Chapter 1

Introduction

The dual representations of planar graphs have many applications in real life world. Among planar graphs, rectangularly dualizable graphs (RDGs) realize very nice geometric representations. Such geometric realizations are rectangular partitions as their duals. A rectangular partition is a partition of a rectangle into n-rectangles. The use of RDGs can be seen in cartogram maps [49, 65], floorplans for VLSI circuits [40, 55] and building architecture [21, 67]. In this thesis, we study planar graphs which can rectangularly be dualized, i.e., can be realized by rectangular partitions. We mainly develop various methods of transformations among rectangular duals.

1.1 Motivation

Constructing an optimal rectangular floorplan (rectangular partition) for VLSI circuit is a challenging problem due to its increasing size. A VLSI system structure can be described by a graph where vertices correspond to component rectangles and edges correspond to their required connections. The theory of rectangular dualizable graphs helps in deciding whether a rectangular floorplan can be realized from a given graph and therefore in this thesis, we study rectangularly dualizable graphs. For a given graph structure of a VLSI circuit, floorplanning is concerned with allocating space to component modules and their interconnections. An embedding method given by Heller [42] enforces interconnection by abutment. Modules are designed in such a way that their connectors exactly match with their neighbors. Adapting this methodology, interconnections are coped with a *clever* design. Due to the advancement of VLSI technology, it is extremely large and on the other hand, an RDG can handle atmost $3n-7$ interconnections [56], where n is the number of modules. Consequently, its graph described by VLSI circuit may not necessarily be planar and hence in modern

VLSI system, component modules and interconnections can not be treated as independent entities. In such a situation, not all interconnections can be enforced by abutment. Linking the remaining interconnections with nonadjacent modules utilize additional routing space.

For practical use, a graph described by a VLSI circuit can be embedded in such a way that most of the interconnections can be made by abutment and the remaining interconnections linking with nonadjacent modules use additional routing space. For instance in Fig. 1.1d, R_7 and R_9, R_2 and R_6 are interconnected through shaded red areas R_{10} and R_{11} respectively. These routing areas are anticipated by introducing crossover vertices. In fact, these vertices are introduced at the intersection of edges if it exists in order to embed a graph as a plane graph at a common point of intersection of edges.

The use of an RDG in floorplanning for VLSI circuits is explained through the following example. Consider a graph described by a VLSI system as shown in Fig. 1.1a. Note that input-output connections between VLSI system and outside world is represented by arrow heads (these are edges adjacent to a vertex at infinity). Although this extended graph is not planar, it is planarized [48] by adding cross over vertices as shown in Fig. 1.1b. In order to satisfy the necessary adjacency requirements, new edges (red edges) have been added in Fig. 1.1c. After these modifications, it is possible to construct a rectangular dual (rectangular floorplan) as shown in Fig. 1.1d where a component rectangle R_i is dualized to a vertex v_i.

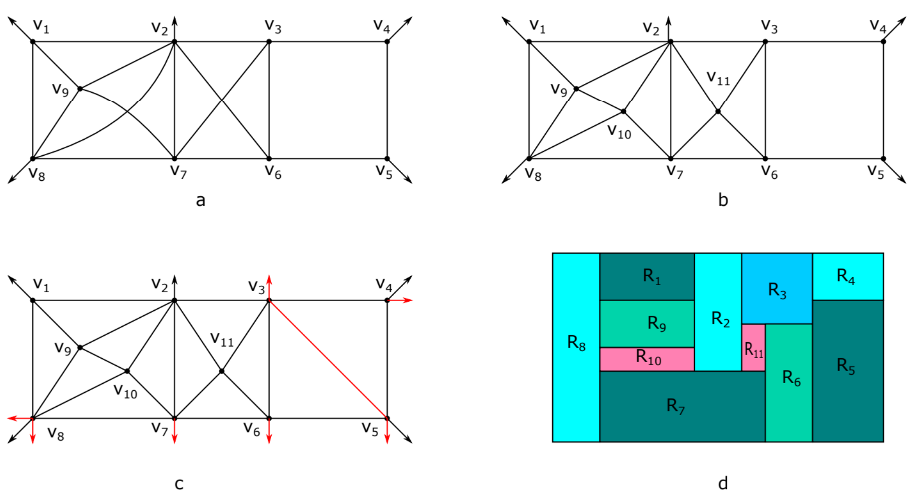

Figure 1.1: A rectangular dual of a plane graph described by a VLSI system.

Another representations of planar graphs are rectangular cartograms. A rectangular cartogram is a rectangular partition wherein rectangles represent the size of geographic regions (of some land, State etc.), and adjacencies and positions of rectangles are chosen to suggest the locations of geographic regions. The size of geographic regions can be proportion to some statistical parameter such as the gross national product, the population, the total birth etc. A cartogram of Germany is depicted in Fig. 1.2a where the districts of the federal states are reset according to their population. Fig. 1.2b is a corresponding rectangular cartogram (weighted rectangular dual). Therefore, the rectangular cartograms [49] are useful to visualize spatial information (it may be economic strength, population etc.) of geographic regions, i.e. they are used to display more than one quantity associated with the same set of geographic regions (in [49], the population, land area and wealth within the United States were shown as cartograms). The visual comparison of multiple cartograms corresponding to the same set of geographic regions can be made easier if each of the cartograms is area-universal.

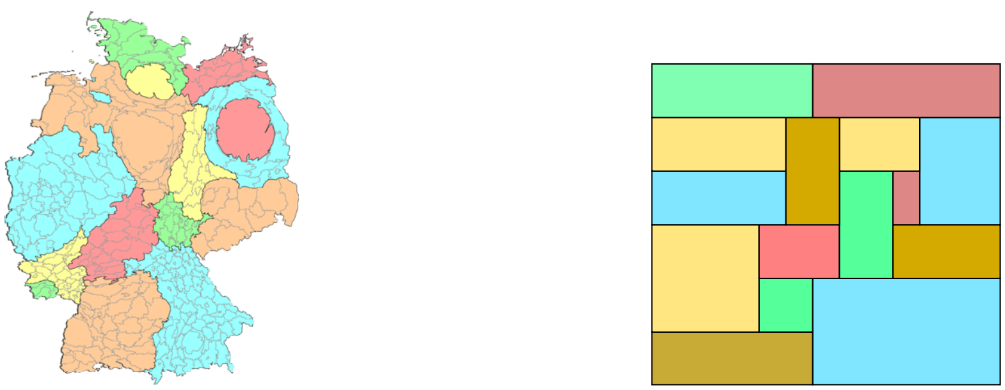

Figure 1.2: (a) A population cartogram of Germany [34] and (b) its rectangular cartogram

1.2 Research Objectives

The thesis focuses on the following objectives:

- To find a necessary and sufficient condition for a plane graph to be rectangularly dualized.
- To find a necessary and sufficient condition for a rectangular dual to transform to another rectangular dual and develop transformation algorithms for the construction.

- To find a necessary and sufficient condition for a plane graph to admit a unique rectangular dual upto combinatorially equivalence.
- To determine a class of planar graphs wherein each planar graph can be realized by an area-universal rectangular dual upto combinatorially equivalence.

1.3 Fundamentals of Rectangular Duals

A *graph* G is a triplet $(V, E, \bullet)$ consisting of a set V of vertices and a set E of edges with the relation $\bullet$ that every edge associates to a pair of vertices [68]. A graph is *planar* if it can be drawn in the Euclidean plane without crossing its edges except endpoints. A *plane graph* refers to a fixed planar embedding with no edge crossings. A plane graph is called a *plane triangulated graph* (PTG) if it has triangular faces. The exterior face may not be triangular in a PTG. In case, the exterior face is triangular, then the PTG is called *plane triangulation*. In this thesis, we consider PTGs with interior triangular faces only. An extended PTG (EPTG) can be obtained by adding edges between the vertices on the exterior and a vertex in the exterior face of the PTG.

A *generic rectangular partition* or simply a *rectangular partition* is a partition of a rectangle into $n-$ rectangles provided that no four of them meet at a point. A *generic rectangular dual* R of a plane graph G is a partition of a rectangle into $n-$ rectangles such that (i) no four of them meet at a point, (ii) rectangles in R are mapped to vertices of G and (iii) two rectangles in R share a common boundary segment if and only if the corresponding vertices are adjacent in G. The point where three or more rectangles of a rectangular dual meet is called a *joint*. It is known that a rectangular dual has 3-joints and 4-joints only where 4-joints are regarded as a limiting case of 3-joints [52]. The term ‘generic rectangular dual’ was introduced by Reading [50] because of not allowing 4-joints in the rectangular dual. In fact, a quadrangle region can always be partitioned into two triangular regions. In such cases, some extra adjacencies allow unrelated components in the given graph to connect, but these connections are not used for interconnection. Hence abiding by common design practice, we consider generic rectangular duals and simply call a generic rectangular dual by a rectangular dual in this thesis.

Two rectangular partitions with n-interior rectangles are *strongly equivalent* if the partitions contain (i) the same adjacency relations among rectangles and (ii) these adjacency relations have the same orientation (vertical or horizontal).

Two well-known classes of rectangular duals are slicible and area-universal rectangular duals. A rectangular dual is *slicible* if it can be recursively subdivided by axis-aligned line segments. Such rectangular duals are also known as *guillotine partitions* or floorplans. If a graph admits a slicible rectangular dual, then the graph is called a *sliceable graph*. Rectangular duals which are not sliceable are called nonslicible rectangular duals. The set of all slicible and nonslicible rectangular duals are *mosaic rectangular duals*. Fig. 1.3 depicts slicible and nonslicible rectangular duals. A rectangular dual is *area-universal* [22] if any assignment of areas to its rectangles can be realized by a combinatorially equivalent rectangular dual. If there associate areas to each of its rectangles, then the rectangular dual is known as a *rectangular cartogram*.

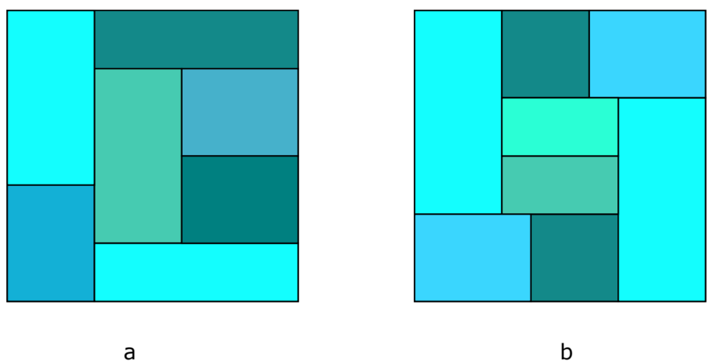

Figure 1.3: (a) a slicible rectangular dual and (b) a non-slicible rectangular dual

1.4 The State of the Art

The main problem in this thesis is to find the methods of transformations among rectangular duals. Before finding these methods of transformations, we first ask for a rectangular dual of a plane graph. We are also dealing with more demanding class: area-universal rectangular duals. We seek an important class of RDGs wherein every RDG can be realized by area-universal rectangular dual upto combinatorial equivalence in polynomial time.

1.4.1 Characterizations of Rectangular Duals

Rectangular partitions are characterized in term of graphs and in this context, they are known as rectangular duals of planar graphs. A plane graph is a *rectangularly dualizable graph* (RDG) if its dual can be embedded as a *rectangular partition*. Every planar graph can be dualized, but not rectangularly dualized, i.e., the class of RDGs is quite restrictive [38, 41, 51]. In 1985, Kozminski and Kinnen [38] derived a necessary and sufficient condition for a plane triangulated graph to be an RDG. In 1988, Lai and Leinward [41] showed that solving the rectangular dualization problem of an RDG is equivalent to a matching problem of a bipartite graph derived from the given graph. This theory relies on the assigned regions to the vertices of a plane graph. But this theory is not easy to implement, i.e., how can we check the assignments of regions to the vertices in an arbitrary given plane graph? In fact, this theory is not implementable until a method for checking assignments of regions to the vertices of an EPTG is known. Rinsma [51] showed through a counter example that it is not always possible for a vertex-weighted outer planar graph having 4 vertices of degree 2 to be an RDG. Besides this property, there are infinite outer planar graphs that are not rectangularly dualized. In fact, an outer planar graph having more than four critical shortcuts can not be rectangularly dualized. For practical reasons, outer planar graphs are not preferable for VLSI circuit's design and architectural complex buildings, i.e., the theory of rectangular dualizable outer planar graphs is limited to architectural small buildings only.

There are some interesting results about slicible rectangular duals, however slicible rectangular duals are not still fully characterized. The result given by Yeap and Sarrafzadeh [74] in 1995 is that all RDGs independent of a separating 4-cycle are always slicible. Then the result was modified to some extent by Dasgupta and Sur-Kolay [18]. They showed that if all 4-separating cycles are maximal in an RDG, then it is slicible. However, Mumford [43] found a critical flaw that invalidates these results. Recently, Kusters and Speckmann [39] introduced a recursively defined class of graphs, so-called rotating pyramids, which contain exactly one separating 4-cycle. It was conjectured that configurations of rotating pyramids can be determined only if a plane graph is slicible and verified the conjecture for the plane graphs that contain exactly one separating 4-cycle. The non-slicible graphs in this class are exactly the graphs that reduce to rotating windmills: rotating pyramids with a specific corner assignment. They proved that rotating windmills are not slicible and argued that all other graphs with exactly one separating 4-cycle are slicible.

1.4.2 Constructive Algorithms for Rectangular Duals

Implementing the theory of rectangular duals, a series of papers provided constructive algorithms.

Kozminski and Kinnen [36] implemented the rectangular dualization theory in quadratic time. Then making use of rectangular dualization theory, Bhasker and Sahani [8] provided a faster algorithm to construct rectangular duals of planar graphs and improved time complexity from quadratic to linear. Using regular edge labelings, a simpler construction method of rectangular duals was given by He [28] and the extension of regular edge labeling for 4-connected planar graphs was given by Kant and He [31].

Rectangular duals have been studied extensively by the VLSI community. Slicible layouts more easily facilitate certain steps in the layout process [47]. For instance, the problem of minimizing the perimeter or area of modules in a rectangular layout according to a given measure can be solved in polynomial time for sliceable layouts, but is NP-complete in general [58].

Slicible rectangular duals are studied without using graph notion and such rectangular duals are known as guillotine partitions or guillotine layouts. In this context, the notion of equivalence is different as in case of a dual graph. In fact, two guillotine partitions are equivalent if they induce the same structure tree [62]. Yao et al. [72] showed that the exact number of guillotine partitions is the n^{th} Schr̈oder number. Ackerman et al. [3] found the asymptotic number of guillotine partitions in higher dimensions.

The class of RDGs in which no RDG has any slicible realization is known as the class of inherently non-slicible graphs [59, 60, 61]. Dasgupta et al. [17] searched a method by which a rectangular dual with minimum number of non-slice cores can be constructed from a given RDG.

1.4.3 Adjacency Preserving Transformations of Rectangular Duals

In general, an RDG may admit a lot of rectangular duals. Rectangular duals of a given RDG thus generated are adjacency preserving. Adjacency preserving transformations of rectangular duals have been studied using graph notion [37, 40, 64]. By these transformations, a number of topologically distinct rectangular duals of a given RDG can be generated. Such transformations generate different regular edge labelings of an extended RDG representing a given rectangular dual. A rectangular dual R naturally induces a labeling of its extended dual graph E(G). If two rectangles of R share a boundary vertical segment, then blue color is assigned to the corresponding edge in E(G) and is directed from left to right otherwise if they share a horizontal segment, red color is assigned to the corresponding edge in E(G) and is directed from bottom to top. Then the orientations of all edges incident to some vertex v_i of an RDG G is a clockwise sequence of these edges composed of four subsequences: vertical edges directed into v_i, followed by horizontal edges directed into v_i and then vertical and horizontal from v_i. Such labeling is called *regular edge labeling*. Corresponding to a distinct regular edge labeling of an RDG, there is a topologically distinct rectangular dual of the RDG. Any arbitrary regular edge labeling of an RDG may not guarantee to admit a rectangular dual. More precisely, an RDG, in general, admits a lot of regular edge labelings. In 1997, Kant and He [31] presented an algorithm that produces a regular edge labeling obtained from assigning the directions of the edges of an RDG. Buchin et al. [13] established an upper bound on the number of edge regular labelings of an RDG. The concept of regular edge labelings is not only important because of their connection to find topological distinct rectangular partitions but also because of their connection to 4-connected plane graphs. Biedl et al. [9] showed that 4-connected plane graphs with at least four vertices on the exterior face can be extended to an irreducible triangulation. Regular edge labelings can then be used to obtain straight-line drawings of these graphs on a small grid [26]. Fusy [26] showed that there is a function $\alpha : V \to Z$ (the set of integers) such that the regular edge labelings of an irreducible triangulation G are in bijection with the α-orientations of the angular graph of G.

Besides the notion of strong equivalence of rectangular partitions, there is also a notion of weak equivalence, where two rectangular partitions are said to be equivalent if the incidence structure among rectangles and maximal line segments is the same. The number of weak equivalence classes can be seen in [2].

The approach considered by Felsner and Zickfeld [24] counts different ways of orienting the edges of planar graphs. The most general among these orientations is an α-orientation given by a planar graph $G = (V, E)$ and a function $\alpha : V \to N$ where an orientation of the edges of G is an α-orientation if every vertex v has out-degree $\alpha(v)$.

They established an upper bound that a planar graph can admit at most $O(3.73^n)$ α-orientations, for a given fixed function α. Regular edge labelings and α-orientations may not seem apparently related, however Fusy [26] showed that there is a bijection α_0 between the regular edge labelings of an irreducible triangulation G and the α_0-orientations of the angular graph of G (the angular graph of G is obtained by adding a vertex in every interior region of G and is connected to the three vertices of that region, then removes all original edges). There are $3n-6$ vertices in the angular graph, implementation of the general bound on α-orientations only gives a bound of $O(51.90^n)$ on the number of regular edge labelings which is far from the bound acheived in [13].

Another orientation of a connected plane graph G is bipolar orientation. This is an acyclic orientation of the edges of G with the property that there are exactly two vertices one with no incoming edges called the source and other with no outgoing edges called the sink. Felsner and Zickfeld [24] found an upper bound on the number of bipolar orientations that any connected planar graph has at most $O(3.97^n)$ bipolar orientations, while also showing that there exists planar graphs with $\Omega(2.91^n)$ bipolar orientations. Note that a regular edge labeling consists of two disjoint bipolar orientations (one on the red edges and one on the blue edges), one might expect that the number of regular edge labelings is related to the number of bipolar orientations. Indeed, any regular edge labeling can be turned into a bipolar orientation by adding a source and a sink vertex and connecting the new source to the red and blue sources and connecting the red and blue sinks to the new sink. However, in this way many regular edge labelings can be mapped to the same bipolar orientation, as some regular edge labelings differ only in edge colors. Conversely, although Kant and He [31] developed an algorithm that produces a regular edge labeling from the directions of the edges, not every bipolar orientation can be turned into a regular edge labeling this way. The reason for this is that bipolar orientations only require each (non-source and non-sink) vertex to have an in-and out-degree of at least one, while regular edge labelings require an in- and out-degree of at least two, one blue and one red.

There is also a connection between Bipolar orientations and α-orientations. Specifically, Rosenstiehl [53] showed that there is a bijection between bipolar orientations of a graph G and 2-orientations (α-orientations where every vertex has out-degree 2) of the angular graph of G. As this angular graph is always a quadrangulation, Felsner and Zickfeld [24] proved an upper bound of $\Omega(1.91^n)$ and a lower bound of $\Omega(1.53^n)$ on the maximum number of 2-orientations that a quadrangulation can have. Although, there exists a bijection between 2-orientations and bipolar orientations, the bounds differ because the number of vertices differs.

There also exist studied many other interesting substructures in planar graphs. Aichholzer et al. [4] listed the known upper bounds for various subgraphs contained in a triangulation: perfect matchings, spanning trees, Hamiltonian cycles, connected graphs etc. Recently, Buchin et al. [12] improved several of these bounds. Some of the techniques used to count these substructures can be helpful to count regular edge labelings.

1.4.4 Rectangulations

Counting of topological distinct rectangular partitions has been a major concern in combinatorics. Recently, trend of finding all partitions of a rectangle into a set of finite number of rectangles without considering prior adjacency relations of rectangles has been focused widely. In fact, these methods do not enumerate all rectangular partitions for a given adjacency list (i.e., for an RDG). This enumeration results into a large solution space. Then it is computationally expensive to pick a desirable solution from such a large solution space. Also, these approaches are restricted to blocks-packing in the minimal rectangular area only. From the context of practicality of these solutions, the other major concerns such as interconnection wire-lengths, aspect ratios etc. are lagged behind.

Earlier, Bloch and Krishnamurti [11] gave two algorithms that counts the number of rectangular partitions. These algorithms have no proofs of the correctness.

There exist special classes of rectangulations which are all based on Avis and Fukuda's reverse search method [7]. Specifically, the CAT (constant amortized time) algorithm described by Nakano [45] for generic rectangulations which does not produce a Gray code [54]. This algorithm further has been adapted by Takagi and Nakano [63] that generates generic rectangulations producing bounds on the number of rectangles that do not touch the outer face. Yoshii et al. [76] described a Gray code for diagonal rectangulations that is based on a generating tree. This Gray code requires at most 3 edges of the rectangulation in each step. Consequently, none of the listings produced by these earlier algorithms corresponds to a walk along the skeleton of the underlying polytope.

There exists a lot of work based on combinatorial properties of rectangulations.

Yao et al. [73] showed that there is one-to-one correspondence between diagonal rectangulations and the Baxter numbers and that guillotine diagonal rectangulations have one-to-one corresponds with the Schröder numbers. Ackerman [2] also established a bijection between diagonal rectangulations and Baxter permutations, which also yields a bijection between guillotine diagonal rectangulations and separable permutations. Shen and Chu [57] found asymptotic estimates for these two rectangulation classes. Moreover, He [27] gave an optimal encoding on diagonal rectangulations having n rectangles using $3n-3$ bits only.

The term 'generic rectangulation' was introduced by Reading [50], who showed a bijection between generic rectangulations and 2-clumped permutations and showed that these permutations are representatives of equivalence classes of a lattice congruence of the weak order on the symmetric group. Earlier, generic rectangulations had been studied under the name 'rectangular drawings' by Amano et al. [6] and by Inoue et al. [6, 25], who gave recursion formulas and asymptotic bounds for finding number of the rectangulations. As a more recent result, more general classes of rectangulations were given by Conant and Michaels [16].

Ackerman et al. [1] considered the setting by letting a set of n points in general position in a rectangle are given, and the goal is to partition the rectangle into smaller rectangles by axis-aligned n walls with the property that different walls passes through different points of the given set. They proved that for every set of points that forms a separable permutation in the plane, the number of possible rectangulations is the $(n+1)$st Baxter number, and for every point set the number of possible guillotine rectangulations is the n^{th} Schröder number. They also described an enumerating and generation procedure based on simple flips and T-flips using reverse search, which was later improved by Yamanaka et al. [70, 71]. Recently, Shekhawat [56] described an algorithm that computes a specific class of rectangular partitions with four rectangles on the exterior.

1.4.5 Rectangular Duals With Area-Universality Characteristic

A more demanding property of rectangular duals is area-universality. A series of papers studies area-universal rectangular duals in the context of rectangular cartograms where geographic regions are represented by rectangles. The position and adjacency relations of these rectangles are chosen to suggest their geographic locations and their areas correspond to the numeric values that the cartogram communicates. Kreveld and Speckmann [65] provided the first algorithm that compute rectangular cartograms. Eppstein et al. [22] presented a numerical algorithm for computing area-universal rectangular duals that approximates the correct areas assigned to its rectangles. A slicible RDG can be realized by a combinatorially equivalent rectangular dual with exactly the specified area assignment. A recent survey on various rectangular cartogram models can be seen in [46].

Not every RDG can be realized by an area-universal rectangular dual [51, 22]. Rinsma [51] described a vertex-weighted outer planar G (area is assigned to each of its vertices) such that no rectangular dual can be realized for G having these weights as rectangles' areas. Thus it is interesting to know when a rectangularly dualizable graph can be realized by an area-universal rectangular dual. Recently, Eppstein et al. [22] derived a necessary and sufficient condition for a rectangular dual to be area-universal: a rectangular dual is area-universal if and only if it is one sided. In general, the algorithm described by Eppstein et al. [22] for the construction of an area-universal rectangular dual for an RDG G (if it admits) is not fully polynomial. In fact, the computational complexity of this algorithm is $O(2^{O(K^2)}n^{O(1)})$ where K is the maximum number of 4 degree vertices in any minimal separation component. For instance, if K is fixed, it runs in polynomial time but in general, it runs in exponential time.

Several heuristic attempts for constructing area-universal rectangular duals can be seen in [69, 66]. The area-universal convex polygonal drawings for biconnected outer planar graphs are given in [14].

Recently, it has been shown that a planar graph G is area-universal if for any area assignment to its inner regions, a straight line drawing of G can be realized [33, 35] and the area-universality characteristics for subgraphs of such graphs can be seen in [10]. The area-universality concept is not limited to rectangular duals. It is also extended to rectilinear duals [32, 5]. More recently, William et al. [23] proposed methods of area-universality for large classes of plane quadrangulations. They showed that these methods work strongly for all plane quadrangulations with up to 13 vertices to prove their area-universality.

1.4.6 Extension of Rectangular Duals

Several papers consider rectilinear duals: a generalization of rectangular duals which is composed of simple (axis-aligned) rectilinear polygons instead of rectangles [5, 15, 29, 75, 80]. Every PTG admits a rectilinear dual where every polygon has eight sides, and eight sides are sometimes necessary [29, 15, 75]. A series of papers studies the question of how many sides are required to respect all adjacency relations and area requirements in general. The first bound was given by Berg et al. [20] showing that forty sides for each polygon is always sufficient. Recently an improved bound was given by Alam et al. [5] proving that eight sides for each polygon is always sufficient.

1.5 Research Gaps and Results

On the basis of the above literature discussion, we found that there exist gaps in the study of rectangular duals.

Investigations in the literature shows that rectangular dualization theory is not well studied. Kozminski and Kinnen [38] found a necessary and sufficient for a separable connected PTG to be an RDG. We have found a critical flaw that invalidates this necessary and sufficient condition. We present a counter example in Chapter 3 showing thereby that it is not true for all separable connected PTGs. Another result on the rectangular dualization theory was given by Lai and Leinward [41]. They derived a necessary and sufficient condition for an EPTG to be an RDG. This theory is not implementable until a method for checking assignments of regions to vertices in the EPTG is known. The work described by Rinsma [51] is given for very restrictive class of RDGs (outer planar graphs).

This motivated us to reconsider rectangular dualization method and we have derived a necessary and sufficient condition for a given PTG to be an RDG. The result derived in this thesis is much simpler and very useful in constructing floorplans for VLSI circuits and architectural buildings.

As we have seen that a series of papers studies transformations methods among rectangular duals preserving adjacency relations. More precisely, a new topological distinct rectangular dual is derived from an existing one without disturbing adjacency relations of component rectangles of the existing one. The result is that both rectangular duals corresponds to the same RDG. Contrary to this, we have given the different methods of transformations among rectangular duals where the set of adjacencies of the new rectangular dual is either subset or superset of the original rectangular dual. By these methods of transformations, one can introduce new adjacency relations to a given rectangular dual to generate a new one and remove possible existing adjacency relations in a rectangular dual until it remains a rectangular dual. We also present transformations algorithms for their constructions.

In general, it is well known that the dual solution space of an RDG is very large. But we have found that there exists a class of RDGs wherein each RDG can be realized by a unique rectangular dual upto combinatorial equivalence. Such class of RDGs is still unknown. Keeping this in mind, we derived a necessary and sufficient for a plane graph to admit a rectangular dual upto combinatorially equivalence. On the basis of a deep investigation through examples, we have conjectured that no biconnected RDG admits a unique rectangular dual.

As we have seen that the existing algorithm [22] for constructing area-universal rectangular dual for a given RDG (if it admits) is, in general, not fully polynomial. We have derived an important class of RDGs wherein each RDG can be realized by an area-universal rectangular dual in polynomial time. Although it apparently looks very simple class, but by a closer examination, it will contribute a lot in the direction of area-universality characterizations. We have further conjectured that an RDG admits an area-universal rectangular dual if and only if it can be split as the union of graphs of this class. This results that every area-universal rectangular dual can be constructed in polynomial time.

1.6 Outline

A brief description of the structured work in this thesis is given as follows:

In Chapter 2, we provide basic facts about RDGs and rectangular duals that would be helpful to understand the subsequent chapters of this thesis. We also describe the methodology, we adapt to derive the results of this thesis.

In Chapter 3, we derive a necessary and sufficient condition for a plane triangulated graph to be an RDG.

In Chapter 4, we introduce a maximal rectangularly dualizable graph (MRDG) and discuss its properties. In particular, we introduce RDG property preserving operations and characterization of an RDG in the term of MRDG. We also show that there always exists an MRDG for a given RDG and provide a polynomial time algorithm for the construction of the MRDG.

In Chapter 5, we introduce the concept of edge-reduction in an RDG and define two types of RDGs: an edge-reducible RDGs and irreducible RDGs. Then we derive a necessary and sufficient condition for an RDG to transform to an edge-reducible RDG. Next we derive a necessary and sufficient condition for an RDG to be edge-irreducible RDG. We also present a polynomial time algorithm for their constructions.

In Chapter 6, we derive a necessary and sufficient condition for an RDG to admit a unique rectangular dual upto combinatorial equivalence.

In Chapter 7, we derive a class of RDG wherein each RDG can be realized by an area-universal rectangular dual. We present a polynomial time algorithm for its construction. We deals with the computational complexity of area-universality. Every RDG of this class is characterized by the fact that every induced subgraph of each of RDGs can be realized by an area-universal rectangular dual upto combinatorial equivalence.

Chapter 8 consists of conclusive summary of the work done in this thesis, several opens problems and various conjectures related to the work described in this thesis.

This document was created with the Win2PDF “print to PDF” printer available at
http://www.win2pdf.com

This version of Win2PDF 10 is for evaluation and non-commercial use only.

This page will not be added after purchasing Win2PDF.

http://www.win2pdf.com/purchase/