Chapter 4

Maximal Rectangularly Dualizable Graphs

In this chapter, for a given RDG, we ask for a new RDG by introducing new adjacencies while preserving all the existing adjacencies among the vertices of the given RDG until no more adjacency can be added to the vertices of the RDG. We first prove that such transformation is always possible and then present an algorithm that constructs the new RDG. As an RDG admits a rectangular dual, therefore transforming an RDG to another RDG is equivalent to finding a transformation between the corresponding rectangular duals.

4.1 Introduction

For a given n, the counting of rectangular duals having n-rectangles has been a fundamental theme in combinatorial geometry [2, 27, 50, 56, 70, 72]. In context of graphs, a series of papers studies transformations of rectangular duals [37, 40, 64] and the goal is to find a topologically distinct rectangular dual of the same RDG. In other words, transformed rectangular dual has the same adjacencies among rectangles, but directions of adjacencies among rectangles are not the same. This means that these transformations preserve adjacencies among rectangles. However, Wang *et al.* [67] studied transformation between rectangular duals by adding (removing) rectangles to (from) a rectangular dual.

Contrary to the existing work for transformation between rectangular duals, in this chapter, we study a method of transformation between rectangular duals with the property that the set of adjacencies of rectangles of transformed rectangular dual is the superset of adjacencies of rectangles of the input rectangular dual from graph notion. In fact, we are here looking for the answer of a major question: can any two given non-adjacent rectangles of a rectangular dual be made adjacent provided adjacencies of the remaining rectangles of the rectangular dual do not get disturbed and the resultant dual is a rectangular dual? To do this, we define a class of maximal RDGs (MRDGs) in this chapter and first develop theory on graphs to construct MRDGs from RDGs. Then we present a polynomial time algorithms for the construction of MRDGs

The class of MRDGs can play an important role in floorplanning because they are rich in adjacency relations. As an application, their respective rectangular duals supply maximal adjacencies of its rectangles. Therefore, it is interesting to construct an MRDG of a given RDG. Intuitively speaking, an MRDG is an RDG having maximal adjacencies of its vertices. In fact, an MRDG with n vertices has $2n-2$ or $3n-7$ edges. We show that there always exists an MRDG for a given RDG. Then we present a polynomial time algorithm that constructs the MRDG for the given RDG by adding new edges among its non-adjacent vertices. The number of such new edges is $2n-2-k$ or $3n-7-k$ where k is the number of edges in the RDG. It would be more interesting if the given RDG is a path graph (the one which has minimal adjacencies) because then it requires the new edges to be added in bulk. Equivalently the new edges in bulk can be added to a Hamiltonian path of the given RDG to realize a new desired form of the RDG. In this way, we can just add those edges of an RDG that are missing in the RDG and we are done. As an RDG admits a rectangular dual, therefore transforming an RDG to another RDG is equivalent to finding a transformation between the corresponding rectangular duals.

The chapter is structured as follows. In Section 4.2, we introduce an MRDG and show that the number of edges in an MRDG is $2n-2$ or $3n-7$ where n represents the number of vertices. MRDGs with $2n-2$ edges are wheel graphs whereas MRDGs with $3n-7$ edges are obtained from the class of maximal plane graphs with the property that they do not have any separating triangle in their interiors by deleting one of their exterior edges. In Section 4.3, we prove that it is always possible to construct an MRDG for a given RDG. In Section 4.4, we present an algorithm for the construction of an MRDG from the given RDG. Finally, we conclude our contribution in Section 4.5.

4.2 Properties of Maximal RDGs

In this section, we introduce an MRDG and drive some important properties of the MRDG.

Definition 4.2.1. An RDG $G = (V, E)$ is an MRDG if there does not exist an RDG $G' = (V, E')$ such that $E \subsetneq E'$. Intuitively, an RDG is an MRDG if adding a new edge to it violates the RDG properties.

For instance, the graph in Fig. 4.1a is an MRDG since adding a new edge to it, its exterior becomes triangular and hence one of rectangles of the corresponding rectangular dual (see 4.1a) transforms to a non-rectangle. Note that R_i is dualized to v_i.

Theorem 4.2.1. The number of edges in an MRDG is $2n-2$ or $3n-7$ where n denotes the number of vertices in the MRDG.

Proof. Let M be an MRDG with n vertices. If $d(v_i) = n-1$ for some vertex $v_i \in M$, then clearly it is a wheel graph W_n. W_n is independent of separating triangle as well as CIPs. By Theorem 2.2.1, it is an RDG. Note that adding a new edge to W_n generates a separating triangle passing through its central vertex and two of its exterior vertices. This implies that it is a maximal RDG. Now by the degree sum formula, the sum of degrees of all vertices of a graph is twice of the number of its edges. This implies that $3(n-1)+(n-1) = 2$ (the number of edges in W_n) and hence the number of edges in W_n is $2n-2$.

We have shown that W_n is an MRDG with $2n-2$ edges and in this case, the number of edges in M is $2n-2$.

Now suppose that $d(v_i) < n-1, \ \forall \ v_i \ \in M$. Consider a maximal plane graph G with n vertices such that there does not exist any separating triangle in its interior. We claim that $M = G-(v_i, v_j)$ for some exterior edge (v_i, v_j) of G. In order to claim this, we need to show that $G-(v_i, v_j)$ is an MRDG with $3n-7$ edges.

By our assumption on G, it is evident that $G-(v_i, v_j)$ has no separating triangle. Suppose that $G-(v_i, v_j)$ has a CIP. Then there is a shortcut (v_s, v_t) in $G-(v_i, v_j)$. This implies that G has a separating triangle $v_s v_t v_e$ in its interior where v_e is its exterior vertex. This is a contradiction to our assumption that G has no separating triangle in its interior. Thus we see that $G-(v_i, v_j)$ has no CIP. Thus By Theorem 2.2.1, it is an RDG.

The number of edges in a maximal plane graph is $3n-6$. This implies that $G-(v_i, v_j)$ has $3n-7$ edges. Note that adding a new edge to $G-(v_i, v_j)$, creates a separating triangle in $G-(v_i, v_j)$ and hence it is an MRDG.

Thus we see that $G-(v_i, v_j)$ is an RDG with $3n-7$ edges and hence M is an MRDG with $3n-7$ edges.

□

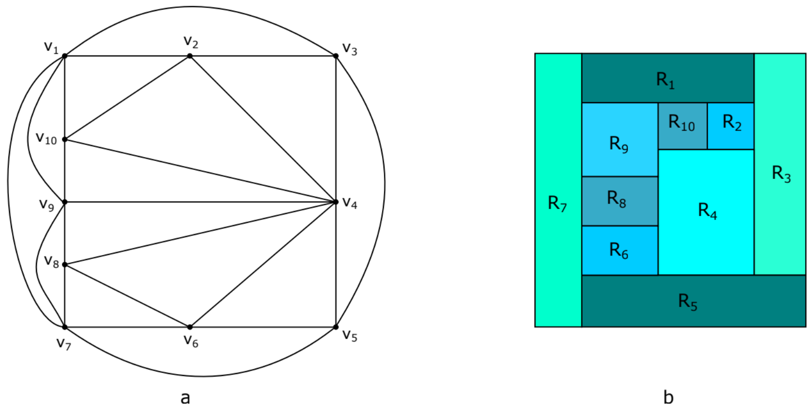

Figure 4.1: (a) An MRDG and (b) corresponding rectangular dual

Theorem 4.2.2. The number of vertices on the outermost cycle of an MRDG with n vertices is 4 or $n-1$.

Proof. Let M be an MRDG with n vertices. If M is a wheel graph W_n, then it is clear that it has $n-1$ vertices on its exterior. Otherwise it is obtained from a maximal plane graph that has no separating triangle in its interior by deleting one of its exterior edges. But a maximal plane graph has 3 vertices on its exterior. Then it is evident that M has 4 vertices on its exterior. □

4.3 MRDG Construction

In this section, we prove that it is always possible to construct an MRDG $M=(V,E)$ for a given RDG $G=(V,E_1)$ such that $E_1 \subsetneq E$.

Before proceeding to the main result, we first need to prove some lemmas. Denote $|N(v_i)\cap N(v_j)|$ by s for any two adjacent vertices v_i and v_j of an RDG G.

Lemma 4.3.1. If $s=0$, then (v_i,v_j) is an exterior edge of G.

Proof. For $s=0$, (v_i,v_j) is a cut-edge (bridge) and hence is an exterior edge. □

Lemma 4.3.2. For all adjacent vertices v_i and v_j, we have $s\leq 2$.

Proof. To the contrary, suppose that $s=3$. Consider a plane embedding G_e of G. Since, $s=3$, $N(v_i)$ and $N(v_j)$ must have 3 common vertices v_k, v_l and v_m which results in 3 cycles $v_iv_jv_k$, $v_iv_jv_l$ and $v_iv_jv_m$ in G_e. Now (v_i, v_j) is a common edge in these 3 cycles. This implies that atleast 2 of 3 cycles would lie on the same side of (v_i, v_j) in G_e. This means that one of the cycles encloses some vertex v_t of the other cycle and hence is not a face in G_e. Therefore its removal results G_e in a disconnected graph and hence it is a separating triangle in G, which is a contradiction to Theorems 2.2.1 and 2.2.2 since G is an RDG. Similarly, if $s \geq 3$, we arrive at the contradiction. □

Lemma 4.3.3. (v_i, v_j) is an interior edge of G if and only if $s = 2$.

Proof. First suppose that $s = 2$. We need to show that (v_i, v_j) is an interior edge in G. To the contrary, suppose that (v_i, v_j) is an exterior edge of G. Let $N(v_i) \cap N(v_j) = \{v_k, v_l\}$. Since (v_i, v_j) is an exterior edge, there exist two triangles $v_iv_jv_k$, $v_iv_jv_l$ in the plane embedding of G such that both lie on the same side of (v_i, v_j). This implies that one of them contains the other and hence is not a region (face) and is a separating triangle. This is a contradiction to Theorems 2.2.1 and 2.2.2 since G is an RDG.

Conversely, suppose that (v_i, v_j) is an interior edge in G. Since G is an RDG, each of its interior regions is triangular. This implies that there exist two triangles $v_iv_jv_k$ and $v_iv_jv_l$ in the plane embedding of G. Hence $N(v_i)$ and $N(v_j)$ have atleast two vertices in common, i.e., $s \geq 2$. By Lemma 4.3.2, we have $s \leq 2$. Hence $s = 2$. □

Corollary 4.3.1. If $s = 1$, then (v_i, v_j) is an exterior edge of G.

Proof. It is the direct consequence of Lemmas 4.3.1 and 4.3.3. □

Lemma 4.3.4. It is always possible to construct a nonseparable (biconnected) RDG from a separable connected RDG by adding edges to it.

Proof. Let $G_1 = (V, E_1)$ be a separable connected RDG such that it has atleast one bridge (cut-edge). Suppose that $L_1 = \{(v_i, v_j) \in E_1 \mid |N(v_i) \cap N(v_j)| = 0\}$ and $L_2 = \{(v_a, v_b) \in E_1 \mid |N(v_a) \cap N(v_b)| = 1\}$. Consider two adjacent edges, (v_i, v_j) from L_1 and (v_j, v_k) from L_2 such that $|N(v_i) \cap N(v_k)| = 1$. Such selection is always possible since both edges belongs to different blocks and $N(v_i) \cap N(v_k) = \{v_j\}$.

Construct a graph $G_2 = (V, E_2)$ where $E_2 = E_1 \cup \{(v_i, v_k)\}$. To prove G_2 is an RDG, we prove the following:

- there does not exist a separating triangle passing through (v_i, v_k) in G_2

  There would be a separating triangle passing through (v_i, v_k) in G_2 if $|N(v_i) \cap N(v_k)| = 2$ in G_2. In this case, $v_j, v_r \in N(v_i) \cap N(v_k)$ such that v_r lies inside the triangle passing through (v_i, v_k). But $(v_i, v_j) \in L_1$ and $(v_j, v_k) \in L_2$. Therefore by Lemma 4.3.1 and Corollary 4.3.1, both (v_i, v_j) and (v_j, v_k) are the exterior edges and v_i and v_k belongs to different blocks in G_1. Hence in G_2, $|N(v_i) \cap N(v_k)| = 1$ is the only possibility.

- the number of critical CIPs in G_2 can not exceed the number of critical CIPs in G_1

  In G_2, a critical CIP can only pass through (v_i, v_k), which already passes through (v_i, v_j) and (v_j, v_k) in G_1. But v_j is a cut vertex which is a contradiction to the fact that a critical CIP never passes through a cut vertex.

Since G_1 is an RDG, each of its region is triangular. The new edge (v_i, v_k) is added with the property that $|N(v_i) \cap N(v_k)| = 1$. By Corollary 4.3.1, (v_i, v_k) is exterior edge in G_2. Therefore, the new region $v_i v_j v_k$ is triangular in G_2. By Theorem 2.2.2, G_2 is an RDG.

After adding (v_i, v_k) to G_1, the edge (v_i, v_j) from L_1 belongs to L_2 since $|N(v_i) \cap N(v_j)| = 1$ ($|N(v_i) \cap N(v_j)| = \{v_k\}$). Therefore a recursive process shows that at the iteration until L_1 is empty, $G_{k+1} = (V, E_{k+1})$ becomes a separable connected RDG with cut-vertices (vertex), but no cut edge where $E_{k+1} = E_k \cup (v_a, v_c)$ such that (v_a, v_b) is from L_1 and (v_b, v_c) is from L_2 with the property $|N(v_a) \cap N(v_c)| = 1$. In this way, we can construct a separable connected RDG having cut-vertices of the given separable connected RDG only.

It now remains to show that it is always possible to construct a nonseparable (biconnected) RDG of the given separable connected RDG $G_1 = (V, E_1)$ having cut-vertices but no cut-edges. Let v_t be its cut-vertex. Since it has no cut-edge, $d(v_t) \geq 4$. A plane embedding of G_1 with exterior cycles C_1 and C_2 sharing a cut vertex v_t is shown in Fig. 4.2a. It is evident from this embedding that there is no separating triangle passing through the new added edges (red edges) in the resultant graph shown in Fig. 4.2b. Since v_t is a cut-vertex, none of the vertices v_1, v_2, v_3 and v_4 in Fig. 4.2, which are adjacent to v_t, can be the endpoints of a shortcut in G_1. This implies that the number of CIPs in the resultant graph (shown in Fig. 4.2b) can not exceed the number of critical CIPs in G_1. This proves the required result. □

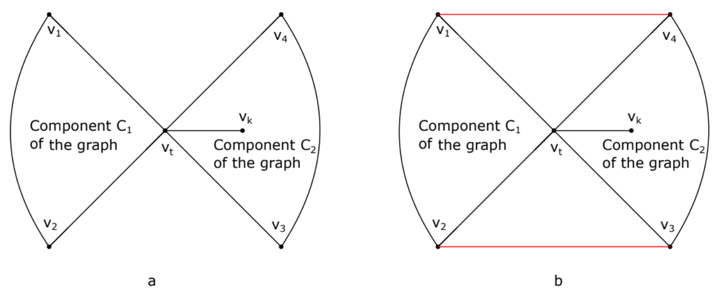

Figure 4.2: Constructing a nonseparable RDG of a separable connected RDG G_1 with a cut-vertex v_t shared by its two components C_1 and C_2.

Remark 4.3.1. It is not straight forward to add edges to a separable connected RDG provided RDG properties do not violate. Randomly adding new edges to an RDG may disturb the RDG properties, i.e., can produce either a CIP or a separating triangle in the resultant graph. In the lie of this, we have shown the procedure of adding edges according to Lemma 4.3.4. For instance, consider a separable connected RDG shown in Fig. 4.3a. It is transformed to a nonseparable graph by adding new edges (red edges) randomly. As a result, the nonseparable graph thus obtained is not an RDG. In fact, it contains a separating triangle $v_4v_5v_7$. On the other hand, red edges are added to the same graph by using Lemma 4.3.4 in order to construct a nonseparable RDG shown in Fig. 4.3b. Thus, Lemma 4.3.4 is helpful in introducing pattern of new edges to be added to a separable connected RDG to be a nonseparable RDG.

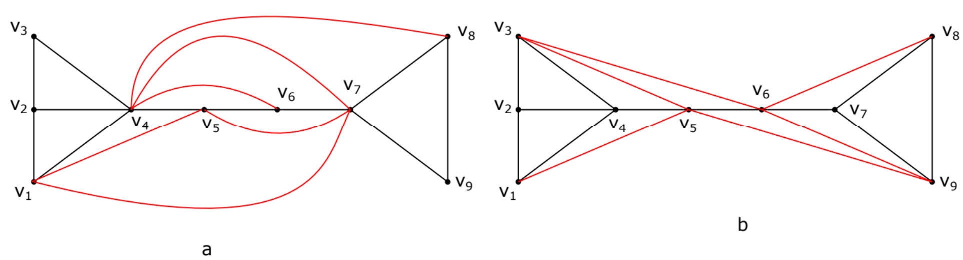

Figure 4.3: (a) A random addition of new edges (red edges) to a separable connected RDG destroy the RDG property because of the presence of a separating triangle $v_4v_5v_7vv_4$ in the resultant graph (b) while addition of new edges (red edges) using Lemma 4.3.4 do not destroy RDG property (neither separating triangle nor any CIP in the resultant graph).

Lemma 4.3.5. It is always possible to construct an MRDG from a biconnected RDG by adding edges to it.

Proof. Let $G_1 = (V, E_1)$ be a biconnected RDG. If $|E_1| = 3|V| - 7$ or $G_1 = W_n$, then G_1 is itself an MRDG and the proof is obvious.

Suppose that $|E_1| < (3|V| - 7)$ and $G_1 \neq W_n$. Assume that $L_2 = \{(v_i, v_j) \in E_1 \mid |N(v_i) \cap N(v_j)| = 1\}$. By Lemma 4.3.1, L_2 is a list of all exterior edges of G_1.

We now prove that there exists atleast a pair of adjacent edges (v_i, v_j) and (v_j, v_k) in L_2 such that $|N(v_i) \cap N(v_k)| = 1$. If such pair does not exist, then $|N(v_a) \cap N(v_c)| = 2$ for each pair (v_a, v_b) and (v_b, v_c) in L_2. In fact, since G_1 is a biconnected graph, by Lemma 4.3.2, we have $|N(v_a) \cap N(v_c)| \in \{1, 2\}$. Let $v_1 v_2 \dots v_p$ be the outermost cycle of G_1. Note that all edges (v_1, v_2), (v_2, v_3) $\dots (v_{p-1}, v_p)$ and (v_p, v_1) are exterior and hence by Lemma 4.3.2, all these edges belongs to L_2. Now if we choose (v_1, v_2) and (v_2, v_3), then $|N(v_1) \cap N(v_3)| = \{v_2, v_c\}$. Again if we choose (v_2, v_3) and (v_3, v_4), then $|N(v_2) \cap N(v_4)| = \{v_3, v_c\}$. Continuing in this way, we see that all the exterior vertices are adjacent to v_c. Observe that the vertex v_c and every adjacent exterior vertices v_i and v_j forms a triangle. Therefore, if G_1 has any other vertex (except $v_1, v_2, \dots, v_p$ and v_c), it would lie inside the triangle $v_i v_j v_c$, which is a separating triangle. This contradicts the fact that G_1 is an RDG. This implies that G_1 cannot have any other vertex (except $v_1, v_2, \dots, v_p$ and v_c) which concludes that G_1 is a wheel graph W_n which is again a contradiction since we assumed that $G_1 \neq W_n$. This proves our claim.

Choose two adjacent edges (v_i, v_j) and (v_j, v_k) from L_2 such that $|N(v_i) \cap N(v_k)| = 1$ and construct a graph $G_2 = (V, E_2)$ where $E_2 = E_1 \cup \{(v_i, v_k)\}$.

Now we show that the number of CIPs in G_2 can not exceed the number of CIPs in G_1. For G_2, there are the following possibilities:

i. None of vertices v_i and v_k is the endpoint of a shortcut in G_1,

ii. One of vertices v_i and v_k is the endpoint of a shortcut in G_1,

iii. Both vertices v_i and v_k are the endpoints of a shortcut in G_1.

These 3 possibilities are shown in Fig. 4.4a-4.4c respectively. In the first case, clearly there is no CIP passing through (v_i, v_k) in G_2. In the second case, $v_i v_k v_{k+1} \dots v_{q-1} v_q$ becomes a CIP in G_2 and $v_i v_j v_k v_{k+1} \dots v_{q-1} v_q$ no longer remains a CIP in G_2. In fact, the edges (v_i, v_j) and (v_j, v_k) of the existing CIP in G_1 are replaced by (v_i, v_k) in G_2. Thus, in this case, the number of CIPs do not get increased. The third case is not possible since $|N(v_i) \cap N(v_k)| = 2$. In fact, $N(v_i) \cap N(v_k) = \{v_j, v_s\}$ and G_2 is obtained from G_1 by adding an edge (v_i, v_k) such that $|N(v_i) \cap N(v_k)| = 1$. This proves our claim.

Now we claim that there does not exist a separating triangle passing through (v_i, v_k) in G_2. Since $|N(v_i) \cap N(v_k)| = 1$, i.e., $N(v_i) \cap N(v_k) = \{v_j\}$. Therefore $v_i v_j v_k$ is the only cycle of length 3 having no vertex inside and passing through (v_i, v_k) in G_2. This shows that $v_i v_j v_k$ is not a separating triangle, it is a new added triangular region (face) in G_2. By Theorem 2.2.1, G_2 is an RDG. A recursive process shows that each $G_i = (V, E_i)$, $(i \geq 3)$ is an RDG where $E_i = E_{i-1} \cup \{(v_a, v_c)\}$ such that $|N(v_a) \cap N(v_c)| = 1$ for some edges $(v_a, v_b), (v_b, v_c)$ belong to L_2 which is defined as $L_2 = \{(v_i, v_j) \in E_{i-1} \mid |N(v_i) \cap N(v_j)| = 1\}$.

It can be noted that the recursive process will terminate when the outermost cycle has four vertices for some RDG G_k. In fact, (v_i, v_j), (v_j, v_k), (v_k, v_l) and (v_l, v_i) are four edges constituted by the four exterior vertices v_i, v_j, v_k and v_l of some RDG G_k. For any two edges (v_a, v_b) and (v_b, v_c), we have $|N(v_a) \cap N(v_c)| = 2$. This terminate our process. On the other hand, there does not exist any other way for adding a new edge such that the resultant graph is an RDG with a new triangular region. Recall that a maximal plane graph has $3|V_1| - 6$ edges where V_1 denotes its vertex set and has all triangular regions including exterior. In our case, every region of G_k is triangular, but exterior is quadrangle. This implies that the number of edges in G_k is $3|V| - 7$ and hence it is an MRDG. This completes the proof of lemma. □

From Lemmas 4.3.4-4.3.5, we conclude that the following main result of the paper.

Theorem 4.3.1. It is always possible to construct an MRDG of a given RDG.

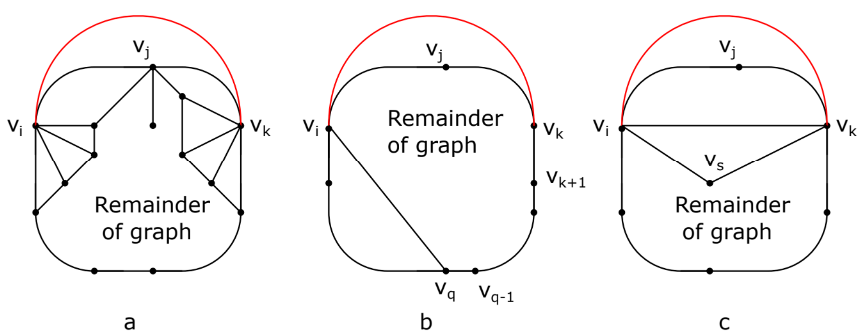

Figure 4.4: Three possible depictions of G_2 obtained from G_1 (consists of black edges) by adding a red edge.

4.4 Algorithm for Constructing of an MRDG

In this section, we present an algorithm that constructs an MRDG for a given RDG. We also analyze its complexity and present an illustrative example.

Algorithm 1 Constructing an MRDG of a given RDG

Input: An RDG $G_1 = (V, E_1)$
Output: An MRDG $M = (V, E)$ for $G_1 = (V, E_1)$ such that $|E_1| < |E|$

1: $L_1 \leftarrow \phi$
2: $L_2 \leftarrow \phi$
3: **for all** $(v_i, v_j) \in E_1$ **do**
4:     $s \leftarrow |N(v_i) \cap N(v_j)|$
5:     **if** $s == 0$ **then**
6:         $L_1 \leftarrow L_1 \cup \{(v_i, v_j)\}$
7:     **else if** $s == 1$ **then**
8:         $L_2 \leftarrow L_2 \cup \{(v_i, v_j)\}$
9:     **else**
10:         **continue**
11:     **end if**
12: **end for**
13: **for all** $(v_i, v_j) \in L_1$ **do**
14:     **if** $(v_j, v_k) \in L_2$ **then**
15:         $L_2 \leftarrow (L_2 \cup \{(v_i, v_j), (v_i, v_k)\}) - \{(v_j, v_k)\}$
16:         $E_1 \leftarrow E_1 \cup \{(v_i, v_k)\}$
17:     **else**
18:         **continue**
19:     **end if**
20: **end for**
21: **for all** $(v_i, v_j), (v_j, v_k) \in L_2$ **do**
22:     **if** $|N(v_i) \cap N(v_k)| == 1$ **then**
23:         $L_2 \leftarrow L_2 \cup \{(v_i, v_k)\} - \{(v_i, v_j), (v_j, v_k)\}$
24:         $E_1 \leftarrow E_1 \cup \{(v_i, v_k)\}$
25:     **else**
26:         **continue**
27:     **end if**
28: **end for**
29: **for all** $(v_i, v_j) \in E_1$ **do**
30:     **if** $(v_i, v_j) \in (E_1 - \{(v_i, v_j)\})$ **then**
31:         $E_1 \leftarrow E_1 - \{(v_i, v_j)\}$
32:     **else**
33:         **continue**
34:     **end if**
35: **end for**
36: **return** M

Since the output of Algorithm 1 is an MRDG having four vertices on its exterior, the corresponding RFP would have four rectangles on the exterior. It may not always be desirable to transform a given RDG to an MRDG. In such a case, we can replace L_2 by $L_2 - A$ where A is the set of edges not to be added to the given RDG. Thus we can obtain the required RDG from a given RDG.

For a better understanding to Algorithm 1, we explain its steps through an example. Consider an RDG $G_1 = (V, E_1)$ shown in Fig. 4.5a. First of all, Algorithm 1 computes two sets L_1 and L_2 (the lines $3 - 12$) from G_2 such that L_1 contains those edges whose endpoints have no common vertex and L_2 contains those edges whose endpoints have exactly one common vertex. Then $L_1 = \{(v_7, v_{10})\}$ and $L_2 = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_6), (v_6, v_8), (v_7, v_8), (v_{10}, v_{12}), (v_{11}, v_{12}), (v_9, v_{11}), (v_9, v_{10}), (v_7, v_1)\}$.

Now it executes the rest of its steps ($13-20$) as follows:

Since for $(v_7, v_{10}) \in L_1$, there is an edge (v_{10}, v_9) belonging to L_2, the loop ($13-20$) adds (v_7, v_9) and (v_7, v_{10}) to L_2, and adds (v_7, v_9) to E_1. Further, it subtracts (v_{10}, v_9) from L_2. Thus, $L_2 = \{(v_7, v_{10}), (v_7, v_9), (v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_6), (v_6, v_8), (v_7, v_8), (v_{10}, v_{12}), (v_{11}, v_{12}), (v_9, v_{11}), (v_7, v_1)\}$ and $E_1 = E_1 \cup \{(v_7, v_9)\}$. Since L_1 has exactly one edge, this loop terminates (in fact, we have transformed the given separable connected RDG to an nonseparable RDG. This method was proved by Lemma 4.3.4) and Algorithm 1 executes the next loop ($21-28$) as follows:

Suppose that Algorithm 1 picks (v_9, v_7) and (v_7, v_1) from L_2. Since $N(v_1) \cap N(v_9) = \{v_7\}$, $|N(v_1) \cap N(v_9)| = 1$. Then it subtracts (v_9, v_7) and (v_7, v_1) from L_2 and adds (v_9, v_1) to both L_2 and E_1 (the lines 23 and 24). Thus $L_2 = \{(v_9, v_1), (v_7, v_{10}), (v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_6), (v_6, v_8), (v_7, v_8), (v_{10}, v_{12}), (v_{11}, v_{12}), (v_9, v_{11})\}$ and $E_1 = E_1 \cup \{(v_9, v_1), (v_7, v_9)\}$.

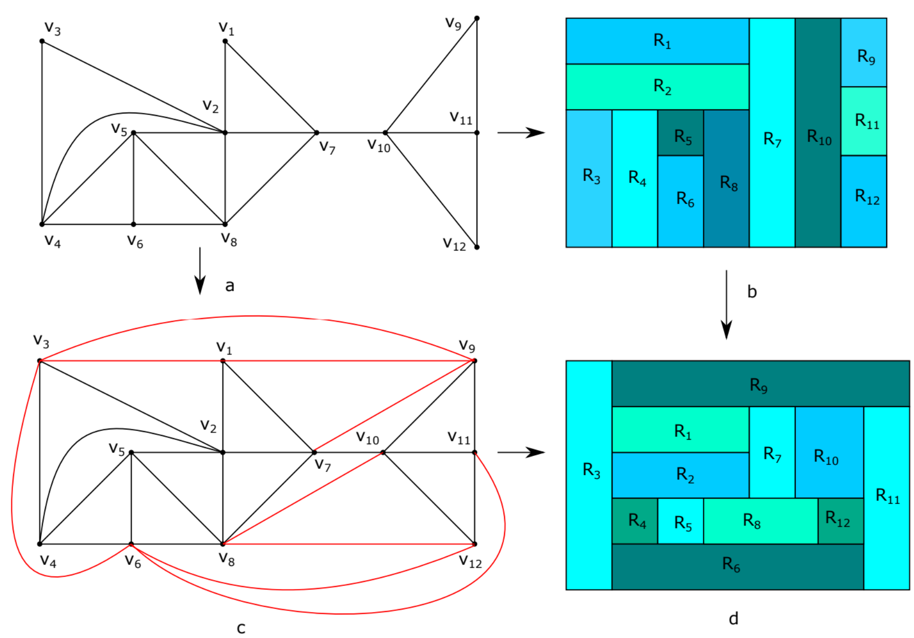

Figure 4.5: (a-b)A given RDG G_1 and its rectangular dual, and (c-d) the derivation of an MRDG M_2 from G_1 and its rectangular dual.

Again it picks (v_{10}, v_7) and (v_7, v_8) from L_2. Since $N(v_{10}) \cap N(v_8) = \{v_7\}$, $|N(v_{10}) \cap$

$N(v_8)| = 1$, it subtracts (v_{10}, v_7) and (v_7, v_8) from L_2, and adds (v_{10}, v_8) to both L_2 and E_1 (the lines 23 and 24).Thus, $L_2 = \{(v_{10}, v_8), (v_9, v_1), (v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_6), (v_6, v_8), (v_{10}, v_{12}), (v_{11}, v_{12}, (v_9, v_{11})\}$ and $E_1 = E_1 \cup \{(v_{10}, v_8), (v_9, v_1), (v_7, v_9)\}$.

Thus a recursive process of this loop adds $\{(v_6, v_{11}), (v_3, v_6), (v_9, v_3), (v_9, v_{11})\}$ to L_2 and $E_1 = E_1 \cup \{(v_9, v_3), (v_6, v_{12}), (v_8, v_{12}), (v_3, v_6), (v_9, v_3), (v_8, v_{10}), (v_9, v_1), (v_3, v_1), (v_9, v_7)\}$ respectively.

Since there has not been added any duplicate edge (multiple edges), the loop $(30-35)$ skips automatically.

Thus we see that the output is an MRDG M_2 shown in Fig. 4.5c where red edges are the new edges which are added to G_1. Note that G_1 admits a rectangular dual R_1 shown in 4.5b and M_2 admits a rectangular dual R_2 shown in 4.5d. Consequently to this, R_1 can be transformed to R_2 (a maximal one).

Analysis of computational complexity

Let v_s be a vertex of the largest degree in the input RDG G. This implies that $|N(v_i| \leq K$ where $d(v_s) = K$. Now we consider each of the following loops:

i. The computational complexity of the lines $3-12$ is $(|N(v_i)||N(v_j)||E_1|) \cong K^2|E_1| \cong \mathrm{O}(n)$.

ii. The computational complexity of the lines $14-20$ is $(|L_1|.|L_2|) \cong |L_2| \cong \mathrm{O}(n)$. In fact, L_1 contains edges whose endpoints are cut vertices. $|L_1| \cong \mathrm{O}(n)$ if the given RDG is a path graph. In that case, L_2 is empty. Both L_1 and L_2 can not be large simultaneously.

iii. The computational complexity of the lines $21-28$ is $(|N(v_i)||N(v_j)| + |L_2|)|L_2| \cong K^2|L_2|^2 \cong \mathrm{O}(n^2)$.

iv. The computational complexity of the lines $29-35$ is $|E_1|^2 \cong \mathrm{O}(n^2)$.

Hence, the computational complexity of Algorithm 1 is quadratic.

Remark 4.4.1. If $|N(v_i)|$ or $|N(v_j)|$ or $|N(v_i)| \times |N(v_i)|$ is near to $|V|$, then the computational complexity of Algorithm 1 becomes $\mathrm{O}(n^3)$. However, in design problems such graphs do not appears quite often. Both $|N(v_i)|$ and $|N(v_j)|$ can not be near to $|V|$ in a plane graph.

Remark 4.4.2. The proof of correctness of the algorithm follows from the above sequence of lemmas.

4.5 Concluding Remarks

In this chapter, we developed a graph based approach for the transformation of rectangular duals. We showed how to transform an RDG into another RDG whose edge set is a superset of the first one in polynomial time and hence proposed a transformation of a rectangular dual to another rectangular dual with the same number of rectangles.

We proved that it is always possible to construct an MRDG from a given RDG. Then we presented an algorithm for its construction from the given RDG. Since adding new edges to an RDG without disturbing RDG property reduce distances among its vertices (usually it is measured by the shortest path between vertices) and hence it is useful in reducing wire-length interconnections among the modules of VLSI floorplans. This method adds new edges to an RDG in bulk if it is a path graph (minimal one which is obvious an RDG). In other words, if we pick a Hamiltonian path of an RDG, then a new desired form of the RDG can be constructed by adding edges in bulk. If it is not possible to make some pair of vertices of a given RDG adjacent in its MRDG without disturbing RDG property, then it would be interesting to find a method that can minimize distances between these vertices. In this case, it is equivalent to finding a minimal spanning tree for routability of interconnections in rectangular floorplans.

Consequently, we can construct an efficient rectangular dual by introducing new adjacencies among rectangles of a rectangular dual.