INFLUENCE LINES

THEIR PRACTICAL USE IN BRIDGE CALCULATION

BY THE SAME AUTHOR

PRACTICAL DESIGN OF SIMPLE STEEL STRUCTURES

VOLUME I.

SHOP PRACTICE, RIVETED CONNECTIONS AND BEAMS, TABLES, ETC.

VOLUME II.

GIRDERS, COLUMNS, TRUSSES, BRIDGES, ETC.

INFLUENCE LINES

THEIR PRACTICAL USE IN BRIDGE CALCULATION

A TEXT-BOOK

SUITABLE FOR CIVIL ENGINEERS, STRUCTURAL ENGINEERS, MUNICIPAL AND COUNTY ENGINEERS AND STUDENTS AT UNIVERSITIES AND TECHNICAL COLLEGES

BY

DAVID S. STEWART B.Sc.(Hons.)(Lond.) Ph.D.(Edin.)
M.Inst.C.E., F.R.S.E., A.R.T.C.

Lecturer in Structural Engineering Drawing and Design, University of Edinburgh,
Formerly a Civil Engineer with Sir William Arrol & Co., Ltd., Bridge Builders, Glasgow

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PREFACE TO THE FIRST EDITION

TEXT-BOOKS on the theory of structures generally include the subject of influence lines, but, as it is usually desired to cover a wide field, the treatment awarded this particular branch of study is often too brief for a beginner. The following text is free from such space considerations and the young designer will find in it complete sets of calculations for the various types of structures he may encounter in the early stages of his career.

The text also recognises the fact that many young engineers shun the subject of mathematics. One reason for this dislike may be due to the dissimilarity in the professional environment of the teacher and his pupil. To the professional mathematician an equation bristling with symbols may be a pellucid answer to an investigation, but to the average engineer the answer to be intelligible must be a numerical one of so many units. Mathematics to the latter individual is only a tool, and one of many at that, which he uses towards the completion of his structure. Because of this aversion the neat methods of the calculus have either been forsaken for, or accompanied by, the arithmetical "long way round." It is hoped thereby that the text will be as easily read by the young practising engineer, who is weak in mathematics, as by the undergraduate completing his final year.

As this book was written for beginners the rather unusual procedure was adopted of having the text criticised by beginners. This often resulted in a more expansive treatment of the debated point and in repetitions being given, for, as it was said, "The student reader has no teacher at his side to cross question and an extra explanatory sentence or so will often save a student hours of thought." Apart from the hyperbole as to the time expended in thought (even Seneca wrote that all complained of time shortage) it is admitted that the criticism is well founded ; while repetition is just as useful in the text-book as it is in the lecture theatre—after all "engineering experience" is but another form of repetition.

Grateful acknowledgement is made to Mr. Wm. Dudgeon, B.Sc. (Hons.), Assoc.M.Inst.C.E., for the help so freely given in preparing the diagrams and to Mr. Leslie Gordon, B.Sc. (Hons.), for his careful scrutiny of the arithmetical work, and to both for the additional help given in checking the proofs. It is also desired to place on record the kindness of the British Standards Institution and of the Ministry of Transport (Roads Department) in permitting quotations from their publications, as acknowledged in the text.

D. S. S.

CONTENTS

ABBREVIATIONS AND SYMBOLS

The following symbols are also explained throughout the text

A	Cross sectional area of a bar.
B.M.	Bending moment (in foot tons).
B.M.$_A$	Bending moment at point A.
B.S.S.	British Standard Specification.
C.L.	(intertwined). Centre line on drawing.
C.P.	Centre pin of an arch.
D	Effective depth of girder.
D.L.	Dead load.
E	Young's modulus.
F	Force (in tons).
H	Horizontal thrust (on an arch).
H_L ; H_R	Horizontal thrust at left ; at right.
I	Impact coefficient.
I.L.	Influence line.
K	A constant in an equation.
L	Span of girder.
L.H.P.	Left-hand pin (of an arch).
L.H.S.	Left-hand side.
L.L.	Live load.
M	Moment.
M_A ; M_B	Moment at A ; at B.
Max.	Maximum.
N, or N.T.	Normal thrust (on an arch).
R	Reaction or resultant.
R_L ; R_R	Reaction at left ; at right.
R.H.P.	Right-hand pin (of an arch).
R.H.S.	Right-hand side.
S	Shear or stress, as per text.
S	Span of an arch, as per text.
S_A ; S_B	Shear at A ; at B.
S_{AB}	Stress in member AB.
T	Superscript, small capital = tons (British or "long" ton of 2,240 lb.).
U.D.D.L.	Uniformly distributed dead load.
U.D.L.	Uniformly distributed load.
U.D.L.L.	Uniformly distributed live load.
V	Vertical component, or shear.
V_L ; V_R	Vertical reaction at left hand ; ditto right hand.
W	Load or work done, depending on text.
dM	Small increment of moment.
dS	Small increment of shear.
f_c	Compressive working stress in tons per sq. in.
f_t	Tensile working stress in tons per sq. in.
ft. or ′	Feet.
ft.^{T} or ′^{T}	Foot tons.
g	Centre of gravity.
h	Height or rise of an arch.
k	Fraction or decimal part (of the span).
n	Neutral point.
p	Length of bridge panel.
sq. in.	Square inches.
u	Unit load.
w	Usually load in tons per ft. run.
x, y, z	Variable distances.
δx	"Delta x" ; a small increment of length x.
δy	"Delta y" ; a small increment of length y.
$\triangle$	(Delta). The amount of deflection.
θ, ϕ	Angles (theta and phi).
$A\hat{B}C$	Angle ABC.
$<$	Less than.
$>$	Greater than.
Σ	(Sigma). The sum of.
$=$	Equal to.

INFLUENCE LINES: THEIR PRACTICAL USE IN BRIDGE CALCULATION

CHAPTER I

REACTION AND SHEAR INFLUENCE LINES (SIMPLE BEAM)

Definition. An influence line is a curve or graph representing some function such as reaction (shear, bending moment, or stress, etc.) which occurs at a particular point or section on a span as a load passes over the span. The curve is drawn on a base line representing the span to scale and the ordinate from the base line to the curve at any point K on the span represents the value of the reaction (or shear, etc.) at the fixed section when the load is placed at the (variable) point K.

Throughout these pages the influence diagrams for reaction, shear, bending moment and stress * are all composed of straight lines because the structures treated are all statically determinate. Curved influence lines for these quantities are only encountered in statically indeterminate structures, *e.g.*, the two pinned arch.†

Further, the influence lines are all drawn for the passage across the span of a unit load of 1T and are thus unit influence lines although briefly termed influence lines. If a 9T or a WT load be rolled across the span the result will be nine times or W times that of unit load, *i.e.*, multiply the effect by the ratio of the loads, or by the load considered as an abstract number.

Reaction at A of the simply supported 40′ 0″ span AB of Fig. 1.

With the wheel at point B the total load passes directly into the right-hand abutment so that R_B (*i.e.*, reaction "R" at point B) is 1T, while the left-hand reaction, R_A, is zero.

Permit the load to travel to point b of Fig. 1 (b) and consider the reactions. By taking moments at B the left-hand reaction is

* See the footnote on page 15.

† In the last chapter curved influence lines occur, but these refer to beam deflections.

found, thus: $1^T \times 10' = R_A \times 40'$ or $R_A = \frac{1}{4}^T$, *i.e.*, when variable x is 10′ 0″ the reaction R_A is $\frac{1}{4}^T$. Similarly when the 1^T load arrives at c both reactions are each $\frac{1}{2}^T$, *i.e.*, variable $x = 20'\ 0''$ and variable $R_A = \frac{1}{2}^T$. When the 1^T load arrives at d the variable $x = 30'\ 0''$ and the corresponding value for $R_A = \frac{3}{4}^T$. Finally, when the 1^T load is placed at point A the value for x is 40′ 0″ and for R_A is 1^T.

It is now seen that these two variables, x and R_A, can be plotted as in Fig. 1 (f); the values for the straight line being :—

x	0	10′	20′	30′	40′
R_A	0	0·25T	0·5T	0·75T	1T

This straight line graph can be much more easily obtained from consideration of the equation of the line. The value for R_A is found, as above, by taking moments at the point B, viz., $R_A \times 40' = 1^T \times x'$ or $R_A = \frac{x^T}{40}$; and this expression is the law governing the graph of Fig. 1 (f). Since x is to the first power the line is a straight one and any value of R_A can be found on substituting the requisite value for x in the equation.

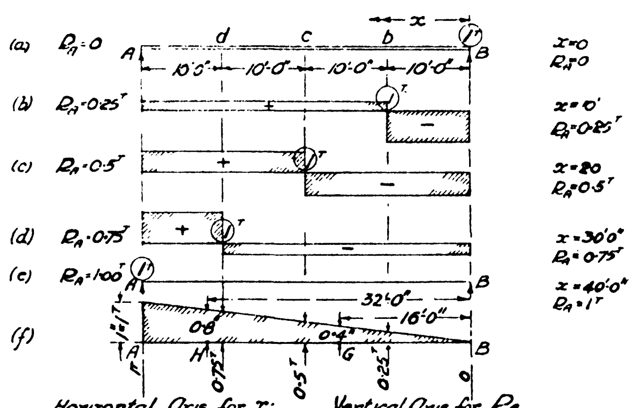


FIG 1

Arithmetical Examples

1. What is the value of the left-hand reaction R_A when the 1^T load is placed at point G, 16′ 0″ into the span, Fig. 1 (f)?

Answer. Measure the ordinate *at the Load*, *i.e.*, at G. This ordinate scales 0·4″ and as the vertical scale is $1'' = 1^T$ the reaction at the left hand is $0{\cdot}4^T$.

2. If the load is placed at H what is the left-hand reaction ?

Answer. Since the ordinate at the load is 0·8″, the reaction is 0·8T.

3. What is the left-hand reaction if a 1T load is placed at G and another at H ?

Answer. The ordinate at G added to the ordinate at H, viz., $0{\cdot}4'' + 0{\cdot}8'' = 1{\cdot}2''$ or 1·2T reaction.

4. If a 9T load be placed at G, what is the reaction R_A ?

Answer. Measure the ordinate at the load, *i.e.*, at G, and multiply this by 9, *e.g.*, $0{\cdot}4'' \times 9 = 3{\cdot}6''$ or 3·6T reaction. (Multiplying by 9 is necessary as the load now considered is nine times larger than the unit load of 1T for which the graph or influence line was originally drawn.)

5. If a 6T load be placed at G and a 4T load at H what is R_A ?

Answer. Ordinate at G multiplied by the load thereat plus the ordinate at H multiplied by its load $= 0{\cdot}4'' \times 6 + 0{\cdot}8'' \times 4 = 5{\cdot}6''$, *i.e.*, $R_A = 5{\cdot}6$T. Check by taking moments about B, $6\text{T} \times 16'\,0'' + 4\text{T} \times 32'\,0'' = R_A \times 40'\,0''$, whence $R_A = 5{\cdot}6$T.

Train of Loads. *Arithmetical Example*

6. What is the reaction on the left at A with the Ministry of Transport Loading (1922), see also Fig. 111, in the position shown in Fig. 2 (b) ? It is assumed that the wheels give up their loads

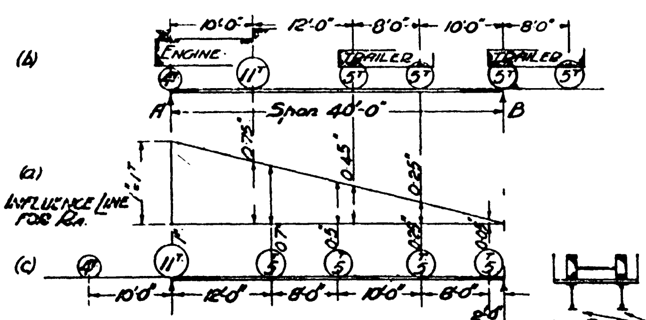


FIG. 2.

to the beams or girders at once, without any distributive effect, and that the vehicles are on the longitudinal centre-line of the bridge as in (d).

Answer. Draw the influence line for the left-hand reaction,

Fig. 2 (a), then multiply each load by its own ordinate and finally sum the results. Thus,

$(4 \times 1'') + (11 \times 0{\cdot}75'') + (5 \times 0{\cdot}45'') + (5 \times 0{\cdot}25'') + 5 \times 0$

and the last load is not on the span, so giving a total length of 15·75″, which is equivalent to a reaction of 15·75T, since the scale used was 1″ = 1T. As a check calculate by moments about point B,

$[(4\text{T}\times 40')+(11\text{T}\times 30')+(5\text{T}\times 18')+(5\text{T}\times 10')]\div 40'=R_A=15{\cdot}75\text{T}.$

7. What is the largest reaction which can occur at A with the above load traversing the span ?

Answer. It is obvious by inspection that by placing the 4T load off the bridge and bringing on to the span the last 5T wheel load that there is an increase in R_A. This position of the load is indicated in Fig. (c).

$$R_A = (11 \times 1'') + (5 \times 0{\cdot}7'') + (5 \times 0{\cdot}5'') + (5 \times 0{\cdot}25'') + (5 \times 0{\cdot}05'') = 18{\cdot}5'' \text{ or } 18{\cdot}5\text{T since } 1'' = 1\text{T}.$$

Checking by moments, $(11\text{T} \times 40') + (5\text{T} \times 28') + (5\text{T} \times 20') + (5\text{T} \times 10') + (5\text{T} \times 2') = 40' \times R_A$

whence $R_A = 18{\cdot}5\text{T}$.

Uniformly Distributed Load (U.D.L.)

Let the self or dead weight of the girder be wT per ft. run (*i.e.*, assuming the girder to be of uniform cross section throughout its full length, Fig. 3) to consider the effect on the reaction at A of the dead load of the 10′ 0″ length Ah.

Since the girder is assumed uniform, then the weight $10w$ of the 10′ 0″ length may be assumed to act at its mid-point or centre of gravity g. Hence the end reaction at A due to a length Ah is

R_A = ordinate at mid-point × (numerical value of load thereat),
= ordinate, scaled in tons × (length Ah in ft. × numerical value of load/ft.),
= bg × Ah × w in tons units,
since ft. cancel and w is a ratio or abstract number ;
= area $achA \times w$, tons.

Hence for a uniformly distributed load covering any length x of the span the result is the diagram area upon x as a base line multiplied by the numerical value of the load per ft. run. The true area of the figure is obtained by measuring the vertical ordinate to its scale and the horizontal base to its scale and then multiplying these two numbers together.

Arithmetical Example

8. Due to the 10′ 0″ length,

$$R_A = (\text{area } Ahca \times w) \text{ tons}$$
$$= (bg \times Ah \times w) \text{ tons}$$

If the girder weighs 0·05T/ft.

then $R_A = 0{\cdot}875\text{T} \times 10 \times 0{\cdot}05$
$= 0{\cdot}4375$ tons.

Due to the complete girder,

$$R_A = (\text{area } AaB \times w) \text{ tons}$$
$$= \tfrac{1}{2}(Aa \times AB \times w) \text{ tons}$$
$$= 0{\cdot}5 \times 1 \times 40 \times 0{\cdot}05 \text{ tons,}$$
$$= 1\text{T}$$

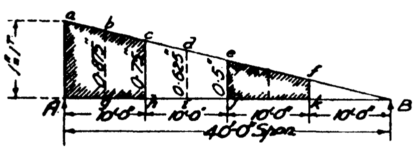


FIG. 3.

Uniformly Distributed Live Load (U.D.L.L.). *Arithmetical Examples*

9. What are the values of the reaction at A when a U.D.L.L. of $\tfrac{3}{4}$T/ft. run covers the span from B to i, and, secondly, from i to A, and, finally, when the U.D.L.L. covers all the span of Fig. 3 ?

Answer. (a) B to i. $R_A = \text{area } Bid \times \tfrac{3}{4}\text{T}$
$= \tfrac{1}{2}(Bi \times id) \times \tfrac{3}{4}\text{T}$
$= \tfrac{1}{2}(25 \times 0{\cdot}625) \times \tfrac{3}{4}\text{T} \quad = 5{\cdot}86\text{T}$

(b) i to A. $R_A = \text{area } idaA \times \tfrac{3}{4}\text{T}$
$= \tfrac{1}{2}(1 + 0{\cdot}625) \times 15 \times \tfrac{3}{4}\text{T} \quad = 9{\cdot}14\text{T}$

(c) B to A. $R_A = \tfrac{1}{2}(40 \times 1) \times \tfrac{3}{4}\text{T} \quad = 15{\cdot}00\text{T}$

Result (c) should be the sum of (a) and (b).

10. Find the maximum reaction or shear at A when the live load of Fig. 4 (a) crosses the 40′ 0″ span girder which weighs $\tfrac{1}{2}$T/ft. run.

Answer. Dead load $R_A = \tfrac{1}{2}(Aa \times AB) \times \tfrac{1}{2}$ tons
$= \tfrac{1}{2}(1\text{T} \times 40) \times \tfrac{1}{2} \quad = 10\text{T}$

Live load

Position (a) $R_A = (4 \times 1\text{T}) + (11 \times 0{\cdot}75\text{T}) + \tfrac{1}{2}(0{\cdot}45\text{T} \times 18) \times \tfrac{3}{4}$
$= 15{\cdot}2875\text{T}$

„ (b) $R_A = (11 \times 1\text{T}) + \tfrac{1}{2}(0{\cdot}7\text{T} \times 28) \times \tfrac{3}{4} \quad = 18{\cdot}35\text{T}$

„ (c) $R_A = \tfrac{1}{2}(1\text{T} \times 40) \times \tfrac{3}{4}$
$= 15{\cdot}00\text{T}$

Maximum reaction is thus D.L. + L.L. (b) $= 28{\cdot}35\text{T}$

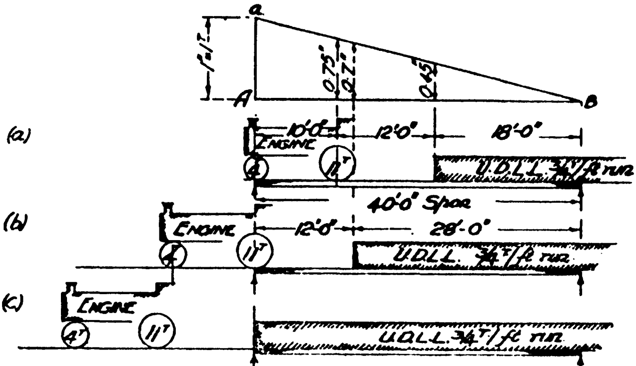


FIG. 4

Right-hand Reaction R_B

From the previous examples it is obvious that the influence line for the right-hand reaction R_B will be as indicated by Fig. 5 (a). However, as the left-hand reaction is termed positive shear it automatically follows, as with shear curves, that the right-hand reaction will be termed negative shear, and in consequence will be plotted below the line as in Fig. 5 (b). The combined influence lines for both reactions can be (and are always) plotted on one single base line after the manner of Fig. 6.

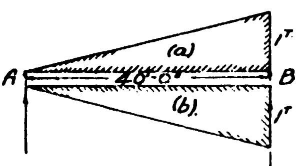
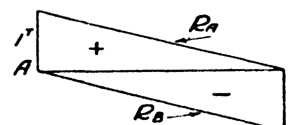


FIG. 5.



REACTION INFLUENCE LINE FOR A SIMPLY SUPPORTED BEAM

FIG. 6.

INFLUENCE LINE FOR SHEAR (SIMPLE BEAM)

With the one ton load stationary at point *a* the reaction at *A* of Fig. 7 is $1^{T} \times x \div L = \frac{x^{T}}{L}$. This upward force or shear acts at *A* and remains constant in value (as shown by the shear diagram of (b)) until point *a* is reached, so that the shear between *A* and *a* is positive $= \frac{x}{L}$. From the figure it is clear that the shear at *C* equals $R_A = \frac{x}{L}$. Therefore, so long as the 1^{T} load lies between *B* and *C* and does not enter into the portion *CA* of the span the shear at C (S_c) equals the reaction at A. This reaction changes, of course, in value according to the position of the wheel load, but $S_c = R_A$ always, with the one ton load between B and C. Therefore, the full line of Fig. (c), which is part of the influence line for R_A, is also part of the influence line for shear at point C.

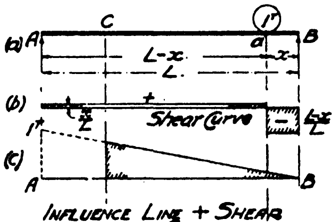


FIG 7

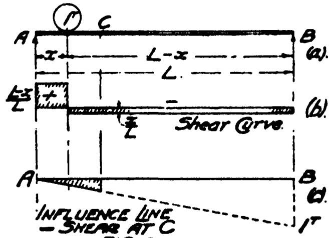


FIG. 8

Let the 1T load now lie between C and A and let x be measured from A instead of from B, Fig. 8. In this position $R_A = 1\text{T}(L-x) \div L$, found by taking moments about B, and the shear at C is this reaction minus the 1T load, diagram (b), *i.e.*,

$$S_c = R_A\text{T} - 1\text{T} = 1\text{T}(L - x) \div L - 1\text{T}$$
$$= -\frac{x\text{T}}{L}$$

Alternatively, it is easier to consider the right-hand reaction at B, since this reaction is also equal to the shear at point C. (By definition the shear is the algebraic summation of all the loads to the left or right of the point considered.) Taking moments at A, $1\text{T} \times x \div L = R_B = S_c$. Again, then, so long as the one ton load lies between C and A the shear at C is of the same value as the reaction B and is negative. As the load travels from C to A the reaction B decreases from $\frac{AC}{L}$ to 0, see influence line for R_B, Fig. 6. Therefore, the shear influence line for point C with the load anywhere between C and A is the full line, shaded, of Fig. 8 (c).

The complete influence line for shear at C is these two lines combined as in Fig. 9.

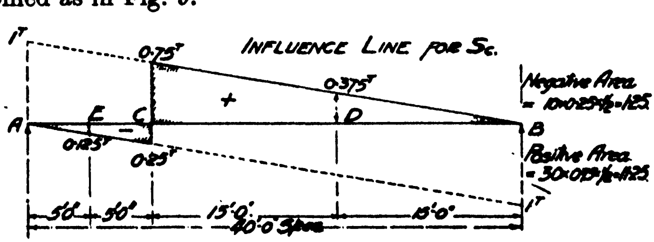


FIG. 9.

Arithmetical Examples

11. Find the shear at point C, 10′ 0″ from the left-hand support of a 40′ 0″ span beam, Fig. 9, when a 1T load is placed at 15′ 0″ from the right-hand support, *i.e.*, D of the figure, then at E and C of the figure.

Answer.

1T at B;	S_c = load × ordinate	$= 1 \times$	0T	= 0T
1T at D;	S_c = ,, × ,,	$= 1 \times$	+ 0·375T	= + 0·375T
1T at E;	S_c = ,, × ,,	$= 1 \times$	− 0·125T	= − 0·125T
1T at C*;	S_c = ,, × ,,	$= 1 \times$	+ 0·75T	= + 0·75T
	or = ,, × ,,	$= 1 \times$	− 0·25T	= − 0·25T

12. If the dead load of the girder is ½T per ft. run, what is the shear at C, S_c?

Answer. As in example (8), S_c = area of the influence line which stands upon the loaded length of base line (*i.e.*, all the span in this case) multiplied by the load per ft.

S_c = (neg. area AEC plus the positive area CDB)½T, Fig. 9,
= (− 1·25 + 11·25)½T = 5T.

13. It is required to find the shear at point C on the 40′ 0″ simply supported span of AB of Fig. 10 due to the given loading.

Answer. The ordinate heights of the influence lines can be either scaled or calculated. The gradient or slope of the curve is $\frac{1}{40}$, *i.e.*, every foot of span is equivalent to an increase in height of $\frac{1}{40} = 0{\cdot}025$.

Load Travelling from B towards A

So long as the 4T wheel is to the right of C there is no negative ordinate to deduct and so lessen the positive shear. Despite this, case (a) does not give a max. value.

(a) $S_C = +0{\cdot}625\text{T} \times 4 + 0{\cdot}375\text{T} \times 11 + 0{\cdot}075\text{T} \times 3 \times \frac{1}{2} \times \frac{3}{4} = +\ 6{\cdot}71\text{T}$

In (b) the 11T wheel is a hair's breadth to the right of C so giving the case of the heaviest concentration at

* Placing the load a hair's breadth to the left or right of C will cause the shear to change in sign and also suddenly in numerical value. If the girder has a plate web the value required for designing is the maximum vertical shear independent of sign and this occurs, theoretically, just a hair's breadth to the right of C, *i.e.*, at C.

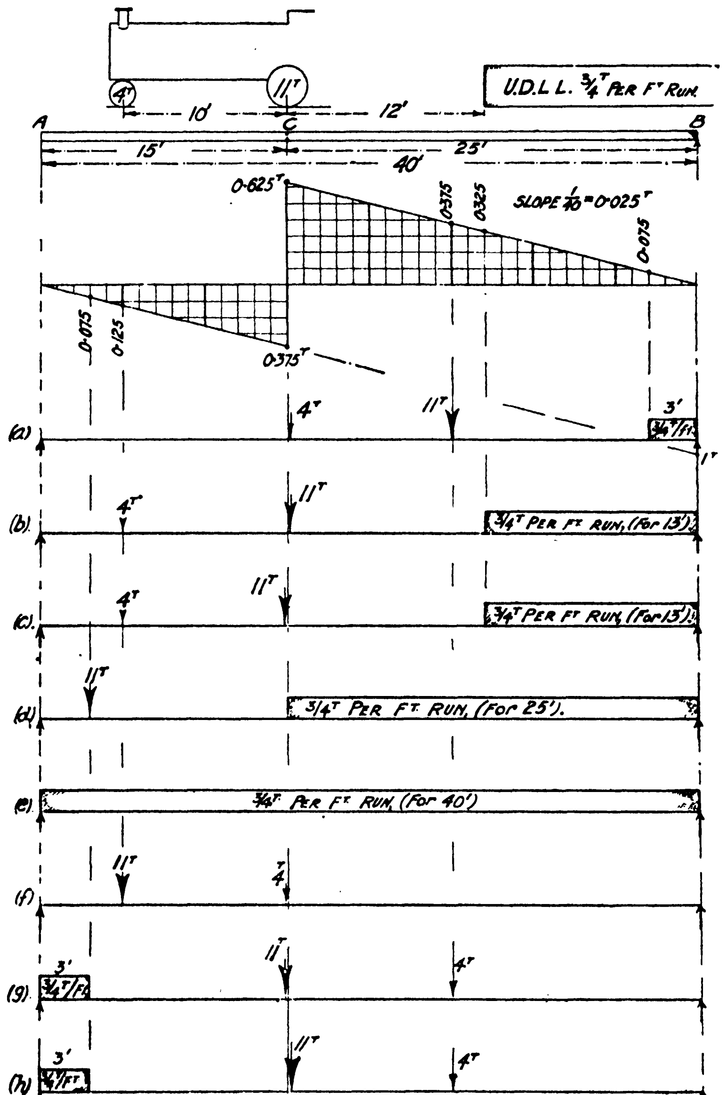


FIG. 10.

the greatest positive ordinate. The 4T wheel has a negative shear value, being in the negative segment AC.

(b) $S_C = -0{\cdot}125^T \times 4 + 0{\cdot}625^T \times 11 + 0{\cdot}325^T \times 13 \times \frac{1}{2} \times \frac{3}{4} = +7{\cdot}96^T$

In (c) the complete load system has moved ever so slightly towards the end A. The 11T wheel has entered the negative segment AC and its ordinate has changed from + 0·625 to − 0·375. The ordinate values at the 4T wheel and the U.D.L.L. remain unaltered because the movement which has taken place is microscopic. This position promises a large, if not the max., negative shear.

(c) $S_C = -0{\cdot}125^T \times 4 - 0{\cdot}375^T \times 11 + 0{\cdot}325^T \times 13 \times \frac{1}{2} \times \frac{3}{4} = -\ \underline{3{\cdot}04^T}$

Cases (d) and (e) obviously do not suggest max. values, but they are given in full.

(d) $S_C = -0{\cdot}075^T \times 11 + 0{\cdot}625^T \times 25 \times \frac{1}{2} \times \frac{3}{4}$
$= -0{\cdot}825^T + 5{\cdot}859^T = +\ 5{\cdot}03^T$

(e) $S_C = -0{\cdot}375^T \times 15 \times \frac{1}{2} \times \frac{3}{4} + 0{\cdot}625^T \times 25 \times \frac{1}{2} \times \frac{3}{4}$
$= -2{\cdot}109^T + 5{\cdot}859^T = +\ 3{\cdot}75^T$

Load Travelling from A towards B

The complete train of loads is now reversed and enters the span at A.

By inspection neither case (f) nor (g) promises a max. value.

(f) $S_C = -0{\cdot}375^T \times 4 - 0{\cdot}125^T \times 11 \quad = -\ 2{\cdot}88^T$

(g) $S_C = +0{\cdot}375^T \times 4 - 0{\cdot}375^T \times 11 - 0{\cdot}075^T \times 3 \times \frac{1}{2} \times \frac{3}{4} = -\ 2{\cdot}71^T$

Inspection suggests that case (h) should be tried for max. positive shear as the negative effect of the U.D.L.L. is small.

(h) $S_C = +0{\cdot}375^T \times 4 + 0{\cdot}625^T \times 11 - 0{\cdot}075^T \times 3 \times \frac{1}{2} \times \frac{3}{4} = +\ \underline{8{\cdot}29^T}$

Dead Load. Let the dead weight of the girder be $\frac{1}{2}^T$ per ft. run as in example 8. In the present example, however, point C is situated 15′ from A.

S_C = (negative area + positive area)$\frac{1}{2}$ tons,
$= (-0{\cdot}375^T \times 15 \times \frac{1}{2} + 0{\cdot}625^T \times 25 \times \frac{1}{2})\frac{1}{2} \quad = +\ 2{\cdot}5^T$

Maximum Combined Shear at C

Max. $+S_C$ = D.L. + L.L. = + $2{\cdot}5^T$ + $8{\cdot}29^T$ of (h) = + $\underline{10{\cdot}79^T}$

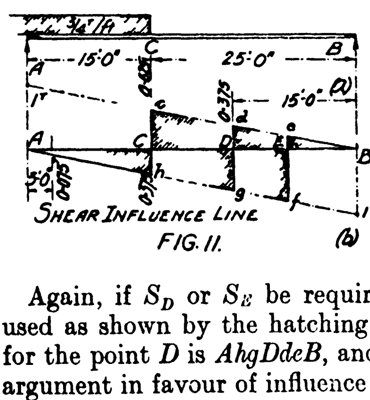
Max. $-S_C$ = „ „ = + $2{\cdot}5^T$ − $3{\cdot}04^T$ of (c) = − $\underline{0{\cdot}54^T}$

Shear diagram, Fig. 12, really combines in one figure the two cases of influence line loading of Fig. 10 (g) and (h). For the loading, stationary as in the figure, there exist two values for the shear at point C; at an infinitesimal distance to one side of C there is a shear of

+ 8·29T and at an infinitesimal distance on the other side of C there occurs the negative shear of 2·71T, either one value or the other, but not the arithmetical sum of these values. These results agree with those for cases (g) and (h). The shear diagram certainly shows a positive shear of 10·54T, but this value refers to point A and not to point C; similarly the − 6·71T ordinate of the shear curve has no relationship whatsoever with point C.

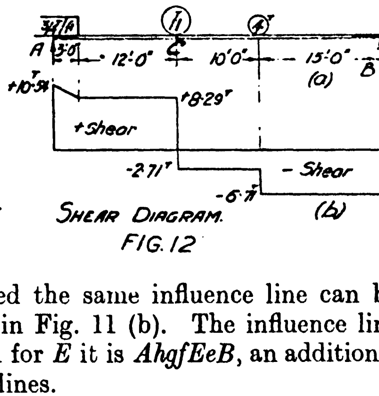
Advantages of Influence Lines

At this stage a brief note upon the advantages of the use of influence lines can be given. From the last example above it is seen that only one simple influence line is required in order to arrive at S_c for any position of the live loading whatsoever, whereas if shear curves be employed in calculating S_c every new position of the load necessitates an entirely new and separate shear curve, and these are by no means comparable with an influence line for simplicity of drawing.



SHEAR INFLUENCE LINE

FIG. 11.



SHEAR DIAGRAM.

FIG. 12

Again, if S_D or S_E be required the same influence line can be used as shown by the hatching in Fig. 11 (b). The influence line for the point D is $AhgDdeB$, and for E it is $AhgfEeB$, an additional argument in favour of influence lines.

However, if a girder carries only dead or static loads there is no advantage in using influence lines, in fact it is the reverse, because one shear curve will give the shears which act at *all* points on the girder, whereas an influence line will show the shear acting at only *one* particular chosen point on the girder.

Rule for Maximum End Shear. Case I. Wheel Loads

Consider, for the time being, only the five wheel loads which are on the span of Fig. 13 (a), viz., loads w_1 to w_5; wheel w_6 will be considered later in the arithmetical example. Now let these five wheels (which keep their set distances apart) move, as a single unit, a small distance δx to the left. This means that w_1 passes off the span and the ordinate at each remaining wheel increases in height by exactly the same amount of δy.

$$\frac{\delta y}{\delta x} = \tan\theta = \frac{1\text{T}}{L\text{ ft.}},\text{ or }\frac{1}{L}\text{ in the figure, whence }\delta y = \frac{\delta x}{L}$$

If δx be equal to a then $\qquad \delta y = \frac{a}{L}$.

It is clear that for a possible maximum value of R_A wheel w_2 should be placed at point A, in other words, the five wheels considered should move a common distance a to the left hand.

Position (a),

$$R_{A1} = w_1y_1 + [w_2y_2 + w_3y_3 + w_4y_4 + w_5y_5]$$

Position (b)

$$R_{A2} = w_2(y_2 + \delta y) + w_3(y_3 + \delta y) + w_4(y_4 + \delta y) + w_5(y_5 + \delta y)$$

Now let $\delta x = a$ and

$$R_{A2} = w_2(y_2 + \frac{a}{L}) + w_3(y_3 + \frac{a}{L}) + w_4(y_4 + \frac{a}{L}) + w_5(y_5 + \frac{a}{L}).$$

$$= [w_2y_2+w_3y_3+w_4y_4+w_5y_5]+\frac{a}{L}(w_2+w_3+w_4+w_5) \quad . \text{ (c)}$$

Then R_{A1} is less than R_{A2} if the Right-Hand Side of (a) is less than the R.H.S. of (c).

i.e., $R_{A1} < R_{A2}$ if $w_1y_1 < \frac{a}{L}(w_2+w_3+w_4+w_5)$, and since $y_1 = 1$ then

„ „ $w_1 <$ „

i.e., „ „ $\frac{w_1}{a} < \frac{(w_2 + w_3 + w_4 + w_5)}{L}$

$\therefore$ $R_{A1} < R_{A2}$ if [the load rolled off $\div$ succeeding wheel space]<[the sum of the loads remaining on the span $\div$ span length].

This rule is of no great utility because with experience one can easily pick out the two or three possible max. cases by inspection and then by actual trial, as simple as the application of the rule, prove which case is the maximum.

Arithmetical Example

14. Loading and span as given by the numerical values of Fig. 13.

$$w_1 \div a = 4 \div 10 = 0{\cdot}4 \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$(w_2 \text{ to } w_5) \div L = 26 \div 40 = 0{\cdot}65 \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$\therefore$ Position 2 is the greater, and much more so since wheel w_6 has now entered.

Now roll w_2 off

$$w_2 \div b = 11 \div 12 = 0{\cdot}917 \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\Sigma\ (w_3 \text{ to } w_6 \text{ now}) \div L = 20 \div 40 = 0{\cdot}5 \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

i.e., a reduction of end shear occurs, and thus it follows that maximum end reaction occurs with w_2 over point A.

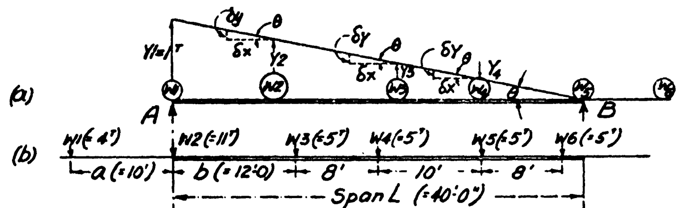


FIG. 13.

Rule for Max. End Shear. Case II. U.D.L.L. of any Length

Maximum positive end shear occurs, by inspection, when the head of the load, advancing on to the span from B, reaches point A.

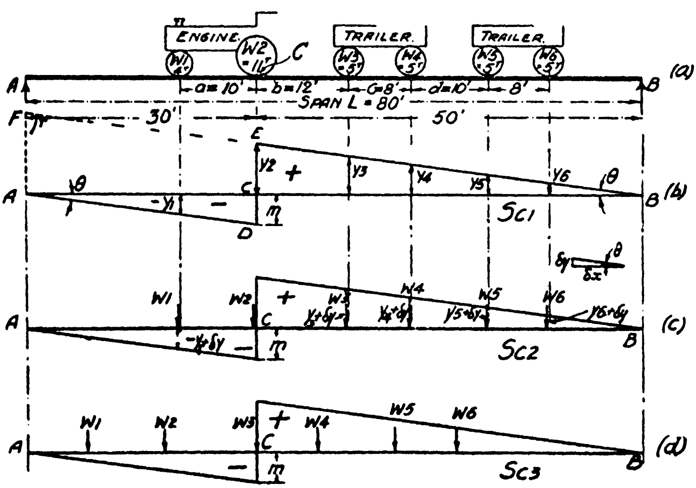


FIG. 14.

Max. Shear at any Point on a Beam. Case I. Wheel Loads

The loads of Fig. 14 roll directly upon the simply supported girder of span AB on which C is any fixed point. $ADCEB$ of (b) is the influence line for shear at C, (S_C).

Then for the loads in positions (a) and (b) the shear at C, S_{C1}, is that given by equation (1) under.

A very slight movement δx, towards the left, position (c), of the entire load system will cause all the ordinates, positive and negative, except that under w_2, to increase by exactly the same amount, $+\ \delta y$, because the slope lines of the influence diagram are parallel and both make the same angle θ with the horizontal. The shear at C, S_{C2}, is now given by equation (2) under.

Now let this very small movement be increased to a horizontal displacement of b, the spacing between the second and third wheels, thus bringing w_3 over the max. positive ordinate at C.

Since $\delta y/\delta x = \tan\theta = AF/AB = 1/L \quad \therefore\ \delta y = \delta x/L$ and if δx be made equal to b then $\delta y = b/L$. Substituting this value for δy, equation (3) is obtained for S_{C3}.

$$S_{C1} = -w_1y_1 + w_2y_2 + [w_3y_3 + w_4y_4 + w_5y_5 + w_6y_6] \quad . \quad . \quad . \quad (1)$$

$$S_{C2} = +w_1(-y_1+\delta y)+w_2(-m)+w_3(y_3+\delta y)+w_4(y_4+\delta y)+ \ldots w_6(y_6+\delta y) \quad . \quad . \quad (2)$$

$$S_{C3} = +\,w_1\left(-\,y_1 + \frac{b}{L}\right)+w_2\left(-\,m + \frac{b}{L}\right)+\,w_3\left(y_3 + \frac{b}{L}\right) + w_4\left(y_4 + \frac{b}{L}\right)+ \ldots w_6\left(y_6 + \frac{b}{L}\right)$$

$$= -\,w_1y_1 - w_2m + [w_3y_3 + w_4y_4 + w_5y_5 + w_6y_6] + \frac{b}{L}(w_1 + w_2 + w_3 + \ldots w_6) \quad . \quad . \quad (3)$$

Then the shear at C for position (b) is numerically less than that for position (d) if the Right-Hand Side of (1) is less than the R.H.S. of (3).

i.e., $S_{C1} < S_{C3}$ if $w_2y_2 < -\,w_2m + \frac{b}{L}(w_1 + w_2 + w_3 + \ \ldots w_6)$

if $(w_2y_2 + w_2m) < \frac{b}{L}\Sigma w.$

if $\frac{w_2(y_2 + m)}{b} < \frac{\Sigma w}{L}.$ But $(y_2 + m) = DE = AF = 1$

if $\frac{w_2}{b} < \frac{\Sigma w}{L}$. . . . . (4)

if [the load rolled past the point ÷ succeeding wheel space]<[the sum of all the loads ÷ span length].

Arithmetical Example

15. To find the max. positive shear at point C on the given span due to the engine and trailers shown thereon advancing from B towards A.

$\frac{\Sigma w}{L}$ $=35 \div 80 = 0{\cdot}438$ (5)

w_1 moved from C and w_2 brought over C, $w_1 \div a = 4 \div 10 = 0{\cdot}4$ (6)

w_2 ,, ,, ,, w_3 ,, ,, $w_2 \div b = 11 \div 12 = 0{\cdot}917$ (7)

w_3 ,, ,, ,, w_4 ,, ,, $w_3 \div c = 5 \div 8 = 0{\cdot}625$ (8)

From (7) it is apparent that max. positive shear occurs at C when w_2 is placed over point C.

Here, again, it is about as quick, and possibly more satisfying, actually to work out the arithmetical values of the shear from the influence lines. Thus the following are the influence line ordinates multiplied by the wheel loads. As an additional exercise the student should verify these values either by direct scaling or by calculating the ordinates.

S_C, w_1 at C.

$4 \times 0{\cdot}625\text{T} + 11 \times 0{\cdot}5\text{T} + 5(0{\cdot}35 + 0{\cdot}25 + 0{\cdot}125 + 0{\cdot}025)\text{T} \quad = 11{\cdot}75\text{T}$

S_C, w_2 at C.

$-4 \times 0{\cdot}25\text{T} + 11 \times 0{\cdot}625\text{T} + 5(0{\cdot}475 + 0{\cdot}375 + 0{\cdot}25 + 0{\cdot}15)\text{T} \quad = 12{\cdot}125\text{T}$

S_C, w_3 at C.

$-4 \times 0{\cdot}1\text{T} - 11 \times 0{\cdot}225\text{T} + 5(0{\cdot}625 + 0{\cdot}525 + 0{\cdot}4 + 0{\cdot}3)\text{T} \quad = 6{\cdot}375\text{T}$

Max. Shear at any Point on a Beam, Case II. U.D.L.L. of any Length

It is clear, without proof, that max. positive shear occurs when the head of a load advancing from B arrives at C, while max. negative shear occurs when the head of a load advancing from A arrives at C.

Stress and Signs. The term *stress influence* line means the influence line for the total stress (force or load) in a member of a structure.

Hence a *stress diagram* is a diagram which gives the total stress in each of the various members of a truss.

Stress intensity, or *stress per sq. in.*, is the total stress in a member divided by the cross-sectional area of that member.

This is the nomenclature adopted by the principal structural engineering firms of this country as shown by their "handbooks," and also by most textbooks on "Structures"; also see the B.S.S. No. 153, par. 19. Similarly, the algebraic signs allotted to *bending moment*, *shear*, *tension*, and *compression* are those used by the foregoing authorities.

CHAPTER II

BENDING MOMENT INFLUENCE LINES (SIMPLE BEAM)

LET AB be the given beam with the unit wheel load running directly upon it, and let C be any given point for which the B.M. influence line is desired, Fig. 15.

First let the unit load travel anywhere between A and C, *i.e.*, x varies from 0 at A to a max. value of a at C. Then $R_B = \frac{1^{\text{T}} \times x'}{L'}$

and $\text{B.M.}_C = R_B{}^{\text{T}} \times b' = x\frac{b}{L}$ ft. tons, ... (1)

a straight line equation as x is to the first power.

Plotting the values for x horizontally and the corresponding values for B.M. vertically, the portion AD of the influence diagram is obtained; for when $x = 0$ then by (1) $\text{B.M.}_C = 0$, and when $x = a$ then $\text{B.M.}_C = \frac{ab}{L}$, the ordinate at D . . . . (2)

Now allow the unit load to pass C and enter the segment CB of the span. If R_B be still considered then $\text{B.M.}_C = R_B \times b - 1^{\text{T}} \times$ lever arm to C. A much simpler expression can be obtained by considering R_A and then $\text{B.M.}_C = R_A \times a$.

The Fig. 15 (b) shows this method with B as the origin and z as the variable measured from B. Then $R_A = \frac{1^{\text{T}} \times z'}{L'}$ and $\text{B.M.}_C =$

$R_A{}^{\text{T}} \times a' = z\frac{a}{L}$ ft. tons . . . . (3)

Again a straight line of minimum value $\text{B.M.}_C = 0$ when $z = 0$ and max. value of $\text{B.M.}_C = \frac{ba}{L}$ when z has its max. value of b . (4)

Plotting this line the portion BD is obtained and it is seen that these two sloping lines meet at a common point D at a height above C of $\frac{ab}{L}$ by (2) and (4). Triangle ABD is therefore the influence line for bending moment at point C as unit load travels across the span.

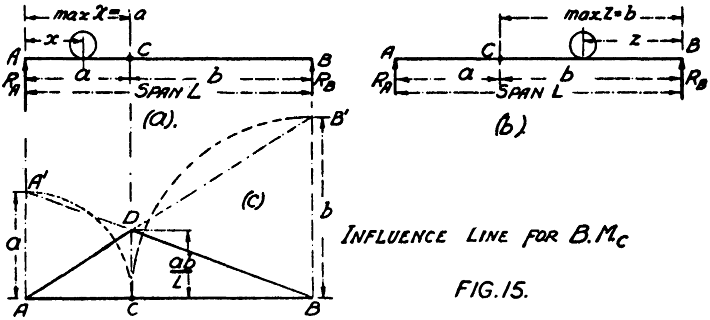


FIG. 15.

A construction often given is as follows. With A as centre and $AC = a$ as radius swing C to A', similarly with B as centre and radius $BC = b$ obtain B'. Next join A to B' and B to A'; then the influence triangle is again ADB. This is easily seen from the similar triangles BCD and BAA' wherein $BC/BA = CD/AA'$ or

$CD = BC \times AA' \div BA = ba/L$ . . . . (5)

i.e., the height of D previously found in (2) and (3).

The disadvantage of this construction is that it predetermines the scale of the vertical ordinates. It will be found preferable to place unit load at the given point, calculate the max. B.M. so created, and then plot this value, viz., ab/L, to any suitable scale instead of one depending entirely upon the linear scale of the span.

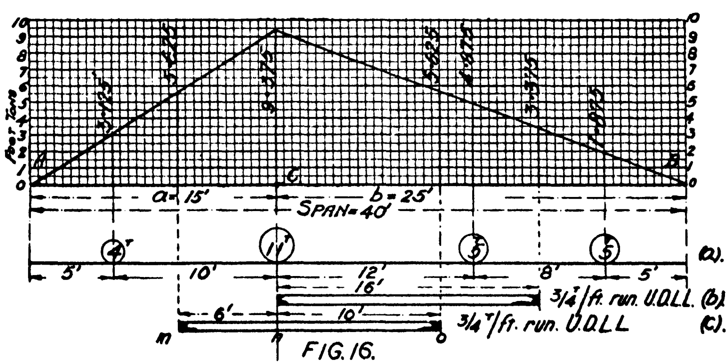


FIG. 16.

Arithmetical Examples

16. Find the $B.M._C$ with the wheel loads in position (a) Fig. 16.

Answer. $4 \times 3{\cdot}125'^{T} + 11 \times 9{\cdot}375'^{T} + 5 \times 4{\cdot}875'^{T} + 5 \times 1{\cdot}875'^{T}$ = 149·375 ft. tons

Check by moments
$R_A=[5\text{T}(5'+13')+11\text{T}\times25'+4\text{T}\times35']\div40'$
$=12{\cdot}625\text{T}$
$\text{B.M.}_C=R_A\text{T}\times15'-4\text{T}\times10'=12{\cdot}625\text{T}\times15'$
$-4\text{T}\times10'$ $\quad\doteq 149{\cdot}375$ ft. tons

17. Find the B.M.$_C$ with the U.D.L.L. in position (b).

Answer. Area of influence diagram × load/ft.
$=\frac{1}{2}(9{\cdot}375+3{\cdot}375)16\times\frac{3}{4}$ ft. tons $\quad= 76{\cdot}5$ ft. tons

18. Now place this U.D.L.L. so that point C divides the load in the same ratio as it divides the span.

$AC/CB = 15/25 = 3/5$ and
$mn/no = 6/10 = 3/5$
B.M.$_C$ = (area of diagram over mo) × $\frac{3}{4}$ ft. tons,
$= [\frac{1}{2}(5{\cdot}625+9{\cdot}375)6+$
$\frac{1}{2}(9{\cdot}375+5{\cdot}625)10]\frac{3}{4}$ ft. tons,
$=(45+75)\frac{3}{4}$ ft. tons $\quad= 90$ ft. tons

This is the max. B.M. which can occur at C due to this 16′ 0″ length of U.D.L.L. See the rule for max. B.M. given under.

19. Find the max. B.M.$_C$ due to dead load of $\frac{1}{2}$T/ft.

Answer. Since the D.L. covers all the span, then the complete area of the diagram must be considered.
B.M.$_C$ due to D.L.$=(\frac{1}{2}\times9{\cdot}375\times40)\frac{1}{2}$ ft. tons $\quad= 93{\cdot}75$ ft. tons

Check.
$15'R_A\text{T}-7{\cdot}5'(15\times\frac{1}{2})\text{T}=15'\times10\text{T}-7{\cdot}5'\times7{\cdot}5\text{T}$ $\quad= 93{\cdot}75$,,

20. Find the total B.M.$_C$ due to the wheel loads of (a) and the dead load combined.

Answer. Add results (16) and (19)
$= 149{\cdot}375 + 93{\cdot}75$ $\quad= 243{\cdot}125$ ft. tons

Rule for Max. B.M. at any Point on a Beam. Case I. Wheel Loads

In Fig. 17 the given beam is AB and C is the fixed point on it; while the loads shown, although moved backwards or forwards, always remain on the span with no load entering or leaving the span. The ordinate CD is the value of the B.M.$_C$ when 1T is placed at C.

$$R_A = 1\text{T}\times b/L = b/L \text{ and B.M.}_C = R_A\times a = ab/L = CD$$

For the loads positioned as shown the B.M.$_C$ is

$$M = w_1y_1 + w_2y_2 + w_3y_3 + W_1Y_1 + W_2Y_2 + W_3Y_3 \quad . \quad . \quad (1)$$

If the entire load system be moved as a rigid unit a tiny distance δx, then each ordinate in the AC segment will either decrease or increase by the same amount δy, while those in the CB segment will either increase or decrease by a constant amount of δY, where δY is different from δy.

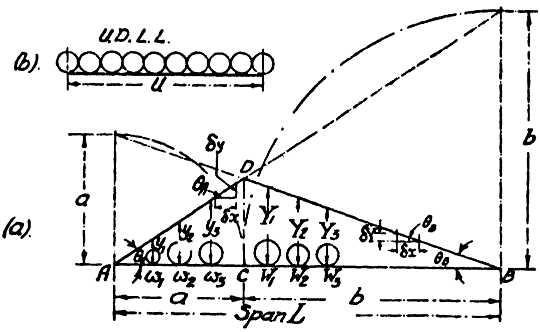


FIG. 17.

The new moment at C will be the original value of M plus or minus an increment of moment δM, *i.e.*, the new moment at C is :—

$$M + \delta M = w_1(y_1 + \delta y) + w_2(y_2 + \delta y) + \dots + W_1(Y_1 + \delta Y) + W_2(Y_2 + \delta Y) + \dots \quad (2)$$

Hence (2) − (1)

$$= \delta M = \delta y(w_1 + w_2 + w_3) + \delta Y(W_1 + W_2 + W_3) \quad . \quad . \quad (3)$$

Dividing both sides of (3) by δx now gives :

$$\frac{\delta M}{\delta x} = \frac{\delta y}{\delta x}(w_1 + w_2 + w_3) + \frac{\delta Y}{\delta x}(W_1 + W_2 + W_3) \quad . \quad . \quad . \quad (4)$$

$$= \tan\theta_A(w_1 + w_2 + w_3) + \tan\theta_B(W_1 + W_2 + W_3)$$

$$= +\frac{b}{L}(w_1 + w_2 + w_3) - \frac{a}{L}(W_1 + W_2 + W_3) \quad . \quad . \quad . \quad (5)$$

since AD has the positive gradient or tangent of $\frac{b}{L}$ and BD the negative tangent of $\frac{a}{L}$.

Equating expression (5) to zero will give a maximum or a minimum value, but if it is desired that no recourse be made to the calculus then consider Fig. 40 (c), page 56. If various values be given to the variable x and the corresponding values of M, the B.M.$_C$, be obtained and plotted, a curve similar to Fig. 40 (c), page 56, or one with hollows and crests like Fig. 33 (c) might be obtained. With the curve rising in Fig. 40 (c) the slope, gradient or tangent to the curve (tan = a small vertical increment ÷ the corresponding horizontal increment = $\delta M \div \delta x$) has a positive value. This value becomes gradually smaller on approaching the crest of the curve where the ratio $\frac{\delta M}{\delta x} = 0$, and on passing this point $\frac{\delta M}{\delta x}$ has a negative value increasing numerically as the crest is left behind. The maximum value for the moment thus happens when $\frac{\delta M}{\delta x}$ is zero. So an equation of condition is established which says that maximum bending moment at C, Fig. 17, occurs *if* the ratio of $\frac{\delta M}{\delta x} = 0$ in equation (5), *i.e.*,

if $$\frac{b}{L}(w_1 + w_2 + w_3) - \frac{a}{L}(W_1 + W_2 + W_3) = 0$$

or *if* $$\frac{w_1 + w_2 + w_3}{a} = \frac{W_1 + W_2 + W_3}{b}$$

or *if* $$\frac{\Sigma w_a}{a} = \frac{\Sigma W_b}{b} \text{ and } \therefore = \frac{\Sigma(w_a + W_b)}{a + b} = \frac{\text{total load}}{\text{span}}$$

In words then. The max. B.M.$_C$ occurs when the average load per ft. to the left of the point equals the average load per ft. to the right of the point, and, therefore, equals the average load per ft. of span.

This rule, applying to all triangular influence lines, is of great value and is constantly used.

Rule for Max. B.M. at any Point on a Beam. Case II. U.D.L.L. of any Length

Case (a). Load longer than the span.

Case (b). Load shorter than the span.

(a) By inspection it is obvious that max. B.M.$_C$ happens when the span is completely covered by the load.

(b) If the load is shorter than the span as in Fig. 17 (b) it may be considered to be made up of a large number of small wheel concentrations and the rule obtained above becomes immediately applicable.

Place the U.D.L. on the span at C so that the load per ft. on the left equals the load per ft. on the right of point C; in other words, the disposition of the load is such that C divides the load in the same ratio as it divides the span.

Arithmetical Examples

21. What is the max. B.M.$_C$ caused by the four wheel loads of Fig. 16 (a)?

First. Assume that the 11^T belongs to the CB segment.
Then $4^T \div AC = 4 \div 15 \qquad = 0{\cdot}27$
and $(11^T + 5^T + 5^T) \div CB = 21 \div 25 \qquad = 0{\cdot}84$
$\therefore$ Max. B.M.$_C$ does not occur with the 11^T in segment CB.

Second. Now assume that the 11^T belongs to the AC segment.
Then $(4^T + 11^T) \div AC = 15 \div 15 \qquad = 1$
and $(5^T + 5^T) \div CB = 10 \div 25 \qquad = 0{\cdot}4$

again max. B.M.$_C$ does not occur with the 11^T in segment AC.

Therefore max. B.M.$_C$ can only take place when the 11^T is placed directly upon point C, for in this position a portion of the 11^T ($5\frac{5}{8}^T$) may be assumed on the AC segment and the remaining portion ($5\frac{3}{8}^T$) allotted to the CB segment.

i.e., $(4 + 5\frac{3}{8}) \div AC = 9\frac{3}{8} \div 15 \qquad = 0{\cdot}625$
and $(10 + 5\frac{5}{8}) \div CB = 15\frac{5}{8} \div 25 \qquad = 0{\cdot}625$

thereby satisfying the rule in that the load/ft. run of each segment is the same.

22. Max. B.M.$_C$ (Fig. 16) due to the U.D.L.L. of $\frac{3}{4}^T$/ft. for 16′ 0″ occurs in the position shown by (c), since point C divides the span and the load in the same ratio.

i.e., $(6 \times \frac{3}{4})^T \div AC = 4{\cdot}5^T \div 15 \qquad = 0{\cdot}3$
and $(10 \times \frac{3}{4})^T \div CB = 7{\cdot}5^T \div 25 \qquad = 0{\cdot}3$

ABSOLUTE MAX. B.M. FOR A SIMPLE GIRDER

Case I. Wheel Loads

Looking back through the bending moment influence lines it will be observed that each was capable of giving the max. B.M. which occurred at the particular fixed point for which the diagram was made, but it did not follow that any one of these B.M.'s was the absolute max. B.M. for the given span. For the design of a beam or plated girder what is required is the absolute max. B.M. and the position of its point of occurrence on the span. These can be found very closely indeed by "trial and error" through taking a series of fixed points, C, distant 1′ apart near the centre of the span. Draw the influence line for each point and so ascertain which point provides the largest B.M. A still closer approximation can be made by taking an additional pair of such points, C, say 6″ on each side of the previously obtained point, which gave the arithmetical max. value, and again taking the greater. This method will give very close approximations to the two desired values, but only after very laborious working. On occasion this method can be successfully used for more complicated influence lines, but in the present instance the following proof and the rule deduced therefrom make the finding of the point of max. B.M. a very simple matter indeed.

The search for these two quantities will be facilitated when it is recalled that max. B.M. occurs under a load and not between loads, and, further, that the point of occurrence must be near the centre line of the span.

In Fig. 18 (b) a series of wheel loads are shown on a span L and it is desired to find the max. B.M. which can happen under the wheel emphasised as W_1. Place the entire load system with the specified wheel near the centre line and let R_L be the resultant of the loads w_1 to w_4 acting at the centre of gravity of these four loads; similarly, R_R is the resultant of the loads marked W_1 to W_4.

The combined resultant of R_L and R_R is R and is the total live load on the span acting through the centre of gravity of the load system. It is, of course, a condition of this proof that no load enters or leaves the span. Distances a, b and c are constants belonging to the loco or load system, while x is the only variable and denotes the distance of W_1 from the centre line "CL."

Reaction at $A = R_A = R\left[\frac{L}{2} + (c - x)\right] \div L$. Then the B.M. under W_1, viz.,

$$M = R_A\left(\frac{L}{2} + x\right) - R_L(a + c)$$

$$= \frac{R}{L}\left(\frac{L}{2} + c - x\right)\left(\frac{L}{2} + x\right) - R_L(a + c)$$

$$= \frac{R}{L}\left(\frac{L^2}{4} + c\frac{L}{2} + cx - x^2\right) - R_L(a + c) \quad . \quad (1)$$

First method, by direct differentiation :—

$$\therefore \quad \frac{dM}{dx} = \frac{R}{L}(c - 2x) \text{ and for a max. value} = 0 \quad . \quad . \quad . \quad . \quad (2)$$

$$\therefore \quad c - 2x = 0 \text{ whence } x = \frac{c}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Therefore max. B.M. under any wheel occurs when the centre line of the span is midway between the centre of gravity of the load system and the wheel considered.

Alternative method. In equation (1) above if x be altered by a tiny amount δx to $(x + \delta x)$ then the moment M under wheel W_1 will alter to :—

$$M+\delta M=\frac{R}{L}\left[\frac{L^2}{4}+c\frac{L}{2}+c(x+\delta x)-(x+\delta x)^2\right]-R_L(a+c) \quad . \quad . \quad (4)$$

then $\delta M = (4) - (1) = \frac{R}{L}(c\delta x - 2x\delta x - \delta x^2)$.

But the square of a very minute quantity, viz., δx^2, is negligible, hence $\delta M = \frac{R}{L}(c\delta x - 2x\delta x)$.

As previously done on page 19, divide throughout by δx, then $\frac{\delta M}{\delta x} = \frac{R}{L}(c - 2x)$, and as explained on the page mentioned there occurs a maximum value for the bending moment under wheel W_1 *if* $\frac{\delta M}{\delta x} = 0$.

i.e., *if* $\frac{R}{L}(c-2x) = 0$, or *if* $c - 2x = 0$ as in (3), whence follows the rule given above.

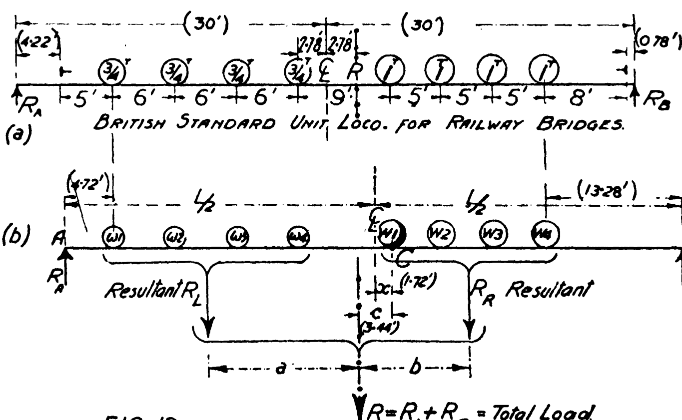


FIG. 18.

Arithmetical Example

23. At what point on the 60′ 0″ span of Fig. 18 does the absolute max. B.M. occur, and what is the value of this B.M. due to the given unit loco ?

Answer. Inspection suggests that max. B.M. should appear under wheel W_1, if not, then under W_2, or possibly w_4. To calculate the position of R take moments about W_4.

$[1^{\text{T}}(5 + 10 + 15) + \frac{3}{4}^{\text{T}}(24 + 30 + 36 + 42)] \div 7^{\text{T}} = 18{\cdot}44'$ from W_4, *i.e.*, $3{\cdot}44'$ to the left of W_1.

Wheel W_1. Place the loco so that the centre line of the span is midway between W_1 and R,

hence $c = 3{\cdot}44'$ and $x = 1{\cdot}72'$, Fig. (b)

$$R_A = (R_L \times 46{\cdot}28 + R_R \times 20{\cdot}78) \div 60$$
$$= (3^{\text{T}} \times 46{\cdot}28' + 4^{\text{T}} \times 20{\cdot}78') \div 60' \quad = \quad 3{\cdot}7^{\text{T}}$$

$$\text{B.M. at } W_1 = R_A \times 31{\cdot}72 - R_L \times 18$$
$$= 3{\cdot}7^{\text{T}} \times 31{\cdot}72' - 3^{\text{T}} \times 18' \text{ in ft. tons} \quad = \quad \underline{63{\cdot}33}$$

Wheel w_4. Dimensions, etc., as in Fig. (a).

$$R_A = (3^{\text{T}} \times 41{\cdot}78' + 4^{\text{T}} \times 16{\cdot}28') \div 60' \quad = \quad 3{\cdot}175^{\text{T}}$$

$$\text{B.M. at } w_4 = R_A^{\text{T}} \times 27{\cdot}22' - \tfrac{3}{4}^{\text{T}}(18' + 12' + 6') \text{ in ft.tons} = \underline{59{\cdot}42}$$

Wheel W_2. Having placed the load so that the centre line is midway between W_2 and R the distances c and x are respectively $= 8{\cdot}44'$ and $4{\cdot}22'$.

$$R_A = 3{\cdot}99^{\text{T}} \text{ and B.M. at } W_2 \text{ in ft. tons} \quad = \quad \underline{62{\cdot}56}$$

The largest bending moment therefore occurs under wheel, or rather axle, W_1 in the position given by point C, Fig. (b).

The hypothetical loco shown is the standard loading proposed by the *British Standards Institution for a single line of way. If the bridge is for a heavy main line then 20 units are suggested, *i.e.*, the B.M. in this case under *axle* W_1 is $20 \times 63{\cdot}33$, or under *wheel* W_1 is $63{\cdot}33 \times 10 =$ in ft. tons $= 633{\cdot}3$

The rule, previously obtained, for max. B.M. at any point on a beam will, of course, also apply to this particular point C (under wheel W_1, Fig. (b)) once it has been found.

Absolute Max. B.M. Case II. U.D.L.L. of any Length

Max. B.M. will, of course, always occur at the centre line of the span. The U.D.L.L. should therefore be placed so that the span centre line is also the centre line of the load.

The method of obtaining this equivalent uniformly distributed live load from a set of wheel loads can hardly be considered under the heading of Influence Lines.†

Absolute max. shear occurs at the ends of a simple girder (since all the load on the bridge must find its way through the end bearings to the abutments) and has been dealt with previously.

* See page 171 and Fig. 110.

† See Vol. II., "Practical Design of Simple Steel Structures," by D. S. Stewart (Constable & Co.), for a method of obtaining the E.U.D.L.L. from an actual train.

CHAPTER III

CANTILEVER PLATE GIRDERS

THE following discussion will be limited to the case where the loads are in direct contact with the girder.

Influence Line for Reaction at A, Fig. 19 (b)

Consider the passage of a unit load from B towards A in the span L, *i.e.*, x varies from 0 to L in value and $R_A = 1^{\text{T}} \times x \div L = x/L$. The maximum ordinate to this straight line is 1^{T} given by $x = L$, or unit load over A; the influence line so far traced out is *aa′b*.

Now permit unit load to enter the cantilever S so that its distance z from B varies from 0 to S. The reaction at A has now to counteract an uplift and is found by taking moments at point B, viz., $R_A \times L = 1^{\text{T}} \times z$ or $R_A = z/L$, again a straight line. This line, commencing from 0 at B, achieves its maximum negative value for R_A with unit load at C when $z = S$ and $R_A = - S/L$. The influence line for R_A is now complete and is *aa′bc′c*.

Influence Line for Reaction at B, Fig. 19 (c)

As the unit load travels from A to B the reaction B increases as in the case of a simple beam of span L ft. With the unit load confined to this particular span the greatest value for R_B happens when the load is over point B, giving $R_B = 1^{\text{T}}$. Passing B and entering the cantilever arm, R_B still increases for (moments at A), $R_B \times L = 1^{\text{T}}(z + L)$ or $R_B = (z + L)/L$, the maximum value taking place when z has its maximum value of S, load at nose C of cantilever, and this being $(S + L)/L$.

Influence Line for Bending Moment at D, Fig. 19 (d)

The curve for span AB is the same as for a simply supported girder of equal span. When the unit load is in portion AD consider R_B when calculating the B.M., *i.e.*, $M_D = 10\ R_B$; and when the load is in portion DB of the span consider R_A so giving $M_D = 30\ R_A$. Similarly when the load is on the cantilever arm, $M_D = 30\ R_A$; numerically this expression will have the negative sign because R_A

is now an uplift. As $R_A = -z \div L$ then $M_D = -30z \div L$, which is a straight line. The values at each end of this curve are: at B, $z = 0$, then $M_D = 0$, and at C when $z = S$ the $M_D = -30\,S/L$, which, for the lengths given on the diagram, works out at $-30 \times 20 \div 40 = -15$ ft. tons.

Influence Line for Shear at *D*, Fig. 19 (e)

The portion of the curve for the span L is the same as for an ordinary girder of equal span. When the unit load enters the cantilever reaction A becomes an uplift and, there being no load between A and D, this R_A is also the shear at D. It follows that the portion bcc' of Fig. (e) is identical to bcc' of Fig. (b).

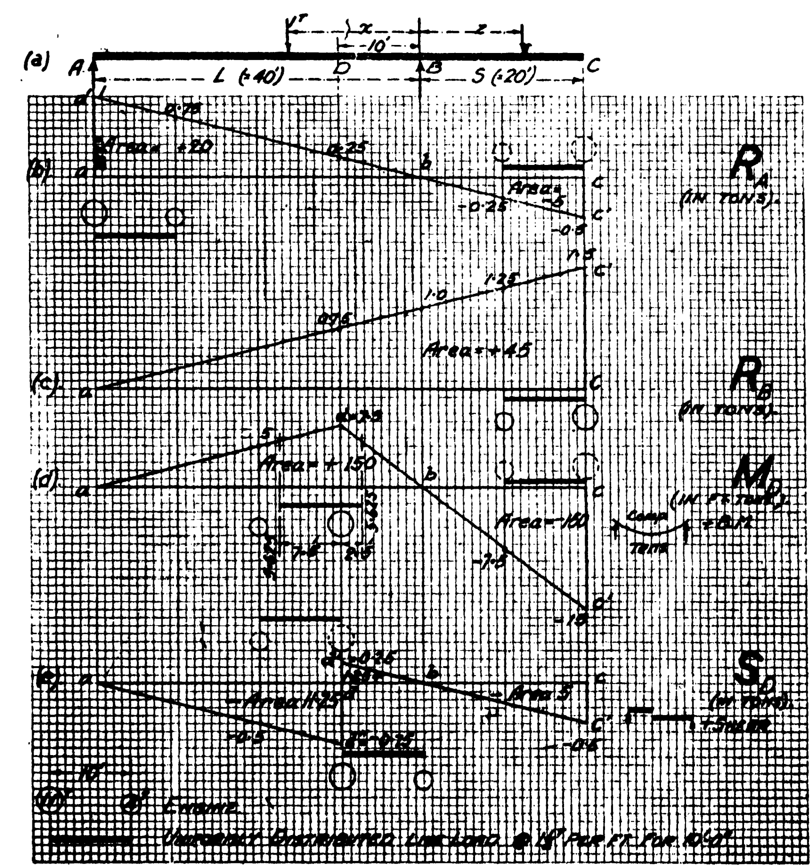


FIG. 19.

Arithmetical Examples

Spans as given in Fig. (a).

Loading. A. Engine: $11^{T} - 10' - 4^{T}$, either wheel leading.

B. U.D. Live L. of $1\frac{1}{2}^{T}$/ft. covering a 10′ length.

C. U.D. Dead L. of $\frac{3}{4}^{T}$/ft. covering all the spans.

Loading A

Max.	11^{T} wheel at	Ordinates.		Results.
$+R_A$	A	$11\times 1^{T}+4\times 0\cdot 75^{T}$	=	$14\cdot 00^{T}$
$-R_A$	C	$-(11\times 0\cdot 5^{T}+4\times 0\cdot 25^{T})$	=	$-6\cdot 50^{T}$
$+R_B$	C	$11\times 1\cdot 5^{T}+4\times 1\cdot 25^{T}$	=	$21\cdot 50^{T}$
$+M_D$	D	$11\times 7\cdot 5'^{T}+4\times 5\cdot 0'^{T}$	=	$102\cdot 50$ ft.^{T}
$-M_D$	C	$-(11\times 15'^{T}+4\times 7\cdot 5'^{T})$	=	$-195\cdot 00$ ft.^{T}
$+S_D$	D	$11\times 0\cdot 25^{T}+4\times 0^{T}$	=	$2\cdot 75^{T}$
$-S_D$	D	$-(11\times 0\cdot 75^{T}+4\times 0\cdot 5^{T})$	=	$-10\cdot 25^{T}$

Loading B

Max.	End of load at	Load/ft. × [area].		
$+R_A$	A	$1\frac{1}{2}[\frac{1}{2}(1+0\cdot 75)10]$	=	$13\cdot 13^{T}$
$-R_A$	C	$-1\frac{1}{2}[\frac{1}{2}(0\cdot 5+0\cdot 25)10]$	=	$-5\cdot 63^{T}$
$+R_B$	C	$1\frac{1}{2}[\frac{1}{2}(1\cdot 5+1\cdot 25)10]$	=	$20\cdot 63^{T}$
$+M_D$	See Fig. (d)	$1\frac{1}{2}[\frac{1}{2}(5\cdot 625+7\cdot 5)10]$	=	$98\cdot 44$ ft.^{T}
$-M_D$	C	$-1\frac{1}{2}[\frac{1}{2}(15+7\cdot 5)10]$	=	$-168\cdot 75$ ft.^{T}
$+S_D$	D	$1\frac{1}{2}[\frac{1}{2}(0\cdot 25)10]$	=	$1\cdot 88^{T}$
$-S_D$	D	$-1\frac{1}{2}[(0\cdot 75+0\cdot 5)10]$	=	$-9\cdot 38^{T}$

Loading C

R_A	load/ft. × sum of areas	$= \frac{3}{4}(20 - 5)$	$= +11\cdot 25^{T}$
R_B	,, ,,	$= \frac{3}{4}(45)$	$= +33\cdot 75^{T}$
M_D	,, ,,	$= \frac{3}{4}(150 - 150)$	$= 0$
S_D	,, ,,	$= \frac{3}{4}(-11\cdot 25+1\cdot 25-5)$	$= -11\cdot 25^{T}$

CANTILEVER BRIDGE WITH SUSPENDED SPAN, FIG. 20

So long as the unit load only travels between points A and C, but does not pass C, the influence lines for R_A, R_B, M_D and S_D, will be identical with those of the previous figure for the length AC.

With unit load just entering the suspended span at C, the reaction from the suspended span R_C is 1^{T}. If it advances 5′ 0″ into the suspended span R_C is $\frac{3}{4}^{T}$; when unit load reaches mid-span at H then $R_C = \frac{1}{2}^{T}$, and if placed midway between H and E the value of R_C is $\frac{1}{4}^{T}$. Finally, R_C is zero when the load arrives at point E. Algebraically this straight line can be expressed as $R_C = y/20$.

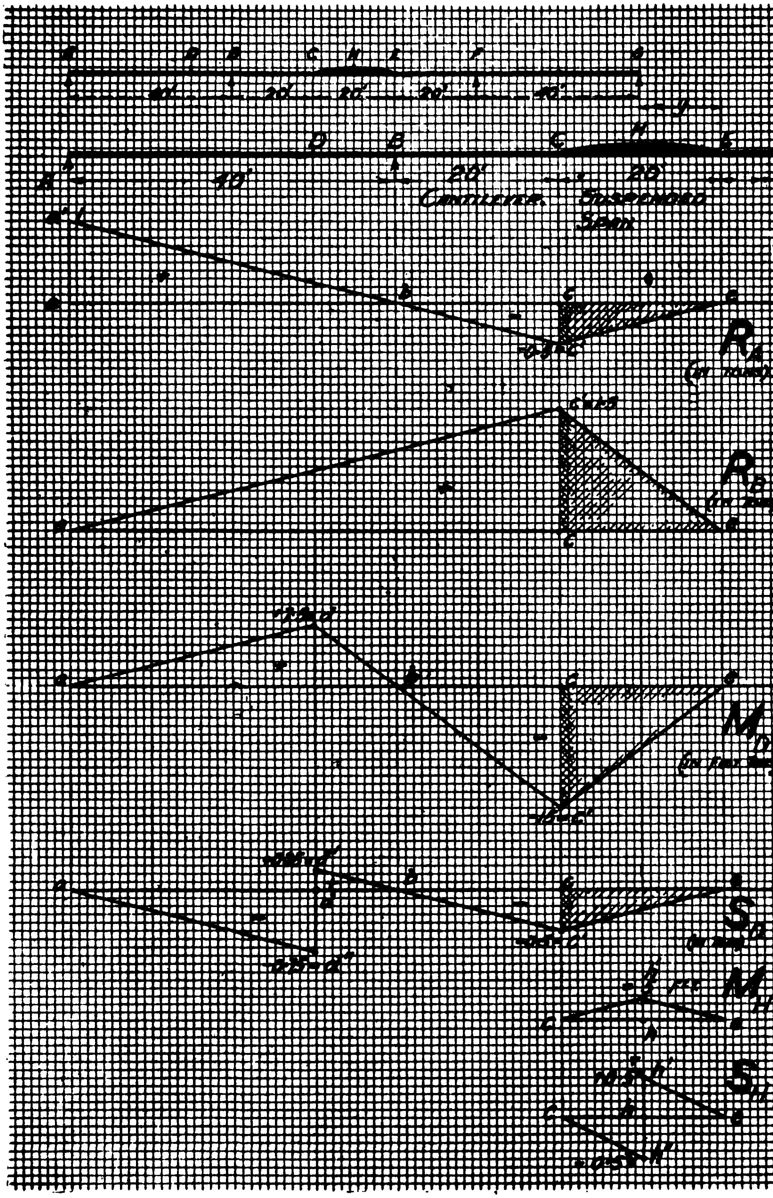


FIG. 20.

Therefore, no matter what arithmetical value the influence line had with unit load placed at C (be it the influence line for R_A, R_B, M_D or S_D, etc.) the values of the vertical ordinates to the influence line die away to zero as unit load approaches point E, because they are functions of, *i.e.*, proportional to, the value when unit load was at C. The complete influence lines are given on the figure with the additions due to the suspended span shown hatched to the right-hand side of CC'.

The influence lines for section EG will be mirror reflections of those for section AC.

To obtain maximum negative reaction at A or maximum negative B.M. at D, fully load length BE with the U.D.L.L. If the U.D.L.L. covers a length of span smaller than length BE, then arrange the U.D.L.L. so that point C divides the length of load in the same ratio as it divides BE ; in this case the loaded length will be placed so that point C lies on its centre line for $-R_A$ and $-M_D$. (In the previous figure max. $+M_D$ was obtained by placing the 10′ load so that $7{\cdot}5' : 2{\cdot}5' = AD : DB = 30' : 10'$.)

The suspended span either rests on simple pedestal bearings at C and E, at the noses of the cantilevers, or else is virtually suspended thereat by single links or tie bars and the span is as freely supported as if it were resting on masonry abutments. Such influence lines as R_C, M_H and S_H are those of a simply supported span and are in no wise dependent upon the cantilevers.

By increasing span AB and decreasing the lengths of the cantilever and suspended spans the uplift at A (loads between B and E) can be reduced, thereby adding to the stability of the structure. The safe bearing pressure on the pier founds or local site conditions, depth of water, etc., would finally settle the position of piers B and F in the river bed. The choice of spans in this example is not altogether a happy one, for, if the bending moment curve be drawn for the spans all fully loaded with a uniformly distributed load of, say, 1^T per ft. run, it will be found that there is a large negative moment at pier B.

DOUBLE CANTILEVER BEAM, FIG. 21

The overhanging girder is continuous from E to F over the two simple supports B and C. A simple span from A to E and another from F to D connect up with the shore abutments. The connections at E and F may be either pinned connections (a) or simple pedestal bearings (b) ; both types are the same in effect because no bending moment can exist at these points E and F. A hinged member cannot withstand a bending or turning moment at the hinge, which adjusts itself like a hinged door till the applied turning or bending moment becomes zero; see Fig. 80, page 116.

Reaction B. From the previous example it is known that the influence line commences with zero value at A and rises to a maximum value with unit load at E. This value, taking moments at C, is given by $R_B \times 40' = 1^T \times (40' + 10')$.

Unit load between B and C: the value of R_B is as for a simply supported beam of span BC.

Unit load between C and F: the greatest value of R_B happens when the load is at point F and is negative, *i.e.*, an uplift whose value is found by taking moments at C. $R_B \times 40 = 1^T \times 10$. From this value of $- 0{\cdot}25^T$ at F the influence line approaches zero value at D as more and more of the load finds its way to the right-hand abutment.

Moment at X. The influence line is straight and falls from 0 at A to a maximum negative value with unit load at E. For this particular position of load the $B.M._X = - 1^T$ at $E \times 20' + R_B \times 10'$, and as the value of R_B is $1{\cdot}25^T$, from diagram (c), so $B.M._X = - 20 + 12{\cdot}5$ or $- 7{\cdot}5$ ft. tons. This bending moment is negative as it tends to make the beam convex upwards at point X.

When unit load is at B it causes no moment at X as the load finds its way immediately into the pier and only uses the girder as a stool or packing.

Between B and C the influence line is that for a simply supported span BC, and is of positive sign since curvature is concave upwards.

With the load going directly into the pier C, when placed there, the value of the influence line for this point is zero, thereafter the M_X gradually increases in negative value as point F is approached. Load at F causes an R_B of $- 0{\cdot}25^T$, Fig. (c), hence M_X = sum of all moments to the left of point X, $= - R_B \times 10'$, $= - 2{\cdot}5$ ft. tons (convex upwards). From this value a straight line goes to zero value at D as the load is gradually taken off the cantilever.

Shear at X. Unit load at E gives, from Fig. (c), an R_B of $1{\cdot}25^T$ and the shear at X, which is the sum of all the vertical forces to the left of point B, is 1^T downwards $+ 1{\cdot}25^T$ upwards $= + 0{\cdot}25^T$. This value is gradually lessened to zero as unit load leaves E and approaches A.

If motion is from E to B then R_B decreases to 1^T (Fig. (c)) when B is reached, at which position of the load there is no shear at X, all the load going directly into the pier, *i.e.*, $- 1^T$ down $+ 1^T$ up $= 0$.

Between B and C the influence line is that of a simple beam span BC.

Load between C and F causes an uplift at B which means that in order to give equilibrium pier B has to exert a downward pull. This

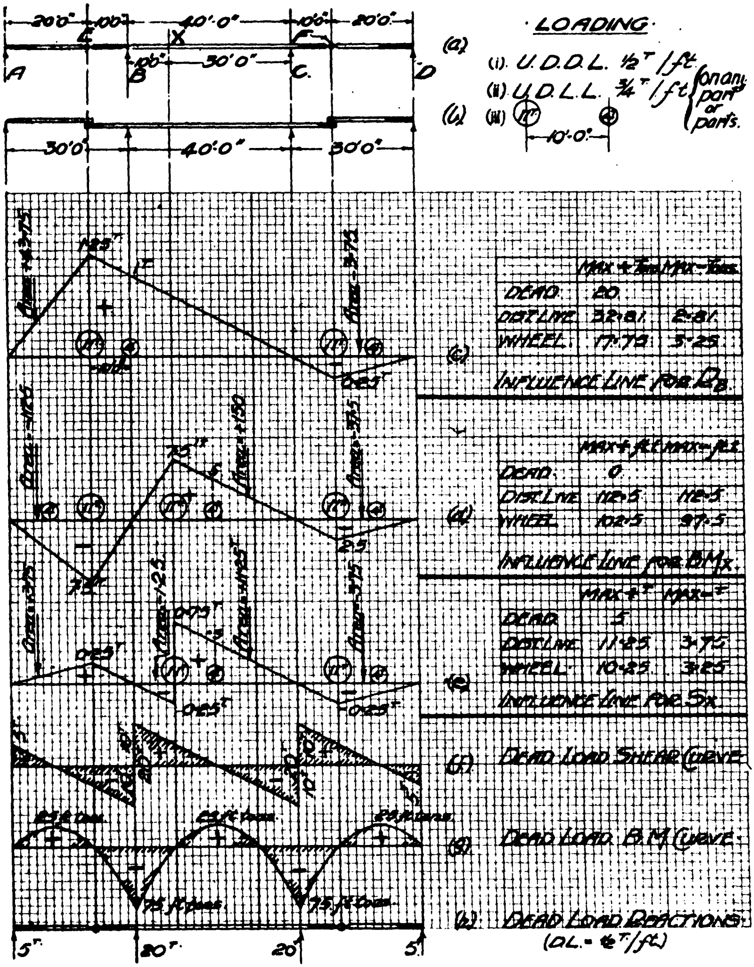


FIG. 21.

R_B is the only force acting to the left of point X, hence $S_X = R_B$ and this applies to all load positions between C and D. It thus follows that the curve for S_X must be the same as that for R_B of Fig. (c) for the length CD.

Shear and bending moment diagrams, Fig. 21 (f) and (g), are drawn for a uniformly distributed dead load of $\frac{1}{2}$T per ft. run for the whole length of the bridge. These diagrams show that there is no bending moment at the pins E and F and also verify two of the values M_X and S_X of the following arithmetical examples.

Arithmetical Examples, Fig. 21

The uniformly distributed dead load acts simultaneously over all the bridge and so all the areas must be taken into account. For example, Fig. (d), dead load M_X = all areas × load/ft.

$$= (-112{\cdot}5 + 150 - 37{\cdot}5)\times\tfrac{1}{2}$$
$$= 0.$$

U.D.L.L.

In this example the uniformly distributed live load of $\frac{3}{4}^{T}$ per ft. run is assumed to act on any part or parts of the bridge, or throughout the complete bridge if need be, in order to give the worst possible condition of loading. In Fig. (d) by loading section BC the max. positive moment is obtained.

$$M_X, \text{max.} + = \text{area} \times \text{load/ft.}$$
$$= +150 \times \tfrac{3}{4} \qquad = +112{\cdot}5 \text{ ft. tons}$$

By loading parts AB and CD simultaneously max. negative moment is obtained, viz.,

$$M_X, \text{max.} - = (-112{\cdot}5 - 37{\cdot}5)\tfrac{3}{4} \qquad = -112{\cdot}5 \text{ ft. tons}$$

Whereas loading all the bridge from A to D reduces the moment to zero, thus,

$$M_X \qquad =(-112{\cdot}5+150-37{\cdot}5)\tfrac{3}{4} = 0$$

Wheel Loads. The positions of these concentrations are shown superimposed upon the influence lines.

Table of Values. From the above explanation the beginner should have no difficulty in obtaining the results set forth in tabular form on the plate.

CHAPTER IV

METHOD OF SECTION AND MOMENTS

In the case of a framed structure the influence lines are often obtained by the Method of Section and Moments of which a brief description will be given before entering into the discussion of the influence lines themselves.

This method of obtaining the stress by sectioning a structure is due to August Ritter, who published the Method of Moments in 1863, and also to Culmann (1821–81). Stress diagrams were invented by J. Clerk Maxwell (1831–79), and their use further extended by Cremona, Mohr and Bow (1873), while influence lines were the invention of Winkler in 1867.

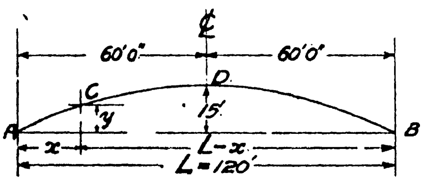


FIG. 22.

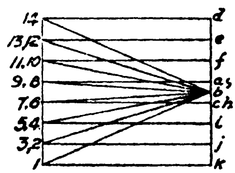


FIG. 23.

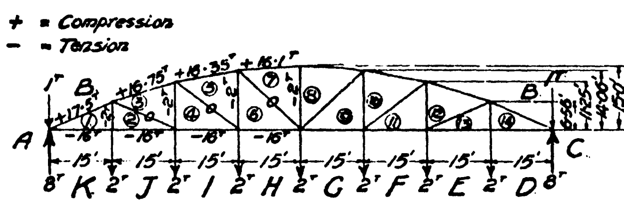


FIG. 24.

Fig. 24 shows a girder the top boom panel points of which lie upon a parabola. The boom itself is not curved, but is built up of straight lengths between the panel points. The property of a girder of this type is that with a uniformly distributed load covering all the span the diagonals carry no stress whatsoever, while the verticals act as suspenders carrying only the bottom panel loads. Given that the span is 120′ 0″ and the height at the centre line is 15′ 0″, and also that the cross girders are at 15′ 0″ centres—to find the heights of the various verticals.

The equation to the parabola *ACDB* of Fig. 22 with *A* as the origin is $x(L - x) = Ky$ . . . . (1)

For the present example L is 120′ and at the centre line $x = 60'$ and $y = 15$, therefore substitute these values in (1) and so find the constant K.

i.e., $60(120 - 60) = K \times 15$, whence $K = 240$ . . . . (2)

The general equation is thus $x(L - x) = 240y$ . . . . (3)

First vertical, substitute $x = 15$ in equation (3),

then $15(120 - 15) = 240y$ whence $y = 6{\cdot}56'$

Second vertical, $x = 30'$

then $30(120 - 30) = 240y$ „ $= 11{\cdot}25'$

Third vertical, $x = 45$

then $45(120 - 45) = 240y$ „ $= 14{\cdot}06'$

In practice the verticals would in all probability be made 6′ 6″ (*i.e.*, 6·5′ instead of 6·56′); 11′ 3″ (*i.e.*, 11·25′); 14′ 0″ (in place of 14·06′). The effect of these slight departures from the vertical lengths is to throw negligible stresses into the diagonals as is borne out by the following calculations; whereas the stress diagram of Fig. 23, which is drawn for the truss of Fig. 24 having the theoretical and calculated heights, shows that there are no stresses existing in the diagonal members, when the U.D.L. completely covers the span.

To Find the Stress in Member *DE*, Fig. 25. "Take a section to cut three bars, including the desired bar, and take moments where the two unwanted bars meet." Consider the stability of that part of the girder which is to the left (or right if desired) of the section line *XX*, since it has fewer forces acting upon it. These forces are R_L and 1^T at *A*, panel loads of 2^T each at *B*, *C* and *D*, and, in addition, three unknown forces acting in each of the cut members *NO*, *NE* and *DE*. Now, as the structure is at rest, the sum of the moments of all the acting forces about any point must come to zero, which is the third law of statics, viz., $\Sigma M = 0$. Further, the moment of a force about a point is the product of the force and its perpendicular lever arm, and if moments be taken at point *N*, the point of intersection of the unwanted and unknown forces *NO* and *NE*, then these forces can have no moments as the lever arms are nil. Two of the three unknown forces are thereby eliminated from the moment equation. Calling clockwise moments positive, then

$$R_L \times 45' - 1^T \times 45' - 2^T(30' + 15' + 0) = \text{force } DE \times ND + \text{force } NO \times 0 + \text{force } NE \times 0.$$

$$(8^T - 1^T)45' - 2^T(45') = DE \times 14' \text{ whence } DE = 16{\cdot}07^T$$

The preponderance on the left-hand side of the equation of the plus sign indicates that the part of the structure shown in heavy line tends to rotate clockwise round the imaginary pin placed at N, due to the overwhelming turning moment of R_L about N. This turning moment tends to extend the restraining bar DE and therefore member DE is in tension. The stress in this bar would have been 16^T if the verticals had not been altered in height from those of Fig. 24.

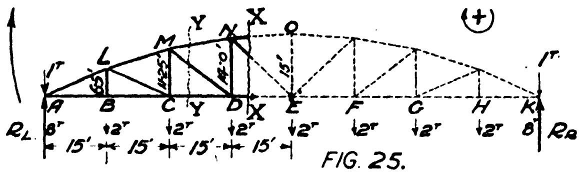


FIG. 25.

Stress in CD. Section line is YY and moment centre where the two unwanted bars MD and MN meet is M, Fig. 25.

$$(8^T - 1^T)30' - 2^T \times 15' = CD \times 11{\cdot}25 \text{ whence } CD = 16^T$$

i.e., approximately the same as DE. The excess on the left-hand side of the plus sign again indicates that the tendency to rotate is in a clockwise direction round M so pulling the restraining member CD, which thus carries a tensile stress of 16^T.

Stress in MD. Take moments at point S, Fig. 26, where the two unwanted bars NM and DC meet, thereby eliminating them from the moment equation when considering the stability of the heavy lined portion of the structure.

The position of point S together with the lengths ST and SA can be obtained graphically, but only approximately. If accurate results are required these lengths must be calculated.

To find the length SA compare the similar triangles, NDS and MCS.

$$\frac{ND}{MC} = \frac{DS}{CS}, \textit{i.e.}, \frac{14}{11{\cdot}25} = \frac{45 + AS}{30 + AS} \qquad \text{whence } AS = 31{\cdot}36'$$

Length MD of the right angle triangle MCD

$$= \sqrt{(MC^2 + CD^2)}$$
$$= \sqrt{(11{\cdot}25^2 + 15^2)}$$
$$= 18{\cdot}75'$$

To find ST compare the similar triangles, SDT and MDC.

$$\frac{ST}{SD} = \frac{MC}{MD}, \textit{i.e.}, \frac{ST}{76{\cdot}36} = \frac{11{\cdot}25}{18{\cdot}75} \qquad \text{whence } ST = 45{\cdot}8'$$

Taking moments at S **and giving the positive sign to clockwise moments :—**

$$-(8^{T}-1^{T})\times SA+2^{T}(SB+SC)=\text{Stress } MD\times ST.$$

i.e., $-7^{T}\times 31\cdot36'+2^{T}(46\cdot36'+61\cdot36')=MD\times 45\cdot8'$

i.e., $(-219\cdot52+215\cdot44)\div45\cdot8=MD$ whence stress $MD=0\cdot09^{T}$

The portion of the structure tends to turn anti-clockwise round the imaginary pin S, but is restrained by the bar MD. The point M tries to move away from D so lengthening bar MD, which must, therefore, be under tension.

This negligible stress would not have occurred if the verticals had been of the correct parabolic heights.

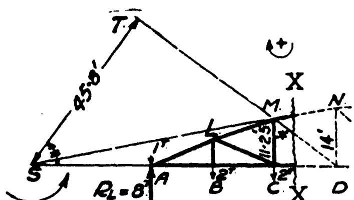


FIG. 26.

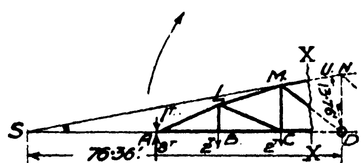


FIG. 27.

Stress in MN, section line is XX and moment centre is D, Fig. 27.

$(8^{T}-1^{T})\times DA-2^{T}(DB+DC)=\text{Stress in } MN\times UD$. (a)

To find the length of DU compare the similar triangles UND and DNS.

$\frac{UD}{DN}=\frac{DS}{SN}$, *i.e.*, $\frac{UD}{14}=\frac{76\cdot36}{77\cdot54}$, since $SN=\sqrt{(SD^2+DN^2)}=77\cdot54'$

Hence $UD=13\cdot8'$

Substitute in equation (a) above

$7^{T}\times45'-2^{T}(30'+15')=MN\times13\cdot8'$ whence stress $MN=16\cdot3^{T}$

The final moment about point D is a clockwise one and tends to make point M travel towards N so shortening MN, which is, therefore, in compression.

Stress in ND, section line is XX and the two unwanted bars NM and ED meet, when produced, at the moment centre S, Fig. 28.

$$-(8^{T}-1^{T})\times31\cdot36'+2^{T}(46\cdot36'+61\cdot36'+76\cdot36')=\text{Stress } ND\times76\cdot36'$$

$$-219\cdot5+368\cdot16=ND\times76\cdot36 \text{ or Stress } ND=1\cdot95^{T}$$

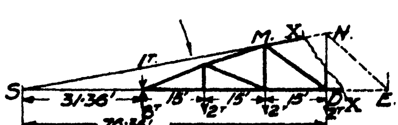


FIG. 28.

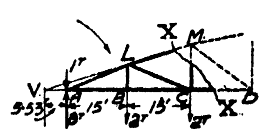


FIG. 29.

The unbalanced turning moment on the left-hand side of the above ɿuation is clockwise, which means that D is being swung away from ', thus extending bar ND whose stress of $1{\cdot}95^{\text{T}}$ is therefore tensile.

Stress in MC, Fig. 29. The unwanted bars ML and DC when ׀roduced meet at point V which lies 5·53′ by calculation from A.

$$-(8^{\text{T}}-1^{\text{T}})\times 5{\cdot}53+2^{\text{T}}(20{\cdot}53'+35{\cdot}53')=\text{stress } MC\times 35{\cdot}53'$$

ᵛhence stress MC is a tensile one of $2{\cdot}06^{\text{T}}$.

PARALLEL FLANGED GIRDER, FIG. 30

The stresses acting in the top and bottom flanges can easily be ›btained by section and moments. Thus the section line XX of Fig. (a) gives point 4 as the moment centre for bar 3,5 and point 5 for the moment centre of bar 4,6. The moment equation for both of these bars will be the same, viz., $5^{\text{T}}\times 20'-2^{\text{T}}\times 10'=\text{Stress}\times 6'$, whence the value of the stress is $13{\cdot}33^{\text{T}}$; *i.e.*, flange members between the same pair of diagonals have the same amount of stress.

The end bar of the bottom flange has no stress (see also Fig. (c)) because the reaction of 5^{T} passes through the moment centre 1, therefore there is no moment about point 1, and, in consequence, no stress in the bar shown in broken line. This bar, since it could be safely left out, is known as a redundant member.

Web Members. The method of taking a section to cut three bars, including the wanted bar, 4,5, Fig. 30 (a), does not work out quite so neatly as it did with the previous girder with non-parallel flanges, because the two unwanted bars, 3,5 and 4,6, will never meet. However, since the stress in 3,5 has been found, its value can be substituted in the appropriate moment equation. The stress in the lower flange cut by section XX can be eliminated from the moment equation by taking a moment centre at some point along its length—no lever arm, no moment.

Regarding the section line XX of Fig. (a), only the forces and that part of the structure to the left of this section line may be taken into account when considering the equilibrium of the broken part of the structure, whereas the moment centre may be taken anywhere, left or right, above or below the section line. Further, since the compression bar 3,5 has been cut the thrust acting on this bar must be applied at its broken end as a push of $13{\cdot}33^{\text{T}}$. Taking point 6 on the bottom flange as the moment centre and calling clockwise moments positive, then

$$5^{\text{T}}\times 30'-2^{\text{T}}\times 20'-\text{Stress of }13{\cdot}33^{\text{T}}\times 6'=\text{Stress in }4{,}5\times 10'.$$

i.e., $150-120=\text{Stress in }4{,}5\times 10$, whence stress is 3^{T} compression since the tendency, due to the moments, is for point 4 to approach point 5.

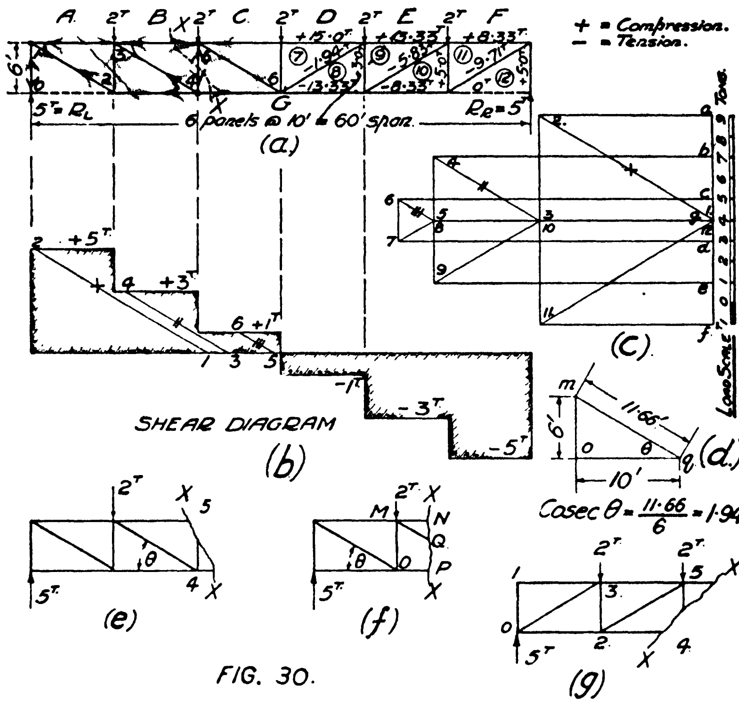


FIG. 30.

Vertical Web Members. A simpler method of solution than the foregoing is obtained by considering the shearing force at any section.

The stress in the end vertical, over the reaction, is 5^T compression because the reaction travels undiminished up to point 1. Thereafter from 1 to 2, down the diagonal, no additional force is encountered, not even at panel point 2, and so the upward shear or thrust in bar 2,3 is 5^T compression. When point 3 is actually reached the upward force or shear is reduced by 2^T leaving an unbalanced upward force of 3^T to travel, as it were, down diagonal 3,4. Since no additional force is met with at point 4 the upthrust on bar 4,5 is 3^T compression. When point 5 is reached there requires to be deducted the downward panel load of 2^T leaving 1^T upwards when the path from 5 to 6 is resumed. Similarly with the right-hand reaction there is an upward force which is gradually reduced until point 6 is reached with an unbalanced load upwards of 1^T. It thus happens that there are two tons travelling, so to speak, upwards in the centre bar, which, therefore, is under compression to that extent. When the upper centre panel point is reached the two tons upwards balances the two tons downwards. This can also be seen from the fact that, as the top boom is at right angles to the mid-panel load of 2^T, the only bar which can directly resist this vertical load is the mid-vertical.

The shear diagram of (b) also shows this, but beginners have difficulty, sometimes, in ascertaining as to which panel shear a vertical participates in ; *e.g.*, bar 2,3 of diagram (a), as to whether it belongs to the first panel shear of 5^T or to the second panel shear of 3^T.

The stress in the vertical is therefore the shear in the corresponding panel. The nature of the stress is found by taking a section to cut three bars such as XX in (e), and finding whether the unbalanced shear acting on the smaller portion of the structure tends to make point 4 move towards point 5, *i.e.*, compression of $5^T - 2^T = 3^T$, or, as in Fig. (g), tends to make point 5 move upwards and away from point 4 due to the upward force of $5^T - 2^T - 2^T = 1^T$ tension, because bar 4,5 is being extended.

Web Diagonals. There is no difficulty in assigning the proper shear values to the diagonals. Diagonal 1,2 is under the 5^T vertical shear of the first panel, while diagonal 3,4 of Fig. (a) is under the panel shear of 3^T, and so on.

Again, take an imaginary section to cut three bars as in Fig. (f). Now the external forces of 5^T and 2^T cause an unbalanced upward shear of 3^T. The horizontal flanges can offer no direct resistance to this force, because they are at right angles to its line of action, and, therefore, it is left entirely to the diagonal member MQ to withstand this upward thrust which is pulling at MQ and so placing it under tension. By the first law of statics, $\Sigma V = 0$, the vertical components of all the forces acting on the body of Fig. (f) must sum to zero,

viz., $5^T - 2^T =$ vertical component of force in MQ. . (a)

i.e., vertical shear in panel
= " " " . . (b)

Turn now to Fig. (d), where the force in the member MQ, of diagram (f), is drawn to scale as mq and it is seen that its vertical component is om. Hence

$$\frac{mq}{mo} = \frac{\text{stress in diag.}}{\text{vert. comp. of ditto}} = \frac{11\cdot66'}{6'} = 1\cdot94 = \text{cosec}\,\theta \quad . \quad . \quad \text{(c)}$$

or stress in diagonal $= \text{the vert. comp.} \times \text{cosec}\,\theta$ . . (d)

$= \text{the vert. shear} \times \text{cosec}\,\theta$ by (b) (e)

To obtain, therefore, the stress value in any diagonal multiply the panel shear by the length of the diagonal and divide by the effective girder depth.

Another proof of this is given by the shear curve (b). The stress in the diagonal 2,1 of Fig. (a) is obtained by drawing in the shear curve (b) a line parallel to the diagonal. Refer now to the stress diagram (c) and it will be observed that the stress 2,1 of (b) is parallel and equal to the line 2,1 of the stress diagram (c), as both diagrams have been drawn to the same scale. Similarly any line drawn in the shear curve at the + 3T portion parallel to the bar 4,3 will give the actual stress 4,3, to the same scale as the shear curve, and this in turn is equal to the length 4,3 of the stress diagram; both lengths are marked with double parallel ticks. Finally, any line in the + 1T portion of the shear curve drawn parallel to 6,5 gives the stress in diagonal 6,5. This is simply a graphical method of multiplying the shearing force by cosec θ, *i.e.*, multiplying the shear by the length of the diagonal and dividing by girder depth.

CHAPTER V

THE EFFECT OF CROSS GIRDERS AND STRINGERS

MAXIMUM LOAD ON A CROSS GIRDER, FIG. 31

A UNIT load situated at x_1 from A creates a reaction at cross girder C of $1^T \times x_1 \div L_1$. When x_1 is 0 then R_C is 0 and when x_1 is L_1 then $R_C = 1^T$. The resulting influence line for the reaction R_C of span L_1 is a straight line rising from 0 at A up to 1^T at C. Similarly the influence line for reaction C of span L_2 is the straight line BD, where CD is also equal to 1^T, and the final combined influence line for R_C is the triangle ADB of apex height 1^T and basal length $L_1 + L_2$.

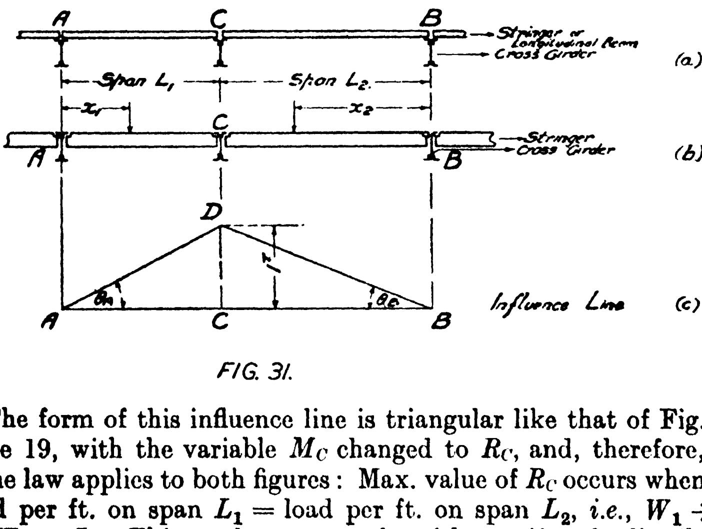


FIG. 31.

The form of this influence line is triangular like that of Fig. 17, page 19, with the variable M_C changed to R_C, and, therefore, the same law applies to both figures: Max. value of R_C occurs when the load per ft. on span L_1 = load per ft. on span L_2, *i.e.*, $W_1 \div L_1 = W_2 \div L_2$. This works out exactly with a uniformly distributed load, but with a series of concentrations it usually happens that one of the wheels must be placed at point C and part of this wheel load considered as belonging to span L_1 and the remainder to L_2 in order to satisfy the mathematical interpretation of the law.

THE EFFECT OF CROSS GIRDERS ON SHEAR

The plate girder of Fig. 32 (a) is hollowed out at CD, and within this hollow is inserted a tiny independent girder or stringer of span CD. The one ton wheel load runs directly upon the top surfaces of both main and subsidiary girders.

Unit Load between B and D

The reaction at A, R_A, is $1^T \times x \div L$ $= x/L$

As previously discussed, the shear throughout the segment AD of the main girder $= R_A$

Therefore the shear in the particular panel CED of the main girder also $= x/L$

Unit Load between A and C

Now measure z from A towards C, but $z \ngtr AC$

Again it follows that the shear in the unloaded segment $CB = -R_B$ $= -z/L$

Therefore the shear in the particular panel CED of the main girder also $= -z/L$

So far, then, the parts of the influence line which can be traced out are Bd' and Ac' of diagram (b), where the ordinates at d and c are 1/5 and $-$ 3/5 respectively.

Unit Load on the Stringer DC

During the passage of the wheel from D to C the main girder receives its load at two fixed points of application, viz., at the wedge-shaped panel points or bearings C and D, instead of the one point of contact under the wheel, of varying position.

In diagram (c) the load has to travel the remaining length DC, *i.e.*, y varies from 0 to p in length. The local reaction on the main girder, $R_C = 1^T \times y \div p = y/p$, and, taking moments at C on the stringer, the reaction $R_D = 1^T(p - y) \div p$.

Considering the main girder as a complete unit the reaction at B, due to the unit load placed as in (c), is

$$R_B = 1^T(4p - y) \div 5p$$

and

$$R_A = 1^T(p + y) \div 5p.$$

Alternatively, the same values for R_A and R_B could have been obtained by considering the 1^T entirely removed and replaced by the local loads R_C and R_D on the main girder. For the load as shown in (c) the shear at any point within the panel CD of the main girder is

$$R_A - R_C = \frac{p + y}{5p} - \frac{y}{p} = \frac{p - 4y}{5p}$$

Since y is the only variable, this expression represents a straight line, whose two terminal values are :—

$$1\text{st}: \text{load at } D\,;\ y = 0 \text{ then } S_{CD} = \frac{p - 0}{5p} = \frac{1}{5} = dd'$$

$$\text{and 2nd}: \text{load at } C\,;\ y = p \text{ then } \quad ,, \quad = \frac{p - 4p}{5p} = -\frac{3}{5} = cc'$$

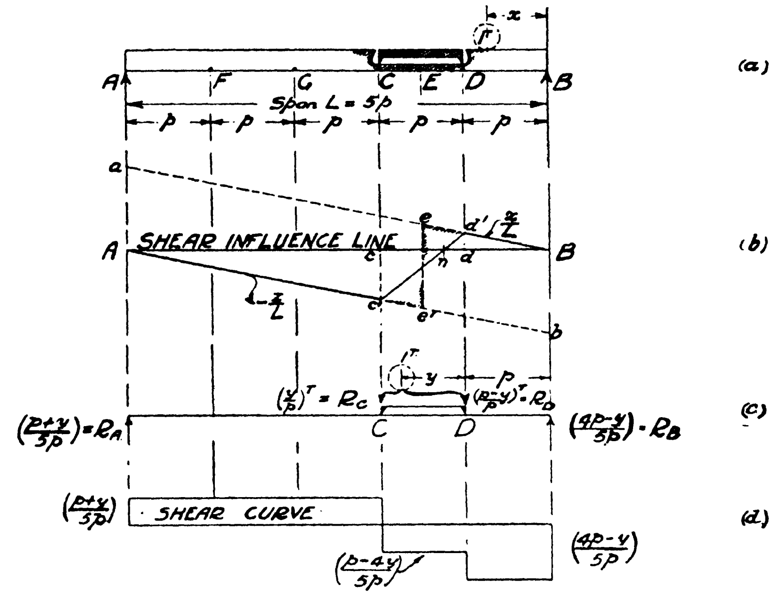


FIG. 32.

But these values are, respectively, also the higher terminal values of the portions of the influence line traced out above, so that the complete influence line for shear within the panel CD of the main girder is $Ac'nd'B$.

The dotted and hatched influence line shown for comparison in (b) refers to point E of an ordinary solid plate girder (without an inserted stringer) in direct contact with the travelling unit load throughout the span.

The shear diagram of (d), drawn for the one position of the load indicated in (c), clearly shows what is meant by the term " shear is constant throughout the panel." If the load position, however, is altered by the merest fraction of an inch the shear curve immediately also alters its shape, and hence the reason of an influence line for moving loads.

NEUTRAL POINT

The point n, where $d'c'$ of the influence line of Fig. 32 (b) crosses the base line AB, is called the neutral point, because a load placed at this point n on stringer CD would cause no shear whatsoever in the panel CD of the main girder.

Triangles cnc' and dnd' are similar, $\therefore \frac{cn}{dn} = \frac{c'n}{d'n}$ . . . . . (1)

Triangles $Ac'n$ and $Bd'n$ are similar, $\therefore \frac{An}{Bn} = \frac{c'n}{d'n}$ . . . . . (2)

It thus follows that $\frac{cn}{dn} = \frac{An}{Bn}$ . . . . (3)

That is, for a parallel flanged girder "the neutral point divides the span in the same ratio as it divides the panel."

The diagram shows a bridge of $5p$ in span, hence, substituting the value of p in (3), $\frac{cn}{p - cn} = \frac{3p + cn}{2p - cn}$

$\therefore \quad 2p.cn - (cn)^2 = 3p^2 - 3p.cn + p.cn - (cn)^2$, *i.e.*, $cn = \frac{3}{4}p$.

Verify by placing unit load at point n on stringer DC, then the local reaction $R_C = \left(1^T \times \frac{p}{4}\right) \div p = \frac{1}{4}^T$ and $R_D = \frac{3}{4}^T$.

Consider the bridge as a complete unit with a 1^T load placed at point n on span AB, then

$$R_A = (1^T \times nB) \div L = \left(1^T \times \frac{5p}{4}\right) \div 5p \qquad = \tfrac{1}{4}^T,$$

and $$R_B = (1^T \times An) \div L = (1^T \times 3\tfrac{3}{4}p) \div 5p \qquad = \tfrac{3}{4}^T$$

Hence for this one position of unit load on the span the shear in panel CD of the main girder $= R_A - R_C = \frac{1}{4}^T - \frac{1}{4}^T = 0$.

The shear in the *stringer*, however, is not zero, but is equal to R_C of $\frac{1}{4}^T$ between C and n and to R_D or $- \frac{3}{4}^T$ between n and D.

MAXIMUM SHEAR IN A PANEL

For convenience of proof the total load W on the span has been divided into three portions, so that W = loadings $A + B + C$.

In Fig. 33, since Fg is parallel to fG, then

$tan\ \theta_1 = tan\ \theta_3 = Ff \div FG = 1 \div \text{span} \qquad = 1/L$ . . (1)

$$Tan\ \theta_2 = \frac{dD}{Cd} = \frac{De - ed}{p} = \frac{1 - ed}{p} = \frac{1 - Cd\ tan\ \theta_1}{p}$$

[and giving *tan* θ_1 its value from (1)]

$$= \frac{1 - p \times \frac{1}{L}}{p} = \frac{L - p}{pL} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The total shear in panel $HJ = \Sigma$ loads $\times$ respective ordinates,

$$i.e., S = A_1a_1 + A_2a_2 + \ . \ . \ . + B_1b_1 + B_2b_2 + \ . \ . \ .$$
$$+ C_1c_1 + C_2c_2 + \ . \ . \ . \ . \ . \quad (3)$$

If an arithmetical result is desired for S then the ordinates should be assigned their proper signs, positive or negative, and arithmetical values.

Now move the load system, as a complete rigid unit, a very small distance δx to the left, or right, of its present position so that no wheel enters or leaves the span. This small movement will cause each ordinate in segment A to be altered in length by exactly the same amount δa; those in segment B by δb; while the C segment ordinates will each have the same change in height of δc. The shear in panel HJ is now

$$S + \delta S = A_1(a_1 + \delta a) + A_2(a_2 + \delta a) + \ . \ . \ . \ B_1(b_1 + \delta b)$$
$$+ B_2(b_2 + \delta b) + C_1(c_1 + \delta c) + C_2(c_2 + \delta c) + \ . \ . \ . \ . \ . \quad (4)$$

Equations (4) — (3) will give the value of the increment of shear,

$$\text{viz., } \delta S = \delta a(A_1 + A_2 + \ . \ . \ .) + \delta b(B_1 + B_2 + \ . \ . \ .) +$$
$$\delta c(C_1 + C_2 + \ . \ . \ .) \quad . \quad . \quad (5)$$

Dividing each side of this equation by δx leaves the equation undisturbed,

$$i.e., \frac{\delta S}{\delta x} = \frac{\delta a}{\delta x}(A) + \frac{\delta b}{\delta x}(B) + \frac{\delta c}{\delta x}(C) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$= tan\ \theta_1 \times A + tan\ \theta_2 \times B + tan\ \theta_3 \times C.$$

But line CD has positive slope while lines Fg and fG have negative slope, therefore *tan* θ_2 is positive and *tan* $\theta_1 =$ *tan* θ_3 is negative.

Now substitute these signs together with the values for the tangents obtainable from (1) and (2) whence :—

$$\frac{\delta S}{\delta x} = -\frac{A}{L} + \frac{B(L - p)}{pL} - \frac{C}{L}$$

$$= \frac{-Ap + BL - pB - pC}{pL} = \frac{-p(A + B + C) + BL}{pL} = \frac{-pW + BL}{pL} \quad (7)$$

The process of differentiating inferred by (7) will give a max. or a min. value when the right-hand side of (7) is equated to zero. The same result can be obtained as follows without direct recourse to the calculus.

Now if the different shear values in the panel were plotted against the corresponding different values for x, or position of the loading on the span, a curve something like that of Fig. (c) might be obtained. The slope of, or the tangent to, this curve at any point is given by $\frac{\delta S}{\delta x}$, *i.e.*, $\frac{\text{a small vertical increment}}{\text{correspond. horiz. increment}}$. Further, when

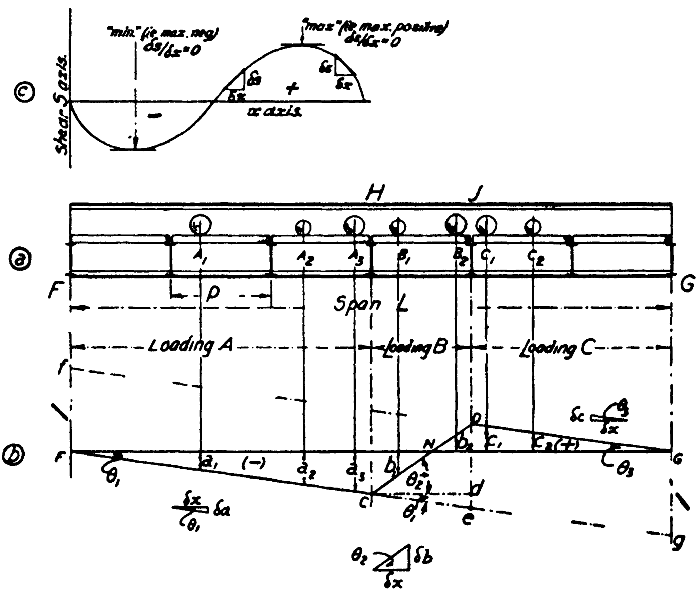


FIG 33.

the curve has achieved a turning point, or point of maximum or minimum value, this ratio of $\frac{\delta S}{\delta x}$, or tangent, is zero. Then considering expression (7) as an equation of condition it can be said that the loads on the span, Figs. (a) and (b), give maximum positive or negative shear in the panel HJ, *if* $\frac{\delta S}{\delta x} = 0$, *i.e.*, *if* $\frac{-pW + BL}{pL} = 0$. Therefore, maximum positive or negative shear occurs in HJ *if*

$$BL = pW \text{ or } if\ \frac{B}{p} = \frac{W}{L} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(a)}$$

$$\text{or } if\ B = W \times \frac{p}{L} = W \div \text{number of panels} \quad . \quad . \quad \text{(b)}$$

Rule for a train of wheels can now be stated: Max. shear (positive or negative) in any panel occurs when the load per ft. in this panel equals the average load per ft. for the complete span, condition (a). Alternatively: The load in the panel considered should be so arranged that it equals the total span load ÷ number of panels, by condition (b).

For a series of wheel concentrations crossing the span the application of the foregoing rules is very tedious and it is more expeditious and just as accurate to resort to trial and error. Usually, by placing heavy wheels where the ordinates are large and keeping wheels out of the segment of opposite shear sign, three trial positions of the load system are sufficient to indicate the position of the loading and the resulting maximum shear.

Single-wheel Load. From Fig. 33 place wheel at D for max. positive and at C for max. negative shear.

Double-wheel Vehicle. For max. + shear place the larger wheel at D and the smaller wheel near C_1; while max. negative shear is given by turning the vehicle round about, so that it faces in the opposite direction, with the larger wheel at point C and the smaller wheel in the region of A_2, depending on the wheel base or spacing.

Dead load covers all the span and the shear in HJ is given by adding the positive and negative triangular areas and multiplying the result by the load per ft.

U.D.L.L. (a) *Length Shorter than NG*

For max. positive shear place the load on segment NG in such a position that the line Dd divides the load in the same ratio as it divides length NG; see rule on page 20.

Max. negative shear: place on segment NF so that line CH divides the load in the same ratio as it divides NF.

U.D.L.L. (b) *Length Longer than Span*

Max. positive shear in HJ occurs with the head of the load at N, the section NG covered, and the remainder of the load on the approach to the right of G.

Max. negative shear in HJ takes place when NF only is loaded.

These positions of the loading satisfy the rule found for max. shear in a panel due to a train of wheel loads.

Condition (b) was—

load on panel HJ = total span load ÷ number of panels

i.e., $NJ \times w^{T}/\text{ft.} = NG \times w^{T}/\text{ft.} \div 6$.

By calculation the neutral point N is so situated that $HN = 0{\cdot}6p$ and $NJ = 0{\cdot}4p$. $\therefore\ 0{\cdot}4\,pw = 2{\cdot}4\,pw \div 6 = 0{\cdot}4\,pw$.

WEB STRESSES

Plate Girder

All that is required when designing the web plate of a plate girder is the maximum arithmetical value of the vertical shear. It is immaterial whether the shear is positive or negative. Example: In a plate girder the depth over the main angles is 4′ 0″; shear is 90T and the working shear stress is 5T/sq. in. To find the web plate thickness "t."

Web area of $48'' \times t''$ at 5T/sq. in. = 90T

whence $t = 90 \div (48 \times 5) = \frac{3}{8}''$

Braced Girders

Examples, Figs. 34 and 35, Pratt and Warren trusses. The Pratt truss, of which one type is shown, has its diagonal members in

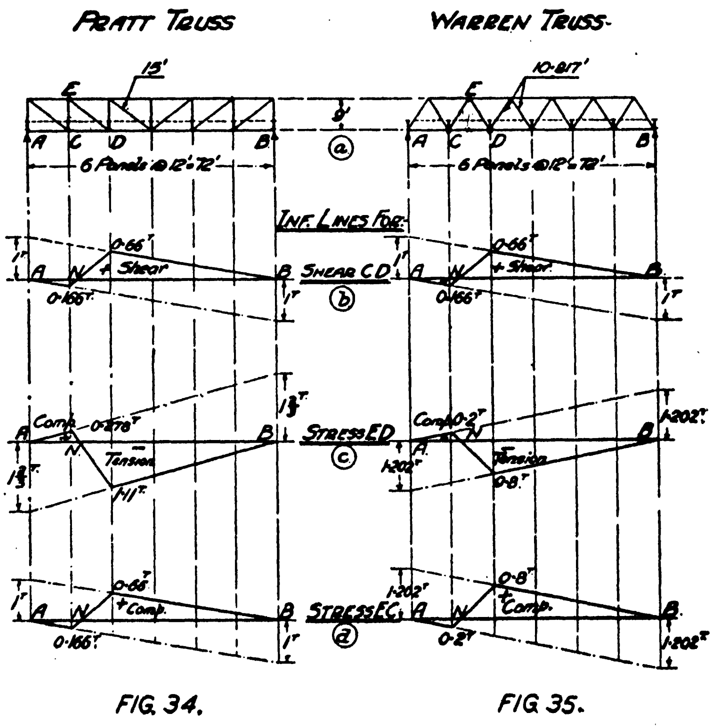


FIG. 34. FIG. 35.

tension and vertical members in compression under dead load. This truss was patented in U.S.A. by Caleb and Thomas Pratt in 1844. The Warren truss web system is composed solely of diagonals which are under tension or compression, alternately, on both sides of the centre line due to dead load; this truss first came into prominence about 1870.

With braced girders both positive and negative shear values are required. Further, in the pages devoted to "Section and Moments" it was shown that the stresses in the diagonals were functions of the vertical shear.

Thus stress in either diagonal ED = shear in panel × (diagonal length ÷ girder depth)

For the Pratt truss stress ED = shear × (15 ÷ 9)
= 1·67 × shear

While the Warren member ED = shear × (10·817 ÷ 9)
= 1·202 × shear.

In both trusses positive shear places bar ED in tension, so multiply the ordinates to the shear influence lines by 1·67 and 1·202, respectively, and plot the results under the base line for negative sign of stress, *i.e.*, tension. Negative shear, obtained by loading length AN, causes compression in bar ED of both trusses and the influence line values are plotted above the base line in both figures marked (c).

Member EC of the Pratt truss, being vertical, has its stress numerically equal to that of the vertical shear and as positive shear creates a compressive or positive stress in EC the influence line for stress of Fig. (d) is identical with that for shear in (b).

Member EC of the Warren truss, not being vertical, has its stress numerically equal to 1·202 × vertical shear, as for the adjoining bar ED. Positive shear creates positive or compressive stress in Warren member EC, and it thus follows that Fig. (d) is a distorted image of (b) to another scale, instead of being identical with it as was the case with the Pratt truss.

Maximum values of stress can now be directly obtained by applying the rules, newly evolved above, for maximum shear.

THE EFFECT OF CROSS GIRDERS ON BENDING MOMENT

Pratt Truss and Plate Girder. Moment Influence Lines, Fig. 36

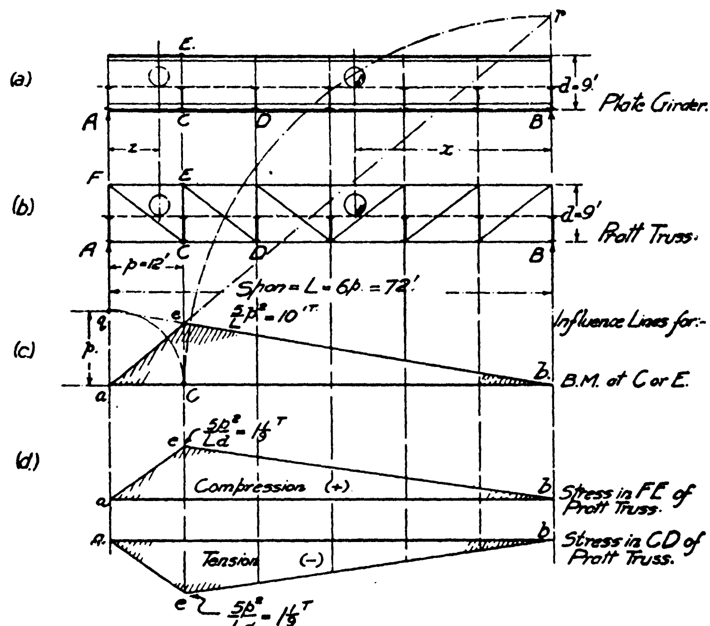
With unit load travelling from B towards C, but not past C, then R_A $= x/L$ (1)

For vertical loads the bending moments at points C and E are identical since they are in the same vertical, *i.e.*, $M_C = M_E = R_A \times AC$ $= px/L$ (2)

This expression of (2) represents a straight line because x is the only variable.

At B the value for x is 0, so that $M_C = M_E \qquad = 0$ (3)
At C ,, ,, $5p$, ,, $M_C = M_E \qquad = 5p^2/L$ (4)
Considering xp/L as simply the mathematical equation of a straight line (so forgetting its connection with influence lines) its value when $x = L$ would be $\qquad = p$ (5)

The ordinate at a would thus be $aq = p$. This value is purely imaginary (since it was specifically stated that $x \ngtr BC$) and so also is the length qe.



NOTE. With regard to the PLATE GIRDER the Influence Lines for:—
B.M. at C or E. is given by diagram (c)
Comp. stress at point E " " " " (d)
Tensile " " " C " " " " (e).

FIG.36.

Let unit load traverse the remaining portion of the span from C to A, *i.e.*, z varies from p to 0 in value.

$M_C = M_E = R_B \times 5p = (1^T \times z \times 5p) \div L \qquad = 5pz/L$ (6)
Unit load at A means that $z = 0$ and therefore $M_C = 0$ (7)
while load at C gives z a value of p and $M_C = 5p^2/L$ (8)

The moment caused by unit load at C is satisfied by both the straight lines given by (2) and (6) and therefore these lines must meet at point e to form the triangular influence curve of aeb.

If z be given the hypothetical value of full span travel of L expression (6) would have the imaginary value of $5p$ (9)
which is the ordinate br in the figure.

This results in the old form and graphical construction of the B.M. influence line which was previously obtained for a beam carrying the wheels in direct contact with the upper flange. This construction offers no advantages and it is better simply to calculate the moment when unit load is applied at point C and to erect ce to represent this moment to any suitable scale.

Pratt Truss. Flange Stresses, Fig. 36

In the truss the stress in FE is desired while FC and AC are the unwanted bars meeting at C, which is, therefore, the moment centre. From the discussion given in " Section and Moments " the

$$\text{Stress in } FE = \text{moments taken at } C \div \text{perp. lever arm } EC$$
$$= M_C \div d, \text{ and is compression.}$$

Similarly the stress in CD is given by $M_E \div d$ and is tension. The influence line for stress, therefore, is given by dividing the moment influence line by the girder depth, or, alternatively, the stress influence line is simply the moment influence line to another vertical scale.

Plate Girder. Flange Stresses

The bending moment diagram of Fig. 37 is for the main plate girder carrying a U.D. load of 1T/ft. run. The stress at any point on the top or bottom flange is obtainable by dividing the B.M.

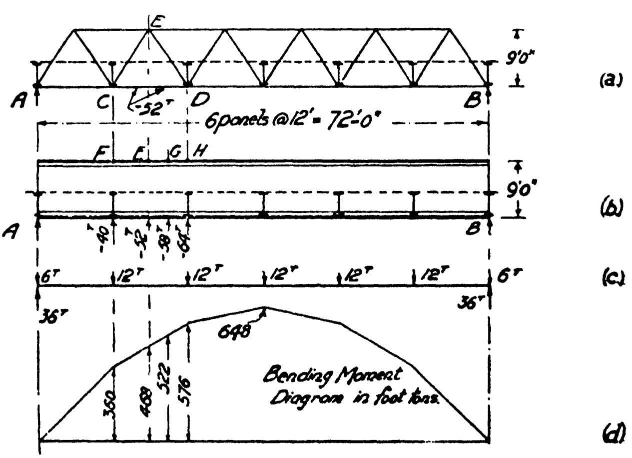


FIG. 37.

occurring at the selected point by the girder's effective depth. It is apparent that the maximum stress occurs at a cross girder, or panel point, and never between such points, and it is for this reason that the influence lines drawn for moment or stress are for the panel points as in Fig. 36. If for any reason, however, the flange stress for some such intermediate point between cross girders as E or G were required the influence line would be of the truncated type of Fig. 39; the apex of the truncated triangle lying on the line through the flange point considered.

With a braced girder, on the other hand, the moment centre is constant in position for a complete panel length of either top or bottom boom. In Fig. 37 the stress of 52^{T} is constant throughout the panel, whereas the plate girder has a total flange stress varying from a minimum of 40^{T} to a maximum of 64^{T} in exactly the same panel length.

The influence lines of Fig. 36 apply then not only to the Pratt truss, but also to the specified points on the flanges of the plate girder.

Warren Truss. Influence Lines with Moment Centres on Bottom Flange, Fig. 38

The moment centre for member FE is point C. Permit unit load to travel between B and C, *i.e.*, x varies from 0 to $5p$.

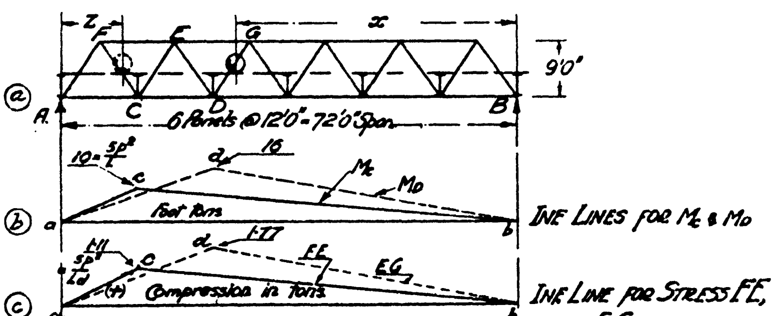


FIG 38.

$M_C = R_A \times AC = (1^{T} \times x \div L) \times p \qquad = px/L \quad (1)$

With load at B the minimum ordinate to this straight line is $\qquad = 0 \quad (2)$

while maximum ordinate occurs when $x = BC = 5p$, viz. $\qquad = 5p^2/L \quad (3)$

Similarly with z as the variable in the remaining length AC, $M_C = R_B \times BC$; giving a minimum ordinate at A $\qquad = 0 \quad (4)$

and a max. ordinate at C of $(z/L) \times BC$ $= 5p^2/L$ (5)

The ordinate at d for the dotted influence line for
$M_D = (4p \div L) \times 2p$ $= 8p^2/L$ (6)

These are identical with the influence lines obtained for the corresponding points on the Pratt truss and plate girder of Fig. 36.

The stress in EF is compression, and is equal to $M_C \div$ perp. lever arm from C to $FE = M_C \div d$, thus providing the stress influence of diagram (c).

Warren Truss. Influence Lines with Moment Centres on Top Flange

In Fig. 39, x is the variable distance as the load travels between B and D, while point E is the moment centre for stress in bar CD.

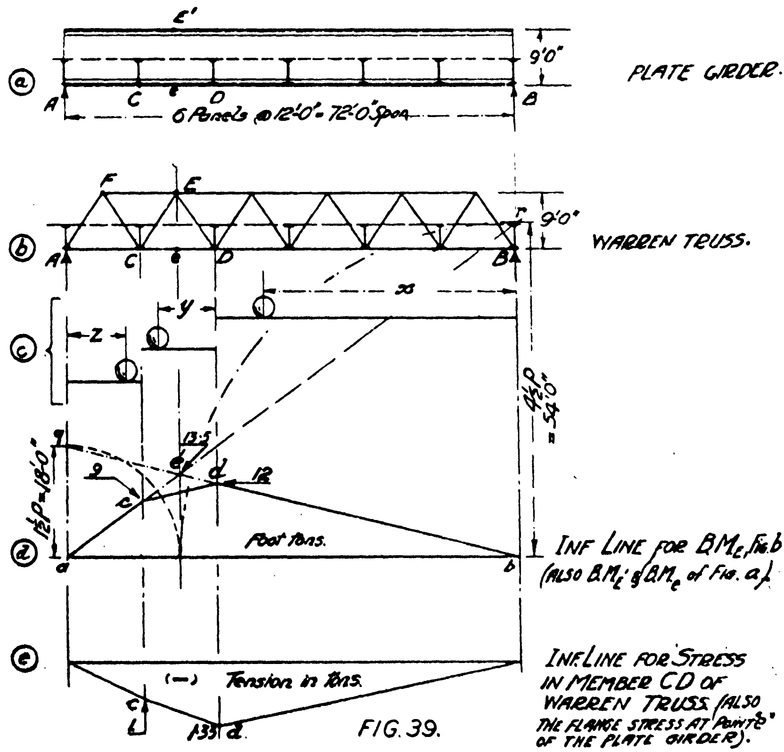


FIG. 39.

Now $R_A = x/L$ and $M_E = R_A \times Ae = (x/L) \times 1\frac{1}{2}p$ $= 1\frac{1}{2}px/L$ (1)

At B the ordinate to this straight line is 0 and at
D it is $= 6p^2/L$ (2)

If the hypothetical value of L be used for x the ordinate at A would be $= 1\frac{1}{2}p$ (3)
so giving the previous construction for aq.

Unit load now travelling from A to C but not beyond C gives $M_E = eB \times R_B = 4\frac{1}{2}p \times z/L$ with a max. value of $= 4\frac{1}{2}p^2/L$ (4)
and an imaginary value at B (when $z = L$) of $= 4\frac{1}{2}p$ (5)

It should be noted that in (1) and (4) the M_E was of the form, $R \times$ distance, with nothing deducted as happens when the load lies between C and D. When the single wheel moves from D to C it does so on a stringer (or tiny internal bridge parallel to the main girder) which causes local reactions at C and D and, therefore, the bending moment $M_E = Ae \times R_A - Ce \times R_C$. The value for R_A, with y as shown, is $R_A = 1^T(4p + y) \div L$ and local $R_c = 1^T \times y \div p$,

$$\text{whence } M_E = Ae \times (4p+y)/L - Ce \times y/p = \left(\frac{3p}{2} \times \frac{4p+y}{L}\right) - \left(\frac{p}{2} \times \frac{y}{p}\right)$$

$$= \frac{12p^2 + 3py - Ly}{2L} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Since the variable y is only to the first power, equation (6) definitely shows that the influence law between points C and D is an absolutely straight line like cd and not a bent line like $ce'd$. If equation (6) is correct it should have the same terminal values as the inner ends of lines ac and bd, already drawn, *i.e.*, when the load is at C, $y = p$ and (6) becomes

$$\frac{12p^2 + 3p^2 - Lp}{2L} = \frac{4\frac{1}{2}p^2}{L} \text{ as in (4)}$$

while for point D, $y = 0$ and (6) becomes

$$\frac{12p^2}{2L} = \frac{6p^2}{L} \text{ as in (2).}$$

This definitely proves that the influence line for a top flange moment centre, such as point E, is a truncated triangle due to the fact that the upper moment centre lies between the cross girders and not vertically above one as with the Pratt truss.

The stress influence diagram immediately follows, since its ordinates are those of the moment influence diagram divided throughout by the girder depth d. The arithmetical values for points c and d are given on the drawing.

MAXIMUM FLANGE STRESS IN A PANEL

When the influence line for moment or stress for a braced girder is of the triangular form, as given by Fig. 36 (c), (d) and (e), and

Fig. 38 (b) and (c), the rule for the position of the load on the span to give maximum effect is that previously enunciated for this shape of influence curve, viz., the load per ft. to the left hand of the moment centre should equal the load per ft. to the right. For a uniformly distributed load crossing the span the bridge should be completely covered by the load, but if the load is shorter than the span then the load should be so placed that the vertical through the apex of the influence triangle divides the load in the same ratio as it divides the span, *i.e.*, load/ft. on left = load/ft. on right, see page 20.

With the truncated form of influence line of Fig. 39 the above rule does not hold and the requisite rule is that derived under.

In Fig. 40 split the load into three portions, calling that portion over the member considered the B portion, and the remaining two parts, the A loading and the C loading, *i.e.*,

$$W = A + B + C \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

For the loads in any position the stress or the bending moment, as the case may be, is $M = \Sigma$ loads $\times$ respective ordinates

$$= A_1a_1 + A_2a_2 + \ . \ . \ . \ B_1b_1 + B_2b_2 + \ . \ . \ . \\ C_1c_1 + C_2c_2 + \ . \ . \ . \ . \ . \quad (2)$$

As explained on page 45, an extremely small movement of the complete load system through a distance δx will give a new bending moment of

$$M + \delta M = A_1(a_1 + \delta a) + A_2(a_2 + \delta a) + \ . \ . \ . \\ B_1(b_1 + \delta b) + \ . \ . \ . \ C_1(c_1 + \delta c) + \ . \ . \ . \ . \ . \quad (3)$$

$$(3) - (2) = \text{increment } \delta M$$

$$= \delta a(A_1 + A_2 + \ . \ . \ .) + \delta b(B_1 + B_2 + \ . \ . \ .) \\ + \delta c(C_1 + C_2 + \ . \ . \ .) \quad . \quad . \quad (4)$$

As for equation (6), page 45, dividing throughout by δx gives

$$\frac{\delta M}{\delta x} = \frac{\delta a}{\delta x}(A) + \frac{\delta b}{\delta x}(B) + \frac{\delta c}{\delta x}(C) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$= \tan \theta_1 . A + \tan \theta_2 . B + \tan \theta_3 . C \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

and giving proper signs to the tangents

$$= \tan \theta_1 . A + \tan \theta_2 . B - \tan \theta_3 . C \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Now refer to Fig. 40 (c) and let the vertical ordinates represent the various values of the bending moment (instead of shear as in Fig. 33 (c)) plotted against the corresponding various distances of x across the span. Once again it is seen that when the curve has reached a maximum value, or turning point, the ratio, slope or tangent, $\delta M/\delta x = 0$. Hence the equation of condition now is that there is maximum moment at a point on the span *if* $\delta M/\delta x = 0$ (8)

or *if* $tan\ \theta_1 . A + tan\ \theta_2 . B - tan\ \theta_3 . C = 0$ . . . . (9)

,, $tan\ \theta_1 . A + tan\ \theta_2 . B = tan\ \theta_3 . C$ . . . . . (10)

If the load can be placed on the span to satisfy condition (10) then the load is in that position which creates maximum bending moment or stress. Each bottom panel of the Warren truss will have its own set of values for the three tangents.

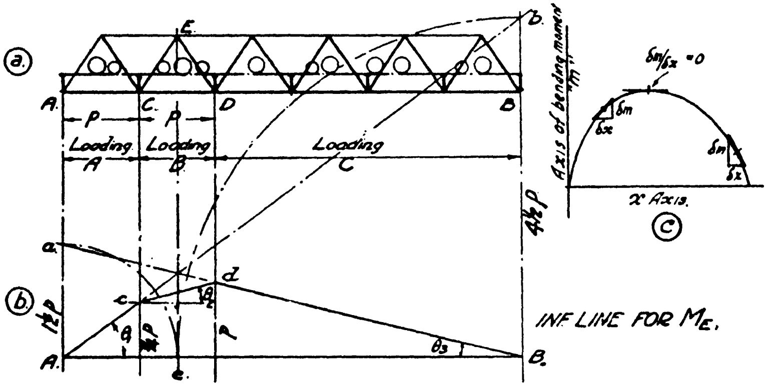


FIG. 40.

As an application of the above rule consider panel *CD* of Fig. 40 and substitute the particular numerical values appertaining to this panel for the three tans θ of expression (10).

$$\frac{4\frac{1}{2}p}{6p} . A + \frac{\frac{1}{2}p}{p} . B = \frac{1\frac{1}{2}p}{6p} . C$$

i.e., $3A + B = C$ . . . . . . (11)

For a U.D. load longer than the span the maximum M or stress occurs with the span fully loaded. This satisfies condition (11) because $3A + B = 4$ panel lengths of load = loaded length C of 4 panels.

If the U.D. load is shorter than the span then adjust the position of the load to satisfy condition (11).

The rule given by expression (10) applies to all influence lines which have the form of a truncated triangle.

THE EFFECT OF CROSS GIRDERS ON END REACTIONS, FIG. 41

Sometimes a cross girder is used at the end of a bridge as illustrated at the ends *A* of the three given girders, and at other times it is missed out altogether and the stringers carried forward on to a specially raised portion of the abutment wall as shown at the ends marked *G*.

When a cross girder is used over the abutment wall the floor load can only reach the abutment through the main bearing at *A*, Fig. (e), and thus the influence line for end reaction at main bearing *A* is *AaG* of (d).

If no cross girder is employed, as at *G*, the stringer delivers part of its load directly into the elevated portion of the abutment without entering the main bearing at *G* at all. Unit load placed at *F* gives $R_G = Ff$; unit load placed midway between *F* and *G* on

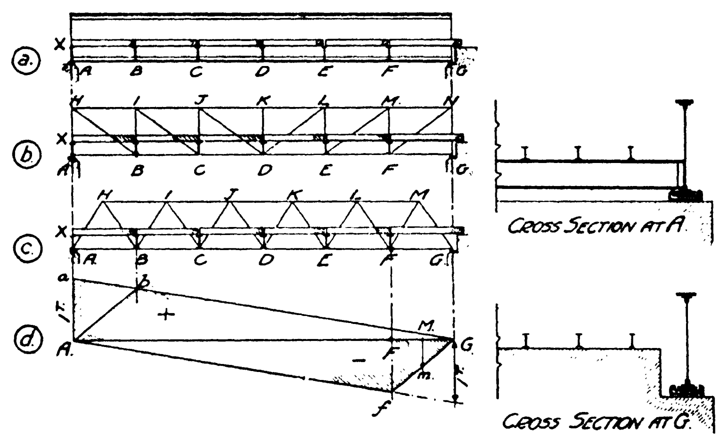


FIG. 41.

the stringer causes a reaction of half a ton at the elevated portion of the abutment (and so does not affect the main cast iron bearing) and the remaining half ton at cross girder *F*. Hence it follows that ordinate *Mm* must be half of *Ff*. The influence line for the main girder reaction R_G is therefore the triangle *AfG*.

Shear in end panel *AB* is given by the influence line *AbG*. This will be clear if it is pointed out that in the Warren truss, for example, the load which comes on to cross girder *A* is immediately transferred into the abutment through the main cast iron bearing at *A* without in any way affecting the main girder. The same argument applies equally well to the girders of (a) and (b), although in these two cases the lower or local part of the main girder in contact with the cross girder has to carry this additional concentration of load. In (b) the inclined path *BH* is travelled and then down from *H* to *X* before any load is encountered, *i.e.*, the load in part *HX* = vertical component of stress in *HB* = shear in panel *AB*.

The influence line for stress in the end vertical *AH* of (b) is the shear influence line *AbG*, and that for stress in the diagonals *HB* of (b) and *AH* and *HB* of (c) is obtained by multiplying the ordinates to *AbG* by (the length of diagonal ÷ depth of girder) as previously explained.

Shear in End Panel *FG* is obviously given by the triangle *AfG*, which was also the influence line for the main girder reaction at *G*; in addition *AfG* is the stress influence line for vertical *NG* of (b). The stress influence lines for the diagonals *FN*, *FM* and *MG* can be obtained by multiplying the ordinates to *AfG* by (length of diagonal ÷ girder depth).

CHAPTER VI

INFLUENCE LINES FOR THE MEMBERS OF A BOWSTRING GIRDER

THE girder of Fig. 25 used in the description of the "Method of Section and Moments" will be considered, and the positions of the various moment centres will be found on referring back to that article.

When handling a new problem in influence lines the safest plan, although laborious, is to calculate the different stresses in the same member when unit load is applied at each panel point in turn on its passage across the span from one abutment to the other.

BOTTOM BOOM

Member *DE*

Moment centre is N and the lever arm $ND = 14'$, Fig. 42.

1^T *at A* is immediately countered by the reaction at A of 1^T, and no stress occurs in the main girder, but only in the end bearing A.

1^T *at B* $\quad R_A = \frac{7}{8}^T$ and $R_K = \frac{1}{8}^T$, see (b), Fig. 43.

Considering the stability of the portion of the girder to the left of the section line XX (see "Section and Moments") then :—

$$R_A \times \text{horiz. length equal to } AD - 1^T \times BD = \text{Stress in } DE \times DN$$

$$\tfrac{7}{8}^T \times 45' - 1^T \times 30' = \text{Stress in } DE \times 14'$$

whence stress in $DE = 0{\cdot}67^T$

The ordinate of 0·67 is plotted immediately under this position at B of the 1^T load. The stress is a tensile one, as previously explained, and is therefore given the negative sign.

Alternatively, it is simpler to consider the stability of the right-hand shaded section of (b), Fig. 43, because there is only one force to work with, namely, the reaction $R_K = \frac{1}{8}^T$.

Moments at N : $R_K \times$ horiz. length DK = Stress $DE \times ND$

i.e., Stress $DE = R_K \times DK \div ND \quad = \frac{1}{8}^T(75' \div 14') \quad = 0{\cdot}67^T$

1^T *at C* $\quad R_K = \frac{1}{4}^T$, see small Fig. (c).

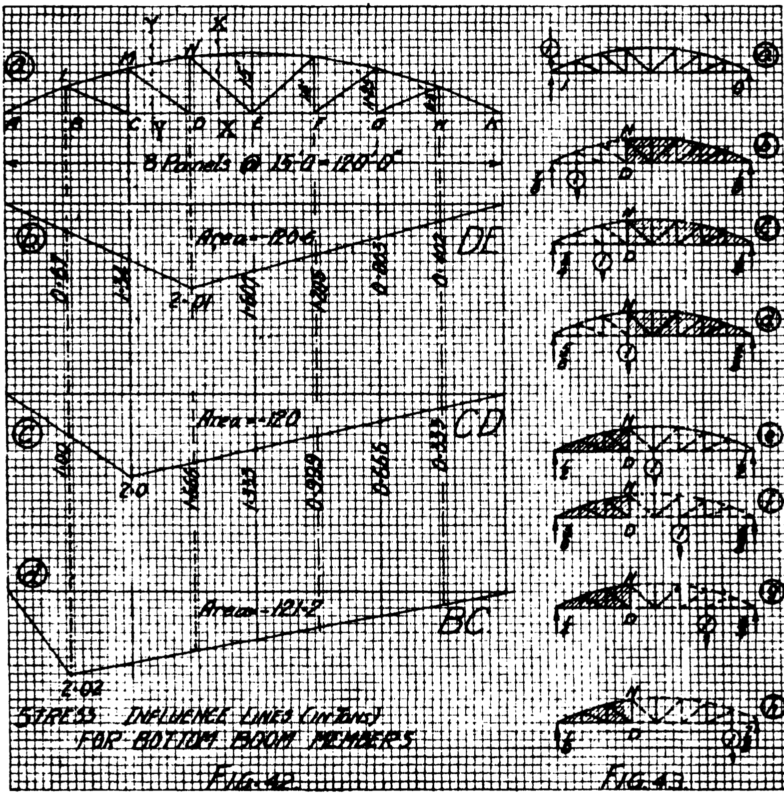
Moments at N : $R_K \times$ horiz. length DK = Stress $DE \times ND$

i.e., Stress $DE = R_K \times DK \div ND$ $= \frac{1}{4}^{\text{T}}(75' \div 14') = 1{\cdot}34^{\text{T}}$

1^{T} *at D* $R_K = \frac{3}{8}^{\text{T}}$, see small Fig. (d).

Stress $DE = R_K \times DK \div ND$ $= \frac{3}{8}^{\text{T}}(75' \div 14') = 2{\cdot}01^{\text{T}}$

The existence of a straight line law is at once apparent.



1^{T} *at E*

See Fig. (e) and consider the stability of the left-hand portion, shown shaded, which has only one force R_A.

Moment centre is still point N.

$R_A \times$ horiz. dist. equal to AD = Stress $DE \times ND$

or Stress $DE = R_A(AD \div ND) = \frac{1}{2}^{\text{T}}(45' \div 14') = 1{\cdot}607^{\text{T}}$

1^{T} *at F*

See Fig. (f) Stress $DE = R_A(AD \div ND) = \frac{3}{8}^{\text{T}}(45' \div 14') = 1{\cdot}205^{\text{T}}$

1^T *at G*

See Fig. (g) Stress $DE = R_A(AD \div ND) = \frac{1}{4}^T(45' \div 14') = 0{\cdot}803^T$

1^T *at H*

See Fig. (h) Stress $DE =$ „ „ $= \frac{1}{8}^T($ „ $) = 0{\cdot}402^T$

Plot the value of $1{\cdot}607^T$ at E and at F the value of $1{\cdot}205^T$, etc., so completing the triangular influence line DE.

If unit load is placed on a stringer between panel points the value of the ordinate under the load can be scaled from the influence line, see previous chapter. The area of the curve newly obtained is $2{\cdot}01 \times$ span of $120 \div 2 = 120{\cdot}6$, negative or tensile stress.

Knowledge born of experience would cause only one point on the curve to be calculated, namely, the apex value under the moment centre, but, nevertheless, it is a useful check to calculate an additional value such as that at C or F, or, preferably, both as checks.

Member *CD*

The influence line for CD is similarly obtained with M as the moment centre, see " Section and Moments," page 35.

Max. ordinate occurs with the load at C, then

$R_K \times CK =$ Stress $CD \times MC$ of $11{\cdot}25'$ or

Stress in $CD = \frac{1}{4}^T \times 90' \div 11{\cdot}25' \qquad = 2^T$

Members *AB* and *BC*

The stress in member LB being at right angles to AB and BC cannot have a horizontal component to affect the stress in AB and BC, which have, therefore, the same stress.

The max. ordinate of the triangular influence line occurs when the unit load is placed at B directly under the common moment centre L.

R_K of $\frac{1}{8}^T \times 105' \div LB$ of $6{\cdot}5' =$ Stress in BC (or AB) $= 2{\cdot}02^T$

Arithmetical Example

The stresses given on page 34 can now be verified by these influence lines. With the bridge loaded with 2^T per panel point the stress in $DE =$ ordinate under each load $\times$ respective load value,

$$= 2(0{\cdot}67 + 1{\cdot}34 + 2{\cdot}01 + 1{\cdot}607 + 1{\cdot}205 + 0{\cdot}803 + 0{\cdot}402) = 16{\cdot}07^T$$

and in CD

$$= 2(1 + 2 + 1{\cdot}666 + 1{\cdot}333 + 0{\cdot}999 + 0{\cdot}666 + 0{\cdot}333) = 16{\cdot}00^T$$

Alternatively the panel point loads of 2^T can be considered as being distributed uniformly over the span

$= 2^T \div$ panel length of 15′ $=$ per ft. run $0{\cdot}133^T$

Then stress in DE = area of $120{\cdot}6 \times 0{\cdot}1\dot{3}$ $= 16{\cdot}07^T$

and „ CD = „ $120 \times 0{\cdot}1\dot{3}$ $= 16{\cdot}00^T$

WEB DIAGONALS

Member LC. Figs. 44 and 45.

The moment centre where the two unwanted bars, cut by XX, meet is S situated 5·53′ to the left of A (previous calculations).

Comparing similar triangles STC and LBC

then $ST/SC = LB/LC$ whence $ST = 14{\cdot}127'$

Clockwise moments are termed positive.

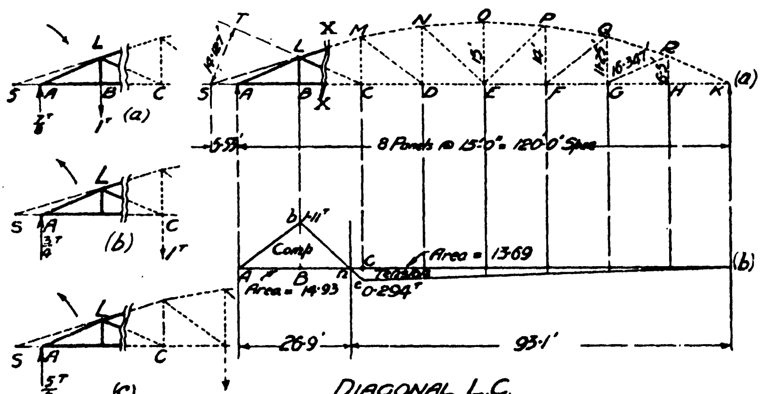


FIG 44 FIG. 45.

1^T *at A*

All the load enters the abutment through the end bearing A and therefore the stress in LC for this load position $= 0$

1^T *at B*

$R_A = \frac{7}{8}^T$ and $R_K = \frac{1}{8}^T$. Consider the stability of the portion to the left of the section line, small Fig. 44 (a).

$$\text{Stress in } LC \times 14{\cdot}127' = -\tfrac{7}{8}^T \times SA + 1^T \times SB$$
$$= -\tfrac{7}{8}^T \times 5{\cdot}53' + 1^T \times 20{\cdot}53'$$
$$= +15{\cdot}691 \text{ ft. tons.}$$

The positive moment of 15·691 would swing the broken smaller part round S in a clockwise direction as indicated by the arrow in (a) so making LC shorter in length, *i.e.*, the stress is compression,

$\therefore$ Stress in $LC = 15{\cdot}691 \div 14{\cdot}127 \qquad = +1{\cdot}11^{T}$

1^{T} *at C*

The unit load having now passed the section line XX drops out of the moment equation for the stability of the smaller portion AB of the structure, see small Fig. (b). Nevertheless the unit load causes a reaction at A of $\frac{3}{4}^{T}$.

Stress in $LC \times ST = -R_A \times SA$. The right-hand side indicates that the unbalanced moment is now counterclockwise and, as indicated by the arrow, member LC is in tension because L moves away from C. So that when unit load lies to the right of XX the stress in LC is tension.

Tensile stress in $LC = R_A(SA \div ST)$

$= R_A(5{\cdot}53' \div 14{\cdot}127') \qquad = -0{\cdot}3914R_A$

For 1^{T} at C the stress $= 0{\cdot}3914 \times \frac{3}{4}^{T} \qquad = -0{\cdot}294^{T}$

1^{T} *at D* $= 0{\cdot}3914 \times \frac{5}{8}^{T} \qquad = -0{\cdot}245^{T}$

1^{T} *at E* $= 0{\cdot}3914 \times \frac{1}{2}^{T} \qquad = -0{\cdot}195^{T}$

And so on, down to zero value at abutment K.

Neutral Point

The position of point n is obtained by comparing the similar triangles Bbn and Ccn of Fig. 45 (b).

$$\frac{bB}{Bn} = \frac{cC}{Cn},\ i.e.,\ \frac{1{\cdot}11}{Bn} = \frac{0{\cdot}294}{15 - Bn} \text{ whence } Bn \qquad = 11{\cdot}9'$$

Note. Since this girder is not parallel flanged the neutral point does not divide the panel in the same ratio as it divides the span.

Compression area

$$= (bB \times An) \div 2 = 1{\cdot}11 \times 26{\cdot}9 \div 2 \qquad = 14{\cdot}93$$

Member MD. Fig. 46

Moment centre and lever arm as on page 36. So long as unit load lies between A and the section line XX the equation is of this form :

$$\text{Stress in } MD \times 45{\cdot}8' = -31{\cdot}36'R_A + 1^{T}(SB \text{ or } SC)$$

Max. value occurs with 1^T at C.

Stress in $MD \times 45{\cdot}8' = -\,31{\cdot}36' \times \frac{3}{4}^T$
$+\,1^T \times 61{\cdot}36' = +\,37{\cdot}84$

This positive or clockwise turning moment moves M towards D and the stress in MD is compressive $= 37{\cdot}84 \div 45{\cdot}8$ $= +\,0{\cdot}826^T$

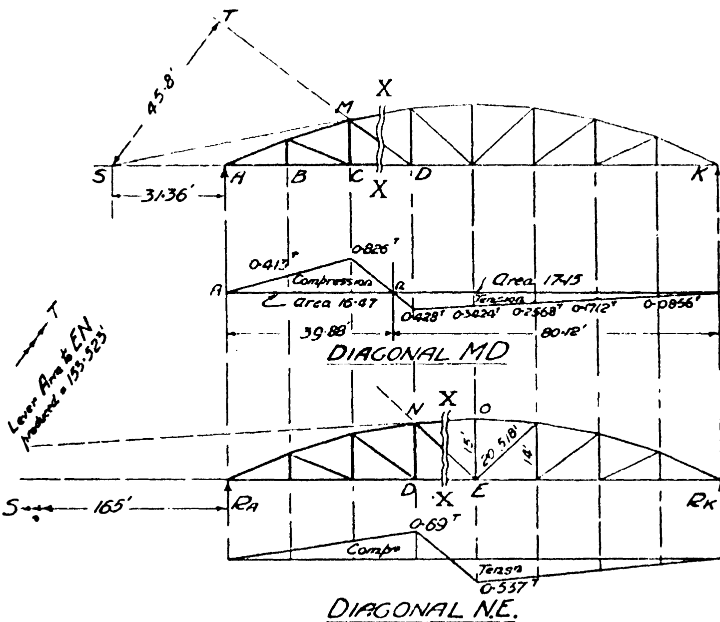


FIG. 46.

Once the unit load is to the right of the section line the equation becomes: Stress $MD \times 45{\cdot}8 = -\,R_A \times 31{\cdot}36$ (a negative moment which places MD in tension).

$\therefore$ Stress $MD = -\,R_A(31{\cdot}36 \div 45{\cdot}8)$ $= -\,0{\cdot}6847 R_A$

A straight line whose minimum value is zero when unit load is at K and whose max. value happens when unit load is at D so giving $R_A = \frac{5}{8}^T$ and Stress $MD = -\,0{\cdot}6847 \times \frac{5}{8}^T$ $= -\,0{\cdot}428^T$

Member NE. Fig. 46. Length $NE = \sqrt{(14^2 + 15^2)} = 20{\cdot}518'$

Moment centre is at S where ON produced meets ED produced.

Comparing similar triangles OES and NDS,

$$\frac{OE}{ND} = \frac{SE}{SD}$$

i.e., $\frac{15'}{14'} = \frac{60' + SA}{45' + SA}$, whence length SA $= 165'$

Comparing triangles STE and NDE,

$$\frac{ST}{SE} = \frac{ND}{NE}$$

i.e., $\frac{ST}{225'} = \frac{14'}{20{\cdot}518'}$, whence lever arm ST $= 153{\cdot}523'$

The influence line then follows, as explained for LC and MD.

Arithmetical Example

Find the stress in bar MD when only panel points A, B and C are loaded each with 2^{T}.

Stress $MD = +\ 2(0 + 0{\cdot}413 + 0{\cdot}826) \qquad = +\ 2{\cdot}48^{T}$

If panel points D to K are loaded with 2^{T} each, then :

Stress $MD = -\ 2(0{\cdot}428 + 0{\cdot}342 + 0{\cdot}257 + 0{\cdot}171 + 0{\cdot}086) = -\ 2{\cdot}57^{T}$

If all panel points carry a 2^{T} load each, then :

Stress $MD = 2[+\ 0{\cdot}826(1 + 0{\cdot}5) - 0{\cdot}428(1 + 0{\cdot}8 + 0{\cdot}6 + 0{\cdot}4 + 0{\cdot}2)] = -\ 0{\cdot}09$

(Inspection of the influence line suggests that simplicity of calculation is obtained by stating the values of the small ordinates in decimal terms of the highest ordinate.)

This agrees with the stress found by direct section and moments, page 36.

During the passage across the span, from K towards A, of a U.D. live load, w^{T} per ft., longer than the span the stress in MD increases from 0 to a maximum tensile value of $-\ 17{\cdot}15w^{T}$ when length Kn only is fully covered by the load. This max. tensile value is immediately reduced in value whenever the head of the load enters the positive section nA, because of the addition of the compressive area now covered.

This diminution of negative stress progresses steadily until the whole span is covered, when the tensile stress in MD is a very small one of $w^{T}(+\ 16{\cdot}47 - 17{\cdot}15) = -\ 0{\cdot}68w^{T}$. When the tail of the load leaves point K and approaches n there is one position of the load system where the negative area covered by the load exactly balances the positive area of 16·47, thereby giving no stress whatsoever in MD.

As the load tail approaches n, remembering that An is fully covered, the tensile area by gradually decreasing in value causes a corresponding increase in compressive stress. Max. compressive stress occurs with the load covering only section nA and the value is $+ 16{\cdot}47w^{\text{T}}$. When the tail of the load leaves n for A the live compressive stress gradually approaches zero as the bridge is freed of its live load.

WEB VERTICALS

Member LB, Fig. 47 (b), is different from the two succeeding members in that it is purely a suspender bar, and is not part of the main web system since it only carries the load from the cross girder at B. This cross girder obtains its load from the two stringers, whose ends it supports. The reaction influence line for end B of small girder AB is the triangle abd; similarly that for end B of small girder CB is the triangle bcd. The total pull or tension on the suspender is due to both these stringer reaction influence lines, hence the resultant influence line is triangle acd of area = 15.

Member MC, Fig. 47 (c), has the moment centre at T. A max. ordinate occurs under C and another of opposite sense under D.

1^{T} *at C*

Stress $MC \times 35{\cdot}53' = - R_A$ of $\frac{3}{4}^{\text{T}} \times 5{\cdot}53'$,
$+ 1^{\text{T}} \times 35{\cdot}53' = + 31{\cdot}38$

A positive moment causing a lengthening of MC,

$\therefore$ Stress $MC = 31{\cdot}38 \div 35{\cdot}53 \qquad = - 0{\cdot}883^{\text{T}}$

1^{T} *at D*

i.e., unit load is now excluded from the moment equation.
Stress $MC \times 35{\cdot}53' = - \frac{5}{8}^{\text{T}} \times 5{\cdot}53'$,
whence stress $MC = + 0{\cdot}097^{\text{T}}$

The negative moment causes a positive stress since point C tends to approach M.

Neutral Point. $\dfrac{0{\cdot}883}{x} = \dfrac{0{\cdot}097}{15 - x}$ or $\qquad x = 13{\cdot}51'$

Member ND, Fig. 47 (d), has point S as the moment centre.

1^T *at D*

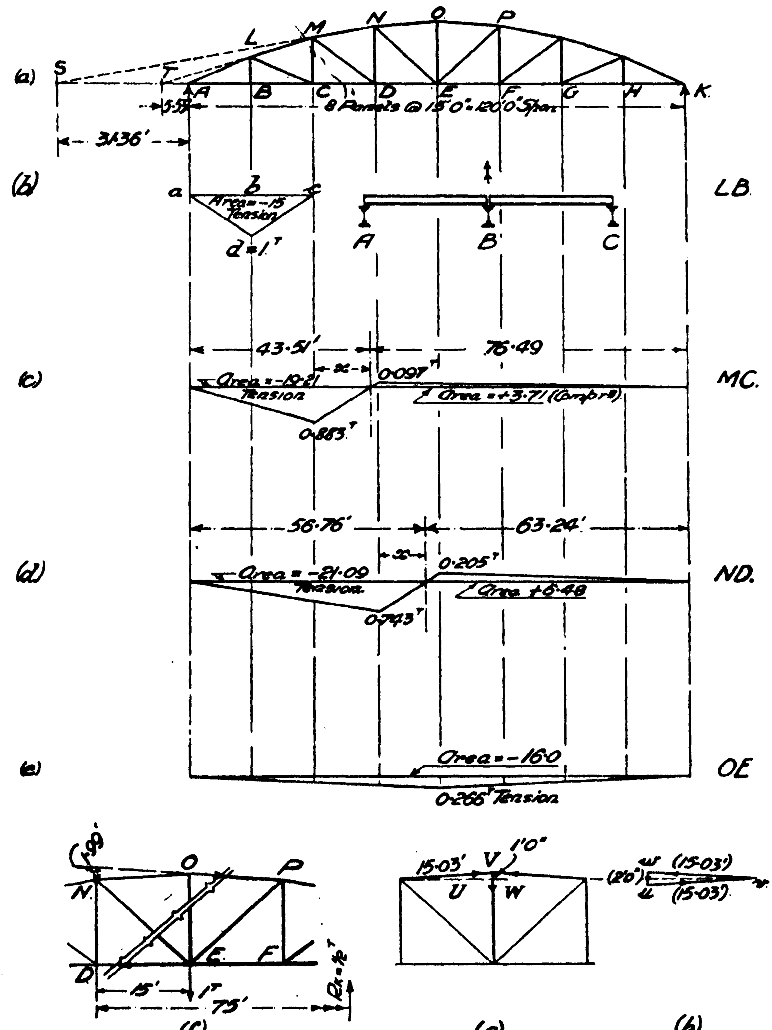
Briefly : $ND \times 76 \cdot 36' = -\frac{5}{8}^T \times 31 \cdot 36' + 1^T \times 76 \cdot 36' = +56 \cdot 76$

$\therefore$ Stress ND is a tensile one, value $= 56 \cdot 76 \div 76 \cdot 36 = -0 \cdot 743^T$

1^T *at E*

$ND \times 76 \cdot 36' = -\frac{1}{2}^T \times 31 \cdot 36'$, whence stress $ND \quad = +\ 0 \cdot 205^T$

Neutral Point. $\dfrac{0 \cdot 743}{x} = \dfrac{0 \cdot 205}{15 - x}$ or $\qquad x = 11 \cdot 76'$



INFLUENCE LINES FOR VERTICAL WEB MEMBERS.

FIG. 47.

Member *OE*, the mid-vertical, is given in Fig. 47 (e).

Any section cutting *OE* cuts through not three but four bars (Fig. (f)). By taking moments on one of the unwanted bars, *e.g.*, at *N*, on *NE*, this unwanted bar is eliminated from further thought, but the stresses in the remaining two broken bars must be taken into consideration. Further, from symmetry, it is clear that max. stress occurs when unit load is at point *E*. Now when unit load is placed at *E* it has been found that the tensile stress in *DE*, corresponding to this load position, is 1·607T—the ordinate under *E* of influence line for stress in *DE*—and the compressive stress in *OP* is 2·004T, similar to that in bar *ON* as given under. If a cut bar is in tension the force acting does so away from the cut end, and if in compression then towards the cut face.

Then moments at *N*, Fig. (f):

$$+ \text{Stress } DE \times ND + \text{Stress } OP \times 1{\cdot}99' + 1^{T} \text{ at } E \times 15' - R_K \times 75' = 15' \times \text{Stress } OE$$

$$+ 1{\cdot}607^{T} \times 14' + 2{\cdot}004^{T} \times 1{\cdot}99' + 15^{T'} - \tfrac{1}{2}^{T} \times 75' = 15' \times \text{Stress } OE$$

i.e., $15' \times$ Stress in *OE* $= + 3{\cdot}99$ ft. tons

The resulting + moment causes a tensile extension in bar *OE*, whence the stress *OE* $= 3{\cdot}99 \div 15 = 0{\cdot}266^{T}$

The influence line is then as given in (e).

Alternatively, consider Fig. (g). At point *O* in the top boom three bars meet and in consequence a triangle of forces can be drawn as in Fig. (h). The stresses in *NO* and *OP* are compressive, and that in the mid-vertical is tensile as given by the force triangle.

Further, vertical *ND* is 14′ and vertical *OE* is 15′ high, giving a height difference of 1′ 0″, as in (g); therefore, in the triangle of Fig. (h) if *wv* is 15·03′ then *wu* is twice a foot in length, viz., 2′ 0″.

But triangle *uwv* is also a triangle of forces, hence

$$\frac{wu}{uv} = \frac{\text{Stress } WU}{\text{Stress } UV} = \frac{2'}{15{\cdot}03'}$$

$$\therefore \quad \text{Stress } WU = \frac{2}{15{\cdot}03} \text{ Stress } UV$$

$$i.e., \quad \text{Stress } OE = \frac{2}{15{\cdot}03} \text{ Stress } NO$$

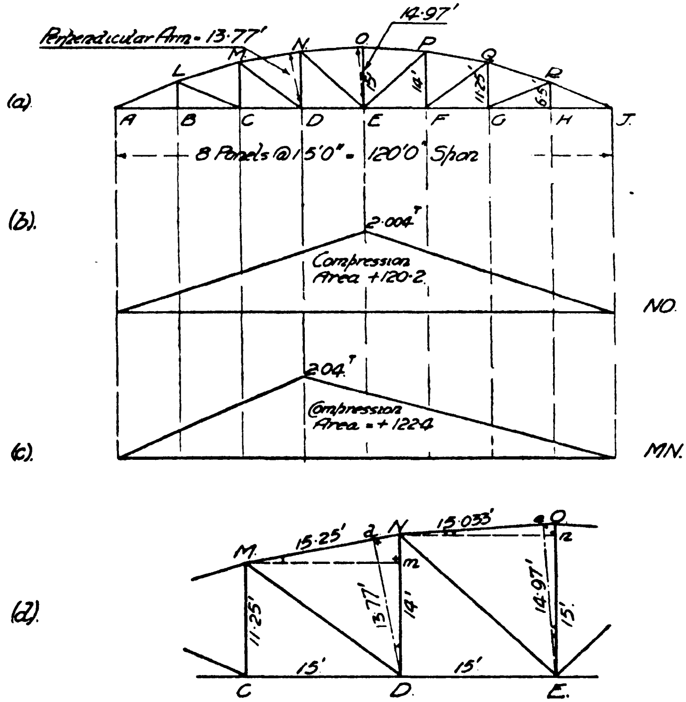
$$= \text{a tensile stress of } 0{\cdot}133 \text{ stress in } NO.$$

That is, whatever is the stress in *NO* the stress in *OE* is 0·133 stress in *NO* and of opposite sign; and therefore the influence line for stress in *OE* must also be 0·133 times that for stress in *NO* but opposite in sign. As will be seen under, the influence line for stress in *NO* is an isosceles triangle of height + 2·004T, and thus the influence line for stress in *OE* is an isosceles triangle of height equal to

$$-0{\cdot}133 \times 2{\cdot}004^{T} = -0{\cdot}266^{T}.$$

Arithmetical Example

With the bridge loaded throughout with 2^{T} panel loads the stress in



INFLUENCE LINES FOR TOP BOOM MEMBERS NO & MN.

FIG. 48.

$$MC = 2[-0{\cdot}883(1+0{\cdot}5)+0{\cdot}097(1+0{\cdot}8+0{\cdot}6+0{\cdot}4+0{\cdot}2)]$$
$$= 2[-1{\cdot}32+0{\cdot}29] \qquad = -2{\cdot}06^{T}$$
$$ND = 2[-0{\cdot}743(1+0{\cdot}66+0{\cdot}33)+0{\cdot}205(1+0{\cdot}75+0{\cdot}5+0{\cdot}25)]$$
$$= 2[-1{\cdot}486 + 0{\cdot}513] \qquad = -1{\cdot}95^{T}$$

These results are identical with those found on page 37.

TOP BOOM

Member *MN*. The moment centre is point *D* and the lever arm can be calculated as on page 36 or from the comparison of the similar triangles NmM and NdD of Fig. 48 (d). The influence line is given by (c) and shows that maximum stress occurs when the bridge is fully loaded.

If all panels carry a 2^T load the value of the stress in

$MN = 2[2{\cdot}04(1+0{\cdot}66+0{\cdot}33+0{\cdot}2+0{\cdot}4+0{\cdot}6+0{\cdot}8)] \qquad = 16{\cdot}32^T$

Member NO. The influence line is found in a similar manner to that for MN.

CHAPTER VII

63-FT. SPAN PLATE GIRDER RAILWAY BRIDGE

THE small cross-section of the bridge given in Fig. 49 shows the usual disposition of the stringers or longitudinal beams. Flat mild steel plates are laid on top of the stringer joists and are riveted to the upper flanges. The thickness of the floor plating is $\frac{7}{16}''$ or $\frac{1}{2}''$ and is adopted without calculation.

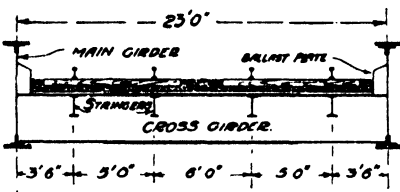


FIG 49.

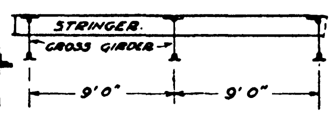
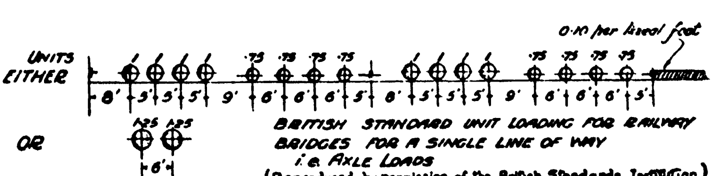


FIG 50.

The floor plating is covered by a 1″ thick coat of asphalte because the upper surface is inaccessible for painting. To protect this bitumastic seal against a carelessly driven pick a 2″ thick layer of small aggregate concrete is laid on top of the asphalte. The ballast, which is in contact with this concrete, is prevented from corroding the web plates of the main girders by subsidiary vertical plates known as ballast plates. The protecting ballast plates when corroded can be easily dismantled and replaced.



BRITISH STANDARD UNIT LOADING FOR RAILWAY BRIDGES FOR A SINGLE LINE OF WAY
i.e. AXLE LOADS
(Reproduced by permission of the British Standards Institution)
FIG. 51.

The complete stress calculations for the steelwork will be given in the following pages. If further information is desired as to the general design of the sections for the girders, stiffeners, bearings, splices, etc., together with working details, see "The Practical Design of Simple Steel Structures," Vol. II.

The live load on the bridge will be taken at 15 units, *i.e.*, 15 times the axle loads of the imaginary train of Fig. 51.*

STRINGERS

Floor Dead Load per ft. run of Span †

Floor plating $\frac{7}{16}''$ tk.;

23′ × 1′ @ 17·85 lb./sq. ft. = 411 lb.

Rivet heads in above; add 5% = 21

Asphalte sealing coat;
23′ × 1′ × (1 ÷ 12) @ 136 lb./cu. ft. = 261

Concrete overlay;
23′ × 1′ × (2 ÷ 12) @ 140 ,, ,, = 537

Ballast, 9″ average tk.;
23′ × 1′ × (9 ÷ 12) @ 90 ,, ,, = 1,553

2,783 lb. (a)

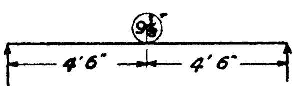


FIG 52

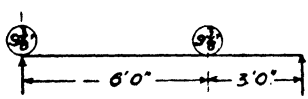


FIG 53.

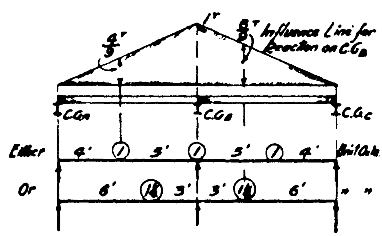


FIG 54.

Single track of 2 rails, chairs, sleepers, etc.,

$1\frac{1}{2}$ cwt./ft. = 168 lb. (b)

Stringer self; dead weight estimated at $\frac{1}{5}$T = 448 lb. (c)

Width supported by internal stringer

= $\frac{1}{2}$ of 6′ + $\frac{1}{2}$ of 5′ = 5′ 6″

Dead load on stringer, item (a);
2,783(5·5′ ÷ 23′) × 9′ long = 5,990 lb.

,, of half track, item (b);
$\frac{1}{2}$ of 168 × 9′ long = 756

,, of self, item (c) = 448

7,194 lb. = 3·21T (d)

* Also see Fig. 110.
† See Weights of Materials, page 168.

Bending Moment, see Figs. 50 and 52

The axle load is 15 units × $1{\cdot}25^T$

$= 18{\cdot}75^T$ or per wheel $= 9{\cdot}375^T$

*Impact factor

$$= \frac{120}{90 + \frac{(n+1)}{2}L} = \frac{120}{90 + \frac{(1+1)}{2}9} = 1{\cdot}21$$

but not to exceed $\underline{1{\cdot}15}$ (e)

Where n = No. of tracks which girder supports or assists in supporting.

L = loaded length in ft. of the track or tracks producing max. stress.

Dead load max. B.M.

$= WL \div 8$ see item (d)

$= 3{\cdot}21^T \times 108'' \div 8$, in inch tons $= 43{\cdot}3$

Live load max. B.M.

$= WL \div 4$ see Fig. 52

$= 9{\cdot}375^T \times 108'' \div 4$, in inch tons $= 253{\cdot}1$

*Impact, item (e)

$= 253{\cdot}1 \times 1{\cdot}15$,, ,, $= 291{\cdot}1$

Total max. B.M. for D.L. + L.L. + I., in inch tons $=$ $\underline{587{\cdot}5}$ (f)

End Shear

Dead load

$= \frac{1}{2}$ of $3{\cdot}21^T$ (item (d)) tons $= 1{\cdot}60$

Live load

$= 9{\cdot}375^T(3'+9') \div 9'$ (see Fig. 53) ,, $= 12{\cdot}5$

Impact

$= 12{\cdot}5^T \times 1{\cdot}15$ (item (e)) ,, $= 14{\cdot}38$

Total max. end shear for D.L.+L.L.+I. ,, $=$ $\underline{28{\cdot}5}$ (g)

* The live load stress is that due to the wheel weights applied as static loads; actually, however, these loads are applied dynamically and not statically to the bridge, and in consequence there is a considerable increment of the static stress. There are several causes of this increment, but the effects are all included in the collective term "impact" (see page 173).

For simplicity the impact formula of the "B.S.S. for Girder Bridges No. 153," 1923 edition, is used here. Although the 1937 edition says that this provisional formula should be discontinued it leaves the fixing of the impact allowance to the discretion of the engineer. (The specifications are published by The British Standards Institution, 28 Victoria Street, S.W.1, post free 2*s*. 2*d*.)

Design stringer to carry a max. B.M. at centre of 587·5 inch tons and an end shear of 28·5 tons.

CROSS GIRDERS

Span 23′ 0″ at 9′ 0″ centres.

Dead Load. Assume each stringer to carry the same amount.

From stringers (see item (d) of stringers)	=	3·21T	
Estimated weight of self = 2·08T. Per panel point = 2·08 ÷ 4	=	0·52T	
(See calculations on page 166.)			
Total panel point dead load	=		3·73T (a)

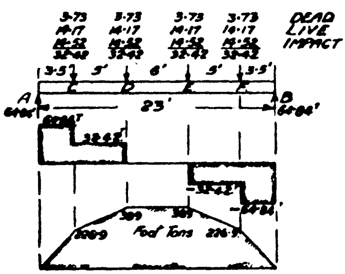


FIG 55.

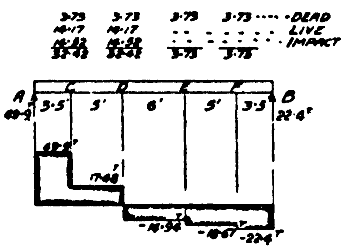


FIG 56.

Live Load. Impact factor $= \dfrac{120}{90+\dfrac{(n+1)}{2}L}$

$$= \frac{120}{90+\dfrac{(2+1)}{2}18} \quad = \quad 1{\cdot}025$$

Live load reaction on cross girder, Fig. 54.

Case 1. $1(\frac{4}{9}+1+\frac{4}{9}) = 1\frac{8}{9}$ unit axle load.
„ 2. $1\frac{1}{4}(2\times\frac{6}{9}) = 1\frac{6}{9}$ „ „

Max. L.L. reaction on C.G. $= 1\frac{8}{9} \times 15$ units per axle $=$ 28·33T

Max. L.L. reaction on C.G. per single rail
$= 28{\cdot}33^{T} \div 2 =$ 14·17T (b)

Max. impact allowance = 14·17T × 1·025 = 14·52T (c)

Max. end shear

$=2(3{\cdot}73^{T}+14{\cdot}17^{T}+14{\cdot}52^{T})$; see Fig. 55 = 64·84T (d)

Max. shear panel CD; „ = 32·42T (e)

Max. B.M.$_C$

$= 3{\cdot}5R_A = 3{\cdot}5' \times 64{\cdot}84^{T}$ „ = ft. tons 226·9 (f)

Max. B.M.$_D$

$=64{\cdot}84^{T}\times 8{\cdot}5' - 32{\cdot}42^{T}\times 5'$ „ = „ 389·0 (g)

The max. shear in centre panel occurs with only one track loaded as in Fig. 56 = 14·9T (h)

This figure of item (h) is required for the design of the riveting in the flanges of the centre panel DE.

Design the cross girder to withstand the bending moments and hears given in Figs. 55 and 56.

MAIN GIRDERS

Span 63′ 0″ centre to centre of bearings, Fig. 57.

Dead Load per Cross Girder	lb.	
See stringers, item (a)		
Floor covering; 9 × 2,783 lb.	= 25,047	
See stringers, item (b)		
Double track; 9 × 168 lb. × 2 off	= 3,024	
See stringers, item (c)		
Stringer self; 448 lb. × 4 off	= 1,792	
See cross girder, item (a)		
Wt. of C.G. = 2·08T	= 4,660	
	34,523	15·4T

Dead load per panel point of main girder $= \frac{1}{2}$ of 15·4T = 7·7T (a)

Estimated weight of main girder, see calculations on page 167. = 16·0T (b)

Live Load. Impact factor $= \dfrac{120}{90 + \dfrac{(n+1)}{2}L}$

(Taking n as 2) $= \dfrac{120}{90 + \dfrac{(2+1)}{2}63} =$ 0·65 (c)

(Taking n as 1) $= \dfrac{120}{90 + \dfrac{(1+1)}{2}63} =$ 0·784

The main girders carry their greatest live load stresses when both tracks are occupied by identical trains crossing the bridge in the same direction and keeping pace with each other—an imaginary condition of loading which results in simplified design calculations. In keeping with this assumption a few designers suggest that n should be taken as 1 in the formula, so giving the greater value of 0·784 for the impact factor or coefficient—this is equivalent to saying that each main girder carries one track.

Usually, however, n is taken as being equal to 2, because, in the words of the specification, each main girder "assists in supporting two tracks." This gives the lower value of 0·65 for the impact factor which is in accordance with practical conditions, because when two separate trains cross a bridge they do so in opposite directions. The two trains are thus out of phase with each other, so producing a damping effect which lessens the impact. The lesser coefficient of 0·65 will be taken.

MAIN GIRDER—SHEAR INFLUENCE LINES, FIG. 57

The diagrams of this figure give the positions of the neutral points, the areas of the curves, and the positions of the axle loads causing maximal shears. These unit axle loads will require to be multiplied by 15 to give the 15 units axle loads, which is the loading on each main girder when both tracks are loaded simultaneously.

(a) **Axle Loads.** The ordinate at each axle is obtained by proportion; thus in the curve for panel AB the ordinate at the leading driving axle adjoining A is $\frac{4'}{AB}$ of the apex value of $\frac{6}{7} = \frac{4}{9}$ of $\frac{6}{7}$. The second axle is at ordinate $\frac{6}{7}$, while the third driving axle is 49′ from end H and its ordinate is $\frac{49'}{BH}$ of the apex value of $\frac{6}{7}$ or $\frac{49}{54}$ of $\frac{6}{7}$, and the ordinate at the fourth driver is $\frac{44'}{BH}$ of $\frac{6}{7}$ or $\frac{44}{54}$ of $\frac{6}{7}$. The $\frac{3}{4}^{T}$ axles, in addition, will have their ordinates multiplied by this figure of $\frac{3}{4}$.

Panel AB, case (a)

$$1^{T} \text{ axles}: \quad 1 \times \tfrac{6}{7}\left(\frac{4}{9} + 1 + \frac{4 + 44 + 49}{54}\right) = 2{\cdot}777$$

$$\tfrac{3}{4}^{T} \quad \text{,,} \quad \tfrac{3}{4} \times \tfrac{6}{7}\left(\frac{17 + 23 + 29 + 35}{54}\right) = \underline{1{\cdot}238}$$

$$\underline{4{\cdot}015}\ (60{\cdot}23^{T})$$

Panel AB, case (b)

1^{T} axles : $1 \times \frac{6}{7}\left(\frac{4}{9} + 1 + \frac{44 + 49}{54}\right) \qquad = 2{\cdot}714$

$\frac{3}{4}^{T}$,, $\frac{3}{4} \times \frac{6}{7}\left(\frac{17 + 23 + 29 + 35}{54}\right) \qquad = 1{\cdot}238$

U.D.L.L. $0{\cdot}1 \times \frac{6}{7}(\frac{12}{54} \times \frac{12}{2}) \qquad = 0{\cdot}114$

$4{\cdot}066$

The latter case gives the greater value.
Max. S_{AB} due to 15 units train $= 4{\cdot}066 \times 15 \qquad = \qquad 61^{T}$

Panel BC. Group the heavy wheels near the apex.

1^{T} axles : $1 \times \frac{5}{7}\left(\frac{2\frac{1}{2}}{7\frac{1}{2}} + \frac{35 + 40 + 45}{45}\right) \qquad = 2{\cdot}143$

$\frac{3}{4}^{T}$,, $\frac{3}{4} \times \frac{5}{7}\left(\frac{8 + 14 + 20 + 26}{45}\right) \qquad = 0{\cdot}810$

3′ 0″ of U.D.L.L. $= \frac{5}{7} \times \frac{3}{45} \times \frac{3}{2} \times 0{\cdot}1 \qquad = 0{\cdot}007$

$2{\cdot}960$

Max. S_{BC} due to 15 units train $= 2{\cdot}96 \times 15 \qquad = \qquad 44{\cdot}4^{T}$

Panel CD, case (a). For max. positive shear keep the loads within the positive segment of the span.

1^{T} axles : $1 \times \frac{4}{7}\left(\frac{26 + 31 + 36}{36} + \frac{1}{6}\right) \qquad = 1{\cdot}571$

$\frac{3}{4}^{T}$,, $\frac{3}{4} \times \frac{4}{7}\left(\frac{5 + 11 + 17}{36}\right) \qquad = 0{\cdot}393$

$1{\cdot}964$

Panel CD, case (b)

1^{T} axles : $1 \times \frac{4}{7}\left(\frac{21 + 26 + 31 + 36}{36}\right) \qquad = 1{\cdot}809$

$\frac{3}{4}^{T}$,, $\frac{3}{4} \times \frac{4}{7}\left(\frac{6 + 12}{36}\right) \qquad = 0{\cdot}214$

$2{\cdot}023$

Max. S_{CD} due to 15 units train $= 2{\cdot}023 \times 15 \qquad = \qquad 30{\cdot}35^{T}$

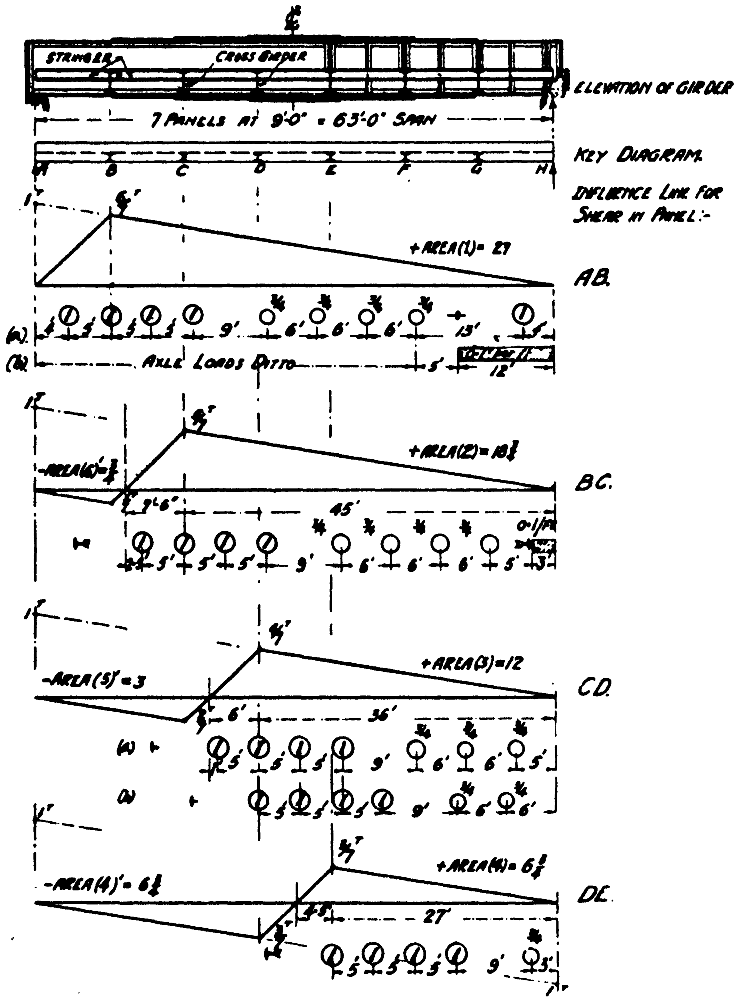


FIG. 57.

Panel DE

$$1^T \text{ axles}: \quad 1 \times \tfrac{3}{7}\left(\frac{12 + 17 + 22 + 27}{27}\right) = 1{\cdot}238$$

$$\tfrac{3}{4}^T \quad ,, \quad \tfrac{3}{4} \times \tfrac{3}{7}\left(\tfrac{3}{27}\right) = 0{\cdot}036$$

$$1{\cdot}274$$

Max. S_{DE} due to 15 units train $= 1{\cdot}274 \times 15 \quad = \quad$ **19·11^T**

(b) U.D.L.L. of $0{\cdot}1^T \times 15$ units. For max. positive shear load only the positive segment of the influence line between H and the neutral point.

Shear = area × load per ft.

S_{AB}	$= 27 \times 0{\cdot}1 \times 15$ units	$40{\cdot}5^T$
S_{BC}	$= 18\frac{3}{4} \times 0{\cdot}1 \times 15$ units	$28{\cdot}13^T$
S_{CD}	$= 12 \times 0{\cdot}1 \times 15$ units	$18{\cdot}00^T$
S_{DE}	$= 6\frac{3}{4} \times 0{\cdot}1 \times 15$ units	$10{\cdot}13^T$

In all cases the wheel concentrations give the greater shears.

(c) D.L. shear from cross girders (at $7{\cdot}7^T$ each panel).
Shear = Σ loads × ordinates at panel points.

$$S_{AB} = 7{\cdot}7\left(\frac{6+5+4+3+2+1}{7}\right) = \underline{23{\cdot}1^T}$$

$$S_{BC} = 7{\cdot}7\left(\frac{-1+5+4+3+2+1}{7}\right) = \underline{15{\cdot}4^T}$$

$$S_{CD} = 7{\cdot}7\left(\frac{-1-2+4+3+2+1}{7}\right) = \underline{7{\cdot}7^T}$$

$$S_{DE} = 7{\cdot}7\left(\frac{-1-2-3+3+2+1}{7}\right) = \underline{0}$$

(d) D.L. shear from main girder self. The D.L. $= 16^T$ or $16^T \div 63$ per ft.
Shear = sum of positive and negative areas × (16 ÷ 63).

S_{AB}	$=(\text{area } 1-0)16/63=(27)\times 16/63$	=	$6{\cdot}86^T$
S_{BC}	$=(A2-A6')16/63 =(18\frac{3}{4}-\frac{3}{4})\times 16/63$	=	$4{\cdot}57^T$
S_{CD}	$=(A3-A5')16/63 =(12-3) \times 16/63$	=	$2{\cdot}29^T$
S_{DE}	$=(A4-A4')16/63 =(6\frac{3}{4}-6\frac{3}{4})\times 16/63$	=	$0{\cdot}0^T$

The thickness of the main web plate and the pitches of the web to flange-angles rivets are calculated from the values tabulated in the final column on page 80.

SUMMARY OF MAXIMAL SHEARS IN TONS

Panel	Axle loads (a)	*Impact = 0·65 × (a) (b)	Dead load from		Total max. shear = (a) + (b) + (c) + (d)
			Cross girder (c)	Main girder (d)	
AB	61	39·65	23·1	6·86	130·61
BC	44·4	28·86	15·4	4·57	93·23
CD	30·35	19·73	7·7	2·29	60·07
DE	19·11	12·42	0·0	0·0	31·53

* As the main girders have plated webs the impact factor obtained for the shear in the end panels is also used for the remaining panels of the girders.

However, had the main girders been Warren trusses, then the two web diagonals in panel *BC* would have had their impact factor calculated on a value for *L* of 52′ 6″ and those in panel *CD* on a value for *L* of 42′ 0″. This follows from the shear influence lines of Fig. 57 which show that the necessary length of span to be loaded to produce maximum stress in a web diagonal in these panels is from the right-hand abutment at *H* up to the neutral point in the panel.

MAIN GIRDER—B.M. INFLUENCE LINES, FIG. 58

The ordinate at each axle is obtained by proportion and the sum of the ordinates is multiplied by 15 as in the case of shear.

The influence line for point *K* is a truncated triangle, because *K* lies between the panel points. The apex value of this triangle is found by placing unit load at *K* actually on the main girder. The moment at $K = R_A \times \frac{1}{2}L = \frac{1}{2}^{T} \times (63' \div 2) = 15\frac{3}{4}$ foot tons. The ordinate at *D* is $15\frac{3}{4}(AD \div AK) = 15\frac{3}{4}(3 \div 3\frac{1}{2}) = 13\frac{1}{2}$, measuring *AD* and *AK* in panel lengths.

The units of panel length are easier to use than feet dimensions. Thus when a girder has all its panels of the same length and the influence line is a triangle then the rule for a max. condition of loading can be restated as—the average load per panel to the left of the point should equal the average load per panel to the right—in place of saying the average load per foot, etc. As a numerical example take point *D* with the loads (b) placed as in the figure and consider the unit load at apex *D* as belonging to neither segment.

The average load per panel in segment

$DH = (2 @ 1^{T} + 3 @ \frac{3}{4}^{T}) \div 4$ panels $= 4{\cdot}25 \div 4 =$ 1·06T

$AD = (1 @ 1^{T} + 2 @ \frac{3}{4}^{T}) \div 3$,, $= 2{\cdot}5 \div 3 =$ 0·83T

If the 1^{T} load at *D* is now considered as belonging to the lesser segment *AD*, the addition to $AD = 1^{T} \div 3 =$ 0·33T

Total average load per panel of segment *AD* = 1·16T

Without unit load at D the average panel load of segment AD is less than that of segment DH and with the unit load at D considered in segment AD the average panel load of this AD segment is larger than that for segment DH and therefore a maximum condition is present. Alternatively, $0{\cdot}82^{T}$ of the unit load at D can be considered as being in AD and the remaining $0{\cdot}18^{T}$ in DH. The average load per panel now balances.

Segment $DH = (4{\cdot}25^{T} + 0{\cdot}18^{T}) \div 4$ panels $= 4{\cdot}43 \div 4 =$ $1{\cdot}11^{T}$

Segment $AD = (2{\cdot}5^{T} + 0{\cdot}82^{T}) \div 3$,, $= 3{\cdot}32 \div 3 =$ $1{\cdot}11^{T}$

(a) Axle Loads

B.M. at K

1^{T} axles : $1 \times \frac{13{\cdot}5}{27}(22+26) + 2 \times 1 \times 13{\cdot}5$ $= 51{\cdot}00$ foot tons

$\frac{3}{4}^{T}$,, $\frac{3}{4} \times \frac{13{\cdot}5}{27}(3 + 9 + 5 + 11 + 17)$ $= 16{\cdot}875$

$67{\cdot}875$

Max. M_K due to 15 units train $= 67{\cdot}875 \times 15 =$ $1{,}018{\cdot}13$

B.M. at D. Case (a)

1^{T} axles : $1 \times \frac{108}{7}\left(\frac{22 + 17}{27} + \frac{36 + 31}{36}\right)$ $= 51{\cdot}00$

$\frac{3}{4}^{T}$,, $\frac{3}{4} \times \frac{108}{7}\left(\frac{4}{27} + \frac{4+10+16+22}{36}\right)$ $= 18{\cdot}43$

$69{\cdot}43$

Max. M_D due to 15 units train $= 69{\cdot}43 \times 15 =$ $1{,}041{\cdot}45$

(Case (b) gives a smaller moment =
$(52{\cdot}43 + 15{\cdot}75) \times 15$ units $= 1{,}022{\cdot}7$.)

B.M. at C

1^{T} axles : $1 \times \frac{90}{7}\left(\frac{8 + 13 + 18}{18} + \frac{40}{45}\right)$ $= 39{\cdot}285$

$\frac{3}{4}^{T}$,, $\frac{3}{4} \times \frac{90}{7}\left(\frac{13 + 19 + 25 + 31}{45}\right)$ $= 18{\cdot}857$

U.D.L.L. $0{\cdot}1 \times \frac{90}{7}\left(\frac{8}{45}\right) \times 8 \times \frac{1}{2}$ = area × load/ft. $= 0{\cdot}914$

$59{\cdot}056$

Max. M_C due to 15 units train $= 59{\cdot}056 \times 15 =$ $885{\cdot}84$

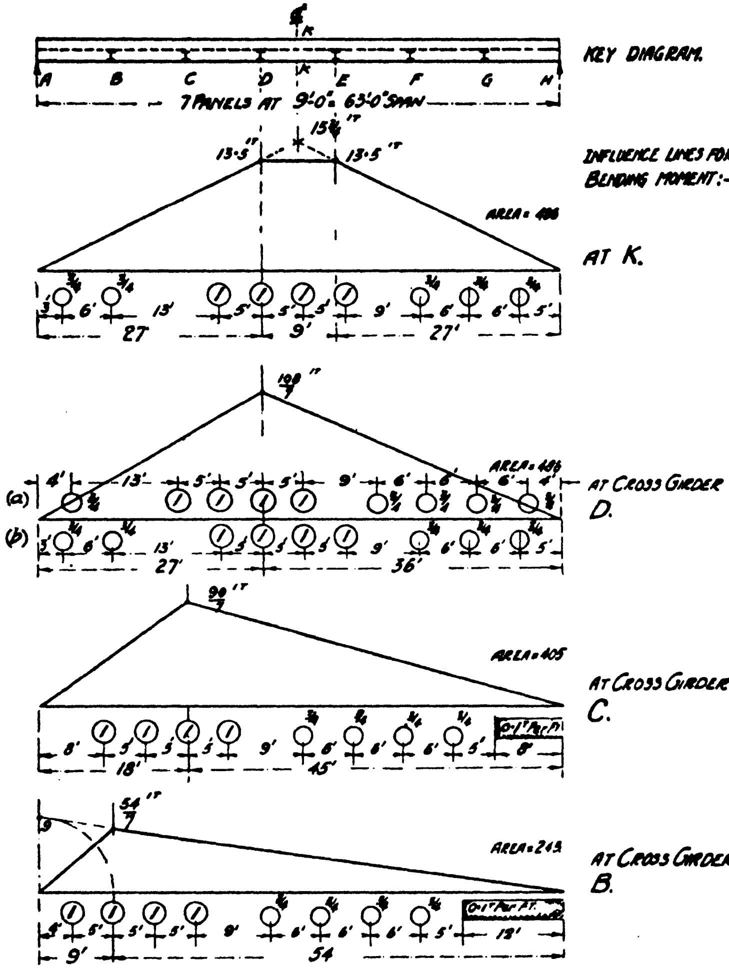


FIG. 58.

B.M. at B

1^T axles : $1 \times \frac{54}{7}\left(\frac{4}{9} + \frac{44 + 49 + 54}{54}\right)$ = 24·429 foot tons

$\frac{3}{4}^T$,, $\frac{3}{4} \times \frac{54}{7}\left(\frac{17 + 23 + 29 + 35}{54}\right)$ = 11·143

U.D.L.L. $0{\cdot}1 \times \frac{54}{7}\left(\frac{12}{54}\right) \times 12 \times \frac{1}{2}$ = area × load/ft. = 1·028

36·600

Max. M_B due to 15 units train = 36·6 × 15 = 549

This B.M. influence line is similar to that for shear in panel *AB* and the apices of the B.M. and Shear I.L.'s are in the ratio of $\frac{54}{7} : \frac{6}{7}$, *i.e.*, as 9 is to 1. Therefore the B.M. corresponding to case (b) loading for the shear, panel *AB*, Fig. 57, is 9 times the respective shear result of 61 (page 77), *i.e.*, 549 as above. This follows because the positions of the wheels, whether for shear or B.M., are governed by the same law, viz., the average load per ft. on the left of the apex equals the average load per ft. on the right of the apex.

Similarly the B.M. for case (a) loading shown on the shear influence line, panel *AB*, Fig. 57, is 9 times the shear result of 60·23, page 76. = 542 foot tons.

(b) U.D.L.L. of $0{\cdot}1^{\mathrm{T}} \times 15$ units. Only the U.D.L.L. now covers the span.

	Foot tons.
Max. M_K = area × 0·1 × 15 = 486 × 0·1 × 15	= 729
Max. M_D = ,, ,, = ,, ,,	= 729
Max. M_C = ,, ,, = 405 × 0·1 × 15	= 607·5
Max. M_B = ,, ,, = 243 × 0·1 × 15	= 364·5

In all cases the wheel concentrations give the greater B.M.

(c) D.L.B.M. from cross girders.

B.M. = loads × respective ordinates.

$$M_K = 7{\cdot}7\left(\frac{9+18+27}{27}\right)2 \times 13{\cdot}5 \qquad = 415{\cdot}8$$

$$M_D = 7{\cdot}7\left(\frac{9+18+27}{27} + \frac{9+18+27}{36}\right)\frac{108}{7} \qquad = 415{\cdot}8$$

$$M_C = 7{\cdot}7\left(\frac{9+18}{18} + \frac{9+18+27+36}{45}\right)\frac{90}{7} \qquad = 346{\cdot}5$$

$$M_B = 7{\cdot}7\left(\frac{9+18+27+36+45+54}{54}\right)\frac{54}{7} \qquad = 207{\cdot}9$$

(d) D.L.B.M. from girder self at $16^{\mathrm{T}} \div 63$ in T/ft. run.

Note.—Since the dead load of main girder self acts directly on the girder, and not through panel points, the proper influence line for M_K is the triangle of apex height $15\frac{3}{7}$, Fig. 58, *K*.

		Foot tons.
M_K = area × load/ft.	$= (15{\cdot}75 \times 63 \div 2)16 \div 63$	$= 126{\cdot}0$
M_D = ,, ,,	$= 486 \times 16 \div 63$	$= 123{\cdot}4$
M_C = ,, ,,	$= 405 \times 16 \div 63$	$= 102{\cdot}86$
M_B = ,, ,,	$= 243 \times 16 \div 63$	$= 61{\cdot}71$

SUMMARY OF MAXIMAL BENDING MOMENTS IN FOOT TONS

Point	Axle loads (a)	*Impact = 0·65 (a). (b)	Dead load from Cross girder (c)	Dead load from Main girder (d)	Total max. B.M. = (a) + (b) + (c) + (d)
K	1018·13	661·78	415·8	126·0	2221·71
D	1041·45	676·94	415·8	123·4	2257·59
C	885·84	575·80	346·5	102·86	1911·00
B	549·0	356·85	207·9	61·71	1175·46

* The influence lines of Fig. 58 clearly indicate that the maximum flange stress at any point occurs when the bridge is completely covered by the load. Similarly with a Warren (or other type of) truss maximum stress in any flange member occurs when the load covers all the span, and so the value to use for *L*, in the impact formula, is the span length of 63′, *i.e.*, the impact factor is constant for the flange members.

The sections of the main flanges and the points of curtailment of the several flange plates can be determined from the values listed in the final column above.

CHAPTER VIII

LIGHT WARREN TRUSS ROAD BRIDGE

Span = 75′ 0″. Clear width of roadway is 20′ 0″.

Dead Load of Floor *

Buckled plates $\frac{5}{16}$″ tk., dished 3″.

Concrete filling is 2″ tk. over plates at kerb to $4\frac{1}{2}$″ at crown, for camber. 1″ sand and 4″ deep granite sets. Stringers and local cross beams supporting buckled plates. Av. wt./sq. ft. of floor = 154 lb.

Dead Wt. of Cross Girder each = $0{\cdot}9^{T}$

Dead Wt. of Main Truss (see calculations, page 167) each = $7{\cdot}0^{T}$

Dead Load per Panel Point

$15' \times 10'$ at 154 lb./sq. ft. = $10{\cdot}3^{T}$, *i.e.*, panel length by half width, plus $\frac{1}{2}$ of $0{\cdot}9^{T}$ plus $\frac{1}{5}$ of $7{\cdot}0^{T}$ = $12{\cdot}15^{T}$

Dead Load/ft. of Main Truss

$12{\cdot}15^{T} \div$ panel length of 15′ = $0{\cdot}81^{T}$

Live Load

Wheel Loads. Two vehicles of 12^{T} each cross the bridge side by side, exactly abreast, in either direction; *i.e.*, the main trusses carry a vehicle each or a set of axle loads.

U.D.L.L. Uniformly distributed live load of 100 lb./sq. ft. of floor surface. Each main truss will carry per ft. run of its length a load of $\frac{1}{2}$ of $20' \times 1'$ at 100 lb./sq. ft. = 1,000 lb. = $0{\cdot}447^{T}$

Design of Bridge

The bridge carries a private road to a mill and the vehicle loads were determined by the weights of merchandise and the tare or dead load of the lorries. The 100 lb./sq. ft. represents pedestrian traffic in the nature of a dense crowd and hence a slowly moving

* See page 168 for "Weights of Materials."

mass without impact; alternatively, it may be considered as an equivalent uniformly distributed load for light traffic, as explained on page 171. The stresses from the actual wheel loads, on the other hand, were increased by 30% * to allow for impact, and the permissible axial tensile stress was 7^T/net sq. in. †Compare this with the Ministry of Transport figure for impact allowance of 50%, which is included in the imaginary wheel loads of Fig. 111, and the working tensile stress of 9^T/net sq. in. as specified in the B.S.S. No. 153, revised September, 1933, and September, 1937.

Further, it is hardly possible for the full wheel loads with impact allowance added (*i.e.*, maximum speed) to occur simultaneously with full pedestrian traffic on the remaining free area of the floor not occupied by the vehicles. Either one type of loading or the other, but not both types, can act on this small span and narrow bridge.

Since the flat edges of the buckle plates rest upon the cross beams, stringers and cross girders, the top flanges of these members are all at the same level. The floor loads thus travel through the buckle plates to the local cross beams of 5′ 0″ span and from there to the stringers, which are tiny 15′ 0″ spans parallel to and between the main trusses. The stringers in turn give up their loads to the cross girders, which deliver their loads as end reactions to the main trusses at all points marked L on the lower boom, see Fig. 66.

Main Bearings. The cross girders at L_0 and L_5 could be left out and the masonry abutments brought up to a higher level to carry the ends of the stringers and floor. This would not affect the influence lines given for the boom or web members because cross girders L_0 and L_5 really deliver their loads directly into the main bearings without influencing the structure of the bridge in any way. What is altered, however, is the amount of the load passing through the main bearings.

Cross Girder at L_0‡. The influence line for the reaction on the end bearing is given by (b) of Fig. 59. Every load on the bridge floor must ultimately find its way into the main truss, and the influence line is exactly the same as if the main truss were a beam of 75′ 0″ span carrying the loads directly on its upper flange. A unit

* See page 178 for Road Impact Formulæ.

† *E.g.*, consider an axial tensile live load stress of 50^T.

Above example:

Design stress $= 50^T + 30\% = 65^T$. Net area required at 7^T/sq. in. $= 9{\cdot}29$ sq. in.

B.S.S. 1923:

" " $= 50^T + 50\% = 75^T$. " " 8^T " $= 9{\cdot}37$ "

B.S.S. 1933:

" " $= 50^T + 50\% = 75^T$. " " 9^T " $= 8{\cdot}33$ "

‡ The notation of U_1, U_2, etc., for the upper panel points and L_1, L_2, etc., for the lower panel points is used by some bridge engineers.

load at D causes an end reaction of $1^T \times 67{\cdot}5' \div 75' = 0{\cdot}9^T$, as is also given by the influence line.

No Cross Girder at L_0. Fig. 59 (c) gives the influence line for the reaction on the end main bearing. Because the stringers rest

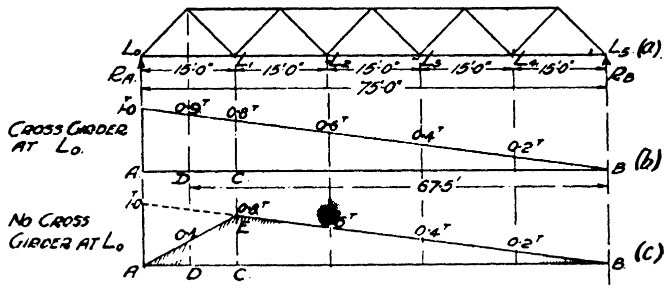


FIG. 59.

directly on the abutment they give up part of their load to cross girder C and the remainder to the masonry without affecting the bearing at A. A unit load at D of Fig. (c) means only $0{\cdot}4^T$ to end A. This can be checked by thinking of the tiny internal stringer bridge with a central load at D. Then $0{\cdot}5^T$ reaction goes directly to the abutment and $0{\cdot}5^T$ to the main truss at point C. So far then the main truss carries not 1^T but only $0{\cdot}5^T$ at C, which means a reaction at bearing A of $0{\cdot}5^T \times CB \div BA$ (*i.e.*, moments at B to find A) $= 0{\cdot}5^T \times 60' \div 75' = 0{\cdot}4^T$, as is also obtained by influence line *c*.

Top Boom or Flange. Moment centres for U_1U_2 and U_2U_3 are respectively L_1 and L_2.

U_2U_3 in Full Detail. (1) Unit load anywhere between L_0 and L_2, *i.e.*, x varies from 0 to 30′ 0″, no other load on the span, Fig. 60.

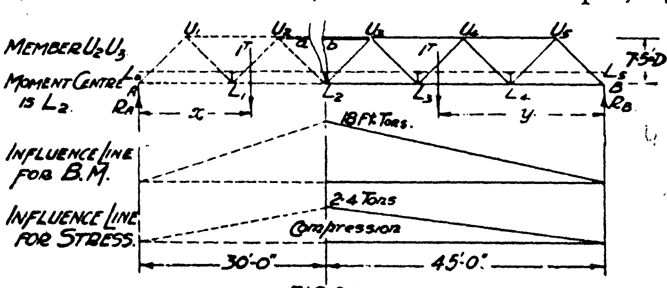


FIG. 60.

B.M. at L_2

$= 45\ R_B = 45(1^T \times x \div 75) \qquad = 0{\cdot}6x$

$= 0$ when x is 0, and when x is a max. of $30' \qquad = 18$ ft.^{T}

Now consider the load between L_5 and L_2, *i.e.*, y varies from 0 to 45′.

B.M. at L_3

$= 30\ R_A = 30(1^T \times y \div 75)$ $= 0{\cdot}4y$

$= 0$ with y equal to 0, and when y is a max. of 45′ $= 18$ ft.^{T}

The B.M. influence curve is thus a triangle of apex height $= 18$ ft.^{T}

These moments tend to close gap *ab*, therefore causing compression in U_2U_3.

The stress influence curve immediately follows on dividing the I.L. values for B.M. by the girder depth of 7·5′. Max. ordinate $= 18 \div 7{\cdot}5$ $= 2{\cdot}4^T$

Alternatively, as is usually done, obtain the I.L. curve for stress without drawing the B.M. I.L., thus :—

Stress U_2U_3

$= \text{B.M.} \div D = 45\ R_B \div 7{\cdot}5 = 0{\cdot}6x \div 7{\cdot}5$ $= 0{\cdot}08x$

$= 0$ when x is 0 and when x is a max. of 30′ $= 2{\cdot}4^T$

$=$ also $30\ R_A \div 7{\cdot}5 = 0{\cdot}4y \div 7{\cdot}5$ $= 0{\cdot}05\dot{3}y$

$= 0$ when y is 0 and when y is a max. of 45′ $= 2{\cdot}4^T$

Bottom Boom or Flange. Moment centre for L_0L_1 is U_1; for L_1L_2 is U_2, etc.

Because these moment centres do not lie vertically above the cross girders the resulting influence curves are truncated triangles as fully explained on page 54. The method will be briefly repeated for bar L_1L_2, Fig. 61.

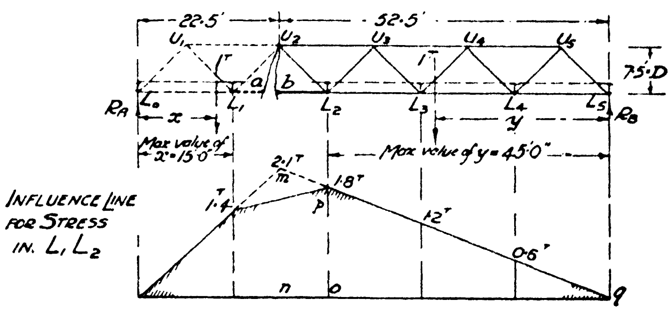


FIG. 61.

Stress $L_1L_2 = \text{B.M. at } U_2 \div D$

With x as the only variable, *i.e.*, unit load between L_0 and L_1, $R_B = (1^T \times x \div 75)$ and B.M. $U_2 = 52{\cdot}5\ R_B$

Stress L_1L_2

$= 52{\cdot}5\ R_B \div 7{\cdot}5 = 52{\cdot}5(x \div 75) \div 7{\cdot}5$ $= 0{\cdot}09\dot{3}x$

$= 0$ when x is 0 and when x is a max. value of 15′ $= 1{\cdot}4^T$

Unit load between L_5 and L_2, *i.e.*, y varies from 0 to 45′ then :—

Stress L_1L_2

$= 22{\cdot}5\ R_A \div 7{\cdot}5 = 22{\cdot}5(y \div 75) \div 7{\cdot}5 \qquad = \quad 0{\cdot}04y$

$= 0$ when y is 0 and when y is a max. value of 45′ $\qquad = \quad 1{\cdot}8^T$

Finally, let the load lie between L_1 and L_2, *i.e.*, Z varies from 0 to 15′ in Fig. 62. The local reaction from the stringer on cross girder L_2 is $1^T \times Z \div 15$, and on the main girder $R_B = 1^T(15 + Z) \div 75$.

Hence the B.M. at U_2

$= 52{\cdot}5\ R_B - 7{\cdot}5\ R_{L_2}$

$= 52{\cdot}5\dfrac{(15 + Z)}{75} - 7{\cdot}5\dfrac{(Z)}{15}$. Dividing by $D = 7{\cdot}5$

gives :—

Stress L_1L_2

$$= \frac{52{\cdot}5(15 + Z)}{75 \times 7{\cdot}5} - \frac{7{\cdot}5Z}{15 \times 7{\cdot}5} = \frac{42 + 0{\cdot}8Z}{30}$$

$$= \frac{42 + 0{\cdot}8 \times 0}{30} \text{ (when the straight line has } Z = 0) \qquad = \quad 1{\cdot}4^T$$

$$= \frac{42 + 0{\cdot}8 \times 15}{30} \text{ (when } Z \text{ has its max. value)} \qquad = \quad 1{\cdot}8^T$$

These values agree with those found above.

A quick method of obtaining this influence line is to calculate the B.M. at U_2 as if there were no stringers. In such a case the B.M. influence line would be a triangle with apex lying on the vertical through U_2, Fig. 61.

1^T at U_2 then $R_B = 1^T \times 22{\cdot}5' \div 75' \qquad = \quad 0{\cdot}3^T$

B.M. at $U_2 \quad = 52{\cdot}5\ R_B = 52{\cdot}5' \times 0{\cdot}3^T$; in ft. tons $\qquad = 15{\cdot}75$

" Flange Stress " $= \text{B.M.} \div D = 15{\cdot}75 \div 7{\cdot}5 \qquad = \quad 2{\cdot}1^T$

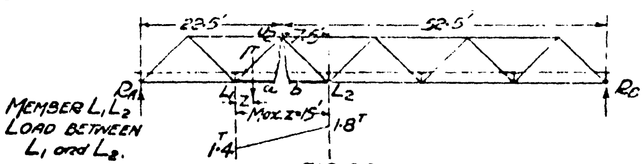


FIG. 62.

This value of 2·1 is the apex of the truncated triangle. The proper influence line is now obtained by beheading this triangle by a line joining the points where the influence triangle meets the verticals dropped from L_1 and L_2, Fig. 61. The arithmetical value at point p is given by the similar triangles mnq and poq :

$mn/nq = po/oq$

or $\ po = mn \times oq \div nq = 2{\cdot}1 \times 45 \div 52{\cdot}5 \qquad = 1{\cdot}8^T$

The stress in member L_1L_2 is tensile because the bending moment, taken about point U_2, tends to widen the gap *ab* in Fig. 61.

Web Members

Stress influence lines for the web members of parallel flanged girders having been dealt with already on page 49, the following short explanation is all that is now necessary.

If a section were cut at *XX* the positive shear of Fig. 63 (a) would force the left-hand portion upwards, placing bar U_2L_1 in compression. It follows that the vertical component of stress U_2L_1 equals this vertical shearing force, because the flange members offer no direct resistance to a force at right angles to themselves. In diagram (c) the length *ab* represents the stress in U_2L_1 to scale, and its vertical component is *ac*.

i.e., $ac = ab \sin\theta =$ vertical shear. Now, since $\dfrac{\text{distance } D}{\text{length } U_2L_1}$ also $= \sin\theta$, then $ab = \dfrac{\text{vert. shear}}{\sin\theta} = \dfrac{\text{vert. shear}}{D \div \text{length } U_2L_1} =$ vert. shear $\times \dfrac{\text{(member's length)}}{\text{girder's depth}}$, whence *ab* or stress in bar $U_2L_1 =$ vert. shear $\times$ 1·414.

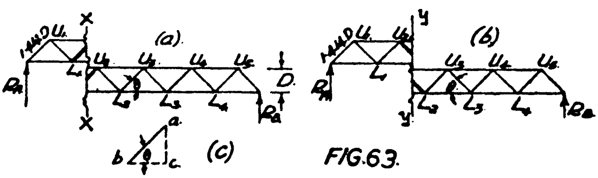


FIG. 63.

Similarly if a section *YY* be taken the positive shearing force places U_2L_2 under a tension whose numerical value equals that of stress U_2L_1 of the same panel, because shear is constant throughout a panel. Had the shear in the panel been negative then U_2L_1 would have been in tension and U_2L_2 in compression.

To ascertain values for the influence line roll unit load from *B*, where $y = 0$, to L_2, where $y = 45$, an action which creates positive shear in panel L_1L_2, Fig. 64.

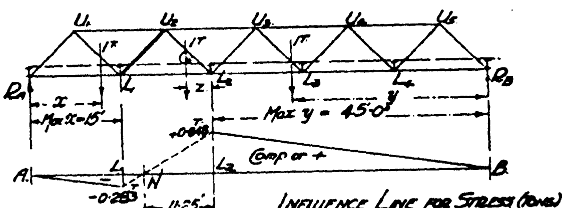
The positive vertical shear in panel $L_1L_2 = R_A = (1^T \times y) \div 75$, which has a min. and a max. value of 0 and 0·6, respectively. Consequently the corresponding stress values for member $U_2L_1 = 0$ and $0{\cdot}6 \times 1{\cdot}414 = 0{\cdot}848^T$ compression and for member $U_2L_2 = 0$ and $0{\cdot}848^T$ tension.

Negative shear in panel L_1L_2 is given by allowing unit load to move from *A* to L_1. Its value $= R_B = (1^T \times x) \div 75$ with a minimum value of 0 and a maximum value of 15 ÷ 75 or 0·2 when x is 15′; the corresponding stress values for U_2L_1 being 0 and $0{\cdot}2 \times 1{\cdot}414 = 0{\cdot}283^{T}$ tension, and for member U_2L_2 they are 0 and $0{\cdot}283^{T}$ compression. A straight line joining point 0·283 to the value 0·848 completes the influence line of Fig. 64.

More fully, however, think of a load between L_1 and L_2. This causes an $R_A = 1^{T}(45 + Z) \div 75$ and a local reaction at L_1 of $Z \div 15$.

Shear in panel L_1L_2

$$= R_A - R_{L1} = \frac{45 + Z}{75} - \frac{Z}{15} = \frac{45 - 4Z}{75} \quad . \quad . \quad . \quad . \quad . \quad (a)$$



INFLUENCE LINE FOR STRESS (TONS) MEMBER U_2L_1.
FIG. 64.

and the stress in the diagonals $= \left(\dfrac{45 - 4Z}{75}\right)1{\cdot}414$

when $Z = 0$ the stress value is $\frac{3}{5}$ of 1·414 $= 0{\cdot}848^{T}$

and $Z = 15$ the stress is $\frac{1}{5}$ of 1·414 $= 0{\cdot}283^{T}$

When conversant with the theory the following is a quick method of deriving the influence line. At point A on Fig. 64 erect above the base line a vertical equal to 1·414 × unit shear, *i.e.*, 1·414, and under the base line at B erect a similar perpendicular. Now join both these points to the opposite ends of the base line AB. Finally, join the intersection points of these sloping lines with the verticals dropped from L_1 and L_2. The values of these two last mentioned points are $\frac{3}{5}$ of 1·414, or 0·848, and $\frac{1}{5}$ of 1·414, or 0·283, see Fig. 67.

Neutral Point for Panel L_1L_2. The point where the stress influence line changes from plus to minus, *i.e.*, the neutral point, can be found by equating (a) to zero since stress is zero.

$$\text{Stress in web members} = 0 = \frac{45 - 4Z}{75}$$

$$\therefore Z = 45 \div 4 = 11{\cdot}25'$$

measured from L_2. This verifies the rule for the position of the neutral point which was: The neutral point divides the panel in the same ratio as it divides the span.

$$\frac{L_1N}{L_2N} = \frac{AN}{BN}, \text{ i.e., } \frac{(15 - Z)}{Z} = \frac{(30 - Z)}{(45 + Z)}$$

or $30Z - Z^2 = 675 - 45Z + 15Z - Z^2$ whence $Z = 11{\cdot}25'$.

Stress U_1L_0. A load placed on the stringers anywhere between L_1 and L_5 must find its way back into the main truss. At L_5 it causes no reaction at A, then as it approaches L_1 the value of R_A rises, Fig. 65.

$$R_A = 1^T \times x \div 75 \quad . \quad . \quad . \quad . \quad . \quad \text{(b)}$$

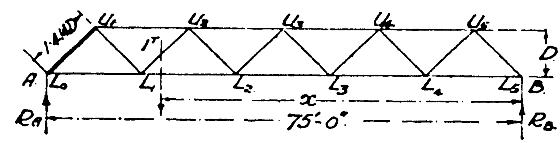


FIG. 65.

The shear in panel L_0L_1 is thus $= R_A$, and therefore the stress in U_1L_0 or $U_1L_1 = 1{\cdot}414\ R_A = 1{\cdot}414x \div 75$.

The max. value occurs when $x = 60'$, *i.e.*, 1^T load at L_1, then stress in U_1L_0 or $U_1L_1 = 1{\cdot}414 \times 60 \div 75 = 1{\cdot}13$; the maximum ordinate for influence lines U_1L_0 and U_1L_1, Fig. 67.

Now let the unit load pass point L_1 and enter the stringers of panel L_0L_1. As it advances into this panel the local reaction or load which finds its way back to L_1 gets smaller and smaller, while the other reaction, either on the cross girder at L_0, or on the masonry if cross girder L_0 is left out, gets greater and greater. In any case the effect upon the main structure, and web members U_1L_0 and U_1L_1 in particular, decreases to zero as was already explained for the Main Bearings, Fig. 59 (c).

Member U_5L_5 is a mirror reflection of U_1L_0 and so are the influence lines; U_1L_1 of U_5L_4, U_2L_1 of U_4L_4, and hence only the influence lines for members up to the centre line need be drawn. Further, U_1L_0 is similar to U_1L_1, but opposite in sign, and so again a saving in drawing the influence lines can be made, see Fig. 67.

Design Stresses, Fig. 68. The stresses due to the several loadings are computed on pages 93 and 96, and are based upon the complete sets of influence lines set out in Figs. 66 and 67.

Since the pedestrian traffic, or U.D.L.L., was taken as a dense, slow-moving crowd of 1 cwt./sq. ft. intensity, no increase for impact requires to be made to the stresses derived from the influence lines.

The stresses induced by the wheel loads as static loads are set forth in Fig. 68 (c), and the 30% allowance which requires to be added is given by (d). The total stresses created by the moving wheel loads are summarised in (e).

The design stress for any one bar is now found by adding to the dead load stress the greater of the two stresses given by diagrams (b) and (e). For each of the web members U_2L_1, U_2L_2, U_3L_2, U_3L_3, U_4L_3 and U_4L_4 the maximum is the summation of the dead load and the wheel load stresses, but for every other member of the truss it is the dead load and the U.D.L.L. Further, there are alternating stresses in web members U_3L_2 and U_3L_3, because as the U.D.L.L. crosses the span each of these members has a complete reversal in stress from + 4·74T to − 4·74T, or *vice versâ*.

Dead Load Stresses. (0·81T/ft. run ; main truss fully loaded.)

Member		Tons.
L_0L_1 and L_5L_4	= (−) area 30 × 0·81	= − 24·3
L_1L_2 and L_4L_3	= ,, ,, 75 × 0·81	= − 60·75
L_2L_3	= ,, ,, 90 × 0·81	= − 72·9
U_1U_2 and U_5U_4	= (+) area 60 × 0·81	= + 48·6
U_2U_3 and U_4U_3	= ,, ,, 90 × 0·81	= + 72·9
U_1L_0 and U_5L_5	= (+) area 42·375 × 0·81	= + 34·32
U_1L_1 and U_5L_4	= (−) ,, ,, ,,	= − 34·32
U_2L_1 and U_4L_4	= area (23·85 − 2·65) × 0·81	= + 17·17
U_2L_2 and U_4L_3	= ,, (− 23·85 + 2·65) × 0·81	= − 17·17
U_3L_2 and U_3L_3	= ,, (+ 10·613 − 10·613) × 0·81	= 0

Alternative Method for Dead Load Stresses.

Panel point load was 12·15T.

L_0L_1 and L_5L_4 12·15 (sum of Inf. Line ordinates at L_0, L_1, L_2, etc.).
= 12·15(0+0·8+0·6+0·4+0·2+0)
= 12·15 × 2 = − 24·3

L_1L_2 and L_4L_3 = 12·15(0+1·4+1·8+1·2+0·6+0)
= 12·15 × 5 = − 60·75

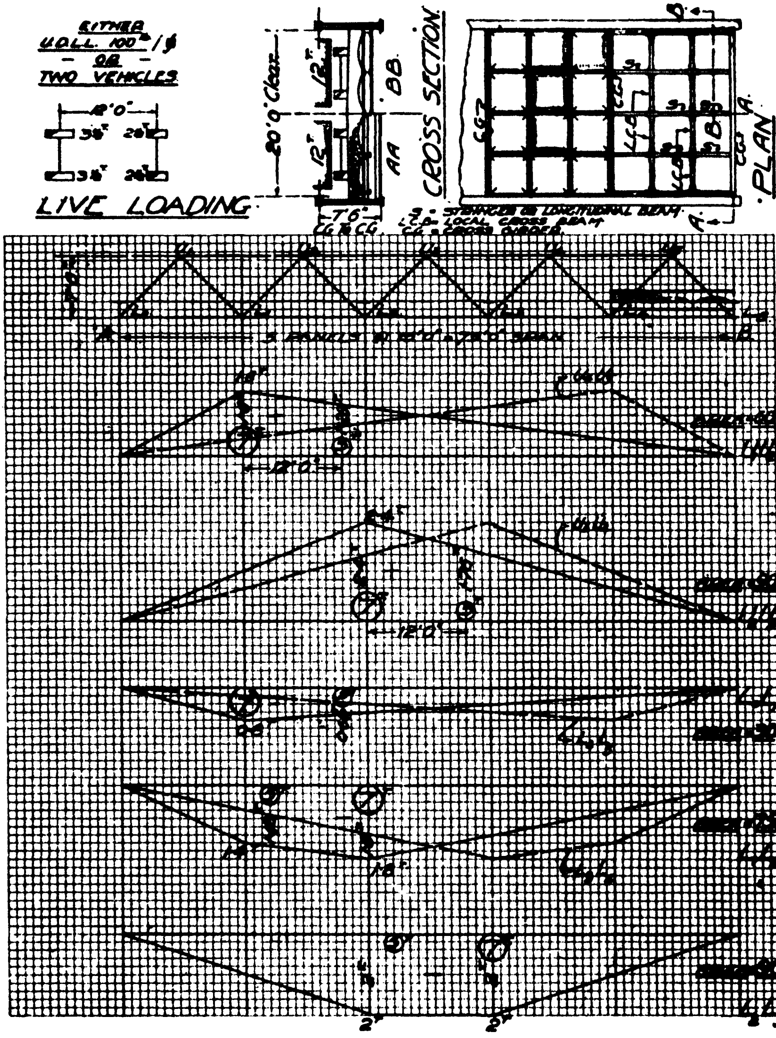
L_2L_3 = 12·15(0 + 1 + 2 + 2 + 1 + 0)
= 12·15 × 6 = − 72·9

U_1U_2 and U_5U_4 = 12·15(0+1·6+1·2+0·8+0·4+0)
= 12·15 × 4 = + 48·6

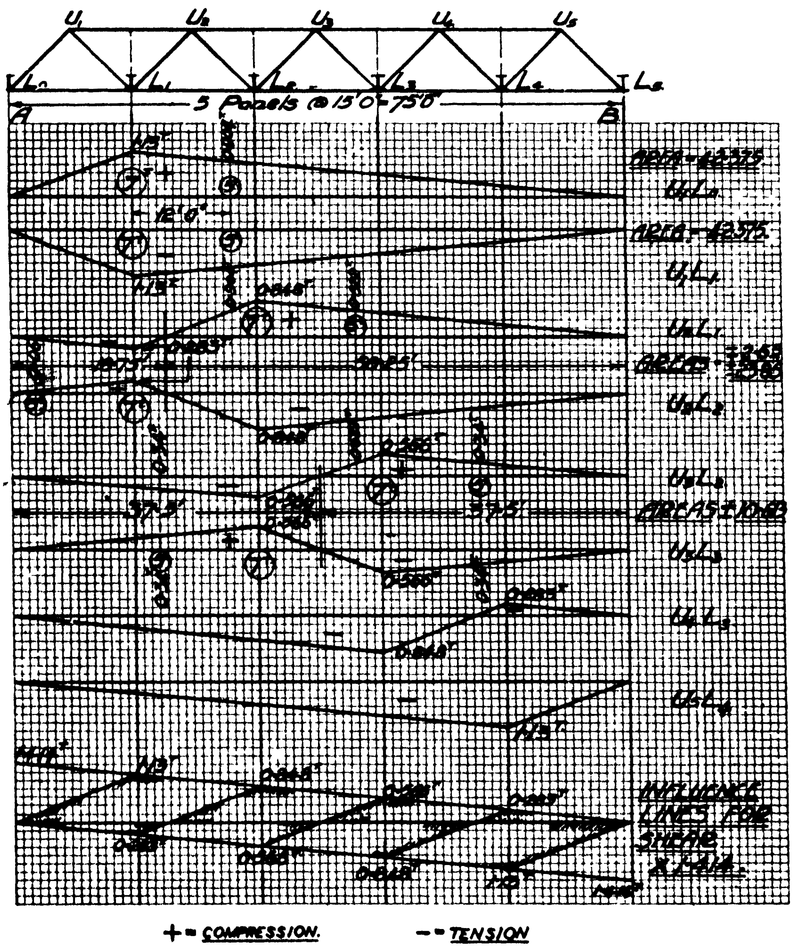
U_2U_3 and U_4U_3 = 12·15(0+1·2+2·4+1·6+0·8+0)
= 12·15 × 6 = + 72·9

U_1L_0 and U_5L_5 = 12·15(0+1·13+0·85+0·57 + 0·28 + 0)
= 12·15 × 2·83 = + 34·36

U_1L_1 and U_5L_4 = ditto = − 34·36



FLANGE MEMBERS.
Fig. 66.



WEB MEMBERS
FIG. 67.

		Tons.
U_2L_1 and U_4L_4	$= 12{\cdot}15(0 - 0{\cdot}283 + 0{\cdot}848 + 0{\cdot}566 + 0{\cdot}283 + 0)$	
	$= 12{\cdot}15 \times 1{\cdot}414$	$= + 17{\cdot}17$
U_2L_2 and U_4L_3	= ditto	$= - 17{\cdot}17$
U_3L_2 and U_3L_3	$= 12{\cdot}15(0 - 0{\cdot}283 - 0{\cdot}566 + 0{\cdot}566 + 0{\cdot}283 + 0)$	
	$= 12{\cdot}15 \times 0$	$= 0$

U.D.L.L. Stresses. ($0{\cdot}447^T$/ft. run of main truss.)

L_0L_1 and L_5L_4	$= - 30 \times 0{\cdot}447$	$= - 13{\cdot}41$
L_1L_2 and L_4L_3	$= - 75 \times 0{\cdot}447$	$= - 33{\cdot}53$
L_2L_3	$= - 90 \times 0{\cdot}447$	$= - 40{\cdot}23$
U_1U_2 and U_5U_4	$= + 60 \times 0{\cdot}447$	$= + 26{\cdot}82$
U_2U_3 and U_4U_3	$= + 90 \times 0{\cdot}447$	$= + 40{\cdot}23$
U_1L_0 and U_5L_5	$= + 42{\cdot}375 \times 0{\cdot}447$	$= + 18{\cdot}94$
U_1L_1 and U_5L_4	= – (,, ,,)	$= - 18{\cdot}94$
U_2L_1 and U_4L_4	$= + 23{\cdot}85 \times 0{\cdot}447$	$= + 10{\cdot}66$
and	$= - 2{\cdot}65 \times 0{\cdot}447$	$= - 1{\cdot}18$
U_2L_2 and U_4L_3	$= - 23{\cdot}85 \times 0{\cdot}447$	$= - 10{\cdot}66$
and	$= + 2{\cdot}65 \times 0{\cdot}447$	$= + 1{\cdot}18$
U_3L_2 and U_3L_3	$= \pm 10{\cdot}613 \times 0{\cdot}447$	$= \pm 4{\cdot}74$

Wheel Load Stresses.

A set of axle loads to each main truss.

L_0L_1 and L_5L_4	$= - (7 \times 0{\cdot}8 + 5 \times 0{\cdot}64)$	$= - 8{\cdot}8$
L_1L_2 and L_4L_3	$= - (7 \times 1{\cdot}8 + 5 \times 1{\cdot}48)$	$= - 20{\cdot}0$
L_2L_3	$= - (7 \times 2 + 5 \times 2)$	$= - 24{\cdot}0$
U_1U_2 and U_5U_4	$= + (7 \times 1{\cdot}6 + 5 \times 1{\cdot}28)$	$= + 17{\cdot}6$
U_2U_3 and U_4U_3	$= + (7 \times 2{\cdot}4 + 5 \times 1{\cdot}76)$	$= + 25{\cdot}6$
U_1L_0 and U_5L_5	$= + (7 \times 1{\cdot}13 + 5 \times 0{\cdot}9)$	$= + 12{\cdot}41$
U_1L_1 and U_5L_4	= – (,, ,,)	$= - 12{\cdot}41$
U_2L_1 and U_4L_4	$= + (7 \times 0{\cdot}848 + 5 \times 0{\cdot}62)$	$= + 9{\cdot}04$
and	$= - (7 \times 0{\cdot}283 + 5 \times 0{\cdot}06)$	$= - 2{\cdot}28$
U_2L_2 and U_4L_3	$= - (7 \times 0{\cdot}848 + 5 \times 0{\cdot}62)$	$= - 9{\cdot}04$
and	$= + (7 \times 0{\cdot}283 + 5 \times 0{\cdot}06)$	$= + 2{\cdot}28$
U_3L_2 and U_3L_3	$= \pm (7 \times 0{\cdot}566 + 5 \times 0{\cdot}34)$	$= \pm 5{\cdot}66$

Vertical Members. The verticals at points L_1, L_2, L_3 and L_4 do not belong to the truss system and are shown in Fig. 68 (f) as having no stress.

Their function is to lower the slenderness ratio $\frac{l}{k}$ of the upper boom and to act as stools to which the cross girders are attached.

The section used for these verticals is usually either an R.S.J. or a built-up section of four angles and a web plate to give the R.S.J. form. The sizes of the rolled sections in this member are usually governed by details and not by calculation. The width of the web

FIG. 68.

plate of the built-up vertical (as for other web members) is governed by the width between the gusset plates of the inverted U-shaped upper boom, and the thickness by the minimum thickness allowed by specification. The dimensions of the angles forming the flanges of these "redundant" verticals are settled thus :—

The outstanding legs in the flanges must be in register with the end angle cleats of the cross girder, because of the common connecting site rivets or bolts, while the thickness of the angles is usually determined by the bearing value of these site rivets or bolts.

CHAPTER IX

PRATT TRUSS ROAD BRIDGE

SPAN = 70′ 0″. Roadway of 22′ 0″ including gutters, *i.e.*, two tracks of 10′ 0″ net, to carry the Ministry of Transport loading for highway bridges.*

The floor of this bridge was designed to carry the M. of T. wheel loads because the 11^{T} wheel loads gave a more severe case of loading on the trough flooring than the U.D.L.L. of 220 lb./sq. ft. plus the knife edge load of 2,700 lb. per lineal foot placed anywhere.† The troughs adopted were 24″ wide, crest to crest, and $7\frac{1}{2}$″ deep, on the supposition that one and a half troughs participated in carrying a wheel load of 11^{T} in addition to the dead load. For a simply supported span of 12′ 0″ the modulus required with f_t at 9^{T}/sq. in. was 49, while that given was 53 ins. cu.

Experiments carried out on the dispersion effects of concentrated loads on trough flooring (Report of the Bridge Stress Committee) show that if contiguous troughs D, C, B, A, B', C', D' have a load W placed at the crest of trough A, then this mid-trough A carries $\frac{4}{16}$ of W. Troughs B and B' immediately on each side of the loaded trough carry $\frac{3}{16}$ W each; troughs C and C' $\frac{2}{16}$ W each, while troughs D and D' take $\frac{1}{16}$ W each.

There are ten troughs on the cross section of the bridge with a crest on the longitudinal centre line—the small cross section of Fig. 69 is only diagrammatic. Immediately on each side of the centre line there is a wheel, one belonging to each vehicle, so that there is a possibility of approximately $\frac{4}{16}$ W from each of the innermost adjacent wheels of each engine, *i.e.*, a total of 0·5 W, coming on the mid-trough.

However, there is the case of the two outermost troughs, parallel and adjacent to the main girders, which are given no relief of stress such as that envisaged above, and, as the working stress was taken at the higher figure of 9^{T} (in place of 8^{T}) per sq. in., it was thought reasonable to take the figure of 0·66 W or one and a half troughs per wheel load. The designer, of course, can adopt the alternative method of reinforcing the outermost troughs and so save material in the internal troughs.

* See "Wheel Loads," page 173, and Fig. 112.

† For the drawings and complete design of this bridge, see Chapter 10, Vol. II, "Practical Design of Simple Steel Structures."

*Floor Dead Load per sq. ft.**

Tar mac. av. thick. 4″ at 140 lb./cu. ft.	= 46·7 lb.
3″ concrete cover on top of troughs at 140 lb./cu. ft.	= 35
$3\frac{1}{2}$″ concrete av. thick. in troughs at 140 lb./cu. ft.	= 40·8
M.S. trough per sq. ft.	= 27
Total per sq. ft. of road surface, say	= 150 lb.

Cross Girder

Dead weight of each built-up or plated R.S.J. = 1·3T
(Either 4 Ls 6″ × 4″ × $\frac{5}{8}$″ + 27″ × $\frac{3}{8}$″ web pl. or 90 lb. R.S.J. + 10″ × $\frac{1}{2}$″ pl. on each flange.)

Main Pratt Truss

Approximate estimate of dead weight = 10T

Dead Load per Panel Point = 11·2T

Since each main truss carries half the roadway, half of each cross girder and its own dead load.
(12′ × 11′ @ 150 lb./sq. ft. + $\frac{1}{2}$ of 1·3T + 10T ÷ 6 full panels = 8·84T + 0·65T + 1·67T)

D.L. per ft. run of Main Truss = 11·2T ÷ 12′ 0″ panel length, say = 0·94T

Live Load

The Ministry of Transport wheel loads which include 50% for impact (June, 1922) as given in Fig. 69. Two complete sets of engines and trailers are supposed to cross the bridge side by side both in the same direction. Hence each truss will carry the *axle* loads given on figure.

Uniformly Distributed Live Load of 220 lb. per sq. ft. for spans of 10′ 0″ to 75′ 0″.
Per ft. run of main truss = 220 lb. × 10′ width × 1′ ÷ 2,240 = 0·98T

Knife Edge Load (in addition to the U.D.L.L.) placed anywhere, 2,700 lb. × 10′ 0″ width ÷ 2,240 = 12·05T

U_1U_2. Moment Centre L_2 — Tons.

			Tons.
D.L.	= 54·05 × 0·94 =		+ 50·81
U.D.L.L.	= 54·05 × 0·98 = 53T		
Knife L.	= 1·544 × 12·05 = 18·6T		
		Total =	+ 71·6
Axle L.	= 8 × 0·873 + 22 × 1·544 + 10(1·152 + 0·888 + 0·559 + 0·296)	=	+ 69·9

* See "Weights of Materials," page 108.

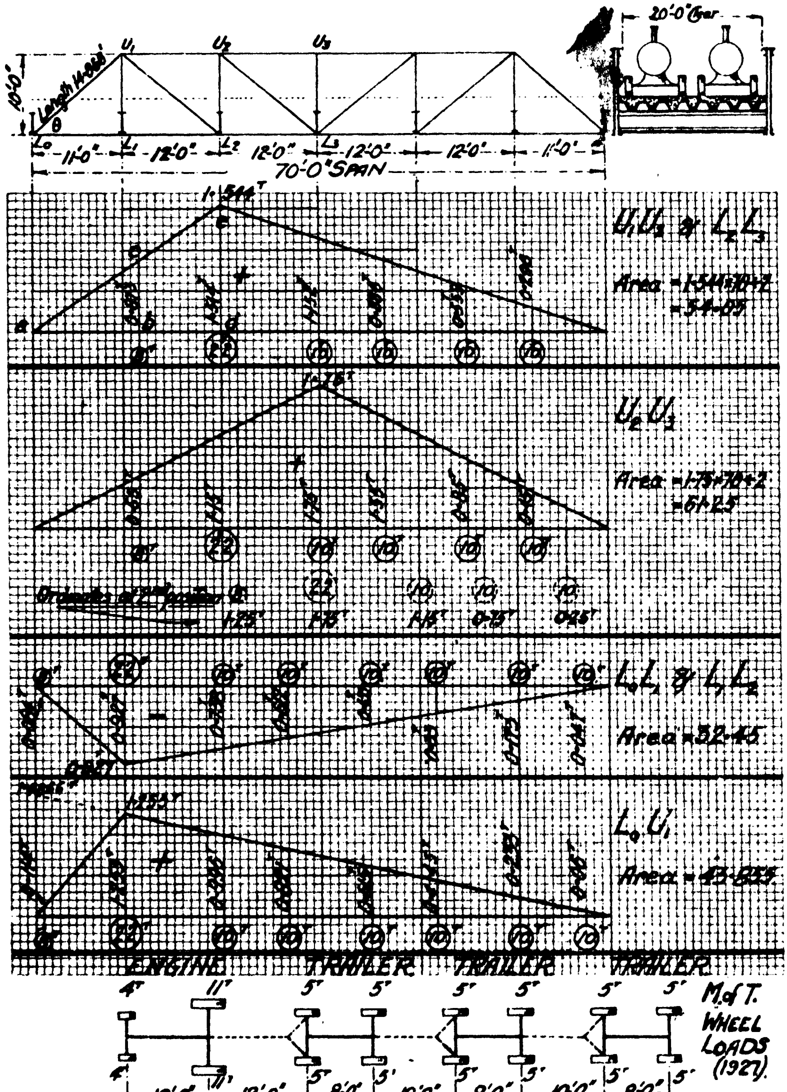

U1
U2
U3
L0
L1
L2
L3
10'-0"
Length 14.866'
θ
11'-0"
12'-0"
12'-0"
12'-0"
12'-0"
11'-0"
70'-0" SPAN
20'-0" over
U1U2 & L2L3
Area = 1.544×70÷2 = 54.05
U2U3
Area = 1.75×70÷2 = 61.25
L0L1 & L1L2
Area = 32.45
L0U1
Area = 43.835
ENGINE
TRAILER
TRAILER
TRAILER
M. of T.
WHEEL
LOADS
(1927).
10'-0"
12'-0"
8'-0"
10'-0"
8'-0"
10'-0"
8'-0"

FIG. 69.

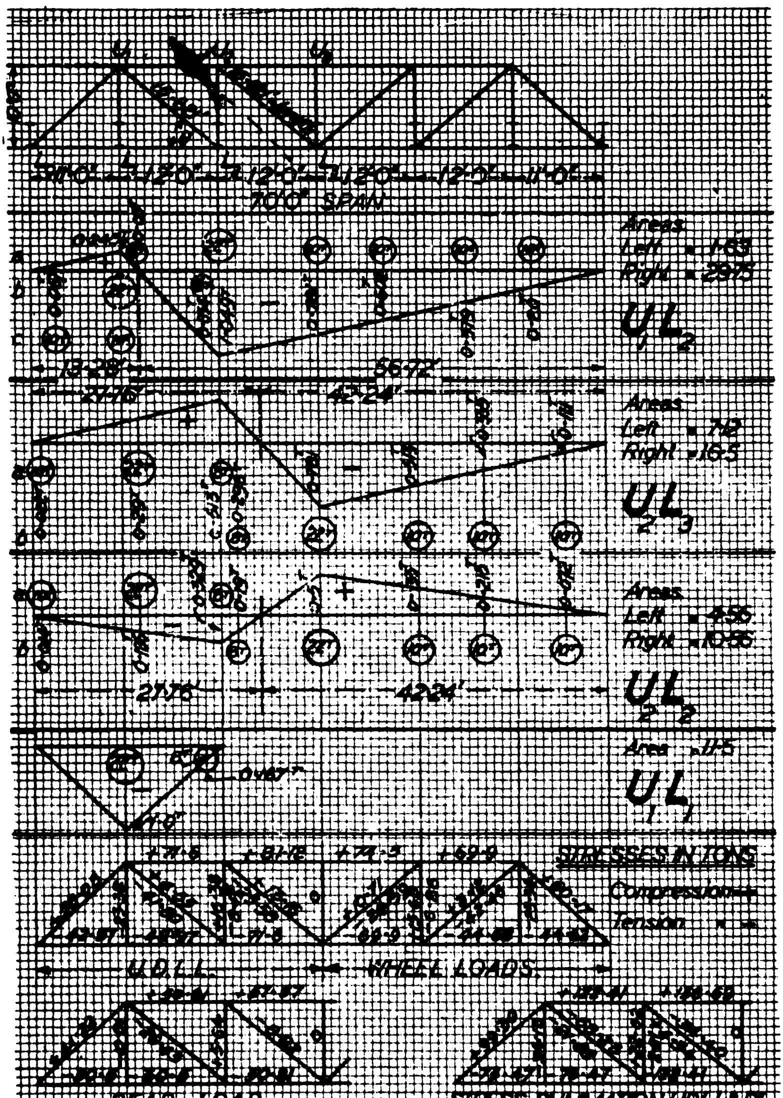

70'0" SPAN
Areas
U.D.L.L.
WHEEL LOADS.
STRESSES IN TONS
Compression +
Tension −
DEAD LOAD
STRESS SUMMATION, U.D.L.L. & D.L.

FIG. 70.

U_2U_3. Moment Centre L_3 — Tons.

D.L. = 61·25 × 0·94 = + 57·57
U.D.L.L. = 61·25 × 0·98 = 60·03$^{\text{T}}$
Knife L. = 1·75 × 12·05 = 21·09$^{\text{T}}$
Total = + 81·12
Axle L. = 8 × 0·65 + 22 × 1·15 + 10(1·75 + 1·35 + 0·85 + 0·45) = + 74·5
2nd position = 8 × 1·25 + 22 × 1·75 + 10(1·15 + 0·75 + 0·25) = + 70·0

L_0L_1 and L_1L_2. Moment Centre U_1

D.L. = 32·45 × 0·94 = − 30·5
U.D.L.L. = 32·45 × 0·98 = 31·8$^{\text{T}}$
Knife L. = 0·927 × 12·05 = 11·17$^{\text{T}}$
Total = − 42·97
Axle L. = 8 × 0·084 + 22 × 0·927 + 10(0·738 + 0·612 + 0·455 + 0·33 + 0·173 + 0·047) = − 44·62

L_2L_3. Moment Centre U_2 (same as L_2 above)

D.L. = − 50·81
U.D.L.L. = 53$^{\text{T}}$
Knife L. = 18·6$^{\text{T}}$
Total = − 71·6
Axle L. = − 69·9

L_0U_1. Vertical shear × cosec θ = vert. shear × (14·866 ÷ 10)

D.L. = 43·855 × 0·94 = + 41·22
U.D.L.L. = 43·855 × 0·98 = 42·98$^{\text{T}}$
Knife L. = 1·253 × 12·05 = 15·10$^{\text{T}}$
Total = + 58·08
Axle L. = 8 × 0·114 + 22 × 1·253 + 10(0·996 + 0·821 + 0·615 + 0·445 + 0·233 + 0·06) = + 60·17

U_1L_2.* Vertical shear × (15·621 ÷ 10)

D.L. = (+ 1·63 − 29·75) × 0·94 (all span loaded) = − 26·43
U.D.L.L. = −29·75×0·98 = −29·16$^{\text{T}}$ (part span loaded)
Knife L. = −1·049×12·05 = −12·64$^{\text{T}}$
Total = − 41·80
Axle L., (a) = 8 × (+ 0·03) − 22 × 1·049 − 10(0·781 + 0·602 + 0·379 + 0·2) = − 42·46

or

* See remarks on pages 104 and 105 regarding this member.

			Tons.
U.D.L.L.	$= +1{\cdot}63 \times 0{\cdot}98 = +1{\cdot}60^T$ (part span loaded)		
Knife L.	$= +0{\cdot}245 \times 12{\cdot}05 = +2{\cdot}95^T$	Total $= +$	4·55
Axle L.	Position (c), trailer leaving span, is more severe than position (b), engine entering span from left-hand abutment.		
	(c) $= 10(0{\cdot}067 + 0{\cdot}245)$	$= +$	3·12
	(b) $= -8 \times 0{\cdot}834 + 22 \times 0{\cdot}245 = -1{\cdot}282^T$.		

U_2L_3.* Vertical shear $\times$ $(15{\cdot}621 \div 10)$

D.L.	$= (+7{\cdot}12 - 16{\cdot}5) \times 0{\cdot}94$ (all span loaded)	$= -$	8·82
U.D.L.L.	$= -16{\cdot}5 \times 0{\cdot}98 = -16{\cdot}17^T$ (part span loaded)		
Knife L.	$= -0{\cdot}781 \times 12{\cdot}05 = -9{\cdot}41^T$	Total $= -$	25·58
Axle L., (b)	$= +8 \times 0{\cdot}298 - 22 \times 0{\cdot}781 - 10(0{\cdot}513 + 0{\cdot}335 \times 0{\cdot}111)$	$= -$	24·39
	or		
U.D.L.L.	$= +7{\cdot}12 \times 0{\cdot}98 = +6{\cdot}98^T$ (part span loaded)		
Knife L.	$= +0{\cdot}513 \times 12{\cdot}05 = +6{\cdot}18^T$	Total $= +$	13·16
Axle L., (a)	$= +8 \times 0{\cdot}513 + 22 \times 0{\cdot}29 + 10 \times 0{\cdot}022$	$= +$	10·71

U_1L_1. Vertical suspender carrying only the load from cross girder L_1

D.L.	$= -11{\cdot}5 \times 0{\cdot}94$	$= -$	10·81
U.D.L.L.	$= -11{\cdot}5 \times 0{\cdot}98 = -11{\cdot}27^T$		
Knife L.	$= -1{\cdot}0 \times 12{\cdot}05 = -12{\cdot}05^T$	Total $= -$	23·32
Axle L.	$= -22 \times 1 - 8 \times 0{\cdot}167$	$= -$	23·34

U_2L_2 carries the vertical shear in panel L_2L_3

D.L.	$= (+10{\cdot}56 - 4{\cdot}56) \times 0{\cdot}94$ (all span loaded)	$= +$	5·64
U.D.L.L.	$= (+10{\cdot}56) \times 0{\cdot}98 = +10{\cdot}35^T$ (part span loaded)		
Knife L.	$= +0{\cdot}5 \times 12{\cdot}05 = +6{\cdot}03^T$	Total $= +$	16·38
Axle L., (b)	$= -8 \times 0{\cdot}19 + 22 \times 0{\cdot}5 + 10(0{\cdot}33 + 0{\cdot}215 + 0{\cdot}072)$	$= +$	15·65
	or		

* See page 105.

			Tons.
U.D.L.L.	$= (-4{\cdot}56) \times 0{\cdot}98 = -4{\cdot}47^{T}$ (part span loaded)		
Knife L.	$= -0{\cdot}329 \times 12{\cdot}05 = -3{\cdot}98^{T}$		
		Total = −	8·45
Axle L., (a)	$= -8 \times 0{\cdot}329 - 22 \times 0{\cdot}186 - 10 \times 0{\cdot}014$	= −	6·864

Maximal Stresses

In the foregoing calculations it is seen that for maximal live and dead load stresses in the boom members all the span should be loaded. The majority of the web members, on the other hand, receive maximum stress with a partially loaded length because the influence lines have negative and positive lengths.

Take U_1L_2 as an example. Maximum positive shear in this panel occurs when only the 56·72′ right-hand length is covered by the live load. As the load advances into the small triangle (of opposite sign) of 13·28′ base the shear is reduced accordingly. Positive shear in panel L_1L_2 means that member U_1L_2 is extended in length, *i.e.*, in tension, hence the influence line for negative stress or tension is drawn under the base line. A unit load at L_2 would cause an end reaction at L_0 of $1^{T} \times 47 \div 70 = 0{\cdot}671^{T}$. There being no other load between L_0 and L_2 the shear in panel L_1L_2 is positive or upwards and $= 0{\cdot}671^{T}$.

The tensile stress in bar U_1L_2 = panel shear × length of U_1L_2 ÷ girder depth

$$= 0{\cdot}671 \times 15{\cdot}621 \div 10 = 1{\cdot}049$$

which is the max. ordinate to the influence line.

Since dead load covers all the span, therefore,

D.L. stress = area difference $(1{\cdot}63 - 29{\cdot}75) \times 0{\cdot}94^{T}$/ft. $= -26{\cdot}43^{T}$

Max. positive L.L. shear, *i.e.*, max. tensile stress in U_1L_2, load only the 56·72′ portion.

Max. negative L.L. shear, *i.e.*, max. compressive stress in U_1L_2, load only the 13·28′ portion.

Further, with the knife-edge load extending right across the bridge, of 20′ 0″ active width, the worst position which this load can occupy is that point on the bridge immediately under the apex of either triangle, *i.e.*, at a max. ordinate.

Wheel Loads

As the *Ministry of Transport has now specified a uniformly distributed live load plus a knife-edge load as being equivalent to, and acceptable in place of, the separate wheel loads of June, 1922, there is no call to employ both loadings. Either is sufficient and the

* See M. of T. loadings, page 173, and Fig. 112.

equivalent U.D.L.L. is preferable because of its simplicity. However, as an exercise in dealing with wheel loads these were also used.

From the stress summation diagram, Fig. 70, it will be observed how remarkably close in agreement are the stresses obtained by both types of loading. The wheel loads were used to design the bridge floor troughs because these carry the live load in its most concentrated form. Thereafter the U.D.L.L. was used for the remainder of the structure since the dispersion effect comes more and more into play as the load travels from tar macadam into troughs and from troughs into cross girders, thence to main trusses and so through the abutments into the foundations.

Alternating Stresses

Referring to the stress summation diagram, Fig. 70, it will be seen that the web members U_2L_2 and U_2L_3 each carry stresses of opposite sign, whereas all the flange or boom members are of constant sign and vary from the minimum or dead load stress to max. value of D.L. + L.L.

U_1L_2. The U.D.L.L. stress in this member varies from $4{\cdot}55^T$ compression to $41{\cdot}8^T$ tension as the load crosses the bridge. However, as the D.L. stress of $26{\cdot}43^T$ tension is always in existence the L.L. compressive stress of $4{\cdot}55^T$ only makes itself felt by reducing $26{\cdot}43^T$ tension to $21{\cdot}88^T$ tension. The variation of the total stress is thus from $-\ 68{\cdot}23^T$ to $-\ 21{\cdot}88^T$ with the L.L. on the bridge, and rises again to $-\ 26{\cdot}43^T$ under dead load only.

U_2L_3. In this case the D.L. figure of $-\ 8{\cdot}82^T$ is not sufficiently large to neutralise the U.D.L.L. figure of $+\ 13{\cdot}16^T$, and so the final stress is of opposite sign $= +\ 13{\cdot}16^T - 8{\cdot}82^T = +\ 4{\cdot}34^T$. This figure in turn decreases to zero as the load changes its position and rises to a maximum of $-\ 34{\cdot}4^T$, falling again to $-\ 8{\cdot}82^T$ when the bridge is empty.

U_2L_2. A similar explanation applies to this member.

Influence Line Ordinates

These given in the examples have been calculated to the "third decimal place." Thus, for member U_1U_2, Fig. 69, by similar triangles *abc* and *ade*, $bc : de :: ab : ad$ or $bc = (ab/ad)de =$ $(13/23)1{\cdot}544 = 0{\cdot}873$.

The usual degree of accuracy employed with influence lines in practice is the "second place of decimals" obtained either by calculation or by direct scaling. A draughtsman can read by the naked eye to 0·01″. The scale adopted for the influence line will settle whether the influence line ordinates should be scaled or calculated.

CHAPTER X

BRACED CANTILEVER AND SUSPENDED SPAN ROAD BRIDGE

The Dead Load of the wearing surface of the bridge floor, the floor steelwork, stringers and cross girders can usually be estimated with considerable accuracy before the design of the main cantilever girders is undertaken. The dead weight of the items mentioned can easily be turned into an equivalent figure of so many tons per ft. run of the bridge span. With main girders varying rapidly in depth, as these cantilevers do, it is clear that their dead weights cannot be so given, but must be distributed throughout the girder span as panel point loads. However, for simplicity, the dead load of the complete bridge will be taken as being equivalent to $\frac{3}{4}^{T}$ per ft. run of span for each main girder.

The Uniformly Distributed Live Load (*U.D.L.L.*) per ft. run of main girder depends only on the constant area of the bridge floor. Thus if the bridge is 30′ 0″ wide, the U.D.L.L. per ft. run of bridge at 2 cwts./sq. ft. $= 3^{T}$, and per ft. run of main girder is $1\frac{1}{2}^{T}$.

The Engines and Trailers. If the roadway be 20′ 0″ wide then each main girder in effect carries one engine with its trailer, *i.e.*, a set of axle loads. If the roadway be 30′ 0″ wide, with no footpaths, then it is possible to pass the engines and trailers across the bridge so that they are nearer one main girder than the other. The effect of this would be to throw, approximately, one and one-third axle loads on to one main girder and two-thirds axle loads on to the other. In the example the loading per main girder is one set of axle loads.

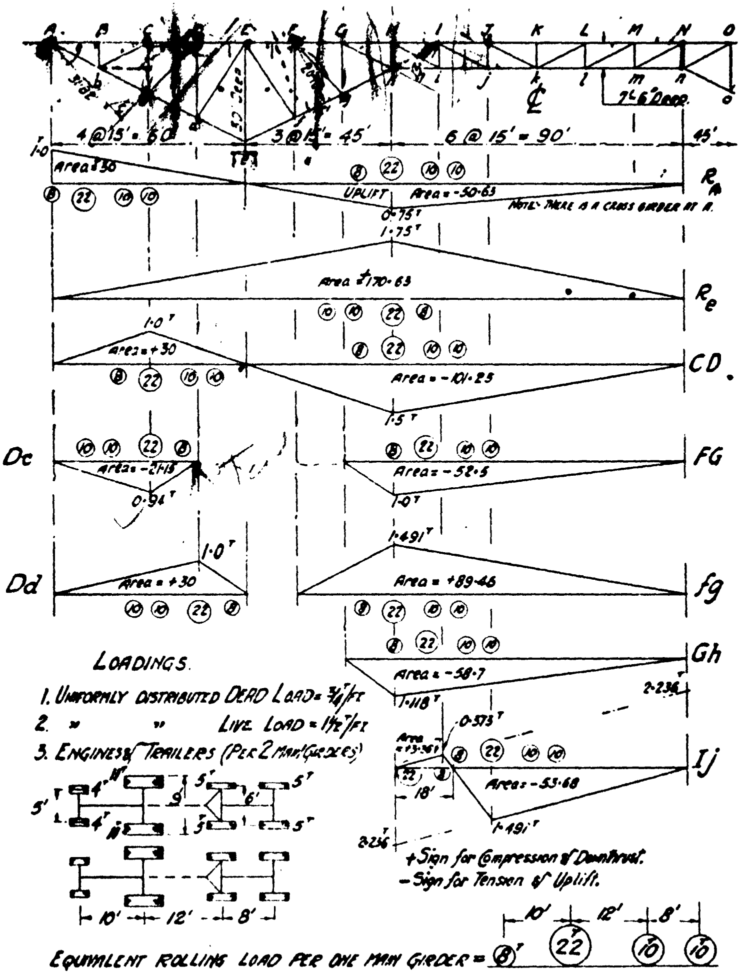
Reactions R_A and R_e. The influence lines for the reactions at the shore abutment A and the pier e are found as explained for the plate girder cantilever on page 25. These influence lines prove extremely useful when calculating the stress influence lines for the bridge main members.

Stress $CD = S_{CD}$. The two unwanted bars cd and cD meet at the moment centre c, giving the lever arm of 15′ to member CD.

1^{T} placed at A goes straight into the abutment $\quad \therefore S_{CD} = 0$

1^{T} „ B gives $R_A = \frac{3}{4}^{T}$, see I.L. for R_A, Fig. 71.

$$\therefore 15\, S_{CD} = 30\, R_A - 1^{T} \times 15 \qquad \therefore S_{CD} = +\tfrac{1}{2}^{T}$$



RESULTS. (TONS).

	R_A	R_e	CD	Dc	FG	Dd	fg	Gh	Ij
(1). U.D. Dead Load.	-15·5	128	-53·4	-15·9	-39·4	+22·5	+67·1	-44·0	-37·7
(2) U.D. Live Load	-75·9, +45	256·0	-151·9, +45	-31·7	-78·8	+45·0	+134·2	-88·1	-80·5, +5·0
(3) Wheel Loads.	-33·5, +37·7	+80·6	-67·0, +36·7	-32·0	-41·8	+37·6	+65·3	-46·7	-56·7, +5·72

FIG. 71.

1^T placed at C gives $R_A = \frac{1}{2}^T$, see I.L. for R_A

$$\therefore 15\, S_{CD} = 30\, R_A - 1^T \times 0 \qquad \therefore S_{CD} = +\, 1^T$$

When unit load passes the section line at C it leaves only one force to be dealt with in the moment equation, viz., R_A as shown in (a), Fig. 72. $\therefore$ 15 $S_{CD} = 30\ R_A$, or $S_{CD} = 2\ R_A$ applies to all wheel positions to the right of C, and whatever value R_A may have the influence line for S_{CD} has twice this numerical value, *i.e.*, I.L. for S_{CD} = twice the ordinate values of the I.L. for R_A.

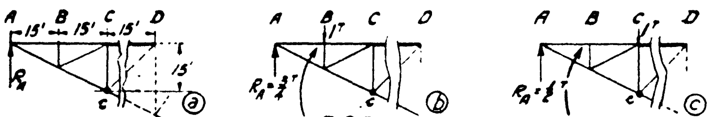


FIG. 72.

As a check try unit load at point F, Fig. 71. From the I.L. for R_A this creates an uplift at A of $\frac{1}{4}^T$, and so the reaction arrow head of Fig. 72 (a) acts downwards now instead of upwards, and 15 $S_{CD} = -\ 30\ R_A$ or $S_{CD} = -\ 2\ R_A = -\ 2 \times \frac{1}{4} = -\ \frac{1}{2}^T$. The broken ends of bar CD now tend to fly apart, indicating that CD is in tension with the load to the right of the pier e.

S_{CD} for U.D.D.L. The dead load is always acting throughout the full length of the bridge, hence S_{CD} dead load

$$= (\text{summation of areas} \times \tfrac{3}{4})^T$$
$$= (+\,30 - 101{\cdot}25)\tfrac{3}{4}^T = -\ 53{\cdot}4^T$$

S_{CD} for U.D.L.L.

Max. compression, apply load to the base of the + 30 triangle.
Max. tension, ,, ,, ,, $-$ 101·25 ,,

S_{CD} for Wheel Loads. Apply the max. axle load at the apex of either positive or negative triangle, as illustrated on the I.L. for CD, Fig. 71, in order to obtain the greatest stress in the member CD due to the vehicles.

Negative stress

$$= 8 \times \tfrac{35}{45} \text{ of } 1{\cdot}5^T + 22 \times 1{\cdot}5^T + 10(\tfrac{70}{90} + \tfrac{78}{90}) \times 1{\cdot}5^T$$
$$= 1{\cdot}5^T(8 \times \tfrac{7}{9} + 22 + \tfrac{1}{9} \times 148) \qquad = 67^T$$

Diagonal cD. On production the unwanted bars DC and dc meet at the moment centre A. By similar triangles ADc' and cDC it follows that $\frac{Ac'}{AD} = \frac{cC}{cD}$, whence $Ac' = AD \times cC \div cD$.

In Fig. 73 (a) the reaction R_A passes through the moment centre at A causing no moment, $\therefore S_{cD} \times 31{\cdot}82 = 1^T \times AC$, whence $S_{cD} = 0{\cdot}94^T$. Unit load at B will cause half this value, and at A the tensile stress in bar cD will be zero.

When unit load arrives at D it causes no stress in cD ; the web members shown in broken line, Fig. (b), are redundant for this load position. Similarly a load placed at E or to the right of E will not cause any direct stress in cD. These statements are easily proved by "Section and Moments," see diagram (a): since there is no load to the left of the section line except R_A there can be no moments about point A and therefore no stress in cD.

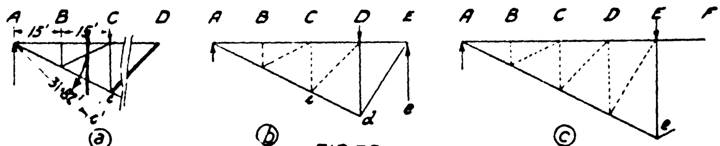


FIG. 73.

Vertical Dd. The section line cuts through bars ED, Dd, and dc, the first and last meeting on continuation at the moment centre A. As for the previous member cD, the stress in dD increases as unit load moves from A to D. $1^T \times AD = S_{dD} \times 45$ gives the max. value when unit load is at D, while zero values are obtained with unit load at A and E or elsewhere on the bridge outside of AE.

Top flange, FG, of the lever arm carries no live load stress so long as the moving unit load is between A and E because with this position of the load the shore arm acts as a simple girder. Neither does a load placed at F affect FG since this load goes down the vertical Ff on its way to the nearest points of support e and A. "Section and Moments" about point g with a lever arm of 15′ verifies this, Fig. 74 (a).

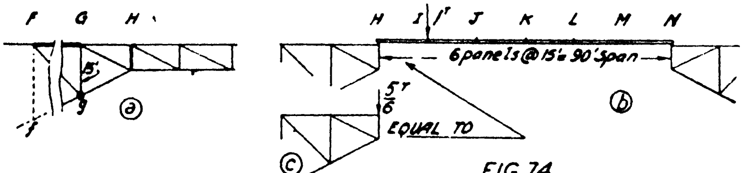


FIG. 74.

1^T at F creates no reaction or other force to the right of the section line and hence no stress in FG.

1^T at G being vertically over the moment centre g also causes no moment; and no stress in FG.

1^T at H. $S_{FG} = \frac{1}{15}(1^T \times 15)$, pulling FG apart $= -1^T$

If $\frac{5}{8}^T$ be placed at H then S_{FG} $= -\frac{5}{8}^T$

and $\frac{3}{8}^T$ placed at H gives a stress in FG $= -\frac{3}{8}^T$

These last two results are also obtained by placing a unit load first at I and then at J, on the suspended span, when R_H is $\frac{5}{6}$T and $\frac{4}{6}$T respectively, see Figs. 74 (b) and (c).

The results exhibit one interesting feature in that the values for the wheels shown on the plate, Fig. 71, are greater than if the 22T load were placed at the max. ordinate, also see Fig. 75.

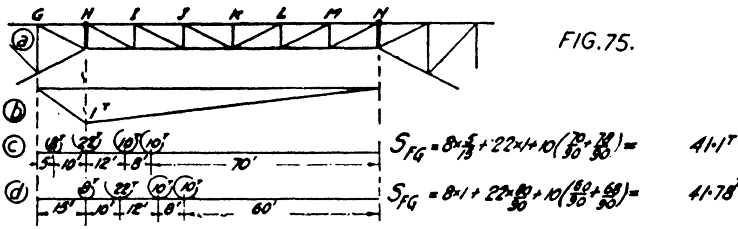


FIG. 75.

Lower flange, fg, if produced meets the upper flange at point I forming a triangle $f'FI$ which is similar to triangle gGI, and from these is obtained the ratio $\frac{Ff'}{FI} = \frac{gG}{gI}$, and so giving $Ff' = 20{\cdot}124'$, see Figs. 71 and 76.

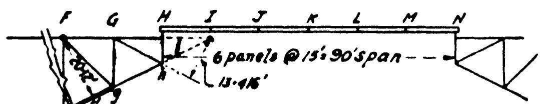


FIG. 76.

1T at F

No moment equation since F is the moment centre. $\therefore S_{fg} = 0$

1T at G

$1^T \times FG = S_{fg} \times 20{\cdot}124$, *i.e.*, $S_{fg} = 15 \div 20{\cdot}124 \qquad = 0{\cdot}745^T$

1T at H

$1^T \times FH =$,, ,, *i.e.*, $S_{fg} = 30 \div 20{\cdot}124 \qquad = 1{\cdot}491^T$

Thereafter, as the single wheel reaches I, J, K, etc., in turn, the load at H gradually decreases in value to $\frac{5}{6}$T, $\frac{4}{6}$T, $\frac{1}{2}$T, etc., respectively, and so the stress in fg has the corresponding values of $\frac{5}{6}$ of 1·49, then $\frac{4}{6}$ of 1·49, etc., down to zero value when the wheel arrives at point N, at which point the wheel load is wholly carried by the right-hand river cantilever.

With the wheels in the position shown on the plate, Fig. 71,

$$S_{fg} = 1{\cdot}491^T[\tfrac{2}{3} \times 8 + 22 + 10(\tfrac{70}{90} + \tfrac{78}{90})] = 1{\cdot}491^T[43{\cdot}777] = 65{\cdot}3^T$$

The broken ends of the bar fg approach each other under load intimating that the stress is compression. The foregoing position of the wheel loads gives a larger stress for fg than is obtained by moving the complete system 10′ to the right hand until the 8^T wheel comes under the apex.

GH of the top flange carries no stress whatsoever and is redundant, because the real cantilever is *AEGhe*, member *Hh* carrying the load from *H* down to nose *h*. Alternatively, by taking moments at *h*, see Fig. 77, and considering the stability of the section to the right of the cutting line, no moment equation is possible for this bar because *h* is directly under *H*, the point of loading.

Web diagonal Gh has its moment centre at *I*, Figs. 71 and 76.

1^T at *G* is to the left of the section line and has therefore no effect on member *Gh*. $\therefore S_{Gh} = 0$

1^T at *H*

$$S_{Gh} \times 13{\cdot}416 = 1^T \text{ at } H \times HI, \therefore S_{Gh} = \frac{1 \times 15}{13{\cdot}416} = 1{\cdot}118^T$$

A straight line from 0 at *G* rising to 1·118 at *H* and then dying away to zero as point *N* is approached is the outline of the influence line.

The wheel loads placed as shown for *Gh*, Fig. 71, give a larger stress to this member than that obtained if the system were moved 10′ 0″ towards the left-hand side until the max. wheel coincides with the max. ordinate.

As shown : $S_{Gh} = 1{\cdot}118^T\left[8 + \frac{80}{90} \times 22 + 10\frac{(68 + 60)}{90}\right] = 46{\cdot}7^T$

If moved : $S_{Gh} = 1{\cdot}118^T\left[\frac{1}{3} \times 8 + 22 + 10\frac{(78 + 70)}{90}\right] = 45{\cdot}96^T$

Ij, *Suspended Span*. The girder *HN* is a simply supported Pratt truss and the influence lines for all its members are found as previously described. This particular influence line should offer no difficulty.

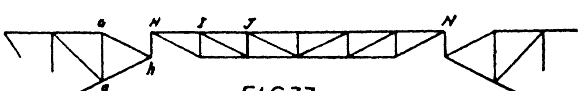


FIG.77.

Members *Hh* and *hi* of the Pratt truss are redundant, while bar *Hh* of the cantilever functions always as a column transferring the truss end reaction on to the nose *h* of the cantilever, and therefore its stress I.L. is the I.L. for reaction at *H* of the suspended span.

SECOND EXAMPLE ON BRACED CANTILEVER

In general layout this example is very similar to the previous one, there being but two differences, first, the depth of the suspended span (which has but little effect on the shape of the influence lines), and, second, the terminal inclination of the bottom boom of the shore arm. This latter change, although small, creates quite large alterations in the stresses which occur in some of the web members of the shore arm: thus, compare the influence lines for the members lettered *Dc* of Figs. 71 and 78.

End reaction R_A, Fig. 78, is obtained in a similar manner to that of Fig. 71, and since the lengths *AE*, *EH* and *HN* are unaltered, the influence lines will be the same.

Stress Dc = S_{Dc}, Fig. 78. The two unwanted bars, *DC* and *dc*, cut by section *ZZ*, meet, on production, at point *O* situated 30′ to the left hand of *A* and the lever arm, *OL*, to the member under discussion, is 60′.

Load between *A* and *C*

1T at *B* creates an R_A of $\frac{3}{4}$T. (See I.L. for R_A.)

Moments at *O* of all forces to the left of section line *ZZ*.

$- R_A \times OA + 1^T \times OB = S_{Dc} \times 60$ . . . . (a)

$- \frac{3}{4}^T \times 30' + 1^T \times 45' = S_{Dc} \times 60'$ (tending to lengthen *Dc*).

$\therefore 22{\cdot}5 \div 60 = S_{Dc}$ (tension) $= - 0{\cdot}375^T$

1T at *C* gives $R_A = \frac{1}{2}^T$ and, as in (a),

$- \frac{1}{2}^T \times 30' + 1^T \times 60' = 60' S_{Dc}$ giving $S_{Dc} = - 0{\cdot}75^T$

Load between *C* and *E*

1T at *D* although excluded from the moment equation (being now to the right of the section line *ZZ*) causes a reaction R_A of $\frac{1}{4}$T.

$\therefore - \frac{1}{4}^T \times 30' = 60' S_{Dc}$ giving $S_{Dc} = + 0{\cdot}125^T$

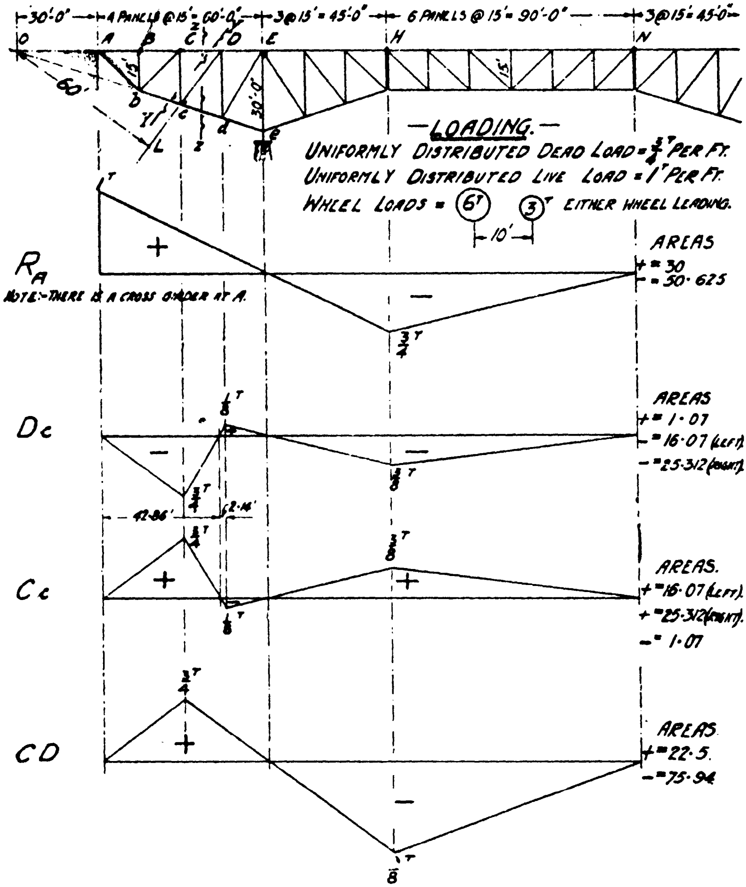
The stress in *Dc* is now compression as R_A, the only force to the left of section *ZZ*, tends to push point *c* up to *D*.

1T at *E* goes straight into the pier and so no stress occurs in *Dc* for this particular position of the unit load.

Load between *E* and *H* obviously causes a steadily increasing uplift at *A* as unit load approaches *H*.

1T at *H* causes an uplift at *A* of $\frac{3}{4}$T. (See I.L. for R_A.) This means, of course, that the abutment holding-down bolts at *A* pull the steelwork down at *A*. Then, moments at *O*:—

$+ \frac{3}{4}^T \times OA = S_{Dc} \times 60'$ *i.e.*, $S_{Dc} = - 0{\cdot}375^T$



RESULTS.	RA.	Dc	Cc	CD
U.D. DEAD LOAD	-15·47	-30·23	+30·23	-40·08
U.D. LIVE LOAD +	30	1·07	41·38	22·5
" " " " −	50·63	41·38	1·07	75·94
WHEEL LOADS +	8·5	0·875	6·0	6·0
" " −	6·5	6·0	0·875	9·75

STRESSES IN TONS.
+ = COMPRESSION or DOWNTHRUST.
− = TENSION or UPLIFT.

FIG. 78.

Load between H and N. As point N is approached by unit load the uplift at A approaches zero and so the stress in Dc approaches zero, giving a straight line from the $-\frac{3}{8}^T$ ordinate at H to zero at N.

Stress Cc. Since the moment centre is still point O and the lever arm OC is also 60′ in length, then the arithmetical values of the influence line will be identical with those newly found for member *Dc*. The natures of the stresses, however, will be opposite to those for *Dc*. For example, unit load at *D* causes R_A to be the only force between *A* and the section lines *YY* or *ZZ*, and R_A pushing the broken end-piece of steelwork upwards tends to open the cut *YY* in bar *Cc* (tension) and close the cut *ZZ* in bar *Dc* (compression).

Stress CD. The influence line for this member is similar in general outline to the corresponding member of the previous example, but the arithmetical values, however, are not in agreement because of the difference in main girder depths.

CHAPTER XI

THREE-PINNED PARABOLIC ARCH RIB

The Arch. The antiquity of the arch is very great, as the first known example dates from about 4,000 B.C. and is that giving entrance to the tomb of the un-named queen of King Mes-Kalam-Dug in the city of Ur.* Despite its common occurrence, the word arch (L. arcus = a bow) is very loosely used ; an arch need not necessarily be a curved structure nor need a curved structure be an arch.

An arch rib when subjected to external loads tends to compress axially, while any tendency to outward lateral movement of its ends is absolutely prevented by corresponding inward-acting horizontal forces.

Fig. 79 (*a*) is a two-pinned arch, while Fig. (b) is an ordinary curved beam, because the right-hand end is free to move laterally and only vertical reactions are called into play. In fact this second figure is used at one stage of the calculations to obtain the necessary additional elastic equations necessary for the solution of the statically indeterminate two-pinned arch.

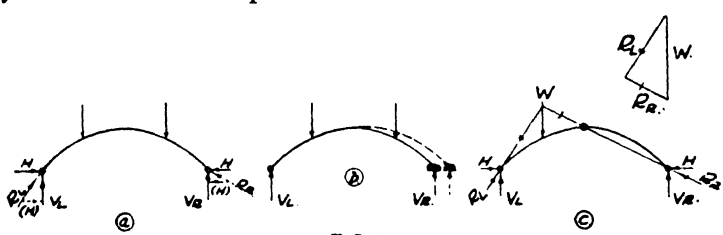


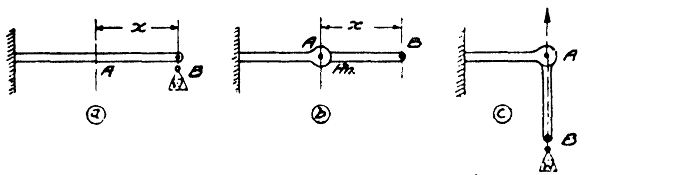
FIG 79.

Fig. 79 (*c*) is a three-hinged arch rib and is statically determinate, *i.e.*, all the reactions can be found by the use of ordinary statics. This is the only type which will be dealt with in these pages.

If the loads are vertical and the basal pins are at the same level, then H_L and H_R have the same value. Also, because no bending moment can exist at a pin (Fig. 80) the line of action of the force R_R must pass through the centre pin until it meets the load line of W. The structure of (c) is now under the action of three forces,

* **Being built of imported limestone blocks, this vaulted tomb is the oldest known stone structure in the world.**

R_L, W, and R_R, which, since there is equilibrium, must meet at a point, and, further, a triangle of forces can be drawn. The intersection point of R_R and W when joined to the L.H.P. (*i.e.*, left-hand pin) gives the line of action of R_L. The values of R_L and R_R can now be obtained from the triangle of forces in which W is drawn to scale. Now knowing the direction and magnitude of R_L and R_R these can be resolved into their respective horizontal and vertical components H and V. If there be several loads on the arch then



Continuous at A, BM$_A$ = Wx. Pin at A. No Load at B. W* Load at B. Arm falls ∴ No Moment at A.

FIG. 80.

each load can be considered separately, the total H acting on a basal pin being the sum of the horizontal thrusts due to each separate load; similarly, the total vertical reaction at either basal pin is the sum of the separate V's.

These forces can also be found arithmetically as well as graphically. Taking moments at the L.H.P., A, Fig. 81, and noting that, because pins A and C are at the same level, H has no moment about A, then :—

$$V_R \times 100' = W \times 25' \quad \therefore \quad V_R = 1^T \times 25' \div 100' = 0{\cdot}25^T$$

Moments at C, then :—

$$V_L \times 100' = 1^T \times 75' \quad \therefore \quad V_L = 1^T \times 75' \div 100' = 0{\cdot}75^T.$$

The vertical reactions are the same as for a simply supported girder of the same span as the arch.

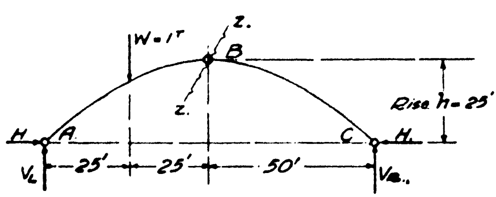


FIG 81.

To find the value of H recourse is made to the method of "Section and Moments" by taking a section ZZ through the C.P. (centre pin), B. Had the arch been continuous at B without a pin there would have been an unknown bending moment at B, but since there is a pin at B there can be no moment existing at this point. Considering the right half of the span : -

$$V_R \times 50' = H \times \text{rise of } 25'$$

$$0{\cdot}25^{T} \times 50' = 25'H \text{ or } H \qquad = 0{\cdot}5^{T}$$

Alternatively, consider the left-hand portion :—

$$V_L \times 50' - 1^{T} \times 25' - H \times 25' = 0$$

$$(0{\cdot}75^{T} \times 50' - 1^{T} \times 25') \div 25' = H \qquad = 0{\cdot}5^{T}$$

Before going further it might be advisable to show the difference between the spandrel-braced arch and the plated arch rib. In the former the superstructure above the bottom chord is an integral part of the arch structure and the failure of one of the main web members, vertical or diagonal, would entail the complete collapse of the structure. In the arch rib the load from the roadway is carried down to the rib through vertical columns or walls as the case may be, and any failure of one of these would mean only a local failure of the roadway, but not of the three-pinned rib which would remain standing.

Influence Lines for V_L and V_R. Figs. 82 and 83

Let unit load travel anywhere between A and C, *i.e.*, x varies from 0 to 100′.

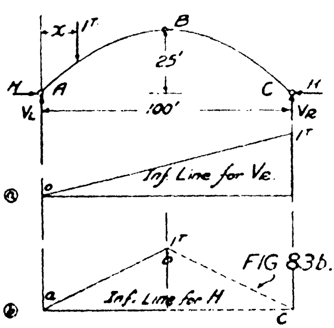


FIG 82.

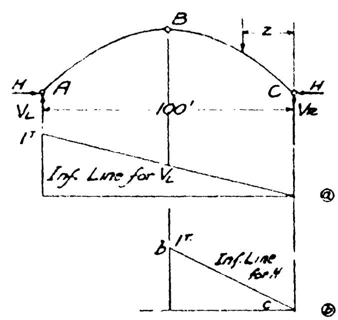


FIG 83.

Moments at A. $100' V_R = 1^{T} \times x'$ $\quad \therefore V_R = 0{\cdot}01x$ . . (i)

This is the equation of a straight line.

Unit load at A, *i.e.*, $x = 0$ then $V_R = 0$

,, ,, B, ,, $x = 50'$,, $V_R = 0{\cdot}5^{T}$

,, ,, C, ,, $x = 100'$,, $V_R = 1^{T}$

which is the standard form of the reaction influence line for a simple beam. Now to find V_L let z be the variable measured from C.

Moments at C. $100' V_L = 1^{T} \times z$ $\therefore V_L = 0{\cdot}01z$ . . (ii)

A straight line similar to that of V_R but other hand, *i.e.*, a mirror reflection.

Influence Line for the Horizontal Thrust *H*. Figs. 82 and 83

Let unit load travel only between A and B, *i.e.*, x varies from 0 to 50′.

Moments at B. $50' V_R = 25' H$ $\therefore H = 2V_R$ . . (iii)

Substitute the value for V_R from (i) then $H = 0{\cdot}02x$ . . (iv)

which is again a straight line (heavy line *ab* of (b), Fig. 82).

Unit load at A, *i.e.*, $x = 0$ then $H = 0$
,, ,, B, ,, $x = 50'$,, $H = 1^{T}$ *

Now let unit load travel between B and C and let the variable be z measured now from C, *i.e.*, z varies from 50′ to 0.

Moments at B. $50' V_L = 25' H$ $\therefore H = 2V_L$ . . (v)

Substitute the value for V_L from (ii) then $H = 0{\cdot}02z$ . . (vi)

also a straight line (heavy line *cb* of (b), Fig. 83).

Unit load at C, *i.e.*, $z = 0$ then $H = 0$
,, ,, B, ,, $z = 50'$,, $H = 1^{T}$ *

Joining these two half paths of unit load travel the complete influence line for H, as unit load travels across the span, is the triangle *abc* of Fig. 82 (b).

Geometrical Properties of the Parabola

The segmental (part of a circle) arch is seldom used in arch bridges because of the heavy and laborious calculations involved due to its awkward mathematical equation, and hence the choice of the parabola.

The equation of a parabolic rib with its origin at the left-hand pin, A, was shown to be (page 34) $x(\text{span} - x) = Ky$, where K is a constant for any particular parabola, x is the variable horizontal distance from origin A and y is the corresponding vertical height to the arch. To find the value for K substitute known values for x and y. Thus at the centre of the span x is 50′ and y is 25′.

$$\therefore 50(100 - 50) = K \times 25 \text{ or } K = 100$$

and the equation of the rib's neutral surface is

$$x(100 - x) = 100y \text{ or } y = x - 0{\cdot}01x^2 \quad \text{. . . . (vii)}$$

The height of the rib axis at a horizontal distance of 25′ from A is

$$y = 25 - 0{\cdot}01 \times 25^2 \qquad = 18{\cdot}75'$$

* The max. ordinate for H is not always 1^{T}, *e.g.*, it is $1{\cdot}25^{T}$ for a rise of 20′.

To find the slope of the tangent to the parabola at a point such as M, Fig. 84

Geometrically it is a property of the parabola that if a tangent be drawn to it at any point, and if this tangent be produced to cut the vertical centre line, or axis of symmetry, then the height of this intersection point above the apex of the parabola is equal to the depth below the apex of the original point M, *i.e.*, $BN = BO$.

The ordinate y at M, or height of M, was 18·75′ $\therefore OB =$ 6·25′

and $\therefore ON = 2 \times 6{\cdot}25' = 12{\cdot}5'$

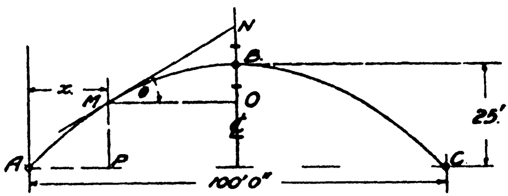


FIG. 84.

Tan of angle $NMO = \tan\theta = ON \div MO = 12{\cdot}5' \div 25' = 0{\cdot}5$

Alternatively, from the calculus :—

$\dfrac{dy}{dx}$ will give the slope or tangent to the curve at M, therefore

differentiating equation (vii), $\dfrac{dy}{dx} = 1 - 0{\cdot}02x$ . . . . (viii)

With $x = 25$ then the value of $\tan\theta = \dfrac{dy}{dx} = 1 - 0{\cdot}02 \times 25 = 0{\cdot}5$

The angle whose tan is 0·5 is 26° 34′

$\therefore \sin\theta = 0{\cdot}4472$ and $\cos\theta = 0{\cdot}8944$.

As a further example let M be situated at 40′ from the L.H.P.

Height $PM = y = x - 0{\cdot}01x^2$ . . . . (vii)

$= 40 - 0{\cdot}01 \times 40^2 \qquad = 24'$

$\therefore OB = 25 - PM \qquad = 1'$

$\therefore \tan\theta = ON \div OM = 2 \times 1 \div 10 \qquad = 0{\cdot}2$

$\sin\theta = 0{\cdot}1962$ and $\cos\theta = 0{\cdot}9805$

Alternatively, by substituting in (viii) $\tan\theta = \dfrac{dy}{dx} = 1 - 0{\cdot}02 \times 40$

$= 0{\cdot}2$

Influence Line for B.M.

Let K be any fixed point on the axis of the rib, for which the moment is desired, and let unit load occupy any position distant x

from A, where $x < AJ$. The total bending moment at K, Fig. 85, is

$$M_K = [V_L \times AJ - 1^T(AJ - x)] \quad - H \times KJ$$
$$= [\text{Vert. moment as for a beam}] - \text{“ Horizontal moment ” of } H \times KJ.$$
$$= \quad [m_V] \quad - m_H$$

and if the unit load be considered to act anywhere in the portion JC of the span, *i.e.*, $x > AJ$, a similar expression is obtained.

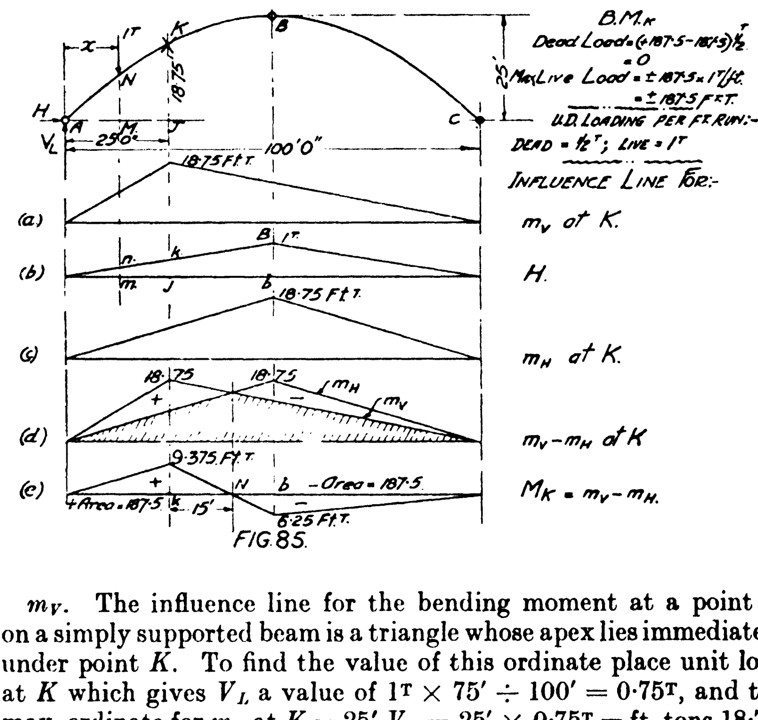


FIG. 85.

m_V. The influence line for the bending moment at a point K on a simply supported beam is a triangle whose apex lies immediately under point K. To find the value of this ordinate place unit load at K which gives V_L a value of $1^T \times 75' \div 100' = 0{\cdot}75^T$, and the max. ordinate for m_V at $K = 25'\ V_L = 25' \times 0{\cdot}75^T =$ ft. tons 18·75.

m_H. The B.M. at the fixed point K is simply $18{\cdot}75' \times$ the appropriate value of H.

m_H at K when 1^T is at $N = 18{\cdot}75' \times$ ordinate nm, diagram (b).

,, ,, ,, $K = 18{\cdot}75' \times$ ordinate kj,

,, ,, ,, $B = 18{\cdot}75' \times$ ordinate Bb

The influence line for m_H is thus also a triangle, but one whose ordinates are 18·75 times greater in value than those of the I.L. for H.

The complete influence line for the moment at K is the I.L. for m_V minus the I.L. for m_H. This subtraction can be accomplished graphically by drawing both curves on the same side of a common base line. This is done in Fig. 85 (d), where the area shown shaded is common to both m_V and m_H and so cancels out, leaving the positive and negative areas, which are not hatched, as the result. These curves are then redrawn as in Fig. 85 (e) on a straight line base, and, being of the same respective heights and base lengths as those of the Fig. (d), must be equal in area and in the height of the same respective ordinates. The position of the neutral point N of Figs. (d) and (e) is found by comparing similar triangles.

$$\frac{\text{Height } 9{\cdot}375}{kN} = \frac{\text{height } 6{\cdot}25}{Nb} = \frac{6{\cdot}25}{25 - kN}, \text{ whence } kN = 15' \, 0''$$

Positive moment at point K will cause the upper fibres or top flange of a plate web or I section to be in compression and the lower fibres to be in tension, as for a plate girder of simply supported span. Negative moment occurring at point K will have the opposite effect, and the upper flange thereat will be under tension and the lower flange under compression as for a beam cantilever. The direct compressive stress occasioned by the normal thrust also acts on these flanges and usually wipes out these small tensile bending stresses.

Let I = total moment of inertia of the radial cross-section of the rib.
A = total area of the radial cross-section.
y_t = distance from neutral axis to top fibres.
y_b = ,, ,, ,, bottom fibres.
f_t = stress intensity on top fibres.
f_b = ,, ,, bottom fibres.
M = bending moment at point K.
N = normal thrust at point K (compression ∴ + sign).

For positive moment at K. $f_t = +\frac{N}{A} + \frac{My_t}{I}$

$$f_b = +\frac{N}{A} - \frac{My_b}{I}$$

For negative moment at K. $f_t = +\frac{N}{A} - \frac{My_t}{I}$

$$f_b = +\frac{N}{A} + \frac{My_b}{I}$$

Normal Thrust and Radial Shear. Fig. 86

Consider the section at K. Diagram (a) shows the external or active forces acting on this portion of the arch. The resultant vertical force acting at K is $V_L - W_1 - W_2$ = vertical shear V as for a simply supported girder. As far as the section at K is concerned diagram (b) illustrates the vertical and horizontal resultant forces. The shear V, however, is vertical, but what is desired is the true or radial shear at right angles to the tangent at point K, so resolve the sum of all the vertical forces, *i.e.*, V, into two components, one along the tangent (*i.e.*, normal to the section) and the other along the radial section.

$ab = V \sin \theta$ = a normal thrust on the section.

$Ka = V \cos \theta$ = a true or radial shear on the section.

But H also has components along both of these directions, so replace H by its components, $H \cos \theta$ and $H \sin \theta$.

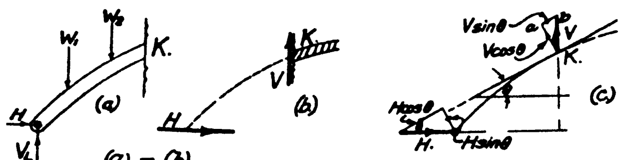


FIG. 86.

The total normal thrust acting on the cut face of the rib at K is $N = V \sin \theta + H \cos \theta$ . . . . (ix)
and the total force tending to shear the rib through in a radial direction at K is $R = + V \cos \theta - H \sin \theta$ . . . . . . (x)

Influence Line for Radial Shear. Fig. 87

From the previous article the vertical shear V was seen to be the same for a point K on either the arch rib of 100′ span or on a simple beam of 100′ span and, similarly, the influence line for V will apply equally well to either of these structures. Then the ordinates of diagram (a) when multiplied by the value of 0·8944 (previously found for cos θ) will give the influence line for $V \cos \theta$.

Similarly, if the values of the ordinates of the H influence line be multiplied by sin θ, = 0·4472, the resulting curve is the $H \sin \theta$ curve for the fixed point K.

Superimposing diagram (b) on (d), and noting that the hatched area cancels out, the result of the graphical subtraction is the two unshaded triangles which are redrawn in (f) on a straight line base.

Influence Line for Normal Thrust. Fig. 88

$N = V \sin \theta + H \cos \theta$.

Multiply the H influence line ordinates by the value of cos θ at point K and the I.L. ordinates of V by the value of sin θ. To add these figures graphically plot one above the base line and the other

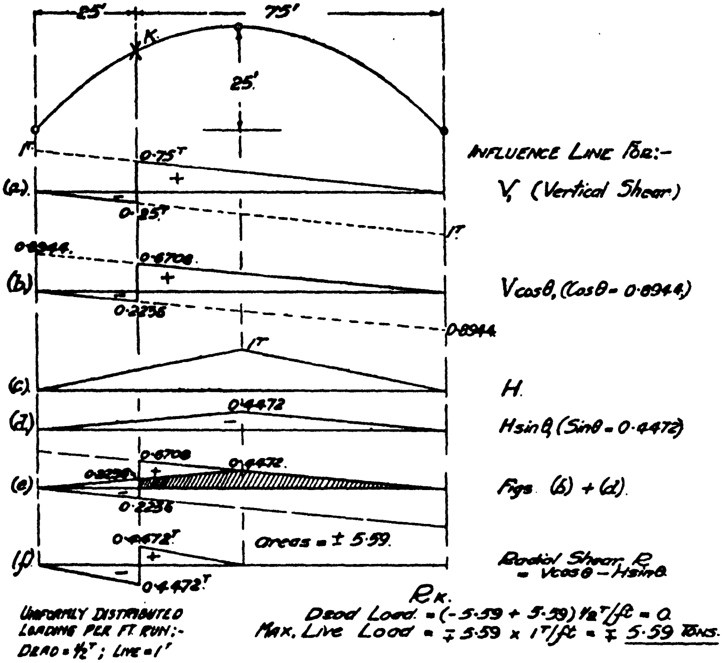

25'
75'
K
25'
INFLUENCE LINE FOR:-
(a)
V (Vertical Shear)
(b)
Vcosθ, (Cosθ = 0·8944)
(c)
H
(d)
Hsinθ (Sinθ = 0·4472)
(e)
Figs. (b) + (d)
(f)
Radial Shear R = Vcosθ − Hsinθ
Areas = ± 5·59
R K
UNIFORMLY DISTRIBUTED LOADING PER FT. RUN:- DEAD = ½T ; LIVE = 1T
Dead Load = (−5·59 + 5·59) ½T/ft = 0
Max. Live Load = ∓ 5·59 × 1T/ft = ∓ 5·59 Tons.

FIG. 87.

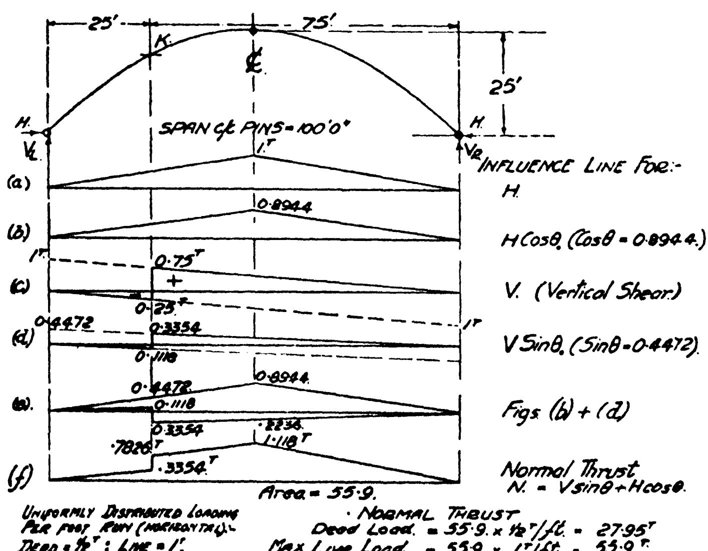
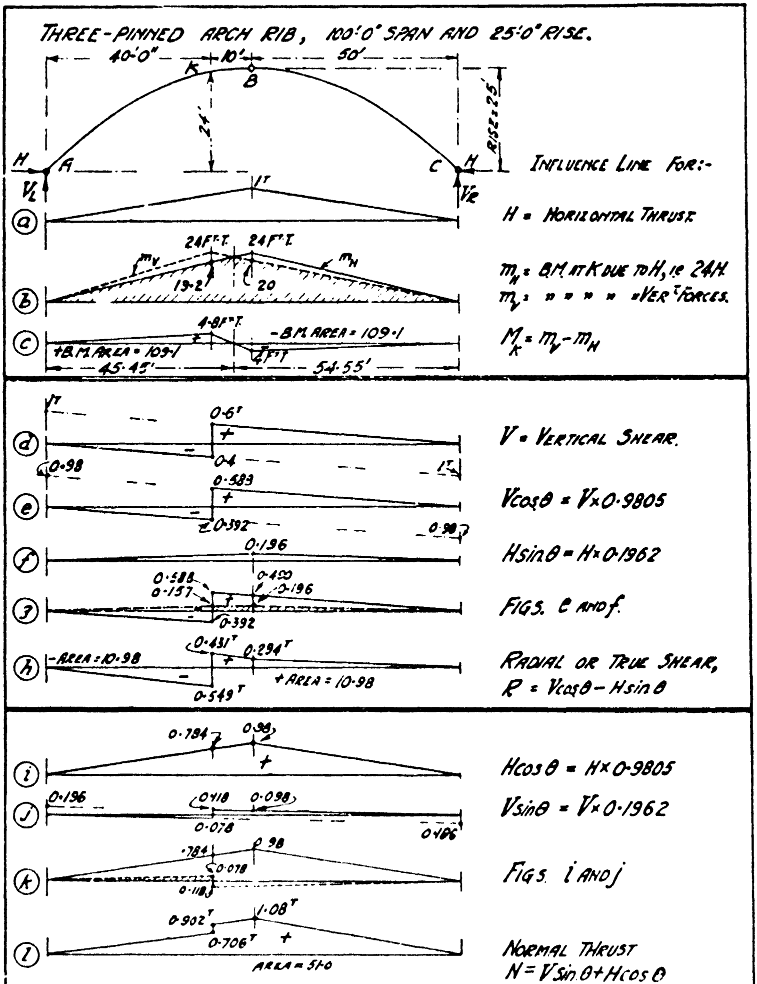

25'
75'
K
℄
25'
SPAN C/L PINS = 100'0"
INFLUENCE LINE FOR:-
(a)
H
(b)
HCosθ (Cosθ = 0·8944.)
(c)
V. (Vertical Shear)
(d)
V Sinθ. (Sinθ = 0·4472)
(e)
Figs (b) + (d)
(f)
Normal Thrust N. = Vsinθ + Hcosθ
Area = 55·9.
UNIFORMLY DISTRIBUTED LOADING PER FOOT RUN (HORIZONTAL):- DEAD = ½T ; LIVE = 1T.
NORMAL THRUST
Dead Load = 55·9 × ½T/ft = 27·95T
Max. Live Load = 55·9 × 1T/ft = 55·9T

FIG 88.



LOADING
UNIFORMLY DISTRIBUTED
(a) DEAD LOAD OF 1/2 T PER FOOT RUN.
(b) LIVE LOAD OF 1 T PER FOOT RUN

RESULTS.

AT POINT K	DEAD LOAD	LIVE LOAD
B.M. in FT TONS	0	±109·1
R. SHEAR IN TONS	0	∓ 10·99
N. THRUST IN TONS	+ 25·5	+ 51·0

FIG. 89.

under. The value of the normal thrust due to a one ton load placed anywhere on the span is the length of the ordinate, under unit load, which is bounded by the upper and lower sloping lines of (e). Diagram (f) is simply diagram (e) placed on a horizontal base line.

Fig. 89 gives the complete set of influence lines for another position on the rib span. In this case point K is taken at 40′ from the left-hand pin, a point whose height and tangent value have already been found. The diagrams of this figure should be self-explanatory in view of the text devoted to the previous examples.

Maximal Values. A uniformly distributed live load (for simplicity adopt 1^T per ft. run) can be so placed on the arch span as to create maximum bending moment at point K. In Fig. 89 the U.D.L.L. should cover the left-hand portion of 45·45′ to give a maximum positive moment of + 109·1 ft. tons. If this be unloaded and the 54·55′ length on the right-hand side be loaded in turn, the moment occurring at K reaches its maximum value for negative moment of − 109·1 ft. tons. (If both parts are loaded simultaneously, *i.e.*, the complete span, there is no moment whatsoever at K.)

In Fig. 85 the left-hand length of 40′ when covered by the U.D.L.L. gives the max. positive moment of 187·5 ft. tons at K, while max. negative moment of the same amount happens when the 60′ portion between N and the R.H.P. is loaded.

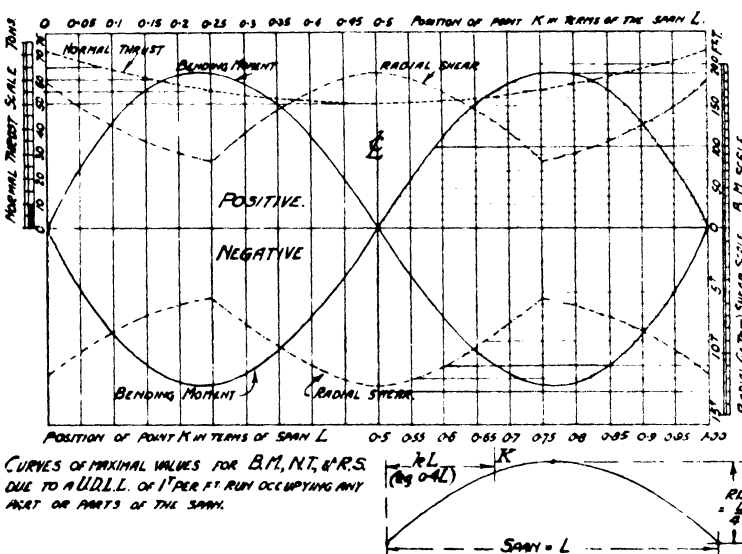


FIG. 90.

Repeating the above for points K at 2′ 6″ or 5′ 0″ intervals across the span permits a curve of maximal bending moments to be drawn. This curve is not a bending moment curve, but a curve giving the values of the maximal positive or negative moments which can occur at any point on the rib and is given in Fig. 90.

Further, by showing the lengths 2′ 6″, 5′ 0″, 7′ 6″, etc., as decimal lengths of the span, *i.e.*, kL, the resulting curve immediately becomes useful for all symmetrical parabolic three-pinned arch ribs of 100 ft. span and any rise whatsoever.

The normal thrust and radial shear values were similarly obtained and the max. values are those given in Fig. 90. Radial shear and bending moment have positive and negative values, but the normal thrust is always positive for this rise to span ratio. All the curves are symmetrical about the centre line through the mid-pin.

Max. B.M. The equation to the curve of maximal bending moments of Fig. 90 can be found from the geometry of Fig. 91. Let the span be S and the rise be h.

Equation of parabola is $x(S - x) = Ky$.

At the centre $x = \frac{S}{2}$, and $y = h$, whence $\frac{S^2}{4h} = K$ and the equation of any parabolic rib similar to the figure is $y = \frac{4hx}{S^2}(S - x)$ . . . (1)

Max. H occurs with unit load at C.P.

$$= R_B \times \frac{S}{2} \div h = \frac{S}{4h} \quad . \quad . \quad . \quad . \quad (2)$$

Max. B.M. at K due to H

$$= y \times H = \frac{4hx}{S^2}(S - x) \times \frac{S}{4h} = \frac{x}{S}(S - x) \quad . \quad . \quad . \quad (3)$$

Max. B.M. at K due to vert. loading

$$= xR_A \text{ with unit load at } K$$

$$= x \times 1(S - x) \div S = \frac{x}{S}(S - x) \quad . \quad . \quad . \quad (4)$$

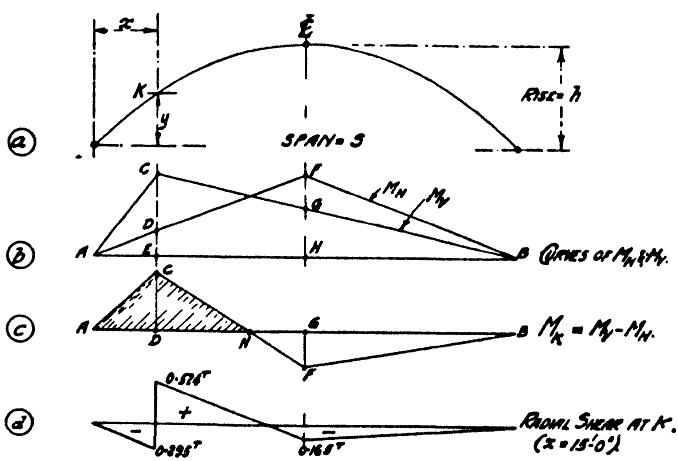


FIG. 91.

Observe that in (b) height CE

$$= FH \text{ always} \qquad = \frac{x}{S}(S - x)$$

$$\frac{DE}{FH} = \frac{AE}{AH} \text{ or } DE = AE \times FH \div AH. \text{ Substituting values, then}$$

$$DE = x \times \frac{x}{S}(S - x) \div \frac{S}{2} \qquad = \frac{2x^2}{S^2}(S - x) \quad . \quad . \quad (5)$$

$$\frac{GH}{CE} = \frac{BH}{BE} \text{ or } GH = CE \times BH \div BE. \text{ Substituting values, then}$$

$$GH = \frac{x}{S}(S - x) \times \frac{S}{2} \div (S - x) \qquad = \frac{x}{2} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$CD = CE - DE = \frac{x}{S}(S-x) - \frac{2x^2}{S^2}(S-x) \quad = \frac{x}{S^2}(S - x)(S - 2x) \quad (7)$$

$$FG = FH - GH = \frac{x}{S}(S - x) - \frac{x}{2} \qquad = \frac{x}{2S}(S - 2x) \quad . \quad . \quad (8)$$

By similar triangles

$$\frac{CD}{DN} = \frac{FG}{GN}, \text{ i.e., } \frac{CD}{DN} = \frac{FG}{AG - AN} \quad = \frac{FG}{\frac{S}{2} - x - DN}$$

$$\therefore DN = \frac{CD\left(\frac{S}{2} - x\right)}{FG + CD}$$

$$= \frac{\frac{x}{S^2}(S - x)(S - 2x)\left(\frac{S}{2} - x\right)}{\frac{x}{2S}(S - 2x) + \frac{x}{S^2}(S - x)(S - 2x)} = \frac{(S - x)(S - 2x)}{(3S - 2x)} \quad (9)$$

Area of shaded triangle $= \frac{1}{2}AN \times CD = \frac{1}{2}(AD + DN)CD$ and substituting

$$= \tfrac{1}{2}\left[x + \frac{(S - x)(S - 2x)}{3S - 2x}\right]\left[\frac{x}{S^2}(S - x)(S - 2x)\right]$$

$$= \frac{x(S - x)(S - 2x)}{2(3S - 2x)} \quad (10)$$

Triangles ACB and AFB of Fig. (b) are equal in area so that in subtracting the common area from each the resulting triangles ACN and BFN of (c) must also be equal in area. Either of these basal lengths should be covered by the U.D.L.L. to give max. B.M.$_K$, where K is any point on the span between the left-hand pin and the centre pin.

And this applies to any similar three-pinned parabolic arch rib of whatever span and rise. For the particular rib under discussion the max. B.M.$_K$ due to a uniformly distributed load of w^T per ft. run placed on any part or parts of the span is

$$\frac{x(100 - x)(100 - 2x)}{2(300 - 2x)} \times w \quad . \quad . \quad . \quad . \quad . \quad (11)$$

By differentiating and equating to zero a cubic expression can be obtained giving the position of point K for max. B.M.: alternatively by trial values find the position of K thus: $x = 23{\cdot}3'$, $23{\cdot}4'$ and $23{\cdot}5'$ gives the respective B.M.$_K$ of 188·302, 188·305 and 188·302 ft. tons, so that the max. moment occurs at 23·4′ from the left-hand pin A.

Radial shear, K lying between 0 and 25′ from the L.H.P. A typical influence line for this case is that given by Fig. 91 (d) and is composed of three triangular areas. The right-hand triangle becomes smaller in area as the quarter point is approached by the point K.

With K situated at the quarter point the right-hand negative triangle has faded away into the straight line of Fig. 87 (f). A load placed anywhere between the C.P. and the R.H.P. will cause no radial shear at point K, 25′ 0″ from the L.H.P.

When K is taken between the quarter point and the C.P. the influence line takes the form indicated by Fig. 89 (h) and is formed of a negative triangle and a positive area composed of a triangle and a trapezium.

It is clear, then, that the quarter span point is a critical or change point in the curve of maximal shears of Fig. 90.

The values from which Fig. 90 was obtained are listed below.

Position in ft.	B.M. ± ft. tons	Rad. S ± tons	Normal T + tons	Position in ft.	B.M. ± ft. tons	Rad. S ± tons	Normal T + tons
0·0	0·0	11·79	70·71	27·5	183·1	6·55	54·83
2·5	39·25	10·75	68·97	30·0	175·0	7·52	53·85
5·0	73·71	9·82	67·27	32·5	163·36	8·47	52·97
7·5	103·45	8·97	65·62	35·0	148·37	9·39	52·20
10·0	128·57	8·22	64·03	37·5	130·21	10·23	51·54
12·5	149·15	7·56	62·50	40·0	109·09	10·98	50·99
15·0	165·28	6·99	61·03	42·5	85·25	11·61	50·56
17·5	177·06	6·51	59·63	45·0	58·93	12·09	50·25
20·0	184·62	6·11	58·31	47·5	30·41	12·39	50·06
22·5	188·05	5·81	57·06	50·0	0·0	12·5	50·00
25·0	187·5	5·59	55·90				

CHAPTER XII

THREE-PINNED SPANDREL-BRACED ARCH

In this example the centre pin is placed on the bottom or rib boom, and there is a clear structural gap left in the top chord immediately above this pin. The value of the horizontal thrust H is obtained by taking moments about this centre pin d and considering (preferably) the stability of the unloaded half of the arch.

In Fig. 92 (b) there are two vertical reactions, V_L and V_R, whose values are the same as if the structure were a simply supported beam of the same span. Then taking moments about the centre pin d and giving the positive sign to clockwise moments :—

$$+H \times h - V_R \times \frac{L}{2} = 0$$

$$\text{or } H = \frac{L}{2h} \times V_R = \frac{120}{2 \times 25} \times V_R = 2{\cdot}4 V_R$$

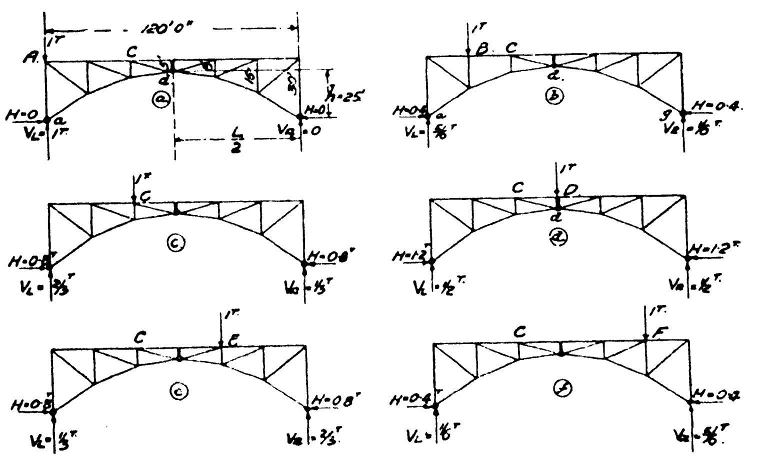


FIG. 92.

(a) Unit load at A gives $V_R = 0 \ \therefore H = 2{\cdot}4 V_R \qquad = 0{\cdot}0$

(b) „ „ B „ $= \frac{1}{6}^{T} \therefore H = 2{\cdot}4 \times \frac{1}{6} \qquad = 0{\cdot}4^{T}$

(c) „ „ C „ $= \frac{1}{3}^{T} \therefore H = 2{\cdot}4 \times \frac{1}{3} \qquad = 0{\cdot}8^{T}$

(d) „ „ D „ $= \frac{1}{2}^{T} \therefore H = 2{\cdot}4 \times \frac{1}{2} \qquad = 1{\cdot}2^{T}$

When unit load arrives at E consider the stability of the left-hand portion of the arch, and this case becomes a mirror reflection of Fig. (c), so that load at E has $0{\cdot}8^{\text{T}}$ as the value of H. Unit load at F has $0{\cdot}4^{\text{T}}$ for H and the final influence line for H is the isosceles triangle of Fig. 93 (a).

Arithmetical Example (1). Find the value of the horizontal thrust when :—

(a) Unit loads are placed simultaneously at points B, C, D and E.

(b) The U.D. Live L. of 1^{T} per ft. covers the right-hand half of the span.

(c) The U.D. Dead L. is $\frac{3}{4}^{\text{T}}$ per ft. of main arch.

Answers. (a) $H = 1(0{\cdot}4 + 0{\cdot}8 + 1{\cdot}2 + 0{\cdot}8)^{\text{T}} \qquad = \ 3{\cdot}2^{\text{T}}$

(b) $H = 1 \times \text{area} = 1(60 \times 1{\cdot}2 \div 2) \qquad = 36{\cdot}0^{\text{T}}$

(c) $H = \frac{3}{4} \times \text{area} = \frac{3}{4}(120 \times 1{\cdot}2 \div 2) \qquad = 54{\cdot}0^{\text{T}}$

Bottom Boom Members, Fig. 93

Fig. (*b*). The perpendicular lever arm from point C on the upper chord to the line of action of H is 30′ and the moment caused by H about C is $30H$ for every value of H. The influence line for the moment about C due to H, *i.e.*, m_H, is given by Fig. (b), whose ordinates are thirty times greater than the corresponding ordinates of the influence line for H. This bending moment is negative because H inclines to make the basal pins meet and make the structure convex upwards.

Arithmetical Example (2). Find the moment at C due to H, *i.e.*, m_H at C, caused by the loading given in the preceding example.

Answers

(a) $m_H = -12 - 24 - 36 - 24 \qquad = \text{ft. tons} \ - \ 96$

(b) $m_H = \text{load} \times \text{area} = 1(60 \times 36 \div 2) \ = \quad ,, \quad -1{,}080$

(c) $m_H = \quad ,, \quad ,, \quad = \frac{3}{4}(120 \times 36 \div 2) = \quad ,, \quad -1{,}620$

i.e., in all three cases the results are thirty times larger, numerically, than those of the previous example.

Fig. (*c*) gives the bending moment at point C due to vertical loads only. Returning to Fig. 92, then in the diagram lettered :—

(a) With unit load at A the internal stress takes the shortest path, Aa, to the abutment and so the bending moment at C $\qquad = 0$

(b) If unit load is placed at B the reaction V_L is $\frac{5}{6}^{\text{T}}$ and the moment at $C = m_V = V_L \times 40' - 1^{\text{T}} \times 20' = \frac{5}{6} \times 40 - 20 \ = 13\frac{1}{3}$

(c) Unit load at point C gives $V_L = \frac{2}{3}$ and at this point C
$m_V = 40V_L - 1^T \times 0 = 40 \times \frac{2}{3}$ $= 26\frac{2}{3}$

(d) 1^T at D gives $V_L = \frac{1}{2}^T$ and $m_V = 40V_L$ $= 20$

(e) 1^T „ E „ $= \frac{1}{3}^T$ „ $=$ „ $= 13\frac{1}{3}$

(f) 1^T „ F „ $= \frac{1}{6}^T$ „ $=$ „ $= 6\frac{2}{3}$

The results are all in ft. tons and are, moreover, all positive moments because the structure tries to take a concave upper surface under the action of the vertical forces. These results are graphed in Fig. 93 (c), and it is apparent that this influence line, m_V, is exactly the same as that for point C placed in a similar position on a simply supported horizontal girder of the same span.

Arithmetical Example (3). Find the moment, m_V, at point C due to the vertical forces only for the loading as given in example 1.

Answers

(a) Unit loads at B, C, D and E, then
$m_V = 13\frac{1}{3} + 26\frac{2}{3} + 20 + 13\frac{1}{3}$ in ft. tons $= + \quad 73\frac{1}{3}$

(b) 1^T per ft. covering right half of span :—
$m_V =$ load $\times$ area $= 1(60 \times 20 \div 2)$ in ft. tons $= + \quad 600$

(c) Dead load of $\frac{3}{4}^T$ per ft. :—
$m_V =$ load $\times$ area $= \frac{3}{4}(120 \times 26\frac{2}{3} \div 2)$ in ft. tons $= + \; 1{,}200$

Fig. (*d*). If Fig. (c) is superimposed on Fig. (b) then the portion which is shaded, being common to both the positive and negative curves, subtracts out leaving as the result of this graphical addition the unshaded triangles.

Fig. (*e*). The unhatched triangular areas from (d) are, in the present figure, plotted on a straight line base with the positive moment above the line.

Arithmetical Example (4). Making use of diagram (e), obtain the total bending moment at point C due to the vertical and horizontal forces acting on the structure, for the three cases of loading of the previous examples.

Answers

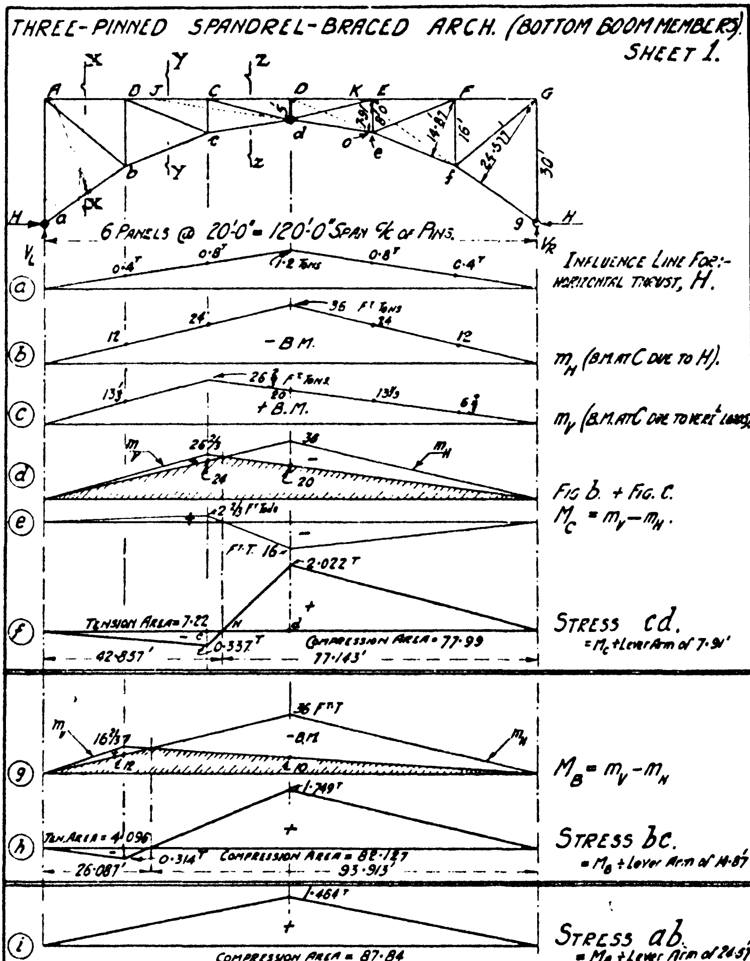
(a) $M_C = +1\frac{1}{3} + 2\frac{2}{3} - 16 - 10\frac{2}{3}$ in ft. tons $= - \quad 22\frac{2}{3}$

(b) $M_C =$ load $\times$ area $= 1(16 \times 60 \div 2)$ in ft. tons $= - \; 480$

(c) $M_C =$ „ „ $= \frac{3}{4}(\frac{1}{2})(2\frac{2}{3} \times 42{\cdot}86 - 16 \times 77{\cdot}14)$
in ft. tons $= - \; 420$

The results for M_C of example 4 equal the sum of the corresponding results for m_H and m_V of examples 2 and 3.

Fig. (*f*). This is the influence line for stress in member dc, and



LOADING:-
UNIFORMLY DISTRIBUTED PER FOOT RUN OF GIRDER,
(a) DEAD LOAD = 3/4 TON,
(b) LIVE LOAD = 1 TON.

STRESSES

MEMBER	DEAD LOAD (Tons).	LIVE LOAD (Tons)	
		COMPRESSION	TENSION
ab	+ 65·88T	87·84T	—
bc	+ 58·52T	82·13T	4·10T
cd	+ 53·08T	77·99T	7·22T

FIG. 93.

it is similar to (e) because it is the M_C influence line divided by the lever arm of 7·91′. The stresses in this member are obtained by "Section and Moments," where ZZ is the section line and the moment centre is C, the point of intersection of the two unwanted bars CD and Cd.

There is too much latitude for inaccuracy in scaling the length of the lever arm from the drawing, and this length should be calculated.

Bar ed produced meets the upper chord or boom at J. Then by similar triangles $\frac{Ee}{Dd} = \frac{EJ}{DJ}$,

i.e., $\frac{8}{5} = \frac{40' + CJ}{20' + CJ}$, whence $CJ = 13{\cdot}33'$ and $JE = 53{\cdot}33'$.

The length Je can now be obtained and is equal to $\sqrt{(JE^2 + Ee^2)}$ $= 53{\cdot}93$

Now comparing the similar triangles Eoe and JEe,

$\frac{Ee}{Eo} = \frac{Je}{JE}$, *i.e.*, $\frac{8}{Eo} = \frac{53{\cdot}93}{53{\cdot}33}$ or lever arm Eo $= 7{\cdot}91'$

Considering the stability of that portion of the cut arch to the left hand of line ZZ by taking moments at point C, it follows that M_C = stress in cut bar $cd \times 7{\cdot}91'$

or $S_{cd} = M_C \div 7{\cdot}91$ as given by Fig. (f).

Next, the nature of the stress is ascertained by observing that the cut ZZ in member cd is closed by the action of H and opened by the action of V_L, or, for this particular member, negative moment causes compression while positive moment entails tension.

Finally, the exact position of the neutral point N, common to all three figures, (d), (e) and (f), is found thus from (f) :—

$$\frac{cN}{Nd} = \frac{\text{height } 0{\cdot}337}{\text{height } 2{\cdot}022}$$

i.e., $\frac{cN}{20 - cN} =$,, whence cN $= 2{\cdot}857'$

Arithmetical Example (5). Ascertain the stresses in member cd created by the loadings given in example 1.

(a) $S_{cd} = -0{\cdot}168 - 0{\cdot}337 + 2{\cdot}022 + 1{\cdot}348$ $= +\ 2{\cdot}87$T

(b) S_{cd} = load × area $= 1(2{\cdot}022 \times 60 \div 2)$ $= +\ 60{\cdot}66$T

(c) S_{cd} = ,, ,, $= \frac{3}{4}(-7{\cdot}22 + 77{\cdot}99)$ $= +\ 53{\cdot}08$T

***Figs.* (*g*) *and* (*h*), S_{bc}.** The influence line for m_H is identical with that of (b), because the H influence line is constant for any one arch and the lever arm from the new moment centre B to the line of action of H is still 30′. The influence line for m_V is the same as for point B on an ordinary simply supported girder of 120′ 0″ span.

The intermediate influence lines are not drawn, while the final stress influence line S_{bc} is derived by dividing the differences in heights at the apices of the unshaded triangles by the new lever arm of 14·87′ from point B to rib member bc.

Fig. (*i*), S_{ab}. The section line is XX and the moment centre is A. Considering the stability of the cut arch to the left of XX it is seen, no matter what position unit load takes on the span, that there are only two forces to consider when taking moments about point A, viz., H and V_L. Further, since V_L passes through A it has no moment about A, whence it follows that

$$S_{ab} \times \text{lever arm of } 24{\cdot}577' = H \times 30'$$

$$\therefore\ S_{ab} = 30H \div 24{\cdot}577 = \text{I.L. for } m_H \div 24{\cdot}577$$

i.e., a simple isosceles triangle of apex height $= 36 \div 24{\cdot}577 = 1{\cdot}464$T.

Stress Table. The loading and the resulting maximal stresses are given at the bottom of the plate. The uniformly distributed live load is advanced on to the span from either abutment so as to take up that position which causes max. positive or negative stress in the member under consideration. The stresses in the final list are obtained by multiplying the requisite areas, given on the diagrams, by the load per ft. run.

Top Boom Members, Fig. 94

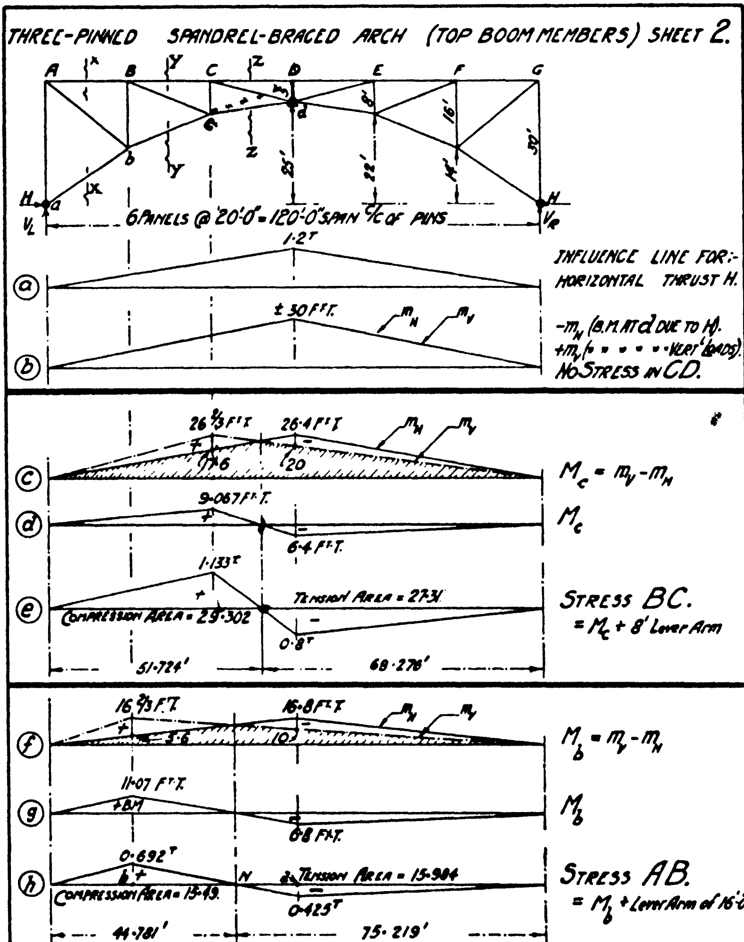
Fig. (*a*) is the same as the previous diagram (a), Fig. 93.

Fig. (*b*). The section line ZZ cuts the two unwanted bars Cd and cd, which meet at the moment centre d. This point is situated 25′ 0″ vertically above the horizontal line of action of H, and in consequence the moment of H about d is now $25H$. The max. ordinate of m_H is $1{\cdot}2 \times 25$, or 30 ft. tons.

The influence line for B.M. at d due to vertical loads is as for an ordinary beam; max. ordinate occurring under d with unit load thereat. In this position $V_L = \frac{1}{2}$T and B.M.$_d = \frac{1}{2}$T $\times$ 60′ = 30 ft. tons.

Thus m_H and m_V are identical influence lines, but being of opposite sign they cancel each other and there is no stress in bar CD, which is, therefore, redundant. That this is correct is verified by observing that the real or effective arch is A, B, C, d, c, b, a, members CD and Dd being redundant as regards the main structure. The double vertical bars at Dd do carry loads, but only the local loads from the dual cross girders at D down to point d.

Fig. (*c*). The moment centre of BC is c situated 22′ above H.



LOADING :-
UNIFORMLY DISTRIBUTED PER FOOT RUN OF GIRDER,
(a) DEAD LOAD = 3/4 TON.
(b) LIVE LOAD = 1 TON.

STRESSES

MEMBER	DEAD LOAD (TONS)	LIVE LOAD (TONS)	
		COMPRESSION	TENSION
AB	-0·37	15·49	15·98
BC	+1·49	29·30	27·31
CD	—	—	—

FIG. 94.

This gives the max. ordinate of the m_H triangle as $22 \times 1{\cdot}2 = 26{\cdot}4$ ft. tons.

The influence line for m_V for point c is the same as that for point C of diagram (d), Fig. 93. These points lie in the same vertical line, and, as only vertical loads are being considered, the same B.M. is encountered at both points. V_L, which causes positive moment about point c, has the effect of closing gap YY in the cut bar BC, while H opens the gap, *i.e.*, positive moment creates positive or compressive stress in the upper boom members while negative moment is associated with tensile stress.

Figs. (*d*) *and* (*e*). These are derived as previously explained.

Ordinate $1{\cdot}133 = 9{\cdot}067 \div$ lever arm Cc of 8′.

Figs. (*f*), (*g*) *and* (*h*) call for no special mention.

The position of the neutral point N is obtained as follows :—

$$\frac{bN}{0{\cdot}692} = \frac{Nd}{0{\cdot}425} = \frac{40 - bN}{0{\cdot}425}, \text{ whence } bN \qquad = 24{\cdot}781'$$

Vertical Web Members, Fig. 95

Fig. (*a*). It was mentioned in the text regarding the previous plate that the two bars at Dd carry, as columns, the local loads applied at their caps by the cross girders. The left-hand cross girder at D supports the right-hand end of the stringers spanning from C to D and the max. column load is the max. reaction at D from these stringers. The influence line for the load on the left-hand vertical Dd is the small right-angle triangle of perpendicular height equal to one ton.

Fig. (*b*). The H influence line is constant for any one arch and is repeated here to facilitate the calculation of the other curves.

Fig. (*c*). The moment centre for bar Cc is point K, situated on the top boom, where cd produced meets BC produced. The length CK has been calculated already, see EJ of the figure on the bottom boom members.

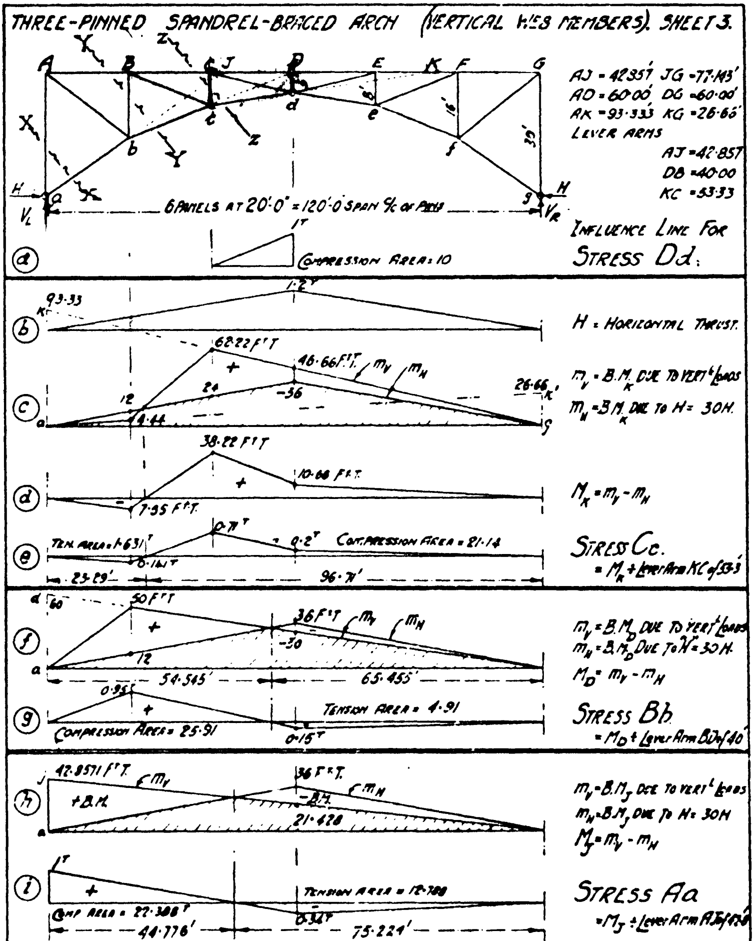
The lever arm of H about K is 30′ and the max. ordinate of the m_H curve is $30' \times 1{\cdot}2^{T}$ $= 36$ ft. tons

m_V *at* K. Possibly the clearest idea of this bending moment is gained by considering unit load to be moved from panel point to panel point on its travel across the span from A to G.

1^{T} at A. No B.M. at K, *i.e.*, m_V at K $= 0$

1^{T} at B. There are two forces to the left of section line ZZ, viz., V_L and 1^{T}, and only one force V_R to the right of ZZ. Consider the simpler case of V_R; then

m_V at $K = KG \times V_R = 26{\cdot}66' \times \frac{1}{6}^{T}$ $= 4{\cdot}44$



LOADING:-
UNIFORMLY DISTRIBUTED PER FOOT RUN
(a) DEAD LOAD = ¼ TON.
(b) LIVE LOAD = 1 TON.

MEMBER	DEAD LOAD (TONS)	LIVE LOAD (TONS). COMPRESSION	TENSION.
Aa	+7.20	22.39	12.79
Bb	+15.75	25.91	4.91
Cc	+14.64	21.14	1.65
Dd	+7.5	10.00	–

FIG. 95.

(If point *a* be joined to point 4·44, in Fig. (c), and produced it will cut the vertical *gk'* on the right at a height of 26·66, *i.e.*, V_R with an imaginary value of 1^T in $KG \times V_R = 26{\cdot}66 \times 1^T$.)

1^T at *C*. Unit load has now passed section line *ZZ*, leaving only V_L to the left of the cut, so examine the stability of portion *ABZZcba* of the arch under vertical forces only.

m_V at $K = AK \times V_L = 93{\cdot}33 V_L = 93{\cdot}33' \times \frac{4}{6}^T = 62{\cdot}22$

1^T at *D* makes $V_L = \frac{1}{2}^T \therefore m_V$ at $K = 93{\cdot}33' \times \frac{3}{6}^T = 46{\cdot}66$

1^T at *E* makes $V_L = \frac{1}{3}^T \therefore m_V$ at $K = 93{\cdot}33' \times \frac{2}{6}^T$

1^T at *F* makes $V_L = \frac{1}{6}^T \therefore m_V$ at $K = 93{\cdot}33' \times \frac{1}{6}^T$

This clearly represents a straight line from 62·22 down to nothing when unit load is at *G*. (If this straight line were continued to cut the vertical through *Aa* the ordinate would be 93·33 = *ak*, diagram (c), *i.e.*, V_L would have the imaginary value of 1^T and the imaginary B.M. would be $93{\cdot}33' \times 1^T$.)

The actual influence line for m_V at *K* is finally completed by joining the upper extremities of the two straight lines newly obtained, *i.e.*, point 4·44 to point 62·22.

The foregoing suggests an easy construction, viz.: At point *a* erect *ak* = *AK* to scale, and at *g* erect perpendicular *gk'* = *GK* to scale, then join the tops of these perpendiculars to the far ends of the base line. Lastly, connect the two points of intersection of the sloping lines with the verticals through the ends *B* and *C* of the panel considered. The similarity with the shear influence line of a parallel flanged braced girder is very apparent.

Fig. (*d*) is the geometrical subtraction of m_H from m_V so giving M_K.

Fig. (*e*) gives the stress influence line obtained by dividing the values at the change points of (d) by the lever arm, *KC* = 53·33', from *K* to bar *Cc*.

Fig. (*f*). The moment centre for *Bb* is point *D* where the unwanted bars *bc* and *AB* meet. When unit load is placed on any of the panel points between *B* and *G* there is only one vertical force V_L to the left of section line *YY* and the $B.M._D$ due to V_L is $AD \times V_L = 60 V_L$. The value of V_L for unit load at *B*, *C* and *D*, etc., is $\frac{5}{6}^T$, $\frac{4}{6}^T$ and $\frac{3}{6}^T$, etc., respectively, and so the I.L. from *B* to *G* runs from a value of $60' \times \frac{5}{6}^T$, *i.e.*, 50 ft. tons, straight down to zero. This line, if continued, would meet the vertical through the L.H.P. at a height above the base line of *ad* = *AD* = 60' to scale. From the apex point of 50 ft. tons the I.L. descends to zero at *a*, because 1^T at *A* causes no moment.

The m_H influence curve for point D is identical with that in (c), as both moment centres D and K are 30′ above the line of action of H, the horizontal thrust.

Fig. (*g*) then follows automatically on using the lever arm BD of 40′ as the divisor.

The horizontal thrust H increases the gap in a cut vertical bar when the broken structure swings round the moment centre, whereas force V_L closes the gap, *i.e.*, negative moment creates tensile stress and positive moment, compressive stress.

Fig. (*h*). Consider the stability of the small cut-part of the arch below section line XX when taking moments about the moment centre J. No matter what position unit load may be given there are only two forces acting on the small triangular piece of structure XaX, viz., V_L and H.

Stress in $Aa = (-H \times 30 + V_L \times AJ) \div AJ$.

Now $30H$ is the same triangular I.L. as in the examples above, while $AJ \times V_L$ simply means 42·85 times the left-hand vertical reaction I.L. for the ordinary girder of 120′ 0″ span, and this V_L influence line varies from 1^{T} at A to zero at G.

Fig. (*i*). The two ordinates of the triangle are 42·85 of Fig. (h) ÷ lever arm of 42·85 or unit stress and (36 − 21·428) ÷ 42·85 gives the other apex of $0{\cdot}34^{\text{T}}$.

The resulting stresses given in the table refer to the loading stated alongside.

Diagonal Web Members, Fig. 96

The dimensions necessary for finding the lengths of the lever arms between the diagonal members and their respective moment centres are given on the plate.

Fig. (*a*). The moment centres are still situated on the top boom and so the influence lines for m_H are constant in outline and similar to those of the previous plate.

Since the section line is ZZ for bar Cd then unit load at D, E, F, or G causes only one vertical force V_L to the left of ZZ, and in consequence the vertical moment is a straight line whose equation is $AK \times V_L = 93{\cdot}33V_L$, where V_L varies from $\frac{1}{2}^{\text{T}}$ down to zero. Max. ordinate of 46·67 occurs under the first position D. Unit load at A, B or C gives V_R only to the right of the section ZZ and the m_V at $K = KG \times V_R = 26{\cdot}66V_R$.

V_R has its max. value of $\frac{1}{3}^{\text{T}}$ when unit load is at C and the corresponding m_V at K with load at C is $\frac{1}{3} \times 26{\cdot}66 = 8{\cdot}88$ as in the figure.

If these straight lines be produced they cut the end verticals in a similar manner to those of Fig. 95 (c).

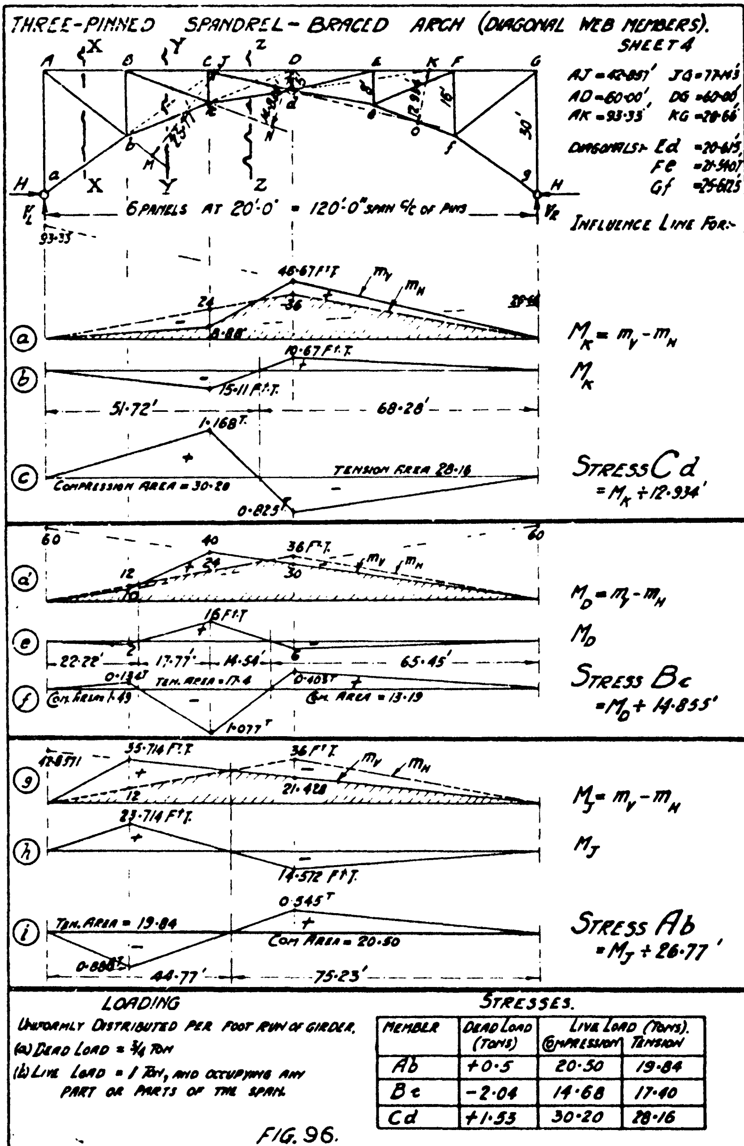

THREE-PINNED SPANDREL-BRACED ARCH (DIAGONAL WEB MEMBERS).
SHEET 4
AJ = 42.857′ JG = 77.143′
AD = 60.00′ DG = 60.00′
AK = 93.33′ KG = 26.66′
DIAGONALS:- Ed = 20.615′, Fe = 21.5407′, Gf = 25.6125′
6 PANELS AT 20′·0″ = 120′·0″ SPAN C/C OF PINS
INFLUENCE LINE FOR:-
$M_K = m_V - m_H$
M_K
STRESS Cd = M_K ÷ 12·934′
COMPRESSION AREA = 30·20
TENSION AREA 28·16
51·72′
68·28′
$M_D = m_V - m_H$
M_D
STRESS Bc = M_D ÷ 14·855′
TEN. AREA = 17·4
COM. AREA = 13·19
22·22′ 17·77′ 14·54′ 65·45′
$M_J = m_V - m_H$
M_J
STRESS Ab = M_J ÷ 26·77′
TEN. AREA = 19·84
COM. AREA = 20·50
44·77′ 75·23′
LOADING
UNIFORMLY DISTRIBUTED PER FOOT RUN OF GIRDER.
(a) DEAD LOAD = 3/4 TON
(b) LIVE LOAD = 1 TON, AND OCCUPYING ANY PART OR PARTS OF THE SPAN.
STRESSES.
MEMBER | DEAD LOAD (TONS) | LIVE LOAD (TONS) COMPRESSION | TENSION
Ab | +0·5 | 20·50 | 19·84
Bc | −2·04 | 14·68 | 17·40
Cd | +1·53 | 30·20 | 28·16

FIG. 96.

Figs. (*b*) ***and*** (*c*) then follow. Note that positive moment opens the break at *ZZ* in diagonal *Cd*, *i.e.*, positive moment gives tensile stress.

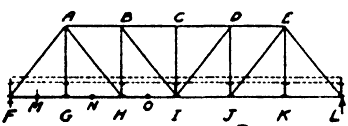
Figs. (*d*), (*e*) ***and*** (*f*). With unit load to the left of section line *YY*, consider V_R when drawing the I.L. for m_V at *D* and with unit load to the right of *YY* consider V_L.

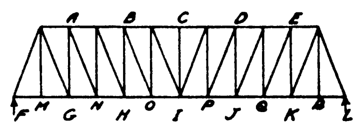
Figs. (*g*), (*h*) ***and*** (*i*). Unit load on any panel point from *B* to *G*, inclusive, gives rise to only one vertical force V_L on the left of *XX*, hence m_V at $J = AJ \times V_L$. The max. value for V_L, unit load at *B*, is $\frac{5}{6}^{T}$ and therefore the max. ordinate to the straight line for m_V as unit load travels from *B* to *G* is $\frac{5}{6} \times AJ = \frac{5}{6} \times 42{\cdot}857 = 35{\cdot}714$. Unit load at *A* being immediately countered by a V_L of 1^T has no moment about *J*, hence join the point 35·714 to the left-hand end of the base line.

CHAPTER XIII

BALTIMORE TRUSS

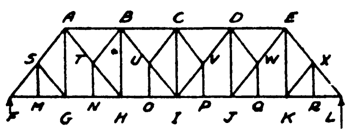
WITH a very large span Pratt truss of the type illustrated by Fig. 97 the floor stringers spanning from *F* to *G*, *G* to *H*, etc., become uneconomically large and heavy. These stringers could be reduced in length if cross girders were placed at *M*, *N* and *O*, etc., but the design would become even more inefficient because the bottom boom, in addition to being subjected to direct axial tensile stress, would now be required to carry the cross bending caused by the mid-panel loads, *M*, *N*, *O*, etc. This * combined direct and lateral loading would entail a deep and heavy bottom boom.



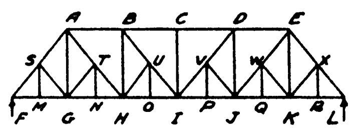


PRATT TRUSSES.

FIG. 97 FIG. 98.



WITH SUB-TIES



WITH SUB-STRUTS.

BALTIMORE TRUSSES.

FIG. 99. FIG. 100.

Halving the panel lengths of Fig. 97, and so doubling the number of web members, produces the Pratt truss of Fig. 98. The stringers are now of an economical length and the bottom boom has only direct primary stress, but the main truss, as a complete unit, is too heavy and rather unsightly due to the increased weight of the web.

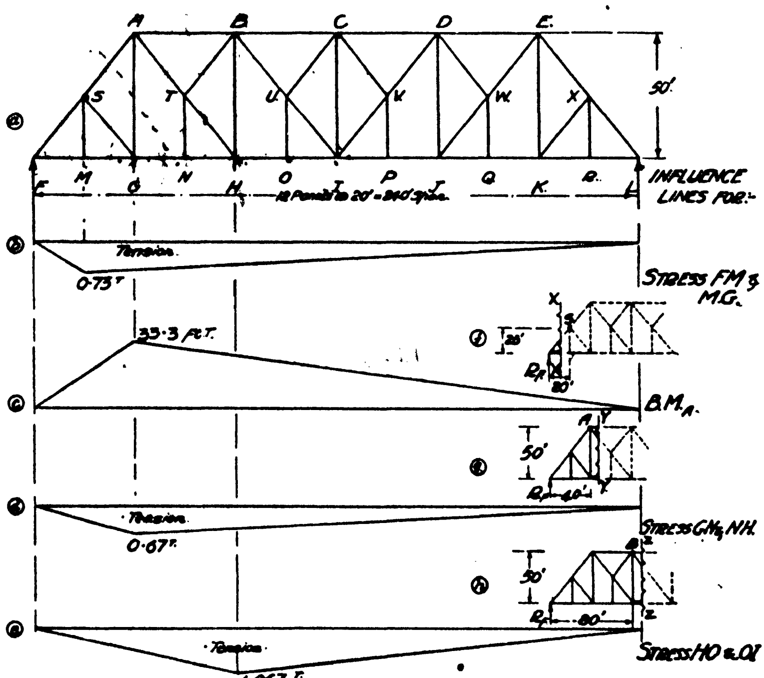
The Baltimore trusses of Figs. 99 and 100 are offspring of the Pratt truss because the main system of Fig. 97 is maintained. The additional cross girders are supported by subsidiary suspenders at *M*, *N*, *O*, etc., and to prevent bending in the main web diagonals secondary ties *TB*, *UC*, etc., are added in Fig. 99, and secondary

* See "Practical Design of Simple Steel Structures," Vol. II.

or sub-struts *TG*, *UH*, etc., in the case of Fig. 100. It will be noted that these additional members will be much lighter than those of Fig. 98 because the sub-ties and sub-struts, engaged in transferring only local panel loads, are in no wise parts of the main web system.

Bottom Boom Members

Member *SM*, being perpendicular to *FG*, has no horizontal component of stress to affect the stress in *FG*, consequently the stress throughout member *FG* is constant, *i.e.*, whatever the stress



BALTIMORE TRUSS BOTTOM BOOM MEMBERS

FIG. 101

FM is then *MG* has the same value. Similarly the stress in *GN* equals that in *NH* and stress in *HO* equals the stress in *OI*, etc.

Member FG. The moment centre for bar *FM* is point *S* in Fig. 101 (f). Reaction $R_F \times 20' =$ stress $FM \times 25'$ or stress $FM = \frac{4}{5}R_F$. Reaction *F* is zero when unit load is at *L*, and therefore stress $FM = 0$. The straight line influence curve increases in value as the unit load leaves *L* and approaches *M* and reaches its maximum value when the load is directly under the moment centre *S*. Taking moments about L to find R_F, and, for simplicity, working in panel lengths, then 1^T at $M \times 11$ panel lengths $= R_F \times 12$ panel lengths, or $R_F = \frac{11}{12}^T$.

The max. stress in FM due to unit load $= \frac{4}{5}$ max. $R_F = \frac{4}{5} \times \frac{11}{12}^T = \frac{11}{15}^T$ or $0{\cdot}73^T$. As unit load leaves M and approaches F the stress in FM rapidly decreases to zero because the load is being transferred either directly to the abutments by the stringer MF or indirectly through a cross girder at F into the main bearings.

Member GH. Fig. 101 (g) shows the section line YY and the moment centre A where the two unwanted bars meet. The influence line for bending moment at point A offers no difficulty, the max. ordinate occurring under point A. The value of this ordinate, 1^T at G, is $R_F \times FG = 40R_F = 40(1^T \times GL \div LF) = 40 \times 200 \div 240 = 33{\cdot}\dot{3}$ ft. tons, as given by diagram (c). The stress influence line for GH will also be a triangle, the height of whose apex is $33{\cdot}\dot{3}$ ft. tons $\div$ effective girder depth of $50' = 0{\cdot}67^T$ as shown by diagram (d).

Member HI. Diagram 101 (h) shows B to be the moment centre. Max. B.M. occurs when unit load is placed at H, under point B, giving a reaction at F of $1^T \times HL \div LF$ or $\frac{2}{3}^T$ and a B.M. of $\frac{2}{3}^T FH = \frac{2}{3}^T \times 80' = 53{\cdot}33$ ft. tons. The B.M. $\div$ the girder depth gives the stress value in HI and the max. value due to unit load traversing the span is $53{\cdot}33 \div 50 = 1{\cdot}067^T$, which is the apex value for the triangle of diagram (e).

Max. Stress in Members FG, GH and HI. Since the stress influence lines for these members are triangular in form then, as given on page 20, max. stress occurs when the average load per ft. of span to the left of the apex (or moment centre) equals the average load per ft. to the right, and, therefore, equals the average load per ft. of span.

Top Boom Members

Member AB. By taking a section line such as XX, Fig. 102 (d), only three members are cut including the wanted bar AB; and the moment centre, where the two unwanted bars AT and GN meet on production, is panel point H.

Now consider the stability of that portion of the structure shown in heavy line, diagram (d). So long as the 1^T load travels between L and N the only external force acting on the heavily lined portion is R_F. Then, taking moments about point H, $FH \times R_F =$ stress $AB \times BH$, *i.e.*, stress $AB = \frac{FH}{BH} R_F = 1{\cdot}6R_F$.

1^T at L means R_F is zero and stress $AB = 1{\cdot}6R_F \quad = 0$

1^T ,, H ,, $0{\cdot}67^T$,, $= 1{\cdot}6 \times 0{\cdot}67 = 1{\cdot}07^T$

1^T ,, N ,, $0{\cdot}75^T$,, $= 1{\cdot}6 \times 0{\cdot}75 = 1{\cdot}2^T$

Whenever unit load leaves N and moves towards G there are two external forces acting on the small piece of structure $FAXXG$, viz., R_F and panel load G.

1^T at G means R_F is $\frac{5}{8}^T$ and the equation about moment centre H is now $FH \times R_F - 1^T$ at $G \times GH =$ stress $AB \times BH$, calling clockwise moments positive; *i.e.*, $80' \times \frac{5}{8}^T - 1^T \times 40' = 50'AB$, whence stress $AB = +\ 0{\cdot}5334^T$.

As the clockwise moment is the greater on the left-hand side of the equation the effect on AB is to close the cut in this bar, *i.e.*, AB is in compression.

R_F has a value of $\frac{1}{12}^T$ when unit load is placed at M and the moment equation about point H is $80' \times \frac{1}{12}^T - 1^T \times 60' = 50'AB$, or stress $AB = +\ 0{\cdot}267^T$.

The complete influence line is given by diagram (b). The only point to note is that when unit load is placed at N it is still outside of the heavily lined portion of the structure even although it has passed the moment centre H. The reaction at F is therefore the only external force to be considered.

Max. Stress in AB. As was previously done in Fig. 40, split the total load of W on the bridge into three distinct portions A, B and C,

$$i.e.,\ W = A + B + C \quad \text{(i)}$$

where the A loading consists of A_1, A_2, A_3, etc., and the B loading of the separate loads B_1, B_2, B_3, and so on.

S, the stress in $AB = \Sigma$ loads $\times$ respective ordinates

$$= A_1a_1 + A_2a_2 + \ldots B_1b_1 + B_2b_2 + \ldots C_1c_1 + C_2c_2 \ldots \quad \text{(ii)}$$

Move the complete load system a small distance δx along the span, no load entering or leaving the span, and the new stress in AB is

$$S + \delta S = A_1(a_1 + \delta a) + A_2(a_2 + \delta a) + \ldots B_1(b_1 + \delta b) + B_2(b_2 + \delta b) + \ldots C_1(c_1 + \delta c) + \ldots \quad \text{(iii)}$$

The change of stress = (iii) – (ii) = δS.

$$\delta S = \delta a(A_1 + A_2 + \ldots) + \delta b(B_1 + B_2 + \ldots) + \delta c(C_1 + C_2 + \ldots) \quad \text{(iv)}$$

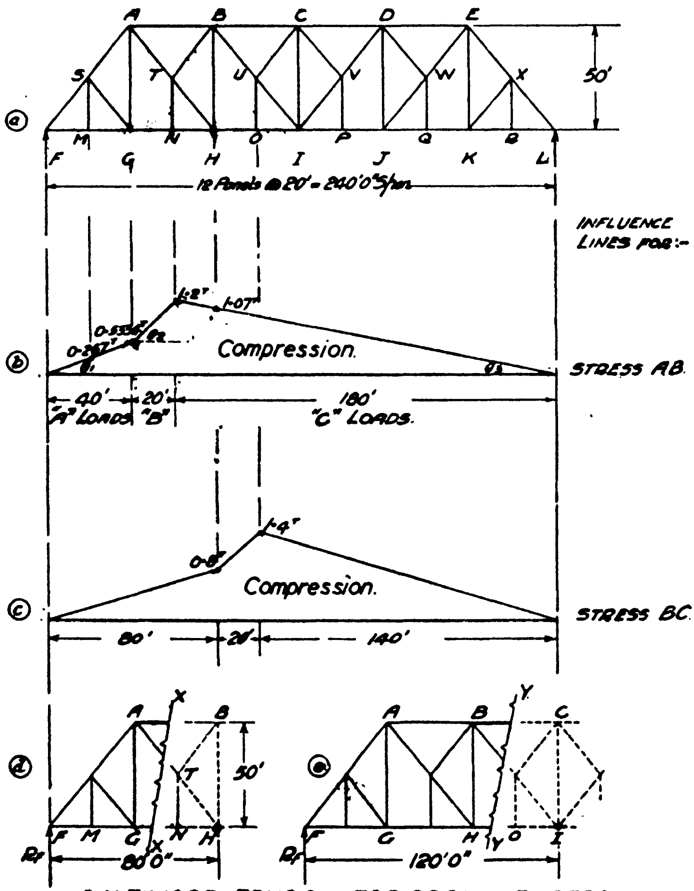
and, as for equation (v), page 55, dividing throughout by δx gives

$$\frac{\delta S}{\delta x} = \frac{\delta a}{\delta x}(A) + \frac{\delta b}{\delta x}(B) + \frac{\delta c}{\delta x}(C) \quad \text{(v)}$$

$$= \tan\theta_1 A + \tan\theta_2 B + \tan\theta_3 C \quad \text{(vi)}$$

But since the slope of the influence line for portion C is opposite to that for portions A and B its tangent will have opposite sign, *i.e.*,

$$\frac{\delta S}{\delta x} = \tan\theta_1 A + \tan\theta_2 B - \tan\theta_3 C \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(vii)}$$



BALTIMORE TRUSS TOP BOOM MEMBERS.

FIG. 102.

Now substitute the arithmetical values for these tangents from Fig. (b) and

$$\frac{\delta S}{\delta x} = \left(\frac{0{\cdot}5334}{40}\right)A + \left(\frac{1{\cdot}2 - 0{\cdot}5334}{20}\right)B - \left(\frac{1{\cdot}2}{180}\right)C$$

$$= 0{\cdot}013335A + 0{\cdot}0\dot{3}B - 0{\cdot}00\dot{6}C$$

For reasons stated against equation (viii), page 56, maximum stress occurs in AB if $\frac{\delta S}{\delta x} = 0$,

i.e., if $0{\cdot}013335A + 0{\cdot}0\dot{3}B - 0{\cdot}00\dot{6}C = 0$

or if $0{\cdot}013335A + 0{\cdot}0\dot{3}B = 0{\cdot}00\dot{6}C$

$= 0{\cdot}00\dot{6}(W - A - B)$ by equation (i)

if $0{\cdot}02A + 0{\cdot}04B = 0{\cdot}00\dot{6}W$

if $2A + 4B = \frac{2}{3}W$

i.e., if $3A + 6B = W$.

For a uniformly distributed load this equation of condition can be easily fulfilled, but with concentrated loads it may be found quicker simply to find the max. stress in AB, directly from the influence line, by trial and error.

Member BC. The section line is YY and the moment centre is I, Fig. 102 (e). So long as unit load is to the right of panel point O only one external force, R_F, acts on the $ABYYHF$ portion of the bridge. Consequently, moments about I, $CI \times$ stress $BC = FI \times R_F$, whence stress BC $= \frac{FI}{CI}R_F$

$= 2{\cdot}4R_F$.

The stress influence line will rise from zero at L to a max. value, at the vertical through O, of $2{\cdot}4R_F = 2{\cdot}4 \times \frac{7}{12} = 1{\cdot}4^T$, since R_F is $\frac{7}{12}^T$ when unit load is placed at O.

Unit load at H means a new type of moment equation, because there are now two external forces acting on the broken piece of bridge, *i.e.*, $CI \times$ stress $BC = FI \times R_F - 1^T \times HI$. As R_F has the value of $\frac{2}{3}^T$ with unit load at H then

$50' \times$ stress $BC = 120' \times \frac{2}{3}^T - 1^T \times 40'$ or stress $BC = 0{\cdot}8^T$

From this point the influence line drops to zero at F, Fig. 102 (c).

Max. Stress in BC. Substituting the new values for the tangents of equation (vii) :—

$$\frac{\delta S}{\delta x} = \frac{0{\cdot}8}{80}A + \frac{1{\cdot}4 - 0{\cdot}8}{20}B - \frac{1{\cdot}4}{140}C$$

$$= 0{\cdot}01A + 0{\cdot}03B - 0{\cdot}01C$$

There is a max. stress in BC if $\frac{\delta S}{\delta x} = 0$

i.e., if $0{\cdot}01A + 0{\cdot}03B - 0{\cdot}01C = 0$

if $A + 3B - C = 0$

or if $A + 3B - (W - A - B) = 0$

or if $2A + 4B = W$.

Web Members

With a cross girder at F the reaction influence line for the main bearing at F is the triangle lff', Fig. 103 (b). However, since the panel load at F goes straight through into the main bearing without affecting the main girder in any way, the influence line for the reaction as it affects the main girder is lfm. This is also the influence line for the reaction when there is no cross girder at F and the end stringers rest directly upon the abutment wall, as previously explained on page 57.

Member FS. The internal forces acting in members FS and FM together with the external reaction form three forces acting at a point, whence it follows that the vertical component of the stress FS, *i.e.*, of S_{FS}, must balance R_F.

$$\therefore\ R_F = S_{FS} \times \sin \widehat{SFM} = S_{FS} \times \frac{MS}{FS} = S_{FS} \times \frac{25}{32{\cdot}015}$$

or $S_{FS} = 1{\cdot}281 R_F$.

Because the influence line for R_F is a triangle then that for S_{FS} is also a triangle, but one whose apex value is 1·281 times that of R_F, *i.e.*, $1{\cdot}281 \times 0{\cdot}9167 = 1{\cdot}174$.

Similar reasoning gives the apex value of the triangular influence line for FM as $R_F \times \cot \widehat{SFM} = R_F \times \frac{20}{25} = 0{\cdot}9167 \times 0{\cdot}8 = 0{\cdot}733$, as was previously obtained for Fig. 101 (b) by the method of Section and Moments.

Member SM is simply a suspender supporting the cross girder at M. As the load travels from G to M the load on SM gradually increases from zero up to 1^T, at which value unit load has arrived at M, and then gradually decreases again to zero as the 1^T load approaches F. As explained on page 66, the influence curve is a triangle and is given by diagram (e), which is also the I.L. for all the suspenders, SM, TN, UO, etc.

Member SG. The load from panel point M travels up MS and bifurcates at point S, one half going down SG as a strut and the other half down SF. The vertical component of S_{SG} is always one half of the load in SM. $\frac{1}{2}S_{SM} = S_{SG} \times \cos \widehat{MSG} = S_{SG} \times \frac{25}{32{\cdot}015}$ or $S_{SG} = 0{\cdot}64 S_{SM}$, and the I.L. for SG is a triangle like that for SM, but one whose apex value is $0{\cdot}64 \times 1^T$. A reference to the stress diagram of Fig. 106 will show that the stresses follow the paths outlined.

Sub-tie DW (and CV, CU and BT). Here again as the load arrives at point W it splits into two equal portions, one half going up tie WD and the remaining half entering the main diagonal.

The vertical component of $S_{DW} = \frac{1}{2}S_{WQ}$

$$\therefore (\tfrac{1}{2}DJ \div DW)S_{DW} = \tfrac{1}{2}S_{WQ}$$

$$\text{or } \frac{25}{32{\cdot}015}S_{DW} = \tfrac{1}{2}S_{WQ}, \textit{ i.e., } S_{DW} = 0{\cdot}64S_{WQ}.$$

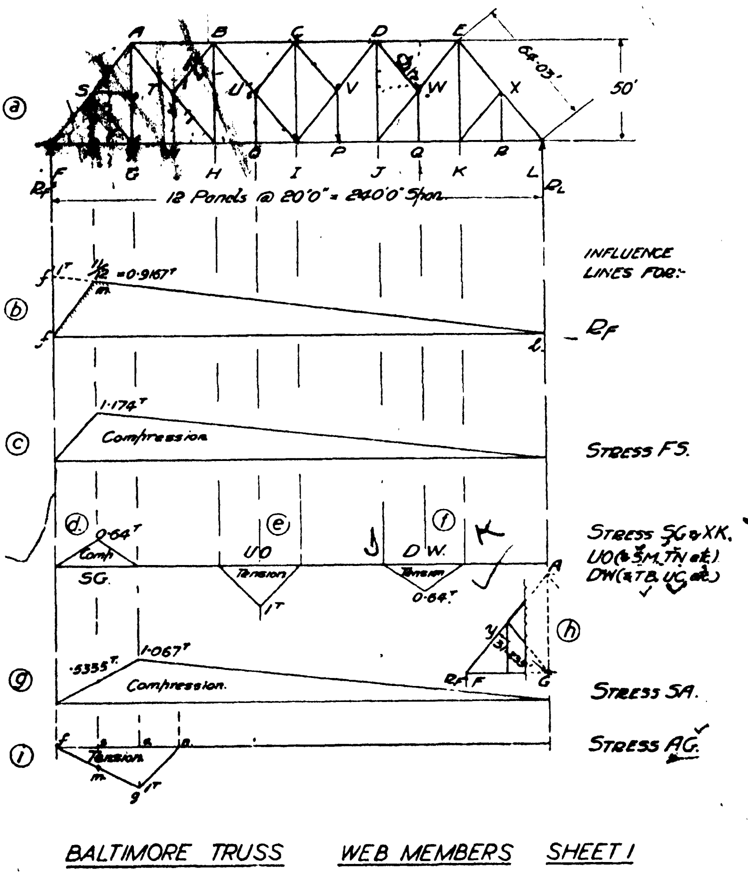


FIG. 103

Therefore the I.L. for S_{DW} is a triangle like that for S_{WQ} and with an apex value of $0{\cdot}64$T. An examination of the stress diagram will verify that the panel load in a suspender takes the routes indicated.

Member SA. Referring to the inset figure, 103 (h), the length of the lever arm yG is obtained by comparing similar triangles AGy and AFG :—

$$\frac{yG}{GA} = \frac{GF}{FA} \quad \therefore yG = \frac{GA \times GF}{FA} = \frac{50 \times 40}{64{\cdot}03} = 31{\cdot}235'.$$

With the moving load confined between L and G there is only one external force, R_F, acting on the broken end-piece of the bridge and, taking moments about G, $31{\cdot}235' S_{SA} = 40' R_F$. The maximum value of this straight line occurs when unit load is placed at G giving R_F a value of $\frac{5}{6}$T and so $S_{SA} = 40' \times \frac{5}{6}\text{T} \div 31{\cdot}235' = 1{\cdot}067$T.

When unit load enters the small end portion of the bridge consider the right-hand reaction R_L and $31{\cdot}235' S_{SA} = 200 R_L$ or $S_{SA} = 6{\cdot}403 R_L$. The max. value to this expression happens with unit load placed at G giving R_L a value of $\frac{1}{6}$T and S_{SA} its maximum ordinate of $6{\cdot}403 \times \frac{1}{6} = 1{\cdot}067$T. These two straight lines form the triangle of diagram (g).

Vertical AG. In Fig. 103 (i) as unit load moves from F to M suspender SM is brought into action and so also is SG, which transfers to panel point G one half of the vertical load in SM. This panel point load at G has then to travel up the main suspender GA to get to the abutments. The influence line so far outlined is from zero at f to $sm = \frac{1}{2}$T, with unit load at M. Approaching G from M more of the load goes to panel point G from the stringer MG and less into the suspender SM until point G is reached when the full panel load of 1T is wholly carried by AG. Now it is known (page 43) that between panel points influence lines must be straight and the curve fmg now traced out is therefore absolutely straight since the panels are all of the same length. As the load travels along stringer GN the reaction at G becomes smaller while that at N increases. With the moving wheel at N the whole load is supported by NT, thence into the main diagonal HA and sub-tie TB without placing any stress whatsoever on main suspender AG, and so the closing line of the curve is gn.

Maximal Stresses, Fig. 103 (*b*) *to* (*i*). Each influence line being a triangle, the rule is that the load per ft. to the left of the apex should be equal to the load per ft. to the right in order to obtain the position of the load which gives the maximum stress.

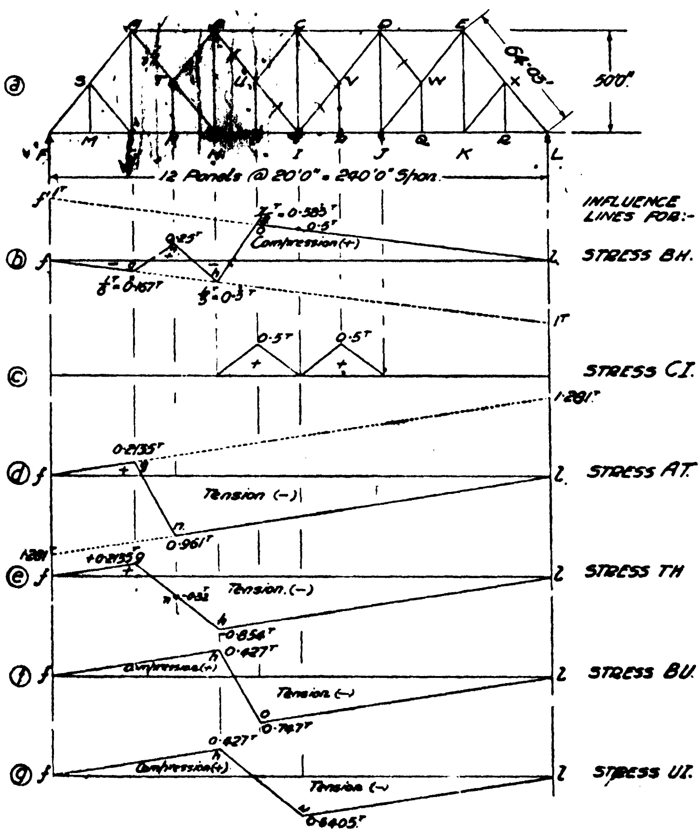
Vertical BH, Fig. 104 (*a*) *and* (*b*)

Portion LI. When unit load lies anywhere between L and I there is no stress in the sub-ties NT and TB, and these, for such a load position, could be well left out as in Fig. 105 (a). The stress influence line is thus of the standard form for a Pratt truss, Fig. 105 (a), and rises from zero value at L to 0·5T at I.

Portion IO. Now referring to Fig. 104, it is clear that if 1T be placed anywhere between I and O the positive vertical shear is R_F and is constant in value between F and O, and consequently the stress in HB is compression and equal arithmetically to R_F. When unit load is at O the value for R_F is $\frac{7}{12}$T or $0{\cdot}58\dot{3}$T. So far, then, the influence line is straight from l to o, Fig. 104 (b).

Portion OH. From point O advance the load to H. If in Fig. 105 (a) one were to leave H and travel up HB, then down BI and so ultimately arrive at L no external load would be encountered, and therefore the stress in HB is equal to the negative shear between H and L, *i.e.*, equal to R_L or $\frac{1}{3}$T. The stress in HB has now changed from compression to tension since R_L tends to push B upwards away from H. The influence line as at present determined is *loh* in Fig. 104 (b).

Portion HN. When unit load is placed at N it, in effect, travels up suspender NT, breaks into two equal parts at T, one going up



BALTIMORE TRUSS WEB MEMBERS SHEET 2

FIG. 104

TA and the other, *TB*, with the result that the panel loads at *A* and *B* are each $\frac{1}{2}$T. Having endowed unit load with the climbing propensity of the primates the path *NT*, *TB* can be removed leaving the $\frac{1}{2}$T loads secured at points *A* and *B* of the simplified truss of Fig. 105 (b). The reaction at *F* is $\frac{3}{4}$T whether unit load at *N* be considered or its equivalent at *A* and *B*. There being no external load at *H*, the vertical component of the force in *AH* must be the same as the force in *HB*, *i.e.*, the vertical shear of $\frac{1}{4}$T. This positive shear subjects vertical *HB* to a compressive stress, and the point *n* of Fig. 104 (b) is another point on the desired influence line ***lohn***.

Portion NF. A load at *G* (or any position between *G* and *F*) affects only the primary members of the Pratt truss of Fig. 105 (b) and the bars *NT* and *TB* could be removed if desired. For this load position the stress in *BH* is due to the shear in panel $GH = R_F - 1^T$ at $G = \frac{5}{6}^T - 1^T = -\frac{1}{6}^T$.

The negative shear causes tensile stress in *BH*.

Alternatively it is due to the reaction R_L. Let the load be anywhere between *F* and *G* at a distance x' from *F*, then the reaction $L = 1^T \times x' \div 240'$.

$\therefore$ stress in *BH* is tensile $= x' \div 240'$, *i.e.*, a straight line.

Unit load at *G* then $S_{BH} = 40 \div 240 \qquad = -\frac{1}{6}^T$

,, ,, *M* ,, $= 20 \div 240 \qquad = -\frac{1}{12}^T$

and the complete influence line is *lohngf*.

Maximum Stress. Max. compressive stress in *BH* with a U.D.L.L. will be obtained by breaking up the live load so as to cover the positive segments, and max. tensile stress in *BH* by covering the bases of the negative triangles whose apices are $-\frac{1}{6}^T$ and $-\frac{1}{3}^T$. However, some specifications do not countenance this partial breaking up of the load and they suggest for max. compressive stress in *BH* that the live load should either (a) extend from *l* to the neutral point between *O* and *H*, or (b) cover all the span. A mathematical criterion can be worked out on the lines previously laid down, viz., finding the equation embracing the various tangents, differentiating, and equating to zero to give a max. value, a procedure possibly of more academic interest than practical utility.

Vertical CI. In the simplified truss of Fig. 105 (b) this member is redundant, and so in Fig. 104 it can only receive stress through the sub-bracing system, which limits the influence base line to a length equal to *HJ*. Unit load approaching *O* from *H* increases the stress in *OU* and *UC* from zero up to a max. value when 1^T is at *O*. In this last position the equivalent panel load at *C* is $\frac{1}{2}$T and this $\frac{1}{2}$T must travel down the centre vertical *CI* on its way into the main diagonals. Shifting unit load from O to I decreases panel load O and increases that at I. When the wheel arrives at I no stress goes up UC to panel point C and so no stress is in CI. The panel load at I

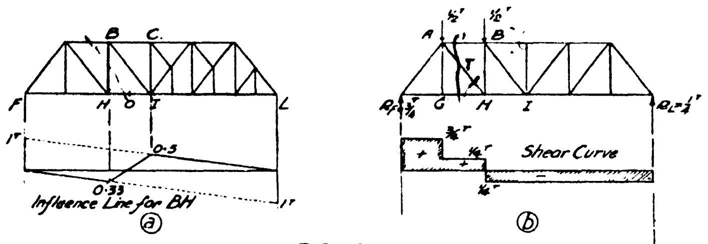


FIG 105

is shared equally between diagonals IB and ID without affecting IC which is now redundant. The same reasoning applies to panel IJ and the stress influence line for CI is the dual triangle of diagram (c).

Diagonal AT. Assume a vertical cutting line midway between G and N. There is only R_F acting on the left-hand portion FAG when unit load is to the right of N.

Vertical component of S_{AT}, the stress in $AT = R_F$

$$\therefore S_{AT} \cos \widehat{GAH} = S_{AT}(AG \div AH) = R_F$$

$$\therefore S_{AT} = (AH \div AG)R_F = (64{\cdot}03 \div 50)R_F = 1{\cdot}281 R_F$$

The influence line for S_{AT} is a straight line from nothing at L to $1{\cdot}281 \times \frac{3}{4}^{T} = 0{\cdot}961^{T}$ at N, since R_F is $\frac{3}{4}^{T}$ when 1^{T} is at N. The stress in AT for this load travel is tensile.

When unit load is between G and F consider R_L, whose value with 1^{T} at G is $\frac{1}{6}^{T}$.

$S_{AT} \cos \widehat{GAH}$ now equals R_L, *i.e.*, $S_{AT} = 1{\cdot}281 R_L$; a straight line from $1{\cdot}281 \times \frac{1}{6}^{T}$, or $0{\cdot}213^{T}$ at G down to zero at F. This portion of the load travel induces compression, while the completed influence line is *fgnl* of diagram (d).

Diagonal TH. Subsidiary bars NT and TB are redundant and can be taken out if unit load is confined to move between L and H, and so the simplified panel $ABHG$ exists as shown by Fig. 105 (b). Stress in TH is tensile and equals $(AH \div AG)R_F = 1{\cdot}281 R_F$. Since R_F is $\frac{2}{3}^{T}$ with the load at H, the max. value of the foregoing straight line is $1{\cdot}281 \times \frac{2}{3}^{T}$ or $0{\cdot}854^{T}$.

Similarly when the load lies between G and F the bars TN and TB are redundant and the stress in AH or TH is $1{\cdot}281 R_L$. The stress in TH is now compression with a max. value of $1{\cdot}281 \times \frac{1}{6}^{T}$ or $0{\cdot}2135^{T}$, when unit load is at G.

So far, then, the influence line traced out is from *l* to *h* and from *f* to *g*.

Now place the unit load at the doubtful point *N*, an action which brings *TN* and *TB* immediately into play without directly affecting the stress in *TH*, because, as we have seen, the load travels up *NT*, bifurcates at *T*, and, taking the dual paths of *TA* and *TB*, deposits $\frac{1}{2}$T panel loads at *A* and *B*.

From the partial influence line *fg* and *lh* of Fig. 104 (e) it is to be observed that the $\frac{1}{2}$T load placed at *A*, relative to the span, causes a

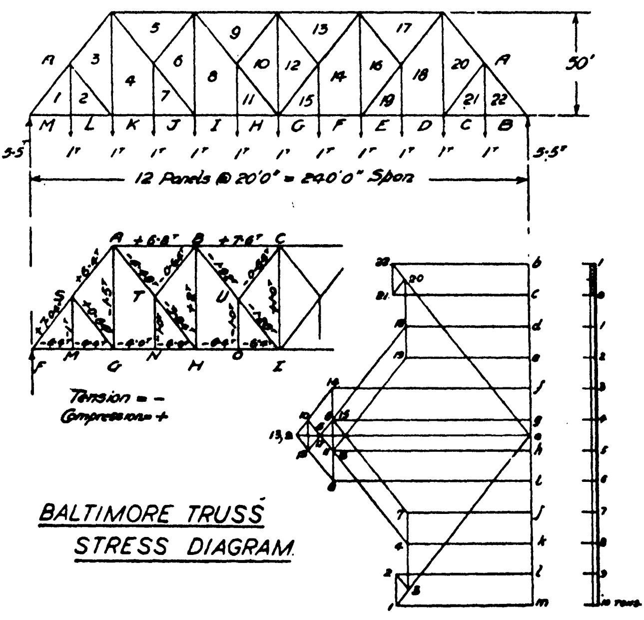


FIG. 106.

compressive stress in *TH* of one-half of the ordinate at *g* ($\frac{1}{2}$ of 0·2135T) while the $\frac{1}{2}$T load at *B* creates a tensile stress in *TH* of one-half the ordinate at *h*, namely $\frac{1}{2}$ of 0·854T. Hence a unit load placed at *N*, a point which is midway between points *G* and *H*, induces a stress in *TH* which is the mean of the stress ordinates at *G* and *H*. In other words, the required stress ordinate at *n* is the mean of the stress ordinates at *g* and *h* and it thus follows that the closing line *gnh* is absolutely straight. Alternatively, when load *N* has been replaced by $\frac{1}{2}$T loads at *A* and *B*, Fig. 105 (b), the bars *TN* and *TB* are redundant and can be removed. Considering this denuded truss, it is seen that the stress in $AH = S_{TH} = 1{\cdot}281$ (shear in panel $GH) = 1{\cdot}281 \times \frac{1}{4}^{T} = 0{\cdot}32^{T}$, tension. But this value of $-0{\cdot}32^{T}$ is the mean of $+0{\cdot}2135^{T}$ and $-0{\cdot}854^{T}$ of Fig. 104 (e), and thus lies on the straight line joining the two last mentioned ordinates. The complete influence line *fgnhl* now derived for S_{TH} is identical with the stress influence line for the complete and unbraced diagonal *AH* of the ordinary Pratt truss.

Members BU and UI are derived in a manner similar to that of *AT* and *TH* respectively.

Max. Stress in Members AT, TH, BU and UI. The laws governing the disposition of the loading to give max. stresses in these members are those given relative to Figs. 33, 34 and 35.

CHAPTER XIV

MISCELLANEOUS TRUSSES

PENNSYLVANIA OR PETTIT TRUSS

THIS truss is a derivative of the Bowstring, the Pratt and the Baltimore types of bridge trusses. The general method of obtaining the stress influence curves follows the lines laid down in the section devoted to the Bowstring girder because many of the moment centres now lie outside of the truss.

The intermediate suspenders are again the agents whereby the panel loads at their lower extremities are transferred to the upper boom panel points. If the panels are of equal length then the amount transferred to each of the top boom panel points on both sides of a suspender is one-half of the suspended load. Although this means that the vertical component of the stress in each of the sub-ties is the same, for unit suspender load, yet the triangular shaped stress influence curves will have different values for the ordinates to the apices because the inclinations of the sub-ties are different. Thus sub-ties CU and BT will have the respective apex heights of $\frac{1}{2}\text{T}[CU \div (CI - UO)]$ and $\frac{1}{2}\text{T}[BT \div (BH - NT)]$; the arithmetical value of the latter is $\frac{1}{2}\text{T}(34 \div 27{\cdot}5) = 0{\cdot}618\text{T}$.

In view of the full explanation given for each member of the Bowstring and the Baltimore trusses no difficulty should be encountered in verifying the typical influence lines given on Fig. 107. A brief summary of the straight line equations as derived for these influence lines is given under.

$$S_{FS} = R_F \operatorname{cosec} \widehat{SFM} = R_F(FS \div SM) \qquad = 1{\cdot}28R_F$$

$$S_{SA} = R_F \times FG \div 31{\cdot}31, \text{ with load between } L \;\&\; G = 1{\cdot}277R_F$$

$$= R_L \times GL \div 31{\cdot}31 \quad \text{,,} \quad \text{,,} \quad G \;\&\; F = 6{\cdot}387R_L$$

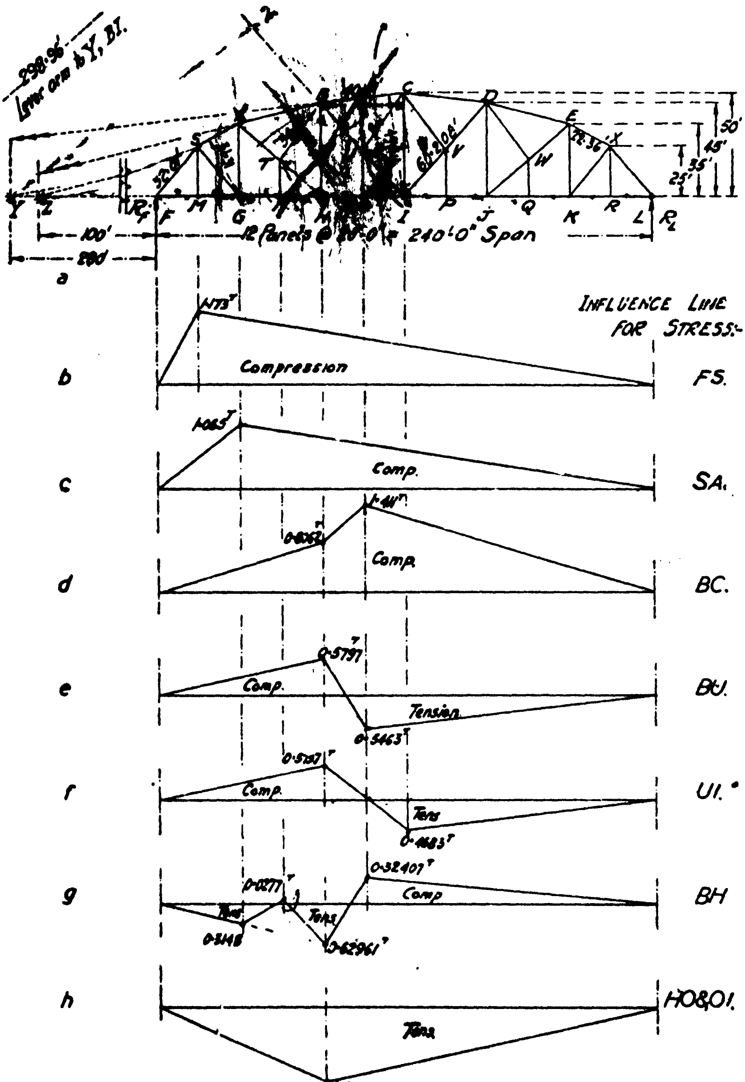
$$S_{BC} = R_F \times FI \div 49{\cdot}61 \quad \text{,,} \quad \text{,,} \quad L \;\&\; O = 2{\cdot}419R_F$$

$$= R_L \times IL \div 49{\cdot}61 \quad \text{,,} \quad \text{,,} \quad H \;\&\; F = 2{\cdot}419R_L$$

Member BU has its moment centre outside of the truss at point Y so giving a lever arm of $298{\cdot}96'$.

$$S_{BU} = R_F \times YF \div 298{\cdot}96, \text{ with load between } L \;\&\; O = 0{\cdot}9366R_F$$

$$= R_L \times YL \div 298{\cdot}96 \quad \text{,,} \quad \text{,,} \quad H \;\&\; F = 1{\cdot}7393R_L$$



PENNSYLVANIA OR PETTIT TRUSS.

FIG. 107.

$S_{UI} = R_F \times YF \div 298{\cdot}96$ with load between L & I $= 0{\cdot}9366R_F$

$= R_L \times YL \div 298{\cdot}96$ „ „ H & F $= 1{\cdot}7393R_L$

Member BH has its moment centre at Z.

$S_{BH} = R_F \times ZF \div ZH$, with load between L & O $= 0{\cdot}\dot{5}R_F$

Member BT is not acting for the foregoing load travel and the cut ends of BH tend to meet, indicating compressive stress in the member.

$S_{BH} = R_L \times ZL \div ZH$, with load at point H $= 1{\cdot}\dot{8}R_L$

With unit load at H member BT is redundant; while the portion of the structure on the right-hand side of the section line tends to move upwards relative to the left-hand segment $FATH$ showing that S_{BH} is now tension.

S_{BH} unit load at N. Replace TN and TB by $\frac{1}{2}^{T}$ load at A and another $\frac{1}{2}^{T}$ at B, then

$$S_{BH} = (R_L \times ZL - \tfrac{1}{2}^{T} \text{ at } B \times ZH) \div ZH$$
$$= (\tfrac{1}{4}^{T} \times 340 - \tfrac{1}{2} \times 180) \div 180$$
$$= -5 \div 180$$

i.e., clockwise moment causing cut ends to meet, compressive stress $= +\,0{\cdot}02\dot{7}^{T}$

$S_{BH} = R_L \times ZL \div ZH$, with load between G & F $= 1{\cdot}\dot{8}R_L$

For the above travel of the load, members TN and TB are redundant.

Members HO and OI have always identical stress; moment centre is at B.

$S_{HO} = R_L \times HL \div BH$, load between F & H $= 3{\cdot}\dot{5}R_L$

$= R_F \times HF \div BH$ „ „ H & L $= 1{\cdot}\dot{7}R_F$

PRATT TRUSS WITH DOUBLE PANELLED STRINGERS

In Fig. 108 (a) there is a cross girder placed at every second panel point of the truss, and it is now proposed to investigate what effect this has upon the influence lines.

Bottom Flange

Member NM. The moment centre is at D. As the unit load travels from R to N the standard piece of influence line rn, of diagram (b), is traced out. The height of ordinate en is found by placing the load at N so giving R_J a value of $\frac{1}{2}^{T}$.

The B.M.$_D$ = $R_J \times AD$ $= \frac{1}{2}AD$

S_{NM} = B.M. ÷ depth $= \frac{1}{2}AD \div DM$

$$= \frac{1}{2} \times \frac{3 \text{ panels}}{1 \text{ panel}} = \frac{3p}{2p} = 1\tfrac{1}{2}^{\text{T}}$$

Similarly portion jl is of standard form with the load between J and L.

Unit load at L gives R_R a value of $\frac{1}{4}^{\text{T}}$ and the

$$\text{B.M.}_D = DI \times R_R = \tfrac{1}{4}DI$$

while the corresponding $S_{NM} = \frac{1}{4}DI \div DM = \frac{1}{4} \times 5p \div p = 1\frac{1}{4}^{\text{T}}$

Finally place unit load on stringer LN. As there is no cross girder at M there can be no panel load at this point. The local reaction from stringer LN at L is $R_L = 1^{\text{T}} \times x \div LN$ $= x \div 2p$

and the main reaction at J is $1^{\text{T}}(NR + x) \div JR$ $= \dfrac{4p + x}{8p}$

B.M.$_D$ = $R_J \times AD$ minus the local reaction or load at $L \times CD$

$$= \frac{4p + x}{8p} \times 3p - \frac{x}{2p} \times p = \frac{12p - x}{8}$$

The S_{NM} = B.M. ÷ depth $= \dfrac{12p - x}{8} \div p = \dfrac{12p - x}{8p}$

a straight line, the two extreme values of which are found by placing :—

(a) $x = 0$ and $S_{NM} = \dfrac{12p}{8p} = 1\frac{1}{2}^{\text{T}}$

(b) $x = LN$ and $S_{NM} = \dfrac{12p - 2p}{8p} = 1\frac{1}{4}^{\text{T}}$

The final influence line is the truncated triangle $jlnr$ of diagram (b).

Member ML. The moment centre is now C.

Load between R and N

1^{T} at N then $S_{ML} = R_J \times AC \div CL = \frac{1}{2}^{\text{T}} \times 2p \div p = 1^{\text{T}}$

Load between L and J

1^{T} at L then $S_{ML} = R_R \times CI \div CL = \frac{1}{4}^{\text{T}} \times 6p \div p = 1\frac{1}{2}^{\text{T}}$

Load between N and L

$$\text{B.M.}_C = R_J \times AC - R_L \times 0 = \frac{1^{\text{T}}(NR + x)}{8p} \times 2p = \frac{4p + x}{4}$$

$$S_{ML} = \frac{4p + x}{4} \div \text{depth } p = \frac{4p + x}{4p}$$

$x = 0$ then $S_{ML} = 4p \div 4p$ $= 1^{\mathrm{T}}$

$x = NL$ then $S_{ML} = (4p + 2p) \div 4p$ $= 1\frac{1}{2}^{\mathrm{T}}$

The triangular figure of diagram (c) is the desired influence line.

Member LK. From R to L the influence line will follow the standard type and $S_{LK} = R_J \times AB \div BK$. The reaction at J is $\frac{3}{4}^{\mathrm{T}}$ when unit load is placed at L, hence $S_{LK} = \frac{3}{4}p \div p$ $= \frac{3}{4}^{\mathrm{T}}$

Now consider unit load on the stringer JL and let y be the variable distance measured from J.

$$R_R = 1^{\mathrm{T}} \times y \div JR = \frac{y}{8p}$$

and

$$R_L = 1^{\mathrm{T}} \times y \div JL = \frac{y}{2p}$$

$$\text{B.M.}_B = BI \times R_R - BC \times R_L = 7p \times \frac{y}{8p} - p \times \frac{y}{2p} = \frac{3y}{8}$$

$$\therefore S_{LK} = \text{B.M.} \div \text{depth} = \frac{3y}{8} \div p = \frac{3y}{8p}$$

The terminal values of this straight line are :—

Load at J, $y = 0$, then S_{LK} $= 0$

Load at L, $y = 2p$, then S_{LK} $= \frac{3}{4}^{\mathrm{T}}$

Member KJ is redundant and has no direct primary stress.

Top Flange Members

Member DE has moment centre at N.

Travel RN. $S_{DE} = R_J \times JN \div EN$ $= 4R_J$

Load at N, then $S_{DE} = 4 \times \frac{1}{2}^{\mathrm{T}}$ $= 2^{\mathrm{T}}$

Travel JN. $S_{DE} = R_R \times NR \div EN$ $= 4R_R$

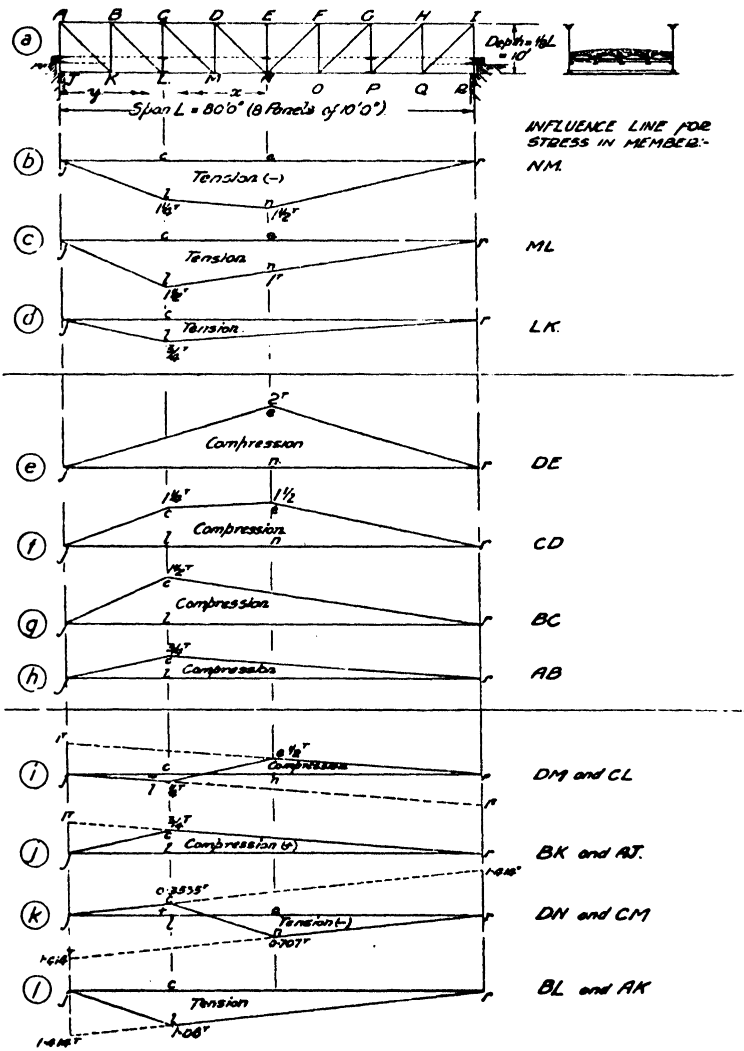
Load at N, then $S_{DE} = 4 \times \frac{1}{2}^{\mathrm{T}}$ $= 2^{\mathrm{T}}$

Member CD. The moment centre is point M. Travel RN conforms to the standard; unit load at N causes a stress in CD of $R_J \times JM \div DM = \frac{1}{2}^{\mathrm{T}} \times 3p \div p$ $= 1\frac{1}{2}^{\mathrm{T}}$

Travel NL.

$$R_J = \frac{RN + x}{RJ} = \frac{4p + x}{8p} \text{ and } R_L = \frac{x}{LN} = \frac{x}{2p}$$

$$S_{CD} = (R_J \times JM - R_L \times LM) \div p = \frac{12p - x}{8p}$$



PRATT TRUSS WITH DOUBLE PANELLED STRINGERS.

FIG 108

Load at N, $x = 0$, then $S_{CD} = 12p \div 8p \quad = 1\frac{1}{2}^{T}$

Load at L, $x = 2p$, then $S_{CD} = (12p - 2p) \div 8p \quad = 1\frac{1}{4}^{T}$

Travel LJ. $R_R = y \div JR \quad = \dfrac{y}{8p}$

$$S_{CD} = R_R \times MR \div p = \frac{y}{8p} \times 5p \div p \quad = \frac{5y}{8p}$$

Load at L, $y = 2p$, then $S_{CD} = 5 \times 2p \div 8p \quad = 1\frac{1}{4}^{T}$

Load at J, $y = 0$, then $S_{CD} \quad = 0$

Members AB and BC are investigated in a manner similar to that given for members KL and LM.

Web Members

Members DM and CL. The load carried by these members is numerically equal to the shear in the stringer panel LN.

Load between R and N, then $S_{DM} = S_{CL} = R_J$

Unit load at N, $S_{DM} = R_J = \frac{1}{2}^{T}$, compression.

Load between J and L, then $S_{DM} = S_{CL} = R_R$

Unit load at L, $S_{DM} = R_R = \frac{1}{4}^{T}$, tension.

No load is applied at M, and from the text given in connection with Fig. 32 it is known that the line joining points e and l must be straight, so giving the influence line *jlernc* of Fig. 108 (i). Alternatively, obtain the straight line equation for the shear in panel LN with unit load placed in the panel at a variable distance x from N, as was done above for several of the other members.

Members BK and AJ will have the same stress because no external load is applied between the panel points J and L. Unit load placed at L causes R_J to have a value of $\frac{3}{4}^{T}$ and so the ordinate *cl* of diagram (j) is found. The line *cr* is drawn in as for the ordinary Pratt truss and so also is *cj* because as unit load travels along stringer LJ more and more of this load finds its way directly into the abutment at J.

Member EN is redundant. The lower portion acts as a stool to the cross girder and carries the load into panel point N.

Diagonals DN and CM. The vertical component of the force in each bar equals the shear in panel LN. $S_{DN} \times \dfrac{DM}{DN} =$ vertical shear $= S_{DM}$. $\therefore S_{DN} = \dfrac{DN}{DM} S_{DM} = 1{\cdot}414 S_{DM}$, *i.e.*, the influence line for S_{DN} is similar in form to that for S_{DM}, but with the ordinates

1·414 times as large. The stress is now opposite in sign to that of S_{DM}, and therefore Fig. (k) is a vertically distorted mirror reflection of Fig. (i).

Diagonals BL and AK. For similar reasons Fig. (l) is a distorted reflection of Fig. (j).

INFLUENCE LINES FOR ROOF TRUSS

Influence lines are of little use in roof truss design because maximum stress in any member occurs when the truss is carrying its full load. Hence one stress diagram gives all the maximal stresses

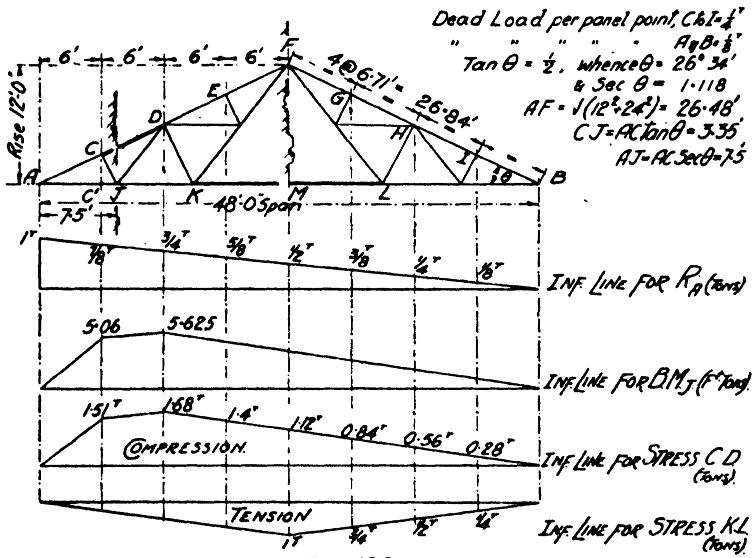


FIG. 109.

simultaneously, whereas to accomplish the same object would, for the French truss above, require no less than fourteen separate influence lines.

As an example on influence lines, however, members *CD* and *KL* will be considered in full detail, see Fig. 109.

It will facilitate the calculations to have the influence line for reaction *A* handy for reference.

B.M. at J.

1^T at C. $\quad \text{B.M.}_J = R_A \times 7{\cdot}5' - 1^T \times C'J$

$$= \tfrac{7}{8}^T \times 7{\cdot}5' - 1^T \times 1{\cdot}5' \qquad = 5{\cdot}06 \text{ ft. tons}$$

1^T at D. $B.M._J = R_A \times 7{\cdot}5'$ (no load between J and A)
$= \frac{3}{4}^T \times 7{\cdot}5'$ $= 5{\cdot}625$ ft. tons

1^T at E. ,, $= R_A \times 7{\cdot}5'$ (no load between J and A)
$= \frac{5}{8}^T \times 7{\cdot}5'$ $= 4{\cdot}69$,,

1^T at F. ,, $= \frac{1}{2}^T \times 7{\cdot}5'$, etc., *i.e.*, a straight line down to zero at B.

But the stress in $CD \times$ perpendicular lever arm to $J =$ external B.M. at J.

$\therefore$ Stress in $CD = B.M._J \div 3{\cdot}35'$.

Stress CD. Divide the foregoing bending moment values by $3{\cdot}35'$ and so obtain the stress influence line.

Stress KL. Th[illegible] moment centre is now F and the section line FM. Consi[illegible] [illegible]load between F and B.

Stress $KL=$ [illegible]	$\div$lever arm FM	$=(R_A \times AM) \div FM$	$= 2R_A$
1^T at I	Stress $KL = 2R_A$	$=2 \times \frac{1}{8}^T$	$= \frac{1}{4}^T$
1^T at H	,, $=$,,	$=2 \times \frac{1}{4}^T$	$= \frac{1}{2}^T$
1^T at G & F	,,	$=2 \times \frac{3}{8}^T$ and $2 \times \frac{1}{2}^T$	$= \frac{3}{4}^T$ and 1^T

When the unit load passes F towards E consider R_B instead of R_A for bending moment and it is seen that the influence line is a symmetrical figure.

Stresses. If the dead load per full panel point is $\frac{1}{4}^T$ then the total force acting in member CD

$$S_{CD} = \tfrac{1}{8} \times 0 + \tfrac{1}{4}(1{\cdot}51 + 1{\cdot}68 + 1{\cdot}4 \ldots + 0{\cdot}28)^T + \tfrac{1}{8} \times 0$$
$$= \tfrac{1}{4} \times 7{\cdot}39^T$$
$$= 1{\cdot}84^T$$

Member KL

$$S_{KL} = \tfrac{1}{8} \times 0 + \tfrac{1}{4}[(\tfrac{1}{4} + \tfrac{1}{2} + \tfrac{3}{4})2 + 1]^T + \tfrac{1}{8} \times 0$$
$$= 1^T.$$

CHAPTER XV

BRIDGE LOADINGS

THE three principal causes of stress in the members of a statically determinate small span bridge are: (a) Dead Load; (b) Live Load; and (c) Impact Effects.

It is upon the combined effect of these three items that the preliminary design of the bridge is based. The permissible (or working) axial tensile stress may vary from 7 to 9 tons per square inch, depending upon the impact formula use[illegible]

Afterwards the stresses due to the undernote[illegible] (d) to (i), as may apply, are calculated and added to the co[illegible]d stresses (a), (b) and (c); but for this summation the working stress is now raised by an amount varying from 15% to 25%, with the result that the sectional areas of the majority of the members, as originally computed, require no enlargement. The reason for the higher permissible stress, when dealing with the total effect of all the stresses, is that the simultaneous occurrence of the worst possible causes of stress will be but seldom in the life of the bridge. The additional items are: (d) Wind Pressure; (e) Longitudinal Forces (tractive effort and braking effect); (f) Centrifugal Effect (where a bridge is situated at a curve on the track); (g) Temperature Effect; (h) Deformation Stresses; and (i) Erection Stresses.

(a) **THE DEAD LOAD** on a bridge is a constant static load and may act with or without items (b) and (c). The first step in the design calculations of any member, main truss or secondary stringer, is to make an estimate of the probable dead weight carried. To facilitate this preliminary estimate a list of the weights of various materials is given under. Where a material on the list has alternative weights placed against it, the heavier weight should be used in the absence of more exact knowledge. A point not to be overlooked is the presence of service mains, sewers, water mains, electric cables, etc., and that these may have to be carried in the future, if not immediately on the erection of the bridge. The young designer often under-estimates the dead weight, possibly because he overlooks connections, splices, secondary bracings, lacing, batten plates, etc., in his preliminary mental picture of the job.

Drawings and data of existing bridges, whether in the office file room or in text-books and current engineering periodicals, prove extremely useful when estimating the dead weight of a bridge.

The beginner must remember that the live load is an important factor and a necessary function to be taken into account when comparing the weights of two bridges.

For preliminary work the following method gives quite satisfactory results. The cross girder of the Plate Girder Railway Bridge, page 71, will be taken in detail as the first example.

Railway Bridge Cross Girder (page 74)

Dead weight of track, floor plating, stringers, etc., but excluding unknown weight of cross girder self (see item (a), page 74) per panel point *C*, *D*, etc., of Fig. 55	=	$3{\cdot}21^{\text{T}}$
Live load and impact at $14{\cdot}17^{\text{T}}$ and $14{\cdot}52^{\text{T}}$ respectively, items (b) and (c), page 74	=	28·69
External load per panel point	=	31·90
The max. B.M. caused by the above external loads, see Fig. 55, is $2 \times 31{\cdot}90^{\text{T}} \times 8{\cdot}5' - 31{\cdot}9^{\text{T}} \times 5'$ in ft. tons	=	382·8
If an effective depth of 2′ 9″ be assumed then the total flange stress, top or bottom, is the B.M. ÷ depth of girder = 382·8 ÷ 2·75	=	139^{T}
If f_t, the working tensile stress, is 8^{T} per net square inch the *net* area of tension flange required at mid-span is $139^{\text{T}} \div 8^{\text{T}}$ per square inch	=	17·38 sq. in.
A steel bar 1″ × 1″ in cross section by 1′ 0″ long weighs		3·4 lb.
Hence weight of *net* section is 17·38 × 3·4 per ft. run	=	59 lb.
Weight of flange of *net* section for 23′ span = 59 × 23	=	1,357 lb.

Approximately the weights of the top flange, the web plate and the bottom flange of a plate girder are the same, but to make allowance for the fact that the *gross* area of the tension flange should have been taken instead of the *net* area and also to cover for the weight of rivet heads, splices, etc., multiply the weight of the net flange by 3½ instead of 3.

Approx. weight of cross girder = 1,357 × $3\frac{1}{2}$ ÷ 2,240 lb.	=	$2{\cdot}12^T$

The figure of $2{\cdot}08^T$ used in the text of item (a), page 74, was run out from the actual scantlings after the girder had been designed, showing that this easy method is reasonably accurate.

Railway Bridge Main Plate Girder (page 75)

Dead weight of track, floor, stringers and cross girders, but excluding unknown weight of self (see item (a), page 75) per panel point	=	$7{\cdot}7^T$
Using the influence line for $B.M._K$, Fig. 58, the resulting B.M. = 7·7(4·5 + 9 + 13·5)2	=	415·8 ft.^{T}

(Alternatively, consider the dead weight as being uniformly distributed at the rate of $7{\cdot}7^T \div 9'$ panel, in tons per ft. run of span. Then approx. max. B.M. $= wl^2 \div 8 = (7{\cdot}7 \div 9) \times 63^2 \div 8 = 424$ ft. tons.)

Max. B.M. due to wheel loads and impact from summary, page 84, = 1,041·5 + 676·9	=	1,718·4 ft.^{T}
Approx. total external moment	=	2,134 ft.^{T}

Assume, meantime, an effective depth of 5·7′ (*i.e.*, approx. span ÷ 11).

Total flange stress = B.M. ÷ depth = 2,134 ÷ 5·7	=	375^T
Net area of tension flange at mid-span = 375 ÷ 8	=	47 sq. in.
Equivalent weight per ft. run = 47 × 3·4	=	160 lb.
Equivalent weight for the 63′ length or span = 160 × 63 ÷ 2,240	=	$4{\cdot}5^T$
Approx. weight of complete girder = $4{\cdot}5^T$ × the constant of $3\frac{1}{2}$	=	16^T

Warren Truss Bridge, Main Girder (page 85)

For a braced girder the constant is 4 instead of $3\frac{1}{2}$, provided that the girder depth is kept to span ÷ 10. When the depth is greater than this, *e.g.*, span ÷ 7, the constant has a value round about $4\frac{1}{2}$.

Dead weight of floor per ft. run = 154 lb./sq. ft. × 10′ ÷ 2,240	=	$0{\cdot}688^T$

From page 85, dead weight of cross girder is 0·9T ÷ 2 per panel or per ft. of main truss = 0·45T ÷ 15	=	0·03
U.D.L.L. per ft. of main truss = 100 lb. × 10′ ÷ 2,240	=	0·446
(U.D.L.L. gives larger B.M. than the wheel loads + impact.)		
Total, excluding weight of self	=	1·164T
Approx. max. B.M. at mid-span = $wl^2 \div 8$ = $1{\cdot}164 \times 75^2 \div 8$	=	818 ft.T
Total flange stress at mid-span = B.M. ÷ depth = 818 ÷ 7·5	=	109T
Net area required at the specified f_t of 7T/sq. in. = 109 ÷ 7	=	15·6 sq. in.
Approx. weight of girder = 15·6 × 3·4 × 75 × constant of 4 ÷ 2,240	=	7T

WEIGHTS OF MATERIALS

Material.	Description.	lb. per cubic foot.
Aluminium	Cast-hammered.	165
Ashes and cinders	Dry, loose.	30–45
Asphalte	Rock.	136
Ballast	Stone or slag	90–100
,,	Gravel.	100–112
Basalt, trap		180
Bitumen		90
Brickwork	Ordinary.	112–125
,,	Pressed.	140
Brass	Cast-rolled.	530
Bronze		510
Cast iron		450
Cement	(Portland) loose.	90
,,	,, set.	183
Cement concrete	Aggregate, breeze.	90
,, ,,	,, brick.	112
,, ,,	,, stone.	140
,, ,,	Reinforced.	150
Clay	Dry, compact.	100
,,	Plastic.	100
Cork		15
Copper	Cast-rolled.	555

WEIGHTS OF MATERIALS—*continued*

Material.	Description.	lb. per cubic foot.
Earth	Loose and dry.	76
,,	Loose and moist	78
,,	Packed and dry.	95
,,	Packed and moist.	96
Granite		160–170
,,	Chippings.	90
,,	Mortar rubble masonry.	155
Granolithic	2 to 1.	140
Gunmetal		549
Lead		710
Limestone		160–180
,,	Broken, 35% voids.	110
,,	Mortar rubble masonry.	150
Macadam	Tar.	140
,,	Rolled.	160
Masonry	Ashlar (freestone).	140–150
,,	Rubble.	140–155
,,	Squared stone.	145–150
Paving	Granite sett.	155–160
,,	Rubber.	60
,,	Whinstone sett.	160–170
,,	Wooden block.	45–56
Phosphor bronze	And manganese bronze.	549
Pitch		73
Sand	Dry, clean.	90
,,	Wet.	115
,,	Wet, compressed.	120
Sandstone		150
,,	Mortar rubble masonry.	140
Snow	Newly fallen.	6
Steel		489·6
Tar		62–64
Timber	Usual constructional.	44
,,	Greenheart.	74
,,	Jarrah and Karri.	60
,,	Oak.	45
,,	Teak.	41
Water	Fresh (in pipes).	62·5
,,	Sea.	64
Wrought iron		480
Floorings	Steel trough.	13–56 lb./sq. ft.
,,	Jack-arch.	140 lb./cu. ft.
,,	Buckled plate $\frac{1}{4}''$ thick.	10·2 lb./sq. ft.
,,	,, ,, $\frac{5}{16}''$,,	12·75 lb./sq. ft.
,,	,, ,, $\frac{3}{8}''$,,	15·3 lb./sq. ft.
,,	,, ,, $\frac{7}{16}''$,,	17·85 lb./sq. ft.
,,	2″ asphalte on 1″ sand on 4″ concrete on $\frac{5}{16}''$ buckled plate.	85–90 lb./sq. ft.

WEIGHTS OF MATERIALS—*continued*

Material.	Description.	
Railway track .	Chairs (weight varies).	45–66 lb. each.
,, ,, .	Fish plates and bolts (weight varies).	46 lb. per set.
,, ,, .	Oak keys.	$1\frac{1}{4}$ lb. each.
,, ,, .	Timber sleepers, 2′ 6″ to 2′ 10″ c/c.	125 lb. each.
,, ,, .	Rails, bullhead.	75 lb. by 5 lb. to 100 lb./yard.
,, ,, .	Rails, flat bottom.	80 lb. by 5 lb. to 120 lb./yard.
,, ,, .	Single track of two rails, chairs, keys, sleepers, etc.	$1\frac{1}{2}$ cwt./ft. run.
Railing (hand) .	Vert. rods and horizontal flats.	15–20 lb./ft. run.
,, .	Latticed flat bars.	20–30 lb./ft. run.
Rivet heads .	Add to weight of steel sections	5%
Steel . . .	A bar 1 sq. in. in cross section weighs	3·4 lb./ft. run.
,, . . .	Or approximately	$\frac{10}{3}$ lb./ft. run.

(b) **THE LIVE LOAD** is the transient load on a structure. With a road bridge consideration has to be given to two types of live loads, viz., pedestrian traffic and vehicular traffic: with a railway bridge only the latter has to be considered.

Pedestrian Traffic. The following equivalent weights are commonly used in bridge design and no addition need be made for the dynamic application of the load. Impact effect, however, would require to be taken into account for the particular case of soldiers marching in formation across a light bridge if the order to break step were not rigidly enforced. It is to be recalled that the denser, and, therefore, the heavier, a crowd is the nearer the live load approaches the static state.

The following loadings are per square foot of road surface.

56 lb. ($\frac{1}{2}$ cwt.). This allowance represents people moving in both directions and is the loading suitable for the design of a light country foot bridge.

*84 lb. ($\frac{3}{4}$ cwt.). At this intensity there is impeded movement. The value of 84 lb. is the commonly accepted one for use in the design of pavements and possibly the roadway of a light bridge, and is taken to include for the passage across the bridge of herds of cattle, flocks of sheep, horses, etc.

* B.S.S. No. 153 for Girder Bridges—revised September, 1937—specifies 84 lb. but adds that "the Engineer may at his discretion vary this loading where special circumstances require it."

100 lb. (to 1 cwt.). This value indicates a dense slow-moving crowd on the pavements and roadway of a small span city-street bridge. A reduction is made in the intensity as the span increases, but these two figures are often taken as the equivalent uniformly distributed load for light traffic.

140 lb. is the estimated weight of a throng of people, but it is to be remembered that self-preservation will not permit of this density over a large area.

181 lb. has been attained experimentally, but only by jamming the arms against the bodies of the individuals taking part, and is a case unlikely to be encountered in practice.

Vehicular Traffic. Even apart from the present indefinite state of our knowledge as to the dynamic effect of moving loads on a structure, it is impossible to state that a bridge has a definite factor of safety against destruction. The stresses in the various members of a bridge not only depend directly upon the value of the wheel loads, but they also depend upon the spacing of the wheels, and both these items vary with almost every vehicle (road or rail) which crosses the bridge.

In the past, engineers, who had to design bridges, studied the traffic needs of their districts and issued various types of design loadings. For roads in one district the specification might demand a vehicle of 6 tons on two axles at 8′ 0″ centres and 5′ 0″ gauge, whereas bridges in the adjoining district and carrying the same road might be designed for a given road roller or a traction engine. Similarly in railway work, previous to the unification of the railways, the usual load considered was two engines of similar type coupled together preceding a uniformly distributed train load, but, naturally enough, the locomotives varied greatly as to axle loads and wheel bases.

Railway Bridges. One of the most important contributions towards standardisation of design loads for railway bridges was that made in the United States of America by Theodore Cooper (1894). Cooper's loadings are composed of double-headed trains preceding uniformly distributed train loads and the class of the loading takes its name from the weight on the driving axles ; thus, *E*—60 means 60,000 lb. (or 60 " kips ") per driving axle. Moreover, the *E*—50 loading has the same wheel spacing as the *E*—60, but the axle loads are 50,000 lb., *i.e.*, the loads are in the ratio of 6 to 5.

The British Standard Unit Loading for Railway Bridges for a single line of way, Fig. 110, is of the same form as Cooper's loadings. Although the Minister of Transport recommends 20 units (*i.e.*, 20 tons) per axle for main line bridges, 18 units seems to be a common figure for the design of main and subsidiary line bridges.

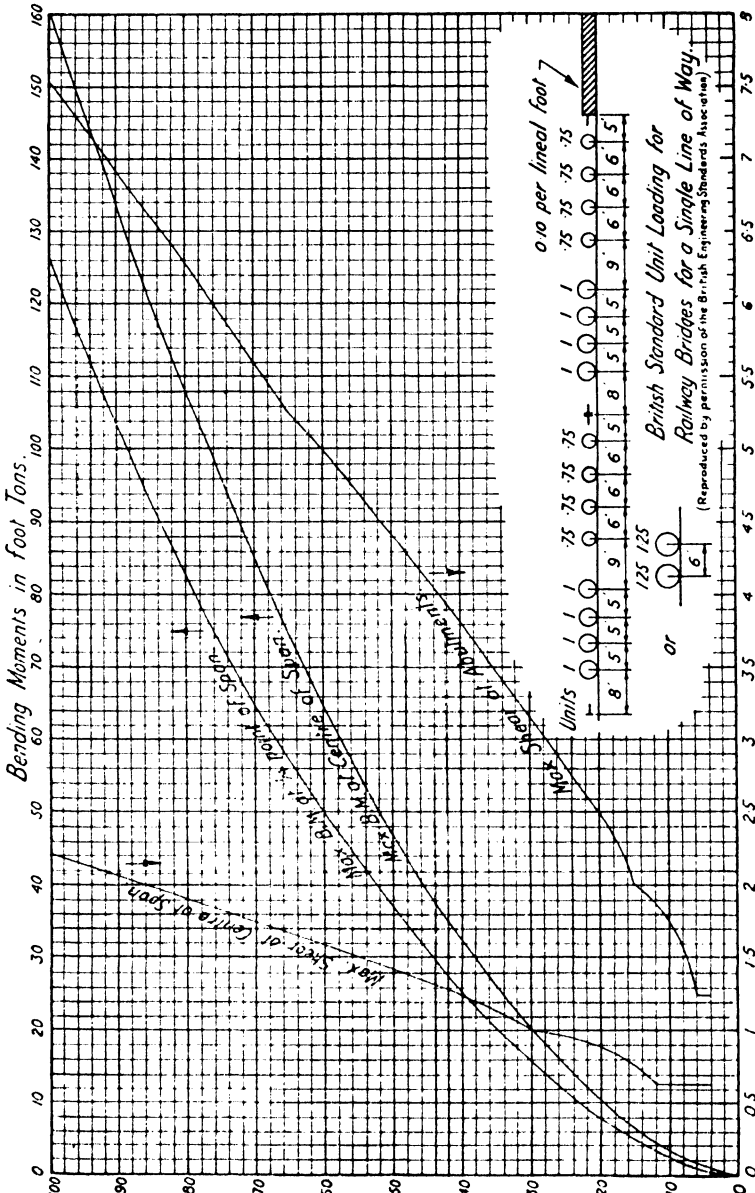

Bending Moments in Foot Tons.
Shear in Tons
Span in Feet.
Max. B.M. at ¼ point of Span
Max. B.M. at Centre of Span
Max. Shear at Abutments
Max. Shear at Centre of Span
0·10 per lineal foot
Units
or
British Standard Unit Loading for
Railway Bridges for a Single Line of Way.
(Reproduced by permission of the British Engineering Standards Association)

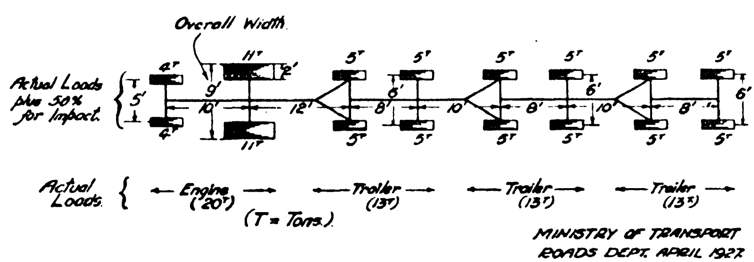
FIG. 110

For a branch line with light traffic a smaller number of units is adopted. Usually in the calculations both tracks on a bridge are assumed to be loaded with double-headed trains and both sets travelling abreast in the same direction. An actual locomotive whose driving wheels may be at 7′ 6″ centres with 20^{T} axle loads may, on a certain length of span, give approximately the same stresses as the hypothetical standard loco with 16 units, or tons, as the driving axle loads at the specified 5′ 0″ axle centres, because, as mentioned above, both axle spacing and axle loads affect maximum bending moment and maximum shear.

It may be interesting to state that the maximum axle load for British railways is in the neighbourhood of 22^{T}, in India it has reached 28^{T}, while in the United States of America axle loads of 36^{T} to 40^{T} have been used in the design of special locomotives.

Road Bridges. Here again an imaginary set of vehicles is taken as the standard loading on every 10′ 0″ width of bridge, as given by Fig. 111. Unlike the railway standard loading, however, the road

STANDARD LOAD FOR HIGHWAY BRIDGES



Note. The bridge shall be assumed to be loaded with such standard trains or parts of standard trains as will produce the maximum stress in any bridge member, provided that in any line of trains there shall not be more than one engine per 75′ 0″ of the span of the bridge, and each standard train shall occupy a width of 10′ 0″. Where the width of the carriageway exceeds a multiple of 10′ 0″, such excess shall be assumed to be loaded with a fraction of the *axle* loads of a standard train. The fraction to be used shall be the excess width in feet divided by ten.

FIG. 111

loads include a 50% allowance for impact. Denuding the road engine of its impact allowance and comparing it with the railway loading it is seen that the axle loads are $14\frac{2}{3}$ tons and 20 tons respectively.

IMPACT. In the previous pages on influence lines the wheel loads were really treated as temporary static loads. The live load system was brought to rest at several positions on its passage across the span and the corresponding stress on any particular member of the bridge was calculated for each position of the load. In effect one could regard the procedure as if the wheels were slowly and gently skidded from one position to the next, without shock or vibration to the structure. Actually the high velocity live load is applied impulsively or dynamically to the structure with a consequently increased stress on the bridge member. This increase in stress over that caused by the live load at rest is termed the impact stress.

When a load is suddenly applied to the end of a light suspended bar the resulting momentary maximum tensile stress is twice that caused by the same load applied gently, provided that the elastic limit is not exceeded and that the inertia effects are negligible. In the case of a bridge, however, the load is not applied so suddenly as in the foregoing case, neither are the inertia effects negligible, nor is a bridge one single unit; on the contrary, a bridge is composed of a very large number of separate pieces of steel averaging well over 100 for every ton of steel in the bridge. Further, one part of a bridge may help a neighbouring member to carry stress. For example, in the majority of through bridges with steel-plated floors the calculated stress in the tensile flanges is seldom attained because of the help afforded by the floor members in this relief of stress. Painstaking experiments on existing bridges under ordinary traffic conditions have definitely proved that this particular dynamical effect of load application is not by any means so large as at one time thought.

Among the other contributions to the stress increment or impact may be listed the dynamic effects of hammer-blow; flats upon wheel treads; rolling, nosing and lurching of the locomotives; rail joints and badly laid tracks.

Hammer-blow. In a steam locomotive there are masses which reciprocate in practically what is the horizontal plane, and others which revolve about the axles and crankshafts. No difficulty is encountered in completely balancing the revolving masses, it is the reciprocating masses which cause the trouble, and the balance is partially obtained by adding revolving masses to the wheels. These revolving masses on the wheels certainly make for smoother running of the locomotive, but they have a very severe effect upon the rails and hence upon the bridge. If the locomotive speed be raised sufficiently high it is possible for these balance weights to lift their wheels off the rails once in every revolution of the wheel. This hammer-blow is somewhat reduced in practice by the locomotive designer balancing only two-thirds or less of the reciprocating masses. It is the two-cylinder locomotives which create the most harm as the modern three- and four-cylinder locomotives cause very little hammer-blow, while electric locomotives are free of this particular trait.

The intensity of the hammer-blow is not proportional to the axle load; a light locomotive may produce hammer-blow effects much in excess of those given by a very much heavier locomotive.

The frequency of the hammer-blow has a maximum value between six and seven per second, and depends upon the diameter of the driving wheels and the speed of the train. Thus for six revolutions per second the driving wheels of diameter 4′ 6″, 5′ 0″, 6′ 0″ and 7′ 0″ would necessitate locomotive speeds of 58, 64, 77 and 90 miles per hour.

Resonance. Practical research has definitely shown that in small span bridges only the percussive effect of the hammer-blow is to be contended with, but in larger spans there is an additional source of stress by reason of the resonance set up. This is an approximate synchronisation between the number of hammer-blows per second given to the bridge by the driving wheels and the natural period or frequency of oscillation of the loaded bridge. A simple analogous case is that of a child on a suspended swing steadily increasing its maximum height above the ground by a series of timed impulses personally applied; and hence the previously mentioned army command to break step when crossing a bridge.

Lurching. Due to the lurching of the locomotive it has been found that the wheel pressure on the rails varies by as much as 40% of the static load value. The greater the locomotive speed and the rougher the track the greater is the lurching effect.

Locomotive Springs. If these are friction locked the locomotive participates in the vertical swaying motion of the bridge, but if they are free to act they may exert a considerable damping action on the bridge oscillations.

Flats on the driving wheels have been known to cause considerable damage to the track. They may cause hammer-blows just as serious as those created by the partial balancing of the reciprocating masses of the engine.

Other causes of impact, but to a lesser extent, are the rail joints and the nosing of the locomotive. In long spans the first are inevitable, but in short spans the joints can either be kept off the bridge or arranged to be near the ends of the span. Still another method is to weld the rail joints, which occur on the span, thereby making the rail a continuous unit on the bridge.*

The impact stresses are not wholly dependent upon the external

* Unbroken lengths of 120′ 0″ are now being used on some British railways, while, by welding the constituent parts together, continuous lengths up to 2,700 ft. have been used on the main railways of Germany.

causes listed above, for they are also contingent upon the internal features of the structure itself. Thus the type of main girders adopted has an effect, and so also have the bearings, the piers or abutments, and the floor. As an example of the latter it need only be mentioned that a ballasted floor has a beneficial effect because of its damping and cushioning qualities.

Rail Impact Formulæ. The impact stresses have to be taken into account in the calculations either directly or indirectly. In the direct method empirical formulæ are used to obtain arithmetical values for the impact stresses, either as separate items by themselves or in conjunction with the live load stresses, while in the indirect method the working stress is varied in value according to the intensity of impact the individual members are thought to carry.

As an example of the latter method, which is now falling into desuetude, a through plate girder railway bridge would have working tensile stresses of 5·5, 5·0 and 4·5 tons per square inch for the 60′ 0″ span main girders, the 23′ 0″ span plate cross girders, and the 10′ 0″ span stringers, respectively.

In the direct method the first and probably the best-known formula is the Pencoyd Formula originated in 1887 by C. C. Schneider, Pres. Am. Soc. C.E., for the Pencoyd Iron Company of America.

It was $I = \dfrac{300}{L + 300}$

where I = coefficient, or factor, of impact by which the live load stresses are multiplied,

and L = loaded length of the span which gives max. live load stress in the member.

If the span is zero then the impact coefficient is unity, *i.e.*, the case of the suspended bar with instantaneous loading. As a further example consider the case of a railway bridge of 200 ft. span and assume that the dead load stress on one of the tension flange members is 250^{T} and the max. live load stress is 500^{T} (*i.e.*, with the live loads at rest). The impact coefficient or factor is $I = \dfrac{300}{200 + 300} = 0{\cdot}6$, since max. B.M. (and flange stress) occurs when the load covers all the span, and the value of the impact stress is $0{\cdot}6\text{L.L.} = 0{\cdot}6 \times 500^{T} = 300^{T}$.

Hence the design stresses for the member are :—

$$\begin{aligned} \text{D.L.} &= 250^{T} \\ \text{L.L.} &= 500^{T} \\ I &= 300^{T} \\ \text{Total} &= \underline{\underline{1{,}050^{T}}} \end{aligned}$$

If $f_t = 8^T$ per square inch then the net tensile area required

$$= 1{,}050 \div 8$$
$$= 131{\cdot}25 \text{ sq. in.}$$

For railway bridges the Pencoyd type of formula has been greatly favoured, and it appears in amended forms in many specifications, thus :—

Date.	Authority.	Formula.	Coefficient *I* for a span of :—	
			Unity.	200′.
1887	Pencoyd . .	$I = \dfrac{300}{L + 300}$	1	0·6
1896	Waddell . .	$I = \dfrac{400}{L + 500}$	0·8	0·57
1916	Waddell . .	$I = \dfrac{165}{nL + 150}$*	1·09	0·47
1923	B.S.S.† . .	$I = \dfrac{120}{\left(\dfrac{n+1}{2}\right)L + 90}$	(1·15)‡	0·41
1925	Indian Ry. Board	$I = \dfrac{65}{L + 45}$	(1·00)§	0·27

* Where n = number of tracks ; taken as 1 in the examples.
‡ Value is 1·31, but is not to exceed 1·15.
§ Value is 1·41, but is not to exceed 1·00.

It will be observed in the case of large spans that the allowance for impact has steadily decreased since this type of formula was originally specified.

The American Railway Engineering Association specifies a formula of somewhat different form ; it is

$$I = \frac{300}{300 + \dfrac{L^2}{100}}$$

where, as hitherto, *I* and *L* are the impact coefficient and the loaded length, respectively.

One objection offered to the foregoing formulæ is that the calculated impact stresses depend upon axle weights and no apparent direct provision is made for the other kinds of impact stresses which have their sources elsewhere than at the axle loads.

† In place of one single formula to embrace all impact effects the revised B.S.S. of 1937 gives a series of formulæ, one for each of the various causes mentioned in the text. Also see footnotes, pages 73 and 181.

Road Impact Formulæ. The impact effects are not nearly so severe on road bridges. Two formulæ of the Pencoyd type in common use in America are :—

(a) $I = \dfrac{100}{L + 300}$

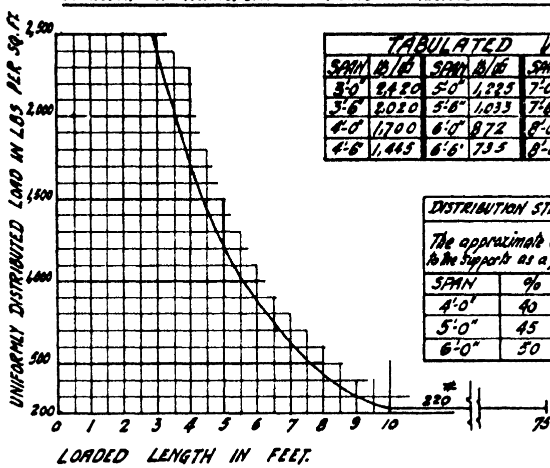
(b) $I = \dfrac{50}{L + 125}$ (Am. Assoc. Highway Officials)

where I and L still have the meanings previously stated.

—— STANDARD LOAD FOR HIGHWAY BRIDGES. ——

EQUIVALENT LOADING CURVE LOADED LENGTHS UP TO 10'-0"

— MINISTRY OF TRANSPORT — ROADS DEPARTMENT — SEPTEMBER 1931. —



TABULATED VALUES							
SPAN	LB/□'	SPAN	LB/□'	SPAN	LB/□'	SPAN	LB/□'
3'-0"	2,420	5'-0"	1,225	7'-0"	625	9'-0"	314
3'-6"	2,020	5'-6"	1,033	7'-6"	525	9'-6"	265
4'-0"	1,700	6'-0"	872	8'-0"	444	10'-0" to 75'-0"	220
4'-6"	1,445	6'-6"	735	8'-6"	374		

DISTRIBUTION STEEL IN SLABS.			
The approximate amounts of steel parallel to the supports as a % of main reinforcement			
SPAN	%	SPAN	%
4'-0"	40	8'-0"	55
5'-0"	45	10'-0"	60
6'-0"	50	Max. 0·50" per ft.	

The uniformly distributed load applicable to the " loaded length " of the bridge or member in question is selected from the curve or table.

The " loaded length " is the length of member loaded in order to produce the most severe stresses. In a freely supported span the " loaded length " would thus be (a) for bending moment, the full span ; (b) for shear at the support, the full span ; (c) for shear at intermediate points, from this point to the farther support.

In arches and continuous spans the " loaded length " can be taken from the influence line curves.

The live load to be used consists of two items : (1) The uniformly distributed load which varies with the loaded length, and which represents the ordinary axle loads of the M.T. standard train, perfectly distributed ; (2) an invariable knife-edge load of 2,700 lb. per ft. of width applied at the section where it will, when combined with the uniformly distributed load, be most effective, *i.e.*, in a freely supported span : (a) for bending moment at midspan, at midspan point ; (b) for shear at the support, at the support ; (c) for shear at any section, at the section.

FIG. 112 (a)

For spans of unity and 200 ft. the values of the multiplying factor I are, respectively, (a) 0·33 and 0·2 ; (b) 0·4 and 0·15.

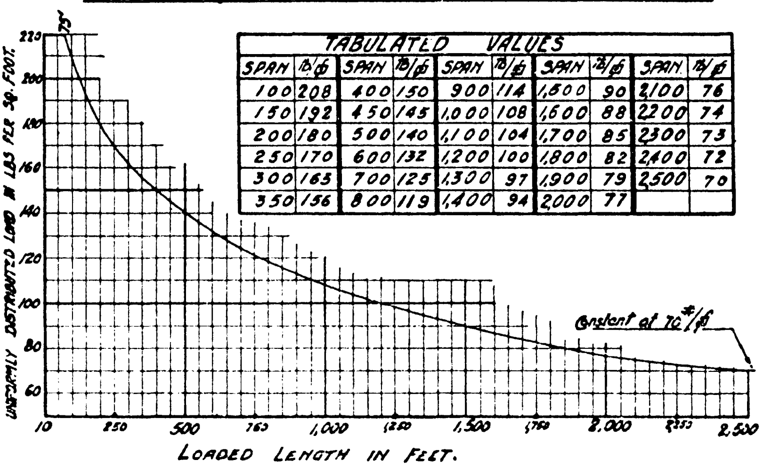
Many specifications took advantage of this comparatively small range of values by simply using a constant impact factor of 25% or

30%. The Warren road bridge of page 85 is an example of this method, and the impact factor used there is 0·3 or 30%. The corresponding impact factors by the above formulæ for this 75′ 0″ span bridge are (a) 0·27, and (b) 0·25. The diagrams of Fig. 68 show how these impact stresses are dealt with in the stress summations.

—STANDARD LOAD FOR HIGHWAY BRIDGES.—

EQUIVALENT LOADING CURVE LOADED LENGTHS FROM 10′0″ UP TO 2,500′0″

— MINISTRY OF TRANSPORT — ROADS DEPARTMENT — SEPTEMBER 1931.—



TABULATED VALUES

SPAN	lb/□	SPAN	lb/□	SPAN	lb/□	SPAN	lb/□	SPAN	lb/□
100	208	400	150	900	114	1,500	90	2,100	76
150	192	450	145	1,000	108	1,600	88	2200	74
200	180	500	140	1,100	104	1,700	85	2300	73
250	170	600	132	1,200	100	1,800	82	2,400	72
300	165	700	125	1,300	97	1,900	79	2,500	70
350	156	800	119	1,400	94	2,000	77		

(*Continued*)

This knife-edge load represents the excess in the M.T. standard train of the heavy axle over the other axles, this excess being undistributed (except laterally as already assumed).

In spans of less than 10′ (*i.e.*, less than the axle spacing) the concentration serves to counteract the over-dispersion of the distributed load.

In slabs the knife-edge load of 2,700 lb. per ft. of width is taken as acting parallel to the supporting members, irrespective of the direction in which the slab spans.

In longitudinal girders, stringers, etc., this concentrated loading is taken as acting transversely to them (*i.e.*, parallel with their supports).

In transverse beams the concentrated loading is taken as acting in line with them (*i.e.*, 2,700 lb. per ft. run of beam).

If longitudinal or transverse members are spaced more closely than at 5′ centres, the live load allocated to them shall be that calculated on a 5′ wide strip. With wider spacing this strip will be equal to the girder spacing.

In all cases, irrespective of span length, one knife-edge load of 2,700 lb. per ft. of width is taken as acting in conjunction with the uniform distributed load appropriate to the span or "loaded length."

FIG. 112 (b)

Formula (b), viz., $I = 50/(L + 125)$, was used in the design calculations for the San Francisco–Oakland Bay Bridge for the impact caused by road vehicles and 13% was added as impact to the stresses produced by electric trains.

A still simpler manner of allowing for impact effect is that of the Ministry of Transport, see Fig. 111,* on page 173. The axle loads of the standard vehicles are all increased by 50% to allow for impact, and in consequence no separate items for impact stresses occur in the stress calculations. The heavy axles are taken at $14\frac{2}{3}$^T net and 22^T gross, inclusive of 50% for impact. But this figure of 22^T total could, from the point of view of some specifications, be regarded as 17^T net, plus an impact allowance of 30% (5·1^T), again giving the total of 22^T.

Some engineers regard this 22^T axle as over heavy for present-day traffic requirements, but as no one can foresee the loads of the near future the use of the standard loading may ultimately save renewals and strengthenings. (On the other hand, if the new working tensile stress of 9^T per square inch be used in place of the previous 8^T per square inch this is equivalent to reducing the axle loads by approximately 11%.) It is worth remembering that an increase of, say, 10% on the initial calculated stress does not necessarily entail a 10% increase in the cost of the finished structure; the extra stress may necessitate an increase of 10% in the cost of the raw material, but possibly very little addition to the fabrication cost. Again there are other considerations which govern the selection of the scantlings of a bridge member such as rigidity, minimum sizes, mercantile sizes, appearance, maintenance, etc. That due attention should be paid to future requirements, subject, of course, to present-day economics, is borne out by the fact that some of the railway bridges in the industrial parts of the United States of America have been rebuilt no less than four times.

As with the railway impact formulæ, criticism is often levelled against the impact allowance and the form for its provision in the

* The Ministry of Transport has issued an equivalent loading curve (Fig. 112) for the standard train of Fig. 111. The accompanying notes issued state that: "Impact is included and therefore need not be considered separately. Explanation is given for its use in applying to various bridge members. It is thought that the bridge designer's work will be simplified by the use of this equivalent loading as it standardises the interpretation and method of application of the standard load, leaving a minimum of calculation for design.

"On bridges whose span exceeds 75 feet a reduction has been made in the intensity of loading as compared with the standard train. This is to allow partly for the lower average weight of vehicles in the larger group, and partly for the lessening effect of impact on the longer spans.

"For spans below 10 feet the equivalent loading makes allowance for bending moments in both directions of which only the main bending moment need be calculated. To allow for continuity in the deck slab or other type of flooring a 0·8 factor should be applied to the free bending moment obtained at midspan, and the same bending moment is assumed to operate at the support. In end panels and at the first interior support no deduction should be made.

"This equivalent loading (or the standard train on which it is based) is the minimum loading recognised by the Ministry of Transport for highway bridges."

M. of T. standard loading for road bridges. The amount of practical research and mathematical investigation which has been devoted to the subject of impact is phenomenal, but no exact solution has been, nor probably ever will be, proposed; all that can be hoped for is a compromise, and the simpler form this compromise takes the cheaper will be the bridge.*

In the days of the *fratres pontifices*,† when a bridge failed, a second, nay, on occasion, phœnix-like, a third, attempt had to be made before stability was achieved. Undoubtedly much water has passed under these bridges since then, *i.e.*, if they are still standing, but even in this scientific age the design of bridges is not by any means a pure science, it is also an art.

* An indication of the difficulties encountered in arriving at a suitable formula for impact is furnished by the statement in the Foreword to the B.S.S. No. 153 (September, 1937). Herein it states that although the provisional formula of the 1923 edition should be discontinued, "agreement could not be obtained upon any one method which should be recommended in its place." (Also see footnotes, pages 73 and 177.)

† *Fratres pontifices* (L. Bridge building brotherhood), a lay or "third order" of monks, *i.e.*, a guild, whose duty it was in mediæval times to safeguard fords, build bridges, and look after the safety of travellers. Peter of Colechurch (d. 1205), who built the famous London Bridge—a stone structure which stood for 600 years—was a member of this order.

CHAPTER XVI

INFLUENCE LINES FOR DEFLECTIONS

THE preliminary paragraphs of this section will be devoted to a brief explanation of graphical integration. This method is an extremely useful and accurate one for obtaining the elastic curve, *i.e.*, the deflection curve of the neutral surface of a loaded beam or girder, no matter what type of loading has been employed. It requires no knowledge of the integral calculus in its application,

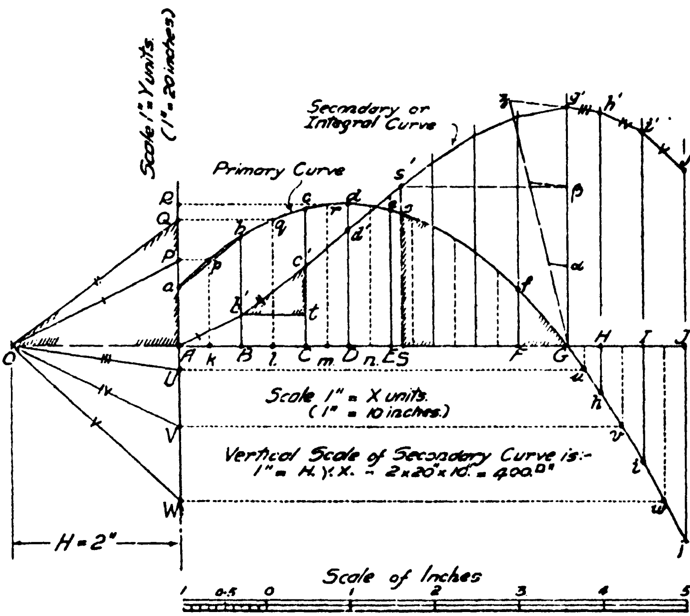


FIG. 113

and for complicated load distribution it is distinctly a quicker method than the mathematical one of evolving the equation of the elastic line. If done with ordinary care the accuracy of the results is of a high order, as will be seen from the mathematical results given alongside the graphical ones. The graphical results have been given as they were found and have not been tampered with in any way. In most cases 1 in. scales have been shown on the drawings to serve as checks on the reduced reproductions.

Given the curve *abcdfhiwj* of Fig. 113, to find the area contained between this curve and the X and Y axes.

Construction. Divide the base line of the curve, or X axis, into several parts, AB, BC, CD, etc., usually made equal for convenience and scaling about $\frac{1}{2}''$ or $\frac{1}{4}''$ each, depending on the size of the drawing. FG and GH are odd lengths made to suit the curve and so simplify the drawing, while lengths AB and BC are made larger than the others in this case in order to show the construction clearly.

Draw the mid-ordinates of all the foregoing divisions, kp, lq, mr, etc., and project points p, q, r, etc., horizontally across to the Y axis so as to give the corresponding points P, Q, R, etc.

Take any pole, or point, O, such that the " polar distance " OA, universally known as H, is, preferably, an integral number of inches, *e.g.*, 1″, 2″, 3″, etc., although lengths such as $1\frac{1}{2}''$, $2\frac{1}{2}''$, etc., will be found useful at times. Join O to P, O to Q, O to R, etc., and draw Ab' parallel to OP, $b'c'$ parallel to OQ, $c'd'$ parallel to OR, and so on, thus giving the secondary or integral curve.

Proof. Now if the divisions AB, BC, etc., are small the chords and the arcs will be coincident, *i.e.*, arc ab becomes practically a straight line, and all such figures as $ABba$ become trapeziums.

Area of trapezium $bcCB = lq.BC = AQ.b't$ . . . . (a)

By construction the triangles AOQ and $tb'c'$ are similar ;

$$\therefore \frac{AQ}{OA} = \frac{tc'}{b't} \text{ (Euclid : Book VI, prop. 4)}$$

$$\therefore AQ.b't = OA.tc'$$

i.e., by (a) area $bcCB = OA.tc' = H.tc'$ . . . . (b)

If, for the primary curve, the vertical scale is $1'' = Y$ units and the horizontal scale is $1'' = X$ units, then

$$\begin{aligned}\text{the } \textit{true} \text{ area of } bcCB = (lq.Y)(BC.X) &= lq.BC.YX \\ &= H.tc'.YX \text{ by (a) and (b)} \\ &= HYX.tc' \quad . \;. \;. \quad \text{(c)}\end{aligned}$$

The vertical scale of the secondary curve is thus $1'' = HYX$, and if an ordinate scales tc' actual inches then the area represented by tc' is $HYX.tc'$.

Similarly, true area of $abBA = HYX.Bb'$
,, ,, $acCA = HYX.Cc'$
,, ,, $adGDA = HYX.Gg'$; *i.e.*, in all cases HYX times the ordinate at the point to the secondary curve.

In the example the vertical scale Y was $1'' = 20''$
the horizontal scale X was $1'' = 10''$
and H was $1'' = 2$ (simply a number)

then the vertical scale to the integral curve is $1'' = HYX$
$= 2 \times 20'' \times 10''$
$= 400$ sq. in.

As the original drawing is reduced in size for reproduction the original inch scale is drawn in, and measuring Gg' with dividers, and reading the dimension on the inch scale, Gg' is found to be 2·83″ long.

Hence area of figure $adGDA$ $= Gg' \times HYX$
$= 2{\cdot}83 \times 400 = 1{,}132$ sq. in.

The equation of the primary curve is $y = -\frac{x^2}{20} + 2x + 14$, and, as a check on the accuracy of the graphical integration, the result by calculation is :

$$\int_{x=0}^{x=46\cdot07} ydx = \left[-\frac{x^3}{60} + x^2 + 14x \right]_0^{46\cdot07}$$

$$= -\frac{97{,}783\cdot6}{60} + 2{,}122\cdot5 + 645$$

$$= 1{,}138 \text{ sq. in.}$$

Negative Areas. The primary curve crosses the X axis at G and the area between the curve and the X axis now becomes negative. The construction of the secondary curve still follows the method previously explained, viz., project the ends of the mid-ordinates across to the Y axis, then join to pole O and draw parallels to the rays from O.

Total area of curve $adGjJHDA = HYX.Jj' = 400 \times 2{\cdot}09$
$= 836$ sq. in.

By calculation it is $\int_0^{60} ydx = \left[-\frac{x^3}{60} + x^2 + 14x \right]_0^{60}$

$$= -3{,}600 + 3{,}600 + 840 = 840 \text{ sq. in.}$$

Deduction. Although of no direct bearing upon the subject under discussion, the following deduction is of use in engineering, viz., to divide the given area $acGCA$ into any number of equal areas.

Construction. Let the required number be three, then divide any line Gz, from G, into three equal parts, and by means of parallels divide Gg' similarly. Project β horizontally along until it cuts the secondary curve at s', and from this point drop the vertical $s'S$. The shaded area $SsfG$ is one-third of the given area, because $\beta g'$, the length of ordinate belonging to the shaded area, is one-third of Gg'.

Relationship between Load, Shear, Bending Moment, Slope and Deflection

Shear. If the intensity of the loading on a beam be w tons per inch run, then the summation of the loading between two points on the span is the change in shear, or as mathematically stated in text-books on strength of materials,

$$\int_{x_1}^{x_2} wdx = \text{change in shearing force } V;$$

i.e., graphically sum the area of the load curve and so obtain the shearing force diagram.

Bending Moment. Similarly, if the shear curve be graphically summed, or integrated, the summation curve is the bending moment diagram,

$$i.e., \quad \int_{x_1}^{x_2} Vdx = \text{change in bending moment } M.$$

The bending moment and shear which act upon a beam are independent of the beam's geometrical properties of shape and depth, and would have the same numerical effect on whatever size of rolled steel joist were used to span the opening, neglecting, of course, the differences in the dead weights of the beams.

It is not so, however, with beam curvature, and the deflection or sag for these, obviously, depend upon the elastic nature of the material, or modulus E, and the geometrical properties of the beam, or moment of inertia I.

The Value for Young's (or "Stretch") Modulus, E. For a simply supported girder carrying a uniformly distributed total load W in tons the elastic deflection in inches at mid-span, due to bending, is given by the formula $\Delta = \frac{5Wl^3}{384EI}$, where l and I are in inch units and E is in tons per square inch.

It is found when a girder is built up of separate plates and angles riveted together, or is a braced structure, that the resulting deflection is larger than that occurring when the beam or girder is a single rolled section of equal moment of inertia. This additional deflection, caused by the "give" or play in the riveted structure, is taken cognisance of by lowering the accepted value of E from 13,000 tons per square inch to 12,000 tons per square inch in the denominator of the above type of formula.

Slope. The graph or curve whose vertical ordinates represent the slope of a beam in radians is thus obtained by integrating the bending moment curve and dividing the result by EI.

$$i.e., \int_{x_1}^{x_2} \frac{M}{EI} dx = \text{change in slope, or } \phi, \text{ in radians.}$$

If the quantities E and I are constant throughout the span, as they usually are, then

$$\frac{1}{EI}\int_{x_1}^{x_2} M dx = \text{change in } \phi$$

Deflection. The deflection in turn is the summation of the ϕ curve, hence

$$\int_{x_1}^{x_2} \phi dx = \text{change in sag or deflection } \Delta.$$

$$i.e., \Delta = \int \phi dx = \int\int \frac{M}{EI} dx dx.$$

Alternatively: If the bending moment diagram $\div EI$ be considered as a load curve the secondary (equivalent to shear) curve is the ϕ curve while the third and final integral curve is the curve of deflection (equivalent to the B.M. curve). This loading of the beam with the B.M. diagram divided by EI is known as the method of elastic weights.

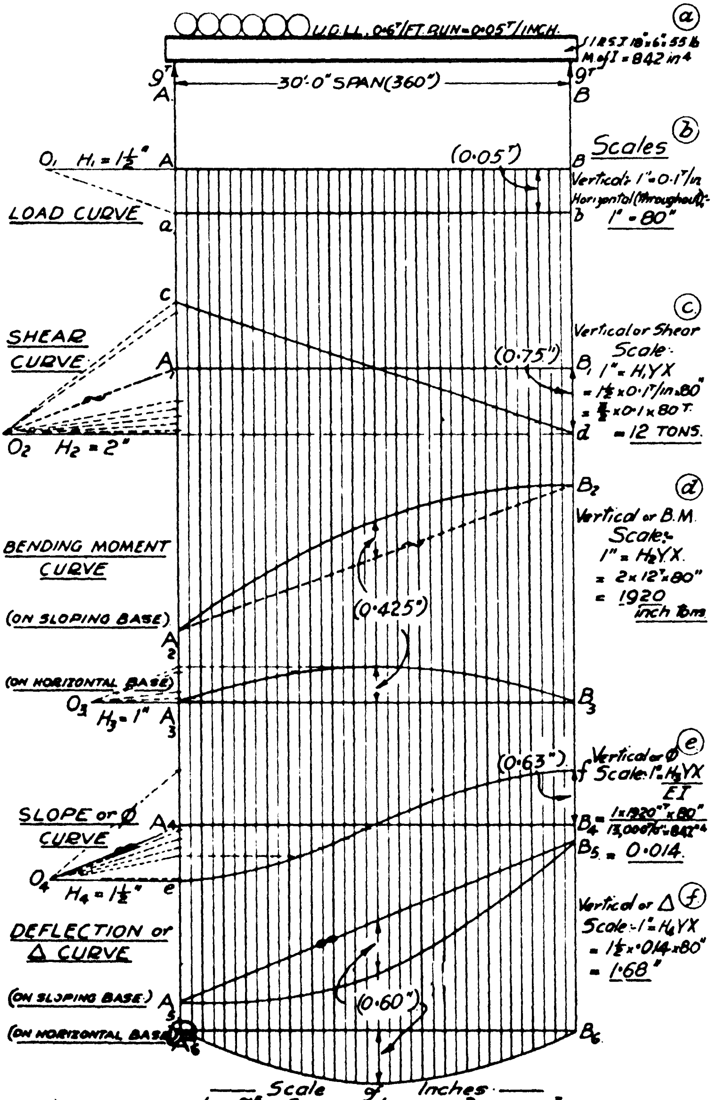
Continuous Integration from Load Curve to Deflection Curve, Fig. 114

(a) Required to find the deflection of an 18″ × 6″ × 55 lb. R.S.J. of 30′ 0″ span carrying a uniformly distributed load of $0{\cdot}6^{T}$ per ft. run of span.

Change all dimensions into ton and inch units, since E is in tons per square inch and the moment of inertia I is 842 inch units to the fourth power, *i.e.*, span 30′ 0″ = 360″ and

$0{\cdot}6^{T}$/foot = $0{\cdot}05^{T}$/inch.

(b) Since the downward load is of constant intensity, ab will be parallel to AB at a depth of 0·5″ or $0{\cdot}05^{T}$, because the vertical scale is 1″ = $0{\cdot}1^{T}$. Polar distance H_1 is $1\frac{1}{2}$″, and since there is only one



(a)
U.D.L.L. 0·6T/FT. RUN = 0·05T/INCH.
9T A
30'-0" SPAN (360")
9T B
(b)
Scales
O1 H1 = 1½" A
(0·05T)
LOAD CURVE
Vertical: 1" = 0·1T/in
Horizontal (throughout): 1" = 80"
(c)
SHEAR CURVE
(0·75")
Vertical or Shear Scale: 1" = H1YX = 1½ x 0·1T/in. 80" = 12 TONS.
O2 H2 = 2"
(d)
BENDING MOMENT CURVE
(ON SLOPING BASE)
(ON HORIZONTAL BASE)
(0·425")
Vertical or B.M. Scale: 1" = H2YX = 2 x 12T x 80" = 1920 inch tons
O3 H3 = 1"
(e)
(0·63")
SLOPE or Φ CURVE
Vertical or Φ Scale: 1" = H3YX / EI = 0·014
O4 H4 = 1½"
(f)
DEFLECTION or Δ CURVE
Vertical or Δ Scale: 1" = H4YX = 1½ x ·014 x 80" = 1·68"
(ON SLOPING BASE)
(ON HORIZONTAL BASE)
(0·60")
Scale Inches

FIG. 114

ray O_1a the secondary, or shear, curve *cd* is a straight sloping line.

(c) For clearness this shear curve was drawn under and not superimposed on the primary curve as was done in the proof.

The base line A_1B_1 has now to be placed in position, which is a simple matter with this particular example because the end reactions are equal to each other, and hence $A_1c = B_1d = 9^T$ or $\frac{3}{4}''$ actual height with a scale of $1'' = 12^T$. Recall that the scale is always $1'' = HYX$ where H, Y and X refer to the immediately preceding curve.

Had the original load intensity curve been unsymmetrical, however, some difficulty would have been encountered in obtaining the arithmetical values of the end reactions at A and B and recourse would have been made to the graphical method outlined under. (d) A polar distance of $H_2 = 2''$ is adopted primarily that these distances can be varied at will. A small H results in a deep integral curve while a large H distance gives a shallow integral curve.

Project the ends of the mid-ordinates to the shear curve across to the Y axis and join to the pole O_2. Commence anywhere, as at A_2, and by drawing parallels to the rays the curve A_2B_2 of (d) is obtained. Since there is no bending moment at the supports of a simple beam the base line must be the straight line joining A_2 to B_2.

In diagram (c) if the ray O_2A_1 be drawn parallel to the closing line A_2B_2 of diagram (d) then the base line of the shear curve is the horizontal line A_1B_1 drawn from point A_1 at the end of the ray. This follows because a ray in a force polygon drawn parallel to the closing line or link of the corresponding funicular polygon gives the reactions on the load line.

It is usual to show a positive bending moment curve on a horizontal base line such as A_3B_3. This curve is obtained from curve A_2B_2 by simply measuring and transferring the various vertical heights by dividers or by markings on a strip of paper. Calculation gives the max. B.M. as $wl^2 \div 8 = 0{\cdot}05^T/\text{inch} \times 360'' \times 360'' \div 8 = 810$ inch tons, and the drawing gives it as $0{\cdot}425'' \times 1{,}920 = 816$ inch tons, or $0{\cdot}7\%$ error.

(e) Now integrate the B.M. curve and obtain the slope curve to scale. The scale of the slope curve is now H_3YX divided by the elastic constants of EI for the beam. Working in inch and ton units, the polar distance being a number, the resulting scale is $1'' = 0{\cdot}014$, a number, or radians.

The formula for the slope at the supports of a beam with uniformly distributed loading is

$$\frac{wl^3}{24EI} = \frac{0{\cdot}05 \times 360 \times 360 \times 360}{24 \times 13{,}000 \times 842} = 0{\cdot}00888$$

while the result by drawing is vertical intercept of 0·63″ × 0·014 = 0·00882; a difference of 0·67%.

As with the shear curve the base line in this particular case can be positioned from symmetry at halfway up, *i.e.*, A_4e must equal B_4f. With unsymmetrical load distribution this is not so and the base line A_4B_4 is fixed by drawing O_4A_4 parallel to the closing line A_5B_5 of the succeeding figure (f), and the required base line is the horizontal A_4B_4 from A_4.

(f) Base line A_4B_4 of (e) may not be known at this stage, so take the pole O_4 horizontally opposite to e and integrate the slope curve to give the deflection curve A_5B_5 on a sloping base line. The base line must be A_5B_5 because there is no deflection at either support, *i.e.*, end ordinates have zero values. A_6B_6 is this curve on a horizontal base line having the same vertical ordinates, or intercepts, as A_5B_5.

The standard formula for max. deflection at mid-span is $\frac{5Wl^3}{384EI}$

i.e., $\Delta = \frac{5 \times (0{\cdot}05 \times 360) \times 360 \times 360 \times 360}{384 \times 13{,}000 \times 842} = 0{\cdot}999''$

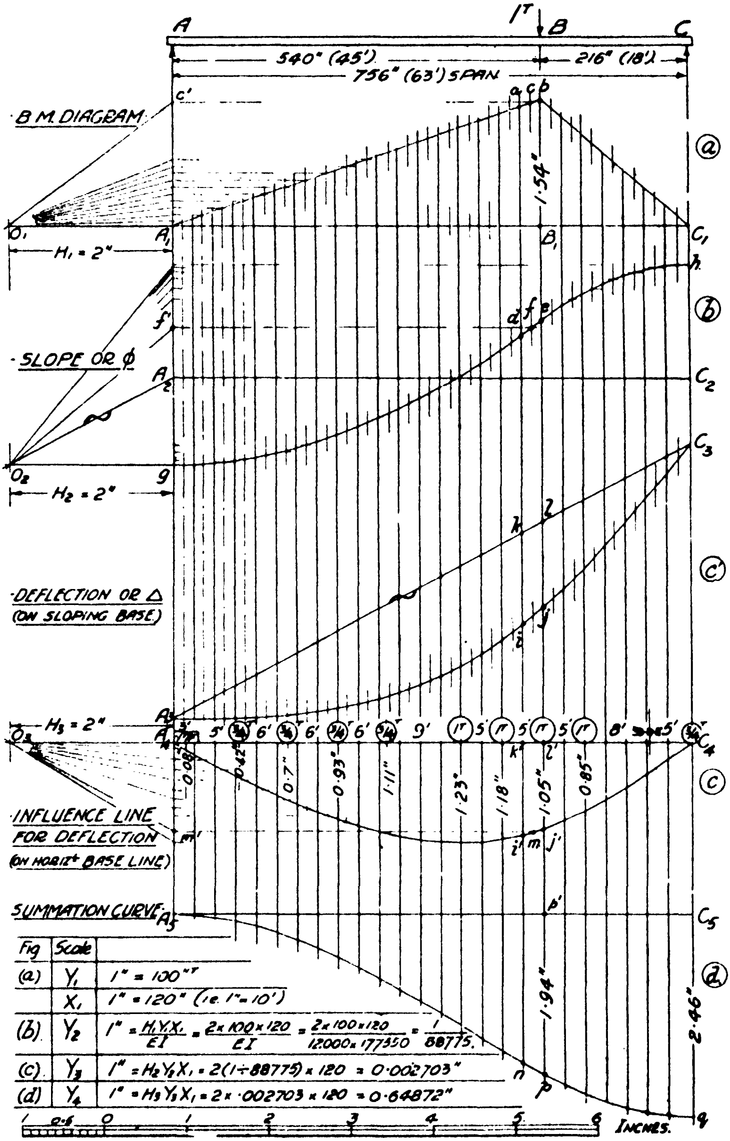
By drawing = 0·6″ vertical intercept × 1·68 = 1·008″, an error of 0·9%.

Maxima and Minima. It will be observed that when a curve changes from plus to minus by cutting the X axis then the succeeding integral curve has a maximum value directly under the point of cutting, *i.e.*, maximum bending moment occurs at the point of zero shear and maximum deflection at the point where the elastic line of the beam is horizontal or has zero slope.

Deflection Caused by Concentrated Loads, Fig. 115

When a girder carries concentrated loads alone, or in addition to distributed loading, the first curve which can be integrated is the shear curve. However, it is recommended that wherever possible the primary curve should be the bending moment one, thereby contributing to the accuracy of the deflection curve by eliminating one or two graphical integrations.

Fig. 115 shows a girder of 63′ 0″ span supporting a one-ton load at 18′ 0″ from one support. The first curve drawn, (a), is the bending moment and by continuous graphical integration follow the slope curve, (b), and then the curve of deflection, (c'). The last curve is



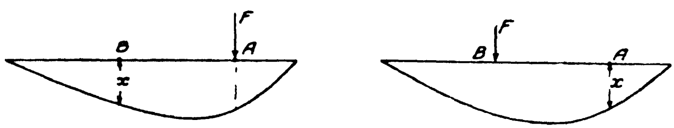
A
B
C
540" (45')
216" (18')
756" (63') SPAN
B.M. DIAGRAM
SLOPE OR φ
DEFLECTION OR Δ
(ON SLOPING BASE)
INFLUENCE LINE
FOR DEFLECTION
(ON HORIZ^L BASE LINE)
SUMMATION CURVE
H₁ = 2"
H₂ = 2"
H₃ = 2"
1.54"
1.94"
2.46"
0.08"
0.42"
0.7"
0.93"
1.11"
1.23"
1.18"
1.05"
0.85"
Fig
Scale
(a) Y₁ 1" = 100"ᵀ
X₁ 1" = 120" (i.e. 1" = 10')
(b) Y₂ 1" = H₁Y₁X₁/EI = 2×100×120/EI = 2×100×120/(12000×177350) = 1/88775.
(c) Y₃ 1" = H₂Y₂X₁ = 2(1÷88775)×120 = 0.002703"
(d) Y₄ 1" = H₃Y₃X₁ = 2×.002703×120 = 0.64872"
INCHES.

FIG. 115.

redrawn on a horizontal base line (*c*) and the vertical intercepts $i'k' = ik$ and $j'l' = jl$, etc. In this example a constant value of 2″ for H is used.

The curves (*c*) and (*b*) are, of course, really smooth flowing curves, but they are drawn for graphical reasons as a series of small straight lines. If desired the curve of deflection (*c*) may be drawn in with the aid of French curves. This curve (*c*), as will now be explained, is the influence line for deflection for the point B on the girder AC. The mathematical equation of this curve is derived on page 197.

Maxwell's Law* of Reciprocal Deflections. A load F placed at any point A on a girder (Fig. 116) will cause another point B to deflect by an amount x, and if the load be transferred from the first point A to the second point B the deflection which will now occur at A is also x.



(a) FIG 116 (b)

Proof. Gradually apply a load F_a to a beam, Fig. 117, from zero to full value and so cause a gradually increasing deflection under the load. If the deflection is y under the load, when fully applied, then the work done, W, is the product of the average force by the distance travelled, viz., $W = \frac{1}{2}F_a.y$.

If, with the load F_a in position, the beam is further deflected by some other external agent, and the load F_a is caused to sink or deflect an additional amount δy, then the additional work, δW, done by F_a is the product of the now unvarying force F_a by the distance, *i.e.*, $\delta W = F_a \delta y$.

In Fig. 117 (a) gradually apply load F_a from zero to full value. The work done is $W = \frac{1}{2}F_a y$ . . . . (1)

Next apply at B an additional and equal load F_b, also gradually from zero to full value. This will further deflect the beam as in diagram (b).

The additional work done on the beam is now :—

Average load of $\frac{1}{2}F_b \times$ distance δx $= \frac{1}{2}F_b.\delta x$

plus the work done at A of constant load

$$F_a \times \delta y = F_a.\delta y$$

i.e., the additional work done $= \frac{1}{2}F_b\delta x + F_a\delta y$ (2)

* James Clerk Maxwell (1831–79), of Penicuik, near Edinburgh. Cavendish Professor of Experimental Physics at Cambridge University.

The total work done in arriving at the final loading of (b) is (1) + (2), *i.e.*, total $W = \frac{1}{2}F_a y + \frac{1}{2}F_b \delta x + F_a \delta y$ . . . . (3)

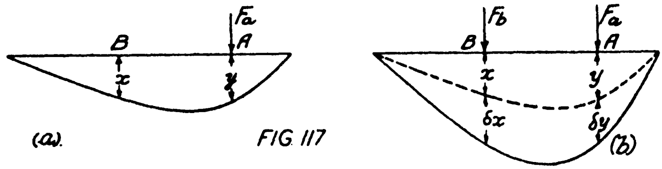


FIG 117

Next gradually apply loads F_a and F_b simultaneously from zero to full value, as in diagram (b), and the same final deflection curve will be obtained. The total deflection at B must again be $x + \delta x$ and at A it is $y + \delta y$. Hence total $W = \frac{1}{2}F_b(x + \delta x) + \frac{1}{2}F_a(y + \delta y)$ (4)

Now the total work done, whether obtained by stage application of one load at a time or with both loads simultaneously, must be the same, *i.e.*, (3) = (4). Dropping the subscripts a and b which were attached to the identical loads F for clarity then :—

$$\tfrac{1}{2}Fy + \tfrac{1}{2}F\delta x + F\delta y = \tfrac{1}{2}Fx + \tfrac{1}{2}F\delta x + \tfrac{1}{2}Fy + \tfrac{1}{2}F\delta y$$

$$\tfrac{1}{2}F\delta y = \tfrac{1}{2}Fx$$

$$\therefore \delta y = x$$

which is Maxwell's law since x occurs at B due to F_a and δy happens at A because of F_b.

Influence Line for Deflection, Fig. 118

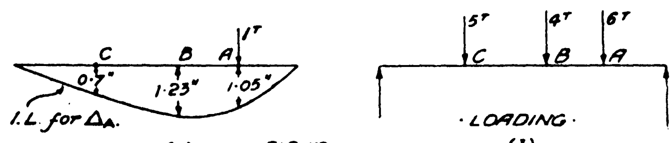


FIG 118

1^T at A causes a deflection at B of 1·23" ∴ by Maxwell's law				
1^T at B	,,	,,	A of 1·23"	
∴ 4^T at B	,,	,,	A of 1·23" × 4 =	4·92"
Similarly 5^T at C	,,	,,	A of 0·7" × 5 =	3·5"
and 6^T at A	,,	,,	A of 1·05" × 6 =	6·3"

i.e., the total deflection at point A due to the loading = 14·72"

63′ 0″ Span Railway Girder, Δ_B, Fig. 115

This is the girder previously dealt with on page 71, and it is desired to ascertain the deflections at a point B, 18′ 0″ from one end, as the British standard loco passes over the bridge. Usually it is at mid-span where the deflection is required, but the mid-span case is a particular one, whereas that now dealt with is the general case.

The construction has been explained already and it only remains to emphasise that unit load must be applied at the point whose deflection is desired.

The influence line of diagram (c) can be graphed directly from the mathematical equations (14) and (15) on page 197 by giving K its value of $\frac{45'}{63'}$, or $\frac{5}{7}$.

Moment of Inertia. The scantlings at mid-span are :—

Top flange.	3 plates 21″ × ⅝″
	2 angles 6″ × 6″ × ½″
Web plate.	78″ × ½″
Bottom flange.	2 angles 6″ × 6″ × ½″
	3 plates 21″ × ⅝″

(It is more usual to make the flange plates an even number of inches wide, such as 20″ or 22″, instead of 21″.)

Recall that it is the gross sectional area which is considered when dealing with deflection because elastic deformation affects all the metal in a member and not only the net area.

It is advisable to work the graphical example in tons and inch units throughout since E is 12,000T/square inch. However, if the solution is being evolved mathematically the arithmetical work is less arduous if E is taken in tons per square foot (12,000 × 144) and all linear dimensions and M. of I. in foot units.

The M. of I. of the girder is 177,550 inches^4.

(In foot units it is $177{,}550 \div 12^4 = 8{\cdot}56$ feet^4.)

Deflection for Unit Axle Loads. The bridge by supporting a double track causes each main girder to carry a set of axle loads. The loads positioned by inspection in diagram (c) will give a deflection which must be very nearly the maximum. One or two additional trials would settle the maximum value. The dimensions stated on the influence line are the actual dimensions as scaled by the decimal inch scale.

Deflection at B = sum of the loads × their respective ordinates :—

¾T wheels, left to right :

$\frac{3}{4}(0{\cdot}42 + 0{\cdot}7 + 0{\cdot}93 + 1{\cdot}11) \times$ scale $0{\cdot}002703 = 0{\cdot}0064$

1T wheels, left to right :

$1(1{\cdot}23 + 1{\cdot}18 + 1{\cdot}05 + 0{\cdot}85) \times$ scale $0{\cdot}002703 = 0{\cdot}0116$

Uniformly distributed load, 3′ 0″ long at end :

$0{\cdot}3(0{\cdot}08) \times 0{\cdot}002703 = 0{\cdot}0001$

Total Δ_B unit axle loads $= \underline{0{\cdot}0181}$

As the design was for 15 units the actual deflection at B for the live load positioned as shown is

$$\Delta_B = 15 \times 0{\cdot}0181 = 0{\cdot}2715'', \textit{ i.e.}, 0{\cdot}27''$$

By calculation (see page 197)

$$\Delta_B \quad = 0{\cdot}2682'', \textit{ i.e.}, 0{\cdot}27''$$

a difference of only 1·2%.

Deflection for the E.U.D.L.L. When dealing with a distributed load it is the area of the influence curve which is required, hence sum or integrate the influence line diagram 115 (c) and obtain the area curve (d). The E.U.D.L.L. for the unit train is $0{\cdot}1^{T}$/foot or $0{\cdot}1 \div 12$ ton/inch and for 15 units it is $0{\cdot}1 \times 15 \div 12 = 0{\cdot}125^{T}$/inch run.

Load covering length AB of 45′;

$$\Delta_B = pp' \times \text{scale} \times 0{\cdot}125$$
$$= 1{\cdot}94 \times 0{\cdot}64872 \times 0{\cdot}125 \qquad = 0{\cdot}1573''$$

Load covering length BC of 18′;

$$\Delta_B = (2{\cdot}46 - 1{\cdot}94) \times 0{\cdot}64872 \times 0{\cdot}125 = 0{\cdot}0422''$$

Load covering all span AC of 63′;

$$\Delta_B = 2{\cdot}46 \times 0{\cdot}64872 \times 0{\cdot}125 \qquad = 0{\cdot}1995''$$

i.e., a deflection of 0·2″

By calculation (integrating equations AB and BC, page 197)

$$\Delta_B = 0{\cdot}1962'', \textit{ i.e.}, \qquad 0{\cdot}2''$$

a difference of 1·7%.

Alternative Derivation (Mathematical) of the Influence Line, Fig. 119

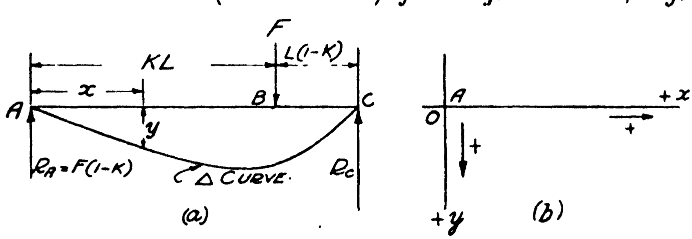


FIG. 119.

Let the fixed point B be situated anywhere on span AC at a distance of K times the span from A, where K is a fraction having any value between nothing and one. The origin is taken at A and positive values of x and y are measured in the directions indicated in the lesser figure.

Text-books on the strength of materials give the derivation of the formula $\frac{M}{EI} = \frac{1}{R} = \frac{d^2y}{dx^2}$, where R is the radius of curvature.

Hence $\frac{dy}{dx} =$ slope ϕ of beam $= \int \frac{M}{EI}dx$ and $y =$ deflection $\Delta = \int\int \frac{M}{EI}dxdx.$

The tangents to the curve as it leaves A are positive, *i.e.*, the gradient of the curve is positive. However, as x increases in value the curve becomes less and less steep, *i.e.*, the tangents become decreasingly positive, or the rate at which the gradient changes is a decreasing one, therefore $\frac{d^2y}{dx^2}$ is negative. But R, the radius of curvature, is essentially a positive quantity, and, hence, with the origin situated at A, the equation should be written as $EI\frac{d^2y}{dx^2} = -M$

$x \leqq KL.$ $M_x = R_A x = F(1 - K)x = Fx - FKx$

$$\therefore EI\frac{d^2y}{dx^2} = -M = -Fx + FKx \quad . \quad . \quad . \quad (1)$$

$$\therefore EI\frac{dy}{dx} = -\tfrac{1}{2}Fx^2 + \tfrac{1}{2}FKx^2 + C_1 \quad . \quad . \quad . \quad (2)$$

$$\therefore EIy = -\tfrac{1}{6}Fx^3 + \tfrac{1}{6}FKx^3 + C_1x + C_2 \quad . \quad . \quad . \quad (3)$$

in (3) when $x = 0, y \doteq 0, \therefore C_2 = 0$ . . . . (4)

$x \geqq KL.$ $M_x = R_A x - F(x - KL) = -FKx + FKL$

$$\therefore EI\frac{d^2y}{dx^2} = -M = FKx - FKL \quad . \quad . \quad . \quad (5)$$

$$\therefore EI\frac{dy}{dx} = \tfrac{1}{2}FKx^2 - FKLx + C_3 \quad . \quad . \quad . \quad (6)$$

$$\text{and } EIy = \tfrac{1}{6}FKx^3 - \tfrac{1}{2}FKLx^2 + C_3x + C_4 \quad . \quad . \quad . \quad (7)$$

At point B the slope, whether calculated by equation (2) or (6), must be the same,

i.e., $EI\left[\frac{dy}{dx}\right]_B = -\tfrac{1}{2}Fx^2 + \tfrac{1}{2}FKx^2 + C_1 = \tfrac{1}{2}FKx^2 - FKLx + C_3$

and $x = KL \therefore -\tfrac{1}{2}FK^2L^2 + C_1 = -FK^2L^2 + C_3$

Whence $C_1 = C_3 - \tfrac{1}{2}FK^2L^2$ . . . . (8)

Again at this point B, with $x = KL$, the deflection, whether obtained by equation (3) or (7), must be the same, *i.e.*,

$$EI[y]_B = -\tfrac{1}{6}Fx^3 + \tfrac{1}{6}FKx^3 + C_1x + C_2 = \tfrac{1}{6}FKx^3 - \tfrac{1}{2}FKLx^2 + C_3x + C_4$$

$$\therefore \quad -\tfrac{1}{6}FK^3L^3 \quad + C_1KL + 0 = \quad -\tfrac{1}{2}FK^3L^3 + C_3KL + C_4$$

and substituting the value for C_1 from equation (8)

$$-\tfrac{1}{6}FK^3L^3 + C_3KL - \tfrac{1}{2}FK^3L^3 = -\tfrac{1}{2}FK^3L^3 + C_3KL + C_4$$

whence $C_4 = -\tfrac{1}{6}FK^3L^3$ . . . . (9)

The deflection y is 0 when $x = L$ and substituting value of C_4 from (9) equation (7) gives

$$0 = \tfrac{1}{6}FKL^3 - \tfrac{1}{2}FKL^3 + C_3L - \tfrac{1}{6}FK^3L^3$$

$\therefore C_3L = \tfrac{1}{3}FKL^3 + \tfrac{1}{6}FK^3L^3$, whence $C_3 = \tfrac{1}{3}FKL^2 + \tfrac{1}{6}FK^3L^2$. (10)

Now substituting the value of C_3 in equation (8)

$C_1 = \tfrac{1}{3}FKL^2 + \tfrac{1}{6}FK^3L^2 - \tfrac{1}{2}FK^2L^2$ . . . . (11)

Having evaluated the constants in equations (3) and (7) these may now be rewritten as :—

(3) $EIy = -\tfrac{1}{6}Fx^3 + \tfrac{1}{6}FKx^3 + \tfrac{1}{3}FKL^2x + \tfrac{1}{6}FK^3L^2x - \tfrac{1}{2}FK^2L^2x$. (12)

(7) $EIy = \tfrac{1}{6}FKx^3 - \tfrac{1}{2}FKLx^2 + \tfrac{1}{3}FKL^2x + \tfrac{1}{6}FK^3L^2x - \tfrac{1}{6}FK^3L^3$. (13)

For unit load $F = 1$ these equations become :—

(12) $EIy = \tfrac{1}{6}(-x^3 + Kx^3 + 2KL^2x + K^3L^2x - 3K^2L^2x)$ . . (14)

(13) $EIy = \tfrac{1}{6}(Kx^3 - 3KLx^2 + 2KL^2x + K^3L^2x - K^3L^3)$ . . (15)

Application of the Mathematical Equation, Fig. 120

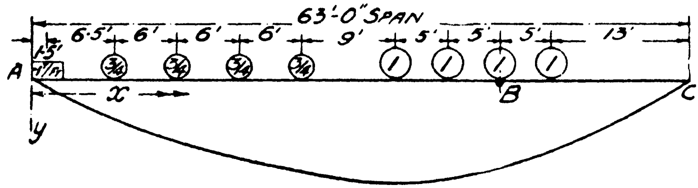


FIG 120

The same example of the 63′ 0″ span railway bridge will be considered : K for the desired point $B = \dfrac{45'}{63'} = \dfrac{5}{7}$.

To simplify the arithmetic all dimensions will be stated in foot units, *i.e.* :—

$L = 63'$

$E = 12{,}000^{\text{T}}/\text{sq. in.} = (12{,}000 \times 144)^{\text{T}}/\text{sq. ft.} = 1{,}728{,}000^{\text{T}}/\text{sq. ft.}$

$I = 177{,}550\ \text{in.}^4 = (177{,}550 \div 12^4)\text{ft.}^4 = 8{\cdot}5623\ \text{ft.}^4$

The only variables are thus x and y.
Portion AB: equation (14) was

$$EIy = \tfrac{1}{6}(-x^3 + Kx^3 + 2KL^2x + K^3L^2x - 3K^2L^2x)$$

$$= \tfrac{1}{6}\left(-x^3+\frac{5}{7}x^3+\frac{2\times5\times63^2}{7}x+\frac{5^3\times63^2}{7^3}x-\frac{3\times5^2\times63^2}{7^2}x\right)$$

$$= \tfrac{1}{6}\left[-\frac{2}{7}x^3 + \frac{5\times63^2}{7}x\left(2+\frac{5^2}{7^2}-\frac{3\times5}{7}\right)\right]$$

$$= -0{\cdot}0476x^3 + 173{\cdot}571x$$

To plot the graph give various values to x; in order to obtain a direct check upon the graphical solution of Fig. 115 the values given will be 1·5, 8, 14, etc., as in Fig. 120.

x	1·5	8	14	20	26	35	40	45
EIy	260	1,364	2,299	3,091	3,676	4,034	3,896	3,473

Portion BC: equation (15) was

$$EIy = \tfrac{1}{6}(Kx^3 - 3KLx^2 + 2KL^2x + K^3L^2x - K^3L^3)$$

$$= \tfrac{1}{6}\left(\frac{5}{7}x^3-\frac{3\times5\times63}{7}x^2+\frac{2\times5\times63^2}{7}x+\frac{5^3\times63^2}{7^3}x-\frac{5^3\times63^3}{7^3}\right)$$

$$= 0{\cdot}11905x^3 - 22{\cdot}5x^2 + 1{,}186x - 15{,}187{\cdot}5$$

When $x = 50$, the value for $EIy = 2{,}744$.

EIy for 3′ 0″ of U.D.L. considered as a point load $= 260 \times 0{\cdot}3 = 78$

EIy for $\tfrac{3}{4}^{T}$ axles $= \tfrac{3}{4}(1{,}364 + 2{,}299 + 3{,}091 + 3{,}676) = 7{,}823$

EIy for 1^{T} axles $= 1(4{,}034 + 3{,}896 + 3{,}473 + 2{,}744) = 14{,}147$

EIy total for train $= 22{,}048$

$$\therefore y = \Delta_B \text{ for unit train} = \frac{22{,}048}{EI} = \frac{22{,}048}{1{,}728{,}000 \times 8{\cdot}562} = 0{\cdot}00149'$$

or in inches $= 0{\cdot}01788''$

and for a 15 units train $\Delta_B = 0{\cdot}01788 \times 15 = 0{\cdot}2682''$

$= 0{\cdot}27''$

Elastic Deformations of a Braced Frame

Since stress intensity = total stress S in tons $\div$ gross sectional area A in sq. in.

$$= \frac{S}{A} \text{ tons per sq. in.}$$

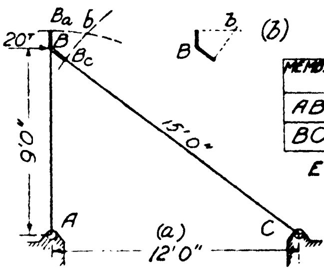
and strain $= \text{elongation } e'' \div \text{original length } L''$

$$= \frac{e}{L} \text{ (a ratio or number)}$$

$$\therefore \text{ Young's modulus, } E = \frac{\text{stress}}{\text{strain}} = \frac{S}{A} \div \frac{e}{L} = \frac{SL}{Ae} \text{ tons per sq. in.}$$

$$\text{and hence elongation } e = \frac{SL}{AE} \text{ inches } \left(i.e., \frac{S^{\text{T}}L''}{A \text{ sq. in. } E^{\text{T}}/\text{sq. in.}}\right)$$

noting that the units, tons and inches " cancel top and bottom " as with numbers.



MEMBER	LENGTH L"	STRESS S^T	AREA $A^{\square}$"	S/A	$e'' = \frac{SL}{AE}$
AB	108	−15	2·5	−6	−0·054
BC	180	+25	6·25	+4	+0·06

$E^* = 12{,}000$ T/□" + = Compression.

FIG. 121.

Williot Diagram.† The horizontal load of 20^{T} in Fig. 121 acting on the vertical cantilever causes the stresses and elongations in the two members as given by the table alongside.*

With point A definitely pinned in position the extension BB_a (shown in heavy line), which occurs in the tension member AB, must move point B away from A in the direction of AB produced. Similarly, because point C is definitely pinned in position, the shortening BB_c in this bar, caused by the compressive stress, must tend to move end B towards C along path BC.

The new position of point B is b obtained by using A as centre

† After the inventor, a French engineer, 1877.

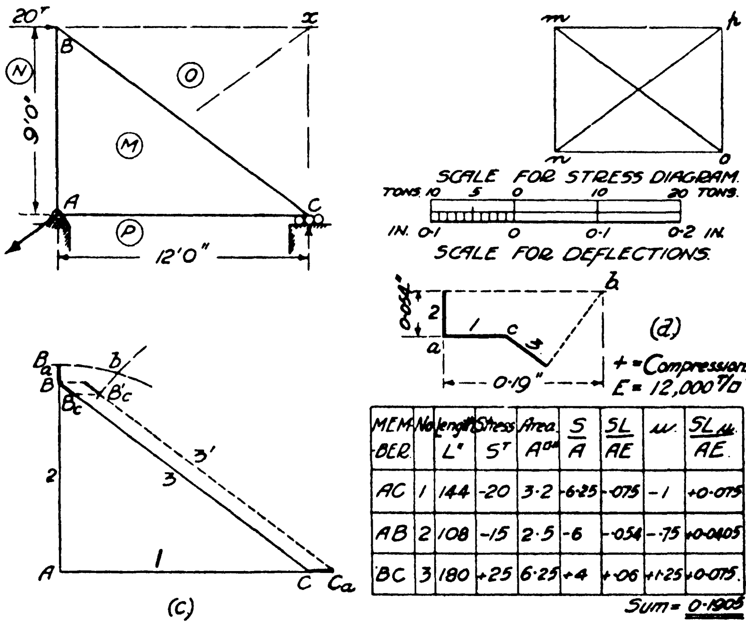
* For elastic deformations only use $E = 13{,}000$.

If the total deformation (*i.e.*, elastic and inelastic) is desired use $E = 12{,}000$. Inelastic deformations are due to shop errors in marking off the lengths of members, play in holes, etc. Some designers prefer to work with E at 13,000 and then multiply the elastic deflection so obtained by a constant which experience has shown to be suitable for the particular type of structure under consideration.

and new length AB_a as radius to cut the arc drawn with C as centre and CB_c as radius.

These extensions are very small in comparison with the original lengths. With bar AB, for example, $e \div L = 0{\cdot}054 \div 108 = 0{\cdot}0005$ or $\frac{1}{2{,}000}$. It is therefore clear that such a construction, as suggested above, is impracticable. In the small figure (b) the elongations alone have been set out and perpendiculars drawn at their terminations in place of arcs to give the new position of the apex b relative to the original position B. The arcs employed in (a) are really perpendiculars because the lengths along the arcs to the cutting point are very small relative to the radii.

In Fig. 122 a tie-bar has been added to the cantilever frame of the previous figure: end A remains pinned in position, but C is free to



MEMBER	No	Length L"	Stress S^T	Area $A^{□"}$	$\frac{S}{A}$	$\frac{SL}{AE}$	u.	$\frac{SLu}{AE}$
AC	1	144	-20	3·2	-6·25	-·075	-1	+0·075
AB	2	108	-15	2·5	-6	-·054	-·75	+0·0405
BC	3	180	+25	6·25	+4	+·06	+1·25	+0·075

FIG 122.

move laterally under deformation. Reaction C must be at right angles to the frictionless plane upon which the free or roller bearing rests and, by the theorem of the triangle of forces, the reaction line of A must pass through x the intersection of the load line and the reaction C line. The stress diagram together with the stresses, etc., are also given on the figure.

To fix the positions of the deformations in space a fixed point for position and a reference line for direction are necessary and these are furnished by point A and line of constant direction AC.

In Fig. 122 with reference to A point B moves up to B_a and the final position of B is somewhere on arc B_a. The tie-bar lengthens and point C moves in the direction AC away from A. The final position of C_a must be on AC produced because end C_a will not swing on an arc with A as centre. So far the new lengths of AB and AC have been plotted and it now remains to complete the third side of the triangle by obtaining the new length of CB. The position of end C of this bar is known and with C_a as centre and the now shortened length $C_aB'_c$ as radius describe the arc to cut that through B_a and obtain the new position of apex B.

As a practical construction, however, plot, to a large scale, the extensions only as in (d). Commence with a fixed point a and from it draw the heavy line 1 = 0·075″ to the right of A and parallel to AC, because the extension of AC tends to pull A in the direction mentioned. Bar AC is the reference line and point C is at its end and requires no arc to be drawn. Now thinking of this newly obtained point C it will be seen that the effect of the compression in CB is to push point C in a south-easterly direction; hence draw heavy line 3 away from c in a S.E. direction parallel to BC and of length 0·06″. The point b in Fig. (c) lies on an arc through B'_c and therefore on a perpendicular from the end of line 3 in (d). Lastly, the extension in AB tends to pull point A due north, so from point a in Fig. (d) draw line 2 due north (*i.e.*, parallel to AB) and away from the point and of length 0·054″.

The perpendicular from the end of 2 intersects that from the end of 3 at b, so fixing the position of b. Scaling Fig. (d) shows that point B has moved eastwards a length of 0·19″ and northwards a distance of 0·054″. This value of 0·19″ is in close agreement with the summation of the last column which now requires to be explained.

Dummy or Unit Load Method. In Fig. 123 it is required to find the vertical deflection of point A due to the various loads marked F.

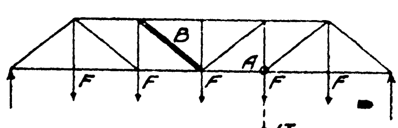


FIG. 123.

Apply a unit load of 1^T at the point A, whose deflection is desired, and let it act in the required direction of the deflection. Let the F loads and the unit load be applied simultaneously from zero to full value: the resulting total stress in any member, such as B, will be S tons due to the F loads and u tons due to the unit load.

i.e., stress in member B $= S + u$

The extension of this member $B = e$ $= \dfrac{(S + u)L}{AE}$

The internal work done on bar B by the external unit load, which was applied from zero to full value,

= average value of $\frac{1}{2}u \times$ total extension $= \dfrac{\frac{1}{2}u(S + u)L}{AE}$

While the internal work done on all the members by the dummy or unit load will be the sum of all such quantities, *i.e.*, $= \Sigma\dfrac{\frac{1}{2}u(S + u)L}{AE}$ (1)

Now point A will have a deflection made up of two component parts, viz., Δ_{AF} due to the F loads and Δ_{Au} due to the unit load, and so the work done at A by the 1^T load = average load of $\frac{1}{2}^T \times$ total distance travelled $= \frac{1}{2}(\Delta_{AF} + \Delta_{Au})$ (2)

The internal work done by the unit load = external work done by unit load

i.e., (1) = (2) or $\frac{1}{2}\Sigma\dfrac{u(S + u)L}{AE} = \frac{1}{2}(\Delta_{AF} + \Delta_{Au})$

i.e., $\Sigma\dfrac{SLu}{AE} + \Sigma\dfrac{u^2L}{AE} = \Delta_{AF} + \Delta_{Au}$ . . . . . . (3)

Δ_{Au} may be evaluated by making the F loads so small that in the limit they have zero value and therefore both S and $\Delta_{AF} = 0$.

Equation (3) will now read $\Sigma\dfrac{u^2L}{AE} = \Delta_{Au}$ . . . . (4)

Having thus established the value for Δ_{Au} the value for Δ_{AF} will be obtained by subtracting (4) from (3), so giving $\Sigma\dfrac{SLu}{AE} = \Delta_{AF}$. (5)

Hence the arithmetical value of the deflection at A, caused by the external loads F, is found by adding together the values of $\dfrac{SLu}{AE}$ as calculated for all the members of the truss ; where u is the stress induced in any member by unit load applied in the direction and at the point of the required deflection.

The application of this method in Fig. 122 ascertains the deflection of only one point in a pre-selected direction and not, like the Williot diagram, the complete displacements of all points. If the deflection of point B is desired in a direction due east then a 1^T load is placed at B acting from left to right. This happens to be the sense and direction of the 20^T load, and therefore the u stresses will be $\frac{1}{20}$ of the S stresses. The dummy or unit load method is an excellent check upon the accuracy of the deflection of one main point as given by the Williot diagram and requires but little additional labour.

Mohr's Rotation Diagram.* In Fig. 125 the bottom boom of the girder will deflect or sag unlike member AC of Fig. 122, and so in the former figure if either member L_0U_1 or L_0L_1 be chosen as the reference line from the fixed point L_0 the Williot diagram will give the deformations relative to this reference line, *which now rotates itself.* The value of the Williot diagram is greatly enhanced by th rotation diagram of Mohr, for by the use of both diagrams th absolute deformation of any panel point on the structure can b ascertained.

If the saw-tooth or north-light truss of Fig. 124 be slightl rotated as a single unit about a distant centre O each radial ar

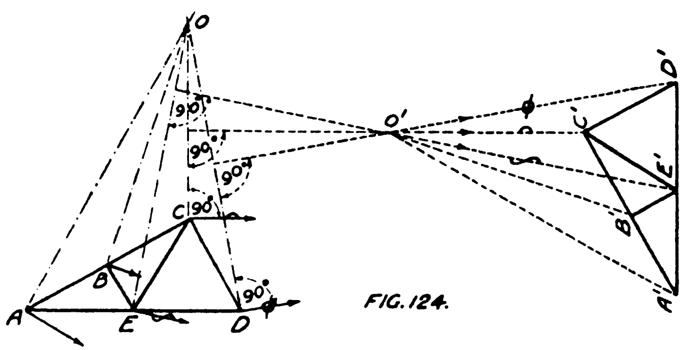


FIG. 124.

OA, OD, etc., will be turned through the same angle θ, but the tangential paths at A, D, etc., will be of different lengths, viz., $\theta.OA$, $\theta.OD$, etc.; the greater the spoke length from hub O the greater will be the linear path travelled.

Take any other point O' and from it scale, as vector quantities, the length and direction of each of the paths travelled by the panel points. Thus $O'D'$ represents the distance and direction travelled by the original point D and $O'A'$ by point A, and so on. If the points $A'C'D'$ be joined up the resulting figure will be found to be similar to the original figure, but with the corresponding bars of the figures at right angles to each other.

* Professor O. Mohr, 1887.

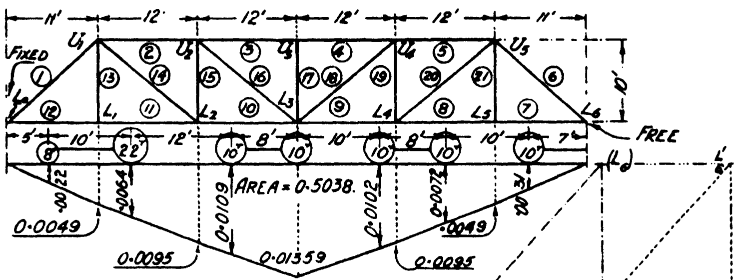
INFLUENCE LINE FOR DEFLECTION FOR A BRACED FRAME

To find the influence line for deflection for a panel point on the lower flange of the road bridge (Fig. 125) whose dead load, live load and impact stresses have been calculated previously, see Chapter IX, page 98.

In the column of the table headed A, given on the figure, are the gross cross-sectional areas of the members as they were finally designed; while the S column contains the stresses due to a unit load applied at the point for which the influence line is required. The two last columns headed u and $\frac{SLu}{AE}$ are not required meantime.

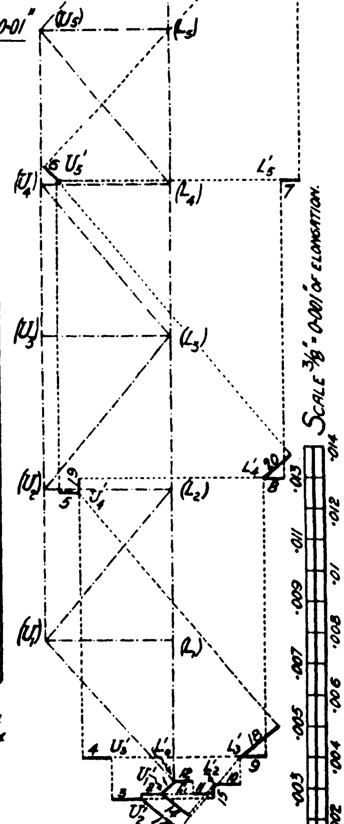
The Williot diagram drawn alongside is for point L_0 as the fixed point and bar L_0U_1 as the reference line. The stress in this member is compression tending to push L_0 along line 1 of the enlargement a distance $L'_0U'_1 = 0{\cdot}000578''$ (*i.e.*, $0{\cdot}578 \times 10^{-3}$) as given in the column headed $\frac{SL}{AE}$. In order to facilitate the tabulation (and scaling) the extension is stated in this column as $\frac{SL}{AE} \times 10^3$. On the base U_1L_0 stands the original triangle $U_1L_0L_1$ and on the corresponding base of heavy line 1 build the respective deformations. Bar 12 is in tension tending to pull L_0 towards the right, therefore heavy line 12 of the diagram is drawn from L'_0, also towards the right, a distance of $0{\cdot}000597''$, while point L'_1 will be on the perpendicular (or arc) from the free end of this line. From U'_1 draw line 13 parallel to U_1L_1, but as this bar has no stress there is no extension to draw, *i.e.*, heavy line 13 is given by point U'_1. The point L'_1 will lie on the perpendicular from the free end of heavy line 13, *i.e.*, on the line drawn due east from U'_1. These two perpendiculars meet at L'_1.

The next triangle to be considered is that standing on U_1L_1 as base line, viz., $U_1L_1L_2$. U_1L_2 is in tension and so U_1 is subjected to a south-easterly pull, therefore from U'_1 heavy line 14 is drawn towards the S.E. and parallel to U_1L_2 for a length$=\frac{SL}{AE}=0{\cdot}001123''$—to scale, of course, viz., 0·00112 on the original $\frac{3}{8}''$ scale for the complete Williot diagram or the same reading on the original 1″ scale belonging to the enlargement. L'_2 will lie on the perpendicular from the free end of heavy line 14. Of the base line U_1L_1 point U_1 has been considered, so now consider the other extremity L_1 and from L'_1 draw heavy line 11 due east parallel to L_1L_2 to represent the extension of $0{\cdot}00065''$ caused by the tension in L_1L_2 tending to

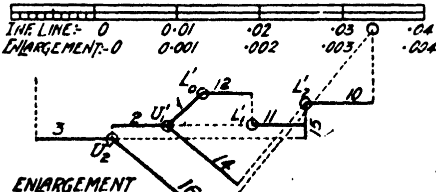


INFLUENCE LINE FOR Δ AT L_3. SCALE 1" = 0·01"

L = LENGTH IN INCHES	A = AREA IN SQ INS. GROSS.							
S = TOTAL STRESS IN TONS.	E = 12,000 T/□"							
POSITION	MEMBER	Nº	L	S	A	$\frac{SL}{AE} \times 10^3$	u	$\frac{SLu}{AE} \times 10^3$
TOP BOOM	U_1U_2	2	144	+1·15	20·85	0·661	+1·15	0·760
	U_2U_3	3	144	+1·75	22·85	0·919	+1·75	1·608
	U_3U_4	4	144	+1·75	22·85	0·919	+1·75	1·608
	U_4U_5	5	144	+1·15	20·85	0·661	+1·15	0·760
BOTTOM BOOM	L_0L_1	12	132	−0·55	10·12	0·597	−0·55	0·329
	L_1L_2	11	144	−0·55	10·12	0·652	−0·55	0·358
	L_2L_3	10	144	−1·15	16·88	0·817	−1·15	0·940
	L_3L_4	9	144	−1·15	16·88	0·817	−1·15	0·940
	L_4L_5	8	144	−0·55	10·12	0·652	−0·55	0·358
	L_5L_6	7	132	−0·55	10·12	0·597	−0·55	0·329
END POSTS	U_1L_0	1	178·4	+0·744	19·10	0·578	+0·744	0·430
	U_5L_6	6	178·4	+0·744	19·10	0·578	+0·744	0·430
WEB VERTICALS	U_1L_1	13	120	0	11·5	0	0	0
	U_2L_2	15	120	+0·5	11·5	0·435	+0·5	0·218
	U_3L_3	17	120	0	11·5	0	0	0
	U_4L_4	19	120	+0·5	11·5	0·435	+0·5	0·218
	U_5L_5	21	120	0	11·5	0	0	0
WEB DIAGONALS	U_1L_2	14	187·4	−0·781	10·86	1·123	−0·781	0·877
	U_2L_3	16	187·4	−0·781	7·62	1·601	−0·781	1·250
	U_4L_3	18	187·4	−0·781	7·62	1·601	−0·781	1·250
	U_5L_4	20	187·4	−0·781	10·86	1·123	−0·781	0·877
								Sum = 13·54.



WILLIOT & MOHR DIAGRAMS.



ENLARGEMENT
SCALE 1" = 0·001"

FIG. 125.

pull L_1 towards the east. At the free end of this line erect a perpendicular and where this perpendicular intersects that from heavy line 14 is the position of L'_2

And so, as in the structure itself, build up the Williot diagram by considering, structurally, triangle after triangle until the last point, L'_6, is obtained.

The Mohr rotation, or correction diagram as it is sometimes termed, is next drawn in and is shown in dash-dot lines with the panel point lettering in brackets; in the text the corresponding lettering is also given within brackets. This figure will have one end at L'_0, the fixed point, and the base $L'_0(L_6)$ vertical therefrom since the latter must be perpendicular to the original boom L_0L_6. The end bearing L_6 will only move horizontally, therefore (L_6) of the Mohr diagram must lie on the horizontal line through L'_6 of the Williot diagram, and thus length $L'_0(L_6)$ of the Mohr figure is fixed. Now divide this vertical line $(L_6)L'_0$ into six parts, recalling that $(L_6)(L_5) : (L_5)(L_4) : (L_4)(L_3) :: 11' : 12' : 12'$, viz., the length ratio of the original panels of the bottom boom. Erect perpendiculars at (L_5), (L_4), (L_3), etc., *i.e.*, these lines are now at right angles to the original verticals. Through (L_6) draw (L_6) (U_5) at right angles to L_6U_5 of the girder until it cuts the perpendicular from (L_5). This intersection is point (U_5). Similarly the line $L'_0(U_1)$ is drawn at right angles to the end post L_0U_1 cutting $(L_1)(U_1)$ at (U_1). Join (U_1) to (U_5) and the remainder of the Mohr diagram falls into position.

The movement of any panel point of the truss is given in length and direction by the line joining the panel point of the Mohr figure to the corresponding panel point of the Williot, *e.g.*, panel point L_5 moves in direction along the line joining (L_5) of the Mohr to L'_5 of the Williot. The vertical deflection of point L_5 is the vertical component, viz., the vertical drop or distance between point (L_5) of the Mohr to the horizontal, line 7 produced, through L'_5 of the Williot: while the horizontal motion is eastwards from (L_5) of the Mohr to the vertical through L'_5 of the Williot, *i.e.*, the length of the horizontal line which appears as if it were lettered $(L_4)L'_5$.

Influence Line. The vertical distances between (L_1) and L'_1, (L_2) and L'_2, (L_3) and L'_3, etc., on scaling are found to be 0·0049″, 0·0095″ and 0·01359″, etc., respectively. Plotting these downwards from a horizontal base line gives the influence line for deflection for point L_3 on the bottom boom as was proved for the simple beam of Fig. 115.

If the influence line for L_4 on the bottom boom is required the procedure is similar save that unit load requires to be applied at L_4. Should the deflection influence line be required for any panel point of the top boom then apply unit load at the desired point and thereafter follow the method outlined.

Returning now to the table, the columns headed u and $\frac{SLu}{AE}$ may be used to check the accuracy of the Williot diagram. The u stresses will be identical to the S stresses, both being caused by the same unit load applied at the same point. By summing the arithmetical values in the $\frac{SLu}{AE}$ column the vertical deflection of L_3 is 0·01354″ as against 0·01359″ by the Williot diagram.

If, in effect, each main girder carries a set of the M. of T. wheel loads, then for the engine and trailers positioned as shown on the influence line, the deflection at L_3

$$= 10(0{\cdot}0031 + 0{\cdot}0072 + 0{\cdot}0102 + 0{\cdot}01359 + 0{\cdot}0109) = 0{\cdot}4499''$$

$$\text{plus } 22 \times 0{\cdot}0064 + 8 \times 0{\cdot}0022 = 0{\cdot}1584''$$

$$\text{Total } \Delta \text{ at } L_3 = 0{\cdot}6083''$$

For the uniformly distributed load of 220 lb. per square foot (equal to 0·98ᵀ per ft. run of girder for the 10′ 0″ width supported) plus the knife-edge load of 2,700 lb. per ft. width (equal to 12·05ᵀ for the 10′ 0″ width) the deflection at L_3

$$= \text{load} \times \text{area} = 0{\cdot}98 \times 0{\cdot}5038 = 0{\cdot}4937''$$

$$\text{plus } 12{\cdot}05 \times \text{mid-ordinate of } 0{\cdot}01359 = 0{\cdot}1638''$$

$$0{\cdot}6575''$$

And for the dead load of 0·94ᵀ per ft. run the deflection at L_3 = load × area = 0·94 × 0·5038 = 0·4736″

The vertical deflection at the mid-point, L_3, is therefore

(a) Due to D.L. + Wheel L.

$$= 0{\cdot}4736'' + 0{\cdot}6083'' = 1{\cdot}08''$$

(b) Due to D.L. + U.D.L.L. + K.E.L.

$$= 0{\cdot}4736'' + 0{\cdot}4937'' + 0{\cdot}1638'' = 1{\cdot}13''$$

Result (b) can be obtained directly, without recourse to an influence line, either by a Williot diagram or by the arithmetical unit load method. The loads applied at the lower panel points would then be: L_1 and L_5 = 22·08ᵀ; L_2 and L_4 = 23·04ᵀ and L_3 = 35·09ᵀ (including knife-edge load). The S stresses would be the stresses due to the foregoing loads, but the u stress would still be due to unit load applied at L_3, the point whose deflection is desired.

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