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EXAMPLES IN POWER DISTRIBUTION

AND

ELECTRIC TRACTION

WITH DIAGRAMS, THEOREMS FORMULAE, WORKED EXAMPLES AND ANSWERS

BY

A. T. DOVER, M.I.E.E., A.Am.I.E.E.

HEAD OF THE ELECTRICAL ENGINEERING DEPARTMENT
BATTERSEA POLYTECHNIC, LONDON

PREFACE

IN arranging the notes and examples of the present volume the author's aim was to provide a small handbook which would be useful to Power Distribution and Traction Engineers and Students, particularly those preparing for University and Diploma final examinations, the final examination (in Electrical Engineering) of the City and Guilds of London Institute, and the Graduateship Examination of the Institution of Electrical Engineers.

With the object of assisting students engaged in private study, the more important theorems and formulae relating to Power Distribution are given in a separate chapter, and those relating to Electric Traction are arranged in chapters appropriate to the classification of the examples in this subject. Neither proofs of theorems nor deduction of formulae are given (except in special cases), as these will be found in suitable textbooks.

As far as possible the examples have been selected from the examination papers of the University of London, City and Guilds of London Institute, and the Institution of Electrical Engineers, but a number of examples have been compiled to suit the needs of particular problems.

The best thanks of the author are due to the Senate of the University of London, the Examinations Board of the City and Guilds of London Institute, and the Council of the Institution of Electrical Engineers. Grateful acknowledgment of assistance is due to his colleagues, Messrs. H. C. Mann, B.Sc. (Eng.), (Lond.), M.I.E.E., and J. G. Fleming, B.Sc. (Eng.), (Lond.), A.M.I.E.E.

<div align="right">A. T. DOVER.</div>

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INSET

ABBREVIATIONS

Alternating current; Direct current	a.c.; d.c.
Amperes	amp., A
Electro-motive force; Potential difference	e.m.f.; p.d.
Watt; Kilowatt	W; kW
Kilogram	kg
Kilogram-metre	kg-m.
Kilometre	km
Kilometres per hour . . .	km.p.h.
Kilowatt-hour	kWh
Ohm; Microhm	Ω; $\mu\Omega$
Pounds-feet	lb.-ft.
Miles per hour	m.p.h.
Miles per hour per second .	m.p.h.p.s.
Revolutions per minute . .	r.p.m.
Root-mean-square . . .	r.m.s.
Volts	V
Yard	yd.

REFERENCES (Examples)

IEE, Graduateship Examination, Institution of Electrical Engineers.

*LU*, Final B.Sc. (Engineering) Examination, University of London.

CG, Final Examination in Electrical Engineering, City and Guilds of London Institute.

BP, Final Diploma and Higher National Certificate Examinations in Electrical Engineering, Battersea Polytechnic.

NOTES

LOGARITHMS, where used, are to the base 10.

VECTOR DIAGRAMS. All diagrams have been drawn for *counter-clockwise* rotation.

E.M.F. vectors are represented by an ordinary _____→ arrow-head.

Flux vectors are represented by a double arrow- _____⇒ head.

Ampere-turn vectors are represented by a solid _____▶ arrow-head.

Current vectors are represented by a closed arrow- _____▷ head.

VECTOR QUANTITIES are denoted by dotted capitals, thus, $\dot{E}$, $\dot{I}$, $\dot{Z}$: their *rectangular components* are denoted by appropriate symbols or numerals, vertical components being compounded with the symbolic operator j ($= \sqrt{-1}$), thus, $\dot{Z} = R + j X$.

WEIGHTS. The British ton (2,240 lb.) is used throughout.

EXAMPLES IN
POWER DISTRIBUTION AND
ELECTRIC TRACTION

PART I

POWER DISTRIBUTION

Systems of distribution. Three systems are in use: (*a*) the *two-wire* system with two conductors of the same cross-section; (*b*) the *three-wire* system with three conductors, two of which —called the "outers"—have the same cross-section, and the other—called the "neutral" or "middle wire"—has a smaller cross-section (usually one-half of that of either outer); (*c*) the *three-phase four-wire* system (which is now adopted for all new schemes) with four conductors, three of which—called the "line" or "phase" conductors—have the same cross-section, and the remaining conductor—called the "neutral"—is in some cases of smaller cross-section, but in many cases has the same cross-section as a phase conductor. With the three-wire system equal voltages are maintained normally between each outer and the neutral, the voltage between the outers being twice that between an outer and the neutral. With the four-wire system equal voltages are also normally maintained between the neutral and each of the phase conductors, but in this case the voltage between any pair of phase conductors is $\sqrt{3}$ times the voltage to neutral. In both cases the neutral is earthed (usually through a resistance) at the power or distributing station.

Distributors and feeders. The supply conductors to which the consumers are actually connected are called *distributors*, and their function is to supply the various consumers at practically equal voltages. A statutory variation of $\pm$ 4 per cent of the declared voltage of distribution is permitted in the supply to consumers.

The distributors are supplied by *feeders*, the function of which is to maintain specified points (called feeding points) of the distributors at definite voltages.

Two-wire direct-current distributor. *Case* 1. *Feeding point at one end.* Fig. 1 shows the circuit diagram. Fig. 2 is a conventional single-line diagram in which only the positive side is shown, and the resistances of the sections between the load points are equal to those of both outgoing and return conductors of the actual distributor (i.e. the negative side or return
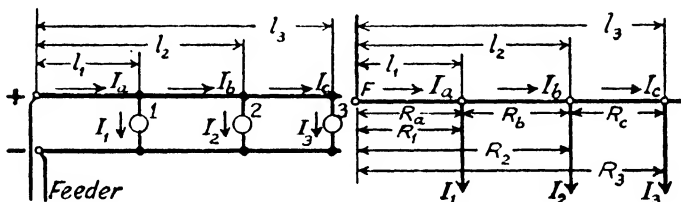

FIG. 1   FIG. 2

CIRCUIT AND CONVENTIONAL DIAGRAMS OF TWO-
WIRE DISTRIBUTOR

is considered to have no resistance). The voltage drop between the feeding point and the extreme load point is given by

$$v = I_a R_a + I_b R_b + I_c R_c + \ldots$$
$$= I_1 R_1 + I_2 R_2 + I_3 R_3 + \ldots \qquad . \qquad . \qquad (1)$$

If the distributor is of uniform cross-section throughout, and R is the resistance per yard of route (i.e. R is the resistance of one yard of outgoing conductor + one yard of return conductor), we have

$$v = R(I_1 l_1 + I_2 l_2 + I_3 l_3 + \ldots) \qquad . \qquad . \qquad (2)$$

where l_1, l_2, l_3, . . . denote the distances, in yards, of the various load points from the feeding point.

Case 2. *Feeding points at both ends and at the same potential.* (Fig. 3.) The load point, or points, having the lowest voltage must be determined before the voltage drop can be calculated.

If the lowest voltage occurs at load point No. 3, we have

$$R_a(I_1 + I_2 + I_c) + R_b(I_2 + I_c) + R_c I_c$$
$$= R_d(I_3 - I_c) + R_e(I_3 - I_c + I_4)$$

whence $\quad I_c R_t = I_3 R_3'' + I_4 R_4'' - (I_1 R_1' + I_2 R_2') \qquad . \qquad (3)$

where R_t denotes the total resistance of the distributor; R_1', R_2', the total resistances between feeding point F_1, and the load points Nos. 1 and 2 respectively; R_3'', R_4'', the resistances between feeding point F_2 and the load points Nos. 3 and 4 respectively.

If the cross-section is uniform throughout

$$I_c l = I_3 l_3'' + I_4 l_4'' - (I_1 l_1' + I_2 l_2') \quad . \quad . \quad (4)$$

where l denotes the distance between the feeding points; l_1', l_2', the distances of the load points Nos. 1 and 2 from feeding point F_1; l_3'', l_4'', the distances of the load points Nos. 3 and 4 from feeding point F_2.

If, in a particular problem, the assumed point of lowest voltage is in error, the value of I_c may be negative, or it may be greater than the adjacent load currents. The solution,
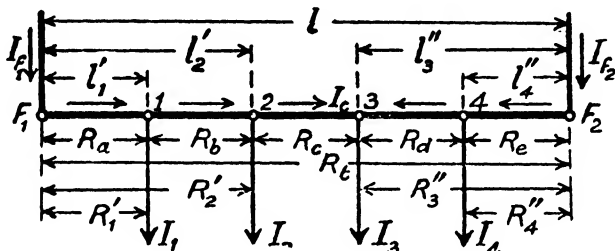

Fig. 3. Conventional Diagram of Distributor with Two Feeding Points

$I_c = 0$, indicates that adjacent load points (e.g. Nos. 2 and 3) are at the same potential.

Equation (4) suggests a method of determining the point, or points, of lowest voltage. For example, for the conditions represented in Fig. 3 to be satisfied we must have

$$(I_4 l_4'' + I_3 l_3'') > (I_1 l_1' + I_2 l_2')$$

or generally, with n loads, the sum of the products $I_n l_n''$, $I_{(n-1)} l_{(n-1)}''$, . . . with respect to the right-hand feeding point must be greater than (or equal to in the special case when two adjacent load points are at the same potential) the sum of the products $I_1 l_1'$, $I_2 l_2'$, . . . with respect to the left-hand feeding point.

Case 3. *Feeding points at both ends and at different potentials.* Let V_1, V_2 denote the voltages at the feeding points, and R_t the total resistance of the distributors. Due to the difference of potential between the feeding points, a circulating current, equal to $(V_1 - V_2)/R_t$, flows through the distributor in addition to the currents due to the loads. The currents in the sections can therefore be obtained by taking the algebraic sum of the circulating current and the currents

which would be obtained if the feeding points were at equal potentials. The calculation of the voltage drop is similar to that for the preceding case.

Alternatively, the currents in the sections may be calculated directly. Thus, the current I_a in section R_a is determined from the equation

$$I_a R_t = (V_1 - V_2) + I_1 R_1'' + I_2 R_2'' + I_3 R_3'' + I_4 R_4'' \quad (1a)$$

where R_1'', R_2'', . . . denote the resistances from the feeding point F_2 to the load points 1, 2, . . .

The current (I_e) in section R_e is

$$I_e = I_1 + I_2 + I_3 + I_4 - I_a,$$

and the currents in sections R_b, R_d, R_c are

$$I_b = I_a - I_1; \ I_d = I_e - I_4; \ I_c = I_b - I_2 = I_3 - I_d.$$

Three-wire direct-current distributor. The voltage drop in each outer and that in the neutral are calculated separately. The calculation of the voltage drop in the neutral is quite straightforward, if due care is exercised in determining the magnitudes and *directions* of the currents in the sections between the load points. Observe that the voltage drop due to currents having a direction towards a neutral feeding point is considered as positive, and those due to currents in the opposite direction as negative. Thus the resultant voltage drop in the neutral is equal to the algebraic sum of the voltage drops in the sections between the load points.

Alternating-current distributors (two-wire and three-wire). These cases differ only from those for the corresponding direct-current distributors when (*a*) the distributor possesses inductance (e.g. when overhead lines are employed), (*b*) the power factors of the loads are below unity.

With the first case and loads of unity power factor, the impedance of the distributor is substituted for the resistance term in the appropriate equations.

With a non-inductive distributor and loads having different power factors, currents and voltage drops are added vectorially instead of arithmetically. But with similar load conditions and an inductive distributor the calculation of voltage drop in a section involves the product of two complex quantities (e.g. current and impedance), and is best effected by complex algebra.*

---

* The principles of complex algebra and its application to the solution of alternating-current circuits are given in the author's *Theory and Practice of Alternating Currents*.

Three-phase four-wire distributor. The calculation of the voltages at the load points in the special case when the loads are balanced is quite straightforward. But in the general case, when the loads are unbalanced and have different power factors, the calculation is involved on account of the phase differences between the voltage drops in the sections of the phase conductors and the neutral. In this case the voltages between each of the phase conductors and the neutral will generally be unequal, and their phase differences will differ from 120 degrees. Moreover, the potential of the load points on the neutral conductor will generally differ from the potential of the neutral point of the system.

Cross-section of distributor for minimum weight of conductors. For a specified overall voltage drop the minimum weight of conductors is obtained when the cross-sections between the load points are proportional to the square root of the currents in the respective sections. For example, if in Fig. 2 the cross-sections taken in order from the feeding point are to be a_a, a_b, a_c, . . .

then $\quad a_a : a_b : a_c \ . \ . \ . = \sqrt{I_a} : \sqrt{I_b} : \sqrt{I_c} \ . \ . \ . \ .$ (5)

The overall voltage drop v being known, the voltage drops in the sections are given by

$$v_a = v l_1 \sqrt{I_a} / (l_1 \sqrt{I_a} + l_2 \sqrt{I_b} + l_3 \sqrt{I_c} + \ . \ . \ .) \quad (6)$$

$$v_b = v_a (l_2 / l_1) \sqrt{(I_b / I_a)}$$
$$= v l_2 \sqrt{I_b} / (l_1 \sqrt{I_a} + l_2 \sqrt{I_b} + l_3 \sqrt{I_c} + \ . \ . \ .) \quad (7)$$

Whence

$$a_a = 2 \rho l_1 I_a / v_a; \ a_b = a_a \sqrt{(I_b / I_a)};$$
$$a_c = a_a \sqrt{(I_c / I_a)} \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (8)$$

where ρ denotes the resistivity of the conductor.

PROOF. Consider for simplicity a distributor of two sections supplying loads I_1, I_2. Then, with the notation of Fig. 2, if v is the overall voltage drop, we have

$$v_a = 2 \rho l_1 (I_1 + I_2) / a_a, \ \text{or} \ a_a = 2 \rho l_1 (I_1 + I_2) / v_a$$
$$v_b = v - v_a = 2 \rho l_2 I_2 / a_b, \ \text{or} \ a_b = 2 \rho l_2 I_2 / (v - v_a)$$

The volume of copper $= 2 l_1 a_a + 2 l_2 a_b$, and its minimum value is obtained by equating to zero the differential coefficient of volume with respect to v_a and solving for v_a. Thus

$$\frac{d}{dv_a} \left(\frac{4 \rho l_1^2 (I_1 + I_2)}{v_a} + \frac{4 \rho l_2^2 I_2}{v - v_a} \right) = - \frac{4 \rho l_1^2 (I_1 + I_2)}{v_a^2} + \frac{4 \rho l_2^2 I_2}{(v - v_a)^2} = 0$$

Whence

$$\frac{v - v_a}{v_a} = \frac{l_2}{l_1} \sqrt{\frac{I_2}{I_1 + I_2}}$$

Hence

$$\frac{a_a}{a_b} = \left(\frac{v - v_a}{v_a}\right)\left(\frac{I_1 + I_2}{I_2}\right)\frac{l_1}{l_2} = \sqrt{\frac{I_1 + I_2}{I_2}} = \sqrt{\frac{I_a}{I_b}}$$

For **minimum weight of conductors in a branched main,** AB, BC, BD, which is fed at A, and in which equal voltage drops (v) occur between the feeding point and the distant ends, the voltage drop in the common portion AB is given by

$$v_1 = vl_1\sqrt{I_1}/(l_1\sqrt{I_1} + \sqrt{(l_2^2 I_2 + l_3^2 I_3)}) \quad . \quad . \quad (9)$$

where I_1, I_2, I_3, are the equivalent currents in the portions AB, BC, BD, taken in order, and l_1, l_2, l_3 are the corresponding lengths; the equivalent current being that current which, flowing through the whole length of a section, produces a voltage drop equal to that due to the actual load currents.

Whence, for a d.c. two-wire system

$$a_1 = 2\rho l_1 I_1/v_1 \, ; \, a_2 = 2\rho l_2 I_2/(v - v_1) \, ; \, a_3 = a_2 l_3 I_3/l_2 I_2 \quad (10)$$

where a_1, a_2, a_3 denote the cross-sections of the portions AB, BC, CD, taken in order.

Current distribution and voltage drop in networks. These are best determined by the application of Kirchhoff's Laws, which may be stated thus—

1. At every junction of two or more branches of an electric circuit the algebraic sum of all currents is zero.

2. In every closed circuit carrying a current the algebraic sum of all e.m.f.s taken in order round the circuit is zero.

Insulation resistance of mains and networks while working. With a non-earthed two-wire system the insulation resistance can be determined by means of a voltmeter of known resistance (or, alternatively, a low reading ammeter and series resistance of about 1,000 ohms or more). Each main is earthed successively through the voltmeter, the readings being V_1 (+ side) and V_2 (− side). If the voltage of the system is V, and the resistance of the voltmeter is R_v, then

insulation resistance of + side
$$= R_v(V - V_1 - V_2)/V_2 \quad . \quad\quad . \quad\quad . \quad (11)$$

insulation resistance of − side
$$= R_v(V - V_1 - V_2)/V_1 \quad . \quad\quad . \quad\quad . \quad (12)$$

insulation resistance of network
$$= R_v(V - V_1 - V_2)/(V_1 + V_2) \quad\quad . \quad\quad . \quad (13)$$

To apply this method to a three-wire system, the normal earth connection is removed from the neutral. One of the outers and the neutral are earthed successively through the voltmeter, the readings being V_1 (outer) and V_0 (neutral). Then if V is the voltage between the neutral and each outer the insulation resistance of the network is given by

$$R = R_v \left(\frac{V}{V_1 - V_0} - 1 \right) \qquad . \qquad . \qquad . \qquad . \qquad (14)$$

The insulation resistance may also be determined without removing the earth connection from the neutral. Thus, if the current in the neutral earthing resistance (R_e) is I, and the voltage between neutral and earth (determined by an electrostatic voltmeter) is V_0,

$$R = (V_0/I) - R_e \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (15)$$

Alternatively, if the earthing resistance is shunted by a resistance of equal value, and I' is the current in the former,

$$R = R_e(I - I')/(2I' - I) \qquad . \qquad . \qquad . \qquad . \qquad (16)$$

Division of current in parallel connected feeders and transformers. These cases become identical in principle when the ratio of transformation is the same for each transformer. Thus, considering the single-phase case, a load current I, at a power factor of $\cos \varphi$, will divide between two feeders or transformers in the following manner—

$$I_1 = I Z_2/(Z_1 + Z_2) \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (17)$$
$$I_2 = I Z_1/(Z_1 + Z_2) \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (18)$$

where $I = I \cos \varphi \mp jI \sin \varphi$, and Z_1, Z_2 are the impedances of the respective feeders or transformers. The calculation is carried through by the symbolic (complex algebra) method, and both the magnitudes and phase differences of the currents I_1, I_2 are determined at the same time.

If the voltage at the receiving end of the feeders, or the common terminal voltage of the transformers, is denoted by V, we have

$$V I_1 = V I Z_2/(Z_1 + Z_2) \qquad . \qquad . \qquad . \qquad . \qquad (19)$$
$$V I_2 = V I Z_1/(Z_1 + Z_2) \qquad . \qquad . \qquad . \qquad . \qquad (20)$$

These equations show how the load (in volt-amperes or kVA) is divided between the feeders or the transformers.

If the no-load e.m.f.s, E_1, E_2, of the transformers are un-equal, and the impedance of the load is Z, the currents supplied by the transformers will be given by

$$I_1 = \frac{E_1 Z_2 + (E_1 - E_2)Z}{Z_1 Z_2 + Z(Z_1 + Z_2)} \qquad . \qquad . \qquad . \qquad . \qquad (21)$$

$$I_2 = \frac{E_2 Z_1 - (E_1 - E_2)Z}{Z_1 Z_2 + Z(Z_1 + Z_2)} \qquad . \qquad . \qquad . \qquad . \qquad (22)$$

WORKED EXAMPLES. 1. A three-wire single-phase distributor cable, AFB, 500 yd. long, is fed at its centre F and loaded as follows; the phase differences of the load currents being given with reference to the p.d.s at the feeding point, and in all cases being lagging—

Red side : 50 A at 25·8° at A ; 10 A at 0° at C, 150 yd. from B ; 20 A at 36·8° at B.

Blue side : 30 A at 36·8° at D, 100 yd. from A ; 30 A at 18·2° at E, 50 yd. from B.

The feeding point is connected directly to the secondary windings of the transformer, which maintains a p.d. of 235 V on each side, and a p.d. of 470 V between the outers.

The resistance of each outer is 0·16 Ω, and that of the neutral is 0·3 Ω.

Calculate the current in each section of the distributor, the current in the secondary windings of the transformers, and the p.d. at each load.

SOLUTION. This problem must be calculated throughout, either by complex algebra or by resolving all currents and p.d.s into in-phase and quadrature components.

The load currents are therefore given symbolically by

$I_A = 50 \cos 25·8° - j50 \sin 25·8° = 45 - j21·8 ; I_C = 26 - j12 ;$
$I_B = 16 - j12 ; I_D = 24 - j18 ; I_E = 28·5 - j9·36.$

Hence the currents in the sections of the distributor are

Red outer : $FA - I_A$; $FC - I_B + I_C$; $CB - I_B$.
Blue outer : $FD - I_D$; $FE - I_E$.
Neutral : $FD - I_A - I_D$; $DA - I_A$; $FC - -(I_E - I_C - I_B)$;

$CE - -(I_E - I_B)$; $EB - I_B$; the negative sign in the cases FC and CE denoting that the direction of the currents in these sections with respect to the neutral point of the transformer is, at a given instant, opposite to that of the currents in the other sections of the neutral.

The currents in the transformer are—

Red side : $I_1 = I_A + I_C + I_B = 71 - j33 \cdot 8$;

$\qquad I_1 = \sqrt{(71^2 + 33 \cdot 8^2)} = 78 \cdot 6$ A.

Blue side : $I_2 = I_D + I_E = 52 \cdot 5 - j27 \cdot 36$;

$\qquad I_2 = \sqrt{(52 \cdot 5^2 + 27 \cdot 36^2)} = 59 \cdot 2$ A.

The p.d.s at the feeding points being $235 + j0$ and $235 + j0$, those at the loads are—

$V_A = (235 + j0) - [I_A(0 \cdot 08 + 0 \cdot 06) + 0 \cdot 09(I_A - I_B)]$

$\qquad = (235 + j0) - (9 \cdot 09 - j3 \cdot 392) = 225 \cdot 9 + j3 \cdot 4.$

$\therefore \; V_A = \sqrt{(225 \cdot 9^2 + 3 \cdot 4^2)} = 225 \cdot 9$ V.

$V_B = (235 + j0) - [0 \cdot 032(I_B + I_C) + I\,(0 \cdot 048 + 0 \cdot 03)]$

$\qquad \quad + [0 \cdot 06\,\{(I_E - I_B) + (I_E - I_B - I_C)\}]$

$\qquad = (235 + j0) - (2 \cdot 208 - j1 \cdot 756) = 232 \cdot 8 + j1 \cdot 75$

$\therefore \; V_B = 232 \cdot 8$ V

$V_C = (235 + j0) - [0 \cdot 032(I_B + I_C) - 0 \cdot 06(I_E - I_C - I_B)]$

$\qquad = (235 + j0) - (0 \cdot 68 - j0 \cdot 6) = 234 \cdot 3 + j0 \cdot 6$

$\therefore \; V_C = 234 \cdot 3$ V

$V_D = (235 + j0) - [0 \cdot 048 I_D - 0 \cdot 09(I_A - I_D)]$

$\qquad = (235 + j0) - (0 \cdot 26 - j0 \cdot 521) = 234 \cdot 7 + j0 \cdot 52$

$\therefore \; V_D = 234 \cdot 7$ V

$V_E = (235 + j0) - [0 \cdot 06\,\{(I_E - I_B - I_C) + (I_E - I_B)\} + 0 \cdot 064 I_E]$

$\qquad = (235 + j0) - (2 \cdot 69 - j0 \cdot 164) = 232 \cdot 3 + j0 \cdot 16$

$\therefore \; V_E = 232 \cdot 3$ V

2. A three-phase, four-wire, overhead-line distributor, AE, 1,600 yd. long, fed at A, has single-phase loads connected between the phase conductors and the neutral as follows, the loads being expressed symbolically in terms of their impedances—

Red phase : $10 + j7$ at D, 1,100 yd. from A.

Blue phase : $15 + j5$ at C, 750 yd. from A.

White phase : $45 + j0$ at B, 250 yd. from A; $30 + j15$ at E, 1,600 yd. from A.

The feeding point is connected directly to the secondary winding of a transformer, the p.d.s of which, expressed in accordance with the B.E.S.A. standard phase rotation, are—

White phase : $V_1 = 240 + j0$.

Blue phase : $V_2 = 240(-0 \cdot 5 - j0 \cdot 866) = -120 - j208.$

Red phase : $V_3 = 240(-0 \cdot 5 + j0 \cdot 866) = -120 + j208.$

Each line conductor has a cross-section of 0·05 sq. in. The resistance per mile of conductor is 0·95 Ω, and the reactance per route mile of loop formed by any phase conductor and the neutral is 0·9 Ω.

Calculate the current and p.d. at each load.

SOLUTION. This problem involves the application of Kirchhoff's Laws. The distributing system is divided into closed loops or meshes, and for each loop the vector sum of all p.d.s in the loop must be zero. Thus, if the p.d.s of the white, blue, and red phases of the transformer are denoted by V_1, V_2, V_3, in order; the impedances of the loads by Z_B, . . . Z_E; the load currents by I_B, . . . I_E; and the impedance of one line conductor between the feeding point and each of the load points, B, . . . E, by Z_1, . . . Z_4, in order, we have

$$V_1 = I_B(Z_B + 2Z_1) \qquad\qquad\qquad + 2I\ Z_1$$
$$V_2 = I_BZ_1 \quad + I_C(Z_C + 2Z_2) + I_DZ_2 \qquad + I_EZ_2$$
$$V_3 = I_BZ_1 \quad + I_CZ_2 \qquad + I_D(Z_2 + 2Z_3) + I_EZ_3$$
$$I_BZ_B = \qquad I_C(Z_2 - Z_1) \quad + I_D(Z_3 - Z_1) \quad + I_E(Z_E + 2(Z_4 - Z_1))$$

From these four equations we can determine the four currents I_B . . . I_E. But it is simpler to eliminate I_B from the first, second, and third equations and solve these for I_C, I_D, I_E. The solution is best effected by determinants,[*] but is tedious, although only arithmetic computation is involved. The results are—

$$I_C = -10·7 - j9·7 ; \ I_D = 1·21 + j19·25 ; \ I_E = 6·14 - j3·2.$$

Substituting these values in the fourth equation, we have

$$I_B = 5·3 + j0·02$$

Whence

$$I_B = \sqrt{(5·3^2 + 0·02^2)} = 5·3 \text{ A} ; \ I_C = \sqrt{(10·7^2 + 9·7^2)} = 14·5 \text{ A} ;$$
$$I_D = \sqrt{(1·21^2 + 19·25^2)} = 19·25 \text{ A} ; I_E = \sqrt{(6·14^2 + 3·2^2)} = 6·92 \text{ A}.$$

The load voltages are obtained by multiplying the load currents by the appropriate load impedances.

Thus

$$V_B = I_BZ_B = 5·34 \times 45 = 238·5 \text{ V} ;$$
$$V_C = I_CZ_C = 14·5\sqrt{(15^2 + 5^2)} = 229·2 \text{ V} ;$$
$$V_D = I_DZ_D = 19·25\sqrt{(10^2 + 7^2)} = 235 \text{ V} ;$$
$$V_E = I_EZ_E = 6·92\sqrt{(30^2 + 15^2)} = 232·2 \text{ V}.$$

[*] Examples of the solution of similar equations are given in the author's *Theory and Practice of Alternating Currents*, p. 285, in connection with the calculation of polyphase circuits.

3. 5,000 kW at 0·8 power factor (lagging) and 30,000 V is received by a substation through two three-phase overhead lines, A, B (which follow different routes) operating in parallel. The current supplied by A is 50 A, and the power obtained from B is 3,000 kW. If each of the lines B has a resistance of 8 Ω and a reactance of 10 Ω, what are the corresponding quantities for the lines A, and what is the current in each line?

SOLUTION. The problem is best calculated by employing complex algebra. First the currents in the lines A, B are determined, thence the ratio of the line impedances is calculated by the application of equations (19), (20). Thus, the load current (I) is

$$I = 5000 \times 10^3/(\sqrt{3} \times 30,000 \times 0\text{·}8) = 120 \text{ A}$$

or $I = 120 \times 0\text{·}8 - j120 \sin(\cos^{-1} 0\text{·}8) = 96 - j72$

The in-phase component of the current in lines A

$$= (5000 - 3000) \times 10^3/(\sqrt{3} \times 30,000) = 38\text{·}5 \text{ A}$$

and the quadrature component $= \sqrt{(50^2 - 38\text{·}5^2)} = 31\text{·}9$ A.

Hence $I_A = 38\text{·}5 - j31\text{·}9$

Whence $I_B = I - I_A = 57\text{·}5 - j40\text{·}1$

From equations (19), (20) we have

$$Z_A/Z_B = I_B/I_A$$

Whence

$$Z_A = (8 + j10)(57\text{·}5 - j40\text{·}1)(38\text{·}5 + j31\text{·}9)/(38\text{·}5^2 + 31\text{·}9^2)$$
$$= 10 + j15$$

i.e.   Resistance $= 10 \Omega$; reactance $= 15 \Omega$.

EXAMPLES I
POWER DISTRIBUTION

1. Compare the three-wire d.c. system with the four-wire, three-phase system at unity power factor with respect to relative economy in copper for equal voltages at consumers' terminals. The distance, power delivered, and current density are to be equal in the two cases. In each system the neutral conductor may be taken as of half the area of any outer conductor.

2. Calculate the relative weights of copper required for a distribution network on the d.c. three-wire and the three-phase four-wire system. Assume in both cases the same voltage at consumers' terminals, the same copper losses, that the loads are balanced, and unity power factor in the three-phase case.   (CG)

3. Compare the relative weights of copper required for a three-wire d.c. distribution system and a four-wire three-phase system on the basis of equal power, equal losses, and equal voltage to neutral.

The power factor in the latter case can be taken as 0·9, and the neutral wire in each case is to be one-half the cross-section of the corresponding outer.   (*BP*)

4. Compare the weight of copper necessary for a three-phase four-wire transmission with that necessary for a d.c. three-wire transmission to supply a lighting load with a given voltage on the lamps, a given distance of transmission and the same percentage total voltage drop. Assume that the reactance drop in a given line is equal to the resistance drop, and neglect the voltage drop in the neutral conductors.   (*LU*)

5. Explain the separate functions of the boosters and balancer in obtaining voltage regulation on a d.c. three-wire feeder, and draw a diagram of connections.

Assuming that the resistance of each outer is R, that the middle wire has half the cross-section of either outer, and that the more heavily-loaded side carries I amperes, calculate the current on the lightly-loaded side which will give a pressure, at the far end, on this side of the system equal to the pressure at the generator end of the feeder. Under what conditions will the pressure at the far end exceed that at the generator end ?   (*LU*)

6. What are the principal methods of locating faults on low-tension distribution networks and on main transmission three-core cables ? What precautions and calculations have to be made when several lengths of different size cables form the main on which the fault has occurred ? What is the equivalent length of main consisting of (*a*) 750 yd., 0·15 sq. in., (*b*) 400 yd. 0·1 sq. in., (*c*) 200 yd., 0·075 sq. in., (*d*) 50 yd., 0·04 sq. in. ? Resistance of 0·1 sq. in. cable is 0·42 Ω per mile.   (*IEE*)

7. Give an account of the tests to be made (*a*) in the station, (*b*) on the network, for locating the position of a fault in a low voltage distribution network.

A fault to earth occurs in a section of distributor cable 250 yd. long. The section is isolated from the supply, and the resistance of the faulty conductor, measured between earth and the testing end, is found to be 12 Ω. When the distant end is earthed, the resistance measured as before is 2·25 Ω. Determine the distance of the earth fault from the testing end of the cable, and the resistance of the fault. The conductor has a resistance of 0·01 Ω per yd.   (*CG*)

8. A voltmeter of resistance R_v is connected alternately between each main and earth. The voltage between the mains of a two-wire d.c. system is V ; the numerical sum of the readings so obtained is V_1. Show that the insulation resistance R of the system is given by $R = R_v(V - V_1)/V_1$. Show also, that if the readings V_1 are in error by p per cent, the corresponding percentage error in the insulation resistance is $p(R + R_v)/R$.   (*LU*)

9. A resistance of 1,000 Ω is connected to earth from one terminal (*A*) of an insulated two-wire d.c. network at 200 V, and found to carry 0·13 A. It is then connected to earth from the other terminal (*B*) and found to carry 0·05 A. A section of the network is then disconnected by means of a double-pole switch, and the measurements repeated with the results 0·137 A and 0·033 A in the
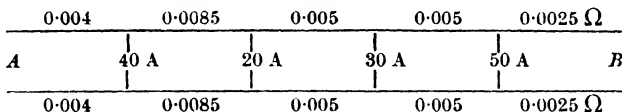
same order. What is the insulation resistance of each part of each pole of the network ?   (*LU*)

10. Explain what is meant by the insulation resistance of a three-wire d.c. current network. In measuring the insulation resistance, the following results were obtained after the Board of Trade earth connection has been removed: Reading on electrostatic voltmeter connected between neutral and earth, 200 V. Milliammeter reading when connected in series with a resistance of 1,000 Ω and earth, 20 mA. The voltages on the sides of the system were maintained the same. Develop a formula giving the insulation resistance of the network, and calculate its value in the above case.   (*CG*)

11. The earth connection having been removed from the middle wire of a three-wire d.c. network, an ammeter in series with 1,000 Ω was connected between the positive wire and earth, and then between the mid-wire and earth; the readings were 0·385 A and 0·185 A respectively. The p.d. between each outer and the middle wire is 240 V. Calculate the fault resistance of the system, and prove any formula used.   (*LU*)

12. A distributor, 1,000 yd. long and of 0·2 Ω resistance (lead and return), is fed at one end (*A*) by a shunt generator giving 220 V at no load and 210 V at its full load of 100 A, the load characteristic being a straight line. The other end (*B*) is fed from a battery in a substation, the discharge e.m.f. being 215 V, and the internal resistance 0·04 Ω. Loads of 75 A and 100 A are taken off at distances of 250 and 750 yd., respectively, from the generator. Find the p.d. at each end of the distributor and at the two loads.   (*LU*)

13. A two-wire distributor is fed at both ends with a 480 V d.c. supply. The resistances and loads are as follows—

	0·004		0·0085		0·005		0·005		0·0025 Ω	
A		40 A		20 A		30 A		50 A		*B*
	0·004		0·0085		0·005		0·005		0·0025 Ω	

Determine the position and value of the maximum drop in volts, and the magnitudes and the directions of the currents in the various parts of the distributor. If the voltage at the feeding point *A* happens to be 480·5 V, while that at *B* is held constant at 480 V, determine the redistribution of currents in the distributor on the assumption that the load currents remain unaltered.   (*LU*)

14. In a developing district the load on a new two-wire distributor, 400 yd. long, is being connected at the rate of 20 A every 10 yd. It is fed at the end from which the loading begins, and the maximum voltage drop must not exceed 4 V. Each core has a resistance of 0·02 Ω per 1,000 yd. (*a*) How many services may be connected before it becomes necessary to feed the cable from the other end ? Under these new conditions of feeding calculate (*b*) the maximum voltage drop and (*c*) the point at which it occurs. (*LU*)

15. A two-wire d.c. distributor, *A*, *B*, 1,000 yd. long, is loaded as follows: 70 A, 200 yd. from *A*; 20 A, 350 yd. from *A*; 65 A, 550 yd. from *A*; 60 A, 100 yd. from *B*; 90 A, 250 yd. from *B*.

The feeding point A is maintained at 226 V, and the feeding point B at 224 V. Find the position of the point of minimum p.d., and the current supplied at each feeding point, if the total resistance of the distributor (lead and return) is 0·08 Ω.   (LU)

16. A two-wire distributor, 500 yd. long, is loaded uniformly at the rate of 0·2 A per yd. The distributor is fed at both ends, and the voltages at the feeding points are 200 V and 196 V. Determine the point in the distributor at which the current is zero. The resistance per yard of each of the conductors is 0·0008 Ω.

17. Design a distributor cable 500 yd. long to be fed with direct current from both ends to supply the following loads at the stated distances from one end. The voltage drop in no case must exceed 2 per cent of the feeder p.d. of 250 V. Specific resistance of copper = 0·67 $\mu\Omega$ per in. cube.

Consumer No.	1	2	3	4	5	6	7	8	9	10
Current	20	20	30	10	20	10	25	40	15	5
Yards	10	25	45	80	120	200	300	400	420	470

(CG)

18. Show that, for a given power (I^2R) loss, the weight of conductors in a distributor is a minimum when the cross-sections between the load points are so chosen that the current density in all sections is constant.

19. Show that, with a "tapered" distributor designed for a given overall voltage drop, the weight of the conductors is a minimum if the cross-sections between the load points, taken in order, are chosen proportional to the square root of the currents in these sections. Show also that under these conditions the current density in any section is proportional to the cross-section of the conductor in that section.

20. A street main is 650 yd. long from A to B, the power being supplied at A. At a point C 250 yd. from A there is a branch main CD 200 yd. long. The equivalent loads which give the same fall of voltage over the sections concerned are—

On the section AC, 40 A at 100 yd from A.
„   „ CB, 15 A „ 300 yd. „ C.
„   „ CD, 20 A „   80 yd. „ C.

Determine the cross-sections of the mains AC, CD, and CB for minimum total copper volume, if the fall of voltage is to be 4 V, between A and B and A and D.   (LU)

21. A two-wire d.c. distributor is laid in the form of an equilateral triangle, each side of which is loaded uniformly, the rates of loading of the three sides being 1, 1, and 2 A per yd. respectively. The feeding points are at the corners of the triangle, and the supply station is at the centre of the triangular area bounded by the distributor. All feeders are straight and have the same pressure drop. Compare the volume of copper required for these feeders with that

for the case where the supply station is located in the position which gives a minimum volume of copper for the feeders, the pressure drop being the same in each case.   (*LU*)

22. A three-wire d.c. distributor cable *AB*, 700 yd. long, is fed from both ends, and the p.d. between either outer and the neutral is maintained constant at 200 V at each end. The cable is loaded as follows: + *side*: 20 A at *C*, 200 yd. from *A*; 40 A at *D*, 200 yd. from *B*; – *side*: 60 A at *E*, 240 yd. from *A*; 20 A at *F*, 160 yd. from *B*.

The resistance of 100 yd. of each outer conductor is 0·05 Ω; that of the neutral is 0·1 Ω per 100 yd. Calculate the current in each section of the positive, neutral, and negative conductors, and the voltage across each load point.   (*LU*)

23. A three-wire single-phase distributor cable *AFB*, 800 yd. long, is fed at its centre, *F*, and loaded as follows, the phase differences of the load currents being given with reference to the p.d.s at the feeding point and in all cases being lagging—

Red side : 20 A at 18·2° at *A*; 60 A at 36·8° at *C*, 280 yd. from *A*; 30 A at 18·2° at *D*, 50 yd. from *B*; 15 A at 25·8° at *B*.

Blue side : 10 A at 25·8° at *A*; 50 A at 0° at *E*, 200 yd. from *A*; 50 A at 36·8° at *G*, 300 yd. from *B*; 15 A at 0° at *B*.

The feeding point is connected directly to the secondary windings of the transformer, and a p.d. of 235 V is maintained on each side, the p.d. between the outers being 470 V.

The resistance of 100 yd. of each outer is 0·032 Ω, and that of 100 yd. of the neutral is 0·06 Ω.

Calculate (1) the current in each section of the distributor, (2) the p.d. at each load, (3) the currents in the secondary windings of the transformer, (4) the power supplied by the transformer.

24. A three-wire single-phase distributor cable *AB*, 700 yd. long, is fed from both ends and loaded as follows, the phase differences of the load currents being given with reference to the p.d.s at the feeding points and in all cases being lagging—

Red side : 20 A at 0° at *C*, 200 yd. from *A*; 40 A at 36·8° at *D*, 200 yd. from *B*.

Blue side : 60 A at 25·8° at *E*, 240 yd. from *A*; 20 A at 36·8° at *F*, 160 yd. from *B*.

The feeding points are connected directly to the secondary windings of transformers in street boxes. Each transformer maintains a p.d. of 200 V on each side of the system, and a p.d. of 400 V between the outers, the p.d.s at each feeding point being in phase with each other.

The resistance of 100 yd. of each outer is 0·05 Ω, and that of 100 yd. of the neutral is 0·1 Ω.

Calculate (1) the current in each section of the distributor, (2) the voltage at each load, (3) the power supplied by each transformer.

25. A three-phase four-wire distributor, *AB*, 400 yd. long, supplies four star-connected balanced loads (all at unity power factor) as follows: 30 A at *C*, 80 yd. from *A*; 35 A at *D*, 190 yd. from *A*; 17·5 A at *E*, 270 yd. from *A*; 10 A at *B*.

If the cable is fed from *A* at 410 V between lines, calculate the

cross-section to give a voltage drop to consumers of not more than 4 per cent. Calculate also the voltage at each load point.

What would be the reduction in the voltage drop if the cable were extended to a second feeding point, 100 yd. beyond B, and fed also at 410 V ? What would be the current at the second feeding point and the voltage at each load point ?

The resistance only need be taken into account in calculating the voltage drop.

26. A three-phase four-wire distributor, XY, supplies lighting loads A, B, and C of 120, 60, and 20 amp. respectively. A is 50, B 120, and C 160 yd. from X. The loads at A and B are balanced on the three phases, and that at C is connected between the red phase and neutral only. If the cable is fed from both ends at equal pressures, the two ends of each core being at the same potential, calculate the voltage drop at each of the three loads connected between the red phase and neutral. The length of the cable is 200 yd., and the resistance per 1,000 yd. of each core is 0·4 Ω.   (BP)

27. A three-phase, four-wire overhead-line distributor, 1,500 yd. long, is fed from one end, A, and loaded with single-phase loads which are connected between a phase conductor and the neutral in the following manner, the phase differences of the load currents being given with reference to the p.d.s at the feeding points, and in all cases being lagging—

Red phase : 10 A at 25·8° at B, 200 yd. from A ; 15 A at 0° at E, 800 yd. from B.

Blue phase : 12 A at 0° at C, 400 yd. from A ; 15 A at 25·8° at F, 700 yd. from C.

White phase : 20 A at 36·8 at D, 700 yd. from A ; 10 A at 18·2° at G, 800 yd. from D.

The feeding point is connected directly to the secondary winding of a transformer which maintains a p.d. of 240 V between each phase conductor and the neutral, and a p.d. of 416 V between any pair of phase conductors.

Calculate (1) the p.d. at each load, (2) the currents in each phase of the transformer, (3) the power supplied by the transformer.

The resistance per mile of each line conductor is 0·65 Ω, and the reactance per route mile of loop formed by any phase conductor and the neutral is 0·86 Ω.

28. A ring main has conductors of 1 sq. in. in cross-section, and is 1 mile long. A d.c. supply at 500 V is provided, and at distances of $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ miles respectively, are tapped off supplies of 250, 500, and 250 A. Calculate (1) the voltage drop at the three points.

In order to reduce this drop a connection is made direct from the supply by a cable of the same section one-third mile long. Calculate (2) the drop under the new conditions. The resistance of one mile of conductor 1 sq. in. in cross-section = 0·044 Ω.   (CG)

29. A ring main supplies four feeding points, B, C, D, and E, from a substation A, the resistances of the cables being as follows : AB, 0·032 Ω; BC, 0·057 Ω; CD, 0·081 Ω; DE, 0·73 Ω; EA, 0·024 Ω. The maximum loads are : B, 110 A; C, 90 A; D, 60 A, E, 120 A. Calculate (1) the current in each section under these conditions.

Calculate also (2) the current distribution if an interconnector of resistance 0·068 Ω is added between C and E.   (BP)

30. A distribution network ABCDEFA with interconnectors AGD and DF is fed at A and loaded as follows: 4 A at B, 3 A at C, 7 A at D, 4 A at E, 5 A at F, 2 A at G. The resistances of the various sections of the network (outgoing and return conductors) are as follow: AB, 0·1 Ω; BC, 0·2 Ω; CD, 0·2 Ω; DE, 0·3 Ω; EF, 0·2 Ω; FA, 0·25 Ω; AG, 0·3 Ω; GD, 0·15 Ω; DF, 0·4 Ω. Determine what must be the resistance of an equalizing conductor between A and D to reduce the voltage drop between these points to 2 V. Show the resulting distribution of currents when the equalizer is in use.   (LU)

31. A three-phase supply of 500 kW is required for a factory 2 miles from the nearest substation. This substation is for transforming 6,600 V to 440 V. The load will have a power factor of 0·8. The load factor is 30 per cent, and the I^2R losses may be taken at 1·3 times those caused by the equivalent steady load.
Calculate the efficiency of the supply (1) with a 0·1 sq. in. cable at 6,600 V, (2) with a pair of 0·3 sq. in. feeders at 440 V. Comment on the comparative results. Take the resistance of a single conductor 1 mile long and 1 sq. in. in cross-section as 0·044 Ω.   (CG)

32. A factory is supplied from a distributing centre through a 500 V three-core cable with a resistance of 0·05 Ω per conductor. The load consists of a synchronous motor which absorbs 100 kW at unity power factor, and other motors totalling 175 kW with an average power factor of 0·6 lagging. The star-connected stator of the synchronous motor has a resistance of 0·08 Ω per phase. If the power factor of this machine is lowered to 0·8 leading, calculate the copper losses in (1) the supply cable and (2) the stator winding. Calculate also (3) the power factor at which the machine should be operated to make the sum of these losses a minimum.   (CG)

33. A factory makes a maximum demand of 630 kW at 0·7 power factor and uses 1,500,000 kWh per annum. Energy is supplied at £6 per annum per kVA of maximum demand, plus 0·75d. per kWh.
It is proposed to raise the power factor to 0·9 by the installation of a phase advancer, the losses in which may be taken as 1 per cent of the maximum demand. Assuming interest and depreciation on the phase advancer at 10 per cent, what maximum capital expenditure upon it will be justified? Discuss whether the installation of the advancer is worth while in this case.   (LU)

34. Calculate the most economical angle of lag to which a consumer's load should be advanced with condensers if the consumer is charged £3 10s. per annum per kVA of maximum demand plus a flat rate per kWh. The cost of the condensers is £4 per kVA. Interest and depreciation, 10 per cent per annum.   (CG)

35. A local authority purchases a supply in bulk at 10,000 V, three-phase, at a price of £4 per kVA demand + 0·4d. per kWh, and distributes 1,000 kVA as three-phase low tension at a power factor of 0·8 (lag) and 1,000 kW as d.c.
(i) What plant should be used for converting to d.c.? (ii) At

what power factor should it be operated to keep the cost of energy
a minimum ?

(iii) Calculate the cost of energy per annum, assuming a load
factor of 30 per cent at (a) unity power factor, (b) 0·8 (lag) power
factor.   (*IEE*)

36. If the tariff in a certain district is £6 per annum per kVA of
maximum demand plus 0·55d. per kWh, calculate the saving in the
cost of energy resulting if the power factor of a 200 h.p. three-phase
motor is raised from 0·8 to 0·9. The supply voltage is 6,000, and
the motor efficiency at full load is 89 per cent. The motor runs at
full load for 1,000 hours per annum.   (*LU*)

37. Calculate the losses per annum incurred in giving, over a
line 10 miles long, a three-phase supply at 10,000 V through a trans-
former of 1,500 kVA capacity, the demand being 1,000 kW at power
factor 0·8 lag, and load factor 25 per cent. The size of line conductor
is 0·1 sq. in., and the resistance per mile is 0·4 Ω. The transformer
has an iron loss of 6 kW, and a copper loss at full load of 12 kW.
                                                                (*IEE*)

38. A three-phase 50-cycle generating station supplies an induc-
tive load of 5,000 kW at a power factor of 0·7 by means of an over-
head transmission line 5 miles long with conductors symmetrically
arranged. The resistance per mile of each wire is 0·61 Ω, and the
self-induction per mile of the loop formed by any two of the con-
ductors taken together is 0·0035 henry. The pressure at the receiving
end is maintained constant at 10,000 V. If a static condenser is
connected across the load to increase the power factor at the re-
ceiving end from 0·7 to 0·9, calculate (a) the value of the capacity
per phase of the condenser, (b) the station voltage when the con-
denser is in use, (c) the station voltage when the condenser is dis-
connected.   (*CG*)

39. A three-phase load of 2,000 kW, power factor 0·71 lagging,
is supplied through an overhead transmission line 10 miles long.
The inductance of a loop formed by two of the conductors is 0·004
henry per mile, and each of the three conductors has a resistance of
0·3 Ω per mile. Condensers are employed to raise the power factor
from 0·71 to 0·87. Calculate (a) the kVA rating of the condenser
bank required, (b) the voltage at the generating station when the
condensers are in circuit and when disconnected. Voltage at re-
ceiving end 11,500 V, 50 frequency.   (*CG*)

40. What considerations determine the cross-section of the con-
ductor in a low voltage system (a) for the feeder cables, (b) for the
distributor cables? Determine the most suitable cross-section for a
two-wire feeder cable 500 yd. long from the following considerations—

Cross-section (sq. in.) . .	0·1	0·15	0·2	0·25	0·3
Resistance (ohms per 1,000 yd.)	0·245	0·167	0·126	0·1	0·081
10% of cable cost (£) . .	27·5	30·0	33·0	37·5	46·0
Maximum safe current (A) .	155	200	240	278	313

Current, 175 A; interest and depreciation, 10 per cent. Cost of

energy wasted, 1d. per kWh, the loss being such as would be produced by a current of 175 A for 4 hours per day.   (CG)

41. Show that when the fall of voltage due to resistance and reactance is small compared with the line voltage, the fall of voltage along a three-phase overhead transmission line per ampere per mile is given by

$$\sqrt{3}\,[R\cos\varphi + X\sin\varphi]$$

where R, X denote the resistance and reactance respectively per mile of conductor, and cos φ is the power factor of the load.

Find the fall of voltage for a three-phase line 30 miles long when 5,000 kVA are delivered at 30,000 V and 0·8 power factor (lagging). The resistance and reactance per mile are 0·72 Ω and 0·6 Ω respectively.   (LU)

42. A substation is fed from a power house by two routes of equal length; on one route the reactance per mile is 50 per cent greater than on the other route, the same size of conductor being used. Show, graphically or analytically, how the load would be divided between the two routes.   (LU)

43. The full output (5,000 kW at power factor 0·8 lagging) of a three-phase hydro-electric station is transmitted to a substation by two routes, the lines being connected in parallel. The respective resistances are 1·5 Ω and 1·0 Ω, and the corresponding reactances are 1·25 Ω and 1·2 Ω. Determine the power transmitted by each route.   (CG)

44. A three-phase load of 1,000 kW is supplied at 6,600 V, 0·8 power factor (lagging) by two feeder cables working in parallel. One cable delivers 670 kW, and the current in each of its conductors is 76 A. Calculate the resistance and reactance per conductor of this cable if the corresponding quantities for the other are 3 Ω and 4 Ω respectively.   (LU)

45. It is desired to increase the power transmitted by an overhead route by adding another exactly similar conductor to the existing two-conductor one-phase line, and working three-phase with the same voltage between lines. There will then be three conductors of the same size arranged at the corners of an equilateral triangle, their distance apart being the same as before. When transmitting 1,000 kW with 6,600 V between the one-phase wires, the total line impedance drop was 10 per cent. Calculate the power that can be transmitted when working three-phase with the same voltage drop (i.e. the voltage at the load is to be the same in both cases).   (CG)

46. Two transformers of equal rating share a load of 180 kW at 0·9 power factor, the current lagging.

At full load the fall of voltage due to resistance in the first transformer is 1 per cent of the normal terminal voltage, and that due to reactance 6 per cent. The corresponding figures for the second transformer are 2 per cent and 5 per cent respectively. Find the load in kW on each transformer.   (LU)

47. Two transformers, A, B, are connected in parallel to supply a load having a resistance of 2 Ω and a reactance of 1·5 Ω. The

equivalent resistances referred to the secondary windings are (*A*) 0·15 Ω, (*B*) 0·1 Ω, and the equivalent reactances are (*A*) 0·5 Ω, and (*B*) 0·6 Ω. If the open-circuit e.m.f.s are (*A*) 207 V, (*B*) 205 V, calculate (1) the voltage at the load, (2) the power supplied to the load, (3) the kVA output of each transformer, (4) the phase difference between the e.m.f. and current in each transformer.

48. Two transformers, *A* and *B*, which are working in parallel have equivalent impedances (referred to the secondary) expressed thus—

$$\text{Impedance of } A = 0\cdot1 + j1.$$
$$\text{Impedance of } B = 0\cdot138 + j1\cdot5.$$

The impedance of the load circuit is $2 + j0\cdot8$, and the open-circuit e.m.f.s of the secondary windings *A*, *B* are in the ratio 100 : 99.

Calculate the reactance which, when connected to the secondary of *B*, will make *A* deliver twice as much current as *B*.   (*LU*)

PART II

ELECTRIC TRACTION

SECTION I

TRACTION MOTORS
(DIRECT CURRENT AND ALTERNATING CURRENT)

Torque-current characteristics of direct-current motors. The torque is given by

$$\mathfrak{T} = (p/a)\Phi Iz/852 = p\Phi \cdot IN/426 \qquad . \qquad . \quad (23)$$

or $\qquad \mathfrak{T}/I = (pN/426)\Phi \qquad . \qquad . \qquad . \qquad . \qquad . \quad (24)$

where $\mathfrak{T}$ is the gross torque (in lb.-ft.), p the number of poles, a the number of circuits in the armature winding, Φ the flux per pole (in megalines), I the current input to the armature, z the number of active armature conductors, N ($= \frac{1}{2}z/a$) the number of turns per armature circuit.

Speed-current and speed-torque characteristics of direct-current motors. The fundamental equation for the e.m.f. generated in the armature is

$$V - IR = (n/60)(p/a)\, z\Phi \times 10^{-2},$$

whence $\qquad n = \dfrac{6000\,(V - IR)}{z\,\Phi\,p/a} = \dfrac{3000\,(V - IR)}{pN\Phi} \qquad . \quad (25)$

or $\qquad n = \dfrac{3000\,(V - IR)}{426\,\mathfrak{T}/I} = 7.05\,\dfrac{(V - IR)}{\mathfrak{T}/I} \qquad . \quad (26)$

where n is the speed in revolutions per minute, V the voltage at the terminals of the motor, R the resistance of the main circuit of the motor, and I, p, a, z, N, Φ have the same significance as above.

Conversion of armature speed and torque into train speed and tractive effort. The train speed S m.p.h., corresponding to an armature speed n r.p.m., a gear ratio γ, and driving wheels D inches diameter is

$$S = (nD/\gamma)60\pi/(12 \times 5280) = 0.00297\, nD/\gamma \, . \quad (27)$$

The tractive effort F (lb.) corresponding to an armature

gross torque $\mathfrak{J}$ (lb.-ft.) and the above gear ratio and wheel diameter is

$$F = 24\ \mathfrak{J}\eta_m\gamma/D \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (28)$$

where η_m is the "mechanical" efficiency of the motor and gearing, i.e. $100\ (1 - \eta_m)$ represents the percentage mechanical and core losses (comprising friction, windage, gear and core losses).

If the gear ratio and wheel diameter are changed to γ' and D' respectively, the new speed (S') and tractive effort (F') are

$$S' = S(\gamma D'/\gamma' D) \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (29)$$
$$F' = F(\gamma' D/\gamma D') \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (30)$$

NOTE. The friction and gear losses are assumed to be unaffected by the change of gear ratio.

Division of load between motors driving uncoupled wheels.
(*a*) *Motors operating in parallel.* With similar speed-current and torque-current characteristics calculated for normal wheels (D), the current inputs to motors A, B, driving wheels of unequal diameters (D_A, D_B) at a given train speed (S), are given by points on the normal speed-current curve corresponding to the speeds SD/D_A, SD/D_B respectively. The tractive efforts are obtained from the points on the normal tractive-effort/current curve corresponding to these currents, but multiplied by the ratio of wheel diameters, i.e. D/D_A for motor A, D/D_B for motor B.

(*b*) *Motors operating in series.* With two similar direct-current motors the terminal voltages corresponding to a current I are

$$V_A = \frac{V - IR}{1 + D_A/D_B} + \frac{IR}{1 + D_B/D_A} \qquad . \qquad . \qquad . \qquad (31)$$

$$V_B = \frac{V - IR}{1 + D_B/D_A} + \frac{IR}{1 + D_A/D_B} \qquad . \qquad . \qquad . \qquad (32)$$

where R denotes the resistance of each motor.

If S is the train speed corresponding to a current I at normal voltage V and with normal wheels D, the actual train speed is

$$S' = S[(V_A - IR)/(V - IR)]D_A/D$$
$$= S[(V_B - IR)/(V - IR)]D_B/D \qquad . \qquad . \qquad (33)$$

Testing. *Correction of resistances to standard hot machine temperature.* For standard annealed copper conductors, the

ratio of the resistances R_1, R_2 of a given winding at temperatures θ_1, θ_2 respectively is

$$R_1/R_2 = (234 \cdot 5 + \theta_1)/(234 \cdot 5 + \theta_2) \qquad . \qquad . \quad (34)$$

for the Centigrade scale, and

$$R_1/R_2 = (390 + \theta_1)/(390 + \theta_2) \qquad . \qquad . \quad (35)$$

for the Fahrenheit scale.

Hence, if R_1 is the measured resistance at a temperature $\theta_1°$ C., the resistance corrected to the standard hot machine temperature (75° C.) is

$$R_2 = R_1 [309 \cdot 5/(234 \cdot 5 + \theta_1)] \qquad . \qquad . \quad (36)$$

Efficiency tests (*direct-current machines*). When two similar machines are tested together and a rheostat load is employed
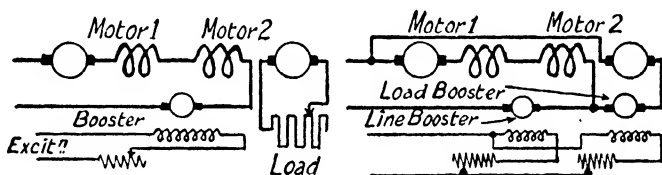
FIG. 4   FIG. 5

CONNECTIONS FOR LOAD TESTS

for the generator, Fig. 4, the efficiency of the motor (determined by the input-output method) is

$$\eta = 1 - (0 \cdot 5/V_1)[(V_0 + I_1 R_1) - (I_2/I_1)(V_2 + I_2 R_2)] . \quad (37)$$

where $V_0 = $ "total" (i.e. motor + generator field) voltage; $V_1 = $ motor terminal voltage; $V_2 = $ generator armature terminal voltage; I_1, $I_2 = $ motor and generator currents; R_1, $R_2 = $ resistances of motor and generator armatures respectively (including brushes).

With the loading-back method (Fig. 5) the efficiency is determined preferably from the motor input and the total losses (as given by the input from the supply system and the outputs from the boosters). Thus

$$\eta = 1 - 0 \cdot 5(V_0 I + V_3 I_2 + I_1^2 R_1 - I_2^2 R_2)/V_1 I_1 \quad (38)$$

where $V_3 = $ terminal voltage of "load" booster, $I = $ current input from supply system, and the other symbols have the same significance as above.

3—(5186)

Correction of speed. Let R denote the resistance of the armature and field windings at a temperature of 75° C., R_1 the resistances during the test, n_1 the test speed, and n the corrected speed, both corresponding to a current I. If the terminal voltage during the test has been held at its normal value V_1, then

$$n_1/n = (V_1 - IR_1)/(V_1 - IR)$$

whence $n = n_1(V_1 - IR)/(V_1 - IR_1)$.   .   .   (39)

If the speed n_1 has been determined at a voltage (V'_1) other than normal, then

$$n = n_1(V_1 - IR)/(V'_1 - IR_1)   .   .   .   (39a)$$

Tractive effort. The tractive effort (F lb.) is calculated from the input (V_1, I_1) efficiency with gearing (η) and armature speed (n r.p.m.). Thus

$$F = \frac{V_1 I_1 \eta \gamma}{nD} \times \frac{12 \times 33,000}{746\pi} = \frac{169\ V_1 I_1 \eta \gamma}{nD}   .   (40)$$

Alternatively, if S is the train speed (m.p.h.) corresponding to n, D, γ

$$F = 0.5\ V_1 I_1 \eta/S   .   .   .   .   .   (41)$$

Torque-current characteristics of single-phase series motors. When the flux is in phase with the current, and both follow sine laws, the mean torque is given by

$$\mathfrak{T}_{mean} = (p/a)(\Phi_m/\sqrt{2})Iz/852 = p(\Phi_m/\sqrt{2})IN/426 .   (42)$$

where Φ_m is the maximum or crest value of the flux, I the r.m.s. value of the current and the other symbols have the same significance as in the direct-current case.

Speed-current characteristics of single-phase series motors. The e.m.f. generated in the armature winding by its rotation in the main flux (called the *dynamic* e.m.f.) is given by

$$E = p(\Phi_m/\sqrt{2})Nn/3000   .   .   .   .   (43)$$

whence $n = 3000E/(pN\Phi_m/\sqrt{2})$.   .   .   .   (44)

The dynamic e.m.f. is determined from the vector diagram of the motor, the simplest of which is shown in Fig. 6. This vector diagram refers to the case where the flux is in phase with the current. The internal e.m.f.s in the motor circuit are: (1) the dynamic e.m.f. (OE, Fig. 6), (2) the e.m.f.s due to the impedances of the windings (of which the in-phase

components are represented by ED and the quadrature components by DF). Hence, if V denotes the terminal voltage; $\cos \varphi$ the power factor at this voltage, and a current I; and R, X, the equivalent resistance* and reactance respectively of the motor at this current, we have

$$E = V \cos \varphi - RI = \sqrt{(V^2 - X^2 I^2)} - RI$$

$$= \sqrt{[V^2 - (V \sin \varphi)^2]} - RI \qquad (45)$$

$$\cos \varphi = (E + RI)/V \qquad . \qquad . \qquad (46)$$

Polyphase induction motors. The fundamental equations for the speed, torque, and output are

(1) Speed

$$n = n_s(1 - s) = f(1 - s)/\tfrac{1}{2}p \ . \qquad (47)$$

(2) Rotor current

$$I_2 = sE_2/\sqrt{(R_2{}^2 + s^2 X_2{}^2)} \qquad . \qquad (48)$$

(3) Rotor power factor

$$\cos \varphi_2 = R_2/\sqrt{(R_2{}^2 + s^2 X_2{}^2)} \qquad . \qquad (49)$$

FIG. 6. SIMPLIFIED VECTOR DIAGRAM FOR SINGLE-PHASE MOTOR

(4) Torque

$$\mathfrak{F} = K\Phi I_2 \cos \varphi_2 = K\Phi s E_2 R_2/(R_2{}^2 + s^2 X_2{}^2) \qquad (50)$$

(5) Power input to rotor

$$P_2 = E_2 I_2 \cos \varphi_2 \qquad . \qquad . \qquad . \qquad . \qquad (51)$$

(6) Mechanical output

$$P_M = (1 - s)P_2 = (1 - s)E_2 I_2 \cos \varphi_2 \qquad . \qquad . \qquad (52)$$

(7) Rotor I^2R loss

$$P_R = sE_2 I_2 \cos \varphi_2 (= P_2 - P_M = sP_2) \qquad . \qquad . \qquad (53)$$

where n is the speed of the rotor in revs. per sec. ($= f/\tfrac{1}{2}p$), n_s the synchronous speed of the revolving field in revs. per sec., f the frequency of the supply current, p the number of

* The equivalent resistance, R, is such that the in-phase voltage component, RI, includes not only the appropriate voltage drops due to the actual resistances, but also the equivalent voltage drop corresponding to additional I^2R losses due to eddy currents and circulating currents, together with the in-phase voltage component corresponding to the stator core loss at the current I.

poles, s the slip ($= (n_s - n)/n_s$), $\Im$ the gross torque, Φ the flux per pole, E_2 the e.m.f. induced in the rotor at standstill, I_2 the current in the rotor, φ_2 the phase difference between e.m.f. and current in rotor, R_2 the resistance per phase of the rotor, X_2 the reactance per phase of the rotor at standstill, and K a constant.

Predetermination of performance of polyphase induction motor. A close approximation to the performance of the
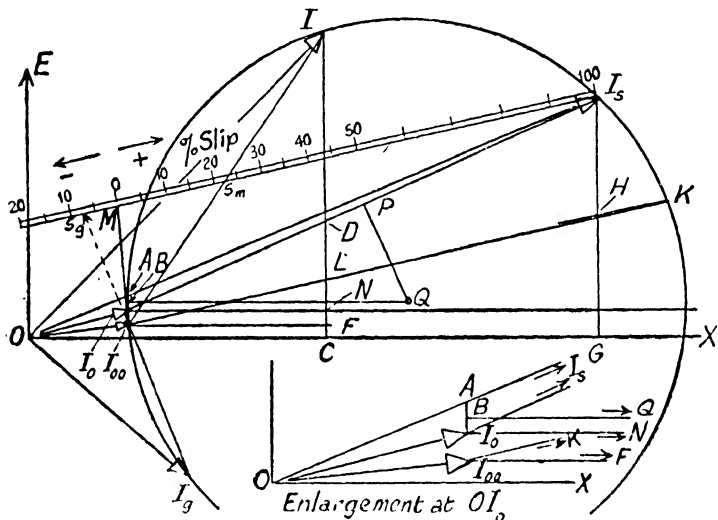
Fig. 7. Circle Diagram for Polyphase Induction Motor

motor may be obtained from the circle diagram (general vector diagram) for the motor. This diagram (Fig. 7) may be drawn from a knowledge of the no-load and short-circuit currents and power factors of the motor together with the resistances of the windings. Thus, taking the vector of the terminal voltage, OE, as the vector of reference, the vectors of the no-load and short-circuit line currents at normal voltage are set off as OI_o and OI_s respectively. The centre of the circle, Q, is determined as follows: Join I_oI_s, bisect and draw a perpendicular PQ. From I_o draw a parallel to OE, intercepting OI_s at A. Bisect I_oA and draw BQ at right angles to cut PQ at Q, which is the centre of the circle, the circumference of which passes through the no-load and short-circuit points I_o and I_s respectively. A vector, such as OI, drawn from O

to any point I on the circumference of the circle represents the stator current in magnitude and phase. The datum lines for the power input and the power output are OX (the horizontal axis) and I_oI_s (the line joining the no-load and short-circuit points). The ordinate IC represents the power input, and the portion ID the power output. Therefore the losses for the current input OI are represented by DC, of which the "no-load" losses (i.e. core loss, friction, and windage) are represented by NC, and the I^2R losses by DN. If the core loss is known the vector of the current input at synchronous speed may be determined. Thus, if CF represents the core loss, the horizontal line drawn through F will cut the circumference at I_{oo}, and OI_{oo} therefore represents the current input at synchronous speed. The point I_{oo} also corresponds to zero torque, and the torque datum line must pass through this point. Another point on the torque datum line is obtained as follows: The ordinate I_sG at the short-circuit point is divided at H, such that $I_sH : HG ::$ stator I^2R loss at short circuit : rotor I^2R loss at short circuit. Join OH and produce so as to cut the circumference at K. Then OK represents the current input when the slip is infinitely great and the torque is zero. Hence, $I_{oo}K$ is the torque datum line. The torque at the current I is represented by the portion IL of the ordinate IC.

The slip scale is constructed on a line I_sM drawn from I_s parallel to the torque datum line, the zero mark being at the point where the tangent at I_{oo} cuts this line, and the 100 mark at I_s. The per cent slip corresponding to the current input I is given by the scale reading at which the line $I_{oo}I$ cuts the slip scale.

Operation of polyphase induction motor as generator. An induction motor connected to a supply system and driven mechanically at speeds above synchronism (i.e. the slip is negative) operates as a generator, the power (within the limit of the maximum output) increasing as the slip increases. The frequency is determined by the supply system to which the induction machine is connected and which supplies the magnetizing current. The performance as generator may be predetermined from the circle diagram constructed from the no-load and short-circuit currents of the machine operating as a motor. The portion of the circumference below the point I_{oo} (synchronous speed) and the horizontal axis OX is the locus of the stator current vector; the horizontal axis is the output datum line; the line $I_{oo}K$ is the torque datum line; and the line I_oI_s is the input datum line. The slip scale is extended to the left of the zero point, and the slip is given by the scale

reading at which the line, produced, joining I_{oo} and the extremity of the current vector OI_g cuts the slip scale.

Cascaded motors. When two motors are operating in cascade (i.e. the stator of one motor is supplied with energy at the slip frequency of the other motor, and the two motors are mechanically coupled so as to rotate at the same speed) the cascade synchronous speed (n_c, revs. per sec.) is

$$n_c = f/\tfrac{1}{2}(p_1 + p_2) \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (54)$$

where f is the supply frequency and p_1, p_2 the numbers of poles of the motors.

The ratio of the mechanical outputs of the cascaded motors when running at a speed n is given approximately (losses in the motors being ignored) by

$$P_M'/P_M'' = n_c/(n_s - n) = p_1/(p_2 + s_2 p_1) . \qquad . \qquad (55)$$

where s_2 is the slip of the second motor.

WORKED EXAMPLES. 1. A 60-h.p.. 500-V trolley-bus motor has the following characteristics at normal voltage, full field—

Amperes . . .	60	80	100	120	140
Speed (m.p.h.) . . .	17·8	15·1	13·35	12·1	11·2
Tractive effort (lb.) . .	700	1,130	1,610	2,100	2,620

Deduce the characteristics when the motor is operating with the field winding shunted 25 per cent. Resistance of motor $= 0.3\,\Omega$.

SOLUTION. The magnetization characteristic is calculated from the internal e.m.f. and speed. Thus

$$K\Phi = (V - IR)/n$$

When the motor is operating with shunted field winding the excitation corresponding to a given armature current (I) is $0.75I$, and the speed S' (m.p.h.) is given by

$$S' = S(V - IR')K\Phi/ (V - IR)K\Phi'$$

where S, Φ, R denote the speed, flux, and motor resistance respectively for full-field operation, and Φ', R' the flux and motor resistance for shunted-field operation.

In the majority of cases the value of $V - IR'$ is within 1 per cent of $V - IR$, so that approximately

$$S' = S(K\Phi/K\Phi')$$

Hence for full-field operation we have—

Armature amperes (I) . .	60	80	100	120	140
Counter-e.m.f. ($= 500 - 0\cdot3I$) .	482	476	470	464	458
Speed (S) (mp.h.) . . .	17·8	15·1	13·35	12·1	11·2
$K\Phi$ [$= (500 - 0\cdot3I)/S$] . .	27·1	31·5	35·2	38·3	40·9

For shunted-field operation we have—

Armature amperes (I) . .	60	80	100	120	140
Field amperes ($= 0\cdot75I$) . .	45	60	75	90	105
$K\Phi'$	21·5	27·1	30·5	33·5	36·1
Counter-e.m.f. ($= 500 - 0\cdot3I$) .	482	476	470	464	458
Speed ($= SK\Phi/K\Phi'$) (m.p.h.).	22·4	17·55	15·4	13·8	12·7
Tractive effort (lb.)* .	556	972	1,395	1,840	2,310

* Tractive effort with shunted field $= (K\Phi'/K\Phi)$ tractive effort with full field.

2. A railway motor has the following characteristics—

Current (amp.) . . .	50	100	150	200
Speed (m.p.h.) . . .	46	30	25·7	23·3
Tractive effort (lb.) . . .	300	1,050	1,860	2,670

Give the approximate speeds and tractive efforts for the same currents when 47 per cent of the main field windings are cut out.

SOLUTION. The approximate magnetization characteristic for the full field winding is given by the current tractive effort per ampere curve. Thus—

Current (amp.) . . .	50	100	150	200
Trac. effort per amp. ($= K\Phi$) .	6	10·5	12·4	13·35

When these values are plotted we can obtain the values of $K\Phi$ for the tapped-field winding and the above armature currents. The new values of the tractive effort are obtained by multiplying the appropriate values of $K\Phi$ by the corresponding armature currents. Thus—

Armature current (I) (amp.) .	50	100	150	200
Equivalent excitation ($= 0\cdot53I$)	26·5	53	79·5	106
Trac. effort per amp. ($= K\Phi_1$) .	3·3	6·4	9	10·8
Tractive effort ($= K\Phi_1 I$) (lb.) .	165	640	1,350	2,016

As no values for the resistances of the armature and field windings are given, the speeds, corresponding to a given armature current, with full field and tapped field may be considered to be proportional to the inverse ratio of the fluxes, i.e.

Speed (full field)/ speed (tapped field) $= K\Phi_1/K\Phi$

Hence the approximate characteristics for tapped-field operation are—

Current Input (amp.) . .	50	100	150	200
Speed (m.p.h.) . .	83·6	49·2	35·4	28·8
Tractive effort (lb.) . .	165	640	1,350	2,016

3. The characteristics of a single-phase series railway motor at 287 V, $16\frac{2}{3}$ cycles, are as follow—

Amperes	800	1,200	1,600	2,000
Speed (r.p.m.) . . .	700	500	410	350
Torque (kg.-m) . . .	280	560	860	1,160
Power factor . . .	0·96	0·932	0·9	0·87

Determine the characteristics for 380 V, $16\frac{2}{3}$ cycles. The equivalent resistance of the motor is 0·012 Ω .

SOLUTION. The speed corresponding to a given current is proportional to the dynamic e.m.f. generated in the armature, i.e. for a current I, $n_1/n_2 = E_1/E_2$. If V_1, V_2, cos φ_1, cos φ_2 denote the corresponding terminal voltages and power factors,

$$E_1 = V_1 \cos \varphi_1 - RI = \sqrt{(V_1^2 - X^2 I^2)} - RI$$
$$E_2 = V_2 \cos \varphi_2 - RI = \sqrt{(V_2^2 - X^2 I^2)} - RI$$

where R, X denote the equivalent resistance and reactance respectively of the motor. Now $XI = V_1 \sin \varphi_1 = V_2 \sin \varphi_2$.

Hence carrying out the calculations in tabular form we have—

Amperes (I) . . .	800	1,200	1,600	2,000
Cos φ . . .	0·96	0·932	0·9	0·87
Sin φ . . .	0·28	0·362	0·436	0·492
RI . . .	9·6	11·4	19·2	24
287 sin φ (= XI) .	80·4	104	125	141
$E_{287} = \sqrt{(287^2 - X^2 I^2)} - RI$	266	253	239	226
$E_{380} = \sqrt{(380^2 - X^2 I^2)} - RI$	362	351	340	328
Speed at 380 V (r.p.m.) .	950	692	583	508
Cos φ at 380 V (%) . .	97·8	96·2	94·6	92·7
Torque (kg.-m.) . .	280	560	860	1,160

EXAMPLES II

TRACTION MOTORS

1. The relationship between the current and gross torque of a series motor—determined by a static test—is as follows—

Current (amp.) . . .	10	20	30	40	50	60	70
Torque (lb.-ft.) . . .	33	95	170	258	346	450	565

Deduce the speed curve of the motor when supplied at a constant voltage of 500 V, resistance of main circuit of motor = 0·5 Ω.

2. The magnetization curve of a four-pole series traction motor —determined by separately exciting the field winding, and connecting a voltmeter across the brushes and driving the armature at a constant speed of 500 r.p.m.—is as follows—

Field amperes	.	.	.	50	100	150	200	250	300
Armature volts	.	.	.	240	380	446	488	522	550

Determine the speed-torque curve for this motor when operating at a constant voltage of 600, having given that the armature has a two-circuit winding with 97 turns per circuit, the resistance of the armature winding and brushes is 0·06 Ω, and the resistance of the field windings is 0·05 Ω.

3. The magnetization curve of a d.c. four-pole series motor, with two-circuit armature winding, was obtained by separately exciting the field winding and loading the armature to take the same currents, the speed being maintained constant at 750 r.p.m. The results were—

Field amperes	.	.	.	10	20	30	40	50	60	70
Armature volts	.	.	.	160	295	375	425	460	485	505

Determine the torque-speed curve for this motor, over the above range of loading, on a line p.d. of 550 V, having given that the total number of conductors on the armature is 660; the resistance of the armature is 0·28 Ω, and that of the field coils is 0·292 Ω.

4. A 500-V, four-pole, wave-wound, d.c. series motor has the following magnetic characteristics—

Amperes	.	.	.	.	20	40	60	80
Flux per pole (megalines)	.	.	1·5	2·325	2·8	3·05

The gross torque-speed variation of the load is as follows—

R.p.m.	.	.	.	.	300	400	500	600	700	800
Torque (lb.-ft.)	.	.	.	96	148	208	283	375	480

The total motor resistance is 0·3 Ω, and the total number of armature conductors is 984.

Find the speed at which the motor will run with this load.

<div align="right">(IEE)</div>

5. A 500 V series motor has a resistance of 1·36 Ω and runs at a speed of 500 r.p.m. when taking a current of 23 A. The magnetization curve is as follows—

Flux per pole (per cent) .	32	57	74	85	94	100	104
Current (amp.) . . .	4	8	12	16	20	24	28

Draw the speed-current curve for this motor, and from it derive the speed-current curve when the field coils have a 50 per cent tapping for speed control.   *(CG)*

6. A motor has characteristics as follow—

Amperes	300	250	200	150	100
Tractive effort (lb.) . .	3,250	2,500	1,780	1,100	500

Deduce the corresponding characteristics for the motor with one-third of its exciting field cut out.   *(IEE)*

7. The field winding of a series motor for a commercial electric (battery) vehicle is arranged in two equal portions for connection either in series or in parallel. When connected in series the characteristics of the motor at a terminal pressure of 80 V are as follow—

Current (amp.) . . .	20	30	40	50
Speed (r.p.m.) . . .	1,000	800	690	625
Torque (lb.-ft.) . . .	10	18	26·5	35·5

Determine the characteristics when the two halves of the field winding are connected in parallel. Resistance of armature = 0·133 Ω. Resistance of field winding (series connection) = 0·107 Ω.

8. The following figures refer to the speed/current and tractive-effort/current characteristics of a d.c. series railway motor—

Current (amp.) . .	100	150	200	250	300	400
Tractive effort (lb.) .	1,050	2,100	3,200	4,250	5,350	7,500
Speed (m.p.h.) . .	32	25·5	22·5	20·5	19	18

Derive corresponding approximate curves when 50 per cent field weakening is obtained by halving the field current. Neglect any change in efficiency.   *(IEE)*

9. The following characteristics refer to a locomotive motor operating at full field, 1,350 V.

Current (amp.) . . .	700	600	500	400	300
Armature speed (r.p.m.) . .	463	488	525	582	680
Torque at shaft (kg.-m.) .	1,865	1,535	1,208	880	573

Calculate the approximate characteristics when operating with (a) 78 per cent, (b) 61 per cent of the field winding in circuit. Resistance of motor 0·167 Ω.

10. The speed-current characteristics of a tramway motor at normal voltage (525 V) are—

Current (amp.)	.	.	.	160	120	80	40
Speed (m.p.h.)	.	.	.	11·5	12·9	15·6	23·5

A tramcar is equipped with two of these motors. Calculate the speed-current characteristic when the motors are operating in series at normal line voltage. Resistance of each motor = 0·42 Ω.

11. The controller for the tramway motors in the preceding example (10) has four series steps, and the resistances of the sections of the rheostats for these steps are

$$R_1 - R_2 = 2\cdot47 \ \Omega, \ R_2 - R_3 = 1\cdot93 \ \Omega, \ R_3 - R_4 = 1\cdot5 \ \Omega.$$

Calculate the speed-current characteristics for each of the series steps and for a line voltage of 525.

12. A tramcar is equipped with two motors which are operating in parallel, and are supplied at a constant voltage of 525 V. Determine the current input when the car is running at a steady speed of 24 m.p.h., and each motor is developing a tractive effort of 360 lb. The resistance of each motor is 0·42 Ω, and the motor friction, axle friction, windage and gear losses per motor at a car speed of 24 m.p.h. total 2,890 watts.

13. A four-pole d.c. series motor has the following characteristics—

Exciting current (amp.)	.	.	25	35	45	55	65	75
Flux per pole (megalines)	.	.	1·7	2·13	2·45	2·7	2·9	3·05

It is direct-coupled to a fan, and when supplied at 500 V the speed, with the four field coils connected in series, is 670 r.p.m., the current being 35 A.

Find by a graphical method the current taken and the speed when the field coils are rearranged in two parallel groups each of two coils in series.

Assume that the power taken by the fan is proportional to the cube of the speed, that the iron loss in the motor remains unchanged at 650 W, and that the motor friction is 800 W at 670 r.p.m., and is proportional to the speed. Resistances: armature 0·4 Ω, each field coil 0·1 Ω   (LU)

14. A 500 V tramcar motor has an efficiency of 88 per cent when running with full field at 660 r.p.m., and developing a torque at the pinion of 180 lb.-ft. Determine the percentage weakening of the field to give a speed of 750 r.p.m. and a torque of 200 lb.-ft., the efficiency under these conditions being 88·2 per cent. The resistances of the armature and field windings are 0·2 Ω and 0·3 Ω respectively, and the combined friction, windage, and core losses may be assumed to be proportional to the speed.

15. A two-axle tramcar is equipped with two standard d.c. series motors. If the wheels on one axle wear much quicker than those on the other axle, how would the motors share the load (a) in series, (b) in parallel?

If, in addition, the motor driving the smaller wheels had a speed characteristic slightly higher than that of the other motor, how would this affect the sharing of the load? State full reasons for the answers given.	(IEE)

16. A motor-coach is equipped with two motors, each having the following characteristics at 550 V when geared to 36 in. wheels, the gear ratio being 3·37—

Amperes	.	.	.	350	300	250	200	150
Speed (m.p.h.)	.	.	.	17·5	18·7	20·1	22·9	27·5
Tractive effort (lb.)	.	.	4,850	3,940	3,030	2,140	1,320

The wheels of one axle (A) of the motor truck of this coach are 35 in. in diameter, and those of the other axle (B) are 33½ in. in diameter. Determine, when the motors are operating in parallel at normal line voltage, the current input to each and the tractive effort at a train speed of 19 m.p.h.	(BP)

17. If the motors of the preceding example (16) are operating in series and the current is 300 A, what is the train speed and the voltage across each motor? Resistance of each motor 0·09 Ω.

18. The following motor characteristic is based on a wheel diameter of 36 in.—

Current (amp.)	.	.	.	80	160	240	320	400
Tractive effort (lb.)	.	.	400	1,350	2,470	3,700	4,950
Speed (m.p.h.)	.	.	.	53	34·5	28·8	25·5	23·2

A motor bogie is fitted with two of these motors, one pair of wheels being 36 in. in diameter, and the other pair 35 in. The motors are operated on the series-parallel system. Suppose the tractive effort at the 36 in. wheels is 3,000 lb., what will be the current and tractive effort of the other motor (a) in full series, and (b) in full parallel.	(IEE)

19. A motor-coach is equipped with two motors, each having the following characteristics at 550 V when the driving wheels are 36 in. in diameter—

Amperes	.	.	.	350	300	250	200	150
Speed (m.p.h.)	.	.	.	17·5	18·7	20·1	22·9	27·5
Tractive effort (lb.)	.	.	4,850	3,940	3,030	2,140	1,320

The wheels of one axle (A) of the motor truck of this coach are 35 in. in diameter, and those of the other axle (B) are 33 in. in diameter. Determine, when the motors are operating in parallel at normal line voltage, the current inputs to each and the tractive efforts at train speeds of 18, 21, 25 m.p.h.	(BP)

20. The motor-coach of an electric train is equipped with two geared motors, having characteristics at 775 V and for 42 in. driving wheels, as follows—

Amperes input per motor . .	250	200	150	100
Speed of car (m.p.h.)	20·2	22	24·7	32·2
Tractive effort (lb.) . .	4,200	3,100	2,065	1,005
Efficiency (per cent) . .	87·5	88	87·7	83·5

The diameters of the driving wheels connected to one motor (A) are 42 in., and those connected to the other motor (B) 40 in. When the motors are operating in parallel at a train speed of 24 m.p.h., determine (a) the power input to each motor, (b) the tractive effort, and (c) output at each pair of driving wheels.

Determine also for series operation and a current input of 150 A (d) the power input to each motor, (e) the train speed.   (BP)

21. The cold resistance of the armature winding of a traction motor is 0·04885 Ω at 7° C. The resistance at the end of the one-hour test is 0·0708 Ω, the air temperature being 15° C. What is the rise in temperature of the armature winding? The resistance coefficient is 1/234·5 at 0° C.   (IEE)

22. (a) The cold resistance of the field windings of a motor is 0·045 Ω at an air temperature of 15° C. What would be the resistance for a temperature rise of 80° C. measured when the air temperature is 20° C.? The resistance coefficient is 1/234·5 at 0° C.

(b) If there is an error of 5 per cent in the cold resistance, what would be the corresponding error in the hot resistance?   (IEE)

23. The cold resistance of the main fields of a motor is 0·0431 Ω measured at an air temperature of 12° C., and the hot resistance is 0·0526 Ω measured at an air temperature of 7° C. What is the temperature rise? The resistance coefficient is 1/234·5 at 0° C.   (IEE)

24. A motor has the following characteristics—

Current (amp.) . . .	50	100	150	200	250
Speed (m.p.h.) . . .	46	30	25·7	23·3	22
Tractive effort (lb.) . .	300	1,050	1,860	2,670	3,500

The size of the car wheel is 40 in., and the gear ratio is 72 : 23. The car wheel is changed to 42 in., and the gear ratio to 75 : 20. Give the new characteristics.   (IEE)

25. The following observations were taken on a pair of similar railway motors during a test, with gears, on a stand, one machine being operated as a motor and the other as a separately-excited generator, its field winding being connected in the motor circuit—

Motor volts	600
Motor (and generator field) amperes . .	275
Generator armature amperes . . .	232
Generator armature volts	525
Generator field volts	20·2
Armature speed (r.p.m.)	500

The resistances of the machines, cold (20° C.) and hot (determined immediately after the test), are—

	Cold	Hot	
Motor armature	0·05	0·0675	Ohm
Motor field	0·057	0·073	,,
Generator armature	0·05	0·065	,,
Brushes	0·01	0·01	,,

Calculate the efficiency, train speed, and tractive effort for a line voltage of 600 V, when the temperature of the windings of the motor is 75° C., the gear ratio being 3·5 : 1 and the diameter of driving wheel being 42 in.

26. The airgap under the pole face of a single-phase series motor is 0·06 in. in length, and has an effective area of 80 sq. in. The exciting winding has five turns and a resistance of 0·005 Ω. Assuming the reluctance of the iron portions of the magnetic circuit to be one-fifth of the reluctance of the airgap, calculate the voltage at the terminals of the exciting winding when a current of 150 A at 25 cycles is passing. (CG)

27. Find the terminal e.m.f. and power factor at full load of a single-phase series motor when operating at a frequency of 25 cycles per second. The motor has 8 poles with a flux of 2 megalines per pole. Other data are as follow: total field turns = 20; ratio of total armature ampere turns to total field ampere turns = 2; speed at full load = 600 r.p.m.; full load current = 600 A; iron loss at full load = 1,800 W; resistance of field windings = 0·004 Ω; leakage reactance = 0·01 Ω; armature and brush resistance = 0·003 Ω; armature reactance, 0·02 Ω. (LU)

28. Calculate the gross torque of the motor referred to in the preceding example (27).

29. A single-phase series motor has characteristics at 380 V, 16⅔ cycles, as follow—

Amperes	1,000	1,400	1,800	2,200
Speed (r.p.m.)	790	628	540	490
Torque (kg.-m.)	420	720	1,010	1,320
Power factor (per cent)	97	95·4	93·8	92·2

Deduce the approximate characteristics at 440 V. Equivalent resistance of motor = 0·012 Ω.

30. The motor referred to in the preceding example (29) forms part of the equipment of a locomotive, the driving wheels being 1,610 mm. in diameter and the gear ratio 2·57 : 1. Determine the characteristics of the locomotive (per motor) at a motor voltage of 380. The efficiency of the motor including gear losses is—

Amperes	1,000	1,400	1,800	2,200
Efficiency (per cent)	84·5	85·5	85·5	84·8

31. A single-phase locomotive, equipped with two 16⅔ cycle compensated series motors, has the following speed characteristic when the voltage per motor is 400 V—

Amperes per motor . . .	1,000	1,500	2,000	2,500
Power factor (per cent) . .	93·8	90·8	87·5	85
Speed (km.p.h.) . . .	31	25	21·5	19

Determine the speed characteristic when the terminal voltage per motor is 225 V. The effective resistance of each motor is 0·025 Ω.

32. A single-phase locomotive with three individually-driven axles is equipped with motors having characteristics at normal voltage and 1,610 mm. wheels as follow—

Amperes	1,000	1,400	1,800	2,200
Speed (m.p.h.). . . .	68	54·6	47·4	43·4

The driving wheels on one axle (A) are 1,600 mm. in diameter, and those on the other axles (B, C) are 1,580 and 1,550 mm. respectively in diameter. What will be the current input to the respective motors when the locomotive is running at a speed of 50 m.p.h. ?

33. The characteristics of a single-phase locomotive motor are as follow for a voltage of 400 and driving wheels of 1,070 mm. diameter—

Amperes	1,000	1,500	2,000	2,500
Speed (km.p.h.) . . .	31	25	21·5	19
Tractive effort (kg.) . . .	3,600	6,360	9,300	12,150

A type $C + C$ locomotive is equipped with two of these motors. The coupled wheels driven by one motor (A) have a diameter of 1,055 mm., and those driven by the other motor (B) have a diameter of 1,040 mm. Determine the tractive effort of the locomotive and the current input to each motor at a speed of 25 km.p.h.

34. A three-phase, 1,200 h.p., 930 V, 45-cycle, locomotive-type induction motor has, at normal voltage and frequency, a no-load current of 300 A at 0·05 power factor, and a short-circuit current of 4,700 A at 0·09 power factor. The equivalent resistance per phase of the rotor is equal to that of the stator. Calculate (1) the full-load current, (2) the power factor and efficiency at full load, (3) the maximum torque, (4) the maximum output, (5) the maximum braking torque when operating as an asynchronous generator at normal voltage and frequency.

35. A car, driven by a three-phase induction motor, ascends a gradient of 1 in 10 at a speed of 8 m.p.h. The frictional resistances are equivalent to a gradient of 1 in 50. The motor on no-load and normal voltage has a power factor of 0·15, and at standstill a power factor of 0·25. The standstill current is thirty times the no-load

current, and the current taken under the given conditions of run-
ning is four times the no-load current. At what speed will the car
run with the same current on a "down" gradient, and what is the
value of this gradient?   (*LU*)

36. Two locomotives, equipped with $16\frac{2}{3}$-cycle, three-phase in-
duction motors, are hauling a train. The rated output of each
locomotive is 1,000 h.p., at which the slip is 4 per cent. One loco-
motive has new wheels of 1·5 metres diameter, and the other has
wheels with 1·5 cm. radial wear. Estimate the distribution of load
between the locomotives when the track conditions demand total
outputs of (*a*) 2,000 h.p., (*b*) 1,000 h.p., (*c*) 100 h.p. How could a
more equal distribution be obtained?   (*IEE*)

37. An electric locomotive is equipped with a three-phase, six-
pole, 1,000 h.p. induction motor, and a three-phase, eight-pole in-
duction motor. They are controlled to operate singly, or with the
eight-pole motor fed in cascade from the six-pole motor. (1) What
economic running speeds are obtainable, and (2) what should be
the rating of the eight-pole motor? (3) When running in full cascade,
what is the contribution of mechanical power from each motor?
The supply frequency is 50.   (*IEE*)

SECTION II

CONTROL

Control of direct-current motors. Speed control is effected by series and parallel groupings of the motors together with in some cases, variation of the flux (which is effected by either shunting or tapping the field winding). Starting is effected by connecting external rheostats in series with the motors.

The operating conditions at starting require the current per motor to vary within definite limits, and usually the initial current peak is lower than the succeeding peaks.

Grading of sections of starting rheostats for series combination of motors. The fundamental equations based upon the circuit diagram of Fig. 8 (a), and the starting conditions represented in Fig. 8 (b), are

$$R_1 = V/I \qquad . \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (56)$$

$$\frac{\Phi_1}{\Phi_2} = \frac{V - I_1 R_2}{V - I_2 R_1} = \frac{V - I_1 R_3}{V - I_2 R_2} = \cdots = \frac{V - 2I_1 R_m}{V - I_2 R_n} \qquad (57)$$

where Φ_1, Φ_2 are the fluxes corresponding to the currents I_1, I_2 respectively, V is the line voltage, n the number of *sections* in the rheostat (the number of steps or notches being $n + 1$), and R_m the resistance of each motor.

All methods, whether analytical or graphical, of calculating the resistances of the sections of the rheostats are based upon these fundamental equations. Observe that the current limits I_1, I_2, together with the corresponding fluxes Φ_1, Φ_2 are involved, which necessitates a knowledge of the magnetization curve (or its equivalent) of the motor.

Author's analytical method.[*] The fundamental equation (57) is rearranged in the form

$$\frac{\Phi_1/I_1}{\Phi_2/I_2} = \frac{V/I_1 - R_2}{V/I_2 - R_1} = \frac{V/I_1 - R_3}{V/I_2 - R_2} = \cdots = \frac{V/I_1 - 2R_m}{V/I_2 - R_n}$$

whence by cross multiplication and reduction the following equations are obtained, giving the resistances of the sections of the rheostat in terms of two grading coefficients, viz.

$$\lambda \,[= (\Phi_1/I_1)/(\Phi_2/I_2)] \text{ and } \gamma (= I_2/I_1).$$

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[*] See *Journ. I.E.E.*, v. 60, p. 867; *Electric Traction*, p. 257.

Thus $R_1 - R_2 = R_1 \left\{ \dfrac{\lambda}{\gamma} \left(\dfrac{1}{\zeta} - \gamma \right) + \left(1 - \dfrac{1}{\zeta} \right) \right\}$.   . (58a)

$R_2 - R_3 = \lambda (R_1 - R_2)$.   .   .   .   . (58b)

$R_3 - R_4 = \lambda (R_2 - R_3)$.   .   .   .   . (58c)

etc.

NOTE. $R_1 = V/I$; $\zeta = I_1/I$. ζ takes into account the initial starting current, and is equal to unity when this current has the same value as the succeeding peaks.
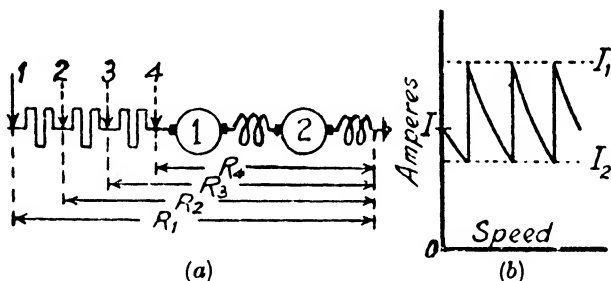
FIG. 8. PERTAINING TO CALCULATIONS OF STARTING
RHEOSTATS

The coefficients λ and γ are inter-related by the general equation

$$R_1 - 2R_m = R_1 \left\{ \frac{\lambda}{\gamma} \left(\frac{1}{\zeta} - \gamma \right) + \left(1 - \frac{1}{\zeta} \right) \right\} \frac{1 - \lambda^n}{1 - \lambda}$$

or

$$\frac{2R_m}{R_1} \left(= \frac{e/I}{V/I} = \frac{e}{V} \right) = 1 - \left\{ \frac{\lambda}{\gamma} \left(\frac{1}{\zeta} - \gamma \right) + \left(1 - \frac{1}{\zeta} \right) \right\} \frac{1 - \lambda^n}{1 - \lambda} \quad (59)$$

e denoting the voltage drop in the motor portion of the circuit (i.e. the *two* motors) corresponding to the current I.

Families of curves, or a general chart, Fig. 9, can be prepared from equation (59) for given values of e/V and n.

The evaluation of λ and γ for a *particular* case (i.e. for given values of e/V, ζ, n, I) involves determining a second relationship between λ and γ from data of the magnetization curve of the motor, and solving for λ and γ. The solution is obtained graphically by plotting the *particular* relationship on the general chart, and determining the point of intersection of this curve with the appropriate general curve as explained in the worked example on page 45.

Alternative analytical method.* In this method the general equation (57) is rearranged with all the terms on one side so as to equate to zero. Thus

$$R_1 \left\{ \frac{\lambda}{\gamma}\left(\frac{1}{\zeta} - \gamma\right) + \left(1 - \frac{1}{\zeta}\right) \right\} \frac{1 - \lambda^n}{1 - \lambda} - R_1 + 2R_m = 0$$

i.e. $\dfrac{V}{I} \left[\left\{ \dfrac{\Phi_1}{\Phi_2}\left(\dfrac{I}{I_1} - \dfrac{I_2}{I_1}\right) + \left(1 - \dfrac{I}{I_1}\right) \right\} \dfrac{1 - [(\Phi_1/I_1)/(\Phi_2/I_2)]^n}{1 - (\Phi_1/I_1)/(\Phi_2/I_2)} - 1 \right]$

$$+ 2R_m = 0 \qquad . \qquad . \qquad . \qquad (60)$$

In a particular case V, I, I_1, Φ_1, R_m are given, and the relationship between Φ_2 and I_2 can be calculated from the magnetization or speed curve. The value of I_2, which satisfies equation (60), is determined by trial, e.g. values are assumed
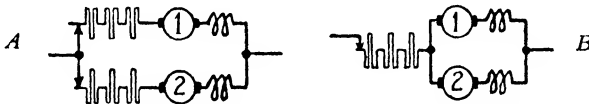
FIG. 10. ALTERNATIVE ARRANGEMENTS OF RHEOSTATS FOR PARALLEL STEPS

for I_2, and the left-hand side of the equation is evaluated for each of these assumed values. The results are plotted against the assumed values of I_2, and the value of the latter corresponding to the zero value of the equation is readily determined.

Grading of rheostats for parallel combination of motors. *Author's method.*† The fundamental equations for the resistances of the parallel steps, based upon the circuit diagram A of Fig. 10, the transition being effected at the current I_2, and the *lower* limit of current being the same for both parallel and series steps, are

$$\Phi_{1p}/\Phi_2 = (V - I_{1p}R_{1p})/(\tfrac{1}{2}V - I_2 R_m)$$

$$. \quad . \quad . \text{ (transition step)} \quad . \quad . \quad (61)$$

$$\frac{\Phi_{1p}}{\Phi_2} = \frac{V - I_{1p}R_{2p}}{V - I_2 R_{1p}} = \frac{V - I_{1p}R_{3p}}{V - I_2 R_{2p}}$$

$$= \ldots = \frac{V - I_{1p}R_m}{V - I_2 R_n{}'} \quad . \quad (62)$$

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* Based upon a method given by Dr. S. Parker Smith in *Journ. I.E.E.*, ·. 58, p. 645. The symbols given in the original paper have been modified o agree with those employed in the present text.

† *Journ. I.E.E.*, v. 60, p. 878.

where I_{1p} is the upper limit of current for the parallel steps; Φ_{1p}, the corresponding flux; R_{1p}, R_{2p}, . . . the circuit resistances (for each motor) on the several steps; n', the number of steps.

Equations (62) reduce to

$$\frac{e'}{V}\frac{I_2}{I} = \frac{(1 + a)(\gamma_p - \tfrac{1}{2}\lambda_p) - \tfrac{1}{2}a}{(1 + a)(1 - \lambda_p)} \qquad . \qquad . \qquad . \quad (63)$$

where

$$a = \lambda_p(1 - \lambda_p{}^{n'})/(1 - \lambda_p); \ \gamma_p = I_2/I_{1p}; \ \lambda_p = (\Phi_{1p}/I_{1p})/(\Phi_2/I_2);$$

and e' is the voltage drop in the motor portion of the circuit (i.e. the voltage drop *per motor*) at the current I.

Families of curves (Fig. 11) can be plotted, for given values of e'/V and n', to give the general relationship between λ_p/γ_p and γ_p. The particular values of λ_p and γ_p for a given case are determined by a process similar to that employed for the series steps.

The resistances of the sections of the rheostat are given by the equations—

$$R_{1p} - R_{2p} = \lambda_p\{(R_1[\tfrac{1}{2}(1 + \lambda_p) - \gamma_p]/\gamma\zeta) + R_m(1 - \lambda_p)\} \quad . \quad (64a)$$

$$R_{2p} - R_{3p} = \lambda_p(R_{1p} - R_{2p}) \qquad . \qquad . \qquad . \qquad . \qquad . \quad (64b)$$

etc., etc.

If, however, the *upper* limit of current is the same for both series and parallel steps, equations (61), (62) become respectively

$$\Phi_1/\Phi_2 = (V - I_1R_{1p})/(\tfrac{1}{2}V - I_2R_m) \quad . \qquad . \quad (65)$$

$$\frac{\Phi_1}{\Phi_{2p}} = \frac{V - I_1R_{2p}}{V - I_{2p}R_{1p}} = \ . \ . \ . \ = \frac{V - I_1R_m}{V - I_{2p}R_n'} \qquad . \qquad . \quad (66)$$

Equations (66) reduce to

$$1 - \lambda_p\left(\frac{1 - \lambda_p{}^{n'}}{1 - \lambda_p}\right)\left(\frac{1}{\gamma_p} - 1\right) - \tfrac{1}{2}\frac{\lambda}{\gamma}\lambda_p n' = \frac{e}{V}(1 - \lambda\lambda_p{}^{n'}) \quad (67)$$

The resistances of the sections of the rheostat are given by

$$R_{1p} - R_{2p} = R_1[\lambda_p/\gamma_p - \lambda_p - \tfrac{1}{2}(1 - \lambda_p)\lambda/\gamma] + R_m\lambda(1 - \lambda_p) \quad (68a)$$

$$R_{2p} - R_{3p} = \lambda_p(R_{1p} - R_{2p}) \qquad . \qquad . \qquad . \qquad . \qquad . \quad (68b)$$

etc., etc.

Control of single-phase series motors. Starting and speed control are both effected by applying definite voltages to the

motor. These voltages are obtained from tappings on the secondary winding of the transformer which supplies the motor.

Calculation of the motor voltage steps for starting single-phase series motor. The method of calculating the voltage steps is based upon the vector diagram of Fig. 6, the speed and e.m.f. equations (44), (45), and the starting conditions represented in Fig. 8 (b). The fundamental equations are

$$V_1 = I \sqrt{(R^2 + X^2)} = \sqrt{(R^2 I^2 + V_1^2 \sin^2 \varphi_1)}$$
$$= \sqrt{(R^2 I^2 + V_n^2 \sin^2 \varphi_n)} \qquad . \qquad . \qquad . \quad (69)$$

$$\frac{\Phi_{m1}}{\Phi_{m2}} = \frac{V_2 \cos \varphi_2' - I_1 R}{V_1 \cos \varphi_1'' - I_2 R} = \frac{V_3 \cos \varphi_3' - I_1 R}{V_2 \cos \varphi_2'' - I_2 R}$$
$$= \ldots = \frac{V_n \cos \varphi'_n - I_1 R}{V_{n-1} \cos \varphi_{n-1}'' - I_2 R} \qquad . \qquad . \quad (70)$$

where I is the initial starting current; I_1, I_2 the upper and lower limits respectively of current throughout the starting period; V_1, V_2, . . . V_n, the voltage steps; $\cos \varphi_1'$, $\cos \varphi_2'$ . . . $\cos \varphi_n'$, the power factors corresponding to the various voltages and the upper limit of current (I_1); $\cos \varphi_1''$, $\cos \varphi_2''$, . . . $\cos \varphi_n''$, the power factors corresponding to the various voltages and the lower limit of current (I_2); R the equivalent resistance of the motor (mean value for the currents I_1 and I_2); and X the reactance of the motor at the current I.

The general equations (70) cannot be reduced to a simple equation—similar to (59)—involving λ, γ, n, V_1, V_n, etc.[*] But if n and V_1 (or I) are not confined to definite values, and V_n together with the current limits are specified (which conditions will usually correspond to practical requirements, particularly with locomotives), a simple solution may be obtained for the voltage steps. Thus, since $\Phi_{m1}/\Phi_{m2} = \lambda/\gamma$, we have

$$V_1 \cos \varphi_1'' = (\gamma/\lambda) V_2 \cos \varphi_2' - R I_2(1/\lambda - 1)$$
$$V_2 \cos \varphi_2'' = (\gamma/\lambda) V_3 \cos \varphi_3' - R I_2(1/\lambda - 1)$$
$$V_{n-1} \cos \varphi_{n-1}'' = (\gamma/\lambda) V_n \cos \varphi_n' - R I_2(1/\lambda - 1)$$

Now
$$V_1 \cos \varphi_1'' = \sqrt{[V_1^2 - (V_1 \sin \varphi_1'')^2]}$$
$$= \sqrt{(V_1^2 - V_n^2 \sin^2 \varphi_n'')};$$
$$V_2 \cos \varphi_2'' = \sqrt{(V_2^2 - V_n^2 \sin^2 \varphi_n'')};$$
$$V_2 \cos \varphi_2' = \sqrt{(V_2^2 - V_n^2 \sin^2 \varphi_n')}; \text{ etc.}$$

[*] Such an equation can be obtained, however, if the magnetization curve is a straight line (i.e. if $\lambda/\gamma = 1 \cdot 0$), but this assumption is not justifiable with single-phase railway motors.

Whence

$$V_{n-1} = \sqrt{\{[\gamma/\lambda \sqrt{(V_n^2 - V_n^2 \sin^2 \varphi_n')} - RI_2(1/\lambda - 1)]^2 + V_n^2 \sin^2 \varphi_n''\}} \qquad . \qquad . \qquad . \qquad (71)$$

$$V_1 = \sqrt{\{[\gamma/\lambda \sqrt{(V_2^2 - V_n^2 \sin^2 \varphi_n')} - RI_2(1/\lambda - 1)]^2 + V_n^2 \sin^2 \varphi_n''\}} \qquad . \qquad . \qquad . \qquad (71a)$$

WORKED EXAMPLES. (1) Calculate the resistances of the sections of the starting rheostat for the four series steps of a tramcar controller to be used with two 50 h.p., 525 V motors having characteristics, at 525 V, as follow—

Amperes.	105	80	70	60
Speed (m.p.h.).	13·7	15·6	16·7	18·2

The initial starting current is to be 77 A, and the upper limit of current 105 A. The resistance of each motor is 0·45 Ω.

SOLUTION. We have $\zeta = I_1/I = 105/77 = 1·36$; voltage drop in the *two* motors at 77 A = 77 × 2 × 0·45 = 69·3 V. Whence $e/V = 69·3/525 = 0·132$.

Next the particular relationship between λ and λ/γ is calculated for a fixed upper limit of 105 A and lower limits of 80 A, 70 A, 60 A. The calculations are arranged in tabular form—

I_1 .	105						
I_2 .			80		70		60
$\gamma (= I_2/I_1)$.		0·762		0·666		0·571	
Voltage drop per motor .	47·2		36		31·5		27
Internal e.m.f. (= E) .	477·8		489		493·5		498
Speed (= S) .	13·7		15·6		16·7		18·2
$\lambda/\gamma (= E_1 S_2/E_2 S_1)$.		1·112		1·18		1·275	
$(\lambda/\gamma - 1)/\zeta$.		0·0822		0·132		0·202	
λ .		0·848		0·787		0·728	

Plotting these results on Fig. 9 we obtain the co-ordinates of the point of intersection with the interpolated curve for $e/V = 0·132$, $n = 3$, as $\lambda = 0·78$, $(\lambda/\gamma - 1)/\zeta = 0·142$. Whence $\lambda/\gamma = 1·193$, $\gamma = 0·654$, and $I_2 = 68·6$ A.

Now $R_1 = 525/77 = 6·82$ Ω.

Hence, substituting in equation (58a) we have

$$R_1 - R_2 = R_1 \{[\lambda(1/\zeta - \gamma)/\gamma] + (1 - 1/\zeta)\} = 6·82 \times 0·362$$
$$= 2·47 \text{ Ω.}$$

and from equations (58b), etc.,

$$R_2 - R_3 = \lambda(R_1 - R_2) = 0·78 \times 2·47 = 1·93 \text{ Ω.}$$
$$R_3 - R_4 = \lambda(R_2 - R_3) = 0·78 \times 1·93 = 1·5 \text{ Ω.}$$

(2) Calculate the resistances of the sections of the starting rheostat for the four parallel steps of the controller to be used with the 50 h.p. tramway motors of the preceding example. Transition is effected at a current of 68·6 A, and the upper limit of current (per motor) is to be 105 A.

SOLUTION. In tramway controllers the parallel steps are arranged in accordance with the circuit diagram, B, of Fig. 10. Hence in determining the resistances of the sections we may calculate as for circuit diagram A—making use of equations (67), (68a), etc.—and halve the values so obtained. Thus, from the preceding example, we have: $R_1 = 6\cdot82\ \Omega$, $\lambda = 0\cdot78$, $\lambda/\gamma = 1\cdot193$, $\zeta = 1\cdot36$, $e'/V = 77 \times 0\cdot45/525 = 0\cdot066$. Substituting these values in equation (67), and noting that $n' = 3$, we have

$$\frac{1}{1\cdot36}\left[1 - \lambda_p\left(\frac{1 - \lambda^3_p}{1 - \lambda_p}\right)\left(\frac{1}{\gamma_p} - 1\right) - 0\cdot596\lambda_p{}^3\right] - 0\cdot066(1 - 0\cdot78\lambda_p{}^3) = 0$$

Calculating the values of this expression for $\lambda_p = 0\cdot8$, $\lambda_p = 0\cdot85$, $\lambda_p = 0\cdot9$—the corresponding values of γ_p being obtained from the preceding example—and plotting, the value of λ_p which satisfies the equation is 0·868. Whence $\gamma_p = 0\cdot798$, $\lambda_p/\gamma_p = 1\cdot088$, $I_{2p} = 83\cdot8$ A.

Hence substituting appropriate values in equations (64), and halving the values so obtained, we have

$$R_{1p} - R_{2p} = 0\cdot376\ \Omega,$$
$$R_{2p} - R_{3p} = 0\cdot327\ \Omega,$$
$$R_{3p} - R_{4p} = 0\cdot284\ \Omega.$$

(3) Calculate the voltage steps for starting a single-phase series motor if the current limits are 1,600 A and 1,100 A (except the initial step), and the maximum voltage is 287 V. The speed and power factor at 287 V, 1,600 A, are 410 r.p.m. and 0·9 respectively. The corresponding quantities for 1,100 A are 530 r.p.m. and 0·94. The equivalent resistance of the motor = 0·012 Ω.

SOLUTION. First calculate the constant terms in equations (71). Thus. $V_n{}^2\sin^2\varphi_n' = 15,625$; $V_n{}^2\sin^2\varphi_n'' = 9,545$; $\gamma = 0\cdot688$; $\gamma/\lambda = 0\cdot853$; $\lambda = 0\cdot807$; $RI_2(1/\lambda - 1) = 3\cdot46$.

NOTE. $\gamma/\lambda = \Phi_{m2}/\Phi_{m1} = E_2n_1/E_1n_2$, where E_1, E_2 are the dynamic e.m.f.s calculated according to the method given on page 25.

Hence substituting appropriate values in equations (71) and solving we obtain $V_n = 287$ V; $V_{n-1} = 237\cdot5$ V; $V_{n-2} = 195$ V; $V_{n-3} = 158$ V; $V_{n-4} = 125\cdot7$ V.

Now $\sqrt{(X^2I_1{}^2 + R^2I_1{}^2)} = 126\cdot2$ V, which is approximately equal to the calculated value for V_{n-4}. Hence with 126 V as the first step the initial starting current will reach the upper limit of 1,600 A.

EXAMPLES III

CONTROL

1. A car driven by two d.c. series motors is taking 58 A from a 500 V line with the motors in full series. The motors are switched into parallel through a resistance; calculate the value of this resistance in order that the transition may be effected without shock. The total resistance of each motor is 0·5 Ω   (*LU*)

2. On a motor-coach there are two motors: the resistance of each motor is 0·15 Ω, the maximum permissible starting current per motor is 400 A, and the average current is 350 A. Assuming the voltage is constant at 600 V during starting, and that for the range considered the flux is proportional to the fourth root of the current, what are the values of the resistances required for the first and second notches in series?   (*IEE*)

3. A motor-coach running on a 600 V supply is equipped with two 300 h.p., 600 V motors, which are controlled on the series-parallel system. The hot resistance of each is 0·08 Ω. The current varies between 380 A and 510 A per motor during acceleration. The corresponding speeds, at 600 V, are 26·4 m.p.h. and 23·3 m.p.h. Determine the resistance required on the first two series notches.

     (*IEE*)

4. On the motor-coach of example (2), transition from series to parallel is effected by the bridge method at a current of 300 A, and the current per motor rises to 400 A at the completion of the transition. What is the value of the resistance of each motor circuit for the first notch in parallel?

5. What values of resistance are required on the first and second parallel notches of the controller for the motor-coach of example (3), if the current limits for these notches are to be the same as those for the series notches, and the transition is effected at a speed of 12·5 m.p.h.?

6. It is desired to arrange for starting a certain locomotive and train so that the tractive effort is within given limits. At the upper limit the speed of the locomotive with motors in full series is 6·8 m.p.h., and the resistance drop in the motor is 12 per cent of its terminal voltage. At the lower limit the speed is 7·1 m.p.h., and the resistance drop 10 per cent of the terminal voltage. How many series notches are necessary in the control?   (*IEE*)

7. The controller for the commercial electric vehicle motor of example 7, page 32, has three rheostatic steps. Calculate the resistances of the sections of the starting rheostat to give an initial starting current of 50 A and current peaks of this value. The supply voltage is 80 V, and the resistance of the motor is 0·24 Ω.

8. The controller for the trolley-bus motor, of which the characteristics are given on page 28, has five rheostatic steps, one full-field step and one shunted-field step. Determine the resistances of the sections of the starting rheostat. The initial starting current is to be 120 A, and the succeeding current peaks 140 A.

9. Calculate the resistances of the three series sections of the starting rheostats for two 40 h.p., 500 V tramway motors, each of

which has a resistance of 0·62 Ω. The current limits during starting are 90 A and 63 A, and the speeds at these currents and 500 V on each motor are 9 and 10·3 m.p.h. respectively.   (*LU*)

10. Calculate the resistances of the series sections of the starting rheostat for the series-parallel control of two railway motors having characteristics as given in Examples II, 19, page 34, the number of steps in the series portion of the controller being five. The initial starting current is 300 A, and the upper and lower limits of current during starting are 350 A and 253 A respectively. The resistance of each motor is 0·1 Ω   (*BP*)

11. Calculate the resistances of the three rheostat sections for the parallel steps of the controller for the tramway motors referred to in example 9. Transition is effected at a current of 63 A, and the current limits for the parallel steps are 90 A and 68·4 A.

12. Calculate the resistances of the rheostat sections for the four parallel steps of the controller for the motors referred to in example 10. Transition is effected by the bridge method at a current of 253 A, and the lower limit of current for the parallel steps is to be the same as that for the series steps.

13. Find, analytically or graphically, the values of the four series and the three parallel rheostat sections suitable for the series-parallel control of two 275 h.p., 550 V d.c. series motors, each having a resistance of 0·15 Ω. The maximum current limit is to be 400 A throughout. A part of the magnetization curve is given below—

Current (amp.) .	300	350	400
Flux (per cent) .	90	95	100

                                                                    (*IEE*)

14. Calculate the resistances of the starting rheostats for the series-parallel control of two 600 V, d.c. railway motors. The controller has five series and four parallel steps, and bridge transition is employed. The initial starting current is 300 A. The upper limit of current is 348·7 A for the series steps, and 326 A for the parallel steps; the lower limit is 251·3 A throughout. Each motor has a resistance of 0·12 Ω and the speeds (at 600 V) corresponding to the currents 348·7, 326, and 251·3 A are 17·9, 18·4, and 20·5 m.p.h. respectively.

15. A 5 h.p., 100 V, 25-cycle, single-phase compensated series motor is to be started in three steps, the reduced voltages being obtained from tappings on an auto-transformer, and the upper limit of current being 55 A. The current-speed-power factor characteristics of the motor at 100 V, 25 cycles, are as follow—

Current (amp.)	.	.	.	70	55	40	35
Speed (r.p.m.)	.	.	.	530	850	1,210	1,400
Power factor .	.	.	.	0·785	0·875	0·94	0·955

The equivalent resistance of the motor is $0.4\,\Omega$. Calculate the voltages of the tappings and the lower limit of current.

16. If the motor of the preceding example (15) is to be started in two steps by means of a series resistance which is short-circuited by a contactor (the operating coil of which is connected across the brushes of the motor) when the speed reaches 550 r.p.m., calculate (1) the value of the starting resistance if the initial starting current is 55 A. Calculate also (2) the current peak when the resistance is short-circuited, and (3) the voltage across the operating coil when the contactor closes.

17. Determine suitable transformer voltage tappings for starting a multiple-unit train equipped with single-phase series motors, each rated at 230 h.p., 240 V, and having an effective resistance of $0.02\,\Omega$. Take current limits of 1,400 A and 1,100 A, except for the lowest tapping; the power factors for these currents on normal voltage are 0·775 and 0·86 respectively.   (*IEE*)

18. Calculate the voltage steps for starting a single-phase 16⅔-cycle locomotive motor, the current limits being 1,750 A and 1,500 A, and the final voltage 380 V. The power factors and speeds, at 380 V, corresponding to the current limits are 0·94, 0·95; and 550 r.p.m., 600 r.p.m. respectively.

SECTION III

Simplified speed-time curves. The symbols denoting the various quantities are shown in Fig. 12, and the units are as follow: distance, miles; time, seconds; speed, miles per hour; acceleration and retardation, miles per hour per second.
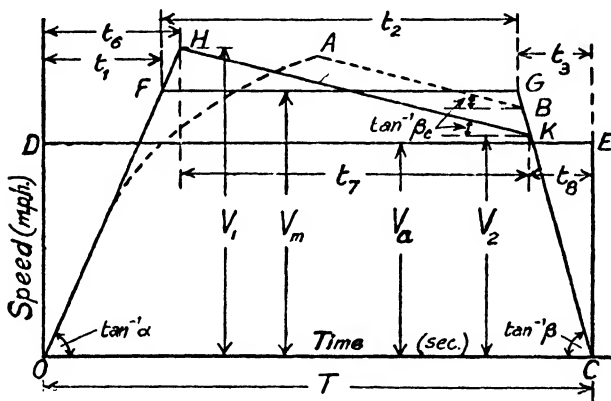
FIG. 12. EQUIVALENT SPEED TIME CURVES

(*OABC*, actual speed-time curve; *ODEC*, equivalent rectangle; *OFGC*, equivalent trapezium; *OHKC* equivalent quadrilateral)

For the *trapezoidal speed-time curve* the equations are—

$$3600D = V_m T - \tfrac{1}{2}V_m^2(1/\alpha + 1/\beta)$$

$$V_m = \left(\frac{\alpha\beta}{\alpha + \beta}\right)T - \sqrt{\left[\left(\frac{\alpha\beta}{\alpha + \beta}\right)^2 T^2 - 7200D\left(\frac{\alpha\beta}{\alpha + \beta}\right)\right]}$$

$$= \sqrt{\left\{\left(\frac{\alpha\beta}{\alpha + \beta}\right)\left[7200D\left(\frac{V_m}{V_a} - 1\right)\right]\right\}} \qquad . \qquad . \quad (72)$$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{7200}{V_m^2}\left(\frac{V_m T}{3600} - D\right) = \frac{7200D}{V_m^2}\left(\frac{V_m}{V_a} - 1\right) . \qquad . \quad (73)$$

NOTES. D = distance of run in miles = area of speed-time diagram. V_a = average or mean running speed = $3600D/T$.

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For the *quadrilateral speed-time curve* the equations are—

$$D = \frac{1}{7200}\left\{ V_1 T\left[1 + \frac{a(\beta + \beta_c) + 2\beta\beta_c}{a(\beta - \beta_c)}\right]\right.$$

$$\left. - \frac{\beta}{\beta - \beta_c}\left[V_1{}^2 \frac{(a + \beta_c)(a + \beta)}{a^2\beta} + \beta_c T^2\right]\right\} \quad . \quad (74)$$

$$V_2 = \beta[V_1(1 + \beta_c/a) - \beta_c T]/(\beta - \beta_c) \quad . \quad . \quad . \quad (75)$$

Tractive force, power, and energy. *Tractive force for acceleration*—

$$F_a = 102W_e a \quad . \quad . \quad . \quad . \quad . \quad . \quad (76)$$

(F_a in lb., W_e (effective or accelerating weight of train) in tons, a in m.p.h.p.s.).

$$W_e = W + 1\cdot 2n_1 W_1 + 0\cdot 49n_2 W_2\gamma^2(r_2/r_1)^2 \quad . \quad (77)$$

where W = dead weight of train, W_1 = weight of each wheel, W_2 = weight of each armature, n_1 = number of axles in train, n_2 = number of motors, γ = gear ratio, r_1 = radius of driving wheel, r_2 = radius of armature.

Tractive force for gradient—

$$F_g = 22\cdot 4WG \quad . \quad . \quad . \quad . \quad . \quad . \quad (78)$$

(F_g in lb. G = percentage gradient = rise or elevation (ft.) per 100 ft. of track.)

Power output from driving axles—

$$P = 0\cdot 002F_t V \quad . \quad . \quad . \quad . \quad . \quad . \quad (79)$$

where P = power in kW, F_t = total tractive effort in lb., V = speed in m.p.h.

Specific energy output for runs made according to trapezoidal or quadrilateral speed-time curves—

Watt-hours per ton mile for acceleration

$$= (0\cdot 0283V_m{}^2/D)W_e/W \quad . \quad . \quad . \quad . \quad (80)$$

Watt-hours per ton mile for train resistance

$$= 1\cdot 99rD'/D \quad . \quad . \quad . \quad . \quad . \quad . \quad (81)$$

where V_m = maximum speed; D = distance of run; D' = distance run with power; r = specific train resistance (lb. per ton of train dead weight).

Calculation of speed-time curve and energy consumption. The point-to-point method is employed, the speed being taken

as the independent variable and time as the dependent variable. Thus an increment of speed is assumed, the mean accelerating tractive effort is determined from the characteristic curves of the motor, from which the mean acceleration and the time increment follow. The instant at which power must be cut off is determined by trial, and the instant of the application of the brakes may be determined either by trial or from the following equation, which is deduced from the equations to the coasting and braking lines—

$$t'' = (\beta T - V' - \beta_c t')/(\beta - \beta_c) \qquad . \qquad . \qquad . \qquad (82)$$

where t', t'' denote the times from the start to the cut off of power and the application of the brakes respectively; T the time from start to stop; V' the speed at the instant of cut off; and β, β_c the braking and coasting retardations respectively.

EXAMPLES IV
SERVICE CHARACTERISTICS, SPEED-TIME CURVES

1. Draw a typical speed-time curve for an electric train for suburban service. State usual values for acceleration and retardation.

On a certain line, with an average of $1\frac{1}{4}$ miles between stops, the schedule speed is 24 m.p.h. If the maximum speed attained is 36 m.p.h., the stops 20 sec., and the retardation 2 m.p.h.p.s., find the acceleration required.   (CG)

2. Distinguish between schedule speed and average speed. On an electrified suburban line the distance between two stations is 1·5 miles. Calculate the maximum speed to be attained to give a schedule speed of 25 m.p.h. if there is a station stop of 25 sec. The acceleration is 1·5 m.p.h.p.s., and the braking is 2 m.p.h.p.s.

(CG)

3. A service of trains is to be run at a schedule speed of 17 m.p.h. over a level route in which the distance between stations is 0·5 mile. The station stops are of 20 sec. duration. Using the simplified (trapezoidal) speed-time curve calculate the acceleration required to run the service, assuming that the braking retardation is 2 m.p.h.p.s., and that the maximum speed is 30 per cent greater than the average speed.   (BP)

4. A motor-coach train weighing 210 tons is accelerated on level track, at a mean acceleration of 1·125 m.p.h.p.s., up to a speed of 26 m.p.h. Calculate (1) the tractive effort required, and (2) the output at the end of the accelerating period. Assume the train resistance at 10 lb. per ton, and the effective weight 10 per cent greater than the dead weight.

5. If the train in the preceding example (4) were started on a gradient of 1 in 200, what would be the acceleration if the tractive effort exerted by the motors is 28,600 lb. ?

6. An electric train has a mean running speed from start to stop of 20 m.p.h.; it accelerates at 1 m.p.h.p.s., and brakes at 1·75 m.p.h.p.s. The mean distance between stations is 3,000 ft.

(1) Draw an approximate speed-time curve for the run, and (2) estimate the energy consumption per ton-mile.   (*IEE*)

7. An electric train weighing 220 tons (dead weight) makes a run on the level between two stations 1·2 miles apart. The initial acceleration is 1·2 m.p.h.p.s., and the braking retardation is 2 m.p.h.p.s. The run is made to a trapezoidal speed-time curve, and the free-running speed is 20 per cent above the average speed. Draw to scale the speed-time curve and calculate the power output (*a*) at the end of the accelerating period, (*b*) during free-running. Calculate also (*c*) the specific energy output for the run. Assume the effective weight of train as 10 per cent greater than the dead weight, and the train resistance at 10 lb. per ton at all speeds.

(*BP*)

8. What is meant by the effective weight of a car or train? A traction motor weighing 6,000 lb., with an armature which is 20 in. in diameter weighing 2,200 lb., drives wheels 44 in. in diameter through 57 : 20 gearing. Calculate, approximately, the effective weight of the motor. ,   (*IEE*)

9. Calculate the tractive effort necessary for the *angular* acceleration of the armature of a motor geared to 36 in. driving wheels (the gear ratio being 4·43 : 1) when the linear acceleration of the latter is 1·2 m.p.h.p.s. Weight of armature, 1,650 lb.; diameter of armature, 17·5 in.   (*BP*)

10. A two-coach train, comprising a motor-coach and trailer of the normal double-bogie pattern, has a dead weight of 80 tons. The four motor armatures weigh 1,800 lb. each, and have a radius of gyration of 7 in. The gear wheels weigh 500 lb. each, and have a radius of gyration of 12 in. The sixteen wheels weigh 1,200 lb. each, have a diameter at the tread of 44 in., and a radius of gyration of 0·38 of this diameter. Calculate the effective weight of the train. Gear ratio, 3·5.   (*IEE*)

11. The following data refer to the speed-time curve of an electric train for the run on level track between two stations on a suburban railway—

Time (sec.) . .	0	18	20	23	28	33	38*	62·6†	76
Speed (m.p.h.) .	0	18·7	20·6	23	26	28·2	30	27·1	0

* Power off.   † Brakes applied

Determine (1) the energy output, in watt-hours, from the driving axles for the run. Determine also (2) the energy output per ton-mile.

The dead weight of the train is 150 tons, and the effective weight is 166 tons. Train resistance may be assumed at a constant value of 10 lb. per ton.   ’   (*BP*)

12. An electric train, having an effective mass of 32 tons per motor, is controlled to give uniform acceleration during the starting period up to a speed of 20 m.p.h., after which the net tractive effort is as follows—

Speed (m.p.h.) . .	20	25	30	35	40	43
Tractive effort (lb.) .	3,480	1,840	880	440	130	0

The distance between stops is 4,000 ft; the train accelerates up to 35 m.p.h. and then coasts until braking is necessary. Draw the speed-time curve and determine the schedule speed, allowing for a stop of 15 sec. at every station. The mean resistance during coasting is 12 lb. per ton of effective mass, and the braking retardation is 3 ft. per sec. per sec.   (*IEE*)

13. An electric train weighing 220 tons has to make a run of 2 miles up a gradient of 1 in 180 at an average speed of 30 m.p.h. The train resistance may be assumed to be 10 lb. per ton during the run, and the allowance for rotational inertia may be taken at $12\frac{1}{2}$ per cent.

Assuming an acceleration of 1 m.p.h.p.s. and a retardation of 2 m.p.h.p.s., calculate the energy consumption of the train.

(*LU*)

14. Assuming a trapezoidal speed-time diagram, find the maximum speed of a 100-ton train running between stations 1 mile apart and maintaining a schedule speed of 20 m.p.h. with 30 sec. stops. The train accelerates at 1·4 m.p.h.p.s. and brakes at 2 m.p.h.p.s. Estimate also the specific energy consumption on level track with a mean tractive resistance of 10 lb. per ton and a motor efficiency of 75 per cent.   (*IEE*)

15. The average distance between stops on a level section of an electrified railway is 1·75 miles. Trains have a schedule speed of 24 m.p.h., the duration of station stops being 20 sec. The acceleration is 1 m.p.h.p.s., and the rate of braking 1·75 m.p.h.p.s. Resistance to traction is 10 lb. per ton. Allowance for rotational inertia 12 per cent. Overall efficiency 70 per cent. Estimate the specific energy consumption in watt-hours per ton-mile for this section.

(*CG*)

16. A motor-coach train has an overall efficiency of 72 per cent, and operates on a section in which the stops are 1,200 yd. apart. The sections between the stations are uniformly graded so that the stations are at the summits of 1 per cent gradients. The resistance to motion is 15 lb. per ton, and the maximum speed 36 m.p.h. The maximum speed is attained by a uniform acceleration of 1 m.p.h.p.s., after which the power is switched off. Calculate the specific energy consumption in watt-hours per ton-mile required to operate this service.   (*LU*)

17. A d.c. electric train, having an effective mass of 14 tons per motor, accelerates uniformly, by series-parallel control, to 28 m.p.h.

The time speed characteristics of the run is given by the following figures—

Time (sec.)	0	54	66	97	131	160	190
Speed (m.p.h.)	0	28	32·5	37·5	40	36	0
Amperes per motor	130	130	90	66	58	0	0

The power is cut off at 40 m.p.h., and braking commences after 160 sec. from the start. Determine (1) the distance travelled, (2) the energy consumption per ton-mile, and (3) the efficiency of conversion from electrical to kinetic energy. The supply voltage is 500.
(*IEE*)

18. A 200-ton motor-coach train starts from rest on a level track with a uniform acceleration of 1·5 m.p.h.p.s., and attains a maximum speed of 27 m.p.h. The tractive resistance is 10 lb. per ton, and the effect of rotational inertia is equivalent to an increase of 10 per cent in the dead weight of the train. Calculate (*a*) the tractive effort during the period of acceleration, (*b*) the approximate kilowatt-hour input to the train during this period if series-parallel control is used, and (*c*) the approximate energy consumption in watt-hours per ton-mile if the distance between stops is 0·4 mile. The power is cut off at the end of the accelerating period and the run completed by coasting. (*CG*)

19. A 15-ton car, driven by two series motors in parallel, takes 120 A from a 500 V line in ascending a gradient of 1 in 15 at a speed of 10 m.p.h. The gear ratio is 4·73 : 1. If this ratio be changed to 5·25 : 1, find (1) the speed of the car, and (2) the current taken when ascending the same gradient. The resistance to traction is 20 lb. per ton, and it can be assumed that, over the required range, the speed of the motor is inversely proportional to the current taken. (*LU*)

20. An electric train weighing 300 tons is equipped with eight motors. If the acceleration is maintained constant, calculate the necessary torque which each motor armature must exert for the train to reach a speed of 35 m.p.h. in 25 sec. when starting on an up grade of 1·5 per cent. The diameter of the driving wheels is 36 in., the single gearing has a ratio of 3·36 with an efficiency of 78 per cent, and the resistance to traction averages 15 lb. per ton. An allowance of 10 per cent should be made for rotational inertia.
(*CG*)

21. Two d.c. motors, each rated at 40 h.p., are driving a car weighing 16 tons at 12 m.p.h. up a gradient of 1 in 40. The tractive resistance is 15 lb. per ton. The resistance of each armature is 0·3 Ω, and that of the field coils of each motor is 0·15 Ω. The motors being in full parallel, and the line pressure 550 V, find the current per motor. The overall efficiency is 75 per cent.
The controller is moved to a tap-field point cutting out 25 per cent of the field turns. Find the alteration in (1) the steady current and (2) the speed on the same gradient. Assume that the flux per pole is proportional to the current in the field windings. (*LU*)

22. An electric train weighs 150 tons and has four motors. The gear ratio is 4·7, with an efficiency of 75 per cent, the diameter of the car wheels being 30 in. With the maximum permissible starting current, each motor exerts an armature torque of 2,200 lb.-ft. Assuming that the train accelerates uniformly, find the time taken to reach a speed of 30 m.p.h. when starting on an up grade of 1 in 100. Allow 10 per cent for rotational inertia and a tractive resistance of 20 lb. per ton.   (*CG*)

23. In an urban electrified railway the stops are 1,000 yd. apart, and the distance between stations is occupied by up and down gradients of 1 in 80, the stations being at the summits. After switching in, the current is kept at a value such that an acceleration of 30 m.p.h. per minute is maintained until a maximum speed of 35 m.p.h. is reached. The motors are then switched off, and the train is brought to rest with a constant retardation. Assuming the average resistance to motion to be 20 lb. per ton, and the overall efficiency of the train equipment to be 75 per cent, calculate the energy required per ton-mile to run the service.   (*CG*)

24. A train driven by d.c. motors and using series-parallel control is accelerated to a speed of 20 m.p.h. in 25 sec. with a tractive effort of 3,000 lb. per motor. Determine the approximate energy, in kilowatt-hours, lost in the starting rheostats of each pair of motors, at each acceleration.   (*IEE*)

25. A 200-ton multiple-unit train has eight motors, each taking an average current during notching of 400 A and developing a gross tractive effort of 3,500 lb. Series-parallel control is used, the final voltage across each motor being 550 V when the speed is 24 m.p.h. The resistance of each motor is 0·1375 Ω. If the train accelerates on level tangent track against a constant tractive resistance of 10 lb. per ton, find for each motor (*a*) the energy lost in the starting rheostat, (*b*) the motor heating loss, and (*c*) the output (including friction, etc.). Estimate also (*d*) the speed at which transition is made from series to parallel connection, and (*e*) the total accelerating time. Allow 10 per cent for the additional inertia of rotating parts.   (*IEE*)

26. A tramcar weighing 15 tons is equipped with two motors having the following characteristics—

Amperes per motor . .	80	70	60	50	40	30	20
Speed of car (m.p.h.) . .	9	9·5	10·3	11·5	13·2	16·2	21
Tractive effort (lb.) . .	1,850	1,550	1,250	940	650	400	200

What is the maximum schedule speed of the car when operating on level track with eight stops per mile, the duration of each stop being 8 sec. ? The initial, or rheostatic, acceleration and the braking retardation are to be 1·5 m.p.h.p.s., and there is no coasting period. The tractive resistance may be assumed to be 25 lb. per ton at all speeds, and the inertia of the rotating parts may be assumed to be equivalent to 10 per cent of the dead weight.   (*LU*)

27. A train weighing 120 tons is fitted with four motors, each having a one-hour rating of 252 A and the following characteristics on the line voltage of 675—

Current (amp.) . . .	80	160	240	320	400
Tractive effort (lb.) . . .	400	1,350	2,470	3,700	4,950
Speed (m.p.h.) . . .	53	34·5	28·8	25·5	23·2

Assume an average current of 35 per cent above the one-hour rating during the rheostatic period, a track resistance of 10 lb. per ton, and braking at 2 m.p.h.p.s.

(a) What is the minimum time in which an average section of 0·75 mile can be accomplished ?

(b) Draw the speed-time curve for an average running speed of 24·5 m.p.h.   (*IEE*)

28. A multiple-unit train of five coaches has a total weight of 245 tons, and an effective rotational inertia equivalent to an addition of 10 per cent to its dead mass. There are twelve motors, and the net tractive effort/speed relation for the whole train (after deduction of tractive resistance) is—

Speed (ft. per sec.) .	0	35	40	45	50	55	70
Tractive effort (lb.) .	43,500	43,500	28,500	18,000	11,000	7,000	0

Draw the speed-time curve for the train on an up grade of 1 in 75 when running between stations 6,000 ft. apart with a running time of 160 sec. Assume a mean tractive resistance of 3,200 lb. during coasting, and a braking retardation of 3·5 ft. per sec. per sec.   (*IEE*)

29. A motor-coach train weighing 180 tons is fitted with six d.c. motors, each having the following characteristics at 600 V—

Amperes . . .	400	300	200	100	70
Speed (m.p.h.) . . .	21·7	23·6	27·6	38·4	52·6
Tractive effort (lb.) . .	4,750	3,320	1,890	665	290

The train does a run of 0·66 mile up a gradient of 1 per cent at an average speed of 21·6 m.p.h. The track resistance may be taken as 10 lb. per ton, and the average accelerating current on the rheostats as 350 A per motor. Draw the speed-time and distance-time curves, and calculate the energy consumption per ton-mile. Ordinary series-parallel control is employed.   (*IEE*)

30. A train has a total weight of 308 tons, and is equipped with eight motors, each of 300 h.p. The characteristics of the motor are—

Current (amp.) . . .	100	200	300	400	500
Speed (m.p.h.) . . .	65	36·5	29·8	26·5	24·7
Tractive effort (lb.) . .	330	1,450	2,740	4,100	5,450

The ratio of effective weight to the dead weight of the train is
1·1 : 1. The mean accelerating current is 415 A per motor. The
braking retardation is 1·75 m.p.h.p.s., and the train resistance can
be assumed as 10 lb. per ton throughout the run. Draw the speed-
time curve for a run on the level of 1·1 mile to be made in 152 sec.
Calculate the r.m.s. current per motor for the run.   (*IEE*)

31. A train weighing 120 tons is hauled by two pairs of motors
operated in series-parallel on an average line voltage of 675. A run
of 0·9 mile is performed in 110 sec. with the following current per
motor—

Time (sec.) . .	0	22·5	25	30	35	41·5 (off)
Current (amp.) . .	340	340	270	212	180	154

What is (1) the energy consumption per ton-mile, and (2) the
r.m.s. current per motor?   (*IEE*)

32. A train has a total weight of 116 tons and is equipped with
four motors each of 275 h.p. The characteristics of the motor are—

Current (amp.) . . .	100	200	300	400	500
Speed (m.p.h.) . . .	51	31·4	26·4	23·9	22·1
Tractive effort (lb.) . .	390	1,600	2,960	4,330	5,690

The ratio of the effective weight to the dead weight of the train
is 1·1 to 1. The mean accelerating current is 425 A per motor.
The braking retardation is 2 m.p.h.p.s., and the train resistance can
be assumed as 10 lb. per ton throughout the run. A run of 0·86
mile is to be made in 115 sec., there being an average up grade of
0·119 per cent. Calculate the r.m.s. current per motor for the run.
(*IEE*)

33. A six-coach train consists of two motor-coaches and four
trailers; its dead weight is 138·5 tons, and its effective weight is
151·7 tons. Each motor-coach is equipped with two 200 h.p., 550 V
d.c. motors, having characteristics at 550 V as follow—

Amperes . . .	350	300	250	200	150	100
Speed (m.p.h.) .	17·5	18·7	20·1	22·9	27·5	37·4
Tractive effort (lb.) .	4,850	3,940	3,030	2,140	1,320	590

This train operates at a schedule speed of 15 m.p.h. with 15 sec.
stops on an underground tube railway having graded track with
stations at the same level. The profile of the track between centres
of station platforms, in the direction of running, is as follows—
210 ft. level; 240 ft. 1 in 30 down; 870 ft. level; 480 ft. 1 in 60 up;
210 ft. level.
Calculate the speed-time curve and energy consumption of the
train. The accelerating current is 300 A per motor, and the braking

retardation (on level track) is 2 m.p.h.p.s. Assume the train resistance at 10 lb. per ton, and the apparent train resistance during coasting (excluding gravitational effect) at 12 lb. per ton.

34. The curves given in the figure are the result of tests made on a seven-car train. Explain and criticize these curves, and describe, how they may be obtained under service conditions. Calculate
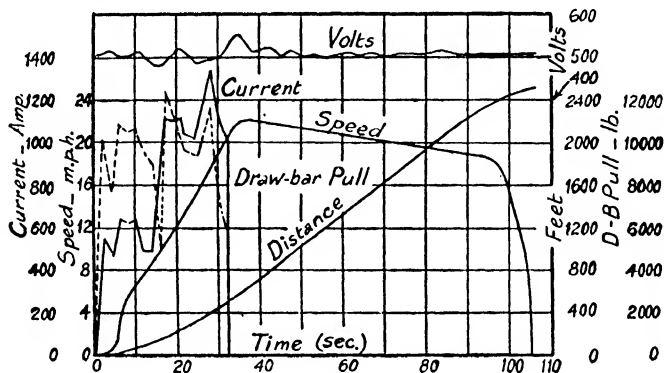
approximately, (*a*) the energy used during the starting period, (*b*) the energy used per train mile, (*c*) the retarding effort during coasting and braking, and (*d*) the efficiency over the starting period. The total weight of the train is 333,000 lb., and the stations may be taken to be equally spaced. There is a 20 sec. stop at each station.

(*LU*)

Magnetic track brakes. Vertical pull (in tons)

$$B^2A/(3\cdot9 \times 10^9) \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (83)$$

where $B =$ flux density at pole faces in lines per sq. cm., $A =$ area (in sq. cm.) of pole faces in contact with rail.

Horizontal drag = vertical pull $\times$ coefficient of friction.

✓ **Limiting tractive effort.** The tractive effort which just causes slipping of the driving wheels is equal to the product of adhesive weight and coefficient of friction.

Weight distribution on driving wheels as affected by horizontal thrust on car body or bogie centres. The thrust at the bogie centres (or the centre of mass of the car body in the
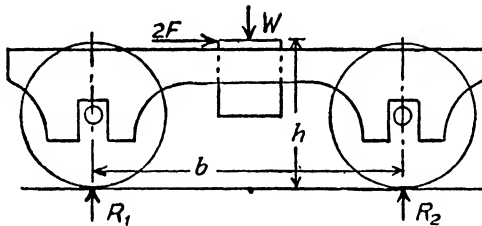
FIG. 13. REACTIONS ON BOGIE TRUCK

case of a rigid truck) due to the tractive effort at the driving wheels causes a lifting action on the front wheels, resulting in an alteration of the static load distribution on the driving wheels. Thus, considering the case represented in Fig. 13 in which W denotes the load on the bolster, F the tractive effort exerted by the driving wheels of each axle, R_1, R_2 the reactions at each pair of wheels, b the wheel base, h the height of the bogie centre, we have

$$\tfrac{1}{2}b(R_2 - R_1) = 2Fh$$

$$R_1 + R_2 = W$$

whence

$$R_1 = \tfrac{1}{2}W - 2Fh/b \qquad . \qquad . \qquad . \qquad . \qquad (84)$$

$$R_2 = \tfrac{1}{2}W + 2Fh/b \qquad . \qquad . \qquad . \qquad . \qquad (85)$$

Impulsive blows due to irregularities in track. If a wheel of diameter D travelling horizontally at velocity V meets an irregularity (such as a high rail joint) in the track which causes a sudden vertical rise, δ, the impulse of the blow is $mV \sin \theta$, where m denotes the uncushioned mass connected to the wheel, and θ is the inclination with respect to the horizontal of the line joining the two points of contract of wheel with obstruction and track.   Now $\sin \theta = \delta/\sqrt{(D\delta)} = \sqrt{(\delta/D)}$. Hence, the impulse is

$$P = mV\sqrt{(\delta/D)} \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (86)$$

Brake rigging. Alternative forms of brake rigging for a motor bogie truck are shown in Fig. 14. With the arrangement of Fig. 14(a) a single pull rod operates the whole of the
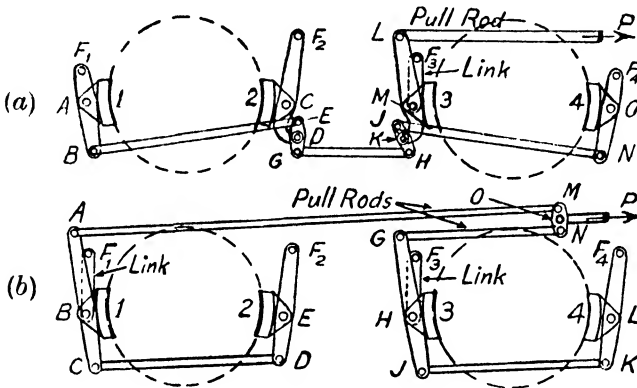
Fig. 14. Alternative Forms of Brake Rigging

brake levers on each side of the truck, but with the arrangement of Fig. 14(b) double pull rods are necessary. In both diagrams F_1, F_2, F_3, F_4 denote fixed pivotal points.

The pressure exerted by each brake block is easily calculated—if friction at the pivots and joints is neglected—from the dimensions of the levers and the pull in the pull rods.

WORKED EXAMPLE. The dimensions of the levers of a brake rigging arranged as in Fig. 14 (a) are as follow, the diameter of the wheels being 42 in.—

F_1A, $8''$; AB, $12''$; F_2C, $15''$; CD, $10''$; ED, DG, $4\frac{1}{4}''$; LM, $15''$; MK, $10''$; JK, KH, $4\frac{1}{4}''$; F_4O, $8''$; ON, $12''$. Determine the pressure exerted by each brake block when a pull of 2,500 lb. is applied to the pull rod. Neglect friction.

SOLUTION. When the brake blocks are pulled up tight, K becomes a fulcrum for lever LK. Hence, the pressure exerted by block No. 3 $= 2500 \times LK/MK = 6250$ lb.

The horizontal component of the forces at K and $N = 2500 \times LM/MK = 3750$ lb. Hence, the pressure exerted by block No. 4 $= 3750 \times F_4N/ON = 6250$ lb.

Similarly, the horizontal component of the forces at D and B is 3750 lb. Hence the pressure exerted by block No. 2 $= 3750 \times F_2D/F_2C = 6250$ lb., and that exerted by block No. 1 $= 3750 \times F_1B/AB = 6250$ lb.

EXAMPLES V

ROLLING STOCK AND LOCOMOTIVES

1. Sketch and describe one good form of magnetic track brake for use on a tramway, and show how it is energized. Calculate (1) the vertical pull and estimate (2) the horizontal drag exerted if the area of each pole is 5 sq. in., and in which the iron is magnetized to an induction density of 18,000 C.G.S. units.   (*LU*)

2. A magnetic track brake has to exert a vertical pull of 2 tons weight when the magnetic flux density is 16,000 C.G.S. lines per sq. cm. Make a dimensional sketch of a good type of brake which will meet this requirement.   (*IEE*)

3. What is meant by the adhesive weight of a locomotive? A locomotive weighs 120 tons, of which 80 tons is adhesive weight. What maximum trailing load will this locomotive haul at a steady speed up a 1 per cent gradient, assuming a track resistance of 10 lb. per ton.   (*IEE*)

4. The coefficient of adhesion for an electric locomotive is 0·25. A locomotive is required to start a trailing load of 600 tons up a gradient of 1 per cent with an acceleration of 0·2 m.p.h.p.s., the track resistance is 10 lb. per ton, and the ratio of effective weight to dead weight of the whole train is 1·1 : 1. What is the minimum adhesive weight required on the locomotive?   (*IEE*)

5. Find the approximate weight of a locomotive to haul a 450-ton goods train up a 1 per cent gradient with an initial acceleration of 0·8 m.p.h.p.s. Coefficient of adhesion, 0·25; tractive resistance, 12 lb. per ton; effect of rotating masses equal to 10 per cent of the dead mass. How many driving axles are necessary if the track can support an axle-loading not exceeding 22 tons?   (*IEE*)

6. If the coefficient of adhesion is 0·22, find the maximum possible rate of acceleration in the case of (1) a 100-ton locomotive hauling a train weighing 200 tons, and (2) a multiple-unit train, if each of the three motor-coaches weighs 50 tons, and each of the two trailer coaches weighs 30 tons. Assume the tractive resistance to be 10 lb. per ton.   (*CG*)

7. What factors limit the trailing load that can be hauled by an electric locomotive?

A four-axle locomotive weighing 36 tons is equipped with four

motors each having characteristics as follow, with a gear rato of
62 : 18—

Current (amp.) . . .	80	160	240	320	400
Tractive effort (lb.) . .	400	1,350	2,470	3,700	4,950
Speed (m.p.h.) . . .	53	34·5	28·8	25·5	23·2

If the gear ratio were changed to 81 : 15, what is the maxi-
mum trailing load which can be started with an acceleration of
0·5 m.p.h.p.s. on level track, assuming 10 lb. per ton train resis-
tance ? The motors are able to stand 50 per cent overload in current
above the one-hour rating of 252 A.   (*IEE*)

8. A single-phase express locomotive is required to haul a 250-ton
train on the level at 55 m.p.h. The route includes long gradients of
2·5 per cent, over which the speed is to be 20 m.p.h. Outline (1)
a suitable arrangement of axles, (2) the number and type of motors
and their rating, (3) the type of drive, and (4) the kilovolt-ampere
rating of the main transformer.   (*CG*)

9. What is the effect on the weight distribution on a motor-bogie
of the thrust on the bogie centre caused by the motors or by braking ?
A bogie has a wheel base of 7 ft., and the height of the bogie centre
above the rail is 3 ft. 6 in. The total weight on the wheels is 25 tons,
and the horizontal thrust at the bogie centre due to the motors is
5 tons. Calculate the approximate loads on the two axles stating
the assumptions made.   (*IEE*)

10. A four-wheel tramcar has 32 in. wheels on a 6 ft. 6 in. wheel
base. The weight of the loaded car is 14 tons, the mass centre being
3 ft. above the axle centres. Estimate the maximum acceleration
which can be obtained without slip of the front wheels. Coefficient
of adhesion, 0·15. Neglect tractive resistance and gradient.
                    (*IEE*)

11. A bogie truck carries a pair of motors driving through gears
in the usual manner. Prove that if each motor exerts tractive effort
T, and h is the height of the king-pin above the rail, and b the wheel
base of the truck, the rear pair of wheels carries greater weight than
the fore pair by $4Th/b$.

Hence show that if the apparent limit of adhesion is 25 per cent
when $h = 3$ ft. and $b = 7$ ft., the actual limit is nearly 32 per cent.
                    (*IEE*)

12. A 36 in. wheel, on an axle driven by a nose-suspended motor,
meets an inequality in the track (equivalent to a sudden rise of
¼ in.) when travelling at 35 m.p.h. If the non-spring-borne weight
on the axle be 1·5 tons. estimate the magnitude of the impulsive
blow impressed on the axle. Discuss briefly the significance of this
in the design of express electric locomotives.   (*IEE*)

13. Sketch a typical form of brake rigging for one bogie truck of
a multiple-unit coach, showing the pair of wheels on one side of the
bogie with four coupled brake blocks. Give approximate dimen-
sions, using 42 in. wheels, in order that the forces on the brake

blocks should be equal. Calculate the total pull required to obtain a force of 2,000 lb. on each brake block, using the dimensions adopted in the sketch.   (*IEE*)

14. The brake rod connected to the piston of the vacuum brake mechanism exerts a maximum pull of 2,500 lb. on each side of the bogie. Arrange the brake rigging to give about 3,000 lb. braking pressure on all brake blocks (two per wheel). State the dimensions of the links used and show detailed calculations.   (*CG*)

Super-elevation of track rails. To prevent derailment at curves the outer rail must be super-elevated, so that the line of action of the resultant force (due to centrifugal force and the weight of the train) is perpendicular to the plane containing the treads of the rails. As this force acts at the centre of mass of the train, its line of action is centrally situated with respect to the rails. Thus with correctly super-elevated track the inclination (θ), with respect to the horizontal, of the plane containing the treads of the rails is equal to the inclination, with respect to the vertical, of the resultant force acting on the train.

Hence, sin θ = super-elevation/track gauge.

tan θ = centrifugal force/weight of train.

As θ is a small angle sin θ = tan θ, approximately.

Therefore, super-elevation

= (track gauge × centrifugal force)/weight of train.

If the radius of the curve (r) is expressed in feet, the train speed (V) in m.p.h., and the gauge is standard (4 ft. $8\frac{1}{2}$ in.), the super-elevation (E) in inches is given by

$$E = 56 \cdot 5 \, (5280V/3600)^2/gr$$

$$= 3 \cdot 8 V^2/r \qquad . \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (87)$$

taking $g = 32 \cdot 2$.

Tramway overhead construction. *Tension and sag in trolley wire.* The tension T (lb.) in a wire weighing w lb. per foot, suspended with a sag δ (ft.) between supports $2l$ (ft.) apart at the same level is

$$T = wl^2/2\delta \qquad . \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (88)$$

The tension T_1 at a temperature θ_1 is given by

$$T_1 = \sqrt{\frac{0 \cdot 166aEw^2l^2}{T_1 + aE[a(\theta_1 - \theta) + \frac{1}{6}(wl/T)^2] - T}} \qquad . \qquad (89)$$

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T, θ, being the tension and temperature respectively at erection; a, the coefficient of linear expansion; a, the cross-sectional area of the wire; E, Young's modulus of elasticity.

Catenary overhead construction. *Tension and sag in cate-nary wire.* For an *even* number of droppers per span the tension is given by

$$T = w'l^2/2\delta \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (90)$$

where w' is the average weight per foot run of the whole suspension (i.e. catenary wire, droppers, and trolley wire).

For an odd number of droppers per span

$$T = (w'l^2/2\delta) + (w_2\lambda(n + \tfrac{3}{4})/2\delta) \qquad . \qquad . \qquad (91)$$

where w_2 is the average weight of a dropper; λ, the horizontal spacing in feet of the droppers; n, the number of droppers per half span, the centre one not being included.

EXAMPLES VI
TRACKWORK AND OVERHEAD CONSTRUCTION

1. Explain what is meant by super-elevation. What is the correct value for a train travelling at 30 m.p.h. on a curve of 500 ft. radius on standard gauge track?

If there were no super-elevation, and the height of the centre of gravity were 5 ft., what would be the ratio of the pressures on the two rails?   (*IEE*)

2. What considerations determine the permissible sag to be given to a trolley wire for an electric tramway? Calculate the sag to be given to a trolley wire 0·4 in. in diameter and weighing 0·484 lb. per foot when erected, with a span of 120 ft., if the stress in the wire when erected is 9,000 lb. per sq. in.   (*BP*)

3. The stress in a trolley wire having an area of 0·109 sq. in. is not to exceed 9,000 lb. per sq. in. when erected in a span of 120 ft. Calculate the tension that must be allowed, and the resulting sag.

Sketch clearly a method of supporting the wires of a double trolley-bus line round a sharp right-angle bend, without the use of centre poles.   (*IEE*)

4. A 120 ft. span of 0·4 in. copper trolley wire is erected with a tension of 1,160 lb., at 65° F. Determine the tension at a temperature of 100° F., having given: Young's modulus of elasticity $= 18 \times 10^6$ lb. per sq. in.; coefficiency of linear expansion per 1° F. $= 9\cdot3 \times 10^{-6}$.

5. If in winter, at a temperature of 22° F., the trolley wire of the preceding example (4) is subjected to a horizontal wind pressure of 20 lb. per sq. ft. (on a flat normal surface), determine the stress. The wind pressure on a cylindrical surface may be taken as 0·6 times the normal pressure on the equivalent projected area.

6. In a single-catenary construction there is a catenary wire and a trolley wire supported at intervals by droppers. Develop an equation giving the minimum permissible sag in the catenary for a given tension in the catenary wire.   (*IEE*)

7. In a catenary system the catenary wire is steel with a maximum permissible tension of 1,000 kg. The weight of the catenary wire is 1 kg. per metre run, the weight of the trolley wire is 1·5 kg. per metre run, and the allowance for droppers and fittings is 20 per cent of the trolley wire weight. If the span is 80 metres, what is the minimum sag of the catenary?   (*IEE*)

8. On an overhead catenary construction with a span of 300 ft. the tension in the catenary wire is 2,000 lb. and the sag is 10 ft. With the same weight per foot run, but the span reduced to 280 ft., what would be (*a*) the sag with the same tension, (*b*) the tension with the same sag?   (*IEE*)

TRACTION FEEDING AND DISTRIBUTING SYSTEMS, ROTARY CONVERTERS, AND RECTIFIERS

Traction distributing and feeding systems. The voltage drop is calculated by methods similar to those given in Section I, but in systems employing an earthed return separate calculations are necessary for the trolley wire and track owing to the different methods of feeding. In many cases the loading along the track is assumed to be uniform; the voltage drop then follows a parabolic law. Thus if the track is loaded at the rate of I amp. per mile, and R is the resistance per mile, the maximum voltage drop V in a section is

$$V = \tfrac{1}{2}IRl^2 \qquad . \qquad . \qquad . \qquad . \qquad . \qquad . \quad (92)$$

where l is the distance in miles, between the feeding point and the point in the track at which the current is zero.

With equally-loaded sections throughout the length of the track, the distance between the negative feeding points for a voltage drop V is equal to $2l$, or to $2\sqrt{(V/\tfrac{1}{2}IR)}$.

Rotary Converters. *Voltage ratio.* The theoretical voltage ratio at no-load, assuming sine wave e.m.f. and distribution of flux, is

$$E_s/E_c = 0.707 \sin (\pi/m) \qquad . \qquad . \qquad . \qquad . \quad (93)$$
$$\scriptstyle \text{r.m.s.}$$

where E_s is the voltage between slip-rings connected to tappings $2\pi/m$ electrical degrees apart, E_c the commutator voltage, and m the number of slip-rings.

Armature current. The r.m.s. value of the resultant current in a conductor $\theta°$ from the mid-point of a pair of tappings is

$$I' = \frac{I_c}{a}\sqrt{\left(1 + \frac{8}{[m\eta \cos \varphi \sin (\pi/m)]^2} - \frac{16 \cos (\theta + \varphi)}{m\pi \eta \cos \varphi \sin (\pi/m)}\right)} \qquad . \qquad . \quad (94)$$

and the average value of the resultant current for the whole armature winding of a six-ring machine ($m = 6$) is

$$I = \frac{I_c}{a}\sqrt{\left(1 + \frac{8}{9\eta^2 \cos^2 \varphi} - \frac{1.62}{\eta}\right)}. \qquad . \qquad . \quad (95)$$

where I_c is the current output from the commutator, a the number of circuits in the armature winding, η the efficiency and $\cos \varphi$ the power factor.

Limiting commutator voltage. This is given by

$$E_c = E_{av}v_c/(10f\tau_c) \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (96)$$

where E_{av} is the mean voltage between segments, v_c the peripheral speed of commutator in feet per minute, τ_c the pitch of the segments in inches, and f the frequency.

Mercury arc rectifiers. *Voltage ratio.* The theoretical voltage ratio, assuming a sine law for the e.m.f.s and ignoring voltage drops, is

$$E_d/E_a = [\sqrt{2} \sin (\pi/m)]/(\pi/m) \qquad . \qquad . \qquad . \qquad . \qquad (97)$$

where E_d is the output voltage, E_a the r.m.s. value of the e.m.f. of each phase of the transformer supplying the rectifier, and $2\pi/m$ the mutual phase difference between the several e.m.f.s.

Anode currents. If the wave-form of the anode currents is rectangular, the r.m.s. value of the current input to an anode is

$$I_a = I_d/a\sqrt{m} \qquad . \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (98)$$

where I_d is the current output from the cathode, m the number of phases in the secondary of the transformer, and a the number of anodes (operating in parallel) supplied by each phase.

EXAMPLES VII

FEEDING AND DISTRIBUTING SYSTEMS, ROTARY
CONVERTERS AND RECTIFIERS

1. In an overhead trolley construction there are two tracks arranged in parallel electrically. Each overhead line has 0·25 sq. in. of copper, and the rails are 90 lb. per yd. (9 sq. in. section) and 10 yd. long. The resistance of each rail joint is equal to that of 1 yd. of rail. The resistance of copper is 0·75$\mu\Omega$ per in. cube, and the resistance of the rail is eleven times that of copper. What is the total resistance per mile of line ?   (*IEE*)

2. In a single-catenary system the cross-sections of the wires are : copper catenary 0·2 sq in., trolley wire 0·1 sq. in. The line is single track only, and the starting current of a locomotive is 2,000 A. What is the voltage drop in the overhead line when the locomotive starts up between two substations 8 miles apart, the distance from the locomotive to the nearest substation being 2 miles ? Resistance of copper is 0·75$\mu\Omega$ per in. cube.   (*IEE*)

3. An overhead trolley wire in a tramway system has a resistance of 0·46Ω per mile, and the maximum permissible voltage drop in

a distributing section is 55 V. The cars on the section have a schedule speed of 10 m.p.h., and there is an interval of 2 min. between the cars. The average current per car is 25 A. Find the length of distributing section corresponding to the maximum voltage drop, assuming the average voltage drop to be 50 per cent of the maximum. (CG)

4. A tramway system delivers energy to the cars from two over-head conductors, each having a resistance of 0.44Ω per mile. Two feeding points, A, B, 1,800 yd. apart, have voltages of 654 and 650 respectively, and there are five cars equally spaced 300 yd. apart. The currents taken by the respective cars are 20, 40, 20, 60, and 80 A, commencing 300 yd. from the end at 654 V. Find the current distribution and the lowest voltage. (IEE)

5. Describe a negative booster for use on a traction system. The resistance of the track rails on a railway is 0.1Ω per mile, and the normal return current is 500 A. What size of copper feeder would be used in parallel with the rails if the voltage drop is limited to 10 V per mile, the resistance of copper being $0.75\mu\Omega$ per in. cube. (IEE)

6. A traction system employs the running rails as return conductors. Describe measures and precautions that may be required to keep the voltage drop within reasonable limits.

It is proposed to take a current of 750 A from the track, at a point 2,000 yd. from the nearest substation, and to use a cable of 0.8 sq. in. cross-section. Make a sketch of the booster connections and determine the capacity of the booster. Resistance of copper $0.75\mu\Omega$ per in. cube. (IEE)

6A. In a section of tramway 3 miles long there is a total current of 300 A., which may be considered to be drawn off uniformly. The track resistance is 0.015Ω per 1,000 yd. To reduce the voltage drop between the ends of the section to 6 V, a negative cable is connected to the distant end of the section. Find the current carried by this feeder. (LU)

7. In a single-phase traction system a train takes P kW at a power factor of cos φ. Derive an expression in terms of P, cos φ, and the impedance (Z) per mile of contact wire and rail, for the distance (D) in miles between the train and a substation for which the voltage on the train shall not be less than $(100 - p)$ per cent of the substation voltage V.

Calculate this distance for a train taking 2,000 kW at a power factor of 0.8 if the total line and rail impedance is 0.2Ω per mile. Voltage at substation, 11,000 V; at the train, 8,500 V. (LU)

8. A d.c. traction feeder AB is l_1 miles long. At B it divides; one part, C, is l_2 miles long and carries a current I_2, the other part, D, is l_3 miles long and carries a current I_3. The total permissible fall of potential, from A to C or A to D, is v volts. Determine the sizes of the conductors in the three parts of the feeder so as to make the total cost of copper a minimum. (LU)

9. On a 1,500 V d.c. railway there are two substations, A and B, 10 miles apart. The first 4 miles of track from A is single, and the remainder is double. The resistance of the single-track rails is

0·05Ω per mile. With 200 A collected by a locomotive running from A to B the maximum voltage drop allowed on the system is 100 V. The overhead line has a constant section over each track, and over the double track the overhead lines and track rails are bonded together at frequent intervals. What is the minimum section of the copper in the overhead line between A and B? Resistance of copper is 0·75μΩ per in. cube.

10. Two substations, A and B, supply power at 600 V and 620 V respectively to a d.c. railway through four feeders (two positive and two negative) from each substation, each feeder having a resistance of 0·01Ω. The resistance of each of the (positive and negative) conductor rails between adjacent feeding points is 0·02Ω. There is one train on the track taking a constant power of 500 kW. Find the point at which the train ceases to derive energy from one of the substations.   (*LU*)

11. Two substations E, F supply power to an electric railway at equidistant intervals along the track at four adjacent feeding points, A, B, C, D. Substation E feeds at A and B; substation F at C and D. The resistance, go and return, of the feeders to each point is 0·01Ω, and of the track conductors between adjacent feeding points 0·02Ω. If substation E has a fixed voltage of 500 V, what must be the voltage regulation of substation F so that when a train taking 300 A is at B the substation E supplies 200 A and F 100 A; and when the train is at C, substation E supplies 100 A and F 200 A? Find also the train voltages at B and C. Assume that no other trains are between A and D, and that no other station is effective.
                                                                 (*IEE*)

12. Two traction substations A and B are spaced 10 miles apart, and maintain at their bus-bars a voltage of 1,500. An electric train taking 500 A runs at a steady speed of 40 m.p.h. from A to B. For the first 5 miles the track is single and the remainder is double. The overhead line of each track has a resistance of 0·1Ω per mile, and the resistance of a single track rail is 0·05Ω per mile. Draw a graph on a time basis showing the load on substation A and the voltage at the train. The overhead lines and the track rails are bonded at frequent intervals on the double-track section.   (*IEE*)

13. Calculate the voltage ratios of the transformers for (1) a six-ring 600 V rotary converter, (2) a six-ring 1,500 V rotary converter. The supply is three-phase at 11,000 V, the primary windings are star-connected, and the secondary windings are connected diametrically to the slip-rings.

14. A 1,000 kW, six-ring, 600 V rotary converter is supplied from a three-phase, 11,000 V system through a transformer with star-connected primary windings and diametrically-connected secondary windings. The full-load efficiency of the rotary converter is 95 per cent, and that of the transformer 98·2 per cent. Calculate the currents in the primary and secondary windings when the rotary converter is running at full load, 0·98 power factor.

15. If the rotary converter in the preceding example (14) has ten poles and a lap winding on the armature, calculate the resultant current in the armature winding.

16. What must be the peripheral speed of the commutator of a 50-cycle, 1,500 V rotary converter if the pitch of the commutator segments is 0·18 in., and the average voltage per segment is 18 V ?

17. A six-phase 500 kW rotary converter working on a traction circuit is required to give 500 V on no-load, rising to 550 V on full load, by means of reactance. The supply pressure at the transformer secondary is kept constant at 370 V. Find the reactance needed in each phase when the converter is over-excited on full load to give a leading power factor of 80 per cent. (*LU*)

18. The transformer (of ratio 6,000 to 370 V) intended to supply a six-phase, 525 V rotary converter, provided with a series winding, has the following short-circuit characteristics : voltage applied to primary, 1,200; current in secondary, 1,000 A.

When the load on the converter is 1,000 kW at 550 V (neglecting drop in the windings and brushes), (1) what is the current in each slip-ring, and (2) what is its phase relation (*a*) to the slip-ring voltage, (*b*) to the primary voltage ? Primary volts, 6,000. (*LU*)

19. A 1,000 kW, six-anode, mercury-arc rectifier, connected so as to operate as a double three-phase rectifier, is required to give an output voltage of 1,500. If the voltage drop in the arc is 25 V, (1) what is the voltage of the secondary windings of the transformer, and (2) how are these windings connected ?

20. If the wave-form of the anode currents of the rectifier in the preceding example (19) is rectangular, calculate (1) the r.m.s. value of the currents in the transformer secondary windings when the rectifier is giving its full-load output of 1,000 kW at 1,500 V, (2) the kilovolt-ampere output from the secondary windings.

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ANSWERS TO THE EXAMPLES

<div align="center">EXAMPLES I</div>

1–2. A.c. copper = 0·924 d.c. copper.*

3. A.c. copper = 1·17 d.c. copper.

4. A.c. copper = 1·32 d.c. copper.*

5. $\frac{1}{3}I$.

6. 1,695 yd., 0·1 sq. in.

7. 69 yd., 11·31Ω.

9. A side: 715Ω, 910Ω. B side: 520Ω, 220Ω.

10. 9,000Ω. $R = (V_0 - iR')/i$ [R = insulation resistance, R' = series resistances, V_0 = voltmeter reading, i = ammeter reading].

11. 200Ω.

12. $V_A = 211\cdot484$, $V_B = 211\cdot407$, $V_{75A} = 211\cdot06$, $V_{100A} = 210\cdot96$.

13. Currents from A end: 57·6, 17·6, – 2·4, – 32·4, – 82·4.
Currents from A end: 27·6, 7·6, 7·6, – 22·4, – 72·4.

14. (a) 31, (b) 1·524 V (with full number of services), (c) at central load point.

15. Currents from A : 151·8, 81·8, 61·8, – 3·2, – 93·2, – 153·2.
Minimum p.d. at third load point from A.

16. 275 yd. from the 200 V feeding point.

17. 0·088 sq. in.

20. AC, 0·292 sq. in.; CD, 0·0448 sq. in.; CB, 0·126 sq. in.

21. Ratio of copper, 1·015.

22. *Currents.* + outer: 25·7 A in AC, 5·7 A in CD, – 34·3 A in DB; neutral: – 18·3 A in AC, 38·3 A in CE, – 21·7 A in ED, 18·3 in DF, – 1·7 A in FB; – outer: – 46·3 A in AE, 13·7 A in EF, 33·7 in FB (minus sign denotes currents in direction BA). *P.d.s* : 201·09 V at C, 196·11 V at D, 189·26 V at E, 197·03 at F.

23. (1) *Currents.* Red outer: 79·2 A in FC, 20 A in CA, 44·9 A in FD, 15 A in DB. Blue outer: 59·2 A in FE, 10 A in EA, 71·6 A in FG, 15 A in GB. Neutral: 34·9 A in FC, – 40 A in CE, 10·2 A in EA, – 98·4 A in FG, 30·1 A in GD, – 6·7 A in DB (minus sign denotes neutral current in direction away from feeding point). (2)

* Cross-section of neutral equal to half that of outer.

P.d.s. Red side: *A*, 231 V; *C*, 231·9 V; *D*, 230·3 V; *B*, 230·1 V. Blue side: *A*, 230·5 V; 229·8 V; *G*, 229 V; *B*, 231·5 V. (3) 123·1 A (Red), 128·6 A (Blue). (4) 25·6 kW (Red), 29·1 kW (Blue).

24. (1) *Currents.* Red outer: 24·4 A in *AC*, 7·66 A in *CD*, 33·4 A in *DB*. Blue outer: 44·5 A in *BE*, 15·6 A in *EF*, 35·3 A in *FB*. Neutral: 20·2 A in *AC*, 37·9 A in *CE*, − 22·7 A in *ED*, 17·3 A in *DF*, − 2·72 A in *FB*. (Minus sign denotes neutral current in direction *BA*). (2) *P.d.s.* Red side: *C*, 200·8 V; *D*, 196·96 V. Blue side: *E*, 190·7 V; *F*, 197·2 V. (3) *A* end: 4·68 kW (Red), 7·94 kW (Blue). *B* end: 5·72 kW (Red), 6·06 kW (Blue).

25. (1) 0·45 sq. in. (2) Load voltages: *C* 232·96, *D* 229·2, *E* 228, *B* 227·3. (3) Voltage drop reduced from 9·7 V to 4·1 V. (4) 35·5 A. (5) Load voltages: *C* 234·5, *D* 232·9, *E* 233·25, *B* 235·06.

26. *A*, 0·92 V; *B*, 1·088 V; *C*, 0·864 V.

27. (1) *B*, 239·2 V; *C*, 234·4 V; *D*, 228 V; *E*, 237·7 V; *F*, 228·2 V; *G*, 224·1 V. (2) (Red) 24·4 A; (Blue) 26·3 A; (White) 30 A. (3) (Red) 5·76 kW; (Blue) 6·12 kW; (White) 6·12 kW.

28. (1) 11 V, 16·5 V, 11 V. (2) 7·14 V, 12·67 V, 9·08 V.

29. (1) *AB*, 189·3 A; *BC*, 79·3 A; *CD*, 10·7 A; *DE*, 70·7 A; *EA*, 190·7 A. (2) *AB*, 149·4 A; *BC*, 39·4 A; *CE*, 22 A; *CD*, 72·6 A; *DE*, 132·6 A; *EA*, 230·6 A.

30. 0·5Ω. Current in equalizer, 4 A. Currents in sections: *AB*, 8·16 A; *BC*, 4·16 A; *CD*, 1·16 A; *DE*, 1·456 A; *EF*, 2·544 A; *FA*, 7·72 A; *FD*, 0·176 A; *AG*, 5·11 A; *GD*, 3·11 A.

31. (i) 99·8 per cent, (ii) 91·6 per cent.

32. (1) 20·13 kW, (2) 5·01 kW, (3) 0·76 leading.

33. £11,530.

34. $[1 - 0.114\sqrt{(1 - \cos^2\varphi_1)}]/\cos\varphi_1 = [1 - 0.114\sqrt{(1 - \cos^2\varphi)}]/\cos\varphi$. $\cos\varphi_1$ = corrected power factor; $\cos\varphi$ = original power factor.

35. (i) Synchronous motor generators. (ii) 0·88 leading. (iii) (*a*) £16,070, (*b*) £16,460.

36. £141.

37. 105,420 kWh.

38. (*a*) 85μF, (*b*) 10,400 V, 10,600 V.

39. (*a*) 850 kVA, (*b*) 12,440 V, 13,100 V.

40. 0·23 sq. in.

41. 4,750 V.

42. 2,610 kW, 2,390 kW.

43. $R = 0.845Ω$, $X = 1.785Ω$.

44. 2,000 kW.

45. 88·7 kW, 91·3 kW.

46. (1) 203 V; (2) 12·8 kW; (3) (*A*) 8·8 kVA, (*B*) 6·6 kVA; (4) (*A*), 32·8°, (*B*) 39°.

47. 0·456Ω.

EXAMPLES II

1.

Current . . .	10	20	30	40	50	60	70
Speed (r.p.m.) . .	1,057	728	604	524	483	422	406

2.

Current . . .	50	100	150	200	250	300
Speed (r.p.m.) . .	1,240	775	654	592	549	516
Gross torque (lb.-ft.).	169	536	942	1,375	1,837	2,320

3.

Current . .	10	20	30	40	50	60	70
Torque (lb.-ft.) .	15·3	56·4	108	164	222	282	345
Speed (r.p.m.) .	2,510	1,346	1,045	907	827	772	730

4. 590 r.p.m.

5.

Current . .	4	8	12	16	20	24	28
Speed, full field (r.p.m.) . .	1,500	905	690	593	531	493	469
Speed, tapped field (r.p.m.) .	3,260	1,610	1,110	884	756	666	510

6.

Current . . .	300	250	200	150	100
Tractive effort (lb.) .	2,670	1,990	1,320	750	337

7.

Current . . .	20	30	40	50
Speed (r.p.m.) . .	1,635	1,195	982	857
Torque (lb.) . .	6·3	12·5	19·5	27·4

8.

Current . .	100	150	200	250	300	400
Tractive effort (lb.).	560	1,245	2,100	3,100	4,200	6,400
Speed (m.p.h.) .	61·1	43	34·2	28·1	24·2	21·1

9.

Current . . .	700	600	500	400	300
(*a*) Speed (r.p.m.) .	497	532	584	654	778
(*a*) Torque (kg-m.) .	1,736	1,410	1,085	785	500
(*b*) Speed (r.p.m.) .	545	592	653	746	915
(*b*) Torque (kg-m.) .	1,582	1,260	970	688	426

10.

Current . . .	160	120	80	40
Speed (m.p.h.) . .	4·9	5·75	7·27	11·35

11.

Current . . .	160	120	88	40
Speed, 1st series (m.p.h.)	—	—	0 (77 A)	5·9
„ 2nd „ „	—	0·18	2·92	8·2
„ 3rd „ „	1·88	3·3	5·4	10
„ 4th „ „	4·9	5·75	7·27	11·35

12. 38·2 A.

13. 55·2 A, 785 r.p.m.

14. 12·3 per cent.

16. Currents, (*A*) 245 A, (*B*) 272 A ; Tractive efforts, (*A*) 3,170 lb., (*B*) 3,550 lb.

17. Speed, 8·42 m.p.h. ; Voltages, (*A*) 269·5 V, (*B*) 280·5 V.

18. (*a*) 275 A, 3,085 lb. ; (*b*) 256 A, 2,780 lb.

19. Currents, (*A*) 305 A, 218 A, 167 A ; (*B*) 262 A, 198 A, 152 A ; tractive efforts, (*A*) 4,150 lb., 2,500 lb., 1,645 lb. ; (*B*) 3,550 lb., 2,290 lb., 1,470 lb.

20. (*a*) 124·9 kW (*A*), 112·4 kW (*B*) ; (*b*) 2,290 lb. (*A*), 2,060 lb. (*B*) ; (*c*) 110 kW (*A*), 98·8 kW (*B*) ; (*d*) 56·8 kW (*A*), 59·5 kW (*B*) ; (*e*) 11·68 m.p.h.

21. 100° C.

22. (*a*) 0·06032Ω, (*b*) 5·02 per cent high, or 4·96 per cent low, according to whether original error is high or low.

23. 60° C.

24.

Current	50	100	150	200	250
Speed (m.p.h.) .	40·3	26·3	22·5	20·4	19·3
Tractive effort (lb.) . .	343	1,200	2,125	3,050	4,000

25. Efficiency 85·2 per cent, train speed 17·8 m.p.h., tractive effort 3,950 lb.

26. 20·8 V.

27. 110 V, cos $\varphi = 0.89$.

28. 638 lb.-ft.

29.

Current	1,000	1,400	1,800	2,200
Speed (r.p.m.) . . .	956	718	643	603
Torque (kg-m.) . .	420	720	1,010	1,320
Power factor (per cent) .	97·8	96·6	95·4	94·2

30.

Current	1,000	1,400	1,800	2,200
Speed (m.p.h.) . .	57·8	46	39·5	35·9
Tractive effort (lb.) . .	2,690	4,720	6,950	9,100

31.

Current	1,000	1,500	2,000	2,500
Speed (km.p.h.) . .	16·5	12·8	10·5	8·8

32. (*A*) 1,615 A, (*B*) 1,580 A, (*C*) 1,540 A.

33. 12,290 kg ; 1,460 A (*A*) ; 1,420 A (*B*).

34. (1) 690 A ; (2) cos $\varphi = 0.845$, $\eta = 0.96$; (3) 3·73 times full-load torque ; (4) 3,270 kW ; (5) 4·25 times full-load torque as motor.

35. 8·28 m.p.h. ; 1 in 6·9 (down). [NOTE. I^2R losses assumed to be divided equally between stator and rotor.]

36. (*a*) 1,130 h.p., 870 h.p. ; (*b*) 620 h.p., 380 h.p. ; (*c*) 180 h.p., – 80 h.p.

37. (1) 1,000 r.p.m., 750 r.p.m., 428 r.p.m. (synchronous speeds);
(2) 750 h.p.; (3) 42·8 per cent from six-pole motor, 57·2 per cent from
eight-pole motor, losses ignored.

EXAMPLES III

1. 2·15Ω.

2. $R_1 - R_{n+1} = 1·2Ω$, $R_2 - R_{n+1} = 0·8Ω$.

3. $R_1 - R_{n+1} = 0·996Ω$, $R_2 - R_{n+1} = 0·763Ω$.

4. $R_{1p} = 0·816Ω$.

5. $R_{1p} = 0·543Ω$, $R_{2p} = 0·328Ω$.

6. 10.

7. $R_1 - R_2 = 0·574Ω$, $R_2 - R_3 = 0·445Ω$, $R_3 - R_4 = 0·346Ω$.

8. $R_1 - R_2 = 1·03Ω$, $R_2 - R_3 = 0·88Ω$, $R_3 - R_4 = 0·754Ω$, $R_4 - R_4$
$= 0·645Ω$, $R_5 - R_6 = 0·552Ω$.

9. $R_1 - R_2 = 1·835Ω$, $R_2 - R_3 = 1·41Ω$, $R_3 - R_4 = 1·09Ω$.

10. $R_1 - R_2 = 0·54Ω$, $R_2 - R_3 = 0·435Ω$, $R_3 - R_4 = 0·353Ω$,
$R_4 - R_5 = 0·286Ω$.

11. $R_{1p} - R_{2p} = 0·46Ω$, $R_{2p} - R_{3p} = 0·375Ω$, $R_{3p} - R_{4p} = 0·3Ω$.

12. $R_{1p} - R_{2p} = 0·29Ω$, $R_{2p} - R_{3p} = 0·246Ω$, $R_{3p} - R_{4p} = 0·208Ω$.

13. $R_1 - R_2 = 0·337Ω$, $R_2 - R_3 = 0·287Ω$, $R_3 - R_4 = 0·244Ω$,
$R_4 - R_5 = 0·208Ω$, $R_{1p} - R_{2p} = 0·23Ω$, $R_{2p} - R_{3p} = 0·2Ω$, $R_{3p} - R_{4p}$
$= 0·17Ω$.

14. $R_1 - R_2 = 0·2955Ω$, $R_2 - R_3 = 0·238Ω$, $R_3 - R_4 = 0·192Ω$,
$R_4 - R_5 = 0·1545Ω$; $R_{1p} - R_{2p} = 0·32Ω$, $R_{2p} - R_{3p} = 0·269Ω$,
$R_{3p} - R_{4p} = 0·226Ω$.

15. $V_1 = 53$ V, $V_2 = 75$ V; 38 A.

16. (1) 1·2Ω; (2) 69 A; (3) 49 V.

17. $V_1 = 139·2$ V, $V_2 = 168$ V, $V_3 = 194$ V, $V_5 = 217·8$ V, V_6
$= 240$ V.

18. $V_1 = 131$ V, $V_2 = 144·5$ V, $V_3 = 158·5$ V, $V_4 = 173$ V, V_5
$= 189$ V, $V_6 = 206$ V, $V_7 = 223$ V, $V_8 = 241$ V, $V_9 = 261$ V,
$V_{10} = 282$ V, $V_{11} = 304$ V, $V_{12} = 328$ V, $V_{13} = 353$ V, $V_{14} = 380$ V.

EXAMPLES IV

1. 0·538 m.p.h.p.s.

2. 31·3 m.p.h.

3. 1·05 m.p.h.p.s.

4. (1) 28,600 lb.; (2) 1,487 kW.

5. 1·025 m.p.h.p.s. up the gradient; 1·225 m.p.h.p.s. down the
gradient.

6. (1)

Time (sec.) .	.	0	24·8	87·8	102
Speed (m.p.h.)	.	0	24·8	24·8	0

(2) 52 Wh per ton-mile.

7. (a) 2,290 kW; (b) 138·4 kW; (c) 52 Wh per ton-mile.

8. 7,800 lb.

9. 207 lb.

10. 87·26 tons.

11. (1) 4·65 kWh; (2) 71·3 Wh per ton-mile.

12.

Time (sec.)	.	0	18·8	24·9	36·9	61·7	70·7	87·3
Speed (m.p.h.)	.	0	20	25	30	35	33·95	0

Schedule speed = 26·7 m.p.h.

13. 60·8 Wh per ton-mile.

14. V_m = 27 m.p.h. Specific energy consumption = 55·7 Wh per ton-mile.

15. 48·5 Wh per ton-mile.

16. 110 Wh per ton-mile.

17. (1) 7,875 ft; (2) 81·1 Wh per ton-mile; (3) 41·2 per cent.

18. (a) 35,650 lb.; (b) 8 kWh (assuming average energy efficiency = 60 per cent); (c) 60·1 Wh per ton-mile.

19. (1) 9·46 m.p.h.; (2) 114·5 A.

20. 4,415 lb.-ft.

21. (1) 66 A to 76·2 A; (2) 12 m.p.h. to 13·85 m.p.h.

22. 27·4 sec.

23. 91·5 Wh. per ton-mile.

24. 0·417 kWh.

25. (a) 1,037 kW sec.; (b) 220 W; (c) 168 kW (max.); (d) 10·55 m.p.h.; (e) 20·7 sec.

26. 9·75 m.p.h.

27.
(a) 90 sec.

(b) Time (sec.)	.	0	22·9	26·3	33·3	44·7	47	96·2	112
Speed (m.p.h.).		0	24·9	28	32	36	37	31·8	

28.

Time (sec.) .	.	0	18·1	21·4	27·3	40·4	81	156·4	160
Speed (ft. p. sec.) .		0	35	40	45	50	54·8	12	0

29.

Time (sec.) .	.	0	24·5	27·2	33·4	51·1	59·3	100·5	110
Speed (m.p.h.)	.	0	22·5	24·5	27·5	31	31·7	19	0
Distance (ft.)	.	0	404	496	734	1,494	1,820	1,535	3,480

Energy consumption = 112·5 Wh per ton-mile.

30.
Time (sec.) .   . 0 29·3   32·3   38·9   47·3   52 136·3   152
Speed (m.p.h.)   . 0 26·2   28·5   32   35 36·3   27·5   0
R.m.s. current per motor = 217 A.

31. (1) 73·2 Wh per ton-mile; (2) 178 A.

32. 210 A.

33. Speed-time curve—
Time (sec.) .   18   20   23·1   28·5   43·2   53·2   58·9   67·2   76·2
Speed(m.p.h.) 18·7 21·1   24   27·1   25·6   22·9   20·4   18   0
Energy consumption: 71·8 Wh per ton-mile.

34. (a) 3·71 kWh; (b) 7·76 kWh per train mile; (c) 0·064 m.p.h.p.s.
1·33 m.p.h.p.s.; (d) 46 per cent.

EXAMPLES V

1. (1) 0·83 ton; 0·208 ton.

2. Pole face area, 30·5 sq. cm.

3. 986 tons (coefficient of adhesion = 0·2).

4. 65 tons.

5. 6.

6. (1) 1·38 m.p.h.p.s.; (2) 3·05 m.p.h.p.s.

7. 400 tons.

8. (1) $2C_0 1$; (2) 3, compensated series, 550 h.p.; (3) individual axle, geared, with universal linkwork couplings; (4) 1,480 kVA.

9. 7·5 tons, 17·5 tons.

10. 2·5 m.p.h.p.s.

12. 9·06 ton-ft.-sec.

13. See Fig. 14(b), AB, 16″; BC, 8½″; F_2E, 15″, ED, 8″.
Pull (P, Fig. 14(b) = 1390 lb.

14. See Fig. 14(b), AB, 14″; BC, 10″; F_2E, 12·6″, ED, 9″.

EXAMPLES VI

1. 6·8″.

2. 9·25″.

8. 980 lb., 9·25″.

4. 752 lb.

5. 14,500 lb. per sq. in.

6. $\delta = \tfrac{1}{2}w'l^2/T$. [$w'$ = equivalent weight per ft. of span, l = half length of span.]

7. 2·24 metres.

8. (a) 8·72 ft.; (b) 1,745 lb.

EXAMPLES VII

1. 0·111Ω.

2. 475 V.

3. 1 mile.

4. Currents (A): A, 104·4 – 84·4 – 44·4 – 24·4 – 35·6 – 115·6 B; Lowest voltage (644·3 V) at 60 A load point.

5. 1·9 sq. in.

6. 38 kW, assuming feeding point to be at same potential as negative bus-bar.

6a. 75 A.

7. $D = pV^2 \cos \varphi \, (100 - p)/(PZ \times 10^7)$; 42·5 miles.

8. $a_1 = [2\rho\sqrt{(I_2 + I_3)}/v](l_1\sqrt{(I_2 + I_3)} + \sqrt{(l_2{}^2I_2 + l_3{}^2I_3)})$;
$a_2 = 2\rho l_2 I_2(l_1\sqrt{(I_2 + I_3)} + \sqrt{(l_2{}^2I_2 + l_3{}^2I_3)})/[v(l_2\sqrt{I_2} + l_3\sqrt{I_3})]$;
$a_3 = a_2 l_3 I_3/l_2 I_2$.

9. 0·2015 sq. in.

10. One-fifth of distance between intermediate feeding points.

11. (a) 501·25 V at 100 A, 498·75 at 200 A; (b) (B) 498·5 V, (C) 497·25 V.

12.

Time (sec.) .	0	1·5	3	4·5	6	7·5	9	12	15
Load on A (kW) .	750	650	550	450	400	250	200	100	0
Volts at train .	1,500	1,435	1,390	1,365	1,360	1,375	1,390	1,435	1,500
Miles from A .	0	1	2	3	4	5	6	8	10

13. (1) 15 : 1; (2) 6 : 1.

14. 843 A; 57·2 A.

15. 95 A.

16. 7,500 ft. per min.

17. 0·0612Ω.

18. (1) 967 A; (2) (a) 22·6°, (b) 14°.

19. (1) 1,200 V; (2) two three-phase star groups with centre-tapped reactance between neutral points.

20. (1) 192 A; (2) 1,387 kVA.

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