R. C. C. DESIGNING MADE EASY

WITH NUMEROUS ILLUSTRATIONS, SOLVED PROBLEMS AND EXAMPLES

BY

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PREFACE

The motto, viz. "R. C. C. design without tears," not only to the average student, but even to the dullest one, has been my constant aim in compiling this text-book. With this end in view I have introduced several novel features in the presentation of the subject matter.

The first is that the explanatory notes which are written in a very clear, lucid manner, are immediately followed by illustrative examples solved step by step and similar practice problems as exercises for the student, at the end of each chapter.

Secondly, an array of long, complicated formulæ forms the nightmare of students, and it has been my experience while examining answer papers that there is, in consequence, a tendency on the part of students to memorise them without remembering the first principles from which they were derived. For this purpose, no formulæ as such are given in the book. All the 90 illustrative examples are solved from the first principles. This principle is observed so strictly that no numbers to equations are given, nor any back references to equations are quoted anywhere in the book.

Thirdly, to set the mind of the student a-thinking, a few important aspects of the subject, which would have ordinarily escaped his attention are prominently brought to his notice in the form of questions and answers at the end of several chapters.

Fourthly, a number of practical hints and tips in many chapters are given, pointing out to the designer, the common mistakes made in construction and how to provide against them, and certain difficulties and how to overcome them.

The book not only covers completely the syllabusses for degree courses of Indian Universities, but more than that an attempt has been made to explain fully the current practice of design of common structures, quoting authority at every step. The contemporary American practice also is shown where the latter differs from the Indian or British practice.

Design in prestressed concrete has made great strides in recent years, and has become an almost common thing on important works, and therefore, though it is not included in the University syllabusses at present, the fundamental principles underlying the design are explained simply and clearly in a special chapter.

Similarly, a chapter on "Plastic Theory" is added at the end. The conventional theory of straight line stress distribution is falling gradually into disrepute, and there are sure indications that the plastic theory based on ultimate loads, and independent of modular ratio may be universally adopted in a not-very-distant future, as it is rational and even simpler than the present standard method. Even at this moment the U. S. S. R. and Brazil have adopted it as the standard theory and the United States of America are fast following suit. Therefore, though a student may for the present earn his living by following the conventional theory, he must know the latest trends in design. A number of designs have therefore been made in that chapter by both the theories for comparison of results.

In order to bring the book in line with the latest researches and development, "Shell Concrete Construction" is also briefly described in the Appendix.

In conclusion I must express my indebtedness to Messrs. N. M. Joshi, B. E., M. S. Prabhavalkar, B. E., and A. Sesha Iyer, B. E. for the valuable help they rendered in checking the mathematical calculations, and to the United Book Corporation, who agreed to offer the book at such a low price for the benefit of students, without stinting on the quality of the paper or printing.

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CHAPTER I

INTRODUCTORY AND GENERAL

DESIGN of R. C. C. structures is based more on the vast experience gained in the past, rather than on any scientific theory backed by mathematics. Our knowledge of what actually happens as regards the stress distribution, particularly in a member forming a part of a rigid framed structure, before failure occurs, is limited even to-day, and changes are gradually taking place, in the current practice and are bound to take place in future as a result of experimental research.

The rules based on the past experience, and laid down in detail for the guidance of the designer, therefore, form the backbone of the current practice in design. These rules were first published in 1915, under the title "Reinforced Concrete Regulations of the London County Council". They were rather very conservative as regards working stresses. This was quite justifiable from the point of safety of structures as the experience then collected was small. As more experience was gained, a Committee, specially appointed for the purpose, of *Building Research Board*, published in 1934 "Code of Practice" which is more scientific and rational. After this, the London County Council revised their old rules in 1938 under the name "*The New By-laws*" and "*The Memorandum*", which though still less scientific, are more influential.

In the present volume the current practice of design is explained entirely with the help of these two very valuable documents, and wherever they differ on small points, the differences are shown. Side by side with this, occasional references have been made to the current American practice, as set forth in A. C. I. (American Concrete Institute's) Code for comparison.

R. C. C. *versus* steel and timber :—The essential difference between a steel or timber structure and an R. C. C. structure is :

(1) Whereas in steel the material is perfectly homogeneous, and in timber more or less so (because of knots,

heartwood, sapwood, etc.), in the case of reinforced concrete the two materials are heterogeneous possessing very widely differing properties.

(2) Whereas in the case of timber separate units such as columns, beams, floor-planks, etc., are framed together, or in the case of steel they are bolted or riveted together, in the case of R. C. C. they are formed into one monolithic continuous whole structure by pouring concrete, with steel running through all the connections and construction joints. This resembles in some respects the modern rigid steel frame with welded joints.

The advantages of R. C. C. construction over other structures are :—

(1) In the monolithic structure if some member is overloaded, the continuity causes re-adjustment of stresses and prevents it from being unduly deformed.

(2) An R. C. C. structure is capable of resisting vibrations, wind-load and earthquake shocks, though it is very difficult to allocate accurately definite loads to the different members for purposes of design.

(3) A combination of steel and concrete makes for economy. For, volume for volume, steel costs about 90 times as much as concrete and for the same cross section steel resists 300 times as much in tension and 30 times as much in compression as concrete. Therefore, though to support a load in tension concrete will cost 7·5 times as much as steel, to support a load in compression it will cost only one-third as much.

(4) R. C. C. is a very good fire-resisting material.

(5) Masonry is strong in compression but very weak in tension. R. C. C. is a sort of masonry which is equally strong both in compression and tension.

Classification of structural members according to stresses:— For purposes of analysis, structural members are divided into three classes :—(1) Those in direct stress. (2) Those in flexural or bending stress. (3) Those with both direct and flexural stresses.

(1) Members in direct stresses may be either in tension or compression. The member taking up direct stress must be straight and the resultant force must act along its axis through the centre of gravity of each cross-section and at right angles to it. An axially loaded column is an instance of a member under direct compressive stress. The king post or king rod in a roof-truss is an instance of a member under direct tension.

(2) A structural member is said to be in bending or under pure flexural stress when the resultant of the normal stresses is a couple. Beams are instances of such members. The flexural stress may be associated with shear stresses acting on the cross sections.

(3) A member is said to be under combined stresses of bending and either direct pull or direct thrust, when the resultant of the normal stresses does not pass through the centre of gravity of the cross section. In other words, the member is under a force at the centre of gravity producing direct stress, and under a couple producing bending about the centre of gravity. Shear forces may also be present. Columns with eccentric loading and arches are instances of such members.

CHAPTER II

MATERIALS AND STRESSES

THE materials employed in R. C. C. are two viz. concrete and steel. Out of these, concrete is the most variable. Its strength depends upon many factors, such as the quality, sizes, grading and proportioning of aggregates, quality and quantity of cement, consistency or the quantity of water used, mixing, placing, consolidation, curing, etc. If anyone of these is neglected, or not properly attended to, the strength is likely to be adversely affected. Strength, density and workability are the three essential requirements of a first class concrete. These could be ensured by using a sufficiently large quantity of cement. But the concrete thus made would be not only uneconomical, but would be open to the danger of excessive shrinkage while hardening and may even crack.

Building By-law No. 14 of the London County Council recognises two grades of concrete, viz. (1) Ordinary grade and (2) High grade, as suitable for R. C. C. work. The Code of Practice gives one more grade, viz. *Special grade* Each of these grades comprises three different mixes. Minimum crushing strengths as shown by tests in cubes at 28 days are prescribed for each grade. The mixes for the grades and the stresses allowed for each are given in the subjoined table. It will be seen from the table that the allowable maximum working stress for flexural compression C is first fixed from the minimum crushing strengths of concrete at 28 days, allowing a factor of safety of 3, and the other stresses are fixed as certain percentages of that. Thus if x is the minimum crushing strength of a particular mix and grade, then the flexural compressive stress, C would be $\frac{x}{3}$; the direct compressive stress would be $0{\cdot}8C$; shear $0{\cdot}1C$ and bond stress $0{\cdot}1C+25$.

TABLE No. 1

CONCRETE MIXES AND THE STRESSES ALLOWED BY L. C. C. BY-LAWS AND CODE OF PRACTICE

Mix	Grade of Concrete	Minimum Crushing Strength at 28 days lbs./in.² x	Permissible Stress lbs./in.²				Modular Ratio according to	
			Flexural Compression $\frac{x}{3}=C$	Direct Compression 0·8C	Shear 0·1C	Bond 0·1C+25	By-laws	Code of Practice
1:2:4 1 Bag Cement 2·5 c.ft. F. A. 5·0 c ft. C.A.	Ordinary	2250	750	600	75	100	15	18
	High	2850	950	760	95	120	15	14
1:1·5:3 1 Bag Cement 1⅞ c.ft. F. A. 3¾ c.ft. C. A.	Ordinary	2550	850	680	85	110	15	16
	High	3300	1100	880	110	135	15	12
1:1:2 1 Bag Cement 1¼ c.ft. F. A. 2½ c.ft. C. A.	Ordinary	2925	975	780	98	123	15	14
	High	3750	1250	1000	125	150	15	11

F. A. = Fine Aggregate; C. A. = Coarse Aggregate.

Note:—There is not much difference between the figures stipulated by the By-laws and Code of Practice as regards the stresses for various mixes in each grade of concrete. However, the By-laws prescribe the same modular ratio viz., 15 for all grades and mixes, based on the moduli of elasticity of 30,000,000 and 2,000,000 lbs./in.² for steel and concrete respectively, whereas the Code of Practice varies it from 11 to 18. The latter is obtained from the following equation:—

$$m \text{ (modular ratio)} = \frac{40{,}000}{\text{crushing strength at 28 days.}}$$

calculated to the nearest whole number, given in the last column of the above table.

Thus for Ordinary Grade 1:2:4 mix

$$m=\frac{40,000}{2250}=17{\cdot}7, \text{ say } 18$$

for High Grade 1:1·5:3 mix

$$m=\frac{40,000}{3300}=12{\cdot}12, \text{ say } 12.$$

Many designers adopt the modular ratio of 18 for ordinary grade (1:2:4 mix of concrete) as recommended by the Code of Practice, but as it is safer to assume it as 15, the latter is adopted throughout in the calculations in this volume unless specially mentioned otherwise. This is in keeping with the current practice with engineers in this country.

Reinforcement:—In great majority of cases, mild steel (0·2 per cent carbon) round bars $\frac{1}{4}$ in. to $1\frac{1}{2}$ in. diameters, satisfying the requirements of B. S. S. No. 15 * is used. Both the By-laws and the Code allow the use of high tension steel satisfying the requirements of B. S. S. No. 785 (1938). Normally the maximum tensile stress allowed is 50 p. c. of the yield point stress; and the maximum compressive stress as mentioned in one of the following two cases:—

Case I:—40 per cent of the yield point stress, when concrete is neglected as in the steel beam theory.

Case II:—Compressive stress in steel = the modular ratio × the stress in the surrounding concrete.

The following table gives the summary of the recommendations of the Code regarding the permissible working stresses in mild steel reinforcement.

* Ultimate tensile strength 28–32 tons/in.^{2} 20 per cent elongation on 8 times the diam. bar, and 24 per cent on 4 times the diam. bar, and standing cold bending to an angle of 180 deg. around a test piece of equal diameter.

TABLE No. 2

PERMISSIBLE WORKING STRESSES IN MILD STEEL REINFORCEMENT

Nature of the Stress	Mild steel complying with B. S. S. No. 15 lbs./in.^{2}	Mild steel complying with B. S. S. 785 with Y. P. stress not less than 44,000 lbs./in.^{2}
[I] *Bending—*		
(a) Tension in longitudinal bars in beams, slabs, columns, subject to bending.	18,000	20,000
(b) Compression in longitudinal bars in beams, slabs, columns subject to bending when the compressive resistance in concrete is taken into account.	Compressive stress in the surrounding concrete × modular ratio, *m*.	$m \times C$
(c) Compression in longitudinal bars in beams when the compressive resistance of concrete is not taken into account.	18,000	20,000
[II] *Direct compression—*		
(a) Compression in longitudinal bars in columns axially loaded.	13,500	15,000
[III] *Shear—*		
Tension in web reinforcement.	18,000	18,000

Note :—The By-laws do not allow more than 18,000 lbs./in.^{2} even in high tensile steel and the maximum stress allowed for II (a) above is 13,500 lbs.

When hard drawn wire complying with B. S. S. No. 165 is used in solid slab (i. e. other than flat slab) the permissible stress according to Code is 25,000 lbs./in.^{2} provided that the area of steel in tension does not exceed one per cent of the effective area of the slab.

Loads :—Live loads, often called "Super-imposed" or simply "Super loads" are divided by the L. C. C. By-laws and the Code of Practice into 8 different classes. The following table gives these loads which are re-arranged to make a reference more convenient.

TABLE No. 3

SCHEDULE OF SUPERIMPOSED LOADS ON SLABS, STAIRCASES, BALCONIES, ETC., BASED ON RECOMMENDATIONS OF BUILDING BY-LAWS AND CODE OF PRACTICE

S. No.		Description	Slabs: Load lb./ft.^{2}	Slabs: Least span or panel width	Slabs: Minimum load distributed over lesser span (ton)	Beams: Load per/ft.^{2} of floor	Beams: Minimum distributed load over lesser span (ton)
		(1)	(2)	(3)	(4)	(5)	(6)
1	Floors	Domestic Rooms	50	11′–0	1/4	40	1
2	Floors	Hotel Bed-rooms, Hospital Wards and rooms	50	11′–0	1/4	40	1
3	Floors	Public spaces in above	100	8′–6″	3/8	100	2
4	Floors	Office, Rooms : Entrance floor, and below Above Entrance floors	 80 80	 10′–6″ 10′–6″	 3/8 3/8	 80 50	 2 2
5	Floors	Churches, Schools, Reading Rooms Art Galleries, Retail Shops, Garages of cars, less than two tons weight	80	10′–6″	3/8	80	2
6	Floors	Assembly Halls, Drill Halls, Dance Halls, Gymnasia, Light Workshops, Theaters, Cinema Houses, Restaurants, Cafes and Grandstands	100	8′–6″	3/8	100	2
7	Floors	Printing Presses, Large Work-shops and Factories	150	5′–6″	3/8	100	2
8	Floors	Ware-houses, Bookshops, Stationary Stores, Garages for cars more than two tons weight	200	4′–3″	3/8	200	2
9	Stairs & Balconies	Staircases Landings, and Corridors, Overhanging Balconies in Buildings other than Domestic buildings Do-in Domestic Buildings	100 80	8′–6″ 8′–6″	3/8 1/4	100 40	2 1
10	Roofs	Roofs inclined at not greater than 20 degrees to Horizontal	50	...	...	30	...
11	Roofs	Roofs inclined at greater than 20 degrees to Horizontal.	15 lbs./ft.^{2} normal to the sloping surface on the windward side. 10 lbs./ft.^{2} normal to leeward slope (not simultaneously).				

Note 1:—When the spans of slabs and beams are less than those mentioned in column 3 of the above table, the minimum alternative loadings, given in columns 4 and 6, must be adopted for slabs and beams respectively. Thus for all spans of slabs less than 11 ft., a minimum load of ¼ ton, distributed over one foot width of the span of domestic rooms, is prescribed.

Note 2:—There is a slight difference between the loadings prescribed by the By-laws and those by the Code of Practice. In the above table, a combination of both is made adopting figures, whichever are on the safer side.

Super-imposed loads on columns:—For the purpose of calculating the total load to be carried on columns, piers, walls and foundations in buildings of more than two storeys high the super-imposed loads, for the roof and the topmost storey shall be calculated in full in accordance with the schedule of loading given in Table No. 3. But for the lower storeys, a reduction of the super-imposed loads may be allowed in accordance with the following table:—

TABLE No. 4

REDUCTION OF SUPER-IMPOSED LOADS ON COLUMNS OF MULTI-STOREYED BUILDINGS

Position of Column	Reduction in loads given in Table No. 3
Topmost storey below roof	Full superimposed load without reduction
Next storey below topmost	10 per cent
Next storey below	20 per cent
Next storey below	30 per cent
Next storey below	40 per cent
Every succeeding storey below	50 per cent

Note:—The above rule is applicable to buildings having superimposed loads not exceeding 100 lbs./ft.^{2}. This means that full load without reduction must be calculated for columns, walls, etc. of buildings of the warehouse and factory types.

In the case of warehouse type buildings, it is necessary to calculate the super-imposed load of materials which are likely to be stored on top of floor slabs.

Wind-load :—By-law No. 6 of the L. C. C. Regulations and Sec. 16 of the Code prescribe that if the height of a building is less than twice its width, the wind-pressure may be neglected, provided that the building is stiffened by cross-walls, partitions, floors, etc.

In other cases, a wind-pressure of 15 lbs. per sq. ft. upon the upper two-thirds of the vertical projection of the surface of such buildings, with an additional pressure of ten lbs. per sq. ft. upon all projections such as chimneys, etc. above the general roof level should be taken.

Dead loads :—The primary dead load is the weight of the concrete structural members, to which should be added weights of all walls, floors, ceilings and roof-finishes, brick-work, stone-work, steel-work, partitions, fixed tanks, machinery and similar permanent construction comprised in the building. The following table gives weights of some of the materials.

TABLE No. 5

SCHEDULE OF WEIGHTS OF BUILDING MATERIALS

Material	Weight	Material	Weight
Timbers	lbs. per cub. ft.	**Metals**	lbs. per cub. ft.
Deodar	35–40	Gun-metal	540
Jarrah	55	Iron-cast	450
Kail	28–32	Iron-wrought	480
Sal	54–62	Lead-cast	708
Teak	38–52	Lead-sheet	710
Metals	lbs. per cub. ft.	Steel cast	492
Aluminium cast	156	Steel rolled	490
Aluminium wrought	174	Zinc	440

TABLE No. 5 (*Continued*)

Material	Weight	Material	Inches	Weight
Masonry, etc.	lbs. per cub. ft.	**Pipes**		lbs./R.ft.
Brick-work in lime or cement	120	Asbestos Cement Soil, waste	$3\frac{1}{2}''$	3·75
Sun-dried brick in mud	100	,,	4″	4·25
		—Do—Rain-water	2″	2–0
Sand-lime brick-work	115	...	3″	3–0
Cement concrete	140	...	4″	4·25
		C. I. drain	4″	17·5
—Do— reinforced	144	,,	6″	25
Lime concrete with stone-metal	120	—Do—light rain-water	$2\frac{1}{2}''$	3·5
Clinker concrete	90	,,	3″	4
—Do— with brick metal	105	,,	4″	6
—Do— breeze	96	—Do—soil	4″	10
Mortar Wet	109	,,	6″	16
Stone rubble masonry	144	Lead — soils waste	2″	4
		,,	3″	5·75
Ashlar masonry	165	,,	4″	7·5
Dry stone masonry	130	**Earths & Shingle**		lbs./cub. ft.
Flooring, Plaster, etc.	lbs/sq.ft.	Earth dry loose		72
1″ Cement tiles	12	—Do—Moist packed		90
Paving stones 1″ to $1\frac{1}{2}''$	12 to 18	Dry rammed		100
Lime plaster $\frac{3}{4}''$	7	Sand dry		90
		,, wet		100
Cement plaster $\frac{3}{4}''$	8			

TABLE No. 5 (*Continued*)

Material	Weight	Material	Weight
Flooring, Plaster, etc.	lbs. per sq. ft.	**Earths & Shingle**	**lbs./cub. ft.**
Mortar bedding		Gravel	120
′ below paving 1″	10	Stone Basalt or Trap	163
Plaster board ⅜″	3	Granite	165
¾″ Moulmein teak-ceiling	2·5	Lime	150
Terrazzo ¾″	9	Sand-stone	160
Granolithic 1″	12		
Window with frame	6		
Door with frame	8		

ILLUSTRATIVE EXAMPLES

Example 1:—Find the total load per sq. ft. on a flat terraced roof slab 4½ in. thick on the top of which are laid cement tiles one inch thick on 1½ in. average cement mortar.

	lbs./s.ft.
Solution: Super load from Table No. 3	50
Tiles 1″	10
Mortar bedding 1½″	15
Slab $\frac{4{\cdot}5}{12} \times 1' \times 144$	54
Total	129
Say	130 lbs./ft.²

Example 2:—If the size of room below the slab in Example 1, is 12 ft.×14 ft. and if there is a beam 6″×10″ in its centre spanning 12 ft. distance, find the load per ft. run on the beam.

Solution:—As the beam is in the centre, it carries the load of half the slab, i. e. 3·5 ft. on either side of it.

$$\text{Slab load} \quad 130 \times 7 = 910$$

$$\text{Beam load} = \frac{6}{12} \times \frac{10}{12} \times 144 = 60$$

Total 970 lbs. per ft. run.

Example 3:—Find the load per ft. run carried by a beam supporting a 5 in. slab on the top of the ground floor of a building used for office. The room is 16×30 ft. The slab is supported by two beams 8″×12″ each, at 10 ft. centres. The floor carries one inch terrazzo on 1½ in. average lime mortar bedding. One of the beams carries a half brick partition 12′ high. Find the load carried by this beam per ft. run.

Solution:—Super-load on floor supported by the beam
=10 ft.×80 lbs. = 800 lbs./ft. run.

B. F. 800 lbs.

Dead Loads :—	1 in. terrazzo	11 lbs./ft.²
	1½ in. mortar bedding	15 ,,
	5 in. slab	60 ,,
	½ in. plaster on soffit	6 ,,
		92 ,,

Dead load of slab	10 × 92	= 920
Wt. of beam	8″ × 12″	= 96
Wt. of partition	$\frac{4\frac{1}{2}}{12}$ × 120 lbs.×12 ft.	= 540
½″ Plaster on both sides	2× 5 lbs. ×12 ft.	= 120
	Total load	2476

Say 2500 lbs./ft. run

Illustrative Example 4

Calculate the load per ft. run on a staircase flight, 4 ft. wide consisting of 12 steps, each with 11 in. tread, 7 in. riser on a slab called "waist", 6 in. thick, carrying granolithic surface at top ½ in. thick.

(i) Super load $= 12 \times \frac{11}{12}$ in. $\times 1' \times 100$ lbs. $= 1100$ lb./ft.

(ii) Dead load: Wt. of triangular step $\frac{11 \times 7}{2}$

$= 38{\cdot}5$

½ in. granolithic $= 6{\cdot}0$

Wt. of waist $6'' + \frac{1}{2}''$ (plaster) $= 78{\cdot}0$

$122{\cdot}5$

Wt. of 12 steps $122{\cdot}5 \times 12$ per ft. strip 1470 lbs./ft.

Total (i)+(ii) $= 2570$ lbs./ft.

Say 2600 lbs. per ft. strip.

Illustrative Example 5

A certain framed building of residential type has eight storeys including the ground floor. Calculate the load carried by a particular column which supports a floor area of 14 ft. $\times$ 12 ft. The ground floor height is 14 ft. and that of each succeeding floor is 10 ft. All the slabs are 5½ in. thick including plaster on soffit, with ¾ in. granolithic finish at top.

Live loads:—

Roof—		30 lbs./s. ft.
7th floor i. e. top floor—		50
6th ,,	·9	
5th ,,	·8	
4th ,,	·7	
3rd ,,	·6	
2nd ,,	·5	
1st ,,	·5	
	4·0	
6th to 1st floor 50×4		= 200 lbs./s. ft.
		280

Brought forward from last page			280
Dead load:	Slab $5\frac{1}{2} \times 12 =$	66	
	Granolithic finish $\frac{3}{4}''$ $=$	9	
		75	
	8 floors $\times$ 75 $=$		600
	Total		880 lbs.

Load on column $= 880 \times 14 \times 12 \qquad = 147840$ lbs.

Wt. of column $= 18 \times 12 \times (14 + 10 \times 7) = 18144$ lbs.

Total 165984 lbs.

Say 166000 lbs.

CHAPTER III

BASIS OF DESIGN

REINFORCED concrete design is based on the following assumptions:—

(1) *That both steel and concrete are perfectly elastic within certain limits*—concrete under compression, and steel, both under tension and compression,—and *that they obey the Hooke's Law* viz., stress is proportional to strain and the modulus of elasticity is constant.

This assumption holds good for steel, which stretches or contracts in proportion to the applied load, and the stress-strain graph is a straight line. In the case of concrete, however, up to a certain limit the deformation caused is entirely recovered when the load is removed. But beyond that limit for subsequent loadings and unloadings, the recovery decreases as time passes, i. e., there is a permanent set. Further, even if the load is not increased, i. e. it remains constant and is allowed to remain long, the deformation goes on increasing as time passes. Thus concrete, though rigid, is a plastic material. Its behaviour, as mentioned above, is called "creep", or "plastic flow".

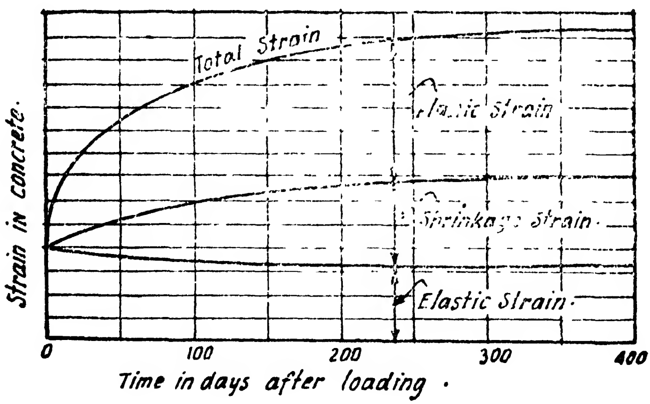


Fig. 1.

Again, in addition to plastic strain or creep due to application of load, there are also strains caused by shrinkage, or loss of water from the concrete, which are of considerable magnitude.

Fig. 1 shows the time-strain relation in concrete at constant temperature, humidity and load. The total strain is made up of these different strains.

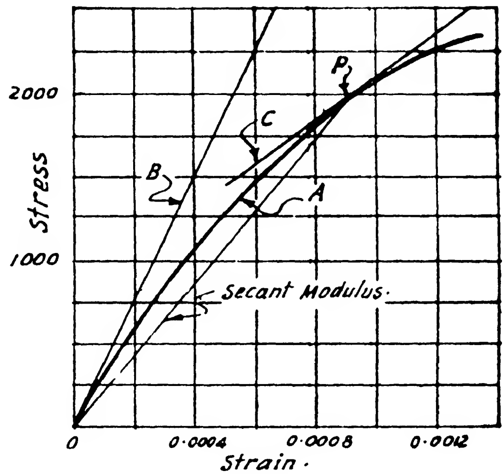


Fig. 2.

A—Stress-strain curve; *B*—Initial tangent modulus; *C*—Tangent at *P*.

Fig. 2 shows a typical stress-strain curve for concrete in compression. The modulus of elasticity, *Ec*, as determined by the slope of the curve at any point on it, decreases as the stress increases. Take for instance a point, P, on the curve and repeat loadings and unloadings several times, taking care not to exceed the load so as to cause a stress greater than that between O and P, and plot the curves. It will be seen that as time passes, each curve will show a tendency to flatten more and more until ultimately it is very nearly the straight line OP. The value of the modulus will then be represented by the slope of OP. This line is called the *Secant Modulus*. It is the ratio of stress to the total strain including plastic strain. It is the common practice to take the secant modulus as the constant modulus of elasticity of concrete in compression at or near the maximum working stresses in concrete.

(2) *That planes of transverse cross-sections remain plane and normal to the longitudinal axis after bending.*

This assumption is realised.

(3) *That the bond, or adhesion between steel and concrete is perfect within certain limits, and that they act in unison.*

This is also realised within reasonable limits. As the co-efficients of expansion of steel and concrete are very nearly the same, temperature and other atmospheric influences also do not affect the bond.

(4) *Concrete is assumed incapable of taking any tension.*

The tensile strength of concrete is about 0·10 of its compressive strength, but at construction joints and at shrinkage cracks it may be much less—even approaching zero. Therefore, the assumption made is correct, and errs, if at all, on the safe side.

(5) *That there are no initial stresses due to shrinkage and temperature.*

This is not strictly correct, as has been already explained. As fresh concrete dries out, it shrinks. The strains due to shrinkage increase with the increase in consistency (water contents), increase in cement, and increase in dryness in atmosphere. The initial stresses, particularly in a member of thin section, may even exceed those produced by loads. Besides, there may be stresses already induced by expansion and contraction under atmospheric temperature conditions.

From the above discussion, particularly with regard to the modulus of elasticity of concrete in compression, it will be seen that to ignore altogether the strains due to creep, shrinkage and temperature, and the stresses caused by them, and to assume a constant value for *Ec* is fundamentally wrong, as it would not give a true analysis of stress distribution.

There are three possible alternatives out of the difficulty:

(1) To assume a fixed value for *m*, that is for *Ec*, as has been the current practice, and scrupulously to avoid compression stresses in concrete going beyond a certain safe maximum limit ($\frac{1}{3}$ the ultimate stress). This is like feeding and treating a person in such a way as to maintain him, at all times, in perfect health, obviously at great cost. If by any chance he falls ill or on that analogy, under a certain combination of circumstances, the stresses caused in a concrete section are excessive, the hypothetical factor of safety falls to the ground.

(2) To adopt a variable modular ratio in design. This is recommended by the Code of Practice, which suggests that *m* should be taken as $\frac{40{,}000}{\text{cube strength}}$. This normally gives a range of values from 11 to 18 (vide Table No. 1 page 5). The current American practice is the same. The A. C. I. (American Concrete Institute) recommend $m = 30{,}000$ divided by the cube strength at 28 days, and assume the modulus $Ec = 1000 \times$ ultimate crushing strength at 28 days.

Theoretically this is quite correct. But there are two practical difficulties in the way: (a) That it makes the mathematical calculations intricate as they depend upon certain co-efficients, and (b) that on small jobs which form more than 95 per cent of concrete construction, there are no means available for making tests of crushing strength of concrete cubes, which are necessary in view of the fact that the strength of concrete is dependent upon a number of factors such as proportion of cement, grading, proportioning of aggregates, quantity of water, mixing, placing, curing, etc.

(3) To base the design on ultimate stresses. Recently this last method has been steadily gaining support in the Engineering world. Tests of actual structures made to destruction in U. S. A. have recently supplied sufficient data to derive empirical formulæ which take into consideration the effect of various strains. The results give an ultimate load, causing failure which, divided by a certain factor of safety gives safe

working loads. This method is being used even in this country in designing columns. It is bound to play an important part in designing other members also in future. The method goes under the name of "Plastic Theory" or "Ultimate stress method". It is briefly described later in a chapter in this volume and a few illustrative examples solved by it are given for comparison.

As the purpose of this book is to explain current standard practice, the first method has been followed throughout, assuming 1:2:4 mix ordinary grade concrete, with ultimate crushing strength of 2250 lbs./in.^{2} and safe working maximum stress of 750 lbs./in.^{2} and mild steel with a safe maximum working stress of 18000 lbs./in.^{2}, are to be used unless specially mentioned otherwise. A constant modular ratio, viz. 15, is used in calculations.

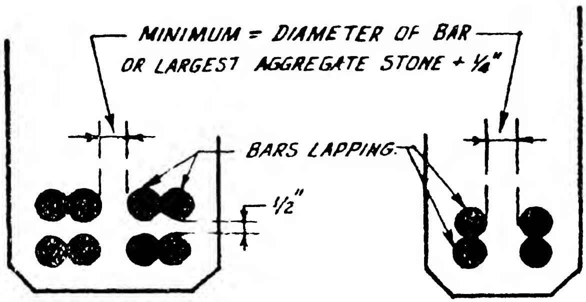
✓**Concrete Cover** :—In order to protect the steel from the effects of atmosphere and possible fire-hazard, it is necessary to provide a concrete cover over and above the effective sectional dimensions of the designed member. This cover is measured from the outside of all reinforcing bars, including transverse ties, spirals, stirrups, and all secondary reinforcement.

Place			Minimum Cover inches
Floors and Walls	...	...	$\frac{1}{2}''$ to $\frac{3}{4}''$
Main bars in beams and columns		...	1″ to 2″

✓**Distance between bars** :—This is very important in heavy beams, particularly on the top of columns, where reinforcement is likely to be crowded and at places where bars are lengthened by splicing (overlapping ends and tying together by winding a wire). Because if the bars are not surrounded by concrete, the necessary bond stress would not be developed. In fact, the width of heavy beams is determined by this factor.

The minimum lateral distance, specified by the By-laws and the Code, between two bars, should be equal to the diameter, or the maximum size of the aggregate plus $\frac{1}{4}$ inch whichever is greater.

The vertical distance between main bars when more than one tier is employed in beams, should be at least half inch



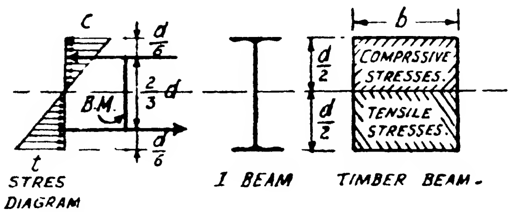
Figs. 3 & 4.

At points where laps or splices are made horizontally the minimum lateral distance should be as shown in Fig. 3. When laps occur vertically the distance is not affected. (See Fig. 4.)

CHAPTER IV

THEORY OF BENDING

DESIGN OF BEAMS OF HOMOGENEOUS MATERIALS



(a) (b) (c)

Fig. 5.

Fig. 5 represents cross-sections of steel and timber beams respectively with breadth b and depth d. If the beams are loaded on the top, compressive stresses are set up in the upper half and tensile stresses in the lower half of the sections. As in both the cases, the material is homogeneous and the sections symmetrical about the neutral axis (N. A.), the latter passes exactly through the middle of the sections. From the stress diagram, shown in Fig. 5 (*a*), it will be seen that the stresses at the neutral axis are zero and that they increase in proportion to the distance from the neutral axis towards the top and bottom, the maximum stresses being at the extreme fibres, viz., c (compressive) at top and t (tensile) at bottom, as shown in the figure, both of equal magnitude.

The relation between the Bending Moment (M) at any section, and the stress (f) developed at a distance y from the N. A. is shown by the equation.

$$\frac{f}{y} = \frac{M}{I} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

in which I is the moment of inertia of the section.

For rectangular beams $I = \frac{bd^3}{12}$ and maximum $y = \frac{d}{2}$. If f is the maximum fibre stress (either compressive c, or tensile t)

$$\frac{f}{\frac{d}{2}} = \frac{M}{\frac{bd^3}{12}} \text{ or } f = \frac{6Md}{bd^3} \text{ or } \frac{M}{\frac{bd^2}{6}} = \frac{M}{Z} \text{ or, } \frac{M}{f} = Z \ldots (2)$$

where $Z=$ modulus of section $= \frac{bd^2}{6}$ for rectangular beams.

For design of timber beams M, the bending moment, is calculated in the usual way and f, the working stress, either for tension or compression, depends upon the kind and quality of the timber (vide table No. 7 below). When these two are known, Z can be worked out. It is then easy to assign suitable values to b and d to make up the required modulus of section $\frac{bd^2}{6}$. It should be borne in mind that d should be within the limits of $\frac{1}{18}$ to $\frac{1}{24}$ of span and the proportion between b and d should be such that d should be between 1·5 and 3 times b.

For design of steel beams the bending moment is calculated in the usual way. The working stress f for mild steel section is $7\frac{1}{2}$ to 8 tons per sq. in. From these, Z can be worked out. Then from tables of properties of I–beams, a suitable section giving the required value of Z should be selected.

TABLE NO. 7

WORKING STRESSES FOR FIRST GRADE TIMBERS

Kind of Timber 1	Weight. lbs./cub. ft. 2	Tension parallel to grains lbs./in.^{2} 3	Compression parallel to grains lbs./in.^{2} 4	Shear parallel to grains lbs./in.^{2} 5	Modulus of Elasticity 6
Burma or Malbar Teak	41	2200	1700	125	1600,000
C. P. Teak	38	1800	1400	120	1207,000
Sal	54	2100	1500	175	1920,000
Chir	35	1600	1200	95	1500,000
Sain	53	2200	1700	155	1600,000
Deodar	36	1300	1200	160	1348,000
Jarrah	55	2300	900	120	1500,000
Douglas Fir	31	1600	1200	100	1700,000
Kail	28	1000	1000	110	986,000

Deflections:—Beams of steel and timber may be strong enough to resist the calculated bending moment and yet may bend too much under the load. Normally beams are designed for a deflection of $\frac{1}{360}$ of the span, but when there is a plastered ceiling below the floor, the deflection should not exceed $\frac{1}{480}$ of span. They, therefore, require to be checked for deflection, and if a greater depth is required for the necessary stiffness than that calculated for resisting B. M., that depth is finally adopted. The general formula for deflection is $\delta = k\,\frac{Wl^3}{48EI}$ where k = co-efficient, δ = maximum deflection, W = load in lbs., l = span in inches, E = modulus of elasticity in lbs./in.^{2}, and I = moment of inertia in inch units. The values of k are given below for different conditions of beams:—

TABLE No. 8

Condition of Beam	Coefficient
Cantilever with load at free end	16
,, ,, load uniformly distributed	6
Beam supported at both ends—(load central)	1
,, ,, load uniformly distributed	$\frac{5}{8}$

The following two illustrative examples will make the procedure of design of timber and steel beams clear.

Illustrative Example 6

Design a teak beam to support a load of 250 lbs./ft. over a span of 14 ft. Permissible deflection $\frac{1}{360}$ of span, safe working stress for teak 1800 lbs./in.^{2} and modulus of elasticity $=1{,}600{,}000$ lbs./in.^{2}.

Solution:—It is not given whether the ends of the beam are simply supported or fixed. However, timber beams are usually designed as simply supported, except in a few cases, such as purlins, battens, etc. with half-lap joints where semicontinuity over supports exists. In the latter cases, they need not be designed for deflection.

$$\text{B. M.} = \frac{wl^2}{8} = \frac{250 \times 14 \times 14 \times 12}{8} = 73500 \text{ in. lbs.}$$

$$f = 1800\ ; \quad z = \frac{bd^2}{6}\ ; \quad M = fz = f \cdot \frac{bd^2}{6}$$

$$\text{or,} \quad bd^2 = \frac{6 \times 73500}{1800} = 245.$$

If $b = 4$ in. and $d = 8$, $bd^2 = 256$

∴ To resist the B. M., the size required is 4″ × 8″.

To check for deflection,

$$\text{deflection } \delta = \frac{L}{360} = \frac{5}{384} \frac{WL^3}{EI}$$

$$\text{I for rect. beams} = \frac{bd^3}{12} = 360 \frac{5}{384} \cdot \frac{WL^2}{E}\ ; \quad (W = 250 \times 14).$$

E for teak = 1,600,000 lbs./in.^{2};

$$\text{or} \qquad bd^3 = \frac{360 \times 5 \times (250 \times 14) \times 14 \times 14 \times 12 \times 12 \times 12}{384 \times 1600{,}000}$$

$$= 3474{\cdot}0.$$

with $b = 4$, and $d = 10$, $bd^3 = 4000$.

Hence **b = 4·0″, d = 10″.**

Illustrative Example 7

Design a steel beam for the same load, allowing the same deflection.

Solution :—f = 8 tons/in.^{2}, E = 13,500 tons/in.^{2}

B. M. = 73,500 in. lbs. as before,

$$fz = M = 73{,}500, \ Z = \frac{73{,}500}{2240 \times 8} = 4{\cdot}10.$$

From tables of properties of steel beams, a rolled steel beam 5″ × 2½″–9 lbs./ft. gives Z = 4·36. A beam of this size is therefore strong enough to resist the B. M., but may not be stiff enough for the permissible deflection; let us check it.

deflection, $\delta = \frac{L}{360} = \frac{5}{384} \frac{WL^3}{E.I.}$; $I = \frac{360 \times 5}{384} \frac{WL^2}{E}$.

substituting E= 13,500 tons/in.^{2}

$$I = \frac{360 \times 5}{384} \times \frac{(250 \times 14) \times 14 \times 14 \times 12 \times 12}{13{,}500 \times 2240} = 15{\cdot}3.$$

From the table of properties of I beams $6'' \times 3'' \times 12$ lbs. per ft. gives I=20·99. The next lower section of $5'' \times 2\frac{1}{2}'' \times 9$ lbs./ft. gives 10·91. Hence adopt **6″ × 3″ @ 12 lbs./ft.**

Note :—Unless an unusually heavy section is adopted, the self-lead of timber and steel beams is not taken for calculations. To meet this, such a section is adopted, as gives a slightly higher modular ratio, to compensate for the self-load.

Unlike timber or steel, reinforced concrete is a combination of two heterogeneous materials and there are two important factors, which make the procedure of design of R. C. C. members different from that adopted above for members of homogeneous material.

These are:—

(i) Concrete is assumed to be incapable of taking any tensile stresses. It follows, therefore, that wherever there is tension in any part of a structural member, adequate quantity of steel must be provided for. This assumption has another important bearing on the distribution of stresses in beam cross-section viz., whereas the compressive stress, in concrete above the neutral axis, varies from zero at the neutral axis to c at the top-surface and is represented by a triangle with its apex at the N. A. as in the case of timber or steel beam, the tensional stress is concentrated near the bottom in the steel, the tension in the concrete below the neutral axis being altogether neglected.

(ii) In steel and timber, the material being more or less homogeneous, the modulus of elasticity for compression as well as for tension is the same. In reinforced concrete, the two different materials have different moduli, and the steel, mostly used for taking up tension has the modulus of elasticity 10 to 18 times, (let us say, m times) that of the other material viz., concrete, employed only for compression. This means that if steel is so thoroughly embedded in concrete that the strain or deformation, caused by an external force in it, is the same as that in the surrounding concrete, *the stress developed in it is 'm' times that developed in the surrounding concrete.*

In other words, if it were possible, to replace the steel of area 'A_T' provided in the beam, illustrated in Fig. 6 by an "equivalent area" of additional concrete, the area of the latter would be 'm' times A_T.

As this steel developing 'm' times the stress in the surrounding concrete, or its imaginary equivalent area of concrete is added only on the lower side of the neutral axis to take up tension, the balance, as far as the stresses are concerned, is naturally upset, and as a consequence the neutral axis no longer remains midway in the section, but moves to assume a position, *depending upon the quantity of steel provided.* These are the basic principles of reinforced concrete design.

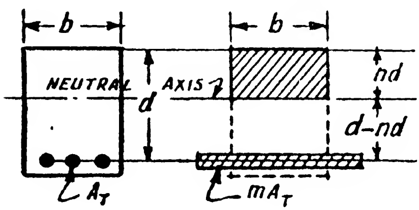


Fig. 6.
Equivalent Area of Concrete.

Illustrative Example 8

A beam has a width of 10 in., an effective depth of 20 in., and longitudinal reinforcement of 2 square inches of steel. If $m = 15$ and the maximum B. M. = 400,000 lb. ins., calculate the maximum fibre stresses imagining the steel to be replaced by m times its area of concrete and treating the beam, then, as of homogeneous material.

Solution:—First calculate the distance of the neutral axis rom the top of the beam. If it is x, from Fig. 6, the area in compression is bx and the distance of its centroid from the neutral axis $\frac{x}{2}$, the moment of this area being $bx \times \frac{x}{2}$. Similarly, the area of equivalent concrete is $m \times A_T = 15 \times 2$ and its lever arm is $d-x$, the moment of this area being $mA_T(d-x)$ or $15 \times 2\,(d-x)$. Equating these,

$$bx \times \frac{x}{2} = mA_T\,(d-x)$$

$$\text{or} \quad 10x \times \frac{x}{2} = 15 \times 2\,(20-x)$$

$$\text{or} \quad 5x^2 + 30x = 600$$

$$x^2 + 6x = 100 \quad \text{or} \quad x^2 + 6x + 9 = 109$$

$$x + 3 = 10{\cdot}47$$

$$x = 7{\cdot}47 \text{ in.}$$

$$\therefore \quad \text{The lever arm} = d - \frac{x}{3} = 20 - \frac{7{\cdot}47}{3} = 17{\cdot}51 \text{ in.}$$

Resisting moments are $C \times 17{\cdot}51$ or $T \times 17{\cdot}51$ of the compressive and tensile stresses respectively which must be equal to B. M. = 400,000 in. lbs.

$$400{,}000 = C \times 17{\cdot}51 \quad \text{i. e.} \quad C = 22844 = \frac{c}{2} \times bnd$$

$$= \frac{c}{2} \times 10 \times 7{\cdot}47$$

$$\therefore \; c = \frac{22844 \times 2}{10 \times 7{\cdot}47} = \mathbf{612}\ \textbf{lb./in.}^2; \; t = \frac{T}{A_T} = \frac{22844}{2}$$

$$= \mathbf{11422}\ \textbf{lb./in.}^2.$$

CHAPTER V

THEORY OF BENDING—R. C. C. BEAMS

Symbols used in flexural calculations :—

A_T = Steel reinforcement for tension.

b = Breadth of the beam.

c = Maximum permissible compressive stress in concrete (lb./in.²).

C = Total compression in beam section (lbs.).

d = Effective depth of a beam or slab in inches.

D = Total or overall depth of a beam or slab in inches.

E_c = Modulus of elasticity of concrete.

E_t = ,, ,, ,, steel.

j = Lever arm constant, so that jd = lever arm.

n = Constant of neutral axis, so that nd = distance of neutral axis in inches from compression edge.

p = Ratio of steel area to concrete area $= \frac{A_T}{bd}$.

p_s = Percentage reinforcement in a section $= \frac{A_T}{bd} \times 100$.

Q = Constant of resisting moment. (R. M.)

r = Ratio of tensile stress in steel to compressive stress in concrete $= \frac{t}{c}$.

t = Maximum permissible tensile stress in steel.

T = Total tension in beam or slab section.

Fig. 7 shows a cross-section of a reinforced concrete beam of breadth, *b*, and effective depth (depth from the compression edge to the c. g. of steel) = *d* in. The beam is reinforced with steel of total sectional area = A_T square inches and suppose it is subjected to a bending moment, M in./lbs.

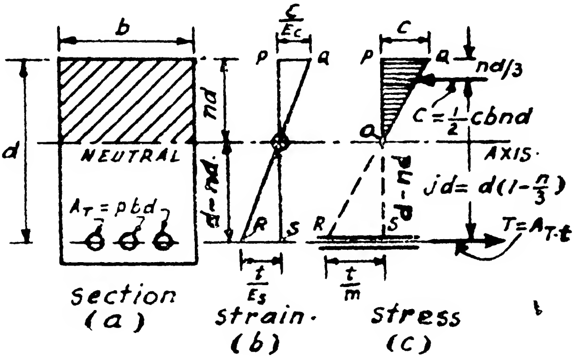


Fig. 7.

Let the neutral axis be at *nd* from the top of the beam. As the beam is subjected to a bending moment, compressive stresses must have been induced in the section above N. A. and tensile stresses below the N. A. with corresponding strains. The latter are shown in the strain diagram, Fig. 7 (*b*). The horizontal distances above the N. A. show compressive strains and those below the N. A., tensile strains. At the place where, steel reinforcement is provided, the strain in the concrete is represented by the horizontal line RS = $\frac{t}{E_s}$. As steel is thoroughly embedded in concrete, the strain in the steel must also be the same. But we have already seen that for equal strains, the stress developed in steel must be *m* times the stress in the surrounding concrete. This is shown in the stress diagram in Fig. 7 (c). The study of the latter shows that the compressive stress varies from a maximum of *c*=PQ at the top in the figure to zero at the neutral axis. At N. A. there is neither a compressive nor tensile stress. Below the neutral axis as the concrete is strained as shown in the strain diagram there must be tensile stresses induced in it but we disregard or neglect them for reasons already mentioned. Hence, in the portion of the triangle in the stress diagram below the N. A. up to the place where reinforcing steel is placed, the diagram is shown hollow with dotted sides. All the tensile stress is concentrated in the steel and its intensity is m times the stress R S. As the triangles OPQ and ORS are similar

$$\frac{RS}{PQ} = \frac{\text{stress in concrete at } d \text{ from the top}}{\text{stress in concrete at top}}$$

$$= \frac{OS}{OP} = \frac{d-nd}{nd} = \frac{1-n}{n}.$$

But as the stress in steel is m times the stress in the surrounding concrete,

$$\frac{\text{stress at } d \text{ from the top} \times m}{\text{stress in concrete at top}} = \frac{\text{stress in steel at } d}{\text{stress in concrete at top}}$$

$$\left.\begin{aligned} &= \frac{m(1-n)}{n} = \frac{t}{c} \\ \text{or} \quad t &= mc \times \frac{(1-n)}{n}. \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (1)$$

As the forces are in equilibrium, the bending moment, M, must be balanced by an equal moment developed inside the materials of the beam to resist it. This moment is obviously the couple consisting of either compressive force above or the tensile force below the N. A. multiplied by the distance between their c. g. s. The total compressive force C is represented by the area of triangle OPQ $\times$ breadth b, i. e. $\frac{c+0}{2} \times nd \times b$ and its point of application is at the centroid of the triangle which is at $\frac{nd}{3}$ from the top.

The total tensile force T equals $A_T \times t$, i. e. the area of reinforcement multiplied by m times the stress in concrete at that distance. The latter is $(d-nd)$. Thus we have

$$C = \tfrac{1}{2}\, cnd \times b = T = A_T \times t$$

The *Lever Arm* (jd) of the couple is the distance between the centroid of the compression area and the centre of gravity of the steel, or

$$jd = d - \frac{nd}{3} = d\left(1 - \frac{n}{3}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

The strength of the couple resisting the bending moment,

$$\left.\begin{aligned} M &= C \text{ or } T \times \text{lever arm} \\ &= \tfrac{1}{2}\, cnd\, b \times d\left(1 - \frac{n}{3}\right) \\ \text{or substituting } jd \text{ for } d\left(1 - \frac{n}{3}\right) \quad &= (\tfrac{1}{2}\, c\, nj)\, bd^2 \\ \text{putting Q for the constant, } M &= Q\, b\, d^2 \end{aligned}\right\} \ldots (3)$$

$$\left.\begin{aligned} &= T \times \text{lever arm} \\ &= A_T \times t \times d\left(1 - \frac{n}{3}\right) \\ \text{or} \quad &= A_T \times t \times j\, d \end{aligned}\right\} \ldots (4)$$

Equations (3) and (4) give the internal moments which the materials in a beam must develop to resist the bending moment produced by external loads. They are, therefore, called *Resisting Moments* (R. M.)—the first is developed in the compression part and the second in the tensile part of the section. *The R. M. of a beam or a slab must never be less than the bending moment, M, at any section.*

Percentage of steel :—Steel is very costly as compared with concrete. Its percentage, therefore, determines the economy of design. The results are therefore often expressed in terms of percentage of steel as follows :—

Considering the cross-section in Fig. 7 (*a*)

the effective area of the beam $= b \times d$

Area of steel $= A_T$.

$$\text{If } p_s = \frac{A_T}{bd} \cdot 100, \quad A_T = \frac{p_s\, bd}{100}$$

$$C = \tfrac{1}{2}\, cbnd = T = t\, A_T = t\,\frac{p_s bd}{100}$$

$$\text{whence } p_s = \frac{100}{2}\,\frac{cn}{t} = \frac{50\, cn}{t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

$$\text{From equation (1)} \quad \frac{c}{t} = \frac{n}{m\,(1-n)}\,;$$

Substituting this in (5) we have

$$p_s = 50n \frac{n}{m(1-n)} = \frac{50n^2}{m(1-n)} \quad \text{.....................} \quad (5A)$$

This shows that the depth of the neutral axis depends only on percentage steel and the value of m, and is independent of either the bending moment or the stresses induced by it.

The R. M. can be expressed in terms of p_s also. Thus we have

$$\text{R. M.} = A_T \, t \, jd$$

$$= p_s \frac{bd}{100} \, t \, jd \ \text{.................}$$

$$= \frac{p_s \, tj}{100} bd^2 \ \text{...................} \quad (6)$$

$$\text{Thus R. M.} = \tfrac{1}{2} cnj \times bd^2 = \frac{p_s \, tj}{100} bd^2 \ \text{...........}$$

Position of Neutral Axis:—Oftentimes if n, or the neutral axis constant, showing the position of the N. A. below the compressive edge is determined, the problem becomes simple. There are several ways of doing this:

(1) When the ratio of the stresses, $c : t$ and the percentage of steel, p_s are known, from equation (5) $p_s = 50n \frac{c}{t}$.

(2) When the ratio of stresses $t : c$ and m are known, from eqn. (1) $\frac{t}{c} = \frac{m(1-n)}{n}$.

(3) When the stresses and m are given, from eqn. (1)

$$\frac{t}{c} = \frac{m(1-n)}{n}.$$

$$nt = cm(1-n)$$

$$n(t+cm) = cm$$

$$n = \frac{cm}{t+cm}.$$

Dividing both the numerator and denominator by m

$$n = \frac{c}{\frac{t}{m}+c} \qquad \text{..............................(7)}$$

(4) When p_s and m are given, we have

$$\tfrac{1}{2}\, cbnd = A_T\, t$$

$$\frac{t}{c} = \frac{bnd}{2A_T}$$

$$= \frac{m\,(1-n)}{n} \quad \text{by eqn. (1)}$$

$$\frac{bnd}{2A_T} = \frac{m\,(1-n)}{n}.$$

Transposing and rearranging

$$bdn^2 + 2m\,A_T n - 2\,A_T m = 0$$

Solving this quadratic equation for n

$$n = \frac{-2m\,A_T \pm \sqrt{4m^2\,A_T^2 + 8bmA_T d}}{2bd}.$$

Dividing both the numerator and denominator by $2bd$. we get

$$n = -m\left(\frac{A_T}{bd}\right) \pm \sqrt{m^2\left(\frac{A_T}{bd}\right)^2 + 2m\left(\frac{A_T}{bd}\right)}.$$

Putting $p = \frac{A_T}{bd}$, $n = -mp \pm \sqrt{m^2p^2+2mp}$.

Since n must always be positive, the negative sign is omitted. This gives

$$n = \sqrt{m^2p^2+2mp} - mp \qquad \text{........................}$$

If p_s is substituted $\qquad$ (8)

$$n = \sqrt{\left(\frac{mp_s}{100}\right)^2 + \frac{2mp_s}{100}} - \frac{mp_s}{100}. \qquad \text{..............}$$

The following illustrative examples will show the application of these important results.

Illustrative Example 9.

Find n to determine the position of the N. A. in a beam and also the lever arm constant, j, when,

(i) $c = 600$ lb./in.^{2} $t = 16000$ lb./in.^{2} and $m = 15$. (Old By-laws)
(ii) $c = 750$,, $t = 18000$ lb./in.^{2} and $m = 15$. (New By-laws)
(iii) $c = 750$,, $t = 18000$,, and $m = 18$. (Code)

Solution:—By equation (1)

$$\frac{t}{c} = \frac{m(1-n)}{n}$$

$$\frac{16000}{600} = \frac{15(1-n)}{n}$$

$$26{\cdot}67n = 15 - 15n$$

$$41{\cdot}6/n = 15$$

$$n = \mathbf{0{\cdot}36}$$

$$j = \left(1 - \frac{n}{3}\right) = \left(1 - \frac{0{\cdot}36}{3}\right)$$

$$= 0{\cdot}88.$$

n could be even more easily determined by the application of equation (7).

$$n = \frac{c}{\frac{t}{m} + c}, \quad \frac{t}{m} = \frac{16000}{15} = 1066$$

$$= \frac{600}{1066+600} = \frac{600}{1666} = \mathbf{0{\cdot}36}$$

(ii) $$n = \frac{750}{\frac{18000}{15} + 750} = \frac{750}{1950} = \mathbf{0{\cdot}385}$$

$$j = 1 - \frac{n}{3} = 1 - \frac{0{\cdot}385}{3} = \mathbf{0{\cdot}87}$$

(iii) $$n = \frac{750}{\frac{18000}{18} + 750} = \frac{750}{1750} = \mathbf{0{\cdot}428}$$

$$j = 1 - \frac{n}{3} = 1 - \frac{0{\cdot}428}{3} = \mathbf{0{\cdot}86.}$$

Illustrative Example 10

Find the constants of resisting moment, Q, with the data given for the cases in the above example.

Solution :—

R. M. = Area of triangular stress diagram $\times b \times$ lever arm

$$= \frac{c}{2} \times nd \times b \times jd \qquad j = \left(1 - \frac{n}{3}\right)$$

Case (i) $\text{R. M.} = \frac{600}{2} \times \cdot 36d \times b \times 0{\cdot}88d$

$= \mathbf{95\ bd^2}$

$\therefore\ Q = \mathbf{95}$ .... (i)

Case (ii) $\text{R. M.} = \frac{c}{2} \times nd \times b \times jd$

$= \frac{750}{2} \times 0{\cdot}385d \times b \times 0{\cdot}87d$

$= \mathbf{126\ bd^2}$

$Q = \mathbf{126}$ .... (ii)

Case (iii) $\text{R. M.} = \frac{750}{2} \times \cdot 428d \times b \times 0{\cdot}86d$

$= \mathbf{138\ bd^2}$

$Q = \mathbf{138}$ .... (iii)

Illustrative Example 11

Find the percentage of steel required in a beam when

(i) $c = 600$ lb./in.^{2} $t = 16{,}000$ in.^{2} and $m = 15$

(ii) $c = 750$,, $t = 18{,}000$,, and $m = 18$

(iii) $c = 750$,, $t = 18{,}000$,, $m = 15$

Solution :—This can be solved in either of the two ways :

(i) R. M. for (i) = $95\ bd^2$ (vide above example)

$\therefore\ 95\ bd^2 = A_T \times t \times jd$; $j = 0{\cdot}88$;

or $95\,bd = A_T \times 16000 \times 0{\cdot}88$

$$\frac{A_T}{bd} = p = \frac{95}{16000 \times {\cdot}88} = 0{\cdot}00675$$

$$p_s = \mathbf{0{\cdot}675}$$

(ii) R. M. $= 138\,bd^2$ (vide above example)

$$138\,bd^2 = A_T \times 18000 \times jd\,;\ j = 0{\cdot}86$$

$$\frac{A_T}{bd} = p = \frac{138}{18000 \times 0{\cdot}86} = 0{\cdot}0089$$

$$p_s = \mathbf{0{\cdot}89}$$

(iii) R. M. $= 126\,bd^2 = A_T \times 18000 \times jd$

$$j = 0{\cdot}87$$

$$126bd^2 = A_T \times 18000 \times 0{\cdot}87d$$

$$\frac{A_T}{bd} = p = \frac{126}{18000 \times 0{\cdot}87} = 0{\cdot}00804$$

$$p_s = \mathbf{0{\cdot}804}.$$

By a still simpler method,

eqn. (5) $p_s = \dfrac{50\,nc}{t}$

$$\text{(i) } p_s = \frac{50 \times {\cdot}36 \times 600}{16000} = 0{\cdot}675$$

$$\text{(ii) } p_s = \frac{50 \times {\cdot}428 \times 750}{18000} = 0{\cdot}89$$

$$\text{(iii) } p_s = \frac{50 \times {\cdot}39 \times 750}{18000} = 0{\cdot}804$$

Illustrative Example 12

The following were or have been the recommendations of different Committees on R. C. C. design for 1:2:4 ordinary mix and mild steel. Tabulate and compare the corresponding value of each for n, j, Q and p_s.

	c lbs./in.^{2}	t lbs./in.^{2}	m Et ÷ Ec
Old L. C. C. By-laws (1915)	600	16,000	15
Code of Practice (current)	750	18,000	18
Current practice in India	750	18,000	15

Solution:—The results have been already worked out above in Examples 9 to 11. We have only to tabulate and compare them.

TABLE NO. 10

Authority	c lbs./in.^{3}	t lbs./in.^{2}	m	n	j	p_s p. c.	Q
Old L. C. C. Regulations	600	16,000	15	0·39	0·88	0·675	95
Code of Practice	750	18,000	18	0·428	0·86	0·89	138
Current Standard Practice and New L. C. C. Regulations	750	18,000	15	0·385	0·87	0·804	126

A scrutiny of the above figures shows that the resisting moment of $95bd^2$ according to the old L. C. C. Regulations was very conservative and uneconomical. The Code of Practice and the New L. C. C. Regulations both assume the same permissible maximum stresses in ordinary grade concrete and mild steel, but by taking $m = 18$ as recommended by the Code, the design requires a higher percentage of steel and the *calculated* strength of a beam or slab is higher. This means that the current practice of taking $m = 15$ makes the design slightly cheaper and about 11% safer than that recommended by the Code of Practice.

Illustrative Example 13

Find the depth of a beam and the reinforcement required in it if it is 8 in. wide and has to resist a B. M. of 185,000 in. lbs. Take $c = 750$ lb./in.^{2}, $t = 18000$ lb./in.^{2} and $m = 15$.

Solution:—The R. M. must be at least equal to the B. M.

$$\therefore \quad 185000 = \tfrac{1}{2}\, cjnbd^2$$

$$= 126\, bd^2 \text{ for the above data}$$

$$d = \sqrt{\frac{185000}{126 \times 8}} = \sqrt{182}$$

$$= 13{\cdot}6 \text{ in. effective}$$

$$D = 15 \text{ in. (say) overall}$$

Reinforcement, $A_T = \dfrac{\text{B. M.}}{t \times jd}$

$$= \frac{185000}{18000 \times \cdot 87 \times 13\cdot 6}$$

$$= 0\cdot 874 \text{ sq. in.}$$

2 Nos. $\frac{3}{4}$ in. ϕ, $A_T = 0\cdot 88$ sq. in.

or 3 Nos. $\frac{5}{8}$ in. ϕ, $A_T = 0\cdot 92$ „

$p_s = \dfrac{0\cdot 874 \times 100}{b \times d} = \dfrac{0\cdot 874 \times 100}{8 \times 13\cdot 6} = 0\cdot 804\ \%$ as shown in Table No. 10 above.

Illustrative Example 14

What will happen if in the above beam, the percentage of steel is

(i) increased to 1·2 %

(ii) decreased to 0·5 %

Solution :—Since the percentage of steel is altered, there will be a corresponding change in the position of the N. A.

By equation (5A) $p_s = \dfrac{50n^2}{m\,(1-n)}$

Case (i) $1\cdot 2 = \dfrac{50n^2}{15\,(1-n)}$

Transposing and rearranging,

$$50n^2 + 18n - 18 = 0$$

$$25n^2 + 9n - 9 = 0$$

$$n = 0\cdot 45$$

Case (ii) $p_s = 0\cdot 5 = \dfrac{50n^2}{15\,(1-n)}$

Solving for n we get

$$n = 0\cdot 32.$$

In Case (i) $n = 0{\cdot}45$ and $j = \left(1 - \frac{0{\cdot}45}{3}\right) = 0{\cdot}85.$

R. M. of the concrete flange $= \frac{1}{2}\, cnjbd^2$

$= \frac{1}{2}\, 750 \times {\cdot}45 \times {\cdot}85 \times 8 \times (13{\cdot}6)^2$

$=$ **212280 in. lbs.**

R. M. of the tensile flange by equation (6)

$$= \frac{p_s\, b \times d}{100} \times t \times (jd)$$

$$= \frac{1{\cdot}2 \times 8 \times 13{\cdot}6}{100} \times 18000 \times {\cdot}85 \times 13{\cdot}6$$

$=$ **271600 in. lbs.**

This shows that so long as the steel was 0·804 per cent, the resisting moment of the beam either in compression flange or in tension flange was 185,000 in. lbs., but when we increased the steel by 50 %, there has been an increase of 14·5 per cent only in the compression flange and nearly 50 per cent in the tension flange. But the latter is of no use. Because the maximum strength of the beam will be equal to that of the weaker of the two, *i. e.*, 212280 in. lbs. If more load is applied, the concrete will fail by being crushed out long before the steel is overstressed.

Case (ii) $p_s = 0{\cdot}5$ per cent

$$p_s = \frac{50n^2}{m\,(1-n)}$$

$$0{\cdot}5 = \frac{50n^2}{15\,(1-n)}$$

$$50n^2 + 7{\cdot}5n - 7{\cdot}5 = 0$$

$$n = 0{\cdot}32$$

$$j = \left(1 - \frac{{\cdot}32}{3}\right) = 0{\cdot}89$$

R. M. of compressive flange $= \frac{1}{2}\, cnjbd^2$

$= \frac{1}{2}\, 750 \times {\cdot}32 \times {\cdot}89 \times 8 \times (13{\cdot}6)^2$

$=$ **158000 in. lbs.**

R. M. of tension flange $= \frac{p_s \cdot b \times d}{100} \times t \times jd$

$= \frac{\cdot 5 \times 8 \times 13{\cdot}6 \times 18000 \times \cdot 89 \times 13{\cdot}6}{100}$

$=$ **118550 in. lbs.**

This shows that by decreasing the steel reinforcement to less than that required normally (0·804 per cent) the resisting moment, even of the compression flange is reduced and that the real strength of the beam is determined by the weaker of the two,—in this instance by the tensile reinforcement. If we go on loading the beam, the tensile stress in steel will reach and pass the safe limit before stress in concrete reaches its permissible value.

Practice Problems on Chapter V

1. Given the ratio of maximum permissible stresses in steel and concrete = (i) 20; (ii) 25 and (iii) 30 and $m = 15$. Find (a) the positions of N. A.; (b) lever arm constant; and (c) percentage reinforcement in each case.

Hints:—For finding out n apply eqn. (1) viz.

$$\frac{t}{c} = m \frac{(1-n)}{n} \text{ or (7) } n = \frac{c}{\frac{t}{m} + c} \cdot$$ From n find j and

for percentage steel use eqn. (5).

	n	j	p_s
Answers:—(i)	0·43	0·86	1·07
(ii)	0·38	0·88	0·75
(iii)	0·33	0·89	0·55.

2. Find out the resisting moments of a rectangular beam section $b \times d$ when $m = 15$ and

	c lb./in.²	t lb./in.²	Answers	
(1)	600	15,000	(i)	98·4 bd^2
(2)	700	17,000	(ii)	116·7 bd^2
(3)	800	20,000	(iii)	131·2 bd^2

Hints :—First find out n, then j, then R. M. $= \frac{1}{2}\, cnjbd^2$.

3. Find the effective depth of a beam 10 in. wide if it is to be subjected to a resisting moment of (i) 284500 in. lbs. (ii) 12,90000 in. lbs. taking c and t as 750 and 18000 lb. per sq. in. and $m = 15$.

Ans. (i) 15 in. (ii) 32 in.

4. Calculate the reinforcement required in sq. in. for the above beams.

Ans. (i) 1·2. (ii) 2·56.

CHAPTER VI

THEORY OF BENDING—R. C. C. BEAMS

(Continued)

Further considerations:—An observant student must have seen, from the foregoing discussion, the fundamental difference in reinforced concrete design, caused by the fact that there are two materials, the quality and the quantity of which could be varied. It is this: viz. whereas in a beam of homogeneous material and symmetrical section the neutral axis is midway between the top and bottom of the section and therefore the total strength, in compression, must always be equal to the total strength in tension, in an R. C. C. beam, they could be made unequal by varying the quantity or quality of one or the other material. For example, we have seen that when the permissible maximum stress of concrete is 750 lbs./in.^{2}, and that of steel is 18000 lbs./in.^{2} and m is 15, the compressive strength of the beam is 126 bd^2 and that if we provide 0·804 per cent steel, the tensile strength is also the same. This makes the design *balanced* and also most economical since both the materials are stressed to their maximum permissible limits.

Suppose now, we reduce the percen[...] of steel, to less than 0·804 per cent, the maximum momen[...] it will safely resist, would be determined by its tensile strength, viz. R. M. $= \frac{p_s bd}{100} \cdot tjd$. in which p_s is less than ·804 per cent. If on the other hand, we provide more steel than ·804 %, the beam would be weaker in compression, and the maximum moment it would resist, would be determined, not by the tensile strength of the section, but by the compressive strength viz. R. M. $= \frac{1}{2}\, cjnbd^2$. The extra steel supplied would be almost a waste.

Thus for every ratio of tensile and compressive stresses and a particular corresponding modular ratio, there is one critical value of p_s and correspondingly one of 'n', which make the design balanced and economical; for $c = 600$, $t = 16{,}000$ and $m = 15$, we have already seen (Vide illustrative Ex. No. 9) the critical value of $n = 0{\cdot}36$. That of p_s is $= \frac{50 \times nc}{t} = 50 \times {\cdot}36 \times \frac{600}{16000} = 0{\cdot}675$ per cent; similarly, the critical values of n and p_s for $c = 750$, $t = 13{,}000$ and $m = 18$ are $n = {\cdot}428$ and $p_s = {\cdot}89$ (vide Table No. 9.)

If we gradually increase the steel, from zero upto ·89 % for the above combination of stresses and modular ratio, the beam would remain weak in tension, and the moment, it will safely resist, would be determined, by its strength in tension. With ·89 per cent of steel, it would be as strong in tension, as in compression. If we keep on increasing the percentage of steel, still further, the beam would be weaker in compression and its safe resisting moment would be determined by its strength in compression. This will be more clearly seen in the following table in which the safe moments are worked out for values of 'n', increasing from 0 to ·8, with corresponding increases in percentage of steel for the stresses $c = 750$, $t = 18{,}000$, and $m = 15$.

TABLE NO. 11.

SAFE MOMENTS FOR DIFFERENT VALUES OF n AND p_s: $c = 750$, $t = 18000$, $m = 15$

n	j	p_s	Compressive strength	Tensile strength
0·1	0·967	0·037	36·3 bd^2	**6·4** bd^2
0·2	0·933	0·167	69·9 bd^2	**28·8** bd^2
0·3	0·90	0·429	101·3 bd^2	**69·4** bd^2
0·385	0·872	0·804	**126·8** bd^2	**126·8** bd^2
0·5	0·833	1·67	**156** bd^2	250·5 bd^2
0·6	0·80	3·00	**180** bd^2	432 bd^2
0·7	0·767	5·44	**201** bd^2	751 bd^2
0·8	0·733	10·67	**219·9** bd^2	140·8 bd^2

The safe resisting moments are shown in bold type.

A curve representing the relation between percentage steel and the safe-resisting moment is plotted in Fig. 8. From the table and the graph, the following facts could be gathered :—

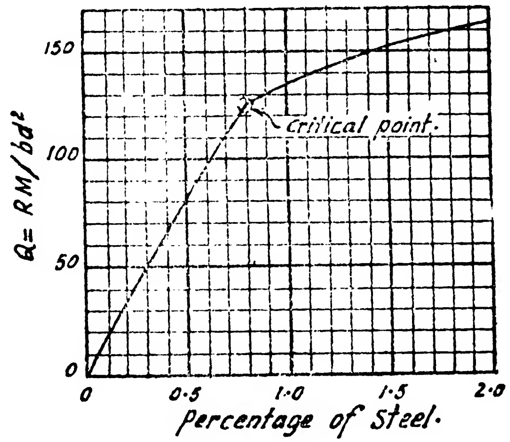


Fig. 8—Percentage steel and strength relation of R. C. C. beams.

(1) That as the percentage of steel increases, *n* also increases, i. e. the N. A. goes lower and lower down.

(2) That for every ratio of *t*:*c*, there is a critical value of percentage steel and a corresponding position of N. A., which make the beam equally strong in compression and tension with the maximum permissible stresses, in both materials.

(3) That the safe resisting moment increases almost uniformly and rapidly as the percentage of steel increases upto the limit of the critical value of the steel, after that it increases at a much slower rate.

(4) That if the percentage of steel is less than the critical value, the safe resisting moment of the beam is determined by the tensile strength, and if more, by the compressive strength of the section.

(5) That the lever arm decreases, but very slightly as n increases.

Some Questions and Answers

Question I:—You have stated above that concrete does not take any tension. But in a beam subjected to deflection, there must be tension below the N. A. and as there is only concrete and no steel, the concrete must be subjected to it. How is this explained?

Answer:—By the statement that concrete does not take any tension, it is not meant that it cannot be subjected to tension. In fact, experiments have proved that the tensile strength of concrete is about one-tenth that of its compressive strength. That is, it may take about 225 lbs./in^2. or more tensile stress, according to its quality before it breaks. Allowing a factor of safety of 3 or 4, it is possible that it may safely bear tensile stresses of about 55 to 75 lbs./in^2 but there are construction joints and shrinkage cracks inside concrete, where its strength in tension is very low, sometimes approaching zero. It is therefore safe *to assume* that concrete is incapable of taking any tension, and *neglect* whatever tension, it may take, when subjected to it, and provide sufficient area of steel, to take the entire tension, as if there were no concrete to help it. However, the concrete between the N. A. and the steel at bottom is subjected to tension, and must be taking part of it and to that extent, our design is safer.

Question II:—You have just now stated that the ultimate breaking strength of ordinary grade concrete is 225 lbs. or thereabout, depending upon its quality. In an R. C. C. beam, there is concrete surrounding steel, and even beyond the steel for the latter's protection, in the cover; when we subject steel to 18,000 lbs./in^2. or even more when it is of high tensile quality, as the modular ratio of concrete is about $\frac{1}{15}$ that of steel, it follows that the concrete in this region must be subjected to $\frac{18000}{15} = 1200$ lbs./in^2. or even more, i. e. to 5 or 6 times its breaking strength, then how is it not broken?

Answer:—Yours is an intelligent question. Yes. The stresses induced in the concrete, at the places mentioned, are far beyond the ultimate limit. What actually takes place is that concrete at these places, is cracked. The cracks, however, are so small, and so distributed, as to be invisible to the naked eye, and are harmless, from the point of view of protection of steel, from corrosion.

Question III:—The bending moment of a beam, supported at ends, is obviously maximum at the centre and reduces towards ends. It is reasonable, therefore, that its thickness should be in proportion to the B. M. This principle cannot be enforced in the case of a timber beam, or a rolled steel I–beam, as the cost of shaping them, to suit the B.M. would be too heavy. However, some steel built-up beams, like plate girders, are designed accordingly. An R. C. C. beam can be given any shape, without much expense. In fact, this is stated as a 'special' advantage of R. C. C. construction. Why is it then not enforced?

Answer:—The principle is correct, and it would have been possible to bring it into practice in R. C. C. beams, were it not for the considerations of shear, which is maximum at ends. It is cheaper to provide the same thickness of concrete at ends, to meet the shear, than to make the section thinner and provide extra steel. We shall discuss this later on.

It is however possible to practise economy, by providing steel in quantities proportional to the B. Ms., and this is actually done. As it is not possible to reduce the diameters of bars, some of them are stopped short, and some bent upwards to help resist shear. Please follow the discussion on shear at a later stage.

Question IV:—We have seen that R. M. $= \frac{1}{2} cjn\, bd^2$ (or $Q\, bd^2$). The latter expression is based on the compressive stress and cross sectional dimensions of beams, and is independent of the steel provided. But from the curve in Fig. 8 expressing the relation between percentage steel and R.M. it is seen that addition of steel after the critical point is reached, i. e. in excess of the economic percentage does increase the

R. M. of the compressive flange. How does it happen when concrete has been already stressed to the maximum ?

Answer:—The reason of the increase in the compressive resistance is that when more steel is added, the neutral axis descends, i. e. its position is lowered, as will be seen from Table No. 11 in which *n* increases from 0·1 to 0·7, as the percentage increases from 0·37 to 5·4 %. As N. A. is lowered down, greater area of concrete is thrown into compression, i. e. is made available for taking compression, and as a result, C, the total compression, increases. It may be noted that the increase in R. M. is not directly proportional to the increase in p_s, and thus addition of steel above economic percentage is not advantageous. See Illustrative Example 14.

CHAPTER VII

DESIGN OF FLOOR SLABS

(SPANNING ONE-WAY)

FLOOR slabs are designed as rectangular beams of 12 in. width. If the ordinary mix of 1:2:4 concrete and mild steel bars are used, it is usual in this country to adopt $c = 750$ lb./in.^{2}, $t = 18000$ lb./in.^{2}, and $m = 15$ as recommended by the L. C. C. By-laws. For the balanced design, with these data, we have already seen that $n = 0{\cdot}39$; $j = 0{\cdot}87$; $p_s = 0{\cdot}804$ per cent and R. M. constant is 126.

Effective span :—This is taken as "the distance from centre to centre of supports" or "the clear distance between supports *plus* the effective depth of the slab" whichever is lesser.

Reinforcement :—For main reinforcement, bars of less than $\frac{1}{4}$ in. diam. should not be used. The maximum diameter is not fixed, but bars of diam. greater than $\frac{5}{8}$ in. are seldom used for several reasons: (i) bars of small diam. are easy to bend; (ii) being closer apart, there is even distribution of steel; (iii) the surface area of smaller bars in contact with the concrete being greater, the bond is better. In a few cases welded fabric of high tensile steel is also used; (iv) the depth of slabs being small, the smaller the bars the greater the effective depth.

Distance between bars :—The distance between main bars should not exceed 12 in. nor should it be more than twice the effective depth of the slab, whichever is lesser. The minimum distance between bars should not be less than the diameter of the bar or $\frac{1}{4}$ in. larger than the largest size of the aggregate, whichever is greater.

Distribution steel :—This is often called "temperature steel", "secondary reinforcement" or "binders." Over and above the area of main steel required by design, 10 per cent of it according to the Bylaws, or 20 per cent according to the Code of Practice, must be provided in all slabs, spanning in one direction in the form of $\frac{3}{16}$ to $\frac{5}{16}$, usually $\frac{1}{4}$ in. diameter bars on the top of the main steel, and tied to the latter at every crossing by pieces of wire. The object is three-fold; (i) to resist partly the temperature and shrinkage stresses, (ii) to bind all the reinforcement together to form a sort of mesh, and (iii) to assist in distributing local loading over as wide an area as possible. It is the current practice in this country to use 20 per cent of steel for binders.

Depth or thickness of slab:—An R. C. C. slab is necessarily massive, and so its own weight forms a large part of the total load for which it is to be designed. It is usual, therefore, to assume certain depth in the beginning as a first approximation for preliminary calculations, and to correct it later, if necessary. The student finds some difficulty in making this assumption. However, there are certain thumb-rules for his guidance, which if strictly enforced, make separate calculations for deflection unnecessary. These are :—

(1) Howsoever short the span may be, the minimum overall thickness of a slab should not be less than 3 in.

(2) For uniformly distributed loads on a slab simply supported at ends the depth should be $\frac{1}{2}$ in. for every foot of span. This depth should be limited to 7 in. for a 14 ft. span. If the span exceeds this, some means to reduce it and eventually the depth, should be adopted, such as providing intermediate beams, or introducing 2-way reinforcement as will be explained in the chapters which follow.

(3) For a slab with fixed ends such as a lintel with considerable weight of wall on ends, or middle spans of a continuous beam, the depth of slab may be taken as equal to $\frac{\text{Span}}{30}$.

(4) For a cantilever slab with uniformly distributed load the thickness at support should be not less than $\frac{\text{Span}}{8}$.

The above rules are applicable when the loads are *uniformly* or *almost uniformly distributed* and *not* for concentrated, point loads.

Bending moments:—It is presumed that the student is familiar with the principles of the theory of structure and that he is able to calculate and draw out bending moment and shear force diagrams both of ordinary and continuous slabs and beams. Great accuracy is not required for R. C. C. design, but a clear conception of the positive and negative bending moments, and more particularly of shears is necessary. For, in R. C. C. work if any failures or even cracks occur they are in most cases due to shear.

The following table gives the common cases of non-continuous beams with loads and conditions of supports mentioned therein.

TABLE No. 12

BENDING MOMENTS OF NON-CONTINUOUS BEAMS AND SLABS

S. No.	Condition of supports	Manner of loading	Maximum B. M. ft. lbs.	Place where it occurs
1	Both ends simply supported	Concentrated load W lb. in the centre	$+\frac{Wl}{4}$	At centre
2	do	w lbs./r. ft. uniformly distributed	$+\frac{wl^2}{8}$	At centre
3	Both ends fixed	W lbs. concentrated at centre	$+\frac{Wl}{8}$	At centre
4	do	do	$-\frac{Wl}{8}$	At ends
5	do	w lbs./r. ft. uniformly distributed	$-\frac{wl^2}{12}$	At ends
6	do	do	Not less than $+\frac{wl^2}{12}$	At centre
7	One end fixed, the other freely supported	do	$-\frac{wl^2}{8}$	At fixed end
8	do	do	$+\frac{wl^2}{14}$	At $\frac{3}{8}$ of span from free end
9	One end fixed, the other freely supported	Concentrated load, W lb. at free end	$-Wl$	At fixed end
10	do	w lbs./r. ft. uniformly distributed	$-\frac{wl^2}{2}$	At fixed end

Note:—Since concrete is plastic and massive, certain allowances are made in favour of R. C. C. beams, perhaps also to compensate for possible defects in the execution, particularly in fixing ends. Thus, for case 6 in the above table, the B. M. for slabs and beams, other than of R. C. C., is $\frac{wl^2}{24}$, while for those of R. C. C. it is taken as $\frac{wl^2}{12}$. Also for case 8, the maximum positive B. M. at $\frac{3}{8}\, l$ from supports is $\frac{9}{128}\, wl^2$ for ordinary beams, whereas for R. C. C. beams it is taken as $\frac{wl^2}{14}$.

In the light of the above preliminary remarks we shall now proceed with design of slabs spanning in one direction.

Illustrative Example 15

Design a slab of a hotel room, 18 ft. long, and 10 ft. wide supported on walls, with one inch cement tiles on one inch mortar bedding.

Solution:—The effective span would be 10′ 6″. The thickness of the slab would be 5 in. by rule 2.

Super load = 10·5 × 50 (Table No. 3)
= 525 lbs.

This is less than the minimum of ¼ ton or 560 lbs. prescribed as an alternative load on small spans. (Vide Table No. 3, page 8). We shall therefore, adopt the latter.

Super load	=	560 lbs.
Self load 10·5 × 5″ × 12″	=	630 ,,
Mortar bedding 1″ 10 × 8	=	80 ,,
Cement tiles 10 × 10	=	100 ,,
Total		1370 ,,

This is the total load W and not w lb./sq. ft.

$$\text{B. M.} = \frac{1370 \times 10{\cdot}5 \times 12}{8} = 21577 \text{ in. lb.}$$

$$\text{say} = 21600$$

$$= 126\, bd^2$$

$$d = \sqrt{\frac{21600}{126 \times 12}} = \sqrt{14{\cdot}3} = 3{\cdot}8 \text{ (about) effective.}$$

Add $\frac{1}{2}$ in. for cover and $\frac{1}{4}''$ for steel

overall D = **4·5** in.

Our assumption of 5″ is not far wrong and errs on the safe side. There is therefore no need of revising calculations.

$$\text{Steel, } A_T = \frac{\text{B. M.}}{t \times jd} = \frac{21600}{18000 \times {\cdot}87 \times 3{\cdot}8} = {\cdot}364 \text{ sq. in.}$$

Using $\frac{1}{2}$ in. bars (area = 0·196 sq. in.)

$$\text{Spacing} = \frac{{\cdot}196 \times 12}{{\cdot}364} = \mathbf{6{\cdot}5} \text{ in.}$$

or, using $\frac{3}{8}$ in. bars (A = 0·11).

$$\text{Spacing} = \frac{0{\cdot}11 \times 12}{0{\cdot}364} = 3{\cdot}63 \text{ in. Say } 3{\cdot}5 \text{ in. } c. \text{ to } c.$$

$$\text{Distribution steel} = \frac{0{\cdot}364}{5} = 0{\cdot}073$$

Using $\frac{1}{4}$ in. bars (A = 0·049/in.²).

$$\text{Spacing} = \frac{0{\cdot}049 \times 12}{0{\cdot}073} = 8 \text{ in. } c. \text{ to } c.$$

Shear:—The maximum shear at ends equals

$$R = \frac{1370}{2} = 685 \text{ lbs.}$$

The effective section of the slab $= 3{\cdot}8 \times 12 \times 0{\cdot}87 = 39{\cdot}2$.

$$\text{Intensity of shear} = \frac{685}{39{\cdot}2} = 17{\cdot}5 \text{ lb./in.}^2$$

This is far less than 75 lbs./in.² which is permissible in concrete. The concrete will therefore safely take all the shear stress; still alternate bars should be bent upwards at $\frac{1}{7}$ span from supports i. e. at 1·5 ft. The B. M. is maximum at centre and reduces towards the ends. We have provided steel to resist the maximum B. M. If the same is continued to the ends it would be wasted. Instead of this if alternate bars i. e. 50 per cent are bent up and brought near the top they would be useful in resisting negative B. M. i. e. tension at top, which might possibly be caused if walls were raised on the top of the ends of the slab resting on the wall below. In the latter event, partial or full fixed-end condition may occur depending on the end fixity.

Thus the design consists of:

d (*effective*) = **3·8 in.**; *D* (overall) = **4·5 in.**

Steel—main = $\frac{1}{2}$ in. ϕ bars @ 6·5 in. *c/c*

,, —binders = $\frac{1}{4}$ in. ϕ bars @ 8 in. *c/c*.

Illustrative Example 16

Design a simply supported slab on the top of a school class-room 12 ft. wide. The floor is topped with flagstones, one in. thick on one in. mortar bedding.

Solution:—From rule 2, the slab would be 6 in. thick. The effective depth = 12·5 ft.

Super load	80	lb./ft.² (Table No. 3)
Self load 12″ × 6′	72	,, ,,
Mortar bedding 1″	8	,, ,,
Flagstone	10	,, ,,
Total	170	,, ,,

$$\text{B. M.} = \frac{wl^2}{8} = \frac{170 \times 12{\cdot}5 \times 12{\cdot}5 \times 12}{8}$$

$$= 39840, \text{ say } 40{,}000 \text{ in. lb.}$$

$$\text{Effective } d = \sqrt{\frac{40{,}000}{126 \times 12}} = \sqrt{26{\cdot}4} = 5{\cdot}14 \text{ in.}$$

5·14 + ·5 cover + ·25 half rod thickness = 5·89
overall D = 6 in. (say)

$$\text{Steel } A_T = \frac{40{,}000}{18000 \times 5{\cdot}14 \times {\cdot}87} = 0{\cdot}5 \text{ sq. in.}$$

Using ½ in. ϕ bars (A = 0·196)

$$\text{Spacing} = \frac{12 \times 0{\cdot}196}{0{\cdot}5} = 4{\cdot}7 \text{ in. say } 4{\cdot}5 \text{ in. } c/c.$$

$$\text{Distribution steel} = \frac{0{\cdot}5}{5} = 0{\cdot}1$$

Using ¼ in. ϕ bars (A = 0·049)

$$\text{Spacing} = \frac{12 \times 0{\cdot}049}{0{\cdot}1} = 5{\cdot}88 \text{ in. say } 6 \text{ in. } c. \text{ to } c.$$

$$\text{Maximum shear} = \frac{170 \times 12{\cdot}5}{2} = 1062{\cdot}5$$

$$\text{Shear intensity} = \frac{1062{\cdot}5}{12 \times 5{\cdot}14 \times {\cdot}87} = 20 \text{ lb./in.}^2$$

which is very safe.

Thus,
effective *d* = 5·1 in.
overall *D* = 6·0 in.
main steel = ½ in. ϕ 4½ in. c. to c.
Binders = ¼ in. ϕ 6 in. c. to c.

Alternate bars should be bent up at 1·8 (say 2 ft.) from supports and brought near the top.

Illustrative Example 17

Design a lintel on the top of a window with a clear opening 6 ft. wide. The lintel should have an over-hanging canopy projecting 2 ft. beyond the outer face of the wall. As 8 in. *khandkees* are used in the outside face of the wall, the lintel requires to be equal to one course thick. The height of the wall above the lintel is 8 ft.

Solution :—Effective span = 6′ 8″ say 6·75 ft. A lintel is supposed to carry the load of the wall enclosed in an equilateral triangle with the effective span as its base.

Height of equilateral triangle

$$= 6{\cdot}75 \sin 60^\circ = 6{\cdot}75 \times \frac{\sqrt{3}}{2} = 5{\cdot}9 \text{ ft.}$$

Weight of stone wall $= \frac{6{\cdot}75 \times 5{\cdot}9}{2} \times 1{\cdot}5 \times 160 = 4779$ lbs.

Wt. of lintel $8'' + \frac{1}{2}''$ mortar joint $= 8{\cdot}5$ in. thick

$$= 6{\cdot}75 \times 1{\cdot}5 \times \frac{8{\cdot}5}{12} \times 144 = 1033 \text{ lb.}$$

Wt. of canopy projecting 2 ft. :—

Thickness at support according to rule 4

$$= \frac{12 \times 2}{8} = 3 \text{ in. say } 3\tfrac{1}{2} \text{ in. with finishing.}$$

Let us assume average D of canopy = 3 in.

Wt. of canopy	$= 6{\cdot}75 \times 2 \times \frac{3}{12} \times 144$	= 486 lb.
Super load on canopy	$= 6{\cdot}75 \times 2 \times 30$	= 405 ,,
	Total load	6703 ,,
	say	6700 lbs.

$$\text{B. M.} = \frac{Wl}{8} = \frac{6700 \times 6{\cdot}75 \times 12}{8} = 67840 \text{ in. lbs.}$$

$$d = \sqrt{\frac{67840}{126 \times 18}} = \text{less than 6 in.}$$

ours is 8 in.

Allowing $\frac{3}{4}$ in. cover and $\frac{1}{4}$ in. for reinforcement we can take d as 7·5 in.

$$A_T = \frac{67840}{18000 \times 7{\cdot}5 \times {\cdot}87} = 0{\cdot}58 \text{ in.}^2$$

If $\frac{1}{2}$ in. rods are used (A = 0·196)

Area of 3 bars = ·588 in.^{2}

Two of these should go straight and the middle one bent up at $\frac{1}{4}$ span = 1·5 ft. from supports and turned horizontally to lie $\frac{3}{4}$ in. below top :—

Design of canopy :—$3\frac{1}{2}$ in. thick at support + $2\frac{1}{2}$ to 3 in. at the free end, average D=3 in.

Effective span = 2′ – 4″ say 2·5 ft; wt. of canopy

$$\text{one ft. strip.} \quad = 2{\cdot}5 \times 1 \times \frac{3}{12} \times 144 = 90 \text{ lbs.}$$

$$\text{super load} \quad = 2{\cdot}5 \times 1 \times 30 \qquad = 75 \text{ lbs.}$$

$$\text{Total} \qquad 165 \text{ lbs.}$$

$$\text{B. M.} = \frac{Wl}{2} = \frac{165 \times 2{\cdot}5 \times 12}{2} = 2475 \text{ in. lbs.}$$

$$\text{Effective } d = \sqrt{\frac{2475}{126 \times 12}} = \text{less than } 1{\cdot}5 \text{ in.}$$

But as it is a cantilever, for deflection it should be 4 in. at support and 3 in. at free end.

$$\text{Steel } A_T = \frac{2475}{18000 \times 2 \times {\cdot}87} = {\cdot}08 \text{ in.}^2$$

Using $\frac{1}{4}$ in. ϕ bars (A=0·049)

$$\text{Spacing} = \frac{0{\cdot}049 \times 12}{0{\cdot}08} = 7 \text{ in. } c \text{ to } c.$$

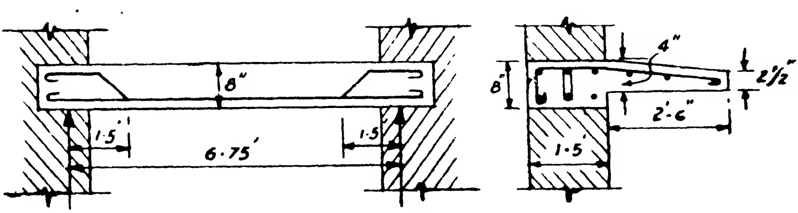


Fig. 9 (a) & 9 (b).

Figs. 9 (a) and 9 (b) show the longitudinal and cross sections of the lintel and canopy.

It should be noted that a lintel has usually the weight of wall on its ends, and so theoretically it should be designed as a fixed beam. But in practice the ends going into the wall are 6 in. to 9 in. only which does not give them the necessary fixity. If it were a fixed beam the maximum B. M. would be $\frac{Wl}{12}$ and it would occur both at top of supports and at bottom at mid-span (See Table No. 12). In that case only $\frac{2}{3}$ of the steel used above would have been required, being placed at bottom of the lintel at mid-span and bent up at $\frac{1}{4}$ span so as to lie near the top from $\frac{1}{8}$ span to middle of supports. Actually a lintel is something between a fixed and a freely supported beam. Hence it is advisable to design it as a simply supported beam and bend part of the reinforcement at top near the supports to take negative B. M.s due to partial fixity.

Illustrative Example 18

Design a cantilever gallery slab of a cinema theatre projecting 6 ft. beyond a brick wall 2′ 3″ thick. There is an R. C. C. parapet 3 ft. high and 4 in. thick on the edge of the free end. The gallery floor is topped with one in. cement tiles on one in. average mortar bedding. The main reinforcement of the slab behind the gallery is projected to form the reinforcement of the gallery.

Solution :—According to rule 4

Effective $d = \frac{6 \times 12}{8} = 9$ in. at support.

Effective span = 6′ 9″

Let the thickness at free end be 4 in.

Super load per ft. width

$= 100 \text{ lbs.} \times 6{\cdot}75 \qquad = 675 \text{ lbs.}$

Self load $= 6{\cdot}75 \times 1' \times \frac{9+4}{2 \times 12} \times 144 \qquad = 526$ „

Total 1201 „

say 1200 lbs. acting at 3′ from support

Wt. of parapet $= 3' \times 1' \times \frac{4''}{12} \times 144 = 144$ lbs.

acting at 5′ 10″, say 6 ft., from support.

$$\text{B. M.} = 1200\times3\times12+144\times6\times12$$
$$= 43200+10368 = 53568 \text{ in. lbs.}$$
$$\text{say} = 53600 \text{ in. lb.}$$

$$\text{Effective } d = \sqrt{\frac{53600}{126\times12}} = 6 \text{ in. approx.}$$

As greater stiffness is required for a cantilever, overall D=9 in. Allowing $\frac{3}{4}$ in. cover + $\frac{1}{4}$ in half diam. of steel, the effective ***d***=8 in.

$$\text{Steel, } A_T = \frac{53600}{18000\times8\times\cdot87} = 0\cdot433 \text{ in.}^2$$

If we use $\frac{1}{2}$ in. ϕ bars (A=0·196)

$$\text{spacing} = \frac{0\cdot196\times12}{0\cdot433} = 5\cdot4 \text{ in. say } \mathbf{5} \text{ in.}$$

$$\text{Distribution steel} = \frac{0\cdot433}{5} = 0\cdot086 \text{ in.}^2$$ $\frac{1}{4}$ in. ϕ bars at say 7 in. *c.* to *c.*

Alternatively, if $\frac{3}{8}$ in. ϕ bars are employed

$$\text{Spacing} = \frac{0\cdot11\times12}{0\cdot433} = 3 \text{ in. (about)}$$

But $\frac{1}{2}$ in. ϕ bars are preferable as explained under Practical Hints at the end of this Chapter.

As it is a cantilever the B. M. is negative. The reinforcing bars must therefore be laid *at the top* under the designed cover of $\frac{3}{4}$ in.

$$\text{Maxi. shear} = 1200+144 = 1344 \text{ lbs.}$$

$$\text{Intensity of shear} = \frac{1344}{12\times8\times\cdot87} = 16 \text{ lbs./in.}^2 \text{ (about)}$$

which is very low.

Illustrative Example 19

Certain considerations require the simply supported floor slab to be used for a retail shop, over an effective span of 10 ft. to be 7 in. deep. Adopting the standard stresses for ordinary grade 1 : 2 : 4 concrete and mild steel, find out

(a) the R. M. of compressive flange ;

(b) the area of steel required per ft. and its percentage;

(c) the economic depth of slab which would have been ordinarily required ;

(d) the maximum stress in concrete;

(e) the quantity of steel which would have been required for economic design for maximum strength

What inferences do you draw from the results ?

Solution :—Super load for retail shop 80 lbs./ft.²

Slab load $= 12 \times 7$	84	,,
Top finish	10	,,
Total	174	,,

$$\text{B. M.} = \frac{wl^2}{8} = \frac{174 \times 10 \times 10 \times 12}{8} = \mathbf{26100} \text{ in. lbs.}$$

R. M. of compressive flange $= 126\, b\, d^2$

D = 7 $d = 7 - {\cdot}5$ cover $- {\cdot}25$ half thickness of steel $= 6{\cdot}25$

$$\text{R. M.} = 126 \times 12 \times 6{\cdot}25^2$$
$$= \mathbf{59000} \text{ in. lbs.}$$

$$\text{Steel required} = \frac{26100}{18000 \times 6{\cdot}25 \times {\cdot}87}$$
$$= \mathbf{0{\cdot}27} \text{ in.}^2$$

$$\text{Percentage of steel} = \frac{{\cdot}27}{12 \times 6{\cdot}25} = \mathbf{{\cdot}036}\,\%.$$

$$\text{Economic depth } d = \sqrt{\frac{26100}{126 \times 12}} = \mathbf{4{\cdot}1} \text{ in.}$$

$$\text{Maximum stress in concrete} = \tfrac{1}{2}\, cbnjd^2 = 26100$$
$$= \tfrac{1}{2}\, c \times 12 \times {\cdot}39 \times {\cdot}87 \times 6{\cdot}25^2$$

$$c = \frac{26100}{79{\cdot}4} = 330 \text{ lbs./in.}^2$$

Steel, A_T (economic) for maximum strength (59,000 in. lb.)

$$= \frac{\cdot 804}{100} \times 12 \times 6{\cdot}25$$

$$= \mathbf{0{\cdot}6} \text{ in.}^2$$

Inferences:—The resistance moment required for the beam is 26100 in. lbs. For this the economic depth was 4·1 in., economic area of steel 0·27 in.^{2}. But as the depth is increased, the concrete is very lightly stressed, viz. 330 lb./in.^{2} instead of 750 and thus it is wasted. The R. M. of compression flange is 59000 in. lbs. and that of tension flange is 26100 in. lbs. Thus the design is uneconomic.

The student must have observed from the examples solved in this chapter that the intensity of shear is in every case very low. *This is true in all cases of slabs.* There is thus no necessity of providing or even for checking for shear in slab design unless the load is very heavy such as a ton per sq. ft. as in slabs of ware houses.

Practical hints on slab design:—Design of cantilever slabs and beams, particularly when they project more than 4 ft., should be rather on a conservative principle, as regards anchoring, depth or thickness near support, and reinforcement. On occasions of public processions on streets, the overhanging balconies and galleries are likely to be overloaded, not only by the floor being crowded with a rush, but also by persons leaning on the railing and thus throwing the load on the edge.

The first consideration when designing is to see that the moment caused by the maximum load on the cantilever is well balanced by sufficient anchoring inside. There have been many bad accidents, in most cases while removing centering on account of the negligence in this matter of simple common-sense. If there is a slab behind the wall, it is the best to project the main steel of the slab with an upward bend and to bring it near the top surface of the balcony. As an additional precaution the wall should further be raised bearing pressure on the fixed end of the balcony.

If the reinforcing bars of the slab run at right angles to the width of the balcony, the main bars of the latter should be anchored at least 2 or 3 ft. into the slab at right angles to the span of the latter. If there is no slab behind, there should be more than sufficient load of the wall bearing on the end of the balcony going into it. A balcony has usually a door opening into it. In that case unless there is a slab behind into which the main bars of the balcony could be anchored, the balcony slab should rest on two cantilever beams with their rear ends fixed into the wall bearing sufficient load on them.

(2) Though $\frac{3}{8}$ in. ϕ bars are better in distributing load evenly than bars of larger diameter, in the case of a cantilever balcony or gallery $\frac{1}{2}$ in. ϕ bars which are stiffer are preferable to guard against the possible danger of their being trodden under feet of workmen while the slab is being concreted. If they are bent down, cracks are sure to appear on the top of the balcony.

TABLE NO. 13

TABLE OF AREAS, CIRCUMFERENCE, WEIGHTS AND SPACING OF MILD STEEL ROUND BARS IN SLABS, BEAMS AND COLUMNS

				Single Bar Areas of round bars in sq. inches for centre to centre spacing in inches of																	
				1	1½	2		2½	3	3½	4	4½	5	5½	6	7	8	9	10	12	
				Steel Area in Beams and Columns when number of bars is																	
Diam. in.	Area sq. in.	Circumference in.	Weight per ft. lb.	12	8	6	5		4		3				2					1	Diam. in.
¼	0·049	0·785	0·167	0·59	0·39	0·29	0·25	0·24	0·20	0·17	0·15	0·13	0·12	0·11	0·10	0·08	0·07	0·07	0·06	0·05	¼
⅜	0·11	1·178	0·376	1·32	0·88	0·66	0·55	0·53	0·44	0·38	0·33	0·29	0·26	0·24	0·22	0·19	0·17	0·15	0·13	0·11	⅜
½	0·196	1·57	0·668	2·36	1·57	1·18	0·98	0·94	0·79	0·67	0·59	0·52	0·47	0·43	0·39	0·34	0·29	0·26	0·24	0·20	½
⅝	0·307	1·96	1·04	3·68	2·45	1·84	1·53	1·47	1·23	1·05	0·92	0·82	0·74	0·67	0·61	0·53	0·46	0·41	0·37	0·31	⅝
¾	0·44	2·36	1·50	5·30	3·53	2·65	2·21	2·12	1·77	1·51	1·32	1·18	1·06	0·96	0·88	0·76	0·66	0·59	0·53	0·44	¾
⅞	0·60	2·75	2·04	7·22	4·81	3·61	3·01	2·89	2·41	2·06	1·80	1·60	1·44	1·31	1·20	1·03	0·90	0·80	0·72	0·60	⅞
1	0·785	3·14	2·67	9·42	6·28	4·71	3·93	3·77	3·14	2·69	2·36	2·09	1·88	1 71	1·57	1·35	1·18	1·05	0·94	0·79	1
1⅛	0·994	3·53	3·38	11·93	7·95	5·96	4·97	4·77	3·98	3·41	2·98	2·65	2·39	2·17	1·99	1·70	1·49	1·33	1·19	0·99	1⅛
1¼	1·23	3·93	4·17	14·73	9·82	7·36	6·14	5·89	4·91	4·21	3·68	3·27	2·95	2·68	2·45	2·10	1·84	1·64	1·47	1·23	1¼
1⅜	1·48	4·32	5·05	17·82	11·88	8·91	7·42	7·13	5·94	5·09	4·45	3·96	3·56	3·24	2·97	2·55	2·23	1·98	1·78	1·48	1⅜
1½	1·77	4·71	6·01	21·21	14·14	10·60	8·84	8·48	7·07	6·06	5·30	4 71	4·24	3·86	3·53	3·03	2·65	2·36	2·12	1·77	1½
1¾	2·40	5·50	8·18	28·86	19·24	14·43	12·03	11·55	9·62	8·25	7·22	6·41	5·77	5·25	4·81	4·12	3·61	3·21	2·89	2·41	1¾
2	3·14	6·28	10·68	37·70	25·13	18·85	15·71	15·08	12·57	10·77	9·42	8·38	7·54	6·85	6·28	5·39	4·71	4·19	3·77	3·14	2

This Table supplies all that is required about m. s. round bars in R. C. C. Design—Areas, circumferences, weights, spacing of bars for slabs and number of bars in beams and columns to give specified steel areas.

CHAPTER VIII

DESIGN OF TWO-WAY REINFORCED SLABS

IF the framing plan of beams and slabs is so arranged that each rectangular panel of slab is supported on all four sides, the tensile steel could be placed in two directions perpendicular to each other horizontally, and the panel treated as spanned in two directions at right angles to each other.

The advantages of the system are :—

(1) Since there is a net-work of reinforcing steel in a closer mesh, the slab is stiffer, even with lesser depth.

(2) There is a saving in concrete and also in steel if the panel approaches a square.

The two-way reinforcing system of slab design is, in fact, statically indeterminate, since, if the spans are different, the deflections must be different, if the strip in the direction of each span is free to deflect. But as the reinforcement in the two-way system is intertwined, both the sets must have the same deflection at their intersection. Again, the deflection has a two-way curvature, unlike that of the one-way slab, the curvature of which is cylindrical. The distribution of load between the two sets of bands is dependent on their relative resistance to deflection. The panel resembles a uniformly loaded net, held tight at all four edges. The stress analysis is very complicated, involving higher mathematics, and the final result, too, is not of much use in commercial designs as the equations are very complicated.

However, a number of empirical equations based on experimental work have been put forward, out of which two are very commonly accepted :

(1) By *Grashof and Rankine* :—If l = long span or length, b = small span or breadth, W = total distributed load, W_1 = share of the total load taken by the short span,

W_2 = share of the total load taken by the long span i. e. $W_1 + W_2 = W$,

$$\text{then} \quad W_1 = W \frac{K^4}{K^4+1}$$

$$\text{and} \quad W_2 = W \left(1 - \frac{K^4}{K^4+1}\right)$$

where $K = \frac{l}{b}$, or the ratio of length to breadth. The relation holds good up to the limit, $K = 2$.

The following table gives the coefficients for various values of K or $l : b$.

TABLE No. 14

GRASHOF AND RANKINE'S FORMULA

K	Part of Total Load Borne by Shorter Span	Part of Total Load Borne by Longer Span
1·00	0·50	0·50
1·10	0·59	0·41
1·25	0·71	0·29
1·30	0·74	0·26
1·50	0·83	0·17
1·60	0·87	0·13
1·75	0·90	0·10
1·90	0·93	0·07
2·00	0·94	0·06

This formula gives very good results for slabs uniformly loaded and freely supported on all four sides.

(2) By *Dr. Marcus* of Germany :—If $w = w_1 + w_2$ lbs./ft.², i. e. the sum of loads shared by the short and long span respectively, the deflections of the strips at right angles to

each other, at the centre of the panel, in both directions must be the same at the point where the two strips cross each other, i. e.

$$\frac{5}{384} \cdot \frac{w_1 b^4}{E I_b} = \frac{5}{384} \cdot \frac{w_2 l^4}{E I_l}$$

where I_b and I_l are the moments of inertia of the short and long span strips; assuming $I_b = I_l$, the above equation reduces to

$$w_1 b^4 = w_2 l^4$$

$$\text{or} \qquad w_1 = w_2 \frac{l^4}{b^4}.$$

$$= (w - w_1) \frac{l^4}{b^4}$$

Solving this for w_1 we get

$$w_1 = w \frac{l^4}{l^4 + b^4}$$

$$\text{and} \qquad w_2 = w \frac{b^4}{l^4 + b^4}.$$

The above are the loads shared by one foot strip parallel to each span. From these loads the B. M.s are calculated in the usual way. Thus the B. M. of unit width strip of short span,

$$M_b = w_1 \frac{l^2}{8} = w \frac{l^4}{l^4 + b^4} \times \frac{l^2}{8}$$

for slabs uniformly loaded and simply supported on all four sides.

Similarly M_l of long span

$$= \frac{w\, b^4}{l^4 + b^4} \times \frac{b^2}{8}.$$

The share of the load taken by the shorter span, b, and the B. M. on the shorter span are evidently greater than that for the longer span, l. The coefficients for end span and intermediate spans are $\frac{1}{10}$ and $\frac{1}{12}$ respectively, which may be substituted for the coefficient, $\frac{1}{8}$, for free B. M. in the above expressions. The following table gives the proportional loads shared by the short and long spans under different conditions of supports :

TABLE NO. 15

COEFFICIENTS OF LOADS OR B. M.S ON SHORT AND LONG SPANS OF SLABS SPANNING IN TWO DIRECTIONS BY DR. MARCUS' METHOD

	Ratio $l:b$	1·0	1·1	1·2	1·3	1·4	1·5	1·6	1·8	2·0
Simply Supported	Load on short span	0·50	0·60	0·67	0·74	0·80	0·84	0·87	0·91	0·94
	Load on long span	0·50	0·40	0·33	0·26	0·20	0·16	0·13	0·09	0·06
Semi-continuous (end span)	Load on short span	0·40	0·48	0·54	0·59	0·64	0·67	0·70	0·73	0·75
	Load on long span	0·40	0·32	0·27	0·21	0·16	0·13	0·10	0·07	0·05
Continuous Intermediate span	Load on short span	0·33	0·40	0·44	0·49	0·54	0·56	0·58	0·61	0·63
	Load on long span	0·33	0·27	0·23	0·18	0·13	0·11	0·09	0·06	0·04

Illustrative Example 20

Design a floor slab on a room of a domestic house with effective spans of 15 and 18 ft. freely supported on walls on all sides with corners unrestricted. For floor finish take 20 lb./ft.^{2}.

Solution :—Super load	50	lbs./ft.^{2}
Floor finish	20	,,
Slab load 5 in. thick	60	,,
Total	130	,,

$\frac{l}{b} = K = \frac{18}{15} = 1{\cdot}20$. As the slab is freely supported, we shall use Grashof-Rankine formula.

$$w_1 = {\cdot}71\, w = {\cdot}71 \times 130 = 92 \text{ lbs./ft.}^2$$

$$w_2 = 130 - 92 = 38 \text{ lbs./ft.}^2$$

$$\text{B. M. on short span} = \frac{92 \times 15 \times 15 \times 12}{8}$$

$$= 31050 \text{ in. lbs.}$$

$$\text{B. M. on long span} = \frac{38 \times 18 \times 18 \times 12}{8}$$

$$= 18400 \text{ in. lbs.}$$

Effective depth for short span

$$d = \sqrt{\frac{31050}{126 \times 12}} = 4{\cdot}6 \text{ in.}$$

$$\text{Steel for short span } A_T = \frac{31050}{18000 \times {\cdot}87 \times 4{\cdot}6} = 0{\cdot}43 \text{ in.}^2$$

Using $\frac{1}{2}$ in. ϕ bars (A = ·196)

$$\text{Spacing} = \frac{0{\cdot}196 \times 12}{0{\cdot}43} = 5{\cdot}4 \text{ in., say } 5 \text{ in. } c/c.$$

$$\text{Effective } d \text{ for long span} = \sqrt{\frac{18400}{126 \times 12}} = 3{\cdot}5 \text{ in.}$$

As the rods for the long span will be placed on the top of those for the short span, the effective depth of short span which we have taken as 4·6 in. will be reduced by 0·5 in., i. e. the available depth for long span is 4·1 in., say 4·0.

$$A_T \text{ for long span} = \frac{18400}{18000 \times 4 \times {\cdot}87} = 0{\cdot}296 \text{ in.}^2$$

$$\text{for } \tfrac{1}{2} \text{ in. } \phi \text{ rods spacing} = \frac{0{\cdot}196 \times 12}{0{\cdot}296} = 8 \text{ in. } c/c.$$

Note that the effective depth is calculated from the B. M. of the short span, as it is maximum. Further, as the rods for the long span are to be placed on the top of those for the short span, the effective depth for the long span is reduced. The steel for the latter is, therefore, designed for this reduced depth. In the present case, it was found to be more than sufficient.

As there are longitudinal and cross bars forming together a sort of mesh in a two-way slab, there is no need of distribution steel.

Illustrative Example 21

Find the reactions on the supports in the above example.

Solution:—A little thinking will convince the student that B. M. is governed by the considerations of deflection. As the shorter span resists and restricts it, the share of the load taken by the short span is naturally greater, and the B. M. of the shorter span is also greater. But the load actually transmitted to the supports by the longer span, which covers a greater area must be greater. Thus the reaction on each support of the longer span, R_l.

$$R_l = \frac{18 \times 92}{2} = \mathbf{828} \text{ lb.}$$

R_s on supports of shorter span

$$= \frac{15 \times 38}{2} = \mathbf{285} \text{ lb./ft.}$$

Illustrative Example 22

Design a two-way intermediate slab 16′ × 16′ effective area, carrying an inclusive load of one cwt. per sq. ft. The slab is continuous on all four sides.

Solution :—As the slab is continuous we shall adopt Dr. Marcus' formula

$$\frac{l}{b} = k = 1. \quad \text{Hence } c \text{ for both spans} = 0{\cdot}33.$$

$$\text{B. M. on one span} = \frac{0{\cdot}33 \times 112 \times 16 \times 16 \times 12}{12}$$

$$= 9557 \text{ in. lbs.}$$

$$\text{Effective depth } d = \sqrt{\frac{9557}{126 \times 12}} = 2{\cdot}54 \text{ in.}$$

Assuming 5 in. overall D, and leaving ¾ in. for cover and half thickness of steel, we get effective d = 4·25 in.

$$A_T = \frac{9557}{18000 \times 4{\cdot}25 \times {\cdot}87} = 0{\cdot}144 \text{ in.}^2$$

By using ⅜ in. ϕ bars at 9 in. centres, we get 0·147 in.^{2} per ft.

The steel for the slab in the other direction will be either above or below this layer. If above, the effective depth will be reduced by ⅜ in. (0·375) and so it will be 4·25 – 0·375 = 3·875 in.

$$A_T = \frac{9557}{18000 \times 3{\cdot}875 \times {\cdot}87} = {\cdot}156 \text{ in.}^2$$

Use ⅜ in. ϕ bars at 8½ in. c/c. A = 0·156 in.^{2} or, as an alternative this steel may be laid below that for the longitudinal strips. In the latter case d = 4·25 + ·375 = 4·6 say,

$$A_T = \frac{9557}{18000 \times 4{\cdot}6 \times {\cdot}87} = 0{\cdot}133 \text{ in.}^2$$

⅜ in. ϕ bars at 10 in c/c. In this case the overall thick ness D will be 4·6 + ·375 + ·5 = 5·5 in. say.

Illustrative Example 23

Compare the quantities of concrete and steel if in the above example the slab is designed with one-way reinforcement.

Solution :—Clear span = 16. Hence thickness of slab by Rule No. 1 = 8 in. Effective span = 16·67.

Load given	112 lb./ft.²
Additional load for 3″ increased thickness $= 1' \times \frac{3}{12} \times 144 = 36$	,,
Total	148 lbs. ,,

$$\text{B. M.} = \frac{wl^2}{12} = \frac{148 \times 16{\cdot}67 \times 16{\cdot}67 \times 12}{12}$$

$$= 41140$$

$$d = \sqrt{\frac{41140}{126 \times 12}} = 5{\cdot}3 \text{ in.}$$

But for necessary stiffness 8 in. is required. Effective d = say 7 in.

$$A_T = \frac{41140}{18000 \times 7 \times {\cdot}87} = {\cdot}385 \text{ in.}^2$$

Using ½ in. ϕ rods at 6 in. c/c A = 0·393

Quantity of main steel = 34 Nos. × 17·5 × ·67

= 396 lb.

$$\text{Distribution steel} = \frac{{\cdot}385}{5} = 0{\cdot}077$$

¼ in. ϕ at 7·5 c/c, quantity = 28 × 17 × ·167

= 78 ,,

Total steel = **474** lbs.

Total concrete $= 16{\cdot}67 \times 16{\cdot}67 \times \frac{8}{12} =$ **186** cft.

(ii) Steel for 2-way slab

Lower layer $\frac{16{\cdot}67 \times 12}{9} = 23 \times 17 \times {\cdot}375 = 146{\cdot}6$ lbs.

Upper ,, $\frac{16{\cdot}67 \times 12}{8{\cdot}5} = 24 \times 17 \times {\cdot}375 = 153$

299·6

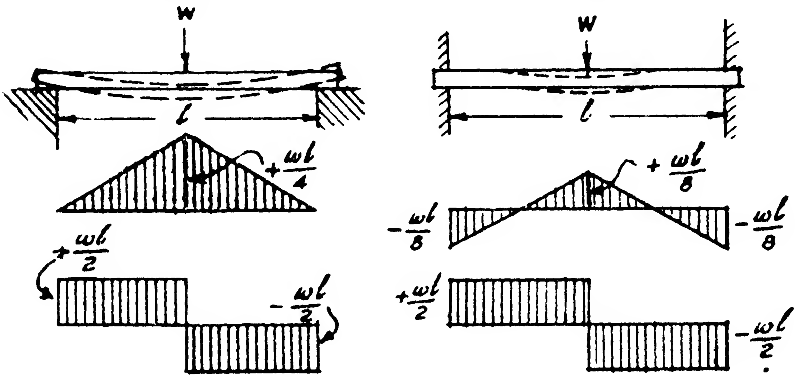
Concrete $16{\cdot}67 \times 16{\cdot}67 \times \frac{5}{12}$ $=$ **116** cft.

Kind of Design	Steel	Concrete
One way steel	474 lb.	186 cft.
Two way steel	300 lb.	116 cft.

CHAPTER IX

CONTINUOUS SLABS AND BEAMS

A SLAB or a beam is said to be continuous when it rests on at least three supports. One of the principal advantages of R. C. construction, viz. the monolithic pouring of concrete to form a monolithic continuous member of a rigid frame is fully utilised in continuous beams in reducing the maximum bending moments, without in most cases, affecting the shear force. This is illustrated in Figs. 10 and 11. In Fig. 10 a simply sup-



Figs. 10 & 11.

ported beam of span l carries a central load W. The B. M. and shear force diagrams in Figs. 10 and 11 are self-explanatory. The maximum B. M. is $\frac{Wl}{4}$. Now, if the ends of the same beam are fixed as shown in Fig. 11, the maximum B.M. is now either $+\frac{Wl}{8}$ or $-\frac{Wl}{8}$ i. e. half of the former. The shear force remains unaffected in both the cases. This means that in the second case the fibre stress is reduced to half. In other words a smaller beam would carry the same load if the principle of continuity is brought into effect.

By casting an R. C. C. beam or slab monolithically over three or more supports, conditions similar to those in a beam with fixed ends are created in all the spans except those at the ends.

There are several methods of calculating the bending moments of continuous beams and slabs viz. by

(1) Theory of three moments.

(2) Slope deflection method.

(3) Strain-energy or principles of work done.

(4) Moment distribution.

(5) Arbitrary approximate coefficients.

Out of these the first four are based on some theory or other and amongst them the theory of three moments is more commonly employed. However, they all involve lengthy calculations particularly when the spans are unequal, and the loads and moments of inertia are variable and though the results obtained are mathematically correct, the theories are based on certain assumptions which are not realised in practice. These are:

(1) That beams and slabs are simply hinged to the knife-edged tops of supports; in other words, the beams and slabs are quite free to deflect under their loads. But in practice the supports have a considerable width at top. Besides it is customary to frame the beams into columns and, as a consequence, the stiffness of the latter restrains them from deflecting. Even the ends of slabs are either anchored into walls or framed into ell-beams. The secondary beams are framed into main beams, the torsional resistance of which, imposes restrictions on the ends of secondary beams, and so on.

(2) The supports are rigid and their tops remain, at all times, at the same level, even after loading. This is very difficult in practice. Even though the support tops may be kept at the same level initially it is possible that some one or other, bearing the heaviest load may sink a little—even a few thousandths of an inch are sufficient to undermine the assumption.

(3) The modulus of elasticity of concrete is constant This is never realised.

(4) Besides these the spans may be unequal, loading may vary, moments of inertia which depend upon the section may vary not only from span to span, but in the same span also, e. g., in a T-beam the section over supports is rectangular and so on. Although an allowance could be made for these variable items while using formula, it makes the calculations too difficult and complicated to be of any use in everyday practice of commercial design.

One alternative method is to use the approximate coefficients recommended by the Memorandum accompanying the By-laws and the Code of Practice. They may perhaps be uneconomical when the loadings are light, but they have been used for many years and have been found to be very convenient and amply safe against even the worst cases. They are given in the subjoined table.

TABLE No. 16

MAXIMUM BENDING MOMENTS ON CONTINUOUS SLABS AND BEAMS.

Near middle of end span	At penultimate supports	At middle of interior span	At other interior supports
$+\frac{wl^2}{10}$	$-\frac{wl^2}{10}$	$+\frac{wl^2}{12}$	$-\frac{wl^2}{12}$

Where w = combined (i. e. sum of live and dead) load on a beam or slab.

These B. Ms. apply to slabs and closely spaced secondary beams and are based on three assumptions, viz.

(1) *Approximately equal spans*:—Spans are said to be approximately equal when they do not differ by more than 15 % of the length of the longer span.

(2) *Approximately uniform loading*:—Loads which normally come on the floors of residential, office and other buildings except those of the factory or warehouse type may be

regarded as uniform for purposes of design with these coefficients.

(3) *Free end supports* :--This assumption is not realised in most cases in practice. The ends of slabs or beams are almost invariably partially fixed by being framed into columns or beams or anchored into walls. Under these circumstances, not only will there be a negative B. M. on the top of end supports, but the latter will affect the B. Ms. of 3 or 4 interior spans next to the end support to an appreciable extent on account of continuity. The moment on the top of the end support caused by the restraint may vary from $-\frac{wl^2}{120}$ to $-\frac{wl^2}{12}$ depending upon the relative flexibility or rigidity of the end support. In normal cases it may safely be taken as $-\frac{wl^2}{24}$. To what extent the B. Ms. of the interior spans would be affected is shown in Fig. 12.

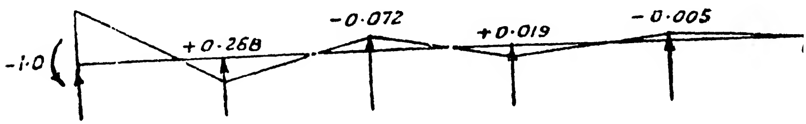


Fig. 12.

In the figure the moment at the left hand end support is taken as unity and other moments are in proportion to it.

The following are the possible worst cases to be considered :—

(I) The maximum + ve B. M. in the centre of one of the spans.

(II) The maximum −ve B. M. on one of the supports.

(III) Hogging (opposite of sagging) action in one of the spans caused by the possible combination of loads on adjacent spans.

(IV) The case of a continuous beam with only two spans which is not covered by the above table.

Let us now consider when these conditions would occur and whether the B. M. coefficients prescribed in the above table are safe enough to meet them.

Case I :—Maximum possible + *ve* B. M. would occur in the middle of a continuous system of slabs or beams when that span is loaded and those adjacent to it on either side are unloaded as shown in Fig. 14. (Middle Fig. at B).

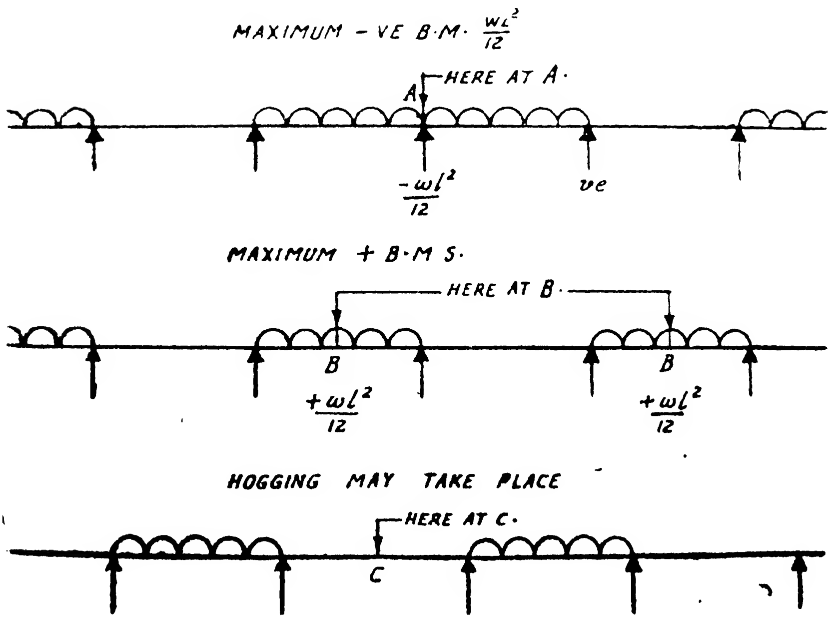
If w_d = dead load / ft.², w_l = live load / ft.²

$W = (w_d + w_l)$ and l = span in. ft.

The maximum positive B. M. at B

$$= \frac{w_d l^2}{24} + \frac{w_l l^2}{12}$$

Suppose $w_d = w_l = \frac{W}{2}$



Figs. 13, 14 & 15.

Then the maximum B. M. $= \frac{l^2}{24}(w_d + 2w_d)$

$$= \frac{3}{24} l^2 (w_d)$$

$$= \frac{3}{24} l^2 \frac{W}{2}$$

$$= \frac{3}{48} Wl^2$$

Against this the maximum B. M. given in the above table is $= \frac{Wl^2}{12}$ or $\frac{4Wl^2}{48}$ which is greater than $\frac{3Wl}{48}$.

Now suppose $w_l = 60 w_d$, then $W = 61 w_d$,

then $\frac{w_d l^2}{24} + \frac{w_l l^2}{12} = \frac{w_d l^2}{24} + \frac{60 w_d l^2}{12}$

$$= \frac{121}{24} w_d l^2$$

Putting $W = 61 w_d$, $= \frac{121}{24} \cdot \frac{W}{61} l^2$

$$= \frac{121}{1464} Wl^2.$$

Against this the maximum B. M. in the above table is $= \frac{Wl^2}{12}$ or $\frac{122}{1464} Wl^2$ which is greater than $\frac{121\ Wl^2}{1464}$. Thus the B. M. given in the table is always greater than the maximum in a worst case.

Case II:—The maximum negative B. M. occurs on the top of an intermediate support when two adjacent spans are loaded and those next to them on either side are unloaded as shown in Fig. 13 at A. The maximum B.M. $= -\left(\frac{w_d l^2}{12} + \frac{w_l l^2}{10}\right)$. For all normal loadings the B. M. recommended in the table is $-\frac{wl^2}{12}$, and it can be proved as above that it is always greater than one under any combination of loading.

The support at A in Fig. 13 carries a very heavy load, viz. half the span load on either side. If it sinks but very slightly, the negative B. M. on its top automatically diminishes.

Case III :—Hogging (opposite of sagging) action is caused in a span when it is unloaded and the spans adjacent to it on either side are loaded as shown in Fig. 15 at C.

The B. M. at the centre of the span

$$= \frac{w_d l^2}{24} - \frac{w_l l^2}{24}.$$

If w_l is greater than w_d, the expression would be negative, and there would be hogging in the span. The remedy suggested is that steel should be supplied also near the top of slabs in such cases. However, such a contingency is of very rare occurrence and perhaps only of academic interest, since no such hogging has been réported to have ever occurred without top steel in the past.

Case IV :—When a slab or a beam is continuous over two spans only, each of approximately equal length, with approximately uniform loading, the maximum bending moments—both positive and negative are shown in Fig. 16. The reaction at top of the central support is 1·25 wl.

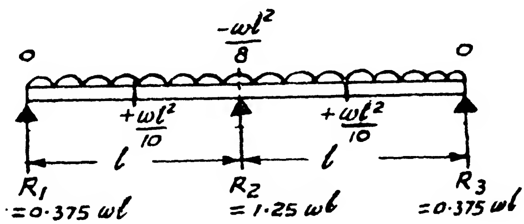


Fig. 16.

Bending moments and reactions on supports of two continuous spans only.

If the end supports are not free, but partially fixed, or restrained, a suitable negative B. M. factor should be adopted as already explained on page 76.

When more exact bending moments are required they may be taken from tables supplied in certain hand-books* giving the coefficients of B. Ms. for continuous beams or slabs from two to five equal spans for conditions of symmetrical loading with uniform moment of inertia. The coefficients are given separately for dead loads and live loads, and also for concentrated loads at $\frac{1}{2}$, $\frac{1}{3}$ or $\frac{1}{4}$ spans.

An important concession is allowed by the Code of Practice, while using the tables referred to above, or even Table No. 16 above, of approximate B. Ms. on continuous slabs and beams. It is this: Any calculated negative B. M. on support may be reduced by 15 % if the maximum positive B. Ms. in the adjacent spans are increased by an amount equal to the numerical value of the reduction (not 15 % of positive moment). This reduction in the support moments, though arbitrary, is justified for several factors wrongly assumed while calculating the B. Ms. such as, partial fixity, width of supports and failure of concrete to behave as a perfect elastic material. The reduction of support moments results also in reducing the congestion of reinforcement at the supports, where satisfactory consolidation of concrete is so often a practical difficulty.

This concession is entirely left to the volition of the designer, who may or may not use it.

Recapitulation :—(1) Continuity in R. C. C. beams is a great advantage, and results in reducing the stresses and thus causes economy by reducing the sizes of beams and thickness of slabs and the reinforcement in them.

(2) As a general rule, when beams are continuous, the negative B. M. on tops of supports are greater than those at mid-spans.

(3) When the loading is approximately uniform and spans are also approximately equal, the coefficients for approx-

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* Please refer to tables supplied in the "Explanatory Handbook on the Code of Practice for R. C." by Scott and Glanville or "Reinforced Concrete Designer's Handbook" by Reynolds.

imate maximum B. Ms. recommended by the Code of Practice and Memorandum for combined dead and live load given in Table No. 16 are quite safe and convenient.

(4) When more exact B. Ms. are required, they may be calculated from the table of coefficients separate for dead and live load supplied in certain handbcoks, or may be worked out from the theorem of three moments, particularly when the spans are unequal, and loads heavy and not uniform, or there are point loads.

(5) The case of a continuous beam over two approximately equal spans and uniform loading should be considered separately, as shown in Fig. 16.

Illustrative Example 24

A floor having more than five spans of 10 ft. each carries a superload of 1 cwt. per sq. ft. Design the slab for the interior span assuming standard stresses for ordinary grade mix of concrete and commercial mild steel; $m = 15$. Adopt approximate B. M. coefficients.

Solution :—Live load = 112 lbs./sq. ft.

Assuming 5″ depth, self load = 60 ,, ,,

Floor finish (Top and Bottom) = 25 ,, ,,

Total 197 ,, ,,

say, 200 ,, ,,

$$\text{B. M.} = \frac{200 \times 10 \times 10 \times 12}{12} = 20{,}000 \text{ in. lbs.}$$

$$\text{Effective } d = \sqrt{\frac{20000}{126 \times 12}} = 3{\cdot}66 \text{ in. say } 3{\cdot}75$$

$$\text{Overall D} = 4{\cdot}5 \text{ in.}$$

$$A_T = \frac{20000}{18000 \times {\cdot}87 \times 3{\cdot}75}$$

$$= 0{\cdot}346$$

Using $\frac{1}{2}$ in. ϕ bars, spacing $= \dfrac{12 \times \cdot 196}{0 \cdot 346}$

$= 6{\cdot}8$ in.

say $= 6\frac{1}{2}$ in. A = 0·364

Distribution steel $= \dfrac{0 \cdot 346}{5} = 0{\cdot}069$

$\frac{1}{4}$ in. ϕ bars at $8\frac{1}{2}$ in. centres.

Illustrative Example 25

A floor slab is continuous over four spans each 9′ – 0″. Carrying a uniform load of 1·5 cwt. per sq. ft., it frames into ell-beams at ends. Find B. Ms. at (1) top of the end support (2) centre of the end span (3) top of the support next to ell-beam.

Solution:—Superload = 168 lb./ft².

Self load of slab D = 5 in. = 60 ,,

Floor finish = 20 ,,

248 lb./ft.²

(1) *B. M. at top of end support*

$$= -\frac{wl^2}{24}$$

$$= -\frac{248 \times 9 \times 9 \times 12}{24}$$

$$= -10040 \text{ in. lbs.}$$

$$A_T = \frac{10040}{18000 \times \cdot 87 \times 4} = 0{\cdot}163 \text{ sq. in. ... (i)}$$

(2) *B. M. at centre of end span*

$$= \frac{wl^2}{10} = \frac{248 \times 9^2 \times 12}{10}$$

$$= 24100 \text{ in. lbs.} \quad \dots \quad \dots \quad \dots (2)$$

To this must be added algebraically the B. M. induced by the end restriction.

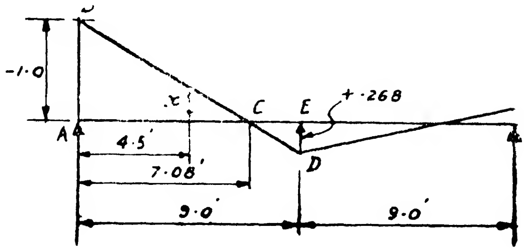


Fig. 17

Let us first find out the distance AC. (Fig. 17).

Since $\triangle^s$ ABC and CDE are similar

$$\frac{AB}{AC} = \frac{DE}{CE}$$

$$\text{or} \qquad \frac{1}{AC} = \frac{0{\cdot}268}{9 - AC}$$

$$\text{or} \quad 1{\cdot}268AC = 9$$

$$AC = 7{\cdot}08$$

Now we want the value of x, the moment caused by end restriction at mid span of AE. By similarity of triangles

$$\frac{AB}{AC} = \frac{x}{(7{\cdot}08 - 4{\cdot}5)}$$

$$\text{or} \quad \frac{1}{7{\cdot}08} = \frac{x}{2{\cdot}58}$$

$$x = 0{\cdot}365 \qquad (1)$$

AB = − 10040 in. lbs. from example solved above.

(i) $\therefore \quad x = -10040 \times 0{\cdot}365 = -3665$ in. lbs. (i)
this is negative.

The B.M. at centre of span is the algebraic sum of 2 and 2 (i)

= 24100 − 3665

= 20435 in. lbs.

$$\text{Effective } d = \sqrt{\frac{20435}{126 \times 12}} = 3{\cdot}7 \text{ in., say } 3{\cdot}75 \text{ in.}$$

$$A_T = \frac{20435}{18000 \times {\cdot}87 \times 3{\cdot}75} = 0{\cdot}36.$$

Using $\frac{1}{2}$ in ϕ bars

$$\text{Spacing} = \frac{0{\cdot}196 \times 12}{0{\cdot}36} = 6{\cdot}5 \text{ in. } c|c.$$

If 50 p. c. (or alternate bars) are bent up at $\frac{1}{5}$ span 0·18 sq. in. of steel would be available at top against 0·163 required as 1 (i) above. There is thus no need of providing extra steel at the top of end support.

(3) *B. M. at penultimate support* DE in Fig. 17.

This will be $-\frac{wl^2}{10}$ plus (algebraically) the B. M. induced by the end restriction

$$= -\frac{248 \times 9 \times 9 \times 12}{10} + 0{\cdot}268 \times 10040$$

$$\doteq -24100 + 2690 = -21410 \text{ in. lbs. say } 21400.$$

We have already taken $d = 3{\cdot}75$ in (2) above

$$\therefore \quad A_T = \frac{21400}{18000 \times {\cdot}87 \times 3{\cdot}75} = 0{\cdot}38 \text{ sq. in.}$$

50 per cent of the $\frac{1}{2}''$ bars provided at bottom in (2) above, i. e. 0·18 sq. in. per ft. will be available for top reinforcement. Thus we want ·38 − ·18 = ·20 sq. in. If $\frac{3}{8}$ in ϕ bars are placed at $6\frac{1}{2}$ in. *c/c* we get 0·204 sq. in.

Thus the reinforcement will be as follows :—$\frac{1}{2}$ in. ϕ bars at $6\frac{1}{2}$ in. *c/c.* at bottom of end span at centre, half of these, i. e. alternate bars would be bent up at $\frac{1}{5}$ span so as to be near the top of end support. The same bars would be also bent up at $\frac{1}{5}$ span and arranged near the top of the penultimate support, where additional $\frac{3}{8}$ in. ϕ extra bars of length equal to ($\frac{1}{4}$ span + width of top support + $\frac{1}{4}$ span with hooks at ends) should be placed at $6\frac{1}{2}$ in. in between the bars at top.

Illustrative Example 26

A floor consists of two span 8′–6″ each carrying a uniform load of 200 lbs./sq. ft. Design the slab for stresses 750. 18000 and $m = 18$.

Solution:—For the above stresses and $m=18$, R.M.$=138\ bd^2$ (vide page 36).

Loads:—	Live load	= 200	lbs./sq. ft.
	Self load D=5 in.	= 60	„
	Floor finish	= 20	„
	Total	280 lbs.	„

(1) B. M. in the centre of span

$$\frac{wl^2}{10} = \frac{280 \times 8{\cdot}5^2 \times 12}{10} = 24270 \text{ in. lbs.}$$

$$\text{Effective } d = \sqrt{\frac{24270}{138 \times 12}} = 3{\cdot}84 \text{ say } 4 \text{ in.}$$

$$\text{Overall D} = 4{\cdot}75 \text{ in.}$$

$$A_T = \frac{24270}{18000 \times {\cdot}86 \times 4} = 0{\cdot}39 \text{ sq. in.}$$

$$\text{Spacing of } \tfrac{1}{2}'' \text{ bars} = \frac{{\cdot}196 \times 12}{0{\cdot}39} = 6 \text{ in. } c/c.$$

$$\text{Distribution steel} = \frac{0{\cdot}39}{5} = 0{\cdot}08 \text{ in.}^2$$

$\frac{1}{4}$ in. ϕ bars @ $7\frac{1}{2}$ in. c./c.

(2) B. M. at top of central support

$$= -\frac{wl^2}{8} = -\frac{280 \times 8{\cdot}5^2 \times 12}{8} = -30345 \text{ in. lbs.}$$

We have already adopted $d = 4$ in.

$$A_T = \frac{30345}{18000 \times {\cdot}86 \times 4} = 0{\cdot}48 \text{ in.}^2$$

If the alternate bars of the reinforcement at bottom at mid-span are bent up and brought near the top surface, they would give $\frac{0{\cdot}39}{2} = 0{\cdot}195$ sq. in. of steel. We require 0·285 sq. in. more to make up 0·48. For this provide $\frac{7}{16}$ in. ϕ extra bars at 6 in. c/c. in between the $\frac{1}{2}$ in. bars at 6 in. Their area per ft. is 0·305, the length of these extra bars will be = ($2 \times \frac{1}{4}$ span + top width and end hooks).

Illustrative Example 27

What will be the B. M. on the top of the central support in the above example if the ends of the slab are partially framed into ell-beams?

Solution:—The result of the partial restraint is to cause some positive B. M. over the middle supports. If this is taken as $-\frac{wl^2}{24}$ on the end support, the B. M. at the central support would be from both sides i. e. $2x$.

$$-\frac{wl^2}{8} + 0{\cdot}536\,\frac{wl^2}{24}\S$$

$$= -30345 + \frac{0{\cdot}536 \times 280 \times 8{\cdot}5^2 \times 12}{24}$$

$$= -30345 + 5410 = \mathbf{24935} \text{ in. lb.}$$

Questions and answers on continuous slabs and beams

Question 1. In the theory of continuous slabs and beams one of the assumptions is that the supports are knife-edged at top so that the deflections are unrestricted. What is the effect of a wide top of a support? Similarly, what would happen if one of the supports sinks or settles down?

Answer:—The effect of a wide support is to reduce the negative B. M. over its top by 5 to 8 per cent depending upon

§ Actually this works out to $0{\cdot}5\,\frac{wl^2}{24}$

the length of the span and width of the support, but at the same time the positive B. Ms. at mid spans are increased by the same amount.

If one of the supports sinks very little, the reaction on its top would be reduced and those of supports on either side of it correspondingly increased. As a consequence the negative B. M. on its top would be reduced and the positive B. Ms. at centres of spans on either side correspondingly increased, and worse still, there would be a rapid increase in the moments on top of supports, not only of those adjacent to it, but a few others on both sides though to a progressively less extent. If it sinks so much that its reaction is zero, the span length would be doubled as if there was no support in the centre, causing a heavy deflection and also very heavy positive B.M. in the centre of the large span with possible development of dangerous cracks. Moments on a series of supports on both sides would also be affected. Hence, if foundations are likely to settle down it is safer to design simply supported slabs on the spans.

Question 2. In continuous slabs there is negative B. M. on the top and sides of supports, where should exactly the bars of bottom reinforcement be cranked and why? Is it necessary to use extra steel bars either at top or bottom? What is the most economical arrangement?

Answer:—The principles to be borne in mind are:—

(1) There must be sufficient steel to resist the B. M. and shear.*

(2) There must be adequate grip length provided where bars are to be spliced or anchored.

(3) All this must be done consistent with economy.

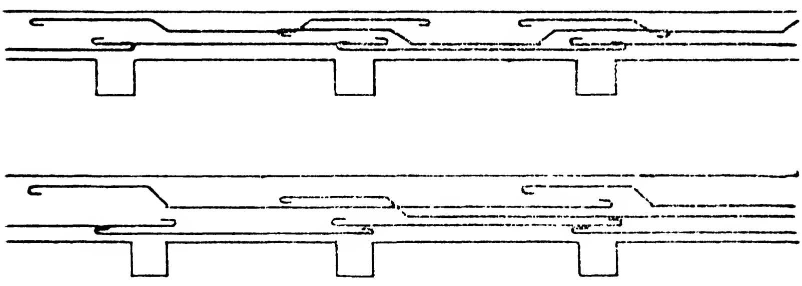
For consideration of B. M. we shall make two cases: (*a*) Intermediate spans made by several supports, and (*b*) end spans.

---

* Discussion on shear force will be made later in special chapters under that caption.

Case (*a*):–In intermediate spans of a continuous system the maximum B. M. is $\pm \frac{wl^2}{12}$ at mid-span or on top of supports. As the B. M. is equal at both these places, the same steel used at bottom at mid-spans could be bent up to keep it on top of supports.

It is economical to use maximum lengths of bars. Excepting one-quarter in. bars, which are of 12 ft. length, the others are usually 18 to 36 ft. long. Still joints are unavoidable. If welded steel fabric rolls are used, they are of 150 or 300 ft. length and they can be simply unrolled and bent at the proper places. This arrangement, viz. the same bars or fabric as at bottom of mid span to be cranked up for negative B. M. on top of supports, does not provide for possible tension which may occur anywhere near the bottom of a slab when traversed by a rolling load as on slabs of road bridges. To meet all these contingencies the following two arrangements are suggested.



Figs. 18 & 19.

In Fig. 18 one bar is cranked at both ends and the alternate bar goes straight at bottom.

In the arrangement suggested in Fig. 19 the bars have one end cranked and the other straight.

Now two points arise: one is where exactly the bar should be cranked up and the other how much over-lap should be provided for splicing rods.

The answer to the first is that theoretically bars should be bent from bottom to top at points where B. M. changes sign, called the points of contraflexure. But these vary according to different conditions of loading. However, we can reasonably assume some position which will be near about.

In the case of simply supported slabs the B. M. does not

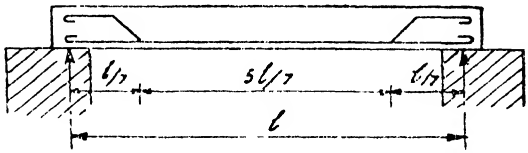
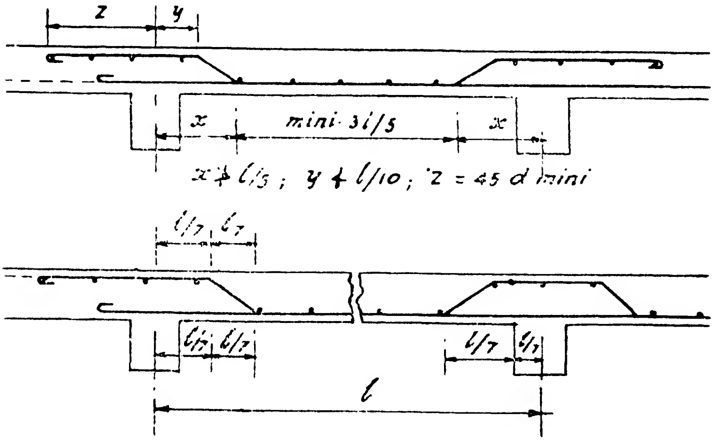


Fig. 20—Cranking of bars in a simply supported span.

change sign, and so theoretically there is no need of cranking the bars upward. But as it is possible that the ends may be partially fixed it is advisable to crank alternate bars at $\frac{l}{7}$ from the supports as shown in Fig. 20 so that if there be any negative B. M. at top they will take care of it.



Figs. 21 & 22. Intermediate spans of a continuous system.

As regards cranking up bars in the intermediate spans of a continuous slab system, there are two most commonly used methods which do not differ much from each other. The first is shown in Fig. 21 in which x should not be greater than $\frac{l}{5}$. In slabs of normal thickness of 3 to 5 in., the bars should preferably be bent at 30 degrees to allow greater flexibility to points of contraflexure, y should not be less than $\frac{l}{10}$ and $z = \frac{l}{4}$ or 45 diams. whichever is greater. The latter is required for purposes of the necessary grip length for anchoring. The other method is shown in Fig. 22 in which the span is divided into 7 parts, three of which are left in the middle and the bars should be cranked up so as to cross the effective depth between $\frac{2l}{7}$ and $\frac{l}{7}$ from the supports. Obviously here too the bars must be bent at 30 deg.

Case 2. End spans. In these the B. M. at centre of span is $\frac{wl^2}{10}$ and that on the top of the penultimate support $-\frac{wl^2}{10}$ i. e. both are 20% more than $\pm\frac{wl^2}{12}$ in intermediate spans. The same steel as in intermediate spans will not therefore suffice unless the slab is made thicker. There are three possible alternatives.

(1) The same thickness as in intermediate spans and extra steel to meet the excess B. M.

(2) Increased thickness of slab for the end spans say ½ in. or so, and

(3) Making the end spans shorter so as to keep B. Ms. equal.

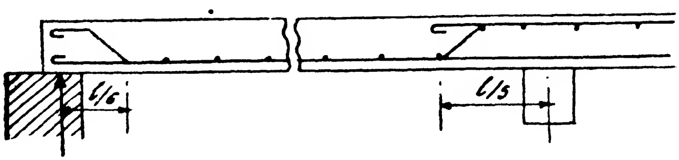


Fig. 23. End span of a continuous system.

In most cases the first alternative is adopted, as we have already done in Illustrative Example 25 by placing extra bars in between the other main bars, and extending them $\frac{l}{4}$ or 45 diameters, whichever is greater, beyond the centre of penultimate support, as snown in Fig. 22.

As regards bending the bars in the end span, on the continuous side the alternate bars should be bent at $\frac{l}{5}$ as shown in Fig. 23. On the free end side they may be cranked up at $\frac{l}{7}$ if the end be free, but it is desirable to bend them at $\frac{l}{6}$ from the supports as shown in the figure to meet slight end restriction.

CHAPTER X

DESIGN OF BEAMS

Singly and Doubly Reinforced

Difference between slab and beam:—There is not much difference between the design of simple (singly reinforced) beams and slabs. A slab is designed for a 12 in. width whereas, a beam is designed for its actual width '*b*'. Other differences are: a slab generally contains economic percentage of steel, while a beam may have much more. Thirdly, a slab is almost invariably strong in shear as the load on it is comparatively light and that too uniformly distributed, while in the case of a beam, which is much heavily loaded it requires to be checked in respect of shear stress, and if the latter goes beyond the permissible limit for concrete, reinforcement for shear must be provided.

There are no thumb-rules, as in the case of slabs for guidance, in respect of the depth of beams, which depends upon the quantity of steel provided. Obviously, the greater the depth, the less is the steel required and more economical is the beam. But usually there are restrictions of head room, and often the economic depth of singly reinforced beams becomes too much. Unless shear and other considerations prevail, $d = 1{\cdot}5\,b$ to $2\,b$ may be adopted.

Effective span of beams:—The effective span of a beam is the same as that of a slab, viz. clear span plus *d* (effective depth) or the distance between the centres of bearing, whichever is lesser.

Protective cover:—The protective cover of concrete in beams must not be less than one inch or the diameter of the main bars, whichever is greater.

Continuous beams:—All the discussion made on pages 73 to 80 in respect of maximum positive and negative bending moments of slabs holds good also in the case of beams, including the approximate coefficients of $\pm \frac{wl^2}{10}$ for B. Ms. at middle of end span, and top of penultimate support, and $\pm \frac{wl^2}{12}$ for middle of intermediate spans and top of intermediate support, and the concession of adjustment of 15 per cent of B. M. envelope between B. Ms. at mid-span and at support allowed by the By-laws and Code.

Hanger bars:—Beams are invariably provided with stirrups. The latter are required for shear and are discussed in the chapter on shear. For hanging these stirrups two bars in the corners at top are required. They are also useful for lifting the assembly of beam bars and placing it in position. The area of these bars, though helpful in increasing the compressive resistance, is not taken into account. These are called *hanger bars*. (See Fig. 24).

Classification of R. C. C. beams:—R. C. C. beams may be classified into four types :—

(1) Singly reinforced beams.

(2) Tee and ell beams used in ribbed floors.

(3) Beams with compressive reinforcement to add to the compressive strength of concrete.

(4) Beams with equal steel in compression and tension in which the resistance of concrete is altogether neglected.

Illustrative Example 28

A simply supported beam over a span of 12 ft. clear carries a uniformly distributed load of 4 cwt. per ft. and a concentrated load of one-half ton at the centre. Design its section and reinforcement.

Solution :—The distance between centres of bearing is not given. We shall therefore assume effective span = 12 + 1·5 = 13·5 ft.

Assuming a section = 10″ × 15″ as the first approximation.

Self load of beam	= 15 × 10	= 150 lbs./ft.
Superload distributed	= 4 × 112	= 448 ,,
Total distributed load		598
say		600
Concentrated load at centre		= 1120 lbs.

B. M. due to distributed load

$$= \frac{wl^2}{8} = \frac{600 \times 13{\cdot}5 \times 13{\cdot}5 \times 12}{8}$$

= 164000 in. lbs. ... ... (i)

B. M. due to concentrated load

$$= \frac{Wl}{4} = \frac{1120 \times 13{\cdot}5 \times 12}{4}$$

= 45360 in. lbs. ... ... (ii)

Total maxi. B. M. = 209360 in. lbs.

$$d = \sqrt{\frac{209360}{126 \times 10}}$$

= 12·9

12·9 + ·375 for $\frac{1}{2}$ diam. of steel + 1 in. cover, overall D = 14·275

say = 15 in., take d = 13·6

$$= \frac{209360}{18000 \times {\cdot}87 \times 13{\cdot}6} = 0{\cdot}95 \text{ sq. in.}$$

4 Nos. $\frac{5}{8}$ in. ϕ bars, A = 1·227 in.²

As the beam is simply supported, theoretically the B. M. at ends will be zero and therefore all these bars need not be taken to the end. Two of them may therefore be bent up at $\frac{1}{7}$ span and brought near the top to resist the shear and small negative B. M. in case the ends may possibly be partially fixed in walls.

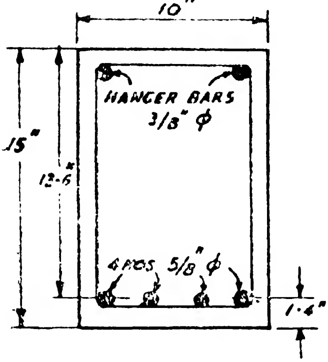


Fig. 24.

In addition to this, two hanger bars, say of $\frac{3}{8}$ in. diam., are required at top to tie the links round for shear. This is discussed in the chapter on shear. The beam section is shown in Fig. 24.

Illustrative Example 29

Design the reinforcement of the beam in the above example if the cross section specified is 9" × 12"

Maxi. B. M. at centre = 209360 in. lbs.

$$A_T = \frac{209360}{18000 \times \cdot 87 \times (12 - 1\cdot 4)}$$

$$= 1\cdot 26 \text{ in.}^2$$

$$3 \text{ Nos. } \tfrac{3}{4} \text{ in. } \phi,\ A = 1\cdot 325 \text{ in.}^2.$$

In the previous example the percentage of steel was 0·804, i. e. at economic limit, in the second it is

$$\frac{1\cdot 325}{9'' \times 10\cdot 6''} = 0\cdot 138\%.$$

Beams with compressive reinforcement:—We have seen in Chapter VI page 43 that the tensile resistance of an R. C. C. beam can be increased easily by increasing the reinforcement, but that it is difficult to increase the compressive resistance. The only ways to do so, we so far knew are: (a) Using high grade or special grade concrete, and (b) Increasing the lever arm, i.e., the depth of the beam. But both these alternatives have their limitations. Oftentimes in practice, beams are subjected to such heavy loads, that far higher stresses are induced in them, than could be resisted even by high or special grade concrete. There are also economic, architectural, and practical limits to the increase in the depth. Again, we have already seen in previous chapters that compressive reinforcement is unavoidable in beams with fixed ends or on top of supports of continuous beams. Under these circumstances, the only way to meet the contingency is to provide steel to take at least part of the compression. Such beams are often called doubly reinforced beams.

The following are a few common cases where a necessity for using compressive steel arises :—

(1) Beams in certain rooms where head room is restricted, e. g. basement and mezzanine floors, or large halls, where too much depth of beams, would reduce the height of ceiling, and give them a stunted appearance.

(2) Top of supports in continuous beams.

(3) Tee, and ell beams in which the narrow stem has to resist compression on top of supports, and it does not alone supply the necessary concrete area.

(4) Beams with very long spans carrying heavy loads, such as under deck bridges, or balconies of theatres, the depths of which have to be restricted, in the first case for fear of obstruction to flow of water, in the latter, of obstruction to view of the stage or screen.

(5) Braces of columns and piles, the former may have to face wind stresses in any direction, and the latter may be handled while lifting and fixing erect with either face subjected to tension.

There are two methods of computing compressive resistance of steel :—

(1) In one, steel reinforcement is provided to add to the resistance supplied by concrete, i. e. the steel and concrete act in conjunction to resist the compressiye stresses developed in a beam.

(2) In the other method the compressive resistance offered by the concrete is altogether ignored, and full reinforcement, equal to the tensile reinforcement, is provided in

the compression flange. Thus the steel at top takes up the entire compression and that at bottom, the entire tension. In other words, the R. C. C. beam behaves as a steel beam with equal area of steel in the top and bottom flanges, the only difference being that there is concrete between the two "flanges".

Additional symbols used :—

A_c = Area of compressive reinforcement in sq. in.

d_c = Distance of the centroid of compression steel from the compressed edge in inches.

f_c = Stress in concrete at c. g. of compressive reinforcement, per sq. in.

a_r = Lever arm of the combined compressive stresses in concrete and in steel in inches.

Design procedure by the first method :—(1) Calculate the R. M. of the concrete in the usual way. R. M. = $\frac{1}{2}\,cbnd \cdot jd$ or 126 bd^2, for $c = 750$, $t = 18000$ and $m = 15$. Subtract the R. M. thus calculated from the B. M. Then compressive steel must be supplied for the balance. Thus if M = B. M., and $(R.\ M.)_c$ = resisting moment of the concrete, then, $(R.\ M.)_s$ i. e. the resisting moment of the compressive steel must equal this or,

$$M - (R.\ M.)_c = (R.\ M.)_s$$

If d_c = distance from the centroid of compressive steel from the edge, i. e. it is the cover of concrete above c. g. of compressive steel, A_c = sum of the cross sectional areas of compressive bars, f_c = stress per sq. in. in concrete at c. g. of compressive steel then stress in steel, must be equal to $m \times$ stress in the surrounding concrete at that place. The total compressive resistance of compressive steel $= A_c \times f_c \times m$.

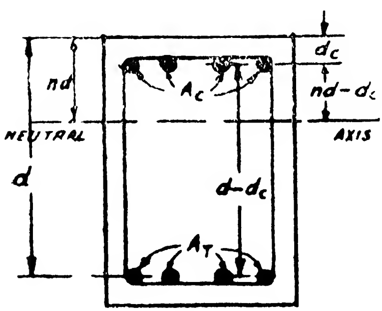


Fig. 25.

As the stress in concrete at $(nd - d_c)$ from the N. A.

$$= \frac{nd - d_c}{nd} \times c$$

the stress in steel

$$= \frac{15(nd - d_c)}{nd} \cdot c$$

Thus the total compressive resistance of steel

$$= \frac{nd - d_c}{nd} \times c \times A_c \,(m-1)$$

since the area of steel added has resulted in displacing an equal area of concrete. As this force is acting at the *c. g.* of the compressive steel, the lever arm is the distance between the *c. g.* of tensile steel, and *c. g.* of compressive steel viz. $d - d_c$ (see Fig. 25). The resistance moment of the compressive steel, or $(R. M.)_s$

$$= A_c \,(m-1) \frac{nd - d_c}{nd} \cdot c \times (d - d_c)$$

Illustrative Example 30

A beam 12 in. × 20 in. (effective d) is subjected to a B. M. of 1,000,000 in. lbs. Calculate the areas of tensile and compressive reinforcement, if the latter is necessary, take the usual stresses and $d_c = \frac{1}{10} d$.

Solution:—Here $nd = \cdot39 \times 20 = 7\cdot8$ in.

$$jd = \cdot87 \times 20 = 17\cdot4 \text{ in.}$$

$$\text{and } d - d_c = 20 - \frac{d}{10} = 18 \text{ in.}$$

$$(R.\ M.)_c = \tfrac{1}{2}\, cbnd.\, jd = \tfrac{1}{2} \times 750 \times 12 \times 7\cdot8 \times 17\cdot4$$
$$= 610800 \text{ in. lb.}$$

As this is less than the B. M., compressive reinforcement is necessary, and it must be supplied for

$$M - (R.\ M.)_c = 1{,}000{,}000 - 610800$$
$$= 389200 \text{ in. lbs.} = (R.\ M.)_s$$

$$f_c = \frac{nd-d_c}{nd} \times c = \frac{7\cdot8-2}{7\cdot8} \times 750 = 558 \text{ lb./in}^2.$$

$$(\text{R. M.})_s = A_c(m-1)\, f_c \times (d-d_c) = 389200 \text{ in. lbs.}$$
$$= A_c \times 14 \times 558 \times 18.$$

$$\therefore \quad A_c = \frac{389200}{14 \times 558 \times 18} = 2\cdot77 \text{ sq. in.}$$

Use 4 nos. 1 in. ϕ bars (A = 3·14.)

Lever arm of the combined compressive stresses

$$a_r = \frac{1,000,000}{3\cdot14 \times 558 \times 14 + \frac{1}{2} \times 750 \times 12 \times 7\cdot8}$$

$$= \frac{1,000,000}{23436 + 35100} = 17\cdot08 \text{ in.}$$

$$A_T = \frac{1,000,000}{18000 \times 17\cdot08} = 3\cdot27.$$

Use 3 Nos. 1 in. ϕ bars	A =	2·356
2 Nos. $\frac{7}{8}$,, ,,	A =	1·202
Total		3·55 sq. in.

Note that the stress in compression steel equals only $15 \times 558 = 8370$ lb./in². This shows that a beam with compressive reinforcement is uneconomical.

Illustrative Example 31

Find the tensile and compressive reinforcement of a beam having $b=9$ in., overall D=27 in. with a minimum protective cover of 2 in. on both sides, to resist a B. M. of 1,500,000 in. lb.

Solution :—Supposing one inch bars are used, the effective $d = 27-(2 \text{ in. cover } +\frac{1}{2}'' \text{ half diam.}) = 24\cdot5$ in. $n = \cdot39 \times 24\cdot5 = 9\cdot6$ in; $jd = \cdot87 \times 24\cdot5 = 21\cdot3$, $(\text{R. M.})_c = 126\, bd^2 = 126 \times 9 \times 24\cdot5^2 = 680,400$ in. lbs. $M-(\text{R. M.})_c = 1,500,000 - 680,400 = 819600$ in.lbs. $= (\text{R. M.})_s$, f_c, stress in concrete at 2·5 in. below top i. e. at $(nd-d\ ) = 9\cdot6-2\cdot5$ or 7·1 in. above N. A.

$$= \frac{7\cdot1}{9\cdot6} \times 750 = 555 \text{ lb./in}^2.$$

The resistance added by the steel, A_c,

$$= 555\,(m-1)\,A_c$$

$$A_c = \frac{819600}{555 \times 14 \times 22{\cdot}0} \quad \text{since } d-dc = 22{\cdot}0$$

$$= 4{\cdot}8 \text{ sq. in.}$$

Use 4 Nos. $1\frac{1}{4}$ in. ϕ bars (A = 4·97 sq. in.) in compression.

a_r, the combined lever arm of the compressive stress in concrete and steel

$$= \frac{1{,}500{,}000}{4{\cdot}97 \times 555 \times 14 + \frac{1}{2}{\cdot}750 \times 9 \times 9\,6}$$

$$= \frac{1500{,}000}{38073+32400} = 21{\cdot}3 \text{ in.}$$

$$A_T = \frac{1{,}500{,}000}{18000 \times 21{\cdot}3} = 3{\cdot}92 \text{ sq. in.}$$

5 Nos. 1 in. ϕ bars, (A = 3·93 sq. in.)

Note that the compressive stress in steel

$$= 555 \times 15 = 8325 \text{ lb./in.}^2 \text{ only,}$$

which is less than half the permissible maximum stress, this proves again that doubly reinforced beam is uneconomical.

Illustrative Example 32

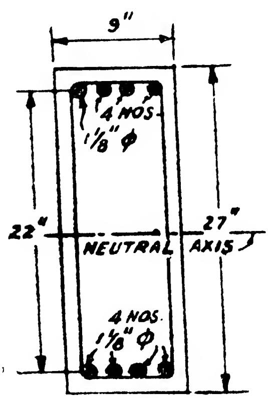


Fig. 26.

Solve the above example by the Steel Beam Theory and compare the quantities of steel required in both.

Solution:—b = 9 in., d effective = 24·5 in., $d - dc$ = 22·0 in. lever arm. As there is to be equal steel at top and bottom, the neutral axis will be mid-way in the section, and the lever arm will be = 22 in. as shown in Fig. 26.

$$\text{B. M.} = \text{R. M.} = A_T \text{ or } A_c \times 18000 \times 22$$

$$A_T = \frac{1,500,000}{18000 \times 22} = 3{\cdot}8 \text{ sq. in.}$$

Use 4 Nos. $1\frac{1}{8}$ in. ϕ bars ($A = 3{\cdot}976$ sq. in.) Total steel in tension and compression $= \mathbf{7{\cdot}6}$ sq. in.

The total steel in the previous example $= 4{\cdot}8 + 3{\cdot}92 = \mathbf{8{\cdot}72}$ sq. in.

The design on the basis of steel beam theory is therefore economical.

The stress in concrete round the reinforcement either compressive or tensile $= \frac{18000}{15} = 1200$ lb./in.^{2} and that at the extreme fibre which is 2·5 in. further from the *c. g.* of steel, or 13·5 in. away from the N. A. is

$$= \frac{1200 \times 13{\cdot}5}{11} = 1473 \text{ lb./in.}^2$$

which is a little less than double the permissible maximum compression in concrete of 1 : 2 : 4 mix ordinary grade.

The reason why the design on the basis of Steel Beam Theory is more economical is that the stress in compressive steel is 18000 lb./in.^{2}, whereas when steel is supposed to act in conjunction with concrete it is less than 600×15 or 9000 lb./in.^{2} maximum. (In the two examples solved above it was 558×15 and 555×15 only respectively).

It is therefore obvious that as the difference between B. M. and (R. M.)$_c$ of the concrete area reaches a certain limit it is more economical to neglect the compressive value of concrete. This limit could be found by assuming that the lever arm is the same in both cases (the error is negligible). In that case,

$$\frac{\text{B. M.}}{t \times jd} = \frac{\text{B. M.} - (\text{R. M.})_c}{f_c\,(m-1) \times jd}$$

$$\text{or} \quad \frac{\text{B.M.} - (\text{R. M.})_c}{\text{B. M.}} = \frac{f_c\,(m-1)}{t} = \frac{600 \times 14}{18000}$$

(Assuming $f_c = 600$)

$$= 0{\cdot}47$$

or $\text{B. M.} - (\text{R. M.})_c = 0{\cdot}47 \text{ B. M.}$

or $(\text{R. M.})_c = 0{\cdot}53 \text{ B. M.}$

This means that when the R. M. of concrete is less than 0·53 times the B. M. it is more economical to disregard the compressive value of concrete and treat the R. C. C. beam virtually as steel beam.

Thus in Example 31 above the $(\text{R. M.})_c$ of 6,80,000 was 45 per cent of the B. M. (1,500,000) i. e. less than 53 per cent and we have seen that the design by Steel Beam Theory was cheaper.

Illustrative Example 33

A column brace has a width of 10 in. and effective depth also of 10 in. It is subjected to a maximum B. M. of 1,50,000 in. lb. If the stresses in concrete and steel are not to exceed 750 and 18000 lb./in.^{2} respectively with $m = 15$ design the compressive and tensile reinforcement. Take $d_c = 1{\cdot}5$ in.

Solution:—As the stress in concrete is not to exceed 750 lbs. steel beam theory method is out of question here,

$$(\text{R. M.})_c = 126\, bd^2 = 126 \times 10 \times 10^2$$
$$= 126000 \text{ in. lbs.}$$

$$(\text{R. M.})_s = \text{B. M.} - (\text{R. M.})_c = 150000 - 126000$$
$$= 24000 \text{ in. lb.}$$

$nd = {\cdot}39 \times 10 = 3{\cdot}9$; $jd = {\cdot}87 \times 10 = 8{\cdot}7$; $d - d_c = 8{\cdot}5$

stress in compressive steel

$$= \frac{nd - d_c}{nd} \times (m-1)\, c$$

$$= \frac{3{\cdot}9 - 1{\cdot}5}{3{\cdot}9} \times 14 \times 750 = 6460 \text{ lb./in}^2.$$

$$A_c = \frac{24000}{6460 \times 8{\cdot}5} = 0{\cdot}436 \text{ sq. in.}$$

Use 4 Nos. $\frac{3}{8}$ in. ϕ bars, A = 0·442 sq. in.

$$\text{combined lever arm } a_r = \frac{150{,}000}{0{\cdot}442 \times 6460 + \frac{1}{2}{\cdot}750 \times 10 \times 3{\cdot}9}$$

$$= \frac{150000}{17442} = 8{\cdot}6 \text{ in.}$$

$$\text{Tension steel, } A_T = \frac{150000}{18000 \times 8{\cdot}6} = 0{\cdot}97.$$

Use 3 Nos. $\frac{3}{4}$ in. ϕ bars A = 1·32 sq. in.

or 4 Nos. $\frac{5}{8}$,, ,, ,, A = 1·228 sq. in.

$$\text{Stress in steel} = \frac{6460}{14} \times 15 = 6930 \text{ lb./in.}^2$$

Here $(R.\ M.)_c$ is $\frac{126000}{150000}$ of B. M. or ·84 of B. M. Hence Steel Beam Theory if applied would have proved uneconomical.

Illustrative Example 34

If the brace in the above example were subjected to 2,50,000 in. lbs. B. M. design the compressive and tensile reinforcement by both the methods, and compare the results. Also find the maximum stress in steel when concrete acts in conjunction with steel and that in concrete in the Steel Beam Theory design.

Solution:—Here B. M. =2,50,000 in. lb. and $(R.\ M.)_a$ =126,000 in. lb. The latter is a little more than 50 per cent i. e. less than 53 per cent. Hence, there will not be much economic advantage if Steel Beam Theory is applied.

(i) *By Steel Beam Theory*

$$A_T \text{ or } A_c = \frac{250000}{18000 \times 8{\cdot}5} = \mathbf{1{\cdot}65} \text{ in.}^2$$

Total reinforcement $= 2 \times 1{\cdot}65 = 3{\cdot}3$ sq. in.

Stress in concrete at 1·5 in below top

$$= \frac{18000}{15} = \mathbf{1200\ lb./in.^2}$$

$$\text{Stress at top} = \frac{1200 \times 5{\cdot}0}{4{\cdot}25} = \mathbf{1412\ lb./in.^2}$$

(ii) *By usual method*

$$(R.\,M.)_s = B.\,M. - (R.\,M.)_c = 250000 - 126000$$

$$= 124000 \text{ in. lb.}$$

$$A_c = \frac{124000}{6460 \times 8{\cdot}5} = 2{\cdot}25 \text{ in.}^2$$

$$a_r = \frac{250000}{2{\cdot}25 \times 6460 + \frac{1}{2}{\cdot}750 \times 10 \times 3{\cdot}9}$$

$$= \frac{250000}{14535 + 14625} = 8{\cdot}6 \text{ in.}$$

$$\text{Tensile steel, } A_T = \frac{250000}{18000 \times 8{\cdot}6} = 1{\cdot}39 \text{ in.}^2$$

Total reinforcement $= 2{\cdot}25 + 1{\cdot}39 = 3{\cdot}64$ in.^{2}.

TABLE No. 17

Method	Total steel sq. in.	Stress in concrete lb./in.^{2} in 1st method	Stress in compressive steel lb./in.^{2} in 2nd method
Steel Beam Theory	3·3	1200	18000
Standard method	3·64	600	6930

Caution:—It should be noted that though the method based on Steel Beam Theory is very simple and allowed by the Building Bylaws and Code of Practice, a certain caution is required in using it indiscriminately, since with increasing percentage of steel the "Steel Beam Theory" gives very high resisting moments as shown in the following table:—

TABLE No. 18

COMPARISON OF BEAM STRENGTHS CALCULATED BY STEEL BEAM THEORY AND THE USUAL METHOD WITH CONCRETE SUPPOSED TO ACT IN CONJUNCTION WITH STEEL.

Percentage of steel $A_T = A_C$	Beam strength based on Steel Beam Theory	Beam strength based on concrete and steel acting together
1 per cent	162 bd^2	201·6 bd^2
1·5 ,,	243 bd^2	239·4 bd^2
2 ,,	324 bd^2	277·6 bd^2
3 ,,	486 bd^2	352·6 bd^2
4 ,,	648 bd^2	426·4 bd^2
5 ,,	810 bd^2	504·0 bd^2

As concrete has to be filled from the top into beams, the top bars of beams designed on steel beam theory are bound to come in the way of filling and compacting concrete particularly on top of columns, where there is unavoidable crowding of reinforcing bars, and therefore air-pockets and even hollows are likely to be formed inside. Consequently the bond between steel and concrete, and anchoring of ends, which are the essence of R. C. C. design and construction, are bound to be imperfect. Under these circumstances it is problematic whether the calculated theoretical high moments even for moderately high percentages of steel based on the steel beam theory would be realised in practice.

Authorities on R. C. C. design agree in advising that the steel beam theory should not be applied to beams with more than 3 per cent reinforcement, without applying a further factor of safety.

When compressive reinforcement is used, the stirrups shall not be spaced further apart than 12 times the diameter of the compressive bars, when the compressive strength of the concrete is taken into account, and when the beam is designed on Steel Beam Theory the spacing of stirrups shall not exceed 8 times the diameter of compressive bars. This closer spacing is specified, because, thin bars, as are used for reinforcement are susceptible to buckle and bulge out, when compressed from both ends. The stirrups or links serve the same purpose as binders do in columns.

Practice Problems

(1) A beam 30 in. deep is subjected to a B. M. of 200,000 in. lb. and the R.M. of concrete is 100,000 in. lb. Given $d_c = 1{\cdot}5$ in. (a) Find the area of compressive reinforcement. (b) Find also the area of reinforcement by steel beam theory method.

Answers :—(a) 4·07 in.^{2} (b) 4·95 in.^{2}

(2) A beam 12 in. × 23 in. (effective) d is subjected to a B. M. of 10,50,000 in. lbs. If $d_c = 2$ in. find the compressive reinforcement.

Ans :—1·46 sq. in.

(3) A beam 8 in. × 16 in. (overall) is subjected to a B. M. of 240,000 in. lbs. If the concrete cover beyond c. g. of steel is 2 in. both at top and bottom, find the compressive and tensile reinforcements.

Ans :—0·53 and 1·39 sq. in.

(4) A beam designed on the principle of steel beam theory has a total reinforcement in tension and compression of 20 sq. in. If $d = 48$ in., $d_c = 3$ in., $c = 750$, $t = 18000$ and $m = 18$, find the R. M. and compressive stress in concrete.

Ans :—R. M. = 8,100,000 in. lb.; $c = 1160$ lb./in.^{2}

CHAPTER XI

DESIGN OF TEE AND ELL BEAMS

It is common experience that a plane iron sheet placed flat on supports deflects very much even under its own weight, but that a corrugated iron sheet of the same weight and thickness can carry a considerable load on the same span in addition without appreciable deflection. Since R. C. C. floor can be poured with the slab and beams at the same time as one unit, the ribs, on the above principle make it much stiffer with a lighter section of slab than the same size rectangular beam having no assistance from the slab on its top.

A ribbed R. C. C. floor is commonly called *tee-beam* floor or flanged floor. The part of the floor projecting downwards is often called a '*rib*', '*web*' or '*stem.*' When there is a flange only on one side, as in the case of outside beams, the shape taken by the beam is like that of an inverted L, and hence it is called an *ell-beam.* Of all the forms of R. C. C. floors such as a solid slab, beam and slab floor, flat or beamless floor, etc., a tee-beam floor is the most economical, since there is a considerable saving in the mass of concrete resulting in saving in dead load and consequently in reinforcing steel also as shown in the example which follows.

A typical tee-beam and ell-beam floor is shown in Fig. 27.

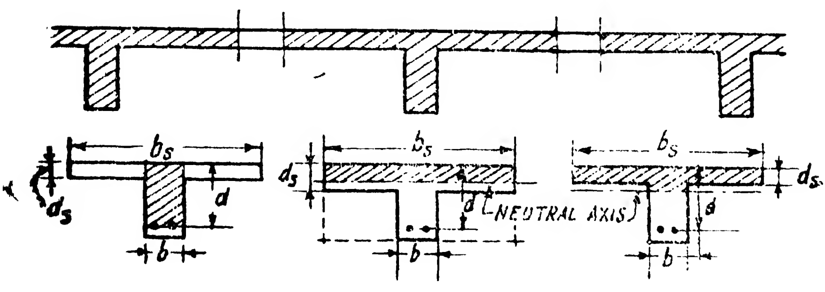


Fig. 27.

The symbols used are the same as before except the following additions:

b_s = width of the flange

d_s = thickness of slab.

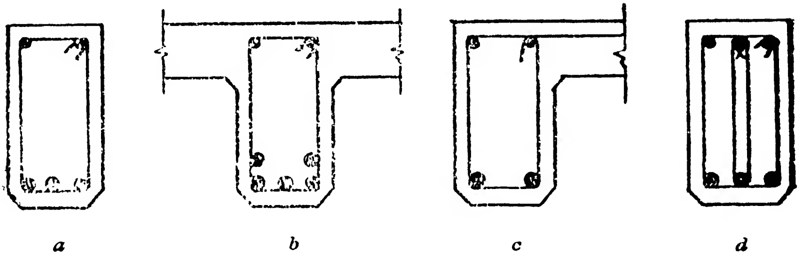


Fig. 28.

a.—rectangular beam ; *b*.—tee beam ; *c*.—ell beam ; *d*.—rectangular beam with double stirrups.

Illustrative Example 35

A tee-beam floor consists of a slab $4\frac{1}{2}$ in. thick and a rib 10 in. wide and 20 in. deep from the top of slab to the bottom of the stem with effective $d = 18$ in. It is reinforced with 8 Nos. m. s. ϕ bars placed in two tiers. Compare the quantity of concrete required with that of a rectangular beam of the same resisting moment if the permissible maximum stresses are 750, and 18000 and $m = 15$.

The tee beam floor is illustrated in Fig. 29.

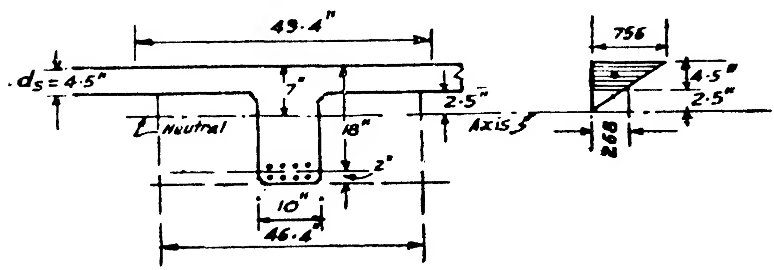


Fig. 29.

Solution:—For the given stresses

$$n = \cdot 39 \quad \therefore \; nd = \cdot 39 \times 18 = 7{\cdot}02 \text{ in.}$$

$$\text{say} \quad = 7{\cdot}00$$

If the small compressive stress in the portion of the rib between the neutral axis and the bottom of the slab is neglected, we have the stress at the top of the slab = 750 lbs./in.^{2} and that at the bottom i. e. at $(7-4{\cdot}5) = 2{\cdot}5$ in. above the N. A.

$$= \frac{2{\cdot}5}{7} \times 750 = 268 \text{ lbs./in.}^2$$

$$\text{Average stress} = \frac{750+268}{2} = 509 \text{ lbs./in.}^2$$

acting at the centroid of the trapezium shown shaded in the stress diagram. Taking moments about the top of the slab, the centroid is at

$$\frac{4{\cdot}5 \times 2{\cdot}68 \times 2{\cdot}25 + \dfrac{4{\cdot}82 \times 4{\cdot}5}{2} \times \dfrac{4{\cdot}5}{3}}{\dfrac{7{\cdot}5 + 2{\cdot}68}{2} \times 4{\cdot}5}$$

$$= \frac{27{\cdot}13 + 16{\cdot}25}{22{\cdot}90} = 1{\cdot}9 \text{ in.}$$

The lever arm $= 18 - 1{\cdot}9 \quad = 16{\cdot}1$ in.

Total tension = area of 8 Nos. 1 in. ϕ bars $\times$ 18000

$= 8 \times \cdot 7854 \times 18000$

$= 113100$ lbs.

The compressive stress in the flange per inch

= average stress $\times \, d_s$.

$= 509 \times 4{\cdot}5 = 2290$ lb./per inch.

The width of the flange required to balance the total tension

$$= \frac{113100}{2290} = \mathbf{49{\cdot}4} \text{ in.} \qquad (1)$$

If instead of the tee beam and slab it were a rectangular beam of the same effective depth viz. 18 in., the breadth required to develop so much resistance would be

$$b_s = \frac{\text{M. R.}}{126d^2} = \frac{113100 \times 16{\cdot}1}{126 \times 18 \times 18}$$

$$= \mathbf{44{\cdot}6} \text{ in.} \qquad (2)$$

This width is shown in broken lines in the figure.

The concrete required for one foot of the tee-beam and slab

$$= 49{\cdot}4 \times 4.5 + 10 \times (20 - 4{\cdot}5)$$

$$= 222{\cdot}3 + 15{\cdot}5 = \mathbf{377} \text{ cub. in.} \qquad (1)$$

The concrete required for the rect. beam of equivalent strength $= 44{\cdot}6 \times 20 =$ **892** cub. in. (2)

Difference = **515** cub. in.

The above is a typical example of a tee-beam floor. Its analysis shows that

(a) The slab and beam are treated as one unit, the slab or flange supplying the area for compression and the narrow, elongated stem providing the lever arm, thus each part contributing to the strength of the floor.

(b) The area of concrete for compression is spread in a narrow band called the flange and is placed as near the top as possible where the stresses are higher, so that the entire area is intensively stressed within permissible limits. In the usual rectangular beam the area at a little above the N. A. is very lightly stressed and does not materially contribute to the compressive strength.

(c) The stem is made narrow so that the concrete below the N. A. which is supposed to take no tensile stress is reduced to a minimum.

(d) This arrangement, viz. of pushing the concrete area for compression towards the top and the steel area towards bottom, both as far away from the N. A. as possible has resulted in increasing the lever arm.

The above are the reasons of economy of tee-beam floor. The analysis further shows that

(e) If the area of steel is increased, the area of concrete which takes the compression must also be increased to balance the design. This could be done either by increasing the depth or the breadth of the flange.§ The latter helps materially, but there are restrictions on the increase as will be shortly discussed.

(f) In the illustrative example worked out above we have neglected the small compressive stress in the stem between the N. A. and the bottom of the flange. The area is $10'' \times 2{\cdot}5 = 25$ sq. in. The average stress measured at half the distance of this i. e. 1·25 from the N. A. is

$$= \frac{750 \times 1{\cdot}25}{7} = 134 \text{ lbs./in.}^2$$

The total compression in that portion $= 25 \times 134 = 3350$ lb. This, compared with the total compression developed in the 49·4 in. of flange, viz. 113100 lbs., is very small and could be safely neglected in practical designs, as it very much simplifies calculations. Besides, the design is on the safe side by neglecting it.

(g) The centroid of the concrete flange stress area is 1·9 in. below the top. If we take it half way down the flange i. e. at 2·25 in., the error of 0·35 in. in the lever arm of 16·1 in. is very small and could not appreciably affect the result. If at all it also makes the design safer to a slight extent. This assumption enables the length of the lever arm to be determined at once and simplifies the design.

The restrictions to the breadth b_s, of the flange referred to in (*e*) above by the By-law and Code of Practice * are:

The breadth of the flange of tee-beams taking compression shall not exceed the least of the following :—

(a) One-third of the effective span of tee-beam, i. e. b_s shall not be greater than $\frac{\text{span}}{3}$.

§ There is a third way of increasing compressive resistance viz. introducing steel to take up additional compression. This has been already discussed in the preceding chapter.

* The American practice is that $bs \not> \frac{\text{span}}{3}$ or spacing or $16ds + b$.

(b) The distance between centres of adjacent ribs, i. e. no part of the slab shall be considered common to two beams; b_s shall not be greater than the spacing of beams.

(c) The breadth of the rib plus 12 times the slab thickness i. e. b_s shall not be greater than $12d_s + b$.

In the case of ell-beams the breadth of the flange shall not exceed the least of:

(a) $\frac{\text{span}}{6}$; (b) $\frac{\text{spacing}}{2} + b$ and (c) $4d_s + b$.

There are two cases into which tee beams fall:

(a) When the neutral axis falls within the slab. (Fig. 27 b).

(b) When the neutral axis falls outside the slab (Fig. 27 c).

The design of tee and ell beams with the N. A. falling within the flange is identical with the design of rectangular beams in which b_s takes the place of b. When the N. A. falls outside the flange the only modification necessary for very approximate practical design is (1) to neglect the small compressive resistance in the area of concrete between the N. A. and the underside of the flange and (2) to take the length of the lever arm $= d - \frac{d_s}{2}$ as already explained in (f) and (g) above.

The following two illustrative examples will make the above points clear:

Illustrative Example 36

Find the R. M. of tee beams placed with centres 8 ft. apart across an effective span of 20 ft. with the flange 7 in. deep, and rib=10 in. by 18 in. measured from the top of the slab to bottom. Take the usual stresses of 750 and 18000 lb. per sq. in. and $m = 15$. Also calculate the reinforcement required.

Solution :—First to find b_s

(1) Spacing of ribs centres = 96 in.

(2) $\frac{\text{span}}{3} = \frac{15 \times 12}{3} = 60$ in.

(3) $12d_s + b = 84 + 10 = 94$ in.

The least of these is 60 in. hence $b_s = 60$ in.

Effective $d = 18 - 1{\cdot}5$ say, for cover $= 16{\cdot}5$ in.

Hence the N. A. is at $nd = {\cdot}39 \times 16{\cdot}5 = 6{\cdot}4$ in.

The N. A. falls *within* the slab. The design is therefore just like that of ordinary rectangular beam.

$$\text{Lever arm } jd = d\left(1 - \frac{n}{3}\right) = 16{\cdot}5\left(1 - \frac{{\cdot}39}{3}\right)$$
$$= 14{\cdot}4 \text{ in.}$$

$$\text{R. M.} = 126\, bd^2$$
$$= 126 \times 60 \times 16{\cdot}5^2$$
$$= 126 \times 60 \times 272$$
$$= \mathbf{2056320} \text{ in. lbs.}$$

$$A_T = \frac{2056320}{18000 \times 14{\cdot}4}$$
$$= 7{\cdot}93 \text{ sq. in.}$$

Provide 8 bars $1\frac{1}{8}$ in. ϕ in two tiers, area $= 7{\cdot}95$ sq. in.

Illustrative Example 37

A floor consists of tee-beams spaced at 8 ft. between centres with a slab 6 in. thick, rib 10 in. × 22 in. measured from the top of slab, to the bottom; The effective span is of 18 ft. The reinforcement consists of 8 bars $1\frac{1}{8}$ in. round in two tiers. Check the moment of resistance in compression.

Solution :—Breadth of flange:

(1) $\frac{\text{Span}}{3} = \frac{18 \times 12}{3} = 72$ in.

(2) Spacing $= 8 \times 12 = 96$ in.

(3) $12d_s + b \cdot = 12 \times 6 + 10 = 82$ in.

72 in. is the least $\therefore$ $b_s = 72$ in.

Effective d = 22 in. minus (one tier $1\frac{1}{8}'' = 1{\cdot}125 + \frac{1}{2}$ distance between the tiers, say ·5 + cover 1 in. = 2·625) say 2·5 = 19·5 in.

The N. A. is at $nd = {\cdot}39 \times (22 - 2{\cdot}5) = 7{\cdot}6$

The N. A. falls *outside* the slab.

$$\text{Lever arm } jd = d - \frac{d_s}{2} = 19{\cdot}5 - 3 = 16{\cdot}5 \text{ in.}$$

Compressive stress at top of slab = 750 lbs./in.²

do at bottom i. e. at (7·6 − 6) or 1·6 from N. A.

$$= \frac{750 \times 1{\cdot}6}{7{\cdot}6} = 158 \text{ lbs./in.}^2$$

$$\text{average} = \frac{750 + 158}{2} = 454 \text{ lbs./in.}^2$$

Compressive stress per inch width of flange

$= 454 \times 6 = 2724$ lbs.

Total tension = Area of 8 bars of $1\frac{1}{8}''$ diam. $\times$ 18000

$= 8 \times 18000 = 144000$ lbs.

Flange width required to balance this

$$= \frac{144000}{2724} = 52{\cdot}8 \text{ say } 53 \text{ in.}$$

As this is less than 72 in., the maximum permissible width, the design is sound.

The moment of resistance

$= 144000 \times 16{\cdot}5$ (lever arm)

= **2376000 in. lbs.**

Procedure in practical design :—(1) In most cases span and loads are known from the framing plan, and therefore, the B. M. can be calculated.

(2) The next step is to determine t, the thickness of the slab which should be between 4 and 6 inches according as the B. M. is light or heavy. With very heavy loads it may be even 7 inches but generally not more than that.

(3) The third step is to determine the size of the rib i. e. its width and depth. Since the resisting moment varies as the square of the depth, the latter is of greater importance. The section of the rib should be sufficiently wide (*a*) to accommodate steel reinforcement in a single tier and sufficiently deep if in more than one tiers and (*b*) sufficient to resist the shear. The latter is, in fact, the deciding factor and is discussed later in the chapter on design of beams. It is only the rib which is supposed to take the entire shear. For its computation the depth of the rib is measured from the top of the slab. As a rule, for accommodation of steel bars the width should be 2½ times the sum of the diameters of bars in one tier. Thus if there are 8 bars of one inch in two tiers i. e. 4 bars in one tier, the minimum width should be $4 \times 2\frac{1}{2} = 10$ in.

The greater the depth, the greater the lever arm and consequently, less the quantity of steel required. However, there may be restrictions on the increase of depth due to head room. Further, the depth may be governed by the consideration of the depth of main beams, spacing of columns, etc. There are also economic considerations, for which alternate rough estimates for different framing plans must be made. Increasing the spacing between columns, requires deeper main beams and either a fewer deeper secondary beams with heavy reinforcement or a greater number of shallower ones with light reinforcement. For rough estimates reinforcement may be taken at 2 to 3 per cent of the rib section of secondary beams and 4 to 5 per cent of the main beams.

Economic depth:— If the ratio of cost of steel to cost of concrete is r, it can be proved with certain reasonable assumptions that the most economical depth of lever arm

$$= \sqrt{\frac{\text{R.M.} \times r}{b \times t}}$$

where b = width of rib and t = stress in lb./in.^{2} in steel. If $r = 60$ as at present and $t = 18000$, the equation reduces to

$$\text{lever arm} = \sqrt{\frac{\text{R. M.}}{300\ b}}$$

As a rough rule the width of the rib should be a minimum of twice the flange thickness, and depth, 4 to 6 times i. e. $b = 2d_s$ and $d = 4d_s$ to $6d_s$ subject to special considerations such as of head room restriction, economic framing lay out, shear, etc. Under head-room restrictions, the shear strength can be increased by increasing the width of the rib.

Another rough rule for guidance is that the depth should be $= \frac{\text{span}}{8}$ for simply supported beams, $\frac{\text{span}}{10}$ for semi-continuous beams and $\frac{\text{span}}{12}$ for continuous beams.

(4) When the depth is decided on, the position of the neutral axis and the length of the lever arm could be easily calculated, the distance of the N. A. below the top of slab is $= nd$, n depends upon the ratio of stresses and the particular modular ratio. For 750 and 18000 lb. stresses and $m=15$, $n = \cdot 39$. For determining the lever arm, the c. g. of the compression flange may be safely taken at mid-depth of the slab.

(5) When this is done, calculate the average compressive stress at mid-depth of the slab and if the small amount of compressive stress in the rib above N. A. is neglected, the product of the depth of flange and average compressive stress gives the compressive stress per inch width of the flange.

(6) Next step is to divide the B. M. already calculated by the compressive stress per inch obtained and find out the flange width required. If this is equal to or less than the least of the three stipulated conditions viz. $\frac{\text{span}}{3}$, spacing, or $12d_s + b$, the design is sound. If not, consider the alternative proposals as follow:—

(a) If there are no restrictions of head room, increase the depth of the rib.

(b) If the difference is not much, a small increase in d_s, the thickness of slab, will help matters considerably.

(c) Failing both the above, provide compression steel at top to take the balance of B. M. The method will be clear from the examples solved hereafter. Sometimes a combination two or more of the above three alternatives is adopted.

(7) The last step in the design is to calculate the area of steel reinforcement. For this divide the B. M. by the product of the permissible tensile stress in steel and the lever arm.

$$A_T = \frac{\text{B. M.}}{t \times jd} = \frac{\text{B. M.}}{18000 \times \cdot 87d}$$ (for the stresses of 750 and 18000 and $m = 15$).

Reinforcement in slab of tee beams:—Both the Bylaws and Code of Practice recommend that reinforcement must be provided in the portion of the slab used as the compression flange, and must be placed transversely to the beam in the top of the slab. Such reinforcement must not be less than 0·3 per cent of the cross sectional area of the slab, and must extend over the full width of the slab used as the compression flange. As this reinforcement is of the nature of the main reinforcement, the spacing of bars should not exceed twice the effective depth of the slab.

Thus for a slab of 4·5 in. thickness, the reinforcement would be

$$\frac{4\cdot5 \times 0\cdot3 \times 12}{100} = 0\cdot162 \text{ sq. in.}$$

If the effective depth of a 4·5 in. slab is taken as 3·75 in., the spacing should not exceed $7\frac{1}{2}$ in. For this particular case $\frac{3}{8}$ in. bars at 7·5 in. give an area of 0·177 or $\frac{5}{16}$ in. bars at $5\frac{1}{2}$ in. give 0·167. The length of these bars placed transversely across the tee-beam would be, if the stem is 9 in. wide, 12 times the thickness of slab plus rib width

$$= 12 \times 4\cdot5 + 9 = 5 \text{ ft. } 3 \text{ in.}$$

Note that this reinforcement is required only in the portion of the tee beam under compression. It is often convenient and economical to bend up the distribution steel bars to provide part of this reinforcement at top.

When a tee beam is continuous over several spans, the thin flange is in tension and the lower portion of the stem is in compression over top of supports. As regards tension there is no difficulty, but the concrete area of the stem is not sufficient to cope with the total compressive force. The remedies suggested are:

(a) Providing a haunch as shown in Fig. 30.

(b) Providing compression reinforcement to supplement the compressive resistance of the concrete. Oftentimes a combination of these two methods is employed.

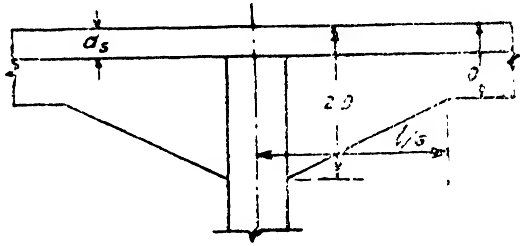


Fig. 30.

A safe practical rule governing the depth and the length of the splay of the haunch is that the depth should be equal to twice the normal depth of the beam at midspan, and the length of the splay should be equal to $\frac{l}{6}$ measured from the centre line of the support, l being the length of the span. When the span is short and beam in consequence shallow, the splay should not exceed 30° with the horizontal.

Illustrative Example 38

A tee beam floor of an office building has a slab 5 in thick topped with 1½ in. mortar screeding and one in. cement tiles; the effective span is 21 ft. between centres of supports. The beams are spaced at 9 ft. centres. Design the floor.

Solution:—The area of floor supported by a beam = 9 × 1 = 9 sq. ft./rft. of span

Dead loads: Slab 5″	= 60 lbs.	
1·5″ mortar screeding	= 12 „	
1″ cement tiles	= 10 „	
Total	82 lbs.	
Per rft. of span	= 9 × 82 = 738 lbs.	

Assume rib section 10″ × 20″ ;
($b=2d_s=10$ in. $d=4d_s=20''$)

load of rib 10″ × 20″	= 200 lbs.	200 lbs.
	Total dead load	938
	say,	940 lbs./rft.

Superload = 80 lbs./sq. ft. (Vide Table No. 3.)

9 × 80	= 720	lbs./rft.
Total load	1660	lbs./rft.

Effective span = 21 ft.

maximum B. M. at mid span $= \dfrac{wl^2}{8}$

$$= \frac{1660 \times 21 \times 21}{8} \times 12$$

$$= 1098000 \text{ in. lbs.}$$

Economic depth of lever arm $= \sqrt{\dfrac{\text{B. M.}}{300\,b}}$

$$= \sqrt{\frac{1098000}{300 \times 10}}$$

$$= 19 \text{ in.}$$

d_s, slab thickness (given) = 5 in.

$$\therefore \quad d - \frac{d_s}{2} = 19 \text{ in.}$$

$\therefore \quad d = 19 + 2{\cdot}5 = 21{\cdot}5$ in. (Effective)

Add 1·5 in. for reinforcement and cover.

$\therefore$ overall depth, D = 23 in.

In the calculations of dead load we have assumed the rib = 10" × 20". It is now 10" × 23".

The increase in dead load = 10 × 3 = 30 lbs./rft.

$$\text{Addition to B. M.} = \frac{30 \times 21 \times 21}{8} \times 12$$

$$= 19845 \text{ say, } 20000$$

Revised B. M. 1098000 + 20000 = 1118000 in. lbs.

The N. A. is at $nd = {\cdot}39 \times 21{\cdot}5 = 8{\cdot}4$ in. below top

$\therefore$ N. A. falls outside the flange.

The average compressive stress occurs at 2·5" below top or at (8·4 − 2·5) = 5·9 in. above the N. A. and is equal to

$$\frac{5{\cdot}9}{8{\cdot}4} \times 750 = 527 \text{ lbs./in.}^2$$

Compressive stress per inch width of flange

$$= 527 \times 5 = 2635 \text{ lbs.}$$

Flange width required to resist the B. M.

$$= \frac{1118000}{2635 \times 19} \text{ (lever arm)}$$

$$= 22{\cdot}3 \text{ in. only.}$$

The permissible flange width :

(1) $\frac{\text{span}}{3} = \frac{21 \times 12}{3} = 84$ in.

(2) Spacing = 9 × 12 = 108 in.

(3) $12d_s + b = 12 \times 5 + 10 = 70$ in.

Our designed width of 22·3 is much less than the least of the above. Hence the design is sound. The slab is very lightly stressed.

Reinforcement. $A_T = \frac{1118000}{18000 \times 19} = 3{\cdot}27$ sq. in.

Either 2 Nos. $1\frac{1}{8}$ in. $\phi = 1{\cdot}99$

and 2 Nos. 1 in. $\phi = 1{\cdot}57$

Total **3·56 in one tier**

or 8 Nos. $\frac{3}{4}$ in. $\phi =$ **3·53 in 2 tiers of 4 each.**

Illustrative Example 39

A T-beam has a flange 4 in. thick the maximum allowable width 42 in. If it has to resist a B. M. of 5,50,000 in. lb., design the reinforcement for $c=750$, $t=18000$, $m=15$.

Solution:—Here the span is not given, nor are there any restrictions to the depth of the beam. Supposing it to be a normal case, we shall assume b, width of rib $=2{\cdot}5\ d_s=10$ in. and $d=4d_s=16$ in.

$nd=16\times{\cdot}39=6.24$ in.

The N. A. falls outside the flange.

$jd={\cdot}87\times16=13{\cdot}92$ in.

For a balanced design Total compression = Total tension

$$= \frac{550{,}000}{13{\cdot}92} = 40{,}000 \text{ lbs.}$$

Average compression in flange

$$= \frac{4{\cdot}24}{6{\cdot}24} \times 750 = 510 \text{ lbs.}$$

Total compression per inch width of flange

$$= 510 \times 4{\cdot}0 = 2040$$

Minimum width of flange $= \frac{40000}{2040} = 19{\cdot}6$ in.

$$A_T = \frac{40000}{18000} = 2{\cdot}22 \text{ sq. in.}$$

4 Nos. $\frac{7}{8}''$ $= 2{\cdot}41$ sq. in.

Illustrative Example 40

A T-beam has a flange 6 in. thick and the maximum flange width 50 in. It resists a B. M. of 3,600,000 in. lb. If the maximum depth permissible is 28 in., calculate the reinforcement required. If compressive steel is required, it must be placed with its centre at 2·5 in. below the compression edge.

Solution:—$d = 28-3=25$. Assuming $b = 2d_s = 12$ in.
$nd = \cdot 39 \times 25 = 9{\cdot}75$ in.

The N. A. falls outside the flange

$jd = \cdot 87 \times 25 = 21{\cdot}75$

Total compression = Total tension

$$= \frac{3{,}600{,}000}{21{\cdot}75} = 167500 \text{ lbs.}$$

Average compressive stress $= \frac{6{\cdot}75}{9{\cdot}75} \times 750 = 519$ lb./in.^{2}

Compressive strength of the flange per inch

$= 519 \times 6 = 3114$ lbs.

Total compressive strength of 50 in. flange

$= 3114 \times 50 = 155700$

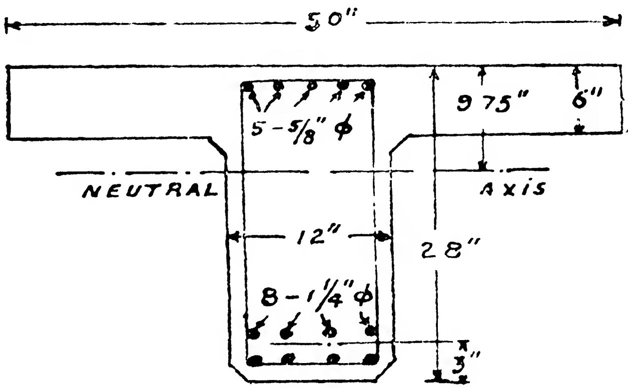


Fig. 31.

Total compression for which reinforcement is necessary

$$= 167500 - 155700$$

$$= 11800 \text{ lbs.}$$

The stress in concrete at 2·5 in. from top

$$= \frac{7{\cdot}25}{9{\cdot}75} \times 750 = 558 \text{ lbs.}$$

one inch compression steel would resist

$$558 \times (15-1) = 7812 \text{ lbs.}$$

$$A_c = \frac{11800}{7812} = 1{\cdot}51 \text{ in.}^2$$

5 Nos. $\frac{5}{8}$ in. ϕ ; A = **1·53** sq. in.

$$A_T = \frac{167500}{18000} = 9{\cdot}31 \text{ sq. in.}$$

8 Nos. $1\frac{1}{4}$ in. ϕ ; A = 9·817 in two rows of 4 each.

Practice Problems on Chapter XI

Unless otherwise mentioned take $c = 750$, $t = 18000$ $m = 15$.

Question 1. A tee-beam has a maximum flange width = 54 in. and thickness = 4 in. It is subjected to a B. M. of 720,000 in. lb. If the effective d = 18 in., calculate the reinforcement to balance the design. What is the maximum compressive stress in concrete?

Answer:—2·5 in.^{2}; $c = 345$ lb./in.^{2}

Question 2. A tee-beam has a maximum flange width = 48 in. and thickness = 4·5 in. If the overall depth is not to exceed 20 in., find the R. M. and the reinforcement.

Answer:—Taking d = 17 in.

R. M. = 1,590,000 in. lb.

$A_T = 6$ in.^{2}

Question 3. Find the R. M.s of both concrete and steel in a tee-beam with $b_s = 60$ in. $d_s = 4$ in. $d = 16$ in. and $A_T = 4{\cdot}5$ in.^{2}

Answer:—R. M. of concrete = 1,713,000.
,, steel = 1,134,000.

Question 4. A tee-beam 10″ wide, 30 in. effective d, has $A_T = 8$ sq. in. The flange is 5 in. thick and 48 in. wide. Compare R. M.s of concrete and steel. What would you do to balance the design?

Answer:—R. M. of steel = 3960,000 in. lb.
,, concrete = 3,900,000 in. lb.
Either (i) Provide compressive reinforcement
or (ii) Increase flange thickness.

Question 5. Find the stresses in concrete and steel of a tee-beam having $b_s = 48$ in., $d_s = 4$ in., (overall) D = 30 in., $b = 12''$, $A_T = 6 - \frac{3}{4}$ in. rods, subject to a B.M. = 1,200,000 in. lb.

Answer:—Taking $nd = 11$ in., lever arm = 26 in.;
$c = 293{\cdot}3$ lb./in.^{2}; $t = 17420$ lb./in.^{2}

CHAPTER XII

SHEAR AND BOND STRESS

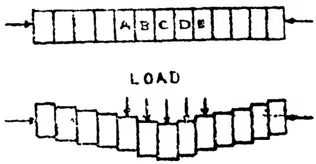
IT HAS been the author's experience while giving instructions in the subject, that the average student finds it difficult to visualise mentally the shear forces in a beam, particularly the diagonal shear. The reason may be, perhaps, that shear does not act in a direction normal to the section as tension and compression do, but along superficial planes. Further that it offers a two-dimensional problem as it acts both longitudinally as well as transversely, i. e. in longitudinal and vertical planes. The author has also found that the average student does not understand the proper significance of the bond stress and anchoring of rods which is the fundamental basis of R. C. C. design. It is therefore proposed to treat these aspects of the subject here in a little greater detail.

Importance of shear force in R. C. C. design:—Shear force is of particular importance in R. C. C. structures, than in those of timber or steel. In the latter two, the homogeneous material of the beams, by its very nature, is capable of developing shear resistance, in whatever part it is called forth. The designer requires no effort to provide for shear. All he has to do is to see that the section provided is adequate. In a R. C. C. member, on the other hand, concrete being very weak in tension or tensional shear, steel reinforcement, adequate in area must be provided, and unless the designer knows the exact nature, location, and the amount of shear stress, he cannot obviously do it.

That shear is the weakest point in an R. C. C. beam is proved by the fact that visible shear cracks in beams as shown in Fig. 36—at least one near the support—are of very common occurrence. Let one minutely observe existing beams and one will find that not a few amongst them cracked diagonally as in the figure. These cracks may be due to either inadequate provision of shear reinforcement, lack of proper bond and anchorage, bad quality of concrete, or a combination of two or more of these. They may not be always dangerous.

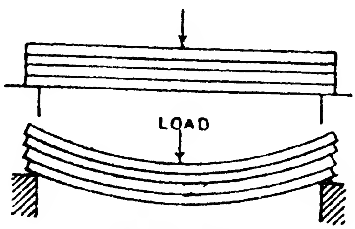
For, in many cases as soon as the unbalanced shear finds relief when a crack is formed, the stresses inside the beam automatically adjust themselves, and it is then possible that no further damage may be caused. Still, the unsightly cracks remain permanently there, casting reflection on the designer and the builder. The student is therefore advised to study the following discussion very carefully.

When a beam, whether of homogeneous material or otherwise, is loaded, along with the compressive stresses set-up in its section on one side of the neutral axis, and tensile stresses on the other, vertical and horizontal shear stresses



Figs. 32 & 33.

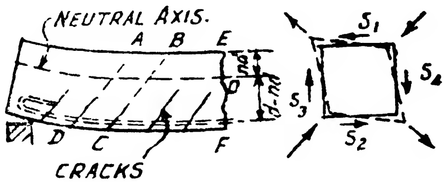
also are simultaneously set up. An idea of the vertical shear can be obtained from the sketch in Fig. 32 in which a beam is imagined to be composed of small rectangular blocks A, B, C, D, E, etc. glued together and further helped to be held together in a line by horizontal pressures applied at ends as shown by arrows. If the "beam" so formed is loaded with a small weight at the centre, or instead of this, one or two central blocks are lightly struck down with a hammer, the blocks will assume some such position as shown in Fig. 33, each block sliding downwards against the sides of the other. In an actual beam this sliding action is not visible, but the forces are there, and a similar action must be taking place inside.



Figs. 34 & 35.

An idea of the horizontal shear can be conceived, if the beam is imagined to be composed of thin planks, piled one upon another like a pack of playing cards. If the beam is supported freely at ends and loaded centrally, as shown in Fig. 34, it will bend. As each plank will retain its original length and yet as the ends do not coincide, as they did before the load was applied, sliding in a horizontal direction must have taken place between each pair of planks, due to the horizontal shear.

We have verified the presence of both the vertical and



Figs. 36 & 37.

horizontal shear. Their combination, resulting in producing diagonal shear can also be proved by a reference to Figs. 36 and 37. Consider a small cube of the beam having sides equal to one inch and its vertical faces subjected to shearing forces equal to s_1 and s_2. If these were the only forces acting, the couple formed would have caused the block to rotate. But as it is stationary there must have been other equal forces acting on it to prevent it from rotating, and obviously these forces must be s_3 and s_4 on the faces at right angles to the former, forming another couple of equal strength. Hence

$$s_1 = s_2 = s_3 = s_4 = \text{let us say } s \text{ lbs./in.}^2$$

There are the two shear forces—one vertical and the other longitudinal of equal intensity at any point in the beam we have discussed above. These tend to distort the material as shown in Fig. 35 (a), s_1 and s_3 on one side and s_2, s_4 on the opposite side, combining and tending to pull the material in the direction TT, and s_1, s_4 on one side and s_2, s_3 on the opposite side compressing the material along the diagonal CC. These are called diagonal tension and diagonal compression.

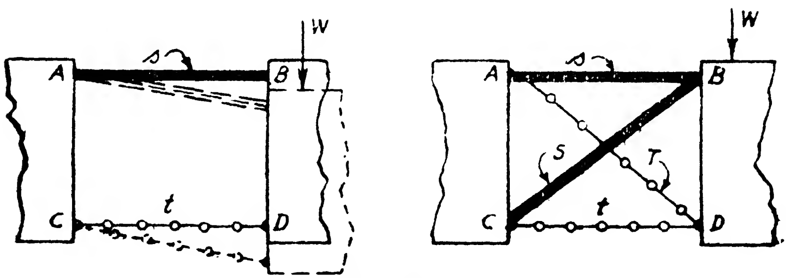
As each acts along planes of the hypotenuse on the area $= \sqrt{2}$ its intensity is equal to

$$\frac{\sqrt{2} \times S}{\sqrt{2}} = S.$$

This means that the vertical shear = horizontal shear = either diagonal compression or diagonal tension.

The presence of these diagonal tension and compression can also be demonstrated experimentally thus:—

Make two vertical cuts across a beam a few inches apart and remove the piece between them, forming a gap between the two ends as shown in Fig. 38. If compression in the top flange, and tension in the bottom flange were



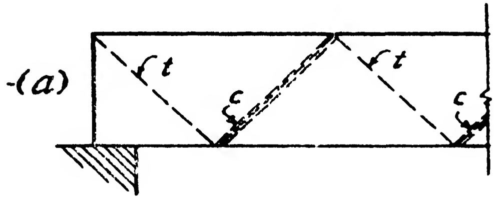
Figs. 38 & 39.

the only forces acting in a beam like this supported at ends, the provision of a cord, *t*, tying the two bottom ends C and D together, and that of a strut S thrust in the gap near the top, to serve as a tension and a compression member respectively as shown in Fig. 38 should have restored the equilibrium of the beam. But it will be found that a small weight W, placed on the right hand section would upset the balance and that portion would assume the position shown dotted in the figure.

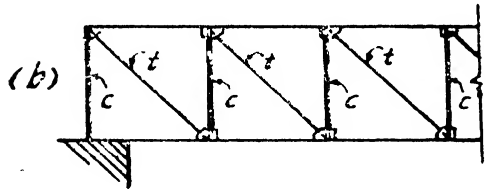
Now lift the right hand portion to its original position and tie a cord or a chain T, diagonally connecting the ends A and D, or insert a strut S diagonally between the ends B and C as shown in Fig. 39 and it will be seen that the stability of the beam is fully restored.

This conclusively proves that there is, in a loaded beam, always diagonal tension at 45° to the horizontal accompanied by diagonal compression at right angles to the tension and we have mathematically proved that these are the resultants of vertical and longitudinal shears and that all these are of equal intensity. These diagonal stresses are sometimes called web stresses also.

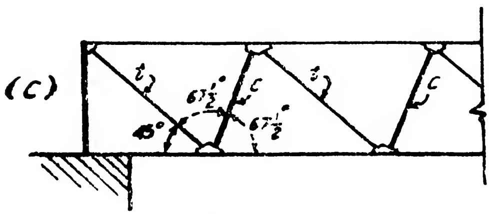
In a timber beam the solid material which is capable of resisting either tension or compression forms, as it were, different zones of tension and zones of compression, inducing these stresses in them when it is loaded (Fig. 40*a*). In a steel beam



(*a*) A timber beam supported at ends with compression and tension zones shown dotted developed inside it.



(*b*) An N-girder showing vertical members shown in thick lines in compression, and inclined members shown in thin lines in tension.



(*c*) A lattice girder with the inclined members shown in thick lines in compression and the other members in tension.

Fig. 40.

of rolled I-section with a solid plate forming the web, the same thing happens. In an N-girder, which is built up by assembling just the necessary parts, the vertical members of the web system take the compression and the diagonal members take the diagonal tension as shown by thick and thin lines in Fig. 40*b*. In a lattice girder, which is also an artificially built up beam, the diagonal members in one direction shown in thick lines take the diagonal compression and those in the other direction, shown by thin lines, the diagonal tension as shown in Fig. 40*c*.

If we build our R. C. C. beam on the analogy of one of these trussed beams say, e. g., a lattice girder which is a perfect beam with just the essential parts assembled together, our R. C. C. beam is bound to be equally efficient.

The essential requisites of a lattice girder (See Fig. 40*c*) are :—

(1) The two flanges—one to take flexural compression and the other, flexural tension.

(2) A web system, consisting of inclined tension members and inclined compression members for taking shear, and

(3) The rivets by which the web members are rigidly fixed to, and between the flanges. Because of the riveted rigid joints, all the different parts act in unison.

Now let us compare our R. C. C. beam and supply whatever is wanting in it, keeping constantly in mind the fact, which we already know, that as concrete is weak in tension we must reinforce it with steel wherever there is tension.

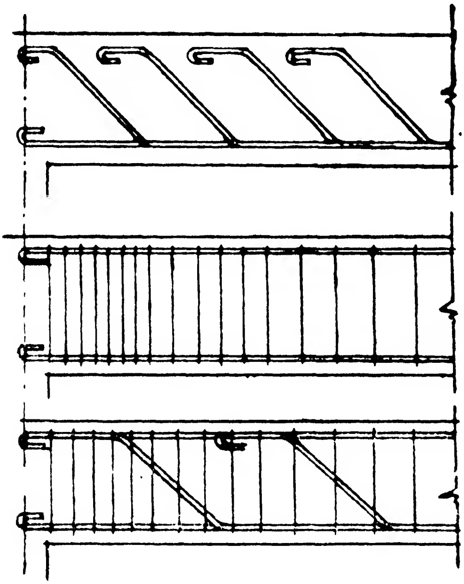
(1) Corresponding to the flanges of a lattice girder, the concrete in an R. C. C. beam on one side of the neutral axis is capable of serving as a compression flange if reinforcing bars are supplied on the opposite side as equivalent of the tension flange.

(2) As regards the web system, concrete is already there to form inside it inclined compression zones, provided inclined steel rods supply the deficiency of tension members; as shown

in Fig. 40*c*. However, mere supplying this deficiency will not do, but,

(3) The rods must be fixed to and between the flanges rigidly by some means which are equivalent to the rivets in a lattice girder and the only means possible in an R. C. C. beam are anchoring their ends by burying in concrete and developing bond stress, as shown in Fig. 41 (*a*).

Another way to achieve the same end is to provide vertical or inclined stirrups and two horizontal bars at top, even though of small diameter, round which and also round bottom



(*a*) Inclined bars with hooks at ends embedded in concrete to serve as riveted web members of a lattice girder.

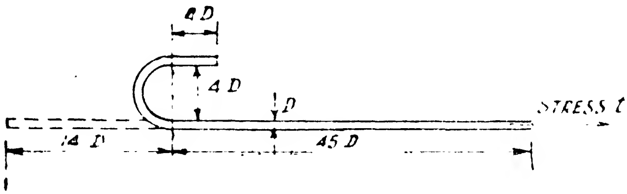
(*b*) Two bars at top and vertical stirrups anchored round serve the same purpose.

(*c*) A combination of inclined bars and stirrups.

Fig. 41.

bars these stirrups could be wound for anchoring as shown in Fig. 41 (b). In this case the two horizontal bars at top, together with the concrete around them, form a sort of compression flange and the stirrups which bind the two flanges together form the tension members of the web system.

Oftentimes when the shear is excessive, a combination of the inclined bars and stirrups is necessary. This is shown in Fig. 41 (*c*). In order to understand the importance of the bond stress and anchoring, suppose there is a steel bar placed on the ground. If it is pulled by one end, it will bodily move.



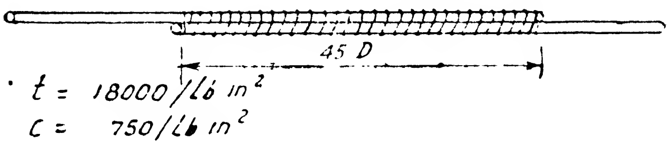


Fig. 42—Standard hook.

Fig. 43—Overlapping two ends of rods in a length of 45 D or "splicing" for lengthening it.

In order that it should not be pulled out but should offer resistance to the pull, it must be embedded in concrete in a sufficient length to develop a resistance by the mutual bond or adhesion between it and the concrete. This length can be calculated. Suppose in Fig. 42 a rod of diameter D, has a length, l, embedded in concrete. The maximum pull permissible in it at 18000 lbs./in.^{2} is $\frac{\pi D^2}{4} \times 18000$. This is resisted by the friction between the surface of the rod and the concrete. This is $= \pi D \times l \times S_b$ where $S_b =$ the permissible bond stress per sq. in. which is 100 lbs. for 1:2:4 concrete. Equating these we get,

$$\frac{\pi D^2}{4} \times 18000 = \pi D \times l \times 100$$

$$l = \frac{18000}{4 \times 100} \cdot D = 45 \text{ D}.$$

Thus the anchorage length = 45 diameters for $t = 18000$ and $S_b = 100$ lbs./in.^{2}. The Bylaws and Code of Practice require that all such ends should be hooked. The Standard hook has the dimensions as shown in Fig. 42. When two pieces of steel rods are to be lengthened, or "spliced" as it is called, their ends are overlapped 45 diameters, tied by a thin wire and embedded in concrete. (See Fig. 43.)

CHAPTER XIII

SHEAR AND BOND STRESS

(Continued)

IN THE last chapter we discussed the nature and the behaviour of the shear in an R. C. C. beam. We shall now proceed to determine its precise location and the strength, so as to enable us to provide reinforcement for it in the design of beams.

It was seen in the preceding chapter that shear is distributed both longitudinally as well as transversely in a beam.

Longitudinal distribution :—Every student of the theory of structures knows that the shear at a section of a beam is the algebraic sum of all the external loads and reaction on any side of it and that it is equal to the rate of change of bending moment at the section. Expressed in mathematical terms,

$S = \dfrac{dM}{dx}$ where M = B. M. and x = distance. It is the differential

of the B.M. Thus shear is maximum when the B.M. is minimum, such as at ends, and that it is minimum or zero when the B.M. is maximum or constant. Figs. 44 to 46 are three typical cases illustrating this relation between B. M. and shear.

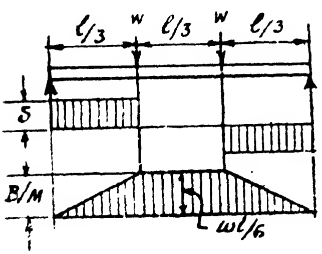


Fig. 44—Simply supported beam with concentrated loads at one-third span from both ends.

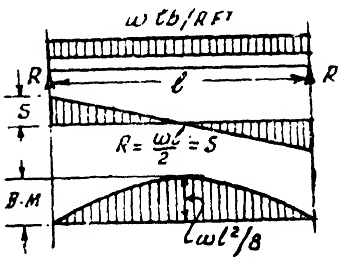


Fig. 45—Simply supported beam with distributed load.

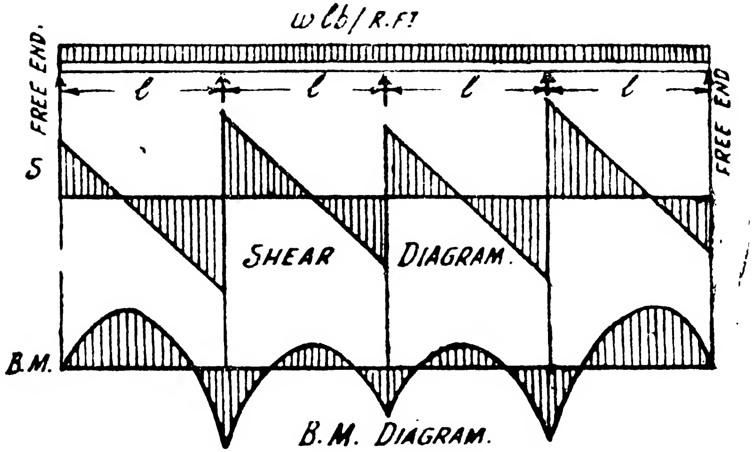


Fig. 46—Continuous beam with uniformly distributed load.

Working shear stresses:—Both the By-laws and the Code allow a maximum shear stress in concrete = 0·1 of the safe permissible compressive stress. Thus for 1:2:4 ordinary grade concrete, the working stress is $= \cdot 1 \times 750 = 75$ lb./in.^{2} It is further stipulated that if the shear intensity $\frac{S}{bjd}$ exceeds this limit, then reinforcement must be provided for the *whole of the shear* and not merely the excess. In other words, concrete is to be altogether neglected as taking part of the shear. Not only this but it is further prescribed that the intensity o shear as worked out by the expression, $\frac{S}{bjd}$ should not be greater than 0·4 times the permissible compressive stress in compression. Most engineers in this country restrict this still further to 0·3 c. In other words if $\frac{S}{bjd}$ exceeds 3×75 or 225 lb./in.^{2}, further increase in reinforcement will not do. Th only course open is to increase the section.

Thus the shear strength of a beam, 10″ × 20″ (effective *d*) would be, without reinforcement $= s \times bjd = 75 \times 10 \times \cdot 87 \times 20$ $= 13050$ lbs.

If the shear is, say, 15000 lbs., then reinforcement must be provided for all this, neglecting concrete and if the total shear is, say, 40,000 lbs. or more, so that the intensity exceeds 225 lb./in.^{2} the section of the beam must be increased either by increasing the width or the depth or both.

For commercial mild steel a working shear stress of 18000 lbs./in.^{2} is allowed, both for inclined bars and stirrups. However, some engineers adopt a stress of 16000 lbs. per sq. in.

Shear strength of inclined bars :—It is the usual practice to bend the bars for shear either at 45° or 30°, the former is more common However, bending at 30° is safer as will be explained later and therefore for shallow beams they should be done at 30°. The vertical shear is obviously resisted by the vertical component of the stress in the inclined bar. If A_s is the combined area of the inclined bars, θ, the angle of inclination of the bars and $t =$ the safe shear stress allowed, $A_s \times t \times \sin\theta$ will be the total shear stress in the inclined bars.

Since $\sin 45° = \frac{1}{\sqrt{2}} = 0{\cdot}707$, and $\sin 30° = \frac{1}{2} = 0{\cdot}5$, the shear strengths of one inch bar, for instance, will be as follow :—

Inclination	Shear strength
45°	$A_s \times 18000 \times \sin 45° = \cdot 7854 \times 18000 \times \cdot 707$ $= 9925$ lbs.
30°	$A_s \times 18000 \times \sin 30° = \cdot 7854 \times 18000 \times 0{\cdot}5$ $= 7020$ lbs.

The following table gives shear strengths of bars of different diameters for both the above inclinations.

TABLE NO. 19

SHEAR VALUE OF INCLINED BARS

for shear stress= 18000 lb./in.²

Diam. of bars inches	Shear value lbs. for inclination 30°	Shear value lbs. for inclination 45°
½	1710	2420
5/8	2700	3820
3/4	3960	5600
7/8	5400	7640
1	7020	9925
1⅛	8910	12,600
1¼	10,980	15,530
1½	15,840	22,400

The B. M. of a beam except in the case of a cantilever and in a few cases of continuous beams, is maximum at mid-span and decreases towards the ends, where the shear is maximum. It is, therefore, possible to stop short at certain points some of the bars from amongst those provided for the maximum B. M. where they could be spared, after continuing them for anchorage to a certain further length. Instead of this they could be, with advantage, utilised by bending as inclined bars for shear. In beams under normal loading, these are sufficient to meet the demands of the shear force. At the most a few vertical stirrups may be required near the ends in addition. In heavy beams such as of warehouse type buildings, or bridges, extra inclined bars may be necessary.

Spacing of inclined bars:—We imagined in the last chapter while discussing the nature of the diagonal tension, a loaded R. C. C. beam to have developed truss action inside it with short concrete struts imagined to be inclined at 45° to the horizontal, equivalent of the compression members of the web system of a lattice girder. The zone of action of an inclined bar between two or these struts extends over the distance on either side of it equal to the horizontal projection of the inclined concrete struts.

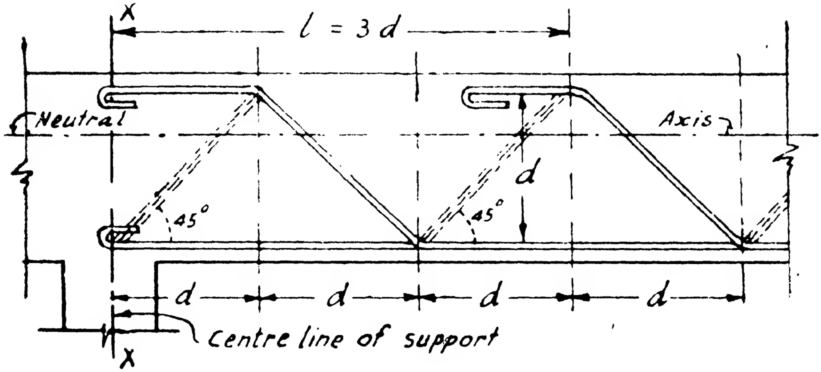


Fig. 47.

Thus if the bar is inclined at 45°, its influence extends over $l = 3d$ between XX and YY as shown in Fig. 47. Any vertical plane in this zone cuts either one diagonal steel tension bar or one imaginary concrete strut. The latter is shown dotted in the figure. As the shear is maximum near the support, the first inclined bar nearest the support should have its bottom bend at a distance not greater than $2\,d$ from the centre of the support if the inclination is of 45°. If it is 30°, then the distance should not exceed $d + d \cot 30°$ or $2{\cdot}73\,d$. If there is shear beyond YY enough to cause an intensity of more than $\frac{1}{10}c$ (75 lbs. per sq. in. for 1:2:4 grade concrete), then another bar should be bent at a distance of $2\,d$ or less, if the inclination is 45° or $2{\cdot}73\,d$ or less if it is 30°, from the bottom bend of the previous inclined bar. In other words, the horizontal distance between any two inclined steel bars should not exceed $2\,d$, or $2{\cdot}73\,d$ according as the inclination is 45° or 30° respectively.

The arrangement of the inclined bars and the probable induced inclined concrete struts inside the beam (the latter shown dotted in Fig. 47) is called "Single Shear" system, as any vertical section in the zone of action cuts either one inclined imaginary concrete strut or one steel inclined bar.

If it is possible to spare more bars out of the tensile reinforcement, to be bent for taking inclined tension, or if the shear force is so heavy that extra inclined bars are required according to the design, what is called "Continuous Shear" system is adopted as shown in Fig. 48. In this system one or

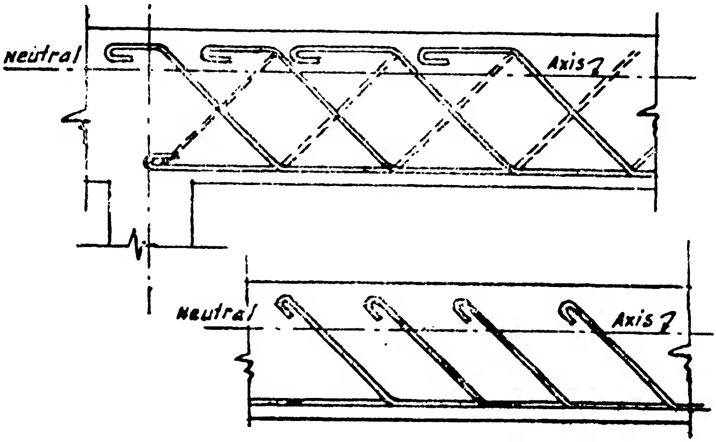


Fig. 48 & 49.

more inclined bars are provided at equal intervals between the inclined steel bars of the single shear system. In Fig. 48 one such additional bar is shown between the inclined bars of single shear system, so that any vertical section in the zone cuts two inclined bars or one inclined bar and one inclined imaginary concrete strut. This continuous shear system obviously induces corresponding equal number of imaginary concrete struts and increases the shear strength to double or treble according as one or two additional inclined steel bars are introduced between those of the single shear system.

For shallow beams (with d less then 1·5 b') the inclined bars should be bent at 30°, since the length of the bar if bent at 45° would be very small. A still better and more economical plan is to provide vertical stirrups to take the whole shear.

The inclined shear bars will not develop full shear strength unless they are sufficiently anchored at both ends to afford them the necessary grip-length to develop sufficient bond stress, and not stopped with a hook at top as in Fig. 49. The correct way is to extend the bars horizontally and provide a hook at ends as shown in Fig. 48. The grip length is measured from the point at which the bars cut the neutral axis. For developing full strength of 18000 lb./in.^{2}, the length of the bars beyond the N. A. must be 45 times the diameter of the bars. Whenever insufficient length is available, either a reduced shear stress say 14000 to 16000 lb./in.^{2} should be taken in calculations or the end of the bar further bent downwards as shown in Figs. 53 (b) and 56.

Spacing of stirrups:—Stirrups, as we have imagined before, are equivalents of the tension members of the web system of a lattice girder, the imaginary inclined short struts of concrete supported by the flanges inside the R. C. C. beam, acting as the compression members. Assuming that the compression in the concrete is inclined at 45° to the neutral axis, and that the pitch of the stirrups is p, the number of stirrups to each

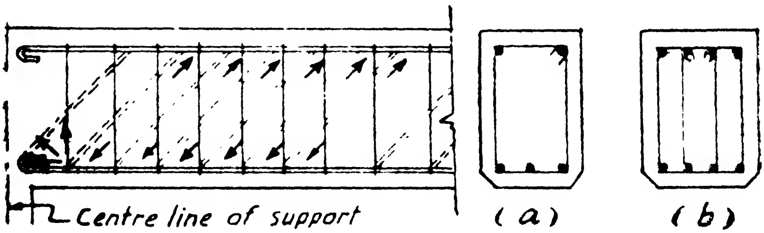


Fig. 50—(a) Single stirrups. (b) Double stirrups.

compression member of the equivalent lattice girder is $= \frac{jd}{p}$

and if A_s = area of the two legs of a stirrup (or four legs, if they are double), t = the permissible safe tensile stress in shear reinforcement and S = total shear

$$S = \frac{tA_s \times jd}{p}$$

or $$p = \frac{t \times A_s \times jd}{S}.$$

It will be seen from Fig. 50 that the spacing of stirrups should never be less than jd horizontally, otherwise a crack may appear at some section between the two stirrups. To serve as a really effective and continuous tensile web system, p should not exceed $\frac{jd}{2}$. This may be attained by reducing the diameter of the stirrup, if necessary.

The vertical tension in stirrups is supposed to combine with the horizontal tension in the longitudinal steel to produce a diagonal resultant as shown by arrows at the left hand corner of the figure to resist the diagonal tension.

Some designers rely entirely on the stirrups and provide them to take up the whole of the shear, when the intensity exceeds the permissible limit of shear for concrete, i. e. they ignore altogether the diagonally bent bars.

Stirrups are really superior to inclined bars in the following respects: (1) They necessarily require two "hanger bars" at top; the latter add to the strength of the "compressive flange". (2) They bind the concrete as links do in columns. (3) They tie the two flanges together throughout the length of the beam. (4) Unlike inclined bars, they offer the least interference to proper concreting, and (5) In beams carrying heavy rolling loads, running in either direction, stirrups effectively function as shear reinforcement. Inclined bars are effective in one direction only.

The minimum diameter of stirrups should be ¼ in. and the maximum ½ in., as thicker bars are difficult to bend.

Even in the portion of the beam where no shear reinforcement is required, stirrups should always be provided at a spacing not exceeding jd, the lever arm.

In doubly reinforced beams, particularly in those designed on the "Steel Beam Theory", the Bylaws and Code require the spacing of stirrups not to exceed 8 times the diam. of the smallest bar in compressive reinforcement. In beams in which the compressive resistance of concrete is not ignored, it should not exceed 12 times the least diam. of compressive steel (See page 105). The object is that if there are no such binders in the form of stirrups at close intervals, the thin bars in compression might bulge out as in the case of columns. A thin, long bar pressed inwards at both ends is bound to buckle.

Illustrative Example 41

Design the shear reinforcement of a simply supported beam 8 in. wide having effective $d = 18$ in. The beam is reinforced with four bars $1\frac{1}{8}$ in. diam. at bottom and carries a maximum shear of 27000 lbs. at ends on an effective span of 20 ft.

Solution:—Intensity of shear $= \frac{S}{bjd} = \frac{27000}{8 \times \cdot 87 \times 18} = 217$ lbs./in.². As this exceeds the limit of 75 lbs./in.², the whole shear of 27000 lbs. must be provided for. But as it is less than 3×75 or 225 lbs./in.², the section is safe. Supposing that 50 per cent of the longitudinal reinforcement, or 2 bars $1\frac{1}{8}$ in. diam. could be spared after meeting the demands of the B. M. to be bent at 30° at $\frac{1}{4}$ span or 5 ft. from the ends.

$$\begin{aligned}\text{Shear value} &= A_s \times t \times \sin\theta \\ &= 2 \times 0{\cdot}99 \times 18000 \times 0{\cdot}5 \\ &= 17800 \text{ lbs.}\end{aligned}$$

Shear for which stirrups are required

$$= 27000 - 17800 = 9200 \text{ lb.}$$

Using $\frac{1}{4}$ in. diam. stirrups, the area of 2 legs $= 2 \times 0{\cdot}049$

$$\begin{aligned}\text{pitch} \quad p &= \frac{A_s \times t \times jd}{S} \\ &= \frac{2 \times 0{\cdot}049 \times 18000 \times \cdot 87 \times 18}{9200} \\ &= 3 \text{ in.}\end{aligned}$$

or if double stirrups (having 4 vertical legs) are used, the spacing will be 6 in. *c/c*.

Since the beam is simply supported and the load is uniformly distributed, the shear diagram is two triangles with 27000 lbs. shear at the ends as the base and zero at the centre as the apex as shown in Fig. 51.

The distance x from the centre upto which the concrete will take care of the diagonal shear without reinforcement could be found out thus:

The shear value of the concrete section

$$= b \times jd \times s$$

$$= 8 \times \cdot 87 \times 18 \times 75 = 9396, \text{ say } 9400 \text{ lb.}$$

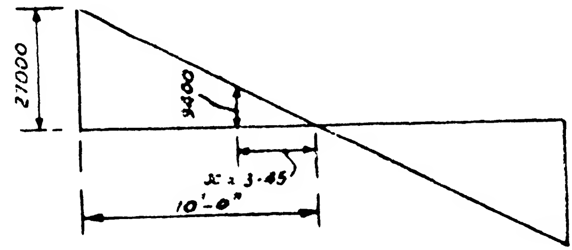


Fig. 51.

If x ft. is the distance from the centre of the beam to the point where the shear is 9400 lbs. by reference to Fig. 51.

$$\frac{x}{9400} = \frac{10}{27000}; \; x = 3{\cdot}45' \text{ or } 3'-5''.$$

i. e. from the centre up to 3′ 5″ towards the ends where there is a shear intensity of 75 lbs./in.^{2}, there is no necessity of any reinforcement. Still, $\frac{1}{4}$ in. stirrups will be provided at less than jd or ·87 × 18 = 15 in. (better 12 in.) even there.

Illustrative Example 42

Design the stirrup system for a beam 12″ × 18″ (effective d) to take the entire maximum shear force = 24000 lbs. at ends.

Solution:—Shear intensity $= \frac{S}{bjd} = \frac{24000}{12 \times \cdot 87 \times 18} = 128$ lbs./in.^{2}

Since this is more than 75, reinforcement is necessary.

Using $\frac{3}{8}$ in. diam, two legged stirrups

$$p = \frac{A_s \times t \times jd}{S} = \frac{2 \times 0 \cdot 11 \times 18000 \times \cdot 87 \times 18}{24000}$$

$$= 2 \cdot 58 \text{ in.}$$

Use four legged (double) stirrups at 5 in. *c/c*.

Illustrative Example 43

A tee beam, 20 ft. span has a flange 4″ thick and a rib 10 in. × 18 in. (overall). It carries 4 numbers of one inch bars at bottom, two of which are bent at 45° for shear. Besides it has a stirrup system of $\frac{3}{8}$ in. bars at 5 inches centres. If the beams are at 8 ft. centres apart, find the safe maximum loading per foot consistent with its shear strength.

Solution:—Overall depth = 18″
Effective depth = 16″

As the flange is 4 in. thick, the lever arm may be taken as $16 - \frac{4}{2} = 14$ in.

Since the section of the beam is fixed, we shall first find out the limiting shear strength of the beam which is

$$= \cdot 3 \, c \, bjd$$
$$= \cdot 3 \times 750 \times 8 \times 14$$
$$= 25200 \text{ lbs.} \quad \text{(i)}$$

Shear strength of two diagonal bars

$$= A_s \times t \times \sin 45$$
$$= 2 \times \cdot 7854 \times 18000 \times 0 \cdot 707$$
$$= 19850 \quad \text{(a)}$$

Shear strength of the stirrup system

$$S = A_s \times t \times \frac{jd}{p}$$

A_s of 2 legs each having ·11 sq. in. area = ·22

$$= A_s \times t \times jd \div p$$

$$= \frac{2 \times \cdot 11 \times 18000 \times 14}{5}$$

$$= 11088 \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(b)}$$

Total strength of the web system

(a) + (b) = 19850 + 11088
= 30938 lbs.

But the limiting shear strength of the section is 25200 lbs. which cannot be exceeded. This is the shear equal to the reaction at end. The total load which the beam can safely bear is twice this or = 50400 lbs.

$$\text{Load per rft.} = \frac{50400}{20}$$

$$= 2520 \text{ lbs.}$$

This includes the dead load also, viz. slab load and self load.

Load of slab = 8′ × 12″ × 4″ = 384 lbs.
Self load of stem, b × d = 8 × (18 − 4) = 112 ,,

Total dead load = 496 ,,

Live load = 2520 − 496
= 2024 lbs. on 8 rft.
= **253** lbs. per sq. ft.

Bond stress:—One of the fundamental assumptions on which the theory of R. C. C. beams is based is that there is a perfect bond or adhesion between steel and concrete, that therefore both these stretch together without breaking this bond. shear, being a stress acting superficially along the surface, is always associated with bond. In fact, they are synonymous, because shear (horizontal) is the stress caused by external load and bond is the equal and opposite stress developed by the friction between the two to resist it. We have already seen in the last chapter that bond and anchorage together do the same function as rivets do in a built-up girder. The anchoring is simply holding the ends fast so as to prevent the rod from moving bodily when a pull is applied. This anchoring is done by the adhesion or bond between the end portion of the rod and the concrete in which it is embedded. Bond, like rivets in a built-up steel girder, is helpful in developing different intensities of stress in different parts of a member. The analogy of a built-up girder, say, an N-girder of 11 bays illustrated in

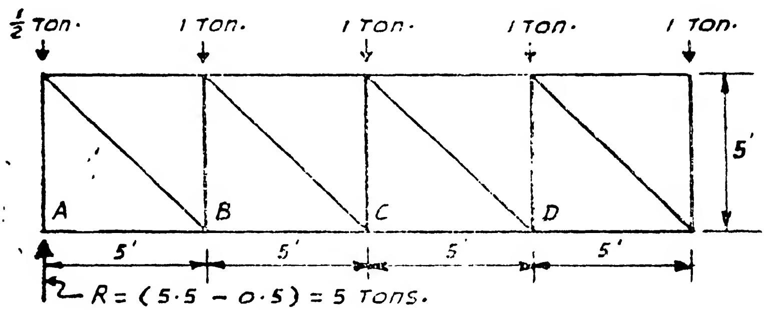


Fig. 52—N-girder having 11 bays of 5 ft. each.

part in Fig. 52, will again make this point clear. From the data given in the illustration it will be clear that

$$\text{Tension T in AB} = \frac{\text{B.M. at B}}{\text{lever arm}} = \frac{\text{R} \times \text{AB}}{\text{lever arm}}$$

$$= \frac{(5{\cdot}5 - {\cdot}5) \times 5'}{5} = \frac{25}{5} = 5 \text{ tons}$$

$$\text{,, \quad ,, in BC} = \frac{\text{B.M. at C}}{\text{lever arm}} = \frac{\text{R} \times \text{AC} - 1 \text{ ton} \times \text{BC}}{\text{lever arm}}$$

$$= \frac{5 \times 10 - 1 \times 5}{5} = 9 \text{ tons.}$$

Therefore, there is an excess pull of 4 tons in BC than in AB. We shall call this a pick-up of tension. As the B.M. increases from zero at A to the maximum at the midspan, there is a progressive pick-up at every joint such as B, C, D, etc. up to the centre. The pick-up of 4 tons from AB to BC is effected in the case of this built-up girder through the medium of the rivets at B, those on left of B are designed for 5 tons, and those on right, for 9 tons.

In an R. C. C. beam, instead of such separate members as AB, BC, etc. joined by rivets, there are long, continuous bars corresponding to the bottom chord of the above girder and there is therefore no possibility of any rivets to transfer the pick-up of stress from one point to the next. This is done by the bond or adhesion between the surface of the bars and the concrete in contact with it.

Expressed mathematically, if M and $M + \delta M$ are the B. Ms. at any two sections δx apart, the tensile force in the steel bars at these sections is $= \frac{M}{\text{lever arm}}$ and $\frac{M+\delta M}{\text{lever arm}}$ respectively. The difference between the two, which we called above *the pick-up*, is resisted by the bond between the bars and the concrete in which it is embedded in a length δx.

$$\therefore \frac{M+\delta M}{\text{lever arm}} - \frac{M}{\text{lever arm}} = \text{sum of perimeter of}$$

bars $\times$ δx $\times$ bond stress

$$\text{or bond stress } s = \frac{\delta M}{\delta x} \times \frac{1}{\text{lever arm} \times \Sigma\odot}$$

$$= \frac{\delta M}{\delta x} \times \frac{1}{jd \times \Sigma\odot}$$

Where S_b = bond stress and $\Sigma\odot$ = sum of the perimeters of longitudinal bars.

But we have seen before that $\frac{\delta M}{\delta x}$ = the rate of change of B. M. that is, shear

$$\therefore \quad s_b = \frac{S}{jd \times \Sigma\odot}$$

Note that this bond stress is the *shear bond* stress caused by the horizontal shear in beams and must be distinguished from the direct bond stress or anchorage which are discussed at the end of the previous chapter in connection with end anchoring and grip length. The permissible limit of the latter is 100 lbs./in². while the Code of Practice and Bylaws specify that the shear bond stress may reach a maximum of 200 lbs. per sq. in. for the ordinary 1:2:4 mix of concrete. This means that whatever tensile stress there may be at a given section of a bar, there must always be a sufficient length of it between the section and the free hooked end of the bar to maintain the *average* bond stress below 100 lbs./in.^{2} To make this more clear, suppose, at three different sections of a beam, the tensile stresses are 8,000, 10,000 and 16,000 lbs. Then the embedded lengths of the tensile reinforcement must be at least 20, 25 and 40 diameters respectively, between the respective section and the free hooked end of the bar. The intensity of 200 lb./in.^{2} of shear bond is allowed only at the peak value of the shear, e. g. at end of beams.

If bond stress at any particular point is found to be less, it can be increased in either of the two ways:

(i) Using bars of smaller diameter to supply the same cross sectional area of steel. This increases the perimeter, i. e. the superficial area of steel.

(ii) Using deformed bars, such as twisted, indented, lugged, etc. instead of plane surfaced round bars.

Additional examples worked out in detail on shear will be found in the next chapter on "Complete Design of Beams."

Steps in designing shear reinforcement:—(1) Calculate the maximum shear, wherever it may be; usually it is near the supports.

(2) Calculate its intensity by dividing the shear by the product of breadth and lever arm.

(3) If the intensity is less than one-tenth of the maximum permissible compressive stress for the concrete (75 lbs./in.^{2} for 1:2:4 ordinary grade) no reinforcement is necessary. Still provide $\frac{1}{4}$ in. stirrups at a spacing of less than the lever arm.

(4) If the intensity exceeds 3 *c* (or 225 lbs./in.^{2} for 1:2:4 ordinary grade concrete) the section must be increased. If it is more than 75, but less than 225, find out at what point or points

some of the longitudinal reinforcement could be spared as inclined bars for shear, after meeting the demands of the bending moment. Still see that at least 25 per cent of the tensile reinforcement is carried past the centre of support for developing bond stress. In the case of heavy beams it is desirable to draw B. M. diagram and shear force diagrams to the same horizontal scale, one below the other, for determining which bars could be spared for shear and where.

(5) Calculate the shear strength of the inclined bars and if the shear calculated as per (1) above exceeds this, provide stirrups at intervals less than the lever arm. If there is no excess, still provide nominal reinforcement in the form of stirrups with a pitch equal to or less than the lever arm.

(6) Verify that the bond stress is within permissible limits. For this calculate the maximum shear force, and divide it by the product of the sum of perimeters of longitudinal tensile bars reaching the ends, and the lever arm, i. e. by $\Sigma O \times jd$. This should be less than 200 lbs./in.^{2}

Questions and Answers

Q:—What is the difference between shear bond stress and anchorage bond stress both of which involve bond between the steel and concrete?

Answer :—Shear bond stress varies as the B. M. at different sections of a beam, and the designer has no control on it except by using thinner or deformed bars.

Extending the bar further does not help. The bond stress in end anchorage, however, can be varied by the designer by using a longer bar. The permissible shear bond stress is 200 lb./in.^{2} while the permissible anchorage bond stress is 100 lbs.

Practice Problems on Shear

1. A beam with 15 ft. effective span and section 10″ × 18″ (effective) carries a uniform load of 480 lbs./rft. exclusive of its own weight and a concentrated load of 7500 lbs. at 5 ft. from one end. Calculate (a) the maximum shear

and (b) its intensity. (c) What is the maximum permissible shear the beam can take with reinforcement?

Ans. (a) 10100 lbs.; (b) 64 lb./in.^{2}; (c) 35235 lbs.

2. Calculate the shear values of the following:—

(1) 2 Nos. $\frac{7}{8}$ in. ϕ bars bent at 30° and 45°.

(2) $\frac{1}{2}$ in. ϕ single stirrups at 6 in. *c/c*.

(3) $\frac{3}{8}$ in. ϕ double stirrups at 5 in. *c/c*.

(4) $\frac{1}{4}$ in. ϕ „ „ „ 3 „ „

Ans. (1) 10800 and 15250 lbs. (2) 1175, (3) 1584, (4) 1175.

3. A beam 12 in. × 26 in. (effective) has 4 Nos. 1 in. ϕ bars bent at 30°, and in addition $\frac{1}{4}$ in. ϕ double stirrups at $4\frac{1}{2}$ in. *c/c*. Calculate its shear strength.

Ans. 28080 + 17689 = 45769 lb.

4. Find the pitch of $\frac{1}{4}$ in. ϕ double vertical stirrups to take maximum shear of 25000 lbs. if the beam section is 12″ × 27″ (effective *d*). Ans. 3·4 in.

5. Calculate the shear intensity of a beam 15″ × 28″ (effective) if the maximum shear is 40000 lb. Design the pitch of 4-legged $\frac{3}{8}$ in. round stirrups.

Ans. (a) 109 lb./in.^{2} (b) 4·8 in.

6. A beam is 20 ft. long between centres of bearings and carries a distributed load of 2400 lbs./rft. including its own weight. If its breadth is 12 in., find its depth so that theoretically no reinforcement should be necessary.

Ans. d = 30·6 in.; D = 32 in.

7. A simply supported T-beam having a flange 70″×5″ and rib 12″ wide carries a total distributed load of 52500 lbs. If the steel in the rib is placed 18 in. below the top of the flange and the N. A. is at 12 in. above the centre of the steel, calculate the shear intensity at the support and determine the pitch of $\frac{3}{8}$″ ϕ single stirrups. Take tensile shear stress = 16000 lb./in.^{2}

Ans. (a) 136 lb./in.^{2} (b) 4·3 in.

8. A beam having 20 ft. effective span and 10″ × 25″ section carries a point load of 30,000 lbs. at the centre. If it is designed on the principles of steel beam theory, find the reinforcement, both for B. M. and shear.

Ans. (a) 8 Nos. $1\frac{1}{8}''$ ϕ in 2 tiers; (b) $\frac{3}{8}$ in. single stirrups at 4 in. *c/c*.

9. A T-beam 12″ × 30″ (overall) carries a uniformly distributed load of 2500 lb./rft. besides its own weight. The effective span is 24 ft. and the reinforcement is 6 Nos. 1 in. bars in two tiers with 3 in. distance between the centres of steel bars. The rods of bottom tier go straight. Verify the shear intensity and find the pitch of $\frac{1}{4}$ in. ϕ stirrups. Take shear stress 16000 lbs./in.^{2}. Draw B. M. and shear diagrams.

Ans. Intensity 102 lbs./in.^{2}; pitch 4·9″ say 4·5.

CHAPTER XIV

COMPLETE DESIGN OF BEAMS

HITHERTO we designed beams in parts, i. e. either for depth and reinforcement or for shear or for bond. We shall now design a few practical beams of different types in every respect.

Illustrative Example 44

A singly reinforced simply supported beam

A simply supported beam over an effective span of 20 ft. carries a uniformly distributed load of one ton per foot in addition to its own weight, besides two point loads each of 6 tons at 5 ft. from each end. Design the beam completely.

Solution:—Assuming a section of rib = 15″ × 24″

Distributed load :—Self load = 15×24 = 360 lbs./ft.

Super load = 2240 ,,

Total 2600 ,,

B. M. due to distributed load

$$= \frac{wl^2}{8} = \frac{2600 \times 20^2 \times 12}{8} = 1560000 \text{ in. lb.}$$

B. M. due to point load

$$= 6 \times 2240 \times 5 \times 12 = 806400 \text{ ,,}$$

Total B. M. 2,366,400 ,,

$$d = \sqrt{\frac{2366400}{126 \times 15}} = 35{\cdot}5 \text{ in.}$$

Overall D = 35·5+1·5 (cover)+·5 (half diam. of steel)

= 37·5 say 38 in.

d = 36 in.

We have assumed it as 24 in. above, i. e. 14 in. less. Extra B. M. due to 15″ × 14″ = 210 lbs./ft.

$$= \frac{210 \times 20^2 \times 12}{8} = 126{,}000 \text{ in. lb.}$$

Total maxi. B. M. = 2366400+126000
= 2492400

Shear due to distributed load is maximum at the ends

$$= \frac{2810 \times 20}{2} = 28100 \text{ lbs.}$$

Shear due to point load is 6 × 2240 = 13440 lbs. and is uniform in five ft. lengths at ends.

$$A_T = \frac{2492400}{18000 \times \cdot 87 \times 36} = 4 \cdot 5 \text{ sq. in.}$$

Use 6 Nos. 1″ ϕ bars (A = 4·712 in.^{2}). The B. Ms. calculated at every 2 ft. from the support separately for distributed and concentrated loads are tabulated below:—

TABLE No. 20

BENDING MOMENTS

Distances from one end	2 ft.	4 ft.	5 ft.	6 ft.	8 ft.	10 ft.
Moments due to distributed load	606960	1079040	1264890	1416240	1618560	1686000
Moments due to point loads	322560	645760	806400	806400	806400	806400
Total Moments	929590	1724160	2071290	2222640	2424960	2492400

These are plotted in Fig. 53(a) in which the parabola drawn in chain dotted lines represents the B. M. diagram for the distributed load and the half trapezium drawn in dotted lines shows the B. M. diagram due to point loads. The full lines diagram at the bottom is the curve for the combined loads. The rectangle *Opfp* represents the R. M. diagram of the steel if all the six bars are carried to the ends and securely anchored there. We have provided six one-inch round *m. s.* bars The R. M. diagram is therefore divided into six equal strips, each strip such as 1, 2, 3, 4, etc. representing the R. M. of one bar.

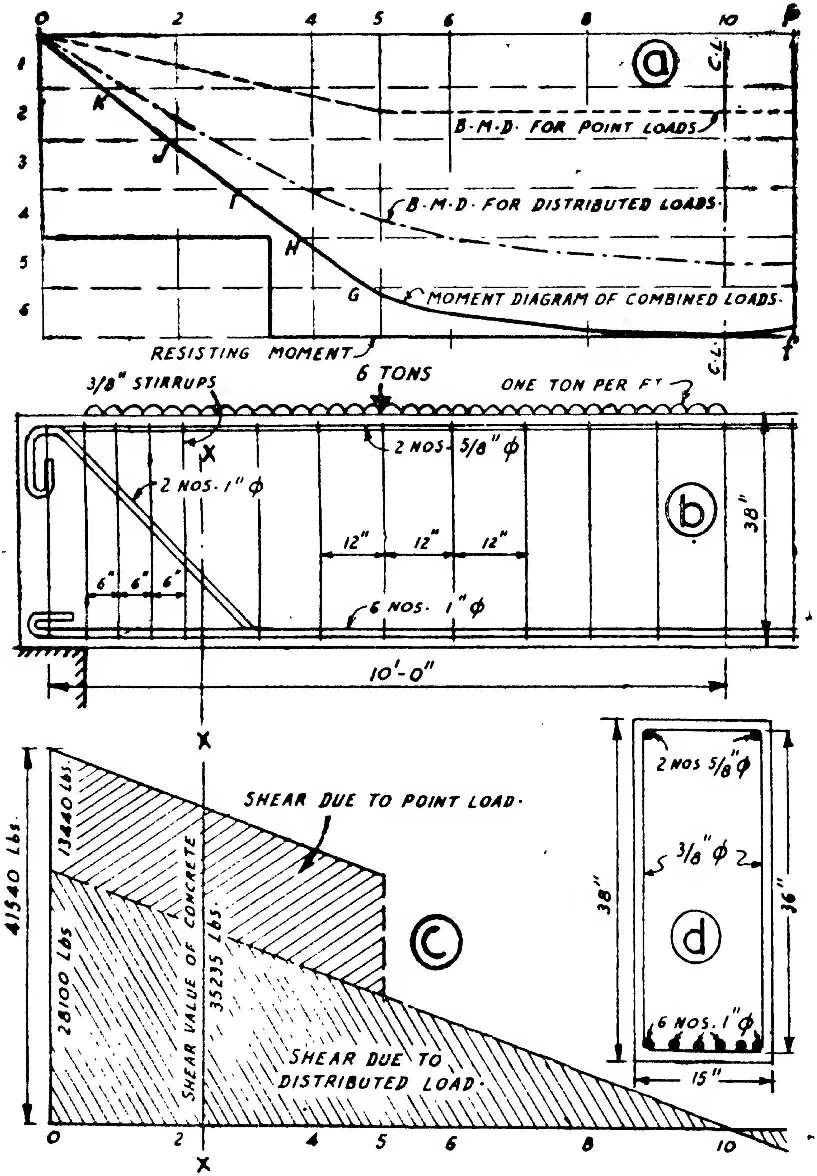


Fig. 53 (a)—The rectangle *opfp* represents R. M. diagram of the six steel bars, while the curve *f* GHIJKO is the envelope of the B. M. of combined loads. Fig. (b) is a longitudinal section of the beam. Note the ends of inclined bars lowered down and bent for increasing grip-length. Fig. (c) Shear diagram and Fig. (d) C. S. of the beam at centre.

For economical design, the R. M. should be equal to or slightly greater than the B. M. It will be seen from the figure in which the R. M. and B. M. are superimposed on one another, that all the six bars are required only in the middle portion of about 5⅓ ft. on either side of the centre line up to G, where the B. M. diagram cuts the strip No. 6. Theoretically the bar No. 6 could be terminated after continuing it a few inches beyond G to develop full bond strength, or it may be bent up at G to meet the requirements of shear. Similarly bars represented by strips 5, 4, 3 and 2 also could be terminated at a few inches beyond H, I, J and K respectively as they are not required for the B. M. but there are two considerations: (1) that at least 25 per cent of the tensile bars must be carried to the ends and anchored well there and (2) that no economy would be caused by cutting short pieces of rods. It is therefore proposed in the present case to bend only two bars Nos. 5 and 6 at 45° at about 3½ ft. from the ends and carry four bars Nos. 1 to 4 straight to the ends. The ends of all the bars would be terminated into hooks. All this is shown in the longitudinal section of the beam (Fig. 53 (b)). As there is not much space left for anchorage of the bent bars near the end, they are extended downwards to increase the grip length.

In short, the B. M. diagram must be fully enclosed by the R. M. envelope.

Shear Force:—The shear forces from 0 at the support to 10 ft. at the centre of the beam are worked out at every 2 ft. and tabulated below:—

TABLE No. 21

SHEAR FORCE

Distances from end	0	2	4	5	6	8	10
Shear due to distributed load	28100	22480	16860	14050	11240	5620	nil
Shear due to point loads	13440	13440	13440	13440	nil	nil	nil
Total	41540	35920	30300	27490	11240	5620	nil

These figures are plotted in Fig. 53 (c). The maximum shear is at the ends and

$$= 41540 \text{ lbs.}$$

maximum shear intensity

$$= \frac{41540}{15 \times \cdot 87 \times 36} = 83 \text{ lb./in}^2.$$

As this exceeds the safe limit of 75 lbs., reinforcement must be supplied to take the whole shear.

The shear value of the concrete section

$$= s.\ b.\ jd$$

$$= 75 \times 15 \times \cdot 87 \times 36$$

$$= 35235 \text{ lbs.}$$

The point x from the end where it occurs can be found by

$$\frac{10-x}{(35235-13440)} = \frac{10}{28100}$$

$$x = \frac{(281000-217950)}{28100} = \frac{63050}{28100} = 2{\cdot}2 \text{ ft.}$$

Up to 7·8 ft. from the centre, the concrete will take care of all the shear. Beyond that point up to the end shear reinforcement must be provided.

If out of 6–one inch bars 4 are taken straight to the supports and 2 bent at 45°, their shear value

$$= 2 \times \cdot 7854 \times 18000 \sin 45^\circ$$

$$= 2 \times \cdot 7854 \times 18000 \times \cdot 707$$

$$= 19850 \text{ lbs.}$$

Still we must provide for 41540 – 19850 = 21690 lbs. If stirrups of $\frac{3}{8}$ in. bars are provided, the pitch

$$= \frac{2 \times \cdot 11 \times 18000 \times \cdot 87 \times 36}{21690}$$

$$= 6{\cdot}13 \text{ in., say, } 6 \text{ in.}$$

The point at 2·2 ft. from the end is marked on the shear diagram in Figs. 53 (b) and (c) by the vertical line XX. On the right hand of this line, the concrete will take care of all the shear. On the left hand of it 2–one in. bars bent at 45° extended horizontally near the top and terminated in hooks at ends as shown in longitudinal section of the beam in Fig. (b) together with $\frac{3}{8}$ in. stirrups at 6 in. centres will resist all the shear.

Beyond XX on the right hand side the stirrups are provided at 12 in. centres, though theoretically they are not required.

Maximum Bond stress—This is equal to

$$\frac{\text{Maximum shear}}{\text{Sum of perimeter} \times jd}.$$

The perimeter of 4 one in. bars is $4 \times 3{\cdot}14$.

$$\text{Stress intensity} = \frac{41540}{4 \times 3{\cdot}14 \times 36 \times {\cdot}87} = 98{\cdot}37$$

This is well within the limit of 200 maximum. The design is thus completed. Fig. 53 (d) shows a cross section.

Illustrative Example 45

A rectangular beam with compressive reinforcement

Design a simply supported rect. beam over an effective span of 20 ft., carrying a load of 2000 lb./ft. including its own weight. The depth of the beam must not exceed 27 in. overall.

Solution :—Maximum B. M. at centre

$$= \frac{wl^2}{8} = \frac{2000 \times 20^2}{8} \times 12$$

$$= 120000 \text{ in. lbs.}$$

Effective depth d = 27 − 2 (cover) = 25 in.

Assuming a breadth of 12 in.

$$\text{R. M.} = 126\,bd^2 = 126 \times 12 \times 25^2$$

$$= 94500 \text{ in. lbs.}$$

Economic percentage of steel = 0·8

$$A_T = \frac{{\cdot}8bd}{100} = \frac{{\cdot}8 \times 12 \times 25}{100}$$

$$= 2{\cdot}4 \text{ sq. in.}$$

Since the R. M. is less than the B. M., we must strengthen the beam by providing compressive reinforcement, and also by adding to the tensile reinforcement for

$$120000 - 94500 = 25500 \text{ in. lbs.}$$

Assuming the c. g. of compression steel to be at 2 in. below the top, the stress in concrete at that place

$$= \frac{nd - 2}{nd} \times 750 = \frac{\cdot 39 \times 25 - 2}{\cdot 39 \times 25} \times 750$$

$$= \frac{9{\cdot}75 - 2}{9{\cdot}75} \times 750 = 596 \text{ lb./in.}^2$$

The stress in steel $= 596 \times 14 = 8344$ lb./in.^{2}

The lever arm = distance between c. gs. of top and bottom steel $= 25 - 2 = 23$ in.

$$A_c = \frac{25500}{8344 \times 23} = 0{\cdot}133 \text{ sq. in.}$$

Use 2 Nos. $\frac{3}{8}$ in. ϕ bars A $= \cdot 022$ sq. in.

Additional tensile reinforcement

$$= \frac{25500}{18000 \times 23} = 0{\cdot}062 \text{ sq. in.}$$

Total tensile steel $= 2{\cdot}4 + 0{\cdot}062 = 2{\cdot}462$ sq. in.

Use 4 Nos. $\frac{15}{16}$ in. ϕ bars A $= 2{\cdot}76$ sq. in. or, 4–1″ bars.

Thus the section and reinforcement will be as shown in Fig. 54.

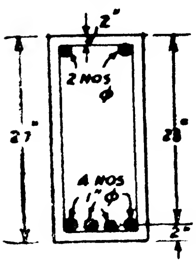


Fig. 54.

Shear :—The maximum shear will occur at ends and will equal $\frac{20 \times 2000}{2}$

$= 20000$ lb.

Intensity of shear $= \frac{20000}{12 \times \cdot 87 \times 25} =$ 77 lb./in.^{2} This is slightly more than the permissible stress of 75 lbs. The excess is so small that if 2 of the 4-1 in. ϕ bars are bent at 45° and in addition $\frac{3}{8}$ in. ϕ stirrups are provided all over the length at 12 in. centres, this will amply meet the situation.

$$\textit{Bond Stress:} \quad = \frac{20000}{2 \times 3{\cdot}14 \times 25 \times {\cdot}87} = 136{\cdot}5 \text{ lb./in.}^2$$

This is less than the permissible stress of 200

Hence the design is safe.

Illustrative Example 46

A simply supported T-beam

A certain framing plan of a building for a restaurant has simply supported beams over a span of 25 ft. between centres of bearings spaced at 8 ft. centres. The slab on their top which is 5 in. thick is cast monolithically with the beams. Design one of the intermediate beams.

Solution:—Wt. of slab (5")	60 lbs./ft.^{2}	
Floor finish	20 ,,	
Total	80 ,,	

The beams are spaced at 8 ft. centres, so that each intermediate beam will support 4 ft. strip on either side.

Dead load of slab	80 × 8	640	lbs./ft.
Superload on slab	100 × 8	800	,,
Assuming beam section 12"× 26"			
Self load of beam		312	,,
	Total	1752	,,
	Say,	1750	,,

$$\text{Maximum B. M. at centre} = \frac{wl^2}{8} = \frac{1750 \times 25^2}{8} \times 12$$

$$= 1664000 \text{ in. lbs.}$$

$$\text{Maximum shear is at end} \quad = \frac{1752 \times 25}{2} = 21900 \text{ lbs.}$$

(*a*) *Flange width*:—

(1) $\frac{1}{3}$ span = $\frac{25}{3} \times 12 =$ 100 in.

(2) Spacing = $8 \times 12 =$ 96 in.

(3) $12 d_s + b =$ $60 + 12 =$ 72 in.

72 in. being the least is adopted.

(*b*) *Depth of beam*:—We have previously assumed a depth = 26 in. overall. Deducting 2 in. for cover, $d = 24$ in. from top of slab to *c. g.* of steel

$$\text{Lever arm} = 24 - \frac{d_s}{2} = 24 - 2{\cdot}5 = 21{\cdot}5 \text{ in.}$$

$$A_T = \frac{1664000}{18000 \times 21{\cdot}5} = 4{\cdot}3 \text{ sq. in.}$$

Use 2 Nos. – $1\frac{1}{4}$ in. ϕ A = 2·454

and 2 Nos. – $1\frac{1}{8}$ in. ϕ A = 1·988

Total 4·442 sq. in.

(*c*) *Stress in concrete*:—The maximum B. M. = 1664000 in. lb., width of flange 72 in., lever-arm = 21·5 and $d_s = 5$ in.

$$\therefore \text{Average stress in flange} = \frac{1664000}{72 \times 5 \times 21{\cdot}5} = 215 \text{ lb./in}^2.$$

This obviously occurs at the middle of flange, i. e. at 2·5 in. below the top or at $\left(nd - \frac{d_s}{2}\right)$ above neutral axis, $nd = {\cdot}39 \times 24 = 9{\cdot}36$ and $\frac{d_s}{2} = 2{\cdot}5$.

The maximum fibre stress in flange

$$= \frac{nd \times 215}{nd - \frac{d_s}{2}} = \frac{9{\cdot}36 \times 215}{9{\cdot}36 - 2{\cdot}5}$$

$$= 293 \text{ lb./in}^2.$$

The concrete is very lightly stressed.

(*d*) *Shear*:—The maximum shear occurs at the support and we have already calculated it to be 21900 lbs. Since the load is uniformly distributed theoretically the shear at the centre is zero. But it may happen that the superimposed load may occupy any part of the slab leaving the remaining part unloaded. The maximum shear at centre would occur when only half the span is loaded as shown in Fig. 55. In that case the shear at the centre would be equal to the end reaction

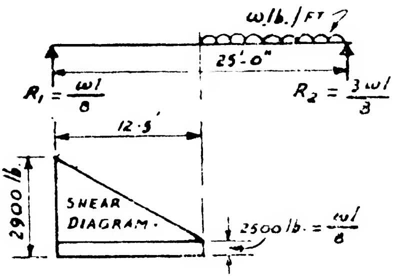


Fig. 55.

$= \frac{wl}{8}$ (see fig.) which is $= \frac{100 \times 8 \times 25}{8} = 2500$ lb.

Then the shear at any point x from the support

$$= 21900 - \frac{21900 - 2500}{12 \cdot 5} x$$

$$= 21900 - 1520\,x \qquad (a)$$

The shear strength of the concrete section $= s \times b \times$ lever-arm $= 75 \times 12 \times 21 \cdot 5 = 19350$ lbs.

Substituting this in the above equation (a)

$$19350 = 21900 - 1520x$$

$$x = 1 \cdot 66 \text{ ft. or } 1' - 8''$$

The concrete section is capable of resisting all the shear up to $1' - 8''$ from the supports. It is therefore only in this length that shear reinforcement is necessary.

If 2 Nos. $1\frac{1}{8}$ in. bars are bent at 45°, their shear resistance

$$= 2 \times \cdot 994 \times 18000 \times \sin 45^\circ$$

$$= 25300 \text{ lb.}$$

We want to provide only for 21900 lb. Hence this is quite ample. Still $\frac{1}{4}$ in. stirrups should be provided in addition at 12 in. centres throughout the length of the beam.

A longitudinal section of the beam with tensile and shear reinforcement is shown in Fig. 56.

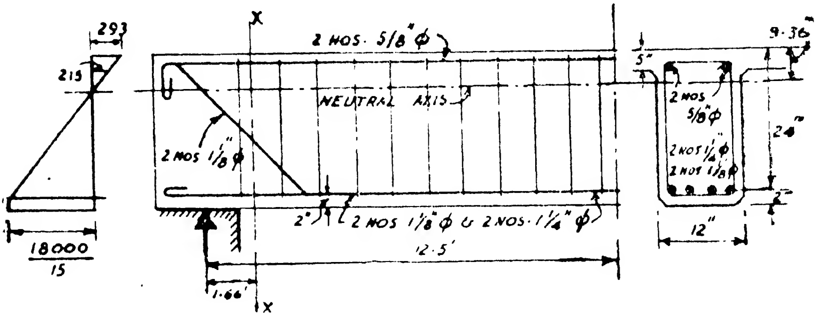


Fig. 56.

Illustrative Example 47

A simply supported rect. beam designed by Theory of Steel Beam.

A simply supported rect. beam has a span of 40 ft. between centres of bearing. Three roof trusses each having a reaction of 2 tons rest on it at intervals of 10 feet. It carries besides a brick wall 15 in. thick including plaster on both sides and 8 ft. high. Design the beam if the æsthetical requirements restrict its depth to a maximum of 40 in.

Solution:—As the beam carries a 15 in. wall, its width must be at least 15 in.

		lbs./ft.
Distributed loads—self load	$15'' \times 40'' =$	600
weight of wall	$15'' \times 8' \times 120 =$	1200
		1800

B. M. due to distributed load

$$= \frac{wl^2}{8} = \frac{1800 \times 40 \times 40 \times 12}{8} = 4320000 \text{ in. lbs.}$$

B. M. due to point loads—

As the loads are symmetrically placed the reaction will be half of the total load = 3 tons.

Maximum B. M. at centre $= (3 \times 2240 \times 20 - 2 \times 2240 \times 10)\ 12$

$- \quad 2 \times 2240 \times 20 \times 12 \qquad = 1075200$

Total B. M. $\quad 5395200$

If the overall depth = 40, effective $d = 37$ in.

$$A_T = \frac{5395200}{18000 \times \cdot 87 \times 37} = 9{\cdot}31 \text{ sq. in.}$$

R. M. of the beam $= 126\ bd^2$

$= 126 \times 15 \times 37^2 = 2587411$ in. lb.

$$\frac{\text{R. M.}}{\text{B. M.}} = \frac{2587}{5395} = \text{less than 48 p. c.}$$

Application of steel beam theory will therefore prove more economical than providing compression steel. (Vide page 102).

Out of 40 in. overall depth, if 2 in. cover is provided both at top and bottom, effective $d = 36$ in.

$A_T \times 18000 \times 36 = 5395200$

$A_T = 8{\cdot}42$ sq. in.

Use 4 Nos. $1\frac{1}{4}$ in. ϕ bars $A = 4{\cdot}908$

and 4 Nos. $1\frac{1}{8}$ in. ϕ ,, $A = 3{\cdot}976$

Total $A_T \quad 8{\cdot}884$

or 7 Nos. $1\frac{1}{4}$ in. ϕ bars $A_T = 8{\cdot}590$

$$\text{percentage of steel} = \frac{8{\cdot}59}{15 \times 36} = 1{\cdot}6$$

This is less than 3 and therefore safe.

(See note on page 105).

The entire shear is caused by dead load and is equal to

$$\frac{1800 \times 40}{2} + \frac{6 \times 2240}{2}$$

$= 42720$ lbs. at ends.

$$\text{Intensity of shear} = \frac{42720}{15 \times \cdot 87 \times 37}$$

$$= 79 \text{ lb./in.}^2$$

This is slightly more than the permissible. Hence provide $\frac{3}{8}$ in. ϕ stirrups.

$$\text{pitch} = \frac{2 \times \cdot 11 \times 16000 \times 36}{42720} \text{ (lever arm)}$$

$$= 2{\cdot}97 \text{ in. say } 3 \text{ in.}$$

As the beam is heavy, a lower shear stress of 16000 lbs./in.^{2} is adopted. 7 rods each $1\frac{1}{4}$ in. would occupy 8·75 in. leaving $(15 - 8{\cdot}75) = 6{\cdot}25$ for 8 spaces. The space between the rods is about ·78 in. This would make proper concreting difficult. Hence it is advisable to place the reinforcement in two tiers, the outer one of 4 bars and inner of 3 bars, or better still, outer tier of 4 Nos. $1\frac{1}{4}$ in. bars, inner of 4 Nos. $1\frac{1}{8}$" ϕ bars with $\frac{3}{8}$ in. stirrups at 3 in. centres as shown in the cross section in Fig. 57. As the stirrups take all the shear, there is no necessity of bending any bars for web reinforcement.

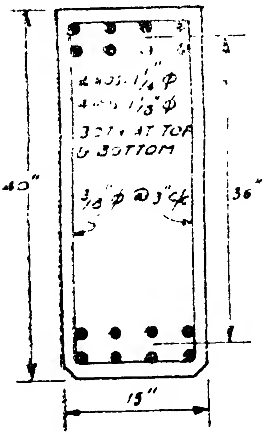


Fig. 57.

Illustrative Example 48

A tee–beam continuous over five spans.

A tee-beam of an office floor has five spans of 18 ft. each. It carries a dead load of 1000 lbs. per foot of slab and partition exclusive of its own weight, and a live load of 1500 lbs. per foot. The flange is 5 in. thick and the beams are spaced at 7' – 6" centres. Design the end span complete in every respect.

Solution:—Dead load due to slab, etc. 1000 lbs./ft.

Rib (assuming $10'' \times (25-5)$	200 „ „	
	1200 „ „	1200
Superincumbent load		1500
	Total	2700

B. M. at centre of end span

$$= \frac{wl^2}{10} = \frac{2700 \times 18^2}{10} \times 12 \text{ (Vide Table No. 16, p. 75)}$$

$$= 1049760 \text{ in. lb. say, } 1050,000.$$

$$\text{Lever arm} = 25 - 2 \text{ (cover)} - \frac{d_s}{2} = 25 - 2 - 2{\cdot}5$$

$$= 20{\cdot}5 \text{ in.}$$

$$A_T = \frac{1050,000}{18000 \times 20{\cdot}5} = 2{\cdot}8 \text{ sq. in.}$$

Provide 2 bars 1″ φ A	= 1·570
and 2 „ $\frac{7}{8}$″ φ A	= 1·202
Total A_T	= 2·772

Since $d = 25 - 2 = 23$, $nd = {\cdot}39 \times 23 = 8{\cdot}97$.

The N. A. falls outside the flange.

Average stress at mid-depth of flange, i. e. at $(8{\cdot}97 - 2{\cdot}5)$ above the N. A.

$$= \frac{6{\cdot}47}{8{\cdot}97} \times 750$$

$$= 540 \text{ lb./in.}^2$$

$$\text{Flange width} = \frac{1050,000}{540 \times 5 \times 20{\cdot}5}$$

$$= 19{\cdot}1 \text{ in.}$$

This is much less than

(i) $\frac{1}{3}$ span $= 72''$; (ii) spacing $= 90''$; (iii) $12 d_s + b = 70''$.

Hence acceptable. The concrete in the flange is very lightly stressed.

Shear :—The shear forces can be accurately calculated with the help of the theory of three moments, and for important works it is advisable to do so. But they can also be calculated approximately by imagining the span to be separate i. e. non-continuous, and compute them as if the beam were simply supported. In the latter case, however, it is usual to adopt a reduced shear stress of 16000 lb./in.^{2} for 1:2:4 ordinary grade concrete.

Adopting the latter method, the shear on the top of the supports in the present example $= \dfrac{2700 \times 18}{2} = 24300$ lbs.

The shear at the centre as explained in Example 46 will be due to live load only and equal to $\dfrac{wl}{8} = \dfrac{1500 \times 18}{2}$

$= 3375$ lbs.

Maximum intensity of shear $= \dfrac{24300}{10 \times 20{\cdot}5} = 106$ lb./in.^{2}

The section is therefore acceptable as the intensity is less than 225 but reinforcement must be provided for the entire shear.

The shear value of the concrete

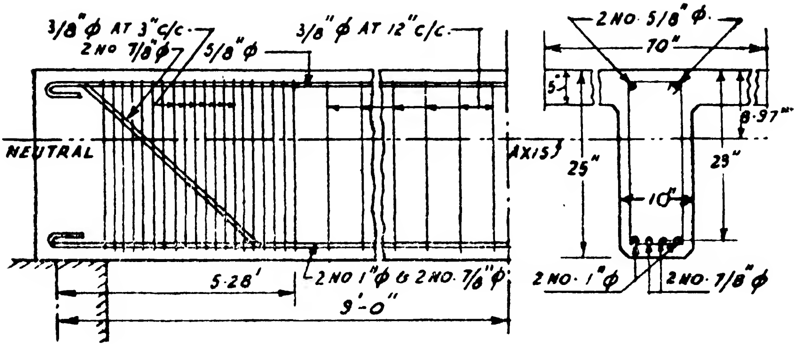
$$= 60 \times 10 \times {\cdot}87 \times 23$$
$$= 12000 \text{ lbs.}$$

The shear varies from 24300 at the support to 3375 at the centre. Shear at any point, x ft. from the support

$$= 24300 - \frac{24300 - 3375}{9} x$$
$$= 24300 - 2325\, x$$

Shear of 12000 lbs. occurs at x

$$12000 = 24300 - 2325\, x$$
$$x = 5{\cdot}28 \text{ ft.}$$



Figs. 58 & 59.

It will be clear that reinforcement for shear is required only in the lengths of 5·28 from the support. If ⅜ in. φ stirrups are provided, their pitch $= \frac{2 \times \cdot 11 \times 16000 \times 20{\cdot}5}{24300} = 3$ in.

In the remaining portion ⅜ in. stirrups may be provided at 12 in. centres.

The end condition is not given but usually it is partially fixed. In that case the negative B. M. may be taken as $\frac{wl^2}{24}$. For this the two ⅞ in. φ bars may be bent at 45°, at $\frac{l}{6}$ or 3 ft. from support and extended horizontally near the top as shown in Figs. 58 and 59.

$$\textit{Bond stress} = \frac{\text{maximum shear}}{\text{perimeter} \times \text{lever arm}}$$

$$= \frac{24300}{2 \times 3{\cdot}14 \times 20{\cdot}5} = 169 \text{ lbs./in}^2.$$

where 3·14 is the perimeter of 1 in. bar. 169 is less than the permissible stress of 200 lbs. and hence is safe.

The design of the sections at centre of intermediate span and at top of intermediate support is similar to the above. Only the B. M.s will be $+\frac{wl^2}{12}$ and $-\frac{wl^2}{12}$ respectively instead of $\pm \frac{wl^2}{10}$ in the end span.

CHAPTER XV

AXIALLY LOADED SHORT COLUMNS

Necessity of Reinforcement :—Theoretically it is uneconomical to use steel in concrete columns. For, we have already seen that for the same strain, steel takes m times the stress in the surrounding concrete, and if $m = 15$ and the allowable compressive stress in concrete $= 600$ lbs./in^2., the reinforcing steel in R. C. C. columns is subjected to 15×600 or 9000 lbs./in.^{2} only when it is capable of taking safely 18,000 lbs. But though plain concrete is very strong in compression, for certain practical reasons reinforcement of steel is necessary. These reasons are: (1) That plain concrete column, even though short, must necessarily be massive. (Minimum section prescribed for plain concrete column under normal load is 12 in. square or round). This not only takes too much space, but also may spoil the architectural appearance. (2) That very frequently columns are subjected to lateral forces such as wind, thrust or pull caused by an inclined member supported by it or a beam framing into it, when deflected. Again, oftentimes the loads coming on their top may be eccentric. Under such circumstances, the columns may be subjected to bending moments, in addition to axial loads. In such cases it is these which may cause tension in some part, and which, therefore, govern the design and make reinforcement essential, rather than the vertical loads.

Types of columns :—Columns are divided according to the manner in which they are strengthened by lateral reinforcement into two categories:

(1) *Tied columns*, in which the longitudinal reinforcement is tied by independent links or hoops, and (2) *Spirally reinforced columns* in which the longitudinal rods are enclosed by closely spaced continuous helical spirals.

There are two classes of columns :—(1) *Short*, when the unsupported length or height does not exceed 15 times the least lateral dimension* and (2) *Long*, when the length is more than 15 times the least width. Thus a column 15 ft. long from the top surface of lower floor to the soffit of the beam supporting the upper floor, is short if the smaller of its sides is $\frac{15 \times 12}{15} = 12$ in. Thus a column, 15″ × 12″ or 12″ × 12″ or 12″ round is short up to 15 ft. height. If the height exceeds 15 ft., or if the smaller dimension is less than 12 in. for the same height, it is termed "long", and a suitable slenderness coefficient is applied to it.

The reinforcement in columns is of two types: (1) *Main* or *longitudinal* reinforcement consisting of vertical bars to share with the concrete the load, and take care of all tensile stresses caused by lateral forces or eccentric loads, in which concrete is weak and (2). *Transverse* or *lateral* reinforcement in the form of either independent links or helically wound spirals around the main reinforcement. The object of the transverse reinforcement is as follows:

(i) To prevent buckling tendency on the part of the longitudinal thin bars pressed at both ends by serving as so many braces.

(ii) To prevent also the concrete from bulging, shearing and cracking in a plane at 45° to the axis in a manner similar to that of a cube of plain concrete in a compression testing machine. The links or the spirals check this tendency. The more closely spaced these links are, as in a spirally bound column, the greater the load the column would safely take. Even a plain concrete column is considerably strengthened by these links or spirals at close intervals.

Design requirements of columns:—These are in respects of (i) Working stresses in concrete and steel. (ii) Cover of concrete. (iii) Effective cross-sectional area. (iv) Main reinforcement. (v) Transverse reinforcement.

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* According to American practice, a long column is one whose length exceeds *10 times* its least cross-sectional dimension.

(i) **Working stresses**:—Both the Bylaw and Code allow a working stress of 600 lbs./in.^{2} in compression for 1:2:4 mix ordinary grade concrete and higher stresses for high grade concretes as given in Table No. 1 page 5.

As regards steel, the Bylaws do not allow more than $m \times c$ stress (maximum of 9000 lbs./in.^{2} when 1:2:4 mix ordinary concrete is used) and further, they do not recognise any extra strength in high tensile steel used in columns, i. e. if used, they do not allow any higher stresses. However they allow a stress of 13500 lbs./in.^{2} only in spiral reinforcement for tension.

The standard practice is, however, to follow the Code of Practice in this respect, which allows 13500 lbs./in.^{2} in mild steel and 15000 lbs./in.^{2} in steels with yield point of at least 44000 lbs./in.^{2} for tension in spiral reinforcement.

(ii) **Cover of concrete**:—The Bylaws require a minimum cover of 1½ in. or equal to the diam. of main steel, whichever is greater.

The Code of Practice requires a cover of 1 in. or equal to the diameter of main steel, whichever is greater.

(iii) Effective cross section:—*For tied columns*, both Bylaws and Code allow the full cross section including the cover as the effective area.

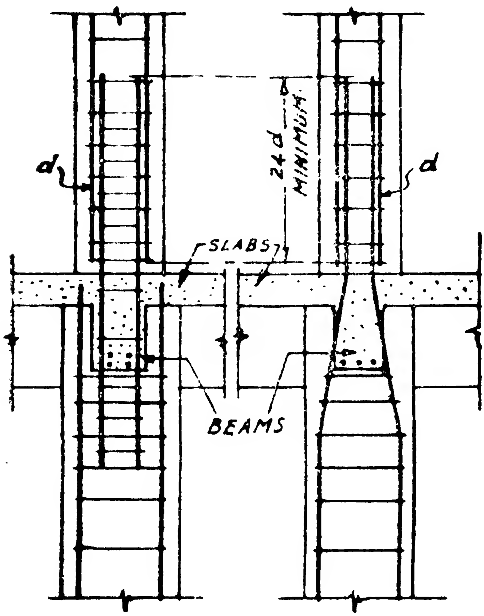
For spirally bound columns both stipulate that only the core area should be counted as effective.

(iv) Main Reinforcement:—(a) *Diameter* :—Both Bylaws and Code prescribe a minimum diameter of ½ in. and maximum of 2 in.

(b) *Percentage* :—The percentage of main reinforcement prescribed is from 0·8 to 8·0 per cent.

The standard practice, however, is to adopt 1 per cent as normal for ordinary columns and a maximum of 5 or at the most 6 per cent for columns when special reasons such as saving in space, too heavy and eccentric loads, architectural needs, etc. prevail.

(c) *Laps in longitudinal rods* :—A minimum lap of 24 diameters of the smaller bar is allowed for lengthening longi-



Figs. 60 and 61—Two different ways of splicing column rods. The joints are made at floor levels and the lap is of 24 diameters of rods of upper column.

tudinal rods. Such joints should only be made at floor levels or beam intersections. At the joints the upper bars are usually smaller than those below. Immediately above and below such a joint, the links should be spaced closer than elsewhere. Figs. 60 and 61 show two such joints. Welding of bars or using screwed coupling is also allowed.

(v) Lateral reinforcement—Independent ties :—(a) ***Diameter*** :—The minimum diameter according to Code is $\frac{3}{16}$ in. and according to Bylaws $\frac{1}{4}$ in. The maximum size is not specified. However for independent ties not more than $\frac{1}{2}$ in. rods are used. For spirals rods up to 1 in. are used.

(b) ***Pitch*** should not be greater than

(1) 12 in.

(2) least lateral dimension.

(3) 12× diam. of smallest longitudinal bar.

(c) *Volume.*—The Code requires nett volume of lateral reinforcement to be not less than 0·4 per cent of total volume of column. Bylaws make no reference to volume.*

Spiral reinforcement:—According to the Code, the pitch of helical binding shall not exceed 3 in. or, ⅓ core diameter whichever is less and not less than 1 in. or 3 × its own diameter whichever is greater.

Design of Short Columns :—*Symbols* :—

W = Load on column.

A_c = Cross-sectional area of concrete.

A_s = Cross-sectional area of steel.

l = Length of column in ft.

D = Diameter of circular column.

d = Diameter of main reinforcement.

c = Permissible compressive stress in concrete.

p = Ratio of area of main steel to that of concrete = $\frac{A_s}{A_c}$.

p_s = Percentage of steel $\frac{A_s}{A_c} \times 100$.

If the applied load is axial, the pressure is uniform throughout the section and the longitudinal steel bars must also share the load in certain proportion, as both the concrete and steel are simultaneously strained. The strains also must be uniform. We have already seen that as the modulus of elasticity of steel is m times that of concrete, for equal strain the stress in steel must be m times the stress in concrete. In other words, every square inch of steel carries m times the load that a square inch of concrete carries or one unit of steel is equivalent to m units of concrete.

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* The standard practice, however, is to use a volume of lateral reinforcement = 0·2 per cent of gross volume of concrete in the case of tied columns, normally, and to increase it to a maximum of 0·5 per cent in columns having longitudinal reinforcement 5 to 6 per cent of the gross volume.

In the examples solved, the Code has been strictly followed, i. e. a maximum of 0·4 per cent is adopted.

∴ The load on a column = load on concrete + m times stress in concrete × the area of steel.

The nett area of concrete in an R. C. C. column = Area of the column − area of steel = $A_c - A_s$.

∴ The load on column, $W = c\,(A_c - A_s) + mc \times A_s$

$= c\,[A_c + A_s(m-1)]$ ....(1)

where W = total load on column, m = modular ratio and c = stress in concrete.

If $p = \frac{A_s}{A_c}$, $W = c[A_c + pA\,(m-1)]$

$= cA_c[1 + p(m-1)]$ ... ... (2)

If $m = 15$, equations (1) and (2) become

$W = c(A_c + 14A_s)$ ... ... (1)

and $W = cA_c(1 + 14p)$ ... ... (2)

Illustrative Example 49

Find the safe load which a column 10 in. square with 4−1⅛ in. ϕ bars will carry.

Solution:

Gross area of column = $10^2 = 100$ sq. in.

Area of steel A_s = 4 sq. in.

Nett area of concrete = 96 sq. in.

$W = 96 \times 600 + 4 \times (15 \times 600)$

$= 57600 + 36000$

$= 93600$ lbs.

or, according to the result (1) above

$= (100 + 14 \times 4) \times 600$

$= 93600$ lbs.

A stress of only 15×600 or 9000 lbs./in.^{2} for steel is assumed in the above calculations according to the recommendations of the Bylaws which is very low.

Recent experiments have shown that under a heavy load continued for a long time, the concrete creeps, (see page 17) resulting in a decrease of unit stress in the concrete, but increase of stress in steel, i. e. more and more load is thrown on the steel, corresponding to the gradual decrease of it on concrete. From this point of view the above formula gives very conservative results. The current practice with British, American and also Indian Engineers is to design columns on the basis of the Plastic Theory* or ultimate stress. Thus, taking yield point stress of steel = about 40,000 lb./in.^{2} and applying a load factor of 3, the formula becomes, $W = 600 (A_c - A_s) + 13500\ A_s$. This, even the Code of Practice seems to have accepted since the working stress in longitudial steel for columns recommended by it is 13500 lb./in.^{2} the safe load in the above example becomes

$$W = c(A_c - A_s) + t \times A_s;\ t = 13500 \text{ lb./in.}^2 \text{ for mild steel.}$$
$$= 600(100 - 4) + 13500 \times 4$$
$$= 57600 + 54000$$
$$= 111600 \text{ lbs. instead of } 93600 \text{ lbs.}$$

Illustrative Example 50

Design the transverse reinforcement for the above column.

Solution:—According to Code 0·4 per cent transverse reinforcement of the volume of concrete is required, i. e.

$$= \frac{\cdot 4}{100} \times 10 \times 10 \times 12$$
$$= 4{\cdot}8 \text{ cub. in. per ft. length}$$

Assuming one inch cover, the length of each link = 4 × 8 = 32 in. and if p be the pitch in inches the number of links per foot $= \frac{12}{p}$.

$$\therefore \quad \frac{\pi d^2}{4} \times 32 \times \frac{12}{p} = 4{\cdot}8$$

$$\text{or } \frac{d^2}{p} = {\cdot}0095$$

* See Chapter XXIV at the end of this volume.

Taking $d = \frac{1}{4}$ or ·25 in.

$$p = \frac{\cdot 25^2}{\cdot 0095} = 6\cdot 5 \text{ in.}$$

Illustrative Example 51

Find the safe load which a 15 in. square column can safely bear if it is reinforced with four bars each of $1\frac{1}{8}$ in. diam. Also design the transverse reinforcement if the cover is $1\frac{1}{2}$ in. thick.

Solution :—Volume of column per foot length

$$= 15 \times 15 \times 12 \qquad = 2700 \text{ cub. in.}$$

Volume of transverse steel per foot

$$= \frac{\cdot 4}{100} \times 2700 \qquad = 10\cdot 8 \text{ cub. in.}$$

The length of a link $= 4 \times 12 = 48$ in.

Using $\frac{3}{8}$ in. ϕ steel for links

$$10\cdot 8 = \cdot 7854 \times \left(\frac{3}{8}\right)^2 \times 48 \times \frac{12}{p}$$

$$p \quad = 5 \text{ in.}$$

$$\text{load, W} = (A_c - A_s)\, 600 + A_s \times 13500$$
$$= (225 - 4)\, 600 + 4 \times 13500$$
$$= 132500 + 54000 = 186600 \text{ lbs.}$$

Illustrative Example 52

Design the longitudinal and transverse reinforcement of a short column 12 in. × 15 in. carrying a load of 75 tons.

Solution :—We know W = load on concrete + load on steel

or $\quad 75 \times 2240 = (15 \times 12 - A_s)\, 600 + A_s \times 13500$
$\qquad = 600 \,\{(180 - A_s) + 22\cdot 5\, A_s\}$

or $\quad \dfrac{75 \times 2240}{600} = (180 - 21\cdot 5\, A_s)$

Solving $\quad A_s = 4\cdot 65$ sq. in.

As it is an oblong column, 6 bars are required. From Table No. 13 page 63, 6 Nos. 1 in. ϕ bars give 4·712 sq. in.

Hence **6 Nos. 1 in.** ϕ bars.

Transverse reinforcement :—As there are six bars, it is desirable to use one more link to bind the two middle bars as shown in Fig. 62. Allowing a cover of $1\frac{1}{2}$ in. the total length of the binding bars would be $42 + 18 = 60$ in. and if $\frac{1}{4}$ in. bar is used, the volume $= 60 \times 0{\cdot}049 = 2{\cdot}94$ in.^{3} According to Code of Practice 0·4 per cent steel must be used.

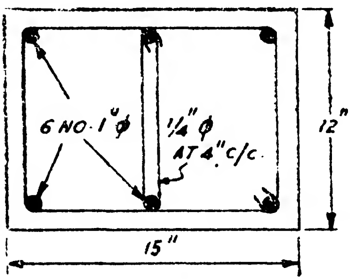


Fig. 62.

Volume of 12 in. length of column

$$= 15 \times 12 \times 12 = 2160 \text{ cub. in.}$$

Volume of transverse steel 0·4 per cent of concrete

$$= 2160 \times {\cdot}004$$

$$= 8{\cdot}64 \text{ in.}^3$$

$$\text{Pitch } p = \frac{2{\cdot}94 \times 12}{8{\cdot}64} = 4{\cdot}1 \text{ in.}$$

say $= 4$ in.

Lateral reinforcement — $\frac{1}{4}$ *in.* ϕ *double links at 4 in. c/c.*

Spirally bound columns :—When columns are reinforced with helically wound rods, the total safe load they can bear is given by the equation

$$W = W_c + W_T + W_{spiral}$$

in which W = load taken by concrete, $= c\,(A_c - A_s)$

$W_T =$ „ „ longitudial rods $= 13500 \times A_s$

$W_{spiral} =$ „ „ the spiral $= 2 \times 13500 \times A_{spiral}$

A_{spiral} = the volume of helical binder per unit length of column. An illustrative example will make this clear.

Illustrative Example 53

A short column of 12 in. external diameter is reinforced with 8 No. $\frac{3}{4}$ in. ϕ bars and $\frac{5}{16}$ in. ϕ helical binders, with a pitch of $1\frac{1}{8}$ in. c/c. Calculate the safe load which it can be subjected to.

Solution:—Area of 8 Nos. $\frac{3}{4}$ in. ϕ rods $=3{\cdot}53$ sq. in.

Assuming $1\frac{1}{2}$ in. cover, diam. of column $=9$ in.

Area of core $= \frac{\pi}{4}\,9^2 = 63{\cdot}62$ sq. in.

Length of one round of binder

$$= 3{\cdot}14 \times 9 = 28{\cdot}26$$

Volume of " " " $= 28{\cdot}26 \times \frac{\pi}{4}\left(\frac{5}{16}\right)^2$

$$= 2{\cdot}16 \text{ cub. in.}$$

The pitch is $1\frac{1}{8}$ in.

∴ Volume of coil per unit (inch) length

$$= \frac{2{\cdot}16}{1\frac{1}{8}} = 1{\cdot}92 \text{ cub. in.}$$

Total load $=$ Load on concrete $+$ load on vertical steel $+$ load taken by spirals

$$= 63{\cdot}62 \times 600 + 3{\cdot}53 \times 13500 + 2 \times 1{\cdot}92 \times 13500$$

$$= 38172 + 47655 + 51840$$

$$= 137667 \text{ lbs.}$$

Illustrative Example 54

A column having 15 in. core diameter bound by a spiral of $\frac{3}{8}$ in. diam. with a pitch of 2 in. has 8 No. $1\frac{1}{8}$ in. ϕ bars. If a rich concrete of 1:1·5:3 mix ordinary grade with a safe compressive stress of 680 lb./in.^{2} is used calculate the safe load the column could carry.

Solution:—Area of main steel $= 8$ sq. in.

Area of core $= {\cdot}7854 \times 15^2 = 196{\cdot}36/\text{in.}^2$

Volume of spiral per ring $= \pi D \times$ area of $\frac{3}{8}''$ ϕ

$$= 3{\cdot}142 \times 15 \times 0{\cdot}11$$

$$= 5{\cdot}18 \text{ in.}^3$$

Since the pitch is 2″

Volume of spiral per inch $= 2{\cdot}59$ cub. in.

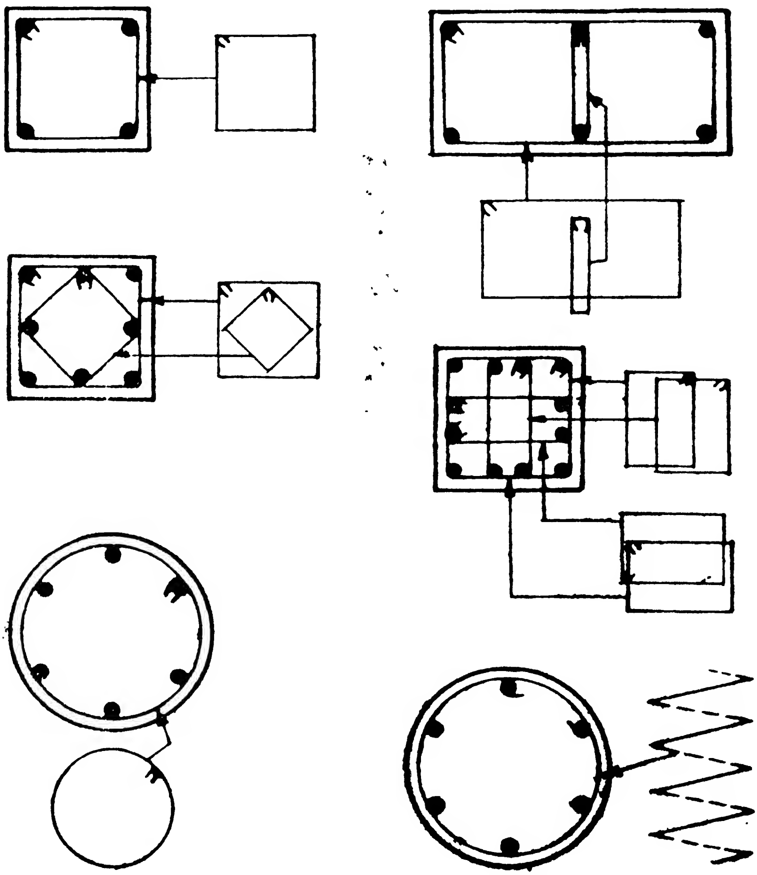
Total load = Load shared by concrete + Load shared by main steel + load shared by spirals

$= 680 \times 196{\cdot}35 + 8 \times 13500 + 2 \times 2{\cdot}59 \times 13500$

$= 133500 + 108000 + 69930$

$= 311430$ lbs. or 139 tons.

Some common sections of columns are given below with their links or spirals:—



Figs. 63 to 69—A few common cros sections of columns with their binders shown separately.

Illustrative Example 55

Design a short round column for an unsupported length of 15 ft. to carry an axial load of 60 tons including self-load. It is to be tied with independent links.

Solution:—In order that a column of 15 ft. unsupported length should fall into the category of short columns its minimum diameter

$$= \frac{15 \times 12}{15} = 12 \text{ in.}$$

Area of 12 in. column $= \cdot 7854 \times 12^2 = 113{\cdot}1$ sq. in.

We know, W = Load taken by concrete + load on steel

$$60 \times 2240 = 600(113{\cdot}1 - A_s) + 13500 \times A_s$$
$$= 600 \} 113{\cdot}1 - A_s + 22{\cdot}5A_s \{$$
$$\text{or } 224 = (113{\cdot}1 + 21{\cdot}5A_s)$$
$$\text{or } 224 - 113{\cdot}1 = 21{\cdot}5A_s$$
$$A_s = \frac{110{\cdot}9}{21{\cdot}5} = 5{\cdot}3 \text{ sq. in.}$$

As the column is round, a minimum number of six bars must be used.

From Table No. 13 page 63, 6 No. $1\frac{1}{8}$ in. ϕ give 5·96 sq. in.

Volume of concrete per foot length.

$$= 113{\cdot}1 \times 12 = 1357{\cdot}2 \text{ cub. in.}$$

Volume of transverse steel

$$= \frac{1357{\cdot}2 \times \cdot 4}{100} = 5{\cdot}43 \text{ cub. in.}$$

Length of one ring = $3{\cdot}14 \times 10$ allowing one in. cover

$= 31{\cdot}4$

$$\text{Pitch}, = \frac{31{\cdot}4 \times \cdot 11 \times 12}{5{\cdot}43} = \frac{41{\cdot}45}{5{\cdot}43} \text{ (using } \tfrac{3}{8} \text{ in. } \phi \text{ links)}$$
$$= 7{\cdot}6 \text{ in. Say, } 7\tfrac{1}{2} \text{ in.}$$

Answer:— 12 in. ϕ column
6 Nos. $1\frac{1}{8}$ in. ϕ bars
$\frac{3}{8}$ in. ϕ links at $7\frac{1}{2}$ in. *c/c*.

Some Practical Hints and Tips

1. Columns are foundation members of a structure and so are more important even than beams and slabs. Utmost care is therefore necessary in their design and construction. A rich mixture, say of 1:1·5:3 is very helpful, particularly when the loads are heavy. It allows a higher stress to be taken in calculations and makes it also economical.

2. Ample cover of concrete (1 in., preferably $1\frac{1}{2}$ in. for columns of normal sizes) should be provided. As the area of cover is included in the effective cross sectional area (except in spirally bound columns) there is no reason why economy in cover should be made.

3. The entire unsupported length of column *e. g.* from top of lower floor to the soffit of the beam under the upper floor should be filled in one operation.

4. Utmost care should be taken to see that the main steel rods are truly in plumb and remain so while filling. This precept is very often overlooked, with adverse results.

5. The most important precept, which unfortunately is honoured more in breach rather than in observance is that the centre line of all the columns one upon another on different floors should be the same. There is a tendency on the part of architects and house-builders, either for greed of space or for appearance to push the upper columns towards the outside to the limit of the forward edge. But this is a dangerous practice, as it causes progressively greater eccentricity.

CHAPTER XVI

AXIALLY LOADED LONG COLUMNS

THERE is practically no difference between the design of short and long columns. The only small difference is that as a long column is likely to bend and buckle, a certain coefficient depending upon the ratio of its length to the least lateral dimension, called the "*slenderness ratio*," is to be applied to reduce the load. Then for that reduced load it is to be designed as a short column.

Additional symbols used :—

I_{ec} = Equivalent moment of inertia of section.

A_{ec} = Equivalent area of concrete.

g = Radius of gyration.

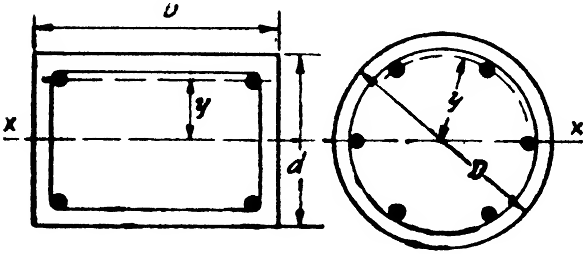
According to the Code of Practice the slenderness ratio of a long column is the ratio of its "effective" length to the least cross sectional dimension. If it is more than 15, it is a long column. But this method cannot be applied to columns of irregular cross section or with re-entrant angles. The method recommended by the Bylaws, though slightly more complicated, is applicable to all cases. In it the slenderness ratio is the ratio of the "effective" length to the least radius of gyration (g) of the section. If it exceeds 50, it is a long column. There are two new technical terms introduced, viz. radius of gyration, and "effective" length. We shall first explain the radius of gyration.

$$\text{Radius of gyration } (g) = \sqrt{\frac{\text{Equivalent moment of inertia}}{\text{Equivalent concrete area}}}$$

$$= \sqrt{\frac{I_{ec}}{A_{ec}}}.$$

We have already discussed A_{ec} in the previous chapter. It is

$$= \text{Area of concrete} + m \times \text{area of steel.}$$



Figs. 70 & 71.

The equivalent moment of inertia of an R. C. section about x-axis is calculated by the following equation:

$$I_{ec} = \tfrac{1}{12}\, bd^3 + (m-1)\, A_s\, y^2 \qquad (1)$$

where b = side || to x-axis and d = side || to y-axis if the A_s = area of total reinforcement and y = the distance of the centroid of steel from the x-axis. If the section is a square with sides = a, or a circle of diam. = D, obviously the equations are respectively,

$$I_{ec} = \tfrac{1}{12}\, a^4 + (m-1)\, A_s\, y^2 \qquad (2)$$

$$\text{and} \quad I_{ec} = \tfrac{1}{64} D^4 + (m-1)\, A_s\, y^2 \qquad (3)$$

Illustrative Example 56

Calculate the radius of gyration of a rectangular column having sides 12″ and 18″ and 4 bars $1\tfrac{1}{8}''\ \phi$ placed with their centres 2 in. inside the surface.

Solution:—Here $b = 18$; $d = 12$; $y = \frac{12}{2} - 2 = 4$,

$$A_s = 4\,\frac{\pi d^2}{4} = 3{\cdot}976$$

$$I_{ec} = \tfrac{1}{12} \times 18 \times 12^3 + 14 \times 3{\cdot}976 \times 4^2$$

$$= 2592 + 891 = 3483 \text{ in.}^4$$

$$A_{ee} = 18 \times 12 + 14 \times 3{\cdot}976$$
$$= 216 + 55{\cdot}7$$
$$= 271{\cdot}7 \text{ in.}^2$$

Radius of gyration $g = \sqrt{\frac{3483}{271{\cdot}7}} = \sqrt{12{\cdot}6} = 3{\cdot}54$ in.

Illustrative Example 57

Calculate the radius of gyration if

(i) the section is a square with sides, a = 16 in.

(ii) „ „ „ a circle of diam. = 16 in.

Take the same reinforcement as in the above example for (i) and 6 bars $1\frac{1}{8}$ in. (ii) all placed with centres 2 in. inside the surface.

Solution:—$a = 16$; $A_s = 3{\cdot}976$; $y = \frac{16}{2} - 2 = 6$

(i) $I_{ee} = \frac{1}{12} \cdot 16^4 + 14 \times 3{\cdot}976 \times 6^2$
$$= 5461{\cdot}3 + 2003{\cdot}9$$
$$= 7465{\cdot}2 \text{ in.}^4$$
$$A_{ee} = 16^2 + 3{\cdot}976 \times 14 = 256 + 55{\cdot}7 = 311{\cdot}7$$
$$g = \sqrt{\frac{7465{\cdot}2}{311{\cdot}7}} = \sqrt{23{\cdot}9}$$
$$= \mathbf{4{\cdot}9} \text{ in.}$$

(ii) $I_{ee} = \frac{1}{64} \pi\, 16^4 + 14 \times 5{\cdot}964 \times 36$
$$= 3215{\cdot}4 + 3005{\cdot}8$$
$$= 6221{\cdot}2 \text{ in.}^4$$

In this $y = 6$; if the six $1\frac{1}{8}$ in. rods are imagined to form a ring of steel, its centre line will be 6 in. in a radial direction from the centre.

$$A_{ee} = \frac{\pi}{4} 16^2 + 14 \times 5{\cdot}964$$
$$= 200{\cdot}9 + 83{\cdot}5 \text{ in}^2.$$
$$g = \sqrt{\frac{6221{\cdot}2}{284{\cdot}4}} = \sqrt{21{\cdot}8} = \mathbf{4{\cdot}64} \text{ in.}$$

In order to simplify calculations sometimes the area of reinforcement is neglected in finding the approximate moment of inertia. But this obviously does not give correct results. The I_{ec} in Example 57 would then be

$$= \frac{1}{12}\, 18 \times 12^3 = 2592 \text{ instead of } 3483$$

$$\text{and } g = \sqrt{\frac{2592}{271 \cdot 7}} = \sqrt{9 \cdot 26} = 3 \cdot 05 \text{ instead of } 3 \cdot 54.$$

The term "effective" length is rather misleading. It is also called "virtual" length or the portion of a vertical column which is likely to bend under its load. A column which has both its ends fixed in position and direction, *e. g.* a column with a deep footing at bottom and whose top end is fixed by four beams in four directions is supposed to have an effective or virtual length which is likely to deviate from the vertical as shown in Fig. 72 equal to ·75l according to Bylaws and the Code. There is at the other end the "lamp-post" type column with its lower end fixed in position and direction and the upper end free. In this case the effective or virtual length is 2l. R. C. C. columns are generally framed into and monolithically cast with beams, and those on the ground floor have their lower ends fixed in footings under ground. Their actual length may therefore safely be regarded as effective length, unless special circumstances prevail to decide otherwise.

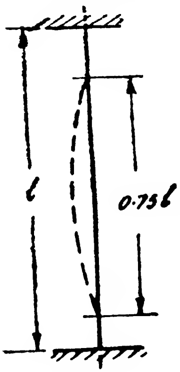


Fig. 72.

The coefficients applied to the load on long columns for treating them as short, for design purposes are called "buckling coefficients" and are obtained by the following empirical formulæ : *

$$K_1 = 1 \cdot 5 - \frac{l}{30d} \qquad \text{......................... (1)}$$

* According to American practice. $K = 1 \cdot 3 - 0 \cdot 03 \frac{l}{d}$.

in which K_1 = buckling coefficient according to the code of practice, l = effective length in inches and d = the least lateral dimension in inches, and

$$K_2 = 1{\cdot}5 - \frac{l}{100g} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

in which K_2 = buckling coefficient according to Bylaws, l has the same meaning and g = least radius of gyration $= \sqrt{\frac{I_{cc}}{A_{cc}}}$.

The following table gives the results worked out from the above formulæ for different slenderness ratios.

TABLE No. 22

Ratio of Effective Length to Least Lateral Dimension (Code of Practice)	Ratio of Effective Length to Least Radius of Gyration (Bylaws)	Buckling Coefficients
15	50	1·0
18	60	0·9
21	70	0·8
24	80	0·7
27	90	0·6
30	100	0·5
33	110	0·4
36	120	0·3
39	130	0·2
42	140	0·1
45	150	0·0

Note:—In the case of a spirally bound column the least lateral dimension and the least radius of gyration are to be measured on the area of the core and not on that of the gross cross section.

Illustrative Example 58

An R. C. C. column 10 in. square is 15 ft. long. The longitudinal reinforcement consists of 4 – 1 in. ϕ bars with independent ties. Find the safe load permissible by the Code.

Solution :—The safe load if it were a short column

$$= c \times \text{Area of concrete} + t \times \text{area of steel}$$
$$= 600\,(10^2 - 3{\cdot}14) + 13500 \times 3{\cdot}14$$
$$= 58116 + 42390$$
$$= 100506.$$

Taking effective length equal to the actual length,

$$\frac{\text{Effective length}}{\text{Least lateral dimension}} = \frac{15 \times 12}{10} = 18$$

As this is more than 15, it is a long column and the buckling coefficient from the above table is 0·9.

$$\therefore \quad \text{Safe load} = 0{\cdot}9 \times 100500$$
$$= 90450 \text{ lbs.}$$

Illustrative Example 59

Find the buckling coefficient by the radius of gyration method and calculate the safe load and design the lateral reinforcement of a column 24 ft. high, 12″ × 18″ section, with 6 – 1·0 in. ϕ bars placed with their centres 2·0 in. inside the surface.

Solution :—Here, $A_s = 4{\cdot}71 \text{ in.}^2 ; \; y = \frac{12}{2} - 2 = 4$ in.

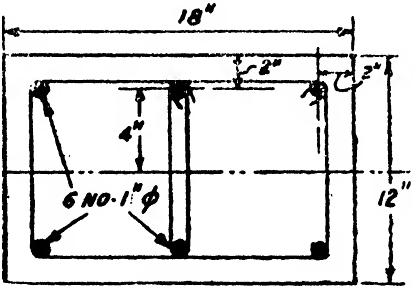


Fig. 73.

$$I_{cc} = \frac{18 \times 12^3}{12} + 14 \times 4{\cdot}71 \times 4^2$$
$$= 2592 + 1062$$
$$= 3654 \text{ in.}^4$$
$$A_{cc} = 18 \times 12 + 14 \times 4{\cdot}71$$
$$= 216 + 66$$
$$= 282$$
$$g = \sqrt{\frac{3654}{282}} = \sqrt{12{\cdot}9} = 3{\cdot}55$$

$$\text{Slenderness ratio} = \frac{24 \times 12}{3{\cdot}55} = 80{\cdot}1$$

$$K = 1{\cdot}5 - \frac{l}{100g} = 1{\cdot}5 - \frac{24 \times 12}{100 \times 3{\cdot}55}$$

$$= 0{\cdot}69$$

$$\text{Safe load } W = 0{\cdot}69(600 \times 18 \times 12 + 13500 \times 4{\cdot}71)$$

$$= {\cdot}69\,(129600 + 63585)$$

$$= \mathbf{133300} \text{ lbs. or } \mathbf{59} \text{ tons.}$$

Transverse Reinforcement:—The minimum required according to Code regulations is 0·4 per cent of the gross volume of concrete

$$= \frac{18 \times 12 \times 12 \times {\cdot}4}{100} = 10{\cdot}4 \text{ cub. in.}$$

With a cover of one in. all round, the length of one binder = 2 (16+10) = 52 in. If $\frac{3}{8}$ in. ϕ binders are used

$$\text{pitch} = \frac{52 \times 0{\cdot}11 \times 12}{10{\cdot}4}$$

$$= 6{\cdot}6 \text{ in. say } 6\tfrac{1}{2} \text{ in.}$$

But in this arrangement the two vertical rods in the middle of the long sides are not bound so well as those at the corners. If special binders are provided to tie these together, the length of one such binder will be 2 × 10 = 20. In all 52 + 20 = 72 in.

If $\frac{3}{8}$ in. binders are used

$$\text{pitch} = \frac{72 \times 0{\cdot}11 \times 12}{10{\cdot}4}$$

$$= 9{\cdot}1 \text{ say } 9 \text{ in.}$$

It is however desirable to use $\frac{1}{4}$ binders (area roughly half that of $\frac{3}{8}''$ ϕ) at $4\frac{1}{2}$ in. c/c.

Illustrative Example 60

Design a square column with independent ties having an effective length of 12 ft. to support an axial load of 80,000 lbs. both by the method recommended by L. C. C. Bylaws and by Code of Practice.

Solution:—As the effective height is 12 ft. and the column is square its side must be equal.

$$\frac{12\times 12}{15} = 9{\cdot}6'', \text{ say } 10''$$

As there are no restrictions on size we shall provide 1 per cent of concrete for longitudinal reinforcement and the minimum steel required by the Regulations viz. 0·4 per cent for lateral reinforcement.

$$\text{Area of concrete} = 10\times 10 = 100 \text{ sq. in.}$$

$$\text{Area of steel} = \frac{1}{100}\times 100 = 1{\cdot}0 \text{ sq. in.}$$

4 – $\frac{5}{8}$ in. ϕ bars give 1·2 sq. in.

$W = 80{,}000$ lbs.

According to the Bylaws,

$$80{,}000 = c \times \text{nett area of concrete} + mc \times A_s.$$

$$= c\,\{(10^2-1{\cdot}2) + 15\times 1{\cdot}2\}$$

$$= 98{\cdot}8 + 18 = 116{\cdot}8$$

$$c = \frac{80000}{116{\cdot}8} = 685 \text{ lb./in.}^2$$

This goes beyond the safe limit. We must therefore either increase the size or increase the reinforcement. Suppose we make the size 12″ × 12″, the area of steel would be 144″ × ·01 = 1·44 (4 Nos. $\frac{11}{16}$ in. ϕ A = 1·484) or we can keep the same area of steel since minimum allowed is ·8 per cent.

$$\text{Then } c = \frac{80000}{142{\cdot}8+18} = 497 \text{ lbs./in.}^2$$

This is safe.

Transverse Reinforcement:—Assuming a cover of 1¼ in. all round the main rods, the length of a tie = 4×9=36 in. Adopting ·4 per cent of steel

$$\text{Volume} = \frac{0{\cdot}4}{100} \times 12 \times 12 \times 12 = 6{\cdot}9 \text{ cub. in.}$$

$$\text{If } p = \text{pitch, } \frac{\pi d^2}{4} \times 36 \times \frac{12}{p} = 6{\cdot}9$$

$$\text{Using } \tfrac{1}{4} \text{ in. links } p = \frac{36 \times 12 \times {\cdot}049}{6{\cdot}9}$$

$$= 3{\cdot}07 \text{ say } 3 \text{ in.}$$

The complete design then becomes:

Size	**12″ × 12″**
Main steel	**4 No. ⅝ in. φ bars**
Links	**¼ in. φ at 3 in. *c/c*.**

(ii) According to Code of Practice.

$$W = 80000 = c \times (100 - 1{\cdot}2) + 13500 \times 1{\cdot}2$$

$$\text{Solving,} \quad c = \frac{63800}{98{\cdot}8} = 645 \text{ lbs./in.}^2$$

This is also slightly more than the permissible safe stress. Hence keep the size 12″ × 12″. There is no necessity then to check the stress in concrete which is bound to be less than 600.

Other part of the design remains as above.

Procedure in practical design:—In practice generally the load is known as it can be calculated, the height is also known and columns are to be designed in respects of their sizes, and longitudinal and lateral reinforcement. The width of the beam on top of the columns generally decides one side of the column if it is oblong rectangular or both sides if it is a square. A square column is more economical, but if the beam or the wall on top of the beam is not sufficiently wide, an oblong rectangular column has to be provided. A circular column requires elaborate shuttering which makes it uneconomical.

As the beams generally run through the columns in a slab-and-girder floor, the column section below the floor must be larger than the beam widths, and that above the floor, smaller. If there are no special restrictions on the size of the column, provide 1 % of steel. In that case the concrete area of a square column would be $0{\cdot}99a^2$ and steel area $\cdot01a^2$. The design can then be started.

Illustrative Example 61

Design a square column to carry 80 tons of load if its unsupported length is 12 ft. high

Solution:—Assuming 1 per cent longitudinal reinforcement

$$80 \times 2240 = \cdot99a^2 \times 600 + \cdot01a^2 \times 13500$$

$$a^2 = 239$$

$$a = 15{\cdot}5 \text{ say } 16 \text{ in.}$$

Slenderness ratio $= \dfrac{12' \times 12}{16} = 9$, this is less than 15. The column is short.

Area of steel $= 2{\cdot}40$ sq. in.

Use 4 bars $\frac{7}{8}$ in. ϕ A $= 2{\cdot}40$ sq. in.

The length of one link, leaving $1\frac{1}{2}$ in. for cover is 52 in. and if $\frac{1}{4}$ in. ϕ links are used, the volume per link $= 52 \times \cdot049$ $= 2{\cdot}55$ cub. in.

Volume of concrete per foot length

$$= 16^2 \times 12 = 3072 \text{ cub. in.}$$

0·4 per cent of this $= 12{\cdot}3$ cub. in.

$$\text{pitch } p = \frac{12{\cdot}3}{2{\cdot}55} = 4{\cdot}8 \text{ in.}$$

say $4\frac{1}{2}$ in.

Illustrative Example 62

Design an axially loaded spirally bound column on the ground floor of an office building having 4 storeys and a terraced roof. The height of the ground floor is 16 ft. and that of the upper floors 12 ft. each. The column carries a panel 10′ × 10′ of a flat, i. e. beamless slab, the entire dead load of which is 85 lbs./ft². The average weight of the column may be taken as 400 lbs./ft.

Solution:—In order that the column should fall in the category of short columns, the slenderness ratio, i. e. ratio of height to $g = 50$,

$$\frac{h}{g} = 50, \; g \text{ for a circle is } \frac{D}{4}.$$

$$\frac{16 \times 12}{\frac{D}{4}} = 50 \text{ or } D = \frac{16 \times 12 \times 4}{50} = 15{\cdot}4 \text{ in.}$$

The actual g would be much more when the reinforcement is taken into account. Hence we shall assume the overall diameter of the column as 15 in. as a first approximation. Allowing $1\frac{1}{2}$ in. cover the core diameter will be 12 in.

		Dead Load	Live Load	Total lbs.
Loads of	Roof	85 × 100 +	100 × 80 × 100 %	= 16500
10′ × 10′ panel	5th floor	85 × 100 +	100 × 80 × 100 %	= 16500
= 100 sq. ft.	4th ,,	85 × 100 +	100 × 80 × 90 %	= 15700
	3rd ,,	85 × 100 +	100 × 80 × 80 %	= 14900
	2nd ,,	85 × 100 +	100 × 80 × 70 %	= 14100
	1st ,,	85 × 100 +	100 × 80 × 60 %	= 13300
			Total	91000

Add wt. of column

Ground floor	16 × 400 = 6400		
4 floors	4 × 12 × 400 = 19200		
	25600		25600
	Total load		116600
		say	120000 lb.

Let us try one per cent main reinforcement.

Its area $= \frac{\pi D^2}{4} \times \frac{1}{100} = \frac{\cdot 7854 \times 12^2}{100}$ (core diam. = 12″)

$= \frac{113 \cdot 04}{100} = 1 \cdot 13$ sq. in.

A minimum of 6 bars are required as it is circular. Use 6 Nos. ½ in. ϕ with A = 1·17. The load, W, which the concrete and steel in this column can bear safely,

$$= 600 \left(\frac{\pi 12^2}{4} - 1 \cdot 17 \right) + 13500 \times 1 \cdot 17$$

$$= 600\,(113 \cdot 04 - 1 \cdot 17) + 13500 \times 1 \cdot 17$$

$$= 67100 + 15800$$

$$= 82900 \text{ lb.}$$

Our load is 1,20,000. Hence the spiral reinforcement must take 120,000 − 82900 = 29100 lbs.

If ⅜ in. ϕ spiral is used, the strength of one turn

$$= 2 \times \text{volume of one ring} \times 13500$$

$$= 2 \times \pi d \times \cdot 11 \times 13500$$

$$= 2 \times 3 \cdot 14 \times 12 \times \cdot 11 \times 13500$$

$$= 112050 \text{ lb. per inch.}$$

We want 29100 lbs.

Hence pitch $= \frac{112050}{29100} = 3 \cdot 8$ in.

According to regulations the pitch should not exceed 3 in. Hence, the complete design is:

Overall diam.	**15 in.**
Core diam.	**12 in.**
Main steel	**4 − ½ in. ϕ bars**
Spiral	**⅜ in. ϕ at 3 in. *c/c*.**

CHAPTER XVII

COLUMNS WITH ECCENTRIC LOADING

COLUMNS are frequently subjected to eccentric loads, such as for instance, that placed on a bracket connected to a column or of beams eccentrically connected to it. These eccentric loads cause a bending moment in addition to the vertical thrust caused by the load. The result is that there is an additional compression in some part due to the eccentricity with perhaps tension in the remaining part. A student of the theory of structure must be aware that when the resultant of pressure falls away from the centre, at a distance e = one-sixth the width at base or at what is called the "middle third" of the width there is neither tension nor compression at the extreme edge, but that the pressure on the nearer edge is double the average, and that if e is greater than one sixth width there is tension at the extreme edge. This is expressed by the formula :

$$P = \frac{W}{D}\left(1 \pm \frac{6e}{D}\right) \text{ where } D = \text{width at base.}$$

This makes two distinct cases in which columns can be divided. (1) When $e \angle \frac{D}{6}$ or ·17 of the width, in which case, there is compression on the entire section and the neutral axis is outside the section, and (2) when there is tension in part of the section and the neutral axis is inside the section. In the case of R. C. C. columns, since the reinforcement helps in providing m times its area as equivalent area of concrete, case (2) occurs in general, when $e > 0{\cdot}27D$. Between these two limits viz. $e = {\cdot}17D$ and $e = {\cdot}27D$ there may or may not be tension in part of the section depending on the mix and the percentage of steel.

Case 1 :—Eccentricity not exceeding ·17 of width, compression on entire section, and neutral axis outside section. This is explained below.

Fig. 74 (a) and (b) show a horizontal and vertical section respectively of a rectangular column of sides, B × D with a direct load P, applied at a distance e away from the central axis on one side. This will cause a direct compressive stress of uniform intensity as shown in the stress diagram (c), and bending stresses consisting of compression on one side and equal tension on the other, of the neutral axis, as shown at (d). When both (c) and (d) are combined, the tensile stresses in (d) are annulled or cancelled by an equal amount of compressive stresses in (c), still their effect as far as bending is concerned, will remain unchanged. The combined stress diagram is shown at (e) from which it will be seen that there is compression throughout the section, the stress on one side being minimum and that on the other maximum. Expressed mathematically the direct compressive stress $= \frac{P}{A_{ec}} = \frac{P}{A\{1+(m-1)p\}}$

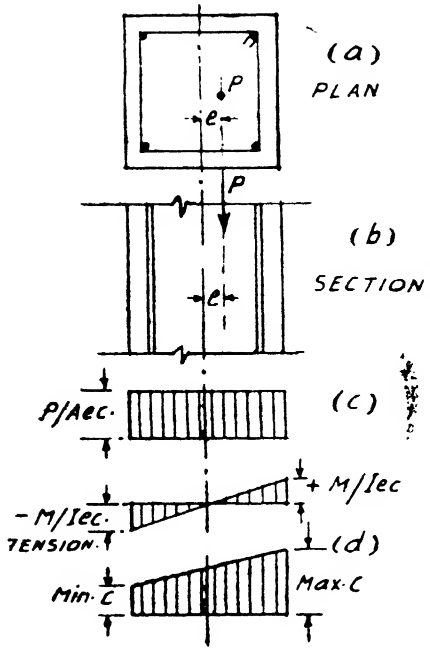


Fig. 74

the bending stress, $f = \frac{My}{I} = \frac{B.M.}{I_{ec}} \times y = \frac{P.e}{I_{ec}} \times \frac{D}{2}$

For rectangular section $= \frac{P \times e}{\frac{1}{12} BD^3 + m(A_s \times x^2)} \times \frac{D}{2}$

where x = distance of the centre of longitudinal reinforcement from the axis of the column.

$$\text{Combined stress} = \frac{P}{A_{ec}} \pm \frac{P \times D}{I_{ec} 2}.$$

Illustrative Example 63

A column, 12 in. square carries a direct load of 44,000 lbs. placed 2 in. away on one side of its central axis. The longitudinal reinforcement consists of 4 – 1 in. ϕ rods placed with centres 2 in. inside the surface. If the height of the column is 15' calculate the maximum and minimum stresses in concrete and steel.

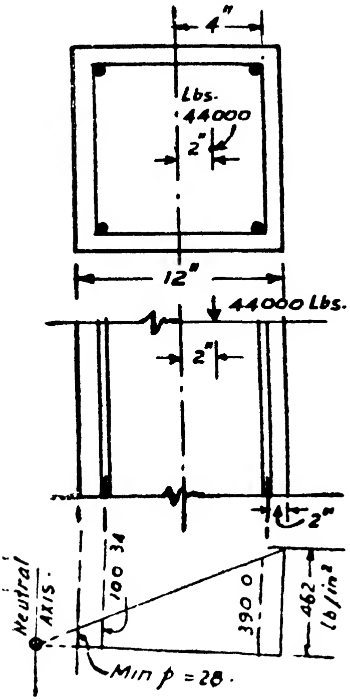


Fig. 75 *a*, *b*, *c*.

Solution:—The eccentricity e, is 2"; $\frac{l}{6}$ or $\frac{12}{6} = 2''$. It is just at the middle third of the base. There will then be little or no tension on the cross section.

Direct load $= 44{,}000$ lbs.
Self wt. $= 15' \times 12'' \times 12'' = 2{,}160$ „

Total 46,160 „

Bending moment $= 44{,}000 \times 2$
$= 88{,}000$ in.lbs.

$I_{ec} = \frac{1}{12} \times 12 \times 12^3 + 14(3{\cdot}14 \times 4^2)$
$= 1728 + 704 = 2432$ in.^{4}

$A_{ec} = 12 \times 12 + (14 \times 3{\cdot}142) = 188$ in.^{2}

Combined stress $= p_1 + p_2$

$$= \frac{46160}{188} \pm \frac{88000 \times 6}{2432}$$

$= 245 \quad \pm 217$

$\therefore$ **Maximum c** $=$ **462** lbs./in ^{2}

Minimum c $=$ **28** lb./in^2

Stress in steel must be m times the stress in surrounding concrete. From the stress diagram (Fig. 75 (c)) the stress increases from 28 to 462 in 12 in. i. e. at the rate of $\frac{434}{12} =$

36·17 lbs. per in. As the centres of steel rods are 2 in. inside the edges on either side

Maximum stress in steel $=15(462-2\times 36{\cdot}17)=$ **5845** lb./in.^{2}.
Minimum stress in steel $=15(28+2\times 36{\cdot}17)=$ **1505** lb./in.^{2}.

As there is compression on the entire section the neutral axis is out-side the section.

Case 2:—Tension on part of the section. Neutral axis within section.

We have seen that the combined stress $=\dfrac{P}{A_{cc}}\pm\dfrac{P\times e\times D}{2\,I_{ec}}$

when the first expression $\dfrac{P}{A_{ec}}<\dfrac{P_e D}{2\,I_{ec}}$, the resultant stress is negative i. e. there is tension in some part and the neutral axis falls within the section.

Illustrative Example 64

A rectangular column 20 in. × 12 in. is reinforced with 4 — 1⅛ in. bars with their centres 1½ in. inside the surface. It is subjected to a direct axial load of 60,000 lbs. and a bending moment of 240,000 in. lbs. Find the minimum and maximum stresses both in concrete and steel.

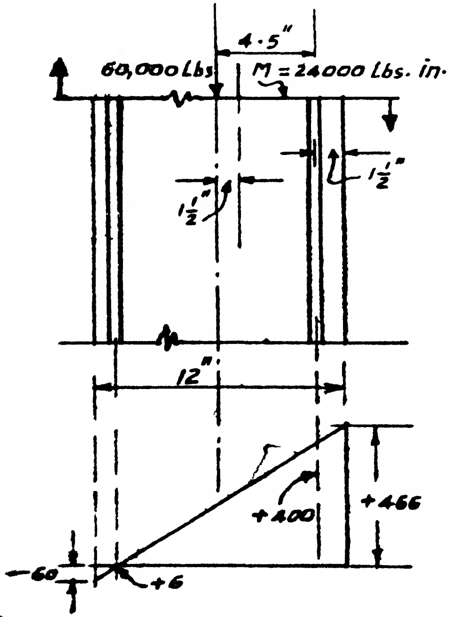


Fig. 76.

Solution:—

$A_{ec}=20\times 12+14\times 4$
$=296$ in.^{2}

$I_{ec}=\frac{1}{12}\cdot 12\times 20^3+14\times 4\times 4{\cdot}5^2.$
$=8000+1134$
$=9134$ in^4.

$e=\dfrac{240{,}000}{60000}=4>\dfrac{20}{6}$

There will therefore be tension in some part.

Combined stress

$=\dfrac{60{,}000}{296}\pm\dfrac{240{,}000\times 10}{9134}$

$=203\pm 263$

max. stress = **466 lb./in.^{2} compression** in concrete

min. stress = − **60 lb./in.^{2} tension** in concrete.

Rate of increase per inch $= \dfrac{60+466}{12} = 44$ lbs./in.

Stress in steel on tension side at 1·5 in. inside the edge

$= (-60+1{\cdot}5\times 44)\times 15 =$ **90 lb./in.^{2} compression**

Stress in steel on compression side

$= (466 - 1{\cdot}5\times 44)\ 15$

= **6000 lb./in.^{2} compression.**

Stress intensities:—The procedure adopted above in finding out the stress values may be expressed in mathematical signs thus :

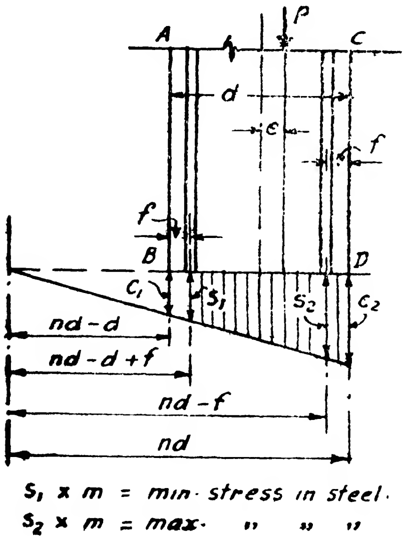


Fig. 77.

If n is the neutral axis constant as in the case of beams so that nd = distance of the N. A. from the opposite edge of the cross section and if f = the concrete cover beyond the centres of longitudinal rods as shown in Fig. 77 and c_1 = maximum stress in concrete and c_2 = minimum, referring to Fig. 77 above

$$\frac{c_2}{c_1} = \frac{nd-d}{nd};$$

$$c_2 = c_1\,\frac{(n-1)}{n}$$

average stress in the column $= \dfrac{c_1+c_2}{2}$

$$= \frac{c_1 + c_1\left(\dfrac{n-1}{n}\right)}{2} = c_1\left(\frac{2n-1}{2n}\right)$$

The compression taken by concrete

$$P_1 = \text{average stress} \times \text{area of concrete}$$

$$= c_1 \frac{(2n-1)}{2n} \times (A - A_s) \;\ldots\ldots\; (\times)$$

Stress intensities in steel:

$$= \text{intensity in surrounding concrete} \times m$$

$$= \frac{nd-f}{nd} \times mc_1 \text{ in rods near face CD}$$

$$= \frac{nd-d+f}{nd} \times mc_1 \text{ in rods near face AB}$$

$$\text{Average stress in steel} = \tfrac{1}{2} \frac{mc_1}{nd} \left\{ (nd-f) + (nd-d+f) \right\}$$

$$\text{Total compression in steel} = \text{average stress} \times A_s$$

$$P_2 = A_s\, mc_1 \left(\frac{2n-1}{2n} \right)$$

$$\text{Total load on column} = P_1 + P_2 = (A_q - A_s) \times c_1 \left(\frac{2n-1}{2n} \right) + A_s mc \left(\frac{2n-1}{2n} \right)$$

$$= c_1 \frac{2n-1}{2n} (A_c - A_s \overline{m-1}) \;\ldots\ldots (\times)$$

This expression is similar to the general formula for the total axial load on an R.C.C. column (vide equation 1 page 173) with the only difference that this is qualified by a constant $= \frac{2n-1}{2n}$ in which n = neutral axis constant.

Practical Design :—It will be seen from the above equation (×) that if n i. e. the constant of neutral axis is calculated, the relation between the total load, P, the cross sectional area A_c and steel area A_s becomes very simple for a given maximum compressive stress c, in concrete and a fixed value of m. But the calculations for finding out the value of n are so involved with so many variables that practical designs are almost invariably made by assuming a section, percentage of steel and value of m with the help of suitable charts. The stresses are then calculated as shown in the illustrative examples given above to see that they are within prescribed limits. *Manning* has given very good charts at the end of Chapter VIII in his book "*Reinforced Concrete Design.*"

Bending in columns:—We have so far considered non-axial, or slightly eccentric loads only on columns. Oftentimes the eccentricity is too much and as a result the bending moments are very high, for instance, an external column of a structure framing into a fairly stiff beam cast monolithically with it. As the beam deflects under its heavy load, its end exerts a pull on the upper end of the column as shown in Fig. 78. Since the column is loaded only at its top, the B. M. in the column will vary uniformly from the value of M_L at the top of the column to M_U of the floor below, at its bottom. As shown in the figure each column has a point of inflection near its centre. The end of the beam transmits part of its bending moment to the column. The exact amount depends upon the relative stiffness of the beam and that of the column at its lower and upper ends. If the beam is very stiff and the column comparatively flexible, the entire B. M. at the end of the beam may be transmitted to the column for which it must then be designed.

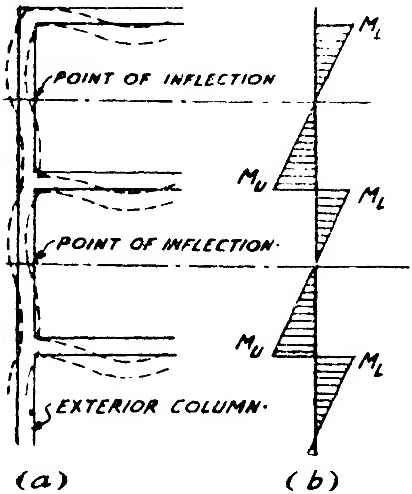


Fig. 78.

The stiffness of a structural member is measured by the ratio of the moment of inertia of its section to its length. In other words, stiffness $= \frac{I_{ea}}{l}$.

Both the Code and Bylaws recommend the formula given in the following table for estimating the moments at the foot of the upper column (M_U) and at head of the lower column

(M_L) separately for the framed structure of only one bay and of more than one bay. The formula appears to be complicated as there are a number of new symbols, but in fact it is very simple.

TABLE No. 23

BENDING IN EXTERNAL COLUMNS

	Frame of one bay only	Frame of two or more bays
Moment at foot of upper column M_L	$Me \dfrac{K_U}{K_U + K_L + \frac{1}{2}K_B}$	$Me \dfrac{K_U}{K_U + K_L + K_B}$
Moment at Head of Lower Columns M_U	$Me \dfrac{K_L}{K_U + K_L + \frac{1}{2}K_B}$	$Me \dfrac{K_L}{K_U + K_L + K_B}$

In the above formulæ

M = B. M. transmitted to column

M_e = B. M. at the end of the beam framing into the external column, on the assumption that its ends are rigidly fixed.

K_B = stiffness of the beam $= \dfrac{I_{ec}}{\text{span}}$

K_U = stiffness of the upper column $= \dfrac{I_{ec}}{l}$

K_L = „ „ „ lower column $= \dfrac{I_{ec}}{l}$

Note 1:—For columns of top storey $K_U = 0$. The B. M. at the end of the beam framing rigidly into the column is obviously $= M_U + M_L$.

Note 2:—For the purposes of this formula the Code of Practice has allowed the moment of inertia to be calculated on the gross area of the section, ignoring the area of reinforcement.

The following example will make the procedure of design clear.

Illustrative Example 65

A framed structure having more than two bays consists of two storeys, the height of first is 16 ft. and that of the upper 12 ft. The exterior lower column carries a total load of 2,40,000 lbs. including its own weight, and also the reaction of beam, etc. The beam framing into it has a span of 16 ft. and a section of 12″ × 22″ including a cover of 2″, and carries a load of 1600 lb./ft. including self-load. Design the exterior column of the lower floor. Assume the upper column to be of the same size as the lower one.

Solution:—For the first approximation we shall take a load 20 per cent in excess of the actual load and design the column as a "short" one.

$$\text{Load W} = 2{,}40{,}000 \times \frac{120}{100} = 288{,}000 \text{ lbs.}$$

We shall assume a 3 per cent reinforcement as it is generally between 1 % and 5 %. Then

$$W = 288000 = 600\,(a^2 + \cdot03a^2 \times 14)$$

Solving we get $a = 18{\cdot}5$ in.

Let us take 18 in. square as the size.

Then the reinforcement $= 18 \times 18 \times 0{\cdot}03 = 9{\cdot}72$ in.^{2}

8 bars $1\frac{1}{4}$ in. ϕ give $A = 9{\cdot}817$ sq. in.

Now the stiffness of the upper column

$$K_U = \frac{I}{l} = \frac{18 \times 18^3}{12} \div 12 \times 12 = 60{\cdot}75$$

The stiffness of the lower column

$$K_L = \frac{I}{l} = \frac{18 \times 18^3}{12} \div 16 \times 12 = 45{\cdot}5$$

The stiffness of the beam

$$K_B = \frac{I}{l} = \frac{12 \times 20^3}{12} \div 16 \times 12 = 41{\cdot}67.$$

End moment of the beam

$$M_o = -\frac{wl^2}{12} = \frac{1600 \times 16 \times 16}{12} \times 12 = 409600 \text{ in. lb.}$$

Moment at the head of the lower column

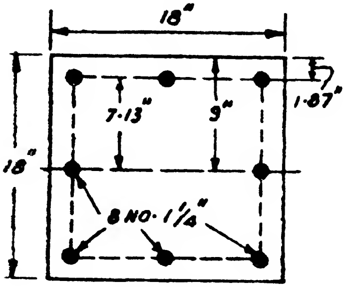


Fig. 79.

$$M_U = M_e \frac{K_L}{K_U + K_L + K_s} = 409600$$

$$\times \frac{45{\cdot}5}{60{\cdot}75 + 45{\cdot}5 + 41{\cdot}67}$$

$$= 409600 \frac{45{\cdot}5}{147{\cdot}92}$$

$$= 124600 \text{ in. lbs.}$$

The reinforcement is 8 bars $1\frac{1}{4}$ in. ϕ. Allowing a cover = diam., of bar the *c. g.* of steel will be $1{\cdot}25 + \frac{1{\cdot}25}{2} = 1{\cdot}87$ in. inside the surface or $9 - 1{\cdot}87 = 7{\cdot}13$ in. from the axis as shown in Fig. 79.

The moment of inertia

$$I_{ec} = \frac{18 \times 18^3}{12} + 14 \times 6 \times 1{\cdot}227 \times 7{\cdot}13^2$$

$$= 8750 + 5270 = 14020 \text{ in.}^4$$

It is possible to find Z_{ec} from I_{ec}

$$Z_{ec} = \frac{I_{ec}}{\frac{D}{2}} = \frac{14020}{9} = 1558 \text{ in.}^3$$

This Z_{ec} is made up of the concrete section and the equivalent area of the steel

$$A_{ec} = 18 \times 18 + 14 \times 9{\cdot}817$$

$$= 461{\cdot}5 \text{ in.}^2$$

$\therefore$ maxi. compressive stress $= \frac{W}{A_{ec}} + \frac{124600}{1558}$

$= 520 + 80 = 600$ lb./in.²

mini. „ „ $= 520 - 80 = 440$ lb./in.²

Average compressive stress $= \frac{600 + 440}{2} = 520$ lb./in.²

Safe load the column would carry if it is "short"

$$W = 520 \times \text{concrete area} + 520 \times m \times \text{steel area}$$
$$= 520\,(18 \times 18 - 9{\cdot}817) + 520 \times 15 \times 9{\cdot}817$$
$$= 163400 + 76600$$
$$= 2{,}40{,}000 \text{ lbs.}$$

$$g = \sqrt{\frac{I_{ec}}{A_{ec}}} = \sqrt{\frac{14020}{461{\cdot}5}} = 5{\cdot}5 \text{ in.}$$

Slenderness ratio

$= \frac{16 \times 12}{5{\cdot}5} = 34{\cdot}8$. As this is less than 50, the column is short, and no reduction in load is called for.

Thus a column **18″ × 18″** with **8 No. 1¼″** ϕ bars will carry the load safely.

It will be observed from the procedure adopted in the illustrative example solved above, that there is no direct method of hitting on the correct size and reinforcement of a column to carry a certain load. A trial and error method has to be employed. To simplify the design, some books have given charts or tables which give a very close approximation to the correct result. For example such graphs are given in G. P. Manning's, "Reinforced Concrete Design" Chapter VIII both for rect. and circular columns.

Problems for Practice

Take c = 600 lbs./in.², $m = 15$ and $t = 18000$ lb./in.² unless otherwise mentioned.

(1) Calculate the permissible load on a column 16″×16″ 12 ft. long reinforced with 4 – 1⅜ in. round bars if it is tied by independent links. Ans. 306,900 lb.

(2) Design the minimum size and longitudinal reinforcement of a column to support an axial load of 100,000 lbs. having an unsupported length of 10 ft.

Ans. 8 in. square, 4 No. 1¼″ ϕ;

(3) Find the maximum safe load on a column 18 in. diam. (16″ core) and 20 ft. high reinforced with 8 – ⅝ in. bars, spirally bound with $\frac{5}{16}$ in. helicals with a pitch of 1¾ in.

Ans. 220,100 lb.

(4) An R. C. C. column 15 in. square reinforced with 4 – 1 in. ϕ bars with a clear cover of 1½ in. is subjected to an axial load of 1,00,000 lbs. and a bending moment of 1,48,000 in. lbs. Calculate the maximum and minimum stresses in concrete.

Ans. 570 and 170 lb./in.^{2}

(5) Find the radius of gyration of

(a) 24″ × 20″ with 8 – 1¼ in. round bars with their centres 2 in. inside the surface.

(b) 21 in. diam. with 12 – $\frac{7}{8}$ in. round bars placed with a cover of 2½ in. beyond centres.

Ans. (a) $A_{ec} = 346 + 79 = 425$ in.^{2}
$I_{ce} = 9540 + 2530 = 12070$ in.^{4}
$g = 5{\cdot}33$.

(6) Design the smallest section for a short column to carry 100 tons of axial load with minimum reinforcement and with 1 : 1·5 : 3 mix concrete ordinary grade (permissible stress $c = 680$ lbs./in.^{2})

(7) A square column with 16 in. sides is reinforced with 4 – 1 in. round bars placed with centres 1½ in. inside the faces. Find the stresses in concrete and steel if they are subjected to an axial load P = 90,000 lbs. and a bending moment M = 160,000 in. lbs.

(8) A rectangular column 18 in. × 12 in. reinforced with 6 – $\frac{7}{8}$ in. round bars with centres 2 in. inside faces carries a load of 75,000 lbs. placed 4 in. away from the central axis. Compute the stresses in concrete and steel.

CHAPTER XVIII

R. C. C. FOUNDATIONS

THE entire load, both structural or dead, and moving or live, is ultimately transferred, through walls, columns or both to the "soil" below the base of the structure. The "soil" may be of clay, or clay mixed with sand or lime, or muram or rock, each having its own safe bearing power. If the intensity of the load exceeds this safe bearing power in a certain part, that part sinks or settles down. If the soil under the entire structure sinks down uniformly, there is no harm, but if some part sinks down more than the other, cracks are formed in the structure and there is a tendency to tilt.

The principles underlying safe foundations are:

(1) That no part of the structure should, under any combination of loading be stressed beyond the safe bearing power of the soil on which the structure is to rest. i. e. the structure should, as it were, "float" on the soil.

(2) That the axis of the load should coincide with the centroid of the area of foundation. In other words, the settlement, if any, should be uniform.

To achieve the first object, the concentrated load is distributed over a large area of the soil by spreading out the "footings" of the structure, so as to reduce the intensity of pressure to within the limit of the safe bearing power.

Even though the column be circular the footing is invariably square.

The design consists of determining

(1) The area of footing

(2) The depth or thickness of footing.

(3) The reinforcement in the footing slab

(1) **Area of footing**:—This should be sufficiently large to keep the intensity of vertical pressure within the safe bearing power of the soil.

If W = Total load including that of the column and its footing, and p_b = safe bearing power in lb./ft.^{2} of the soil,

$$\text{Area of footing} = \frac{W}{p_b} \text{ sq. ft,}$$

$$\text{Side of square footing} = \sqrt{\frac{W}{p_b}} \text{ ft.}$$

Illustrative Example 66

A square concrete column carries a load of 60,000 lb. including its own weight. The soil which consists of dry clay mixed with sand is capable of safely bearing 1·25 tons per sq. ft. Find the area of footing.

Solution:—p_b = 2240 × 1·25 = 2800 lb./ft.^{2}

Deduct wt. of footing (assuming 1 ft. thick) = 144

Net p_b = 2656

Load to be supported = 60,000 lb.

$$\text{Area of footing} = \frac{60000}{2656} = 22{\cdot}6 \text{ ft.}^2$$

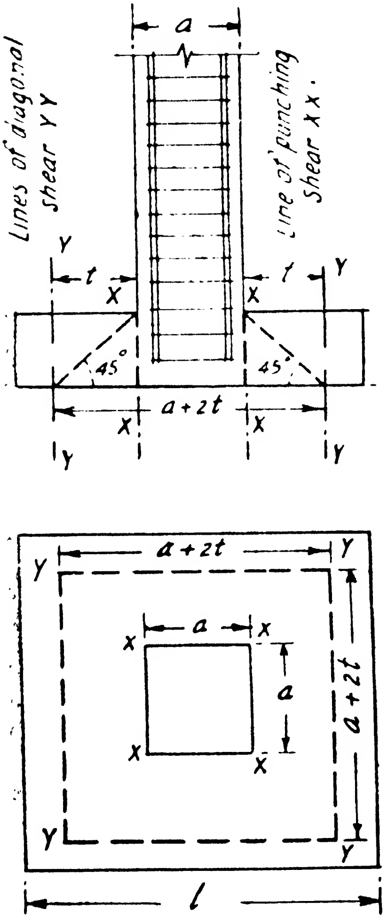
$$\text{Side of square footing} = \sqrt{22{\cdot}60} = 4{\cdot}7' \text{ or } 4'-9''$$

(2) The depth or thickness of footing is determined by one of the three:

(a) Punching shear (b) Diagonal shear and (c) Bending moment, whichever gives the maximum result.

A footing is, as a rule, strong in resistance to B. M. Hence the depth is generally determined by the punching shear or diagonal shear whichever gives greater result.

Punching Shear:—The thickness or depth of a footing must not be so small as to cause the danger of the column punching a hole through it as shown by the dotted vertical lines XX in Fig. 80. Assuming failure to occur along the lines XX, the area over which the punching shear would act is the product of the perimeter of the column and the depth or thickness of the footing i. e. $4a \times t$ (see Figs. 80 and 81) or $\pi D \times t$ if the column is round of diameter D.*



Figs. 80 & 81–Plan and section of footing.

The intensity of the upward reaction of the soil below the footing caused by the load W, on the column is $\frac{W}{l^2}$ and acts on the area $l^2 - a^2$ with a tendency to lift the footing, while the load W on the column presses the column down. These two forces opposite to each other in direction cause a shear which acts along the sides of the column, through thickness t of the footing. Equating these two,

$$\frac{W}{l^2} \times (l^2 - a^2) = 4a\,t \times s.$$

where a^2 = cross sectional area of the column, $4a$ = perimeter of the column, and s = unit shear. The permissible value of punching shear which according to regulations is twice the ordinary shear i. e. 150 lbs./in². for 1:2:4 ordinary grade of concrete.

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* Some authorities disregard punching shear altogether, as, according to them it is highly improbable that a hole may be punched straight through, instead of at an angle of 45° or 60°. The Code of Practice has made no mention of punching shear.

Illustrative Example 67

A column 12 in. square carries a load of 120,000 lb. including its own weight and that of the footing. If the safe bearing power of the soil is $2\frac{1}{2}$ tons/ft.², design the footing area and the depth for punching shear.

Solution:— $p_b = 2240 \times 2{\cdot}5 = 5600$ lbs./ft².

Deduct wt. of footing (21″ assumed)

$144 \times 1{\cdot}75 = 252$ „ „

Net $p_b = 5348$ „ „ say 5350

Area of footing $= \frac{120000}{5350} = 22{\cdot}5$ ft.²

Side of square footing $= \sqrt{22{\cdot}5} = 4{\cdot}75$ ft.; $l^2 = 22{\cdot}56$ ft.².

Total punching shear $= \frac{W}{l^2} \times (l^2 - a^2)$.

$$= \frac{120000}{22{\cdot}56} \times (22{\cdot}56 - 1^2).$$

This must be equal to $4a\ t \times s$

$$t = \frac{120000}{22{\cdot}50} \times \frac{21{\cdot}56}{4 \times 12 \times 150}$$

$= 16{\cdot}3$ in. effective.

As this will remain under ground, a minimum cover of 3 in. is required. Adopt overall T = say, 21 in.

(3) *Diagonal Shear* :—This is the same type of shear we have already discussed while designing web reinforcement in beams. It is measured at the vertical planes lying at 45° from the column face as shown at Y—Y by dotted lines in Fig. 80. Obviously the planes are at a distance of t from the faces on all sides (some authorities take the critical section for diagonal shear at the vertical plane at the intersection of 60 degree planes with the bottom i. e. at a distance of $\frac{t}{\sqrt{3}}$ from the column face.)

The unit diagonal shear should not exceed $\frac{1}{10}$ of the safe compressive stress in concrete i. e. for 1 : 2 : 4 mix ordinary grade concrete, it should not exceed 75 lb./in.². As no reinforcement either in the form of inclined bars or stirrups is provided in column footings, the thickness should be ample. Diagonal shear is calculated in the same manner as punching shear, only, the perimeter is taken at a distance of t instead of at the faces as in the case of punching shear. Thus,

Total shear S = unit upward soil pressure × area at the footing outside the section at t from the faces.

$= p_b \{ l^2 - (a + 2t)^2 \}$ and this acts on the area $= 4(a + 2t) \times \cdot 87\, t$.

$$\therefore \text{ The intensity of shear } = \frac{p_b \times \{ l^2 - (a + 2t)^2 \}}{4(a + 2t) \times \cdot 87 \times t}$$

Illustrative Example 68

Calculate the intensity of diagonal shear in the above example taking $p_b = 5320$ lb./ft.^{2} and verify if the depth adopted to meet punching shear is sufficient.

Solution:—Here $a = 12$ in., W = 120,000 lb. $p_b = 5320$ lb./ft.^{2} $t = 17$ in. (provisional) ; $l = 4' - 9''$.

$$\therefore \quad a + 2t = 12 + 34 = 46 \text{ in.} = 3{\cdot}83 \text{ ft.}$$

$$\text{Diagonal shear, } S = p_b \{l^2 - (a + 2t)^2\}$$

$$= 5320\,(4{\cdot}75^2 - 3{\cdot}83^2)$$

$$= 5320 \times 7{\cdot}51$$

$$\text{Intensity of shear, } s = \frac{5320 \times 7{\cdot}51}{4\,(a + 2t) \times \cdot 87 \times 17}$$

$$= \frac{5320 \times 7{\cdot}51}{4 \times 3{\cdot}83 \times \cdot 87 \times 17}$$

$$= 173 \text{ lb./in.}^2$$

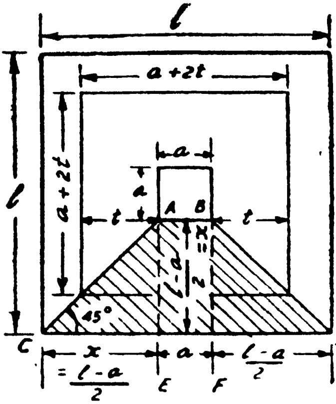


Fig. .

This is far in excess of the permissible shear of 75 lb./in.^{2} We must therefore increase the effective depth of footing. Let us assume it 1′–9″.

Now $a + 2t = 1 + 2 \times 1{\cdot}75 = 4{\cdot}5$.

Intensity of shear,

$$s = \frac{5320\,(4{\cdot}75^2 - 4{\cdot}5^2)}{4 \times 4{\cdot}5 \times {\cdot}87 \times 21} = 37{\cdot}5 \text{ lb./in}^2.$$

This is quite safe. Hence **t = 1·75 ft.**

Bending Moment:—For purposes of calculating the B. M. the base area is divided into four trapeziums like the one A B C D shown hatched in Fig. 82. Each trapezium is regarded as a cantilever of depth t fixed at its support near the faces of the column and of span equal to AE with a uniform load $= p_b$ / ft.^{2} of the soil reaction pressing upwards from below. The area of the trapezium consists of two triangles at sides ACE and BDF and the rectangle in the middle AB × AE. Taking moments about AB

$$\text{B. M.} = (2 \times \triangle \text{ACE} \times \tfrac{2}{3}\text{AE} + \text{AB}{\cdot}\text{AE} \times \tfrac{1}{2}\text{AE})\, p_b.$$

$$\text{Now AE} = \text{BF} = \text{CE} = \text{DF} = x$$

$$\therefore \quad \text{B. M.} = \left(2\frac{\text{AE}^2}{2} \times \frac{2}{3}\text{AE} + \frac{\text{AB} \times \text{AE}^2}{2}\right) p_b$$

$$= \left(\frac{2x^3}{3} + \frac{ax^2}{2}\right) p_b.$$

The width on which this B. M. is supposed to act and in which, therefore, reinforcement must be provided is the column width, plus twice the effective depth, plus half the remaining width of the footing, if any, i. e.

$$a + 2t + \frac{l - (a+2t)}{2}.$$

The following example will make the design procedure clear.

Illustrative Example 69

Calculate the B. M. and design the necessary reinforcement for the column in the above example.

Solution:—Here $a = 12''$; t (effective) = 21 in.

$$x = \frac{l-a}{2} = \frac{4{\cdot}75-1}{2} = 1{\cdot}875;\ a+2t = 4{\cdot}5 \text{ ft};\ p_b = 5320$$

$$\text{B. M.} = \left(2\times\frac{1{\cdot}875^2}{2}\times\frac{2}{3}{\cdot}1{\cdot}875 + 1\times1{\cdot}875\times\frac{1{\cdot}875}{2}\right)5320$$

$$= \left(\frac{2\times1{\cdot}875^3}{3} + \frac{1{\cdot}875^2}{2}\right)5320 = 30218 \text{ ft. lb.}$$

$$= 362616 \text{ in. lb.}$$

This acts on the width $= 12''+2\times21+\dfrac{57-54}{2}$

$$= 55{\cdot}5 \text{ in.}$$

$$d \text{ for B. M.} = \sqrt{\frac{362616}{126\times55{\cdot}5}} = \text{about } 7{\cdot}5.$$

We have adopted 21 in. to satisfy the requirements of diagonal shear and hence, is ample

$$A_T = \frac{362616}{18000\times{\cdot}87\times21} = 1{\cdot}1 \text{ sq. in.}$$

This area of steel is to be spread over a width of 55·5 in.

$\therefore$ Area of steel per foot width $= \dfrac{1{\cdot}10}{55{\cdot}5} = 0{\cdot}02$ in.^{2} This is very small. We may require more steel to meet the requirements of bond stress. Provide $\frac{5}{8}$ in. ϕ bars at $6\frac{1}{2}$ in. *c/c*. (A=0·204). The number of rods is 10 at right angles to each other in both directions. But as there is only $1\frac{1}{8}$ in. width remaining on either side we shall use one more bar that is 11 bars in all, in one direction and 11 at rt. angles to these, with hooks at end, forming a mesh or grid tied by pieces of wires at every intersection.

Bond Stress:—This is very important in footing design and should always be checked. The American practice is to use deformed bars in footing slab to make it strong in bond. The permissible bond stress is the same as in beams and slabs, viz. $0{\cdot}1\,c + 25$, i. e. 100 lb./in.² for 1 : 2 : 4 mix. ordinary grade concrete.

The bond stress is calculated on $\frac{1}{4}$ of the load causing punching shear and is resisted by the friction between the surface of bars and the surrounding concrete. Thus if S = punching shear, and $\Sigma 0$ = the sum of perimeters of bars

$$\text{the intensity of bond stress} = \frac{\frac{1}{4}S}{\Sigma 0 \cdot jd}$$

and this should not exceed 100 lb./in.² for 1 : 2 : 4 ordinary grade concrete.

Illustrative Example 70

Check the bond stress in the above example.

Solution:—$\dfrac{\text{punching shear}}{4} = \dfrac{120000}{22{\cdot}56} \times \dfrac{21{\cdot}56}{4}$ (vide Ex. 68).

The area on which it acts = perimeter of 11 – $\frac{3}{8}''$ bars $\times {\cdot}87 \times 21 = 11 \times 1{\cdot}18 \times {\cdot}87 \times 21$

$$\therefore \text{ Intensity of bond stress} = \frac{120{,}000 \times 21{\cdot}56}{22{\cdot}56 \times 4 \times 11 \times 1{\cdot}18 \times {\cdot}87 \times 21}$$

$$= 119 \text{ lb./in.}^2$$

The permisssible bond stress is 100 lbs /in.². Hence use $\frac{5}{16}$ in. ϕ bars at $4\frac{1}{2}$ in c/c. Number of bars is 14 instead of 11— $\frac{3}{8}$in. bars. Even if this is not sufficient reduce the spacing still further to add one or two more bars and thus keep the bond stress within 100 lb.

After the excavation for the footing is made to the proper depth a lean concrete (1 : 4 : 8) is spread at the bottom 3 to 6 in. thick. On the top of this is placed the grid with the two-way reinforcement with hooks at the ends and wired at every intersection. On this is erected at the centre a short length of the assembly of the vertical column bars, the lower ends of which are bent at right angles and wired to the grid, (see Fig. 83). This short length of assembly of vertical bars serves as dowel bars, to which the upper assembly of the vertical column bars are spliced later after the concrete is poured into the footing, and allowed to harden. The overlap for splicing must be of 24 diameters of the vertical bars.

Very often the footing is made with a sloping top to save concrete as shown in the figure. In that case the effective depth for the diagonal shear is measured or calculated from the base to the top at a distance $= t$ from the face of the column.

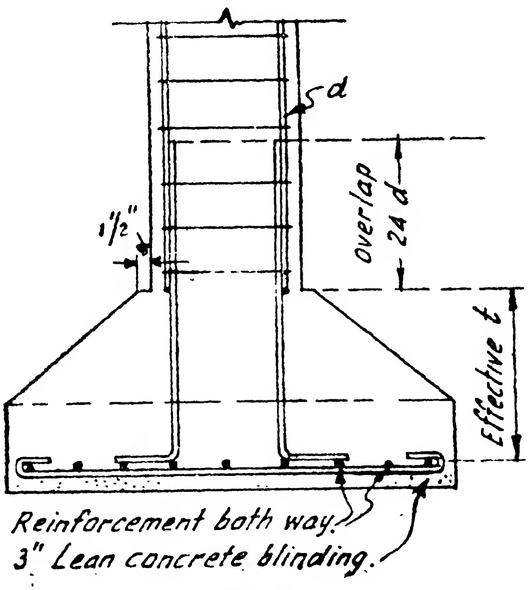


Fig. 83.

Illustrative Example 71

Design a suitable sloped-top square footing for a 15 in. square column, carrying a central load of 60 tons. The soil below has a maximum safe bearing power of 1·25 tons per sq. ft.

Solution:—$p_b = 2800$, $W = 60 \times 2240$, $a = 15$.

Assuming 18 in. depth of footing

$$\text{nett } p_b = 2800 - 1{\cdot}5 \times 144$$
$$= 2584 \text{ lb./ft.}^2$$

$$\text{Area of footing} = \frac{60 \times 2240}{2584}$$
$$= 52{\cdot}01 \text{ sq. ft.}$$

$$\text{Side of square footing} = \sqrt{52{\cdot}01} = \text{say } 7' - 3''$$

$$\text{Nett } p_b = \frac{60 \times 2240}{7{\cdot}25^2} = \frac{60 \times 2240}{52{\cdot}56}$$
$$= 2560 \text{ lb./ft.}^2$$

$$\text{Punching shear} = 2560\ (l^2 - a^2)$$
$$= 2560\ (7{\cdot}25^2 - 1{\cdot}25^2)$$
$$= 2560 \times 51.$$

$$\text{Intensity of punching shear} = \frac{2560 \times 51}{4 \times 15 \times 18} = 121 \text{ lb./in.}^2$$

This being less than the permissible 150 lb./in². is safe.

$$\textit{Diagonal Shear} = 2560\ \{l^2 - (a+2t)^2\} \qquad a + 2t = 51 \text{ in.}$$
$$= 2560\ (7{\cdot}25^2 - 4{\cdot}25^2) = 2560 \times 34{\cdot}5$$

$$\text{Intensity of shear} = \frac{2560 \times 34{\cdot}5}{4\,(a+2t) \times {\cdot}87 \times 15}$$ since effective d at 18 in. from face is 15 in.

$$= 32{\cdot}3 \text{ lb./in.}^2$$

As this is less than the permissible 75 lb. it is safe

$$\textit{Bending Moment} = \left\{ 2 \times \frac{3^2}{2} \times \frac{2}{3} \times 3 + 1{\cdot}25 \times 3 \times \frac{3}{2} \right\} 2560 \text{ ft. lb.}$$
$$= 2560 \times 23{\cdot}62 \times 12 \text{ in. lb.}$$

$$\text{The width on which this acts} = 15 + 2 \times 18 + \frac{87 - 51}{2}$$
$$= 69 \text{ in.}$$

$$\text{checking depth } d = \sqrt{\frac{2560 \times 23{\cdot}62 \times 12}{126 \times 69}}$$
$$= 9{\cdot}2 \text{ in.}$$

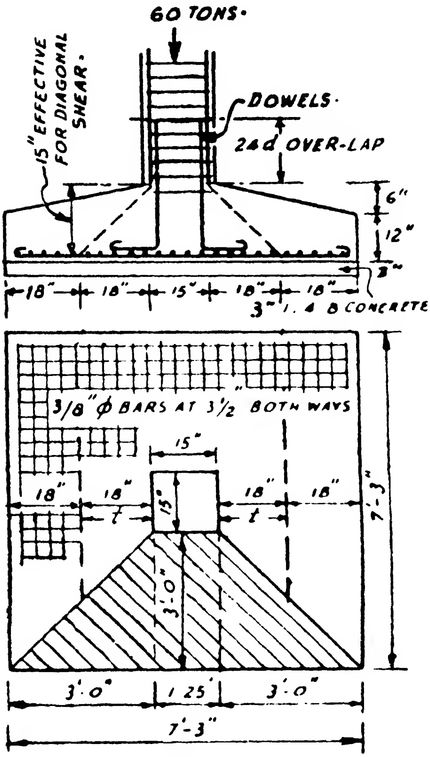


Fig. 84.

We have adopted 18 in. which is ample.

$$\text{steel, } A_T = \frac{2560 \times 23{\cdot}62 \times 12}{18000 \times {\cdot}87 \times 18}$$

$$= 2{\cdot}58 \text{ sq. in.}$$

This area is to be spread over 69 in. width. Steel area per foot $= \frac{2{\cdot}58}{69} \times 12 = 0{\cdot}45$ in.^{2}.

Use ½ in. ϕ bars at 5 in. *c/c* A=0·47. In a width of 69 in. 15 rods in each direction with hooks at ends will be required.

Bond stress:—Total bond = ¼ punching shear

$$= \frac{2560 \times 51}{4} = 32640 \text{ lbs.}$$

and is resisted by the perimeters of 15 Nos. ½ in. ϕ bars $\times \cdot 87 \times 18$.

$$\text{Intensity of bond stress} = \frac{32640}{15 \times 1{\cdot}57 \times {\cdot}87 \times 18}$$

$= 89$ lb. This is safe.

A plan and vertical section of the footing is shown in Fig. 84.

Wall footing:—A wall footing is very simple. It is designed as a double cantilever beam with the wall in the centre, the weight on the wall including its self load pressing down while the reaction of the soil pressure tending to lift the cantilevers upwards are both supposed to be uniformly distributed. If the wall is of concrete built monolithic with the footing, the B. M. is maximum at the face of the wall, but if it is of stone or brick the maximum moment occurs at the centre under the wall.

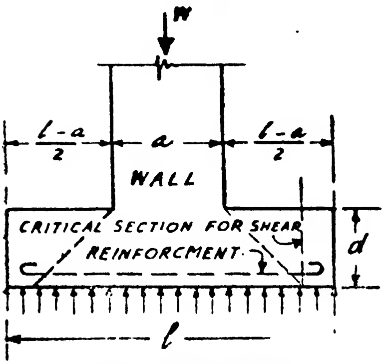


Fig. 85—Wall footing.

$$\text{Maxi. B. M. at wall face} = p_b \cdot \frac{l-a}{2} \cdot \frac{l-a}{4} = p_b \frac{(l-a)^2}{8}$$

The vertical shear at the points where the 45° planes meet the base is the measure of diagonal shear as in the case of independent footings of columns, i. e. at a distance $= t$ from the face.

The bond stresses are usually very high and should always be checked. The critical section for bond is at the face of wall as in the case of columns.

Illustrative Example 72

Design the footing for a wall 2 ft. wide carrying a load of 12 tons per foot including load of wall and footing. The soil under the footing can safely bear a maximum pressure of 2 tons per sq. ft.

Solution:—Width of footing $= \frac{12}{2} = 6$ ft.

The footing will project 2 ft. on either side.

$l = 6'$, $a = 2'$, $p_b = 4480$ lb./ft.^{2}

$$\text{B. M.} = \tfrac{1}{8}\cdot 4480 \times (6-2)^2 \times 12$$
$$= 8960 \times 12 = 107520 \text{ in. lb.}$$

$$d = \sqrt{\frac{107520}{126 \times 12}} = 8{\cdot}4 \text{ in.} \qquad \text{(i)}$$

This depth must be checked for shear.

Shear at 8·4″ from the wall face

$$= \frac{24-8{\cdot}4}{12} \times 4480 = 5824 \text{ lb.}$$

$$\text{Intensity} = \frac{5824}{12 \times {\cdot}87 \times 8{\cdot}4} = 62{\cdot}5 \text{ lb./in.}^2$$

As this is less than the permissible viz. 75 lb./in.^{2} it is acceptable.

Make $d = 9$ in. and provide 3″ of cover

Overall D = 12 in.

$$A_T = \frac{107520}{18000 \times {\cdot}87 \times 9} = {\cdot}764$$

Use ½ in. ϕ bars at 3″ c/c. (A = ·785)

$$\text{Bond stress} = \frac{2 \times 4480}{4} = 2490 \text{ lb.}$$

$$\text{Intensity} = \frac{2490}{3{\cdot}14 \times \frac{12}{3} \times \frac{1}{2} \times {\cdot}87 \times 9} = 45 \text{ lb./in.}^2$$

Being less than 100, this is acceptable.

Continuous footings:—Very often in places where land is very valuable the faces of exterior columns come very close to the building line or boundary line of the property, and there is no room for extending the footing on that side without encroaching on the street or the adjoining property. A continuous footing under the exterior columns solves the problem in such cases. The design of such a footing is very simple, if it is regarded, and in fact it is, an inverted continuous beam with uniformly distributed load in the form of upward soil pressure. As the beam is inverted, the beam supports the columns, instead of the latter supporting the beam as usual. The placing of reinforcement, therefore, will be just the opposite of the usual practice, i. e. under the columns the steel rods will be at the *bottom* of the beam and between the columns, they will be at the *top*. Excepting this the design is just like that of an ordinary continuous beam. The following example will indicate the procedure of design.

Illustrative Example 73

Design a continuous footing for exterior columns of a building lining a city street each carrying 150,000 lbs. They are 16 ft. apart between centres and are 20 in. square with their centres 1′ – 4″ from the building line. The soil below has a safe bearing capacity of 4250 lbs./ft.^{2} Take usual stresses and $m = 15$.

Solution:—We shall consider a portion of the footing between two columns.

As a first approximation assume the weight of the footing between two columns to be 10 °/₀ of the load on the columns. Then the total load

$$= 150000 + 15000 = 165{,}000 \text{ lb.}$$

$$\text{Area of footing} = \frac{165000}{4250} = 38{\cdot}8 \text{ sq. ft. say } 39$$

As the columns are 16 ft. apart, the minimum width of footing $= \frac{39}{16} = 2{\cdot}44$ ft. The centre line of the columns is 1·33 ft. from the building line. We shall therefore adopt a width of 2·66 ft. of the footing, so that the edge of the footing is just on the building line, and that the centre line of columns and that of the footing will coincide.

As the inverted beam of the footing is continuous, the B. M. below the column will be $+\frac{Wl}{12}$ and that in the centre of span $-\frac{Wl}{12}$ i. e. just the opposite in signs of B. Ms. of the usual beam supported on columns.

$$M = \pm\frac{Wl}{12} = \pm\frac{150000\times16\times12}{12} = 2{,}400{,}000 \text{ in./lb.}$$

$$d = \sqrt{\frac{2{,}400{,}000}{126\times32}} \qquad (b = 2{\cdot}66 \text{ ft. or } 32 \text{ in.})$$

$$= 24{\cdot}4 \text{ in.}$$

However, to keep the shear and bond stress within permissible limits we shall adopt $d = 27$ in. and with a cover of 3″ *at top* D = 30 in.

$$A_T = \frac{2{,}400{,}000}{18000\times{\cdot}87\times27} = 5{\cdot}7 \text{ in.}^2$$

5·7 in.² in a width of 32 i. e. 2·14 in.² in 12 in.
or 10 No. $\frac{7}{8}$ in. ϕ (A = 6·013).

This is provisional, since we may have to revise this while considering bond stress.

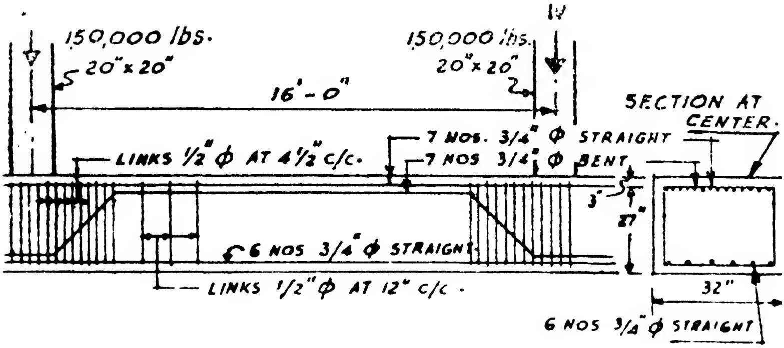


Fig. 86.

$$\text{Shear stress intensity} = \frac{150000}{2}\times\frac{1}{32\times{\cdot}87\times27}$$

$$= \text{nearly } 100 \text{ lb./in.}^2$$

As this exceeds the permissible stress of 75 lb. shear reinforcement is necessary. Shear value of the concrete $= 32\times\cdot87\times75 = 56400$ lbs. If this occurs at a distance of x ft. from the centre

$$\frac{x}{56400} = \frac{8}{75000}, \quad x = 6 \text{ ft.}$$

Reinforcement must be supplied only in the 2 ft. portions at ends. If 50 per cent of the main rods i. e. 5 – $\frac{7}{8}$ in. ϕ rods are bent at 45°, their shear value

$$= 18000 \times 5 \times 0{\cdot}6 \times 0{\cdot}707$$

$$= 37800 \text{ lbs.}$$

In this equation 5 is the number of rods, 0·6 is the area of $\frac{7}{8}$ in. ϕ bar and 0·707 is sin 45°.

Stirrups must be supplied for 75000 – 37800 = 37200 lb.

If $\frac{1}{2}$ in. ϕ bars are used

$$\text{pitch} = \frac{18000\times\cdot4\times27\times\cdot87}{37200} = 4{\cdot}54 \text{ in. say } 4\tfrac{1}{2} \text{ in.}$$

Bond stress = The maximum vertical shear = 75000 lbs.

Intensity of bond stress

$$= \frac{75000}{(5\times\frac{7}{8}\times3{\cdot}14)\times\cdot87\times27} = 232 \text{ lb./in.}^2$$

The maximum allowed with hooks at ends is 200. We must therefore use smaller bars. If 13 – $\frac{3}{4}$ in. ϕ bars are used and 7 out of them carried to the end after bending 6 for shear.

$$\text{Intensity of bond} = \frac{75000}{(7\times\frac{3}{4}\times3{\cdot}14)\times\cdot87\times27} = 193 \text{ lb.}$$

This is in order.

A longitudinal section of the footing is shown in Fig. 86. 7 – $\frac{3}{4}''$ ϕ bars are placed at top and 6 at bottom and in addition 7 – $\frac{3}{4}''$ ϕ bars are placed at bottom under the columns which are bent at 45° close to the inner faces of the columns and extended horizontally at the top in the centre. This arrangement will provide 13 bars for + ve and 14 for − ve B. M.; $\frac{1}{4}$ in. ϕ stirrups are provided at $4\frac{1}{2}$ in. on both sides of the columns in 2 ft. length and at 12 in. *c/c.* in the intermediate length.

The weight of the footing

$$= 32 \times 16 \times 30 = 15360 \text{ lbs.}$$

We have assumed it to be 15000 which is very close to this.

Combined footings:—Cases frequently occur when a column is so placed that its face is flush with the property line or building line. A continuous footing is also not practicable as the footing cannot be extended even an inch on that side. The construction of a single footing base to support the exterior and one or more interior columns is the answer to this problem. This is called a *combined footing.* When designing such a footing the principles to be observed are:—

(a) The base area should be so shaped as to be symmetrical about the centre line of the loads. A trapezoid, or a rectangle, or a series of rectangles each below its column, joined together, allows facility of calculating the position of the centroid of the area accurately.

(b) The bearing power of the soil should obviously be equal to or greater than the intensity of reaction of the combined load.

(c) The centroid of the base area should coincide with the vertical axis of the combined load.

(d) The effect of extending the footing longitudinally beyond the columns results in causing continuity in the footing and though it increases the cantilever B. M., it reduces at the same time, the B. M. at the mid-span between the columns.

The design procedure is just on the same lines as those of the continuous footing. The following example will illustrate it.

Illustrative Example 74

Design a combined footing to support two columns—the exterior, 18″ × 12″ carrying a load of 2,00,000 lbs. and interior

18" × 18" carrying 300,000 lbs. The columns are 16 ft. apart between centres and the soil below has the safe bearing power of 3400 lbs./ft.^{2}

Solution:—Total load on footing = 3,00,000 + 2,00,000 + load of the footing, assuming the latter to be 12 % of the column load = 5,00,000 + 60,000 = 560,000 lbs.

$$\text{Area of footing} = \frac{560{,}000}{3400} = 164\cdot7, \text{ say } 165 \text{ sq. ft.}$$

The distance of the *c. g.* of the two column loads from the centre of the outer column $= \frac{300{,}000 \times 16}{500{,}000} = 9\cdot6$ ft. Its distance from the right hand edge of the footing is 9·6 + ·5 = 10·1 ft. Since the *c. g.* of load must coincide with the centroid of the base area, we must extend the footing to 10·1 ft. also on the left hand side. It goes 2·95 ft. beyond the left hand face of the interior column as shown in Fig. 87 (a).

The width of the footing

$$= \frac{\text{Area of footing}}{\text{length}} = \frac{165}{20\cdot2} = 8\cdot17 \text{ say } 8'-3''$$

Soil pressure per rft.

$$= \frac{5{,}00{,}000}{20\cdot2} = 24800 \text{ lbs.}$$

The shear force at critical points can now be calculated and shear diagram drawn. Thus shear at the left hand face of the interior column i. e. at 2·95 ft. from the left hand edge

$$= -(24800 \times 2\cdot95) = -73160 \text{ lbs.}$$

Shear at right hand face of the same column

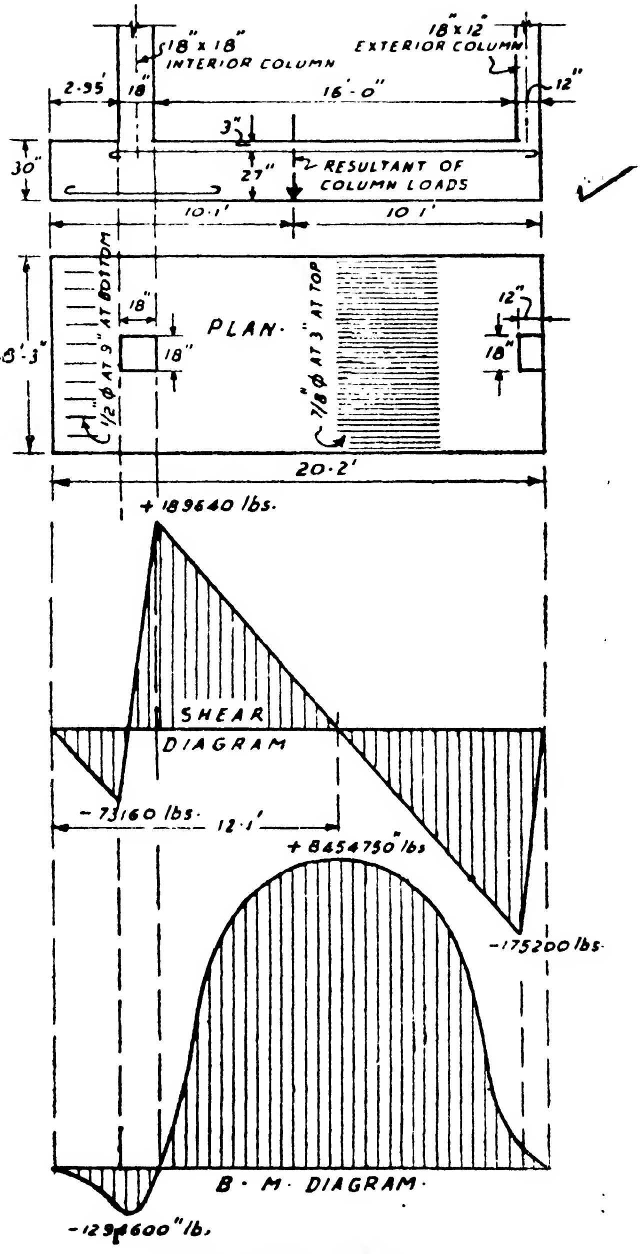
$$= 3{,}00{,}000 - (2\cdot95 + 1\cdot5) \times 24800$$
$$= 189640 \text{ lbs.}$$

Similarly shear at left hand face of exterior column

$$= -2{,}00{,}000 + 24800 \times 1 = -175200 \text{ lbs.}$$

Shear will be zero at x ft. from the left hand edge of footing when $3{,}00{,}000 = 24800\,x$

$$x = 12\cdot1 \text{ ft.}$$


18" x 18"
INTERIOR COLUMN
18" x 12"
EXTERIOR COLUMN
2·95'
18"
16'-0"
12"
3"
30"
27"
RESULTANT OF
COLUMN LOADS
10·1'
10·1'
1/2" Φ AT 9" AT BOTTOM
PLAN
7/8" Φ AT 3" AT TOP
18"
18"
12"
18"
8'-3"
20·2'
+ 189640 lbs.
SHEAR
DIAGRAM
- 73160 lbs.
12·1'
+ 8454750" lbs
-175200 lbs.
B · M · DIAGRAM
-1294600" lb.

Figs. 87—(a) (b) (c) & (d).

These figures are plotted in Fig. 87 (c). From the shear the B. M. at critical points can be calculated, thus :

B. M. will be maximum at 12·1 from left hand edge where shear is zero and $= \left(3{,}00{,}000 \times 8{\cdot}4 - \frac{24800 \times 12{\cdot}1^2}{2}\right) 12$

$= 8454192$ in. lb.

B. M. on the left hand face of inner column

$= -2{\cdot}95 \times 24800 \times \frac{2{\cdot}95}{2} \times 12$

$= -1294600$ in. lbs.

From these figures the B. M. diagram is drawn in Fig. 87(d)

Depth of footing for the maximum B. M.

$d = \sqrt{\frac{8454200}{126 \times 99}}$ $b = 8' - 3''$ or 99 in.

$= 26$ in. say 27.

Adding 3 in. for cover on the tension side i. e. at top overall D = 30 in.

$A_T = \frac{8454200}{18000 \times {\cdot}87 \times 27} = 20$ in.^{2} in 99 in. width

$A_T = \frac{20}{99} \times 12 = 2{\cdot}42$ in.^{2} in one ft. width.

Provide $\frac{7}{8}$ in. ϕ bars at 3 in. c/c. (A=2·41).

In all 34 No. of bars will be required in 99 in.

Shear :—The critical section for shear is at *d* from the right hand face of the interior column, i. e. at 27 in. or 2·25 ft.

$S = 189640 - \frac{27}{12} \times 24800$

$= 133831$ lbs.

Intensity $= \frac{133831}{99 \times {\cdot}87 \times 27} = 58$ lb./in.^{2}

This is acceptable. No stirrups are necessary.

Bond stress :—Total bond stress = 189640.

$$\text{Intensity} = \frac{189640}{2{\cdot}75 \times 34 \times {\cdot}87 \times 27} = 81{\cdot}5 \text{ lbs.}$$

This is also acceptable.

In the above equation 2·75 is the perimeter of one $\frac{7}{8}$ in. ϕ bar and 34 is the number of bars.

The above is the design of the footing beam between the two columns. There is a portion on the left hand side of the interior column 2·95 ft. long which must be designed as a cantilever. The maximum B. M. and shear on the left hand face of the column are 1294600 in. lb. and 73160 lb. respectively (Fig. 87 *d* and *c*)

$$d = \sqrt{\frac{1294600}{126 \times 99}} = 10{\cdot}1 \text{ in.}$$

Our 27 in. *d* is ample.

$$A_T = \frac{1294600}{18000 \times {\cdot}87 \times 27} = 3{\cdot}06 \text{ in.}^2 \text{ in 99 in. width.}$$

$$A_T = \frac{3{\cdot}06}{99} \times 12 = 0{\cdot}37/12 \text{ in.}$$

$\frac{5}{8}$ in. ϕ bars at 10 in. c/c give 0·368. 10 spaces or 11 bars would be required in 99 in. These will be placed at the bottom since there is tension at bottom.

Bond Stress = Shear = 73160 lb.

$$\text{Intensity} = \frac{73160}{11 \times 3{\cdot}14 \times \frac{5}{8} \times {\cdot}87 \times 27} = 145 \text{ lbs/in.}^2$$

With hooked ends this is within permissible limit, hence acceptable.

Shear at 27 in from the face = 24800 × $(2{\cdot}95 - \frac{27}{12})$ = 17360 lb.

$$\textit{Shear Intensity} = \frac{17360}{99 \times {\cdot}87 \times 27} = 7{\cdot}2 \text{ lbs./in.}^2$$

which is very low.

Transverse section of the footing beam :—We have so far designed the beam longitudinally. The beam is 8′ 3″ wide and the columns are 1′–6″ in the centre in the direction of its width. Thus there are side widths of the beam $= \frac{8{\cdot}25 - 1{\cdot}5}{2}$ or 3·375 ft. or 40·5 in. acting as cantilevers on either side of the columns which must be reinforced.

The upward pressure on the footing of the inner column $= \frac{3{,}00{,}000}{8{\cdot}25} = 36360$ lb. Its moment at the face

$$= 36360 \times \frac{3{\cdot}375^2}{2} \times 12$$

$$= 2484840 \text{ in. lb.}$$

The width of the cantilever parallel to the footing on which this B. M. acts can be safely assumed to be = width of the column + ½ effective depth of footing i. e. $18'' + \frac{27}{2} =$ 31·5 say 32″. $d = \sqrt{\frac{2484840}{126 \times 32}} = 24{\cdot}8.$

The depth of 27 in. already adopted by us is sufficient.

$$A_T = \frac{2484840}{18000 \times {\cdot}87 \times 27} = 5{\cdot}87 \text{ in.}^2 \text{ in 32 in.}$$

$$= \frac{5{\cdot}87}{32} \times 12 = 2{\cdot}21 \text{ in.}^2 \text{ per ft. width}$$

Provide 1 in. ϕ rods at 4″ *c/c.* (A = 2·36), 9 rods will be required in 32 in. width.

Shear stress :—This is measured at a distance of *d* or 27 in. from the face.

Intensity $= \frac{36360}{32 \times {\cdot}87 \times 27}\left(\frac{40{\cdot}5}{12} - \frac{27}{12}\right)$

$= 55$ lb./in.^{2} which is safe.

Bond stress = shear at the face of column

$= \frac{40{\cdot}5}{12} \times 36360. = 3030$

Intensity $= \frac{3030}{9 \times 3{\cdot}14 \times {\cdot}87 \times 27} = 184$ lbs./in.^{2}

With standard hooks at ends this is permissible.

Exterior column :—The upward pressure of the soil

$$= \frac{2,00,000}{8 \cdot 25} = 24240 \text{ lb. per rft.}$$

$$\text{B. M.} = 24240 \times \frac{3 \cdot 375^2}{2} \times 12 = 1656560 \text{ in. lb.}$$

$$d = \sqrt{\frac{1656560}{126 \times 32}} = 20 \cdot 2 \text{ in.}$$

Our depth of 27 in. is ample

$$A_T = \frac{1656560}{18000 \times \cdot 87 \times 27} = 3 \cdot 91 \text{ in.}^2 \text{ in 32 in. width}$$

$$= \frac{3 \cdot 91}{32} \times 12 = 1 \cdot 47 \text{ in.}^2 \text{ per foot.}$$

Provide $\frac{3}{4}$ in. ϕ bars at $3\frac{1}{3}$ in. *c/c* (A=1·52). 10 rods would be required in 32 in. width. Shear at *d* or 2·25 ft. from the face

$$= 24240 \times (3 \cdot 375 - 2 \cdot 25) = 24240 \times 1 \cdot 125.$$

$$\text{Intensity} = \frac{24240 \times 1 \cdot 125}{32 \times \cdot 87 \times 87} = 37 \text{ lb./in.}^2\text{; this is quite low.}$$

Bond stress = Shear at face = 24240 × 3·375

$$\text{Intensity of ,,} = \frac{24240 \times 3 \cdot 375}{10 \times 3 \cdot 14 \times \frac{3}{4} \times \cdot 87 \times 27} = 156 \text{ lb. in.}^2\text{.}$$ This is within permissible limits.

Pile foundation :—When foundations are unreliable, such as those in made ground, piles are used. R. C. C. piles are either pre-cast or cast *in-situ*. They are 12 to 18 in. in diameter and are driven at 2′ 6″ to 3 ft. between centres. The bearing capacity of each pile is known from the weight of the hammer and the height of its fall. The design of pile footing is similar to that on soil, the only difference being that instead of an upward uniformly distributed pressure of the soil, the resistance of each pile is taken as an isolated force. The footing is extended to include as many piles as are required to balance the column load.

An R. C. C. pad is provided on the top of piles in which the tops of piles are embedded to a depth of 4 to 6 in. This pad is called a pile cap. On the top of the cap is laid the footing with a grid of two-way steel rods at bottom. The depth of the footing is determined by the shear. The critical section for the latter is taken at $\frac{d}{2}$ from the face of the column. The minimum depth of the footing exclusive of the cap should be 12 in.

The design procedure will be apparent from the follow-ing typical illustrative example.

Illustrative Example 76

Design a footing for a column 18 in. square exerting a load of 100,000 lb. Each pile has the bearing capacity of 25000 lb. Take the usual stresses and $m = 15$.

Solution:—Assume the weight of the footing to be 20 % of the column load.

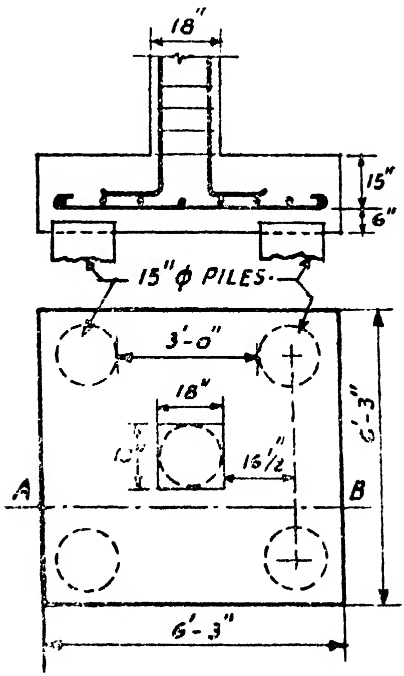


Fig. 88.

Total load $= 100{,}000 + 20{,}000 = 120{,}000$ lb.

No. of piles required $= \frac{120000}{25}$

$=$ more than 4, say 5.

They may be arranged symmetrically as shown in Fig. 88.

Each pile is of 15″ diam. The distance between the piles is 3′—0″. The load on each pile

$$= \frac{100{,}000}{5} = 20{,}000 \text{ lb.}$$

B. M. at the face of the column

$$= 20{,}000 \times 16{\cdot}5 = 330{,}000 \text{ in. lb.}$$

16·5 in. is the distance from the centre of any one of the four corner piles to the face of the column. The width on which this B. M. acts is taken = width of the column + depth of footing. Let us assume the latter to be 15 in. then width $= 18 + 15 = 33$ in.

$$d = \sqrt{\frac{330000}{126 \times 33}} = \text{about } 9 \text{ in.}$$

But a minimum depth of 12 in. must be provided and we have provisionally adopted 15 in. Let us see if this satisfies the shear.

As the critical section for shear is at $\frac{d}{2}$ from the column face,

$$b = 2 \times \frac{d}{2} + \text{width of column}$$

$$= 2 \times \frac{15}{2} + 18 = 33 \text{ in.}$$

Shear intensity $= \frac{25000}{33 \times {\cdot}87 \times 15} = 51$ lb./in.^{2}

This is acceptable. Hence the assumed depth of 15 in. is also approved.

$$A_T = \frac{330000}{18000 \times \cdot 87 \times 15} = 1 \cdot 41 \text{ in.}^2 \text{ on } 33 \text{ in.}$$

$$= \frac{1 \cdot 41}{33} \times 12 = 0 \cdot 51 \text{ in.}^2/\text{foot.}$$

Provide ½ in. ϕ bars at 4½ in. c/c. (A=0·52)

8 rods will be required in 33 in. width.

$$\text{Bond stress} = \frac{25000}{3 \cdot 14 \times \frac{1}{2} \times 8 \times \cdot 87 \times 15} = 155 \text{ lb./in.}^2$$

This is acceptable as it is less than 200 lb.

A plan and cross section of the pile footing are shown in Fig. 88. The footing is taken 6′ – 3″ square. The thickness of pile cap is 6 in. and above that a footing of 15 in. designed depth is adopted.

Wt. of the footing = 6·25 × 6·25 × 1·75 × 144 = about 10000 lb. Our assumption of 20,000 lb. is on the safe side.

Practice Problems

1. Design a footing for a 14 in. square column carrying a load of 2,80,000 lb. The bearing capacity of the soil is 4000 lb./ft.² and μ = ·4.

2. Design a footing for a 15 in. square column carrying a load of 1,60,000 lb. including its own wt. The foundation has a safe bearing power of 1½ tons per sq. ft. and coefficient of friction = 0·45.

3. A 20″ × 20″ column founded on a soil capable of bearing safely 3000 lb./ft.² carries a load of 250,000 lb. If the coefficient of friction is 0·5 design the footing.

4. Design a sloped top footing for a column 18″ × 18″ carrying an axial load of 50 tons and a B. M. of 2,60,000 in. lb. The bearing capacity of the foundation is 5000 lbs./ft.² and μ = 0·52.

5. Design a continuous footing beam for three columns 16 in. square placed with their centres 24 in. from the building line. Each carries an axial load of 75 tons. The distance between their centres is 18 ft. The soil can safely bear 4000 lbs./ft.^{2}

6. Design a combined footing for two columns—the exterior 15″ × 10″ carrying 60 tons and the interior 20 ft. away between centres 15 in. sq. carrying 100 tons. The soil can safely bear 3 tons/ft.^{2}

CHAPTER XIX

CANTILEVER RETAINING WALL

SINCE reinforced concrete is a material capable of taking tensile stresses, it is particularly suited to the construction of retaining walls. A thin section of R. C. C. wall costing comparatively much less, serves the purpose more satisfactorily of an otherwise heavy, expensive masonry wall.

The object of a retaining wall is to retain, or hold back a mass of earth by giving it a lateral support. Walls of cellars, toe walls of roads and railway embankments, wing walls of bridges are some of the common instances of retaining walls.

If a mass of earth is left exposed to weather, its sides will slip, and assume a stable slope. The inclination of this slope to the horizontal is called the "angle of repose." This angle depends upon the granular nature, weight, and particularly the moisture contents of the material as will be seen from the following figures :—

TABLE No. 24

RELATION BETWEEN ANGLE OF REPOSE AND MOISTURE

Condition of clay	Weight lb./ft.^{3}	Angle of repose degrees
Dry	120 to 125	30
Moist	130 to 140	45
Wet*	135 to 140	15

If an attempt is made to hold the material at a steeper slope than its angle of repose, the extra material heaped up,

* This shows the necessity of draining the earthen mass as efficiently as possible by means of "weep holes" placed 4 to 5 ft. vertically and 10 ft. horizontally through the concrete wall by providing a hollow packing of rubble and gravel at their inner faces.

exerts a lateral pressure. The retaining wall is to be designed, for this pressure. The greater the moisture contents, the less is the angle of repose, and the greater is the lateral pressure.

A number of theories for computing this pressure have been advanced—amongst them, the one by Rankine, is commonly accepted in this country. This is not a place to go into the theory. Only its result is given expressed by the following formulae :—

Case I—Surface of the soil horizontal :—The horizontal pressure, or the thrust per sq. ft. at a depth h below the surface

$$ph = wh\,\frac{1 - \sin\phi}{1 + \sin\phi}$$

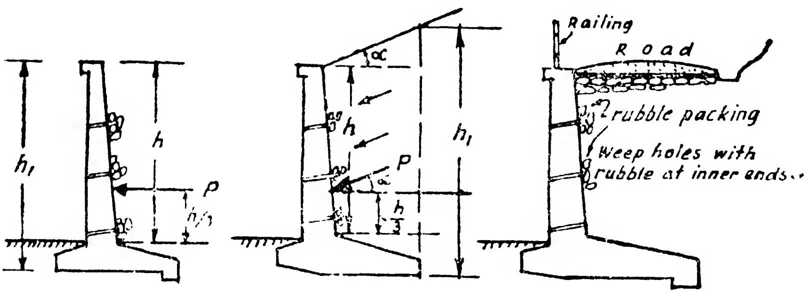
where w = wt. of soil per cub. ft. and

ϕ = angle of repose of the soil.

The total thrust per foot square on a wall of H ft. height,

$$P = \tfrac{1}{2}\,wH^2\,\frac{1 - \sin\phi}{1 + \sin\phi}$$

The stress diagram as shown in Fig. 96 being a triangle, the point of application is at $\frac{H}{3}$ from the base.



Figs. 89, 90 & 91.

Case I

Cantilever type retaining wall with surface of soil horizontal.

Case II

(a) Retaining wall with a surcharge of soil.

(b) Retaining wall with a surcharge of road way.

Note the "weep" holes shown for draining water through the rubble, packed hollow at their inner mouths.

Case II:—Surface sloping at an angle α (See Fig. 90). The extra weight of the mass of earth above the horizontal surface is called "surcharge". The side thrust at a depth h

$$= ph = wh \cos\alpha \frac{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi}}{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\phi}}$$

where ϕ = angle of repose as before. The direction of the pressure is *not horizontal*, but inclined at α to the horizontal, i. e. it is parallel to the slope of surcharge as shown in the figure.

Sometimes the surcharge consists of some superload such as a road or railway line as shown in Fig. 91 (b). In such a case the dead load as well as moving load in the road or railway line is added to the total pressure calculated as above. If w_1 = additional superload per sq. ft. the wall will be subjected to additional lateral pressure $= w_1 \frac{1-\sin\phi}{1+\sin\phi}$

Illustrative Example 77

Calculate the maximum thrust intensity and total pressure on a retaining wall 15 ft. high with a vertical back and level earth surface if it retains dry sand weighing 96 lb./ft.^{3} with an angle of repose of 30°.

Solution:—If p = equivalent fluid pressure, it is equal to

$$w \frac{(1-\sin\phi)}{(1+\sin\phi)}$$

$$= 96 \frac{(1-\frac{1}{2})}{(1+\frac{1}{2})} = 96 \times \tfrac{1}{2} \times \tfrac{2}{3}.$$

$$= 32 \text{ lb /ft.}^2$$

Maximum $p = ph = 32 \times 15 = 480$ lb./ft.^{2}

Total P $= \frac{1}{2} ph^2 = \frac{1}{2} \cdot 32 \times 15^2$

$= 3600$ lbs. per ft. run vertical of wall

and acts at $\frac{h}{3} = 5$ ft. from the base.

Illustrative Example 78

A retaining wall 12 ft. high has to resist a thrust of moist earth. The surface is sloping at an angle of 15° above its top. If it weighs 110 lb./ft.^{3} calculate :

(a) The total horizontal pressure per vertical running ft. of the wall. Take $\phi = 30°$.

(b) If, instead of a sloping surface the surcharge consists of a road-way having the intensity of uniformly distributed load of 500 lb./ft.^{2}

Solution :—Here $p = w \cos\alpha \dfrac{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi}}{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\phi}}$

$\cos 15° = 0{\cdot}966$

$\cos 30° = 0{\cdot}857$

$$= 110 \times 0{\cdot}966 \frac{0{\cdot}966 - \sqrt{0{\cdot}966^2 - 0{\cdot}857^2}}{0{\cdot}966 + \sqrt{0{\cdot}966^2 - 0{\cdot}857^2}}$$

$$= 110 \times 0{\cdot}966 \times 0{\cdot}367$$

$$= 39{\cdot}0 \text{ lbs.}$$

$$P = \tfrac{1}{2} ph^2 = \tfrac{1}{2}{\cdot}39 \times 12^2$$

$$= 2808 \text{ lbs. acting at an angle of } 15°.$$

The horizontal $P = P\cos\alpha$

$$= 2808 \times {\cdot}966 = \mathbf{2713} \text{ lb.}$$

(b) When the surcharge is 500 lb./ft.^{2}

The additional thrust $= 500 \dfrac{(1-\sin\phi)}{(1+\sin\phi)} \times h$

$$= 500 \times \tfrac{1}{3} \times 12$$

$= 2000$ lb. acting horizontally at the mid-point of the wall, i. e. at 6 ft. from either base or top.

The thrust due to earth with level surface

$$= 110 \times \tfrac{1}{3} = 37 \text{ lb./ft.}^2$$

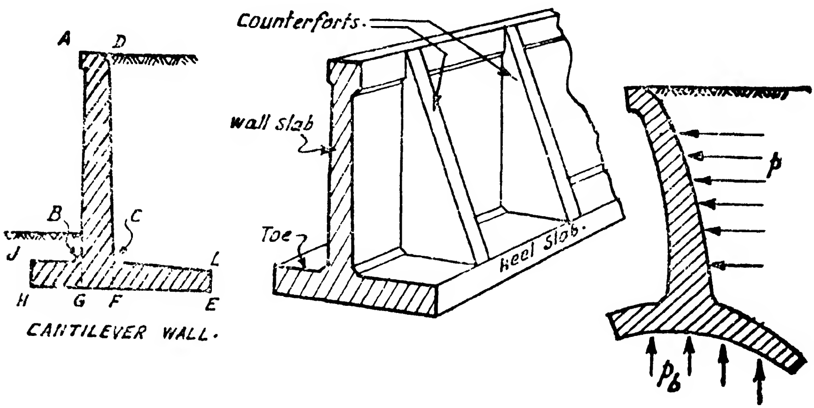
Total thrust due to this

$$= \tfrac{1}{2} ph^2 = \tfrac{1}{2} \times 37 \times 12^2 = 2664.$$

Combined thrust

$$= 2664 + 2000 = \mathbf{4664} \text{ lb./ft.}$$

Types of Retaining walls :—Since the function of a retaining wall is to support a mass of soil which has a tendency to spread out, it is possible to design retaining walls in either of the two ways:—(a) as a *cantilever* with the lower end fixed in a broad base and the upper end free with a cross section resembling a T-square as shown in Fig. 92. This is by far a very common method. (b) as a vertical wall supported by buttresses or counterforts at intervals, as shown in Fig. 93



Figs. 92, 93 & 94.

In Fig. 92—ABCD—Stem; JBGH—Toe; CLEF—Heel; JLEH—Base slab.

the wall is designed as a continuous slab supported or held at its upper (i. e. inner) surface by the counterforts designed as cantilever beams, both cast monolithically. This is called a counterfort retaining wall. We shall discuss the cantilever type first.

Fig. 92 denotes the different component parts of a cantilever type retaining wall. They are:

(1) *The stem.* It is subjected to the horizontal thrust of the mass of earth behind. It is designed as a cantilever.

(2) *The heel.* It supports the mass of earth above it and is also subjected to the upward pressure of the soil from below. It is also designed as a cantilever with its one end fixed at the bottom of the stem.

(3) *The toe.* This is shorter than the heel and is subjected to the upward pressure of the soil from below. Its own weight as well as that of the small quantity of earth on its top are usually neglected. It is also designed as a cantilever.

The tendency of these component parts to bend under the various forces is shown in Fig. 94.

The Design.—This is based on the following principles:—

(1) That the structure must be stable against being overturned by the horizontal thrust of the mass of earth. For this the resultant of the total vertical and horizontal forces must intersect the base at such a place as to leave sufficient factor of safety against overturning. Some authorities recommend a factor of safety of at least 1·5.

(2) That the wall must not slide forward by the action of the thrust. The resisting force is the friction between the soil and the bottom surface of the foot slab, which must be greater than the thrust.

In order to increase the resistance against sliding oftentimes a concrete projection is monolithically cast at the bottom of the foot slab as shown in Fig. 89 or an upward sloping shape is given to the bottom of the heel as in Figs. 90 and 91.

(3) The maximum intensity of vertical pressure caused by the wt. of the structure and the load of the mass of earth on the heel must not exceed the safe bearing power of the soil below.

(4) All the component parts, viz. the stem, toe, and heel must be strong enough to develop bending, shearing and bond stresses to resist the external forces.

Preliminary to design, certain dimensions proved by experience are adopted by the rule of the thumb. They are:

Stem:—Its thickness at top should be a minimum of 6 in. and may be up to 18 in. with a heavy surcharge like that of a combined live and dead load of a road or railway line. The thickness at the bottom is usually ½ in. per foot length *plus* the top thickness or slightly more.

The *average* thickness of the base should be the same as that of the stem at bottom or 2 or 3 in. more for high walls.

The width of the base is 0·4 to 0·6 of the total height above foundations.

The length of the toe should be 0·2 to 0·3 of the base width.

Design procedure:—This will be clear from the following few practical examples solved.

Illustrative Example 79

Design a cantilever retaining wall subjected to a pressure due to 15 ft. of earth bank above ground. The foundations

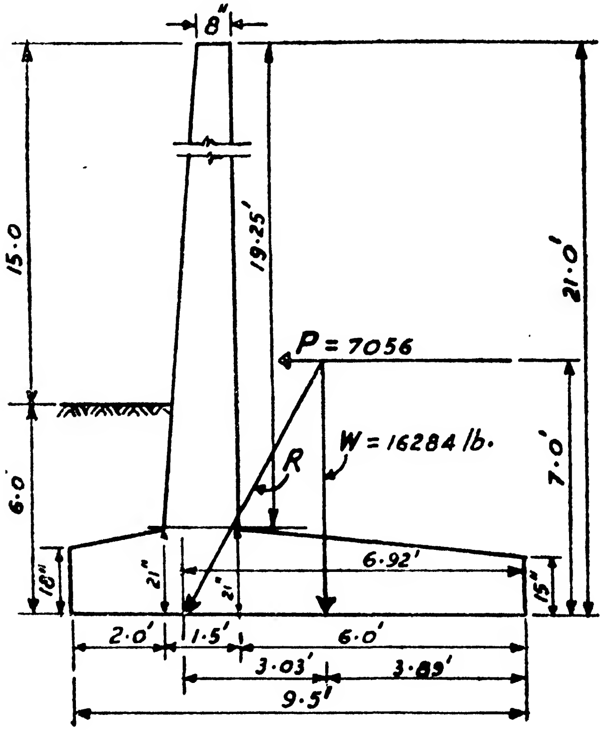


Fig. 95.

are 6 ft. below ground and consist of a soil capable of bearing 2 tons per sq. ft. The angle of repose of the earth is 30° and weight 96 lbs./cft. The cœfficient of friction = 0·45.

Solution:—The total height of bank above foundations is 21 ft. We shall therefore assume the following dimensions by the rules of the thumb given above

Top width = 8 in. Bottom width of stem $8+\frac{21}{2}$ = say 18″

Width of base, B, = ·45×h,=·45 × 21 = 9·45 say = 9·5 ft.

Length of toe = 0·2 of B = 0·2 × 9·5 = 1·9 say 2 ft.

Length of heel = 9·5 − 2 − 1·5 = 6 ft.

The average thickness of heel = thickness at bottom of stem = 18″. 15″ at free end, 21″ at support.

The section of the proposed wall with these dimensions is shown in Fig. 95

(1) *Testing stability of the wall against overturning*:—

$$P = w\frac{(1-\sin\phi)}{(1+\sin\phi)} = 96 \times \tfrac{1}{3} = 32 \text{ lb./ft.}^2$$

Total horizontal thrust, P

$= \frac{1}{2} ph^2 = \frac{1}{2} \times 32 \times 21^2 = 7056$ lbs. per vertical foot strip, acting at $\frac{21}{3} = 7$ ft. from the base.

The vertical forces are:

(1) Load of the wall, consisting of the weight of the vertical stem and horizontal base.

(2) The load of the mass of earth resting on the top of the heel.

These and their moments about the heel are tabulated below:—

TABLE No. 25

Description of the load	Load per foot vertical strip of width, lbs.	Lever arm measured from the edge of the heel	Moments ft. lb.
W_1, wt. of stem	$\frac{8''+18''}{2\times12}\times 19\,25\times 1'\times 144=3000$	$6+\frac{1\cdot5}{2}=6\cdot75$	20225
W_2, wt. of base slab	$9\cdot5'\times 1\cdot5'\times 1'\times 144=2052$	$\frac{9\cdot5}{2}=4\cdot75$	9447
W_3, wt. of earth	$6\times 19\cdot50\times 1\times 96=11232$	$\frac{6}{2}=3$	33696
	16284		63368

The distance of the *c. g.* of all the vertical forces from the edge of the heel $=\frac{63368}{16284}=3\cdot89$ ft.

If this is combined with the horizontal thrust of 7056 acting at 7 ft. above the base, the resultant R, will intersect the base at x when

$$x=\frac{7056}{16284}\times 7=3\cdot03 \text{ beyond } 3\cdot89 \text{ ft.}$$

i. e. at 6·92 from the edge of the heel. This is well within the base and is therefore safe. The horizontal thrust which would cause the resultant R to pass through the edge of the toe i. e. overturn the wall is

$$9\cdot5-3\cdot89=\frac{x}{16284}\times 7=13050 \text{ lbs.}$$

$$\text{Factor of safety}=\frac{13050}{7056}=1\cdot85 \text{ which is ample.}$$

(2) *Checking pressure on foundation*:—The resultant cuts the base at 6·92. i. e. the eccentricity is $=6\cdot92-\frac{9\cdot5}{2}=2\cdot17$ ft.

which is greater than $\frac{9\cdot5}{6}$ or 1·6. There will, therefore, be some tension on part of the heel.

The maximum pressure at toe

$$= \frac{\text{Load}}{\text{area}} + \frac{\text{Load} \times \text{eccentricity}}{\text{modulus of section}}$$

$$= \frac{16284}{9{\cdot}5 \times 1} + \frac{16284 \times 2{\cdot}17}{\frac{1 \times 9{\cdot}5^2}{6}} = 1714 + 2350$$

$$= 4064 \text{ lbs./ft.}^2$$

As this is less than the safe bearing pressure of the soil given viz. 2 tons or 4480 lbs./ft.^{2} the design is safe.

The minimum pressure at the heel

$$= 1714 - 2350 = -636 \text{ lb./ft.}^2 \text{ tension.}$$

(3) *Stability against sliding*:—The horizontal thrust is 7056 lb./ft. run. This is resisted by the friction between the base and the foundation soil underneath caused by the vertical load.

Frictional resistance $= \mu \times W = 0{\cdot}45 \times 16284 = 7328$ lbs. which is more than the horizontal thrust, still if necessary it can be increased still further by casting a 6″ × 9″ projection underneath the heel or giving an upward slope to the bottom surface of the toe as shown in Fig. 91.

(4) *Design of component parts.*

(a) *Stem*:—The bending moment at any point h_x ft. below the top

$= \frac{1}{2} p h_x \times \frac{h_x}{3}$ as the stress diagram is a triangle. $= \frac{1}{6} p h_x^3$ $\frac{1}{6} p$ is a constant. Thus the B. M. varies as the cube of the height.

Maxi. B. M. at 19·5 ft.

$$= \tfrac{1}{6} \cdot 32 \times 19{\cdot}5^3 \times 12 = 474200 \text{ in. lbs.}$$

The B. M. at half this height will be $(\frac{1}{2})^3$ or $\frac{1}{8}$ of this.

The B. M.s at different heights are worked out and tabulated as follows :—

TABLE No. 26

Depth below top and its ratio to h	Cube of ratio	B. M. in in. lb.	Effective thickness in inches.	A_T, steel area
$h = 19{\cdot}5$	1	474200	16·0	1 9
$\frac{3}{4}h = 14{\cdot}6$	$\frac{27}{64}$	200000	13·5	·95
$\frac{1}{2}h = 9{\cdot}75$	$\frac{1}{8}$	59275	11·0	0·35
$\frac{1}{4}h = 4{\cdot}9$	$\frac{1}{64}$	7410	8 5	0 056

Note:—While working out the figures in the above table a concrete cover of 2 in. is assumed and economic percentage of steel is taken.

In order to provide $A_T = 1{\cdot}9$ in.^{2} at the junction of wall with the base use 1 in. ϕ bars at 5 in. *c/c*. (A = 1·89 sq. in.) At $\frac{3}{4}h$ or 14·6 ft. below the surface half this steel area is re-

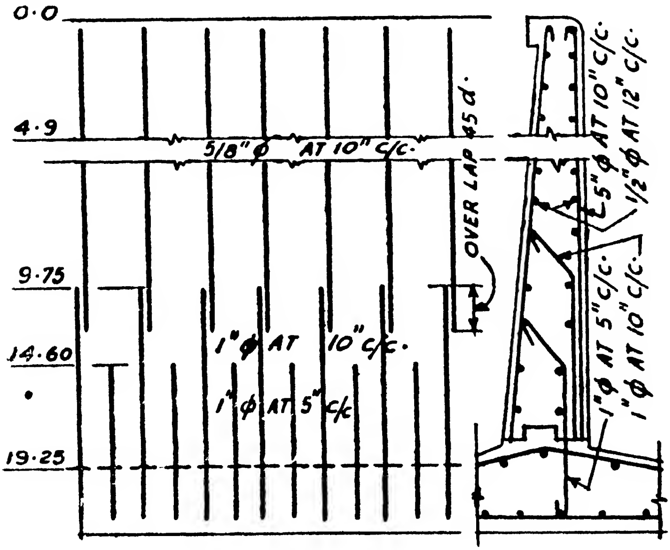


Fig 96.

quired i. e. 1 in. ϕ bars spaced at 10 in. centres will do, but the lower bars must be extended to provide the necessary grip length. Then at 9·75 ft. below the surface, only ·35 sq. in. area is required. Here we shall provide $\frac{5}{8}$ in. ϕ bars at 10 in. centres (A = 0·368) and these may be continued to the top. The arrangement is shown in Fig. 96. Theoretically this much quantity of steel is not required as we approach the top, but in the first place in a slab, reinforcement must be placed at a spacing not exceeding 12 diams. and secondly the labour and wastage involved in cutting and splicing the bars does not justify it.

It should be noted that though the bars are vertical, they are under bending and not under vertical thrust like those in a column. For this reason the lap for splicing must be 45 diameters and not 24.

Distribution steel.—This may be supplied on the basis of the main steel at $\frac{3}{4}$ h i. e.

$$\frac{\cdot 95}{5} = 0\cdot 19$$

Use $\frac{1}{2}$ in. ϕ bars at 12 in. centre (A = ·196).

Shear.—The maximum shear $= \frac{1}{2}\, ph^2$

$$= \frac{1}{2} \times 32 \times 19\cdot 5^2$$

$$= 6080 \text{ lbs.}$$

Intensity of shear $= \dfrac{6080}{12 \times \cdot 87 \times 16} = 36\cdot 5$ lb./in.^{2} which is quite safe.

Bond stress $= \dfrac{6080}{3\cdot 14 \times 1 \times \frac{12}{5} \times \cdot 87 \times 16}$

$= 52$ lbs./in.^{2} which is quite safe.

In the above equation $(3\cdot 14 \times 1)$ is the perimeter of one 1 in. bar and $\frac{12}{5}$ is the number of bars in one ft. width.

(b) *Design of the toe*:—

The span = 2 ft. Its own load and that of the small depth of soil on its top are neglected. It is therefore to be designed as a cantilever with the up ward reaction of the soil at the bottom.

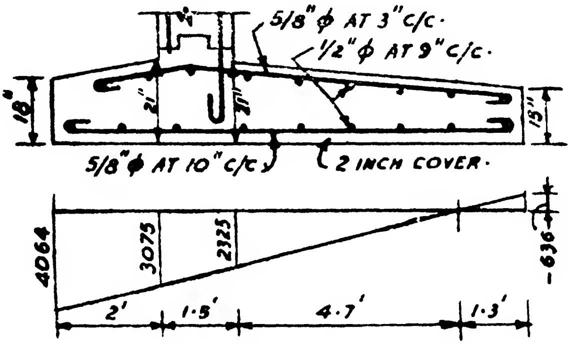


Fig. 97.

We have already found that the pressure at the toe is +4064 lb./ft.^{2} and that at the heel is −636. The difference is 4064 + 636 = 4700 in a length of 9·5 or 494·7 lbs. per ft. Thus the pressure at the inner end of the toe at B is 4064 − 2 × 494·7 = 3075 lbs.

$$\text{The B. M.} = 3075 \times 2 \times \frac{2}{2} + (4064 - 3075) \times \frac{2}{2} \times \frac{2}{3} \times 2$$

$$= 6150 + 1319 = 7469 \text{ ft. lb.}$$

$$= 89628 \text{ in. lb. say } 90{,}000$$

$$d = \sqrt{\frac{90000}{126 \times 12}} = \text{about 8 in.}$$

Ours is more than 16 in. and thus ample.

$$\text{Steel} = \frac{90000}{18000 \times \cdot 87 \times 16} = \cdot 36 \text{ sq. in.}$$

Use ⅝ in. ϕ bars at 10 in. *c/c*. A = ·368 *at the bottom* as shown in Fig. 97.

$$\textit{Shear} = \frac{4064 + 3075}{2} \times 2 = 7140 \text{ lb.}$$

Shear intensity $= \frac{7140}{12 \times \cdot 87 \times 18} = 38$ lb./in.². Very low.

Bond $= \frac{7140}{3 \cdot 14 \times \frac{5}{8} \times \frac{12}{10} \times \cdot 87 \times 18}$

$= 193$ lb./in.² which is safe with standard hooks at ends.

(c) *Design of heel.*—The span of the cantilever is 6 ft. It is subjected to

(1) Its own load $= 6 \times 1' \times 1 \cdot 5$ (average) $\times 144 = 1296$ lbs.
and the B. M. $= 1296 \times \frac{6}{2} \times 12 = 45456$ in. lb.

(2) Load of earth
$= 6 \times 19 \cdot 5 \times 1 \times 96 = 11232$ lbs.
and the B. M. $= 11232 \times 3 \times 12 = 404352$ in. lbs.

(3) Reaction of upward soil pressure. From the stress diagram in Fig. 97. it will be seen that the pressure between D and E in a distance of about 1·3 ft. is negative that is tension. As the soil is incapable of applying tension it may be neglected. At E the pressure is zero and at C, 4·7 ft. away from it, it is $= 494 \cdot 7 \times 4 \cdot 7 = 2325$ lbs./ft.²

$$\text{The B. M.} = -\frac{2325+0}{2} \times 4 \cdot 7 \times \frac{4 \cdot 7}{3} \times 12$$
$$= -102765 \text{ in. lbs.}$$

$$\text{The combined B. M.} = 45456 + 404352 - 102765$$
$$= 347043 \text{ in. lbs. say } 347000$$

$$d = \sqrt{\frac{347000}{126 \times 12}} = \text{about } 15 \text{ in.}$$

Ours is 21 in. overall, or 19 in. effective and is safe.

$$A_T = \frac{347000}{18000 \times \cdot 87 \times 19} = 1 \cdot 2 \text{ sq. in.}$$

Use $\frac{3}{4}$ in. ϕ bars at 3 in. *c/c*. A $= 1 \cdot 23$ in.² *at top* of the heel as shown Fig. 97.

Shear.—This is also due to the two loads on one side and the upward soil pressure on the opposite side.

Load due to self weight $= \frac{15+21}{2} \times 12 \times 6 =$ 1296 lbs.

,, ,, earth $= 6 \times 19{\cdot}5 \times 1 \times 96 =$ 11332 ,,

Total 12628 ,,

,, upward soil pressure

$= \frac{2325+0}{2} \times 4{\cdot}7 =$ − 5463 ,,

Nett shear 7165 lb.

Intensity of shear $= \frac{7165}{19 \times {\cdot}87 \times 12}$

= 36 lbs./in². which is low.

Bond stress $= \frac{7165}{3{\cdot}14 \times 0{\cdot}306 \times \frac{12}{8} \times {\cdot}87 \times 19}$

= 56·7 lbs./in.² which is safe.

Distribution steel— $\frac{1{\cdot}2}{5} =$ 0·24 in.² $\frac{1}{4}$ in. ϕ bars at 9 in. *c/c.* (A = 0·262).

The completely designed wall with the reinforcement is shown in Fig. 96 and 97.

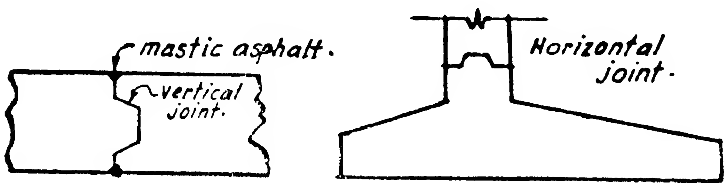
Practical Hints:—(1) An increase of moisture in the earthen mass retained by the wall increases the horizontal thrust. Ample provision should, therefore, be made of weep holes, 4 to 5 ft. vertically and 10 ft. horizontally by preferably laying stoneware pipes at a slight inclination in the concrete wall with hand-packed hollow rubble and gravel at heir inner mouths.

(2) As the base slab and the back of the wall are liketf to remain continually in contact with moi st earth a cover o. 3 in. concrete should be provided at the bottom of the basl slab and 2 in. on the back side of the vertical wall.

(3) The footing is poured first, then the wall, with a joiny at their junction as shown in dotted lines in Fig. 98 with a key to add to the shear resistance. The vertical wall steeel

though in several lengths spliced together, is bound to be of considerable length apiece. The best course is to provide dowels at the junction section anchored 45 diam. into the footing and projecting at least 40 diameters into the vertical wall. The wall forms may then be erected, wall steel being placed later in the forms resting on the footing.

(4) Vertical expansion joints should be provided at every 20 to 25 ft. of the wall as shown in Fig. 98 and to prevent leakage mastic asphalt should be filled in the joint near the surfaces.



Figs. 98 & 99.

Practice Problems:—(1) Design a cantilever retaining wall 10 ft. high above ground and 4 ft. below resting on a soil with a safe bearing capacity of 2 tons/ft.^{2}. There is a surcharge of ground sloping at $1\frac{1}{2}$ to 1. The earth to be retained weighs 100 lbs./ft.^{3} and $\theta = 35°$ and $\mu = 0{\cdot}48$.

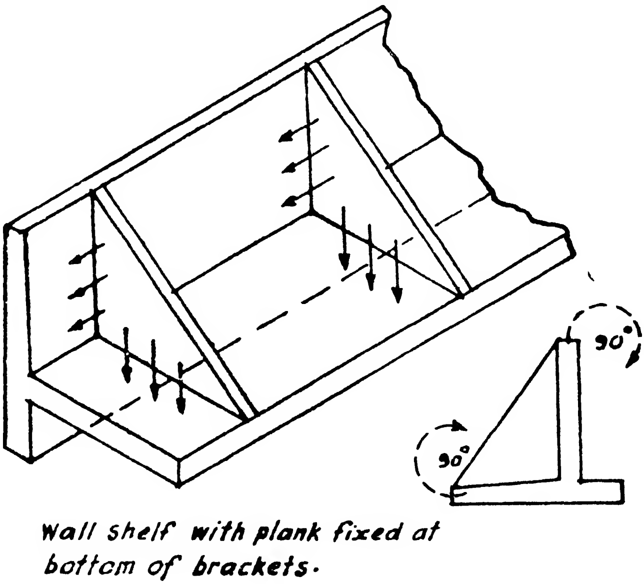
(2) Design a cantilever retaining wall of a total height of $16'-6''$ on a soil bearing a safe load of 3 tons/ft.^{2}. The retained earth weighs 105 lb./ft.^{3} and angle of repose is 30°. The coefficient of friction is 0·45.

CHAPTER XX

COUNTERFORTED RETAINING WALL

A CANTILEVER retaining wall is distinctly cheaper for heights below 15 ft. Between 15 and 20 ft. both types practically cost the same. As the height increases to more than 20 ft. the thickness of the stem of a cantilever retaining wall increases with sudden increase in cost, and then the counterforted wall has an economic advantage.

In a cantilever wall all the component parts are cantilevers whereas, in a counterforted retaining wall the counterforts and the toe act as cantilevers, and the vertical wall and the heel slab are continuous slabs, "supported" or rather anchored into the counterfort beams. The latter are triangular in elevation and are T-beams, the vertical slab serving as their flange.



Figs. 100 & 101.

The structure is similar to a shelf supported on wall brackets which are also usually triangular in elevation. In the case of a shelf, the shelf-board is usually on the top of the brackets, whereas here the slab is at the bottom cast monolithically with, and fixed, by anchors to the counterforts. In fact, if the retaining wall is rotated through 90 deg. in a clockwise direction as shown in Fig. 101 it becomes a shelf like the one pictured in Fig. 100. The only difference being that here the brackets are on the top of the shelf-board.

From the analogy of the shelf it will be clear that as both the vertical and horizonal forces first act on the slabs, viz. heel slab and the vertical wall slab respectively and through them on the counterforts, the junction between the slabs and the respective sides of the counterfort must be adequately strengthened by means of steel ties, otherwise they will be torn asunder.

Illustrative Example 80

Design a counterfort retaining wall 25 ft. high above foundations. The angle of repose of the earth retained is 30 deg. and weight 105 lbs./ft.^{3} There is an additional surcharge load of 250 lbs./ft.^{2} Take $p_b = 3$ tons/ft.^{2} and $\mu = 0{\cdot}45$.

Solution:—Assume width of base, $B = {\cdot}5h = 12{\cdot}5'$. The spacing of counterforts is usually 8 to 12 ft. Let us assume it 9 ft. Length of toe $= {\cdot}25B = {\cdot}25 \times 12{\cdot}5$ say $3'-2''$; thickness of wall slab is usually $8''$ to $12''$. Let us take it $= 10''$; thickness of heel slab $24''$ (average) and that of counterforts $= 12''$.

$$\text{Earth pressure, } P = \frac{wh^2}{2}\left(\frac{1-\sin\phi}{1+\sin\phi}\right)$$

$$= \frac{105}{2} \times 25^2 \times \frac{1}{3}$$

$$= 10940 \text{ lbs.} \qquad (1)$$

This acts horizontally at $\frac{h}{3} = 8{\cdot}33$ ft. above the base.

$$\text{Pressure due to surcharge load} = 250\left(\frac{1-\sin\phi}{1+\sin\phi}\right)h$$

$$= 250 \times \frac{1}{3} \times 25$$

$$= 2080 \text{ lbs.} \qquad (2)$$

acting at $\frac{25}{2}$ or 12·5 ft. horizontally above the base. The moment of the first about the base

$$= 10940 \times 8{\cdot}33 = 91170 \text{ ft. lb.}$$

Moment of (2) $= 2080 \times 12{\cdot}5 = 26000$ „

117170 „

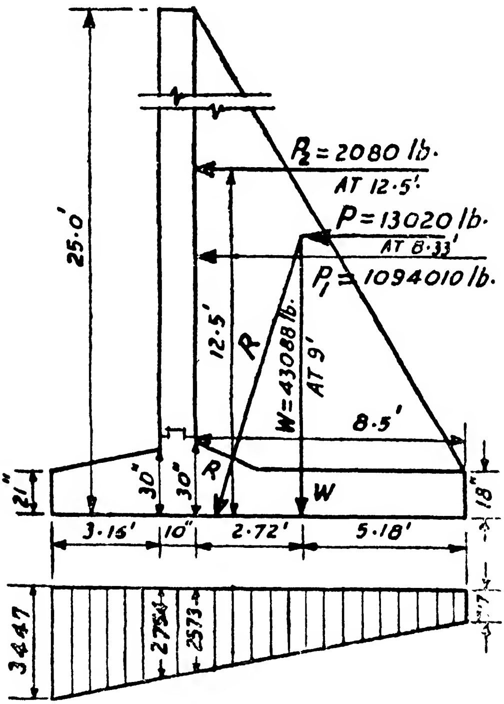


Fig. 102 & 103.

If the resultant of these is acting at x above the base,

$$x = \frac{117170}{10940+2080} = 9 \text{ ft.}$$

(1) *Checking stability of wall against overturning*:—We have already found out that the horizontal force,

P = 10940 + 2080 = 13020 lbs. acting at 9 ft. above the base.

The vertical forces are:

(a) wt. of the stem or wall slab.
(b) wt. of the base slab.
(c) wt. of the counterforts.
(d) wt. of earth on the heel.

These and their moments are tabulated below:—

TABLE NO. 27

VERTICAL LOADS AND THEIR MOMENTS

Load and its measurements	Weight lb.	Lever Arm ft.	Moment about the heel ft. lb.
W_1 wall slab, $23 \times 1' \times \frac{10''}{12} \times 144$	2760	$8'-11''$ or 8·92	24619
W_2 base slab, $12{\cdot}5 \times 1' \times 2' \times 144$	3600	6·25	22500
W_3 counterfort $\frac{8{\cdot}5+0}{2} \times 1' \times 23' \times 144$	14076	$8{\cdot}5 \times \frac{2}{3} = 5{\cdot}66$	79670
W_4 earth mass $8{\cdot}5 \times 1' \times 23' \times 105$	20527	$\frac{8{\cdot}5}{2} = 4{\cdot}25$	87240
W_5 surcharge $8{\cdot}5 \times 1' \times 250$	2125	$\frac{8{\cdot}5}{2} = 4{\cdot}25$	9031
Total	43088		223060

If $\bar{x}$ is the distance of the resultant from the heel

$$x = \frac{223060}{43088} = 5{\cdot}18 \text{ ft.}$$

The resultant of the two forces viz. horizontal force $P = 13020$ lb. at 9 ft. above the base, and vertical force of 43088 at 5·18 ft. from the heel meets the base at $\bar{x}$ where $\bar{x} = \frac{9 \times 13020}{43088} = 2{\cdot}72$ ft. beyond 5·18 ft. from the edge i. e. at 7·90 from the heel as shown in Fig. 102. As this is well within the base the wall is safe against overturning. The horizontal thrust which would cause the resultant to pass just through the edge would be

$$= 12{\cdot}5 - 5{\cdot}18 = \frac{x \times 9}{43088}$$

$$x \quad = 35284 \text{ lbs.}$$

$$\text{Factor of safety} \quad = \frac{35284}{13020} = 2{\cdot}7$$

The wall is therefore very stable against overturning.

(2) *Checking stability against sliding*:—The maximum horizontal thrust = 13020 lb. and W = 43088 lbs. The frictional resistance

$$= 43088 \times 0{\cdot}45 = 19390 \text{ lbs.}$$

This is nearly 150 per cent of the horizontal thrust. The design is therefore safe in this respect.

(3) *Checking pressure on foundation*:—The eccentricity $= 7{\cdot}90 - 6{\cdot}25 = 1{\cdot}65$. As this is less than $\frac{l}{6}$ or $\frac{12{\cdot}5}{6} = 2{\cdot}1$ there will be compression on the entire section and no tension anywhere.

$$\text{Maxi. pressure at toe} = \frac{\text{Load}}{\text{area}} + \frac{\text{Load} \times e}{\text{modulus of section}}$$

$$= \frac{43088}{12{\cdot}5 \times 1} + \frac{43088 \times 1{\cdot}65}{\frac{12{\cdot}5^2 \times 1}{6}}$$

$$= 3447 + 2730$$

$$= 6177 \text{ lbs./ft}^2$$

As this is less than the bearing power of soil given, viz. 3 tons or 6720 lb./ft.^{2} the design is safe.

The minimum pressure at the heel

$$= 3447 - 2730 = 717 \text{ lb./ft.}^2$$

(see Fig. 103)

(4) *Design of toe*:—From these figures the stress diagram of pressures on the foundation is drawn in Fig. 103. The variation per foot is $= \dfrac{3447-717}{12{\cdot}5} = 218{\cdot}2$. The pressures at the left and right hand faces of the wall slab are 2764 and 2573 respectively. The toe is subjected to the upward reaction of the soil pressure varying from 2754 at support to 3447 at the free end. Its own weight and that of the small earth above it are neglected.

The B. M. on the cantilever of the toe

$$= \left\{2754 \times 3{\cdot}17 \times \frac{3{\cdot}17}{2} + \frac{3447-2754}{2} \times 3{\cdot}17^2 \times \frac{2}{3}\right\} \times 12$$

= 202644 in. lb.

$$d = \sqrt{\frac{202644}{126 \times 12}} = 11{\cdot}5$$

our (24 − 3″ cover =) 21 in. is quite ample

$$A_T = \frac{202644}{18000 \times {\cdot}87 \times 21} = 0{\cdot}62 \text{ in}^2.$$

Provide $\frac{5}{8}$ in. ϕ bars at 6 in. *c/c* (A = ·614)

$$\text{Shear intensity} = \frac{\dfrac{3447+2754}{2} \times 3{\cdot}17}{12 \times {\cdot}87 \times 21} = \frac{10613}{219}$$

= 48 lb./in². which is quite safe.

$$\text{Bond stress} = \frac{10613}{3{\cdot}14 \times \frac{5}{8} \times \frac{12}{6} \times {\cdot}87 \times 21}$$

= 147 lbs.

As this is less than the permissible 200 lbs. this is acceptable. Still the ends should be hooked. In the above equation $3{\cdot}14 \times \frac{5}{8}$ is the perimeter of one $\frac{5}{8}$″ bar and $\frac{12}{6}$ is the number of bars in 12 in. with a pitch of 6 in.

(5) *Design of heel slab*:—This is to be designed as a continuous slab supported by, or rather anchored to, the counterforts at 9 ft. intervals. It is subjected to the following loads:—

Loads pressing down:— (1) Self-weight

(2) Wt. of the mass of 23′ of earth

(3) Surcharge load

Loads pressing upward:—(4) Upward reaction of soil varying from 717 lbs. at the heel to 2573 lb./ft.^{2} near inner face of wall slab. See stress diagram in Fig. 103.

(1) Self wt. $= 12'' \times 24'' = 288$ lb./ft^2.

(2) Wt. of earth $= 23 \times 105 = 2415$ „

(3) Surcharge $= 250 = 250$ „

Total downward load $= 2953$ „

(4) Upward soil pressure in one ft. inside the edge of the heel

$$= 717 + \frac{218{\cdot}2}{2} = 826$$

Nett load $2953 - 826 = 2127$ lbs./ft.^{2}

B. M. at the middle of the end span and that at the penultimate support

$$= \pm \frac{wl^2}{10} = \pm \frac{2127 \times 9 \times 9 \times 12}{10} = 206744 \text{ in. lbs.}$$

Our effective depth of 21 in. will be more than ample and need not be calculated. The reinforcement also will be light.

$$A_T = \frac{206744}{18000 \times {\cdot}87 \times 21} = 0{\cdot}63$$

Provide $\frac{5}{8}$ in. ϕ bars at 6 in. *c/c*.

Bond stress $= 2127 \times 9 = 19143$ lbs.

$$\text{Intensity} = \frac{19143}{3{\cdot}14 \times \frac{5}{8} \times \frac{12}{6} \times {\cdot}87 \times 21} = 270 \text{ lb./in.}^2$$

As this is more than the permissible viz. 200 lbs, use $\frac{1}{2}$ in. ϕ bars at 3·5 in. *c/c*. The intensity of bond stress then is 190 lb./in.^{2} which will do if the ends are hooked.

We have designed one foot strip near the heel, of the end span. The load on the strip near the inner face of the wall slab will be much lighter since the vertical downward loads will remain the same while the upward soil pressure will increase resulting in reducing the downward load still further.

The load on the heel slab is light here because the resultant of the horizontal and vertical loads fell close to the centre of the base in the above example. Oftentimes when it falls beyond the middle third there is negative pressure i. e. tension at the heel and the slab has to be designed for the full downward load.

(6) *Design of vertical wall slab*:—This slab also is to be designed as a continuous slab supported by, or rather, anchored to the counterforts at 9 ft. intervals. It is subjected to horizontal pressures due to

(1) Mass of earth $= \left(\frac{1-\sin\theta}{1+\sin\theta}\right) wh = \frac{1}{3}\,105\,h = 35 \times h$

(2) Surcharge thrust $= W\left(\frac{1-\sin\theta}{1+\sin\theta}\right) = \frac{250}{3} = 88{\cdot}3$

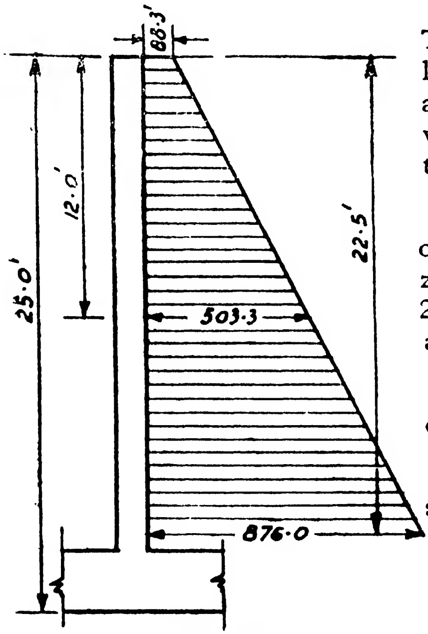


Fig. 104.

The surcharge thrust of 88·3 lb./ft.^{2} is constant throughout and that due to mass of earth varies as h. These are plotted in Fig. 104.

For illustrating the method of design we shall take a horizontal strip between 22 and 23 ft. depth from the top i.e. average depth of 22·5 ft.

Pressure due to mass of earth = 22·5×35 = 787·5

Pressure due to mass of surcharge

= 250 × $\frac{1}{3}$ = 88·3

Total 875·8

say 876

B. M. $= \pm \frac{wl^2}{10}$ or $\pm \frac{wl^2}{12}$ according as it is in the middle of the end span, top of penultimate support or in the intermediate spans or supports. Let us design it for the end span.

$$B.\,M. = \frac{876\times9\times9\times12}{10} = 85147 \text{ in. lb.}$$

$$d = \sqrt{\frac{85147}{126\times12}} = 7{\cdot}5 \text{ in.}$$

We have adopted D = 10″ with a cover of 2 in. Our effective depth of 8 in. is sufficient.

$$A_T = \frac{85147}{18000\times{\cdot}87\times8} = 0{\cdot}69$$

Provide $\frac{5}{8}$ in. ϕ bars at 5 in. *c/c*. (A = 0·736).

$$\text{Shear intensity} = \frac{876\times9}{2}\div12\times8\times{\cdot}87$$

= about 48 lb./in.² This is acceptable.

$$\text{Bond stress intensity} = \frac{876\times9}{2}\div(3{\cdot}14\times\tfrac{5}{8})\times\tfrac{12}{5}\times{\cdot}87\times8$$

In this equation $3{\cdot}14\times\frac{5}{8}$ is the perimeter of one bar, $\frac{12}{5}$ = no. of bars per ft.

= 121·4 lb./in.²

This is also acceptable with hooked ends.

For intermediate spans the steel may be reduced to $\frac{10}{12}$ in the proportion of the B. M.

Slab at any depth is to be designed in a similar manner. The following table gives the B. Ms. and reinforcement at different levels.

TABLE NO. 28

BENDING MOMENTS AND STEEL IN THE END SPAN

Depth below top ft.	P lbs.	Bending Moment in. lb.	Reinforcement	
			Area	Diam. and spacing of rods
22·5	876	85147	0·69	$\frac{5}{8}$″ ϕ at 5 in. c/c
18·0	718	69800	0·56	$\frac{5}{8}$″ ϕ at 6½ in. c/c
12·0	503	48900	0·39	½″ ϕ at 6 in. c/c
6	298	28770	0·24	$\frac{3}{8}$″ ϕ at 5½ in. c/c

As all the thrust on the vertical slab is to be ultimately transferred to the counterforts the two must be rigidly joined together. The reaction of the slab between 22 and 23 ft. below top to be transmitted to the counterfort in the end span $= \frac{Wl}{2} = \frac{876 \times 9}{2} = 3942$ lbs.

$$\text{Area of steel required} = \frac{3942}{18000} = 0{\cdot}219 \text{ in.}^2$$

Provide 2 Nos. $\frac{3}{8}$ in. ϕ bars (A = 0·22). These will be in the form of a bar with hooks at ends in the one ft. width of slab at bottom connecting the vertical slabs with the counterfort at the end.

For intermediate counterforts the reaction will be double this as it will be from spans on either side. Here provide two-legged i. e. U-shaped $\frac{3}{8}$ in. bars at 6 in. *c/c*.

As one goes towards the top with reduction in depth, the reactions will also be reduced and still smaller tension bars or U-shaped links with hooks will be required to connect the vertical slab with the counterforts.

Design of counterforts :—The counterfort is to be designed as a cantilever beam with horizontal thrusts due to (1) mass of earth and (2) surcharge load pressing against the vertical wall, which is ultimately transferred to the counterfort through U-shaped ties.

The maximum thrust due to earth at 22·5 ft. below the top, $p, = \frac{wh^2}{2}\left(\frac{1 - \sin 30^\circ}{1 + \sin 30^\circ}\right)$ where $h = 22{\cdot}5$ and $w = 105$

$= \frac{105}{2} \times 22{\cdot}5^2 \times \frac{1}{3}$ acting at $\frac{22{\cdot}5}{3}$ from the top of heel slab.

The thrust due to surcharge

$$= 250 \times \left(\frac{1 - \sin 30^\circ}{1 + \sin 30^\circ}\right) \times h \text{ acting at } \frac{h}{2}$$

$$= 250 \times \tfrac{1}{3} \times 22{\cdot}5$$

$$\text{B. M.} = \left(\frac{105}{2} \times \frac{22{\cdot}5^2}{3} \times \frac{22{\cdot}5}{3} + 250 \times \tfrac{1}{3} \times 22{\cdot}5 \times \frac{22{\cdot}5}{2}\right) 9 \times 12$$

$= (66444 + 21092)\ 9 \times 12$

$= 9453888$ in. lb. say 9454000.

The depth near the support where this B. M. occurs need not be calculated as it is ample viz.,

$= 8{\cdot}5' \times 12 + 10''$ (wall slab) $-$ say $5''$ (3 in. for cover and $2''$ as the depth rapidly decreases)

$= 107$ in.

$$A_T = \frac{9454000}{18000 \times {\cdot}87 \times 107} = 5{\cdot}65 \text{ in.}^2$$

As reinforcement is to be used at the bottom along the slope of the hypotenuse and not at right angles to the direction of the pressure as in normal beams, to make effective steel area $= 5{\cdot}65$ sq. in. we must use $5{\cdot}65 \sec \theta =$

$$5{\cdot}65 \times \frac{\sqrt{22{\cdot}5^2 + 8{\cdot}5^2}}{22{\cdot}5}$$

$$= 5{\cdot}65 \times \frac{24{\cdot}3}{22{\cdot}5} = 6 \text{ sq. in.}$$

Use 5 Nos. $1\tfrac{1}{4}$ in. ϕ (A $= 6{\cdot}136$) or better still 4 Nos. $1\tfrac{3}{8}$ in. ϕ (A$=5{\cdot}94$) as the width is only 12 in.

For distribution steel use $\tfrac{1}{2}''$ ϕ at 9 in. c/c.

$$\text{Shear (maximum)} = \left(\frac{250}{3} \times 22{\cdot}5 + \frac{105}{3} \times \frac{22{\cdot}5^2}{2}\right) \times 9$$

$= 96570$ lbs.

As the main reinforcing bars are inclined along the slope of the hypotenuse, their horizontal component which resists the shear has the value

$$= 5{\cdot}94 \times 18000 \times \frac{8{\cdot}5}{24{\cdot}3}$$

$= 37400$ lb.

In this equation 5·94 is the area of steel bars, 18000 lbs. is the permissible shear stress per sq. in. of steel, 8·5 is the depth and 24·3 is the length along the hypotenuse.

Thus the nett shear the concrete has to resist

$$= 96570 - 37400 = 59170 \text{ lbs.}$$

Intensity of shear

$$= \frac{59170}{10 \times \cdot 87 \times 107} = 63 \text{ lb./in.}^2 \quad \text{This is acceptable.}$$

$$\text{Bond stress} = \frac{96570}{(4 \times 3{\cdot}14 \times 1{\cdot}375)} \times \cdot 87 \times 107.$$

$$= 59 \text{ lb./in.}^2 \text{ which is safe.}$$

In the above equation, the terms inside the bracket show the perimeter of 4 – $1\frac{3}{8}''$ ϕ bars and 107 is the effective depth at 22·5 ft. below top.

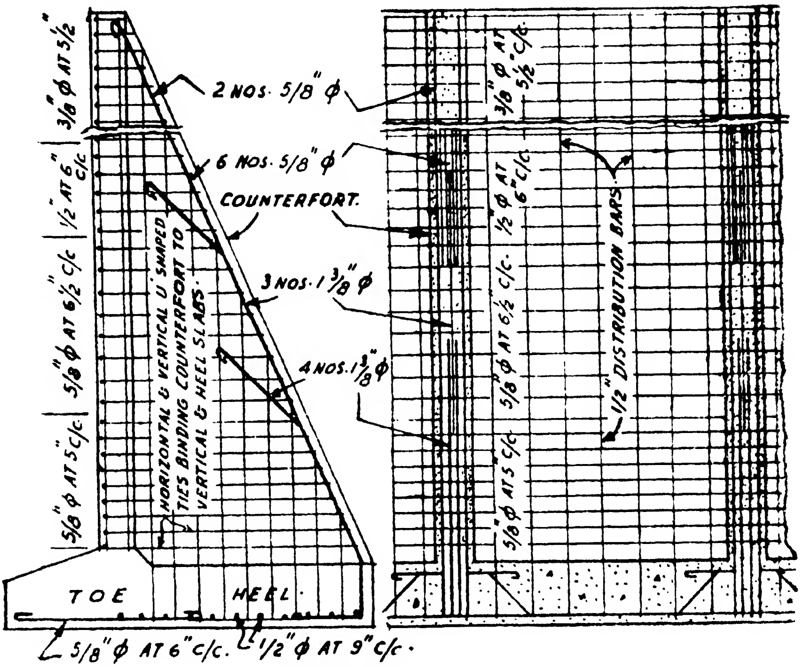
In a similar manner the B. Ms. at different depths below the surface may be calculated and reinforcement designed. The following table shows these figures at four different depths.

TABLE NO. 28

B. M.S AND REINFORCEMENT AT DIFFERENT DEPTHS IN A COUNTERFORT

Depth below surface ft.	Bending Moment in. lb.	Effective depth inches.	Reinforcement	
			Area sq. in.	No. and diameter in.
22·5	9454000	107	6·0	4 Nos. $1\frac{3}{8}$ in. ϕ A = 5·94
18·0	5132600	83	4·27	3 Nos. ,, ,, A = 4·45
12·0	1412640	57	1·73	6 Nos. $\frac{5}{8}$,, A = 1·84
6·0	217080	31	0·49	2 Nos. $\frac{5}{8}$,, A = 0·61

Figs. 105 and 106 show a side elevation and a rear elevation of the counterfort respectively with the concrete cover removed so that the reinforcing bars are seen.



Figs. 105 & 106.

CHAPTER XXI

R. C. C. STAIRCASES

STAIRCASES are designed as slabs, but the method of procedure is slightly different. They may be either transverse (parallel to nosings) or longitudinal (parallel to flight). In transverse slabs, again, they may be either cantilevered with one end fixed into a wall and the other free, or, they may be freely supported with ends resting in grooves in side walls or on stringers either of R. C. C. or rolled sections. The thinnest part of the stair is called the " waist " and its depth is measured normal to the slope of the flight.

Some authorities take only the waist as the depth of the slab for design purposes, the saw tooth projections in that case, being merely a dead load. In this case a slight filling of concrete in the right angles between the upper surfaces of the tread and lower edge of the riser as shown in Fig. 110 materially helps in increasing the depth of the waist by an inch to 1½ inches.

Other authorities take the effective depth as half the maximum slab thickness, from the top of nosing to the soffit, measured normal to the soffit.

The following few examples of each type will illustrate the current practice in stair design.

1. **Horizontal or transverse span** (*a*) *Cantilever type*

Illustrative Example 81

Design a staircase of cantilever type projecting 4 ft. beyond wall face, including 3 in. thick × 2′ 9″ high R. C. C. parapet on the edge of the free end. The tread is 11 in. inclusive of one in. nosing and rise 7 in.

Solution: Dead load :— $\frac{10\times7}{2}$ = 35 lb.

Waist (assume 3 in.)

Sloping length 13·5×3″ = 40·5 lb.

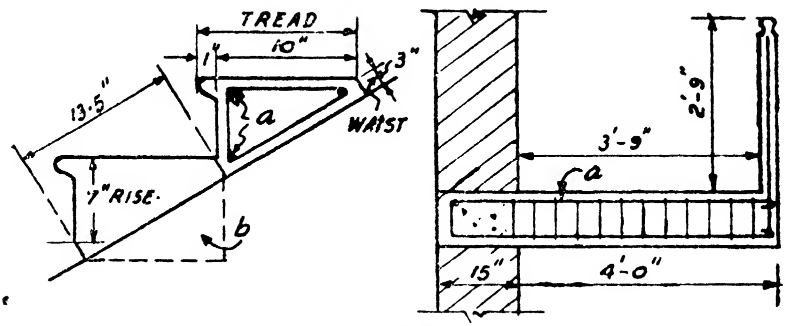
Top finish = 5·5 lb.

Live load:— 100 × $\frac{11}{12}$ = 92·5 lb.

Total 173·5 lb.

acting at 2 ft. from the face of wall.

Parapet load = $\frac{3}{12}$ ×2·75×144=99 lb. acting at say 4 ft. from the support.



Figs. 107 & 108.

B. M. = 173·5 × 4 × 2 × 12 + 99 × 4 × 12
= 16608 + 4752 = 21360 in. lb.

This acts at right angles to the tread. When resolved, the component at rt. angles to the waist slab = 21360 cos 30° = 18580 lb. The slope of the staircase is assumed to be 30°.

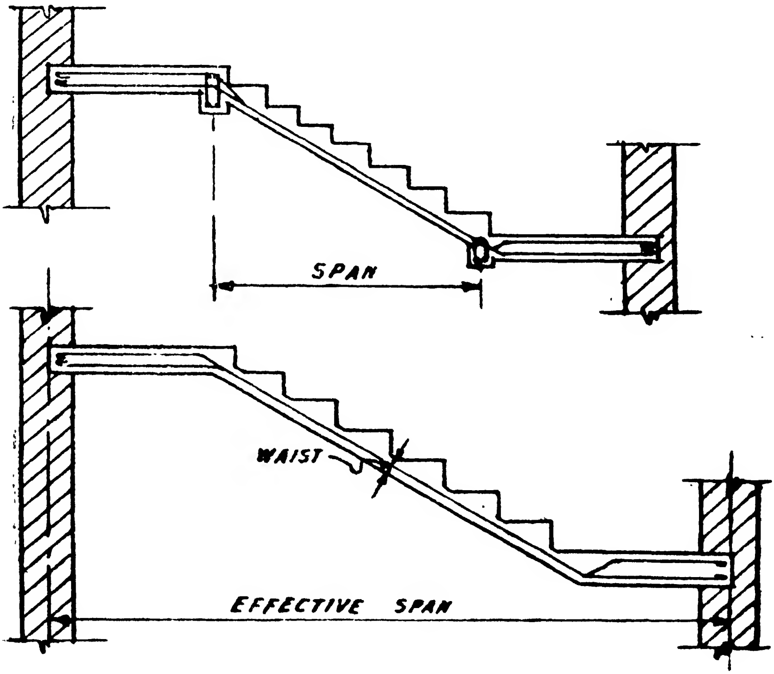
Note :—Some authorities do not allow for the slope and take the entire load as if it is acting at rt. angles to the waist slab.

$$\text{Effective } d = \sqrt{\frac{18580}{126 \times 12}} = 3\text{·}5 \text{ in.}$$

$$A_T = \frac{18580}{18000 \times \text{·}87 \times 3\text{·}5} = 0\text{·}34 \text{ in.}^2$$

Use 2 Nos. $\frac{1}{2}$ in. ϕ (A = ·392). These rods should be at the top. One of them viz. bar "*a*" shown in Fig. 107 should be U-shaped, to which $\frac{1}{4}$ in. ϕ temperature bars should be tied longitudinally at 12 in. intervals. The end of each step going into the wall should be rectangular as shown dotted at *b*. Though the intensity of shear is small a few triangular links as shown in figure should be tied round the bars at intervals of, say, 9 in.

(*b*) *Simply supported type*:—The procedure of design in this case is similar to the above, only the B. M. will be $\frac{wl^2}{8}$ instead of $\frac{wl^2}{2}$ in the case of the cantilever stair designed



Figs. 109 & 110.

above and the steel will be at bottom. As the B. M. is ½ of the above, the reinforcement will be lighter.

If the steps are supported on stringer beams below the soffit, the two end beams may be designed as ell-beams with flange width equal to half the width of the staircase.

2. **Longitudinal or sloping span:**—A sloping flight of stairs supported at top and bottom is designed as a slab with partially fixed ends $\left(\text{B. M.} = \frac{wl^2}{10}\right)$.

It is the common practice to provide transverse beams at top and bottom as shown in Fig. 109 at the junction of landing and sloping flight in which case the horizontal distance between the centres of the landing beams is the span.

It is not, however, unusual to omit the transverse beams altogether as shown in Fig. 110 as the horizontal thrust is very small. In any case the span is measured along the horizontal distance and not along the slope of the flight.

In a longitudinally spanning flight of stairs the thickness of the slab required to resist the bending moment determines the thickness of the waist which is from 4 to 6 in.

Illustrative Example 82

A flight of stairs supported at top and bottom by transverse beams has a span = 9 ft. measured horizontally between centres of beams. The tread is 11 in. including one in. nosing and riser 7 in. Design the staircase.

Solution :—Assume a waist = 5 in. thick

Dead load $= \frac{10 \times 7}{2} = 35{\cdot}0$ lb. per ft. run of step

Waist $= 12{\cdot}5 \times 5'' = 62{\cdot}5$ „ „

Top finish $11 \times \frac{1}{2} = 5{\cdot}5$ „ „

Live load $100 \times \frac{11}{12} = \frac{92{\cdot}0}{195}$ „ „

$$\text{B. M.} = \frac{wl^2}{10} = \frac{195 \times 9 \times 9 \times 12}{10}$$

$$= 18960 \text{ in. lbs.}$$

$$d = \sqrt{\frac{18960}{126 \times 12}}$$

$$= 3{\cdot}6.$$

The waist of 5 in. assumed provides a cover of 1·4 in. and 3·6″ is acceptable. $A_T = \dfrac{18960}{18000 \times \cdot 87 \times 3{\cdot}6} = 0{\cdot}36 \text{ in.}^2$

Use ½ in. bars at 6 in. centres A= 0·392. Alternate bars should be bent up. Use ¼ – in. bars at 9 in. centres transversely for distribution steel.

CHAPTER XXII

A CIRCULAR TANK

A CYLINDRICAL tank is usually meant and designed for storing a fluid. The latter exerts at any point a uniform radial pressure in all directions in diametrical plane at right angles to the curved surface of the cylinder. The pressure increases as the depth below the surface and is equal to $w \times h$ lb. per sq. ft., where w = wt. of the fluid per cub. ft. and h = the depth below the surface of the fluid. The total pressure on the curved surface A C B (Fig. 111) which is the same as that in the diametrical plane is = Dwh and is resisted by the two sides of the tank, and therefore the total tension in each side of the tank at any depth h is

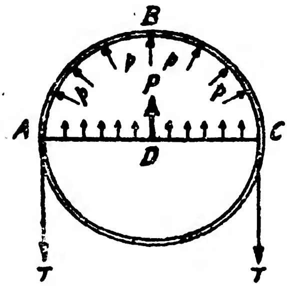


Fig. 111.

$$T = \frac{Dwh}{2} \text{ lb.}$$

Working stresses: (*a*) *Concrete*:—Since concrete is also subjected to tension along with the reinforcing steel, a stress of 100 lb./in². for 1:2:4 mix and 150 lb./in². for 1:1·5:3 mix ordinary grade concretes are allowed on the composite section i. e. on the equivalent area of concrete (A_{eq}) of the section. If a higher stress is adopted, it is possible that fine hair-cracks may be developed through which water might leak.

For compression in concrete a stress of 600 lb./in². is adopted.

(*b*) *Steel*:—Though a low stress is adopted as above in concrete, shrinkage cracks are unavoidable. Besides there will be construction joints. At these places the entire tension comes on the circumferential steel. To prevent these cracks and joints opening and forming a source of leakage a tensile stress of 12000 lb./in.^{2} is adopted. At other places where there is a bending moment a stress of 16000 lbs./in.^{2} is adopted. With the stress of $c = 600$ in concrete and $t = 16000$ in steel, lb./in^2. and $m = 15$, the $R.M. = 95\ bd^2$ and the lever arm $= 0{\cdot}88d$.

For determining the thickness of the shell of the cylinder an empirical rule is :

$$t = 3 + \frac{h}{3}$$

Where t = thickness in inches and h = depth of water in ft.

Illustrative Example 83

Find the thickness and reinforcement of a circular R. C. C. tank to store 22500 gallons of water with 8 ft. depth ignoring the fact that the sides are fixed to the bottom.

Solution:— $\frac{22500}{6{\cdot}25} = 3600$ cub. ft.

Floor area $= \frac{3600}{h} = \frac{3600}{8} = 450$ sq. ft.

Diameter $D = \sqrt{\frac{450 \times 4}{3{\cdot}14}} = 23{\cdot}8$ say 24 ft.

Maximum tension in sides, $T = \frac{D\,wh}{2}$ lb./ft.^{2}

$$= \frac{24 \times 62{\cdot}5 \times 8}{2} = 6000 \text{ lbs.}$$

Area of steel $= \frac{6000}{12000} = 0{\cdot}5$ in.^{2}

Minimum thickness of shell

$$t = 3 + \frac{h}{3} = 3 + 2{\cdot}67 = 5{\cdot}67 \text{ say } 6 \text{ in.}$$

Also t is determined by the composite stress in concrete thus,

$$T = \text{tensile stress in concrete} \times A_{eo}$$

$$6000 = 100\,(12'' \times t + {\cdot}5 \times 14)$$

$$\text{or } t = \frac{5300}{1200} = 4{\cdot}4 \text{ in. (effective).}$$

Since 6″ is the greater of the two it will be adopted.

Reinforcement will be ½ in. φ bars at 4½ in. (A=·524). As the pressure varies from 6000 lb./ft.² at bottom to zero at top the hoop steel will also vary proportionately, that at the top being nominal. However, the thickness will remain the same throughout.

Restraint at bottom:—We imagined in the above example that the bottom end of the tank also was free like the top end, to expand circumferentially. But actually the sides at the bottom are rigidly fixed to the base, and therefore circumferential elongation is restrained in a certain portion near the bottom. Had there been no restraint, and had the material forming the sides been perfectly elastic the sides would have assumed the shape of the pressure diagram as shown by

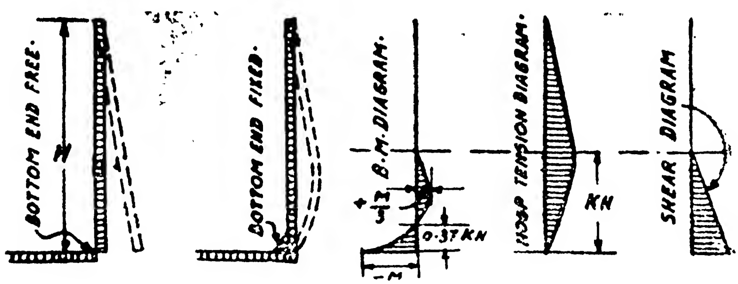


Fig. 112 a, b, c, d, e.

straight dotted lines in Fig. 112 (a) But as the sides are prevented from expanding at the bottom they will assume the shape shown by the dotted curved lines in Figure 112 (b). In consequence there will be a sort of cantilever action in a portion near the bottom and the bending moment caused will take some such shape as shown in Fig. 112 (c). A scrutiny of the B. M. diagram suggests the following points:

(1) That there is a negative B. M. from the bottom to a short distance above the base (0·37 KH shown in Fig. 112 (c) in which there is cantilever action and very small hoop or ring tension.

(2) That the hoop tension varies from zero at the bottom to a maximum at a certain distance above the base, KH shown in Fig. 112 (d) from where it has a straight line variation up to the top where it is zero. See Fig. 112 (d).

(3) That the B. M. changes sign at some distance above the bottom, this distance being 0·37 of the distance of the maximum hoop tension above the base, where the restraint moment is zero.

(4) That the maximum positive B. M. is $\frac{1}{5}$ of the negative B. M. at the bottom.

The values of the restraint moment and the distance at which the maximum circumferential tension occurs depend upon a number of factors and the calculations are complicated. Besides, they are seldom used in practice. The following table gives these values which are based on experimental tests and though not quite accurate are approximately correct and are satisfactory for all practical purposes.

TABLE NO. 29

	F				K			
$\frac{H}{t}$ →	10	20	30	40	10	20	30	40
H ÷ D 0·1	0·075	0·047	0·036	0·028	...	...	...	...
0·2	0·046	0·028	0·022	0·015	...	0·50	0·45	0·40
0·3	0·032	0·019	0·014	0·010	0·55	0·43	0·38	0·33
0·4	0·024	0·014	0·010	0·007	0·50	0·39	0·35	0·30
0·5	0·020	0·012	0·009	0·006	0·45	0·37	0·32	0·27
1·0	0·012	0·006	0·005	0·003	0·37	0·28	0·24	0·21
2·0	0·006	0·005	0·002	0·002	0·30	0·22	0·19	0·16
3·0	0·004	0·003	0·002	0·001	0·28	0·21	0·18	0·15
4·0	0·004	0·002	0·002	0·001	0·27	0·20	0·17	0·14

Maximum negative restraint moment $= F \times p \times H^2$

$= F\, w\, H^3 \quad \ldots \text{(i)}$

,, hoop tension $= (1-k)\, p \times \frac{D}{2}$

$$= \frac{(1-k)wHD}{2} \quad \ldots \quad \ldots \quad \text{(ii)}$$

Distance above the bottom where maximum hoop tension occurs $= K \times H \quad \ldots \quad \ldots \quad \text{(iii)}$

Distance of point of contraflexure above floor $= 0{\cdot}37\, K \times H \quad \ldots \quad \ldots \quad \text{(iv)}$

Maximum positive restraint moment = $\frac{1}{5}$ of negative moment $= \frac{1}{5} F\, w\, H^3 \ldots \quad \text{(v)}$

where w = wt. of 1 cub. ft. of fluid in lb.

H = maximum depth of fluid in ft.

D = Diameter of tank in ft.

t = Thickness of tank in inches.

Illustrative Example 84

Design a circular R. C. C. tank resting on ground, of 20 ft. diameter to store water to a maximum depth of 10 ft.

Solution :—H = 10′ ; D = 20′ ; w = 62·5 lb./ft.³

$$\therefore \quad t = 3 + \frac{H}{3} = 6{\cdot}3'' \text{ say 6 in.}$$

$$\frac{H}{D} = \frac{10}{20} = 0{\cdot}5 ; \quad \frac{H}{t} = \frac{10}{\cdot 5} = 20$$

Maximum $p = wH = 62{\cdot}5 \times 10 = 625$ lb./ft.².

From the above table F = 0·012 and K = 0·37.

Maximum restraint moment by eqn. (i)

$$F\, w\, H^3 = 0{\cdot}012 \times 62{\cdot}5 \times 10^3 \text{ ft. lb.}$$

$$= 750 \times 12 = 9000 \text{ in. lbs.}$$

Maximum hoop tension by eqn. (ii)

$$= (1-k)\frac{wHD}{2} = (1-0{\cdot}37)\,\frac{62{\cdot}5 \times 10 \times 20}{2}$$

$= 3937{\cdot}5$ lbs.

Position of maximum hoop tension by eqn. (iii)

$$= K \times H = 0.37 \times 10 = 3.7' \text{ above base.}$$

$$\text{Hoop reinforcement} \quad = \frac{3937.5}{12000} = 0.33 \text{ in.}^2$$

Use ½ in. ϕ bars at 7 in. vertical distance between centres, and as the maximum hoop tension is at 3·7 ft. from the bottom continue this reinforcement up to 5′ 3″ (10 rows at 7 in. apart or 9 spaces). The distribution of steel in the upper 5 ft. of tank wall is shown in the table below :—

TABLE No. 30

DISTRIBUTION OF HOOP STEEL

Place	Mean depth below surface ft.	Hoop tension lb. per foot height $wHD/2$	Steel per ft. ht. sq. in.	Diameter and spacing in.
Floor to 5′	7·5	3937	0·33	½ in. ϕ @ 7″
5′ to 6′ above floor	4·5	2812	0·23	⅜ in. ,, 5¼″
6′ to 7′ above floor	3·5	2185	0·18	⅜ in. ,, 7″
7′ to 8′ above floor	2·5	1562	0·13	⅜ in. ,, 9″
8′ to 9′ above floor	1·5	937	0·08	¼ in. ,, 7″
9′ to 10′ and above for one foot free-board.	·5	312	0·02	

As there is a bending moment due to the restraint at bottom (see Fig. 112c) vertical reinforcement is also required. The maximum restraint moment is 9000 in. lbs. thickness

$$t \text{ or } d = \sqrt{\frac{9000}{95 \times 12}} = \sqrt{8} = 2.82 \text{ in.} \quad \ldots \quad \ldots \quad \ldots \quad (a)$$

We have adopted 6 in. which is ample. Taking effective *d* or thickness 4 in.

$$A_T = \frac{9000}{16000 \times .88 \times 4} = 0.16 \quad \ldots \quad \ldots \quad (b)$$

Provide $\frac{3}{8}$ in. ϕ bars at 8 in. *c/c.* (A=·166). These will be placed 1 in. inside the *inner* face as due to the cantilever action the B. M. is negative. Half of these i. e. alternate bars will be bent at the point of contraflexure at ·37 KH or or 3·7 ft. above the floor and brought one in. inside the *outer* surface to resist the positive B. M. As the vertical reinforcement is light it is not worth while to reduce it in proportion to the decreasing positive B. M.

The maximum shear is at the bottom and equal to

$$\frac{w \times H \times KH}{2} = 62{\cdot}5 \times 10 \times {\cdot}37 \times 10 = 2312{\cdot}5 \text{ lb. per ft.}$$

of perimeter. Shear above 3·7′ is zero. (See Fig. 112d).

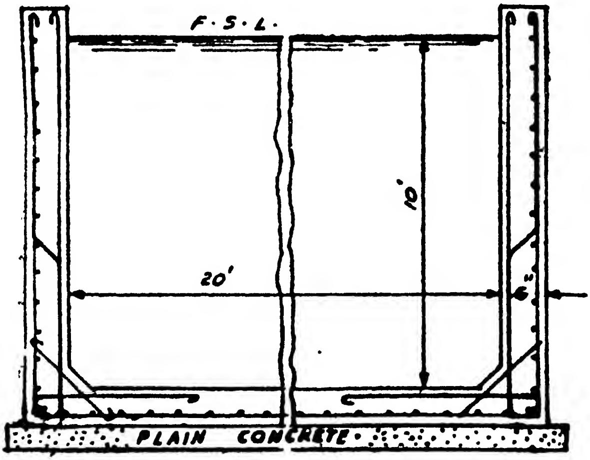


Fig. 113.

$$\text{Intensity of shear} = \frac{2312{\cdot}5}{12'' \times t \times {\cdot}87} = \frac{2312{\cdot}5}{12 \times 6 \times {\cdot}87} = 36{\cdot}7 \text{ lb./in.}^2$$

This being less than 75 is acceptable.

The floor:—As the tank is resting on ground (i. e. not resting on columns) it must be designed like a column footing, with the wt. of walls and water pressing down and the reaction of the soil pressing upwards. In the present case the thickness of the floor will be 6 in. and the reinforcement $\frac{3}{8}$ in.

ϕ bars at 6 in. *c/c.* both ways with hooks at ends as in a column footing.

A vertical section of the tank is shown in Fig. 113.

Practical Hints:—A tank which leaks is useless. Hence every care must be taken to make it water-tight. These precautions are:

(1) As far as possible use a rich mixture. 1:1·5:3 is preferable to 1:2:4. Even in the former mix there should be a little more proportion of sand to minimise shrinkage cracks

(2) Adopt a minimum thickness of 5 in. even for the smallest tank.

(3) As far as possible avoid construction joints, particularly the vertical joints. When in large tanks they are unavoidable take every care by (*a*) using extra dowel bars (*b*) scraping the old surface (*c*) using a slightly richer mixture at the joint and (*d*) making either a grooved or rebated joint.

(4) The working stresses given above should in no case be exceeded both in concrete and in steel.

(5) Where splicing is required, provide laps of 40 to 50 diameters with ends hooked and the laps in successive rings should be staggered.

CHAPTER XXIII

PRESTRESSED REINFORCED CONCRETE

NOW that the advantages of prestressed reinforced concrete have been universally recognised and that it is being increasingly used at great economy, it is necessary that the student should be familiar with the fundamental principles of its design.

Ordinary reinforced concrete has the following disabilities:—

(1) As the tensile strength of concrete is very low and uncertain, the area of concrete in a beam on the tension side of the N. A. has to be neglected in design. Consequently not only that much quantity of concrete is wasted but as it adds to the dead load some reinforcement also is wasted.

(2) As tensile stresses in the concrete in the area mentioned above cannot be avoided by ordinary means, cracks, though unimportant from the point of view of strength in ordinary positions, may be harmful in exposed situations and in places where watertightness is required, they are positively harmful as they reduce the thickness of impermeable concrete.

(3) We have already seen that shear force is of great importance in beams; very often it determines the dimensions of beams. If it is high a very large beam is required, but the latter again increases the shear force and thus it is a vicious circle.

(4) Shrinkage of concrete, while hardening, may result in cracks by initial stresses of considerable magnitude.

(5) Lastly though concrete is very strong in compression, and could be made still stronger by attending carefully to its mix and other things *e. g.* high grade and special grade, its full advantage cannot be taken. For, if the size of a beam is reduced beyond a certain limit, the quantity of reinforcement

required becomes uneconomical. If high tensile steel is employed to overcome this difficulty, its strain, which may be 4 to 6 times that of mild steel would cause wide cracks in the concrete under working loads.

Prestressed reinforced concrete is a panacea for all these disabilities, and has the further advantages that (*a*) it effects a saving of steel to the extent of 60 to 80 per cent, as tensile stresses are very much reduced and shear stresses are practically eliminated. (*b*) In ordinary reinforced concrete the two materials react to load within their limits of resistance as separate materials; in prestressed concrete both react together as if they formed a homogeneous material. (*c*) The section and consequently the dead load is much reduced. This results in reducing the deflection of beams and in making longer spans possible.

For taking the maximum advantage of prestressing, both the materials must be high grade, developing maximum strength—concrete possessing a crushing strength of 7000 to 8000 lb./in ^{2} and steel 80 to 100 tons/in.^{2} ultimate tensile strength.

Prestressed concrete is now commonly used for pipes, water tanks, simply supported beams, girder and arch bridges, railway sleepers, and other precast, independent units. It has not so far proved economical for continuous beams.

The principle underlying prestressing a member is to induce in it, before it is subjected to load, stresses of a nature opposite to those which would be developed when it is loaded, so that the prestress of tension would partly or wholly neutralise the compressive stress developed as a result of loading, and the prestress of compression would neutralise the tensile stress. In other words, the effect of loading is minimised.

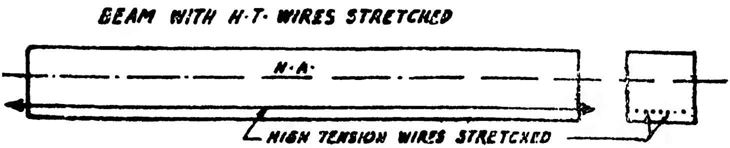
One characteristic of prestressed beams is that even if they are overloaded to such an extent as to produce cracks, on removal of the load, as the strains would disappear, the cracks would be automatically closed, provided the load has not been so heavy as to cause a stress in steel beyond its yield point.

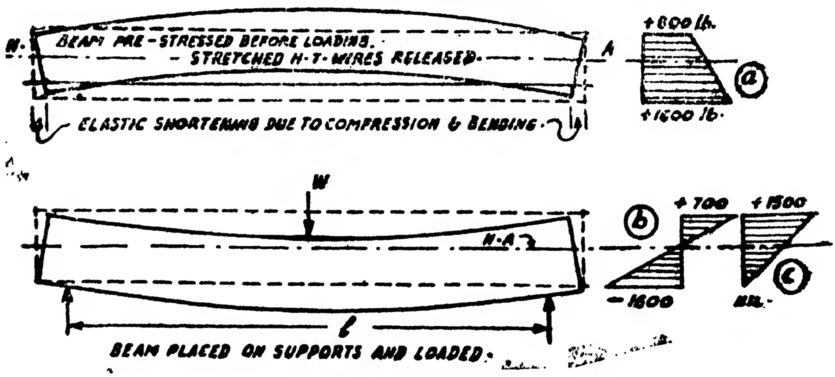
In theory, prestressing is a very simple affair. All that is required is to stretch very high tensile steel wires at the bottom of a beam, for instance, and allow them to react on the surrounding concrete after the latter has hardened. As a result of this the concrete is compressed.

This compressive force acts at just a few inches above the bottom of the beam i. e. away from the axis through centroid. In other words it is eccentric. We have already seen while discussing eccentrically loaded columns or their footings that if the eccentricity exceeds one-sixth of the width there is tension induced in part of the section. Thus by controlling the tensile stress applied for stretching steel wires and by controlling also its eccentricity, it is possible to induce either compression throughout the section or tension in part section and compression in the remaining section of the beam.

To illustrate this let us take an example of a beam shown in Fig. 114. While in mould the wires which are at about 2 in. above the bottom are stretched and in this state concrete is poured and allowed to harden. During all this time the stretching force is kept constant.

Fig. 114.





Figs. 115 & 116.

After this, the stretching force is released; the result is that when the stretched wire tries to assume its normal position it exerts a compressive stress on the surrounding concrete by the mutual bond between the steel and concrete and there is an elastic shortening in concrete as shown in dotted lines in Fig 115 and the beam has a tendency to deflect upwards (chamber) as shown in full lines. Fig. 115 (a) shows the stress diagram with a minimum compression at top of 800 lbs./in^2. and a maximum of 1600 lbs./in^2. at bottom, these being caused by the stretched steel pressing eccentrically on the concrete.

Let the beam be now placed on two end supports and loaded as shown in Fig. 116. It will, now, under the central load have a tendency to deflect downwards as usual. This is shown in full lines in Fig. 116 and there will be compression induced above, and tension below the neutral axis. This is shown in stress diagram in Fig. 116 (b) the maximum compressive stress at top being 700 lbs./in^2. and the maximum tensile stress at bottom 1600 lbs./in^2. But there were already the compressive stresses shown in stress diagram 115 (a) of 800 and 1600 lbs./in.^{2} at top and bottom respectively. When the stress diagrams 115 (a) and (b) are superimposed their algebraic sum will be the final stresses shown in stress diagram 116 (c), these being 1500 lbs./in.^{2} compression at top and no stress at bottom.

If F, be the total tensile stress in the reinforcement, applied at a distance e from the centre, A_{ec}, the equivalent area of cross setion of the beam, and y_1 and y_2 the distances from the axis through centroid of the area to the tensile and compressive edges respectively as shown in Fig.117 and I_{ec}=equivalent moment of inertia.

the extreme fibre stress at the bottom of a prestressed

beam $= \frac{F}{A_{ec}} + \frac{Fey_1}{I_{ec}}$ ... ... (1)

and the fibre stress at the top

$$= \frac{F}{A_{ec}} - \frac{Fey_2}{I_{ec}} \quad \text{... ... ...} \quad (2)$$

If the latter is a tensile stress $\frac{Fey_2}{I_{ec}}$ would be greater than $\frac{F}{A_{ec}}$.

The above are the stresses in the prestressed beam prior to loading, placed flat on the ground i. e. no stresses due to its own weight are induced in it. These are shown in the stress diagram in Fig. 117 (a). If there is tension at top then the stresses will be represented by diagram (b).

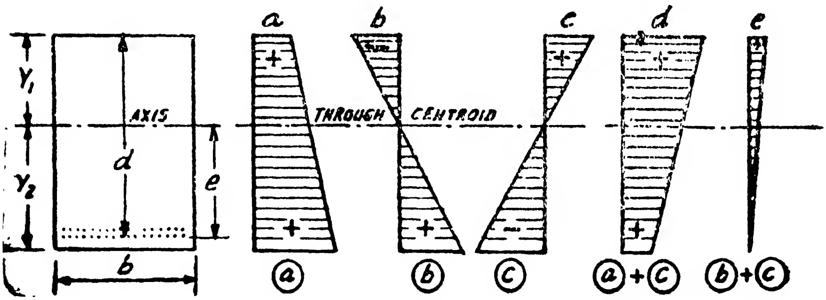


Fig. 117.

When the beam is placed on two end supports and loaded, the combined effect of the dead and live loads would be to induce compressive stress above the N. A. and tensile, below it, as shown in Fig. 117 (c). The extreme compressive stress $= \frac{My_2}{I_{ec}}$ and extreme tensile stress $= \frac{My_1}{I_{ec}}$.

When both these stress diagrams viz. (a) and (c), or (b) and (c) are combined we get resulting stress distribution diagrams either (d) which is a combination of (a) and (c) or of (e) which is combination of (b) and (c) as the case may be.

F (the stretching force) and A (the area of concrete section) are so selected as to produce as far as possible compression over the entire section or some tension on one face within permissible limit. The maximum stresses resulting from the combination of prestresses and load stresses should in no event exceed the permissible limits.

There are two methods of prestressing concrete: (1) Prestressing with the help of the bond between concrete and steel and (2) Prestressing without bond. In the first, as has been already described, before concrete is poured in a mould the steel wires are stretched and the tensile stress is released only after the concrete has hardened. In the second, the reinforcement in the form of a cable of wires is greased or inserted in a sheath in order to prevent bond with the concrete and when the concrete is hardened the cable is stretched and the reaction is taken on bearing plates at ends which compress the concrete surrounding the cable. In the first method the bond between steel and concrete takes up the reaction of the stretched wires. The stresses in the concrete and steel, therefore vary with the bond which is maximum at midspan and nil at extreme ends. In the second method the reaction remains constant throughout. However, as far as the result is concerned the difference is unimportant.

Not all the force employed in stretching reinforcement is useful in prestressing concrete. Part of it is lost in shrinkage, part in plastic yield of concrete and to a small extent also in plastic yield of steel.

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CHAPTER XXIV

PLASTIC THEORY

ACCORDING to the current practice of design of R. C. C. structures, only the elastic strains caused by the application of load are considered. However, we have already seen in Chapter III that there are, besides the elastic strains, strains due to shrinkage which have been produced even before the application of load, and strains due to plastic flow or creep, caused by loads sustained for a long time. The latter go on increasing indefinitely as time passes and are often greater than the elastic strains.

Further, according to the current practice of design, the calculations are based on the assumption that the stress varies as the strain and that their relation is expressed by a straight line. This assumption is true up to the limit of the working stresses. However, from the point of view of the safety and stability of the structure, the ultimate strength and ultimate moment distribution in the members and their monolithic frame, just before failure occurs is of greater importance than the knowledge of the stress distribution of the individual

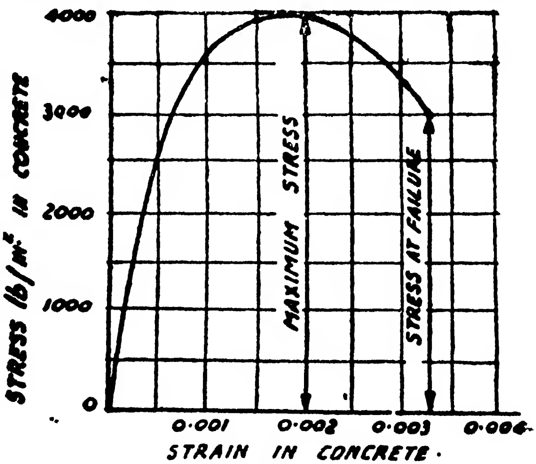

Fig. 118.

members at working loads. During the past decade a school of engineers has arisen, which is not satisfied with the present method of design. This, according to them, is based on wrong assumptions. They rely more upon the results of tests of actual members carried to destruction. The method they adopt takes into account shrinkage strains, elastic strains, and some of the strains due to plastic flow.

When a beam made of say, the ordinary grade concrete of 1:2:4 mix is tested to failure the stress strain relation shows a typical curve as shown in Fig. 118. A close scrutiny of it reveals the following facts:—

(1) Up to the limit of the present working stresses and a little beyond, the graph is a straight line, indicating that the stress varies as the strain.

(2) After the strain reaches 0·001 it increases faster than the stress and the curve assumes the form of something like a parabola.

(3) The stress reaches a maximum value, for strains between 0·0015 and 0·0020.

(4) As the strain increases still further the stress decreases and failure occurs at the most severely strained section. At this moment the strain is about 0·0030.

According to the straight line stress distribution theory,

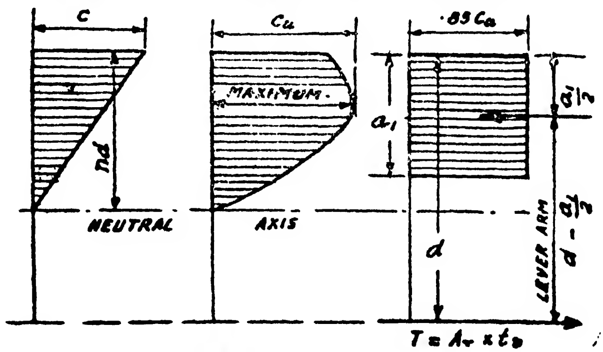

Fig. 119 (a) (b) & (c).

the maximum stress occurs at the extreme fibre of the section as shown in Fig. 119(a). But tests carried to failure have conclusively shown that the maximum stress occurs in particles some distance below the top of the beam as shown in Fig. 119(b), and this can be easily explained by the behaviour of the beam at failure as indicated by the curve in the above figure which shows that the maximum stress is reached not at the time of the failure, but a little earlier and that the immediate cause of the failure is not any increase in the stress, but increase in strain.

The advocates of the plastic theory take for the stress diagram, a rectangle with its width representing a constant stress = 0·85 c_u (c_u = ultimate compressive stress) and depth a_1, as shown in Fig. 119(c) (as against a triangle with its base representing maximum stress c and depth = nd in the working stress theory). The total compression $C = 0{\cdot}85\, c_u \times a_1 \times b$ acting at $\frac{a_1}{2}$ from the top.

$$\text{Since } T = C$$

or $A_T\, t_y = {\cdot}85\, c_u\, a_1 b$, t_y = yield point tensile stress of steel.

Putting $p\, bd$ for A_T, $p\, b\, d\, t_y = {\cdot}85\, c_u\, a_1\, b$

$$a_1 = \frac{t_y\, pd}{{\cdot}85\, c_u}$$

$$\text{B. M.} = {\cdot}85\, c_u\, a_1\, b \left(d - \frac{a_1}{2}\right)$$

Tests have shown that $a_1 = {\cdot}537d$

$$\therefore \quad \text{Lever arm} = d - \frac{a_1}{2} = d - \frac{{\cdot}537d}{2}$$

$= 0{\cdot}732d$ for ordinary grade concrete of 1 : 2 : 4 mix.

For *under-reinforced* beams i. e. for beams which may fail by failure of steel,

Ultimate Resisting Moment, $RM_u = T \times \text{lever arm}$

$$= p\, b\, d\, t_y \times {\cdot}732d$$

$$= {\cdot}732\, p\, t_y\, b\, d^2 \quad \text{.............(1)}$$

and $$A_T = \frac{M}{{\cdot}732\, t_y\, d} \quad \text{...............(1a)}$$

For *over-reinforced* beam, which will fail by the crushing of concrete

$$\text{R. M.}_u = C \times \text{lever arm}$$

$$= \cdot 85\ c_u\ a_1\ b \times \cdot 732d$$

Putting $0{\cdot}537d$ for a_1 $\quad = \cdot 85\ c_u \times \cdot 537d \times b \times \cdot 732d$

$$= \cdot 333\ c_u\ bd^2 \text{ or } \frac{c_u\ bd^2}{3} \quad \ldots\ldots\ldots\ldots (2)$$

For a *balanced design* or for equal strength in tension in steel and compression in concrete

$$A_T \cdot t_y = \cdot 85\ c_u\ a_1\ b$$

$$p\ b\ d\ t_y = \cdot 85\ c_u\ a_1\ b$$

$$p = \frac{\cdot 85\ c_u\ a_1}{t_y \cdot d} \quad \ldots\ldots\ldots\ldots (3)$$

Putting $\cdot 537\ d$ for a_1 $\quad = \dfrac{\cdot 85\ c_u}{t_y} \cdot \dfrac{\cdot 537d}{d}$

$$= \frac{\cdot 456\ c_u}{t_y} \quad \ldots\ldots\ldots\ldots (4)$$

and A_T $\quad = 0{\cdot}456\ \dfrac{c_u}{t_y} \cdot bd \quad \ldots\ldots\ldots\ldots (5)$

This p or the ratio of steel to the concrete in beam section in equation (3) is much more than that required according to the design by the working stress theory. Generally speaking plastic theory gives smaller beam section with more steel.

It is worth noting that in the working stress theory, steel in excess of that required for balancing the design increases the strength (R. M.) of the beam though at a much reduced rate as will be seen from the percentage steel-strength curve in Fig. 8 (page 45). In the plastic theory on the other hand, there is no advantage at all.

Under-reinforcing an advantage:—On the contrary there are some advantages in under-reinforcing a beam. The first is that with less steel used, the beam would be economical. The second advantage is that overloading will cause a gradual yielding of the steel at the ultimate stress, t_y, with a warning in the form of a noticeable deflection and cracks on the tension side of the beam and this could be remedied in time by either reducing the load or strengthening the beam in some way.

Permissible stresses:—In plastic theory designs, the permissible live plus dead load is taken as *four-tenths* of the ultimate load. This is called a *load factor* corresponding to the factor of safety in the working stress design. This means permissible $c_u = 0{\cdot}4 \times$ ultimate crushing strength of the concrete at 28 days. For ordinary grade concrete of 1:2:4 mix, $c_u = {\cdot}4 \times 2000 = 800$ lb./in.^{2} and $t_y = {\cdot}4 \times 44000 = 17600$ say 18,000 lb./in.^{2} which is the same as in working stress design.

Calculations for determining the ultimate shear strength of beams by the plastic theory are very complicated. Until simplified methods for computing it are evolved, the plastic theory is confined for the present to design of beams and slabs as far as bending moments only are concerned.

The following few typical illustrative examples*—one of each kind, will demonstrate the application of the theory to practical designs of a slab, rect. beam, T-beam and a column.

Illustrative Example 85

A slab continuous over three equal spans of 10 ft. each carries a load of 1¼ cwt. per sq. ft. in addition to its own load. Design it by both the methods for comparison.

Solution:—Assuming thickness of slab 5 in.

wt. of the slab (5 in.)	=	60 lbs.
Superload 1¼ cwt.	=	140 ,,
Tiles and mortar bedding	=	20 ,,
Total		220 ,,

* In the examples which follow the symbols c_u and t_y are used to mean permissible stresses in concrete and steel which are four-tenths of the respective ultimate stresses, to distinguish them from c and t of standard theory.

Maxi. B. M. at penultimate support

$$= \frac{wl^2}{10} = \frac{220 \times 10 \times 10}{10} \times 12$$

$$= 26400 \text{ in. lbs.}$$

By standard or working stress theory:—

$$d = \sqrt{\frac{26400}{126 \times 12}} = 4{\cdot}25$$

$$\text{overall D} = 4{\cdot}25 + {\cdot}25 + {\cdot}5 = \mathbf{5 \text{ in.}} \qquad \text{(i)}$$

$$A_T = \frac{26400}{18000 \times {\cdot}87 \times 4{\cdot}25}$$

$$= \mathbf{0{\cdot}39} \text{ sq. in.} \qquad \text{(ii)}$$

By plastic theory—

$$26400 = \tfrac{1}{3}\, c_u b d^2$$

$$= \tfrac{1}{3}\, 800 \times 12 \times d^2$$

$$d = \sqrt{\frac{26400}{3200}} = \sqrt{8{\cdot}25}$$

$$= 2{\cdot}85$$

$$\text{overall D} = 2{\cdot}85 + {\cdot}25 + {\cdot}5 = \mathbf{3{\cdot}6} \qquad \text{(i)}$$

This would be 3·5 in. or even less if the B. M. is recalculated with $d = 3{\cdot}5$ instead of 5 in. assumed for calculating the dead load.

$$A_T = 0{\cdot}456 \frac{c_u}{t_y} . \, b\, d$$

$$= 0{\cdot}456 \frac{800}{18000} \times 12 \times 2{\cdot}85$$

$$= \mathbf{0{\cdot}68} \qquad \text{(ii)}$$

TABLE No. 31

COMPARISON BETWEEN STANDARD THEORY AND PLASTIC THEORY

	Thickness in.	Steel sq. in.
Standard theory	5	0·39
Plastic theory	3·5	0·68

Illustrative Example 86

A beam 12 in. × 24 in. (effective), simply supported at ends carries 2500 lbs. load per ft. over a span of 16 ft. Design the reinforcement and compare it with that designed by plastic theory.

By standard theory:—

Solution:—$M = \frac{wl^2}{8} = \frac{2500 \times 16 \times 16 \times 12}{8}$

$= 960{,}000$ in./lbs.

R. M $= 126\, bd^2$

$= 126 \times 12 \times 24 \times 24$

$= 870912$ in. lbs.

M - RM $= 89{,}088$ in lbs.

Compressive reinforcement must be provided for this. Supposing it is placed with its *c. g.* at 2·5 in. below the top, the stress in concrete at that depth

$$= \frac{nd - 2{\cdot}5}{nd} \times 750 = \frac{{\cdot}39 \times 24 - 2{\cdot}5}{{\cdot}39 \times 24} \times 750$$

$$= \frac{6{\cdot}86}{9{\cdot}36} \times 750 = 586 \text{ lbs./in.}^2$$

Stress in compressive steel at that place.

$= $ stress in concrete $\times 14$

$= 586 \times 14 = 8204$ lb./in.^{2}

$$A_c = \frac{89088}{8204 \times 21{\cdot}50 \text{ (lever arm)}}$$

Lever arm $= 24 - 2{\cdot}5 = 21{\cdot}5$

$= 5{\cdot}05.$

Tensile reinforcement

$$A_T = \frac{960000}{18000 \times {\cdot}87 \times 24} = 2{\cdot}56 \text{ sq. in.}$$

Total reinforcement $5{\cdot}05 + 2{\cdot}56 = 7{\cdot}61$ sq. in.

By plastic theory—

$$M = \tfrac{1}{8}\, c_u\, bd^2$$

$$960000 = \tfrac{1}{8}\, 800\, bd^2$$

$$\therefore \quad bd^2 = 3600$$

$$\text{If} \quad b = 12'' \; d = 17{\cdot}6$$

This shows that not only there is no need of compressive reinforcement but the section could be safely reduced to 12″×17·6 or 18″.

Tensile reinforcement

$$A_T = 0{\cdot}456 \frac{c_u}{t_y} \cdot bd$$

$$= \frac{0{\cdot}456 \times 800 \times 12 \times 18}{18000}$$

$$= 4{\cdot}38 \text{ sq. in.}$$

TABLE NO. 32

COMPARISON BETWEEN DESIGNS BY STANDARD THEORY AND PLASTIC THEORY

	Section	Steel		
		Comp. sq. in.	Tensile sq. in.	Total sq. in.
Standard theory	12″×24″	5·05	2·56	7·61
Plastic theory	12″×18″	nil	4·38	4·38

The design by plastic theory is in this case much economical.

Tee Beams:—For applying plastic theory to tee-beams the compressive stress is assumed at $c = 0{\cdot}85\, c_u$ for a depth $a_1 = 0{\cdot}537\, d$. When the thickness of the flange, d_s is greater than or almost equal to 0·537 d the beam is designed as a rectangular one of width b_s and depth d_s with a constant compressive stress of $0{\cdot}85\, c_u$. In that case

$$M = 0{\cdot}85\, c_u\, b_s\, d_s \left(d - \frac{d_s}{2}\right)$$

$$= 0{\cdot}85\, c_u\, \frac{d_s}{d}\left(1 - \frac{d_s}{2d}\right) b_s d^2 \ \ldots \quad \ldots \qquad (\text{i})$$

and $$A_T = \frac{M}{t_y\left(d - \frac{d_s}{2}\right)} \quad \ldots \quad \ldots \quad \ldots \qquad (\text{ii})$$

If the flange thickness, d_s is much less than $0{\cdot}537\,d$, the area in the stem upto a depth of $0{\cdot}537\,d$ is included. Thus the total compression area $A = (b_s - b)\,d_s + b \times 0{\cdot}537\,d$

Taking moments about the top of the beam the distance of the centre of gravity

$$\bar{x} = \frac{(b_s - b)\,d_s \times \frac{d_s}{2} + {\cdot}537d \times b \times \frac{{\cdot}537d}{2}}{(b_s - b)\,d_s + b \times {\cdot}537d}$$

$$= \frac{d}{2}\,\frac{\left[(b_s-b)\left(\frac{d_s}{d}\right)^2 + {\cdot}537b^2\right]}{\left[(b_s-b)\,\frac{d_s}{d} + {\cdot}537\,b\right]} \quad \ldots \quad \ldots \ (\text{a})$$

The bending moment

$$M = 0{\cdot}85\, c_u\,(d - \bar{x}) \times A_c \quad \ldots \quad \ldots \quad \ldots \ (\text{b})$$

and steel area $$A_T = \frac{M}{t_y\,(d - \bar{x})} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \ (\text{c})$$

From the indications of the tests, Mr. Whitney, the originator of the plastic theory recommends that b_s should not be greater than $8\,d_s + b$, instead of $12d_s + b$ allowed by the standard theory.

Illustrative Example 87

Design an intermediate tee-beam of a continuous system of beams of equal spans of 12 ft. each, bearing a load of 1600 lbs. per ft. including the dead load of the slab which is 4 in. thick. The overall dimensions of the stem are 8" × 25".

Solution:—

Slab and upper load	=	1600
Beam load 25×8	=	200
Total		1800

$$-\text{ve B. M.} = -\frac{wl^2}{10}$$

$$= -\frac{1800 \times 12 \times 12 \times 12}{10}$$

$$= -311000 \text{ in. lbs.}$$

By working stress theory

Assuming cover = 2·5, effective d = 22·5

nd = 0·39 × 22·5 = 8·77; jd = ·87 × 22·5 = 19·58.

On top of supports the bottom of the stem will be in compression.

R. M. in compression $= 126\ bd^2$

$= 126 \times 8 \times 22{\cdot}5^2 = 510{,}000$ in. lbs.

As this is more than the B. M. there is no need of compression reinforcement.

$$A_T = \frac{311000}{18000 \times 19{\cdot}58}$$

$= \mathbf{0{\cdot}88}$ sq. in.

$$+\text{ve B. M.} = \frac{wl^2}{12} = \frac{1800 \times 12 \times 12 \times 12}{12}$$

$= 259200$ in. lbs,

The depth of N. A. is 8·77 below top and the flange thickness is 4 in. The N. A. is out side the slab.

The average compressive stress in the flange i. e. at 2″ below top $= \frac{8{\cdot}77 - 2}{8{\cdot}77} \times 750 = 580$ lbs./in.²

Equating B. M. to R. M.

$$259200 = 580 \times 4 \times b_s\,(22{\cdot}5 - 2)$$

$b_s = \mathbf{55{\cdot}5}$ in.

A_T for $+ve$ B. M. $= \dfrac{259200}{18000 \times 22{\cdot}5}$

$= \mathbf{0{\cdot}64}$ **sq. in.**

Solution by plastic theory—

$-ve$ B. M. $= 311000$ in. lbs.

$$M = \tfrac{1}{3}\, c_u\, bd^2$$

$$311000 = \tfrac{1}{3}\, 800 \times 8 \times d^2$$

$$d^2 = \frac{3 \times 311000}{800 \times 8} = 146$$

d $= \mathbf{12{\cdot}1}$ in. let it be = say 14 in.

$$-ve\ A_T = \frac{M}{0{\cdot}732 \times t_y d} = \frac{311000}{{\cdot}732 \times 18000 \times 14}$$

$= \mathbf{1{\cdot}7}$ **sq. in.**

Design of mid-span section

The width of the slab, $b_s = 8\, ds + b = 8 \times 4 + 8 = 40$ **in.**
$0{\cdot}537\, d = 0{\cdot}537 \times 14 = 7{\cdot}52.$

This is much greater than flange thickness, 4″. We **must therefore** consider the compression in the stem.

Total compression area, $A_c = (b_s - b)ds + b \times {\cdot}537d$
$= (40 - 8)\, 4 + 8 \times {\cdot}537 \times 14$
$= 128 + 60{\cdot}16$
$= 188{\cdot}16$ sq. in.

Taking moments about the top of beam, the **distance of c. g.**

$$\bar{x} = \frac{d}{2}\ \frac{\left[(b_s - b)\left(\frac{d_s}{d}\right)^2 + {\cdot}288\, b\right]}{\left[(b_s - b)\ \frac{d_s}{d} + {\cdot}537\, b\right]}$$

$$\text{or} = \frac{14}{2}\ \frac{\left[(40-8)\left(\frac{4}{14}\right)^2 + {\cdot}288 \times 8\right]}{\left[(40-8)\ \frac{4}{14} + {\cdot}537 \times 8\right]}$$

$$= 2{\cdot}54$$

$$M = 0{\cdot}85\, c_u\, (d - \bar{x})\, A_c$$

$$= 0{\cdot}85 \times 800\, (14 - 2{\cdot}54)\, 179{\cdot}55$$

$$= 1398200 \text{ in. lbs.}$$

This is the resisting moment of the beam in compression and is several times the positive B. M., viz. 259200 in. lb. Hence, the beam even with $b = 40$ and $d = 14$ is amply strong. $A_T = \dfrac{259200}{{\cdot}732 \times 18000 \times 14} = 1{\cdot}38$ sq. in.

If we take b_s = 20 instead of 40 in.

$$A_c = 12 \times 4 + 8 \times {\cdot}537 \times 14$$

$$= 118{\cdot}14$$

$$\bar{x} = \frac{14}{2} \times \frac{(12 \times (\frac{4}{14})^2 + {\cdot}288 \times 8}{(12 \times \frac{4}{14} + {\cdot}537 \times 8)}$$

$$= 7 \times \frac{3{\cdot}26}{7{\cdot}73} = 2{\cdot}95$$

$$M = {\cdot}85 \times 800 \times 11{\cdot}05 \times 118{\cdot}14$$

$$= 934000 \text{ in. lb.}$$

Even this is more than sufficient. Comparative results are given below.

TABLE No. 33

COMPARISON BETWEEN DESIGNS BY STANDARD METHOD AND BY THE PLASTIC THEORY

	Stem width in.	Effective depth in.	Flange width in.	+*ve* steel sq. in.	−*ve* steel sq. in.
Working stress theory	8	22	54·5	0·64	0·88
Plastic theory	8	14	20·0	1·38	1·70

***Columns*:—It is not uncommon that columns are designed by plastic theory, even when other structural members are designed by the orthodox working stress theory. This is due to the fact that for sustained load on a column, the concrete gradually creeps throwing progressively more and more load**

on the steel (please see the remarks on page 174). Even the Code of Practice has recognised this fact, and has based the axial load on columns on tests carried to failure. The strength of concrete in columns is taken by the Code equal to 0·8 (instead of 0·85 in plastic theory) of the crushing strength of concrete at 28 days and the maximum stress of steel is taken at the yield point. Thus the load at failure i. e.

Ultimate load W_u = ·8 × ultimate stress in concrete × area of concrete + $t_y \times A_s$.

$= \cdot 8 \times 2250\ (ab - A_s) + 40{,}000 \times A_s$.

Applying a load factor of 3 to both, safe working load

$= 600\ (ab - A_s) + 13333\ A_s$.

Instead of 13,333 lbs./in.^{2}, the Code of Practice has addopted a round figure of 13500 lbs.

Safe W $= 600\ (A_c - A_s) + 13500\ A_s$.

For illustrative examples please refer to solved examples Nos. 49 to 52 on pages 174 to 176.

It should be noted that the modular ratio of steel to concrete which is the main cause of controversy amongst engineers, does not come in anywhere in plastic theory designs.

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APPENDIX

SHELL CONCRETE CONSTRUCTION

SHELL construction is perhaps the most striking development of R. C. C. The name indicates clearly a roof structure of very thin section resembling the shell of an egg. In fact the ratio of the thickness to span viz. $\frac{1}{650}$ actually achieved (shell $1\frac{9}{16}$ in. thick on a span of 85 ft. for the dome at Frankfurt on Main), is less than that of an egg shell.

Principle:—The usual stone or brick arch transmits the load only in the direction of the curvature, but cannot carry load horizontally at right angles to it, for which purlins resting on the curved surface of the arches or trusses are required. An R. C. C. barrel vault, on the other hand, if properly reinforced, owing to the curved surface and continuity of the slab, transmits loads in both directions i. e. the slab acts also as a longitudinal beam and makes purlins in the conventional vaulted roofs unnecessary. If a vaulted R. C. C. roof is suitably restrained, distributed loads such as the dead load of the vault, wind load, snow load, etc. do not produce bending moments, i. e. the forces are axial in any cross section. The equilibrium is maintained by the so-called *membrane* stresses.

Design:—The condition for developing such stresses is the maintenance of the shape of the shell by means of a rigid frame. The latter consists of columns, tie beams at springing and edges and in some cases ribs for stiffening the barrel—all forming with the shell one whole unit cast monolithically. In the conventional construction, as the span increases beyond a certain limit, the dead load, viz. of the main and secondary beams and sheathing increases by leaps and bounds. In the shell roof construction, on the other hand, not only the increase in dead load is very small, due to the thin shell as the span increases, but in addition the shell itself acts as a load bearing member.

The calculations for the design tend to get complicated even with certain reasonable assumptions. The compressive, tensile, and shear stresses are calculated for each point of the shell, and these are converted into principal stresses. Reinforcement is then provided according to the magnitude and direction of these principal stresses. The reinforcement consists of ¼" to ½" ϕ bars arranged in two-way diagonal meshes, and distribution steel both at bottom and top. All this is accommodated in a thickness of 2 to 3½ in. including ½ to ¾ in. minimum cover at bottom and top,

Types:—There are two types:

(1) *Domes* curved in more than one direction.

(2) *Vaults* curved in one direction only. If an area of rectangular shape is to be covered by a dome, the surface of the latter may be curved in both directions, formed by a generating curve, which is moved along another curve. If the area is very large, two, three or any number of domes or vaults may be constructed in shell joined to each other. In this way an area of 82 acres has been covered by a shell roof in U. S. A.

Another line of development is that of polygonal domes formed by the intersection of cylindrical shells. For covering a square area an octagonal dome formed by four cylindrical shells is suitable. This resembles a fully opened umbrella with the central pole removed, placed erect on a flat ground, the eight points or ends of ribs on which it rests representing columns. The ridges formed by the intersection of triangular shells replace the ribs in a barrel vault.

The largest span of shell vault so far built is 294 ft. in U. S. A. for a naval hanger with 84 ft. clear height.

Tests:—Elaborate tests made at the Hamburg Dock Shed showed that the deflection did not exceed 1/10,000th of the span.

The Fronton Recoletos in Madrid of which the shell roof is formed by two cylindrical shells of 3¼ in. thickness intersecting each other at right angles covers an area of 180′×107′.

The roof lights of reinforced glass-concrete extend over the whole length. During the Civil War in Spain, a shell penetrated the roof forming a hole of 6 ft. diam. without in the least affecting the stability of the structure.

Shell construction is suitable only for distributed loads. Heavy point loads are not admissible unless special provision is made.

2:1 as the proportion of length to span gives most economical results. The maximum permissible is 5 : 1.

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