A COMPLETE COURSE IN

ELEMENTARY AERODYNAMICS

WITH EXPERIMENTS AND EXAMPLES

By the Same Author

AERODYNAMICS

An advanced text-book on the theory and practice of the subject.

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ELEMENTARY AERODYNAMICS

WITH EXPERIMENTS AND EXAMPLES

BY

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PREFACE

In an advanced book on Aerodynamics extensive use may be made of mathematical methods. Difficulties besetting this approach to the subject are readily surmounted by undergraduate engineers and others possessing some mathematical aptitude. However, occasions are multiplying when a wide but less exacting knowledge of the subject suffices, and then the science must be described more frequently in terms of experiment.

This book presupposes a very limited knowledge of algebra, trigonometry, and mechanics. The experiments relied upon require only simple apparatus; but if they cannot actually be carried out, a careful consideration of them is necessary to grasp the subject.

Various important parts of Aerodynamics, whose adequate investigation calls for advanced methods, have been introduced qualitatively. Thus the final chapter presents a brief account of the famous Lanchester–Prandtl theory of wings without attempting its scientific analysis. Some other matters have been omitted altogether, being regarded as too difficult or specialised. Of this nature, for instance, are recent developments foreshadowed in some measure by the Schneider Trophy Races.

Applications of aerodynamical principles and methods are illustrated by examples worked in the text, covering an appreciable range of difficulty without making demands on ingenuity. Additional questions of comparable standard can be found in the aeronautical examination papers of the University of Cambridge (Engineering Studies, II and III), the University of London (B.Sc., Part I), the Institution of Civil Engineers, the Institution of Mechanical Engineers, the Royal Aeronautical Society, etc.

Examples 12, 25, 58, 96–7 and 102, though involving no essential difficulty, lead on to further study and, together with Articles 50, 170, 202 and 218–21, need only be scanned upon a first reading.

Reference has been made to Davy's "Interpretive History of Flight" and other admirable works in connection with the few historical allusions in the text.

The author gratefully acknowledges his indebtedness to The English Universities Press (especially to Mr. A. H. Tyas in their behalf) and to the Printers for facing up to the manifold difficulties attending war-time production.

N. A. V. PIERCY.

Cambridge.

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Chapter I

INTRODUCTION

1. AERODYNAMICS is the principal science of Aeronautical Engineering. Its chief concern is to ensure the safety of aircraft in flight and to improve their operational efficiency so far as may be achieved by a judicious shaping of external form and distribution of mass.

The subject surveys the stable equilibrium of aircraft in steady or accelerated flight and their reaction to disturbance; the ease and efficacy of control in all circumstances; and also the maximum air loads arising on exposed parts, with a view to securing requisite structural strength. Many questions are largely determined by considering the character of the air flow relative to the craft. These notably include the manner in which lift is best generated to support the flying weight; minimising resistance to motion through the air; and providing propulsive force to overcome such resistance as remains.

Both safety and performance depend in part on the physical properties of the atmosphere, which have therefore to be ascertained at all flying altitudes.

Unlike some other studies of Aeronautical Engineering, Aerodynamics is in no sense a development of a subject previously comprehended within general engineering science. It has little or nothing in common with Hydraulics, whilst the difficult modern engineering subject of Fluid Mechanics has itself been fostered by the expansion of Aerodynamics. The latter brings to engineering, indeed, an array of new fundamentals, phenomena and methods of investigation, some originating in the subject and others transplanted from Mathematics or Physics and given new life.

The student equipped with engineering knowledge, elementary or more advanced, encounters on turning to Aerodynamics a certain break with previous studies, arising partly from the novelty of the subject matter and no less from a change in method. The subject is already developed to an exceptional degree, but remains in vigorous growth, susceptible to discoveries and inventions of benefit to Aviation. In these circumstances it is as well that no so-called engineering theory of the subject exists to make its study 'cut and dried'.

In all engineering there is room for intuition, born of an instinctive visualisation of how things will work. Aerodynamics is no exception, and calls for engineering instincts that are supplementary to those a student has already formed. These soon develop, whether the subject is studied mathematically or, as in this book, experimentally. The initial difficulty experienced on this score implies that Aerodynamics is not a subject to 'con over' but rather to study deeply, even at the cost of covering less ground.

Finally it is evident that the success of aerial transport results from many endeavours of different kinds, of which our subject deals with only one, although the most important and fundamental. Parallel progress in other fields of Aeronautical Engineering—in aero-engines, structural plans, materials and modes of construction, aerodromes and airports, and so on—influences the scope and orientation of aerodynamical preoccupations, modifying these from time to time. Existing aerodynamical knowledge is very extensive, and a short course of study must be selective, giving preference to matters of immediate aeronautical interest; for instance, it is justifiable at the present time almost to neglect the biplane and triplane in order to consider the monoplane more effectively.

This chapter describes a few of the considerations determining the present treatment of the subject. The brief references to some outstanding events of the past no more than touch upon the history of Aerodynamics and flying, for which many excellent books and papers may be consulted.

2. Static and Aerodynamic Lift

The first requirement for human flight is a lift that will support a considerable weight freely in the atmosphere. Such a lift can be secured by two dissimilar means.

The one method exploits Archimedes' Principle, by which every body in the atmosphere experiences an upward force equal in magnitude to the weight of the air it displaces. For ordinary bodies this force is negligible; thus it lightens an 11-stone man by only 3 ozs. But the man weighs no more than the air in a room 10 feet high and wide and 20 feet long, and a tenuous body of twice this volume and the same weight could lift him.

Speculation was already rife in the 17th century as to the

›ossibility of securing a sufficient margin of buoyancy from con-ainers enclosing either a partial vacuum or air rendered light by ıeating. During the following century the second of these ›chemes was successfully demonstrated at full scale, and the ›roblem was also solved in a more practical way by employing ›nvelopes inflated with hydrogen.

The method is aptly described as 'lighter-than-air', the term ·eferring to the lifting agent and excluding the load attached. 3eing evidently statical, the force of buoyancy supporting the .oad is called *static lift*.

The other method is purely aerodynamical. It is frequently listinguished as the 'heavier-than-air' method, but the weight ɔf air displaced is in fact quite negligible. *Aerodynamic lift* is the reaction resulting from a rate of change of downward momentum imposed upon continually renewed air. Whether the action is achieved by flapping or fixed wings, horizontal propellers, or paddle-wheels, the lifting agent must be maintained in quick motion through the air, which the latter resists, however idealised the action. This method therefore calls for a continuous expenditure of power, on which the lift depends.

Aerodynamic lift excited attention in most remote times. The earliest projects naturally favoured beating wings in imitation of birds and insects, but the horizontal propeller is some four-and-a-half centuries old, whilst the aeroplane itself was invented in the early years of the 19th century.

The practical utilisation of aerodynamic lift was delayed until the present century owing to (*a*) the problem of making flight by this means even tolerably safe, (*b*) the tardy advent of the light-weight prime mover. The second of these difficulties also impeded lighter-than-air aeronautics, but much less implacably because the maintenance of static lift, unlike that of aerodynamic lift, absorbs no power.

3. The First Ascents

The story of the first ascents into the atmosphere provides one of the most exciting pages of aeronautical history. In 1783 the idea of obtaining static lift from an imprisoned volume of heated air was put into practice by the brothers Montgolfier of Lyons. Their audacious contraption consisted of a giant inverted paper bag, strengthened against bursting by a fabric cover and filled with the hot smoke of a damp straw fire which was maintained at the open mouth during flight. The apparatus, as will be seen from Fig. 1, somewhat resembled an ecclesiastical hat, the passengers being accommodated in the brim.

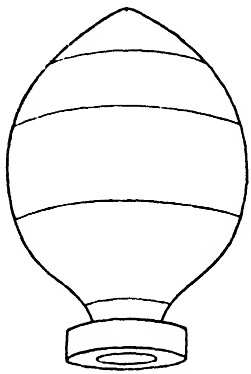


FIG. 1.—HOT-AIR BALLOON, IN WHICH THE FIRST ASCENT WAS MADE IN 1783.

The displacements of these 'Montgolfieres' ranged from 30,000 to 60,000 cu. ft.

Fire-fighting equipment seems to have comprised a bowl of water and a sponge. But a more insidious risk than this blatant one arose from lack of knowledge of the atmosphere. Apart from other perils that might conceivably lurk in the air, there was the chance of being caught in a current and hurled violently upward or downward. All flying depends upon freedom from vertical surging on a large scale in the atmosphere, and the reason for confidence in this respect was unknown at that time.

However, sending up a trial load of live-stock resulted only in such damage as could be traced to animal spirits, and thereafter as many as seven passengers together made aerial trips in safety. Montgolfières, as these hot-air balloons were called, were abandoned only when an ingenious but unwise attempt to combine them with hydrogen balloons led to disaster.

4. Hydrogen Balloons

Cavendish isolated hydrogen in 1766. Its exceeding lightness was so quickly harnessed to aeronautics that a first ascent was made by its means before the end of the same year that witnessed the Montgolfières. Balloons filled with hydrogen were clearly more efficient and utilitarian. They permitted an aerial crossing of the English Channel in 1785, 124 years before this was accomplished by aeroplane. Almost immediately they were also put to scientific use for investigations into the state of the

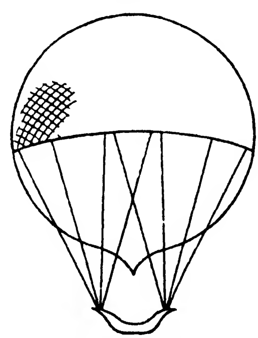


FIG. 2.—THE FIRST HYDROGEN BALLOON, 1783.

Displacement, 9000 cu. ft.

atmosphere, which are still continued with the aid of balloons sent up to great altitudes.

Other matters of interest regarding gas-filled envelopes include their design to carry a given weight to a prescribed altitude or 'ceiling', and their tendency to ride level at lower altitudes. Detailed consideration of these problems is postponed to the next chapter, but evidently the envelope must be only partly filled at low altitude to permit expansion of the gas without loss at increased altitudes.

5. Early Dirigibles

A balloon is incapable of motion relative to the surrounding air except in a vertical sense; free of contact with the ground or sea, it drifts with the wind and cannot be steered. To overcome the disadvantage and enable a journey to be made in a direction different from that in which the wind is blowing requires a horizontal pull or thrust. Sails being useless and oars requiring to be too large, it was proposed at one time to harness flocks of trained birds to balloons, like teams of dogs to sledges. But more practical ideas prevailed in the 19th century, and a power plant driving a propeller was incorporated.

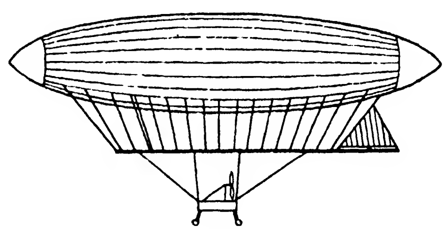


FIG. 3.—THE FIRST AIRCRAFT—GIFFARD'S STEAM-DRIVEN DIRIGIBLE, 1852.
Displacement, 88,000 cu. ft.; H.P., 3; speed, 5 m.p.h.

The aircraft resulting from the combination of a gas-filled envelope, an engine and propellor, and a rudder to steer with, is appropriately called a dirigible balloon, or simply a dirigible. However, the elongation of the gas-bag into a streamline shape to reduce resistance to motion and render a small thrust effective brought the name airship early into use.

The first successful dirigible, and therefore the first aircraft in the true sense of the word, was Giffard's airship (Fig. 3), which cruised under its own steam at some 5 miles per hour in the absence of wind. It appeared in 1852, which seems rather late when one recalls that Watt's inventions relating to the steam engine were worked out during the period of the isolation of hydrogen and the first ascents. But a glance at early prime movers in a museum will suffice to explain how much easier was the task of making a large envelope and filling it with gas than that of evolving an engine of useful power which the gas-bag could lift. Giffard's power unit represented a considerable advance, but it still weighed per horse-power more than a cwt. for each lb. of the modern aero engine.

As each prime mover was invented during the 19th century it was quickly tried out in the air. Dirigibles propelled by electric motor or gas engine were flown within a dozen years or so of these inventions becoming practical. The first successful petrol- or benzene-fuelled airship (Santos-Dumont, 1898, Fig. 4) appeared only thirteen years after Benz succeeded in driving the first corresponding motor-car round in a circle nearly four times before breakdown occurred. Santos-Dumont's diminutive power plant was of about the same size as Giffard's (3–4 H.P.), but weighed 80 per cent. less. The establishment of the internal-combustion engine led to many small airships being built; Santos-Dumont alone is credited with fourteen.

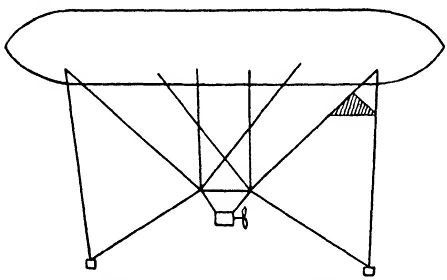


FIG. 4.—SANTOS-DUMONT'S SMALL PETROL-DRIVEN AIRSHIP, 1898.

The ballast-bags slung fore and aft were employed to adjust trim.

6. The Three Types of Airship

A difficulty with the early dirigibles lay in maintaining the streamline shape of the envelope in face of variation in volume of the gas on change of altitude. Occasionally, transverse baffles were fitted to prevent surging of the gas towards nose or tail, which, by concentrating the lift at one or other extremity, could cause a partially deflated envelope to stand on end. Even so, the longitudinal compression arising from the fans of wires supporting the gondola could make a limp envelope buckle like a weakly inflated inner tube from a tyre.

The difficulty was met for small aircraft by fitting internal ballonets filled with air. These collapsed on increase of altitude to allow the gas to expand inwardly, and they could be inflated again to keep the envelope taut at lower altitudes when the gas contracted. An airship of this kind is distinguished as 'non-rigid'.

Two constructional changes were introduced at the end of the century, leading to the 'semi-rigid' and 'rigid' types.

In the semi-rigid type the gondola containing the engine and other loads was suspended from a long girder, concealed within the envelope and extending along its length, which relieved the envelope from end-wise compression from the suspending wires and in other ways helped to preserve its shape.

The rigid type was introduced by Zeppelin, and large airships of recent times have all taken this form. Its external shape is independent of the volume of the gas, being secured by a framework covered with fabric. The hull so formed is divided by a

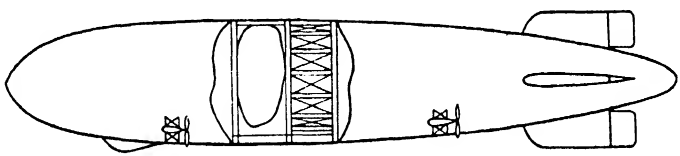


FIG. 5.—LARGE RIGID AIRSHIP WITH FABRIC CUT AWAY OVER TWO CELLS, SHOWING (*right*) EXTERNAL FRAMEWORK, (*left*) GAS-BAG.

succession of bulkheads into a string of numerous cells (Fig. 5), each containing a gas-bag, which, in modern examples, may be many times as large as a balloon. The several power units and other loads are supported directly by the great hull.

Only lack of powerful and light engines prevented greater development of airships during the Victorian era. Otherwise trouble would have been encountered through the use of hydrogen, which is inflammable in air; the danger has since proved real, and this gas will probably never be used again in Civil Aviation. Substitution of the inert gas helium entails little increase of volume to support a given weight; in round numbers each ton of static lift requires 33,000 cubic feet of hydrogen and 36,000 cubic feet of helium; but the supply of the latter is restricted. It will be seen that an airship weighing 200 tons would require a volume exceeding 7 million cubic feet, leading to a hull some 800 feet in length and more than 135 feet in maximum diameter.

7. Size and Speed

It was early recognised that as airships become larger both lift and structural weight tend to increase as the cube of the linear dimension, whilst surface area increases only as the square of the same. Resistance to motion is roughly proportional to the area, and so large sizes promise a greater proportionate useful lift at the same speed through the saving of engine, fuel and fixed weights. The development of the aero engine has enabled this theoretical advantage of the large airship to be tried out in modern times. But results have so far proved to be rather indifferent from the point of view of Civil Aviation, examples of up to 200 tons weight failing to evince exceptional weight-carrying capacity compared with other types of aircraft. The reason is that the construction of vast hulls of sufficient strength and light weight presents a severe engineering problem for solution.

A comparison will be effected shortly between airships and other aircraft, but the following points may be noted. A large airship, 6–7 million cubic feet in volume, has a surface area of the order of 6 acres. Hence, however perfect the streamline form of the hull, the speed through still air is necessarily slow and seldom approaches 100 miles per hour. It follows that an airship can make only poor progress against strong headwinds and must seek out favourable weather conditions. This it is largely in a position to do by virtue of the exceptionally long time it can remain in the air without re-fuelling. But the time required for an ocean crossing, say, against the direction of the prevalent wind becomes long and uncertain.

8. The Beginnings of Aerodynamics

Aerodynamics originated with Leonardo da Vinci, who at the close of the 15th century planned and engaged in comprehensive researches with a view to exploring the subject. He invented the helicopter and the parachute. But much more remarkable was his realisation that the elements of a science had to be established before flying could reasonably be essayed. His work remained unpublished until the end of the 18th century, when it was found to comprehend some of the basic principles of the subject.

Little further progress was made during this interval of nearly 300 years, but then the subject was revived creatively by Sir

George Cayley in a series of researches spread over the first half of the 19th century. These illuminated many different facets of aeronautics, but his most enduring and notable achievement was elucidating and applying the principle of the fixed wing, used on aeroplanes and flying-boats.

Cayley invented the aeroplane. It is true, of course, that there existed no engine to make level flight possible by this or any other aerodynamic means, but the fact that his prototypes derived power from gravity, the flight path being inclined for this purpose slightly downward from the horizon, detracts in no way from the invention. His first small glider flew in 1804. A year or two afterwards he was experimenting with almost a full-scale glider which flew with "perfect steadiness, safety and steerage . . . from the top of a hill . . ." It was large enough to give rise to conjecture as to whether a man ever perched himself upon it, but apparently this never occurred.

These experiments were epoch-making in demonstrating not only the aerodynamic lift of fixed wings but also how an aircraft could be shaped and loaded to fly by their means. A measure of stability was secured by fitting the gliders with vertical and horizontal fins, which were made adjustable for the purpose of steering. Above all, Cayley discovered the principle of the 'lateral dihedral', the upward inclination of the wings from their roots to form a flat vee, a device that is still outstanding amongst the many inventions that go to the making of a modern aeroplane.

9. Early Laboratory Experiments

It is particularly instructive to consider briefly one other aspect of this early work. To judge of the possibilities of aviation it was necessary to estimate the resistance of the air to the motion of the aircraft. Calculation being impossible, Cayley resorted to laboratory experiment, introducing the 'whirling arm' for the purpose.

This apparatus, whose occasional use still survives, consists of a long horizontal arm which can be whirled round a vertical axis located at one end, so that a model secured to the other end is forced through the air in a wide circle at a speed which can be varied. The lift and resistance of the model are measured directly, though rather inconveniently, by suspending it from the arm through a balance, whilst its speed through the air can be ascertained by a stop-watch after deducting an allowance for the swirl of air set up by the rotating apparatus as a whole.

The whirling arm came to be much used during the second half of the century, notably by Professor Langley in the U.S.A. and Sir Hiram Maxim in England, but then began to give place to the more convenient wind tunnel, in which the force components are measured on a stationary model suspended in an artificial wind. Thus the wind tunnel, employed on so magnificent a scale today, was already preferred by Horatio Phillips for his researches, *circa* 1870–90, on the most favourable forms of section to employ for aeroplane wings.

10. The Main Problem

The period between Cayley's original researches and those of Langley and Phillips was characterised by designs for full-scale aeroplanes and the construction of power-driven models, notably those of Henson and Stringfellow in Britain and Pénaud in France. All these efforts and projects received scant support, for the world remained incredulous, though it must be admitted that the inventors were over-sanguine of a ready success.

Meritorious though these inventions were, they suffered in varying degree from lack of an assured measure of inherent stability enabling them to fly safely without constant attention, or alternatively of an adequate system of aerodynamic controls, such as would have sufficed in skilled hands. Without the removal of this defect, either in the one way or the other, mechanical flight remained impracticable, for a gust of wind could bring the aircraft fluttering or crashing to the ground. Thus the aeroplane, in contrast to the airship, waited upon a matter of even greater urgency than the provision of a light engine.

This position still existed at the close of the century, but was by then clearly appreciated. Attempts to surmount the difficulty proceeded along three different lines: (*a*) adventurous experiments with man-carrying gliders, (*b*) advance in aerodynamical theory, (*c*) *ad hoc* experiments on models in the wind tunnel.

A gallant band of pioneer pilots engaged perilously in (*a*); mention must be made of Lilienthal, Pilcher and Chanute, among others. Progress under (*b*) was achieved by Dr. Lanchester, who embarked upon a theory of dynamical stability which aimed at no less than such a judicious shaping of the aircraft and distribution of its mass as would cause it to respond safely, without assistance from the pilot, to any moderate disturbance encountered during flight. This fundamental line of enquiry involved a mathematical step of great complexity, only finally negotiated by Professor Bryan in 1911. Lastly, (*c*) was the method favoured by the Brothers Wright, and it led to an instant success. They did not concern themselves with achieving inherent stability for the aeroplane they had in mind to build, but concentrated simply on making it controllable in average weather with reasonable skill. Their wind-tunnel experiments on models were correlated with flights on full-scale gliders, so that their plan also utilised the method (*a*).

11. The First Mechanical Flights

Wilbur and Orville Wright brought off the first successful aeroplane flights at Kitty Hawk, North Carolina, on December 17,

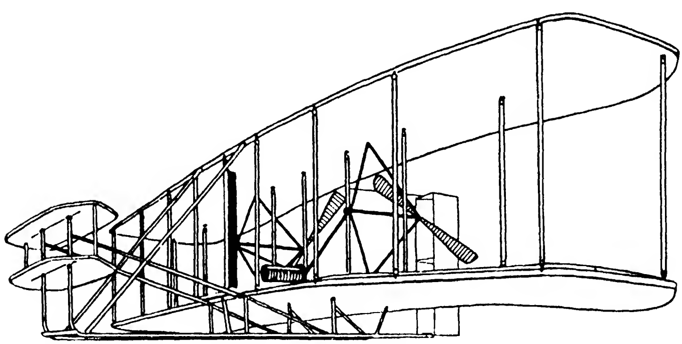


Fig. 6.—The Wright Brothers' Biplane, in which the First Mechanical Flights were made in 1903.

1903. Their craft was a biplane of 40 feet span weighing 1000 lb. all up (see Fig. 6). It was fitted with a biplane elevator in front for longitudinal control and with a twin rudder behind for steering, and the wings could be warped to roll the machine round its longitudinal axis. A 12 horse-power motor drove two propellers in opposite directions, through chains, and gave a speed relative to the air of about 32 miles per hour. Launched from a rail under its own power and in the presence of a boisterous

20 miles-per-hour wind, it was flown several times, finally for nearly a minute, and safely landed.

This historic achievement cannot be overrated. It was typically of an engineering character, being due to a general soundness of design and construction, permitting that touch of brilliance, the warping of the wings, to demonstrate its effectiveness. A tail plane, already suggested by others, would have been preferable to the front stabiliser, but the latter sufficed and was retained for some time afterwards. Lighter engines had been planned or constructed, but the one used did not peter out, and provided a sufficient margin of power; moreover, special care had been taken to ensure that the transmission would stand the strain. Better wing sections were known at the time, but those employed avoided some earlier mistakes and allowed of a light construction, and the whole featherweight structure withstood a moderately severe test without fracture. Finally the wing-warping, whose function was to be discharged later on by *ailerons*, demonstrated the advantage of powerful means of aerodynamic control in the lateral sense.

Full accounts of the achievement exist, of the painstaking experiments, both model and full-scale, leading up to it, and of the remaining difficulties surmounted soon afterwards. The first aeroplane to fly was not conceived from a flash of inspiration. It resulted from concentrated study and attention to detail by two level-headed men, crowning not only their own wise preparations, but also a century of high endeavour to which their predecessors had consecrated time, fortune and life in face of every discouragement. The most enduring aspect of the enterprise is that it made clear the need in aeronautics, as plain today as then, for intense care, a composite excellence, and a policy of leaving nothing to chance. Especially significant was the exploitation of experiments on models. The method was soon adopted far and wide, tunnels being constructed in Britain (notably at the National Physical Laboratory), the U.S.A., Russia, France and other countries.

12. Wing-loading and Stalling Speed

Discussion of the sphere of aviation and of the interplay between Aerodynamics and other aeronautical sciences must refer to two fundamentals, which will be explained in detail in later chapters, but may at once be perceived in a general way. The first to be considered is the dependence of aeroplanes on wing area.

The lift of given wings fixed rigidly to an aeroplane body depends primarily on the speed and the inclination of the sections of the wings to the direction of motion. The amount of lift generated is adjusted to balance the weight of the aircraft by suitably raising or depressing the tail end of the body by contrary motion of the *elevators*, movable parts of the tail-plane of a modern aeroplane. Reduction of speed, which in itself rapidly decreases lift, is compensated by lowering the tail, thus increasing the inclination of the sections and the lifting capacity of the wings. But this compensation cannot be continued indefinitely,

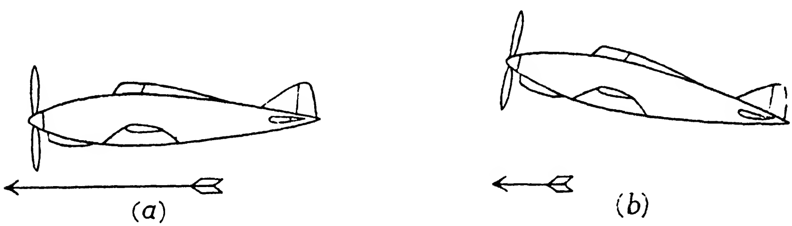


FIG. 7.—MONOPLANE FLYING STRAIGHT AND LEVEL.
(*a*) At a high speed and small incidence, (*b*) at a low speed near the stall.

there being a limiting angle for each aeroplane at which the wings reach their maximum capacity; it is called the *stalling angle*.

Fig. 7 shows an aeroplane flying (*a*) at a small angle appropriate to high speed, (*b*) nearly at its stalling angle. The speed at which sufficient lift is generated is much reduced in the second case, as indicated by the shorter arrow; it is the minimum speed of which the aeroplane is capable, and is called the *stalling speed*.

Apart from certain devices which improve maximum lifting capacity, the stalling speed of an aeroplane is directly proportional to the square-root of the weight and inversely proportional to the square-root of the wing area, defined as the product of the span of the wings and their mean width. The total weight divided by this area is called the *wing-loading*, i.e. the wing-loading is the average weight carried per unit area of the wings.

Thus the stalling speed is approximately proportional to the square-root of the wing-loading.

The wings of the Wright biplane were so lightly loaded (2 lb. per square foot) that the stalling speed was only some 26 miles per hour, and other early aeroplanes stalled at not a great deal more. It is found that in skilled hands an aeroplane can be landed at its stalling speed, so that there is an advantage in keeping this low. However, familiarity and prepared landing grounds make landing speeds of 65–70 miles per hour quite feasible for skilled pilots today. Such high modern landing speeds make possible much larger wing-loadings, which can still further be increased by the use of landing flaps and monoplane wings. A wing-loading of 32 lb. per square foot, sixteen times as great as that of the Wright biplane, is now by no means excessive. The approximate square-root law would then indicate a landing speed of over 100 miles per hour, i.e. a four-fold increase of that for the Wright biplane. But the substitution of monoplane wings, which have a greater lifting capacity than biplane wings, reduces this to 85—90 miles per hour, and then landing flaps decrease it further to 70 miles per hour.

Summing up, the wing area of modern monoplanes, weight for weight, need be only a very small fraction—say between one-tenth and one-twentieth—of that employed for the earliest biplanes.

13. The Power Required for Aerodynamic Lift

It has already been mentioned that aerodynamic lift, unlike static lift, absorbs engine power. The second fundamental of heavier-than-air aviation to be considered is how this demand depends on speed. Attention will be focused upon an aeroplane in straight and level flight through a still atmosphere, but the results also apply in principle to other means by which aerodynamic lift may be secured.

Soon after the Wrights' achievement Lanchester published his remarkable theory of the aerodynamic lift and drag of wings. Though of too revolutionary a character to be generally credited until many years afterwards, this theory as improved by Professor Prandtl is now universally accepted. Its details are complicated, however, and a description of them must be deferred until a later chapter. On the other hand, the aspect of immediate consequence becomes apparent by considering a hypothetical system governed by the same broad principle.

The lift is the reaction that results from giving downward momentum at a certain rate to the air flown through. In the actual case the large mass of air concerned is affected unequally. But for present purposes the wings are assumed to act uniformly on a mass m of air per second, giving it a uniform downward velocity v. Then the rate of change of downward momentum is mv and, if L denotes the lift, $L = mv$.

Now, before receiving the downward velocity v the air, being at rest, had no kinetic energy. But after the action each mass m has acquired kinetic energy to the amount $\frac{1}{2}mv^2$. In the case of level flight through a still atmosphere this energy can come only from a continuous combustion of fuel, and it accounts for part of the power expended by the engine.

Consider the effect of changing the horizontal speed V of the aeroplane. Since flight is both straight and level, the lift L of the wings is equal to the weight of the aircraft, which, neglecting loss of fuel, is constant. Hence L is constant and, therefore, also the product mv. But with change of speed m will vary directly as V. Therefore v must vary inversely as V. Energy is given to the atmosphere at the rate $\frac{1}{2}mv^2$ per second, i.e. at a rate which is proportional to the product of m and v^2. Therefore on change of speed this rate varies as $V \times 1/V^2$, i.e. as $1/V$.

This important result leads to inferences which conflict with common experience, increase of speed usually demanding more power. The minimum flying speed of almost every aeroplane is determined by its wing-loading, as explained in the preceding article. The power required to generate the lift necessary for straight level flight is a maximum at this slowest speed, but, although an aeroplane has occasionally been fitted with too small an engine to fly as slowly as its wing-loading would permit, there usually exists an ample margin of power. Nevertheless, if the lifting capacity of wings were greatly increased by a new invention and small wing-loadings adopted to realise very low landing speeds, the power required might well become prohibitive before the stall; the circumstances of these aeroplanes would then resemble those of autogyros, whose minimum flying speeds are determined by the power available.

The power required for aerodynamic lift is expended by the airscrew, or propeller, in doing work against a certain part of the resistance of the wings to motion, a part which would not exist if the wings had no lift, and is called the *induced drag*. Since the associated power varies as $1/V$, the induced drag of a given aeroplane in straight level flight varies as $1/V^2$.

The given wings have also a drag of another kind, which, together with the drag of the body and other exposed parts of the aeroplane, varies with speed in a normal manner, i.e. roughly as V^2. Thus the power absorbed by this second part of the drag, often called the *total parasitic drag*, varies roughly as V^3; it is small at low speeds and rapidly becomes great at high speeds.

Combining the two demands made by a particular aeroplane in straight level flight, the total power it requires can evidently be written in the form

$$\frac{A}{V} + BV^3,$$

approximately, where A and B are constant coefficients. These coefficients differ greatly in magnitude, A being very large and B very small. At low speeds with landing flaps retracted, the second term is so small compared with the first that its augmentation with increase of speed is overshadowed by the much more important decrease of the first term, and thus the total power required rapidly diminishes. But, on progressively increasing speed, a stage is reached when the second term becomes as important as the first, increasing the power required as much as the first term reduces it, and thereafter the second term takes charge.

Thus each aeroplane has a certain speed at which the power it requires to fly is a minimum. So weak were the engines of early aeroplanes that they could do little more than fly at this one speed, which was very low. The modern aeroplane has a much higher speed for minimum power and its engine permits it to fly much faster or slower; in short, it possesses a wide *speed range*. Towards the upper limit of its speed range the power required for lift is less than 10 per cent. of the total power, which mounts rapidly, nearly in proportion to the cube of speed, as is generally characteristic of other means of transport. Compared with those other means, the aeroplane starts with an initial advantage in respect of high speeds, which it owes in the end to an inability to fly slowly at all.

14. Comparative Efficiency of the Lifting Systems

A cursory review of aircraft development since the first mechanical flight reveals three outstanding features: the ascendancy of aeroplanes, the triumph of the monoplane after a long period of suppression in favour of the biplane, and a great increase in the speed of aeroplanes. Preliminary discussion of these broad issues is useful, and the present article deals with the first.

The ornithopter, or flapping-wing aircraft, can be lightly dismissed in spite of its long and disastrous innings, for no advantage has yet been demonstrated in favour of this imitative method. There remain the three systems of lift, picturesquely called the spinning-top, kite and gas-bag methods. The first was conceived in a remote age, but has taken practical shape only in recent times, in the autogyros of J. de la Cierva and in experimental helicopters. The third has been tried repeatedly throughout nearly a century. Why has the kite method soared so high above its competitors?

The permanent interest of a lifting system centres in its suitability and operational efficiency in relation to aerial transport. Regarding suitability there is involved a question of special purposes. Each system permits the performance of some duties which would be impossible with one or both of the others; for example, an airship can remain for a very long time in the air without refuelling, whilst an autogyro can land in a confined space, but an aeroplane can do neither. In short, circumstances arise in which any one of the three systems must be employed, whatever its efficiency, if aerial service is required.

In regard to operational efficiency account must be taken of (*a*) dead weight, which detracts from useful lift, (*b*) size and (*c*) speed. Factors neglected affecting efficiency in a wider sense include durability and ease of handling and repair, but such are beyond the scope of our subject.

In the following comparison between the envelope of an airship, the rotor of an autogyro and the wing of an aeroplane, (*a*) is met by referring to definite aircraft, the weights of whose lifting systems are known. Difficulty arises in connexion with (*b*), since data are available for only small autogyros, whilst airships become efficient only at large sizes; as a compromise, the envelope selected is from a recent airship of only $5\frac{1}{2}$ tons gross lift which proved especially serviceable. In respect of (*c*), a familiar characteristic of autogyros is their ability to fly very slowly, but this useful feature is associated with a top speed that is economically too small for aeroplanes. In these circumstances the wing considered is taken from a particularly slow aeroplane.

Perhaps the most generally familiar measure of transport efficiency is miles-per-gallon. However, the load transported is

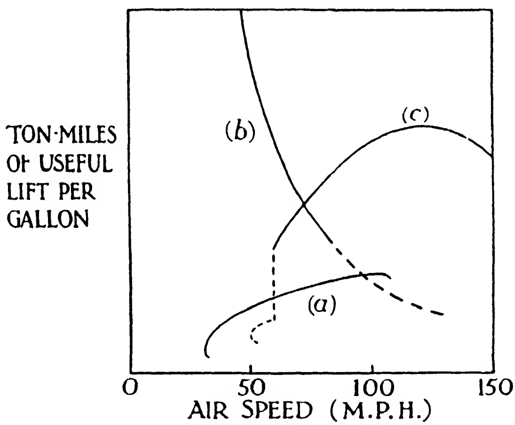


FIG. 8.—COMPARATIVE EFFICIENCIES OF LIFTING SYSTEMS.
(*a*) Autogyro rotor, (*b*) small airship envelope, (*c*) slow aeroplane wings.

equally significant, and its variation is allowed for by substituting ton-miles-per-gallon. Finally, the load reckoned should be useful; i.e. the dead weight should be subtracted from the total load. Thus in Fig. 8 the ton-miles-per-gallon of useful lift are plotted * against the true air speed, defined as the speed in a still atmosphere.

On the basis of the figure the rotor is upon the whole the least efficient, but it is adapted to unique services—e.g. aerial taxi work and private flying of short range—and moreover is still in course of technical development. The wonderful efficiency of

* Further data regarding the figure may be obtained from "Aircraft Efficiencies", a paper read before The Institution of Mechanical Engineers in 1938.

the airship hull at speeds below 60 miles per hour is illusory, since the drift of head-winds must be subtracted in full. At 100 miles per hour, the efficiency of airships and autogyros is of the same order as that of a corridor train with a normal complement of passengers and reasonable luggage. The wing selected surpasses the small hull at 75 miles per hour, but below this speed requires excessive power for its aerodynamic lift. At twice this speed the drag of the wing is largely frictional, depending primarily on the skin area exposed, like that of the airship hull, and its great advantage is due to the comparatively small area of this ‘ wetted surface ’.

With increase of size an airship hull improves aerodynamically, as already described in Article 7. Increase of size at first also improves a wing, owing to better use that can be made of the materials of construction, but once an economical scale has been reached from this point of view the dead weight increases as the cube of the linear size, and the lift only as the square. Thus eventually a limitation to the size of aeroplanes is reached, but this already exceeds 100 tons gross weight. Large aeroplanes and flying-boats employ greater wing-loadings, thus scoring again off the airship by reducing their wetted surface. For much larger sizes than contemplated in the figure, the curves for the hull and the wing still cross in the manner depicted, though at a higher speed.

The conclusion is that, in practical sizes and above a quite moderate speed, the wing easily surpasses in efficiency any other lifting system known.

15. Monoplane Supremacy

Amongst the designs and models of the 19th century will be found monoplane, biplane, triplane and even multiplane wing arrangements. The biplane predominated for three or four years after the first flight, but then the monoplane reappeared, and for a time the two types grew up together.

The English Channel was first crossed by aeroplane in 1909 by Blériot, using a monoplane of 600 lb. weight with a wing-loading of 4 lb. per square foot and driven by an engine of some 24 horse-power. Interest in the achievement has, perhaps, more to do with airmanship and engine reliability than with Aerodynamics.

The monoplane attained in 1911 to a high peak of aerodynamical development in the French 'Pantalette', illustrated in Fig. 9 (*a*). In this small craft will be seen anticipated many features of recent aeroplanes: low wings, widened and thickened at the roots to permit of internal bracing against the bending moments arising from the lift; an enclosed streamlined body or fuselage; and a trousered, or faired, undercarriage. No biplane produced for many years afterwards could bear comparison with this early design, yet a long period soon ensued in which monoplanes fell into disuse.

The same figure shows at (*b*) the 1912 type of the same Antoinette series of monoplanes. It will be seen that the Pantalette

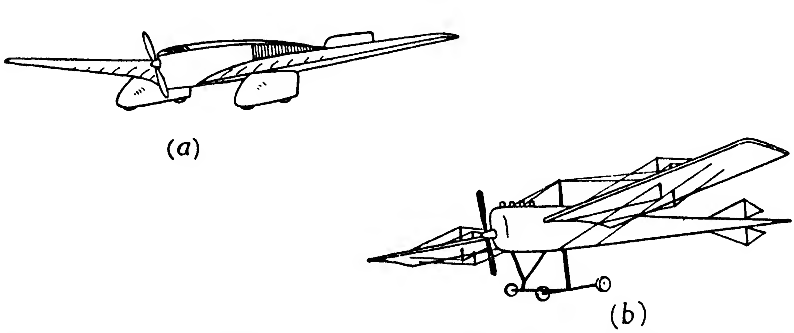


FIG. 9.—(*a*) THE 'PANTALETTE' OF 1911, A STREAMLINED CANTILEVER MONOPLANE OF THE ANTOINETTE SERIES. (*b*) THE ANTOINETTE MONOPLANE OF 1912, WITH EXTERNAL BRACING AND NOSE-WHEEL UNDERCARRIAGE.

was unique, subsequent monoplanes continuing to have their wings supported, as before, by a fan of lift wires radiating from the body or a strong point beneath it. The lengthwise compression of the wings induced by these wires was the principal objection to the monoplane, it being much less severe in the biplane.

Thus the Pantalette was long ahead of its time, and for two reasons. Firstly, there existed no aero-engine of sufficient power to produce a speed that would give effect to the elimination of parasitic drag which the design achieved; the speed range was, in fact, so limited that the drag was largely of that different kind associated with the production of lift. Secondly, in general, wings constructed on the full cantilever principle, i.e. with only internal bracing against bending moments, weigh much more than the externally braced wings of biplanes, and the unavoidable increase of landing speed was prohibitive at that time, when landing grounds were rough and familiarity with high speeds had yet to be attained. Use of full cantilever construction in spite of its large dead weight has since been justified by the much greater wing-loadings which higher landing speeds and landing flaps make possible.

The above considerations, together with an exaggerated predilection for very thin wing sections such as cannot be used in cantilever construction, explain the preference for biplanes during the primary period of aero-engine development. They fail to account for its continuance, however, once engines had become powerful and light, moderately high aircraft speeds a matter of course, and the now familiar landing flaps had been invented. These flaps permitted the wing-loading of a monoplane to be nearly double that of a biplane not so equipped, without landing speed being increased. They thus enabled the two wings of a monoplane to do almost the duty of the four wings of an unflapped biplane, but though invented long previously, they remained practically unused until 1934, improvement of aircraft performance being left largely to the rapidly developing aero-engine. In that year the London–Melbourne race demonstrated the advantage they confer, and flaps were soon incorporated generally. Much smaller wing surfaces became feasible and the reduction of wetted surface brought higher speeds into view. These raised aerodynamical blemishes into prominence, owing to the importance of reducing parasitic drag, and external wing bracing clearly had to be dispensed with. Stripped of this, the biplane is structurally inferior to the monoplane.

The modern preference for the latter is so marked as to make it unnecessary to consider the properties of the biplane in detail, although special purposes may lead to its reappearance in modified form. Triplanes and quadruplanes have received considerable attention in the past without showing to special advantage.

16. Theory and Experiment

It will be seen that though Aerodynamics has sometimes lagged behind the development of aviation, it has often leaped far ahead. Important new scientific theories and inventions have often in the past been pigeon-holed because aviation could not utilise them at the time.

In Aerodynamics the frequent difficulty of calculating results directly from first principles, or with the aid of only a few common coefficients, leads to extensive demands on experiment. During the first twenty years of mechanical flight progress was achieved almost entirely by testing models in wind tunnels. This experimental era, as it may almost be called, was highly successful, revealing many principles and phenomena, amassing data of general utility and, above all, establishing methods by which aeroplanes could be made aerodynamically safe.

A small wind tunnel can be constructed and equipped at little expense, and will re-demonstrate, qualitatively but convincingly, much of this accumulated knowledge. Such an apparatus forms the nucleus of an Aeronautical Laboratory, and will be described in a later chapter.

Unfortunately, Aerodynamics is not especially suitable for the direct experimental method. Designing aircraft from even precise wind-tunnel data calls for costly apparatus and an advanced technique, because large corrections are often required before model experiments can be interpreted in terms of full-scale flight. The process of interpretation calls for aid from theoretical reasoning. More generally, progress in Aerodynamics is most rapid and reliable when theory and experiment join hand in hand.

Chapter II

AEROSTATICS

17. Dry air is a nearly uniform mixture of oxygen, nitrogen and other gases which are chemically indifferent to one another. Though rapidly becoming less dense with increase of altitude, air extends throughout the regions of the atmosphere used for flying. At higher altitudes the heavier gases of the mixture gradually fail to rise until, at some 50 or 60 miles up, hydrogen or helium predominates.

In relation to the Earth, the atmosphere can only be regarded as a skin. Nevertheless its height is sufficient for the air and separated gases vertically above each square foot of the Earth's surface at sea-level to weigh nearly a ton.

A force of nearly a ton weight presses down, therefore, each square foot of the top side of a board held horizontally at sea-level. Yet it cannot be felt, and evidently the board is also pressed upward by an approximately equal force arising from the atmosphere on its lower side. This compensation is made possible by the fact that the forces are caused by fluid pressure.

Fluid pressures arise ultimately from external causes, such as gravity, centrifugal force, heat, or the action of a pump. They are transmitted through the fluid, and so communicated to the surfaces of immersed bodies or enclosing envelopes by molecular agitation.

The sides and edges of a board held in any position in the atmosphere, for instance, are continuously bombarded at short range by the free and fast-moving molecules of the air. If the board be withdrawn, the parts of the gas brought again into mutual contact bombard one another, and the same pressure results across the area of contact as was formerly sustained by each side of the board.

Each small portion of a bulk of gas at rest bombards every adjacent portion, irrespective of direction, maintaining an equal pressure in all directions. Thus a change of pressure arising from an external cause is transmitted in all directions and locally equalised.

A continuous pressure is measured by the force exerted on unit area of a surface over which it is uniformly distributed. Thus if p, P and A denote the pressure, force and area, respectively, $p = P/A$. More convenient means exist, however, for determining the magnitude of a fluid pressure experimentally than that of weighing the force exerted on a material surface of known area. Immersing a material surface in a fluid at rest makes no difference to the fluid pressure, but only defines the magnitude and (as we shall see) the direction of the force arising therefrom on the surface.

These preliminary conceptions can be illustrated by two simple experiments. Let an exhaust pump be connected to a large biscuit tin, or similar box, which has been made air-tight. The crushing forces due to the external atmospheric pressure will soon become apparent on proceeding to remove a proportion of the molecules from within. If the action be arrested in time, the box can be at least partly restored to its original shape by heating, the enclosed molecules making up for their reduced numbers by an increased activity. Now let the biscuit tin be filled with water and a long upright tube be firmly connected to a small hole in its lid. Filling up the tube with additional water will cause the sides, lid and bottom of the box to bulge outward. In this second experiment molecules are not added to those already enclosed, for the liquid is incompressible. But a considerable pressure is applied over a small area at the foot of the tube and is transmitted equally throughout the water in the box, presenting each side of the latter with a large additional bursting force to support.

The pressure is called aerostatic if the fluid is everywhere apparently at rest; that is to say, if no collection of many molecules, such as could be perceived with a microscope, move together. We proceed to investigate the aerostatic pressure. It is important to note that the following laws do not in general apply to fluids which are in motion.

18. The pressure exerted on a material surface by a fluid at rest acts perpendicularly to the surface at every point.

This law is established by the consideration that otherwise the force arising from the pressure on some part of the surface would have a tangential component, and the fluid, being unsupported against a force parallel to the surface, would thereby be caused to move.

It follows that the pressure on a cylindrical or spherical surface acts everywhere radially.

19. The pressure in a fluid at rest is constant at any one horizontal level.

Imagine a narrow cylinder AB, Fig. 10, suspended in a fluid with its axis horizontal, and let its weight be adjusted to have the same value per unit volume as that of the surrounding fluid. Evidently, the fluid being at rest, the cylinder will remain suspended without support or movement.

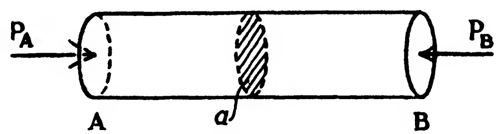


FIG. 10.

Consider its equilibrium in the horizontal sense. Neither the pressure on the cylindrical part of its surface nor its weight can give rise to a horizontal component of force. Hence the horizontal forces P_A and P_B arising from the pressure on the two ends must be equal. The diameter of the cylinder is assumed to be so small that the variation of the pressure over each end can be neglected. Denote these pressures by p_A and p_B and the cross sectional area by a. Then $P_A = p_A \times a$ and $P_B = p_B \times a$, whence, since $P_A = P_B$, $p_A = p_B$.

There is no restriction on the altitude, length or orientation of the cylinder, whence follows the generalisation expressed in the law.

20. The pressure at any point in a fluid acts equally in all directions.

The pressure on an immersed sphere acts in all directions, being radially directed at every point, but it does not act equally at all points, owing to differences in horizontal level. If the sphere be reduced to a minute size, however, all parts of its surface will be sensibly at the same level, and then the pressure will act not only radially in all directions but also equally upon all parts. Eventually the sphere can be regarded as a point.

21. The pressure in a gas at rest decreases upward through a small displacement by the product of the vertical displacement and the local weight per unit volume of the gas.

In Fig. 11 the point A is vertically above the point B and both are situated in a bulk of air or other gas at rest. The pressure is p_A at the horizontal level of A and p_B at that of B. The vertical displacement h is assumed to be sufficiently small for the weight w per unit volume of the gas to be sensibly constant between the two levels.

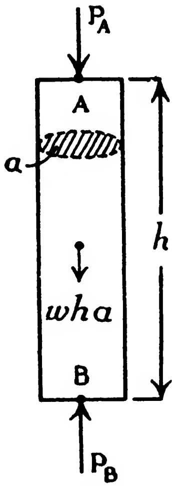


FIG. 11.

Imagine a cylinder of this same weight w per unit volume, height h and cross-sectional area a, disposed with the ends of its axis at A and B. It will evidently remain suspended without movement. The pressure on the cylindrical surface, acting radially and being constant round each cross-section, can give rise to no resultant force, and the cylinder will be in vertical equilibrium under its own weight, wha, and the forces $P_A = p_A a$ and $P_B = p_B a$ caused by the pressures on its ends. Hence

$$wha + p_A a = p_B a$$

i.e.,

$$p_A = p_B - wh \quad . \quad . \quad . \quad . \quad (1)$$

The restriction that h must be sufficiently small for this expression to apply to a gas is necessitated by the fact that the weight of upper portions compresses lower portions, increasing their weight per unit volume, so that w is not constant, but decreases upward. Little error arises with vertical displacements that do not greatly exceed 100 feet. Thus (1) can be applied to the air in a moderately high building and to the gas in a balloon or airship. But the expression is inapplicable to large changes of altitude in the atmosphere.

22. The expression (1) applies without restriction to liquids at rest on account of their incompressibility. The change of pressure for a given vertical displacement is the same for all depths below the free surface of the liquid.

It follows that if an additional pressure is applied to any part of a bulk of liquid it will be transmitted equally to all parts. For the pressure is constant at the horizontal level of the point of application, and so we first note that the pressure is increased uniformly at that level as it would be in case of a gas. But since the weight per unit volume of the liquid is sensibly independent of the pressure (unlike that of a gas), the change of pressure between this and any other level remains the same, being equal to the product of the weight per unit volume and the vertical displacement. Hence the same increase of pressure occurs at the second level as at the first.

In particular, the atmospheric or other gas pressure acting on the free surface of a liquid is transmitted equally to all parts. Thus the pressure at a point distant z below the free surface of a liquid is equal to the sum of the pressure acting on its free surface and the product of the weight per unit volume of the liquid and the depth z of the point.

Example 1.—Assuming the atmospheric pressure to be 2116 lb. per sq. ft. at sea-level and air to weigh 0·0765 lb. per cu. ft. estimate the pressure at an altitude of 200 ft.

Immediately from (1) the pressure required is

$$2116 - 0{\cdot}0765 \times 200 = 2101 \text{ lb. per sq. ft.,}$$

approximately.

Example 2.—A tube 10 ft. high is filled with water weighing 62·4 lb. per cu. ft. What is the pressure tending to burst the tube at its foot?

There is no bursting pressure at the top of the tube. The increase of water pressure between the free surface and the foot of the column $= 62{\cdot}4 \times 10 = 624$ lb. per sq. ft. A corresponding increase in the external atmospheric pressure, amounting to about $\frac{3}{4}$ lb. per sq. ft. from the previous example, is comparatively so small that it can be neglected. Hence the bursting pressure can be given as 624 lb. per sq. ft. This would be the same whatever the inclination of the tube to the vertical provided its height was the same.

It will be realised that the pressure in the water at the foot of the column, and therefore the pressure that acts on the interior surface of the tube in this region, is not the above but $624 + 2116 = 2740$ lb. per sq. ft.; but the atmospheric pressure transmitted through the water is slightly more than compensated by the pressure of the air external to the tube at its base.

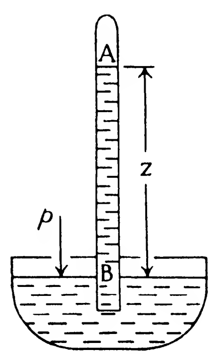


FIG. 12.

23. Pressure Head

Fig. 12 represents the familiar apparatus of a glass tube some 3 feet long sealed at the upper end and open at the lower end, which is immersed in a basin of mercury. Before inversion, the tube is filled with mercury, which is allowed to run out into the basin as much as it will against the atmospheric pressure p, leaving a vacuum above the free surface A.

The pressure at B, any point within the tube level with the surface of the mercury in the basin, is equal to p. But, if B is at a depth z below A and the mercury weighs w lb. per unit volume, the pressure at B is equal to wz. Hence $p = wz$.

Though variable, the height of the column of mercury supported at sea-level is found to be about $2\frac{1}{2}$ feet, and mercury weighs 850 lb. per cubic foot, approximately. These numbers give $p = 850 \times 2\frac{1}{2} = 2125$ lb. per square foot. A somewhat less value is adopted as standard, viz. that appropriate to a 'barometric height' of 760 millimetres of mercury at a temperature of 15° C. At this temperature mercury weighs 13·593 grams per cubic centimetre, giving for the standard pressure: $13{\cdot}593 \times 76 = 1033$ grams per square centimetre $= 2116$ lb. per square foot.

Water weighs 62·4 lb. per cubic foot, approximately. So if water took the place of mercury in the above experiment the height of the column supported would be, under standard conditions, $2116 \div 62{\cdot}4 = 34$ feet, nearly.

The height of a column of specified liquid which a given pressure will support is called the *pressure head*. Measuring the pressure head is a convenient method of determining the magnitude of a moderate fluid pressure, water being substituted for mercury when the pressure is light. Experiments in Aerodynamics are often concerned with differences of pressure, and these are assessed in much the same way, as illustrated in the following example.

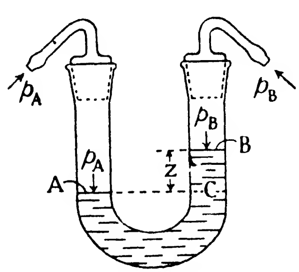


FIG. 13.

Example 3.—A U-tube, Fig. 13, is partly filled with water, and unequal pressures are transmitted to the free surfaces by the air above them, causing a head of 5 inches of water. What is the difference between the pressures?

In the figure, C is at the same horizontal level as A, and therefore the pressure at C = that at A = p_A. But the pressure at C $= p_B + wz$

$$= p_B + \frac{5}{12} \times 62{\cdot}4 = p_B + 26 \text{ lb. per sq. ft.}$$

Therefore the pressure difference $p_A - p_B = 26$ lb. per sq. ft.

24. Pressure Gauges

The pressure differences of experimental Aerodynamics are often small, and various modifications of the familiar U-tube gauge of Fig. 13 are used to increase accuracy of measurement. Two simple adaptations are shown at (*a*) and (*b*) in Fig. 14.

Referring to (*a*), let θ be the angle between the inclined limb and the horizon and l the displacement of the meniscus B of the

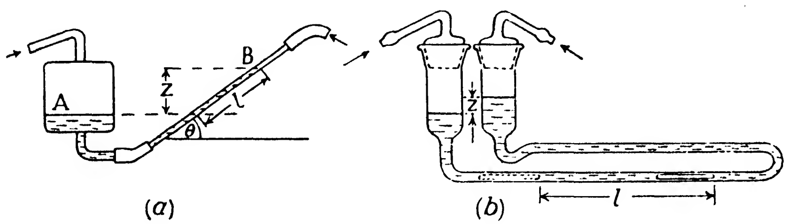


FIG. 14.

liquid along this limb due to the air pressure applied to the wide liquid surface A exceeding that above B. Provided the area of B is very small compared with that of A, as illustrated, the displacement of A associated with the rise of B will be negligible. The head $z = l \sin \theta$ and for $\theta = 30°$, for example, the displacement of B is twice the head, providing a greater length for measurement. But θ must be determined accurately by means of a protractor and spirit level.

In the gauge illustrated at (*b*) in the same figure an air bubble is used to indicate the displacement of liquid along an approximately horizontal tube of small bore connecting two wide, equal pressure chambers. If l is the displacement of the bubble caused by a head z, l may be many times greater than z, for $l = z \times$ the ratio of the area of the free surface of the liquid in each pressure chamber to the cross-sectional area of the connecting tube. This ratio can be determined experimentally.

Both these gauges are preferably filled with methylated spirit or a similar liquid. The bubble of (*b*) takes a considerable time to reach its proper position, and the narrow tube must be reasonably clean. A convenient attachment to (*a*) is a lens that is slideable along a scale fixed parallel to the sloping tube.

Manometers of many other and more accurate types exist, some permitting a head of less than one-thousandth of an inch of water to be detected with ease.

The Chattock gauge is illustrated schematically in Fig. 15. It comprises a modified U-tube AB carried on a frame F which can be tilted about pivots P by means of the micrometer screw S. The U-tube is filled to the level L with a saline solution, which also completely fills the central and enclosed tube T. The closed vessel surrounding T is filled up with castor oil. A water–oil meniscus M, or bubble, is formed at the open mouth of T and is kept under observation through a microscope attached to F. An excess of pressure in A tends to bubble water through the oil from A into B, but is prevented from doing so by tilting F through a small angle as indicated at (*b*) in the figure. The amount by which the water level in B must be raised above that in A to prevent any deformation of the bubble relative to cross-hairs in the eyepiece of the microscope gives the head *z* required to balance the pressure difference between A and B. Its precise measurement is effected by noting the number of graduations through which it is necessary to turn a micrometer wheel integral with S, complete revolutions being indicated on a scale at the side.

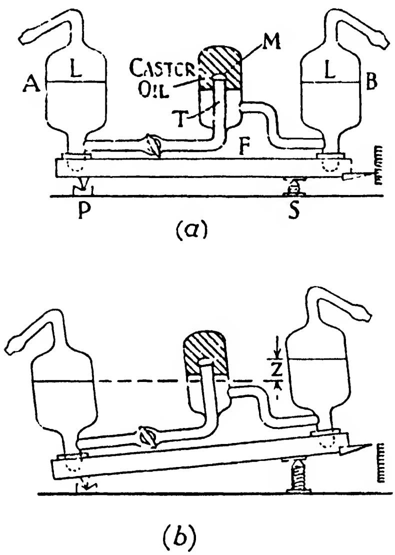


FIG. 15.—CHATTOCK GAUGE.

In another type a long, plain U-tube is pivoted midway between the pressure chambers so that it can swing freely, like a scales. Water is allowed to run from the one pressure chamber into the other and the amount displaced is weighed, whence the pressure head causing the displacement can be calculated from a knowledge of the diameters of the pressure chambers.

Liquid gauges are unsuitable for use on aircraft for several reasons, of which the most important is that the aircraft is subject to accelerations which would apparently vary the weight per unit volume of the liquid. Pressure differences occurring in flight are usually large enough to be indicated with sufficient accuracy by a diaphragm instrument working on the principle of the aneroid barometer. An example is the familiar air-speed indicator.

25. Static Lift at Constant Altitude

Referring again to Article 21 and Fig. 11, if the cylinder there considered were thin-walled and weightless, and could be completely exhausted of fluid without collapse, a load equal to wha would evidently require to be attached to balance the upthrust due to the resultant pressure force $P_B - P_A$. But wha is equal to the weight of the fluid displaced by the cylinder. Thus the upthrust due to the pressures, called the buoyancy, is equal to the weight of the fluid displaced by the cylinder.

This result can be seen in several ways to apply to a body of any shape immersed in any fluid, and is then known as the Principle of Archimedes. Thus we note, for instance, that the volume of a shaped body may be made up by a large number of cylinders of different sizes so that the sum of their volumes is equal to the volume of the body. Now, the quantity $wha = w \times$ the volume of the cylinder and applies to each of the cylinders and therefore to all added together. Hence the buoyancy of the body $= w \times$ the volume of the body, provided w may be assumed constant.

A balloon makes a large proportion of the above buoyancy available as useful lift by employing a light gas imprisoned in an approximately spherical envelope to displace a suitable weight of air. The pressure of the gas within the envelope is approximately the same as that of the surrounding air, relieving the envelope of any appreciable bursting pressure, so that its fabric need not be strong and will weigh little. Neglecting the weight of the fabric altogether gives for the static lift: the weight of the air displaced less the weight of the gas.

26. The above leads to simple formulæ for the static lift L exerted by a gas-filled envelope of volume V'.

Writing W for the weight of air displaced and W' for the weight of the gas,

$$L = W - W',$$

neglecting the weight of the envelope. But if w and w' are the weights per unit volume of the surrounding air and the gas, respectively, $W = wV'$ and $W' = w'V'$, and the above formula gives

$$L = W\left(1 - \frac{w'}{w}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

At equal pressures and temperatures the value of w'/w is 0·0695 for hydrogen and 0·138 for helium. Hence

$$L = 0{\cdot}9305\, W \text{ for hydrogen}$$

and

$$L = 0{\cdot}862\, W \text{ for helium.}$$

Again, substituting wV' for W in the last formulæ gives for the volume of gas required to secure a given static lift L,

$$V' = \frac{L}{0{\cdot}9305\, w} \text{ for hydrogen}$$

and

$$V' = \frac{L}{0{\cdot}862\, w} \text{ for helium.}$$

Thus using helium instead of hydrogen increases the volume required in the ratio 0·9305/0·862 = 1·08, nearly. The increase of 8 per cent. is a small matter in consideration of the non-inflammable properties of the heavier gas.

The numbers given refer to the gases in the pure state. Contamination soon occurs by diffusion through the envelope, leading to a less lift for a given volume, or the need for a greater volume to provide a given lift.

Example 4.—Find the diameter of a spherical balloon filled with pure hydrogen for a lift of 1 ton at 10,000 feet altitude, where the air weighs 0·0565 lb. per cu. ft.

$$V' = \frac{2240}{0{\cdot}9305 \times 0{\cdot}0565} = 42{,}600 \text{ cu. ft., approximately.}$$

The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$. Equating to the value found for V' leads to

$$r^3 = \frac{3}{4\pi} \times 42{,}600,$$

whence

$$r = 21 \text{ ft. } 8 \text{ in.},$$

i.e. the diameter required is 43 ft. 4 in.

27. Unit Density

Units and dimensions are discussed in Chapter IV. As there mentioned, the units of mass, length and time are initially open to separate choice. In engineering generally, the lb.–foot–second system is employed. A different choice is made in Aeronautics and Fluid Mechanics regarding the unit of mass, the *slug* being preferred to the lb. The slug is the mass of a body weighing g lb. As will be apparent from the foregoing, the lb. is a convenient unit of mass in Aerostatics, but this is not the case in Aerodynamics, where the slug saves much useless labour.

Whilst retaining the lb. throughout Aerostatics would effect some slight simplification, the formulæ noted in the remainder of this chapter will, on the other hand, be referred to in Aerodynamics. To avoid confusion, therefore, the slug is introduced forthwith.

The density of a substance is defined as its mass per unit volume. In contrast with liquids, the density of a gas depends acutely upon pressure and temperature. Thus even at a given altitude the density of atmospheric air changes continually with the weather, and a numerical value must refer to specified conditions. The conditions accepted as standard for sea-level assume the mercury barometer to read 760 mm. and the thermometer 15° C., when air weighs 0·0765 lb. per cubic foot, as already mentioned. Its density in aerodynamical units under these standard conditions is thus $0{\cdot}0765/g$ slug per cubic foot, g being the acceleration due to gravity, which we accept at 32·2 feet per second per second, neglecting a small variation with altitude and latitude.

Air density is denoted by the Greek letter ρ (rho) and its standard sea-level value is distinguished by adding the suffix 0. The following should be remembered :

$$\rho_0 = 0{\cdot}00238 = \frac{1}{420} \text{ slug per cubic foot} \quad . \quad . \quad (3)$$

With increase of altitude the air becomes cold and its pressure decreases, while the density diminishes nearly as rapidly. This variation is clearly of great significance in Aerodynamics, but before discussing it we have to review the laws governing the expansion or compression of air, which behaves in this connexion like a perfect gas.

D

28. Equation of State

The pressure, density and temperature at any position in a bulk of gas are related to one another by the equation

$$\frac{p}{\rho} = gB\tau \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

In this important equation, p is the pressure in lb. per square foot; ρ is the density in slugs per cubic foot; τ (the Greek letter tau) denotes the absolute temperature in degrees Centigrade, obtained by adding 273 to the reading of a Centigrade thermometer; and B, the 'gas constant', is an absolute constant for each gas, having the values 96 for air, 696 for helium and 1381 for hydrogen, the units being feet per degree Centigrade.

Example 5.—Assuming that $p = 2116$ lb. per sq. ft., what is the density of air at a temperature of 15° C.?

We have $\tau = 15° + 273° = 288°$ C. Hence (4) gives

$$\rho = \frac{p}{gB\tau} = \frac{2116}{32{\cdot}2 \times 96 \times 288}$$
$$= 0{\cdot}00238 \text{ slug per cu. ft.,}$$

in agreement with (3).

Example 6.—If conditions at sea-level are standard, what is the density at an altitude of 20,000 ft., given that the pressure there is reduced by 54 per cent. and the temperature by 39·6° C.?

With suffix 0 distinguishing sea-level values, (4) gives

$\rho_0 = p_0/gB\tau_0$ at sea-level and $\rho = p/gB\tau$ at the altitude.

Dividing the second expression by the first leads to

$$\frac{\rho}{\rho_0} = \frac{p}{p_0} \cdot \frac{\tau_0}{\tau}.$$

By the question, $p/p_0 = 0{\cdot}46$ and $\tau_0/\tau = 288/248{\cdot}4$, whilst ρ_0 is known. Hence

$$\rho = 0{\cdot}46 \times 1{\cdot}16 \times 0{\cdot}00238$$
$$= 0{\cdot}00127 \text{ slug per cu. ft.}$$

29. Boyle's and Charles' Laws

The relationship (4) is obtained by combining the Laws of Boyle and Charles. Boyle's Law states that if the pressure changes while the temperature remains constant, the density will change in direct proportion to the pressure. It can therefore be written

$$p = \text{constant} \times \rho \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Charles' Law states, in effect, that if the temperature changes while the pressure remains constant, the density will vary inversely as the absolute temperature since the coefficient of expansion is approximately the same (viz. 0·00366 = 1/273) for all ordinary gases and temperatures.

The first is sometimes called the isothermal law, since the temperature remains constant; it will be found to have a special interest in connexion with high altitudes. But otherwise the two laws are more useful as combined in the Equation of State, which facilitates calculation of the pressure, density or temperature of a given gas for which B is known, once two of the quantities have been determined.

30. Adiabatic Law

Expansions and compressions take place in the gas when a balloon or airship rises or descends, and in atmospheric air during a change of altitude or the passage of an aircraft nearby. Isothermal adjustments of pressure and density have only a restricted interest in our subject. For when a gas is compressed it becomes warmed, as is familiar in the use of the tyre pump, gaining an amount of heat equivalent to the work done during compression. Similarly, an expansion causes a loss of heat, resulting in a fall in temperature. Hence, for the action to be isothermal, heat must be conducted away from the gas in the first case and supplied to it in the second. The thermal conductivity of gases is poor, and such transference of heat takes time.

Usually, expansions or compressions occur so quickly that transference of heat during the process can be neglected altogether. In these circumstances the pressure and density are related to one another by the Adiabatic Law:

$$p = \text{constant} \times \rho^{1 \cdot 4} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

This law should be compared with Boyle's, from which it differs widely. It indicates that in an adiabatic compression, for example, the increase of density for a given change of pressure is much reduced by the gas holding its heat. Similarly, in an adiabatic expansion the decrease of density is reduced by drop of temperature. These effects are of particular interest in Aerodynamics. It is a convenience to note that $1{\cdot}4 = 7/5$.

Example 7.—If the pressure of a gas is halved by an adiabatic expansion, how is its density changed?

Denoting initial conditions by suffix 1 and final conditions by suffix 2, we have

$$p_1 = \text{constant} \times \rho_1{}^{7/5} \text{ and } p_2 = \text{constant} \times \rho_2{}^{7/5}.$$

Dividing the second expression by the first gives, since the constant cancels,

$$\frac{\rho_2}{\rho_1} = \left(\frac{p_2}{p_1}\right)^{5/7}.$$

So if $p_2/p_1 = \frac{1}{2}$, taking logs:

$$\log \frac{\rho_2}{\rho_1} = \frac{5}{7} \log \tfrac{1}{2} = \bar{1}{\cdot}785,$$

whence $\rho_2/\rho_1 = 0{\cdot}61$. This ratio would have been $\frac{1}{2}$ had the expansion been isothermal.

The Equation of State at once gives the change of temperature accompanying adiabatic expansion or compression. If the pressure changes from p_1 to p_2 and the density from ρ_1 to ρ_2, the absolute temperature will change from τ_1 to τ_2 according to:

$$\frac{\tau_2}{\tau_1} = \left(\frac{\rho_2}{\rho_1}\right)^{2/5} = \left(\frac{p_2}{p_1}\right)^{2/7} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Example 8.—In being pushed out of the way by a fast aeroplane at low altitude, some atmospheric air momentarily loses one-quarter of its pressure. How cold does it become?

$p_2/p_1 = \frac{3}{4}$. τ_1 may be assumed to be $15° + 273° = 288°$.

Hence $\tau_2/288 = (\frac{3}{4})^{2/7}$

or $\log (\tau_2/288) = (2/7) \log \frac{3}{4} = \bar{1}{\cdot}963,$

whence $\tau_2 = 265°$ C., or 8° C. below the temperature of ice.

The question is numerically practical; the diverted air would pass close over the top of the wings of the aeroplane; and so it is seen that temperature in that region may be quite cold.

31. Structure of the Atmosphere

The atmosphere, so far as flying is concerned, consists of two dissimilar parts. A lower layer, called the *troposphere*, extends from sea-level to altitudes which vary from 4 miles at the poles to 9 miles at the equator. In temperate latitudes the height of the troposphere is about 7 miles. This layer is characterised by storms and vertical currents of air which induce, upon the average, an approximately constant fall of temperature with increase of altitude. Above the troposphere lies the *stratosphere*, so called because vertical winds gradually cease, leading to a stratified condition and eventually to the separation of the various gases of which air is composed, the light gases mounting high and the heavy ones remaining at lower levels. Only the lower levels of the stratosphere are of aeronautical interest, and there the composition of the air may be taken as standard. Through these lower levels the stratosphere is characterised, in contrast with the troposphere, by a more or less uniform temperature, changing little, that is to say, from one altitude to another. Therefore, it is often called the isothermal part of the atmosphere. Between $-50°$ and $-58°$ C. appears to be representative of the constant temperature. No sudden break occurs, of course, between the troposphere and the stratosphere; the two layers merge gradually the one into the other.

Regular flying in the stratosphere has only recently begun. The very low pressure necessitates air being pumped to the passengers and engines and the cabins being sealed and strengthened against bursting. Again, the low temperature calls for artificial heating, and altogether the air-conditioning apparatus tends to be weighty. Nevertheless, aviation in this upper layer appears to have a bright future for several reasons, amongst which may be mentioned at once an ability to fly over the tops of storms.

The troposphere remains of the greater immediate interest, and the precise manner in which the pressure, density and temperature of the air change through it from one altitude to another is by no means always the same. The performance of an aircraft depends on these quantities, and so will vary from day to day, and even from hour to hour, though the altitude of flight be constant. To judge of the capabilities of a given aircraft it is consequently necessary to correct the performance observed with casual values of the above quantities to what it would become under average, or at least specified, conditions.

To this and other ends, a certain *standard atmosphere* has been adopted internationally for the troposphere. It represents average conditions over Western Europe, so that its consideration presents a fair picture of the structure of the troposphere in this region whilst ignoring the vagaries that occur from time to time.

32. The Standard Atmosphere

The drop in temperature per 1000 feet increase of altitude, which is the same for all heights within the troposphere, is called the *temperature lapse rate*. In the standard atmosphere the temperature at sea-level is taken as 15° C. and the lapse rate as 1·98° C. Thus at 30,000 feet altitude, for example, the standard temperature is $15° - 30 \times 1{\cdot}98° = -44{\cdot}4°$ C. It can be shown that this lapse rate is consistent with the following relationship between the pressure p and the density ρ :

$$p = \text{constant} \times \rho^{1\cdot235} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

which should be compared with (5) and (6).

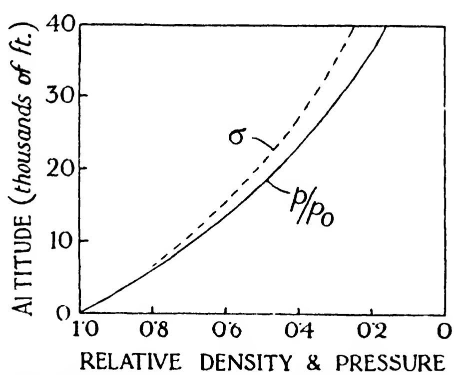


FIG. 16.—RELATIVE DENSITY AND PRESSURE IN THE STANDARD ATMOSPHERE.

Fig. 16 indicates how p and ρ vary with altitude. For convenience there are introduced the terms *relative pressure* and *relative density*. The first is defined as the ratio of the pressure at any altitude to the standard pressure at sea-level, which is 2116 lb. per square foot. Similarly, the second is defined as the ratio of the density at any altitude to the standard density at sea-level, which is 0·00238 slug per cubic foot. The relative density is in such frequent use that it is given a special symbol, the Greek letter σ (sigma). It will be noticed from the figure that the density at 30,000 feet altitude has only three-eighths of its value at sea-level and that the pressure falls off even more rapidly. Some accurate numbers are given in Table I.

TABLE I

The Standard Atmosphere

Altitude (ft.)	Temperature (° C.)	p/p_0	$\rho/\rho_0 = \sigma$
0	15·0	1·000	1·000
5,000	5·1	0·832	0·862
10,000	− 4·8	0·688	0·738
15,000	−14·7	0·564	0·629
20,000	−24·6	0·459	0·534
25,000	−34·5	0·371	0·448
30,000	−44·4	0·297	0·375
35,000	−54·3	0·235	0·310
40,000	−54·3	0·185	0·244

33. The Ceiling of a Balloon or Airship

The formula (2) of Article 26 for the static lift L of a balloon or airship can be re-written in the form

$$L = W'\left(\frac{w}{w'} - 1\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

W' denotes the weight of the gas and w/w' is equal to the ratio of the density of air to that of the gas, which is now seen to be constant if both are at the same pressure and temperature.

It follows that the lift of a balloon remains the same at all altitudes provided (*a*) no gas is lost, (*b*) the pressure and temperature of the gas remain sensibly the same as those of the surrounding air. The fabric of a gas-bag is not strong enough to support any appreciable bursting pressure. Hence the second condition reduces to that of equality of temperature. This condition is seldom satisfied in practice, but for the present we shall assume it to be fulfilled. It then follows that on ascent the gas expands in just the same way as does the surrounding air. The greater and greater volume that a given mass must occupy is provided for by only partly filling the envelope at sea-level and leaving an open vent at the bottom. But eventually the gas will fill the envelope completely, so that it becomes taut, and then the balloon can ascend no higher without losing gas and, therefore, lift. Thus the ceiling of a balloon is decided by the extent to which the envelope can be left limp at sea-level.

Example 9.—An observation balloon is required to ascend to 30,000 ft. What reserve capacity must the envelope possess at sea-level?

For constancy of lift, and equality of temperature between the gas and surrounding air, the equation (2) gives

$$W = g\rho V' = \frac{\text{Lift}}{1 - \frac{w'}{w}}, = \text{constant},$$

i.e., the volume of the gas is inversely proportional to the density of the air, and therefore to its relative density σ. Between 30,000 ft. and sea-level, Table I shows that σ increases 8/3 times. Therefore the volume of the gas decreases in this proportion; i.e., to 3/8ths of its volume at the high altitude. Hence 5/8ths of the capacity of the bag must be reserved at sea-level for expansion.

34. Why a Balloon Maintains Altitude

A balloon is not in the circumstances of a ship, floating only partly immersed in its supporting fluid. Indeed, the lift is the same for all altitudes below the ceiling, provided no gas is lost and the temperatures of the gas and surrounding air remain equal. While this condition holds, equilibrium is always maintained between the lift and the weight supported, and at what height the balloon would ride without continual adjustment by jettisoning ballast or valving gas is quite uncertain. But the thermal conductivity of gases is so poor, as already mentioned, that heat is transferred only slowly through the envelope. For this reason the assumption of equal temperatures within and outside is artificial, and we shall now see that an important consequence is to give the balloon a measure of stability in regard to vertical displacement, so that it tends to ride at constant altitude.

To fix ideas, assume that a balloon has been kept at a certain altitude sufficiently long for equality of temperatures to have been established, and enquire into the effect of then suddenly decreasing its altitude. The gas is compressed by the increased pressure of the atmosphere at the lower altitude, and the temperature rises in accordance with (7), for it is evident that no appreciable amount of heat will be lost during the quick compression, which will therefore accord closely with the Adiabatic Law. But Fig. 17 shows that the balloon is now surrounded by

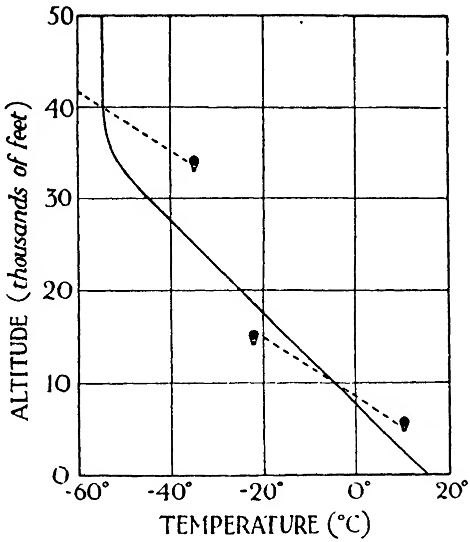


Fig. 17.

——— Mean atmospheric temperature.
- - - - Temperature of gas in quickly displaced balloon.

warmer air than before. Which is the hotter at the lower level, the air or the gas? An answer to this question is most clearly provided by considering any numerical example.

Example 10.—A balloon riding initially at 10,000 ft. altitude descends quickly through 5000 ft. What is the temperature of the gas immediately after the compression?

Let properties of the gas be distinguished at the first altitude by suffix 1 and at the second by suffix 2. Then p_1, τ_1 and p_2 are all given by Table I, for they have the same values as those of the surrounding air. Inserting these values in (7) gives approximately, since $p_2/p_1 = 0{\cdot}832/0{\cdot}688, = 1{\cdot}21$, nearly, and $\tau_1 =$ 268·2° C.,

$$\tau_2 = (1{\cdot}21)^{2/7} \times 268{\cdot}2 = 283{\cdot}2^\circ \text{ C.},$$

showing a rise in temperature of 15° C. But the corresponding rise in the temperature of the atmosphere is only 5 × 1·98 =

9·9° C. Hence immediately after the descent the gas is more than 5° C. warmer than the surrounding air.

Such examples verify that displacing a balloon vertically changes the temperature of its gas much more than the temperature of the atmosphere changes in respect of altitude, as illustrated in Fig. 17. The increased temperature immediately after descent causes the gas to occupy a larger volume than it would do at the temperature of the surrounding air; a greater weight of air is displaced and the static lift increases, becoming greater than the weight of the balloon. Therefore, the balloon is accelerated upwards and tends automatically to regain the altitude from which it was forced down. Similarly, a balloon whose altitude is quickly increased loses lift and, if no ballast has been thrown overboard, descends again to its original level. We conclude that a balloon, unless heated by the sun, tends to maintain altitude so long as the state of the atmosphere at that altitude remains constant.

35. Stability of the Atmosphere

The stability of a balloon in respect of vertical displacement has been considered at some length because the same considerations apply to bulks of air in the atmosphere, and the same conclusions are reached and have a wider significance than in application to balloons and airships. Local heating produces upward and downward currents in the atmosphere, but only, as a rule, of a gentle character easily negotiated by aircraft. Upward currents, indeed, are naturally helpful to flying except when they occur in the form of gusts. Otherwise, large or small bulks of air, forming part of the atmosphere, tend to remain at constant altitude for the same reason as does a balloon. Briefly stated, the reason is that the pressure through the troposphere varies with the density raised to a power whose index is substantially less than 1·4, being in the neighbourhood of 1·235, upon the average, over Western Europe. If this were not so, the atmosphere would be liable to great upward or downward surges of air, which would make flying impossible. It follows without further calculation that the isothermal reaches of the stratosphere are still more stable than is the troposphere.

Chapter III

THE NATURE OF FLOW PAST BODIES

36. A BODY of small height immersed in stationary air is exposed to a practically uniform pressure which acts everywhere perpendicular to its surface. For ordinary volumes the buoyancy, equal to the weight of air displaced, is negligibly small. Ignoring this, there is no resultant force due to the fluid.

If the body is set in motion through the air, the pressure to which it is subjected changes in magnitude, increasing on some parts of the surface and decreasing on others, whilst everywhere deviating from its original direction. Resolving the pressure acting at any point into components normal and tangential to the surface, the first component is called the *normal pressure* and the second the *intensity of skin friction*. Thus in Fig. 18 PS, PS represent the equal and normal static pressures of still air acting at any points P, P on the surface of a stationary body, whilst PR, PR indicate the unequal values and inclined directions of the pressures acting at the same points when the body is in motion in the direction of the arrow V. The new pressures PR give rise to unequal normal pressures PN and intensities of skin friction PF.

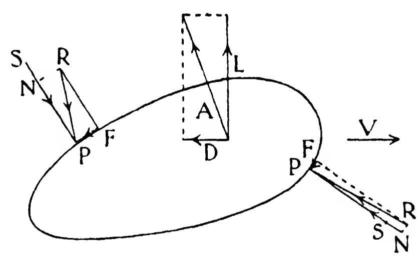


FIG. 18.

37. Aerodynamic Force

The unequal distribution of pressure over the body yields a resultant force, due to the motion through the fluid, called the *aerodynamic force*. Its magnitude, direction and line of action depend on a number of factors having to do with the shape, size, attitude and speed of the body, as well as the physical state of the air. It may be directed wholly backward, constituting simply a resistance to the motion, or, at the other extreme, nearly perpendicularly to this direction.

For most purposes, the aerodynamic force A, Fig. 18, is resolved into components D and L, parallel and perpendicular, respectively, to the direction of motion. The first component is called the *drag* and the second usually the *lift*, but in some circumstances the *cross-wind force*.

38. Direction of Lift

Interest in the lift of a body centres as a rule in its effect on the lift of an aeroplane of which it will form part. The direction of lift is then defined as perpendicular to the direction of motion of the aeroplane and the line joining its wing-tips. Thus aerodynamic lift is not necessarily vertical, as is static lift, nor even directed vaguely upward. If an aeroplane is climbing or gliding along a flight path inclined at a certain angle to the horizon, its lift is inclined backward or forward, respectively, by the same angle from the vertical. If it is turning, so that the outer wing-tip is higher than the inner one, the lift is inclined inwardly from the vertical towards the centre of the turning circle.

Only in straight level flight, and when any wind that may be present is blowing horizontally, is the lift vertical. The lift is reckoned positive in these circumstances if the aircraft is the right way up; if the aircraft is flying straight and level but upside down, there must evidently exist an upward component of aerodynamic force to balance the weight, but the lift is negative.

Any cross-wind force on an aircraft is perpendicular to the lift and the drag; examples occur when an aeroplane side-slips or its rudder is used. However, the terms lift and cross-wind force are often employed alternatively in a less stringent manner to denote a component perpendicular to the direction of motion of the aerodynamic force arising on an isolated body.

39. Aerofoils

Any body which is asymmetric in form or moves at an inclined attitude will generate a lift (or cross-wind force). For example, an aerodynamic lift can be added to the static lift of an airship hull by holding its tail down during level motion; the only reason for not doing so, indeed, is that such an extra lift is preferably kept in reserve to maintain a change of altitude until the gas adjusts its temperature, and as a margin of safety against loss of gas.

Wings, however, have been intensively developed in shape to generate lift with a minimum of drag consistently with other common requirements, aerodynamical and structural. They are studied in wind tunnels by means of scale models, called aerofoils. Aerofoil investigations are fundamental also to the shaping of airscrews and the rotors of autogyros and helicopters, all of which may be described as twisted aerofoils.

A unique aerofoil shape for all purposes is impracticable. Good aerofoils do not differ widely from one another in lifting capacity, but chiefly in the attitude, called incidence, required to produce a given lift under set conditions. Their shapes and properties are discussed in later chapters. For the present it is sufficient to note that an aerofoil experiences lift almost entirely as a modification of its normal pressures; skin friction over the aerofoil surface affects lift indirectly, but its direct contribution is negligible.

40. The study of drag is more complicated than that of lift. It arises in general from both the normal pressures and skin friction, and varies greatly from one shape of body to another.

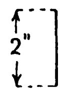
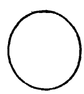
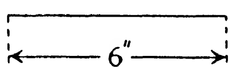
Table II gives in the third column the drags in lb. per 10 feet run of a number of long strips or cylinders, of the sections shown, which are supposed to be moving in the direction from left to right of the table at a velocity of 150 feet per second through still air in the standard sea-level state. All except the flat plates have the same maximum width of section, viz. 2 inches. (*b*) represents the normal plate (*a*) turned through a right-angle; (*e*) represents the symmetrical aerofoil (*d*) reduced to small thickness. Uniformly changing size or speed would alter each drag differently, and there would still remain an enormous variation of drag from one shape to another.

41. The Three Kinds of Drag

It is convenient to distinguish between three different kinds of drag—viz., *induced drag, form drag* and *skin friction.* A body may experience one, two, or all three.

Induced drag has already received some attention in Article 13, and further discussion is deferred until later chapters. It arises solely from the continuous generation of aerodynamic lift under three-dimensional conditions, as by a wing, and is non-existent

TABLE II

	Section of Strip	*D (lb)	Skin Friction
(a)		86	0
(b)		0·3	100%
(c)		53	5%
(d)		3	40%
(e)		0·5	100%

*D = Drag (lb) of 10 ft. run resisting motion through still air at a velocity of 150 ft. per sec.

(Further particulars relating to the drag of long strips and cylinders having the sections shown in Table II are given in the following articles :—

Normal plates, Articles 69 and 102.
Flat plates, Articles 69 and 119.
Circular cylinders, Articles 46, 69 and 101.
Streamline cylinders, Articles 43 and 102.)

in the absence of such lift. It is associated with a modification of the normal pressures on the lifting body.

The generation of 3-dimensional lift does not account for the whole of the normal pressure variation arising over the surface of a body in motion. If the attitude of the body be such as to generate no lift, the normal pressures still, in general, yield a drag, called the form drag. Thus form drag is that part of the drag resulting from the normal pressures that is not due to the generation of lift under three-dimensional conditions.

Skin friction is caused by the tractions along the surface of a body, assuming such small roughness as it may possess to have no effect, when the body is said to be *aerodynamically smooth.* At high speeds, skin friction is considerably augmented by slight roughness and is then called *skin drag.*

42. Limiting Cases

Column 4 of Table II gives approximately for each strip or cylinder the fraction of the total drag due to skin friction. The remainder is form drag, induced drag being absent throughout.

Tractions exist along the surfaces of the normal plate (*a*), but cannot contribute to drag, since they act at right angles to the direction of motion. The large form drag arises from an increase of normal pressure on the face of the plate and a reduction on the back; it is put to practical use in aeroplane landing flaps considered as air-brakes. A still larger drag is experienced by a U-shaped strip arranged to cup the air, and finds three-dimensional application in the parachute. But usually the need is to abate drag.

The drags of the flat plates, which are supposed to be very thin and sufficiently smooth, are pure skin frictions. Actually, the normal pressure of the air is hardly modified by the passage of such a plate, but it could not contribute to drag in any case. For shapes between the two extremes of (*a*) and (*e*) the drag is partly form and partly frictional.

43. Streamlining or Fairing

The very small drag of (*b*) compared with that of (*a*) suggests that skin friction is of a nature to cause relatively little drag, and this is true. But it must not be inferred that skin friction is unimportant; in a first-class aeroplane flying at high speeds some 60 per cent. of the total horse-power is expended in overcoming it. The correct deduction is that skin friction constitutes an irreducible minimum of drag under given conditions, and a thick body should be shaped to approach this minimum as closely as possible by reducing its form drag.

Shaping to this end is called fairing or streamlining. A comparison of (*d*) with (*c*) in Table II illustrates the process and its effect. In some large biplanes of the past the inter-plane struts were circular metal tubes fitted along the front and back with shaped strips of spruce, the whole being bound together with fabric and reproducing some such section as (*d*). The spruce 'fairings' added little or nothing to the strength of the struts, but greatly reduced their drags. Similarly, a normal disk is faired by adding a hemisphere to its front and back faces, to form a sphere, but the latter can itself be faired by elongation to resemble an airship envelope. Passable streamline shapes can be contrived in this way by eye, and a great saving in drag effected if the original shape is poor. The 'wetted surface' is enlarged, but the increase of skin friction on this account is justified until the 'lines' of the body become 'fine'.

Fineness and Thickness Ratios. Considering a body of compact cross-section perpendicular to the direction of motion, such as an airship envelope, an aeroplane fuselage or a flying-boat hull, the ratio of its length to its maximum width is called its *fineness ratio*. Thus the fineness ratio of a thin normal disk is zero, of a sphere unity, whilst that of (*d*), regarded for the moment as the longitudinal section of a solid of revolution, is 3.

For rather flat bodies which extend mainly across the direction of motion, such as wings, tail-planes and struts, a term which is essentially the reciprocal of fineness ratio is preferred—viz., the *thickness ratio*. This is defined with reference to a section of the body perpendicular to its length and parallel to the direction of motion, and is given by the ratio of the maximum width of the section to the length of the section. Thus the thickness ratio of a very thin flat plate is zero, whilst that of (*d*), regarded again as the section of an aerofoil extending perpendicular to the page, is 1/3.

The fineness ratio of the envelope of the airship R 101 was 5¼, approximately; the thickness ratios of wing sections suitable for full cantilever monoplanes usually range between 0·1 and 0·2;

biplanes of the past commonly employed wing sections having thickness ratios of about 0·06.

Determining the best lines for sections, or profiles as their boundaries are often called, is a highly technical problem. The use of french curves is inadvisable owing to sudden changes of curvature being left in the profile, and designing from mathematical formulæ is preferred. The final choice of a profile often involves a compromise between aerodynamical and other considerations.

Fig. 19 gives as an example the results of some experiments on struts of various thickness ratios. Down to a thickness ratio of about 0·25 the decrease of form drag due to better streamlining exceeds the increase of skin friction due to a larger wetted surface; below this value the reverse is true. The optimum thickness ratio is therefore clear so far as drag is concerned. However, since the strength of a long streamline strut varies with the cube of the width and linearly with the length of its section, a slightly larger thickness ratio might be preferred in order to save weight. Still other considerations would affect the final choice.

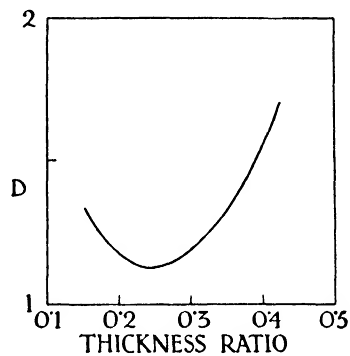


FIG. 19.—D = DRAG IN LB., AT A SPEED OF 100 FT. PER SEC., OF A STRUT 10 FT. LONG AND 2 IN. MAXIMUM WIDTH OF SECTION.

44. Streamlines

It will be appreciated that streamlining is a process of wide application and requires careful study. This is best begun by considering the streamlines.

A *streamline* is an imaginary line drawn in the fluid such that, at the instant considered, the component of the fluid velocity perpendicular to it is everywhere zero. Thus adjacent fluid is flowing tangentially to it. Restriction to a given instant of time is necessary, because the streamline may vary in shape either quickly or slowly.

In the present article the flow will be assumed to be steady, so that the streamlines retain a constant shape at all times. The velocity of the air flow past an obstacle varies greatly from one point to another in magnitude and direction. The criterion of steadiness is absence of change at any one point geometrically fixed relatively to the obstacle.

The aerodynamic force on an aircraft or component part is evidently identically the same whether the aircraft is flying straight and level at a certain speed through still air or is itself stationary in a uniform horizontal head wind of the same speed. The streamlines of flow in the latter case seen from the ground are the same as those that would be viewed by an observer moving with the aircraft in the former case, and are said to be relative to the aircraft or body.

An immersed body disturbs the oncoming head wind supposed, which must divide to flow past it. The disturbance is propagated upstream to some distance in front of the body, persists far behind, and also spreads deeply in the lateral sense. Thus the disturbed wind flows past in a widely curved manner, as depicted in Fig. 20, for example, which shows the streamlines of the flow past a faired strut held stationary in the wind. The strut is assumed to be long, so that the motion can be regarded as two-dimensional.

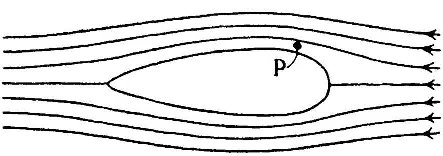


Fig. 20.—Streamlines of Flow Past a Strut Section, the Wake being Neglected.

The flow between any two neighbouring streamlines may be regarded as confined within a channel, of unit depth perpendicular to the page, for fluid cannot cross the streamlines. Where the channel narrows, the velocity increases, since an equal *mass* of air passes every cross-section in unit time. It will be illustrated that, unless exceptionally high speeds are attained, the compressibility of the air can be neglected, and then the same *volume* of air passes every cross-section in unit time. The velocity consequently varies along the channel inversely as its width.

In a uniform flow the velocity is the same in magnitude and direction at all points and the streamlines are parallel to one another. Drawing them at equal distances apart, as in Fig. 21 (*a*), and regarding them as forming a number of parallel channels all of the same width and unit depth, an equal volume of air per second passes all cross-sections. In Fig. 20 the streamlines have similarly been made equidistant far in front of the strut where the wind is almost uniform. As a result the mean velocity near any point P can at once be compared not only with that near another point in its own channel, but also with the velocities at points in different channels. By means of this artifice the pattern of streamlines conveys not only the direction of the disturbed flow, but also the widespread changes in the magnitude of the velocity.

Example 11.—Assuming that the strut of Fig. 20 is in a wind of 100 ft. per sec., estimate the velocity at the point P in the figure.

By measurement, the ratio of the distance apart of neighbouring streamlines in the region of P to that far in front of the strut is 0·7. Hence the local velocity is 100/0·7 = 143 ft. per sec.

(Other examples of the present kind will show that, though the method is valuable as conveying readily a first estimate of the velocity variation, accuracy cannot be expected unless the streamlines are closely and carefully drawn.)

45. Conversely, the foregoing conceptions can be used to construct the streamlines of a steady two-dimensional flow from appropriate knowledge of the distribution of fluid velocity, which may be obtained experimentally. One streamline must be known to start with, and this may be the profile of the section of the body causing the disturbance. The principle of the method is illustrated in the following example.

Example 12.—A wind is blowing steadily over and parallel to a flat roof. The air is at rest on the surface, and its velocity increases in proportion to distance above the surface up to a height H. Draw the streamline at height H and two others between it and the roof.

(The problem is to determine the position of the two additional streamlines so that, per unit width of the wind, the same volume of air flows in unit time between the roof and the first streamline as between the first streamline and the second, and so on.)

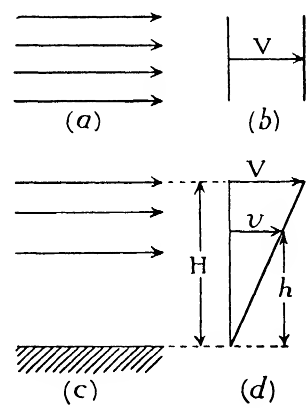


FIG. 21.

A diagram of the linearly increasing velocity is given at (*d*) in Fig. 21. Let V be the velocity at height H. Per unit width, the volume of air passing in unit time between the roof and H is the mean velocity $\times$ the height, i.e., $\frac{1}{2}V \,.\, H$. The corresponding flow between each pair of neighbouring streamlines is one-third of this, viz. $VH/6$. Thus the volume passing per second between the roof and the first streamline will be $VH/6$, and that between the roof and the second streamline $VH/3$.

From the velocity diagram, the velocity at any height h less than H is

$$v = \frac{h}{H} \,.\, V$$

and the volume per second passing between the roof and h is

$$\tfrac{1}{2}vh = \tfrac{1}{2}\frac{h}{H} V \,.\, h = \frac{V}{2H}h^2.$$

Equating this to $VH/6$ gives for the position of the first streamline

$$h = H/\sqrt{3} = 0{\cdot}577\ H.$$

Again, equating the expression to $VH/3$ gives for the position of the second streamline

$$h = H\sqrt{2/3} = 0{\cdot}817\ H.$$

The streamlines are shown at (*c*) in the figure. Those at (*a*), appropriate to the velocity diagram (*b*)—i.e., to uniform flow—have been drawn for the same flow between each pair of streamlines. Thus they are separated by the distance 0·167 H. Imagining the linearly increasing wind near the surface of the roof to underlie the uniform wind, separating it from the roof, the pattern (*c*) would be continued upward by the pattern (*a*) and, counting from the surface of the roof, the distances between neighbouring streamlines would be proportional to : 0·577, 0·24, 0·183, and then 0·167 indefinitely.

46. Eddying Flow

Unsteadiness develops very easily in the flow past a body, greatly increasing its drag. Shaping of sections is especially directed, therefore, towards delaying the tendency and weakening unavoidable ' eddying ', while confining this to the back part of the flow. Success makes for a thin wake, comparatively free from violent fluctuations of velocity. Attention is then focussed on restricting an insidious and finely grained unsteadiness that arises very close to the surface of the body.

Fig. 22 depicts the kind of wake that exists behind a long normal plate or cylinder of bluff section. The streamlines reveal

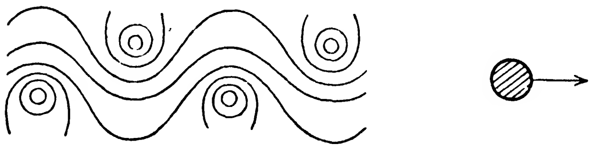


FIG. 22.—THE STRONGLY EDDYING WAKE CAUSED BY A CIRCULAR CYLINDER AT ORDINARY SIZES AND SPEEDS.

a double row of staggered vortices, strongly whirling eddies of air, which are continually shed by the body, from one side of it and the other alternately, forming an ever lengthening ‘ vortex street ’. More than 90 per cent. of the drag of a bluff body can sometimes be traced to their generation.

The wake including this regular or periodic type of unsteady flow is often wider than the long plate or cylinder producing it. With a better streamlined section, or sometimes even with a poor section at large sizes and speeds, the eddies become smaller and more numerous, lose their regular disposition and form a narrower wake. Further improvement in streamlining leads, especially with large sizes and speeds, to a thin wake of weak and irregular eddying. These changes are associated with the great saving of form drag made possible by fairing, as described in Article 43.

Thus *eddying flow* can be strongly marked and regular, or weak and indefinite. But the term is usually reserved to indicate a flow whose unsteadiness is on a sufficiently large scale to be readily perceived.

47. Turbulence

To gather a fair preliminary idea of what is meant by a *turbulent flow,* imagine an eddying flow with its vortices broken up and reduced to minute proportions so that all semblance of pattern is lost. The innumerable tiny eddies dart about in a chaotic manner, jostling one another and creating much resistance to flow. The velocity at any point fluctuates quickly, though through only a small range, but the unsteadiness is so finely grained that details can be revealed only by the most intimate measurements. Nevertheless, its effects on drag and other phenomena are large and of great practical importance.

An interesting experiment can be made with water pouring through a long straight glass tube, a little ink being fed into the stream, as indicated in Fig. 23. At sufficiently low speeds the flow is steady and the ink extends as a dark line along the tube, more or less parallel to its axis as shown at (*a*). On increasing the speed a stage is reached when the flow suddenly changes from streamline to turbulent and the dark line breaks up and is lost, the ink becoming mixed with the water. If the pressure drop along the pipe is also measured, it will be found that the resistance to flow is much increased by the change.

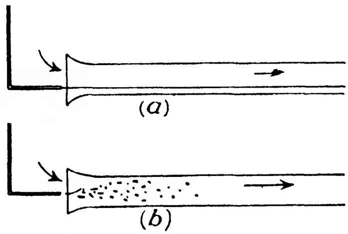


FIG. 23.—FLOW THROUGH A PIPE.
(*a*) Steady, (*b*) Turbulent.

Turbulence arises near the surfaces of aeroplane wings and other components, as described in the next article, and increases their drag. But it may also exist in the undisturbed wind. Natural winds are singularly free from turbulence except near the ground, although they possess much eddying. Artificial winds, on the other hand, such as are used for aerodynamical experiments, are usually turbulent, though largely free from eddying. A wind containing little or no turbulence is often described as ‘ smooth ’.

48. The Boundary Layer

The flow past a streamlined body is essentially of a two-fold character. Air which never closely approaches the surface flows in a comparatively straightforward manner, as we shall see. Separating this outer flow from the body, and merging into the wake behind, there is a ‘ boundary layer ’ in which the flow is of an altogether different kind. Near the nose of the body the boundary layer is almost too thin to measure, but it gradually thickens backward over the surface; 6 feet behind the leading edge of a flat plate in a smooth wind of 100 feet per second, for example, the boundary layer would be about 0·2 inch thick.

Within this enveloping layer the air is subjected to an intense shearing action. No slipping can occur between the surface of the body and the fluid that touches it; molecular attraction prevents this. Air coming in contact with the body is immediately brought to rest and, escaping again, impedes the motion of air which is trying to brush past a little distance away. Close to the surface the flow is consequently sluggish. Yet, at a small fraction of an inch away—i.e., at the outer edge of the boundary layer—obstruction is not felt at all and the air is able to flow with unimpaired velocity.

Fig. 24 illustrates at (a) a small part of the boundary layer 2 feet or so behind the nose of a smooth aeroplane wing. The

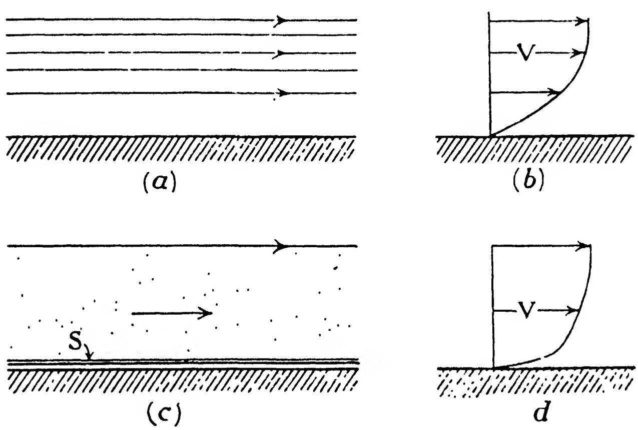


FIG. 24.

(*a*) Laminar boundary layer. (*c*) Turbulent boundary layer. S, viscous sub-layer. (*b*) and (*d*) corresponding velocity diagrams.

magnification is considerable, the height of the diagram corresponding to perhaps a millimetre in the actual case. The air is supposed to be moving and the wing stationary. This part of the boundary layer is steady and streamlines are shown. The diagram at (b) indicates the rapid manner in which the velocity of the air increases from zero on the surface of the wing to several hundred feet per second at a distance of a millimetre or so away. The streamlines may be imagined to divide the layer into a number of thin laminæ which slide over one another. The steady part of the boundary layer is therefore called the *laminar* part.

In the same figure, (c) illustrates with much less magnification the thicker boundary layer farther back along the wing where the flow is turbulent. Streamlines cannot be drawn owing to the unsteadiness of the motion, but the diagram (d) gives the average velocity at different distances from the surface.

The velocity increases very rapidly through a very thin film of air lying on the surface of the wing at the foot of the turbulent boundary layer and called the *viscous sub-layer.* It is proportionately much thinner than is shown in the figure, and may be regarded as laminar again.

49. Tractions and Skin Friction

Referring first to laminar flow, the rate of shear or the sliding over one another of adjacent laminae is resisted by *viscosity*, a physical property which all fluids possess and is so marked a feature of thick oil and the like. The viscosity of air is exceedingly small, but shearing is maintained at such a high rate close to the surface of a body that considerable resistance arises there. This resistance may be regarded as a dragging force acting between adjacent laminæ of air. It is called *traction* and builds up into the *surface traction*, or intensity of skin friction, which drags on the surface of the body itself.

Much more intense traction is spread through a turbulent boundary layer than through a laminar one, but the resistance to flow is caused differently—viz., by a continuous mixing of sluggish and fast-moving air. This greater traction is built up and communicated to the viscous sub-layer, which in turn transmits it to the surface.

The intensity of skin friction is not constant over the surface of a body, but varies widely, according to the local steepness of the ' velocity profile ', Fig. 24 (b), in laminar flow. It increases considerably where the flow becomes turbulent. Taking all parts of the body into account, the components of the surface tractions resolved parallel to the direction of motion add up to the total force of skin friction.

It will be seen that skin friction is quite unlike the friction between two dry surfaces, such as arises when one board slides along another. If, however, a layer of oil separated the boards, the relative motion would produce a similar shearing action and resistance, owing to the adherence of the oil to the surfaces combined with its viscosity.

50. Irrotational Flow

The practically tractionless flow outside the boundary layer and wake is called *irrotational* because no element of air in this region turns about its own centre as it moves along. This widely used mathematical term, and especially the converse, are rather difficult to understand. In the following qualitative description we begin with the latter, considering a small square element of air moving within a thin laminar boundary layer.

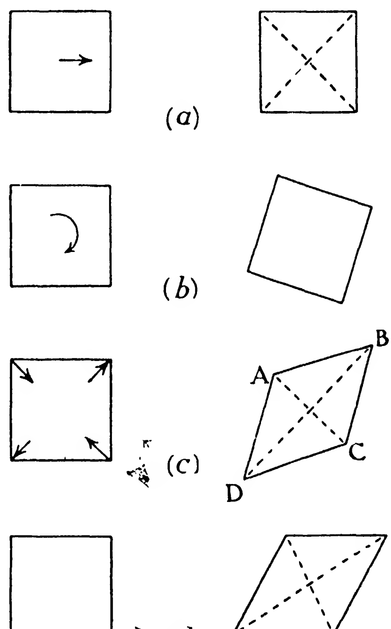


FIG. 25.—THE COMPONENT MOTIONS OF AN ELEMENT OF AIR.

Reverting to the analogy of thin strata of air sliding over one another, the relative velocity of adjacent laminæ and, consequently, the traction between them diminish from the surface of the body outward. The traction is more intense on the inner face of a lamina or stratum than on the outer face, tending to turn an element of the stratum. The element has, indeed, an angular velocity, but its rotation is masked by a simultaneous distortion of shape due to some parts of the element moving faster than others.

Thus an element of the air within a laminar boundary layer has three components of motion—viz., translation, rotation and distortion. The separate effects of these component motions during a brief instant of time are illustrated in Fig. 25 at (a), (b) and (c) respectively, the element moving from the left- to the right-hand column. It will be seen that while the angular velocity alone would turn a small square element into the position shown at (b), the distortion alone—i.e., in the absence of rotation —would change it into an oppositely canted lozenge, as at (c). The way in which the element actually moves is obtained by combining the three motions with the result shown at (d). At first glance the figure (c) may appear to show rotation, but the side AB has rotated in one sense just as much as the side BC has rotated in the opposite sense, and proceeding in this way shows that upon the whole no rotation has occurred; this is summed up by noting that the diagonals of the element have retained their original directions. The rotation included in (d) becomes clearly apparent in the same way; (d) is (c) turned in a clockwise sense—i.e., combined with the rotation (b).

Though the air close to a surface possesses angular velocity, it evidently does not move ' like roller bearings '; if it did, the resulting drag would be large, not small, for the body would have to keep on manufacturing new roller bearings to replace those shed continually behind (cf. Fig. 22).

Elements of air in the outer flow possess no angular velocity, though they may become distorted; hence the term irrotational, as already mentioned. An irrotational flow is sometimes described as one in which there is no ' vorticity '; this comes to the same thing, since the further mathematical term means twice the angular velocity. Still another name is ' potential flow '.

Resulting from the lack of tractions in the fluid, the pressure in an irrotational flow acts equally in all directions at any one point, just as in a fluid at rest. The pressure in the irrotational part of a disturbed motion may vary greatly from one point to another, and this variation is related to an associated change of velocity by the following important theorem.

51. Bernoulli's Equation

The theorem now to be explained is of the greatest utility in Aerodynamics. Its application is restricted to the irrotational part of the flow, but this limitation is by no means derogatory, because the two very dissimilar parts into which every aerodynamical flow resolves itself can be treated to a large extent separately. When Daniel Bernoulli first propounded his theorem in 1738 he employed a method of reasoning which is easy to remember but rather difficult to comprehend. The following non-mathematical derivation avoids this disadvantage, and forms the basis of one of the several mathematical proofs which further reading will reveal.

The assumptions should first be clear. We follow the fluid along any streamline, or rather within a narrow imaginary tube or pipe, enclosing this streamline, the imaginary pipe-wall being itself formed of streamlines (Fig. 26), so that no fluid can get in or out except at the ends. The fluid may be flowing in any steady and incompressible manner, which may be three-dimensional, subject to certain restrictions which will be stated. The assumption of incompressibility —i.e., uniformity of density—is not actually necessary, but it makes for simplicity; in a later article we shall illustrate, by means of an example, the effect of taking the compressibility of the air into account and infer that it is unnecessary to do so except at high speeds. The restrictions are as follows:—First, no tractions must act on the fluid in the pipe or 'stream-tube'; this we have seen to be realised closely in non-boundary layer flow. Second, the fluid must be flowing freely in the sense that no work is done by it or upon it. For example, if the stream-tube passes through the disk of rotation of a windmill or autogyro rotor, then Bernoulli's equation will not apply from one face of the disk to the other, because the air does work in the course of making the windmill or rotor rotate. Again, the equation will not apply without restriction to a stream-tube that crosses the disk of rotation of an airscrew, because the airscrew in imparting momentum to the air does work upon it. These restrictions can be summed up in the statement that the mechanical energy of the air per unit mass must remain constant.

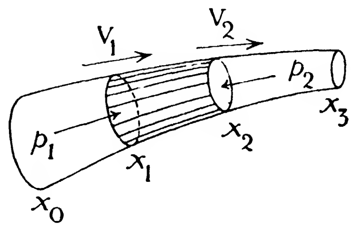


FIG. 26.—STREAM-TUBE.

The pressure p and the velocity V are supposed to be changing from point to point along the stream-tube, whose cross-sectional area therefore varies along its length, because in unit time the same mass of air must pass every cross-section. But, the stream-tube being everywhere very narrow, the pressure and velocity are sensibly constant over any one cross-section.

We are going to consider the motion of a block of air which fills the tube and is of short length $x_2 - x_1$ (Fig. 26), the x's being measured along the bent axis of the tube from any point upstream. Different pressures, p_1 and p_2, act on the two faces of the element, which are of different areas because the velocities,

V_1 and V_2, at x_1 and x_2, respectively, are different. The familiar dynamical law governing this motion is:

$$\textit{force} = \textit{mass} \times \textit{acceleration} \quad . \quad . \quad . \quad . \qquad \text{(i)}$$

The first step is to calculate the force on the element, which arises from the pressure field. Let the pressure vary along the tube in any continuous manner, as shown for example in Fig. 27. First consider a narrow element-cylinder AB of uniform cross-sectional area a and short length l. Let p_A denote the pressure on the end A and p_B that on the end B. Then the force on the cylinder in the x-direction is $p_A a - p_B a$, which can be written

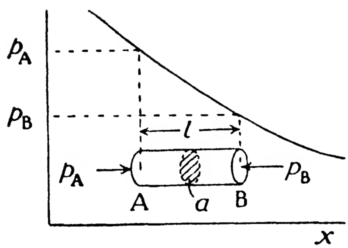


FIG. 27.

$$\frac{p_A - p_B}{l} \times al$$

$$= \frac{p_A - p_B}{l} \times \textit{volume of cylinder} \quad . \quad . \quad . \qquad (10)$$

This same expression gives the force due to the pressure field on any body, of whatever shape, provided it is so small that the pressure varies along its length in a sensibly linear manner—i.e., provided $(p_A - p_B)/l$ remains constant as l varies. For the volume of the body can be made up of a large number of element-cylinders of different lengths and very small sections, as described in Article 25.

Turning now to the tapered element in Fig. 26, the force on it in the x-direction is therefore

$$\frac{p_1 - p_2}{x_2 - x_1} \times \textit{volume.}$$

The mass of the element is $\rho \times$ *volume*, ρ being the density of the air, which has been assumed constant, and the acceleration is the increase of velocity, $V_2 - V_1$, divided by the time in which this increase occurs. This time is equal to the length $x_2 - x_1$ divided by the mean velocity: $\frac{1}{2}(V_2 + V_1)$. Hence the acceleration is

$$(V_2 - V_1) \times \frac{\frac{1}{2}(V_2 + V_1)}{x_2 - x_1}$$

$$= \frac{1}{2}\,\frac{V_2^2 - V_1^2}{x_2 - x_1}.$$

Hence the equation (i) gives

$$\frac{p_1 - p_2}{x_2 - x_1} \times volume = \rho \times volume \times \tfrac{1}{2} \frac{V_2^2 - V_1^2}{x_2 - x_1},$$

reducing to

$$p_1 - p_2 = \tfrac{1}{2}\rho(V_2^2 - V_1^2) \quad . \quad . \quad . \quad . \quad (11)$$

Considering instead a contiguous element to the left, extending upstream from x_1 to x_0, the same reasoning will give

$$p_0 - p_1 = \tfrac{1}{2}\rho(V_1^2 - V_0^2).$$

Again, considering a contiguous element to the right, extending from x_2 to x_3, gives

$$p_2 - p_3 = \tfrac{1}{2}\rho(V_3^2 - V_2^2).$$

Adding the last three equations together leads immediately to

$$p_0 - p_3 = \tfrac{1}{2}\rho(V_3^2 - V_0^2).$$

Extension along the streamline can proceed without limit in either direction in the same manner. Hence finally the equation (11) is seen to hold good when the positions denoted by the suffixes 1 and 2 are located anywhere on the streamline.

With this understanding, (11) expresses Bernoulli's theorem. It can be written in various ways. The best form to remember is

$$p_1 + \tfrac{1}{2}\rho V_1^2 = p_2 + \tfrac{1}{2}\rho V_2^2 \quad . \quad . \quad . \quad . \quad (12)$$

Immediately from this expression it follows that

$$p + \tfrac{1}{2}\rho V^2 = \text{constant} \quad . \quad . \quad . \quad . \quad (13)$$

for all points on the streamline, since points 1 and 2 can be located anywhere along it. This is the form in which the theorem is usually stated.

52. Interpretation

In the preceding article Bernoulli's equation has been obtained for flow along a streamline. Restricted to any one streamline, the equation holds good whether the flow is irrotational or not, provided the viscous tractions are negligible. The proviso can be stated in the alternative form that the theorem applies to an inviscid fluid. But this statement is unhelpful, because no such fluid exists, whilst large tractions arise even in a fluid of small viscosity like air, as we have seen, in regions where the velocity changes very rapidly across the flow.

Absence of tractions and free flow enable the air to proceed on its path without loss or gain of mechanical energy, and the adjustments of pressure and velocity that take place accord with a conservative interchange of pressure energy and kinetic energy. The value of the total mechanical energy per unit mass is related to the constant in equation (13). Though invariable along any particular streamline, this constant will change, in general, from one streamline to another. An example is provided by the slip-stream behind an airscrew, which adds energy in an unequal manner to the air passing through its disk of rotation.

But in a uniform wind the energy per unit mass is the same for all streamlines. As the stream flows past an immersed bódy, this equality still holds good, or very nearly so, away from the boundary layer and wake. In other words, the constant of equation (13) has the same value for all streamlines in the irrotational part (cf. Article 50) of the disturbed motion emerging from a uniform stream.

The application of Bernoulli's equation in these circumstances is especially simple, for the points 1, 2, etc., of the preceding article need not then be located on the same streamline. These favourable circumstances are fortunately so widely representative of aerodynamical motions that we can afford to postpone discussion of more troublesome cases, which involve changes in the constant, until they arise.

Referring, then, to a free and approximately irrotational flow, not passing through an airscrew or into a wake or the like, the first point Bernoulli's equation makes is that the pressure increases when the velocity decreases, and vice-versa. This very natural law occasionally proves a stumbling-block in a first reading, owing to a feeling that the pressure should go up when the velocity increases. But its truth can be verified at once by blowing through a paper tunnel supported on a table (Fig. 28 (a)); the tunnel at once collapses, as shown at (b) due to the pressure within decreasing.

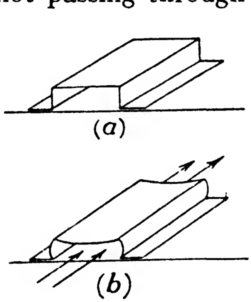


FIG. 28.

The next point is that if the increase of velocity is known the decrease of pressure can be calculated, and vice-versa. In the slug–foot–second system (cf. Article 27) the units are : density, slugs per cubic foot; velocity, feet per second; pressure, lb. per square foot. For many calculations it is convenient to rearrange equation (11) in the form

$$\frac{p_1 - p_2}{\frac{1}{2}\rho V_1^2} = \left(\frac{V_2}{V_1}\right)^2 - 1 \quad . \quad . \quad . \quad . \quad (14)$$

Example 13.—A 150-m.ph. wind flows past a wing. At a certain position outside the boundary layer the velocity has double its undisturbed value. What is the pressure drop there?

Let suffix 1 distinguish undisturbed values and suffix 2 those at the point concerned. Since 22 ft. per sec. = 15 m.p.h., V_1 = 220 ft. per sec. and $\frac{1}{2}\rho V_1^2 = \frac{1}{2} \times 0{\cdot}00238 \times 220 \times 220 = 57{\cdot}6$ lb. per sq. ft. By the question, $V_2/V_1 = 2$, whence (14) gives

$$p_1 - p_2 = 57{\cdot}6(4 - 1) = 172{\cdot}8 \text{ lb. per sq. ft.}$$

Example 14.—An air duct of circular section has a long divergent part expanding from 10 ft. diameter at A to 15 ft. diameter at B, followed by a convergent nozzle contracting to $7\frac{1}{2}$ ft. diameter at C. The speed of the wind through the duct is increased until a gauge shows a pressure difference of 1 in. head of water between A and B. Neglecting energy losses, estimate the speed at C.

Since the divergent part is long the duct can be assumed to run full—i.e., without breaking away from the walls—so that the velocity along it varies inversely as the cross-sectional area.

Distinguishing positions along the duct by appropriate suffixes, Bernoulli's Theorem gives

$$p_A + \tfrac{1}{2}\rho V_A^2 = p_B + \tfrac{1}{2}\rho V_B^2$$

i.e., $\frac{1}{2}\rho(V_A^2 - V_B^2) = p_B - p_A.$

Now $V_A = (7\frac{1}{2}/10)^2 V_C, = 0{\cdot}5625\ V_C,$

$V_B = (7\frac{1}{2}/15)^2 V_C = 0{\cdot}25\ V_C$

and $p_B - p_A = 62{\cdot}4/12 = 5{\cdot}2$ lb. per sq. ft.

Substitution gives

$$\tfrac{1}{2}\rho \times 0{\cdot}254\ V_C^2 = 5{\cdot}2$$

i.e., $$V_C = \sqrt{\frac{5{\cdot}2 \times 840}{0{\cdot}254}} = 131 \text{ ft. per sec.}$$

Example 15.—The convergent nozzle of Example 14 is 15 ft. long and the static pressure decreases linearly along it. A streamline body, having a volume of $\frac{1}{2}$ cu. ft., is held within the nozzle at a position where the wind velocity is known. In free flight at this same velocity the drag of the body would be 0·2 lb. What is it in the convergent stream?

Using the notation and results of Example 14,

$$p_B - p_C = \tfrac{1}{2}\rho(V_C^2 - V_B^2) = \frac{15}{32}\rho V_C^2$$

$$= 19{\cdot}2 \text{ lb. per sq. ft.,}$$

since $V_B/V_C = \frac{1}{4}$ and $V_C = 131$ ft. per sec.

Thus the decreasing pressure gradient is $19{\cdot}2/15 = 1{\cdot}28$ lb. per cu. ft. and the increase of drag on this account is $1{\cdot}28 \times$ the volume of the body—i.e., 0·64 lb. Hence the total drag of the body in the convergent stream is

$$0{\cdot}2 + 0{\cdot}64 = 0{\cdot}84 \text{ lb.}$$

53. The Stagnation Pressure

An immediate deduction from Bernoulli's equation is that the maximum possible increase of pressure in a disturbed wind, provided compressibility can be neglected, is equal to $\frac{1}{2}\rho V^2$, V being the undisturbed velocity. To see this, equation (11) is written as

$$p_1 - p = \tfrac{1}{2}\rho V^2 - \tfrac{1}{2}\rho V_1^2,$$

p being the undisturbed pressure. Now clearly ρV_1^2 cannot possibly be negative. Hence the maximum pressure rise occurs when $V_1 = 0$ and has the value $\frac{1}{2}\rho V^2$.

When a stream divides to flow past an immersed body there must be a dividing streamline which, except in most unusual circumstances, abuts on the body at some point near its nose. The fluid is brought to rest at this point, which is therefore called the front stagnation point. The known pressure increase there is called alternatively the stagnation pressure, the impact pressure or the dynamic pressure. Being of constant occurrence, it will be denoted by the symbol q. So we note for reference

$$q = \tfrac{1}{2}\rho V^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

54. The Pitot Tube

If an open-mouthed tube faces directly upstream and its other end is sealed so that air cannot flow through, the dividing streamline terminates within the mouth of the tube at some point denoted by suffix 1, say, and $V_1 = 0$. Thus if p, V refer to the undisturbed stream, Bernoulli's equation gives

$$p_1 = p + q$$

for the pressure in the tube. This is called the *pitot pressure* or *pitot head*. A pitot tube can be of almost any size or shape. Diameters as small as 0·02 inch are used for fine experimental work.

55. The Pitot-static Tube

It is continually required to measure q, and for this purpose the pitot-static tube has been developed. It consists essentially of a pitot tube connected to one side of a pressure gauge whose other side is connected to another tube, called the static tube, so designed that the pressure within it is the same as that of the oncoming air. Thus the gauge records the pitot pressure less the static pressure—i.e., $p + q - p = q$.

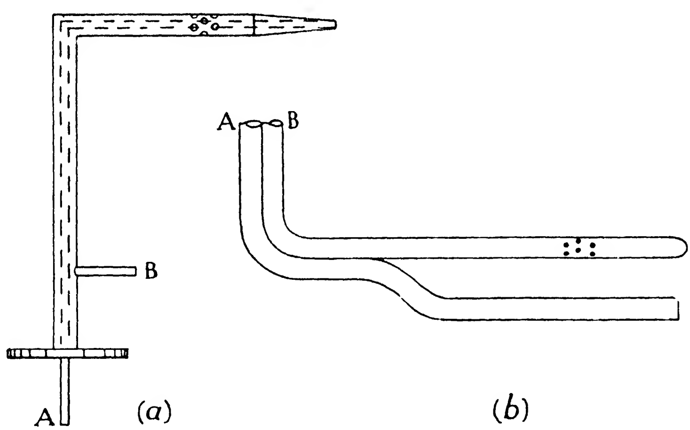


FIG. 29.—PITOT-STATIC TUBES.
A, pitot tube; B, static tube.

A concentric type developed at the National Physical Laboratory is shown at Fig. 29 (a), the static tube surrounding the pitot tube and having for outlet a number of small perforations at a chosen position. The gauge is connected between A and B. As will be inferred from the preceding article, no difficulty arises from the pitot tube, but the design of the static tube rests ultimately on experiments with the whirling arm (Article 9). Once a reliable design has been worked out by this means it can be used as a standard, with which other pitot-static tubes can be compared in the wind tunnel. Another type is shown at (b).

Example 16.—Find the head of water supported by a pitot-static tube in a wind of (*a*) 100 m.p.h., (*b*) 10 m.p.h., the air being in the standard sea-level state.

(*a*) In lb. per sq. ft. the pressure difference $= q = \frac{1}{2}\rho V^2$, if V is expressed in ft. per sec. Therefore it comes to

$$\tfrac{1}{2} \times 0{\cdot}00238 \times \left(100 \times \frac{22}{15}\right)^2 = 25{\cdot}6 \text{ lb. per sq. ft.}$$

Then from Article 23 the head of water is

$$\frac{25{\cdot}6}{62{\cdot}4} \times 12 = 4{\cdot}92 \text{ in.}$$

(*b*) The head is evidently reduced in the ratio $(10/100)^2$. Thus it is 0·0492 in.

The example illustrates that, while the pitot-static tube is suitable for the measurement of moderate and high speeds, it gives, at exceptionally low speeds, a head that is too small to observe accurately. Special gauges have been developed to cope with the difficulty in a laboratory (Article 24), but for speeds lower than 10 feet per second other methods of measurement are in use.

56. Measurement of Aircraft Speed

The pitot-static tube is employed on aircraft to determine the *air speed*—i.e., the speed relative to the wind. The tube is exposed in a position affected as little as possible by other parts of the craft, and is connected by piping to a pressure gauge of the aneroid barometer type in the pilot's cock-pit. The dial of the gauge, over which a needle moves, is graduated in miles per hour by applying to the gauge a sequence of pressure heads calculated from the knowledge that the pressure difference operating the needle will be equal to q. But evidently the graduations can only be true for a particular value of ρ—namely, that for which they were marked. In other words, the gauge will register the *true air speed* only at a certain altitude.

The speed registered is called the *indicated air speed* and is denoted by V_i. Let the gauge be graduated for a density ρ_0. Then for a known pressure difference, $p_1 - p$, expressed in lb. per square foot,

$$V_i = \frac{15}{22}\sqrt{\frac{p_1 - p}{\frac{1}{2}\rho_0}} \text{ m.p.h.}$$

Interpreting ρ_0 as the standard sea-level value of the air density gives

$$V_i = (15/22)\sqrt{840} \times \sqrt{p_1 - p}$$
$$= 19{\cdot}8\sqrt{p_1 - p} \text{ m.p.h., approximately} \quad . \quad . \quad (16)$$

At another altitude, where the density is ρ and the true air speed is V_t, we have for a given reading of the gauge

$$\tfrac{1}{2}\rho V_t^2 = \tfrac{1}{2}\rho_0 V_i^2,$$

whence

$$V_t = V_i\sqrt{\frac{\rho_0}{\rho}} = \frac{V_i}{\sqrt{\sigma}} \quad . \quad . \quad . \quad . \quad . \quad (17)$$

where σ is the relative density of the air at the altitude.

Example 17.—The speed of an aeroplane at 20,000 ft. altitude is 150 m.p.h., A.S.I. (Note: the letters A.S.I. are often used in this way to convey that the speed concerned is the indicated air speed.) What is the true air speed?

From Table I, Article 32, $\sigma = 0{\cdot}534$, whence $V_t = V_i/\sqrt{\sigma} = 150/0{\cdot}731 = 205{\cdot}2$ m.p.h.

Sources of Error.—The above use of the pitot-static tube is subject to various errors, as follows, for which corrections should be made.

(1) A 'position error' arises from two causes. First, it is difficult to find a convenient location for the tube where interference from other parts of the aircraft can be neglected. Usually, such interference changes the velocity both in magnitude and direction. Second, an aeroplane flies at various attitudes to the direction of motion, ranging through some 20°, and a pitot-static tube is affected by inclination to the stream. Supposing that it is set tangentially to the local stream at a mean attitude, its pressure difference may be increased 2–3 per cent. above normal when inclined in either direction by 10°. But the interference itself is also affected by change of attitude of the aircraft.

(2) An error of a different kind, often termed the 'instrument error', may arise casually from imperfections of manufacture, or systematically from the use of a particular design of pitot-static tube in which the pressure in the static tube differs from the true static pressure of the surrounding air—i.e., the pressure difference is not exactly equal to q.

Both the errors (1) and (2) can be determined by flying the aeroplane to and fro over a measured course and comparing the indicated speed with the mean speed estimated by timing. This calibration can be carried out at various speeds.

(3) Any pitot-static tube leads to an over-estimation of exceptionally high speeds. The error is due to compressibility, as described in the next Article. It can be eliminated by graduating the dial from a more complicated formula.

Advantage of the Pitot-Static Tube on Aircraft.—Other methods are available for the measurement of air speed. But the use of the pitot-static tube on aeroplanes has a great advantage, as follows.

Whilst the pilot (or navigator) requires to know the true air speed in order to estimate his position on a course and the like, the indicated air speed is of vital concern in connexion with the immediate condition of flight. The reason is that the aerodynamic force on the aircraft depends on the value of q, and therefore V_i, whilst how it depends on V_t could be gauged only by making a calculation involving σ. Thus, for example an aeroplane stalls (cf. Article 12) at all altitudes at an approximately constant value of V_i, which is immediately visible to the pilot.

Example 18.—An aeroplane lands at its stalling speed. On an aerodrome at sea-level the landing speed is 64 m.p.h. What is it on an aerodrome situated at an altitude of 5000 ft.?

The value of V_i will be the same in both cases. At low altitude ($\sigma = 1$), $V_t = V_i = 64$ m.p.h. On the high aerodrome, since $\sigma = 0{\cdot}862$, $V_t = V_i/\sqrt{\sigma} = 64/0{\cdot}928 = 69$ m.p.h.

Ground Speed.—The navigation of an aircraft depends upon a knowledge of the speed relative to the Earth's surface, called the ground speed. This is obtained by adding the speed of the wind vectorially to the true air speed. A head or following wind is simply subtracted or added, respectively. Other cases are solved immediately by the triangle of velocities.

Example 18 would appear to be restricted to the unlikely circumstance of a windless day. But actually the landing speed of an aeroplane is always stated as an air speed. Since landing occurs into the wind, the ground speed on landing is always less than the landing speed quoted for the aircraft.

Example 19.—An aeroplane flies due North between two aerodromes 500 miles apart. The indicated air speed is 180 m.p.h., the altitude is such that the relative density of the air is 0·81, and there is an East wind of 40 m.p.h. Estimate the time required for the journey.

The true air speed $= 180/\sqrt{0{\cdot}81} = 200$ m.p.h. Setting out the triangle of velocities shows that the aeroplane must head East of North at an angle whose sine is 40/200, i.e., 11° 32′. This gives a ground speed of 200 cos 11° 32′ = 196 m.p.h., nearly. Thus the flying time is 500/196 = 2·55 hours.

57. Compressibility Effect

The formulæ (12) and (13) express Bernoulli's theorem for incompressible flow. But air is compressible, and the question arises as to what effect this may have. There was theoretically no need to neglect variations of density, but including these would have entailed mathematical treatment. In this article will be given a brief description only of the results of such a more advanced analysis.

Pressure in the air stream cannot change without an accompanying variation of density, and pressure variations on a high-speed aeroplane may exceed one-third of an atmosphere, while they are still larger near the tips of an airscrew. The whole cycle of change occupies only a small fraction of a second; there is no time for the air to gain or lose heat, and accordingly it expands and contracts adiabatically (cf. Article 30). Thus the pressure and density are related by formula (6), showing that the latter varies less than the former, whilst the temperature also varies as given by the equations (7).

The result of taking compressibility into account will be illustrated in an important case—viz., the pressure difference of a pitot-static tube. For all but exceedingly high speeds, the more accurate expression of this pressure difference is, for low altitudes where the speed of sound is 1100 feet per second,

$$p_1 - p = q\left[1 + \left(\frac{V}{2200}\right)^2\right]. \quad . \quad . \quad . \quad (18)$$

V, the undisturbed speed, being in feet per second, as before. Neglecting the compressibility of the air gives $p_1 - p = q$, as we have seen, and the error incurred can therefore be indicated by simple calculations as in the following table :—

Speed (m.p.h.) . .	75	150	225	300	450
,, (ft./sec.) . .	110	220	330	440	660
Error, per cent. . .	$\frac{1}{4}$	1	$2\frac{1}{4}$	4	9

It will be seen that at low and moderate speeds, say up to 150 m.p.h., the error is negligible. But then it begins to increase rather rapidly and becomes important at over 300 m.p.h. The pitot-static tube does not provide the worst case in Aerodynamics, but in all instances there is no need to take the compressibility of the air into account and modify the simple expressions of Bernoulli's theorem until high speeds occur.

The general question naturally arises as to whether the compressibility of the air is an advantage or a disadvantage to flying. Without going into this wide question thoroughly, it may be remarked that until the maximum velocities attained by the disturbed air reach 500–600 m.p.h. at low altitudes, and 100 m.p.h. less at very high altitudes, there is no great effect. At higher speeds, compressibility begins to exert an adverse influence on flying, until eventually it prevents further increase of speed. This final stage is distinguished by Bernoulli's equation no longer applying to the flow past the aircraft owing to the formation of shock waves.

58. Bernoulli's Equation and Lift

The applications of Bernoulli's equation in Aerodynamics are almost unlimited. We complete the few instances studied by enquiring what it has to say in explanation of the lift of a wing or aerofoil in a wind.

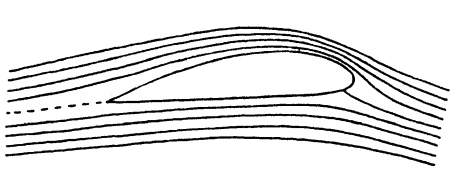


FIG. 30.—STREAMLINES OF FLOW PAST A LIFTING AEROFOIL.

Fig. 30 shows the streamlines past a long aerofoil, far from its wing-tips. Inspection of the way in which these streamlines first approach one another and then diverge again to their normal distance apart, appropriate to the undisturbed stream, gives a picture of how the lifting aerofoil modifies the fluid velocity, for the velocity is inversely proportional to the distance apart of neighbouring streamlines, as we have seen. The velocity change penetrates deeply into the disturbed stream, but is most marked in the vicinity of the aerofoil surface —at the edge of the boundary layer, in fact.

Applying Bernoulli's equation to this velocity field would evidently yield a widespread modification of the pressure, which on the whole is reduced by the aerofoil. The pressure drop (and also the pressure increase under the nose where the velocity is reduced) is built up to a maximum, like the velocity, at the edge of the boundary layer. Theory and experiment then show that it is propagated without further change through the boundary layer to the skin of the aerofoil as a normal pressure.

A detailed working out in this way of the pressure alterations thus transmitted to the upper and lower surfaces of the aerofoil results in Fig. 31. Referring first to the lower surface, the pressure change for a short distance behind the nose pushes up the aerofoil, but further aft it pulls the aerofoil down. Turning now to the upper surface, the pressure drop over the entire profile pulls the aerofoil up so strongly that on the whole there is sufficient lift.

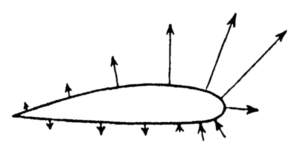


FIG. 31.—NORMAL PRESSURE DISTRIBUTION ROUND A LIFTING AEROFOIL.

Further details and other cases are dealt with in later chapters, but it is as well to appreciate at once the following outstanding features.

(1) The pressure changes given neglect the compressibility of the air. They would be slightly modified at high speeds on the lines indicated in Article 57, and the lift of the aerofoil accordingly. But this variation of density, incidental to compressibility, which cannot be avoided with air, is entirely subsidiary. Except at high speeds, the lift of the aerofoil in no way depends on it. To say, for example, that the lift of an aerofoil depends essentially on the rarefication of the air above, and the compression of the air below, is incorrect, implying a misapprehension of Bernoulli's equation; the streamlines and pressure changes shown in the figures would occur equally if the fluid were water, which is sensibly incompressible.

(2) These pressure changes are generated in the outer, irrotational part of the flow and merely transmitted through the boundary layer. We have omitted from discussion the tractions

which are generated within the boundary layer, as described earlier, because they do not contribute appreciably to lift.

(3) A pressure distribution that gives an upward lift depends on greater velocities existing, upon the whole, in the part of the divided stream which passes over the aerofoil than in the part which passes beneath it. Therefore, an aerofoil can lift only if it tends to throw, as it were, the oncoming air over its upper surface. This throwing action, making the streamlines crowd rather more closely together above than below, will be referred to again in a later chapter. It is not unique to wings, being exploited in several ball-games, in order to make the ball soar, duck or swerve in flight. The advantage of the aerofoil lies in the fact that it achieves the effect fairly strongly without being spun, and has little form drag.

Example 20.—A shaft, whose diameter is 6 in., is rotated at 1440 revs. per min. in air originally at rest. Assuming that the circulatory flow, generated by absence of slip between the shaft and adjacent air together with the action of viscosity, is irrotational, find the reduction of normal pressure on the surface due to the rotation.

The peripheral speed of the shaft is $2\pi \times \frac{1}{4} \times 1440/60$ ft. per sec. The velocity v of immediately adjacent air is the same, whence $v = 12\pi$ ft. per sec. Air at a very large distance from the shaft will not be affected by the rotation of the latter and will remain at rest. Let its pressure be P. Also let the normal pressure on the surface of the shaft before rotation—i.e., when $v = 0$—be p_0, and that during rotation be p. Then, since the motion is irrotational, Bernoulli's equation can be applied and gives

$$P + 0 = p_0 + 0$$
$$P + 0 = p + \tfrac{1}{2}\rho v^2,$$

whence
$$p_0 - p = \tfrac{1}{2}\rho v^2 = \tfrac{1}{2} \times 0{\cdot}00238 \times 144\pi^2$$
$$= 1{\cdot}69 \text{ lb. per sq. ft.}$$

This pressure reduction is uniform round the section and may be accounted for by the centrifugal force of the revolving air.

Example 21.—The shaft of Example 20 is horizontal and exposed in a horizontal wind perpendicular to its axis. Before rotation of the shaft, the local air velocity just outside the thin boundary layer at the top A and bottom B of the section is 88 ft. per sec. The shaft is revolved in the wind at 1440 revs. per min., when the maximum air velocity at A increases to $88 + 12\pi$, whilst that at B decreases to $88 - 12\pi$ ft. per sec. Find the pressure difference between A and B.

Let the undisturbed wind have a pressure P and a velocity V. Then

$$P + \tfrac{1}{2}\rho V^2 = p_A + \tfrac{1}{2}\rho v_A{}^2$$
$$P + \tfrac{1}{2}\rho V^2 = p_B + \tfrac{1}{2}\rho v_B{}^2,$$

whence
$$p_B - p_A = \tfrac{1}{2}\rho(v_A{}^2 - v_B{}^2) = \tfrac{1}{2}\rho(125{\cdot}7^2 - 50{\cdot}3^2)$$
$$= 15{\cdot}8 \text{ lb. per sq. ft.}$$

The pressure is no longer uniform round the section and evidently the shaft will exert a lift.

Chapter IV

THE THEORY OF MODEL EXPERIMENT

59. ALL the data in Table II, Article 40, except those relating to flat plates, are necessarily experimental and could not be calculated. The same remark applies to most of the information regarding aerodynamic force. Given such data, relating as in the table to a specified size and speed and to air in a standard state, can the drags of the same shapes be deduced for different sizes, speeds and altitudes? If so, the often insuperable difficulties of calculation from first principles can be circumvented by carrying out measurements on models in a laboratory and translating the results into terms of full-scale flight.

This question in one guise or another continually confronts the designer who wishes to utilise experiments on models in wind tunnels to forecast, for instance, the speed his aircraft will attain. A fair answer is as follows. The deduction is simple and reliable under certain conditions. When, as often happens, these conditions cannot be complied with, a straightforward application of wind-tunnel results may lead to large errors, and making the transition from model to full-scale is then fraught with difficulty.

The present chapter studies the conditions under which experimental results can be applied directly to full-scale flight. The matter is of immediate importance, since it controls in large measure the arrangement of laboratory work. The enquiry may be stated as follows. The effects on aerodynamic force of peculiarities in the shape of an aircraft or its components being too complicated for calculation, it is agreed to carry out experiments on models made accurately to scale. Having made this provision regarding shape, it is desired to know the dynamical conditions under which measurements are to be obtained and how the latter are to be interpreted.

The method of study is an application of the Principle of Physical Dimensions, which will first be considered briefly in general terms.

60. Physical Dimensions

Quantities occurring in Aerodynamics depend on one or more of the basic conceptions: mass, length and time, and are measured in terms of the units in which the latter are expressed. The fundamental units of mass, length and time are independent of one another, and the magnitude of the particular unit selected in each case is open to arbitrary choice. In Aerodynamics the magnitudes adopted are the slug, the foot and the second, respectively. Whatever the choice made, these fundamental units are respectively denoted by M, L and T.

The unit of any dependent quantity is derived by defining the quantity explicitly in terms of the three fundamentals. Often the unit is self-evident—e.g. one foot per second, in the aerodynamical system, for the unit of velocity. But sometimes the precise manner in which a quantity depends on mass, length and time is not immediately apparent. Elucidation is then effected by establishing its ' physical dimensions '.

In illustration of the method, consider again a velocity. Its magnitude is directly proportional to a length traversed, and inversely proportional to the time occupied in traversing that length. This relationship is expressed by writing its dimensions

$$\frac{L}{T}.$$

The magnitude of a given velocity is specified by multiplying L/T by a pure number, or non-dimensional coefficient, which depends on the magnitude of the velocity and also on the fundamental units adopted for length and time. For example, if the magnitude of a velocity is such as to require the coefficient 88 in the foot–second system, then a velocity of one-half that magnitude will require the coefficient 30 in the mile–hour system. But the fundamental nature of a velocity, whatever its magnitude and the convention adopted in measurement, is completely revealed by its dimensions L/T.

To express the physical character of quantities in this way, it is clear that one or more of the dimensions M, L and T will often require to be raised to a power. For instance, the magnitude of an area is measured in terms of that of the square of unit side (whatever the unit of length may be), and therefore direct dependence of a quantity on area will be indicated by the dimension L^2. Again, an acceleration is a change of velocity that occurs in a time; its magnitude is directly proportional to the former and inversely proportional to the latter. A change of velocity has the same dimensions as a velocity and, therefore, the dimensions of acceleration are

$$\frac{L}{T} \times \frac{1}{T} = \frac{L}{T^2}.$$

The quantities so far considered are evidently independent of mass, whence the absence of M from their dimensions. The dimensions of density are

$$\frac{M}{L^3}$$

since the density of a given bulk of matter is directly proportional to its mass and inversely proportional to its volume, and volume has plainly the dimensions L^3. In the aerodynamical system the unit of density is one slug per cubic foot, and the magnitude of the density of a given material is specified by stating how many slugs of it occupy a cubic foot of space; this number is $1/g$-times the number of pounds, as already mentioned (Article 27). It is necessary to take this into account in visualising the magnitudes that numbers are intended to convey, but density as a property which all materials possess is clear from its dimensions M/L^3 alone.

A force is gauged by the rate at which it increases momentum, and the dimensions of momentum are evidently ML/T. Thus the dimensions of force are ML/T^2—i.e., those of a mass × an acceleration.

Example 22.—Verify that work, potential energy and kinetic energy all have the same dimensions.

The amount of work done is proportional to the product of the force employed and the distance through which it acts. Thus its dimensions are $ML/T^2 \times L = ML^2/T^2$. The potential energy of a body is increased by the product of its weight (which is a force) and the height through which it is raised (which is a length), and therefore its dimensions are the same as those of work. The kinetic energy of a body is proportional to the product of its mass and the square of its velocity. Hence its dimensions are $M \times (L/T)^2$—i.e., ML^2/T^2.

Example 23.—What are the dimensions of horse-power?

Unit horse-power is 33,000 foot-pounds of work done in one minute. Thus horse-power measures the rate of doing work, and its dimensions are those of work divided by that of time—i.e., $ML^2/T \div T = ML^2/T^3$.

Rather more complicated is the question of the dimensions of the coefficient of viscosity, μ. This quantity is defined with reference to the following particular fluid motion. Two infinite parallel plates are supposed to be separated by a thin flat layer of fluid. One of the plates is held stationary whilst the other is moved with a uniform velocity in its own plane. The fluid sticks to both plates, being dragged along by the moving plate but retarded by the stationary one, and through the layer there is constantly taking place a shearing action which is resisted by viscosity, as described in Article 49. Consequently a force must be applied to the moving plate to maintain its motion, whilst an equal force must be exerted in the opposite direction on the other plate to keep it still. Now it can be proved that, provided the motion becomes steady, for a given fluid the magnitude of the force per unit area on either plate is directly proportional to the relative velocity and inversely proportional to the distance separating the plates. Hence, the coefficient of viscosity is defined as this force per unit area divided by the relative velocity and multiplied by the thickness of the fluid layer.

TABLE III.

Quantity	Dimensions
Mass	M
Length	L
Time	T
Area	L^2
Volume	L^3
Velocity	L/T
Acceleration	L/T^2
Angular velocity	T^{-1}
Density	M/L^3
Force	ML/T^2
Pressure	M/LT^2
Moment	ML^2/T^2
Momentum	ML/T
Energy	ML^2/T^2
Power	ML^2/T^3
Viscosity	M/LT
Kinematic viscosity	L^2/T

Thus the dimensions of μ come to the same as those of

$$\frac{\text{force}}{\text{area}} \times \frac{1}{\text{velocity}} \times \text{distance},$$

which are

$$\frac{\mathsf{ML}}{\mathsf{T}^2} \times \frac{1}{\mathsf{L}^2} \times \frac{\mathsf{T}}{\mathsf{L}} \times \mathsf{L} = \frac{\mathsf{M}}{\mathsf{LT}}.$$

In the aerodynamical system the units of μ are slugs per foot-second.

Proceeding on these lines, the above Table III can be verified and extended as desired.

61. Application to Equations

Two quantities can be equated to one another only if they are of the same kind. We are justified in framing, for example, the equation

$$N \text{ feet per second} = 150 \text{ miles per hour},$$

giving

$$N = 150 \times \frac{\text{miles}}{\text{hour}} \times \frac{\text{second}}{\text{feet}}$$

$$= 150 \times \frac{5280}{3600} = 220.$$

For, although the units on the two sides are different, each side has the same dimensions—viz., L/T. But no number of feet per second could possibly be equated to a number of lb. per square foot, for example, there being no basis of comparison.

What is evidently true of two single quantities also holds for two groups of quantities, or for a single quantity on the one hand and a group on the other. They can be equated only if there exists a basis of comparison, and this question is decided by whether each side of the equation has the same dimensions.

The principle is of wide utility. An equation may not be right if the dimensions of the two sides are the same, but it is unquestionably wrong if they are not, and thus comparing the dimensions of the two sides provides a check. The method can also be used constructively in forecasting the form of a desired equation or formula, as described in the following examples.

A familiar example of the latter use is in application to the simple pendulum. From inspection, the frequency of its oscillation is likely to depend only on its mass m, its length l and the acceleration g due to gravity. A first attempt at a formula for the frequency $\sim$ is therefore

$$\sim = C\, m^p l^q g^r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(i)}$$

where C is a constant and p, q, r are unknown indices. The principle asserts that the right-hand side must have the same dimensions as that of the left-hand side, which is $1/\mathsf{T}$, i.e., since C is non-dimensional,

$$\mathsf{M}^p \times \mathsf{L}^q \times \left(\frac{\mathsf{L}}{\mathsf{T}^2}\right)^r = \frac{1}{\mathsf{T}}.$$

This requirement can be satisfied only if (a) $p = 0$, since M does not appear on the right-hand side of the dimensional equation; (b) $q + r = 0$, or $q = -r$, for a similar reason regarding L; (c) $r = \frac{1}{2}$, in order to make the dimensions in T correct. Hence (i) is solved to the extent :—

$$\sim = C\, l^{-1/2} g^{1/2} = C\sqrt{\frac{g}{l}}.$$

Only the constant coefficient remains for determination in some other way, e.g., by experiment. It will be observed that the method corrected a mistaken notion that the frequency depended on the mass of the bob, but it cannot go further and point out any mistake of omission. If experiment clearly showed that C was not a constant, we should have to go back and find out for ourselves what physical factor had been left out in the first place. Repairing the omission and applying the principle anew would yield, of course, a different formula.

Example 24.—Assuming that the drag of an airship envelope of a particular shape depends only on its velocity (V), its volume (vol.) and the density of the air, precisely how is the drag (D) affected by these three factors?

A formula is required for D in terms of ρ, V and the volume. Introducing a constant coefficient C, write this first as

$$D = C\, \rho^p V^q\, (\text{vol.})^r$$

and then in the dimensional form

$$\frac{\mathsf{ML}}{\mathsf{T}^2} = \left(\frac{\mathsf{M}}{\mathsf{L}^3}\right)^p \left(\frac{\mathsf{L}}{\mathsf{T}}\right)^q (\mathsf{L}^3)^r.$$

Solving for the indices in order to secure the same dimensions on the right as on the left gives

$$\begin{aligned} &(\mathsf{M}) \quad . \; . \; . \quad p = 1 \\ &(\mathsf{L}) \quad . \; . \; . \quad 1 = -3 + q + 3r \\ &(\mathsf{T}) \quad . \; . \; . \; -2 = -q \end{aligned}$$

so that $r = 2/3$.

Hence the required formula is

$$D = C \, . \, \rho V^2(\text{vol.})^{2/3}.$$

Thus the assumption in the question is consistent with the so-called velocity-squared law. It is known, however, that the assumption makes an important omission, and including the missing factor destroys that law. Nevertheless, the formula obtained is sufficiently accurate for restricted changes of density, speed and volume.

Example 25.—Obtain a formula for the drag of a flying-boat hull during the run prior to take-off, assuming it to depend only on the waves continually formed on the surface of the water.

It is reasonable to suppose that the formation of waves will depend in some way on the density ρ of the water, the velocity V and linear size l of the hull, and the acceleration g due to gravity, the last factor being included because water is continually being raised against it. The formula is written in the general form

$$D = C\rho^p V^q l^r g^s$$

and re-writing it in dimensional form and solving as far as possible for the indices, as indicated above, gives

$$p = 1, \; q = 2 - 2s, \; r = 2 + s.$$

It is not possible to eliminate s because there are 4 unknowns and only 3 equations can be formed, viz., those in respect of M, L and T. Thus the formula cannot be reduced farther than to

$$D = C \, . \, \rho V^{2-2s} l^{2+s} g^s = C \, . \, \rho V^2 l^2 \, . \left(\frac{V^2}{lg}\right)^{-s}.$$

Nothing whatever has been discovered concerning the index s, and, at first sight, the investigation may appear to have achieved little. But this is far from true, for it has indicated a rule by which the drag may be inferred from an experiment with a scale model of the hull. If the drag of the model is measured at such a speed that the quantity in the brackets has the same value for the model as for the hull at the full-scale speed, then this quantity raised to the power $-s$ will be the same in the two cases, whatever s may be, and the full-scale drag will be greater than that of the model precisely in the ratio of the cubes of the sizes, since

$$D_1/D_2 = V_1^2 l_1^2/V_2^2 l_2^2 \text{ while } V_1^2/V_2^2 = l_1/l_2.$$

The formula has been used for many years in Naval Architecture, and forms the basis of tests in ship's tanks on flying-boat models. A ship's tank is a long trough of water along which the model can be pulled by means of a carriage travelling on rails.

When the ratio V^2/lg (or V^2/l, since g is a constant) has the same value for the model and the hull, they are said to be moving at corresponding speeds. The ratio is very convenient, since the speed of the model is required to be less than that of the hull; thus if the model is to one-sixteenth scale, its corresponding speed is only one-quarter of the full-scale speed. But wave-making accounts for only part of the water-resistance of the hull; the other part is akin to an aerodynamical drag and much less amenable.

62. Returning now to the main problem enunciated in Article 59, it is tackled in just the same way as have been the simpler questions already illustrated. Search is made for a general formula expressing the aerodynamic force that arises on all bodies of a constant geometrical shape in terms of other essential variables.

The formula must contain all the factors, except shape, that affect aerodynamic force. What these factors are can mostly be arrived at by simple experiments; for instance, the following:—Choose two bodies of different size but geometrically similar in shape—e.g., two rectangular boards, one twice the length, width and thickness of the other. The requirement of geometrical similarity includes constancy of attitude to the direction of motion, so decide, for simplicity, to move the boards broadside-on. Waving each through the air reveals two factors on which the aerodynamic force depends—viz., size and velocity. Now move either board deeply immersed through water. The greater resistance felt indicates that a third factor is the density of the fluid. Finally, movement through thick oil or treacle shows that there must be a fourth factor because the density is little different from that of water. This last factor is the viscosity of the fluid.

It is important that no essential factor be omitted. Further consideration suggests that the compressibility of the air might be added. Its effect could not be felt by waving a board, but that it will influence the aerodynamic force is suggested by Article 57. On the other hand, that article concludes that the pressures will be affected only at exceptionally high speeds. For simplicity, therefore, and as representing usual conditions, we can omit compressibility as a factor on the understanding that speeds will not become unduly high.

Considering, then, a series of geometrically similar bodies—all, that is, of one and the same shape, whatever that shape may be—differing widely in size, let them move at any agreed constant attitude but with different speeds through air at various altitudes. They may equally well be considered to move through different fluids, liquid and gaseous, provided only that in the case of gas the velocities are not exceptionally high and that in the case of liquids no surface waves are formed.

The size of the body will be denoted by l, which may be any agreed dimension—e.g., the length, or width—the velocity by V; the density of the fluid by ρ; and its viscosity by μ (the Greek letter mu). The aerodynamic force will be denoted by A.

63. Derivation of the Formula

If A is to stand alone on the left-hand side of the formula, the right-hand side must be so arranged as to have the dimensions of a force—viz., $\mathsf{ML/T^2}$—because A is a force. In other words, the factors l, V, ρ, μ must be combined together in such a manner as to haye collectively these dimensions. There may occur a number of terms on the right-hand side of the formula, separated by + and − signs; in that case the requirement must be satisfied by each term separately and it will be sufficient to deal with any one of them.

The right-hand side, or the typical term, is written in the form

$$\rho^p l^q V^r \mu^s \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\mathrm{i})$$

and the indices are suitably determined, so far as this is possible.

Written dimensionally, the product gives

$$\left(\frac{\mathsf{M}}{\mathsf{L^3}}\right)^p \times \mathsf{L}^q \left(\frac{\mathsf{L}}{\mathsf{T}}\right)^r \times \left(\frac{\mathsf{M}}{\mathsf{LT}}\right)^s$$

$$= \mathsf{M}^{p+s} \times \mathsf{L}^{q+r-3p-s} \times \mathsf{T}^{-r-s}.$$

For this to come to ML/T^2, we must have

$$1 = p + s$$
$$1 = q + r - 3p - s$$
$$-2 = -r - s$$

Solving these simultaneous equations for the indices, the first gives $p = 1 - s$ and the last $r = 2 - s$, so that substituting for r and p in the second gives $1 = q + 2 - s - 3(1 - s) - s$, i.e., $2 = q + s$. Hence the indices in (i) must be related as follows :

$$p = 1 - s$$
$$q = r = 2 - s.$$

This is as far as Dimensional Theory can guide us. There are four unknowns—viz., p, q, r and s—and only three equations can be constructed for their evaluation—viz., those in respect of M, L and T. One of the unknowns, therefore, must be left undetermined. However, the aid rendered is extremely effective, as will shortly be seen.

The required formula for A is

$$A = \rho^{1-s} l^{2-s} V^{2-s} \mu^{s},$$

or, on the right-hand side, the algebraic sum of a number of terms of the same form. The formula can be arranged as

$$A = \rho V^2 l^2 \cdot \left(\frac{\rho V l}{\mu}\right)^{-s} \quad . \quad . \quad . \quad . \quad . \quad (19)$$

This equation is of far-reaching importance, forming the backbone, as it were, of experimental Aerodynamics. Expressed in rather better form, as will be possible after preliminary discussion, it is known as Rayleigh's formula, having been obtained and expounded by the late Lord Rayleigh in the early days of the British Advisory Committee for Aeronautics, now the Aeronautical Research Committee.

64. Preliminary Discussion

To save repetition, it will be remembered that the following remarks apply to any one geometrical shape of body only, as assumed in the analysis. It will also be realised that Dimensional Theory cannot discriminate between the aerodynamic force and any component of it, for all components have the same dimensions. Thus (19) holds equally with lift or drag written in place of A, although the unknown index s would be varied thereby. Two vital conclusions follow at once from the analysis :

(1) However A (or the lift, drag or other component of A) varies with size for a given shape, it must also vary in the same manner with V. It follows that size and speed are often interchangeable, a concept which is put to practical use.

(2) A cannot vary as the square of the speed, or as the area of the body, for if it did we should have $s = 0$—i.e., A would be independent of the viscosity, which is demonstrably absurd. Thus whenever A appears in experiment to vary with V^2, as it often does, we know this can be only an approximation, good or bad, holding through the range of the particular experiments and liable to break down beyond that range. For example, the drag of spheres increases through a wide range approximately in proportion to V^2, but then, with little warning, increase of speed actually reduces their drag.

The Reynolds Number.—The composite quantity $\rho Vl/\mu$ forms a single variable of the greatest significance in Aerodynamics. It is non-dimensional; an attempt to find its dimensions gives

$$\frac{\mathsf{M}}{\mathsf{L}^3} \times \frac{\mathsf{L}}{\mathsf{T}} \times \mathsf{L} \times \frac{\mathsf{LT}}{\mathsf{M}}$$

and all are seen to cancel. In other words, it is a pure number, whose value in any case of motion is easily calculated, as will be described later. It is called the Reynolds number, after Professor Osborne Reynolds, who first discovered its significance.

The ratio μ/ρ is called the kinematic coefficient of viscosity of the fluid, or simply the *kinematic viscosity* It occurs so frequently that a special symbol has been assigned to it—viz., ν (the Greek letter nu). In these terms the Reynolds number becomes Vl/ν. Both forms of expression are in use.

It is essential to understand clearly that the Reynolds number is a single variable in the sense that, though its value is continually required, there is no significance whatever in how this value is contributed to by ρ, V, l, μ or ν separately. Thus, if we require to double the Reynolds number, we may double the speed or double the size or halve the kinematic viscosity, just as we please, a concept which again is of great practical utility. The expression for the Reynolds number—viz., $\rho Vl/\mu$ or Vl/ν—must be remembered in order to calculate its value in any given case. But, since it is a single variable, there is no advantage in retaining these details in (19). Therefore, the Reynolds number is usually written simply as R.

The Index s.—Nothing whatever has been determined or revealed by the dimensional analysis regarding the form of the remaining index s. Lengthy experience with a few geometrical shapes indicates that this index is of a most complicated form. It is useless to attempt its determination. There is consequently no advantage in retaining it in (19). All we can say from (19) is that the aerodynamic force, or any component of it, arising on bodies of any single geometrical shape, is equal to the product of $\rho V^2 l^2$ and a factor which is not a constant, even for that one shape, but whose value depends in some unknown way solely upon the value of the Reynolds number. It is for experiment to obtain the factor (when it can) for any particular Reynolds number that may be of interest. This limitation of knowledge is suitably reflected by writing in lieu of $(\rho V l/\mu)^{-s}$ the symbol $f(R)$, which stands for the words, 'a function of the Reynolds number'.

65. Rayleigh's Formula

The expression (19) is written for brevity, with the above understanding,

$$A = \rho V^2 l^2 \,.\, f(R) \qquad (20)$$

This is the form in which Rayleigh's formula is perhaps best remembered, being the form which is most convenient when only size and speed are changed and the air remains in a constant state.

Another form, especially convenient if the state of the air varies widely, is obtained as follows. The formula preceding (19) can clearly be re-written :

$$A = \frac{\mu^2}{\rho} \,.\, \rho^{2-s} l^{2-s} V^{2-s} \mu^{s-2}$$
$$= \frac{\mu^2}{\rho}\left(\frac{\rho V l}{\mu}\right)^{2-s},$$

leading in the same way as before to

$$A = \frac{\mu^2}{\rho} \,.\, f_1(R) \qquad (21)$$

The suffix is added to f to denote that the function of R is different from that occurring for the same geometrical shape in (20).

66. Calculation of the Reynolds Number

This article sets out rules for calculating the Reynolds number of any aerodynamical motion. It is nearly always a large number, often running into millions owing to the fact that μ amounts only to a few ten-millionths of a slug per foot-second. Since ρ is also rather small, there is an arithmetical convenience in evaluating R in the form

$$R = \frac{Vl}{\mu/\rho} = \frac{Vl}{\nu},$$

combining μ with ρ as a first step.

ρ. The density is that of the undisturbed stream, or of the air before disturbance by the body moving through it. At 15° C. and 760 mm. pressure, $\rho = \rho_0 = 0{\cdot}00238$ slug per cubic foot. This value is therefore appropriate to very low altitudes in the standard atmosphere. At higher altitudes in this atmosphere, $\rho = 0{\cdot}00238\ \sigma$, and the relative density σ is given by Table I or Fig. 16. For air in any other state, ρ is calculated from equation (4), Article 28, and a knowledge of the pressure and temperature.

Example 26.—Air enclosed in a vessel at 5 atmospheres pressure has a temperature of 47° C. What is its density? (Note: large static pressures are often expressed, as illustrated, in terms of the atmospheric pressure, by which is meant the standard sea-level pressure of 2116 lb. per sq. ft.)

Directly, equation (4) gives from the question, since $\tau = 273 + 47 = 320°$ C.,

$$5 \times 2116 = gB\tau\rho = 32{\cdot}2 \times 96 \times 320 \times \rho,$$

whence $\rho = 0{\cdot}0107$ slug per cu. ft.

Rather more neatly, the equation gives, with suffix 0 distinguishing standard sea-level values,

$$p_0 = gB\tau_0\rho_0 \text{ and } p = gB\tau\rho,$$

whence, dividing the second of these expressions by the first and substituting from the question,

$$\frac{\rho}{\rho_0} = \frac{p}{p_0} \cdot \frac{\tau_0}{\tau} = 5 \times \frac{288}{320} = 4{\cdot}5.$$

This gives the same answer as before.

μ. The coefficient of viscosity μ of a given gas depends only on the temperature; by *Maxwell's Law* it is independent of the pressure and density. For air at 15° C.,

$$\mu_0 = 3{\cdot}72 \times 10^{-7} \text{ slug per foot–second} \quad . \quad . \quad (22)$$

and it varies for this gas directly as $\tau^{3/4}$. Fig. 32 shows how its value decreases through the standard atmosphere to a constant value in the stratosphere.

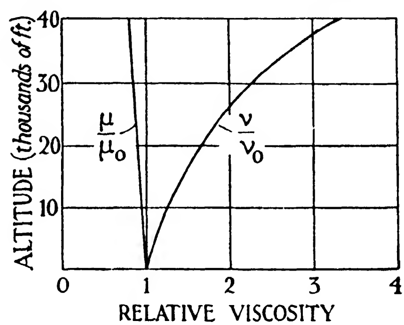


FIG. 32.—VARIATION OF THE VISCOSITY μ AND THE KINEMATIC VISCOSITY ν THROUGH THE STANDARD ATMOSPHERE.

Example 27.—Determine the viscosity of the air of Example 26.

The compression has no effect and, since τ is 320° C. for this air, the value given in (22), which is for $\tau = 288°$ C., is to be increased in the ratio

$$(320/288)^{3/4} = 1{\cdot}082.$$

Hence the required value is $4{\cdot}03 \times 10^{-7}$ slug per ft.–sec.

ν. The calculation of R is more concerned, however, with the ratio μ/ρ—i.e., the kinematic coefficient of viscosity ν, which depends on both the temperature and the density. For air at 15° C. and 760 mm. pressure,

$$\nu_0 = 0{\cdot}000156 \text{ square feet per second} \quad . \quad . \quad (23)$$

Corresponding to this value, it is useful to note that the Reynolds number under standard sea-level conditions is given closely by

$$R = 6400\ Vl \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

Fig. 32 also shows how ν increases with altitude in the standard atmosphere, the gain through a diminishing density more than offsetting the loss through a falling temperature.

The value of the kinematic viscosity under other conditions is most readily derived from the standard sea-level value (23), for

$$\frac{\nu}{\nu_0} = \frac{\mu/\rho}{\mu_0/\rho_0} = \frac{\rho_0}{\rho} \cdot \frac{\mu}{\mu_0}$$

$$= \frac{1}{\sigma}\left(\frac{\tau}{288}\right)^{3/4} \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

Or again, since by equation (4) of Article 28

$$\frac{\rho_0}{\rho} = \frac{p_0}{p} \cdot \frac{\tau}{\tau_0}$$

(cf. Example 26), ν/ν_0 can be expressed in terms of the relative pressure thus

$$\frac{\nu}{\nu_0} = \frac{p_0}{p}\left(\frac{\tau}{288}\right)^{7/4} \quad . \quad . \quad . \quad . \quad . \quad (26)$$

Example 28.—Determine the kinematic viscosity of the air of Example 26.

Directly from (26),

$$\nu = \frac{0{\cdot}000156}{5}\left(\frac{320}{288}\right)^{7/4} = 0{\cdot}0000312 \times 1{\cdot}202$$

$$= 0{\cdot}0000375 \text{ sq. ft. per sec.}$$

Alternatively from (25), since by Example 26 $1/\sigma = 0{\cdot}00238/0{\cdot}0107 = 0{\cdot}222$ whilst by Example 27 $\mu/\mu_0 = 1{\cdot}082$,

$$= 0{\cdot}000156 \times 0{\cdot}222 \times 1{\cdot}082 \text{ sq. ft. per sec.,}$$

giving the same answer as before.

V. The velocity V in the expression for the Reynolds number is the true air speed expressed in feet per second. Thus, if only V_i, the indicated air speed in miles per hour, is known, and the altitude is such that the relative density of the air is σ, the required velocity is obtained from (17),

$$V = \frac{22}{15}\frac{V_i}{\sqrt{\sigma}}.$$

If, on the other hand, the body is held stationary in a wind, V is the velocity in feet per second of the undisturbed wind.

An exceptional case of practical interest occurs when the Reynolds number is required for a small body situated close to a large one—e.g., for a streamline object under a wing. Considering for a moment the small body to be removed, the local wind velocity in the region it occupied will differ from the undisturbed velocity of the wind, being modified by the presence of the wing; Example 11, Article 44, illustrates the effect. The Reynolds number of the small object is to be specified on this locally modified velocity. A corresponding local variation of the kinematic viscosity may usually be ignored.

Example 29.—An aeroplane is flying at an indicated air speed of 270 m.p.h. at an altitude where the relative density is 0·81. A small component part is exposed in a region where the pressure is increased by 23 lb. per sq. ft. in the absence of the part. What velocity should be used in calculating the Reynolds number of the part?

Denoting by V_1 the relative velocity of the aeroplane and by V_2 the relative velocity of the body, we have by Bernoulli's Theorem

$$p_1 + \tfrac{1}{2}\rho V_1^2 = p_2 + \tfrac{1}{2}\rho V_2^2$$

or

$$\frac{p_2 - p_1}{q} = 1 - \left(\frac{V_2}{V_1}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad \text{(i)}$$

where $q = \tfrac{1}{2}\rho V_1^2$, ρ being the density at the altitude.

Using the data in the question,

$$V_1 = \frac{22}{15}\frac{270}{\sqrt{0{\cdot}81}} = 440 \text{ ft. per sec.}$$

$$q = \tfrac{1}{2} \times 0{\cdot}81 \times 0{\cdot}00238 \times (440)^2 = 186{\cdot}6 \text{ lb. per sq. ft.}$$

Substituting in (i),

$$\frac{23}{186{\cdot}6} = 1 - \left(\frac{V_2}{440}\right)^2,$$

whence

$$\frac{V_2}{440} = \sqrt{0{\cdot}877} = 0{\cdot}936.$$

Thus the required velocity = 412 ft. per sec.

l. The length *l*, on which the Reynolds number is further specified, may be any agreed dimension of the body concerned, but in Aerodynamics it is usual to choose the maximum length in approximately the direction of motion. Thus for an aeroplane fuselage, *l* would be the extreme length from nose to tail and would be reckoned the same for flight on an even keel or with the tail held down, in spite of the fact that in the latter case the length measured in the direction of motion might actually be slightly less. For a wing of tapered or shaped plan-form, the mean length of the wing section, called the mean 'chord', is conventionally chosen. Any unusual case is dealt with by adding a statement as to what convention has been adopted regarding this length. *l* must be expressed in feet.

67. Examples

Once the foregoing simple rules are understood, no difficulty will be experienced in calculating R. The following examples illustrate the process and give some idea of the magnitudes of the numbers to be expected.

Example 30.—An airship, 800 ft. in length, cruises at 75 m.p.h. at an altitude of 10,000 ft. Find the Reynolds number of its hull.

From Table I, $\sigma = 0{\cdot}738$, giving $\sqrt{\sigma} = 0{\cdot}859$, whence

$$V = \frac{22}{15} \times \frac{75}{0{\cdot}859} = 128 \text{ ft. per sec.}$$

Assuming the atmosphere to be in the standard condition, the temperature of the air is $-4{\cdot}8°$ C. by Table I, giving $\tau = 268{\cdot}2°$ C. and then

$$\nu = \frac{0{\cdot}000156}{0{\cdot}738} \times \left(\frac{268{\cdot}2}{288}\right)^{3/4} = 0{\cdot}00020 \text{ sq. ft. per sec.}$$

Hence

$$R = \frac{128 \times 800}{0{\cdot}0002} = 512 \text{ millions.}$$

Example 31.—The mean length of section (mean chord) of the wings of a flying-boat is 15 ft. Find their Reynolds number at a speed of 240 m.p.h. at low altitude.

From (24), since 240 m.p.h. = 352 ft. per sec.,

$$\text{R} = 6400 \times 352 \times 15 = 33{\cdot}8 \text{ millions.}$$

Example 32.—A one-thirtieth scale model of the wings of the flying boat of Example 31 is tested in a wind tunnel working at 100 ft. per sec. What is the Reynolds number of the test?

$$\text{R} = 6400 \times 100 \times 15/30 = 0{\cdot}32 \text{ million} = 320{,}000.$$

Example 33.—The section of a streamline tube exposed on an aeroplane, whose speed is 150 m.p.h., measures $\frac{1}{2}$ in. across the direction of motion and has a fineness ratio of 3. Find its Reynolds number at low altitude ignoring interference from the aeroplane.

The length of the section is $1\frac{1}{2}$ in. = 0·125 ft. The velocity is 220 ft. per sec. Hence

$$\text{R} = 6400 \times 220 \times 0{\cdot}125 = 176{,}000.$$

From these examples will be noted the wide range of Reynolds numbers occurring in Aerodynamics; their large magnitudes, leading to habitual expression in millions; and also the great difference between the Reynolds number of a large aircraft component in full-scale flight and that of a small model under test in atmospheric air.

68. Measurement of f(R)

Having seen how to assess the Reynolds number of any given aerodynamical motion, we proceed to discuss further the important formulæ of Article 65. The first of these gives

$$\frac{A}{\rho V^2 l^2} = f\left(\frac{\rho V l}{\mu}\right) = f(R) \quad . \quad . \quad . \quad . \quad (27)$$

showing that the value of $f(R)$ can be measured for any experimentally suitable value of R; for A, ρ, V, l and μ can all be determined and both R and $A/\rho V^2 l^2$ evaluated. The last quantity is a pure number, like the first, writing out its dimensions giving

$$\frac{\mathsf{ML}}{\mathsf{T}^2} \div \left(\frac{\mathsf{M}}{\mathsf{L}^3} \times \frac{\mathsf{L}^2}{\mathsf{T}^2} \times \mathsf{L}^2\right).$$

It is a non-dimensional coefficient, but not a constant one, its value varying, though the geometrical shape of the body be kept constant. By stating that it is a function of the Reynolds number for any one shape, the formula asserts that its variation then depends solely on the variation of R. The formula has nothing to say as to the manner in which these two numbers, the coefficient and R, vary together, but the limited claim it makes, if justified, is nevertheless of outstanding significance. For it follows at once that if R remain constant, so will also the coefficient, however size, speed or the physical properties of the fluid may vary.

The claim is so important that it is as well to recapitulate the restrictions on which it rests. Regarding first the series of bodies considered, geometrical similarity must be strictly conserved; it must extend, for instance, to the attitude of the body and the degree of roughness of its surface. Gravitational effects on the motion of the fluid have been excluded, whence it follows, for instance, that if one of the bodies is moved through liquid contained in a tank, then immersion must be so deep that no waves are formed on the surface of the liquid. If the fluid is air or another gas, the velocity must not be so great as to call compressibility into play. Finally, if the relative motion between the fluid and the body is secured by holding the latter stationary in a stream or wind, instead of moving it through fluid initially at rest, then the oncoming stream or wind must be steady and uniform before disturbance by the body. The last requirement is not easy to satisfy, but some of the most modern wind tunnels succeed in providing very good approximations to steady and uniform streams.

By complying with these conditions and measuring A, ρ, V, l, μ in a series of experiments carried out on some geometrical shape, a series of values of $f(R)$ can be obtained for a series of values of R. If the formula is adequate and its claim justified, we know that the value of $A/\rho V^2 l^2$ will always be the same for any one value of R; if, for example, one of the bodies is four times as large as another and is tested in the same fluid at one-quarter of the speed of the other, then the coefficient will be the same for both because R is the same. It follows that plotting all the ascertained values of the coefficient against R will yield a single curve for that particular shape. This curve will be the graphical representation of $f(R)$.

The construction of a curve of this type, covering a wide range of the Reynolds number, involves extensive and careful experimental work and often the use of large or costly apparatus. Nevertheless, the research has been carried out for a few shapes of body. The results justify the formula, all results lying close to a single curve for each shape, as predicted.

The conclusion regarding aerodynamic force applies equally to any component of it—e.g., the lift or drag—but the curve will be different for each component. Changing the shape or attitude of the body changes all such curves.

69. Illustrations

Fig. 33 gives graphical representations of $f(R)$ for bodies of four shapes, viz. (*a*) long plates normal to the wind, (*b*) long circular cylinders normal to the wind, (*c*) spheres and (*d*) thin plates parallel to the wind. In each case A is a drag, and for (*a*), (*b*) and (*d*) A is the drag of a length l across the stream, where l is the maximum width of the body. The Reynolds numbers are also specified on this length. There is no need for the symbol l in the coefficient $A/\rho V^2 l^2$ to have the same meaning but only that l^2 shall denote a representative area of the body. For ease of future reference l^2 is made equal to the area S defined

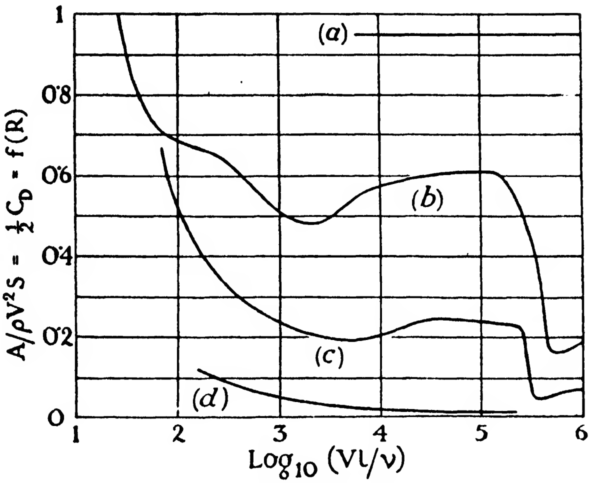


FIG. 33.—EXAMPLES OF $f(R)$.

(a) Long normal plates, (b) long circular cylinders, (c) spheres, (d) thin flat plates. Let l be the maximum width of a long plate or cylinder. Then for (a), (b), (d), $S = l^2$. For (c), $S = \frac{\pi}{4}l^2$, where l is the diameter of the sphere. [Note : Some experiments suggest rather larger values for (a).]

below the figure, the coefficient then becoming one-half the conventional 'coefficient of drag C_D'—i.e., $\frac{1}{2}C_D$. It is plotted against *log R* instead of R in order to open out the horizontal scale when R is small and compress it when R is large.

Referring to (b), the flow is steady for R less than 100. The regular eddying wake depicted in Fig. 22 makes its appearance at approximately that Reynolds number and continues until R reaches a value of about 200,000. But then the 'vortex street' breaks up into a pack of small vortices, the wake narrows and the drag coefficient rapidly decreases, as shown.

Little variation appears in $f(R)$ for normal plates at ordinary Reynolds numbers. The curve (d) for flat plates rises and falls in a marked manner at Reynolds numbers ranging from $\frac{1}{2}$ million to 10 millions, though the scale of the present figure is unsuitable for illustration.

It will be seen that $f(R)$ is nearly constant for (b) and (c) between $R = 20,000$ and $R = 200,000$. Some years ago, when the highest Reynolds numbers of experiment reached to only a small fraction of a million, rapid change in the value of $f(R)$ seemed generally to subside within their limited range, and it was thought that further changes could be ignored. Were this idea justifiable it would simplify aircraft design, for measurements made on models could be applied directly to full scale with no more precaution than that of avoiding very small Reynolds numbers in the experiments. But instances could be multiplied to show, like those given, that the notion cannot be justified; important changes continue to occur in $f(R)$ even at the Reynolds numbers of large full scale.

70. Use of a f(R) Curve

An experimental curve such as that just described supplies very complete information. To put it to practical use, all that is necessary is to work out the Reynolds number in the given circumstances, read off from the curve the corresponding value of the coefficient, and finally calculate directly from this coefficient the required aerodynamic force.

Example 34.—A long wire, $\frac{1}{16}$ in. diameter, is exposed on an aeroplane. What is its drag per ft. run at 240 m.p.h., A.S.I., at an altitude where $\sigma = 0{\cdot}64$ and the temperature is $-14°$ C.?

Ignoring interference from the aeroplane,

$$V = 440 \text{ ft. per sec.}$$

By (25), since $\tau = 259°$ C.,

$$\nu = 0{\cdot}000225 \text{ sq. ft. per sec.}$$

In order to use Fig. 33, l must be the diameter of the wire, i.e.,

$$l = \tfrac{1}{16}/12 = 0{\cdot}00521 \text{ ft.}$$

These give

$$R = \frac{Vl}{\nu} = \frac{440 \times 0{\cdot}00521}{0{\cdot}000225} = 10{,}190.$$

Fig. 33 shows the corresponding value of $A/\rho V^2 l^2$ to be 0·57. Also

$$\rho V^2 = 0{\cdot}64 \times 0{\cdot}00238 \times 440^2 = 295 \text{ lb. per sq. ft.}$$

Hence the drag of one foot length, away from the ends of the wire, is

$$0{\cdot}57 \times \rho V^2 l^2 \times 1/l$$
$$= 0{\cdot}57 \times 295 \times 0{\cdot}00521 = 0{\cdot}875 \text{ lb.}$$

Other examples, in which lift and drag coefficients are employed, are given later on.

71. Isolated Tests in Atmospheric Air

The large error which may arise if a coefficient determined at one Reynolds number is applied to calculate the aerodynamic force at a very different Reynolds number has already been illustrated. On the other hand, building up an adequate $f(R)$ curve for each shape of body used in flying is far too ambitious a scheme. Preceding articles suggest immediately, however, a method by which this labour may be avoided, without loss of accuracy, in the case of a body whose full-scale size is small.

Knowing beforehand the Reynolds number of the body in flight, it may be possible to carry out a single experiment on a scale model of the body at the same Reynolds number. When this can be done, the single coefficient determined in the experiment will evidently apply exactly to the full-scale case; failure to do so would mean only that the experimental technique needed improvement.

If the experiment is made in air of the same condition as that through which the flight occurs, so that the kinematic viscosity is the same, then for R to be constant the product Vl must have the same value in the two cases. The experimental velocity V_M will usually be much less than the flight speed V_F, and then the model must be correspondingly larger than the aircraft component; for example, if $V_F = 400$ feet per second whilst V_M is limited to 100 feet per second, the model will require to be four times as large as the actual body. Having regard to economic restrictions on the size of experimental apparatus, it will be obvious that such magnified models can be handled only in respect of small aircraft parts and that the method cannot be applied to large parts, such as wings and fuselages.

At first sight it may appear that an alternative method, suitable for the larger components or complete aeroplanes, would be to experiment with models of reduced size at speeds correspondingly greater than those of flight. This would just have been possible in the early days of mechanical flight when aeroplanes were of small size and their speeds seldom much more than 50 miles per hour. Had the funds been available, arrangements could then have been made to test a one-sixth scale model of a complete aeroplane in an artificial wind of 300 miles per hour. But the method has long ceased to be applicable apart from economic considerations, owing to the growth of aircraft sizes and speeds; the latter are now so high that any substantial further increase called for in experiment would introduce compressibility effects.

72. Isolated Tests in Compressed Air

There is one proved method, however, by which complete aircraft or their main components can be tested in experiments on small models at full-scale Reynolds numbers, provided the latter are limited at full scale by either rather small values of the product Vl or a substantial increase in ν due to high altitude. This method consists in suspending the model in a stream of highly compressed air which circulates in a wind tunnel of specialised construction, known as the compressed air tunnel (and commonly referred to as the C.A.T.).

The apparatus is briefly described in the next chapter, but the principle will at once be evident. Writing the Reynolds number in the form

$$R = \frac{\rho V l}{\mu},$$

we note that by Maxwell's Law μ is independent of the pressure, while by Boyle's Law ρ is directly proportional to the pressure for constant temperature. Hence R increases, for constant temperature, in direct proportion to the compression. The following example illustrates the scope of the artifice.

Example 35.—A one-fifteenth scale model of an aeroplane is tested in a C.A.T. working at 100 ft. per sec., 25 atmospheres pressure and 15° C. Supposing the aeroplane to be flying at 30,000 ft. altitude, at what speed will its Reynolds number be the same as that of the model test?

Denoting model conditions by suffix M and full-scale conditions by suffix F, it is required that

$$\frac{\rho_F V_F l_F}{\mu_F} = \frac{\rho_M V_M l_M}{\mu_M},$$

and the condition gives

$$V_F = \frac{\rho_M}{\rho_F} \cdot \frac{l_M}{l_F} \cdot \frac{\mu_F}{\mu_M} \cdot V_M.$$

From Table I, $\rho_F = 0{\cdot}375\ \rho_0$, whilst from the question $\rho_M = 25\ \rho_0$. Also $\tau_F = 228{\cdot}6°$ C. Hence

$$V_F = \frac{25}{0{\cdot}375} \times \frac{1}{15} \times \left(\frac{228{\cdot}6}{288}\right)^{3/4} \times 100$$

$$= 374 \text{ ft. per sec.}$$

In the above example the model might have a span of 4 feet, when that of the aeroplane would be 60 feet. The true air speed of 255 miles per hour (= 374 feet per second) corresponds to an indicated air speed of $255 \times \sqrt{0{\cdot}375} = 156$ miles per hour. Thus full-scale Reynolds numbers of practical interest can be realised by the method. It would be used much more widely but for the large cost of the apparatus.

73. Dynamical Similarity

Two or more motions which satisfy the conditions summarised in Article 68 and for which also the Reynolds number is the same are said to be dynamically similar. The great *Principle of Dynamical Similarity* is far-reaching, and the above provides only a single instance of its application. Discussion of the principle is beyond the scope of this book, but its aid is so frequently invoked in Aerodynamics that some consideration, however inadequate, can hardly be postponed altogether to more advanced reading. There occur in Aerodynamics other motions of geometrically similar bodies for which the formula (20) requires fundamental extension and the criterion of dynamical similarity becomes different. Nevertheless, the conditions at present assumed are those most commonly experienced. The following description is restricted to this important though particular case.

The question immediately arising is: What is meant by dynamical similarity? When in addition to being dynamically similar the motions are also steady, the meaning is easily investigated, as follows.

Consider two geometrically similar bodies of different sizes suspended in uniform streams of the same or different fluids having different undisturbed velocities, subject always to the provision that R has the same value in the two cases. Then it can be proved that the streamlines of the two disturbed streams are geometrically similar. In other words, making the two sets of streamlines visible would reveal the one set to be a magnified version of the other. If the Reynolds number were not the same, this would not be true, and the two pictures would differ from one another.

The proof presents no difficulty and provides an interesting exercise on the method of Article 63. Precisely on the lines of that article, search is made for a general formula for the angle of the tangent to the streamline at any representative point in the flow. Now, angles are non-dimensional quantities, and so it is found that the angular deflection of the streamline is simply a function of the Reynolds number. Since this is true for all points, the streamlines cannot change in shape if the Reynolds number remains constant; they are subject only to a uniform expansion or contraction to suit the size of the body. Of course, admitting time as a factor instead of assuming steadiness of flow would introduce a complication.

Extension of geometrical similarity to the streamlines enables other details to be followed with ease. Let attention be focused on any pair of 'corresponding points'—i.e., any point in the flow past one body together with the particular point in the other flow which is geometrically similarly situated with reference to the other body.

By measuring the distance separating adjacent streamlines at one of these points, we can easily find the local velocity (cf. Example 11, Article 44). The local velocity can similarly be determined at the corresponding point in the other flow. In this way it is verified that, since the two sets of streamlines are identical except for a change of linear scale, adjacent streamlines approach or are displaced from one another proportionately, and the local velocities at any pair of corresponding points are related in the ratio of the undisturbed velocities.

Having investigated the dynamical similarity existing in regard to the velocity distributions, we can go on to compare the pressures at corresponding points outside the two boundary layers by means of Bernoulli's Theorem. They are found to be related in the ratio of the two values of ρV^2, V denoting the undisturbed velocity.

Proceeding on these lines we could eventually relate the aerodynamic force on the one body to that on the other. But this step may be achieved at less trouble, as described in the next article, by a method which is not restricted to steady flow as is the above argument.

74. Application to Aerodynamic Force

Considering again two dynamically similar motions governed by the formula (20), let them be distinguished from one another by the suffixes 1 and 2. Then the formula gives for the ratio of the two aerodynamic forces :

$$\frac{A_1}{A_2} = \frac{\rho_1 V_1{}^2 l_1{}^2 \,.\, f(R)_1}{\rho_2 V_2{}^2 l_2{}^2 \,.\, f(R)_2}.$$

Owing to geometrical similarity the same function of R applies to the two cases. And for dynamical similarity R itself is also the same. Therefore $f(R)$ has the same value. Hence

$$\frac{A_1}{A_2} = \frac{\rho_1 V_1{}^2 l_1{}^2}{\rho_2 V_2{}^2 l_2{}^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(i)}$$

Equating the Reynolds numbers,

$$\frac{\rho_1 V_1 l_1}{\mu_1} = \frac{\rho_2 V_2 l_2}{\mu_2}$$

whence

$$\frac{V_1{}^2 l_1{}^2}{V_2{}^2 l_2{}^2} = \left(\frac{\mu_1 \rho_2}{\mu_2 \rho_1}\right)^2.$$

Substituting in (i),

$$\frac{A_1}{A_2} = \frac{\rho_2}{\rho_1}\left(\frac{\mu_1}{\mu_2}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

This result follows equally, and more directly, from the formula (21) and, since $\nu = \mu/\rho$, may be expressed in terms of the kinematic viscosity as

$$\frac{A_1}{A_2} = \frac{\rho_1}{\rho_2}\left(\frac{\nu_1}{\nu_2}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

It will be observed that in order to relate the aerodynamic forces (or any given components of them) in two dynamically similar motions the only data required relate to the physical properties of the fluids. Some illuminating deductions follow.

75. Examples

The expressions (28) and (29) reveal the following in regard to two dynamically similar motions.

(*a*) If the fluid is the same, then $\rho_1 = \rho_2$, $\mu_1 = \mu_2$ and therefore

$$A_1 = A_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

i.e., the aerodynamic force is exactly the same. This covers experiments in atmospheric air relating to aircraft in flight at low altitude. For example, it is just possible to determine the stalling speed (supposed low) of a very small aeroplane, weighing some $\frac{3}{4}$ ton, by means of an experiment under dynamically similar conditions on a small model in a wind tunnel of ordinary size working at a high speed, say 250 miles per hour. The above result shows that the lift of the small model would also be $\frac{3}{4}$ ton.

(*b*) If the fluid is air at the same temperature in the two cases, then by Maxwell's Law $\mu_1 = \mu_2$, whence

$$\frac{A_1}{A_2} = \frac{\rho_2}{\rho_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

i.e., the aerodynamic force is inversely proportional to the density, or, since the temperature is constant, to the pressure. This result covers the application of experiments in compressed air to flight through atmospheric air of the same temperature, i.e., to flight at low altitude, in practice.

Example 36.—A $2\frac{1}{2}$ ton aeroplane in straight level flight at low altitude is tested under dynamically similar conditions by means of a model in a stream of compressed air, whose pressure is 24 atmospheres and temperature 15° C. What lift will the model exert?

The relationships deduced for A apply equally to any given component such as lift. The lift of the aeroplane is evidently $2\frac{1}{2} \times 2240 = 5600$ lb. Hence that of the model will be

$$5600/24 = 233\tfrac{1}{3} \text{ lb.}$$

(*c*) When experiments are carried out in atmospheric air at low altitude under dynamically similar conditions to flight at high altitude, both ρ and μ are different, but, since the fluid is air in the two cases, μ varies as $\tau^{3/4}$ and the expression (28) simplifies to

$$\frac{A_1}{A_2} = \frac{\rho_2}{\rho_1}\left(\frac{\tau_1}{\tau_2}\right)^{3/2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

Example 37.—A small body is exposed on an aircraft flying at 30,000 ft. altitude. A model is tested under dynamically similar conditions in an atmospheric wind tunnel and its drag measured to be $\frac{3}{4}$ lb. Deduce the drag of the body in flight.

With suffixes F and M distinguishing full-scale and model conditions respectively, directly from (32) and Table I:

$$\frac{A_F}{A_M} = \frac{1}{0{\cdot}375}\left(\frac{228{\cdot}6}{288}\right)^{3/2}$$

whence the required drag is

$$\frac{3}{4} \times \frac{8}{3} \times 0{\cdot}707 = 1{\cdot}41 \text{ lb.}$$

(The model would probably be larger than the body, perhaps twice as large, but this information is not required to answer the question.)

76. Aerodynamic Scale

The flow past bodies of given shape, even if so unobtrusive as flat plates, changes in nature as the Reynolds number reaches greater and greater values; Article 69 provides an illustration (Fig. 33) and others could be given. These changes may occur suddenly at certain Reynolds numbers or develop gradually. The aerodynamic force is affected by them and in this sense can be said to depend on the 'bigness' of the motion. A statement of linear size is meaningless in this connexion unless accompanied by information as to speed and kinematic viscosity. Equally, the speed alone conveys little, unless it be so high as to give warning of the imminence of compressibility effects. Only the Reynolds number can provide a proper scale on which bigness, in the aerodynamical sense, can be gauged. Considering for example the three motions:

(*a*) an aeroplane model, 3 feet long, in an airstream of 100 feet per second velocity and 25 atmospheres pressure;

(*b*) a geometrically similar aeroplane, 18 feet long, in low altitude flight at 300 miles per hour;

(*c*) an aeroplane of the same shape but 72 feet long, flying at 30,000 feet altitude at a true air speed of 150 miles per hour;

the smallest is the last, not the first; the ascending order of magnitude aerodynamically is (*c*) (*a*) (*b*), not (*a*) (*b*) (*c*).

The Reynolds number is consequently called the ***aerodynamic scale***, or simply the scale. To say, for instance, that a proposed aeroplane will have a scale of so-many millions is to mean that its Reynolds number, with l conventionally specified on the mean chord, will be of that magnitude.

When geometrical similarity is implied in the term scale, the criterion of dynamical similarity is that the motions occur at the same scale. This condition can often be satisfied by one expedient or another, as we have seen, and then the difficulties regarding the assessment of aerodynamic force are met half-way, for the motions are at least the same, whatever their nature may be.

Unfortunately, circumstances of practical urgency continually arise in which securing dynamical similarity for the purpose of determining aerodynamic force is theoretically impossible, or else beyond the world's existing resources, or (still more frequently) would involve delay in construction due to the rarity of suitable apparatus. In these circumstances we have to experiment up to as large a scale as possible, and make the best of it. Changes due to increase of scale are called scale effects, and those occurring between the ultimate experimental scale available and that of the aircraft may be called residuary scale effects. Uncorrected for, they may lead to the aircraft having a better or a worse performance than that forecast. Thus it is essential to investigate and assess them in any given case. For a new aeroplane of very large scale, the model experiments may be supplemented and extended by flying experiments carried out on a small edition, or a ' mock-up ', of the aircraft eventually in view. Otherwise the means available comprise (*a*) advanced theory, (*b*) accumulated experimental data of a general character, (*c*) practical experience—i.e., accumulated data regarding other full-scale aeroplanes.

The estimation of residuary scale effects, often tentative, is in any case too involved for present consideration. Geometrical similarity is lost, except in the rough sense that a wing, for example, will be compared with other wings, of as like a shape as possible, and not with bodies of an entirely different shape, such as fuselages. A wider use is therefore made in practice of the term scale than that above described; two wings, for instance, are still said to have the same scale, though they differ in shape or inclination to the direction of motion, provided their Reynolds numbers are the same. Of course, dynamical similarity then no longer occurs at any scale.

Chapter V

USING THE WIND TUNNEL

77. EXPERIMENTS in a wind tunnel form an important part of a course in Aerodynamics. Their first object is to illustrate the principles of the subject and demonstrate phenomena bearing upon aviation; their second object is to cultivate an instinctive knowledge of the ' habits ' of flowing air. To be *en rapport* with the susceptibilities of air flow, to be able to anticipate its response to a given disturbance, is an incomparable advantage alike to the experimental engineer, scientist or mathematician, the designer and the pilot.

Occasionally it is desirable to make the flow visible by means of smoke, or to observe its effects on tufts and streamers, but as a rule there is need for accurate measurements; to take an example, a deviation of the air stream amounting to 2° is only just perceptible to the eye, but an aeroplane responds noticeably to less than one-tenth of this angular change.

Precision is not easy to achieve in aerodynamical experiments, which call for careful preparation and skill. Nevertheless, it is readily possible for the student to arrive during his short apprenticeship at the stage of taking representative observations, even with rather crude apparatus and models.

This chapter does not describe the magnificent aerodynamical laboratories that now exist, nor the polished technique which has persuaded the wind tunnel to reveal so much that long remained inscrutable. Its aim is to outline only such apparatus and methods as can be encompassed in a small laboratory devoted to education rather than research. Systematic investigations with this modest equipment form the subject of Chapter VI; the few experiments now described are intended only to illustrate the operation of the apparatus and the reduction of results.

78. The Air Stream

The first requirement of an Aeronautical Laboratory is a satisfactory artificial wind, and the following properties are desirable.

The part of the stream used for measurements, commonly called the working section, should preferably be 4 feet or more in width; many interesting experiments can be made in air streams 2–3 feet wide, but others only with difficulty, if at all. The advantage of the rather larger size is operational; it permits of more robust and relatively compact apparatus; the gain in aerodynamic scale is only of interest in much larger sizes.

The aerodynamic scale being small, and the wind far too slow to illustrate compressibility effects, a high speed is not worth striving after. A maximum of 100 feet per second is adequate, but the speed must be adjustable and range upwards from some 20 feet per second, for scale effects are investigated more easily by varying the speed of the stream than the size of the model.

At each operational speed the air stream should be approximately uniform in three respects. Its velocity should be nearly constant in magnitude and direction over about four-fifths of the working section; there should be immunity from gusts or other easily perceived unsteadinesses; and there should be as much freedom as possible from turbulence in the sense of Article 47. These qualities are rather elusive, especially the last named, and wind tunnels have been developed primarily to secure an acceptable approximation to them.

79. Lay-out of Wind Tunnels

Wind tunnels are said to be *atmospheric* when they use air at approximately the same pressure as that of the surrounding

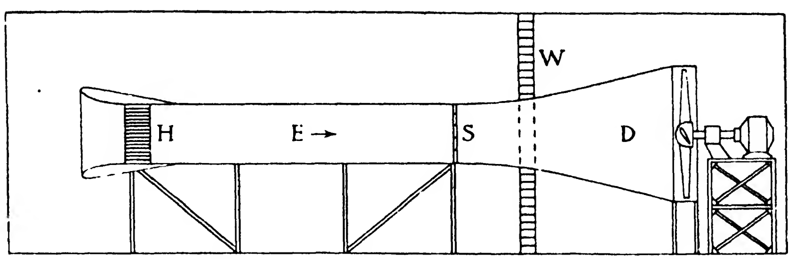


FIG. 34.—STRAIGHT-THROUGH OR OPEN-RETURN WIND TUNNEL.

atmosphere. With few exceptions all come within this category and are broadly of three kinds.

In the *open-return* or *straight-through* type shown diagrammatically in Fig. 34, the used air circulates back to the mouth of the tunnel through the laboratory itself. The *closed-return* type conveys the return stream within a divergent duct D, Fig. 35. Either type may be intended by the description *an enclosed-*

section tunnel; for in both the working stream is enclosed within a wooden or metal shell, whose section may be square, rectangular, circular, oval or of some other shape. But another variation provides for a working stream that is unrestrained by an enclosing wall and takes instead the form of a jet springing from a delivery nozzle N, Fig. 36, to a collector C. This is known as the *open-jet* type. Most examples arise from the 'race course' design illustrated, but the straight-through type can also be suitably modified; it was in this manner, in fact, that the open jet was introduced by Eiffel in 1909.

Before comparing one type of wind tunnel with another, some common features may be noticed.

Apart from a few specialised tunnels, the air streams are horizontal, and then it is advantageous for them to be level to a high degree of accuracy. The working stream is induced by a tractor airscrew so that the eddying discharge from the latter shall have least opportunity to affect the model. This discharge is straightened and brought to a low speed before being rapidly accelerated again into the working section, through a honeycomb H. The sole function of the grill or net screen S is to prevent loose objects from damaging the airscrew (its fitment does not relieve the student from the prime duty of verifying, on each and every occasion before starting up the driving motor, that nothing whatever can work loose and be caught up by the wind). The size of a tunnel is specified by the width—or the width and height, if these are not the same—of the working section, marked E in the figures. The electric motor is best coupled direct to the airscrew shaft. For control of speed it is usually equipped with generous field and armature resistances, to which may be added a carbon plate resistance for fine adjustment. The Ward-Leonard system is a great convenience. When a large battery exists and a tunnel may sometimes be engaged on difficult research, it is an advantage to arrange for the motor to be switched on special occasions from the mains to the battery.

80. Open-Return Tunnel

The design depicted in Fig. 34 was developed many years ago at the National Physical Laboratory, Teddington, and the Royal Aircraft Establishment, Farnborough. Examples of various sizes are still in use at the N.P.L. and elsewhere amongst numerous tunnels of more recent construction. Given a large room, this type is easy and inexpensive to construct. To accommodate the 4-foot size, the room needs to be some 55 feet long and 300 square feet in vertical cross-sectional area. Forming the return channel for the flow, it should be kept reasonably clear of furniture and especially clean; swirling dust is injurious to health besides being a nuisance and affecting some experiments.

Air enters the intake from all directions and is straightened and spread evenly across the tunnel by suitably adjusting the position of the honeycomb H. If the room is sufficiently high, a larger intake resembling the nozzle N of Fig. 35 may be preferred.

The skin friction of the tunnel wall gives rise to a boundary layer of sluggish flow penetrating only a short distance from the wall into the stream but gradually thickening along the tunnel. The stream in a parallel-sided tunnel slightly converges, therefore, since just as much air must pass S in unit time as H, and greater difficulty is experienced in flowing close to the wall at S than at H. Beyond S the air stream is gradually enlarged in section along the regenerative cone D and thereby slowed down before being engaged by the airscrew. The spinning and eddy-packed discharge from the airscrew is delivered back to the room through the honeycomb wall W or other perforated baffle. This step is unnecessary in case of a small tunnel in a large room.

Neglecting for the moment the honeycomb and screen, which oppose skin friction and form drag to the main stream, and so detract from its mechanical energy, we may apply Bernoulli's Theorem along a streamline which threads the tunnel but avoids the vicinity of the wall. Then we find that the pressure decreases rather suddenly through the intake, decreases again but only slightly and gradually through the parallel part of the tunnel, and finally increases fairly rapidly along the divergent cone, until the air arrives at the airscrew with a pressure little less than that in a sheltered corner of the laboratory. The airscrew makes up the relatively small deficit and communicates a little additional pressure to make the air flow back to the intake again. Thus the kinetic energy generated in the vicinity of the intake at the expense of the pressure energy is largely recovered before the used stream is returned to the room, being transformed again into pressure energy in accordance with the conservative principle expressed in Bernoulli's Theorem, and the return flow passes slowly back along the laboratory, wasting little energy and providing time for remaining eddies from the airscrew to die away.

Example 38.—A straight-through enclosed-section wind tunnel, 4 ft. 6 in. in diameter and fitted with a regenerative cone diverging to 9 ft. diameter, is working at 100 ft. per sec. Neglecting the boundary layer and other sources of energy loss and assuming the stream to fill the cone, determine the pressure difference in inches of water between a sheltered corner of the laboratory and (*a*) the working section, (*b*) the wide end of the cone.

Let suffixes 1, 2, 3 refer to the laboratory, the working section and the wide end of the cone, respectively. By Bernoulli's Theorem,

$$p_1 + q_1 = p_2 + q_2 = p_3 + q_3,$$

and it can be assumed that $V_1 = 0$, giving $q_1 = 0$.

Hence (*a*),

$$p_1 - p_2 = q_2 = \tfrac{1}{2}\rho V_2^2 = \tfrac{1}{2} \times 0{\cdot}00238 \times 100^2$$
$$= 11{\cdot}9 \text{ lb. per sq. ft.}$$

and the corresponding pressure head is

$$\frac{11{\cdot}9}{62{\cdot}4} \times 12 = 2{\cdot}29 \text{ inches of water.}$$

Again, the mean velocity of the stream is inversely proportional to its cross-sectional area, or, neglecting the boundary layer, $V_3 = V_2 \times (4\tfrac{1}{2}/9)^2 = \tfrac{1}{4}V_2$. Hence (*b*),

$$p_1 - p_3 = q_3 = \tfrac{1}{16}q_2$$
$$= 0{\cdot}744 \text{ lb. per sq. ft.}$$
$$= 0{\cdot}143 \text{ inches of water.}$$

In the practical case, the honeycomb involves a further pressure drop and prevents application of the method to determine the wind speed directly from $p_1 - p_2$. The screen and walls also give the airscrew work to do.

The regenerative action of the divergent cone greatly reduces the power required to drive a tunnel of the present type, but this remains considerable; it exceeds 35 horse-power for the 4-foot size at a speed of 100 feet per second and varies as the square of the size and the cube of the speed. Another disadvantage arises from fierce little whirlpools of air, which mature continually on the floor and ceiling of a low room in the neighbourhood of the intake and dart down the tunnel like snakes, momentarily upsetting the velocity distribution across the working section. However, the type still has several advantages, including the possibility of employing a very large airscrew, thereby minimising noise.

81. Closed-Return Tunnels

Returning the stream within a divergent duct can save two-thirds of the power required to maintain a wind of given section

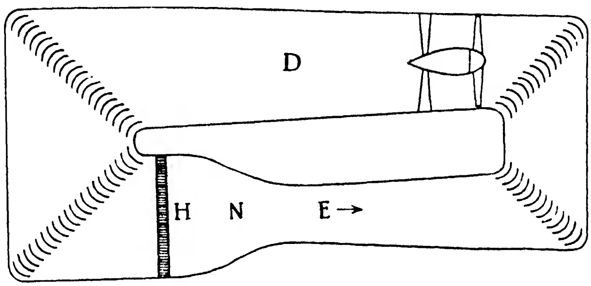


FIG. 35.—ENCLOSED-SECTION TUNNEL OF RACE-COURSE TYPE.

and speed. The closed-return type is also more economical in first cost when laboratory space has to be built, for a large clear room is not required. Little difference exists between the designs of the open- and enclosed-jet forms; in fact a single tunnel can be rapidly convertible from the one into the other; and one description almost suffices for both.

Essential features are as follows. The air received from the jet or working section has to be guided round at least four corners in such a manner as to preserve an approximately uniform distribution of velocity across each section. This is achieved by cascades of aerofoils or other guide vanes stretching diagonally across the corners as shown in the figures. The airscrew may be located in the collector C or the divergent duct D, but must be sufficiently far removed from the front end of the working section to prevent its eddies from being carried through. It communicates spin to the air stream, which is removed by a fixed windmill or set of radial helical guide vanes. Where the cross-sectional area is a maximum, and the speed, therefore, a minimum, the air passes through a large honeycomb of small mesh to enter the convergent nozzle N, which leads immediately to the working section or delivers the jet.

The sudden contraction of the stream by this nozzle has a beneficial effect in steadying the flow. Thus the *contraction ratio* of the tunnel, defined by the ratio of the maximum cross-sectional area of the stream to that of the issuing jet, is a matter of prime significance. Few modern tunnels rely upon a contraction ratio of less than 5, and a larger value is necessary if exceptional steadiness is desired. A large contraction ratio tends to a long tunnel, for it is important that the return flow completely fill the divergent duct and not separate from the walls, to ensure which the divergence must be gradual unless special means are employed to keep the return stream under intimate control. A long tunnel increases the cost of construction, but has the advantage of preventing the wake of a high drag model from being carried completely round the circuit, a defect which cannot arise with an open return.

Turning to the open jet, the first point to notice is that the pressure within it is approximately the same as that of the surrounding air. If a closed-return tunnel has a jet open to the laboratory, its ducts must sustain a small bursting pressure.

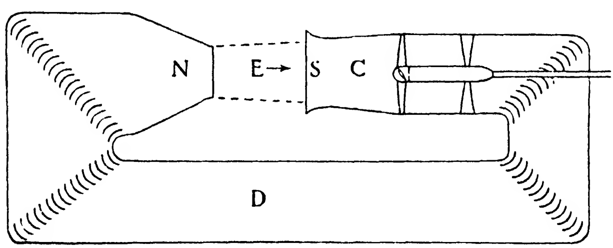


FIG. 36.—OPEN-JET WIND TUNNEL.

Example 39.—An open-jet, race-course tunnel has a contraction ratio of 6. Neglecting energy losses, estimate the bursting pressure, at a speed of 100 ft. per sec., sustained by the duct at its widest part.

Denoting the position concerned by suffix 1 and the jet by suffix 2, Bernoulli's Theorem and the question give

$$p_1 + q_1 = p_2 + q_2$$

i.e.,
$$
\begin{aligned}
p_1 - p_2 &= q_2(1 - q_1/q_2) \\
&= \tfrac{1}{2}\rho V_2^2[1 - (V_1/V_2)^2] \\
&= 11{\cdot}9[1 - (1/6)^2] \\
&= 11{\cdot}57 \text{ lb. per sq. ft.}
\end{aligned}
$$

This is a bursting pressure because p_2 = the pressure of the surrounding air. Energy losses would increase the pressure difference in the actual case.

An open jet is deformed by a model suspended within it, whence troubles tend to arise in accurate work. It is especially suitable for testing model airscrews, elastic models exhibiting the torsional-flexural vibration known as flutter, experiments in connexion with the stability of model aeroplanes, and also (in very large sizes) full-scale aeroplanes or aero-engines, which require hoisting into position by means of a lift or crane. Many tunnels employing open jets have been built and are in operation, but a tendency exists to reserve the type whenever possible to such uses as those mentioned, where accessibility and an uninterrupted view are at a premium.

82. Brief reference to two national tunnels will give some idea of the impressive development reached in aerodynamical apparatus.

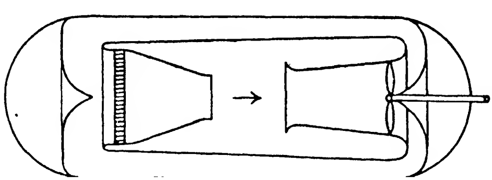


FIG. 37.—COMPRESSED AIR TUNNEL.

Fig. 37 illustrates in outline the compressed-air tunnel at Teddington. It has a jet 6 feet in diameter, unenclosed immediately by a wall, though not, of course, open to the laboratory. Indeed, the complete apparatus, apart from the 450-H.P. driving motor, is boxed within a cylindrical steel shell with dished ends, 18 feet in diameter and 2½ inches thick. The return flow is annular in section and accommodated just within the outer shell. The model having been suspended from electrically controlled measuring apparatus and the manhole closed, a large compressor plant pumps up the shell to a pressure of 25 atmospheres (370 lb. per square inch). The dense air, weighing nearly 2 lb. per cubic foot, attains a speed of 90 feet per second in the jet. The theory of compressed-air tunnels has been given in Chapter IV. Models have to be strongly made and smoothly finished, and Reynolds numbers are reached equal to those of small full scale. Variable density tunnels may also be of the race-course pattern.

The horse-power required to drive an enclosed return tunnel is roughly equal to $Cb(V/105)^3$, where C is the cross-sectional area of the working section in square feet, b the pressure there in atmospheres, and V the velocity in feet per second. If b is considerably less than unity—i.e., if the tunnel be partially exhausted—high speeds can be attained with the same power. Very high air speeds can be induced through a small nozzle for a short time by exhausting a compressed-air tunnel, or similar vessel, through an annular orifice surrounding the nozzle.

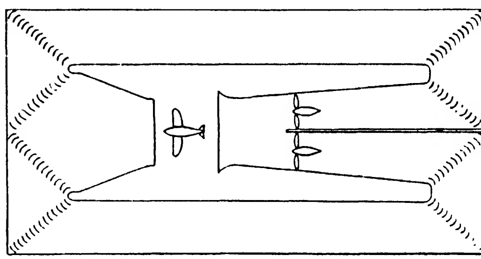


Fig. 38.—Full-scale Wind Tunnel.

The second tunnel selected is the giant atmospheric one at Langley Field, U.S.A. (Fig. 38). The open jet, 60 feet wide and 30 feet high, is sufficiently large for testing many full-scale aeroplanes. Twin motors, totalling 8000 H.P., drive two side-by-side airscrews more than 35 feet in diameter and produce a speed in the jet of 175 feet per second. The return flow is conveyed in double divergent ducts; in fact, the lay-out suggests two enormous wind tunnels so joined together and merged as to form one gigantic whole.

A larger aerodynamic scale is attained in the smaller of these tunnels, for while the product Vl is only some 20 times the greater in the large tunnel, the reciprocal of the kinematic viscosity is 25 times the greater in the small tunnel. The latter is also less costly to run. On the other hand, the full-scale tunnel permits investigation of a number of problems for which a small model cannot be prepared—e.g., engine cooling and slipstream effects.

83. Measurement of Tunnel Speed

When a tunnel is first installed, adjustments are made to secure as uniform a distribution of velocity as possible over the working section. Success is proved by measurements with a pitot-static tube at a network of points 2 or 3 inches apart. The results of this final exploration may be noted collectively by stating the mean speed across a wide central part of the stream in terms of the speed at its centre.

Subsequent use of a pitot-static tube in the tunnel when a model is under test is avoided by the following artifice. A smooth flat plate is let into the side of the tunnel, flush with the interior surface and several feet upstream of the working section, and a small hole drilled through it is connected permanently with a pressure gauge, which thus records continuously the difference of static pressure between the atmosphere of the laboratory and the wind opposite the plate. This pressure difference is calibrated by comparing it, through the entire speed range, with that given by a pitot-static tube occupying the position intended for the centre of span of the model. The direct readings of the permanent gauge are plotted against the corresponding values of $\frac{1}{2}\rho V^2$ obtainable from the pressure heads given by the pitot-static tube.

The curve gives the undisturbed value of $\frac{1}{2}\rho V^2$, corresponding to any reading of the gauge and without further use of the pitot-static tube, provided the model in the tunnel does not modify the pressure near the plate. The plate is therefore kept away from the floor and roof of the tunnel and located well upstream of the working section.

Example 40.—A pitot tube is moved upstream along the axis of a straight-through wind tunnel until it emerges therefrom and finally touches the end wall of the laboratory. Describe the pressure changes occurring within it.

As the pitot tube is displaced from the working section towards the honeycomb its pressure remains constant, though the velocity of the wind decreases slightly. From the downstream to the upstream side of the honeycomb the pitot head increases because the skin friction and form drag of the honeycomb detract from the mechanical energy of the stream. The pitot head then remains constant between the upstream side of the honeycomb and the wall of the laboratory, though in the latter position the velocity is reduced to zero and the pitot registers merely the static pressure in the laboratory.

84. The Aerodynamic Balance

Most wind-tunnel experiments are concerned with the measurement of aerodynamic force. The model is suspended for this purpose from one or more balances installed outside the working section or jet, and the experimental problem is to determine the aerodynamic force in magnitude, direction and line of action. Magnitude and direction are obtained by measuring the component forces—i.e., the lift, drag and cross-wind force, which have already been defined. The line of action is deduced from measurements of component moments about suitably located pivotal axes.

The latter components, called the pitching, rolling and yawing moments, are indicated in Fig. 39 with reference to an aeroplane in normal flight, the pivotal axes then passing through the centre of gravity of the aeroplane. A positive pitching moment tends to raise the nose and depress the tail. A rolling moment tends to roll the aircraft round the longitudinal axis and a yawing moment to turn it away from the direction of flight; these two moments are reckoned positive if they are suitable to begin a right-hand turn—i.e., if they depress and retard the right-hand or starboard wing-tip.

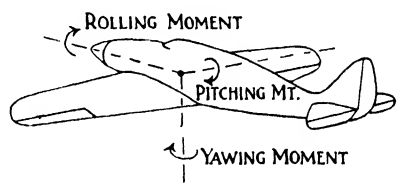


FIG. 39.

In the general case of an asymmetrically disposed model, the three force and three moment components will all require measurement. A single balance capable of measuring the six quantities simultaneously, i.e., without changing the rigging of the model, is necessarily of complicated construction. Apparatus available to students is seldom able to cope with more than three of the quantities at the same time.

An especially simple aerodynamic balance which measures simultaneously only the lift (or cross-wind force) and drag—and is frequently called, therefore, a *lift-drag balance*—is sufficient for many purposes if supplemented by a plain steelyard. We begin by considering one of many forms that a lift-drag balance may take and its use in particularly straightforward circumstances.

85. Light Lift-Drag Balance

The crudest form of a serviceable horizontal type is illustrated schematically in Fig. 40. The base plate (not shown) supports

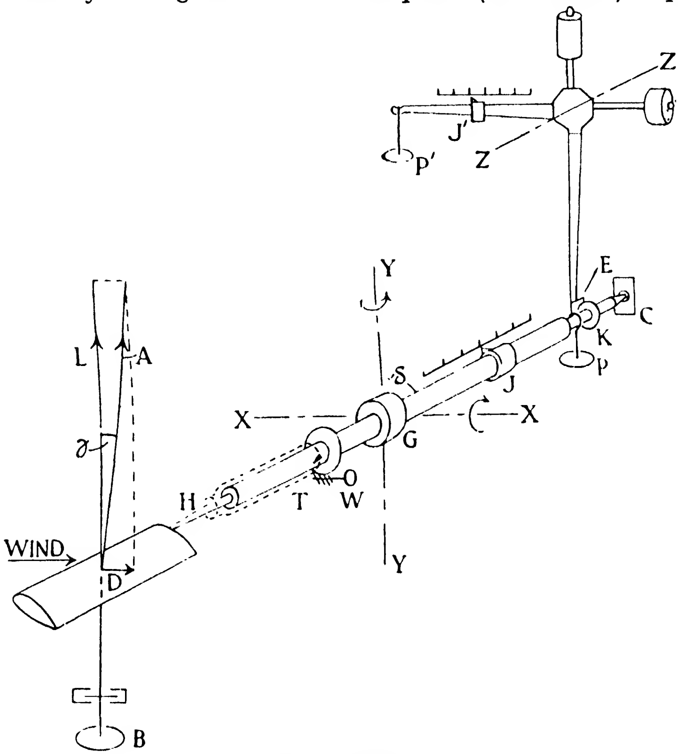


FIG. 40.—LIGHT LIFT-DRAG BALANCE OF HORIZONTAL TYPE.

a steelyard S at G in such a way that it can pivot about the horizontal axis XX, which is parallel to the wind, and also about the vertical axis YY. S is divided at W so that the tapered spindle H, which enters the wind tunnel through a clearing hole T in the wall, can be turned about its own axis. To illustrate one application of the balance, H is shown screwed firmly into the end of a square-tipped aerofoil, whose attitude to the wind can be changed accurately without stopping the tunnel by means of a worm-gear at W. A large angle plate with a vernier can be used instead of the worm-gear, but angle changes should be observed to within 0·1 degree.

The lift component L of the aerodynamic force A turns the steelyard or lift beam about XX and, if amounting to no more than 2 or 3 lb., is weighed by taking known weights off the scale-pan P and adjusting the jockey or rider J. Larger lifts are counterbalanced for the most part by placing known weights in another scale-pan B, hung beneath the stream by a wire which passes freely through a hole in the floor of the tunnel, and the process described for small lifts is then used for final adjustment. The movement of B is restrained by stops so that a sudden cessation of the wind cannot break the model away from H.

The drag D turns the lift beam about the axis YY, operating a subsidiary drag balance by pressing (in the example illustrated) a knife-wheel K on the hardened steel plate E. The drag balance pivots about the axis ZZ and is provided with its own scale-pan P′ and rider J′ to enable D to be weighed.

The beam S is readily given stability about both its axes of swing. But the aerodynamic force on a model in a wind tunnel is frequently too unsteady for a leisurely weighing, in spite of dampers applied at B and near P, and a quick-release stop at the end of S is therefore an advantage.

A fitment of this kind is indicated in Fig. 41. Terminating the steelyard is a long cone O, which passes freely through a hole in the small plate C and whose point moves in juxtaposition to a cross-hair fixed near to C. Lift and drag are weighed simultaneously, and a correct balance is obtained by observing the displacement of the point of the cone in relation to the cross-hair. Fully retracted by the spring F, C acts as a circular stop preventing the maximum displacement of the end of the steelyard from exceeding one millimetre, say, and it is readily slideable over the cone to prevent swinging and to centralise the point in relation to the cross-hair. Thus the stop is employed in rather the same way as is the lifting handle of an ordinary physical balance, but more quickly and frequently.

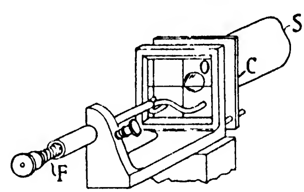


Fig. 41.—Quick Release Stop for Lift-drag Balance.

This balance, being particularly simple to construct and use, has taken more elaborate forms. Some of the improvements effected apply to aerodynamic balances in general and may be mentioned briefly. Unsteadiness is often violent when a model is near its stalling angle and damping must be heavy in these and similar circumstances, but it should be light when small forces are to be measured. Thus damping should be variable, and the electro-magnetic method has an advantage in this respect over the plunger working in oil. Elastic pivots are usually preferred in aerodynamic balances to knife-edges or points; these commonly take the form of crossed strips of clock-spring, whilst in the present instance a thin metal diaphragm has been used with success in place of a gymbals at G. A fixed guard-tube, shown dotted in Fig. 40, is required to shroud the greater part of the tapered spindle H from the wind and, with an enclosed-section tunnel, there tends to be an inward leak of air along it. To prevent the leak, the balance may be boxed in a draught-cupboard, or H may enter the tunnel through an oil seal, or the flexible diaphragm mentioned may be specially adapted. A drag balance can be brought into position alongside a lift balance by use of bell-crank transmission.

An aerofoil with a model landing flap attached may have a lift of 15 lb. at 100 feet per second in a 4-foot tunnel, and yet a minimum drag of less than 0·02 lb. at 30 feet per second. To measure this small drag with good accuracy is well within the compass of the balance described. But occasion frequently arises to measure aerodynamic forces which are much smaller still and then a more sensitive balance is called for.

86. Calibration of Lift-Drag Balance

To calibrate a lift-drag balance, first determine the values of the displacements of the two riders J and J′ in terms of lb. weight on the scale-pans P and P′, respectively. Then extend H and hang from it, exactly at the centre of the tunnel, a temporary scale-pan. Centralise the conical point or other indicator by adjusting the weights in P, P′ and the positions of the riders. Place known weights in the temporary scale-pan and note the weights required to be added to P, including any movement of J, to centralise again. This calibrates the lift-beam. The drag beam may be dealt with similarly by attaching the temporary scale-pan to a thread passed over a pulley in such a way as to exert a horizontal force on the end of H, see Fig. 42 (*a*). Another method, which eliminates the friction of the pulley, is shown at (*b*) in the same figure. The scale-pan is slung from a pin joint between the two fine wires BD and AD, the first of which is

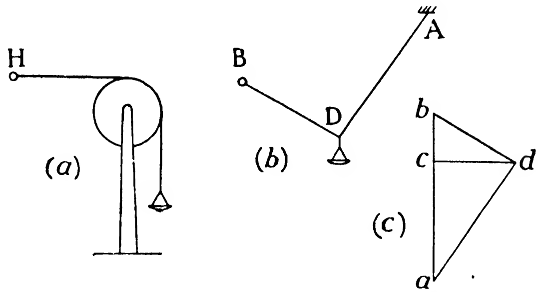


FIG. 42.—CALIBRATING THE DRAG BEAM.

attached freely to the end of H and the second to a fixed point, such as the top of a retort stand. Having obtained the zeros of the balance, a known weight is put in the temporary scale-pan and readings for a renewed balance are observed on both the lift and drag beams. The inclinations to the horizon of the two wires are then carefully measured and the force polygon shown at (*c*) is constructed, where *ab* represents the known weight and so defines the scale of the diagram. On this scale, *bc* should agree with the lift recorded on the balance; if it does not do so, the angles have been wrongly determined. Assuming success, *cd* gives the horizontal force exerted at the centre of the tunnel on H.

87. A Simple Aerofoil Test

To carry out a test on a square ended aerofoil with the simple arrangement shown in Fig. 40, proceed as follows. Verify that the centre of span of the model is at the centre of the wind tunnel. To remove backlash from the worm-gear or other turning device, increase the angle of incidence positively to the most negative incidence at which data are required.

Incidence and its Measurement.—Several conventions have been used in the past to define the angle of incidence (α) of an aerofoil. Thus if the lower surface of the section has a concavity, the incidence may be defined as the angle between the direction of the undisturbed wind and the tangent touching the lower surface at two points. In these circumstances the incidence is easily determined, for a steel rule can be held against the lower surface of the aerofoil by a spring clip and the difference of the heights of its two ends above the floor of the tunnel carefully measured, whence the angle follows from a table of sines. Unfortunately in the present connexion, modern aerofoils are usually bi-convex in section. The angle of incidence is then defined by the line joining the nose to the tail or, more accurately, by the line joining the centres of curvature of the nose and tail of the section. In the case of a square-ended aerofoil of this type, scratches may be made on the nose and tail at the outer end consistently with the above definition, and the heights of these scratches above the floor determined. But now, owing to the short length of the section, the heights should be found with accuracy; an error not exceeding 0·005 inch is possible with practice.

Having estimated the angle of incidence of the model, centralise the balance by adjusting the weights in the various pans and the positions of the riders. Note these zero readings. Obtain also a zero reading of the pressure gauge used to measure the tunnel speed. Verify that nothing has been left in the tunnel and then start up. Allow two or three minutes for the electrical resistances in circuit with the motor to warm up and the pressure gauge to settle down. Then adjust the balance weights and obtain readings of lift, drag and the pressure drop in the tunnel. Increase the incidence by 1° or 2° and repeat all three measurements; it is not necessary to shut down the tunnel, for the zeros will not have changed. Repeat this process until as many angles as desired have been tested; from − 4° to + 18° is usually sufficient at small scale. Finally, shut down the tunnel and, after allowing time for the residual wind to stop and the gauge again to settle down, repeat all zeros and check the incidence to make sure no mistake has occurred.

Correction of Readings.—First, by the subtraction of zeros from gross readings, obtain net readings of lift, drag and tunnel pressure. The observations of lift and drag will have been obtained at slightly different tunnel speeds, owing chiefly to other demands on the electric mains and to further heating of the electrical resistances, and it is seldom worth while to avoid this by continuous adjustment of a finely variable electrical resistance. In a few minutes by slide-rule, all the net readings of lift and drag can be corrected to a selected gauge reading, for which the corresponding tunnel speed is known, the forces being assumed for this purpose to vary directly with the tunnel pressure—i.e., with the square of the speed. Now detach the model from H and measure the drag of the latter at the selected tunnel speed. If use has been made of the supplementary scale-pan B of Fig. 40, estimate the drag of the wire from Fig. 33 and add to that of H. The parasitic drag so obtained is to be subtracted from the observations of total drag already adjusted to the selected tunnel speed. The corrected lifts and drags can now be plotted against angle of incidence. Alternatively, they may first be converted to coefficients, as described in the next article.

88. Coefficients of Lift and Drag

Following directly from Article 68, non-dimensional coefficients of lift and drag can be obtained as

$$k_L = \frac{\text{Lift}}{\rho V^2 S}, \quad k_D = \frac{\text{Drag}}{\rho V^2 S},$$

where S denotes some representative area of the body. In the case of aerofoils, this area is conventionally taken as that of the plan-form of the aerofoil at 0° incidence. For a fuselage it would be the maximum cross-sectional area. These coefficients, said to be in the 'k-system', were uniformly employed for many years in this country, and so appear in most of the literature published here prior to about 1937. More recently, this country has largely adopted the alternative 'C-system' of U.S.A. and elsewhere, in which the coefficients are just double those of the k-system. Since $C_L = 2k_L$, etc., we may define

$$C_L = \frac{\text{Lift}}{qS}, \quad C_D = \frac{\text{Drag}}{qS} \quad . \quad . \quad . \quad . \quad (33)$$

where $q = \frac{1}{2}\rho V^2$. To evaluate these formulæ the forces must be expressed in lb., q in lb. per square foot, and S in square feet. As explained in Chapter IV, the coefficients for any aerofoil at a given incidence are not constants but functions of the Reynolds number, in other words they are subject to scale effects.

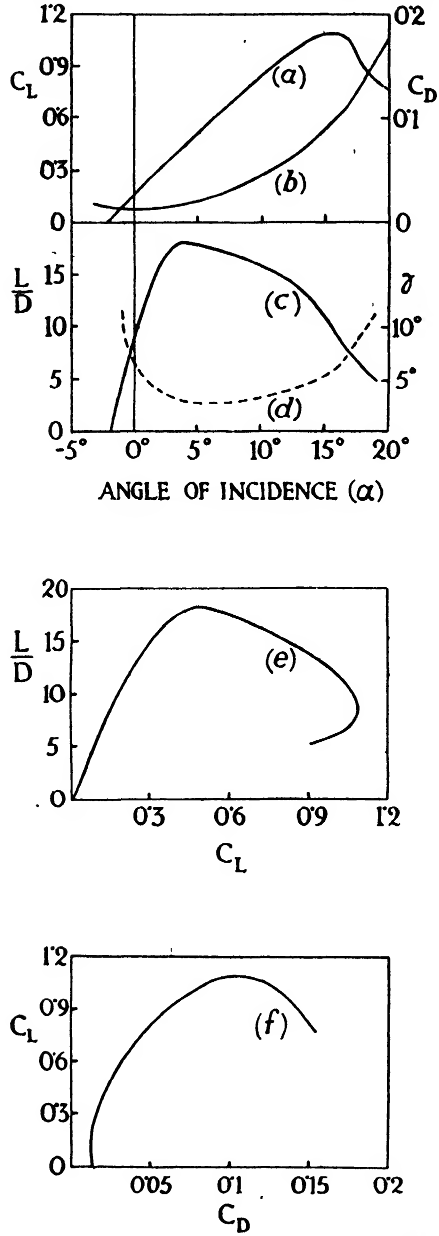


FIG. 43.—AEROFOIL CHARACTERISTICS AT A SMALL SCALE.

89. Representation of Results

Fig. 43 gives at (*a*) a typical lift curve for an aerofoil at small scale. It will be seen that for this aerofoil, and at the scale of the test, the angle of zero lift is $-2°$. From this angle, the lift coefficient increases closely in proportion to the increase of incidence up to about 12°, but ever more slowly afterwards until C_L reaches the maximum of 1·1 at 15° incidence. The aerofoil then stalls. The corresponding drag curve is shown at (*b*). As the incidence increases from $-2°$, C_D decreases a little at first, to a minimum of 0·01 in the present example, and then increases. At angles approaching the stall, and especially when this occurs, C_D increases rapidly.

For curve (*c*), C_L is divided by C_D at each angle of incidence, giving the *lift-drag ratio*, written L/D. This quantity is closely related to the efficiency of the aerofoil as a lifting device. At the most efficient incidence in the present case, $L/D = 18$; or in words, the lift is 18 times as great as the drag. If we know the lift in lb. at any incidence, the drag in lb.

is obtained at once, whatever the speed or altitude, by dividing the lift by the L/D at that incidence. The reciprocal of the L/D is the tan of the angle γ between the lift and the aerodynamic force, see Fig. 40. Hence γ can be plotted against α, as at (*d*), Fig. 43.

A specially convenient graph for many practical purposes is that shown at (*e*), in which the angle of incidence has been eliminated by plotting corresponding values of L/D and C_L. We can find C_L for an aeroplane in straight level flight from the wing-loading and indicated air speed. Then a knowledge of the corresponding L/D gives at once the drag of the wings. Another well known mode of exhibiting the results with α eliminated is shown at (*f*). This last curve, obtained by plotting C_L against C_D, is often called the *polar* for the aerofoil.

90. Suspension of Models

The design of aerodynamic balances and the arrangement of experiments are largely controlled by an urgent need to disturb the flow round the model as little as possible. The arrangement shown in Fig. 40 is singularly free from objection. If, after measuring the combined drag of the model and the exposed part of the holder H, the drag of H is determined without the model present as described, the second measurement can reliably be subtracted from the first to obtain the drag of the aerofoil alone. In these circumstances there is said to be no 'interference' between the two bodies, the aerofoil and the holder.

This simplicity is rare. If two bodies have drags X and Y when isolated, their combined drag when joined together seldom has the value X + Y, but some other value Z. The difference between X + Y and Z is a measure of the interference. This difference may become important if either body appreciably disturbs the flow round the other; for example, a good airship model 6 inches in diameter may have its drag increased by 20 per cent. if a spindle of the size of a pencil is screwed into its side, whilst the drag of the spindle will also be increased by the presence of the model. The first effect is due to a splitting of the delicate flow close to the surface of the streamline body, the second to a general increase of wind speed in the neighbourhood of the spindle.

The interference can usually be determined if it is small.

Thus referring to Fig. 44, an aerofoil A can be tested for drag as supported by two fine streamlined holders B and C and also with each removed in turn (care is necessary to ensure that removing either holder does not twist the aerofoil). Subtracting the drag with B removed from that with both B and C present gives the effective increase of drag due to attaching B; and subtracting this effective increase from the drag measured when A is supported by B alone gives the true drag of A.

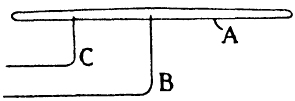


FIG. 44.

This method is used to estimate the effective drag of any attachment to the body, such as even a wire. It sometimes fails as follows, for example. Referring again to Fig. 44, B may appreciably advance, at a small aerodynamic scale, the stalling angle of the aerofoil. C alone may also advance this angle. The two effects are not strictly additive, and so neither can be determined accurately. Again, it is unlikely that the drag of the airship model already mentioned would have its drag increased just twice as much by the attachment of two spindles to its side instead of one.

There is in general only one unobjectionable place for a supporting spindle—viz., screwed into the tail of the body so that it lies along the mean direction of motion in an already eddying region. A holding spindle arranged in this way is called a *sting*. The nodding weight of the body, or its angular deflection under air load, is usually too great for this means of support alone, which is assisted, therefore, by two fine suspending wires of circular or streamline section. The practice applies equally to aerofoils, whose wing-tips are often too thin to receive an endwise spindle of sufficient diameter for the stiffness required. Testing an aerofoil for lift and drag with three-point suspension requires care, but has the advantage of yielding the pitching moment without further work.

91. Test of Aerofoil Shaped in Plan

Fig. 45 illustrates the combination of a lift-drag balance with an overhead steelyard for the purpose of testing an aerofoil for lift, drag and pitching moment. The supplementary 'roof balance' S_1, installed above the tunnel or jet, pivots about the axis X_1X_1. It is loaded through vertical wires, which descend into the wind from a winding device attached to the steelyard so that they can be raised or lowered.

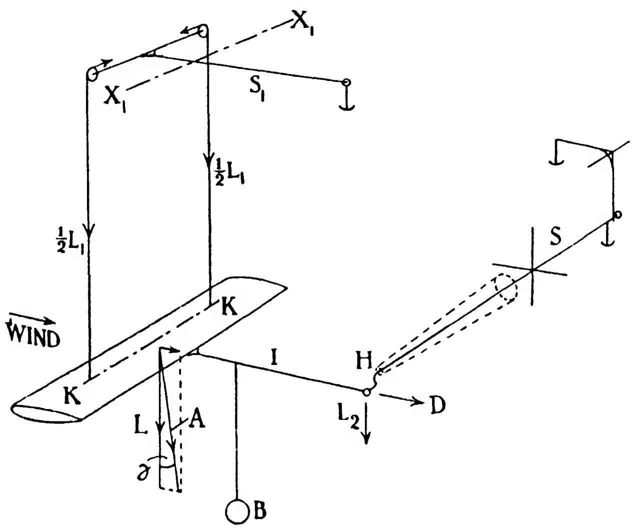


FIG. 45.—COMBINATION OF LIFT-DRAG BALANCE AND OVERHEAD STEELYARD TO DETERMINE LIFT, DRAG AND PITCHING MOMENT.

The aerofoil is suspended upside-down from these wires, which are flexible, so that the model can pivot freely about the line KK joining their ends. Set firmly into its back at the centre of span is a sting I, whose tail end is pivoted freely on the end of H, the prolongation of the lift beam of the lift-drag balance (S) already described and which now extends to the centre of the tunnel and terminates in an elbow. The weight B, hung beneath the tunnel by a wire from I or the aerofoil, serves to keep the lift wires taut when the wind is off and safeguards the aerofoil against breaking loose should the lift become negative.

The greater part L_1 of the total lift L of the aerofoil is measured by S_1 and the remainder, L_2, by the lift-drag balance. Usually the lift imparted to the sting and the exposed part of H is negligible and then

$$L = L_1 + L_2.$$

Provided the lift wires are truly vertical, the lift-drag balance also measures the whole of the drag D of the aerofoil together with the effective drag of the wires, the sting and the exposed part of H. The total parasitic drag d is assessed as follows. The effective drag of the wires is estimated experimentally by adding an extra lift wire for this purpose; part of this drag is supported by S_1 and only the part that is supported by H must be taken into account. The effective drag of the exposed part of H is found with the model still in its proper position upstream, so as to include effects of the wake; for this purpose the tail of the sting is detached from H and temporarily supported by other means. Finally, the effective drag of the sting is determined in a separate experiment, in which the aerofoil is supported by a second sting, or in a different way, so that it can be tested for drag with and without the original sting in place. Hence D is finally obtained from

$$D = (D + d) - d.$$

The figure shows the aerodynamic force A acting on the aerofoil some distance behind KK, the pivot line joining the lower ends of the lift wires. The leverage of A about this line is denoted by a. The pitching moment equals $A \times a$ and is counterbalanced, and therefore measured, by the product of the force exerted by the end of H and its leverage about KK. Approximately, if b denotes the horizontal distance of the pivot on the end of H from KK,

$$A \times a = L_2 \times b.$$

In order, however, that this moment should convey a definite meaning, the location of KK must be stated; it is usually chosen to be at one-quarter of the chord from the nose of the aerofoil, the ' chord ' being the distance from the nose to the tail of the section. In the circumstances of the figure, the pitching moment is negative, since it would tend to depress the nose of the aerofoil if the latter were in the right-way-up position.

Any inclination of the lift wires from the vertical must affect the drag reading on the lift-drag balance. The angle between the directions of the lift and aerodynamic force is denoted by γ (see Fig. 45). If the lift wires were inclined forward from the vertical by this angle—i.e., if they were parallel to A—the horizontal component of the force generated in them would nearly balance the whole of the drag and little would be registered by the lift-drag balance. γ may be less than 3°, in which case an inclination of the wires to the vertical amounting to 3° may lead to a 100 per cent. error in the drag observation; to measure D to 1 per cent. requires the wires to be vertical to within 0·03°, a very small angle. The difficulty may be overcome as follows.

H is terminated in a crank, so that turning H about its own axis by operating the worm-gear, or other incidence-changing device of the lift-drag balance, displaces the tail of the sting upstream or downstream, facilitating adjustment of the lift wires to the vertical. A simple and effective static test shows when this adjustment has been achieved with accuracy. With the wind off, a 'hook weight' (resembling a question-mark in shape) is hung on the model. This increases the tension in the lift wires by approximately the value of the weight, which is preferably known and may be 1 lb. Only if the wires are truly vertical will their increased tension be found to have no effect on the drag balance. It will be appreciated that this static test is urgently necessary, however carefully the model may have been set up. It is frequently convenient to allow the lift wires to become slightly inclined. The drag observation is then corrected from the measured lift in them and the statically determined effect due to a hook weight of 1 lb.

92. Procedure and Results

Unless the compound balance described in the last article has been elaborately constructed or calibrated, an aerofoil test will take much longer to complete than by the method of Article 87, and one will proceed as follows, remembering that the zeros of the balances are subject to continual change.

Having set the aerofoil approximately at the most negative angle of incidence required, use the elbow or crank at the end of H to set the lift wires truly vertical. Then take all three zeros and determine the incidence accurately. Start the tunnel and obtain readings of L_1, L_2 and $D + d$ at a measured speed. Without stopping the tunnel, let out the lift wires so as to increase the incidence and repeat the readings. Stop the tunnel, find the new zeros and angle of incidence, and then hang the hook weight on the aerofoil and obtain three more readings as due to this weight—viz., L_{1w}, L_{2w} and D_w, say, after subtraction of zeros. With w written for the value of the hook weight,

$L_{1w} = w - L_{2w}$, and it follows that for the second set of readings with the wind on, the drag beam zero must have changed by $D_w \times (L_1/L_{1w})$. This correction is applied to the apparent value of $D + d$ in that test. Change the incidence, adjust the lift wires to the vertical again, and repeat.

To reduce the pitching moment to a coefficient we must introduce an additional factor of dimensions L because the moment has the dimensions of force × length. The convention in the case of the pitching moment is to choose the chord of the aerofoil section, denoted by c. Thus the required coefficient in the C-system, c being expressed in feet and the moment in lb. feet is,

$$C_M = \frac{Moment}{qSc} \qquad \qquad (34)$$

Fig. 46 gives an example of a pitching moment coefficient curve referred to the quarter-chord point. The fact that the coefficient is negative throughout indicates that the aerodynamic force never reaches so far forward as one-quarter of the chord from the nose in the present case. It will also be observed that the pitching moment by no means vanishes at the angle of incidence appropriate to zero lift.

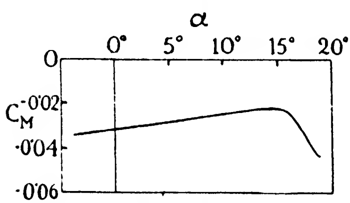


FIG. 46.

93. Large-Scale Testing in Small Tunnels

The largest chord an aerofoil may have for a reliable test at all angles of incidence in a 4-foot enclosed-section is about 5 inches. Hence the Reynolds number specified on the chord, or the aerodynamic scale, is little more than ¼ million at 100 feet per second. A much larger scale can be attained in the same tunnel, however, for angles of incidence approximating to that for zero lift, providing the effects of a finite span can be ignored. An aerofoil, whose chord may exceed the width of the tunnel, but whose span is 8–12 inches less than this width, is so suspended as to ride with small clearance between shoulders or distance-pieces, having the same section as that of the aerofoil and fixed to the tunnel walls. These shoulders separate the aerofoil from the confused and sluggish flow adjacent to the walls, and the test is thus made under an imitation of two-dimensional conditions of flow. The scale may reach as much as 2½–3 millions. Besides imparting information as to scale effects at small incidences, the method is valuable as permitting the effects of ridges and other details of practical construction to be investigated.

A model of this description may easily weigh 30–40 lb. and its lift at zero incidence come to a like amount. Its drag, for a clean shape, will be less than 2 lb., but may amount to much more if severe blemishes are tested. The balance arrangements so far described are inadequate for the present purpose owing chiefly to a difficulty in retaining sufficient sensitivity at low speeds. The type of balance described in principle in the next article has given much useful service in the present connexion and has been found to have other applications.

94. Referring to Fig. 47, the heavy aerofoil is carried upside-down by a bar EE which threads through its quarter-chord point and is firmly secured to its ends. This bar passes freely through the shoulder-pieces mentioned in the preceding article (omitted for clearness from the figure) and through the tunnel walls, and terminates in knife-edges or elastic pivots AA which prevent any displacement of the model within the balance due to drag. These pivots are carried on the top ends of a stiff frame HHH, which encircles the lower half of the working section of the tunnel and rides vertically upright on pivots BB below the floor by virtue of the adjustable counterpoises UU. Thus the greater part of the weight of the model and a part L_1 of the downward lift L it exerts can be weighed in the scale-pan P_1 of the lower steelyard S_1, which is situated below the floor of the tunnel and pivots about XX.

The remainder of the weight and lift are dealt with by a vertical wire attached to the trailing edge of the model and passing freely through the floor and roof. The upper end of this wire is supported by an overhead steelyard S_2, pivoting about ZZ above the roof of the tunnel. S_2 can be used for weighing, but is more conveniently given a bias by a counterpoise W, so that the part L_2 of the lift can be determined in the scale-pan P_2. The axis ZZ is vertically above the axes AA and BB, whilst S_2 is parallel to the chord-line of the aerofoil; hence the back wire remains truly vertical as the angle of incidence is varied by raising or lowering the stop O_2. The stops O_1 and O′ are fixed.

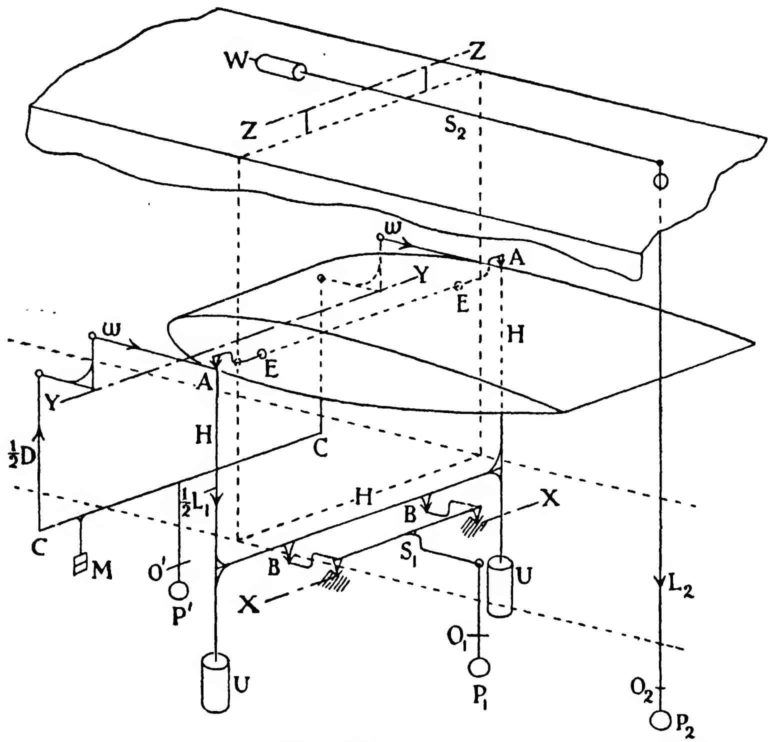


FIG. 47.—BALANCE FOR LARGE-SCALE TESTING UNDER TWO-DIMENSIONAL CONDITIONS.

The drag is taken by the horizontal wires w, w, fastened to the two top ends of HHH and operating twin bell-crank levers, which are pivoted outside the side walls on the axis YY. Vertical wires attached to the other ends of these levers support a horizontal connecting bar CC, below the tunnel, from which depends the scale-pan P′ for measuring the drag. A bracket on CC carries a cross-hair M for observation through a lens or microscope.

95. Testing a Heavy Fuselage Model

The same balance can be used to determine the cross-wind force, drag and yawing moment of a heavy model of an aeroplane body, or fuselage, at small angles of yaw. The arrangement is indicated in Fig. 48, the model being on its side. A sting I is fastened to the tail of the body and also secured to the middle

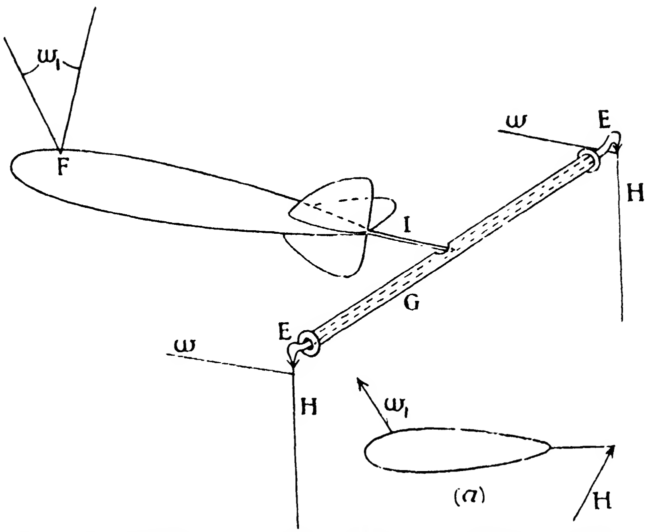


Fig. 48.—Arrangement for Testing a Heavy Model of Low Drag.

of the bar EE, which is now enclosed within a guard-tube G fixed between the tunnel walls. (It is easy to devise means by which the back end of I can be pushed into a hole in EE and locked there, using a small aperture in G for the purpose.) The drag is measured through the wires w, w as before. The model is additionally suspended from a point F near the nose by V-wires w_1 depending from the roof balance illustrated in Fig. 45. Thus the vee is across the wind tunnel and prevents swaying, which the sting is too weak to resist. Also the angle of yaw can be varied by means of the winding drum on the roof balance.

Good sensitivity is obtained in spite of the heavy weight of the model for a reason which will be apparent from the inset diagram (*a*), which shows, greatly exaggerated for clearness, a displacement of the model downstream during a test. If, following such a displacement, the wires w_1 and the arms of the frame HHH became inclined to the vertical by equal angles and the centre of gravity of the model were situated midway between the two points of suspension, the upstream component of the tension in w_1 would be counterbalanced by the downstream component of the compression in HHH, and the displaced model would be in equilibrium. Actually, the first component exceeds the second because the centre of gravity is nearer the wires than it is to EE. Thus the model is statically stable in respect of horizontal displacement. Without the compensation of HHH, this stability would, of course, be far too great to permit the small drag of a heavy model to be determined with sufficient accuracy. The compensation may be varied by adjusting the counterpoises U, U (Fig. 47).

96. Systematic Errors

Apart from errors due to false readings, unsuitable measuring apparatus, inadvisable suspensions and the like, wind tunnel experiments are liable to over-ruling errors which arise from five causes, as follows: (*a*) the restricted lateral extent of the artificial wind, (*b*) the throttling of the stream by the obstruction presented by the model, (*c*) turbulence possessed by the stream on arrival at the working section, (*d*) inclination of the working stream to the horizontal and (*e*) the variation in static pressure along the tunnel prior to the introduction of the model.

Adequate treatment of (*a*)–(*c*) is beyond the scope of this book and brief general remarks must suffice in their case. Regarding (*a*), the walls of an enclosed-section tunnel constrain the direction of flow, whose streamlines would be bent by the model to a greater extent in their absence. The effect is felt notably by aerofoils and models of complete aeroplanes, whose performance is flattered thereby. More or less the opposite occurs with an open jet. It is not difficult to calculate approximate theoretical corrections which suffice in most cases. (*b*) need not worry us; the throttling effect of even a comparatively large body is much less than appears to the eye. (*c*) is important and involves a difficult correction. It is usually because their oncoming streams possess turbulence in a varying degree that different wind tunnels sometimes yield estimations of the same aerodynamic force which fail to agree. There is hardly any limit to the care that should be taken in the construction of a wind tunnel to decrease this initial turbulence, for the atmosphere, though subject to much large-scale unsteadiness, is singularly free from the finely-grained turbulence that is found to increase the drag of well streamlined bodies.

There remain (*d*) and (*e*), both of which often make large corrections necessary and must be considered in detail.

In most wind tunnels, however carefully they have been aligned, the undisturbed working stream deviates a little from the horizontal. In the open-return type it is often directed slightly downward owing to the return flow being less obstructed near the ceiling of the laboratory than near the floor. The acute effect of such deviation on the measured drag of a model of high lift-drag ratio will be appreciated from the part of Article 91 which deals with the urgent need for truly vertical lift wires and an accurately horizontal stream. If in the case there considered, the angle γ (Fig. 45) being 3°, the wires were truly vertical, but the oncoming wind were inclined downward at this angle to the horizontal, the component parallel to the wind of the force in the wires might again counterbalance the whole of the drag, and the apparent drag, registered on the drag balance, would be zero; the error in the estimate of drag would be 100 per cent. minus. With the model in its natural instead of the inverted position, the error would be 100 per cent. plus. So large a deviation of the undisturbed stream never occurs, but one of $\frac{1}{2}$° is common, and then the error exceeds 16 per cent. for $\gamma = 3°$. Obviously, a true estimate can be obtained by testing first one way up and then the other and adopting the mean, but this practice in every case would be arduous, and it is better to apply a correction to the readings obtained.

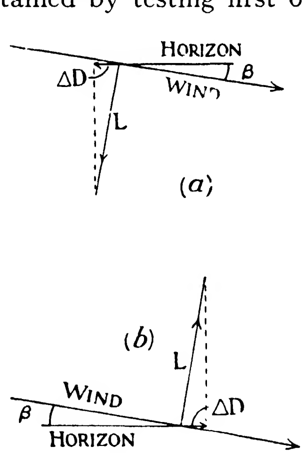


FIG. 49.

In Fig. 49 the wind is supposed directed downward at the small angle β to the horizon. The lift L is, by definition, perpendicular to the wind. If the model is upside-down (*a*), so that the lift is downward, the force component in the horizontal direction is reduced by ΔD. If the model is in natural position (*b*), it is increased by an equal amount. Since β is small, $\Delta D = \beta L$. Fig. 50 shows at A, A the lift and drag curves for the first arrangement and at B, B those for the second, the value assumed for β being 1°. The true drag curve, shown dotted, is obtained from either of those observed by applying the above correction. The true lift curve, also dotted, results directly by increasing or decreasing, respectively, the observed incidences by β.

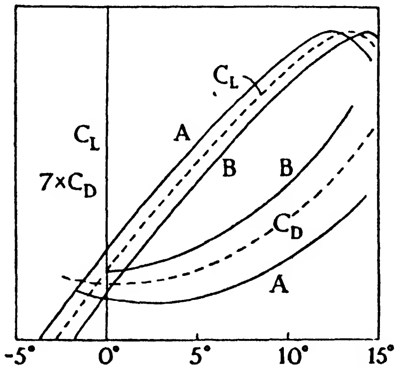


FIG. 50.—LIFT AND DRAG COEFFICIENTS PLOTTED AGAINST INCIDENCE.

– – – True values, ——— apparent values when the wind is inclined at 1° to the horizon.

Turning finally to (*e*), the pressure drop along a parallel-sided tunnel, arising as described in Article 80, can be determined by measuring the pressure drop between two positions A and B, widely distant l apart. A sensitive pressure gauge and a high tunnel speed should be employed and care taken to achieve a reliable estimate. If p_A is the static pressure at the upstream and p_B that at the downstream position, and l is expressed in feet, the rate of fall of pressure along the stream can be taken to be

$$\frac{p_A - p_B}{l} \text{ lb. per cubic foot,}$$

that is to say, the *pressure gradient* may be assumed to be constant for a given tunnel speed. It may also be taken to vary in proportion to ρV^2 when the tunnel speed is changed.

On account of this gradient there arises on a body in the stream an increment of drag that is in no way connected with aerodynamic force and is absent in free flight. Thus the drag measured in the tunnel requires to be decreased in order to remove this artificial increment due solely to the apparatus. For streamlined bodies the increment can be estimated by the method explained in Article 51, when it comes to

$$\frac{p_A - p_B}{l} \times \text{the volume of the body.}$$

More adequate analysis provides estimates 10–30 per cent. greater. The gradient is small; in a 4-foot tunnel of the type illustrated in Fig. 34 a value of about $0{\cdot}01q$ would be expected. Therefore the correction is important only in the case of low-resistance bodies of considerable volume.

97. Summary of Correction Formulæ

To correct for a falling pressure gradient P, decrease the measured drag coefficient by $P \times$ the volume of the body $\div qS$, where S is the conventional area on which C_D is specified.

To allow for a downward inclination of the wind by a small angle β to the horizon, the aerofoil being inverted so that its lift is exerted downward, increase the measured incidence by β and add βC_L to the measured drag coefficient. Use the same corrections but with opposite sign on reversing the position of the model, or if the lift is upward. β must be expressed, of course, in circular measure.

The following are added without attempt at derivation to enable allowance to be made for the limited width of the experimental stream, the cross-sectional area of which is denoted by C.

For an enclosed section tunnel, add to the incidence measured in the tunnel

$$\frac{1}{8} \cdot \frac{S}{C} C_L \quad \ldots \ldots \quad (35)$$

and add to the drag coefficient observed in the tunnel

$$\frac{1}{8} \cdot \frac{S}{C} C_L^2 \quad \ldots \ldots \quad (36)$$

For an open jet use the same corrections but with sign changed. The resulting incidence and drag coefficient refer to free flight conditions at the same aerodynamic scale, but these corrections are required only for 3-dimensional aerofoils having exposed wing-tips; they do not apply to an aerofoil which stretches completely across the tunnel or is tested between shoulder-pieces under approximately 2-dimensional conditions. The formulæ are approximately correct only for streams of compact, not elongated, sections.

98. Examples

Example 41.—A gauge connected between two static pressure holes drilled through the side of a wind tunnel, one hole 10 ft. downstream from the other, shows a pressure drop of 0·192 in. water at a speed of 100 ft. per sec. How does the falling pressure gradient affect the drag coefficient in this tunnel of a fuselage model, whose maximum cross-sectional area is 0·2. sq. ft. and whose volume is 0·36 cu. ft.?

In lb. per sq. ft. the pressure difference is $62{\cdot}4 \times 0{\cdot}192/12 = 1{\cdot}00$, giving a pressure gradient of $1{\cdot}00/10 = 0{\cdot}1$ lb. per cu. ft. At 100 ft. per sec. the drag of the model will be increased on this score by $0{\cdot}1 \times$ its volume $= 0{\cdot}036$ lb. But at this speed $qS = \frac{1}{2}\rho V^2 S = 11{\cdot}9 \times 0{\cdot}2 = 2{\cdot}38$ lb. Thus the drag coefficient will be increased by

$$0{\cdot}036/qS = 0{\cdot}015.$$

Example 42.—A large aerofoil section, of mean thickness 4 in., gives a drag coefficient of 0·011 when tested under 2-dimensional conditions in a wind tunnel along which there is a falling pressure gradient $= 0{\cdot}005q$. Estimate the true drag coefficient.

Let P denote the pressure gradient, S the area (span × chord) of the model and t its mean thickness. The drag coefficient is to be reduced by

$$\frac{P \times volume}{qS} = \frac{PSt}{qS} = \frac{P}{q} \,.\, t$$
$$= 0{\cdot}005 \times \frac{4}{12} = 0{\cdot}0017.$$

Thus the true drag coefficient is $0{\cdot}011 - 0{\cdot}0017 = 0{\cdot}0093$.

Example 43.—A rectangular aerofoil, of 36 in. span and 6 in. chord, is tested upside-down at a certain incidence giving a good lift-drag ratio in a 4 ft. 6 in. diameter enclosed section wind tunnel working at 100 ft. per sec. The oncoming wind is inclined downward at $\beta = 0{\cdot}4°$ to the horizon. If the lift, measured vertically, is 14·28 lb., what is the true lift coefficient?

On setting out the triangle of forces (cf. Fig. 49) the true lift is found (since $\cos \beta =$ unity) to be less than that measured by $\beta \times$ the drag. The correction can be neglected since both β and the drag are small. We have

$$qS = \tfrac{1}{2}\rho V^2 \,.\, S = 11{\cdot}9 \times 1{\cdot}5 = 17{\cdot}85 \text{ lb.}$$

Hence $C_L =$ measured lift$/qS = 14{\cdot}28/17{\cdot}85 = 0{\cdot}800$.

Example 44.—If the aerofoil chord is inclined at 7·7° to the horizon in the test of Example 43, at what incidence would the aerofoil produce the same lift coefficient in free flight?

To the measured incidence must be added the angle of the stream and also $SC_L/8C$ expressed in degrees.

C, the cross-sectional area of the stream $= \pi(2\frac{1}{4})^2 = 15{\cdot}9$ sq. ft., giving

$$\frac{SC_L}{8C} = \frac{1{\cdot}5 \times 0{\cdot}80}{8 \times 15{\cdot}9} = 0{\cdot}00943 = 0{\cdot}54^\circ.$$

Thus the required incidence is

$$7{\cdot}7^\circ + 0{\cdot}4^\circ + 0{\cdot}54^\circ = 8{\cdot}6^\circ.$$

Example 45.—If, in the test of Example 43, the drag registered by the aerodynamic balance is 0·714 lb., what would be the drag coefficient of the aerofoil in free flight at the same lift coefficient?

The observed drag coefficient $= 0{\cdot}714/qS = 0{\cdot}0400$. To this must be added βC_L on account of the downwardly directed wind, and $SC_L{}^2/8C$, to allow for the constraint of the tunnel wall. We have

$$\beta = 0{\cdot}4^\circ = 0{\cdot}007 \text{ radians, giving } \beta C_L = 0{\cdot}0056,$$

$$\frac{SC_L{}^2}{8C} = \frac{1{\cdot}5 \times 0{\cdot}64}{8 \times 15{\cdot}9} = 0{\cdot}00755.$$

Hence, under free flight conditions,

$$C_D = 0{\cdot}0400 + 0{\cdot}0056 + 0{\cdot}00755 = 0{\cdot}0532.$$

99. It is useful to observe that the formulæ (35) and (36) of Article 97 can be expressed in terms of lift, drag and the stagnation pressure q, and so are not restricted to monoplane aerofoils. Thus if $\Delta\alpha$ is the increment of incidence due to the narrow stream and L is the lift of the monoplane, biplane or other model,

$$\Delta\alpha = \frac{1}{8}\cdot\frac{S}{C}\cdot\frac{L}{qS} = \frac{1}{8C}\cdot\frac{L}{q} \quad . \quad . \quad . \quad . \quad (37)$$

Similarly, if ΔD is the corresponding increment of drag, from (36)

$$\Delta D = \frac{1}{8}\cdot\frac{S}{C}\left(\frac{L}{qS}\right)^2\cdot qS = \frac{1}{8C}\cdot\frac{L^2}{q} \quad . \quad . \quad . \quad (38)$$

Example 46.—The lift of a triplane model in an open-jet wind tunnel, whose cross-sectional area is $12\frac{1}{2}$ sq. ft., is 15 lb. at a stagnation pressure of 10 lb. per sq. ft. How much of the measured drag is due to the limited size of the jet?

Directly from (38)

$$\Delta D = \frac{15^2}{8 \times 10 \times 12\frac{1}{2}} = 0{\cdot}225 \text{ lb.}$$

This is included in the balance reading because the drag of a lifting 3-dimensional model is greater in an open jet than in free flight.

Chapter VI

EXPERIMENTAL STUDIES

100. This chapter describes and explains groups of experiments illustrative of the principles, phenomena and methods of Aerodynamics within the scope of a small Aeronautical Laboratory. Some of these studies replace mathematical investigations which are possible in more advanced treatises. Thus the chapter does not comprise merely an optional experimental course, but forms an integral part of the treatment of the subject in this book. It is hoped that the student will have access to a wind tunnel to carry out a selection of the experiments described, and some practical details are included accordingly. But even if no such opportunity exists, the matters treated will still be found easier to grasp under the simplified conditions contemplated in experiment than in the technically involved circumstances of full-scale flight. A few specialised experiments relating to wings and airscrews are reserved to later consideration.

101. Circular Cylinder

A round tube, say $1\frac{1}{2}$ inches in diameter, stretches completely across a wind tunnel, being mounted in such a way that it can be turned about its axis through angles which are indicated outside the tunnel. The ends are sealed, and a small hole drilled midway along the length communicates through the cylinder with a pressure gauge whose other limb is connected through a tee-piece to the permanent static pressure hole in the side of the tunnel.

At some definite speed, chosen in accordance with the sensitivity of the gauge available, measurements of pressure are made with the hole in the cylinder facing directly upstream and at intervals of from 5° to 10° away from this position. The cylinder is then removed and a pitot-static tube mounted in its place to verify the dynamic pressure $q = \frac{1}{2}\rho V^2$ and the static pressure p_0 of the undisturbed stream.

From these data is evaluated the variation round the cylinder of the normal pressure coefficient p/q, where p denotes the increase of pressure above p_0 expressed in lb. per square foot. Fig. 51 (a) shows this non-dimensional coefficient plotted radially outward from a circle representing the cylindrical surface. The coefficient is unity with the hole facing directly upstream, zero at 30° away on either side of this position, and negative round the remainder of the section. At (*b*) in the same figure the values of the coefficient round the profile are plotted on the diameter *MN* transverse to the wind as base.

Finally, the drag per unit length of the cylinder is determined at the same speed by means of a balance, a shortened length being arranged for this purpose to ride with small clearance between shoulder-pieces of the same section fixed to the tunnel walls. This measurement enables the drag coefficient C_D to be evaluated.

Analysis of Results.—Referring first to Fig. 51 (*a*), consider an element AB of the cylindrical profile. The force per unit

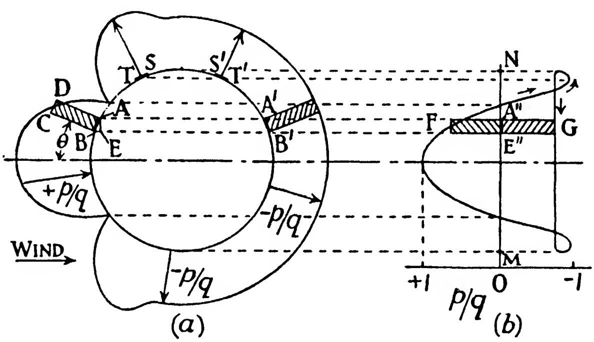


FIG. 51.—PRESSURE DIAGRAM FOR CIRCULAR CYLINDER.

length of the cylinder on this element is $f = \text{AB} \times p$ and so f/q can be represented by the area of the rectangle ABCD. The force f makes the small contribution to form drag $d = f \times \cos\theta$. Now, if AE is the projection of AB in the direction of motion, the angle BAE $= \theta$ and AB $\cos\theta =$ AE. Therefore $d =$ AE $\times p$.

In the diagram (*b*), A″E″ is the projection of AB and is equal to AE, whilst A″F is equal to p/q. Therefore d/q is represented to the same scale as before by the hatched area A″E″F. Similarly the contribution of the opposite element A′B′ to positive drag is proportional to the hatched area A″GE″.

Thus D_F the total form drag per unit length of the cylinder is proportional to the sum of all such strips—i.e., to the area enclosed within the curve plotted at (*b*) provided the areas of the small loops are reckoned negative. The need for the proviso is clear from the diagram (*a*); the element S′T′ increases drag by a smaller increment than that by which the opposite element ST reduces drag (if the area is measured by tracking a planimeter round the curve as indicated by the arrows, the areas of the small loops will be subtracted automatically from the large area).

Let the net area of the diagram (*b*) come to x square inches, and suppose that 1 inch of the horizontal scale represents a units of pressure coefficient and 1 inch of the vertical scale represents b units of length (feet). Then $x \times ab$ is equal to the actual value of D_F/q. Remembering that D_F is the form drag of unit length of the cylinder, the associated part of the drag coefficient, C_{DF} is equal to $(D_F/q) \div$ diameter in feet (cf. Article 88).

C_{DF} is found to be a little less than C_D. The difference represents that part of the drag which is due to skin friction. To determine the skin friction of any body directly is experimentally difficult. But an indirect method will now be plain; it is only necessary to find the drag which arises from the normal pressures and to subtract this from the total drag found by weighing.

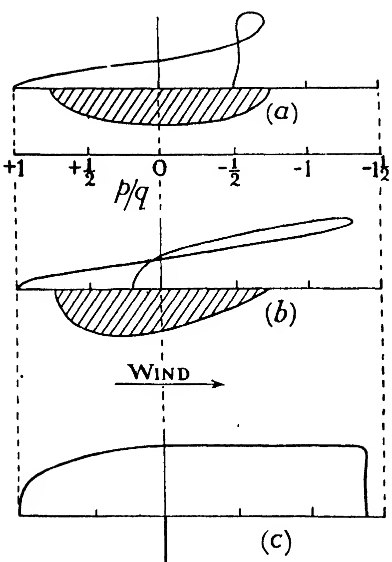


FIG. 52.—FORMS OF PRESSURE DIAGRAM FOR (*a*) AN ELLIPTIC CYLINDER, (*b*) A SYMMETRICAL AEROFOIL AND (*c*) THE NORMAL PLATE.

Only half the diagram and section are shown in each case.

102. Comparison with Other Sections

Repetition of the foregoing experiment with cylinders of other than circular section takes longer to do because a separate hole must be drilled for each position at which the pressure is required and all holes except the one under immediate observation carefully sealed. Fig. 52 gives the pressure distributions round (*a*) an elliptic cylinder of fineness ratio 3, (*b*) a symmetrical aerofoil or streamline strut section of fineness ratio

2½, (*c*) the normal plate—i.e., a long plate broadside-on to the wind. Only half the diagram is shown in each case, and only half the section of the body. The pressure distribution depends on the Reynolds number, which is less for (*a*) than for Fig. 51, whilst that for Fig. 51 is less than that for (*b*) of the present figure; nevertheless, the diagrams are fairly typical of the shapes concerned.

Evaluating the form drag from such diagrams for a number of different bodies by the method of the preceding article and subtracting it from the total drag shows much less variation in the resulting skin friction than might be expected, verifying that the improvement achieved by streamlining (cf. Table II, Article 40) is due to elimination of form drag. Streamlining expands the negative loops of the pressure diagram, which are absent from that of the normal plate and only small in the case of the circular and elliptic cylinders. This results from increasing the pressure (i.e. restricting its reduction) over the back part of the profile. Thus the kinetic energy, generated near the shoulders of the section at the expense of pressure energy, is transformed to a larger extent into pressure energy again before the air passes the body. The action resembles that secured by fitting a regenerative cone to a wind tunnel (cf. Article 80).

Many tests illustrating the striking advantage of streamlining can be carried out rapidly by direct weighings on the aerodynamic balance. Thus a normal disk, first tested alone, if successively fitted with a long nose, a still longer tail, and finally both fairings, when it will resemble an airship envelope, may show a reduction of drag from first to last approaching 98 per cent.

103. The Wake

The advantageous modification of the pressure distribution by streamlining is associated with a narrowing of the wake behind the cylinder or other body. This is easily verified by an experimental method based on Article 52.

A pitot tube of small diameter is supported from the tunnel floor or roof through a screw, or other device, by means of which it can be traversed across the wake behind the body. Connexion is made to a pressure gauge, the other limb of which may communicate with another pitot tube held stationary in a position clear of the wake. Outside the wake the flow is irrotational and the constant of Bernoulli's equation does not change as the pitot tube crosses the streamlines. But this condition breaks down at the edge of the wake, which is at once revealed, therefore, by a fall in the pitot pressure.

If, in addition to providing means for traversing the pitot tube fairly accurately across the middle of the wind tunnel, arrangements are also made to slide the model along it, the edges of a length of the wake can be mapped out in a very short time. Fig. 53 shows at (*a*) those for a circular cylinder; the fiercely eddying motion between is described in Article 46. At (*b*) is indicated the much thinner and only weakly eddying wake behind an aerofoil at moderate incidence and at (*c*) the steady wake behind a flat plate at a Reynolds number of 100,000.

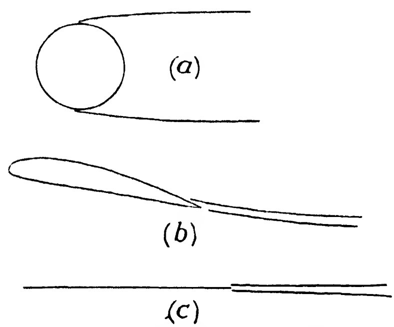


FIG. 53.—EXAMPLES OF WAKES.

This method is important as determining at once what part of the flow past a body is irrotational. If measurements are also made at a number of points across the wake itself at some distance behind the body, an estimate can be made of the drag of the latter, a development which is used both in model experiment and in full-scale flying tests.

The *pitot boundary*, as the region where the pitot pressure begins to fall is sometimes called, extends upstream from the wake to envelop the body in a boundary layer (Article 48). This may be shown by the same method, but calls for finer work since the boundary layer is thin.

104. Breakaway

Where the boundary layer ends and the wake begins marks the *points of breakaway*, so called because the flow ceases to follow the profile of the body. With a circular cylinder at small scales breakaway occurs in front of the shoulders of the section, at about 0·43 of the diameter behind the extreme nose. With the elliptic cylinder (*a*) of Fig. 52 it recedes to about 0·6 of the chord behind the nose. With the streamline cylinder of the same figure it is pressed back farther still, to about 0·8 of the chord behind the nose. These positions are subject, however, to scale effect. Separation does not occur on a flat plate except in the presence of an adverse pressure gradient.

It is easy to perceive the reason for this phenomenon. Chapter III describes how the pressure changes are generated in a wide-spread manner through the outer and irrotational part of the flow, attaining a maximum at the edge of the boundary layer and being transmitted through that layer without modification. The pressures on the body are therefore related to the velocities at the edge of the boundary layer by Bernoulli's equation. Consider an element of air which passes close to the stagnation point of the streamline strut (*b*), for example, of Fig. 52. Its velocity is reduced there nearly to zero, its pressure being raised by nearly the amount q. But as it passes on towards the upper shoulder of the section, through a region of falling pressure, it quickly gathers speed; no energy is lost in this process so long as the element remains in the irrotational part of the flow. Sooner or later, however, it enters the boundary layer and has to do work against the shearing action going on there, losing energy. Behind the shoulder of the section, it is called upon to push its way against a rising pressure. Eventually it arrives at a position, well in front of the trailing edge, with all its kinetic energy used up. Left to itself it would stop. But other elements are coming up behind in similar circumstances, and all are pushed out into the stream.

Summing up, streamlining delays breakaway, enabling the thin boundary layer to extend farther towards the tail and narrowing the wake; this induces an improved pressure distribution, which decreases form drag.

THE PRESSURE ROUND AN AEROFOIL SECTION

105. The Model

The distribution of pressure round a lifting aerofoil is particularly important and repays careful study. It is necessary to distinguish 2-dimensional conditions, when the aerofoil is simply a special case of the long cylinder, from the very different 3-dimensional circumstances of aeroplane wings. Primary interest centres in the effects of changing the angle of incidence and restricting the span. An aerofoil of rectangular plan-form and constant section suffices for the 3-dimensional experiments and the same model can be made 2-dimensional, when required, by fitting extensions of like section and incidence between its wing-tips and the tunnel walls. If a 4-foot tunnel is used, suitable dimensions for the short aerofoil are 25 inches span and 5 inches chord. To save labour in carrying out a number of tests, a judicious choice of incidences should be made as will be described. Some difficulty arises in fitting the model for pressure observations owing to a need for very closely spaced pressure holes round the nose. 'Mass production' of pressure observations by using multiple tube manometers, though easy to devise, tends to inaccuracy unless the tubes can be accommodated within the aerofoil until they pass out of the tunnel.

One construction assumes an aerofoil well made in hard wood to a length equal to the width of the tunnel, from which length the above extensions are cut. On either side of the centre of span of the short aerofoil so obtained, a narrow thick-walled brass tube, carefully bent to shape, is let into the wood, one into the upper and the other into the lower surface as indicated in Fig. 54, so that they stand just 'proud' of the profile. These tubes are closed at the ends A and B, where, as also near the trailing edge, they can be fastened firmly in their grooves. They are then filed down in place to the correct shape, using the thick wall, and the grooves made good with wax, which is scraped to shape. Fine pressure holes are drilled into the tubes, very closely together round the nose and in staggered formation there, but widely spaced elsewhere. The central part of the aerofoil is covered with a wide strip of oiled silk. The open ends of the tubes, projecting behind the trailing edge, are connected by bicycle valve tubing each to a pressure gauge, the other limb of which communicates through a tee-piece to the hole in the side of the tunnel.

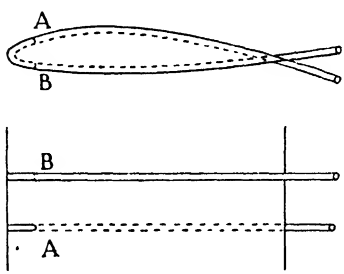


FIG. 54.—PRESSURE TUBES; THE CENTRE OF SPAN IS MIDWAY BETWEEN A AND B IN THE PLAN VIEW.

To obtain readings a hole is pricked through the oiled silk into each tube. After observing the pressures at these points for all incidences, the holes are re-sealed, care being taken to avoid blocking up the tubes in the process, and new perforations made farther along the tubes. It is essential to test for leak before each prick through; this is contrived by slightly increasing the internal pressure in each tube and verifying that the gauges will remain steady. Leaks are to be anticipated, and precautions taken beforehand to insure against them are well worth while.

Since each pair of holes in turn will be tested at all the chosen incidences, whilst eventually the pressures at all holes will be required at each one of these incidences, it is important that the angle changes be always accurately the same. To adjust so many angles by direct measurement is impracticable. If a fitment for changing angles rapidly and accurately is attached to the lift-drag balance, the model will naturally be carried on the balance throughout the investigation. Otherwise, special provisions should be made.

The reduction of the readings to pressure coefficients is effected precisely as described for the circular cylinder. But it is now necessary to prepare an accurate drawing of the aerofoil section and mark on it the precise position of the centre of each pressure hole. A suitable method of plotting is explained in the next article.

106. Two-Dimensional Aerofoil

Fig. 55 shows the section of an aerofoil set at a moderate incidence in a wind stream which it completely spans. At (*a*) the pressure readings, reduced to coefficients, are plotted normally outward from the profile on the same plan as was adopted in the first instance with the circular cylinder. Referring to the diagram (*b*), the base AB is the projection in the direction of the wind of the total depth of the aerofoil section at the given incidence. The pressure holes are projected in the same direction on to AB and the pressure coefficients re-plotted on AB as base.

Under the 2-dimensional conditions assumed, the net area of the diagram (*b*) is proportional to the form drag of the aerofoil per unit of span. Careful note should be made of the restriction of this statement to 2-dimensional flow; the corresponding net area for the central or any other section of a lifting 3-dimensional aerofoil—i.e., one with exposed wing-tips in the stream or of

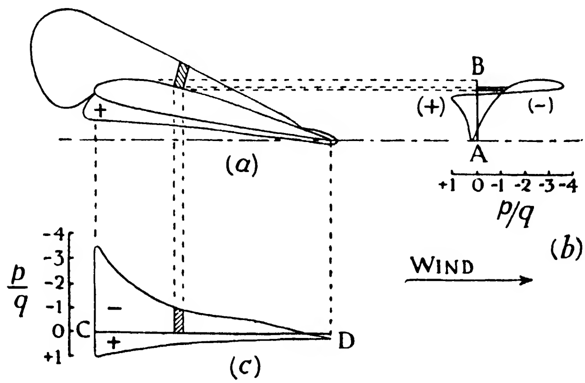


FIG. 55.—ESTIMATION OF LIFT AND FORM DRAG OF AEROFOIL FROM PRESSURE EXPLORATION.

non-uniform section—includes besides form drag the local induced drag.

The method of deducing the form drag, already explained in detail for the circular cylinder, can clearly be applied to estimate the lift from the normal pressures. The hatched area shown in (*a*) (Fig. 55) is proportional to the force arising on the element of the profile concerned, per unit of span, and is directed along the normal to the profile at the centre of the element. Just as resolving this force parallel to the direction of the undisturbed wind gives an increment of drag, so resolving it in the direction perpendicular to the wind and to the span gives an increment of lift. But this process is equivalent to resolving the length of the element in the new direction and multiplying that projected length by the observed pressure change. Thus the increment of lift is correctly shown hatched in the diagram (*c*), where the observed pressure is plotted on the projection CD of the chord of the aerofoil section, and whose area is proportional to the total lift per unit span.

Comparison with Weighed Forces.—Supposing estimations of lift and form drag to have been deduced from the pressure readings, the true lift and total drag can now be obtained by direct weighing in a lift-drag balance at the same speed. To preserve 2-dimensional conditions, the aerofoil will be arranged to ride closely between extensions fixed to the tunnel walls, as already described. Differences are to be looked for because the balance readings will include the effects of skin friction, which cannot be obtained from the normal pressures.

The measured lift per unit of span is found to agree closely with the estimate previously made, showing that skin friction only negligibly affects lift, so far as we are now concerned. The measured drag is substantially greater than the form drag and the difference is the frictional drag, or simply the skin friction.

Profile Drag.—The total drag of an aerofoil under 2-dimensional conditions is called its profile drag. Thus

$$\textit{Profile Drag} = \textit{Form Drag} + \textit{Skin Friction} \quad . \quad (39)$$

Slope of Lift Curve.—One of the distinguishing features of a 2-dimensional aerofoil is the rapidity with which the lift coefficient increases with incidence. This is readily determined by direct weighing. Up to moderate angles the rate of increase of C_L with incidence will be found to be about 0·094 per degree.

107. Improved Representation

The diagram (*c*) suggests a method of plotting the normal pressures round an aerofoil section, but not a perfect one because, as the angle of incidence increases, the projection of the chord continually shortens and the pressure holes project into continually changing positions along the shortening base. This leads to needless labour when several angles of incidence are to be analysed.

The disadvantage is avoided by plotting the pressures on the chord itself as base, a convention usually adopted, therefore, as in the next article. The area of the new diagram that now takes the place of (*c*) is no longer proportional to the lift but to the component of the pressure force perpendicular to the chord. Again, projecting the pressure holes parallel to the chord, instead of parallel to the wind direction, and plotting the pressures on the thickness of the aerofoil section as base, yields a new pressure diagram in place of (*b*), and the area of the new diagram is proportional to the component of the pressure force parallel to the chord. Let these two components be *Z* and *X*, respectively, and let α be the angle of incidence defined by the chord. Then per unit of span

$$\left.\begin{aligned} L &= Z \cos \alpha - X \sin \alpha \\ D &= Z \sin \alpha + X \cos \alpha \end{aligned}\right\} \quad . \quad . \quad . \quad . \qquad (40)$$

where L and D are the lift and drag components of the pressure force.

108. Typical Pressure Diagrams

Pressure distributions round aerofoil sections vary from one shape to another at the same incidence or the same lift coefficient.

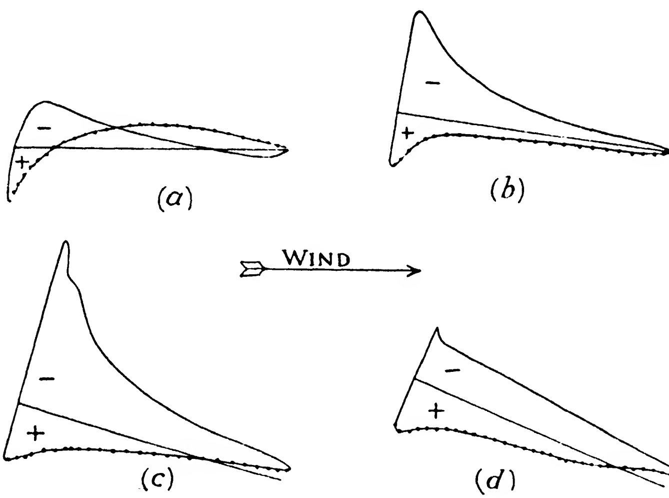


FIG. 56.—TYPICAL PRESSURE DIAGRAMS FOR AN AEROFOIL SECTION AT VARIOUS INCIDENCES: (a) HIGH SPEED, (b) CLIMBING, (c) LOW SPEED, (d) PAST THE STALL

———— Upper surfaces, –·–·–·–·–·– lower surface.

It is more profitable, therefore, to consider salient features than a specific example. Fig. 56 illustrates a sequence of pressure diagrams referring to aerofoils in general and not to one in particular. The method of plotting is that explained in the preceding article.

At lifting incidences the front stagnation point, where $p = q$, occurs just under the nose; it recedes with increase of C_L, but only slightly so. From this point the pressure falls very steeply round the nose to the upper surface, but only slowly along the lower surface. The maximum pressure drop is usually between $-\frac{3}{4}q$ and $-q$ at small lift coefficients, but may amount to $-3q$ or $-4q$ at larger ones. The pressure towards the trailing edge is unpredictable and of little present interest until the stall approaches. But then the pressure drop over the back of the upper surface can be expected to increase, as shown at (*c*), in association with a failure of the sharp pressure drop round the nose to maintain its former rapid increase with incidence. This is the beginning of the stall. Further development at still greater incidence results in a much changed diagram after the stall, as shown at (*d*), indicating poor lift and high form drag.

109. Inferences and Applications

Aerofoil sections formerly possessed flat or even concave lower surfaces, and the pressure was then increased over practically the whole of the lower surface, so that this surface exerted considerable lift, although a much greater proportion was derived from the pressure drop over the upper surface. But modern wing sections are usually bi-convex, and then the pressure is reduced below the aerofoil as well as above it, except for the forward part of the lower surface. The latter as a whole may exert no lift at all, or even a negative lift. In either case, still more lift is required from the upper surface.

The main force component Z, perpendicular to the chord, acts through the centroid of any pressure diagram shown in Fig. 56, and is differently located along the chord at different incidences. This characteristic can be modified by re-shaping the section and is evidently important as affecting the balance or trim of the aeroplane in flight. It suggests the definition of a *centre of pressure*, as will be discussed in a later article.

The diagrams also show that the maximum intensity of lift is often far greater than its mean intensity, which for straight level flight is equal to the wing-loading w (cf. Article 12). Since lift $= C_L qS$, for such flight $w = C_L q$. Thus, to take an example, if $C_L = 1$ and the maximum pressure drop $= -3q$, the maximum lift per square foot may locally approach or surpass $3w$, depending upon the inclination of the profile at the position where the maximum pressure drop occurs and the local contribution from the lower surface. It follows that the ' ribs ' of a wing (see

Fig. 57) fitted partly to transmit the pressure variation on its skin to the girders, or 'spars', which in turn support the body, must sustain large local air loads. An important application of pressure diagrams is evidently concerned with specifying just what worst load distribution the ribs must be structurally designed to carry.

FIG. 57.

Example 47.—A thin flat plate of 4 in. chord, tested at 4° incidence under 2-dimensional conditions, gives a lift of $\frac{1}{2}$ lb. per unit length. Estimate the form drag of unit length.

Neglecting the contribution of skin friction to lift, the force Z normal to the chord of the plate $= \frac{1}{2}/\cos 4°$ in lb. per unit length, for the surface of the plate is everywhere inclined at 4° to the wind and the normal pressures have no component parallel to the chord. Hence, whatever the distribution of these pressures along the chord, the form drag of unit length is $Z \sin 4° = \frac{1}{2} \tan 4° = 0{\cdot}035$ lb.

It follows from this example that the lift–drag ratio of a thin flat plate at any incidence α would be equal to $\cot \alpha$ in the absence of skin friction. This gives high values for very small incidences at which, however, skin friction in fact takes charge, so that the values are actually small. At 5°, $\cot \alpha = 11{\cdot}4$, and the lift–drag ratio of a good aerofoil at this incidence and a high Reynolds number might be double, including the skin friction. Such calculations are often made to illustrate the superiority of the aerofoil over the flat plate; another advantage is the higher maximum lift coefficient of the former.

Example 48.—If the drag measured on the balance in the test of Example 47 is 0·06 lb. per unit length of the plate, estimate the mean intensity of skin friction.

From the known form drag, the drag due to skin friction $= 0{\cdot}06 - 0{\cdot}035 = 0{\cdot}025$ and acts parallel to the wind. The skin friction itself acts parallel to the surface of the plate, and thus the frictional force per unit length is $0{\cdot}025/\cos 4° = 0{\cdot}025$ lb. closely. The wetted surface per unit length is $0{\cdot}333 \times 2$ sq. ft., whence the mean intensity of skin friction $= 0{\cdot}025/0{\cdot}667 = 0{\cdot}0375$ lb. per sq. ft.

Example 49.—If at a greater incidence (α) the lift coefficient of the above plate is 0·8 and the pressure diagrams for both the upper and lower surfaces are triangular, the pressure change, due to the motion, varying uniformly on each side from a maximum at the leading edge to zero at the trailing edge, what proportion of the lift is due to the upper surface?

The maximum pressure increase under the nose is necessarily equal to q. Let nq be the magnitude of the pressure drop above the nose, and let c be the chord of the plate. Then, since the area of a triangle is one-half the product of the base and height, the normal forces arising on the lower and upper surfaces per unit length are $\frac{1}{2}cq$ and $\frac{1}{2}cnq$, respectively. Thus the total lift, neglecting skin friction, is $\frac{1}{2}cq(n + 1) \cos \alpha$. But $C_L =$ lift per unit length$/cq$. Hence and from the question, $\frac{1}{2}(n + 1) \cos \alpha = 0{\cdot}8$, giving $n = 0{\cdot}6$, approximately.

The ratio of the upper surface lift to the total lift is $n/(n + 1)$ and, ignoring the difference between $\cos \alpha$ and unity, this gives 0·375 for the fraction required.

EFFECTS OF RESTRICTED SPAN

110. Span-Grading of Lift

The pressures observed round middle sections of a rectangular aerofoil of restricted span apply approximately only to positions far removed from the wing-tips. Fig. 58 shows at (*a*) the theoretical variation of the lift per unit of span along a rectangular aerofoil set at a moderate angle of incidence. The dimensions assumed are those suggested in Article 105; for a shorter span the distribution would be more graded, for a longer one more nearly uniform. The dotted line (*b*) applies to wings which are half-tapered in plan. In practice, irregularities occur near the wing-tips. These curves give the *span-grading* of lift, which is not to be confused with the 'span-loading', a term used to denote the whole lift of the 3-dimensional wing divided by its span. The span-grading (*b*) is mostly due to plan-form.

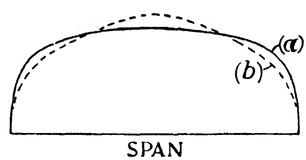


FIG. 58.—SPAN-GRADING DIAGRAMS FOR (*a*) RECTANGULAR WING, (*b*) HALF-TAPERED WING.

The span-grading can only be obtained experimentally by observing the pressures round a sequence of sections between the centre of span and a wing-tip. This rather long job verifies that for a wing, as for a 2-dimensional aerofoil, the lift is given by the pressures and is hardly contributed to by skin friction. Other aerodynamical applications will be described later on. The span-grading is also of structural interest since it specifies the bending moments the spars must be able to resist at their roots—i.e., where they join the body.

111. Effect on Incidence

On comparing the pressure diagram for a 2-dimensional aerofoil with that for the centre of span of a rectangular aerofoil of the same section and at the same incidence, a large difference is found to exist; the latter diagram is of a shape suggesting a smaller incidence and would evidently yield a less lift per unit span, illustrating a theoretical conclusion—viz., that the first action of a restricted span is to diminish the effective incidence of all sections of the aerofoil.

To determine the overall consequence of this action, select an incidence—α, say—for which the lift coefficient C_L of the 3-dimensional aerofoil is known. Now find, by use of the lift–drag balance, the incidence, α_0, say—at which the aerofoil gives the same lift coefficient under 2-dimensional conditions. It will be found that α_0 is substantially less than α; with the dimensions suggested in Article 105 the difference $\alpha - \alpha_0$ would amount to about $3C_L{}^\circ$ in a 4-ft. tunnel having an enclosed working section. Repeating at other lift coefficients well below the stall will reveal the angle of zero lift to be approximately the same for the two aerofoils and $\alpha - \alpha_0$ to be proportional to C_L.

112. Induced Drag

Induced drag is experienced by a 3-dimensional aerofoil as a modification of its pressure distribution and varies along the span, as also does the effective incidence. To verify the statement for the centre of span, evaluate the local lift per unit of span from the pressure diagram at incidence α. Find, by weighing, the incidence of the 2-dimensional aerofoil which gives the same lift per unit of span. Now determine the pressures round the 2-dimensional aerofoil and compare with the first diagram, when a marked difference will be found to exist except at small lift coefficients.

It is of more interest, however, to investigate the total induced drag, and this can be done by direct weighing. Let C_D be the drag coefficient of the 3-dimensional aerofoil at any lift coefficient C_L, well below the maximum, and C_{DO} be the drag coefficient of the 2-dimensional aerofoil at the same lift coefficient. Then the difference is called the *coefficient of induced drag* and is written C_{Di}. Thus we have

$$C_D = C_{Di} + C_{DO} \quad . \quad . \quad . \quad . \quad . \qquad (41)$$

and the induced drag D_i is given by $C_{Di}qS$, where S is the area of the aerofoil. Alternatively, we say

$$\textit{Total Drag} = \textit{Induced Drag} + \textit{Profile Drag} \quad . \qquad (42)$$

Although this statement is beyond criticism, the question of whether D_i or C_{Di} can reliably be evaluated in the manner described depends upon whether the coefficients of form drag and skin friction are sensibly the same for a 2-dimensional and a 3-dimensional aerofoil of the same section at the same lift coefficient and scale. This question is of practical importance, and numerous experiments have been carried out to investigate it. Their results justify the assumption being made provided (1) the incidence is considerably less than the stall and (2) the aerofoil has a span considerably greater than its chord.

The following law may be verified approximately from balance readings over a range of moderate incidences with the aerofoil working in turn under 2- and 3-dimensional conditions: C_{Di} is proportional to C_L^2 for a given aerofoil of limited span.

113. Varying Aspect Ratio

The *aspect ratio* of a rectangular aerofoil is defined as the ratio of the span to the chord. To include modern wings, which for aerodynamical and structural reasons are usually shaped or tapered in plan-form, the definition is generalised as the ratio of the span to the mean chord. It is always denoted by A and this symbol will be employed, there being no danger of confusion arising from the use of the same symbol for aerodynamic force. A slightly more useful expression is derived by multiplying each side of the ratio by the span, giving

$$A = \frac{(\text{span})^2}{\text{mean chord} \times \text{span}} = \frac{(\text{span})^2}{\text{wing area}} \quad . \quad . \qquad (43)$$

The aspect ratio of the aerofoil suggested for test in Article 105 is 5. The value commonly employed for experimental work is 6. Aeroplane wings often have values between 7 and 8, although monoplanes and biplanes have been constructed with wings of larger aspect ratios. An aerofoil stretched completely across a wind tunnel, or otherwise arranged to simulate 2-dimensional conditions, is frequently referred to as of *infinite aspect-ratio.*

Increasing the aspect ratio of a wing confers aerodynamical advantages, but scope is restricted by structural and constructional disadvantages and any practical case calls for a carefully determined compromise. The chief aerodynamical advantages are readily obtained mathematically from Aerofoil or Wing Theory, which, however, is beyond the scope of this book. But they can be verified by simple experiments, which will now be described.

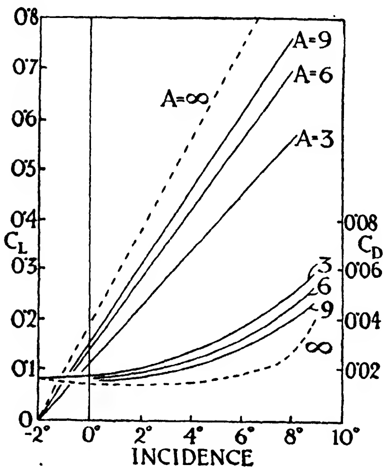


FIG. 59.—EFFECTS OF VARYING ASPECT RATIO.

It is convenient to use three rectangular aerofoils, exact copies of one another and each of aspect ratio 3, say; a chord of $3\frac{1}{2}$ inches is then suitable for a 4-foot tunnel. They are carefully fitted with detachable inset lugs so that two, and finally all three, can be joined rigidly together, end to end, at exactly the same angle of incidence. By this means, lift and drag curves are obtained in the balance for aspect ratios of 3, 6 and 9, the aerofoil section being constant throughout. These curves will be very different in the three cases; Fig. 59 gives an example, the dotted curves referring to infinite aspect ratio.

Aspect Ratio Formulæ.—Points to notice from these results are as follows. The angles of zero lift are about the same. The slope of the lift curve—i.e., the rate of increase of lift cofficient with increase of incidence—rapidly increases with increase of aspect ratio, particularly when the latter is small; the dotted lift curve defines the limit to this increase. The slope enters acutely into aerodynamical and structural calculations regarding aeroplane wings and an approximate formula is:

$$\text{Increase of } C_L \text{ per degree} = \frac{0{\cdot}094\,A}{1{\cdot}9 + A} \quad . \quad . \quad . \quad (44)$$

For convenience, $1{\cdot}9 + A$ is replaced by $2 + A$ in Examples. A much greater incidence—α, say—becomes necessary to secure a given lift coefficient as aspect ratio is reduced. If α_0 denotes the corresponding incidence under 2-dimensional conditions, the experiments will show approximately that $\alpha - \alpha_0$ is inversely proportional to A. Combining this result with the law noted at the end of Article 111 gives $\alpha - \alpha_0$ proportional to C_L/A. It is often sufficiently accurate to take

$$\alpha - \alpha_0 \text{ (in degrees)} = 20\,C_L/A \quad . \quad . \quad . \quad (45)$$

The coefficient of profile drag is assumed to be independent of A at flying incidences (Article 112). Thus the difference between any of the full-line curves and that shown dotted is due to the coefficient of induced drag, which is found to vary in inverse proportion to the aspect ratio at a given lift coefficient. Combining this result with another law stated at the end of the preceding article gives C_{Di} proportional to $C_L{}^2/A$. An approximate formula is

$$C_{Di} = 0{\cdot}35\,C_L{}^2/A \quad . \quad . \quad . \quad . \quad . \quad (46)$$

$\frac{1}{3}$ may often be substituted for $0{\cdot}35$.

All the above approximate formulæ are restricted to incidences appreciably less than the stalling angle. Again, they apply to free flight conditions and neglect effects due to the narrow wind stream of experiment. They can be verified approximately directly from balance results or, more accurately, by introducing the tunnel corrections mentioned in Article 97.

Example 50.—Find the aspect ratio of an aerofoil of elliptic plan-form having a span of 3 ft. and a maximum chord of 6 in.

The wing area $= \frac{1}{4}\pi \times 3 \times \frac{1}{2} = 3\pi/8$ sq. ft. Hence the aspect ratio is $3 \times 3 \times 8/3\pi = 7{\cdot}64$, by (43).

Example 51.—A wing of aspect ratio 8 has an angle of zero lift of $-2°$. At what incidence will it develop a lift coefficient of $1{\cdot}2$?

From (44) with $A = 8$, the increase of C_L per degree $= 0{\cdot}094 \times 8/10 = 0{\cdot}075$. Thus the increase of incidence required from the angle of zero lift = 1·2/0·075 = 16°, giving for the actual incidence 16° − 2° = 14°.

Example 52.—At $C_L = 0{\cdot}8$ an aerofoil tested under 2-dimensional conditions gives a lift–drag ratio of 24. Estimate its lift–drag ratio at the same lift coefficient for an aspect ratio of 7.

The profile drag coefficient = 0·8/24 = 0·0333. By (46) the approximate induced drag coefficient = $\frac{1}{3} \times 0{\cdot}8 \times 0{\cdot}8/7 = 0{\cdot}0305$. Thus the total drag coefficient = 0·0638 and the 3-dimensional lift–drag ratio = 0·8/0·0638 = 12·5.

EFFECTS OF CAMBER

114. Centre of Pressure

Although the distribution of pressure over a body in a wind is required in a number of connexions, information as to its gross effect is frequently sufficient. This is given by the magnitude, direction, and line of action of the aerodynamic force, how to determine which has been described in Chapter V. For a body symmetrically disposed in straight line flight, we measure the lift L, the drag D and the pitching moment M. Then the aerodynamic force $= \sqrt{(L^2 + D^2)}$ and is inclined to the direction of L by the angle $\gamma = \tan^{-1} (D/L)$; and its leverage a about the point to which M is referred is given by $a = M/\sqrt{(L^2 + D^2)}$. Use of these formulæ enables the above line of action to be marked correctly on a drawing of the body. To avoid confusion as to the pivotal point or line chosen for the measurement of M, we might state the leverage of the aerodynamic force about, say, the nose of the body, as measured on the marked drawing. However, since both this force and its leverage vary as the incidence changes, it is often more convenient to specify the intersection of the aerodynamic force with some line or plane drawn within the body—e.g., its longitudinal axis. Such a point of intersection is called the *centre of pressure* and written C.P. It lacks the physical significance of, for example, the centre of pressure of a plane lamina immersed in water, but is no less helpful on that account.

In the case of an aerofoil, the C.P. is conventionally located on the chord-line, the line passing through the centres of curvature of the extreme nose and tail of the section. We can then with little error regard L and D as acting through this point, as depicted in Fig. 60. If the C.P. is distant x behind the nose of a wing of chord c, a non-dimensional *centre of pressure coefficient*, $k_{C.P.}$, is defined as x/c. Thus $k_{C.P.}$ varies between 0 and 1 unless the C.P. is off the aerofoil, as occurs at vanishing lift coefficients.

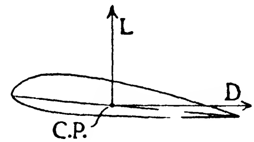


FIG. 60.

For an aerofoil of symmetrical section, $k_{C.P.}$ is approximately the same for all flying incidences and equal to $\frac{1}{4}$—i.e., the C.P. is then located at the 'quarter-chord point'. With usual wing sections, however, the C.P. shows a 'travel' as incidence varies. An example of this travel may now be determined by the method described in Articles 91–2.

In the typical case illustrated in Fig. 61, the C.P. is situated a large distance behind the aerofoil at an incidence just exceeding that for zero lift, so that the lift coefficient, though very small, is positive. The explanation lies in the fact that the aerofoil still experiences a pitching couple when its lift vanishes. As incidence increases, the C.P. moves forward, at first rapidly and then slowly, reaching a most forward position shortly before the stall occurs, after which it moves back.

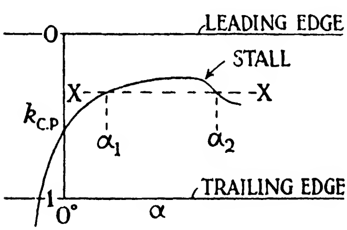


FIG. 61.—EXAMPLE OF CENTRE OF PRESSURE CURVE

Such a C.P. travel is unstable at flying incidences. For imagine the aerofoil to be pivoted and balanced about a line such as XX. If the incidence were set at α_1, the aerodynamic force would pass through XX, giving rise to no pitching moment about that line. But the state of equilibrium would resemble that of a pencil momentarily balanced upright on its point; the slightest angular displacement would at once develop. Thus if the disturbance slightly increased the incidence, the C.P. would move forward and produce a pitching moment tending to raise the nose of the aerofoil still more. The aerofoil would continue to pitch until the incidence reached α_2. But at this angle, past the stall, a small increase of incidence would cause the C.P. to move back and produce a righting moment. Thus the whole of the curve covering incidences below the stall, in which the C.P. moves forward with increase of α, indicates an unstable travel, whilst the part beyond the stall shows a stable travel.

115. Direct Determination of C.P.

An approximate method of arriving at the C.P. curve for a rectangular aerofoil without first measuring the lift, drag and pitching moment is described below. Besides being rapid, it carries conviction to the student, who is enabled to feel directly the surprisingly heavy moment sometimes generated about a pivotal line corresponding to a possible position of the centre of gravity of an aeroplane employing the aerofoil shape for wings.

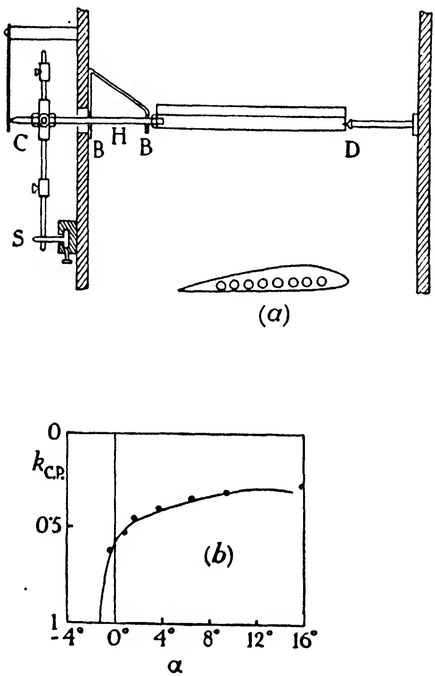


FIG. 62.—APPARATUS FOR DIRECT DETERMINATION OF C.P. OF AEROFOIL. (*b*) TEST IN A 2-FT. TUNNEL.

The curve is accurately known for the aerofoil chosen, the points are direct observations.

The simple apparatus is indicated schematically in Fig. 62. The crosshead C, located just outside the tunnel, is integral with the spindle H, which passes into the stream and is supported in the bearings BB. This spindle is securely fastened—e.g., by screw and lock-nut—to one end of the aerofoil at a chosen position along the chord. The other end of the aerofoil is freely supported at the same position along the chord by a fine peg D, working in a clearing hole drilled into that end, the peg being rigidly supported from the opposite wall of the tunnel. A guess being made as to the incidence at which the C.P. will be situated on the chosen pivotal line, the crosshead is swung to that angle by adjusting the position of the movable stop S, which restricts angular play to a minimum, and weights on C are adjusted to give static balance. With the wind on, the bottom end of C will bear on one side or the other of S, which is further moved until that end bears indifferently on either side of the stop. If this new angle differs substantially from that anticipated, the apparatus may be found, on stopping the tunnel, to be slightly out of static balance; in these circumstances the experiment is repeated, the incidence first found to give balance in the wind being regarded as a first approximation. Finally, the pivotal line is changed and the incidence determined for a second position of the C.P., and so on. To facilitate the process, one end of the aerofoil may be closely packed with screwed holes for the spindle H, as shown at (*a*), and the other end with fine clearing holes for the peg. The reliability of the method is illustrated at (*b*) from an actual test in a 2-ft. tunnel.

116. Restriction of C.P. Travel

Reducing the unstable movement of the C.P. of aeroplane wings results in a lighter structure and a smaller tail-plane, thus increasing useful load and decreasing parasitic drag. Several methods are available for shaping the wing section to this end. But all have concomitant disadvantages unless employed with restraint and discretion and, as happens so often in Aerodynamics, a compromise usually becomes necessary. The various means are readily verified by testing four or five rectangular aerofoils in the apparatus described in the preceding article or by the method of Articles 91–2. Suitable sections for these aerofoils will be plain from the following discussion.

It has been mentioned that a symmetrical aerofoil section has a practically stationary C.P., but this form of wing is inefficient and, for most flying purposes, 'camber' is required. Two variables then become significant: the amount of the camber and the shape of the camber-line.

The camber-line of a section is found as follows. On a drawing of the section a number of points are marked, each being an equal distance from both the upper and lower surfaces measured perpendicularly to them. The curve drawn through all such points—i.e., the median line of the section—is called the *camber-line*. Thus the camber-line terminates at the centres of curvature of the nose and tail of the aerofoil section where it meets the chord-line, but is usually prolonged to reach the profile, as shown in Fig. 63.

The amount of camber, or more simply the *camber*, is defined by the maximum height δ of the camber-line above the chord-line, expressed in terms of the chord c. Thus

$$camber = \delta/c.$$

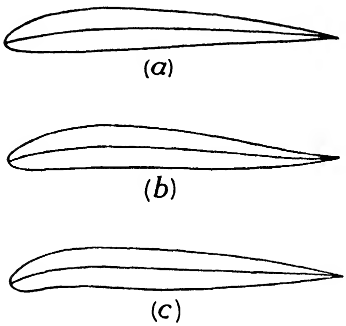


FIG. 63.—EXAMPLES OF CAMBER-LINES.

Since δ is small compared with c, the camber of a section is commonly stated as a percentage of the chord. Usual values lie between 1 per cent. and 3 per cent.

For a given shape of camber-line—e.g., a circular arc—the adverse pitching moment at the angle of zero lift is proportional to the camber. Aerofoils made to test this theoretical result, or to illustrate the exaggerated C.P. travel normally induced by large cambers, may conveniently be given a circular arc camber-line which, though a bad shape from the point of view of restricting C.P. travel, constitutes a standard of reference.

It has long been known that reflexure of the tail of the camber-line, as indicated in Fig. 63 (*b*), greatly reduces the unstable C.P. travel and, if emphasised, will eventually make the travel stable through flying incidences. The chief disadvantage of this method lies in an accompanying loss of maximum lift coefficient.

More recently, experiments in the U.S.A. have shown that the C.P. travel can be controlled by displacing the crest of the camber-line forward, and this method has been widely adopted in practice. Little travel remains if the crest occurs at one-sixth of the chord from the nose, as shown in Fig. 63 (*c*), but so extreme a displacement is usually inadvisable in view of other considerations. If a model of, say, 2 per cent. camber, with a circular-arc camber-line, and another having precisely the same camber but with the crest moved forward from midway along the chord to a distance of about $\frac{1}{4}c$ from the nose, be tested, comparison will verify a reduction of C.P. travel in the latter case.

117. Camber and the Lift Curve

The following can easily be verified by direct balance measurements on a few rectangular aerofoils of different cambers. Their camber-lines should be shaped on a single plan and have no reflexure at the tail—e.g., all may be circular arcs.

With a symmetrical section zero lift occurs at zero incidence as defined by the inclination of the chord-line to the wind. The principal effect of adding camber is to cause zero lift to occur at a negative incidence. This affects the setting of the wings on the fuselage of an aeroplane, so that the latter may normally fly on an even keel, but is otherwise of little significance; a section can be designed to have an angle of no lift of $-15°$, but no advantage accrues; in fact the model would give a bad performance in flight at full scale. The slope of the lift curve—i.e., the increase of C_L per degree of incidence, remains sensibly unchanged. It follows that if the maximum lift coefficient is about the same, stalling occurs at a smaller incidence, a feature which is again of no practical interest. It is possible by careful design to produce a highly cambered aerofoil which will stall rather late, giving a maximum lift coefficient at small scale of approximately 2. Though of passing interest as showing what can be done with models, such a result is of very doubtful value in connexion with full-scale predictions; a wing of the same section is likely to stall several degrees earlier at a lift coefficient less than 1·5, the value already suggested as suitable to expect from a well designed section of moderate camber under full-scale conditions. In short, the effects of camber variation on maximum lift as ordinarily obtained at small model scales are of dubious value when viewed in relation to full-scale flight.

Example 53.—Estimate the centre of pressure coefficient for the aerofoil of Fig. 56 (*d*).

The normal pressure curve plotted on the chord is seen to be approximately triangular. The centroid of a triangle is located at one-third of its height from its base. Thus the centre of pressure coefficient is about 0·33, probably a little more.

Example 54.—A rectangular aerofoil, whose span is 2 ft. and chord 4 in., gives in the wind tunnel a pitching moment of $-0{\cdot}09$ lb. ft. about the quarter-chord point when the lift coefficient is

0·3 and $q = 15$ lb. per sq. ft. Determine (*a*) the centre of pressure coefficient, (*b*) the moment coefficient.

(*a*) $qS = 15 \times 2/3 = 10$ lb., whence the lift $= 0{\cdot}3qS = 3$ lb. Ignoring the small moment of drag, the distance of the C.P. behind the quarter-chord point is therefore $0{\cdot}09/3 = 0{\cdot}03$ ft. $=$ 0·36 in. Thus its distance behind the leading edge is 1·36 in., and the C.P. coefficient $= 1{\cdot}36/4 = 0{\cdot}34$.

(*b*) $qSc = 10/3 = 3{\cdot}33$ lb. ft. This gives for the pitching moment coefficient : $-\ 0{\cdot}09/3{\cdot}33 = -\ 0{\cdot}027$.

Example 55.—The aerofoil of Example 54 is a one-twentieth scale model of the wings of a monoplane which flies at the same lift coefficient at 2½-times the tunnel speed. Ignoring tunnel and scale effects, what is the pitching moment of the aeroplane's wings about their quarter-chord point?

With geometrical similarity, constant density and negligible tunnel and scale effects, the moment is proportional to V^2l^3, V denoting speed and l size. Hence the full-scale pitching moment is

$$-\ 0{\cdot}09 \times (2\tfrac{1}{2})^2 \times (20)^3 = -\ 4500 \text{ lb. ft.}$$

SCALE EFFECTS

118. Scale Effects on Aerofoils

An important experiment is to test an aerofoil at the lowest and highest speeds available. The incidence should range from a value giving a small negative lift to one that exceeds the stalling angle by several degrees. The chord may be limited to one-tenth of the width of the stream, the maximum thickness of the section to one-seventh of the chord and the aspect ratio to 6. A substantial variation of aerodynamic scale is secured by change of speed; the alternative of using two models of very different sizes is not so satisfactory on account of the different tunnel effects and the difficulty of ensuring that the models shall be exactly similar to one another.

The forces on the model vary greatly in magnitude, causing very different strains in the suspension which tend to prevent a true correspondence of incidences. To guard against this source of error, either the incidence should be checked by a telescope when the wind is on or, after shutting down the tunnel, the model should be loaded with weights, in imitation of the previous air load, and the angular deflection resulting therefrom carefully measured and allowed for.

The results obtained will depend upon the section, the scales actually tested and the smoothness of flow in the tunnel. But the following salient features can be expected in a small wind tunnel.

Increase of scale will have little effect on the lift coefficients at normal flying incidences, but the lift curve will be straightened towards the stall, which will occur later and at a higher lift coefficient. The lift coefficients beyond the stall may also be greater. Corresponding changes will be reflected in the drag curve at large incidences. But, apart from such modification due to a delay of the stall, the drag coefficients at small incidences will be lower. As a result, the maximum lift-drag ratio will be

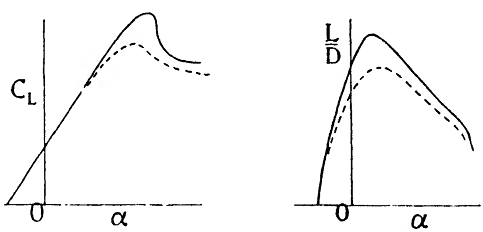


FIG. 64.—SCALE EFFECTS ON AN AEROFOIL IN A SMALL WIND TUNNEL.
- - - - - Low speed, ———— high speed.

considerably higher and occur at a smaller incidence or lift coefficient. Fig. 64 illustrates these effects.

It must not be assumed that these modifications would develop in the same way on increasing the scale much further, or for all aerofoils, or in all wind tunnels. Thus an aerofoil tested in a large, good tunnel may give a smaller minimum drag coefficient than would prevail at small full scale. The maximum lift coefficient observed in a bad wind tunnel, and, in any case, at small scale with a few aerofoils, may be greater than could be expected from an actual wing of similar section. The present experiments demonstrate, however, that important scale effects do exist.

119. Scale Effect on Profile Drag

One of the most important scale effects is that on the minimum drag coefficient of an aeroplane wing. This coefficient is contributed to by induced drag, skin friction and form drag, but the first is immune from scale effect, which modifies only the sum of the last two—viz., the profile drag. Tests may therefore be carried out under 2-dimensional conditions, which, together with the small incidence, enable an exceptionally wide range of scale to be covered in a small wind tunnel.

Fig. 65 illustrates the scale effect on a thin wing at small incidence from the smallest Reynolds numbers of experiment to those of large full scale. C_{DO} is the profile drag coefficient.

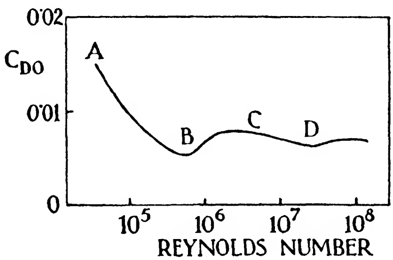


FIG. 65.—SCALE EFFECT ON PROFILE DRAG FOR SMALL THICKNESS RATIO.

The part AB of the curve corresponds to steady or laminar flow of air in the boundary layer (cf. Article 48). The improvement through this initial range is largely due to decrease of the skin friction coefficient. If it could persist to full scale, a very low coefficient would eventually be attained. But at B the flow in the back part of the boundary layer becomes turbulent, causing a great increase in the intensity of skin friction on the adjoining part of the wing. The aerodynamic scale at which turbulence begins to set in is called the *transition Reynolds number.*

As the aerodynamic scale increases further, turbulence spreads forward in the boundary layer, subjecting a still greater part of the profile to the more intense skin friction associated with this kind of flow, and so increasing the coefficient of skin friction for the wing as a whole. C_{DO} mounts rapidly in consequence, for any progressive reduction in the form drag of the thin streamline shape assumed can do little to offset the disadvantage described.

At C, almost the whole of the boundary layer has become turbulent. Thereafter, the skin friction begins to decrease again. An easing of form drag also helps, but only in a subsidiary manner, for the form drag of a good thin wing at a large scale amounts to only a small fraction of the skin friction. Still another change is in store, for finally the improvement at high scale is arrested, as indicated at D, by slight roughness of the surface of the wing. Aircraft surfaces have to be exceedingly smooth to escape being penalised in this way at high speeds, though with small models a corresponding phenomenon appears only in the compressed air tunnel.

It will be seen that the profile drag coefficient experiences much modification between a small-scale test and full-scale flight. The sequence of changes has been described with special reference to wings because it is most important, and can most conveniently be verified by experiments, in their case. But in principle it applies also to tail-planes and, in fact, to all exposed parts of an aircraft that are sufficiently well streamlined for skin friction to form a large part of their resistance.

An interesting range for experiments to cover is from a scale less than that at B to one that is greater than that at C. The largest Reynolds number would then be some 25 times as great as the smallest. This range is too wide for a single model, but can be managed by employing two models of geometrically similar sections, one five times as large as the other, if the tunnel speed can be varied from, say, 20 to 100 feet per second. Feasibility also depends, however, upon the transition Reynolds number being no more than $\frac{1}{2}$ million.

This Reynolds number—viz., that at B—depends partly upon the condition of the undisturbed wind, transition being accelerated by a rough stream and delayed by an exceptionally smooth one; the roughness or unsteadiness in the wind that matters in the present connexion is of the fine-grained kind described in Article 47. A tunnel wind is always less smooth in this sense than a natural wind. In drawing Fig. 65 a fairly steady tunnel has been assumed. Tunnels available to students are often sufficiently rough to bring on transition from laminar to turbulent flow in the boundary layer appreciably earlier. With a tunnel of this common kind, the chord of the larger of the two models need be no more than 4–5 feet. Such a size is not too large for a 4-foot tunnel, since a thin symmetrical section at zero incidence will be used. In the unlikely circumstance of a specially smooth tunnel, the stream can be lightly roughened, if need be, by a mesh screen.

Example 56.—Two geometrically similar aerofoils A and B, A being twice as large as B, are tested at various speeds V in a wind tunnel for maximum lift coefficient C_{Lm} with the following results:

V (ft. per sec.)	20	40	60	80	100
C_{Lm} (A) . .	1·00	1·08	1·14	1·18	1·20
C_{Lm} (B) . .	—	1·00	1·06	1·08	1·11

Investigate these coefficients for experimental error.

The results may be examined by plotting against the Reynolds number or, since the fluid is constant, against the product Vl, for coefficients obtained with the two models should be the same at the same value of Vl.

Let c denote the chord of A. Then the data lead to the following table:

Vc . .	$20c$	$30c$	$40c$	$50c$
C_{Lm} (A) .	1·00	1·04*	1·08	1·112*
C_{Lm} (B) .	1·00	1·06	1·08	1·11

the values marked with an asterisk being read from a fair curve drawn through the readings with model A. The observation on the smaller model at 60 ft. per sec. is probably in error. Either this should be repeated or the larger model tested at 30 ft. per sec.

Example 57.—The wings of a monoplane have a span of 30 ft. and a mean chord of 5 ft. A section of the actual wing, complete with de-icing equipment, is tested at zero incidence and under 2-dimensional conditions in a wind tunnel through a wide range of speed. The observations lead to the following formula for the profile drag coefficient:

$$C_{DO} = 0{\cdot}2/R^{1/5},$$

R being the Reynolds number specified on the chord. Estimate the lift–drag ratio of the wings of the monoplane when flying at a lift coefficient of 0·3 and at a speed of 300 ft. per sec.

The full-scale Reynolds number is (cf. Article 66) $300 \times 5 \times 6400 = 9{,}600{,}000$, giving

$$R^{1/5} = 25, \text{ approximately, and } C_{DO} = 0{\cdot}2/25 = 0{\cdot}008.$$

The aspect ratio $= 30/5 = 6$, so that by (46) the induced drag coefficient

$$C_{Di} = (0{\cdot}3)^2/18 = 0{\cdot}005, \text{ approximately.}$$

Hence the drag coefficient is obtained as

$$C_D = C_{Di} + C_{DO} = 0{\cdot}013$$

and the required lift–drag ratio $= 0{\cdot}3/0{\cdot}013 = 23{\cdot}1$.

Example 58.—A thin flat plate, having sharpened leading and trailing edges and a chord of 1 ft. 9 in., is tested under 2-dimensional conditions and at zero incidence in a wind tunnel working at 50 ft. per sec. and the drag per ft. run perpendicular to the stream is found to be 0·0185 lb. The test is repeated with a wire gauze screen placed in front of the plate, whose drag is found to increase 3 times. It is known for flat plates in general that the frictional drag coefficient has approximately the values :

$$\frac{8}{3\sqrt{R}} \text{ if the boundary layer is laminar, and}$$

$$\frac{3}{20\, R^{1/5}} \text{ if it is entirely turbulent.}$$

Comment on the state of the boundary layer flow in the two tests.

For unit length of the plate in the test the value of $qS = \frac{1}{2}\rho V^2 \times 1\frac{3}{4} = 5{\cdot}2$ lb. Hence the observations gave for the two drag coefficients : $0{\cdot}0185/5{\cdot}2 = 0{\cdot}00356$ in the first case, and 0·0107 in the second.

The Reynolds number of both tests was $50 \times 1\frac{3}{4} \times 6400 = 560{,}000$, giving $\sqrt{R} = 750$ and $R^{1/5} = 14$, approximately. Hence the two drag coefficients measured can be written in the form

$$\frac{0{\cdot}00356 \times 750}{\sqrt{R}} = \frac{2{\cdot}67}{\sqrt{R}} \text{ for the first case, and}$$

$$\frac{0{\cdot}0107 \times 14}{R^{1/5}} = \frac{0{\cdot}15}{R^{1/5}} \text{ for the second.}$$

Comparison with the formulae given in the question shows that the boundary layer was entirely laminar without the screen and entirely turbulent with the screen in position, for otherwise the drag coefficients would have had intermediate values.

TAIL-PLANE AND CONTROLS

120. Stabilizing Action

The first duty of the tail-plane of an aeroplane is to convert the unstable travel of the C.P. of the cambered wings into a stable travel for the wings and tail-plane combined. On a large modern craft its task is increased by the long engine nacelles and body and the powerful airscrews.

The principle involved is almost self-evident. Suppose an aeroplane flying at a certain speed has its incidence suddenly increased. The C.P. of the wings moves forward, tending to augment the change, as already described, and additional unstable pitching moments arise from other parts of the craft. But the incidence, and therefore the lift, of the tail-plane is increased, and this member is given such a size and leverage about the centre of gravity of the aeroplane as will overcome all opposition and pitch the craft back to its original incidence.

Investigation of the precise manner in which statical stability is secured will lead to an interesting discussion in the next chapter, but an important factor must be determined by experiment. Geometrically, the change of angle of the tail-plane is the same as that of the wings, for both are fixed to the fuselage, but aerodynamically it is much reduced by downwash, as will now be described.

121. Downwash

The flow in the region occupied by a tail-plane is deflected downward partly by the wing sections and partly by disturbances, called vortices, which trail behind the wing, particularly behind its tips. The angle through which the air is deflected from these causes is called the *downwash angle* and denoted by ε. Its value under given conditions is not the same at all points, but increases in front of the tail-plane and decreases above, below and behind it. But in all feasible positions for the tail-plane the variation from point to point is slow, and can be ignored.

For a given lift coefficient of the wings, the downwash angle at the tail-plane depends upon the span and plan-form of the wings, so that experiments must be made with a correctly shaped model. It will be assumed that a specific design of wing has been selected.

When the wing incidence is varied, the downwash angle at any point varies in proportion to the lift coefficient. This law is readily established experimentally as follows. Let the aerofoil be supported from the tunnel walls by long spindles located some three chords upstream from the lift–drag balance. Let the latter hold an exploring plate, in imitation of a small tail-plane, directly behind the centre of span. As a first test, the exploring plate may be kept constantly parallel to the floor of the tunnel whilst the aerofoil is rotated through a full range of measured incidences. The negative lift of the plate will be found to vary and, indeed, to reproduce in miniature an approximation to the lift curve of the aerofoil when the lift of the plate is plotted against the incidence of the former. In a second test, ε may be determined directly for each incidence of the aerofoil by turning the plate until its lift is zero and noting the angle required. The curve connecting ε with C_L will be a straight line throughout the range of flying incidences.

There exists no reliable method of calculating ε from first principles, whilst at the same time experimental determination is affected in a complicated manner by the limited width and height of the tunnel stream. Corrections for the latter must be left to more advanced reading.

After these corrections have been made, ε is usually found to amount to between one-third and one-half of the incidence of the aerofoil *reckoned from the angle of zero lift*. Thus the effective change of incidence of the tail-plane of an aeroplane may amount to little more than one-half its geometrical change. This necessitates, of course, a larger tail-plane than would otherwise be required.

The law of linear variation of ε with C_L is intimately related to the fact that a wing can exert upward lift only as a reaction to the rate at which it impresses downward momentum on the air. Further discussion of this important matter is best deferred, however, pending a more general view of the type of flow.

Example 59.—The tail-plane of a complete model aeroplane is a flat plate. It exerts no lift when the wings are at zero incidence, and the wings have no lift when their incidence is $-3°$. The downwash angle increases at one-third of the rate at which the wing incidence increases. If the tail-plane has an aspect ratio of 4, find its lift coefficient when the wings are at 9° incidence.

The downwash angle $\varepsilon = 0°$ when the wing incidence $\alpha = -3°$, whence and by the question, $\varepsilon = 3°/3 = 1°$ when $\alpha = 0°$. Since the tail-plane has an uncambered section and its lift $= 0$ when $\alpha = 0$, its incidence relative to the undisturbed wind must be 1° greater than α.

Thus when α is changed to 9° the incidence of the tail-plane relative to the undisturbed wind becomes 10°, for both wings and tail-plane turn together with the fuselage.

But when α is changed to 9° the increase of α from its value for $\varepsilon = 0$ is 12°. Hence and by the question again, ε becomes $12/3 = 4^\circ$. Hence the effective incidence of the tail-plane $= 10^\circ - 4^\circ = 6^\circ$.

By (44) the increase of the lift coefficient of the tail-plane per degree of effective incidence is $0{\cdot}094 \times 4/(2 + 4) = 0{\cdot}0627$. Therefore its lift coefficient with the model aeroplane in its final position is $0{\cdot}0627 \times 6 = 0{\cdot}376$.

122. Control Surfaces

It will be shown that with an entirely fixed tail-plane only one speed is possible in level flight. To vary the speed, the lift of the tail-plane must be changed. In a few early aeroplanes, tail lift was altered by quickly changing the incidence of the whole tail-plane, which was small. But for many years the tail-plane has comprised a fixed front part, to which is hinged a movable back part, called the elevator, and the necessary adjustment is made by swinging the elevator up or down according to whether the tail lift requires to be decreased or increased.

The rudder of an aeroplane works in the same way. It is usually hinged to the back of a fixed fin and varies the cross-wind force on the tail unit when turned about an approximately vertical axis.

The wings again incorporate similar control surfaces. It is frequently necessary in flight to generate a rolling moment by increasing the lift on one side of an aeroplane and decreasing it on the other. Many early aeroplanes adopted the Wrights' scheme of warping the wings to this end, but the method was soon superseded by fitting ailerons along the outer rear parts of the wings—i.e., providing for the back part of one wing in this region to be hinged up whilst that of the other is hinged down, the two ailerons being inter-connected.

All these controls, therefore, employ the device of turning a movable part of an otherwise fixed surface. Considering the surface as a whole, the device results in not only a change of incidence, but also a crude adjustment of camber, and the lift produced is greater than would be obtained by turning the hinged part alone in the absence of the fixed part. For example, depressing an elevator gives it positive lift and at the same time induces positive lift on the fixed part of the tail-plane, the alteration amounting to an increase of both camber and incidence of the tail-plane as a whole.

To facilitate the investigation of the longitudinal trim of an aeroplane, as in the next chapter, it is convenient to have a record of the variation of the lift coefficient of a complete tail-plane with (*a*) variation of the incidence of the fixed part, (*b*) change of elevator angle. A sequence of lift curves may usefully be obtained on a model tail-plane, therefore, with various elevator settings. Detaching the elevator and testing the two parts of the tail-plane separately enables the effect of the one part on the other to be analysed.

Aerodynamic Balancing.—Considering any control flap on an aeroplane, a large force may arise on it with a centre of pressure located at about one-third of the width of the flap from its front edge. If the flap be hinged at this front edge and directly connected to a lever in the pilot's cockpit, a large hinge moment may be transmitted. Minimising this moment is a necessity with large aircraft and is also desirable with smaller sizes in order to save fatigue. The straightforward method is to set the hinge-line back as indicated in Fig. 66. The part of the flap in advance of the hinge-line then has a throw and is specially shaped to avoid high form drag arising in use. In order to prevent the flap from developing a torsional vibration in flight, known as flutter, its centre of gravity must be brought at least as far forward as the hinge-line, and setting the latter back facilitates the necessary weighting.

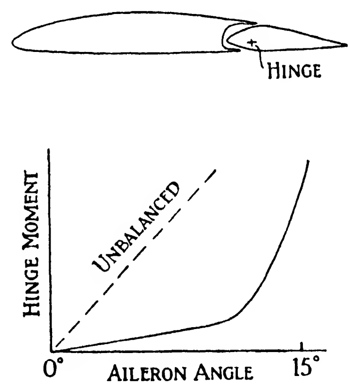


FIG. 66.—REDUCTION OF HINGE MOMENT OF AILERON BY A BACKWARD LOCATION OF ITS HINGE-LINE.

Experiments to determine a suitable location for the hinge-line provide an example of the many direct uses of the wind tunnel to the designer. The overriding condition to be observed is that in no circumstances of flight must the C.P. of the flap reach as far forward as the hinge-line, and the remaining hinge moment must continuously increase as the flap is turned, so that the pilot can feel an increasing resistance to operation. Complying with this condition usually results in the control 'going hard' at some moderate angle, as shown in the figure.

Other systems of aerodynamic balancing are known. Large control flaps may be operated by a small rudder fixed to them, a narrow strip known as a *control balance and trimming tab*.

THE STALL, THE SLOT AND THE FLAP

123. All bodies have a stalling angle beyond which lift fails to increase. If they extend mainly in the direction of motion, the stalling angle is large, but the maximum lift is still restricted, whilst the associated induced drag is high. Efficiently lifting wings stall at less than 20°, when a sharp drop often occurs in the lift coefficient. A large lift coefficient is required from the wings for slow flight, during which they are liable to be stalled by an upgust or by depressing an aileron, and the consequent loss of lift and lateral control are of first importance, especially during the crucial operation of landing.

The problem of stalling, anti-stalling devices and lift augmentation is evidently of outstanding practical interest. A number of simple experiments that can readily be performed in a wind tunnel facilitates future discussion in terms of flight.

124. Nature of Stalling

The sequence of events leading to the partial failure of lift and rapid increase of drag which characterise stalling will be described with reference to a thin convex section such as that of a modern aeroplane wing. There are two points of breakaway, one on the upper and one on the lower surface, and at small incidences both are located well back towards the trailing edge, leading to a narrow wake. As incidence increases, the lower point remains approximately stationary or may even recede slightly towards the trailing edge, but the upper point creeps forward, gradually thickening the wake. This progression is accelerated until, when the stall ultimately occurs, the flow makes little effort to follow the upper surface, but springs away early to form a deep wake.

The description applies more particularly to a small scale and results in a rather gentle type of stall. The striking effect on the thickness of the wake is readily demonstrated with a traversing pitot tube on the lines suggested in Article 103; Fig. 67 is drawn from such a test at a small Reynolds number. Probing the eddying flow over the back of the stalled aerofoil with a wire carrying a streamer of unspun silk will show evidence of return flow, a feature of a bluff shape, which the aerofoil has now become. The experiment may be extended to measure the much reduced wind speed some three chords behind the aerofoil. Then will be perceived the poor circumstances of a tail unit behind a stalled wing and the desirability of a high rudder to retain directional control under extreme conditions.

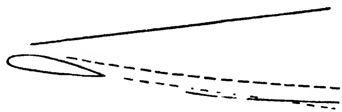


FIG. 67.—WAKE BEHIND AEROFOIL.
– – – – – Before stalling,
———— after stalling.

At larger scales, or even with a small model in some wind tunnels, turbulence in the boundary layer delays breakaway and produces a higher lift coefficient at an increased incidence. The stall is then often the more abrupt.

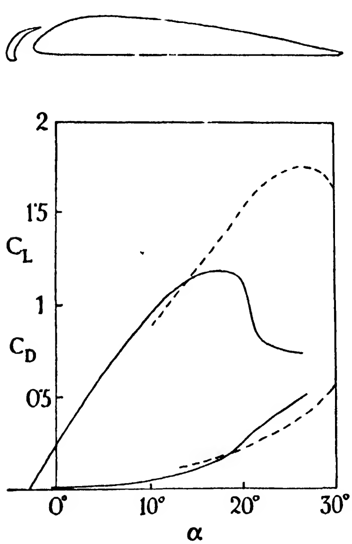


FIG. 68.—USE OF NOSE SLOT TO DELAY STALLING.
———— Slot closed,
– – – – – slot open.

125. Nose Slot

The stalling of a wing is greatly delayed by the *nose slot.* This device involves fitting a narrow guide vane (or slat, or auxiliary aerofoil) a short distance in front of the nose, as indicated in Fig. 68. The vane is retracted to form the leading edge of the wing in normal flight, and is advanced to expose a slot only when stalling would otherwise be imminent.

The same figure gives typical lift and drag curves with the slot open and closed. The slot prolongs the original lift curve by perhaps 50 per cent. before the stall occurs. The large incidence for maximum lift prevents application of the device to the reduction of landing speeds, since it would entail a high and heavy undercarriage and discomfort to passengers. This difficulty could be overcome by mounting the wings in trunnions so that their incidence could be increased without pitching the fuselage, an expedient which has been demonstrated in full-scale flight, but not adopted. However, it is desirable to have as much drag as possible on landing, and the nose slot tends to diminish drag. The last characteristic explains the unrivalled value of the nose slot in connexion with lateral control near the stall, when the wing raised by depressing its aileron and opening its slot is desired to have less drag than the other wing, which is lowered, so that the aeroplane will turn in a direction suitable for the bank. The auxiliary aerofoil may be linked in such a manner as to open a slot only when the aileron behind it is depressed.

Experiments to test the efficacy of the device are easily performed on any aerofoil. For simplicity, the vane may consist of a long narrow strip of soft copper plate, bent to the shape of the nose and mounted thereon by pliable lugs. The aerofoil is carried on a lift-drag balance at an incidence past its stall, and the position and incidence of the vane are adjusted by trial and error until a large increase of lift and reduction of drag result. The subject is also particularly suitable for visual tests with smoke, a low wind speed being employed. The principle has a wide application outside Aeronautics, and smoke tests will show that a fairly smooth flow can be induced by its means round even a right-angled corner, for instance.

126. Cut Slot

The stalling of a wing by the depression of its aileron can also be delayed by so shaping the front part of the aileron and locating its hinge that, on being depressed considerably, it opens a slot through the body of the wing, Fig. 69 (*a*). Such a channel is called a *cut slot*. A cut slot can be arranged to remain closed for small positive angles of the aileron and when the latter is raised.

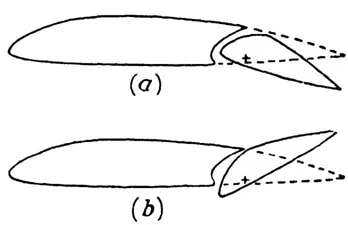


FIG. 69.—AILERON WITH CUT SLOT.

It may be contended that raising such an aileron mutilates the lower surface of the wing (Fig. 69 (*b*)) and so increases form drag. This effect can be mitigated, if desired, by shaping the underside of the nose of the aileron, but is often considered an advantage, assisting the turning action described in the preceding article. An aileron that deliberately increases form drag on being raised is often described as of the 'Frise' type.

The cut slot can also be used between the tail-plane and the elevator. It should then open to induce flow from above to below because the elevator requires to be raised through much greater angles than it requires to be depressed.

The easiest way to test the anti-stalling properties of a cut slot is to fit it along the whole length of an aerofoil, whose back part is then hinged, and to use the lift–drag balance. A special interest in this experiment lies in demonstrating that such a flap with a cut slot can assist take-off and climbing by increasing lift at little sacrifice in drag. Another method is to test a model fitted with ailerons on the cut-slot principle. The aerofoil is pivoted about a line parallel to the wind and passing through its centre of span, a fitment being incorporated to permit variation of incidence. A counterpoise beneath the tunnel is hung by a wire from one of the wing-tips, and the other is attached to a lift beam, so that the rolling moment due to deflecting the ailerons can be measured with the slot on the one side open or sealed. A strikingly more powerful moment will be found in the former case at high incidence. (The slot should be designed with some care, making use on the drawing-board of a cardboard templet of the proposed aileron section secured by a pin, representing the hinge, whose position can be adjusted. A relatively wider passage should be provided on a model than would be opened on an aeroplane, owing to scale effect.)

127. Flaps

Some early aeroplanes and seaplanes exploited the contrivance of hinging upwards and downwards the back parts of their wings as a whole in order to improve performance and especially to reduce landing speeds. But in the latter connexion the so-called hinged flaps were not depressed through a sufficiently large angle. When this angle is 60–90° the gain in maximum lift coefficient is considerable and the greatly increased form drag is a help, not a hindrance, to landing. For maximum effectiveness from this point of view there should be no leak between the flap and the body of the wing.

Scientifically, the hinged flap was superseded in 1921 as a landing device by the *split flap*, although, as has already been remarked, the latter did not come into general use until some 12 years later. The split flap is more effective than its predecessor and has other advantages. It is exposed by letting down the back part of the lower surface of a wing only, leaving the upper part undisturbed (Fig. 70). The same figure indicates the effects on the lift and drag curves of the wing with a large flap angle. Points to notice are as follows :

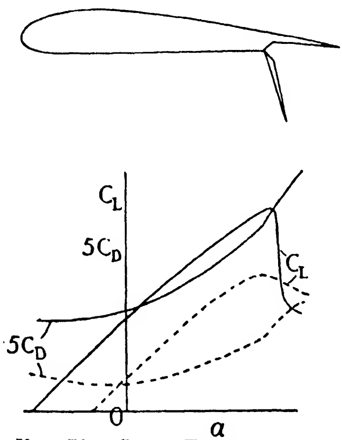


FIG. 70.—SPLIT FLAT; EFFECTS ON LIFT AND DRAG.
——— Flap down,
- - - - - flap retracted.

(1) Most of the advantage in lift is attained when the flap angle reaches 60°, but a larger angle gives appreciably greater drag.

(2) Lift is increased by approximately the same amount at all incidences up to the stalling angle, which is scarcely changed.

(3) The lift past the stall is little changed, so that the loss of lift at the stall is much more serious with the flap down than with it retracted.

(4) The increase of drag is so large that, in spite of increased lift, there is a remarkable reduction of lift–drag ratio.

The advantages these qualities confer in landing are discussed later. But it may be noted that (2) enables the device to be employed without a high undercarriage; (3) prevents full exploitation and leads to flaps being fitted along only the inner half or two-thirds of the span; (2) and (4) enable a confined landing ground to be approached slowly and on an even keel at a steep angle to the horizon. Since the London–Melbourne race, many improvements have been incorporated in Dayton Wright's original invention.

Verifying experiments should be carried out at as large a scale as possible, but the high maximum lift coefficients realised at full scale, sometimes exceeding 2·3, cannot be expected in a small tunnel.

128. Lateral Spread of Stall

The stalling of a wing section has been described in Article 124 and its effect on the pressure distribution in Article 108. Considering a complete wing, the stall does not in general occur simultaneously at all sections, but begins at certain parts of the span and spreads to other parts as incidence is further increased. With a rectangular plan-form, middle sections fail first, often several degrees earlier than those towards the tips; with a sharply tapered plan-form the converse is true. Plan-forms can be shaped to encourage evenness of stalling, but the problem is more complicated than one of plan-form alone.

The complete lift curve of an aerofoil obtained from an aerodynamic balance averages all sections and conveys no information as to which have failed and which remain unstalled, a question of both aerodynamical and structural interest. Extensive pressure explorations are required to establish the much changed lift-grading curves (cf. Article 110) that hold at large incidences. The distribution of the stall is readily made visible, however, by glueing a large number of very short streamers over the back half of the upper surface. These flicker, often turning upstream, under the disturbed regions, as will be anticipated from the experiment with a single streamer at the end of a wire in the 2-dimensional case.

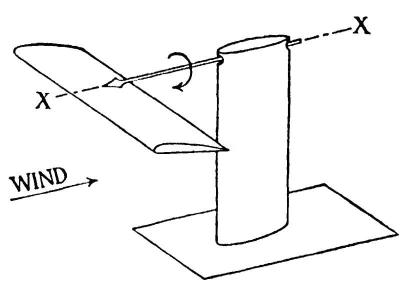


FIG. 71.—APPARATUS FOR DEMONSTRATING AUTOROTATION IN A SLOW 2-FT. WIND TUNNEL.

129. Autorotation

An experiment of exceptional interest in both the design and operation of aircraft is as follows. Let an aerofoil of rectangular or any other symmetrical plan-form be pivoted so that it can rotate freely about an axis XX through its centre of span and parallel to the wind (Fig. 71). At any incidence well below the stall, its span will assume indifferently any direction, provided no spin exists in the oncoming wind. Give the aerofoil a sharp angular velocity about XX and it will quickly come to rest again. But a range of larger incidences can be found, near to or past the stall, at which the aerofoil, if only slightly disturbed, will gather rotational speed until it is spinning quickly about XX. It is then said to *autorotate*.

The phenomenon is easily explained. For clearness, consider the rotating aerofoil at an instant when it is right-way-up, with its span horizontal. Let v be the vertical velocity of the downwardly moving section at a distance y from the axis and let V be the undisturbed velocity of the wind. Then from the triangle of velocities (Fig. 72 (*a*)), the incidence of the section is increased by the amount $\Delta\alpha = v/V$, approximately. If any difficulty is experienced in seeing this, one may reflect that the average time t taken by particles of air to pass the wing section, of chord c, is c/V, whilst during this time the tail of the section moves down a distance $vt = vc/V$. Hence, so far as the air is concerned, the section might just as well be stationary and inclined at an incidence increased by the amount vt/c, approximately—i.e., v/V. Similarly, the corresponding section on the other side of the aerofoil, at an equal distance from the axis, has its incidence decreased by the same amount. But, if ω is the angular velocity of the aerofoil, $v = \omega y$, and it is seen that this change of incidence increases or decreases uniformly along each half-span; it is a maximum at the wing-tips and zero at the centre of span.

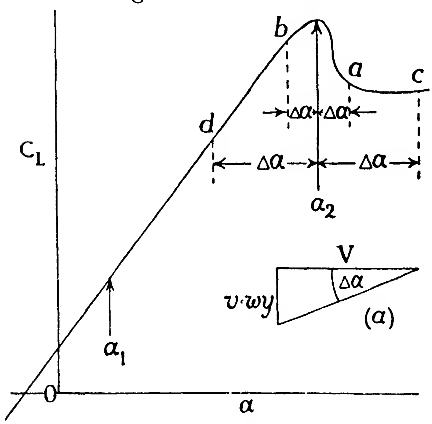


FIG. 72.

Suppose first that the incidence at the centre of span has a small value α_1 (Fig. 72). Then the downwardly moving sections, having their incidence increased, exert more lift, whilst the upwardly moving sections along the other half of the span exert less lift, and a stable rolling moment results resisting the rotation.

This moment is surprisingly large in a practical case and partly explains the need for long ailerons; they have much resistance to overcome when rolling is required at speed.

Now suppose the incidence at the centre of span to have a large value α_2 appropriate to flight near the stall. Then the lift of all sections is reduced, whether they are moving upward or downward. For small values of $\Delta\alpha$—i.e., for small values of $\omega y/V$ and especially, therefore, at small angular velocities or for sections near the roots of the wings—the lift of the downwardly moving element is reduced much more than that of the corresponding upwardly moving one (cf. points *a* and *b* of Fig. 72), and such a pair of elements exerts an unstable rolling moment, tending to increase the angular velocity. Considering, on the other hand, pairs of elements well out towards the wing-tips, for which $\Delta\alpha$ is large, these exert a stabilising moment, though a weak one (cf. points *c* and *d* of the figure). The ensuing motion depends on whether the stable or the unstable moments are the more powerful. At a small angular velocity the latter predominate and the angular velocity increases. But this makes the former group more effective, until a stage is finally reached when there is no resultant couple on the aerofoil. By then it may be rotating quickly, and it will continue to do so indefinitely in the wind.

Autorotation is easy to demonstrate even in a diminutive wind tunnel, for which the simple apparatus shown in Fig. 71 would be suitable, provided the aerofoil has been mounted at an autorotating incidence. With little complication the incidence can be made adjustable from outside the tunnel without interfering mechanically with the rotation, and a sudden decrease of incidence can then be shown to bring the aerofoil quickly to rest.

A larger tunnel permits of more ambitious experiments. These begin with checking a forecast of the rotational speed, obtained by a process of graphical integration from the lift curve of the aerofoil under test. In a 4-foot tunnel of considerable speed the apparatus should be exceptionally sturdy and well secured. The foregoing account is incomplete; there may also be large yawing moments; and a viciously stalling model can rip itself free from, or twist up, fragile supporting gear, or tear the latter from weak fastenings.

130. Practical Significance

Apart from several points of interest in connexion with a more advanced study of Aerodynamics, the bearing of the above experiment on flying is plain. An aeroplane may develop autorotation when flying near its stalling angle, the wings also losing lift as a whole. The descending spiral flight that ensues is called the *ordinary spin* and is not dangerous provided the corkscrew path is not allowed to tighten unduly. It can be stopped by decreasing incidence, which involves a dive. Room for the recovery does not exist when landing, and so, to avoid risk of autorotation in these circumstances, an aerodrome is approached at a ' coming-in ' speed that is considerably greater than the stalling speed of the aeroplane concerned.

The stalling speed is reached just before touching down, and then a small upgust on one wing may start the phenomenon, whilst the ailerons may not be able to arrest it. The restricted rolling of the aeroplane is known as ' wing-dropping '. Reasons for limiting the lateral extent of landing flaps and avoiding wings which stall first at their tips will now be apparent.

Chapter VII

EQUILIBRIUM IN STEADY FLIGHT

131. For an aircraft to fly steadily it must be in equilibrium with respect to all the forces and couples acting externally upon it. The weight and airscrew thrust must just balance the resultant aerodynamic force, and the control surfaces must be so adjusted that no uncompensated moment remains to produce pitch, roll or yaw.

These conditions cannot be fulfilled with fixed controls for any appreciable length of time. The weight is not constant, being continuously decreased by consumption of fuel. The aircraft itself is disturbed continually by small changes of wind. It is nevertheless of great interest to ignore such variations and examine the equilibrium of the aircraft flying with assumed steadiness in a number of circumstances typical of those it will normally encounter. The aircraft is then regarded as of constant weight and so designed as to respond in a stable manner to casual disturbance, so that the condition of flight for which its controls have been set is quickly restored.

STRAIGHT LEVEL FLIGHT

132. Flight in a straight level path is naturally of greatest interest and will be studied at some length. The lateral controls will be assumed so disposed as to keep the span horizontal and perpendicular to the direction of motion.

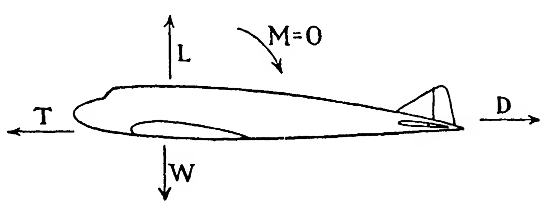


FIG. 73.

Referring to Fig. 73, the conditions for equilibrium in this form of flight are: total lift (L) = total weight (W); total drag (D) = total horizontal component of airscrew thrust (T), and the algebraic sum (M) of all the pitching moments about the centre of gravity of the aeroplane = 0. These conditions are briefly noted by writing down the appropriate equations of equilibrium :

$$W = L, \qquad T = D, \qquad M = 0 \quad . \quad . \quad . \quad (47)$$

The aeroplane can fly steadily only if all three equations are satisfied and if, in addition, the equilibrium noted in the third equation is stable in regard to small angular disturbance.

We have to enquire what these conditions imply and how—and within what limitations—they can be fulfilled. We shall take each equation in turn and discuss its main points in detail, assuming meanwhile that the other two equations are satisfied automatically.

133. The Lift Equation

L denotes the algebraic sum of all lifts generated on the aircraft. Contributions arise, for instance, from the tail-plane and also from the airscrews, whose axes can lie in the direction of motion only at one speed. All these small and variable lifts are commonly negligible, however, compared with that of the wings. Thus, if S denotes the wing area (in square feet) and q the stagnation pressure (in lb. per square foot), a close approximation to the first equation of (47) is

$$W = L = C_L q S \quad . \quad . \quad . \quad . \quad . \quad . \quad (48)$$

An alternative form is obtained by writing w for the wing-loading W/S, viz.,

$$w = C_L q \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (49)$$

W is to be expressed in lb., ρ in slugs per cubic foot and V in feet per second.

Considering application to a given aeroplane in straight level flight, the following implications of these formulæ should be noticed. For a given indicated air speed V_i (cf. Article 56) the value of q is constant, no matter what the density may be. Thus C_L and V_i are connected by a single relationship which does not change with altitude. For any possible value of C_L the stagnation pressure is obtained from $q = w/C_L$ and then the indicated air speed follows from the formula

$$V_i = 19{\cdot}8\sqrt{q} \text{ miles per hour} \quad . \quad . \quad . \quad (50)$$

Calculations do not depend on whether the landing flaps are up or down, since lift coefficients are always specified on S and ignore any increase of area that may result from lowering some types of flap. C_L being inversely proportional to q, the minimum value of q, and therefore of V_i, occurs at the maximum value of C_L. The aeroplane stalls at the same value of V_i at all altitudes. Any pair of corresponding values of C_L and q being known—e.g., those at the stall—the value of q for any other lift coefficient, or *vice versa*, can be found at once, for

$$q = q_1 \cdot C_{L1}/C_L \quad . \quad . \quad . \quad . \quad . \quad (51)$$

The following examples illustrate further the use of these simple formulæ.

Example 60.—The wing-loading of an aeroplane is 33 lb. per sq. ft. and the maximum lift coefficient of its wings with flaps down is 2·2. What is the stalling speed (V_s)?

By (49), $33 = 2{\cdot}2q$, whence $q = 15$ and, by (50), $V_s = 76{\cdot}7$ m.p.h. (Note that the stalling speed is given as an indicated speed; this is the form in which the pilot requires to know it.)

Example 61.—Assuming that the aeroplane of Example 60 is to fly at a ground speed of 330 m.p.h. at an altitude where the relative density $\sigma = 0{\cdot}64$ and there exists a following wind of 43 m.p.h., what must be the lift coefficient?

The true air speed = 330 − 43 = 287 m.p.h. The indicated air speed $= 287\sqrt{\sigma} = 230$ m.p.h., nearly. By Example 60 the lift coefficient at $V_i = 76{\cdot}7$ m.p.h is 2·2. Therefore the required $C_L = 2{\cdot}2 \times (76{\cdot}7/230)^2 = 0{\cdot}245$.

Example 62.—A monoplane of 10,500 lb. weight is to have wings of aspect ratio 7 and is to fly at an indicated air speed of 300 m.p.h. with a lift coefficient of 0·15. Determine the necessary span.

Let c be the mean chord. Then the span $= 7c$ and the wing area $= 7c^2$. Now 300 m.p.h. = 440 ft. per sec., whence $q = \frac{1}{2} \times 0{\cdot}00238 \times 440^2 = 230{\cdot}4$ lb. per sq. ft. With these values (48) gives

$$10{,}500 = 0{\cdot}15 \times 230{\cdot}4 \times 7c^2.$$

So $c^2 = 10{,}000/230{\cdot}4$—i.e., $c = 6{\cdot}59$ ft. Thus the span = 46 ft. $1\frac{1}{2}$ in.

134. Application to Landing Speed

The calculation of the minimum speed at which an aeroplane can actually land in a quiet atmosphere is unreliable for two reasons. As described in Article 130, an aeroplane approaching a landing ground is kept well above the stalling speed to ensure control and particularly to guard against wing-dropping. Its coming-in incidence is about the same as its incidence when standing on the ground—i.e., 4° or 5° less than the stalling angle. The aeroplane touches down during an unsteady motion, in course of which the flight path is flattened out and the incidence at first increased. To some extent the process is individual to the pilot and especially low landing speeds are achieved by exceptional skill. A second difficulty arises from the fact that the near presence of the ground affects the lift coefficient of the wings in a way which is imperfectly understood. This effect is most marked, of course, with aeroplanes which have especially low wings.

Practical experience shows, however, that in skilled hands landing speeds differ little from stalling speeds. For this reason the landing speed of an aeroplane is usually identified with its stalling speed, and this practice will be followed here. Lack of operational ability or unfamiliarity with a given type would make a somewhat higher landing speed advisable.

With this understanding, Table IV can be verified readily. It relates to a monoplane weighing 5 tons, having wings of aspect ratio 7 fitted with flaps which raise the maximum lift coefficient by 50 per cent.—i.e., to about 2·25. Quantities are given in round figures for ease of inspection.

TABLE IV.

Landing speed (m.p.h.)	Required wing area (sq. ft.)	Required span (ft.)	Wing-loading (lb./sq. ft.)
80	303	46	37
70	396	53	28
60	539	61½	21
50	776	74	14½
40	1212	92	9
30	2155	123	5

A biplane of the same weight without landing flaps would have a maximum lift coefficient amounting to little more than one-half the value assumed for the table. Thus for the same landing speed its span and chord would be nearly as large as those of the monoplane equipped with flaps and its wing loading little more than one-half the values given.

Inferences drawn from such calculations and associated aeronautical considerations may be summarised as follows. It is impossible for aeroplanes to achieve really low landing speeds. In order to land in the neighbourhood of 40 miles per hour, early aeroplanes carried less than 5 lb. per square foot, and were thus characterised by enormous wings for their weight. The landing speeds of modern main-line civil transport aeroplanes are upwards of 70 miles per hour and correspond to wing-loadings of 28 lb. per square foot, or more. Larger and better-prepared aerodromes will doubtless make higher landing speeds and wing-loadings feasible for such aeroplanes, as they already are for flying-boats. On the other hand, landing speeds of under 50 miles per hour still remain of interest in connexion with private or inexpert flying and alighting on confined or rough surfaces. Biplanes with landing flaps may be used especially in these connexions.

135. The Total Drag

The drag of an aeroplane can be divided into two parts: the drag of the wings (D_w) and the remainder, which will be denoted by D_B. The latter is sometimes called the body drag, but a better term is the American one, the *extra-to-wing drag*, for it includes the drag of the fin and rudder, tail-plane, engine nacelles and other exposed parts or attachments. Still another name employed occasionally is the 'parasitic drag', but care is then necessary to see just what is intended, for part of the wing drag is sometimes included under this heading.

Either D_w or D_B, or both, are increased by airscrew slipstreams. This effect is neglected in the present chapter; rules for taking it into account, by which present formulæ can be corrected, are given in Chapter IX. The total drag in the absence of slipstream effects is often called the *glider drag* of the aeroplane.

The *wing drag* for any value of q can be obtained by first calculating the appropriate value of the lift coefficient, and then reading off the corresponding value of the drag coefficient from the ' polar ' connecting C_D with C_L for the wings concerned (see Fig. 74), and finally using the formula

$$D_w = C_D q S.$$

An alternative, and usually more convenient, method is to note that D_w = the lift of the wings ÷ their lift–drag ratio. Writing r for the last quantity and remembering that $L = W$ for straight level flight, we then have

$$D_w = W/r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (52)$$

To evaluate for any value of q we first find the lift coefficient from $C_L = w/q$, or otherwise, and read off the corresponding value of r from a curve connecting r with C_L (see Fig. 75).

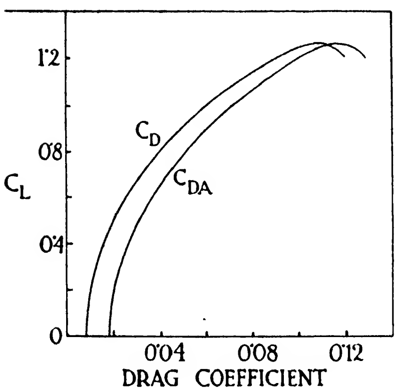


FIG. 74.

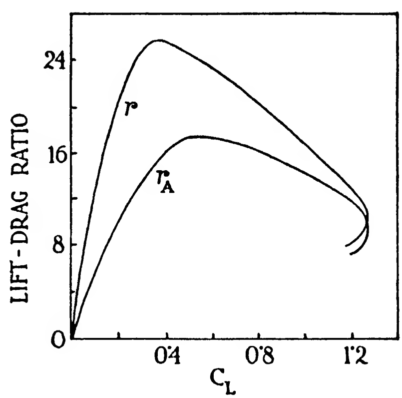


FIG. 75.

The extra-to-wing drag is difficult to determine accurately. It is estimated by experiments on some parts and advanced calculations regarding others, whilst allowances are also made for the interference between one component and another and for aerodynamic scale. These matters have been treated in previous chapters so far as scope allows, and it is now assumed that a particular value of D_B—viz., D_B'—has been estimated for a particular value of q—viz., q'. The approximation is also adopted that D_B is proportional to q; this neglects further modifications due to change of scale and incidence; errors on this score are minimised by estimating D_B' for the speed and altitude with which subsequent calculations will be principally concerned.

On this understanding,

$$D_B = D_B' \times q/q' = D_B' \times C_L'/C_L \quad . \quad . \quad . \quad (53)$$

since C_L is inversely proportional to q for straight level flight.

Summing up, the total drag D is given approximately by

$$D = \frac{W}{r} + D_B' \cdot \frac{C_L'}{C_L} \quad . \quad . \quad . \quad . \quad . \quad (54)$$

and q/q' may be substituted for C_L'/C_L if more convenient. With the assumptions made, this expression is independent of altitude.

A second method will now be described which permits variation of extra-to-wing drag with incidence to be taken into account when known.

A series of values of q are evaluated from a series of values of C_L by the relation $q = w/C_L$, and the corresponding incidences α are read from a curve connecting C_L with α. At each q and α the extra-to-wing drag D_B is estimated for the altitude at which the aeroplane will fly. Choice of altitude fixes the aerodynamic scale for each estimate of D_B, for the scale is proportional to the Reynolds number, which depends on the true air speed and not the indicated air speed. The estimates of D_B are then divided by qS to yield a series of drag coefficients C_{DB}, all based on the *wing* area. These are added to the values of C_D for the wings at the various incidences, giving a total drag coefficient $C_{DA} = C_D + C_{DB}$.

The overall drag coefficient C_{DA} may be plotted against C_L, yielding a ' polar ' for the complete aeroplane (Fig. 74). To use this curve to find the total drag D at any indicated air speed V_i, the first step is to calculate the value of $q = \frac{1}{2} \times 0{\cdot}00238 \times V_i^2 \times (22/15)^2 = 0{\cdot}00256\, V_i^2$, independent of altitude. The next step is to find C_L, which is equal to w/q for straight level flight. Then C_{DA} is read from the polar. Finally

$$D = C_{DA}qS = C_{DA}qW/w \quad . \quad . \quad . \quad . \quad (55)$$

The following alternative method of handling the estimates of C_{DA} often saves labour and will have a special interest later on. Each value of C_{DA} is divided into the corresponding value of C_L, giving a corresponding value of the overall lift–drag ratio, which will be denoted by r_A to distinguish it from the lift–drag ratio of the wings only. Then r_A is plotted against C_L (Fig. 75). In use, the total drag at any indicated air speed is found by first determining the value of C_L, as before, and dividing the corresponding value of r_A, as read from the curve, into the lift. For straight level flight the lift is equal to the weight, whence

$$D = W/r_A \qquad (56)$$

136. The Drag Equation

Always neglecting slipstream effects, and also scale effects due to a varying difference between V_i and the true air speed, each aeroplane with flaps retracted (or with them in any one position) possesses a unique curve connecting the total drag D in straight level flight with the indicated air speed. Remaining the same for all altitudes with the approximations mentioned, this curve also requires little correction to apply to climbing and gliding at moderate angles to the horizon.

To see how the curve is constructed, it is best to consider a specific example, and calculations will relate to the following aeroplane :

> Weight : 5 tons; wing-loading : 25 lb. per square foot; wing characteristics : as detailed in first three columns of Table V below (flaps retracted); extra-to-wing drag : 240 lb. at an indicated air speed of 150 miles per hour.

Referring to the table, α denotes the incidence in degrees, C_L the lift coefficient and r the lift–drag ratio of the wings. q, the stagnation pressure in lb. per sq. ft. $= 25/C_L$. V_i, the indicated air speed in m.p.h. $= 19{\cdot}8\sqrt{q}$. D_w, the drag of the wings in lb. $= 11{,}200/r$. D_B, the extra-to-wing drag $= 240\ q/57{\cdot}6$, 57·6 lb. per square foot being the value of q at 150 m.p.h. D, the total drag in lb., is the sum of D_w and D_B.

TABLE V.

α	C_L	r	q	V_i	D_w	D_B	D
− 0·4	0·1	11·0	250	313	1018	1042	2060
+ 0·9	0·2	19·5	125	221	574	521	1095
3·6	0·4	26·0	62·5	157	431	260	691
6·0	0·6	23·5	41·7	128	477	174	651
10·0	0·9	18·5	27·8	104	605	116	721
14·3	1·2	13·0	20·8	90	862	87	949

The results are exhibited in Fig. 76, where the full line is the drag curve required. The dotted curves show how this drag is made up through the speed range; a large proportion is due to the wings at low speeds but only about one-half at high speeds. The total drag diminishes to a minimum of just under 651 lb. at an indicated air speed of about 130 m.p.h. The curve might have been extended to a minimum speed of approximately 81 m.p.h., but this has not been done as the landing flaps would then normally be in use.

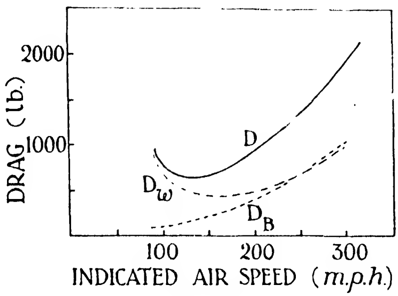


Fig. 76.

Referring to the second equation of equilibrium for level flight, viz., $T = D$, the engine throttle must be so adjusted at each speed that the airscrew gives a thrust equal to the total drag shown in the figure. If the power unit is not capable of providing this thrust, then level flight is not possible at the speeds at which the failure occurs.

137. The Moment Equation

We proceed to consider the moment equation, $M = 0$. In flight at any wing incidence α, unstable pitching moments about the C.G. of the aeroplane arise from the wings (Article 114) and other parts, e.g., the fuselage and engine nacelles. The equation requires, in the first instance, all these to be cancelled by a pitching moment generated in the opposite sense by the tail-plane. But the equilibrium also requires to be at least statically stable; that is to say, should the incidence α casually increase or decrease during flight, the tail-plane moment must increase more rapidly than the unstable moment, overruling the latter and bringing the incidence back to α again. Moreover, this stabilising action must take place at all incidences that may occur during flight. The resultant pitching moment can be made zero at a particular incidence in a variety of ways, but only a method that accords with this wider exigency can be accepted. Thus it is necessary to consider the entire speed range.

The matter is not complicated if approached methodically, but there have to be taken into account a number of factors which operate simultaneously. These are explained in the following articles, where are also introduced special terms and quantities which simplify the calculations and are in common use.

138. Effective Tail Incidence

The incidence of the tail-plane, denoted by α', is defined in such a manner as to be independent of the movement of the elevators relative to the front part of the tail-plane which is fixed to the fuselage. Depressing the elevators actually increases the tail-plane incidence as would be defined in the ordinary way by the line joining the leading edge to the trailing edge. But such modification is specifically excluded from affecting α', its aerodynamical effect being treated separately. Thus α' is defined, in the first place, with reference to the chord-line of the section when the elevators are in the neutral position with regard to the fixed part of the tail-plane. Again, the relative wind in the vicinity of the tail-plane is deflected downward by the wings through the angle of downwash ε. So finally α' is defined with reference to this deflected air stream (Fig. 77).

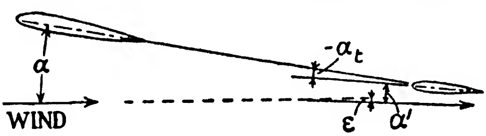


FIG. 77.

The chord-line of the tail-plane (with elevators neutral) may be so fixed as to be parallel to that of the wings. In this case, if the wing incidence is α, $\alpha' = \alpha - \varepsilon$, for the downwash evidently reduces the effective incidence of the tail-plane.

Generally, however, the tail-plane is fixed to the fuselage at some small angle to the wings. The angle between the chord-line of the tail-plane and that of the wings is called the *tail-setting angle* and is denoted by α_t. It is reckoned positive if the nose of the tail-plane is tilted up. Thus in Fig. 77 there is a negative tail-setting angle. When the tail-setting angle is negative, as shown, the aeroplane is said to possess a *geometrical longitudinal dihedral*.

Formally, the tail-plane incidence is given by

$$\alpha' = \alpha + \alpha_t - \varepsilon \qquad (57)$$

But the correct sign must be attached to α_t. For example, if the wings are at 8° incidence and the tail-plane is fixed nose-down relative to the wings by 2°, whilst the downwash angle is $3\frac{1}{2}°$, then $\alpha' = 8° - 2° - 3\frac{1}{2}° = 2\frac{1}{2}°$.

For a given aeroplane, α_t usually remains constant at all speeds. As the speed of flight is reduced, α of course increases and so also does ε, though much less quickly; it was found by experiment in the preceding chapter that ε increases at between one-third and one-half the rate at which α increases.

139. The Tail Lift

Strictly speaking, the lift of the tail-plane is perpendicular to the deflected air stream and so is inclined backward a little from the vertical in straight level flight. But this slight inclination is ignored, and the tail lift, denoted by L_t, is assumed to be perpendicular to the direction of flight. It is calculated from

$$L_t = C_{Lt} q S_t \qquad . \quad . \quad . \quad . \quad . \quad (58)$$

where C_{Lt} is the lift coefficient of the tail-plane, S_t its area and $q = \frac{1}{2}\rho V^2$. V is less than the speed of the aircraft if the tail-plane is situated within the low velocity wake behind the wings, but for simplicity it will be assumed to be outside the wake.

The lift coefficient of the tail-plane depends partly on α' and partly on the elevator angle, which is denoted by η (the Greek letter eta). This angle specifies the rotation of the elevators away from their neutral position relative to the fixed part of the tail-plane. It is reckoned positive if the elevators are depressed, as shown in Fig. 78. An increase of α' does not imply an increase of η.

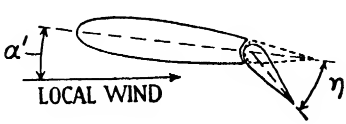


Fig. 78.—The Elevator Angle.

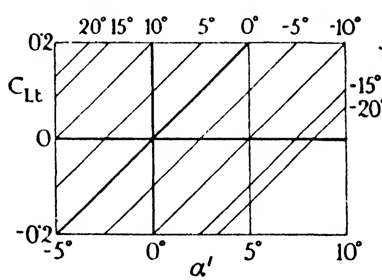


Fig. 79.—Lift Curves for a Tail-plane.

Fig. 79 shows a series of lift curves for a tail-plane—C_{Lt} plotted against α'—each curve being characterised by a particular elevator setting. The curves are restricted in range in order to exclude stalling effects. Such a chart is used as follows. A tail-plane of known area is required to give a certain lift at a particular value of q. The formula (58) at once gives the required lift coefficient. The value of α' is then estimated and, finally, the elevator angle necessary for the required lift coefficient at the known incidence α' is spotted by interpolation on the chart. Examples of this process will be given shortly.

140. Tail-Volume Ratio

Calculations are facilitated by employing a non-dimensional quantity called the *tail-volume ratio* and denoted by τ, which will now be described. Let l be the distance of the centre of pressure of the tail-plane behind the centre of gravity of the aeroplane. Then the corrective pitching moment due to the tail $= L_t \times l$. But for constant values of C_{Lt} and q, L_t is proportional to S_t, and the moment is proportional to $S_t \times l$. In words, the tail-plane area and its leverage about the centre of gravity of the aeroplane are equally effective in producing a pitching moment under given conditions, provided S_t is not so enlarged at the expense of l as to bring in secondary effects, such as a change of downwash angle.

The 'tail-volume', $S_t \times l$, is thus an especially significant variable in the present connexion when referring to a particular size of aeroplane. To achieve independence of size, it is divided by the volume $S \times c$, where S is the wing area and c the mean chord of the wings. Hence the tail-volume ratio

$$\tau = \frac{S_t l}{Sc} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (59)$$

It is a non-dimensional fraction, often about one-third in value.

141. The C_{Lt} for Equilibrium

We now assume that all the pitching moments acting on the aeroplane, other than the tail moment, are added together algebraically, and, from the gross unstable moment so obtained, there is derived a composite C.P. travel curve (Article 114).

The method of carrying out the calculation has been described in the article mentioned. The curve gives the position of the C.P. along the wing chord, when the tail-plane is ignored, for any incidence α of the wings. Appropriate to α we also know the lift coefficient C_L of the wings and therefore the value of q for straight level flight.

In order to avoid unnecessary arithmetic, we assume that the centre of gravity (C.G.) of the aeroplane is situated on the wing chord, and that the C.P. of the tail-plane lies on the wing chord produced (Fig. 80).

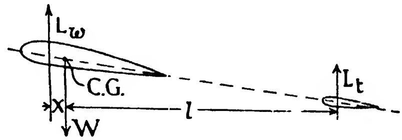


FIG. 80.

Let c be the wing chord and x the distance of the C.P. of the wings, etc., in front of the C.G. Then the unstable pitching moment $= L_w \times x$, approximately, L_w denoting the wing lift. For equilibrium, this moment must be balanced by the contrary moment $L_t \times l$—i.e.,

$$L_t l = L_w x.$$

Divide both sides of this equation by the product $qS_t l$, obtaining

$$C_{Lt} = x \cdot \frac{L_w}{q} \cdot \frac{1}{S_t l}.$$

Now multiply the numerator and the denominator of the right-hand side by Sc and the following formula will result for the tail-plane lift coefficient :

$$C_{Lt} = \frac{x}{c} \cdot \frac{C_L}{\tau} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (60)$$

This simple formula applies to all sizes, speeds and altitudes. x/c is a positive or negative fraction depending upon the wing incidence or, more directly, upon C_L. It is negative at high speeds, when the C.P. is behind the C.G. and a negative (or downward) lift is then required from the tail.

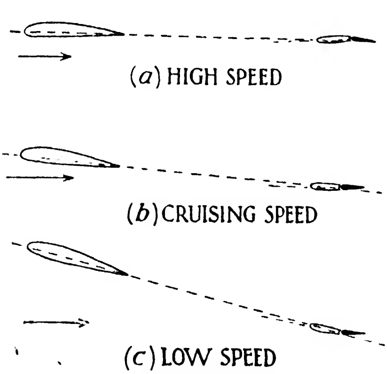


FIG. 81.—ELEVATOR SETTINGS.

142. The Elevator Setting

Fig. 81 shows an aeroplane under three typical conditions of flight. Cambered wings and a tail-plane of symmetrical section are assumed. The tail-setting angle is taken as $-2°$. Depicted at (a) is a high-speed case; α may be 0°, but there is still a downwash angle, and α' is more negative than $-2°$.

x is negative, requiring L_t to be negative, but not sufficiently so as to pitch the aeroplane to a larger incidence. Therefore, the amount of negative tail lift is decreased to the required amount by depressing the elevators; in other words, the elevators hold the tail up. (The elevator angle is exaggerated in the figure for clearness.)

Indicated at (*b*) is a particular cruising speed case. No lift is required from the tail and, at the same time, $\alpha' = 0°$. Therefore the elevators are neutral.

(*c*) is typical of low speeds. The positive tail lift necessary for equilibrium is abundantly available from the tail-plane, α' being positive and fairly large. But the tail-plane is prevented from pitching the aeroplane to a smaller incidence by the raised elevators, which hold it down.

It will now begin to appear that to each and every speed of a given aeroplane in straight level flight there corresponds a particular elevator setting. Only with one elevator setting will the algebraic sum of all the pitching moments acting on the aeroplane vanish at the precise incidence which will provide the lift coefficient that the particular speed demands from the wings.

143. The Elevator Curve

This important result is further elucidated by considering an example, and a simplified case will be chosen so that the arithmetic can almost be done mentally. The calculations are set out in Table VI below. For the aeroplane considered τ is taken as equal to 1/3, so that by (60) $C_{Lt} = 3C_L(x/c)$. The tail-setting angle α_t is taken as equal to $-2°$. The downwash angle ε is assumed to increase at one-third the rate at which the wing incidence α increases. Finally, Fig. 79 gives a number of lift curves for the tail-plane tested alone with various elevator settings.

TABLE VI

(1) α (deg.)	(2) C_L	(3) $\Delta\alpha$ (deg.)	(4) ε (deg.)	(5) α' (deg.)	(6) x/c	(7) C_{Lt}	(8) η (deg.)	(9) V_i (m.p.h.)
0	0·2	$2\frac{2}{3}$	0·9	−2·9	−0·12	−0·072	2·2	250
4	0·5	$6\frac{2}{3}$	2·2	−0·2	0	0	0·4	158
8	0·8	$10\frac{2}{3}$	3·6	2·4	0·02	0·048	−2·4	125
16	1·4	$18\frac{2}{3}$	6·2	7·8	0·04	0·168	−7·2	94

Referring to the table, the first two columns give the lift curve of the wings. Remembering that $\varepsilon = 0$ when the wings are at their angle of no lift, which is $-2\frac{2}{3}°$, the third column gives the increase of wing incidence ($\Delta\alpha$) from this angle. The angle of downwash follows in column 4, being given by $\Delta\alpha/3$. The effective incidence of the tail-plane, column 5, is obtained from $\alpha' = \alpha + \alpha_t - \varepsilon$, remembering that $\alpha_t = -2°$. The values of x/c given in column 6 represent the unstable travel of the C.P. without the tail-plane, x being positive if the C.P. is located upstream from the C.G. of the aeroplane. The required values of C_{Lt}, column 7, follow at once from (60). Now we have α' and C_{Lt} fixed and known at all the wing C_L's and have only to find an appropriate elevator setting in each case. This is done from Fig. 79 by interpolation, and the resulting elevator angles are given in column 8. The last column, 9, gives possible values of the indicated air speed—viz., those corresponding to a wing-loading of 32 lb. per square foot.

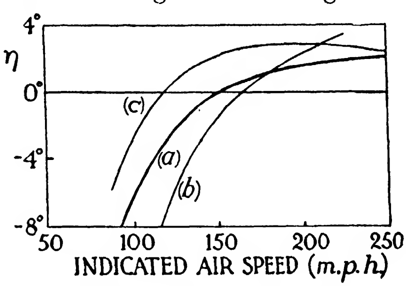


FIG. 82.—ELEVATOR CURVES.
(a) Satisfactory, (b) insufficient control, (c) dangerous.

Fig. 82 (*a*) shows the elevator angle plotted against the indicated air speed. In order to draw this curve accurately, additional angles required evaluation, the table giving only a few specimen rows of the calculation.

144. Statical Stability.

The shape of the curve just calculated signifies that the aeroplane possesses statical stability through the speed range. This quality is a first step towards safety in flight. Verification is as follows.

Consider first a flying speed of 152 m.p.h., for which the elevators of the aeroplane to which the curve relates must be neutral. If the speed inadvertently changes, the tail-plane will clearly bring it back to 152 m.p.h., for the curve shows that to enable a different speed to persist the elevators must be depressed or raised to weaken the power of the tail-plane. Then consider flight at some other speed, for example a higher one, the elevators being depressed in order to hold the tail up sufficiently. If now the speed inadvertently increases, momentarily reducing the wing incidence, the tail-plane will reduce the speed and increase the wing incidence each to its first value, for only if the elevators were depressed further could the change of speed and incidence be maintained. In this way it is verified that statical stability exists throughout the range of speed covered by the calculations.

The argument can evidently be shortened as follows. Begin again with a speed of 152 m.p.h., when the elevators are in the neutral position, and consider a progressive increase, or decrease, of speed. The curve shows that either can only be secured by the elevators fighting increasingly hard against the fixed part of the tail-plane, progressively weakening its power.

145. Unsatisfactory Elevator Curves

Changing the foregoing data arbitrarily and working out another example may easily lead to an unacceptable elevator curve. Two such curves are shown at (*b*) and (*c*) in Fig. 82.

First considering curve (*b*), stability is still signified, but the elevators are not strong enough to secure the lowest speeds of which the aeroplane is otherwise capable. The aeroplane cannot be stalled, and, with a less margin than that shown in the figure, this might be an advantage in some connexions—e.g., inexpert flying; the landing speed would become rather high for the wing-loading, but could be reduced again by suitably lightening the load.

The curve (*c*) reveals a serious defect, the aeroplane becoming unstable at high speeds. Supposing flight at 190 m.p.h. to be disturbed in such a way as to increase the speed appreciably, the tail-plane would fail to effect a recovery. At higher speeds an elevator angle exists which enables the third of the equations (47) to be satisfied, but not the proviso noted immediately following these equations, and so steady flight is impossible. An elevator curve of this highly dangerous type is avoided by locating the centre of gravity of the aeroplane sufficiently far forward, but this artifice must be used sparingly or other troubles ensue.

GLIDING, CLIMBING AND TURNING

146. Suppose that during straight level flight, and without adjustment of the elevator setting, the engine throttle is closed or opened, decreasing or increasing the airscrew thrust, and thus preventing the second of the equations (47) from continuing to be satisfied. Flight continues to be straight, but cannot remain level. If the throttle opening is decreased, the path slopes downward to the horizon, so that a component of the weight of the craft can make up for the deficit of thrust. The aeroplane is then said to glide, although this term is usually reserved for the specially interesting case that occurs when the thrust is reduced to zero. If the throttle opening is increased, on the other hand, the flight path becomes upwardly inclined to the horizon and the excess of thrust is balanced by a fraction of the weight. The aeroplane is said to climb. The circumstances of the aeroplane are shown in Fig. 83 (a) and (b), θ being the angle of glide or climb. Resolving perpendicular to the flight path, we have in each case

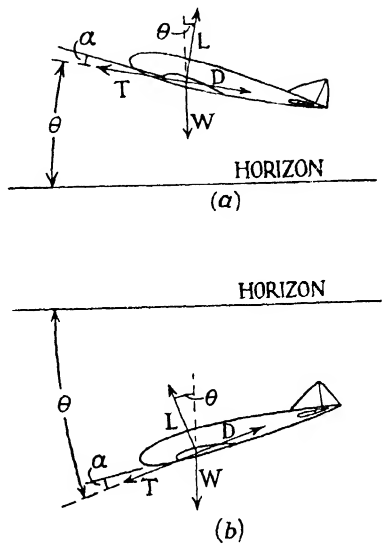


FIG. 83.—(a) CLIMBING, (b) GLIDING.

$$W \cos \theta = L \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (61)$$

showing that the lift is reduced, though only slightly for small values of θ. Resolving along the flight path,

$$\left.\begin{aligned} T + W \sin \theta &= D \text{ (gliding)} \\ \text{or} \qquad T &= D + W \sin \theta \text{ (climbing)} \end{aligned}\right\} \quad . \quad . \quad (62)$$

These are the new conditions for equilibrium, together with the requirement that the pitching moment must vanish in the stable manner that has already been investigated for straight level flight.

Regarding the last requirement, we continue to assume that the axis of the airscrew passes close to the centre of gravity of the aeroplane and to ignore drag arising from the slipstream, so that, since for moderate values of θ the lift is little changed, the unstable pitching moment is scarcely affected by the throttle opening. In these circumstances the elevator curve obtained for straight level flight applies with little modification to gliding and climbing and a given elevator angle corresponds to almost the same speed.

This verifies the description given of the effect of varying the opening of the engine throttle during straight level flight. Speed is determined by the elevator setting because the resultant pitching moment must be reduced to zero. Increasing or decreasing the thrust does not make the aeroplane fly faster or slower unless the elevators are changed, but only inclines the flight path, one way or the other, to the horizon.

Conditions are different during a turn or a steep glide and these cases must be considered as they arise.

The range of angles at which it is possible for a given aeroplane to climb is largely determined by the power equipment, and further investigation is postponed to the next chapter. There is no such restriction, however, on the angle at which the aeroplane can glide, for T may be less than the drag, or zero, or even negative. The most important case of gliding, and what is usually intended by the term, is when $T = 0$. This case is developed in the following articles.

147. Gliding Angles and Speeds

With $T = 0$ the conditions for straight flight become $W \cos \theta = L$ and $W \sin \theta = D$. Dividing the second of these equations by the first gives

$$\tan \theta = \frac{D}{L} = \frac{1}{r_A} \quad . \quad . \quad . \quad . \quad . \quad . \quad (63)$$

where r_A denotes the overall lift–drag ratio of the aeroplane, for D represents the total drag.

This result must be regarded as a hard fact. When $T = 0$ and there is no vertical wind, an aeroplane can glide only in accordance with it. In order to glide at a chosen angle θ, the elevators must be set to make $M = 0$ at a wing incidence that will give to the whole aeroplane a lift–drag ratio $= 1/\tan \theta$.

The curve (a) of Fig. 84 gives the overall lift–drag ratio, r_A, of a certain aeroplane with flaps retracted plotted against its lift coefficient. The curve has been extended to very large incidences, which are marked. Choosing a suitable value of θ leads to a line such as AB, representing the required value of r_A. This line makes two intersections with the lift-drag curve, indicating

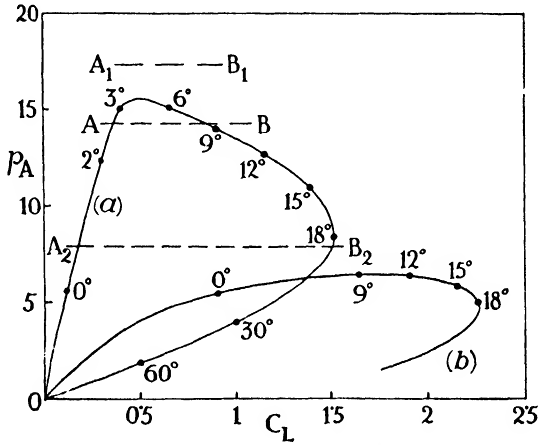


FIG. 84.—OVERALL L/D RATIO (p_A) PLOTTED AGAINST LIFT COEFFICIENT FOR A MONOPLANE WITH (a) FLAPS RETRACTED, (b) SPLIT FLAPS OPEN.

that two lift coefficients are possible. Thus there are two alternative incidences for gliding at θ to the horizon, and two corresponding speeds given by

$$q = w \cos \theta / C_L \quad . \quad . \quad . \quad . \quad . \quad (64)$$

where w is the wing-loading. The equation (64) is the counterpart for gliding flight of (49). For flat glides the difference between $\cos \theta$ and unity may be neglected, when the equations come to the same, but for steeper glides the speed associated with a given lift coefficient is reduced.

If too small a value is chosen for θ, the corresponding value of r_A will yield a line such as A_1B_1, giving no intersection with the lift-drag curve, and therefore indicating that a glide at this angle is impossible without help from the engine. If, on the other hand, a rather large value is chosen for θ, displacing the line to A_2B_2, say, two intersections with the lift-drag curve are again secured, but B_2, yielding the larger lift coefficient, and therefore the lower of the two possible speeds on the glide, has a restricted aerodynamical interest, for it is beyond the stall. The aeroplane using it would be liable to autorotation (Article 130), which would lead to a spin. It could be employed for a spiral descent at altitude, so that space existed for recovery by way of a preliminary dive, but would be dangerous at low altitudes. The intersection A_2 may entail a very fast gliding speed.

Since θ depends only on r_A, the lift coefficients possible for a given angle of glide are the same for all altitudes. Thus the possible indicated air speeds are always the same, for a given aeroplane, but the true air speeds vary inversely as the square root of the air density. Gliding right down from 30,000 feet at a constant angle, the speed will ultimately decrease to 61 per cent. of its initial value.

Example 63.—Assuming the curve (*a*) of Fig. 84 and that the wing-loading is 25 lb. per sq. ft., at what speeds will the aeroplane glide at 4° to the horizon, the airscrew thrust being zero?

Since $\theta = 4°$, $1/\tan\theta = 14{\cdot}3$ and this must be the value of r_A. The overall lift–drag curve then gives $C_L = 0{\cdot}35$ or $0{\cdot}85$. With $w = 25$ and the standard value of ρ, the equation (64) gives

$$V = 145\sqrt{\cos\theta/C_L},$$

where V is in ft. per sec. Substituting leads at once to $V = 245$ or 157. Therefore the alternative speeds are 167 and 107 m.p.h. These are the indicated air speeds. Approximately, the faster speed corresponds to 2½° incidence and the slower to 8½° incidence.

Example 64.—What is the minimum angle at which the aeroplane of Example 63 could glide without help from the airscrew?

The minimum value of θ clearly corresponds with the maximum value of r_A, which is 15·7 in the present case. Thus $\tan\theta = 1/15{\cdot}7$, giving $\theta = 3{\cdot}65°$. The corresponding value of C_L is 0·53. Hence the indicated air speed comes to 135 m.p.h. The incidence is about 4½°.

In aerodynamical questions we often have to employ ‘ trial and error ’ or graphical methods of solution. The following example provides an illustration.

Example 65.—Still assuming the aeroplane of Example 63, and that $T = 0$, at what gliding angle will a speed of 400 m.p.h. be reached?

The difficulty here is that if we substitute the speed in equation (64) we are left with two unknowns, θ and C_L, whilst the value of r_A is not given. The substitution yields (see also Example 63)

$$\sqrt{C_L} = 145 \cdot \frac{15}{22} \cdot \frac{1}{400} \cdot \sqrt{\cos\theta}$$

i.e., $$C_L = 0{\cdot}061 \cos\theta.$$

It is required to find the value of θ which will satisfy this equation and also the curve of Fig. 84.

Bearing in mind the wing-loading, the high speed must result in a small value for C_L; in fact, putting $\cos\theta = 1$ gives $C_L = 0{\cdot}061$ and the coefficient must be less on the glide since the lift is reduced. Assume, therefore, some smaller values and construct the following table.

(1) C_L	(2) r_A	(3) θ°	(4) $0{\cdot}061 \cos\theta$
0·060	3·18	17·45	0·0582
0·050	2·65	20·7	0·0570
0·058	3·07	18·05	0·0580

Column (2) is obtained from column (1) by means of the lift–drag curve, arranging r_A to be proportional to C_L, as closely applies to small lift coefficients. Column (3) is obtained from column (2) by the method already illustrated. Column (4) then follows, giving the right-hand side of the equation above. The first two rows were worked first. Evidently the true value of θ lies between the first two values because the figure in column (1) is first greater and then less than the corresponding figure in the last column. The lift coefficient for the third row was then estimated. It transpired to be correct, otherwise a closer estimate would have been framed on the further experience. For a graphical solution, four reasonably assumed C_L's would have been worked as above and columns (1) and (4) plotted against column (3). The intersection of the two curves would have indicated the true value of θ.

Thus the answer to the question is 18°, closely. The incidence is − 1°.

148. Applications of Minimum Gliding Angle

If the power unit of a single-engined aeroplane breaks down, or the fuel becomes exhausted during flight, a small minimum gliding angle increases the ground area from which to select a suitable place for a forced landing. Let the failure occur at altitude h, Fig. 85. Then the distance which can be traversed in the absence of wind is $x = h/\tan\theta$, and the above area is equal to πx^2 whether a wind exists or not (except that it is greater if the wind has an upward component,

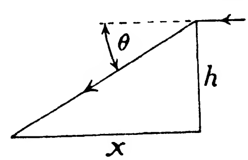


FIG. 85.

and *vice versa*). The idle airscrew will exert considerable drag, unless its blades can be feathered, but on the other hand the aeroplane will be relieved of drag due to its slipstream. Aeroplanes disabled at high altitude can cover a long distance before having to land. Motorless gliders can be designed for especially small minimum gliding angles and speeds and can reach high altitudes by using the strong rising wind currents associated with storms.

Example 66.—An aeroplane weighs 10,000 lb. Its wings, which are loaded to 32·5 lb. per sq. ft., have the characteristics given by the first three columns of Table V, p. 186. It can just attain an indicated air speed of 225 m.p.h. at an altitude of 15,000 ft., the power unit providing a thrust of 1000 lb. Supposing the engine to fail at this altitude and neglecting airscrew effects, estimate the distance the aeroplane can cover in the absence of wind before landing at sea-level.

The question is solved once the maximum lift–drag ratio (r_A) of the complete aeroplane is known and this may be found by considering straight level flight through a range of speeds.

For an indicated air speed of 225 m.p.h. $q = 130$ lb. per sq. ft. Thus the lift coefficient at this speed is $32{\cdot}5/130 = 0{\cdot}25$. Plotting the third column of Table V against the second gives 22·2 for the corresponding lift–drag ratio of the wings, leading to $10{,}000/22{\cdot}2 = 450$ lb. for their drag (D) and leaving $1000 - 450 = 550$ lb. for the extra-to-wing drag D_B.

D_B will vary inversely as C_L at other speeds and the following table is readily constructed for straight level flight :

C_L	r	D_w	D_B	Total drag	r_A
0·25	22·2	450	550	1000	10·0
0·4	26	385	344	729	13·7
0·6	23·5	426	229	655	15·3
0·9	18·5	541	153	694	14·4

Plotting the last column against the first, with the help of another row if required, indicates a maximum lift–drag ratio for the aeroplane of 15·4, giving for the minimum gliding angle (θ) $\tan\theta = 0{\cdot}065$.

Hence the ground distance that can be covered in the absence of wind is

$$\frac{15{,}000}{5280 \times 0{\cdot}065} = 44 \text{ miles, nearly.}$$

149. Sinking Speed.

The rate of descent during a glide is equal to $V \sin \theta$ (see Fig. 85), and is sometimes called the *sinking speed.* Its minimum value for a given aeroplane occurs at a greater incidence than that for maximum lift–drag ratio, since the consequent diminution of V is more important than the increase of $\sin \theta$. This value is easily obtained by constructing a table as follows. A few promising lift coefficients are assumed, and r_A for each is read from the lift–drag curve for the aeroplane. This gives a sequence of values of θ, for which, knowing the wing-loading, the speeds can be evaluated. Hence $V \sin \theta$ can be plotted against C_L and the minimum value found. For the aeroplane of Examples 63–65 it comes to about 11 feet per second. If the wing-loading were reduced from 25 lb. per square foot to 4 lb. per square foot whilst retaining the lift–drag curve, which could be managed if the aeroplane were changed to a glider, the minimum sinking speed would be less than $4\frac{1}{2}$ feet per second. Upward currents of this magnitude are common in the atmosphere. Thus a lightly loaded glider of good lift–drag ratio can expect to find conditions enabling it to ' soar ', by which is meant that it is carried up by the rising currents rather faster than it glides down through them.

Such reflections help one to realise the intimate way in which atmospheric changes affect aeroplane flight. In a wind having an upward component of 11 feet per second the aeroplane of Examples 63–65 could fly level at 100 m.p.h. with the engine throttle practically closed. Up-currents of this magnitude are encountered only locally, but those which are prevalent give substantial assistance to maintaining altitude under conditions of flight in which the value of $V \sin \theta$ is rather low; the effect is the same as if the lift–drag ratio were much higher, and the performance of the aircraft is therefore flattered. It follows that only with difficulty can the lift–drag ratio of an aeroplane be determined by observing the gliding angle in full-scale flight experiments, for up-currents cannot be avoided by changing the direction of flight, and must be estimated. It is also not easy to ensure the airscrew thrust being accurately zero. Article 96 includes consideration of the same difficulty in connexion with wind-tunnel experiments.

Example 67.—With $T = 0$ an aeroplane is observed to glide at 4° to the horizon, the speed being 120 m.p.h. There is present an up-current of $3\frac{1}{2}$ ft. per sec. What is the true gliding angle at this speed?

Since 120 m.p.h = 176 ft. per sec., the observed sinking speed is 176 sin 4° = 12·3 ft. per sec. Without the upcurrent this would be 15·8 ft. per sec., with a change of forward speed that can be neglected since $\cos\theta$ is nearly equal to unity. Hence the true gliding angle θ' is closely given by $176 \sin \theta' = 15{\cdot}8$—i.e., $\theta' = 5{\cdot}15°$. The true lift–drag ratio $= 1/\tan\theta' = 11{\cdot}1$. Without the correction the value would have appeared to be 14·3.

150. Effect of Opening Split Flaps

Fig. 84 (*b*) refers again to the aeroplane of Article 147, and shows the profound change in the lift–drag curve due to opening the landing flaps fully. These are of the split type (Article 127). Reasonable modifications in their design would alter the curve shown, but not fundamentally.

It is first seen that the lift–drag ratio varies comparatively little between 2° and the stall, which occurs at about the same incidence as with flaps closed. The maximum value of 6·5 gives a minimum gliding angle of 8·75°. At zero incidence and just before the stall the value is 5·5 leading to 10·3°. The aeroplane requires holding to a considerable negative incidence if required to glide much more steeply unstalled. At an incidence of $-7\frac{1}{2}°$, $r_A = 3$ and the lift coefficient is about 0·37, giving a speed of 158 m.p.h. for gliding at 18° to compare with the speed of 400 m.p.h. estimated with the flaps closed (cf. Example 65). Flaps can be used, therefore, for gliding steeply without great increase of speed.

Interest centres chiefly, however, in the way they affect coming-in conditions. As already mentioned, an aeroplane glides down to a landing ground at a considerably smaller incidence than its stalling angle, in order to retain lateral control and insure against wing-dropping, and 14° might be suitable in the present case. Fig. 84 (*a*) then gives $r_A = 11{\cdot}7$ and $C_L = 1{\cdot}3$, giving $\theta = 4{\cdot}9°$ and $V = 87$ m.p.h. But with the flaps open, $r_A = 6{\cdot}2$ and $C_L = 2{\cdot}07$, giving $\theta = 9{\cdot}2°$ and $V = 69$ m.p.h. Thus opening the flaps in this instance decreases the coming-in speed by 18 m.p.h. and increases the gliding angle by 88 per cent. The latter effect is important in case of a confined landing ground surrounded by high obstructions, providing more room for the landing run.

151. Equilibrium on a Turn

Fig. 86 shows an aeroplane turning in a horizontal circular path of radius R. The wings are 'banked' to the horizon at the *angle of bank* ϕ in order that the centrifugal force F can be balanced by a component of the lift. Resolving vertically and horizontally and writing V for the speed,

$$W = L\cos\phi \quad . \quad . \quad (65)$$

$$F = \frac{WV^2}{gR} = L\sin\phi,$$

$$D = T.$$

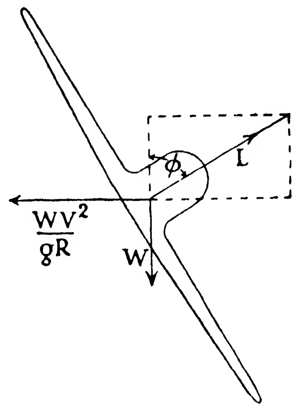


FIG. 86.

Since ϕ may be a large angle, the first equation shows that the lift of the wings may require to be several times as great as the weight of the aeroplane. To find ϕ, divide the second equation by the first, obtaining

$$\tan\phi = \frac{V^2}{gR} \quad . \quad . \quad . \quad . \quad . \quad . \quad (66)$$

which shows that for a given value of the ratio V^2/R the angle of bank is the same for all aeroplanes. V is the true air speed in feet per second and R is in feet.

Example 68.—An aeroplane whose wing-loading is 30 lb. per sq. ft. is turning horizontally at 20,000 ft. altitude, the indicated air speed being 200 m.p.h. and the angle of bank 60°. What are (*a*) the radius of the turning circle, (*b*) the lift coefficient?

(*a*) Since σ, the relative density of the air $= 0{\cdot}534$, and $\tan 60° = 1{\cdot}732$, (66) gives

$$R = \frac{200^2 \times (22/15)^2}{32{\cdot}2 \times 1{\cdot}732 \times 0{\cdot}534} = 2890 \text{ ft.}$$

(*b*) $L = W/\cos\phi$, whence $C_L = L/qS = w/q\cos\phi$.

From the question, $q = \frac{1}{2} \times 0{\cdot}00238 \times (200 \times 22/15)^2 = 102\frac{1}{2}$ lb. per sq. ft. Hence

$$C_L = \frac{30}{102\frac{1}{2} \times \frac{1}{2}} = 0{\cdot}585.$$

Applying the method of Article 143 shows that the elevator angles will correspond to different speeds because the lift is increased in the ratio $1/\cos \phi$. When an aeroplane in straight level flight is put into a turn at the same speed, for example, the elevators must be used to increase the wing incidence by the amount necessary to secure the greater lift required.

The outer wing of a circling monoplane will have more lift and drag than the inner one because it is moving the faster. Thus a rolling moment and a yawing moment arise which must be balanced by use of the ailerons and rudder.

The fourth control of the aircraft—viz., the engine throttle—will also be called into play to adjust the airscrew thrust so that the third of equations (65) will be satisfied. Considering again the example of an aeroplane being put into a turn at constant speed, the extra-to-wing drag may be assumed, as an approximation, to remain the same, but the wing drag, now obtained by dividing $W/\cos \phi$ by the new lift–drag ratio corresponding to the new lift coefficient, will increase.

It is easy to choose conditions in which the engine is too small to enable the third of equations (65) to be satisfied. The aeroplane will then spiral downwards, a component of gravity making up for the deficit of thrust. Alternatively, the thrust may be greater than this equation requires, when the aeroplane will execute a climbing turn.

152. Minimum Radius of Turn

First let the circling flight be horizontal, so that $L/W = 1/\cos \phi$ and $R = V^2/g \tan \phi$. For any chosen value of V, R will be a minimum when $\tan \phi$ is a maximum—i.e., when $\cos \phi$ is a minimum and L/W is a maximum. Now increasing ϕ increases D, as we have seen, and for horizontal flight we must have $T = D$. Therefore, the increase of ϕ will be limited by the possible increase of T if the speed of the aeroplane is well on towards its maximum speed in straight level flight. But if the speed is low compared with this maximum speed and a sufficient increase of T is available, then the increase of ϕ will be limited by the stalling of the wings, which will prevent any further increase of the ratio L/W.

Example 69.—A fast aeroplane, whose wing-loading is 28·8 lb. per sq. ft. and whose maximum lift coefficient is 1·5 with flaps closed, is in straight level flight at 150 m.p.h. with engines throttled. Find the minimum radius of a horizontal turn at this speed on the assumption that ample airscrew thrust is available.

For the straight flight, $C_L = w/q$, where w is the wing-loading and $q = \frac{1}{2}\rho V^2 = 57 \cdot 6$ lb. per sq. ft. Hence $C_L = 0 \cdot 5$ and $W = 0 \cdot 5qS$, S being the wing area.

The maximum lift L on the turn $= 1 \cdot 5qS$ and, since q remains unchanged, the maximum value of the ratio $L/W = 1 \cdot 5/0 \cdot 5$. Thus $1/\cos \phi = 3$, giving $\tan \phi = 2 \cdot 83$. Hence, since 150 m.p.h. = 220 ft. per sec.,

$$\text{the minimum radius} = 220^2/2 \cdot 83g = 531 \text{ ft.}$$

In making a quick change of direction of flight, a limited reduction of altitude and speed may be permissible, and then the equations $L \cos \phi = W$ and $T = D$ need not be satisfied. The wings can be banked at 90° so that the whole of their maximum lift opposes the centrifugal force. Ignoring the reduction of speed gives $L = WV^2/gR$. On this basis the data of Example 69 would yield a minimum radius of 501 feet, but the calculation assumes that the wings have sufficient time to stall.

Chapter VIII

PRINCIPLES OF PERFORMANCE

153. By the performance of an aeroplane is meant its capabilities in the standard atmosphere regarding such matters as speed, rate of climb, the altitude to which it can ascend, the load it can take on board, and range. Several systems exist for predicting accurately the performance of a given aeroplane. Without detailing these, the chapter will describe the main principles underlying such systems and the improvement of performance.

Civil Aviation is concerned rather with all-round merit than outstanding achievement of a particular kind. Nevertheless, some main-line services have to face exacting conditions that can be ignored by others, and modifications of design that foster particular qualities, though not to the degree of record-breaking, are of interest. A quality of importance to passenger transport is fuel economy at fairly high speeds and the question of efficiency in this sense is prominent.

In Chapter VII it was possible to deduce some of the capabilities of an aeroplane from considerations of equilibrium. To proceed further we examine the balance of power between the demands of the aeroplane on the one hand and the output available from the power plant on the other.

The power demanded by the aeroplane will be studied by two methods. The first develops naturally from that employed in the last chapter, the wing drag being separated from the remaining drag. The second method discriminates between induced drag on the one hand and form drag and skin friction, wherever arising, on the other. In applying the latter method we revert to the ideas of Dr. Lanchester, who not only discovered induced drag, but also forecast its essential influence on aviation.

Regarding the power available we shall be content to make reasonable assumptions, but a descriptive knowledge of its variation is essential.

The quantity $TV/550$, T being the airscrew thrust in lb. and V the true air speed in feet per second, gives the rate in horse-power at which useful work is done by the power plant. It is called the thrust horse-power and written T.H.P. It is equal to the brake horse-power of the engine or engines, written B.H.P., multiplied by the airscrew efficiency, which is denoted (like the elevator angle) by η. Thus

$$\text{T.H.P.} = \eta \times \text{B.H.P.} = TV/550 = TV_{\text{m.p.h.}}/375 \quad (67)$$

The B.H.P. of which an engine aspirating air of constant density is capable depends upon its rotational speed. This speed must not exceed the designed maximum, or excessive wear will result, but can be reduced as required by the throttle. Gearing is interposed between engine and airscrew so that the latter rotates more slowly. The airscrew is carefully designed to allow the engine to develop its maximum rotational speed, and therefore its full B.H.P., at full throttle. With an airscrew of fixed pitch this can only be managed for chosen conditions of flight; under other conditions, either the airscrew slows down the engine or the throttle must be used to prevent the engine racing, leading to a loss of B.H.P. in either case. Moreover, the efficiency of the airscrew diminishes at the lower speeds. If it has been chosen to produce full T.H.P. at or near top speed, much less will be available at climb and take-off. Use of variable pitch, described in a later chapter, greatly reduces such losses.

With increase of altitude the B.H.P. of a normally aspirated engine decreases rather faster than the pressure of the air, which itself decreases faster than the density. In the modern aero engine this is compensated up to a pre-arranged altitude, called the *supercharged height*, by compressing the air supplied for combustion. This altitude may amount to many thousands of feet, but above it the power falls away in a manner depending upon the type of supercharger.

In brief, the T.H.P. available from a given plant is not constant, but depends upon altitude and speed. For small changes the maximum thrust available varies in inverse proportion to the true air speed, but this assumes constant T.H.P., and is therefore far from true for large changes.

154. For steady level flight, whether straight or otherwise, $T = D$, the total drag, and it is therefore necessary that

$$\text{T.H.P.} = DV/550 \quad . \quad . \quad . \quad . \quad . \quad (68)$$

The quantity on the right is called the *horse-power required*, being the horse-power necessary to overcome the drag. The throttle will ensure that the power plant does work at the correct rate provided that the T.H.P. does not exceed the maximum value of $\eta \times$ B.H.P. available under the particular conditions of working.

If so much T.H.P. cannot be provided and the elevators are not adjusted to decrease the speed, the flight path will slope downward, so that the extra work can be done by gravity. If, on the other hand, an excess of T.H.P. is supplied, this excess will be absorbed in raising the weight of the aeroplane against gravity—i.e., in climbing. It follows that the maximum rate of climb will result when there is a maximum excess of T.H.P. over and above that required to do work against the drag.

The maximum level speed of the aircraft will occur when the horse-power required to overcome the drag absorbs the full T.H.P. available at the speed. Probably the T.H.P. will then be a maximum for the altitude concerned.

155. Horse-Power Required Curve

Denoting by H the horse-power required for straight level flight, the methods of Article 135 at once give

$$H = \left(\frac{W}{r} + D_B' \cdot \frac{C_L'}{C_L}\right)\frac{V}{550} \quad . \quad . \quad . \quad . \quad (69)$$

V is the true air speed in feet per second and the other symbols are defined in the article cited, where also the approximation achieved is discussed. If, alternatively, the variation of r_A, the lift–drag ratio of the complete aeroplane, is known, the formula (56) gives

$$H = \frac{W}{r_A} \cdot \frac{V}{550} \quad . \quad . \quad . \quad . \quad . \quad . \quad (70)$$

The correction of these formulæ for slipstream effects will be explained in a later chapter.

The 5-ton aeroplane studied in Article 136 provides material for an example. Referring to Table V, the speed is given as an indicated air speed, which is equal to the true air speed in m.p.h. at low altitude. Instead of changing to feet per second, we multiply the final column of that table by $V_i/375$ instead of $V/550$, but it is important to remember that the result will apply to low altitudes only. This result is given in the third column of Table VII, the first two columns being copied from the earlier table.

TABLE VII

α (degrees)	V_i (m.p.h.)	H.P. required (H)
− 0·4	313	1719
+ 0·9	221	645
3·6	157	289
6·0	128	222
10·0	104	200
14·3	90	228

H is plotted against V_i in Fig. 87. It will be seen that this 5-ton aeroplane can fly at low altitude with some 200 thrust horse-power, though the speed is then only about 104 m.p.h. The speed for minimum H is less than that for minimum drag (i.e., about 130 m.p.h.) because the drag curve is rather flat in this region and decrease of speed more important than the associated increase of drag. To fly at 313 m.p.h. would require an exceptionally large engine for an aeroplane of the size—viz., one of 2000 B.H.P., allowing reasonably for the airscrew efficiency. If the aeroplane were intended for Civil Aviation, no more than 1000 B.H.P. would be contemplated. The airscrew efficiency might be 83 per cent. at maximum speed, giving 830 T.H.P. available. This value is marked in the figure and, from the intersection with the curve for H, it is seen to yield a speed of 245 m.p.h. The maximum speed attainable at low altitude with an engine of different horse-power is determined in the same way.

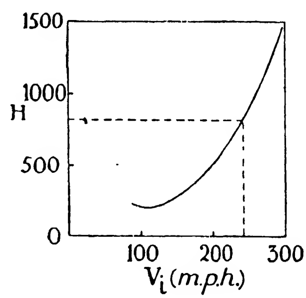


FIG. 87.—ESTIMATING MAXIMUM SPEED.

Given the wing data—i.e., corresponding values of r and C_L—and the extra-to-wing drag at some speed or lift coefficient, the above method of obtaining a first estimate of the maximum level speed, involving the working out of Table V and extension to Table VII, becomes rapid with a little practice. There is no shorter method. Approaching the problem algebraically leads to a cubic equation which must be solved either graphically or by trial and error, and a special advantage of the tabular method is that mistakes of arithmetic are easily detected.

156. Rate of Climb

The above curve is reproduced in Fig. 88 with an extension to show the horse-power required at low speeds with split flaps open. Also plotted is a possible T.H.P. available curve, assuming a variable pitch airscrew. The excess of T.H.P. available over and above that required for straight level flight is measured by the height of the hatched area. This excess, called the *reserve horse-power* and written H_R, is the T.H.P. available for doing work against gravity—i.e., for producing a rate of climb—which is conventionally expressed in feet per minute. Hence

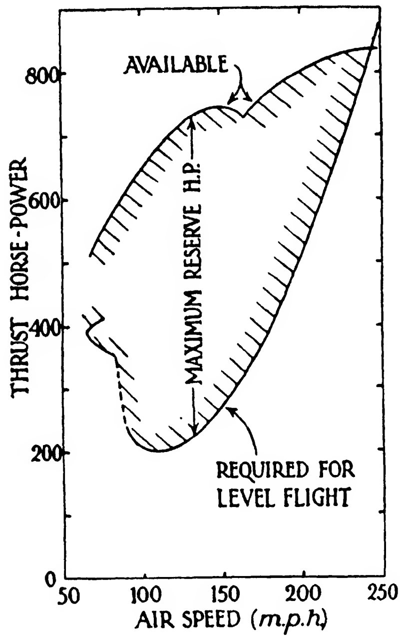


FIG. 88.—THE HORSE-POWER FOR CLIMB.

$$H_R = \frac{W \times \textit{rate of climb}}{33,000} \quad (71)$$

and the rate of climb possible at each forward speed of the aeroplane can be calculated at once. The maximum rate of climb occurs when the speed of the aeroplane is such as to provide maximum reserve horse-power. For the case illustrated, this speed is 135 m.p.h.; H_R is then 495 and, since $W = 11,200$ lb., the maximum rate of climb $= 33,000 \times 495/11,200 = 1458$ feet per minute. Three-quarters of this rate would usually be sufficient in Civil Aviation, but for some military purposes it might require to be doubled, when a larger engine would be necessary.

157. Angle of Climb

The condition for climbing at a maximum angle to the horizon is rather different and does not imply climbing at the greatest rate. Fig. 89 shows how the angle of

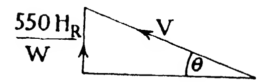


FIG. 89.—THE ANGLE OF CLIMB.

climb, θ, can at once be calculated from a knowledge of the rate of climb and the aircraft speed, for

$$\sin\theta = \frac{\textit{rate of climb}}{60\ V} \quad . \quad . \quad . \quad . \quad . \quad . \quad (72)$$

V being expressed in feet per second. If V is reduced below its value for maximum rate of climb, θ increases because the rate of climb does not diminish so rapidly, at first, as does V. To estimate the maximum angle, we may assume a few suitable speeds, work out θ for each speed and plot the results.

The foregoing treatment is not quite exact, in that the values of V and D are carried over from straight level flight for which $L = W$, whilst both are actually reduced during climb, when accurately $L = W\cos\theta$, as we have seen. But the error is small in Civil Aviation, amounting to only some $1\frac{1}{2}$ per cent. for $\theta = 10°$, and is on the right side for safety, its neglect leading to an under-estimate. Corrections must be made, however, for fast-climbing military aeroplanes.

Retaining the assumption that θ is small, the following expression results for the total T.H.P., H_T, required by an aeroplane of weight W to fly at a true air speed V feet per second and at the same time climb at a vertical velocity v feet per second.

$$\begin{aligned} H_T &= H + H_R \\ &= \frac{DV}{550} + \frac{Wv}{550} \\ &= \frac{WV}{550}\left(\frac{1}{r_A} + \frac{v}{V}\right) = \frac{WV}{550}\left(\frac{1}{r_A} + \theta\right) \quad . \quad (73) \end{aligned}$$

To use this expression, r_A is found from C_L, which in turn is found from W, S, V and the true value of ρ.

158. Examples.

The following examples all refer to flight at low altitude. Wherever possible, results are given in round numbers only.

Example 70.—An aeroplane weighing 10,000 lb. has wings of 49 ft. span and aspect ratio 7, a power unit providing 815 T.H.P., and the following overall lift–drag ratios at the lift coefficients stated :

C_L . . .	0·16	0·25	0·50	1·00	1·40
r_A . . .	8·1	12·5	16·0	13·5	10·0

Estimate the maximum speed in straight level flight.

S, the wing area $= 49 \times 49/7 = 343$ sq. ft. Since $L = W$, the stagnation pressure $q = 10{,}000/343\ C_L$, leading to

$$V = \sqrt{\frac{840 \times 10{,}000}{343 \times C_L}} = \frac{157}{\sqrt{C_L}} \text{ ft. per sec.}$$

$H = DV/550$, where $D = 10{,}000/r_A$ and V is now known from C_L. The following table is constructed by means of these formulæ :

C_L . .	0·16	0·25	0·50	1·00	1·40
V . .	393	314	222	157	133
D . .	1235	800	625	741	1000
H . .	882	457	253	211	242

Plotting the last row against the second shows that $H = 815$ at $V = 385$ ft. per sec. Thus the top speed is 263 m.p.h. The curve is shown in Fig. 90 (a).

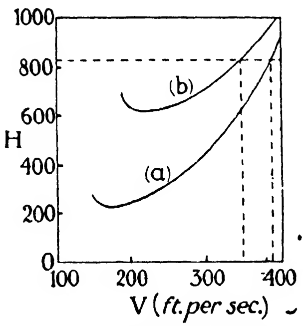


FIG. 90.—INCREASE OF POWER REQUIRED ON A TURN.

Example 71.—The aeroplane of Example 70 flies ' all out ' in a level circular path with an angle of bank (ϕ) of 60°. Find the radius of the turn.

We now have $L = W/\cos\phi = 20{,}000$. Hence, by the same method as before, $V = 222/\sqrt{C_L}$ and $D = 20{,}000/r_A$, giving the new table :

C_L . .	0·25	0·50	1·00	1·40
V . .	444	313	222	187
D . .	1600	1250	1481	2000
H . .	1290	712	598	680

Plotting the last row against the second (Fig. 90 (b)), shows that $H = 815$ at $V = 350$ ft. per sec., approximately—i.e., 239 m.p.h. The required radius of turn $= V^2/g \tan\phi = (350)^2/32{\cdot}2 \times 1{\cdot}732 = 2196$ ft.

Example 71 has been worked as if the results of Example 70 did not exist, but labour could have been saved by using the latter as follows. At constant C_L the lift is proportional to V^2, whence V is proportional to $\sqrt{\sec\phi}$. But r_A is unchanged, so that D is proportional to L. Hence H, which is proportional to DV, is proportional to $\sqrt{\sec^3\phi}$.

Example 72.—The aeroplane of Example 71, whilst circling at full power with an angle of bank of 60°, is put into straight flight without change of speed or the engine throttle. At what rate will it climb?

We cannot use the above short cut because the incidence will change. For 350 ft. per sec., which is the constant speed concerned, $q = \frac{1}{2}\rho V^2 = 145{\cdot}8$ lb. per sq. ft. and the lift coefficient for straight flight is (ignoring the small difference between cos θ and unity, θ being the angle the straight flight path makes with the horizon):

$$C_L = \frac{W}{qS} = \frac{10{,}000}{145{\cdot}8 \times 343} = 0{\cdot}200,$$

the value for the wing area being taken from Example 70. By plotting r_A against C_L from the data there given, it is also found that the value of r_A corresponding to the above value of C_L is about 10·2. Hence in straight flight at 350 ft. per sec. $D = 10{,}000/10{\cdot}2 = 980$ lb. and $H = 980 \times 350/550 = 624$.

The reserve horse-power that becomes available for climbing, on changing to straight flight at the same speed, is thus $815 - 624 = 191$. Hence the rate of climb

$$= 191 \times 33{,}000/10{,}000 = 630 \text{ ft. per min.}$$

Example 73.—A 20-ton flying-boat has a wing-loading (w) of 32 lb. per sq. ft. and its wings have the following coefficients at some incidences:

C_L . . .	0·2	0·4	0·7	1·0
C_D . . .	0·010	0·015	0·032	0·063

Its rate of climb is 1200 ft. per min. at 150 m.p.h., its power units then giving a total of 2800 T.H.P. Estimate the maximum level speed for 3000 T.H.P.

We have first to find the extra-to-wing drag, D_B, from the data given for 150 m.p.h.

At 150 *m.p.h.*—$q = \frac{1}{2}\rho V^2 = 57{\cdot}6$ lb. per sq. ft., $C_L = w/q = 32/57{\cdot}6 = 0{\cdot}555$. Plotting the polar given in the question—i.e., C_L against C_D—shows that the drag coefficient corresponding to this lift coefficient is 0·022. Also from the question the wing area $S = 20 \times 2240/w = 1400$ sq. ft.

Still at 150 m.p.h., the wing drag $D_w = C_D qS = 0{\cdot}022 \times 57{\cdot}6 \times 1400 = 1774$ lb. The T.H.P. expended on climbing as such

$$= \frac{20 \times 2240 \times 1200}{33{,}000} = 1629.$$

Thus the T.H.P. that overcomes the drag at this speed = 2800 − 1629 = 1171. Writing D for this total drag, we therefore have

$$1171 = \frac{D \times 150}{375},$$

giving $D = 2928$ lb.

Thus the extra-to-wing drag at 150 m.p.h. $= D - D_w = 1154$ lb.

At other speeds D_B is assumed to vary directly as q or inversely as C_L. Thus at all speeds, $D_B = 1154 \times 0{\cdot}555/C_L = 640/C_L$.

The following table can now be constructed :

C_L . . .	0·2	0·4	0·7	1·0
$q = w/C_L$.	160	80	45·7	32
$qS/1000$ . .	224	112	64	44·8
$D_w = C_D qS$.	2240	1680	2048	2822
$D_B = 640/C_L$.	3200	1600	914	640
$D = D_w + D_B$.	5440	3280	2962	3462
$V = 29\sqrt{q}$.	367	259	196	164
$H = DV/550$.	3627	1545	1055	1032

Plotting H against V shows that $H = 3000$ when V is 345 ft. per sec. = 235 m.p.h. This, then, is the maximum speed of the flying-boat in level flight.

Example 74.—The wings of a small aeroplane, which weighs 2000 lb., have a drag of 100 lb. at an indicated air speed of 80 m.p.h. Find the T.H.P. required for an angle of climb of 5° at this speed, given that a ¼-scale model of the aeroplane without its wings, tested in a wind tunnel at 240 m.p.h., has a drag of 36 lb.

At 80 m.p.h., the full-scale extra-to-wing drag $= 36 \times 16 \times 1/9 = 64$ lb. Thus the total drag = 164 lb., neglecting interference between the body and the wings. This gives an overall lift–drag ratio of $2000/164 = 1/0{\cdot}082$. $\theta = 5° = 0{\cdot}0873$ radian. Hence from (73) :

$$\text{T.H.P. required} = \frac{WV}{375}\left(\frac{1}{r_A} + \theta\right) \quad \text{(for } V \text{ in m.p.h.)}$$
$$= 426{\cdot}7(0{\cdot}082 + 0{\cdot}0873)$$
$$= 72\tfrac{1}{4}.$$

159. Change of Altitude

It was seen in Article 136 that, neglecting subsidiary effects, each aeroplane possesses a unique curve connecting total drag D in straight level flight with indicated air speed V_i, a curve which does not change with altitude. But the corresponding curve connecting H, the T.H.P. required, with V_i depends upon the altitude, for H is proportional to DV, where V is the true air speed and $V = V_i/\sqrt{\sigma}$. Hence both H and V increase with altitude in the ratio $1/\sqrt{\sigma}$ for constant indicated air speed.

Curves giving the thrust horse-power required by a certain aeroplane of 20 tons weight for straight level flight at various altitudes are shown plotted against the true air speed (m.p.h.) in Fig. 91. A wide range is illustrated, the highest altitude being nearly two miles into the stratosphere. Such curves are derived immediately from that for low-altitude flight by dividing the ordinates and abscissae of points on the latter curve by $\sqrt{\sigma}$.

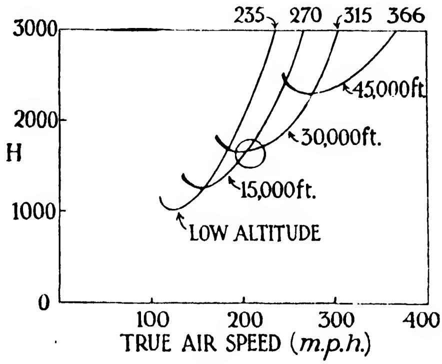


Fig. 91.—High and Low Altitude Flying.

The increase of power required is not so important a matter as is the decrease of power available from the engines. To fix ideas, a maximum of 3000 T.H.P. is assumed for the aeroplane of the figure, giving a speed of 235 m.p.h. at low altitude. With normally aspirated engines the power would fall at 15,000 feet to within the region enclosed by the circle shown towards the middle of the figure. The speed would be reduced to 205 m.p.h., though the power absorbed would be 500 T.H.P. less than for the same speed at low altitude. Moreover, only 500 additional T.H.P. would serve to restore the speed to 235 m.p.h., which would be attained at 15,000 feet with 27 per cent. less power than at low altitude.

The aerodynamical economy achieved by high flying increases with the altitude. Still referring for illustration to the particular aircraft of Fig. 91, if the engines were supercharged to 15,000 feet the speed there would be 270 m.p.h. If this altitude were the maximum for the supercharging system fitted, the power would fall to the region of the circle again at 30,000 feet. But an additional 500 T.H.P. would enable the speed of 270 m.p.h. to be achieved with 25 per cent. less power than at 15,000 feet and with very much less than at sea-level. Increasing the power from 2500 to 3000 T.H.P. increases the speed of this aeroplane by 50 m.p.h. at 45,000 feet as compared with 17 m.p.h. at sea-level.

Such comparisons are flattering because they make no allowance for extra dead weight, due to the air compressing and conditioning plant and strengthening the cabin against bursting. However, when proper correction is made on this score there still appears a considerable advantage in high flying, apart from passing over the tops of storms.

The advantage can be described in general terms as follows. An aeroplane at low altitude is encumbered with large reserves of wing area and engine power in order to provide for a reasonable landing speed and a satisfactory rate of climb. These reserves are largely mobilised at a suitably high altitude. The wing area then corresponds to a high stalling speed (for the aeroplane of Fig. 91 it would be 170 m.p.h. at 45,000 feet), whilst the horse-power not actually being utilised would produce only two or three hundred feet per minute of climb. Neither fact implies a disadvantage; low stalling speed and rapid climb reappear on descending to low altitudes, where they are particularly needed. The reduced air density permits cruising at incidences near to that for minimum horse-power without loss of true air speed, indeed with a gain in this respect compared with low altitude flying.

160. Ceiling

The maximum altitude to which a given aircraft can ascend is called its *absolute ceiling*. The rate of climb diminishes with increase of altitude, partly on account of the greater power required at any incidence to overcome drag, but more acutely on account of the decrease of power available above the supercharged height. The *service ceiling* is defined as the altitude at which the rate of climb falls to 100 feet per minute. The absolute ceiling corresponds to zero rate of climb. At the latter the aircraft can fly level at only one speed, the power available being insufficient for faster or slower flight; any other speed entails a power glide and loss of altitude.

To find the ceiling in a given case, an estimate may first be framed from the general particulars of the aeroplane and its power equipment. Thrust horse-power available and required curves being set out for this provisional altitude, inspection will suggest the incidence (or lift coefficient) at which these two curves will just touch one another on increasing or decreasing the provisional altitude, as may be required, and further calculations will refer to this incidence only. The curve of horse-power required in Fig. 92 is for the appropriate incidence so determined. The other two curves in the figure are obtained from particulars of the engines and airscrews and illustrate the falling T.H.P. available with (a) a normally aspirated and (b) a supercharged engine. The points A, B mark the service ceilings for a certain weight and the intersections A′, B′ the absolute ceilings. With normally aspirated engines the rate of climb decreases approximately linearly with increase of altitude, facilitating rapid estimation of either the absolute or service ceiling.

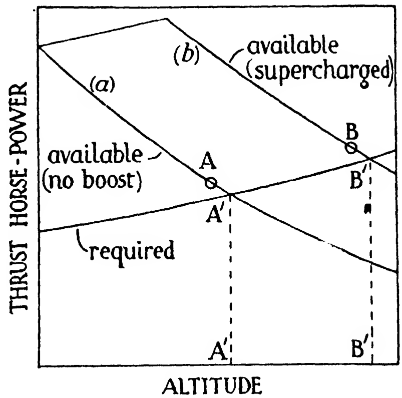


FIG. 92.—ESTIMATION OF CEILING.

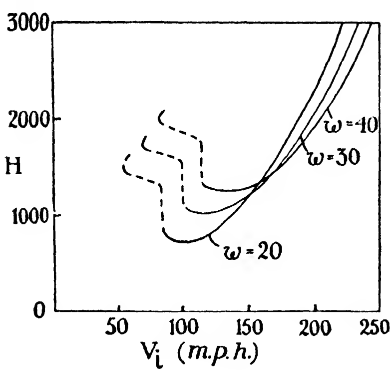


FIG. 93.—CHANGE OF WING AREA.

161. Change of Wing-Loading

The curves in Fig. 93 relate to three aeroplanes which are identical in weight and shape except that they have different wing-areas. The three wing-loadings are 20, 30 and 40 lb. per square foot. A low altitude is assumed throughout. The dotted extensions refer to split flaps opened; otherwise the flaps are closed. The effect of increasing the wing-loading on the minimum flying speed, which may be identified with the landing speed, has been described in Article 134. Another effect is to increase the minimum power required for level flight and also the speed at which it occurs. A result is to reduce the rate of climb and increase the difficulty of take-off in starting a flight, for ease of which a rather low wing-loading is desirable. But the power required is reduced at high speeds, which are hardly attainable without heavy wing-loadings. There are several ways of explaining these contrasting effects; the simplest, and one which we are already in a position to understand fully, is as follows. Consider the change of drag due to reducing the wing area of an aeroplane at any low speed. Keeping the speed constant prevents the extra-to-wing drag from changing. But the drag of the wings, obtained by dividing the flying weight by their lift–drag ratio r, increases because the lift coefficient increases in the range where the curve of r plotted against C_L is falling (cf. Fig. 43). Repeating this comparison at some constant high speed shows the reverse to happen, because the increase of C_L then occurs within the range in which r is increasing with C_L.

Comparison of Fig. 91 with Fig. 93 shows some effective resemblance between increasing altitude and reducing wing area. It is restricted by the fact that in the former case the extra-to-wing drag contributes the same fraction of the total drag at all altitudes for any one indicated air speed, whilst in the latter case this fraction increases as the wings become smaller. Both figures depict aerodynamical effects only, neglecting variation of useful load, the total weight being kept constant. But while this limitation flatters the aeroplane in relation to increase of altitude, as already described, the reverse holds in respect of increase of wing-loading, for smaller wings can be made lighter to sustain the same total load, thus increasing the useful load.

When account is duly taken of the lighter wings, the advantage of heavy wing-loadings at high speeds becomes striking. Some years ago, when aircraft speeds were being increased rapidly, wing-loadings were increased in roughly the same proportion. Theoretically the latter should have increased the faster, as we shall see later, but restriction arose from increased landing speeds, in spite of the general adoption of the split flap, and difficulties of take-off. With long non-stop flights, however, the landing weight is much reduced by consumption of fuel on the journey, and the question of take-off is the more urgent. Various schemes have therefore been suggested for 'assisting' take-off when the wing-loading is especially heavy and the power available not superabundant. In the Short–Mayo system, for example, the long-range aircraft is carried into the air on the back of a larger parent aircraft, the engines of both being used to fly in combination until a suitable speed and altitude are reached for the smaller aeroplane to fly away on its own.

162. Change of Weight

The investigation of the preceding paragraph is complicated by the fact that the shape of the aeroplane changes as the wings get smaller or larger, whence the overall lift-drag ratio r_A varies though the lift coefficient remain constant. The present article is concerned with the effect of varying the weight of a given aeroplane, as by taking more fuel or pay-load on board and, since the shape does not change, r_A remains constant for any one lift coefficient. The problem is therefore simpler, and it is evidently no less important. Many practical questions concerning change of weight also involve change of altitude. Little complication results from considering both variations together, as follows.

Attention is confined to any given aeroplane in straight level flight at any one lift coefficient C_L. Then the lift $L = C_L qS = W$ and, whatever W may be, the wing area S is constant. Consider the effect of altering the weight W by adding or subtracting disposable load. Two results follow from C_L being kept constant: q must vary in proportion to W, and V_i in proportion to $\sqrt{W}$; the overall lift–drag ratio is constant, and therefore D, the total drag, varies as W. Now, H, the horse-power required, is proportional to DV, where V is the true air speed and $= V_i/\sqrt{\sigma}$. Hence H varies as $W\sqrt{W/\sigma}$.

This result applies to any lift coefficient, and a curve of H plotted against V for a particular weight W_0 at low altitude is rapidly modified to apply to another weight W_1 at a higher altitude by increasing all the V's and H's respectively in the ratios

$$\sqrt{\frac{1}{\sigma}\cdot\frac{W_1}{W_0}} \quad \text{and} \quad \sqrt{\frac{1}{\sigma}\left(\frac{W_1}{W_0}\right)^3} \quad . \quad . \quad . \quad . \qquad (74)$$

The curves of Fig. 94 all refer to one and the same aeroplane at low altitude. The middle curve applies to the aeroplane when it is loaded to a total weight of 18 tons. For the lower curve the weight is reduced by one-third, for the upper one it is increased by one-third. Increasing the weight makes less difference to the horse-power required at high than at low speeds. It rapidly increases the minimum H.P. and consequently reduces the rate of climb and the ceiling. Take-off is made more difficult, and in an extreme case of over-loading the power equipment would not be sufficient to get the aircraft into the air. Even a light aeroplane cannot take-off unless the engine provides sufficient power; instances of failure on this score were not infrequent thirty years ago, when aero engines were very weak.

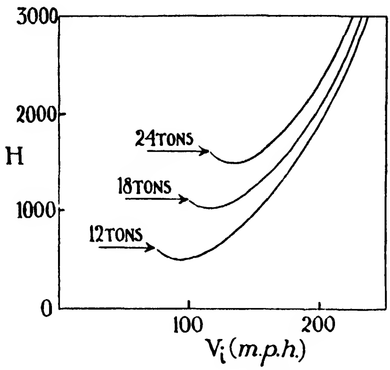


FIG. 94.—CHANGE OF WEIGHT.

The wing-loading for the middle curve of the figure is 30 lb. per square foot and that for the others is 20 and 40 lb. per square foot. Thus the wing-loadings correspond with those assumed in Fig. 93. But there they are varied in another way—viz., by keeping the weight constant and varying the size of the wings only. A family of such curves exists for each of the weights of Fig. 94.

163. Examples

Example 75.—At the beginning of a long non-stop flight a flying-boat cruises at a certain incidence at a speed of 180 m.p.h., using 3000 T.H.P. Towards the end of the flight the weight is reduced by 19 per cent. owing to consumption of fuel. What are then the speed and the T.H.P. required for this incidence?

Directly by (74), the speed will be reduced in the ratio $\sqrt{(1 - 0{\cdot}19)} = 0{\cdot}9$—i.e., to 162 m.p.h.—and the T.H.P. will be reduced in the ratio $\sqrt{(0{\cdot}81)^3} = 0{\cdot}729$—i.e., to 2187.

Example 76.—At the indicated air speed for maximum rate of climb, the engine and airscrew of a small aeroplane are capable of the following thrust horse-powers at the altitudes noted:

Altitude (ft.) .	0	10,000	15,000	20,000
T.H.P. . .	300	216	182	152

When lightly loaded to 5000 lb. total weight, the aeroplane has a service ceiling of 20,000 ft. What is its service ceiling with 1500 lb. of additional load on board?

$W = 5000$ *lb.*—The T.H.P. in reserve for the remaining rate of climb of 100 ft. per min., defining the service ceiling $= 5000 \times 100/33{,}000 = 15{\cdot}2$. Since the power unit is capable of 152 T.H.P. at this altitude, the T.H.P. required for level flight there $= 152 - 15{\cdot}2 = 136{\cdot}8$. The relative density of the air at this ceiling $= 0{\cdot}534$, from Table I, p. 39, giving $\sqrt{\sigma} = 0{\cdot}7308$. Hence level flight at the same incidence at sea-level requires $136{\cdot}8 \times 0{\cdot}7308 = 100$ T.H.P.

$W = 6500$ *lb.*—The T.H.P. in reserve at the service ceiling must now be $6500 \times 100/33{,}000 = 19{\cdot}7$. The weight is increased by the factor 1·3 and the T.H.P. for level flight is increased on this score in the ratio $\sqrt{(1{\cdot}3)^3} = 1{\cdot}482$, or in the ratio $1{\cdot}482/\sqrt{\sigma}$ at various altitudes. Hence the new service ceiling will be reached when σ is such as to make $148{\cdot}2/\sqrt{\sigma} + 19{\cdot}7 =$ the T.H.P. available, which itself depends upon the altitude as stated in the question.

A solution will be obtained graphically. The following table is constructed for the new weight:

Altitude (ft.) .	0	10,000	15,000	20,000
$1/\sqrt{\sigma}$ . . .	1·0	1·164	1·261	1·369
$148{\cdot}2/\sqrt{\sigma} + 19{\cdot}7$.	168	192	207	223

The last row of this table and the second row of the table given in the question are plotted against altitude in Fig. 95. The inter-section of the two curves gives for the answer: 12,500 ft., approximately.

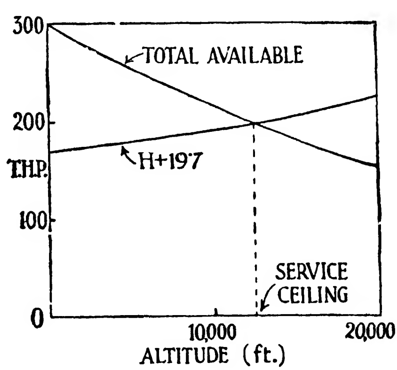


FIG. 95.

Example 77.—An aeroplane weighs 10 tons without pay-load and then has a rate of climb of 1650 ft. per min. at low altitude, the power units providing a total of 1600 T.H.P. What pay-load may be taken on board if the rate of climb is not to be less than 1000 ft. per min. at 10,000 ft. altitude, assuming that the power available is maintained to this height?

At low altitude when the weight is 10 tons, the T.H.P. absorbed in climbing alone = 22,400 × 1650/33,000 = 1120, whence the T.H.P. required for level flight = 1600 − 1120 = 480.

Let W' be the required weight in tons and write W for $10 + W'$, i.e., for the new total weight. At 10,000 ft. $1/\sqrt{\sigma} = 1{\cdot}164$. Hence the T.H.P. required for level flight at the altitude = $480(W/10)^{3/2}/\sqrt{\sigma} = 17{\cdot}7 \times W^{3/2}$. The T.H.P. required for climbing at the minimum rate specified = $W \times 2240 \times 1000/33{,}000 = 67{\cdot}9W$. Hence W is given by the following equation

$$17{\cdot}7\ W^{3/2} + 67{\cdot}9\ W = 1600$$

or, dividing both sides for convenience of working by W,

$$17{\cdot}7\sqrt{W} + 67{\cdot}9 = 1600/W.$$

This equation will be solved by trial and error. Referring to the table below, the first value assumed for W—viz., 16 tons—was chosen for a trial run because it is a possible value and the arithmetic can almost be done mentally. It was evidently much too large and so the value of 12 tons was next chosen. This came out to be a little too small and was adjusted to 12·3 in a third trial. The value 12·3 transpired to be so nearly correct as to be acceptable; otherwise a fourth row would have been added.

W (tons)	$\sqrt{W}$	$17{\cdot}7\sqrt{W}$	L.H.S. of equation	R.H.S.
16	4	70·8	138·7	100·0
12	3·464	61·3	129·2	133·3
12·3	3·507	62·1	130·0	130·1

The pay-load is 12·3 − 10 = 2·3 tons.

164. Alternative Method

So far, the extra-to-wing drag has been assumed to vary only with q, no account being taken of variation with incidence. The approximation is usually justifiable in primary calculations. It becomes poor near the stalling angle, but then the extra-to-wing drag is small compared with the wing drag, variation of which with incidence is correctly allowed for.

When knowledge exists of the variation of extra-to-wing drag with incidence, this drag is best added to the wing drag in the manner of Article 135, resulting in an overall drag coefficient C_{DA} specified on the wing area. Experimental data will be expressed automatically in this form when obtained from wind tunnel tests on a complete model.

Either C_L may be divided by C_{DA}, yielding the overall lift-drag ratio r_A, which may be plotted against C_L, or C_L may be plotted directly against C_{DA} in polar form (Article 135). In the first alternative the formula (73) is used in the way already explained. For the second alternative the formula is re-written as

$$H_T = \frac{WV}{550C_L}(C_{DA} + \theta C_L) \quad . \quad . \quad . \quad (75)$$

The true air speed is V in feet per second, and is found from the indicated air speed appropriate to W and C_L. Writing w for the wing-loading, $q = w/C_L$ and $V = 29\sqrt{q/\sigma}$, so that

$$V = 29\sqrt{w/\sigma C_L}.$$

It is hardly worth while substituting for V in the formula for H_T, since a knowledge of V is usually required.

Whichever mode of calculation is adopted, a curve of H_T can be plotted against V for any assumed small angle of climb θ. Plotting also the curve of T.H.P. available against V will show at a glance through what speed range, if any, steady climbing at θ is possible.

Example 78.—Below are given the overall lift-drag ratios and the T.H.P. available for a slow 5-ton aeroplane at various indicated air speeds.

V_i (m.p.h.) .	85	120	140	160
r_A	13·0	15·5	14·4	12·6
T.H.P. available .	420	580	640	685

Find the speed for an angle of climb of 4° at 5000 ft. altitude.

At the altitude, $\sigma = 0{\cdot}862$—i.e., $V = 1{\cdot}077V_i$ in m.p.h. This yields the first row of the table below. WV (ft. per sec.)/550 = 11,200V (m.p.h.)/375 = 29·9V (m.p.h.); hence the second row. The third row follows from the values of r_A given and the fact that $\theta = 0{\cdot}0698$ radians. The fourth row is obtained by multiplying together the numbers in the second and third rows and gives, by (73), the T.H.P. required (to the nearest 5 H.P.).

V (m.p.h.) . .	91½	129	151	172
$WV/375$ . .	2740	3850	4510	5150
$1/r_A + \theta$ . .	0·147	0·134	0·139	0·149
T.H.P. required .	400	515	625	770

The T.H.P.'s. available and required are plotted in Fig. 96 and are equal at about 86 and 155 m.p.h., true air speed. Within this speed range the aircraft can climb more steeply still, if desired, but outside it only less steeply.

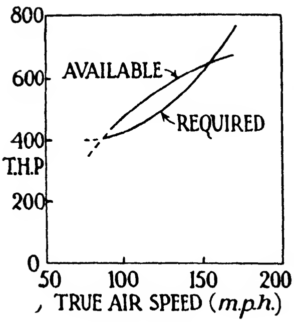


FIG. 96.

165. Induced Drag Method

The last article refines upon methods of preceding articles. At the other extreme, we may neglect variation with incidence of the profile drag (Article 106) of the wings, besides that of the extra-to-wing drag. This approximation is often permissible through a restricted speed range, corresponding to incidences not far removed from that for maximum lift–drag ratio. First estimates of performance follow very simply. Another advantage is that, though precision is lost at high and low speeds, some of the principles underlying the attainment of good performance are revealed particularly clearly.

The total drag D is still dealt with in two parts, but these differ essentially from the wing and extra-to-wing drag so far used. The profile drag of the wings is now added to the extra-to-wing drag, thus collecting together all skin frictions and form drags. The sum will be called the total parasitic drag, written D_P. Subject to scale effect and variation due to change of incidence or airscrew slipstreams, all of which are neglected, D_P varies proportionately with q for a given aeroplane, no matter what load the aeroplane takes on board or discharges, or whether it is gliding, climbing, circling or simply flying straight and level.

This leaves only the induced drag D_i to comprise the other part of the total drag. It is calculated approximately from the formula (46) of Article 113, as follows, A denoting the aspect ratio.

$$\begin{aligned} D_i &= C_{Di} q S \\ &= \frac{0{\cdot}35}{A} C_L{}^2 q S \\ &= \frac{0{\cdot}35}{q} \cdot \frac{L^2}{AS} \quad \text{(because } C_L{}^2 = L^2/(qS)^2\text{)} \end{aligned}$$

$$= \frac{0{\cdot}35}{q}\left(\frac{\text{Lift}}{\text{Span}}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (76)$$

since by (43) $1/A = S/\text{span}^2$.

The coefficient 0·35 is not constant, but varies a little with the plan-form of the wings, being a minimum for the elliptic and approximately half-tapered plan-forms. The value given is probably correct within $\pm$ 5 per cent. for all ordinary plan-forms.

The formula is of fundamental importance. It states that, for a given aeroplane, D_i varies directly as the square of the lift and inversely as q. If the wings are changed for others of similar shape but a different span, then, however the chord or wing area may be adjusted, D_i changes inversely as the square of the span for the same values of L and q. The ratio of lift to span is called the *span-loading*. Comparing different aeroplanes in flight at different speeds and altitudes, subject only to the restriction that the plan-forms of their wings show little diversity, D_i varies from one to another directly as the square of the span-loading and inversely as the square of the indicated air speed. The term span-loading must not be confused with the span-grading, which describes another matter—viz., the way in which the lift varies along the span (Article 110). Again, the span-loading is the lift, not the weight, per foot of span; no difference occurs in straight level flight, but in other forms of flight the span-loading may be several times as great as the weight divided by the span.

The contrary way in which D_i and D_P vary during flight supplies the reason for their separation. For straight level flight, the expression for the total drag of a given aeroplane now takes the form

$$D = D_i + D_P$$

$$= \frac{a}{q} + bq \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (77)$$

where the coefficient a has approximately the value $0{\cdot}35(W/\text{span})^2$ and the coefficient b is assessed as already described for D_B, except that it has to include the profile drag of the wings. If the form of flight changes from being straight and level, a will also change.

166. Examples

The examples in this article relate to an aeroplane weighing 20,000 lb., for which, in straight level flight at an indicated air speed of 150 m.p.h., $D_i = 500$ lb. and $D_P = 750$ lb., making $D = 1250$ lb. The horse-power required is denoted as usual by H.

Example 79.—What is the power required by the above aeroplane for a speed of 180 m.p.h. at low altitude?

As ρ is constant, D_i varies as $1/V^2$ and D_P as V^2 and, since $180/150 = 1{\cdot}2$, $D_i = 500/1{\cdot}44 = 347$, $D_P = 750 \times 1{\cdot}44 = 1080$, giving $D = 1427$ lb. Hence $H = 1427 \times 180/375 = 685$.

Example 80.—The aeroplane flies at 150 m.p.h. in a level circular path with an angle of bank (ϕ) of 45°. What power is required?

The lift $L = W/\cos\phi = W\sqrt{2}$ and D_i varies as L^2. Thus $D_i = 2 \times 500$ whilst D_P remains at 750. Hence $D = 1750$ lb. and $H = 1750 \times 150/375 = 700$.

Example 81.—If 1500 T.H.P. is available, what is the maximum speed of the aeroplane at low altitude?

With V expressed in m.p.h.,

$$H = \frac{DV}{375} = \left[500\left(\frac{150}{V}\right)^2 + 750\left(\frac{V}{150}\right)^2\right]\frac{V}{375}.$$

H is required to be equal to 1500. So we must have at top speed

$$\left(\frac{150}{V}\right)^2 + 1{\cdot}5\left(\frac{V}{150}\right)^2 = \frac{1125}{V}.$$

Solving by trial and error,

V (m.p.h.)	$(150/V)^2$	$1{\cdot}5(V/150)^2$	L.H.S.	R.H.S. $= 1125/V$
225	0·444	3·375	3·819	5·000
250	0·360	4·167	4·527	4·500

The speed of 225 m.p.h. first assumed is far too low but the second assumption of 250 m.p.h. is so nearly correct as to make further approximation unnecessary. The aeroplane will fly at just under this speed.

Example 82.—Evaluate the coefficients in the drag equation (77) for this aeroplane in straight level flight.

For $V_i = 150$ mp.h., $q = \frac{1}{2} \times 0{\cdot}00238 \times 220^2 = 57{\cdot}6$ lb. per sq. ft. $D_i = a/q$ generally and when $D_i = 500$ lb. q has the above value. Hence $500 = a/57{\cdot}6$, giving $a = 28{,}800$. Similarly

$b = 750/57{\cdot}6 = 13$. Therefore, to apply to the aeroplane concerned, the equation becomes

$$D = \frac{28{,}800}{q} + 13\,q.$$

167. Weight and Climb

The principles governing analysis by the earlier method apply equally, of course, to the alternative one. It is often convenient in the present connexion to keep the indicated air speed constant, for then only the induced drag changes.

If W increase from W_0 to W_1, D_i for straight level flight will increase from D_{i0} to

$$D_{i1} = D_{i0}(W_1/W_0)^2$$

at the same value of V_i, and the T.H.P. required will increase by the amount

$$(D_{i1} - D_{i0}) \times V(\text{m.p.h.})/375$$

$$= D_{i0}\left[\left(\frac{W_1}{W_0}\right)^2 - 1\right]\frac{V_i}{375\sqrt{\sigma}} \quad . \quad . \quad . \quad . \quad (78)$$

To find the rate of climb appropriate to $W = W_1$ at the given indicated air speed, the foregoing increment of power required is subtracted from the reserve horse-power when $W = W_0$, giving a corrected reserve T.H.P. $= H_{R1}$, say. Then approximately,

$$\text{rate of climb} = 33{,}000\ H_{R1}/W_1.$$

During climb at an angle θ lift is reduced in the ratio $\cos\theta : 1$ (Article 146), whence induced drag is reduced in the square of this ratio, making slightly more T.H.P. available for climbing. A closer estimate of climb is therefore obtained by suitably reducing the H.P. required to overcome drag at the same speed in straight level flight.

168. Change of Span

A complex question arising in the design of an aeroplane is choice of span. Span is varied without change of wing-loading by suitably adjusting the wing chord—i.e., by altering the aspect ratio of the wings.

Increasing the aspect ratio decreases induced drag, improving take-off, climb, ceiling, extreme range and performance at low indicated air speed generally, when induced drag forms a large fraction of the total drag. It also reduces the requisite tail volume (Article 140), thus effecting some economy in the weight and drag of the body and tail plane. On the other hand, increase of span, together with the associated reduction of the chord and thickness of the wings, increases the weight of the latter and reduces their storage capacity. The second disadvantage is of practical importance in connection with the retraction of the undercarriage, the fitting of fuel tanks in the wings, and the like.

Final choice of span is therefore a technical matter to be decided with reference to the particular class of aeroplane concerned. The following examples no more than illustrate it in a preliminary manner. The aeroplane of Article 166 is assumed, and enquiry made into some first effects of changing the aspect ratio of its wings whilst keeping the wing-loading constant.

Example 83.—The span of the aeroplane described in Article 166 is increased by 20 per cent., keeping the wing area constant. The total weight is increased 3 per cent. thereby. What is the improvement in climb at 150 m.p.h. if a total of 1400 T.H.P. is available?

Before the modification the T.H.P. required for level flight at the given speed $= 1250 \times 150/375 = 500$; the reserve T.H.P., $H_R = 1400 - 500 = 900$ and the rate of climb $= 900 \times 33{,}000/20{,}000 = 1485$ ft. per min.

The modification decreases the induced drag from 500 lb. to

$$500 \times (1{\cdot}03/1{\cdot}20)^2 = 368 \text{ lb.}$$

Since D_P is unchanged, the total drag become $750 + 368 = 1118$ lb. H is reduced to $1118 \times 150/375 = 447$, making $H_R = 1400 - 447 = 953$. This gives a rate of climb $= 953 \times 33{,}000/20{,}600 = 1527$ ft. per min.

This simple calculation accordingly gives an improvement in the rate of climb amounting to 42 ft. per min. The estimate is flattering, however, because either the disposable load must be reduced by 3 per cent. of the total weight or the area of the wings must be increased by 3 per cent. to keep the landing speed the same. The latter step would slightly increase D_P.

Example 84.—It is estimated that reducing the span of the aeroplane described in Article 166 by 20 per cent. decreases the total weight by 3 per cent. The wings are reduced in area to maintain the original landing speed and this is estimated to reduce the total parasitic drag by 2 per cent. Determine the effect upon the maximum speed if 1500 T.H.P. is available.

The induced drag is increased in the ratio $(0{\cdot}97/0{\cdot}80)^2 = 1{\cdot}47$. The total parasitic drag is reduced in the ratio $1{\cdot}47/1{\cdot}50$. Therefore, the second equation of Example 81 which is to be satisfied by the top speed, is modified to

$$1{\cdot}47\left(\frac{150}{V}\right)^2 + 1{\cdot}47\left(\frac{V}{150}\right)^2 = \frac{1125}{V}.$$

Solving by trial and error gives a top speed of just under 248 m.p.h.—i.e., the sacrifice on the maximum speed is only 2 m.p.h.

169. Application to Minimum Gliding Angle

Fig. 97 shows the induced and total parasitic drags of the aeroplane of Article 166 in straight level flight plotted against the indicated air speed. In this fairly typical case the induced drag amounts to 10 per cent. of the whole at about 233 m.p.h.—i.e., D_i is then one-ninth of D_P. At one-third of this speed the proportionate contributions to the whole drag are reversed. D_i and D_P are equal at 138 m.p.h., and at this speed the total drag is a minimum, being equal to 1224 lb. The maximum lift-drag ratio is therefore 16·3, since the lift = the weight = 20,000 lb., and the minimum gliding angle is $3\frac{1}{2}°$.

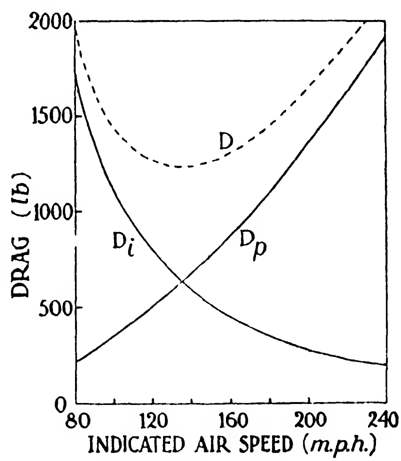


FIG. 97.—INDUCED AND TOTAL PARASITIC DRAGS.

The student acquainted with the methods of the Differential Calculus can obtain the minimum drag and the speed at which it occurs without the labour of plotting, as follows. Writing the expression for the total drag D in the general form

$$D = a/q + bq,$$

differentiating with respect to q and equating to zero for a minimum gives

$$\frac{dD}{dq} = -\frac{a}{q^2} + b = 0,$$

whence

$$q = \sqrt{\frac{a}{b}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (79)$$

Substituting this value of q in the original expression for D gives

$$\text{Minimum } D = \sqrt{ab} + \sqrt{ab}$$
$$= 2\sqrt{ab} \quad . \quad . \quad . \quad . \quad . \quad . \quad (80)$$

Both parts of the minimum drag are equal and their sum is readily calculated.

170. The Basis of Aerodynamic Efficiency

In Chapter I the efficiencies of various lifting agents were compared on the basis of 'ton-miles-per-gallon' of useful or disposable lift, sometimes called the *net efficiency*. The *aerodynamic efficiency* is not concerned with what proportion of the total lift of an aeroplane is absorbed by its tare weight, and the useful work done is conceived to be the gross weight × the distance transported. The energy expended in this transportation is proportional to the gallons of fuel consumed, which is proportional to the brake horse-power (B.H.P.) actually exerted by the power plant and the time of the flight. Thus the aerodynamic efficiency is proportional to

$$\frac{\text{Gross weight} \times \text{distance flown}}{\text{B.H.P.} \times \text{time of flight}} = \frac{W \times V}{\text{B.H.P.}}$$

Now, the B.H.P. = the T.H.P. divided by η, the efficiency of the airscrews, whilst the T.H.P. is proportional to the thrust $T \times V$. Again, $T = D$, the total drag, whilst for straight level flight $W = L$, the lift. And so the aerodynamic efficiency is proportional to

$$\frac{LV}{DV/\eta} = \eta r_A \quad . \quad . \quad . \quad . \quad . \quad . \quad (81)$$

If the gross weight is expressed in tons and V in m.p.h., the quantity $WV/\text{B.H.P.} = \frac{1}{8}\eta r_A$ with an error of less than $\frac{1}{2}$ per cent. The last quantity is one of the oldest expressions for efficiency. In the early days of mechanical flight η was less than $\frac{3}{4}$ and r_A less than 8, so that a value of unity, corresponding to an efficiency on this basis of 100 per cent., was not easy to obtain. With modern aircraft the value of 2 is readily exceeded at several times as great a speed. For this reason $\frac{1}{6}\eta r_A$ is seldom used today to express the aerodynamic efficiency of an aeroplane. But no other gauge has arrived to take its place and, though no longer providing the target of 100 per cent. at which to aim, it is nevertheless still of great value as giving, within $\frac{1}{2}$ per cent., the number of ton-miles (gross) per B.H.P.-hour.

Whether the coefficient of $\frac{1}{6}$ is retained or dropped, the product ηr_A is a measure of aerodynamic efficiency in the operational sense. Postponing discussion of η to a later chapter, the efficiency becomes proportional to r_A—i.e., it is greatest when the drag for a given weight is least. This result is verified to be consistent with the fundamental idea of efficiency, as follows. Consider the transportation of a given load from one point to another through a still atmosphere. The total work done is equal to the drag multiplied by the distance and is a minimum when the drag is a minimum. However, the presence of wind modifies the practical aspect of this conclusion.

Further discussion is greatly clarified by expressing r_A in the form

$$r_A = \frac{\text{Lift}}{D_i + D_P}.$$

Dividing numerator and denominator by qS and denoting by C_{DP} the drag coefficient of the total parasitic drag expressed on the wing area, this becomes

$$r_A = \frac{C_L}{C_{Di} + C_{DP}}.$$

Substituting $0{\cdot}35\ C_L{}^2/A$ for C_{Di} from formula (46), Article 113, A being the aspect ratio, and dividing numerator and denominator by C_L,

$$r_A = \frac{1}{\dfrac{0{\cdot}35}{A} C_L + \dfrac{C_{DP}}{C_L}} \quad . \quad . \quad . \quad . \quad (82)$$

For straight level flight $C_L = w/q$, w being the wing-loading. Making the substitution, we have for this condition

$$r_A = \frac{1}{\dfrac{0{\cdot}35}{A} \cdot \dfrac{w}{q} + C_{DP}\dfrac{q}{w}} \quad . \quad . \quad . \quad . \quad (83)$$

The maximum value of r_A for a given aeroplane occurs when the total drag is a minimum—i.e., when the two terms in the denominator are equal to one another (Article 169). Equating these terms gives

$$C_L = \sqrt{\frac{C_{DP}A}{0{\cdot}35}} = \frac{w}{q} \quad . \quad . \quad . \quad . \quad . \quad (84)$$

for straight level flight. Substituting this value of C_L or w/q gives for the maximum efficiency

$$r_A \text{ (max.)} = \tfrac{1}{2}\sqrt{\frac{A}{0{\cdot}35 \times C_{DP}}} \quad . \quad . \quad . \quad (85)$$

Any of these expressions for r_A when multiplied by the airscrew efficiency η, or $\frac{1}{6}\eta$, according to the convention adopted, gives the aerodynamic efficiency of the aeroplane, the aeroplane being defined by A, C_{DP} and the appropriate value of the numerical coefficient 0·35, which is subject to minor variation by altering the plan-form of the wings. The efficiency then depends upon C_L or, for straight level flight, upon w/q. The maximum efficiency of the aeroplane is given by (85), and occurs when C_L or w/q is given by (84).

Comparing different aeroplanes all having wings of a single type so that variation of the numerical coefficient is not involved, the maximum efficiency varies directly as the square-root of the aspect ratio and inversely as the square-root of the coefficient of total parasitic drag.

It is easily found from (84) and (85) that the optimum indicated air speed is equal to

$$16{\cdot}5\sqrt{\frac{r_A(\text{max.}).w}{A}} \text{ m.p.h.}$$

For most civil aeroplanes this speed will lie between $22\sqrt{w}$ and $28\sqrt{w}$ and is unduly low for main-line air services operating at ordinary altitudes. In Fig. 98 the lower range is plotted as a dotted curve, the higher range as a full-line curve. The scale at the right-hand side of the figure indicates the landing speeds that correspond with the wing-loadings, assuming ordinary split flaps.

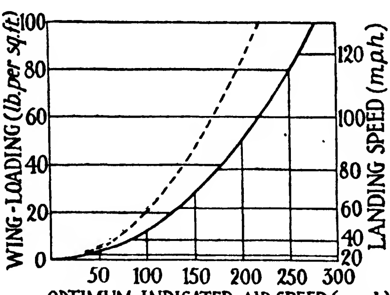


FIG. 98.—WING-LOADING FOR MAXIMUM EFFICIENCY.

It will be seen that even under favourable conditions a wing-loading of 25 lb. per square foot implies an optimum flying speed of only 140 m.p.h., and that to secure maximum aerodynamic efficiency at 250 m.p.h. would demand a wing-loading of 80 lb. per square foot, involving a landing speed of 115 m.p.h., and a severe problem in assisted take-off. The loss of efficiency due to flying considerably faster than the optimum speed, or due to employing a considerably larger wing area than would give the optimum wing-loading, is at first small. The precise amount of the loss for any excess depends upon the shape of the aeroplane, but a fairly typical case is illustrated in Fig. 99. Approximately, the wing-loading may be decreased by 40 per cent. or the speed increased by 27 per cent. for a reduction in the efficiency amounting to 10 per cent. These adjustments are insufficient, however, for they still leave the wing-loading high and the speed low, and a greater loss is usually incurred.

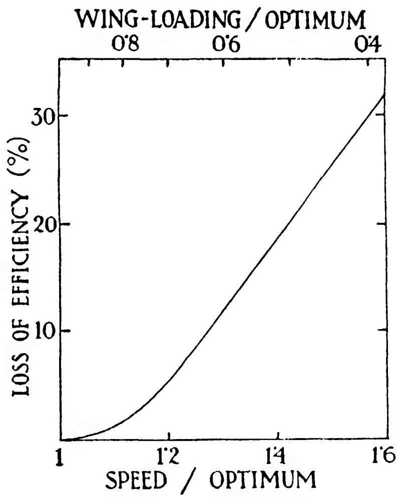


FIG. 99.—LOSS OF EFFICIENCY AT PRACTICAL WING-LOADINGS.

Some general conclusions can be drawn from this discussion. Progress in the operational speeds of first-class aeroplanes has outstripped progress in the development of means for landing them reasonably slowly. Until flaps or other landing devices improve substantially, aeroplanes must continue to employ wastefully large wings, especially at low altitudes. If the desired improvement were forthcoming regarding landing conditions, assisted take-off would become an urgent matter. Meanwhile, at least, flying at high altitudes recommends itself as providing a reasonably high true air speed with a low value of q, and therefore incurring little loss in aerodynamic efficiency with a moderate wing-loading, suitable for both landing and take-off.

Example 85.—A flying-boat, 100 ft. in span, weighs 40,000 lb. and has 4 1000-H.P. engines which give a speed of 250 m.p.h. in straight level flight at low altitude, the airscrew efficiency being 82 per cent. Estimate the aerodynamic efficiency at 175 m.p.h. if the airscrew efficiency is then 75 per cent.

At the top speed the total drag $= 4 \times 1000 \times 0{\cdot}82 \times 375 \div 250 = 4920$ lb. and, since $q = 160$ lb. per sq. ft., the induced drag D_i

$$= \frac{0{\cdot}35}{160}\left(\frac{40{,}000}{100}\right)^2 = 350 \text{ lb., by formula (76),}$$

so that the total parasitic drag $D_P = 4920 - 350 = 4570$ lb.

At 175 m.p.h., which is 0·7 of the top speed, D_i increases to $350 \times 1/0{\cdot}49 = 714$ lb., and D_P decreases to $4570 \times 0{\cdot}49 = 2239$ lb. Thus at the lower speed the total drag $= 2953$ lb. and the overall lift–drag ratio $= 40{,}000/2953 = 13{\cdot}55$. Finally

$$\eta r_A = 0{\cdot}75 \times 13{\cdot}55 = 10{\cdot}16, \text{ or } \eta r_A/6 = 1{\cdot}7, \text{ nearly.}$$

Chapter IX

PROPULSION AND AIRSCREWS

171. Ideal Propulsion

In order to fly otherwise than downward through the air an aircraft is supplied with a quantity of energy, and the duty of the propelling device is to convert as much of this as possible into work usefully done in overcoming the drag of the aircraft and raising its weight. Many forms which a propeller might take are too wasteful of either energy or lift. The airscrew possesses advantages in these respects, but is by no means ideal. The whirling blades have form drag and skin friction, and work badly towards the tips even when tip-speeds are only moderate; vortices cast from them spin the air behind to no purpose and spoil streamline flow over aircraft components on which they play; and finally the central part of the disk area is obstructed by a boss or other obstacle, providing drag instead of thrust. Nevertheless, the airscrew survives its disabilities with considerable success.

It is valuable to investigate in general terms, and supposing all disabilities absent, an ideal scheme of propulsion. The results will apply qualitatively to actual systems, and can afterwards be modified and amplified to take account of additional problems which the propelling device presents. The idealised propeller is called vaguely the 'actuator'; the term 'ideal airscrew' is also used when a screw propeller is particularly in mind.

The actuator is supposed to occupy a disk through which air passes, and to engage a certain mass m of air per second equal to the product of the disk area, the relative velocity and the density of the air. This mass is accelerated backward, purely parallel to the direction of motion and equally as to all parts. The reaction experienced, equal in magnitude to the rate at which backward momentum is imparted to the mass of air, is the forward thrust T and, owing to the evenly distributed action, T is uniformly spread over the disk.

Let the actuator propel an aircraft through a still atmosphere at the velocity V, and let v be the uniform backward velocity given to the air engaged. Then the actuator does useful work at the rate TV, and $T = mv$.

The prime mover, or other source of power, must supply the actuator with energy at the rate TV and, in addition, make good any wastage. Such wastage occurs even under ideal conditions because the air engaged cannot receive momentum without also acquiring kinetic energy. Thus the actuator puts less work to a useful purpose in propelling the aircraft than it receives, dissipating the remainder in the form of kinetic energy imparted to the atmosphere. The atmosphere gets infinitesimally warmer as a result, but this does not help the propulsion.

Efficiency is defined in the present and similar connexions as the ratio of the useful work to the total work or, as it is often described, the ratio of the output to the input. The actuator, or ideal airscrew or ideal propeller, is said to have an ideal efficiency, which will be denoted by η_i. Let E be the kinetic energy dissipated per unit length of the flight path. Then EV is the kinetic energy lost in unit time and

$$\eta_i = \frac{TV}{TV + EV} = \frac{T}{T + E}.$$

This expression is readily simplified. For the mass of air engaged per unit length of the flight path is m/V, so that $E = \frac{1}{2}(m/V)v^2$. Hence, since $T = mv$,

$$\eta_i = \frac{mv}{mv + \frac{1}{2}(m/V)v^2} = \frac{1}{1 + \frac{1}{2}(v/V)} \quad . \quad . \quad (86)$$

Efficiencies are usually expressed as percentages. The ideal efficiency never reaches 100 per cent., but it is greater the less the value of v/V. Now $v = T/m$ and, for constant density of the air, m is approximately proportional to the product of the disk area and the forward speed. Thus v is proportional, at constant altitude, to the thrust per square foot of the disk and inversely proportional to V. Hence high ideal efficiency is obtained with a lightly loaded actuator disk at high speed.

The problem of ideal propulsion is solved once v can be determined, but this is not immediately possible because m is greater than appears at first sight, owing to the air crossing the actuator disk at a faster speed than the aircraft speed.

172. In developing the theory further the actuator is assumed to be located without axial movement in a uniform head wind of velocity V. Thrust is then derived from increasing the momentum of the air crossing its disk. The increase of velocity implied cannot be added suddenly to this air, but occupies a space of, say, two or three disk diameters and is begun well in front of the disk. The stream of air engaged accordingly takes the form of a jet stretching in front of, and behind, the airscrew. The density of the air is constant throughout, and the cross-sectional area of the jet is inversely proportional to the velocity, which is uniform over any cross-section. The part behind the actuator is called the *slipstream*, and the narrowest part the *vena contracta*. Only here, a diameter or so along the slipstream, is the fully augmented velocity reached.

Let the total increase of velocity be bV. Then b, a coefficient to be determined, is called the *slipstream factor*, and the final velocity of the slipstream is $V(1 + b)$.

Let the part of this increase which is added in front of the disk be aV. Then a is called the *inflow factor*, and the velocity through the disk is $V(1 + a)$.

It will be convenient to have symbols for the stagnation or dynamic pressure far in front of the actuator, through the disk of rotation, and at the *vena contracta*. So we define : $q = \frac{1}{2}\rho V^2$, $q_A = \frac{1}{2}\rho[V(1 + a)]^2$ and $q_S = \frac{1}{2}\rho[V(1 + b)]^2$.

The impulsive action of the ideal actuator is evidenced by a sudden increase of the static pressure from the uniform value of p_1 on the face of the disk to the uniform value of p_2 on its back. The pressure difference $p_2 - p_1$ is equal, of course, to the thrust per square foot of the disk.

The pressure p_1 is evidently less than the undisturbed pressure p, because Bernoulli's equation must apply in front of the disk and q_A is greater than q. But in ideal propulsion the flow in the slipstream is assumed also to be irrotational; there is assumed to be no spin in the slipstream. The streamlines all become parallel again at the *vena contracta*, and the air possesses no centrifugal force to support a pressure gradient. Hence the pressure at the position of the *vena contracta* is the same within the slipstream as outside it, and is equal to the undisturbed pressure p. It follows that the pressure and velocity in the slipstream are related to one another by Bernoulli's equation and, since q_S is greater than q_A, that p_2 is greater than p.

Bernoulli's equation cannot be applied between any point outside the slipstream and any point within it, because the actuator does work on the air in forcing it into the slipstream against the sudden pressure rise $p_2 - p_1$. The pitot head of the air engaged is increased by this amount at the disk.

The foregoing considerations are illustrated schematically in Fig. 100, where the velocity, pressure and pitot head are plotted along the stream. As already mentioned, $p_2 - p_1$ can be found from the thrust and the disk area; bV, the added velocity, can be found from the thrust and a knowledge of the mass of air dealt with per second, on the lines indicated in the preceding article and as will be further explained; and finally the flow will be completely understood once the relationship between the factors a and b is established.

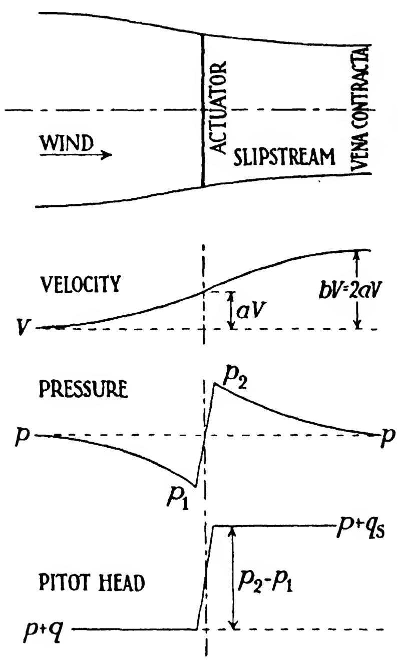


Fig. 100.

173. Relation of *a* to *b*

Consider any streamline which crosses the actuator disk. Applying Bernoulli's equation between any point far upstream and the point of intersection on the face of the disk gives

$$p_1 + q_A = p + q.$$

Applying this equation between points on the back of the disk and at the *vena contracta* gives, since the pressure at the latter position is again equal to p,

$$p_2 + q_A = p + q_S.$$

Subtracting the first of these expressions from the second,

$$\begin{aligned} p_2 - p_1 &= q_S - q \\ &= \tfrac{1}{2}\rho V^2[(1 + b)^2 - 1] \\ &= qb(2 + b). \end{aligned}$$

Let A denote the area of the actuator disk. Then the mass m of air crossing the disk per second is $\rho AV(1 + a)$. The velocity finally added to m is bV. Hence, as in the preceding article,

$$T = \rho AV(1 + a) \,.\, bV$$
$$= 2Aqb(1 + a).$$

But $T = A(p_2 - p_1)$. Hence

$$p_2 - p_1 = 2qb(1 + a).$$

We now have two alternative expressions for the pressure difference. Equating them gives

$$2 + 2a = 2 + b,$$

i.e.,
$$a = \tfrac{1}{2}b \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (87)$$

Thus one-half of the total increase of velocity has been added to the stream by the time it crosses the actuator disk.

This important result is not restricted to ideal propellers, but is also closely true of airscrews. There is no need to retain the symbol b for the slipstream factor; if the inflow factor is a, the slipstream factor is $2a$.

Example 86.—A propelling device, 2 ft. in diameter, is located in a wind tunnel working at 50 ft. per sec. and gives a thrust of 5 lb. Regarding it as an ideal actuator, determine (*a*) the velocity at the disk of rotation, (*b*) the pressure on each side of the disk relative to the undisturbed pressure of the wind.

The disk area $A = \pi$ sq. ft., whence

$$T = 5 = \pi\rho V(1 + a) \,.\, 2aV$$
$$= 4\pi q(a + a^2).$$

Since $V = 50$ ft. per sec., $q = 2{\cdot}975$ lb. per sq. ft., whence

$$a + a^2 = 0{\cdot}134,$$

giving $a = 0{\cdot}12$, approximately.

Hence (*a*) $V_A = 50 \times 1{\cdot}12 = 56$ ft. per sec.

This result gives $q_A = \frac{1}{2}\rho(56)^2 = 3{\cdot}73$ lb. per sq. ft. and, since $V_S = V(1 + 2a) = 1{\cdot}24V = 62$ ft. per sec., $q_S = \frac{1}{2}\rho(62)^2 = 4{\cdot}575$ lb. per sq. ft. Therefore, applying Bernoulli's equation and noting that $p_S = p$,

$$p_1 - p = q - q_A = -\,0{\cdot}755 \text{ lb. per sq. ft.},$$
$$p_2 - p = q_S - q_A = +\,0{\cdot}845 \text{ lb. per sq. ft.}$$

Hence (*b*), compared with the undisturbed pressure p, the pressure on the face of the disk is reduced by 0·755 lb. per sq. ft. = 0·145 inches of water, and the pressure on the back of the disk is increased by 0·845 lb. per sq. ft. = 0·163 inches of water.

It may be noted that according to these estimates the thrust = $\pi(p_2 - p_1) = 1{\cdot}6\ \pi = 5{\cdot}027$ lb., providing a satisfactory check considering the round numbers used.

174. Evaluation of Ideal Efficiency

Referring back to Article 171, the velocity v there introduced is now seen to amount to $2aV$. Thus $\frac{1}{2}v/V = a$ and the expression (86) of that article reduces to

$$\eta_i = \frac{1}{1 + a} \quad . \quad . \quad . \quad . \quad . \quad . \quad (88)$$

It is of interest to derive this result alternatively for the case of an actuator stationary in a head wind. Relative to the head wind, useful work is done at the rate TV. But $T = m \,.\, 2aV$. Therefore this rate $= m \,.\, 2aV^2$. The rate at which energy is supplied to the ideal actuator is equal to the rate at which the wind increases its kinetic energy. This rate $= \frac{1}{2}m[V(1 + 2a)]^2 - \frac{1}{2}mV^2 = 2mV^2 \,.\, (1 + a)a$. The ideal efficiency is therefore given by

$$\eta_i = \frac{m \,.\, 2aV^2}{2mV^2(1 + a)a}$$

which agrees with (88).

To evaluate the ideal efficiency in a given case the first step is to find a. This is obtained from a knowledge of the thrust, speed and disk area; for $T = m \,.\, 2aV$ and $m = \rho AV(1 + a)$. Expressing A in terms of the diameter D of the actuator, we have

$$T = \rho\frac{\pi}{4}D^2V^2(1 + a)2a$$

whence

$$a + a^2 = \frac{T}{\pi D^2 q} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (89)$$

The inflow factor is small compared with unity and a^2 frequently less than 10 per cent. of a. So the quadratic equation for a is best solved by trial and error, neglecting a^2 for a first approximation. Once a is found, the efficiency follows immediately by (88).

Example 87.—An airscrew, of 12 ft. 6 in. diameter, delivers 1160 thrust H.P. at a speed of 174 m.p.h. What is its ideal efficiency?

The thrust $T = 1160 \times 375/174 = 2500$ lb. $q = 77{\cdot}5$ lb. per sq. ft. $(\pi/4)D^2 = 122{\cdot}7$ sq. ft. $= A$. Working from first principles:

$$T = m \,.\, 2aV = \rho AV(1 + a) \,.\, 2aV = 4Aq(a + a^2)$$

giving $$a + a^2 = \frac{T}{4Aq} = \frac{2500}{4 \times 122{\cdot}7 \times 77{\cdot}5} = 0{\cdot}0657.$$

		(1)	(2)	(3)
Approximation:		(1)	(2)	(3)
Assumed values:	a :	0·0657	0·0614	0·0619
	a^2 :	0·0043	0·0038	0·0038
Results:	$a + a^2$:	0·0700	0·0652	0·0657

Hence $a = 0{\cdot}0619$ and $\eta_i = 1/1{\cdot}0619 = 94{\cdot}2$ per cent.

The above example is set out in full to illustrate that such problems are readily and conveniently worked without reference to derived formulæ, of which there are many in the present subject. The expressions necessary for a solution are easily constructed as required. A formal method of successive approximation is given in which the excess of each value of $a + a^2$ is subtracted from the value assumed for a to provide an ensuing approximation. Slight experience in making a judicious estimate for the first approximation shortens this work.

175. Airscrew Slipstream Calculations

Let S be the cross-sectional area of the *vena contractor*. Then, since the same mass of air crosses this area in unit time as crosses the airscrew disk and the flow is incompressible, $AV(1 + a) = SV(1 + 2a)$ or, if d is the diameter of the *vena contracta* and D that of the airscrew, $d/D = \sqrt{(S/A)}$—i.e.,

$$\frac{d}{D} = \sqrt{\frac{1 + a}{1 + 2a}}$$

This represents only a small contraction at the narrowest part of the slipstream; thus for the above example the value of d/D is $\sqrt{(1{\cdot}0657/1{\cdot}1314)} = 0{\cdot}97$; and contraction is often neglected.

The drag of a body, such as an engine nacelle or aeroplane fuselage, exposed in a slipstream is clearly increased thereby.

The effect is complicated, but a sufficient approximation for many purposes is obtained by assuming that the whole length of the body is subjected to the final velocity appropriate to the narrowest part of the slipstream. The glider drag of the body—i.e., that for $T = 0$—is then increased by the factor

$$\frac{V^2(1+2a)^2}{V^2} = 1 + 4(a + a^2) = 1 + \frac{T}{Aq} \quad . \quad . \quad (90)$$

by (89), A being the disk area of the airscrew. This is, however, an under-estimate, and a closer approximation can be obtained by substituting a smaller value for A to reflect the inefficiency of airscrew tips.

The calculations of Chapters VII and VIII can be corrected for the effect by estimating the glider drag of all aircraft parts exposed within the slipstream and increasing this part of the drag by the above factor. In a given case the increase of drag is proportional to T/q, and is therefore most marked during climbing and take-off, when the airscrew is heavily loaded and the speed is low.

Estimates framed in this way, even including a suitable reduction in A, still do not go far enough in the case of first-class aeroplanes, since slipstreams induce turbulent flow over smooth and well-shaped aircraft surfaces, increasing their drag coefficients besides the air speed. As some compensation for airscrew slipstream losses, the higher local speed increases the power of the tail unit and controls during take-off.

Example 88.—A single-engined aeroplane weighing 11,000 lb. has an airscrew 12 ft. in diameter. The glider drag of parts affected by the slipstream is 250 lb. at 150 m.p.h. Form a lower estimate of the reduction in the rate of climb at this speed, as due to the slipstream, assuming 800 thrust H.P. to be available.

$T = 800 \times 375/150 = 2000$ lb.; $q = \frac{1}{2}\rho V^2 = 57{\cdot}6$ lb. per sq. ft.; A (without reduction) $= (\pi/4)144 = 113$ sq. ft. These give $T/Aq = 0{\cdot}307$.

The minimum increase of drag due to the slipstream is thus $250 \times 0{\cdot}307 = 76{\cdot}75$ lb. and the increment of H.P. absorbed in this way is $76{\cdot}75 \times 150/375 = 30{\cdot}7$. Now 30·7 H.P. would climb the aeroplane at the rate $30{\cdot}7 \times 33{,}000/11{,}000 = 92$ ft. per min. This represents, therefore, a lower estimate of the loss in rate of climb incurred (it would probably amount to some 7 per cent.).

THE AIRSCREW

176. Thrust-Grading

Considering the disk of rotation of an airscrew, the area of a narrow annulus of mean radius r and width Δr is $2\pi r \times \Delta r$. Let t be the thrust per unit area of the disk at radius r. Then the thrust of the annulus is $2\pi r \times \Delta r \times t$. The *thrust-grading* of an airscrew disk at radius r is defined as the thrust of a narrow annulus of that radius per unit of its width, and is therefore $2\pi rt$. It will be denoted by T_0'.

With an ideal airscrew t is uniform over the disk. Thus the thrust-grading is proportional to r and can be represented by the triangular diagram (*a*) of Fig. 101. The area of any such diagram is proportional to the total thrust T. In the present case, for instance, if the diameter of the disk is D the maximum value of T_0' is $2\pi \cdot \frac{1}{2}D \cdot t$ and the area of the diagram is $\frac{1}{2} \cdot \frac{1}{2}D \cdot 2\pi \cdot \frac{1}{2}D \cdot t = (\pi D^2/4) \times t$, which evidently $= T$.

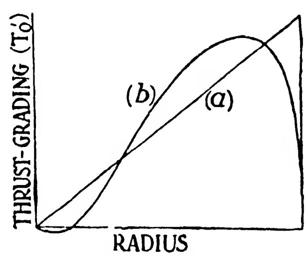


FIG. 101.—THRUST-GRADING FOR (*a*) IDEAL PROPELLER, (*b*) PRACTICAL AIRSCREW.

The perfect triangular thrust-grading diagram cannot be realised with a practical airscrew, since flow is obstructed at inner radii and fails to be accelerated backward in the desired manner at outer radii. Inner and outer annuli consequently produce a much-diminished thrust, becoming negative in the former case, and these losses require to be compensated by increasing the thrust of intermediate annuli. The thrust-grading diagram is accordingly modified for a practical airscrew from (*a*) to (*b*) of Fig. 101.

The two diagrams (*a*) and (*b*) enclose equal areas and so represent equal thrusts on the same diameter, but equal ideal efficiencies are not implied because departure from the triangular form involves loss of efficiency. The loss arises mainly at outer radii and is reduced by increasing the number of blades.

The inflow factor of a real airscrew depends on the radius. If T_0' is the thrust-grading at radius r, the mass of air per second crossing a narrow annulus of this radius per unit of its width is $\rho \cdot 2\pi r \cdot V(1 + a)$; the velocity eventually added is $2aV$; whence

$$T_0' = 8\pi rq(a + a^2) \quad . \quad . \quad . \quad . \quad (91)$$

The thrust is transmitted equally to the boss by the blades. If their number is B, then the thrust-grading diagram with its ordinates reduced in the ratio $1/B$ applies to each blade separately. The thrust-grading of each blade will be denoted by T', so that $T_0' = BT'$. It is important to note that T_0' in (91), as elsewhere, denotes the thrust-grading for all the blades added together.

177. Velocity and Force Diagram for a Blade Section

Consider an airscrew which rotates at n revolutions per second and propels an aircraft at the velocity V feet per second. With a certain reservation, the resultant velocity relative to the wind of a blade section at radius r is compounded of a translational speed of $V(1 + a)$ and a circumferential speed of $2\pi rn$. The reservation is that a real airscrew causes the slipstream to spin slowly, reducing the latter component of relative velocity. The spin sometimes requires to be determined but its effects are often small, and it will be neglected throughout this chapter.

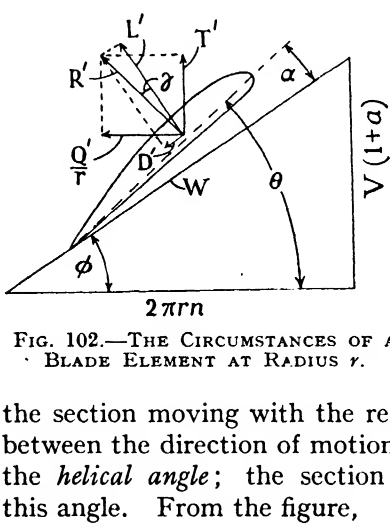


FIG. 102.—THE CIRCUMSTANCES OF A BLADE ELEMENT AT RADIUS r.

Fig. 102 gives the velocity diagram in these circumstances, the section moving with the resultant velocity W. The angle ϕ between the direction of motion and the disk of rotation is called the *helical angle*; the section traces out a screw-path having this angle. From the figure,

$$\tan\phi = \frac{V(1 + a)}{2\pi rn} \quad . \quad . \quad . \quad . \quad . \quad . \quad (92)$$

Thus ϕ is large for a section near the root of a blade, but much smaller for a section near its tip. Consider for illustration the case: $n = 25$ and $V(1 + a) = 100\pi$. At a radius of 1 foot, $\tan\phi = 100\pi/50\pi$—i.e., $\phi = 63\frac{1}{2}°$, nearly; at a radius of 3 feet $\tan\phi = 2/3$—i.e., $\phi = 33{\cdot}7°$; whilst at a radius of 6 feet ϕ is less than $18\frac{1}{2}°$.

Similarly, the resultant velocity is little greater than the translational speed at small radii, but it is much greater at large radii. In the above example, where the relative translational velocity is 314 feet per second, the resultant velocity is only $314/\sin 63\frac{1}{2}^\circ = 351$ feet per second at a radius of 1 foot, but it is $314/\sin 18\frac{1}{2}^\circ = 990$ feet per second, nearly, at a radius of 6 feet.

The blade is required to yield a large thrust with as little resistance to motion as possible. It is therefore formed of an aerofoil, twisted to take account of the variation of ϕ (Fig. 103). The section at radius r, shown in Fig. 102, is set at a suitable angle of incidence α to the direction of motion.

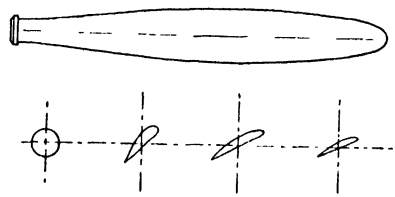


FIG. 103.—SECTIONS OF AN AIRSCREW BLADE.

The local lift L' and drag D' per unit radius that are appropriate to the aerofoil section, its incidence and its resultant velocity are also marked; L' is perpendicular to W and D' is parallel thereto. It is sometimes convenient to deal with the resultant aerodynamic force per unit radius, which is denoted by R' and is inclined backward from L' by the angle γ, given by $\tan \gamma = D'/L'$. Thus R' is inclined to the direction of thrust by the angle $\phi + \gamma$.

The thrust-grading T' is marked in the figure. T' is equal to the resolved part of L' less the resolved part of D'. If L' and D' are resolved parallel to the disk of rotation, a force per unit length results which, when multiplied by r, gives the torque per unit length at that radius. Denoting this *torque-grading* by Q', the force-grading is given by Q'/r. The torque is about the airscrew axis and has to be overcome by the engine.

178. Variation of Efficiency Along the Blade

In dealing with the real airscrew, whose whole torque Q can be estimated or measured, we can write down at once an expression for the overall efficiency. If the whole thrust is T, useful work is done at the rate TV, as before. But now it is known that work must be done by the engine at the rate: torque $\times$ angular velocity—i.e., at the rate $2\pi nQ$. Thus, using the symbol η without a suffix for the overall or *actual efficiency*,

$$\eta = \frac{TV}{2\pi nQ} \quad . \quad . \quad . \quad . \quad . \quad . \quad (93)$$

It is important to note that this expression duly includes losses arising from kinetic energy gained by the atmosphere.

Just as the area of the thrust-grading diagram is proportional to the thrust, so the area of a corresponding torque-grading diagram is proportional to the torque. If these areas are known, therefore, and also V and n, the overall efficiency follows. Alternatively, it can be determined from experiment, as will be described shortly.

With the ideal airscrew the efficiency is the same at all radii. But this is not true of the real airscrew, and it is of interest to see how the efficiency of the latter varies along the blades. The local efficiency at radius r is usually called the efficiency of the blade element, and is given by

$$\eta_e = \frac{T'V}{2\pi n Q'}$$

where the thrust- and torque-grading refer to that radius.

From Fig. 102,

$$T' = R' \cos(\phi + \gamma) \quad . \quad . \quad . \quad . \quad (94)$$

$$\frac{Q'}{r} = R' \sin(\phi + \gamma),$$

whence

$$\frac{T'}{Q'} = \frac{1}{r \tan(\phi + \gamma)}$$

and

$$\eta_e = \frac{V}{2\pi r n} \cdot \frac{1}{\tan(\phi + \gamma)}.$$

Now

$$V/2\pi r n = \tan\phi/(1 + a).$$

Hence finally

$$\eta_e = \frac{1}{1 + a} \cdot \frac{\tan\phi}{\tan(\phi + \gamma)} \quad . \quad . \quad . \quad (95)$$

If the blades had no skin friction or form drag, we should have $\gamma = 0$, and the expression for the efficiency of the element would take the same form as that for the ideal efficiency. But the value of a would not be the same for all radii, and the efficiency of the whole airscrew would therefore still be less than its ideal efficiency.

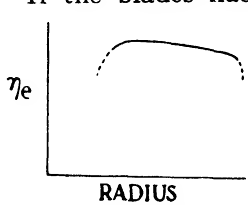


FIG. 104.—VARIATION OF EFFICIENCY ALONG BLADE.

The expression (95) includes all losses, and plotting it against the radius shows η_e to increase from the root outwards to a maximum at a certain radius and then to decrease again (Fig. 104). The maximum occurs when ϕ is approximately equal to 45°. Airscrews are therefore designed to develop a strong thrust per foot run of blade at round about the radius which makes $\phi = 45°$, so that this large part of the thrust will be produced with high efficiency.

179. Lift and Drag Coefficients on the Blades

Let a value be specified for the thrust-grading T_0' at a certain radius, which it is required the blades shall produce at given values of V, ρ and n. The inflow factor can be determined as in Article 176, formula (91), and the triangle of velocities constructed for the given radius, as in Fig. 102. Let B represent the number of blades to the airscrew. Then the value of R' required from each blade is given by

$$R' = \frac{1}{B} \cdot \frac{T_0'}{\cos(\phi + \gamma)} \quad . \quad . \quad . \quad . \quad (96)$$

and is determined once γ is known from the effective lift–drag ratio of the sections at the radius concerned.

A close approximation to R' is usually

$$R' = C_L \cdot \tfrac{1}{2}\rho W^2 c,$$

where c is the chord of the blade section at radius r, called the *blade width*. Accurately, $\sqrt{(C_L^2 + C_D^2)}$ should be substituted for C_L, but C_D^2 is always negligible in comparison with C_L^2 except near the root and tip of the blade or in case of a stalled section.

To realise the value of R' required by (96), both C_L and c can be adjusted. Let it be assumed that a suitable blade width has been decided upon, thus fixing the value the lift coefficient must have. Then the question arises as to what incidence α the blade section must be given in order to develop the required lift coefficient.

An aerofoil of similar section can be tested in a wind tunne at the same Reynolds number. But should it be tested under two- or three-dimensional conditions? As found experimentally in Article 111, the relation between lift coefficient and incidence is very different in the two cases.

Associated with this question is the equally vital one of what to take for the drag coefficient, on which γ depends. Should C_D include skin friction and form drag only, as in the two-dimensional case, or induced drag as well?

The blade section should be set at the incidence appropriate to two-dimensional flow. This does not mean that an airscrew blade works under two-dimensional conditions. But increasing the translational component of velocity from V to $V(1 + a)$, as we have done, decreases the effective incidence of the blade section in the same way as changing from two- to three-dimensional conditions decreases that of an aerofoil. The allowance is already made in the diagram of velocities, and cause for further increase of the two-dimensional incidence arises only from the slow spin of the slipstream, which we are neglecting for simplicity.

Similarly, no induced drag should be included in C_D. Waste of power from this cause does occur in an airscrew, but it is precisely the waste of which account is taken in the theory of ideal propulsion. For the airscrew blade at radius r, the loss is already represented in the calculation of η_e by the reducing factor $1/(1 + a)$.

It follows that, away from the root and tip of the blade, and provided the section is not stalled, the value to be assumed for the drag coefficient will be small, making γ small. It is usually possible to assume $\gamma = 1°$, subject to a minimum value for C_D (0·010, or rather less).

180. Blade Angle and Pitch

If an airscrew is put on a table with its axis vertical, the angle between the table and the chord-line of the section at any radius is called the *blade angle* at that radius. It is denoted by θ and, from Fig. 102,

$$\theta = \phi + \alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad (97)$$

If θ is not adjustable during flight—i.e., if the blades are rigidly connected to the boss—the airscrew is said to be of fixed pitch. This case will be discussed first, leaving that of variable pitch until later.

We may suppose that we have designed an airscrew of a certain diameter so as to be especially favourable under top speed conditions in straight level flight, represented by particular values of the aircraft speed, V feet per second, and the airscrew speed, n revolutions per second. At a sufficient number of radii, we have found the helical angles ϕ, arrived at the necessary incidences α, and so laid down the blade angles θ, to which the airscrew has been made.

Under the designed conditions, the airscrew has a certain advance per revolution—viz., V/n feet—which may be vaguely called pitch. Working at this pitch the airscrew will always be equally satisfactory, however V and n may change. But changing the conditions of working in such a way that V/n remains constant, though experimentally interesting, does not reflect the varying conditions of flight. Changing to maximum climb, for example, will cause both V and n to decrease, but V far more than n. Thus at any radius ϕ decreases and, θ remaining constant, the incidence α increases; α may increase so much as to stall the blade. The advance per revolution—that is to say, the working pitch—has become very different from the designed value. Let us go to the other extreme and imagine a fast power glide. V will have increased greatly, but not n; the engine will have had to be throttled to prevent it from rotating at a speed greater than that for which it is designed. Thus the pitch is now greatly increased. Hence it will be seen that pitch, in the above sense, is a rather indefinite term.

It is useful to define a pitch which conveys a definite picture to the mind of what the airscrew concerned looks like, enabling one of stated pitch to be selected on inspection from amongst others. This is called the *geometrical pitch*, and is most easily understood with reference to the fast glide. Let the axial advance per revolution be increased until $\phi = \theta$. Then $\alpha = 0$, but this does not imply zero thrust; the blade sections are cambered and will still have a lift coefficient at zero incidence. Owing to variation of the inflow factor along the blade, ϕ cannot be made equal to θ at all radii simultaneously. We therefore decide to make $\alpha = 0$ at some chosen radius, either 0·7 or 0·75 of the maximum radius. The value of V/n for this condition is the geometrical pitch required. The inflow factor will be very small for this condition. Neglecting it gives, denoting the geometrical pitch by P, $\tan\theta = \tan\phi = V/2\pi rn$—i.e.,

$$P = V/n = 2\pi r \,.\, \tan\theta \quad . \quad . \quad . \quad . \quad (98)$$

and θ can be measured on the actual airscrew by use of a protractor.

This pitch has little aerodynamical significance because the camber of the sections varies from one airscrew to another. It is found, however, that every one design of airscrew develops zero thrust at a particular value of V/nD, no matter what V, n or D (the diameter) may be. This value of V/nD, unique to that particular design, is called the *experimental mean pitch.* The quantity V/nD is non-dimensional, and the different pitch P is also preferably expressed non-dimensionally in the form P/D in order to allow for variation of diameter in a specified design.

Fig. 105 illustrates the section at a certain radius of a fixed pitch airscrew blade under various working conditions. The

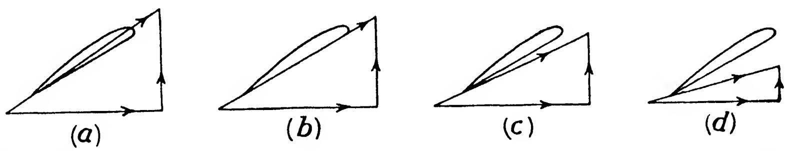


FIG. 105.—INCIDENCE OF A BLADE SECTION OF A FIXED PITCH AIRSCREW: (*a*) AT EXPERIMENTAL MEAN PITCH (ZERO THRUST), (*b*) GEOMETRICAL PITCH (POWER GLIDE), (*c*) NORMAL AIRCRAFT SPEED (MAXIMUM EFFICIENCY), (*d*) DURING CLIMB AT MAXIMUM ANGLE.

example is a flat-backed airscrew, in which incidence and blade angle are specified by the flat undersurface of the section. θ is constant, but α increases from a negative to a large positive value as the helical angle ϕ diminishes.

When working at its experimental mean pitch, an airscrew cleaves through the air without giving it backward momentum. Even at the pitch P there will be comparatively little backward momentum if the blade sections are only gently cambered. To develop a large thrust, the airscrew must advance a considerably less distance per revolution. The decrease of such axial advance is sometimes called the *slip*, and is usually expressed non-dimensionally in terms of P. The effect of slip is to decrease the helical angle ϕ and so increase the incidence of the blade sections, whence the greater thrust is obtained. But persisting in the adjustment eventually stalls the blade sections. No additional thrust then results, and increase of torque, due to the large form drag of the sections, leads to a low efficiency.

181. The Complete Airscrew

An airscrew design can be tested by means of a model in a wind tunnel. The model airscrew is best driven by an electric motor situated directly behind it and enclosed in a fairing, which should not be unreasonably large compared with the airscrew.

For such reasons $2\frac{1}{2}$ feet is found to be a suitable minimum diameter for the airscrew, indicating a 4-foot tunnel. However, qualitative results can be obtained with smaller models.

The suspension from the balance usually enables both the thrust and the torque to be measured directly. The suspending wires may pass through clearing holes in the fairing and the latter be supported independently from the tunnel walls. Interference between the airscrew and the fairing still gives trouble, however, unless the latter is small and situated well downstream. The rotational speed of the airscrew is conveniently determined by use of the stroboscope. Only a limited number of rotational speeds can then be tested, but the tunnel speed can always be varied to secure a continuous change of V/n.

Measurements of the thrust T and torque Q are reduced to non-dimensional coefficients by the formulæ

$$k_T = \frac{\text{Thrust}}{\rho n^2 D^4}, \qquad k_Q = \frac{\text{Torque}}{\rho n^2 D^5} \quad . \quad . \quad . \quad (99)$$

The units are: T, lb.; Q, lb. feet; ρ, slug per cubic foot; n, revolutions per second; D, feet. These coefficients are evaluated for a sequence of values of V/nD, a quantity of such common occurrence that it is given for brevity the symbol J. For any particular design of airscrew working at any particular value of J, the thrust and torque coefficients have constant values, whatever the values of V, n and D, subject to scale effects and provided tip speeds are not excessively high. Thus curves giving k_T and k_Q plotted against J are representative of all airscrews of the given design.

Typical curves of this kind are given in Fig. 106. There is also added a curve showing the variation with J of the efficiency, which is simply related to the coefficients as follows. We have

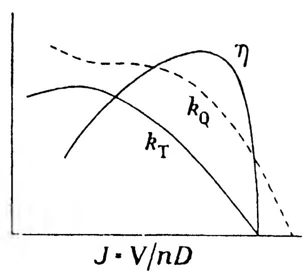


FIG. 106.—PERFORMANCE CURVES FOR A COMPLETE AIRSCREW.

$$\eta = \frac{TV}{2\pi n Q} = \frac{k_T \rho n^2 D^4 \,.\, V}{2\pi n \,.\, k_Q \rho n^2 D^5}$$

$$= \frac{k_T}{k_Q} \cdot \frac{J}{2\pi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (100)$$

Various characteristics of the curves will be noticed. The torque is not zero when the thrust is zero. But, if J were much further increased, leading to a considerable negative thrust, the torque would change sign—i.e., the airscrew would begin to work as a windmill. In this state an airscrew acts as an air-brake. The bend occurring in the curve for k_T when a small J is reached indicates the stalling of the blade sections, already described. Maximum efficiency occurs at a reasonable slip and can be arranged to correspond with the top speed of the aircraft, or a somewhat less speed if preferable. But a greatly reduced forward speed, as for take-off or climbing, then involves considerable loss of efficiency owing to the low value of J. The maximum efficiency is a good deal less than the ideal efficiency under the same conditions, but it is nevertheless remarkably high considering the disabilities to which an airscrew is liable.

Example 89.—A model airscrew, 3 ft. in diameter, tested in a wind tunnel working at 120 ft. per sec., is found to develop a thrust coefficient of 0·11 and a torque coefficient of 0·022 at 2400 r.p.m. What is its efficiency?

$$J = \frac{V}{nD} = \frac{120}{(2400 \div 60) \times 3} = 1,$$

whence the expression (100) gives

$$\eta = \frac{0{\cdot}11}{0{\cdot}022} \times \frac{1}{2\pi} = 0{\cdot}796, \text{ or } 79{\cdot}6 \text{ per cent.}$$

182. Choice of Pitch

In view of the simple relationship (100), only two of the three curves need be retained, and it is usually most convenient to have those for k_Q and η. The brake horse-power of the engine is then given by

$$\begin{aligned} \text{B.H.P.} &= 2\pi nQ/550 \\ &= 2\pi\,.\,k_Q\rho n^3D^5/550 \quad . \quad . \quad . \quad (101) \end{aligned}$$

whilst the thrust horse-power made available is given by

$$\text{T.H.P.} = \eta \times \text{B.H.P.} \quad . \quad . \quad . \quad . \quad . \quad (102)$$

Fig. 107 illustrates the k_Q- and η-curves for a family of three airscrews, all of the same design except for an increasing pitch.

It will be noticed that the maximum efficiency increases considerably with pitch within the range illustrated.

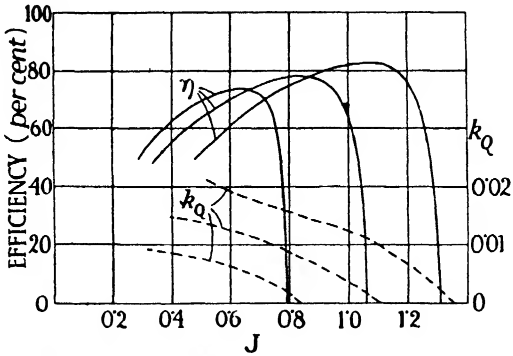


FIG. 107.—VARIATION OF AIRSCREW CHARACTERISTICS WITH PITCH.

The reason for such initial increase can be understood readily. Mention has already been made (Article 178) of the fact that the maximum efficiency along an airscrew blade occurs close to the radius for which $\phi = 45°$. Accepting this value so that $\tan \phi = 1$, and neglecting the inflow factor, leads to the approximate result that $V = 2\pi rn$ at the radius for maximum efficiency, and hence

$$J = \frac{V}{nD} = \frac{2\pi rn \cdot \tan \phi}{nD} = \frac{2\pi r}{D}$$
$$= \pi \frac{r}{\frac{1}{2}D}.$$

Now the thrust-grading diagram shows that the thrust delivered by the airscrew is heavy at about two-thirds of the maximum radius but light at one-third of the way along the blades. The above result then shows that if the airscrew works at $J = 2\pi/3$ a large part of the thrust will be delivered under the condition of maximum local efficiency, but if it works at $J = \pi/3$ only a small part of the thrust will be delivered with this advantage, the radius for maximum local efficiency being too small in the second case.

A large working value of J is much more easily achieved by increasing V than by reducing n. Thus the high speeds of modern aircraft account in large part for the high efficiencies of modern airscrews compared with those of fifteen years ago, a result that is evident qualitatively from the simple theory of ideal propulsion.

183. Variable Pitch

The essential difficulty in the way of employing high pitch at top speed is that the airscrew stalls very badly at take-off and may even stall at climb. The difficulty is removed in the variable pitch airscrew, whose pitch can be suitably reduced, when required, by turning the blades so as to reduce the blade angle θ. The effect is very similar to that produced by decreasing the pitch in the family of airscrews illustrated in Fig. 107, with reference to which the action will accordingly be described.

It must first be appreciated that the B.H.P. of an aero engine depends acutely on its rotational speed and that, at full throttle, this speed is determined by the airscrew. If the torque of the airscrew is excessive, the engine is slowed down, and fails, perhaps by a wide margin, to develop its full B.H.P. The equation (101) states that k_Q must have a precisely correct value for such failure to be avoided. The first point in designing an airscrew is to ensure that k_Q will be correct in this sense at the appropriate value of J.

We will assume the above condition to have been fulfilled at the top speed of the aircraft and with the airscrew at its largest pitch, the value of J corresponding to maximum efficiency. Without changing pitch, let the aircraft speed be halved; k_Q is greatly increased, n falls away, and much of the B.H.P. of which the engine is capable is lost. Even if the efficiency were maintained, therefore, the T.H.P. would be reduced in the same proportion. But actually the efficiency drops very considerably, and the T.H.P. is all the more reduced thereby.

If, however, the pitch is suitably reduced, the original value of k_Q is realised at one-half the original value of J; the full value of n is again reached, and consequently the full B.H.P. This alone represents a great gain. But in addition, as reference to Fig. 107 shows, the efficiency is substantially increased, though not to its original value. Therefore, referring to the equation (102), both the factors on the right are increased, and the T.H.P. increases on both counts.

It is readily possible for the take-off thrust of a power unit to be doubled in this way. An example below illustrates conservatively the effects on climb. Use of variable pitch greatly enlarges the scope of twin-engined aeroplanes by enabling a high ceiling to be maintained on one engine. Another circumstance in which it shows to advantage is when the aeroplane will normally fly for long periods at very different altitudes.

184. Examples

The following examples frequently lead on from one another in order to save useless repetition. It will be clear that each could be worked *ab initio.*

Example 90.—A 3-bladed airscrew rotates at 1350 r.p.m. and propels an aircraft at 225 m.p.h. Determine the helical angle at a radius of 4 ft., where the thrust-grading is 220 lb. per ft. on each blade.

$$T'_0 = 220 \times 3 = 2\pi \, . \, 4 \, . \, \rho V(1 + a) \, . \, 2aV$$

$$= 32\pi q(a + a^2).$$

$$q = \tfrac{1}{2}\rho(330)^2 = 129{\cdot}6 \text{ lb. per sq. ft.}$$

Hence $a = 0{\cdot}048$, approximately, and $V(1 + a) = 346$ ft. per sec.

$$2\pi rn = 8\pi \times 22\tfrac{1}{2} = 565 \text{ ft. per sec.}$$

These give $\tan \phi = V(1 + a)/2\pi rn = 346/565 = 0{\cdot}612.$

Therefore $\phi = 31{\cdot}45°.$

Example 91.—If the two-dimensional lift–drag ratio of the blade sections is 50, determine the local efficiency of the above airscrew at 4 ft. radius.

$$\tan \gamma = 1/50 = 0{\cdot}02\text{—i.e., } \gamma = 1{\cdot}15°.$$

This gives $\phi + \gamma = 32{\cdot}6°$ and $\tan(\phi + \gamma) = 0{\cdot}640.$

Hence
$$\eta_e = \frac{1}{1 + a} \cdot \frac{\tan \phi}{\tan(\phi + \gamma)} = \frac{1}{1{\cdot}048} \cdot \frac{0{\cdot}612}{0{\cdot}640}$$

$$= 91{\cdot}2 \text{ per cent.}$$

Example 92.—If the local lift coefficient is to be 0·80, find the necessary blade width (c) of the above airscrew at 4 ft. radius.

The resultant velocity $W = V(1 + a) \div \sin \phi = 346/0{\cdot}522 =$ 663 ft. per sec., giving $\tfrac{1}{2}\rho W^2 = 523$ lb. per sq. ft. Then by the formula (96) and the question data,

$$C_L \cos(\phi + \gamma) \, . \, \tfrac{1}{2}\rho W^2 c = 220$$

i.e.,
$$C_L c = \frac{220}{0{\cdot}8425 \times 523} = 0{\cdot}5.$$

Hence $c = 0{\cdot}5/0{\cdot}8 = 0{\cdot}625$ ft. $= 7\tfrac{1}{2}$ in.

Example 93.—The blade sections concerned in the above examples have an angle of no lift of -2°. What is the blade angle at 4 ft. radius?

Two-dimensional conditions are to be assumed, whence, from Article 106, C_L increases at the rate 0·094 per degree. Thus the increase of incidence from the angle of no lift $= 0{\cdot}8/0{\cdot}094 = 8{\cdot}5^\circ$. The incidence α is therefore $8{\cdot}5 - 2 = 6{\cdot}5^\circ$.

This gives for the blade angle

$$\theta = \phi + \alpha = 31{\cdot}45^\circ + 6{\cdot}5^\circ = 38^\circ, \text{ nearly.}$$

Example 94.—What is the pitch of the above airscrew?

It may be assumed that 4 ft. is about 0·7 of the maximum radius, so that the pitch P can be assessed from the foregoing results. This gives at once

$$P = 2\pi r \tan\theta = 8\pi \tan 37{\cdot}95^\circ$$
$$= 19 \text{ ft. } 7 \text{ in.}$$

Example 95.—The airscrew of an aeroplane is 12 ft. in diameter and has the following characteristics:

J . .	0·95	1·1	1·3	1·4
k_Q . .	0·021	0·020	0·017	0·014
η . .	0·75	0·795	0·83	0·83

At what aircraft speed will the engine develop 900 B.H.P. at 1200 airscrew r.p.m.?

Since $n = 20$ r.p.s., and $Q = k_Q \rho n^2 D^5$, the general expression

$$\text{B.H.P.} = 2\pi n Q/550$$

gives $$900 = 2\pi \,.\, k_Q (20)^3 \rho (12)^5/550$$

whence $$k_Q = 0{\cdot}0166.$$

Plotting k_Q against J from the data in the question shows the corresponding value of J to be 1·315. Since $J = V/nD$,

$$V = JnD = 1{\cdot}315 \times 20 \times 12$$
$$= 316 \text{ ft. per sec. or } 215 \text{ m.p.h.}$$

Example 96.—Referring to Example 95 and further assuming that the B.H.P. delivered by the engine is proportional to $n^{3/4}$, what will be the rotational speed when $J = 0{\cdot}95$?

Since we can write B.H.P. $= kn^{3/4}$ and B.H.P. $= 900$ when $n = 20$, the constant coefficient $k = 900/(20)^{3/4} = 95{\cdot}1$, giving for all permissible speeds

$$\text{B.H.P.} = 95{\cdot}1 \times n^{3/4}.$$

for the power of which the engine is capable. But the power required by the airscrew is

$$\text{B.H.P.} = 2\pi n \, . \, k_Q \rho n^2 D^5/550.$$

Hence, since $\rho D^5 = 592$,

$$\frac{2\pi \times 592}{550} k_Q n^3 = 95{\cdot}1 \times n^{3/4}$$

i.e.,

$$n^{9/4} = 14{\cdot}07/k_Q.$$

At $J = 0{\cdot}95$, $k_Q = 0{\cdot}021$ and the formula gives $n = 18{\cdot}2$ r.p.s.

Example 97.—Supposing the aeroplane to climb at $J = 0{\cdot}95$, what would be the speed and the T.H.P. available?

As in Example 95, the speed

$$V = JnD = 0{\cdot}95 \times 18{\cdot}2 \times 12$$
$$= 207 \text{ ft. per sec. or } 141 \text{ m.p.h.}$$

This speed is not required in order to evaluate the T.H.P. For

$$\text{B.H.P.} = 95{\cdot}1 \times n^{3/4} = 838$$

on substituting 18·2 for n. Hence, from the data in Example 95,

$$\text{T.H.P.} = \eta \times \text{B.H.P.} = 0{\cdot}75 \times 838$$
$$= 629.$$

It will be noted that the T.H.P. available at 215 m.p.h. when $n = 20$ r.p.s. is about 747. A large proportion of the difference of 118 T.H.P. would be recovered by employing variable pitch.

Chapter X

STABLE FLIGHT

185. The Inherently Stable Aeroplane

An aeroplane is equipped with four flying controls : the engine throttle, the elevators, the ailerons and the rudder. Simultaneous settings of these can be found to give equilibrium in respect of all the forces and moments acting externally upon the aeroplane in some chosen form of steady flight. Imagine the settings to be made and the flight started, and then let the aeroplane be left to its own devices. Will it continue to move in the same way? If it is disturbed, by a gust for instance, will it return to the original form of flight?

Some attention has already been given to this matter in connexion with longitudinal trim (Article 144). It was found that in addition to equilibrium in respect of the forces and pitching moments, an aeroplane must possess a statical stability in regard to change of incidence. No indication was obtainable, however, as to the strength of the statical stability required. This difficulty cannot be met by providing a large margin—a device often employed in engineering to cover uncertainties and emergencies—because the aeroplane would become restive and uncomfortable, a butt for every trivial vagary of the atmosphere, fatiguing to fly in and obstinate to control.

Sudden change of incidence is only one of many disturbances the aeroplane may suffer; it can be displaced in other ways whose statical aspect is of much less interest. Connexions arise, indeed, in which too much stability of this kind actually prevents unaided recovery.

In considering the response of an aeroplane, therefore, it is advisable to resist analogies with a weathercock. Both have a pivot about which to rotate—the centre of gravity in the one case and the hinge in the other—and the initial movement of each is to turn into a changed relative wind. But there the likeness largely ends, and it does not carry us far.

An aeroplane can be so designed that, in face of reasonable changes of wind such as commonly occur, it will continually renew the form of flight for which its controls are set without further operation of the controls, by either a pilot or an automatic device. It is then said to be inherently stable.

When disturbed, it interrupts its course of more or less steady flight to execute a spontaneous manoeuvre. Though starting as a turn into the wind, the disturbed motion may well become complicated; again, it may last for only a few seconds or for a minute or two; moreover, it requires space. Quickly or slowly, however, the manoeuvre dies away and the original form of flight is recovered. This is not to say that the initial flight path is regained, nor necessarily the same direction of motion. Suppose the aeroplane to be flying due north at a certain speed and altitude when disturbed. After recovery it will be in straight level flight at the same speed but not at precisely the same altitude nor exactly, perhaps, in the northerly direction. The essential function of the stability is to recover the original speed of flight.

The inherent stability of the aeroplane is dynamical, and the foregoing description gives a preliminary idea of what this term implies. To achieve such stability careful attention must be directed both to the shaping of external form and to the distribution of mass. As already suggested, there may be little margin for error. The process—unless experience pure and simple is relied upon—is partly analytical, partly experimental, and on both sides soon becomes difficult and complicated. Our aim in this book will be to illustrate further the meaning and significance of dynamical stability by considering in turn some of its outstanding aspects.

186. Experiments with Model Gliders

To offset its difficulties, the subject provides a fascinating field for unique experiments which every student can explore. Many interesting effects, which make a considerable call on the imagination to visualise from a description, are readily demonstrable in a large room by means of little gliders made of paper or mica. The most striking results are obtained on the border-line between stability and instability, the only just stable models making persistent but prolonged efforts to regain the forms of flight on which they were launched.

The models should be very small, and the simplest construction is preferable if only to ensure a light weight and slow speed, and to make damage of little account. Proportions and loading for desired effects can be calculated beforehand by mathematical skill, but much can be achieved in its absence by a little general knowledge and a great deal of patience. A few suggestions are made in place below. A model should not be called upon to contend with initial conditions that are quite unsuited to its shape and loading. Until experience is gained, therefore, it should be launched by means of some simple mechanical contrivance.

187. Longitudinal and Lateral Stability

The stability of an aeroplane in straight flight, whether the flight path is level or inclined to the horizon, is divisible into two separate parts. No error arises, therefore, from considering these in turn.

The first part, called the *longitudinal stability*, is brought into play by a purely symmetric disturbance. If the cause of trouble is a gust, it may be head-on, tail-on, vertical, or inclined in the vertical plane of symmetry, but it must not have a lateral component nor affect one side of the aeroplane more than the other. If the disturbance arises from a displacement and return of the controls, only the engine throttle and the elevators, and not the ailerons or rudder, may be involved. The ensuing motion is concerned with changes that occur in the following: incidence; inclination of the flight path to the horizon; speed; lift, drag and thrust; angular velocity (originally zero) about the transverse axis through the centre of gravity of the aeroplane—i.e., pitching—and the pitching moment (see Fig. 39).

The remaining part of the stability of an aeroplane is called its *lateral stability*. It is brought into play by an asymmetric disturbance such as arises from a gust affecting one wing more than the other, or a flick of the ailerons. It involves roll (inclination of the span to the horizon) and rolling; yaw (horizontal inclination of the longitudinal axis of the aircraft to the direction of motion) and yawing; side-slipping (motion parallel to the span); rolling moment; yawing moment and cross-wind force (Art. 38). An attempt is sometimes made to separate off part of lateral stability and distinguish it as 'directional stability', but this additional step is unjustifiable and may lead to confusion.

LONGITUDINAL STABILITY

188. The Short Oscillation

Imagine that an aeroplane in straight level flight at velocity V runs into an upgust of relatively small velocity v. Incidence is immediately increased by the amount v/V, and the tail-plane quickly pitches the aeroplane to an incidence which is too small. It will then proceed to pitch the aeroplane back, but again possibly overshoot the mark, and so on. There are the ingredients here for an oscillatory motion of the aeroplane around the transverse axis through its centre of gravity, and the question arising is whether this oscillation is suitably damped. Will the amplitude decrease, whether quickly or slowly, or will it increase in course of time?

The above component of the unsteady motion induced by a symmetric disturbance is called the *short oscillation*. If it develops at full scale its period is likely to be between 3 and 5 seconds. The tail-plane has more to do than provide a righting moment; it has also to damp out the pitching set up. This task becomes arduous only at very low speeds near the stall. Increasing the tail volume ratio (Article 140) to prevent the oscillation increasing in amplitude near the stall results in very heavy damping at ordinary speeds, when the motion is so dead-beat that only a fraction of a complete oscillation occurs. This fraction leaves the aeroplane, whose incidence has suddenly been increased by an upgust during straight level flight, with its nose pointing a little downward, the pitching into the relative wind having occupied perhaps a second.

The consequence of this initial adjustment of a stable aeroplane is traced in the next article. Meanwhile, it may be noted that the short oscillation is not easy to demonstrate on a glider model. Success is most readily secured, perhaps, with a rudimentary model of an 'all-wing' aeroplane having a little 'sweep-back'. In all experiments on longitudinal stability steps must be taken, of course, to ensure sufficient immunity from lateral instability.

189. The Phugoid Oscillation

The short oscillation having produced a nose-down attitude to the ground, the aeroplane proceeds to dive and gather speed.

The increased speed generates more lift, which becomes greater than the weight. Thus arises an upward margin of vertical force which gives an upward acceleration to the descending aeroplane and prevents the dive from becoming more than a shallow one. From the trough of the shallow dive, the aeroplane climbs towards its original flight path, which it overshoots. It then begins another shallow dive, and so on.

This second oscillation of longitudinal stability is long, slow and lightly damped. Thus at about 100 m.p.h. the period may be nearly $\frac{1}{2}$ minute and the aeroplane may travel $1\frac{1}{2}$ miles before the initial amplitude is decreased by 50 per cent. It is called the *phugoid oscillation*, after Lanchester, who gave the following simple explanation as applying to really low speeds. With certain assumptions, which are then justified, the oscillation is governed by an alternating exchange between the kinetic energy of the aeroplane and its potential energy—i.e., between that part of its total energy which depends upon the square of the speed and that part which depends upon height above sea-level. It is not surprising, therefore, that the variation of drag between the crest and the trough of the wave plays an important part in the damping. The up and down oscillation of the centre of gravity of the aeroplane and the variation of its speed are accompanied by a slow angular oscillation about the transverse axis through the centre of gravity.

The phugoid oscillation is very easy to demonstrate in a model. The period will be about $V/7$ seconds—i.e., the length of a complete oscillation will be about $V^2/7$ feet—so the model should be lightly loaded and glide at a large lift coefficient in order to make V small. Other advisable features are a far forward position of the centre of gravity and as much moment of inertia about the transverse axis as is compatible with the light weight. If in addition the tail-plane is kept rather small, it should be possible to make the oscillation increase noticeably in amplitude. These features are not desirable in an aeroplane, of course.

Another kind of disturbed motion takes the place of the phugoid oscillation at high speeds. However, the foregoing sufficiently illustrates the nature of the longitudinal stability of an aeroplane, and we proceed to enquire briefly into the character of its lateral stability.

LATERAL STABILITY

190. Rolling Moment due to Rolling

In turning to lateral stability we begin by supposing that an asymmetric disturbance produces rolling. Experiments described in Article 129 show that rolling during flight at small or moderate incidences is quickly damped out. The stable rolling moment is readily estimated for wings of rectangular plan-form.

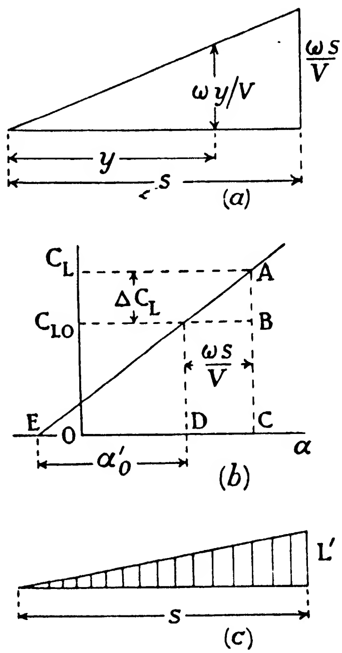


FIG. 108.—ROLLING: (*a*) INCREASE OF INCIDENCE and (*c*) INCREASE OF LIFT ALONG DESCENDING WING.

Fig. 108 shows at (*a*), from the article quoted, the increase of incidence along the descending wing of a monoplane of span $2s$, whose speed is V and which has an angular velocity of roll ω. The increase reaches a maximum of $\omega s/V$ radians at the wing-tip. If α'_0 is the undisturbed incidence (or that at the centre of span) measured from the angle of zero lift in radians, and the corresponding normal lift coefficient is C_{L0}, the increase ΔC_L of the lift coefficient near the wing-tip is, from Fig. 108 (*b*),

$$\frac{\omega s}{V} \times \frac{1}{\alpha'_0} \times C_{L0}$$

since $AB/BC = CD/DE$.

The additional lift L' per foot run of span in the region of the wing-tip is $\Delta C_L qc$, where c is the constant chord. Therefore

$$L' = \frac{\omega s}{V\alpha'_0} \times C_{L0}qc \quad . \quad . \quad . \quad . \quad . \quad (\text{i})$$

But if W is the weight of the aeroplane, in straight level flight $C_{L0}q(sc) = \frac{1}{2}W$. Hence

$$L' = \frac{1}{2}\frac{\omega}{V\alpha'_0}W \quad . \quad . \quad . \quad . \quad . \quad . \quad (\text{ii})$$

The additional lift of the descending wing is distributed as shown in Fig. 108 (c), giving a total of $\frac{1}{2}sL'$. Since the additional lift passes through the centroid of the triangle, the centre of pressure is at $2s/3$ from the centre of span, whence the rolling moment due to this wing is $s^2L'/3$. The rising wing has its lift reduced in a similar way and contributes an equal moment in the same direction. Thus the total rolling moment is $2s^2L'/3$ or, substituting for L' from (ii) and remembering that s is the semi-span,

$$\text{Rolling moment due to rolling} = \frac{1}{12}\frac{\omega}{V\alpha'_0} W\,(\text{span})^2 \quad . \quad (103)$$

The large magnitude of the moment is illustrated in the following example and very quickly reduces the initial rolling to a small angular velocity.

Example 98.—A monoplane weighing 5 tons and having rectangular wings 56 ft. in span begins to roll with an angular velocity of 0·2 radian per sec. when in flight at a speed of 225 m.p.h. The flight incidence is $\frac{1}{2}°$ and the angle of zero lift $-$ 2·1°. Estimate the initial rolling moment.

$V = 330$ ft. per sec. and $\alpha'_0 = 2{\cdot}6° = 0{\cdot}0454$ radian. Thus (103) gives for the rolling moment

$$\frac{1}{12} \times \frac{0{\cdot}2}{330 \times 0{\cdot}0454} \times 11{,}200 \times (56)^2 = 39{,}070 \text{ lb. ft.}$$

An alternative expression can be obtained by introducing the symbol k for the slope of the lift curve (Article 113), so that $C_{L0} = k\alpha'_0$, and substituting in (i). We then have for this rolling moment

$$\frac{2s^2L'}{3} = \frac{(\text{span})^2}{6} \times \frac{\omega s}{V} \times kqc$$

$$= \frac{k \times S \times (\text{span})^2}{24} \,.\, \omega\rho V \quad . \quad . \quad (104)$$

since the wing area $S = 2sc$ and $q = \frac{1}{2}\rho V^2$. If ω is expressed in radians per second, however, the value of k obtained from (44) must be increased in the ratio $180/\pi$ to give the increase of C_L per radian.

Example 99.—Re-estimate the rolling moment of Example 98, assuming that the incidences there given are unknown but that the wing-loading is 25 lb. per sq. ft.

To solve the question in this rather more practical form, we have to find the aspect ratio A in order to evaluate k from (44).

The total wing area $S = 5 \times 2240/25 = 448$ sq. ft. and, since the span $= 56$ ft., the mean chord $= 448/56 = 8$ ft., giving $A = 7$.

Then (44) leads to, with 2 substituted for 1·9,

$$k = \frac{0{\cdot}094 \times 7}{9} \times \frac{180}{\pi} = 4{\cdot}19$$

and the rolling moment is

$$\frac{4{\cdot}19 \times 448 \times (56)^2}{24} \times \frac{330}{5 \times 420} = 38{,}540 \text{ lb. ft.}$$

The above formulæ are restricted to rectangular plan-forms; the rolling moments for practical plan-forms of other shapes are less. The formulæ are also based on the assumption that the rolling wings are nowhere stalled; the unstable moments arising at large incidences have been discussed in Articles 129–30.

191. The Lateral Dihedral

The foregoing moment, which stops rolling at ordinary flight incidences in a fraction of a second, leaves the aeroplane rolled or banked and, as it is still flying nearly straight, a sideslip ensues, during which the aeroplane is righted by the lateral dihedral, as follows.

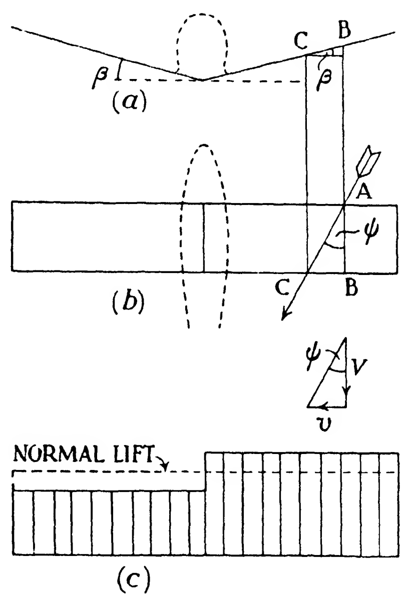


FIG. 109.—MONOPLANE WITH LATERAL DIHEDRAL. AT (c) IS SHOWN THE REDISTRIBUTION OF LIFT ALONG THE SPAN DUE TO A SIDESLIP TO THE RIGHT.

Fig. 109 (a) represents the back view of an aeroplane flying into the page. Each wing is inclined upward to the line joining the wing-tips by an angle β, and 2β is called in this book the *dihedral angle* (the dihedral angle is often defined alternatively by β).

Fig. 109 (b) is a plan view of the aeroplane, rectangular wings being assumed for simplicity. AB represents any chord line and, in the absence of sideslip, the relative wind V passes over the leading edge at A and the trailing edge at B. But if the aeroplane sideslips towards the right so that the relative wind has a component v from that direction, the relative wind makes an angle ψ with AB and passes over the trailing edge at C. Both β and ψ are assumed to be small angles, so that sines and tans are sensibly the same as circular measures and cosines differ little from unity.

Referring to the figure, C is lower than B by BC $\times\ \beta$. But BC $=$ AC $\times\ \psi$. Thus v produces an increase of incidence to the changed relative wind amounting to BC $\times\ \beta$/AC $=$ AC $\times\ \psi \times \beta$/AC $= \psi\beta$. But $\psi = v/V$. Therefore the increase of incidence $= \beta(v/V)$. This increase is the same all along the right-hand wing which is leading into the sideslip and, ignoring blanketing by the fuselage, there is an equal decrease of incidence along the left-hand wing. Hence a rolling moment arises in the counter-clockwise sense, equally contributed to by each wing. The description and the figure regard, for convenience, the span as horizontal, but this is unnecessary and the same moment arises at any angle of bank if the velocity of sideslip is the same.

As in the preceding article, if α'_0 is the incidence in undisturbed flight, measured from the angle of zero lift, and C_{L0} the corresponding lift coefficient, the increase ΔC_L of the wing leading into the sideslip is obtained, assuming the lift curve to be straight, from $\Delta C_L \div C_{L0} = \beta v/V \div \alpha'_0$—i.e.,

$$\Delta C_L = \frac{\beta}{\alpha'_0} \times \frac{v}{V} \times C_{L0} \quad . \quad . \quad . \quad . \quad \text{(i)}$$

If S is the total wing area of the monoplane, the increase of lift on the leading wing is $\Delta C_L q \times \frac{1}{2}S$ and there is an equal decrease on the following wing. The distribution is as indicated in Fig. 109 (c), so that the distance between the centres of pressure of the two changes of lift is the same as that between the two halves of the normal lift—i.e., $\frac{1}{2}$ $\times$ span. Hence the total rolling moment is $\frac{1}{4}\Delta C_L qS$ $\times$ span, or substituting for ΔC_L from (i) and noting that $C_{L0}qS = W$ the weight of the monoplane,

$$\text{Rolling moment due to sideslip} = \tfrac{1}{4}\frac{\beta}{\alpha'_0}\cdot\frac{v}{V}\cdot W \times \text{span} \; . \quad (105)$$

An alternative form equivalent to the expression (104) for the rolling moment due to rolling is readily obtained. Writing k as before for the slope of the lift curve, so that $C_{L0} = k\alpha'_0$ and $\Delta C_L = k\beta v/V$, gives in place of the right-hand side of (105)

$$\frac{k \times S \times \text{span}}{8} \times \beta\rho V v \quad . \quad . \quad . \quad . \qquad (106)$$

The examples below refer back to those of the preceding article, enabling a comparison to be made between the two rolling moments.

Example 100.—The monoplane of Example 98 has a dihedral angle of 6°. After a roll at a speed of 225 m.p.h. it sideslips at 33 ft. per sec. Estimate the rolling moment.

Since $\alpha'_0 = 2{\cdot}6°$ and $v/V = 33/330 = 0{\cdot}1$, (105) gives directly

$$\frac{5 \times 2240 \times 56}{4} \times \frac{3}{2{\cdot}6} \times 0{\cdot}1 = 18{,}100 \text{ lb. ft.}$$

Example 101.—As in Example 99, re-estimate the above rolling moment assuming that the incidences are unknown but that the wing-loading is 25 lb. per sq. ft.

From the example quoted, $k = 4{\cdot}19$ and, since $\beta = 0{\cdot}0524$ radian, (106) gives for the rolling moment

$$\frac{4{\cdot}19 \times 448 \times 56}{8} \times 0{\cdot}0524 \times \frac{1}{420} \times 330 \times 33 = 17{,}900 \text{ lb. ft.}$$

192. Discussion

A point of fundamental interest made clear by the formulæ (104) and (106) is that the rolling moments for a given aeroplane in particular circumstances are proportional to V, not to V^2.

The above simple calculations of rolling moment due to rolling are reliable. This may be checked by experiment, but the apparatus is a little complicated.

The above estimates of rolling moment due to sideslip, on the other hand, are too generous; blanketing of the following wing and other losses considerably reduce the moment. Fortunately, this moment is easily measured in a wind tunnel, for the effect of sideslip is reproduced by giving the model an angle of yaw = v/V. The same experiment will allow for a shaped or tapered plan-form, although such allowance is readily estimated in the present instance, as illustrated in the following example.

Example 102.—How much is the rolling moment in Example 100 reduced by substituting wings of the same span and area but which give an elliptic lift-grading curve?

The assumption is made that the changed lift on each wing in the sideslip will also be elliptically distributed. Change of lift on each wing is the same and decrease of moment is due only to the centres of pressure being nearer the centre of span. The centroid of a quadrant of an ellipse, of semi-axes a, b, is distant $(4/3\pi) \times b$ from a. Thus the centre of pressure on each wing is $0{\cdot}4244 \times$ *semi-span* from the axis. For (105) this distance is taken as $\frac{1}{2} \times$ *semi-span*. Hence the rolling moment is reduced in the ratio $0{\cdot}4244/\frac{1}{2}$, i.e., by some 15 per cent.

193. The Lateral Oscillation

We have seen that if a downward gust strikes the starboard wing, say, that wing drops, though the clock-wise rolling is fiercely opposed, and the sideslip to the right results in an anti-clockwise rolling moment by virtue of the lateral dihedral. This moment rolls the aeroplane back—rather slowly, or it would be overcome by an opposing moment due to rolling. The aeroplane passes through the span-level position; the original bank and sideslip are reversed in direction; the lateral dihedral raises in turn the left-hand wing in a clock-wise recovery; and so on.

There may ensue an oscillation in rolling combined with a ‘ tail wag ’, the whole being called the *lateral oscillation* or (popularly) the dutch roll. It is appreciably slower than the short oscillation of longitudinal stability.

The lateral oscillation is readily demonstrated with a model glider. Having given the wings an exaggerated dihedral, the area of the fin and rudder may be reduced gradually until the oscillation appears.

194. The Directional Fins

The effect of a lateral dihedral could be achieved by a large vertical fin above the wings. The fuselage, engine nacelles and propellers act in a sideslip as vertical fins located forward. Finally, the fin properly so called and the rudder hinged to it together provide a vertical fin at the back of the aeroplane.

In a sideslip all these fins contribute to cross-wind force. If the centre of pressure of the resultant cross-wind force is well behind the centre of gravity of the aeroplane, the latter is said to have strong weathercock stability. The degree of weathercock stability may be reduced or changed to weathercock instability by modifying the area of the fin and rudder.

A far forward centre of pressure—i.e., marked weathercock instability—makes the aeroplane turn away from the direction of motion with increasing rapidity. The catastrophic motion that ensues is illustrated by a model fitted with a vertical fin in front of its wings and having a larger area than that of the fin and rudder at the tail.

Consider the contrasting case of a far-back position of the centre of pressure. A sideslip to the right then produces a turning to the right; the left-hand wing lifts more strongly than the right-hand wing owing to its greater speed; the bank and therefore the sideslipping and turning all increase. The aeroplane spirals in a rather downward direction and is said to have *spiral instability*. The defect, which is seen to arise from marked weathercock stability, is readily demonstrated by means of a model glider having little lateral dihedral and a large rudder.

It appears that for inherent stability the centre of pressure of the crosswind force should not be far removed from the centre of gravity. Whether it should be located slightly in front of or behind the latter in a given case cannot be decided from elementary considerations. However, a large rudder provides an especially useful control, and for this reason some degree of spiral instability is tolerated in most aeroplanes, since it is slow to develop and easy to correct by the pilot.

Chapter XI

LOAD FACTORS ON WINGS

195. EACH structural part of an aircraft is designed to withstand several times the load, called the *normal load*, which it sustains in straight level flight. In the structural sense of the term, the 'load factor' is equal to the ratio of the load that would produce fracture to the normal load. But included in this load factor is a 'factor of safety', often amounting to 2, to cover faults of design, fabrication and material. To arrive at the strength for which a part should be designed, the factor of safety is applied to the maximum load which the part may ever be called upon actually to sustain.

Over-loadings are caused in various ways, but we are concerned only with those which may arise during flight. Even so, the subject is a wide one and more particularly of interest to structural design. The brief investigation below is accordingly restricted to the external forces and moments arising on wings.

The normal load on the wings of an aeroplane in the direction of the lift is equal to the flying weight W. If the wings exert at any time a different lift L, the ratio L/W is the *aerodynamical load factor*. However, the word 'aerodynamical' is usually omitted and, to save confusion, we note that the maximum value of L/W would be multiplied by the factor of safety in specifying the reputed strength of the wings.

196. The Pilot as a 'Safety Valve'

Aerodynamical over-loading may be steady or transient. In aeroplanes of the aerobatic class, the first kind can be severe and the second could be fantastically so but for the limited physical endurance of the pilot. An aeroplane can be designed to fly without a pilot on board, its controls being operated by radio from the ground. If such an aeroplane were directed into a terminal nose dive and its elevators raised suddenly, the wings would immediately break off; to prevent fracture they might require to be more than 50 times as strong as is necessary for steady flight. But in like circumstances with a pilot at the controls the elevators would be employed with restraint and the load factor prevented from exceeding 4 or 5. Such easing can be relied upon, since the pilot experiences in his person just the same load factor as he allows to come on to the aircraft; his head weighs upon his neck just as excessively as the body of the aeroplane weighs upon its wings; and experience shows that few pilots can tolerate an actual load factor greater than that mentioned.

It may be thought on this basis that specifying a suitable strength for an aircraft part is a simple matter; that it is only necessary to decide upon an aerodynamical load factor consistent with human endurance and double it. But the problem is more complicated, for two reasons. Some over-loads are not directly transmitted to the pilot, who remains unaware therefore of their intensity. Again, the majority of aeroplanes are not manoeuvred at the limit of the pilot's endurance; passenger-carrying aircraft, for instance, are studiedly flown with as little sensation as possible; and the strength required then depends upon uncontrollable disturbances and exigencies.

197. Steady Load Factors

Instances of steady load factors are provided by climbing or gliding and maintained turning. The first two have little interest unless the dive is very steep, and even then practical concern is with the strength of the fuselage rather than that of the wings. In climbing or gliding, the lift $L = W \cos \theta$. Thus the load factor on the wings, the ratio of the actual lift to that in straight level flight, comes to $L/W = \cos \theta$ and is fractional and commonly little less than unity.

Level circling.—If ϕ is the angle of bank, the requirement of a level path in circling is that $L \cos \phi = W$, giving $L/W = 1/\cos \phi$. This factor may be fairly severe but is limited by the fact that making such a turn without change of speed increases the induced drag in the ratio $1/\cos^2 \phi$, absorbing all the power available before ϕ becomes very large.

Example 103.—An aeroplane is in straight level flight at a speed at which the induced drag forms one-half of the whole and the power required is one-quarter of the total available. What maximum load factor can arise on the wings in level turning at the same speed?

The total parasite drag will remain unchanged to a first approximation—i.e., it will continue to absorb one-eighth of the total power available. Therefore the power overcoming induced drag may be increased 7 times. Thus the maximum possible value of $1/\cos\phi$ at this speed is $\sqrt{7}$—i.e., the maximum load factor is 2·65.

Spiral descent.—The restriction of the load factor by the power available in level circling is easily removed by permitting the aeroplane to lose height slowly so that gravity can help the engines. A helical angle of descent of a few degrees allows the drag to be doubled. Provided this angle is small, so that the difference between its cosine and unity can be neglected, the load factor is still $1/\cos\phi$, but the maximum value is now determined by the stalling of the wings. Let C_{LMAX} denote the maximum lift coefficient with flaps retracted and C_{L0} that for straight level flight at the same speed. Then, since $W = C_{L0}qS$ and the maximum lift $L = C_{LMAX}qS$, the load factor is

$$L/W = C_{LMAX}/C_{L0}.$$

The value of the maximum lift coefficient varies little from 1·5 from one design of wing to another, and the above ratio is seen to depend on the square of the speed, at constant altitude, or more generally on the stagnation pressure and the wing-loading. Thus, writing w for the wing-loading W/S, $C_{L0} = w/q$ and the maximum load factor can be written approximately :

$$1{\cdot}5\, q/w \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (107)$$

For a lightly loaded aeroplane flying fast at low altitude, it soon becomes greater than the pilot could ordinarily withstand.

Example 104.—What maximum load factor is incurred if an aeroplane is allowed to descend on a turn at 160 m.p.h., given that it stalls in straight level flight with flaps retracted at 80 m.p.h.?

Since C_L varies as $1/V^2$ in straight level flight and the aeroplane can so fly at $\frac{1}{2} \times 160$ m.p.h., the maximum lift coefficient is 4 times that appropriate to 160 m.p.h. Therefore the maximum load factor is 4.

Example 105.—Estimate the load factor that would be set up if an aeroplane with a wing-loading of 30 lb. per sq. ft. and flying at 250 m.p.h. were turned as quickly as possible at the same speed. Assume that the wings have time to stall.

$q = \frac{1}{2}\rho V^2 = \frac{1}{2}(250 \times 22/15)^2/420 = 160$ lb. per sq. ft., and (107) gives for the maximum load factor $240/30 = 8$. The pilot would make a wider turn, of course, and this factor would never develop in practice.

198. Discussion in Terms of Acceleration

Turning, though uniform, is not a steady motion, the aeroplane being continuously accelerated towards the centre of the turn to prevent its 'flying off at a tangent'. If R is the radius of the turn, the acceleration is V^2/R. Thus considering for simplicity level circling with a load factor of 4, so that $\cos \phi = \frac{1}{4}$, we have $\phi = 75\frac{1}{2}°$, giving $\tan \phi = V^2/gR = 3{\cdot}87$—i.e., the acceleration towards the centre is $3{\cdot}87g$. As for the complete aeroplane, an object of mass m contained therein must have exerted upon it a force mV^2/R directed towards the centre of the turn to counterbalance its centrifugal force. The weight mg of the object must also be supported to prevent its falling downward with the acceleration g. Thus the total force to be exerted comes to $mg\sqrt{(3{\cdot}87^2 + 1)} = 4mg$, directed upward to the horizon at the angle $14\frac{1}{2}°$. If the object is a bag resting on the floor of the fuselage, the force is exerted by the floor and, the aeroplane being correctly banked, there is no component of force tending to slide the bag across the floor. The equal and opposite reaction to this force is the resultant of the weight and centrifugal force of the bag, and may be called its *apparent weight*. Thus the circling of the aeroplane can be said to have apparently altered the weight of the bag in two ways, first by increasing it four times, and second by changing its direction from the vertical to within 15° of the horizontal. The first alteration might make the bag impossible to lift by a passenger, who would be unconscious of the second, however, unless he took bearings by looking through a window. If the passenger succeeded in 'lifting' the bag (as he would view the operation) and then let it 'drop', it would not fall directly downward but in the direction perpendicular to the floor, and it would gather speed 4 times as fast as if it were dropped to the floor of a room.

Steady motions are so common, and our movements and conceptions so regulated therefore by the acceleration g due to gravity, that an effort is often needed to imagine in detail the new world, in this sense, created by a manoeuvring aeroplane of the aerobatic class. The changes illustrated above apply equally to each particle of fluid in a cup of tea, say. No drop would spill though the rim of the cup were nearly vertical in a properly banked turn, but the tea would be singular to drink since it might apparently weigh one-third as much as mercury. The same considerations apply, of course, to the fuel in the petrol tanks. Always assuming a just bank, the liquid surface remains parallel to the designed bottom of the tank; there is no tendency to flow towards one end or the other; but the fluid pressure on the designed bottom may be one-third of an atmosphere in place of that due to, say, 2½ feet head of petrol.

Attention has so far been restricted to a constant acceleration, such as an aeroplane can maintain. But more generally the (positive or negative) accelerations to which an aeroplane is liable are transient; the resultant over-loading is of the kind called 'live' in Engineering; and the action is not confined to the direction of lift.

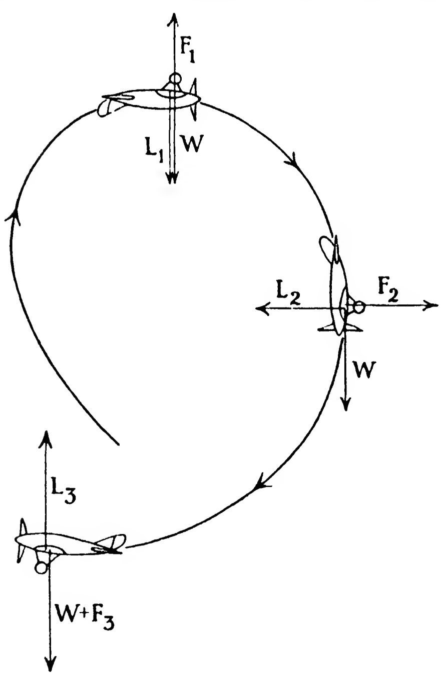


FIG. 110.—NORMAL FORCES DURING A LOOP.

199. Loop

Fig. 110 shows an aeroplane of weight W at various positions round a loop —viz., at the top, about half-way down and during the flatten out. The tangent to the flight path is horizontal in the first and last positions, distinguished by the suffixes 1 and 3, respectively, and vertical in the intermediate position, denoted by suffix 2. L is written for positive lift (cf. Article 38), F for the centrifugal force WV^2/gR and R for the radius of the flight path.

Resolving perpendicular to the flight path,

$$L_1 + W = F_1; \quad L_2 = F_2; \quad L_3 = F_3 + W.$$

The normal lift $= W$, giving for the load factors :

$$\frac{L_1}{W} = \frac{F_1}{W} - 1; \quad \frac{L_2}{W} = \frac{F_2}{W}; \quad \frac{L_3}{W} = \frac{F_3}{W} + 1.$$

But since $F/W = V^2/gR$, these can be written

$$\frac{V_1^2}{gR_1} - 1, \quad \frac{V_2^2}{gR_2}, \quad \frac{V_3^2}{gR_3} + 1 \quad . \quad . \quad . \quad (108)$$

respectively. Intermediate positions are solved similarly.

Example 106.—The speed of an aeroplane is 100 m.p.h. at the top of a loop and 250 m.p.h. at the bottom. Find the radius of the flight path at the two positions for the load factors to be 0 and 5, respectively.

For the top, (108) gives $0 = V_1^2/gR_1 - 1$; i.e., $R_1 = (146{\cdot}7)^2/32{\cdot}2 = 668$ ft.

Similarly, at the bottom of the loop $5 = V_3^2/gR_3 + 1$; i.e., $R_3 = \frac{1}{4}(366{\cdot}7)^2/32{\cdot}2 = 1044$ ft.

200. Up-Gust

If an aeroplane flying straight and level at velocity V is struck by an upward gust of vertical velocity v, incidence is momentarily increased by v/V radians and the lift coefficient from, say, C_{L0} to $C_{L0} + \Delta C_L$ (Fig. 111). Drag at once exceeds the airscrew thrust, and backward acceleration arises, but the inertia of the aeroplane prevents immediate change of speed. Again, the tail-plane will rapidly reduce the excessive incidence. But meanwhile there exists a transient load factor given by

$$\frac{L}{W} = \frac{(C_{L0} + \Delta C_L)qS}{C_{L0}qS} = 1 + \frac{\Delta C_L}{C_{L0}}.$$

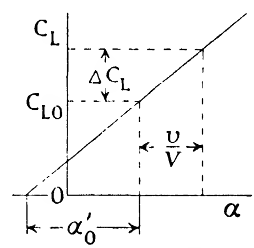


FIG. 111.—INCREASE OF LIFT COEFFICIENT DUE TO AN UPGUST OF VELOCITY v.

Let α'_0 be the original incidence reckoned from the angle of zero lift and assume the lift curve to be straight. Then from the figure $\Delta C_L/C_{L0} = (v/V) \div \alpha'_0$ and the load factor becomes

$$\frac{L}{W} = 1 + \frac{v}{V\alpha'_0} \quad . \quad . \quad . \quad . \quad . \quad (109)$$

If the angle of zero lift is unknown, the momentary increase of lift coefficient can be estimated from v/V and the slope of the lift curve (cf. Articles 190 and 191).

A special case arises when the aeroplane is flying very slowly at so large a lift coefficient that the increase of incidence due to the gust carries the wings beyond their stalling angle. It is usual to assume in these circumstances that stalling would take too long to develop for the momentary over-load to be appreciably affected, and therefore to estimate the latter for a lift curve that is extended past the stall without change of slope. A different treatment is necessary, of course, for stalled flight.

Example 107.—An aeroplane encounters an upgust of 33 ft. per sec. when flying at 150 m.p.h. with an incidence of $4\frac{1}{2}°$. The angle of zero lift of the wings is $-2°$. What is the load factor?

With the above notation, $\alpha'_0 = (4\frac{1}{2} + 2) \times \pi/180 = 0{\cdot}113$ radian. Since 150 m.p.h. = 220 ft. per sec., $v/V = 0{\cdot}15$ radian. Hence by (109)

$$\frac{L}{W} = 1 + \frac{0{\cdot}15}{0{\cdot}113} = 2{\cdot}33.$$

Example 108.—A monoplane of aspect ratio 8 and whose wing-loading is 25·7 lb. per sq. ft. encounters an upgust of 30 ft. per sec. at a horizontal velocity of 300 ft. per sec. What load factor arises on the wings?

The undisturbed lift coefficient $= 25{\cdot}7/q$ and $q = \frac{1}{2} \times 0{\cdot}00238(300)^2 = 107$ lb. per sq. ft., giving $C_{L0} = 0{\cdot}240$. The sudden increase of incidence $= 30/300 = 0{\cdot}1$ radian $= 5{\cdot}7°$.

For wings of aspect ratio 8 the formula (44) of Article 113 shows that C_L increases at the rate $0{\cdot}8 \times 0{\cdot}094 = 0{\cdot}075$ per degree. Hence the momentary increase of lift coefficient $= 0{\cdot}075 \times 5{\cdot}7 = 0{\cdot}428$.

Thus the transient lift coefficient $= 0{\cdot}240 + 0{\cdot}428 = 0{\cdot}668$ and the load factor is

$$0{\cdot}668/0{\cdot}240 = 2{\cdot}78.$$

201. Sideslip

In Article 191 it appeared that a monoplane, having a lateral dihedral angle 2β, and sideslipping at velocity v during flight at velocity V, generates an increase of lift coefficient on the wing leading into the sideslip given by

$$\Delta C_L = \frac{\beta}{\alpha'_0} \times \frac{v}{V} \times C_{L0},$$

C_{L0} being the original lift coefficient appropriate to an undisturbed incidence α'_0 reckoned from the angle of zero lift. Hence the load factor imposed on this wing is $(C_{L0} + \Delta C_L)/C_{L0}$—i. e.,

$$1 + \frac{\beta}{\alpha'_0} \times \frac{v}{V} \quad . \quad . \quad . \quad . \quad . \quad . \quad (110)$$

The load factor on the wing following into the sideslip is fractional.

Example 109.—A monoplane having a dihedral angle of 8° is suddenly yawed through 18°, the forward speed being 300 ft. per sec. Estimate the load factor on the advanced wing if the incidence for zero lift is $-2{\cdot}2°$ and that for undisturbed flight at the given speed is 1°.

As explained in Article 191, the angle of yaw $\psi = v/V$, approximately, and in the present instance $\psi = 0{\cdot}314$ radian. The angle β (4°) is 0·07 radian and, reckoned from the angle of zero lift, the incidence prior to the sideslip is $3{\cdot}2° = 0{\cdot}056$ radian. Hence (110) gives for the load factor on the leading wing

$$1 + \frac{0{\cdot}07 \times 0{\cdot}314}{0{\cdot}056} = 1{\cdot}4.$$

202. Rolling

During a roll the descending wing experiences an increased lift and the ratio L/W for this wing is readily calculated from Article 190. It is not of direct interest, however, as the extra lift is not uniformly distributed as in a sideslip, but increases towards the wing-tip. The rising wing has its lift similarly reduced, so that the total lift remains approximately unchanged, and the load factor is not felt by the pilot. The essential effect of the greater lift of the descending wing in the case of full cantilever construction is to increase the bending moments along that wing. The effect in the case of an externally braced wing is more complicated.

Restricting attention to an internally braced wing of rectangular plan-form and ignoring decrease in the lift-grading towards the tip, the normal bending moment at the root is $\frac{1}{2}W \times$ one-quarter of the span of the aeroplane, approximately. The additional moment on this wing only, due to rolling at the angular velocity ω and speed V, is one-half the total rolling moment given by (103) of Article 190. Hence the load factor to be applied to the root bending moment is

$$\frac{\dfrac{W \times span}{8} + \dfrac{1}{24} \times \dfrac{\omega}{V\alpha'_0} W\,(span)^2}{\dfrac{W \times span}{8}}$$

$$= 1 + \frac{1}{3}\frac{\omega}{V\alpha'_0} \times span \quad . \quad . \quad . \quad . \quad . \quad . \quad (111)$$

Example 110.—A monoplane, 50 ft. in span, has cantilever wings of rectangular plan-form and $-2°$ zero lift incidence. During flight at 225 m.p.h. and at an incidence of 1·3° it rolls with an angular velocity of ½ radian per sec. What is the load factor on the root bending moment of the descending wing?

We have $V = 330$ ft. per sec. and $\alpha'_0 = 3{\cdot}3° = 0{\cdot}0576$ radian. Hence the factor is

$$1 + \frac{0{\cdot}5 \times 50}{3 \times 330 \times 0{\cdot}0576} = 1{\cdot}44.$$

Chapter XII

OUTLINE OF THE LANCHESTER–PRANDTL THEORY OF WINGS

203. Irrotational Circulation Round Spinning Cylinder

IMAGINE a long circular cylinder supported with small clearance between parallel walls (Fig. 112), so that it can rotate freely about its axis XX. Let it be set spinning at a uniform rate and consider the air flow generated well away from the retarding influence of the parallel walls. Air touching the surface of the cylinder will rotate exactly with it (Article 48), while viscosity will rapidly increase the depth of the layer affected until even distant air follows round, although only slowly.

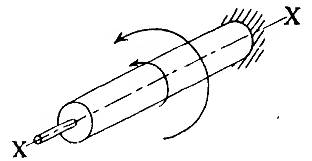


FIG. 112.—SPINNING CYLINDER.

The flow finally set up is, in general, turbulent. But under certain conditions, too exacting for easy experiment with air as fluid, it becomes steady and irrotational, a state of great interest in Aerodynamics. The air then circulates round the cylinder in concentric circular streamlines with a constant velocity round any one path. Since the flow is irrotational, the pressure is related to the velocity by Bernoulli's equation and is, therefore, also constant round any one concentric circle. We shall show that under these conditions the velocity is inversely proportional to the radius.

Consider a thin concentric shell of the revolving air, of inner radius r_1 where the velocity is v_1, and outer radius r_2 where the velocity is v_2, and let m be its mass per unit axial length. Its mean radius is $\frac{1}{2}(r_1 + r_2)$, its mean velocity is $\frac{1}{2}(v_1 + v_2)$, and, therefore, its total centrifugal force is

$$\frac{1}{2}m \frac{(v_1 + v_2)^2}{r_1 + r_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(i)}$$

This force is uniformly distributed round the shell and acts radially outward. It is balanced by a uniformly distributed force directed radially inward and due to a greater pressure being exerted on the outer surface of the shell by the surrounding air than is exerted on its inner surface by the enclosed air.

Let the pressure be p_1 at r_1 and p_2 at r_2. Since the thickness of the shell is $r_2 - r_1$, the radial pressure gradient is $(p_2 - p_1)/(r_2 - r_1)$ and, by Article 51, the total inward force is equal to the product of the gradient and the volume of the shell. If ρ denotes the density of the air, the volume of unit length of the shell is m/ρ, and the total inward force on this length is

$$\frac{m}{\rho} \cdot \frac{p_2 - p_1}{r_2 - r_1}$$

But Bernoulli's equation gives

$$p_2 - p_1 = \tfrac{1}{2}\rho(v_1^2 - v_2^2)$$

and hence the total inward force is, by substitution,

$$\tfrac{1}{2}m\frac{v_1^2 - v_2^2}{r_2 - r_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(ii)}$$

Now the shell neither expands under the centrifugal force nor contracts under the pressure force. Therefore, the forces (i) and (ii) must be equal. Equating them gives

$$\frac{v_1 + v_2}{r_1 + r_2} = \frac{v_1 - v_2}{r_2 - r_1}$$

i.e.,

$$v_1 r_1 = v_2 r_2.$$

Considering a larger thin shell, of radii r_2 and r_3, leads in the same way to $v_2 r_2 = v_3 r_3$, and so on. Thus, finally, for a fluid revolving irrotationally round a long circular cylinder the velocity is everywhere inversely proportional to the radius.

The product of v and the length $2\pi r$ of the path along which the tangential velocity is v provides a simple instance of a quantity called the *circulation* and denoted by K. In the present case

$$K = 2\pi r v \quad . \quad . \quad . \quad . \quad . \quad . \quad (112)$$

and is the same for all concentric circles because rv is constant. Having the dimensions of a velocity multiplied by a length, the units of circulation are square feet per second.

204. Circulation in General

An aerofoil at a lifting incidence in a wind generates a circulation round its sections without being spun. Fig. 113 shows the streamlines of the circulation, imagining it to persist in the absence of the wind. Outer paths, everywhere distant from the section, are nearly circular in shape, but inner paths have elongated forms. The circulatory velocity is no longer constant along each path, but varies, being high near the nose and tail of the section and low elsewhere. In these circumstances the circulation round a path is equal to the product of its length and the mean tangential velocity along it. Circulations of aerodynamical interest only exist in the presence of a wind, and a more general definition of circulation, as follows, becomes necessary.

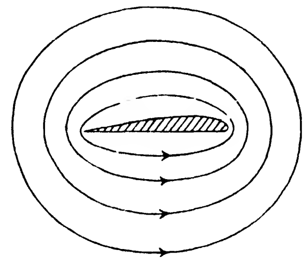


FIG. 113.—CIRCULATION ROUND AEROFOIL.

Let any imaginary circuit be drawn in a fluid motion, such as C in Fig. 114. At any point A on this circuit let V_R, the resultant air velocity, meet the tangent to the circuit at an angle θ. Denoting by v_c the component of V_R along the tangent, $v_c = V_R \cos \theta$. This component is reckoned positive if it is in the counter-clockwise direction when the wind is coming from the right; thus it is positive along the upper part of the circuit C in the figure, but negative along the lower part. The circulation K round the circuit may be defined as equal to the length of the circuit multiplied by the mean value of v_c, taking due account of its changing sign.

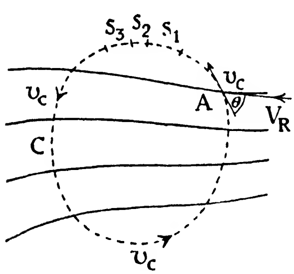


FIG. 114.

This definition can be put another way. Let the circuit be sub-divided into a number of parts of lengths s_1, s_2, s_3, etc. These lengths need not be equal to one another; they are preferably short where v_c is changing rapidly, and *vice-versa*; but they must form an endless chain completely round the circuit. Let the tangential component of the air velocity be v_{c1} at s_1, v_{c2} at s_2, and so on. Then the circulation round the circuit is given by

$$K = v_{c1}s_1 + v_{c2}s_2 + v_{c3}s_3 + \ldots \qquad (113)$$

where the summation is to extend round the complete circuit and each value of v_c is to be given its proper sign.

205. Constancy of the Circulation

Round a circuit which encloses only fluid in irrotational motion, as in Fig. 114, the circulation is always zero. But if the circuit encloses the section of a body, a circulation may exist though the flow be irrotational, as already described for the spinning cylinder, and the circulation round every wide circuit which can be drawn to enclose the section is the same.

This is easily demonstrated, as follows, for the spinning cylinder of Article 203. In Fig. 115 the curve Ccc represents part of any circuit completely enclosing the cylinder whose axis passes through O. The streamlines are concentric circles. The air velocity has the values v_1 at radius r_1, v_2 at radius r_2, etc., and $v_1 r_1 = v_2 r_2 = v_3 r_3$ and so on. The tangential component of this velocity along Ccc is v_c and tends to be large when v is large—i.e., at a small radius, and *vice-versa*. Draw a large number of radial lines, inclined to one another at such small angles θ_1, θ_2, θ_3, etc., that v_c can be regarded as constant along each of the short lengths cc intercepted. Hence approximate the curve Ccc by the serrated circuit AAAaa, which also completely encloses the section and is made up of short circular arcs centred at O, such as aa, and short lengths of the radial lines, such as Aa.

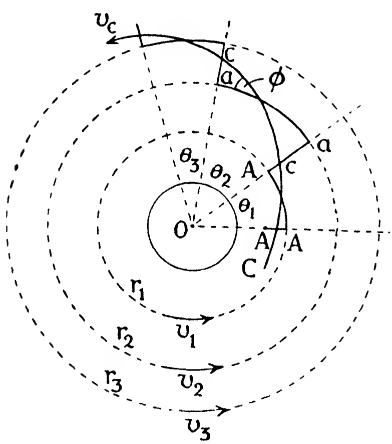


FIG. 115.

Consider the elements cc and aa of the two circuits and let cc intersect aa at an angle ϕ. If θ_2, see figure, is sufficiently small,

$$\text{cc} = \frac{\text{aa}}{\cos\phi} = \frac{r_2\theta_2}{\cos\phi}.$$

If v_c is the tangential velocity component along cc,

$$v_c = v_2 \cos\phi.$$

Therefore

$$v_c \times \text{cc} = v_2 r_2 \theta_2.$$

Now the circulation K round the original circuit is obtained by sub-dividing it into a number of parts such as cc and adding together the products of the lengths of all parts and their respective tangential velocity components. Hence :

$$K = v_1r_1\theta_1 + v_2r_2\theta_2 + v_3r_3\theta_3 + \ . \ . \ .$$

where the summation extends completely round the cylinder. But all the products v_1r_1, v_2r_2, etc., are equal. Writing vr for their common value,

$$K = vr(\theta_1 + \theta_2 + \theta_3 + \ . \ . \ .).$$

The sum of all the angles $= 2\pi$. Hence finally

$$K = 2\pi rv, \text{ a constant,}$$

whatever the shape of Ccc.

The proof for a non-circular circulation in the presence of a wind is more complicated, but the same result can be verified experimentally in a wind tunnel. Let a long aerofoil of small chord be mounted at a lifting incidence, reaching between the walls of the tunnel, and choose any circuit which is everywhere distant from the boundary layer and cuts directly across the wake ; usually a rectangular circuit such as PQRS, as shown in Fig. 116 (a), will be preferred. Subdivide the circuit into a number of parts and determine the resultant air velocity V_R in magnitude and direction at the middle point of each part. For this purpose a pitot-static tube and ‘ yaw-meter ’ are required. One form of the latter, Fig. 116 (c), consists of two fine tubes inclined at 90° to each other and open at the ends near O, their other ends being connected to a sensitive pressure gauge. The

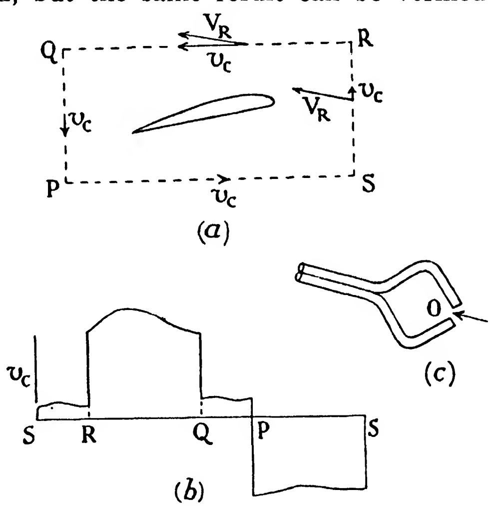


FIG. 116.—EXPERIMENTAL DETERMINATION OF THE CIRCULATION ROUND AN AEROFOIL IN A WIND.

meter is turned about O until the gauge shows the pressures within the tubes to be equal. The meter is set before introducing the aerofoil and the angle change due to the latter measured. Pitot and static tubes are attached to the meter and orientate with it.

In this way is found the velocity component v_c tangential to each part of the circuit. The direction for positive circulation is SRQPS if the wind comes from the right in the figure. It is clear, therefore, that contribution to the circulation will be positive along RQ, probably positive along SR and QP, and certainly negative along PS. In Fig. 116 (b) the circuit is opened out into the straight line SRQPS and the values of v_c are plotted on this base, giving the curve shown. The area under the curve is proportional to the circulation, provided the part of the area below the base is reckoned negative. If the shape of the circuit is changed, still keeping it widely clear of the aerofoil, and the experiment repeated, approximately the same value will be measured for the circulation at the same speed of the wind and incidence. The measurements require to be be made with care, however.

Thus in stating the circulation round a body it is unnecessary to specify what circuit is considered, for, with irrotational flow, the circulation is the same for all wide circuits which enclose the section.

206. Circulation and Lift

Fig. 117 illustrates the flow past a rapidly spinning circular cylinder in a wind. The picture is easily verified in a wind tunnel by the use of smoke. The action of the circulation is clear. Without it, there would be the same average flow above the cylinder as below, and the same average reduction of pressure round the upper shoulder of the section as round the lower shoulder. But the circulation captures some of the air which would otherwise flow beneath the cylinder and causes it to flow over the top, increasing the velocity above at the expense of that

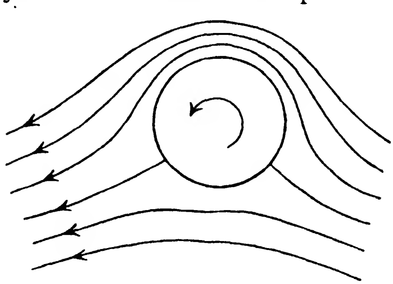


FIG. 117.—SPINNING CYLINDER IN WIND.

below. Bernoulli's equation then shows that the pressure is reduced to a greater extent above the cylinder than below it, and a lift results.

The cylinder may be mounted in a wind tunnel in bearings carried by a balance, to which is also fixed a small driving motor. Starting the wind without spinning the cylinder will verify the absence of mean lift. But spinning the cylinder rapidly in the wind will give it a considerable lift, which can be measured.

The circulation K can also be determined by exploring the velocity in magnitude and direction round an embracing circuit, as described in the preceding article. If L' is the lift per foot run of the cylinder in a wind of undisturbed velocity V and density ρ, it will be found that

$$L' = K\rho V \quad . \quad . \quad . \quad . \quad . \quad . \quad (114)$$

The units are: K, square feet per second; ρ, slug per cubic foot; V, feet per second; L', lb. per foot.

Fig. 118 shows at (a) the flow that would occur past a wing section at an appreciable incidence in the absence of a circulation, and at (b) the actual flow as modified by the circulation which the wing generates. The same action takes place as has been described with reference to the spinning cylinder, and with the same result. An aerofoil can be tested for lift under two-dimensional conditions at some chosen incidence, as described in Article 106, and its circulation determined as above. If the experiments are carried out carefully under suitable conditions, the result expressed in (114) is again found to hold.

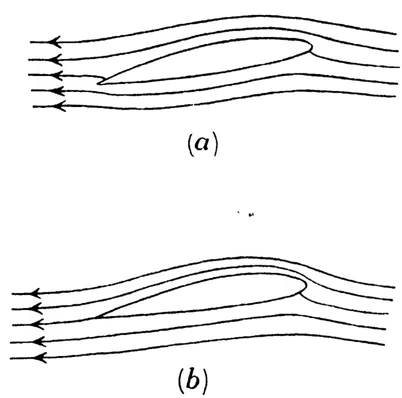


FIG. 118.—STREAMLINES OF FLOW PAST AN AEROFOIL (*a*) WITHOUT CIRCULATION, (*b*) WITH A CIRCULATION APPROPRIATE TO THE LIFT.

207. Discussion of the Lift Formula

The expression (114) is one of the most important formulæ in Aerodynamics. It can be shown theoretically, and experimentally, to hold for every isolated body which provides two-dimensional lift. It also holds, except near the extremities of span, for the sections of a three-dimensional body such as a wing or an airscrew blade, provided it is then interpreted locally. It is not accurately true for the sections of a very low wing monoplane on landing, owing to insufficient isolation from the ground, but such exceptional cases may be ignored.

The first claim of the formula is that if $K = 0$, then $L' = 0$—i.e., an aerodynamic lift cannot exist without a circulation, no matter what ρ and V may be. The next claim is that, for constant speed and density, the lift per foot run is proportional to the circulation, no matter what the size or shape of the body. Again, for constant lift per foot run the circulation is inversely proportional to the product of the density and the true air speed. These claims are fully justified and form the starting point of the Lanchester–Prandtl theory of aerodynamic lift and induced drag.

A simple and useful formula connecting the lift coefficient of a two-dimensional aerofoil with the circulation round it is derived as follows. By definition, if c is the chord,

$$C_L = \frac{\text{Lift per foot of span}}{\frac{1}{2}\rho V^2 c}.$$

Substituting $K\rho V$ for the numerator gives

$$C_L = 2\frac{K}{Vc} \qquad . \quad . \quad . \quad . \quad . \quad (115)$$

This formula may be applied to a strip of the span of a three-dimensional wing, K being interpreted as the circulation round the local wing section and C_L as the lift coefficient of the strip.

Example 111.—The sections of an aeroplane wing near the root have a chord of 12 ft. What is the circulation round them at a speed of 150 m.p.h. if the local lift coefficient is 0·5?

$V = 220$ ft. per sec. $Vc = 2640$ sq. ft. per sec. Hence the above formula gives $K = \frac{1}{2} \times 0{\cdot}5 \times 2640 = 660$ sq. ft. per sec.

208. Review of Aerodynamic Lift

The foregoing gives the most fundamental explanation of two-dimensional lift and the formula (114) reduces it to its simplest terms. In earlier chapters an aerofoil was seen to experience lift as a variation, due to relative motion, of the normal pressures acting on its surface. This pressure variation was seen to arise from the irrotational flow outside the boundary layer and to accord with Bernoulli's equation, implying a faster flow above the section than below it. Now it is found that the last requirement for lift is achieved in the presence of a wind by a circulation.

The theory is indifferent as to how the circulation may be produced. Provided it exists in a certain strength, there will result a certain lift per foot of span for a given density of the air and translational velocity, whether the agent generating the circulation is large or small, mechanical or otherwise.

Lift from spin, the method long practised in golf and other ball games and later applied to sailing by substituting rotating funnels for the sails of a ship lacks interest in Aeronautics, partly owing to the mechanical complication involved, but more fundamentally on account of the high form drag associated with it. Any body pulled through the air at a suitable angle of incidence will generate a circulation without being spun, but usually only a weak one. The advantage of a well-shaped aerofoil is that it generates a comparatively strong circulation at an incidence just below the stall and yet offers almost negligible form drag at small incidences, when high speeds make a weak circulation sufficient for the lift required. A great deal more circulation than a wing will produce would be an advantage at low speeds, and many attempts have been made to combine the spinning and non-spinning methods, usually by fitting an aerofoil with a rapidly rotating nose. But so far only the flap or slot have proved effective, and neither succeeds quite as well as desired.

In a later section of this chapter will be described the important and far-reaching effects of changing from two-dimensional to three-dimensional conditions: from an aerofoil stretched across a wind tunnel to a free wing. It will be found that the circulation then induces an extra drag which is absent from the two-dimensional aerofoil. Though the circulation round the latter is set up by an action of viscosity (there would be no lift if the air were inviscid), yet with a wing or aerofoil this action appears to take place once and for all unless the speed, altitude or weight supported be changed. If the virtue of wings had never been inferred from bird flight, aeroplanes might conceivably have relied on spinning cylinders instead, and then a small fraction of the large power required for flight could have been traced to maintaining the rotation of the cylinders, on which the circulation would have depended. That some such wastage of power is associated with maintaining the necessary circulation round a wing has often been suspected, but not demonstrated. We may accordingly regard the part of the force arising on a two-dimensional aerofoil in a wind due to the circulation round its sections, as exactly perpendicular to the direction of the relative motion of translation. This force therefore involves no drag, and the drag of such an aerofoil comprises only form drag and skin friction.

When the lift coefficient of a wing is changed, the circulation must be adjusted as stated in the formula (115). Large adjustments are therefore necessary in changing from climbing to straight level flight at full power, turning into a circular path, and so on. These are effected by suitable modifications of the angle of incidence. Even in straight level flight at constant altitude a continuous adjustment is, in fact, necessary, owing to a continuous reduction of weight due to consumption of fuel. The wings are assumed, however, to derive neither benefit nor disadvantage from this alteration of the circulation round their sections except as will be found essentially from the three-dimensional effect.

VORTICES

209. Nature of a Vortex

In Article 203 there was no need for the cylindrical core round which the air revolved to be solid; it might equally have been formed of a shaft or column of the air itself in rotation. Such a rotating fluid core is called a *vortex*. Round it the air or other fluid circulates irrotationally as already described for the spinning cylinder, the velocity being inversely proportional to the radius in the two-dimensional case. This law ceases to hold at the edge of the vortex, but the internal motion is of little interest, and may be regarded vaguely as a more or less uniform rotation, though it is in fact more complicated.

The strength of a vortex is measured by the circulation round it. An example of a strong vortex occurring in nature is the whirlwind; another is the whirlpool. Far weaker vortices are habitually generated by the wind in blowing past bluff obstacles. In the case of a long cylinder of poorly streamlined section, their axes are parallel to the axis of the cylinder (Article 46). We shall find that every lifting wing produces a pair of very long vortices whose axes are approximately parallel to the direction of motion.

When the core of a vortex is small or the circulation large, the rotational speed at the centre is high; up to 15,000 revolutions per minute are said to have been observed behind the wing-tips of a short aerofoil in a wind tunnel. The centrifugal force of the air then produces a considerable pressure reduction in the central region. A large pressure drop of this kind accounts for the waterspout at sea and the dangerous sucking dimple of the whirlpool. If a circular cylinder is drawn with its axis vertical through a long tank of water, the vortices in the wake cause a succession of dimples in the water surface. The corresponding pressure drop in the vortices trailing behind an aeroplane condenses water-vapour present in the air, forming plumes or streamers that can sometimes be seen.

A vortex cannot come to a free end in the atmosphere, since it would expose its low pressure region to end-flow, and must either abut on a supporting surface or else form a closed loop. An instance of the latter is provided by the familiar smoke-ring, which is simply a vortex loop or vortex ring.

The fact that a smoke-ring remains distinguishable implies that it is formed always of the same particles of fluid, and this is true generally of a vortex, except that viscosity causes immediately adjacent fluid to give up its irrotational state of motion and become part of the vortex. Though the thickness of a smoke-ring varies from part to part, its strength is constant along its length—i.e., the circulations round all sections are the same.

210. The Bound Vortex

Reverting to the two-dimensional case, there is an essential difference, as follows, between a long straight vortex and the spinning cylinder. When the wind blows on the latter, the circulation may be modified but is not blown away; it remains attached to the cylinder. But an isolated straight vortex is blown helplessly along by the wind.

Fitting a long flat plate, which has no circulation, with a large number of small spinning cylinders, would produce a circulation round it. Similarly, the circulation round an aerofoil is conceived as caused by innumerable fine vortices which envelop it and are enchained to it. A vortex which is prevented in this sense from being blown along by the wind is said to be 'bound'.

Thus a spinning cylinder in a wind can be described as a bound vortex. The formula (114) gives the lift, K denoting the vortex strength. This formula applies equally to a two-dimensional aerofoil, but makes no mention of the aerofoil chord, on which the lift depends only indirectly—i.e., only in so far as the chord may be required to generate the necessary circulation. Thus an aerofoil, too, is often regarded simply as a bound vortex.

The student acquainted with the theory of Electro-magnetism will perceive a striking analogy between a bound vortex in a wind and a wire conveying an electric current through a magnetic field, leading to a mathematical similarity in some respects between the aeroplane and the electric motor. Fig. 119 shows the streamlines past a strong bound vortex in a wind, the wake being neglected; the circulation is much greater than an aerofoil can achieve at the implied speed. The picture equally represents the 'lines of force' round a conducting wire coincident with the vortex and situated in a uniform magnetic field, whose own lines of force are parallel to the direction of the undisturbed wind in the aerodynamical case. Perpendicular to this direction and to the wire an electro-magnetic force arises on the latter analogously to the aerodynamic lift on the bound vortex.

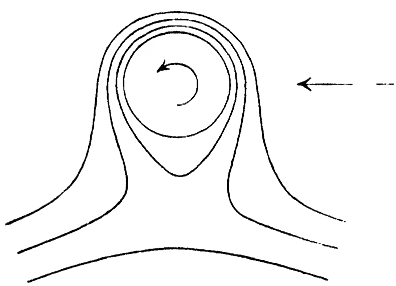


FIG. 119.

211. Vortex Pair

When two or more vortices are present, the resultant induced velocity at any point is obtained by combining, by the triangle of velocities, the separate velocities which would be induced at that point by each of the vortices alone. An example of outstanding interest is provided by what is called the *vortex pair*. This consists of two parallel rectilinear vortices at some distance apart, of equal strengths and revolving in opposite senses. The strength of each will be denoted by K and the distance apart by l.

A vortex pair moves through the fluid because, although neither has any velocity due to itself, each has, by the formula (112), a velocity equal to $K/2\pi l$ due to the other. With the rotations as shown in Fig. 120 the resulting motion of the pair is downward; it is slow in most cases of aerodynamical interest and can usually be neglected.

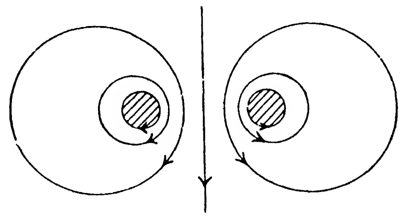


FIG. 120.—STREAMLINES OF A VORTEX PAIR.

Owing to the opposite rotations, the induced velocity appropriate to either separately is increased between the vortices and reduced beyond them. This is indicated in the figure, the streamlines being drawn on the convention described in Article 44, so that the resultant velocity at any position is inversely proportional to the local distance between adjacent streamlines. Since the vortices are themselves moving, it is necessary to add that the streamlines shown are relative to the vortices and move downward with them.

Considering a point midway between the vortices, and thus distant $\frac{1}{2}l$ from either, the induced velocity due to each is, by the formula (112), $K/2\pi\frac{1}{2}l = K/\pi l$ and has the same direction. Thus the resultant velocity at this point amounts to double this quantity—i.e., to $2K/\pi l$.

212. The Bent Vortex

A two-dimensional or rectilinear vortex is one which is not only straight but also extends indefinitely in both directions. Three-dimensional vortices are more complicated and can be referred to in this book only in general terms. The simplest case, and fortunately the one of most interest, is the following.

Imagine for a moment a straight vortex, which is long in one direction only, to terminate in the atmosphere. The velocity induced at a given radius decreases as the end is approached and is no longer inversely proportional to the radius, except opposite the end itself.

The exception mentioned is important in connexion with wings and airscrews. Level with the end of the vortex, the induced velocity v is inversely proportional to the radius r from the axis of the vortex, just as in the two-dimensional case, but has precisely one-half of its two-dimensional value—i.e.,

$$v = K/4\pi r \quad . \quad . \quad . \quad . \quad . \quad (116)$$

The induced velocity does not cease abruptly at the end, but suffers further reduction only gradually as the end is passed.

How in these circumstances can the strength of the vortex, which is measured by the circulation round its sections, remain constant all the way along its length? The answer to this question follows from the fact that for clearness of statement we have supposed a fictitious arrangement. A vortex cannot come to a free end. In the absence of a supporting wall to make it two-dimensional, a straight length must be joined to another part of the same vortex. The two parts might, for instance, be joined at right angles to one another, like the two strokes of the letter L, and then what has been called the end would be the juncture of the two strokes. The description which has been given applies to either arm of the bent vortex considered separately. Both arms produce velocities, and those due to the second combine with those due to the first in such a way as to maintain the circulation. The circulation is still irrotational, but it is not two-dimensional, its paths requiring to thread round the corner.

213. The 'Horse-shoe' Vortex

The difficulty just discussed would arise equally at the free ends of a vortex shaped like the letter L; these ends, too, must lead to other parts of the same vortex. As a first step the L may be converted to a U, and then, assuming the up and down strokes to be very long, we have a pair of vortices joined together by a cross vortex. This is known in Aeronautics, not very suitably perhaps, as the *simple horse-shoe configuration.*

Far away from the cross vortex, the arrangement becomes indistinguishable from a vortex pair. As the cross vortex is approached, each of the parallel arms behaves separately as described in the preceding article. Thus the two together can be regarded in this sense as a terminating vortex pair.

THE WING

214. The Uniform Lift Approximation

Imagine a re-entrant vortex loop to be drawn out into a very elongated form. Let the scale be large, so that the distance between the long parallel sides of the rectangular loop is about three-quarters of the span of a monoplane, whilst the other dimension may be a few miles. Imagine the two wings of the monoplane to be squeezed into one of the short sides, BC, of the loop (Fig. 121), and the projecting wing-tips cut off. The result is a first approximation to the actual vortex system generated by the aeroplane.

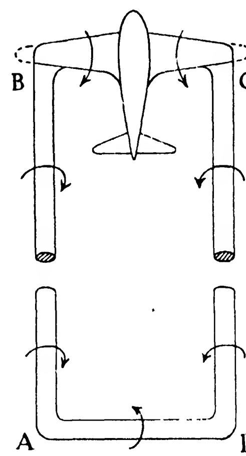


FIG. 121.—FIRST APPROXIMATION TO THE TRAILING VORTEX SYSTEM GENERATED BY A MONOPLANE.

The part BC of the vortex is 'bound' to the wing. The circulation round this part is not added to, but is identified with, the circulation required to give the wings a lift equal to the weight of the monoplane at a certain speed and altitude. The same circulation occurs round every part of the complete vortex loop. Thus the lift per foot of span of the monoplane is constant along the wings. In this respect the mutilated wings differ from the actual wings, which would have a lift-grading (cf. Article 110), and just so much of the wing-tips has been cut off as will make the total lift correct in spite of this change.

The part BC is dragged along with the aeroplane at a velocity V, say. The part AD of the loop is incapable of following and is left behind; it is often called the *starting vortex*. Hence the trailing vortices BA and CD are lengthened at the rate V feet per second. After a few seconds the trailing vortices are so long that the starting vortex is too far away to affect the wings; it may therefore be ignored and we have the simple horse-shoe configuration.

215. Qualitative Description

A two-dimensional aerofoil has no trailing vortex system. The consequences of restricting the span are determined by the effects on the wing of the trailing vortices thereby introduced. These vortices induce a downwash in the air flown through by the wings. In straight level flight, therefore, the wings fly through air which is moving slowly downward.

It is immediately apparent from Article 96 that drag is increased thereby; it was there found that deflecting the relative wind downward through an angle β adds the amount βL to the drag, L being the lift. Thus induced drag can be said to arise as follows. The force due to circulation round the wing sections in a wind acts perpendicularly to the stream (cf. Article 208). But the stream is bent downwards in the neighbourhood of the wings by the trailing vortex system. This inclines the force backward from the vertical in straight level flight, giving a backwardly directed component parallel to the direction of motion. A more fundamental definition of induced drag is given at the end of this chapter, but it is not so useful.

Again, it is apparent why incidence must be increased compared with two-dimensional conditions, as found experimentally in Article 112. To realise a two-dimensional value of the lift coefficient for any fore-and-aft strip of the wing, the incidence requires to be increased by the angle through which the relative wind is deflected downward in the vicinity of that strip by the trailing vortices.

216. The Wing-Tip Vortices

The actual vortex system behind a wing is more complicated because the trailing vortices are formed only in part in the immediate neighbourhood of the aeroplane. They appear in full strength only some distance behind.

A pair of vortices is generated without delay just inside the wing-tips. They are accordingly called the *wing-tip vortices.* Their strength may amount to one-half that of the final trailing vortices, but this fraction is subject to large variation. They form the nuclei round which the trailing vortices mature, a process which pulls them toward one another.

The existence of wing-tip vortices can be demonstrated in a wind tunnel in many different ways—e.g., as follows.

(*a*) In Fig. 122 A represents half the span of a lifting aerofoil in plan-view (the other half exists in place, of course, but is not shown). B represents a short length of one of the wing-tip vortices. CD is an imaginary line parallel to the span and one or two chords behind the aerofoil. A yaw-meter is tracked along CD, which extends beyond the wing-tip, measuring the downwash angle ε which is shown plotted in the figure. The part fg of the downwash curve relates to the vortex itself, indicating approximately uniform rotation; at least 5000 revolutions per minute can be expected. The part gh shows upwash beyond the wing-tip, in great strength near the vortex core where a downwash angle of $-60°$ may be found. The upwash is caused chiefly by the circulation round B, but not solely. For successful observations, the yaw-meter should be of fine construction to avoid pushing the vortex out of the way, and the precise level of CD adjusted by experiment to cut through the centre of the core.

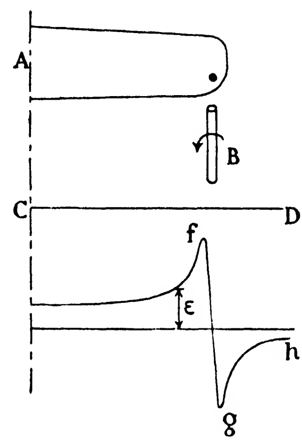


FIG. 122.

(*b*) The vortices can be made visible by the condensation of steam fed into the wind tunnel (cf. Article 209) or by smoke. In the latter case the dense smoke of titanium tetrachloride may be used, drops of the liquid being put on the model or suspended from a glass rod, but the speed must be low.

(*c*) The upper surface of the aerofoil in the region of the wing-tip may be closely explored for pressure (cf. Article 105). The readings, if sufficiently numerous, will show a deep pressure drop over a small area, indicated in the figure by a black dot on the aerofoil, where the vortex approaches the surface.

217. The rest of the rotating air that is absorbed into the fully matured trailing vortices springs like a mane from all along the trailing edge of the wings.

One way of seeing how these innumerable thin vortices arise is as follows. Owing to the greater velocity above the wing than below it, the streamlines over the upper surface will be bent inward towards the centre of span to a greater extent than are those beneath the lower surface, which with some sections will even be bent outward. At the trailing edge, therefore, where the two streams unite again, the upper one tends to slide inwardly over the lower one, producing rotation through viscosity about axes roughly parallel to the direction of motion.

The resulting layer of fine vortices extending from one wing-tip vortex to the other is known as the *vortex sheet*. It is naturally in two halves, for the rotations are in opposite senses behind the two semi-spans. Each half curls and rolls up with its wing-tip vortex. But this process is slow and neglecting it altogether leads to the following approximate representation of the vortex sheet.

218. Stepped Lift Distribution

The wing-tip vortices are omitted from Fig. 123 (*a*), leaving only the trailing vortex sheet. This is represented by a large number of vortex pairs, of which three are shown, numbered 1, 2 and 3, symmetrically disposed about the centre of span and parallel to the direction of motion. Each pair is terminated by a cross vortex bound to the wing. Thus the complete system, including the wing, is represented by a number of ' horse-shoe ' vortices.

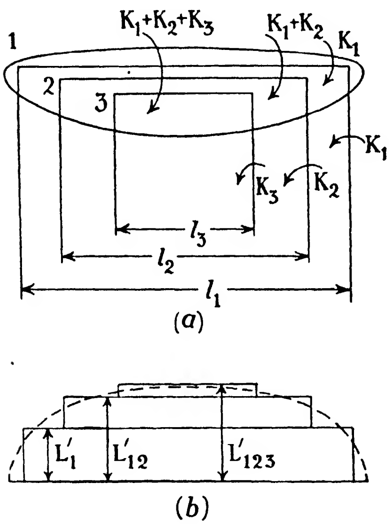


FIG. 123.

The strengths of the vortex pairs being K_1, K_2, K_3, as marked in the figure, the circulation round the wing sections is K_1 between points 1 and 2 on the span, $K_1 + K_2$ between points 2 and 3, and $K_1 + K_2 + K_3$ along the central part. Remembering that L', the lift per foot run, is given generally by $K\rho V$, the lift is distributed along the span in accordance with Fig. 123 (b), where $L_1' = K_1\rho V$, $L_{12}' = (K_1 + K_2)\rho V$ and $L_{123}' = (K_1 + K_2 + K_3)\rho V$. A given lift-grading curve (Article 110), such as that shown dotted at (b) in the figure, can be approximated to by a judicious arrangement of this stepped distribution. A close approximation will result from a large number of horse-shoe vortices, and then the equal number of trailing vortex pairs will approximate to the vortex sheet. It is seen in this way that the nature of the vortex sheet depends on the lift-grading curve.

219. The Elliptic Wing

The term elliptic applied to a wing means that the lift-grading curve is a semi-ellipse. The wing is not necessarily elliptic in plan-form, though this plan-form is usually considered to give the best approximation to elliptic loading.

It will be evident from Article 214 that if a wing has wing-tip vortices part of its lift must be uniformly distributed along the span. It follows that the elliptic wing has no wing-tip vortices. This, however, is not its distinguishing feature amongst theoretical wings; its unique characteristic is found in the nature of its vortex sheet, which induces a uniform downwash velocity along the whole span of the wing. We assume this mathematical result and may therefore calculate the downwash at any position along the span that may be convenient, and the convenient position is obviously the centre of span.

Let the elliptic lift-grading curve be fitted by a number of horse-shoe vortices as explained in the preceding article. The following investigation will again deal only with three, although more would be required for a close fit. In arranging the approximation, we are able to adjust K_1, K_2, K_3 and also l_1, l_2, l_3. Let K be the circulation round the centre of span of the elliptic wing and l the span, and let it be assumed that the best approximation is obtained by making

$$K_1 = a_1 K, \quad K_2 = a_2 K, \quad K_3 = a_3 K,$$

and

$$l_1 = b_1 l, \quad l_2 = b_2 l, \quad l_3 = b_3 l.$$

The coefficients having once been determined will remain unchanged if the semi-ellipse is increased in height or length. The first variation implies an increase of K at constant speed and density, since at the centre of span $L' = K\rho V$, and the second an increase of span.

The approximation achieved regarding the total lift L is

$$L = \rho V(K_1 l_1 + K_2 l_2 + K_3 l_3),$$

each term on the right representing one of the rectangular areas in Fig. 123 (b). This expression can be written

$$L = K\rho V l(a_1 b_1 + a_2 b_2 + a_3 b_3)$$
$$= K\rho V l \times \text{constant} \quad . \quad . \quad . \quad . \quad . \quad \text{(i)}$$

The downwash velocity v at the centre of span due to all the terminating vortex pairs is, by Article 212,

$$\begin{aligned} v &= \frac{K_1}{\pi l_1} + \frac{K_2}{\pi l_2} + \frac{K_3}{\pi l_3} \\ &= \frac{K}{\pi l}\left(\frac{a_1}{b_1} + \frac{a_2}{b_2} + \frac{a_3}{b_3}\right) \\ &= \frac{K}{l} \times \text{constant.} \end{aligned}$$

Hence, if V denotes the velocity of the wing,

$$\frac{v}{V} \text{ varies as } \frac{K}{Vl}.$$

But by (i)

$$K \text{ varies as } \frac{L}{\rho V l}.$$

Therefore, writing β for v/V,

$$\beta \text{ varies as } \frac{1}{\rho V^2} \cdot \frac{L}{l^2}$$

i.e.,

$$\beta = \frac{C}{q} \cdot \frac{L}{l^2} \qquad \qquad (117)$$

where C is a constant and $q = \frac{1}{2}\rho V^2$.

The circumstances of the wing are indicated in Fig. 124. The downwash velocity v deflects the stream at the wing through the small angle $\beta = v/V$ and increases its speed from V to V_R. Since v is assumed constant along the span, the figure may refer not merely to a strip but to the whole wing. R represents the total resultant force, omitting skin friction and form drag. By Articles 206 and 208 and the expression (i), $R = \text{constant} \times K\rho V_R l$ and is inclined backward from the direction of the lift L by the angle β.

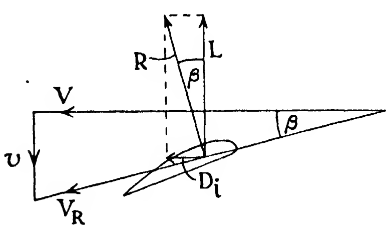


FIG. 124.

Hence

$$\begin{aligned} L = R\cos\beta &= \text{constant} \times K\rho V_R l \,.\, V/V_R \\ &= \text{constant} \times K\rho V l, \end{aligned}$$

and the total induced drag is

$$\begin{aligned} D_i &= R\sin\beta = L\tan\beta \\ &= L\beta. \end{aligned}$$

Substituting for β from (117) gives

$$D_i = \frac{C}{q}\left(\frac{L}{l}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (118)$$

220. Discussion of the Formulæ

The quantity L/l, the total lift divided by the span, is called the *span-loading*. The coefficient C can be evaluated mathematically to the value $1/\pi$ for accurately uniform downwash along the whole span. The induced drag and the necessary increase β of incidence can then be proved to have minimum values for a given span-loading and stagnation pressure (or indicated air speed). D_i and β are increased by any change of the lift-grading curve from a semi-ellipse. The elliptic wing is a theoretical ideal which is particularly useful, however, because practical wings give little more induced drag. The increase commonly varies between 5 and 15 per cent. Adopting 10 per cent. gives $C = 0{\cdot}35$, as in Articles 113 and 165. The angle β will be in degrees instead of circular measure if C in (117) is multiplied by $180/\pi$, giving 20, approximately, as in Article 113.

The formulæ ignore the chord except in so far as it is implied in the angle of incidence; the three-dimensional effects depend fundamentally on the span-loading, not on the lift coefficient. Nevertheless, a sufficient chord is necessary with a given span in order to generate a strong circulation at the minimum flying incidence, whence the aspect ratio A becomes a conception of practical importance. The formulæ are easily re-written in terms of A, and then C_L appears. Multiplying the numerator and denominator of the right-hand side of (117) by c, the mean chord, and noting that $span/c = A$ and $span \times c =$ the wing area, gives immediately :

$$\beta = \frac{C}{A} C_L \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (119)$$

Similarly,

$$C_{Di} = \frac{C}{A} C_L^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (120)$$

as in Article 113.

221. Applications

A number of important applications of the induced drag formula has been given in Chapter VIII, and there are many others. Those which involve no mathematics are often concerned with the effects of increasing the aspect ratio from, say, A to A', and the last two formulæ give for the effects of this increase :

$$\text{Decrease of incidence} = C\left(\frac{1}{A} - \frac{1}{A'}\right) C_L$$

$$\text{Decrease of drag coefficient} = C\left(\frac{1}{A} - \frac{1}{A'}\right) C_L^2.$$

These expressions are often called the reduction formulæ. To use them, the lift coefficient may be kept constant.

Example 112.—An aerofoil of aspect ratio 6 gives a lift–drag ratio of 15 at a lift coefficient of 0·5. What lift–drag ratio would result at the same lift coefficient from increasing the aspect ratio to 9?

Originally, the drag coefficient $= 0{\cdot}5/15 = 0{\cdot}0333$. Using the above formulæ, $1/A = 1/6 = 0{\cdot}1667$; $1/A' = 1/9 = 0{\cdot}1111$; $C_L^2 = 0{\cdot}25$. Hence the decrease of drag coefficient

$$= C(0{\cdot}1667 - 0{\cdot}1111)0{\cdot}25 = 0{\cdot}0139C.$$

Putting $C = 0{\cdot}35$ gives for this decrease 0·00487. So the new drag coefficient $= 0{\cdot}0285$, and the new lift–drag ratio $= 0{\cdot}5/0{\cdot}0285 = 17{\cdot}5$.

Example 113.—Referring to the preceding example, what change of incidence would be required to maintain the lift coefficient?

There would be required a decrease of incidence, given by

$$C(0{\cdot}1667 - 0{\cdot}1111)0{\cdot}5 = 0{\cdot}0097 \text{ radian}$$

with 0·35 assumed for C. This comes to a little less than 0·6°.

As instances of applications requiring mathematical treatment may be mentioned the corrections for tunnel constraint noted at the end of Chapter V. These arise because the tunnel walls decrease the velocity of downwash, but detailed discussion of them is beyond the scope of this book.

222. The Residual Air Flow

Well behind a wing in flight, the trailing vortex system has wrapped up into a vortex pair. The final result, therefore, of the passage of the aeroplane is to generate in the part of the atmosphere flown through the motion depicted in Fig. 120. The distance between the centres of the vortices is about three-quarters of the aeroplane's span. A length of V feet of the air flow is created per second, V being equal to the speed of the aeroplane. A large mass of air is therefore affected in each second.

As noted in Article 211, the vortices move slowly downward, and the air flows downward faster between them than it flows upward beyond them. The vertical momentum given to the mass of air involved is unevenly distributed, some parts having downward momentum and others upward momentum. A balance can be struck between these two components of momentum, though not by elementary means, and it is found that a length of V feet of the complete air flow round the vortices has, upon the whole, a downward momentum equal in magnitude to the lift of the aeroplane. Thus three-dimensional lift eventually appears as the reaction to the rate of change of downward momentum given to the air in continuously generating a vortex pair.

But the air cannot receive momentum without having its kinetic energy increased. The velocity midway between the vortices may readily exceed 5 feet per second, and so the generation by the passage of the aeroplane of a large amount of kinetic energy in the atmosphere each second is apparent from the figure. The wastage of energy must be made good by the engine, and may account, at low speeds when the circulation is large, for one-half of its power.

In expending this engine power the airscrew does work against a certain component part of the drag—viz., the induced drag D_i. Let E represent the kinetic energy given to the atmosphere per unit length of the flight path. Then the kinetic energy given per second is VE, whilst the corresponding part of the work done by the airscrew is VD_i, whence $D_i = E$. Thus induced drag is equal to the kinetic energy given to the atmosphere per unit length of the flight path.

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