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ELEMENTS OF
ENGINEERING THERMODYNAMICS

BY

JAMES A. MOYER
Late Director of the Massachusetts Department of University Extension
formerly Professor of Mechanical Engineering in the
Pennsylvania State College

JAMES P. CALDERWOOD
Late Professor and Head of the Department of Mechanical
Engineering in the Kansas State College of
Agriculture and Applied Science

ANDREY A. POTTER
Dean of the Schools of Engineering and Director
of the Engineering Experiment Station at
Purdue University

SIXTH EDITION

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PREFACE TO SIXTH EDITION

In preparing the material for the sixth edition, the authors kept before them the original intent, namely, to stress the fundamental principles of engineering thermodynamics as a foundation for the more advanced and practical applications of the theory. The first edition was intended particularly for use in those technical colleges which could give special courses in the subjects of advanced thermodynamics as well as in steam turbines, internal combustion engines, heating, refrigeration, and other applications of thermodynamics; and the new edition retains the same objectives.

Many of the ideas and suggestions on the subject of thermodynamics, as brought out at the conference of mechanical engineering teachers held at Purdue University in 1929, under the auspices of the Society for the Promotion of Engineering Education, have been incorporated in this revision. The new material includes the thermodynamic treatment of uniform flow processes and new vapor tables. Reversible and irreversible processes are given additional attention.

All chapters were carefully checked, several were entirely rewritten, and suggestions made by users of the book were considered in bringing the treatment up to date. More space was devoted to cycles, heat transfer, refrigeration, and air conditioning. Symbols and abbreviations were changed to conform to the most recent practice. New problems and new tables have been added. Illustrations have been increased from fifty-four to eighty.

The authors are indebted particularly to Dr. W. J. Cope of the Mechanical Engineering staff of Purdue University for his assistance in the preparation of this revision and for much of the new material included.

THE AUTHORS

April, 1941

PREFACE TO FIRST EDITION

This treatise is an extension of a briefer work entitled " Engineering Thermodynamics " by James A. Moyer and James P. Calderwood. The additions and changes are made to suit the needs of those who had successful experience in using the original publication and found it desirable to add supplementary material to make it sufficiently inclusive for special institutional requirements. A great deal of the new material is supplied by A. A. Potter.

This book is intended to bring out the fundamental principles of Engineering Thermodynamics, and is particularly intended for use in technical colleges where it is possible to give special courses on the subjects of steam turbines, internal combustion engines, refrigeration and other applications of thermodynamics.

The new material includes the theory of the hot air engine and internal combustion engine cycles. The appendix includes logarithmic tables, the properties of gases, the properties of steam, ammonia, sulphur dioxide and carbon dioxide.

Every engineering student should become familiar with standard works on the subject of thermodynamics. This book should consequently be supplemented by references to standard works on this subject. A representative list of such reference books is given in a table of the appendix.

The authors are particularly indebted to the following professors and instructors in Mechanical Engineering Departments who gave valuable suggestions and criticisms: Edwin A. Fessenden, Pennsylvania State College; J. E. Emswiler, University of Michigan; G. L. Christensen, Michigan College of Mines; Roy B. Fehr, formerly of Pennsylvania State College; H. L. Seward, Yale University; J. R. Wharton, University of Missouri; and J. J. Wilmore, Alabama Polytechnic Institute.

J. A. Moyer
J. P. Calderwood
A. A. Potter

CONTENTS

SYMBOLS

A = area in square feet, also used to represent the reciprocal of the mechanical equivalent of heat, $\frac{1}{778}$.

Btu = British thermal unit (= 778 ft-lb).

E_a = available energy in Btu per pound.

F = force in pounds.

J = mechanical equivalent of heat = 778 (a constant).

M = molecular weight.

N = revolutions per minute; also used to represent explosions per minute.

P = pressure in general or pressure in pounds per square foot.

Q = quantity of heat in Btu.

R = thermodynamic constant for gases; for air it is 53.3 in pounds (in foot-pound units per deg fahr).

S^* = total entropy.

T = absolute temperature, in fahr deg = $460 + t$.

U^* = total internal energy in Btu.

V^* = total volume in cubic feet.

$\mathbf{V}$ = velocity in feet per second.

W = work done in foot-pounds.

a = area in square inches.

c = constant of integration.

c_p = specific heat at constant pressure in Btu per pound per degree.

c_v = specific heat at constant volume in Btu per pound per degree.

e = efficiency; also subscript to represent base of natural logarithms.

g = acceleration due to gravity = 32.2 feet per second per second.

h^* = enthalpy per pound.

hp = horsepower.

k = ratio of specific heats $= \dfrac{c_p}{c_v}$.

log = logarithm to base 10.

$\log_e$ = logarithm to natural base e (Naperian).

n = exponent of polytropic expansion in the expression PV^n = constant.

p = pressure in pounds per square inch (psi).

p_m = mean effective pressure in pounds per square inch.

r = ratio of expansion.

* To indicate the various conditions of a fluid, the subscript f indicates any property of a saturated liquid, the subscript fg indicates the change of any property between the saturated liquid and saturated vapor state at constant pressure, and the subscript g indicates any property of a dry saturated vapor. When no subscript is used the term indicates any condition of the vapor.

rpm = revolutions per minute.
s^* = entropy per pound.
t = temperature in fahr deg.
u^* = internal energy per pound.
v^* = specific volume in cubic feet per pound.
w = weight of a substance in pounds.
x = quality of steam expressed as a decimal.
z = potential mechanical energy.
d = density.
Φ = relative humidity.

ELEMENTS OF ENGINEERING THERMODYNAMICS

CHAPTER I

THERMODYNAMIC PRINCIPLES AND DEFINITIONS

Thermodynamics. Thermodynamics deals with energy and its transformation from one form into another. Since energy and its transformation are the foundation of other branches of natural science, a more restricted definition is required when thermodynamics is applied to engineering.

In engineering the transformation of energy into work is of first importance; and since heat energy is the usual source from which work is derived, engineering thermodynamics deals largely with heat energy and its transformation into work. The primary object in the study of engineering thermodynamics is to consider these factors as they affect the efficiency of heat power machinery.

Engineering thermodynamics makes it possible to predict the performance of steam engines and steam turbines when operating under conditions of increased pressure, of higher vacuum and expansions, of reheating the steam between stages, of extracting heat for feed water heating and of using binary fluids.

By means of calculations based upon the principles of thermodynamics it is possible to determine the effect of increasing the compression pressures of a gas engine mixture before it is ignited, the saving in work by increasing the stages of an air compressor, the result of incomplete cooling upon the efficiency of an air compressor, and similar problems.

Another important service which the study of engineering thermodynamics renders is that of showing what maximum efficiency is attainable for any heat engine operating under a given set of conditions. It sometimes happens that tests indicate an efficiency very much better than is usually obtained with any of the present types of engines. In such cases thermodynamic calculations will show conclusively whether the results secured are possible. The ability to interpret correctly the results of experiments performed on all kinds of heat engines requires a knowledge of the basic principles of thermodynamics.

Energy. A body is said to possess energy if it is capable of doing work. Such a definition arises from the fact that most forms of energy can be transformed into work. On the other hand, it is entirely possible that some forms of energy exist that may not be transformed into work.

Many illustrations of energy in various forms may be given. Food contains energy which through the functioning of the various organs of the animal body is transformed into many different forms. Coal contains energy which may be transformed into heat energy by combustion. A moving body possesses energy due to its motion. A rock elevated above the ground possesses energy due to its position with respect to the ground. Electricity is energy which may be transformed into heat by means of a resistance, into work with a motor, or into light by an electric lamp.

Forms of Energy. The forms of energy which enter into thermodynamic processes are: potential mechanical energy, kinetic mechanical energy, work, internal energy and heat.

Potential Mechanical Energy. Potential mechanical energy (z) may be defined as that energy possessed by a body due to its elevation above some arbitrary datum plane. Thus, steam flowing into a turbine has potential mechanical energy with respect to the steam entering the condenser if the condenser is at a lower level than the turbine. Potential mechanical energy is thus measured by the weight of the substance times its height in feet above the datum plane, and its unit of measure is in foot-pounds. In most gas or vapor calculations this type of energy is small and is usually neglected.

Kinetic Mechanical Energy. Kinetic mechanical energy $\left(\frac{V^2}{2g}\right)$ is the energy of a substance due to its motion. In foot-pound units, kinetic mechanical energy is expressed $\frac{wV^2}{2g}$, where w is the weight of the substance flowing, V is the velocity in feet (usually per second), and g is the gravitational unit ($g = 32.16$ ft per sec per sec at sea level).

Pressure. Force per unit of area is called pressure. Pressure is due to a molecular bombardment and depends upon the number of molecules present per unit of volume and upon their mean molecular velocity. The pressure exerted by a gas or vapor is expressed in the English system in pounds per square inch (psi), pounds per square foot, inches of mercury (in. Hg) or atmospheres (atm). In the metric system, pressure is expressed in kilograms per square centimeter or millimeters of mercury.

Gages read pressures above or below atmospheric pressure. Those recording pressures below atmospheric are called vacuum gages. The absolute pressure is the sum of the gage and atmospheric pressures. Thus, if a gage reads 75 lb pressure per square inch (psi), and the barometer is 29.65 in. of mercury, the atmospheric or barometric pressure is, in pounds per square inch,

$$29.65 \times 0.491 = 14.56.$$

(0.491 is the weight of a cubic inch of mercury at 70 deg fahr).

The absolute pressure is:

$$75 + 14.56 = 89.56 \text{ psi.}$$

If a vacuum gage reads 28.5 in. and the barometric pressure is 29.65 in., the absolute pressure is:

$$(29.65 - 28.5) \times 0.491 = 0.564 \text{ psi.}$$

In thermodynamic equations the unit of pressure is usually expressed in pounds per square foot and is called specific pressure.

Volume. By *specific volume*, v, is meant the amount of space occupied by a unit weight of a substance, expressed in cubic feet or in cubic meters. Thus, the volume of 1 lb of steam at atmospheric pressure is its *specific volume* and is equal to 26.80 cu ft.

Work. Work is done by a force during a given displacement and is independent of the time. The foot-pound (ft-lb) is the unit of work in the English system. Thus, when a body weighing 1 lb is raised through a distance of 1 ft, the resulting work is a foot-pound. Similarly, the product of the pressure in pounds per square foot and the volume change in cubic feet is equal to the work in foot-pounds.

Work being the product of two dimensions, it may be represented graphically by the area of a closed figure, the coördinates of which are force and distance, or pressure and volume. Thus, *A* (**Fig. 1**) represents the position of a body between *F* (force) and *S* (space) coördinates, *O* being the origin or zero point. The distance *AD* from the *OS* line represents the force acting and *AA'*, measured from the *OF* line, is the distance of the body from the zero point. If the motion of the body while being acted upon by the force is represented by the path *AB*, the area *ABCD* represents the work done.

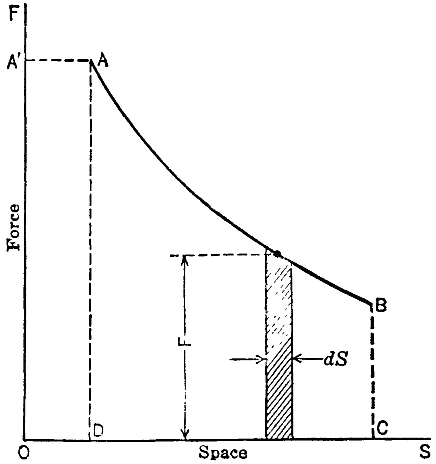

FIG. 1. Work Diagram.

The work is

$$W = \int_{S_A}^{S_B} F\,dS \tag{1}$$

and if the equation of the path is known, this may be integrated and solved.

If the coördinates are pressure (P) and volume (V)

$$W = \int P\,dV. \tag{2}$$

Internal Energy. Internal energy (U) may be defined as that energy contained within a substance due to its molecular activity. In general the internal energy of a substance is made up of two parts: (1) the internal kinetic energy due to molecular velocity which manifests itself by temperature, and (2) the internal potential energy arising from molecular disgregation.

If ΔK represents the internal kinetic energy of a substance and ΔP the internal potential energy of disgregation, the total internal energy of the substance is then

$$\Delta U = \Delta K + \Delta P.$$

The internal kinetic energy is due to the velocity of the molecules; and, although these molecules may be moving in different directions and with different velocities, their total effect is measurable from their mean velocity when referred to the total molecules of which the body is composed. This form of energy is called internal kinetic energy since it is a form of energy due to motion. It is the physical source of temperature.

The internal potential energy is due to a change in position of the vibrating molecules against the forces that hold them together. In the case of gases only a negligible part of the internal energy resides within the substance as internal potential energy. In the case of changing the state of aggregation of a substance, the reverse of this is true. In the boiling of water or melting of ice no change of temperature occurs, and the internal energy absorbed in such a process is used in overcoming the cohesion of the molecules.

Power. Power is the rate of doing work, or the work done divided by the time required to do it. In the English system, the unit of power is the horsepower (hp). It is the power required to raise 550 lb through a vertical distance of 1 ft in 1 sec, or 33,000 lb 1 ft in 1 min. To obtain the horsepower, the work in foot-pounds per minute must be obtained and the result divided by 33,000. *One horsepower is equal to 746 watts.* The French horsepower (cheval) is 542.5 ft-lb per sec.

Indicated Horsepower. The term indicated horsepower (ihp) is applied to the rate of doing work by a gas or a vapor in the cylinder of an engine. It is obtained by means of an engine indicator.

If p_m represents the mean effective pressure (average unbalanced pressure) in psi, as shown by the indicator diagram, and a the effective area of the piston in square inches, the total force exerted on the piston is $p_m a$. If the piston has a stroke of L feet, the work per stroke is $p_m aL$ and the work per minute is $p_m aLN$, where N represents the number of power[1] strokes per minute. The indicated horsepower is:

$$\text{ihp} = \frac{\text{Work per minute}}{33{,}000} = \frac{p_m aLN}{33{,}000}. \tag{3}$$

---

[1] It is necessary to specify that only power strokes are to be counted, because in a single acting steam engine, only one stroke in a revolution is effective for doing work, and in some gas engines only one stroke in two revolutions is effective.

Brake Horsepower (bhp) represents the actual power which an engine can deliver for the purposes of work. The difference between indicated and brake horsepower of an engine represents the horsepower lost in friction. The brake horsepower can be measured, by some form of friction brake, which absorbs the power measured and is called an absorption dynamometer, or by a transmission dynamometer. In either type of dynamometer, if F is the effective pull in pounds, L the lever arm, in feet, through which the pull is exerted, and N the number of revolutions of the shaft per minute, the brake horsepower is

$$\text{bhp} = \frac{2\,\pi FLN}{33{,}000}. \tag{4}$$

Heat. Heat is a form of energy. Every substance is composed of minute particles or molecules which are in constant motion or vibration. The heat of a body depends upon the vibratory motion of these small particles or molecules. The more intense the heat of a body, the greater is the vibration velocity of the molecules.

Transfer of Heat. Heat cannot of its own accord transfer itself from a body of lower temperature to that of a higher. Heat may pass from a body of higher temperature to that of a lower temperature in three different ways: by conduction, by radiation, or by convection.

The transfer of heat between the different particles of the same body is called conduction. Conduction may be internal or external. In internal conduction, heat is transmitted between the molecules of the same body, while in the case of external conduction heat is transmitted from one body to another when the two are in contact. The transfer of heat by conduction depends upon the difference in the temperature between the molecules of the different parts of the body, the material of which the body is composed, the size of the body, and the length of time during which the flow of heat occurs. The particles composing the body must be at different temperatures in order that the heat may flow by conduction. Different materials do not conduct heat equally well. The more dense the material the better its conductive power. Some substances such as cork or dry wood are poor conductors of heat. Metals are the best conductors of heat.

Radiation is the transfer of heat from one body to another through space when the two are not in contact. A steam boiler receives some of the heat by radiation from the glowing fuel on the grate and from the heated fire brick of the furnace. Rough, dark, and opaque surfaces are good radiators of heat, while smooth, highly polished, and light surfaces are good reflectors of heat, but poor radiators.

Convection is the transfer of heat by the movement of the heated particles within a gaseous or fluid substance. Various heating and ventilating systems for buildings depend upon the convection of heat for their operation. Efficient operation of steam boilers depends upon convection for the heat transfer. The draft produced by chimneys also depends upon convection heat. The loss of heat from a body by convection depends upon the shape of the body and the velocity of the convection currents.

Temperature. The heat intensity of a substance or body, or its tendency to transmit heat, is measured by its temperature. The temperature does not indicate the quantity of heat, but is a measure only of the sensible heat or of the degree of coldness or of hotness of a body. If the molecules of a body are vibrating rapidly, the temperature of a body is high; if the molecules vibrate slowly, the temperature is low.

Thermometric Scales. The two thermometric scales which are most commonly used are the Fahrenheit (F) and the Centigrade (C).

On the Fahrenheit scale the melting point of ice is taken at 32°F, and the boiling of water at atmospheric pressure 212°. Thus the Fahrenheit degree is $\frac{1}{180}$ of the temperature interval between the melting of ice and the boiling of water.

On the Centigrade scale the position of the mercury for the melting of ice is marked 0°, and that for the boiling of water at atmospheric pressure 100°. The Centigrade degree is then $\frac{1}{100}$ of the temperature interval between the melting of ice and the boiling of water. The relations between these scales are:

$$\text{Deg cent} = \tfrac{5}{9}\,[\text{deg fahr} - 32]. \tag{5}$$

$$\text{Deg fahr} = \tfrac{9}{5}\,\text{deg cent} + 32. \tag{6}$$

Mercury thermometers, as ordinarily constructed, have the space above the mercury under a vacuum. Such thermometers cannot be used for the measurement of temperatures exceeding 550 deg fahr, as the vacuum reduces the boiling point of mercury. The range of mercury thermometers can be increased to about 1000 deg fahr by filling the space above the mercury with some inert gas like nitrogen.

For the measurement of very high temperatures thermoelectric pyrometers are best suited.

Absolute Zero. In the graduation of liquid thermometers, the vaporization of the liquid at high temperatures and its freezing at low temperatures limits the thermometric range. The fundamental scale of temperature measurement is based on Thomson's absolute thermometric scale, which is independent of the nature of any thermometric substance.

The absolute zero is the temperature at which the molecular or internal kinetic energy becomes zero; in other words, the condition under which molecular motion ceases.

The absolute zero is 459.69 deg (practically 460 deg) below the zero on the Fahrenheit scale and 273.16 (practically 273.0 deg) below the zero on the Centigrade scale.

Calling the absolute temperature T and the temperature as measured by a thermometer t,

$$\text{On the Fahrenheit scale } T = t + 460. \tag{7}$$

$$\text{On the Centigrade scale } T = t + 273. \tag{8}$$

Units of Heat. A heat unit is the amount of energy required to raise the temperature of 1 unit weight of water 1 deg. In the English system of measures the heat unit is the British thermal unit (Btu), which is defined as the amount of heat required to raise the temperature of 1 lb of water 1 deg on the Fahrenheit scale. The heat unit in the metric system is the calorie (cal)[1] which is defined as the heat required to raise the temperature of 1 kilogram of water 1 deg on the Centigrade scale.

To define the British thermal unit more correctly it is necessary to state at what temperature the rise of 1 deg on the Fahrenheit scale is to occur, because the specific heat of water is slightly variable. This heat unit (Btu) is sometimes defined as the

---

[1] Since 1 kg = 2.204 lb and 1 deg cent = $\frac{9}{5}$ deg fahr, 1 kg-cal = $\frac{9}{5} \times 2.204 = 3.968$ Btu.

amount of energy required to raise the temperature of 1 lb of water 1 deg fahr at the condition of maximum density of water, that is, between 39 and 40 deg fahr. Other definitions are based on the amount of energy required to raise the temperature of water 1 deg fahr between 60 and 61 deg fahr. Still another definition, which is now accepted, defines a British thermal unit as one one-hundred-eightieth ($\frac{1}{180}$) of the amount of energy required to raise the temperature of 1 lb of water from 32 to 212 deg fahr.

Specific Heat. The addition of the same quantity of energy will not, as a rule, produce the same temperature changes in equal weights of different substances. For this reason it is necessary to allow for the relative heat capacity or the specific heat (c) of a substance. Specific heat is defined as the ratio of the heat energy added, to the temperature change produced in a unit weight of a substance. In the English system the specific heat is the number of British thermal units (Btu) required to raise the temperature of a pound of the substance 1 deg fahr.

Thus if c is constant and Q is the quantity of heat energy added to 1 lb of a substance, the temperature change is,

$$t_2 - t_1 = \frac{Q}{c},$$

or

$$Q = c(t_2 - t_1). \tag{9}$$

Since the specific heat is usually a variable,

$$Q = \int_{t_1}^{t_2} c\,dt. \tag{10}$$

The following problem illustrates the application of equation (10):

The specific heat of a substance in terms of instantaneous temperature is expressed by the equation,

$$c = 0.24112 + 0.000009\,t \text{ Btu per lb deg fahr.}$$

What amount of heat is required to raise the temperature of 1 lb of the substance from 0 deg to 100 deg fahr?

Solution. Since the specific heat is variable,

$$Q = \int_{t_1}^{t_2} c\,dt,$$

substituting the value of c and integrating,

$$
\begin{aligned}
Q &= \int_{t_1}^{t_2} (0.24112 + 0.000009\, t)\, dt \\
&= 0.24112\, [t]_{0^\circ}^{100^\circ} + 0.000009 \left[\frac{t^2}{2}\right]_{0^\circ}^{100^\circ} \\
&= 0.24112\, (100) + 0.000009\, (5000) \\
&= 24.112 + 0.045 = 24.157 \text{ Btu per lb}
\end{aligned}
$$

The specific heat of gases and vapors varies with the conditions under which the heat energy is applied. When heat energy is applied to a gas or a vapor held in a closed vessel, with no change in volume, no work is performed, and, therefore, all the energy added is used to increase the temperature. This is the condition in a boiler when no steam is being drawn off. In this case the symbol c_v represents the specific heat during heat application at *constant volume.* If, on the other hand, the heating is done while the pressure is kept constant and the volume is allowed to change permitting expansion and the performance of work, the symbol c_p is used and represents the specific heat during heat application at *constant pressure.*

When the problem deals with w pounds of a substance instead of a unit weight, equation (10) becomes,

$$
Q = w \int_{t_1}^{t_2} c\, dt. \tag{11}
$$

In many calculations an average constant value of c_p or c_v is used, as illustrated by equation (9). A table of specific heats for common gases is given in Table I of the Appendix.

Work of Flow. In a process in which the working medium is flowing at constant pressure, work energy of flow is involved. In order to exemplify the reasoning necessary in determining the amount of energy involved, assume a diverging section as illustrated in **Fig. 2**, through which the working medium is passing. Let 1 and 2 be the entrance and exit sections of the stream respectively. At 1 a constant pressure of P_1 is maintained, and at 2 a constant pressure of P_2 is maintained. In order to maintain the constant pressure of P_1 at section 1 additional fluid must be added to replace that which passes through the nozzle, while to maintain the constant pressure P_2 at section 2 the fluid must be discharged at such a rate that it will give place to that which enters the section 2. The quantity of energy and the weight of the fluid passing into section 1 must be accounted for at section 2, provided no energy or weight is added to or abstracted from the working medium between the two sections.

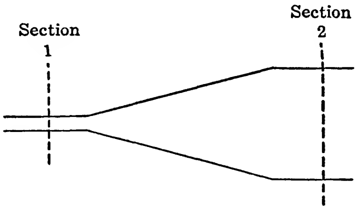

FIG. 2. Expanding Nozzle.

In order to maintain the constant pressure of P_1 at section 1 and of P_2 at section 2, the work of flow must be considered. Since each pound of the working medium which enters at section 1 must be replaced by the flow of an equal weight of the working medium, the working medium entering the section 1 is propelled into that section by a force equal to the initial pressure P_1 acting over the area of the stream. This force acts through the space traveled, and for each pound of fluid entering the nozzle the space movement is equal to the specific volume of the fluid or its volume per pound divided by the area of the stream. The work energy of flow in foot-pound units at entrance is thus

$$F_1 \times S_1 = P_1 \times A_1 \times \frac{\text{volume per pound at } P_1 \text{ conditions}}{A_1} = P_1 v_1 \tag{12}$$

where

v_1 = volume of fluid in cubic feet per pound at P_1 conditions.
P_1 = pressure in pounds per square foot at section 1.

In considering the work of flow at section 2, the outlet, the same conditions prevail, and the work of flow in foot-pounds at section 2 can therefore be written

$$P_2 v_2 \tag{13}$$

where

v_2 = volume of fluid in cubic feet per pound at P_2 conditions.
P_2 = pressure in pounds per square foot at section 2.

The Heat Engine. The heat engine is a machine which converts the stored chemical energy of solid, liquid or gaseous fuel into work. This conversion depends on burning the fuel and so producing a variation in the pressure, volume and temperature of a gas or a vapor medium. It can be accomplished in two ways:

First, by " external combustion," in which case the fuel is burned outside of the engine cylinder; the heat developed by the combustion of the fuel is conducted to the working substance or heat medium through walls; this working substance does work on a piston in the case of a reciprocating engine or on a vane or " blade " in a turbine. To this class belong steam engines of the reciprocating, turbine or rotary types and also external combustion hot air engines. Thus, in the case of the steam engine, the fuel, which may be coal, wood, petroleum or gas, is burned outside of the boiler shell and the resulting energy is transmitted by conduction through the metal of the shell to the working substance, which is water. When enough energy has been added to the water to produce a change in its physical state, water vapor or steam is formed at the required pressure. This vapor, which may be dry, wet or superheated, when allowed to act on the piston of the engine, will do work.

Another method of converting heat energy into mechanical energy is by burning the fuel rapidly or slowly inside of an engine cylinder or in a communicating vessel, the products of combustion being allowed to act directly on the piston of the engine. To this class belong gas, petroleum, and alcohol engines which are called " internal combustion " engines.

Prime Mover. The term *prime mover* is applied to the main power-generating unit in a power plant. Thus the steam turbine or the steam engine is the *prime mover* of the steam power plant; in the internal combustion power plant the gas or oil engine is the *prime mover.*

Thermal Efficiency of a Heat Engine. By thermal efficiency (e) is meant the ratio of the heat energy converted into work (AW) to the heat energy supplied to the engine (Q_1), or

$$e = \frac{AW}{Q_1}. \tag{14}$$

Since only a part of the heat energy supplied to an engine can be converted into work, the above ratio is a fraction always less than unity.

The Working Substance. In every type of heat engine a *working substance* is used. In the steam power plant this *working substance* is water vapor (steam) which absorbs heat energy and transforms a portion of this energy into work; in the internal combustion heat engine the *working substance* is a mixture of air and gaseous fuel. In a *binary heat engine* two working substances are used, each functioning through its most practical temperature range. Mercury and steam are examples of working substances used in a binary heat engine.

Mechanical Equivalent of Heat. There is a definite quantitative relation between work expended and heat energy produced. This relation between heat energy and work is called the mechanical equivalent of heat and is designated by J. In the English system,

$$J = 778 \text{ ft-lb}$$

or

$$1 \text{ Btu} = 778 \text{ ft-lb.}$$

In the metric system, $J = 427$ kilogram-meters

or

$$1 \text{ cal} = 427 \text{ kilogram-meters.}$$

The reciprocal of J, or the heat equivalent of work, is designated by A, where $A = \frac{1}{778}$ in the English system or $\frac{1}{427}$ in the metric system.

Since power is the rate of doing work there is a quantitative relation between power developed and heat expended. As a result the heat equivalent of one horsepower-hour is

$$\frac{33{,}000 \times 60}{778} = 2{,}545 \text{ Btu.} \tag{15}$$

Similarly, the heat equivalent of a kilowatt-hour is

$$\frac{2{,}545}{0.746} = 3{,}411 \text{ Btu.} \tag{16}$$

First Law of Thermodynamics. The statement of the definite relation between heat energy and mechanical work is known as the first law of thermodynamics. It is usually expressed:

Heat energy and mechanical energy are mutually convertible and heat energy requires for its production and produces by its disappearance mechanical work in the ratio of the mechanical equivalent of heat.

This law is a statement of the *conservation* of energy as regards the equivalence of mechanical work and heat.

Second Law of Thermodynamics. *The temperature of the source of heat above surrounding objects furnishes a measure of the availability of the heat for conversion into work.*

As the first law has to do with the *ratio* of thermodynamic transformations, so the second law treats of the availability of heat for conversion into mechanical energy. Analogous to this is the case of a water power plant where the height above a practical discharge level measures the availability of a source of supply.

This law is based upon experiment and states as regards heat engines the limits to their possible performance, which would be otherwise unlimited, if only the " first law " of thermodynamics is considered. It means, also, that no heat engine converts or can convert into work all of the heat energy supplied to it. A very large part of the heat energy supplied is necessarily rejected by the engine in the form of unused heat. Thus the second law is a statement of the principle of the *degradation* of energy.

Energy Equations. The formation of energy equations upon which the solution of thermodynamic processes depends is based upon the principle of the conservation of energy. This principle states that energy can neither be created nor destroyed, but may be transformed into other forms without loss.

Energy Relationships in a Non-Flow Process. A non-flow process is one in which the working medium is confined in a cylinder behind a movable piston. The effect of heat application merely causes an expansion of the medium, and hence the energies involved in such a process are those associated with a body at rest.

If a quantity of heat energy ΔQ is imparted to the working medium in such a process, the following effects may be produced:

1. The moving molecules upon receipt of additional energy will be speeded up increasing the internal kinetic energy, evidenced by a rise of temperature (ΔK).

2. The more rapidly moving molecules will go farther before being stopped by the forces holding them together (cohesion). As each goes farther before reversal, they will on the whole be farther apart and energy will be invested in thus overcoming the attractive forces to attain the new position. This will be energy of position or internal potential energy (ΔP).

3. The new position of the molecules involves a change of the size of the body as a whole which requires that something must be displaced to make room for the new volume. This is a work process which the body must perform on external bodies — external work (ΔAW).

These effects may be stated

$$\Delta Q = \Delta K + \Delta P + \Delta AW,$$

in which ΔK represents the energy utilized in increasing the internal kinetic energy; ΔP represents the energy utilized in overcoming the internal forces, and ΔAW represents the energy utilized in overcoming the external resistance.

ΔK plus ΔP represents that portion of the energy that was utilized in increasing the internal energy of the working medium and may be designated ΔU.

Thus, for the addition of an infinitesimal quantity of heat energy,

$$dQ = dU + A\,dW$$

or

$$_1Q_2 = \int_{U_1}^{U_2} dU + A\int_{V_1}^{V_2} P\,dV. \tag{17}$$

In equation (17), which represents the effect of heat application, the internal energy change $\int_{U_1}^{U_2} dU$ depends on the physical states of the body, while the external work $\int_{V_1}^{V_2} P\,dV$ depends on the character of the path in the work diagram.

Energy Relationships in a Uniform Flow Process. A uniform flow process, as in the case of steam flowing through a turbine, is one in which the working medium flows uniformly into the process and leaves in a similar manner. The energies entering into such a process are those associated with a body in motion.

In order to exemplify the theory used in developing an equation representing the application of heat energy to a uniform flow process, consider a process as indicated in **Fig. 3.** In this process 1 lb of the working substance is flowing into the process at section 1 under a condition of constant pressure (P_1) and at a higher elevation than section 2. A quantity of heat energy (Q) is added to the working substance between sections 1 and 2 and work equal to AW is done by the working substance. The working substance leaves the process under constant pressure (P_2) at section 2.

The energy in Btu in a pound of the working substance at section 1 is the sum of the following energies:

1. The potential mechanical energy (Az_1) at section 1 due to its elevation above the datum plane.
2. The internal energy (u_1) at section 1.
3. The work of flow (AP_1v_1) at section 1.
4. The kinetic mechanical energy $\left(\frac{AV_1^2}{2g}\right)$ at section 1.

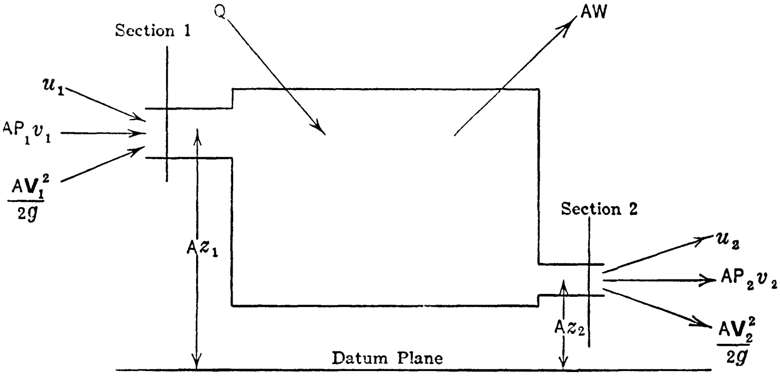

FIG. 3. Uniform Flow Process.

The energy in a pound of the working substance leaving the process is the sum of the following energies:

1. The potential mechanical energy (Az_2) at section 2.
2. The internal energy (u_2) at section 2.
3. The work of flow (AP_2v_2) at section 2.
4. The kinetic mechanical energy $\left(\frac{AV_2^2}{2g}\right)$ at section 2.

The complete process may then be expressed by using these symbols as follows:

$$Az_1 + u_1 + AP_1v_1 + \frac{AV_1^2}{2g} + Q$$

$$= AW + Az_2 + u_2 + AP_2v_2 + \frac{AV_2^2}{2g} \qquad (18)$$

The terms internal energy and work of flow always appear together in a uniform flow process and their sum is called *enthalpy* (h). Enthalpy by definition is then, in Btu per lb

$$h \text{ (enthalpy)} = u + APv.$$

Equation (18) may then be written

$$Az_1 + h_1 + \frac{AV_1^2}{2g} + Q = AW + Az_2 + h_2 + \frac{AV_2^2}{2g}. \qquad (19)$$

Neglecting the potential mechanical energy (z), equation (19) becomes

$$h_1 + \frac{AV_1^2}{2g} + Q = AW + h_2 + \frac{AV_2^2}{2g}. \qquad (20)$$

Inasmuch as there are so many forms of energy entering into an equation of a uniform flow process, the resulting equation is likely to appear cumbersome. Fortunately, in most engineering situations some of the forms of energy can be neglected because of their smallness, with the result that the final equation is much simplified.

Consider the case of the nozzle to exemplify this simplification. When a fluid such as a gas or vapor flows through a nozzle, there is no energy removed as work, the pressure of the fluid falls, its volume and velocity both increase. The increase in velocity is usually large as compared with the entrance velocity, with the result that little error is introduced by assuming in such a case that the entrance velocity and resulting kinetic mechanical energy are zero. The potential mechanical energy, owing to the fact that there is little if any difference in elevation between the entrance and exit from the nozzle, can likewise be called zero. Since the nozzle is of short length and the flow of the fluid rapid the quantity of energy as heat added to or abstracted from the fluid in its passage from entrance to exit is small and may be neglected.

Eliminating these items, the energy equation of a uniform flow process as applied to a nozzle would then be made up of the following items at entrance to the nozzle:

1. Enthalpy (h_1).

In the case of the exit from the nozzle the following energies would be considered:

1. Enthalpy (h_2).
2. Kinetic energy $\left(\frac{AV_2^2}{2\,g}\right)$.

The final equation applicable to nozzle flow would therefore be

$$h_1 = h_2 + \frac{AV_2^2}{2\,g}$$

or

$$\frac{AV_2^2}{2\,g} = h_1 - h_2. \tag{21}$$

In the case of the piston-type engine, where the inlet and outlet velocities are negligible or about equal and the heat energy supplied (Q) is considered zero, the equation becomes

$$AW = h_1 - h_2. \tag{22}$$

In the case of heat transfer apparatus, such as boilers and condensers, where no work is done and the entrance and exit velocities are small or about equal, the equation becomes

$$Q = h_2 - h_1. \tag{23}$$

In the case of throttling calorimeters (page 89) and valves where no heat energy is added, no work done and the entrance and exit velocities are small or about equal, the equation becomes

$$h_1 = h_2. \tag{24}$$

PROBLEMS

1. Given a process in which $p = (1000 + 50V)$ psi abs, where V is in cu ft. How many ft-lb of work is done when V changes from 50 to 100 cu ft?

2. A 50-hp electric motor has an efficiency of 92 per cent. How many Btu per hr must be carried away by the ventilation of the air?

3. The specific heat of a substance is expressed by the equation,

$$c = 0.52901 + 0.00059\,t.$$

(a) What heat energy is required to raise the temperature of 5 lb of the substance from 10 to 20 deg fahr? (b) What is the specific heat at 20 deg fahr?

4. If 100 Btu were abstracted from one pound of air initially at 120 deg fahr under such conditions that the specific heat = 0.241 what would be the final temperature?

5. The mean specific heat of gasoline is 0.70, that of water 1.0, and that of machine oil 0.40. (a) How many Btu will be required to heat a mixture of these from 50 deg fahr to 120 deg fahr if the mixture contains 12 lb gasoline, and 4.5 lb machine oil? (b) What is the specific heat of the mixture?

6. Calculate the indicated horsepower of a 12-in.-diameter by 13-in.-stroke steam engine which operates at a speed of 265 rpm. The mean effective pressure of the head end is 27.5 psi and of the crank end 27.8 psi. The diameter of the piston rod is $1\frac{5}{8}$ in.

7. Calculate the brake horsepower developed by an engine as measured by a prony brake, the effective pull being 32 lb at 250 rpm and the lever arm 32 in. long.

8. Prove that 1 hp developed for 1 hour is equivalent to the consumption of 2545 Btu of heat in the same period.

9. Prove that 1 kw (1000 watts) equals 3411 Btu per hour.

10. One pound of fuel has a heating value of 14,500 Btu. How many foot-pounds of work is it capable of producing, if all this heat energy is converted into work?

11. An engine developed 15,560 ft-lb of work. How much heat energy in Btu was theoretically required?

12. Write the general equation of uniform flow for a steam turbine assuming the following: the entrance and exit velocities are equal, no heat is added to or abstracted from the steam during its passage through the turbine, and the potential mechanical energies at exit and entrance are equal. Prove that in such a case, the work generated by the turbine (AW) equals the difference between the entrance and exit enthalpies.

13. A heat engine receives 100,000 Btu of heat energy in the form of fuel and during the same period 30,000 Btu are converted into work. What percentage (thermal efficiency) of the heat energy received by the engine was converted into work?

14. A gas engine receives 20,000 Btu of heat energy from a fuel, and during the same period 3,112,000 ft-lb of work are developed. What is the thermal efficiency of the engine?

15. It is claimed that a certain motor generates 300,000 ft-lb of work per hour and during this period receives 400 Btu of heat energy in the form of fuel. Are such results possible?

16. An oil engine uses 0.74 lb of fuel per bhp per hour. Calculate the thermal efficiency of this engine if the oil has a calorific value of 18,600 Btu per lb.

17. An engine receives 200 Btu of heat energy per minute and exhausts during the same period 100 Btu. If no losses of heat occur within the cylinder,

(*a*) How many Btu of heat energy are being transformed into work?
(*b*) What number of foot-pounds does this heat produce?
(*c*) What horsepower is being developed?
(*d*) What is the thermal efficiency of the engine?

18. In the manufacture of certain explosives, acids are mixed with an oxidizable substance. During the process the mixture must be constantly agitated by a stirring mechanism to maintain uniform conditions, and the temperature of the mixture must be kept below a certain predetermined value to prevent explosion. If during the process of manufacture 5000

ft-lb of work are delivered per minute to the agitator, and 1000 Btu are generated during the same period by the chemical reaction, how much heat must be absorbed per minute to maintain the temperature of the mixture constant?

19. Tests of a gas turbine show a thermal efficiency of 18 per cent. How many Btu are supplied per hp delivered by the machine?

20. The thermal efficiency of an oil engine is 27 per cent. If the calorific value of the fuel oil is 18,500 Btu per lb, what weight of oil is burned per kw-hr?

21. An internal combustion engine has a thermal efficiency of 26 per cent. If the calorific value of the fuel used is 19,200 Btu per lb, what weight of oil is burned per hp-hr?

22. During an engine test the barometer reads 28.9 in. Hg, the average steam pressure was 206.3 psi gage, and the exhaust 25.7 in. Hg vacuum. Find the absolute pressure of steam supply and exhaust.

23. A 10-hp steam engine rejects 318,000 Btu per hr. How many Btu are supplied per hp-hr and what is the thermal efficiency of the unit?

24. If a steam turbine develops 175 Btu of useful work for each lb of steam supplied, calculate the steam consumption of the turbine in lb per hp-hr, and in lb per kwhr.

CHAPTER II

PROPERTIES OF PERFECT GASES

In thermodynamics the working substance, or heat energy medium, through which the heat engine converts heat into work, is in the condition of either a gas or a vapor. The laws governing the action of the two classes of substances differ. For this reason the subject of thermodynamics is divided into the thermodynamics of gases and the thermodynamics of vapors.

A *gas* may be defined as a fluid which remains in the gaseous state when subjected to moderate changes in pressure or in temperature. Oxygen, hydrogen, nitrogen, air and carbon dioxide are examples of gases.

Vapors are fluids which are readily transformed into liquids by a moderate reduction in temperature or increase in pressure. Examples of vapors are steam and ammonia.

Relation between Pressure, Volume and Temperature of a Perfect Gas. In practically all heat engines, work is done by changes of volume of a fluid, and the amount of work performed depends only on the relation of pressure to volume during such change and not at all on the form of the vessel containing this fluid.

Extensive experimentation has led to the recognition of certain laws regarding pressure, volume, and temperature changes. These laws are followed so very closely by gases under such moderate conditions of temperature and pressure that the stability of the gases is not greatly modified by moderate changes in volume. Gases lose their stability when subjected to pressures in which the molecules are crowded close together. From experiment a set of laws has been formulated which would be followed exactly by a perfectly stable gas. Such a gas is designated as a *perfect gas.* In thermodynamic calculations all gases are ordinarily considered as perfect gases. Vapors, however, do not follow the laws of perfect gases.

Figure 4 shows a vessel containing a perfect gas and surrounded by a jacket filled with cracked ice. Its temperature will, therefore, be at 32 deg fahr. This vessel has a tightly fitting piston *P* of which the lower flat side has an area of 1 sq ft. In the position shown the piston is 2 ft from the bottom of the vessel, so that the volume between the piston and the bottom of the vessel is 2 cu ft. The pressure on the gas is that due to the piston and the weights shown. Assume this total weight is 100 lb and that the air pump connected to the top of the vessel maintains a vacuum above the piston. Then the pressure on the gas below the piston is 100 lb per sq ft. If now the weights are increased to make the pressure on the gas 200 lb per sq ft the piston will sink down until it is only 1 ft from the bottom of the vessel, provided the ice absorbs the heat equivalent of the work done by the weight in compressing the gas and thus maintain a condition of constant gas temperature. If the temperature is not maintained constant, it will be necessary to apply a total weight greater than 200 lb to reduce the volume to 1 cu ft because of the tendency of gases to expand with increase in temperature. Similarly, if the weight on the gas were reduced to 50 lb and the vessel were made high enough, the lower side of the piston would then be 4 ft from the bottom of the vessel.

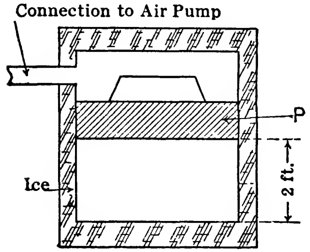

FIG. 4. Constant Temperature Apparatus for Demonstrating Relation Between Pressure and Volume of Gas.

Examination of the foregoing data shows that *if the temperature is constant the product of absolute pressure and volume is a constant,* and in this particular case it is always equal to 200. These facts are expressed by *Boyle's Law,* which can be stated as follows:

Boyle's Law. *If a given weight of gas is compressed or expanded at constant temperature, the pressure varies inversely as the volume, or the product of pressure and volume remains a constant.* Thus, if P_1, V_1 are the initial pressure and volume and P_2, V_2 the final pressure and volume,

$$P_1V_1 = P_2V_2 = P_nV_n. \tag{25}$$

Boyle's law does not hold for very high pressures.

Charles' Laws. The laws of thermodynamics dealing with volume and pressure changes corresponding to temperature variations are expressed by *Charles' Laws,* which can be stated as follows:

(1) *Under constant pressure the volume of a given mass of gas varies directly as the absolute temperature.*

(2) *Under constant volume the absolute pressure of a given mass of gas varies directly as the absolute temperature.*

These fundamental principles may also be stated thus:

$$\text{With } \textit{pressure constant,}\ \frac{V_1}{V_2} = \frac{T_1}{T_2}, \tag{26}$$

$$\text{With } \textit{volume constant,}\ \frac{P_1}{P_2} = \frac{T_1}{T_2}, \tag{27}$$

where V_1 and V_2 are respectively the initial and final volumes, P_1 and P_2 are the initial and final absolute pressures, and T_1 and T_2 are the absolute temperatures corresponding to the pressures and volumes of the same subscripts.

The following problem shows applications of Charles' laws:

A gas has a volume of 2 cu ft, a pressure of 14.7 psi abs and a temperature of 60 deg fahr.

(a) What will be the volume of this gas if the temperature is increased to 120 deg fahr, the pressure remaining constant?

(b) What will be the pressure if the temperature is increased as in (a) but the volume remains constant?

Solution. (a) Since the pressure remains constant and the substance is a gas, the volume varies directly as the absolute temperature.

Letting V_1 and T_1 be the initial conditions and V_2 and T_2 be the final conditions, then

$$\frac{V_1}{V_2} = \frac{T_1}{T_2} \quad \text{or} \quad \frac{2}{V_2} = \frac{60 + 460}{120 + 460},$$
$$V_2 = 2.23 \text{ cu ft.}$$

(b) Since the volume remains constant,

$$\frac{P_1}{P_2} = \frac{T_1}{T_2} \quad \text{or} \quad \frac{14.7}{P_2} = \frac{60 + 460}{120 + 460},$$
$$P_2 = 16.39 \text{ psi abs.}$$

Combination of Boyle's and Charles' Laws. Equations (25), (26), and (27) cannot often be used as they stand, because it does not often happen that any one of the three variables (P, V and T) remains constant. A more general law must be developed, therefore, allowing for variations in all of the terms P, V and T. This is accomplished by combining the above equations.

Assume a pound of gas of which the initial conditions of pressure, volume and temperature are represented by P_1, V_1 and T_1, while the corresponding final conditions are given by P_2, V_2 and T_2. The first step is in changing the volume from V_1 to V_2 and the pressure from P_1 to some intervening pressure P_2' while the temperature T_1 remains constant. This change can be expressed by Boyle's law equation (25).

With constant temperature (T_1),

$$\frac{V_1}{V_2} = \frac{P_2'}{P_1},$$

from which, by solving,

$$P_2' = \frac{P_1 V_1}{V_2}, \tag{28}$$

where P_2' is the resulting pressure of the gas when its volume is changed from V_1 to V_2, with the temperature remaining constant at T_1.

The second step is in the change in pressure from P_2' to P_2 and in temperature from T_1 to T_2, while the volume remains constant at V_2. This step is expressed as follows:

With constant volume (V_2),

$$\frac{P_2'}{P_2} = \frac{T_1}{T_2},$$

which may be written

$$P_2 = \frac{P_2' T_2}{T_1}. \tag{29}$$

Substituting now the value of P_2' from (28) in (29), we have

$$P_2 = \frac{P_1 V_1 T_2}{V_2 T_1},$$

which may be arranged to read,

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} = \frac{P_n V_n}{T_n}. \tag{30}$$

The following problem shows the application of equation (30): A quantity of air at atmospheric pressure has a volume of 2000 cu ft when the barometer reads 28.80 in. Hg and the temperature is 40 deg cent. What will be the volume of this air at a temperature of 0 deg cent when the barometer reads 29.96 in. Hg?

Solution. Volume, pressure, and temperature vary in this case as in the following equation,

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}.$$

Letting $P_1,\ V_1,\ T_1 =$ initial conditions,

$P_2,\ V_2,\ T_2 =$ final conditions,

then

$$\frac{28.80 \times 2000}{40 + 273} = \frac{29.96 \times V_2}{0 + 273},$$

$$V_2 = 1677 \text{ cu ft.}$$

Now, since P_2, V_2 and T_2 in equation (30) represent any simultaneous condition of the gas, we may also write the following more general relations:

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} = \frac{P_3V_3}{T_3} = \text{a constant.} \tag{31}$$

For a *unit weight* of a gas

$$\frac{Pv}{T} = R, \quad \text{or} \quad Pv = RT, \tag{32}$$

where v is the *specific volume* and R is the *gas constant.*

The Law of Perfect Gases. Equation (32) expresses the law connecting the relation between pressure, volume and temperature of a perfect gas. In this equation v is the *specific volume,* or the volume occupied by a unit weight of a gas at the absolute pressure P and absolute temperature T; R is the *gas constant* in foot-pounds per degree and depends on the density of the gas and on the units of measurement adopted.

By means of equation (32) if the pressure, volume and temperature of a gas for one given condition are known, the value of R can be determined.

Example 1. If the volume of air at freezing point and atmospheric pressure is 12.39 cu ft per lb calculate the value of R in the English units.

Solution. $R = \dfrac{P_0 v_0}{T_0} = \dfrac{14.7 \times 144 \times 12.39}{32 + 460} = 53.3.$

Example 2. If 1 lb of air occupies 5 cu ft at a temperature of 200 deg fahr, find the corresponding pressure.

Solution. Using the value of R for air as calculated above

$$P_1 = \frac{RT_1}{v_1} = \frac{53.3(460 + 200)}{5} = 7036 \text{ lb per sq ft abs}$$

In calculating R by equation (32) care must be taken not to confuse the units of measurement. It must also be remembered that the method, as illustrated in the above examples, gives the value of R for 1 lb. of air, or for 1 unit weight of the gas in question; for w pounds, the value of the constant is wR, or:

$$PV = wRT, \tag{33}$$

where P = absolute pressure in pounds per square foot,
V = volume in cubic feet,
w = weight of gas in pounds,
R = the gas constant for 1 lb of gas (in foot-pound system of units),
T = the absolute temperature in Fahrenheit degrees.

This equation is applicable to any perfect gas within the limits of pressure and temperature employed in engineering practice. The " thermodynamic " state of a gas is known when its pressure, volume, temperature, weight and composition are known; when any four of these quantities are known the fifth can be determined by equation (33).

Heat and Its Effect Upon a Gas in a Non-Flow Process. In equation (17) it was shown that in general the effect of adding heat energy to a substance is to increase the internal energy and to overcome the external resistances, thus producing work. This law can be stated as follows:

$$dQ = dU + AP\,dV.$$

This statement is general, any of the terms of which may on occasion become zero or negative. Negative values of the external work would be interpreted as work done upon the working substance; negative internal energy, as a decrease in internal energy; and negative heat energy, as heat energy withdrawn from the working substance.

External Work. The external work or the work done by a gas in its expansion is represented graphically by **Fig. 5.** The area under the expansion line **BC** is proportional to the work done in the expansion. If the initial condition of the gas at **B** as regards pressure and volume is represented by P_1 and V_1 and the final condition at **C** by P_1 and V_2 (expansion being at constant pressure), the force P moved through a distance of abscissas $(V_2 - V_1)$ is a measure of the work done. The area under the line **BC** divided by the horizontal length $(V_2 - V_1)$ is the average value of the force P. If, further, and in general, we represent the area under **BC** by the symbol α, then we can write,

$$\frac{\alpha}{V_2 - V_1} = \text{average value of } P,$$

whether or not P is constant.

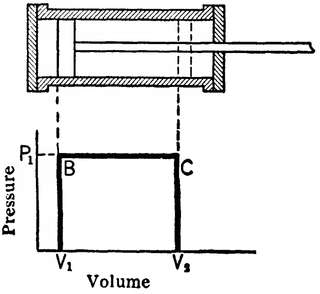

Fig. 5. External Work of Expansion.

The same principle applies whether **BC** is a straight line, as shown in **Fig. 5**, or any curve, as will be shown later.

Internal Energy. The amount of heat added to a substance when no work is performed is utilized in increasing the internal energy of that substance. On the other hand, when heat is added while work is being performed, the internal energy is increased only by the difference between the heat added and the work done.

As a result, two methods are available in determining the heat energy required to produce an internal energy change. One is to measure the heat required to produce a process in which no work is done and no heat is lost. All the heat added to the working medium would then appear as internal energy. If such a process is to be carried on experimentally, a piston would be clamped in a perfectly insulated cylinder so that no movement and consequently no work is done. In such a process, the heat would be supplied under constant volume conditions. Since no work is done, the change of internal energy equals the heat energy supplied, or

$$\text{Change of internal energy} = \int c_v\, dT \quad \text{(in Btu per pound)},$$

where T is the absolute temperature and c_v the specific heat at constant volume. The value of c_v takes into account only that heat required to raise the temperature, since under constant volume conditions no external work is done; and, therefore, in dealing with internal energy, c_v is always used for gases.

Increase in internal energy in Btu (for 1 lb of a gas) is then (at constant c_v)

$$u_2 - u_1 = c_v(T_2 - T_1). \tag{34}$$

The other method of determining the internal energy is to measure the heat required to produce a process in which the external work done is calculated or measured. The internal energy is then the heat required to produce such a process minus the heat equivalent of the work done. If the internal energy is to be determined experimentally by using this method, the insulated cylinder and piston can be arranged in such a way that the pressure of the piston upon the gas is constant. The heat energy supplied to produce the process is then at constant pressure, and the change in internal energy is

$$u_2 - u_1 \text{ (for 1 lb of a gas)} = \int c_p\, dt - A\int P\, dv.$$

Over a finite range this equation becomes (at constant c_p)

$$u_2 - u_1 \text{ (for 1 lb of a gas)} = c_p(T_2 - T_1) - AP_1(v_2 - v_1) \tag{35}$$

Joule's Law. Joule's Law is a statement that *the internal energy of a gas is a function of temperature only.* In the case of ideally perfect gases, such as thermodynamic equations must deal with, it is assumed, when a gas expands without doing external work and without taking in or giving out heat (and, therefore, without changing its stock of internal energy), that its temperature does not change. It was for a long time supposed that when a real gas expanded without doing work, and without taking in or giving out heat, that its temperature did not change. This fact was based on the famous experiments of Joule. Later investigations by Lord Kelvin and Joule have shown that this statement is not exactly correct as all known gases show a change in temperature under these conditions. This change in temperature, characteristic of real gases, is known as the " Joule-Thomson " effect.

Relation of Specific Heats and the Gas Constant. If heat is added at *constant pressure*, then for a constant specific heat

$$Q = wc_p(T_2 - T_1). \tag{36}$$

Also, by equation (34), the increase in internal energy when heat is added (at constant c_v)

$$\begin{aligned} &= wc_v(T_2 - T_1). \\ \text{External work} &= P(V_2 - V_1) \text{ ft-lb.} \\ &= \frac{P(V_2 - V_1)}{778} \text{Btu.} \end{aligned}$$

Since the heat added = increase in internal energy + external work

$$wc_p(T_2 - T_1) = wc_v(T_2 - T_1) + \frac{P(V_2 - V_1)}{778}. \tag{37}$$

By equation (33),

$$P_2V_2 = wRT_2 \quad \text{and} \quad P_1V_1 = wRT_1.$$

Substituting these values in (37) since $P_1 = P_2$,

$$wc_p(T_2 - T_1) = wc_v(T_2 - T_1) + w\frac{R(T_2 - T_1)}{778}.$$

Simplifying
$$c_p = c_v + \frac{R}{778}$$

$$c_p - c_v = \frac{R}{778} = AR. \tag{38}$$

Equation (38) shows that the difference between the two specific heats is equal to the gas constant R, which when measured in foot-pounds represents the external work done by 1 lb of a gas when its temperature is increased by 1 deg under *constant pressure.*

Example. The specific heat of air at constant pressure (c_p) is 0.2375 Btu and $R = 53.3$ ft-lb. Calculate the specific heat at constant volume (c_v).

Solution.

$$c_v = c_p - AR = 0.2375 - \frac{53.3}{778} = 0.1690 \text{ Btu.}$$

Ratio of the Two Specific Heats $\left(\frac{c_p}{c_v}\right)$. The constant representing the ratio of the two specific heats of a perfect gas is represented by k where

$$k = \frac{c_p}{c_v} = \frac{c_p}{c_p - AR} = \frac{1}{1 - \frac{AR}{c_p}}. \tag{39}$$

Example. Calculate the value of k for air. $c_p = 0.2375$ Btu. $R = 53.3$ ft-lb.

Solution. $$k = \frac{1}{1 - \frac{AR}{c_p}} = \frac{1}{1 - \frac{53.3}{778 \times 0.2375}} = 1.405.$$

Values of the Specific Heats. Regnault after conducting experiments on the specific heat of hydrogen, oxygen, air and carbon dioxide concluded that any gas has a constant specific heat under varying pressures and temperatures. Recent experiments tend to show that the specific heats of substances vary with the pressure and temperature. The variability in the values of the specific heats does not influence to any very great extent most thermodynamic computations.

In the application of thermodynamics to internal combustion engines the exact values of the specific heats are of considerable importance.

Tables 1 and 2 in the Appendix give constants for various gases.

Avogadro's Law. *Equal volumes of all gases at the same temperature and pressure contain the same number of molecules.* This means that the volume of a molecule is the same for all gases at the same temperature and pressure, or that the densities of any two gases at the same temperatures and pressures are proportional to their molecular weights.

If v is the specific volume of a gas, d its density, and m its molecular weight,

$$\frac{v_1}{v_2} = \frac{d_2}{d_1} = \frac{m_2}{m_1}. \tag{40}$$

The Mol. The term mol refers to the weight of a definite number of molecules of a gas, which, in the case of oxygen, is 32 lb. It follows from Avogadro's Law that *the volume of* 1 *mol is the same for all gases, when they are at the same temperature and pressure.* The mol is the molecular weight, m, of the gas. Thus,

$$1 \text{ mol} = m \text{ lb}. \tag{41}$$

For oxygen, $m = 32$; thus 1 mol of oxygen = 32 lb.
For ammonia, $m = 17$; thus 1 mol of ammonia = 17 lb.

Since the weight of 1 cu ft of oxygen = 0.08927 lb at 32 deg fahr and 14.7 psi abs, the molecular volume of oxygen $= \dfrac{32}{0.08927} =$ 358.7 cu ft.

Accordingly, from Avogadro's law, *the volume of any gas at* 32 *deg fahr and* 14.7 *psi abs* = 358.7 *cu ft per mol.*

Equation (32) may be written

$$Pv_m = RTm.$$

Since the volume of 1 mol at 32 deg fahr and 14.7 psi abs = 358.7

$$mR = \frac{14.7 \times 144 \times 358.7}{492} = 1544$$

$$R = \frac{1544}{\text{molar weight}}. \tag{42}$$

The constant in equation (42) is not exactly true for real gases because of their departure from perfect gases, but for engineering calculations it is sufficiently accurate to calculate R for any gas from this equation.

Dalton's Law. *If several gases are contained in a closed vessel, the pressure exerted by the mixture on the walls of the vessel is equal to the sum of the pressures exerted by the constituent gases.* This law of "partial pressures" holds true if the gases contained in a closed vessel have no chemical affinity and each gas behaves as if the other gases were not present. If P_m is the total pressure in a mixture and P_1, P_2, P_3 etc. are the pressures of the constituent gases, Dalton's Law may be expressed:

$$P_m = P_1 + P_2 + P_3 + \cdots + P_n. \tag{43}$$

If the weights of the constituent gases are w_1, w_2, $w_3 \cdots w_n$, and if these are perfect gases then from equation (33):

$$P_1 V_m = w_1 R_1 T_1, \quad \text{or} \quad P_1 = \frac{w_1 R_1 T_1}{V_m}$$

$$P_2 V_m = w_2 R_2 T_2, \quad \text{or} \quad P_2 = \frac{w_2 R_2 T_2}{V_m}$$

$$P_3 V_m = w_3 R_3 T_3, \quad \text{or} \quad P_3 = \frac{w_3 R_3 T_3}{V_m}$$

Hence in a mixture in which the final conditions of temperature and volume are represented by T_m and V_m, the total pressure is:

$$P_m = \sum P = \frac{(w_1 R_1 T_1 + w_2 R_2 T_2 + w_3 R_3 T_3 + \cdots + w_n R_n T_n)}{V_m}$$

$$= \frac{\sum (wR) T_m}{V_m}. \tag{44}$$

Equation (44) is the characteristic law of gas mixtures and may be written:

$$P_m = \frac{w_m R_m T_m}{V_m}. \tag{45}$$

$$R_m = \frac{\sum (wR)}{w_m}. \tag{46}$$

The specific heats of mixtures may be expressed:

$$c_{p_m} = \frac{w_1 c_{p_1} + w_2 c_{p_2} + \cdots + w_n c_{p_n}}{w_m} = \frac{\sum (w c_p)}{w_m}. \tag{47}$$

$$c_{v_m} = \frac{\sum (w c_v)}{w_m}. \tag{48}$$

PROBLEMS

1. Air at constant pressure with an initial volume of 2 cu ft and temperature of 60 deg fahr is heated until the volume is doubled. What is the resulting temperature in degrees Fahrenheit?

2. Air is cooled at constant volume. The initial pressure is 30 psi abs and the initial temperature is 101 deg fahr. The final condition has a temperature of 50 deg fahr. What is the final pressure?

3. One pound of hydrogen is cooled at constant pressure from a volume of 1 cu ft and temperature of 300 deg fahr to a temperature of 60 deg fahr. What is the resulting volume?

4. A tank whose volume is 50 cu ft contains air at 105 psi abs pressure and temperature of 80 deg fahr. How many pounds of air does the tank contain?

5. An automobile tire has a mean diameter of 34 in. and a width of 4 in. It is pumped to 80 psi gage pressure at a temperature of 60 deg fahr and the atmospheric pressure is 14.6 psi abs.

(a) How many pounds of air does the tire contain?
(b) Assuming no change of volume, what would be the gage pressure of the tire if placed in the sun at 100 deg fahr?

6. A gas tank is to be made to hold 0.25 lb of acetylene when the pressure is 250 psi gage, the atmospheric pressure 14.4 psi abs, and the temperature of the gas 70 deg fahr. What will be its volume in cubic feet? The specific volume of acetylene gas is 13.78 cu ft at 32 deg fahr and 29.92 in. of mercury pressure.

7. A quantity of air at a temperature of 70 deg fahr and a pressure of 15 psi abs has a volume of 5 cu ft. What is the volume of the same air when the pressure is changed at constant temperature to 60 psi abs?

8. How many pounds of air are required for the conditions in problem 7?

9. The volume of a quantity of air is 10 cu ft at a temperature of 60 deg fahr when the pressure is 15 psi abs. What is the pressure of this air when the volume becomes 60 cu ft and the temperature 60 deg fahr?

10. How many pounds of air are required for the conditions in problem 9?

11. A tank contains 200 cu ft of air at a temperature of 60 deg fahr and a pressure of 200 psi abs.

(a) What weight of air is present?
(b) How many cubic feet will this air occupy at 14.7 psi abs and at a temperature of 100 deg fahr?

12. The volume of a quantity of air at 70 deg fahr and at a pressure of 14.2 psi abs is 20 cu ft. What is the temperature of this air when the volume becomes 5 cu ft and the pressure 80 psi abs?

13. A mixture of gases is compressed from 80 cu ft to 20 cu ft according to the equation $p = \frac{2400}{V} - \frac{V}{8}$, where p is expressed in psi abs and V in cu ft. How much work is necessary?

14. If 650 Btu are supplied to a metallic bar which expands, raises a weight and does 77,800 ft-lb of external work, has the internal energy increased or decreased and by how much?

15. A tank contains one cu ft of oxygen at 2500 psi abs and 90 deg fahr. How many lb of oxygen does it contain? If the maximum safe pressure for the tank is 3500 psi abs what is the maximum safe temperature to which the oxygen may be heated?

16. Calculate the values of c_p and c_v for helium if $R = 386.3$ and $k = 1.667$.

17. A tank filled with 200 cu ft of air at 15 psi abs and 60 deg fahr is heated to 150 deg fahr.

(a) What will be the resulting air pressure in the tank?
(b) How many Btu will be required to heat the air?

18. A tank contains 200 cu ft of air at 60 deg fahr and 40 psi abs. If 500 Btu of heat are added to it, what will be the resulting pressure and temperature?

19. Calculate the molar volume of any gas at 212 deg fahr and 75 psi gage pressure.

20. If R for air is 53.3 calculate the molar weight of air.

21. Utilizing equation (42) calculate the value of R for oxygen and hydrogen.

22. Calculate the molecular weight of air at 32 deg fahr and 14.7 psi abs.

23. Calculate the *gas constant*, R, of a mixture of equal parts by weight of air and marsh gas at atmospheric pressure and temperature of 70 deg fahr.

24. Calculate the specific heat, c_p, of a mixture of equal parts by weight of air and carbon monoxide.

25. A gasoline engine requires 15 lb of air for each lb of gasoline. How many cfm of air will have to be supplied to an 80-hp engine which burns 44 lb of gasoline per hr, the air to be supplied 100 deg fahr and 14.5 psi abs?

CHAPTER III

THERMODYNAMIC PROCESSES FOR GASES

As was pointed out in Chapter I, there are two distinct types of thermodynamic processes, non-flow processes and uniform flow processes. Since the method of solution of problems dealing with the two processes differs, they will be taken up in order.

NON-FLOW PROCESSES FOR GASES

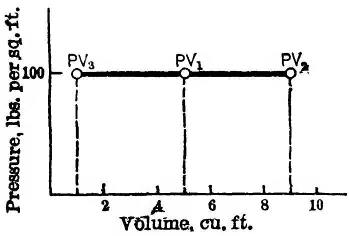

FIG. 6. Diagram of Expansion and Compression at Constant Pressure.

The equation of the perfect gas in the form $PV = wRT$ for the expansion or compression of gases has three related variables, (1) *pressure,* (2) *volume* and (3) *temperature.* For a given weight of gas with any two of these variables known the third is fixed. As regards the analysis of the action of heat engines, the pressure and volume relations are most important, and graphical diagrams, called pressure-volume or P-V diagrams, are frequently needed to assist in the analysis. The indicator diagram is a pressure-volume diagram drawn autographically by the mechanism of an engine indicator. A pressure-volume diagram is shown in **Fig. 6,** in which the vertical scale of coördinates represents *pressures* and the horizontal, *volumes.* Assume that the pressure and volume of a pound of a gas are given by the coördinates P and V_1, which are plotted in the middle of the diagram. It will be assumed further that the pressure remains constant in the changes to be indicated. Now if the gas is expanded until its volume becomes V_2, then its condition as regards pressure and volume would be represented by PV_2. If, on the other hand, the gas had been compressed while a constant pressure was maintained, its final condition would be represented by the point

PV_3 to the left of PV_1. Similarly, any path whether straight or curved extending from the initial condition of the gas at PV_1 will represent an expansion when drawn in the direction away from the zero of volumes and will represent a compression when tending toward the same zero.

It has been shown (page 4) that areas on such diagrams represent the product of pressure and volume, and, therefore, work or energy. Thus in **Fig. 6** the area under the curve PV_3 to PV_2 represents on the scales given 100 (lb per sq ft) $\times$ $(9 - 1)$ cu ft or 800 ft-lb irrespective of whether it is an expansion or a compression from the initial condition.

Most of the thermodynamic processes to be studied in heat engine diagrams can be exactly or approximately represented by an equation in the form

$$PV^n = \text{a constant}, \tag{49}$$

where the index n, as experimentally determined, has varying numerical values, but is constant for any one curve. When the processes are represented by straight lines, the areas of simple rectangles and triangles only need be calculated to find the work done. The two most common forms of curves to be dealt with in expansions represent:

(1) Expansion with addition of heat at such a rate as to maintain the temperature of the gas constant throughout the expansion. Such an expansion is called *isothermal.*

(2) Expansion in which work is done by the gas without the addition or abstraction of heat. To do this work some of the internal energy contained in the gas must be transformed in proportion to the amount of work done. Such an expansion is called *adiabatic.*

The following problems show the application of the foregoing principles to various types of expansions:

1. Energy Changes at Constant Pressure. One pound of air having an initial temperature of 60 deg fahr is expanded to 100 deg fahr under constant pressure. Find:

(a) External work during expansion;

(b) Heat required to produce the expansion.

Solution. The heat added equals the increase in internal energy plus the external work done. In solving for the heat added or required during any expansion it is necessary only to find the external work (which is equal to the area under the expansion curve) and add to it the heat needed to increase the internal energy.

The external work $= W = P_1(V_2 - V_1)$.
Its equivalent is: $wR(T_2 - T_1)$.
$W = 1 \times 53.3\,[(100 + 460) - (60 + 460)] = 2132$ ft-lb.

The increase in internal energy:

$$\begin{aligned} U_2 - U_1 &= wc_v(T_2 - T_1) \\ U_2 - U_1 &= 1 \times 0.169\,[(100 + 460) - (60 + 460)] \\ &= 6.76 \text{ Btu.} \end{aligned}$$

Heat required $(Q) = 6.76 + \frac{2132}{778} = 9.50$ Btu.

Another method of computing the heat required to produce the expansion is:

$$\begin{aligned} Q &= wc_p(T_2 - T_1). \\ Q &= 1 \times 0.237\,[(100 + 460) - (60 + 460)] \\ &= 9.50 \text{ Btu.} \end{aligned}$$

2. Energy Changes at Constant Volume. One pound of air having an initial temperature of 60 deg fahr is heated at constant volume until the final temperature is 100 deg fahr. Find:

(a) External work;
(b) Heat required.

Solution.

Heat added (Q) = increase in internal energy + external work.
External work = 0.

Then

$$\begin{aligned} \text{Heat added} &= \text{increase in internal energy} + 0 \\ &= wc_v(T_2 - T_1) + 0 \\ &= 1 \times 0.169\,[(100 + 460) - (60 + 460)] + 0 \\ &= 6.76 \text{ Btu.} \end{aligned}$$

3. Energy Changes at Constant Temperature (Isothermal). In an isothermal expansion or compression the temperature of the working substance is kept constant throughout the process. The form of the isothermal curve on pressure-volume coordinates depends upon the substance. In the case of perfect gases Boyle's law (equation 25) applies, or

$$PV = C = \text{a constant.} \tag{50}$$

Equation (50) is that of a rectangular hyperbola. It is the special case of the general equation $PV^n = \text{constant}$ (49), in which the exponent $n = 1$ and is represented by **Fig. 7.**

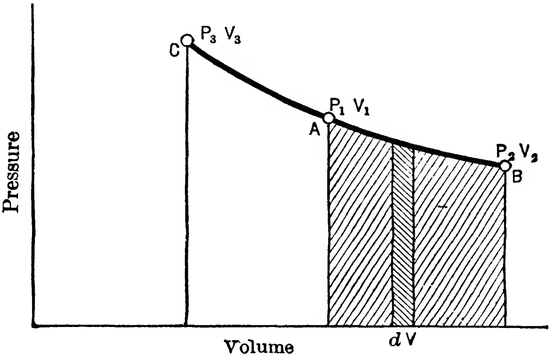

FIG. 7. Work Done During Isothermal Expansion and Compression.

The external work performed is shown graphically by the shaded area under the curve between **A** and **B** (**Fig. 7**). The two vertical lines close together in the figure are the limits of a narrow closely shaded area and indicate an infinitesimal volume change dV. Work done during this small change of volume is:

$$dW = P\,dV,$$

and for a finite change of volume of any size as from V_1 to V_2 the work done, W (foot-pounds), is:

$$W = \int_{V_1}^{V_2} P\,dV. \tag{51}$$

For the integration of this form it is necessary to substitute P in terms of V. Assume that P and V are values of pressure and volume for any point on the curve of expansion of a gas of which the equation is

$$PV = C.$$

Then
$$P = \frac{C}{V}.$$

Substituting this value of P in equation (51),

$$W = \int_{V_1}^{V_2} \frac{C}{V} dV = C \int_{V_1}^{V_2} \frac{dV}{V}.$$

$$W = C (\log_e V_2 - \log_e V_1). \tag{52}$$

Since the initial conditions of the gas are P_1 and V_1,

$$PV = C = P_1V_1,$$

and substituting this value of C in equation (52),

$$W = P_1V_1 (\log_e V_2 - \log_e V_1)$$

or

$$W = P_1V_1 \log_e \frac{V_2}{V_1} \text{ (in foot-pounds).} \tag{53}$$

Units of weight do not enter in equations (52) and (53). For a certain weight of gas under the same conditions, since

$$P_1V_1 = wRT \text{ (in foot-pounds)} \quad \text{and} \quad \frac{V_2}{V_1} = \frac{P_1}{P_2},$$

then the work for w pounds is:

$$W = wRT \log_e \frac{V_2}{V_1} = wRT \log_e \frac{P_1}{P_2} \text{ (in ft-lb).} \tag{54}$$

Often the ratio $\frac{V_2}{V_1}$ is called the *ratio of expansion* and is represented by r. Making this substitution we have,

$$W = wRT \log_e r. \tag{55}$$

Equations (52) to (55) refer to an expansion from P_1V_1 to P_2V_2. If, on the other hand, the work done is the result of a compression from P_1V_1 to P_3V_3 the curve of compression would be from A to C and the area under it would be its graphical representation. Equations (53), (54) and (55) would represent the work done for compression the same as for expansion, except that the expression would have a *negative* value; that is, work is to be done *upon* the gas to decrease its volume.

The isothermal expansion or compression of a perfect gas causes no change in its stock of internal energy since the temperature T is constant. During such an expansion the gas must *take in an amount of heat just equal to the work it does,* and conversely during an isothermal compression it must *reject an amount of heat just* *equal to the work spent upon it.* This quantity of heat Q (in Btu) is, from equation (54),

$$Q = \frac{wRT}{778} \log_e \frac{V_2}{V_1}. \qquad (56)$$

The following problem shows the application of the foregoing formulas to isothermal expansions:

Air having a pressure of 100 lb psi abs and a volume of 1 cu ft expands isothermally to a volume of 4 cu ft. Find:

(a) External work of the expansion;
(b) Heat required to produce the expansion;
(c) Pressure at end of expansion.

Solution. (a) Since the expansion is isothermal, then from equation (53),

$$\text{External work,} \quad W = P_1V_1 \log_e{}^* \frac{V_2}{V_1}$$
$$= 100 \times 144 \times 1 \times 1.3848$$
$$= 19{,}941 \text{ ft-lb.}$$

(b) Since the heat added equals increase in internal energy plus external work, and since the temperature remains constant (requiring therefore no heat to increase the internal energy), the internal energy equals zero and the heat added equals the work done.

$$\text{Then: Heat added} = \text{external work} = \frac{19{,}941}{778} = 25.6 \text{ Btu.}$$

(c) Since $P_1V_1 = P_2V_2,$

then $100 \times 1 = P_2 \times 4,$

$$p_2 = 25 \text{ psi abs.}$$

If a gas expands and does external work without receiving a supply of heat from an external source, it must derive the amount of heat needed to do the work from its own stock of internal energy. This process is then necessarily accompanied by a lowering of temperature and the expansion obviously is not isothermal.

4. Adiabatic Expansion and Compression. In the adiabatic mode of expansion or compression the working substance neither

---

* 2.3 × log base 10 = log base *e*. Tables of natural logarithms are given in the Appendix.

receives nor rejects heat as it expands or is compressed. A curve which shows the relation of pressures to volumes in such a process is called an adiabatic curve (see **Fig. 8**). In any adiabatic process the substance is neither gaining nor losing heat by conduction, radiation or internal chemical action. Hence the work which a gas does in an adiabatic expansion is all done at the expense of its stock of internal energy, and the work which is done upon a gas in an adiabatic compression all goes to increase its internal energy. Ideally adiabatic action could be secured by a gas expanding, or being compressed, in a cylinder which in all parts is a perfect nonconductor of heat. The compression of a gas in a cylinder is approximately adiabatic when the process is very rapidly performed, but when done so slowly that the heat has time to be dissipated by conduction the compression is more nearly isothermal. **Figure 8** shows on a pressure-volume diagram the relation between an isothermal and an adiabatic form of expansion or compression from an initial condition P_1V_1 at **A** to final conditions at **B** and **C** for expansions, and at **D** and **E** for compressions.

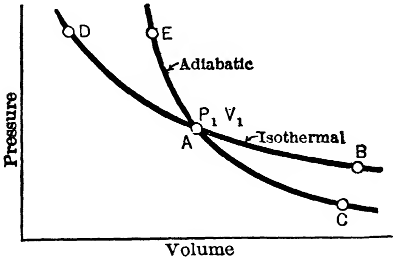

FIG. 8. Isothermal and Adiabatic Expansion Lines.

In order to derive the pressure-volume relation for a gas expanding adiabatically, consider the fundamental equation (page 15):

Heat added = increase in internal energy + external work, or:

$$Q = wc_v(T_2 - T_1) + A\int P\,dV.$$

In the adiabatic expansion no heat is added to or taken away from the gas by conduction or radiation, and, therefore, the left-hand member of the above equation becomes zero. Furthermore, since equation (33) can always be applied to perfect gases, the following simultaneous equations may be written:

$$0 = wc_v\,dT + AP\,dV, \tag{57}$$

$$PV = wRT. \tag{58}$$

When P, V and T vary, as they do in adiabatic expansion, equation (58) may be differentiated as follows:

$$P\,dV + V\,dP = wR\,dT, \tag{59}$$

and

$$dT = \frac{P\,dV + V\,dP}{wR}.$$

Substituting the value of dT in (57),

$$AP\,dV + wc_v\left(\frac{P\,dV + V\,dP}{wR}\right) = 0,$$

$$ARP\,dV + c_vP\,dV + c_vV\,dP = 0.$$

To separate the variables divide by PV:

$$AR\frac{dV}{V} + c_v\frac{dV}{V} + c_v\frac{dP}{P} = 0.$$

Collecting terms,

$$(AR + c_v)\frac{dV}{V} + c_v\frac{dP}{P} = 0.$$

Since from equation (38)

$$AR + c_v = c_p,$$

then

$$c_p\frac{dV}{V} + c_v\frac{dP}{P} = 0.$$

Integrating

$$c_p \log_e V + c_v \log_e P = \text{a constant} = c$$

$$\frac{c_p}{c_v}\log_e V + \log_e P = c,.$$

from which

$$\log_e PV^{\frac{c_p}{c_v}} = c. \tag{60}$$

From equation (39),

$$\frac{c_p}{c_v} = k.$$

Therefore

$$PV^k = \text{a constant}. \tag{61}$$

Following the method used for obtaining an expression for the work done in an isothermal expansion (equation 53), the work done, W (in foot-pounds), for a change of volume from V_1 to V_2,

$$W = \int_{V_1}^{V_2} P\,dV. \tag{62}$$

For purposes of integration, P can be substituted in terms of V as outlined below. In the expression $PV^k = c$, a constant, where P and V are values of pressure and volume for any point on the curve of expansion of a gas of which the initial condition is given by the symbols P_1 and V_1, we can then write,

$$P = \frac{c}{V^k}\cdot \tag{63}$$

And substituting (63) in (62),

$$\begin{aligned} W &= \int_{V_1}^{V_2} \frac{c}{V^k}\,dV = c\int_{V_1}^{V_2} \frac{dV}{V^k} \\ &= c\left[\frac{V^{-k+1}}{-k+1}\right]_{V_1}^{V_2} \\ &= c\left[\frac{V_2^{1-k} - V_1^{1-k}}{1-k}\right]. \end{aligned} \tag{64}$$

Since $PV^k = c = P_1V_1^k = P_2V_2^k$, there can be substituted for c in (64) the values corresponding to the subscripts of V as follows:

$$W = \frac{P_2V_2^kV_2^{1-k} - P_1V_1^kV_1^{1-k}}{1-k},$$

$$W = \frac{P_2V_2 - P_1V_1}{1-k},$$

or

$$W = \frac{P_1V_1 - P_2V_2}{k-1}\ \text{(ft-lb)}. \tag{65}$$

Since

$$PV = wRT,$$

$$W = \frac{wR(T_1 - T_2)}{k-1}\ \text{(ft-lb)}. \tag{66}$$

The development of equations (65) and (66) is entirely general, and equations for any value of n would have the same form.

The adiabatic case developed is for the special condition where $n = \frac{c_p}{c_v} = k$.

Change of Internal Energy During Adiabatic Processes. Since in an adiabatic expansion no heat is conducted to or away from the gas, the work is done at the expense of the internal energy and, therefore, the latter decreases by an amount equivalent to the amount of work performed. This loss in internal energy is readily computed by equations (65) or (66). The result must be divided by 778 in order to be in Btu.

During an adiabatic compression the reverse effect occurs, i.e., there is a gain in internal energy and the same formulas apply, the result coming out negative, because work has been done on the gas.

Volume, Pressure and Temperature Changes in an Adiabatic Expansion of a Perfect Gas. Since P, V and T vary during an adiabatic expansion, it will be necessary to develop formulas for obtaining these various quantities. It will be remembered that equation (33) applies to perfect gases at all times. Therefore, in the case of an adiabatic expansion or compression the two simultaneous equations may be written:

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}. \tag{67}$$

$$P_1V_1^k = P_2V_2^k. \tag{68}$$

By means of these two equations the final conditions of pressure, volume and temperature can be determined, having given two initial conditions and one final condition.

For instance, having given V_1, V_2 and T_1, to find T_2, divide (67) by (68), member for member. Then

$$\frac{V_1}{T_1V_1^k} = \frac{V_2}{T_2V_2^k},$$

$$\frac{T_2}{T_1} = \frac{V_2^{1-k}}{V_1^{1-k}},$$

$$T_2 = T_1\left(\frac{V_2}{V_1}\right)^{1-k},$$

or

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{k-1} \tag{69}$$

In like manner the following formulas can be obtained:

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}, \tag{70}$$

$$P_2 = P_1\left(\frac{V_1}{V_2}\right)^{k}, \tag{71}$$

$$P_2 = P_1\left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}}, \tag{72}$$

$$V_2 = V_1\left(\frac{P_1}{P_2}\right)^{\frac{1}{k}}, \tag{73}$$

$$V_2 = V_1\left(\frac{T_1}{T_2}\right)^{\frac{1}{k-1}} \tag{74}$$

It should be noted that the above formulas can be used for any expansion of a perfect gas following PV^n = a constant, provided k in the formulas is replaced by n.

It is also to be noted that these equations can be used for any system of units so long as the same system of units is employed throughout an equation and both pressure and temperature are absolute.

There are many cases of expansions which are neither adiabatic nor isothermal and which are not straight lines on P-V diagrams. It will be observed from the equations in the discussion of the internal work done by an expanding gas and for the change of internal energy, that if in the general equation PV^n = a constant the exponent or index n is less than k, the work done is greater than the loss in internal energy. In other words for such a case, the expansion lies above an adiabatic and the gas must be taking in heat as it expands. On the other hand, if n is greater than k the work done is less than the loss of internal energy.

Example. Given a quantity of pure air in a cylinder at a temperature of 60 deg fahr ($T_1 = 460 + 60 = 520$ deg fahr abs) which is suddenly (adiabatically) compressed to half its original volume. Then $\frac{V_1}{V_2} = \frac{2}{1}$, and taking k as 1.405, the temperature immediately after compression is completed, T_2, is calculated by equation (69) as follows:

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{k-1} = 520\left(\frac{2}{1}\right)^{1.405-1} = 520 \times 2^{0.405} = 688 \text{ deg fahr abs,}$$

or t_2 in ordinary Fahrenheit is 688 − 460 or 228 deg fahr.

The work done in an adiabatic compression of 1 lb of this air is calculated by equation (66):

$$W = \frac{wR(T_1 - T_2)}{k - 1} = \frac{53.3(520 - 688)}{1.405 - 1} = \frac{53.3(-168)}{0.405} = -22{,}110$$

ft-lb per lb of air compressed. The negative sign means that work was done on the gas, or the gas was compressed. If the sign were positive it would indicate an expansion.

As the result of this compression the internal energy of the gas is increased by $\frac{22{,}110}{778}$ Btu, but if the cylinder is a conductor of heat, as in practice it always is, the whole of this heat will become dissipated in time by conduction to the surrounding air and other bodies, and the internal energy will gradually return to its original value as the temperature of the gas comes back to the initial temperature of 60 deg. fahr.

The *polytropic expansion* ($PV^n = c$) in equation (49) differs from the adiabatic expansion ($PV^k = c$) in equation (61) in that heat may be added or removed during the polytropic process. Ordinarily the value of n in a polytropic expansion is greater than unity and less than k or $\left(\frac{c_p}{c_v}\right)$.

UNIFORM FLOW PROCESSES FOR GASES

All of the theory developed for non-flow processes is applicable to uniform flow processes with some additional considerations. In order to exemplify these principles let us consider the centrifugal blower as used in the handling of air or other perfect gases. In this case, air or other gas is drawn into the machine, compressed, and discharged at some higher delivery pressure by the external work applied to the shaft. Assuming that the shaft work input or work energy required to operate the blower is of chief importance and it is desired to evaluate the various energy changes in the process which must be supplied by the shaft work, we have the following energy changes that must be considered. These will be developed on a basis of 1 lb of the fluid flowing, and the subscripts 1 and 2 will be used to designate entrance and exit conditions respectively.

(1) The change in *mechanical potential energy* z in foot-pounds if the inlet and outlet are at appreciable differences of height is

$$z_2 - z_1. \tag{75}$$

(2) The change in internal energy in foot-pounds is

$$778(u_2 - u_1) = c_v(t_2 - t_1)778. \tag{76}$$

(3) The change in mechanical kinetic energy in foot-pounds is

$$\frac{V_2^2 - V_1^2}{2g}. \tag{77}$$

(4) The change in flow work in foot-pounds is

$$P_2v_2 - P_1v_1. \tag{78}$$

(5) The work of compression is determined as in the case of a non-flow process. If it is assumed that no heat energy is added to or abstracted from the fluid and the work of compression is stored in the fluid as increase in internal energy, the compression is adiabatic and the work of compression in foot-pounds is, according to equations (62), (65) and (66):

$$W = \int P\,dv = \frac{P_1v_1 - P_2v_2}{k-1} = \frac{R(T_1 - T_2)}{k-1}. \tag{79}$$

(6) If the work of compression is isothermal the energy required in foot-pounds would be as stated in equations (54) and (55):

$$W = P_1v_1 \log_e \frac{v_2}{v_1} = RT \log_e r. \tag{80}$$

In formulating the general energy equation applicable to this case, it is easily seen that the shaft work W equals the changes in the various forms of energy plus any heat taken from the process. This equation may then be written

$$\text{Shaft work } (W) = (z_2 - z_1) + 778(u_2 - u_1) + \left(\frac{V_2^2 - V_1^2}{2g}\right) + (P_2v_2 - P_1v_1) + Q. \tag{81}$$

As will be noted from equation (81), the solution of problems dealing with uniform flow processes involves more variables than are encountered in processes of the non-flow type. As a result, considerably more data, either assumed or determined experimentally, are needed for the solution of problems dealing with uniform flow processes.

If it is assumed that the work of compression is adiabatic where the heat taken from the process is zero and the work of compression appears in the working substance as an increase in internal energy, equation (81) may be written

$$\text{Shaft work } (W) = (z_2 - z_1) + \left(\frac{P_2v_2 - P_1v_1}{k - 1}\right) + \left(\frac{V_2^2 - V_1^2}{2g}\right) + (P_2v_2 - P_1v_1). \tag{82}$$

In the case of isothermal compression, heat energy equal to the work of compression must be abstracted from the working substance. Since in isothermal compression no temperature changes occur, and P_1v_1 equals P_2v_2, there is no change in internal energy or work of flow. Equation (81) representing the shaft work in such a case would then be

$$\text{Shaft work } (W) = (z_2 - z_1) + \left(\frac{V_2^2 - V_1^2}{2g}\right) + \left(P_1v_1 \log_e \frac{v_2}{v_1}\right). \tag{83}$$

PROBLEMS

1. Three pounds of air are to be compressed from a volume of 2 to 1 cu ft at a constant temperature of 60 deg fahr. How many Btu of heat must be rejected from the air?

2. An air compressor has a cylinder volume of 2 cu ft. If it takes air at 14.4 psi abs and 70 deg fahr and compresses it isothermally to 100 psi abs, find:

(a) The weight of the air in the cylinder at the beginning of the compression stroke.
(b) The final volume of the compressed air.
(c) The foot-pounds of work done upon the gas during compression.
(d) The Btu absorbed by the air in increasing the internal energy.
(e) The Btu to be abstracted from the cylinder.

3. Data the same as in problem 2 but the compression is to be adiabatic. Find:

(a) The final volume of the compressed air.
(b) The final temperature of the compressed air.

(c) The foot-pounds of work required to compress this air.

(d) The Btu absorbed by the air in increasing the internal energy.

(e) The Btu to be abstracted from the gas.

4. A pound of air at 32 deg fahr and atmospheric pressure is compressed to 4 atmospheres (abs). What will be the final volume and the work of compression if the compression is (a) isothermal, (b) adiabatic?

5. Plot the curve $PV^n = C$, when $n = 1.35$. Initial pressure is 460 psi gage, initial volume 0.5 cu ft, and final volume 8 cu ft.

6. Air at 100 psi abs and a volume of 2 cu ft expands along an $n = 1$ curve to 25 psi abs pressure. Find:

(a) The work done by the expansion.

(b) The heat to be supplied.

7. A quantity of air at 100 psi abs pressure has a temperature of 80 deg fahr. It expands isothermally to a pressure of 25 psi abs when it has a volume of 4 cu ft. Find (a) the weight of air present, (b) the work of the expansion in foot-pounds, (c) the heat required in Btu.

8. Two cubic feet of air at a pressure of 100 psi abs expand adiabatically to a pressure of 25 psi abs. What is the final volume?

9. One cubic foot of air at 60 deg fahr and a pressure of 15 psi abs is compressed without loss or addition of heat to 100 psi abs pressure. Find the final temperature and volume.

10. Five pounds of nitrogen are heated from 40 deg fahr to 240 deg fahr. Calculate the Btu required if the process is (a) at constant volume, (b) at constant pressure.

11. Ten pounds of air at 100 deg fahr and 15 psi abs are compressed adiabatically in a non-flow process to 200 psi abs. Calculate: (a) the work done, (b) the heat supplied, (c) the change in internal energy.

12. An air compressor requires 7 hp to drive it and the water jackets carry away 200 Btu per minute. How much is the internal energy of the compressed air increased or decreased per minute?

13. Five pounds of air are compressed from 13.5 psi abs to 108 psi abs. If the process takes place isothermally at 140 deg fahr, how much work is done? How much heat is added or abstracted, and what is the change in internal energy?

14. Air at a temperature of 380 deg fahr and a pressure of 147 psi abs expands according to an equation $PV^{1.3} = C$ until its pressure is 14.7 psi abs. Find the final temperature. Determine the work done (in ft-lb per lb air) during the process.

15. Electric generators are cooled by circulating air through the windings. If the efficiency of the generator is 97.0 per cent what weight of air must be circulated per min for a generator of 110,000 kw, at full load, when the allowable temperature rise is 40 deg fahr? Consider the heat exchange to take place at constant air pressure.

16. Calculate the same quantities as those required in Problem 15, except that hydrogen is used as the cooling medium in place of air, and the generator efficiency is 98.0 per cent.

17. One lb of air is compressed reversibly and adiabatically from 70 deg fahr and 14 psi abs to 84 psi abs. How much work was done?

18. Air at a pressure of 30 psi abs is compressed until its pressure is 120 psi abs and its temperature 280 deg fahr. If n for this process is 1.32, what was the original temperature? How much work was done?

19. A rigid tank having a volume of 100 cu ft contains air. The pressure gage reads 120 psi and its temperature is 40 deg fahr. If the tank is heated until the pressure gage reads 170 psi (a) What is the change in internal energy? (b) How much heat was added in Btu? (c) What is the final temperature? (d) How much external work was done in ft-lb?

20. The lubricating oil in a certain air compressor must not be heated above 450 deg fahr. What is the maximum allowable pressure in the cylinder if " n " for compression is 1.30, p is 15 psi abs and t = 70 deg fahr?

CHAPTER IV

CYCLES OF HEAT ENGINES USING GAS

In the heat engine, the working substance or heat medium passes through a series of thermodynamic processes converting heat into mechanical work. The series of such changes by the repetition of which the conversion of heat energy into work occurs forms the heat engine cycle.[1] The heat engine cycle usually consists of four events which are: heating, cooling, expansion, and compression.

In comparing heat engine cycles consideration must be given to their thermal efficiencies (equation 14), their relative mean effective pressures (a function of the power developed) and their volume displacements or maximum volume changes during the cycle (a function of the size of the engine per unit of power developed).

Available Energy of Heat Engine Cycles. This is the amount of heat which is transformed into mechanical energy and is the difference between the heat supplied and the heat rejected during the ideal cycle.

1. THE CARNOT CYCLE

Important conclusions regarding theoretically perfect heat engines are to be drawn from the consideration of the action of an ideal engine in which the working substance is a perfect gas which is made to go through a cycle of changes involving both isothermal and adiabatic expansions and compressions. This ideal cycle of operations was invented and first explained in 1824 by Carnot, a French engineer. This cycle gave the first theoretical basis for comparing heat engines with an ideally perfect engine. The ideal

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[1] A thermodynamic machine performing a cycle in which heat energy is changed into work is called a heat engine, and one performing a cycle in which heat is transferred from a medium at a low temperature to one at a higher temperature is called a refrigerating machine.

Carnot cycle requires an engine, illustrated in **Fig. 9**, which consists of the following parts:

(1) A piston and cylinder, as shown in **Fig. 9**, composed of perfectly non-conducting material, except the *cylinder-head* (left-hand end of the cylinder) which is a good conductor of heat. The space in the cylinder between the piston and the cylinder-head is occupied by the working substance, which can be assumed to be a perfect gas.

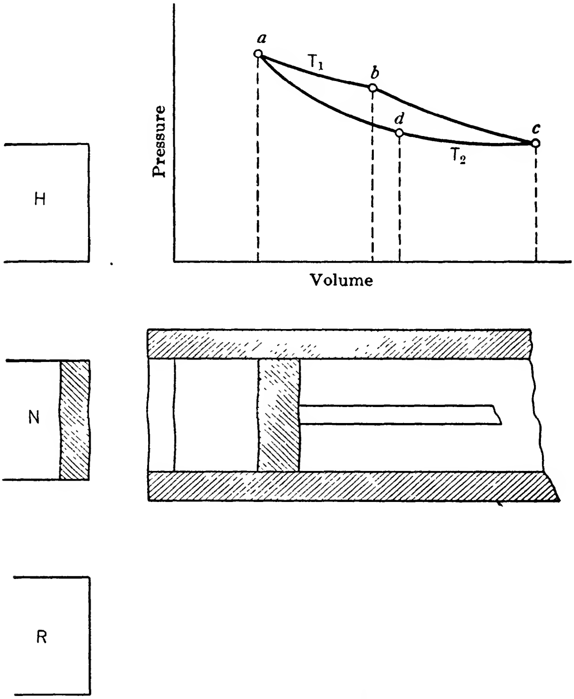

Fig. 9. Apparatus and Diagram Illustrating a Reversible (Carnot) Cycle.

(2) *A hot body* **H** of unlimited heat capacity, always kept at a temperature T_1.

(3) A perfectly *non-conducting cover* **N**.

(4) *A refrigerating or cold body* **R** of unlimited heat-receiving capacity, which is kept at a constant temperature T_2 (lower than T_1).

It is arranged that **H**, **N** or **R** can be applied, as required, to the cylinder-head. Assume that there is a charge of 1 lb of gas in the cylinder between the piston and the cylinder-head, which at the beginning of the cycle, with the piston in the position shown, is at the temperature T_1, has a volume V_a, and has a pressure P_a. The subscripts attached to the letters V and P refer to points on the pressure-volume diagram shown in **Fig. 9.** This diagram shows, by curves connecting the points **a**, **b**, **c** and **d**, the four steps in the cycle.

The operation of this cycle will be described in four parts as follows:

(1) Apply the hot body or heater **H** to the cylinder-head at the left-hand side of the figure. The addition of heat energy to the gas will cause it to expand *isothermally* along the *curve ab*, because the temperature will be maintained constant during the process at T_1. The pressure drops to P_b when the volume becomes V_b. During this expansion external work has been done in advancing the piston and the heat equivalent of this work has been obtained from the hot body **H.**

(2) Remove the hot body **H** and at the same time attach to the cylinder-head the *non-conducting cover N.* During this time the piston has continued to advance toward the right, doing work without receiving any heat from an external source, so that the expansion of the gas in this step has been done at the expense of the stock of internal energy in the gas along the *adiabatic curve bc.* The temperature has continued to drop in proportion to the loss of heat to the value T_2. Pressure is then P_c and the volume is V_c.

(3) Take away the non-conductor **N** and apply the refrigerator **R.** Then force the piston back into the cylinder. The gas will be compressed *isothermally* at the temperature T_2. In this compression, work is being done on the gas, and heat is developed, but all of it goes into the refrigerator **R**, in which the temperature is always maintained constant at T_2. This compression is continued up to a point **d** in the diagram, so selected that a further compression (adiabatic) in the next (fourth) stage will cause the volume, pressure and temperature to reach their initial values as at the beginning of the cycle.[1]

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[1] Briefly the third stage of the cycle must be stopped when a point **d** is reached, so located that an adiabatic curve (PV^k = constant) drawn from it will pass through the "initial" point **a.**

(4) Substitute for the refrigerator **R**, the non-conducting cover **N**. Then continue the compression of the gas without the addition of any heat. It will be the *adiabatic curve da.* The pressure and the temperature will rise and, if the point **d** has been selected properly, when the pressure has been brought back to its initial value P_a the temperature will also have risen to its initial value T_1. The cycle is thus finished and the gas is ready for a repetition of the same series of processes comprising the cycle.

To define the Carnot cycle completely we must determine how to locate algebraically the proper place to stop the third step (the location of **d**). During the second step (adiabatic expansion from **b** to **c**) by applying equation (69), the following temperature and volume relations exist:

$$\frac{T_1}{T_2} = \left[\frac{V_c}{V_b}\right]^{k-1};$$

also for the adiabatic compression the fourth step can be similarly stated,

$$\frac{T_1}{T_2} = \left[\frac{V_d}{V_a}\right]^{k-1}.$$

Hence,

$$\left[\frac{V_c}{V_b}\right]^{k-1} = \left[\frac{V_d}{V_a}\right]^{k-1}.$$

Simplifying and transposing,

$$\frac{V_b}{V_a} = \frac{V_c}{V_d} \tag{84}$$

$\frac{V_b}{V_a}$ is the *ratio of expansion r* for the isothermal expansion in the first step of the cycle. This has been shown (equation 84) to be equal to $\frac{V_c}{V_d}$ in the isothermal compression in the third step in order that the adiabatic compression occurring in the fourth step shall complete the cycle.

A summary of the heat transfers to and from the working gas in the four steps of the Carnot cycle is as follows:

(ab). Heat taken in from hot body, by equation (55), is:

$$AwRT_1 \log_e \frac{V_b}{V_a}. \tag{85}$$

(bc). No heat taken in or rejected.

(cd). Heat rejected to refrigerator, by equation (55), is:

$$AwRT_2 \log_e \frac{V_d}{V_c} = -\, AwRT_2 \log_e \frac{V_c}{V_d} \cdot ^{*} \tag{86}$$

(da). No heat taken in or rejected.

Hence, *the heat equivalent of the net amount of work done, AW,* by the gas in this cycle, being the mechanical equivalent of the *excess* of heat taken in over that rejected, is the *algebraic sum* of (85) and (86):

$$AW = AwR\left(T_1 \log_e \frac{V_b}{V_a} - T_2 \log_e \frac{V_c}{V_d}\right) = AwR(T_1 - T_2) \log_e \frac{V_b}{V_a} \cdot \tag{87}$$

The thermal or heat efficiency of a cycle is defined as the ratio of

$$\frac{\text{Heat equivalent of work done}}{\text{Heat taken in}} = \frac{\text{Available energy}}{\text{Heat taken in}} .$$

The heat equivalent of the available energy (work done) is, by equation (87),

$$AwR(T_1 - T_2) \log_e \frac{V_b}{V_a},$$

and the heat taken in is, by equation (85),

$$AwRT_1 \log_e \frac{V_b}{V_a} .$$

The ratio above representing the efficiency is:

$$e = \frac{AwR(T_1 - T_2) \log_e \frac{V_b}{V_a}}{AwRT_1 \log_e \frac{V_b}{V_a}} = \frac{T_1 - T_2}{T_1} . \tag{88}$$

The efficiency of the Carnot cycle as expressed by equation (88) is the maximum possible theoretical efficiency which may be obtained with any heat engine working between the temperature limits T_1 and T_2. This equation can be used as a standard for the comparison of the efficiencies of actual heat engines working between two temperature limits.

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* $\text{Log} \frac{V_d}{V_c} = -\log \frac{V_c}{V_d}$.

Reversible Cycles. A reversible cycle is a cycle made up of reversible processes. A process is said to be reversible when its direction can be reversed and all parts of the system returned to the original state. The quantity of energies involved in the reversed process is equal but opposite in direction of flow to that when operated direct. A heat engine which is capable of discharging to the "source of heat," when running in the reverse direction from that of its normal cycle, the same quantity of heat that it would take from this source when it is running direct and doing work, is said to operate with its cycle reversed, or, in other words, the engine is thermodynamically reversible. A reversible heat engine is one which, if made to follow its indicator diagram in the reverse direction, will require the same horsepower to drive it as a refrigerating machine as the engine will deliver when running direct, assuming that the quantity of energy involved is the same in the two cases. An engine following Carnot's cycle is a reversible engine. The thermodynamic idea of reversibility in engines is of great value because no heat engine can be more efficient than a reversible engine when both work between the same limits of temperature.

If an engine operating on Carnot's cycle is reversed in its action, so that the same indicator diagram shown in **Fig. 9** is traced in the opposite direction, the reversed cycle, when beginning as before at point **a** with a perfect gas at the temperature T_1, consists of the following stages:

(1) When the non-conductor **N** is applied and the piston is advanced toward the right by the source of power performing the reversed cycle, the gas expands, tracing the adiabatic curve **ad**, with constant lowering of temperature which at the point **d** is T_2.

(2) When the non-conductor **N** is removed, the refrigerator **R** is applied and the piston continues on its outward stroke. The gas expands isothermally at the constant temperature T_2, tracing the curve **dc**. During this stage the gas is taking heat from the refrigerator **R**.

(3) When the refrigerator **R** is removed and the non-conductor **N** is again applied, which is on the back stroke of the engine, the gas is compressed, and on the indicator diagram another adiabatic curve **cb** is traced. At the point **b** the temperature is T_1.

(4) When the non-conductor **N** is removed and the hot body **H** is again applied, with the compression continuing along the isothermal curve **ba**, heat is discharged to the hot body **H**, while the temperature is maintained constant at T_1.

The cycle has now been traced in a reverse direction from the beginning back to the starting point at **a**, and is complete. During this process an amount of work represented by the area of the indicator diagram, equivalent in foot-pounds to

$$wR \log_e \frac{V_b}{V_a} (T_1 - T_2) \text{ (see equation 87)},$$

has been converted into heat. First, heat was taken from the refrigerator **R**, represented in amount by

$$AwRT_2 \log_e \frac{V_c}{V_d},$$

and second, heat was rejected to the hot body **H** in the amount

$$AwRT_1 \log_e \frac{V_a}{V_b} \quad \text{or} \quad - AwRT_1 \log_e \frac{V_b}{V_a}.$$

As in the direct operation of Carnot's cycle no heat is given or lost in the first and third stages outlined above. The algebraic sum of these two quantities, since $\frac{V_b}{V_a} = \frac{V_c}{V_d}$, gives the net amount of work done, W, on the gas, and, therefore, the net amount of heat transferred from the cold body **R** to the hot body **H** or,

$$AW = AwRT_2 \log_e \frac{V_b}{V_a} - AwRT_1 \log_e \frac{V_b}{V_a} = -AwR \log_e \frac{V_b}{V_a} (T_1 - T_2). \tag{89}$$

Since the result is the same as that given by equation (87), although opposite in sign because it is the work of compression, in the reverse cycle the same amount of heat is given to the hot body **H** as was taken from it in the direct operation of the same cycle, and the same amount of heat is now taken from the refrigerator **R** as was in the other case given to it.

Carnot's Principle. It was first proved conclusively by Carnot that no other heat engine can be more efficient than a reversible engine when both work between the same temperature limits. To illustrate this principle, assume that there are two engines

A and **B**. Of these, **A** is reversible and **B** is not. In their operation both take heat from a hot body or heater **H** and reject heat to a refrigerator or cold body **R**. Let Q_H be the quantity of heat which the reversible engine **A** takes in from the hot body **H** for each unit of work performed, and let Q_R be the quantity of heat per unit of work which it discharges to the refrigerator **R**.

For the purpose of this discussion, assume that the non-reversible engine **B** is more efficient than the reversible engine **A**. Under these circumstances it is obvious that the engine **B** will take in less heat than **A** and it will reject correspondingly less heat to **R** per unit of work performed. Let the heat acquired by the non-reversible engine **B** from the hot body **H** be designated by a quantity less than Q_H, or $Q_H - X$, and the heat rejected by **B** to the refrigerator **R** by $Q_R - X$. Now if the non-reversible engine **B** is working direct (converting heat into work) and is made to drive the reversible engine **A** according to its reverse cycle (converting work into heat), then for every unit of work done by the engine **B** in driving the reversible engine **A**, the quantity of heat mentioned above, that is, $Q_H - X$, would be taken from the hot body **H** by the non-reversible engine **B** and, similarly, the quantity of heat represented by Q_H would be returned to the hot body **H** by the reverse action of the cycle of operations performed by **A**. This follows because the engine **A** is reversible and returns, therefore, to **H**, when operating on the reverse cycle, the same amount of heat as it would take in from **H** when working on its direct cycle. By this arrangement the hot body **H** would be continually receiving heat, in the amount represented by X for each unit of work performed. At the same time the non-reversible engine **B** discharges to the refrigerator **R** a quantity of heat represented by $Q_R - X$, while the reversible engine **A** removes from the refrigerator **R** a quantity represented by Q_R. As a result of this last operation the cold body will be losing continually per unit of work performed a quantity of heat equal to X. The combined performances of the two engines, one working direct as a normal heat engine and the other, according to its reverse cycle, as a compressor or what might be called a " heat pump," gives a constant removal of heat from the refrigerator **R** to the hot body **H**, and as a result a degree of infinite coldness must finally be produced in the refrigerator.

If we assume that there is no mechanical friction, this combined machine, consisting of a normal heat engine and a compressor, will require no power from outside the system. For this reason the assumption that the non-reversible engine **B** can be more efficient than the reversible engine **A** has brought us to a result which is impossible from the standpoint of experience as embodied in the statement of the " Second Law of Thermodynamics "; that is, it is impossible to have a self-acting engine capable of transferring heat, infinite in quantity, from a cold body to a hot body. Accordingly, no non-reversible engine, as **B** for example, can be more efficient than a reversible engine **A** when both engines operate between the same temperature limits. More briefly, when the source of heat and the cold receiver are the same for both a reversible heat engine and any other engine, then the reversible engine must have a higher possible efficiency; and if both engines are reversible it follows that neither can be more efficient than the other.

A reversible engine is perfect from the viewpoint of efficiency; that is, it transforms into work a greater part of the heat which it takes in than does a non-reversible engine operating under the same conditions.

Reversible Processes. The study of ideally reversible processes is important in that it makes possible the development of laws that would otherwise be difficult. On the other hand, the practical application of this study is often confusing. There are certain processes which are obviously reversible, others that are definitely irreversible, while a classification of the remainder is difficult.

Undoubtedly the ideal isothermal and adiabatic processes, as used in the Carnot cycle, are reversible. But even these in their practical application are always irreversible. Some of the more common thermodynamic processes that result in irreversibility are:

1. The transition of heat energy from a hot body to a cold body.
2. Friction.
3. Turbulence in the working medium.
4. Failure of a working medium to be in thermal equilibrium.

Hence, reversible processes are ideal processes which can only be approached in practice. As a result, most thermodynamic theory is based upon the ideal or reversible process. Those irreversible processes that are met with in practice are treated as special processes.

Degradation of Energy. Carnot's cycle being one of maximum efficiency is the basis of several important thermodynamic conclusions.

(1) That the transformation of heat energy into work can never be perfect unless the temperature of the cold body or refrigerator is at the absolute zero of temperature. Since the temperature of the cold body or refrigerator is directly or indirectly fixed by the temperature of surrounding objects and since this temperature can never be, practically speaking, as low as the absolute zero of temperature, the transformation of a given quantity of heat energy into work can never be complete. In any heat engine, therefore, there must always be the discharge of heat energy to the cold body or refrigerator.

(2) The degree of efficiency with which the transformation may take place is dependent upon the initial absolute temperature. The higher the initial temperature of the heat energy as compared with that of the cold body, the greater is its possible efficiency in the production of work. Conversely, the lower the temperature of the heat energy, the less its possible efficiency and the less desirable it is as energy in the production of work.

(3) It, therefore, seems impossible to transform heat energy into work perfectly, and as a result all heat transformations into work are accompanied by the discharge of heat energy at a lower or less desirable temperature. It thus seems logical to assume that *all heat energy in its transformation tends to flow from a higher to a lower level.* Although this conclusion is especially true of heat energy, undoubtedly other forms of energy follow the same law. This statement is embodied in the principle of the *degradation of energy* which states that "every natural process is accompanied by a certain degradation of energy or thermodynamic degeneration."

2. HOT-AIR ENGINE CYCLES

The hot-air engine is a non-explosive type of external combustion heat engine, the working substance being atmospheric air which undergoes no change in its physical state. This type of prime mover was invented about 100 years ago. It is little used on account of the difficulty in transmitting heat through the metallic walls to the dry gas (air) and the hot-air engine is very bulky in proportion to its power. The mechanical efficiency of the hot-air engine is also low.

The advantages of the hot-air engine are ease of operation and safety. Also, on account of the slow rate of heat transmission, its speed is low, rendering it suitable for driving small pumps and for other domestic uses where small powers are required and where the fuel economy is also a matter of minor importance.

Hot-air engine cycles are divided into two groups:

Group I. External combustion hot-air engines with closed cycles and constant-volume temperature changes.

Group II. External combustion hot-air engines with open cycles and constant-pressure temperature changes.

In the hot-air engine an attempt was made to put in practice a cycle of the ideal Carnot type with the addition of a regenerator. The regenerator was a " storage-battery " of heat, its function being to absorb, store and return heat rapidly, replacing the adiabatic expansion curves of the Carnot cycle by lines of constant volume in *group I* and by lines of constant pressure in *group II.* The regenerator consists of a chamber containing strips of metal, coils of wire, or any other heat-absorbing material arranged in such a manner as to present a very large surface to the air passing through it.

The Stirling Engine. The Stirling engine belongs to *group I,* the regenerator being so arranged that the pressure drops with the temperature at such a rate as to keep the volume constant. It is an external combustion engine and its cycle of operation is closed, the same air is used over and over again, any loss by leakage being supplied by a small force-pump. The engine consists, essentially, of a pair of displacer cylinders, a double-acting working cylinder, a regenerator and a refrigerator. In the earlier forms the displacer cylinder had a double wall, the regenerator and refrigerator being placed in the annular space surrounding the displacer cylinder. In the later forms the regenerator and refrigerator were arranged separate from the displacer cylinder but in direct communication with its top and bottom. A plunger worked in the displacer cylinder, this plunger being filled with a non-conducting material such as brick-dust. The engine derived its heat by conduction from a furnace which was placed beneath the displacer cylinder.

In the cycle represented by **Fig. 10**, the action of the engine is carried out as follows:

1. The substance, air, having a volume V_a, a pressure P_a and a temperature T_2 receives heat at constant volume by passing through the regenerator, and receives heat also from the furnace. As a result of the heat addition, its pressure is increased to P_b, and its temperature to T_1, this process being represented by the constant volume line *ab*, **Fig. 10**. During this event the plunger is moving up.

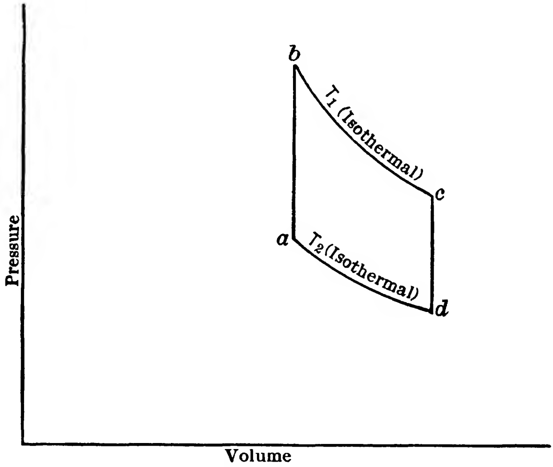

Fig. 10. Stirling Engine Cycle.

2. The pressure under the working piston increases as a result of the addition of heat and the expansion of the air follows, producing the working stroke. This expansion is represented by *bc* and is an isothermal, the air receiving heat from the furnace plates and the temperature remaining T_1.

3. The plunger descends, displacing the hot air and forcing the same through the regenerator and refrigerator. As a result of this, the temperature drops from T_1 to T_2 and the pressure is decreased from P_c to P_d, the volume remaining constant as shown by the line *cd*.

4. The working piston descends compressing the air. This compression, represented by *da,* is isothermal, the air being cooled during the process by the regenerator and refrigerator to remain at the temperature T_2.

Referring to **Fig. 10** of the Stirling cycle, the useful work W is in foot-pounds,

$$W = \left(P_b V_b \log_e \frac{V_c}{V_b}\right) - \left(P_d V_d \log_e \frac{V_d}{V_a}\right). \tag{90}$$

The mean effective pressure is in pounds per square foot,

$$P_m = \frac{W}{V_d - V_a}. \tag{91}$$

The horsepower developed can be represented by:

$$\text{hp.} = \frac{W \times N}{33{,}000}, \tag{92}$$

where N = revolutions (cycles) per minute.

Since the expansion and compression processes are isothermal, the efficiency of the Stirling cycle is:

$$e = \frac{P_b V_b \log_e \frac{V_c}{V_b} - P_d V_d \log_e \frac{V_d}{V_a}}{P_b V_b \log_e \frac{V_c}{V_b}}.$$

Since $V_a = V_b$ and $V_c = V_d$, also $wRT_1 = P_b V_b$ and $wRT_2 = P_d V_d$,

$$e = \frac{wRT_1 \log_e \frac{V_c}{V_b} - wRT_2 \log_e \frac{V_c}{V_b}}{wRT_1 \log_e \frac{V_c}{V_b}} = \frac{T_1 - T_2}{T_1}. \tag{93}$$

The Ericsson Engine. This engine belongs to *group II,* the regenerator changing the temperature at constant pressure. This engine consists of five parts: a compressing pump, a receiver, a regenerator, a refrigerator and a working cylinder. The ideal cycle for this engine is represented by **Fig. 11,** the order of events being as follows:

Atmospheric air is drawn into the compressing pump as shown by *d′d.* This air is compressed isothermally to *a,* during the return stroke of the pump; it is then forced into a receiver as shown by aa'. Thus $d'daa'$ represents the pump cycle.

As the working cylinder begins its forward or up stroke, the compressed air is admitted into the working cylinder at constant pressure. As this admission is through the regenerator, the entering air takes up heat increasing in temperature from T_2 to T_1 and in volume from a to b. As soon as the supply of compressed air is cut off, the air expands isothermally along bc to atmospheric pressure, while heat is being supplied by a furnace at the bottom of the working cylinder. During the return stroke of the working piston, the air is discharged at constant pressure through the regenerator, giving up its heat and cooling to the temperature T_2. In **Fig. 11**, $a'bcd'$ is the cycle of the working cylinder, the net work being represented by $abcd$.

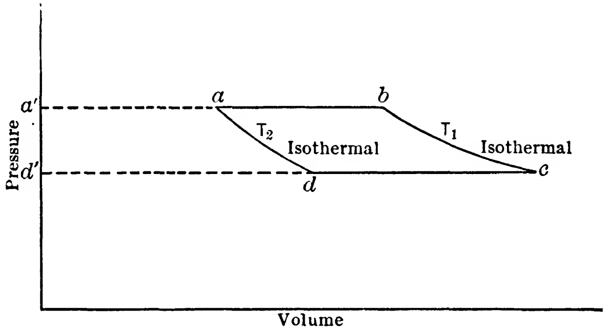

FIG. 11. Ericsson Engine Cycle.

The Ericsson cycle has the same efficiency as the Stirling cycle, the regenerator process being carried out at constant pressure instead of at constant volume.

3. INTERNAL COMBUSTION ENGINE CYCLES

The internal combustion engine utilizes as its working substance a mixture of air and gas, or air and petroleum vapor. Combustion of the mixture takes place inside the engine cylinder, or in a communicating vessel, and the heat generated is converted into work. As the specific heat and the specific volume of the mixture do not differ much from that of air, the cyclical analysis and the theory of the internal combustion engine are developed on the assumption that the working substance is air. The internal combustion engine cycles can be divided into the following groups:

Group I. Engines without compression.

Group II. Compression engines.

Group I. Internal Combustion Engines Without Compression

The Lenoir Engine. An engine of this group, invented by Pierre Lenoir in 1860, was the first successful practical gas engine. The action of this type of engine is as follows:

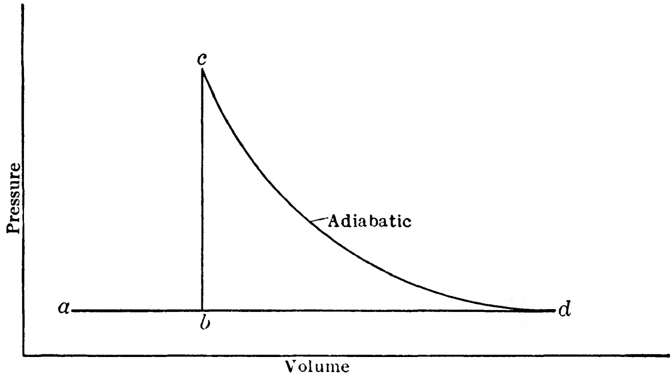

Fig. 12. Ideal Lenoir Engine Cycle.

As the piston leaves the dead center on its forward stroke it draws into the cylinder a charge of gas and air which is proportioned at the admission valve to form an explosive mixture. Admission is cut off somewhat before half-stroke and this is followed by the ignition of the mixture by means of an electric spark. The explosion produces a rapid rise in pressure above atmospheric, thus forcing the piston to the end of the stroke. The exhaust valve opens near the end of the stroke and the burnt products are expelled during the return stroke of the piston. A fly-wheel carries the piston over during the exhaust stroke as well as also during the suction stroke. The above series of operations takes place on each side of the piston, producing two impulses for each revolution.

Figure 12 represents the cycle of operations of the Lenoir engine on P-V coordinates. The drawing in of the mixture of gas and air at atmospheric pressure is represented by *ab*. Ignition takes place at *b* and is followed by the constant volume combustion line *bc*. The working stroke, an adiabatic expansion, takes place to atmospheric pressure, as shown by *cd*, and the products of combustion are rejected during the return stroke of the piston *da*.

Calling P, V, T the absolute pressure in pounds per square foot, the specific volume in cubic feet per pound of mixture, and the absolute temperature in degrees Fahrenheit respectively; also using subscripts b, c, d to designate the points at the corners *bcd* of **Fig. 12**, the following expressions will be obtained:

If the heat developed by the complete combustion of the mixture be designated by Q_1, then

$$Q_1 = wc_v(T_c - T_b). \tag{94}$$

Since the combustion of the mixture takes place at constant volume,

$$V_c = V_b \tag{95}$$

$$T_c = T_b + \frac{Q_1}{wc_v}. \tag{96}$$

$$P_c = P_b \frac{T_c}{T_b}. \tag{97}$$

The expansion *cd* being adiabatic ($n = k$),

$$V_d = V_c\left(\frac{P_c}{P_d}\right)^{\frac{1}{k}}. \tag{98}$$

$$T_d = T_c\left(\frac{V_c}{V_d}\right)^{k-1} \tag{99}$$

If Q_2 is the heat rejected by the engine after performing its cycle of operations

$$Q_2 = wc_p(T_d - T_b). \tag{100}$$

The available energy is:

$$W = J(Q_1 - Q_2). \tag{101}$$

The mean effective pressure is the average unbalanced pressure on the piston of the engine in pounds per unit area, and using symbols and units as on page 63,

$$P_m = \frac{W}{V_d - V_b} \tag{102}$$

The horsepower developed would be found by the equation:

$$\text{hp} = \frac{W \times N}{33,000}, \tag{103}$$

where N designates the number of explosions per minute.

The cycle efficiency for the conditions of the problem is:

$$e = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} = 1 - k\frac{T_d - T_b}{T_c - T_b}. \tag{104}$$

Group II. Compression Engines

The first compression engine was patented as early as 1799 by Philip Lebon. The thermodynamic advantages of compression before ignition will be evident from the demonstrations which follow. A mechanical advantage due to compression is the reduced shock due to the explosion and the resulting improved balance of parts.

1. The Otto Internal Combustion Engine Cycle. The majority of modern commercial internal combustion engines operate upon the Otto internal combustion engine cycle, which was suggested by Beau de Rochas in 1862, and which was made a practical success by Nicholas A. Otto in 1878. The term " engine cycle " is applied to the series of events which are essential for carrying out the transformation of heat into work. The Otto internal combustion engine cycle requires four strokes of the piston and comprises five events, which are: suction, compression, ignition, expansion, and exhaust.

The action of an internal combustion engine working on the four-stroke Otto cycle is illustrated in **Fig. 13.**

1. Suction of the mixture of air and gas through the inlet valve takes place during the complete outward stroke of the piston, the exhaust valve being closed. This stroke of the piston is called the suction stroke.

2. On the return of the piston, both the inlet and exhaust valves remain closed, and the mixture is compressed between the piston and the closed end of the cylinder. This is called the compression stroke. Just before the compression stroke of the piston is completed, the compressed mixture is ignited by a spark and rapid combustion or explosion takes place.

3. The increased pressure within the cylinder due to the rapid combustion of the mixture drives the piston on its second forward stroke, which is the power stroke. This power stroke, or working stroke, is the only stroke in the cycle during which power is generated. Both valves remain closed until the end of the power stroke, when the exhaust valve opens and provides communication between the cylinder and the atmosphere.

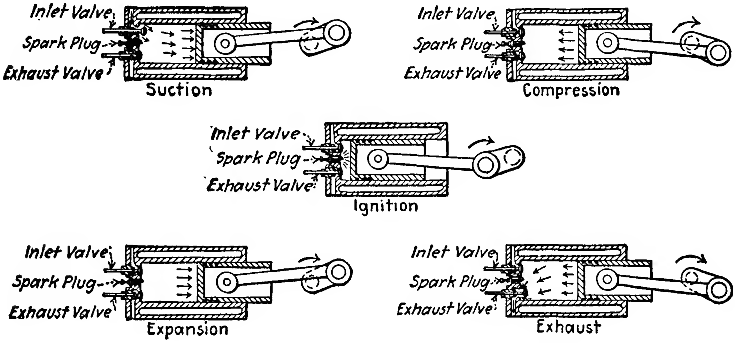

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Fig. 13. The Events in the Otto Cycle.

4. The exhaust valve remains open during the fourth stroke called the exhaust stroke (**Fig. 13**), during which the burned gases are driven out from the cylinder by the return of the piston.

The simplest type of internal combustion engine operating on the Otto four-stroke cycle is the gasoline engine which is illustrated in **Fig. 14.** The fuel from the liquid fuel tank *T* is supplied to the mixing valve or carburetor through the fuel regulating valve *G*. The air, through the air pipe *A*, enters the same carburetor and is thoroughly mixed with the fuel. The mixture of air and vaporized fuel enters the engine cylinder *C* through the inlet valve *V* as the piston *P* moves on the suction stroke. The mixture is then compressed, and ignited by an electric spark produced at the spark plug *Z*, by current furnished from the battery *B*. The ignition of the mixture is followed by the power stroke. The reciprocating motion of the piston *P* is communicated, through the connecting rod *R* to the crank *N*, and is changed into rotary motion at the crankshaft *S*. The crankshaft *S*, while driving the machinery to which it is connected, also turns the valve-gear shaft, sometimes called the two-to-one shaft, through the gears *X* and *Y*. The gear *Y* turns once for every two revolutions of the crank, and near the end of the power stroke opens the exhaust valve *E* by means of the rod *D* pivoted at *O*.

In larger engines, the valve-gear shaft also opens and closes the admission valve *V* and operates the fuel pump and ignition system. As the temperature resulting from the ignition of the explosive mixture is usually over 2000 deg fahr, some method of cooling the walls of the cylinder must be used in order to facilitate lubrication, to prevent the moving parts from being twisted out of shape, and to avoid the ignition of the explosive mixture at the wrong time of the cycle. One method of cooling gasoline engines is to jacket the cylinder *J*, that is, to construct a double-walled cylinder and circulate water between the two walls, through the jacket space. The base *U* supports the various parts of the engine; the flywheel *W* carries the engine through the idle strokes. Besides the above details, every gasoline engine is usually provided with lubricators *L* for the cylinder and bearings, and with a governor for keeping the speed constant at variable loads.

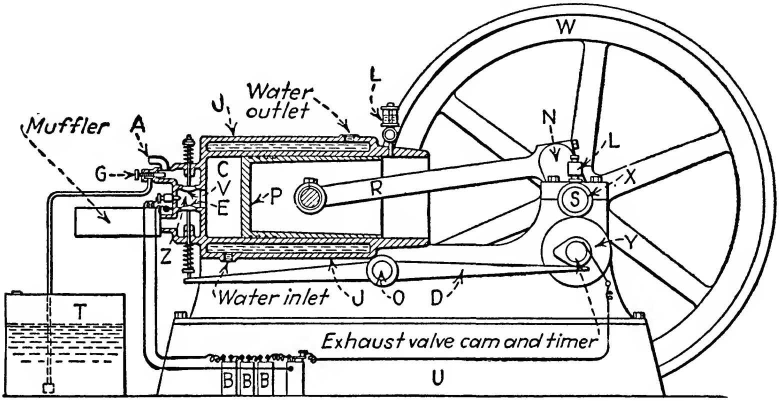

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FIG. 14. Parts of a Gasoline Engine.

An indicator diagram, taken from a four-stroke cycle internal combustion engine, using gasoline as fuel, is illustrated in **Fig. 15**. *IB* is the suction stroke, *BC* the compression stroke, *CD* shows the ignition event, *DE* the power stroke, and *EI* is the exhaust stroke. The direction of motion of the piston during every stroke is illustrated in each case by arrows. Lines *AF* and *AG* were added to the indicator diagram; *AF* is the atmospheric line, while *AG* is the line of pressures. From Fig. 15 it will be noticed that part of the suction stroke occurs at a pressure lower than atmospheric. The reason for this is that a slight vacuum is created in the cylinder by the piston moving away from the cylinder head. The vacuum helps to draw the mixture of fuel and air into the cylinder.

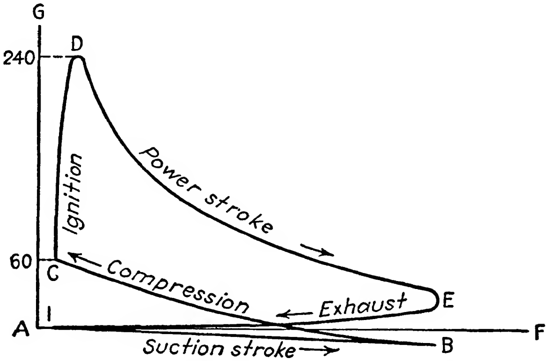

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FIG. 15. Gasoline Engine Indicator Card.

Modern internal combustion engines, operating on the Otto four-stroke cycle, will convert 14 to 30 per cent of the heat available in the fuel into work. The Lenoir engines, in which the mixture was not compressed previous to ignition, converted only about 4 per cent of the heat available in the fuel into work.

The Ideal, or Air Standard Otto Cycle. The ideal, or air standard diagram for the Otto cycle is illustrated in **Fig. 16**, the operation being as follows:

A mixture of gas and air is drawn in during the complete forward stroke of the piston, as shown by *a'a*. The return of the piston compresses the mixture along the adiabatic curve *ab*. Explosion of the compressed charge takes place at *b*, with the consequent combustion at constant volume to *c*; *cd* is the adiabatic expansion producing the second forward stroke. The exhaust valve opens at *d*, cooling the gases to the exhaust pressure *a*, and rejecting them to the atmosphere.

Calling P, V, T the absolute pressure in pounds per square foot, the specific volume in cubic feet per pound of mixture, and the absolute temperature in degrees Fahrenheit, respectively; also using subscripts a, b, c, and d to designate the points in **Fig. 16**, the following expressions will be obtained: The heat added during the combustion from b to c is:

$$Q_1 = wc_v(T_c - T_b). \qquad (105)$$

The heat rejected from d to a is:

$$Q_2 = wc_v(T_d - T_a). \qquad (106)$$

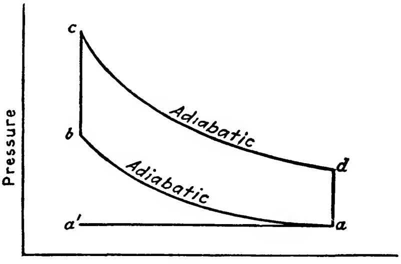

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Fig. 16. Ideal Otto cycle.

The cycle efficiency is:

$$e = \frac{Q_1 - Q_2}{Q_1} = \frac{wc_v(T_c - T_b) - wc_v(T_d - T_a)}{wc_v(T_c - T_b)}$$

$$= 1 - \frac{T_d - T_a}{T_c - T_b}. \qquad (107)$$

Since the expansion and compression are adiabatic, the following relations will hold:

$$T_aV_a^{k-1} = T_bV_b^{k-1}, \text{ and } T_cV_c^{k-1} = T_dV_d^{k-1}.$$

Since

$$V_c = V_b \text{ and } V_a = V_d,$$

$$\frac{T_d}{T_a} = \frac{T_c}{T_b}; \text{ also } \frac{T_d - T_a}{T_c - T_b} = \frac{T_a}{T_b}.$$

Since

$$\frac{T_a}{T_b} = \left(\frac{V_b}{V_a}\right)^{k-1} = \left(\frac{P_a}{P_b}\right)^{\frac{k-1}{k}},$$

equation (107) may be expressed as:

$$e = 1 - \frac{T_a}{T_b} = 1 - \left(\frac{V_b}{V_a}\right)^{k-1} = 1 - \left(\frac{P_a}{P_b}\right)^{\frac{k-1}{k}}. \qquad (108)$$

Equation (108) shows that the efficiency of engines operating on the Otto cycle depends upon the pressure to which the mixture of fuel and air is compressed before ignition. Theoretically, the greater the compression pressure, the better is the economy. Practical considerations and the danger of preignition limit the compression pressures for various fuels to the following values in pounds per square inch: gasoline, 60 to 90 lb; kerosene, 50 to 80 lb; alcohol, 120 to 180 lb; natural gas, 80 to 120 lb; producer gas, 120 to 160 lb; blast-furnace gas, 120 to 190 lb.

From the above values of practical compression pressures, it is evident that with fuels high in hydrocarbons lower compression pressures should be employed than with fuels which are low in these constituents.

The Two-Stroke Cycle Engine. The internal combustion engine working on the four-stroke cycle requires two complete revolutions of the crankshaft, or four strokes of the piston to produce one power stroke. The other three are only idle strokes, but power is required to move the piston through these strokes, and this has to be furnished by storing extra momentum in heavy flywheels. The Otto cycle can be modified so that the five events can be carried out during only two strokes of the piston by precompressing the mixture of fuel and air in a separate chamber, and by having the events of expansion, exhaust, and admission occur during the same stroke of the piston. In large two-stroke cycle engines, the air and fuel for the mixture are compressed and delivered separately by auxiliary pumps driven from the main engine shaft. The precompression of the mixture in the case of small two-stroke cycle engines is accomplished by having a tightly closed crankcase, or by closing the crank end of the cylinder and by providing a stuffing box for the piston rod.

The main features of the two-stroke cycle internal-combustion engine are illustrated in **Fig. 17**. On the upward stroke of the piston P, a partial vacuum is created in the crankcase C, and the explosive mixture of fuel and air is drawn in through a valve at A. At the same time, a mixture previously taken into the upper end of the cylinder W is compressed. Near the end of the compression stroke, the mixture is fired from a spark produced at the spark plug S. The explosion of the mixture drives the piston on its downward or working stroke. The piston descending compresses the mixture in the crankcase to about 6 or 8 lb above atmospheric, the admission valve at A being closed as soon as the pressure in the crankcase exceeds atmospheric. When the piston is very near the end of its downward stroke, it uncovers the exhaust port at *E* and allows the burned gases to escape into the atmosphere. The piston continuing on its downward stroke next uncovers the port at *I*, allowing the slightly compressed mixture in the crankcase *C* to rush into the working part of the cylinder *W*. Thus, two full strokes of the piston complete one cycle.

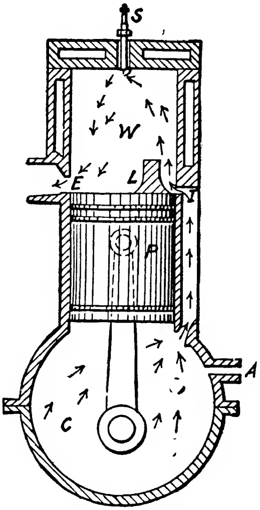

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Fig. 17. Small Two-Stroke Cycle Engine.

The distinctive feature of the two-stroke cycle engine is the absence of valves. The transfer port *I* from the crankcase *C* to the working part of the cylinder *W*, as well as the exhaust port *E*, are opened and closed by the piston.

Large two-stroke cycle engines are often made double acting and have the same number of power impulses per revolution as the single-cylinder steam engine. The proper amounts of gas and air are delivered to each end of the piston at the correct time by auxiliary pumps. An admission valve is provided at each end of the cylinder. The exhaust takes place through ports near the middle of the cylinder, which are uncovered by the piston at the end of each working stroke.

To offset the advantages resulting from the absence of valves, less weight, and greater frequency in working strokes, the two-stroke cycle engine is usually less economical in fuel consumption and is not so dependable as is the four-stroke cycle engine. As the inlet port *I* (**Fig. 17**) is opened, while the exhaust of the gases takes place at *E*, there is always some chance that part of the fresh mixture will pass out through the exhaust port. Closing the exhaust port too soon will cause a decrease in power and efficiency, on account of the mixing of the inert burned gases with the fresh mixture. By carefully proportioning the size and location of the ports, and by providing the piston with a lip at *L* (**Fig. 17**) to direct the incoming mixture toward the cylinder head, the above losses may be decreased. In large two-stroke cycle engines, an effort is made to eliminate the above loss by forcing a current of air through the cylinder by the air pump, while the exhaust port remains open. In any case, the scavenging of the cylinder of the waste gases is not as thorough in the two-stroke cycle as in the four-stroke cycle engine, where one complete stroke of the piston is allowed for the removal of the exhaust gases. The four-stroke cycle engine has also the advantage of wider use and longer period of development.

2. The Brayton Cycle. This cycle is often called the Joule cycle after its inventor, or the Brayton cycle after George B. Brayton, who in 1872 designed an engine with gradual constant pressure combustion. In this engine a mixture of gas and air is first compressed in a separate pump and forced into a receiver. On the way from the receiver to the engine cylinder the mixture is ignited by a gas jet which burns steadily without sudden explosion, producing temperature and volume changes at constant pressure. After expansion the piston drives out the products of combustion at atmospheric pressure.

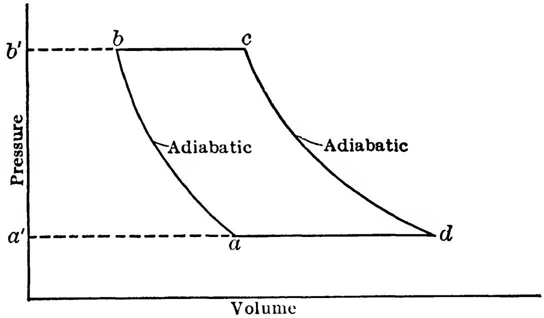

FIG. 18. Brayton Engine Cycle.

The cycle of operations for the Brayton engine is represented by **Fig. 18.** $a'a$ represents the supply of the combustible mixture to the pump where it is compressed adiabatically to b and forced into a receiver. bc represents the burning of the compressed mixture at constant pressure. As the mixture enters the working cylinder, it expands adiabatically along cd. This is followed by the rejection of the burnt gases along the atmospheric line da.

The heat added during the constant pressure combustion from b to c is:

$$Q_1 = wc_p(T_c - T_b). \tag{109}$$

The heat rejected from d to a is:

$$Q_2 = wc_p(T_d - T_a). \tag{110}$$

The cycle efficiency is

$$e = \frac{Q_1 - Q_2}{Q_1} = \frac{wc_p(T_c - T_b) - wc_p(T_d - T_a)}{wc_p(T_c - T_b)} = 1 - \frac{T_d - T_a}{T_c - T_b}. \tag{111}$$

Since the expansion and compression phases of the cycle are adiabatic and $P_c = P_b$; $P_d = P_a$, therefore,

$$\frac{T_d}{T_a} = \frac{T_c}{T_b} \quad \text{and} \quad \frac{T_d - T_a}{T_c - T_b} = \frac{T_a}{T_b}.$$

Hence, the efficiency:

$$e = 1 - \frac{T_a}{T_b} = 1 - \left(\frac{V_b}{V_a}\right)^{k-1} = 1 - \left(\frac{P_a}{P_b}\right)^{\frac{k-1}{k}}. \tag{112}$$

Equation (112) is the same as equation (108) and shows that *the cycle efficiencies of the Otto and Brayton cycles are the same,* under the same initial conditions, *and depend on the amount of compression of the charge before explosion.*

The Brayton cycle has been applied to a gas turbine of recent design. This gas turbine uses a low grade fuel oil and has attained an actual efficiency of about 18 per cent.

3. The Diesel Cycle. This cycle is carried out in four strokes of equal length, as in the case of the Otto cycle. Atmospheric air is drawn in during the complete forward stroke of the piston as shown by *Ia* (**Fig. 19**). The return of the piston compresses the air adiabatically to b, a pressure of about 500 psi. At the end of the compression stroke a charge of the liquid fuel is injected in a finely divided form by an auxiliary pump or compressor and burns nearly at constant pressure when it comes in contact with the highly compressed and consequently high temperature air. The supply and combustion of the fuel, as represented by the combustion line *bc*, is cut off at one-tenth to one-sixth the working stroke, depending upon the load. Expansion takes place during the balance of the stroke as shown by the adiabatic expansion line *cd*. Release occurs at *d* with the consequent drop in pressure, and the burnt gases are rejected during the fourth stroke of the cycle.

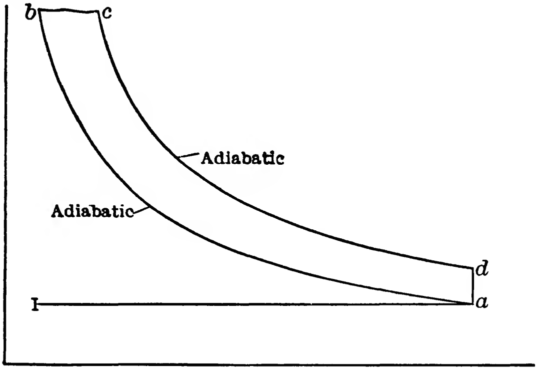

FIG. 19. Diesel Engine Cycle.

The heat added (Q_1) during the combustion of the fuel is:

$$Q_1 = wc_p(T_c - T_b). \tag{113}$$

The heat rejected from *d* to *a* is

$$Q_2 = wc_v(T_d - T_a). \tag{114}$$

The cycle efficiency is then,

$$\begin{aligned} e &= \frac{Q_1 - Q_2}{Q_1} = \frac{wc_p(T_c - T_b) - wc_v(T_d - T_a)}{wc_p(T_c - T_b)} \\ &= 1 - \frac{c_v}{c_p}\left(\frac{T_d - T_a}{T_c - T_b}\right) = 1 - \frac{1}{k}\left(\frac{T_d - T_a}{T_c - T_b}\right). \end{aligned} \tag{115}$$

An indicator diagram taken from a Diesel oil engine is shown in **Fig. 20**. Air is drawn into the cylinder during the suction stroke *AB*. The return of the piston compresses the air to a pressure of about 470 psi during the stroke *BC*. The fuel oil is then gradually introduced by means of an oil pump, to an amount depending upon the load, and burns during *CD*, the first part of the third stroke. This is followed by the expansion of the gases within the cylinder to the end of the third stroke along *DE*. At

E, the exhaust valve opens and the burned gases are exhausted from the cylinder during the fourth stroke *EA*.

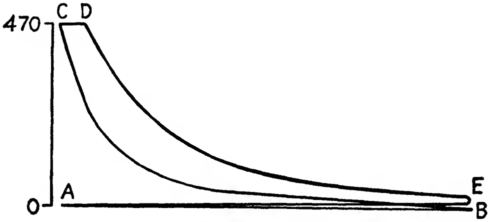

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FIG. 20. Indicator Card from Diesel Oil Engine.

4. The Dual Cycle. The dual cycle is a combination of the Otto and Diesel cycles. Atmospheric air is drawn in during the complete forward stroke of the piston (*Ia* in **Fig. 21**). The return of the piston compresses the air adiabatically to *b*. At the end of the compression stroke the fuel is injected and burns partly at constant volume (*bc*) and partly at constant pressure (*cd*). Expansion takes place adiabatically during the third stroke of the cycle (*de*). Release occurs at *e* with the consequent drop in pressure and the burnt gases are rejected during the fourth stroke of the cycle.

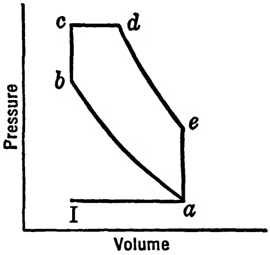

FIG. 21. Dual Cycle.

The heat added Q_1 during the combustion of the fuel is:

$$Q_1 = wc_v(T_c - T_b) + wc_p(T_d - T_c). \tag{116}$$

The heat rejected from *e* to *a* is:

$$Q_2 = wc_v(T_e - T_a) \tag{117}$$

$$e = 1 - \frac{Q_2}{Q_1} = 1 - \frac{wc_v(T_e - T_a)}{wc_v(T_c - T_b) + wc_p(T_d - T_c)}. \tag{118}$$

Expressing all temperatures in terms of T_a:

$$T_b = T_a\left(\frac{V_a}{V_b}\right)^{k-1}$$

$$T_c = T_b\left(\frac{P_c}{P_b}\right) = T_a\left(\frac{V_a}{V_b}\right)^{k-1}\left(\frac{P_c}{P_b}\right)$$

$$T_d = T_c\left(\frac{V_d}{V_c}\right) = T_a\left(\frac{V_d}{V_c}\right)\left(\frac{V_a}{V_b}\right)^{k-1}\left(\frac{P_c}{P_b}\right).$$

$$T_e = T_d\left(\frac{V_d}{V_e}\right)^{k-1} = T_a\left(\frac{V_d}{V_b}\right)^{k}\left(\frac{P_c}{P_b}\right).$$

Substituting the above temperature equalities in equation (118), the air standard efficiency of the dual cycle becomes:

$$e = 1 - \frac{\left(\frac{V_d}{V_b}\right)^k - \left(\frac{P_b}{P_c}\right)}{\left(\frac{V_a}{V_b}\right)^{k-1}\left[1 - \frac{P_b}{P_c} + k\left(\frac{V_d}{V_b} - 1\right)\right]}. \tag{119}$$

Thus the air standard efficiency of the dual cycle increases with the compression ratio $\frac{V_a}{V_b}$, as well as with the change in pressure during ignition $\frac{P_b}{P_c}$. An increase of the ratio of volume at end of combustion $\frac{V_d}{V_b}$ will result in a decrease in efficiency.

PROBLEMS

1. A Carnot cycle delivering 30 hp operates with a source of heat energy at 2040 deg fahr and a refrigerator at 90 deg fahr. Calculate Q_1 and Q_2 in Btu per min. What is the efficiency of the engine?

2. A Carnot cycle has at the beginning of the expansion stroke a pressure of 75 psi abs, a volume of 2 cu ft and a temperature of 200 deg fahr. The volume at the end of the isothermal expansion is 4 cu ft. The temperature at the end of the adiabatic expansion is 30 deg fahr. Find:

(a) Heat added to cycle operating with air.
(b) Efficiency of cycle.
(c) Net work of cycle.

3. A cycle made up of two isothermal and two adiabatic processes has a pressure of 100 psi abs, a temperature of 200 deg fahr, and a volume of 1 cu ft at the beginning of the isothermal expansion. At the end of the adiabatic expansion the pressure is 10 psi abs and the volume is 8 cu ft. Find:

(a) Efficiency of cycle operating with air.
(b) Heat added to cycle.
(c) Net work of cycle.

4. In a Carnot cycle the heat is added at a temperature of 400 deg fahr and rejected at 70 deg fahr. The working substance is 1 lb of air which has a volume of 2 cu ft at the beginning and a volume of 4 cu ft at the end of the isothermal expansion. Find:

(a) Volume at end of isothermal compression.
(b) Heat added to cycle.
(c) Heat rejected from cycle.
(d) Net work of cycle.

5. The temperature limits of a heat engine cycle are 2500 deg fahr and 150 deg fahr. The engine is supplied with 10,000 Btu per hp-hr developed at the shaft. What is the ratio of the actual thermal efficiency of the engine to that of the best conceivable engine operating between the same temperature limits?

6. A Stirling hot-air engine having a working piston 16 in. in diameter and a stroke of 4 ft makes 28 rpm. The upper temperature is 650 deg fahr, while the lower temperature is 150 deg fahr. Assuming that the volume of the working cylinder is one-half that of the displacer cylinder and $P_d = 14.7$ psi, calculate the pressures and volumes at each point of the cycle (Fig. 10); also calculate the mean effective pressure, the horsepower developed, and the cycle efficiency. Consult Goodenough's *Principles of Thermodynamics* for the operation of the Stirling hot-air engine.

7. Plot the Stirling engine cycle from the results of problem 6.

8. Deduce the formula for the efficiency of the Ericsson cycle.

9. Compare the cycle efficiencies of internal combustion engines working on the Otto cycle using the fuels and compression pressures indicated below:

Gasoline, 75; producer gas, 150; and blast furnace gas, 200 psi gage pressure. (Barometer = 30 in.)

10. Calculate the theoretical pressure, volume and temperature at each point of an Otto cycle, as well as the horsepower and cycle efficiency for the following conditions of Fig. 16. Assume that the pressure after compression is 180 psi gage, that $N = 500$, that 80 Btu are added during combustion and that n in $PV^n = 1.4$:

$$P_a = 14.7 \text{ psi abs}, \quad V_a = 13.5 \text{ cu ft},$$
$$T_a = 70 + 460 = 530 \text{ deg fahr abs}.$$

11. An Otto cycle engine develops 30 hp for one hour on a fuel consumption of 15 lb, the fuel having a calorific value of 19,000 Btu per lb. The engine clearance is 18 per cent of the piston displacement. Calculate the actual thermal efficiency and the air-standard thermal efficiency.

12. An engine operating on the air-standard Otto cycle has a clearance of 20 per cent of the piston displacement. If the engine develops 100 hp, how much heat energy must be supplied in Btu per min?

13. Calculate the efficiency of the Brayton cycle when the compression is 470 psi (Barometer = 30 in.).

14. An engine operating on the Diesel cycle has a clearance of 8 per cent of the piston displacement, $T_a = 70$ deg fahr, and a cut-off ratio of 10 per cent

of the piston displacement, i.e., heat energy is supplied while piston travels 10 per cent of its stroke. Find the air-standard cycle efficiency.

15. A Diesel engine uses 1.5 pounds of air at 14.5 psi abs and 100 deg fahr at the start of compression. The pressure at the end of compression is 600 psi abs. Heat energy is added during 6 per cent of the stroke. Find: (a) per cent of clearance, (b) temperature at the end of combustion, (c) air-standard cycle efficiency.

16. Calculate the efficiency of an air-standard dual cycle, which operates with a compression ratio of 12 to 1; $t_a = 212$ deg fahr, $p_a = 14.7$ psi abs, $Q_1 = 500$ Btu, $p_c = 1000$ psi abs, $M = 1$ lb air.

CHAPTER V

PROPERTIES OF VAPORS

Saturated and Superheated Vapors. As was explained in the chapter on the properties of perfect gases, a vapor is a fluid which can be readily transformed into a liquid by a moderate reduction in temperature or increase in pressure. It is essentially a gas near its point of condensation.

At every pressure, there is a fixed temperature, called the temperature of vaporization, at which a liquid can be changed into a vapor by the addition of heat. A vapor at the temperature of vaporization is called a **saturated vapor.** When a vapor is heated so that its temperature is greater than the vaporization temperature corresponding to its pressure, it is said to be **superheated vapor.** Superheated vapors when far removed from their vaporization temperature behave nearly according to the laws of perfect gases.

Theory of Vaporization. When heat energy is transmitted to a liquid, its temperature will rise with only a slight volume change until the temperature of vaporization is reached. This is always a definite temperature for any given pressure, and it depends upon the character of the liquid. Thus the temperature of vaporization of water at the atmospheric pressure of 14.7 psi abs is 212 deg fahr, while that at a pressure of 150 psi abs is 358.4 deg fahr. On the other hand, the vaporization temperature of ammonia liquid at a pressure of 150 psi abs is 79 deg fahr.

When the temperature of vaporization of a liquid is reached, any further heat transmitted to the liquid causes the liquid to change to a vapor. During this process, if the pressure remains constant, the temperature remains constant and no temperature increase will occur until all the liquid has been changed to a vapor.

In the vaporization of the liquid, since the temperature remains constant, no energy is absorbed in increasing the mean velocity of the molecules. A change of state of aggregation as well as considerable change in volume takes place during vaporization, with the result that most of the energy absorbed in the process is used in tearing the molecules apart, so that it resides within the vapor as latent or potential energy.

When vaporization is incomplete the vapor is termed wet saturated vapor. In this condition some of the liquid is present in the vapor.

A dry saturated vapor carries with it no liquid that has not been evaporated. The percentage dryness of a vapor is called its *quality*. Thus the quality of steam is 0.97 when 1 lb of it consists of 97 per cent dry saturated steam and 3 per cent water. Whereas the volume of a vapor increases with the quality, the temperature is constant throughout the whole process of vaporization for any given pressure. Thus the temperature of water vapor corresponding to 150 psi abs is 358.4 deg fahr, no matter whether the quality is 0.10, 0.75, or 1.00.

After complete vaporization, the pressure still remaining constant, the absorption of further energy by the vapor will cause an increase in temperature as well as an increase in volume. In this condition the substance is superheated. A superheated vapor may then be defined as one whose temperature is higher than that of saturated vapor at the same pressure.

Vapor Tables. The exact quantities of energy required to produce vaporization under various conditions, as well as the relations existing between pressure, volume, and temperature of saturated and superheated vapors have been determined experimentally. These experimental results have been expressed in the form of empirical equations from which vapor tables have been computed. Tables showing the properties of dry saturated and superheated steam, and of ammonia, are given in the appendix. This text should be supplemented by the more complete tables, such as those recently prepared by Joseph H. Keenan and Frederick G. Keys and published by John Wiley and Sons. The steam tables and Mollier diagram given in the Appendix are abridged forms of Keenan and Keys' complete tables.

In these tables for saturated steam will be found, corresponding to the pressure of the vapor in pounds per square inch absolute or inches of mercury for low pressures, the vaporization temperature (t) in degrees Fahrenheit, the specific volume of the saturated liquid (v_f), the increase of specific volume during evaporation (v_{fg}), the specific volume of the dry saturated steam (v_g), the enthalpy of the liquid (h_f), the enthalpy of evaporation (h_{fg}), the total enthalpy (h_g), the entropy of the liquid (s_f), the entropy of evaporation (s_{fg}) and the total entropy of the saturated steam (s_g).

Relation Between Temperature, Pressure, and Volume of Saturated Steam. The important relations of temperature, pressure, and volume were first determined in a remarkable series of experiments conducted by the French engineer, Regnault, and it is on the basis of his data, first published in 1847, that the earlier steam tables were computed. Later experimenters have found, however, that these data are somewhat in error, especially for values near the dry saturated condition.

It should be observed that in vapor tables the pressure is given in terms of pounds per square inch, whereas in nearly all thermodynamic calculations the pressure must be used in pounds per square foot.

The specific volume of dry saturated steam (v_g) is equal to the sum of the volume of water (v_f) and of the increase in volume during vaporization (v_{fg}), or

$$v_g = v_f + v_{fg}. \tag{120}$$

Enthalpy of the Liquid (Water) (h_f). The essentials of the process of making steam have been described in a general way. If a pound of water which is initially at some temperature t_0 is heated at a constant pressure P (pounds per square foot) to the boiling point t corresponding to this pressure, and then converted into steam, energy will first be absorbed in raising the temperature of the water from t_0 to t, and then in producing vaporization. During the first stage, while the temperature is rising, the amount of energy taken in is approximately $(t - t_0)$ heat units, that is, British thermal units (Btu), because the specific heat of water is approximately unity.

When cold water from which steam is to be generated is at a lower pressure than that maintained in the boiler, manifestly a pressure change is encountered in raising the temperature of the water to the boiling point under the conditions existing in the boiler. The liquid can conceivably be heated in two ways: first, by heating the water under its own vapor pressure; and, second, by pumping the water into the boiler where further energy is added by heating at constant pressure to complete the process. In either case the final energy as measured from an arbitrary datum plane would be the same. In the first, the entire change is produced by heating; in the second (the usual case), a portion of the energy will be supplied by the pump and the remainder by the furnace as heat. The enthalpy of the liquid (h_f) as used in this text and in the steam tables is the energy of the liquid expressed in Btu per pound under its own vapor pressure as compared with that of saturated water at 32 deg fahr, which is taken as zero by general consent.

During the heating of the liquid, before any vaporization has occurred, the change in volume of the liquid is negligible. As a result, practically all the energy absorbed by the liquid appears as internal kinetic energy, the energy required to overcome the external resistance being negligibly small.

Enthalpy of Vaporization (h_{fg}). In the second stage of the formation of steam, the water at the vaporization temperature t corresponding to the pressure is changed into steam at that temperature and pressure. During this process, there is no rise in temperature, but considerable energy is required to produce this evaporation or vaporization. As the pressure is constant the heat energy absorbed by the vapor during this process is equal to the enthalpy of evaporation. The symbol h_{fg} is used to designate this energy, and its value varies with the particular pressure at which steaming occurs, being smaller at high pressures than at low.

Since the temperature remains constant during evaporation, the internal kinetic energy remains constant and all energy absorbed during the process except that used in overcoming the external resistance resides within the vapor in potential form.

Total Enthalpy (h_g). The energy required to produce the process as represented by the first and second stages in the formation of a pound of steam is called the total enthalpy and is represented by the symbol h_g. Other terms used by various writers are total heat, heat content and thermodynamic potential. Using the symbols already defined, we can write, per pound of the steam,

$$h_g = h_f + h_{fg}. \qquad (121)$$

In analyzing the energies which exist in a vapor there is internal kinetic energy due to molecular velocity, internal potential energy due to the work which was absorbed by the vapor in tearing the molecules apart against an internal structural resistance, and external work which was required because of the increase in volume against an external resistance. The internal kinetic energy and the internal potential energy reside within the vapor and can be classed as internal energy. Total enthalpy of a vapor can, therefore, be defined as and is mathematically equal to the internal energy plus the energy expended as external work, all with reference to liquid water at the freezing point.

Wet Steam. In all problems studied thus far, dealing with saturated steam, it has been assumed that the steaming process was complete and that the water had been converted completely into steam. In actual practice it is not at all unusual to have steam leaving boilers which is not perfectly and completely vaporized; in other words, the boilers are supplying to the engines a mixture of steam and water. This mixture is called *wet steam.* It is steam which carries in suspension minute particles of water, which remain thus in suspension for some time. The temperature of wet steam is always the same as that of dry saturated steam as given in the steam tables, so long as any steam remains uncondensed.

The weight of moisture or water in a pound of wet steam is called the *degree of wetness;* and when this is expressed as a per cent, it is called the *percentage of moisture* or "*per cent wet.*" If in a pound of wet steam there is 0.04 lb of water in suspension or entrained, the steam is 4 per cent wet or 96 per cent dry.

Another term, called the *quality of steam,* which is usually expressed by the symbol x, is also frequently used to represent the condition of wet steam. *Quality of steam* may be defined as *the proportion of dry or completely evaporated steam in a pound of wet steam.* To illustrate from the example above, if there is 0.04 lb of water in a pound of wet steam, the quality of steam would be **1 − 0.04 or 0.96.**

Accordingly, the enthalpy of evaporation of a pound of wet steam is xh_{fg}. Similarly, the total enthalpy of a pound of wet steam above 32 deg fahr is $h_f + xh_{fg}$. The volume of a pound of wet steam is $v_f + xv_{fg}$ or $(1 - x)v_f + xv_g$. Since the term $(1 - x)v_f$

is negligibly small, if the steam is reasonably dry, the volume of a pound of wet steam can be expressed approximately as xv_g. The internal energy in a pound of wet steam is

$$u_{xg} = h_f + xh_{fg} - APxv_g, \tag{122}$$

where P is the absolute pressure in pounds per square foot at which the steam is formed.

Superheated Steam. When the temperature of steam is higher than that corresponding to saturation as taken from the steam tables, and is, therefore, higher than the vaporization temperature corresponding to the pressure, the steam is said to be superheated. In this condition its properties approach those of a gas as its degree of superheat is increased.

There are tables of the properties of superheated steam just as there are tables of saturated steam. When dealing with dry saturated steam there is always only one possible temperature and only one specific volume to be considered. With superheated steam, on the other hand, for a given pressure there may be any temperature above that of saturated steam, and corresponding to each temperature there will be, of course, definite values for specific volume and enthalpy. Like a perfect gas the specific volume of superheated steam increases with the increase in temperature. These properties have been the subject of extended investigation over a period of many years, the most generally accepted values being given in the tables (Appendix) for superheated steam.

External Work Done in the Formation of Steam. In the generation of steam from water at 32 deg fahr and at a given pressure an increase in volume of both the water and steam generated occurs as the heating progresses. As a result, a part of the heat energy used in the generation of steam is expended in doing the external work required by the increase in the volume under the existing pressure. The amount of energy required to do the external work in foot-pounds is equal to P, the absolute pressure in pounds per square foot, times the change in volume in cubic feet occurring during the process.

Internal Energy of Steam. It was explained in the preceding paragraph that when steam is generated a part of the heat added does not appear as internal energy of steam but is stored in the surrounding medium as potential energy due to work done. The internal energy of steam may be calculated from its enthalpy. Since enthalpy is defined as the internal energy plus APv, the internal energy may be calculated by substracting APv from the enthalpy. Thus,

$$u = h - APv. \tag{123}$$

The following problem illustrates the calculation of internal energy of dry saturated steam, in which case

$$u_g = h_g - APv_g.$$

Example. A boiler is evaporating water into dry saturated steam at a pressure of 300 psi abs. What is the total internal energy per pound of the steam above water at 32 deg fahr?

Solution. From the steam tables, the enthalpy and volume are found to be 1202.8 Btu and 1.5414 cu ft respectively. From equation (123) the total internal energy above 32 deg fahr is

$$u_g = 1202.8 - \frac{300 \times 144 \times 1.5433}{778} = 1117.1 \text{ Btu.}$$

Subcooled Liquids. Liquids may exist at temperatures below that corresponding to their saturation pressures. Thus, the saturation temperature of steam or the boiling point of water at atmospheric pressure is 212 deg fahr, but water may exist at atmospheric pressure at any temperature between 212 and 32 deg fahr. A liquid that exists at a temperature below that of the saturation point is sometimes called *subcooled.*

The determination of the properties of a subcooled liquid from vapor tables necessitates the calculation of a *correction factor.* Thus, if 1 lb of water exists at atmospheric pressure and a temperature of 60 deg fahr, the enthalpy of the liquid as found from vapor tables corresponding to this temperature is 28.06 Btu while the absolute pressure is 0.2563 psi. Undoubtedly, the enthalpy under the condition of the atmospheric pressure would have been greater than 28.06 Btu because of the excess pressure.

In calculating the correction to be applied to the enthalpy as found in the vapor tables in such a case some assumptions must be made. Owing to the incompressible nature of liquids the volume is assumed to be unaffected by the excess pressure. Furthermore, since the internal energy is fixed by the temperature, the enthalpy of the subcooled liquid is assumed to be larger than that found in vapor tables corresponding to the temperature by an amount equal to the difference in the APv products, or

$$h_{f_1} = h_f + A(P_1 - P_f)v_f. \qquad (124)$$

Thus, to determine the corrected enthalpy of water if the condition of actual pressure is atmospheric and the temperature is 60 deg fahr, the following data are found corresponding to 60 deg fahr temperature from the steam tables (Appendix): $h_f = 28.06$ Btu, $p_f = 0.2563$ psi abs, and $v_f = 0.01604$ cu ft. The corrected enthalpy of the liquid is then

$$\begin{aligned} h_{f_1} \text{ (subcooled)} &= 28.06 + \frac{(14.696 - 0.2563) \times 144 \times 0.01604}{778} \\ &= 28.09 \text{ Btu.} \end{aligned}$$

It will be noted that this correction produces a very small change in the enthalpy, and for the usual engineering calculations, especially those in which the pressure difference is not excessive, no correction is applied. On the other hand, when the pressure difference is large, considerable error may be introduced by neglecting this correction.

The above method is based upon the assumption that water is incompressible. This is approximately correct for low temperatures, but at high temperatures the error may be considerable, and the correction should be made as indicated in Table 4 of the Keenan and Keys tables.

Drying of Steam by Throttling or Wire-drawing. When steam expands by passing through a very small opening, as, for example, through a valve only partly open in a steam line, the pressure is considerably reduced. This effect is called *throttling* or *wire-drawing.* The result of expansion of this kind when the pressure is reduced and no work is done is that if the steam is initially wet it will be drier and if it is initially dry or superheated the degree of superheat will be increased.

The theory of this action is best understood by considering it as a uniform flow process. Equation (19) shows the energies entering into such a process, combining the terms internal energy and work of flow into enthalpy (h). In modifying this equation to meet the conditions of a throttling process, the heat supplied (Q) and the work (AW) are assumed to be zero and no change takes place in the potential mechanical or the kinetic energy. With these assumptions, the enthalpy at entrance equals the enthalpy at exit. Since the enthalpy of dry and saturated steam is considerably less at low pressures than at high and since the enthalpies in such a process are equal, the same enthalpy content at the lower pressures corresponds to a drier or even a superheated condition, and so tends to dry out or superheat the steam. When steam is initially wet, the action of the throttling tends to produce drier steam and may even produce superheated steam. This theory may be expressed by the following equation. For throttling

$$(h_1) = (h_2); \tag{125}$$

and for wet steam:

$$(h_f + xh_{fg})_1 = (h_f + xh_{fg})_2. \tag{126}$$

The drying action or quality after the throttling is

$$x_2 = \frac{(h_f + xh_{fg})_1 - (h_f)_2}{(h_{fg})_2}. \tag{127}$$

Determination of the Moisture in Steam. Unless the steam used in the power plant is superheated it is said to be either dry or wet, depending on whether or not it contains water in suspension. The general types of steam calorimeters used to determine the amount of moisture in the steam may be classified under four heads:

1. Throttling or superheating calorimeters.
2. Separating calorimeters.
3. Condensing calorimeters.
4. Electric steam calorimeters.

Throttling or Superheating Calorimeters. The type of steam calorimeter used most operates by passing a sample of the steam through a very small orifice, in which it is superheated by throttling. One type of throttling calorimeter is illustrated in **Fig. 22.** It consists of an orifice **O** discharging into a chamber **C**, into which a thermometer **T** is inserted, and a mercury manometer is usually attached to the cock **V₃** for observing the pressure in the calorimeter.

If the entering steam contains too much moisture the drying action due to the throttling in the orifice may not be sufficient to superheat. It may be stated in general that unless there are about 5 to 10 deg fahr of superheat in the calorimeter, or, in other words, unless the temperature on the low pressure side of the orifice is at least about 5 to 10 deg fahr higher than that corresponding to the pressure in the calorimeter, there may be some doubt as to the accuracy of results.[1] The working limits of throttling calorimeters vary with the initial pressure of the steam. For 35 psi abs pressure the calorimeter ceases to superheat when the percentage of moisture exceeds about 2 per cent; for 150 lb abs pressure when the moisture exceeds about 5 per cent; and for 250 lb abs pressure when it is in excess of about 6 per cent. For any given pressure the exact limit varies *slightly*, however, with the pressure in the calorimeter.

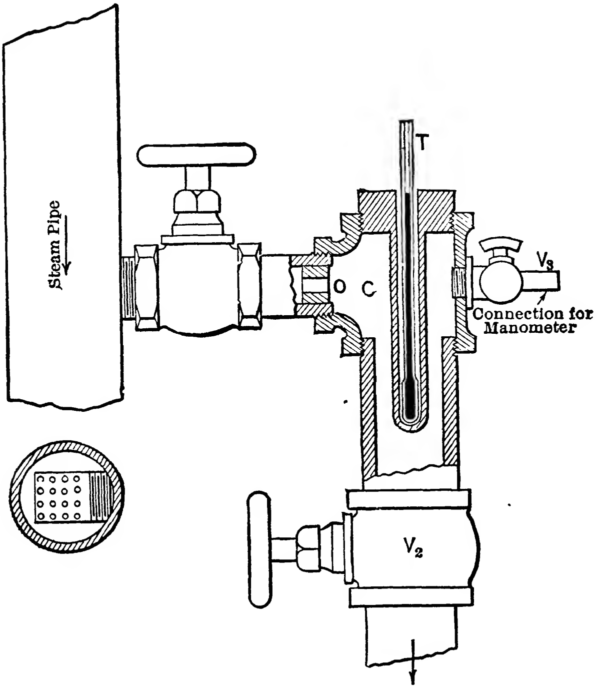

FIG. 22. Simple Throttling Steam Calorimeter.

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[1] See Moyer's *Power Plant Testing* (4th ed.), pages 72–74.

The quality or relative dryness of wet steam is easily calculated by the following method and symbols:

p_1 = steam pressure in main, pounds per square inch absolute.
p_2 = steam pressure in calorimeter, pounds per square inch absolute.
h_1 = total enthalpy of the steam corresponding to the inlet pressure p_1.
h_2 = total enthalpy of the steam corresponding to the pressure p_2 and temperature in the calorimeter.
x_1 = initial quality of steam.

The total enthalpy of a pound of wet steam flowing into the orifice is

$$h_1 = h_{f_1} + x_1 h_{fg_1};$$

and after expansion, assuming all the moisture is evaporated, the total enthalpy of the same weight of steam is h_2.

Then, assuming no heat loss,

$$h_{f_1} + x_1 h_{fg_1} = h_2$$

$$x_1 = \frac{h_2 - h_{f_1}}{h_{fg_1}}. \qquad (128)$$

The following example shows the calculations for finding the quality of steam from the observations taken with a throttling calorimeter:

Example. Steam at a pressure of 100 psi abs passes through a throttling calorimeter. In the calorimeter the temperature of the steam becomes 250 deg fahr and the pressure 14.696 psi abs. Find the quality.

Solution. The enthalpy of the steam in the calorimeter at 14.696 psi abs pressure and 250 deg fahr as determined from superheated steam tables is 1169.0.

The enthalpy of the steam before entering the calorimeter is $h_f + x_1 h_{fg}$. At 100 psi abs pressure, this is 298.40 + 888.8 x_1 Btu per pound. Since the enthalpy of the steam per pound in the calorimeter is obviously the same as that in the line before it entered the instrument,

$$298.40 + 888.8\,x_1 = 1169.0$$
$$x_1 = 0.979$$

or the steam is 2.1 per cent wet.

Separating Calorimeters. Throttling calorimeters cannot be used for the determination of the quality of steam when for comparatively low pressures the moisture is in excess of 2 per cent, and when for boiler pressures commonly used it exceeds 5 per cent. For higher percentages of moisture than these low limits separating calorimeters are most generally used. In these instruments the water is removed from the sample of steam by *mechanical separation* just as it is done in the ordinary steam separator installed in the steam mains of a power plant. This mechanical separation depends for its action on changing very abruptly the direction of flow, and reducing the velocity of the wet steam. Since water is nearly 300 times as heavy as steam at the usual pressures delivered to the engine, the moisture will be deposited because of its greater inertia. A calibrated gage glass is employed to indicate the amount of moisture collected while the calorimeter is in operation.

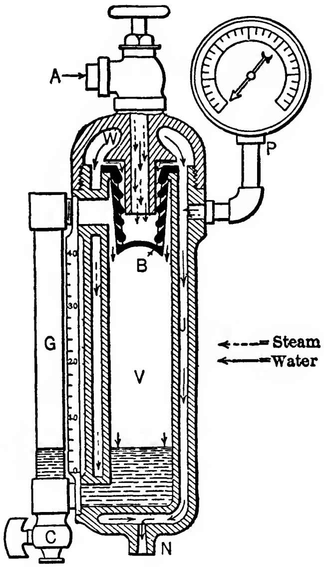

FIG. 23. Separating Steam Calorimeter.

Figure 23 illustrates a form of separating calorimeter having a steam jacketing space which receives live steam at the same temperature as the sample. Steam is supplied through a pipe **A**, discharging into a cup **B**. Here the direction of the flow is changed through nearly 180 deg, causing the moisture to be thrown outward through the meshes in the cup into the vessel **V**. The dry steam passes upward through the spaces between the webs **W**, into the top of the outside jacketing chamber **J**, and is finally discharged from the bottom of this steam jacket through the nozzle **N**. This nozzle is considerably smaller than any other section through which the steam flows, so that there is no appreciable difference between the pressures in the calorimeter proper and in the jacket. The scale opposite the gage glass **G** is graduated to show, in hundredths of a pound, at the temperature corresponding to steam at ordinary working pressures, the variation of the level of the water accumulating. A steam pressure gage **P** indicates the pressure in the jacket **J**, and since the flow of steam through the nozzle **N** is roughly proportional to the pressure, another scale in addition to the one reading pressures is provided at the outer edge of the dial. A petcock **C** is used for draining the water from the instrument, and by weighing the water collected corresponding to a given difference in the level in the gage **G** the graduated scale can be readily calibrated. Too much reliance should not be placed on the readings for the flow of steam as indicated by the gage **P** unless it is frequently calibrated. Usually it is very little trouble to connect a tube to the nozzle **N** and condense the steam discharged in a large pail nearly filled with water. When a test for quality is to be made by this method the pail nearly filled with cold water is carefully weighed, and then at the moment when the level of the water in the water gage **G** has been observed the tube attached to the nozzle **N** is immediately placed under the surface of the water in the pail. The test should be stopped before the water gets so hot that some weight is lost by "steaming." The gage **P** is generally calibrated to read pounds of steam flowing in 10 minutes. For best accuracy it is desirable to use a pail with a tightly fitting cover into which a hole just the size of the tube has been cut.

If W is the weight of dry steam flowing through the orifice **N** and w is the weight of moisture separated, the quality of the steam is

$$x = \frac{W}{W + w}. \tag{129}$$

Electric Steam Calorimeter. In this type of calorimeter electric current is used to dry the steam. In the operation of the electric steam calorimeter the steam is first superheated to some temperature T, requiring an electric input of E_T watts. The current is then decreased until the steam is just dry, requiring an input of E_0 watts. By applying a constant K determined by experiments for a series of pressures and degrees of superheat, the following equation is obtained:

$$h = \frac{KE_0}{E_T - E_0} \tag{130}$$

where h is the enthalpy required to dry a pound of steam.

If h_{fg} is the heat of vaporization,

$$x = \frac{h_{fg} - h}{h_{fg}}. \tag{131}$$

Vapors as Refrigerating Media. Ammonia is generally used as the refrigerating medium in connection with mechanical refrigeration. Carbon dioxide and sulphur dioxide are also used to a limited extent as refrigerating media.

The value of a substance as a refrigerating medium depends upon its enthalpy of evaporation, its vaporization temperature, its cost and its chemical properties. The greater the enthalpy of evaporation of a refrigerating medium, the more energy it will be capable of abstracting by evaporation. Upon the vaporization temperature of the refrigerant at different pressures depends the degree of cold it can produce as well as its practicability for use in hot climates where low temperature cooling water cannot be secured.

PROBLEMS

1. Dry and saturated steam has a pressure of 100 psi abs. What is the temperature of the steam, the volume per pound, the enthalpy of the liquid, the enthalpy of evaporation, and the total enthalpy above 32 deg fahr per lb of this steam?

2. Dry and saturated steam has a temperature of 300 deg fahr. What is its pressure, enthalpy of the liquid, enthalpy of evaporation, and total enthalpy?

3. A closed tank contains 9 cu ft of dry and saturated steam at a pressure of 150 psi abs.

(a) What is its temperature?
(b) What is the weight of the steam in the tank?

4. A boiler generates dry and saturated steam at a pressure of 200 psi abs. The feed water enters the boiler at 60 deg fahr.

(a) What is the temperature of the steam?
(b) How many Btu are required to generate 1 lb of this steam from feed water at 60 deg fahr into the steam at the pressure stated at the beginning of this problem?

5. One pound of dry and saturated steam is at a pressure of 250 psi abs.

(a) What is its internal energy of evaporation?
(b) What is its total internal energy above 32 deg fahr?
(c) How much external work was done during its formation from 32 deg fahr?

6. Dry and saturated steam is generated in a boiler and has a temperature of 400 deg fahr. The feed water enters the boiler at 200 deg fahr.

(a) What pressure is carried in the boiler?
(b) What is the total enthalpy required to generate 1 lb of this steam?

7. Using table 4 (Keenan and Keyes), find the volume, enthalpy, and entropy for one lb of steam at 1500 psi abs and 200 deg fahr.

8. One pound of ammonia at 120 psi abs has an enthalpy of 600 Btu. What is its quality?

9. A pound of sulphur dioxide has a pressure of 27.1 psi abs and a quality of 90 per cent. What is its volume, enthalpy, entropy, and temperature?

10. A pound of carbon dioxide at a temperature of 40 deg fahr has an enthalpy 90 Btu. What is its pressure, quality and entropy?

11. During an engine test the following observations were taken with a throttling calorimeter: Gage pressure in the steam pipe 125.3 psi, temperature in the calorimeter 260 deg fahr, gage pressure in calorimeter 5.3 psi, barometer 29.92 in. Hg. Find the per cent moisture in the steam, using tables.

12. Saturated water at 400 deg fahr is throttled to 20 psi abs through an expansion valve. What per cent of the liquid was evaporated owing to throttling?

13. A tank contains 9 cu ft of steam at 100 psi abs pressure which has a quality of .095. How many pounds of steam does the tank contain?

14. Two pounds of steam have a volume of 8 cu ft at a pressure of 100 psi abs. What is the quality? What is the enthalpy above 32 deg fahr?

15. One pound of steam having a quality of 0.95 has a temperature of 325 deg fahr. What is the pressure?

16. One pound of steam at a pressure of 200 psi abs has a temperature of 400 deg fahr.

(a) Is it superheated or saturated?
(b) What is the change in enthalpy in generating such steam from water at 32 deg fahr?
(c) What is its volume?
(d) How much internal energy above 32 deg fahr does it contain?

17. One pound of steam at a pressure of 200 psi abs has a volume of 3.056 cu ft.

(a) Is it saturated or superheated?
(b) What is its temperature?
(c) How much is its enthalpy above 32 deg fahr?
(d) What is its total internal energy?

18. Steam in a steam pipe has a pressure of 110.3 psi gage. A thermometer in the steam registers 385 deg fahr. Atmospheric pressure is 14.696 psi abs. Is the steam superheated, and if superheated how many degrees?

19. Steam at a pressure of 200 psi abs passes through a throttling calorimeter. After expansion into the calorimeter the temperature of this steam is 250 deg fahr and the pressure 14.696 psi abs. What is the quality?

20. Steam at a temperature of 325 deg fahr passes through a throttling calorimeter. In the calorimeter the steam has a pressure of 14.696 psi abs and a temperature of 250 deg fahr. What is the quality?

21. Steam at 150 psi abs pressure passes through a throttling calorimeter. Assuming that the lowest condition in the calorimeter for measuring the quality is 10 deg fahr superheat and the pressure in the calorimeter is 14.696 psi abs, what is the largest percentage of wetness the calorimeter is capable of measuring under the above conditions?

22. Prepare a chart by means of which can be determined the largest percentages of wetness a throttling calorimeter will measure at all pressures from 50 to 300 psi abs.

23. Steam is throttled through a valve from 400 psi abs and quality 97 per cent to 25 psi abs. Assuming the same velocity on each side of the valve, uniform flow and no heat losses, determine the increase in the specific volume of the steam.

24. A closed tank has a volume of 100 cu ft and contains 120 lb of steam at a temperature of 450 deg fahr. What is the quality and internal energy of the steam?

CHAPTER VI

ENTROPY

Entropy. In the development of engineering thermodynamics, a property termed entropy has been evolved. Its multitude of uses in physics and chemistry as well as in engineering has resulted in a large number of definitions and defining phrases which at first appear unrelated. Closer study, however, shows that these are but special applications of the same basic idea.

In engineering, entropy has two principal uses or applications: first, a coordinate of a diagram in which heat transfer is shown graphically; and, second, a factor or index of the unavailability of heat energy for conversion into work.

If heat energy (Q) is to be graphically represented as an area to a definite scale and if the absolute temperature (T) is selected as the ordinate, then the abscissa (S) must be so related that

$$T \times S = Q.$$

The coördinate thus established termed entropy (S) is mathematically expressed by

$$S = \frac{Q}{T},$$

or more generally

$$dS = \frac{dQ}{T}. \tag{132}$$

Strictly speaking, entropy which is a relation between heat energy and absolute temperature applies only to frictionless or mechanically reversible processes.

In analyzing entropy as a mathematical statement, the absolute temperature or denominator of the fraction can never be negative. Therefore, a positive entropy change signifies a positive quantity of heat energy added during a process; a negative entropy change signifies heat rejected during a process, and a zero entropy change signifies no heat energy added or rejected.

Entropy as a Coordinate. Considering further the use of entropy as a coordinate, suppose that the heat energy (Q) involved in a process, such as a mechanically reversible expansion or compression, is divided up into a number of small increments dQ (**Fig. 24**), and that each small increment of heat energy is divided by the absolute temperature at which the infinitesimal quantity of heat transfer occurs. As a result there will be a series of these expressions $\frac{dQ}{T}$ or entropy changes which, when added, will give the total entropy change during the process or the abscissas of the entropy diagram. If the ordinates in the diagram are in absolute temperatures the area of the diagram may be expressed

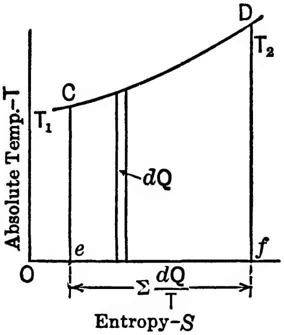

FIG. 24. Analysis of Entropy Diagram.

$$\text{Area under curve} = \int T\,dS.$$

Substituting the differential in terms of entropy, the equation becomes

$$\text{Area under curve} = \int T\,\frac{dQ}{T} = \int dQ = Q.$$

In a temperature-entropy diagram such as **Fig. 24**, where the ordinates are absolute temperatures and the abscissas are entropies, the area under any line CD gives the quantity of heat added to the substance in passing from a temperature T_1 and an entropy $S_1 = Oe$ to a temperature T_2 and an entropy $S_2 = Of$.

Entropy Changes of Gases at Constant Pressure. The heat supplied or abstracted during a *constant pressure* expansion or compression of a gas may be stated:

$$dQ = wc_p\,dT,$$

or

$$Q = wc_p(T_2 - T_1).$$

From equation (132) the change in entropy is

$$S_2 - S_1 = \int \frac{dQ}{T}.$$

Assuming the specific heat c_p constant, the change in entropy becomes:

$$\begin{aligned} S_2 - S_1 &= wc_p \int_{T_1}^{T_2} \frac{dT}{T} \\ &= wc_p\,(\log_e T_2 - \log_e T_1) \\ &= wc_p \log_e \frac{T_2}{T_1}. \end{aligned} \qquad (133)$$

Entropy Changes of Gases at Constant Volume. The heat supplied or abstracted during a constant volume change of a gas may be stated:

$$dQ = wc_v\,dT,$$

or

$$Q = wc_v(T_2 - T_1).$$

Assuming, as before, that the specific heat under *constant volume* (c_v) is constant and substituting in equation (132), the change in entropy is

$$\begin{aligned} S_2 - S_1 &= wc_v \int_{T_1}^{T_2} \frac{dT}{T} \\ &= wc_v\,(\log_e T_2 - \log_e T_1) \\ &= wc_v \log_e \frac{T_2}{T_1}. \end{aligned} \qquad (134)$$

Entropy Changes of Gases at Constant Temperature. During an isothermal change the temperature remains constant. The heat supplied or abstracted is equal to the heat equivalent of the work done, which in terms of weight, temperature and volume, is

$$wRT \log_e \frac{V_2}{V_1}.$$

Since the temperature remains constant, the change in entropy is

$$\begin{aligned} S_2 - S_1 &= \frac{wRT \log_e \dfrac{V_2}{V_1}}{778\,T} \\ &= \frac{wR}{778} \log_e \frac{V_2}{V_1}. \end{aligned} \qquad (135)$$

Entropy Changes During Reversible Adiabatic Processes of Gases. The heat supplied or abstracted during an adiabatic expansion or compression is, by definition, zero. The entropy change in such a process is

$$S_2 - S_1 = \int_{T_1}^{T_2} \frac{dQ}{T} = 0. \qquad (136)$$

Thus, there is no entropy change during a reversible adiabatic process. The term *isentropic* (equal entropy) is frequently used to designate a reversible adiabatic process.

Irreversible Processes. Irreversible adiabatic processes are those in which not only no heat is added or abstracted but also a portion or all of the energy involved in the process may reappear in the working medium as energy at the end of the process. In such a process the entropy does not remain constant but increases, and the degree of irreversibility may be measured by the increase of entropy at the end of the process.

The flow of steam in a nozzle and in the throttling calorimeter are two practical examples of irreversible processes.

In the steam nozzle, energy in the fluid is transformed into kinetic energy. The expansion in the nozzle is adiabatic in the sense that no heat is added or abstracted. Owing to friction and turbulence of flow, not all the energy available from the process is transformed into kinetic energy, and that portion necessary to overcome friction and to produce *turbulence* appears as unavailable energy at the end of the process. As a result, the energy transformed into kinetic energy is less than it would have been had the process been reversible without friction or turbulence; and, likewise, the energy remaining in the working substance is greater than with non-turbulent flow and its entropy at the end of the process has been increased.

In the case of the throttling calorimeter discussed in the previous chapter, the thermodynamic process involved is adiabatic but the apparatus is so constructed that no work energy is transmitted from the working substance. As a result, the energy contained in the working substance as it enters the process equals that contained as it is leaving the process. The throttling process is one of maximum irreversibility in that the work energy taken from the working substance is zero and the change of entropy is a maximum.

In the solution of thermodynamic problems dealing with irreversibility a definite knowledge of the degree of irreversibility is necessary. This information must be obtained from experimental data. When no accurate knowledge is available as to the extent of irreversibility, calculations are usually made on the assumption that the process is reversible.

Entropy Changes during a Carnot Cycle. The study of heat-engine cycles is often simplified by the use of entropy. The study of the Carnot cycle is a good illustration. A pound of the working substance is first expanded isothermally. On a *T-S* (temperature-entropy) diagram (**Fig. 25**), this process would be represented by line **AB**, where the temperature remains constant at T_1, and where the entropy increases from **Oe** to **Of**, because of the addition of heat that is required to keep the temperature constant.

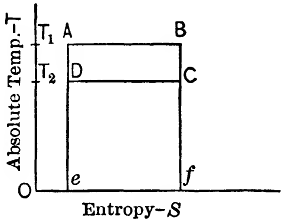

FIG. 25. Entropy Diagram of Carnot Cycle.

The next process is adiabatic expansion from T_1 to T_2. Heat is neither added to nor abstracted from the substance during this expansion. Hence the entropy remains constant, as indicated by **BC**.

The substance is isothermally compressed along **CD**, the temperature remaining constant at T_2 and the entropy decreasing because of the abstraction of heat.

The last process of the cycle is the adiabatic compression from **D** to **A**, no heat being added or abstracted, and the entropy, therefore, remaining constant.

The total heat supplied during the cycle is represented by the area, *ABfe* and equals

$$(S_f - S_e)T_1.$$

The heat abstracted during the cycle is represented by the area, *CDef*, and equals

$$(S_f - S_e)T_2.$$

Since the work of the cycle, in terms of heat, equals the heat added minus the heat abstracted,

$$\begin{aligned}\text{Work} &= \text{Area } ABfe - \text{area } CDef\\ &= \text{Area } ABCD.\end{aligned}$$

In terms of entropy,

$$\begin{aligned}\text{Work} &= (S_f - S_e)T_1 - (S_f - S_e)T_2 \\ &= (T_1 - T_2)(S_f - S_e). \qquad (137)\end{aligned}$$

The efficiency of the cycle is

$$\begin{aligned}e &= \frac{\text{Area } ABCD}{\text{Area } ABfe} \\ &= \frac{(T_1 - T_2)(S_f - S_e)}{T_1(S_f - S_e)} = \frac{T_1 - T_2}{T_1}. \qquad (138)\end{aligned}$$

From the foregoing discussion two important conclusions may be drawn in regard to the use of the T-S diagram:

1. If any reversible process be represented by a curve on a T-S diagram, the *heat involved during the process is equal to the area under the curve*, that is, between the curve and the axis of absolute zero temperature.

2. If a cycle of reversible processes be represented on a T-S diagram by a closed figure, the net work done is equal to the enclosed area, that is, the enclosed area measures the amount of heat that was converted into work.

Entropy Changes During an Otto Cycle. **Figure 26** represents the Otto cycle on T-S coordinates with the different events lettered the same as in **Fig. 16.** **ab** is the adiabatic compression which is followed by the combustion of the mixture **bc.** **cd** is the adiabatic expansion, and the gases are exhausted along **da.**

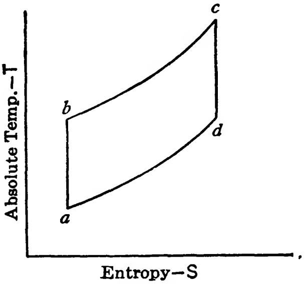

Fig. 26. Entropy Diagram of Otto Cycle.

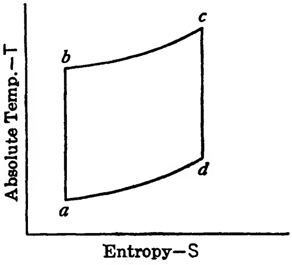

Fig. 27. Entropy Diagram of Diesel Cycle.

Entropy Changes During a Diesel Cycle. The Diesel cycle on T-S coordinates is illustrated in **Fig. 27.** (Compare **Figs. 19** and

27.) Air is compressed adiabatically along the constant entropy line **ab**; **bc** is the combustion phase; adiabatic expansion is represented by **cd**; and the burnt gases are rejected along **da**.

Entropy Changes During a Dual Cycle. The dual cycle on T-S coordinates is illustrated in **Fig. 28.** (Compare **Figs. 28** and **21.**)

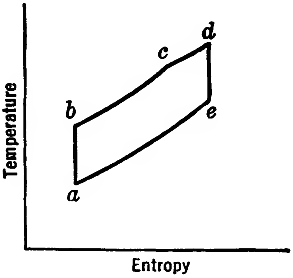

FIG. 28. Entropy Diagram of a Dual Cycle.

Entropy as a Measure of Unavailable Energy. In the transformation of heat energy into work, only a part of the original energy is available, the remainder being discharged to the cold body as unavailable energy.

If Carnot's principle is used, which would imply the most efficient transformation and the least energy made unavailable, entropy may be used to calculate the minimum amount of the unavailable energy.

Let dQ_1 = small quantity of heat energy originally available at temperature T_1.

dQ_2 = unavailable energy that must be discharged to cold body at temperature T_2.

$$dS = \frac{dQ_1}{T_1} = \text{entropy of original heat energy.}$$

Then, since

$$dQ_1 - dQ_2 = \frac{T_1 - T_2}{T_1} \times dQ_1$$

$$dQ_2 = \frac{dQ_1 T_2}{T_1}$$

$$= dS\, T_2. \qquad (139)$$

The change of entropy thus becomes a factor which multiplied by the lowest available temperature of the exhaust gives that portion of the energy which must be lost, that is, inherently unavailable under the conditions.

Temperature-Entropy Diagrams for Steam. T-S diagrams find particularly useful application in steam engineering, as indicated by the following:

1. Graphical analysis of heat transfers in a steam engine cylinder.
2. Determination of quality of steam during adiabatic expansion.
3. Calculations of steam engine cycle efficiencies.
4. Steam turbine calculations.

A temperature-entropy diagram for steam is shown in **Fig. 29.** The various shaded areas represent the heats added to water at 32 deg fahr to vaporize it completely at the pressure P_1. The area **ABCD**, neglecting the feed pump work, is the heat added to the water to bring it to the temperature of vaporization, corresponding to the pressure P_1 on the assumption that the water is in a saturated condition during the change. This energy may be designated as the enthalpy h_f. Further heating produces evaporation at the constant temperature T_1 corresponding to the pressure P_1, and is represented by the area under the line **CE.** When vaporization is complete, the latent heat, or the heat of vaporization (h_{fg}), is the area **DCEF.** If, after all the water is vaporized, more heat is added, the steam becomes superheated, and the additional heat required would be represented by an area **EGHF.**

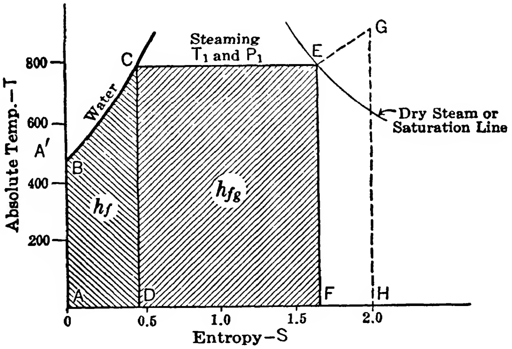

Fig. 29. Temperature-Entropy Diagram of Steam.

Calculation of Entropy for Steam. In order to lay off the increase in entropy as abscissas in the heat diagram for steam, it is necessary to determine various values. For convenience, liquid water at 32 deg fahr has been adopted as the arbitrary starting point for calculating increase of entropy, as well as for the other thermal properties of steam. Referring to **Fig. 30** the entropy of water is seen to be 0 at 32 deg fahr. In order to raise the temperature from T_0 to T_1, h_f heat units are required, and by equation (132), *the entropy of* 1 *lb of the liquid* s will be

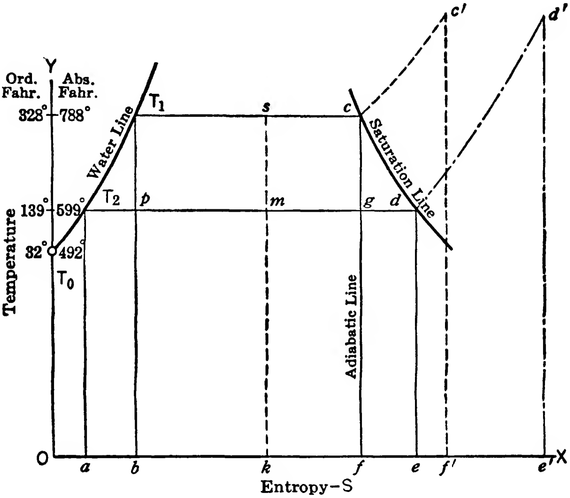

FIG. 30. Diagram for Calculation of Entropy of Steam.

$$s_f = \int_{T_0}^{T_1} \frac{dh_f}{T}, \tag{140}$$

or, assuming the specific heat of water to be unity we can write

$$s_f = \int_{T_0}^{T_1} \frac{dT}{T}$$

$$= \log_e T_1 - \log_e T_0 = \log_e \frac{T_1}{T_0}. \tag{141}$$

In order to evaporate water into steam at the boiling point T_1, the latent heat, h_{fg}, must be added at constant temperature T_1.

The increase of entropy during the steaming process, s_{fg}, is represented by $\frac{h_{fg}}{T}$ and is equal to, in this case,

$$s_{fg_1} = \frac{h_{fg_1}}{T_1}. \tag{142}$$

Further heating would produce superheated steam, and the change in entropy would be

$$\begin{aligned} \text{Entropy of superheat} &= \int_{T_{\text{sat}}}^{T_{\text{sup}}} \frac{c_p\,dT}{T} \\ &= c_p \log_e \frac{T_{\text{sup}}}{T_{\text{sat}}}, \end{aligned} \tag{143}$$

where c_p = mean value of the specific heat of the superheated steam at constant pressure.
T_{sup} = absolute temperature of the superheated steam.
T_{sat} = absolute temperature of the saturated steam.

The total entropy of steam, which is the sum of the entropies, corresponding to the various increments making up the total, depends upon the quality of the steam.

For dry saturated steam at the temperature T_1 the total entropy above 32 deg fahr is equal to the sum of the entropy of the liquid (s_{f_1}) and the entropy of evaporation (s_{fg_1}) for dry saturated steam. This may be written

$$s_{g_1} = s_{f_1} + s_{fg_1}. \tag{144}$$

Similarly, for wet steam whose quality is x, the total entropy is

$$s_{x_1} = s_{f_1} + x s_{fg_1}. \tag{145}$$

For superheated steam whose final temperature is T_{sup} and whose temperature corresponding to saturation is T_1, the total entropy is,

$$S_{\text{sup}} = s_{f_1} + s_{fg_1} + \int_{T_{\text{sat}}}^{T_{\text{sup}}} \frac{c_p\,dT}{T}. \tag{146}$$

For convenience, most vapor tables give the values of the entropy of the liquid (s_f)*, the entropy of the evaporation* (s_{fg})*, the total entropy of dry saturated vapor* (s_g) *and the total entropy of superheated vapor.*

Referring to **Fig. 30,** the line $\mathbf{T_1c}$ represents the increase of entropy due to the latent heat added during the steaming process. If this steaming process had stopped at some point such that the steam was wet, having a quality x, this condition of the steam could be denoted by the point **s**, where

$$x = \frac{\mathbf{T_1s}}{\mathbf{T_1c}}. \tag{147}$$

This relation is obvious, for a distance along $\mathbf{T_1c}$ represents the entropy of steam, which is proportional to the latent heat added, which in turn is proportional to the amount of dry steam formed from 1 lb of water. In like manner, $\frac{T_2\mathbf{m}}{T_2\mathbf{d}}$ would be the quality of steam at the point **m** that would be formed along $T_2\mathbf{d}$ at the temperature T_2 and the appropriate pressure.

It is thus apparent that any point on a T-S diagram will give full information in regard to the steam. The proportional distances on a line drawn through the given point between the water line and the dry steam line and parallel to the X-axis give the quality of the steam as shown above. The ordinate of the point gives the temperature and corresponding pressure, while its position relative to other lines, such as constant volume and constant enthalpy lines which can be drawn on the same diagram, will give further important data.

Temperature-Entropy Diagram for the Steam Power Plant. **Figure 31** illustrates the heat process occurring when feed water is received in the boilers of a power plant at 100 deg fahr, is heated and converted into steam at a temperature of 400 deg fahr, and then loses heat in doing work. When the feed water first enters the boiler its temperature must be raised from 100 deg fahr to 400 deg fahr before any steaming begins. The heat added to the liquid is indicated by the area **MNCD.** This area represents approximately the difference between the enthalpies of the liquid (375 − 68) or about 307 Btu. The horizontal or entropy scale shows that the difference in entropy between water at 100 deg fahr and 400 deg fahr is about 0.436.

The curve **NC** is constructed by plotting from the steam tables the values of the entropy of the liquid for a number of different temperatures between 100 deg fahr and 400 deg fahr.

If water at 400 deg fahr is converted into steam at that temperature, the curve representing the change is necessarily a constant temperature line and therefore a horizontal, **CE**. Provided the evaporation has been complete, the heat added in the steaming process is the latent heat or the heat of evaporation of steam (h_{fg}) at 400 deg fahr, which is 826.0 Btu.

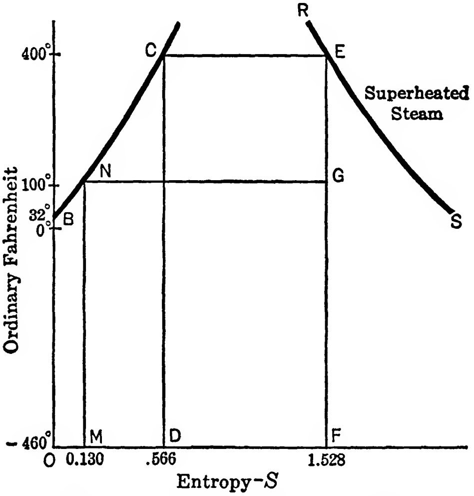

Fig. 31. Temperature-Entropy Diagram for Steam Power Plant.

The change in entropy during evaporation is, then, the heat added (826.0) divided by the absolute temperature at which the change occurs (400 + 460 = 860 deg fahr abs) or

$$s_{fg} = \frac{826.0}{860} = 0.9608.$$

The total entropy of steam completely evaporated at 400 deg fahr is, therefore, 0.130 + 0.437 + 0.9608 = 1.527. To represent this final condition of the steam, the point **E** is plotted as shown in the figure. The point **E** is shown located on another curve **RS**, which is determined by plotting a series of points calculated the same as **E**, but for different pressures. The area **MNCEF** represents the total heat added to a pound of feed water at 100 deg fahr, to produce steam at 400 deg fahr. Area **OBCEF** represents the total heat required to form 1 lb of steam at 400 deg fahr.

If the steam generated in the boiler is allowed to expand adiabatically (isentropically) in an engine cylinder or in a turbine nozzle to a temperature of 100 deg fahr, this expansion is represented by the line **EG**. **GN** represents the condensation of the exhaust steam. The area **MNGF** $[T_2(S_G - S_N) = 560(S_E - S_N)]$ represents the heat rejected to the condenser Q_2 (unavailable) and the area **NCEG** $(Q_1 - Q_2)$ represents the heat which has disappeared by being converted into work in the engine.

The Mollier Chart. Although the temperature-entropy diagram is extremely useful in analyzing various heat processes, its use in representing some conditions of steam is not so convenient as that of the Mollier chart. This chart is fully illustrated in Plate 1 of the Appendix; a small section of the diagram is shown in **Fig. 32.** In this chart the ordinates represent the total enthalpy of the steam above 32 deg fahr and the abscissas the entropy. The saturation curve marks the boundary between the superheated

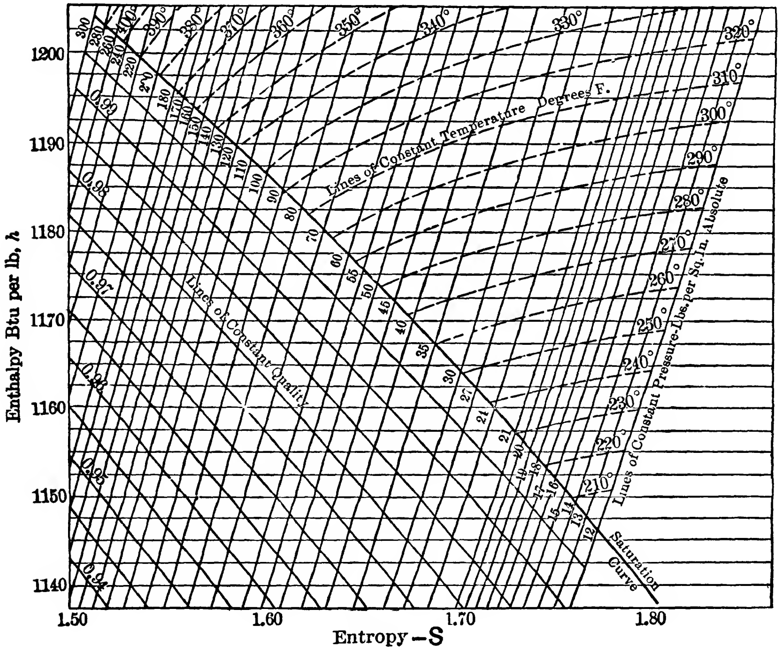

Fig. 32. Mollier Diagram for Determining Quality of Steam.

and saturated regions. In the region of wet steam, lines of constant quality and pressure are drawn and in the superheated regions, isotherms and lines of equal superheats appear. In both the superheated and saturated regions, lines of constant pressure are drawn and the absolute pressures in pounds per square inch of the various curves are labeled.

The Mollier chart is a convenient device for obtaining the approximate enthalpy of a vapor if its pressure and quality are known. Reversible adiabatic processes are represented on such a chart by a line perpendicular to the entropy axis. The chart makes possible the rapid estimating of the quality and enthalpy of the vapor during reversible adiabatic processes. Processes involving constant enthalpy are represented on a Mollier chart by a line perpendicular to the total heat or enthalpy axis.

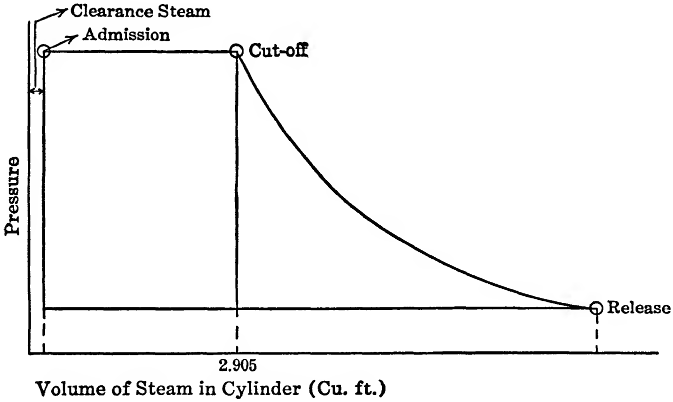

Fig. 33. Illustrative Indicator Diagram of Engine Using Steam with Expansion.

By means of the Mollier chart (Appendix or **Fig. 32**) the curve for adiabatic expansion may be drawn on a pressure-volume diagram (**Fig. 33**) when the initial quality of the steam at cut-off is known. Assume that there is 1 lb of wet steam at a pressure of 150 psi abs and quality 0.965 dry in the engine cylinder at cut-off. The volume of a pound of dry saturated steam at this initial pressure of 150 psi is, from the steam tables, 3.010 cu ft. At 0.965 quality it will be $1(0.965 \times 3.015) = 2.909$ cu ft. Other points in the adiabatic expansion curve can readily be plotted after determining the quality by the method given on page 119.

Graphical Determination of Quality of Steam by Throttling Calorimeter and Mollier Diagram. It will be remembered that the throttling calorimeter depends for its action upon the fact that the enthalpy of steam which expands without doing work remains the same, the energy in excess of that required to keep the steam dry and saturated going to superheat the steam. Suppose that steam enters the calorimeter at a pressure of 150 psi abs, and

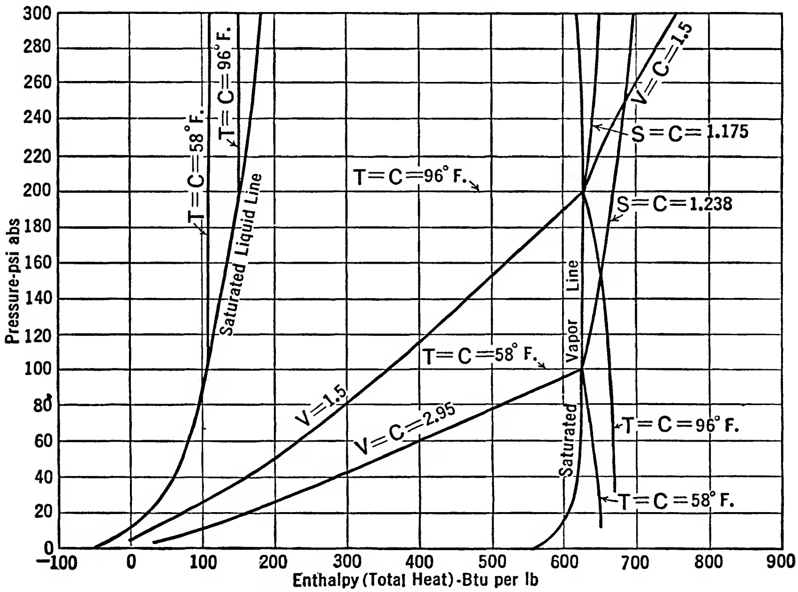

Fig. 34. *P-h* Diagram for Ammonia.

is throttled down to 17 psi abs, the actual temperature being 240 deg fahr. Since the saturation temperature for steam at 17 psi pressure is 219.4, the steam in the calorimeter is superheated 240 deg − 219.4 deg or 20.6 deg. In order to find the quality of the live steam refer to the "*Mollier Diagram*" (Appendix or **Fig. 32**) and find the intersection of the 20.6 deg superheat line with the 17-lb pressure line. From this point follow a horizontal line (line of constant enthalpy or total heat) to the left until it intersects the 150-lb pressure line. This point of intersection is found to lie on the 0.965 quality line.

Equation (128) gives the following result in close agreement with the diagram:

$$x = \frac{1163.1 - 330.44}{863.1} \quad \text{or} \quad 0.965.$$

Figure 34 shows a portion of a *P-h* diagram for ammonia. The saturation curve, lines of constant properties, etc., may be followed in the *P-h* diagram (**Fig. 34**) as they were described in connection with the Mollier chart (**Fig. 32**). The *P-h* diagram is especially valuable in refrigeration processes.

PROBLEMS

1. A Carnot cycle operates between 1500 deg fahr and 500 deg fahr. The entropy change during the heating process is 2 units.
   (a) Determine the work per cycle.
   (b) Draw the *T-S* diagram and show the significance of the areas.

2. Hydrogen at a constant pressure of 28 psi abs and a temperature of 70 deg fahr is heated at constant pressure until the temperature is 300 deg fahr. Then it is allowed to expand at constant temperature until the pressure is 14.0 psi abs. How much heat was added per mol of gas and what was the change in entropy?

3. Nitrogen expands from a pressure of 280 psi abs and a temperature of 340 deg fahr until its pressure is 14.0 psi abs. How much work is done per mol if the path followed is $PV^{1.3}$ = constant? What is the change in entropy? in internal energy?

4. The volume of 10 lb of oxygen changes from 20 to 100 cu ft during an isothermal change at 100 deg fahr. What is the change in entropy? Is heat added or abstracted during the process and how much?

5. During the isothermal compression of 2 lb of nitrogen at 200 deg fahr the entropy decreased 0.4 units. How much work was done?

6. Dry and saturated steam has a pressure of 100 psi abs. Determine the entropy of the liquid, of evaporation, and of the steam.

7. Steam at 150 psi abs has a quality of 0.90.
   (a) What is the entropy of the liquid?
   (b) What is the entropy of evaporation?
   (c) What is the total entropy of the steam?

8. Steam having a temperature of 300 deg fahr has an entropy of evaporation of 1.1900. What is its quality?

9. Steam having a pressure of 200 psi abs has a total entropy of 1.5400.
   (a) What is the total entropy of dry and saturated steam under the given pressure?
   (b) Is the steam wet or dry?
   (c) What is its quality?

10. Steam having a pressure of 125 psi abs is superheated 100 deg fahr.

(a) What is the total entropy of dry and saturated steam under the given pressure?
(b) What is the entropy of the superheat?
(c) What is its total entropy?

11. Steam having a pressure of 160 psi abs has a total entropy of 1.5906.

(a) What is the total entropy of dry and saturated steam under the given pressure?
(b) Is the above steam saturated or superheated? How can one tell?
(c) How much superheat has the steam?

12. A boiler generates steam of 0.90 quality at a temperature of 350 deg fahr with the feed water admitted at 90 deg fahr. What is the increase in entropy?

13. Plot a temperature-entropy diagram for 1 lb of water vapor for the absolute pressures of 15, 50, 100, 150, 200, 250 psi abs. The diagram should show the liquid line, the 90 per cent quality line, the dry saturated line, and the 50 and 100 deg superheat curves for each of the given pressures.

14. What is the entropy of steam 92 per cent dry at a pressure of 14.696 psi abs?

15. With a quality of 0.90, what is the entropy of evaporation of steam at a pressure of 30 psi abs?

16. What is the total entropy of steam 94 per cent dry at a pressure of 100 psi abs?

17. Using the Mollier chart (Appendix), what is the enthalpy of 1 lb of steam at a pressure of 190 psi abs if the steam has a temperature of 600 deg fahr? Verify the result by steam tables.

CHAPTER VII

THERMODYNAMIC PROCESSES OF VAPORS

Vapors, like gases, can be expanded or compressed, but the laws governing their thermodynamic relations are more complex. The solution of practical problems dealing with vapor processes is complicated by the fact that the process may be either uniform flow or non-flow, and either reversible or irreversible. In any of these cases the condition of the vapor may be wet, dry or superheated.

Non-Flow Processes. In the non-flow processes using vapors as the working medium, the energies entering into the process consist of the heat energy (Q), the internal energy (U) and the external work (AW).

The energy equation involved in an expansion or a compression of this type is

$$Q = \int du + A \int P\,dv. \tag{148}$$

The external work $A \int P\,dv$ performed during the expansion or compression of vapors is calculated, as in the case of gases, from the area under the expansion or compression curve. External work, being a function of pressure and volume, is independent of the working medium.

The general energy equation (148) is applicable to any vapor process of the non-flow type.

The following problems show the application of equation (148) to various non-flow reversible processes.

1. *Constant Volume Processes*

Example. Assume that 1 lb of steam at a pressure of 14.696 psi abs and 50 per cent dry, receives heat under constant volume raising the pressure to 30 psi abs.

Find (a) the volume of the steam, (b) the quality of the steam at the end of the process, (c) the work done, (d) the heat required.

Solution. The following values are found from the steam tables at 14.696 psi abs: $v_g = 26.80$, $h_f = 180.07$, $h_{fg} = 970.3$; at 30 psi abs $v_g = 13.746$, $h_f = 218.82$, $h_{fg} = 945.3$.

(a) Using the approximate method for the determination of this value, the volume of the steam at the beginning of the process will be

$$0.50 \times 26.80 = 13.40 \text{ cu ft.}$$

(b) Since the volume of the steam at the end of the process if dry and saturated would be 13.746 and since the actual volume of the steam as found in (a) is less than 13.746 cu ft, the steam is wet at the end of the process and its quality is

$$x_2 = \frac{13.40}{13.746} = 0.975 \quad \text{or} \quad 97.5 \text{ per cent dry.}$$

(c) Since the volume is constant during the process, the work done is

$$W = 0.$$

(d) The heat required is

$$\begin{aligned} Q &= U_2 - U_1 + AW \\ &= U_2 - U_1 + 0 \\ &= U_2 - U_1. \end{aligned} \tag{149}$$

The internal energy at the end of the process is

$$\begin{aligned} u_2 &= h_f + x_2 h_{fg} - APx_2 v_g \\ &= 218.82 + 0.975 \times 945.3 - \frac{30 \times 144 \times 13.40}{778} \\ &= 1065.4 \text{ Btu.} \end{aligned}$$

The internal energy at the beginning of the process is

$$\begin{aligned} u_1 &= h_f + x_1 h_{fg} - APx_1 v_g \\ &= 180.07 + 0.50 \times 970.3 - \frac{14.696 \times 144 \times 13.40}{778} \\ &= 629.8 \text{ Btu.} \end{aligned}$$

The heat required to produce the process is

$$Q = 1065.4 - 629.8 = 435.6 \text{ Btu.}$$

2. *Constant Pressure Processes*

To exemplify the solution of problems dealing with constant pressure processes consider the following problem as an illustration.

Example. One pound of steam at a pressure of 150 psi abs and 50 per cent dry is heated under constant pressure until it becomes dry and saturated.

(a) What is the volume of the steam at the initial condition?
(b) What is the volume of the steam at the final condition?
(c) What is the work done during the process?
(d) How much heat is required?

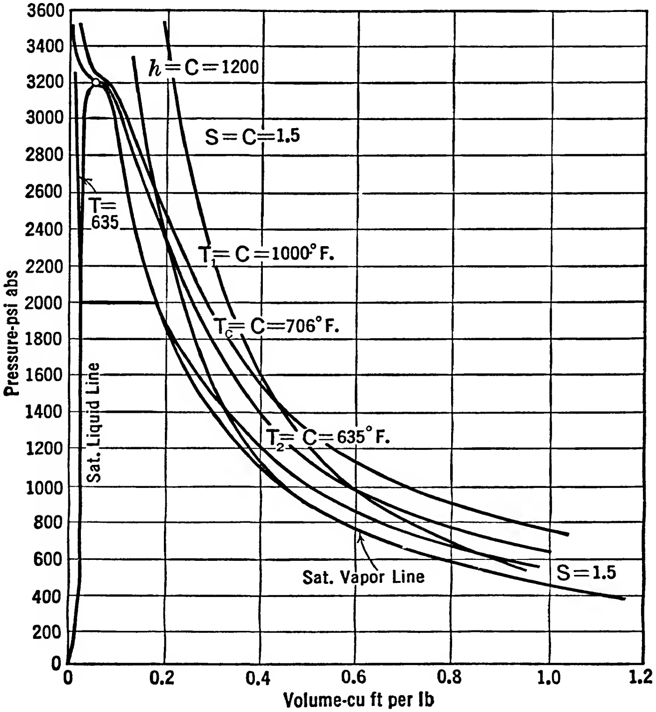

FIG. 35. *P-V* Diagram for Steam.

Solution. (a) The volume of 1 lb of dry saturated steam at 150 psi abs is found from the steam tables to be 3.015 cu ft. Since the steam at the initial condition is 50 per cent dry, the volume of the steam present at the beginning of the process is

$$3.015 \times 0.50 = 1.507 \text{ cu ft.}$$

(b) Since the steam is dry and saturated at the end of the process its volume will be 3.015 cu ft.

(c) The work done during the expansion is

$$AW = AP(v_2 - v_1) = \frac{150 \times 144(3.015 - 1.507)}{778} = 41.8 \text{ Btu.}$$

(d) The heat required may be determined from the equation

$$Q = U_2 - U_1 + AW. \tag{150}$$

In a constant pressure process the heat energy (Q) may be expressed in terms of enthalpy as follows:

Since
$$\begin{aligned} Q &= U_2 - U_1 + AP(v_2 - v_1) \\ &= (U_2 + APv_2) - (U_1 + APv_1). \end{aligned}$$

Since in general enthalpy

$$(h) = u + APv$$

$$Q = h_2 - h_1 \tag{151}$$

where h_1 and h_2 are the enthalpies of the vapor at the initial and final conditions of the process.

Thus, using the data as given in the previous problem, the heat energy to produce the process could have been determined from the enthalpies as follows:

$$Q = h_2 - h_1.$$

As taken from the steam tables the enthalpy (h_2) is 1194.1. The enthalpy (h_1) is, then,

$$330.51 + 0.50 \times 863.6 = 762.31.$$

Therefore,

$$Q = 1194.1 - 762.31 = 431.79 \text{ Btu.}$$

Isothermal Processes for Steam. Isothermal expansion of steam occurs at constant pressure, in the conversion of water into steam in a boiler. Isothermal changes for wet steam are, therefore, lines of uniform pressure. On a pressure-volume diagram an isothermal line is, consequently, represented by a horizontal line parallel to the axis of abscissas. As steam becomes superheated and the isothermal change continues, the pressure decreases as the volume increases; for highly superheated vapors the isothermal curve approaches a rectangular hyperbola. On a *T-S* diagram, the isothermal line is represented by a line of constant temperature, i.e., by a line parallel to the *S*-axis.

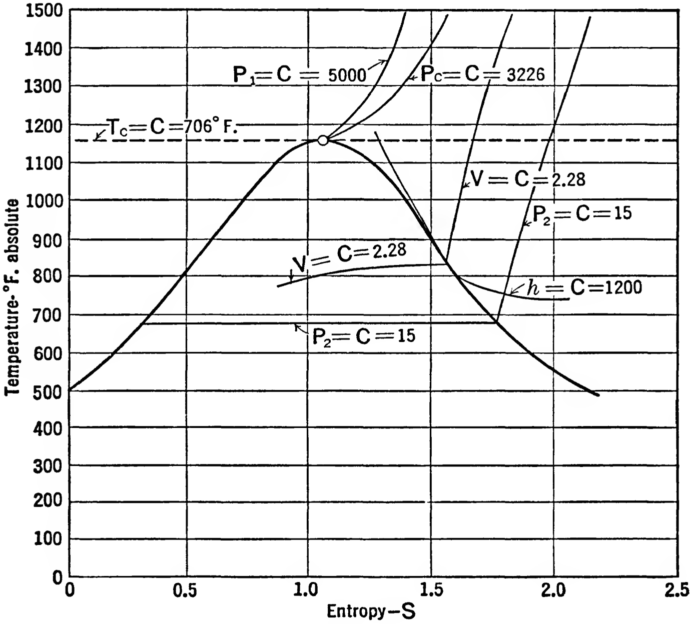

FIG. 36. *T-S* Diagram for Steam.

Adiabatic Processes. No heat is added or abstracted during an adiabatic process. The energy equation for an adiabatic process of the non-flow type is

$$Q = U_2 - U_1 + AW = 0,$$

from which,

$$AW = U_1 - U_2. \qquad (152)$$

The work done during a non-flow adiabatic process can then be determined from the change in the internal energy during the process. In order to calculate changes in internal energy, the corresponding changes in quality must be known, and these may be found from the entropy.

Quality of Steam During a Reversible or Isentropic Type of Adiabatic Process. In a reversible or isentropic type of adiabatic process no entropy change takes place during the process; and, therefore, on a *T-S* diagram this condition is represented by a straight vertical line such as **cgf** or **smk** (**Fig. 30**).

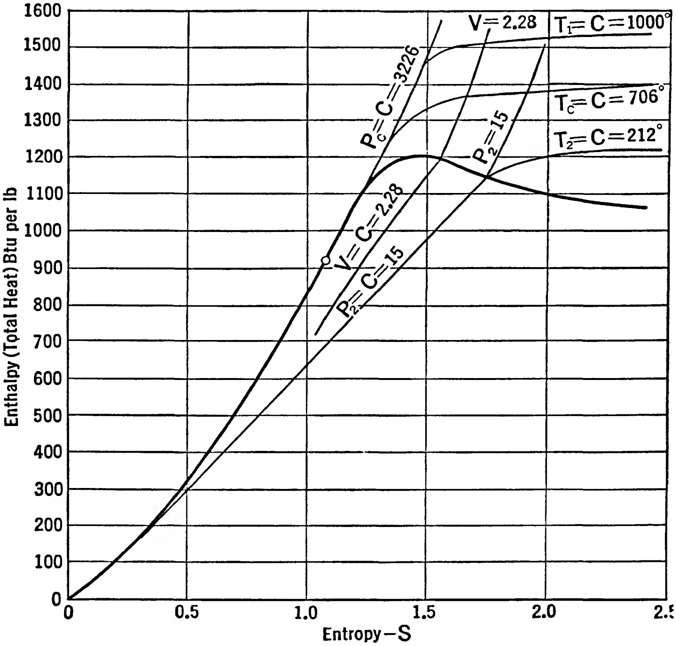

FIG. 37. *h-S* Diagram for Steam.

Since in the reversible or isentropic type of adiabatic process the entropy remains constant, the following equations can be written:

Total entropy at initial condition equals total entropy at any other condition, or

$$S_1 = S_2.$$

For wet steam this becomes:

$$s_{f_1} + x_1 s_{fg_1} = s_{f_2} + x_2 s_{fg_2}. \tag{153}$$

The initial conditions of the steam being known, the quality of the steam at any time during such a process can readily be determined. Thus, assume that the initial pressure is 100 psi abs, the steam is dry and saturated, and the final pressure after the expansion is 20 psi abs.

From the steam tables the total entropy for the dry steam at 100 psi is 1.6026. The entropies of the liquid and evaporation at the final condition of 20 psi are, respectively, 0.3356 and 1.3962.

Therefore, $$0.3356 + 1.3962\,x_2 = 1.6026$$

when $$x_2 = 0.908.$$

For a rapid and convenient means of checking the above result, the " total heat-entropy " (Mollier) diagram in the Appendix can be used. From the intersection of the 100 psi line and that of unit quality (" saturation line ") is dropped a vertical line (line of constant entropy) to the 20 psi line. This latter intersection is found to be between the 9 and 10 per cent moisture lines, thus giving a quality of 0.905.

Example. One pound of steam has a temperature of 400 deg fahr at a pressure of 100 psi abs and expands isentropically to 20 psi abs.

1. *What is the quality at the first condition?*

The steam at the initial condition is superheated and its total entropy taken from the steam tables is 1.6518.

The total entropy at the final condition is

$$s_{f_2} + x_2 s_{fg_2} = 0.3356 + 1.3962\,x_2.$$

Since entropy is constant during this reversible type of expansion

$$0.3356 + 1.3962\,x_2 = 1.6518.$$

from which $$x_2 = 0.943.$$

2. *How much work is done* **during** *the expansion?*

$$W = 778\,(U_1 - U_2).$$

The internal energy at the initial condition is

$$U_1 = 1227.6 - \frac{100 \times 144(4.937)}{778}$$
$$= 1227.6 - 91.5 = 1136.1 \text{ Btu}.$$

The internal energy at the final condition is

$$U_2 = 196.16 + 0.943 \times 960.1 - \frac{20 \times 144(0.01683 + 0.943 \times 20.089)}{778}$$

$$= 196.16 + 905.37 - 70.09$$

$$= 1031.44 \text{ Btu.}$$

The work of expansion, therefore, equals

$$1136.1 - 1031.44 = 104.66 \text{ Btu.}$$

or

$$104.66 \times 778 = 81,425 \text{ ft-lb.}$$

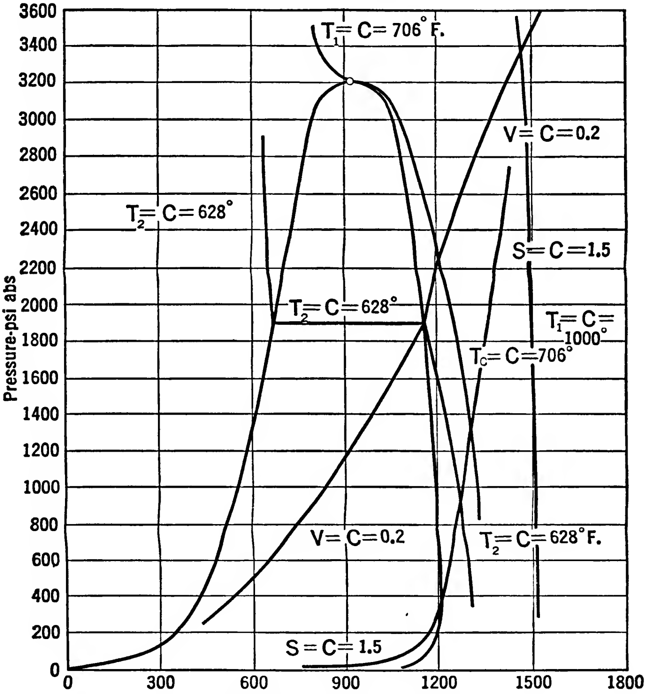

FIG. 38. *P-h* Diagram for Steam.

The various processes just described are illustrated in **Figs. 35, 36, 37,** and **38,** plotted to *P-V, T-S, h-S,* and *P-h* coordinates. These charts were prepared for steam, but similar charts may be secured for other vapors. Lines of constant pressure, volume, temperature, etc., may be traced on each plane and in different regions.

Non-Flow Irreversible Processes. The irreversibility of most processes is caused by internal disturbances, usually due to friction or turbulence, which require energy for overcoming or producing them. The effect of irreversibility when compared with a reversible process manifests itself by a decrease in the energy transformed into useful work and by an increase in the internal energy of the working substance at the end of the process. The flow of steam through an orifice or a partially closed valve can be classed as an irreversible process. Many such processes are found in the practical application of thermodynamics. It could, therefore, be concluded that no practical process is reversible, and that a reversible process is an ideal that may be approached, but seldom, if ever, attained.

Reversible and Irreversible Uniform Flow Processes. A uniform flow process is one in which the expansion or compression is a reversible or irreversible adiabatic process. Of the various types of expansions and compressions met with in the practical application of the thermodynamic theory of vapors, that of the reversible and irreversible adiabatic is the most important. The flow of vapor through a nozzle, a turbine, an orifice or a throttling calorimeter is in every case a practical application of uniform flow in which either a reversible or irreversible adiabatic process is involved.

As developed in a previous chapter, the energy equation of a uniform flow process per pound of the working substance is

$$\frac{A\mathrm{V}_1^2}{2g} + Az_1 + u_1 + AP_1v_1 + Q_{(in)}$$
$$= AW_{(out)} + \frac{A\mathrm{V}_2^2}{2g} + Az_2 + u_2 + AP_2v_2. \qquad (154)$$

Since h (enthalpy) equals u plus $\frac{Pv}{778}$, and Q equals zero for an adiabatic process, this equation for a reversible or irreversible adiabatic process in terms of enthalpy and changes of energy may be written thus,

$$AW_{(out)} = A\left(\frac{\mathrm{V}_1^2 - \mathrm{V}_2^2}{2g}\right) + A(z_1 - z_2) + h_1 - h_2. \qquad (155)$$

Assuming that the change in velocity and mechanical potential energy is negligibly small, the equation becomes

$$AW_{(out)} = h_1 - h_2. \tag{156}$$

Thus, in a process in which the change in mechanical potential energy and velocity is small, the heat equivalent of the work of the adiabatic process equals the difference between the enthalpies at the beginning and the end of the process. In order to determine the value of the enthalpy at the end of the process the quality must be known, and this, in a reversible or isentropic process, is determined from the constant value of entropy that exists during such a process.

Example. One pound of dry and saturated steam at a pressure of 160 psi abs expands adiabatically and reversibly in a steam turbine exhausting at atmospheric pressure. Assuming that the change in velocity and mechanical potential energy is negligibly small, what energy should be converted into work?

Solution. From reference to the steam tables and the fact that the entropies should be constant during such a process the quality at the end of the process is

$$1.5640 = 0.3120 + 1.4446\, x_2$$
$$x_2 = 0.866.$$

The enthalpy at the beginning of the process is

$$h_1 = 1195.1 \text{ Btu}.$$

The enthalpy at the end of the process is

$$h_2 = 180.07 + 0.866 \times 970.3 = 1020.3 \text{ Btu}.$$

The heat equivalent of the energy available is

$$AW = 1195.1 - 1020.3 = 174.8 \text{ Btu}.$$

In case the expansion or compression is irreversible the effect of the irreversibility would manifest itself in an increase of entropy at the end of the process, increasing the quality and the enthalpy.

Continuing the previous problem but including the effect of irreversibility, let it be assumed that the entropy at the end of the process is increased to 1.6120. The quality at the end of the process is then

$$1.6120 = 0.3120 + 1.4446\, x_2$$
$$x_2 = 0.90.$$

The heat equivalent of the available work of the process is then

$$AW = 1195.1 - (180.01 + 0.90 \times 970.3) = 141.7 \text{ Btu.}$$

The loss in available energy due to the irreversibility of the process is

$$174.8 - 141.7 = 33.1 \text{ Btu.}$$

PROBLEMS

1. A closed tank contains 50 lb steam at 700 psi abs and 700 deg fahr. This is cooled until the final pressure is 100 psi abs. Find the final quality and the heat abstracted.

2. A rigid closed tank having a volume of 100 cu ft contains 30 lb of steam at 100 psi abs. If this is heated until the steam is saturated, what is the final pressure? How much heat is required?

3. A closed tank contains 20 lb of steam at a temperature of 200 deg fahr and a quality of 15 per cent. If this steam is heated until it is dry and saturated, determine the final pressure. How much heat is required?

4. A closed tank contains 40 lb of steam at 150 psi abs and 400 deg fahr. This is cooled until the final pressure is 100 psi abs. How much heat was removed? What is the volume of the tank?

5. Ten pounds of saturated water at 300 deg fahr are heated at constant pressure by the addition of 10,127 Btu. Find the final temperature or quality and the work done. Sketch the process on T-S coordinates.

6. Five pounds of steam at 100 psi abs and 400 deg fahr are cooled at constant pressure until the final quality is 40 per cent. Compute the heat removed and the change in total volume. Show the process on T-S coordinates.

7. One pound of steam at a pressure of 150 psi abs has a quality of 0.73. What work is done and what heat is required to double its volume at constant pressure?

8. If steam at 200 psi abs, 95 per cent dry, is caused to expand adiabatically and reversibly to 230 deg fahr, what are the properties of this steam at the lower point (that is, final total entropy, entropy of evaporation, quality, and volume)?

9. What will be the final enthalpy of dry saturated steam that is expanded adiabatically from 150 psi abs down to 10 psi abs?

10. Steam having a quality of 0.20 dry is compressed along a reversible adiabatic curve from a pressure of 20 psi abs to a pressure corresponding to a temperature of 295 deg fahr. What is the final quality?

11. Determine the final quality of the steam and find the quantity of work performed by 2 lb of steam in expanding adiabatically from 250 psi abs pressure to 100 psi abs, the steam being initially dry and saturated, and the expansion reversible.

12. One pound of steam at 150 psi abs, and 100 deg fahr superheat, expands adiabatically and reversibly. What is the pressure when the steam becomes dry and saturated? What is the work done during the expansion?

13. One pound of dry and saturated steam at 14.696 psi abs pressure is compressed adiabatically and reversibly to 100 psi abs pressure. What is the quality at the end of the compression and the negative work done?

14. One pound of steam at 100 psi abs pressure has a quality of 0.80. It expands along an $n = 1$ curve to a pressure of 14.696 psi abs. Find: (a) the volumes at the beginning and at the end of the expansion, (b) the quality at the end of the expansion, (c) the work done during the expansion, (d) the heat supplied to produce the expansion.

15. One pound of steam at 100 psi abs and 50 per cent wet expands isothermally until it is dry and saturated. Calculate the heat supplied and the work done.

16. Five pounds of water are heated at constant pressure of 3000 psi abs from 100 deg fahr to 900 deg fahr. Compute the heat added and the work done.

17. Five pounds of steam at 300 psi abs and 97 per cent quality are expanded adiabatically and reversibly to 150 psi abs. Using the Mollier diagram, find the change in enthalpy and the final quality. Show the process on *H-S* coordinates.

18. One pound of steam at 200 psi abs and 400 deg fahr expands isentropically to 50 psi abs. Using the Mollier diagram, find the final quality and the final enthalpy. How much heat was added or abstracted? Show the process on *H-S* coordinates.

19. One pound of steam at 150 psi abs expands along an $n = 1$ curve to 14.696 psi abs. If the steam at 14.696 lb is dry and saturated, (a) What must be the quality at the initial conditions? (b) What work will be done by the expansion? (c) What heat must be supplied?

20. Steam at a pressure of 100 psi abs having a quality of 0.50 expands adiabatically and reversibly to 14.696 psi abs. What is the quality at the end of the expansion?

21. One pound of steam at a pressure of 100 psi abs has a volume of 4.594 cu ft and expands adiabatically to 10 psi abs. On account of irreversibility, the entropy increases during the process to 1.7874.

(a) What is the quality at the beginning and at the end of the process?
(b) What is the available work of the process?
(c) What would have been the energy available had the process been reversible?
(d) What is the energy made unavailable by the irreversible process?

22. Steam having a pressure of 140 psi abs and a temperature of 400 deg fahr flows through a steam turbine exhausting at 10 psi abs.

(a) What will be the quality of the steam at the exhaust if the process is assumed reversible, and the changes in velocity and mechanical potential energy negligibly small?
(b) What will be the energy available for work per pound of the steam flowing in such a case?

23. Steam at 600 psi abs pressure and at a temperature of 700 deg fahr expands isentropically to 28 in. vacuum, the barometer pressure being 29.92 in.

Calculate the final quality and check your results by means of the Mollier diagram.

24. Steam is supplied to a turbine at 500 psi abs and 640 deg fahr. It is throttled in the governor to 100 psi abs, after which it is expanded isentropically to 4 psi abs.

(a) Compute the steam consumption of the turbine in lb per hp-hr, neglecting the pump.

(b) Draw an *H-S* diagram to illustrate the process and indicate thereon the loss in available energy due to throttling.

25. Steam at 300 psi abs and 520 deg fahr is throttled in a turbine governor to 100 psi abs, after which it is expanded isentropically to a back pressure of 2 in. Hg abs.

(a) Compute the theoretical steam consumption in lb per hp-hr, neglecting the boiler feed pump.

(b) Draw an *H-S* diagram and show by the diagram the loss in the available energy due to throttling.

CHAPTER VIII

VAPOR CYCLES

The Carnot Cycle. The Carnot cycle for an engine using vapor instead of gas is illustrated in **Fig. 39** and **Fig. 40**. In the discussion which follows it is assumed that the working substance is a liquid at point **a** and dry and saturated vapor at point **b**. With these assumptions the cycle on pressure-volume coordinates differs from the Carnot cycle using gas in that the isothermal processes **ab** and **cd** are lines of constant pressure (**Fig. 39**). It may also be well to call attention to the facts that the Carnot cycle is made up of non-flow processes; and, since it is an ideal cycle, its processes are considered reversible.

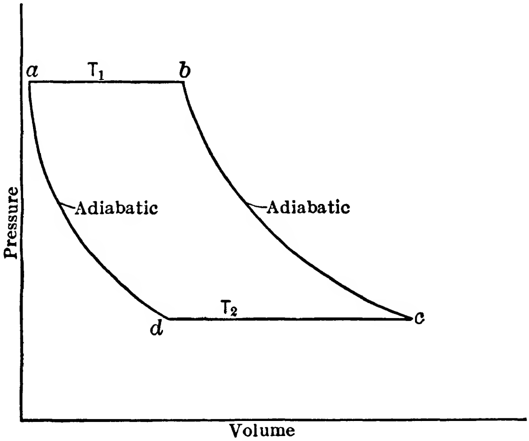

FIG. 39. Pressure-Volume Diagram of Carnot Cycle Engine Using Vapor.

The apparatus required to carry out this ideal cycle is the same as that described in Chapter IV. The working substance may be a vapor in any condition, but in the following discussion it is assumed to be in the liquid state at the point **a**. It receives heat and is completely vaporized at constant temperature (T_1) until the state **b** is reached. The heat source is then removed and the reversible adiabatic expansion **bc** is carried on until the medium reaches the temperature T_2. From **c** to **d**, heat is abstracted from the vapor at such a rate that the temperature remains constant at T_2. This is followed by the reversible adiabatic compression **da** until the working substance is at the initial state **a**. The area **abcd**, in both **Fig. 39** and **Fig. 40**, represents the work of the cycle.

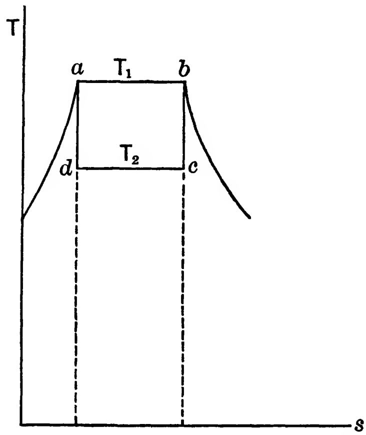

Fig. 40. Temperature-Entropy Diagram of Carnot Cycle Engine Using Vapor.

From the temperature-entropy diagram, **Fig. 40**, the heat energies involved in the cycle in terms of entropy are

$$Q_1 = T_1(s_b - s_a)$$
$$Q_2 = T_2(s_c - s_d)$$

Since

$$s_c - s_d = s_b - s_a$$
$$Q_2 = T_2(s_b - s_a).$$

The work of the cycle is then

$$\begin{aligned} AW &= Q_1 - Q_2 \\ &= T_1(s_b - s_a) - T_2(s_b - s_a) \\ &= (T_1 - T_2)(s_b - s_a). \end{aligned}$$

The efficiency of the cycle is

$$e = \frac{(T_1 - T_2)(s_b - s_a)}{T_1(s_b - s_a)} = \frac{T_1 - T_2}{T_1}. \qquad (157)$$

Equation (157) shows that the efficiency of the Carnot cycle when using a vapor as the working substance is the same as that for a perfect gas and is consequently dependent only upon its initial and final temperatures.

The Carnot cycle may be used as the basis for comparing the performances of heat engines using vapors, but more generally the ideal cycles which are more closely followed by actual power plants are used. The most important of these are the *Rankine*, the *reheating* and the *regenerative* cycles.

The Rankine Vapor Cycle.[1] This cycle is used as a standard for comparing the efficiencies of steam prime movers in which there is no reheating or extraction of steam. It differs from the Carnot cycle in that its principle of operation is essentially one of uniform flow, and since it is an ideal cycle the expansion and compression processes are considered reversible.

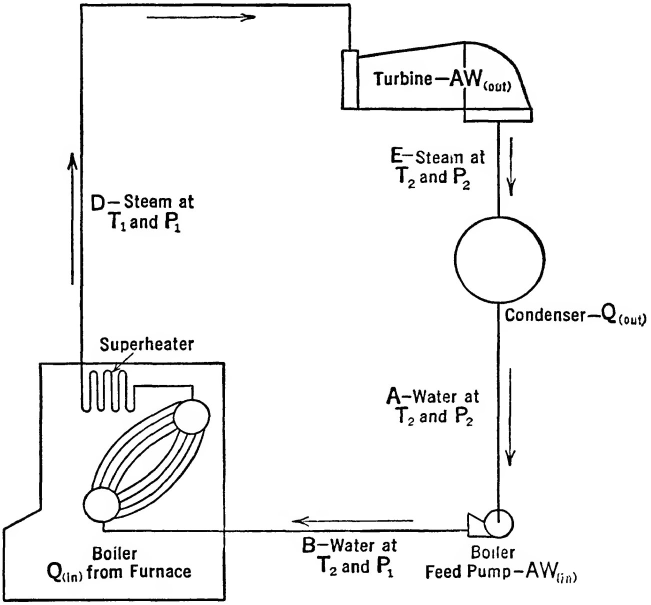

FIG. 41. Arrangement of Equipment in Reproducing Rankine Cycle.

The Rankine cycle requires the following equipment for its operation:

1. A boiler in which the vapor admitted to the engine is generated at constant pressure.

2. An engine having a non-conductive cylinder without clearance, into which the steam is admitted, isentropically expanded to the exhaust pressure, and then exhausted, thereby transforming heat energy into work or shaft energy.

---

[1] Also known as the Clausius cycle, having been published simultaneously and independently by Clausius.

3. A condenser in which the vapor exhausted from the engine is condensed into a liquid at a pressure corresponding to that of the exhaust.

4. A pump operated from the engine in which the liquid from the condenser is forced into the boiler.

5. Necessary non-conductive piping to connect the various parts of the equipment.

Figure 41 illustrates diagrammatically the arrangement of the mechanical equipment in reproducing the Rankine cycle. The pressures and enthalpies of the medium as they enter and leave various parts of the cycle are indicated. **Figures 42, 43,** and **44** illustrate the Rankine cycle on *P*-*V*, *T*-*S*, and *h*-*S* coordinates.

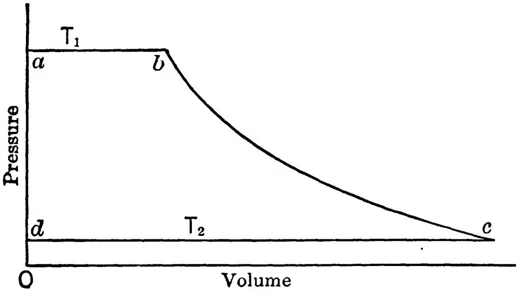

FIG. 42. Pressure-Volume Diagram of Ideal Rankine Cycle.

The Rankine vapor cycle, assuming steam as the working medium, is composed of the following processes:

1. The feed water received from the condenser at the pressure (P_2) of the exhaust enters the pump and is isentropically compressed and discharged into the boiler at a pressure (P_1) of the admission steam (**Fig. 41**).

2. The water in the boiler is raised from the temperature at which it enters to the temperature of the admission steam.

3. Evaporation takes place in the boiler at constant pressure. This process is represented by (**ab**), **Fig. 43**, if the steam that is generated is dry and saturated, and by (**abb′**) if the steam that is generated is superheated.

4. The working substance expands isentropically to the back pressure in the prime mover (**bc** or **b′c′**, **Fig. 43**).

5. The exhaust of the steam occurs at constant pressure (**cd** or **c′d**, **Fig. 43**).

6. Exhaust steam is condensed to liquid at exhaust pressure in the condenser, thus discharging water at a temperature corresponding to the exhaust pressure (P_2) to the pump.

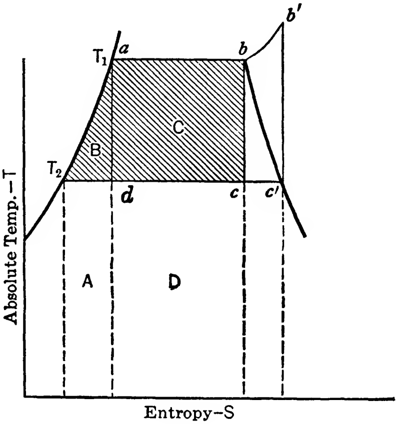

FIG. 43. Temperature-Entropy Diagram of Ideal Rankine Cycle.

The energies entering into these processes are best studied by considering the entire process as one of uniform flow. This equation can be simplified in this case, for the mechanical potential energy change and the kinetic energy change are both so small that they may be neglected. The equation then in terms of enthalpy (h), heat (Q) and work (AW) is

$$Q_{(in)} = h_2 - h_1 + AW_{(out)}. \tag{158}$$

Equation (158) is general, and the subscripts 1 and 2 apply to the entrance to and the exit from any part of the cycle under the conditions assumed. Care must be exercised, therefore, in applying this theory to the various processes of which the Rankine cycle consists, to insert in each case the proper values of the energies involved.

In the case of the pump, the amount of heat $Q_{(in)}$ may be assumed to be zero. The pump work is *in* rather than *out*, consequently it changes its sign as it is positive or negative. The enthalpy of the liquid entering the pump is h_1, and the enthalpy of the liquid leaving pump or entering the boiler is h_2. Equation (158) as it applies to the pumping process becomes

$$0 = h'_{f_2} - h_{f_2} + AW_{(in)}, \tag{159}$$

where h'_{f_2} is the enthalpy of the feed water entering the boiler, h_{f_2} is the enthalpy of the feed water entering the pump, and $AW_{(in)}$ is the heat equivalent of the pump work.

The pump work may be determined from equation (159), but the chief use of the equation is in determining the effect of the pump work on the feed water entering the boiler.

During the compression of the water in the pump, the volume of the water is assumed to remain constant. The heat equivalent of the pump work in terms of pressure change and volume discharged is

$$AW_{(pump)} = A(P_1 - P_2)v_{f_2}, \tag{160}$$

where P_1 = pressure in pounds per square foot of the boiler pressure.

P_2 = pressure in pounds per square foot at the exhaust pressure.

v_{f_2} = volume in cubic feet per pound of the saturated liquid at the pressure P_2.

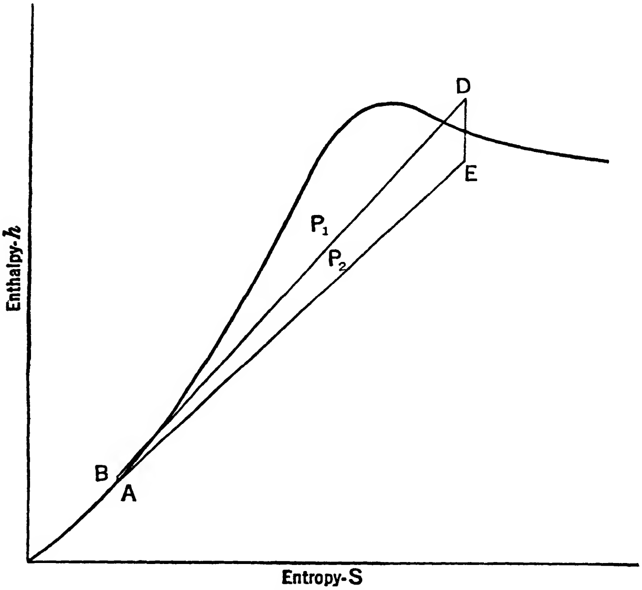

FIG. 44. Enthalpy-Entropy Diagram of Ideal Rankine Cycle.

Equation (160) when used to determine the enthalpy in the feed water entering the boiler is

$$h'_{f_2} = h_{f_2} + A(P_1 - P_2)v_{f_2}. \tag{161}$$

In the case of the boiler, no work (AW) is done, h_2 is the enthalpy of the steam leaving the boiler and entering the engine and h'_{f_2} is the enthalpy of the liquid entering the boiler (h'_{f_2} in equation (161)). The heat $Q_{(in)}$ which is absorbed by the working substance and delivered to the engine, may be determined from equation (158), and with these conditions becomes

$$Q_{(in)} = h_1 - h'_{f_2}, \tag{162}$$

where h_1 = enthalpy of steam leaving boiler.
h'_{f_2} = enthalpy of the feed water entering boiler.

Equation (162) may be modified by introducing h'_{f_2} from equation (161), becoming

$$Q_{(in)} = h_1 - (h_{f_2} + A(P_1 - P_2)v_{f_2}). \tag{163}$$

In the case of the condenser, the work (AW) may be assumed to be zero. The enthalpy in the condensed steam leaving the condenser and entering the pump is h_{f_2}, and the enthalpy in the exhaust steam from the engine is h_2. The heat extracted from the steam or that absorbed by the condenser is, from equation (158),

$$Q_{(out)} = h_2 - h_{f_2}, \tag{164}$$

where h_2 = enthalpy of the exhaust steam.
h_{f_2} = enthalpy of the condensed steam leaving condenser.

In the case of the engine, the heat added or abstracted is zero, h_1 is the enthalpy of the entering steam and h_2 is the enthalpy of the exhaust steam. The work of the engine cycle (AW) may then be calculated from equation (158), which becomes

$$AW_{(out)} = h_1 - h_2, \tag{165}$$

where h_1 = enthalpy of entering steam.
h_2 = enthalpy of exhaust steam.

Assuming that the pump is operated from the engine, the net or shaft work delivered by the Rankine cycle is

$$\begin{aligned}\text{Shaft work} &= \text{engine cycle work} - \text{pump work}\\ &= h_1 - h_2 - A(P_1 - P_2)v_{f_2}.\end{aligned} \tag{166}$$

The efficiency of the Rankine cycle is then

$$\begin{aligned} e &= \frac{\text{Heat equivalent of shaft work}}{\text{Heat input}}\\ &= \frac{h_1 - h_2 - A\,(P_1 - P_2)v_{f_2}}{h_1 - (h_{f_2} + A(P_1 - P_2)v_{f_2})}.\end{aligned} \tag{167}$$

Example. One pound of steam at a pressure of 160 psi abs having a quality of 0.95 performs the work of a Rankine cycle. The exhaust pressure is 5 psi abs. Determine:

1. What is the shaft work?
2. What is the heat required?
3. What is the efficiency?

Solution. 1. Assuming a reversible adiabatic expansion in the engine cylinder, the quality of the steam at exhaust is

$$0.5204 + 0.95 \times 1.0436 = 0.2347 + 1.6094\, x_2$$
$$x_2 = 0.794.$$

$$\begin{aligned}\text{Shaft work} &= h_1 - h_2 - \text{pump work}\\ &= (335.93 + 0.95 \times 859.2) - (130.13 + 0.794 \times 1001.0)\\ &\quad - \frac{(160 - 5)144 \times 0.01640}{778} = 226.8 \text{ Btu.}\end{aligned}$$

2. The heat required to produce the cycle is

$$Q_{(in)} = h_1 - (h_{f_2} + \text{pump work}) = 335.93 + 0.95 \times 859.2 - \left(130.13 + \frac{(160 - 5)144 \times 0.0164}{778}\right) = 1021.6 \text{ Btu.}$$

3. The efficiency of the cycle is

$$e = \frac{226.8}{1021.6} = 0.222 \quad \text{or} \quad 22.2 \text{ per cent.}$$

The Rankine Cycle with Irreversible Adiabatic Expansion. In some types of calculations, especially in turbine practice and other types of practical processes where internal disturbances create an irreversible process, the effect of such irreversibility may be included in the theory of the Rankine cycle. Irreversibility manifests itself in such a process by an increase in the enthalpy of the vapor exhausted from the engine. Reference to the previous equations developed for the Rankine cycle having reversible adiabatic expansion will reveal no changes in the pump work, no changes in the heat supplied by the boiler, a decrease in the work of the engine cycle because of the increased value of the enthalpy of the exhaust vapor, an increase in the amount of energy absorbed by the condenser, and a decrease in the efficiency of the cycle.

The irreversible process requires data upon the amount of the enthalpy of the exhaust steam, and these may be obtained from a knowledge of the increased entropy due to irreversibility or to the energy made unavailable because of irreversibility which increases the enthalpy of the exhaust steam by an equal amount.

The Rankine Cycle with Incomplete Expansion. In the reciprocating steam engine cycle it is impracticable to expand the steam down to the back-pressure line. The theoretical cycle of the steam engine with incomplete expansion is similar to that of the Rankine cycle with complete expansion, except that the adiabatic expansion terminates at a pressure higher than that of the back pressure; that is, the expansion is incomplete.

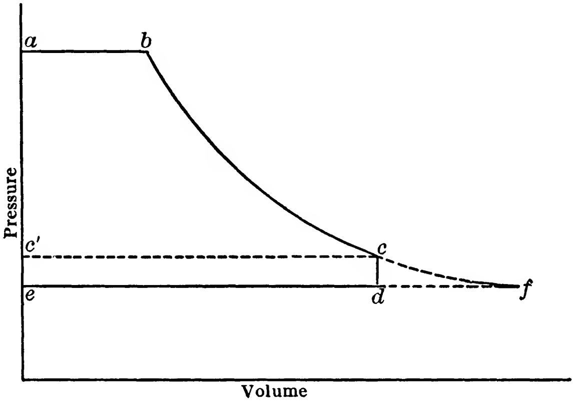

FIG. 45. Pressure-Volume Diagram of Rankine Cycle with Incomplete Expansion.

The pressure-volume and temperature-entropy diagrams of this modified cycle using dry and saturated steam are shown in **Figs. 45** and **46.**

The equipment required in the ideal case for the incomplete cycle is the same as for the Rankine cycle with complete expansion. The engine is without clearance and has a perfectly insulated cylinder. Steam is admitted to the engine cycle (**Figs. 45** and **46**) at constant pressure and temperature along **ab**. Cut-off occurs at point **b** and the steam expands along a reversible adiabatic curve to point **c** where the pressure is p', the release pressure. Part of the steam is discharged and condensed at constant volume **cd**. The remainder is exhausted during the back-pressure stroke **de**. Line **ea** (**Fig. 46**) represents the rise in temperature of the feed water from T_2 to T_1.

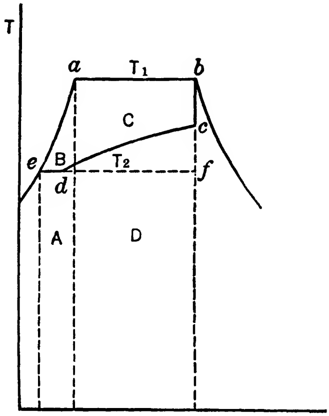

Fig. 46. Temperature-Entropy Diagram of Rankine Cycle with Incomplete Expansion.

The work of the engine cycle can best be calculated by dividing **Fig. 45** into the two areas **abcc′** and **c′cde**. The area **abcc′** is that of the theoretical or ideal Rankine engine cycle, while **c′cde** is that of a rectangle whose work can be calculated from the pressure and volume changes. The work of the modified engine cycle is equal to the sum of the two parts.

The heat equivalent of the work of the Rankine engine as represented by the area **abcc′** is

$$h_1 - h'.$$

The heat equivalent of the work represented by area **c′cde** is

$$A(P' - P_2)(v' - 0).$$

The heat equivalent of the work of the Rankine engine cycle is

$$AW_{(engine\ cycle)} = h_1 - h' + A(P' - P_2)v'. \qquad (168)$$

The heat equivalent of the work of the Rankine modified cycle, which includes the effect of the pump work, is

$$AW_{(complete\ cycle)} = h_1 - h' + A(P' - P_2)v' - A(P_1 - P_2)v_{f_2}. \qquad (169)$$

The heat received from the boiler and required to produce the cycle, being the same as that for the Rankine cycle with complete expansion, is

$$Q_1 = h_1 - (h_{f_2} + A(P_1 - P_2)v_{f_2}). \qquad (170)$$

The efficiency of the modified vapor cycle is

$$e = \frac{h_1 - h' + A(P' - P_2)v' - A(P_1 - P_2)v_{f_2}}{h_1 - (h_{f_2} + A(P_1 - P_2)v_{f_2})}. \qquad (171)$$

Frequently the terms containing v_{f_2} as a factor are relatively small and may be omitted in numerical estimates. The Rankine cycle with incomplete expansion (**Figs. 45** and **46**) is less efficient than the ideal cycle (**Figs. 42** and **43**) because of the failure to expand the steam completely. This loss is represented graphically in **Figs. 45** and **46** by the area **cdf**. The loss **cdf** is largely offset in the actual engine, as the cycle with complete expansion requires a larger cylinder and involves greater losses due to friction.

The incomplete Rankine cycle is sometimes used as a standard in preference to the ideal Rankine cycle when efficiencies of engines are compared.

Example. One pound of steam at a pressure of 160 psi abs having a quality of 0.95 passes through an incomplete Rankine cycle. The release pressure is 14.696 psi abs, and the exhaust pressure is 5 psi abs. What is (a) the work of the cycle, (b) the heat required to produce the cycle, and (c) the efficiency of the cycle?

Solution. Assuming a reversible adiabatic expansion between the inlet and terminal pressure, the quality of the steam at the release pressure is given by

$$0.5204 + 0.95 \times 1.0436 = 0.3120 + 1.4446\,x'$$
$$x' = 0.83.$$

The volume of the steam at the terminal pressure is

$$0.01672 + 0.83 \times 26.80 = 22.26 \text{ cu ft.}$$

The enthalpy of the entrance steam is

$$335.93 + 0.95 \times 859.2 = 1152.17 \text{ Btu.}$$

The enthalpy of the steam at the terminal pressure is

$$180.07 + 0.83 \times 970.3 = 985.42 \text{ Btu.}$$

The heat equivalent of the engine cycle work is

$$1152.17 - 985.42 + \frac{(14.696 - 5)144 \times 22.26}{778} = 206.65 \text{ Btu.}$$

The heat equivalent of the pump work is

$$\frac{(160 - 5) \times 144 \times 0.01640}{778} = 0.47 \text{ Btu.}$$

The heat equivalent of the net or shaft work is

$$206.65 - 0.47 = 206.18 \text{ Btu.}$$

The heat supplied the cycle inclusive of the effect of the pump work is

$$1152.17 - (130.13 + 0.47) = 1021.57 \text{ Btu.}$$

The efficiency of the cycle is

$$e = \frac{206.18}{1021.57} = 0.202 \quad \text{or} \quad 20.2 \text{ per cent.}$$

The Practical or Actual Steam Engine Cycle. In the steam engine designed for practical operation it is impracticable to expand the steam down to the back-pressure line; and, furthermore, some mechanical clearance must be provided. The result is that in the indicator diagram from the actual steam engine, a clearance volume, and both incomplete expansion and incomplete compression appear as shown in **Fig. 47.** In order to calculate the theoretical efficiency of this practical cycle, it is necessary to assume that the expansion line **cd** and the compression line **fa** are adiabatic. Knowing the cylinder feed of steam per stroke and the pressure and volume relations as determined from the indicator diagram, the theoretical thermal efficiency can be calculated by obtaining the net area of the diagram (expressed in Btu) and dividing by the heat supplied per cycle.

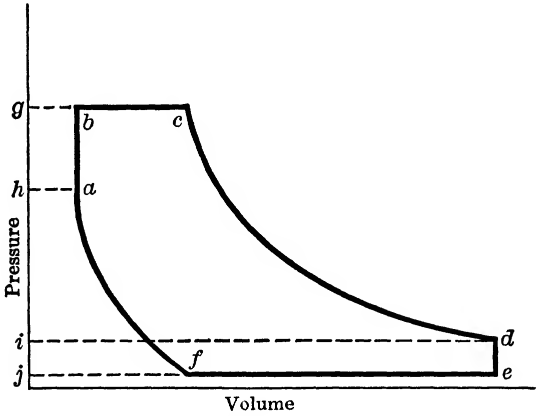

Fig. 47. Indicator Diagram of Practical Engine Cycle.

The actual efficiency of the steam engine is usually determined by dividing the heat equivalent of a horsepower by the heat in the steam required to produce a horsepower. Since 1 hp per hour is equal to $\frac{33{,}000 \times 60}{778}$ or 2545 Btu, the actual efficiency of a steam engine is

$$e = \frac{2545}{w_{hp}(h_1 - h_{f_2})} \tag{172}$$

In equation (172) w_{hp} is the steam rate or the steam consumption per horsepower per hour, h_1 is the enthalpy of the steam at the initial pressure and quality as it enters the engine, h_{f_2} is the enthalpy of the feed water corresponding to saturated liquid at the exhaust pressure.

In terms of kilowatt hours (kwhr) of output the actual efficiency would be:

$$e = \frac{3413}{w_{kw}(h_1 - h_{f_2})} \tag{173}$$

In equation (173)w_{kw} is the steam rate per kwhr.

When comparing an actual steam prime mover with a Rankine cycle engine (complete expansion) operating between the same limits the ratio of efficiencies is used. Thus $e_{actual} \div e_{Rankine}$ is called the engine efficiency, or Rankine cycle efficiency ratio.

$$\text{Engine efficiency} = \frac{2545}{w_{hp}(h_1 - h_2)} = \frac{3413}{w_{kw}(h_1 - h_2)} \tag{174}$$

The denominator of equations (172) and (173) is termed the heat rate of the prime mover for the given conditions.

The Reheating Vapor Cycle. Steam plant performance can be improved by reheating the steam after it has passed partially through the prime mover. The reheating process consists of returning the partially expanded steam to the boiler plant where it is heated in a reheater located either within the furnace walls of the main boiler or in a furnace which is separately fired. High pressure and high temperature steam is being used to an increasing extent in place of furnace gases for reheating the partially expanded steam. Irrespective of the reheating system used the reheated steam is returned to the prime mover to complete the expansion therein. It is found that the improved quality of the low pressure steam after expansion increases the plant efficiency and reduces prime mover maintenance sufficiently to justify the cost of reheating.

Figure 48 illustrates the arrangement of equipment for a reheating vapor cycle.

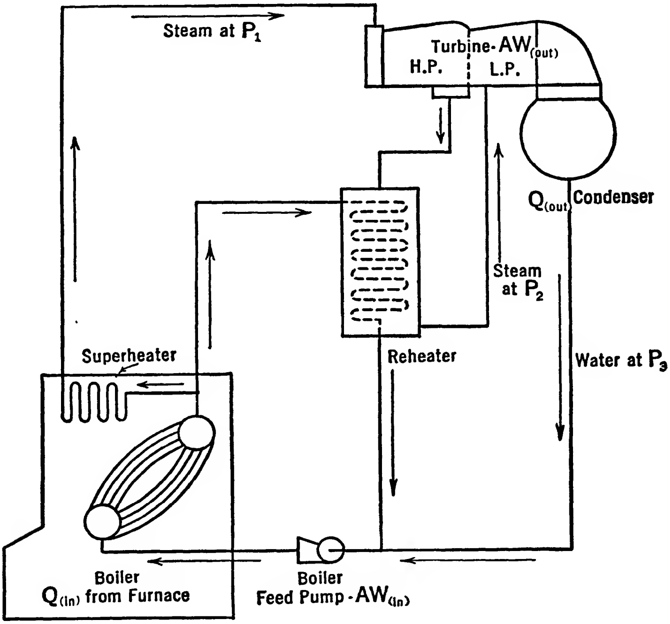

FIG. 48. Arrangement of Equipment for a Reheating Vapor Cycle.

The ideal vapor reheating cycle is illustrated on *P-V*, *T-S*, and *h-S*, coordinates in **Figs. 49, 50,** and **51.**

Superheated steam is admitted to the prime mover at **b**, is partially expanded from **b** to **c** and is reheated at constant pressure from **c** to **d**. The reheated steam is then returned to the prime mover without loss (in the state **d**) and is expanded isentropically to **e**. The remainder of the cycle is similar to that of the Rankine vapor cycle. Either or both **c** and **e** in Fig. 50 may be in the superheat field, the dry saturation field, or in the wet field.

Since such a cycle in its ideal case is composed of two Rankine cycles with complete expansion, the heat equivalent of the engine cycle work is their sum. The energy available for the production of work by the engine cycle, using subscripts to the symbols as in **Figs. 49** and **50** to designate their condition, is

$$h_b - h_c + h_d - h_e. \tag{175}$$

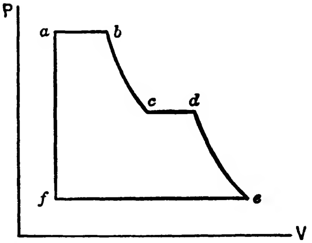

Fig. 49. Pressure-Volume Diagram of Reheating Cycle.

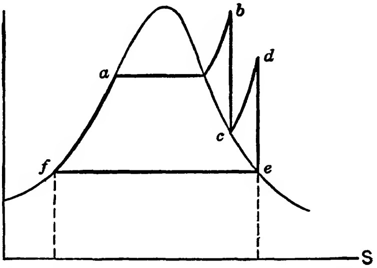

Fig. 50. Temperature-Entropy Diagram of Reheating Cycle.

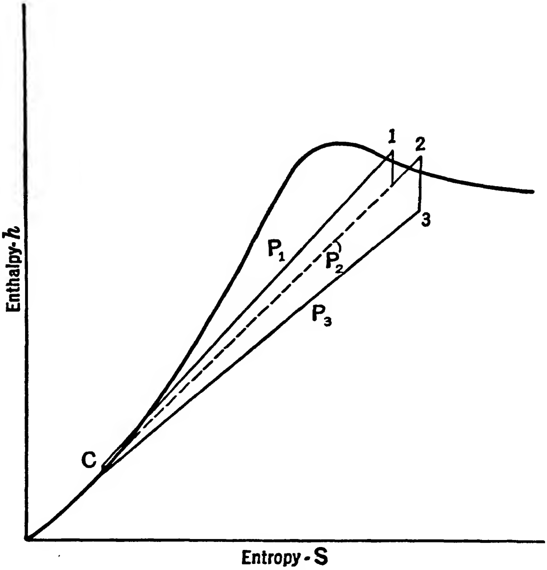

Fig. 51. Enthalpy-Entropy Diagram of Reheating Cycle.

The heat equivalent of the pump work, as in the Rankine cycle with complete expansion, is

$$AW_{(pump)} = A(P_1 - P_3)v_{f_3}.$$

The heat required to produce the cycle, using subscripts to symbols as in equation (170), is

$$
\begin{aligned}
Q_1 &= h_b - (h_{f_2} + \text{pump work}) + h_d - h_c \\
&= h_b - (h_{f_2} + A(P_1 - P_2)v_{f_2}) + h_d - h_c. \qquad (176)
\end{aligned}
$$

The efficiency of the reheating cycle is

$$
e = \frac{h_b - h_c + h_d - h_e - A(P_1 - P_3)v_{f_2}}{h_b - (h_{f_2} + A(P_1 - P_3)v_{f_2}) + h_d - h_c}. \qquad (177)
$$

The Regenerative Vapor Cycle. In the regenerative cycle, steam is partially expanded in the prime mover, after which a portion is extracted or bled at one or more stages intermediate between the initial steam pressure and the exhaust. The steam bled from the prime mover is utilized in heating the feed water to the saturation temperature corresponding to the bleeder pressure, the extraction and heating taking place at constant pressure. The arrangement of equipment for a regenerative vapor cycle is illustrated in **Fig. 52.**

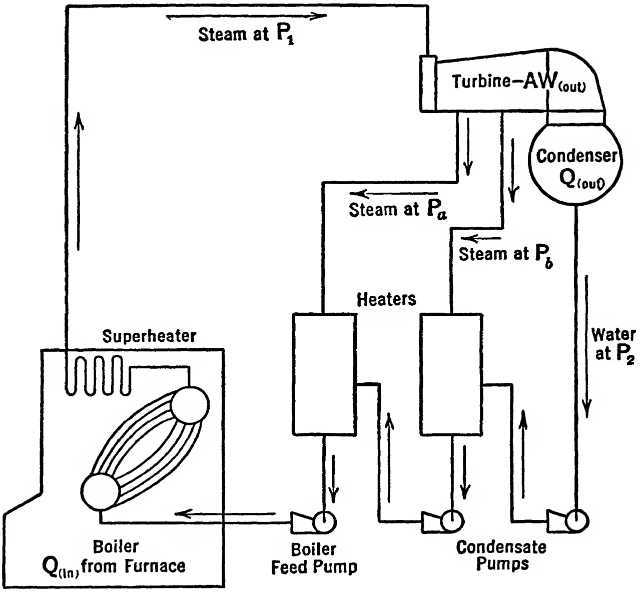

FIG. 52. Arrangement of Equipment for a Regenerative Vapor Cycle.

In the ideal regenerative cycle, it is assumed that an infinitely large number of bleeder heaters are used. **Figure 53** illustrates on temperature-entropy coordinates such a cycle, the extraction of the steam beginning when the expanded steam reaches a dry and saturated condition. In the cycle illustrated, superheated steam at the condition **b** expands in the prime mover isentropically to the point **c**. Since the heat extracted equals the heat absorbed in the heating of the feed water, **cd** is drawn parallel to the liquid line **ef**. With no extraction of steam the expansion would have continued to **d′** and the Rankine cycle with complete expansion would have been produced.

In analyzing such a cycle several conclusions may be drawn from reference to **Fig. 53**. The heat extracted and graphically represented by the area under the curve **cd** equals the heat used in raising the temperature of the feed water from **e** to **f**. The length of **ed** equals **fc**, and consequently the entropy change $s_c - s_f$ equals $s_d - s_e$.

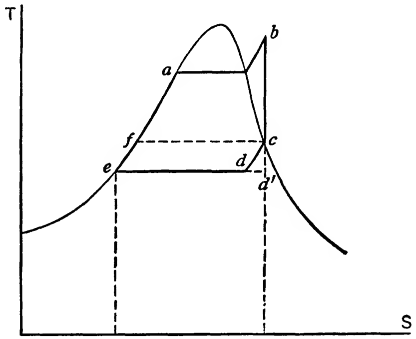

Fig. 53. Temperature-Entropy Diagram of Ideal Regenerative Cycle with Infinite Number of Bleeders.

The enthalpy of the steam at entrance is represented by h_b.

The enthalpy of the steam exhausted equals the enthalpy of the liquid at **e** plus the heat represented by the entropy change from **e** to **d** plus the heat represented by the change from **d** to **c**. The enthalpy of the steam exhausted, using subscripts to designate the conditions as shown in **Fig. 53**, is

$$h_e + T_2(s_d - s_e) + (h_c - h_d).$$

But $h_c - h_d = h_f - h_e$ and $T_2(s_d - s_e) = T_2(s_c - s_f)$.

Substituting these values, the enthalpy of the exhaust steam is

$$h_f + T_2(s_c - s_f).$$

Neglecting the pump work, the energy made available for the production of work is

$$h_b - [h_f + T_2(s_c - s_f)]. \qquad (178)$$

The efficiency of the ideal regenerative cycle, neglecting pump work, is

$$e = \frac{h_b - [h_f + T_2(s_c - s_f)]}{h_b - h_f}. \tag{179}$$

The Practical Regenerative Vapor Cycle. Some gain in plant efficiency may be expected by the use of a regenerative cycle with infinite extraction, but owing to practical difficulties such a cycle would be impossible. The regenerative cycle with infinite extraction could be approached in practice by the use of a large number of bleeder heaters, but the gain in thermal efficiency per heater diminishes as the number of heaters is increased. Moreover, the increased fixed charges and greater plant complexity overshadow the thermodynamic advantages when a prime mover is bled at too many points. In practice even very large plants rarely employ more than four heaters, and smaller plants set the limit at two.

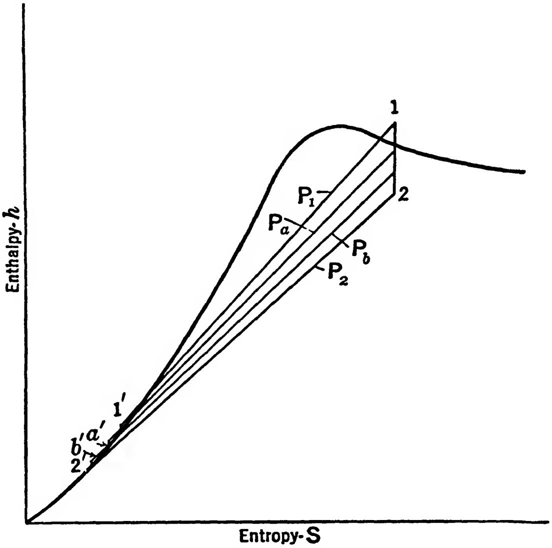

FIG. 54. Enthalpy-Entropy Diagram of Regenerative Cycle using Vapor.

The regenerative vapor cycle with superheated steam and using bleeder heaters is illustrated on *h*-*S* and *T*-*S* coordinates in **Figs. 54 and 55.** In this case it is assumed that the prime mover is bled at points 1 and 2. Since the quantity of steam passing through the prime mover decreases after each bleeding, the diagram is constructed so that its net area represents the work per pound of entering steam.

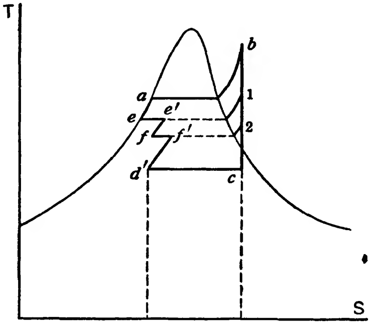

Fig. 55. Temperature-Entropy Diagram of Regenerative Cycle using Vapor.

In the practical operation of the regenerative cycle different methods are used; consequently, in developing thermodynamic theory for the cycle it is necessary to know in detail how the cycle is operated. In the following discussion the cycle is assumed to operate with open heaters as follows:

1. The steam in the main unit expands isentropically from the inlet pressure to the condenser pressure.

2. The condensed steam leaves the condenser at a temperature corresponding to the exhaust of the main unit.

3. The steam extracted from the various stages and entering the heaters is condensed under constant pressure and leaves as water at a temperature corresponding to saturated vapor at the pressure in each stage, and the water passing through each heater is raised to the same temperature.

4. The condensed steam from each heater joins with the heated water and is delivered to the next heater. The water leaving one heater and entering another is, therefore, the sum of the extracted steam and the entering feed water.

Letting h represent the enthalpy of the steam or liquid at the states represented by the subscripts in **Fig. 55**, the weight of steam to be extracted in the first stage is found

$$w_1h_1 + (w - w_1)h_f = wh_e, \qquad (180)$$

where w_1 = weight of steam to be extracted from first stage.
$(w - w_1)$ = weight of feed water entering heater to produce w pounds of steam.
w = weight of steam originally entering prime mover.
h_e = enthalpy of the feed water leaving heater.
h_f = enthalpy of the feed water entering heater.

From equation (180) the weight of steam extracted in the first stage per pound of the original steam entering the main unit is

$$w_1 = \frac{h_e - h_f}{h_1 - h_f}. \tag{181}$$

Similarly, the equation for the second-stage heater may be written

$$w_2 h_2 + (w - w_1 - w_2)h_d' = (w - w_1)h_f' \tag{182}$$

where w_2 = weight of steam extracted in the second stage.
h_f' = enthalpy of the liquid leaving the second-stage heater.
h' = enthalpy of the liquid from condenser.

From equation (182) the weight of steam extracted in the second stage per pound of the original steam is

$$w_2 = (1 - w_1)\frac{h_f' - h_d'}{h_2 - h_d'}. \tag{183}$$

The available energy or work of such a regenerative cycle, neglecting the pump work, is

$$w(h_b - h_1) + (w - w_1)(h_1 - h_2) + (w - w_1 - w_2)(h_2 - h_c), \tag{184}$$

where w = original weight of steam entering the prime mover.
w_1 and w_2 = weight of steam extracted at stages 1 and 2 respectively.
h = enthalpy of the vapor at the states represented by the subscripts (**Fig. 55**).

The heat carried into such a cycle from the boiler is, neglecting the pump work,

$$w(h_b - h_e);$$

and the efficiency, neglecting the pump work, is

$$e = \frac{w(h_b - h_1) + (w - w_1)(h_1 - h_2) + (w - (w_1 + w_2))(h_2 - h_c)}{w(h_b - h_e)}. \tag{185}$$

Example. A steam turbine operates on a pressure of 300 psi abs and 700 deg fahr temperature. The absolute pressure of the exhaust is 1 in. of mercury. Steam is bled from two stages at 100 and 24 psi abs pressure respectively. Neglecting the pump work: (1) How many pounds of steam should be extracted from each stage per pound of steam admitted to the turbine? (2) What would be the work of the cycle and its efficiency?

Solution. From the Mollier chart and steam tables the following values are obtained:

Pressure	Enthalpy Btu per lb	Enthalpy of Liquid Btu per lb
300 psi	1368.3	
100 psi	1248.5	298.40
24 psi	1130	206.14
1 in. mercury	900	47.05

The weight of steam extracted from the first stage is

$$w_1 = \frac{298.40 - 206.14}{1248.5 - 206.14} = 0.0885 \text{ lb.}$$

The weight of steam extracted from the second stage is

$$w_2 = (1.0 - 0.0885)\frac{206.14 - 47.05}{1130 - 47.05} = 0.1337 \text{ lb.}$$

The available work of the cycle, neglecting the pump work, is

$$1(1368.3 - 1248.5) + (1 - 0.0885)(1248.5 - 1130) + (1.0 - 0.0885 - 0.1337)(1130 - 900) = 406.5 \text{ Btu.}$$

The efficiency of the cycle, neglecting the pump work, is

$$e = \frac{406.5}{1368.3 - 298.40} = 38 \text{ per cent.}$$

The Reheating-Regenerative Vapor Cycle. In some power plants a combination of the reheating and regenerative cycles has been used. While the efficiency of this cycle is high the complications in power plant layout are only justified in special cases. A complete discussion of reheating and regenerative cycles will be found in Vol. 45, *Transactions American Society of Mechanical Engineers.*

The Binary Vapor Cycle. In all cycles the temperature range influences the efficiency. With steam as the working substance the lower temperature is limited by the temperature of the condenser and the upper temperature by the available structural materials. In order to avoid excessive initial pressures while maintaining a high temperature range experiments have been conducted with installations which make use of more than one working substance, each fluid functioning through its most suitable temperature range. The operation is such that the condensation of the first working substance, after flowing through the prime mover, vaporizes the next fluid and this vapor is used in another prime mover.

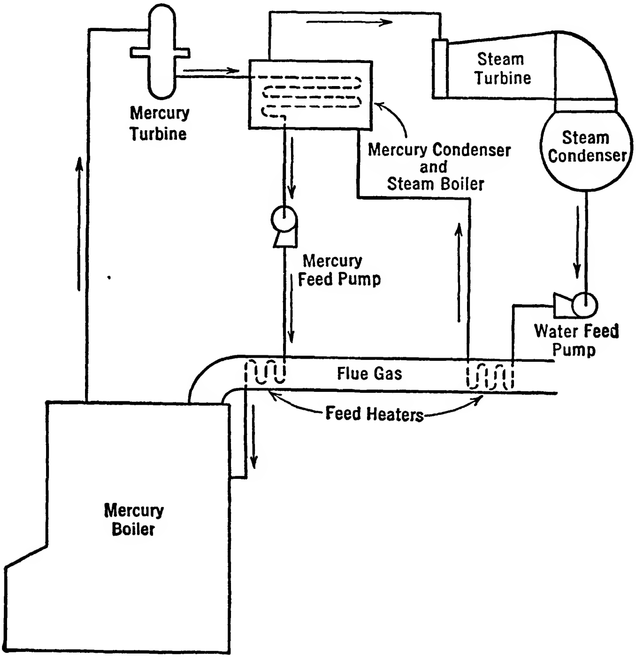

FIG. 56. Arrangement of Equipment for a Binary Cycle.

Steam and sulphur dioxide have been tried in a binary cycle. The exhaust steam of a reciprocating engine entering a surface condenser at a pressure of about 2.5 psi was used to evaporate sulphur dioxide. The pressure of the SO_2 thus vaporized was about 200 psi abs and was used in a second engine.

In the United States mercury and steam are being investigated on a large commercial scale. Mercury may be vaporized in a boiler at temperatures much higher than those for steam. It is then carried through a turbine and does useful work exhausting into a surface condenser, which becomes a steam boiler where the latent heat of mercury is used to make steam. The condensed mercury liquid is carried back to the boiler. High initial temperatures may be obtained with mercury without excessive pressures and the latent heat of condensation may be utilized at temperatures suitable for steam making. Thus mercury at a pressure of 45 psi abs has a vaporization temperature of about 800 deg fahr and at a pressure of 180 psi its vaporization temperature is about 1000 deg fahr. If a mercury turbine is operated so that its exhaust vacuum is 28 in., this corresponds to a temperature of about 456.8 deg fahr. If there is a temperature difference of 21 deg fahr in the mercury condenser, then for every 9 lb of condensed mercury at a temperature of about 435.8 deg fahr 1 lb of water may be vaporized into steam at a pressure of 300 psi abs. The steam thus generated may also be superheated by the furnace gases of the mercury boiler and used to operate a steam turbine. The arrangement of equipment for a binary (mercury-water) cycle is illustrated in **Fig. 56.**

Details concerning the mercury-steam binary power plant will be found in papers by W. L. R. Emett before the American Institute of Electrical Engineers in 1913 and before the American Society of Mechanical Engineers in 1924.

PROBLEMS

1. Determine the heat absorbed, the steam consumption and the efficiency of an ideal steam engine operating on the Carnot cycle. Assume that the cylinder contains water at the beginning of the cycle and dry steam at cut-off and that the initial pressure is 300 psi abs and the exhaust pressure 28 in. vacuum (bar press 30 in.).

2. Assume that 1 lb of steam at a pressure of 160 psi abs and a quality of 0.95 performs an ideal Rankine cycle, being exhausted at a pressure of 5 psi abs. Compute the quality of the steam exhausted, the engine cycle work, the pump work, the shaft work and the efficiency of the cycle.

3. What is the available energy (engine cycle work) in an ideal Rankine cycle if the steam initially at 200 psi abs pressure, superheated 200 deg fahr, goes through such a cycle with a back pressure of 1 psi abs?

4. One pound of steam at a pressure of 100 psi abs with a quality of 0.90 performs an ideal Rankine cycle exhausting at a back pressure of 2 psi abs. What are the net work and the efficiency of the cycle?

5. Two pounds of steam at a pressure of 140 psi abs and a volume of 8.82 cu ft perform an ideal Rankine cycle. The exhaust pressure is 40 psi abs. What are the net work and the efficiency of the cycle?

6. One pound of steam at a pressure of 160 psi abs and a quality of 0.95 passes through a modified Rankine cycle (incomplete expansion). The terminal pressure is 11 psi abs and the exhaust pressure 5 psi abs. What is the efficiency of the cycle?

7. One pound of steam at a pressure of 200 psi abs and 200 deg fahr superheat passes through a modified Rankine cycle. The terminal pressure is 15 psi abs and the exhaust pressure 10 psi abs. Calculate the net work and the efficiency of the cycle.

8. Two pounds of steam at a pressure of 140 psi abs and a volume of 8.82 cu ft pass through a modified Rankine cycle. The terminal pressure is 20 psi abs and the exhaust pressure 16 psi abs. Calculate the efficiency, the net work, and the heat added to the cycle.

9. An engine operating on the Rankine cycle with complete expansion is supplied with steam at 160 psi abs and 98 per cent quality, and exhausts at 16 psi abs.

(a) Compute the thermal efficiency of the cycle, neglecting the boiler feed pump.

(b) Draw a T-S diagram and show by the areas the heat supplied, the heat equivalent of the work done and the heat rejected to the condenser.

10. A steam engine, operating on the Rankine cycle with complete expansion, is supplied with steam at 168 psi abs and 98 per cent quality. The engine exhausts at a back pressure of 14.7 psi abs.

(a) Neglecting the boiler feed pump, determine the steam consumption in pounds per hour.

(b) Compute the heat rejected to the condenser in Btu per lb of wet steam.

(c) Draw a flow diagram of the cycle.

11. A steam engine operates on the Rankine cycle with complete expansion. It is supplied with steam at 200 psi abs and 440 deg fahr and exhausts at 20 psi abs.

(a) Neglecting the boiler feed pump, determine for one pound of steam the heat supplied, heat converted into work, heat rejected to the condenser, and the efficiency of the cycle.

(b) Draw a flow diagram of the cycle and indicate the enthalpy at exit from the superheater, engine, and condenser.

12. A steam engine, operating on the Rankine cycle with incomplete expansion, is supplied with steam at 220 psi abs and 400 deg fahr. The steam is expanded isentropically to 70 psi abs and exhausted at 16 psi abs.

(a) Neglecting the boiler feed pump, compute the thermal efficiency of the cycle.

(b) Determine the enthalpy of the steam entering the condenser.

13. A steam locomotive is supplied with steam at 260 psi abs and 500 deg fahr and operates on the Rankine cycle with incomplete expansion. Steam is expanded isentropically to 60 psi abs and is exhausted at a back pressure of 20 psi abs.

(a) Compute the thermal efficiency of the cycle, neglecting the effect of the boiler feed pump.

(b) Draw a *P-V* diagram to show the work done by the steam in the engine and the loss in work due to failure to expand isentropically to the back pressure.

14. A steam turbine operating on the reheating cycle is supplied with steam at 800 psi abs and 680 deg fahr. The steam is expanded isentropically to 350 psi abs, reheated at constant pressure to 720 deg fahr and expanded isentropically to 2 in. Hg abs.

(a) Compute the thermal efficiency of the cycle considering the work of the boiler feed pump.

(b) Draw a *T-S* diagram and indicate by areas the heat supplied, heat equivalent of work and heat rejected to the condenser.

15. A steam turbine operating on the reheating cycle is supplied with steam at 1200 psi abs and 800 deg fahr. The steam is expanded isentropically to 300 psi abs, then heated at constant pressure of 300 psi abs to 720 deg fahr, and expanded isentropically to 29 in vacuum.

(a) Compute the thermal efficiency of the cycle, including the work of the pump.

(b) Draw a flow diagram for the cycle and name the equipment through which the fluid passes in completing the cycle.

16. A steam turbine operates at a pressure of 600 psi abs and 640 deg fahr and exhausts at a vacuum of 29.2 in. (30 in. bar). Steam is bled from two stages at 140 and 35 psi abs respectively. Neglecting the pump work, calculate:

(a) Pounds of steam extracted from each stage per pound of steam admitted to the turbine.

(b) Thermal efficiency of the cycle.

17. Draw *T-S* diagrams for the ideal Rankine cycle with superheated steam at 700 deg fahr and with initial pressures of 1200 and 1500 psi abs and an exhaust pressure of 1 psi abs and compare them with Fig. 43.

18. A steam turbine plant (reheating cycle) operates under the following conditions: Steam pressure 600 psi abs, steam temperature 700 deg fahr, exhaust 29 in. vacuum (30 in. bar press). If one reheat is used to 700 deg fahr at 80 psi abs pressure, calculate the cycle efficiency of this plant and compare this with the efficiency of an ideal Rankine cycle operating without reheating (problem 17).

CHAPTER IX

FLOW OF FLUIDS

Problems embracing the measurement of the flow of gases and vapors through nozzles and orifices depend for their solution upon the following:

(1) That the volume in cubic feet per second of the fluid flowing is equal to the product of the area of the stream in square feet and the velocity of the stream in feet per second.

(2) That in uniform flow the same weight entering any section of the stream must be discharged at any other section during the same period of time. This principle is sometimes spoken of as the continuity of mass.

(3) That the energy entering any section of the stream is equal to the energy passing any other section, provided no heat is added or removed betweeen these sections by external sources.

(4) That the velocity created at any section is due to the transformation of available energy into kinetic energy.

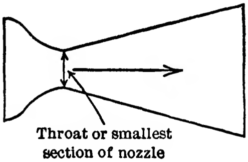

Fig. 57. Typical Nozzle for Expanding Gases and Vapors.

Forms of Nozzles. Nozzles and orifices are used in the measurement of the flow of fluids. A nozzle is illustrated in **Fig. 57** and may be defined as a channel to transform the maximum of energy into velocity.

An orifice is illustrated in **Fig. 58.** It is frequently an opening in a thin plate and differs from the nozzle in that the energy transformation into velocity is not the principal consideration. The orifice is used where the velocity created is not important; it finds its greatest application in meters designed to measure the weight of the flow of fluids.

Fig 58. Typical Orifice for Expanding Gases and Vapors.

In considering further details of the nozzle it is evident that when a fluid such as a gas or vapor flows through a nozzle its pressure falls, and its volume and velocity both increase. In order that the expansion may take place properly, the areas at the various sections must be designed to accommodate the new volumes with due consideration for the new velocities.

If the velocity increases more rapidly than the volume as the fluid passes from one section to the succeeding one, the nozzle must be convergent between the two points. On the other hand, if the volume increases more rapidly than the velocity, the cross-sectional area of the nozzle must be greater at the second than at the first point, the nozzle being divergent between the two points.

A nozzle designed for the expansion of any expansive fluid like gases or vapors is made up of a converging and diverging section. This is due to the fact that, near the inlet end of the nozzle, the velocity increases more rapidly than the volume, whereas near the outlet end of the nozzle the reverse is true. There are, consequently, three important sections to a nozzle: the entrance, the throat or section of minimum area, and the exit.

The expansion of the fluid in a nozzle follows definite physical laws. If the pressure at the mouth of the nozzle is sufficiently low, the pressure at the throat is a definite and fixed value, termed the critical pressure, and is dependent only upon the initial pressure. Thus a nozzle discharging into a region whose pressure is equal to or above the critical pressure need be only of the converging type. On the other hand, a nozzle discharging into a region whose pressure is below that of the critical pressure must be of the converging-diverging type.

General Equations for the Flow of Fluids. Considering a steady flow of the fluid so that there is no accumulation of material at any point, or assuming that the same weight (w) is passing all points at any time:

$$w_1 = w_2.$$

Since *volume of flow* (V) equals area (A) × velocity ($\mathbf{V}$), and

$$w = \frac{\text{volume of flow } (V)}{\text{specific volume } (v)}$$

$$w_1 = \frac{V_2}{v_2} = \frac{A_1\mathbf{V}_1}{v_1}$$

$$w_2 = \frac{V_2}{v_2} = \frac{A_2\mathbf{V}_2}{v_2}$$

$$\frac{A_1\mathbf{V}_1}{v_1} = \frac{A_2\mathbf{V}_2}{v_2}. \qquad (186)$$

In the above equations the subscripts 1 and 2 represent the inlet and any other section respectively; w is the pounds of flow per second, V is the cubic feet of volume passing any section per second, $\mathbf{V}$ is the velocity in feet per second, A is the area in square feet, and v is the specific volume of the fluid in cubic feet per pound.

The velocity of flow is determined from the energy changes taking place during the process. Since the process is one of uniform flow the energy equation is

$$Q_{(in)} + Az_1 + \frac{A\mathbf{V}_1^2}{2g} + U_1 + AP_1v_1 =$$

$$AW_{(out)} + Az_2 + \frac{A\mathbf{V}_2^2}{2g} + U_2 + AP_2v_2. \qquad (187)$$

The Velocity of Vapor Flow. By reason of the fact that no shaft work (AW) is done and that the fluid neither gains nor loses heat, both the AW and Q terms are zero. Likewise, the mechanical potential energy (Az) terms may be disregarded; then,

$$A\left(\frac{\mathbf{V}_2^2 - \mathbf{V}_1^2}{2g}\right) = (U_1 + AP_1v_1) - (U_2 + AP_2v_2)$$

or

$$A\left(\frac{\mathbf{V}_2^2 - \mathbf{V}_1^2}{2g}\right) = h_1 - h_2.$$

Assuming that the initial velocity ($\mathbf{V}_1$) is negligibly small this equation for vapor flow becomes

$$\mathbf{V}_2 = 223.8\sqrt{h_1 - h_2}. \qquad (188)$$

To determine the enthalpy of the vapor at section 2 the quality x_2 must be known. In the case of reversible adiabatic expansions, the quality may be determined from the constant entropy that exists in such a process. In the case of irreversibility where turbulence and friction are considered the degree of irreversibility must be known.

Example. Dry and saturated steam at a pressure of 130 psi abs expands in a nozzle to 16 psi abs. Calculate the velocity acquired.

Solution. Assuming the expansion of the steam through the nozzle to be a reversible adiabatic process, the quality of the steam at the outlet from the nozzle is determined from the entropies. Thus, from the steam tables, the entropy at 130 psi abs is 1.5808, and this is equal to the entropy at the lower pressure, or

$$1.5812 = 0.3184 + 1.4313\, x_2,$$

from which

$$x_2 = 0.882.$$

From reference to the steam tables the enthalpies of the steam at the two conditions are

$$h_1 = 1191.7 \text{ Btu.}$$
$$h_2 = 184.42 + 0.882 \times 967.6 = 1038.1 \text{ Btu.}$$

The velocity created is

$$\begin{aligned} V_2 &= 223.8\sqrt{1191.7 - 1038.1} \\ &= 2775 \text{ ft per sec.} \end{aligned}$$

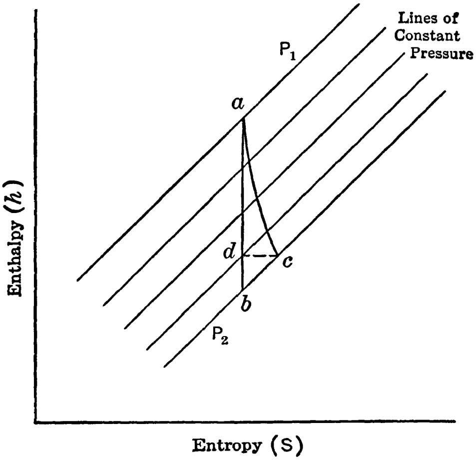

FIG. 59. Section of Mollier Diagram.

The Effect of Irreversibility on the Velocity of Vapor Flow. The effect of irreversibility on the energy transformed into velocity is most conveniently represented graphically on a Mollier chart.

Figure 59 represents a section of the Mollier chart of which the vertical axis is enthalpy and the horizontal axis is entropy. The line **ab** represents a reversible adiabatic process from the initial pressure P_1 to the final pressure P_2. The curve **ac** drawn between the same pressure limits represents an irreversible adiabatic process or one with increasing entropy. The length of the curve **ab** represents the energy theoretically available for producing velocity and **ad** that actually transformed. The distance **db** represents the loss in the transformation of the energy into velocity due to friction and turbulence and **dc** the increase in entropy. The amount of energy represented by **db** (**Fig. 59**) is often termed the reheat. As compared with the reversible type of adiabatic process, the irreversible type produces less velocity and the entropy and the enthalpy of the vapor at the end of the process are increased.

The effect of irreversibility may be accounted for by including the nozzle efficiency in equation (188). If that efficiency be considered equal to the ratio of the increase of kinetic energy of the jet in an actual nozzle to the increase of kinetic energy in the corresponding ideal nozzle, then:

$$e_n = \frac{\frac{A}{2g}[(V_2')^2 - V_1^2]}{\frac{A}{2g}(V_2^2 - V_1^2)}$$

Since

$$(V_2')^2 = \frac{2g}{A}(h_1 - h_2') + V_1^2$$

and

$$V_2^2 = \frac{2g}{A}(h_1 - h_2) + V_1^2$$

$$e_n = \frac{(h_1 - h_2')}{(h_1 - h_2)} \qquad (189)$$

In equation (189) h_2' represents the actual enthalpy at the end of the actual nozzle expansion (point c in **Fig. 59**).

From the foregoing it is evident that $(h_1 - h_2')$ = actual enthalpy change = $e_n(h_1 - h_2)$. Thus equation (188) may be modified to read:

$$V_2^1 = 223.8\sqrt{(h_1 - h_2)e_n}, \qquad (190)$$

when $$V_1 = 0$$

for the determination of actual exit velocity.

The Velocity of Gas Flow. The same general theory developed for determining the velocity of vapor flow is applicable to that of gas flow. Making the same assumptions as for vapor flow, the following energy equation in terms of kinetic energy, internal energy and work of flow may be written:

$$A\left(\frac{V_2^2 - V_1^2}{2g}\right) = U_1 - U_2 + AP_1v_1 - AP_2v_2. \qquad (191)$$

Since in the theory of perfect gases

$$U_1 - U_2 = c_v(T_1 - T_2)$$

also $$A(P_1v_1 - P_2v_2) = AR(T_1 - T_2).$$

Equation (191) may be written

$$A\left(\frac{V_2^2 - V_1^2}{2g}\right) = (c_v + AR)(T_1 - T_2)$$
$$= c_p(T_1 - T_2).$$

Assuming V_1 negligibly small

$$V_2 = \sqrt{\frac{1}{A} \times 2g \times c_p(T_1 - T_2)}. \qquad (192)$$

In equation (192) the term $c_p(T_1 - T_2)$ is the change in enthalpy. Since Pv equals RT, equation (192) may be expressed in terms of pressures and volumes, thus

$$V_2 = \sqrt{\frac{1}{A} \times 2g \times \frac{c_p}{R}(P_1v_1 - P_2v_2)}.$$

Since $$c_p - c_v = AR \quad \text{and} \quad \frac{c_p}{c_v} = k$$

$$\frac{c_p}{A \times R} = \frac{c_p}{c_p - c_v} = \frac{1}{1 - \frac{c_v}{c_p}} = \frac{1}{1 - \frac{1}{k}} = \frac{k}{k - 1}.$$

Therefore,

$$V_2 = \sqrt{2g \times \frac{k}{k - 1}(P_1v_1 - P_2v_2)}. \qquad (193)$$

In problems dealing with the flow of fluids the usual data known are the initial conditions (P_1v_1 or T_1) and the pressure (P_2) at any other section. Equation (193) for velocity may be rearranged so that only these values appear.

Since in a reversible type of adiabatic process

$$P_1v_1^k = P_2v_2^k$$

$$V_2 = \sqrt{2\,g \times \frac{k}{k-1} \times P_1v_1\left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]}. \qquad (194)$$

Weight of Flow. The weight of flow (w) of a fluid passing through any nozzle can be calculated if the pressure (P_1) and the temperature (T_1) or volume (v_1) of the fluid at the initial conditions are known, together with the area (A_2) and the pressure (P_2) at any other section of the nozzle.

Fundamentally, the weight of the fluid passing all sections of the nozzle is the same. This weight may in general be expressed

$$w = A_2 \times \frac{1}{v_2} \times V_2. \qquad (195)$$

Assuming a reversible adiabatic process in the passage of the fluid through the nozzle and that the working medium is a gas,

$$P_1v_1^k = P_2v_2^k \quad \text{and} \quad P_1v_1 = RT_1,$$

the value of $\frac{1}{v_2}$ may then be expressed

$$\frac{1}{v_2} = \frac{P_2^{\frac{1}{k}} \times P_1}{P_1^{\frac{1}{k}} \times RT_1} \quad \text{or} \quad \frac{1}{v_2} = \frac{P_2^{\frac{1}{k}}P_1^{\frac{k-1}{k}}}{RT_1}.$$

The velocity of flow as stated in equation (194) is

$$V_2 = \sqrt{2\,g \times \frac{k}{k-1} \times P_1v_1\left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]}.$$

Substituting these values and equation (194) for V_2, equation (195) becomes

$$w = A_2 \times \frac{P_1^{\frac{k-1}{k}} P_2^{\frac{1}{k}}}{RT_1} \times \sqrt{2\,g \times \frac{k}{k-1} \times P_1v_1\left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right]}. \qquad (196)$$

Simplifying this equation by bringing all terms other than A_2 under the radical, it becomes

$$w = A_2\sqrt{2\,g \times \frac{k}{k-1} \times \frac{P_1}{v_1}\left[\left(\frac{P_2}{P_1}\right)^{\frac{2}{k}} - \left(\frac{P_2}{P_1}\right)^{\frac{k+1}{k}}\right]}. \qquad (197)$$

The effect of irreversibility during the expansion of gases in nozzles may be accounted for in a manner similar to that outlined for vapors. Thus for a gas,

$$e_n = \frac{c_p(T_1 - T_2')}{c_p(T_1 - T_2)} \qquad (198)$$

In the above equation $c_p(T_1 - T_2') = e_n c_p(T_1 - T_2)$. In the application of this concept, T_2 may be found for the adiabatic expansion from P_1 to P_2 and T_2' (needed for the calculation of the actual specific volume at exit) may be calculated as $T_2' = T_2 + (T_1 - T_2)\,(1 - e_n)$.

Critical Pressure. Equation (197) for the weight of flow through a nozzle shows that the weight of discharge may be calculated at any section of the nozzle provided the area and pressure at the section are known. Since the weight of flow is equal at all sections any area could be used in the determination, but in most cases the area at the throat is selected.

If we assume that the elastic fluid is flowing through a properly designed converging-diverging nozzle in which P_1 is the pressure of the fluid entering the nozzle and P_2 that at any other section, equation (197) represents mathematically the weight of flow.

To determine the critical pressure or pressure at the throat of the nozzle, various values can be assigned to the area A_2 and the resulting pressure when the value of A_2 becomes a minimum can be determined. This value may be determined by equating the first derivative of equation (197) to zero, which gives the equation

$$\frac{P_2}{P_1} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}. \qquad (199)$$

From this equation the *critical pressure* is found to be 0.525 of the inlet absolute pressure when the substance is *air* and the value of k is taken as 1.4. For steam the critical pressure varies from 0.58 for *saturated steam* to 0.55 for *highly superheated steam.*

Two important rules may be stated from this theory:

1. When the pressure on the downstream side of the nozzle is equal to or lower than the critical pressure, the pressure at the throat of the nozzle and also the weight of discharge from the nozzle are independent of the downstream pressure and are dependent only upon the initial absolute pressure condition. In formulas for weight of flow under this condition the only pressure included is the inlet pressure (P_1), the fixed ratio $\frac{P_2}{P_1}$ being incorporated in a coefficient.

2. When the pressure on the downstream side of the nozzle is higher than the critical pressure, the pressure at the throat of the nozzle is considered as equal to the downstream pressure and as a result the weight of discharge from the nozzle is dependent upon the initial and downstream pressures. In the formula for the weights of flow under this condition both the inlet (P_1) and downstream (P_2) pressures must be included.

Determination of Weight of Vapor Flow. The application of the theory in the determination of the weight of vapor flow is illustrated by the following examples:

Example. The area at the throat (A_0) of a suitably designed nozzle is 0.54 sq in. What is the weight of flow (w) of dry and saturated steam per second from this nozzle when the initial pressure (P_1) is 140 psi abs and the discharge pressure (P_2) is 15 psi abs?

Solution. The pressure at the throat of the nozzle is

$$140 \times 0.58 = 81.2 \text{ psi abs.}$$

From the steam tables the following values are found: At 140 psi abs, h_{g_1} is 1193.0 Btu and s_{g_1} is 1.5751. At the throat condition of 81.2 psi abs, s_{f_0} is 0.4545, s_{fg_0} is 1.1650, the specific volume v_{g_0} is 5.396, h_{f_0} is 283.09 and h_{fg_0} is 900.3.

Assuming an isentropic type of adiabatic expansion, the quality at the throat is

$$1.5751 = 0.4545 + 1.1650\, x_0,$$

from which

$$x_0 = 0.961.$$

The velocity at the throat is then

$$\begin{aligned} V_0 &= 223.8\sqrt{1193.0 - (283.09 + 0.961 \times 900.3)} \\ &= 1495 \text{ ft per sec.} \end{aligned}$$

The specific volume of the steam at the throat is then

$$0.961 \times 5.396 = 5.186 \text{ cu ft per lb.}$$

Since weight of flow for the throat section may be expressed

$$w = A_0 \times \frac{1}{v_0} \times V_0$$

the weight of flow in this case is

$$\begin{aligned} w &= \frac{0.54}{144} \times \frac{1}{5.186} \times 1495 \\ &= 1.08 \text{ lb per sec.} \end{aligned}$$

Example. If the discharge pressure in the previous problem had been 100 psi abs, what would have been the weight of flow per second?

Solution. Since the discharge pressure is now higher than the critical pressure of 81.2 psi abs, the pressure at the throat of the nozzle would be 100 psi abs. The quality of the steam at the throat would be

$$1.5751 = 0.4740 + 1.1286\, x_0$$

from which $x_0 = 0.975.$

The velocity at the throat would then be

$$\begin{aligned} V_0 &= 223.8\sqrt{1193.0 - (298.40 + 0.975 \times 888.8)} \\ &= 1183 \text{ ft per sec.} \end{aligned}$$

The specific volume of the steam at the throat would be

$$0.975 \times 4.432 = 4.32 \text{ cu ft per lb.}$$

The weight of flow per second would be

$$w = \frac{0.54}{144} \times \frac{1}{4.32} \times 1183 \quad \text{or} \quad 1.026 \text{ lb per sec.}$$

Orifice Flow. The functions of the nozzle and orifice are identical; the essential difference between them is that the orifice is short in comparison with its area and a nozzle is longer. Since the orifice is essentially a nozzle of infinitely small length the theory of the weight of flow applicable to the nozzle should apply to the orifice. For the nozzle, the weight of flow is determined with an accurate knowledge of the pressure existing at the section of the stream under consideration. Failure to know accurately the pressure may introduce considerable error in calculations for the weight of discharge. In the case of the orifice, although its area may be accurately known the pressure existing at this section is doubtful.

In a well-designed nozzle there is a close agreement between the theoretical or ideal flow and the actual flow. In other words, the coefficient of discharge is close to unity.

In an orifice, because of the probable errors introduced, the coefficient of discharge is smaller. There would, likewise, be a difference in the coefficient of discharge for an orifice with well-rounded entrance and one with sharp edges.

In the calculation of the weight of flow through an orifice there appears to be no satisfactory theoretical method and special formulas are usually resorted to. *Fliegner's formula* for air which follows is typical. In Fliegner's formula the flow of air under comparatively high pressure is measured by means of a pressure and temperature observation made on the inlet side of a sharp-edged orifice and pressure observation on the outlet side. The method requires the use of two pressure gages or manometers on opposite sides of the orifice and a thermometer for obtaining the temperature T_1 at the initial or higher pressure P_1. The weight of flow per second (w) may then be calculated from the following formulas:

$$w = 0.530 fA \frac{P_1}{\sqrt{T_1}} \text{ when } P_1 \text{ is greater than } 2\,P_2, \qquad (200)$$

in which f is the coefficient of discharge determined by calibration. Fliegner's formula as given by equation (200) can be derived from equation (197) by substituting

$$g = 32.2, \quad k = 1.4, \quad \frac{P_1}{v_1} = \frac{P_1^2}{RT_1}, \quad \text{and} \quad \frac{P_2}{P_1} = 0.53.^{1,2}$$

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[1] See *University of Illinois Engineering Experiment Station Bulletin*, 207. In this pamphlet, data are given of the flow of air through circular orifices with rounded approach when discharging into the atmosphere.

[2] See "Fluid Meters Report," A.S.M.E.

PROBLEMS

1. Air at a temperature of 100 deg fahr and a pressure of 100 psi abs flows through a nozzle against a back pressure of 20 psi abs. Assuming the initial velocity to be zero, what will be the velocity of discharge?

2. If the area at the throat of the nozzle in problem 1 is 0.0025 sq ft and the coefficient of discharge is unity, how many pounds of air will be discharged per minute?

3. A nozzle is supplied with steam at 160 psi abs and 98 per cent dry, the discharge being to the atmosphere. Assuming an isentropic expansion, find the area of the throat of the nozzle, if the weight of steam discharged is 0.5 lb per sec. Bar 14.7 psi abs. Specify the units in your answer.

4. Find the area of the smallest section of a nozzle to deliver 60 lb of steam per min against 200 psi abs, if the steam supplied to the nozzle is at a pressure of 350 psi abs and 520 deg fahr. Assume isentropic expansion.

5. Steam at 300 psi abs and 560 deg fahr is supplied to a turbine nozzle which discharges against 60 psi abs. Assume isentropic expansion and find the discharge quality and velocity.

6. If during the nozzle expansion of problem 5, 10 per cent of the energy released by isentropic expansion is diverted to overcoming the friction resistance to flow, what will be the discharge quality and velocity?

7. The area of a nozzle at its smallest section is 0.5 sq in. It is supplied with steam at 240 psi abs and 480 deg fahr, and discharges into a condenser at 28 in. vacuum (bar = 30 in. Hg). How many pounds of steam will be discharged per hour, assuming adiabatic expansion?

8. Calculate the weight of air discharged per minute through a nozzle which has a minimum area of 1 sq in., if the air is supplied at 58 psi abs and 100 deg fahr and is discharged to the atmosphere (bar = 14.7 psi abs).

9. Steam at a pressure of 150 psi abs and 3 per cent wet flows through a nozzle against a back pressure of 20 psi abs. Calculate the velocity at the throat of the nozzle and at its exit.

10. A safety valve is to be designed for a 200-hp. boiler generating steam at 150 psi abs pressure and 5 per cent moisture. Assuming that the safety valve should have a capacity such that it will release the boiler of all steam when generating double its rated capacity, what will be the smallest sectional area of the valve? (Feedwater temperature = 200 deg fahr.)

11. Calculate the dimensions of a nozzle to deliver 800 lb of steam per hour with a steam pressure of 600 psi abs and a steam temperature of 700 deg fahr, if the pressure at the end of the nozzle is 130 psi abs.

12. Design a nozzle to deliver 400 lb of steam per hour, initial pressure 200 psi abs, final pressure atmospheric (bar = 28.62 in.), temperature of steam 600 deg fahr.

13. Determine the weight of air discharged per minute from a nozzle of 1.5 sq in. throat area. p_1 = 105 psi abs, p_2 = 15 psi abs, t_1 = 85 deg fahr. Assume isentropic expansion.

14. Air is expanded in a nozzle from a pressure of 180 psi abs and 200 deg fahr to 100 psi abs.

(a) Calculate the enthalpy change for isentropic expansion.
(b) Calculate the minimum nozzle area for a flow of 1 lb per sec.

15. Superheated steam at 120 psi abs and 400 deg fahr flows through a pipe line. A leak occurs at one of the pipe joints so that some steam escapes into the atmosphere. Estimate the weight of steam leaking in lb per hour, if the leakage area is 0.15 sq in. (bar = 14.7 psi abs).

16. An orifice was calibrated for use in measuring the flow of steam. The actual weight discharged was 31.8 lb per min, when dry saturated steam at 20 psi abs was expanded to 14.5 psi abs through the 2 sq in. minimum area. What was the ratio of the actual to the theoretical discharge velocity?

CHAPTER X

APPLICATIONS OF THERMODYNAMICS TO COMPRESSED AIR, REFRIGERATING MACHINERY AND AIR CONDITIONING

COMPRESSED AIR

Air when compressed may be used as the working medium in an engine, in exactly the same way as steam. Furthermore, it may be employed as an agent for the transmission of power and can be distributed from a central station for the purpose of operating engines and various pneumatic tools.

Air Compressors. The most common type of machine used for the compression of air is that known as a piston-compressor and consists of a cylinder provided with valves and a piston.

The work performed in the air-cylinder of a compressor can best be studied from an indicator diagram. If the compression is effected very slowly in a conducting cylinder, so that the air within may lose heat by conduction to the atmosphere as fast as heat is generated by compression, the process will be isothermal, at the temperature of the atmosphere. Also if the compressed air is distributed for use in compressed air motors[1] or engines without a change of temperature, and the process of expansion in the compressed air motors or engines is indefinitely slow and consequently isothermal, then (if the loss caused by friction in pipes is neglected) there would be no waste of power in the whole process of transmission. The indicator diagram would then be the same per pound of air in the compressor as in the air motor, although the course of the cycle would be the reverse — that is, it would retrace itself.

Adiabatic compression and expansion take place approximately if the expansion and compression are performed very quickly, or when the air is not cooled during compression — in such a case the temperature of the air rises. The theoretical indicator diagram of the compressor, **Fig. 60**, is **FCBE** and that of the air engine,

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[1] Compressed air motors are similar to steam engines, but use compressed air instead of steam.

Fig. 61, is **EABCD**. **CB** and **AB** are both adiabatic lines. The change of volume of the compressed air from that of **EB** to **EA** occurs through its cooling in the distributing pipes, from the temperature produced by adiabatic compression down to the temperature of the atmosphere.

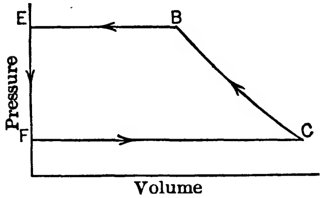

Fig. 60. Pressure-Volume Diagram of Compressor.

Suppose the diagrams of the compressor and of the air engine are superimposed as in **Fig. 62**, and then an imaginary isothermal line is drawn between the points **A** and **C**.

The use of adiabatic compression causes a waste of power which is measured by the area **ABC**, while the use of the adiabatic expansion in the air engine involves a further waste, shown by the area **ACD**.

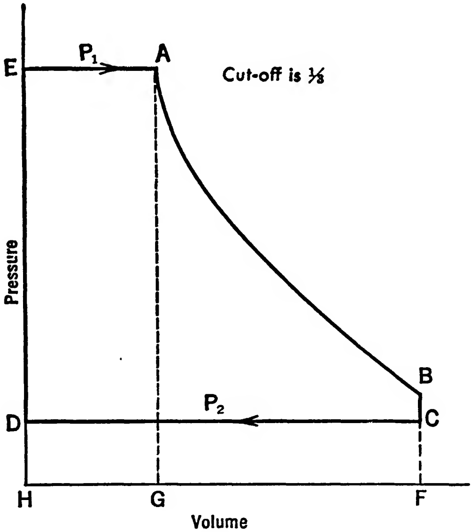

Fig. 61. Pressure-Volume Diagram of Air Engine.

Work of Compression. Assuming no clearance in the compressor and isothermal compression, the pressure-volume diagram will be similar to **Fig. 60**. If the subscript 1 represents the inlet conditions and the subscript 2 the discharge conditions, the work of the cycle in foot-pound units will be

$$W = P_1V_1 + P_1V_1 \log_e \frac{V_2}{V_1} - P_2V_2,$$

which, since $P_1V_1 = P_2V_2$ in an isothermal process, becomes

$$W = P_1V_1 \log_e \frac{V_2}{V_1}. \tag{201}$$

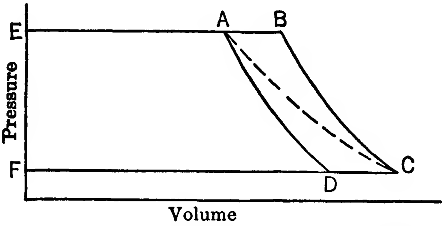

FIG. 62. Superimposed Diagrams of Figs. 60 and 61.

The net work done by an air engine may be calculated from a study of the areas in **Fig. 61** as follows:

$$\begin{aligned}
\text{area } EAGH &= W = +\, P_1V_A \\
\text{area } ABFG &= W = +\, \frac{P_1V_A - P_BV_B}{n-1} \\
\text{area } CDHF &= W = -\, P_2V_c \quad \text{or} \quad -\, P_2V_B.
\end{aligned}$$

In practice the compression cannot be made strictly isothermal, as the operation of the piston would have to be too slow. The difference between isothermal and adiabatic compression (and expansion) can be shown graphically as in **Figs. 63** and **64.** In these illustrations the terminal points are correctly placed for a certain ratio for both compression and expansion. Note that in the compression diagram (**Fig. 63**) the area between the two curves, **ABC**, represents the work lost in compression owing to temperature rise, and the area between the two curves, **ACMNF** (in **Fig. 64**), shows the work lost due to temperature drop during the expansion. The isothermal curve **AC** is the same for both cases.

The temperature of the air may be prevented from rising during the compression by injecting water into the cylinder or by water-jacketing the cylinder, and in this way the compression curve changes. The curves which would have been PV = a constant, if isothermal, and $PV^{1.4}$ = a constant, if adiabatic, would be very much modified. In perfectly adiabatic conditions the exponent "n" = 1.405 for air, but in practice the compressor cylinders are water-jacketed, and thereby part of the heat of compression is conducted away, so that "n" becomes less than 1.40. This value of "n" varies with conditions; generally the value is between 1.2 and 1.3.

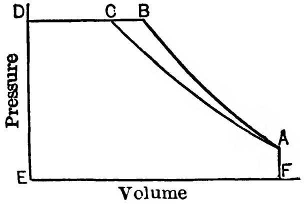

FIG. 63. Pressure-Volume Diagram Showing Work Lost Due to Heating During Compression.

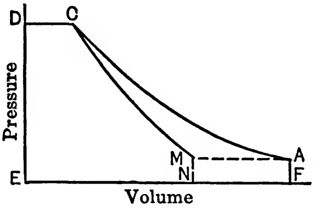

FIG. 64. Pressure-Volume Diagram Showing Work Lost Due to Cooling During Expansion.

When the compression curve follows the law, PV^n equals a constant, the work of compression (**Fig. 60**) is

$$W = -\frac{P_2V_2 - P_1V_1}{n-1} - P_2V_2 + P_1V_1$$

$$= \frac{n}{n-1}(P_1V_1 - P_2V_2), \qquad (202)$$

in which the subscripts 1 and 2 represent the inlet and discharge conditions respectively.

Since $\quad P_1V_1^n = P_2V_2^n; \quad V_2 = V_1\left(\frac{P_1}{P_2}\right)^{\frac{1}{n}};$

substituting this value of V_2 in equation (202)

$$W = \frac{n}{n-1}P_1V_1\left(1 - \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}\right). \qquad (203)$$

The Effect of Clearance upon Volumetric Efficiency. It is usually impracticable to construct a compressor without clearance, consequently the indicator diagram differs from that of the ideal case. At the end of the discharge stroke, the clearance volume is filled with compressed air. When the piston moves on its outward stroke, the clearance air expands and the suction valves of the compressor are held shut until the piston has moved a sufficient distance to permit the entrapped air to expand to atmospheric pressure. When that point is reached any further movement of the piston opens the suction valves and external air is drawn into the cylinder. Thus the entire stroke of the compressor piston is not effective in pumping air. The ratio of the apparent volume of free air drawn in, as shown by the indicator card to the volume swept by the piston, or the piston displacement of the cylinder, is termed the *apparent volumetric efficiency.* True volumetric efficiency is the ratio of the air actually drawn in to the piston displacement.

Figure 65 illustrates an ideal compressor diagram with clearance. The air entrapped in the clearance space (V_c) equals

$$V_c = CV_s, \tag{204}$$

where V_s = volume swept or piston displacement of the cylinder.
C = percentage of clearance expressed as a decimal.

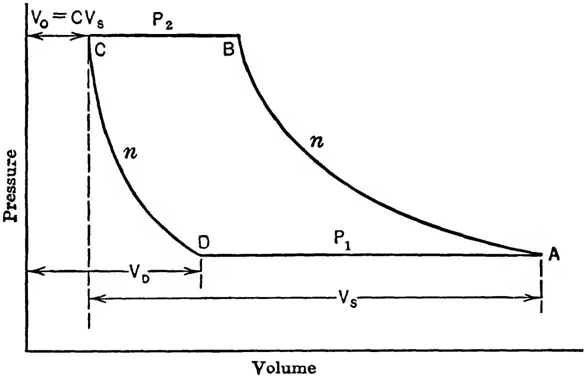

Fig. 65. Pressure-Volume Diagram of Ideal Air Compressor with Clearance.

The air in **Fig. 65** expands to V_D at which point the inlet valves open. The air drawn into the cylinder is represented by the difference in volume between V_A and V_D. This air, as well as the clearance air, is compressed to point B, while the compressed air is discharged from points B to C. Knowing the per cent of clearance, the volume swept by the piston (V_s), and the initial and final pressure, the volumetric efficiency (V_{ef}) may be determined from the following equations:

$$V_{ef} = \frac{V_A - V_D}{V_s}.$$

$$P_2(CV_s)^n = P_1V_D^n.$$

$$V_D = \left(\frac{P_2}{P_1}\right)^{\frac{1}{n}} CV_s,$$

since

$$V_A = V_s + CV_s,$$

$$V_A - V_D = V_s + CV_s - \left(\frac{P_2}{P_1}\right)^{\frac{1}{n}} CV_s$$

$$= \left[1 + C\left(1 - \left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}\right)\right] V_s.$$

Therefore the volumetric efficiency is

$$e = \frac{\left[1 + C\left(1 - \left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}\right)\right] V_s}{V_s}$$

$$= 1 + C\left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}\right]. \qquad (205)$$

Two-Stage Compression. The problem of economy, obviously, becomes one of abstracting the energy added to the air during the process of compression, causing rise of temperature and increase of volume. As previously mentioned, this is partially accomplished by water-jacketing the cylinders, and also by water injection. Nevertheless, owing to the short interval within which the compression takes place, and the comparatively small volume of air actually in contact with the cylinder walls, very little cooling really occurs. The practical impossibility of proper cooling to prevent waste of energy leads to the alternative of discharging air from one cylinder after partial compression has been effected, into a so-called *inter-cooler,* intended to absorb the energy added during the first compression, and then compressing the air to the final pressure in another cylinder. This operation is termed *two-stage* compression and when repeated one or more times for high pressures the term *multi-stage* compression applies.

Referring to **Fig. 66** and assuming the compression in a two-stage compressor to be adiabatic for each cylinder, the compression curve is represented by the broken line **ABDE**. The compression proceeds adiabatically in the first or low-pressure cylinder to **B**; the air is then taken to a cooler and cooled under practically constant pressure to its initial temperature, and its volume reduced from **HB** to **HD**; it is then introduced to the second or high-pressure cylinder and compressed adiabatically along the line **DE** to the final pressure condition that is desired. It is seen that the combined compression curve approaches the isothermal line **FA**.[1] The isothermal condition is obviously desired and, in consequence, air-machines are built to approach that condition as nearly as possible.

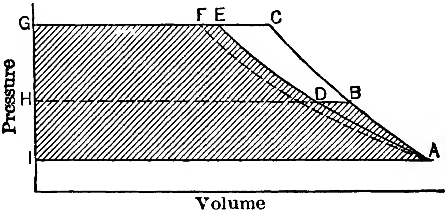

FIG. 66. Indicator Diagram of Two-Stage Air Compressor.

Referring to **Fig. 66**, the work of each stage from equation (203), representing the initial, intermediate and discharge pressures respectively by P_1, P', P_2, will be

$$W \text{ (1st stage)} = \frac{n}{n-1} P_1V_1\left[1 - \left(\frac{P'}{P_1}\right)^{\frac{n-1}{n}}\right].$$

$$W \text{ (2nd stage)} = \frac{n}{n-1} P'V'\left[1 - \left(\frac{P_2}{P'}\right)^{\frac{n-1}{n}}\right].$$

The total work of compression is

$$W \text{ (total)} = W \text{ (1st stage)} + W \text{ (2nd stage)}$$

$$= \frac{n}{n-1}\left\{P_1V_1\left[1 - \left(\frac{P'}{P_1}\right)^{\frac{n-1}{n}}\right] + P'V'\left[1 - \left(\frac{P_2}{P'}\right)^{\frac{n-1}{n}}\right]\right\}$$

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[1] The line **FE** represents further cooling.

with perfect cooling

$P_1V_1 = P'V'$; and V' corresponding to point **D** (**Fig. 66**)

then

$$W\ (\text{total}) = \frac{n}{n-1} \times P_1V_1\left[1 - \left(\frac{P'}{P_1}\right)^{\frac{n-1}{n}} + 1 - \left(\frac{P_2}{P'}\right)^{\frac{n-1}{n}}\right]. \qquad (206)$$

The work of compression as expressed by equation (206) becomes a minimum when

$$\left(\frac{P_2}{P'}\right)^{\frac{n-1}{n}} + \left(\frac{P'}{P_1}\right)^{\frac{n-1}{n}} \qquad (207)$$

is a maximum. Since the initial and final pressures P_1 and P_2 are fixed, the intermediate pressure (P') can be found by differentiating equation (207) and equating the differential coefficient to zero; then

$$\frac{d}{dP'}\left[\left(\frac{P_2}{P'}\right)^{\frac{n-1}{n}} + \left(\frac{P'}{P_1}\right)^{\frac{n-1}{n}}\right] = \frac{1-n}{n}\left[\frac{(P')^{\frac{1-2n}{n}}}{(P_2)^{\frac{1-n}{n}}}\right] + \frac{n-1}{n}\left[\frac{(P')^{-\frac{1}{n}}}{(P_1)^{\frac{n-1}{n}}}\right] = 0,$$

or by transposing,

$$\frac{n-1}{n}\left[\frac{(P')^{-\frac{1}{n}}}{(P_1)^{\frac{n-1}{n}}}\right] = \frac{n-1}{n}\left[\left(P_2\right)^{\frac{n-1}{n}}\left(P'\right)^{\frac{1-2n}{n}}\right],$$

therefore, $$(P')^2 = P_1P_2. \qquad (208)$$

Substituting for P' in equation (206), the value from equation (208)

$$W = \frac{2n}{n-1} \times P_1V_1\left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{2n}}\right]. \qquad (209)$$

It will also be found that the work is equally divided between the high- and low-pressure cylinders. For compressing air to high pressures, three and four stage compressors are used.

The form of equation (209) becomes a general equation for *any number of stages* provided the work is equally divided between stages and can be expressed as

$$W = \frac{Xn}{n-1} \times P_1V_1\left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{Xn}}\right] \qquad (210)$$

where X is the number of stages.

REFRIGERATING MACHINERY

Refrigerating Machines or Heat Pumps. By a refrigerating machine or heat pump is meant a machine which carries heat from a cold to a hotter body. This, as the Second Law of Thermodynamics asserts, cannot be done by a self-acting process, but it can be done by the expenditure of mechanical work. Any heat engine will serve as a refrigerating machine if it can be made to trace its indicator diagram backward, so that the area of the diagram represents work spent on, instead of done by, the working substance. Heat is then taken in from the cold body and heat is rejected to the hot body.

Take the Carnot cycle, using air as the working substance (**Fig. 67**), and let the cycle be performed in the order **dcba**, so that the area of the diagram is negative, and represents work spent upon the working substance. In the stage **dc**, which is isothermal expansion in contact with the cold body **R** (as in **Fig. 9**), the air takes in a quantity of heat from **R** equal to $wRT_2 \log_e r$ [equation (55)], and in stage **ba** it gives out to the hot body **H** a quantity of heat equal to $wRT_1 \log_e r$. There is no transfer of heat in stages **cb** and **ad**. Thus **R**, the cold body, is constantly being drawn upon for heat and can therefore be maintained at a temperature lower than its surroundings. In an actual refrigerating machine operating with air, the cold body **R** consists of a coil of pipe through which brine circulates while " working " air is brought into contact with the outside of the pipe. The brine is kept, by the action of

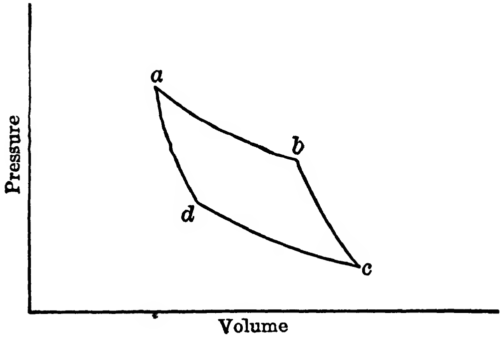

FIG. 67. Pressure-Volume Diagram of Carnot Cycle Using Air.

the machine, at a temperature below 32 deg fahr and is used in its turn to extract heat by conduction from the water which is to be frozen to make ice. The " cooler " **H**, which is the relatively hot body, is kept at as low a temperature as possible by means of circulating water, which absorbs the heat rejected to **H** by the " working " air.

The size of an air refrigerating machine is very large as compared with its performance. The use of a regenerator, as in Stirling's engine (**Fig. 10**), may be resorted to in place of the two adiabatic stages in the Carnot cycle, with the advantage of making the machine much less bulky. Refrigerating machines using air as the working substance, with a regenerator, were introduced by Dr. A. C. Kirk and have been widely used.[1] The working air is completely enclosed, which allows it to be in a compressed state throughout, so that even its lowest pressure is much above that of the atmosphere. This makes a greater mass of air pass through the cycle in each revolution of the machine, and hence increases the performance of a machine of given size. In all air refrigerating machines the temperature range must be high to produce a given refrigerating effect.

In another class of refrigerating machines the working substance, instead of being air, consists of a liquid and its vapor, and the action proceeds by alternate evaporation under a low pressure and condensation under a relatively high pressure. A liquid must be chosen which evaporates at the lower extreme of temperature under a pressure which is not so low as to make the bulk of the engine excessive. Ammonia, ether, sulphurous acid, and other volatile liquids have been used. Ether machines are inconveniently bulky and cannot be used to produce intense cold, for the pressure of that vapor is only about 1.3 psi abs at 4 deg fahr, and to make it evaporate at any temperature nearly as low as this would require the cylinder to be excessively large in proportion to the performance. This would not only make the machine clumsy and costly, but would involve much waste of power in mechanical friction. The tendency of the air outside to leak into the machine is another practical objection to the use of

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[1] See Kirk, On the Mechanical Production of Cold, *Proc. Inst. of C. E.*, vol. XXXVII, 1874. Also lectures on Heat and its Mechanical Applications, in the same proceedings for 1884.

such a low pressure. With ammonia a distinctly lower limit of temperature is practicable: the pressures are rather high and the apparatus is compact. Carbonic acid has been used as a refrigerant in small machines. The objection to carbonic acid is that the pressures are very high as compared with ammonia (see Appendix). The critical temperature of carbonic acid is less than 90 deg fahr. The properties of several types of refrigerants are illustrated in **Fig. 68.**

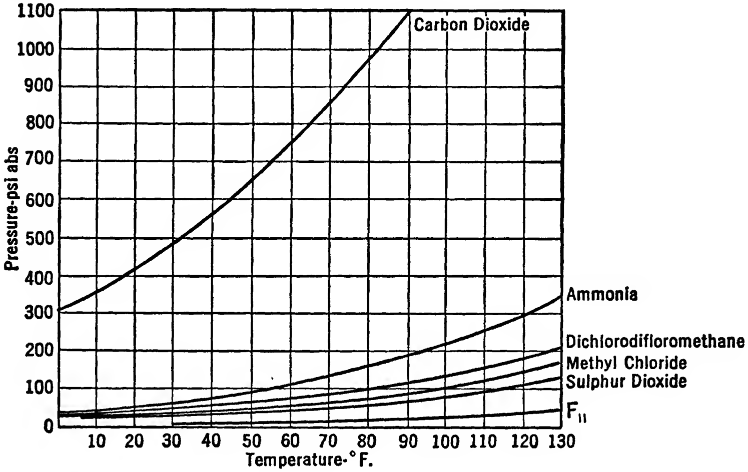

FIG. 68. Properties of Refrigerants.

Unit of Refrigeration. The capacity of a refrigerating machine is usually expressed in tons of refrigeration or ice melting effect per 24 hours. As the latent heat of fusion of 1 lb of ice is about 144 Btu, the heat withdrawn per ton of refrigerating effect is: 2000 × 144 = 288,000 Btu per 24 hours, or 200 Btu per minute.

Systems of Mechanical Refrigeration. The standard systems of mechanical refrigeration are:[1]

(1) The *dense-air system,* so-called because the air which is the medium is never allowed to fall to atmospheric pressure, so as to reduce the size of the cylinders and pipes through which a given weight is circulating.

(2) The *compression system,* using ammonia, carbon dioxide

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[1] Moyer and Fittz's *Refrigeration,* 2d ed., page 9, and Lucke's *Engineering Thermodynamics,* page 1148.

or sulphur dioxide, and so-called to distinguish it from the third system, because a compressor is used to raise the pressure of the vapor and deliver it to the condenser after removing it from the evaporator.

(3) The *steam jet system.* This system makes use of water vapor and is favored under certain conditions for air-conditioning systems.

(4) The *absorption system,* using ammonia, and so-called because a weak water solution removes vapor from the evaporator by absorption, the richer aqua ammonia so formed being pumped into a high-pressure chamber called a generator. This generator is in communication with the condenser, where the ammonia is discharged from the liquid solution to the condenser by heating the generator, to which the rich solution is delivered by the pump.

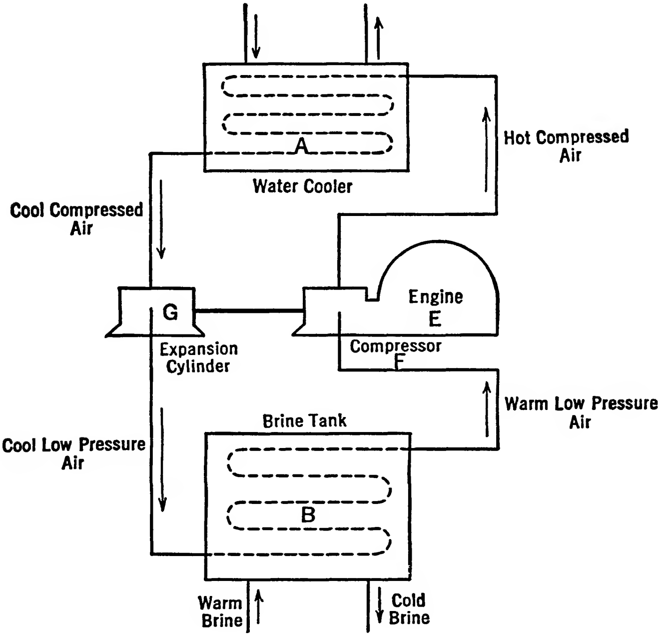

FIG. 69. The Dense-Air System of Refrigeration.

No matter what system is used, a circulating fluid, usually water or air, is used to receive the heat, the temperature of which limits the highest temperature allowable in the system and indirectly the highest pressure.

The Air System of Refrigeration. The *dense* or *closed air system* is illustrated in **Fig. 69**, in which air, previously freed of moisture, is continuously circulated. The engine cylinder **E** furnishes power[1] to drive the compressor cylinder **F**. This cylinder delivers hot-compressed air into a cooler **A**, which is then passed on to the expansion cylinder **G**, which in turn sends cold low-pressure air first through the refrigerating coils in the brine tank **B** and then back to the compressor cylinder **F**; thus the air cycle is completed. The courses of the circulating water and also of the brine are shown.

The dense-air cycle in a pressure-volume diagram is represented in **Fig. 70**, in which **BC** is the delivered volume of hot-compressed air; **CM** is the volume of cooled air admitted into the expansion cylinder; **MB** the reduction in volume due to the water cooler; **MN** the expansion; **NA** the refrigeration or heating of the air by the brine, and **AB** the compression. This operation is but a reproduction of that previously described.

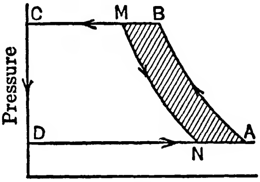

FIG. 70. Pressure-Volume Diagram of Dense-Air Cycle of Refrigeration.

The work W_c expended in the compression cylinder F is $DABC$, that done by the expansion cylinder G is $W_e = DCMN$. The shaded area $MBAN$ represents the work which must be supplied by the engine E.

If w pounds of air are passing through the refrigerating machine per minute, the heat withdrawn from the cold room or absorbed by the brine along NA (**Fig. 70**) is

$$Q_{NA} = wc_p(t_a - t_n). \tag{211}$$

The work expended in compressing w pounds of air, assuming polytropic compression, is, by equation (203),

$$W_c = \frac{wnP_av_a}{n-1}\left[1 - \left(\frac{P_b}{P_a}\right)^{\frac{n-1}{n}}\right] = wP_av_a\,\frac{n}{n-1}\left[1 - \left(\frac{P_b}{P_a}\right)^{\frac{n-1}{n}}\right].$$

---

[1] Since the work done by the expansion of the cool-compressed air in the expansion cylinder is less than that necessary for the compressing of the air taken from the brine coils through the same pressure conditions, a means must be employed to make up for the difference, and for this purpose the engine cylinder is used.

If the compression is adiabatic,

$$W_c = wP_a v_a \frac{k}{k-1}\left[1 - \left(\frac{P_b}{P_a}\right)^{\frac{k-1}{k}}\right], \tag{212}$$

since $\dfrac{T_b}{T_a} = \left(\dfrac{P_b}{P_a}\right)^{\frac{k-1}{k}}$ also $AR = c_p - c_v = c_p\left(\dfrac{k-1}{k}\right)$

$$W_c = \frac{wc_p}{A}(t_b - t_a).$$

If the expansion is complete in the expansion cylinder, the work done in expansion is

$$W_e = \frac{wc_p}{A}(t_m - t_n). \tag{213}$$

The net work of the engine E required to produce refrigeration becomes

$$W = W_c - W_e = \frac{wc_p}{A}(t_b - t_a - t_m + t_n). \tag{214}$$

Since, from equation (211),

$$w = \frac{Q_{NA}}{c_p(t_a - t_n)}\ .$$

$$W = \frac{Q_{NA}}{A}\left(\frac{t_b - t_a - t_m + t_n}{t_a - t_n}\right). \tag{215}$$

The Vapor Compression System of Refrigeration. The compression system for ammonia or similar condensable vapors is shown in **Fig. 71.** The figure illustrates the essential members of a complete compression refrigerating system. **B** represents the direct-expansion coil in which the working medium is evaporated; **F**, the compressor or pump for increasing the pressure of the gasified ammonia; **W**, the condenser for cooling and liquefying the gasified ammonia; and **V**, a throttling valve by which the flow of liquefied ammonia under the condenser pressure is controlled as it flows from the receiver **R** to the expansion coils; **B**, the brine tank in which a materially lower pressure is maintained by the pump or compressor in order that the working medium may boil at a sufficiently low temperature to take heat from and consequently refrigerate the brine which is already cooled.

In analyzing the operation of the individual parts of the compression system several important facts are evident.

In the expansion coils the liquid refrigerant is boiling, and the heat absorbed from the brine theoretically equals that required to evaporate the refrigerant. Since the temperature of boiling liquid or refrigerant is dependent upon the pressure, the suction pressure at the compressor **F** is dependent upon the temperature to be maintained in the brine. Assuming that the temperatures of the brine and of the refrigerant are equal, the temperature of the brine is dependent upon the suction pressure of the compressor. Since a saturated vapor is assumed in the expansion coils, the suction pressure to be maintained at the compressor is, for a given refrigerant, that corresponding to the saturated temperature of the refrigerant.

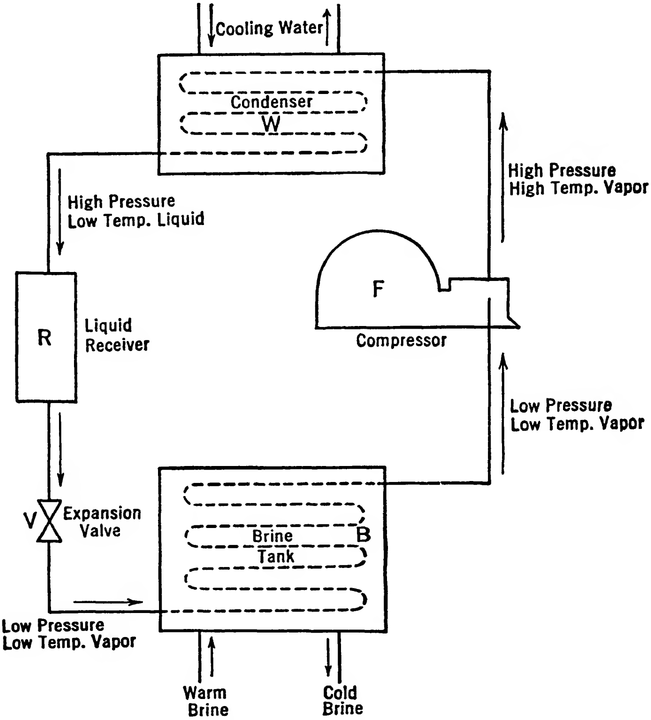

FIG. 71. The Compression System of Refrigeration.

The heat absorbed by the refrigerant, which equals the heat taken from the brine, is, for each pound of refrigerant circulated

$$h_1 - h_{f_1} \tag{216}$$

where h_1 is the enthalpy of the dry and saturated vapor leaving the expansion coils and h_{f_1} is the enthalpy of the refrigerant entering the expansion coils.

In the condenser, the gasified refrigerant returns to the liquid state. The condensation in the condenser is, for a given temperature in the condenser, dependent upon the pressure. The pressure of discharge from the compressor, consequently, is dependent upon the temperature in the condenser, and the discharge pressure to be maintained by the compressor **F** is that corresponding to the saturated temperature maintained in the condenser for the given refrigerant.

The heat absorbed by the condenser, which equals that taken from the refrigerant, is for each pound of refrigerant circulated

$$h_2 - h_{f_2}, \tag{217}$$

where h_2 is the enthalpy of the refrigerant leaving the compressor or entering the condenser, and h_{f_2} is the enthalpy of the liquid refrigerant leaving the condenser.

The operation of the compressor **F** (**Fig. 71**) is theoretically a reversed Rankine cycle. If 1 lb of the refrigerant is passed through the system the work (W) of the compressor is

$$W = h_2 - h_1, \tag{218}$$

where h_1 is the enthalpy of the vapor at the entrance to the compressor and h_2 is the enthalpy of the vapor discharged from the compressor.

The vapor entering the compressor may be treated as though it were dry and saturated. The temperature-entropy (T-S) diagram (**Fig. 72**) illustrates this case. The vapor discharged by the compressor is then superheated and the condition of the discharged vapor may be determined, assuming a reversible adiabatic process, by equating the entropies.

The horsepower (hp) of the compressor if w pounds of vapor are circulated per minute, is:

$$\text{hp} = \frac{w(h_2 - h_1)778}{33{,}000} \tag{219}$$

In the case of the discharge through the throttling or expansion valve of the system, since it is a uniform flow process with all energies negligibly small except the enthalpies, before and after the process, the enthalpies are equal. Thus,

$$h_{f_2} = h_1, \tag{220}$$

where h_{f_2} is the enthalpy of the liquid leaving the condenser (assuming no change in temperature before entering the expansion valve) and h_1 is the enthalpy of the vapor leaving the expansion valve or entering the condensor.

Thus, the enthalpy of the vapor entering the expansion coil may be taken as the enthalpy of the liquid leaving the condenser if no change in temperature of the liquid occurs between the condenser and entrance to the expansion valve.

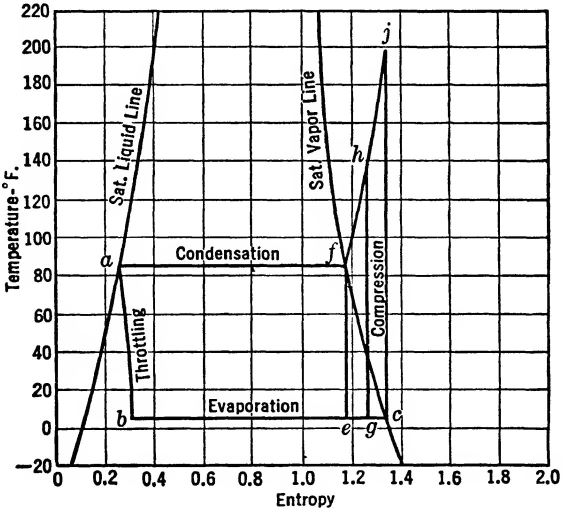

FIG. 72. *T-S* Diagram for Vapor Compression Cycle:

Ammonia compressors are operated in various ways depending upon the condition of the refrigerant entering the compressor. In the *dry system* the ammonia entering the compressor is a dry vapor as illustrated as *cj* in **Fig. 72.** In the *wet system* the ammonia enters the compressor in the wet state (cf. Fig. 72), the energy added during the compression being used to evaporate the liquid ammonia into a vapor.

The Steam Jet System of Refrigeration. The steam jet refrigeration process is illustrated in **Fig. 73.** Steam from the boiler is expanded through a nozzle into a chamber **A** above the evaporator tank. The high velocity jet thus produced entrains vapor from the evaporator, compressing and delivering it through the medium of the steam ejector to the condenser. In the condenser both the motive steam and evaporation from the evaporator tank are condensed and eventually returned to the steam boiler.

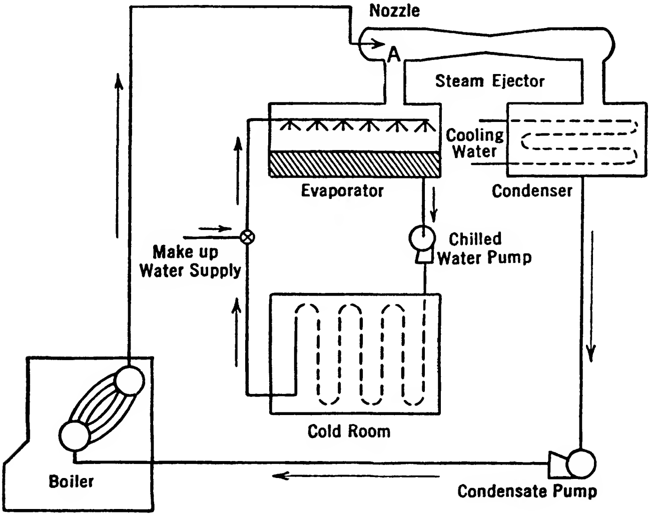

FIG. 73. The Steam Jet System of Refrigeration.

The entraining action of the motive steam jet, by continuously removing vapor from the evaporator tank, causes continuous evaporation of a small part of the water in that tank. Such evaporation is accompanied by the absorption of heat from the rest of the water in the tank, cooling it to moderate temperatures suitable for comfort cooling processes. The cooled water produced is circulated to heat exchangers or sprays in air conditioning units and, together with the required make-up water, is returned to the evaporator tank.

Advantages claimed for this system, in connection with air conditioning, are summertime use of a heating plant that would otherwise be idle, a safe working fluid (steam), used at pressures

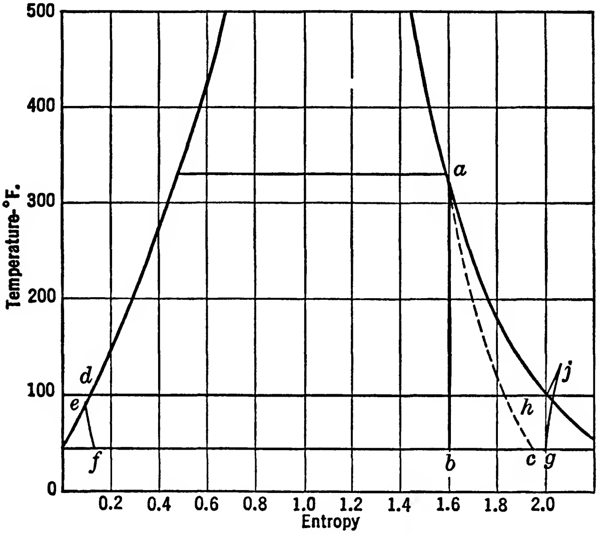

FIG. 74. *T-S* Diagram for the Steam Jet System of Refrigeration.

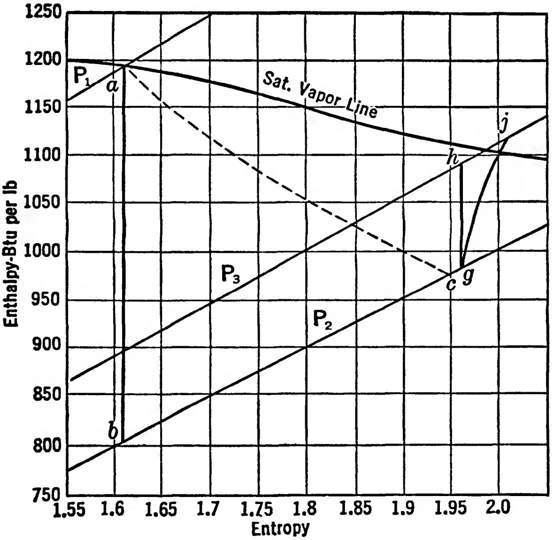

FIG. 75. *h-S* Diagram for the Steam Jet System of Refrigeration.

and temperatures familiar to heating plant operators, and negligible explosion hazard in the cooling circuit since all but the motive steam is flowing at sub-atmospheric pressures.

Figure 74 illustrates the steam jet system on **T-S** coordinates and **Fig. 75** illustrates the same process on the **h-S** plane. The **a-c** line in both cases represents the actual expansion of motive steam, considering the efficiency of the nozzle and the exchange of momentum between motive steam and evaporation molecules. The **a-b** line represents the ideal isentropic process. After the mixture of the motive steam with steam from the evaporator the combined steam is in the condition indicated by point **g.** Compression of the mixture in the ejector tube follows the path **g-j** to the condenser pressure. The usual design includes further compression stages with additional steam jets and ejector tubes to raise the non-condensable gases (air, CO_2, etc.) to atmospheric pressure.

The Vapor Absorption System of Refrigeration. The absorption system depends upon the fact that anhydrous ammonia possesses the property of forming aqua ammonia. The amount of ammonia that water will absorb depends upon the temperature of the water; the colder the water, the greater are its absorptive powers.

The absorption system (**Fig. 76**) differs from the vapor compression system in that the absorption system replaces the compressor by an absorber **A** (**Fig. 76**), where the ammonia vapor is changed into aqua ammonia, a pump **P** which transfers the aqua ammonia from the absorber through a heat exchanger **E** (**Fig. 76**) to the generator **G**, where the aqua ammonia is heated. From the generator (**G**) the vapor passes to an analyzer (**N**), to a rectifier (**R**), used to reduce the water vapor concentration in the system. From the rectifier the NH_3 liquid flows to the receiver (**D**). Both systems have a condenser where the ammonia is cooled and liquefied and an expansion valve or throttling valve by means of which the flow of liquid ammonia to the expansion coils is controlled. In the absorption system the anhydrous ammonia vapor flows from the expansion coils to the absorber, in which the anhydrous vapor comes in contact with weak aqua ammonia. The weak ammonia absorbs the ammonia vapor. From the absorber the ammonia is pumped into the generator, where it is heated by steam coils. The vapor driven off in the generator passes to

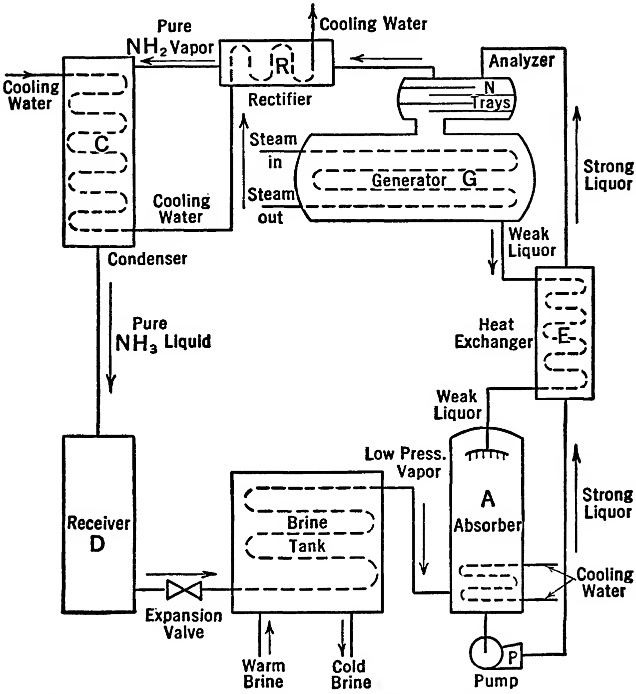

FIG. 76. The Vapor Absorption System of Refrigeration.

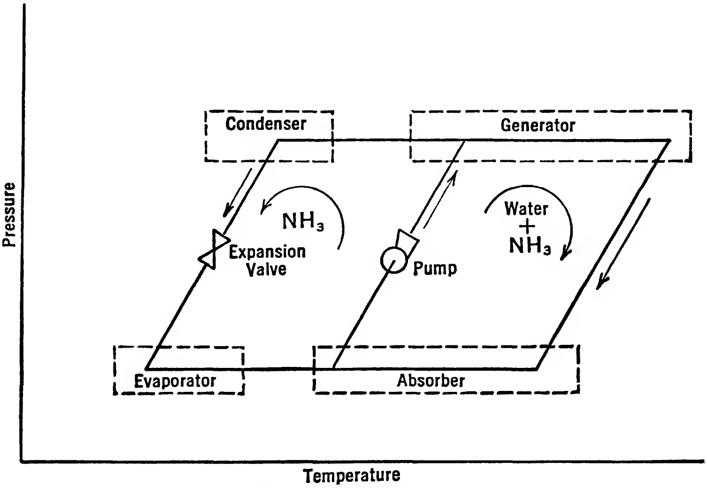

FIG. 77. *P-T* Diagram for Absorption Refrigeration System.

the condenser and from there through the expansion or throttle valve to the expansion coils which are located in the brine tank or in the refrigerating room.

The absorption system is usually provided with a rectifier to dry the gas thoroughly before it enters the condenser, an exchanger which heats the strong ammonia by cooling the weak ammonia, and with other auxiliary equipment to reduce the heat losses in the system.

The absorption system on **P-T** coordinates is illustrated in **Fig. 77.**

Coefficient of Performance of Refrigerating Machines. In estimating the merits of a refrigerating machine the coefficient of performance may be expressed as:

$$\frac{\text{Heat extracted from the cold body}}{\text{Work expended}}$$

When the limits of temperature T_1 and T_2 are assigned it is very easy to show by a slight variation of the argument used in Chapter IV that no refrigerating machine can have a higher coefficient of performance than one which is reversible according to the Carnot method. For let a refrigerating machine **S** be driven by another **R** which is reversible and is used as a heat-engine in driving **S.** Then if **S** had a higher coefficient of performance than **R** it would take from the cold body more heat than **R** (working reversed) rejects to the cold body, and hence the double machine, although purely self-acting, would go on extracting heat from the cold body in violation of the Second Law of Thermodynamics. Reversibility, then, is the test of perfection in a refrigerating machine just as it is in a heat-engine.

When a reversible refrigerating machine takes in all its heat, namely Q_c at T_2, and rejects all, namely, Q_a at T_1, if the heat equivalent of the work done is represented by $W = Q_a - Q_c$, then the coefficient of performance is, as already defined,

$$\frac{Q_c}{W} = \frac{Q_c}{Q_a - Q_c} = \frac{T_2}{T_1 - T_2}. \tag{221}$$

Hence, the smaller the range of temperature, the better is the performance. To cool a large mass of any substance through a few degrees requires much less expenditure of energy than to cool one-fifth of the mass through five times as many degrees, although the amount of heat extracted is the same in both cases. If it is desired to cool a large quantity of a substance it is better to do this by the direct action of a refrigerating machine working through the desired range of temperature, than to cool a portion through a wider range and then let this mix with the rest. This is only another instance of a wide, general principle, that any mixture of substances at different temperatures is thermodynamically wasteful because the interchange of heat between them is irreversible. An ice-making machine, for example, should have its lower limit of temperature only so much lower than 32 deg fahr as will allow heat to be conducted to the working fluid with sufficient rapidity from the water that is to be frozen.

The possibility of using a refrigerating system as a year-round means for comfort air conditioning has been suggested many years ago. A few units are in actual use in warm climates. The operation of such a system may be briefly described as follows: During the hot summer months air for circulation in the rooms to be cooled is forced through a nest of coils in which cooled brine or the refrigerant is flowing. The air may thus be cooled and dehumidified for greater comfort. Heat absorbed from the air is rejected to the condenser, as is done in any refrigerating process. When winter temperatures require heating for comfort, the room air is forced past the condenser coils absorbing heat from them. The expansion coils in this case may be exposed to the colder outdoor air absorbing heat from it. When the winter operation of the unit is considered the coefficient of performance may be written:

$$\frac{Q_a}{Q_a - Q_c} = \frac{T_1}{T_1 - T_2}. \tag{222}$$

Air-Water Vapor Mixtures. Atmospheric air is a mixture of gases and water vapor. The gases in the mixture, principally nitrogen and oxygen with traces of argon, carbon dioxide, hydrogen and other gases, may be treated as a single gas with an apparent dry molecular weight of 29. The water vapor, which varies widely in amount, is usually present as superheated steam at low pressures. Such vapor, at an absolute pressure of 1 psi or less and moderately superheated, behaves like a gas, and many necessary calculations of its properties may be made on the gas law basis.

Some properties of an air-water vapor mixture needed for air conditioning or similar calculations are the barometric pressure, the dry and wet bulb temperatures, and the average specific heats at constant pressure for the dry air and the vapor. From these data, by use of the laws for ideal gases and the properties of low pressure saturated steam, the specific volume, the relative humidity, and the enthalpy of the gases and vapors in an atmospheric air mixture may be calculated. The barometric pressure is obtained by the use of a mercury barometer. The mercury in a glass thermometer gives the dry bulb temperature. The wet bulb temperature is determined by means of a psychrometer.

At the pressurês and temperatures of air conditioning practice the specific heat at constant pressure for dry air may be taken as 0.24, while that for water vapor may be used as 0.45.

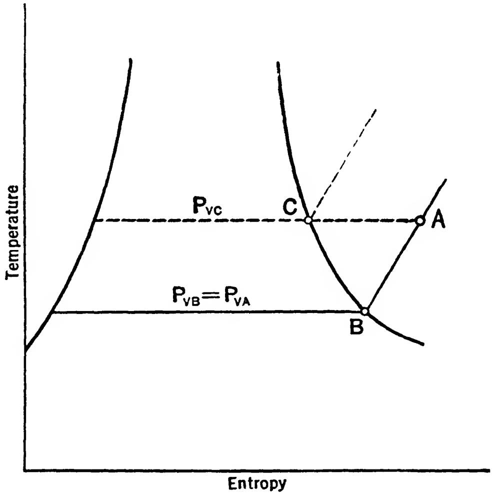

FIG. 78. *T-S* Diagram for Air-Water Vapor Mixtures.

Figure 78 represents, to T-S coordinates, the condition of water vapor in atmospheric air under ordinary conditions (point A). The vapor is superheated at some pressure p_v and some dry bulb temperature $t_a = t_c = t_{\text{mixture}}$. If the air-water vapor mixture were cooled from *A* to *B* the vapor pressure would still be p_v, but the temperature t_b would now be t_{mixture}. At point *B*, when the vapor present is in the dry saturated state the air-water vapor mixture is said to be saturated. At any other point, such as A, the mixture is said to have a relative humidity. Relative humidity (Φ) is defined as either the ratio of the actual partial pressure of the water vapor in the air, p_{va}, to the saturation pressure at the dry bulb temperature, p_{vc}, or the ratio of the actual density (d_a) to the density of saturated vapor at the dry bulb temperature (d_c).

Carrier's[1] equation for the relation between vapor pressures and wet and dry bulb temperatures may also be used in the following form to obtain the actual partial pressures of water vapor in air.

$$p_v = p_v' - \frac{(B - p_v')(t - t')}{2800 - 1.3t}. \qquad (223)$$

In equation (223) p_v is the actual partial pressure of the vapor (in Hg), p_v' is the saturation pressure at wet bulb temperature (in Hg), B is the barometric pressure (in Hg), t = dry bulb temperature (deg fahr), t' = wet bulb temperature (deg fahr).

Air conditioning processes for air-water vapor mixtures may be studied by reference to **Figs. 79** and **80.** When such a mixture is heated at constant total pressure the condition of vapor and of gases changes from that represented by point A to the new condition represented by A'. No new method of calculation is introduced, the properties at A' are determined in the same manner as those at A. However, if the mixture is cooled at constant total pressure from point A new features are presented. If the process continues until the vapor is saturated (point B, **Fig. 79**) the temperature t_b is termed the dew point. The relative humidity at B is 100 per cent and any further cooling will result in the precipitation of some of the vapor as liquid. At B the wet and dry bulb and mixture temperatures are all the same, and the partial vapor pressure is the pressure for saturated vapor at temperature t_b. If cooling at constant total pressure be continued below the dew point temperature, moisture will be precipitated. The partial pressure of the vapor will decrease (**Fig. 79**) and that of the gases will rise (**Fig. 80**). If the initial quantity of water vapor per pound of dry air be considered, the process of continued cooling may be thought of as following the line BD' (**Fig. 79**). In practice, however, the more important properties are those con-

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[1] "Rational Psychrometric Formulae," W. H. Carrier, *Trans. A.S.M.E.* Vol. 33, 1911.

cerned with the saturated vapor remaining mixed with the gases at t_d. Such vapor has the saturation properties found in steam tables at temperatures t_d. Furthermore, the air-water vapor mixture at t_d, when t_d is a temperature below the dew point for the mixture, is saturated, i.e., it has 100 per cent relative humidity.

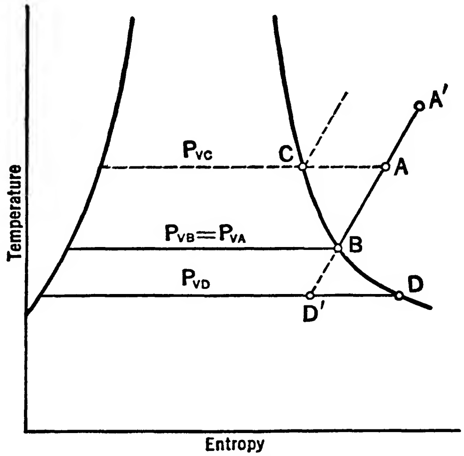

FIG. 79. Vapor in Air-Water Vapor Mixture Processes.

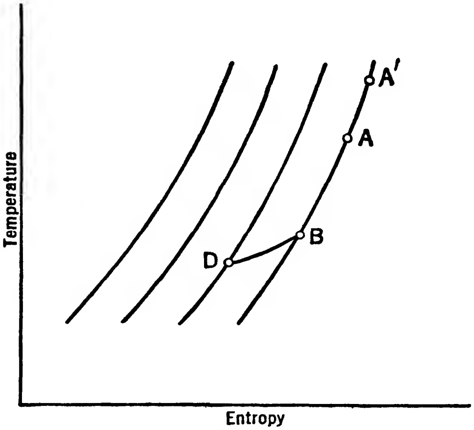

FIG. 80. Gases in Air-Water Vapor Mixture Processes.

The amount of vapor in atmospheric air is usually calculated on the basis of pounds of vapor per pound of dry air. Since the density of the vapor $d_{v\,\text{actual}}$ = relative humidity $\times\, d_v$ for saturated vapor at the dry bulb temperature, and since $d_v = 1/V_v$, $d_{v\,\text{actual}} = \Phi \div v_c$ at the dry bulb temperature. When **Figs. 79** and **80** represent the mixture conditions, then the weight of water vapor per pound of dry air may be written,

$$\frac{v_{\text{air}} \text{ per lb at } t_a \times \Phi}{V_{\text{vapor}} \text{ at } t_c}. \tag{224}$$

The enthalpy of an air-water vapor mixture may be calculated as follows:[2]

$$h = c_{pa}(t - 0) + Mh_s = 0.24(t - 0) + Mh_s \tag{225}$$

h = enthalpy of the mixture, t = dry bulb temperature deg fahr, M = weight of vapor per pound dry air, h_s = enthalpy of vapor in the mixture. The quantity h_s, derived from the properties of vapor in steam tables, is generally used as $h_s = 1059.2 + 0.45\,t$. Thus:

$$h = 0.24(t - 0) + M(1059.2 + 0.45\,t) \tag{226}$$

Calculations for mixtures of air and water vapor are usually made with the aid of a psychrometric chart (Plate 2 in the Appendix).

PROBLEMS

1. If 200 cu ft of free air per min (sea level) are compressed isothermally and then delivered into a receiver, the internal pressure of which is 102.9 psi abs, find the theoretical horsepower required.

2. What will be the net work done in foot-pounds per stroke by an air compressor displacing 3 cu ft per stroke and compressing air from atmospheric pressure to a pressure of 75 psi abs? (Isothermal compression.)

3. What horsepower will be needed to compress adiabatically 1500 cu ft of free air per minute to a pressure of 58.8 psi abs, when n equals 1.4?

4. A compressed-air motor without clearance takes air at a condition of 200 psi (gage) and operates under a cut-off at one-fourth stroke. What is the work in foot-pounds that can be obtained per cubic foot of compressed air, assuming a free air pressure of 14.5 psi abs and n equal to 1.41?

5. Find the theoretical horsepower developed by 3 cu ft of air per minute having a pressure of 200 psi abs, if it is admitted and expanded in an air engine with one-fourth cut-off. The value of n is 1.2. (Neglect clearance.)

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[2] A.S.H.V.E. *Guide*.

6. Compare the work required to compress 1 lb of air from atmospheric pressure and a temperature of 60 deg fahr to a pressure of 200 psi abs in (a) a one-stage and (b) a two-stage compressor. Assume $n = 1.25$.

7. Air enters a single-stage compressor at 80 deg fahr and 14.5 psi abs, and leaves at 145 psi abs after compression, according to the equation $PV^{1.33} = C$. If 30 lb per min are handled, calculate the horsepower of the compressor. Sketch a P-V diagram.

8. What is the saving in horsepower when 1000 cfm of air is compressed from 14.0 psi abs to 460 psi abs by a three-stage compressor instead of single-stage compression, assuming compression in all the cylinders of both compressors to follow the line $PV^{1.3} = C$, and the air is cooled between the cylinders to the initial temperature.

9. A double-acting single-stage air compressor, without clearance, with a bore of 15 in. and a stroke of 18 in. turns at 270 rpm. The air is compressed from 14.0 psi abs to 112 psi abs. When the driving motor develops 230 hp, calculate the ratio of the theoretical to the actual hp ($n = 1.33$).

10. A compressed-air motor, without clearance, takes in air at 180 psi abs and operates at a cut-off of $\frac{1}{3}$ of the stroke. Calculate the work done per cycle per cu ft of compressed air, if the value of n for the expansion is 1.3, and the exhaust pressure is 16 psi abs.

11. How many hp will be required to compress 65 lb of air per min from 15 psi abs and 75 deg fahr to 300 psi abs, when the compression process is isothermal and the mechanical efficiency of the compressor is 85 per cent?

12. Liquid NH_3 at 160 psi abs is throttled through the expansion valve to a pressure of 40 psi abs.

(a) Draw a T-S diagram to illustrate the process.
(b) Find the temperature and quality below the expansion line.

13. Liquid NH_3 at 82.6 deg fahr is throttled through the expansion valve in a refrigeration cycle to a pressure of 10.0 psi abs. Determine the temperature and quality below the expansion line.

14. Dry saturated NH_3 at 30 psi abs is compressed isentropically to 180 psi abs.

(a) If 10 lb of NH_3 are compressed per minute, compute the theoretical horsepower required to compress the ammonia.
(b) Draw a T-S diagram for the refrigeration cycle and show by an area the heat absorbed in the evaporator.

15. Liquid SO_2 at 70 deg fahr is throttled in an expansion valve to 10.35 psi abs and flows through an evaporator from which it leaves at 10.35 psi abs and 100 per cent quality.

(a) Determine the quality after the expansion valve.
(b) If 10 lb of SO_2 are circulated per minute, compute refrigerating effect in tons.

16. Twenty pounds of NH_3 are compressed per minute to 180 psi abs and 200 deg fahr after which the ammonia is condensed at constant pressure of 180

psi abs and sub-cooled to 78.8 deg fahr. Cooling water enters the condenser at 60 deg fahr and leaves at 70 deg fahr for water, C = 1.0.

(a) Determine the weight of cooling water circulated per minute.
(b) Draw a *T-S* diagram of the refrigeration cycle and indicate by an area the heat rejected to the condenser.

17. Ten pounds of NH_3 leave the evaporator per minute at 30 psi abs and 100 per cent quality and are compressed isentropically to 160 psi abs.

(a) Compute the horsepower required to compress the ammonia.
(b) Draw a *T-S* diagram of the refrigeration cycle and show by areas the heat absorbed in the refrigerator and heat rejected to the condenser.

18. If the temperature of atmospheric air is 90 deg fahr when the barometer is 14.7 psi abs and the relative humidity is 50 per cent, what is the partial pressure of the dry gas?

19. Atmospheric air at 90 deg fahr (dry bulb) and 75 deg fahr (wet bulb) temperature is cooled at constant total pressure to 50 deg fahr (bar 29.92 in. Hg). Find the initial relative humidity of the mixture, the partial pressures, and the dew point.

20. From the data of problem 19, find the pounds of water vapor condensed per pound of dry air during the cooling process from $t_a = 90$ deg fahr to $t_d = 50$ deg fahr.

21. From the data of the previous problem, find the enthalpy of the mixture in Btu per lb of dry air at the initial conditions.

APPENDIX

TABLE I

SPECIFIC HEAT OF GASES AND VAPORS

(Taken from Smithsonian Physical Tables)

Substance	Range of Temp.° C.	c_p	Authority	$\frac{c_p}{c_v}$	Authority	Calculated c_v
Air..........	−30–10	0.23771	Regnault			
	0–100	0.23741	"			
	0–200	0.23751	"			
	20–100	0.2389	Wiedemann			
	Mean	0.23788	....	1.4066	Various	0.1691
Alcohol, (ethyl)	108–220	0.4534	Regnault	1.136	Jaeger, Neyreneuf	0.3991
Alcohol, (methyl)	101–223	0.4580	Regnault			
Ammonia. .	23–100	0.5202	Wiedemann			
	27–200	0.5356	"			
	24–216	0.5125	Regnault			
	Mean	0.5228	. ...	1.31	Cazin, Wüllner	0.3991
Benzene....	34–115	0.2990	Wiedemann			
	35–180	0.3325	"			
	116–218	0.3754	Regnault			
Carbon Dioxide	−28–77	0.1843	Regnault			
	15–100	0.2025	"			
	11–214	0.2169	"			
	Mean	0.2012	....	1.300	Wüllner Röntgen,	0.1548
Carbon Monoxide	23–99	0.2425	Wiedemann			
	26–198	0.2426	"	1.403	Cazin, Wüllner	0.1729
Ether.......	69–224	0.4797	Regnault			
	27–189	0.4618	Wiedemann			
	25–111	0.4280	"			
	Mean	0.4565	....	1.029	Müller	0.4436
Hydrogen...	−28–9	3.3996	Regnault			
	12–198	3.4090	"			
	21–100	3.4100	Wiedemann			
	Mean	3.4062	....	1.410	Cazin	2.419
Nitrogen	0–200	0.2438	Regnault	1.410	Cazin	0.1729
Sulphur Dioxide	16–202	0.1544	Regnault	1.26	Cazin Müller	0.1225
Water......	128–217	0.4805	Regnault			
	100–125	0.3787	Gray, Macfarlane,			
	Mean	0.4296	....	1.300	Various	0.3305

Table II

DENSITY OF GASES

Referred to standard atmospheric pressure and temperature of 32 deg fahr
(Taken from Smithsonian Physical Tables)

Gas	Specific gravity	Grams per cubic centimeter	Pounds per cubic foot
Air	1.000	0.001293	0.08071
Ammonia	0.597	0.000770	0.04807
Carbon dioxide	1.529	0.001974	0.12323
Carbon monoxide	0.967	0.001234	0.07704
Coal gas	0.320–0.740	0.000414–0.000957	0.02583–0.05973
Hydrogen	0.0696	0.000090	0.00562
Hydrogen sulphide	1.191	0.001476	0.09214
Marsh gas	0.559	0.000727	0.04538
Nitrogen	0.972	0.001257	0.07847
Oxygen	1.105	0.001430	0.08927
Sulphur dioxide	2.247	0.002785	0.17386

TABLE III

DRY SATURATED STEAM: PRESSURE TABLE *

Abs Press., Lb Sq In.	Temp., F	Specific Volume		Enthalpy			Entropy			Internal Energy		Abs Press., Lb Sq In.
		Sat. Liquid	Sat. Vapor	Sat. Liquid	Evap	Sat. Vapor	Sat. Liquid	Evap	Sat. Vapor	Sat. Liquid	Sat. Vapor	
p	t	v_f	v_g	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g	u_f	u_g	p
1.0	101.74	0.01614	333.6	69.70	1036.3	1106.0	0.1326	1.8456	1.9782	69.70	1044.3	1.0
2.0	126.08	0.01623	173.73	93.99	1022.2	1116.2	0.1749	1.7451	1.9200	93.98	1051.9	2.0
3.0	141.48	0.01630	118.71	109.37	1013.2	1122.6	0.2008	1.6855	1.8863	109.36	1056.7	3.0
4.0	152.97	0.01636	90.63	120.86	1006.4	1127.3	0.2198	1.6427	1.8625	120.85	1060.2	4.0
5.0	162.24	0.01640	73.52	130.13	1001.0	1131.1	0.2347	1.6094	1.8441	130.12	1063.1	5.0
6.0	170.06	0.01645	61.98	137.96	996.2	1134.2	0.2472	1.5820	1.8292	137.94	1065.4	6.0
7.0	176.85	0.01649	53.64	144.76	992.1	1136.9	0.2581	1.5586	1.8167	144.74	1067.4	7.0
8.0	182.86	0.01653	47.34	150.79	988.5	1139.3	0.2674	1.5383	1.8057	150.77	1069.2	8.0
9.0	188.28	0.01656	42.40	156.22	985.2	1141.4	0.2759	1.5203	1.7962	156.19	1070.8	9.0
10	193.21	0.01659	38.42	161.17	982.1	1143.3	0.2835	1.5041	1.7876	161.14	1072.2	10
14.696	212.00	0.01672	26.80	180.07	970.3	1150.4	0.3120	1.4446	1.7566	180.02	1077.5	14.696
15	213.03	0.01672	26.29	181.11	969.7	1150.8	0.3135	1.4415	1.7549	181.06	1077.8	15
20	227.96	0.01683	20.089	196.16	960.1	1156.3	0.3356	1.3962	1.7319	196.10	1081.9	20
25	240.07	0.01692	16.303	208.42	952.1	1160.6	0.3533	1.3606	1.7139	208.34	1085.1	25
30	250.33	0.01701	13.746	218.82	945.3	1164.1	0.3680	1.3313	1.6993	218.73	1087.8	30
35	259.28	0.01708	11.898	227.91	939.2	1167.1	0.3807	1.3063	1.6870	227.80	1090.1	35
40	267.25	0.01715	10.498	236.03	933.7	1169.7	0.3919	1.2844	1.6763	235.90	1092.0	40
45	274.44	0.01721	9.401	243.36	928.6	1172.0	0.4019	1.2650	1.6669	243.22	1093.7	45
50	281.01	0.01727	8.515	250.09	924.0	1174.1	0.4110	1.2474	1.6585	249.93	1095.3	50
55	287.07	0.01732	7.787	256.30	919.6	1175.9	0.4193	1.2316	1.6509	256.12	1096.7	55
60	292.71	0.01738	7.175	262.09	915.5	1177.6	0.4270	1.2168	1.6438	261.90	1097.9	60
65	297.97	0.01743	6.655	267.50	911.6	1179.1	0.4342	1.2032	1.6374	267.29	1099.1	65
70	302.92	0.01748	6.206	272.61	907.9	1180.6	0.4409	1.1906	1.6315	272.38	1100.2	70
75	307.60	0.01753	5.816	277.43	904.5	1181.9	0.4472	1.1787	1.6259	277.19	1101.2	75
80	312.03	0.01757	5.472	282.02	901.1	1183.1	0.4531	1.1676	1.6207	281.76	1102.1	80
85	316.25	0.01761	5.168	286.39	897.8	1184.2	0.4587	1.1571	1.6158	286.11	1102.9	85
90	320.27	0.01766	4.896	290.56	894.7	1185.3	0.4641	1.1471	1.6112	290.27	1103.7	90
95	324.12	0.01770	4.652	294.56	891.7	1186.2	0.4692	1.1376	1.6068	294.25	1104.5	95
100	327.81	0.01774	4.432	298.40	888.8	1187.2	0.4740	1.1286	1.6026	298.08	1105.2	100
110	334.77	0.01782	4.049	305.66	883.2	1188.9	0.4832	1.1117	1.5948	305.30	1106.5	110

120	341.25	0.01789	3.728	312.44	877.9	1190.4	0.4916	1.0962	1.5878	312.05	1107.6	**120**
130	347.32	0.01796	3.455	318.81	872.9	1191.7	0.4995	1.0817	1.5812	318.38	1108.6	**130**
140	353.02	0.01802	3.220	324.82	868.2	1193.0	0.5069	1.0682	1.5751	324.35	1109.6	**140**
150	358.42	0.01809	3.015	330.51	863.6	1194.1	0.5138	1.0556	1.5694	330.01	1110.5	**150**
160	363.53	0.01815	2.834	335.93	859.2	1195.1	0.5204	1.0436	1.5640	335.39	1111.2	**160**
170	368.41	0.01822	2.675	341.09	854.9	1196.0	0.5266	1.0324	1.5590	340.52	1111.9	**170**
180	373.06	0.01827	2.532	346.03	850.8	1196.9	0.5325	1.0217	1.5542	345.42	1112.5	**180**
190	377.51	0.01833	2.404	350.79	846.8	1197.6	0.5381	1.0116	1.5497	350.15	1113.1	**190**
200	381.79	0.01839	2.288	355.36	843.0	1198.4	0.5435	1.0018	1.5453	354.68	1113.7	**200**
250	400.95	0.01865	1.8438	376.00	825.1	1201.1	0.5675	0.9588	1.5263	375.14	1115.8	**250**
300	417.33	0.01890	1.5433	393.84	809.0	1202.8	0.5879	0.9225	1.5104	392.79	1117.1	**300**
350	431.72	0.01913	1.3260	409.69	794.2	1203.9	0.6056	0.8910	1.4966	408.45	1118.0	**350**
400	444.59	0.0193	1.1613	424.0	780.5	1204.5	0.6214	0.8630	1.4844	422.6	1118.5	**400**
450	456.28	0.0195	1.0320	437.2	767.4	1204.6	0.6356	0.8378	1.4734	435.5	1118.7	**450**
500	467.01	0.0197	0.9278	449.4	755.0	1204.4	0.6487	0.8147	1.4634	447.6	1118.6	**500**
550	476.94	0.0199	0.8424	460.8	743.1	1203.9	0.6608	0.7934	1.4542	458.8	1118.2	**550**
600	486.21	0.0201	0.7698	471.6	731.6	1203.2	0.6720	0.7734	1.4454	469.4	1117.7	**600**
650	494.90	0.0203	0.7083	481.8	720.5	1202.3	0.6826	0.7548	1.4374	479.4	1117.1	**650**
700	503.10	0.0205	0.6554	491.5	709.7	1201.2	0.6925	0.7371	1.4296	488.8	1116.3	**700**
750	510.86	0.0207	0.6092	500.8	699.2	1200.0	0.7019	0.7204	1.4223	598.0	1115.4	**750**
800	518.23	0.0209	0.5687	509.7	688.9	1198.6	0.7108	0.7045	1.4153	506.6	1114.4	**800**
850	525.26	0.0210	0.5327	518.3	678.8	1197.1	0.7194	0.6891	1.4085	515.0	1113.3	**850**
900	531.98	0.0212	0.5006	526.6	668.8	1195.4	0.7275	0.6744	1.4020	523.1	1112.1	**900**
950	538.43	0.0214	0.4717	534.6	659.1	1193.7	0.7355	0.6602	1.3957	530.9	1110.8	**950**
1000	544.61	0.0216	0.4456	542.4	649.4	1191.8	0.7430	0.6467	1.3897	538.4	1109.4	**1000**
1100	556.31	0.0220	0.4001	557.4	630.4	1187.8	0.7575	0.6205	1.3780	552.9	1106.4	**1100**
1200	567.22	0.0223	0.3619	571.7	611.7	1183.4	0.7711	0.5956	1.3667	566.7	1103.0	**1200**
1300	577.46	0.0227	0.3293	585.4	593.2	1178.6	0.7840	0.5719	1.3559	580.0	1099.4	**1300**
1400	587.10	0.0231	0.3012	598.7	574.7	1173.4	0.7963	0.5491	1.3454	592.7	1095.4	**1400**
1500	596.23	0.0235	0.2765	611.6	556.3	1167.9	0.8082	0.5269	1.3351	605.1	1091.2	**1500**
2000	635.82	0.0257	0.1878	671.7	463.4	1135.1	0.8619	0.4230	1.2849	662.2	1065.6	**2000**
2500	668.13	0.0287	0.1307	730.6	360.5	1091.1	0.9126	0.3197	1.2322	717.3	1030.6	**2500**
3000	695.36	0.0346	0.0858	802.5	217.8	1020.3	0.9731	0.1885	1.1615	783.4	972.7	**3000**
3206.2	705.40	0.0503	0.0503	902.7	0	902.7	1.0580	0	1.0580	872.9	872.9	**3206.2**

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* Abridged from "Thermodynamic Properties of Steam" by Joseph H. Keenan and Frederick G. Keyes. Copyright, 1937, by Joseph H. Keenan and Frederick G. Keyes.
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TABLE IV

DRY SATURATED STEAM: TEMPERATURE TABLE *

Temp., F	Abs Press., Lb Sq In.	Specific Volume			Enthalpy			Entropy			Temp., F
		Sat. Liquid	Evap.	Sat. Vapor	Sat. Liquid	Evap.	Sat. Vapor	Sat. Liquid	Evap	Sat. Vapor	
t	p	v_f	v_{fg}	v_g	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g	t
32	0.08854	0.01602	3306	3306	0.00	1075.8	1075.8	0.0000	2.1877	2.1877	32
35	0.09995	0.01602	2947	2947	3.02	1074.1	1077.1	0.0061	2.1709	2.1770	35
40	0.12170	0.01602	2444	2444	8.05	1071.3	1079.3	0.0162	2.1435	2.1597	40
45	0.14752	0.01602	2036.4	2036.4	13.06	1068.4	1081.5	0.0262	2.1167	2.1429	45
50	0.17811	0.01603	1703.2	1703.2	18.07	1065.6	1083.7	0.0361	2.0903	2.1264	50
60	0.2563	0.01604	1206.6	1206.7	28.06	1059.9	1088.0	0.0555	2.0393	2.0948	60
70	0.3631	0.01606	867.8	867.9	38.04	1054.3	1092.3	0.0745	1.9902	2.0647	70
80	0.5069	0.01608	633.1	633.1	48.02	1048.6	1096.6	0.0932	1.9428	2.0360	80
90	0.6982	0.01610	468.0	468.0	57.99	1042.9	1100.9	0.1115	1.8972	2.0087	90
100	0.9492	0.01613	350.3	350.4	67.97	1037.2	1105.2	0.1295	1.8531	1.9826	100
110	1.2748	0.01617	265.3	265.4	77.94	1031.6	1109.5	0.1471	1.8106	1.9577	110
120	1.6924	0.01620	203.25	203.27	87.92	1025.8	1113.7	0.1645	1.7694	1.9339	120
130	2.2225	0.01625	157.32	157.34	97.90	1020.0	1117.9	0.1816	1.7296	1.9112	130
140	2.8886	0.01629	122.99	123.01	107.89	1014.1	1122.0	0.1984	1.6910	1.8894	140
150	3.718	0.01634	97.06	97.07	117.89	1008.2	1126.1	0.2149	1.6537	1.8685	150
160	4.741	0.01639	77.27	77.29	127.89	1002.3	1130.2	0.2311	1.6174	1.8485	160
170	5.992	0.01645	62.04	62.06	137.90	996.3	1134.2	0.2472	1.5822	1.8293	170
180	7.510	0.01651	50.21	50.23	147.92	990.2	1138.1	0.2630	1.5480	1.8109	180
190	9.339	0.01657	40.94	40.96	157.95	984.1	1142.0	0.2785	1.5147	1.7932	190
200	11.526	0.01663	33.62	33.64	167.99	977.9	1145.9	0.2938	1.4824	1.7762	200
210	14.123	0.01670	27.80	27.82	178.05	971.6	1149.7	0.3090	1.4508	1.7598	210
212	14.696	0.01672	26.78	26.80	180.07	970.3	1150.4	0.3120	1.4446	1.7566	212
220	17.186	0.01677	23.13	23.15	188.13	965.2	1153.4	0.3239	1.4201	1.7440	220
230	20.780	0.01684	19.365	19.382	198.23	958.8	1157.0	0.3387	1.3901	1.7288	230
240	24.969	0.01692	16.306	16.323	208.34	952.2	1160.5	0.3531	1.3609	1.7140	240
250	29.825	0.01700	13.804	13.821	216.48	945.5	1164.0	0.3675	1.3323	1.6998	250
260	35.429	0.01709	11.746	11.763	228.64	938.7	1167.3	0.3817	1.3043	1.6860	260
270	41.858	0.01717	10.044	10.061	238.84	931.8	1170.6	0.3958	1.2769	1.6727	270
280	49.203	0.01726	8.628	8.645	249.06	924.7	1173.8	0.4096	1.2501	1.6597	280
290	57.556	0.01735	7.444	7.461	259.31	917.5	1176.8	0.4234	1.2238	1.6472	290

300	67.013	0.01745	6.449	6.466	269.59	910.1	1179.7	0.4369	1.1980	1.6350	**300**
310	77.68	0.01755	5.609	5.626	279.92	902.6	1182.5	0.4504	1.1727	1.6231	**310**
320	89.66	0.01765	4.896	4.914	290.28	894.9	1185.2	0.4637	1.1478	1.6115	**320**
330	103.06	0.01776	4.289	4.307	300.68	887.0	1187.7	0.4769	1.1233	1.6002	**330**
340	118.01	0.01787	3.770	3.788	311.13	879.0	1190.1	0.4900	1.0992	1.5891	**340**
350	134.63	0.01799	3.324	3.342	321.63	870.7	1192.3	0.5029	1.0754	1.5783	**350**
360	153.04	0.01811	2.939	2.957	332.18	862.2	1194.4	0.5158	1.0519	1.5677	**360**
370	173.37	0.01823	2.606	2.625	342.79	853.5	1196.3	0.5286	1.0287	1.5573	**370**
380	195.77	0.01836	2.317	2.335	353.45	844.6	1198.1	0.5413	1.0059	1.5471	**380**
390	220.37	0.01850	2.0651	2.0836	364.17	835.4	1199.6	0.5539	0.9832	1.5371	**390**
400	247.31	0.01864	1.8447	1.8633	374.97	826.0	1201.0	0.5664	0.9608	1.5272	**400**
410	276.75	0.01878	1.6512	1.6700	385.83	816.3	1202.1	0.5788	0.9386	1.5174	**410**
420	308.83	0.01894	1.4811	1.5000	396.77	806.3	1203.1	0.5912	0.9166	1.5078	**420**
430	343.72	0.01910	1.3308	1.3499	407.79	796.0	1203.8	0.6035	0.8947	1.4982	**430**
440	381.59	0.01926	1.1979	1.2171	418.90	785.4	1204.3	0.6158	0.8730	1.4887	**440**
450	422.6	0.0194	1.0799	1.0993	430.1	774.5	1204.6	0.6280	0.8513	1.4793	**450**
460	466.9	0.0196	0.9748	0.9944	441.4	763.2	1204.6	0.6402	0.8298	1.4700	**460**
470	514.7	0.0198	0.8811	0.9009	452.8	751.5	1204.3	0.6523	0.8083	1.4606	**470**
480	566.1	0.0200	0.7972	0.8172	464.4	739.4	1203.7	0.6645	0.7868	1.4513	**480**
490	621.4	0.0202	0.7221	0.7423	476.0	726.8	1202.8	0.6766	0.7653	1.4419	**490**
500	680.8	0.0204	0.6545	0.6749	487.8	713.9	1201.7	0.6887	0.7438	1.4325	**500**
520	812.4	0.0209	0.5385	0.5594	511.9	686.4	1198.2	0.7130	0.7006	1.4136	**520**
540	962.5	0.0215	0.4434	0.4649	536.6	656.6	1193.2	0.7374	0.6568	1.3942	**540**
560	1133.1	0.0221	0.3647	0.3868	562.2	624.2	1186.4	0.7621	0.6121	1.3742	**560**
580	1325.8	0.0228	0.2989	0.3217	588.9	588.4	1177.3	0.7872	0.5659	1.3532	**580**
600	1542.9	0.0236	0.2432	0.2668	617.0	548.5	1165.5	0.8131	0.5176	1.3307	**600**
620	1786.6	0.0247	0.1955	0.2201	646.7	503.6	1150.3	0.8398	0.4664	1.3062	**620**
640	2059.7	0.0260	0.1538	0.1798	678.6	452.0	1130.5	0.8679	0.4110	1.2789	**640**
660	2365.4	0.0278	0.1165	0.1442	714.2	390.2	1104.4	0.8987	0.3485	1.2472	**660**
680	2708.1	0.0305	0.0810	0.1115	757.3	309.9	1067.2	0.9351	0.2719	1.2071	**680**
700	3093.7	0.0369	0.0392	0.0761	823.3	172.1	995.4	0.9905	0.1484	1.1389	**700**
705.4	3206.2	0.0503	0	0.0503	902.7	0	902.7	1.0580	0	1.0580	**705.4**

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Table V

Properties of Superheated Steam *

Abs Press., Lb Sq In. (Sat. Temp.)		Temperature—Degrees Fahrenheit												
		200	300	400	500	600	700	800	900	1000	1100	1200	1400	1600
	v.......	392.6	452.3	512.0	571.6	631.2	690.8	750.4	809.9	869.5	929.1	988.7	1107.8	1227.0
1	h.......	1150.4	1195.8	1241.7	1288.3	1335.7	1383.8	1432.8	1482.7	1533.5	1585.2	1637.7	1745.7	1857.5
(101.74)	s.......	2.0512	2.1153	2.1720	2.2233	2.2702	2.3137	2.3542	2.3923	2.4283	2.4625	2.4952	2.5566	2.6137
	v.......	78.16	90.25	102.26	114.22	126.16	138.10	150.03	161.95	173.87	185.79	197.71	221.6	245.4
5	h.......	1148.8	1195.0	1241.2	1288.0	1335.4	1383.6	1432.7	1482.6	1533.4	1585.1	1637.7	1745.7	1857.4
(162.24)	s.......	1,8718	1.9370	1.9942	2.0456	2.0927	2.1361	2.1767	2.2148	2.2509	2.2851	2.3178	2.3792	2.4363
	v.......	38.85	45.00	51.04	57.05	63.03	69.01	74.98	80.95	86.92	92.88	98.84	110.77	122.69
10	h.......	1146.6	1193.9	1240.6	1287.5	1335.1	1383.4	1432.5	1482.4	1533.2	1585.0	1637.6	1745.6	1857.3
(193.21)	s.......	1.7927	1.8595	1.9172	1.9689	2.0160	2.0596	2.1002	2.1383	2.1744	2.2086	2.2413	2.3028	2.3598
	v.......	......	30.53	34.68	38.78	42.86	46.94	51.00	55.07	59.13	63.19	67.25	75.37	83.48
14.696	h.......	......	1192.8	1239.9	1287.1	1334.8	1383.2	1432.3	1482.3	1533.1	1584.8	1637.5	1745.5	1857,3
(212.00)	s.......	......	1.8160	1.8743	1.9261	1.9734	2.0170	2.0576	2.0958	2.1319	2.1662	2.1989	2.2603	2.3174
	v.......	......	22.36	25.43	28.46	31.47	34.47	37.46	40.45	43.44	46.42	49.41	55.37	61.34
20	h.......	......	1191.6	1239.2	1286.6	1334.4	1382.9	1432.1	1482.1	1533.0	1584.7	1637.4	1745.4	1857.2
(227.96)	s.......	......	1.7808	1.8396	1.8918	1.9392	1.9829	2.0235	2.0618	2.0978	2.1321	2.1648	2.2263	2.2834
	v.......	......	11.040	12.628	14.168	15.688	17.198	18.702	20.20	21.70	23.20	24.69	27.68	30.66
40	h.......	......	1186.8	1236.5	1284.8	1333.1	1381.9	1431.3	1481.4	1532.4	1584.3	1637.0	1745.1	1857.0
(267.25)	s.......	......	1.6994	1.7608	1.8140	1.8619	1.9058	1.9467	1.9850	2.0212	2.0555	2.0883	2.1498	2.2069
	v.......	......	7.259	8.357	9.403	10.427	11.441	12.449	13.452	14.454	15.453	16.451	18.446	20.44
60	h.......	......	1181.6	1233.6	1283.0	1331.8	1380.9	1430.5	1480.8	1531.9	1583.8	1636.6	1744.8	1856.7
(292.71)	s.......	......	1.6492	1.7135	1.7678	1.8162	1.8605	1.9015	1.9400	1.9762	2.0106	2.0434	2.1049	2.1621
	v.......	......	......	6.220	7.020	7.797	8.562	9.322	10.077	10.830	11.582	12.332	13.830	15.325
80	h.......	......	......	1230.7	1281.1	1330.5	1379.9	1429.7	1480.1	1531.3	1583.4	1636.2	1744.5	1856.5
(312.03)	s.......	......	......	1.6791	1.7346	1.7836	1.8281	1.8694	1.9079	1.9442	1.9787	2.0115	2.0731	2.1303
	v.......	......	......	4.937	5.589	6.218	6.835	7.446	8.052	8.656	9.259	9.860	11.060	12.258
100	h.......	......	......	1227.6	1279.1	1329.1	1378.9	1428.9	1479.5	1530.8	1582.9	1635.7	1744.2	1856.2
(327.81)	s.......	......	......	1.6518	1.7085	1.7581	1.8029	1.8443	1.8829	1.9193	1.9538	1.9867	2.0484	2.1056
	v.......	......	......	4.081	4.636	5.165	5.683	6.195	6.702	7.207	7.710	8.212	9.214	10.213
120	h.......	......	......	1224.4	1277.2	1327.7	1377.8	1428.1	1478.8	1530.2	1582.4	1635.3	1743.9	1856.0
(341.25)	s.......	......	......	1.6287	1.6869	1.7370	1.7822	1.8237	1.8625	1.8990	1.9335	1.9664	2.0281	2.0854

140 (353.02)	*v*	......	......	3.468	3.954	4.413	4.861	5.301	5.738	6.172	6.604	7.035	7.895	8.752
	h	......	......	1221.1	1275.2	1326.4	1376.8	1427.3	1478.2	1529.7	1581.9	1634.9	1743.5	1855.7
	s	......	......	1.6087	1.6683	1.7190	1.7645	1.8063	1.8451	1.8817	1.9163	1.9493	2.0110	2.0683
160 (363.53)	*v*	......	......	3.008	3.443	3.849	4.244	4.631	5.015	5.396	5.775	6.152	6.906	7.656
	h	......	......	1217.6	1273.1	1325.0	1375.7	1426.4	1477.5	1529.1	1581.4	1634.5	1743.2	1855.5
	s	......	......	1.5908	1.6519	1.7033	1.7491	1.7911	1.8301	1.8667	1.9014	1.9344	1.9962	2.0535
180 (373.06)	*v*	......	......	2.649	3.044	3.411	3.764	4.110	4.452	4.792	5.129	5.466	6.136	6.804
	h	......	......	1214.0	1271.0	1323.5	1374.7	1425.6	1476.8	1528.6	1581.0	1634.1	1742.9	1855.2
	s	......	......	1.5745	1.6373	1.6894	1.7355	1.7776	1.8167	1.8534	1.8882	1.9212	1.9831	2.0404
200 (381.79)	*v*	......	......	2.361	2.726	3.060	3.380	3.693	4.002	4.309	4.613	4.917	5.521	6.123
	h	......	......	1210.3	1268.9	1322.1	1373.6	1424.8	1476.2	1528.0	1580.5	1633.7	1742.6	1855.0
	s	......	......	1.5594	1.6240	1.6767	1.7232	1.7655	1.8048	1.8415	1.8763	1.9094	1.9713	2.0287
220 (389.86)	*v*	......	......	2.125	2.465	2.772	3.066	3.352	3.634	3.913	4.191	4.467	5.017	5.565
	h	......	......	1206.5	1266.7	1320.7	1372.6	1424.0	1475.5	1527.5	1580.0	1633.3	1742.3	1854.7
	s	......	......	1.5453	1.6117	1.6652	1.7120	1.7545	1.7939	1.8308	1.8656	1.8987	1.9607	2.0181
240 (397.37)	*v*	......	......	1.9276	2.247	2.533	2.804	3.068	3.327	3.584	3.839	4.093	4.597	5.100
	h	......	......	1202.5	1264.5	1319.2	1371 5	1423.2	1474.8	1526.9	1579.6	1632.9	1742.0	1854.5
	s	......	......	1.5319	1.6003	1.6546	1.7017	1.7444	1.7839	1.8209	1.8558	1.8889	1.9510	2.0084
260 (404.42)	*v*	......	......	......	2.063	2.330	2.582	2.827	3.067	3.305	3.541	3.776	4.242	4.707
	h	......	......	......	1262.3	1317.7	1370.4	1422.3	1474.2	1526.3	1579.1	1632.5	1741.7	1854.2
	s	......	......	......	1.5897	1.6447	1.6922	1.7352	1.7748	1.8118	1.8467	1.8799	1.9420	1.9995
280 (411.05)	*v*	......	......	......	1.9047	2.156	2.392	2.621	2.845	3.066	3.286	3.504	3.938	4.370
	h	......	......	......	1260.0	1316.2	1369.4	1421.5	1473.5	1525.8	1578.6	1632.1	1741.4	1854.0
	s	......	......	......	1.5796	1.6354	1.6834	1.7265	1.7662	1.8033	1.8383	1.8716	1.9337	1.9912
300 417.33)	*v*	......	......	......	1.7675	2.005	2.227	2.442	2.652	2.859	3.065	3.269	3.674	4.078
	h	......	......	......	1257.6	1314.7	1368.3	1420.6	1472.8	1525.2	1578.1	1631.7	1741.0	1853.7
	s	......	......	......	1.5701	1.6268	1.6751	1.7184	1.7582	1.7954	1.8305	1.8638	1.9260	1.9835
350 (431.72)	*v*	......	......	......	1.4923	1.7036	1.8980	2.084	2.266	2.445	2.622	2.798	3.147	3.493
	h	......	......	......	1251.5	1310.9	1365.5	1418.5	1471.1	1523.8	1577.0	1630.7	1740.3	1853.1
	s	......	......	......	1.5481	1.6070	1.6563	1.7002	1.7403	1.7777	1.8130	1.8463	1.9086	1.9663
400 (444.59)	*v*	......	......	......	1.2851	1.4770	1.6508	1.8161	1.9767	2.134	2.290	2.445	2.751	3.055
	h	......	......	......	1245.1	1306.9	1362.7	1416.4	1469.4	1522.4	1575.8	1629.6	1739.5	1852.5
	s	......	......	......	1.5281	1.5894	1.6398	1.6842	1.7247	1.7623	1.7977	1.8311	1.8936	1.9513

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TABLE V — *Continued*

PROPERTIES OF SUPERHEATED STEAM *

Abs Press. Lb Sq In. (Sat. Temp.)		Temperature—Degrees Fahrenheit													
		500	**550**	**600**	**620**	**640**	**660**	**680**	**700**	**800**	**900**	**1000**	**1200**	**1400**	**1600**
	v.......	1.1231	1.2155	1.3005	1.3332	1.3652	1.3967	1.4278	1.4584	1.6074	1.7516	1.8928	2.170	2.443	2.714
450	h.......	1238.4	1272.0	1302.8	1314.6	1326.2	1337.5	1348.8	1359.9	1414.3	1467.7	1521.0	1628.6	1738.7	1851.9
(456.28)	s.......	1.5095	1.5437	1.5735	1.5845	1.5951	1.6054	1.6153	1.6250	1.6699	1.7108	1.7486	1.8177	1.8803	1.9381
	v.......	0.9927	1.0800	1.1591	1.1893	1.2188	1.2478	1.2763	1.3044	1.4405	1.5715	1.6996	1.9504	2.197	2.442
500	h.......	1231.3	1266.8	1298.6	1310.7	1322.6	1334.2	1345.7	1357.0	1412.1	1466.0	1519.6	1627.6	1737.9	1851.3
(467.01)	s.......	1.4919	1.5280	1.5588	1.5701	1.5810	1.5915	1.6016	1.6115	1.6571	1.6982	1.7363	1.8056	1.8683	1.9262
	v.......	0.8852	0.9686	1.0431	1.0714	1.0989	1.1259	1.1523	1.1783	1.3038	1.4241	1.5414	1.7706	1.9957	2.219
550	h.......	1223.7	1261.2	1294.3	1306.8	1318.9	1330.8	1342.5	1354.0	1409.9	1464.3	1518.2	1626.6	1737.1	1850.6
(476.94)	s.......	1.4751	1.5131	1.5451	1.5568	1.5680	1.5787	1.5890	1.5991	1.6452	1.6868	1.7250	1.7946	1.8575	1.9155
	v.......	0.7947	0.8753	0.9463	0.9729	0.9988	1.0241	1.0489	1.0732	1.1899	1.3013	1.4096	1.6208	1.8279	2.033
600	h.......	1215.7	1255.5	1289.9	1302.7	1315.2	1327.4	1339.3	1351.1	1407.7	1462.5	1516.7	1625.5	1736.3	1850.0
(486.21)	s.......	1.4586	1.4990	1.5323	1.5443	1.5558	1.5667	1.5773	1.5875	1.6343	1.6762	1.7147	1.7846	1.8476	1.9056
	v.......	......	0.7277	0.7934	0.8177	0.8411	0.8639	0.8860	0.9077	1.0108	1.1082	1.2024	1.3853	1.5641	1.7405
700	h.......	......	1243.2	1280.6	1294.3	1307.5	1320.3	1332.8	1345.0	1403.2	1459.0	1513.9	1623.5	1734.8	1848.8
(503.10)	s.......	......	1.4722	1.5084	1.5212	1.5333	1.5449	1.5559	1.5665	1.6147	1.6573	1.6963	1.7666	1.8299	1.8881
	v.......	......	0.6154	0.6779	0.7006	0.7223	0.7433	0.7635	0.7833	0.8763	0.9633	1.0470	1.2088	1.3662	1.5214
800	h.......	......	1229.8	1270.7	1285.4	1299.4	1312.9	1325.9	1338.6	1398.6	1455.4	1511.0	1621.4	1733.2	1847.5
(518.23)	s.......	......	1.4467	1.4863	1.5000	1.5129	1.5250	1.5366	1.5476	1.5972	1.6407	1.6801	1.7510	1.8146	1.8729
	v.......	......	0.5264	0.5873	0.6089	0.6294	0.6491	0.6680	0.6863	0.7716	0.8506	0.9262	1.0714	1.2124	1.3509
900	h.......	......	1215.0	1260.1	1275.9	1290.9	1305.1	1318.8	1332.1	1393.9	1451.8	1508.1	1619.3	1731.6	1846.3
(531.98)	s.......	......	1.4216	1.4653	1.4800	1.4938	1.5066	1.5187	1.5303	1.5814	1.6257	1.6656	1.7371	1.8009	1.8595
	v.......	......	0.4533	0.5140	0.5350	0.5546	0.5733	0.5912	0.6084	0.6878	0.7604	0.8294	0.9615	1.0893	1.2146
1000	h.......	......	1198.3	1248.8	1265.9	1281.9	1297.0	1311.4	1325.3	1389.2	1448.2	1505.1	1617.3	1730.0	1845.0
(544.61)	s.......	......	1.3961	1.4450	1.4610	1.4757	1.4893	1.5021	1.5141	1.5670	1.6121	1.6525	1.7245	1.7886	1.8474
	v.......	......	......	0.4532	0.4738	0.4929	0.5110	0.5281	0.5445	0.6191	0.6866	0.7503	0.8716	0.9885	1.1031
1100	h.......	......	......	1236.7	1255.3	1272.4	1288.5	1303.7	1318.3	1384.3	1444.5	1502.2	1615.2	1728.4	1843.8
(556.31)	s.......	......	......	1.4251	1.4425	1.4583	1.4728	1.4862	1.4989	1.5535	1.5995	1.6405	1.7130	1.7775	1.8363
	v.......	......	......	0.4016	0.4222	0.4410	0.4586	0.4752	0.4909	0.5617	0.6250	0.6843	0.7967	0.9046	1.0101
1200	h.......	......	......	1223.5	1243.9	1262.4	1279.6	1295.7	1311.0	1379.3	1440.7	1499.2	1613.1	1726.9	1842.5
(567.22)	s.......	......	......	1.4052	1.4243	1.4413	1.4568	1.4710	1.4843	1.5409	1.5879	1.6293	1.7025	1.7672	1.8263

	v	......	......	0.3174	0.3390	0.3580	0.3753	0.3912	0.4062	0.4714	0.5281	0.5805	0.6789	0.7727	0.8640
1400	h	......	......	1193.0	1218.4	1240.4	1260.3	1278.5	1295.5	1369.1	1433.1	1493.2	1608.9	1723.7	1840.0
(587.10)	s	......	......	1.3639	1.3877	1.4079	1.4258	1.4419	1.4567	1.5177	1.5666	1.6093	1.6836	1.7489	1.8083
	v	......	......	......	0.2733	0.2936	0.3112	0.3271	0.3417	0.4034	0.4553	0.5027	0.5906	0.6738	0.7545
1600	h	......	......	......	1187.8	1215.2	1238.7	1259.6	1278.7	1358.4	1425.3	1487.0	1604.6	1720.5	1837.5
(604.90)	s	......	......	......	1.3489	1.3741	1.3952	1.4137	1.4303	1.4964	1.5476	1.5914	1.6669	1.7328	1.7926
	v	......	......	......	......	0.2407	0.2597	0.2760	0.2907	0.3502	0.3986	0.4421	0.5218	0.5968	0.6693
1800	h	......	......	......	......	1185.1	1214.0	1238.5	1260.3	1347.2	1417.4	1480.8	1600.4	1717.3	1835.0
(621.03)	s	......	......	......	......	1.3377	1.3638	1.3855	1.4044	1.4765	1.5301	1.5752	1.6520	1.7185	1.7786
	v	......	......	......	......	0.1936	0.2161	0.2337	0.2489	0.3074	0.3532	0.3935	0.4668	0.5352	0.6011
2000	h	......	......	......	......	1145.6	1184.9	1214.8	1240.0	1335.5	1409.2	1474.5	1596.1	1714.1	1832.5
(635.82)	s	......	......	......	......	1.2945	1.3300	1.3564	1.3783	1.4576	1.5139	1.5603	1.6384	1.7055	1.7660
	v	......	......	......	......	......	......	0.1484	0.1686	0.2294	0.2710	0.3061	0.3678	0.4244	0.4784
2500	h	......	......	......	......	......	......	1132.3	1176.8	1303.6	1387.8	1458.4	1585.3	1706.1	1826.2
(668.13)	s	......	......	......	......	......	......	1.2687	1.3073	1.4127	1.4772	1.5273	1.6088	1.6775	1.7389
	v	......	......	......	......	......	......	......	0.0984	0.1760	0.2159	0.2476	0.3018	0.3505	0.3966
3000	h	......	......	......	......	......	......	......	1060.7	1267.2	1365.0	1441.8	1574.3	1698.0	1819.9
(695.36)	s	......	......	......	......	......	......	......	1.1966	1.3690	1.4439	1.4984	1.5837	1.6546	1.7163
	v	......	......	......	......	......	......	......	......	0.1583	0.1981	0.2288	0.2806	0.3267	0.3703
3206.2	h	......	......	......	......	......	......	......	......	1250.5	1355.2	1434.7	1569.8	1694.6	1817.2
(705.40)	s	......	......	......	......	......	......	......	......	1.3508	1.4309	1.4874	1.5742	1.6452	1.7080
	v	......	......	......	......	......	......	......	0.0306	0.1364	0.1762	0.2058	0.2546	0.2977	0.3381
3500	h	......	......	......	......	......	......	......	780.5	1224.9	1340.7	1424.5	1563.3	1689.8	1813.6
	s	......	......	......	......	......	......	......	0.9515	1.3241	1.4127	1.4723	1.5615	1.6336	1.6968
	v	......	......	......	......	......	......	......	0.0287	0.1052	0.1462	0.1743	0.2192	0.2581	0.2943
4000	h	......	......	......	......	......	......	......	763.8	1174.8	1314.4	1406.8	1552.1	1681.7	1807.2
	s	......	......	......	......	......	......	......	0.9347	1.2757	1.3827	1.4482	1.5417	1.6154	1.6795
	v	......	......	......	......	......	......	......	0.0276	0.0798	0.1226	0.1500	0.1917	0.2273	0.2602
4500	h	......	......	......	......	......	......	......	753.5	1113.9	1286.5	1388.4	1540.8	1673.5	1800.9
	s	......	......	......	......	......	......	......	0.9235	1.2204	1.3529	1.4253	1.5235	1.5990	1.6640
	v	......	......	......	......	......	......	......	0.0268	0.0593	0.1036	0.1303	0.1696	0.2027	0.2329
5000	h	......	......	......	......	......	......	......	746.4	1047.1	1256.5	1369.5	1529.5	1665.3	1794.5
	s	......	......	......	......	......	......	......	0.9152	1.1622	1.3231	1.4034	1.5066	1.5839	1.6499
	v	......	......	......	......	......	......	......	0.0262	0.0463	0.0880	0.1143	0.1516	0.1825	0.2106
5500	h	......	......	......	......	......	......	......	741.3	985.0	1224.1	1349.3	1518.2	1657.0	1788.1
	s	......	......	......	......	......	......	......	0.9090	1.1093	1.2930	1.3821	1.4908	1.5699	1.6369

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* Abridged from "Thermodynamic Properties of Steam," by Joseph H. Keenan and Frederick G. Keyes, Copyright, 1937, by Joseph H. Keenan and Frederick G. Keyes.
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TABLE VI

PROPERTIES OF SATURATED SULPHUR DIOXIDE

(Condensed from tables prepared by United States Bureau of Standards)

Temperature, °F.	Pressure (abs) lb per sq in.	Specific volume, cu ft per lb	Enthalpy, Btu per lb			Entropy		
			Liquid	Evaporation	Vapor	Liquid	Evaporation	Vapor
t	p	v_g	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g
−40	3.136	22.42	0.00	178.61	178.61	0.0000000	0.42562	0.42562
−20	5.883	12.43	5.98	175.09	181.07	0.01366	0.39826	0.41192
0	10.35	7.280	12.44	170.63	183.07	0.02795	0.37122	0.39917
10	13.42	5.682	15.80	168.07	183.87	0.03519	0.35787	0.39306
20	17.18	4.487	19.20	165.32	184.52	0.04241	0.34466	0.38707
30	21.70	3.581	22.64	162.38	185.02	0.04956	0.33163	0.38119
40	27.10	2.887	26.12	159.25	185.37	0.05668	0.31873	0.37541
50	33.45	2.348	29.61	155.95	185.56	0.06370	0.30599	0.36969
60	40.93	1.926	33.10	152.49	185.59	0.07060	0.29345	0.36405
70	49.62	1.590	36.58	148.88	185.46	0.07736	0.28110	0.35846
80	59.68	1.321	40.05	145.12	185.17	0.08399	0.26897	0.35291
90	71.25	1.104	43.50	141.22	184.72	0.09038	0.25693	0.34731
100	84.52	0.9262	46.90	137.20	184.10	0.09657	0.24516	0.34173

TABLE VII

PROPERTIES OF SATURATED CARBON DIOXIDE

(Condensed from tables prepared by United States Bureau of Standards)

Temperature, °F.	Pressure (abs) lb per sq in.	Specific volume, cu ft per lb	Enthalpy, Btu per lb			Entropy		
			Liquid	Evaporation	Vapor	Liquid	Evaporation	Vapor
t	p	v_g	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g
−40	145.87	0.607	−38.5	136.5	98.0	−0.0850	0.2400	0.1550
−20.0	215.02	0.409	−27.7	127.5	99.8	−0.0619	0.2277	0.1658
0.0	305.76	0.280	−16.7	117.5	100.8	−0.0381	0.2173	0.1792
10.0	360.4	0.235	−11.2	112.2	101.0	−0.0264	0.2124	0.1860
20.0	421.8	0.199	−5.5	106.3	100.8	−0.0144	0.2071	0.1927
30.0	490.6	0.168	0.4	99.7	100.1	−0.0021	0.2012	0.1991
40.0	567.3	0.143	6.6	91.8	98.4	0.0099	0.1934	0.2033
50.0	652.7	0.122	12.9	83.2	96.1	0.0220	0.1852	0.2072
60.0	747.4	0.093	19.4	74.0	93.4	0.0345	0.1767	0.2112
70.0	852.5	0.077	26.6	62.7	89.3	0.0482	0.1665	0.2147
80.0	969.3	0.063	35.6	44.0	79.6	0.0649	0.1464	0.2113
88.0	1072.1	Critical point						

TABLE VIII

PROPERTIES OF SATURATED AMMONIA

(Condensed from tables prepared by United States Bureau of Standards)

Pressure (abs) lb per sq in.	Temperature, °F.	Specific volume, cu ft per lb	Enthalpy, Btu per lb			Entropy		
			Liquid	Evaporation	Vapor	Liquid	Evaporation	Vapor
p	t	v_g	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g
10.0	−41.34	25.81	− 1.4	598.5	597.1	−0.0034	1.4310	1.4276
20.0	−16.64	13.50	25.0	581.2	606.2	0.0578	1.3122	1.3700
30	− 0.57	9.236	42.3	569.3	611.6	0.0962	1.2402	1.3364
40	11.66	7.047	55.6	559.8	615.4	0.1246	1.1879	1.3125
50	21.67	5.710	66.5	551.7	618.2	0.1475	1.1464	1.2939
60	30.21	4.805	75.9	544.6	620.5	0.1668	1.1119	1.2787
70	37.70	4.151	84.2	538.2	622.4	0.1835	1.0823	1.2658
80	44.40	3.655	91.7	532.3	624.0	0.1982	1.0563	1.2545
90	50.47	3.266	98.4	526.9	625.3	0.2115	1.0330	1.2445
100	56.05	2.952	104.7	521.8	626.5	0.2237	1.0119	1.2356
110	61.21	2.693	110.5	517.0	627.5	0.2348	0.9927	1.2275
120	66.02	2.476	116.0	512.4	628.4	0.2452	0.9749	1.2201
130	70.53	2.291	121.1	508.1	629.2	0.2548	0.9584	1.2132
140	74.79	2.132	126.0	503.9	629.9	0.2638	0.9430	1.2068
150	78.81	1.994	130.6	499.9	630.5	0.2724	0.9285	1.2009
160	82.64	1.872	135.0	496.1	631.1	0.2804	0.9148	1.1952
170	86.29	1.764	139.3	492.3	631.6	0.2881	0.9019	1.1900
180	89.78	1.667	143.3	488.7	623.0	0.2954	0.8896	1.1850
190	93.13	1.581	147.2	485.2	632.4	0.3024	0.8778	1.1802
200	96.34	1.502	150.9	481.8	632.7	0.3090	0.8666	1.1756
225	103.87	1.336	159.7	473.6	633.3	0.3246	0.8405	1.1651
250	110.80	1.202	168.0	465.8	633.8	0.3388	0.8167	1.1555
275	117.22	1.091	175.6	458.4	634.0	0.3519	0.7947	1.1466
300	123.21	0.999	182.9	451.1	634.0	0.3642	0.7741	1.1383

Table IX

PROPERTIES OF SUPERHEATED AMMONIA

(Condensed from tables prepared by United States Bureau of Standards)

v = specific volume in cubic feet per pound; H = enthalpy in Btu per pound; S = total entropy.

Pressure (abs) lb per sq in. (p)	Temperature of saturated vapor, °F. (t)		Temperatures, degrees Fahrenheit								
			20	40	60	80	100	150	200	250	300
20	−16.6	v	14.78	15.45	16.12	16.78	17.43	19.05	20.66	....	....
		H	626.4	637.0	647.5	658.0	668.5	694.7	721.2	....	....
		S	1.4138	1.4356	1.4562	1.4760	1.4950	1.5399	1.5817	....	....
30	−0.6	v	9.731	10.20	10.65	11.10	11.55	12.65	13.73	14.81	....
		H	623.5	634.6	645.5	656.2	666.9	693.5	720.3	747.5	....
		S	1.3618	1.3845	1.4059	1.4261	1.4456	1.4911	1.5334	1.5727	....
40	11.7	v	7.203	7.568	7.922	8.268	8.609	9.444	10.27	11.08	....
		H	620.4	632.1	643.4	654.4	665.3	962.3	719.4	746.8	....
		S	1.3231	1.3470	1.3692	1.3900	1.4098	1.4561	1.4987	1.5382	....
50	21.7	v	....	5.988	6.280	6.564	6.843	7.521	8.185	8.840	9.489
		H	...	629.5	641.2	652.6	663.7	691.1	718.5	746.1	774.0
		S	....	1.3169	1.3399	1.3613	1.3816	1.4286	1.4716	1.5118	1.5500
60	30.2	v	...	4.933	5.184	5.428	5.665	6.239	6.798	7.348	7.892
		H	....	626.8	639.0	650.7	662.1	689.9	717.5	745.3	773.3
		S	....	1.2913	1.3152	1.3373	1.3581	1.4058	1.4493	1.4897	1.5281
70	37.7	v	....	4.177	4.401	4.615	4.822	5.323	5.807	6.282	6.750
		H	....	623.9	636.6	648.7	660.4	688.7	716.6	744.5	772.7
		S	....	1.2688	1.2937	1.3166	1.3378	1.3863	1.4302	1.4710	1.5095
80	44.4	v	...	....	3.812	4.005	4.190	4.635	5.063	5.481	5.894
		H	....	....	634.3	646.7	658.7	687.5	715.6	743.7	772.1
		S	....	....	1.2745	1.2981	1.3199	1.3692	1.4136	1.4547	1.4933
90	50.5	v	....	....	3.353	3.529	3.698	4.100	4.484	4.859	5.288
		H	....	....	631.8	644.7	657.0	686.3	714.7	743.0	771.5
		S	....	....	1.2571	1.2814	1.3038	1.3539	1.3988	1.4401	1.4789
100	56.1	v	....	....	2.985	3.149	3.304	3.672	4.021	4.361	4.695
		H	....	....	629.3	642.6	655.2	685.0	713.7	742.2	770.8
		S	....	....	1.2409	1.2661	1.2891	1.3401	1.3854	1.4271	1.4660

TABLE IX. — *Continued*

Pressure (abs) lb per sq in. (*p*)	Temperature of saturated vapor, °F. (*t*)	Temperatures, degrees Fahrenheit								
		20	40	60	80	100	150	200	250	300
110	61.2 *v*	....	.. .	....	2 837	2 981	3.321	3.642	3.954	4 259
	H	....	....	....	640 5	653 4	683 7	712.8	741 5	770.2
	S	....	....	....	1.2519	1.2755	1.3274	1 3732	1 4151	1.4543
120	66.0 *v*	....	. .	.	2 576	2 712	3.029	3 326	3 614	3.895
	H	....	....	. ..	638 3	651.6	682 5	711 8	740.7	769.6
	S	....	.. .	..	1 2386	1.2628	1 3157	1 3620	1 4042	1.4435
130	70 5 *v*	....	.	..	2.355	2 484	2 781	3.059	3.326	3.587
	H	....	.	.. .	636 0	649 7	681 2	710.9	739 9	769.0
	S	....		. ..	1 2260	1.2509	1.3048	1 3516	1 3941	1.4336
140	74.8 *v*	.. .	....	....	2 166	2.288	2 569	2 830	3 080	3.323
	H	....	....	. ..	633 8	647 8	679 9	709 9	739 2	768.3
	S	....	. ..	....	1.2140	1 2396	1.2945	1 3418	1 3846	1.4243
150	78 8 *v*	....	.. .	....	....	2 118	2 385	2 631	2 866	3 095
	H	....	....	....	....	645 9	678 6	708 9	738.4	767.7
	S	.. .	..	..	...	1.2289	1.2849	1.3327	1 3758	1.4157
160	82.6 *v*	....	...	....	....	1 969	2 224	2 457	2 679	2.895
	H	....	....	....	....	643 9	677 2	707.9	737.6	767.1
	S	....	....	..	. .	1.2186	1.2757	1 3240	1.3675	1.4076
180	89 8 *v*	....	....	....	...	1.720	1.955	2.167	2.367	2.561
	H	....	....	...	....	639 9	674 5	705 9	736.1	765.8
	S	. ..	....	. .	....	1.1992	1.2586	1.3081	1.3521	1.3926
200	96 3 *v*	....	....	....	....	1.520	1.740	1 935	2 118	2.295
	H	....	....	....	....	635.6	671 8	703.9	734 5	764.5
	S	....	....	....	....	1.1809	1.2429	1.2935	1.3382	1.3791
220	102.4 *v*	....	....	....	....	....	1.564	1.745	1.914	2.076
	H	....	....	....	....	....	669.0	701.9	732 9	763.2
	S	....	....	....	....	....	1.2281	1 2801	1.3255	1.3668
240	108.1 *v*	. ..	....	....	....	....	1.416	1 587	1 745	1.895
	H	....	....	....	....	....	666 1	699.8	731.3	762.0
	S	....	....	....	....	....	1 2145	1.2677	1.3137	1.3554

TABLE X. — NAPERIAN LOGARITHMS

Reproduced by permission from Goodenough's "Properties of Steam and Ammonia."

$e = 2.7182818$ $\log e = 0.4342945 = M$

	0	1	2	3	4	5	6	7	8	9
1.0	0.0000	0.00995	0.01980	0.02956	0.03922	0.04879	0.05827	0.06766	0.07696	0.08618
1.1	0.09531	0.1044	0.1133	0.1222	0.1310	0.1398	0.1484	0.1570	0.1655	0.1739
1.2	0.1823	0.1906	0.1988	0.2070	0.2151	0.2231	0.2311	0.2390	0.2469	0.2546
1.3	0.2624	0.2700	0.2776	0.2852	0.2927	0.3001	0.3075	0.3148	0.3221	0.3293
1.4	0.3365	0.3436	0.3507	0.3577	0.3646	0.3716	0.3784	0.3853	0.3920	0.3988
1.5	0.4055	0.4121	0.4187	0.4253	0.4318	0.4382	0.4447	0.4511	0.4574	0.4637
1.6	0.4700	0.4762	0.4824	0.4886	0.4947	0.5008	0.5068	0.5128	0.5188	0.5247
1.7	0.5306	0.5365	0.5423	0.5481	0.5539	0.5596	0.5653	0.5710	0.5766	0.5822
1.8	0.5878	0.5933	0.5988	0.6043	0.6098	0.6152	0.6206	0.6259	0.6313	0.6366
1.9	0.6418	0.6471	0.6523	0.6575	0.6627	0.6678	0.6729	0.6780	0.6831	0.6881
2.0	0.6931	0.6981	0.7031	0.7080	0.7129	0.7178	0.7227	0.7275	0.7324	0.7372
2.1	0.7419	0.7467	0.7514	0.7561	0.7608	0.7655	0.7701	0.7747	0.7793	0.7839
2.2	0.7884	0.7930	0.7975	0.8020	0.8065	0.8109	0.8154	0.8198	0.8242	0.8286
2.3	0.8329	0.8372	0.8416	0.8459	0.8502	0.8544	0.8587	0.8629	0.8671	0.8713
2.4	0.8755	0.8796	0.8838	0.8879	0.8920	0.8961	0.9002	0.9042	0.9083	0.9123
2.5	0.9163	0.9203	0.9243	0.9282	0.9322	0.9361	0.9400	0.9439	0.9478	0.9517
2.6	0.9555	0.9594	0.9632	0.9670	0.9708	0.9746	0.9783	0.9821	0.9858	0.9895
2.7	0.9933	0.9969	1.0006	1.0043	1.0080	1.0116	1.0152	1.0188	1.0225	1.0260
2.8	1.0296	1.0332	1.0367	1.0403	1.0438	1.0473	1.0508	1.0543	1.0578	1.0613
2.9	1.0647	1.0682	1.0716	1.0750	1.0784	1.0818	1.0852	1.0886	1.0919	1.0953
3.0	1.0986	1.1019	1.1053	1.1086	1.1119	1.1151	1.1184	1.1217	1.1249	1.1282
3.1	1.1314	1.1346	1.1378	1.1410	1.1442	1.1474	1.1506	1.1537	1.1569	1.1600
3.2	1.1632	1.1663	1.1694	1.1725	1.1756	1.1787	1.1817	1.1848	1.1878	1.1909
3.3	1.1939	1.1969	1.2000	1.2030	1.2060	1.2090	1.2119	1.2149	1.2179	1.2208
3.4	1.2238	1.2267	1.2296	1.2326	1.2355	1.2384	1.2413	1.2442	1.2470	1.2499
3.5	1.2528	1.2556	1.2585	1.2613	1.2641	1.2669	1.2698	1.2726	1.2754	1.2782
3.6	1.2809	1.2837	1.2865	1.2892	1.2920	1.2947	1.2975	1.3002	1.3029	1.3056
3.7	1.3083	1.3110	1.3137	1.3164	1.3191	1.3218	1.3244	1.3271	1.3297	1.3324
3.8	1.3350	1.3376	1.3403	1.3429	1.3455	1.3481	1.3507	1.3533	1.3558	1.3584
3.9	1.3610	1.3635	1.3661	1.3686	1.3712	1.3737	1.3762	1.3788	1.3813	1.3838
4.0	1.3863	1.3888	1.3913	1.3938	1.3962	1.3987	1.4012	1.4036	1.4061	1.4085
4.1	1.4110	1.4134	1.4159	1.4183	1.4207	1.4231	1.4255	1.4279	1.4303	1.4327
4.2	1.4351	1.4375	1.4398	1.4422	1.4446	1.4469	1.4493	1.4516	1.4540	1.4563
4.3	1.4586	1.4609	1.4633	1.4656	1.4679	1.4702	1.4725	1.4748	1.4770	1.4793
4.4	1.4816	1.4839	1.4861	1.4884	1.4907	1.4929	1.4951	1.4974	1.4996	1.5019
4.5	1.5041	1.5063	1.5085	1.5107	1.5129	1.5151	1.5173	1.5195	1.5217	1.5239
4.6	1.5261	1.5282	1.5304	1.5326	1.5347	1.5369	1.5390	1.5412	1.5433	1.5454
4.7	1.5476	1.5497	1.5518	1.5539	1.5560	1.5581	1.5602	1.5623	1.5644	1.5665
4.8	1.5686	1.5707	1.5728	1.5748	1.5769	1.5790	1.5810	1.5831	1.5851	1.5872
4.9	1.5892	1.5913	1.5933	1.5953	1.5974	1.5994	1.6014	1.6034	1.6054	1.6074
5.0	1.6094	1.6114	1.6134	1.6154	1.6174	1.6194	1.6214	1.6233	1.6253	1.6273
5.1	1.6292	1.6312	1.6332	1.6351	1.6371	1.6390	1.6409	1.6429	1.6448	1.6467
5.2	1.6487	1.6506	1.6525	1.6544	1.6563	1.6582	1.6601	1.6620	1.6639	1.6658
5.3	1.6677	1.6696	1.6715	1.6734	1.6752	1.6771	1.6790	1.6808	1.6827	1.6845
5.4	1.6864	1.6882	1.6901	1.6919	1.6938	1.6956	1.6974	1.6993	1.7011	1.7029
5.5	1.7047	1.7066	1.7884	1.7102	1.7120	1.7138	1.7156	1.7174	1.7192	1.7210
5.6	1.7228	1.7246	1.7263	1.7281	1.7299	1.7317	1.7334	1.7352	1.7370	1.7387

TABLE X. — *Continued.* NAPERIAN LOGARITHMS

	0	1	2	3	4	5	6	7	8	9
5.7	1.7405	1.7422	1.7440	1.7457	1.7475	1.7492	1.7509	1.7527	1.7544	1.7561
5.8	1.7579	1.7596	1.7613	1.7630	1.7647	1.7664	1.7681	1.7699	1.7716	1.7733
5.9	1.7750	1.7766	1.7783	1.7800	1.7817	1.7834	1.7851	1.7867	1.7884	1.7901
6.0	1.7918	1.7934	1.7951	1.7967	1.7984	1.8001	1.8017	1.8034	1.8050	1.8066
6.1	1.8083	1.8099	1.8116	1.8132	1.8148	1.8165	1.8181	1.8197	1.8213	1.8229
6.2	1.8245	1.8262	1.8278	1.8294	1.8310	1.8326	1.8342	1.8358	1.8374	1.8390
6.3	1.8405	1.8421	1.8437	1.8453	1.8469	1.8485	1.8500	1.8516	1.8532	1.8547
6.4	1.8563	1.8579	1.8594	1.8610	1.8625	1.8641	1.8656	1.8672	1.8687	1.8703
6.5	1.8718	1.8733	1.8749	1.8764	1.8779	1.8795	1.8810	1.8825	1.8840	1.8856
6.6	1.8871	1.8886	1.8901	1.8916	1.8931	1.8946	1.8961	1.8976	1.8991	1.9006
6.7	1.9021	1.9036	1.9051	1.9066	1.9081	1.9095	1.9110	1.9125	1.9140	1.9155
6.8	1.9169	1.9184	1.9199	1.9213	1.9228	1.9242	1.9257	1.9272	1.9286	1.9301
6.9	1.9315	1.9330	1.9344	1.9359	1.9373	1.9387	1.9402	1.9416	1.9430	1.9445
7.0	1.9459	1.9473	1.9488	1.9502	1.9516	1.9530	1.9544	1.9559	1.9573	1.9587
7.1	1.9601	1.9615	1.9629	1.9643	1.9657	1.9671	1.9685	1.9699	1.9713	1.9727
7.2	1.9741	1.9755	1.9769	1.9782	1.9796	1.9810	1.9824	1.9838	1.9851	1.9865
7.3	1.9879	1.9892	1.9906	1.9920	1.9933	1.9947	1.9961	1.9974	1.9988	2.0001
7.4	2.0015	2.0028	2.0042	2.0055	2.0069	2.0082	2.0096	2.0109	2.0122	2.0136
7.5	2.0149	2.0162	2.0176	2.0189	2.0202	2.0215	2.0229	2.0242	2.0255	2.0268
7.6	2.0281	2.0295	2.0308	2.0321	2.0334	2.0347	2.0360	2.0373	2.0386	2.0399
7.7	2.0412	2.0425	2.0438	2.0451	2.0464	2.0477	2.0490	2.0503	2.0516	2.0528
7.8	2.0541	2.0554	2.0567	2.0580	2.0592	2.0605	2.0618	2.0631	2.0643	2.0656
7.9	2.0668	2.0681	2.0694	2.0707	2.0719	2.0732	2.0744	2.0757	2.0769	2.0782
8.0	2.0794	2.0807	2.0819	2.0832	2.0844	2.0857	2.0869	2.0881	2.0894	2.0906
8.1	2.0919	2.0931	2.0943	2.0956	2.0968	2.0980	2.0992	2.1005	2.1017	2.1029
8.2	2.1041	2.1054	2.1066	2.1078	2.1090	2.1102	2.1114	2.1126	2.1138	2.1150
8.3	2.1163	2.1175	2.1187	2.1199	2.1211	2.1223	2.1235	2.1247	2.1258	2.1270
8.4	2.1282	2.1294	2.1306	2.1318	2.1330	2.1342	2.1353	2.1365	2.1377	2.1389
8.5	2.1401	2.1412	2.1424	2.1436	2.1448	2.1459	2.1471	2.1483	2.1494	2.1506
8.6	2.1518	2.1529	2.1541	2.1552	2.1564	2.1576	2.1587	2.1599	2.1610	2.1622
8.7	2.1633	2.1645	2.1656	2.1668	2.1679	2.1691	2.1702	2.1713	2.1725	2.1736
8.8	2.1748	2.1759	2.1770	2.1782	2.1793	2.1804	2.1815	2.1827	2.1838	2.1849
8.9	2.1861	2.1872	2.1883	2.1894	2.1905	2.1917	2.1928	2.1939	2.1950	2.1961
9.0	2.1972	2.1983	2.1994	2.2006	2.2017	2.2028	2.2039	2.2050	2.2061	2.2072
9.1	2.2083	2.2094	2.2105	2.2116	2.2127	2.2138	2.2148	2.2159	2.2170	2.2181
9.2	2.2192	2.2203	2.2214	2.2225	2.2235	2.2246	2.2257	2.2268	2.2279	2.2289
9.3	2.2300	2.2311	2.2322	2.2332	2.2343	2.2354	2.2364	2.2375	2.2386	2.2396
9.4	2.2407	2.2418	2.2428	2.2439	2.2450	2.2460	2.2471	2.2481	2.2492	2.2502
9.5	2.2513	2.2523	2.2534	2.2544	2.2555	2.2565	2.2576	2.2586	2.2597	2.2607
9.6	2.2618	2.2628	2.2638	2.2649	2.2659	2.2670	2.2680	2.2690	2.2701	2.2711
9.7	2.2721	2.2732	2.2742	2.2752	2.2762	2.2773	2.2783	2.2793	2.2803	2.2814
9.8	2.2824	2.2834	2.2844	2.2854	2.2865	2.2875	2.2885	2.2895	2.2905	2.2915
9.9	2.2925	2.2935	2.2946	2.2956	2.2966	2.2976	2.2986	2.2996	2.3006	2.3016
10.0	2.3026									

TABLE XI. — LOGARITHMS

Reproduced by permission from Goodenough's "Properties of Steam and Ammonia."

Nat. Nos.	0	1	2	3	4	5	6	7	8	9	Proportional Parts.								
											1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	8	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7

TABLE XI. — *Continued.* LOGARITHMS

Nat. Nos.	0	1	2	3	4	5	6	7	8	9	Proportional Parts.								
											1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

TABLE XII

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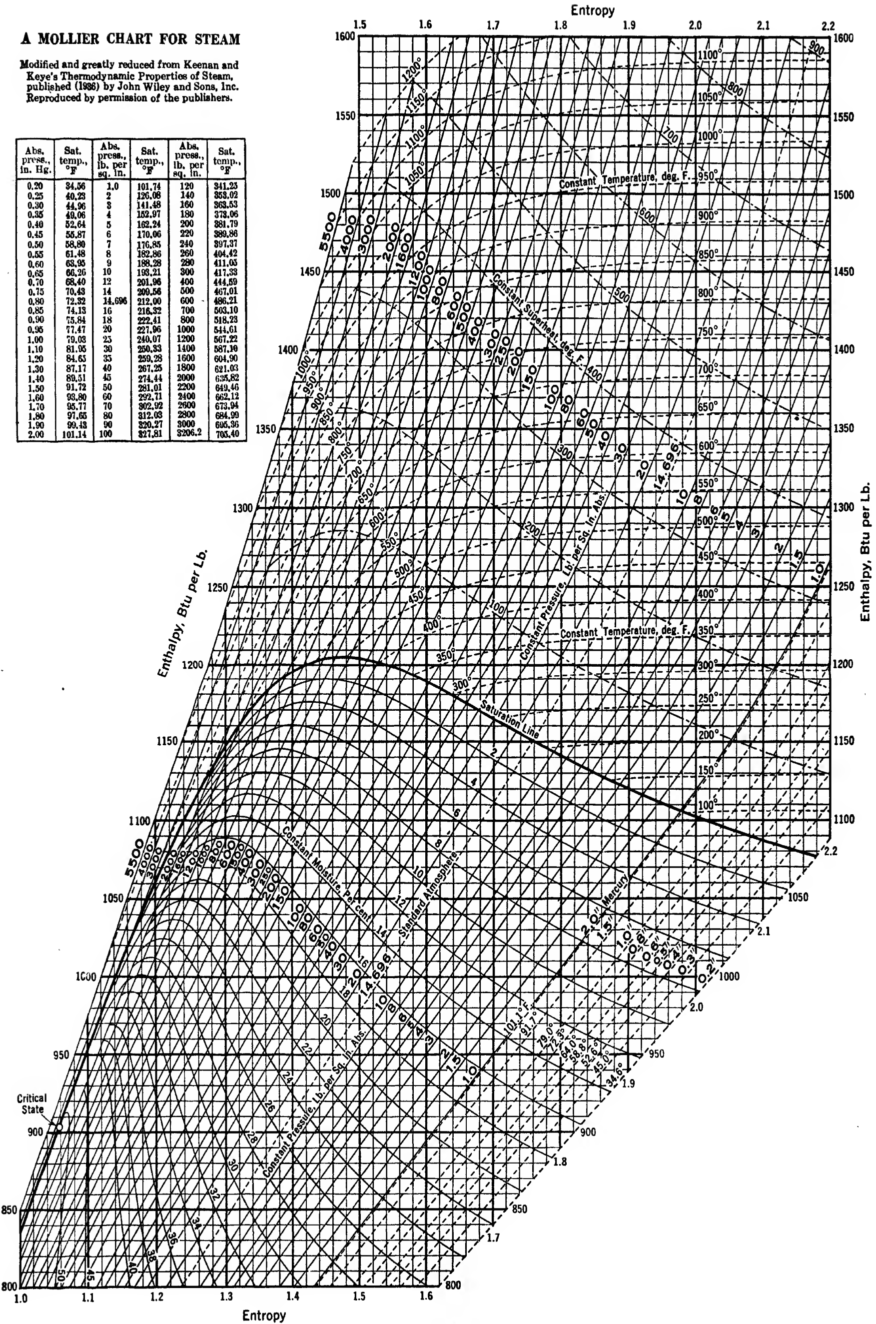
A MOLLIER CHART FOR STEAM

Modified and greatly reduced from Keenan and Keye's Thermodynamic Properties of Steam, published (1936) by John Wiley and Sons, Inc. Reproduced by permission of the publishers.

Abs. press., in. Hg.	Sat. temp., °F	Abs. press., lb. per sq. in.	Sat. temp., °F	Abs. press., lb. per sq. in.	Sat. temp., °F
0.20	34.56	1.0	101.74	120	341.25
0.25	40.23	2	126.08	140	353.02
0.30	44.96	3	141.48	160	363.53
0.35	49.06	4	152.97	180	373.06
0.40	52.64	5	162.24	200	381.79
0.45	55.87	6	170.06	220	389.86
0.50	58.80	7	176.85	240	397.37
0.55	61.48	8	182.86	260	404.42
0.60	63.95	9	188.28	280	411.05
0.65	66.26	10	193.21	300	417.33
0.70	68.40	12	201.96	400	444.59
0.75	70.43	14	209.56	500	467.01
0.80	72.32	14.696	212.00	600	486.21
0.85	74.13	16	216.32	700	503.10
0.90	75.84	18	222.41	800	518.23
0.95	77.47	20	227.96	1000	544.61
1.00	79.03	25	240.07	1200	567.22
1.10	81.95	30	250.33	1400	587.10
1.20	84.63	35	259.28	1600	604.90
1.30	87.17	40	267.25	1800	621.03
1.40	89.51	45	274.44	2000	635.82
1.50	91.72	50	281.01	2200	649.46
1.60	93.80	60	292.71	2400	662.12
1.70	95.77	70	302.92	2600	673.94
1.80	97.65	80	312.03	2800	684.99
1.90	99.43	90	320.27	3000	695.36
2.00	101.14	100	327.81	3206.2	705.40



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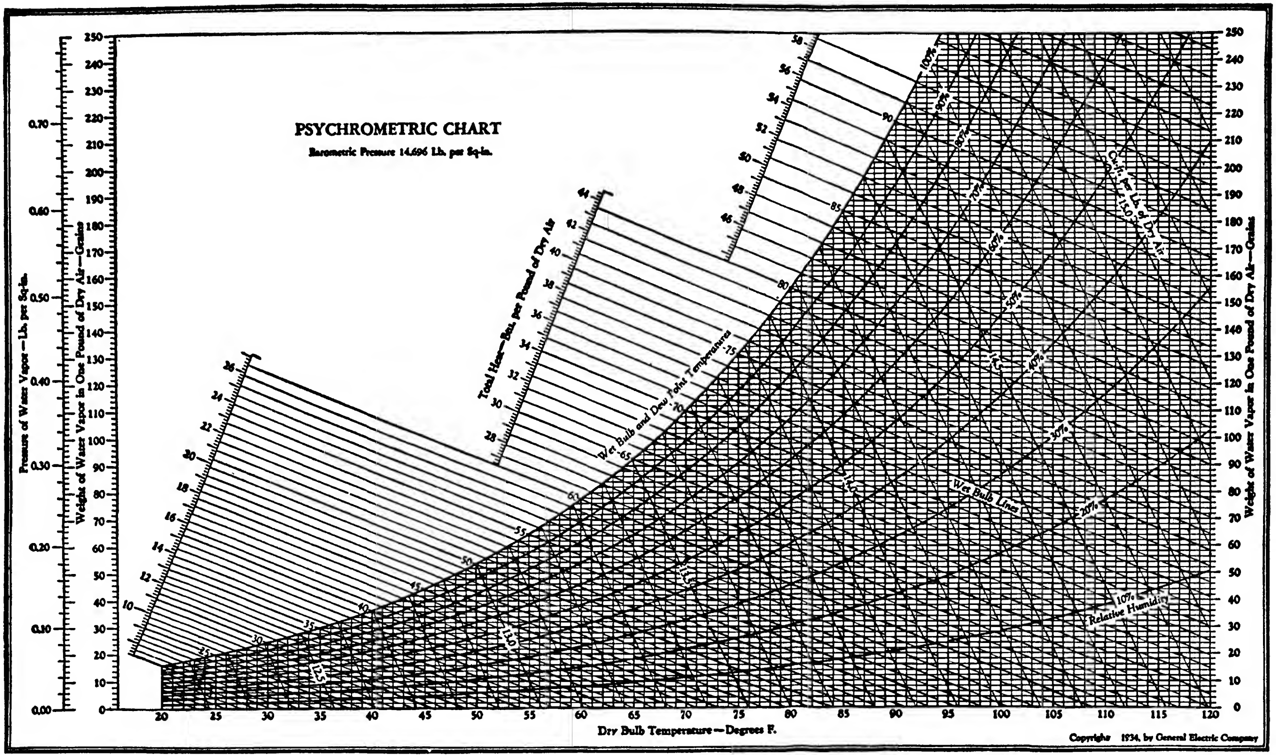

PLATE 2. PSYCHROMETRIC CHART.

Courtesy General Electric Company.

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