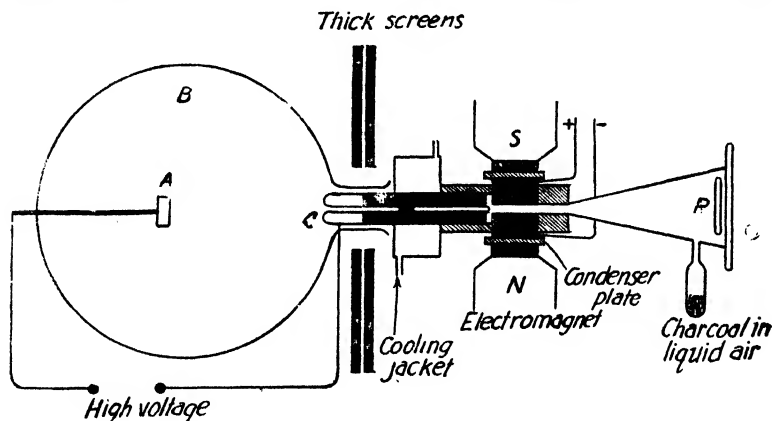
Fig. 23·2

them in simultaneous parallel electric and magnetic fields. His apparatus is shown in fig. 23·2. The anode A, the perforated cathode C, the condenser plates L and the photographic P are all enclosed in an air tight glass chamber. The glass chamber is first thoroughly evacuated and then the gas, the masses of whose atoms are to be measured, is introduced into the bulb B. The pressure on the right of the cathode C is maintained very low compared to that in B to diminish the scattering of the charged atoms. The electrons on their way from the cathode C to the anode A ionize the gaseous atoms. The positively charged atoms are attracted towards the cathode and emerge out of it as a fine beam. Those positively charged atoms which are produced just near the anode have the maximum velocity given by

$$e\mathrm{V} = \tfrac{1}{2}\, mv^2_{max}, \qquad . \quad . \quad . \quad (23\cdot1)$$

where V is the potential difference between the cathode and the anode; whereas those which are produced very near the cathode have the minimum velocity since they fall through only a fraction of the total potential difference. If the atoms of the gas have different masses, we have a beam of positively

charged atoms emerging out of the hole with varying velocity and mass.

An electric field is applied between the plates of the condenser along the axis of Z. The magnetic field has its lines of force parallel to the electric field. As the magnetic field is along the axis of Z and the positively charged atoms are travelling along the axis of Z, the magnetic force will be along the axis of Y. As soon as the positively charged atom comes between the plates of the condenser it is subjected simultaneously to the action of. the electric and the magnetic force along the axes of Z and Y respectively. All the atoms of the species having the mass m_1 with a greater velocity are deflected less and those with a smaller velocity are deflected more. A simple mathematical analysis shows that the path of the particles having the same $\frac{e}{m}$ is a parabola.

3. Theory of the method of parallel fields. (a) Deflection in an electric field. Let an ion of charge e and mass m
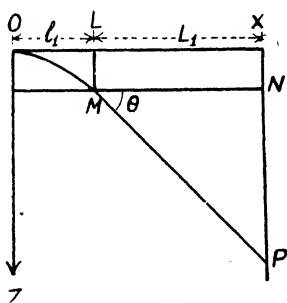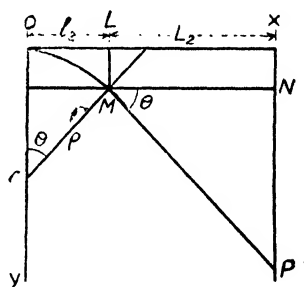
Fig. 23·3 (a)   Fig. 23·3 (b)

travel along the axis of x with the velocity v. Let it be subjected to an electric force along the axis of Z over the distance $OL = l_1$, fig 23·3 (a). The equations of motion of the ion are

$$m\ddot{x} = 0 \qquad \qquad \cdot \ \cdot \ \cdot \ (23 \cdot 2)$$
$$\text{and} \quad m\ddot{z} = Ee, \qquad \cdot \ \cdot \ \cdot \ (23 \cdot 3)$$

where E is the electric intensity. Integrating (23·2) and remembering that when $t = 0$, $x = 0$ we have

$$x = vt. \qquad \qquad \ldots \quad (23 \cdot 4)$$

From eq. (23·3) by integration we have

$$z = \frac{Ee}{2m} t^2, \qquad \qquad \ldots \quad (23 \cdot 5)$$

where the constant of integration is zero.

Eliminating t from eqs. (23·4) and (23·5) we have

$$x^2 = \frac{2mv^2}{Ee} z, \qquad \qquad \ldots \quad (23 \cdot 6)$$

which is a parabola.

This case is identical with that of a body moving in a gravitational field.

In the absence of the electric field the particle will travel straight and strike the photographic plate at $\times$. It describes the arc of a parabola so long as it is under the action of the electric field and then it flies off along a tangent and strikes the plate at P. The total deflection on the plate is

$$Z = XN + PN.$$

Now $XN = LM = \dfrac{Eel_1^2}{2mv^2}$ (since it is the value of the z co-ordinate at $x = l_1$).

Also $PN = MN \tan \theta = L_1 \tan \theta = L_1 \left(\dfrac{dz}{dx} \right)_{x=l}^{x=l} = L_1 \dfrac{Eel_1}{mv^2}.$

Hence
$$Z = \frac{Ee}{mv^2} \left(\frac{l_1^2}{2} + l_1 L_1 \right)$$

$$= A \frac{Ee}{mv^2}, \qquad \qquad \ldots \quad (23 \cdot 7)$$

where A is a constant depending upon the geometry of the apparatus.

(b) Deflection in a magnetic field. In the magnetic field the particle is constantly being acted upon by a force perpendicular to its direction of motion. From dynamics we know that

it will describe a circular path whose radius of curvature ρ is given by

$$\frac{He v}{c} = \frac{m v^2}{\rho}$$

or

$$\rho = \frac{m v c}{H e}, \qquad \qquad \cdots \quad (23\cdot 8)$$

where e is charge in *e.s.u.*

The total magnetic deflection is, fig $(23\cdot 3)(b)$,

$$Y = XN + PN$$
$$= LM + PN.$$

Now, from the property of a circle, LM is given by

$$LM.(2\rho - LM) = OL^2.$$

Hence neglecting LM in comparison to 2ρ we have

$$LM = \frac{l_2^2}{2\rho}.$$

$$PN = MN \tan \theta = MN. \sin \theta = L_2 \frac{l_2}{\rho}, \text{ if } \theta \text{ is small.}$$

Hence using eq. $(23\cdot 8)$ for ρ,

$$Y = \frac{l_2^2}{2\rho} + \frac{L_2 l_2}{\rho} = \frac{He}{mvc} \left(\frac{l_2^2}{2} + I_2 l_2 \right)$$

$$= B \frac{He}{mvc}, \qquad \qquad \cdots \quad (23\cdot 9)$$

where B is a constant depending upon the geometry of the apparatus.

Eliminating v from eqs. $(23\cdot 7)$ and $(23\cdot 9)$ we get the equation for the particle under the simultaneous action of the two fields,

$$y^2 = \frac{B^2}{A} \cdot \frac{H^2}{Ec^2} \cdot \frac{e}{m} \cdot z. \qquad \cdots \quad (23\cdot 10)$$

Eq. $(23\cdot 10)$ represents a parabola with its axis along the axis of z. The latus rectum of the parabola is $\dfrac{B^2}{A} \cdot \dfrac{H^2}{Ec^2} \cdot \dfrac{e}{m}$.

Atoms with different e/m will describe different parabolas. Points on the same parabola correspond to different velocities. With the magnetic field in one direction we shall have only one limb of the parabola; the other limb can be photographed by reversing the polarity of the electromagnet.

Interpretation of the photographs. The photographs shown in fig. 23·4 were taken when the spectrograph contained only hydrogen, oxygen and mercury vapour.

(a)

H^+

H_2^+

O^+
O_2^+
Hg^+
Hg^+
O_2^+
O^+

F_M

F_E

F_M

H_2^+

H^+

(b)

C^{++}
O^{++}
C^-
O^+
CO^+
Hg^{++}
Hg^+
Hg^+
Hg^{++}
CO^+
O^+
C^+
O^{++}
C^{++}
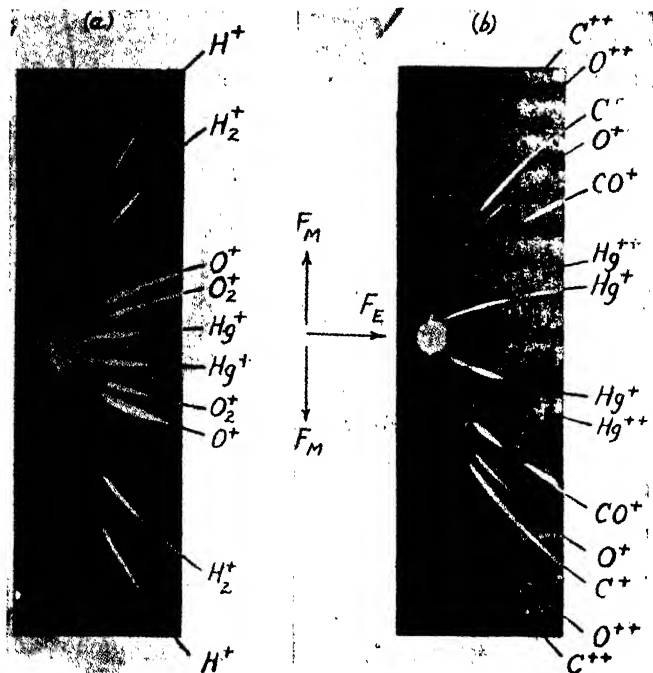
Thomson parabolas.   [From Classical and Modern Physics: H. E. White]

Fig. 23·4

From eq. (23·10) we see that particles having different values of $\frac{e}{m}$ lie on different parabolas. The particle having the largest value of $\frac{e}{m}$ will have the largest parabola because the latus rectum is the largest. It is quite natural, therefore, to identify the largest parabola in the figure with H^+, assuming of course that H^{++} does not exist. Our assumption is justified because no photograph ever revealed the presence of H^{++}. The conclusion is significant because it leads us to believe that neutral hydrogen atoms have only one electron. The next largest parabola will be of ionized

hydrogen molecule H^+_2 if it exists in the molecular form. Knowing the strength of the electric and magnetic fields and assuming that each particle carries a unit positive charge Thomson calculated the masses of the two largest parabolas and found them to correspond with the masses of H^+ and H^+_2. Once the mass corresponding to any one parabola is known with certainty, others can be identified easily by comparison. In this way the presence of O^+_1 O^+_2 and Hg^+ was detected in the photograph. Thomson also found very faint parabolas corresponding to doubly ionized oxygen. The faintness was due to the fact that the number of O^{++} was very small to give appreciable intensity. When experiments were performed with He gas it was observed that the helium atom never carried more than two units of positive charge. This observation again leads to the conclusion that neutral helium atom carries only two electrons.

Isotopes. By his determinations of the atomic weights of the elements, the chemist was led to believe that the atomic weights of all elements are nearly integers. The small departure from the integral values was considered to be merely an experimental error. We know now that the methods of a chemist are far too crude. Prout gave the hypothesis that all elements are made of hydrogen atoms as building stones. This hypothesis received a rude shock when Dumas and Stas, by a careful determination of the atomic weights came to the conclusion that the departure of the atomic weights from the integral values is not an experimental error but a reality. This was explained by Prout on the artificial assumption that some fraction of the hydrogen atom was the building stone, though no experiment ever revealed the presence of these fractional hydrogen atoms. The whole situation was cleared when Thomson subjected neon gas to mass analysis.

Curiously enough Thomson found for the neon atoms two parabolas one was faint and the other strong. The fainter parabola gave the mass 22 and the stronger one the mass 20 for the neon atoms. The atomic weight of neon was known to be

20·2. This led Thomson to believe that neon gas is actually a mixture of two kinds of atoms, 90% of which have a mass 20 and the other 10% a mass 22. The parabola for mass 22 was faint because of the low percentage of the atoms producing it. It is impossible to separate by any chemical means these two kinds of atoms in the neon gas as they are identical in all chemical properties. This was the reason why the chemical determination of the atomic weights gave the value 20·2 for the neon atoms. Atoms having the same chemical properties but differing in atomic weights were given the name *isotopes* by Soddy. After the discovery of the neon isotopes, many others were discovered among other elements. The element mercury has as many as eight isotopes.

5. Aston's mass spectrograph. Thomson's mass spectrograph has two very important defects in it : (1) as the atoms of a particular mass are distributed over the entire length of the parabola it makes it very difficult to observe very rare isotopes and (2) the parabolas are too thick to be suitable for any precision measurement. To remove these defects Aston devised another mass spectrograph which has the advantages of focussing all the atoms of the same mass but different velocities at the same point. This increases the intensity of the spot and makes the determination of the masses of the rare isotopes possible. The image on the photographic plate is sharper compared to that in the Thomson's instrument.

Aston's mass spectrograph is illustrated in fig. 23·5. The gas to be investigated is introduced in the space A where the postive ions are produced by electron collision and are accelerated towards the perforated cathode S_1 . The slits S_1 and S_2 confine the beam to a narrow pencil. As the beam emerges out of the slit S_2 it contains positively charged atoms of varying velocities and masses. For the sake of simplicity we shall assume that there are only two types of masses in the beam. The argument which we are going to develop would be perfectly general and would apply to any number of masses. As the beam enters the electric field E between the

plates of the condenser all the particles are subjected to the same downward force E*e* (assuming that they carry only a unit positive
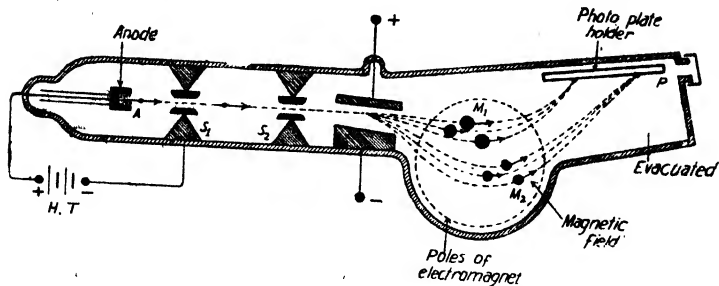
Fig 23·5   Aston's mass-spectrograph

charge).   Of the two masses M_1 and M_2 the heavier one will be deflected less as is obvious from eq. (23·7).   Thus the electric field divides the original beam into two beams M_1 and M_2, ($M_1 > M_2$).   Again in each beam those masses which are travelling faster will be deviated less than those which are travelling slower since the slow moving particles are under the influence of the electric field for a longer time.   The two beams then enter the magnetic field applied perpendicular to the plane of the paper and away from the reader.   The deviation in the magnetic field would be governed by the relation

$$\frac{Hev}{c} = \frac{mv^2}{\rho}$$

o.· $$\rho = \frac{mvc}{He}. \qquad \cdot \quad \cdot \quad \cdot \quad (23·11)$$

From eq. (23·11) we see that the slower particles will have a smaller radius of curvature than those which are moving faster, with the result that all particles having the same mass but different velocities will meet at some common point.   The same would hold good for all the particles of the group M_2.   The mathematical problem is then to determine other elements whose isotopes have previously the locus of the foci of the particles of different masses. This locus would determine the place where the photographic plate P must be placed to record the mass spectra.

The deviation θ of atoms of mass m in the electric field is given by eq. (23·7)

$$i.e. \quad \theta = \frac{Ae}{mv^2}, \qquad \qquad \text{. . . } (23·12)$$

as E does not vary it has been included in the constant A. The deviation ϕ in the magnetic field is given by eq. (23·9)

$$\phi = B\frac{e}{mv}. \qquad \qquad \text{. . . } (23·13)$$

From eqs.(23·12) and (23·13), on differentiation, we have

$$\left.\begin{array}{l} \dfrac{d\theta}{\theta} + \dfrac{2dv}{v} = 0 \\[2mm] \dfrac{d\phi}{\phi} + \dfrac{dv}{v} = 0 \end{array}\right\}, \qquad \text{. . . } (23·14)$$

where $\frac{e}{m}$ has been kept constant.

From eq. (23·14) we have

$$\frac{d\theta}{\theta} = \frac{2d\phi}{\phi}. \qquad \qquad \text{. . . } (23·15)$$

The width of the beam after it has travelled a distance $(a+b)$ without meeting magnetic field would have been $(a+b)d\theta$, fig 23·6; but the magnetic field annuls this width and brings the beam to focus at F after it has travelled a distance b. Hence,

Fig. 23·6

$$(a+b)d\theta = b\,d\phi$$

or

$$\frac{a+b}{b} = \frac{d\phi}{d\theta} = \frac{\phi}{2\theta}$$

or

$$b(\phi - 2\theta) = a.\,2\theta, \qquad \text{. . . } (23·16)$$

using eq. (23·15)

Let the photographic plate be set at an angle α to the direction of the incident beam. The perpendicular

$$LM = a\sin(\alpha + \theta) = b\sin(\phi - \alpha - \theta).$$
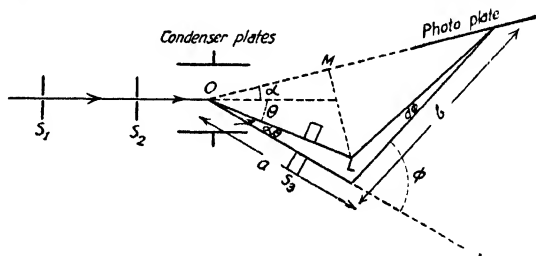
If the angles are small we have

$$a(\alpha+\theta)=b(\phi-\alpha-\theta) \qquad . \quad . \quad . \quad (23\cdot17)$$

Comparing $(23\cdot17)$ with $(23\cdot16)$ we see that

$$\alpha=\theta,$$

i.e., the plate should be inclined at an angle equal to that of the electric deflection with the direction of the incident beam.

A typical photograph obtained by Aston by introducing hydrochloric acid (HCl) carbon monoxide (CO) and sulphur dioxide (SO_2) in his mass spectrograph is shown in fig. $23\cdot7$.
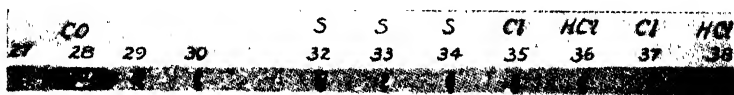
Photograph obtained from Aston's mass spectrograph
[From Classical and Modern Physics: H.E. White]
Fig. $23\cdot7$

From the photograph we see that sulphur has three isotopes of mass 32, 33, and 34, and chlorine has two isotopes of mass 35 and 37. We also observe that there is a linear shift of atoms and molecules, differing in mass by one unit. For studying the isotopes of an element it is introduced into the spectrograph together with other elements whose isotopes have been known previously and which serve as the reference points. Aston determined successfully the masses of a large number of elements in the periodic table. In his latest mass spectrograph he claims an accuracy of 1 part in 100,000.

6. The Bainbridge mass spectrograph. The demands of the present day nuclear physics and its application to the astrophysical problems are for higher

Bainbridge mass spectrograph
Fig. $23\cdot8$

and higher precision in the determination of the masses of the

24 E
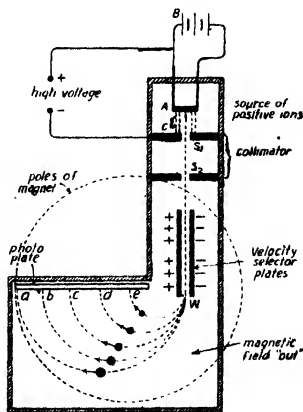
isotopes. For a large number of elements the masses are known correctly up to the fourth place of the decimal. The first attempt towards high precision was of Bainbridge. The essential feature of Bainbridge mass spectrograph is the "velocity selector." The mass spectrograph is illustrated schematically in fig. 23·8.

The positive ions generated in the region between A and C are accelerated towards the cathode by the potential difference between the anode A and the cathode C. The positive ion beam is collimated into a fine pencil by slits S_1 and S_2. The beam emerges at S_2 containing ions with different masses and varying velocities ; it then enters into the narrow space between the two condenser plates. The electric field deflects the beam to the right. The magnetic field is applied perpendicular to the plane of the paper and in a direction such that it deflects the beam to the left. The dotted circular region shows the pole pieces of the electromagnet. As soon as the beam enters the region between the condenser plates it is acted upon simultaneously by two forces : (1) the electric force which is the same for all particles whatever may be their velocity tries to deflect the beam to the right and (2) the magnetic force which is different for particles possessing different velocities tries to deflect the beam to the left. Only those positive ions will be able to pass through the condenser plates undeviated which have a particular velocity v for which the electric and magnetic forces are equal in magnitude *i.e.* for which the relation

$$\frac{Hev}{c} = -\, eE, \qquad \qquad . \quad . \quad . \quad (23·18)$$

would be satisfied. Positive ions with a greater velocity than this will be deflected towards the left under a greater magnetic force and will be absorbed by the positive condenser plate, and those which have a smaller velocity will be deflected towards the right under the greater electric force, and will be absorbed by the negative plate. Thus the function of the velocity selector is to select positive ions of only one velocity, satisfying the relation (23·18)

As the beam emerges out of the velocity selector it contains positive ions of different masses but having the same velocity. The

positive ions under the influence of the magnetic field will describe
circular paths, the radii of curvature of which are given by

$$\frac{Hev}{c} = \frac{mv^2}{\rho}$$

or $$\rho = \frac{mvc}{He}.$$ (23·19)

Since $\dfrac{vc}{He}$ is a constant the radius of curvature ρ will be directly
proportional to the mass. The heavier atoms will have a larger ρ
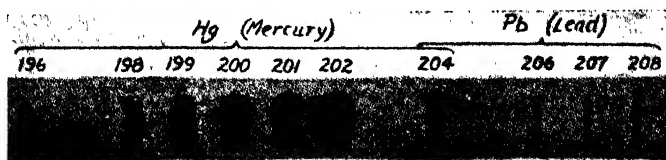
Fig. 23·9 Photograph obtained from the mass-spectrograph of Bainbridge
and Jordan. [From Classical and Modern Physics: Prof. H.E. White]

and will come to a focus at a more distant point on the photographic
plate p. A typical photograph taken for Hg isotopes with this
apparatus is shown in fig. 23·9.

The original Bainbridge's mass spectrograph has now been im-
proved for greater precision.

7. Unit Mass. Giauque and Johnson discovered from the band
spectrum analysis of the diatomic oxygen molecule that there are
three stable isotopes of oxygen with masses 16, 17, and 18. Their
relative abundance is 99·76, 0·04, and 0·20% respectively. The
unit of atomic mass is now taken to be $\frac{1}{16}$ th of the mass of oxygen
isotope 16. The unit mass is equal to $1·66 \times 10^{-24}$ gm. The mass
of an electron is 9.11×10^{-28} gm. *i.e.* $\dfrac{9·11 \times 10^{-28}}{1·66 \times 10^{-24}} = 0·00055$ mass unit.

8. The packing fraction. The mass number A of an atom is
the nearest whole number to its isotopic weight M. The mass
defect $\triangle$ is the difference between the isotopic weight and the whole
number i.e.

$$\triangle = M - A.$$

$\triangle$ is a very small quantity which never exceeds $0 \cdot 1$ mass unit. It is positive in the case of very light and very heavy elements and negative for a large number of isotopes. The packing fraction has been defined by Aston as

$$f = 10^4 \frac{\triangle}{M}. \qquad \cdot \quad \cdot \quad \cdot \quad (23 \cdot 20)$$

Mass Number
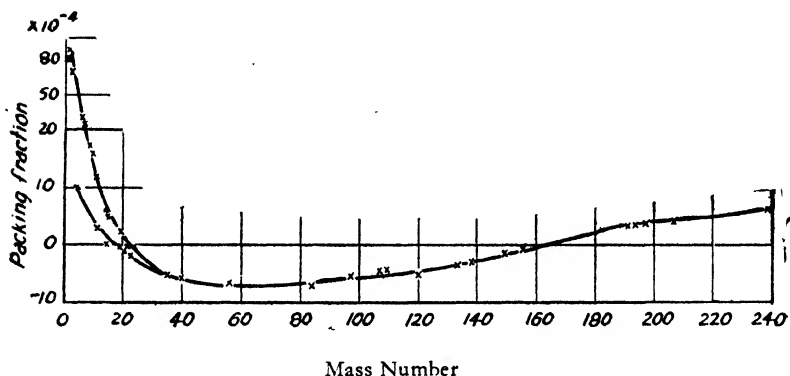
Fig. 23·10

The factor of 10^4 has been introduced so that f may be a number of convenient size. (For packing fractions of the isotopes see fig. $23 \cdot 10$).

9. Characteristics of the isotopes. A close examination of the isotopes and their relative abundances leads one to the following general conclusions :

(1) The elements of an even atomic number usually possess a much larger number of isotopes than the elements of an odd atomic number which never possess more that two stable isotopes.

(2) The number of isotopes with even mass number is much larger than that with odd mass number.

(3) Nuclei with even values of N (neutrons) occur more frequently than those with odd values of N. Nuclei with odd values both of N and Z are extremely rare.

(4) Elements with even atomic number make up about 87 per cent of the crust of the earth. In these elements the abundance

of isotopes of even mass number is the greatest. We therefore arrive at a very important conclusion that *nuclei having an even number of neutrons and protons seem to form a large fraction of all existing matter.*

(5) If we plot a graph with atomic number as abscissa and $N = A - Z$ as ordinate we shall observe that the stable isotopes of elements are not scattered at random on the N, Z diagram, but are contained within a rather narrow range. Therefore the mass number is roughly a function of the atomic number. In fact the mass number is always either equal to or greater than twice the atomic number.

CHAPTER XXIV
ATOMIC STRUCTURE

1. Bohr's theory of Hydrogen spectrum. In the year 1913 Niels Bohr of Copenhagen, fig. 24·1, first successfully explained the various series observed in the line spectrum of hydrogen. His theory marks the beginning of a new era in spectrosocopy and atomic structure.

In his explanation Bohr was guided by the Rutherford picture of the atom according to which the number of electrons around the positively charged nucleus is equal to the atomic number. Bohr, therefore, concluded that the hydrogen atom was made up of one electron, of charge $-e$ and one proton of charge $+e$, fig. 24·2. Since the hydrogen nucleus is nearly 1800 times heavier than the electron Bohr assumed it to be at rest and the electron in motion.
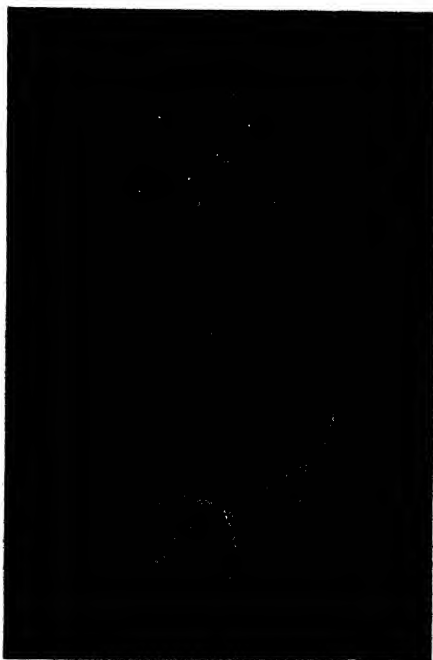
Professor Niels Bohr
Fig. 24·1

2. Bohr's first postulate. Bohr's first postulate was that the electron moves in circular orbits around the nucleus under

the action of a Coulomb field of force. The Coulomb force of attraction between the electron and the nucleus is

$$F = \frac{-Ze^2}{a^2};$$ . . . (24·1)

where Ze is the charge on the nucleus ($Z=1$, for hydrogen) and a the radius of the circular orbit. For equilibrium the force of attraction must be equal to the centripetal force $\frac{mv^2}{a}$. We, therefore, have
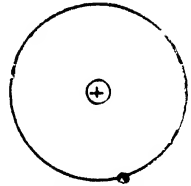
Fig. 24·2

$$\frac{Ze^2}{a^2} = \frac{mv^2}{a},$$ . . . (24·2)

where m is the mass of the electron and v is its velocity. The total energy W of the atom is the sum of its kinetic and potential energies, so that

$$W = \tfrac{1}{2}mv^2 - \frac{Ze^2}{a}.$$ . . . (24·3)

From eqs. (24·2) and (24·3) we have

$$W = -\frac{Ze^2}{2a}.$$ . . . (24·4)

Bohr's second postulate. Bohr's second postulate was that only those orbits were possible for the electron for which the total angular momentum of the atom was equal to an integral multiple of $\frac{h}{2\pi}$, that is

$$m\, a^2\omega = \frac{nh}{2\pi}.$$ . . . (24·5)

Eq. (24·5) is known as *quantum condition.*

In eq. (24·5) ω is the angular velocity of the electron and h the Planck's constant.

Bohr's third postulate. According to classical electrodynamics the electron in any one of its orbits would radiate out energy and would finally fall into the nucleus. That is, on the classi-

cal theory the atom of hydrogen would collapse contrary to the fact that the hydrogen atom is very stable. To get rid of this fundamental difficulty Bohr made the third postulate that the electron cannot radiate so long as it is in any one of its stationary orbits. The radiation is emitted only when the electron jumps from one stationary state to another stationary state, and the frequency of the radiation is given by the relation (law of conservation of energy)

$$h\bar{\nu} = W_n - W_m, \qquad \cdot \quad \cdot \quad \cdot \quad (24 \cdot 6)$$

where W_n is the energy in the nth state of the atom (n^{th} orbit) and W_m that in the mth state. Eq. (24·6) is known as *Bohr's frequency condition*.

From eqs. (24·2) and (24·5) we have

$$a_n' = \frac{h^2}{4\pi^2 m Z e^2} n^2, \qquad \cdot \quad \cdot \quad \cdot \quad (24 \cdot 7)$$

where a_n denotes the radius of the nth orbit.

The energy of the atom in its nth stationary state is given by

$$W_n = -\frac{2\pi^2 m e^4 Z^2}{n^2 h^2}, \qquad \cdot \quad \cdot \quad \cdot \quad (24 \cdot 8)$$

from (24·4) using (24·7).

Hence the frequency $\bar{\nu}_{n_1 n_2}$ of the radiation as the atom jumps from the n_2^{th} state to the n_1^{th} state is given by

$$\bar{\nu}_{n_1 n_2} = W_{n_1} - W_{n_2}$$

$$= \frac{2\pi^2 m e^4 Z^2}{h} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \quad \cdot \quad \cdot \quad \cdot \quad (24 \cdot 9)$$

The frequency ν in wave numbers is given by the relation

$$\nu = \frac{\bar{\nu}}{c} = \frac{1}{\lambda}. \qquad \cdot \quad \cdot \quad \cdot \quad (24 \cdot 10)$$

Hence

$$\nu_{n_1 n_2} = \frac{2\pi^2 m e^4 Z^2}{c h^3} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \cdot \quad \cdot \quad \cdot \quad (24 \cdot 11)$$

or

$$\nu_{n_1 n_2} = R Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right), \quad \cdot \quad \cdot \quad \cdot \quad (24 \cdot 12)$$

where $R = \dfrac{2\pi^2 m e^4}{c h^3}$ is the Rydberg's constant.

3. The hydrogen series.

For the case of hydrogen $Z = 1$, eq. $(24 \cdot 12)$ becomes

$$v_{n_1 n_2} = R\left(\frac{1}{n_1{}^2} - \frac{1}{n_2{}^2}\right). \qquad \ldots \quad (24 \cdot 13)$$

The entire spectrum of hydrogen is explained by eq. $(24 \cdot 13)$
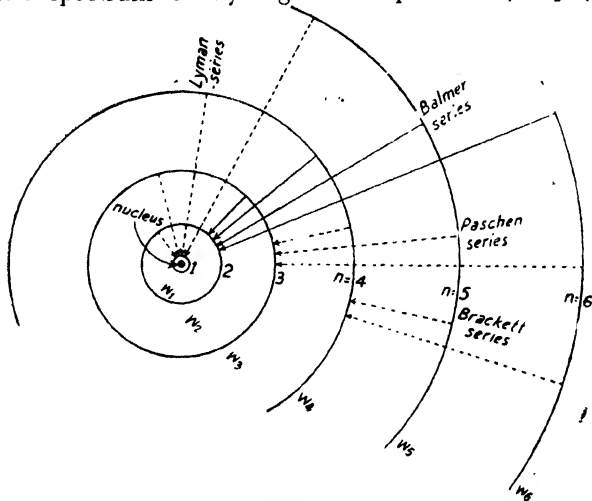
Fig. 24·3

The various series in the hydrogen spectrum are given below :

$$v = R\left(\frac{1}{1^2} - \frac{1}{n_2{}^2}\right), n_2 = 2, 3 \ldots \text{Lyman series} \quad . \quad . \quad (24 \cdot 14)$$

The Lyman series lies in the extreme ultraviolet.

$$v = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right), \quad n = 3, 4 \ldots \text{Balmer series}, \quad . \quad . \quad (24 \cdot 15)$$

which lies in the visible range of the spectrum. It is the members of the Balmer series that we observe in an ordinary hydrogen discharge tube in the laboratory.

$$v = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right), n = 4, 5, \ldots \text{Paschen series}, \quad . \quad . \quad (24 \cdot 16)$$

which lies in the near infra red.

$$v = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right), \quad n = 5, 6, \text{ Brackett series}, \quad . \quad . \quad (24 \cdot 17)$$

which lies in the extreme infra-red.

$$v = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right), n = 6, 7, \text{ Pfund series.} \quad . \quad . \quad (24 \cdot 18)$$

The different transitions are represented in fig 24·3. The customary energy level diagram is shown in fig. 24·4.

When Bohr gave out his theory only certain spectral lines of the Balmer series were known. The success of the theory lay not only in explaining the lines of the Balmer series but in predicting other new series which were later on discovered by other investigators. The chief characteristic of the theory is that it predicts certain discrete stationary states of the atom quite foreign to the old classical ideas. These stationary states are not merely hypothetical but that
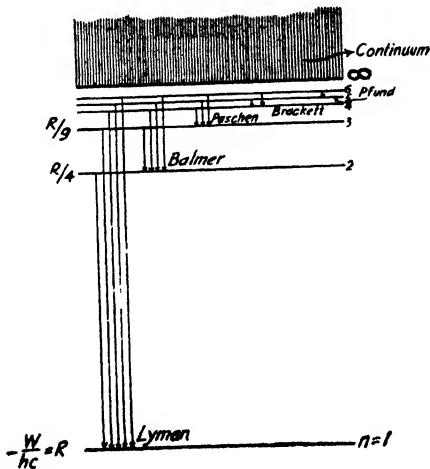
Fig. 24·4

they actually exist has been verified by the classic experiments of Hertz. Though this theory was remarkably successful in explaining certain features of the H spectrum, it failed completely when applied to Helium the next element in the periodic table.

EXAMPLES.

Example   1.

(a) Calculate the radius of the first Bohr orbit.

By putting $n = 1$ in eq. (24·7) we get the radius of the first Bohr orbit

$$a = -\frac{h^2}{4\pi^2 m e^2}.$$

Given $h = 6.547 \times 10^{-27}$ erg. sec., $e = 4.77 \times 10^{-10}$ e.s.u. and $m = 9.035 \times 10^{-28}$ gm.

Substituting these values we have

$$a = 0.528 \times 10^{-8} \text{ c.m.} = 0.528 \text{ Å}$$

(b) Calculate the ionization potential of hydrogen atom.

The ionization potential is the amount of energy required to detach the electron completely from the normal hydrogen atom. In other words it is the energy with which the electron is bound to the atom. Putting $n_1 = 1$ and $n_2 = \infty$ in eq. (24·9) the ionization energy I in ergs. is

$$I = h\nu = \frac{2\pi^2 m\epsilon^4}{h^3}$$

$$= \frac{2 \times \pi^2 \times 9\cdot035 \times 10^{-28}\ (4\cdot77 \times 10^{-10})^4}{(6\cdot547 \times 10^{-27})^2}\ \text{ergs.}$$

$$= \frac{2 \times \pi^2 \times 9\cdot035 \times 10^{-24}(4\cdot77)^4 \times 10^{-40}}{(6\cdot547)^2 \times 10^{-54}} + 0\cdot63 \times 10^{12}\ \text{ev.}$$

$$= 13\ \text{ev, since 1 erg} = 0\cdot63 \times 10^{12}\ \text{ev. approx.}$$

Example 2. How much time would the electron in a normal hydrogen atom take in its spiral course in reaching the nucleus ?

The total energy of the electron in the normal hydrogen atom is

$$W = -\frac{e^2}{a} + \tfrac{1}{2}mv^2 = -\frac{e^2}{2a}\ .$$

Since the electron is under acceleration the amount of energy radiated per second is given by $\tfrac{2}{3}\dfrac{e^2}{c^3}f^2$, where $f = \dfrac{e^2}{ma^2}$ is the acceleration.

Hence $$-\frac{dW}{dt} = \tfrac{2}{3}\frac{e^6}{m^2c^3a^4}\ .$$

But $$-\frac{dW}{dt} = -\frac{e^2}{2a^2}\frac{da}{dt},\ \text{from } W = -\frac{e^2}{2a}\ .$$

Hence $$-\ \tfrac{4}{3}\bigg/\frac{e^4}{m^2c^3}\ dt = a^2 da.$$

Integrating, $\dfrac{4e^4}{m^2c^3}\ t = a_{\text{H}}{}^3 - a^3$, where a_{H} is the radius at $t=0$.

The atom will disappear in time t given by

$$t = \frac{a^3_{\mathrm{H}} \cdot m^3 c^3}{4e^4} = \frac{1}{4} \cdot \left(\frac{a_{\mathrm{H}}}{e^2/mc^2}\right)^3 \cdot \frac{e^2}{mc^3} = \frac{a_0}{4\mathbf{c}} \cdot \left(\frac{a_{\mathrm{H}}}{a_0}\right)^3$$

where a_0 (the classical radius) $= \frac{e^2}{mc^2}$. Substituting for $a_{\mathrm{H}} = \frac{h^2}{4\pi^2 m e^2}$,

we get $t = 10^{-8}$ second.

4. Discovery of Deuteron or Heavy hydrogen.

In Bohr's elementary theory of the hydrogen spectrum the nucleus of the atom was taken stationary. If the motion of the nucleus is taken into account the expression for the Rydberg's constant takes the form,

$$R = \frac{2\pi^2 me^4}{h^3}\left(1 - \frac{m}{M}\right)$$

$$= R_\infty \left(1 - \frac{m}{M}\right),$$

where M is the mass of the nucleus.

If hydrogen consists of two isotopes of mass M and M', the frequencies v and v' of the corresponding lines are given by

$$v = R_\infty \left(1 - \frac{m}{M}\right)\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \left.\vphantom{\begin{array}{c}1\\1\end{array}}\right\} \text{ for the lines of the}$$

$$v' = R_\infty \left(1 - \frac{m}{M'}\right)\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \left.\vphantom{\begin{array}{c}1\\1\end{array}}\right\} \text{Balmer series.}$$

Hence
$$\frac{v'}{v} = \frac{1 - \dfrac{m}{M'}}{1 - \dfrac{m}{M}}.$$

or
$$\frac{v' - v}{v} = m \frac{\delta M}{MM'}.$$

If we take $M' = 2$ and $M = 1$ then

$$\frac{\delta v}{v} = \frac{m}{2M} = \frac{1}{2 \cdot 1836}.$$

But $$\frac{\delta v}{v} = -\frac{\delta \lambda}{\lambda}$$

Hence $$\delta \lambda = -\frac{\lambda}{2 \cdot 1836}.$$

From the above expression we can calculate the shift due to the heavy isotope for H_ι, H_3, $H\gamma$, etc. The calculated shifts are given below.

	H_ι (6562·85 Å)	H_β (4861·3 Å)	$H\gamma$ (4340·49Å)
$\delta \lambda$ (cal)	1·793	1·326	1·185
$\delta \lambda$ (ob,)	1·791	1·313	1·176

The above calculations were first verified experimentally by Urey in 1932. In their experiment dry hydrogen was introduced in a discharge tube. The spectrum was photographed in the second order of a 21 ft. concave grating. The exposure given was 4000 times of that required for photographing ordinary Balmer lines. They found that each Balmer line was accompanied by a faint component. The wave-length shift agreed with the calculated value.

For the discovery of heavy hydrogen Urey was awarded the Nobel prize in chemistry for the year 1934.

5. Sommerfeld's elliptic orbits. Sommerfeld extended

Bohr's theory by assuming that the electron moves round the nucleus, in gen eral in elliptic orbits.

The motion of an electron in an elliptic orbit with the nuc leus at one focus re presents a system with two degrees of free dom. The polar co-

Fig. 24.5 Elliptic orbits.

ordinates of the electron e are (r, ϕ), fig. 24·5. The kinetic energy of the electron is given by
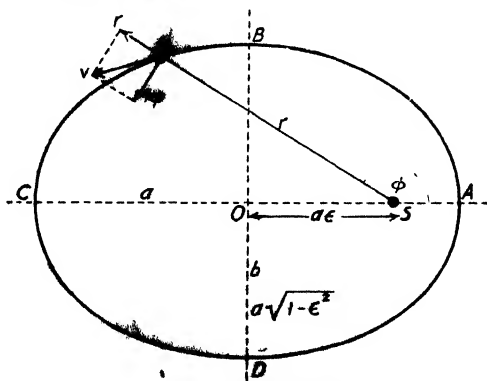
$$T = \tfrac{1}{2}m(\dot{r}^2 + r^2\dot{\phi}^2). \qquad \text{. . . . (24·19)}$$

The potential energy is

$$V = -\frac{Ze^2}{r}. \qquad \text{. . . (24·20)}$$

The radial momentum is

$$p_r = m\dot{r} \qquad \text{. . . (24·21)}$$

and the angular momentum is

$$p_\varphi = mr^2\dot{\phi}. \qquad \text{. . . (24·22)}$$

The quantum conditions for these two momenta are :

$$\oint p_\varphi \, d\phi = kh \qquad \text{. . . (24·23)}$$

and

$$\oint p_r \, dr = rh , \qquad \text{. . . (24·24)}$$

where both k and r are integers and are called the *azimuthal* and *radial quantum numbers*, respectively. (The general quantum condition is $\int p dq = nh$, where p is the generalized momentum and q the generalized co-ordinate).

Since the angular momentum p_ϕ is constant during motion (from Kepler's second law of planetary motion) we have from eq. (24·23)

$$p_\varphi. \oint d\varphi = kh$$

or

$$p_\phi = \frac{kh}{2\pi}. \qquad \text{. . . (24·25)}$$

This is identical with Bohr's quantum condition (24·5) for circular orbits.

The equation of an ellipse in polar co-ordinates is

$$\frac{1}{r} = \frac{1 + \varepsilon\cos\varphi}{a(1-\varepsilon^2)}, \qquad \text{. . . (24·26)}$$

where ε is the eccentricity of the ellipse and a the semi-major axis. The following relations will be found useful.

$$\left.\begin{array}{l} \text{Perihelian } SA = a\,(1-\varepsilon) \\ \text{Aphelian } SA = a\,(1+\varepsilon) \\ \text{Semiminor axis } b = a\,(1-\varepsilon^2)^{1/2} \end{array}\right\} \qquad \text{. . . (24·27)}$$

Taking the log of each side of (24·26) and differentiating we have

$$\frac{1}{r}\frac{dr}{d\varphi} = \frac{\varepsilon\sin\varphi}{1 + \varepsilon\cos\varphi} \qquad \text{. . . (24·28)}$$

Now

$$p_r = m\dot{r} = m\frac{dr}{d\varphi}\frac{d\varphi}{dt} = \frac{p_\phi}{r^2}\frac{dr}{d\varphi}, \qquad \text{. . . (24·29)}$$

since $p_\phi = mr^2\dot{\varphi}$.

Therefore, using (24·28)

$$p_r \, dr = \frac{p_\varphi}{r^2}\left(\frac{dr}{d\phi}\right)^2 \, d\varphi = p_\varphi \varepsilon^2 \frac{\sin^2\varphi}{(1 + \varepsilon \cos \varphi)^2} \, d\varphi \quad . \quad . \quad . \quad (24·30)$$

The phase integral (24·24) becomes

$$\oint p_r \, dr = p_\varphi \varepsilon^2 \int_0^{2\pi} \frac{\sin^2\varphi}{(1 + \varepsilon \cos \varphi)^2} \, d\varphi = rh. \quad . \quad . \quad . \quad (24·31)$$

Substituting (24·25) in (24·31) we have

$$\frac{\varepsilon^2}{2\pi} \int_0^{2\pi} \frac{\sin^2\varphi}{(1 + \varepsilon \cos \varphi)^2} \, d\varphi = \frac{r}{k} \quad . \quad . \quad . \quad . \quad (24·32)$$

On integration, we get

$$\frac{1}{\sqrt{1 - \varepsilon^2}} - 1 = \frac{r}{k} \quad . \quad . \quad . \quad (24·33)$$

or

$$(1 - \varepsilon^2) = \frac{k^2}{(k + r)^2} = \frac{b^2}{a^2} . \quad . \quad . \quad . \quad (24·34)$$

Since k and r are both integers we have

$$k + r = n, \quad . \quad . \quad . \quad (24·35)$$

where $n = 1, 2$, etc., is the so-called *total quantum number*.

From eq. (24·34) we see that out of all the classically possible ellipses, only those are allowed for which the ratio of the major to the minor axes is that of two integers.

The total energy W of the electron is

$$W = \tfrac{1}{2} m\dot{r}^2 + \tfrac{1}{2} mr^2\dot{\varphi}^2 - \frac{Ze^2}{r}, \quad . \quad . \quad . \quad (24·36)$$

or

$$W = \frac{1}{2m}\left(p_r^2 + \frac{p_\phi^2}{r^2}\right) - \frac{Ze^2}{r} \quad . \quad . \quad . \quad (24·37)$$

or

$$W = \frac{p_\phi^2}{2mr^2}\left[\left(\frac{1}{r}\frac{dr}{d\varphi}\right)^2 + 1\right] - \frac{Ze^2}{r} \quad . \quad . \quad . \quad (24·38)$$

Eq. (24·38), on using (24·26) and (24·28), becomes

$$W = \frac{p_\phi^2}{ma^2(1 - \varepsilon^2)^2}\left(\frac{1 + \varepsilon^2}{2} + \varepsilon \cos \phi\right) - \frac{Ze^2(1 + \varepsilon \cos \varphi)}{a(1 - \varepsilon^2)} . \quad . \quad (24·39)$$

Since the atomic system is conservative the total energy W is constant and is independent of the time and the angle φ which varies with time. Therefore, equating the coefficient of cos φ in (24·39) to zero, we have

$$\frac{p^2_\phi}{ma^2(1-\varepsilon^2)^2} = \frac{Ze^2}{a(1-\varepsilon^2)}, \qquad \cdot \quad \cdot \quad \cdot \quad (24\cdot40)$$

from which

$$a = \frac{p^2_\phi}{me^2Z(1-\varepsilon^2)} \cdot \qquad \cdot \quad \cdot \quad \cdot \quad (24\cdot41)$$

Using (24·25) and (24·34) the expression for a becomes

$$a = \frac{h^2}{4\pi^2me^2Z}(k+r)^2 = a_0 \frac{n^2}{Z}, \qquad \cdot \quad \cdot \quad (24\cdot42)$$

where a_0 is the Bohr radius.
The semi-minor axis is

$$b = a\sqrt{1-\varepsilon^2} = \frac{h^2}{4\pi^2me^2Z} k(k+r) = \frac{a_0\,kn}{Z} \cdot \quad \cdot \quad \cdot \quad (24\cdot43)$$

Substituting the value of a from eq. (24·41) the total energy expression (24·39) becomes

$$W = \frac{Ze^2}{a(1-\varepsilon^2)}\left\{\frac{1+\varepsilon^2}{2}-1\right\} = \frac{-Ze^2}{2a} \quad \cdot \quad \cdot \quad (24\cdot44)$$

(Here cos φ has been put equal to zero since W holds for all time). Using for a the expression (24·42) we have

$$W = \frac{-2\pi^2\,me^4\,Z^2}{h^2\,(k+r)^2} = -\frac{2\pi^2me^4Z^2}{n^2h^2} \quad \cdot \quad \cdot \quad (24\cdot45)$$

The expression (24·45) for the energy is identical with that obtained by Bohr for circular orbits.

Though Sommerfeld's modification of Bohr's theory has introduced no new energy states for the hydrogen atom, Sommerfeld's theory differs in the important respect that the electron can now move in different orbits corresponding to the same energy. For example, corresponding to $n=4$, the azimuthal quantum number k may take one of the four values 1, 2, 3 or 4, while the radial quantum number r takes on the values 3, 2, 1, and 0 respectively. The value $k=0$ is excluded because for $k=0$ the elliptic motion of the electron reduces to a linear motion along a straight line with the nucleus at one end. If such a motion is allowed it would

mean that the electron would collide with the nucleus
and the atom would be an unstable [structure, contrary to
experience. Hence Sommerfeld excluded such states of the
atom.

Each quantized orbit of the atom is now determined by two
out of the three quantum numbers n, k and r, (since $n=k+r$).
By convention each quantum state is defined by n and k. The
radial quantum number is not used.

6. Space quantization. So far we have discussed the mo-
tion of the electron in two dimensions, but the electron motion is
three dimensional. We shall get three quantum numbers corres-
ponding to the three degrees of freedom. The third quantum
number does not change the size or shape of the Bohr-Sommerfeld
orbits but simply determines their orientation with respect to some
direction in space.

In order to fix the direction in space let us suppose that the
atom is placed in a magnetic
field H, directed along the axis
of z, fig. 24·6. The direction
of H is a fixed direction in
space. Let α denote the angle
between H and the normal
to the orbital plane. In the
magnetic field the orbit will
precess about H just as a
mechanical top precesses in
a gravitational field. The
angular velocity of this
precession, called *Larmor pre-
cession*, is

$$\omega_L = H. \frac{e}{2\,mc}.$$
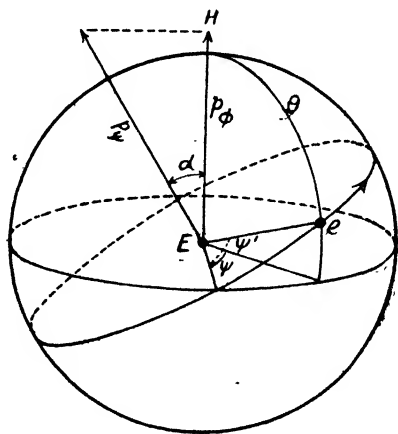
Fig. 24·6

As the orbit precesses the orbital angular momentum p_ψ,
represented as a vector along the normal to the orbital plane,

25 E

generates a cone about the vertical axis. We shall see in the sequence that the angle α cannot take a continuous set of values but that only certain discrete values are allowed, i.e. the orbit is *space quantized.* This space quantization is not hypothetical, but has actually been experimentally established by the classic experiment of Stern and Gerlach. If the field H is gradually reduced to zero, the angle α remains constant, since it is independent of H; while ω_L decreases to zero.

The atom is therefore, left with the orbital plane in one of a discrete set of orientations in space, but the energy is identical with that in a field-free atom.

In the two dimensional problem the kinetic energy in terms of co-ordinates r and ψ is

$$T = \tfrac{1}{2} m \dot{r}^2 + \tfrac{1}{2} m r^2 \dot{\psi}^2 = \tfrac{1}{2} (p_r \dot{r} + p_\psi \dot{\psi}) \quad . \quad . \quad (24\cdot46)$$

with the quantum conditions

and
$$\left. \begin{array}{c} \oint p_\psi d\psi = kh \\ \oint p_r dr = rh \end{array} \right\} . \quad . \quad . \quad . \quad (24\cdot47)$$

In three dimensions the kinetic energy in terms of co-ordinates (r, θ, φ) is

$$T = \tfrac{1}{2} (p_r \dot{r} + p_\theta \dot{\theta} + p_\phi \dot{\phi}). \quad . \quad . \quad . \quad (24\cdot48)$$

The quantum conditions are

$$\oint p_r dr = rh, \ \oint p_\theta d\theta = th \ \text{and} \ \oint p_\phi d\varphi = mh, \quad . \quad . \quad (24\cdot49)$$

where r, t and m are integers.

The potential energy in both cases is $-\dfrac{Ze^2}{r}$.

Since the total energy in both cases is the same,

$$p_r \dot{r} + p_\psi \dot{\psi} = p_r \dot{r} + p_\theta \dot{\theta} + p\varphi \dot{\varphi} \quad . \quad . \quad . \quad (24\cdot50)$$

or
$$\oint p_\psi \dot{\psi} = \oint p_\theta \dot{\theta} + \oint p_\phi \dot{\varphi} \quad . \quad . \quad . \quad (24\cdot51)$$

i.e.,
$$k = t + m. \quad . \quad . \quad . \quad (24\cdot52)$$

As the total angular momentum p_ψ is constant, p_ϕ, its projection on the φ axis, is also constant, so that

$$p_\varphi = p_\psi \cos \alpha \quad . \quad . \quad . \quad (24\cdot53)$$

or $\qquad\qquad\qquad \cos \measuredangle = \dfrac{p_\phi}{p_\psi} = \dfrac{m}{k}$ $\qquad$ . . . (24·54)

m is called the *magnetic quantum number*. It is the projection of the azimuthal quantum number in the direction of field,

$$\text{or} \qquad m = k \cos \alpha ,$$

with $m = \pm 1, \pm 2, \ldots \qquad \pm k$. $\qquad$ . . . (24·55)

The space quantization for $k=2$ is represented in fig 24·7.
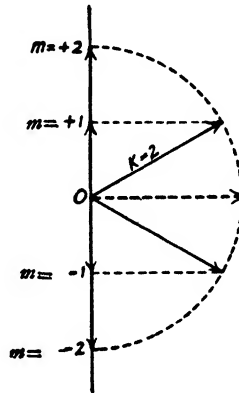
Fig. 24·7 Space quantization diagram for Bohr-Sommerfeld orbit with $k=2$.

From wave-mechanical treatment of the atom, k the azimuthal quantum number is to be replaced by the *orbital quantum number* $=k-1$. Henceforth we shall speak of only l which can take all integer values including zero. The magnetic quantum number m_l can take values $0, \pm 1, \pm 2, \ldots \pm l$. This is found to be experimentally true.

We have seen in the previous article that corresponding to the total quantum number $n=4$, the azimuthal qunantum number can take values $k=1, 2, 3,$ or 4 (or now $l=0, 1, 2, 3, 4$) but the energy is unchanged. These nine quantum states $(ml = 2l + 1 = 2 \times 4 + 1)$ of the atom for each of which the energy is the same are said in wave mechanics to be *degenerate*. The degeneracy is $(2l+1)$ times. This degeneracy is however removed when the atom is placed in a magnetic field. In the presence of the magnetic field

the original $(2l+1)$ coincident levels will now split up, each having a different energy.

7.   Bohr magnetor.   An electron of charge e e.s.u. moving in an orbit with period T constitutes a current I given by

$$I = \frac{e}{Tc}. \qquad \qquad . \quad . \quad . \quad (24\cdot56)$$

From the conception of the magnetic shell the magnetic moment μ of the circuit is

$$\mu = \text{area} \times I, \qquad . \quad . \quad . \quad (24\cdot37)$$

since the strength of the shell I is defined as the magnetic moment per unit area of the shell.

Now the area of the Kepler ellipse is given by

$$\text{area} = T \cdot \tfrac{1}{2} r^2 \dot{\phi} = \frac{T}{2m} \, mr^2 \dot{\phi}$$

$$= \frac{T}{2m} \cdot p_\phi$$

$$= \frac{T}{2m} \cdot l \frac{h}{2\pi}, \quad . \quad . \quad (24\cdot58)$$

since $p_\phi = l \frac{h}{2\pi}$, where l is the orbital quantum number, $(l=k-1)$.

Hence, $$\mu = l \frac{eh}{4\pi mc}. \qquad \qquad . \quad . \quad . \quad (24\cdot59)$$

According to $(24\cdot59)$ the magnetic moment of a hydrogen atom is always an integral multiple of $\frac{eh}{4\pi mc}$. This unit of magnetic moment is called the *Bohr Magneton* and is equal to $0\cdot918 \times 10^{-20}$ erg. gauss^{-1}.

8.   The spinning electron.   So far we have taken the three external degrees of freedom of the electron into account, and have seen that there are three quantum numbers, n, l and m_e to define an electron in an orbit.   In order to account for certain features of the alkali spectra it was postulated by two Dutch physicists, Uhlenbeck and Goudsmidt that there was one more degree of freedom

of the electron manifested in its spin. The spin angular momen-
tum of the electron is $\frac{sh}{2\pi} = \frac{1}{2}\frac{h}{2\pi}$. This half integral spin is not
to be taken as a quantum number that takes different values like
n and l but as an inherent and fixed property of the electron.
The total angular momentum due to single electron in an atom
is made up of two parts, one due to the motion of the centre of
mass of the electron around the nucleus in an orbit, and the other
due to the spin motion of the electron about an axis through its
centre of mass. In a magnetic field the spin is either parallel or
anti-parallel to the field; it has no other direction. The *spin
magnetic quantum number* m_s has therefore the values $\pm\frac{1}{2}$.

9. The Pauli exclusion principle and atomic structure.
The distribution of the electrons in an atom is governed by *Pauli's
exclusion principle. According to this fundamental law no two electrons
in the same atom can have all their four quantum numbers n, l, m_e and m_s
identical.*

In the spectroscopic notation an electron is called an s, a p or
a d *electron* according to the scheme,

s	p	d	f	g	h
$l=0$	1	2	3	4	5

The number of electrons that have the same n and l is indicated
by a superscript. For example, $2s^2 3p^4$ denotes 2 s electrons with
$n=2$ and $l=0$ and 4 p electrons with $n=3$ and $l=1$. Electrons
having the same n are said to belong to the same *shell*. Each
shell is divided into *subshells* according to the values of l.

The shells are designated according to the following scheme :

$n=$	1	2	3	4	5	6
X-ray designation :	K	L	M	N	O	P

The number of electrons in any atom in the K and L shells is given
in the table below, as determined by the Pauli principle.

TABLE

Shell	n	l	m_l	m_s	Number of electrons
K	1 1	0 0	0 0	$+\frac{1}{2}$ $-\frac{1}{2}$	2
L	2 2 2 2 2 2 2 2	0 0 1 1 1 1 1 1	0 0 1 1 0 0 −1 −1	$+\frac{1}{2}$ $-\frac{1}{2}$ $+\frac{1}{2}$ $-\frac{1}{2}$ $+\frac{1}{2}$ $-\frac{1}{2}$ $+\frac{1}{2}$ $-\frac{1}{2}$	8

We see from the above table that in the K shell there cannot be more than 2 electrons, and in the L shell not more than 8 electrons if the Pauli principle is not to be violated. In the subshell, $n=2$, $l=1$. there can be only six electrons and in the subshell $n=2$, $l=0$. there can be only two electrons. Any subshell may contain up to $2(2l+1)$ electrons and no more. This can be easily seen : Corresponding to any value of l there are $(2l+1)$ values of m_l $(-l$ to $l)$ and corresponding to each value of m_e there are two values of m_s. Hence the total number is $2(2l+1)$. The number of electrons in the various types of closed subshells are therefore as follows :

$$
\begin{array}{ccccc}
s & p & d & f & g \\
2 & 6 & 10 & 14 & 18
\end{array}
$$

The maximum number of electrons in a whole shell is $2n^2$,

since
$$
\sum_{l=0}^{l=n-1} 2(2l+1) = 2n^2,
$$

l taking all values from 0 to $n-1$.

Take for example the sodium atom whose atomic number is $Z=11$. The electron configuration of the atom is $1s^2 \ 2s^2 \ 2p^6 \ 3s^1$

The outermost s electron is called the *valence electron. It is the valence electron which determines the chemical properties of sodium.* The $3s$ shell can accommodate one more electron. The sodium atom will therefore readily combine with another atom with which it can share one electron. We know that alkali metals readily combine with halogens forming corresponding salts. This great chemical activity of alkali metals with halogen can be easily understood on the electronic configuration. The electronic configuration of chlorine $(Z=17)$ is

$$1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^5.$$
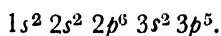
There are 5 electrons in the $3p$ shell of chlorine atom. There is still a vacancy for one electron to complete the $3p$ subshell which can accommodate six electrons. The vacancy is filled up by the $3s$ electron of the sodium atom. After the reaction the sodium and the chlorine atoms form a close structure. Sodium is electropositive. because it parts easily with its loosely bound $3s$ valence electron, whereas chlorine is electronegative because it easily takes up one electron to fill up the vacancy in the $3p$ shell. Take another example of neon whose electronic configuration is

$$1s^2\ 2s^2\ 2p^6.$$
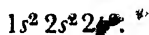
Neon therefore, forms a close structure since its $2p$ shell is all filled up. It is therefore chemically an inert gas. The same is true for gases like helium, argon, krypton and xenon.

The whole periodic table can be explained on the electronic configuration of elements.

CHAPTER XXV
X-RAYS

1. The discovery of X-rays. The discovery of what are called X-rays which are of enormous industrial and scientific importance was accidentally made by Professor Röntgen (1845—1923) at Würzberg in 1895 when studying the discharge of electricity through gases. Röntgen found that a discharge tube even after completely covering it with black paper made a fluorescent screen (a screen coated with a fluorescent material such as zinc sulphide or barium platinocyanide) to glow brightly in dark. He also found that heavy objects interposed between the screen and the tube stopped the fluorescence proving that the fluorescence was due to some radiations starting from the tube. Because of their unknown nature at that time Röntgen called them X—rays.

It was soon discovered by Röntgen that these unknown rays came from that part of the discharge tube which was situated opposite to the negative electrode and that they travelled in straight lines casting sharp shadows of objects upon a fluorescent screen. It was also found that dense matter like bones absorb them more than light substances like flesh and that they affect photographic plates. As we now know this important property of X-rays is being extensively utilized in medicine to alleviate human ills. The discovery of X-rays attracted wide interest and scientists all the world over started detailed study of their properties.

2. Production of X-rays. X-ray tube. The type of tube with which Röntgen made his original discovery is shown in fig. 25·1 The tube was evacuated by a pump so that a potential difference of the order of 30 kilovolts was needed to pass a discharge through it. The cathode rays shooting perpendicularly from the cathode were stopped by the broad end of the tube with the result that X-rays were emitted. It was soon discovered that any substance stopping the cathode rays becomes the source of

X-rays but their intensity increases as the atomic weight of the
substance increases. To have
a point source of X-rays
plane cathode was replaced
by a concave cathode and
the target-cathode distance
was so adjusted that the
cathode rays were focussed
on the target. The target
was made of a substance of

Fig. 25.1

high melting point to save it from destruction by heat generated
by the cathode ray impact.
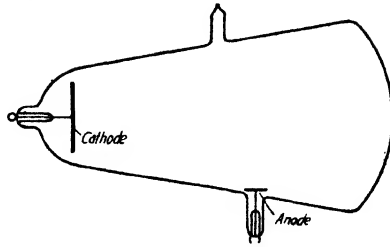
Since Röntgen's time various forms of X-ray tubes have been
designed, the essential features, however, of all of them are the
same. An X-ray tube is an evacuated vessel, either permanently
sealed or kept evacuated by a vacuum pump connected to it, con-
taining a cathode and an anode. The vacuum is of the order of
10^{-4} mm. of mercury. The cathode is connected to the negative
terminal of a high tension generator capable of supplying 100
kilovolts or more and the anode to its positive terminal which
may be earthed. The cathode rays accelerated by the difference
of potential between the cathode and the anode are stopped by the
latter giving rise to X-rays. X-ray tubes may be divided into two
main classes—the gas tube and the electron tube. In the former
type the electrons are obtained by the ionization of the residual
gas in the tube while in the latter type they are obtained from
heated wire of tun-
gsten or platinum.

A typical gas
X-ray tube is repre-
sented in fig. 25·2.
In scientific labor-
atories demountable
gas tubes are now
generally used. Such

Fig. 25.2

tubes are evacuated by a high vacuum pump continuously
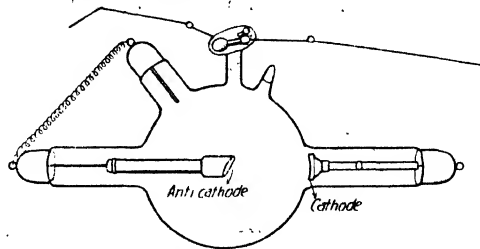
operating. The main advantage of such demountable tubes is that they can be cleaned from inside and any desired material for the anticathode may be used. The intensity of X-rays is controlled by suitably adjusting the gas pressure in the tube by a needle valve and by regulating the voltage put upon the tube. The principal defect of a gas tube is that the tube current and the tube voltage cannot be controlled independently.

Upto 1913 the gas tube was the most common tube in use. In this year Coolidge (assistant director of the general electric research laboratory at Schenectady, N. Y.) in-
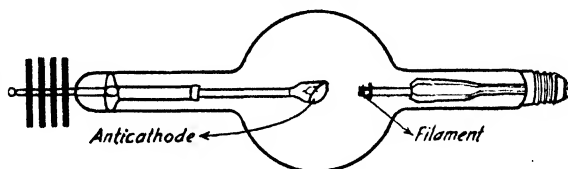
Fig. 25·3

vented the electron type of tube now known as Coolidge tube, fig. 25·3. On account of its great efficiency this tube has now found universal use. It consists of a glass bulb highly evacuated and fitted with a spiral of tungsten as cathode and an anticathode generally of tungsten or molybdenum. The spiral is surrounded by a metal cylinder which focusses the electron beam upon the anticathode. The outside of the anticathode carries a number of fins to cool this part. The electron tube which is to be used for scientific investigations is usually run at a stretch for a long time. This necessitates water cooling arrangement for the anticathode and the cathode. Such tubes are generally demountable. The chief advantage of a Coolidge tube is that the tube current and the tube voltage can be controlled independently.

When large tube currents are used the anticathode is considerably heated and very efficient cooling is necessary. The rotating anode tube represented in fig. 25·4 was designed for this purpose. It is used when very short exposures, e.g. in radiograph of the heart, are desired. The cathode is a straight filament in the plane of the paper and the anode, a bevelled tungsten disc which is rotated at a high speed by a rotating magnetic field applied to

the rotor. As a result of this arrangement each part of the target is heated due to bombardment by the electrons for a very short
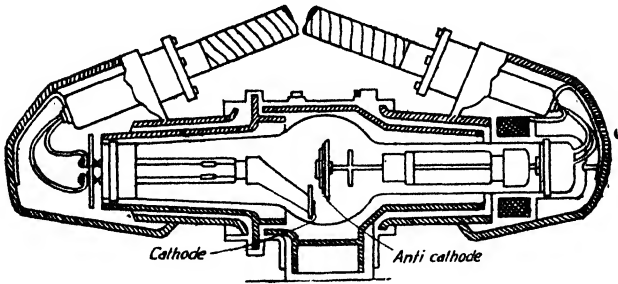
Fig. 25·4

time in each revolution and so gets sufficient time for cooling during each revolution. When seen edgewise the strip of the target which is bombarded acts practically as a point focus.

Recently special tubes have been designed for use with very high voltages. Thus, Lauritsen (Phys. Rev. 36, 988, 1930) has constructed for radiographic purposes a tube capable of working at 1100 kilovolts. It consists of a central discharge chamber made of metal the length of which is about seven feet. To each end are attached insulating glass cylinders of total length of about eleven feet. It is a demountable type of tube evacuated by continuous working of high vacuum pumps. Thick lead screens protect the operator from the harmful effect of stray x-rays. The working beam of x-rays is taken out of a small aperture in the screen.

High tension equipment. In addition to the x-ray tube another essential part of the equipment for production of x-rays is the high potential source. The voltage applied to the x-ray tube determines the penetrating power of x-rays according to the Einstein equation $eV = h\nu$, ν being the highest frequency or the shortest wavelength generated by applying the voltage V to the x-ray tube. Inserting the values of the electronic charge e and the Planck's constant h in the foregoing relation we get

$$\lambda = 12,336/V \qquad \qquad . \quad . \quad . \quad (25.1)$$

From eq. 25·1 we can readily calculate the shortest wave-length of x-rays generated by the voltage V put upon the x-ray tube. Thus to produce X-rays of wave-length 0·1 $\overset{o}{A}$ a potential of th order of 125 kilovolts must be employed. For ordinary x-ray work in a laboratory a high potential source yielding 40 to 50 kilo volts and about 20 to 30 milliamperes is quite adequate.

Of the various sources for high tension the induction coils and transformers are the most commonly used. Induction coils
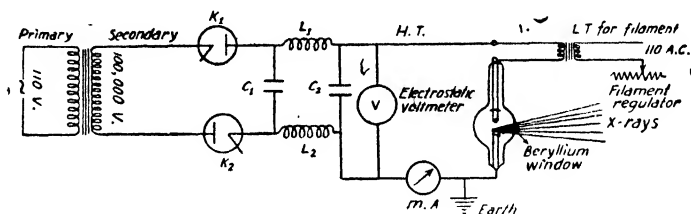
Fig. 25·5

are generally driven by direct current in the primary with some sort of interruptor in series. The most satisfactory source of high tension is a transformer with kenotrons as the rectifying valves. Suitable inductances and capacities used in the secondary circuit smooth out the pulsations of the secondary output. The general scheme of connections is represented in fig. 25.5. The voltage adjustment is made by means of an auto-transformer and a resistance regulator in the primary cicuit.

3. General properties of X-rays. The most important properties of X-rays are listed below :

(1) They can be refracted. Refractive index is less than unity. (2) They can be reflected by smooth surfaces provided the glancing angle between the beam and the surface is small (ordinarily less than 30°). (3) They can be polarized. (4) They can be diffracted by slits and gratings. (5) They show interference fringes. (6) They produce ionization in gases. (7) They eject photoelectrons from matter on which they fall. (8) They are scattered by substances. (9) They produce fluorescent X-rays from matter. (10) They cause luminescence of substances like zinc sulphide,

barium platinocyanide, etc.   (11) They are characterized by great penetrating power. (12) They can cause burns or kill living matter under proper dosage and are recently found to have profound effects on plants.

4. Wave nature of X-rays. Diffraction of X-rays by gratings. It was recognized early that X-rays are electromagnetic radiations like ordinary light but characterized by wavelength considerably shorter than that of the latter.   After a series of preliminary demonstrations of diffraction of X-rays by slits by various workers Walter and Robinou, Larsson and Kellström and others have given very good estimates of X-ray wave-lengths. Precise measurements of X-ray wave-lengths were soon made by ruled gratings similar to those used for visible light.   The grating formula to be used is exactly of the same form as that for visible region, viz., $(e + d)$ $(\sin i + \sin \theta) = n\lambda$, where i is the angle of incidence and θ the angle of diffraction.   The most remarkable consequence of these grating measurements is that they constitute the most accurate method of absolute measurement of X-ray wavelengths.   Using this method in a special vacuum spectrograph, Thibaud was successful in measuring wave-lengths as long as 144×10^{-8} cm ; using concave gratings Osgood measured X-ray wave-lengths as long as 211×10^{-8} cm.

During recent years the technique of high vacuum concave grating X-ray spectrography has been considerably developed by Siegbahn.   The X-ray spectrum and a comparison aluminium spark spectrum are photographed on the same plate so that X-ray wave-lengths can be accurately measured with the help of known aluminium lines.   The longest X-ray line measured in this way is 407×10^{-8} cm. These grating experiments have not only completely established the wave nature of X-rays but have given the most accurate and absolute values for their wave-lengths.

5. Diffraction of X-rays by crystals. Laue's discovery. Long before the successful termination of the experiments on direct diffraction by optical gratings Laue and his collaborators

discovered that crystals because of the periodic arrangement of their atomic or molecular constituents (the average distance between atoms in a solid is of the order of 10^{-8} cm.) are the most suitable natural gratings for diffracting x-rays.

In an optical grating we have a set of lines regularly arranged along a given direction. This is a one dimensional grating. In the diffraction of X-rays by crystals, however, we have to take into account the regular arrangement of atoms or molecules or still better of diffraction centres along three axes x, y, z i.e., a space lattice : a crystal is to be considered as a three-dimensional diffraction grating analogous to the one-dimensional optical grating. If a, b, c are the grating constants—distances between two successive diffraction centres—along the three coordinate axes, α_0, β_0, γ_0 the angles made by the direction of the incident ray with the coordinate axes and α, β, γ the corresponding angles made by the diffracted ray with the coordinate axes we have for the conditions of reinforcement the equations

$$a\,(\cos\alpha - \cos\alpha_0) = h_1\lambda \qquad \cdots \quad (25{\cdot}2)$$

$$b\,(\cos\beta - \cos\beta_0) = h_2\lambda \qquad \cdots \quad (25{\cdot}3)$$

$$c\,(\cos\gamma - \cos\gamma_0) = h_3\lambda, \qquad \cdots \quad (25{\cdot}4)$$

where λ is the wave-length and h_1, h_2, h_3, whole numbers representing the orders. These are the well-known Laue equations comprising the essential features of the theory of space lattice. Eq. $(25{\cdot}2)$ is satisfied by all generators of a cone of semi-angle α with its axis coinciding with the axis of x. Thus for a given angle of incidence α_0 there will be a series of concentric cones surrounding the x-axis, each cone being made up of diffracted rays of one order. Similarly eqs. (25.3) and $(25{\cdot}4)$ represent a set of cones with semi angles β and γ with axes y and z respectively. In order that a strong diffracted beam may appear in the direction α, β, γ it is necessary that all the three equations should be satisfied simultaneously. This means that the line of intersection of the cones $(25{\cdot}2)$ and $(25{\cdot}3)$ should lie on the cone $(25{\cdot}4)$, a require- ment which considerably limits the number of diffracted rays.

The wave-length of the diffracted ray can be easily deduced from eqs. (25·2), (25·3) and (25·4). We have

$$\cos \alpha = \cos \alpha_0 + h_1 \frac{\lambda}{a}, \cos \beta = \cos \beta_0 + h_2 \frac{\lambda}{b},$$

$$\cos \gamma = \cos \gamma_0 + h_3 \frac{\lambda}{c}.$$

Squaring both sides and adding we get

$$\cos{}^2\alpha + \cos{}^2\beta + \cos{}^2\gamma = \cos{}^2\alpha_0 + \cos{}^2\beta_0 + \cos^2\gamma_0$$
$$+ 2\lambda \left(\cos \alpha_0 \frac{h_1}{a} + \cos \beta_0 \frac{h_2}{b} + \cos \gamma_0 \frac{h_3}{c}\right)$$
$$+ \lambda^2 \left(\frac{h_1^2}{a^2} + \frac{h_2^2}{b^2} + \frac{b_3^2}{c^2}\right).$$

Since $\cos{}^2\alpha + \cos{}^2\beta + \cos{}^2\gamma = \cos{}^2\alpha_0 + \cos{}^2\beta_0 + \cos{}^2\gamma_0 = 1$ this equation gives

$$\lambda = - 2 \frac{\cos \alpha_0 \dfrac{h_1}{a} + \cos \beta_0 \dfrac{h_2}{b} + \cos \gamma_0 \dfrac{h_3}{c}}{\left(\dfrac{h_1^2}{a^2} + \dfrac{h_2^2}{b^2} + \dfrac{h_3^2}{c^2}\right)} \quad . \quad . \quad . \quad (25·5)$$

From Laue's equations another deduction can be made. If 2θ is the angle between the incident and diffracted rays we have

$$\cos 2\theta = \cos \alpha \cos \alpha_0 + \cos \beta \cos \beta_0 + \cos \gamma \cos \gamma_0. \qquad (25·6)$$

Squaring and adding equations (25·2), (25·3), (25·4) we have

$$(\cos \alpha - \cos \alpha_0)^2 + (\cos \beta - \cos \beta_0)^2 + (\cos \gamma - \cos \gamma_0)^2$$
$$= 2 - 2 \cos 2\theta$$
$$= 4 \sin{}^2\theta$$
$$= \left(\frac{h_1^2}{a^2} + \frac{h_2^2}{b^2} + \frac{h_3^2}{c^2}\right) \lambda^2$$

and, therefore,

$$\sin \theta = \frac{\lambda}{2} \sqrt{\frac{h_1^2}{a^2} + \frac{h_2^2}{b^2} + \frac{h_3^2}{c^2}}. \qquad . \quad . \quad . \quad (25·7)$$

In the most simple types of crystals viz. cubic crystal, $a=b=c$ and therefore eqs. (25·5) and (25·7) reduce to

$$\lambda = - 2a \frac{h_1 \cos \alpha_0 + h_2 \cos \beta_0 + h_3 \cos \gamma_0}{h_1^2 + h_2^2 + h_3^2} \quad . \quad . \quad . \quad (25·8)$$

and $\sin \theta = \dfrac{\lambda}{2a} \sqrt{h_1^2 + h_2^2 + h_3^2}$ \qquad . \quad . \quad . \quad (25·9)

In the experimental arrangement of Laue a fine heterogeneous

beam of X-rays was passed
on from a slit B through
a crystal at H aldng one of
its axes. The emergent rays
were allowed to fall on a
photographic plate D placed
a few centimetres away
from the crystal, fig 25·6.
On developing, a pattern
like that represented in fig.
25·7 was obtained. The
central spot was due to
undeviated direct beam of
X-rays while the other spots
represent the crystal sym-
metry about the axis along
which the beam was allowed
to pass. It is important to
note that although the in-

Müller spectrograph
[From crystalline state: W. L. Bragg]
Fig. 25.6

cident X-rays were not monochromatic the diffracted images do not
form a continuous band as in the case of an optical grating. This is
because of the requirement that eqs. (25·2) to (25·4)) are to be
satisfied simultaneously. These spots are called Laue spots ; they
vary with the nature of the diffracting crystal. This experiment of
Laue, Friedrich and Knipping opened up a vast field for the deter-
mination of X-ray wave-lengths and the study of crystal structure,
subjects of tremendous importance in pure and applied physics.

Bragg's equation. As an alternative a simple conception of
diffraction of X-rays by crystals was given by Bragg. For a given
incident beam deviated through an angle 2θ as a result of diffrac-
tion it is always possible to find a set of parallel planes containing
all the atoms in a crystal, such that the incident ray undergoes
an apparent regular reflection as if by a pile of mirrors, the incident
and reflection glancing angles being each equal to θ given by eq.

(25.7). These reflexion planes are not necessarily the natural cleavage planes of the crystal.
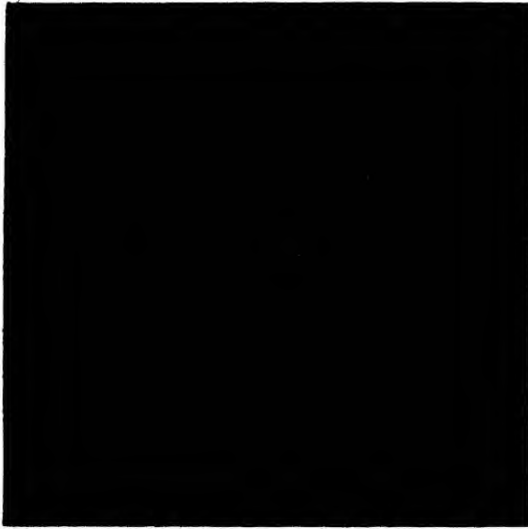
Laue Spots of NaCl
Fig. 25·7

Consider a set of parallel planes of atoms in a crystal successive ones of which are represented by the lines p p, p_1p_1, p_2 p_2, d being the perpendicular distance between any two successive planes, fig 25·8. A parallel beam of monochromatic X-rays of wave-length λ proceeds in the direction IA making an angle θ with the planes. The line I I_1 represents the trace of the in-

Fig. 25·8

cident wave front which on encountering the atoms in the crystal gives rise to scattered waves. If the paths IAR and $I_1A_1R_1$ for waves scattered by the atoms A, A_1 are equal they will reinforce each other in this direction. From the figure it is evident that this

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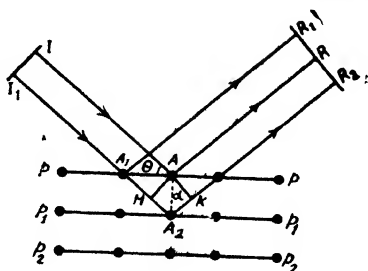
condition is satisfied. Similarly any other atom lying in the plane pp will reinforce the beam in this direction. The rays scattered by atoms in two successive planes will reinforce each other if their difference of path is some whole multiple of λ. Dropping perpendiculars AH and AK on the rays I_1A_2 and A_2R_2 respectively we see that the required difference of path is $(HA_2+A_2K)=2d$ sin θ. Therefore for reinforcement we have the condition

$$2d \sin \theta = n\lambda. \qquad \ldots \quad (25 \cdot 10)$$

The waves scattered by all the atoms of the crystal will thus reinforce in the direction θ defined by eq. (25·10). The integer n is called the order of reflexion. It may be noted here that Bragg's law requires a small correction due to the refractive index of X-rays being slightly less than unity but for all ordinary purposes it may be neglected.

A relation between d, the index numbers h_1, h_2, h_3 of the diffracted ray and the grating constants a, b, c of the crystal can be easily found out by comparing eqs. (25·7) and (25·10) with each other. We have

$$d = n \bigg/ \sqrt{\frac{h_1{}^2}{a^2}+\frac{h_2{}^2}{b^2}+\frac{h_3{}^2}{c^2}}. \qquad \ldots \quad (25 \cdot 11)$$

Defining the integers h, k, l by putting $nh=h_1$, $nk=h_2$ and $nl=h_3$, where n is the greatest common divisor of h_1, h_2, h_3, eq. (25·11) becomes

$$d = 1 \bigg/ \sqrt{\frac{h^2}{a^2}+\frac{k^2}{b^2}+\frac{l^2}{c^2}}. \qquad \ldots \quad (25 \cdot 12)$$

The letters h, k, l are the *Miller indices* which describe the orientation of the reflexion planes in the crystal with respect to its axes. This can be proved as follows. Take the origin in one of the reflexion planes. If λa, μb, νc are the intercepts of the next plane on the crystal axes, λ, μ, ν being fractions and a, b, c the lengths of the three edges of a unit cell, which is the building block by repitition of which in three dimensions the crystal is built up, the plane is given by the equation

$$\frac{x}{\lambda a}+\frac{y}{\mu b}+\frac{3}{\nu c}=1.$$

The perpendicular distance from the origin and therefore d the perpendicular distance between the set of planes is given by

$$d = 1 \Big/ \sqrt{\frac{1}{\lambda^2 a^2} + \frac{1}{\mu^2 b^2} + \frac{1}{\nu^2 c^2}}.$$

Writing $h = \frac{1}{\lambda}$, $k = \frac{1}{\mu}$, $l = \frac{1}{\nu}$ which by definition are called the Miller indices, the foregoing equation becomes

$$d = 1 \Big/ \sqrt{\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}}$$

which is the same as eq. 25·12.

Bragg's X-ray spectrometer. Bragg realised that the cleavage planes of crystals should be rich in atoms and so should give
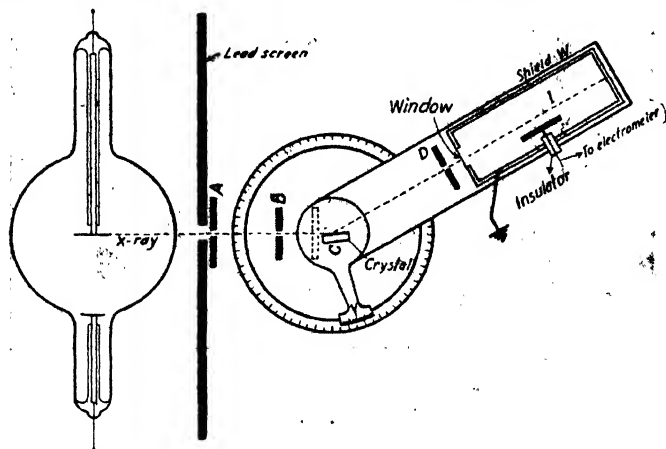
Fig 25·9 (a) Principle of Bragg x-ray spectrometer

most intense spots. He, therefore, tried the experiment of reflecting an X-ray beam from the cleavage face of a crystal and found that a spot was recorded on the photographic plate at the angle of reflection. For investigating the intensity of X-rays at various angles and thus accurately measuring the glancing angles for a given crystal Bragg constructed an ionization spectrometer the principle of which is shown in fig. 25·9 (a) and the perspective in

fig. 25·9 (b).   In this instrument the collimator of the usual
optical spectrometer is replaced by two or more adjustable parallel
slits A, B while the telescope is replaced by an ionization chamber
I or by a photographic plate, fig 25·9 (a).   The crystal is moun-
ted on the turn-table C with its cleavage face vertical and passing
through the centre of rotation.   The ionization chamber or the
photographic plate holder and turn-table are capable of rotation,
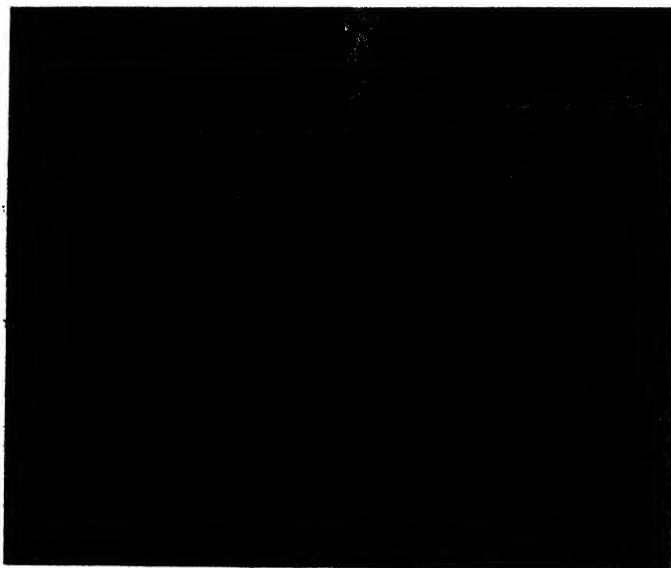
Bragg x-ray spectrometer [From the Crystalline State:
Professor W. L. Bragg.]
Fig. 25·9 (b)

the angles being accurately read upon a circular scale fitted with
a vernier.   The collecting electrode in the ionization chamber
passes through the metallic shield W (earth connected) and is
connected to an electroscope. To increase the ionization current
the ionization chamber is filled with methyl iodide vapour.

The method consists in turning the crystal through a certain
angle and the ionization chamber through twice this angle and
observing the ionization current which represents the intensity of

X-ray reflection. The ionization current is then plotted as ordinate and the corresponding glancing angle as the abscissa. Plots similar to
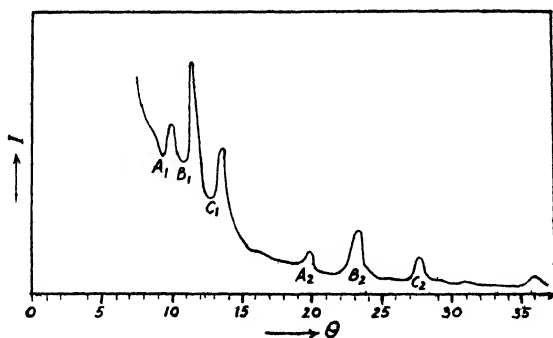
Fig. 25·10

that shown in fig. 25·10 were obtained. The record represented in this figure was obtained with platinum as the anticathode material in the X-ray tube and rock salt as the reflecting crystal. It is seen that instead of varying uniformly with angle the ionization current assumes large values at certain sharply defined glancing angles.

The curve can be explained by the Bragg equation, $2d \sin \theta = n\lambda$. According to this equation a change of θ should alter the wavelength of X-rays reflected by the crystal. The peaks A_1, B_1, C_1, must therefore represent three X-ray lines in the first order. This supposition is justified and proved to be true by the fact that the second order peaks A_2, B_2, C_2 appear at angles exactly twice those corresponding to A_1, B_1, C_1 as required by the Bragg equation ; also their relative intensities are the same in the two orders.

If the source of X-rays is a nickel anticathode instead of platinum it is found that two lines instead of three appear and at different glancing angles. If, however, the crystal is changed the same lines appear but at different angles indicating that the grating space varies with the crystal. These facts prove beyond doubt that the X-ray lines represented by the peaks are truly characteristic of the material of the anti-cathode of the X-ray tube: We get true X-ray spectra characteristic of the anti-cathode material diffracted by a crystal grating.

One more feature of the ionization curve must be noted. The peaks of the characteristic radiations have a continuous background corresponding to continuous radiation emitted by the X-ray tube along with the characteristic radiations.

Determination of X-ray wave-length. When the glancing angles corresponding to the peaks A_1, B_1, C_1, are measured as above the wave-lengths are at once determined if the grating constant d of the crystal is known. In this way the wave-lengths of the characteristic lines of various metals can be found out by using anticathodes made of these metals.

The analysis of Laue spots and Bragg reflections by NaCl crystal leads one to the conclusion that this crystal has cubic symmetry, the sodium and chlorine atoms being arranged alternately at the corners of a cubic lattice as shown in fig. 25·11, the black dots representing sodium and the white dots chlorine. It is seen that the sodium atoms lie on a face-centred cubic lattice ; the chlorine atoms also lie on a similar lattice displaced half the edge of the unit cell along each axis relative to the sodium lattice. The distance d between the successive atoms planes can be easily calculated with the help of the density of the crystal and the value of the Avogadro constant N. On the average, every atom has associated with it a volume d^3. The mass of this is ρd^3, where $\rho = 2·17$ is the density of rock salt. Now one gram molecule, i.e. $(23·05 + 35.45)$ grams of rock salt contain N molecules and so the average mass per atom is $\dfrac{58·50}{2N}$ grams. Equating this quantity with ρd^3 we obtain

$$d = 2·814 \times 10^{-8} \text{cm.}$$

The accuracy of this value is governed by the accuracy with which N is known. Similarly the grating space for calcite is $3·029 \times 10^{-8}$ cm.
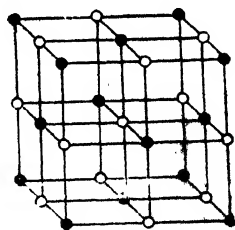
Fig. 25·11 NaCl lattice

Substituting this value of d in the Bragg equation and putting for θ the values obtained from the ionization curve we get the wavelengths. Thus in fig. 25·10 for the peak B_1, $\theta = 11\cdot4^{\circ}$ when $n=1$ and so $\lambda_{B_1} = 1\cdot12 \times 10^{-5}$ cm. The wave-lengths obtained by crystal gratings agree closely with those determined directly by optical gratings indicating the correctness of the fundamental structures attributed to rock salt and calcite.

6. Crystals. Space lattices. Crystal systems. The regular

geometrical form of crystals is due to atoms arranged in a pattern repeating periodically in three dimensions. The regular repetition of the unit of structure is similar to the repetition of a pattern on a wall paper. The atomic array in a crystal is described with respect
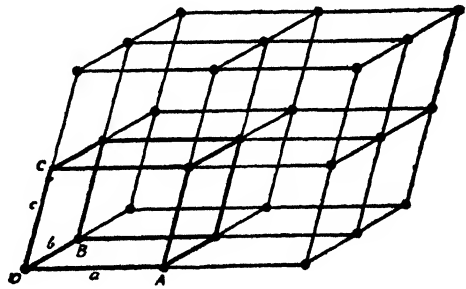
Fig. 25·12

to a three-dimensional network of straight lines as shown in fig 25·12. Such a network divides space into equal-sized prisms standing side by side with all faces in contact without leaving any empty space. The points of intersection of the lines constitute a *space lattice*. In a crystal these lattice points may be actually occupied by atoms or they may be points about which several atoms are grouped together. The manner in which the network of reference lines is drawn is arbitrary, because unit prisms of many different shapes can be drawn through the points of a space lattice. It is not necessary that lattice points lie only at the corners of the unit prisms ; they may lie not only at prism corners but at the centres of prisms and their faces. *The most important property of a space lattice is that every point of a space lattice has identical surroundings.*

There are in all 14 space lattices but almost an infinite number of *crystal structures* that is, methods of building up crystals out of piles of atoms. The points in each space lattice are described with refere-
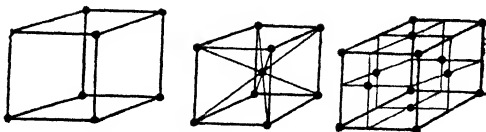
Fig. 25·13

nce to some appropriate set of axes with their origin at one of the lattice points. There are seven different systems of axes in use in crystallography each having special peculiarities regarding axial lengths and angles. The *seven systems are triclinic, mono-clinic, orthorhombic, tetragonal, hexagonal, rhombohedral and cubic.* Out of these the cubic system is the simplest and has three types of lattices (1) simple, (2) body centred and (3) face centred. In this system the axial lengths are equal $(a=b=c)$ and the axes are mutually perpendicular to each other as shown in fig. 25·13.

The net-work of lines through the lattice points divides it into perfectly identical prisms called *unit cells*. These are the building blocks of the crystal.

A system of notation for specifying the orientation of crystal faces and planes within the crystal has been found necessary. Miller indices are universally adopted for this purpose. To determine the miller indices of a plane the following procedure is adopted. (1) Measure the intercepts of the plane on the three edges of a unit cell in terms of the dimensions of the unit cell and not in centimetres. For example a plane that cuts the x-axis at a distance $\frac{a}{3}$ from the origin has intercept $\frac{1}{3}$ on the x-axis and similarly for the y and z axes. (2) Take the reciprocals of these numbers (3) Reduce them to the smallest whole numbers having the the same ratio and enclose these in parantheses, (h, k, l). Thus the plane having intercepts 1, 1, 1 has indices (111) ; a plane with intercepts 1, ∞, 2 has reciprocal intercepts 1, 0, $\frac{1}{2}$ and so Miller indices (201).

7. Methods of crystal analysis. There are three different standard methods of diffraction used in crystal analysis : (1) Laue method, (2) rotating crystal method and (3) powder method.

Laue method. In this method a single crystal is held stationary in a beam of white radiation. Due to variations in λ various diffraction spots called Laue spots are obtained on a photographic plate arranged as shown in fig 25·7. The indices of reflecting planes can be found out from the measurements of the spots.

Rotating crystal method. In this method a single small crystal suitably mounted is made to rotate continuously about an axis perpendicular to the beam of X-rays passing through a a fine cylindrical slit. Diffracted beam flashes out when a reflecting plane comes into a position satisfying the equation $2d \sin \theta = n\lambda$. A series of layer lines of spots are obtained from which the indices of the planes are determined.

Powder method. In this method which is due to Debye, Scherrer and Hull a single crystal is not necessary and so this method is specially useful when substances are more easily obtainable in the form of a powder, a sheet or a wire. This method is found to be of great help in industry. In this method a thin pencil of homogeneous X-rays of known wave-length is allowed to fall upon the specimen of polycrystalline material which may be in the form of a wire, a thin foil or a screen of the powdered substance. The

Fig. 25·14

specimen is mounted in a suitable camera which can hold either a flat photographic plate or a thin film bent into the form of a cylinder whose axis is perpendicular to the X-ray beam and passes through the specimen. The essentials of a cylindrical camera are indicated in fig. 25·14,
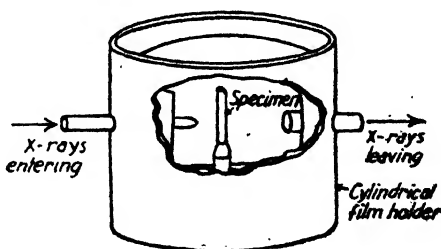
Since the substance is in the form of a powder the reflecting planes are all oriented at random. Conical diffraction beams
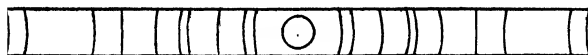
Fig. 25·15

from the various crystal planes coaxial with the X-ray beam emerge in directions determined by the Bragg equation $2d \sin \theta = n\lambda$. A flat plate placed perpendicular to the incident X-ray beam will record concentric circles like those in electron diffraction while a cylindrical film will show a number of lines on both sides of the central spot. Fig. 25·15 shows the general nature of the pattern on a cylindrical film; while that on a flat plate is shown in figure 25·16. If $2l$ is the maximum diameter of a ring in the
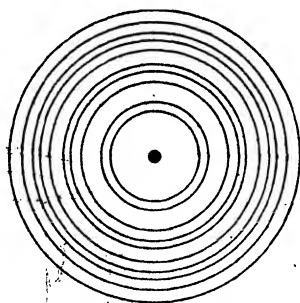
Fig. 25·16

pattern for a given specimen and r the radial distance from the specimen to the film the angle θ in radians is $l/2r$. Knowing θ for the various rings in the pattern the indices of the planes can be found out. In the case of cubic crystals θ is given by

$$\sin \theta = \frac{n\lambda}{2a}(h^2 + k^2 + l^2)^{\frac{1}{2}},$$

where a is the length of the cube edge. Thus knowing h, k, l we obtain the value of a since λ is given.

8. X-ray spectra. Continuous spectrum. In a previous article we saw that the target of an X-ray tube emits continuous radiation (also called white radiation) together with the radiations characteristic of the anticathode material. We will now describe concisely the fundamental peculiarities of these radiations.

Fig. 25·17 shows the distribution of energy in the continuous radiation from a tungsten target obtained by Urey, at different
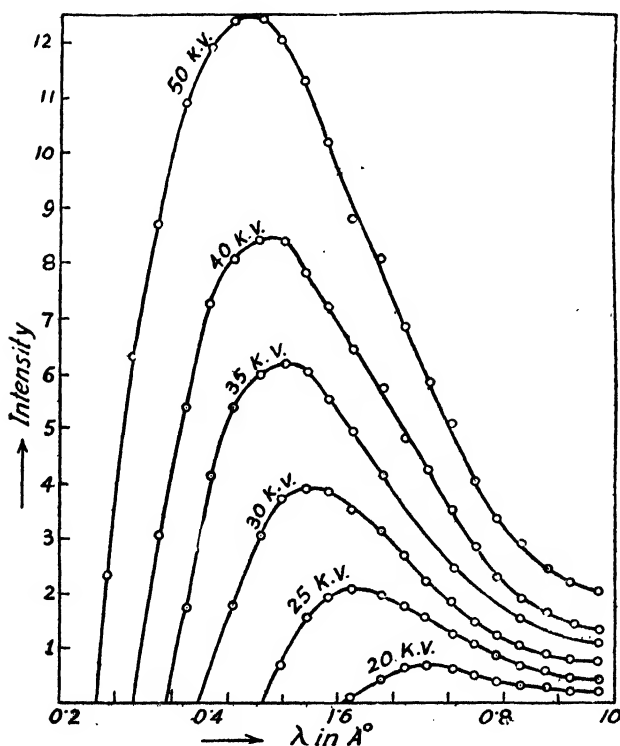
Fig. 25·17

tube voltages. The most important features of this figure are:
(1) Each spectrum ends *abruptly* at a certain minimum wave-

length, (2) the intensity is maximum at a certain wave-length, (3) there is a gradual decrease in intensity beyond this maximum on the long wave-length side of the spectrum and (4) as the tube voltage increases the short wave-length limit and the point of maximum intensity shift towards the shorter wave-length side and the intensity of each wave-length increases. The radiation is generated when the electron starting from the cathode with high velocity is stopped by the anticathode. The short wave-length limit is given by the quantum relation

$$eV = h\,\nu_{max}$$
$$= hc/\lambda_{min}$$

as already mentioned.

The total energy emitted by an X-ray tube increases with the atomic number of the anticathode material and roughly as the square of the applied voltage. The wave-length corresponding to maximum intensity is about 1·5 times the minimum wave-length. For radiographic purposes an anticathode of high atomic number usually tungsten, is employed and the tube is worked at high voltages.

The principal features of characteristic spectrum. As observed before, the continuous spectrum is accompanied by a line spectrum characteristic of the anticathode material when the X-ray tube is worked at a suitable high voltage. In the pioneer work of Barkla based on absorption methods the characteristic radiations were classified as K and L groups, K being harder than the L. In 1913 Moseley took up an extensive quantitative study of the characteristic X-rays emitted by 38 different elements. The apparatus used by him was an X-ray spectrograph fitted with potassium ferrocyanide as the analysing crystal ($d = 4\cdot227 \times 10^{-8}$ cm.). Moseley found that the spectrum lines photographed by him fell into definite K and L groups identical with the K and L groups of Barkla and Sadler.

Researches of Moseley also showed that the K and L radiations consist of a number of lines. For example in the K series for molybdenum there are lines K_α, K_β, $K\gamma$ in the decreasing order of wave-length.

Fig. 25·18 Moseley's law.

Moseley showed that if $\sqrt{\dfrac{\nu}{R}}$ value of an X-ray line is plotted against the atomic number of the element which emits this line a straight line is obtained. This is known as Moseley's law and is shown in fig. 25·18. It can be expressed by a formula

$$\sqrt{\nu} = k(Z - \sigma),$$

where k and σ are constant, Z the atomic number and ν the frequency of the line. Moseley showed by these graphs that the K_α line is represented by the formula

$$\nu_\alpha = k\left(Z - 1\right)^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right) \qquad \ldots \quad (25.13)$$

and the K_β line by the formula

$$\nu_\beta = R\left(Z - 1\right)^2\left(\frac{1}{1^2} - \frac{1}{3^2}\right). \qquad \ldots \quad (25.14)$$

Similar relations were shown to be true for other lines.

Moseley's graph enables an investigator to discover and identify missing elements in the periodic table. In this way elements of atomic number 43, 61, 72, 75, 85 and 87 were discovered and identified. In contrast with optical spectra X-ray spectra are simple and so, very suitable for identification work.

Since the time of Moseley our knowledge of X-ray spectra has been considerably extended in quality and quantity. Now we know that there are K, L, M, N, O, etc. series of which K consists of shortest wave-lengths. Each spectral series consists of a number of lines each obeying Moseley's law.

Corresponding to X-ray emission spectra the absorption spectra in the X-ray region consist of sharp edges. The K absorption edge is single and lies at a wave-length just shorter than the shortest emission line in the K-series. The L absorption spectrum consists of three absorption edges L_1, L_2, L_3 and the M absorption spectrum of five edges, M_1, M_2, M_3, M_4, M_5. Like emission lines the frequencies of the absorption edges obey Moseley's law. It may be mentioned that absorption edges are often attended by extensive fine structure.

9. Interpretation of X-ray spectra. We have already seen that X-ray wave-lengths are of the order 10^{-8} cm. in contrast with the order 10^{-5} cm. for optical wave-lengths. Thus X-ray frequencies are thousand times greater than optical frequencies. This leads to the inevitable conclusion that X-rays arise out of transitions of electrons in inner shells. The primary condition for the emission of X-ray lines is thus the creation of a vacancy in one of the inner shells of an atom. Such a vacancy is created when an electron from the cathode impinges on the electrons of the anticathode atoms with an energy greater than the energy with which these electrons are bound in the various shells. This ejection may also be effected by allowing X-rays of sufficient frequency to fall upon the anticathode of the X-ray tube. A charateristic X-ray line is emitted when an electron from an outer shell falls into an inner shell in which a vacancy is created. The K-series lines will be emitted when the K-shell vacancy is filled up by transitions from L_1, M_1, etc., shells. According to Bohr's frequency condition the wave numbers of the K-series will be given by

$$\nu = R\, Z'^2 \left(\frac{1}{1^2} - \frac{1}{n^2} \right), \ n > 1.$$

Similarly M→L, N→L, etc., transitions give L-series lines with a similar formula.

Such transitions are represented in the simplified energy level diagram, fig. 25·19. In constructing such diagrams it is customary to assign zero energy to the atom in its normal state. A complete diagram must take into account the existence of various subshells.

When the K, L, M, etc., electrons absorb energy equal to that with which they are bound in their respective shells, from a continuous X-radiation, we get the corresponding absorption edges.

Fig. 25·19

10. Applications of X-rays

Perhaps no other rays have been more usefully applied to human service than X-rays. Radiography (shadow photographs by X-rays) is now widely used in medicine for diagnosing diseases like tuberculosis of the lungs, defects in the digestive tract, fractures in bones, etc., while recently it is being increasingly applied in industrial diagnosis. With its help welds, metal castings and things like that are rapidly and most satisfactorily examined for cracks, flaws, etc. Radiography also finds applications in the examination of rubber tyres, golf balls, wood and suspected contrabands. Also because of the fact that old paintings were coloured with pigments consisting of inorganic substances which absorb X-rays to a greater extent than the organic dyes used in modern paintings, old pieces of art and painting can be easily examined for securing evidence for retouching, etc. X-rays have also found wide application in studying crystal structure of substances and metal alloys. Recently X-rays have found useful application in differentiating artificial from real pearls.
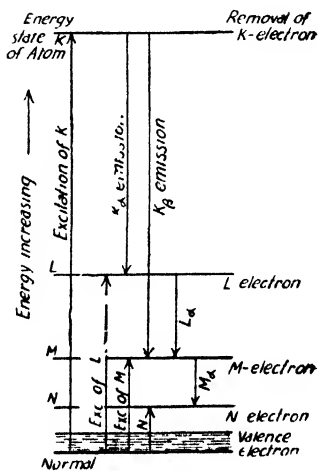
On purely scientific side X-rays have proved most fruitful in the investigations on the structure of atoms. They have been most usefully employed in chemical analysis of mixtures.

In the biological field important genetic effects of X-rays have been found. Under X-ray treatment chromosomes show important changes leading to the new hereditary characteristics in the progeny. As a typical example white corn plants without chlorophyll are obtained as a result of mutation induced by X-rays.

The above is only a brief account of some of the most important applications of X-rays. For a more detailed account the student should consult special technical books.

CHAPTER XXVI
PHOTOELECTRICITY

1. Discovery of photoelectricity. Hertz, in 1887, while engaged in his classic experiments on the existence of electro-magnetic waves observed that the passage of an electric discharge between two electrodes was rendered much easier when ultra-violet light was allowed to fall on the cathode. He could not under-stand the phenomenon. A year later Hallwachs explained that the cause of the above phenomenon was the emission of negative electricity from the cathode, thus rendering the passage of the spark easier. That it is so, he showed by allowing ultraviolet light to fall on insulated charged metallic plates, which lost their electri-city when they were negatively charged, but the light had no effect at all when they were positively charged.

In 1899 Lenard showed that the negative electricity was no-thing but a stream of electrons. He measured their $\frac{e}{m}$. His most important observations were : (1) that the number of photo-elec-trons was directly proportional to the intensity of the incident light and (2) that the electrons were emitted with all velocities from zero to a certain maximum; the maximum velocity was independent of the intensity of light but was directly proportional to the frequency. These two important observations of Lenard led Einstein to develop the quantum theory of light.

2. Einstein's photoelectric equation. In 1905, Einstein incorporated Lenard's observations in his famous equation

$$\tfrac{1}{2}mv^2_{max} = h\nu - \phi, \qquad . \quad . \quad (26\cdot1)$$

where m is the mass of the electron, v_{max} is its maximum velocity, ν the frequency of light, h the Planck's constant and ϕ the energy required for an electron to escape from the surface of the emitting material. Einstein postulated that radiant energy exists in the form of indivisible *light quanta* each of energy $h\nu$. When a quantum of

27 E

energy $h\nu$ falls on a metal it gives its entire energy to the electron inside it. The electron requires a certain amount of energy ϕ (*work function*) to be released from the metal ; it then comes out with a kinetic energy whose maximum value cannot exceed $h\nu - \phi$. Eq. (26·1) is merely an expression of the fact that energy is conserved before and after impact.

3. Millikan's experiment. In 1912 Einstein's equation was experimentally verified by Richardson and Hughes. Later on in 1916, Millikan repeated the experiments with great care by eliminating all possible sources of error. He achieved a high degree of accuracy. His apparatus consists of a highly evacuated glass vessel at the centre of which are mounted blocks of lithium, sodium and potassium on a rotating wheel Q, (fig. 26·1). R is a sharp cutter,
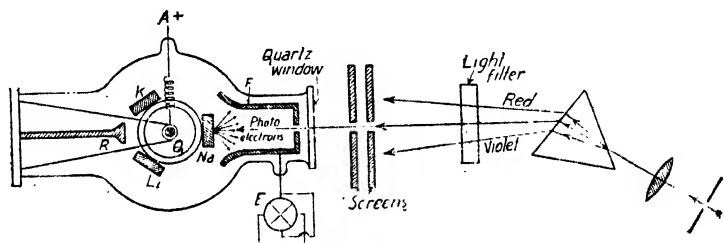
Fig 26·1 Millikan's apparatus

operated from outside by an electro-magnetic device, to scrap a thin slice from the metallic blocks so that each time a fresh surface is exposed to the incident monochromatic radiation. F is a Faraday cylinder to collect the charge. A variable retarding potential is applied between F and Q. The Faraday cylinder is connected to an electrometer E.

The experiment consists in observing the current in the electrometer E for different retarding potentials. The current in the electrometer will be zero for that retarding potential for which all the electrons are repelled back. Contact potential differences are measured by the Kelvin
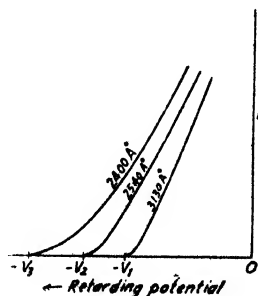
Fig 26·2

method. Millikan's observations for different wave-lengths are shown in fig. 26·2.

He further plotted V_0 (the max. retarding potential) against frequency and got a straight line.

Now $$\frac{eV_0}{300} = \frac{1}{2}m v^2_{max}. \qquad \qquad \text{. . . (26·2)}$$

From (26·1) and (26·2) we have

$$\frac{eV_0}{300} = hv - \phi. \qquad \qquad \text{. . . (26·3)}$$

Theoretically a plot of V_0 against v should give a straight line whose slope is equal to $\frac{300h}{e}$. There is a very close agreement between the theoretical and the experimental values.

Millikan's value of h is $6·57 \times 10^{-27}$ erg. sec., whereas the present accepted value is $6·547 \times 10^{-27}$ erg. sec. The frequency v_0 below which there will be no emission of electrons is called *photo-electric threshold.* For sodium $v_0 = 4·39 \times 10^4$ or $\lambda_0 = 6800$ Å.

4. Photoelectric cells. The photoelectric cell consists of an
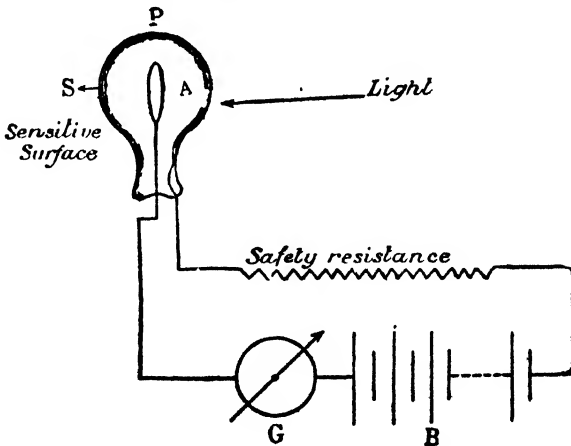
Fig. 26·3

evacuated glass bulb, the inside of which is coated with a thin film of an alkali metal such as Cs or K except a small window for allowing light to pass. At the centre of the cell there is a thin metallic wire

which acts as an anode the coated surface being the cathode, fig 26·3.

When light falls on the metallic surface electrons are emitted and are attracted by the central anode. A current is then established in the circuit. The greater the intensity of light, the more copious is the emission of electrons and hence more current in the circuit. Any variation in the intensity of light produces a corresponding change in the current. A cell which is completely evacuated is called a vacuum cell. Vacuum cells are extremely accurate in their response, the photoelectric current being strictly proportional to the intensity of the light.

In order to increase the current in a cell a small quantity of gas is introduced in it. Such a cell is called a gas-filled cell. The increase in current is due to the process of ionization by collision. In such cells there is always a time lag between the reception of the light and the production of the photo-current. However, the lag is so small that it can be safely used with light of rapidly fluctuating intensity. The gafilled cells with caesium oxide coating are very widely used in industry. For caesium oxide the threshold is far in the red, and therefore the cell has a high red sensitivity. The photo-electric cell has been put to a large number of uses such as microphotometry, talkies and television.

5. The dualistic nature of light. The familiar phenomena of interference, diffraction and polarisation of light lead us to the conclusion that light is a wave motion. On the other hand for the explanation of the phneomena of photoelectricity and the compton effect we have to postulate that light consists of photons each of energy $h\nu$. The dual concept of radiation was a great problem for the physicists. We shall see in Ch.XXIX that there exists a similar dualism in matter.

CHAPTER XXVII
NATURAL RADIOACTIVITY

Discovery of radioactivity. In 1896, Becquerel discovered that salts of uranium emitted spontaneously a radiation which could pass through several thicknesses of matter and affect a photographic plate. It also possessed the property of ionizing the gases. At that time nothing more could be known about the mysterious nature and origin of these rays. They were called the Becquerel rays.

2. Discovery of Radium. Piere and Mme Curie asked the question: "Was the activity confined only to uranium or there were other elements which could emit a similar radiation" ? They started a systematic search for new radioactive elements, and found that a particular sample of mineral known as pitch-blende showed more activity in proportion to its uranium content than ordinary uranium. Obviously, they were led to the conclusion that the extra activity of the mineral must be due to the presence of an unknown element more radioactive than uranium itself. They started with thirty tons of the mineral and went on separating the constituents by applying the methods of group separation used in analytical chemistry. In the end they found that most of the activity was confined to a residue separated with barium. As barium was inactive the obvious conclusion was that the active part must belong to a higher homologue of Ca, Sr and Ba series. The active part, on separation from barium was given the name radium. Out of 30 tons of pitch-blende only 2 milligrams of radium were isolated. Both Mme Curie and her husband were awarded the Nobel prize for their discovery of radium.

3. The general properties of Becquerel rays. Rutheford showed that these rays consist of three distinct types of radiations. known as the α, β and γ rays. Their nature is revealed by a

simple experiment, fig. 27·1. A small sample of radium is placed at the bottom of a small drill-hole made in a block of lead. A narrow pencil of the rays emerges out of the hole. A magnetic field is applied at right angles to the plane of the paper and is directed downwards. The γ-rays remain undeflected. The β-rays are deflected to the right showing that they carry a negative charge. The α-rays are deflected to the left indicating a positive charge.
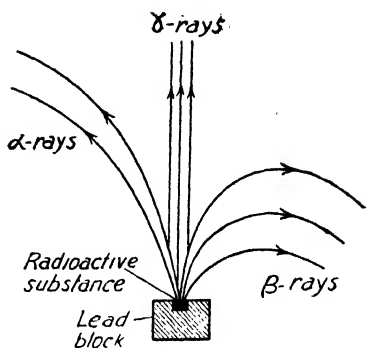
Fig. 27·1

Rutherford by a series of experiments was able to show that α-rays are positively charged particles, each of which has a mass four times that of the hydrogen atom and a positive charge of two electronic units. It is, in fact a nucleus of Helium atom. The β-rays are nothing but electrons. γ-rays turned out to be light waves of very high frequency, higher than that of X-rays. These radiations have the property of penetrating through matter. The α-rays are the least penetrating: they are easily stopped by an aluminium sheet of thickness 0·81 mm. The β-rays are nearly 100 times more penetrating whereas the γ-rays can pass through as much as 30 cms. of iron. They ionize the gases through which they pass.

Radioactive transformations. When a radioactive element of mass number A and charge number Z emits an α-particle of mass 4 and charge $2e$ a new element of mass number A−4 and charge number Z−2 is formed. The new element moves back two places in the periodic table. On the other hand with the emission of a β-particle of charge e, a new element is formed with charge number Z+1, the mass number remaining practically unchanged. In this case the new element moves one place forward in the periodic table. These are the displacement laws of Fajans and Soddy. This process of transformation continues through a succession of

stages, each unstable atom giving rise to another. At each stage
the new element formed is different in chemical and physical
properties from its parent. For example, the sequence of transfor-
mations which occur in uranium is shown in fig 27·2 which is self-
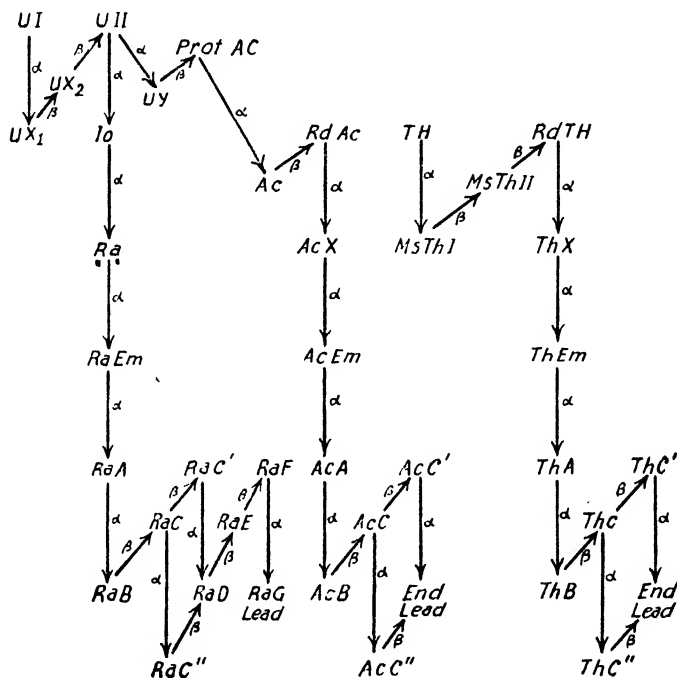
Fig. 27·2

explanatory. The fact that each new element is different in chemi-
cal and physical properties from its parent can but be illustrated by
the example of radium which is the fifth element in the uranium
transformation series. In the pure state the element radium is a
metal with chemical properties resembling those of barium. It
breaks up with the emission of an α-particle and gives rise to a heavy
radioactive gas, the radium emanation, now called *radon*. This
gas is chemically inert and belongs to the family of the inert gases.
The end product of the uranium series, which shows no trace of
activity, is an isotope of lead of mass 206. The elements actinium
and thorium undergo a similar sequence of transformations, fig. 27.2.

The end products of thorium and actinium series are the isotopes of lead of mass 208 and 207 respectively. It is a striking fact that the final product of the transformations of all three series is in each case an isotope of lead.

Rutherford and Soddy's hypothesis of radioactive disintegration. Consider a large number of atoms of radium. Every year a few of them will break but one cannot say which atom will break first. It is entirely a question of chance. If λ denotes the probability of breaking up of a nucleus, then the probability that a selected nucleus will disintegrate during the elementary interval of time dt is λdt. And if there are N atoms the number dN breaking up in time dt is $N\lambda dt$,

$$dN = -N\lambda dt. \qquad \ldots \quad (27\cdot1)$$

(The negative sign is put to indicate that the number is decreasing.). Integrating $(27\cdot1)$ within the limits $t=0$ and $t=t$ we have

$$N = N_0\, e^{-\lambda t}, \qquad \ldots \quad (27\cdot2)$$

where N_0 is the number of atoms at time $t=0$.
The disintegration constant λ is therefore the reciprocal of the time during which the original number of atoms falls to $\dfrac{1}{e}$ of its value. The *half-value* period T is defined as the time in which the number of atoms present is reduced to half its value. Putting $\dfrac{N}{N_0} = \frac{1}{2}$ in eq. $(27\cdot2)$ we have

$$T = \frac{1}{\lambda} \log_e 2 = \frac{0\cdot693}{\lambda}. \qquad \ldots \quad (27\cdot3)$$

The half-value period for Ra is about 1600 years, and for uranium it is 4500×10^6 years, but is only one-millionth of a second for one of the products of radium known as radium C'. It is characteristic of every radioactive body.

The number of atoms which disintegrate between t and $t+dt$ is dN. Each of these atoms has had a life t. The average life τ of the atoms is given by

$$\tau = \frac{1}{N_0} \int_0^\infty dN t$$

$$= \frac{1}{N_o} \int_o^\infty N_o e^{-\lambda t} \, t \, \lambda dt = \frac{1}{\lambda} \cdot \quad . \quad . \quad . \quad (27 \cdot 4)$$

Thus the average life is inversely proportional to the decay constant.

Nature of ionization. The most important property of the radiations from radioactive bodies is to impart electrical conductivity to the gases through which they pass. It is necessary, therefore, to understand the mechanism of the process of ionization. Let us take a parallel plate condenser, whose plates are connected as shown in fig. 5·6. The radioactive material is placed on the lower plate. The voltage on the plates can be varied, and the quadrant electro-meter is used to measure the current.

Mechanism of ionization. The molecules of a gas consist of centres of positive charges, around which electrons move in closed shells. The electrons, in the outermost shell are those which are most loosely bound and can, therefore, be stripped off from the molecule. When a fast moving particle passes through a gas, which is nothing more than a huge assembly of such molecules, it collides with one of the electrons in the shell and imparts a fraction of its energy to the electron. The energy is sufficient to expel it from the molecule. The residual molecule forms the positive ion. At ordinary pressures the expelled electron attaches itself instantaneously to a neutral molecule forming a negative ion of the same mass as the positive ion. At low pressures the expelled electron may exist for some time without colliding with a molecule and becoming attached to it. During this interval it will acquire a huge velocity under the action of an external electric field.

In the absence of an electric field the oppositely charged ions attract one another and recombine to form neutral molecules which do not take part in conduction. The rate of recombination depends on the number of collisions between oppositely charged ions, and this is proportional to the square of the number of ions present. A state of equilibrium is reached when the rate of production of ions by the radiation is equal to the rate of recombination.

Variation of current with voltage. At moderate pressures, as the voltage of the condenser plates is increased the current in the circuit increases and finally reaches a saturation value beyond which there is no increase in current with the increase in voltage. The variation of current with voltage is shown in fig. 27·3. At low voltages the electric
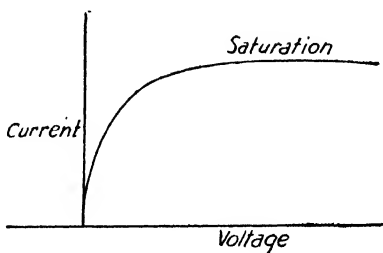
Fig. 27·3

intensity is small and the velocity acquired by the ions is small. Hence most of them recombine on the way. The current is therefore small. The maximum value of the current is obtained when the field is sufficiently strong to remove all the ions before any appreciable recombination has taken place. The saturation current measures the rate of production of the ions and therefore the intensity of the ionizing radiation.

When the pressure is low the graph assumes a different form, fig. 27·4. After a certain voltage any voltage increase increases the current very rapidly and ultimately a spark passes. We have seen that at low pressures a certain time elapses before an electron attaches itself to a neutral molecule. During
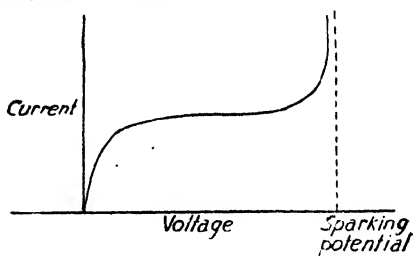
Fig. 27·4

this interval it acquires enormous velocity. The kinetic energy thus acquired by the electron is sufficient to produce more ions and electrons by impact. This process of ion and electron magnification continues until a spark passes. Thus by adjusting the pressure and the voltage the number of ions produced in a gas can be multiplied many thousand times by collision. This affords a very easy method to detect a single ionizing particle.

7. The Geiger-Müller tube counter. On the basis of the above principle Geiger and Müller devised a counter for detecting the individual ionizing particles. The instrument is very simple but of great importance. It consists of a copper cylinder C, open
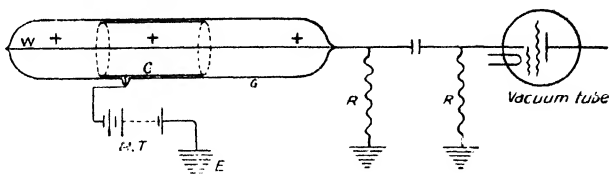
Fig. 27·5 (*a*)

at both the ends. and fitted inside a thin walled glass cylinder G with a fine tungsten wire W stretched along its entire length, fig. 27·5 (*a*). Gases such as air, oxygen or argon are filled inside the cylinder at a pressure of 5 to 15 cms. of Hg. C is connected to the
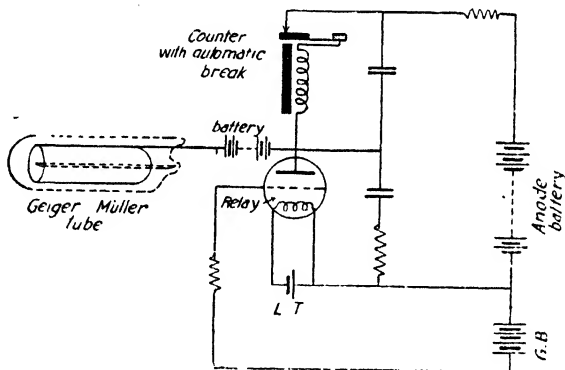
Fig. 27·5 (*b*). Geiger-Müller counter

negative terminal of a source of high potential (700 to 1000 volts) while the positive terminal is connected through a high resistance R (10^8 to 10^{10} ohms) to the central wire. When an ionizing particle passes through the counter, ions are produced and at a suitable pressure and voltage the current shoots up as explained above. Thus the passage of one ionizing particle through the tube momentarily places on the central wire a huge negative charge,

which in turn is impressed on the control grid of a vacuum tube which amplifies the current. The amplified current may be made to operate a counting device. Thus the passage of a particle is recorded automatically. A suitable circuit due to Wynn William is shown in fig. 27·5 (*b*).

8. Wilson chamber. Another very important instrument to detect ionizing particles was devised by C. T. R. Wilson in 1912, and is known as Wilson cloud chamber. The principle on which it is based is very simple. If air saturated with water vapour is expanded suddenly, a mist of water drops is formed, because of sudden adiabatic expansion the temperature falls and the saturated vapour condenses on the dust nuclei present in the air. If the space is saturated and the water vapour is free from dust no condensation takes place. When an ionizing particle passes through saturated air completely free from dust, it produces positively and negatively charged ions in equal number which serve as centres of condensation. The path of the particle is observed on illuminating the space with light.

The Wilson chamber consists of a glass cylinder C, closed at one end by a glass plate and at the other end by a piston P, fig. 27·6. The surface of the piston is wetted with water to keep the air in the chamber saturated with water vapour· An electric field is applied bet-

Fig. 27·6.   Wilson cloud chamber.

ween the metallic ring R and the piston to remove the ions formed. Light from the source S, after being rendered parallel

by a lens L, illuminates the chamber . The piston P is worked by either a mechanical or an electromagnetic device. The photograph of the track of the ionizing particle is taken by the camera M placed above the glass plate.

The importance of this instrument can be judged from the fact that Wilson was awarded the Nobel prize for his discovery. We shall see later on that all artificial transmutations are studied in the Wilson chamber. It is the very soul of cosmic ray research. It is responsible for the discovery of positron and heavy electron. By placing it in a magnetic field one can determine the energy and the mass of the ionizing particle from the curvature and the specific ionization.

Determination of energy by knowing the curvature. The Wilson chamber is placed in a powerful magnetic field. By knowing the strength of the magnetic field and the curvature of the track one can calculate the energy of the particle.

If a particle of rest mass m and charge e e.m.u. moves perpendicularly to a magnetic field of strength H gauss the curvature ρ in cms. is given by

$$\mathrm{H}ev = \sqrt{\frac{m_0 v^2}{1 - \beta^2}}, \qquad \cdots \quad (27.5)$$

where v is the velocity of the particle. From the special theory of relativity we know that mass varies with velocity,

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}, \quad \beta = v/c.$$

The total energy of the particle is given by

$$\mathrm{E} = \frac{m_0 c^2}{\sqrt{1 - \beta^2}}, \qquad \cdots \quad (27.6)$$

from special theory of relativity.

From (27.5) we have

$$\mathrm{H}e = \frac{m_0 v}{\rho} \left(1 - \beta^2\right)^{-\frac{1}{2}} = \frac{m_0 c \beta}{\rho} \left(1 - \beta^2\right)^{-\frac{1}{2}}$$

Squaring the above expression we have

$$(\mathrm{H}\rho e)^2 (1 - \beta^2) = m_0^2 c^2 \beta^2$$

or $\qquad \beta^2 \left\{ m_0^2 c^2 + (\mathrm{H}\rho e)^2 \right\} = (\mathrm{H}\rho e)^2$

Hence $\beta = H\rho \left\{ \dfrac{m_0^2 c^2}{e^2} + (H\rho)^2 \right\}^{-\frac{1}{2}}$ . . . (27·7)

From eqs. (27·6) and (27·7) we have

$$E = \frac{H\rho ce}{\beta} = ec \left\{ \left(\frac{m_0 c}{e}\right)^2 + (H\rho)^2 \right\}^{\frac{1}{2}} \quad \ldots \quad (27\cdot8)$$

Special Case.

For cosmic ray electrons $\dfrac{m_0 c}{e} << H\rho$,

and so eq. (27·4) reduces to a simplified form,

$$E = eH\rho c \text{ ergs} = 300\,H\rho, ev. \quad . \quad . \quad . \quad (27\cdot9)$$

Relation (27·9) is of importance in cosmic ray work.

9. The nature of ⍺-particles. ⍺-particles are corpuscular in nature and carry a positive charge. The value of $\dfrac{e}{m}$ for ⍺-rays has been determined by a method similar to that used for determining $\dfrac{e}{m}$ for cathode rays. The value of $\dfrac{e}{m}$ is 4823 e.m units. They are completely absorbed in a few centimetres of air or by thin foils of matter. When they fall on a phosphorescent zinc sulphide screen they produce scintillations. Each particle produces scintillation and this affords a simple and direct method for counting.

⍺-particle as a helium nucleus. The following experiment proves conclusively that ⍺-particle is a helium nucleus.

T is a thin-walled tube through which ⍺-particles from radium and its disintegration products can pass into a highly evacuated glass bulb G, fig. 27·7. After a few days the gas is compressed in the capillary C. On standing, the ⍺-particle catches two electrons and forms a helium atom. The gas in the capillary on excitation by a discharge between T_1 and T_2 emits a complete helium spectrum.

Fig. 27·7, ⍺-particle as a helium nucleus
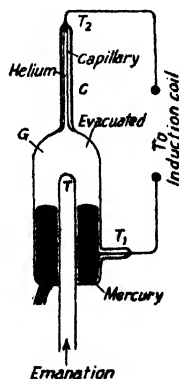
10. The range of ⍺-particles. If we examine the tracks of ⍺-particles from a radioactive substance in a cloud-chamber photogaph, we shall at once notice that all the tracks excepting one or two are nearly of the same length. The maximum distance travelled is called the range of the

α-particle in that gas at a definite temperature and pressure. Thus, the particles from a given substance all have a definite range, characteristic of that substance. Therefore, if we know the range we can identify the substance. The range of α-particles from radium C is 7 cms. (in air temp. 15°C, pressure 76 cms). The α-particle as it travels through air gradually loses its energy by ionization, and finally ends its journey abruptly because beyond this distance the energy has fallen to such a low value that it is insufficient to produce further ionization.

It has been found that the range R of an α-particle and its velocity v are related by the expression

$$v^3 = a R, \qquad \qquad \ldots \quad (27\cdot10)$$

where a is a constant.

Since $\qquad \qquad E = \frac{1}{2}mv^2$, we have

$$E \propto R^{\frac{2}{3}}. \qquad \qquad \ldots \quad (27\cdot11)$$

Another interesting thing which one observes is that some of the α-ray tracks have a fork-like shape. This is due to the fact that when the α-particle passes very near to a nucleus it is repelled by the charge of the nucleus and its course is changed.

11. The Bragg curve. With the help of a shallow ionization chamber Bragg in-vestigated the *specific ionization* along the track of an α-parti-cle. The specific ionization is the number of ions pro-duced by α-particle per unit path. The curve obtained by

Fig 27·8 Bragg curve.

him is shown in fig 27·8. The shape of the curve shows that the specific ionization increases as the particle slows down and finally reaches a maximum value near the end of its path and then suddenly drops to zero. The rate of production of ions is in-versely proportional to the velocity. This can be proved if we

assume, according to Bragg, that the ionization poduced is
proportional to the loss of energy of the α-particle.

Now $$E = AR^{\frac{2}{3}}.$$

Therefore $$\frac{\partial E}{\partial R} = \frac{2}{3} \frac{A}{R^{1/3}} = \frac{B}{v}, \qquad \cdots \quad (27 \cdot 12)$$

where B is some constant.

12. Scattering of α-particles. In his classic experiments
on the scattering of α-particles Rutherford observed that
α-particles are deflected from their straight course when they
pass through sheets of metals. A general explanation of the
above phenomenon was given by Sir J. J. Thomson who supposed
that the deviation of the particles from their straight course must be
due to the electric field of the positively charged centre inside the
atom. By ascribing to the positive core a radius half or one third
of the radius of the atom, he could explain small angle scattering of
α-rays. His theory, however, broke down because it failed comp-
letely to explain the important observation of H. Geiger and E.
Marsden that a small number of α-particles were scattered through
angles as large as 90° or even higher.

Large angle. In Rutherford's experiments α-rays from
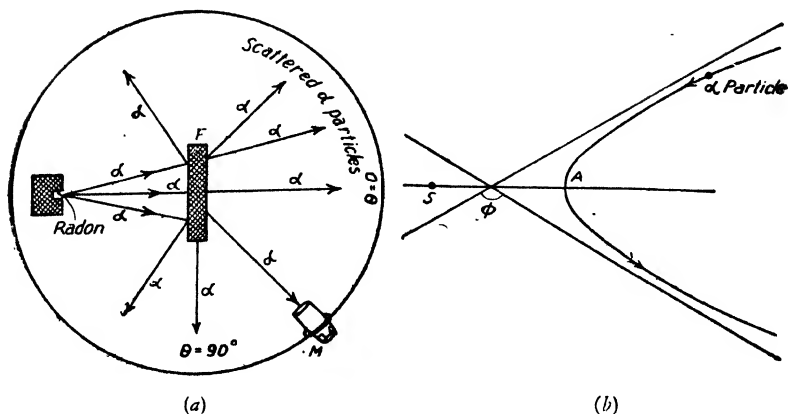
Fig 27·9 Rutherford's α-ray scattering experiment

radon were directed towards a metallic foil F and the individual particles scattered off at different angles θ were observed by a microscope M by the scintillations they produced on a zincsulphide screen, fig 27·9.

Rutherford repeated the scattering experiments with foils of copper, silver and gold. He developed a mathematical theory based on the simple fact that the path of a particle projected with a certain velocity towards a centre of repulsion is a hyperbola. Assuming that the force of repulsion is $\dfrac{2Ze^2}{r^2}$, where Ze is the charge on the positive core and $2e$ that of an α-particle, the distance S A of the nearest approach, fig, 27·9 (b), is given by

$$d = \frac{2\,Ze^2}{m_\chi\,v^2}\left(1 + \operatorname{cosec}\frac{\phi}{2}\right), \qquad \ldots \quad (27\cdot13)$$

where v is the velocity with which an α-particle is projected towards S and m_χ the mass of the particle. For platinum,

$$d = 1\cdot73 \times 10^{-12}\left(1 + \operatorname{cosec}\frac{\phi}{2}\right) \qquad \ldots \quad (27\cdot14)$$

28 E

The mathematical theory as developed by Rutherford was subjected by him to a severe experimental test and was found to hold good. He arrived at the following conclusions : (1) that the whole of the positive charge of an atom is concentrated within a small space of the diameter of the order of 10^{-13} cm., (2) that the entire mass of the atom is confined in this small space and (3) that the amount of positive charge in atomic units is approximately equal to half the atomic weight.

Thus the experiments of Rutherford laid the foundations of the nuclear theory of the atom, which was later on developed by N. Bohr. According to the nuclear theory the positive charge of an atom is concentrated within a small space called the nucleus, and the electrons revolve round it. The atomic core has a radius of the order of 10^{-8} cm.

13. The β-ray spectrum. β-rays consist of electrons emitted by the nucleus during radio-active disintegration. Their energies vary from very low values to several million electron-volts, thus forming an energy spectrum. The nature of the β-ray spectrum is studied by a magnetic spectrograph shown diagrammatically in fig 27·10. S a radioactive

Fig 27·10 β-ray spectrograph

element emitting β-rays is placed in a highly evacuated chamber C. EP is a photographic plate to record the spectrum. The shaded portion between S and the plate is a thick lead block to absorb the β-rays which do not pass through the slit AB. A uniform strong magnetic field is applied normal to the plane of the diagram. The principle of the instrument is based on the fact that all the electrons having the same velocity decsribe circular orbits of the same radiu of curvature ρ and hence come to a focus at the same point on the
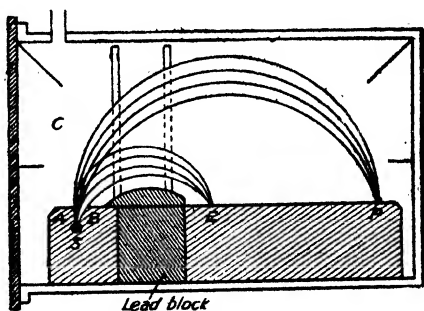
photographic plate. If v denotes the velocity of the electron, its radius of curvature is given by the equation

$$\frac{Hev}{c} = \frac{m_0 \, v^2}{\rho \sqrt{1-\beta^2}}, \qquad \cdot \quad \cdot \quad \cdot \quad (27\cdot15)$$

where m_0 is the rest mass of the electron.

The relativistic expression for the kinetic energy T of the electron is

$$T = m_0 \, c^2 \left(\frac{1}{\sqrt{1-\beta^2}} - 1 \right) \qquad \cdot \quad \cdot \quad \cdot \quad (27\cdot16)$$

From eqs. $(27\cdot15)$ and $(27\cdot16)$ we have

$$H\rho = \frac{1}{c}\sqrt{T^2 + 2m_0 \, c^2 \, T}. \qquad \cdot \quad \cdot \quad \cdot \quad (27\cdot17)$$

Knowing H and ρ the kinetic energy of the β-particles can be determined.

A study of the β-ray spectrum shows that there are sharp lines superimposed on a continuous background. The present explanation is that β-rays of varying velocities are emitted from the nucleus. The continuous background is due to the electrons which originate in the nucleus. When a β-particle has been emitted from the nucleus, it is left in an excited state and a β-ray quantum is emitted as the nucleus falls back to a lower state. The β-ray photon collides with the planetary electrons round the nucleus, say in the K level, ejecting one of them as a secondary β-particle. These secondary β-rays are of fixed energies since they are bound in various shells with definite energies. The line spectrum is due to those secondary electrons. One should not, however, suppose that these electrons are originally present inside the nucleus. They are created somehow the mechanism of which is not yet well understood.

1. Discovery of nuclear disintegration of elements.
Rutherford was engaged for a long time in studying the range of
∝-particles in different gases. His apparatus is shown diagrammati-
cally in fig. 28·1. T is a long glass tube provided with a thin
aluminium foil F at one end and through the other end passes a rod
having at its one end a source R of ∝-particles. The distance of R
from F can be altered. The scintillations produced by the particles
on the fluorescent screen S are observed by the microscope M.
The tube T is first thoroughly evacuated and the desired gas
introduced in it.
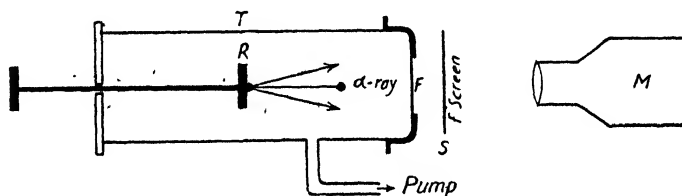
Fig. 28·1

While engaged in these experiments, Rutherford in 1919 made
a great discovery. He found that on introducing nitrogen gas in
the tube T, scintillations could be observed at a distance of 40
cms or more from the source. The maximum range of particles
ever observed from the source fell far too short of this distance.
Obviously the question arose in his mind ; what could be the
cause of these long range particles ? They could not be electrons
or ∝-rays, because they are incapable of producing visible scintill-
ations. He repeated the above experiment in a magnetic field.
The deflections of these long range particles revealed that they
had the mass and charge of protons.

Rutherford was so sure of the results of his experiments that
he at once put forward an explanation. A high speed ∝-particle

may make a " head on " collision with the nitrogen nucleus and be captured. The resulting nucleus being unstable disintegrates into a fast proton. The reaction can be represented as :

$$_2He^4 + _7N^{14} \rightarrow _9F^{18} \rightarrow _8O^{17} + _1H^1 \quad . \quad . \quad . \quad (28 \cdot 1)$$
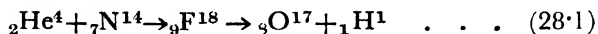
This reaction as studied in the Wilson chamber is shown in fig 28·2.
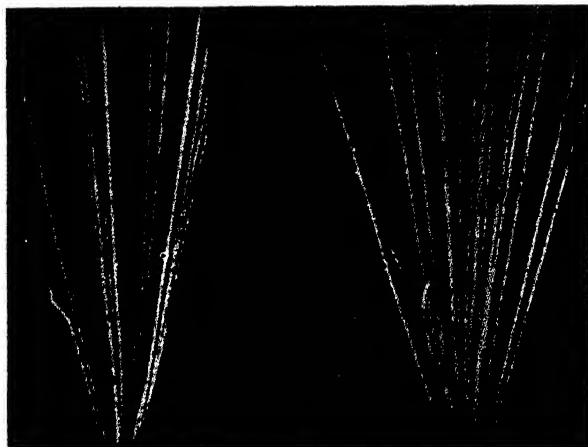
Fig. 28·2 Disintegration of N-nucleus by ∝-particles
[By courtesy of Prof. P. M. S. Blackett.]

Thus when an ∝-particle of mass 4 and charge 2 collides with a nitrogen nucleus of mass 14 and charge 7, an unstable nucleus of fluorine of mass 18 and charge 9 is formed, which instantaneously breaks into an oxygen nucleus of mass 17 and charge 8 and a proton.

This process of producing new stable nuclei out of other stable nuclei is called *artificial transmutation* of elements. Thus Rutherford for the first time realized the dream of Alchemists. Ever since this discovery a large number of elements have been transmuted by scientists in the laboratory. The subject of artificial transmutation forms an important branch of the present day physics.

Another reaction studied with fluorine gas is

$$_9F^{19} + _2He^4 \rightarrow _{10}Ne^{22} + _1H^1 \quad . \quad . \quad . \quad (28 \cdot 2)$$
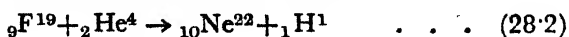
The projectiles used in these reactions are α-particles, protons, neutrons and deuterons. Neutron was discovered by Chadwick and deuteron by Urey.

2. Neutron. In 1932 Bothe and Becker observed that when α-particles from polonium bombarded beryllium a very penetrating radiation came out. They measured its penetrating power by using a G-M counter and came to the conclusion that the radiation consisted of γ-rays. According to them the following transmutation occurred

$$_4Be^9 + _2He^4 \rightarrow {_6}C^{13} + \gamma\text{-ray} \quad . \quad . \quad . \quad (28\cdot3)$$
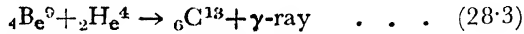
The experiments of Bothe and Becker were repeated by Irene Curie and her husband Joliot. They found that the penetrating radiation, coming out of Be, had a remarkable property of ejecting protons when passed through a paraffin block. Assuming the radiation to be γ-rays they postulated that the γ-rays inside the paraffin block which contains large quantities of hydrogen suffered Compton effect ; thus ejecting out a proton.

Chadwick's experiment. Chadwick was not satisfied with the conclusion of the Joliots. He repeated the experiments and carefully measured the range of protons coming out of the paraffin block. Later he replaced the paraffin block by a small ionization chamber, filled it with nitrogen gas and measured the recoil of the nitrogen nuclei under bombardment by the new radiation. Chadwick's experimental arrangement is indicated in fig. 28·3.
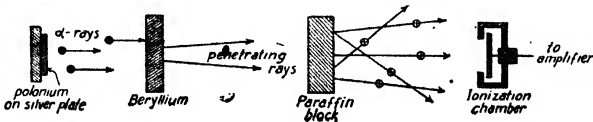
Fig. 28·3 Discovery of neutron by Chadwick

From the theory of the Compton effect we know that the maximum energy of the recoil of a nucleus of mass M_r after scattering of a light quantum of energy $h\nu$ is given by

$$E_r = \frac{2h\nu}{2 + \dfrac{M_r c^2}{h\nu}}, \qquad \bullet \quad \bullet \quad \bullet \quad (28\cdot4)$$

Knowing the range of the protons, E_r was found on calculation to

be 5.7×10^6 ev.   On substituting this value in the above equation we get $h\nu$ to be equal to 55×10^6 ev. From his experiments with the recoil of the nitrogen nuclei, Chadwick found E_r to be 1.6×10^6 ev. On substituting this value as before in equation (28·4) he got $h\nu$ to be $= 11 \times 10^7$ ev. What can be the reason of this discrepancy, he argued?   He postulated that the α-ray hypothesis must be incorrect unless the firmly established laws of conservation of energy and momentum were relinquished.   These laws were found to hold good in all nuclear reactions; and must also hold good in the reactions of Joliots, and if they do the γ-ray hypothesis must be abandoned.   Moreover it appears highly improbable to derive such high energies from light nuclei like Be.   It was found that these radiations did not produce any tracks in a Wilson cloud chamber.   Chadwick, convinced of his results, suggested that the penetrating radiations were not α-rays but material particles carrying no electric charge.   These are now called *neutrons*.   The penetrating rays are both γ-rays and neutrons.   The true reaction is therefore

$$_2\mathrm{He}^4 + _4\mathrm{Be}^9 \rightarrow _6\mathrm{C}^{12} + _0 n^1 + \gamma\text{-rays} \quad . \quad . \quad (28\cdot5)$$
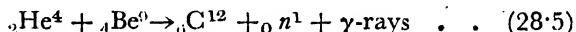
Chadwick was awarded the Nobel prize for his discovery of the neutron.

3.   Mass of the neutron.   From the cloud-chamber experiments the velocity of the recoil nitrogen nuclei set in motion by the neutrons was found to be 4.7×10^8 cms/sec. For protons the value is 3.3×10^9 cms/sec.   Applying the laws of conservation of momentum and energy the mass of the neutron can be determined. For a head on collision, applying the laws of conservation of momentum and energy, we have

$$\mathrm{M}_n v = \mathrm{M}_n v' + \mathrm{M}_p v_p, \qquad . \quad . \quad (28\cdot6)$$

where M_n is the mass of the neutron, v, the initial velocity of the neutron, v' the final velocity of the neutron, v_p the velocity and M_p the mass of the proton.

Also, $\qquad \tfrac{1}{2}\mathrm{M}_n v^2 = \tfrac{1}{2}\mathrm{M}_n v'^2 - \tfrac{1}{2}\mathrm{M}_p v_p^2 \qquad . \quad . \quad (28\cdot7)$

From eqs. (28·6) and (28·7) we have

$$v_p = \frac{2\,\mathrm{M}_n}{\mathrm{M}_n + \mathrm{M}_p} v \qquad . \quad . \quad (28\cdot8)$$

Similarly, for the nitrogen nuclei

$$v_N = \frac{2 M_n}{M_n + M_N} v, \qquad \qquad . \quad . \quad (28 \cdot 9)$$

where M_N is the mass of the nitrogen nucleus.

From eqs. $(28 \cdot 8)$ and $(28 \cdot 9)$ we have

$$\frac{v_p}{v_N} = \frac{M_n + M_N}{M_n + M_p} = \frac{3 \cdot 3 \times 10^9}{4 \cdot 7 \times 10^8}, \qquad . \quad . \quad . \quad (28 \cdot 10)$$

Since $M_p = 1 \cdot 008$ mass units and $M_N = 14$ mass units.

Hence $\qquad M_n = 1 \cdot 15$ mass units approx. $\qquad . \quad . \quad . \quad (28 \cdot 11)$

The present value of the mass is $1 \cdot 00893$. A more accurate determination of the mass of the neutron is from a reaction by Chadwick and Goldhaber.

$$_1H^2 + h\nu \rightarrow {}_0n^1 + {}_1H^1 + Q. \qquad . \quad , \quad . \quad (28 \cdot 12)$$

Intense γ-rays from Th.c" disintegrate a deuteron into a proton and a neutron. Since a neutron and a proton have nearly the same mass, their kinetic energies after bombardment will have the same value. The kinetic energy of the protons is known from its track in the Wilson chamber; and was found to be $0 \cdot 24 \times 10^6 ev$., γ-rays from Thc" have energy equal to $2 \cdot 6 \times 10^6 ev$. The reaction energy, Q is the difference between the kinetic energies of the products on the two sides of eq. $(28 \cdot 12)$. Since $_1H^2$ is at rest,

$$Q = 2 \cdot 6 \times 10^6 - 2 \times 0 \cdot 24 \times 10^6 \text{ ev.}$$
$$= 2 \cdot 1 \times 10^6 ev = 0 \cdot 0023 \text{ mass units}$$

Hence $\qquad {}_0n^1 = {}_1H^2 + 0 \cdot 0023 - {}_1H^1$
$$= 2 \cdot 0147 + 0 \cdot 0023 - 1 \cdot 0081$$
$$= 1 \cdot 0089 \qquad \qquad . \quad . \quad (28 \cdot 13)$$

The mass of the neutron is, therefore, $1 \cdot 0089$ mass units.

Since neutrons have no charge they leave no tracks in a cloud chamber. On account of their neutral character they experience no force of repulsion when thrown towards a nucleus. They are more efficient in producing artificial transmutations than any other particle.

Equivalence of mass and energy. According to Einstein's special theory of relativity mass and energy are equivalent. His mass-energy equation is

$$E = Mc^2, \qquad \qquad . \quad . \quad (28 \cdot 14)$$

where m is the mass, c is the velocity of light and E the energy equivalence of the mass. Therefore mass and energy are inter-convertible. If a mass m is annihilated a definite amount of energy E would be available. The amount of energy available from 1 gm. of matter is

$E=1 \times (3 \times 10^{10})^2$ ergs $= 9 \times 10^{20}$ ergs $= 2 \cdot 1 \times 10^{13}$ calories.

The magnitude of the energies involved are enormous. The energy liberated in the annihilation of one atomic mass unit, that is, the energy equivalent to $\frac{1}{16}$ of the mass of an ordinary oxygen atom is

$$E = 1 \cdot 6609 \times 10^{-24} \times 9 \times 10^{20} \text{ ergs},$$
$$= 1 \cdot 6609 \times 9 \times 10^{-4} \times 6285 \cdot 10^{12} \text{ ev},$$
$$= 931 \times 10^6 \text{ ev}, \qquad \qquad . \quad . \quad (28 \cdot 15)$$

The reader must commit this number to memory because it is frequently used in calculating the energies in nuclear reactions.

4. Conservation of energy in nuclear disintegration. The laws of conservation of energy and momentum are so fundamental in nature that they must hold good everywhere. They do so in nuclear disintegrations. The total energy before impact must be equal to the total energy after impact. For an illustration let us take the reaction first studied by Rutherford.

$$_2He^4 + {_7N^{14}} + E_1 = {_8O^{17}} + {_1H^1} + E_2, \quad . \quad . \quad (28 \cdot 16)$$
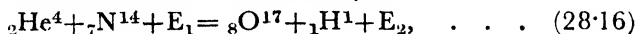
where E_1 is the total kinetic energy of the nuclei before disinteg-ration and E_2 that after disintegration.

The kinetic energy of α-particles from Ra C' is 7·7 Mev or 0·0083 mass unit. Total energy on the left of eq. (28·16) is

$$_2He^4 = 4 \cdot 0040.$$
$$_7N^{14} = 14 \cdot 0075.$$
$$E_1 = 0 \cdot 0083.$$

$$\overline{\text{Total} = 18 \cdot 0198.}$$

and that on the right

$$_8O^{17} = 17 \cdot 0045$$
$$_1H^1 = 1 \cdot 0081$$
$$E_2 = 0 \cdot 0072$$

$$\overline{\text{Total} = 18 \cdot 0198}$$

Thus we see that the energy on the left is equal to the energy on the right. This is true in all the reactions. The nuclear reactions thus prove the validity of Einstein's fundamental relation $E = mc^2$.

5. Transmutation of lithium by protons.

In 1932, C o c k c r o f t and Walton succeeded for the first time in disintegrating lithium atoms with p r o t o n s accelerated by a high voltage generator at the Caven-

Fig. 28·4 (a)

dish laboratory. The principle of their experimental arrangement is shown diagrammatically in fig. 28·4 (a) and the result in fig. 28·4 (b)
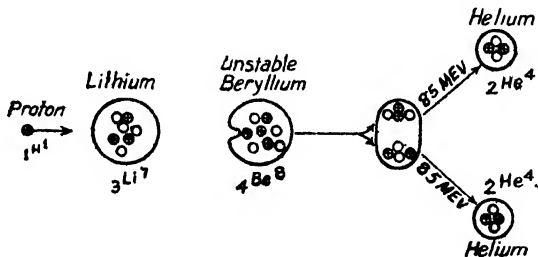
Fig. 28·4 (b) Cock croft—Walton experiment.

Electrons from the hot filament F ionize the hydrogen atoms. The protons thus formed are accelerated by a potential of 150,000 volts. The fast proton then impinges on a lithium target producing two ⍺-particles. The reaction is

$$_3L^7 + {}_1H^1 + E_1 = 2\,{}_2He^4 + E_2 \quad . \quad . \quad (28·17)$$
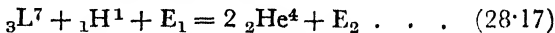
Knowing the range of the tracks of ⍺-particles, the energy E_2 can be calculated. It was found to be 17 Mev.

$$_3Li^7 = 7·0182.$$
$$_1H^1 = 1·0081.$$
$$E_1 = 0·0002.$$

$$\text{Total} = 8·0265.$$

Hence $E_2 = 8 \cdot 0265 - 8 \cdot 0078 = 0 \cdot 0187$ mass units

$$= 17 \cdot 4 \text{ Mev.}$$

The calculated value is therefore in good agreement with the experimentally determined value of $17 \cdot 0$ Mev. In these experiments it was found that the yield of α-particles increased with the energy of the incident protons.

6. Disintegration by protons.

$$_3\text{Li}^7 + {}_1\text{H}^1 \rightarrow {}_4\text{Be}^8 + \gamma\text{-rays.} \qquad (28 \cdot 18)$$
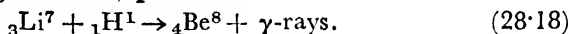
Here a proton is simply captured by a lithium nucleus forming beryllium which emits γ-rays. The γ-ray intensity is maximum for protons of energies $0 \cdot 40$ and $0 \cdot 48$ Mev showing that there is a resonance capture.

Another interesting reaction is with boron

$$_5\text{B}^{11} + {}_1\text{H}^1 \rightarrow {}_4\text{Be}^8 + {}_2\text{He}^4 \rightarrow 3{}_2\text{He}^4 \qquad (28 \cdot 19)$$

the beryllium nucleus is in an excited state and breaks to give three α-particles. This is a case of multiple reaction.

7. The Cyclotron.

E. O. Lawrence, an American physicist, devised for the first time an " atomic machine gun" to produce high energy particles. The importance of the machine lay in the fact that it could produce very high energy particles without using very high voltages. The principle of this machine is illustrated in figs. $28 \cdot 5$ (a) and (b). The " Dees " (D_1 and D_2) consist of a

(a)   (b)
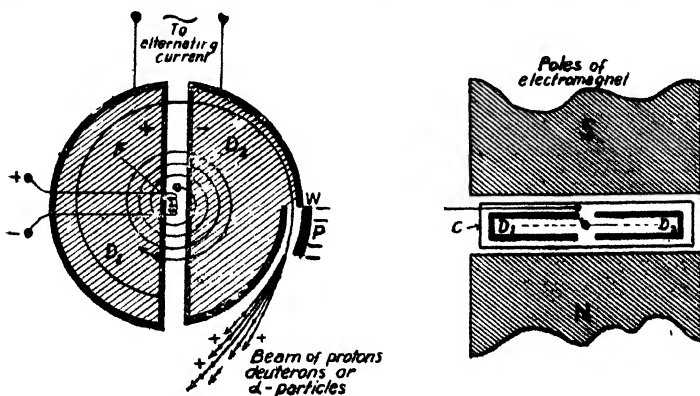
Fig. $28 \cdot 5$. Principle of cyclotron

pill-box shaped copper chamber split into two parts. They are placed in an evacuated chamber C, between the poles of a powerful electro-magnet, and are connected to a high frequency alternating current generator(short wave radio transmitter). Electrons coming from a heated filament F ionize the gas atoms inside the chamber. When the gas is hydrogen, protons are produced. In order to understand the principle of the machine clearly, let us suppose that at any instant D_1 is charged positively and D_2 negatively. The protons will be accelarated towards D_2, but being at the same time under the action of a magnetic field will be bent in a circular path. As soon as the proton completes its first semi-circular journey the potential on the D's is reversed so that D_1 now becomes negatively charged and D_2 positively charged. The protons will be now attracted by D_1 and its kinetic energy will be increased by an equal amount. As the proton now possesses a higher velocity it moves in a semi circle of greater radius. At each reversal of the potential on the dees the proton is speeded up and moves in circles of larger and larger radii until it reaches near the plate P where it emerges in the form of a powerful proton beam. The path of the proton is a spiral. The main principle which makes cyclotron work is the fact that the time taken by a charged particle to make a half revolution within the Dees is independent of its velocity. The strength of the magnetic field and the frequency of the oscillator are so adjusted that the time taken by the ions to travel half the circle is equal to half the period of the oscillator. That is why cyclotron is also known as *magnetic resonance accelerator.*

The time t for the ions to complete a semi-circle of radius ρ is

$$t = \frac{\pi \rho}{v}, \qquad \qquad \text{. . . (28·20)}$$

where v is the velocity of the ion.

Also $$\frac{mv^2}{\rho} = Hev. \qquad \qquad \text{. . . (28·21)}$$

From the above two equations we have

$$t = \frac{\pi}{H} \times \frac{m}{e}, \qquad \qquad \text{. . . (28·22)}$$

where H is the strength of the magnetic field in gauss, m the mass of the ion and e its charge in e.m.u.   The angular velocity ω of the ion is given by

$$\omega = \frac{v}{\rho} = \frac{He}{m} \qquad \qquad \ldots \quad (28\cdot23)$$

which is a constant for a given ion and field strength.   By changing H, t can be changed.   For resonance $2t$ must be equal to the period of the oscillator.   If f is the frequency of the oscillator, then

$$f = \frac{1}{2t} = \frac{He}{2\pi m} . \qquad \ldots \quad (28\cdot24)$$

Also $\qquad\qquad eV \times 10^8 = \frac{1}{2} mv^2, \qquad \ldots \quad (28\cdot25)$

where V is the potential on the D's.   Substituting the value of v from (28·21) in (28·25)

$$V = \frac{e}{m} \times \frac{(H\rho)^2}{2 \times 10^8}. \qquad \ldots \quad (28\cdot26)$$

From eq. (28·26) we see that it is possible to increase the energy of the emergent beam by increasing the field strength. But an upper limit to H is set by eq.(28·24) according to which the frequency of the oscillator has to be increased with the increase in H for resonance.

It is still possible to increase the energy by increasing ρ i.e., by increasing the dimensions of the cyclotron. There is therefore a tendency these days towards the construction of cyclotrons of large dimensions. At Berkley 60 inch cyclotron, the largest in the world has been constructed. With this cyclotron, deuterons, having final energy equal to 19 million electron volts have been obtained. A cyclotron assembly in the University of California is shown in fig. 28·5 (c).

Applications of cyclotron. It is a most powerful source of supply of high energy deuterons, protons, α-particles and neutrons. Apart from its academic value in the study of nuclear disintegra-

tions it has been extensively used in the manufacture of radio-sodium,
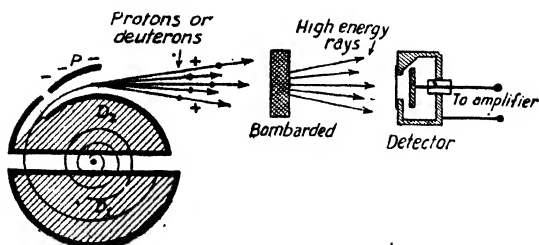
Fig. 28·6

radio-phosphorous and radio-iron. The last three have played
a very important part in stimulating research. Radio-iron
has been used in studying iron metabolism in relation to anaemia ;
and radio-phosphorous has been used in studying exchanges of
phosphorus in plants and on phosphorus metabolism. Radio-
sodium is used in modern hospitals for treating cancer patients.

Cyclotron has been used to provide a prolific source of
neutrons. A powerful beam of deuterons, fig. 28·6, from cyclo-
tron is directed towards a beryllium target which then emits
neutrons,

$$_4Be^9 + _1H^2 \rightarrow _5B^{10} + _0n^1 + Q, \quad . \; . \; , \; . \; . \; . \; . \quad (28·27)$$
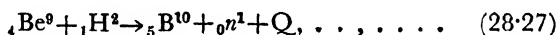
where Q denotes the energy available from the loss in mass alone.
Using the Berkley cyclotron, the energy of the bombarding de-
uteron is 19 Mev. Therefore the total available energy becomes
23 Mev, 1 M ev going to the recoil boron nucleus and approxi-
mately 22 Mev to the neutron.

Recently Alvarez has used the Berkley Cyclotron to produce
a beam of monochromatic neutrons. As a very powerful neutron
source, the cyclotron has been used in many of the recent investi-
gations in the fission of the uranium nucleus.

8. Deuteron transmutations. He

$$\left. \begin{array}{l} _7N^{14} + _1H^2 \rightarrow _6C^{12} + _2He^4 \\ _8O^{16} + _1H^2 \rightarrow _7N^{14} + _2He^4 \\ _3Li^6 + _1H^2 \rightarrow _3Li^7 + _1H^1 \\ _3Li^6 + _1H^2 \rightarrow 2 _2He^4 \\ _1H^2 + _1H^2 \rightarrow _2He^3 + _0n^1 \end{array} \right\} \qquad (28·28)$$
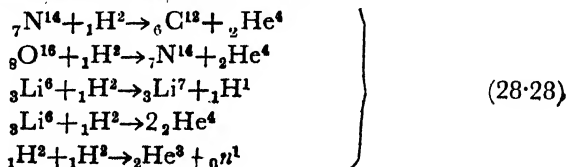
9. Photo-disintegration.

$$_1H^2 + \gamma\text{-rays} \rightarrow {}_1H^1 + {}_0n^1 \qquad \Big\} \quad , \quad \cdot \cdot \; (28\cdot29)$$
$$_4Be^9 + \gamma\text{-rays} \rightarrow {}_4Be^8 + {}_0n^1$$
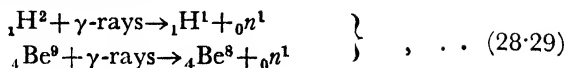
10. Transmutations by fast neutrons.

The first evidence of nuclear disintegration by fast neutrons was obtained by Feather in 1932. He allowed a beam of neutrons from beryllium to enter a Wilson-cloud chamber containing pure nitrogen gas. He observed a heavy track of the recoil nucleus and a long track of a fast proton, both originating at the same point. From energy and momentum considerations he concluded that the reaction taking place was.

$$_7N^{14} + {}_0n^1 \rightarrow {}_6C^{14} + {}_1H^1 \qquad \cdot \; \cdot \; \cdot \quad (28\cdot30)$$
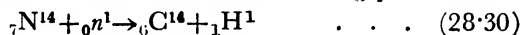
Examples of other neutron disintegrations are :

$$\left. \begin{array}{l} _9F^{19} + {}_0n^1 \rightarrow {}_7N^{16} + {}_2He^4 \\ _{10}Ne^{2J} + {}_0n^1 \rightarrow {}_8O^{17} + {}_2He^4 \\ _0n^1 + {}_{13}Al^{27} \rightarrow {}_{11}Na^{24*} + {}_2He^4 \\ \text{and} \quad {}_6C^{12} + {}_0n^1 \rightarrow {}_4Be^9 + {}_2He^4 \end{array} \right\} \qquad (28\cdot31)$$
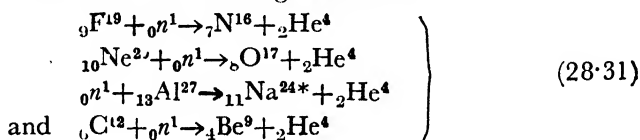
* radio—sodium.

11. Slow neutrons.

In 1934 Fermi made a very remarkable observation that the activity induced by fast neutrons in silver, cadmium and rhodium, etc. was enhanced when the neutron source was surrounded by very thick paraffin blocks or materials which are rich in hydrogen. Fermi assumed that on account of elastic collisions with hydrogen nuclei the neutrons were continually slowed down until at a distance of several centimetres from the source most of them have lost all their energy. Whatever energy they possess is the energy of thermal collision with atoms and molecules of the gas.

The slow neutrons are very efficient in producing disintegrations. A neutron experiences no repulsive force when it approaches a nucleus, and hence its chance of penetration into, and of being captured by a nucleus is relatively large.

Some of the slow neutron reactions are given below :

$$\left. \begin{array}{l} _5B^{10} + {}_0n^1 \rightarrow {}_3Li^7 + {}_2He^4 \\ _3Li^6 + {}_0n^1 \rightarrow {}_2He^4 + {}_1H^3 \\ _7N^{14} + {}_0n^1 \rightarrow {}_6C^{14} + {}_1H^1 \\ _1H^1 + {}_0n^1 \rightarrow {}_1H^2 + \gamma \end{array} \right\} \qquad \cdot \; \cdot \; \cdot \quad (28\cdot32)$$
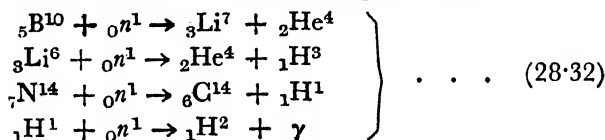
The last reaction is of importance. It is a case of a simple capture of neutron. The energy of the γ-ray photon is equal to the binding energy of the deuteron($_1H^2$), the energy of the slow neutrons being negligibly small. Let us find out the binding energy of the deuteron :

$$n_0{}^1 = 1{\cdot}0090$$
$$_1H^1 = 1{\cdot}0081$$
$$\overline{\text{Total} = 2{\cdot}0171}$$
$$_1H^2 = 2{\cdot}0147$$

$$\text{Difference} = 0{\cdot}0024 \quad \text{mass units.}$$

Therefore, the difference in mass is 0·0024 mas unit which is equal to 2 2 Mev energy. The energy of the γ-ray photon is therefore 2·2 Mev The confirmation of the fact that the binding energy of deuteron is 2.2 Mev comes from another evidence. It has been found that γ-rays of energy less than 2.2 Mev are not able to break the deuteron, while those with energy greater than this can easily break it.

✓ **12. Structure of the nucleus. Introduction.** A nucleus consists of neutrons and protons. The total number of neutrons and protons is equal to the atomic weight of nucleus or more exactly to its mass number A *i.e.*, the integer nearest to the atomic weight. The number of protons is equal to the nuclear charge Z. The number of neutrons is

$$N = A - Z.$$

It is certain that there are no electrons inside the nucleus.

A deuteron nucleus conists of one neutron and one proton. In an oxygen nucleus $_8O^{16}$ there are eight neutrons and eight protons. How is it that these eight protons having similar charges do not fly apart? On the other hand they remain cemented inside the nucleus. Not only there is a force of attraction between protons and protons inside the nucleus but there is a stronger attraction between neutrons and protons and also a force of attraction between neutrons. Here is a force coming into operation which does not fit in with the classical ideas. Before we discuss the nature of these strange forces let us estimate the energy with which the particles are bound inside the nucleus.

Nuclear binding energies. Take for example a deuteron.

$$\text{Neutron mass } _0n^1 = 1\cdot0090$$
$$\text{Proton mass } _1H^1 = 1\cdot0081$$
$$_0n^1 + {}_1H^1 = 2\cdot0171$$
$$\text{Deuteron mass } _1H^2 = 2\cdot0147$$

$$\text{Difference} = 0\cdot0024 \text{ mass unit}$$
$$= 2\cdot2 \text{ Mev.}$$

We thus see that the combined mass of neutron and proton inside the nucleus is less than the combined mass of neutron and proton when outside. This difference of mass is the energy with which the particles are bound inside the deuteron nucleus. To break a deuteron nucleus into a proton and a neutron will therefore involve an expenditure of energy equal to 2.2 Mev. This has actually been experimentally verified by bombarding deuteron with γ-rays of energy 2.2 Mev. Again for example, take the helium nucleus $_2He^4$.

$$2_0n^1 + 2_1H^1 = 4\cdot0342$$
$$_2He^4 = 4\cdot0039$$
$$\text{Difference} = 0\cdot0303 \text{ mass unit}$$
$$= 28 \text{ Mev.}$$

Or the binding energy per particle inside a helium nucleus is 7 Mev. This is the reason that α-particles are very stable.
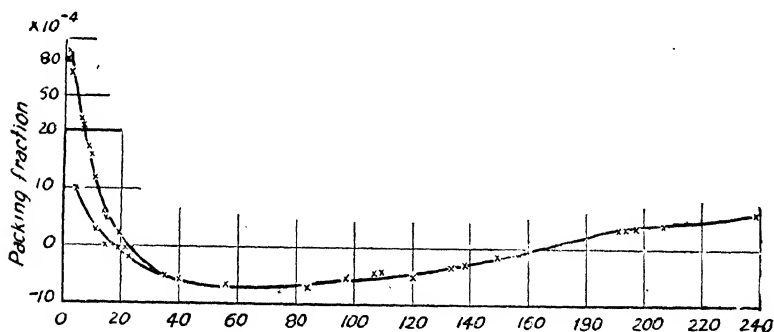
Fig. 28·7 Packing fractions.

A curve of packing fractions as a function of mass number A is shown in fig. 28·7. Below A=30 two curves are drawn, since

29 E C

there is a tendency for the packing fraction to be larger when the atomic number is odd than when it is even. The packing fraction decreases up to about $A=60$ after which it increases again. This means that the binding energy per particle inside the nucleus increases up to $A=60$ after which it slowly decreases.

Nature of nuclear forces. The existence of deuteron proves conclusively that an attraction of some sort exists between a neutron and a proton. In a heavy nucleus this neutron-proton force must, obviously, be greater than the electrostatic repulsion of the protons. There is also a force of attraction between a pair of neutrons and a pair of protons ; but this force is much weaker than the force between a neutron and a proton.

If every particle in the nucleus is supposed to interact with every other, the interaction energy, and therefore the binding energy would be roughly proportional to the number of interacting pairs, *i.e.* to the square of the number of particles in the nucleus. But it is found experimentally that the binding energies of nuclei increase only linearly with increasing number of particles. This fact shows that nuclear forces are similar to chemical forces, where the total chemical binding energy is proportional to the number of particles present.

The nature of nuclear forces is very similar to the homopolar binding in chemical valence. Two hydrogen atoms attract each other very strongly ; the binding, is however, not electrostatic in nature. The binding between two hydrogen atoms is brought about by the exchange of the two electrons of the two hydrogen atoms. The forces resulting from this exchange are called exchange forces, the theory of which was given by Heitler and London. The H_2 molecule is a saturated molecule since no third H-atom would attach to H_2 molecule. An assembly of many hydrogen atoms e.g. a drop of liquid hydrogen, has therefore an energy approximately equal to that of the corresponding number of hydrogen molecules and therefore proportional to the number of atoms present. The binding energy of a hydrogen droplet will, however, be slightly greater than that of the separated molecules because there exists a very weak binding called the Van der Waal binding between H_f molecules.

Since the binding energy of a nucleus is proportional to the number of particles inside the nucleus, we assume from an analogy with homopolar forces, that nuclear forces show saturation. An α-particle corresponds to a saturated molecule, the binding energy of which is 28 Mev, or 7 Mev per elementary particle. The binding energy of nuclei which have the highest packing fractions, viz., those with Z round 30, is $8\frac{1}{2}$ Mev per elementary particle. In an analogy this means that 7 Mev of these $8\frac{1}{2}$ are due to the "chemical binding energy of the molecule", $_2He^4$, while the remaining $1\frac{1}{2}$ Mev are to be attributed to Van der Waal binding between He particles.

If the force between a neutron and a proton is an exchange force; what is that which exchanges? An electron passes from neutron to proton so that the former neutron is transformed into a proton and the former proton into a neutron. This process can be considered as an exchange of the coordinates of neutron and proton. Thus our "exchange forces" mean that neutron and proton interchange their position when they interact. Together with electron a neutrino must pass over to conserve the momentum. The passage is of course, not meant literally in the sense that the electron and neutrino are bound in the neutron and then pass bodily to the proton. We might assume that electron and neutrino are just "created" for a very short time, and are then reabsorbed.

The present view is however, that the exchange is not due to an electron but due to a meson.

The nuclear potential barrier. Suppose that a proton is approaching a nucleus. As the particle approaches closer and closer the coulomb force of repulsion becomes greater and greater. Graphically the increasing repulsion is represented in fig. 28·8 by the solid curve. The force of repulsion cannot continue to increase all the way to zero separation, since, as we have seen before, for very small separation there is a
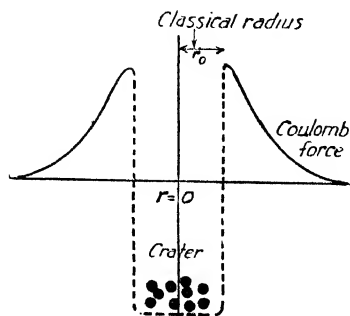
Fig 28·8. Potential barrier.

strong attraction between neutrons and protons; hence the force of repulsion must eventually give place to a force of attraction. Gamow proposed therefore that at close approach the coulomb law breaks down and the curve turns over and drops almost straight down as shown by the dotted curve in the figure. Such a curve is called the *potential barrier*. The peak of the barrier corresponds to the classical radius of the nucleus $r_0 \approx 9 \times 10^{-13}$ cm. A charged particle before it can penetrate the nucleus has to have enough energy to cross the peak. An interesting question arises in this connection : α-particle coming out of the radioactive Uranium nucleus has an energy 6.6×10^{-5} erg, but the height of the potential barrier of the Uranium nucleus is certainly greater than 12×10^{-6} ergs. It appears difficult to imagine how such an α-particle is able to escape from the nucleus. It must, rather remain for ever inside the nucleus, since it has not sufficient energy to surmount the barrier. This apparent difficulty disappears if we assume the wave aspect of the α-particle. Gamow, has shown from wave mechanics that there is always a finite probability for the leakage of the α-particle through the barrier. This effect is known as *tunnelling effect* in quantum mechanics.

Bohr's liquid drop model of the nucleus. In 1937 Niels Bohr proposed that a nucleus should be considered as a drop of liquid rather than as a gas because the interaction between the nuclear particles is large and therefore the fluctuations of the density very improbable. The motions of neutrons and protons inside the nucleus is analogous to that of the molecules in a liquid drop. The nuclear forces of short range, which are analogous to the cohesive forces between molecules in a drop of liquid must tend to produce similar effects of surface tension in nuclei. The short range forces try to keep the nuclear matter together, whereas the electrostatic forces of repulsion between protons try to annul the effect of short range forces. As the nuclear charge increases the coulomb force of repulsion increases but the surface tension diminishes.

More violent the motion of the nuclear particles the greater will be the *temperature* of the nuclear matter in analogy with the liquid

drop. To explain disintegration, analogy is drawn that the ejection of a particle from the nucleus is like the evaporation of a liquid molecule from a drop of liquid. Since a rise in temperature brings about a more rapid evaporation of the liquid, so an increase in the motions within the nucleus gives rise to a higher probability of disintegration.

The mechanism of nuclear disintegration can now be easily understood as the liquid drop model. In a stable nucleus, the particles within are moving with very little kinetic energy corresponding to a low temperature state. When a fast moving particle from outside penetrates the potential barrier it is accelerated towards the centre of the nucleus and acquires a very high kinetic energy before it collides with its new friends inside the nucleus. Soon the energy becomes divided between the many particles and the nucleus takes on a higher temperature state which we may call the compound state of the nucleus. Now as the particles move about inside there is always a probability that some particle will be hit by several particles, giving it enough energy to escape through the potential barrier. After the ejection of the particle the nucleus drops down to a lower temperature state.

13. Energy generation in stars. Nuclear reactions are not only a special feature of terrestrial laboratories, but rather they are going on with far greater efficiency in cosmic laboratories such as the interior of the stars. It is these reactions which keep our sun, one of the many stars, shining and thus keep us alive.

The sun is radiating energy at the rate of 4×10^{33} ergs/sec. and has been doing so for 2×10^9 yrs. The sun is a great spendthrift. Any theory concerning the stellar energy should be able to explain, firstly, the enormous rate of energy production, and secondly, the long period of time over which the sun is known to have been emitting energy.

Helmholtz thought that the energy production in the sun is due to its gravitational contraction. Even if the sun started with an infinite diameter the energy set free during the contraction to its present size would only be sufficient for 3×10^7 years at the

present rate of radiation. Hence, we are forced to give up Helmholtz contraction hypothesis. In the year 1938 Bethe first explained the energy production in the sun on the hypothesis of nuclear reactions.

We have seen before that in the laboratory nuclear transmutations are produced by accelerating particles such as protons to high energies with the help of a cyclotron. A proton of about 0·8 microerg would be able to penetrate a carbon nucleus. However out of a million accelerated protons there will still be only one which will make a successful hit ; the others will be slowed down by collisions with atoms without producing transmutations. The number of particles which can be accelerated in terrestrial laboratories is too small. In the big cyclotron of Dr. E. O. Lawrence at Berkeley only 0·1 milligram of protons goes through the machine in a whole days work. Thus the nuclear reactions in our laboratories are not efficient.

In stars we have just the opposite case. It has been shown by Eddington that the temperature at the centre of the sun is $1·9 \times 10^{7}$ ° and density 80 gms./cm.³ At such a high temperature in the interior all the protons have high kinetic energies $(\approx \kappa T)$. Assuming 35% hydrogen by weight in the sun, there are a billion billion tons of fast protons. These protons will have such high energies all the time, they are not slowed down by collisions with other atoms because all have equally high energies. Therefore the efficiency of the energy production will be much better, every nuclear transmutation that occurs gives a net gain of energy because it is no longer necessary to shoot a million projectiles at the target in order to have one hit.

Since the protons and the reacting nuclei are positively charged there will be a force of repulsion between them. It has been shown by Gamow that the probability per collision for a particle of charge $Z_1 e$ to penetrate the potential barrier of a target nucleus of charge $Z_2 e$ is given by

$$W = e^{\displaystyle -2^{3/2} \times (M)^{\frac{1}{2}} \frac{\pi^2 Z_1 Z_2 e^2}{h E^{\frac{1}{2}}}} , \qquad \cdot \quad \cdot \quad \cdot \quad (28.33)$$

where M is the reduced mass $\left(= \dfrac{M_1 M_2}{M_1 + M_2} \right)$ and E the relative kinetic energy of the colliding particles. Eq. 28·33 is of great importance. We see that the probability diminishes exponentially as the charge on the reacting nuclei increases but it increases with the increase of the kinetic energy E. A reaction between hydrogen and lithium is much more probable than a reaction between helium and lithium. In the table below we give the average life time of some elements in contact with hydrogen at a temperature of 2×10^7 °C.

TABLE

Element	$_2He^4$	$_4Be^9$	$_6C^{12}$	$_8O^{16}$	$_{14}Si^{30}$	Pb	$_1H^2$
Average life	6 days	15 mins	$2·5 \times 10^6$ yrs.	10^{12} yrs.	3×10^{20} yrs.	10^{60} yrs.	2 sec.

From an inspection of the above table we see that only the lightest elements may be appreciably transformed at this temperature during a time equal to the estimated present age of the stars ($\sim 10^9$ yrs). If instead of hydrogen we take helium as the bombarding particle, we get extremely large values of the life time for even the lightest elements. Hence we can discard helium and any other element heavier than hydrogen as the possible missile for the nuclear reactions. After much consideration Bethe hit upon the carbon-nitrogen chain reaction which gives the correct energy evolution in the sun.

C. N. Chain reaction of Bethe. In the carbon nitrogen reaction proposed by Bethe, carbon and nitrogen merely serve as catalysts for the combination of four protons and two electrons into a helium nucleus. His reactions are :

$$_6C^{12} + {}_1H^1 \rightarrow {}_7N^{13} + \gamma \text{ rays.}$$

$$_7N^{13} \rightarrow {}_6C^{13} + e^+ \quad \text{(positron)}$$

$$_6C^{13} + {}_1H^1 \rightarrow {}_7N^{14} + \gamma \text{ rays.}$$

$$_7N^{14} + {}_1H^1 \rightarrow {}_8O^{15} + \gamma \text{ rays.}$$

$$_8O^{15} \rightarrow {}_7N^{15} + e^+ \quad \text{(positron)}$$

$$_7N^{15} + {}_1H^1 \rightarrow {}_6C^{12} + {}_2He^4.$$

$$(28\cdot34)$$
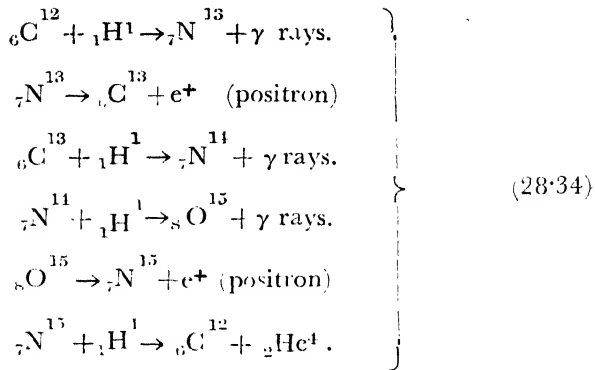
The catalyst $_6C^{12}$ is reproduced. The abundance of carbon and nitrogen remains practically unchanged.

Mass of 4H atoms = 4·03250

,,   ,, $_2He^4$ = 4·00390

Difference   = 0·02860 mass units.

≈ 43 × 10⁻⁶ ergs.

Out of 43 microergs generated 3 microergs go to neutrinos the remaining 40×10^{-6} ergs are available from each cycle. It can be shown from Bethe's theory that each carbon cycle takes 6×10^5 yrs. Thus each carbon nucleus will produce 40×10^{-6} ergs every 6×10^5 yrs, i.e. 25×10^{-20} ergs per second. The amount of carbon present in the sun is nearly 10% by weight. Hence the energy evolution is

$$\frac{6 \times 10^{23} \times 0 \cdot 1}{12} 25 \times 10^{-20} \simeq 10^3 \text{ergs/gm. sec} \ldots \ldots (28 \cdot 35)$$

This result is just what is necessary to explain the observed luminosity of the sun. Since the carbon reaction depends very strongly on the temperature (as T^{18}) and the temperature as well as the density decrease rapidly from the centre of the sun outwards, the average energy production will only be a fraction of the production at the centre.

14. Discovery of induced radioactivity. In 1933, F. Joliot and I. Curie Joliot were first successful in producing radio-
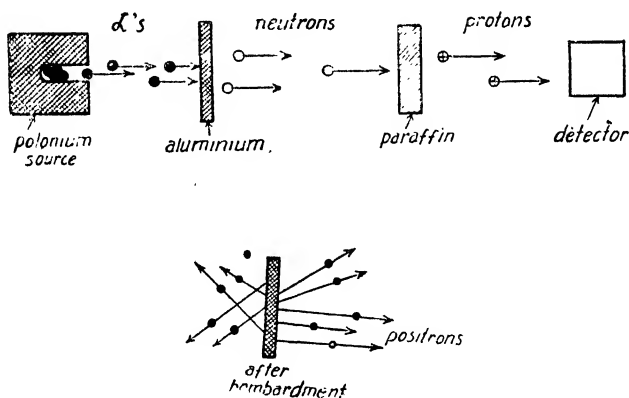
Fig. 28·9 Artificial radioactivity

activity by artificial means. They had long been studying trans-mutations produced by particles from naturally radioactive elements. In one of their experiments, fig 28.9, they bombarded aluminium with α-particles from polonium and studied the energies of the ejected neutrons by measuring the range of the recoil protons from paraffin. To their surprise they discovered that when the polonium source was removed, even then the counter continued detecting some radiations. What could be the reason of this activity in the absence of the polonium source ?

By using an expansion chamber they found that the activity was due to positrons coming out from the aluminium block. The energy distribution of positrons was similar to that of electrons from natural radioactive substances. The half-life of aluminium was found to be 3 minutes and 15 seconds.

The aluminium target was dissolved in hydrochloric acid and stable elements near aluminium were added to it. On chemical separation it was found that the activity was confined to the precipitate of phosphorus, whereas other precipitates were inactive. This proved conclusively that radio-phosphorus was formed. The reaction taking place can be represented as

$$\left. \begin{array}{l} {}_{13}Al^{27} + {}_2He^{4} \rightarrow {}_{15}P^{30} + {}_0n^{1} \\ {}_{15}P^{30} \rightarrow {}_{11}Si^{30} + e^{+} \end{array} \right\} \qquad (28.36)$$
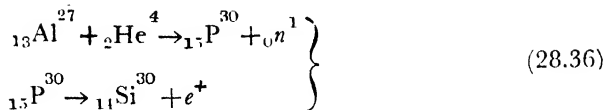
The phosphorus nucleus is unstable and emits positrons with the formation of a stable silicon nucleus.

Radio-sodium. Lawrence on bombarding sodium with deuterons from a cyclotron produced radioactive sodium which was found to emit γ-rays and electrons. The half-life of radio-sodium was found to be as much as 15 hours. The reaction is

$$\left. \begin{array}{l} {}_{11}Na^{23} + {}_1H^{2} \rightarrow {}_{11}Na^{24} + {}_1H^{1} \\ {}_{11}Na^{24} \rightarrow {}_{12}Mg^{24} + {}_{-1}e + \gamma\ ray \end{array} \right\} \qquad (28\cdot37)$$
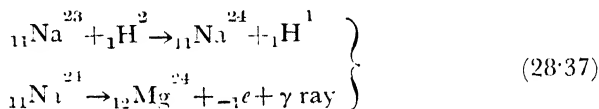
Radio-sodium, with its long half-life, has proved to be of great importance in medical research. It has been used as a means of tracing chemicals passing through human body and plants.

The other important reactions are :

$$\left. \begin{array}{l} {}_{15}P^{31} + {}_1H^{2} \rightarrow {}_{15}P^{32} + {}_1H^{1} \\ {}_{15}P^{32} \rightarrow {}_{16}S_i^{32} + {}_1e \\ {}_6C^{12} + {}_1H^{2} \rightarrow {}_7N^{13} + {}_0n^{1} \\ {}_7N^{13} \rightarrow {}_6C^{13} + {}_{+1}e \\ {}_{20}Ca^{42} + {}_0n^{1} \rightarrow {}_{19}K^{42} + {}_1H^{1} \\ {}_{19}K^{42} \rightarrow {}_{20}Ca^{42} + {}_{-1}e \end{array} \right\} \qquad (28\cdot38)$$
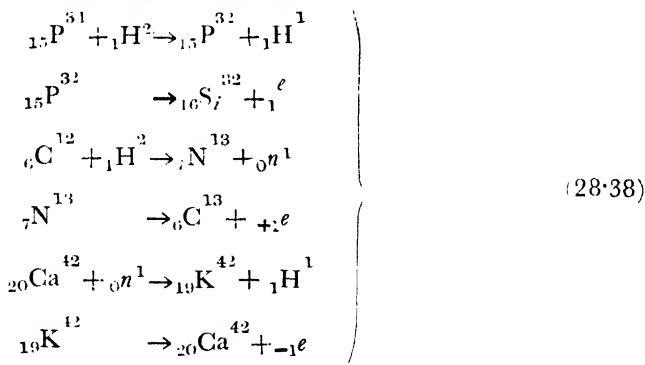
15. Transuranic elements. In 1934 Fermi and his collaborators observed that when uranium was bombarded with neutrons a large number of radioactive nuclei with different half-lives were produced emitting electrons. From chemical tests they concluded that elements with atomic number greater than 92 were formed. Thus they proved the existence of the so-called

transuranic elements. Later work by Hahn, Meitner and Strassmann showed that uranium under neutron bombardment formed three different active isotopes which decayed by successive disintegration according to the following scheme.

$$_{92}U + _0n^1 \rightarrow _{92}U \ (10 \ \text{sec.}) \xrightarrow{\beta} \text{Eka} \ _{93}\text{Re} \ (2 \cdot 2 \ \text{min})$$

$$\xrightarrow{\beta} \text{Eka} \ _{94}\text{Os} \ (59 \ \text{mins}) \xrightarrow{\beta} \text{Eka} \ _{95}\text{Ir} \ (66 \ \text{hrs.}) \xrightarrow{\beta} \text{Eka.}$$

$$_{96}\text{Pt} \ (2 \cdot 5 \ \text{hrs.}) \xrightarrow{\beta} \text{Eka} \ _{97}\text{Au}$$

$$_{92}U + _0n^1 \rightarrow _{92}U \ (40 \ \text{sec}) \xrightarrow{\beta} \text{Eka} \ _{93}\text{Re} \ (16 \ \text{mins})$$

$$\xrightarrow{\beta} \text{Eka} \ _{94}\text{Os} \ (5 \cdot 7 \ \text{hrs}).$$

$$_{92}U + _0n^1 \rightarrow _{92}\mathbf{U} \ (23 \ \text{min's}) \xrightarrow{\beta} \text{Eka} \ _{93}\text{Re} \ (?)$$

$$(28 \cdot 39)$$
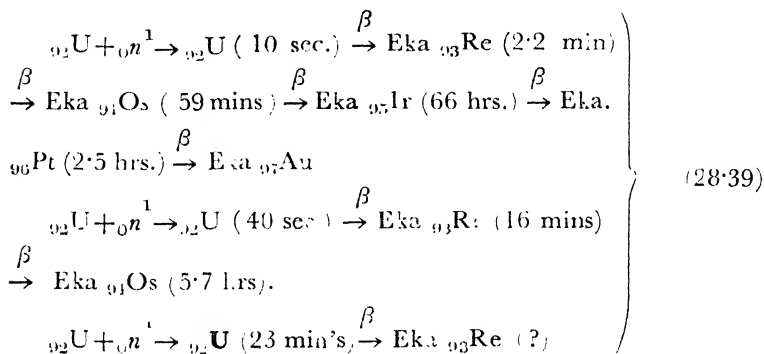
A great difficulty arose when Curie and Savitch showed that there is formed in irradiated uranium a radio-element having the chemical properties of a rare earth and having a half-life of 3·5 hrs. Assuming that it was another transuranic element they stressed the difficulty of finding a place for an element having chemical properties like La in the region of the periodic table somewhat beyond U.

16 Nuclear fission. Curie and Savitch missed a great
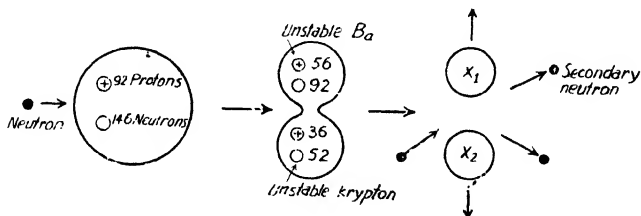
Fig. 28·10

discovery which came to Hahn and Strassmann. They found that three radioactive bodies were formed in uranium under neutron bombardment which were chemically similar to Ba. Their further work showed that it was impossible to separate them from Ba. Hahn and Strassmann were thus forced to conclude

" that isotopes of Ba $(Z=56)$ are formed as a consequence of the bombardment of uranium $(Z=92)$ with neutrons." Thus a uranium nucleus splits up into two fragments of medium atomic weight Ba and Kr, fig. 28·10. Here was an entirely new type of disintegration. To the process, in which a heavy nucleus breaks up into fragments, the name of nuclear fission has been given.

This startling discovery of nuclear fission cre ted so much interest in the scientific world that within less than a year since its discovery nearly hundred papers appeared on the subject.

The fragments Ba and Kr are unstable on account of their high neutron content in proportion to protons. They, therefore, try to stabilize by undergoing β-disintegration until a right ratio of protons to neutrons is reached. The various disintegration sequences are given below.

$$_{92}U + _0n^1 \rightarrow _{56}Ba^{41} + _{36}Kr^{63}$$
$$\downarrow \beta \qquad \downarrow \beta$$
$$_{57}La \qquad _{37}Rb.$$
$$\downarrow \beta \qquad \downarrow \beta$$
$$_{58}Ce \qquad _{38}Sr$$
$$\downarrow \beta$$
$$_{39}Y$$
$$\downarrow \beta$$
$$_{40}Zr. \qquad . \quad . \quad . \quad . \quad (28{\cdot}40)$$
$$\downarrow \beta$$
$$_{41}Nb.$$
$$\downarrow \beta$$
$$_{42}Mo.$$
$$\downarrow \beta$$
$$_{43}Ma.$$
$$\downarrow \beta$$
$$_{44}Ru.$$
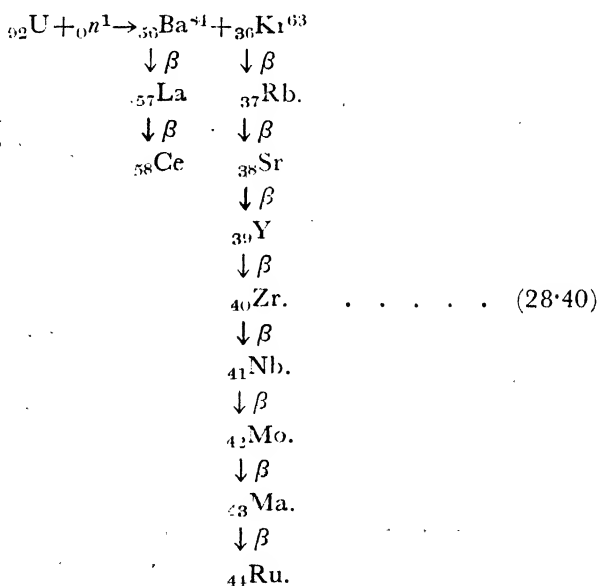
All these disintegration products and a host of others have been identified by actual chemical means. And the various half-lives which were formerly attributed to the so-called trans-uranic elements actually bel ng to some of these products. Abelson has

been able to identify radioactive Sn, Sb, Te and I in the fission products. The whole process is a very complex one.

Energy of fission products. The breaking up of a heavy uranium nucleus into too lighter nuclei releases an enormous amount of energy, since the sum of the masses of practically all possible pairs of lighter nuclei is less than that of the original nucleus. Taking the final reaction to be

$$_{92}U^{233} + _0n^1 \rightarrow _{58}Ce^{140} + _{44}Ru^{99} + \triangle E, \quad . \quad . \quad . \quad (28\cdot41)$$
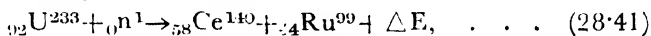
the energy release is given by

$$\triangle E = c^2 (M_0 - M_1 - M_2),$$

where M_0, M_1 and M_2 are the isotopic masses of the original nucleus and the two final nuclei respectively. Introducing mass numbers, A's and packing fractions f's we have

$$\triangle E = 931\{A_0 (1+f_0) - A_1 (1+f_1) - A_2(1+f_2)\} \text{ Mev.}$$
$$= 931\{239 (1 + 0\cdot00056) - 140(1 - 0\cdot0003) - 99(1 - 0\cdot00059)\}$$
$$= 218 \text{ Mev.} \qquad\qquad\qquad . \quad . \quad . \quad (28\cdot42)$$

Subtracting from this the energy of β-disintegration we get
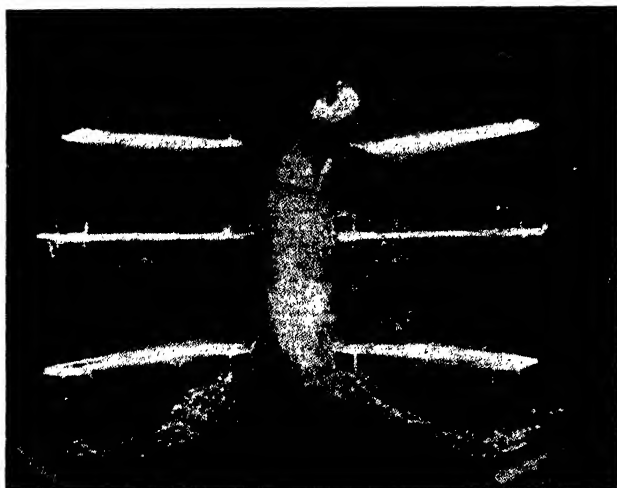
Fig. 28·11 Nuclear fission. [By courtesy of Prof. H. E. White from Classical and Modern physics]

the kinetic energy of the two fragments. A more exact calculation shows that the kinetic energy of the two fragments is 200

Mev. Nuclei having such enormous kinetic energy would be expected to produce heavy ionization tracks in a cloud chamber. The conservation of momentum demands that the two fragments must fly apart in opposite directions. All these expectations have been fulfilled by cloud-chamber pictures obtained by Joliot and Corson, fig. 28.11.

We have seen that the fission fragments are unstable on account of their high neutron content. The uranium nucleus after capturing a neutron becomes highly unstable. There is a large preponderance of neutrons in it. Joliot, therefore, suggested, that the neutron excess will be partially reduced by the direct liberation of neutrons occurring at the instant of fission or as an evaporation from the resulting newly formed nuclei, the latter process appears to be more probable on account of the large neutron-excess of the fragments. The emission of the secondary neutrons was experimentally verified by Von. Halbab, Joliot and Kowarski. They observed that several neutrons are probably emitted during each fission.

Could not these extra neutrons that were being erupted in turn become involuntary bullets flying from one exploding uranium nucleus into the heart of another, causing fresh fission? If this cumulative process goes on, was there a dangerous possibility of a very violent explosion?

F. Joliot found that the chain reaction, instead of building up to a grand climax runs down and stops like an unwound clock. The reason is not far to seek. The enormous energy generated in the solid material would ultimately increase the energy of the thermal neutrons, thus reducing their efficiency of bringing about fission. The reaction stops as the temperature rises.

It has been shown by Bohr that the thermal neutrons can cause fission only in the isotope U^{235} which forms about 0.7% of natural uranium. The fission of U^{238} occurs only when the impinging neutrons are fast. Experiments on samples of U^{235} have shown that Bohr's idea was correct.

Mechanism of nuclear fission. The whole process of nuclear fission becomes intelligible on the liquid drop model of

the nucleus. The nuclear forces of short range, which are analogous to the cohesive forces between molecules in a drop of liquid, must tend to produce similar effects of surface tension in nuclei. The short range forces try to keep the nuclear matter together, whereas the electrostatic forces of repulsion between protons try to annul the effect of the short range forces. As the nuclear charge increases the effect of repulsive forces increases. A rough calculation shows that the surface tension of the nuclear matter becomes zero for the atomic number $Z = 100$. Uranium nucleus having $Z = 92$ has a very little stability of form; and if we could agitate it by bombarding it, it may break into fragments, just as a liquid drop breaks up into two droplets when violently agitated. The condition for breaking is that the energy of deformation must be greater than the energy which tries to keep a nucleus together.

The classical theory of nuclear fission. The total energy of a nucleus is made up of two parts : (1) non electrical and (2) electrical. The non-electrical energy consists of two parts, volume energy and surface energy, the former is equal to γA and the latter equal to $\alpha A^{2/3}$, where A is the mass number of the nucleus, γ a constant and $\alpha = 10$ Mev. The volume energy is independent of the shape of the nucleus so long as the density remains constant.

The Coulomb (or electrical) energy of repulsion between the protons of the nucleus is

$$E = \frac{3}{5} \frac{(Ze)^2}{r}, \qquad \qquad . \quad . \quad . \quad (28\cdot43)$$

where Z is the atomic number of the nucleus and r its radius. The radius of any nucleus of mass number A is

$$r = 1\cdot4 \times 10^{-13} \, A^{1/3} \qquad \qquad . \quad . \quad . \quad (28\cdot44)$$

A nucleus can thus be likened to an electrically charged liquid drop, with the only difference that its charge is distributed uniformly over the volume and not over the surface, as in the case of ordinary conducting substances. The surface tension tends to keep the nucleus together, whereas the Coulomb repulsion tends to oppose it. The nucleus tends to break into two

drops of equal size as soon as the increase of surface energy (equal to the work done against the cohesive forces) becomes smaller than the resulting decrease of electrical energy.

Where the nucleus breaks into two equal drops the Coulomb energy is

$$2 \times \frac{3}{5} \left(\frac{Ze}{2} \right)^2 \Big/ r_0 \left(\frac{A}{2} \right)^{1/3},$$

corresponding to the two drops at ∞.

Before breaking, the Coulomb energy is $E = \frac{3}{5} \frac{(Ze)^2}{r_0 A^{1/3}}$. The ratio of the two energies is $2^{1/3}/2$. The Coulomb energy is decreased in the ratio $\frac{2^{1/3}}{2}$ E. The two daughters must fly apart with a kinetic energy equal to the mutual electrical energy at the instant of splitting which is approximately

$$T = \frac{(Ze/2)^2}{2r_0 (A/2)^{1/3}} = \frac{(Ze/2)^2}{2 \times \frac{1}{2^{1/3}} r}$$

$$= \frac{5 \times 2^{1/3}}{24} E \quad \text{approx.} \qquad . \quad . \quad . \quad (28 \cdot 45)$$

The surface energy of the daughters is

$$U' = 2 \prec \left(\frac{A}{2} \right)^{2/3}.$$

Before breaking the surface energy was

$$U = \prec (A)^{2/3}.$$

The surface energy is, therefore increased in the ratio $2^{1/3}$. Hence the condition of spontaneous splitting into two equal parts can be written in the form

$$\triangle W = E \left(1 - \frac{2^{1/3}}{2} - \frac{5 \times 2^{1/3}}{24} \right) - U \left(2^{1/3} - 1 \right) \geqslant 0$$

or $\qquad \triangle W = 0 \cdot 12E - 0 \cdot 26 U \geqslant 0 \qquad . \quad . \quad . \quad (28 \cdot 46)$

Putting here $U = 4\pi r^2 \prec = 9 \cdot 6 A^{2/3}$ Mev

and $\quad E = \frac{3}{5} \times \frac{(Ze)^2}{r} \times \frac{1}{1 \cdot 59 \times 10^{-5}}$ Mev $= \frac{9}{5} \frac{Z^2 e}{r} \times 10^{-4}$ Mev

$$\triangle W = \left(7 \cdot 4 \times 10^{-2} \frac{Z^2}{A} - 2 \cdot 50 \right) A^{2/3} \geqslant 0. \qquad . \quad . \quad . \quad (28 \cdot 47)$$

In the case of uranium, $Z=92$ and $A=238$ and so
$$\triangle W = (2 \cdot 62 - 2 \cdot 50) A^{2/3} \geqslant 0$$
Hence uranium should be able to split up into two equal parts ($Z=46$, $A=119$), each of them possessing a small excess energy

$$\frac{\triangle W}{2} = 0 \cdot 06 \, A^{2/3} = 2 \cdot 3 \text{ Mev.}$$

For Thorium $\frac{\triangle W}{2} = 0 \cdot 04 \, A^{2/3} = 1 \cdot 6$ Mev.

For lighter nuclei $\triangle W$ comes out to be negative; for example for Hg ($Z=80$, $A=200$)
$$\triangle W = -0 \cdot 13 \times (200)^{2/3}.$$
Hence they will be stable against fission. The important feature of the inequality (28·47) consists in the fact that with a slight readjustment of the numerical coefficients it actually easures the stability (with respect to fission) of all the known elements, and excludes the possibility of the existence of elements with higher Z and A.

CHAPTER XXIX
THE ELECTRON WAVES

1. De Broglie relation. It was long known that there existed a close analogy between the laws of mechanics and those of optics. Corresponding to the Maupertius *principle of least action* in mechanics there was the Fermat's *principle of least path* in optics. The former principle states that the path described between two points A and B by a particle under the influence of forces is such that the line-integral of the mechanical velocity v is a minimum, *i.e.*

$$\int_A^B v\,ds = \text{minimum}, \qquad \ldots \quad (29\cdot1)$$

where ds is an element of the path. All the fundamental equations of mechanics can be deduced from this principle. Fermat's principle, on the other hand, states that the optical path of a ray of light between two points A and B is such that the time taken is a minimum, *i.e.*

$$\int_A^B \frac{ds}{u} = \text{minimum}, \qquad \ldots \quad (29\cdot2)$$

where u is the wave velocity.

We see that the two fundamental principles are formally analogous, the mechanical velocity v on one hand corresponding to the reciprocal of the wave velocity u on the other. This close analogy between the laws of mechanics and optics suggested De Broglie that a wave-motion can be associated with a material particle. In 1924, in his doctorate thesis submitted to the University of Paris, this young physicist put forward a bold hypothesis, which later on revolutionized our conceptions of matter. As a pure hypothesis he equated the mechanical momentum of the mass (namely mv) to the momentum $\dfrac{h}{\lambda}$ of the waves associated with it, *i.e.* he put

$$\frac{h}{\lambda} = mv \quad \text{or} \quad \lambda = \frac{h}{mv}. \qquad \ldots \quad (29\cdot3)$$

For electrons $\dfrac{e\mathrm{V}}{300} = \frac{1}{2}mv^2$, if V is in volts.

Hence

$$\lambda = \sqrt{\frac{150}{\mathrm{V}}} \ \overset{\circ}{\mathrm{A}} \ . \quad \ldots \quad (29\cdot4)$$

An electron with energy equal to 150 eV has a wave-length of 1 $\overset{\circ}{\mathrm{A}}$ associated with it.

2. The experiment of Davisson and Germer. The
hypothesis of De Broglie was first experimentally verified by Davisson and Germer in 1927. In their experiment electrons
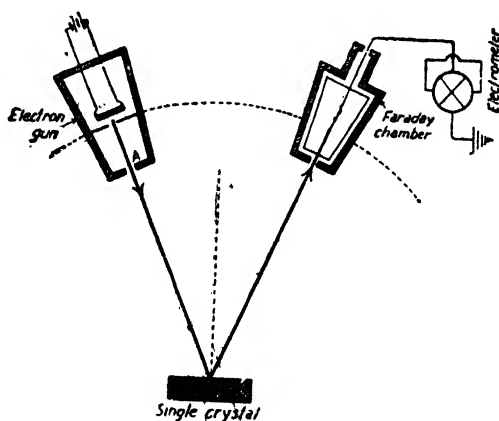
Fig. 29·1. Experiment of Davisson and Germer.

from a hot filament F are accelerated towards an anode A, and after passing through fine holes, they emerge from the "electron gun" as a narrow beam, fig. 29·1. The potential on the anode can be varied to obtain any desired velocity of electrons. The electrons, after reflection from the surface of a nickel crystal, are caught in a Faraday chamber connected to an electrometer. The

"electron gun", the Faraday chamber and the nickel crystal are all contained in a highly evacuated chamber. The first two can move along the arc of a circle. The Faraday chamber is surrounded by a shield connected to a negative potential equal to the accelerating potential in the electron gun. The purpose of the retarding potential is to eliminate all those electrons which have lost their energy in reflection from the crystal and also the secondary electrons produced by the impact of the primary beam.

On moving the Faraday chamber along the arc of the circle Davisson and Germer found that there are certain preferred directions along which the reflection of electrons is most copious. For different accelerating potentials corresponding to different electron velocities, they measured the angles of selective reflections.

This selective reflection cannot be understood on the concept of corpuscular nature of the electron. There is no *a priori* reason why electrons when treated as particles should be scattered in a preferred direction by the ions in the lattice planes of the crystal, when travelling with a certain speed. The whole phenomenon
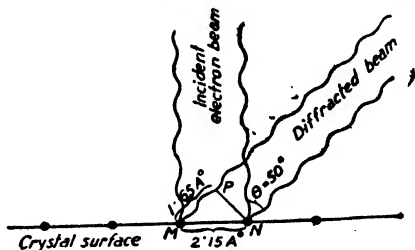
Fig 29·2

becomes readily intelligible if we consider the electrons as waves. The surface of the crystal acts as an ordinary diffraction grating since in the crystal surface there is a regular arrangement of atoms, the rows of atoms being analogous to the lines of a diffraction grating. The separation between two rows corresponds to the grating element. Let us consider a beam of electrons falling normally on the crystal surface, fig 29.2. The electron waves are reflected from the rows M and N of atoms. They will reinforce only in that direction where the path difference between the two adjacent rays is an integral multiple of the wave-length, *i.e.*

$$d \sin \theta = n\lambda \qquad \qquad \cdot \quad \cdot \quad \cdot \quad (29,5)$$

For all other directions there will occur a destructive inter-
ference. As an exprimental observation it was found that for 54 volt
electrons the angle of diffraction was 50°. From X-ray data
$d = 2\cdot15$Å. Taking $n = 1$ *i.e.* the first order spectrum

$$\lambda = 2\cdot15 \times \sin 50°$$

$$= 1\cdot65 \text{ Å}$$

The theoretical wave-length according to the De Broglie relation is

$$\lambda = \sqrt{\frac{150}{54}} = 1\cdot66\text{Å}.$$

Thus there is a close agreement between the theoretical and the
experimental values.

3. G. P. Thomson's experiment. The wave nature of the

electron was veri-
fied by G. P.
Thomson in 1928
by using an entire-
ly different method
from that of Davi-
sson and Germer.
The method was

Fig. 29·3. G. P. Thomson's experiment
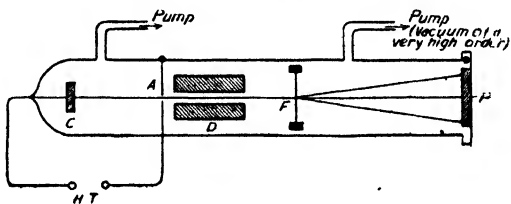
analogous to the Debye Scherrer method for studying crystal struc-
ture by X-rays. In his experiments electrons from the cathode C
are accelerated towards the perforated anode A by a high potential
of nearly 50,000 volts. The elect-
ron beam after being collimated by
the capillary hole in the metal
block D falls on a gold foil F. The
thickness of the foil is of the order
of 10^{-3} cm. The electrons after
being diffracted fall on the photo-
graphic plate P, fig 29·3.

If the electrons were asssumed
to be corpuscular in character the
foil would have scattered them in all
directions; and there would have

Fig 29·4. Electron diffraction rings

been a general illumination. Instead of a continuous distribution of light the photographic plate revealed a number of fine rings, fig 29·4. The nature of the pattern depends upon the material of the foil. The radii of the rings decrease with the increase in the velocity of electrons showing that the wave-length decreases with the increase in velocity.

If we take the electron as a wave the explanation is quite straightforward. The metallic foil consists of numerous micro-crystals all oriented at random. Obviously, some of the crystals will be tilted at such an angle with respect to the incident electron beam that the Bragg relation is satisfied for an atomic plane (h, k, l) in it. Waves reflected from this set of planes will produce constructive interference giving a black spot on the photo-plate. In this multitude of crystals there will be also others satisfying the above condition, but rotated about the axis of the incident electrons. As a result, a complete ring appears on the photographic plate. Reflections from another plane having the Miller indices (h', k', l') will similarly produce another ring.

4. The dualistic nature of matter. The experiments on the nature of cathode rays prove conclusively that an electron is corpuscular in character. At the same time the experiments of G. P. Thomson, Davisson and Germer prove that an electron behaves as a wave. Thus we are faced with two apparently contradictory view points. How are we going to reconcile these two hypotheses ? Nature is dual in character. There is no escape from it.

Schrödinger attempted to explain the dual character of the electron by considering it as a *wave packet*. Consider two sound waves of slightly different wave-lengths travelling in the same direction but having the same wave-velocity u. The amplitude of the resultant wave at any point in space would be given by the principle of super-position. We shall have maxima and minima travelling past any point with the wave velocity u. This will occur only in a nondispersive medium; whereas in a dispersive medium where the velocity of the waves is a function of the wave-length, these maxima and minima do not travel with the wave velocity u

but with velocity v which is different from u. This can be shown by a simple mathematical treatment. Now, let us consider a large number of infinite wave trains, each with slightly different wave-length and velocity. It can be shown by a suitable selection of the waves that the vibrations cancel every where except over a very small region in space. In this region they combine to form a hump, called the *wave packet*. As the waves progress with the *phase velocity* u which is an average of the individual wave velocities the packet moves forward with the velocity v known as the *group velocity*. The relation between the group velocity and the phase velocity is

$$v = \frac{dv}{d\left(\dfrac{v}{u}\right)}. \qquad \ldots \quad (29 \cdot 6)$$

If the electron is to be associated with a wave packet it becomes imperative to postulate the existence of phase waves because there can be no wave packet without the phase waves. They serve as a guide for the electron. Their average wave-length is given by $\lambda = \dfrac{h}{mv}$. Both the phase waves and the wave packet constitute the reality of a particle. The corpuscular and the wave properties are, therefore, not contradictory but complimentary. When the particle displays its corpuscular properties it has revealed its one half, the other half remaining hidden ; "an electron cannot at the same time behave as a particle and a wave, it acts as one or the other in any experiment that can be devised." Naturally, a question arises in which experiment will it behave as a wave and in which as a particle? The electron is a very accommodating person. If the experiment devised is such that it will reveal only the wave property, the electron will show its wave aspect; and if the experiment is such that it will reveal only the corpuscular property the electron will show its particle aspect. The electron is a docile creature.

5. The Schrödinger wave equation. Schrödinger by first deriving the wave equation of the phase waves laid the foundation of a newer mechanics known as the *wave mechanics*. He

starts by considering the equation for the propagation of light waves. The one-dimensional wave equation is

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2}, \qquad \cdots \quad (29 \cdot 7)$$

where ψ is wave displacement and c its velocity.

Also $\qquad \psi = \psi(x)\, e^{i2\pi\nu t}, \qquad \cdots \quad (29 \cdot 8)$

where ψ is a function of x alone

Substituting $(29 \cdot 8)$ in $(29 \cdot 7)$ we have

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{4\pi^2 \nu^2}{c^2} \psi = 0$$

or $\qquad\qquad \dfrac{\partial^2 \psi}{\partial x^2} + \dfrac{4\pi^2}{\lambda^2} \psi = 0. \qquad \cdots \quad (29 \cdot 9)$

The wave equation of the particle is obtained by substituting in $(29 \cdot 9)$ $\lambda = \dfrac{h}{mv}$. We have then

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{4\pi^2 m^2 v^2}{h^2} \psi = 0 \qquad \cdots \quad (29 \cdot 10)$$

If the particle moves in a field of force

$$\tfrac{1}{2}\, mv^2 = E - V, \qquad \cdots \quad (29 \cdot 11)$$

where E is the total energy of the particle and V its potential energy. Substituting $(29 \cdot 11)$ in $(29 \cdot 10)$ we have

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{8\pi^2 m}{h^2}\Big(E - V\Big)\psi = 0 \qquad \cdots \quad (29 \cdot 12)$$

This is Schrödinger's wave equation of a particle.

CHAPTER XXX

COSMIC RAYS

1. Discovery of cosmic rays. During the study of the radiations emitted by radioactive bodies it was discovered by various
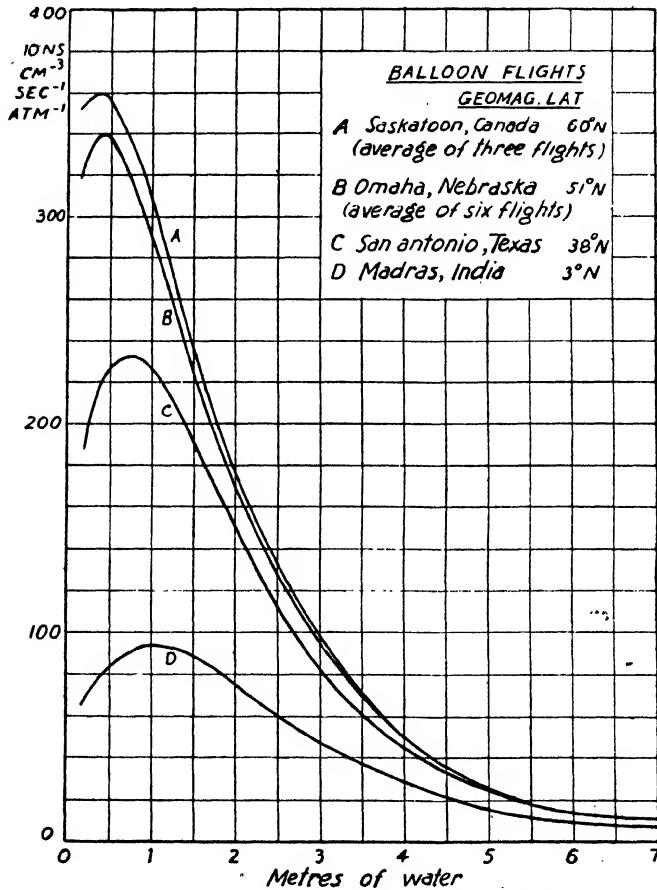
Fig 30·1 Variation ef cosmic ray intensity with altitude.
[By courtesy of Prof. R. A. Millikan]

investigators that their electroscopes continued to discharge, although at a very slow rate, even if their instruments were isolated

from all known sources of radiations arising from radioactive materials. It was found that this natural leak of the electroscopes persisted even after surrounding them by tons of lead. In 1910, Gockel, a Swiss physicist, carrying an electroscope ascended nearly three miles in a baloon and found to his great surprise that there was an actual increase in intensity at the greater heights. This important observation was confirmed by the baloon flights of Hess and Kolhörster, who found that at 9000 metres the ionization reached a value about forty times higher than that at sea level. Kolhörster attributed this effect to some unknown radiation falling on the earth from outside and being gradually absorbed in the atmosphere. These radiations possessed a far greater penetrating power than any radiation of earthly origin. These are now known as cosmic rays.

2. Variation of cosmic ray intensity with altitude and latitude. The instruments used in the measurement of cosmic ray intensity are the ionization chamber and the G. M. counter. The cosmic ray intensity measurements into the high mountains and into the atmosphere and deep into the mines have shown that the intensity increases rapidly with height until it reaches maximum and then begins to decrease. The general trend of the intensity with increasing altitude at four different latitudes is shown in fig 30·1.

The altitude is expressed in terms of the height in metres of a
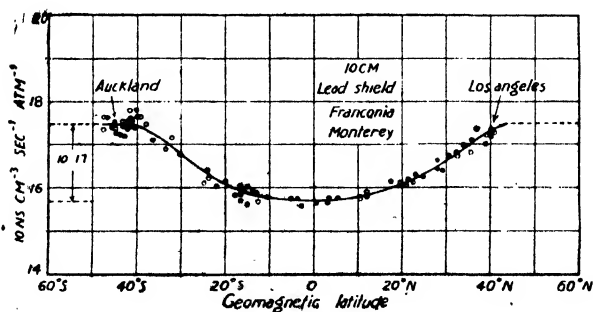
Fig . 30·2

Variation of cosmic ray intensity with geomanetic latitude.
[By courtesy of Prof. R. A. Millikan].

water column which would exert a pressure equal to the atmos-

pheric pressure at that altitude; sea level corresponds to about ten metres of water equivalent. For the same height the cosmic ray intensity is greater for higher magnetic latitudes than for the lower latitudes. This change in intensity with latitude is the result of the fact that the earth itself is a huge magnet. The incoming cosmic ray particles, most of which are electrically charged, are deflected from their straight-line path by the magnetic force of the earth's field. Many of the incoming particles are deflected into curved paths but are still able to reach the earth ; others are deviated to such an extent that travelling in great circular arcs they approach to within several hundred miles of the earth's surface, but miss striking it altogether and then continue their journey through space in a new direction. Assuming the particles to be electrons or positrons the minimum energy required to reach the earth at latitude 50^0 is 3×10^9 ev. Any other particle with energy less than this shall not be able to reach the earth's surface. At the equator the minimum energy required is nearly 2×10^{10} ev. The cosmic ray intensity as a function of geomagnetic latitude is shown in fig 30.2.

The variation of cosmic ray intensity with direction has been studied by using a Geiger-Müller telescope, fig 30·3.

A, B and C are three G. M. counters arranged in one line. They will be actuated simultaneously only by a particle travelling in the direction XY and if it passes through all of them; such an arrangement is called a G. M. T.
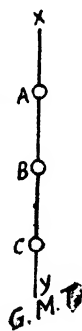
It has been found that the number of particles coming from the West is greater than that coming from the East. This is known as East-West effect. This effect is more pronounced at the equator. Theoretical calculations show that for the positively charged particle coming from the West at least an energy of 10^{10} ev is required while 6×10^{10} ev is the minimum energy for the particle coming from the East. That is West is favoured at the expense of the East if the original particles are positively charged. This leads us to conclude that the particles reaching the earth's atmosphere are mainly positive.

Fig. 30·3

3. Interaction of cosmic rays with matter. The Wilson-cloud chamber has played a fundamental part in revealing to us the nature of cosmic rays. When a cosmic ray particle passes through a cloud chamber it makes its presence known by leaving behind a trail of ions along its path. By actually counting the number of ions per cm. of path we can know the specific ionization. The energy of the particle is revealed by measuring its curvature when the cloud-chamber is placed in a powerful magnetic field.

To study the interaction of cosmic rays with matter the common technique is to place a block of lead in the cloud-chamber placed in a powerful magnetic field. A host of new facts have been revealed by such experiments. Fig 30·4 shows that a particle may readily pass through such a plate with apparently no measurable diminution in energy. In fig. 30·5 an electron of an atom in the plate is knocked by the high speed cosmic ray particle as it passed
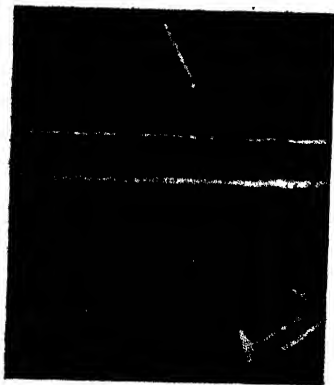
Fig. 30·4. A cosmic-ray particle passes downward through a plate of lead about ½ inch thick.
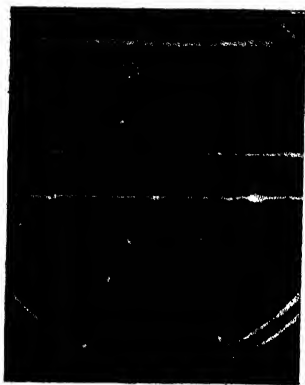
Fig. 30·5. The track of a cosmic-ray particle passing through a plate of carbon and striking an electron in the carbon. The struck electron produces a sharply curved circular track shown below and above the carbon plate.

through the plate. The large curvature of the struck electron shows that its energy was smaller compared to the incident particle. It did however attain sufficient speed to enable it to emerge from the

plate, to pass through it in the opposite direction, and then emerge again from its upper side, but with diminished speed as shown by the increased curvature. Fig 30·6 shows the production of several electrons ; some of them are positively charged and others negatively charged, as shown by the direction of their curvatuie. This phenomenon in which several particles are observed at one time

Fig. 30·6. A shower of positive and negative cosmic-ray electrons.
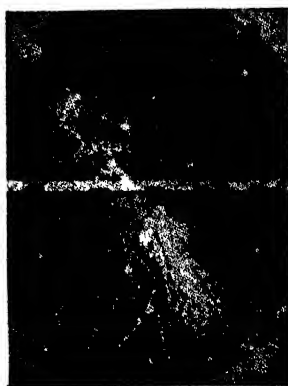
Fig. 30·7. A large shower of more than one hundred positive and negative electrons.

is called an *electron shower*. Electron showers sometime rise to the magitude of veritable *cloud bursts* as shown in fig. 30·7.

A quantitative study of the shower phenomenon was first made by Rossi whose experimental arrangement is indicated in fig. 30·8. P is a lead block the thickness of which can be varied. The counters A, B, C are actuated simultaneously. Rossi measured the number of coincidences as a function of the thickness of the block.
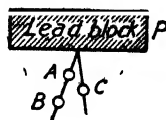
Fig. 30·8

Fig. 30·9 is a typical curve for Pb. It has a maximum at about 1·5 cm. of Pb and then decreases rapidly. After 5 cms an almost constant value is reached. At 15 or 20 cms the curve has a flat second maximum. An interpretation of the Rossi curve shows that the shower producing radiation consists of two components

(1) soft and (2) hard. The soft component of the incident radiation is much more efficient in producing showers than the hard component.

Briefly the mechanism of shower production is this. A fast cosmic ray electron, as it passes near an atomic nucleus, is accelerated and radiates a high energy photon with energy comparable with the energy of the incident electron. This photon on interaction with a nucleus produces an electron pair. Each of the secondary electrons now behaves in a similar way to the primary electron, that is, it will emit light quanta which again produce pairs and so on.

Fig. 30·9 Rossi curve

This cumulative process of pair production continues as long as the energy of the photon and the electron is greater than the critical energy. The moment this energy limit is crossed no further multiplication can take place ; the electrons then lose their energy steadily by ionization and the light quanta by Compton scattering. The above mechanism was first proposed and worked out theoretically by Bhabha and Heitler.
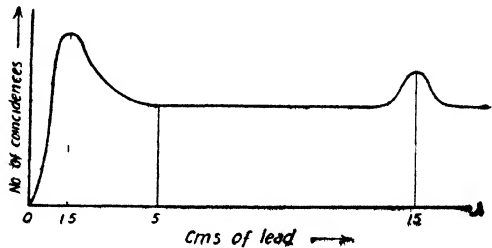
4. The Positron. In 1932 Carl Anderson of the California Institute of Technology while photographing the cosmic ray tracks in a vertical cloud-chamber traversed by a 6 mm plate and placed in a powerful magnetic field came accross a rare photograph, fig. 30·10.

By knowing the curvature Anderson found that the corresponding energies of the particle above and below the Pb plate are 63×10^6 and 22×10^6 ev. From the direction of the curvature he nccoluded that the path was that of a positively charged particle

moving downwards and losing energy in passing through 6 mm of Pb. Another possibility is that of a negatively charged particle moving upwards and gaining energy in passing through the lead plate. He ruled out the latter possibility because there was no such process known by which a negatively charged particle would triple its energy by passing through 6 mms. of Pb. If it is a positively charged particle, could it not be a proton ? If it were a proton the respective energies on the opposite side of the absorber would be 2×10^6 and 3×10^5 ev and the specific ionization for the proton of 3×10^5 ev energy is much greater than that of an electron. Also the range of such a proton is only 0·5 cm in air
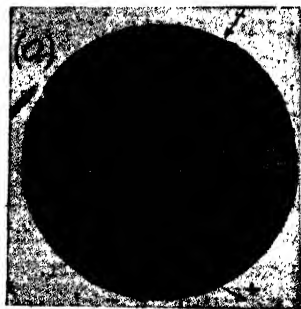
Fig. 30·10 Positron track [By courtesy of Prof. C. D. Anderson]

whereas the track in the chamber was actually 5 cms. If we assume the charge on the particle to be unity the loss of energy in passing through the absorber requires it to have a mass less than that of a proton. Again the specific ionization is definitely that of a particle with charge less than $2e$. In the light of this evidence Anderson was forced to conclude that the track was of a particle with a positive charge and mass equal to that of an electron. This led to the discovery of another fundamental particle, the *positron*, whose existence was theoretically predicted by Dirac. Anderson was awarded the Nobel prize for this great discovery.

Positron can be produced by bombarding Be with γ-rays. γ-rays from ThC″ in the field of a nucleus materialise into an electron and a positron. The positrons do not occur free in nature because their average life time is very short, of the order of a billionth of a second or less. When a positron meets an electron, both particles will suffer the fate of complete annihilation and in their stead a pair of corpuscles of radiant energy, each of one-half million electron volts, will remain. There is no reason to believe, however, that a positron, if removed from a region

densely populated by negative electrons, may not live hundreds
of millions of years, instead of perishing in billionth of a second.

5. The heavy electron or mesotron. The cosmic
ray intensity measurements have revealed the presence of
particles not only capable of passing through several feet of
lead but through several hundred feet of hard rock. These
highly penetrating particles were initially thought of as
electrons and protons. Theoretical investigations have shown
that the processes of shower formation would place such severe
limitations on the ranges of both electrons and protons of all
energies that they could not be used to account for the highly
penetrating particles. To account for the nature of highly
penetrating particles one has to assume the existence of a new
kind of particle, hitherto unknown to physics. This particle has
been called a *heavy electron* or a *mesotron*.

During their study of cosmic ray tracks some experimenters
came across certain rare photographs. Con-
siderations of specific ionization and energy
of the tracks forced them to conclude that
the tracks were of unknown particles having
mass intermediate between a proton and an
electron. But there was no unanimity as to
the exact magnitude of the mass. Fig. 30 11
shows a typical experimental arrangement
of Street and Stevenson to observe the tracks
of such heavy electrons. G. M. counters
1, 2, 3 and bank 4 are so arranged that the
Wilson chamber C will not be expanded
unless an ionising particle traverses 1, 2, 3
but not 4. L is a lead block 10 cms in thickness to absorb elec-
trons. C is the chamber which can be expanded one second after the
particle has traversed 1, 2, 3 but not 4. The experimental arrange-
ment is such that it allows the track of the penetrating particle
to be photographed near the end of its journey ; and the ions have
diffused enough to permit a comparison of specific ionization of
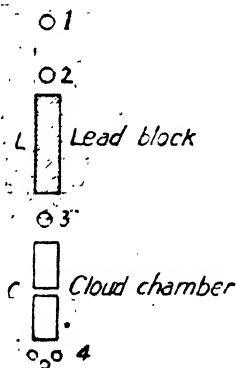
Fig. 30·11

different tracks. Out of the several photographs two were such where ionization along the track was heavier than for the others. We quote below the results of Street and Stevenson. "The value of $H\rho$ is 9.6×10^4; for the track shown in fig 30·12 (a) the range is 6·0 cms in air, while that of a proton with this value of $H\rho$ it would be 0·9 cm. The estimated mass is $130 \pm 25\%$ electron mass. The track shown in fig 30.12 (b) has a specific ionization of 4 times that of an electron and has an $H\rho$ value of
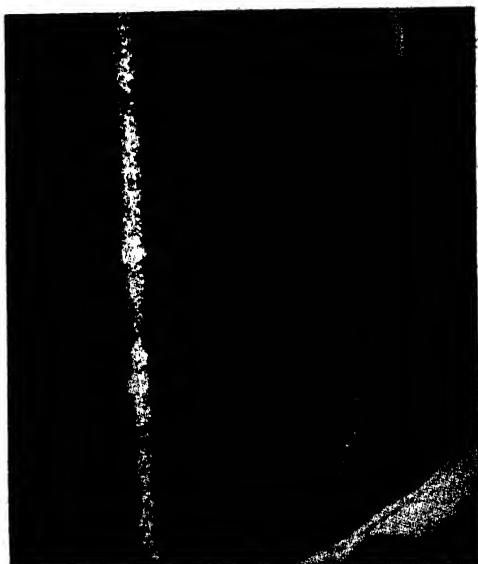
(a)   (b)

Fig. 30·12. Meson and proton tracks. [By courtsey of American Journal of Physics.

$1·8 \times 10^4 \pm 10\%$. This leads to an estimated mass of 1900 times the electron mass. This track is evidently due to a proton". The heavy electron has a very short life. It disintegrates in about one millionth of a second into an electron and a neutrino. This would lead to the conclusion that the heavy electrons cannot themselves come from outside the earth but are created as secondaries in the atmosphere by primary particles. The origin of cosmic rays is still a mystery.

31 EC

CHAPTER XXXI
TERRESTRIAL MAGNETISM AND ATMOSPHERIC ELECTRICITY

1. Magnetic elements of earth's field. The magnetic behaviour of our globe has been the subject of qualitative and quantitative study for the last several centuries. Exact knowledge of the earth's field is of importance not only to the pure scientific investigator but also to the sea and air navigator. Such an exact knowledge means exact information about the magnitude and direction of this field at every time and place. It is found convenient to express this field in terms of some experimentally determinable components called the *elements* of the magnetic field. These are (1) *the declination* (2) *the horizontal component of the earth's field* and (3) *the Dip.*

Declination. This is the angle between the geographical and magnetic meridians at a place. The geographical meridian is a vertical plane passing through the geographical north and south while the magnetic meridian is a vertical plane passing through the magnetic axis or the line joining the north and south pole of a freely suspended magnet. For nautical purposes the declination is called the variation of the compass, meanig thereby its variation from the true north-south direction.

Dip. Mount a magnetic needle with a fine straight axle passing through its centre of gravity on a pair of horizontal edges. It can then rotate in a vertical plane and if this plane is made to coincide with the magnetic meridian the needle instead of remaining horizontal will set with its magnetic axis along a line inclined to the horizontal. This line represents the direction of the total magnetic field of the earth at the place. The angle θ between this direction and the horizontal is called the dip. The dip angle can be determined accurately by a dip circle.

Resultant field. The axis of the dip needle gives us the direction of this field. If I denote its magnitude and V and H its vertical and horizontal components we have the following relations:

$$V/H = \tan \theta \; ; \; R^2 = V^2 + H^2 \qquad . \quad . \quad . \quad (31\cdot1)$$

The horizontal component H is capable of accurate measurement and so is θ. Eq. $(31\cdot1)$ gives the vertical component and the resultant R. The earth's magnetic field at a given place is thus completely determined in magnitude and direction.

2. Experimental determination of declination, H and dip.

For the study of terrestrial magnetism knowledge of declination, H and θ is essential. The determination of H by magnetometer has already been described. The declination is most easily obtained with an accurate magnetometer such as the kew magnetometer.

The dip angle can be measured on a reliable dip circle such as that of the kew pattern shown is fig. 31.1 (a). The dip needle is a thin steel magnet $N_1 N_2$ with a fine

Fig. $31\cdot1$

steel axle resting on two horizontal agate knife edges K_1 and K_2, fig. $31\cdot1$ (b).
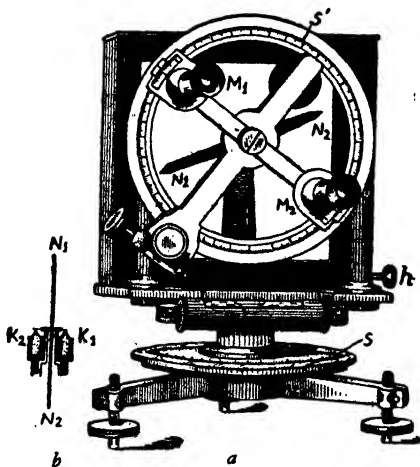
The needle may be raised and lowered by the milled head h. The body of the instrument can be rotated about the vertical axis and its azimuth read upon the horizontal scale S. The dip angle can be measured on the vertical scale by observing the ends of the needle S by the vernier microscopes M_1, M_2.

The instrument is first levelled and then rotated about the vertical axis so that the needle assumes the position $90°-90°$. The plane in which the needle rotates is then perpendicular to the magnetic meridian; the former is made to coincide with the latter

by turning the instrument through 90°. The dip angle is immediately found by reading the two ends of the needle.

The following procedure should be adopted for an accurate determination of the dip angle: (1) the two ends of the needle should be read for correcting the error arising from the non-coincidence of the centre of rotation of the needle and the centre of the circular scale. (2) The instrument should be rotated through 180° about the vertical axis and the previous reading repeated. This is to allow for the effect of any departure of the 0-0 line of the vertical scale from the horizontal. Such a departure will give the apparent dip too great in one position and too small by an equal amount in the second position. (3) As the magnetic and the geometric axes of the needle may not coincide the needle should be turned over its bearing and all the previous readings repeated. (4) The axis of rotation of the needle may not pass through the centre of gravity of the needle. This will exert a small couple due to gravity causing an error in the dip angle. To correct this, the needle is remagnetized in the opposite direction so that the end of the needle which was up is now down and vice versa. All the previous eight readings are repeated. The mean of the total sixteen readings gives the true dip.

3. Magnetic maps. The magnetic elements described before are determined at various places over the surface of the earth and their values are plotted in charts giving us what are called magnetic maps, fig. 31.2. These give us at a glance the most important peculiarities of the magnetic behaviour of the earth. The most convenient method of representation is to draw lines passing through places corresponding to constant values of the various magnetic elements. We thus get *isogonal lines* or lines of constant declination, *isoclinal lines* or lines of constant dip and *isodynamic lines* or lines of constant horizontal intensity. A close scrutiny of these various lines brings out certain important peculiarities. They are breifly described below.

The *isogonals* are found to converge towards two points upon the surface of the earth viz., the two *geographical poles* and two other

points called the *magnetic poles.* There are two chief agonic lines
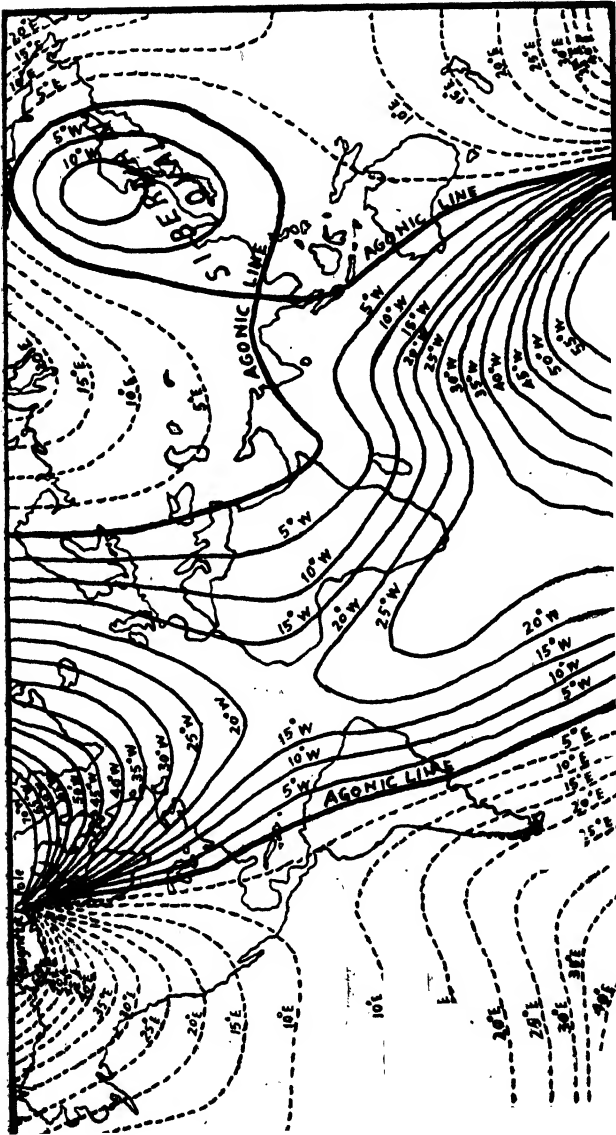
Fig. 31·2

or lines of no declination. At all points of these the compass points towards the geographical poles. One of these agonic lines passes from the magnetic north pole to the geographical south pole via America and the Atlantic ocean. The other agonic line passes from the geographical north pole to the magnetic south pole via Eastern Europe, Arabia, the Indian ocean and Australia. Along a line joining the magnetic and geographical north poles the declination is 180°. Similar is the case with the magnetic and geographic south poles.

East of the American agonic line the declination is westerly i.e. the compass needle points west of the true north. West of the American agonic line the declination is easterly. The isogonals are not all regular curves. In eastern Asia they are most irregular. Here we come across a district surrounded by a closed agonic line. This is called the *Siberian oval*. Within this closed line the declination is westerly.

The isoclinals are more regular than the isogonals. They are approximately circles with their poles situated at the magnetic poles. There is a line of no dip. This is called *magnetic equator*. It crosses the geographic equator once in the Atlantic and once in the Pacific ocean. It lies south of the geographic equator in the American hemisphere. The other isoclinals are nearly parallel to the magnetic equator. At points called magnetic poles the dip is 90°. Sir James Ross found in 1831 the North magnetic pole in longitude 96°,43′ W, latitude 72°,25′N: while the South magnetic pole was found in 1909 by the party of Sir Earnest Shackleton in longitude 155°,16′,E, latitude 72°,25′,S. The magnetic poles are thus not exactly at the opposite ends of the earth's diameter. The approximate magnetic axis of the earth makes an angle of about of 17° with the rotation axis and misses the earth's centre by about 700 miles.

4. Variation of Terrestrial field. It is very interesting to note that the magnetic field of the earth at all places is not constant but is changing. The change is found to resolve itself into a number of periods and sudden and irregular changes called magnetic storms.

A very brief summary of the important changes is given below:
Secular variation. From the available records it has been found that at all points the declination is undergiong a change possessing a long period of the order of 960 years. The magnetic north pole describes a circle of about 17° radius.

Annual variation. There is another variation of declination occurring simultaneouly in opposite direction in the northern and southern hemispheres. The period of this is one year. The maximum easterly deviation occurs in August and the westerly one in February.

Daily variation. The diurnal variation is found to depend upon the Sun and the Moon. For the solar influence it has been observed that there is a twenty-four hour period change in the earth's field. It has also been discovered that this daily variation is not constant for all days and is less in winter than in summer. It is also indtcated that about three-fourth of this solar effect arises from overhead causes and one-fourth from induced currents in the earth.

Eleven year period. A surprising parallelism between the eleven year sunspots frequency and the daily variation in magnetic elements has been observed.

Lunar variations. The daily variation is also seen to depend upon the moon. A careful analysis of the daily variation curves shows that lunar variation is of symmetrical semidiurnal type similar to a tidal variation. As in the case of the solar diurnal variation analysis of observational data indicated that three-fourth or two-thirds of the lunar variation arises from overhead influences and one-fourth or one-third from induced currents in the earth.

Magnetic storms. In addition to the long and short periodic variations in the terrestrial magnetic field there are found erratic variations or disturbances in the magnetic fleld. They may occur at any time, may be confined to a localised region or may be world wide. They defy complete classification. The world-wide disturbances called magnetic storms are usually accompanied by displays of auororas.

5.   Theories of terrestrial magnetism.   Modern analysis of magnetic measurements on the surface of the earth lead to the conclusion that major portion of the magnetic field at the earth's surface has its origin within the earth, a very small part having its origin outside. As there are no magnetic substances outside the earth this small part having an external origin must be due to electric current the origin of which is not known precisely.

The earth as a uniformly magnetized sphere.   To a first approximation the normal magnetic field of the earth may be explained by regarding the earth as a uniformly magnetized sphere of intensity of magnetization I.   To an external point such a sphere behaves as a dipole of moment $M = \frac{4}{3} \pi r^3$ I, where r is the radius of the sphere.   In a place of northern latitude λ the vertical and horizontal components of the force are respectively $2M/r^3 \sin \lambda$ and $\frac{M}{r^3} \cos\lambda$.   The dip angle θ is therefore given by $\tan \theta = \frac{2M}{r^3} \sin \lambda / \frac{M}{r^3} \cos \lambda = 2 \tan\lambda$.

At the equator the observed magnetic intensity is found to correspond to $I = 0.08$ C.G.S units.   According to Schuster the maximum value of I impartable to a sphere is 100.   This means that a small fraction of the earth's volume is magnetized strongly or a large portion magnetized to about one-thousandth of the full intensity.   Under ordinary conditions iron (preponderating component of the earth) loses its property of magnetiztion at temperatures existing inside the earth at a depth of about 20 kilometres. Thus according to the hypothesis of permanent magnetization, the earth's field is due to a strongly magnetized thin shell near the earth's surface.   This produces difficulties in the explanation of the observed secular variations of the earth's magnetic field.   It must also be pointed out that the magnetic poles of the earth are not situated at the two extremities of a diameter and that their positions are continually shifting.   The theory of permanent terrestrial magnetism is therefore still incomplete.

Magnetic variations.   The analysis of Gauss has shown that slow variations in the earth's magnetic field are due to

changes of some sort in the crust or in the interior of the earth and that the more quick changes originate partly in the higher atmosphere of the earth.

As already stated the rapid changes are of two types : periodic (solar and lunar diurnal variations) and erratic (storms). A very brief summary of the various theories proposed to explain these important variations is given below.

The theories of the solar diurnal variation of the terrestrial magnetic field of the earth are (1) *the dynamo,* (2) *the diamagnetic* and (3) *the drift current.* The dynamo theory as first proposed by Balfour Stewart and later extended by Schuster conceived that the solar diurnal variation is caused by horizontal movements of conducting regions of the upper atmosphere across the vertical component of the earth's magnetic field. This induces an electromotive force given by $E = -v\,H$, where v is the velocity and H the field. The induced electromotive force produces electric currents causing variations in the magnetic field. Calculations on this theory lead to a very high wind velocity of the order of 2900 kms/hour; besides it requires a world-wide wind system. Such high winds are taken to be improbable. It may be said, however, that the dynamo theory has neither been proved nor disproved.

Gunn pointed out that the ionization in the upper regions is diamagnetic and worked out a diamagnetic theory of the solar diurnal variation. Assuming a certain distribution of intensity of magnetization it was shown that the calculated fields agreed closely with the observed effects. This was confirmed by Chapman. The diamagnetic theory requires a number of charged particles in the high atmosphere larger than the number indicated by ionospheric measurements. Thus its position is nearly the same as that of the dynamo theory.

An interesting theory of magnetic storms is what is called the corpuscular theory proposed by Chapman. According to this the sun emits in the beam particles of like electric charge. These plunging into the outer atmosphere cause downward movement of the conducting region. By dynamo action an induced eastward

current around the earth is produced. This causes an increase in the horizontal component of the earth's magnetic intensity constituting the first phase of the storm. Due to electrostatic repulsion of the charged particles the downward motion is followed by an upward movement causing an induced westward current. This is the second phase of the storm. Lindemann has pointed out that it is impossible for such a highly charged beam to approach the earth on account of electrostatic repulsion. Corpuscular theory of magnetic storms has not been finally accepted.

There is another interesting theory of magnetic storms and the associated auroras which has been recently developed. This is the ultra-violet theory. In this theory it is assumed that the sun sometimes radiates out excess of energy for a short time. Such a flare in solar energy lying in the ultra-violet region increases the number of ions in the upper atmosphere. In their downward motion they cause an increase in the eastward current producing an increase in the horizontal component of the magnetic field as in the initial phase of the storm. In the second phase the heating effect of the solar flare asserts itself. Due to heat the upper atmosphere expands. The ionized layers move upward causing diminution in the magnetic intensity; when the upward movement ceases, further possible increase in the magnetic intensity is stopped by the hindering action imposed by neutral molecules upon the movement of charged ions. This causes decrease in the magnetic intensity. As the flare disappears the storm ionization diminishes and the atmosphere and the magnetic condition assume their normal feature. The ultra-violet theory has been also applied to the auroral displays. For a full account of this interesting theory the student should reach the original papers [Hulburt Phys. Rev. 31, (1929), 34 (1929), 36 (1930), Marris and Hulburt Phys. Rev. 33 (1929)].

6. Atmospheric and terrestrial electricity. Ionization of the atmosphere. As early as 1887 it was established that the earth's atmosphere possesses electrical conductivity. About the year 1901 Elster and Geitel and C. T. R. Wilson show-

ed that this conductivity is due to the presence of positive and negative ions in the atmosphere. These ions are produced by radioactive matter in the surface of the earth and in air and cosmic and ultra-violet rays. The conducting layers situated at several miles above the ground constituting what is called the ionosphere play a very great part in the transmission of radio waves. Close to the earth the conductivity is greater over ground than that on the sea.

Fine weather electricity. In 1752 Lemonnier found that there is a definite electric field in the air above the earth in fine weather. The direction of the electric field is always vertically downwards towards the earth indicating that the earth is negatively charged and the atmosphere positively charged. Now, the field $F = -dV/_{dh} = 4\pi\sigma$, where V is the potential, h the height and σ the surface density of electric charge. Thus F can be determined by measuring either $-\dfrac{dV}{dh}$, the potential gradient or σ the surface density of electrical charge on the earth. The average value for $\dfrac{dV}{dh}$ is about 100 volts/metre and that for σ, $2\cdot7 \times 10^{-4}$ e.s.u/cm^2. In the foregoing definitions it is assumed that the surface of the earth is perfectly flat, free from all sorts of projections in the form of trees and buildings. The total fine weather electrical charge on the earth works out to about 5×10^5 coulombs. The value of σ is found directly by using Wilson electroscope and the potential gradient by finding the potential difference between two insulated conductors at different heights above the ground. It has been found that the potential gradient diminishes as the height above sea-level increases. At a height of about 15 kilometres the potential is about 3×10^5 volts above the ground value. This potential may be taken to be that of the whole upper atmosphere since beyond 15 kilometres the conductivity of the atmosphere is very great. The potential of the highly conducting "Kennelly-Heaviside" layer about 80 kilometres above the ground may thus be taken to be 3×10^5 volts.

Important variations of the potential gradient with locality, time and season have been obtained indicating that the potential gradient is not a fundamental quantity.

Due to the electric field towards the earth positive ions will be moving downwards and negative ions upwards with the result that there is a downward conduction current. In addition to this there is also a convection current which arises as follows : If at any point there is an excess of one kind of ions over the other (space charge) mechanical transference of charge due to wind will take place. If v is the vertical component of wind velocity directed upwards and ρ the space charge, the convection current is given by $v\rho$ and the total current towards the earth becomes (conduction current$-v\rho$). It has been found that the total fine weather current is about 2×10^{-16} amps/sq.cm. The whole current flowing to the earth from the atmosphere thus comes out to be about 1000 amperes. Unlike potential gradient this current is almost independent of locality, time and season and so is more fundamental.

Maintenance of earth's electrical charge. Thunderstorms. It has been mentioned above that the whole charge on the globe is about $-500,000$ coulombs and that the conduction current is 2×10^{-16} amps/cm.² directed to the ground. This current is sufficient to neutralize the earth's charge in about ten minutes. But in addition to this conduction current there is a precipitation current due to rain, snow, etc., which is estimated at about 400 amps. for the whole earth raising the total air-earth current to about 1400 amps. This would neutralize the earth's -5×10^5 coulombs in about six minutes time; but it is found that the earth practically maintains its charge. This indicates that some agency is constantly replenishing the earth's charge. On the basis of recent experiments it is thought that the most probable replenishing agency is the thunderstorm.

The occurrence of thunderstorms at one place is a rare phenomenon but it is estimated that over the whole globe some 1800 thunderstorms are occurring every moment. Recent measure-

ments indicate that the average thunderstorm develops and spends about two million kilowatts. Thus for the whole earth thunderstorms are spending electrical energy at the rate of some 4×10^9 kilowatts. This enormous power is of great importance in problems like the ionization of the atmosphere, maintenance of earth's negative charge and of the positive charge on the upper atmosphere.

7. Electricity of thunderstorms. It is now generally thought that the thunder cloud electricity is generated in the cloud itself. There are different theories regarding the origin and distribution of electrical charges in a thunder cloud.

Wilson's theory. According to Wilson the thunder cloud is made up of large negatively charged drops and small droplets carrying positive charge. The larger velocity of the bigger drops under gravity leads to the development of free negative charge at the bottom of the cloud and free positive charge at its top, the two regions of free charges being separated by a neutral region. Due to the increasing attraction between the two opposite free charges as the latter grow up the rate of their separation diminishes. Finally the electric field becomes so strong that a discharge may pass making the whole cloud electrically neutral if the charges are equal. Immediately after this lightning flash the large drops with negative charge once more separate out under gravity leading to electrical conditions suitable for another lightning flash. Wilson found that the charge on the negatively charged large drops is less than 30 e.s.u.

The process by which large negatively charged drops and small positively charged drops arise is as follows. In a cloud there are always some positive and negative ions and an electric field downwards. Now consider a water drop large in comparison to the positive and negative ions. Due to the downward electric field there arise polarisation charges on the drop negative on the upper side and positive on the bottom side. As it falls downwards under gravity it encounters the small positive ions moving more slowly downwards and the small negative ions

receding upwards. Due to repulsion of positive ions the bottom part of the drops captures negative ions. The upper part of the drop is not able to capture positive ions to any appreciable extent since they have been already repelled away by the drops and moreover the latter is receding from the ions. Innumerable repititions of this event will endow the drop with a considerable amount of resultant negative charge. The small droplets in the cloud moving with velocity of the same order as that of the ions are unable to make similar selective capture of negative ions and soon acquire a net positive charge.

Simpson's theory. Simpson has given quite a different picture of the origin and distribution of electrical charges in a thunder cloud. According to this theory the origin of the electrical charges in a thunder cloud lies in the breaking of water drops by the ascending current of air. The theory is based upon the following experimental facts : (1) water drops 5 mms in diameter and less never fall in a current of air moving upward with a velocity of 8 metres per second, (2) drops of size bigger than this are unstable and break up into smaller drops and (3) as a result of disruption, larger drops acquire positive charge and smaller droplets negative charge. In a thunder cloud the smaller droplets are carried to the top by the ascending air while the larger drops descend to the bottom. Thus the top of a thunder cloud is negatively charged while the bottom is positively charged.

Decisive experimental evidence in favour of one or the other view regarding the distribution and origin of thunder cloud electricity is still wanting ; perhaps both the processes are simultaneously at work in a thunder cloud.

PHYSICAL CONSTANTS

—:o:—

Avogadro number $N_0 = (6.0228_3 \pm 0.0011) \times 10^{-23}$ mole^{-1}

Electronic charge $\quad e = (1.60203_3 \pm 0.00034) \times 10^{-20}$ abs. e.m.u.

$\qquad\qquad\quad e^1 = (4.8025_1 \pm 0.0010) \times 10^{-10}$ abs. e.s.u.

Specific electronic

charge $\quad\quad e/m = (1.7592 \pm 0.0005) \times 10^7$ abs. e.m.u.g.$^{-1}$

$\qquad\qquad e'/m = (5.2736_6 \pm 0.0015) \times 10^{17}$ abs. e.s.u.g.$^{-1}$

Planck's const. $\quad h = (6.624_2 \pm 0.0024) \times 10^{-27}$ erg. sec.

$\qquad\qquad\quad h/e = (4.1349_0 \pm 0.0007_1) \times 10^{-7}$ erg. sec. abs. e.m.u.$^{-1}$

$h/e^1 = h/ec = (1.3793_3 \pm 0.0002_3) \times 10^{-17}$ erg. sec. abs. e.s.u.$^{-1}$

Boltzmann const. $\quad K = (1.38047_4 \pm 0.00026) \times 10^{-16}$ erg. deg.$^{-1}$

Gas constant $\qquad R = (8.31436 \pm 0.00038) \times 10^7$ erg.

$\qquad\qquad\qquad = (1.986467 \pm 0.00021$ Cal$_{15}$

Velocity of light $\qquad = (2.99776 \pm 0.00004) \times 10^{10}$ cm. sec.$^{-1}$

Gravitation const. $\quad G = (6.670 \pm 0.005) \times 10^{-8}$ dyne. cm.g.$^{-2}$

Standard atmos. $\quad A_0 = (1.013246 \pm 0.000004) \times 10^6$ dyne. cm.$^{-2}$ atmos

Joule equivalent $\quad$ J is $= 4.1855 \pm 0.0004$ abs-joule. Cal$^{-1}_{15}$

Joule quivalent $\quad$ J' is $= 4.1847 \pm 0.0003$ int.-joule. Cal$^{-1}_{15}$
(electrical)

Faraday constant. $\quad F = 96514 \pm 10$ abs-conl. g-equiv.$^{-1}$

Mass of electron $\quad m = (9.10660 \pm 0.0032) \times 10^{-28}$ g

Mass of proton $\quad H^+ = (1.67248_2 \pm 0.00031) \times 10^{-24}$ g

Mass of H-atom $\quad H = (1.673393 \pm 0.00031) \times 10^{-24}$ g

Mass of alpha particle

$\qquad\quad He^{++} \quad = (6.6442_2 \pm 0.0012) \times 10^{-24}$ g

Atomic Masses.

$\qquad\qquad H^1 = 1.00813 \quad \pm 0.000017$

$\qquad\qquad H^2 = 2.01473 \quad \pm 0.000019$

$$He_4 = 4\cdot00389 \quad \pm\ 0\cdot00007$$
$$C^{12} = 12\cdot00386 \ \pm\ 0\cdot00004$$
$$N^{14} = 14\cdot00753 \ \pm\ 0\cdot00005$$
$$O^{16} = 16\cdot0000$$
$$O^{17} = 17\cdot0045$$
$$O^{18} = 18\cdot0049$$

Mass of neutron $= 1.00893$

$$\frac{\lambda}{mc} = 2\cdot417 \ \times\ 10^{-10} \text{ cm.}$$

$$\frac{e^2}{mc^2} = 2\cdot807 \ \times\ 10^{-13} \text{ cm.}$$

$$mc^2 = 0\cdot5107 \times \text{ MEV.}$$

1 mass unit $= 931 \times 10^6$ ev.

Electron $= (5\cdot4862_4 \pm 0\cdot0017) \times 10^{-4}$

Acceleration due to gravity (standard)

$$g_0 = 980\cdot665 \text{cm. sec}^{-2}$$

Density of Hg $D_0 = 13\cdot59504_0 \pm 0\cdot00005_7$ g. cm^3

Electrochemical equivalent of Ag.

$$E_{Ag} = 1\cdot11800 \times 10^{-3} \text{ g int} - \text{coul}^{-1}$$
$$E_{Ag} = (1\cdot11807 \pm 0\cdot00012) \times 10^{-3} \text{g. abs} - \text{coul.}$$

Rydberg const. H $R_H = 109677\cdot581_2 \pm 0\cdot007_5$ cm^{-1}

D $R_D = 109707\cdot419_3 \pm 0\cdot007_5$ cm^{-1}

He $R_{He} = 109722\cdot263 \pm 0\cdot012$ cm^{-1}

$R = 109737\cdot303 \pm 0\cdot017$ cm^{-1}

Energy in ergs of one
electron volt (abso-
lute) $E_0 = (1\cdot60203_3 \pm 0\cdot00034) \times 10^{-12}$ erg.

$= 23052\cdot_{85} \pm 3\cdot_2$ Cal Mole^{-1}

Wave length for 1 elec-
tron-volt (absolute) $= (12395\cdot_4 \pm 2\cdot_1) \times 10^{-3}$ cm volt

Wave number for 1
electron-volt (abso-
lute) $S_0 = \dfrac{1}{\lambda_0} = 8067\cdot4_9 \pm 1\cdot4$ cm^{-1} volt^{-1}

Fine structure const.

$$1/\alpha = hc/2 \ e^{12} = 137\cdot030_2 \pm 0\cdot016$$

Magnetic   m o m e n t
(Bohr Magneton) $= \dfrac{eh}{4\pi\, mc} = (0.9273_{45} \pm 0.0003_7) \times 10^{-20}$ erg. gauss^{-1}

$= 5585.2_4 \pm 1.6$ erg. gauss^{-1} mole^{-1}

Mass of proton/Mass of electron $= 1836.5_{61} \pm 0.5_6$

INDEX

E R R A T A

4 line 21; read $1\cdot66 \times 10^{-24}$ gm as mass of proton.

92, line 5; read fig. $12\cdot12$ (*c*) in place of Fig. $13\cdot1$

· 192, line 5; read v_0 in place of ω_0 in equation $13\cdot13$

Page 260, line 6; replace $\times$ sign by $+$ sign.

Pa 315, line 2; replace Hev/c by Hev.

Page 318, para 4; read ρ in place of ρ_1

· 3/8, line 2; read $\frac{a_0}{4c}$ in place of $\frac{a_0}{4e}$

378; line 4; replace $t=10^{-8}$ sec. by $t=1\cdot35 \times 10^{-11}$ sec.